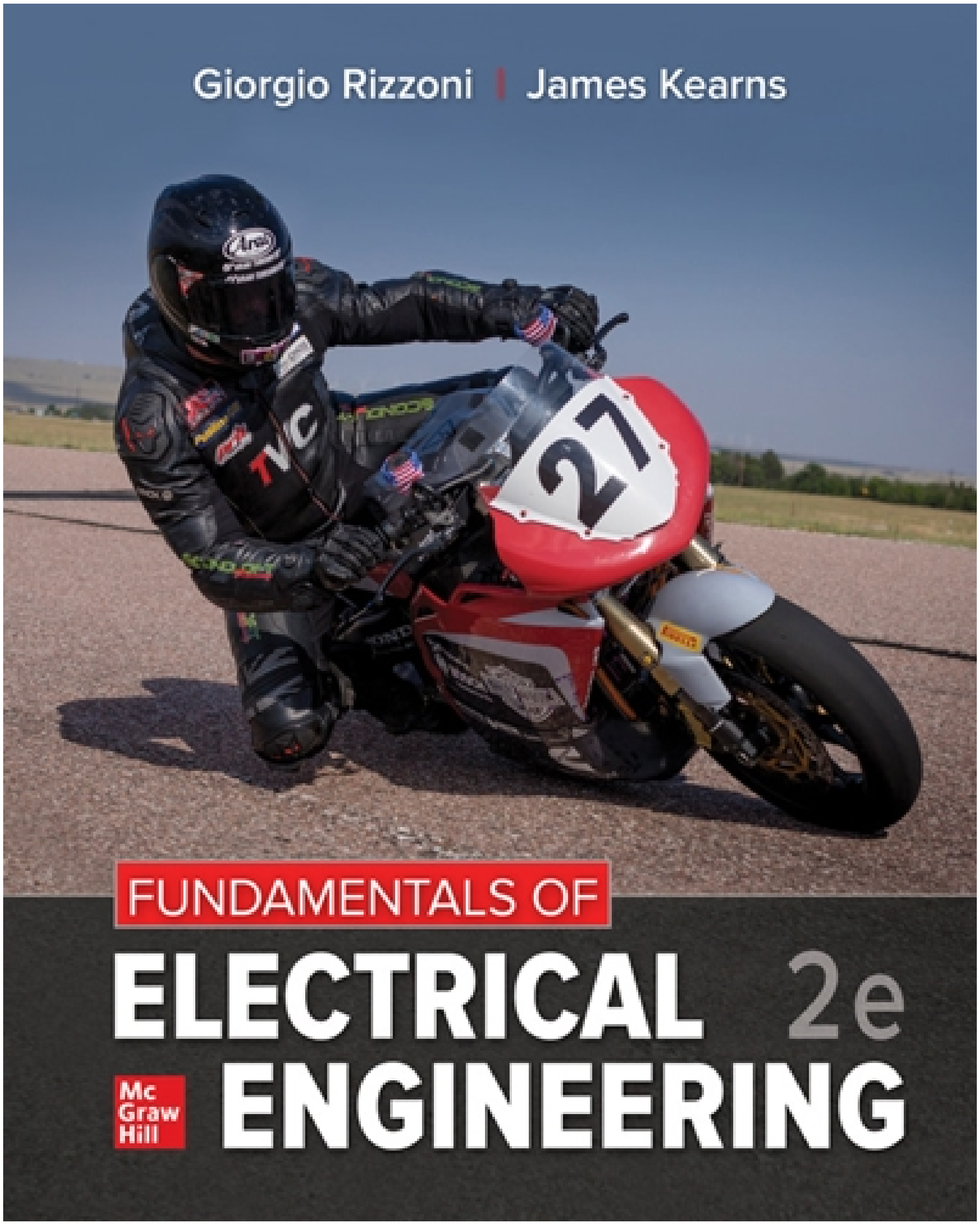
Solutions for Fundamentals of Electrical Engineering 2nd Edition by Rizzoni



Solutions

Chapter 1: Fundamentals of Electrical Circuits – Instructor Notes

Chapter 1 develops the foundations for the first part of the book. Coverage of the entire chapter would be typical in an introductory course. The first four sections provide the basic definitions (Section 1.1) and cover Kirchhoff's Laws, instantaneous power and the passive sign convention (Sections 1.2, 1.3 and 1.4); A special feature, *Focus on Problem Solving: The Passive Sign Convention* (p. 22) and ten examples illustrate these important topics. A second feature, that will recur throughout the first six chapters, is presented in the form of sidebars. *Make The Connection: Hydraulic Analog of a Voltage Source* (p. 28) and *Make The Connection: Hydraulic Analog of a Current Source* (p. 29) present the concept of analogies between electrical and other physical domains.

Sections 1.5 and 1.6 introduce the *i-v* characteristic and the resistance element. Tables 1.1 (p. 31) and 1.2 and 1.3 (p. 33) summarize the resistance of copper wire for various gauges, the resistivity of common materials and standard resistor values, respectively. The sidebar *Make The Connection: Hydraulic Analog of Electrical Resistance* (p. 31) continues the electric-hydraulic system analogy.

Sections 1.7 and 1.8 introduce the critically important concepts of elements in series and parallel and the associated analytic methods of voltage and current division. Section 1.8 introduces another special feature of the book, the *Focus on Measurements* box. The first three of these boxes appear at the end of that section and are focused on practical aspects of a *Resistive Throttle Position Sensor*, *Resistance Strain Gauges*, and *The Wheatstone Bridge and Force Measurements*. These three boxes demonstrate the common role of voltage division in the construction of electrical sensors.

Finally, Sections 1.9 and 1.10 introduce basic but important concepts related to ideal and non-ideal models of practical sources, and common measuring instruments.

The Instructor will find that although the material in Chapter 1 is quite basic, it is possible to give an applied flavor to the subject matter by emphasizing a few selected topics in the examples presented in class. In particular, a lecture could be devoted to *resistance devices*, including the resistive displacement transducer of *Focus on Measurements: Resistive Throttle Position Sensor* (pp. 46-47), the resistance strain gauge of *Focus on Measurements: Resistance Strain Gauges* (p. 48), and *Focus on Measurements: The Wheatstone Bridge and Force Measurements* (pp. 49-50). The instructor wishing to gain a more in-depth understanding of resistance strain gauges can find detailed analyses in many common references.

Early motivation for the application of circuit analysis to problems of practical interest to the non-electrical engineer can be found in these *Focus on Measurements* boxes. The Wheatstone bridge material can also serve as an introduction to a laboratory experiment on strain gauges and the measurement of force. Finally, Example 1.16 on the Impact of a Practical Voltmeter at the end of Section 1.10 describes a quick but interesting laboratory experiment.

The homework problems include a variety of practical examples, with emphasis on instrumentation. Problem 1.39 illustrates the result of placing two practically equivalent batteries in parallel; problems 1.43 relates to wire gauges; problem 1.44 discusses one common method of resistor construction; problem 1.45 illustrates analysis related to fuses; problem 1.53 introduces the thermistor; problem 1.54 discusses circuit design involving a moving coil meter; problem 1.57 describes a procedure for measuring the internal resistance of an ammeter; problem 1.60 illustrates the potential loading effect of a practical voltmeter; and problems 1.62 and 1.63 illustrate calculations related to strain gauge bridges.

It has been the author's experience that providing the students with an early introduction to practical applications of electrical engineering to their own disciplines can increase the interest level in a course significantly.

Learning Objectives for Chapter 1

Students will learn to…

1. Identify the principal *features of electric circuits or networks*: nodes, loops, meshes, and branches. *Section 1.1*.
2. Apply *Kirchhoff's Laws* to simple electrical circuits. *Sections 1.2-1.3*.
3. Apply the *passive sign convention* to compute the power consumed or supplied by circuit elements. *Section 1.4*.
4. Identify *sources* and *resistors* and their i-υ characteristics. *Sections 1.5–1.6*.
5. Apply *Ohm's law* and *voltage and current division* to calculate unknown voltages and currents in simple series, parallel, and series-parallel circuits. *Sections 1.6–1.8*.
6. Understand the impact of internal resistance in practical models of voltage and current sources as well as of voltmeters, ammeters, and wattmeters. *Sections 1.9–1.10*.

Sections 1.2-1.3: Charge, Current, and Kirchhoff's Current Law; Voltage and Kirchhoff's Voltage Law

Problem 1.1

A free electron has an initial potential energy per unit charge (voltage) of 17 kJ/C and a velocity of 93 Mm/s. Later, its potential energy per unit charge is 6 kJ/C. Determine the change in velocity of the electron.

Solution:

Known quantities:

Initial Coulombic potential energy, $V_i = 17kJ/C$; initial velocity, $U_i = 93M\frac{m}{s}$; final Coulombic potential energy, $V_f = 6kJ/C$.

Find:

The change in velocity of the electron.

Assumptions:

$\Delta PE_g << \Delta PE_c$

Analysis:

Using the first law of thermodynamics, we obtain the final velocity of the electron:

$Q_{heat} - W = \Delta KE + \Delta PE_c + \Delta PE_g + ...$

Heat is not applicable to a single particle. W=0 since no external forces are applied.

$\Delta KE = -\Delta PE_c$

$\frac{1}{2}m_e(U_f^2 - U_i^2) = -Q_e(V_f - V_i)$

$U_f^2 = U_i^2 - \frac{2Q_e}{m_e}(V_f - V_i)$

$= \left(93\,M\,\frac{m}{s}\right)^2 - \frac{2\left(-1.6\times10^{-19}\,C\right)}{9.11\times10^{-37}\,g}\left(6kV - 17kV\right)$

$= 8.649\times10^{15}\,\frac{m^2}{s^2} - 3.864\times10^{15}\,\frac{m^2}{s^2}$

$U_f = 6.917\times10^{7}\,\frac{m}{s}$

$\left|U_f - U_i\right| = 93\,M\,\frac{m}{s} - 69.17\,M\,\frac{m}{s} = 23.83\,M\,\frac{m}{s}.$

Problem 1.2

The units for voltage, current, and resistance are the volt (V), the ampere (A), and the ohm (Ω), respectively. Express each unit in fundamental MKS units.

Solution:

Known quantities:

MKSQ units.

Find:

Equivalent units of volt, ampere and ohm.

Analysis:

$$\text{Voltage} = \text{Volt} = \frac{\text{Joule}}{\text{Coulomb}} \quad V = \frac{J}{C}$$

$$\text{Current} = \text{Ampere} = \frac{\text{Coulomb}}{\text{second}} \quad a = \frac{C}{s}$$

$$\text{Resistance} = \text{Ohm} = \frac{\text{Volt}}{\text{Ampere}} = \frac{\text{Joule} \times \text{second}}{\text{Coulomb}^2} \quad \Omega = \frac{J \cdot s}{C^2}$$

$$\text{Conductance} = \text{Siemens or Mho} = \frac{\text{Ampere}}{\text{Volt}} = \frac{C^2}{J \cdot s}$$

Problem 1.3

A particular fully charged battery can deliver 2.7 x 10^6 coulombs of charge.

a. What is the capacity of the battery in ampere-hours?
b. How many electrons can be delivered?

Solution:

Known quantities:

$q_{Battery} = 2.7 \cdot 10^6$ C.

Find:

The current capacity of the battery in ampere-hours
The number of electrons that can be delivered.

Analysis:

There are 3600 seconds in one hour. Amperage is defined as 1 Coulomb per second and is directly proportional to ampere-hours.

$$2.7 \cdot 10^6 C \cdot \frac{1hr}{3600s} = 750\ AH$$

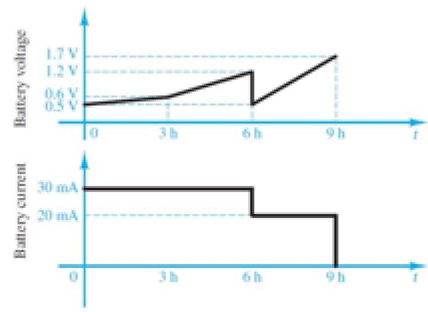
a) The charge of a single electron is $-1.602 \cdot 10^{-19}$ C. The negative sign is negligible. Simple division gives the solution:

$$2.7 \cdot 10^6 C \Big/ \frac{1.602 \cdot 10^{-19} C}{1\ electron} = 1.685 \cdot 10^{25}\ electrons$$

Problem 1.4

The charge cycle shown in Figure P1.4 is an example of a three-rate charge. The current is held constant at 30 mA for 6 h. Then it is switched to 20 mA for the next 3 h. Find:

a. The total charge transferred to the battery.
b. The energy transferred to the battery.
Hint: Recall that energy w is the integral of power, or P = dw/dt.



Solution:

Known quantities:

See Figure P1.4

Find:

a) The total charge transferred to the battery.
b) The energy transferred to the battery.

Analysis:

Current is equal to $\frac{Coulombs}{Second}$, therefore given the current and a duration of that current, the transferred charge can be calculated by the following equation:

$$A \cdot t = C$$

The two durations should be calculated independently and then added together.

$$0.030A \cdot 21600s = 648C$$
$$0.020A \cdot 10800s = 216C$$
$$648C + 216C = \mathbf{864C}$$

P=V·I, therefore, an equation for power can be found by multiplying the two graphs together.
First separate the voltage graph into three equations:

0 h → 3 h :$V = 9.26 \cdot 10^{-6}t + 0.5$

3 h → 6 h :$V = 5.55 \cdot 10^{-5}t$

6 h → 9 h :$V = 1.11 \cdot 10^{-4}t - 1.6$

Next, multiply the first two equations by 0.03A and the third by 0.02A.

0 h → 3 h :$P = 2.77 \cdot 10^{-7}t + 0.015$

3 h → 6 h :$P = 1.66 \cdot 10^{-6}t$

6 h → 9 h :$P = 2.22 \cdot 10^{-6}t - 0.032$

Finally, since Energy is equal to the integral of power, take the integral of each of the equations for their specified times and add them together.

0 h → 3 h :$E = \left[\frac{2.77 \cdot 10^{-7}t^2}{2} + 0.015t\right] \Big|_{0}^{10800}$ = 178.2 J

3 h → 6 h :$E = \left[\frac{1.66 \cdot 10^{-6}t^2}{2}\right] \Big|_{10800}^{21600}$ = 290.43 J

6 h → 9 h :$E = \left[\frac{2.22 \cdot 10^{-6}t^2}{2} + 0.032t\right] \Big|_{21600}^{32400}$ = 992.95 J

$$\boldsymbol{E_{Total} = 1462\,J}$$

Problem 1.5

Batteries (e.g., lead-acid batteries) store chemical energy and convert it to electric energy on demand. Batteries do not store electric charge or charge carriers. Charge carriers (electrons) enter one terminal of the battery, acquire electrical potential energy, and exit from the other terminal at a lower voltage. Remember the electron has a negative charge! It is convenient to think of positive carriers flowing in the opposite direction, that is, conventional current, and exiting at a higher voltage. All currents in this course, unless otherwise stated, are conventional current. (Benjamin Franklin caused this mess!) For a battery with a rated voltage = 12 V and a rated capacity = 350 A-h,
determine
a. The rated chemical energy stored in the battery.
b. The total charge that can be supplied at the rated

Solution:

Known quantities:

Rated voltage of the battery; rated capacity of the battery.

Find:

The rated chemical energy stored in the battery
The total charge that can be supplied at the rated voltage.

Analysis:

a)

$$\Delta V \equiv \frac{\Delta PE_c}{\Delta Q} \quad I = \frac{\Delta Q}{\Delta t}$$

$$\text{Chemical energy} = \Delta PE_c = \Delta V \cdot \Delta Q = \Delta V \cdot \left(I \cdot \Delta t\right)$$

$$= 12\ V\ 350\ \text{A} - \text{hr}\ 3600\frac{\text{s}}{\text{hr}} = 15.12\ \text{MJ}.$$

As the battery discharges, the voltage will decrease below the rated voltage. The remaining chemical energy stored in the battery is less useful or not useful.
b) ΔQ is the total charge passing through the battery and gaining 12 J/C of electrical energy.

$$\Delta Q = I \cdot \Delta t = 350\ a\ hr = 350\frac{\text{C}}{\text{s}}\ \text{hr} \cdot 3600\frac{\text{s}}{\text{hr}} = 1.26\ \text{MC}.$$

Problem 1.6

What determines:
a. The current through an ideal voltage source?
b. The voltage across an ideal current source?

Solution:

Known quantities:

Resistance of external circuit.

Find:

Current supplied by an ideal voltage source
Voltage supplied by an ideal current source.

Assumptions:

Ideal voltage and current sources.

Analysis:

a) An ideal voltage source produces a constant voltage at or below its rated current. Current is determined by the power required by the external circuit (modeled as R).

$$I = \frac{V_S}{R} P = V_S \cdot I$$

b) An ideal current source produces a constant current at or below its rated voltage. Voltage is determined by the power demanded by the external circuit (modeled as R).

$$V = I_S \cdot R \quad P = V \cdot I_S$$

A real source will overheat and, perhaps, burn up if its rated power is exceeded.

Problem 1.7

An automotive battery is rated at 120 A-h. This means that under certain test conditions it can output 1 A at 12 V for 120 h (under other test conditions, the battery may have other ratings).
a. How much total energy is stored in the battery?
b. If the headlights are left on overnight (8 h), how much energy will still be stored in the battery in the morning? (Assume a 150-W total power rating for both headlights together.)

Solution:

Known quantities:

Rated discharge current of the battery; rated voltage of the battery; rated discharge time of the battery.

Find:

Energy stored in the battery when fully recharging
Energy stored in the battery after discharging

Analysis:

a)
$$\text{Energy} = \text{Power} \times \text{time} = (1A)(12V)(120\text{hr})\left(\frac{60\,\text{min}}{\text{hr}}\right)\left(\frac{60\,\text{sec}}{\text{min}}\right)$$

$$\boxed{w = 5.184 \times 10^6 \text{ J}}$$

b) Assume that 150 W is the combined power rating of both lights; then,

$$w_{\text{used}} = (150W)(8hrs)\left(\frac{3600\,\text{sec}}{hr}\right) = 4.32 \times 10^6 \text{ J}$$

$$w_{stored} = w - w_{used} = 864 \times 10^3 \text{ J}$$

Problem 1.8

A car battery kept in storage in the basement needs recharging. If the voltage and the current provided bythe charger during a charge cycle are shown in FigureP1.8,

a. Find the total charge transferred to the battery.

b. Find the total energy transferred to the battery.

Solution:

Known quantities:

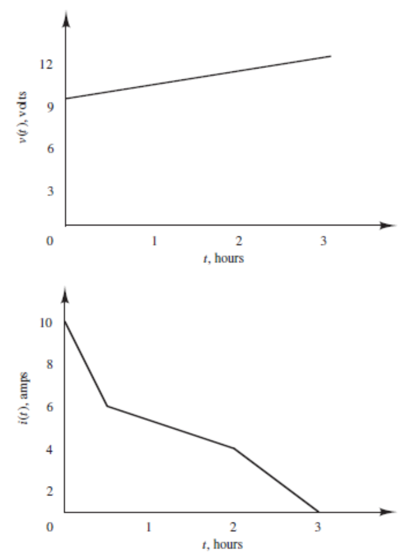
Recharging current and recharging voltage

Find:

Total transferred charge
Total transferred energy

Analysis:

a)



$$Q = \text{area under the current-time curve} = \int I dt$$

$$= \frac{1}{2}(4)(30)(60) + 6(30)(60) + \frac{1}{2}(2)(90)(60) + 4(90)(60) + \frac{1}{2}(4)(60)(60) = 48{,}600\text{C}$$

$$\boxed{Q = 48{,}600\text{C}}$$

b) $\frac{dw}{dt} = p$ so $w = \int p dt = \int vi dt$

$$v = 9 + \frac{3}{10800}t \quad \text{V}, \quad 0 \le t \le 10800\text{s}$$

$$i_1 = 10 - \frac{4}{1800}t \quad \text{A}, \quad 0 \le t \le 1800\text{s}$$

$$i_2 = 6 - \frac{2}{5400}t \quad \text{A}, \quad 1800 \le t \le 7200\text{s}$$

$$i_3 = 12 - \frac{4}{3600}t \quad \text{A}, \quad 7200 \le t \le 10800\text{s}$$

where $i = i_1 + i_2 + i_3$

Therefore,

$$w = \int_0^{1800} vi_1\,dt + \int_{1800}^{7200} vi_2\,dt + \int_{7200}^{10800} vi_3\,dt$$

$$= \left(90t + \frac{t^2}{720} - \frac{t^2}{100} - \frac{t^3}{4.86\times10^6}\right)\Bigg|_0^{1800}$$

$$+ \left(60t + \frac{t^2}{1080} - \frac{t^2}{600} - \frac{t^3}{29.16\times10^6}\right)\Bigg|_{1800}^{7200}$$

$$+ \left(108t + \frac{t^2}{600} - \frac{t^2}{200} - \frac{t^3}{9.72\times10^6}\right)\Bigg|_{7200}^{10800}$$

$$= 132.9\times10^3 + 380.8\times10^3 - 105.4\times10^3 + 648\times10^3 - 566.4\times10^3$$

$$\boxed{Energy = 489.9\ kJ}$$

Problem 1.9

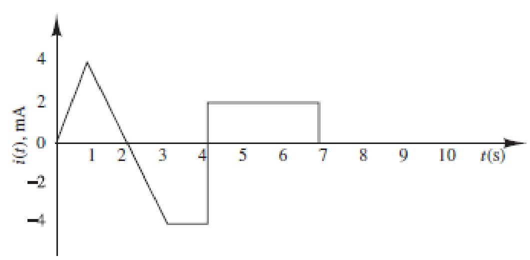
Suppose the current through a wire is given by the curve shown in Figure P1.9.

a. Find the amount of charge, q, that flows through the wire between t1 = 0 and t2 = 1 s.
b. Repeat part a for t2 = 2, 3, 4, 5, 6, 7, 8, 9, and 10 s.
c. Sketch q(t) for 0 ≤ t ≤ 10 s.

Solution:

Known quantities:

Current-time curve



Find:

Amount of charge during 1st second
Amount of charge for 2 to 10 seconds
Sketch charge-time curve

Analysis:

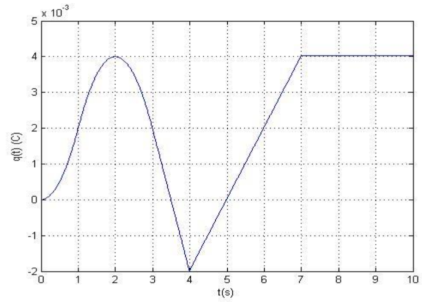
a) $i = \frac{4\times10^{-3}t}{1}$

$$Q_1 = \int_0^1 i\,dt = \int_0^1 4\times10^{-3}t\,dt = 4\times10^{-3}\frac{t^2}{2}\Bigg|_0^1 = 2\times10^{-3}\ \frac{\text{amp}}{\text{sec}} = 2\times10^{-3}\ \text{Coulombs}$$

b) The charge transferred from $t=1$ to $t=2$ is the same as from $t=0$ to $t=1$.

$Q_2 = 4\times10^{-3}$ Coulombs

The charge transferred from $t=2$ to $t=3$ is the same in magnitude and opposite in direction to that from $t=1$ to $t=2$. $Q_3 = 2\times10^{-3}$ Coulombs



$t=4$

$Q_4 = 2\times10^{-3} - \int_3^4 4\times10^{-3}dt = 2\times10^{-3} - 4\times10^{-3} = -2\times10^{-3}$ Coulombs

$t = 5,6,7$

$Q_5 = -2\times10^{-3} + \int_4^5 2\times10^{-3}dt = 0$

$Q_6 = 0 + \int_5^6 2\times10^{-3}dt = 2\times10^{-3}$ Coulombs

$Q_7 = 2\times10^{-3} + \int_6^7 2\times10^{-3}dt = 4\times10^{-3}$ Coulombs

$t = 8,9,10s$

$Q = 4\times10^{-3}$Coulombs

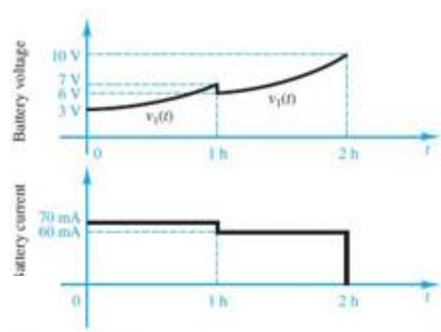
Problem 1.10

The charge cycle shown in Figure P2.10 is an example of a two-rate charge. The current is held constant at 70 mA for 1 h. Then it is switched to 60 mA for the next 1 h. Find:

a. The total charge transferred to the battery.
b. The total energy transferred to the battery.
Hint: Recall that energy w is the integral of power, or P = dw/dt. Let:

$$v_1 = 5 + e^{t/5194.8}\ V$$

$$v_2 = \left(6 - \frac{4}{e^1 - 1}\right) + \frac{4}{e^2 - e^1} * e^t\ V$$



Solution:

Known quantities:

See Figure P1.10

Find:

a) The total charge transferred to the battery.
b) The energy transferred to the battery.

Analysis:

Current is equal to $\frac{Coulombs}{Second}$, therefore given the current and a duration of that current, the transferred charge can be calculated by the following equation:

$$A \cdot t = C$$

The two durations should be calculated independently and then added together.

$$0.070A \cdot 3600s = 252C$$
$$0.060A \cdot 3600s = 216C$$
$$648C + 216C = \mathbf{468C}$$

P=V·I, therefore, an equation for power can be found by multiplying the two graphs together. First separate the voltage graph into three equations:

0 h → 1 h :$V = 5 + e^{t/5194.8}V$

1 h → 2 h : $V = \left(6 - \frac{4}{e^{1}-1}\right) + \frac{4}{e^{2}-e^{1}} * e^{t} V$

Next, multiply the first equation by 0.07A and the second by 0.06A.

0 h → 1 h : $P = 0.35 + 0.07e^{t/5194.8}$

1 h → 2 h : $P = 0.06\left(6 - \frac{4}{e^{1h}-1}\right) + 0.06\frac{4}{e^{2h}-e^{1h}} * e^{t} V$

Finally, since Energy is equal to the integral of power, take the integral of each of the equations for their specified times and add them together.

0 h → 1 h : $E = \left[0.35t + 363.64e^{t/5194.8}\right]\Big|_{0}^{3600}$ = 1623.53 J

1 h → 2 h : $E = [0.36t + 2.88 * 10^{-3128} * 2.72^{t}]\Big|_{3600}^{7200}$ = 1296.24 J

$$\boldsymbol{E_{Total} = 2919.77\, J}$$

Problem 1.11

The charging scheme used in Figure P1.11 is an example of a constant-current charge cycle. The charger voltage is controlled such that the current into the battery is held constant at 40 mA, as shown in Figure P1.11. The battery is charged for 6 h. Find:

a. The total charge delivered to the battery.

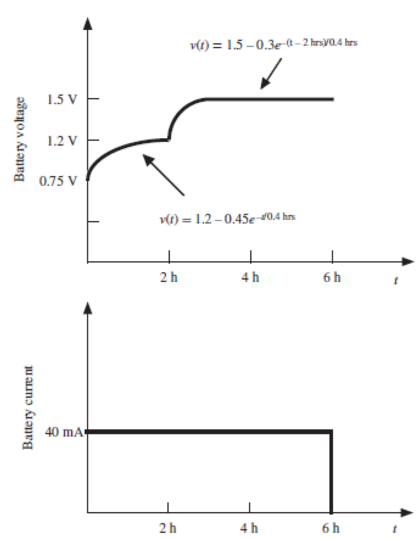
b. The energy transferred to the battery during the charging cycle.

Hint: Recall that the energy, w, is the integral of power, or P = dw/dt.

Solution:

Known quantities:

Current-time curve and voltage-time curve of battery recharging



Find:

Total transferred charge

Total transferred energy

Analysis:

a) $40mA = 0.04A$

Q = area under the current-time curve $= \int I dt = (0.04)(6)(3600) = 864$ C

$\boxed{Q = 864\,C}$

b) $\frac{dw}{dt} = P$ so

$$w = \int Pdt = \int vidt = (3600)\int_0^2 vidt + (3600)\int_2^4 vidt$$

$$= (3600)\int_0^2 \left(1.2 - 0.45e^{-t/0.4}\right)(0.04)dt + (3600)\int_2^4 \left(1.5 - 0.3e^{-(t-2)/0.4}\right)(0.04)dt$$

$$= 1{,}167\,J$$

$$\boxed{Energy = 1{,}167J}$$

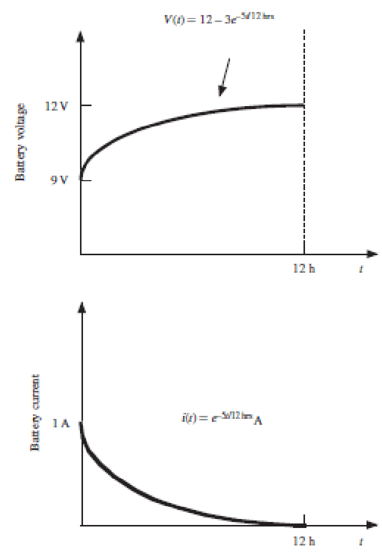
Problem 1.12

The charging scheme used in Figure P1.12 is called a tapered-current charge cycle. The current starts at the highest level and then decreases with time for the entire charge cycle, as shown. The battery is charged for 12 h. Find:

a. The total charge delivered to the battery.

b. The energy transferred to the battery during the charging cycle.

Hint: Recall that the energy, w, is the integral of power, or P = dw/dt.



Solution:

Known quantities:

Current-time curve and voltage-time curve of battery recharging

Find:

Total transferred charge

Total transferred energy

Analysis:

$$Q = \text{area under the current - time curve} = \int Idt = (3600)\int_0^{12} e^{-5t/12}dt = 8{,}564\ \text{C}$$

a)

$$\boxed{Q = 8{,}564\text{C}}$$

b) $\frac{dw}{dt} = P$ so

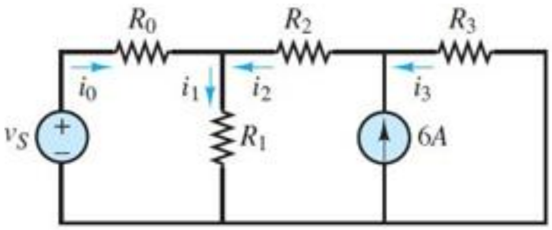
$$w = \int Pdt = \int vidt = (3600)\int_0^2 \left(12 - 3e^{-5t/12}\right)\left(e^{-5t/12}\right)dt$$

$$= 8{,}986\text{J}$$

$$\boxed{\text{Energy} = 8{,}986\text{J}}$$

Problem 1.13

Use KCL to determine the unknown currents in Figure P1.13.



Solution:

Known quantities:

$i_o = 2$ A, $i_2 = -7$ A

Find:

i_1
i_3

Analysis:

a) Use KCL at the node between R_o, R_1, and R_2.

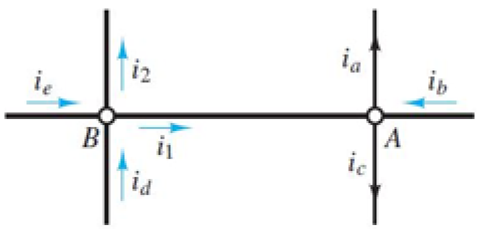
$$i_0 - i_1 + i_2 = 0$$
$$i_1 = i_0 + i_2$$
$$\boldsymbol{i_1 = -5A}$$

b) Use KCL at the node between R_2, R_3, and the current source.

$$6A + i_3 - i_2 = 0$$
$$i_3 = i_2 - 6A$$
$$\boldsymbol{i_3 = -13A}$$

Problem 1.14

Use KCL to find the currents i_1 and i_2 in Figure P1.14.



Solution:

Known quantities:

i_a = 3 A, i_b = -2 A, i_c = 1 A, i_d = 6 A, i_e = -4 A

Find:

i_1
i_2

Analysis:

a) Use KCL at Node A.

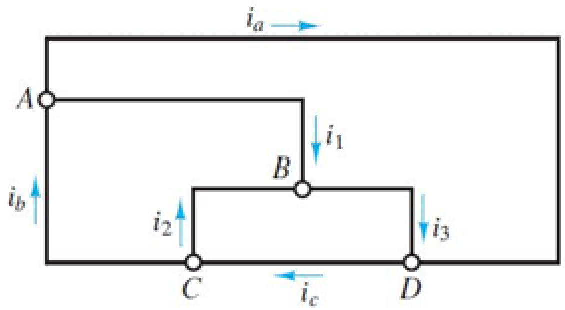
$$i_1 + i_b - i_a - i_c = 0$$
$$i_1 = i_a - i_b + i_c$$
$$\boldsymbol{i_1 = 6A}$$

b) Use KCL at Node B.

$$i_e + i_d - i_1 - i_2 = 0$$
$$i_2 = i_e + i_d - i_1$$
$$\boldsymbol{i_2 = -4A}$$

Problem 1.15

Use KCL to find the current i_1, i_2, and i_3 in the circuit of Figure P1.15.



Solution:

Known quantities:

i_a = 2 mA, i_b = 7 mA, i_c = 4 mA

Find:

i_1
i_2
i_3

Analysis:

a) Use KCL at Node A.

$$i_b - i_a - i_1 = 0$$
$$i_1 = i_b - i_a$$
$$\boldsymbol{i_1 = 5mA}$$

b) Use KCL at Node C.

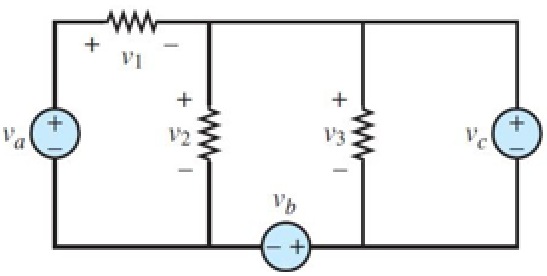
$$i_c - i_2 - i_b = 0$$
$$i_2 = i_c - i_b$$
$$\boldsymbol{i_2 = -3mA}$$

c) Use KCL at Node D.

$$i_3 + i_a - i_c = 0$$
$$i_3 = i_2 - i_a$$
$$\boldsymbol{i_3 = 2mA}$$

Problem 1.16

Use KVL to find the voltages v_1,v_2, and v_3 in Figure P1.16.



Solution:

Known quantities:

$V_a = 2$ V, $V_b = 4$ V, $V_c = 5$ V

Find:

V_1
V_2
V_3

Analysis:

a) Use KVL at the third loop.

$$V_3 - V_c = 0$$
$$V_3 = V_c$$
$$\boldsymbol{V_3 = 5V}$$

b) Use KVL at the second loop.

$$V_2 - V_3 - V_b = 0$$

$$V_2 = V_3 + V_b$$
$$\boldsymbol{V_2 = 9V}$$

c) Use KCL at the first loop.

$$V_a - V_1 - V_2 = 0$$
$$V_1 = V_a - V_2$$
$$\boldsymbol{V_1 = -7V}$$

Problem 1.17

Use KCL to determine the current i_1, i_2, i_3, and i_4 in the circuit of Figure P1.17.

Solution:

Known quantities:

i_a = -2 A, i_b = 6 A, i_c = 1 A, i_d = -4 A

Find:

i_1
i_2
i_3
i_4

Analysis:

a) Use KCL at Node A.

$$i_1 - i_a - i_c = 0$$
$$i_1 = i_a + i_c$$
$$\boldsymbol{i_1 = -1A}$$

b) Use KCL at Node B.

$$i_2 - i_1 - i_b = 0$$
$$i_2 = i_1 + i_b$$
$$\boldsymbol{i_2 = 5A}$$

c) Use KCL at Node C.

$$i_3 - i_2 - i_d = 0$$
$$i_3 = i_2 + i_d$$
$$\boldsymbol{i_3 = 1A}$$

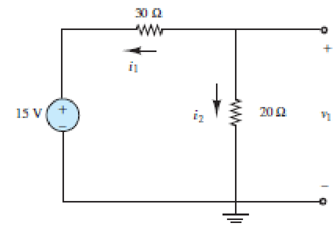
d) Use KCL at Node D.

$$i_c + i_4 - i_3 = 0$$
$$i_4 = i_3 - i_c$$
$$\boldsymbol{i_4 = 0A}$$

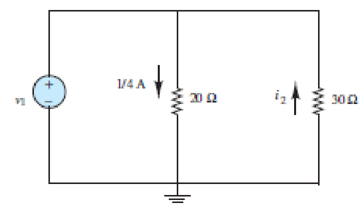
Section 1.4 Power and the Passive Sign Convention

Problem 1.18

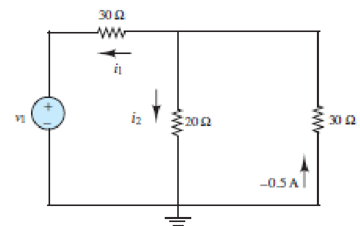
In the circuits of Figure P1.18, the directions of current and polarities of voltage have already been defined. Find the actual values of the indicated currents and voltages.



(a)



(b)



(c)

Solution:

Known quantities:

Circuit shown in Figure P1.18.

Find:

Voltages and currents in every figure.

Analysis:

(a) Using $I = \frac{15}{30+20}$ (clockwise current) : $I_1 = -0.3A; I_2 = 0.3A; V_1 = 6V$

(b) The voltage across the 20 Ω resistor is $\frac{20}{4} = 5\text{V}$; since the current flows from top to bottom, the polarity of this voltage is positive on top. Then it follows that $V_1 = 5V$ and $I_2 = \frac{5}{30} = \text{-}0.167\text{A}$ (the negative sign follows from the direction of I_2 in the drawing).

(c) Since -0.5A pointing upward is the same current as 0.5A pointing downward, the voltage across the 30 Ω resistor is

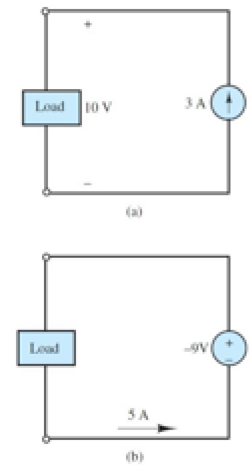
$V_{30\Omega} = 15V$ (positive on top); and $I_2 = \frac{15}{20} = 0.75\text{A}$,

since $V_{30\Omega}$ is also the voltage across the 20 Ω resistor. Finally,

$I_1 = \text{-}(I_2 + 0.5) = \text{-}1.25A$,and $V_1 = \text{-}30\, I_1 + 15 = 52.5V$

Problem 1.19

Find the power delivered by each source in Figure P1.19.



and

Solution:

Known quantities:

Circuit shown in Figure P1.19.

a) Power delivered by the 3A Current Source
b) Power delivered by the -9V Voltage Source

Analysis:

a) Follow the counterclockwise current:

$$P = (+3A) \cdot (+10V)$$

$$\boldsymbol{P = +30W \text{supplied}}$$

b) Follow the counterclockwise current:

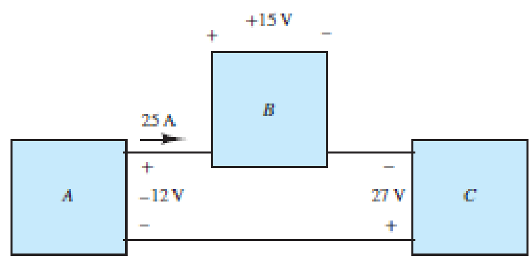
$$P = (+5A) \cdot (-9V)$$

$$\boldsymbol{P = -45W \text{supplied}}$$

Problem 1.20

Determine whether each element in Figure P2.20 is supplying or dissipating power, and how much.

Solution:



Known quantities:

Circuit shown in Figure P1.20.

Find:

Determine power dissipated or supplied for each power source.

Analysis:

Element A:

$P = -vi = -(-12V)(25A) = 300W$ (dissipating)

Element B:

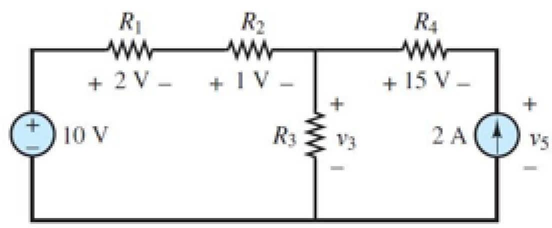
$P = vi = (15V)(25A) = 375W$ (dissipating)

Element C:

$P = vi = (27V)(25A) = 675W$ (supplying)

Problem 1.21

In the circuit of Figure P1.21, find the power absorbed by R_4 and the power delivered by the current source.



Solution:

Known quantities:

Circuit shown in Figure P1.21.

Find:

a) Power absorbed by R_4
b) Power delivered by the current source

Analysis:

a) Follow the counterclockwise current in the rightmost loop:

$$P = (2A) \cdot (-15V)$$

$$\boldsymbol{P = -30W \text{absorbed}}$$

$$\boldsymbol{P = +30W \text{supplied}}$$

b) Use KVL at the leftmost loop to find V_3:

$$10V - 2V - 1V - V_3 = 0$$

$$V_3 = 7V$$

Use KVL at the rightmost loop to find V_5:

$$7V - 15V - V_5 = 0$$

$$V_5 = -8V$$

The current source has a -8V drop across it. Use this to calculate the power dissipated using the proper sign convention.

$$\boldsymbol{(+2A) \cdot (-8V) = -16W \text{supplied}}$$

Problem 1.22

For the circuit shown in Figure P1.22:
a. Determine whether each component is absorbing or delivering power.
b. Is conservation of power satisfied? Explain your answer.

Solution:

Known quantities:

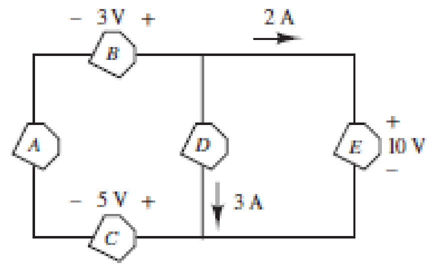
Circuit shown in Figure P1.22.

Find:

Determine power absorbed or power delivered
Testify power conservation

Analysis:

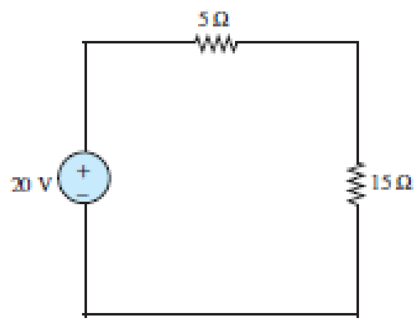
By KCL, the current through element B is 5A, to the right.
By KVL, $-v_a - 3 + 10 + 5 = 0$.
Therefore, the voltage across element A is
$v_a = 12V$ (positive at the top).



A supplies $(12V)(5A) = 60W$
B supplies $(3V)(5A) = 15W$
C absorbs $(5V)(5A) = 25W$
D absorbs $(10V)(3A) = 30W$
E absorbs $(10V)(2A) = 20W$
Total power supplied = $60W + 15W = 75W$
Total power absorbed = $25W + 30W + 20W = 75W$
Tot. power supplied = Tot. power absorbed
$\therefore$ conservation of power is satisfied.

Problem 1.23

For the circuit shown in Figure P1.23, determine the power absorbed by the 5 Ω resistor.



Solution:

Known quantities:

Circuit shown in Figure P1.23.

Find:

Power absorbed by the 5Ω resistance.

Analysis:

The current flowing clockwise in the series circuit is $i = \frac{20V}{20\hat{o}} = 1A$

The voltage across the 5 □ resistor, positive on the left, is $v_{5\hat{o}} = (1A)(5\hat{o}) = 5V$

Therefore, $P_{5\hat{o}} = (5V)(1A) = 5\ W$

Problem 1.24

For the circuit shown in Figure P1.24, determine which components are supplying power and which are dissipating power. Also determine the amount of power dissipated and supplied.

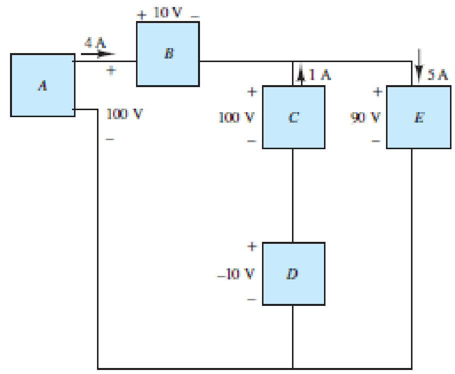
Solution:

Known quantities:

Circuit shown in Figure P1.24.

Find:

Determine power absorbed or power delivered and corresponding amount.



Analysis:

If current direction is out of power source, then power source is supplying, otherwise it is absorbing.

A supplies $(100V)(4A) = 400$ W

B absorbs $(10V)(4A) = 40$ W

C supplies $(100V)(1A) = 100W$

D supplies $(-10V)(1A) = -10W$, i.e absorbs $10W$

E absorbs $(90V)(5A) = 450W$

Problem 1.25

For the circuit shown in Figure P1.25.determine which components are supplying power and which are dissipating power. Also determine the amount of power dissipated and supplied.

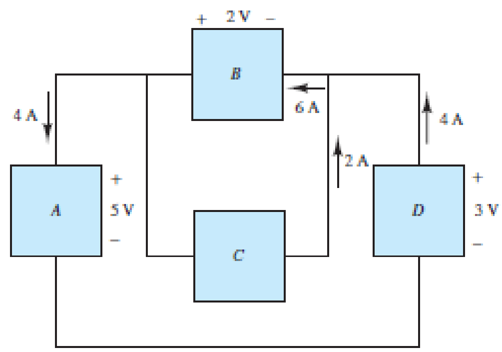
Solution:

Known quantities:

Circuit shown in Figure P1.25.

Find:

Determine power absorbed or power delivered and corresponding amount.



Analysis:

If current direction is out of power source, then power source is supplying, otherwise it is absorbing.

A absorbs $(5V)(4A) = 20W$

B supplies $(2V)(6A) = 12W$

D supplies $(3V)(4A) = 12W$

Since conservation of power is satisfied, Tot. power supplied = Tot. power absorbed

Total power supplied = $12\text{W} + 12\text{W} = 24\text{W}$

$\therefore$ Cabsorbs $24W - 20W = 4W$

Problem 1.26

If an electric heater requires 23 A at 110 V, determine

a. The power it dissipates as heat or other losses.

b. The energy dissipated by the heater in a 24-hperiod.

c. The cost of the energy if the power companycharges at the rate 6 cents/kWh.

Solution:

Known quantities:

Current absorbed by the heater; voltage at which the current is supplied; cost of the energy.

Find:

Power consumption
Energy dissipated in 24 hr.
Cost of the Energy

Assumptions:

The heater works for 24 hours continuously.

Analysis:

a) $P = VI = 110\,V\,(23\,A) = 2.53\times 10^3 \frac{J}{A}\frac{A}{s} = 2.53\text{ KW}$

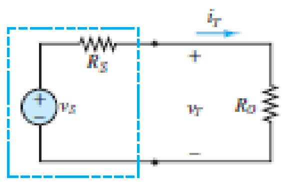
b) $W = Pt = 2.53\times 10^3 \frac{\text{J}}{\text{s}} \times 24\text{ hr}\times 3600\frac{\text{s}}{\text{hr}} = 218.6\text{ MJ}$

c) $\text{Cost} = (\text{Rate})\times W = 6\frac{\text{cents}}{\text{kW}-\text{hr}}(2.53\text{ kW})(24\text{ hr}) = 364.3\text{ cents} = \$3.64$

Sections 1.5-1.6: *i-v* Characteristics and Sources; Resistance and Ohm's Law

Problem 1.27

In the circuit shown in Figure P1.27, determine the terminal voltage v_T of the source, the power absorbed by $R_o = R_L$ and the efficiency of the circuit. Efficiency is defined as the ratio of load power to source power.



Solution:

Known quantities:

Circuit shown in Figure P1.27 with voltage source, $V_s = 12V$; internal resistance of the source, $R_s = 5k\Omega$; and resistance of the load, $R_L = 7k\Omega$.

Find:

The terminal voltage of the source; the power supplied to the circuit, the efficiency of the circuit.

Assumptions:

Assume that the only loss is due to the internal resistance of the source.

Analysis:

$$KVL: \quad -V_S + I_T R_S + V_T = 0 \qquad OL: \quad V_T = I_T R_L \quad \therefore \quad I_T = \frac{V_T}{R_L}$$

$$-V_S + \frac{V_T}{R_L} R_S + V_T = 0$$

$$V_T = \frac{V_S}{1+\frac{R_S}{R_L}} = \frac{12\ V}{1+\frac{5\ k\Omega}{7\ k\Omega}} = 7\ V \quad or \quad VD: V_T = \frac{V_S R_L}{R_S + R_L} = \frac{12\ V\ 7\ k\Omega}{5\ k\Omega + 7\ k\Omega} = 7\ V.$$

$$P_L = \frac{V_R^2}{R_L} = \frac{V_T^2}{R_L} = \frac{(7\ V)^2}{7\times 10^3\ \frac{\text{V}}{\text{A}}} = 7\ \text{mW}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{R_L}}{P_{R_S} + P_{R_L}} = \frac{I_T^2 R_L}{I_T^2 R_S + I_T^2 R_L} = \frac{7\ k\Omega}{5\ k\Omega + 7\ k\Omega} = 0.5833 \quad or \quad 58.33\%$$

.

Problem 1.28

A 24-V automotive battery is connected to two headlights that are in parallel, similar to that shown in Figure 1.11. Each headlight is intended to be a 75-W load; however, one 100-W headlight is mistakenly installed. What is the resistance of each headlight? What is the total resistance seen by the battery?

Solution:

Known quantities:

Headlights connected in parallel to a 24-V automotive battery; power absorbed by each headlight.

Find:

Resistance of each headlight; total resistance seen by the battery.

Analysis:

Headlight no. 1:

$$P = v \times i = 100\ \text{W} = \frac{v^2}{R} \quad \text{or}$$

$$R = \frac{v^2}{100} = \frac{576}{100} = 5.76\ \Omega$$

Headlight no. 2:

$$P = v \times i = 75\ \text{W} = \frac{v^2}{R} \quad \text{or}$$

$$R = \frac{v^2}{75} = \frac{576}{75} = 7.68\ \Omega$$

The total resistance is given by the parallel combination:

$$\frac{1}{R_{TOTAL}} = \frac{1}{5.76\ \Omega} + \frac{1}{7.68\ \Omega}$$ or $R_{TOTAL} = 3.29\ \Omega$

Problem 1.29

What is the equivalent resistance seen by the battery of Problem 1.28 if two 15-W taillights are added (in parallel) to the two 75-W (each) headlights?

Solution:

Known quantities:

Headlights and 24-V automotive battery of problem 2.13 with 2 15-W taillights added in parallel; power absorbed by each headlight; power absorbed by each taillight.

Find:

Equivalent resistance seen by the battery.

Analysis:

The resistance corresponding to a 75-W headlight is:

$$R_{75W} = \frac{v^2}{75} = \frac{576}{75} = 7.68\ \Omega$$

For each 15-W tail light we compute the resistance:

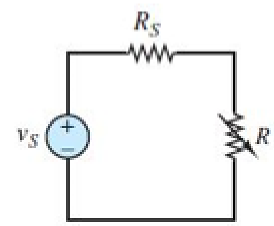
$$R_{15W} = \frac{v^2}{15} = \frac{576}{15} = 38.4\ \Omega$$

Therefore, the total resistance is computed as:

$$\frac{1}{R_{TOTAL}} = \frac{1}{7.68\Omega} + \frac{1}{7.68\Omega} + \frac{1}{38.4\Omega} + \frac{1}{38.4\Omega} \text{ or } R_{TOTAL} = 3.2\ \Omega$$

Problem 1.30

For the circuit shown in Figure P1.30, determine the power absorbed by the variable resistor R, ranging from 0 to 30 Ω. Plot the power absorption as a function of R. Assume that v_S = 15 V, R_S = 10 Ω.



Solution:

Known quantities:

v_s=15V, Rs=10 Ohms, and the circuit in Figure P1.30.

Find:

R

Analysis:

Use ohms law to find an equation for P as a function of R:

$$P_R = V_R * I_R$$

The voltage across R is equal to the source voltage minus the voltage across R_s:

$$V_R = 15V - V_{Rs}$$

V_{RS} is determined by the current through the loop which can be found by adding the resistors in series:

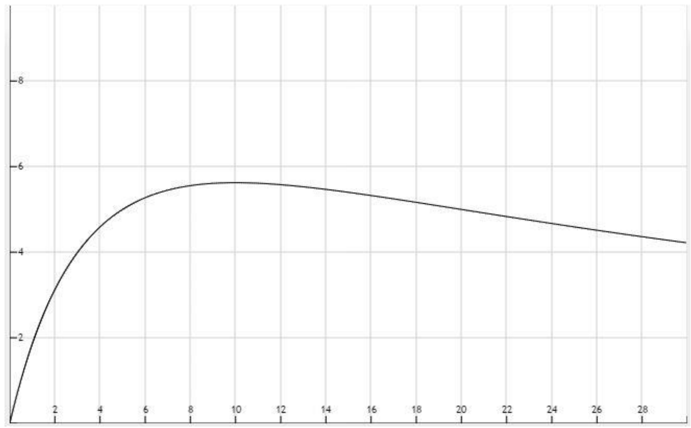
$$I_R = \frac{15V}{(R_s + R)}$$

$$V_{RS} = 10\Omega * I_R$$

Simplify:

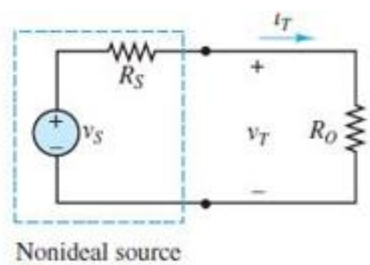
$$P_R = \left[15V - \left(10\Omega * \frac{15V}{10\Omega + \mathrm{R}}\right)\right] * \left[\frac{15V}{10\Omega + \mathrm{R}}\right]$$

Plot:



Problem 1.31

Refer to Figure P1.27 and assume that v_S = 15 V and R_S = 100 _. For i_T = 0, 10, 20, 30, 80, and 100 mA:
a. Find the total power supplied by the ideal source.
b. Find the power dissipated within the non-ideal source.
c. How much power is supplied to the load resistor?
d. Plot the terminal voltage v_T and power supplied to the load resistor as a function of terminal current i_T .



Solution:

Known quantities:

v_s=15V, Rs=100 Ohms, i_T= 0, 10, 20, 30, 80, 100 mA.
The circuit in Figure P1.27.

Find:

a) The total power supplied by the ideal source
b) The power dissipated within the non-ideal source
c) How much power is supplied to the load resistor
d) Plot v_T and power supplied to R_0 as a function of i_T.

Analysis:

a) The power supplied by the ideal source is equal to the current through the loop times the 15V of the supply. From current lowest to highest the power supplied would be:

$$0W \quad 0.15W \quad 0.3W \quad 0.45W \quad 1.2W \quad 1.5W$$

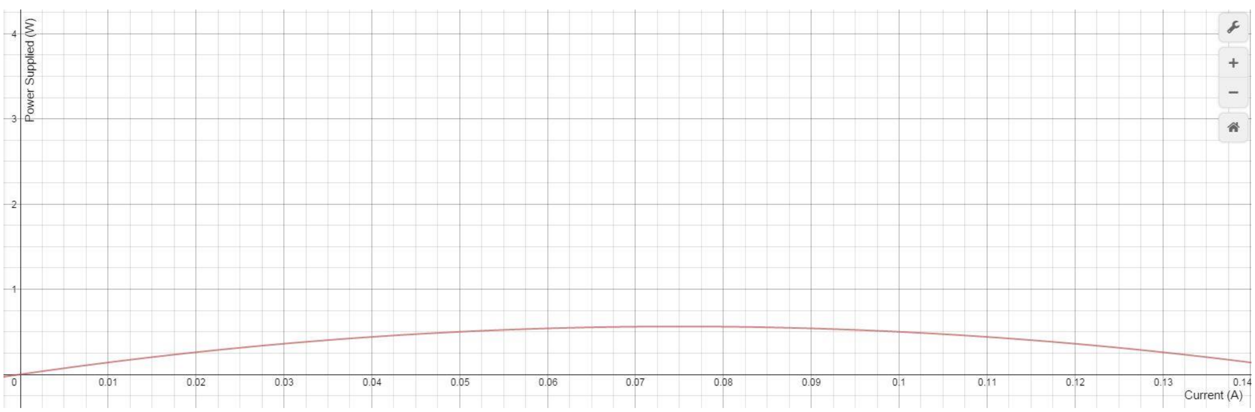
b) The power dissipated within the non-ideal source is the power dissipated by R_s which can be found using $P=i^2*r$. From current lowest to highest the power dissipated would be:

$$0W \quad 0.01W \quad 0.04W \quad 0.09W \quad 0.64W \quad 1W$$

c) The power supplied to the load resistor is equal to the total power supplied minus the power dissipated by the non-ideal source. From current lowest to highest the power supplied would be:

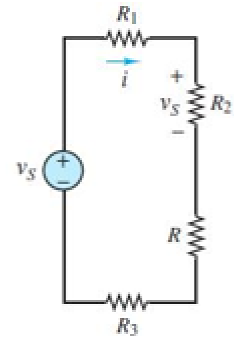
$$0W \qquad 0.14W \qquad 0.26W \qquad 0.36W \qquad 0.56W \qquad 0.5W$$

d) For the v_T plot Ohm's Law can be used to find the voltage drop across R_s which is equal to V_T. For the power plot, the data from part c can be used directly.



Problem 1.32

In the circuit in Figure P1.32, assume $v_2 = v_s/6$ and the power delivered by the source is 150 mW. Also assume that $R_1 = 8$ kΩ, $R_2 = 10$ kΩ, $R_3 = 12$ kΩ. Find R, v_s, v_2, and i.



Solution:

Known quantities:

R_1=8kΩ, R_2=10kΩ, R_3=12kΩ and the circuit in Figure P1.32.

Find:

R,v_s, v_2, and i.

Analysis:

Use ohms law to find v_s and i:

$$v_2 = \frac{v_s}{6} = R_2 * i$$

Also:

$$v_s * i = 150mW$$

So:

$$\boldsymbol{i = \sqrt[2]{\frac{150mW}{R_2 * 6}} = 1.6mA}$$

Since we know i, v_s can easily be found:

$$v_s * i = 150mW$$
$$\boldsymbol{v_s = 94.87V}$$

$$\boldsymbol{v_2 = \frac{v_s}{6} = 15.81V}$$

Use Ohm's law to find Req:

$$\frac{v_s}{i} = R_{eq} = 60k\Omega$$

Use Req to find R:

$$R_{eq} = R_1 + R + R_2 + R_3$$
$$R = R_{eq} - (R_1 + R_2 + R_3) = 30k\Omega$$

Problem 1.33

A GE SoftWhite Longlife lightbulb is rated as follows:

P_R = rated power = 60 W

P_{OR} = rated optical power = 820 lumens (lm) (average)

1 lumen = 1/680W

Operating life = 1,500 h (average)

V_R = rated operating voltage = 115 V

The resistance of the filament of the bulb, measured with a standard multimeter, is 16.7 Ω. Whenthe bulb is connected into a circuit and is operating atthe rated values given above, determine

a. The resistance of the filament.

b. The efficiency of the bulb.

Solution:

Known quantities:

Rated power; rated optical power; operating life; rated operating voltage; open-circuit resistance of the filament.

Find:

The resistance of the filament in operation
The efficiency of the bulb.

Analysis:

a)

$$P = VI \quad \therefore I = \frac{P_R}{V_R} = \frac{60\ \text{V}A}{115\ \text{V}} = 521.7\ \text{mA}$$

$$R = \frac{V}{I} = \frac{V_R}{I} = \frac{115\ \text{V}}{521.7\ \text{mA}} = 220.4\ \Omega$$

OL:

b)

Efficiency is defined as the ratio of the useful power dissipated by or supplied by the load to the total power supplied by the source. In this case, the useful power supplied by the load is the optical power. From any handbook containing equivalent units: 680 lumens=1 W

$P_{o,out}$ = Optical Power Out $= 820\ \text{lum}\ \dfrac{\text{W}}{680\ \text{lum}} = 1.206\ \text{W}$

$\eta =$ efficiency $= \dfrac{P_{o,out}}{P_R} = \dfrac{1.206\ \text{W}}{60\ \text{W}} = 0.02009 = 2.009\,\%$.

Problem 1.34

An incandescent lightbulb rated at 100 W will dissipate 100 W as heat and light when connected across a 110-V ideal voltage source. If three of these bulbs are connected in series across the same source, determine the power each bulb will dissipate.

Solution:

Known quantities:

Rated power; rated voltage of a light bulb.

Find:

The power dissipated by a series of three light bulbs connected to the nominal voltage.

Assumptions:

The resistance of each bulb doesn't vary when connected in series.

Analysis:

When connected in series, the voltage of the source will divide equally across the three bulbs. The across each bulb will be 1/3 what it was when the bulbs were connected individually across the source. Power dissipated in a resistance is a function of the voltage squared, so the power dissipated in each bulb when connected in series will be 1/9 what it was when the bulbs were connected individually, or 11.11 W:

Ohm's Law: $P = IV_B = I^2 R_B = \frac{V_B^2}{R_B}$ $\quad V_B = V_S = 110\,\text{V}$ $\quad R_B = \frac{V_B^2}{P} = \frac{(110\,\text{V})^2}{100\,\text{V}A} = 121\,\Omega$

Connected in series and assuming the resistance of each bulb remains the same as when connected individually:

KVL: $-V_S + V_{B1} + V_{B2} + V_{B3} = 0$ $\quad$ OL: $-V_S + IR_{B3} + IR_{B2} + IR_{B1} = 0$

$$I = \frac{V_S}{R_{B1} + R_{B2} + R_{B3}} = \frac{110\,\text{V}}{121 + 121 + 121\,\text{V}/A} = 303\,\text{mA}$$

$$P_{B1} = I^2 R_{B1} = (303\,\text{m}A)^2 (121\,\text{V}/A) = 11.11\,\text{W} = \frac{1}{9}100\,\text{W}.$$

Problem 1.35

An incandescent lightbulb rated at 60 W will dissipate 60 W as heat and light when connected across a 100-V ideal voltage source. A 100-W bulb will dissipate 100 W when connected across the same source. If the bulbs are connected in series across the same source, determine the power that either one of the two bulbs will dissipate.

Solution:

Known quantities:

Rated power and rated voltage of the two light bulbs.

Find:

The power dissipated by the series of the two light bulbs.

Assumptions:

The resistance of each bulb doesn't vary when connected in series.

Analysis:

For the two bulbs in series KVL and KCL require

$$100\text{V} = V_{100} + V_{60} \quad \text{and} \quad I_{100} = I_{60}$$

The resistance of each bulb when connected individually across a 100V source is

$$R_{100} = \frac{(100V)^2}{100W} = 100\Omega \quad \text{and} \quad R_{60} = \frac{(100V)^2}{60W} \cong 167\Omega$$

Assume that the resistance of each bulb is the same when operated in series as when operated alone. Then

$$V_{100} = I_{100}R_{100} = I_{60}(100) \quad \text{and} \quad V_{60} = I_{60}R_{60} = I_{60}(167)$$

Plug into the KVL equation to find

$$100\text{V} \cong I_{60}(100 + 167) \quad \rightarrow \quad I_{60} \cong \frac{100}{267}\text{A} \cong I_{100}$$

The power absorbed by each bulb is

$$P_{100} = I_{100}^2 R_{100} \cong 14.0\text{W}$$

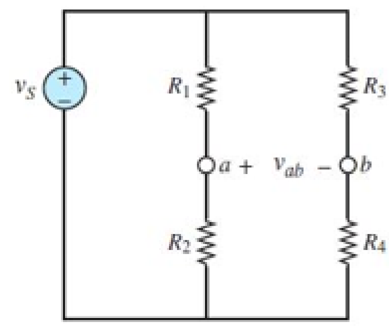
And

$$P_{60} = I_{60}^2 R_{60} \cong 23.4\text{W}$$

Notes: 1. It's strange but it's true that a 60 W bulb connected in series with a 100 W bulb will dissipate more power than the 100 W bulb. 2. If the power dissipated by the filament in a bulb decreases, the temperature at which the filament operates and therefore its resistance will decrease. This fact made the assumption about the resistance necessary.

Problem 1.36

Refer to Figure P1.36, and assume that v_S = 12 V, R_1 = 5 Ω, R_2 = 3 Ω, R_3 = 4 Ω, and R_4 = 5 Ω. Find:
a. The voltage v_{ab}.
b. The power dissipated in R_2.



Solution:

Known quantities:

Circuit of Figure P2.36. R_1=5Ω, R_2=3Ω, R_3=4Ω, R_4=5Ω, v_s=12V.

Find:

v_{ab} and the power dissipated in R_2.

Analysis:

Find the current through each resistor pair by combining them in series:

$$R_1 + R_2 = 8\Omega$$
$$R_3 + R_4 = 9\Omega$$

Since there is 12V across both of them Ohm's law is used to find the currents:

$$12V = 8\Omega * I_a$$
$$I_a = 1.5A$$
$$12V = 9\Omega * I_b$$
$$I_b = 1.33A$$

Now find the voltage drop across R_1 and R_3:

$$V_a = 12V - (I_a * R_1) = 4.5V$$
$$V_b = 12V - (I_b * R_3) = 6.66V$$
$$\boldsymbol{V_{ab} = -2.17V}$$

An alternate and more efficient approach is to apply voltage division to find the voltage across R_2 and R_4, respectively:

$$v_2 = v_s \frac{R_2}{R_1 + R_2} = 4.5V$$

$$v_4 = v_s \frac{R_4}{R_3 + R_4} = \frac{20}{3} V$$

It is implied in the calculations that the polarities of v_2 and v_4 are high (+) to low (-) from above to below each resistor in the figure. Apply KVL to find:

$$v_{ab} = v_2 - v_4 = \frac{13.5}{3} - \frac{20}{3} = -\frac{6.5}{3} V \cong -2.16V$$

P_{R2} is equal to I_b times V_a:

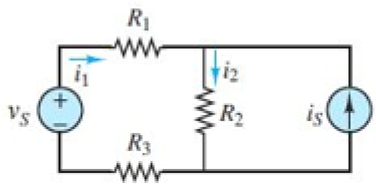
$$\boldsymbol{P_{R2} = v_2 i_2 = \frac{v_2^2}{R_2} = 6.75W}$$

Problem 1.37

Refer to Figure P1.37, and assume that V_S = 7 V, I_S = 3A, R_1 = 20 Ω, R_2 = 12 Ω, and R_3 = 10 Ω. Find:

a. The currents i_1 and i_2.

b. The power supplied by the voltage source v_S.



Solution:

Known quantities:

Circuit of Figure P2.37. R_1=20Ω, R_2=12Ω, R_3=10Ω,

I_s=3A, v_s=7V.

Find:

i_1, i_2, and P_v.

Analysis:

Use KVL of the rightmost loop:

$$v_s - R_1 * i_1 - R_2 * i_2 - R_3 * i_1 = 0$$
$$v_s = R_1 * i_1 + R_2 * i_2 + R_3 * i_1$$

Use KCL where i_1 meets i_S:

$$i_1 + i_s - i_2 = 0$$
$$i_1 + i_s = i_2$$

Combine the two equations to solve for i_1:

$$v_s = R_1 * i_1 + R_2 * (i_1 + i_s) + R_3 * i_1$$
$$\boldsymbol{i_1 = -0.69A}$$

Solve for i_2:

$$i_1 + i_s = i_2$$
$$\boldsymbol{i_2 = 2.31A}$$

Power delivered equals the $v_s$times i_1:

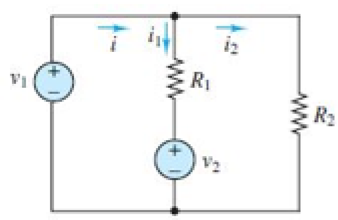
$$\boldsymbol{P_v = v_s * i_1 = -4.83W}$$

Problem 1.38

Refer to Figure P1.38, and assume v1 = 15 V, v2 = 6 V, R1 = 18Ω, R2 = 10Ω. Find:

a. The currents i_1 and i_2.

b. The power delivered by the sources v_1 and v_2.



Solution:

Known quantities:

Circuit of Figure P2.38. R_1=18Ω, R_2=10Ω, v_1=15V, v_2=6V.

Find:

i_1, i_2, P_{v1},P_{v2}.

Analysis:

Use KVL of the rightmost loop:

$$v_2 + R_1 * (-i_1) - R_2 * i_2 = 0$$
$$v_2 = R_1 * i_1 + R_2 * i_2$$

Use KCL where i_1 meets i_s:

$$i - i_1 - i_2 = 0$$
$$i_1 + i_2 = i$$

The assumption can be made that v_1 is equal to the voltage across R_2.

$$v_2 = R_2 * i_2$$
$$\boldsymbol{i_2 = 0.6A}$$

The assumption can be made that the voltage drop across R_1 is equal to the difference between v_1 and v_2.

$$v_1 - v_2 = R_1 * i_1$$
$$\boldsymbol{i_1 = 0.5A}$$

Solve for i:

$$i_1 + i_2 = i$$
$$i = 1.1A$$

Power delivered equals the voltage source times the current through it:

$$\boldsymbol{P_1 = v_1 * i = 16.5W}$$
$$\boldsymbol{P_2 = v_2 * (-i_1) = -3W}$$

Problem 1.39

Consider NiMH hobbyist batteries depicted in Figure P1.39.
a. If V_1 = 12.0 V, R_1 = 0.15 Ω and R_o = 2.55Ω find the load current I_o and the power dissipated by the load.
b. If battery 2 with V_2 = 12 V and R_2 = 0.28 Ω is placed in parallel with battery 1, will the load current I_o increase or decrease? Will the power dissipated by the load increase or decrease? By how much?

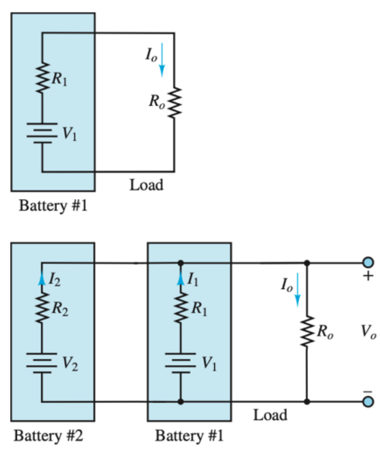


Figure P1.39

Solution:

Known quantities:

Schematic of the circuit in Figure P1.39.

Find:

If $V_1 = 12.0V, R_1 = 0.15\Omega, R_L = 2.55\Omega$, the load current and the power dissipated by the load

If a second battery is connected in parallel with battery 1with $V_2 = 12.0V, R_2 = 0.28\Omega$, determine the variations in the load current and in the power dissipated by the load due to the parallel connection with a second battery.

Analysis:

a) $I_L = \frac{V_1}{R_1 + R_L} = \frac{12}{0.15+2.55} = \frac{12}{2.7}$ = 4.44 A

$P_{Load} = I_L^2 R_L$ = 50.4 W.

b) with another source in the circuit we must find the new power dissipated by the load. To do so, we write KVL twice using mesh currents to obtain 2 equations in 2 unknowns:

$$\begin{cases} I_2 R_2 + V_1 - V_2 - I_1 R_1 = 0 \\ (I_1 + I_2) R_L + I_2 R_2 = V_2 \end{cases} \Rightarrow \begin{cases} 0.28 \cdot I_2 - 0.15 \cdot I_1 = 0 \\ 2.55 \cdot (I_1 + I_2) + 0.28 \cdot I_2 = 12 \end{cases}$$

Solving the above equations gives us:

$I_1 = 2.95$ A, $I_2 = 1.58$ A $\Rightarrow I_L = I_1 + I_2 = 4.53$ A $\Rightarrow$ $P_{Load} = I_L^2 R_L = 52.33$ W

The power dissipated increased by roughly 4%.

Problem 1.40

With no load attached, the voltage at the terminals of a particular power supply is 50.8 V. When a 10-W load is attached, the voltage drops to 49 V.

a. Determine v_S and R_S for this non-ideal source.

b. What voltage would be measured at the terminals in the presence of a 15-Ω load resistor?

c. How much current could be drawn from this power supply under short-circuit conditions?

Solution:

Known quantities:

Open-circuit voltage at the terminals of the power source is 50.8 V; voltage drop with a 10-W load attached is 49 V.

Find:

The voltage and the internal resistance of the source

The voltage at its terminals with a 15-Ω load resistor attached

The current that can be derived from the source under short-circuit conditions.

Analysis:

(a) $v_S = 50.8V$ is the open-circuit voltage. The power absorbed by the load is

$$P_o = i_T^2 R_o = \frac{v_T^2}{R_o} = \frac{(49V)^2}{R_o} = 10W \quad \rightarrow \quad R_o \approx 240\Omega$$

And thus

$$i_T \approx \sqrt{\frac{10W}{240\Omega}} \approx 204mA$$

Apply Ohm's law to solve for R_S.

$$R_S = \frac{(50.8 - 49)V}{204mA} \approx 8.8\Omega$$

(b)

$$v_T = i_T R_o = \frac{v_S}{(R_S + R_o)} R_o = \frac{R_o}{(R_S + R_o)} v_S \approx 32V$$

(c)

$$i_{SC} = \frac{v_S}{R_S} \approx 5.76A$$

Problem 1.41

A 220-V electric heater has two heating coils that can be switched such that either coil can be used independently or the two can be connected in series or parallel, yielding a total of four possible configurations. If the warmest setting corresponds to 2,000-W power dissipation and the coolest corresponds to 300 W, find
a. The resistance of each of the two coils.
b. The power dissipation for each of the other two possible arrangements.

Solution:

Known quantities:

Voltage of the heater, maximum and minimum power dissipation; number of coils, schematics of the configurations.

Find:

The resistance of each coil
The power dissipation of each of the other two possible arrangements.

Analysis:

(a) For the parallel connection, $P = 2000$ W. Therefore,

$$2000 = \frac{(220)^2}{R_1} + \frac{(220)^2}{R_2} = (220)^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

or,

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{5}{121}.$$

For the series connection, $P = 300$ W. Therefore, $300 = \frac{(220)^2}{R_1 + R_2}$ or, $\frac{1}{R_1 + R_2} = \frac{3}{484}$.

Solving, we find that $R_1 = 131.6\Omega$ and $R_2 = 29.7\Omega$.

(b) the power dissipated by R_1 alone is:

$$P_{R_1} = \frac{(220)^2}{R_1} = 368W$$

and the power dissipated by R_2 alone is

$$P_{R_2} = \frac{(220)^2}{R_2} = 1631W$$

Section 1.7-1.8: Resistors in Series and Voltage Division; Resistors in Parallel and Current Division

Problem 1.42

For the circuits of Figure P1.42, determine the resistor values (including the power rating) necessary to achieve the indicated voltages. Resistors are available in 1/8-, 1/4-, 1/2-, and 1-W ratings.

Solution:

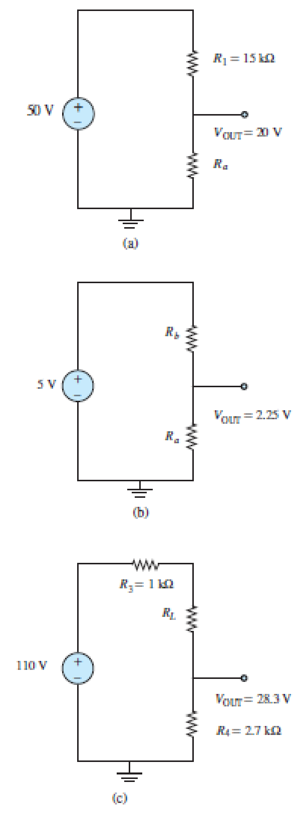
Known quantities:

Circuits of Figure 1.42.

Find:

Values of resistance and power rating

Analysis:



(a)
$$20 = \frac{R_a}{R_a + 15{,}000}(50)$$

$$R_a(50 - 20) = 20(15) \times 10^3$$

$$R_a = 10\text{k}\Omega$$

$$Pa = I^2R = \left(\frac{50}{25000}\right)^2(10{,}000) = 40 \text{ mW}$$

$$P_{R_a} = \frac{1}{8}\text{W}$$

$$P_1 = I^2R = 60\ mW$$

$$P_{R_1} = \frac{1}{8}\text{W}$$

(b)
$$2.25 = 5 \times \left(\frac{270}{270 + R_b}\right)$$

$$R_b = 330\Omega$$

$$P_{R_b} = \frac{1}{8}\text{W}$$

$$P_{R_2} = \frac{1}{8}\text{W}$$

c)
$$28.3 = 110 \times \left(\frac{2.7 \times 10^3}{2.7 \times 10^3 + 1 \times 10^3 + R_L}\right)$$

$$R_L = 6.8k\Omega$$

$$P_{R_L} = 1\text{W}$$

$$P_{R_3} = \frac{1}{8}\text{W}$$

$$P_{R_4} = \frac{1}{2}\text{W}$$

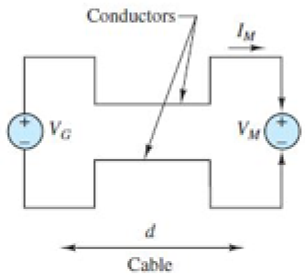
Problem 1.43

At an engineering site, a 1-hp motor is placed a distance *d* from a portable generator, as depicted in Figure P1.43. The generator can be modeled as an ideal DC source V_G = 110 V. The nameplate on the motor gives the following rated voltages and full-load currents:

V_{Mmin} = 105 V → I_{MFL} = 7.10 A

V_{Mmax} = 117 V → I_{MFL} = 6.37 A

If *d* = 150m and the motor must deliver its full-rated power, determine the minimum AWG conductors that must be used in a rubber-insulated cable. Assume that losses occur only in the wires.



Solution:

Known quantities:

Figure P1.43.

Find:

The minimum AWG wire that will meet the specifications.

Assumptions:

Assume an ideal DC source.

Analysis:

The circuit that this setup is describing is essentially a voltage source with a resistor, an inductor, and another resistor all in series.

350m is equal to 1148.29 feet. This will help with the resistance later.

Assume the voltage of the motor is 105V. Use Ohm's Law to calculate the Current through the first conductor:

$$I_M = \frac{110V - 105V}{2R}$$

I_M has to be 7.1 A or greater in order for the motor to draw full power:

$$7.1A = \frac{2.5V}{R}$$

$$R = 0.352\Omega$$

Use the value of R along with the chart to find the correct AWG:

$$\frac{0.352\Omega}{1.14829} = 0.307\Omega@1000ft$$

This means that **4 AWG** wire must be used.

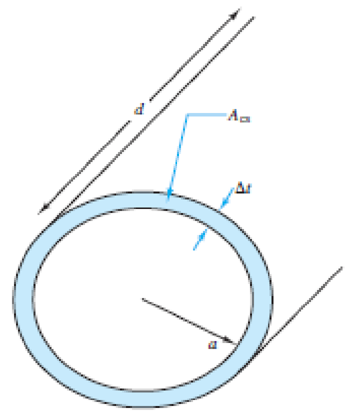
Problem 1.44

Cheap resistors are fabricated by depositing a thin layer of carbon onto a nonconducting cylindrical substrate (see Figure P1.44). If such a cylinder has radius *a* and length *d*, determine the thickness of the film required for a resistance R if

a = 1 mm   R = 33 kΩ   σ = 1/ρ= 2.9 MS/m   d = 9 mm

Neglect the end surfaces of the cylinder and assume that the thickness is much smaller than the radius.

Solution:



Known quantities:

Figure P1.50. Diameter of the cylindrical substrate; length of the substrate; conductivity of the carbon.

Find:

The thickness of the carbon film required for a resistance R of 33 kΩ.

Assumptions:

Assume the thickness of the film to be much smaller than the radius

Neglect the end surface of the cylinder.

Analysis:

$$R = \frac{d}{\sigma \cdot A} \cong \frac{d}{\sigma \cdot 2\pi a \cdot \Delta t}$$

$$\Delta t = \frac{d}{R \cdot 2\pi a \cdot \sigma} = \frac{9 \cdot 10^{-3}\ m}{33 \cdot 10^{3}\ \Omega \cdot 2.9 \cdot 10^{6}\ \frac{S}{m} \cdot 2\pi \cdot 1 \cdot 10^{-3}\ m}$$

Problem 1.45

The resistive elements of fuses, lightbulbs, heaters, etc., are nonlinear (i.e., the resistance is dependent on the current through the element). Assume the resistance of a fuse (Figure P1.45) is given by $R = R_0[1 + A(T - T_0)]$ with:

$T - T_0 = kP$; $T_0 = 25°C$; $A = 0.7[°C]^{-1}$; $k = 0.35°C/W$; $R_0 = 0.11\ \Omega$; and *P* is the power dissipated in the resistive element of the fuse. Determine the rated current at which the circuit will melt (that is, "blow") and thus act as an open-circuit. (*Hint:* The fuse blows when *R* becomes infinite.)

Solution:

Known quantities:

Figure P1.45. The constants A and k; the open-circuit resistance.

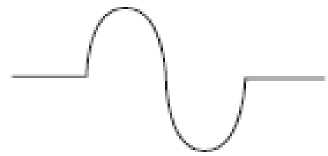
Find:

The rated current at which the fuse blows, showing that this happens at:

$$I = \frac{1}{\sqrt{AkR_0}}.$$

Assumptions:

Here the resistance of the fuse is given by:



Fuse

$R = R_0[1 + A(T - T_0)]$

where T_0, room temperature, is assumed to be 25°C.

We assume that:

$T - T_0 = kP$

where P is the power dissipated by the resistor (fuse).

Analysis:

$$R = R_0(1 + A \cdot \Delta T) = R_0(1 + AkP) = R_0(1 + AkI^2R)$$

$$R - R_0AkI^2R = R_0$$

$$R = \frac{R_0}{1 - R_0AkI^2} \quad \to \infty$$

when $I - R_0AkI^2 \to 0$

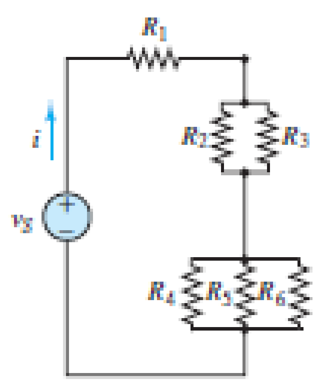
$$I = \frac{1}{\sqrt{AkR_0}} = (0.7\frac{m}{°C}\ 0.35\frac{°C}{Va}\ 0.11\frac{V}{a})^{-\frac{1}{2}}$$ =6.09 A.

Problem 1.46

Use KCL and Ohm's law to determine the current in each of the resistors R_4, R_5, and R_6 in the circuit of Figure P1.46. V_S = 10 V, R_1 = 20 Ω, R_2 = 40 Ω, R_3 = 10 Ω, R_4 = R_5 = R_6 = 15 Ω.

Solution:

Known quantities:



Circuit shown in Figure P1.46 with voltage source, $V_s = 10V$ and resistors, $R_1 = 20\Omega, R_2 = 40\Omega, R_3 = 10\Omega, R_4 = R_5 = R_6 = 15\Omega$.

Find:

The current in the 15-Ω resistors.

Analysis:

Since the 3 resistors must have equal currents,

$$I_{15\Omega} = \frac{1}{3} \cdot I$$

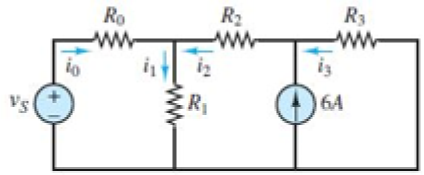
and,

$$I = \frac{V_S}{R_1 + R_2 \parallel R_3 + R_4 \parallel R_5 \parallel R_6} = \frac{10}{20 + 8 + 5} = \frac{10}{33} = 303$$ mA

Therefore, $I_{15\Omega} = \frac{10}{99} = 101$ mA

Problem 1.47

Refer to Figure P1.13. Assume $R_0 = 1\ \Omega$, $R_1 = 2\ \Omega$, $R_2 = 3\ \Omega$, $R_3 = 4\ \Omega$, and $v_S = 10$ V. Use KCL and Ohm's law to find the unknown currents.



Solution:

Known quantities:

Circuit shown in Figure P1.13

$R_0=1\Omega$, $R_1=2\Omega$, $R_2=3\Omega$, $R_3=4\Omega$, $v_s=10V$

Find:

a) i_0 b) i_1 c) i_2 d) i_3

Analysis:

Use KVL at mesh A:

$$v_s - (i_0 \cdot R_0) - (i_1 \cdot R_1) = 0$$

$10 = (i_0 \cdot 1) + (i_1 \cdot 2)$ Eq. 1

Use KVL at middle + right mesh:

$(i_1 \cdot R_1) = -(i_2 \cdot R_2) - (i_3 \cdot R_3)$ Eq. 2

Use KCL at node A:

$$i_0 + i_2 - i_1 = 0$$

$i_1 = i_0 + i_2$ Eq. 3

Use KCL at node B:

$$i_3 + 6A - i_2 = 0$$

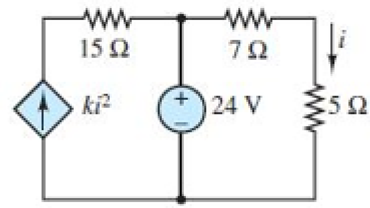
$i_2 = i_3 - 6A$ Eq. 4

Use algebra to solve the four equations with four unknowns:

$$\boldsymbol{i_0 = 1.83A}$$
$$\boldsymbol{i_1 = 4.09A}$$
$$\boldsymbol{i_2 = 2.26A}$$
$$\boldsymbol{i_3 = -3.74A}$$

Problem 1.48

Use KCL and Ohm's law to find the power delivered by the voltage source in Figure P1.48. Assume k=0.25A/A^2.



Solution:

Known quantities:

Circuit shown in Figure P1.48
k=0.25A/A^2

Find:

Power supplied by the voltage source.

Analysis:

The 7 Ohm and 5 Ohm resistor are in series so i can be found by using Ohm's law:

$$24V = 12 * i$$
$$i = 2A$$

The dependent current source current can therefore be found using i:

$$i_d = 0.25 * (2A)^2$$
$$i_d = 1A$$

Use KCL to determine the current through the voltage source:

$$i_v + i - i_d = 0$$
$$i_v = 1A$$

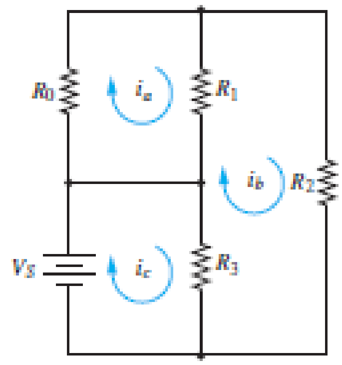
Use P=v*I to determine the power supplied by the voltage source:

$$P = 24V * 1A$$
$$\mathbf{P = 24W}$$

Problem 1.49

Assuming $R_0 = 2\ \Omega$, $R_1 = 1\Omega$, $R_2 = 4/3\ \Omega$ $R_3 = 6\ \Omega$and $V_S = 12$ V in the circuit of Figure P1.49, use Kirchhoff's voltage law and Ohm's law to find i_a, i_b, and i_c.

Solution:



Known quantities:

Schematic of the circuit shown in Figure P1.49 with resistors $R_0 = 2\Omega, R_1 = 1\Omega, R_2 = 4/3\Omega, R_3 = 6\Omega$ and voltage source $V_S = 12$V.

Find:

The mesh currents i_a, i_b, i_c
The current through each resistor.

Analysis:

Applying KVL to mesh (a), mesh (b) and mesh (c):

$$\begin{cases} i_a R_0 + (i_a - i_b) R_1 = 0 \\ (i_a - i_b) R_1 - i_b R_2 + (i_c - i_b) R_3 = 0 \\ V_S = (i_c - i_b) R_3 \end{cases} \Rightarrow \begin{cases} 2 i_a + (i_a - i_b) = 0 \\ (i_a - i_b) - \frac{4}{3} i_b + 6(i_c - i_b) = 0 \\ 6(i_c - i_b) = 12 \end{cases}$$

Solving the system we obtain:

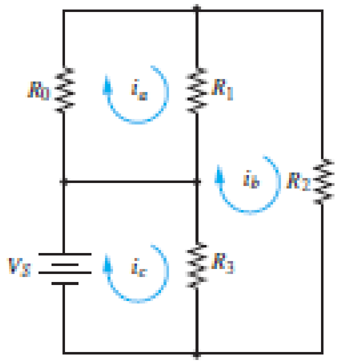
$$\begin{cases} i_a = 2 \text{ A} \\ i_b = 6 \text{ A} \\ i_c = 8 \text{ A} \end{cases} \Rightarrow \begin{cases} I_{R_0} = i_a = 2 \text{ A} & \text{(positive in the direction of } i_a) \\ I_{R_1} = i_b - i_a = 4 \text{ A} & \text{(positive in the direction of } i_b) \\ I_{R_2} = i_b = 6 \text{ A} & \text{(positive in the direction of } i_b) \\ I_{R_3} = i_c - i_b = 2 \text{ A} & \text{(positive in the direction of } i_c) \end{cases}$$

Problem 1.50

Refer to Figure P1.49 and assume R_0 = 2Ω, R_1 = 2 Ω, R_2 = 5 Ω, R_3 = 4 A, and V_S = 24 V. Use KCL and Ohm's law to find:

a. i_a, i_b, and i_c

b. The voltage across each resistor.



Solution:

Known quantities:

Schematic of the circuit shown in Figure P1.49 with resistors $R_0 = 2\Omega, R_1 = 2\Omega, R_2 = 5\Omega, R_3 = 4\Omega$ and voltage source $V_S = 24$V.

Find:

The mesh currents i_a, i_b, i_c

The current through each resistor.

Analysis:

Applying KVL to mesh (a), mesh (b) and mesh (c):

$$\begin{cases} i_a R_0 + (i_a - i_b) R_1 = 0 \\ (i_a - i_b) R_1 - i_b R_2 + (i_c - i_b) R_3 = 0 \\ V_S = (i_c - i_b) R_3 \end{cases} \Rightarrow \begin{cases} 2 i_a + 2(i_a - i_b) = 0 \\ 2(i_a - i_b) - 5 i_b + 4(i_c - i_b) = 0 \\ 4(i_c - i_b) = 24 \end{cases}$$

Solving the system we obtain:

$$\begin{cases} i_a = 2 \text{ A} \\ i_b = 4 \text{ A} \\ i_c = 10 \text{ A} \end{cases} \Rightarrow \begin{cases} V_{R_0} = R_0 i_a = 4 \text{ V} & (\oplus \text{ up}) \\ V_{R_1} = R_1 (i_b - i_a) = 4 \text{ V} & (\oplus \text{ down}) \\ V_{R_2} = R_2 i_b = 20 \text{ V} & (\oplus \text{ up}) \\ V_{R_3} = R_3 (i_c - i_b) = 24 \text{ V} & (\oplus \text{ up}) \end{cases}$$

Problem 1.51

Assume that the voltage source in the circuit of Figure P1.49 is now replaced by a current source I_S, and $R0$ = 1 Ω, $R1$ = 3 Ω, $R2$= 2Ω, $R3$ = 4 Ω, and I_S = 12 A, directed positively upward. Use KVL and Ohm's law to determine the voltage across each resistor.

Solution:

Known quantities:

Schematic of the circuit shown in Figure P1.49 with resistors $R_0 = 1\Omega, R_1 = 3\Omega, R_2 = 2\Omega, R_3 = 4\Omega$ and of the current source $I_S = 12$A.

Find:

The voltage across each resistance.

Analysis:

Applying KVL to mesh (a), mesh (b) and mesh (c):

$$\begin{cases} i_a R_0 + (i_a - i_b)R_1 = 0 \\ (i_a - i_b)R_1 - i_b R_2 + (i_c - i_b)R_3 = 0 \\ i_c = I_S \end{cases} \Rightarrow \begin{cases} i_a + 3(i_a - i_b) = 0 \\ 3(i_a - i_b) - 2i_b - 4i_b + 48 = 0 \\ i_c = 12 \text{ A} \end{cases}$$

Solving the system we obtain:

$$\begin{cases} i_a = \dfrac{16}{3} \text{ A} \\ i_b = \dfrac{64}{9} \text{ A} \\ i_c = 12 \text{ A} \end{cases} \Rightarrow \begin{cases} V_{R_0} = R_0 i_a = 5.33 \text{ V} & (\oplus \text{ up}) \\ V_{R_1} = R_1(i_b - i_a) = 5.33 \text{ V} & (\oplus \text{ down}) \\ V_{R_2} = R_2 i_b = 14.22 \text{ V} & (\oplus \text{ up}) \\ V_{R_3} = R_3(i_c - i_b) = 19.55 \text{ V} & (\oplus \text{ up}) \end{cases}$$

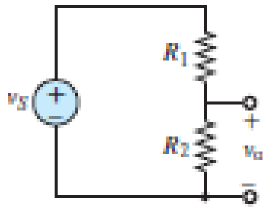
Problem 1.52

The voltage divider network of Figure P1.52 is expected to provide $v_{out} = v_S/2$. However, in practice, the resistors may not be perfectly matched; that is, their tolerances are such that the resistances are unlikely to be identical. Assume v_S = 10 V and nominal resistance values of R_1 = R_2 = 5 kΩ.
a. If the resistors have +/-10 percent tolerance, find the expected range of possible output voltages.
b. Find the expected output voltage range for a tolerance of +/-5 percent.

Solution:

Known quantities:

Schematic of voltage divider network shown of Figure P1.52.



Find:

The worst-case output voltages for ± 10percent tolerance
The worst-case output voltages for ± 5percent tolerance

Analysis:

a) 10% worst case: low voltage
R_2 = 4500 Ω, R_1 = 5500 Ω

$$v_{OUT,MIN} = \frac{4500}{4500+5500}5 = 2.25V$$

10% worst case: high voltage

R_2 = 5500 Ω, R_1 = 4500 Ω

$$v_{OUT,MAX} = \frac{5500}{4500+5500}5 = 2.75V$$

b) 5% worst case: low voltage

R_2 = 4750 Ω, R_1 = 5250 Ω

$$v_{OUT,MIN} = \frac{4750}{4750+5250}5 = 2.375V$$

5% worst case: high voltage

R_2 = 5250 Ω, R_1 = 4750 Ω

$$v_{OUT,MAX} = \frac{5250}{5250+4750}5 = 2.625V$$

Sections 1.9-1.10: Practical Voltage and Current Sources; Measurement Devices

Problem 1.53

A thermistor is a nonlinear device that changes its terminal resistance value as its surrounding temperature changes. The resistance and temperature generally have a relation in the form of $R_{th}(T) = R_0e^{-\beta(T-T_0)}$, where R_{th} = resistance at temperature T, (Ω), R_0 = resistance at temperature T_0 = 298 K, (Ω), β = material constant, (K^{-1}), T, T_0 = absolute temperature, (K).

a. If R_0 = 300Ω and β = −0.01 K^{-1}, plot R_{th} as a function of the surrounding temperature T for 350 ≤ T ≤ 750.

b. If the thermistor is in parallel with a 250-Ω resistor, find the expression for the equivalent resistance and plot $R_{th}(T)$ on the same graph for part a.

Solution:

Known quantities:

Parameters $R_0 = 300\,\Omega$ (resistance at temperature T_0 = 298 K), and $\beta = -0.01\text{ K}^{-1}$, value of the second resistor.

Find:

a) Plot $R_{th}(T)$ versus T in the range $350 \le T \le 750$[°K]
b) The equivalent resistance of the parallel connection with the 250-Ω resistor; plot $R_{eq}(T)$ versus T in the range $350 \le T \le 750$[°K] for this case on the same plot as part a.

Assumptions:

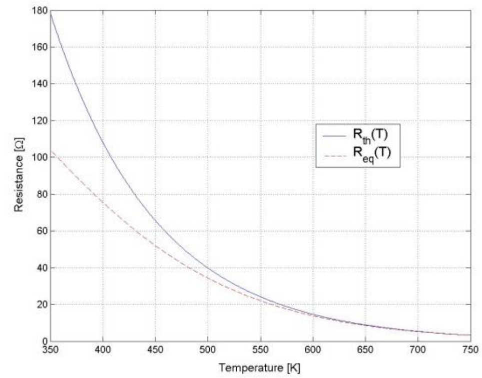
$$R_{th}(T) = R_0 e^{-\beta(T-T_0)}.$$

Analysis:

a) $R_{th}(T) = 300e^{-0.01\cdot(T-298)}$

b) $R_{eq}(T) = R_{th}(T) \parallel 250\Omega = \dfrac{1500\, e^{-0.01\,(T-298)}}{5+6\, e^{-0.01\,(T-298)}}$

The two plots are shown below.

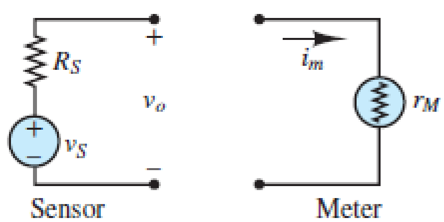


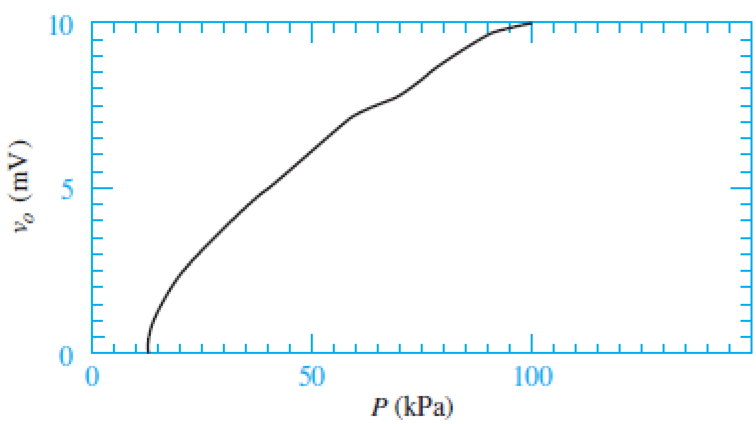
In the above plot, the solid line is for the thermistor alone; the dashed line is for the thermistor-resistor combination.

Problem 1.54

A moving-coil meter movement has a meter resistance r_M= 200 Ω, and full-scale deflection is caused by a meter current I_m= 10 μA. The meter is to be used to display pressure, as measured by a sensor, up to a maximum of 100 kPa. Models of the meter and pressure sensor are shown in Figure P1.54 along with the relationship between measured pressure and the sensor output v_T.

a. Devise a circuit that will produce the desired behavior of the meter, showing all appropriate connections between the terminals of the sensor and the meter.
b. Determine the value of each component in the circuit.
c. What is the linear range, that is, the minimum and maximum pressure that can accurately be measured?





Solution:

Known quantities:

Meter resistance of the coil; meter current for full scale deflection; max measurable pressure.

Find:

a) The circuit required to indicate the pressure measured by a sensor
b) The value of each component of the circuit; the linear range
c) The maximum pressure that can accurately be measured.

Assumptions:

Sensor characteristics follow what is shown in Figure P2.78

Analysis:

a) A full-scale deflection should occur at a pressure of 100kPa. Thus, at this pressure, the meter current should be $10\mu A$. Since the meter coil resistance is 200Ω the voltage across the coil should be $(10\mu A)(200\Omega) = 2mV$.

Assume that the sensor data was measured under open-circuit conditions such that $v_S = v_o = 10mV$ at 100kPa. Therefore, when the sensor and meter are connected the voltage across and current through R_S should be $8mV$ and $10\mu A$, respectively.

b) Consequently:

$$R_S = \frac{8mV}{10\mu A.} = 800\Omega$$

And

$$i_m = \frac{v_S}{800\Omega + 200\Omega}$$

c) By observation, $i_m \propto v_S$ so the deflection of the meter will be linear with pressure as long as v_S is linear with pressure. From the graph of v_S versus pressure that range appears to be roughly 25kPa to 100kPa.

Problem 1.55

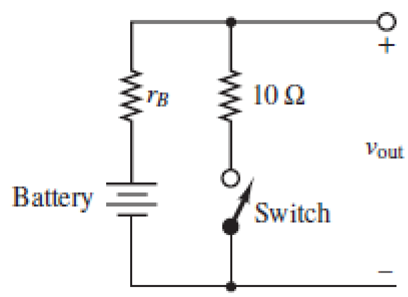
The circuit of Figure P1.55 is used to measure the internal resistance r_B of a battery.

a. A fresh battery is being tested, and it is found that the voltage V_{out} , is 2.28 V with the switch open and 2.27 V with the switch closed. Apply voltage division to find the internal resistance of the battery.
b. The same battery is tested one year later, and V_{out} is found to be 2.2 V with the switch open but 0.31 V with the switch closed. Apply voltage division to find the internal resistance of the battery.

Solution:

Known quantities:

Schematic of the circuit shown in Figure P1.55; voltage at terminals with switch open and closed for fresh battery; same voltages for the same battery after 1 year.



Find:
The internal resistance of the battery in each case.

Analysis:

a) With the switch open $V_B = V_{oc} = V_{out} = 2.28V$ is the open-circuit voltage. Apply voltage division to find the internal resistance.

$$V_{out} = \left(\frac{10}{10 + r_B}\right) V_{oc}$$

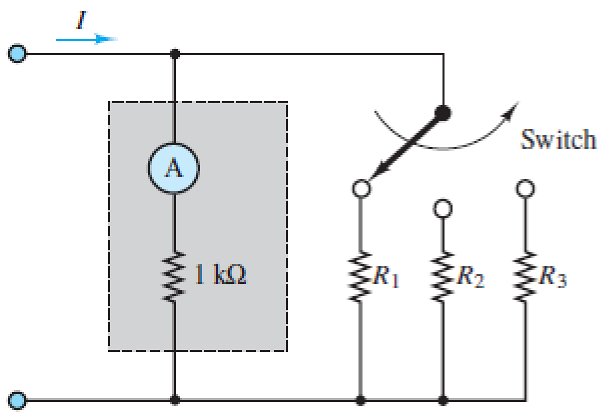
$$r_B = 10\left(\frac{V_{oc}}{V_{out}} - 1\right) = 10\left(\frac{2.28}{2.27} - 1\right) = 0.044\Omega$$

b) Apply voltage division again using the data from 1 year later.

$$r_B = 10\left(\frac{2.28}{0.31} - 1\right) = 63.55\Omega$$

Problem 1.56

Consider the practical ammeter, depicted in Figure P1.56, consisting of an ideal ammeter in series with a 1kΩ resistor. (An ideal ammeter acts like a short-circuit.) The meter sees a full-scale deflection when the current through it is 30 μA. Depending upon the setting of the rotary switch, the ammeter will read full-scale when the current I equals 10mA, 100mA and 1A. Apply current division to determine the appropriate values of R_1, R_2, and R_3.



Solution:

Known quantities:

Ammeter shown in Figure P1.56; Current for full-scale deflection; desired full scale values.

Find:

Value of the resistors required for the given full scale ranges.

Analysis:

$$R_{eq} = R_a \| R = \frac{R_a * R}{R_a + R}$$

Apply current division:

$$\frac{I_a}{I} = \frac{R_{eq}}{R_a} = \frac{R}{R_a + R}$$

Solve for R when $R_a = 1k\Omega$, $I_a = 30\mu A$ and $I = 10, 100, 1000$ mA.

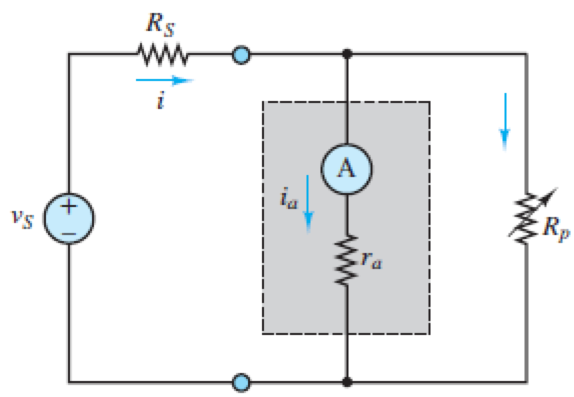
$$R_1 = R_a \frac{I_a}{I - I_a} = 10^3 \frac{3 * 10^{-6}}{10^{-2} - 3 * 10^{-6}} \cong 0.3\Omega$$

$$R_2 = R_a \frac{I_a}{I - I_a} = 10^3 \frac{3 * 10^{-6}}{10^{-1} - 3 * 10^{-6}} \cong 0.03\Omega$$

$$R_3 = R_a \frac{I_a}{I - I_a} = 10^3 \frac{3 * 10^{-6}}{10 - 3 * 10^{-6}} \cong 0.003\Omega$$

Problem 1.57

A circuit that measures the internal resistance of a practical ammeter is shown in Figure P1.57, where R_S = 50,000 Ω, v_S = 12 V, and R_p is a variable resistor that can be adjusted at will.
a. Assume that r_a << 50,000 Ω. Estimate the current *i*.
b. If the meter displays a current of 150 μA when R_p = 15 Ω, find the internal resistance of the meter r_a.



Solution:

Known quantities:

Schematic of the circuit shown in Figure P1.57; for part b: value of R_p and current displayed on the ammeter.

Find:

The current *i*; the internal resistance of the meter.

Assumptions:

$r_a << 50\ k\Omega$

Analysis:

a) Assuming that $r_a << 50\ k\Omega$

$$i \approx \frac{V_s}{R_s} = \frac{12}{50000} = 240\ \mu\text{A}$$

b) With the same assumption as in part a)

$$i_{meter} = 150 \cdot (10)^{-6} = \frac{R_p}{r_a + R_p} i$$

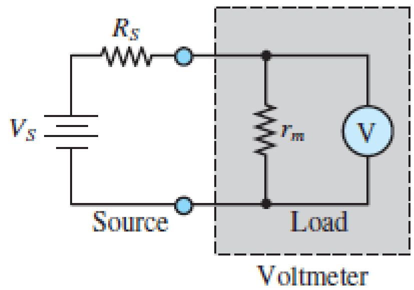
or:

$$150 \cdot (10)^{-6} = \left(\frac{15}{r_a + 15}\right)(240 \times 10^{-6}).$$

Therefore, $r_a = 9\ \Omega$.

Problem 1.58

A practical voltmeter has an internal resistance r_m. What is the value of r_m if the meter reads 11.81V when connected as shown in Figure P1.58?



Solution:

Known quantities:

Voltage read at the meter; schematic of the circuit shown in Figure P1.58 with source voltage, $V_s = 12V$ and source resistance, $R_s = 25k\Omega$.

Find:

The internal resistance of the voltmeter.

Analysis:

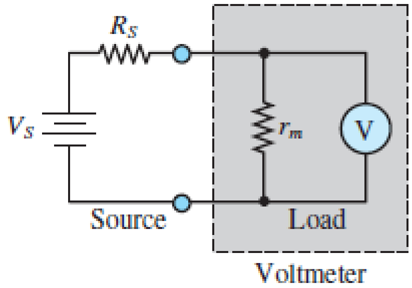
Apply voltage division to find:

$$V = 11.81 = \frac{r_m}{r_m + R_s}(12)$$

Therefore, $r_m = 1.55\ \mathrm{M}\Omega$.

Problem 1.59

Using the circuit of Figure P1.58, find the voltage that the meter reads if V_S = 24 V and R_S has the following values: $R_S = 0.2r_m$, $0.4r_m$, $0.6r_m$, $1.2r_m$, $4r_m$, $6r_m$, and $10r_m$. How large (or small) should the internal resistance of the meter be relative to R_S?



Solution:

Known quantities:

Circuit shown in Figure P1.58 with source voltage, $V_s = 24V$; and ratios between R_s and r_m.

Find:

The meter reads in the various cases.

Analysis:

By voltage division:

$$V = \left(\frac{r_m}{r_m + R_s}\right)(24)$$

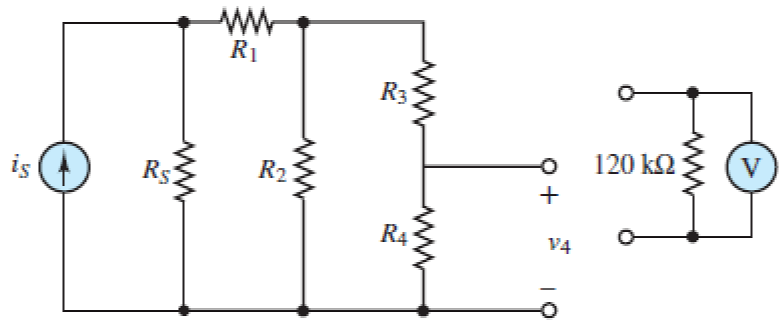
R_S	V
0.2 r_m	20 V
0.4 r_m	17.14 V
0.6 r_m	15 V
1.2 r_m	10.91 V
4 r_m	4.8 V
6 r_m	3.43 V
10 r_m	2.18 V

For a voltmeter, it is desired that $r_m >> R_s$.

Problem 1.60

A voltmeter is used to determine the voltage across a resistive element in the circuit of Figure P1.60. The instrument is modeled by an ideal voltmeter in parallel with a 120kΩ resistor, as shown. The meter is placed to measure the voltage across R_4. Assume R_1 = 8 kΩ, R_2 = 22 kΩ, R_3 = 50 kΩ, R_S = 125 kΩ, and I_S = 120 mA. Find the voltage across R_4 with and without the voltmeter in the circuit for the following values:

a. R_4 = 100 Ω
b. R_4 = 1 kΩ
c. R_4 = 10 kΩ
d. R_4 = 100 kΩ



Solution:

Known quantities:

Schematic of the circuit shown in Figure P1.60, values of the components.

Find:

The voltage across R_4 with and without the voltmeter for the following values:

a) $R_4 = 100\Omega$
b) $R_4 = 1k\Omega$
c) $R_4 = 10k\Omega$
d) $R_4 = 100k\Omega$.

Assumptions:

The voltmeter behavior is modeled as that of an ideal voltmeter in parallel with a 120- kΩ resistor.

Analysis:

We develop first an expression for V_{R_4} in terms of R_4. Next, using current division:

$$\begin{cases} I_{R_1} = I_S\left(\dfrac{R_S}{R_S + R_1 + R_2 \,||\, (R_3 + R_4)}\right) \\ I_{R_4} = I_{R_1}\left(\dfrac{R_2}{R_2 + R_3 + R_4}\right) \end{cases}$$

Therefore,

$$I_{R_4} = I_S\left(\frac{R_S}{R_S + R_1 + R_2 \,||\, (R_3 + R_4)}\right)\cdot\left(\frac{R_2}{R_2 + R_3 + R_4}\right)$$

$$V_{R_4} = I_{R_4} R_4$$

$$= I_S\left(\frac{R_S R_4}{R_S + R_1 + R_2 \,||\, (R_3 + R_4)}\right)\cdot\left(\frac{R_2}{R_2 + R_3 + R_4}\right)$$

$$= \frac{66000 \cdot R_4}{R_4 + 2.1352 \cdot 10^6}$$

Without the voltmeter:

a) V_{R_4} = 3.09 V
b) V_{R_4} = 30.47 V
c) V_{R_4} = 269.9 V
d) V_{R_4} = 1260 V

To find the voltage drop across R_4 with a 120-kΩ resistor across R_4 simply replace R_4 above with $R_4||120K$.

$$I_{R_4} = I_S\left(\frac{R_S}{R_S + R_1 + R_2\|[R_3 + (R_4\|120K)]}\right)\left(\frac{R_2}{R_2 + R_3 + (R_4\|120K)}\right)$$

$$V_{R_4} = I_{R_4}R_4 = I_S\left(\frac{R_S}{R_S + R_1 + R_2\|[R_3 + (R_4\|120K)]}\right)\left(\frac{R_2}{R_2 + R_3 + (R_4\|120K)}\right)R_4$$

$$V_{R_4} = (0.120)\left(\frac{125K}{125K + 8K + (22K)\|[50K + (R_4\|120K)]}\right)\left(\frac{22K}{22K + 50K + (R_4\|120K)}\right)R_4$$

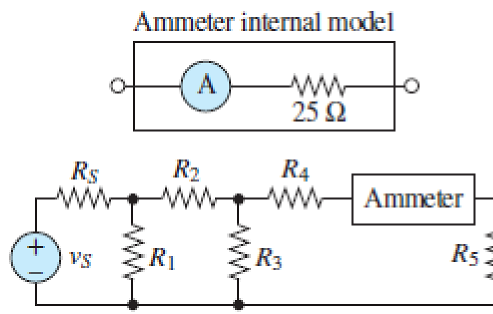
With the voltmeter:

a) V_{R_4} = 3.08 V
b) V_{R_4} = 30.22 V
c) V_{R_4} = 251.6 V
d) V_{R_4} = 940.9 V.

Problem 1.61

An ammeter is used as shown in Figure P1.61. The ammeter model consists of an ideal ammeter in series with a resistance. The ammeter model is placed in the branch as shown in the figure. Find the current through R_5 both with and without the ammeter in the circuit for the following values, assuming that R_S = 20 Ω, R_1 = 800 Ω, R_2 = 600 Ω, R_3 = 1.2 kΩ, R_4 = 150 Ω, and V_S = 24 V.

a. R_5 = 1 kΩ
b. R_5 = 100 Ω
c. R_5 = 10 Ω
d. R_5 = 1 Ω



Solution:

Known quantities:

Schematic of the circuit shown in Figure P1.61 and the values of the components.

Find:

The current through R_5 with and without the ammeter, for the following values of R_5:

a) $R_5 = 1k\Omega$
b) $R_5 = 100\Omega$
c) $R_5 = 10\Omega$

d) $R_5 = 1\Omega$.

Analysis:

First, find the Norton equivalent of the network to the left of R3, including R3. The Norton equivalent resistance is

$$R_N = R_3 \| [R_2 + (R_1 \| R_S)] \cong 408.6\Omega$$

Replace the branch containing the ammeter with a short-circuit (a wire) and solve for that short-circuit current. R_3 plays no part in the calculation since it is in parallel with the short-circuit and with the short-circuit in place R_1 and R_2 are in parallel. Apply voltage division to find the voltage across $R_1 \| R_2$.

$$v_2 = \frac{(R_1 \| R_2)}{R_S + (R_1 \| R_2)} v_S \cong 22.7V$$

The short-circuit current is the current through R_2 so

$$i_{sc} = \frac{v_2}{R_2} \cong 37.8mA$$

Attach the ammeter branch to the Norton equivalent network and apply current division to find the current through the ammeter.

$$i_m = \frac{R_N}{R_N + (R_4 + R_m + R_5)} i_{sc} \cong \frac{408.6}{583.6 + R_5}(37.8)mA$$

The current through that branch without the ammeter in place is

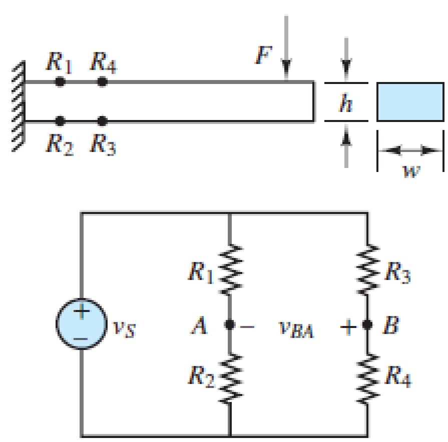
$$i_m = \frac{R_N}{R_N + (R_4 + R_5)} i_{sc} \cong \frac{408.6}{558.6 + R_5}(37.8)mA$$

Use the above equations fill in the following table:

R_5	With meter	Without meter
1kΩ	9.75 mA	9.91 mA
100Ω	22.59 mA	23.45 mA
10Ω	26.02 mA	27.16 mA
1Ω	26.42 mA	27.60 mA

Problem 1.62

Figure P1.62 shows an aluminum cantilevered beam loaded by the force *F*. Strain gauges R_1, R_2, R_3, and R_4 are attached to the beam as shown in Figure P1.62 and connected into the circuit shown. The force causes a tension stress on the top of the beam that causes the length (and therefore the resistance) of R_1 and R_4 to increase and a compression stress on the bottom of the beam that causes the length (and therefore the resistance) of R_2 and R_3 to decrease. The result is a voltage of 50 mV at node *B* with respect to node *A*. Determine the force if R_o = 1 kΩ, V_S = 12 V, *L* = 0.3 m, *w* = 25 mm, *h* = 100 mm, and *Y* = 69 GN/m².



Solution:

Known quantities:

Schematic of the circuit and geometry of the beam shown in Figure P1.62, characteristics of the material, reads on the bridge.

Find:

The force applied on the beam.

Assumptions:

Gage Factor for strain gauge is 2.

Analysis:

R_1 and R_2 are in series; R_3 and R_4 are in series.

Voltage Division: $v_{R_2} = \frac{v_S R_2}{R_1 + R_2} = \frac{v_S (R_0 - \Delta R)}{R_0 + \Delta R + R_0 - \Delta R} = \frac{v_S (R_0 - \Delta R)}{2R_0}$

Voltage Division: $v_{R_4} = \frac{v_S R_4}{R_3 + R_4} = \frac{v_S (R_0 - \Delta R)}{R_0 - \Delta R + R_0 + \Delta R} = \frac{v_S (R_0 + \Delta R)}{2R_0}$

KVL: $-v_{R_2} - v_{BA} + v_{R_4} = 0$

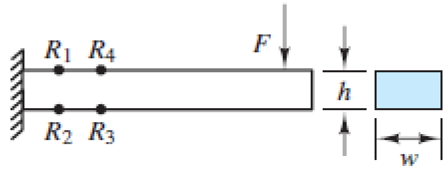
$$v_{BA} = v_{R_4} - v_{R_2} = \frac{v_S (R_0 + \Delta R)}{2R_0} - \frac{v_S (R_0 - \Delta R)}{2R_0} = \frac{v_S 2\Delta R}{2R_0} = v_S GF\varepsilon = \frac{v_S 2*6*LF}{wh^2 Y}$$

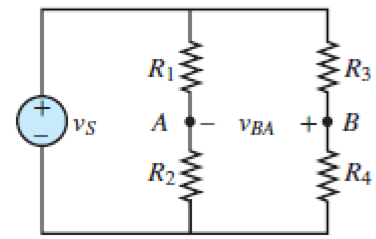
assuming GF=2 for aluminum.

$$F = \frac{v_{BA} wh^2 y}{v_s 12L} = \frac{0.050\,V (0.025\,m)(0.100\,m)^2\, 69 \times 10^9 \frac{N}{m^2}}{12\,V\left(12\right) 0.3\,m} = 19.97\,\text{kN}.$$

Problem 1.63

Refer to Figure P1.62 but assume that the cantilevered beam loaded by a force *F* is made of steel. Strain gauges R_1, R_2, R_3, and R_4 are attached to the beam and connected in the circuit shown. The force causes a tension stress on the top of the beam that causes the length (and therefore the resistance) of R_1 and R_4 to increase and a compression stress on the bottom of the beam that causes the length (and therefore the resistance) of R_2 and R_3 to decrease. The result is a voltage v_{BA} across nodes *B* and *A*. Determine this voltage if $F = 1.3$ MN, $R_o = 1$ kΩ, $V_S = 24$ V $L = 1.7$ m, $w = 3$ cm, $h = 7$ cm, $Y = 200$ GN/m^2





Solution:

Known quantities:

Schematic of the circuit and geometry of the beam shown in Figure P1.62, characteristics of the material, reads on the bridge.

Find:

The force applied on the beam.

Assumptions:

Gage Factor for strain gauge is 2.

Analysis:

R_1 and R_2 are in series; R_3 and R_4 are in series.

Voltage Division: $v_{R_2} = \dfrac{v_S R_2}{R_1 + R_2} = \dfrac{v_S(R_0 - \Delta R)}{R_0 + \Delta R + R_0 - \Delta R} = \dfrac{v_S(R_0 - \Delta R)}{2R_0}$

Voltage Division: $v_{R_4} = \dfrac{v_S R_4}{R_3 + R_4} = \dfrac{v_S(R_0 - \Delta R)}{R_0 - \Delta R + R_0 + \Delta R} = \dfrac{v_S(R_0 + \Delta R)}{2R_0}$

KVL: $-v_{R_2} - v_{BA} + v_{R_4} = 0$

$$v_{BA} = v_{R_4} - v_{R_2} = \frac{v_S(R_0 + \Delta R)}{2R_0} - \frac{v_S(R_0 - \Delta R)}{2R_0} = \frac{v_S 2\Delta R}{2R_0} = v_S GF e = \frac{v_S 2*6*L*F}{wh^2 y}$$

Assuming GF=2 for aluminum.

$$F = 1.3\times 10^6 \text{ N} = \frac{v_{BA} wh^2 y}{V_s 12L} = \frac{v_{BA}(0.03 \text{ m})(0.07 \text{ m})^2\, 200\times 10^9 \frac{N}{m^2}}{24\, V(12)1.7\, m}$$

$$v_{BA} = \frac{1.3\times 10^6 \text{ N}\times 24\, V(12)1.7\, m}{(0.03 \text{ m})(0.07 \text{ m})^2\, 200\times 10^9 \frac{N}{m^2}} = 21.6 \text{ mV}$$