Solutions for Shigley's Mechanical Engineering Design 11th Edition by Budynas

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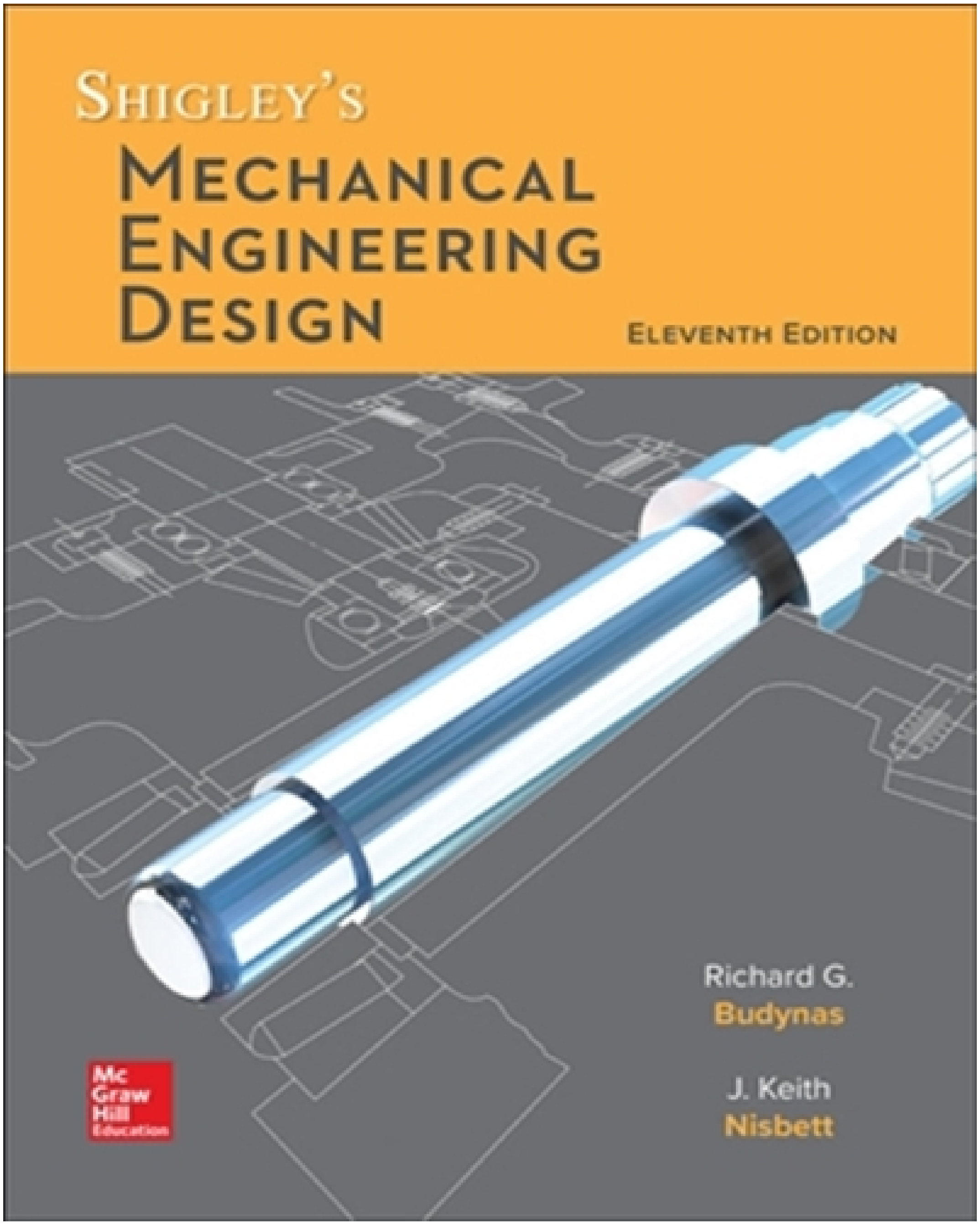


Solutions

Chapter 2

2-1 From Tables A-20, A-21, A-22, and A-24c,

(**a**) UNS G10200 HR: S_{ut} = 380 (55) MPa (kpsi), S_{yt} = 210 (30) MPa (kpsi) *Ans.*

(**b**) SAE 1050 CD: S_{ut} = 690 (100) MPa (kpsi), S_{yt} = 580 (84) MPa (kpsi) *Ans.*

(**c**) AISI 1141 Q&T at 540°C (1000°F): S_{ut} = 896 (130) MPa (kpsi), S_{yt} = 765 (111) MPa (kpsi) *Ans.*

(**d**) 2024-T4: S_{ut} = 446 (64.8) MPa (kpsi), S_{yt} = 296 (43.0) MPa (kpsi) *Ans.*

(**e**) Ti-6Al-4V annealed: S_{ut} = 900 (130) MPa (kpsi), S_{yt} = 830 (120) MPa (kpsi) *Ans.*

2-2 (**a**) Maximize yield strength: Q&T at 425°C (800°F) *Ans.*

(**b**) Maximize elongation: Q&T at 650°C (1200°F) *Ans.*

2-3 Conversion of kN/m^3 to kg/ m^3 multiply by 1(10^3) / 9.81 = 102

AISI 1018 CD steel: Tables A-20 and A-5

$$\frac{S_y}{\rho} = \frac{370\left(10^3\right)}{76.5\left(102\right)} = 47.4 \text{ kN}\cdot\text{m/kg} \qquad \textit{Ans.}$$

2011-T6 aluminum: Tables A-22 and A-5

$$\frac{S_y}{\rho} = \frac{169\left(10^3\right)}{26.6\left(102\right)} = 62.3 \text{ kN}\cdot\text{m/kg} \qquad \textit{Ans.}$$

Ti-6Al-4V titanium: Tables A-24c and A-5

$$\frac{S_y}{\rho} = \frac{830\left(10^3\right)}{43.4\left(102\right)} = 187 \text{ kN}\cdot\text{m/kg} \qquad \textit{Ans.}$$

ASTM No. 40 cast iron: Tables A-24a and A-5.Does not have a yield strength. Using the ultimate strength in tension

$$\frac{S_{ut}}{\rho} = \frac{42.5\left(6.89\right)\left(10^3\right)}{70.6\left(102\right)} = 40.7 \text{ kN}\cdot\text{m/kg} \qquad \textit{Ans}$$

2-4 AISI 1018 CD steel: Table A-5

$$\frac{E}{\gamma} = \frac{30.0\left(10^6\right)}{0.282} = 106\left(10^6\right) \text{ in} \qquad \textit{Ans.}$$

2011-T6 aluminum: Table A-5

$$\frac{E}{\gamma} = \frac{10.4\left(10^6\right)}{0.098} = 106\left(10^6\right) \text{ in} \qquad \textit{Ans.}$$

Ti-6Al-6V titanium: Table A-5

$$\frac{E}{\gamma} = \frac{16.5\left(10^6\right)}{0.160} = 103\left(10^6\right) \text{ in} \quad \textit{Ans.}$$

No. 40 cast iron: Table A-5

$$\frac{E}{\gamma} = \frac{14.5\left(10^6\right)}{0.260} = 55.8\left(10^6\right) \text{ in} \quad \textit{Ans.}$$

2-5 $2G(1+v) = E \quad \Rightarrow \quad v = \dfrac{E-2G}{2G}$

Using values for E and G from Table A-5,

Steel: $$v = \frac{30.0 - 2\left(11.5\right)}{2\left(11.5\right)} = 0.304 \quad \textit{Ans.}$$

The percent difference from the value in Table A-5 is

$$\frac{0.304 - 0.292}{0.292} = 0.0411 = 4.11 \text{ percent} \quad \textit{Ans.}$$

Aluminum: $$v = \frac{10.4 - 2\left(3.90\right)}{2\left(3.90\right)} = 0.333 \quad \textit{Ans.}$$

The percent difference from the value in Table A-5 is 0 percent *Ans.*

Beryllium copper: $$v = \frac{18.0 - 2\left(7.0\right)}{2\left(7.0\right)} = 0.286 \quad \textit{Ans.}$$

The percent difference from the value in Table A-5 is

$$\frac{0.286 - 0.285}{0.285} = 0.00351 = 0.351 \text{ percent} \quad \textit{Ans.}$$

Gray cast iron: $$v = \frac{14.5 - 2\left(6.0\right)}{2\left(6.0\right)} = 0.208 \quad \textit{Ans.}$$

The percent difference from the value in Table A-5 is

$$\frac{0.208 - 0.211}{0.211} = -0.0142 = -1.42 \text{ percent} \quad \textit{Ans.}$$

2-6 (**a**) $A_0 = \pi (0.503)^2/4 = 0.1987 \text{ in}^2$, $\sigma = P_i / A_0$

For data in the elastic range, from Eq. (2–2), $\varepsilon = (l - l_0) / l_0 = \Delta l / l_0 = \Delta l / 2$

For data in the plastic range, from Eq. (2–8), $\varepsilon = \dfrac{A_0 - A}{A}$

On the next two pages, the data and plots are presented. Figure (*a*) shows the linear part of the curve from data points 1 to 7. Figure (*b*) shows data points 1 to 12. Figure (*c*) shows the complete range. **Note**: The exact value of A_0 is used without rounding off.

(**b**) From Fig. (*a*) the slope of the line from a linear regression is E = 30.5 Mpsi *Ans.*

From Fig. (*b*) the equation for the dotted offset line is found to be

$$\sigma = 30.5(10^6)\varepsilon - 61\,000 \qquad (1)$$

The equation for the line between data points 8 and 9 is

$$\sigma = 7.60(10^5)\varepsilon + 42\,900 \qquad (2)$$

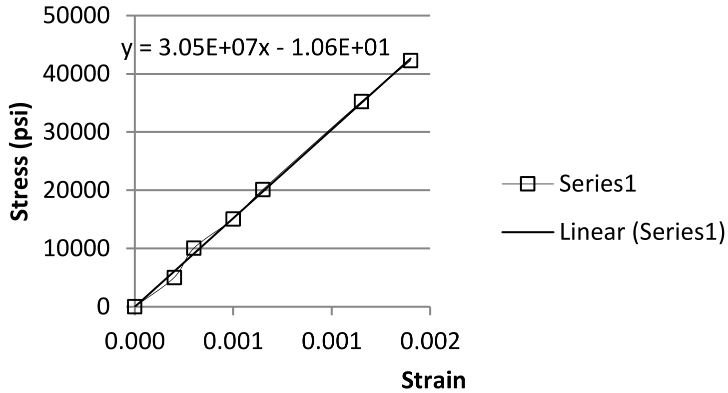
Solving Eqs. (1) and (2) simultaneously yields σ = 45.6 kpsi which is the 0.2 percent offset yield strength. Thus, S_y = 45.6 kpsi *Ans.*

The ultimate strength from Figure (*c*) is S_u = 85.6 kpsi *Ans.*

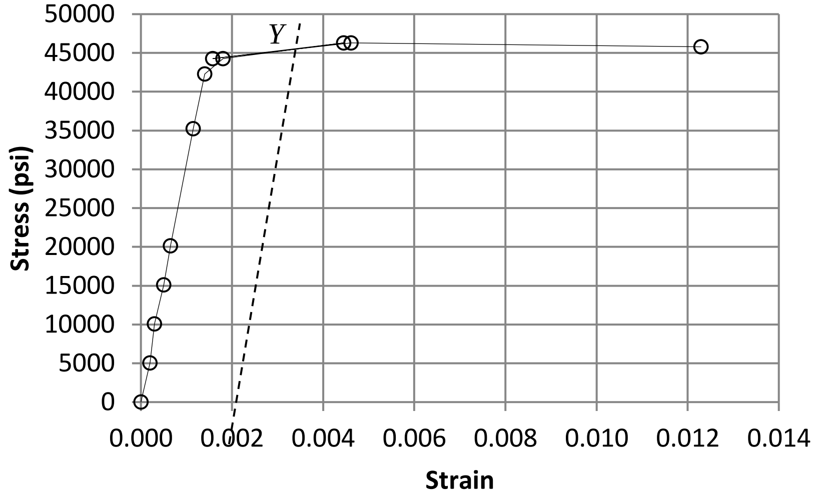
The reduction in area is given by Eq. (2-25) as

$$R = \frac{A_0 - A_f}{A_0} = \frac{0.1987 - 0.1077}{0.1987} = 0.458 = 45.8\ \% \qquad \textit{Ans.}$$

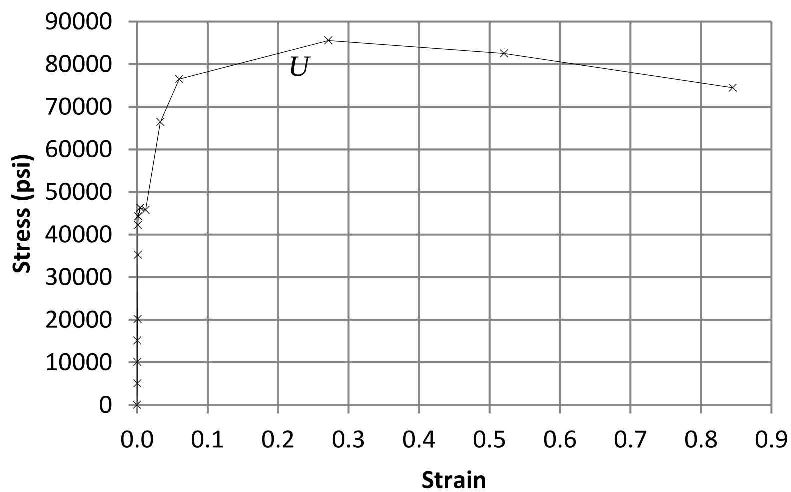
Data Point	P_i	Δl, A_i	ε	σ
1	0	0	0	0
2	1000	0.0004	0.00020	5032
3	2000	0.0006	0.00030	10065
4	3000	0.001	0.00050	15097
5	4000	0.0013	0.00065	20130
6	7000	0.0023	0.00115	35227
7	8400	0.0028	0.00140	42272
8	8800	0.0036	0.00180	44285
9	9200	0.0089	0.00445	46298
10	8800	0.1984	0.00158	44285
11	9200	0.1978	0.00461	46298
12	9100	0.1963	0.01229	45795
13	13200	0.1924	0.03281	66428
14	15200	0.1875	0.05980	76492
15	17000	0.1563	0.27136	85551
16	16400	0.1307	0.52037	82531
17	14800	0.1077	0.84506	74479



(*a*) Linear range



(*b*) Offset yield



(*c*) Complete range

(**c**) The material is ductile since there is a large amount of deformation beyond yield.

(**d**) The closest material to the values of S_y, S_{ut}, and R is SAE 1045 HR with S_y = 45 kpsi, S_{ut} = 82 kpsi, and R = 40 %. *Ans.*

2-7 To plot $\tilde{\sigma}$ vs. $\tilde{\varepsilon}$, the following equations are applied to the data.

Eq. (2-4) $$\tilde{\sigma} = \frac{P}{A}$$

Eq. (2–9)
$$\tilde{\varepsilon} = \ln\frac{l}{l_0} \quad \text{for } 0 \le \Delta l \le 0.0028 \text{ in} \qquad (0 \le P \le 8400 \text{ lbf})$$
$$\tilde{\varepsilon} = \ln\frac{A_0}{A} \quad \text{for } \Delta l > 0.0028 \text{ in} \qquad (P > 8400 \text{ lbf})$$

where $$A_0 = \frac{\pi(0.503)^2}{4} = 0.1987 \text{ in}^2$$

The results are summarized in the following table and plot. The last 5 points of data are used to plot log $\tilde{\sigma}$ vs log $\tilde{\varepsilon}$.

The curve fit gives m = 0.2306

$$\log \sigma_0 = 5.1852 \Rightarrow \sigma_0 = 153.2 \text{ kpsi} \qquad \textit{Ans.}$$

The true strain corresponding to 20% cold work is given by Eq. (2–28) as

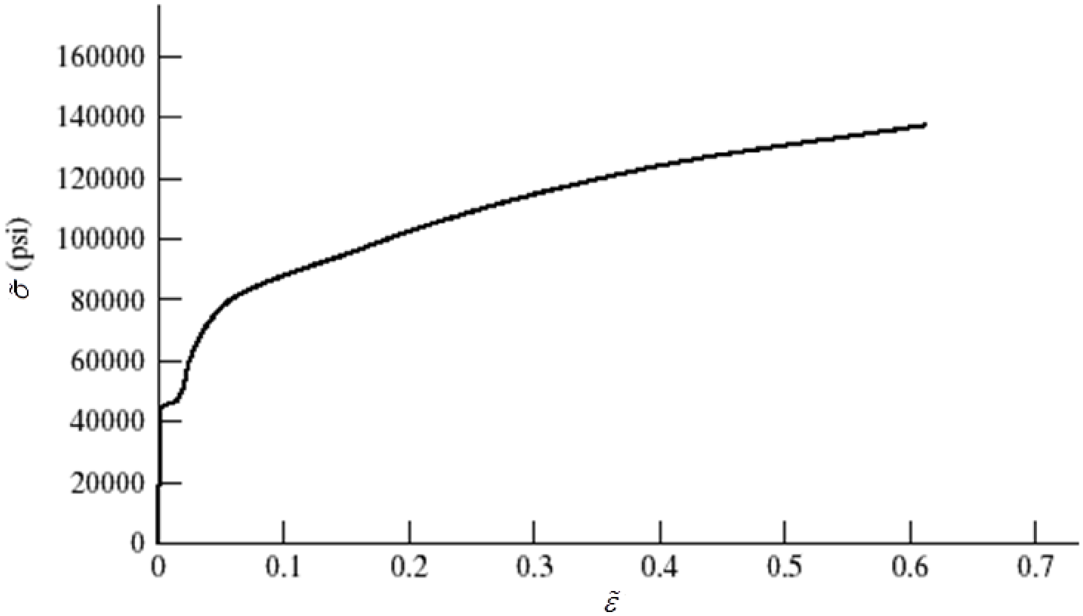
$$\tilde{\varepsilon} = \ln\left(\frac{1}{1-W}\right) = \ln\left(\frac{1}{1-0.2}\right) = 0.2231$$

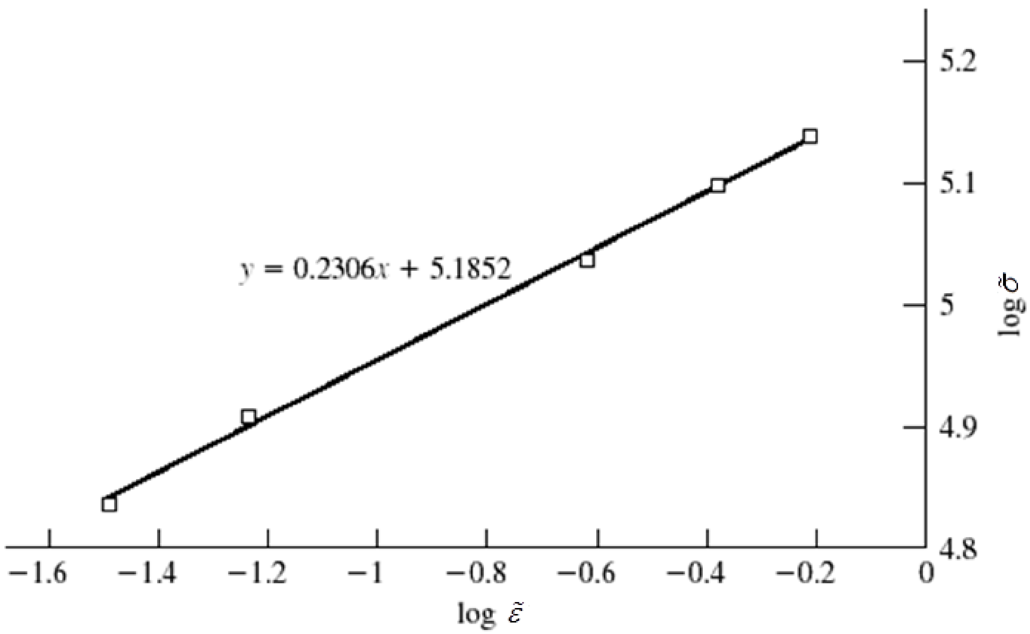
Eq. (2-30): $S_{y'} = \sigma_0 \tilde{\varepsilon}^m = 153.2(0.2231)^{0.2306} = 108.4$ kpsi *Ans.*

Eq. (2-32), with $S_u = 85.6$ from Prob. 2-6,

$$S_u' = \frac{S_u}{1-W} = \frac{85.6}{1-0.2} = 107 \text{ kpsi} \qquad \textit{Ans.}$$

P	Δl	A	$\tilde{\varepsilon}$	$\tilde{\sigma}$	$\log\ \tilde{\varepsilon}$	$\log\ \tilde{\sigma}$
0	0	0.198 7	0	0		
1 000	0.000 4	0.198 7	0.000 2	5 032.71	-3.699	3.702
2 000	0.000 6	0.198 7	0.000 3	10 065.4	-3.523	4.003
3 000	0.001 0	0.198 7	0.000 5	15 098.1	-3.301	4.179
4 000	0.001 3	0.198 7	0.000 65	20 130.9	-3.187	4.304
7 000	0.002 3	0.198 7	0.001 15	35 229	-2.940	4.547
8 400	0.002 8	0.198 7	0.001 4	42 274.8	-2.854	4.626
8 800		0.198 4	0.001 51	44 354.8	-2.821	4.647
9 200		0.197 8	0.004 54	46 511.6	-2.343	4.668
9 100		0.196 3	0.012 15	46 357.6	-1.915	4.666
13 200		0.192 4	0.032 22	68 607.1	-1.492	4.836
15 200		0.187 5	0.058 02	81 066.7	-1.236	4.909
17 000		0.156 3	0.240 02	108 765	-0.620	5.036
16 400		0.130 7	0.418 89	125 478	-0.378	5.099
14 800		0.107 7	0.612 45	137 419	-0.213	5.138





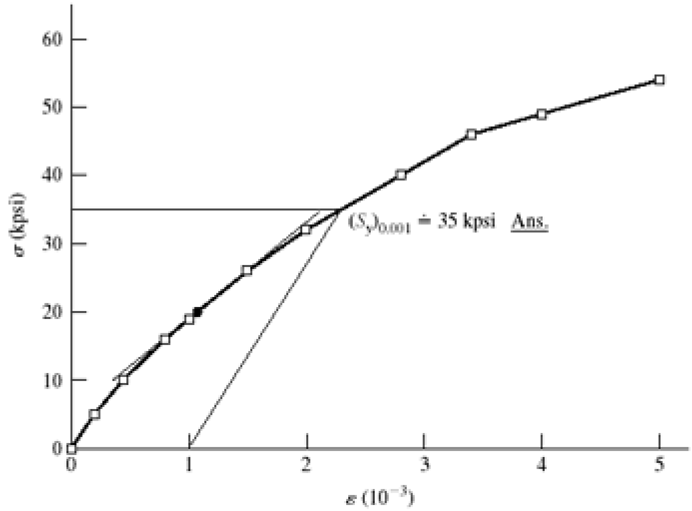
2-8 Tangent modulus at σ = 0 is

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} \approx \frac{5000-0}{0.2\left(10^{-3}\right)-0} = 25\left(10^{6}\right) \text{ psi} \qquad \textit{Ans.}$$

At σ = 20 kpsi

$$E_{20} \approx \frac{(26-19)(10^3)}{(1.5-1)(10^{-3})} = 14.0(10^6)\ \text{psi} \qquad \textit{Ans.}$$

ε (10^{-3})	σ (kpsi)
0	0
0.20	5
0.44	10
0.80	16
1.0	19
1.5	26
2.0	32
2.8	40
3.4	46
4.0	49
5.0	54



2-9 $W = 0.20$,

(**a**) Before cold working: Annealed AISI 1018 steel. Table A-22, S_y = 32 kpsi, S_u = 49.5 kpsi, σ_0 = 90.0 kpsi, $m = 0.25$, $\tilde{\varepsilon}_f$ = 1.05

After cold working: Eq. (2-23), $m = \tilde{\varepsilon}_u = 0.25$

Eq. (2-28): $$\tilde{\varepsilon}_{20} = \ln\left(\frac{1}{1-W}\right) = \ln\left(\frac{1}{1-0.2}\right) = 0.223 < \tilde{\varepsilon}_u$$

Eq. (2-30): $$S_y' = \sigma_0 \tilde{\varepsilon}_{20}^m = 90(0.223)^{0.25} = 61.8\ \text{kpsi}$$ *Ans.* 93% increase *Ans.*

Eq. (2-32): $$S_u' = \frac{S_u}{1-W} = \frac{49.5}{1-0.20} = 61.9\ \text{kpsi}$$ *Ans.* 25% increase *Ans.*

(**b**) Before: $$\frac{S_u}{S_y} = \frac{49.5}{32} = 1.55$$ After: $$\frac{S_u'}{S_y'} = \frac{61.9}{61.8} = 1.002$$ *Ans.*

Lost most of its ductility.

2-10 $W = 0.20$,

(**a**) Before cold working: AISI 1212 HR steel. Table A-22, S_y = 28 kpsi, S_u = 61.5 kpsi, σ_0 = 110 kpsi, $m = 0.24$, $\tilde{\varepsilon}_f$ = 0.85

After cold working: Eq. (2-23), $m = \tilde{\varepsilon}_u = 0.24$

Eq. (2-28): $$\tilde{\varepsilon}_{20} = \ln\left(\frac{1}{1-W}\right) = \ln\left(\frac{1}{1-0.2}\right) = 0.223 < \tilde{\varepsilon}_u$$

Eq. (2-30): $S_y' = \sigma_0 \tilde{\varepsilon}_{20}^m = 110(0.223)^{0.24} = 76.7$ kpsi *Ans.* 174% increase *Ans.*

Eq. (2-32): $S_u' = \dfrac{S_u}{1-W} = \dfrac{61.5}{1-0.20} = 76.9$ kpsi *Ans.* 25% increase *Ans.*

(**b**) Before: $\dfrac{S_u}{S_y} = \dfrac{61.5}{28} = 2.20$ After: $\dfrac{S_u'}{S_y'} = \dfrac{76.9}{76.7} = 1.002$ *Ans.*

Lost most of its ductility.

2-11 $W = 0.20$,

(**a**) Before cold working: 2024-T4 aluminum alloy. Table A-22, $S_y = 43.0$ kpsi, $S_u = 64.8$ kpsi, $\sigma_0 = 100$ kpsi, $m = 0.15$, $\tilde{\varepsilon}_f = 0.18$

After cold working: Eq. (2-23), $m = \tilde{\varepsilon}_u = 0.15$

Eq. (2-28): $\tilde{\varepsilon}_{20} = \ln\left(\dfrac{1}{1-W}\right) = \ln\left(\dfrac{1}{1-0.2}\right) = 0.223 > \tilde{\varepsilon}_f$

Material fractures. *Ans.*

2-12 For $H_B = 275$, Eq. (2-36), $S_u = 3.4(275) = 935$ MPa *Ans.*

2-13 Gray cast iron, $H_B = 200$.

Eq. (2-37), $S_u = 0.23(200) - 12.5 = 33.5$ kpsi *Ans.*

From Table A-24, this is probably ASTM No. 30 Gray cast iron *Ans.*

2-14 Eq. (2-36), $0.5H_B = 100 \Rightarrow H_B = 200$ *Ans.*

2-15 For the data given, converting H_B to S_u using Eq. (2-36)

H_B	S_u (kpsi)		S_u^2 (kpsi)
230	115		13225
232	116		13456
232	116		13456
234	117		13689
235	117.5		13806.25
235	117.5		13806.25
235	117.5		13806.25
236	118		13924
236	118		13924
239	119.5		14280.25
$\Sigma S_u =$	1172	$\Sigma S_u^2 =$	137373

Eq. (1-6)

$$\bar{S}_u = \frac{\sum S_u}{N} = \frac{1172}{10} = 117.2 \approx 117 \text{ kpsi} \quad Ans.$$

Eq. (1-7),

$$s_{S_u} = \sqrt{\frac{\sum_{i=1}^{10} S_u^2 - N\bar{S}_u^2}{N-1}} = \sqrt{\frac{137373 - 10(117.2)^2}{9}} = 1.27 \text{ kpsi} \qquad Ans.$$

2-16 For the data given, converting H_B to S_u using Eq. (2-37)

H_B	S_u (kpsi)		S_u^2 (kpsi)
230	40.4		1632.16
232	40.86		1669.54
232	40.86		1669.54
234	41.32		1707.342
235	41.55		1726.403
235	41.55		1726.403
235	41.55		1726.403
236	41.78		1745.568
236	41.78		1745.568
239	42.47		1803.701
$\Sigma S_u =$	414.12	$\Sigma S_u^2 =$	17152.63

Eq. (1-6)

$$\bar{S}_u = \frac{\sum S_u}{N} = \frac{414.12}{10} = 41.4 \text{ kpsi} \quad \textit{Ans.}$$

Eq. (1-7),

$$s_{S_u} = \sqrt{\frac{\sum_{i=1}^{10} S_u^2 - N\bar{S}_u^2}{N-1}} = \sqrt{\frac{17152.63 - 10(41.4)^2}{9}} = 1.20 \qquad \textit{Ans.}$$

2-17 (**a**) Eq. (2-16) $\quad u_R \approx \dfrac{45.6^2}{2(30)} = 34.7 \text{ in}\cdot\text{lbf}/\text{in}^3 \quad$ *Ans.*

(**b**) $A_0 = \pi(0.503^2)/4 = 0.19871 \text{ in}^2$

P	Δl	A	$(A_0/A) - 1$	ε	$\sigma = P/A_0$
0	0			0	0
1 000	0.000 4			0.000 2	5 032
2 000	0.000 6			0.000 3	10 070
3 000	0.001 0			0.000 5	15 100
4 000	0.001 3			0.000 65	20 130
7 000	0.002 3			0.001 15	35 230
8 400	0.002 8			0.001 4	42 270
8 800	0.003 6			0.001 8	44 290
9 200	0.008 9			0.004 45	46 300
9 100		0.196 3	0.012 28	0.012 28	45 800
13 200		0.192 4	0.032 80	0.032 80	66 430
15 200		0.187 5	0.059 79	0.059 79	76 500
17 000		0.156 3	0.271 34	0.271 34	85 550
16 400		0.130 7	0.520 35	0.520 35	82 530
14 800		0.107 7	0.845 03	0.845 03	74 480

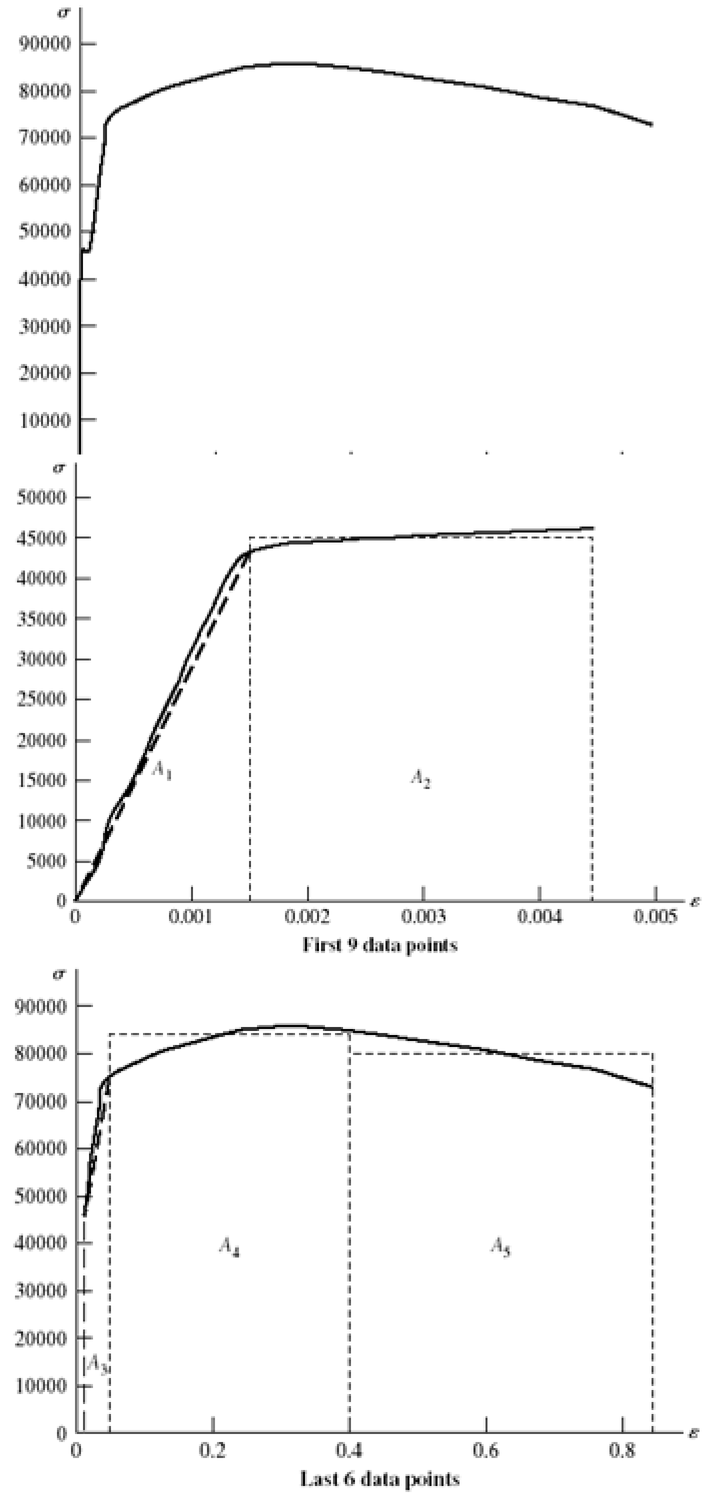
From the figures on the next page,

$$u_T \approx \sum_{i=1}^{5} A_i = \frac{1}{2}(43\ 000)(0.001\ 5) + 45\ 000(0.004\ 45 - 0.001\ 5)$$

$$+\frac{1}{2}(45\ 000 + 76\ 500)(0.059\ 8 - 0.004\ 45)$$

$$+81\ 000(0.4 - 0.059\ 8) + 80\ 000(0.845 - 0.4)$$

$$\approx 66.7(10^3)\text{in}\cdot\text{lbf/in}^3 \quad \textit{Ans.}$$


σ
90000
80000
70000
60000
50000
40000
30000
20000
10000
σ
50000
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
A_1
A_2
0
0.001
0.002
0.003
0.004
0.005
ε
First 9 data points
σ
90000
80000
70000
60000
50000
40000
30000
20000
10000
0
A_3
A_4
A_5
0
0.2
0.4
0.6
0.8
ε
Last 6 data points

2-18, 2-19 These problems are for student research. No standard solutions are provided.

2-20 Appropriate tables: Young's modulus and Density (Table A-5); 1020 HR and CD (Table A-20); 1040 and 4140 (Table A-21); Aluminum (Table A-24); Titanium (Table A-24c)

Appropriate equations:

For diameter, $$\sigma = \frac{F}{A} = \frac{F}{(\pi/4)d^2} = S_y \quad \Rightarrow \quad d = \sqrt{\frac{4F}{\pi S_y}}$$

Weight/length = ρA, Cost/length = \$/in = (\$/lbf) Weight/length,
Deflection/length = $\delta/L = F/(AE)$

With F = 100 kips = 100(10^3) lbf,

Material	Young's Modulus	Density	Yield Strength	Cost/lbf	Diameter	Weight/ length	Cost/ length	Deflection/ length
units	Mpsi	lbf/in^3	kpsi	\$/lbf	in	lbf/in	\$/in	in/in
1020 HR	30	0.282	30	0.27	2.060	0.9400	0.25	1.000E-03
1020 CD	30	0.282	57	0.30	1.495	0.4947	0.15	1.900E-03
1040	30	0.282	80	0.35	1.262	0.3525	0.12	2.667E-03
4140	30	0.282	165	0.80	0.878	0.1709	0.14	5.500E-03
Al	10.4	0.098	50	1.10	1.596	0.1960	0.22	4.808E-03
Ti	16.5	0.16	120	7.00	1.030	0.1333	\$0.93	7.273E-03

The selected materials with minimum values are shaded in the table above. *Ans.*

2-21 First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three would favor steel, cast iron, or maybe a less common ferrous material. The expectation would likely be hot-rolled steel. If it is desired to confirm this, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 7.95 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{7.95 \text{ lbf}}{[\pi(1 \text{ in})^2/4](36 \text{ in})} = 0.281 \text{ lbf/in}^3$$

which agrees well with the unit weight of 0.282 lbf/in^3 reported in Table A-5 for carbon steel. Nickel steel and stainless steel have similar unit weights, but surface finish and darker coloring do not favor their selection. To select a likely specification from Table A-20, perform a Brinell hardness test, then use Eq. (2-36) to estimate an ultimate strength of

$S_u = 0.5H_B = 0.5(200) = 100$ kpsi. Assuming the material is hot-rolled due to the rough surface finish, appropriate choices from Table A-20 would be one of the higher carbon steels, such as hot-rolled AISI 1050, 1060, or 1080. *Ans.*

2-22 First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three favor a softer, non-ferrous material like aluminum. If it is desired to confirm this, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 2.90 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{2.9 \text{ lbf}}{[\pi(1 \text{ in})^2/4](36 \text{ in})} = 0.103 \text{ lbf/in}^3$$

which agrees reasonably well with the unit weight of 0.098 lbf/in^3 reported in Table A-5 for aluminum. No other materials come close to this unit weight, so the material is likely aluminum. *Ans.*

2-23 First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three favor a softer, non-ferrous copper-based material such as copper, brass, or bronze. To further distinguish the material, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 9 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{9.0 \text{ lbf}}{[\pi(1 \text{ in})^2/4](36 \text{ in})} = 0.318 \text{ lbf/in}^3$$

which agrees reasonably well with the unit weight of 0.322 lbf/in^3 reported in Table A-5 for copper. Brass is not far off (0.309 lbf/in^3), so the deflection test could be used to gain additional insight. From the measured deflection and utilizing the deflection equation for an end-loaded cantilever beam from Table A-9, Young's modulus is determined to be

$$E = \frac{Fl^3}{3Iy} = \frac{100(24)^3}{3\left(\pi(1)^4/64\right)(17/32)} = 17.7 \text{ Mpsi}$$

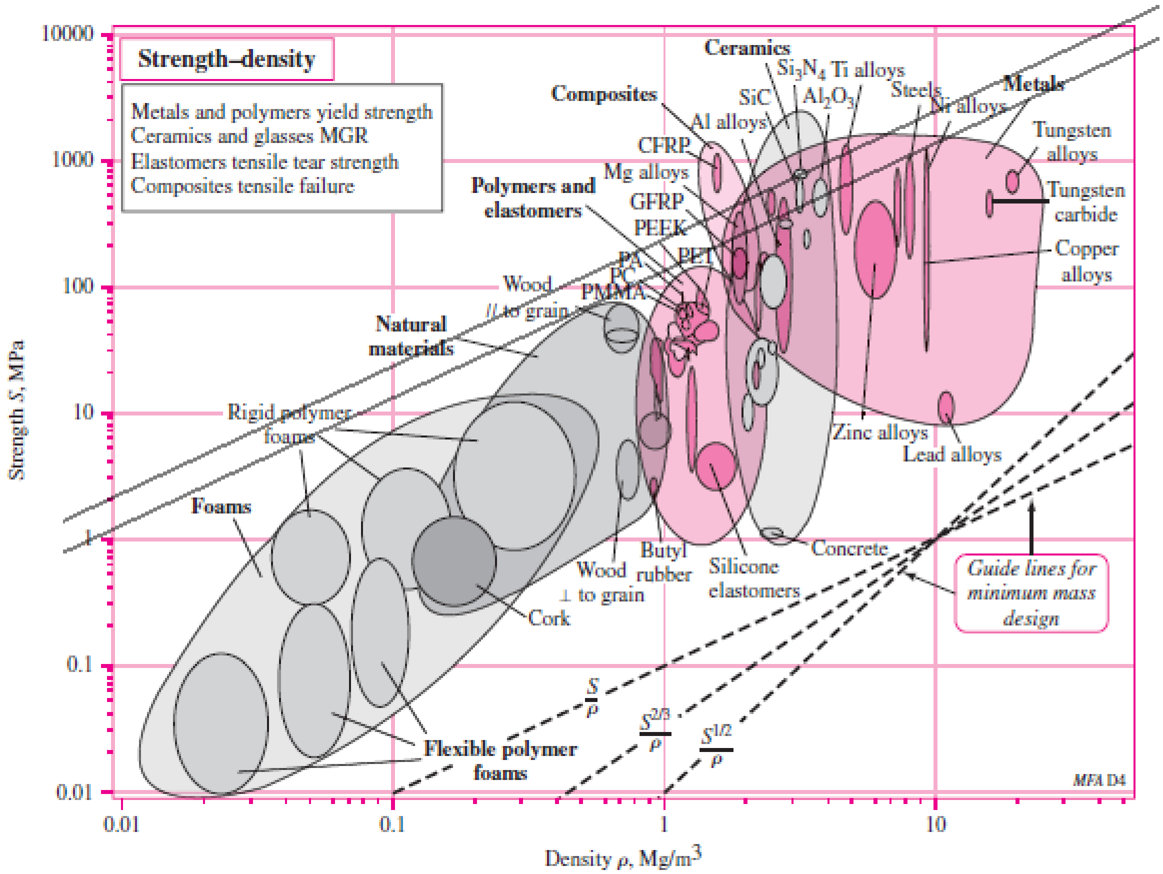
which agrees better with the modulus for copper (17.2 Mpsi) than with brass (15.4 Mpsi). The conclusion is that the material is likely copper. *Ans.*

2-24 This problem is for student research. No standard solution is provided.

2-25 For strength, $\sigma = F/A = S \quad \Rightarrow \quad A = F/S$
For mass, $m = Al\rho = (F/S)\, l\rho$

Thus, $f_3(M) = \rho/S$, and maximize S/ρ $(\beta = 1)$

In Fig. (2-27), draw lines parallel to S/ρ



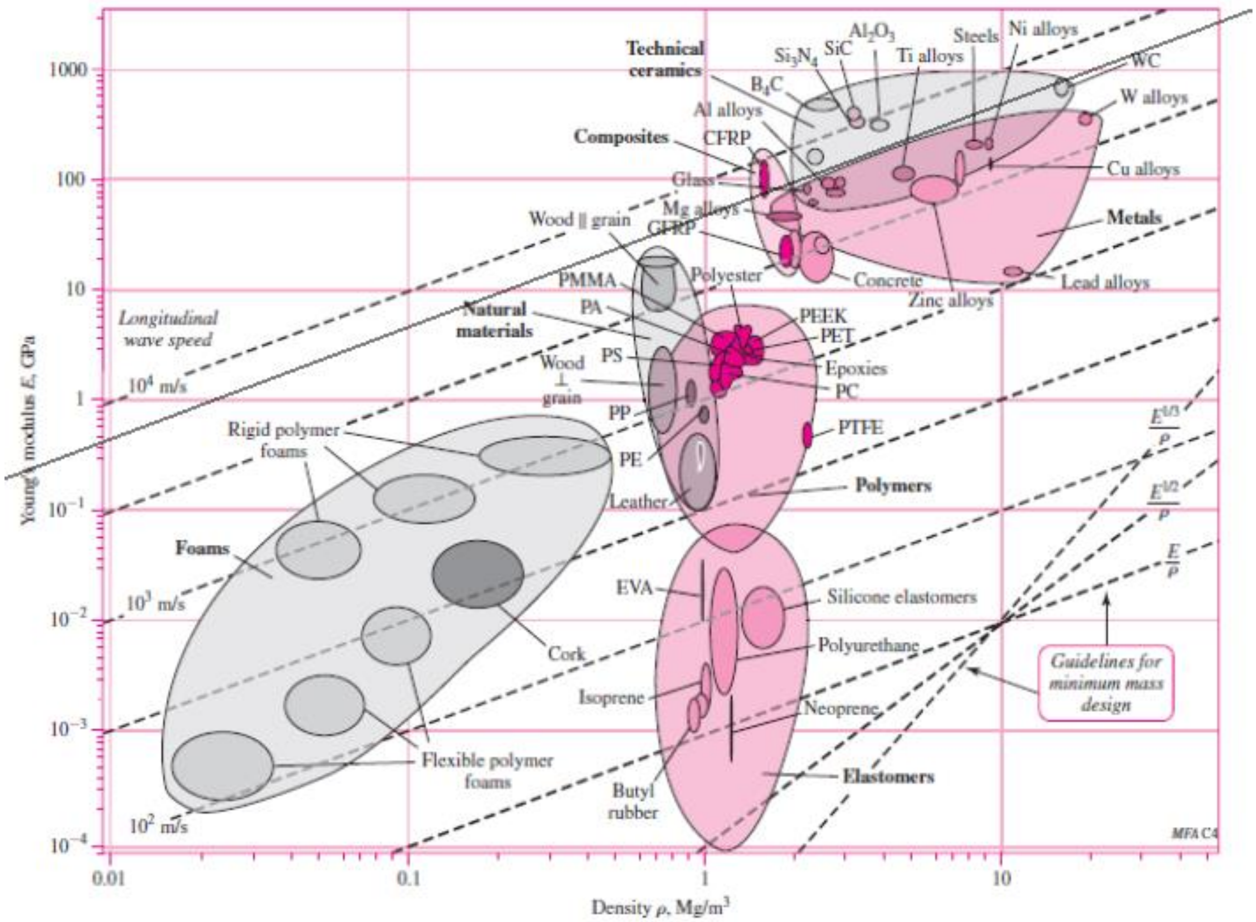
The higher strength aluminum alloys have the greatest potential, as determined by comparing each material's bubble to the S/ρ guidelines. *Ans.*

2-26 For stiffness, $k = AE/l \Rightarrow A = kl/E$

For mass, $m = Al\rho = (kl/E)\, l\rho = kl^2\, \rho/E$

Thus, $f_3(M) = \rho/E$, and maximize E/ρ $(\beta = 1)$

In Fig. (2-24), draw lines parallel to E/ρ



From the list of materials given, **tungsten carbide** (WC) is best, closely followed by aluminum alloys. They are close enough that other factors, like cost or availability, would likely dictate the best choice. Polycarbonate polymer is clearly not a good choice compared to the other candidate materials. *Ans.*

2-27 For strength,

$$\sigma = Fl/Z = S \qquad (1)$$

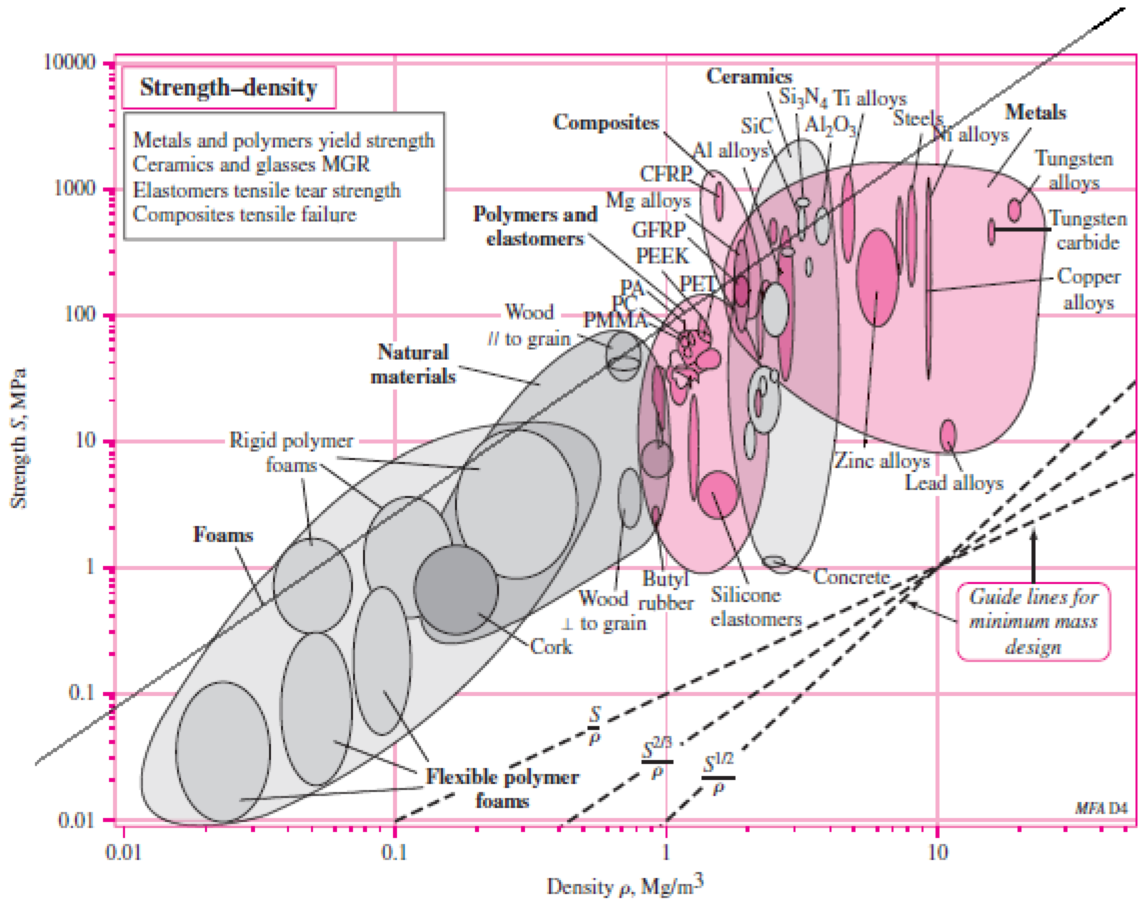
where *Fl* is the bending moment and *Z* is the section modulus [see Eq. (3-26*b*)]. The section modulus is strictly a function of the dimensions of the cross section and has the units in^3 (ips) or m^3 (SI). Thus, for a given cross section, $Z = C\,(A)^{3/2}$, where *C* is a number. For example, for a circular cross section, $C = \left(4\sqrt{\pi}\right)^{-1}$. Then, for strength, Eq. (1) is

$$\frac{Fl}{CA^{3/2}} = S \quad \Rightarrow \quad A = \left(\frac{Fl}{CS}\right)^{2/3} \qquad (2)$$

For mass, $$m = Al\rho = \left(\frac{Fl}{CS}\right)^{2/3} l\rho = \left(\frac{F}{C}\right)^{2/3} l^{5/3}\left(\frac{\rho}{S^{2/3}}\right)$$

Thus, $f_3(M) = \rho / S^{2/3}$, and maximize $S^{2/3}/\rho$ $(\beta = 2/3)$

In Fig. (2-27), draw lines parallel to $S^{2/3}/\rho$



From the list of materials given, a higher strength **aluminum alloy** has the greatest potential, followed closely by high carbon heat-treated steel. Tungsten carbide is clearly not a good choice compared to the other candidate materials. *Ans.*

2-28 Equation (2-41) applies to a circular cross section. However, for any cross section *shape* it can be shown that $I = CA^2$, where C is a constant. For example, consider a rectangular section of height h and width b, where for a given scaled shape, $h = cb$, where c is a constant. The moment of inertia is $I = bh^3/12$, and the area is $A = bh$. Then $I = h(bh^2)/12 = cb\,(bh^2)/12 = (c/12)(bh)^2 = CA^2$, where $C = c/12$ (a constant).

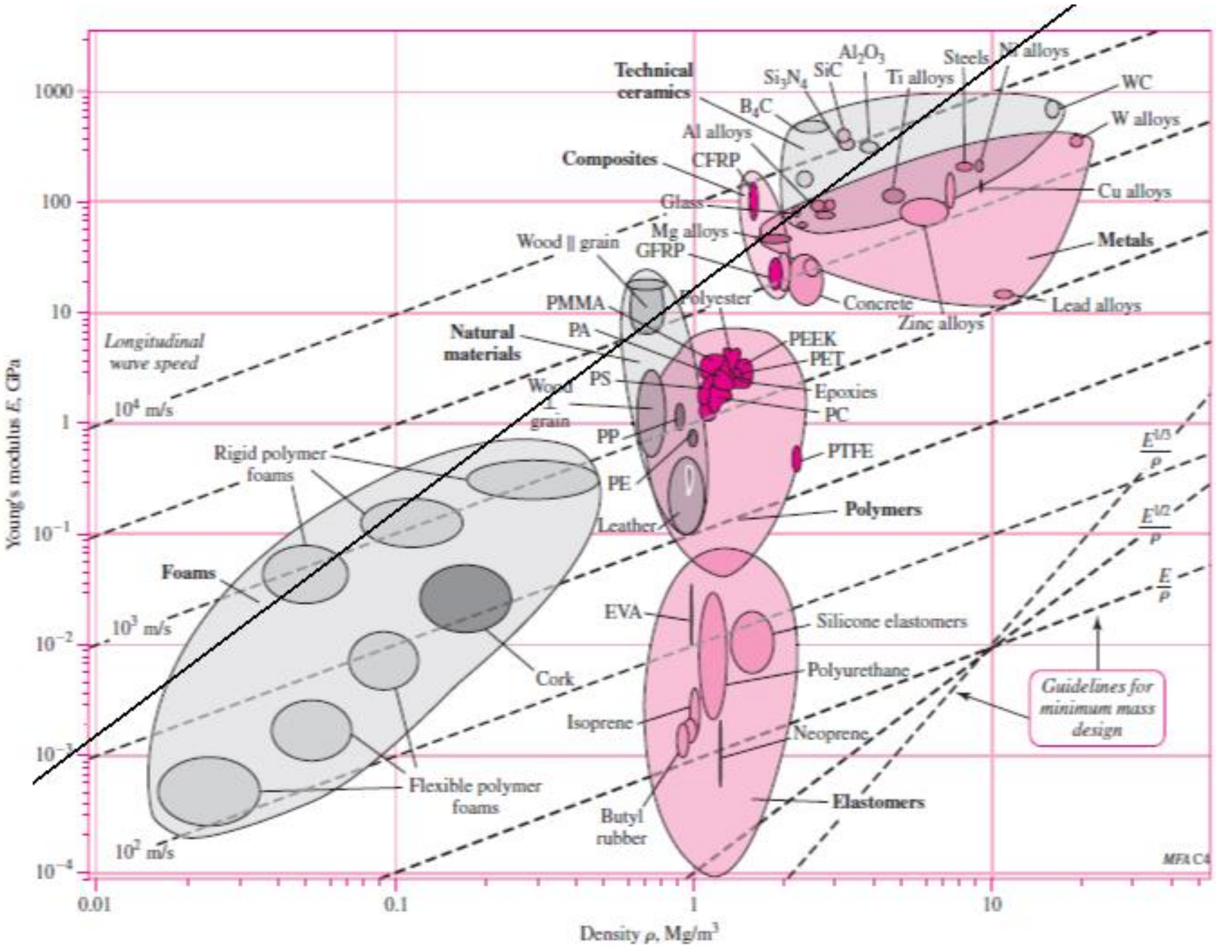
Thus, Eq. (2-42) becomes

$$A = \left(\frac{kl^3}{3CE} \right)^{1/2}$$

and Eq. (2-44) becomes

$$m = Al\rho = \left(\frac{k}{3C} \right)^{1/2} l^{5/2} \left(\frac{\rho}{E^{1/2}} \right)$$

Thus, minimize $f_3(M) = \dfrac{\rho}{E^{1/2}}$, or maximize $M = \dfrac{E^{1/2}}{\rho}$. From Fig. (2-24)



From the list of materials given, **aluminum alloys** are clearly the best followed by steels and tungsten carbide. Polycarbonate polymer is not a good choice compared to the other candidate materials. *Ans.*

2-29 For stiffness, $k = AE/l \quad \Rightarrow \quad A = kl/E$

For mass, $m = Al\rho = (kl/E)\, l\rho = kl^2\, \rho/E$

So, $\quad f_3(M) = \rho/E$, and maximize E/ρ. Thus, $\beta = 1$. *Ans.*

2-30 For strength, $\sigma = F/A = S \quad \Rightarrow \quad A = F/S$

For mass, $m = Al\rho = (F/S)\, l\rho$

So, $f_3(M) = \rho/S$, and maximize S/ρ. Thus, $\beta = 1$. *Ans.*

2-31 Equation (2-41) applies to a circular cross section. However, for any cross section *shape* it can be shown that $I = CA^2$, where C is a constant. For the circular cross section, $C = (4\pi)^{-1}$ [see Eq. (2-41)]. Another example, consider a rectangular section of height h and width b, where for a given scaled shape, $h = cb$, where c is a constant. The moment of inertia is $I = bh^3/12$, and the area is $A = bh$. Then $I = h(bh^2)/12 = cb\,(bh^2)/12 = (c/12)(bh)^2 = CA^2$, where $C = c/12$, a constant.

Thus, Eq. (2-42) becomes

$$A = \left(\frac{kl^3}{3CE}\right)^{1/2}$$

and Eq. (2-44) becomes

$$m = Al\rho = \left(\frac{k}{3C}\right)^{1/2} l^{5/2} \left(\frac{\rho}{E^{1/2}}\right)$$

So, minimize $f_3(M) = \dfrac{\rho}{E^{1/2}}$, or maximize $M = \dfrac{E^{1/2}}{\rho}$. Thus, $\beta = 1/2$. *Ans.*

2-32 For strength,

$$\sigma = Fl/Z = S \qquad (1)$$

where Fl is the bending moment and Z is the section modulus [see Eq. (3-26*b*)]. The section modulus is strictly a function of the dimensions of the cross section and has the units in^3 (ips) or m^3 (SI). The area of the cross section has the units in^2 or m^2. Thus, for a given cross section, $Z = C\,(A)^{3/2}$, where C is a number. For example, for a circular cross section, $Z = \pi d^3/(32)$and the area is $A = \pi d^2/4$. This leads to $C = \left(4\sqrt{\pi}\right)^{-1}$. So, with $Z = C\,(A)^{3/2}$, for strength, Eq. (1) is

$$\frac{Fl}{CA^{3/2}} = S \quad \Rightarrow \quad A = \left(\frac{Fl}{CS}\right)^{2/3} \qquad (2)$$

For mass,

$$m = Al\rho = \left(\frac{Fl}{CS}\right)^{2/3} l\rho = \left(\frac{F}{C}\right)^{2/3} l^{5/3} \left(\frac{\rho}{S^{2/3}}\right)$$

So, $f_3(M) = \rho/S^{2/3}$, and maximize $S^{2/3}/\rho$. Thus, $\beta = 2/3$. *Ans.*

2-33 For stiffness, $k = AE/l$, or, $A = kl/E$.

Thus, $m = \rho Al = \rho(kl/E)l = kl^2\rho/E$. Then, $M = E/\rho$ and $\beta = 1$.

From Fig. 2-24, lines parallel to E/ρ for ductile materials include steel, titanium, molybdenum, aluminum alloys, and composites.

For strength, $S = F/A$, or, $A = F/S$.

Thus, $m = \rho Al = \rho F/Sl = Fl\rho/S$. Then, $M = S/\rho$ and $\beta = 1$.

From Fig. 2-27, lines parallel to S/ρ give for ductile materials, steel, aluminum alloys, nickel alloys, titanium, and composites.

Common to both stiffness and strength are steel, titanium, aluminum alloys, and composites. *Ans.*

2-34 See Prob. 1-13 solution for $\bar{x} = 122.9$ kcycles and $s_x = 30.3$ kcycles. Also, in that solution it is observed that the number of instances less than 115 kcycles predicted by the normal distribution is 27; whereas, the data indicates the number to be 31.

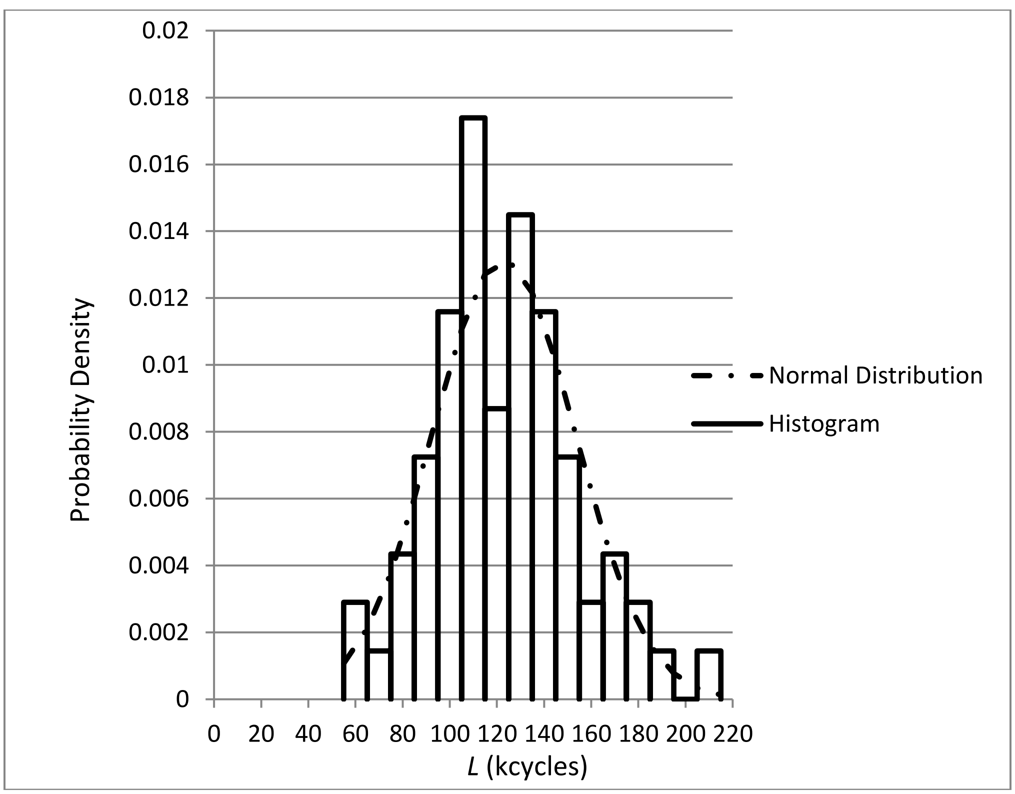
From Eq. (1-4), the probability density function (PDF), with $\mu = \bar{x}$ and $\hat{\sigma} = s_x$, is

$$f(x) = \frac{1}{s_x\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\bar{x}}{s_x}\right)^2\right] = \frac{1}{30.3\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-122.9}{30.3}\right)^2\right] \qquad (1)$$

The discrete PDF is given by $f/(Nw)$, where $N = 69$ and $w = 10$ kcycles. From the Eq. (1) and the data of Prob. 1-13, the following plots are obtained.

Range midpoint (kcycles)	Frequency	Observed PDF	Normal PDF
x	f	$f/(Nw)$	$f(x)$
60	2	0.002898551	0.001526493
70	1	0.001449275	0.002868043
80	3	0.004347826	0.004832507
90	5	0.007246377	0.007302224
100	8	0.011594203	0.009895407
110	12	0.017391304	0.012025636
120	6	0.008695652	0.013106245
130	10	0.014492754	0.012809861
140	8	0.011594203	0.011228104
150	5	0.007246377	0.008826008
160	2	0.002898551	0.006221829
170	3	0.004347826	0.003933396
180	2	0.002898551	0.002230043
190	1	0.001449275	0.001133847
200	0	0	0.000517001
210	1	0.001449275	0.00021141

Plots of the PDF's are shown below.



It can be seen that the data is not perfectly normal and is skewed to the left indicating that the number of instances below 115 kcycles for the data (31) would be higher than the hypothetical normal distribution (27).