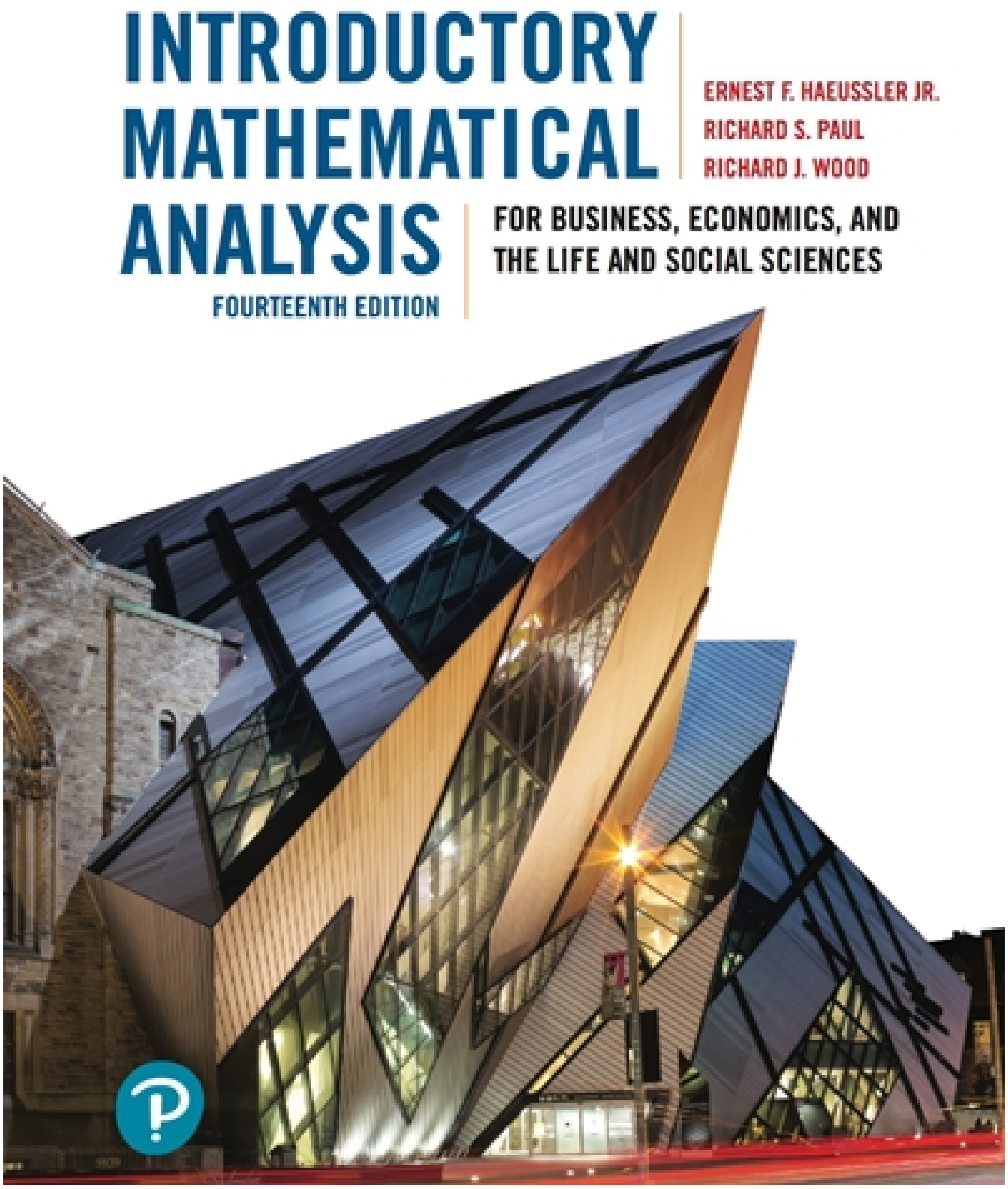
Solutions for Introductory Mathematical Analysis for Business Economics and the Life and Social Sciences 14th Edition by Haeussler



Solutions

Chapter 2

Apply It 2.1

1. a. The formula for the area of a circle is πr^2, where r is the radius.
$a(r) = \pi r^2$

b. The domain of $a(r)$ is all real numbers.

c. Since a radius cannot be negative or zero, the domain for the function, in context, is $r > 0$.

2. a. The formula relating distance, time, and speed is $d = rt$ where d is the distance, r is the speed, and t is the time. This can also be written as $t = \frac{d}{r}$. When $d = 300$, we have $t(r) = \frac{300}{r}$.

b. The domain of $t(r)$ is all real numbers except 0.

c. Since speed is not negative, the domain for the function, in context, is $r > 0$.

d. Replacing r by x: $t(x) = \frac{300}{x}$.

Replacing r by $\frac{x}{2}: t\left(\frac{x}{2}\right) = \frac{300}{\frac{x}{2}} = \frac{600}{x}$.

Replacing r by
$\frac{x}{4}: t\left(\frac{x}{4}\right) = \frac{300}{\frac{x}{4}} = \frac{1200}{x}$.

e. When the speed is reduced (divided) by a constant, the time is scaled (multiplied) by the same constant; $t\left(\frac{r}{c}\right) = \frac{300c}{r}$.

3. a. If the price is \$18.50 per large pizza, $p = 18.5$.
$18.5 = 26 - \frac{q}{40}$
$-7.5 = -\frac{q}{40}$
$300 = q$
At a price of \$18.50 per large pizza, 300 pizzas are sold each week.

b. If 200 large pizzas are being sold each week, $q = 200$.
$p = 26 - \frac{200}{40}$
$p = 26 - 5$
$p = 21$
The price is \$21 per pizza if 200 large pizzas are being sold each week.

c. To double the number of large pizzas sold, use $q = 400$.
$p = 26 - \frac{400}{40}$
$p = 26 - 10$
$p = 16$
To sell 400 large pizzas each week, the price should be \$16 per pizza.

4. Revenue = price · quantity = pq

From the table, the weekly revenue is:

$pq = 500 \cdot 11 = 5500$
$pq = 600 \cdot 14 = 8400$
$pq = 700 \cdot 17 = 11{,}900$
$pq = 800 \cdot 20 = 16{,}000$

Problems 2.1

1. The functions are not equal because $f(x) \geq 0$ for all values of x, while $g(x)$ can be less than 0. For example, $f(-2) = \sqrt{(-2)^2} = \sqrt{4} = 2$ and $g(-2) = -2$, thus $f(-2) \neq g(-2)$.

2. The domain of G is implicitly $[-3, \infty)$ while that of H is is implicitly $(-\infty, \infty)$, so $G \neq H$. (We note, however, that for all x in the domain of $G, G(x) = H(x)$.)

3. The functions are not equal because they have different domains. $h(x)$ is defined for all nonzero real numbers, while $k(x)$ is defined for all real numbers.

4. The functions are equal. For $x = 3$ we have $f(3) = 2$ and $g(3) = 3 - 1 = 2$, hence $f(3) = g(3)$. For $x \neq 3$, we have

$$f(x) = \frac{x^2 - 4x + 3}{x - 3} = \frac{(x-3)(x-1)}{x-3} = x - 1.$$

Note that we can cancel the $x - 3$ because we are assuming $x \neq 3$ and so $x - 3 \neq 0$. Thus for $x \neq 3, f(x) = x - 1 = g(x)$.

$f(x) = g(x)$ for all real numbers and they have the same domains, thus the functions are equal.

5. The denominator is zero when $x = 1$. Any other real number can be used for x.
Answer: all real numbers except 1

6. Any real number can be used for x.

Answer: all real numbers

7. We require both $\frac{x-2}{x+1} \geq 0$ and $x + 1 \neq 0$. $x - 2 > 0$ on $[2, \infty)$, $x - 2 = 0$ for $x = 2$, and $x - 2 < 0$ on $(-\infty, 2)$. $\frac{1}{x+1} > 0$ on $(-1, \infty)$, $\frac{1}{x+1}$ is undefined for $x = -1$, and $\frac{1}{x+1} < 0$ on $(-\infty, -1)$. It follows that $\frac{x-2}{x+1}$ being the product of $x - 2$ and $\frac{1}{x+1}$ is positive on $(-\infty, -1) \cup (2, \infty)$ and is 0 at $x = 2$. It follows that the domain of $\frac{x-2}{x+1}$ is $(-\infty, -1) \cup [2, \infty)$

8. For $\sqrt{z-1}$ to be real, $z - 1 \geq 0$, so $z \geq 1$. We exclude values of z for which $\sqrt{z-1} = 0$, so $z - 1 = 0$, thus $z = 1$.

Answer: all real numbers > 1

9. Any real number can be used for z.
Answer: all real numbers

10. We exclude values of x for which

$x + 3 = 0$

$x = -3$

Answer: all real numbers except -3

11. We exclude values of x where

$2x + 7 = 0$

$2x = -7$

$x = -\frac{7}{2}$

Answer: all real numbers except $-\frac{7}{2}$

12. We require $2 - 3x \geq 0$ equivalently $x \leq -2/3$. The domain of g is $(-\infty, -2/3]$.

13. We exclude values of y for which $y^2 - 4y + 4 = 0$. $y^2 - 4y + 4 = (y-2)^2$, so we exclude values of y for which $y - 2 = 0$, thus $y = 2$.

Answer: all real numbers except 2.

14. We exclude values of x for which

$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x = -3,\ 2$$

Answer: all real numbers except -3 and 2

15. We exclude all values of x for which

$3x^2 - 5x - 2 = 0$

$(3x+1)(x-2) = 0$

$x = -\frac{1}{3}, 2$

Answer: all real numbers except 2 and $-\frac{1}{3}$

16. $r^2 + 1$ is never 0.

Answer: all real numbers

17. $f(x) = 3 - 5x$;

$f(0) = 3, f(2) = 3 - 5(2) = -7$,

$f(-2) = 3 - 5(-2) = 13$.

18. $H(s) = 5s^2 - 3$

$H(4) = 5(4)^2 - 3 = 80 - 3 = 77$

$H\left(\sqrt{2}\right) = 5\left(\sqrt{2}\right)^2 - 3 = 10 - 3 = 7$

$H\left(\frac{2}{3}\right) = 5\left(\frac{2}{3}\right)^2 - 3 = \frac{20}{9} - 3 = -\frac{7}{9}$

19. $G(x) = 2 - x^2$

$G(-8) = 2 - (-8)^2 = 2 - 64 = -62$

$G(u) = 2 - u^2$

$G(u^2) = 2 - (u^2)^2 = 2 - u^4$

20. $F(x) = -7x + 1$

$F(s) = -7s + 1$

$F(t+1) = -7(t+1) + 1 = -7t - 6$

$F(x+3) = -7(x+3) + 1 = -7x - 20$

21. $\gamma(u) = 2u^2 - u$

$\gamma(-2) = 2(-2)^2 - (-2) = 8 + 2 = 10$

$\gamma(2v) = 2(2v)^2 - (2v) = 8v^2 - 2v$

$\gamma(x+a) = 2(x+a)^2 - (x+a)$

$= 2x^2 + 4ax + 2a^2 - x - a$

22. $h(v) = \dfrac{2}{\sqrt{4v}}$; $h(36) = \dfrac{2}{\sqrt{(4)(36)}} = \dfrac{2}{(2)(6)} = \dfrac{1}{6}$,

$h\left(\dfrac{1}{4}\right) = \dfrac{2}{\sqrt{4(1/4)}} = 2/1 = 2,$

$h(1-x) = \dfrac{2}{\sqrt{4(1-x)}}$

23. $f(x) = x^2 + 2x + 1$

$f(1) = 1^2 + 2(1) + 1 = 1 + 2 + 1 = 4$

$f(-1) = (-1)^2 + 2(-1) + 1 = 1 - 2 + 1 = 0$

$f(x+h) = (x+h)^2 + 2(x+h) + 1$

$= x^2 + 2xh + h^2 + 2x + 2h + 1$

24. $H(x) = (x+4)^2$

$H(0) = (0+4)^2 = 16$

$H(2) = (2+4)^2 = 6^2 = 36$

$H(t-4) = [(t-4)+4]^2 = t^2$

25. $k(x) = \dfrac{x-5}{x^2+1}$

$k(5) = \dfrac{5-5}{5^2+1} = 0$

$k(2x) = \dfrac{2x-5}{(2x)^2+1} = \dfrac{2x-5}{4x^2+1}$

$k(x+h) = \dfrac{(x+h)-5}{(x+h)^2+1} = \dfrac{x+h-5}{x^2+2xh+h^2+1}$

26. $k(x) = \sqrt{x-3}$

$k(4) = \sqrt{4-3} = \sqrt{1} = 1$

$k(3) = \sqrt{3-3} = \sqrt{0} = 0$

$k(x+1) - k(x) = \sqrt{(x+1)-3} - \sqrt{x-3}$

$= \sqrt{x-2} - \sqrt{x-3}$

27. $f(x) = x^{2/5}$;

$f(0) = 0^{2/5} = 0,$

$f(243) = (243^{1/5})^2 = 3^2 = 9,$

$f\left(\dfrac{-1}{32}\right) = (-1/32)^{2/5}$

$= (-(1/32)^{1/5}))^2 = (-1/2)^2 = 1/4$

28. $g(x) = x^{2/5}$

$g(32) = 32^{2/5} = \left(\sqrt[5]{32}\right)^2 = (2)^2 = 4$

$g(-64) = (-64)^{2/5} = \left(\sqrt[5]{-64}\right)^2$

$= \left(\sqrt[5]{-32}\sqrt[5]{2}\right)^2 = \left(-2\sqrt[5]{2}\right)^2 = 4\sqrt[5]{4}$

$g(t^{10}) = (t^{10})^{2/5} = t^4$

29. $f(x) = 4x - 5$

a. $f(x+h) = 4(x+h) - 5 = 4x + 4h - 5$

b. $\dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{(4x+4h-5) - (4x-5)}{h} = \dfrac{4h}{h} = 4$

30. $f(x) = \dfrac{x}{3}$

a. $f(x+h) = \dfrac{x+h}{3}$

b. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\frac{x+h}{3} - \frac{x}{3}}{h} = \dfrac{\frac{h}{3}}{h} = \dfrac{1}{3}$

31. $f(x) = x^2 + 2x$

a. $f(x+h) = (x+h)^2 + 2(x+h)$

$= x^2 + 2xh + h^2 + 2x + 2h$

b. $\dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{(x^2 + 2xh + h^2 + 2x + 2h) - (x^2 + 2x)}{h}$

$= \dfrac{2xh + h^2 + 2h}{h} = 2x + h + 2$

32. $f(x) = 2x^2 - 5x + 3$;

a. $f(x+h) = 2(x+h)^2 - 5(x+h) + 3$

$= 2x^2 + (4h-5)x + (2h^2 - 5h + 3),$

b. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{4hx + (2h^2 - 5h)}{h}$

$= 4x + (2h - 5)$ for $h \neq 0$

33. $f(x) = 3 - 2x + 4x^2$

a. $f(x+h) = 3 - 2(x+h) + 4(x+h)^2$
$= 3 - 2x - 2h + 4(x^2 + 2xh + h^2)$

b. $\frac{f(x+h) - f(x)}{h} = \frac{3 - 2x - 2h + 4x^2 + 8xh + 4h^2 - (3 - 2x + 4x^2)}{h}$
$= \frac{-2h + 8xh + 4h^2}{h}$
$= -2 + 8x + 4h$

34. $f(x) = x^3$

a. $f(x+h) = (x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

b. $\frac{f(x+h) - f(x)}{h} = \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h} = \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$

35. $f(x) = \frac{1}{x-1}$

a. $f(x+h) = \frac{1}{x+h-1}$

b. $\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h} = \frac{\frac{x-1-(x+h-1)}{(x-1)(x+h-1)}}{h} = \frac{-1}{(x-1)(x+h-1)}$

36. $f(x) = \frac{x+8}{x}$

a. $f(x+h) = \frac{(x+h)+8}{x+h} = \frac{x+h+8}{x+h}$

b. $\frac{f(x+h) - f(x)}{h} = \frac{\frac{x+h+8}{x+h} - \frac{x+8}{x}}{h} = \frac{x(x+h)\left(\frac{x+h+8}{x+h} - \frac{x+8}{x}\right)}{x(x+h)h} = \frac{x(x+h+8) - (x+h)(x+8)}{x(x+h)h}$
$= \frac{x^2 + xh + 8x - x^2 - hx - 8x - 8h}{x(x+h)h} = \frac{-8h}{x(x+h)h} = -\frac{8}{x(x+h)}$

37. $f(x) = 3x + 7$;
$\frac{f(2+h) - f(2)}{h} = \frac{(3(2+h) + 7) - (3(2) + 7)}{h} = 3$ for $h \neq 0$

38. $\frac{f(x)-f(2)}{x-2} = \frac{2x^2-x+1-(8-2+1)}{x-2}$
$$= \frac{2x^2-x+1-7}{x-2}$$
$$= \frac{2x^2-x-6}{x-2}$$
$$= 2x+3$$

39. $9y-3x-4=0$

The equivalent form $y = \frac{3x+4}{9}$ shows that for each input x there is exactly one output, $\frac{3x+4}{9}$. Thus y is a function of x. Solving for x gives $x = \frac{9y-4}{3}$. This shows that for each input y there is exactly one output, $\frac{9y-4}{3}$. Thus x is a function of y.

40. $x^4-1+y=0$

The equivalent form $y=-x^4+1$ shows that for each input x there is exactly one output, $-x^4+1$. Thus y is a function of x. Solving for x gives $x = \pm\sqrt[4]{1-y}$. If, for example, $y=-15$, then $x=\pm 2$, so x is not a function of y.

41. $y=7x^2$

For each input x, there is exactly one output $7x^2$. Thus y is a function of x. Solving for x gives $x = \pm\sqrt{\frac{y}{7}}$. If, for example, $y=7$, then $x=\pm 1$, so x is not a function of y.

42. Solving for y we get $y=\pm\sqrt{1-x^3}$. The solution is not unique so the equation does not define y as a function of x. Solving for x we get $x=\sqrt[3]{1-x^2}$. The solution is unique so the equation defines x as a function of y.

43. Yes, because corresponding to each input r there is exactly one output, πr^2.

44. **a.** $f(a) = a^2a^3 + a^3a^2 = a^5 + a^5 = 2a^5$

b. $f(ab) = a^2(ab)^3 + a^3(ab)^2$
$$= a^2a^3b^3 + a^3a^2b^2$$
$$= a^5b^3 + a^5b^2$$
$$= a^5b^2(b+1)$$

45. Weekly excess of income over expenses is

$7200 - 4900 = 2300$.

After t weeks the excess accumulates to $2300t$. Thus the value of V of the business at the end of t weeks is given by $V = f(t) = 50{,}000 + 2300t$.

46. Depreciation at the end of t years is $0.02t(30{,}000)$, so value V of machine is

$V = f(t) = 30{,}000 - 0.02t(30{,}000)$, or

$V = f(t) = 30{,}000(1-0.02t)$.

47. Each (nonnegative) q determines a unique P. So P is a function of q. The dependent variable is P and the independent variable is q.

48. Charging \$600,000 per film corresponds to $p = 600{,}000$.

$600{,}000 = \frac{1{,}200{,}000}{q}$

$q = 2$

The actor will star in 2 films per year. To star in 4 films per year the actor should charge

$p = \frac{1{,}200{,}000}{4} = \$300{,}000$ per film.

49. The function can be written as $q = 48p$. At \$8.39 per pound, the coffee house will supply $q = 48(8.39) = 402.72$ pounds per week. At \$19.49 per pound, the coffee house will supply $q = 48(19.49) = 935.52$ pounds per week. The amount the coffee house supplies increases as the price increases.

50. **a.** $f(0) = 1 - 1 = 0$

b. $$f(100) = 1 - \left(\frac{200}{300}\right)^3 = 1 - \left(\frac{2}{3}\right)^3 = 1 - \frac{8}{27} = \frac{19}{27}$$

c. $$f(800) = 1 - \left(\frac{200}{1000}\right)^3 = 1 - \left(\frac{1}{5}\right)^3 = 1 - \frac{1}{125} = \frac{124}{125}$$

d. Solve

$$0.5 = 1 - \left(\frac{200}{200+t}\right)^3$$

$$\left(\frac{200}{200+t}\right)^3 = 0.5$$

$$\frac{200}{200+t} = \sqrt[3]{0.5}$$

$$200 = 200\sqrt[3]{0.5} + t\sqrt[3]{0.5}$$

$$t = \frac{200 - 200\sqrt[3]{0.5}}{\sqrt[3]{0.5}} \approx 51.98$$

Half the group was discharged after 52 days.

51. **a.** $$f(1000) = \frac{\left(\sqrt[3]{1000}\right)^4}{2500} = \frac{10^4}{2500} = \frac{10{,}000}{2500} = 4$$

b. $$f(2000) = \frac{\left[\sqrt[3]{1000(2)}\right]^4}{2500} = \frac{\left(10^3\sqrt[3]{2}\right)^4}{2500} = \frac{10{,}000\sqrt[3]{2^4}}{2500} = 4\sqrt[3]{2^3 \cdot 2} = 8\sqrt[3]{2}$$

c. $$f(2I_0) = \frac{(2I_0)^{4/3}}{2500} = \frac{2^{4/3}I_0^{4/3}}{2500} = 2\sqrt[3]{2}\left[\frac{I_0^{4/3}}{2500}\right] = 2\sqrt[3]{2}f(I_0)$$

Thus $f(2I_0) = 2\sqrt[3]{2}f(I_0)$, which means that doubling the intensity increases the response by a factor of $2\sqrt[3]{2}$.

52. $P(28) = 14 + \sqrt{25} = 14 + 5 = 19$;
$P(52) = 26 + \sqrt{49} = 26 + 7 = 33$

53. **a.** Domain: 3000, 2900, 2300, 2000
$f(2900) = 12, f(3000) = 10$

b. Domain: 10, 12, 17, 20
$g(10) = 3000, g(17) = 2300$

54. **a.** −18.97

b. −581.77

c. −18.51

55. **a.** −5.13

b. 2.64

c. −17.43

56. **a.** 1,997,723.57

b. 1,287,532.35

c. 2,964,247.40

57. **a.** $f(11.7) = 6.94$

b. $f(-73) = 40.28$

c. $f(0) = 0.67$

Apply It 2.2

5. **a.** Let n = the number of visits and $p(n)$ be the premium amount.
$p(n) = 125$

b. The premiums do not change regardless of the number of doctor visits.

c. This is a constant function.

6. **a.** $d(t) = 3t^2$ is a quadratic function.

b. The degree of $d(t) = 3t^2$ is 2.

c. The leading coefficient of $d(t) = 3t^2$ is 3.

7. The price for n pairs of socks is given by

$$c(n) = \begin{cases} 3.5n & 0 \le n \le 5 \\ 3n & 5 < n \le 10. \\ 2.75n & 10 < n \end{cases}$$

8. Think of the bookshelf having 7 slots, from left to right. You have a choice of 7 books for the first slot. Once a book has been put in the first slot, you have 6 choices for which book to put in the second slot, etc. The number of arrangements is $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 7! = 5040$.

Problems 2.2

1. yes

2. $f(x) = \frac{x^3 + 7x - 3}{3} = \frac{1}{3}x^3 + \frac{7}{3}x - 1$, which is a polynomial function.

3. $g(x) = \frac{5}{3x+1}$ cannot be written as a sum of multiples of nonnegative integral powers of x, so g is not a polynomial.

4. yes

5. yes

6. yes

7. no

8. $g(x) = 2x^{-5} = \frac{2}{x^5}$ expresses g as a quotient of polynomials and thus shows that g is rational.

9. all real numbers

10. all real numbers

11. all real numbers

12. all x such that $1 \le x \le 3$

13. F is a polynomial of degree 4 with leading coefficient 5.

14. a. 2

b. 9

15. a. 7

b. 1

16. a. 0

b. 9

17. $f(x) = 8$

$f(2) = 8$

$f(t + 8) = 8$

$f\left(-\sqrt{17}\right) = 8$

18. $g(20) = |2(20) + 1| = |41| = 41$;

$g(5) = |2(5) + 1| = |11| = 11$;

$g(-7) = |2(-7) + 1| = |-13| = 13$

19. $F(12) = 2$

$F\left(-\sqrt{3}\right) = -1$

$F(1) = 0$

$F\left(\frac{18}{5}\right) = 2$

20. $f(3) = 4$

$f(-4) = 3$

$f(0) = 4$

21. $G(8) = 8 - 1 = 7$

$G(3) = 3 - 1 = 2$

$G(-1) = 3 - (-1)^2 = 2$

$G(1) = 3 - (1)^2 = 2$

22. $F(3) = 3^2 - 3(3) + 1 = 1$

$F(-3) = 2(-3) - 5 = -11$

$F(2)$ is not defined.

23. $9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362{,}880$

24. $(3 - 3)! = 0! = 1$

25. $(4 - 2)! = 2! = 2 \cdot 1 = 2$

26. $6! \cdot 2! = (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(2 \cdot 1)$

$= (720)(2)$

$= 1440$

27. $\frac{n!}{(n-1)!} = \frac{n \cdot (n-1)!}{(n-1)!} = n$

28. $\frac{9!}{4!(9-4)!} = \frac{9.8.7.6.5.4.3.2.1}{(4.3.2)(5.4.3.2)} = \frac{9.8.7.6}{4.3.2}$
$= 3.2.7.3 = 126$

29. Let i = the passenger's income and
$c(i)$ = the cost for the ticket.
$c(i) = 2.50$
This is a constant function.

30. Let w = the width of the prism, then
$w + 3$ = the length of the prism, and
$2w - 1$ = the height of the prism. The formula for the volume of a rectangular prism is
$V = \text{length} \cdot \text{width} \cdot \text{height}$.
$V(w) = (w+3)(w)(2w-1) = 2w^3 + 5w^2 - 3w$
This is a cubic function.

31. a. $C = 850 + 3q$

b. $1600 = 850 + 3q$
$750 = 3q$
$q = 250$

32. The interest is Prt, so principal and interest amount to $f(t) = P + Prt$, or $f(t) = P(1 + rt)$. Since $f(t) = at + b$ where $a(= Pr)$ and $b(= P)$ are constants, f is a linear function of t.

33.
$$c(j) = \begin{cases} 0.075 & \text{if } 0 \le j \le 44{,}701 \\ 0.11 & \text{if } 44{,}701 < j \le 89{,}401 \\ 0.13 & \text{if } 89{,}401 < j \le 138{,}586 \\ 0.145 & \text{if } 138{,}586 < j \end{cases}$$

34. For a committee of five, there are 5 choices for who will be member A. For each choice of member A, there are 4 choices for member G. Once members A and G have been chosen, there are 3 choices for member M, two choices for member N, then one choice for member S once members A, G, M, and N have been chosen. Thus, there are $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120$ ways to label the members.

35. $P(2) = \frac{3!\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^2}{2!(1!)} = \frac{6\left(\frac{1}{16}\right)\left(\frac{3}{4}\right)}{2(1)} = \frac{9}{64}$

36. $P(5) = \frac{5!\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0}{5!(0!)} = \frac{5!\left(\frac{1}{1024}\right)(1)}{5!(1)}$
$= \frac{1}{1024}$

37. a. all T such that $30 \le T \le 39$

b. $f(30) = \frac{1}{24}(30) + \frac{11}{4} = \frac{5}{4} + \frac{11}{4} = \frac{16}{4} = 4$
$f(36) = \frac{1}{24}(36) + \frac{11}{4} = \frac{6}{4} + \frac{11}{4} = \frac{17}{4}$
$F(39) = \frac{4}{3}(39) - \frac{175}{4} = 52 - \frac{175}{4} = \frac{33}{4}$

38. a. $f(2.14) = 0.11(2.14)^3 - 15.31 = -14.23$

b. $f(3.27) = 0.42(3.27)^4 - 12.31 = 35.71$

c. $f(-4) = 0.11(-4)^3 - 15.31 = -22.35$

39. a. 1182.74

b. 4985.27

c. 252.15

40. a. 19.12

b. −62.94

c. 57.69

41. a. 2.21

b. 9.98

c. −14.52

Apply It 2.3

9. The customer's price is
$(c \circ s(x) = c(s(x)) = c(x+3) = 2(x+3)$
$= 2x + 6$

10. $g(x) = (x+3)^2$ can be written as $g(x) = a(l(x)) = (a \circ l)(x)$ where $a(x) = x^2$ and $l(x) = x + 3$. Then $l(x)$ represents the length of the sides of the square, while $a(x)$ is the area of a square with side of length x.

Problems 2.3

1. $f(x) = x + 3, g(x) = x + 5$

a. $(f + g)(x) = f(x) + g(x)$
$= (x + 3) + (x + 5)$
$= 2x + 8$

b. $(f + g)(0) = 2(0) + 8 = 8$

c. $(f - g)(x) = f(x) - g(x)$
$= (x + 3) - (x + 5)$
$= -2$

d. $(fg)(x) = f(x)g(x)$
$= (x + 3)(x + 5)$
$= x^2 + 8x + 15$

e. $(fg)(-2) = (-2)^2 + 8(-2) + 15 = 3$

f. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{x+3}{x+5}$

g. $(f \circ g)(x) = f(g(x))$
$= f(x + 5)$
$= (x + 5) + 3$
$= x + 8$

h. $(f \circ g)(3) = 3 + 8 = 11$

i. $(f \circ g)(x) = f(g(x))$
$= f(x + 3)$
$= (x + 3) + 5$
$= x + 8$

j. $(f \circ g)(3) = 3 + 8 = 11$

2. $f(x) = 2x, g(x) = 6 + x$

a. $(f + g)(x) = f(x) + g(x)$
$= (2x) + (6 + x)$
$= 3x + 6$

b. $(f - g)(x) = f(x) - g(x)$
$= (2x) - (6 + x)$
$= x - 6$

c. $x(f - g)(4) = (4) - 6 = -2x$

d. $x(fg)(x) = f(x)g(x) = 2x(6 + x) = 12x + 2x^2$

e. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{2x}{6+x}$

f. $\frac{f}{g}(2) = \frac{2(2)}{6+2} = \frac{4}{8} = \frac{1}{2}$

g. $(f \circ g)(x) = f(g(x))$
$= f(6 + x)$
$= 2(6 + x)$
$= 12 + 2x$

h. $(g \circ f)(x) = g(f(x)) = g(2x) = 6 + 2x$

i. $(g \circ f)(2) = 6 + 2(2) = 6 + 4 = 10$

3. $f(x) = x^2 - 1, \quad g(x) = x^2 + x$

a. $(f + g)(x) = f(x) + g(x)$
$= (x^2 - 1) + (x^2 + x)$
$= 2x^2 + x - 1$

b. $(f - g)(x) = f(x) - g(x)$
$= (x^2 - 1) - (x^2 + x)$
$= -x - 1$

c. $(f - g)\left(-\frac{1}{2}\right) = \frac{1}{2} - 1 = -\frac{1}{2}$

d. $(fg)(x) = f(x)g(x)$
$= (x^2 - 1)(x^2 + x)$
$= x^4 + x^3 - x^2 - x$

e. $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$
$= \frac{x^2 - 1}{x^2 + x}$
$= \frac{(x+1)(x-1)}{x(x+1)}$
$= \frac{x-1}{x}, x \neq -1$

f. $\frac{f}{g}\left(-\frac{1}{2}\right) = \frac{-\frac{1}{2} - 1}{-\frac{1}{2}} = \frac{-\frac{3}{2}}{-\frac{1}{2}} = 3$

g. $(f \circ g)(x) = f(g(x))$
$= f(x^2 + x)$
$= (x^2 + x)^2 - 1$
$= x^4 + 2x^3 + x^2 - 1$

h. $(g \circ f)(x) = g(f(x))$
$= g(x^2 - 1)$
$= (x^2 - 1)^2 + (x^2 - 1)$
$= x^4 - 2x^2 + 1 + x^2 - 1$
$= x^4 - x^2$

i. $(g \circ f)(-3) = (-3)^4 - (-3)^2 = 72$

4. a. $(f + g)(x) = 2x^2 + 5 + 3 = 2x^2 + 8$

b. $(f+g)(1/2) = 2(1/2)^2 + 8 = 1/2 + 8 = 17/2$

c. $(f - g)(x) = 2x^2 + 5 - 3 = 2x^2 + 2$

d. $(fg)(x) = (2x^2 + 5)(3) = 6x^2 + 15$

e. $(fg)(2) = 6(2)^2 + 15 = 24 + 15 = 39$

f. $\frac{f}{g}(x) = \frac{2x^2 + 5}{3}$

g. $(f \circ g)(x) = 2(3)^2 + 5 = 23$

h. $(f \circ g)(100.003) = 23$

i. $(g \circ f)(x) = g(f(x) = 3$

5. $f(g(2)) = f(4 - 4) = f(0) = 0 + 6 = 6$

$g(f(2)) = g(12 + 6) = g(18) = 4 - 36 = -32$

6. $(f \circ g)(p) = f(g(p))$

$$= f\left(\frac{p-2}{3}\right)$$

$$= \frac{4}{\frac{p-2}{3}}$$

$$= \frac{12}{p-2}$$

$$(g \circ f)(p) = g(f(p)) = g\left(\frac{4}{p}\right) = \frac{\frac{4}{p} - 2}{3}$$

$$= \frac{4 - 2p}{3p}$$

7. $(F \circ G)(t) = F(G(t))$

$$= F\left(\frac{2}{t-1}\right)$$

$$= \left(\frac{2}{t-1}\right)^2 + 7\left(\frac{2}{t-1}\right) + 1$$

$$= \frac{4}{(t-1)^2} + \frac{14}{t-1} + 1$$

$(G \circ F)(t) = G(F(t))$

$$= G(t^2 + 7t + 1)$$

$$= \frac{2}{(t^2 + 7t + 1) - 1}$$

$$= \frac{2}{t^2 + 7t}$$

8. $(F \circ G)(t) = F(G(t))$
$= F(2t^2 - 2t + 1)$
$= \sqrt{2t^2 - 2t + 1}$

$(G \circ F)(t) = G(F(t))$
$= G\left(\sqrt{t}\right)$
$= 2\left(\sqrt{t}\right)^2 - 2\left(\sqrt{t}\right) + 1$
$= 2t - 2\sqrt{t} + 1$

9. $(f \circ g)(v) = \frac{2}{(\sqrt{3v+1})^2 - 3} = \frac{2}{3v + 1 - 3}$

$$= \frac{2}{3v - 2}$$

$$(g \circ f)(v) = \sqrt{3\left(\frac{2}{v^2 - 3}\right) + 1} = \sqrt{\frac{v^2 + 3}{v^2 - 3}}$$

10. $(f \circ f)(x) = f(f(x))$
$= f(x^2 + 2x - 1)$
$= (x^2 + 2x - 1)^2 + 2(x^2 + 2x - 1) - 1$
$= x^4 + 4x^3 + 4x^2 - 2$

11. Let $g(x) = 11x$ and $f(x) = x - 7$. Then
$h(x) = g(x) - 7 = f(g(x))$

12. Let $g(x) = x^2 - 2$ and $f(x) = \sqrt{x}$. Then
$h(x) = \sqrt{x^2 - 2} = \sqrt{g(x)} = f(g(x))$

13. Let $g(x) = x^2 + x + 1$ and $f(x) = \frac{3}{x}$. Then

$h(x) = \frac{3}{x^2 - x + 1} = \frac{3}{g(x)} = f(g(x))$

14. Let $f(x) = 7x^2 - 5x + 1$ and $g(x) = 4x^2 + 7x$. Then $(f \circ g)(x) = f(g(x)) = 7(4x^2 + 7x)^2 - 5(4x^2 + 7x) + 1 = h(x)$

15. Let $g(x) = \frac{x^2 - 1}{x + 3}$ and $f(x) = \sqrt[4]{x}$.

Then $h(x) = \sqrt[4]{g(x)} = f(g(x))$.

16. Let $g(x) = 3x - 5$ and $f(x) = \frac{2 - x}{x^2 + 2}$. Then $h(x) = \frac{2 - (3x - 5)}{(3x - 5)^2 + 2} = f(g(x))$.

17. a. The revenue is \$9.75 per pound of coffee sold, so $r(x) = 9.75x$.

b. The expenses are $e(x) = 4500 + 4.25x$.

c. Profit = revenue – expenses.

$(r - e)(x) = 9.75x - (4500 + 4.25x)$
$= 5.5x - 4500.$

18. $v(x) = \frac{4}{3}\pi(3x - 1)^3$ can be written as $v(x) = f(l(x)) = (f \circ l)(x)$ where $f(x) = \frac{4}{3}\pi x^3$ and $l(x) = 3x - 1$. Then $l(x)$ represents the radius of the sphere, while $f(x)$ is the volume of a sphere with radius x.

19. $r = g(q) = g(f(m)) = 24\frac{(20m - m^2)}{2} = 12(20m - m^2)$ is revenue from output of m employees.

20. $(f \circ g)(E) = f(g(E))$

$= f(7202 + 0.29E^{3.68})$
$= 0.45(7202 + 0.29E^{3.68} - 1000)^{0.53}$
$= 0.45(6202 + 0.29E^{3.68})^{0.53}$

This represents status based on years of education.

21. a. 14.05

b. 1169.64

22. a. –0.13

b. 18.85

23. a. 194.47

b. 0.29

24. a. $f(g(2.17)) = f(1/(2.17)^3$
$\approx f(0.097863512) \approx 2/1.097863512$
≈ 1.82

b. $g(f(2.17)) = g(2/3.17)$
$\approx g(0.630914826)$
$\approx 1/(0.630914826)^3 \approx 3.98$

Problems 2.4

1. $f^{-1}(x) = \frac{x}{3} - \frac{7}{3}$

2. $g^{-1}(x) = \frac{x}{5} + \frac{3}{5}$

3. $F^{-1}(x) = 2x + 14$

4. $f^{-1}(x) = \frac{\sqrt{x}}{4} + \frac{5}{4}$

5. $A(r) = 4\pi r^2$, for $r \geq 0$ gives the surface area of a sphere of radius r. Solving $A = 4\pi r^2$ for r gives $r = \sqrt{\frac{A}{4\pi}}$. Thus if a sphere is given to us and somehow its surface area is known to be A then it's radius r is given by the last equation. Said otherwise $A^{-1}(x) = \sqrt{\frac{x}{4\pi}}$

6. $r(V) = \sqrt[3]{\frac{3V}{4\pi}}$

7. $f(x) = 5x + 12$ is one-to-one, for if $f(x_1) = f(x_2)$ then $5x_1 + 12 = 5x_2 + 12$, so $5x_1 = 5x_2$ and thus $x_1 = x_2$.

8. $g(x) = (3x + 4)^2$ is not one-to-one, because $g(x_1) = g(x_2)$ does not imply $x_1 = x_2$. For example, $g\left(-\frac{1}{3}\right) = g\left(-\frac{7}{3}\right) = 9$.

9. $h(x) = (5x + 12)^2$, for $x \geq -\frac{5}{12}$, is one-to-one.
If $h(x_1) = h(x_2)$ then $(5x_1 + 12)^2 = (5x_2 + 12)^2$.
Since $x \geq -\frac{5}{12}$ we have $5x + 12 \geq 0$, and thus $(5x_1 + 12)^2 = (5x_2 + 12)^2$ only if $5x_1 + 12 = 5x_2 + 12$, and hence $x_1 = x_2$.

10. $F(-11) = |-1| = 1 = |1| = |-9+10| = F(-9)$ shows that F is not one-to-one.

11. The inverse of $f(x) = (4x - 5)^2$ for $x \geq \frac{5}{4}$ is $f^{-1}(x) = \frac{\sqrt{x}}{4} + \frac{5}{4}$, so to find the solution, we find $f^{-1}(23)$.

$f^{-1}(23) = \frac{\sqrt{23}}{4} + \frac{5}{4}$

The solution is $x = \frac{\sqrt{23}}{4} + \frac{5}{4}$.

12. The inverse of $f(x) = 2x^3 + 1$ is $f^{-1} = \sqrt[3]{\frac{x-1}{2}}$, so the solution is $f^{-1}(129) = 4$.

13. From $p = \frac{1{,}200{,}000}{q}$, we get $q = \frac{1{,}200{,}000}{p}$.
Since $q > 0$, p is also greater than 0, so q as a function of p is $q = q(p) = \frac{1{,}200{,}000}{p}$, $p > 0$.

$$\begin{aligned} p(q(p)) &= p\left(\frac{1{,}200{,}000}{p}\right) \\ &= \frac{1{,}200{,}000}{\frac{1{,}200{,}000}{p}} \\ &= 1{,}200{,}000 \cdot \frac{p}{1{,}200{,}000} \\ &= p \end{aligned}$$

Similarly, $q(p(q)) = q$, so the functions are inverses.

14. From $p = \frac{q}{48}$, we get $q = 48p$. Since $q > 0$, p is also greater than 0, so q as a function of p is $q = q(p) = 48p$, $p > 0$.

$q(p(q)) = q\left(\frac{q}{48}\right) = 48 \cdot \frac{q}{48} = q$

$p(q(p)) = p(48p) = \frac{48p}{48} = p$

Thus, $p(q)$ and $q(p)$ are inverses.

15. We show that $f(x) = 10^x$ is one-to-one. If $a \neq b$, we may as well assume that $b > a$, and this means, precisely, that for some $e > 0$ we have $a + e = b$. Now for such e, $10^e > 1$ and we have $10^b = 10^{a+e} = 10^a \cdot 10^e > 10^a \cdot 1 = 10^a$. So $10^a \neq 10^b$ and f is one-to-one. It follows that f has an inverse. (In fact $f^{-1}(x)$ is known as $\log(x)$ and will be studied in detail in Chapter 4.)

Apply It 2.5

11. Let y = the amount of money in the account. Then, after one month, $y = 7250 - (1 \cdot 600) = \$6650$, and after two months $y = 7250 - (2 \cdot 600) = \$6050$. Thus, in general, if we let x = the number of months during which Rachel spends from this account, $y = 7250 - 600x$. To identify the x-intercept, we set $y = 0$ and solve for x.

$y = 7250 - 600x$

$0 = 7250 - 600x$

$600x = 7250$

$x = 12\frac{1}{12}$

The x-intercept is $\left(12\frac{1}{12}, 0\right)$.

Therefore, after 12 months and approximately 2.5 days Rachel will deplete her savings. To identify the y-intercept, we set $x = 0$ and solve for y.

$y = 7250 - 600x$

$y = 7250 - 600(0)$

$y = 7250$

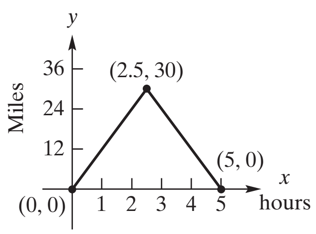
The y-intercept is (0, 7250).

Therefore, before any months have gone by, Rachel has \$7250 in her account.

12. Let y = the cost to the customer and let x = the number of rides he or she takes. Since the cost does not change, regardless of the number of rides taken, the equation $y = 24.95$ represents this situation. The graph of $y = 24.95$ is a horizontal line whose y-intercept is (0, 24.95). Since the line is parallel to the x-axis, there is no x-intercept.

13. The formula relating distance, time, and speed is $d = rt$, where d is the distance, r is the speed, and t is the time. Let x = the time spent biking (in hours). Then, $12x$ = the distance traveled. Brett bikes $12 \cdot 2.5 = 30$ miles and then turns around and bikes the same distance back to the rental shop. Therefore, we can represent the distance from the turn-around point at any time x as $|30 - 12x|$. Similarly, the distance from the rental shop at any time x can be represented by the function $y = 30 - |30 - 12x|$.

x	0	1	2	2.5	3	4	5
y	0	12	24	30	24	12	0



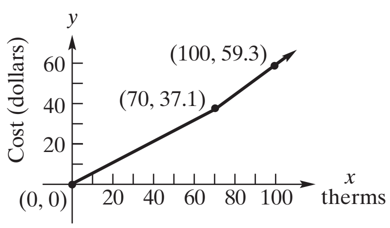
14. The monthly cost of x therms of gas is

$$y = \begin{cases} 0.53x, & \text{if } 0 \le x \le 70 \\ 0.53(70) + 0.74(x - 70), & \text{if } x > 70 \end{cases}$$

or

$$y = \begin{cases} 0.53x, & \text{if } 0 \le x \le 70 \\ 0.74x - 14.7, & \text{if } x > 70 \end{cases}$$

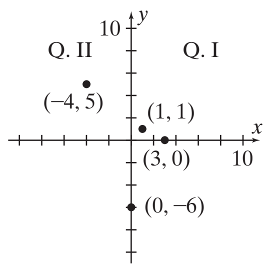
x	0	10	30	50	70	80	90	100
x	0	5.3	15.9	26.5	37.1	44.5	51.9	59.3



Problems 2.5

1. $(-1, -3)$ is in 3'rd quadrant; $(4, -2)$ is in 4'th quadrant; $\left(-\frac{2}{5}, 4\right)$ is in 2'nd quadrant; $(6, 0)$ is on the positive x-axis.

2.



3. a. $f(0) = 1, f(2) = 2, f(4) = 3, f(-2) = 0$

b. Domain: all real numbers

c. Range: all real numbers

d. $f(x) = 0$ for $x = -2$. So a real zero is -2.

4. a. $f(0) = 2, f(2) = 0$

b. Domain: all $x \ge 0$

c. Range: all $y \ge 2$

d. $f(x) = 0$ for $x = 2$. So a real zero is 2.

5. a. $f(0) = 0, f(1) = 1, f(-1) = 1$

b. Domain: all real numbers

c. Range: all nonnegative real numbers

d. $f(x) = 0$ for $x = 0$. So a real zero is 0.

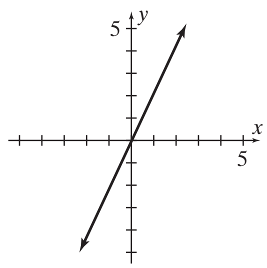
6. a. $f(0) = 0, f(2) = 1, f(3) = 3, f(4) = 2$

b. Domain: all x such that $0 \le x \le 4$

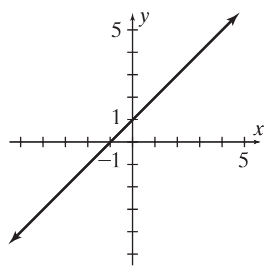
c. Range: all y such that $0 \le y \le 3$

d. $f(x) = 0$ for $x = 0$. So a real zero is 0.

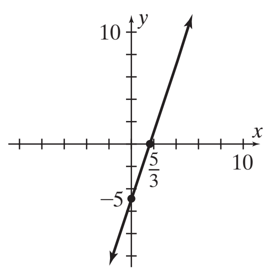
7. $y = 2x$
If $y = 0$, then $x = 0$. If $x = 0$, then $y = 0$.
Intercept: $(0, 0)$
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



8. $y = x + 1$
If $y = 0$, then $x = -1$.
If $x = 0$, then $y = 1$.
Intercepts: $(-1, 0)$, $(0, 1)$
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



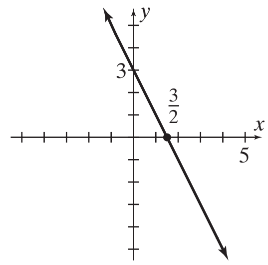
9. $y = 3x - 5$
If $y = 0$, then $0 = 3x - 5$, $x = \frac{5}{3}$.
If $x = 0$, then $y = -5$. Intercepts: $\left(\frac{5}{3}, 0\right)$, $(0, -5)$
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



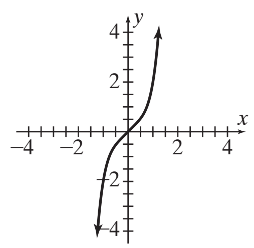
10. $y = 3 - 2x$

If $y = 0$, then $0 = 3 - 2x$, $x = \frac{3}{2}$.

If $x = 0$, then, $y = 3$. Intercepts: $\left(\frac{3}{2}, 0\right)$, $(0, 3)y$ is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



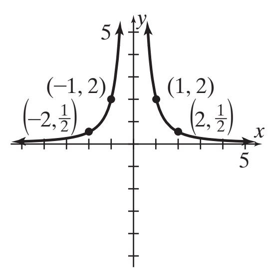
11. $(0, 0)$ is the only intercept of $y = x^5 + x$.



y is a function of x. It is a one-to-one function.
Both its domain and its range are $(-\infty, \infty)$.

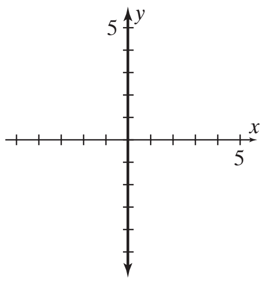
12. $y = \frac{2}{x^2}$

If $y = 0$, then $0 = \frac{2}{x^2}$, which has no solution.
Thus there is no x-intercept. Because $x \neq 0$, Not one-to-one.
Domain: all real numbers except 0
Range: all real numbers > 0

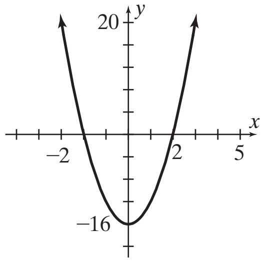


13. $x = 0$

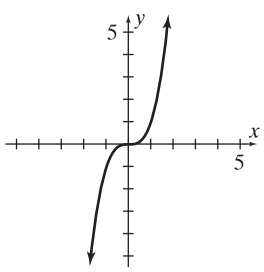
If $y = 0$, then $x = 0$. If $x = 0$, then y can be any real number. Intercepts: every point on y-axis
y is not a function of x.



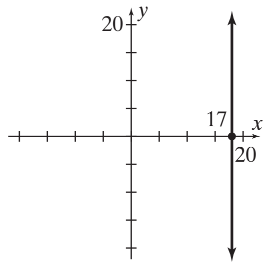
14. $y = 4x^2 - 16$
If $y = 0$, then $0 = 4x^2 - 16 = 4(x^2 - 4)$,
$0 = 4(x + 2)(x - 2)$, $x = \pm 2$.
If $x = 0$, then $y = -16$.
Intercepts: $(\pm 2, 0)$, $(0, -16)$
y is a function of x. Not one-to-one.
Domain: all real numbers
Range: all real numbers ≥ -16



15. $y = x^3$
If $y = 0$, then $0 = x^3$, $x = 0$. If $x = 0$, then $y = 0$.
Intercept: (0, 0). y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers

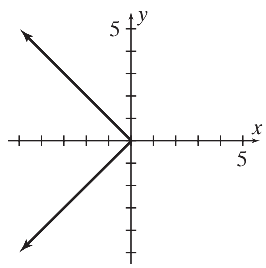


16. $(17, 0)$ is the only intercept of $x = 17$.

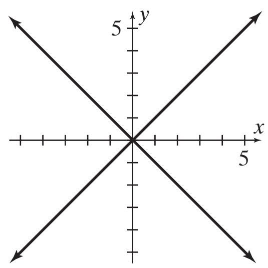


y is not a function of x.

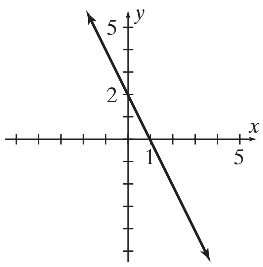
17. $x = -|y|$
If $y = 0$, then $x = 0$. If $x = 0$, then $0 = -|y|$, $y = 0$.
Intercept: (0, 0)
y is not a function of x.



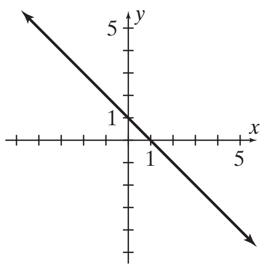
18. $x^2 = y^2$
If $y = 0$, then $x^2 = 0$, $x = 0$. If $x = 0$, then $0 = y^2$, $y = 0$. Intercept: $(0, 0)y$ is not a function of x.



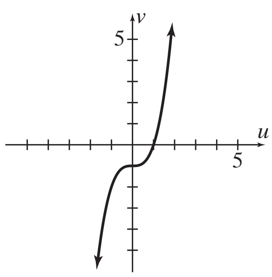
19. $2x + y - 2 = 0$
If $y = 0$, then $2x - 2 = 0$, $x = 1$. If $x = 0$, then $y - 2 = 0$, $y = 2$. Intercepts: (1, 0), (0, 2)
Note that $y = 2 - 2x$. y is a function of x.
One-to-one.
Domain: all real numbers
Range: all real numbers



20. $x + y = 1$
If $y = 0$, then $x = 1$. If $x = 0$, then $y = 1$.
Intercepts: (1, 0), (0, 1)
Note that $y = 1 - x$.
y is a function of x. One-to-one.
Domain: all real numbers
Range: all real numbers



21.



has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$.
$(0, -1)$ is the only intercept.

22. $f(x) = 5 - 2x^2$. If $f(x) = 0$, then $0 = 5 - 2x^2$

$2x^2 = 5$

$x^2 = \frac{5}{2}$

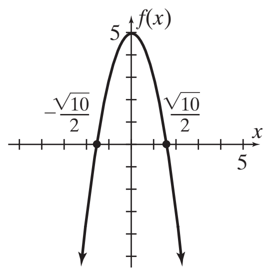
$x = \pm\sqrt{\frac{5}{2}} = \pm\frac{\sqrt{10}}{2}$.

If $x = 0$, then $f(x) = 5$.

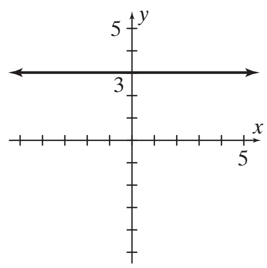
Intercepts: $\left(\pm\frac{\sqrt{10}}{2}, 0\right)$, (0, 5)

Domain: all real numbers

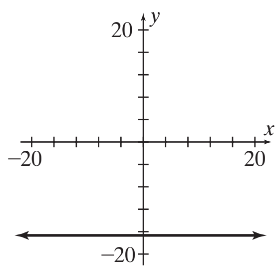
Range: all real numbers ≤ 5



23. $y = h(x) = 3$
Because y cannot be 0, there is no x-intercept. If $x = 0$, then $y = 3$. Intercept: (0, 3)
Domain: all real numbers
Range: 3

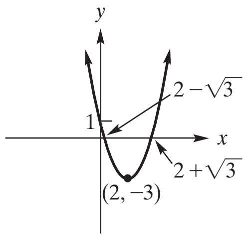


24. $g(s) = -17$
Because $g(s)$ cannot be 0, there is no s-intercept.
If $s = 0$, then $g(s) = -17$.
Intercept: $(0, -17)$
Domain: all real numbers
Range: -17

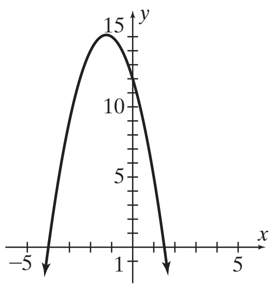


25. $y = h(x) = x^2 - 4x + 1$

If $y = 0$, then $0 = x^2 - 4x + 1$, and by the quadratic formula, $x = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$. If $x = 0$, then $y = 1$. Intercepts: $(2 \pm \sqrt{3},\ 0)$, (0,1)
Domain: all real numbers
Range: all real numbers ≥ -3

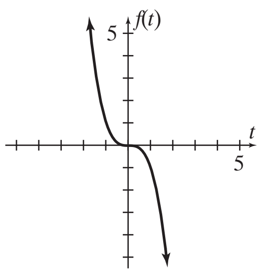


26.

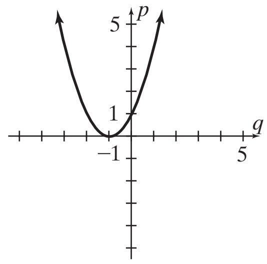


has domain $(-\infty, \infty)$ and range $(-\infty, 121/8]$.
Intercepts are (0, 12), (−4, 0), and (3/2, 0).

27. $f(t) = -t^3$
If $f(t) = 0$, then $0 = -t^3, t = 0$.
If $t = 0$, then $f(t) = 0$. Intercept: (0, 0)
Domain: all real numbers
Range: all real number

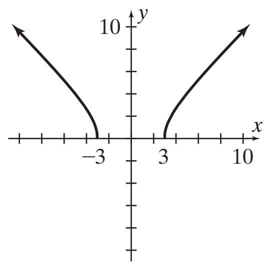


28. $p = h(q) = 1 + 2q + q^2$
If $p = 0$, then $1 + 2q + q^2 = 0, (1 + q)^2 = 0$, so $q = -1$. If $q = 0$ then $p = 1$.
Intercepts: (-1, 0), (0, 1)
Domain: all real numbers
Range: all real numbers ≥ 0



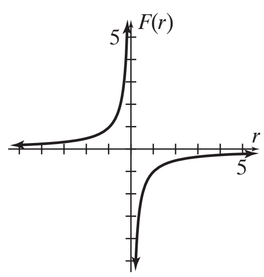
29. $s = f(t) = \sqrt{t^2 - 9}$

Note that for $\sqrt{t^2 - 9}$ to be a real number, $t^2 - 9 \geq 0$, so $t^2 \geq 9$, and $|t| \geq 3$. If $s = 0$, then $0 = \sqrt{t^2 - 9}, 0 = t^2 - 9$, or $t = \pm 3$. Because $|t| \geq 3$, we know $t \neq 0$, so no s-intercept exists.
Intercepts: $(-3, 0), (3, 0)$
Domain: all real numbers $t \leq -3$ and ≥ 3
Range: all real numbers ≥ 0

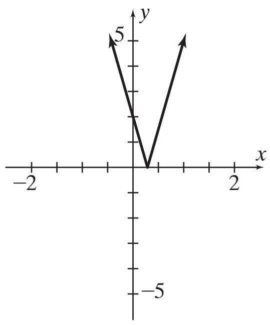


30. $F(r) = -\frac{1}{r}$

If $F(r) = 0$, then $0 = -\frac{1}{r}$, which has no solution.
Because $r \neq 0$, there is no vertical-axis intercept.
Intercept: none.
Domain: all real numbers $\neq 0$
Range: all real numbers $\neq 0$

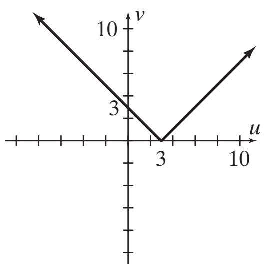


31.

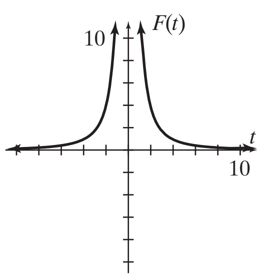


has domain $(-\infty, \infty)$ and range $[0, \infty)$.
Intercepts are $(0, 2)$ and $(2/7, 0)$.

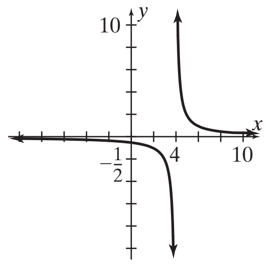
32. $v = H(u) = |u - 3|$
If $v = 0$, then $0 = |u - 3|$, $u - 3 = 0$, so $u = 3$.
If $u = 0$, then $v = |-3| = 3$.
Intercepts: $(3, 0)$, $(0, 3)$.
Domain: all real numbers
Range: all real numbers ≥ 0



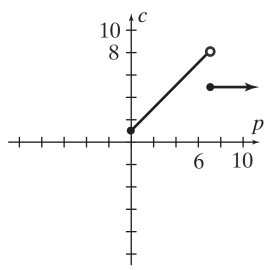
33. $F(t) = \dfrac{16}{t^2}$
If $F(t) = 0$, then $0 = \dfrac{16}{t^2}$, which has no solution.
Because $t \neq 0$, there is no vertical-axis intercept.
No intercepts
Domain: all nonzero real numbers
Range: all positive real numbers



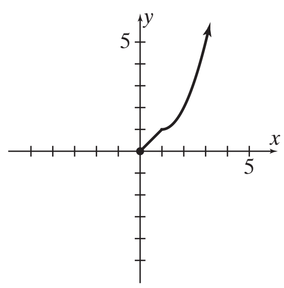
34. $y = f(x) = \dfrac{2}{x - 4}$
Note that the denominator is 0 when $x = 4$.
Thus $x \neq 4$. If $y = 0$, then $0 = \dfrac{2}{x - 4}$, which has no solution. If $x = 0$, then $y = -\dfrac{1}{2}$.
Intercept: $\left(0, -\dfrac{1}{2}\right)$
Domain: all real numbers except 4
Range: all real numbers except 0



35. Domain: all real numbers ≥ 0
Range: all real numbers $1 \leq c < 8$

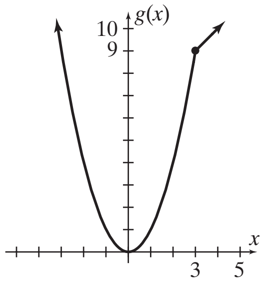


36.

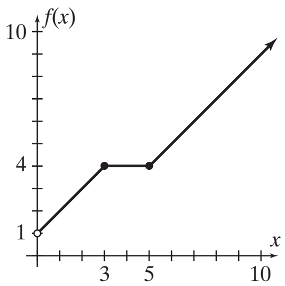


Both domain and range are $[0, \infty)$.

37. Domain: all real numbers
Range: all real numbers ≥ 0



38. Domain: all positive real numbers
Range: all real numbers > 1



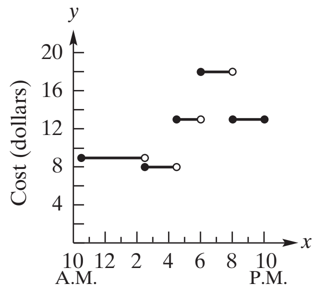
39. From the vertical-line test, the graphs that represent functions of x are (a), (b), and (d).

40. From the horizontal line test, the graphs which represent one-to-one functions of x are (c) and (d).

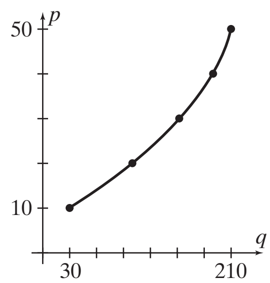
41. Write $D = D(n)$ for her debt after n payments. From the given data, $D = D(n) = 8700 - 300n$. The intercepts are $(0, 8700)$ and $(29, 0)$. The first is her initial debt load; the second is the number of months it takes her to become free of debt.

42. The cost of an item as a function of the time of day, x is

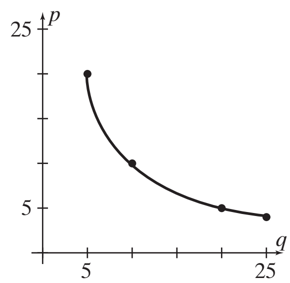
$$y = \begin{cases} 9, & \text{if } 10:30 \text{ A.M.} \leq x < 2:30 \text{ P.M.} \\ 8, & \text{if } 2:30 \text{ P.M.} \leq x < 4:30 \text{ P.M.} \\ 13, & \text{if } 4:30 \text{ P.M.} \leq x < 6:00 \text{ P.M.} \\ 18, & \text{if } 6:00 \text{ P.M.} \leq x < 8:00 \text{ P.M.} \\ 13, & \text{if } 8:00 \text{ P.M.} \leq x < 10:00 \text{ P.M.} \end{cases}$$



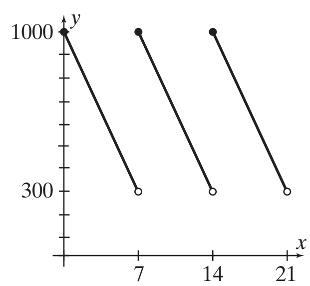
43. As price increases, quantity supplied increases; p is a function of q.



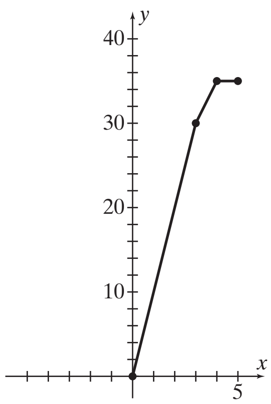
44. As price decreases, quantity increases; p is a function of q.



45.



46.



No, she needs to keep training. She can run for 3 hours at a rate of 10 km/hr and can then run for another hour at a rate of 5 km/hr. She stops after a total of 4 hours, having covered 35 km which is less than the distance of a full marathon.

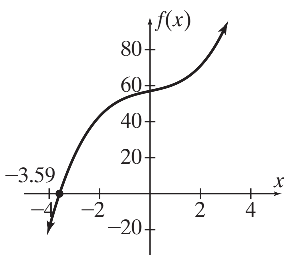
47. 0.39

48. −0.50, 0.57

49. −0.61, −0.04

50. 0.62, 1.73, 4.65

51.

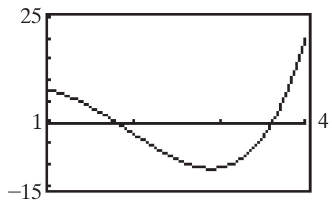


52. No real zeros

53. −1.70, 0

54. −0.49, 0.52, 1.25

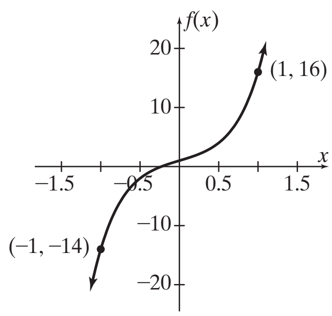
55.



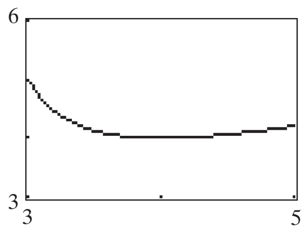
a. maximum value of $f(x)$: 19.60

b. minimum value of $f(x)$: −10.86

56.



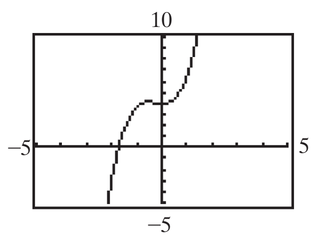
57.



a. maximum value of $f(x)$: 5

b. minimum value of $f(x)$: 4

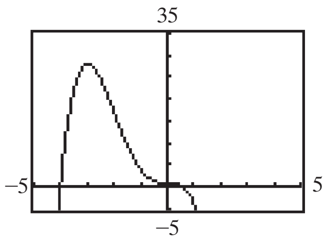
58.



a. range: $(-\infty, \infty)$

b. intercepts: (−1.73, 0), (0, 4)

59.

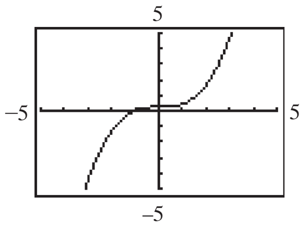


a. maximum value of $f(x)$: 28

b. range: $(-\infty, 28]$

c. real zeros: -4.02, 0.60

60.

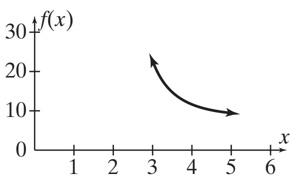


a. range: $(-\infty, \infty)$

b. intercepts: $(0, 0.29)$, $(-1.03, 0)$

c. real zero: -1.03

61.



Problems 2.6

1. $y = 5x$

Intercepts: If $y = 0$, then $5x = 0$, or $x = 0$; if $x = 0$, then $y = 5 \cdot 0 = 0$.
Testing for symmetry gives:

x-axis: $-y = 5x$
$y = -5x$

y-axis: $y = 5(-x) = -5x$

origin: $-y = 5(-x)$
$y = 5x$

line $y = x$: (a, b) on graph, then $b = 5a$, and $a = \frac{1}{5}b \neq 5b$ for all b, so (b, a) is not on the graph.

Answer: $(0, 0)$; symmetry about origin

2. The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, -9)$. There is symmetry about the y-axis.

3. $2x^2 + y^2x^4 = 8 - y$

Intercepts: If $y = 0$, then

$2x^2 = 8$, $x^2 = 4$, or $x = \pm 2$;

if $x = 0$, then $0 = 8 - y$, so $y = 8$.

Testing for symmetry gives:

x-axis: $2x^2 + (-y)^2x^4 = 8 - (-y)$
$2x^2 + y^2x^4 = 8 + y$

y-axis: $2(-x)^2 + y^2(-x)^4 = 8 - y$
$2x^2 + y^2x^4 = 8 - y$

origin: $2(-x)^2 + (-y)^2(-x)^4 = 8 - (-y)$
$2x^2 + y^2x^4 = 8 + y$

line $y = x$: (a, b) on graph, then
$2a^2 + b^2a^4 = 8 - b$, but
$2b^2 + a^2b^4 = 8 - a$ will not
necessarily be true, so (b, a) is not on the graph.

Answer: $(\pm 2, 0)$, $(0, 8)$; symmetry about y-axis

4. $x = y^3$

Intercepts: If $y = 0$, then $x = 0$; if $x = 0$, then $0 = y^3$, so $y = 0$.

Testing for symmetry gives:

x-axis: $x = (-y)^3 = -y^3$

y-axis: $-x = y^3$
$x = -y^3$

origin: $-x = (-y)^3$
$x = y^3$

line $y = x$: (a, b) on graph, then $a = b^3$, and $b = \sqrt[3]{a} \neq a^3$ for all a, so (b, a) is not on the graph.

Answer: $(0, 0)$; symmetry about origin

5. $25x^2 + 144y^2 = 169$

Intercepts: If $y = 0$, then $25x^2 = 169$, $x^2 = \frac{169}{25}$, so $x = \pm\frac{13}{5}$;

If $x = 0$, then $144y^2 = 169$ $y^2 = \frac{169}{144}$, and $y = \pm\frac{13}{12}$.

Testing for symmetry gives:

x-axis: $25x^2 + 144(-y)^2 = 169$
$25x^2 + 144y^2 = 169$

y-axis: $25(-x)^2 + 144y^2 = 169$
$25x^2 + 144y^2 = 169$

origin: Since the graph has symmetry about x- and y-axes, there is also symmetry about the origin.

line $y = x$: (a, b) on graph, then $25a^2 + 144b^2 = 169$, and $a^2 = \frac{1}{25}(169 - 144b^2)$.$(b, a)$ on graph, then $25b^2+144a^2=169$ and

$$a^2 = \frac{1}{144}(169 - 25b^2) \neq \frac{1}{25}(169 - 144b^2)$$

for all b, so (b, a) and (a, b) are not always both on the graph. Not symmetric about $y = x$.

Answer: $\left(\pm\frac{13}{5}, 0\right)$, $\left(0, \pm\frac{13}{12}\right)$ symmetry about x-axis, y-axis, and origin.

6. $y = 57$

Intercepts: Because $y \neq 0$, there is no x-intercept; if $x = 0$, then $y = 57$.

Testing for symmetry gives:

x-axis: $(-y) = 57$
$y = -57$

y-axis: $y = 57$

origin: $(-y) = 57$
$y = -57$

line $y = x$: (a, b) on graph, then $b = 57$, but a can be any value, so $(b, a) = (57, a)$ is not necessarily on the graph.

Answer: (0, 57); symmetry about y-axis

7. The only intercept is $(-7, 0)$. The graph is symmetric about the x-axis.

8. $y = |2x| - 2$

Intercepts: If $y = 0$, then $|2x| = 2$, $2|x| = 2$, $|x| = 1$,so $x \pm 1$; if $x = 0$, then $y = -2$.

Testing for symmetry gives:

x-axis: $-y = |2x| - 2$
$y = -|2x| + 2$

y-axis: $y = |2(-x)| - 2$
$y = |2x| - 2$

origin: $-y = |2(-x)| - 2$
$y = -|2x| + 2$

line $y = x$: (a, b) on graph, then $b = |2a| - 2$ and $a = \pm\frac{b+2}{2} \neq |2b| - 2$ for all b, so (b, a) is not on the graph.

Answer: $(\pm 1, 0)$, $(0, -2)$; symmetry about y-axis

9. $x = -y^{-4}$

Intercepts: Because $y \neq 0$, there is no x-intercept; if $x = 0$, then $0 = -\frac{1}{y^4}$, which has no solution.

Testing for symmetry gives:

x-axis: $x = -(-y)^{-4}$
$x = -y^{-4}$

y-axis: $-x = -y^{-4}$
$x = y^{-4}$

origin: $-x = -(-y)^{-4}$
$x = y^{-4}$

line $y = x$: (a, b) on graph, then $a = -b^{-4}$ and $b = (-a)^{-1/4} \neq -a^{-4}$ for all a, so (b, a) is not on the graph.

Answer: no intercepts; symmetry about x-axis

10. $y = \sqrt{x^2 - 36}$

Intercepts: If $y = 0$, then $\sqrt{x^2 - 36} = 0$,
$x^2 - 36 = 0, x^2 = 36$, so $x = \pm 6$;
if $x = 0$, then $y = \sqrt{-36}$, which has no real root.
Testing for symmetry gives:

x-axis: $-y = \sqrt{x^2 - 36}$
$y = -\sqrt{x^2 - 36}$

y-axis: $y = \sqrt{(-x)^2 - 36}$
$y = \sqrt{x^2 - 36}$

origin: $-y = \sqrt{(-x)^2 - 36}$
$y = -\sqrt{x^2 - 36}$

line $y = x$: (a, b) on graph, then $b = \sqrt{a^2 - 36}$
or $b^2 = a^2 - 36$ and
$a^2 = b^2 + 36 \neq b^2 - 36$ for all b,
so (b, a) is not on the graph.

Answer: $(\pm 6, 0)$; symmetry about y-axis

11. $x - 4y - y^2 + 21 = 0$

Intercepts: If $y = 0$, then $x + 21 = 0$, so $x = -21$;
if $x = 0$, then $-4y - y^2 + 21 = 0$,
$y^2 + 4y - 21 = 0, (y + 7)(y - 3) = 0$, so $y = -7$ or $y = 3$.

Testing for symmetry gives:

x-axis: $x - 4(-y) - (-y)^2 + 21 = 0$
$x + 4y - y^2 + 21 = 0$

y-axis: $(-x) - 4y - y^2 + 21 = 0$
$-x - 4y - y^2 + 21 = 0$

origin: $(-x) - 4(-y) - (-y)^2 + 21 = 0$
$-x + 4y - y^2 + 21 = 0$

line $y = x$: (a, b) on graph, then
$a - 4b - b^2 + 21 = 0$ and
$a = b^2 + 4b - 21$, but
$b = a^2 + 4a - 21$ will not necessarily be true, so (b, a) is not on the graph.

Answer: $(-21, 0), (0, -7), (0, 3)$; no symmetry

12. The only intercept is $(0, 0)$. The graph is symmetric about $y = x$.

13. $y = f(x) = \dfrac{x^3 - 2x^2 + x}{x^2 + 1}$

Intercepts: If $y = 0$, then
$\dfrac{x^3 - 2x^2 + x}{x^2 + 1} = \dfrac{x(x - 1)^2}{x^2 + 1} = 0$, so $x = 0, 1$;
if $x = 0$, then $y = 0$.

Testing for symmetry gives:

x-axis: Because f is not the zero function, there is no x-axis symmetry

y-axis: $y = \dfrac{(-x)^3 - 2(-x)^2 + (-x)}{(-x)^2 + 1}$
$y = \dfrac{-x^3 - 2x^2 - x}{x^2 + 1}$

origin: $-y = \dfrac{(-x)^3 - 2(-x)^2 + (-x)}{(-x)^2 + 1}$
$y = \dfrac{x^3 + 2x^2 + x}{x^2 + 1}$

line $y = x$: (a, b) on graph, then
$b = \dfrac{a^3 - 2a^2 + a}{a^2 + 1}$, but
$a = \dfrac{b^3 - 2b^2 + b}{b^2 + 1}$ is not necessarily true, so (b, a) is not on the graph.

Answer: $(1, 0), (0, 0)$; no symmetry of the given types

14. $x^2 + xy + y^2 = 0$

Intercepts: If $y = 0$, then $x^2 = 0$, so $x = 0$;
if $x = 0$, then $y^2 = 0$, so $y = 0$.

Testing for symmetry gives:

x-axis: $x^2 + x(-y) + (-y)^2 = 0$
$x^2 - xy + y^2 = 0$

y-axis: $(-x)^2 + (-x)y + y^2 = 0$
$x^2 - xy + y^2 = 0$

origin: $(-x)^2 + (-x)(-y) + (-y)^2 = 0$
$x^2 + xy + y^2 = 0$

line $y = x$: (a, b) on graph, then $a^2 + ab + b^2 = 0$ and $b^2 + ba + a^2 = 0$, so (b, a) is on the graph.

Answer: $(0, 0)$; symmetry about origin, symmetry about $y = x$

15. $y = \dfrac{2}{x^3 + 27}$

Intercepts: If $y = 0$, then $\dfrac{2}{x^3 + 27} = 0$, which has no solution; if $x = 0$, then $y = \dfrac{2}{27}$.

Testing for symmetry gives:

x-axis: $-y = \dfrac{2}{x^3 + 27}$

$y = -\dfrac{2}{x^3 + 27}$

y-axis: $y = \dfrac{2}{(-x)^3 + 27}$

$y = \dfrac{2}{-x^3 + 27}$

origin: $-y = \dfrac{2}{(-x)^3 + 27}$

$-y = \dfrac{2}{-x^3 + 27}$

$y = \dfrac{2}{x^3 - 27}$

line $y = x$: (a, b) on graph, then $b = \dfrac{2}{a^3 + 27}$ and

$a = \sqrt[3]{\dfrac{2}{b} - 27} \neq \dfrac{2}{b^3 + 27}$ for all b, so (b, a) is not on the graph.

Answer: $\left(0, \dfrac{2}{27}\right)$; no symmetry

16. $y = \dfrac{x^4}{x + y}$

Intercepts: If $y = 0$, then $\dfrac{x^4}{x} = 0$, which has no solution; if $x = 0$, then $y = \dfrac{0}{y}$, which has no solution.

Testing for symmetry gives:

x-axis: $-y = \dfrac{x^4}{x + (-y)}$

$y = \dfrac{x^4}{-x + y}$

y-axis: $y = \dfrac{(-x)^4}{(-x) + y}$

$y = \dfrac{x^4}{-x + y}$

origin: $-y = \dfrac{(-x)^4}{(-x) + (-y)}$

$y = \dfrac{x^4}{x + y}$

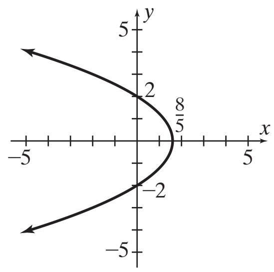
line $y = x$: (a, b) on graph, then $b = \dfrac{a^4}{a + b}$,

and $a + b = \dfrac{a^4}{b}$, but $a + b = \dfrac{b^4}{a}$

will not necessarily be true, so (b, a) is not on the graph.

Answer: no intercepts; symmetry about origin

17. The intercepts are $(0, -2)$, $(0, 2)$, and $(8/5, 0)$. The graph is symmetric about the x-axis.



18. $x - 1 = y^4 + y^2$ or $x = y^4 + y^2 + 1$

Intercepts: If $y = 0$, then $x = 1$; if $x = 0$, then $y^4 + y^2 = -1$, so no y-intercept

Testing for symmetry gives:

x-axis: $x - 1 = (-y)^4 + (-y)^2$

$x - 1 = y^4 + y^2$

y-axis: $-x = y^4 + y^2 + 1$

$x = -y^4 - y^2 - 1$

origin: $-x = (-y)^4 + (-y)^2 + 1$

$x = -y^4 - y^2 - 1$

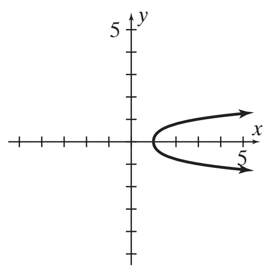
line $y = x$: (a, b) on graph, then

$a = b^4 + b^2 + 1$ and

$b \neq a^4 + a^2 + 1$

for all a so (b, a) is not on the graph.

Answer: $(1, 0)$; symmetry about x-axis.



19. $y = f(x) = x^3 - 4x$

Intercepts: If $y = 0$, then $x^3 - 4x = 0$,

$x(x+2)(x-2) = 0$, so $x = 0$ or $x = \pm 2$; if $x = 0$, then $y = 0$.

Testing for symmetry gives:

x-axis: Because f is not the zero function, there is no x-axis symmetry.

y-axis: $y = (-x)^3 - 4(-x)$

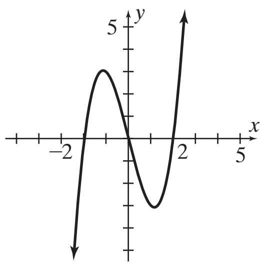
$y = -x^3 + 4x$

origin: $-y = (-x)^3 - 4(-x)$

$y = x^3 - 4x$

line $y = x$: (a, b) on graph, then $b = a^3 - 4a$, but $a = b^3 - 4b$ will not necessarily be true, so (b, a) is not on the graph.

Answer: (0, 0), (±2, 0); symmetry about origin.



20. $2y = 5 - x^2$

Intercepts: If $y = 0$, then $5 - x^2 = 0$, so $x = \pm\sqrt{5}$. If $x = 0$, $y = \frac{5}{2}$.

Testing for symmetry gives:

x-axis: $2(-y) = 5 - x^2$

$2y = -5 + x^2$

y-axis: $2y = 5 - (-x)^2$

$2y = 5 - x^2$

origin: $2(-y) = 5 - (-x)^2$

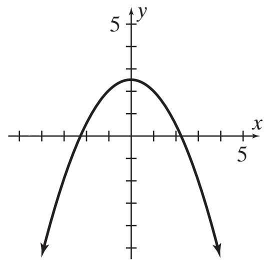
$2y = -5 + x^2$

line $y = x$: (a, b) on graph, then $2b = 5 - a^2$.

(b, a) on graph, then $2a = 5 - b^2$.

(a, b) and (b, a) are not both on the graph.

Answer: $\left(\pm\sqrt{5}, 0\right)$, $\left(0, \frac{5}{2}\right)$; symmetry about y-axis



21. $|x| - |y| = 0$

Intercepts: If $y = 0$, then $|x| = 0$ so $x = 0$; if $x = 0$, then $-|y| = 0$, so $y = 0$.

Testing for symmetry gives:

x-axis: $|x| - |-y| = 0$

$|x| - |y| = 0$

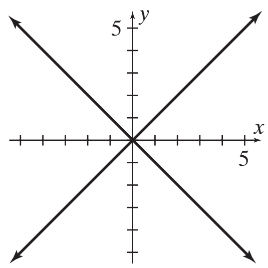
y-axis: $|-x| - |y| = 0$

$|x| - |y| = 0$

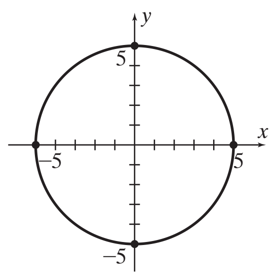
origin: Since there is symmetry about the x- and y-axes, symmetry about origin exists.

line $y = x$: (a, b) on graph, then $|a| - |b| = 0$, thus $|a| = |b|$, and $|b| - |a| = 0$, so (b, a) is on the graph.

Answer: (0, 0); symmetry about x-axis, y-axis, origin, line $y = x$.



22. The intercepts are $(0, -5)$, $(0, 5)$, $(-5, 0)$, and $(5, 0)$. The graph is symmetric with respect to both the x- and y-axes and hence with respect to the origin. It is also symmetric with respect to the line $y = x$.



23. $9x^2 + 4y^2 = 25$

Intercepts: If $y = 0$, then $9x^2 = 25$, $x^2 = \frac{25}{9}$, so $x = \pm\frac{5}{3}$; if $x = 0$, then $4y^2 = 25$, so $y = \pm\frac{5}{2}$.

Testing for symmetry gives:

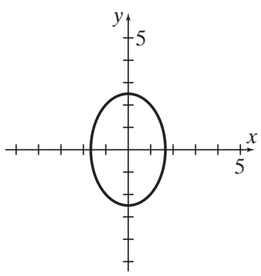
x-axis: $9x^2 + 4(-y)^2 = 25$
$9x^2 + 4y^2 = 25$

y-axis: $9(-x)^2 + 4y^2 = 25$
$9x^2 + 4y^2 = 25$

origin: Since there is symmetry about x- and y-axes, symmetry about origin exists.

line $y = x$: (a, b) on graph, then
$9a^2 + 4b^2 = 25$
$b^2 = \frac{1}{4}(25 - 9a^2)$. (b, a) on graph,
then $9b^2 + 4a^2 = 25$ and
$b^2 = \frac{1}{9}(25 - 4a^2)$, so (a, b) and (b, a) are not always both on the graph.

Answer: $\left(\pm\frac{5}{3}, 0\right)$, $\left(0, \pm\frac{5}{2}\right)$; symmetry about x-axis, y-axis, origin



24. $x^2 - y^2 = 4$

Intercepts: If $y = 0$, then $x^2 = 4$, so $x = \pm 2$; if $x = 0$, then $-y^2 = 4$, $y^2 = -4$, which has no real roots.

Testing for symmetry gives:

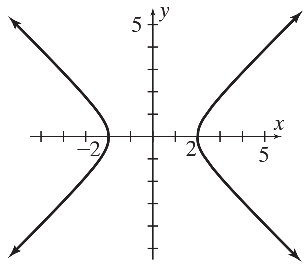
x-axis: $x^2 - (-y)^2 = 4$
$x^2 - y^2 = 4$

y-axis: $(-x)^2 - y^2 = 4$
$x^2 - y^2 = 4$

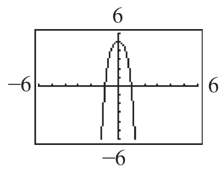
origin: Since there is symmetry about x-and y-axes, symmetry about origin exists.

line $y = x$: (a, b) on graph, then $a^2 - b^2 = 4$ and $a^2 = 4 + b^2 \neq b^2 - 4$ for all b, so (b, a) is not on the graph.

Answer: $(\pm 2, 0)$; symmetry about x-axis, y-axis, origin.



25.



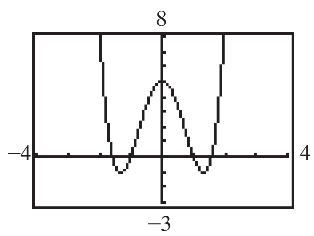
$y = f(x) = 5 - 1.96x^2 - \pi x^4$. Replacing x by $-x$ gives $y = 5 - 1.96(-x)^2 - \pi(-x)^4$ or $y = 5 - 1.96x^2 - \pi x^4$, which is equivalent to original equation. Thus the graph is symmetric about the y-axis.

a. Intercepts: $(\pm 0.99, 0)$, $(0, 5)$

b. Maximum value of $f(x)$: 5

c. Range: $(-\infty, 5]$

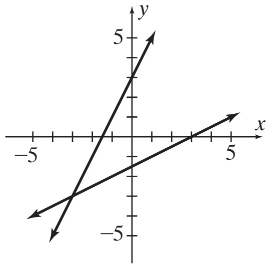
26.



$y = f(x) = 2x^4 - 7x^2 + 5$. Replacing x by $-x$ gives $y = 2(-x)^4 - 7(-x)^2 + 5$ or $y = 2x^4 - 7x^2 + 5$, which is equivalent to original equation. Thus the graph is symmetric about y-axis.

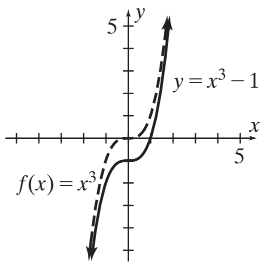
Real zeros of f: $\pm 1, \pm 1.58$

27.

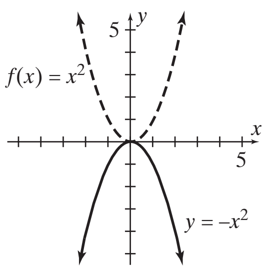


Problems 2.7

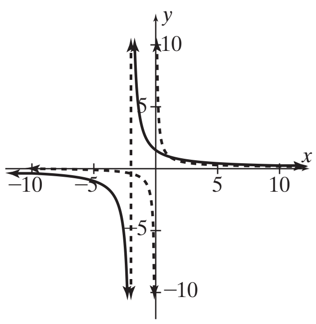
1.



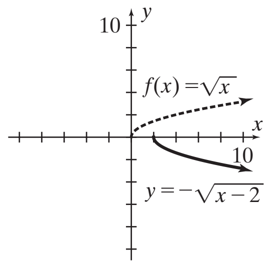
2.



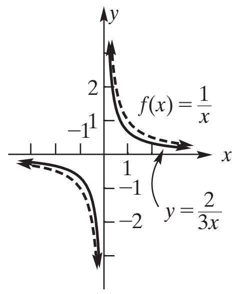
3. The required graph is obtained by translating the graph of $y = 1/x$ 2 units to the left and the streching the resulting graph vertically away from the x-axis by a factor of 3.



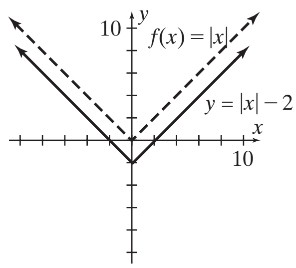
4.



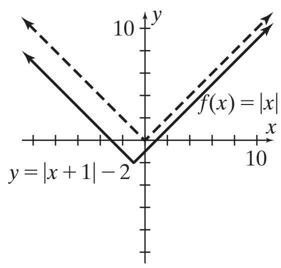
5.



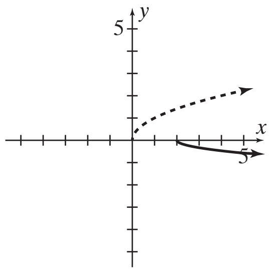
6.



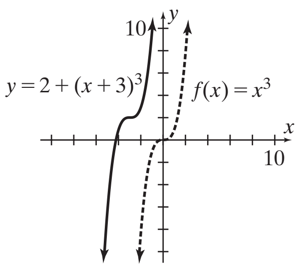
7.



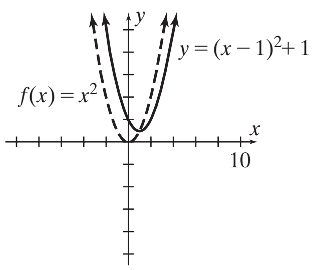
8. Translate the graph of $y = \sqrt{x}$ 2 units to the right; shrink the resulting graph vertically towards the x-axis by a factor of $1/3$; and reflect the result about the x-axis.



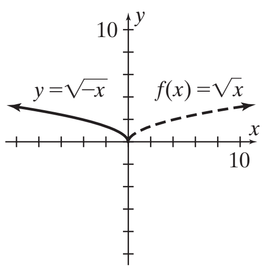
9.



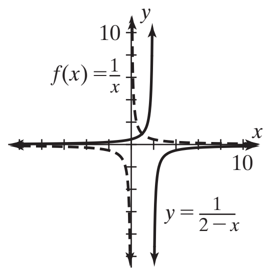
10.



11.



12.



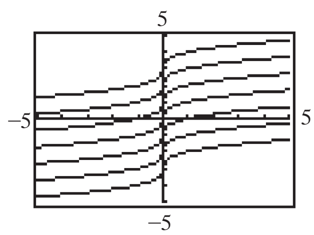
13. Translate the graph of $y = f(x)$5 units to the right and 1 unit up; shrink the result by a factor of 1/2 vertically towards the x-axis; and then reflect about the x-axis.

14. Shift one unit left, four units down, and stretch by a factor of 2 away from the x-axis.

15. Reflect about the y-axis and translate 5 units downward.

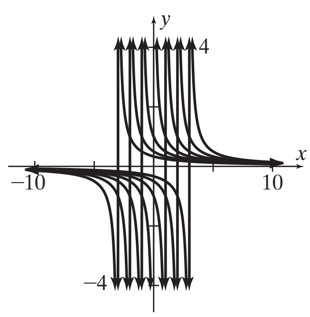
16. Shrink horizontally toward the y-axis by a factor of 3.

17.

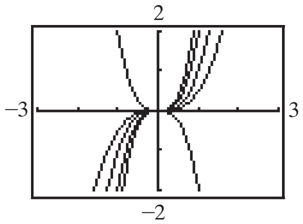


Compared to the graph for $k = 0$, the graphs for $k = 1, 2$, and 3 are vertical shifts upward of 1, 2, and 3 units, respectively. The graphs for $k = -1, -2$, and -3 are vertical shifts downward of 1, 2, and 3 units, respectively.

18.



19.



Compared to the graph for $k = 1$, the graphs for $k = 2$ and 3 are vertical stretches away from the x-axis by factors of 2 and 3, respectively. The graph for $k = \frac{1}{2}$ is a vertical shrinking toward the x-axis.

Apply It 2.8

15. a. $c(500, 700) = 160 + 2(500) + 3(700) = 160 + 1000 + 2100 = 3260$

The cost of manufacturing 500 12-ounce and 700 20-ounce mugs is \$3260.

b. $c(1000, 750) = 160 + 2(1000) + 3(750) = 160 + 2000 + 2250 = 4410$

The cost of manufacturing 1000 12-ounce mugs and 750 20-ounce mugs is \$4410.

Problems 2.8

1. $f(1, 2) = 4(1) - (2)^2 + 3 = 4 - 4 + 3 = 3$

2. $f(2, \ -1) = 3(2)^2(-1) - 4(-1) = -12 + 4 = -8$

3. $g(3, 0, -1) = 2(3)[3(0) + (-1)] = -6$

4. $g(1, b) = 1 + b + b^3 = 0$

5. $h(-3, 3, 5, 4) = \dfrac{-3(3)}{5^2 - 4^2} = \dfrac{-9}{25 - 16}$
$= \dfrac{-9}{9} = -1$

6. $h(1, 5, 3, 1) = (1)(1) = 1$

7. $g(4, 8) = 2(4)(4^2 - 5) = 2(4)(11) = 88$

8. $g(4, 9) = (4^2)\sqrt{9} + 9 = 16 \cdot 3 + 9 = 57$

9. $F(6, 0, -5) = 17$

10. $F(1, \ 0, \ 3) = \dfrac{2(1)}{(0 + 1)(3)} = \dfrac{2}{3}$

11. $f(a + h, \ b) = [(a + h) + b]^2$
$= a^2 + 2ab + 2ah + b^2 + 2bh + h^2$

12. $f(r + t, \ r) = (r + t)^2 r - 3r^3 = r\left(t^2 + 2rt - 2r^2\right)$

13. $f(200, \ 200, \ 50) = \dfrac{(200)(200)}{50} = 800$

14. We must evaluate $P(2, 7) = \dfrac{7!}{2!5!}\left(\dfrac{1}{4}\right)^2\left(\dfrac{3}{4}\right)^5 = \dfrac{7 \cdot 6}{2}\left(\dfrac{3^5}{4^7}\right) = \dfrac{7 \cdot 3^6}{4^7} = \dfrac{7 \cdot 3^6}{2^{14}} \approx 0.31146240$

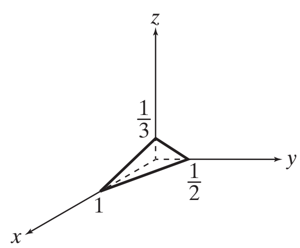
15. A plane parallel to the x, z-plane has the form $y =$ constant. Because (0, 2, 0) lies on the plane, the equation is $y = 2$.

16. A plane parallel to the y, z-plane has the form $x =$ constant. Because (−2, 0, 0) lies on the plane, the equation is $x = -2$.

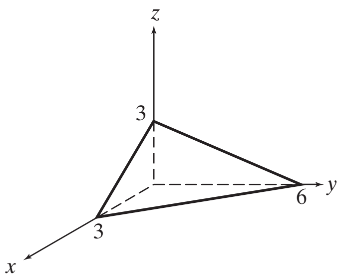
17. A plane parallel to the x, y-plane has the form $z =$ constant. Because (2, 7, 6) lies on the plane, the equation is $z = 6$.

18. A plane parallel to the y, z-plane has the form $x =$ constant. Because (96, −2, 2) lies on the plane, the equation is $x = 96$.

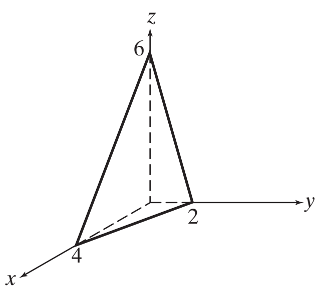
19.



20. $2x + y + 2z = 6$ can be put in the form $Ax + By + Cz + D = 0$, so the graph is a plane. The intercepts are (3, 0, 0), (0, 6, 0), and (0, 0, 3).

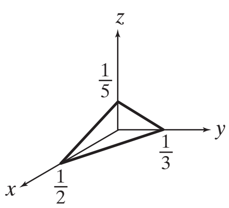


21. $3x + 6y + 2z = 12$ can be put in the form $Ax + By + Cz + D = 0$, so the graph is a plane. The intercepts are (4, 0, 0), (0, 2, 0), and (0, 0, 6).

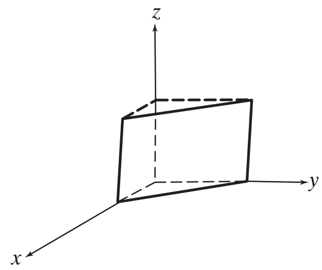


22. $2x + 3y + 5z = 1$ can be put in the form $Ax + By + Cz + D = 0$, so the graph is a plane.

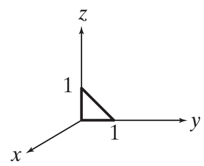
The intercepts are $\left(\frac{1}{2},\ 0,\ 0\right)$, $\left(0,\ \frac{1}{3},\ 0\right)$, and $\left(0,\ 0,\ \frac{1}{5}\right)$.



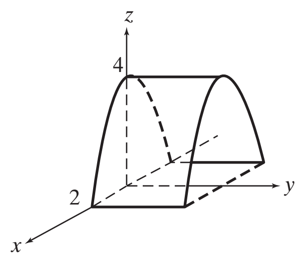
23. $3x + y = 6$ can be put in the form $Ax + By + Cz + D = 0$, so the graph is a plane. There are only two intercepts: $(2, 0, 0)$ and $(0, 6, 0)$. The x, y-trace is $3x + y = 6$, which is a line. For any fixed value of z, we obtain the line $3x + y = 6$.



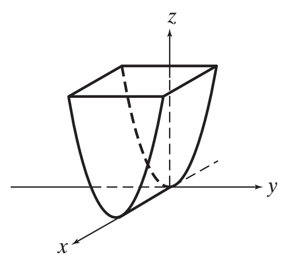
24.



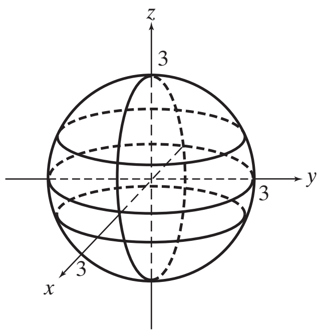
25. $z = 4 - x^2$. The x, z-trace is $z = 4 - x^2$, which is a parabola. For any fixed value of y, we obtain the parabola $z = 4 - x^2$.



26. $y = z^2$. The y, z-trace is $y = z^2$, which is a parabola. For any fixed value of x, we obtain the parabola $y = z^2$.

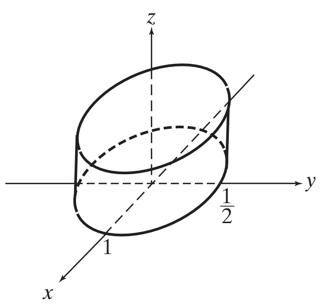


27. $x^2 + y^2 + z^2 = 9$. The x, y-trace is $x^2 + y^2 = 9$, which is a circle. The x, z-trace is $x^2 + z^2 = 9$, which is a circle. The y, z-trace is $y^2 + z^2 = 9$, which is a circle. The surface is a sphere.



28. $x^2 + 4y^2 = 1$

The x, y-trace is $x^2 + 4y^2 = 1$, which is an ellipse. For any fixed value of z, we obtain the ellipse $x^2 + 4y^2 = 1$.



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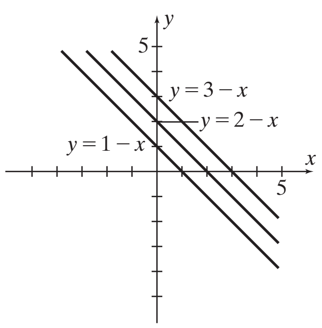
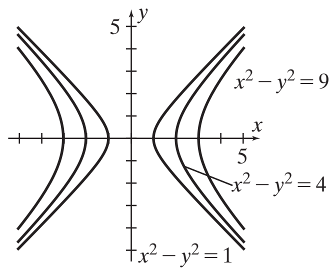


30. $z = x^2 - y^2$

Choose $z = 1, 4$, and 9 for the curves.



Chapter 2 Review Problems

1. Denominator is 0 when

$$x^2 - 6x + 5 = 0$$
$$(x-1)(x-5) = 0$$
$$x = 1, \quad 5$$

Domain: all real numbers except 1 and 5.

2. all real numbers

3. all real numbers

4. all real numbers

5. We require x in $[2, \infty)$ for the numerator to be defined and $x \neq 3$ for the denominator to be different from 0. The domain is $[2, \infty) - \{3\}$.

6. $s - 5 \geq 0$

$s \geq 5$

Domain: all real numbers s such that $s \geq 5$.

7. $f(x) = 2x^2 - 3x + 5$

$f(0) = 2(0)^2 - 3(0) + 5 = 5$

$f(-2) = 2(-2)^2 - 3(-2) + 5 = 8 + 6 + 5 = 19$

$f(5) = 2(5)^2 - 3(5) + 5 = 50 - 15 + 5 = 40$

$f(\pi) = 2\pi^2 - 3\pi + 5$

8. $h(x) = 7$; all function values are 7.

Answer: 7, 7, 7, 7

9. $G(x) = \sqrt[4]{x-3}$

$G(3) = \sqrt[4]{3-3} = \sqrt[4]{0} = 0$

$G(19) = \sqrt[4]{19-3} = \sqrt[4]{16} = 2$

$G(t+1) = \sqrt[4]{(t+1)-3} = \sqrt[4]{t-2}$

$G(x^3) = \sqrt[4]{x^3-3}$

10. $F(-1) = \dfrac{3(-1)+2}{(-1)-5} = \dfrac{-1}{-6} = 1/6,$

$F(0) = \dfrac{3(0)+2}{(0)-5} = \dfrac{2}{-5} = -2/5,$

$F(4) = \dfrac{3(4)+2}{(4)-5} = \dfrac{14}{-1} = -14,$

$F(x+2) = \dfrac{3(x+2)+2}{(x+2)-5} = \dfrac{3x+8}{x-3}$

11. $h(u) = \dfrac{\sqrt{u+4}}{u}$

$h(5) = \dfrac{\sqrt{5+4}}{5} = \dfrac{\sqrt{9}}{5} = \dfrac{3}{5}$

$h(-4) = \dfrac{\sqrt{-4+4}}{-4} = \dfrac{0}{-4} = 0$

$h(x) = \dfrac{\sqrt{x+4}}{x}$

$h(u-4) = \dfrac{\sqrt{(u-4)+4}}{u-4} = \dfrac{\sqrt{u}}{u-4}$

12. $H(t) = \dfrac{(t-2)^3}{5}$

$H(-1) = \dfrac{(-1-2)^3}{5} = -\dfrac{27}{5}$

$H(0) = \dfrac{(0-2)^3}{5} = -\dfrac{8}{5}$

$H\left(\dfrac{1}{3}\right) = \dfrac{\left(\frac{1}{3}-2\right)^3}{5} = \dfrac{\left(-\frac{5}{3}\right)^3}{5}$
$= \left(-\dfrac{125}{27}\right)\left(\dfrac{1}{5}\right) = -\dfrac{25}{27}$

$H(x^2) = \dfrac{(x^2-2)^3}{5}$

13. $f(4) = 4 + 16 = 20$

$f(-2) = -3$

$f(0) = -3$

$f(1)$ is not defined.

14. $f\left(-\dfrac{1}{2}\right) = -\left(-\dfrac{1}{2}\right) + 1 = \dfrac{1}{2} + 1 = \dfrac{3}{2}$

$f(0) = 0^2 + 1 = 1$

$f\left(\dfrac{1}{2}\right) = \left(\dfrac{1}{2}\right)^2 + 1 = \dfrac{1}{4} + 1 = \dfrac{5}{4}$

$f(5) = 5^3 - 99 = 125 - 99 = 26$

$f(6) = 6^3 - 99 = 216 - 99 = 117$

15. **a.** $f(x+h) = 1 - 3(x+h) = 1 - 3x - 3h$

b. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{(1-3x-3h)-(1-3x)}{h}$
$= \dfrac{-3h}{h} = -3 \quad \text{for } h \neq 0$

16. **a.** $f(x+h) = 11(x+h)^2 + 4$
$= 11x^2 + 22xh + 11h^2 + 4$

b. $\dfrac{f(x+h)-f(x)}{h}$
$= \dfrac{(11x^2 + 22xh + 11h^2 + 4) - (11x^2 + 4)}{h}$
$= \dfrac{22xh + 11h^2}{h} = 22x + 11h$

17. **a.** $f(x+h) = 3(x+h)^2 + (x+h) - 2$

b. $\dfrac{f(x+h)-f(x)}{h}$
$= \dfrac{3(x+h)^2 + (x+h) - 2 - (3x^2 + x - 2)}{h}$
$= \dfrac{3x^2 + 6xh + 3h^2 + x + h - 2 - 3x^2 - x + 2}{h}$
$= \dfrac{6xh + 3h^2 + h}{h}$
$= 6x + 3h + 1$

18. a. $f(x+h) = \dfrac{7}{(x+h)+1} = \dfrac{7}{x+h+1}$

b. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{7}{x+h+1} - \frac{7}{x+1}}{h}$

$= \dfrac{\frac{7(x+1)-7(x+h+1)}{(x+h+1)(x+1)}}{h}$

$= \dfrac{-7h}{(x+h+1)(x+1)h}$

$= \dfrac{-7}{(x+h+1)(x+1)}$

19. $f(x) = 3x - 1, g(x) = 2x + 3$

a. $(f+g)(x) = f(x) + g(x)$
$= (3x-1) + (2x+3) = 5x + 2$

b. $(f+g)(4) = 5(4) + 2 = 22$

c. $(f-g)(x) = f(x) - g(x)$
$= (3x-1) - (2x+3) = x - 4$

d. $(fg)(x) = f(x)g(x) = (3x-1)(2x+3)$
$= 6x^2 + 7x - 3$

e. $(fg)(1) = 6(1)^2 = 7(1) - 3 = 10$

f. $\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x-1}{2x+3}$

g. $(f \circ g)(x) = f(g(x)) = f(2x+3)$
$= 3(2x+3) - 1 = 6x + 8$

h. $(f \circ g)(5) = 6(5) + 8 = 38$

i. $(g \circ f)(x) = g(f(x)) = g(3x-1)$
$= 2(3x-1) + 3 = 6x + 1$

20. a. $(f+g)(x) = x^3 + 2x + 1$

b. $(f-g)(x) = x^3 - 2x - 1$

c. $(f-g)(-6) = -216 + 12 - 1 = -205$

d. $(fg)(x) = x^3(2x+1) = 2x^4 + x^3$

e. $\dfrac{f}{g}(x) = \dfrac{x^3}{2x+1}$

f. $\dfrac{f}{g}(1) = 1/3$

g. $(f \circ g)(x) = (2x+1)^3 = 8x^3 + 12x^2 + 6x + 1$

h. $(g \circ f)(x) = 2x^3 + 1$

i. $(g \circ f)(2) = 17$

21. $f(x) = \dfrac{1}{x^2}, g(x) = x + 1$

$(f \circ g)(x) = f(g(x)) = f(x+1) = \dfrac{1}{(x+1)^2}$

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{x^2}\right) = \dfrac{1}{x^2} + 1 =$
$\dfrac{1+x^2}{x^2}$

22. $f(x) = \dfrac{x-2}{3}, g(x) = \dfrac{1}{\sqrt{x}}$

$(f \circ g)(x) = f(g(x)) = \dfrac{\frac{1}{\sqrt{x}} - 2}{3} = \dfrac{1 - 2\sqrt{x}}{3\sqrt{x}}$

$(g \circ f)(x) = g(f(x)) = \dfrac{1}{\sqrt{\frac{x-2}{3}}} = \sqrt{\dfrac{3}{x-2}}$

23. $f(x) = \sqrt{x+2}, g(x) = x^3$

$(f \circ g)(x) = f(g(x)) = f(x^3) = \sqrt{x^3 + 2}$

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x+2}\right) = \left(\sqrt{x+2}\right)^3$
$= (x+2)^{3/2}$

24. $f(x) = 2, g(x) = 3$

$(f \circ g)(x) = f(g(x)) = f(3) = 2$

$(g \circ f)(x) = g(f(x)) = g(2) = 3$

25. Only intercept is $(0, 0)$. $(-y) = 2(-x) + (-x)^3$ is equivalent to $-y = -2x - x^3$ is equivalent to $y = 2x + x^3$ so symmetric about the origin (but no other symmetries among those to be tested).

26. $\dfrac{x^2y^2}{x^2+y^2+1}=4$

Intercepts: If $y=0$, then $0=4$, which is impossible; if $x=0$, then $0=4$, which is impossible.

Testing for symmetry gives:

x-axis: $\dfrac{x^2(-y)^2}{x^2+(-y)^2+1}=4$

$\dfrac{x^2y^2}{x^2+y^2+1}=4$, which is the original equation.

y-axis: $\dfrac{(-x)^2y^2}{(-x)^2+y^2+1}=4$

$\dfrac{x^2y^2}{x^2+y^2+1}=4$, which is the original equation.

origin: $\dfrac{(-x)^2(-y)^2}{(-x)^2+(-y)^2+1}=4$

$\dfrac{x^2y^2}{x^2+y^2+1}=4$, which is the original equation.

line $y=x$: (a, b) on graph, then $\dfrac{a^2b^2}{a^2+b^2+1}=4$

and $b^2=\dfrac{4(a^2+1)}{a^2-4}$. (b, a) on graph,

then $\dfrac{b^2a^2}{b^2+a^2+1}=4$ and

$b^2=\dfrac{4(a^2+1)}{a^2-4}$, so (a, b) and (b, a) are both on the graph.

Answer: no intercepts; symmetry about x-axis, y-axis, origin, and $y=x$.

27. $y=4+x^2$

Intercepts: If $y=0$, then $0=4+x^2$, which is never true.

If $x=0$, then $y=4$.

Testing for symmetry gives:

x-axis: $-y=4+x^2$

$y=-4-x^2$, which is not the original equation.

y-axis: $y=4+(-x)^2$

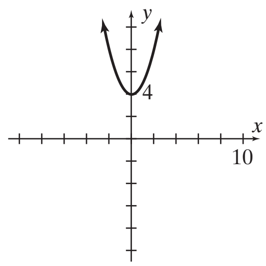
$y=4+x^2$, which is the original equation.

origin: $-y=4+(-x)^2$

$y=-4-x^2$, which is not the original equation.

line $y=x$: (a, b) on graph, then $b=4+a^2$ and $a=\pm\sqrt{b-4}\neq 4+b^2$ for all b, so (b, a) is not on the graph.

Answer: $(0, 4)$; symmetry about y-axis.



28. $y=3x-7$

Intercepts: If $y=0$, then $0=3x-7$, or $x=\dfrac{7}{3}$.

If $x=0$, then $y=-7$.

Testing for symmetry gives:

x-axis: $-y=3x-7$

$y=-3x+7$, which is not the original equation.

y-axis: $y=3(-x)-7$

$y=-3x-7$, which is not the original equation.

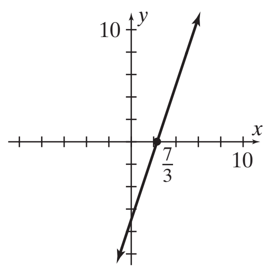
origin: $-y=3(-x)-7$

$y=3x+7$, which is not the original equation.

line $y=x$: (a, b) on graph, then $b=3a-7$ and

$a=\dfrac{1}{3}(b+7)\neq 3b-7$ for all b, so (b, a) is not on the graph.

Answer: $(0, -7)$, $\left(\dfrac{7}{3}, 0\right)$; no symmetry of the given types



29. $G(u) = \sqrt{u+4}$

If $G(u) = 0$, then $0 = \sqrt{u+4}$.

$0 = u + 4$,

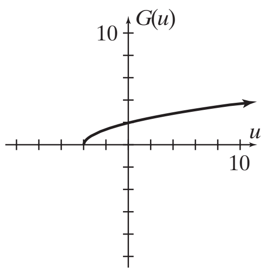
$u = -4$

If $u = 0$, then $G(u) = \sqrt{4} = 2$.

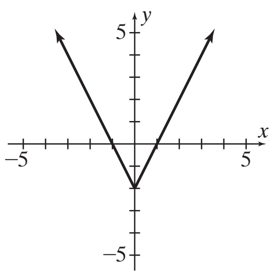
Intercepts: $(0, 2)$, $(-4, 0)$

Domain: all real numbers u such that $u \geq -4$

Range: all real numbers ≥ 0



30.



Domain is $(-\infty, \infty)$; range is $[-2, \infty)$; intercepts are $(0, -2)$, $(-1, 0)$ and $(1, 0)$.

31. $y = g(t) = \dfrac{2}{|t-4|}$

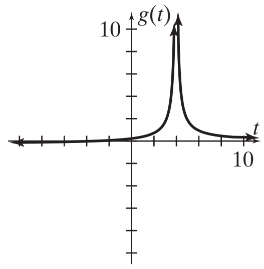
If $y = 0$, then $0 = \dfrac{2}{|t-4|}$, which has no solution.

If $t = 0$, then $y = \dfrac{2}{4} = \dfrac{1}{2}$.

Intercept: $\left(0, \dfrac{1}{2}\right)$

Domain: all real numbers t such that $t \neq 4$

Range: all real numbers > 0

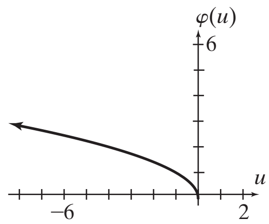


32. $v = \phi(u) = \sqrt{-u}$

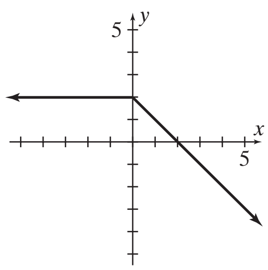
If $\phi(u) = 0$, then $0 = \sqrt{-u}$,

$u = 0$.

If $u = 0$, $\phi(u) = 0$.
Intercept: (0, 0)
Domain: all reals ≤ 0
Range: all reals ≥ 0



33. Domain: all real numbers.
Range: all real numbers $\leq$ g2



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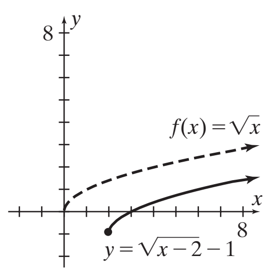
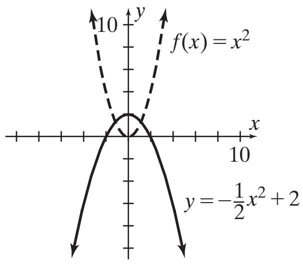


35. From the graph of $f(x) = x^2$, get the graph of $y = -\frac{1}{2}x^2 + 2$ by shrinking the graph by a factor of 1/2 towards the x-axis, reflecting the result in the x-axis, and translating that result up by 2.



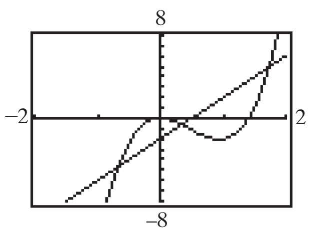
36. For 2006, $t = 5$. Hence
$S = 150{,}000 + 3000(5) = \$165{,}000$.
S is a function of t.

37. From the vertical-line test, the graphs that represent functions of x are (a) and (c).

38. a. 729

b. 359.43

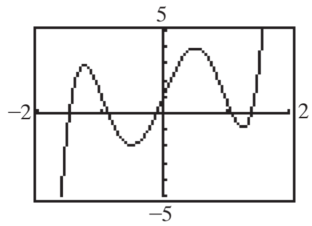
39.



$-0.67; 0.34, 1.73$

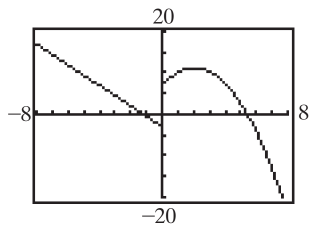
40. A graph of $f(x) = x^3 + x + 1$ will suggest that $x^3 + x + 1 = 0$ has exactly one real root and since $f(-1) = -1 < 0$ and $f(0) = 1 > 0$, the graph further suggests that the unique root, call it r lies in the interval $(-1, 0)$. Since $f(-1/2) = -1/8 - 1/2 + 1 = 3/8 > 0$, the same reasoning suggests that r lies in the interval $(-1, -1/2)$. We chose $-1/2$ as the midpoint of $(-1, 0)$. This leads us to investigate $f(-3/4)$ because $-3/4$ is the midpoint of $(-1, -1/2)$. We have $f(-3/4) = -27/64 - 3/4 + 1 = 15/64 > 0$ So r is in $(-1, -3/4)$. Now $f(-7/8) = -279/512 < 0$ So r is in $(-7/8, -3/4)$ and next we check $f(-13/16)$. Clearly, we can continue this process until the test interval has a length less than the desired degree of accuracy. The process converges rapidly!

41.



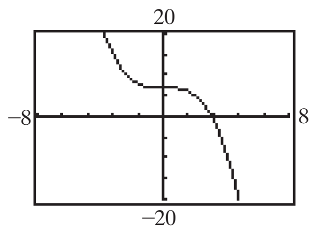
$-1.50, -0.88, -0.11, 1.09, 1.40$

42.



$(-\infty, \infty)$

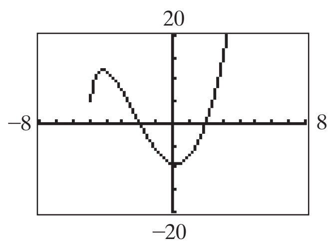
43.



a. $(-\infty, \infty)$

b. $(1.92, 0)$, $(0, 7)$

44.

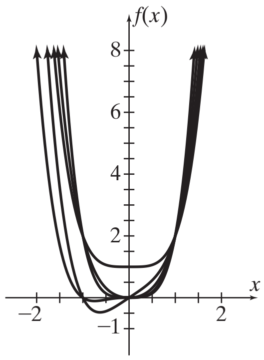


a. -9.03

b. all real numbers ≥ -9.03

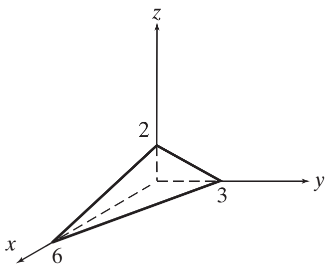
c. $-5, \pm 2$.

45.



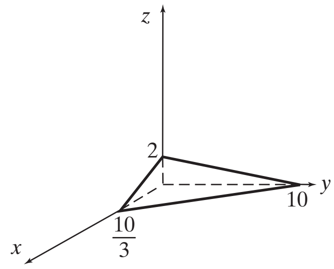
We note that if k is even then $(-x)^k = x^k$ and if k is odd then $(-x)^k = -x^k$. Assume k is even. Replacing x by $-x$ in $y = x^4 + x^k$ we get $y = (-x)^4 + (-x)^k = x^4 + x^k$ the original equation. So for k even we get symmetry about the y-axis. In this case we cannot get symmetry about the origin because that would entail symmetry about the x-axis which does not hold for any function different from the function constantly zero. Assume k is odd. Replacing x by $-x$ and y by $-y$ in the equation produces $-y = (-x)^4 + (-x)^k = x^4 - x^k$ equivalently $y = -x^4 + x^k$ which is not equivalent to the original equation. So for k odd we get no symmetries of the types under consideration.

46. $x + 2y + 3z = 6$ can be put in the form $Ax + By + Cz + D = 0$, so the graph is a plane. Intercepts: (6, 0, 0), (0, 3, 0), (0, 0, 2)



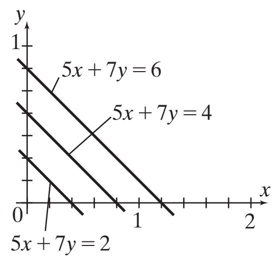
47. $3x + y + 5z = 10$ can be put in the form $Ax + By + Cz + D = 0$, so the graph is a plane.

Intercepts: $\left(\frac{10}{3},\ 0,\ 0\right)$, (0, 10, 0), (0, 0, 2)

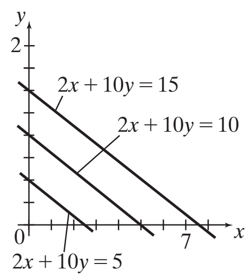


48. $P = 5x + 7y$

Choose $P = 2, 4$, and 6 for the curves.



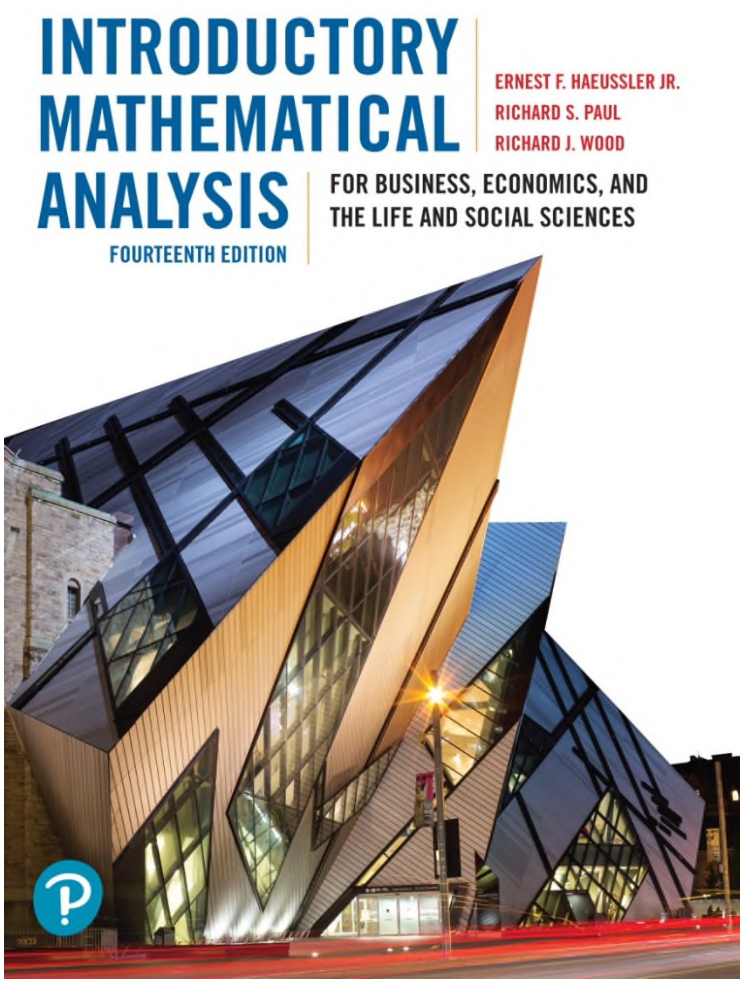
49. $C = 2x + 10y$
Choose $C = 5$, 10, and 15 for the curves.



Introductory Mathematical Analysis

For Business, Economics, and The Life and Social Sciences

Fourteenth Canadian Edition



Chapter 2

Functions and Graphs

Chapter Objectives

- To understand what functions and domains are.
- To introduce different types of functions.
- To introduce addition, subtraction, multiplication, division, and multiplication by a constant.
- To introduce inverse functions and properties.
- To graph equations and functions.
- To study symmetry about the *x*- and *y*-axis.
- To be familiar with shapes of the graphs of six basic functions.

Chapter Outline

2.1) Functions

2.2) Special Functions

2.3) Combinations of Functions

2.4) Inverse Functions

2.5) Graphs in Rectangular Coordinates

2.6) Symmetry

2.7) Translations and Reflections

2.1 Functions (1 of 5)

- A **function** assigns each input number to one output number.
- The set of all input numbers is the **domain** of the function.
- The set of all output numbers is the **range**.

To say that two functions $f, g: X \to Y$ are equal, denoted $f = g$, is to say that

1. The domain of f is equal to the domain of g.
2. For every x in the domain of f and g, $f(x) = g(x)$.

2.1 Functions (2 of 5)

Example 1 – Determining Equality of Functions

Determine which of the following functions are equal.

a. $f(x) = \dfrac{(x+2)(x-1)}{(x-1)}$

b. $g(x) = x+2$

c. $h(x) = \begin{cases} x+2 & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$

d. $k(x) = \begin{cases} x+2 & \text{if } x \neq 1 \\ 3 & \text{if } x = 1 \end{cases}$

2.1 Functions (3 of 5)

Example 1 – Continued

Solution: Observe that the domains of g, h, and k are equal to each other, but that of f is different. So by requirement 1 for equality, $f \neq g$, $f \neq h$ and $f \neq k$.

By definition, $g(x) = h(x) = k(x)$ for all $x \neq 1$.
Note that $g(1) = 3$, $h(1) = 0$, and $k(1) = 3$.
We conclude that $g = k$ and $g \neq h$ (and $h \neq k$).

2.1 Functions (4 of 5)

Example 3 – Finding Domain and Function Values

Let $g(x) = 3x^2 - x + 5$. Note that the domain of g is all real numbers.

a. Find $g(z)$.

Solution: $g(z) = 3z^2 - z + 5$

b. Find $g(r^2)$.

Solution: $g(r^2) = 3(r^2)^2 - r^2 + 5 = 3r^4 - r^2 + 5$

c. Find $g(x+h)$.

Solution: $g(x+h) = 3(x+h)^2 - (x+h) + 5$

$= 3(x^2 + 2xh + h^2) - x - h + 5$

$= 3x^2 + 6xh + 3h^2 - x - h + 5$

2.1 Functions (5 of 5)

Example 5 – Demand Function

Suppose that the equation $p = 100 / q$ describes the relationship between the price per unit p of a certain product and the number of units q of the product that consumers will buy (that is, demand) per week at the stated price. This equation is called a *demand equation* for the product. If q is an input, then to each value of q there is assigned at most one output p:

$$q \mapsto \frac{100}{q} = p.$$

This function is called a **demand function.**

2.2 Special Functions (1 of 4)

Example 1 – Constant Function

Let $h:(-\infty,\infty)\rightarrow(\infty,\infty)$ be given by $h(x)=2$. The domain of h is $(-\infty,\infty)$, the set of all real numbers. All function values are 2. For example, $h(10)=2,\ h(-387)=2,\ h(x+3)=2.$

We call h a *constant function*. More generally, a function of the form $h(x)=c$, where c is a *constant*, is called a **constant function**.

2.2 Special Functions (2 of 4)

Example 3 – Rational Functions

a. $f(x) = \frac{x^2 - 6x}{x+5}$ is a rational function, since the numerator and denominator are each polynomials.

b. $g(x) = 2x + 3$ is a rational function, since $2x + 3 = \frac{2x+3}{1}$.

Example 5 – Absolute-Value Function

The function $f(x) = |x|$ is called the absolute-value function.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

2.2 Special Functions (3 of 4)

The symbol $r!$, with r a positive integer, is read "r **factorial**".
It represents the product of the first r positive integers:

$$r! = 1 \cdot 2 \cdot 3 \cdots r$$

We also define $0! = 1$.

2.2 Special Functions (4 of 4)

Example 7 – Genetics

Suppose two black guinea pigs are bred and produce exactly five offspring. Under certain conditions, it can be shown that the probability P that exactly r of the offspring will be brown and the others black is a function of r, $P = P(r)$, where

$$P(r) = \frac{5!\left(\frac{1}{4}\right)^{r}\left(\frac{3}{4}\right)^{5-r}}{r!(5-r)!} \qquad r = 0,\ 1,\ 2,\ \ldots,\ 5$$

Find the probability that exactly three guinea pigs will be brown.

Solution: We want to find $P(3)$. We have

$$P(3) = \frac{5!\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)^{2}}{3!\ 2!} = \frac{120\left(\frac{1}{64}\right)\left(\frac{9}{16}\right)}{6(2)} = \frac{45}{512}.$$

2.3 Combinations of Functions (1 of 5)

In general, for any functions $f,\ g : X \to (-\infty, \infty)$, we define the *sum* $f + g$, the *difference* $f - g$, the *product* fg, and the *quotient* $\dfrac{f}{g}$ as follows:

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x) \cdot g(x)$$
$$\frac{f}{g}(x) = \frac{f(x)}{g(x)} \quad \text{for } g(x) \neq 0$$

2.3 Combinations of Functions (2 of 5)

Example 1 – Combining Functions

If $f(x) = 3x - 1$ and $g(x) = x^2 + 3x$, find a. $(f + g)(x)$,

b. $(f - g)(x)$, c. $(fg)(x)$, d. $\dfrac{f}{g}(x)$, e. $((1/2)f)(x)$

Solution

a. $(f + g)(x) = f(x) + g(x) = (3x - 1) + (x^2 + 3x) = x^2 + 6x - 1$

b. $(f - g)(x) = f(x) - g(x) = (3x - 1) - (x^2 + 3x) = -1 - x^2$

2.3 Combinations of Functions (3 of 5)

Example 1 – Continued

Solution, continued

c. $(fg)(x) = f(x)g(x) = (3x-1)(x^2+3x) = 3x^3+8x^2-3x$

d. $\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x-1}{x^2+3x}$

e. $((1/2)f)(x) = (1/2)(f(x)) = (1/2)(3x-1)$

- We can also combine two functions by first applying one function to an input and then applying the other function to the output of the first.
- This is called **composition**.

2.3 Combinations of Functions (4 of 5)

For functions $g: X \to Y$ and $f: Y \to Z$, **the composite of f with g is the function $f \circ g: X \to Z$ defined by**

$$(f \circ g)(x) = f(g(x))$$

Where the domain of $f \circ g$ is the set of all those x in the domain of g such that $g(x)$ is in the domain of f.

2.3 Combinations of Functions (5 of 5)

Example 3 – Composition

If $F(p) = p^2 + 4p - 3$, $G(p) = 2p + 1$, and $H(p) = |p|$, find

a. $F(G(p))$

b. $F(G(H(p)))$

c. $G(F(1))$

Solution:

a. $F(2p+1) = (2p+1)^2 + 4(2p+1) - 3 = 4p^2 + 12p + 2$

b. $F(G(H(p))) = (F \circ (G \circ H))(p) = ((F \circ G) \circ H)(p)$

$= (F \circ G)(H(p)) = (F \circ G)(|p|) = 4|p|^2 + 12|p| + 2$

c. $G(F(1)) = G(1^2 + 4 \cdot 1 - 3) = G(2) = 2 \cdot 2 + 1 = 5$

2.4 Inverse Functions (1 of 4)

A function that satisfies, for all a and b,
if $f(a) = f(b)$ then $a = b$,
is called a **one-to-one** function.

A function has an inverse, written $f^{-1}(x)$, precisely if it is one-to-one.
In general,

$f^{-1}(f(x)) = x$ for all x in the domain of f

and

$f(f^{-1}(y)) = y$ for all y in the range of f

Note that the range of f can be different from the domain of f.

2.4 Inverse Functions (2 of 4)

Example 1 – Inverses of Linear Functions

Show that a linear function (a function of the form $f(x) = ax + b$, where $a \neq 0$) is one-to-one. Find the inverse of $f(x)$ and show that it is also linear).

Solution: Assume that $f(u) = f(v)$, that is, $au + b = av + b$.

This gives $au = av$, and since $a \neq 0$, it follows that $u = v$.

Thus, $f(x)$ is one-to-one.

Consider $g(x) = \frac{x-b}{a}$. We have

$$(f \circ g)(x) = f(g(x)) = a\frac{x-b}{a} + b = (x-b) + b = x$$

and $(g \circ f)(x) = g(f(x)) = \frac{(ax+b)-b}{a} = \frac{ax}{a} = x$

It follows that g is the inverse of f.

Since $g(x) = f^{-1}(x) = \frac{x-b}{a} = \frac{1}{a}x + \frac{-b}{a}$, we conclude that $f^{-1}(x)$ is linear.

2.4 Inverse Functions (3 of 4)

Example 3 – Inverses Used to Solve Equations

Many equations take the form $f(x)=0,$ where f is a function. If f is a one-to-one function, then the equation has $x = f^{-1}(0)$ as its unique solution:

$f(x)=0$

$f^{-1}(f(x)) = f^{-1}(0)$ (applying $f^{-1}(x)$ to both sides)

$x = f^{-1}(0)$ ($f^{-1}(f(x)) = x$ by definition of the inverse)

Therefore $x = f^{-1}(0)$ is the only possible solution.

2.4 Inverse Functions (4 of 4)

Example 5 – Finding the Inverse of a Function

To find the inverse of a one-to-one function f, solve the equation $y = f(x)$ for x in terms of y, obtaining $x = g(y)$. Then $f^{-1}(x) = g(x)$. Find $f^{-1}(x)$ if $f(x) = (x-1)^2$, for $x \geq 1$.

Solution: Let $y = (x-1)^2$, for $x \geq 1$. Then $x - 1 = \sqrt{y}$ and hence, $x = \sqrt{y} + 1$. It follows that $f^{-1}(x) = \sqrt{x} + 1$.

2.5 Graphs in Rectangular Coordinates (1 of 5)

- A **rectangular coordinate system** allows us to specify and locate points in a plane. It also provides a geometric way to graph equations in two variables.

An x-intercept of the graph of an equation in x and y is a point where the graph intersects the x-axis.

A y-intercept is a point where the graph intersects the y-axis.

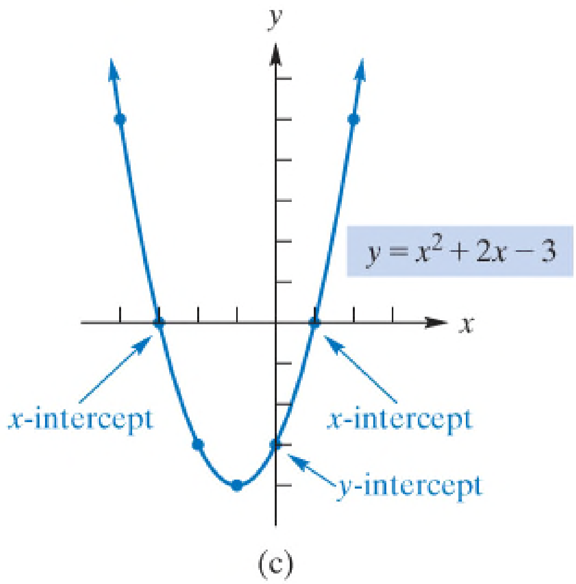


FIGURE 2.9 Graphing $y = x^2 + 2x - 3$.

2.5 Graphs in Rectangular Coordinates (2 of 5)

Example 1 – Intercepts of a Graph

Find the x- and y-intercepts of the graph of $y = 2x + 3$, and sketch the graph.

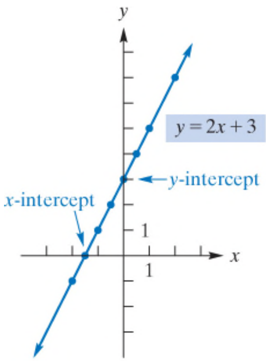
Solution: If $y = 0$,

then $0 = 2x + 3$ so that $x = -\frac{3}{2}$.

Thus, the x-intercept is $\left(-\frac{3}{2},\ 0\right)$.

If $x = 0$, then

$y = 2(0) + 3 = 3$, so the y-intercept is $(0,\ 3)$.



x	0	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1	−1	2	−2
y	3	0	4	2	5	1	7	−1

FIGURE 2.10 Graph of $y = 2x + 3$.

2.5 Graphs in Rectangular Coordinates (3 of 5)

Example 3 – Intercepts of a Graph

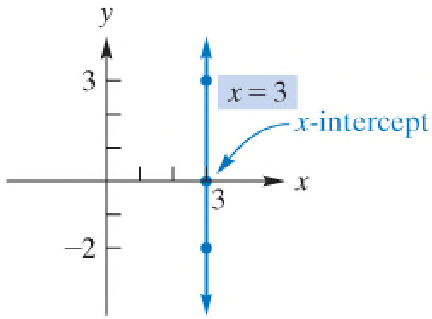
Determine the intercepts of the graph $x = 3$, and sketch the graph.

Solution: We can write $x = 3$ as $x = 3 + 0y$.

Here y can be any value, but x must be 3.

The x-intercept is $(3, 0)$,

and there is no y-intercept because x cannot be 0.



x	3	3	3
y	0	3	−2

FIGURE 2.12 Graph of $x = 3$.

2.5 Graphs in Rectangular Coordinates (4 of 5)

Example 5 – Graph of the Absolute-Value Function

Graph $p = G(q) = |q|$.

Solution: We use the independent variable q to label the horizontal axis. The function-value axis can be labeled either $G(q)$ or p. Note the sharp corner at the origin.

q	0	1	−1	3	−3	5	−5
p	0	1	1	3	3	5	5

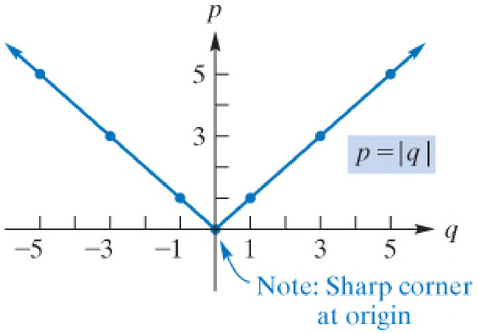


FIGURE 2.14 Graph of $p = |q|$.

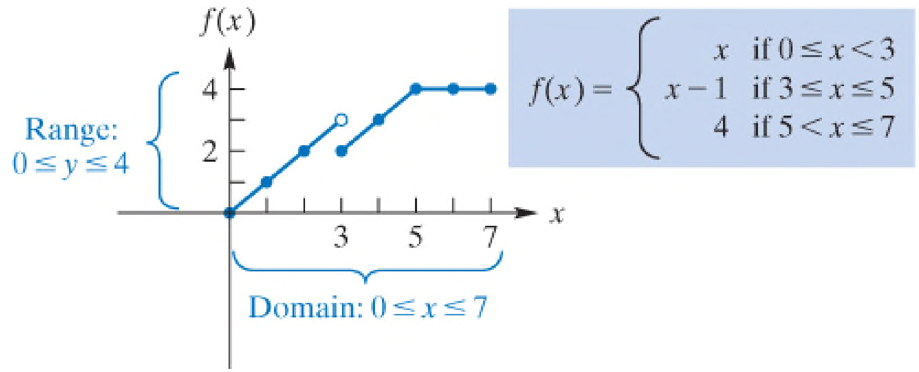
2.5 Graphs in Rectangular Coordinates (5 of 5)

Example 7 – Graph of a Case-Defined Function

Graph the case-defined function

$$f(x)=\begin{cases} x & \text{if } 0\le x<3 \\ x-1 & \text{if } 3\le x\le 5 \\ 4 & \text{if } 5<x\le 7 \end{cases}$$

Solution:



x	0	1	2	3	4	5	6	7
$f(x)$	0	1	2	2	3	4	4	4

FIGURE 2.17 Graph of a case-defined function.

2.6 Symmetry (1 of 5)

A graph is **symmetric about the y-axis** if and only if $(-a,\ b)$ lies on the graph when $(a,\ b)$ does.

Example 1 – y-axis Symmetry

Use the preceding definition to show that the graph of $y = x^2$ is symmetric about the y-axis.

Solution: Suppose $(a,\ b)$ is any point on the graph of $y = x^2$.

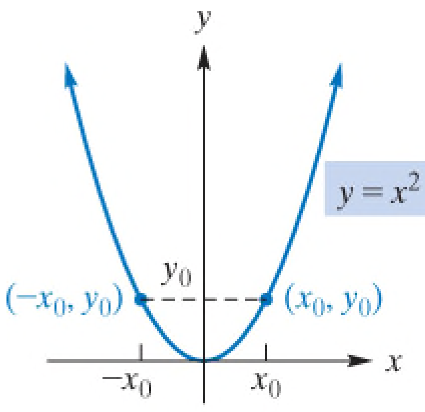


FIGURE 2.25 Symmetry about the y-axis.

Then $b = a^2$. Moreover, consider the point $(-a,\ b) : (-a)^2 = a^2 = b$.

This shows that $(-a,\ b)$ is also on the graph.

2.6 Symmetry (2 of 5)

A graph is **symmetric about the x-axis** if and only if $(x, -y)$ lies on the graph when (x, y) does.

A graph is **symmetric about the origin** if and only if $(-x, -y)$ lies on the graph when (x, y) does.

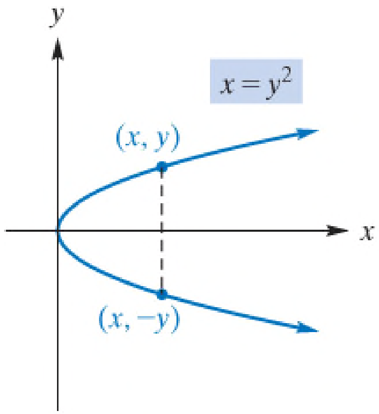


FIGURE 2.26 Symmetry about the x-axis.

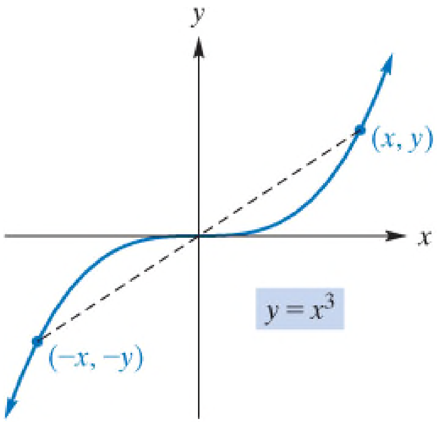


FIGURE 2.27 Symmetry about the origin.

2.6 Symmetry (3 of 5)

Example 3 – Graphing with Intercepts and Symmetry

Test $y = f(x) = 1 - x^4$ for symmetry about the x-axis, the y-axis, and the origin. Then find the intercepts and sketch the graph.

Solution - Symmetry:

x-axis: Replacing y by $-y$ in $y = 1 - x^4$ gives $-y = 1 - x^4$, equivalently, $y = -1 + x^4$ which is not equivalent to the given equation. The graph is *not* symmetric about the x-axis.

y-axis: Replacing x by $-x$ in $y = 1 - x^4$ gives $y = 1 - (-x)^4$, equivalently, $y = 1 - x^4$ which is the given equation. The graph *is* symmetric about the y-axis.

Origin: Replacing x by $-x$ and y by $-y$ in $y = 1 - x^4$ gives $-y = 1 - (-x)^4$, equivalently, $y = -1 + x^4$ which is not equivalent to the given equation. The graph is *not* symmetric about the origin.

2.6 Symmetry (4 of 5)

Example 3 – Continued

Solution - Intercepts:

We set $y = 0$ in $y = 1 - x^4$.

Then $1 - x^4 = 0$

$(1 - x^2)(1 + x^2) = 0$

$(1 - x)(1 + x)(1 + x^2) = 0$

$x = 1$ or $x = -1$.

x	y
0	1
$\frac{1}{2}$	$\frac{15}{16}$
$\frac{3}{4}$	$\frac{175}{256}$
1	0
$\frac{3}{2}$	$-\frac{65}{16}$

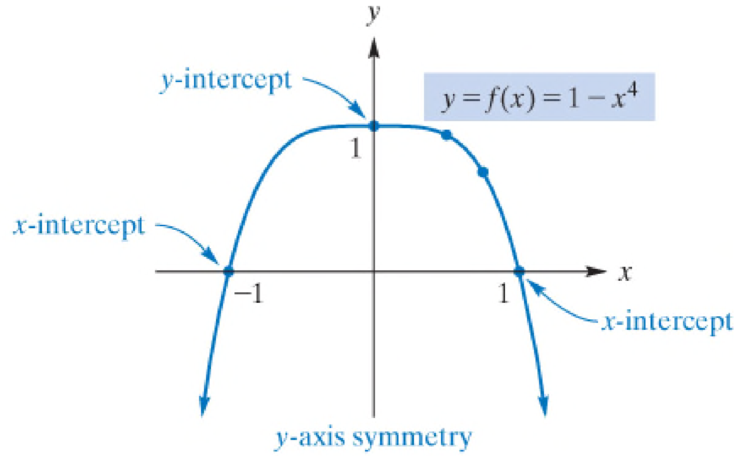


FIGURE 2.29 Graph of $y = 1 - x^4$.

The x-intercepts are therefore $(1,\ 0)$ and $(-1,\ 0)$.

We set $x = 0$, then $y = 1$, so $(0,\ 1)$ is the only y-intercept.

2.6 Symmetry (5 of 5)

A graph is **symmetric about the line** $y = x$ if and only if $(b,\ a)$ lies on the graph when $(a,\ b)$ does.

Example 5 – Symmetry about the Line y = x

Use the preceding definition to show that $x^2 + y^2 = 1$ is symmetric about the line $y = x$.

Solution: Interchanging the roles of x and y produces $x^2 + y^2 = 1$, which is equivalent to $x^2 + y^2 = 1$. Thus $x^2 + y^2 = 1$ is symmetric about the line $y = x$.

2.7 Translations and Reflections (1 of 3)

- Some functions and their associated graphs occur so frequently that we find it worthwhile to memorize them. Below are six such basic functions.

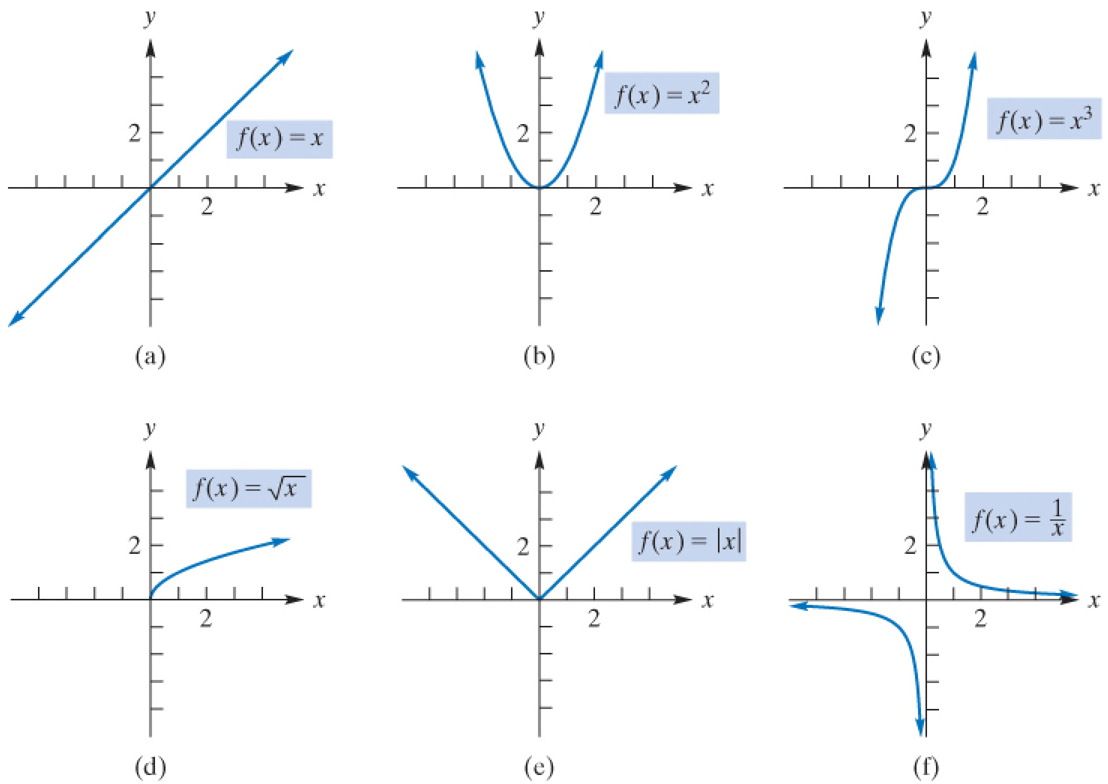


FIGURE 2.32 Six basic functions.

2.7 Translations and Reflections (2 of 3)

- The table below gives a list of basic types of transformations:

Table 2.2 Transformations, c > 0

Equation	How to Transform Graph of $y = f(x)$ to Obtain Graph of Equation
$y = f(x) + c$	shift c units upward
$y = f(x) - c$	shift c units downward
$y = f(x - c)$	shift c units to right
$y = f(x + c)$	shift c units to left
$y = -f(x)$	reflect about x-axis
$y = f(-x)$	reflect about y-axis
$y = cf(x) \quad c > 1$	vertically stretch away from x-axis by a factor of c
$y = cf(x) \quad c < 1$	vertically shrink toward x-axis by a factor of c

2.7 Translations and Reflections (3 of 3)

Example 1 – Horizontal Translation

Sketch the graph of $y = (x-1)^3$.

Solution: We observe that x is replaced with $x-1$. Thus, this function has the form $y = f(x-c)$, where $c = 1$.

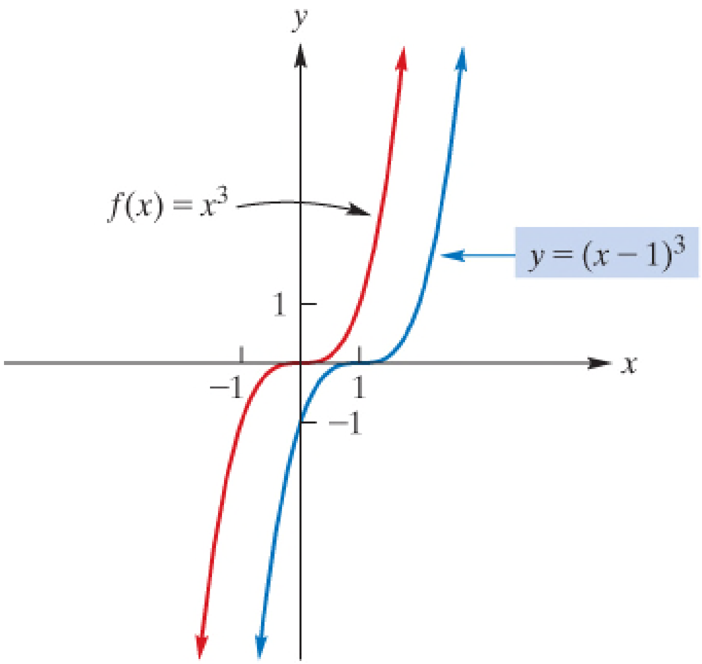


FIGURE 2.34 Graph of $y = (x-1)^3$.

2.8 Functions of Several Variables (1 of 7)

For sets X and Y we can construct the new set $X \times Y$ whose elements are **ordered pairs** (x, y) with x in X and y in Y.

A function $f : X \times Y \to Z$ is a rule that assigns to each element (x, y) in $X \times Y$ at most one element of Z, denoted by $f((x, y))$. We agree to write $f(x, y)$.

In general, a function $f : X_1 \times X_2 \times \cdots \times X_n \to Y$ provides the notion of a Y-valued function of n-variables. An element of the domain of f is an **ordered** n**-tuple** $(x_1, x_2, \ldots, x_n)$, with x_i in X_i for $i = 1, 2, \ldots, n$, for which $f(x_1, x_2, \ldots, x_n)$ is defined.

The **graph** of f is the set of all ordered $n+1$-tuples of the form $(x_1, x_2, \ldots, x_n, f(x_1, x_2, \ldots, x_n))$, where $(x_1, x_2, \ldots, x_n)$ is in the domain of f.

2.8 Functions of Several Variables (2 of 7)

Example 1 – Functions of Two Variables

a. $a(x, y) = x + y$ is a function of two variables. Some function values are

$a(1, 1) = 1 + 1 = 2$

$a(2, 3) = 2 + 3 = 5$

We have $a : (-\infty, \infty) \times (-\infty, \infty) \to (-\infty, \infty)$.

b. $m(x, y) = xy$ is a function of two variables. Some function values are

$m(2, 2) = 2 \cdot 2 = 4$

$m(3, 2) = 3 \cdot 2 = 6$

The domain of both a and m is all of $(-\infty, \infty) \times (-\infty, \infty)$.

2.8 Functions of Several Variables (3 of 7)

Example 3 – Temperature-Humidity Index

On hot and humid days, many people tend to feel uncomfortable. In the United States, the degree of discomfort is numerically given by the temperature-humidity index, THI, which is a function of two variables, t_d and t_w: THI $= f(t_d, t_w) = 15 + 0.4(t_d + t_w)$ where t_d is the dry-bulb temperature and t_w is the wet-bulb temperature of the air. Evaluate the THI when $t_d = 90$ and $t_w = 80$.

Solution: We want to find $f(90, 80)$:

$f(90, 80) = 15 + 0.4(90 + 80) = 15 + 68 = 83.$

A similar measurement, called the Humidex, is used in Canada.

2.8 Functions of Several Variables (4 of 7)

The set of all ordered triples of real numbers can be pictured as providing a 3-**dimensional rectangular coordinate system**.
The three mutually perpendicular real-number lines are called the x-, y-, and z-axes, and their point of intersection is called the origin of the system.

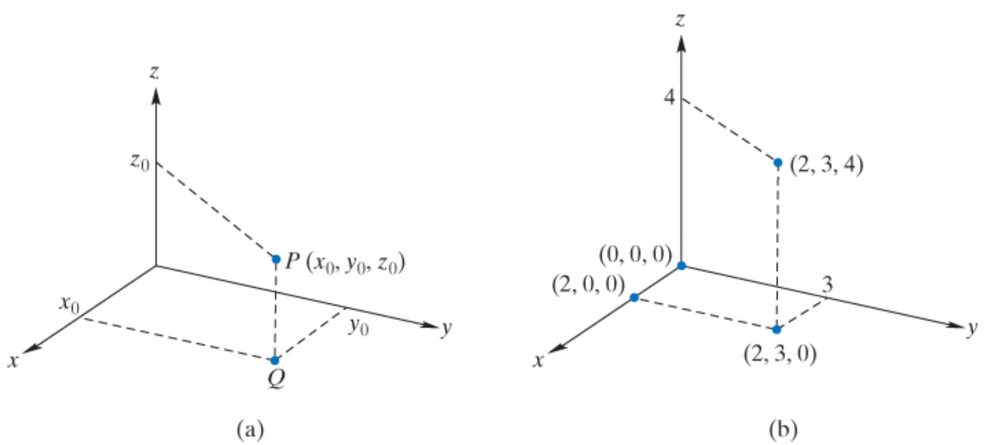


FIGURE 2.37 Points in space.

2.8 Functions of Several Variables (5 of 7)

A "coordinate plane" is a plane containing two coordinate axes.

For example, the plane determined by the x- and y-axes is the x, y-plane.

Below are some sketches of planes parallel to coordinate planes.

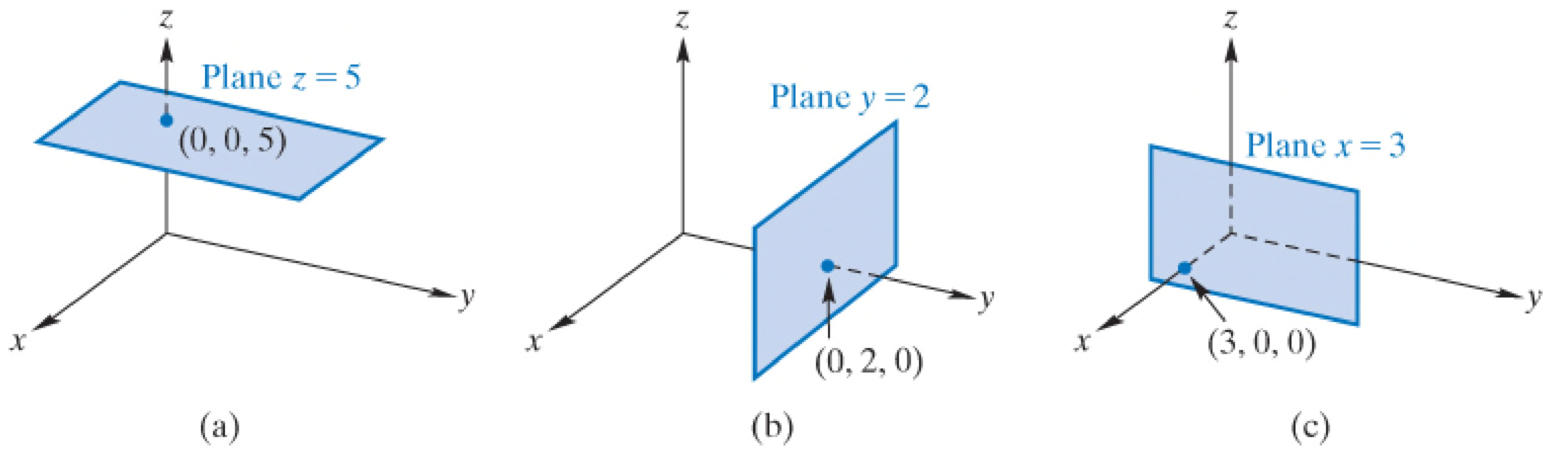


FIGURE 2.39 Planes parallel to coordinate planes.

Example 5 – Sketching a Surface

Sketch the surface $2x + z = 4$.

Solution: This equation has the form of a plane. The x- and z-intercepts are $(2,\ 0,\ 0)$ and $(0,\ 0,\ 4)$, and there is no y-intercept.

Setting $y = 0$ gives the x, z-trace $2x + z = 4$, which is a line in the x, z-plane.

The plane appears to the right.

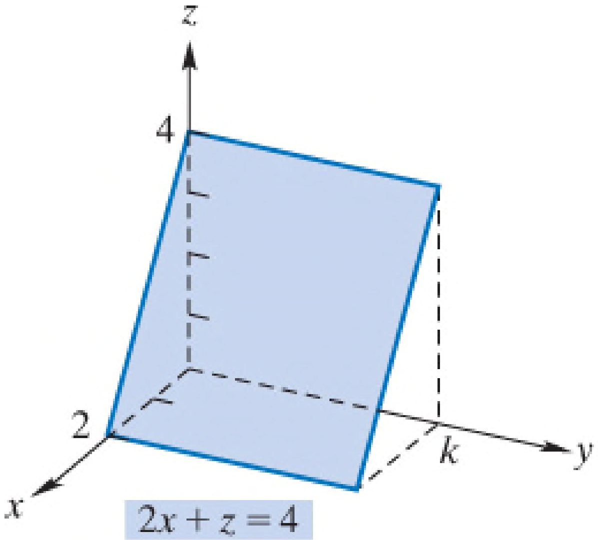


FIGURE 2.41 The plane $2x + z = 4$.

2.8 Functions of Several Variables (7 of 7)

Example 7 – Sketching a Surface

Sketch the surface $x^2 + y^2 + z^2 = 25.$

Solution: Setting $z = 0$ gives the x, y-trace $x^2 + y^2 = 25,$ which is a circle of radius 5. Similarly, the y, z- and x, z-traces are the circles $y^2 + z^2 = 25$ and $x^2 + z^2 = 25$ respectively.

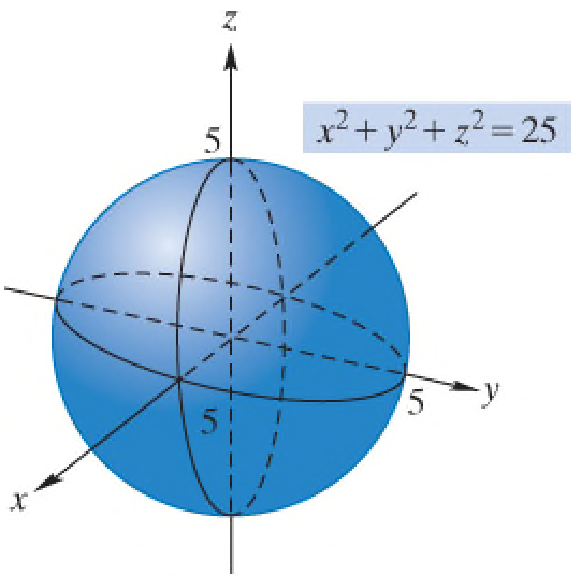


FIGURE 2.43 The surface $x^2 + y^2 + z^2 = 25.$

The surface appears to the right; it is a sphere.

Pearson