Solutions for Introductory Algebra for College Students 7th Edition by Blitzer

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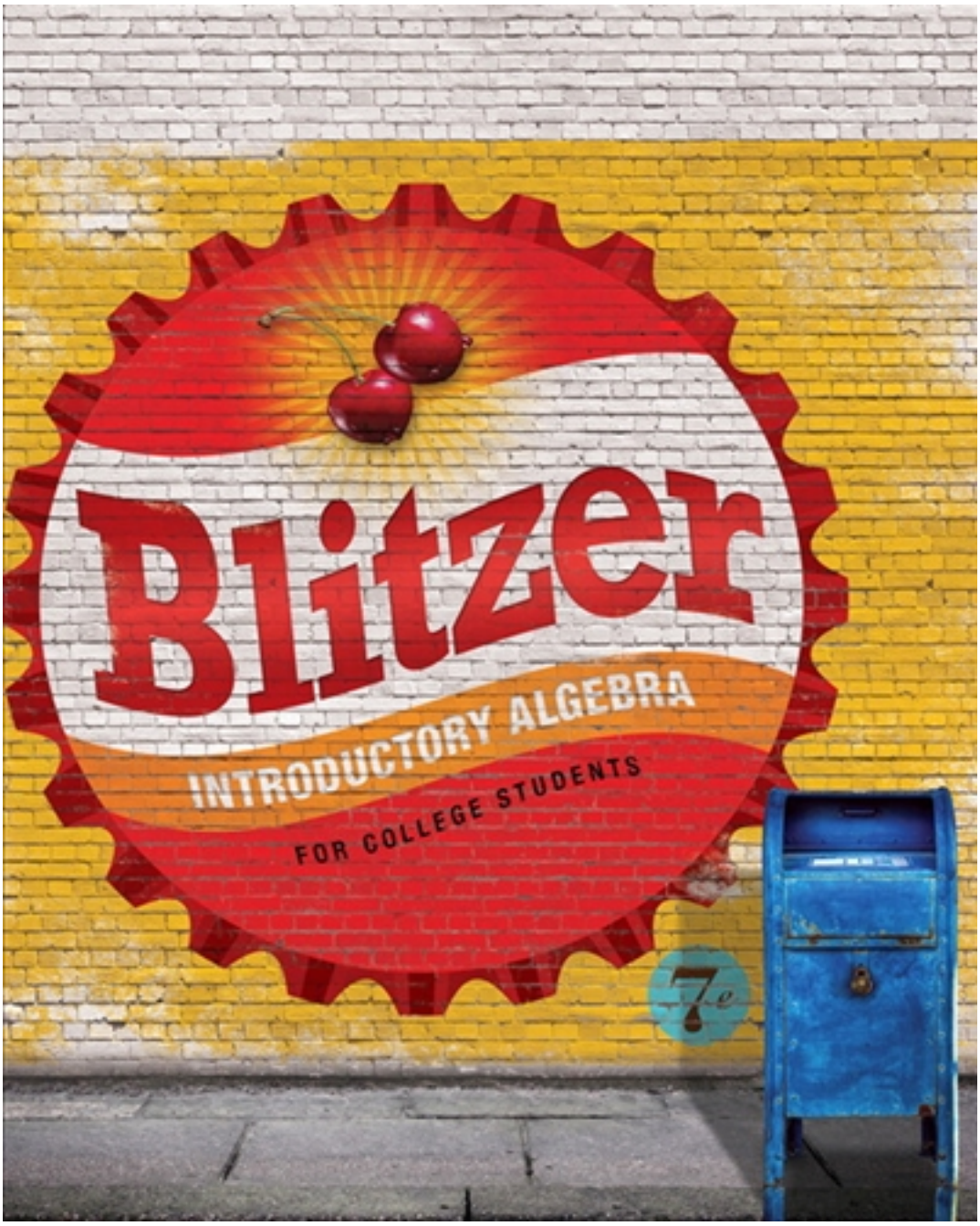


Solutions

Chapter 2
Linear Equations and Inequalities in One Variable

2.1 Check Points

1. $x-5=12$

$x-5+5=12+5$

$x+0=17$

$x=17$

Check:

$x-5=12$

$17-5=12$

$12=12$

The solution set is $\{17\}$.

2. $z+2.8=5.09$

$z+2.8-2.8=5.09-2.8$

$z+0=2.29$

$z=2.29$

Check:

$z+2.8=5.09$

$2.29+2.8=5.09$

$5.09=5.09$

The solution set is $\{2.29\}$.

3. $-\frac{1}{2}=x-\frac{3}{4}$

$-\frac{1}{2}+\frac{3}{4}=x-\frac{3}{4}+\frac{3}{4}$

$-\frac{2}{4}+\frac{3}{4}=x$

$\frac{1}{4}=x$

Check:

$-\frac{1}{2}=x-\frac{3}{4}$

$-\frac{1}{2}=\frac{1}{4}-\frac{3}{4}$

$-\frac{1}{2}=-\frac{2}{4}$

$-\frac{1}{2}=-\frac{1}{2}$

The solution set is $\left\{\frac{1}{4}\right\}$.

4. $8y+7-7y-10=6+4$

$y-3=10$

$y-3+3=10+3$

$y=13$

Check:

$8y+7-7y-10=6+4$

$8(13)+7-7(13)-10=6+4$

$104+7-91-10=10$

$111-101=10$

$10=10$

The solution set is $\{13\}$.

5. $7x=12+6x$

$7x-6x=12+6x-6x$

$x=12$

Check:

$7(12)=12+6(12)$

$84=12+72$

$84=84$

The solution set is $\{12\}$.

6. $3x-6=2x+5$

$3x-2x-6=2x-2x+5$

$x-6=5$

$x-6+6=5+6$

$x=11$

Check:

$3x-6=2x+5$

$3(11)-6=2(11)+5$

$33-6=22+5$

$27=27$

The solution set is $\{11\}$.

7. $V+900=60A$

$V+900=60(50)$

$V+900=3000$

$V+900-900=3000-900$

$V=2100$

At 50 months, a child will have a vocabulary of 2100 words.

2.1 Concept and Vocabulary Check

1. solving

2. linear

3. equivalent

4. $b + c$

5. subtract; solution

6. adding 7

7. subtracting $6x$

2.1 Exercise Set

1. linear

2. linear

3. not linear

4. not linear

5. not linear

6. not linear

7. linear

8. linear

9. not linear

10. not linear

11.
$$x-4=19$$
$$x-4+4=19+4$$
$$x+0=23$$
$$x=23$$
Check:
$$x-4=19$$
$$23-4=19$$
$$19=19$$
The solution set is $\{23\}$.

12.
$$y-5=-18$$
$$y-5+5=-18+5$$
$$y=-13$$
Check:
$$-13-5=-18$$
$$-18=-18$$
The solution set is $\{-13\}$.

13.
$$z+8=-12$$
$$z+8-8=-12-8$$
$$z+0=-20$$
$$z=-20$$
Check:
$$z+8=-12$$
$$-20+8=-12$$
$$-12=-12$$
The solution set is $\{-20\}$.

14.
$$z+13=-15$$
$$z=-15-13$$
$$z=-28$$
Check:
$$-28+13=-15$$
$$-15=-15$$
The solution set is $\{-28\}$.

15.
$$-2=x+14$$
$$-2-14=x+14-14$$
$$-16=x$$
Check:
$$-2=-16+14$$
$$-2=-2$$
The solution set is $\{-16\}$.

16.
$$-13=x+11$$
$$-13-11=x$$
$$-24=x$$
Check:
$$-13=-24+11$$
$$-13=-13$$
The solution set is $\{-24\}$.

17.
$$-17=y-5$$
$$-17+5=y-5+5$$
$$-12=y$$
Check:
$$-17=-12-5$$
$$-17=-17$$
The solution set is $\{-12\}$.

18. $-21 = y - 4$

$-21 + 4 = y$

$-17 = y$

Check:

$-21 = -17 - 4$

$-21 = -21$

The solution set is $\{-17\}$.

19. $7 + z = 11$

$z = 11 - 7$

$z = 4$

Check:

$7 + 4 = 11$

$11 = 11$

The solution set is $\{4\}$.

20. $18 + z = 14$

$z = 14 - 18$

$z = -4$

Check:

$18 + (-4) = 14$

$14 = 14$

The solution set is $\{-4\}$.

21. $-6 + y = -17$

$y = -17 + 6$

$y = -11$

Check:

$-6 - 11 = -17$

$-17 = -17$

The solution set is $\{-11\}$.

22. $-8 + y = -29$

$y = -29 + 8$

$y = -21$

Check:

$-8 + (-21) = -29$

$-29 = -29$

The solution set is $\{-21\}$.

23. $x + \frac{1}{3} = \frac{7}{3}$

$x = \frac{7}{3} - \frac{1}{3}$

$x = 2$

Check:

$2 + \frac{1}{3} = \frac{7}{3}$

$\frac{6}{3} + \frac{1}{3} = \frac{7}{3}$

$\frac{7}{3} = \frac{7}{3}$

The solution set is $\{2\}$.

24. $x + \frac{7}{8} = \frac{9}{8}$

$x = \frac{9}{8} - \frac{7}{8}$

$x = \frac{2}{8} = \frac{1}{4}$

Check:

$\frac{1}{4} + \frac{7}{8} = \frac{9}{8}$

$\frac{2}{8} + \frac{7}{8} = \frac{9}{8}$

$\frac{9}{8} = \frac{9}{8}$

The solution set is $\left\{\frac{1}{4}\right\}$.

25. $t + \frac{5}{6} = -\frac{7}{12}$

$t = -\frac{7}{12} - \frac{5}{6}$

$t = -\frac{7}{12} - \frac{10}{12} = -\frac{17}{12}$

Check:

$-\frac{17}{15} + \frac{5}{6} = -\frac{7}{12}$

$-\frac{17}{12} + \frac{10}{12} = -\frac{7}{12}$

$-\frac{7}{12} = -\frac{7}{12}$

The solution set is $\left\{-\frac{17}{12}\right\}$.

26. $t+\frac{2}{3}=-\frac{7}{6}$

$t=-\frac{7}{6}-\frac{2}{3}$

$t=-\frac{7}{6}-\frac{4}{6}=-\frac{11}{6}$

Check:

$-\frac{11}{6}+\frac{2}{3}=-\frac{7}{6}$

$-\frac{11}{6}+\frac{4}{6}=-\frac{7}{6}$

$-\frac{7}{6}=-\frac{7}{6}$

The solution set is $\left\{-\frac{11}{6}\right\}$.

27. $x-\frac{3}{4}=\frac{9}{2}$

$x-\frac{3}{4}+\frac{3}{4}=\frac{9}{2}+\frac{3}{4}$

$x=\frac{21}{4}$

Check:

$\frac{21}{4}-\frac{3}{4}=\frac{9}{2}$

$\frac{18}{4}=\frac{9}{2}$

$\frac{9}{2}=\frac{9}{2}$

The solution set is $\left\{\frac{21}{4}\right\}$.

28. $x-\frac{3}{5}=\frac{7}{10}$

$x=\frac{7}{10}+\frac{3}{5}$

$x=\frac{7}{10}+\frac{6}{10}=\frac{13}{10}$

Check:

$\frac{13}{10}-\frac{3}{5}=\frac{7}{10}$

$\frac{13}{10}-\frac{6}{10}=\frac{7}{10}$

$\frac{7}{10}=\frac{7}{10}$

The solution set is $\left\{\frac{13}{10}\right\}$.

29. $-\frac{1}{5}+y=-\frac{3}{4}$

$y=-\frac{3}{4}+\frac{1}{5}$

$y=-\frac{15}{20}+\frac{4}{20}=-\frac{11}{20}$

Check:

$-\frac{1}{5}+\left(-\frac{11}{20}\right)=-\frac{3}{4}$

$-\frac{4}{20}-\frac{11}{20}=-\frac{3}{4}$

$-\frac{15}{20}=-\frac{3}{4}$

$-\frac{3}{4}=-\frac{3}{4}$

The solution set is $\left\{-\frac{11}{20}\right\}$.

30. $-\frac{1}{8}+y=-\frac{1}{4}$

$y=-\frac{1}{4}+\frac{1}{8}$

$y=-\frac{2}{4}+\frac{1}{8}=-\frac{1}{8}$

Check:

$-\frac{1}{8}+\left(-\frac{1}{8}\right)=-\frac{1}{4}$

$-\frac{2}{8}=-\frac{1}{4}$

$-\frac{1}{4}=-\frac{1}{4}$

The solution set is $\left\{-\frac{1}{8}\right\}$.

31. $3.2+x=7.5$

$3.2+x-3.2=7.5-3.2$

$x=4.3$

Check:

$3.2+4.3=7.5$

$7.5=7.5$

The solution set is $\{4.3\}$.

32. $-2.7+w=-5.3$

$w=-5.3+2.7$

$w=-2.6$

Check:

$-2.7+(-2.6)=-5.3$

$-5.5=-5.3$

The solution set is $\{-2.6\}$.

33. $x+\frac{3}{4}=-\frac{9}{2}$

$x+\frac{3}{4}-\frac{3}{4}=-\frac{9}{2}-\frac{3}{4}$

$x=-\frac{21}{4}$

Check:

$-\frac{21}{4}+\frac{3}{4}=-\frac{9}{2}$

$-\frac{18}{4}=-\frac{9}{2}$

$-\frac{9}{2}=-\frac{9}{2}$

The solution set is $\left\{-\frac{21}{4}\right\}$.

34. $r+\frac{3}{5}=-\frac{7}{10}$

$r=-\frac{7}{10}-\frac{6}{10}$

$=-\frac{13}{10}$

Check:

$-\frac{13}{10}+\frac{3}{5}=-\frac{7}{10}$

$-\frac{13}{10}+\frac{6}{10}=-\frac{7}{10}$

$-\frac{7}{10}=-\frac{7}{10}$

The solution set is $\left\{-\frac{13}{10}\right\}$.

35. $5=-13+y$

$5+13=y$

$18=y$

Check:

$5=-13+18$

$5=5$

The solution set is $\{18\}$.

36. $-11=8+x$

$-11=8+x$

$-19=x$

Check:

$-11=8+(-19)$

$-11=-19$

The solution set is $\{-19\}$.

37. $-\frac{3}{5}=-\frac{3}{2}+s$

$-\frac{3}{5}+\frac{3}{2}=s$

$-\frac{6}{10}+\frac{15}{10}=s$

$\frac{9}{10}=s$

Check:

$-\frac{3}{5}=-\frac{3}{2}+\frac{9}{10}$

$-\frac{6}{10}=-\frac{15}{10}+\frac{9}{10}$

$-\frac{6}{10}=-\frac{6}{10}$

The solution set is $\left\{\frac{9}{10}\right\}$.

38. $\frac{7}{3}=-\frac{5}{2}+z$

$\frac{7}{3}+\frac{5}{2}=z$

$\frac{14+15}{6}=z$

$z=\frac{29}{6}$

Check:

$\frac{7}{3}=-\frac{5}{2}+\frac{29}{6}$

$\frac{14}{6}=-\frac{15}{6}+\frac{29}{6}$

$\frac{14}{6}=\frac{14}{6}$

The solution set is $\left\{\frac{29}{6}\right\}$.

39. $830+y=520$

$y=520-830$

$y=-310$

Check:

$830-310=520$

$520=520$

The solution set is $\{-310\}$.

40. $-90+t=-35$

$t=-35+90$

$t=55$

Check:

$-90+55=-35$

$-35=-35$

The solution set is $\{55\}$.

41. $r+3.7=8$

$r=8-3.7$

$r=4.3$

Check:

$4.3+3.7=8$

$8=8$

The solution set is $\{4.3\}$.

42. $x+10.6=-9$

$x=-9-10.6$

$x=-19.6$

Check:

$-19.6+10.6=-9$

$-9=-9$

The solution set is $\{-19.6\}$.

43. $-3.7+m=-3.7$

$m=-3.7+3.7$

$m=0$

Check:

$-3.7+0=-3.7$

$-3.7=-3.7$

The solution set is $\{0\}$.

44. $y+\frac{7}{11}=\frac{7}{11}$

$y=\frac{7}{11}-\frac{7}{11}$

$y=0$

Check:

$0+\frac{7}{11}=\frac{7}{11}$

$\frac{7}{11}=\frac{7}{11}$

The solution set is $\{0\}$.

45. $6y+3-5y=14$

$y+3=14$

$y=14-3$

$y=11$

Check:

$6(11)+3-5(11)=14$

$66+3-55=14$

$14=14$

The solution set is $\{11\}$.

46. $-3x-5+4x=9$

$x-5=9$

$x=14$

Check:

$-3(14)-5+(14)=9$

$-42-5+56=9$

$-49+56=9$

$9=9$

The solution set is $\{14\}$.

47. $7-5x+8+2x+4x-3=2+3\cdot 5$

$x+12=17$

$x=5$

Check:

$7-5(5)+8+2(5)+4(5)-3=2+3\cdot 5$

$17=17$

The solution set is $\{5\}$.

48. $13-3r+2+6r-2r-1=3+2\cdot 9$

$(-3r+6r-2r)+(13+2-1)=3+18$

$r+14=21$

$r+14-14=21-14$

$r=7$

Check:

$13-3(7)+2+6(7)-2(7)-1=3+2\cdot 9$

$13-21+2+42-14-1=3+18$

$21=21$

The solution set is $\{7\}$.

49. $7y+4=6y-9$

$7y-6y+4=-9$

$y=-9-4$

$y=-13$

Check:

$7(-13)+4=6(-13)-9$

$-91+4=-78-9$

$-87=-87$

The solution set is $\{-13\}$.

50. $4r-3=5+3r$

$4r-3-3r=5+3r-3r$

$r-3=5$

$r-3+3=5+3$

$r=8$

Check:

$4(8)-3=5+3(8)$

$32-3=5+24$

$29=29$

The solution set is $\{8\}$.

51. $12-6x=18-7x$

$12+x=18$

$x=6$

Check:

$12-6(6)=18-7(6)$

$12-36=18-42$

$-24=-24$

The solution set is $\{6\}$.

52. $20-7s=26-8s$

$20-7s+8s=26-8s+8s$

$20+s=26$

$20-20+s=26-20$

$s=6$

Check:

$20-7(6)=26-8(6)$

$20-42=26-48$

$-22=-22$

The solution set is $\{6\}$.

53. $4x+2=3(x-6)+8$

$4x+2=3x-18+8$

$4x+2=3x-10$

$4x-3x+2=-10$

$x+2=-10$

$x=-10-2$

$x=-12$

Check:

$4(-12)+2=3(-12-6)+8$

$-48+2=3(-18)+8$

$-46=-54+8$

$-46=-46$

The solution set is $\{-12\}$.

54. $7x+3=6(x-1)+9$

$7x+3=6x-6+9$

$7x+3=6x+3$

$x+3=3$

$x=0$

Check:

$7(0)+3=6(0-1)+9$

$0+3=6(-1)+9$

$3=-6+9$

$3=3$

The solution set is $\{0\}$.

55. $x-\square=\triangle$

$x-\square+\square=\triangle+\square$

$x=\triangle+\square$

56. $x+\square=\triangle$

$x+\square-\square=\triangle-\square$

$x=\triangle-\square$

57. $2x+\triangle=3x+\square$

$\triangle=3x-2x+\square$

$\triangle=x+\square$

$\triangle-\square=x+\square-\square$

$\triangle-\square=x$

58. $6x-\triangle=7x-\square$

$6x-\triangle-6x=7x-\square-6x$

$-\triangle=x-\square$

$-\triangle+\square=x-\square+\square$

$\square-\triangle=x$

59. $x-12=-2$

$x=-2+12$

$x=10$

The number is 10.

60. $x-23=-8$

$x-23+23=-8+23$

$x=15$

The number is 15.

61. $\frac{2}{5}x-8=\frac{7}{5}x$

$-8=\frac{7}{5}x-\frac{2}{5}x$

$-8=\frac{5}{5}x$

$-8=x$

The number is -8.

62. $3-\frac{2}{7}x=\frac{5}{7}x$

$3-\frac{2}{7}x+\frac{2}{7}x=\frac{5}{7}x+\frac{2}{7}x$

$3=\frac{7}{7}x$

$3=x$

The number is 3.

63. $S=1850,\ M=150$

$C+M=S$

$C+150=1850$

$C=1850-150$

$C=1700$

The cost of the computer is \$1700.

64. $C=520, S=650$

$C+M=S$

$520+M=650$

$M=650-520$

$M=130$

The markup is \$130.

65. **a.** $p-0.8x=25$

$p-0.8(30)=25$

$p-24=25$

$p=49$

According to the formula, 49% of U.S. college freshman had an average grade of A in high school in 2010.
This overestimates the value given in the bar graph by 1%.

b. $p-0.8x=25$

$p-0.8(40)=25$

$p-32=25$

$p=57$

According to the formula, 57% of U.S. college freshman had an average grade of A in high school in 2020.

66. **a.** $p-0.8x=25$

$p-0.8(20)=25$

$p-16=25$

$p=41$

According to the formula, 41% of U.S. college freshman had an average grade of A in high school in 2000.
This underestimates the value given in the bar graph by 2%.

b. $p-0.8x=25$

$p-0.8(50)=25$

$p-40=25$

$p=65$

According to the formula, 65% of U.S. college freshman had an average grade of A in high school in 2030.

67. a. According to the line graph, the U.S. diversity index was about 55 in 2010.

b. 2010 is 30 years after 1980.

$$I - 0.7x = 34$$
$$I - 0.7(30) = 34$$
$$I - 21 = 34$$
$$I = 55$$

According to the formula, the U.S. diversity index was 55 in 2010.
This matches the line graph very well.

68. a. According to the line graph, the U.S. diversity index was about 47 in 2000.

b. 2000 is 20 years after 1980.

$$I - 0.7x = 34$$
$$I - 0.7(20) = 34$$
$$I - 14 = 34$$
$$I = 48$$

According to the formula, the U.S. diversity index was 48 in 2000.
This matches the line graph very well.

69. – 71. Answers will vary.

72. The adjective linear means that the points lie on a line.

73. does not make sense; Explanations will vary.
Sample explanation: It does not matter whether the number is added beside or below, as long as it is added to both sides of the equation.

74. makes sense

75. makes sense

76. makes sense

77. false; Changes to make the statement true will vary.
A sample change is: If $y - a = -b$, then $y = a - b$.

78. false; Changes to make the statement true will vary.
A sample change is: If $y + 7 = 0$, then $y = -7$.

79. true

80. false; Changes to make the statement true will vary.
A sample change is: If $3x = 18$, then $x = \frac{18}{3} = 6$.

81. Answers will vary. An example is: $x - 100 = -101$

82.
$$x - 7.0463 = -9.2714$$
$$x = -9.2714 + 7.0463$$
$$x = -2.2251$$

The solution set is $\{-2.2251\}$.

83.
$$6.9825 = 4.2296 + y$$
$$6.9825 - 4.2296 = y$$
$$2.7529 = y$$

The solution set is $\{2.7529\}$.

84. $\frac{9}{x} - 4x$

85.
$$-16 - 8 \div 4 \cdot (-2) = -16 - 2 \cdot (-2)$$
$$= -16 + (-2)(-2)$$
$$= -16 + 4$$
$$= -12$$

86.
$$3\left[7x - 2(5x - 1)\right] = 3\left[7x - 10x + 2\right]$$
$$= 3\left[-3x + 2\right]$$
$$= -9x + 6 \text{ or } 6 - 9x$$

87. $5 \cdot \frac{x}{5} = \frac{5}{1} \cdot \frac{x}{5} = x$

88. $\frac{-7y}{-7} = y$

89.
$$3x - 14 = -2x + 6$$
$$3(4) - 14 = -2(4) + 6$$
$$12 - 14 = -8 + 6$$
$$-2 = -2, \text{ true}$$

Yes, 4 is a solution of the equation.

2.2 Check Points

1. $\frac{x}{3}=12$

$3\cdot\frac{x}{3}=12\cdot 3$

$1x=36$

$x=36$

Check:

$\frac{x}{3}=12$

$\frac{36}{3}=12$

$12=12$

The solution set is $\{36\}$.

2. a. $4x=84$

$\frac{4x}{4}=\frac{84}{4}$

$1x=21$

$x=21$

The solution set is $\{21\}$.

b. $-11y=44$

$\frac{-11y}{-11}=\frac{44}{-11}$

$1x=-4$

$x=-4$

The solution set is $\{-4\}$.

c. $-15.5=5z$

$\frac{-15.5}{5}=\frac{5z}{5}$

$-3.1=1z$

$-3.1=z$

The solution set is $\{-3.1\}$.

3. a. $\frac{2}{3}y=16$

$\frac{3}{2}\left(\frac{2}{3}y\right)=\frac{3}{2}\cdot 16$

$1y=24$

$y=24$

The solution set is $\{24\}$.

b. $28=-\frac{7}{4}x$

$-\frac{4}{7}\cdot 28=-\frac{4}{7}\left(-\frac{7}{4}x\right)$

$-16=1x$

$-16=x$

The solution set is $\{-16\}$.

4. a. $-x=5$

$-1x=5$

$(-1)(-1x)=(-1)5$

$1x=-5$

$x=-5$

The solution set is $\{-5\}$.

b. $-x=-3$

$-1x=-3$

$(-1)(-1x)=(-1)(-3)$

$1x=3$

$x=3$

The solution set is $\{3\}$.

5. $4x+3=27$

$4x+3-3=27-3$

$4x=24$

$\frac{4x}{4}=\frac{24}{4}$

$x=6$

The solution set is $\{6\}$.

6. $-4y-15=25$

$-4y-15+15=25+15$

$-4y=40$

$\frac{-4y}{-4}=\frac{40}{-4}$

$y=-10$

The solution set is $\{-10\}$.

7. $2x-15=-4x+21$

$2x+4x-15=-4x+4x+21$

$6x-15=21$

$6x-15+15=21+15$

$6x=36$

$\frac{6x}{6}=\frac{36}{6}$

$x=6$

The solution set is $\{6\}$.

8. a. The bar graph indicates that the median weekly earnings for men with a bachelor's degree and higher in 2013 was \$1395. Since 2013 is 33 years after 1980, substitute 33 into the formula for n.

$M=29n+427$

$M=29(33)+427$

$M=957+427$

$M=1384$

The formula indicates that the median weekly earnings for men with a bachelor's degree and higher in 2013 was \$1384. The formula underestimates by \$11.

b. $M=29n+427$

$1442=29n+427$

$1442-427=29n+427-427$

$1015=29n$

$\frac{1015}{29}=\frac{29n}{29}$

$35=n$

The formula estimates that 35 years after 1980, or in 2015, the median weekly earnings for men with a bachelor's degree and higher will be \$1442.

2.2 Concept and Vocabulary Check

1. *bc*

2. divide

3. multiplying; 7

4. dividing; -8

Alternatively, multiplying; $-\frac{1}{8}$

5. multiplying; $\frac{5}{3}$

6. multiplying/dividing; -1

7. subtracting 2; dividing; 5

2.2 Exercise Set

1. $\frac{x}{6}=5$

$6\cdot\frac{x}{6}=6\cdot 5$

$1x=30$

$x=30$

Check:

$\frac{30}{6}=5$

$5=5$

The solution set is $\{30\}$.

2. $\frac{x}{7}=4$

$7\cdot\frac{x}{7}=7\cdot 4$

$x=28$

Check:

$\frac{28}{7}=4$

$4=4$

The solution set is $\{28\}$.

3. $\frac{x}{-3}=11$

$-3\cdot\frac{x}{-3}=-3(11)$

$1x=-33$

$x=-33$

Check:

$\frac{-33}{-3}=11$

$11=11$

The solution set is $\{-33\}$.

4. $\dfrac{x}{-5}=8$

$-5\cdot\dfrac{x}{-5}=8(-5)$

$x=-40$

Check:

$\dfrac{-40}{-5}=8$

$8=8$

The solution set is $\{-40\}$.

5. $5y=35$

$\dfrac{5y}{5}=\dfrac{35}{5}$

$y=7$

Check:

$5(7)=35$

$35=35$

The solution set is $\{7\}$.

6. $6y=42$

$\dfrac{6y}{6}=\dfrac{42}{6}$

$y=7$

Check:

$6(7)=42$

$42=42$

The solution set is $\{7\}$.

7. $-7y=63$

$\dfrac{-7y}{-7}=\dfrac{63}{-7}$

$y=-9$

Check:

$-7(-9)=63$

$63=63$

The solution set is $\{-9\}$.

8. $-4y=32$

$\dfrac{-4y}{-4}=\dfrac{32}{-4}$

$y=-8$

Check:

$-4(-8)=32$

$32=32$

The solution set is $\{-8\}$.

9. $-28=8z$

$\dfrac{-28}{8}=\dfrac{8z}{8}$

$-\dfrac{7}{2}=z$

Check:

$-28=8\left(-\dfrac{7}{2}\right)$

$-28=-\dfrac{56}{2}$

$-28=-28$

The solution set is $\left\{-\dfrac{7}{2}\right\}$. or $\left\{-3\dfrac{1}{2}\right\}$.

10. $-36=8z$

$\dfrac{-36}{8}=\dfrac{8z}{8}$

$-\dfrac{9}{2}=z$

Check:

$-36=8\left(-\dfrac{9}{2}\right)$

$-36=-36$

The solution set is $\left\{-\dfrac{9}{2}\right\}$.

11. $-18=-3z$

$\dfrac{-18}{-3}=\dfrac{-3z}{-3}$

$6=z$

Check:

$-18=-3(6)$

$-18=-18$

The solution set is $\{6\}$.

12. $-54=-9z$

$\dfrac{-54}{-9}=\dfrac{-9z}{-9}$

$6=z$

Check:

$-54=-9(6)$

$-54=-54$

The solution set is $\{6\}$.

13. $-8x = 6$

$$\frac{-8x}{-8} = \frac{6}{-8}$$

$$x = -\frac{6}{8} = -\frac{3}{4}$$

Check:

$$-8\left(-\frac{3}{4}\right) = 6$$

$$\frac{24}{4} = 6$$

$$6 = 6$$

The solution set is $\left\{-\frac{3}{4}\right\}$.

14. $-8x = 4$

$$\frac{-8x}{-8} = \frac{4}{-8}$$

$$x = -\frac{4}{8} = -\frac{1}{2}$$

Check:

$$-8\left(-\frac{1}{2}\right) = 4$$

$$4 = 4$$

The solution set is $\left\{-\frac{1}{2}\right\}$.

15. $17y = 0$

$$\frac{17y}{17} = \frac{0}{17}$$

$$y = 0$$

Check:

$$17(0) = 0$$

$$0 = 0$$

The solution set is $\{0\}$.

16. $-16y = 0$

$$\frac{-16y}{-16} = \frac{0}{-16}$$

$$y = 0$$

Check:

$$-16(0) = 0$$

$$0 = 0$$

The solution set is $\{0\}$.

17. $\frac{2}{3}y = 12$

$$\frac{3}{2}\left(\frac{2}{3}y\right) = \frac{3}{2}(12)$$

$$1y = \frac{3}{2}\cdot\frac{12}{1} = \frac{36}{2}$$

$$y = 18$$

Check:

$$\frac{2}{3}(18) = 12$$

$$\frac{36}{3} = 12$$

$$12 = 12$$

The solution set is $\{18\}$.

18. $\frac{3}{4}y = 15$

$$\frac{4}{3}\left(\frac{3}{4}y\right) = \frac{4}{3}(15)$$

$$1y = \frac{4}{3}\cdot\frac{15}{1} = \frac{60}{3}$$

$$y = 20$$

Check:

$$\frac{3}{4}(20) = 15$$

$$\frac{3}{4}\cdot\frac{20}{1} - 15$$

$$\frac{60}{4} = 15$$

$$15 = 15$$

The solution set is $\{20\}$.

19. $28 = -\frac{7}{2}x$

$$-\frac{2}{7}(28) = -\frac{2}{7}\left(-\frac{7}{2}x\right)$$

$$-\frac{56}{7} = 1x$$

$$-8 = x$$

Check:

$$28 = -\frac{7}{2}(-8)$$

$$28 = \frac{56}{2}$$

$$28 = 28$$

The solution set is $\{-8\}$.

20. $20 = -\frac{5}{8}x$

$-\frac{8}{5}(20) = -\frac{8}{5}\left(-\frac{5}{8}x\right)$

$-\frac{160}{5} = 1x$

$-32 = x$

Check:

$20 = -\frac{5}{8}(-32)$

$20 = \frac{160}{8}$

$20 = 20$

The solution set is $\{-32\}$.

21. $-x = 17$

$-1x = 17$

$-1(-1x) = -1(17)$

$x = -17$

Check:

$-(-17) = 17$

$17 = 17$

The solution set is $\{-17\}$.

22. $-x = 23$

$-1x = 23$

$-1(-1x) = -1(23)$

$x = -23$

Check:

$-(-23) = 23$

$23 = 23$

The solution set is $\{-23\}$.

23. $-47 = -y$

$-47 = -1(-y)$

$-1(-47) = -1(-1)(-y)$

$47 = y$

Check:

$-47 = -y$

$-47 = -(47)$

$-47 = -47$

The solution set is $\{47\}$.

24. $-51 = -y$

$\frac{-51}{-1} = \frac{-y}{-1}$

$51 = y$

Check:

$-51 = -51$

The solution set is $\{51\}$.

25. $-\frac{x}{5} = -9$

$5\left(-\frac{x}{5}\right) = 5(-9)$

$-x = -45$

$x = 45$

Check:

$-\frac{45}{5} = -9$

$-9 = -9$

The solution set is $\{45\}$.

26. $-\frac{x}{5} = -1$

$-5\left(-\frac{x}{5}\right) = -5(-1)$

$x = 5$

Check:

$-\frac{5}{5} = -1$

$-1 = -1$

The solution set is $\{5\}$.

27. $2x - 12x = 50$

$(2-12)x = 50$

$-10x = 50$

$\frac{-10x}{-10} = \frac{50}{-10}$

$x = -5$

Check:

$2(-5) - 12(-5) = 50$

$-10 + 60 = 50$

$50 = 50$

The solution set is $\{-5\}$.

28. $8x-3x=-45$

$8x+(-3x)=-45$

$5x=-45$

$\frac{5x}{5}=\frac{-45}{5}$

$x=-9$

Check:

$8(-9)-3(-9)=-45$

$-72+27=-45$

$-45=-45$

The solution set is $\{-9\}$.

29. $2x+1=11$

$2x+1-1=11-1$

$2x=10$

$\frac{2x}{2}=\frac{10}{2}$

$x=5$

Check:

$2(5)+1=11$

$10+1=11$

$11=11$

The solution set is $\{5\}$.

30. $2x+5=13$

$2x+5-5=13-5$

$2x=8$

$\frac{2x}{2}=\frac{8}{2}$

$x=4$

Check:

$2(4)+5=13$

$8+5=13$

$13=13$

The solution set is $\{4\}$.

31. $2x-3=9$

$2x-3+3=9+3$

$2x=12$

$\frac{2x}{2}=\frac{12}{2}$

$x=6$

Check:

$2(6)-3=9$

$12-3=9$

$9=9$

The solution set is $\{6\}$.

32. $3x-2=9$

$3x-2+2=9+2$

$3x=11$

$\frac{3x}{3}=\frac{11}{3}$

$x=\frac{11}{3}$

Check:

$3\left(\frac{11}{3}\right)=9$

$11-2=9$

$9=9$

The solution set is $\left\{\frac{11}{3}\right\}$.

33. $-2y+5=7$

$-2y+5-5=7-5$

$-2y=2$

$\frac{-2y}{2}=\frac{2}{-2}$

$y=-1$

Check:

$-2(-1)+5=7$

$2+5=7$

$7=7$

The solution set is $\{-1\}$.

34. $-3y+4=13$

$-3y+4-4=13-4$

$-3y=9$

$\frac{-3y}{-3}=\frac{9}{-3}$

$y=-3$

Check:

$-3(-3)+4=13$

$9+4=13$

$13=13$

The solution set is $\{-3\}$.

35. $-3y-7=-1$

$-3y-7+7=-1+7$

$-3y=6$

$\frac{-3y}{-3}=\frac{6}{-3}$

$y=-2$

Check:

$-3(-2)-7=-1$

$6-7=-1$

$-1=-1$

The solution set is $\{-2\}$.

36. $-2y-5=7$

$-2y-5+5=7+5$

$-2y=12$

$\frac{-2y}{-2}=\frac{12}{-2}$

$y=-6$

Check:

$-2(-6)-5=7$

$12-5=7$

$7=7$

The solution set is $\{-6\}$.

37. $12=4z+3$

$12-3=4z+3-3$

$9=4z$

$\frac{9}{4}=\frac{4z}{4}$

$\frac{9}{4}=z$

Check:

$12=4\left(\frac{9}{4}\right)+3$

$12=9+3$

$12=12$

The solution set is $\left\{\frac{9}{4}\right\}$.

38. $14=5z-21$

$14+21=5z-21+21$

$35=5z$

$\frac{35}{5}=\frac{5z}{5}$

$7=z$

Check:

$14=5(7)-21$

$14=35-21$

$14=14$

The solution set is $\{7\}$.

39. $-x-3=3$

$-x-3+3=3+3$

$-x=6$

$x=-6$

Check:

$-(-6)-3=3$

$6-3=3$

$3=3$

The solution set is $\{-6\}$.

40. $-x-5=5$

$-x-5+5=5+5$

$-x=10$

$x=-10$

Check:

$-(-10)-5=5$

$10-5=5$

$5=5$

The solution set is $\{-10\}$.

41. $6y=2y-12$

$6y+12=2y-12+12$

$6y+12=2y$

$6y+12-6y=2y-6y$

$12=-4y$

$\frac{12}{-4}=\frac{-4y}{-4}$

$-3=y$

Check:

$6(-3)=2(-3)-12$

$-18=-6-12$

$-18=-18$

The solution set is $\{-3\}$.

42. $8y=3y-10$

$8y-3y=3y-10-3y$

$5y=-10$

$\frac{5y}{5}=\frac{-10}{5}$

$y=-2$

Check:

$8(-2)=3(-2)-10$

$-16=-6-16$

$-16=-16$

The solution set is $\{-2\}$.

43. $3z=-2z-15$

$3z+2z=-2z-15+2z$

$5z=-15$

$\frac{5z}{5}=\frac{-15}{3}$

$z=-3$

Check:

$3(-3)=-2(-3)-15$

$-9=6-15$

$-9=-9$

The solution set is $\{-3\}$.

44. $2z=-4z+18$

$2z+4z=-4z+18+4z$

$6z=18$

$\frac{6z}{6}=\frac{18}{6}$

$z=3$

Check:

$2(3)=-4(3)+18$

$6=-12+18$

$6=6$

The solution set is $\{3\}$.

45. $-5x=-2x-12$

$-5x+2x=-2x-12+2x$

$-3x=-12$

$\frac{-3x}{3}=\frac{-12}{-3}$

$x=4$

Check:

$-5(4)=2(4)-12$

$-20=-8-12$

$-20=-20$

The solution set is $\{4\}$.

46. $-7x = -3x - 8$

$-7x + 3x = -3x - 8 + 3x$

$-4x = -8$

$\frac{-4x}{-4} = \frac{-8}{-4}$

$x = 2$

Check:

$-7(2) = -3(2) - 8$

$-14 = -6 - 8$

$-14 = -14$

The solution set is $\{2\}$.

47. $8y + 4 = 2y - 5$

$8y + 4 - 2y = 2y - 5 - 2y$

$6y + 4 = -5$

$6y + 4 - 4 = -5 - 4$

$6y = -9$

$\frac{6y}{6} = \frac{-9}{6}$

$y = -\frac{3}{2}$

Check:

$8\left(-\frac{3}{2}\right) + 4 = 2\left(-\frac{3}{2}\right) - 5$

$-12 + 4 = -3 - 5$

$-8 = -8$

The solution set is $\left\{-\frac{3}{2}\right\}$.

48. $5y + 6 = 3y - 6$

$5y + 6 - 3y = 3y - 6 - 3y$

$2y + 6 = -6$

$2y + 6 - 6 = -6 - 6$

$2y = -12$

$\frac{2y}{2} = \frac{-12}{2}$

$y = -6$

Check:

$5(-6) + 6 = 3(-6) - 6$

$-30 + 6 = -18 - 6$

$-24 = -24$

The solution set is $\{-6\}$.

49. $6z - 5 = z + 5$

$6z - 5 - z = z + 5 - z$

$5z - 5 = 5$

$5z - 5 + 5 = 5 + 5$

$5z = 10$

$\frac{5z}{5} = \frac{10}{5}$

$z = 2$

Check:

$6(2) - 5 = 2 + 5$

$12 - 5 = 2 + 5$

$7 = 7$

The solution set is $\{2\}$.

50. $6z - 3 = z + 2$

$6z - 3 - z = z + 2 - z$

$5z - 3 = 2$

$5z - 3 + 3 = 2 + 3$

$5z = 5$

$\frac{5z}{5} = \frac{5}{5}$

$z = 1$

Check:

$6(1) - 3 = 1 + 2$

$6 - 3 = 3$

$3 = 3$

The solution set is $\{1\}$.

51. $6x + 14 = 2x - 2$

$6x - 2x + 14 = -2$

$4x = -2 - 14$

$4x = -16$

$x = -4$

Check:

$6(-4) + 14 = 2(-4) - 2$

$-24 + 14 = -8 - 2$

$-10 = -10$

The solution set is $\{-4\}$.

52. $9x+2=6x-4$

$9x+2-6x=6x-4-6x$

$3x+2=-4$

$3x+2-2=-4-2$

$3x=-6$

$\frac{3x}{3}=\frac{-6}{3}$

$x=-2$

Check:

$9(-2)+2=6(-2)-4$

$-18+2=-12-4$

$-16=-16$

The solution set is $\{-2\}$.

53. $-3y-1=5-2y$

$-3y+2y-1=5$

$-y=5+1$

$-y=6$

$y=-6$

Check:

$-3(-6)-1=5-2(-6)$

$18-1=5+12$

$17=17$

The solution set is $\{-6\}$.

54. $-3y-2=-5-4y$

$-3y-2+4y=-5-4y+4y$

$y-2=-5$

$y-2+2=-5+2$

$y=-3$

Check:

$-3(-3)-2=-5-4(-3)$

$9-2=-5+12$

$7=7$

The solution set is $\{-3\}$.

55. $\frac{x}{\square}=\triangle$

$\square\cdot\frac{x}{\square}=\triangle\cdot\square$

$x=\triangle\square$

56. $\Delta=\square x$

$\frac{\Delta}{\square}=\frac{\square x}{\square}$

$\frac{\Delta}{\square}=x$

57. $\triangle=-x$

$\triangle(-1)=-x(-1)$

$-\triangle=x$

58. $\frac{-x}{\square}=\Delta$

$-\square\cdot\frac{-x}{\square}=-\square\cdot\Delta$

$x=-\square\cdot\Delta$

59. $6x=10$

$\frac{6x}{6}=\frac{10}{6}$

$x=\frac{10}{6}=\frac{5}{3}$

The number is $\frac{5}{3}$.

60. $-6\cdot x=20$

$\frac{-6x}{-6}=\frac{20}{-6}$

$x=-\frac{10}{3}$

The number is $-\frac{10}{3}$.

61. $\frac{x}{-9}=5$

$\frac{x}{-9}(-9)=5(-9)$

$x=-45$

The number is -45.

62. $\frac{x}{-7}=8$

$-7\cdot\frac{x}{-7}=-7\cdot 8$

$x=-56$

The number is -56.

63. $4x-8=56$

$4x-8+8=56+8$

$4x=64$

$\frac{4x}{4}=\frac{64}{4}$

$x=16$

The number is 16.

64. $3x-10=23$

$3x-10+10=23+10$

$3x=33$

$\frac{3x}{3}=\frac{33}{3}$

$x=11$

The number is 11.

65. $-3x+15=-6$

$-3x+15-15=-6-15$

$-3x=-21$

$\frac{-3x}{-3}=\frac{-21}{-3}$

$x=7$

The number is 7.

66. $-5x+11=-29$

$-5x+11-11=-29-11$

$-5x=-40$

$\frac{-5x}{-5}=\frac{-40}{-5}$

$x=8$

The number is 8.

67. $M=\frac{n}{5}$

$2=\frac{n}{5}$

$5(2)=5\left(\frac{n}{5}\right)$

$10=n$

If you are 2 miles away from the lightning flash, it will take 10 seconds for the sound of thunder to reach you.

68. $M=\frac{n}{5}$

$3=\frac{n}{5}$

$5(3)=5\left(\frac{n}{5}\right)$

$15=n$

If you are 3 miles away from the lightning flash, it will take 15 seconds for the sound of thunder to reach you.

69. $M=\frac{A}{740}$

$2.03=\frac{A}{740}$

$740(2.03)=740\cdot\frac{A}{740}$

$1502.2=A$

The speed of the Concorde is 1502.2 miles per hour.

70. $M=\frac{A}{740}$

$3.3=\frac{A}{740}$

$740(3.3)=740\cdot\frac{A}{740}$

$2442=A$

The speed of the SR-71 Blackbird is 2442 miles per hour.

71. **a.** The bar graph indicates the median weekly earnings, in 2013, for men with some college or an associate's degree is \$858. Since 2013 is 33 years after 1980, substitute 33 into the formula for n.

$M=15n+358$

$M=15(33)+358$

$M=853$

The formula indicates the median weekly earnings, in 2013, for men with some college or an associate's degree is \$853. The formula underestimates by \$5.

b. $M=15n+358$

$1033=15n+358$

$675=15n$

$45=n$

The formula indicates the median weekly earnings for men with some college or an associate's degree will reach \$1033 45 years after 1980, or in 2025.

72. a. The bar graph indicates the median weekly earnings, in 2013, for women with some college or an associate's degree is \$657. Since 2013 is 33 years after 1980, substitute 33 into the formula for n.

$W = 13n + 231$

$W = 13(33) + 231$

$W = 660$

The formula indicates the median weekly earnings, in 2013, for women with some college or an associate's degree is \$660. The formula overestimates by \$3.

b. $W = 13n + 231$

$777 = 13n + 231$

$546 = 13n$

$42 = n$

The formula indicates the median weekly earnings for women with some college or an associate's degree will reach \$777 42 years after 1980, or in 2022.

73. – 75. Answers will vary.

76. does not make sense; Explanations will vary. Sample explanation: The addition property of equality is not necessary for this equation.

77. does not make sense; Explanations will vary. Sample explanation: When you subtract 12 from $12 - 3x$, you should obtain $-3x$, not positive $3x$.

78. makes sense

79. does not make sense; Explanations will vary. Sample explanation: To determine the price in 2009, substitute 69 in for n and simplify.

80. false; Changes to make the statement true will vary. A sample change is: If $7x = 21$, then $\frac{7x}{7} = \frac{21}{7} = 3$.

81. false; Changes to make the statement true will vary. A sample change is: If $3x - 4 = 16$, then $3x = 20$.

82. false; Changes to make the statement true will vary. A sample change is: If $3x + 7 = 0$, then $3x = -7$ and $x = \frac{-7}{3}$.

83. true

84. Answers will vary. Start by selecting the integer answer and set x equal to this value. Then, multiply both sides of this equation by -60 (since we will divide both sides of the equation by -60 to solve). For example, suppose we want the solution to be 3. We set x equal to this value and write $x = 3$. Now multiply both sides of the equation by -60.

$x = 3$

$-60 \cdot x = -60 \cdot 3$

$-60x = -180$

So, our equation is $-60x = -180$ and the solution is 3 (an integer).

85. Answers will vary. As an example, start with an integer solution, such as 10, and set it equal to x. That is, we have $x = 10$. The solution was obtained by multiplying both sides by $\frac{4}{5}$. To undo this, we multiply both sides of our equation by the reciprocal, $\frac{5}{4}$. This gives, $\frac{5}{4}x = \frac{5}{4}(10)$

$\frac{5}{4}x = \frac{25}{2}$

Therefore, an example equation would be $\frac{5}{4}x = \frac{25}{2}$.

86. $3.7x - 19.46 = -9.988$

$3.7x = -9.988 + 19.46$

$3.7x = 9.472$

$\frac{3.7x}{3.7} = \frac{9.472}{3.7}$

$x = 2.56$

The solution set is $\{2.56\}$.

87. $-72.8y - 14.6 = -455.43 - 4.98y$

$-72.8y - 14.6 + 4.98y = -455.43 - 4.98y + 4.98y$

$-67.82y - 14.6 = -455.43$

$-67.82y - 14.6 + 14.6 = -455.43 + 14.6$

$-67.82y = -440.83$

$\frac{-67.82y}{-67.82} = \frac{-440.83}{-67.82}$

$y = 6.5$

The solution set is $\{6.5\}$.

88. $(-10)^2 = (-10)(-10) = 100$

89. $-10^2 = -1 \cdot 10^2 = -1(10)(10) = -100$

90. $x^3 - 4x = (-1)^3 - 4(-1)$
$= -1 + 4$
$= 3$

91. $13 - 3(x+2) = 13 - 3x - 6$
$= -3x + 7$

92. $2(x-3) - 17 = 13 - 3(x+2)$
$2(6-3) - 17 = 13 - 3(6+2)$
$2(3) - 17 = 13 - 3(8)$
$6 - 17 = 13 - 24$
$-11 = -11,$ true
Yes, 6 is a solution of the equation.

93. $10\left(\frac{x}{5} - \frac{39}{5}\right) = 10 \cdot \frac{x}{5} - 10 \cdot \frac{39}{5}$
$= 2x - 78$

2.3 Check Points

1. Simplify the algebraic expression on each side.
$-7x + 25 + 3x = 16 - 2x - 3$
$-4x + 25 = 13 - 2x$
Collect variable terms on one side and constant terms on the other side.
$-4x + 25 = 13 - 2x$
$-4x + 25 + 2x = 13 - 2x + 2x$
$-2x + 25 = 13$
$-2x + 25 - 25 = 13 - 25$
$-2x = -12$
Isolate the variable and solve.
$\frac{-2x}{-2} = \frac{-12}{-2}$
$x = 6$
The solution set is $\{6\}$.

2. Simplify the algebraic expression on each side.
$8x = 2(x+6)$
$8x = 2x + 12$
Collect variable terms on one side and constant terms on the other side.
$8x - 2x = 2x - 2x + 12$
$6x = 12$
Isolate the variable and solve.
$\frac{6x}{6} = \frac{12}{6}$
$x = 2$
The solution set is $\{2\}$.

3. Simplify the algebraic expression on each side.
$4(2x+1) - 29 = 3(2x-5)$
$8x + 4 - 29 = 6x - 15$
$8x - 25 = 6x - 15$
Collect variable terms on one side and constant terms on the other side.
$8x - 6x - 25 = 6x - 6x - 15$
$2x - 25 = -15$
$2x - 25 + 25 = -15 + 25$
$2x = 10$
Isolate the variable and solve.
$\frac{2x}{2} = \frac{10}{2}$
$x = 5$
The solution set is $\{5\}$.

4. Begin by multiplying both sides of the equation by 12, the least common denominator.
$\frac{x}{4} = \frac{2x}{3} + \frac{5}{6}$
$12 \cdot \frac{x}{4} = 12\left(\frac{2x}{3} + \frac{5}{6}\right)$
$12 \cdot \frac{x}{4} = 12 \cdot \frac{2x}{3} + 12 \cdot \frac{5}{6}$
$3x = 8x + 10$
$3x - 8x = 8x - 8x + 10$
$-5x = 10$
$\frac{-5x}{-5} = \frac{10}{-5}$
$x = -2$
The solution set is $\{-2\}$.

5. First apply the distributive property to remove the parentheses, and then multiply both sides by 100 to clear the decimals.
$0.48x + 3 = 0.2(x-6)$
$0.48x + 3 = 0.2x - 1.2$
$100(0.48x + 3) = 100(0.2x - 1.2)$
$48x + 300 = 20x - 120$
$48x + 300 - 300 = 20x - 120 - 300$
$48x = 20x - 420$
$48x - 20x = 20x - 20x - 420$
$28x = -420$
$\frac{28x}{28} = \frac{-420}{28}$
$x = -15$
The solution set is $\{-15\}$.

6. $3x+7=3(x+1)$
$3x+7=3x+3$
$3x-3x+7=3x-3x+3$
$7=3$
The original equation is equivalent to the false statement $7=3$.
The equation has no solution. The solution set is $\{\ \}$.

7. $3(x-1)+9=8x+6-5x$
$3x-3+9=3x+6$
$3x+6=3x+6$
$3x-3x+6=3x-3x+6$
$6=6$
The original equation is equivalent to $6=6$, which is true for every value of x.
The equation's solution is all real numbers or $\{x|x \text{ is a real number}\}$.

8. $D=\frac{10}{9}x+\frac{53}{9}$
$10=\frac{10}{9}x+\frac{53}{9}$
$9\cdot 10=9\left(\frac{10}{9}x+\frac{53}{9}\right)$
$90=10x+53$
$90-53=10x+53-53$
$37=10x$
$\frac{37}{10}=\frac{10x}{10}$
$3.7=x$
$x=3.7$
The formula indicates that if the low-humor group averages a level of depression of 10 in response to a negative life event, the intensity of that event is 3.7. This is shown as the point whose corresponding value on the vertical axis is 10 and whose value on the horizontal axis is 3.7.

2.3 Concept and Vocabulary Check

1. simplify each side; combine like terms

2. 30

3. 100

4. inconsistent

5. identity

6. inconsistent

7. identity

2.3 Exercise Set

1. $5x+3x-4x=10+2$
$8x-4x=12$
$4x=12$
$\frac{4x}{4}=\frac{12}{4}$
$x=3$
The solution set is $\{3\}$.

2. $4x+8x-2x=20-15$
$10x=5$
$x=\frac{5}{10}=\frac{1}{2}$
The solution set is $\left\{\frac{1}{2}\right\}$.

3. $4x-9x+22=3x+30$
$-5x+22=3x+30$
$-5x-3x+22=30$
$-8x+22=30$
$-8x=30-22$
$-8x=8$
$\frac{-8x}{-8}=\frac{8}{-8}$
$x=-1$
The solution set is $\{-1\}$.

4. $3x+2x+64=40-7x$
$5x+64=40-7x$
$12x+64=40$
$12x=-24$
$x=-2$
The solution set is $\{-2\}$.

5. $3x+6-x=8+3x-6$
$2x+6=2+3x$
$2x+6-2=2+3x-2$
$2x+4=3x$
$2x+4-2x=3x-2x$
$4=x$
The solution set is $\{4\}$.

6. $3x+2-x=6+3x-8$
$2x+2=3x-2$
$2x+2-3x=3x-2-3x$
$-x+2=-2$
$-x+2-2=-2-2$
$-x=-4$
$x=4$
The solution set is $\{4\}$.

7. $4(x+1)=20$
$4x+4=20$
$4x=20-4$
$4x=16$
$\frac{4x}{4}=\frac{16}{4}$
$x=4$
The solution set is $\{4\}$.

8. $3(x-2)=-6$
$3x-6=-6$
$3x=0$
$x=0$
The solution set is $\{0\}$.

9. $7(2x-1)=42$
$14x-7=42$
$14x=49$
$x=\frac{49}{14}=\frac{7}{2}$
The solution set is $\left\{\frac{7}{2}\right\}$.

10. $4(2x-3)=32$
$8x-12=32$
$8x=44$
$x=\frac{44}{8}=\frac{11}{2}$
The solution set is $\left\{\frac{11}{2}\right\}$.

11. $38=30-2(x-1)$
$38=30-2x+2$
$38=32-2x$
$38-32=-2x$
$6=-2x$
$\frac{6}{-2}=\frac{-2x}{-2}$
$-3=x$
The solution set is $\{-3\}$.

12. $20=44-8(2-x)$
$20=44-16+8x$
$20=28+8x$
$-8=8x$
$-1=x$
The solution set is $\{-1\}$.

13. $2(4z+3)-8=46$
$8z+6-8=46$
$8z-2=46$
$8z-2+2=46+2$
$8z=48$
$\frac{8z}{3}=\frac{48}{8}$
$z=6$
The solution set is $\{6\}$.

14. $3(3z+5)-7=89$
$9z+15-7=89$
$9z+8=89$
$9z=81$
$z=9$
The solution set is $\{9\}$.

15. $6x-(3x+10)=14$
$6x-3x-10=14$
$3x-10=14$
$3x-10+10=14+10$
$3x=24$
$\frac{3x}{3}=\frac{24}{3}$
$x=8$
The solution set is $\{8\}$.

16. $5x-(2x+14)=10$
$5x-2x-14=10$
$3x-14=10$
$3x=24$
$x=8$
The solution set is $\{8\}$.

17. $5(2x+1)=12x-3$
$10x+5=12x-3$
$10x-10x+5=12x-10x-3$
$5=2x-3$
$5+3=2x-3+3$
$8=2x$
$\frac{8}{2}=\frac{2x}{2}$
$x=4$
The solution set is $\{4\}$.

18. $3(x+2)=x+30$
$3x+6=x+30$
$2x+6=30$
$2x=24$
$x=12$
The solution set is $\{12\}$.

19. $3(5-x)=4(2x+1)$
$15-3x=8x+4$
$15-3x-8x=8x+4-8x$
$15-11x=4$
$15-11x-15=4-15$
$-11x=-11$
$\frac{-11x}{-11}=\frac{-11}{-11}$
$x=1$
The solution set is $\{1\}$.

20. $3(3x-1)=4(3+3x)$
$9x-3=12+12x$
$-3-3=12$
$-3x=15$
$x=-5$
The solution set is $\{-5\}$.

21. $8(y+2)=2(3y+4)$
$8y+16=6y+8$
$8y+16-16=6y+8-16$
$8y=6y-8$
$8y-6y=6y-8-6y$
$2y=-8$
$y=-4$
The solution set is $\{-4\}$.

22. $8(y+3)=3(2y+12)$
$8y+24=6y+36$
$2y+24=36$
$2y=12$
$y=6$
The solution set is $\{6\}$.

23. $3x+3=7x-14-3$
$3x+3=7x-17$
$3x+3-3=7x-17-3$
$3x=7x-20$
$3x-7x=7x-20-7x$
$-4x=-20$
$\frac{-4x}{-4}=\frac{-20}{-4}$
$x=5$
The solution set is $\{5\}$.

24. $5x-4(x+9)=2x-3$

$5x-4x-36=2x-3$

$x-36=2x-3$

$x=2x+33$

$-x=33$

$x=-33$

The solution set is $\{-33\}$.

25. $5(2x-8)-2=5(x-3)+3$

$10x-40-2=5x-15+3$

$10x-42=5x-12$

$10x-42+42=5x-12+42$

$10x=5x+30$

$10x=5x+30-5x$

$5x=30$

$\frac{5x}{5}=\frac{30}{5}$

$x=6$

The solution set is $\{6\}$.

26. $7(3x-2)+5=6(2x-1)+24$

$21x-14+5=12x-6+24$

$21x-9=12x+18$

$21x=12x+27$

$9x=27$

$x=3$

The solution set is $\{3\}$.

27. $6=-4(1-x)+3(x+1)$

$6=-4+4x+3x+3$

$6=-1+7x$

$6+1=-1+7x+1$

$7=7x$

$\frac{7}{7}=\frac{7x}{7}$

$1=x$

The solution set is $\{1\}$.

28. $100=-(x-1)+4(x-6)$

$100=-x+1+4x-24$

$100=3x-23$

$123=3x$

$41=x$

The solution set is $\{41\}$.

29. $10(z+4)-4(z-2)=3(z-1)+2(z-3)$

$10z+40-4z+8=3z-3+2z-6$

$6z+48=5z-9$

$6z+48-48=5z-9-48$

$6z-5z=5z-57-5z$

$z=-57$

The solution set is $\{-57\}$.

30. $-2(z-4)-(3z-2)=-2-(6z-2)$

$-2z+8-3z+2=-2-6z+2$

$-5z+10=-6z$

$z+10=0$

$z=-10$

The solution set is $\{-10\}$.

31. $\frac{x}{5}-4=-6$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 5.

$5\left(\frac{x}{5}-4\right)=5(-6)$

$5\cdot\frac{x}{5}-5\cdot 4=-30$

$x-20=-30$

$x-20+20=-30+20$

$x=-10$

The solution set is $\{-10\}$.

32. $\frac{x}{2}+13=-22$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 2.

$\frac{x}{2}+13=-22$

$2\left(\frac{x}{2}+13\right)=2(-22)$

$2\cdot\frac{x}{2}+2\cdot 13=-44$

$x+26=-44$

$x+26-26=-44-26$

$x=-70$

The solution set is $\{-70\}$.

33. $\frac{2x}{3}-5=7$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 3.

$$3\left(\tfrac{2}{3}x-5\right)=3(7)$$
$$3\cdot\tfrac{2}{3}x-3\cdot5=21$$
$$2x-15=21$$
$$2x-15+15=21+15$$
$$2x=36$$
$$\frac{2x}{2}=\frac{36}{2}$$
$$x=18$$

The solution set is $\{18\}$.

34. $\frac{3x}{4}-9=-6$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 4.

$$4\left(\frac{3x}{4}-9\right)=4(-6)$$
$$4\cdot\frac{3x}{4}-4\cdot9=-24$$
$$3x-36=-24$$
$$3x=12$$
$$x=4$$

The solution set is $\{4\}$.

35. $\frac{2y}{3}-\frac{3}{4}=\frac{5}{12}$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 12.

$$12\left(\frac{2y}{3}-\frac{3}{4}\right)=12\left(\frac{5}{12}\right)$$
$$12\left(\frac{2y}{3}\right)-12\left(\frac{3}{4}\right)=5$$
$$8y-9=5$$
$$8y-9+9=5+9$$
$$8y=14$$
$$\frac{8y}{8}=\frac{14}{8}$$
$$y=\frac{14}{8}=\frac{7}{4}$$

The solution set is $\left\{\frac{7}{4}\right\}$.

36. $\frac{3y}{4}-\frac{2}{3}=\frac{7}{12}$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 12.

$$12\left(\frac{3y}{4}-\frac{2}{3}\right)=12\left(\frac{7}{12}\right)$$
$$12\left(\frac{3y}{4}\right)-12\left(\frac{2}{3}\right)=7$$
$$9y-8=7$$
$$9y=15$$
$$y=\frac{15}{9}=\frac{5}{3}$$

The solution set is $\left\{\frac{5}{3}\right\}$.

37. $\frac{x}{3}+\frac{x}{2}=\frac{5}{6}$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 6.

$$6\left(\frac{x}{3}+\frac{x}{2}\right)=6\left(\frac{5}{6}\right)$$
$$2x+3x=5$$
$$5x=5$$
$$\frac{5x}{5}=\frac{5}{5}$$
$$x=1$$

The solution set is $\{1\}$.

38. $\frac{x}{4}-\frac{x}{5}=1$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 20.

$$20\left(\frac{x}{4}-\frac{x}{5}\right)=20(1)$$
$$5x-4x=20$$
$$x=20$$

The solution set is $\{20\}$.

39. $20-\frac{z}{3}=\frac{z}{2}$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 6.

$$6\left(20-\frac{z}{3}\right)=6\left(\frac{z}{2}\right)$$
$$120-2z=3z$$
$$120-2z+2z=3z+2z$$
$$120=5z$$
$$\frac{120}{5}=\frac{5z}{5}$$
$$24=z$$

The solution set is $\{24\}$.

40. $\frac{z}{5}-\frac{1}{2}=\frac{z}{6}$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 30.

$$30\left(\frac{z}{5}-\frac{1}{2}\right)=30\left(\frac{z}{6}\right)$$
$$6z-15=5z$$
$$z-15=0$$
$$z=15$$

The solution set is $\{15\}$.

41. $\frac{y}{3}+\frac{2}{5}=\frac{y}{5}-\frac{2}{5}$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 15.

$$15\left(\frac{y}{3}+\frac{2}{5}\right)=15\left(\frac{y}{5}+\frac{2}{5}\right)$$
$$15\left(\frac{y}{3}\right)+15\left(\frac{2}{5}\right)=15\left(\frac{y}{5}\right)+15\left(-\frac{2}{5}\right)$$
$$5y+6=3y-6$$
$$5y+6-3y=3y-6-3y$$
$$2y+6=-6$$
$$2y+6-6=-6-6$$
$$2y=-12$$
$$\frac{2y}{2}=\frac{-12}{2}$$
$$y=-6$$

The solution set is $\{-6\}$.

42. $\frac{y}{12}+\frac{1}{6}=\frac{y}{2}-\frac{1}{4}$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 12.

$$12\left(\frac{y}{12}+\frac{1}{6}\right)=12\left(\frac{y}{2}-\frac{1}{4}\right)$$
$$y+2=6y-3$$
$$-5y+2=-3$$
$$-5y=-5$$
$$y=1$$

The solution set is $\{1\}$.

43. $\frac{3x}{4}-3=\frac{x}{2}+2$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 8.

$$8\left(\frac{3x}{4}-3\right)=8\left(\frac{x}{2}+2\right)$$
$$8\left(\frac{3x}{4}\right)-8\cdot 3=8\left(\frac{x}{2}\right)+8\cdot 2$$
$$6x-24=4x+16$$
$$6x-24-4x=4x+16-4x$$
$$2x-24=16$$
$$2x-24+24=16+24$$
$$2x=40$$
$$\frac{2x}{2}=\frac{40}{2}$$
$$x=20$$

The solution set is $\{20\}$.

44. $\frac{3x}{5}-\frac{2}{5}=\frac{x}{3}+\frac{2}{5}$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 15.

$$15\left(\frac{3x}{5}-\frac{2}{5}\right)=15\left(\frac{x}{3}+\frac{2}{5}\right)$$
$$9x-6=5x+6$$
$$4x-6=6$$
$$4x=12$$
$$x=3$$

The solution set is $\{3\}$.

45. $\frac{x-3}{5}-1=\frac{x-5}{4}$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 20.

$$20\left(\frac{x-3}{5}-1\right)=20\left(\frac{x-5}{4}\right)$$
$$4(x-3)-20=5(x-5)$$
$$4x-12-20=5x-25$$
$$4x-5x-32=5x-5x-25$$
$$-x-32=-25$$
$$-x-32+32=-25+32$$
$$-x=7$$
$$-1(-x)=-1(7)$$
$$x=-7$$

The solution set is $\{-7\}$.

46. $\frac{x-2}{3}-4=\frac{x+1}{4}$

To clear the equation of fractions, multiply both sides by the least common denominator (LCD), which is 12.

$$12\left(\frac{x-2}{3}\right)-12(4)=12\left(\frac{x+1}{4}\right)$$
$$4(x-2)-48=3(x+1)$$
$$4x-8-48=3x+3$$
$$4x-56=3x+3$$
$$x-56=3$$
$$x=59$$

The solution set is $\{59\}$.

47. $3.6x=2.9x+6.3$

To clear the equation of decimals, multiply both sides by 10.

$$10(3.6x)=10(2.9x+6.3)$$
$$36x=29x+63$$
$$7x=63$$
$$x=9$$

The solution set is $\{9\}$.

48. $1.2x-3.6=2.4-0.3x$

To clear the equation of decimals, multiply both sides by 10.

$$10(1.2x-3.6)=10(2.4-0.3x)$$
$$12x-36=24-3x$$
$$12x=60-3x$$
$$15x=60$$
$$x=4$$

The solution set is $\{4\}$.

49. $0.92y+2=y-0.4$

To clear the equation of decimals, multiply both sides by 100.

$$100(0.92y+2)=100(y-0.4)$$
$$92y+200=100y-40$$
$$92y=100y-240$$
$$-8y=-240$$
$$y=30$$

The solution set is $\{30\}$.

50. $0.15y-0.1=2.5y-1.04$

To clear the equation of decimals, multiply both sides by 100.

$$100(0.15y-0.1)=100(2.5y-1.04)$$
$$15y-10=250y-104$$
$$15y=250y-94$$
$$-235y=-94$$
$$y=0.4$$

The solution set is $\{0.4\}$.

51. $0.3x-4=0.1(x+10)$

$$0.3x-4=0.1x+1$$

To clear the equation of decimals, multiply both sides by 10.

$$10(0.3x-4)=10(0.1x+1)$$
$$3x-40=x+10$$
$$3x=x+50$$
$$2x=50$$
$$x=25$$

The solution set is $\{25\}$.

52. $0.1(x+80)=14-0.2x$

$0.1x+8=14-0.2x$

To clear the equation of decimals, multiply both sides by 10.

$10(0.1x+8)=10(14-0.2x)$

$x+80=140-2x$

$x=60-2x$

$3x=60$

$x=20$

The solution set is $\{20\}$.

53. $0.4(2z+6)+0.1=0.5(2z-3)$

$0.8z+2.4+0.1=z-1.5$

$0.8z+2.5=z-1.5$

To clear the equation of decimals, multiply both sides by 10.

$10(0.8z+2.5)=10(z-1.5)$

$8z+25=10z-15$

$8z=10z-40$

$-2z=-40$

$z=20$

The solution set is $\{20\}$.

54. $1.4(z-5)-0.2=0.5(6z-8)$

$1.4z-7-0.2=3z-4$

$1.4z-7.2=3z-4$

To clear the equation of decimals, multiply both sides by 10.

$10(1.4z-7.2)=10(3z-4)$

$14z-72=30z-40$

$14z=30z+32$

$-16z=32$

$z=-2$

The solution set is $\{-2\}$.

55. $0.01(x+4)-0.04=0.01(5x+4)$

$0.01x+0.4-0.04=0.05x+0.4$

$0.01x+0.36=0.05x+0.4$

To clear the equation of decimals, multiply both sides by 100.

$100(0.01x+0.36)=100(0.05x+0.4)$

$x+36=5x+40$

$x=5x+4$

$-4x=4$

$x=-1$

The solution set is $\{-1\}$.

56. $0.02(x-2)=0.06-0.01(x+1)$

$0.02x-0.04=0.06-0.01x-0.01$

$0.02x-0.04=-0.01x+0.05$

To clear the equation of decimals, multiply both sides by 100.

$100(0.02x-0.04)=100(-0.01x+0.05)$

$2x-4=-x+5$

$2x=-x+9$

$3x=9$

$x=3$

The solution set is $\{3\}$.

57. $0.6(x+300)=0.65x-205$

$0.6x+180=0.65x-205$

To clear the equation of decimals, multiply both sides by 100.

$100(0.6x+180)=100(0.65x-205)$

$60x+18,000=65x-20,500$

$60x=65x-38,500$

$-5x=-38,500$

$x=7700$

The solution set is $\{7700\}$.

58. $0.05(7x+36)=0.4x+1.2$

$0.35x+1.8=0.4x+1.2$

To clear the equation of decimals, multiply both sides by 100.

$100(0.35x+1.8)=100(0.4x+1.2)$

$35x+180=40x+120$

$35x=40x-60$

$-5x=-60$

$x=12$

The solution set is $\{12\}$.

59. $3x-7=3(x+1)$

$3x-7=3x+3$

$3x-7-3x=3x+3-3x$

$-7=3$

The original equation is equivalent to the false statement $-7=3$, so the equation is inconsistent and has no solution. The solution set is $\{\ \}$.

60. $2(x-5)=2x+10$
$2x-10=2x+10$
$2x-10-2x=2x+10-2x$
$-10=10$
The original equation is equivalent to the false statement $-10=10$, so the equation is inconsistent and has no solution.
The solution set is $\{\ \}$.

61. $2(x+4)=4x+5-2x+3$
$2x+8=2x+8$
$2x-8-2x=2x+8-2x$
$8=8$
The original equation is equivalent to the true statement $8=8$, so the equation is an identity and the solution set is all real numbers
$\{x \mid x \text{ is a real number}\}$.

62. $3(x-1)=8x+6-5x-9$
$3x-3=3x-3$
$3x-3-3x=3x-3-3x$
$-3=-3$
The original equation is equivalent to the true statement $-3=-3$, so the equation is an identity and the solution set is all real numbers
$\{x \mid x \text{ is a real number}\}$.

63. $7+2(3x-5)=8-3(2x+1)$
$7+6x-10=8-6x-3$
$6x-3=5-6x$
$6x+6x-3=5-6x+6x$
$12x-3=5$
$12x-3+3=5+3$
$12x=8$
$\frac{12x}{12}=\frac{8}{12}$
$x=\frac{2}{3}$
The solution set is $\left\{\frac{2}{3}\right\}$.

64. $2+3(2x-7)=9-4(3x+1)$
$2+6x-21=9-12x-4$
$6x-19=-12x+5$
$18x-19=5$
$18x=24$
$x=\frac{24}{18}=\frac{4}{3}$
The solution set is $\left\{\frac{4}{3}\right\}$.

65. $4x+1-5x=5-(x+4)$
$-x+1=5-x-4$
$-x+1=1-x$
$-x+1+x=1-x+x$
$1=1$
The original equation is equivalent to the true statement $1=1$, so the equation is an identity and the solution set is all real numbers
$\{x \mid x \text{ is a real number}\}$.

66. $5x-5=3x-7+2(x+1)$
$5x-5=3x-7+2x+2$
$5x-5=5x-5$
$5x-5-5x=5x-5-5x$
$-5=-5$
The original equation is equivalent to the true statement $-5=-5$, so the equation is an identity and the solution set is all real numbers
$\{x \mid x \text{ is a real number}\}$.

67. $4(x+2)+1=7x-3(x-2)$
$4x+8+1=7x-3x+6$
$4x+9=4x+6$
$4x-4x+9=4x-4x+6$
$9=6$
Since $9=6$ is a false statement, the original equation is inconsistent and has no solution. The solution set is $\{\ \}$.

68. $5x-3(x+1)=2(x+3)-5$

$5x-3x-3=2x+6-5$

$2x-3=2x+1$

$2x-3-2x=2x+1-2x$

$-3=1$

Since $-3=1$ is a false statement, the original equation is inconsistent and has no solution. The solution set is $\{\ \}$.

69. $3-x=2x+3$

$3-x+x=2x+x+3$

$3=3x+3$

$3-3=3x+3-3$

$0=3x$

$\frac{0}{3}=\frac{3x}{3}$

$0=x$

The solution set is $\{0\}$.

70. $5-x=4x+5$

$5-x-4x=4x+5-4x$

$-5x+5=5$

$-5x=0$

$\frac{-5x}{-5}=\frac{0}{-5}$

$x=0$

The solution set is $\{0\}$.

71. $\frac{x}{3}+2=\frac{x}{3}$

Multiply by the LCD, which is 3.

$3\left(\frac{x}{3}+2\right)=3\left(\frac{x}{3}\right)$

$x+6=x$

$x-x+6=x-x$

$6=0$

Since $6=0$ is a false statement, the original equation has no solution. The solution set is $\{\ \}$.

72. $\frac{x}{4}+3=\frac{x}{4}$

Multiply by the LCD, which is 4.

$4\left(\frac{x}{4}+3\right)=4\left(\frac{x}{4}\right)$

$x+12=x$

$x+12-x=x-x$

$12=0$

Since $12=0$ is a false statement, the original equation has no solution. The solution set is $\{\ \}$.

73. $\frac{x}{2}-\frac{x}{4}+4=x+4$

Multiply by the LCD, which is 4.

$4\left(\frac{x}{2}-\frac{x}{4}+4\right)=4(x+4)$

$4\left(\frac{x}{2}\right)-4\left(\frac{x}{4}\right)+16=4x+16$

$2x-x+16=4x+16$

$x+16=4x+16$

$x-x+16=4x-x+16$

$16=3x+16$

$16-16=3x+16-16$

$0=3x$

$\frac{0}{3}=\frac{3x}{3}$

$0=x$

The solution set is $\{0\}$.

74. $\frac{x}{2}+\frac{2x}{3}+3=x+3$

Multiply both sides by the LCD which is 6.

$6\left(\frac{x}{2}+\frac{2x}{3}+3\right)=6(x+3)$

$3x+4x+18=6x+18$

$7x+18=6x+18$

$x+18=18$

$x=0$

The solution set is $\{0\}$.

75. $\frac{2}{3}x = 2 - \frac{5}{6}x$

Multiply both sides by the LCD which is 6.

$$6\left(\frac{2}{3}x\right) = 6(2) - 6\left(\frac{5}{6}x\right)$$
$$2(2x) = 12 - 5x$$
$$4x = 12 - 5x$$
$$4x + 5x = 12 - 5x + 5x$$
$$9x = 12$$
$$\frac{9x}{9} = \frac{12}{9}$$
$$x = \frac{12}{9} = \frac{4}{3}$$

The solution set is $\left\{\frac{4}{3}\right\}$.

76. $\frac{2}{3}x = \frac{1}{4}x - 8$

Multiply both sides by the LCD which is 12.

$$12\left(\frac{2}{3}x\right) = 12\left(\frac{1}{4}x - 8\right)$$
$$8x = 3x - 96$$
$$5x = -96$$
$$x = -\frac{96}{5}$$

The solution set is $\left\{-\frac{96}{5}\right\}$.

77. $0.06(x+5) = 0.03(2x+7) + 0.09$

$0.06x + 0.3 = 0.06x + 0.21 + 0.09$

$0.06x + 0.3 = 0.06x + 0.3$

To clear the equation of decimals, multiply both sides by 100.

$100(0.06x + 0.3) = 100(0.06x + 0.3)$

$6x + 30 = 6x + 30$

$30 = 30$

The original equation is equivalent to the true statement $30 = 30$, so the equation is an identity and the solution set is all real numbers $\{x \mid x \text{ is a real number}\}$.

78. $0.04(x-2) = 0.02(6x-3) - 0.02$

$0.04x - 0.08 = 0.12x - 0.06 - 0.02$

$0.04x - 0.08 = 0.12x - 0.08$

To clear the equation of decimals, multiply both sides by 100.

$100(0.04x - 0.08) = 100(0.12x - 0.08)$

$4x - 8 = 12x - 8$

$4x = 12x$

$-8x = 0$

$x = 0$

The solution set is $\{0\}$.

79. $\frac{x}{\square} + \Delta = \$$

$\frac{x}{\square} + \Delta - \Delta = \$ - \Delta$

$\frac{x}{\square} = \$ - \Delta$

$\square\left(\frac{x}{\square}\right) = \square(\$ - \Delta)$

$x = \square\$ - \square\Delta$

80. $\frac{x}{\square} - \Delta = -\$$

$\frac{x}{\square} - \Delta + \Delta = -\$ + \Delta$

$\frac{x}{\square} = -\$ + \Delta$

$\square \cdot \frac{x}{\square} = \square \cdot (-\$ + \Delta)$

$x = \square \cdot (-\$ + \Delta)$

$x = -\square\$ + \square\Delta$

$x = \square\Delta - \square\$$

81. First solve the equation for x.

$$\frac{x}{5}-2=\frac{x}{3}$$
$$\frac{x}{5}-\frac{x}{5}-2=\frac{x}{3}-\frac{x}{5}$$
$$-2=\frac{5x}{15}-\frac{3x}{15}$$
$$-2=\frac{2x}{15}$$
$$15(-2)=15\left(\frac{2x}{15}\right)$$
$$-30=2x$$
$$\frac{-30}{2}=\frac{2x}{2}$$
$$-15=x$$

Now evaluate the expression x^2-x for $x=-15$.

$$x^2-x=(-15)^2-(-15)$$
$$=225+15$$
$$=240$$

82. First solve the equation for x.

$$\frac{3x}{2}+\frac{3x}{4}=\frac{x}{4}-4$$
$$4\left(\frac{3x}{2}+\frac{3x}{4}\right)=4\left(\frac{x}{4}-4\right)$$
$$6x+3x=x-16$$
$$9x=x-16$$
$$8x=-16$$
$$x=-2$$

Now evaluate the expression x^2-x for $x=-2$.

$$x^2-x=(-2)^2-(-2)$$
$$=4+2$$
$$=6$$

83.
$$\frac{1}{3}x+\frac{1}{5}x=16$$
$$\text{LCD}=15$$
$$15\left(\frac{1}{3}x\right)+15\left(\frac{1}{5}x\right)=15(16)$$
$$5x+3x=240$$
$$8x=240$$
$$\frac{8x}{8}=\frac{240}{8}$$
$$x=30$$

The number is 30.

84.
$$\frac{2}{5}x+\frac{1}{4}x=13$$
$$20\left(\frac{2}{5}x+\frac{1}{4}x\right)=20(13)$$
$$8x+5x=260$$
$$13x=260$$
$$\frac{13x}{13}=\frac{260}{13}$$
$$x=20$$

The number is 20.

85.
$$\frac{3}{4}x-3=\frac{1}{2}x$$
$$4\left(\frac{3}{4}x\right)-4(3)=4\left(\frac{1}{2}x\right)$$
$$3x-12=2x$$
$$3x-2x-12=2x-2x$$
$$x-12=0$$
$$x-12+12=0+12$$
$$x=12$$

The number is 12.

86.
$$\frac{7}{8}x-30=\frac{1}{2}x$$
$$8\left(\frac{7}{8}x-30\right)=8\left(\frac{1}{2}x\right)$$
$$7x-240=4x$$
$$-240=-3x$$
$$\frac{-240}{-3}=\frac{-3x}{-3}$$
$$80=x$$

The number is 80.

87.
$$F=10(x-65)+50$$
$$250=10(x-65)+50$$
$$250-50=10(x-65)+50-50$$
$$200=10x-650$$
$$200+650=10x-650+650$$
$$850=10x$$
$$\frac{850}{10}=\frac{10x}{10}$$
$$85=x$$

A person receiving a \$250 fine was driving 85 miles per hour.

88. $F = 10(x-65)+50$

$400 = 10x - 650 + 50$

$400 = 10x - 600$

$1000 = 10x$

$100 = x$

A person receiving a $400 fine was driving 100 miles per hour.

89. $\frac{W}{2} - 3H = 53$

$\frac{W}{2} - 3(6) = 53$

$\frac{W}{2} - 18 = 53$

$\frac{W}{2} - 18 + 18 = 53 + 18$

$\frac{W}{2} = 71$

$2 \cdot \frac{W}{2} = 2 \cdot 71$

$W = 142$

According to the formula, the healthy weight of a person of height 5'6" is 142 pounds. This is 13 pounds below the upper end of the range shown in the bar graph.

90. $\frac{W}{2} - 3H = 53$

$\frac{W}{2} - 3(12) = 53$

$\frac{W}{2} - 36 = 53$

$\frac{W}{2} - 36 + 36 = 53 + 36$

$\frac{W}{2} = 89$

$2 \cdot \frac{W}{2} = 2 \cdot 89$

$W = 178$

According to the formula, the healthy weight of a person of height 6' is 178 pounds. This is 6 pounds below the upper end of the range shown in the bar graph.

91. $p = 15 + \frac{5d}{11}$

$201 = 15 + \frac{5d}{11}$

$201 - 15 = 15 + \frac{5d}{11} - 15$

$186 = \frac{5d}{11}$

$11(186) = 11\left(\frac{5d}{11}\right)$

$2046 = 5d$

$\frac{2046}{5} = d$

$409.2 = d$

He descended to a depth of 409.2 feet below the surface.

92. $p = 15 + \frac{5d}{11}$

$20 = 15 + \frac{5d}{11}$

$5 = \frac{5d}{11}$

$11(5) = 11\left(\frac{5d}{11}\right)$

$55 = 5d$

$11 = d$

The pressure is 20 pounds per square foot at a depth of 11 feet.

93. – 97. Answers will vary.

98. makes sense

99. makes sense

100. does not make sense; Explanations will vary. Sample explanation: Though 5 is a solution, the complete solution is all real numbers.

101. does not make sense; Explanations will vary. Sample explanation: For this equation it would have been sufficient to multiply by 10.

102. false; Changes to make the statement true will vary. A sample change is: The solution of the equation is all real numbers.

103. false; Changes to make the statement true will vary. A sample change is: The equation $2y + 5 = 0$ is equivalent to $2y = -5$.

104. true

105. false; Changes to make the statement true will vary.

A sample change is: The equation $x+\frac{1}{3}=\frac{1}{2}$ is equivalent to $6\cdot x+6\cdot\frac{1}{3}=6\cdot\frac{1}{2}$ or $6x+2=3$.

106.
$$\begin{aligned} f &= 0.432h-10.44\\ 16 &= 0.432h-10.44\\ 16+10.44 &= 0.432h-10.44+10.44\\ 26.44 &= 0.432h\\ \frac{26.44}{0.432} &= \frac{0.432h}{0.432}\\ 61.2 &\approx h \end{aligned}$$

The woman's height was about 61 inches or 5 feet 1 inch, so the partial skeleton could be that of the missing woman.

107.
$$\begin{aligned} \frac{2x-3}{9}+\frac{x-3}{2} &= \frac{x+5}{6}-1\\ 18\left(\frac{2x-3}{9}+\frac{x-3}{2}\right) &= 18\left(\frac{x+5}{6}-1\right)\\ 18\left(\frac{2x-3}{9}\right)+18\left(\frac{x-3}{2}\right) &= 18\left(\frac{x+5}{6}\right)-18\cdot 1\\ 2(2x-3)+9(x-3) &= 3(x+5)-18\\ 4x-6+9x-27 &= 3x+15-18\\ 13x-33 &= 3x-3\\ 13x-33-3x &= 3x-3-3x\\ 10x-33 &= -3\\ 10x-33+33 &= -3+33\\ 10x &= 30\\ \frac{10x}{10} &= \frac{30}{10}\\ x &= 3 \end{aligned}$$

The solution set is $\{3\}$.

108.
$$\begin{aligned} 2(3x+4) &= 3x+2\left[3(x-1)+2\right]\\ 6x+8 &= 3x+2(3x-3+2)\\ 6x+8 &= 3x+2(3x-1)\\ 6x+8 &= 3x+6x-2\\ 6x+8 &= 9x-2\\ 6x+8-9x &= 9x-2-9x\\ -3x+8 &= -2\\ -3x+8-8 &= -2-8\\ -3x &= -10\\ \frac{-3x}{-3} &= \frac{-10}{3}\\ x &= \frac{10}{3} \end{aligned}$$

The solution set is $\left\{\frac{10}{3}\right\}$.

109. $-24<-20$ because -24 lies further to the left on a number line.

110. $-\frac{1}{3}<-\frac{1}{5}$ because $-\frac{1}{3}$ lies further to the left on a number line.

111.
$$\begin{aligned} -9-11+7-(-3) &= -9-11+7+3\\ &= -20+10\\ &= -10 \end{aligned}$$

112. **a.**
$$\begin{aligned} T &= D+pm\\ T-D &= pm \end{aligned}$$

b.
$$\begin{aligned} T-D &= pm\\ \frac{T-D}{p} &= \frac{pm}{p}\\ \frac{T-D}{p} &= m \end{aligned}$$

113.
$$\begin{aligned} 4 &= 0.25B\\ \frac{4}{0.25} &= \frac{0.25B}{0.25}\\ 16 &= B \end{aligned}$$

The solution set is $\{16\}$.

114. $1.3 = P \cdot 26$

$$\frac{1.3}{26} = \frac{P \cdot 26}{26}$$

$$0.05 = P$$

The solution set is $\{0.05\}$.

2.4 Check Points

1. $A = lw$

$$\frac{A}{w} = \frac{lw}{w}$$

$$\frac{A}{w} = l$$

2. $2l + 2w = P$

$$2l + 2w - 2w = P - 2w$$

$$2l = P - 2w$$

$$\frac{2l}{2} = \frac{P - 2w}{2}$$

$$l = \frac{P - 2w}{2}$$

3. $T = D + pm$

$$T - D = pm$$

$$\frac{T - D}{p} = \frac{pm}{p}$$

$$\frac{T - D}{p} = m$$

$$m = \frac{T - D}{p}$$

4. $\frac{x}{3} - 4y = 5$

$$3\left(\frac{x}{3} - 4y\right) = 3 \cdot 5$$

$$3 \cdot \frac{x}{3} - 3 \cdot 4y = 3 \cdot 5$$

$$x - 12y = 15$$

$$x - 12y + 12y = 15 + 12y$$

$$x = 15 + 12y$$

5. Use the formula $A = PB$: A is P percent of B.

$$\underbrace{\text{What number}}_{A} \ \underbrace{\text{is}}_{=} \ \underbrace{\text{9\%}}_{0.09} \ \underbrace{\text{of}}_{\cdot} \ \underbrace{\text{50?}}_{50}$$

$$A = 4.5$$

6. Use the formula $A = PB$: A is P percent of B.

$$\underbrace{9}_{9} \ \underbrace{\text{is}}_{=} \ \underbrace{\text{60\%}}_{0.60} \ \underbrace{\text{of}}_{\cdot} \ \underbrace{\text{what number?}}_{B}$$

$$\frac{9}{0.60} = \frac{0.60B}{0.60}$$

$$15 = B$$

7. Use the formula $A = PB$: A is P percent of B.

$$\underbrace{18}_{18} \ \underbrace{\text{is}}_{=} \ \underbrace{\text{what percent}}_{P} \ \underbrace{\text{of}}_{\cdot} \ \underbrace{\text{50?}}_{50}$$

$$18 = P \cdot 50$$

$$\frac{18}{50} = \frac{50P}{50}$$

$$0.36 = P$$

To change 0.36 to a percent, move the decimal point two places to the right and add a percent sign.

$$0.36 = 36\%$$

8. Use the formula $A = PB$: A is P percent of B.

Find the price decrease: $\$940 - \$611 = \$329$

$$\underbrace{\text{The price decrease}}_{329} \ \underbrace{\text{is}}_{=} \ \underbrace{\text{what percent}}_{P} \ \underbrace{\text{of}}_{\cdot} \ \underbrace{\text{the original price?}}_{940}$$

$$329 = P \cdot 940$$

$$\frac{329}{940} = \frac{940P}{940}$$

$$0.35 = P$$

To change 0.35 to a percent, move the decimal point two places to the right and add a percent sign.

$$0.35 = 35\%$$

9. a.

Year	Tax Paid the Year Before	increase/decrease	Taxes Paid This Year
1	\$1200	20% decrease: $0.20 \cdot \$1200 = \$240$	$\$1200 - \$240 = \$960$
2	\$960	20% increase: $0.20 \cdot \$960 = \$192$	$\$960 + \$192 = \$1152$

The taxes for year 2 will be \$1152.

b. The taxes for year 2 are less than those originally paid.
Find the tax decrease: $\$1200 - \$1152 = \$48$

$$\overbrace{48}^{\text{The tax decrease}} \overbrace{=}^{\text{is}} \overbrace{P}^{\text{what percent}} \overbrace{\cdot}^{\text{of}} \overbrace{1200}^{\text{the original tax?}}$$

$$48 = P \cdot 1200$$

$$\frac{48}{1200} = \frac{1200P}{1200}$$

$$0.04 = P$$

To change 0.04 to a percent, move the decimal point two places to the right and add a percent sign.
$0.04 = 4\%$
The overall tax decrease is 4%.

2.4 Concept and Vocabulary Check

1. isolated on one side

2. $A = lw$

3. $P = 2l \times 2w$

4. $A = PB$

5. subtract b; divide by m

2.4 Exercise Set

1. $d = rt$ for r

$$\frac{d}{t} = \frac{rt}{t}$$

$$\frac{d}{t} = r \text{ or } r = \frac{d}{t}$$

This is the distance traveled formula:
distance = rate · time.

2. $d = rt$ for t

$$\frac{d}{r} = \frac{rt}{r}$$

$$\frac{d}{r} = t \text{ or } t = \frac{d}{r}$$

This is the motion formula:
distance = rate · time.

3. $I = Prt$ for P

$$\frac{I}{rt} = \frac{Prt}{rt}$$

$$\frac{I}{rt} = P \text{ or } P = \frac{I}{rt}$$

This is the formula for simple interest: interest = principal · rate · time.

4. $I = Prt$ for r

$$\frac{I}{Pt} = \frac{Prt}{PT}$$

$$\frac{I}{Pt} = r \text{ or } r = \frac{I}{Pt}$$

This is the formula for simple interest: interest = principal · rate · time.

5. $C = 2\pi r$ for r

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$

$$\frac{C}{2\pi} = r \text{ or } r = \frac{C}{2\pi}$$

This is the formula for finding the circumference of a circle if you know its radius.

6. $C = \pi d$ for d

$$\frac{C}{\pi} = \frac{\pi d}{\pi}$$

$$\frac{C}{\pi} = d \text{ or } d = \frac{C}{\pi}$$

This is the formula for finding the circumference of a circle if you know its diameter.

7. $E = mc^2$

$$\frac{E}{c^2} = \frac{mc^2}{c^2}$$

$$\frac{E}{c^2} = m \text{ or } m = \frac{E}{c^2}$$

This is Einstein's formula relating energy, mass, and the speed of light.

8. $V = \pi r^2 h$ for h

$$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$

$$\frac{V}{\pi r^2} = h \text{ or } h = \frac{V}{\pi r^2}$$

This is the volume of a cylinder.

9. $y = mx + b$ for m

$$y - b = mx$$

$$\frac{y-b}{x} = \frac{mx}{x}$$

$$\frac{y-b}{x} = m \text{ or } m = \frac{y-b}{x}$$

This is the slope-intercept formula for the equation of a line.

10. $y = mx + b$ for x

$$y - b = mx$$

$$\frac{y-b}{m} = \frac{mx}{m}$$

$$\frac{y-b}{m} = x \text{ or } x = \frac{y-b}{m}$$

This is the slope-intercept formula for the equation of a line.

11. $T = D + pm$ for D

$$T - pm = D + pm - pm$$

$$T - pm = D$$

$$D = T - pm$$

12. $P = C + MC$ for M

$$P - C = C + MC - C$$

$$P - C = MC$$

$$\frac{P-C}{C} = \frac{MC}{C}$$

$$\frac{P-C}{C} = M \text{ or } M = \frac{P-C}{C}$$

This is the business math formula for mark-up based on cost.

13. $A = \frac{1}{2}bh$ for b

$$2A = 2\left(\frac{1}{2}bh\right)$$

$$2A = bh$$

$$\frac{2A}{h} = \frac{bh}{h}$$

$$\frac{2A}{h} = b \text{ or } b = \frac{2A}{h}$$

This is the formula for the area of a triangle: area = $\frac{1}{2}$ · base · height.

14. $A=\frac{1}{2}bh$ for h

$2A=2\left(\frac{1}{2}bh\right)$

$2A=bh$

$\frac{2A}{b}=\frac{bh}{b}$

$\frac{2A}{b}=h$ or $h=\frac{2A}{b}$

This is the formula for the area of a triangle: area = $\frac{1}{2}\cdot$ base $\cdot$ height.

15. $M=\frac{n}{5}$ for n

$5M=5\left(\frac{n}{5}\right)$

$5M=n$ or $n=5M$

16. $M=\frac{A}{740}$ for A

$740M=740\left(\frac{A}{740}\right)$

$740M=A$ or $A=740M$

17. $\frac{c}{2}+80=2F$ for c

$\frac{c}{2}+80-80=2F-80$

$\frac{c}{2}=2F-80$

$2\left(\frac{c}{2}\right)=2(2F-80)$

$c=4F-160$

18. $p=15+\frac{5d}{11}$ for d

$11p=11\left(15+\frac{5d}{11}\right)$

$11p=165+5d$

$11p-165=5d$

$\frac{11p-165}{5}=d$ or $d=\frac{11p-165}{5}$

19. $A=\frac{1}{2}(a+b)$ for a

$2A=2\left[\frac{1}{2}(a+b)\right]$

$2A=a+b$

$2A-b=a+b-b$

$2A-b=a$ or $a=2A-b$

This is the formula for finding the average of two numbers.

20. $A=\frac{1}{2}(a+b)$ for b

$2A=2\left[\frac{1}{2}(a+b)\right]$

$2A=a+b$

$2A-a=b$ or $b=2A-a$

This is the formula for finding the average of two numbers.

21. $S=P+Prt$ for r

$S-P=P+Prt-P$

$S-P=Prt$

$\frac{S-P}{Pt}=\frac{Prt}{Pt}$

$\frac{S-P}{Pt}=r$ or $r=\frac{S-P}{Pt}$

This is the formula for finding the sum of principle and interest for simple interest problems.

22. $S=P+Prt$ for t

$S-P=Prt$

$\frac{S-P}{Pr}=\frac{Prt}{Pr}$

$\frac{S-P}{Pr}=t$ or $t=\frac{S-P}{Pr}$

This is the formula for finding the sum of principle and interest for simple interest problems.

23. $A=\frac{1}{2}h(a+b)$ for b

$$2A=2\left[\frac{1}{2}h(a+b)\right]$$
$$2A=h(a+b)$$
$$2A=ha+hb$$
$$2A-ha=ha+hb-ha$$
$$2A-ha=hb$$
$$\frac{2A-ha}{h}=\frac{hb}{h}$$
$$\frac{2A-ha}{h}=b \quad \text{or} \quad b=\frac{2A}{h}-a$$

This is the formula for the area of a trapezoid.

24. $A=\frac{1}{2}h(a+b)$ for a

$$2A=2\left[\frac{1}{2}h(a+b)\right]$$
$$2A=h(a+b)$$
$$\frac{2A}{h}=\frac{h(a+b)}{h}$$
$$\frac{2A}{h}=a+b$$
$$\frac{2A}{h}-b=a+b-b$$
$$\frac{2A}{h}-b=a \quad \text{or} \quad a=\frac{2A}{h}-b$$

This is the formula for finding the area of a trapezoid.

25. $Ax+By=C$ for x

$$Ax+By-By=C-By$$
$$Ax=C-By$$
$$\frac{Ax}{A}=\frac{C-By}{A}$$
$$x=\frac{C-By}{A}$$

This is the standard form of the equation of a line.

26. $Ax+By=C$ for y

$$Ax+By-Ax=C-Ax$$
$$By=C-Ax$$
$$\frac{By}{B}=\frac{C-Ax}{B}$$
$$y=\frac{C-Ax}{B}$$

This is the standard form of the equation of a line.

27. $A=PB;\ P=3\%=0.03, B=200$

$$A=PB$$
$$A=0.03\cdot 200$$
$$A=6$$

3% of 200 is 6.

28. $A=PB; P=8\%=0.08, B=300$

$$A=PB$$
$$A=0.08(300)=24$$

29. $A=PB;\ P=18\%=0.18,\ B=40$

$$A=PB$$
$$A=0.18\cdot 40$$
$$A=7.2$$

18% of 40 is 7.2.

30. $A=PB; P=16\%=0.16, B=90$

$$A=PB$$
$$A=0.16(90)=14.4$$

16% of 90 is 14.4

31. $A=PB;\ A=3, P=60\%=0.6$

$$A=PB$$
$$3=0.6\cdot B$$
$$\frac{3}{0.6}=\frac{0.6B}{0.6}$$
$$5=B$$

3 is 60% of 5.

32. $A=PB; A=8, P=40\%=0.4$

$$A=PB$$
$$8=0.4\cdot B$$
$$\frac{8}{0.4}=\frac{0.4B}{0.4}$$
$$20=B$$

8 is 40% of 20.

33. $A=PB;\ A=40.8, P=24\%=0.24$

$$A=PB$$
$$40.8=0.24\cdot B$$
$$\frac{40.8}{0.24}=\frac{0.24B}{0.24}$$
$$170=B$$

24% of 170 is 40.8.

34. $A = PB; A = 51.2, P = 32\% = 0.32$

$A = PB$

$51.2 = 0.32 \cdot B$

$\frac{51.2}{0.32} = \frac{0.32B}{0.32}$

$160 = B$

51.2 is 32% of 160.

35. $A = PB;\ A = 3,\ B = 15$

$A = PB$

$3 = P \cdot 15$

$\frac{3}{15} = \frac{P \cdot 15}{15}$

$0.2 = P$

$0.2 = 20\%$

3 is 20% of 15.

36. $A = PB; A = 18; B = 90$

$A = PB$

$18 = P \cdot 90$

$\frac{18}{90} = \frac{P \cdot 90}{90}$

$0.2 = P$

$0.2 = 20\%$

18 is 20% of 90.

37. $A = PB;\ A = 0.3,\ B = 2.5$

$A = PB$

$0.3 = P \cdot 2.5$

$\frac{0.3}{2.5} = \frac{P - 2.5}{2.5}$

$0.12 = P$

$0.12 = 12\%$

0.3 is 12% of 2.5

38. $A = PB; A = 0.6, B = 7.5$

$A = PB$

$0.6 = P \cdot 7.5$

$\frac{0.6}{7.5} = \frac{P \cdot 7.5}{7.5}$

$0.08 = P$

$0.08 = 8\%$

0.6 is 8% of 7.5.

39. The increase is $8 - 5 = 3$.

$A = PB$

$3 = P \cdot 5$

$\frac{3}{5} = \frac{P \cdot 5}{5}$

$0.60 = P$

This is a 60% increase.

40. The increase is $9 - 5 = 4$.

$A = PB$

$4 = P \cdot 5$

$\frac{4}{5} = \frac{5P}{5}$

$0.80 = P$

This is an 80% increase.

41. The decrease is $4 - 1 = 3$.

$A = PB$

$3 = P \cdot 4$

$\frac{3}{4} = \frac{4P}{4}$

$0.75 = P$

This is a 75% decrease.

42. The decrease is $8 - 6 = 2$.

$A = PB$

$2 = P \cdot 8$

$\frac{2}{8} = \frac{8P}{8}$

$0.25 = P$

This is a 25% decrease.

43. $y = (a + b)x$

$\frac{y}{(a+b)} = \frac{(a+b)x}{(a+b)}$

$\frac{y}{a+b} = x$ or $x = \frac{y}{a+b}$

44. $y = (a - b)x$

$\frac{y}{(a-b)} = \frac{(a-b)x}{(a-b)}$

$\frac{y}{a-b} = x$ or $x = \frac{y}{a-b}$

45. $y=(a-b)x+5$

$y-5=(a-b)x+5-5$

$y-5=(a-b)x$

$\dfrac{y-5}{a-b}=\dfrac{(a-b)x}{a-b}$

$\dfrac{y-5}{a-b}=x \quad \text{or} \quad x=\dfrac{y-5}{a-b}$

46. $y=(a+b)x-8$

$y+8=(a+b)x-8+8$

$y+8=(a+b)x$

$\dfrac{y+8}{(a+b)}=\dfrac{(a+b)x}{(a+b)}$

$\dfrac{y+8}{a+b}=x \quad \text{or} \quad x=\dfrac{y+8}{a+b}$

47. $y=cx+dx$

$y=(c+d)x$

$\dfrac{y}{c+d}=\dfrac{(c+d)x}{c+d}$

$\dfrac{y}{c+d}=x \quad \text{or} \quad x=\dfrac{y}{c+d}$

48. $y=cx-dx$

$y=(c-d)x$

$\dfrac{y}{(c-d)}=\dfrac{(c-d)x}{(c-d)}$

$\dfrac{y}{c-d}=x \quad \text{or} \quad x=\dfrac{y}{c-d}$

49. $y=Ax-Bx-C$

$y=(A-B)x-C$

$y+C=(A-B)x-C+C$

$y+C=(A-B)x$

$\dfrac{y+C}{A-B}=\dfrac{(A-B)x}{A-B}$

$\dfrac{y+C}{A-B}=x \quad \text{or} \quad x=\dfrac{y+C}{A-B}$

50. $y=Ax+Bx+C$

$y-C=Ax+Bx+C-C$

$y-C=Ax+Bx$

$y-C=(A+B)x$

$\dfrac{y-C}{(A+B)}=\dfrac{(A+B)x}{(A+B)}$

$\dfrac{y-C}{A+B}=x \quad \text{or} \quad x=\dfrac{y-C}{A+B}$

51. a. $A=\dfrac{x+y+z}{3}$ for z

$3A=3\left(\dfrac{x+y+z}{3}\right)$

$3A=x+y+z$

$3A-x-y=x+y+z-x-y$

$3A-x-y=z$

b. $A=90, x=86, y=88$

$z=3A-x-y$

$z=3(90)-86-88=96$

You need to get 96% on the third exam to have an average of 90%

52. a. $A=\dfrac{x+y+z+w}{4}$ for w

$4A=4\left(\dfrac{x+y+z+w}{4}\right)$

$4A=x+y+z+w$

$4A-x-y-z=x+y+z+w-x-y-z$

$4A-x-y-z=w$

b. $w=4A-xy-z; x=76, y=78, z=79$

$w=4A-x-y-z$

$w=4(80)-76-78-79$

$w=87$

You need to get 87% on the fourth exam to have an average of 80%.

53. a. $d = rt$ for t

$$\frac{d}{r} = \frac{rt}{r}$$

$$\frac{d}{r} = t$$

b. $t = \frac{d}{r};\ d = 100, r = 40$

$$t = \frac{100}{40} = 2.5$$

You would travel for 2.5 $\left(\text{or } 2\frac{1}{2}\right)$ hours.

54. a. $F = \frac{9}{5}C + 32$ for C

$$5F = 5\left(\frac{9}{5}C + 32\right)$$

$$5F = 9C + 160$$

$$5F - 160 = 9C$$

$$\frac{5F - 160}{9} = \frac{9C}{9}$$

$$\frac{5F - 160}{9} = C$$

b. $C = \frac{5F - 160}{9};\ F = 59$

$$C = \frac{5F - 160}{9}$$

$$C = \frac{5(59) - 160}{9}$$

$$C = \frac{295 - 160}{9}$$

$$C = \frac{135}{9} = 15$$

$59°\text{F} = 15°\text{C}$

55. $0.29 \cdot 1800 = 522$

522 workers stated that religion is the most taboo topic to discuss at work.

56. $0.14 \cdot 1800 = 252$

252 workers stated that politics is the most taboo topic to discuss at work.

57. a. This is the equivalent of asking: 5.85 is 5% of what number?

$$A = P \cdot B$$

$$5.85 = 0.05 \cdot B$$

$$\frac{5.85}{0.05} = \frac{0.05B}{0.05}$$

$$117 = B$$

117 million households in the United States.

b. This is the equivalent of asking: \$332,960 is 180% of what number?

$$A = P \cdot B$$

$$332{,}960 = 1.8 \cdot B$$

$$\frac{332{,}960}{1.8} = \frac{1.8B}{1.8}$$

$$184{,}978 \approx B$$

The average income in 1975, for the richest 5% of American households, was about \$184,978.

58. a. This is the equivalent of asking: 35.1 is 30% of what number?

$$A = P \cdot B$$

$$35.1 = 0.3 \cdot B$$

$$\frac{35.1}{0.3} = \frac{0.3B}{0.3}$$

$$117 = B$$

117 million households in the United States.

b. This is the equivalent of asking: \$16,095 is 107% of what number?

$$A = P \cdot B$$

$$16{,}095 = 1.07 \cdot B$$

$$\frac{16{,}095}{1.07} = \frac{1.07B}{1.07}$$

$$15{,}042 \approx B$$

The average income in 1975, for the poorest 30% of American households, was about \$15,042.

59. This is the equivalent of asking: 540 is what% of 1500?

$$A = P \cdot B$$

$$540 = P \cdot 1500$$

$$\frac{540}{1500} = \frac{1500P}{1500}$$

$$0.36 = P$$

36% of those surveyed said the police departments did a poor job at holding officers accountable.

60. This is the equivalent of asking: 105 is what% of 1500?

$$A = P \cdot B$$
$$105 = P \cdot 1500$$
$$\frac{105}{1500} = \frac{1500P}{1500}$$
$$0.07 = P$$

7% of those surveyed said the police departments did an excellent job at holding officers accountable.

61. $A = PB;\ A = 7500, B = 60{,}000$

$$A = PB$$
$$7500 = P \cdot 60{,}000$$
$$\frac{7500}{60{,}000} = \frac{P \cdot 60{,}000}{60{,}000}$$
$$0.125 = P$$

The charity has raised 0.125 = 12.5% of its goal.

62. This question is equivalent to, "225,000 is what percent of $500,000?"

$$A = PB$$
$$225{,}000 = P \cdot 500{,}000$$
$$\frac{225{,}000}{500{,}000} = \frac{P \cdot 500{,}000}{500{,}000} \quad 0.45 = P$$

The charity has raised 45% of the goal.

63. $A = PB;\ p = 15\% = 0.15, B = 60$

$A = 0.15 \cdot 60 = 09$

The tip was $9.

64. $\$3502 + 0.28(35{,}000 - \$23{,}000)$

$= \$3502 + 0.28(\$12{,}000)$

$= \$3502 + \$3360$

$= \$6862$

The income tax on a taxable income of $35,000 is $6862.

65. a. The sales tax is 6% of $16,800.

$0.06(16{,}800) = 1008$

The sales tax due on the car is $1008.

b. The total cost is the sum of the price of the car and the sales tax.

$\$16{,}800 + \$1008 = \$17{,}808$

The car's total cost is $17,808.

66. a. The sales tax is 7% of $96.

$0.07(96) = 6.72$

The sales tax due on the graphing calculator is $6.72.

b. The total cost is the sum of the price of the calculator and the sales tax.

$\$96 + \$6.72 = \$102.72$

The calculator's total cost is $102.72.

67. a. The discount is 12% of $860.

$0.12(860) = 103.20$

The discount amount is $103.20.

b. The sale price is the regular price minus the discount amount:

$\$860 - \$103.20 = \$756.80$

68. a. The discount amount is 40% of $16.50.

$0.4(16.50) = 6.60$

The discount amount is $6.60.

b. The sale price is the regular price minus the discount amount.

$\$16.50 - \$6.60 = \$9.90$

The sale price is $9.90.

69. The decrease is $840 – $714 = $126.

$$A = P \cdot B$$
$$126 = P \cdot 840$$
$$\frac{126}{840} = \frac{P \cdot 840}{840}$$
$$0.15 = P$$

This is a 0.15 = 15% decrease.

70. The decrease is $380 – $266 = $114.

$$A = P \cdot B$$
$$114 = P \cdot 380$$
$$\frac{114}{380} = \frac{P \cdot 380}{380}$$
$$0.30 = P$$

This is a 0.30 = 30% decrease.

71. Investment dollars decreased in year 1 are $0.30 \cdot \$10,000 = \$3000$. This means that \$10,000 – \$3000 = \$7000 remains. Investment dollars increased in year 2 are $0.40 \cdot \$7000 = \$2800$. This means that \$7000 + \$2800 = \$9800 of the original investment remains. This is an overall loss of \$200 over the two years.

$$A = P \cdot B$$
$$200 = P \cdot 10,000$$
$$\frac{200}{10,000} = \frac{P \cdot 10,000}{10,000}$$
$$0.02 = P$$

The financial advisor is not using percentages properly. Instead of a 10% gain, this is a 0.02 = 2% loss.

72. No; the first sale price is 70% of the original amount and the second sale price is 80% of the *first sale price*. The second sale price would be obtained by the following computation:

$$A = P_2\left(P_1(B)\right)$$
$$= 0.80(0.70B)$$
$$= 0.56B$$

The second sale price is 56% of the original price, so there is 44% reduction overall.

73. – 74. Answers will vary.

75. makes sense

76. does not make sense; Explanations will vary. Sample explanation: Sometimes you will solve for one variable in terms of other variables.

77. does not make sense; Explanations will vary. Sample explanation: \$100 is more than enough because 20% of \$80 is $0.20 \cdot \$80 = \$16$.

78. does not make sense; Explanations will vary. Sample explanation: Since the sale price cannot be negative, the percent decrease cannot be more than 100%.

79. false; Changes to make the statement true will vary. A sample change is: If $ax + b = 0$, then $ax = -b$ and $x = \frac{-b}{a}$.

80. false; Changes to make the statement true will vary. A sample change is: If $A = lw$, then $w = \frac{A}{l}$.

81. false; Changes to make the statement true will vary. A sample change is: If $A = \frac{1}{2}bh$, then $\frac{2A}{h} = b$.

82. true

83. $Q = \frac{100M}{C}$ for C

$$CQ = C\left(\frac{100M}{C}\right)$$
$$CQ = 100M$$
$$\frac{CQ}{Q} = \frac{100M}{Q}$$
$$C = \frac{100M}{Q}$$

84.

$$5x + 20 = 8x - 16$$
$$5x + 20 - 8x = 8x - 16 - 8x$$
$$-3x + 20 = -16$$
$$-3x + 20 - 20 = -16 - 20$$
$$-3x = -36$$
$$\frac{-3x}{-3} = \frac{-36}{-3}$$
$$x = 12$$

Check:

$$5(12) + 20 = 8(12) - 16$$
$$60 + 20 = 96 - 16$$
$$80 = 80$$

The solution set is $\{12\}$.

85. $5(2y-3)-1=4(6+2y)$

$10y-15-1=24+8y$

$10y-16=24+8y$

$10y-16-8y=24+8y-8y$

$2y-16=24$

$2y-16+16=24+16$

$2y=40$

$\frac{2y}{2}=\frac{40}{2}$

$y=20$

Check:

$5(2\cdot 20-3)-1=4(6+2\cdot 20)$

$5(40-3)-1=4(6+40)$

$5(37)-1=4(46)$

$185-1=184$

$184=184$

The solution set is $\{20\}$.

86. $x-0.3x=1x-0.3x=(1-0.3)x=0.7x$

87. $\frac{13}{x}-7x$

88. $8(x+14)$

89. $9(x-5)$

Mid-Chapter Check Point - Chapter 2

1. Begin by multiplying both sides of the equation by 4, the least common denominator.

$\frac{x}{2}=12-\frac{x}{4}$

$4\left(\frac{x}{2}\right)=4(12)-4\left(\frac{x}{4}\right)$

$2x=48-x$

$2x+x=48-x+x$

$3x=48$

$\frac{3x}{3}=\frac{48}{3}$

$x=16$

The solution set is $\{16\}$.

2. $5x-42=-57$

$5x-42+42=-57+42$

$5x=-15$

$\frac{5x}{5}=\frac{-15}{5}$

$x=-3$

The solution set is $\{-3\}$.

3. $H=\frac{EC}{825}$

$H\cdot 825=\frac{EC}{825}\cdot 825$

$825H=EC$

$\frac{825H}{E}=\frac{EC}{E}$

$\frac{825H}{E}=C$

4. $A=P\cdot B$

$A=0.06\cdot 140$

$A=8.4$

8.4 is 6% of 140.

5. $\frac{-x}{10}=-3$

$10\left(\frac{-x}{10}\right)=10(-3)$

$-x=-30$

$-1(-x)=-1(-30)$

$x=30$

The solution set is $\{30\}$.

6. $1-3(y-5)=4(2-3y)$

$1-3y+15=8-12y$

$-3y+16=8-12y$

$-3y+12y+16=8-12y+12y$

$9y+16=8$

$9y+16-16=8-16$

$9y=-8$

$\frac{9y}{9}=\frac{-8}{9}$

$y=-\frac{8}{9}$

The solution set is $\left\{-\frac{8}{9}\right\}$.

7. $S = 2\pi rh$

$$\frac{S}{2\pi h} = \frac{2\pi rh}{2\pi h}$$

$$\frac{S}{2\pi h} = r$$

8. $A = P \cdot B$

$12 = 0.30 \cdot B$

$$\frac{12}{0.30} = \frac{0.30 \cdot B}{0.30}$$

$40 = B$

12 is 30% of 40.

9. $\frac{3y}{5} + \frac{y}{2} = \frac{5y}{4} - 3$

To clear fractions, multiply both sides by the LCD, 20.

$$20\left(\frac{3y}{5}\right) + 20\left(\frac{y}{2}\right) = 20\left(\frac{5y}{4}\right) - 20(3)$$

$$4(3y) + 10y = 5(5y) - 60$$

$$12y + 10y = 25y - 60$$

$$22y = 25y - 60$$

$$22y - 25y = 25y - 25y - 60$$

$$-3y = -60$$

$$\frac{-3y}{-3} = \frac{-60}{-3}$$

$$y = 20$$

The solution set is $\{20\}$.

10. $2.4x + 6 = 1.4x + 0.5(6x - 9)$

$2.4x + 6 = 1.4x + 3x - 4.5$

$2.4x + 6 = 4.4x - 4.5$

To clear decimals, multiply both sides by 10.

$10(2.4x + 6) = 10(4.4x - 4.5)$

$24x + 60 = 44x - 45$

$24x = 44x - 105$

$-20x = -105$

$$\frac{-20x}{-20} = \frac{-105}{-20}$$

$x = 5.25$

The solution set is $\{5.25\}$.

11. $5z + 7 = 6(z - 2) - 4(2z - 3)$

$5z + 7 = 6z - 12 - 8z + 12$

$5z + 7 = -2z$

$5z - 5z + 7 = -2z - 5z$

$7 = -7z$

$$\frac{7}{-7} = \frac{-7z}{-7}$$

$-1 = z$

The solution set is $\{-1\}$.

12. $Ax - By = C$

$Ax - By + By = C + By$

$Ax = C + By$

$$\frac{Ax}{A} = \frac{C + By}{A}$$

$$x = \frac{C + By}{A} \text{ or } \frac{By + C}{A}$$

13. $6y + 7 + 3y = 3(3y - 1)$

$9y + 7 = 9y - 3$

$9y - 9y + 7 = 9y - 9y - 3$

$7 = -3$

Since this is a false statement, there is no solution or $\{\ \}$.

14. $10\left(\frac{1}{2}x + 3\right) = 10\left(\frac{3}{5}x - 1\right)$

$10\left(\frac{1}{2}x\right) + 10(3) = 10\left(\frac{3}{5}x\right) - 10(1)$

$5x + 30 = 6x - 10$

$5x - 5x + 30 = 6x - 5x - 10$

$30 = x - 10$

$30 + 10 = x - 10 + 10$

$40 = x$

The solution set is $\{40\}$.

15. $A = P \cdot B$

$50 = P \cdot 400$

$$\frac{50}{400} = \frac{P \cdot 400}{400}$$

$0.125 = P$

50 is 0.125 = 12.5% of 400.

16. $\dfrac{3(m+2)}{4} = 2m+3$

$4\cdot\dfrac{3(m+2)}{4} = 4(2m+3)$

$3(m+2) = 4(2m+3)$

$3m+6 = 8m+12$

$3m-3m+6 = 8m-3m+12$

$6 = 5m+12$

$6-12 = 5m+12-12$

$-6 = 5m$

$\dfrac{-6}{5} = \dfrac{5m}{5}$

$-\dfrac{6}{5} = m$

The solution set is $\left\{-\dfrac{6}{5}\right\}$.

17. The increase is 50 – 40 = 10.

$A = P\cdot B$

$10 = P\cdot 40$

$\dfrac{10}{40} = \dfrac{P\cdot 40}{40}$

$0.25 = P$

This is a 0.25 = 25% increase.

18. $12w-4+8w-4 = 4(5w-2)$

$20w-8 = 20w-8$

$20w-20w-8 = 20w-20w-8$

$-8 = -8$

Since −8 = −8 is a true statement, the solution is all real numbers or $\{x \mid x \text{ is a real number}\}$.

19. a. $B = -\dfrac{5}{2}a+82$

$B = -\dfrac{5}{2}(14)+82$

$= -35+82$

$= 47$

According to the formula, 47% of 14-year-olds believe that reading books is important.
This underestimates the actual percentage shown in the bar graph by 2%

b. $B = -\dfrac{5}{2}a+82$

$22 = -\dfrac{5}{2}a+82$

$2(22) = 2\left(-\dfrac{5}{2}a+82\right)$

$44 = -5a+164$

$-120 = -5a$

$24 = a$

According to the formula, 22% of 24-year-olds will believe that reading books is important.

2.5 Check Points

1. Let x = the number.

$6x-4 = 68$

$6x-4+4 = 68+4$

$6x = 72$

$x = 12$

The number is 12.

2. Let x = the median starting salary, in thousands of dollars, for English majors.
Let $x+18 =$ the median starting salary, in thousands of dollars, for computer science majors.

$x+(x+18) = 100$

$x+x+18 = 100$

$2x+18 = 100$

$2x = 82$

$x = 41$

$x+18 = 59$

The average salary for English majors is \$41 thousand and the average salary for computer science majors is $\$41+\$18 = \$59$.

3. Let x = the page number of the first facing page.
Let $x+1 =$ the page number of the second facing page.

$x+(x+1) = 145$

$x+x+1 = 145$

$2x+1 = 145$

$2x+1-1 = 145-1$

$2x = 144$

$x = 72$

$x+1 = 73$

The page numbers are 72 and 73.

4. Let x = the number of eighths of a mile traveled.

$$2+0.25x=10$$
$$2-2+0.25x=10-2$$
$$0.25x=8$$
$$\frac{0.25x}{0.25}=\frac{8}{0.25}$$
$$x=32$$

You can go 32 eighths of a mile. That is equivalent to $\frac{32}{8}=4$ miles.

5. Let x = the width of the swimming pool.
Let $3x=$ the length of the swimming pool.

$$P=2l+2w$$
$$320=2\cdot 3x+2\cdot x$$
$$320=6x+2x$$
$$320=8x$$
$$\frac{320}{8}=\frac{8x}{8}$$
$$40=x$$
$$x=40$$
$$3x=120$$

The pool is 40 feet wide and 120 feet long.

6. Let x = the original price.

Original price		minus		the reduction (40% of original price)		is		the reduced price, \$564
x		$-$		$0.4x$		$=$		564

$$x-0.4x=564$$
$$0.6x=564$$
$$\frac{0.6x}{0.6}=\frac{564}{0.6}$$
$$x=940$$

The original price was \$940.

2.5 Concept and Vocabulary Check

1. $4x-6$

2. $x+215$

3. $x+1$

4. $125+0.15x$

5. $2\cdot 4x+2x$ or $2x+2\cdot 4x$

6. $x-0.35x$ or $0.65x$

2.5 Exercise Set

1.
$$x+60=410$$
$$x+60-60=410-60$$
$$x=350$$

The number is 350.

2.
$$x+43=107$$
$$x+43-43=107-43$$
$$x=64$$

The number is 64.

3.
$$x-23=214$$
$$x-23+23=214+23$$
$$x=237$$

The number is 237.

4.
$$x-17=96$$
$$x-17+17=96+17$$
$$x=113$$

The number is 113.

5.
$$7x=126$$
$$\frac{7x}{7}=\frac{126}{7}$$
$$x=18$$

The number is 18.

6.
$$8x=272$$
$$\frac{8x}{8}=\frac{272}{8}$$
$$x=34$$

The number is 34.

7.
$$\frac{x}{19}=5$$
$$19\left(\frac{x}{19}\right)=19(5)$$
$$x=95$$

The number is 95.

8.
$$\frac{x}{14}=8$$
$$14\left(\frac{x}{14}\right)=14(8)$$
$$x=112$$

The number is 112.

9. $4+2x=56$
$4-4+2x=56-4$
$2x=52$
$\frac{2x}{2}=\frac{52}{2}$
$x=26$
The number is 26.

10. $5+3x=59$
$3x=54$
$x=18$
The number is 18.

11. $5x-7=178$
$5x-7+7=178+7$
$5x=185$
$\frac{5x}{5}=\frac{185}{5}$
$x=37$
The number is 37.

12. $6x-8=298$
$6x=306$
$x=51$
The number is 51.

13. $x+5=2x$
$x+5-x=2x-x$
$5=x$
The number is 5.

14. $x+12=4x$
$12=3x$
$4=x$
The number is 4.

15. $2(x+4)=36$
$2x+8=36$
$2x=28$
$x=14$
The number is 14.

16. $3(5+x)=48$
$15+3x=48$
$3x=33$
$x=11$
The number is 11.

17. $9x=30+3x$
$6x=30$
$x=5$
The number is 5.

18. $5+4x=x+35$
$5+3x=35$
$3x=30$
$x=10$
The number is 10.

19. $\frac{3x}{5}+4=34$
$\frac{3x}{5}=30$
$3x=150$
$x=50$
The number is 50.

20. $\frac{3x}{4}-3=9$
$\frac{3x}{4}=12$
$3x=48$
$x=16$
The number is 16.

21. Let x = the number of years spent watching TV.
Let $x+19$ = the number of years spent sleeping.
$x+(x+19)=37$
$x+x+19=37$
$2x+19=37$
$2x=18$
$x=9$
$x+19=28$
Americans will spend 9 years watching TV and 28 years sleeping.

22. Let x = the number of years spent eating.
Let $x+24$ = the number of years spent sleeping.
$x+(x+24)=32$
$x+x+24=32$
$2x+24=32$
$2x=8$
$x=4$
$x+24=28$
Americans will spend 4 years eating and 28 years sleeping.

23. Let x = the average salary, in thousands, for an American whose final degree is a bachelor's.
Let $2x-70=$ the average salary, in thousands, for an American whose final degree is a master's.

$$\begin{aligned} x+(2x-70) &= 173 \\ x+2x-70 &= 173 \\ 3x-70 &= 173 \\ 3x &= 243 \\ x &= 81 \\ 2x-70 &= 92 \end{aligned}$$

The average salary for an American whose final degree is a bachelor's is \$81 thousand and for an American whose final degree is a master's is \$92 thousand.

24. Let x = the average salary, in thousands, for an American whose final degree is a bachelor's.
Let $2x-45=$ the average salary, in thousands, for an American whose final degree is a doctorate.

$$\begin{aligned} x+(2x-45) &= 198 \\ x+2x-45 &= 198 \\ 3x-45 &= 198 \\ 3x &= 243 \\ x &= 81 \\ 2x-45 &= 117 \end{aligned}$$

The average salary for an American whose final degree is a bachelor's is \$81 thousand and for an American whose final degree is a doctorate is \$117 thousand.

25. Let x = the number of the left-hand page.
Let $x + 1$ = the number of the right-hand page.

$$\begin{aligned} x+(x+1) &= 629 \\ x+x+1 &= 629 \\ 2x+1 &= 629 \\ 2x+1-1 &= 629-1 \\ 2x &= 628 \\ \frac{2x}{2} &= \frac{628}{2} \\ x &= 314 \end{aligned}$$

The pages are 314 and 315.

26. Let x = the number of the left-hand page.
Let $x + 1$ = the number of the right-hand page.

$$\begin{aligned} x+(x+1) &= 525 \\ 2x+1 &= 525 \\ 2x &= 524 \\ x &= 262 \end{aligned}$$

The smaller page number is 262. The larger page number is 262 + 1 = 263.

27. Let x = the first consecutive odd integer (Babe Ruth).
Let $x+2=$ the second consecutive odd integer (Roger Maris).

$$\begin{aligned} x+(x+2) &= 120 \\ x+x+2 &= 120 \\ 2x+2 &= 120 \\ 2x &= 118 \\ x &= 59 \\ x+2 &= 61 \end{aligned}$$

Babe Ruth had 59 home runs and Roger Maris had 61.

28. Let x = the first consecutive even integer (Hank Greenberg).
Let $x+2=$ the second consecutive even integer (Babe Ruth).

$$\begin{aligned} x+(x+2) &= 118 \\ x+x+2 &= 118 \\ 2x+2 &= 118 \\ 2x &= 116 \\ x &= 58 \\ x+2 &= 60 \end{aligned}$$

Hank Greenberg had 58 home runs and Babe Ruth had 60.

29. Let x = the number of miles you can travel in one week for \$320.

$$\begin{aligned} 200+0.15x &= 320 \\ 200+0.15x-200 &= 320-200 \\ 0.15x &= 120 \\ \frac{0.15x}{0.15} &= \frac{120}{0.15} \\ x &= 800 \end{aligned}$$

You can travel 800 miles in one week for \$320.
This checks because \$200 + 0.15(\$800) = \$320.

30. Let x = the number of miles you can travel in one week for \$395.

$$\begin{aligned} 180+0.25x &= 395 \\ 180+0.25x-180 &= 395-180 \\ 0.25x &= 215 \\ \frac{0.25x}{0.25} &= \frac{215}{0.25} \\ x &= 860 \end{aligned}$$

You can travel 860 miles in one week for \$395.

31. Let x = the number of years after 2014.

$$37,600 + 1250x = 46,350$$
$$1250x = 8750$$
$$\frac{1250x}{1250} = \frac{8750}{1250}$$
$$x = 7$$

7 years after 2014, or in 2021, the average price of a new car will be $46,350.

32. Let x = the number of years after 2014.

$$11.3 + 0.2x = 12.3$$
$$0.2x = 1$$
$$\frac{0.2x}{0.2} = \frac{1}{0.2}$$
$$x = 5$$

5 years after 2014, or in 2019, the average age of vehicles on U.S. roads will be 12.3 years.

33. Let x = the width of the field.
Let $4x =$ the length of the field.

$$P = 2l + 2w$$
$$500 = 2 \cdot 4x + 2 \cdot x$$
$$500 = 8x + 2x$$
$$500 = 10x$$
$$\frac{500}{10} = \frac{10x}{10}$$
$$50 = x$$
$$x = 50$$
$$4x = 200$$

The field is 50 yards wide and 200 yards long.

34. Let x = the width of the field.
Let $5x =$ the length of the field.

$$P = 2l + 2w$$
$$288 = 2 \cdot 5x + 2 \cdot x$$
$$288 = 10x + 2x$$
$$288 = 12x$$
$$\frac{288}{12} = \frac{12x}{12}$$
$$24 = x$$
$$x = 24$$
$$5x = 120$$

The field is 24 yards wide and 120 yards long.

35. Let x = the width of a football field.
Let $x + 200 =$ the length of a football field.

$$P = 2l + 2w$$
$$1040 = 2(x + 200) + 2 \cdot x$$
$$1040 = 2x + 400 + 2x$$
$$1040 = 4x + 400$$
$$640 = 4x$$
$$160 = x$$
$$x = 160$$
$$x + 200 = 360$$

A football field is 160 feet wide and 360 feet long.

36. Let x = the width of a basketball court.
Let $x + 13 =$ the length of a basketball court.

$$P = 2l + 2w$$
$$86 = 2(x + 13) + 2 \cdot x$$
$$86 = 2x + 26 + 2x$$
$$86 = 4x + 26$$
$$60 = 4x$$
$$15 = x$$
$$x = 15$$
$$x + 13 = 28$$

A basketball court is 15 meters wide and 28 meters long.

37. As shown in the diagram,
let x = the height and $3x$ = the length.
To construct the bookcase, 3 heights and 4 lengths are needed.
Since 60 feet of lumber is available,

$$3x + 4(3x) = 60$$
$$3x + 12x = 60$$
$$15x = 60$$
$$x = 4$$
$$3x = 12$$

The bookcase is 12 feet long and 4 feet high.

38. As shown in the diagram,
let x = the length of a shelf and $x + 3$ = the height of the bookcase,
4 shelves and 2 heights are needed.
Since 18 feet of lumber is available,

$$4x+2(x+3)=18.$$
$$4x+2x+6=18$$
$$6x+6=18$$
$$6x=12$$
$$x=2$$
$$x+3=5$$

The length of each shelf is 2 feet and the height of the unit is 5 feet.

39. Let x = the price before the reduction.

$$x-0.20x=320$$
$$0.80x=320$$
$$\frac{0.80x}{0.80}=\frac{320}{0.80}$$
$$x=400$$

The price before the reduction was \$400.

40. Let x = the price before the reduction.

$$x-0.30x=98$$
$$0.70x=98$$
$$\frac{0.70x}{0.70}=\frac{98}{0.70}$$
$$x=140$$

The DVD player's price before the reduction was \$140.

41. Let x = the last year's salary.

$$x+0.08x=50,220$$
$$1.08x=50,220$$
$$\frac{1.08x}{1.08}=\frac{50,220}{1.08}$$
$$x=46,500$$

Last year's salary was \$46,500.

42. Let x = the last year's salary.

$$x+0.09x=42,074$$
$$1.09x=42,074$$
$$\frac{1.09x}{1.09}=\frac{42,074}{1.09}$$
$$x=38,600$$

Last year's salary was \$38,600.

43. Let x = the price of the car without tax.

$$x+0.06x=23,850$$
$$1.06x=23,850$$
$$\frac{1.06x}{1.06}=\frac{23,850}{1.06}$$
$$x=22,500$$

The price of the car without sales tax was \$14,500.

44. Let x = the nightly cost without tax.

$$x+0.08x=172.80$$
$$1.08x=172.80$$
$$\frac{1.08x}{1.08}=\frac{172.80}{1.08}$$
$$x=160$$

The nightly cost without tax is \$160.

45. Let x = the number of hours of labor.

$$63+35x=448$$
$$63+35x-63=448-63$$
$$35x=385$$
$$\frac{35x}{35}=\frac{385}{35}$$
$$x=11$$

It took 11 hours of labor to repair the car.

46. Let x = the number of hours of labor.

$$532+63x=1603$$
$$532+63x-532=1603-532$$
$$63x=1071$$
$$\frac{63x}{63}=\frac{1071}{63}$$
$$x=17$$

It took 17 hours of labor to repair the sailboat.

47. – 50. Answers will vary.

51. does not make sense; Explanations will vary.

52. makes sense

53. makes sense

54. does not make sense; Explanations will vary.
Sample explanation: It is correct to use $x+2$ for the second consecutive odd integer because any odd integer is 2 more than the previous odd integer. In other words, adding 2 to the first odd integer will skip over the even integer and take you to the next odd integer.

55. false; Changes to make the statement true will vary. A sample change is: This should be modeled by $x-10=160$.

56. false; Changes to make the statement true will vary. A sample change is: This should be modeled by $x-0.35x=780$.

57. true

58. true

59. Let x = the number of inches over 5 feet.

$$W = 100+5x$$
$$135 = 100+5x$$
$$135-100 = 100-100+5x$$
$$35 = 5x$$
$$\frac{35}{5} = \frac{5x}{5}$$
$$7 = x$$

The height 5' 7" corresponds to 135 pounds.

60. Let x = the number of minutes.
Note that \$0.55 is the cost of the first minute and $\$0.40(x-1)$ is the cost of the remaining minutes.

$$0.55+0.40(x-1) = 6.95$$
$$0.55+0.4x-0.40 = 6.95$$
$$0.4x+0.15 = 6.95$$
$$0.4x+0.15-0.15 = 6.95-0.15$$
$$0.4x = 6.80$$
$$\frac{0.4x}{0.4} = \frac{6.80}{0.4}$$
$$x = 17$$

The phone call lasted 17 minutes.

61. Let x = the woman's age.
Let $3x$ = the "uncle's" age.

$$3x+20 = 2(x+20)$$
$$3x+20 = 2x+40$$
$$3x-2x+20 = 2x-2x+40$$
$$x+20 = 40$$
$$x+20-20 = 40-20$$
$$x = 20$$

The woman is 20 years old and the "uncle" is $3x$ = 3(20) = 60 years old.

62. Let x = weight of unpeeled bananas.
Let $\frac{1}{8}x$ = the weight of banana peel and $\frac{7}{8}x$ = the weight of peeled banana.
The information in the cartoon translates into the equation.

$$x = \frac{7}{8}x+\frac{7}{8}$$

To solve this equation, first eliminate fractions by multiplying both sides by the LCD, which is 8.

$$8x = 8\left(\frac{7}{8}x+\frac{7}{8}\right)$$
$$8x = 8\left(\frac{7}{8}x\right)+8\left(\frac{7}{8}\right)$$
$$8x = 7x+7$$
$$8x-7x = 7x+7-7x$$
$$x = 7$$

The unpeeled banana weighs 7 ounces.

63.

$$\frac{4}{5}x = -16$$
$$\frac{5}{4}\left(\frac{4}{5}x\right) = \frac{5}{4}(-16)$$
$$x = -20$$

Check:

$$\frac{4}{5}(-20) = -16$$
$$\frac{4}{5}\cdot\frac{-20}{1} = -16$$
$$\frac{-80}{5} = -16$$
$$-16 = -16$$

The solution set is $\{-20\}$.

64.

$$6(y-1)+7 = 9y-y+1$$
$$6y-6+7 = 9y-y+1$$
$$6y+1 = 8y+1$$
$$6y+1-1 = 8y+1-1$$
$$6y = 8y$$
$$6y-8y = 8y-8y$$
$$-2y = 0$$
$$y = 0$$

Check:

$$6(0-1)+7 = 9(0)-0+1$$
$$6-10+7 = 0-0+1$$
$$1 = 1$$

The solution set is $\{0\}$.

65. $V = \frac{1}{3}lwh$ for w

$$V = \frac{1}{3}lwh$$
$$3V = 3\left(\frac{1}{3}lwh\right)$$
$$3V = lwh$$
$$\frac{3V}{lh} = \frac{lwh}{lh}$$
$$\frac{3V}{lh} = w \quad \text{or} \quad w = \frac{3V}{lh}$$

66. $A = \frac{1}{2}bh$

$$30 = \tfrac{1}{2}\cdot 12h$$
$$30 = 6h$$
$$\frac{30}{6} = \frac{6h}{6}$$
$$5 = h$$

67. $A = \frac{1}{2}h(a+b)$

$$A = \tfrac{1}{2}(7)(10+16)$$
$$A = \tfrac{1}{2}(7)(26)$$
$$A = 91$$

68. $x = 4(90-x)-40$

$$x = 360-4x-40$$
$$x = 320-4x$$
$$5x = 320$$
$$x = 64$$

The solution set is $\{64\}$.

2.6 Check Points

1. $A = 24, b = 4$

$$A = \frac{1}{2}bh$$
$$24 = \frac{1}{2}\cdot 4\cdot h$$
$$24 = 2h$$
$$12 = h$$

The height of the sail is 12 ft.

2. Use the formulas for the area and circumference of a circle. The radius is 20 ft.

$$A = \pi r^2$$
$$A = \pi(20)^2$$
$$= 400\pi$$
$$\approx 1256 \text{ or } 1257$$

The area is 400π ft^2 or approximately 1256 ft^2 or 1257 ft^2.

$$C = 2\pi r$$
$$C = 2\pi(20)$$
$$= 40\pi$$
$$\approx 126$$

The circumference is 40π ft or approximately 126 ft.

3. The radius of the large pizza is 9 inches, and the radius of the medium pizza is 7 inches.

large pizza:

$$A = \pi r^2 = \pi(9 \text{ in.})^2 = 81\pi \text{ in.}^2 \approx 254 \text{ in.}^2$$

medium pizza:

$$A = \pi r^2 = \pi(7 \text{ in.})^2 = 49\pi \text{ in.}^2 \approx 154 \text{ in.}^2$$

For each pizza, find the price per inch by dividing the price by the area.

Price per square inch for the large pizza

$$= \frac{\$20.00}{81\pi \text{ in.}^2} \approx \frac{\$20.00}{254 \text{ in.}^2} \approx \frac{\$0.08}{\text{in.}^2}$$

Price per square inch for the medium pizza

$$= \frac{\$14.00}{49\pi \text{ in.}^2} \approx \frac{\$14.00}{154 \text{ in.}^2} \approx \frac{\$0.09}{\text{in.}^2}.$$

The large pizza is the better buy.

4. Smaller cylinder: $r = 3$ in., $h = 5$ in.

$$V = \pi r^2 h$$
$$V = \pi(3)^2\cdot 5$$
$$= 45\pi$$

The volume of the smaller cylinder is $45\pi \text{in.}^3$.

Larger cylinder: $r = 3$ in., $h = 10$ in.

$$V = \pi r^2 h$$
$$V = \pi(3)^2\cdot 10$$
$$= 90\pi$$

The volume of the smaller cylinder is $90\pi \text{in.}^3$.

The ratio of the volumes of the two cylinders is

$$\frac{V_{\text{larger}}}{V_{\text{smaller}}} = \frac{90\pi \text{in.}^3}{45\pi \text{in.}^3} = \frac{2}{1}.$$

So, the volume of the larger cylinder is 2 times the volume of the smaller cylinder.

5. Use the formula for the volume of a sphere. The radius is 4.5 in.

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(4.5)^3$$
$$= 121.5\pi$$
$$\approx 382$$

The volume is approximately 382 in.^{3}. Thus the 350 cubic inches will not be enough to fill the ball. About 32 more cubic inches are needed.

6. Let $3x$ = the measure of the first angle.
Let x = the measure of the second angle.
Let $x - 20$ = the measure of the third angle.

$$3x + x + (x - 20) = 180$$
$$5x - 20 = 180$$
$$5x = 200$$
$$x = 40$$
$$3x = 120$$
$$x - 20 = 20$$

The three angle measures are 120°, 40°, and 20°.

7. *Step 1* Let x = the measure of the angle.

Step 2 Let $90 - x$ = the measure of its complement.

Step 3 The angle's measure is twice that of its complement, so the equation is $x = 2 \cdot (90 - x)$.

Step 4 Solve this equation

$$x = 2 \cdot (90 - x)$$
$$x = 180 - 2x$$
$$x + 2x = 180 - 2x + 2x$$
$$3x = 180$$
$$x = 60$$

The measure of the angle is 60°.

Step 5 The complement of the angle is $90° - 60° = 30°$, and 60° is indeed twice 30°.

2.6 Concept and Vocabulary Check

1. $A = \frac{1}{2}bh$

2. $A = \pi r^2$

3. $C = 2\pi r$

4. radius; diameter

5. $V - lwh$

6. $V = \pi r^2 h$

7. 180°

8. complementary

9. supplementary

10. $90 - x$; $180 - x$

2.6 Exercise Set

1. Use the formulas for the perimeter and area of a rectangle. The length is 6 m and the width is 3 m.

$$P = 2l + 2w$$
$$= 2(6) + 2(3) = 12 + 6 = 18$$
$$A = lw = 6 \cdot 3 = 18$$

The perimeter is 18 meters, and the area is 18 square meters.

2. Use the formulas for the perimeter and area of a rectangle. The length is 4 ft and the width is 3 ft.

$$P = 2l + 2w$$
$$P = 2(4) + 2(3)$$
$$P = 8 + 6 = 14$$

The perimeter is 14 ft.

$$A = lw$$
$$A = 4 \cdot 3 = 12$$

The area is 12 ft^2.

3. Use the formula for the area of a triangle. The base is 14 in and the height is 8 in.

$$A = \frac{1}{2}bh = \frac{1}{2}(14)(8) = 56$$

The area is 56 square inches.

4. Use the formula for the area of a triangle. The base is 30 m and the height is 33 m.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(30)(33) = 495$$

The area is 495 m^2.

5. Use the formula for the area of a trapezoid. The bases are 16 m and 10 m and the height is 7 m.

$$A = \frac{1}{2}h(a+b)$$

$$= \frac{1}{2}(7)(16+10) = \frac{1}{2}\cdot 7 \cdot 26 = 91$$

The area is 91 square meters.

6. Use the formula for the area of a trapezoid. The bases are 37 meters and 26 meters and the height is 18 meters.

$$A = \frac{1}{2}h(a+b)$$

$$A = \frac{1}{2}(18)(37+26)$$

$$A = \frac{1}{2}\cdot 18 \cdot 63 = 567$$

The area is 567 m^2.

7. $A = 1250,\ w = 25$

$$A = lw$$

$$1250 = l \cdot 25$$

$$50 = l$$

The length of the swimming pool is 50 feet.

8. $A = 2450; w = 35$

$$A = lw$$

$$2450 = l \cdot 35$$

$$70 = l$$

The length of the swimming pool is 70 ft.

9. $A = 20, b = 5$

$$A = \frac{1}{2}bh$$

$$20 = \frac{1}{2}\cdot 5 \cdot h$$

$$20 = \frac{5}{2}h$$

$$\frac{2}{5}(20) = \frac{2}{5}\left(\frac{5}{2}h\right)$$

$$8 = h$$

The height of the triangle is 8 feet.

10. $A = 30, b = 6$

$$A = \frac{1}{2}bh$$

$$30 = \frac{1}{2}\cdot 6 \cdot h$$

$$60 = 6h$$

$$10 = h$$

The height is 10 ft.

11. $P = 188,\ w = 44$

$$188 = 2l + 2(44)$$

$$188 = 2l + 88$$

$$100 = 2l$$

$$50 = l$$

The length of the rectangle is 50 cm.

12. $P = 208, w = 46$

$$P = 2l + 2w$$

$$208 = 2l + 2(46)$$

$$208 = 2l + 92$$

$$116 = 2l$$

$$58 = l$$

The length of the rectangle is 58 cm.

13. Use the formulas for the area and circumference of a circle. The radius is 4 cm.

$$A = \pi r^2$$

$$A = \pi(4)^2$$

$$= 16\pi$$

$$\approx 50$$

The area is $16\pi \text{ cm}^2$ or approximately 50 cm^2.

$$C = 2\pi r$$

$$C = 2\pi(4)$$

$$= 8\pi$$

$$\approx 25$$

The circumference is 8π cm or approximately 25 cm.

14. Use the formula for the area and circumference of a circle. The radius is 9m.

$A = \pi r^2$

$A = \pi(9)^2$

$= 81\pi$

≈ 254

The area is 81π m^2 or approximately 254 m^2.

$C = 2\pi r$

$C = 2\pi(9)$

$= 18\pi$

≈ 57

The circumference is 18π m or approximately 57 m.

15. Since the diameter is 12 yd, the radius is $\frac{12}{2} = 6$ yd.

$A = \pi r^2$

$A = \pi(6)^2$

$= 36\pi$

≈ 113

The area is 36π yd^2 or approximately 113 yd^2.

$C = 2\pi r$

$C = 2\pi \cdot 6$

$= 12\pi$

≈ 38

The circumference is 12π yd or approximately 38 yd.

16. Since the diameter is 40 ft, the radius is $\frac{40}{2} = 20$ ft.

$A = \pi r^2$

$A = \pi(20)^2$

$= 400\pi$

≈ 1257

The area is 400π ft^2 or approximately 1257 ft^2.

$C = 2\pi r$

$C = 2\pi \cdot 20$

$= 40\pi$

≈ 126

The circumference is 40π ft or approximately 126 ft.

17. $C = 2\pi r$

$14\pi = 2\pi r$

$\frac{14\pi}{2\pi} = \frac{2\pi r}{2\pi}$

$7 = r$

The radius is 7 in. and the diameter is 2(7 in) = 14 in.

18. $C = 2\pi r$

$16\pi = 2\pi r$

$\frac{16\pi}{2\pi} = \frac{2\pi r}{2\pi}$

$8 = r$

The radius is 8 in. and the diameter is $2 \cdot 8 = 16$ in.

19. Use the formula for the volume of a rectangular solid. The length and width are each 3 inches and the height is 4 inches.

$V = lwh$

$V = 3 \cdot 3 \cdot 4$

$= 36$

The volume is 36 in.^{3}.

20. Use the formula for the volume of a rectangular solid. The length is 5 cm and width and height are each 3 cm.

$V = lwh$

$V = 5 \cdot 3 \cdot 3$

$= 45$

The volume is 45 cm^3.

21. Use the formula for the volume of a cylinder. The radius is 5 cm and the height is 6 cm.

$V = \pi r^2 h$

$V = \pi(5)^2 6$

$= \pi(25)6$

$= 150\pi$

≈ 471

The volume of the cylinder is 150π cm^3 or approximately 471 cm^3.

22. Use the formula for the volume of a cylinder. The radius is 6 cm and the height is 8 cm.

$V = \pi r^2 h$

$V = \pi(6)^2 \cdot 8$

$= 288\pi$

≈ 905

The volume is 288π cm^3 or approximately 905 cm^3.

23. Use the formula for the volume of a sphere. The diameter is 18 cm, so the radius is 9 cm.

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(9)^3$$
$$= 972\pi$$
$$\approx 3052$$

The volume is 972π cm^3 or approximately 3052 cm^3.

24. Use the formula for the volume of a sphere. The diameter is 24 in., so the radius is 12 in.

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(12)^3$$
$$= 2304\pi$$
$$\approx 7238$$

The volume is $2304\,\pi$ in^3 or approximately 7238 in^3.

25. Use the formula for the volume of a cone. The radius is 4 m and the height is 9 m.

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi(4)^2 \cdot 9$$
$$= 48\pi$$
$$\approx 151$$

The volume is 48π m^3 or approximately 151 m^3.

26. Use the formula for the volume of a cone. The radius is 5 m and the height is 16 m.

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi(5)^2 \cdot 16$$
$$= \frac{400}{3}\pi$$
$$\approx 419$$

The volume is $\frac{400}{3}\pi$ m^3 or approximately 419 m^3.

27. $$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$
$$\frac{V}{\pi r^2} = h$$

28. $$V = \frac{1}{3}\pi r^2 h$$
$$3V = 3\left(\frac{1}{3}\pi r^2 h\right)$$
$$3V = \pi r^2 h$$
$$\frac{3V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$
$$\frac{3V}{\pi r^2} = h \text{ or } h = \frac{3V}{\pi r^2}$$

29. Smaller cylinder: $r = 3$ in, $h = 4$ in.

$$V = \pi r^2 h = \pi(3)^2 \cdot 4 = 36\pi$$

The volume of the smaller cylinder is $36\pi in^3$.

Larger cylinder: $r = 3(3 \text{ in}) = 9$ in, $h = 4$ in.

$$V = \pi r^2 h = \pi(9)^2 \cdot 4 = 324\pi$$

The volume of the larger cylinder is 324π in.^{3}.

The ratio of the volumes of the two cylinders is

$$\frac{V_{\text{larger}}}{V_{\text{smaller}}} = \frac{324\pi}{36\pi} = \frac{9}{1}.$$

So, the volume of the larger cylinder is 9 times the volume of the smaller cylinder.

30. Smaller cylinder; $r = 2$ in., $h = 3$ in.

$$V = \pi r^2 h$$
$$V = \pi(2)^2 \cdot 3$$
$$V = 12\pi$$

The volume of the smaller cylinder is 12π in^3.

Large cylinder: $r = 4(2 \text{ in.}) = 8$ in., $h = 3$ in.

$$V = \pi r^2 h$$
$$V = \pi(8)^2 \cdot 3$$
$$V = 192\pi$$

The volume of the larger cylinder is 192π in.^{3}.

The ratio of the volumes of the two cylinders is

$$\frac{V_{\text{Larger}}}{V_{\text{Smaller}}} = \frac{192\pi}{12\pi} = \frac{16}{1},$$ so the volume of the larger cylinder is 16 times the volume of the smaller cylinder.

31. The sum of the measures of the three angles of any triangle is $180°$.

$$x + x + (x + 30) = 180$$
$$3x + 30 = 180$$
$$3x = 150$$
$$x = 50$$
$$x + 30 = 80$$

The three angle measures are $50°, 50°, \text{and } 80°$.

32. The sum of the measures of the three angles of a triangle is 180°.

$$\begin{aligned} x+3x+(x+40) &= 180 \\ 5x+40 &= 180 \\ 5x &= 140 \\ x &= 28 \\ 3x &= 84 \\ x+40 &= 68 \end{aligned}$$

The three angle measures are 28°, 84°, and 68°.

33.
$$\begin{aligned} 4x+(3x+4)+(2x+5) &= 180 \\ 9x+9 &= 180 \\ 9x &= 171 \\ x &= 19 \\ 3x+4 &= 61 \\ 2x+5 &= 43 \end{aligned}$$

The three angle measures are 76°, 61°, and 43°.

34.
$$\begin{aligned} x+4x+5x &= 180 \\ 10x &= 180 \\ x &= 18 \\ 4x &= 72 \\ 5x &= 90 \end{aligned}$$

The three angle measures are 18°, 72°, and 90°.

35. Let x = the measure of the smallest angle.
Let $2x$ = the measure of the second angle.
Let $x + 20$ = the measure of the third angle.

$$\begin{aligned} x+2x+(x+20) &= 180 \\ 4x+20 &= 180 \\ 4x &= 160 \\ x &= 40 \\ 2x &= 80 \\ x+20 &= 60 \end{aligned}$$

The three angle measures are 40°, 80°, and 60°.

36. Let x = the measure of the smallest angle.
Let $3x$ = the measure of the second angle.
Let $x + 30$ = the measure of the third angle.

$$\begin{aligned} x+3x+(x+30) &= 180 \\ 5x+30 &= 180 \\ 5x &= 150 \\ x &= 30 \\ 3x &= 90 \\ x+30 &= 60 \end{aligned}$$

The three angle measures are 30°, 90°, and 60°.

37. If the measure of an angle is $58°$, the measure of its complement is $90° - 58° = 32°$.

38. If the measure of an angle is 41°, the measure of its complement is 90° – 41° = 49°.

39. If the measure of an angle is $88°$, the measure of its complement is $2°$.

40. If the measure of an angle is 2°,the measure of its complement is 90° – 2° = 88°

41. If the measure of an angle is $132°$, the measure of its supplement is $180° - 132° = 48°$.

42. If the measure of an angle is 93°, the measure of its supplement is 180° – 93° = 87°.

43. If the measure of an angle is $90°$, the measure of its supplement is $180° - 90° = 90°$.

44. If the measure of an angle is 179.5°, the measure of its supplement is 180° – 179.5° = 0.5°

45. *Step 1* Let x = the measure of the angle.

Step 2 Let $90 - x$ = the measure of its complement.

Step 3 The angle's measure is $60°$ more than that of its complement, so the equation is $x = (90 - x) + 60$.

Step 4 Solve this equation

$$\begin{aligned} x &= 90 - x + 60 \\ x &= 150 - x \\ 2x &= 150 \\ x &= 75 \end{aligned}$$

The measure of the angle is 75°.

Step 5 The complement of the angle is 90° - 75° = 15°, and 75° is 60° more than 15°.

46. *Step 1* Let x = the measure of the angle.

Step 2 Then $90 - x$ = the measure of its complement.

Step 3 The angle's measure is 78° less than that of its complement, so the equation is $x = (90 - x) - 78$.

Step 4 Solve this equation

$$x = 90 - x - 78$$
$$x = 12 - x$$
$$2x = 12$$
$$x = 6$$

The measure of the angle is 6°.

Step 5 The complement of the angle is 90° - 6° = 84°, and 6° is 78° less than 84°.

47. *Step 1* Let x = the measure of the angle.

Step 2 Then $180 - x$ = the measure of its supplement.

Step 3 The angle's measure is three times that of its supplement, so the equation is $x = 3(180 - x)$.

Step 4 Solve this equation

$$x = 3(180 - x)$$
$$x = 540 - 3x$$
$$4x = 540$$
$$x = 135$$

The measure of the angle is $135°$.

Step 5 The measure of its supplement is $180° - 135° = 45°$, and $135° = 3(45°)$, so the proposed solution checks.

48. *Step 1* Let x = the measure of the angle.

Step 2 Then $180 - x$ = the measure of its supplement.

Step 3 The angle's measure is 16° more than three times that of its supplement, so the equation is $x = 3(180 - x) + 16$.

Step 4 Solve this equation

$$x = 3(180 - x) + 16$$
$$x = 540 - 3x + 16$$
$$x = 556 - 3x$$
$$4x = 556$$
$$x = 139$$

The measure of the angle is 139°.

Step 5 The measure of its supplement is 180° - 139° = 41°, and 139° = 3(41°) + 16°, so the proposed solution checks.

49. *Step 1* Let x = the measure of the angle.

Step 2 Let $180 - x$ = the measure of its supplement, and, $90 - x$ = the measure of its complement.

Step 3 The measure of the angle's supplement is 10° more than three times that of its complement, so the equation is $180 - x = 3(90 - x) + 10$.

Step 4 Solve this equation

$$180 - x = 3(90 - x) + 10$$
$$180 - x = 270 - 3x + 10$$
$$180 - x = 280 - 3x$$
$$2x = 100$$
$$x = 50$$

The measure of the angle is $50°$.

Step 5 The measure of its supplement is $130°$ and the measure of its complement is $40°$. Since $130° = 3(40°) + 10°$, the proposed solution checks.

50. *Step 1* Let x = the measure of the angle.

Step 2 Let $180 - x$ = the measure of its supplement, and, $90 - x$ = the measure of its complement.

Step 3 The measure of the angle's supplement is 10° more than three times that of its complement, so the equation is $180 - x = 3(90 - x) + 10$.

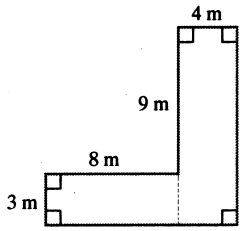
Step 4 Solve this equation

$$\begin{aligned} 180 - x &= 2(90 - x) + 52 \\ 180 - x &= 180 - 2x + 52 \\ 180 - x &= 232 - 2x \\ 180 - x + 2x &= 232 - 2x + 2x \\ 180 + x &= 232 \\ x &= 52 \end{aligned}$$

The measure of the angle is $52°$.

Step 5 The measure of its supplement is 128° and the measure of its complement is 38°. Since 128° = 2(38°) + 52°, the proposed solution checks.

51. Divide the shape into two rectangles.

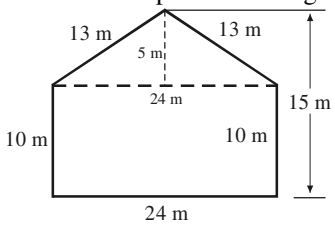


$$A_{\text{entire figure}} = A_{\text{bottom rectangle}} + A_{\text{side rectangle}}$$

$$\begin{aligned} A_{\text{entire figure}} &= 3 \cdot 8 + 4(9 + 3) \\ &= 24 + 4(12) \\ &= 24 + 48 \\ &= 72 \end{aligned}$$

The area of the figure is 72 square meters.

52. Divide the shape into a triangle and a rectangle.

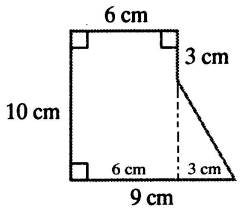


$$A_{\text{entire figure}} = A_{\text{rectangle}} + A_{\text{triangle}}$$

$$\begin{aligned} A_{\text{entire figure}} &= lw + \frac{1}{2}bh \\ &= 10(24) + \frac{1}{2}(24)(15 - 10) \\ &= 240 + \frac{1}{2}(24)(5) \\ &= 240 + 60 = 300 \end{aligned}$$

The area of the figure is 300 m^2.

53. Divide the shape into a rectangle and a triangle.



$$A_{\text{entire figure}} = A_{\text{rectangle}} + A_{\text{triangle}}$$

$$\begin{aligned} A_{\text{entire figure}} &= lw + \frac{1}{2}bh \\ &= 10(6) + \frac{1}{2}(3)(10 - 3) \\ &= 60 + \frac{1}{2}(3)(7) \\ &= 60 + 10.5 = 70.5 \end{aligned}$$

The area of the figure is 70.5 cm^2.

54. Subtract the area of the two smaller circles from the area of the larger circle. Note that the radius of the large circle is 4 and note that the two smaller circles are the same size.

$$\begin{aligned} A_{\text{shaded}} &= A_{\text{larger circle}} - 2 \cdot A_{\text{smaller circle}} \\ &= \pi R^2 - 2 \cdot \pi r^2 \\ &= \pi(4)^2 - 2 \cdot \pi(2)^2 \\ &= \pi(16) - 2 \cdot \pi(4) \\ &= 16\pi - 8\pi \\ &= 8\pi \end{aligned}$$

The shaded area is 8π cm^2.

55. Subtract the volume of the three hollow portions from the volume of the whole rectangular solid.

$$\begin{aligned} V_{\text{cement block}} &= V_{\text{rectangular solid}} - 3 \cdot V_{\text{hollow}} \\ &= LWH - 3 \cdot lwh \\ &= (8)(8)(16) - 3 \cdot (4)(6)(8) \\ &= 1024 - 576 \\ &= 448 \end{aligned}$$

The volume of the cement block is 448 cubic inches.

56. Subtract the volume of the smaller cylinder from the volume of the larger cylinder.

$$\begin{aligned} V_{\text{shaded}} &= V_{\text{larger cylinder}} - V_{\text{smaller cylinder}} \\ &= \pi R^2 h - \pi r^2 h \\ &= \pi\left(\frac{6}{2}\right)^2 \cdot 10 - \pi\left(\frac{2}{2}\right)^2 \cdot 10 \\ &= \pi(3)^2 \cdot 10 - \pi(1)^2 \cdot 10 \\ &= 90\pi - 10\pi \\ &= 80\pi \end{aligned}$$

The volume of the shaded region is 80π cubic inches.

57. The area of the office is $(20 \text{ ft})(16 \text{ ft}) = 320 \text{ ft}^2$. Use a proportion to determine how much of the yearly electric bill is deductible.

Let x = the amount of the electric bill that is deductible.

$$\begin{aligned} \frac{320}{2200} &= \frac{x}{4800} \\ 2200x &= (320)(4800) \\ 2200x &= 1{,}536{,}000 \\ \frac{2200x}{2200} &= \frac{1{,}546{,}000}{2200} \\ x &\approx 698.18 \end{aligned}$$

$698.18 of the yearly electric bill is deductible.

58. **a.** The area of the lot is $(500 \text{ ft})(200 \text{ ft}) = 100{,}000 \text{ ft}^2$.

The area of the house is $(100 \text{ ft})(60 \text{ ft}) = 6000 \text{ ft}^2$.

The area of the shed is $(20 \text{ ft})(20 \text{ ft}) = 400 \text{ ft}^2$.

The area of the driveway is $(20 \text{ ft})(100 \text{ ft}) = 2000 \text{ ft}^2$.

Therefore, the area of the lawn is $100{,}000 - 6000 - 400 - 2000 = 91{,}600 \text{ ft}^2$.

Since each bag of fertilizer covers 4000 square feet and $\frac{91{,}600}{4000} = 22.9$, 23 bags of fertilizer will be needed.

b. The cost of the fertilizer is $23(\$25) = \$575$.

59. The radius of the large pizza is $\frac{1}{2} \cdot 14 = 7$ inches, and the radius of the medium pizza is $\frac{1}{2} \cdot 7 \text{ inches} = 3.5$ inches.

large pizza:

$$\begin{aligned} A &= \pi r^2 = \pi(7 \text{ in.})^2 \\ &= 49\pi \text{ in.}^2 \approx 154 \text{ in.}^2 \end{aligned}$$

medium pizza:

$$\begin{aligned} A &= \pi r^2 = \pi(3.5 \text{ in.})^2 \\ &= 12.25 \text{ in.}^2 \approx 38.465 \text{ in.}^2 \end{aligned}$$

For each pizza, find the price per inch by dividing the price by the area.

Price per square inch for the large pizza $= \frac{\$12.00}{154 \text{ in.}^2} \approx \frac{\$0.08}{\text{in.}^2}$ and the price per square inch for the medium pizza $= \frac{\$5.00}{28.465 \text{ in.}^2} \approx \frac{\$0.13}{\text{in.}^2}$.

The large pizza is the better buy.

60. The radius of the large pizza is $\frac{1}{2} \cdot 16 \text{ inches} = 8$ inches, and the radius of each small pizza is $\frac{1}{2} \cdot 10$ inches = 5 inches.

Large pizza:

$$A = \pi r^2 = \pi(8 \text{ in.})^2 = 64\pi \text{ in.}^2 \approx 201 \text{ in.}^2$$

Small pizza:

$$A = \pi r^2 = \pi(5 \text{ in.})^2 = 25\pi \text{ in.}^2 \approx 79 \text{ in.}^2$$

The area of one large pizza is about 201 in.^2 and the area of two small pizzas is about $2(79 \text{ in.}^2) = 158 \text{ in.}^2$. Since the price of one large pizza is the same as the price of two small pizzas and the large pizza has the greater area, the large pizza is the better buy. (Because the prices are the same, it is not necessary to find the prices per square inch in this case.)

61. The area of the larger circle is

$$A = \pi r^2 = \pi \cdot 50^2 = 2500\pi \text{ ft}^2.$$

The area of the smaller circle is

$$A = \pi r^2 = \pi \cdot 40^2 = 1600\pi \text{ ft}^2.$$

The area of the circular road is the difference between the area of the larger circle and the area of the smaller circle.

$$A = 2500\pi \text{ ft}^2 - 1600\pi \text{ ft}^2 = 900\pi \text{ ft}^2$$

The cost to pave the circular road is $\$0.80(900\pi) \approx \$2262$.

62. The area of the rectangular portion of the floor is (60 ft)(40 ft) = 2400 ft^2.
Since the radius of each semicircle is 20 ft and the two semicircles together make one circle, the area of the two semicircular portion of the floor is $\pi(20\text{ ft})^2 = 400\pi\text{ ft}^2$.
Therefore, the area of the dance floor is $2400\text{ ft}^2 + 400\pi\text{ ft}^2$.
Since the flooring costs \$10.00 per square foot, the cost of hardwood flooring for the dance floor will be about $\$10(2400+400\pi) = \$36{,}566$.

63. To find the perimeter of the entire window, first find the perimeter of the lower rectangular portion. This is the bottom and two sides of the window, which is 3 ft + 6 ft + 6 ft = 15 ft. Next, find the perimeter or circumference of the semicircular portion of the window. The radius of the semicircle is $\frac{1}{2}\cdot 3\text{ft} = 1.5\text{ft}$, so the circumference is
$\frac{1}{2}\cdot 2\pi r \approx 3.14(1.5) = 4.7\text{ ft}$.
So, approximately 15 ft + 4.7 ft = 19.7 ft of stripping would be needed to frame the window.

64. The circumference of the garden is $2\pi(30\text{ ft}) = 60\pi$ ft.
Since 6 in. = $\frac{1}{2}$ ft., the number of plants needed is
$\frac{60\pi}{\frac{1}{2}} = 2(60\pi) = 120\pi \approx 377$.
To the nearest whole number, 377 plants are needed.

65. First, find the volume of water when the reservoir was full.
$V = lwh = 50\cdot 0\cdot 20 = 30{,}000$
The volume was 30,000 yd^3.
Next, find the volume when the height of the water was 6 yards.
$V = 50\cdot 30\cdot 6 = 9000$
The volume was 9000 yd^3. The amount of water used in the three-month period was 30,000 yd^3 – 9000 yd^3 = 21,000 yd^3.

66. The volume of the foundation is (4 yd)(3 yd). (2 yd) = 24 yd^3. Since each truck holds 6 yd^3 of dirt, $\frac{24}{6} = 4$ truckloads will be needed. Since the charge to remove the dirt is \$10 per load, the cost to have all the dirt hauled away is $4(\$10) = \$40$.

67. For the first can, the diameter is 6 in. so the radius is 3 in. and $V = \pi r^2 h = \pi(3)^2\cdot 5 = 45\pi \approx 141.3$.
The volume of the first can is 141.3 in^3. For the second can, the diameter is 5 in., so the radius is 2.5 in. and $V = \pi r^2 h = \pi(2.5)^2\cdot 6 = 37.5\pi \approx 117.75$.
The volume of the second can is 117.75 in^2. Since the cans are the same price, the can with the greater volume is the better buy. Choose the can with the diameter of 6 inches and height of 5 inches.

68. The volume of each tunnel is $V = \frac{1}{2}\pi r^2 h$
$V = \frac{1}{2}\pi(4)^2\cdot 50{,}000$
$V = 400{,}000\pi$
The volume of each tunnel is $400{,}000\pi$ m^3., so the volume of all three tunnels, which is the total amount of dirt that had to be removed, is $3(400{,}000\pi) = 1{,}200{,}000\pi\text{ m}^3 \approx 3{,}769{,}900\text{ m}^3$.

69. Find the volume of a cylinder with radius 3 feet and height 2 feet 4 inches.
2 ft 4 in = $2\frac{1}{3}$ feet = $\frac{7}{3}$ feet
$V = \pi r^2 h$
$= \pi(3)^2\left(\frac{7}{3}\right) = \pi\cdot 9\cdot\frac{7}{3} = 21\pi \approx 65.94$
The volume of the tank is approximately 65.94 ft^3. This is a little over 1 ft^3 smaller than 67 ft^3 so it is too small to hold 500 gallons of water. Yes, you should be able to win your case.

70. – 78. Answers will vary.

79. does not make sense; Explanations will vary. Sample explanation: Though the heights of the books are proportional to the data, the widths are also changing. This cause the larger values to be visually exaggerated.

80. does not make sense; Explanations will vary. Sample explanation: The sum of the three angles of the triangle must be 180°, but these three values total 181°.

81. does not make sense; Explanations will vary.
Sample explanation: If the radius is doubled, the area is multiplied by 4.

$$A_{\text{radius } x} = \pi r^2$$
$$= \pi(x)^2$$
$$= \pi x^2$$

$$A_{\text{radius } 2x} = \pi r^2$$
$$= \pi(2x)^2$$
$$= 4\pi x^2$$

82. makes sense

83. true

84. true

85. false; Changes to make the statement true will vary. A sample change is: $90°$ does not have a complement.

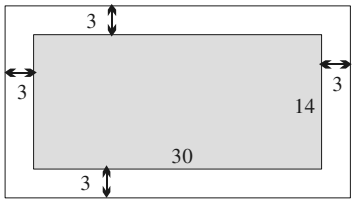
86. true

87. Area of smaller deck $= (8 \text{ ft})(10) = 80 \text{ ft}^2$.
Area of larger deck $= (12 \text{ ft})(15) = 180 \text{ ft}^2$.
Find the ratio of the areas.

$$\frac{A_{\text{larger}}}{A_{\text{smaller}}} = \frac{180\ ft^2}{80\ ft^2} = \frac{2.25}{1} \text{ or } 2.25:1$$

The cost will increase 2.25 times.

88. Consider the following diagram:



The area of the outer rectangle (pool plus path) is $(36 \text{ ft})(20 \text{ ft}) = 720 \text{ ft}^2$. The area of the inner rectangle (pool only) is $(30 \text{ ft})(14 \text{ ft}) = 420 \text{ ft}^2$.
Therefore, the area of the walk is
$720\text{ft}^2 - 420\text{ft}^2 = 300\text{ft}^2$.
Since the cost to resurface the path is \$2 per square foot, the total cost of resurfacing the path is 300(\$2) = \$600.

89. Let x = the radius of the original sphere.
Let $2x$ = the radius of the larger sphere.
Find the ratio of the volumes of the two spheres.

$$\frac{A_{\text{larger}}}{A_{\text{original}}} = \frac{\frac{4}{3}\pi(2x)^3}{\frac{4}{3}\pi x^3} = \frac{8x^3}{x^3} = \frac{8}{1} \text{ or } 8:1$$

If the radius of a sphere is doubled, the volume increases 8 times.

90. If the length, width, and height of a rectangular solid are each multiplied by 10, the volume will be multiplied by $10 \cdot 10 \cdot 10 = 1000$. The volume of the car will be 1000 times that of the model.

91. The angles marked $2x$ and $2x+40$ in the figure are supplementary, so their sum is $180°$.

$$2x + (2x + 40) = 180$$
$$2x + 2x + 40 = 180$$
$$4x + 40 = 180$$
$$4x = 10$$
$$x = 35$$

The angle of inclination is $35°$.

92. $P = 2s + b$ for s

$$P - b = 2s$$
$$\frac{P-b}{2} = \frac{2s}{2}$$
$$\frac{P-b}{2} = s \text{ or } s = \frac{P-b}{2}$$

93. $\frac{x}{2} + 7 = 13 - \frac{x}{4}$
Multiply both sides by the LCD, 4.

$$4\left(\frac{x}{2} + 7\right) = 4\left(13 - \frac{x}{4}\right)$$
$$2x + 28 = 52 - x$$
$$2x + 28 + x = 52 - x + x$$
$$3x + 28 = 52$$
$$3x + 28 - 28 = 52 - 28$$
$$3x = 24$$
$$\frac{3x}{3} = \frac{24}{3}$$
$$x = 8$$

The solution set is $\{8\}$.

94. $\left[3\left(12 \div 2^2 - 3\right)^2\right]^2$

$$= \left[3\left(12 \div 4 - 3\right)^2\right]^2$$
$$= \left[3(3-3)^2\right]^2 = \left(3 \cdot 0^2\right)^2 = 0^2 = 0$$

95. $x + 3 < 8$
$2 + 3 < 8$
$5 < 8$, true
2 is a solution to the inequality.

96. $4y-7\ge 5$

$4(6)-7\ge 5$

$24-7\ge 5$

$17\ge 5$, true

6 is a solution to the inequality.

97. $2(x-3)+5x=8(x-1)$

$2x-6+5x=8x-8$

$7x-6=8x-8$

$7x-8x-6=8x-8x-8$

$-x-6+6=-8+6$

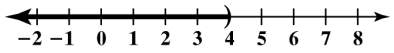
$-x=-2$

$x=2$

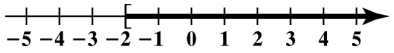
The solution set is $\{2\}$.

2.7 Check Points

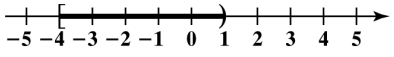
1. a.



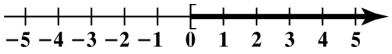
b.



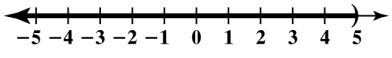
c.



2. a. $[0,\infty)$



b. $(-\infty,5)$

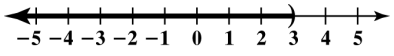


3. $x+6<9$

$x+6-6<9-6$

$x<3$

The solution set is $(-\infty,3)$ or $\{x|x<3\}$.



4. $8x-2\ge 7x-4$

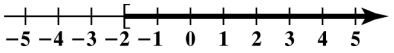
$8x-7x-2\ge 7x-7x-4$

$x-2\ge -4$

$x-2+2\ge -4+2$

$x\ge -2$

The solution set is $[-2,\infty)$ or $\{x|x\ge -2\}$.

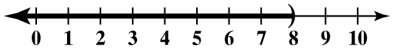


5. a. $\frac{1}{4}x<2$

$4\cdot\frac{1}{4}x<4\cdot 2$

$x<8$

The solution set is $(-\infty,8)$ or $\{x|x<8\}$.

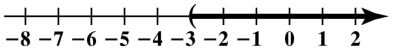


b. $-6x<18$

$\frac{-6x}{-6}>\frac{18}{-6}$

$x>-3$

The solution set is $(-3,\infty)$ or $\{x|x>-3\}$.



6. $5y-3\ge 17$

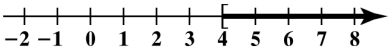
$5y-3+3\ge 17+3$

$5y\ge 20$

$\frac{5y}{5}\ge\frac{20}{5}$

$y\ge 4$

The solution set is $[4,\infty)$ or $\{y|y\ge 4\}$.



7. $6-3x\le 5x-2$

$6-3x-5x\le 5x-5x-2$

$6-8x\le -2$

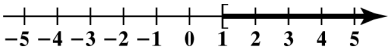
$6-6-8x\le -2-6$

$-8x\le -8$

$\frac{-8x}{-8}\ge\frac{-8}{-8}$

$x\ge 1$

The solution set is $[1,\infty)$ or $\{x|x\ge 1\}$.



8. $2(x-3)-1 \le 3(x+2)-14$

$2x-6-1 \le 3x+6-14$

$2x-7 \le 3x-8$

$2x-3x-7 \le 3x-3x-8$

$-x-7 \le -8$

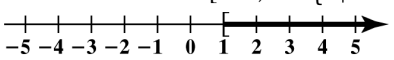
$-x-7+7 \le -8+7$

$-x \le -1$

$\frac{-x}{-1} \ge \frac{-1}{-1}$

$x \ge 1$

The solution set is $[1,\infty)$ or $\{x \mid x \ge 1\}$.



9. $4(x+2) > 4x+15$

$4x+8 > 4x+15$

$4x-4x+8 > 4x-4x+15$

$8 > 15$, false

There is no solution or $\{\ \}$.

10. $3(x+1) \ge 2x+1+x$

$3x+3 \ge 3x+1$

$3x-3x+3 \ge 3x-3x+1$

$3 \ge 1$, true

The solution is $(-\infty,\infty)$ or $\{x \mid x \text{ is a real number}\}$.

11. Let x = your grade on the final examination.

$\frac{82+74+78+x+x}{5} \ge 80$

$\frac{234+2x}{5} \ge 80$

$5\left(\frac{234+2x}{5}\right) \ge 5 \cdot 80$

$234+2x \ge 400$

$234-234+2x \ge 400-234$

$2x \ge 166$

$x \ge 83$

To earn a B you must get at least 83% on the final examination.

12. Let $x =$ the number of people you invite to the picnic.

$95+35x \le 1600$

$35x \le 1505$

$\frac{35x}{35} \le \frac{1505}{35}$

$x \le 43$

To can invite at most 43 people to the picnic.

2.7 Concept and Vocabulary Check

1. $(-\infty,5)$

2. $(2,\infty)$

3. $< b+c$

4. $< bc$

5. $> bc$

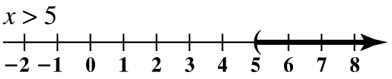
6. subtracting 4; dividing; -3; direction; $>$; $<$

7. $\varnothing$ or the empty set

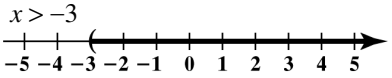
8. $(-\infty,\infty)$

2.7 Exercise Set

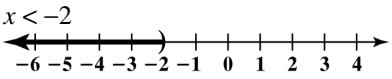
1. $x > 5$



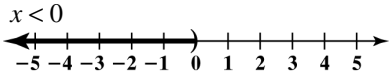
2. $x > -3$



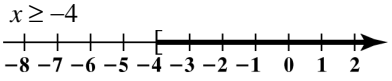
3. $x < -2$



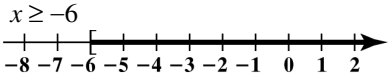
4. $x < 0$



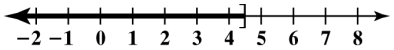
5. $x \ge -4$



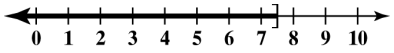
6. $x \ge -6$



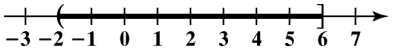
7. $x \le 4.5$



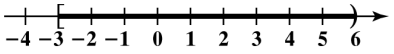
8. $x \le 7.5$



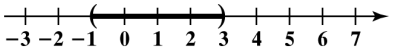
9. $-2 < x \le 6$



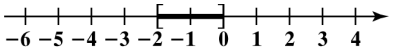
10. $-3 \le x < 6$



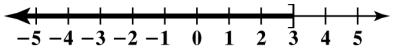
11. $-1 < x < 3$



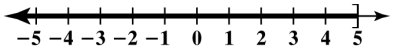
12. $-2 \le x \le 0$



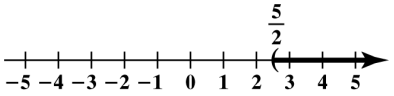
13. $(-\infty, 3]$



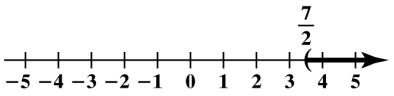
14. $(-\infty, 5]$



15. $\left(\frac{5}{2}, \infty\right)$



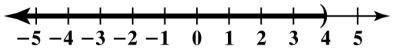
16. $\left(\frac{7}{2}, \infty\right)$



17. $(-\infty, 0]$

18. $(-\infty, 1]$

19. $(-\infty, 4)$



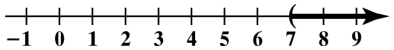
20. $(-\infty, 5)$

21. $x - 3 > 4$

$x - 3 + 3 > 4 + 3$

$x > 7$

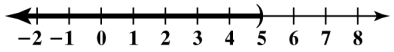
$(7, \infty)$



22. $x + 1 < 6$

$x < 5$

$(-\infty, 5)$

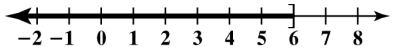


23. $x + 4 \le 10$

$x + 4 - 4 \le 10 - 4$

$x \le 6$

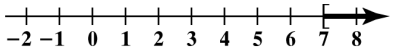
$(-\infty, 6]$



24. $x - 5 \ge 2$

$x \ge 7$

$[7, \infty)$



25. $y - 2 < 0$

$y - 2 + 2 < 0 + 2$

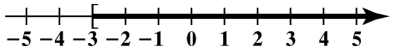
$y < 2$

$(-\infty, 2)$

26. $y + 3 \ge 0$

$y \ge -3$

$[-3, \infty)$

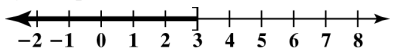


27. $3x + 4 \le 2x + 7$

$3x - 2x \le 7 - 4$

$x \le 3$

$(-\infty, 3]$

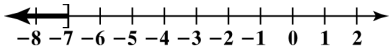


28. $2x+9 \le x+2$

$2x-x \le 2-9$

$x \le -7$

$(-\infty,-7]$

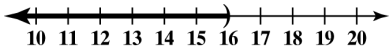


29. $5x-9<4x+7$

$5x-4x<7+9$

$x<16$

$(-\infty,16)$

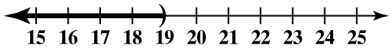


30. $3x-8<2x+11$

$3x-2x<11+8$

$x<19$

$(-\infty,19)$

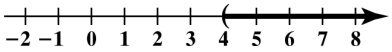


31. $7x-7>6x-3$

$7x-6x>-3+7$

$x>4$

$(4,\infty)$

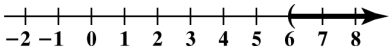


32. $8x-9>7x-3$

$8x-7x>-3+9$

$x>6$

$(6,\infty)$



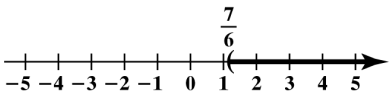
33. $x-\frac{2}{3}>\frac{1}{2}$

$x-\frac{2}{3}+\frac{2}{3}>\frac{1}{2}+\frac{2}{3}$

$x>\frac{3}{6}+\frac{4}{6}$

$x>\frac{7}{6}$

$\left(\frac{7}{6},\infty\right)$

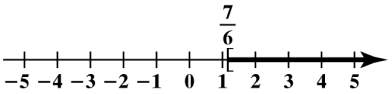


34. $x-\frac{1}{3} \ge \frac{5}{6}$

$x \ge \frac{5}{6}+\frac{1}{3}$

$x \ge \frac{7}{6}$

$\left[\frac{7}{6},\infty\right)$



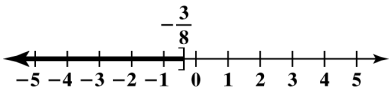
35. $y+\frac{7}{8} \le \frac{1}{2}$

$y+\frac{7}{8}-\frac{7}{8} \le \frac{1}{2}-\frac{7}{8}$

$y \le \frac{4}{8}-\frac{7}{8}$

$y \le -\frac{3}{8}$

$\left(-\infty,-\frac{3}{8}\right]$



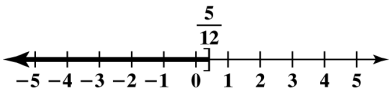
36. $y+\frac{1}{3} \le \frac{3}{4}$

$y \le \frac{3}{4}-\frac{1}{3}$

$y \le \frac{9}{12}-\frac{4}{12}$

$y \le \frac{5}{12}$

$\left(-\infty,\frac{5}{12}\right]$



37. $-15y+13>13-16y$

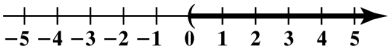
$-15y+13+16y>13-16y+16y$

$y+13>13$

$y+13-13>13-13$

$y>0$

$(0,\infty)$

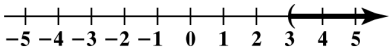


38. $-12+17>20-13y$

$-12+13y>20-17$

$y>3$

$(3,\infty)$



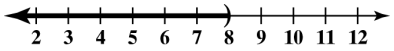
39. $\frac{1}{2}x<4$

$2\left(\frac{1}{2}x\right)<2(4)$

$1x<8$

$x<8$

$(-\infty,8)$

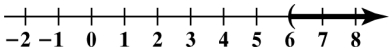


40. $\frac{1}{2}x>3$

$2\left(\frac{1}{2}x\right)>2(3)$

$x>6$

$(6,\infty)$

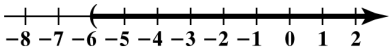


41. $\frac{x}{3}>-2$

$3\left(\frac{x}{3}\right)>3(-2)$

$x>-6$

$(-6,\infty)$

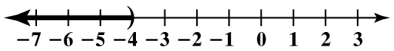


42. $\frac{x}{4}<-1$

$4\left(\frac{x}{4}\right)<4(-1)$

$x<-4$

$(-\infty,-4)$

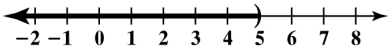


43. $4x<20$

$\frac{4x}{4}<20$

$x<5$

$(-\infty,5)$

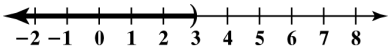


44. $6x<18$

$\frac{6x}{6}<\frac{18}{6}$

$x<3$

$(-\infty,3)$

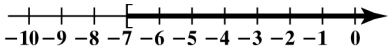


45. $3x\ge-21$

$\frac{3x}{3}\ge\frac{-21}{3}$

$x\ge-7$

$[-7,\infty)$

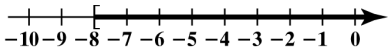


46. $7x\ge-56$

$\frac{7x}{7}\ge\frac{-56}{7}$

$x\ge-8$

$[-8,\infty)$

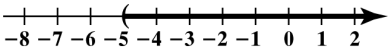


47. $-3x<15$

$\frac{-3x}{-3}>\frac{15}{-3}$

$x>-5$

$(-5,\infty)$

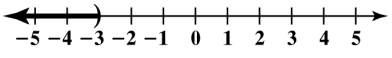


48. $-7x>21$

$\frac{-7x}{-7}<\frac{21}{-7}$

$x<-3$

$(-\infty,-3)$

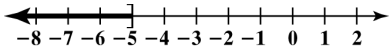


49. $-3x \ge 15$

$$\frac{-3x}{-3} \le \frac{15}{-3}$$

$$x \le -5$$

$(-\infty, -5]$

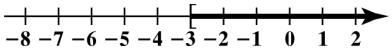


50. $-7x \le 21$

$$\frac{-7x}{-7} \ge \frac{21}{-7}$$

$$x \ge -3$$

$[-3, \infty)$



51. $-16x > -48$

$$\frac{-16x}{-16} < \frac{-48}{-16}$$

$$x < 3$$

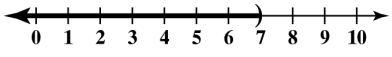
$(-\infty, 3)$

52. $-20x > -140$

$$\frac{-20x}{-20} < \frac{-140}{-20}$$

$$x < 7$$

$(-\infty, 7)$



53. $-4y \le \frac{1}{2}$

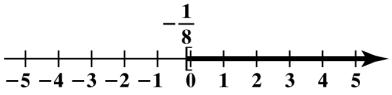
$$2(-4y) \le 2\left(\frac{1}{2}\right)$$

$$-8y \le 1$$

$$\frac{-8y}{-8} \ge \frac{1}{-8}$$

$$y \ge -\frac{1}{8}$$

$\left[-\frac{1}{8}, \infty\right)$

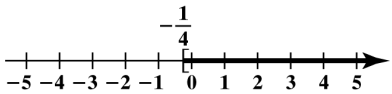


54. $-2y \le \frac{1}{2}$

$$\left(-\frac{1}{2}\right)(-2y) \ge \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$y \ge -\frac{1}{4}$$

$\left[-\frac{1}{4}, \infty\right)$

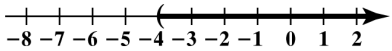


55. $-x < 4$

$$-1(-x) > -1(4)$$

$$x > -4$$

$(-4, \infty)$

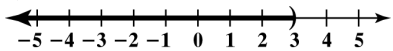


56. $-x > -3$

$$-1(-x) < -1(-3)$$

$$x < 3$$

$(-\infty, 3)$



57. $2x - 3 > 7$

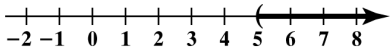
$$2x - 3 + 3 > 7 + 3$$

$$2x > 10$$

$$\frac{2x}{2} > \frac{10}{2}$$

$$x > 5$$

$(5, \infty)$



58. $3x + 2 \le 14$

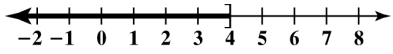
$$3x + 2 - 2 \le 14 - 2$$

$$3x \le 12$$

$$\frac{3x}{3} \le \frac{12}{3}$$

$$x \le 4$$

$(-\infty, 4]$



59. $3x+3<18$

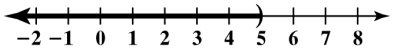
$3x+3-3<18-3$

$3x<15$

$\frac{3x}{3}<\frac{15}{3}$

$x<5$

$(-\infty,5)$



60. $8x-4>12$

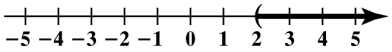
$8x-4+4>12+4$

$8x>16$

$\frac{8x}{8}>\frac{16}{8}$

$x>2$

$(2,\infty)$



61. $3-7x\le 17$

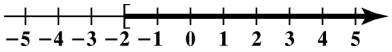
$3-7x-3\le 17-3$

$-7x\le 14$

$\frac{-7x}{-7}\ge\frac{14}{-7}$

$x\ge -2$

$[-2,\infty)$



62. $5-3x\ge 20$

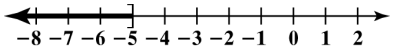
$5-3x-5\ge 20-5$

$-3x\ge 15$

$\frac{-3x}{-3}\le\frac{15}{-3}$

$x\le -5$

$(-\infty,-5]$



63. $-2x-3<3$

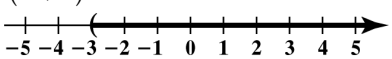
$-2x-3+3<3+3$

$-2x<6$

$\frac{-2x}{-2}>\frac{6}{-2}$

$x>-3$

$(-3,\infty)$



64. $-3x+14<5$

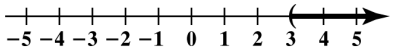
$-3x+14-14<5-14$

$-3x<-9$

$\frac{-3x}{-3}>\frac{-9}{-3}$

$x>3$

$(3,\infty)$



65. $5-x\le 1$

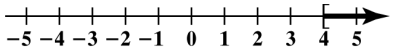
$5-x-5\le 1-5$

$-x\le -4$

$-1(-x)\ge -1(-4)$

$x\ge 4$

$[4,\infty)$



66. $3-x\ge -3$

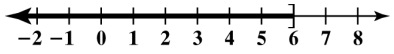
$3-x-3\ge -3-3$

$-x\ge -6$

$-1(-x)\le -1(-6)$

$x\le 6$

$(-\infty,6]$



67. $2x-5>-x+6$

$2x-5+x>-x+6+x$

$3x-5>6$

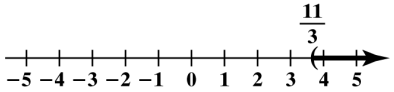
$3x-5+5>6+5$

$3x>11$

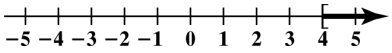
$\frac{3x}{3}>\frac{11}{3}$

$x>\frac{11}{3}$

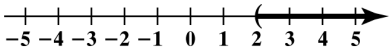
$\left(\frac{11}{3},\infty\right)$



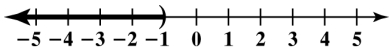
68.
$$6x-2\ge 4x+6$$
$$6x-2-4x\ge 4x+6-4x$$
$$2x-2\ge 6$$
$$2x-2+2\ge 6+2$$
$$2x\ge 8$$
$$\frac{2x}{2}\ge\frac{8}{2}$$
$$x\ge 4$$
$$[4,\infty)$$



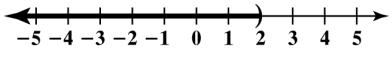
69.
$$2y-5<5y-11$$
$$2y-5-5y<5y-11-5y$$
$$-3y-5<-11$$
$$-3y-5+5<-11+5$$
$$-3y<-6$$
$$\frac{-3y}{-3}>\frac{-6}{-3}$$
$$y>2$$
$$(2,\infty)$$



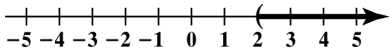
70.
$$4y-7>9y-2$$
$$4y-7-9y>9y-2-9y$$
$$-5y-7>-2$$
$$-5y-7+7>-2+7$$
$$-5y>5$$
$$\frac{-5y}{-5}<\frac{5}{-5}$$
$$y<-1$$
$$(-\infty,-1)$$



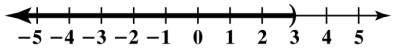
71.
$$3(2y-1)<9$$
$$6y-3<9$$
$$6y-3+3<9+3$$
$$6y<12$$
$$\frac{6y}{6}<\frac{12}{6}$$
$$y<2$$
$$(-\infty,2)$$



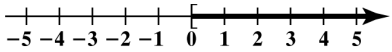
72.
$$4(2y-1)>12$$
$$8y-4>12$$
$$8y-4+4>12+4$$
$$8y>16$$
$$\frac{8y}{8}>\frac{16}{8}$$
$$y>2$$
$$(2,\infty)$$



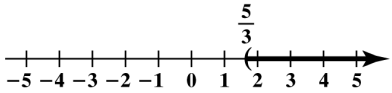
73.
$$3(x+1)-5<2x+1$$
$$3x+3-5<2x+1$$
$$3x-2<2x+1$$
$$3x-2-2x<2x+1-2x$$
$$x-2<1$$
$$x-2+2<1+2$$
$$x<3$$
$$(-\infty,3)$$



74.
$$4(x+1)+2\ge 3x+6$$
$$4x+4+2\ge 3x+6$$
$$4x+6\ge 3x+6$$
$$4x+6-3x\ge 3x+6-3x$$
$$x+6\ge 6$$
$$x+6-6\ge 6-6$$
$$x\ge 0$$
$$[0,\infty)$$



75.
$$8x+3>3(2x+1)-x+5$$
$$8x+3>6x+3-x+5$$
$$8x+3>5x+8$$
$$8x+3-5x>5x+8-5x$$
$$3x+3>8$$
$$3x+3-3>8-3$$
$$3x>5$$
$$x>\frac{5}{3}$$
$$\left(\frac{5}{3},\infty\right)$$



76. $7-2(y-4)<5(1-2y)$

$7-2y+8<5-10y$

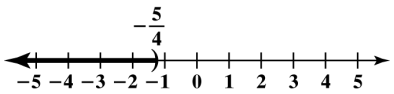
$15-2y<5-10y$

$-2y<-10-10y$

$8y<-10$

$y<-\frac{5}{4}$

$\left(-\infty,-\frac{5}{4}\right)$



77. $\frac{x}{3}-2\ge 1$

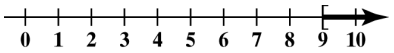
$\frac{x}{3}-2+2\ge 1+2$

$\frac{x}{3}\ge 3$

$3\left(\frac{x}{3}\right)\ge 3(3)$

$x\ge 9$

$[9,\infty)$



78. $\frac{x}{4}-3\ge 1$

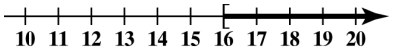
$\frac{x}{4}-3+3\ge 1+3$

$\frac{x}{4}\ge 4$

$4\left(\frac{x}{4}\right)\ge 4(4)$

$x\ge 16$

$[16,\infty)$



79. $1-\frac{x}{2}-1>4-1$

$-\frac{x}{2}>3$

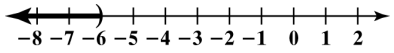
$2\left(-\frac{x}{2}\right)>2(3)$

$-x>6$

$-1(-x)<-1(6)$

$x<-6$

$(-\infty,-6)$



80. $1-\frac{x}{2}<5$

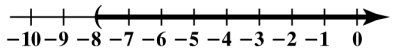
$1-\frac{x}{2}-1<5-1$

$-\frac{x}{2}<4$

$-2\left(-\frac{x}{2}\right)>-2(4)$

$x>-8$

$(-8,\infty)$



81. $4x-4<4(x-5)$

$4x-4<4x-20$

$4x-4+4<4x-20+4$

$4x<4x-16$

$4x-4x<4x-16-4x$

$0<-16$

The original inequality is equivalent to the false statement $0<-16$, so the inequality has no solution.

The solution set is $\{\ \}$.

82. $3x-5<3(x-2)$

$3x-5<3x-6$

$3x-5-3x<3x-6-3x$

$-5<-6$

The original inequality is equivalent to the false statement $-5<-6$, so the inequality has no solution.

The solution set is $\{\ \}$.

83. $x+3<x+7$

$x+3-x<x+7-x$

$3<7$

The original inequality is equivalent to the true statement $3<7$.

The solution is the set of all real numbers, written $\{x|x \text{ is a real number}\}$ or $(-\infty,\infty)$.

84. $x+4<x+10$

$x+4-x<x+10-x$

$4<10$

The original inequality is equivalent to the true statement $4<10$

The solution is the set of all real numbers, written $\{x|x \text{ is a real number}\}$ or $(-\infty,\infty)$.

85. $7x\le 7(x-2)$

$7x\le 7x-14$

$7x-7x\le 7x-14-7x$

$0\le -14$

Since $0\le -14$ is a false statement, the original inequality has no solution.

The solution set is $\{\ \}$.

86. $3x+1\le 3(x-2)$

$3x+1\le 3x-6$

$3x+1-3x\le 3x-6-3x$

$1\le -6$

Since $1\le -6$ is a false statement, the original inequality has no solution.

The solution set is $\{\ \}$.

87. $2(x+3)>2x+1$

$2x+6>2x+1$

$2x+6-2x>2x+1-2x$

$6>1$

Since $6>1$ is a true statement, the original inequality is true for all real numbers the solution set is $\{x|x \text{ is a real number}\}$ or $(-\infty,\infty)$.

88. $5(x+4)>5x+10$

$5x+20>5x+10$

$5x+20-5x>5x+10-5x$

$20>10$

Since $20>10$ is a true statement, the original inequality is true for all real numbers. The solution set is $\{x|x \text{ is a real number}\}$ or $(-\infty,\infty)$.

89. $5x-4\le 4(x-1)$

$5x-4\le 4x-4$

$5x-4+4\le 4x-4+4$

$5x\le 4x$

$5x-4x\le 4x-4x$

$x\le 0$

$(-\infty,0]$

90. $6x-3\le 3(x-1)$

$6x-3\le 3x-3$

$6x-3+3\le 3x-3+3$

$6x\le 3x$

$3x\le 0$

$6x-3x\le 3x-3x$

$x\le 0$

$(-\infty,0]$

91. $3x+a>b$

$3x>b-a$

$\frac{3x}{3}>\frac{b-a}{3}$

$x>\frac{b-a}{3}$

92. $-2x-a\le b$

$-2x-a+a\le b+a$

$-2x\le b+a$

$\frac{-2x}{-2}\ge\frac{b+a}{-2}$

$x\ge\frac{b+a}{-2}$

93. $y\le mx+b$

$y-b\le mx$

$\frac{y-b}{m}\ge\frac{mx}{m}$

$\frac{y-b}{m}\ge x$ or $x\le\frac{y-b}{m}$

94. $y > mx + b$

$y - b > mx + b - b$

$y - b > mx$

$\frac{y-b}{m} > \frac{mx}{m}$

$\frac{y-b}{m} > x$ or $x < \frac{y-b}{m}$

95. x is between -2 and 2, so $|x| < 2$.

96. x is between -3 and 3, so $|x| < 3$.

97. x is less than -2 or greater than 2, so $|x| > 2$.

98. x is greater than 3 or less than -3, so $|x| > 3$.

99. weird, cemetery, accommodation

100. weird

101. supersede, inoculate

102. supersede, inoculate

103. harass

104. cemetery, accommodation, harass

105. a. $p = -0.4x + 16$

$= -0.4(30) + 16$

$= -12 + 16$

$= 4$

According to the formula 4% of U.S. college freshman had an average grade of C in high school. This is the same as the bar graph.

b. $p = -0.4x + 16$

$1.2 = -0.4x + 16$

$-14.8 = -0.4x$

$= 37$

37 years after 1980, or from 2017 onward.

106. a. $p = -0.4x + 16$

$= -0.4(20) + 16$

$= -8 + 16$

$= 8$

According to the formula 8% of U.S. college freshman had an average grade of C in high school. The formula overestimates by 1%.

b. $p = -0.4x + 16$

$0.8 = -0.4x + 16$

$-15.2 = -0.4x$

$= 38$

38 years after 1980, or from 2018 onward.

107. a. Let x = your grade on the final exam.

$\frac{86 + 88 + x}{3} \geq 90$

$3\left(\frac{86 + 88 + x}{3}\right) \geq 3(90)$

$86 + 88 + x \geq 270$

$174 + x \geq 270$

$174 + x - 174 \geq 270 - 174$

$x \geq 96$

You must get at least a 96% on the final exam to earn an A in the course.

b. $\frac{86 + 88 + x}{3} < 80$

$3\left(\frac{86 + 88 + x}{3}\right) < 3(80)$

$86 + 88 + x < 240$

$174 + x < 240$

$174 + x - 174 < 240 - 174$

$x < 66$

If you get less than a 66 on the final exam, your grade will be below a B.

108. a. If you get 100 on the final, your average will be $\frac{88 + 78 + 86 + 100}{4} = \frac{354}{4} = 88.$

Since $88 < 90$ and it is not possible to get more than 100 on the final, an A in the course is not possible.

b. Let x = your grade on the final exam.

$\frac{88 + 78 + 86 + x}{4} \geq 80$

$4\left(\frac{88 + 78 + 86 + x}{4}\right) \geq 4(80)$

$88 + 78 + 86 + x \geq 320$

$252 + x \geq 320$

$252 + x - 252 \geq 320 - 252$

$x \geq 68$

You must get at least 68 to get a B in the course.

109. Let x = number of miles driven.

$$80+0.25x \le 400$$
$$80+0.25x-80 \le 400-80$$
$$0.25x \le 320$$
$$\frac{0.25x}{0.25} \le \frac{320}{0.25}$$
$$x \le 1280$$

You can drive up to 1280 miles.

110. Let x = the number of miles driven.

$$60+0.50 \le 600$$
$$60+0.50x-60 \le 600-60$$
$$0.50 \le 540$$
$$\frac{0.50x}{0.50} \le \frac{540}{0.50}$$
$$x \le 1080$$

You can drive up to 1080 miles.

111. Let x = number of cement bags.

$$245+95x \le 3000$$
$$245+95x-245 \le 3000-245$$
$$95x \le 2755$$
$$\frac{95x}{95} \le \frac{2755}{95}$$
$$x \le 29$$

Up to 29 bags of cement can safely be listed on the elevator in one trip.

112. Let x = the number of cement bags.

$$265+65x \le 2800$$
$$265+65x-265 \le 2800-265$$
$$65x \le 2535$$
$$\frac{65x}{65} \le \frac{2535}{65}$$
$$x \le 39$$

Up to 39 bags of cement can safely be lifted on the elevator in one trip.

113. – 116. Answers will vary.

117. makes sense

118. makes sense

119. makes sense

120. makes sense

121. false; Changes to make the statement true will vary. A sample change is: The inequality $x-3>0$ is equivalent to $x>3$.

122. false; Changes to make the statement true will vary. A sample change is: The statement "x is at most 5" is written $x \le 5$.

123. false; Changes to make the statement true will vary. A sample change is: The inequality $-4x<-20$ is equivalent to $x>5$.

124. true

125. Let x = number of miles driven.
Weekly cost for Basic Rental: \$260.
Weekly cost for Continental: $\$80 + 0.25x$
The cost for Basic Rental is a better deal if $80+0.25x>260$.
Solve this inequality.

$$80+0.25x-80>260-80$$
$$0.25x>180$$
$$\frac{0.25x}{0.25}>\frac{180}{0.25}$$
$$x>720$$

Basic Car Rental is a better deal if you drive more than 720 miles in a week.

126. Let x = the number of hours a person works out at the fitness club yearly.
Yearly cost at first club (in dollars)
$=500+1x=500+x$
Yearly cost at second club $=440+1.75x$
The first club will be cheaper if
$500+x<440+1.75x$
Solve this inequality.

$$500+x-1.75x<440+1.75x-1.75x$$
$$500-0.75x<440$$
$$500-0.75x-500<440-500$$
$$-0.75x<-60$$
$$\frac{-0.75x}{-0.75}>\frac{-60}{-0.75}$$
$$x>80$$

The first club will be cheaper if the person works out more than 80 hours a year.

127.

$$1.45-7.23x>-1.442$$
$$1.45-7.23x-1.45>-1.442-1.45$$
$$-7.23x>-2.892$$
$$\frac{-7.23x}{-7.23}<\frac{-2.892}{-7.23}$$
$$x<0.4$$

$(-\infty, 0.4)$

128. $126.8-9.4y \le 4.8y-34.5$

$126.8-9.4y-4.8y \le 4.8y+34.5-4.8y$

$126.8-14.2y \le 34.5$

$126.8-14.2y-126.8 \le 34.5-126.8$

$-14.2y \le -92.3$

$\frac{-14.2y}{-14.2} \ge \frac{-92.3}{-14.2}$

$y \ge 6.5$

$[6.5, \infty)$

129. $A = PB$, $A = 8$, $P = 40\% = 0.4$

$A = PB$

$8 = 0.4B$

$\frac{8}{0.4} = \frac{0.4B}{0.4}$

$20 = B$

8 is 40% of 20.

130. Let x = the width of the rectangle.

Let $x+5$ = the length of the rectangle.

$P = 2l + 2w$

$34 = 2(x+5) + 2 \cdot x$

$34 = 2x + 10 + 2x$

$34 = 4x + 10$

$34 - 10 = 4x + 10 - 10$

$24 = 4x$

$6 = x$

$x = 6$

$x + 5 = 11$

The width is 6 inches and the length is 11 inches.

131. $5x + 16 = 3(x+8)$

$5x + 16 = 3x + 24$

$5x + 16 - 3x = 3x + 24 - 3x$

$2x + 16 = 24$

$2x + 16 - 16 = 24 - 16$

$2x = 8$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

Check: $5(4) + 16 = 3(4+8)$

$20 + 16 = 3(12)$

$36 = 36$, true

The solution is set is $\{4\}$.

132. $x - 4y = 14$

$2 - 4(-3) = 14$

$2 + 12 = 14$

$14 = 14$, true

Yes, the values make it a true statement.

133. $x - 4y = 14$

$12 - 4(1) = 14$

$12 - 4 = 14$

$8 = 14$, false

No, the values make it a false statement.

134. $y = \frac{2}{3}x + 1$

$y = \frac{2}{3}(-6) + 1$

$y = -4 + 1$

$y = -3$

Chapter 2 Review Exercises

1. $x - 10 = 22$

$x - 10 + 10 = 22 + 10$

$x = 32$

The solution is set is $\{32\}$.

2. $-14 = y + 8$

$-14 - 8 = y + 8 - 8$

$-22 = y$

The solution is set is $\{-22\}$.

3. $7z - 3 = 6z + 9$

$7z - 3 - 6z = 6z + 9 - 6z$

$z - 3 = 9$

$z - 3 + 3 = 9 + 3$

$z = 12$

The solution is set is $\{12\}$.

4. $4(x+3)=3x-10$
$4x+12=3x-10$
$4x+12-3x=3x-10-3x$
$x+12=-10$
$x+12-12=-10-12$
$x=-22$
The solution is set is $\{-22\}$.

5. $6x-3x-9+1=-5x+7x-3$
$3x-8=2x-3$
$3x-8-2x=2x-3-2x$
$x-8=-3$
$x-8+8=-3+8$
$x=5$
The solution is set is $\{5\}$.

6. $\frac{x}{8}=10$
$8\left(\frac{x}{8}\right)=8(10)$
$x=80$
The solution is set is $\{80\}$.

7. $\frac{y}{-8}=7$
$-8\left(\frac{y}{-8}\right)=-8(7)$
$y=-56$
The solution is set is $\{-56\}$.

8. $7z=77$
$\frac{7z}{7}=\frac{77}{7}$
$z=11$
The solution is set is $\{11\}$.

9. $-36=-9y$
$\frac{-36}{-9}=\frac{-9y}{-9}$
$4=y$
The solution is set is $\{4\}$.

10. $\frac{3}{5}x=-9$
$\frac{5}{3}\left(\frac{3}{5}x\right)=\frac{5}{3}(-9)$
$1x=-15$
$x=-15$
The solution is set is $\{-15\}$.

11. $30=-\frac{5}{2}y$
$-\frac{2}{5}(30)=-\frac{2}{5}\left(-\frac{5}{2}y\right)$
$-12=y$
The solution is set is $\{-12\}$.

12. $-x=25$
$-1(-x)=-1(25)$
$x=-25$
The solution is set is $\{-25\}$.

13. $\frac{-x}{10}=-1$
$10\left(\frac{-x}{10}\right)=10(-1)$
$-x=-10$
$-1(-x)=-1(-10)$
$x=10$
The solution is set is $\{10\}$.

14. $4x+9=33$
$4x+9-9=33-9$
$4x=24$
$\frac{4x}{4}=\frac{24}{4}$
$x=6$
The solution is set is $\{6\}$.

15. $-3y-2=13$
$-3y-2+2=13+2$
$-3y=15$
$\frac{-3y}{-3}=\frac{15}{-3}$
$y=-5$
The solution is set is $\{-5\}$.

16. $5z+20=3z$

$5z+20-3z=3z-3z$

$2z+20=0$

$2z+20-20=0-20$

$2z=-20$

$\frac{2z}{2}=\frac{-20}{2}$

$z=-10$

The solution is set is $\{-10\}$.

17. $5x-3=x+5$

$5x-3-x=x+5-x$

$4x-3=5$

$4x-3+3=5+3$

$4x=8$

$\frac{4x}{4}=\frac{8}{4}$

$x=2$

The solution is set is $\{2\}$.

18. $3-2x=9-8x$

$3-2x+8x=9-8x+8x$

$3+6x=9$

$3+6x-3=9-3$

$6x=6$

$\frac{6x}{6}=\frac{6}{6}$

$x=1$

The solution is set is $\{1\}$.

19. a. 2012 is 5 years after 2007.

$p=0.9n+15$

$p=0.9(5)+15=19.5$

According to the formula, 19.5% of Americans were religiously unaffiliated in 2012.
The formula underestimates the actual value given in the bar graph by 0.1%.

b. $p=0.9n+15$

$24=0.9n+15$

$9=0.9n$

$\frac{9}{0.9}=\frac{0.9n}{0.9}$

$10=n$

If trends continue, 24% of Americans will be religiously unaffiliated in 10 years after 2007, or in 2017.

20. $5x+9-7x+6=x+18$

$-2x+15=x+18$

$-2x+15-x=x+18-x$

$-3x+15=18$

$-3x+15-15=18-15$

$-3x=3$

$\frac{-3x}{-3}=\frac{3}{-3}$

$x=-1$

The solution is set is $\{-1\}$.

21. $3(x+4)=5x-12$

$3x+12=5x-12$

$3x+12-5x=5x-12-5x$

$-2x+12=-12$

$-2x+12-12=-12-12$

$-2x=-24$

$\frac{-2x}{-2}=\frac{-24}{-2}$

$x=12$

The solution is set is $\{12\}$.

22. $1-2(6-y)=3y+2$

$1-12+2y=3y+2$

$2y-11=3y+2$

$2y-11-3y=3y+2-3y$

$-y-11=2$

$-y-11+11=2+11$

$-y=13$

$y=-13$

The solution is set is $\{-13\}$.

23. $2x-8+3x+15=2x-2$

$5x+7=2x-2$

$5x+7-2x=2x-2-2x$

$3x+7=-2$

$3x+7-7=-2-7$

$3x=-9$

$\frac{3x}{3}=\frac{-9}{3}$

$x=-3$

The solution is set is $\{-3\}$.

24. $-2(y-4)-(3y-2)=-2-(6y-2)$

$$-2y+8-3y+2=-2-6y+2$$
$$-5y+10=-6y$$
$$-5y+10+6y=-6y+6y$$
$$10+y=0$$
$$10+y-10=0-10$$
$$y=-10$$

The solution is set is $\{-10\}$.

25. $\frac{2x}{3}=\frac{x}{6}+1$

To clear fractions, multiply both sides by the LCD, which is 6.

$$6\left(\frac{2x}{3}\right)=6\left(\frac{x}{6}+1\right)$$
$$6\left(\frac{2x}{3}\right)=6\left(\frac{x}{6}\right)+6(1)$$
$$4x=x+6$$
$$4x-x=x+6-x$$
$$3x=6$$
$$\frac{3x}{3}=\frac{6}{3}$$
$$x=2$$

The solution is set is $\{2\}$.

26. $\frac{x}{2}-\frac{1}{10}=\frac{x}{5}+\frac{1}{2}$

Multiply both sides by the LCD, which is 10.

$$10\left(\frac{x}{2}-\frac{1}{10}\right)=10\left(\frac{x}{5}+\frac{1}{2}\right)$$
$$10\left(\frac{x}{2}\right)-10\left(\frac{1}{10}\right)=10\left(\frac{x}{5}\right)+10\left(\frac{1}{2}\right)$$
$$5x-1=2x+5$$
$$5x-1-2x=2x+5-2x$$
$$3x-1=5$$
$$3x-1+1=5+1$$
$$3x=6$$
$$\frac{3x}{3}=\frac{6}{3}$$
$$x=2$$

The solution is set is $\{2\}$.

27. Multiply both sides by 100 to clear the decimals.

$$0.5x+8.75=13.25$$
$$100(0.5x+8.75)=100(13.25)$$
$$50x+875=1325$$
$$50x=450$$
$$x=9$$

The solution set is $\{9\}$.

28. First apply the distributive property to remove the parentheses, and then multiply both sides by 100 to clear the decimals.

$$0.1(x-3)=1.1-0.25x$$
$$0.1x-0.3=1.1-0.25x$$
$$100(0.1x-0.3)=100(1.1-0.25x)$$
$$10x-30=110-25x$$
$$10x=140-25x$$
$$35x=140$$
$$\frac{35x}{35}=\frac{140}{35}$$
$$x=4$$

The solution set is $\{4\}$.

29. $3(8x-1)=6(5+4x)$

$$24x-3=30+24x$$
$$24x-3-24x=30+24x-24x$$
$$-3=30$$

Since $-3 = 30$ is a false statement, the original equation is inconsistent and has no solution or $\{\ \}$.

30. $4(2x-3)+4=8x-8$

$$8x-12+4=8x-8$$
$$8x-8=8x-8$$
$$8x-8-8x=8x-8-8x$$
$$-8=-8$$

Since $-8 = -8$ is a true statement, the solution is the set of all real numbers, written $\{x \mid x \text{ is a real number}\}$.

31. $H = 0.7(220 - a)$

$133 = 0.7(220 - a)$

$133 - 154 = 154 - 154 - 0.7a$

$-21 = -0.7a$

$\frac{-21}{-0.7} = \frac{-0.7a}{-0.7}$

$30 = a$

If the optimal heart rate is 133 beats per minute, the person is 30 years old.

32. $I = Pr$ for r

$\frac{I}{P} = \frac{Pr}{P}$

$\frac{I}{P} = r$ or $r = \frac{I}{P}$

33. $V = \frac{1}{3}Bh$ for h

$3V = 3\left(\frac{1}{3}Bh\right)$

$3V = Bh$

$\frac{3V}{B} = \frac{Bh}{B}$

$\frac{3V}{B} = h$ or $h = \frac{3V}{B}$

34. $P = 2l + 2w$ for w

$P - 2l = 2l + 2w - 2l$

$P - 2l = 2w$

$\frac{P - 2l}{2} = \frac{2w}{2}$

$\frac{P - 2l}{2} = w$ or $w = \frac{P - 2l}{2}$

35. $A = \frac{B + C}{2}$ for B

$2A = 2\left(\frac{B + C}{2}\right)$

$2A = B + C$

$2A - C = B + C - C$

$2A - C = B$ or $B = 2A - C$

36. $T = D + pm$ for m

$T - D = D + pm - D$

$T - D = pm$

$\frac{T - D}{p} = \frac{pm}{p}$

$\frac{T - D}{p} = m$ or $m = \frac{T - D}{p}$

37. $A = PB$; $P = 8\% = 0.08$, $B = 120$

$A = 0.08 \cdot 120$

$A = 9.6$

8% of 120 is 9.6

38. $A = PB$; $A = 90$, $P = 45\% = 0.45$

$90 = 0.45B$

$\frac{90}{0.45} = \frac{0.45B}{0.45}$

$200 = B$

90 is 45% of 200.

39. $A = PB$; $A = 36$, $B = 75$

$36 = P \cdot 75$

$\frac{36}{75} = \frac{P \cdot 75}{75}$

$0.48 = P$

36 is 48% of 75.

40. Increase = Percent · Original

First, find the increase: 12 – 6 = 6

$6 = P \cdot 6$

$\frac{6}{6} = \frac{P \cdot 6}{6}$

$1 = P$

The percent increase is 100%.

41. Decrease = Percent · Original

First, find the decrease: 5 – 3 = 2

$2 = P \cdot 5$

$\frac{2}{5} = \frac{P \cdot 5}{5}$

$0.4 = P$

The percent decrease is 40%.

42. Increase = Percent · Original

First, find the increase: 45 – 40 = 5

$5 = P \cdot 40$

$\frac{5}{40} = \frac{P \cdot 40}{40}$

$0.125 = P$

The percent increase is 12.5%.

43. Investment dollars lost last year were $0.10 \cdot \$10,000 = \$1000$. This means that \$10,000 – \$1000 = \$9000 remains. Investment dollars gained this year are $0.10 \cdot \$9000 = \$900$. This means that \$9000 + \$900 = \$9900 of the original investment remains. This is an overall loss of \$100.

decrease = percent · original

$$100 = P \cdot 10,000$$

$$\frac{100}{10,000} = \frac{P \cdot 10,000}{10,000}$$

$$0.01 = P$$

The statement is not true. Instead of recouping losses, there is an overall 1% decrease in the portfolio.

44. a. $r = \frac{h}{7}$

$$7r = 7\left(\frac{h}{7}\right)$$

$7r = h$ or $h = 7r$

b. $h = 7r;\ r = 9$

$h = 7(9) = 63$

The woman's height is 63 inches or 5 feet, 3 inches.

45. $A = P \cdot B$

$$91 = 0.26 \cdot B$$

$$\frac{91}{0.26} = \frac{0.26 \cdot B}{0.26}$$

$$350 = B$$

The average U.S. household uses 350 gallons of water per day.

46. Let x = the unknown number.

$$6x - 20 = 4x$$

$$6x - 20 - 4x = 4x - 4x$$

$$2x - 20 = 0$$

$$2x - 20 + 20 = 0 + 20$$

$$2x = 20$$

$$x = 10$$

The number is 10.

47. Let $x =$ Buffett's net worth.

Let $x + 14 =$ Gate's net worth.

$$x + (x + 14) = 148$$

$$x + x + 14 = 148$$

$$2x + 14 = 148$$

$$2x = 134$$

$$x = 67$$

$$x + 14 = 81$$

In 2014 Buffett's net worth was \$67 billion and Gate's net worth was \$81 billion.

48. Let x = the smaller page number.

Let $x + 1$ = the larger page number.

$$x + (x + 1) = 93$$

$$2x + 1 = 93$$

$$2x = 92$$

$$x = 46$$

The page numbers are 46 and 47.

49. Let x = the percentage of females.

Let $x + 2 =$ the percentage of males.

$$x + (x + 2) = 100$$

$$x + x + 2 = 100$$

$$2x + 2 = 100$$

$$2x + 2 - 2 = 100 - 2$$

$$2x = 98$$

$$x = 49$$

$$x + 2 = 51$$

For Americans under 20, 49% are female and 51% are male.

50. Let x = number of years after 2001.

$$7284 + 328x = 12,204$$

$$328x = 4920$$

$$\frac{328x}{328} = \frac{4920}{328}$$

$$x = 15$$

According to this model, the U.S. will spend\$12,204 per pupil 15 years after 2001, or in 2016.

51. Let x = the number of checks written.

$$6 + 0.05x = 6.90$$

$$6 + 0.05x - 6 = 6.90 - 6$$

$$0.05x = 0.90$$

$$\frac{0.05x}{0.05} = \frac{0.90}{0.05}$$

$$x = 18$$

You wrote 18 checks that month.

52. Let x = the width of the field.
Let $3x$ = the length of the field.

$$P = 2l + 2w$$
$$400 = 2 \cdot 3x + 2 \cdot x$$
$$400 = 6x + 2x$$
$$400 = 8x$$
$$\frac{400}{8} = \frac{8x}{8}$$
$$50 = x$$
$$x = 50$$
$$3x = 150$$

The field is 50 yards wide and 150 yards long.

53. Let x = the original price of the table.

$$x - 0.25x = 180$$
$$0.75x = 180$$
$$\frac{0.75x}{0.75} = \frac{180}{0.75}$$
$$x = 240$$

The table's price before the reduction was \$240.

54. Find the area of a rectangle with length 6.5 ft and width 5 ft.

$$A = lw = (6.5)(5) = 32.5$$

The area is 32.5 ft^2.

55. Find the area of a triangle with base 20 cm and height 5 cm.

$$A = \frac{1}{2}bh = \frac{1}{2}(20)(5) = 50$$

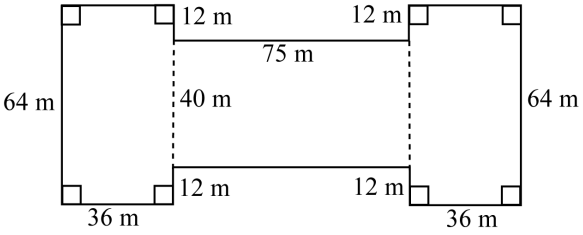
The area is 50 cm^2.

56. Find the area of a trapezoid with bases 22 yd and 5 yd and height 10 yd.

$$A = \frac{1}{2}h(a+b)$$
$$= \frac{1}{2}(10)(22+5)$$
$$= \frac{1}{2} \cdot 10 \cdot 27 = 135$$

The area is 135 yd^2.

57. Notice that the height of the middle rectangle is $64 - 12 - 12 = 40$ m.



Using $A = lw$ we must find the sum of areas of the middle rectangle and the two side rectangles.

$$A = (40)(75) + 2 \cdot (64)(36)$$
$$= 3000 + 2 \cdot 2304$$
$$= 3000 + 4608$$
$$= 7608$$

The area is 7608 m^2.

58. Since the diameter is 20 m, the radius is $\frac{20}{2} = 10$ m.

$$C = 2\pi = 2\pi(10) = 20\pi \approx 63$$
$$A = \pi r^2 = \pi(10)^2 = 100\pi \approx 314$$

The circumference is 20π m or approximately 63 m; the area is 100π m^2 or approximately 314 m^2.

59. $A = 42, b = 14$

$$A = \frac{1}{2}bh$$
$$42 = \frac{1}{2} \cdot 14 \cdot h$$
$$42 = 7h$$
$$6 = h$$

The height of the sail is 6 ft.

60. Area of floor:

$$A = bh = (12\,\text{ft})(15\,\text{ft}) = 180\,\text{ft}^2$$

Area of base of stove:

$$A = bh = (3\,\text{ft})(4\,\text{ft}) = 12\,\text{ft}^2$$

Area of bottom of refrigerator:

$$A = bh = (3\,\text{ft})(4\,\text{ft}) = 12\,\text{ft}^2$$

The area to be covered with floor tile is

$$180\,\text{ft}^2 - 12\,\text{ft}^2 - 12\,\text{ft}^2 = 156\,\text{ft}^2.$$

61. First, find the area of a trapezoid with bases 80 ft and 100 ft and height 60 ft.

$$A = \frac{1}{2}h(a+b)$$

$$= \frac{1}{2}(60)(80+100) = 5400$$

The area of the yard is 5400 ft^2. The cost is \$0.35(5400) = \$1890.

62. The radius of the medium pizza is $\frac{1}{2}\cdot 14 \text{ inches} = 7 \text{ inches}$, and the radius of each small pizza is $\frac{1}{2}\cdot 8 \text{ inches} = 4 \text{ inches}$.

Medium pizza:

$$A = \pi r^2 = \pi(7 \text{ in.})^2$$

$$= 49\pi \text{in.}^2 \approx 154 \text{ in.}^2$$

Small pizza:

$$A = \pi r^2 = \pi(4 \text{in.})^2$$

$$= 16\pi \text{in.}^2 \approx 50 \text{ in.}^2$$

The area of one medium pizza is approximately 154 in.^2 and the area of two small pizzas is approximately $2(50) = 100 \text{ in.}^2$. Since the price of one medium pizza is the same as the price of two small pizzas and the medium pizza has the greater area, the medium pizza is the better buy. (Because the prices are the same, it is not necessary to find price per square inch in this case.)

63. Find the volume of a rectangular solid with length 5 cm, width 3 cm, and height 4 cm.

$A = lwh = 5\cdot 3\cdot 4 = 60$

The volume is 60 cm^3.

64. Find the volume of a cylinder with radius 4 yd and height 8 yd.

$$V = \pi r^2 h$$

$$= \pi(4)^2\cdot 8 = 128\pi \approx 402$$

The volume is $128\pi \text{ yd}^3 \approx 402 \text{ yd}^3$.

65. Find the volume of a sphere with radius 6 m.

$$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(6)^3 = \frac{4}{3}\cdot\pi\cdot 216$$

$$= 288\pi \approx 905$$

The volume is $288\,\pi \text{ m}^3 \approx 905 \text{ m}^3$.

66. Find the volume of each box.

$V = lwh = (8\text{m})(4\text{m})(3\text{m}) = 96\text{m}^3$

The space required for 50 containers is $50(96 \text{ m}^3) = 4800 \text{ m}^3$.

67. Since the diameter of the fish tank is 6 ft, the radius is 3 ft.

$V = \pi r^2 h = \pi(3)^2\cdot 3 = 27\pi \approx 84.82$

The volume of the tank is approximately 85 ft^3. Divide by 5 to determine how many fish can be put in the tank.

$\frac{84.82}{5} \approx 16.96$

There is enough water in the tank for 16 fish. Round down to 16, since 0.96 of a fish cannot be purchased.

68. The sum of the measures of the angles of any triangle is 180°, so $x + 3x + 2x = 180$.

$$x + 3x + 2x = 180$$

$$6x = 180$$

$$x = 30$$

If x = 30, then $3x$ = 90 and $2x$ = 60, so the angles measure 30°, 60°, and 90°.

69. Let x = the measure of the second angle.
Let $2x$ +15 = the measure of the first angle.
Let x + 25 = the measure of the third angle.

$$x + (2x+15) + (x+25) = 180$$

$$4x + 40 = 180$$

$$4x = 140$$

$$x = 35$$

If x = 35, then $2x + 15 = 2(35) + 15 = 85$ and $x + 25 = 35 + 25 = 60$. The angles measure 85°, 35°, and 60°.

70. If the measure of an angle is 57°, the measure of its complement is $90° - 57° = 33°$

71. If the measure of an angle is 75°, the measure of its supplement is $180° - 75° = 105°$.

72. Let x = the measure of the angle.
Let 90 – x = the measure of its complement.

$$x = (90 - x) + 25$$

$$x = 115 - x$$

$$2x = 115$$

$$x = 57.5$$

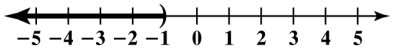
The measure of the angle is 57.5°.

73. Let x = the measure of the angle.
Let $180 - x$ = the measure of its supplement.

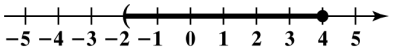
$$180 - x = 4x - 45$$
$$180 - 5x = -45$$
$$-5x = -225$$
$$x = 45$$

If $x = 45$, then $180 - x = 135$. The measure of the angle is $45°$ and the measure of its supplement is $135°$.

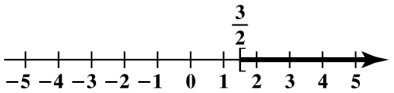
74. $x < -1$



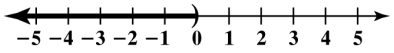
75. $-2 < x \le 4$



76. $\left[\frac{3}{2}, \infty\right)$

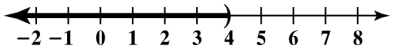


77. $(-\infty, 0)$



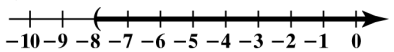
78.
$$2x - 5 < 3$$
$$2x - 5 + 5 < 3 + 5$$
$$2x < 8$$
$$\frac{2x}{2} < \frac{8}{2}$$
$$x < 4$$

$(-\infty, 4)$



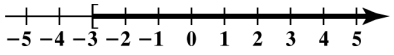
79.
$$\frac{x}{2} > -4$$
$$2\left(\frac{x}{2}\right) > 2(-4)$$
$$x > -8$$

$(-8, \infty)$



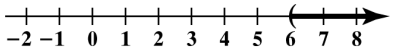
80.
$$3 - 5x \le 18$$
$$3 - 5x - 3 \le 18 - 3$$
$$-5x \le 15$$
$$\frac{-5x}{-5} \ge \frac{15}{-5}$$
$$x \ge -3$$

$[-3, \infty)$



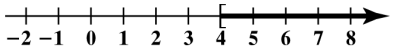
81.
$$4x + 6 < 5x$$
$$4x + 6 - 5x < 5x - 5x$$
$$-x + 6 < 0$$
$$-x + 6 - 6 < 0 - 6$$
$$-x < -6$$
$$-1(-x) > -1(-6)$$
$$x > 6$$

$(6, \infty)$



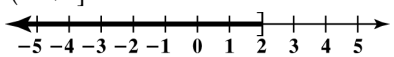
82.
$$6x - 10 \ge 2(x + 3)$$
$$6x - 10 \ge 2x + 6$$
$$6x - 10 - 2x \ge 2x + 6 - 2x$$
$$4x - 10 \ge 6$$
$$4x - 10 + 10 \ge 6 + 10$$
$$4x \ge 16$$
$$\frac{4x}{4} \ge \frac{16}{4}$$
$$x \ge 4$$

$[4, \infty)$



83.
$$4x + 3(2x - 7) \le x - 3$$
$$4x + 6x - 21 \le x - 3$$
$$10x - 21 \le x - 3$$
$$10x - 21 - x \le x - 3 - x$$
$$9x - 21 \le -3$$
$$9x - 21 + 21 \le -3 + 21$$
$$9x \le 18$$
$$\frac{9x}{9} \le \frac{18}{9}$$
$$x \le 2$$

$(-\infty, 2]$



84. $2(2x+4) > 4(x+2) - 6$

$4x + 8 > 4x + 8 - 6$

$4x + 8 > 4x + 2$

$4x + 8 - 4x > 4x + 2 - 4x$

$8 > 2$

Since $8 > 2$ is a true statement, the original inequality is true for all real numbers, and the solution set is $\{x \mid x \text{ is a real number}\}$.

85. $-2(x-4) \le 3x + 1 - 5x$

$-2x + 8 \le -2x + 1$

$-2x + 8 + 2x \le -2x + 1 + 2x$

$8 \le 1$

Since $8 \le 1$ is a false statement, the original inequality has no solution. The solution set is $\{\ \}$.

86. Let x = the student's score on the third test.

$$\frac{42+74+x}{3} \ge 60$$

$$3\left(\frac{42+74+x}{3}\right) \ge 3(60)$$

$42 + 74 + x \ge 180$

$116 + x \ge 180$

$116 + x - 116 \ge 180 - 116$

$x \ge 64$

The student must score at least 64 on the third test to pass the course.

87. Let x = the number of people you invite to the picnic.

$350 + 55x \le 2000$

$55x \le 1650$

$$\frac{55x}{55} \le \frac{1650}{55}$$

$x \le 30$

To can invite at most 30 people to the party.

Chapter 2 Test

1. $4x - 5 = 13$

$4x + 5 + 5 = 13 + 5$

$4x = 18$

$$\frac{4x}{4} = \frac{18}{4} = \frac{9}{2}$$

$$x = \frac{9}{2}$$

The solution set is $\left\{\frac{9}{2}\right\}$.

2. $12x + 4 = 7x - 21$

$12x + 4 - 7x = 7x - 21 - 7x$

$5x + 4 = -21$

$5x + 4 - 4 = -21 - 4$

$5x = -25$

$$\frac{5x}{5} = \frac{-25}{5}$$

$x = -5$

The solution set is $\{-5\}$.

3. $8 - 5(x-2) = x + 26$

$8 - 5x + 10 = x + 26$

$18 - 5x = x + 26$

$18 - 5x - x = x + 26 - x$

$18 - 6x = 26$

$18 - 6x - 18 = 26 - 18$

$-6x = 8$

$$\frac{-6x}{-6} = \frac{8}{-6}$$

$$x = -\frac{8}{6} = -\frac{4}{3}$$

The solution set is $\left\{-\frac{4}{3}\right\}$.

4. $3(2y-4)=9-3(y+1)$

$$6y-12=9-3y-3$$
$$6y-12=6-3y$$
$$6y-12+3y=6-3y+3y$$
$$9y-12=6$$
$$9y-12+12=6+12$$
$$9y=18$$
$$\frac{9y}{9}=\frac{18}{9}$$
$$y=2$$

The solution set is $\{2\}$.

5. $\frac{3}{4}x=-15$

$$\frac{4}{3}\left(\frac{3}{4}x\right)=\frac{4}{3}(-15)$$
$$x=-20$$

The solution set is $\{-20\}$.

6. $\frac{x}{10}+\frac{1}{3}=\frac{x}{5}+\frac{1}{2}$

Multiply both sides by the LCD, 30.

$$30\left(\frac{x}{10}+\frac{1}{3}\right)=30\left(\frac{x}{5}+\frac{1}{2}\right)$$
$$30\left(\frac{x}{10}\right)+30\left(\frac{1}{3}\right)=30\left(\frac{x}{5}\right)+30\left(\frac{1}{2}\right)$$
$$3x+10=6x+15$$
$$3x+10-6x=6x+15-6x$$
$$-3x+10=15$$
$$-3x+10-10=15-10$$
$$-3x=5$$
$$\frac{-3x}{-3}=\frac{5}{-3}$$
$$x=-\frac{5}{3}$$

The solution set is $\left\{-\frac{5}{3}\right\}$.

7. $9.2x-80.1=21.3x-19.6$

To clear the equation of decimals, multiply both sides by 10.

$$10(9.2x-80.1)=10(21.3x-19.6)$$
$$92x-801=213x-196$$
$$92x=213x+605$$
$$-121x=605$$
$$\frac{-121x}{-121}=\frac{605}{-121}$$
$$x=-5$$

The solution set is $\{-5\}$.

8. $N=2.4x+180;\ N=324$

$$2.4x+180=324$$
$$2.4x+180-180=324-180$$
$$2.4x=144$$
$$\frac{2.4x}{2.4}=\frac{144}{2.4}$$
$$x=60$$

The US population is expected to reach 324 million 60 years after 1960, in the year 2020.

9. $V=\pi r^2h$ for h

$$\frac{V}{\pi r^2}=\frac{\pi r^2h}{\pi r^2}$$
$$\frac{V}{\pi r^2}=h \text{ or } h=\frac{V}{\pi r^2}$$

10. $l=\frac{P-2w}{2}$ for w

$$2l=2\left(\frac{P-2w}{2}\right)$$
$$2l=P-2w$$
$$2l-P=P-2w-P$$
$$2l-P=-2w$$
$$\frac{2l-P}{-2}=\frac{-2w}{-2}$$
$$\frac{2l-P}{-2}=w \quad \text{or} \quad w=\frac{P-2l}{2}$$

11. $A=PB;\ P=6\%=0.06,\ B=140$

$$A=0.06(140)$$
$$A=8.4$$

6% of 140 is 8.4.

12. $A = PB$; A=120, $P = 80\% = 0.80$

$120 = 0.80B$

$\frac{120}{0.80} = \frac{0.80B}{0.80}$

$150 = B$

120 is 80% of 150.

13. $A = PB$; $A = 12,\ B = 240$

$12 = P \cdot 240$

$\frac{12}{240} = \frac{P \cdot 240}{240}$

$0.05 = P$

12 is 5% of 240.

14. Let x = the unknown number.

$5x - 9 = 306$

$5x - 9 + 9 = 306 + 9$

$5x = 315$

$\frac{5x}{5} = \frac{315}{5}$

$x = 63$

The number is 63.

15. Let x = the average number of vacation days for Americans.

Let $x + 29$ = the average number of vacation days for Italians.

$x + (x + 29) = 55$

$x + x + 29 = 55$

$2x + 29 = 55$

$2x = 26$

$x = 13$

$x + 29 = 42$

Americans average 13 vacation days and Italians average 42 vacation days.

16. Let x = number of monthly text messages.

$15 + 0.05x = 45$

$0.05x = 30$

$x = \frac{30}{0.05}$

$x = 600$

You can send 600 text messages.

17. Let x = the width of the field.

Let $2x =$ the length of the field.

$P = 2l + 2w$

$450 = 2 \cdot 2x + 2 \cdot x$

$450 = 4x + 2x$

$450 = 6x$

$\frac{450}{6} = \frac{6x}{6}$

$75 = x$

$x = 75$

$2x = 150$

The field is 75 yards wide and 150 yards long.

18. Let x = the book's original price.

$x - 0.20x = 28$

$0.80x = 28$

$x = \frac{28}{0.80}$

$x = 35$

The price of the book before the reduction was $35.

19. Find the area of a triangle with base 47 meters and height 22 meters.

$A = \frac{1}{2}bh = \frac{1}{2}(47)(22) = 517$

The area of the triangle is 517 m^2.

20. Find the area of a trapezoid with height 15 in, lower base 40 in and upper base 30 in.

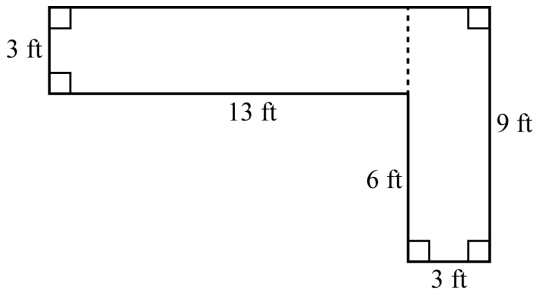
$A = \frac{1}{2}h(a + b)$

$= \frac{1}{2}(15)(40 + 30)$

$= \frac{1}{2} \cdot 15 \cdot 70 = 525$

The area is 525 in^2.

21. Notice that the height of the side rectangle is $6+3=9$ ft.



Using $A=lw$ we must find the sum of areas of the upper rectangle and the side rectangle.

$$A=(3)(13)+(3)(9)$$
$$=39+27$$
$$=66$$

The area is 66 ft^2.

22. Find the volume of a rectangular solid with length 3 in, width 2 in, and height 3 in.

$V=lwh=3\cdot 2\cdot 3=18$

The volume is 18 in^3.

23. Find the volume of a cylinder with radius 5 cm and height 7 cm.

$$V=\pi r^2 h$$
$$=\pi(5)^2\cdot 7=\pi\cdot 25\cdot 7$$
$$=175\pi\approx 550$$

The volume is 175π cm^3 or approximately 550 cm^3.

24. The area of the floor is $A=(40\,\text{ft})(50\,\text{ft})=2000\,\text{ft}^2$.

The area of each tile is $A=(2\,\text{ft})(2\,\text{ft})=4\,\text{ft}^2$.

The number of tiles needed is $\dfrac{2000\,\text{ft}^2}{4\,\text{ft}^2}=500$.

Since there are 10 tiles in a package, the number of packages needed is $\dfrac{500}{10}=50$.

Since each package costs \$13, the cost for enough tiles to cover the floor is $50(\$13)=\$650$.

25. $A=56, b=8$

$$A=\frac{1}{2}bh$$
$$56=\frac{1}{2}\cdot 8\cdot h$$
$$56=4h$$
$$14=h$$

The height of the sail is 14 feet.

26. Let x = the measure of the second angle.
Let $3x$ = the measure of the first angle.
Let $x-30$ = the measure of the third angle.

$$x+3x+(x-30)=180$$
$$5x-30=180$$
$$5x=210$$
$$x=42$$
$$3x=126$$
$$x-30=12$$

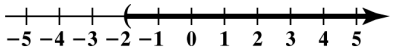
The measure of the first angle is $126°$.
The measure of the second angle is $42°$.
The measure of the third angle is $12°$.

27. Let x = the measure of the angle.
Let $90-x$ = the measure of its complement.

$$x=(90-x)+16$$
$$x=106-x$$
$$2x=106$$
$$x=53$$

The measure of the angle is $53°$.

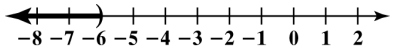
28. $(-2,\infty)$



29. $(-\infty,3]$

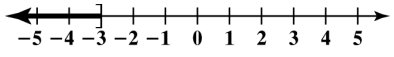
30. $$\frac{x}{2}<-3$$
$$2\left(\frac{x}{2}\right)<2(-3)$$
$$x<-6$$

$(-\infty,-6)$

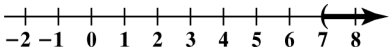


31. $$6-9x\geq 33$$
$$6-9x-6\geq 33-6$$
$$-9x\geq 27$$
$$\frac{-9x}{-9}\leq\frac{27}{-9}$$
$$x\leq -3$$

$(-\infty,-3]$



32. $4x-2>2(x+6)$
$4x-2>2x+12$
$4x-2-2x>2x+12-2x$
$2x-2>12$
$2x>14$
$x>7$
$(7,\infty)$



33. Let x = the student's score on the fourth exam.

$$\frac{76+80+72+x}{4}\geq 80$$
$$4\left(\frac{76+80+72+x}{4}\right)\geq 4(80)$$
$$76+80+72+x\geq 320$$
$$228+x\geq 320$$
$$x\geq 92$$

The student must score at least 92 on the fourth exam to have an average of at least 80.

34. Let x = the width of the rectangle.

$2(20)+2x>56$
$40+2x>56$
$40-40+2x>56-40$
$2x>16$
$x>8$

The perimeter is greater than 56 inches when the width is greater than 8 inches.

Cumulative Review Exercises (Chapters 1-2)

1. $-8-(12-16)=-8-(-4)=-8+4=-4$

2. $(-3)(-2)+(-2)(4)=6+(-8)=-2$

3. $(8-10)^3(7-11)^2=(-2)^3(-4)^2$
$=-8(16)=-128$

4. $2-5[x+3(x+7)]$
$=2-5(x+3x+21)$
$=2-5(4x+21)$
$=2-20x-105$
$=-103-20x$

5. The rational numbers are
$-4,-\frac{1}{3},0,\sqrt{4}\,(=2)$, and 1063.

6. $\frac{5}{x}-(x+2)$

7. $-10{,}000<-2$ since $-10{,}000$ is to the left of -2 on the number line.

8. $6(4x-1-5y)=6(4x)-6(1)-6(5y)$
$=24x-6-30y$

9. $A=-0.9n+80$
$A=-0.9(0)+80$
$A=80$
According to the formula, 80% of seniors had used alcohol in 2000.
This is the same as the actual value shown in the bar graph.

10. $A=-0.9n+80$
$62=-0.9n+80$
$-18=-0.9n$
$\frac{-18}{-0.9}=\frac{-0.9n}{-0.9}$
$20=n$
If trends continue, 62% of seniors will use alcohol 20 years after 2000, or 2020.

11. $5-6(x+2)=x-14$
$5-6x-12=x-14$
$-7-6x=x-14$
$-7-6x-x=x-14-x$
$-7-7x=-14$
$-7-7x+7=-14+7$
$-7x=-7$
$\frac{-7x}{-7}=\frac{-7}{-7}$
$x=1$
The solution set is $\{1\}$.

12. $\frac{x}{5}-2=\frac{x}{3}$

Multiply both sides by the LCD, 15.

$$15\left(\frac{x}{5}-2\right)=15\left(\frac{x}{3}\right)$$
$$15\left(\frac{x}{5}\right)-15(2)=15\left(\frac{x}{3}\right)$$
$$3x-30=5x$$
$$3x-30-3x=5x-3x$$
$$-30=2x$$
$$\frac{-30}{2}=\frac{2x}{2}$$
$$-15=x$$

The solution set is $\{-15\}$.

13. $V=\frac{1}{3}Ah$ for A

$$V=\frac{1}{3}Ah$$
$$3V=3\left(\frac{1}{3}Ah\right)$$
$$3V=Ah$$
$$\frac{3V}{h}=\frac{Ah}{h}$$
$$\frac{3V}{h}=A \text{ or } A=\frac{3V}{h}$$

14. $A = PB$; $A = 48$, $P = 30\% = 0.30$

$$48=0.30B$$
$$\frac{48}{0.30}=\frac{0.30B}{0.30}$$
$$160=B$$

48 is 30% of 160.

15. Let x = the width of the parking lot.

Let $2x-10=$ the length of the parking lot.

$$P=2l+2w$$
$$400=2(2x-10)+2\cdot x$$
$$400=4x-20+2x$$
$$400=6x-20$$
$$400+20=6x-20+20$$
$$420=6x$$
$$\frac{420}{6}=\frac{6x}{6}$$
$$70=x$$
$$x=70$$
$$2x-10=130$$

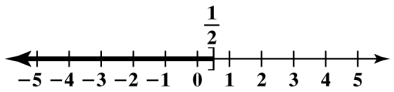
The parking lot is 70 yards wide and 130 yards long.

16. Let x = number of gallons of gasoline.

$$0.40x=30{,}000$$
$$\frac{0.40x}{0.40}=\frac{30{,}000}{0.40}$$
$$x=75{,}000$$

75,000 gallons of gasoline must be sold.

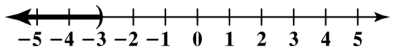
17. $\left(-\infty,\frac{1}{2}\right]$



18. $3-3x>12$

$$3-3x-3>12-3$$
$$-3x>9$$
$$\frac{-3x}{-3}<\frac{9}{-3}$$
$$x<-3$$

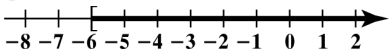
$(-\infty,-3)$



19. $5-2(3-x)\le 2(2x+5)+1$

$$5-6+2x\le 4x+10+1$$
$$2x-1\le 4x+11$$
$$2x-1-4x\le 4x+11-4x$$
$$-2x-1\le 11$$
$$-2x-1+1\le 11+1$$
$$-2x\le 12$$
$$\frac{-2x}{-2}\ge\frac{12}{-2}$$
$$x\ge -6$$

$[-6,\infty)$



20. Let x = value of medical supplies sold.

$$600+0.04x>2500$$
$$600+0.04x-600>2500-600$$
$$0.04x>1900$$
$$\frac{0.04x}{0.04}>\frac{1900}{0.04}$$
$$x>47{,}500$$

You must sell more than \$47,500 worth of medical supplies.

Mini Lecture 1.1

Introduction to Algebra: Variables and Mathematical Models

Learning Objectives:

1. Evaluate algebraic expressions.
2. Translate English phrases into algebraic expressions.
3. Determine whether a number is a solution of an equation.
4. Translate English sentences into algebraic equations.
5. Evaluate formulas.

Examples:

1. Evaluate each expression for $x = 5$.

   a. $4(x-3)$ b. $\dfrac{6x-15}{3x}$

2. Evaluate each expression for $x = 3$ and $y = 6$.

   a. $5(x+y)$ b. $\dfrac{2x+3y}{2y}$

3. Write each English phrase as an algebraic expression. Let x represent the number.
   a. the difference of a number and six
   b. eight more than four times a number
   c. four less than the quotient of a number and twelve

4. Determine whether the given number is a solution of the equation.
   a. $x - 8 = 12$; 20 b. $4x - 7 = 9$; 3 c. $3(y - 5) = 6$; 7

5. Write each English sentence as an equation. Let x represent the number.
   a. The product of a number and seven is twenty-one.
   b. The difference of twice a number and three is equal to twenty-seven.
   c. Six less than three times a number is the same as the number increased by twelve.

Teaching Notes:

- It may be helpful to draw students' attention to the word "evaluate." Help them see the letters v – a – l – u . This will help them remember that evaluate means to find the value of an expression.
- Students often make mistakes with the phrase "less than" so they should be cautioned about the order of the subtraction.
- Translating from English to algebra is an important skill that will be used often.

Answers: 1a. 8 b. 1 2a. 45 b. 2 3a. $x-6$ b. $4x+8$ c. $\dfrac{x}{12}-4$ 4a. yes b. not a solution c. yes 5a. $7x = 21$ b. $2x - 3 = 27$ c. $3x - 6 = x + 12$

Mini Lecture 1.2

Fractions in Algebra

Learning Objectives:

1. Convert between mixed numbers and improper fractions.
2. Write the prime factorization of a composite number.
3. Reduce or simplify fractions.
4. Multiply fractions.
5. Divide fractions.
6. Add and subtract fractions with identical denominators.
7. Add and subtract fractions with unlike denominators.
8. Solve problems involving fractions in algebra.

Examples:

1. Convert each mixed number to an improper fraction.

a. $3\frac{7}{10}$ b. $8\frac{3}{7}$ c. $5\frac{2}{3}$ d. $9\frac{1}{4}$

2. Convert each improper fraction to a mixed number.

a. $\frac{13}{8}$ b. $\frac{12}{11}$ c. $\frac{25}{3}$ d. $\frac{37}{7}$

3. Give the prime factorization of each of the following composite numbers.

a. 24 b. 48 c. 90 d. 108

4. What makes a number a prime?

5. Reduce the following fractions to lowest terms by factoring each numerator and denominator and dividing out common factors.

a. $\frac{10}{12}$ b. $\frac{32}{48}$ c. $\frac{24}{50}$ d. $\frac{77}{98}$

6. Perform the indicated operation. Always reduce answer, if possible.

a. $\frac{3}{4}+\frac{1}{6}$ b. $8\frac{1}{8}+3\frac{1}{3}$ c. $\frac{7}{10}-\frac{3}{8}$

d. $10\frac{11}{12}-4\frac{1}{4}$ e. $\left(\frac{7}{9}\right)\left(\frac{18}{19}\right)$ f. $\left(6\frac{2}{3}\right)\left(2\frac{1}{4}\right)$

g. $\frac{7}{8}\div\frac{3}{4}$ h. $5\frac{3}{8}\div 2\frac{1}{4}$

Teaching Notes:

- When teaching factorization, it is often helpful to review divisibility rules.
- To add or subtract fractions, you must have a LCD.
- To divide fractions, multiply by the reciprocal of the divisor.
- To multiply or divide mixed numbers, change to improper fractions first.

Answers: 1. a. 37/10 b. 59/7 c. 17/3 d. 37/4 2. a. 1 5/8 b. 1 1/11 c. 8 1/3 d. 5 2/7
3. a. $2\cdot 2\cdot 2\cdot 3$ b. $2\cdot 2\cdot 2\cdot 2\cdot 3$ c. $2\cdot 3\cdot 3\cdot 5$ d. $2\cdot 2\cdot 3\cdot 3\cdot 3$
4. a number whose only factors are 1 and itself 5. a. 5/6 b. 2/3 c. 12/25 d. 11/14 6. a. 11/12
b. $\frac{275}{24}$ or 11 11/24 c. 13/40 d. $\frac{20}{3}$ or 6 2/3 e. 14/19 f. 15 g. 7/6 or 1 1/6 h. 43/18 or 2 7/18

Mini Lecture 1.3
The Real Numbers

Learning Objectives:

1. Define the sets that make up the set of real numbers.
2. Graph numbers on a number line.
3. Express rational numbers as decimals.
4. Classify numbers as belonging to one or more sets of the real numbers.
5. Understand and use inequality symbols.
6. Find the absolute value of a real number.

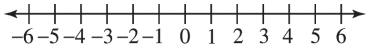
Examples:

1. Answer the following questions about each number:

   Is it a natural number? Is it rational?
   Is it a whole number? Is it irrational?
   Is it an integer? Is it a real number?

   a. 18 b. –3.5 c. $\sqrt{5}$ d. 0 e. $-\frac{3}{4}$ f. π g. –5 h. 0.45

2. Graph each number on the number line.



   a. 5.5 b. $-\frac{16}{4}$ c. $2\frac{1}{4}$ d. –3.2

3. Express each rational number as a decimal.

   a. $\frac{7}{8}$ b. $\frac{9}{11}$ c. $\frac{5}{3}$ d. $\frac{1}{4}$

4. Use $>$ or $<$ to compare the numbers.

   a. 18 □ –20 b. –16 □ –13 c. –4.3 □ –6.2

   d. $\frac{4}{7}$ □ $\frac{8}{11}$ e. $-\frac{3}{5}$ □ $\frac{2}{3}$

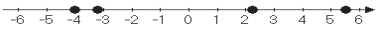
5. Give the absolute value.

   a. $|8|$ b. $|-5|$ c. $|-3.2|$ d. $|22|$

Teaching Notes:

- Make sure the students have minimal understanding of square roots.
- Absolute value is ALWAYS POSITIVE because it measures distance from zero.
- Remind students that a number cannot be rational and irrational.
- To change a rational number to a decimal, divide the numerator by the denominator.

Answers: 1. a. natural, whole, integer, rational , real b. rational, real c. irrational, real d. whole, integer, rational, real e. rational, real f. irrational, real g., integer, rational, real h. rational, real 2. See below 3. a. 0.875 b. 0.81 c. 0.6 d. 0.25 4. a. > b. < c. > d. < e. < 5. a. 8 b. 5 c. 3.2 d. 22



Mini Lecture 1.4
Basic Rules of Algebra

Learning Objectives:

1. Understand and use the vocabulary of algebraic expressions.
2. Use commutative properties.
3. Use associative properties.
4. Use the distributive property.
5. Combine like terms.
6. Simplify algebraic expressions.

Examples:

1. Fill in the blanks.

Algebraic Expression	# of terms	coefficients	like terms
a. $6y - 3x - 4y + 8$	_____	_____	_____
b. $5x^2 + 2y - 2x^2 + 9 - 3y$	_____	_____	_____
c. $6x^2 - 9y + 4x + 8 - y + 5$	_____	_____	_____

2. Name the property being illustrated and then simplify if possible.
   a. $6(x + 2) = 6x + 12$ _____
   b. $(9 \cdot 12)5 = 9(12 \cdot 5)$ _____
   c. $(x + 4) + 8 = x + (4 + 8)$ _____
   d. $(2)(3.14)(5) = 2(5)(3.14)$ _____

3. Simplify.
   a. $6x - x + 2x =$ _____ b. $3a - 8 + 2a + 10 =$ _____
   c. $6(x + 3) - 5 =$ _____ d. $2(x - 4) - (x - 2) =$ _____
   e. $5(y - 2) + 3(4 - y) =$ _____

Teaching Notes:

- A coefficient is the number factor of a term.
- Like terms have the very same variables raised to the same exponents.
- When applying the commutative property, only the order changes.
- The commutative property holds for addition and multiplication only.
- When applying the associative property the grouping changes.
- The associative property holds for addition and multiplication only.
- When combining like terms, add or subtract the coefficients, the variable part remains the same.
- Always use parentheses when substituting a value for a variable.

Answers: 1 a. 4; 6, –3, –4, 8; $6y$ and $-4y$ b. 5; 5, 2, –2, 9, –3; $5x^2$ and $-2x^2$; $2y$ and $-3y$
c. 6; 6, –9, 4, 8, –1, 5; $9y$ and $-y$; 8 and 5 2. a. distributive b. associative of multiplication
c. associative of addition d. commutative of multiplication 3. a. $7x$ b. $5a + 2$ c. $6x + 13$
d. $x - 6$ e. $2y + 2$

Mini Lecture 1.5
Addition of Real Numbers

Learning Objectives:

1. Add numbers with a number line.
2. Find sums using identity and inverse properties.
3. Add numbers without a number line.
4. Use addition rules to simplify algebraic expressions.
5. Solve applied problems using a series of additions.

Examples:

1. Find each sum using a number line.

   a. $3 + -5$ b. $-4 + -6$ c. $-1 + 2$ d. $5 + 4$

2. Add without using a number line.

   a. $-7 + -11$ b. $-0.4 + -3.2$ c. $-\frac{4}{5} + -\frac{3}{10}$ d. $-15 + 4$

   e. $7.1 + 8.5$ f. $-8 + 25$ g. $-6.4 + 6.1$ h. $\frac{5}{8} + -\frac{3}{4}$

3. Simplify the following.

   a. $-30x + 5x$ b. $-2y + 5x + 8x + 3y$ c. $-2(3x + 5y) + 6(x + 2y)$

4. Write a sum of signed numbers that represents the following situation. Then, add to find the overall change.

   If the stock you purchased last week rose 2 points, then fell 4, rose 1, fell 2, and rose 1, what was the overall change for the week?

Teaching Notes:

- When adding numbers with like signs, add and take the sign.
- When adding numbers with unlike signs, subtract the smaller absolute value from the larger absolute value, and the answer will have the sign of the number with the larger absolute value.

Answers: 1. a. –2 b. –10 c. 1 d. 9 2. a. –18 b. –3.6 c. $-\frac{11}{10}$ or $-1\frac{1}{10}$ d. –11 e. 15.6
f. 17 g. –0.3 h. $-\frac{1}{8}$ 3. a. $-25x$ b. $13x + y$ c. $2y$
4. $2 + (-4) + 1 + (-2) + 1 = -2$; fell 2 points

Mini Lecture 1.6
Subtraction of Real Numbers

Learning Objectives:

1. Subtract real numbers.
2. Simplify a series of additions and subtractions.
3. Use the definition of subtraction to identify terms.
4. Use the subtraction definition to simplify algebraic expressions.
5. Solve problems involving subtraction.

Examples:

1. Subtract by changing each subtraction to addition of the opposite first.

   a. $6 - 12$ b. $-15 - 15$ c. $13 - 21$ d. $\frac{2}{5} - \frac{5}{6}$

   e. $4.2 - 6.8$ f. $25 - (-25)$ g. $-51 - (-13)$ h. $14 - (-13)$

2. Simplify.

   a. $-16 - 14 - (-10)$ b. $-20.3 - (-40.1) - 18$

   c. $15 - (-3) - 10 - 18$ d. $-11 - 21 - 31 - 41$

3. Identify the number of terms in each expression; then name the terms.

   a. $4x - 6y + 12 - 3y$ b. $16 - 2x - 15$

   c. $15a - 2ab + 3b - 6a + 18$ d. $5y - x + 3y - 14xy$

4. Simplify each algebraic expression.

   a. $8x + 7 - x$ b. $-11y - 14 + 2y - 10$

   c. $15a - 10 - 12a + 12$ d. $25 - (-3x) - 15 - (-2x)$

5. Applications.

   a. The temperature at dawn was –7 degrees but fortunately the sun came out and by 4:00 p.m. the temperature had reached 38 degrees. What was the difference in the temperature at dawn and 4:00 p.m.?

   b. Express 214 feet below sea level as a negative integer. Express 10,510 above sea level as a positive integer. What is the difference between the two elevations?

Teaching Notes:

- Say the problem to yourself. When you hear the word "minus", immediately make a "change-change". That means to "change" the subtraction to addition and "change" the sign of the number that follows to its opposite.
- Remember, the sign in front of a term goes with the term.
- The symbol "–"can have different meanings:
  1. subtract or "minus" only when it is between 2 terms
  2. the opposite of
  3. negative

Answers: 1. a. –6 b. –30 c. –8 d. $-\frac{13}{30}$ e. –2.6 f. 50 g. –38 h. 27 2. a. –20 b. 1.8 c. –10 d. –104 3. a. 4 terms; $4x$,$-6y$, 12, $-3y$ b. 3 terms; 16, $-2x$, –15 c. 5 terms; $15a$, $-2ab$, $3b$, $-6a$, 18 d. 4 terms; $5y$, $-x$, $3y$, $-14xy$ 4. a. $7x + 7$ b. $-9y - 24$ c. $3a + 2$ d. $5x + 10$ 5. a. 45 degrees b. –214 feet. 10,500 feet; 10, 724 feet

Mini Lecture 1.7
Multiplication and Division of Real Numbers

Learning Objectives:

1. Multiply real numbers.
2. Multiply more than two real numbers.
3. Find multiplicative inverses.
4. Use the definition of division.
5. Divide real numbers.
6. Simplify algebraic expressions involving multiplication.
7. Determine whether a number is a solution of an equation.
8. Use mathematical models involving multiplication and division.

Examples:

1. Multiply.

   a. $(3)(-4)$ b. $(-6)(-5)$ c. $(-8)(0)$ d. $(-3.2)(-1.1)$ e. $\left(-\frac{3}{4}\right)\left(\frac{2}{9}\right)$

   f. $(-5)(2)(-1)$ g. $(-2)(2)(-3)(-3)$

2. Find the multiplicative inverse of each number.

   a. -8 b. $\frac{2}{5}$ c. -7 d. $\frac{1}{4}$

3. Use the definition of division to find each quotient.

   a. $-49 \div 7$ b. $\frac{-24}{-4}$

4. Divide or state that the expression is undefined.

   a. $\frac{-18}{0}$ b. $-\frac{4}{5} \div \frac{20}{25}$ c. $-32.4 \div 8$ d. $0 \div -8$

5. Simplify.

   a. $-3(2x)$ b. $9x + x$ c. $-12a + 4a$ d. $-(5x - 3)$

   e. $-2(3y + 4)$ f. $2(3x + 4) - (4x - 6)$

Teaching Notes:

- The product of an even number of negative numbers is positive.
- The product of an odd number of negative numbers is negative.
- Any product using zero as a factor will equal zero.
- The quotient of two real numbers with different signs is negative.
- The quotient of two real numbers with same signs is positive.
- Division of a non-zero number by zero is undefined.
- Any non-zero number divided into 0 is 0.

Answers: 1. a. -12 b. 30 c. 0 d. 3.52 e. $-\frac{1}{6}$ f. 10 g. -36 2. a. $-\frac{1}{8}$ b. $\frac{5}{2}$ c. $-\frac{1}{7}$ d. $\frac{4}{1}$
3. a. -7 b. 6 4. a. undefined b. -1 c. -4.05 d. 0 5.a. $-6x$ b. $10x$ c. $-8a$ d. $-5x + 3$
e. $-6y - 8$ f. $2x + 14$

Mini Lecture 1.8
Exponents and Order of Operations

Learning Objectives:

1. Evaluate exponential expressions.
2. Simplify algebraic expressions with exponents.
3. Use order of operations agreement.
4. Evaluate mathematical models.

Examples:

1. Identify the base and the exponent, then evaluate.
   a. 3^4 b. $(-4)^3$ c. -8^2 d. $(-8)^2$

2. Evaluate.
   a. 13^2 b. 2^5 c. $(-3)^3$ d. 5^2

3. Simplify if possible.
   a. $6x^2 - x^2$ b. $5y^3 + 2y - 3y^3$ c. $6a^2 + 2a - 4a^2 - 6a$
   d. $10p^3 - 8p^2$

4. Simplify by using the order of operations.
   a. $30 \div 2 \cdot 3 - 52$ b. $14 - (33 \div 11) + 4$
   c. $(5 + 2)^2$ d. $10 - 7(32 \div 8) + 5 \cdot 3$
   e. $\left(\frac{1}{4}\right) + \left(\frac{1}{3}\right)^2$ f. $15 - 3[8 - (-12 \div 2^2) - 4^2]$
   g. $\frac{16 + 4^2 \div 8}{-2 - (-5)}$ h. $22 + 5(x + 7) - 3x - 10$

5. 5. Evaluate each expression for the given value.
   a. $-a - a^2$ if $a = -3$ b. $-a - a^2$ if $a = 3$ c. $4x^2 - x + 3x$ if $x = -1$

6. Use the formula for perimeter of a rectangle, $P = 2w + 2l$ to find the perimeter of a rectangle if the length is 28 cm and the width is 15 cm.

Teaching Notes:

- If the negative sign is part of the base, it will be inside the parentheses.
- **NEVER** multiply the base and the exponent together.
- The exponent tells how many times to write the base as a factor.
- Always use parentheses when substituting a value for a variable.
- The Order of Operations must be followed on every problem.

Answers: 1. a. 81 b. –64 c. –64 d. 64 2. a. 169 b. 32 c. –27 d. 25 3. a. $5x^2$ b. $2y^3 + 2y$ c. $2a^2 - 4a$ d. $10p^3 - 8p^2$ 4. a. –7 b. 15 c. 49 d. –3 e. $\frac{13}{36}$ f. 30 g. 6 h. 2x + 47 5. a. –6 b. –12 c. 2 6. 86 cm

Mini Lecture 2.1
The Addition Property of Equality

Learning Objectives:

1. Identify linear equations in one variable.
2. Use the addition property of equality to solve equations.
3. Solve applied problems using formulas.

Examples:

1. Identify the linear equations in one variable.
   a. $x + 7 = 10$
   b. $x^2 - 2 = 7$
   c. $\frac{3}{x} = 5$
   d. $|x + 1| = 6$
2. Solve the following equations using the addition property of equality. Be sure to check your proposed solution.
   a. $x + 2 = 17$

   b. $-12 = x - 9$

   c. $x - \frac{1}{2} = 4$

   d. $3x - 2x = 8$

   e. $5x + 1 = 4(x - 2)$

   f. $x + 3.5 = 4.8$

   g. $2x + 5 = x - 2$

   h. $3x + 5 = 2x + 5$

   3. If Sue is 2 years older than John then we will use S to represent Sue's age and J to represent John's age. Use the equation $S = J + 2$ to find John's age if Sue is 41.

Teaching Notes:

- Solving an equation is the process of finding the number (or numbers) that make the equation a true statement. These numbers are called the solutions, or roots, or the equation.
- To apply the addition property of equality, one must add the same number or expression to both sides of the equation.
- Equivalent equations are equations that have the same solution.

Answers: 1. a. linear b. not linear c. not linear d. not linear 2. a. 15 b. -3 c. $4\frac{1}{2}$ or $\frac{9}{2}$ d. 8 e. -9 f. 1.3 g. -7 h. 0 3. 39

Mini Lecture 2.2
The Multiplication Property of Equality

Learning Objectives:

1. Use multiplication property of equality to solve equations.
2. Solve equations in the form $-x = c$.
3. Use addition and multiplication properties to solve equations.
4. Solve applied problems using formulas.

Examples:

1. Multiply both sides of the equation by the reciprocal of the coefficient of the variable to solve for the variable.

   a. $\frac{x}{3} = 6$ b. $\frac{x}{-2} = -7$ c. $\frac{y}{15} = -10$ d. $8 = \frac{x}{-3}$

2. Divide both sides of the equation by the coefficient of the variable to solve for the variable.

   a. $6x = 18$ b. $-2x = -14$ c. $15y = -10$ d. $24 = -3x$

Both of the above methods of isolating the variable are effective for solving equations.

3. Solve each equation by multiplying or dividing.

   a. $18y = -108$ b. $\frac{3}{5}x = 12$ c. $124 = \frac{x}{3}$ d. $-7x = -63$

4. Multiply or divide both sides of each equation by –1 to get a positive x.

   a. $-x = -7$ b. $82 = -x$ c. $-a = -\frac{3}{7}$ d. $14 = -x$

5. Solve each equation using both the addition and multiplication properties of equality.

   a. $3x - 5 = 13$ b. $18 - 6x = 14 - 2x$ c. $23 = 2a - 7$

   d. $-6y - 21 = 21$ e. $33 - x = 3x - 11$ f. $\frac{2}{3}x - 6 = 12$

Teaching Notes:

- Remind students that reciprocals always have the same sign.
- When students see $-x$ they must realize the coefficient is –1.

Answers: 1. a. $x = 18$ b. $x = 14$ c. $y = -150$ d. $x = -24$ 2. a. $x = 3$ b. $x = 7$ c. $y = -\frac{2}{3}$ d. $x = -8$

3. a. $y = -6$ b. $x = 20$ c. $x = 372$ d. $x = 9$ 4. a. $x = 7$ b. $x = -82$ c. $a = \frac{3}{7}$ d. $x = -14$

5. a. $x = 6$ b. $x = 1$ c. $a = 15$ d. $y = -7$ e. $x = 11$ f. $x = 27$

Mini Lecture 2.3
Solving Linear Equations

Learning Objectives:

1. Solve linear equations.
2. Solve linear equations containing fractions.
3. Solve linear equations containing decimals.
4. Identify equations with no solution or infinitely many solutions.
5. Solve applied problems using formulas.

Examples:

1. $3x + 2x + 8 = -7 + x + 11$

2. $6x = 3(x + 9)$

3. $5(2x - 1) - 15 = 3(4x + 2) + 4$

4. $\frac{x}{5} = \frac{2x}{3} + \frac{7}{15}$

5. $1.2x + 1.8 = 0.6x$

6. $1.3x + 1.7 = -1 - 1.4x$

7. $2x + 9 = 2(x + 4)$

8. $4(x + 2) + 5 = 5(x + 1) + 8$

9. Use the formula $P = 4s$ to find the length of a side of a square whose perimeter is 32 in.

Teaching Notes:

- Simplify the algebraic expression on each side of the equal sign.
- Collect variable terms on one side of the equal sign and all constant terms on the other side of the equal sign.
- Isolate the variable and solve.
- Check your solution in the original expression.

Answers: 1. –1 2. 9 3. –15 4. –1 5. –3 6. –1 7. inconsistent, no solution 8. 0 9. 8 inches

Mini Lecture 2.4
Formulas and Percents

Learning Objectives:

1. Solve a formula for a variable.
2. Use the percent formula.
3. Solve applied problems involving percent change.

Examples:

1. Solve the formula for the indicated variable by isolating the variable.

   a. $A = \dfrac{B_1 + B_2}{2}$ for B_1

   b. $P = a + b + c$ for c

   c. $A = \pi r^2 h$ for h

   d. $4p + H = M$ for p

   e $Ax + By = C$ for A

   f. $y = mx + b$ for b

2. Translate each question into an equation using the percent formula, $A = PB$, then solve the equation.

   a. What is 15 percent of 60?

   b. 62% of what number is 31?

   c. What percent of 132 is 33?

   d. 60 is what percent of 500?

3. The average, or mean A of the three exam grades, x, y, z, is given by formula $A = \dfrac{x + y + z}{3}$.

   a. Solve the formula for z.

   b. If your first two exams are 75% and 83% ($x = 75$, $y = 83$), what must you get on the third exam to have an average of 80%?

Teaching Notes:

- Many students have trouble solving formulas for a letter and need to be reminded the same steps are used when solving for a letter in a formula as are used when solving any equation for a variable.
- When changing a decimal to a percent, move the decimal point two places to the right and use the % symbol.
- When changing a percent to a decimal, move the decimal point two places to the left and drop the % symbol.
- When translating English into a mathematical equation, the word "is" translates to equals and the word "of" means multiply.

Answers: 1. a. $B_1 = 2A - B_2$ b. $c = P - a - b$ c. $h = \dfrac{A}{\pi r^2}$ d. $p = \dfrac{M - H}{4}$ e. $A = \dfrac{C - By}{x}$ f. $b = y - mx$ 2. a. $x = 0.15(60)$; 9 b. $0.62x = 31$; 50 c. $x \cdot 132 = 33$; 25% d. $60 = x \cdot 500$; 12% 3. a. $z = 3A - x - y$ b. 82%

Mini Lecture 2.5
An Introduction to Problem Solving

Learning Objectives:

1. Translate English phrases into algebraic expressions.
2. Solve algebraic word problems using linear equations.

Examples:

1. Translate each English phrase into an algebraic expression. Let "x" represent the unknown.
   a. Three times a number decreased by 11.
   b. The product of seven and a number increased by 2.
   c. Eight more than a number.

2. Translate each sentence into an algebraic equation and then solve the equation.
   a. Twice a number less five is eleven.
   b. Five times the sum of a number and eight is 30.

3. Identify all unknowns, set up an equation, and then solve.
   a. Bill earns five dollars more per hour than Joe. Together their pay for one hour totals $21. How much does each man earn per hour?
   b. Two consecutive even integers equal 42. Find the integers.

Teaching Notes for solving algebraic equations:

- Make sure to familiarize all students with basic mathematical terms and the proper way to translate to algebraic terms.
- First, read the problem carefully and assign a variable for one of the unknown quantities.
- Write expressions if necessary for any other unknown quantities in terms of same variable.
- Write an equation for the stated problem.
- Solve the equation and answer the question.
- Check the solution in the original stated problem.

Answers: 1. a. $3x - 11$
b. $7x + 2$
c. $x + 8$

2. a. $2x - 5 = 11$
$x = 8$
b. $5(x + 8) = 30$
$x = -2$

3. a. x = Joe
$x + 5$ = Bill
$x + (x + 5) = 21$
x = \$8 (Joe)
$x + 5$ = \$13 (Bill)

b. $x = 1^{st}$ even integer
$x + 2 = 2^{nd}$ even integer
$x + (x + 2) = 42$
$x = 20$
$x + 2 = 22$

Mini Lecture 2.6
Problem Solving in Geometry

Learning Objectives:

1. Solve problems using formulas for perimeter and area.
2. Solve problems using formulas for a circle's area and circumference.
3. Solve problems using formulas for volume.
4. Solve problems involving the angles of a triangle.
5. Solve problems involving complementary and supplementary angles.

Examples:

1. A triangular flower bed has an area of 48 square feet and a height of 12 feet. Find the base of the flower bed.

2. The diameter of a fish pond is 6 feet. Find the area and circumference of the fish pond. First express answer in terms of π, then round both answers to the nearest square foot and foot respectively.

3. Which is the better buy: a 3 liter bottle of soft drink for \$2.99 or a 1.2 liter bottle for \$1.10?

4. Find the volume of a cylinder with a radius of 2 inches and height of 6 inches. Give answer in π form and then round answer to nearest cubic inch.

5. A volleyball has a radius of 3 inches. Find how much air is needed to fill the ball. Give answer in π form and then round answer to nearest cubic inch.

6. Given a right triangle and knowing that the two acute angles are complementary, find the measure of each if one angle is twice the measure of the other.

Teaching Notes:

- Make sure to emphasize the formulas outlined in the section.
- Write formula, substitute the given values, and solve for the unknown.

Answers: 1. base = 8 ft. 2. area = 9π ft^2, 28 ft^2; circumference = 6π ft., 19 ft. 3. 1.2 liter bottle 4. 24π in^3, 75 ft^3 5. 36π in^3, 113 in^3 6. 30°, 60°

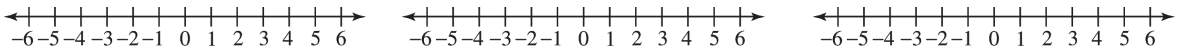
Mini Lecture 2.7
Solving Linear Inequalities

Learning Objectives:

1. Graph the solutions of an inequality on a number line.
2. Use interval notation.
3. Understand properties used to solve linear inequalities.
4. Solve linear inequalities.
5. Identify inequalities with no solution or true for all real numbers.
6. Solve problems using linear inequalities.

Examples:

1. Graph each inequality on the number line.
   a. $x \geq -4$ b. $x < 3$ c. $-1 \leq x < 5$

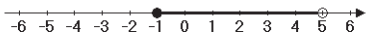


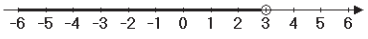
2. Solve each inequality. Write answers in set builder notation and interval notation.
   a. $4x - 3 \leq 5$ b. $6 - x \geq 3$ c. $6x - 12 < 8x - 14$

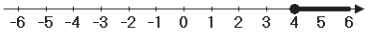
3. Solve each inequality and give the solution in set builder notation: Graph solution on a number line.
   a. $\frac{1}{5}x > -3$ b. $4(6 - 2x) \geq 12 - 4x$ c. $12x - 3 \geq 4(3x + 2)$
   d. $5(x - 3) \geq 5x - 15$ e. $20 < 3x + 5$ f. $2(x - 5) > 5x + 3$

Teaching Notes:

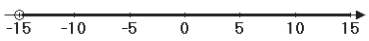
- When graphing the solution of an inequality:
  Use a solid dot when the end point is included in the solution. ($\geq$ or $\leq$)
- When graphing the solution of an inequality:
  Use an open dot when the end point is not included in the solution. ($>$ or $<$)
- When an inequality is multiplied or divided by a negative value, the inequality symbol must be reversed.

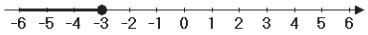
Answers: 1. a. 

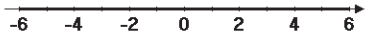
b. 

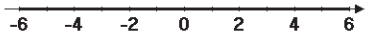
c. 

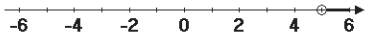
2. a. $\{x \mid x \leq 2\}$ $(-\infty, 2]$ b. $\{x \mid x \leq 3\}$ $(-\infty, 3]$ c. $\{x \mid x > 1\}$ $(1, \infty)$

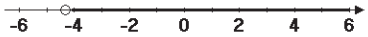
3. a. $x > -15$ $\{x \mid x > -15\}$ 

b. $x \leq 3$ $\{x \mid x \leq 3\}$ 

c. No Solution $\{\ \}$ or Ø 

d. All Real Numbers $\{x \mid x$ is a real number$\}$ 

e. $x > 5$ $\{x \mid x > 5\}$ 

f. $x > -\frac{13}{3}$ $\left\{x \mid x > -\frac{13}{3}\right\}$ 

Mini Lecture 3.1

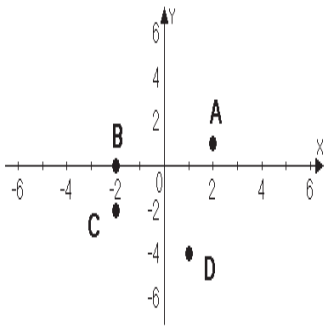
Graphing Linear Equations in Two Variables

Learning Objectives:

1. Plot ordered pairs in the rectangular coordinate system.
2. Find coordinates of points in the rectangular coordinate system.
3. Determine whether an ordered pair is a solution of an equation.
4. Find solutions of an equation in two variables.
5. Use point plotting to graph linear equations.
6. Use graphs of linear equations to solve problems.

Examples:

1. Plot the given points in a rectangular coordinate system. Indicate in which quadrant each point lies.
   a. (–2, –4)
   b. (3, 1)
   c. (–2, 3)
   d. (5, –2)
2. Give the ordered pairs that correspond to the points labeled.



3. Determine if the ordered pair is a solution for the given equation.
   a. $2x + 3y = 10$ (2, 2) b. $3x - y = 5$ (–1, 2)

4. Find five solutions for $y = 2x - 1$ by completing the table of values.
   a.

x	$y = 2x - 1$	(x, y)
–2		
–1		
0		
1		
2		

   b. Plot the ordered pairs to graph the line $y = 2x - 1$.

5. Find five solutions for $y = -x + 1$ by completing the table of values.

a.

x	$y = -x + 1$	(x, y)
–2		
–1		
0		
1		
2		

b. Plot the ordered pairs to graph the quadratic equation $y = -x + 1$.

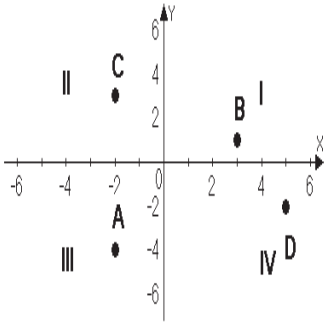
6. a. Your cell phone contract has a base charge of \$10 per month and a \$.03 per minute charge for nation-wide calling. Create a table of values for $y = .03x + 10$. Use 0, 60, 120, 180, 240 for x.

b. Plot the ordered pairs to graph the above equation.

Teaching Notes:

- When graphing linear equations $(y = mx + b)$ on a coordinate plane, the ordered pairs will form a line when connected.
- It is very important to be able to plot points accurately. Students often have problems with points in the form (0, b).

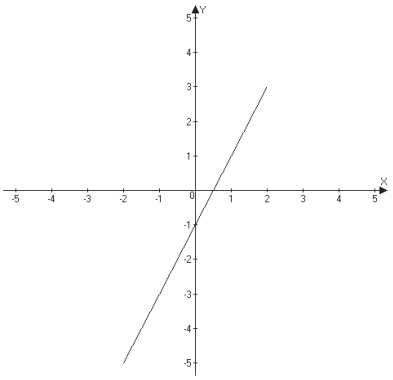
Answers: 1.



2.a. (2, 1) b. (–2, 0) c. (–1, –2) d. (1, –4)

3.a. yes b. no

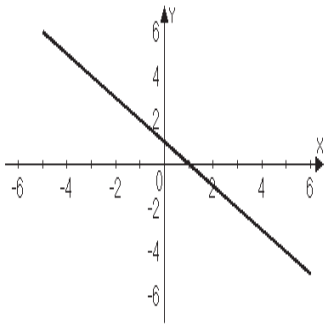
b.



4. a. (–2, –5) (–1, –3) (0, –1) (1, 1) (2, 3)

5. a. (–2, 3) (–1, 2) (0, 1) (1, 0) (2, –1)

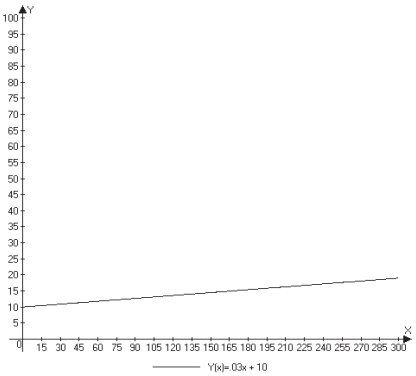
b.



b.

6. a.

x	$y = 0.03x + 10$	(x, y)
0	$y = 0 + 10$	(0, 10)
60	$y = 1.8 + 10$	(60, 11.80)
120	$y = 3.6 + 10$	(120, 13.60)
180	$y = 5.4 + 10$	(180, 15.40)
240	$y = 7.2 + 10$	(240, 17.20)



Mini Lecture 3.2

Graphing Linear Equations Using Intercepts

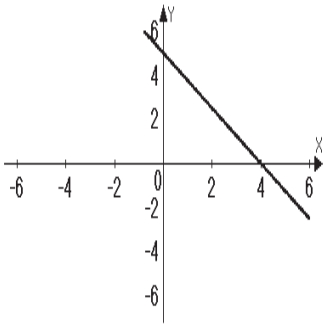
Learning Objectives:

1. Use a graph to identify intercepts.
2. Graph a linear equation in two variables using intercepts.
3. Graph horizontal and vertical lines.

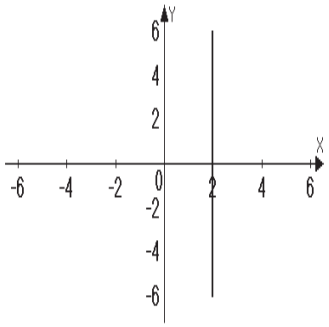
Examples:

1. Identify the x and y-intercepts of each line.

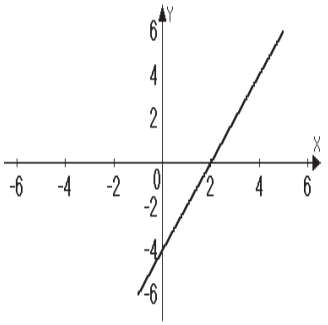
a.



b.



c.



2. Find the x-intercept of the graphs of each of the following equations by substituting 0 in for y and solving for x.

   a. $4x + 7y = 12$ b. $y = 3x - 3$ c. $x - 2y = -8$

3. Find the y-intercept of the graphs of each of the following equations by substituting 0 in for the x and solving for y.

   a. $3x + 2y = -12$ b. $y = 2x + 7$ c. $5x - y = 3$

4. Graph each of the following equations by finding the x and y-intercepts and a check point. Label the intercepts.

   a. $2x - 4y = 12$ b. $5x + 3y = -15$ c. $y = 2x + 6$

5. Graph each equation on the coordinate plane.

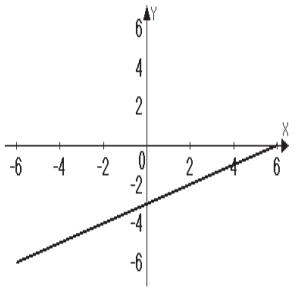
   a. $y + 8 = 12$ b. $x = -3$

Teaching Notes:

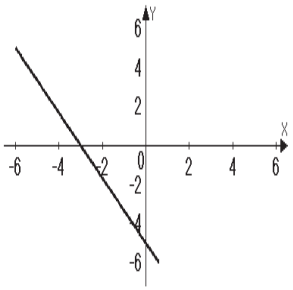
- Intercepts are points not just numbers.
- The *x*-intercept is the point where a graph crosses the *x* axis. The value of *y* is always zero at the *x*-intercept.
- The *y*-intercept is the point where a graph crosses the *y* axis. The value of *x* is always zero at the *y*-intercept.
- When an equation is in standard form and *a* and *b* are factors of *c*, then finding intercepts is a good method to choose for graphing.
- A table is often useful to find intercepts.
- A vertical line has no *y*-intercept, unless it is the *y*-axis ($x = 0$).
- A horizontal line has no *x*-intercept, unless it is the *x*-axis ($y = 0$).

Answers: 1. a. *x*-intercept (4,0); *y*-intercept (0,5) b. *x*-intercept (2,0); *y*-intercept (none)
c. *x*-intercept (2,0); *y*-intercept (0, –4) 2. a. (3,0) b. (1,0) c. (–8,0) 3. a. (0, –6) b. (0,7) c. (0, –3)

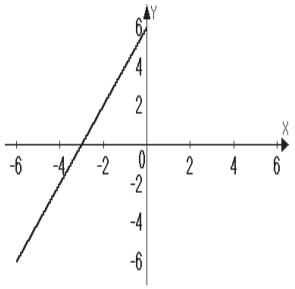
4. a.



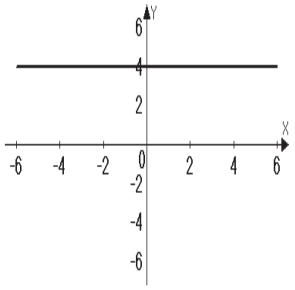
b.



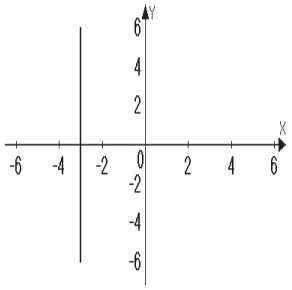
c.



5. a.



b.



Mini Lecture 3.3
Slope

Learning Objectives:

1. Compute a line's slope.
2. Use slope to show that lines are parallel.
3. Use slope to show that lines are perpendicular.
4. Calculate rate of change in applied situations.

Examples:

1. Using the formula for slope $m = \frac{y_2 - y_1}{x_2 - x_1}$, find the slope of the line passing through each pair of points.
   a. (2, 4) (–3, 1)

   b. (–4, 2) (3, –1)

   c. (1, 5) (2, 5)

   d. (–8, 3) (–8, 1)

2. Determine whether a line passing through points (1, 4) and (5, 6) is parallel or perpendicular to a line passing through points (1, –5) and (0, –3).

3. Determine whether a line passing through points (3, 4) and (5, 2) is parallel or perpendicular to a line passing through points (–3, 5) and (–1, 3).

4. Determine whether a line passing through points (5, –8) and (3, 2) is parallel or perpendicular to a line passing through the points (–6, 3) and (–1, 4).

5 Property taxes have continued to increase year after year. Given that in 1990 a home's taxes were $1200 and that same home's taxes were $2600 in 2010. If x represents the year and y the real estate tax, calculate the slope and explain the meaning of your answer.

Teaching Notes:

- Slope is defined as $\frac{rise}{run}$ $\left(\frac{horizontal\ change}{vertical\ change}\right)$.
- $m = \frac{y_2 - y_1}{x_2 - x_1}$, where m represents slope and comes from the French verb "*monter*" meaning to rise or ascend.
- Four slope possibilities:
  1. $m > 0$, positive slope, rises from left to right
  2. $m < 0$, negative slope, falls from left to right
  3. $m = 0$, line is horizontal
  4. m is undefined, line is vertical

Answers: 1.a. $\frac{3}{5}$ b. $-\frac{3}{7}$ c. 0 d. undefined 2. Perpendicular 3. Parallel 4. Perpendicular 5. slope is $\frac{70}{1}$; taxes went up \$70 per year.

Mini Lecture 3.4

The Slope-Intercept Form of the Equation of a Line

Learning Objectives:

1. Find a line's slope and y-intercept of a line from its equation.
2. Graph lines in slope-intercept form.
3. Use slope and y-intercept to graph $Ax + By = C$.
4. Use slope and y-intercept to model data.

Examples:

1. Find the slope and y-intercept of each line with the following equations: (Write the y-intercept as a point.)

   a. $y = \frac{2}{3}x - 4$ b. $y = -3x + 2$ c. $y = -1$

   d. $y = \frac{1}{2}x$ e. $y = 4x - 5$ f. $y = -\frac{3}{4}x + 8$

2. Put each equation in slope-intercept form by solving for y. (Isolate y) Then name the slope and y-intercept.

   a. $2x + y = -6$ b. $-4x - 3y = 6$ c. $x - 2y = 8$

   d. $5y = 10x + 4$ e. $x + y = 10$ f. $3x - 4y = 7$

3. Graph each equation using the slope and the y-intercept.

   a. $4x - 2y = 6$ b. $y = -\frac{1}{2}x + 2$ c. $3x - y = -3$

Teaching Notes:

- In an equation in the form $y = mx + b$, m is the slope of line and b is the y-coordinate of the y-intercept.
- To graph: use the y-intercept as the starting point. Then use the slope to plot at least two more points.
- Remember, the slope must be in fraction form, $\frac{rise}{run}$. If the slope is an integer, it can be put over 1 to form a fraction.

Answers: 1. a. slope $\frac{2}{3}$; y-intercept $(0, -4)$ b. slope -3; y-intercept $(0, 2)$ c. slope 0; y-intercept $(0, -1)$

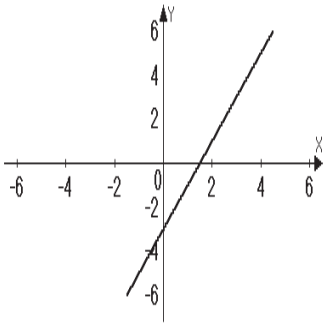
d. slope $\frac{1}{2}$; y-intercept $(0, 0)$ e. slope 4, y-intercept $(0, -5)$ f. slope $-\frac{3}{4}$; y-intercept $(0, 8)$

2. a. y = –2x – 6; slope –2; y-intercept $(0, -6)$ b. $y = -\frac{4}{3}x - 2$; slope $-\frac{4}{3}$; y-intercept $(0, -2)$

c. $y = \frac{1}{2}x - 4$; slope $\frac{1}{2}$; y-intercept $(0, -4)$ d. $y = 2x + \frac{4}{5}$; slope 2; y-intercept $(0, \frac{4}{5})$

e. $y = -x + 10$; slope -1, y-intercept $(0, 10)$ f. $y = \frac{3}{4}x - \frac{7}{4}$; slope $\frac{3}{4}$, y-intercept $\left(0, -\frac{7}{4}\right)$

3. a.



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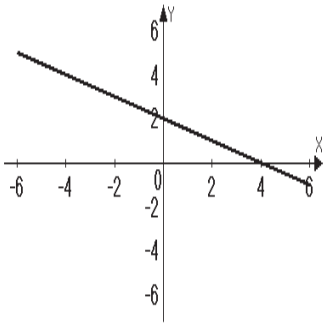

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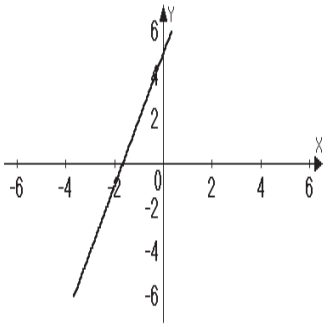


Mini Lecture 3.5

The Point-Slope Form of the Equation of a Line

Learning Objectives:

1. Use the point-slope form to write equations of a line.
2. Write linear equations that model data and make predictions.

Examples:

1. Write the point-slope form and the slope-intercept form of the equation of the line with slope 3 that passes through the point (–1, 4).
2. Write the point-slope form and the slope-intercept form of the equation of the line through the points (1, 4) (–2, 3).
3. The cost of graphing calculators over time has decreased. In 2000, one particular brand sold for $136, in 2011 that same calculator sold for $92. Use the coordinates of the two points (2000, 136) (2011, 92) to write the equation in point-slope and slope-intercept form.

Teaching Notes:

- Distinguish between 2 forms of an equation of a line: point-slope (4.3) and slope-intercept (4.4).
- Point-slope equation with the slope of m and passing through point (x_1, y_1) is $y - y_1 = m(x - x_1)$.
- Use the point-slope equation when given at least one point and the slope.
- If given two points and asked to find the equation of the line, one must first find the slope, section 3.3 and then use the slope and one of the given points in the point-slope equation.

Answers: 1. $y = 4(x + 1); y = 3x + 7$ 2. $y - 4 = \frac{1}{3}(x - 1);\ y = \frac{1}{3}x + \frac{11}{3}$

3. $y - 136 = -4(x - 2000);\ y = -4x + 8136$ or $y - 92 = -4(x - 2011);\ y = -4x + 8136$

Mini Lecture 3.6

Linear Inequalities in Two Variables

Learning Objectives:

1. Determine whether an ordered pair is a solution of an inequality.
2. Graph linear inequalities in two variables.
3. Solve applied problems involving linear inequalities in two variables.

Examples:

Determine which ordered pairs are solutions to the given inequalities.

1. $2x + 3y < 10$
   a. $(-1, 4)$ b. $(4, -1)$ c. $(0, 3)$ d. $(3, 2)$

2. $y \geq -x + 3$
   a. $(4, 7)$ b. $(-3, 0)$ c. $(5, 2)$ d. $(-1, -1)$

3. $4x - 2y \leq 8$
   a. $(0, 2)$ b. $(2, 0)$ c. $(-2, -2)$ d. $(1, -5)$

Put each inequality in slope-intercept form then graph the boundary line and shade the appropriate half plane.

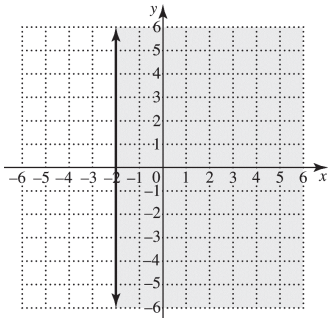
4. a. $2x + 7 \geq 3$ b. $3x - 3y > 6$ c. $y \geq \frac{1}{4}x - 3$
   d. $x < y$ e. $4x + 6y \leq -12$ f. $x \geq 4$

Teaching Notes:

- An inequality has many solutions, therefore shading must be used to show the solutions.
- The graphed line is a boundary and it divides the coordinate plane into two half planes, one of which will be shaded.
- Remind students to use a solid line if the points on the line are included in the solution ($\geq$ or $\leq$).
- Remind students to use a broken or dashed line if points on the line are not included in the solution ($>$ or $<$).
- Students tend to forget to reverse the inequality symbol when multiplying or dividing both sides by a negative number.
- Remind students to use a straight edge when graphing lines.

Answers: 1.no; yes; yes; no 2. yes; no; yes; no 3. yes; yes; yes; no

4. a.



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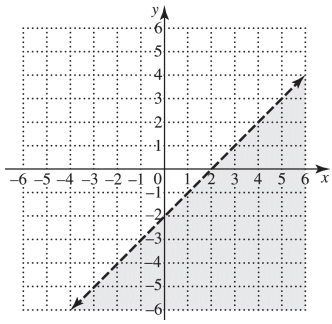
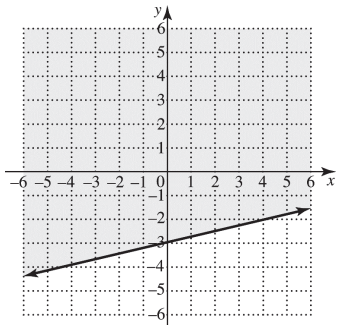
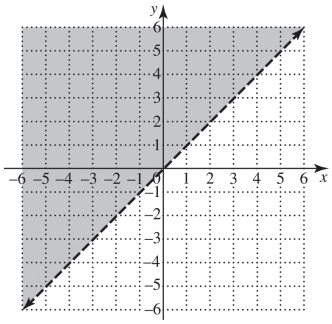


Boundary line dashed

c.

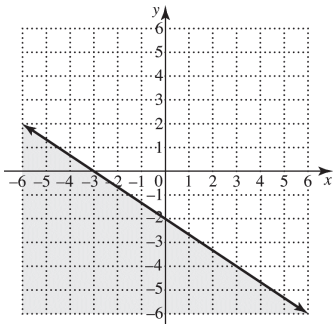


d.

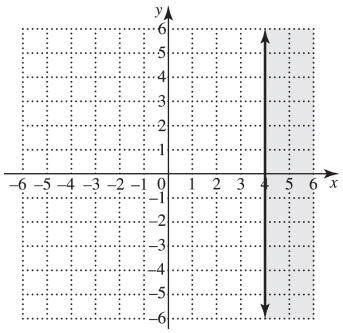


Boundary line dashed

e.



f.



Mini Lecture 4.1

Solving Systems of Linear Equations by Graphing

Learning Objectives:

1. Decide whether an ordered pair is a solution of a linear system.
2. Solve systems of linear equations by graphing.
3. Use graphing to identify systems with no solution or infinitely many solutions.
4. Use graphs of linear systems to solve problems.

Examples:

1. Consider the system.
   $x + y = -3$
   $2x + y = 1$

   Determine if each ordered pair is a solution of the system.
   a. (4, 7) b. (4, –7)

2. Solve the following systems by graphing. State the solution (the intersection point) as an ordered pair (x, y) or state if there is no solution, or state if there are an infinite number of solutions.

   a. $2x + y = -3$
      $y = -2x - 3$

   b. $2x + y = 3$
      $3x - 2y = 8$

   c. $x + 2y = 6$
      $x + 2y = 2$

Teaching Notes:

- When graphing a system of linear equations, there are three possible outcomes:
  1. The two lines can intersect at one point, meaning there is one solution to the system.
  2. The two lines can be parallel to one another, meaning there is no solution to the system.
  3. The two lines are identical or coincide, meaning there are infinitely many solutions to the system.
- When two lines are parallel the system is inconsistent and has no solution.
- When two lines are coinciding, they are called dependent equations and have infinitely many solutions.

Answers:

1. a. not a solution b. yes, a solution 2. a. infinitely many solutions b. (2, –1) c. lines parallel, no solution

Mini Lecture 4.2

Solving Systems of Linear Equations by the Substitution Method

Learning Objectives:

1. Solve linear systems by the substitution method.
2. Use the substitution method to identify systems with no solution or infinitely many solutions.
3. Solve problems using the substitution method.

Examples:

Solve each system using the substitution method. If there is no solution or an infinite number of solutions, so state.

1. a. $x + y = 3$, $y = x + 5$ b. $3x - 2y = 5$, $x = 4y - 5$ c. $7x + 6y = -9$, $y = -2x + 1$ d. $5x - 6y = -4$, $x = y$

2. a. $x + 3y = 4$, $x - 2y = -1$ b. $-2x - y = -3$, $3x + y = 0$ c. $8x - y = 15$, $3x + 4y = 10$ d. $3x - 5y = 12$, $x + 2y = 4$

3. a. $3x + 5y = -3$, $x - 5y = -5$ b. $2x - 4y = -4$, $x + 2y = 8$ c. $7x - 6y = -1$, $x - 2y = -1$ d. $2x - y = 1$, $4x + y = 8$

4. a. $6x + 3y = 1$, $y = -2x - 5$ b. $4x - 4y = 8$, $x - y = 2$ c. $4x - 2y = 8$, $2x - y = 4$ d. $y = -3x + 2$, $6x + 2y = 1$

Teaching Notes:

- Students like to follow specific steps so give them a list of steps to use for solving systems by substitution. Begin with: Isolate a variable with a coefficient of 1 first.
- Many students think they must solve for y. Stress that it does not matter whether the variable solved for is x or y.
- Use colored pens or markers to underline in one equation what will be substituted in the other equation.
- If a graphing calculator is being used in the class, graphing on the calculator is a good way to check solutions.

Answers: 1. a. $(-1, 4)$ b. $(3, 2)$ c. $(3, -5)$ D. $(4, 4)$ 2. a. $(1, 1)$ b. $(-3, 9)$ c. $(2, 1)$ d. $(4, 0)$ 3. a. $(-2, \frac{3}{5})$ b. $(3, \frac{5}{2})$ c. $(\frac{1}{2}, \frac{3}{4})$ d. $(\frac{3}{2}, 2)$ 4. a. No solution b. Infinite solutions c. Infinite solutions d. No solution

Mini Lecture 4.3

Solving Systems of Linear Equations by the Addition Method

Learning Objectives:

1. Solve linear systems by the addition method.
2. Use the addition method to identify systems with no solution or infinitely many solutions.
3. Determine the most efficient method for solving a linear system.

Examples:

Solve the following systems by the addition method.

1. $x + y = 10$
   $x - y = 8$

2. $4x + 3y = 7$
   $-4x + y = 5$

3. $3x - y = 8$
   $x + 2y = 5$

4. $2w - 3z = -1$
   $3w + 4z = 24$

5. $4x - 5y = 8$
   $-4x + 5y = -8$

6. $2x = 5y + 4$
   $2x - 5y = 6$

Teaching Notes:

- When solving a system of linear equations there are three methods:
  Graphing (4.1)
  Substitution (4.2)
  Addition (4.3)
- Any of the three methods will work when solving a system and produce the correct answer.
- Teach students how to determine which of the three methods is the most efficient when solving a system of equations.

Answers: 1. (9, 1) 2. $\left(-\frac{1}{2}, 3\right)$ 3. (3, 1) 4. (4, 3) 5. infinitely many solutions 6. no solution

Mini Lecture 4.4

Problem Solving Using Systems of Equations

Learning Objectives:

1. Solve problems using linear systems.

Examples:

Use variables to represent unknown quantities. Write a: Let x = and y = statement for each problem. (Do not solve).

1. The sum of two numbers is 14. One number is six times larger than the other. Find the two numbers.
2. Three pairs of socks and two pairs of mitten cost $42. One pair of the same kind of socks and four pair of the mittens cost $24. Find out how much one pair of socks and one pair of mittens cost.
3. John has $5 bills and $10 bills in his wallet. He has a total of $80. He has twice as many $5 bills as $10 bills. How many $5 bills and how many $10 bills does he have?

Now, for problems 4 – 6, write a system of equations that models the conditions of each problem. (Do not solve).

4.
5.
6.

Solve each of the following using a system of equations.

7. The sum of two numbers is 11. The second number is 1 less than twice the first number. Find the two numbers.
8. Alexis has $1.65 in nickels and quarters. She has 9 coins altogether. How many coins of each kind does she have?
9. Paul invested $12,000 in two accounts. One account paid 4% interest and one account paid 5% interest. At the end of the year his money had earned $560 in interest. How much did he invest in each account?
10. A department store receives 2 shipments of bud vases and picture frames. The first shipment of 5 bud vases and 4 picture frames costs $62. The second shipment of 10 bud vases and 3 picture frames cost $84. Find the cost of a vase and a picture frame.

Teaching Notes:

- Stress the importance of reading the problem several times before beginning. Reading aloud really helps.
- Have students write a Let x= and y = statements for each word problem before trying to write the system of equations.
- Help students look at the system they have created and determine which method of solving will work best.
- Remind students to make sure their answers make sense for the given situation.
- Try to build confidence with word problems.

Answers: 1. Let x = one number; let y = the other number. 2. Let x = cost of 1 pair of socks; let y = cost of 1 pair of mittens. 3. Let x = number of $5 bills; let y = number of $10 bills

4. $x + y = 14$, $x = 6y$ 5. $3x + 2y = 42$, $x + 4y = 24$ 6. $5x + 10y = 80$, $x = 2y$

7. The numbers are 4 and 7 8. 3 nickels, 6 quarters 9. $4000 invested @ 4% and $8000 invested @ 5% 10. bud vases $6, picture frames $8

Mini Lecture 4.5
Systems of Linear Inequalities

Learning Objectives:

1. Use mathematical models involving systems of linear inequalities.
2. Graph the solution sets of systems of linear inequalities.

Examples:

1. Graph the solution set of each system.

   a. $y < -x + 3$
   $y \geq x - 4$

   b. $y > -\frac{1}{2}x + 3$
   $x \geq 3$

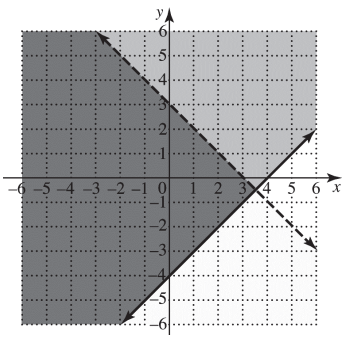
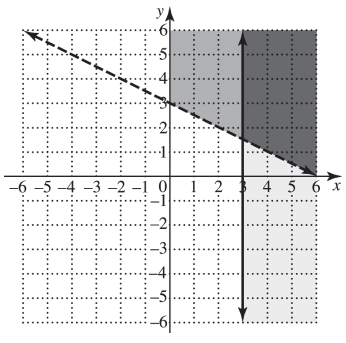
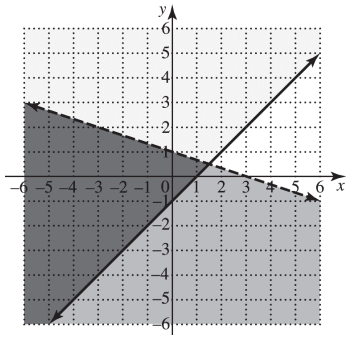
   c. $y \geq x - 1$
   $y < -\frac{1}{3}x + 1$

2. Name one point that is a solution for each system of linear inequalities in examples 1a, 1b, and 1c.
   a. b. c.

Teaching Notes:

- When the inequality symbol is > or <, the line should be dashed (- - - - -).
- When the inequality symbol is ≥ or ≤, the line should be solid (_____).
- When graphing inequalities, it is easy to see the overlap of the graphs if different colored pencils are used to graph each inequality.

Answers:

1. a. 
b. 
c. 

2. a. Answers will vary b. Answers will vary c. Answers will vary