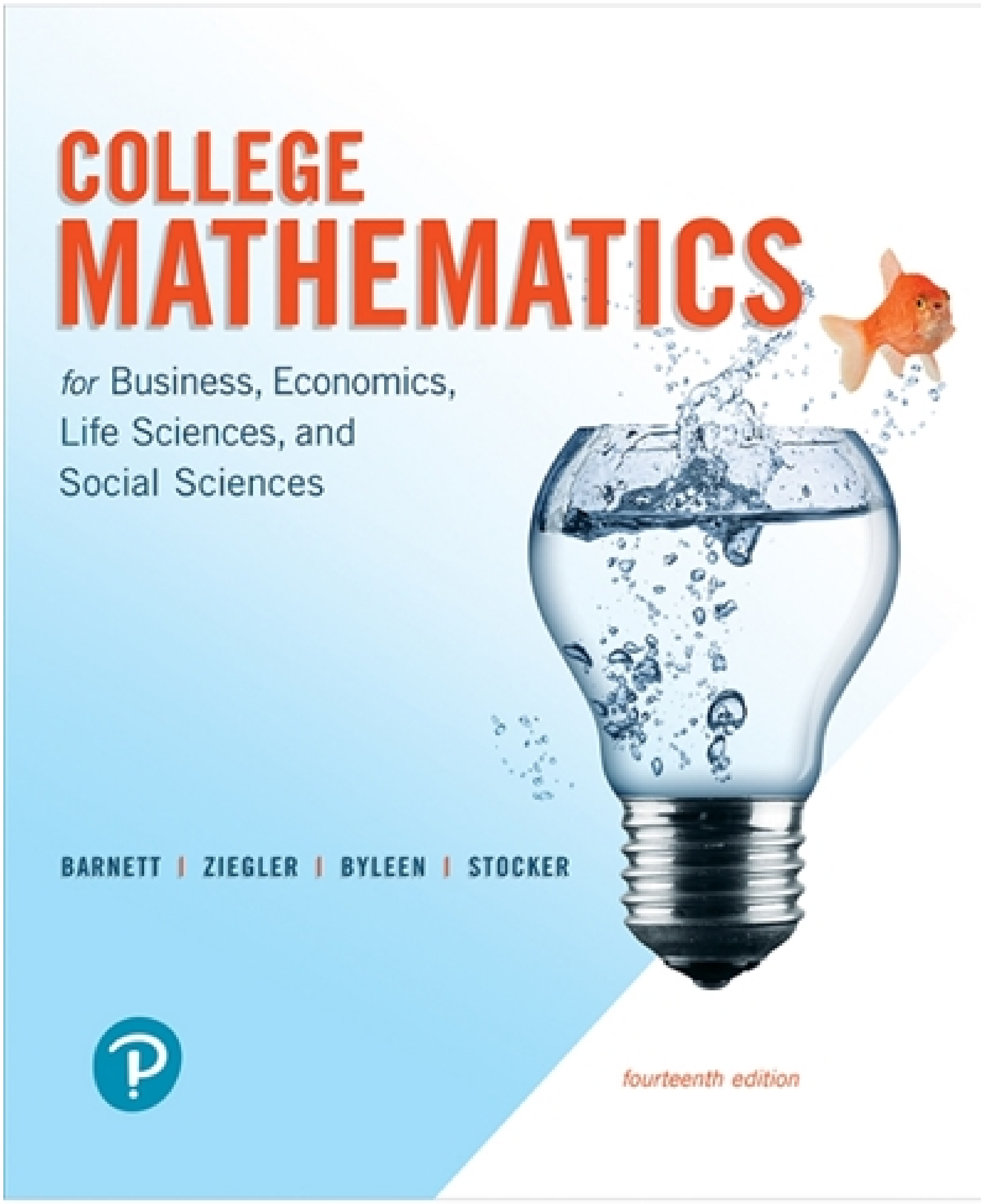
Solutions for College Mathematics for Business Economics Life Sciences and Social Sciences 14th Edition by Barnett

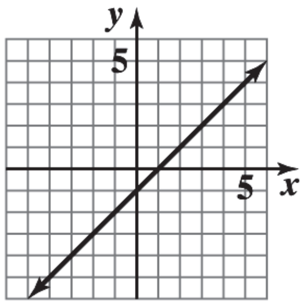


Solutions

2 FUNCTIONS

EXERCISE 2-1

2.



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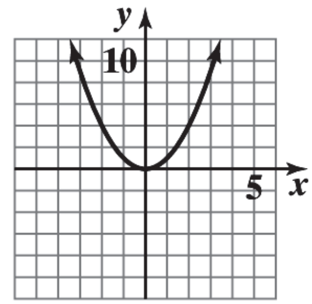

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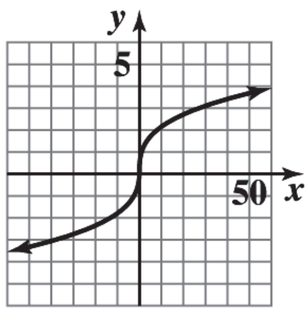

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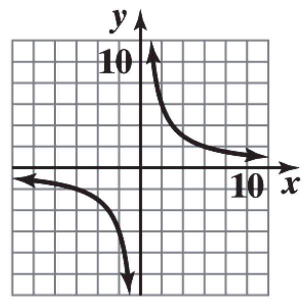


10. The table specifies a function, since for each domain value there corresponds one and only one range value.

12. The table does not specify a function, since more than one range value corresponds to a given domain value.
(Range values 1, 2 correspond to domain value 9.)

14. This is a function.

16. The graph specifies a function; each vertical line in the plane intersects the graph in at most one point.

18. The graph does not specify a function. There are vertical lines which intersect the graph in more than one point. For example, the y-axis intersects the graph in two points.

20. The graph does not specify a function.

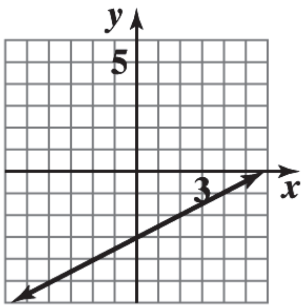
22. $y = 4x + \frac{1}{x}$ is neither linear nor constant.

24. $2x - 4y - 6 = 0$ is linear.

26. $x + xy + 1 = 0$ is neither linear nor constant.

28. $\frac{y-x}{2} + \frac{3+2x}{4} = 1$ simplifies to $y = \frac{1}{2}$ constant.

30.



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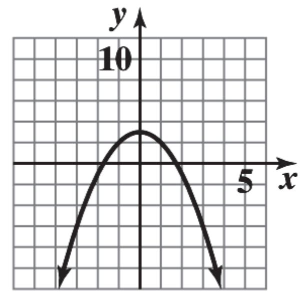

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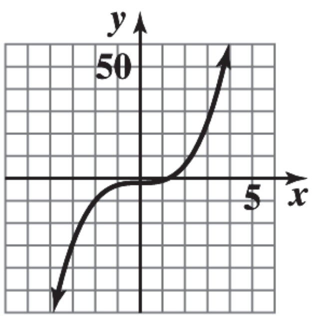

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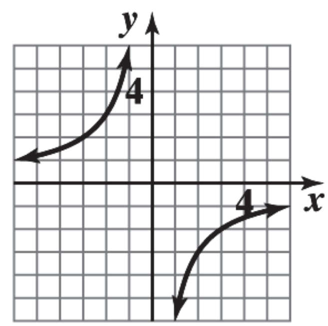
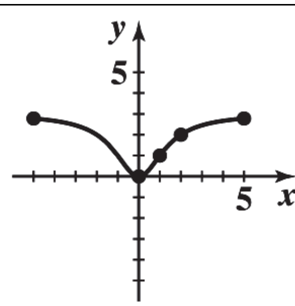


38. $f(x) = \dfrac{3x^2}{x^2+2}$. Since the denominator is bigger than 1, we note that the values of f are between 0 and 3. Furthermore, the function f has the property that $f(-x) = f(x)$. So, adding points $x = 3$, $x = 4$, $x = 5$, we have:

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$F(x)$	2.78	2.67	2.45	2	1	0	1	2	2.45	2.67	2.78

The sketch is:



40. $y = f(4) = 0$

42. $y = f(-2) = 3$

44. $f(x) = 4$ at $x = 5$.

46. $f(x) = 0$ at $x = -5, 0, 4$.

48. Domain: all real numbers.

50. Domain: all real numbers except $x = 2$.

52. Domain: $x \geq -5$ or $[-5, \infty)$.

54. Given $6x - 7y = 21$. Solving for y we have: $-7y = 21 - 6x$ and $y = \frac{6}{7}x - 3$.

This equation specifies a function. The domain is R, the set of real numbers.

56. Given $x(x+y)=4$. Solving for y we have: $xy+x^2=4$ and $y=\frac{4-x^2}{x}$.

This equation specifies a function. The domain is all real numbers except 0

58. Given $x^2+y^2=9$. Solving for y we have: $y^2=9-x^2$ and $y=\pm\sqrt{9-x^2}$.
This equation does not define y as a function of x. For example, when $x=0$, $y=\pm 3$.

60. Given $\sqrt{x}-y^3=0.$. Solving for y we have: $y^3=\sqrt{x}$ and $y=x^{1/6}$.
This equation specifies a function. The domain is all nonnegative real numbers, i.e., $x\geq 0$.

62. $f(-3x)=(-3x)^2-4=9x^2-4$

64. $f(x-1)=(x-1)^2-4=x^2-2x+1-4=x^2-2x-3$

66. $f(x^3)=(x^3)^2-4=x^6-4$

68. $f(\sqrt[4]{x})=\left(x^{1/4}\right)^2-4=x^{1/2}-4=\sqrt{x}-4$

70. $f(-3)+f(h)=(-3)^2-4+h^2-4=5+h^2-4=h^2+1$

72. $f(-3+h)=(-3+h)^2-4=9-6h+h^2-4=5-6h+h^2$

74. $f(-3+h)-f(-3)=\left[(-3+h)^2-4\right]-\left[(-3)^2-4\right]=(9-6h+h^2-4)-(9-4)=-6h+h^2$

76. (A) $f(x+h)=-3(x+h)+9=-3x-3h+9$

(B) $f(x+h)-f(x)=(-3x-3h+9)-(-3x+9)=-3h$

(C) $\frac{f(x+h)-f(x)}{h}=\frac{-3h}{h}=-3$

78. (A) $f(x+h)=3(x+h)^2+5(x+h)-8$
$=3(x^2+2xh+h^2)+5x+5h-8$
$=3x^2+6xh+3h^2+5x+5h-8$

(B) $f(x+h)-f(x)=\left(3x^2+6xh+3h^2+5x+5h-8\right)-\left(3x^2+5x-8\right)$
$=6xh+3h^2+5h$

(C) $\frac{f(x+h)-f(x)}{h}=\frac{6xh+3h^2+5h}{h}=6x+3h+5$

80. (A) $f(x+h)=x^2+2xh+h^2+40x+40h$

(B) $f(x+h)-f(x)=2xh+h^2+40h$

(C) $\frac{f(x+h)-f(x)}{h}=2x+h+40$

82. Given $A = lw = 81$.

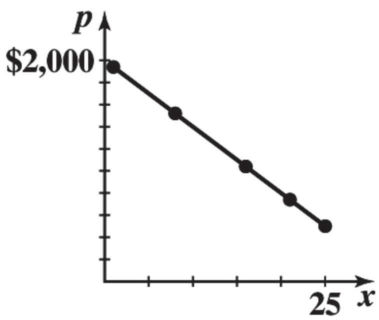
Thus, $w = \frac{81}{l}$. Now $P = 2l + 2w = 2l + 2\frac{81}{l} = 2l + \frac{162}{l}$.

The domain is $l > 0$.

84. Given $P = 2\ell + 2w = 160$ or $\ell + w = 80$ and $\ell = 80 - w$.

Now $A = \ell w = (80 - w)w$ and $A = 80w - w^2$.
The domain is $0 \le w \le 80$. [Note: $w \le 80$ since $w > 80$ implies $\ell < 0$.]

86. (A)



(B) $p(11) = 1{,}340$ dollars per computer
$p(18) = 920$ dollars per computer

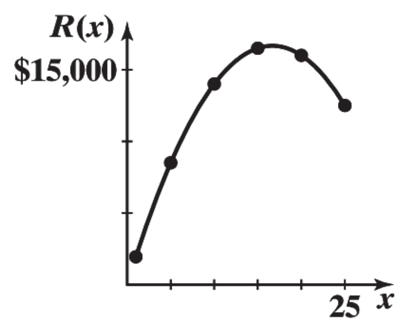
88. (A) $R(x) = xp(x)$
$= x(2{,}000 - 60x)$ thousands of dollars

Domain: $1 \le x \le 25$

(B) Table 11 Revenue

x(thousands)	$R(x)$(thousands)
1	\$1,940
5	8,500
10	14,000
15	16,500
20	16,000
25	12,500

(C)



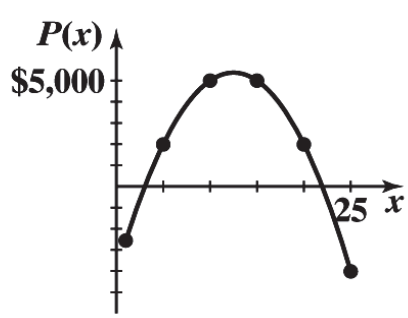
90. (A) $P(x) = R(x) - C(x)$
$= x(2{,}000 - 60x) - (4{,}000 + 500x)$ thousand dollars
$= 1{,}500x - 60x^2 - 4{,}000$

Domain: $1 \le x \le 25$

(B) Table 13 Profit

x (thousands)	$P(x)$ (thousands)
1	–\$2,560
5	2,000
10	5,000
15	5,000
20	2,000
25	–4,000

(C)



92. (A) Given $5v - 2s = 1.4$. Solving for v, we have:
$v = 0.4s + 0.28$.
If $s = 0.51$, then $v = 0.4(0.51) + 0.28 = 0.484$ or 48.4%.

(B) Solving the equation for s, we have:
$s = 2.5v - 0.7$.
If $v = 0.51$, then $s = 2.5(0.51) - 0.7 = 0.575$ or 57.5%.

EXERCISE 2-2

2. $f(x) = 1 + \sqrt{x}$ Domain: $[0, \infty)$; range: $[1, \infty)$.

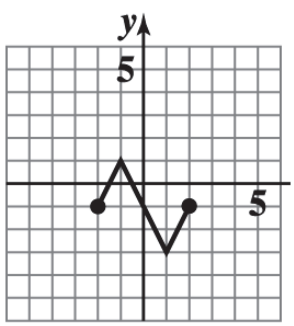
4. $f(x) = x^2 + 10$ Domain: all real numbers; range: $[10, \infty)$.

6. $f(x) = 5x + 3$ Domain: all real numbers; range: all real numbers.

8. $f(x) = 15 - 20|x|$ Domain: all real numbers; range: $(-\infty, 15]$.

10. $f(x) = -8 + \sqrt[3]{x}$ Domain: all real numbers; range: all real numbers.

12.



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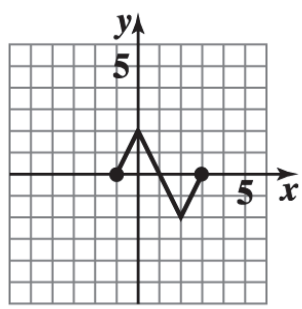

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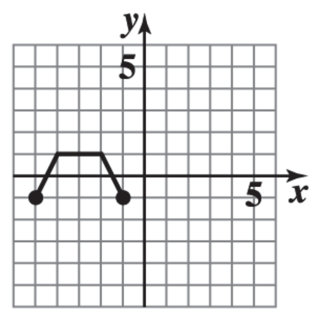

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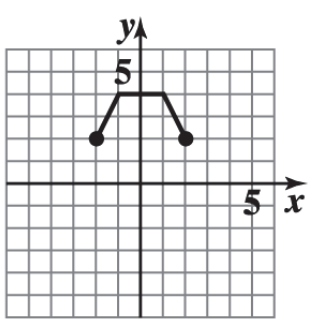

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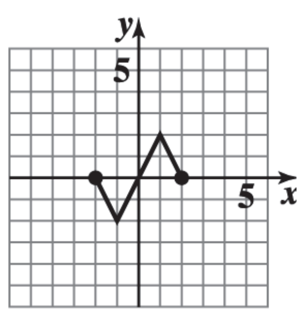

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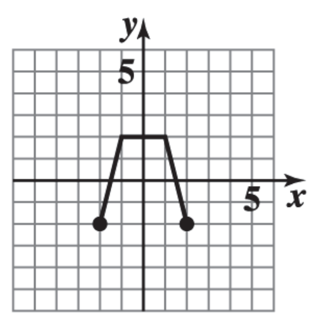

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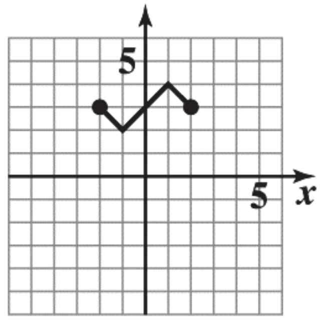

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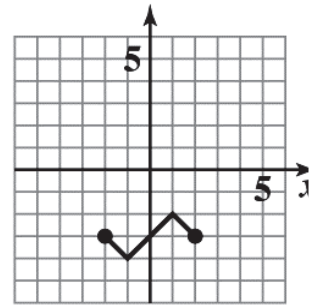
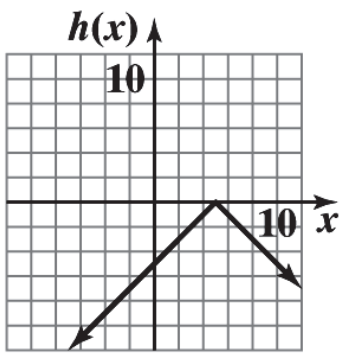
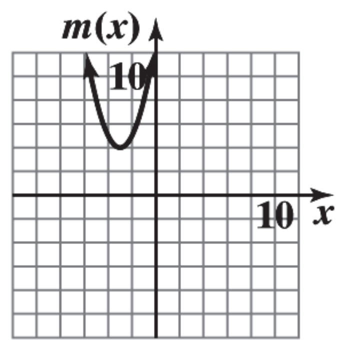


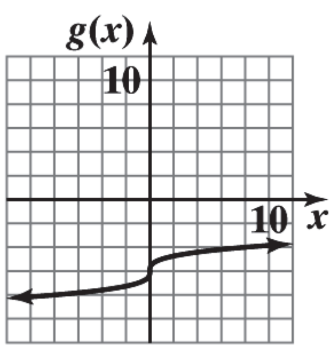
28. The graph of $h(x) = -|x - 5|$ is the graph of $y = |x|$ reflected in the x axis and shifted 5 units to the right.



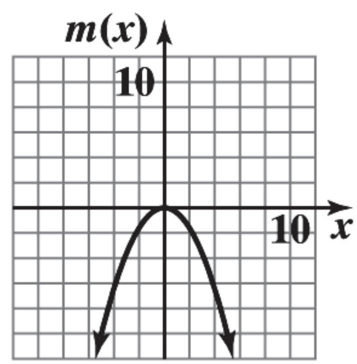
30. The graph of $m(x) = (x + 3)^2 + 4$ is the graph of $y = x^2$ shifted 3 units to the left and 4 units up.



32. The graph of $g(x) = -6 + \sqrt[3]{x}$ is the graph of $y = \sqrt[3]{x}$ shifted 6 units down.



34. The graph of $m(x) = -0.4x^2$ is the graph of $y = x^2$ reflected in the x axis and vertically contracted by a factor of 0.4.



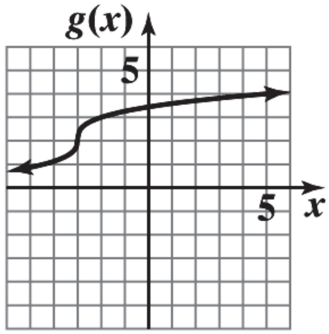
36. The graph of the basic function $y = |x|$ is shifted 3 units to the right and 2 units up. $y = |x - 3| + 2$

38. The graph of the basic function $y = |x|$ is reflected in the x axis, shifted 2 units to the left and 3 units up. Equation: $y = 3 - |x + 2|$

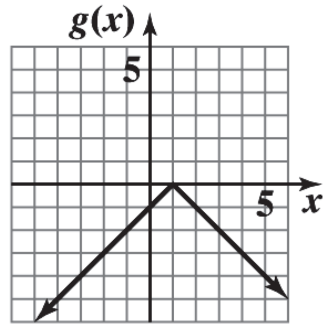
40. The graph of the basic function $\sqrt[3]{x}$ is reflected in the x axis and shifted up 2 units. Equation: $y = 2 - \sqrt[3]{x}$

42. The graph of the basic function $y = x^3$ is reflected in the x axis, shifted to the right 3 units and up 1 unit. Equation: $y = 1 - (x - 3)^3$

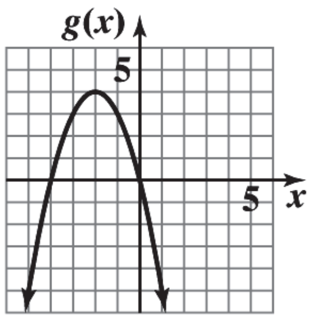
44. $g(x) = \sqrt[3]{x+3} + 2$



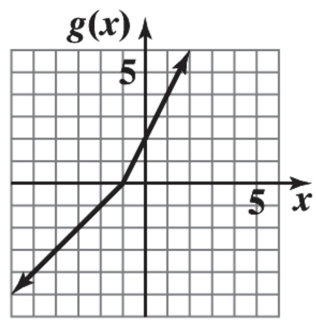
46. $g(x) = -|x - 1|$



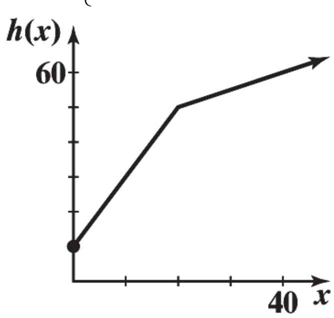
48. $g(x) = 4 - (x + 2)^2$



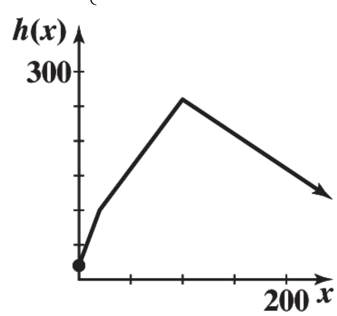
50. $g(x) = \begin{cases} x+1 & \text{if } x < -1 \\ 2+2x & \text{if } x \geq -1 \end{cases}$



52. $h(x) = \begin{cases} 10+2x & \text{if } 0 \leq x \leq 20 \\ 40+0.5x & \text{if } x > 20 \end{cases}$



54. $h(x) = \begin{cases} 4x+20 & \text{if } 0 \leq x \leq 20 \\ 2x+60 & \text{if } 20 < x \leq 100 \\ -x+360 & \text{if } x > 100 \end{cases}$



56. The graph of the basic function $y = x$ is reflected in the x axis and vertically expanded by a factor of 2. Equation: $y = -2x$

58. The graph of the basic function $y = |x|$ is vertically expanded by a factor of 4. Equation: $y = 4|x|$

60. The graph of the basic function $y = x^3$ is vertically contracted by a factor of 0.25. Equation: $y = 0.25x^3$.

62. Vertical shift, reflection in y axis.
Reversing the order does not change the result. Consider a point (a, b) in the plane. A vertical shift of k units followed by a reflection in y axis moves (a, b) to $(a, b + k)$ and then to $(-a, b + k)$. In the reverse order, a reflection in y axis followed by a vertical shift of k units moves (a, b) to $(-a, b)$ and then to $(-a, b + k)$. The results are the same.

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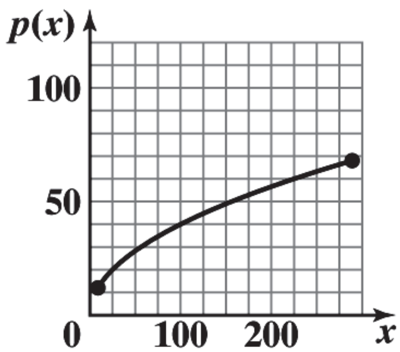
64. Vertical shift, vertical expansion.
Reversing the order can change the result. For example, let (a, b) be a point in the plane. A vertical shift of k units followed by a vertical expansion of h $(h > 1)$ moves (a, b) to $(a, b + k)$ and then to $(a, bh + kh)$. In the reverse order, a vertical expansion of h followed by a vertical shift of k units moves (a, b) to (a, bh) and then to $(a, bh + k)$; $(a, bh + kh) \neq (a, bh + k)$.

66. Horizontal shift, vertical contraction.
Reversing the order does not change the result. Consider a point (a, b) in the plane. A horizontal shift of k units followed by a vertical contraction of h $(0 < h < 1)$ moves (a, b) to $(a + k, b)$ and then to $(a + k, bh)$. In the reverse order, a vertical contraction of h followed by a horizontal shift of k units moves (a, b) to (a, bh) and then to $(a + k, bh)$. The results are the same.

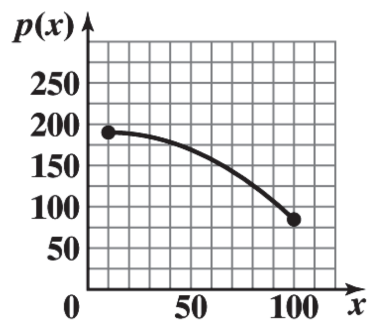
68. (A) The graph of the basic function $y = \sqrt{x}$ is vertically expanded by a factor of 4.

(B)



70. (A) The graph of the basic function $y = x^2$ is reflected in the x axis, vertically contracted by a factor of 0.013, and shifted 10 units to the right and 190 units up.

(B)

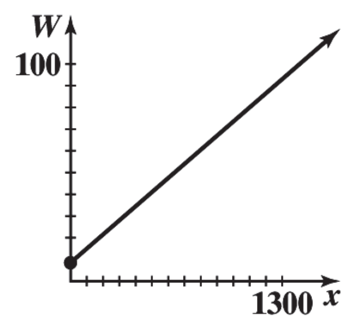


72. (A) Let x = number of kwh used in a winter month. For $0 \le x \le 700$, the charge is $8.5 + .065x$. At $x = 700$, the charge is \$54. For $x > 700$, the charge is $54 + .053(x - 700) = 16.9 + 0.053x$.

Thus,

$$W(x) = \begin{cases} 8.5 + .065x & \text{if } 0 \le x \le 700 \\ 16.9 + 0.053x & \text{if } x > 700 \end{cases}$$

(B)

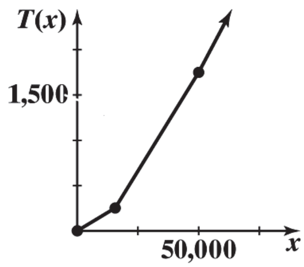


74. (A) Let x = taxable income.
If $0 \le x \le 12{,}500$, the tax due is \$$.02x$. At $x = 12{,}500$, the tax due is \$250. For $12{,}500 < x \le 50{,}000$, the tax due is $250 + .04(x - 12{,}500) = .04x - 250$. For $x > 50{,}000$, the tax due is $1{,}250 + .06(x - 50{,}000) = .06x - 1{,}250$.

Thus,

$$T(x) = \begin{cases} 0.02x & \text{if } 0 \le x \le 12{,}500 \\ 0.04x - 250 & \text{if } 12{,}500 < x \le 50{,}000 \\ 0.06x - 1{,}250 & \text{if } x > 50{,}000 \end{cases}$$

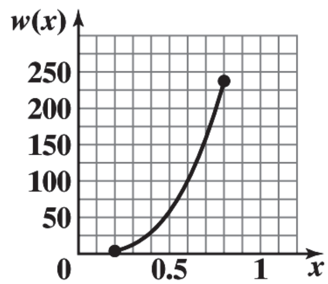
(B)



(C) $T(32{,}000) = \$1{,}030$
$T(64{,}000) = \$2{,}590$

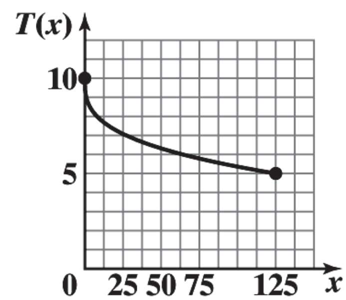
76. (A) The graph of the basic function $y = x^3$ is vertically expanded by a factor of 463.

(B)



78. (A) The graph of the basic function $y = \sqrt[3]{x}$ is reflected in the x axis and shifted up 10 units.

(B)



EXERCISE 2-3

2. $x^2 + 16x$ (standard form)
$x^2 + 16x + 64 - 64$ (completing the square)
$(x+8)^2 - 64$ (vertex form)

4. $x^2 - 12x - 8$ (standard form)
$(x^2 - 12x) - 8$
$(x^2 - 12x + 36) + 8 - 36$
(completing the square)
$(x-6)^2 - 44$ (vertex form)

6. $3x^2 + 18x + 21$ (standard form)

$3(x^2 + 6x) + 21$

$3(x^2 + 6x + 9 - 9) + 21$ (completing the square)

$3(x+3)^2 + 21 - 27$

$3(x+3)^2 - 6$ (vertex form)

8. $-5x^2 + 15x - 11$ (standard form)

$-5(x^2 - 3x) - 11$

$-5(x^2 - 3x + \frac{9}{4} - \frac{9}{4}) - 11$ (completing the square)

$-5(x - \frac{3}{2})^2 - 11 + \frac{45}{4}$

$-5(x - \frac{3}{2})^2 + \frac{1}{4}$ (vertex form)

10. The graph of $g(x)$ is the graph of $y = x^2$ shifted right 1 unit and down 6 units; $g(x) = (x-1)^2 - 6$.

12. The graph of $n(x)$ is the graph of $y = x^2$ reflected in the x axis, then shifted right 4 units and up 7 units;

$n(x) = -(x-4)^2 + 7$.

14. (A) g (B) m (C) n (D) f

16. (A) x intercepts: –5, –1; y intercept: –5 (B) Vertex: (–3, 4)
(C) Maximum: 4 (D) Range: $y \le 4$ or $(-\infty, 4]$

18. (A) x intercepts: 1, 5; y intercept: 5 (B) Vertex: (3, –4)
(C) Minimum: –4 (D) Range: $y \ge -4$ or $[-4, \infty)$

20. $g(x) = -(x+2)^2 + 3$

(A) x intercepts:

$$\begin{aligned} -(x+2)^2 + 3 &= 0 \\ (x+2)^2 &= 3 \\ x + 2 &= \pm\sqrt{3} \\ x &= -2 - \sqrt{3}, -2 + \sqrt{3} \end{aligned}$$

y intercept: –1

(B) Vertex: (–2, 3) (C) Maximum: 3 (D) Range: $y \le 3$ or $(-\infty, 3]$

22. $n(x) = (x-4)^2 - 3$

(A) x intercepts:

$$\begin{aligned} (x-4)^2 - 3 &= 0 \\ (x-4)^2 &= 3 \\ x - 4 &= \pm\sqrt{3} \\ x &= 4 - \sqrt{3}, 4 + \sqrt{3} \end{aligned}$$

y intercept: 13

(B) Vertex: (4, –3) (C) Minimum: –3 (D) Range: $y \ge -3$ or $[-3, \infty)$

24. $y = -(x-4)^2 + 2$

26. $y = [x-(-3)]^2 + 1$ or $y = (x+3)^2 + 1$

28. $g(x) = x^2 - 6x + 5 = x^2 - 6x + 9 - 4 = (x-3)^2 - 4$

(A) x intercepts:

$$\begin{aligned} (x-3)^2 - 4 &= 0 \\ (x-3)^2 &= 4 \\ x - 3 &= \pm 2 \\ x &= 1, 5 \end{aligned}$$

y intercept: 5

(B) Vertex: (3, –4) (C) Minimum: –4 (D) Range: $y \ge -4$ or $[-4, \infty)$

30. $s(x) = -4x^2 - 8x - 3 = -4\left[x^2 + 2x + \frac{3}{4}\right] = -4\left[x^2 + 2x + 1 - \frac{1}{4}\right]$

$= -4\left[(x+1)^2 - \frac{1}{4}\right] = -4(x+1)^2 + 1$

(A) x intercepts: $-4(x + 1)^2 + 1 = 0$

$4(x + 1)^2 = 1$

$(x + 1)^2 = \frac{1}{4}$

$x + 1 = \pm\frac{1}{2}$

$x = -\frac{3}{2}, -\frac{1}{2}$

y intercept: -3

(B) Vertex: $(-1, 1)$ (C) Maximum: 1 (D) Range: $y \le 1$ or $(-\infty, 1]$

32. $v(x) = 0.5x^2 + 4x + 10 = 0.5[x^2 + 8x + 20] = 0.5[x^2 + 8x + 16 + 4]$

$= 0.5[(x + 4)^2 + 4]$

$= 0.5(x + 4)^2 + 2$

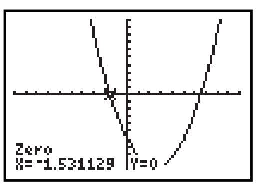
(A) x intercepts: none
y intercept: 10

(B) Vertex: $(-4, 2)$ (C) Minimum: 2 (D) Range: $y \ge 2$ or $[2, \infty)$

34. $g(x) = -0.6x^2 + 3x + 4$

(A) $g(x) = -2$: $-0.6x^2 + 3x + 4 = -2$

$0.6x^2 - 3x - 6 = 0$

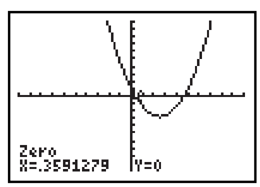


$x = -1.53, 6.53$

(B) $g(x) = 5$: $-0.6x^2 + 3x + 4 = 5$

$-0.6x^2 + 3x - 1 = 0$

$0.6x^2 - 3x + 1 = 0$

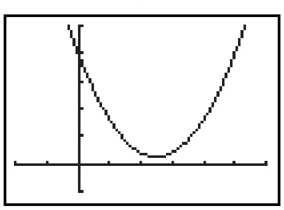


$x = 0.36, 4.64$

(C) $g(x) = 8$: $-0.6x^2 + 3x + 4 = 8$

$-0.6x^2 + 3x - 4 = 0$

$0.6x^2 - 3x + 4 = 0$



No solution

36. Using a graphing utility with $y = 100x - 7x^2 - 10$ and the calculus option with maximum command, we obtain 347.1429 as the maximum value.

38. $m(x) = 0.20x^2 - 1.6x - 1 = 0.20(x^2 - 8x - 5)$
$= 0.20[(x - 4)^2 - 21] = 0.20(x - 4)^2 - 4.2$

(A) x intercepts: $0.20(x - 4)^2 - 4.2 = 0$
$(x - 4)^2 = 21$
$x - 4 = \pm\sqrt{21}$
$x = 4 - \sqrt{21} = -0.6,\ 4 + \sqrt{21} = 8.6;$

y intercept: -1

(B) Vertex: $(4, -4.2)$ (C) Minimum: -4.2 (D) Range: $y \geq -4.2$ or $[-4.2, \infty)$

40. $n(x) = -0.15x^2 - 0.90x + 3.3 = -0.15(x^2 + 6x - 22) = -0.15[(x + 3)^2 - 31] = -0.15(x + 3)^2 + 4.65$

(A) x intercepts: $-0.15(x + 3)^2 + 4.65 = 0$
$(x + 3)^2 = 31$
$x + 3 = \pm\sqrt{31}$
$x = -3 - \sqrt{31} = -8.6, -3 + \sqrt{31} = 2.6;$

y intercept: 3.30

(B) Vertex: $(-3, 4.65)$ (C) Maximum: 4.65 (D) Range: $x \leq 4.65$ or $(-\infty, 4.65]$

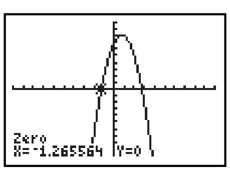
42. $(x+6)(x-3) < 0$

Therefore, either $(x+6) < 0$ and $(x-3) > 0$ or $(x+6) > 0$ and $(x-3) < 0$. The first case is impossible. The second case implies $-6 < x < 3$. Solution set: $(-6, 3)$.

44. $x^2 + 7x + 12 = (x+3)(x+4) \geq 0$

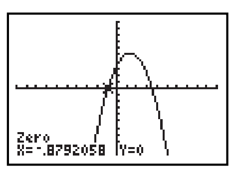
Therefore, either $(x+3) \geq 0$ and $(x+4) \geq 0$ or $(x+3) \leq 0$ and $(x+4) \leq 0$. The first case implies $x \geq -3$ and the second case implies $x \leq -4$. Solution set: $(-\infty, -4] \cup [-3, \infty)$.

46.



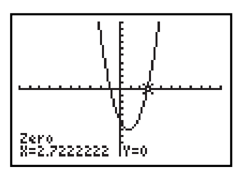
$x = -1.27, 2.77$

48.



$-0.88 \leq x \leq 3.52$

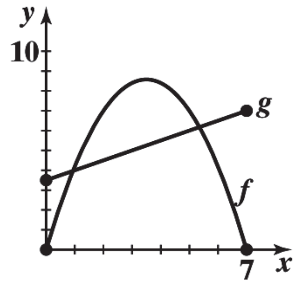
50.



$x < -1$ or $x > 2.72$

52. f is a quadratic function and max $f(x) = f(-3) = -5$
Axis: $x = -3$
Vertex: $(-3, -5)$
Range: $y \leq -5$ or $(-\infty, -5]$
x intercepts: None

54. (A)



(B) $f(x) = g(x)$: $-0.7x(x-7) = 0.5x + 3.5$

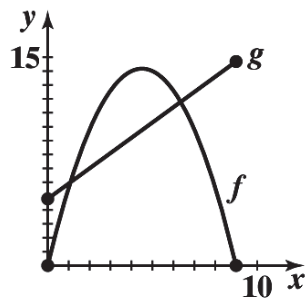
$$-0.7x^2 + 4.4x - 3.5 = 0$$

$$x = \frac{-4.4 \pm \sqrt{(4.4)^2 - 4(0.7)(3.5)}}{-1.4} = 0.93, 5.35$$

(C) $f(x) > g(x)$ for $0.93 < x < 5.35$

(D) $f(x) < g(x)$ for $0 \le x < 0.93$ or $5.35 < x \le 7$

56. (A)



(B) $f(x) = g(x)$: $-0.7x^2 + 6.3x = 1.1x + 4.8$

$$-0.7x^2 + 5.2x - 4.8 = 0$$

$$0.7x^2 - 5.2x + 4.8 = 0$$

$$x = \frac{-(-5.2) \pm \sqrt{(-5.2)^2 - 4(0.7)(4.8)}}{1.4} = 1.08, 6.35$$

(C) $f(x) > g(x)$ for $1.08 < x < 6.35$

(D) $f(x) < g(x)$ for $0 \le x < 1.08$ or $6.35 < x \le 9$

58. The graph of a quadratic with no real zeros will not intersect the x–axis.

60. Such an equation will have $b^2 - 4ac = 0$.

62. Such an equation will have $\frac{k}{a} < 0$.

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64. $ax^2+bx+c=a(x-h)^2+k$

$$=a(x^2-2hx+h^2)+k$$
$$=ax^2-2ahx+ah^2+k$$

Equating constant terms gives $k=c-ah^2$. Since h is the vertex, we have $h=-\frac{b}{2a}$. Substituting then gives

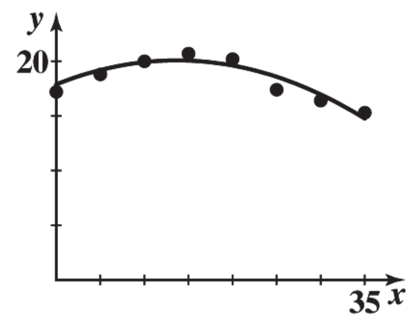
$$k=c-ah^2=c-a\left(\frac{b^2}{4a^2}\right)=c-\frac{b^2}{4a}$$
$$=\frac{4ac-b^2}{4a}$$

66. $f(x)=-0.0117x^2+0.32x+17.9$

(A)

x	Mkt Share	$f(x)$
5	18.8	19.2
10	20.0	19.9
15	20.7	20.1
20	20.2	19.6
25	17.4	18.6
30	16.4	17
35	15.3	14.8

(B)



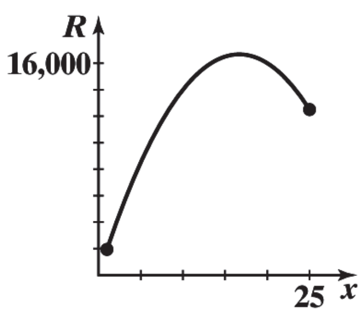
(C) For 2025, $x=45$ and $f(45)=-0.0117(45)^2+0.32(45)+17.9=8.6\%$

For 2028, $x=48$ and $f(48)=-0.0117(48)^2+0.32(48)+17.9=6.3\%$

(D) Market share rose from 18.8% in 1985 to a maximum of 20.7% in 1995 and then fell to 15.3% in 2010.

68. Verify

70. (A)



(B) $R(x)=2{,}000x-60x^2$

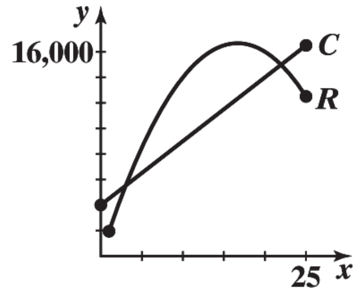
$$=-60\left(x^2-\frac{100}{3}x\right)$$
$$=-60\left[x^2-\frac{100}{3}x+\frac{2500}{9}-\frac{2500}{9}\right]$$
$$=-60\left[\left(x-\frac{50}{3}\right)^2-\frac{2500}{9}\right]$$
$$=-60\left(x-\frac{50}{3}\right)^2+\frac{50{,}000}{3}$$

16.667 thousand computers (16,667 computers); 16,666.667 thousand dollars ($16,666,667)

(C) $2000-60(50/3)=$ $1,000

72. (A)

$$p\left(\frac{50}{3}\right) = 2{,}000 - 60\left(\frac{50}{3}\right) = \$1{,}000$$



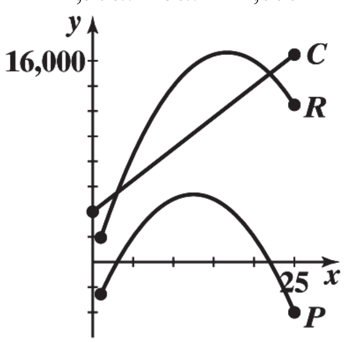
(B) $R(x) = C(x)$

$$x(2{,}000 - 60x) = 4{,}000 + 500x$$
$$2{,}000x - 60x^2 = 4{,}000 + 500x$$
$$60x^2 - 1{,}500x + 4{,}000 = 0$$
$$6x^2 - 150x + 400 = 0$$
$$x = 3.035, \; 21.965$$

Break-even at 3.035 thousand (3,035) and 21.965 thousand (21,965)

(C) Loss: $1 \le x < 3.035$ or $21.965 < x \le 25$;
Profit: $3.035 < x < 21.965$

74. (A) $P(x) = R(x) - C(x)$
$= 1{,}500x - 60x^2 - 4{,}000$

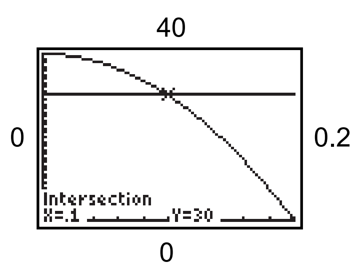


(B) and (C) Intercepts and break-even points: 3,035 computers and 21,965 computers

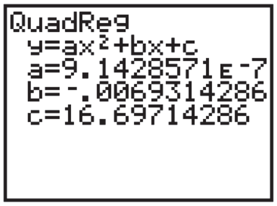
(D) Maximum profit is $5,375,000 when 12,500 computers are produced. This is much smaller than the maximum revenue of $16,666,667.

76. Solve: $f(x) = 1{,}000(0.04 - x^2) = 30$

$$40 - 1000x^2 = 30$$
$$1000x^2 = 10$$
$$x^2 = 0.01$$
$$x = 0.10 \text{ cm}$$



78.



For $x = 2{,}300$, the estimated fuel consumption is
$y = a(2{,}300)^2 + b(2{,}300) + c = 5.6$ mpg.

EXERCISE 2-4

2. $f(x) = x^2 - 5x + 6$

(A) Degree: 2

(B) $x^2 - 5x + 6 = 0$
$(x-2)(x-3) = 30$
$x = 2, 3$

x-intercepts: $x = 2, 3$

(C) $f(0) = 0^2 - 5(0) + 6 = 6$
y-intercept: 6

4. $f(x) = 30 - 3x$

(A) Degree: 1

(B) $30 - 3x = 0$
$3x = 30$
$x = 10$

x-intercept: 10

(C) $f(0) = 30 - 3(0) = 30$
y-intercept: 30

6. $f(x) = 5x^6 + x^4 + x^8 + 10$

(A) Degree: 8

(B) $f(x) \geq 10$ for all x.
No x-intercepts.

(C) $f(0) = 5(0)^6 + (0)^4 + (0)^8 + 10 = 10$
y-intercept: 10

8. $f(x) = (x-5)^2(x+7)^2$

(A) Degree: 4

(B) $(x-5)^2(x+7)^2 = 0$
$x = 5, -7$
x-intercepts: $x = 5, -7$

(C) $f(0) = (0-5)^2(0+7)^2 = 1{,}225$
y-intercept: 1,225

10. $f(x) = (2x-5)^2(x^2-9)^4$

(A) Degree: 10

(B) $(2x-5)^2\left(x^2-9\right)^4 = 0$
$x = \dfrac{5}{2}, -3, 3 \quad x = -3, \tfrac{1}{2}$
x-intercepts: $-3, 5/2, 3$

(C) $f(0) = [2(0)-5]^2[(0)^2-9]^4 = 5^2 9^4 = 164{,}025$
y-intercept: 164,025

12. (A) Minimum degree: 2

(B) Negative – it must have even degree, and positive values in the domain are mapped to negative values in the range.

14. (A) Minimum degree: 3

(B) Negative – it must have odd degree, and positive values in the domain are mapped to negative values in the range.

16. (A) Minimum degree: 4

(B) Positive – it must have even degree, and positive values in the domain are mapped to positive values in the range.

18. (A) Minimum degree: 5

(B) Positive – it must have odd degree, and large positive values in the domain are mapped to positive values in the range.

20. A polynomial of degree 7 can have at most 7 x intercepts.

22. A polynomial of degree 6 may have no x intercepts. For example, the polynomial $f(x) = x^6 + 1$ has no x intercepts.

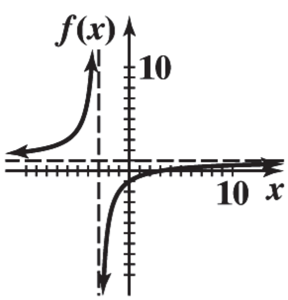
24. (A) Intercepts:

x-intercept(s):	y-intercept:
$x - 3 = 0$	$f(0) = \frac{0-3}{0+3} = -1$
$x = 3$	
(3, 0)	(0, –1)

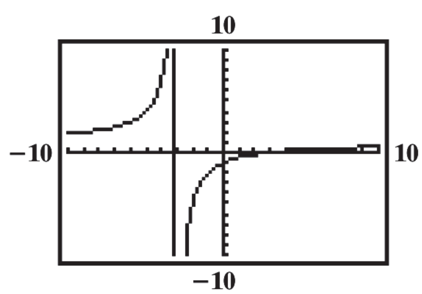
(B) Domain: all real numbers except $x = -3$

(C) Vertical asymptote at $x = -3$ by case 1 of the vertical asymptote procedure on page 57.
Horizontal asymptote at $y = 1$ by case 2 of the horizontal asymptote procedure on page 57.

(D)



(E)



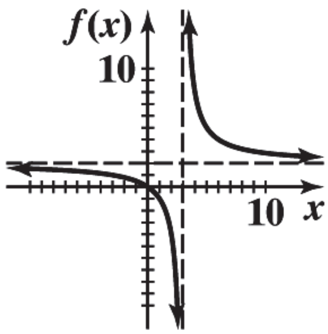
26. (A) Intercepts:

x-intercept(s):	y-intercept:
$2x = 0$	$f(0) = \frac{2(0)}{0-3} = 0$
$x = 0$	
(0, 0)	(0, 0)

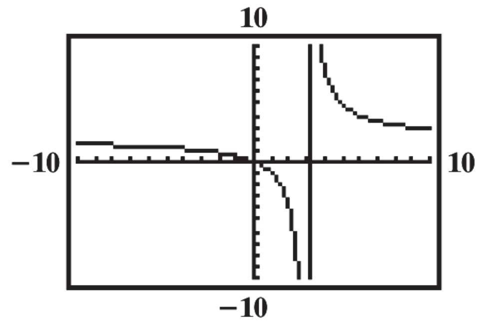
(B) Domain: all real numbers except $x = 3$.

(C) Vertical asymptote at $x = 3$ by case 1 of the vertical asymptote procedure on page 57.
Horizontal asymptote at $y = 2$ by case 2 of the horizontal asymptote procedure on page 57.

(D)



(E)



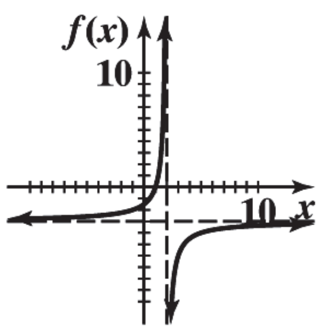
28. (A) Intercepts:

x-intercept:	y-intercept:
$3 - 3x = 0$ $x = 1$ $(1, 0)$	$f(0) = \dfrac{3 - 3(0)}{0 - 2} = -\dfrac{3}{2}$ $\left(0, -\dfrac{3}{2}\right)$

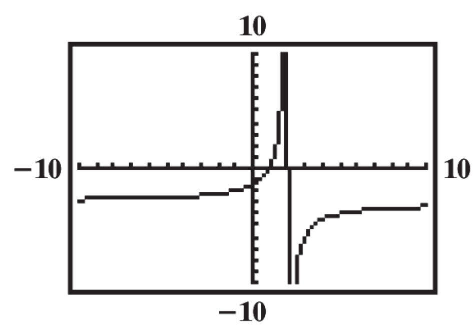
(B) Domain: all real numbers except $x = 2$

(C) Vertical asymptote at $x = 2$ by case 1 of the vertical asymptote procedure on page 57.
Horizontal asymptote at $y = -3$ by case 2 of the horizontal asymptote procedure on page 57.

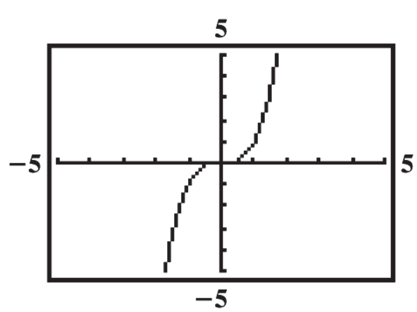
(D)

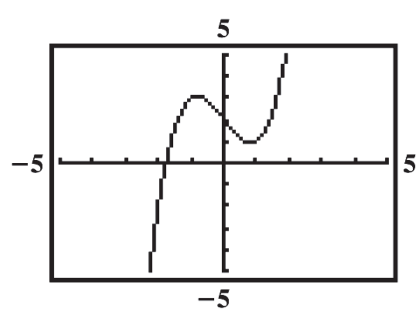


(E)

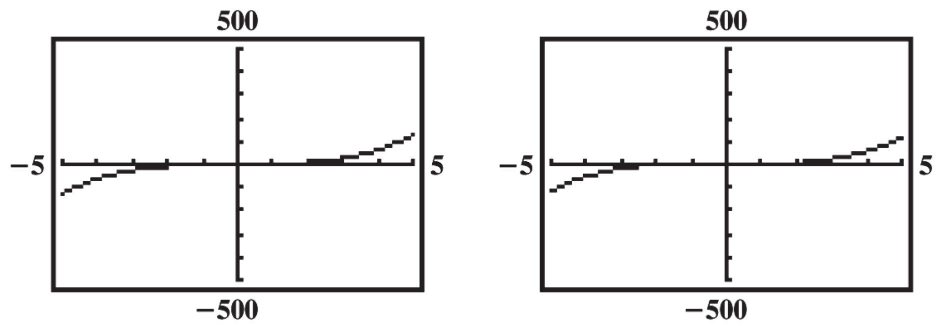


30. (A)

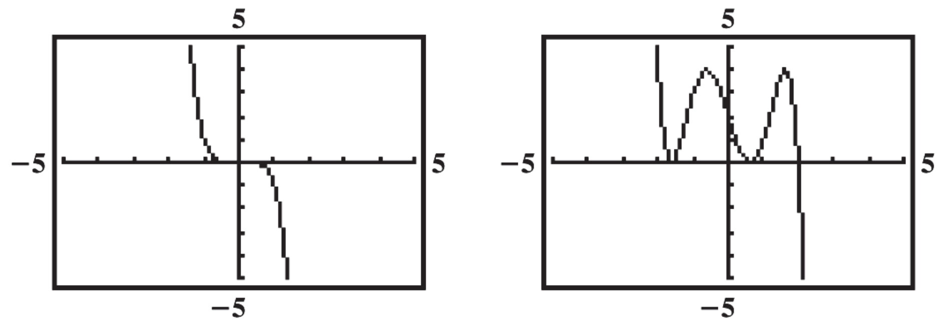




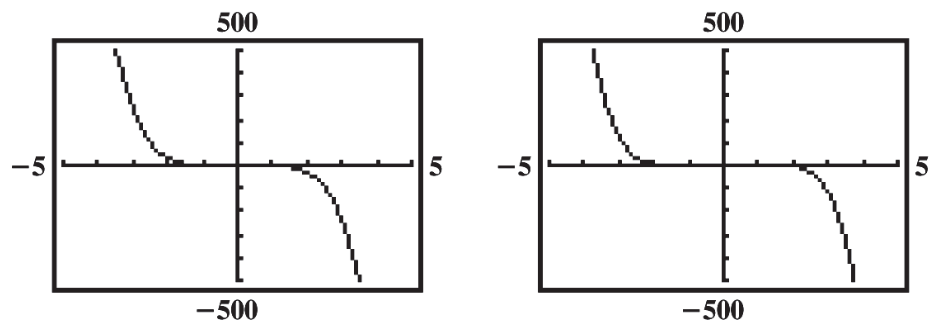
(B)



32. (A)



(B)



34. $y = \frac{6}{4}$, by case 2 for horizontal asymptotes on page 57.

36. $y = -\frac{1}{2}$, by case 2 for horizontal asymptotes on page 57.

38. $y = 0$, by case 1 for horizontal asymptotes on page 57.

40. No horizontal asymptote, by case 3 for horizontal asymptotes on page 57.

42. Here we have denominator $(x^2 - 4)(x^2 - 16) = (x-2)(x+2)(x-4)(x+4)$. Since none of these linear terms are factors of the numerator, the function has vertical asymptotes at $x = 2$, $x = -2$, $x = 4$, and $x = -4$.

44. Here we have denominator $x^2 + 7x - 8 = (x-1)(x+8)$. Also, we have numerator $x^2 - 8x + 7 = (x-1)(x-7)$. By case 2 of the vertical asymptote procedure on page 57, we conclude that the function has a vertical asymptote at $x = -8$.

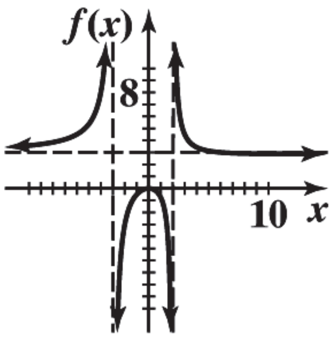
46. Here we have denominator $x^3 - 3x^2 + 2x = x(x^2 - 3x + 2) = x(x-2)(x-1)$. We also have numerator $x^2 + x - 2 = (x+2)(x-1)$. By case 2 of the vertical asymptote procedure on page 57, we conclude that the function has vertical asymptotes at $x = 0$ and $x = 2$.

48. (A) Intercepts:

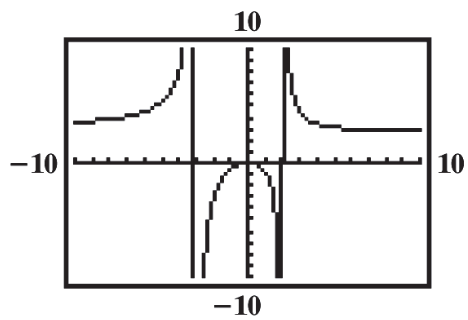
x-intercept(s):	y-intercept:
$3x^2 = 0$ $x = 0$ (0, 0)	$f(0) = 0$ (0, 0)

(B) Vertical asymptote when $x^2 + x - 6 = (x-2)(x+3) = 0$; so, vertical asymptotes at $x = 2$, $x = -3$.
Horizontal asymptote $y = 3$.

(C)



(D)

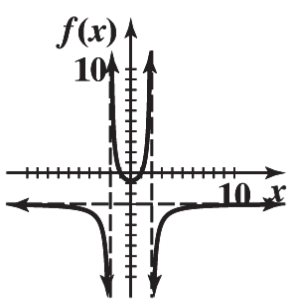


50. (A) Intercepts:

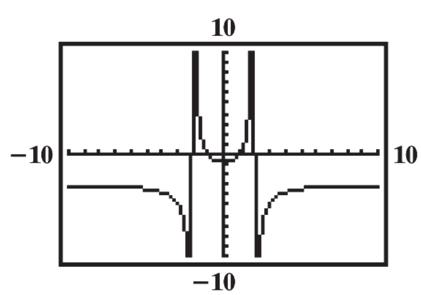
x-intercept(s):	y-intercept:
$3 - 3x^2 = 0$ $3x^2 = 3$ $x = \pm 1$ (1,0), (–1,0)	$f(0) = -\frac{3}{4}$ $\left(0, \frac{-3}{4}\right)$

(B) Vertical asymptotes when $x^2 - 4 = 0$; i.e. at $x = 2$ and $x = -2$.
Horizontal asymptote at $y = -3$

(C)



(D)

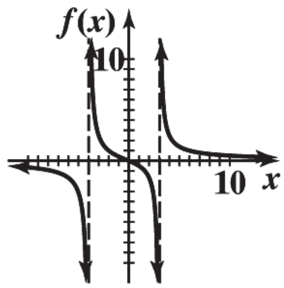


52. (A) Intercepts:

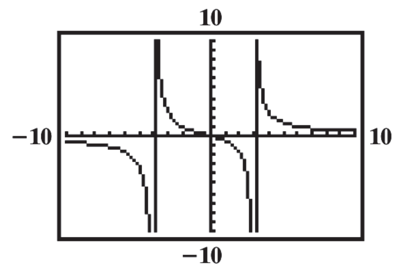
x-intercept(s):	y-intercept:
$5x - 10 = 0$ $x = 2$ (2,0)	$f(0) = \frac{-10}{-12} = \frac{5}{6}$ (0,5/6)

(B) Vertical asymptote when $x^2 + x - 12 = (x+4)(x-3) = 0$; i.e. when $x = -4$ and when $x = 3$. Horizontal asymptote at $y = 0$.

(C)



(D)



54. $f(x) = -(x+2)(x-1) = -x^2 - x + 2$

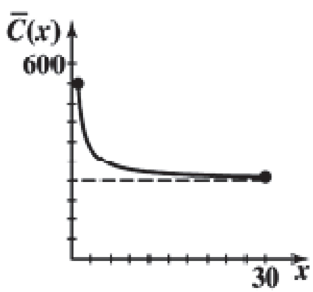
56. $f(x) = x(x+1)(x-1) = x\left(x^2 - 1\right) = x^3 - x$

58. (A) We want $C(x) = mx + b$. Fix costs are $b = \$300$ per day. Given $C(20) = 5{,}100$ we have

$$m(20) + 300 = 5{,}100$$
$$20m = 4800$$
$$m = 240$$
$$C(x) = 240x + 300$$

(B) $\bar{C}(x) = \dfrac{C(x)}{x} = \dfrac{240x + 300}{x} = 240 + \dfrac{300}{x}$

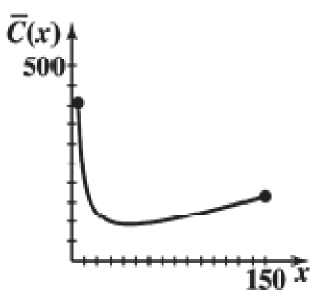
(C)



(D) Average cost tends towards \$240 as production increases.

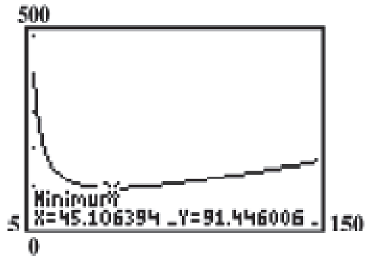
60. (A) $\bar{C}(x) = \dfrac{x^2 + 2x + 2{,}000}{x}$

(B)



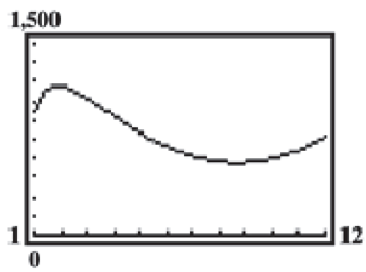
(C) A daily production level of $x = 45$ units per day, results in the lowest average cost of $\bar{C}(45) = \$91.44$ per unit

(D)



62. (A) $\bar{C}(x) = \dfrac{20x^3 - 360x^2 + 2{,}300x - 1{,}000}{x}$

(B)



(C) A minimum average cost of \$566.84 is achieved at a production level of $x = 8.67$ thousand cases per month.

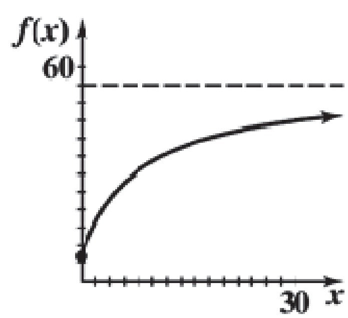
64. (A) Cubic regression model

CubicReg
y=ax³+bx²+cx+d
a=.0902777778
b=-1.87202381
c=10.14484127
d=241.5714286

(B) $y(21) = 583$ eggs

66. (A) The horizontal asymptote is $y = 55$.

(B)



68. (A) Cubic regression model

CubicReg
y=ax³+bx²+cx+d
a=4.4444444E-5
b=-.0065833333
c=.2471031746
d=2.073809524

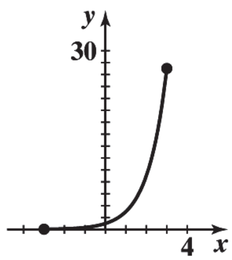
(B) This model gives an estimate of 2.5 divorces per 1,000 marriages.

EXERCISE 2-5

2. A. graph g B. graph f C. graph h D. graph k

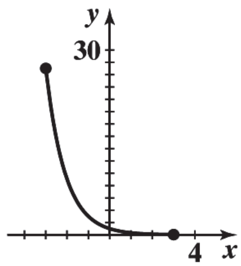
4. $y = 3^x; [-3, 3]$

x	y
–3	$\frac{1}{27}$
–1	$\frac{1}{3}$
0	1
1	3
3	27



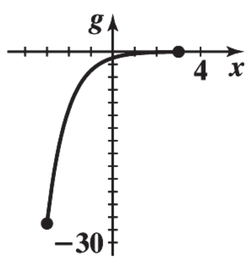
6. $y = 3^{-x}; [-3, 3]$

x	y
–3	27
–1	3
0	1
1	$\frac{1}{3}$
3	$\frac{1}{27}$



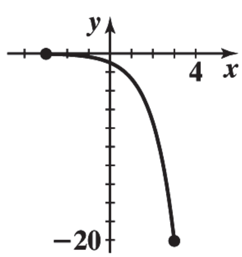
8. $g(x) = -3^{-x}; [-3, 3]$

x	$g(x)$
–3	–27
–1	–3
0	–1
1	$-\frac{1}{3}$
3	$-\frac{1}{27}$



10. $y = -e^x; [-3, 3]$

x	y
–3	≈ –0.05
–1	≈ –0.37
0	–1
1	≈ –2.72
3	≈ –20.09



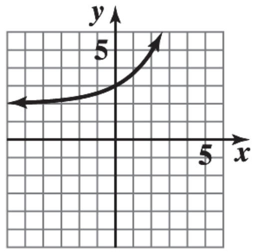
12. The graph of g is the graph of f shifted 2 units to the right.

14. The graph of g is the graph of f reflected in the x axis.

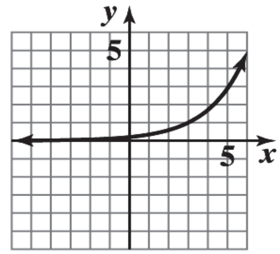
16. The graph of g is the graph of f shifted 2 units down.

18. The graph of g is the graph of f vertically contracted by a factor of 0.5 and shifted 1 unit to the right.

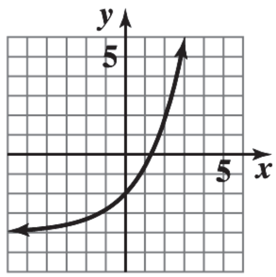
20. (A) $y = f(x) + 2$



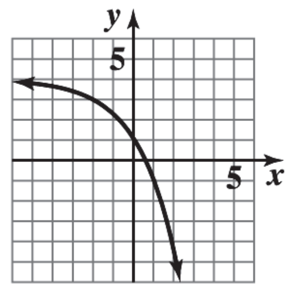
(B) $y = f(x - 3)$



(C) $y = 2f(x) - 4$

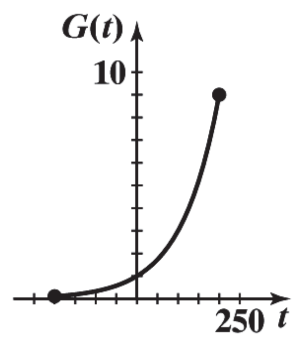


(D) $y = 4 - f(x + 2)$



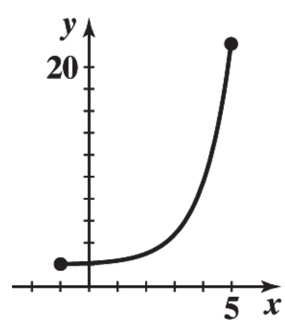
22. $G(t) = 3^{t/100}$; [−200, 200]

x	$G(t)$
−200	$\frac{1}{9}$
−100	$\frac{1}{3}$
0	1
100	3
200	9



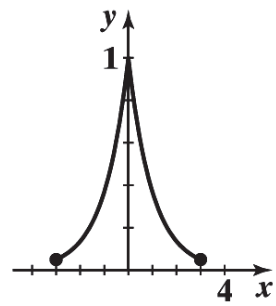
24. $y = 2 + e^{x-2}$; [−1, 5]

x	y
−1	≈ 2.05
0	≈ 2.14
1	≈ 2.37
3	≈ 4.72
5	≈ 22.09



26. $y = e^{-|x|}$; $[-3, 3]$

x	y
–3	≈ 0.05
–1	≈ 0.37
0	1
1	≈ 0.37
3	≈ 0.05



28. $a = 2,\ b = -2$ for example. The exponential function property: For $x \neq 0$, $a^x = b^x$ if and only if $a = b$ assumes $a > 0$ and $b > 0$.

30.
$$3^{x+4} = 3^{2x-5}$$
$$x + 4 = 2x - 5$$
$$-x = -9$$
$$x = 9$$

32.
$$5^{x^2-x} = 5^{42}$$
$$x^2 - x = 42$$
$$x^2 - x - 42 = 0$$
$$(x-7)(x+6) = 0$$
$$x = -6, 7$$

34.
$$(3x+4)^4 = (52)^4$$
$$3x + 4 = 52$$
$$3x = 48$$
$$x = 16$$

36.
$$(2x+1)^2 = (3x-1)^2$$
$$4x^2 + 4x + 1 = 9x^2 - 6x + 1$$
$$5x^2 - 10x = 0$$
$$x(x-2) = 0$$
$$x = 0, 2$$

38.
$$(4x+1)^4 = (5x-10)^4$$
$$(4x+1)^2 = (5x-10)^2$$
$$4x + 1 = \pm 5(x-2)$$
$$4x + 1 = 5(x-2),\ x = 11$$
$$4x + 1 = -5(x-2),\ x = 1$$

40.
$$10xe^x - 5e^x = 0$$
$$e^x(10x - 5) = 0$$
$$10x - 5 = 0 \quad (\text{since } e^x \neq 0)$$
$$x = \frac{1}{2}$$

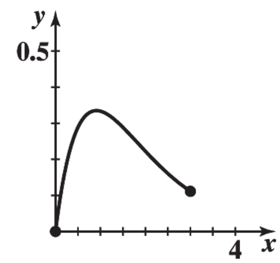
42.
$$x^2e^{-x} - 9e^{-x} = 0$$
$$e^{-x}(x^2 - 9) = 0$$
$$(x^2 - 9) = 0 \quad (\text{since } e^{-x} \neq 0)$$
$$x = -3, 3$$

44. $e^{4x} + e > 0$ for all x;
$e^{4x} + e = 0$ has no solutions.

46.
$$e^{3x-1} - e = 0$$
$$e^{3x-1} = e^1$$
$$3x - 1 = 1$$
$$x = 2/3$$

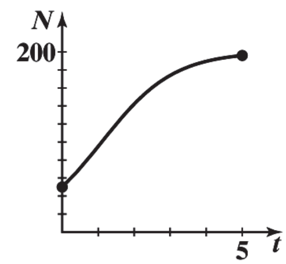
48. $m(x) = x(3^{-x}); [0, 3]$

x	$m(x)$
0	0
1	$\frac{1}{3}$
2	$\frac{2}{9}$
3	$\frac{1}{9}$



50. $N = \dfrac{200}{1+3e^{-t}}$; $[0, 5]$

x	N
0	50
1	≈ 95.07
2	≈ 142.25
3	≈ 174.01
4	≈ 184.58
5	≈ 196.04



52. $A = Pe^{rt}$

$A = (24{,}000)e^{(0.0435)(7)}$

$A = (24{,}000)e^{0.3045}$

$A = (24{,}000)(1.35594686)$

$A = \$32{,}542.72$

54. (A) $A = P(1+\frac{r}{m})^{mt}$

$A = 4000(1+\frac{0.06}{52})^{(52)(0.5)}$

$A = 4000(1.0011538462)^{26}$

$A = 4000(1.030436713)$

$A = \$4121.75$

(B) $A = P(1+\frac{r}{m})^{mt}$

$A = 4000(1+\frac{0.06}{52})^{(52)(10)}$

$A = 4000(1.0011538462)^{520}$

$A = 4000(1.821488661)$

$A = \$7285.95$

56. $A = P(1+\frac{r}{m})^{mt}$

$40{,}000 = P(1+\frac{0.055}{365})^{(365)(17)}$

$40{,}000 = P(1.0001506849)^{6205}$

$40{,}000 = P(2.547034043)$

$P = \$15{,}705$

58. (A) $A = P(1+\frac{r}{m})^{mt}$

$A = 10{,}000(1+\frac{0.0135}{4})^{(4)(5)}$

$A = 10{,}000(1.003375)^{20}$

$A = 10{,}000(1.069709)$

$A = \$10{,}697.09$

(B) $A = P(1+\frac{r}{m})^{mt}$

$A = 10{,}000(1+\frac{0.0130}{12})^{(12)(5)}$

$A = 10{,}000(1.00108333)^{60}$

$A = 10{,}000(1.067121479)$

$A = \$10{,}671.21$

(C) $A = P(1+\frac{r}{m})^{mt}$

$A = 10,000(1+\frac{0.0125}{365})^{(365)(5)}$

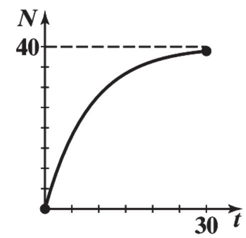
$A = 10,000(1.000034245)^{1825}$

$A = 10,000(1.06449332)$

$A = \$10,644.93$

60. $N = 40(1-e^{-0.12t})$; $[0, 30]$

x	N
0	0
10	≈ 27.95
20	≈ 36.37
30	≈ 38.91



The maximum number of boards an average employee can be expected to produce in 1 day is 40.

62. The exponential regression model

```
ExpReg
 y=a*b^x
 a=3.996184237
 b=1.523286295
```

(B) $y(10) = 268.8$ exabytes per month

64. (A) $I(50) = I_o e^{-0.00942(50)} \approx 62\ \%$

(B) $I(100) = I_o e^{-0.00942(100)} \approx 39\ \%$

66. (A) $P = 204e^{0.0077t}$.

(B) Population in 2030:
$P(15) = 204e^{0.0077(15)} \approx 229$ million.

68. (A) $P = 7.4e^{0.0113t}$

(B) Population in 2025: $P(10) = 7.4e^{0.0113(10)} \approx 8.29$ billion

Population in 2033: $P(18) = 7.4e^{0.0113(18)} \approx 9.07$ billion

EXERCISE 2-6

2. $\log_2 32 = 5 \Rightarrow 32 = 2^5$

4. $\log_e 1 = 0 \Rightarrow e^0 = 1$

6. $\log_9 27 = \frac{3}{2} \Rightarrow 27 = 9^{3/2}$

8. $36 = 6^2 \Rightarrow \log_6 36 = 2$

10. $9 = 27^{2/3} \Rightarrow \log_{27} 9 = \frac{2}{3}$

12. $M = b^x \Rightarrow \log_b M = x$

14. $\log_{10} \frac{1}{1000} = \log_{10} 10^{-3} = -3$

16. $\log_{10} 10,000 = \log_{10} 10^4 = 4$

18. $\log_2 \frac{1}{64} = \log_2 2^{-6} = -6$

20. $\ln(-1)$ is not defined.

22. $\ln(e^{-1}) = -1$

24. $\log_b FG = \log_b F + \log_b G$

26. $\log_b w^{15} = 15\log_b w$

28. $\dfrac{\log_3 P}{\log_3 R} = \log_R P$

30. $\log_{10} x = 1$

$x = 10^1 = 10$

32. $\log_b \dfrac{1}{25} = 2$

$b^2 = \dfrac{1}{25}$

$b = \dfrac{1}{5}$

34. $\log_{49} 7 = y$

$49^y = 7$

$y = 1/2$

36. $\log_b 10{,}000 = 2$

$b^2 = 10{,}000$

$b = 100$

38. $\log_8 x = \dfrac{5}{3}$

$x = 8^{5/3} = \left(8^{1/3}\right)^5 = 2^5 = 32$

40. False; an example of a polynomial function of odd degree that is not one–to–one is $f(x) = x^3 - x$. $f(-1) = f(0) = f(1) = 0$.

42. False; the graph of every function (not necessarily one–to–one) intersects each vertical line at most once.

For example, $f(x) = \dfrac{1}{x-1}$ *is a one-to-one function which does not intersect the vertical line* $x = 1$.

44. False; $x = -1$ is in the domain of *f*, but cannot be in the range of *g*.

46. True; since *g* is the inverse of *f*, then (*a*, *b*) is on the graph of *f* if and only if (*b*, *a*) is on the graph of *g*. Therefore, *f* is also the inverse of *g*.

48. $\log_b x = \dfrac{2}{3}\log_b 27 + 2\log_b 2 - \log_b 3$

$\log_b x = \log_b 27^{2/3} + \log_b 2^2 - \log_b 3$

$\log_b x = \log_b 9 + \log_b 4 - \log_b 3$

$\log_b x = \log_b \dfrac{(9)(4)}{3}$

$\log_b x = \log_b 12$

$x = 12$

50. $\log_b x = 3\log_b 2 + \dfrac{1}{2}\log_b 25 - \log_b 20$

$\log_b x = \log_b 2^3 + \log_b 25^{1/2} - \log_b 20$

$\log_b x = \log_b 8 + \log_b 5 - \log_b 20$

$\log_b x = \log_b \dfrac{(8)(5)}{20}$

$\log_b x = \log_b 2$

$x = 2$

52. $\log_b(x+2)+\log_b x=\log_b 24$

$$\log_b(x+2)x=\log_b 24$$
$$\log_b(x^2+2x)=\log_b 24$$
$$x^2+2x=24$$
$$x^2+2x-24=0$$
$$(x+6)(x-4)=0$$
$$x=-6,4$$

Since the domain of $\log_b$ is $(0,\infty)$, omit the negative solution. Therefore, the solution is $x=4$.

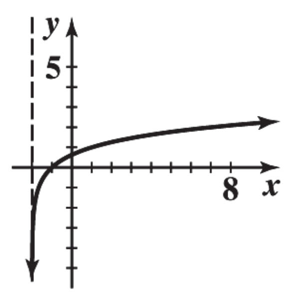
54. $\log_{10}(x+6)-\log_{10}(x-3)=1$

$$\log_{10}\frac{x+6}{x-3}=1$$
$$10^1=\frac{x+6}{x-3}$$
$$10(x-3)=x+6$$
$$10x-30=x+6$$
$$x=4$$

56. $y=\log_3(x+2)$

$$3^y=x+2$$
$$3^y-2=x$$

x	y
$-\frac{53}{27}$	-3
$-\frac{17}{9}$	-2
$-\frac{5}{3}$	-1
-1	0
1	1
7	2
25	3



58. The graph of $y=\log_3(x+2)$ is the graph of $y=\log_3 x$ shifted to the left 2 units.

60. The domain of logarithmic function is defined for positive values only. Therefore, the domain of the function is $x-1>0$ or $x>1$. The range of a logarithmic function is all real numbers. In interval notation the domain is $(1,\infty)$ and the range is $(-\infty,\infty)$.

62. (A) $\log 72.604=1.86096$ (B) $\log 0.033041=-1.48095$

(C) $\ln 40,257=10.60304$ (D) $\ln 0.0059263=-5.12836$

64. (A) $\log x=2.0832$

$$x=\log^{-1}(2.0832)$$
$$x=121.1156$$

(B) $\log x=-1.1577$

$$x=\log^{-1}(-1.1577)$$
$$x=0.0696$$

(C) $\ln x = 3.1336$
$x = \ln^{-1}(3.1336)$
$x = 22.9565$

(D) $\ln x = -4.3281$
$x = \ln^{-1}(-4.3281)$
$x = 0.0132$

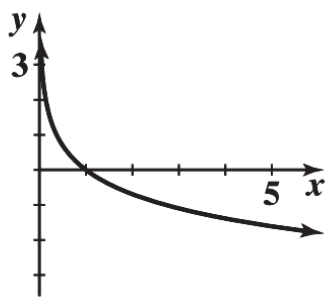
66. $10^x = 153$
$\log 10^x = \log 153$
$x = 2.1847$

68. $e^x = 0.3059$
$\ln e^x = \ln 0.3059$
$x = -1.1845$

70. $1.02^{4t} = 2$
$\ln 1.02^{4t} = \ln 2$
$4t \ln 1.02 = \ln 2$
$t = \dfrac{\ln 2}{4 \ln 1.02}$
$t = 8.7507$

72. $y = -\ln x;\ \ x > 0$

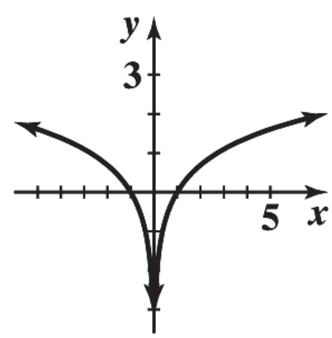
x	y
0.5	≈ 0.69
1	0
2	≈ -0.69
4	≈ -1.39
5	≈ -1.61



Based on the graph above, the function is decreasing on the interval $(0, \infty)$.

74. $y = \ln|x|$

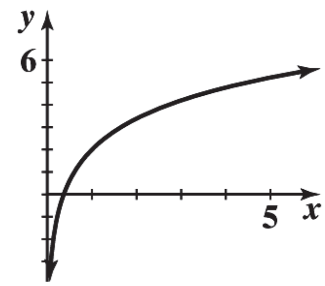
x	y
-5	≈ 1.61
-2	≈ 0.69
1	0
2	≈ 0.69
5	≈ 1.61



Based on the graph above, the function is decreasing on the interval $(-\infty, 0)$ and increasing on the interval $(0, \infty)$.

76. $y = 2\ln x + 2$

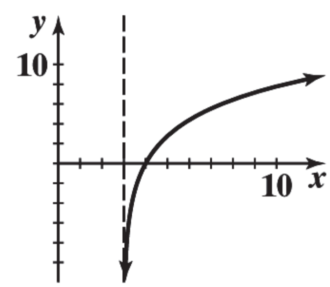
x	y
0.5	≈ 0.61
1	2
2	≈ 3.39
4	≈ 4.77
5	≈ 5.22



Based on the graph above, the function is increasing on the interval $(0, \infty)$.

78. $y = 4\ln(x-3)$

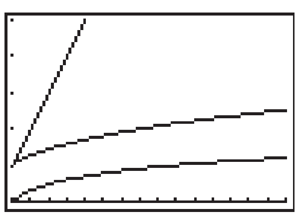
x	y
4	0
6	≈ 4.39
8	≈ 6.44
10	≈ 7.78
12	≈ 8.79



Based on the graph above, the function is increasing on the interval $(3, \infty)$.

80. It is not possible to find a power of 1 that is an arbitrarily selected real number, because 1 raised to any power is 1.

82.



A function f is "smaller than" a function g on an interval $[a, b]$ if $f(x) < g(x)$ for $a \le x \le b$. Based on the graph above, $\log x < \sqrt[3]{x} < x$ for $1 < x \le 16$.

84. Use the compound interest formula: $A = P(1+r)^t$. The problem is asking for the original amount to double, therefore $A = 2P$.

$$2P = P(1+0.0958)^t$$
$$2 = (1.0958)^t$$
$$\ln 2 = \ln(1.0958)^t$$
$$\ln 2 = t\ln(1.0958)$$
$$\frac{\ln 2}{\ln 1.0958} = t$$
$$7.58 \approx t$$

It will take approximately 8 years for the original amount to double.

86. Use the compound interest formula: $A = P(1+\frac{r}{m})^{mt}$.

(A)
$$\begin{aligned} 7500 &= 5000(1+\tfrac{0.08}{2})^{2t} \\ 1.5 &= (1.04)^{2t} \\ \ln 1.5 &= \ln(1.04)^{2t} \\ \ln 1.5 &= 2t\ln(1.04) \\ \frac{\ln 1.5}{2\ln 1.04} &= t \\ 5.17 &\approx t \end{aligned}$$

It will take approximately 5.17 years for \$5000 to grow to \$7500 if compounded semiannually.

(B)
$$\begin{aligned} 7500 &= 5000(1+\tfrac{0.08}{12})^{12t} \\ 1.5 &= (1.0066667)^{12t} \\ \ln 1.5 &= \ln(1.0066667)^{12t} \\ \ln 1.5 &= 12t\ln(1.0066667) \\ \frac{\ln 1.5}{12\ln 1.0066667} &= t \\ 5.09 &\approx t \end{aligned}$$

It will take approximately 5.09 years for \$5000 to grow to \$7500 if compounded monthly.

88. Use the compound interest formula: $A = Pe^{rt}$.

$$\begin{aligned} 41{,}000 &= 17{,}000e^{0.0295t} \\ \tfrac{41}{17} &= e^{0.0295t} \\ \ln\tfrac{41}{17} &= \ln e^{0.0295t} \\ \ln\tfrac{41}{17} &= 0.0295t \\ \frac{\ln\frac{41}{17}}{0.0295} &= t \\ 29.84 &\approx t \end{aligned}$$

It will take approximately 29.84 years for \$17,000 to grow to \$41,000 if compounded continuously.

90. Equilibrium occurs when supply and demand are equal. The models from Problem 85 have the demand and supply functions defined by $y = 256.4659159 - 24.03812068\ln x$ and $y = -127.8085281 + 20.01315349\ln x$, respectively. Set both equations equal to each other to yield:

$$\begin{aligned} 256.4659159 - 24.03812068\ln x &= -127.8085281 + 20.01315349\ln x \\ 384.274444 &= 44.05127417\ln x \\ \frac{384.274444}{44.05127417} &= \ln x \\ e^{384.274444/44.05127417} &= e^{\ln x} \\ 6145 &\approx x \end{aligned}$$

Substitute the value above into either equation.

$$y = 256.4659159 - 24.03812068 \ln x$$
$$y = 256.4659159 - 24.03812068 \ln(6145)$$
$$y = 256.4659159 - 24.03812068(8.723394022)$$
$$y = 46.77$$

Therefore, equilibrium occurs when 6145 units are produced and sold at a price of \$46.77.

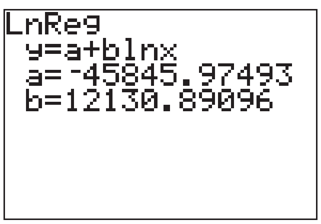
92. (A) $N = 10\log\dfrac{I}{I_0} = 10\log\dfrac{10^{-13}}{10^{-16}} = 10\log 10^3 = 30$

(B) $N = 10\log\dfrac{I}{I_0} = 10\log\dfrac{3.16\times 10^{-10}}{10^{-16}} = 10\log 3.16\times 10^6 \approx 65$

(C) $N = 10\log\dfrac{I}{I_0} = 10\log\dfrac{10^{-8}}{10^{-16}} = 10\log 10^8 = 80$

(D) $N = 10\log\dfrac{I}{I_0} = 10\log\dfrac{10^{-1}}{10^{-16}} = 10\log 10^{15} = 150$

94.



2024: $t = 124$; $y(124) \approx 12{,}628$. Therefore, according to the model, the total production in the year 2024 will be approximately 12,628 million bushels.

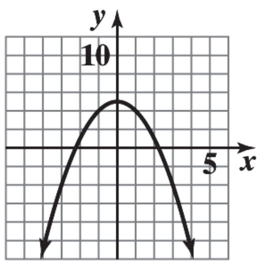
96.

$$A = A_0 e^{-0.000124t}$$
$$0.1A_0 = A_0 e^{-0.000124t}$$
$$0.1 = e^{-0.000124t}$$
$$\ln 0.1 = \ln e^{-0.000124t}$$
$$\ln 0.1 = -0.000124t$$
$$18{,}569 \approx t$$

If 10% of the original amount is still remaining, the skull would be approximately 18,569 years old.

CHAPTER 2 REVIEW

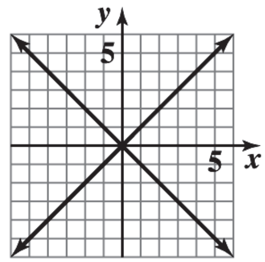
1.



(2-1)

2. $x^2 = y^2$:

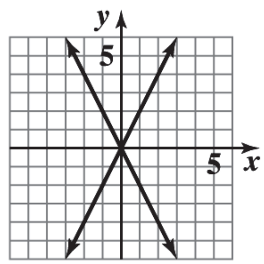
x	−3	−2	−1	0	1	2	3
y	±3	±2	±1	0	±1	±2	±3



(2-1)

3. $y^2 = 4x^2$:

x	−3	−2	−1	0	1	2	3
y	±6	±4	±2	0	±2	±4	±6



(2-1)

4. (A) Not a function; fails vertical line test

(B) A function

(C) A function

(D) Not a function; fails vertical line test (2-1)

5. $f(x) = 2x - 1,\ g(x) = x^2 - 2x$

(A) $f(-2) + g(-1) = 2(-2) - 1 + (-1)^2 - 2(-1) = -2$

(B) $f(0) \cdot g(4) = (2 \cdot 0 - 1)(4^2 - 2 \cdot 4) = -8$

(C) $\dfrac{g(2)}{f(3)} = \dfrac{2^2 - 2 \cdot 2}{2 \cdot 3 - 1} = 0$

(D) $\dfrac{f(3)}{g(2)}$ not defined because $g(2) = 0$ (2-1)

6. $u = e^v$
$v = \ln u$ (2-6)

7. $x = 10^y$
$y = \log x$ (2-6)

8. $\ln M = N$
$M = e^N$ (2-6)

9. $\log u = v$
$u = 10^v$ (2-6)

10. $\log_3 x = 2$
$x = 3^2 = 9$ (2-6)

11. $\log_x 36 = 2$
$x^2 = 36$
$x = 6$ (2-6)

12. $\log_2 16 = x$
$2^x = 16$
$x = 4$ (2-6)

13. $10^x = 143.7$
$x = \log 143.7$
$x \approx 2.157$ (2-6)

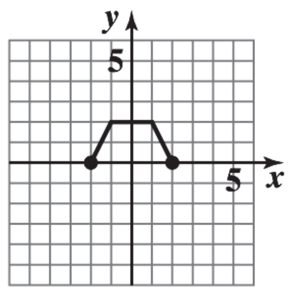
14. $e^x = 503{,}000$
$x = \ln 503{,}000 \approx 13.128$ (2-6)

15. $\log x = 3.105$
$x = 10^{3.105} \approx 1273.503$ (2-6)

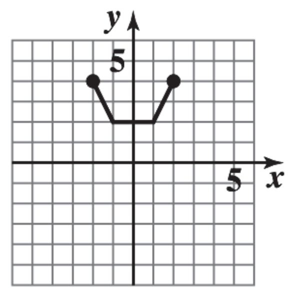
16. $\ln x = -1.147$
$x = e^{-1.147} \approx 0.318$ (2-6)

17. (A) $y = 4$ (B) $x = 0$ (C) $y = 1$ (D) $x = -1$ or 1
(E) $y = -2$ (F) $x = -5$ or 5 (2-1)

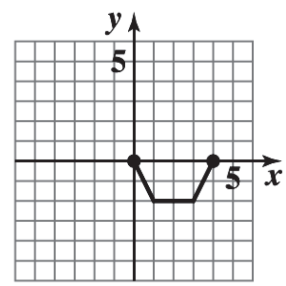
18. (A)



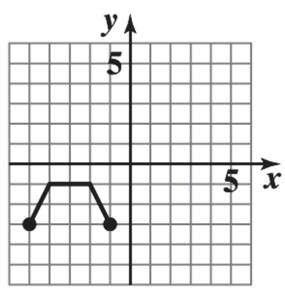
(B)



(C)



(D)



(2-2)

19. $f(x) = -x^2 + 4x = -(x^2 - 4x)$
$= -(x^2 - 4x + 4) + 4$
$= -(x - 2)^2 + 4$ (vertex form)

The graph of $f(x)$ is the graph of $y = x^2$ reflected in the x axis, then shifted right 2 units and up 4 units.
(2-3)

20. (A) g (B) m (C) n (D) f (2-2, 2-3)

21. $y = f(x) = (x + 2)^2 - 4$
(A) x intercepts: $(x + 2)^2 - 4 = 0$; y intercept: 0
$(x + 2)^2 = 4$
$x + 2 = -2$ or 2
$x = -4, 0$

(B) Vertex: $(-2, -4)$ (C) Minimum: -4 (D) Range: $y \geq -4$ or $[-4, \infty)$ (2-3)

22. $y = 4 - x + 3x^2 = 3x^2 - x + 4$; quadratic function. (2-3)

23. $y = \frac{1+5x}{6} = \frac{5}{6}x + \frac{1}{6}$; linear function. (2-1, 2-3)

24. $y = \frac{7-4x}{2x} = \frac{7}{2x} - 2$; none of these. (2-1), (2-3)

25. $y = 8x + 2(10 - 4x) = 8x + 20 - 8x = 20$; constant function (2-1)

26. $\log(x + 5) = \log(2x - 3)$
$x + 5 = 2x - 3$
$-x = -8$
$x = 8$ (2-6)

27. $2 \ln(x - 1) = \ln(x^2 - 5)$
$\ln(x - 1)^2 = \ln(x^2 - 5)$
$(x - 1)^2 = x^2 - 5$
$x^2 - 2x + 1 = x^2 - 5$
$-2x = -6$
$x = 3$ (2-6)

28. $9^{x-1} = 3^{1+x}$
$(3^2)^{x-1} = 3^{1+x}$
$3^{2x-2} = 3^{1+x}$
$2x - 2 = 1 + x$
$x = 3$ (2-5)

29. $e^{2x} = e^{x^2-3}$
$2x = x^2 - 3$
$x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$
$x = 3, \ -1$ (2-5)

30. $2x^2e^x = 3xe^x$
$2x^2 = 3x$
$2x^2 - 3x = 0$
$x(2x - 3) = 0$
$x = 0, \ 3/2$ (2-5)

31. $\log_{1/3} 9 = x$
$\left(\frac{1}{3}\right)^x = 9$
$\frac{1}{3^x} = 9$
$3^x = \frac{1}{9}$
$x = -2$ (2-6)

32. $\log_x 8 = -3$
$x^{-3} = 8$
$\frac{1}{x^3} = 8$
$x^3 = \frac{1}{8}$
$x = \frac{1}{2}$ (2-6)

33. $\log_9 x = \frac{3}{2}$
$9^{3/2} = x$
$x = 27$ (2-6)

34. $x = 3(e^{1.49}) \approx 13.3113$ (2-5)

35. $x = 230(10^{-0.161}) \approx 158.7552$ (2-5)

36. $\log x = -2.0144$
$x \approx 10^{-2.0144} \approx 0.0097$ (2-6)

37. $\ln x = 0.3618$
$x = e^{0.3618} \approx 1.4359$ (2-6)

38. $35 = 7(3^x)$

$3^x = 5$

$\ln 3^x = \ln 5$

$x \ln 3 = \ln 5$

$x = \dfrac{\ln 5}{\ln 3} \approx 1.4650$ (2-6)

39. $0.01 = e^{-0.05x}$

$\ln(0.01) = \ln(e^{-0.05x}) = -0.05x$

Thus, $x = \dfrac{\ln(0.01)}{-0.05} \approx 92.1034$

(2-6)

40. $8{,}000 = 4{,}000(1.08)^x$

$(1.08)^x = 2$

$\ln(1.08)^x = \ln 2$

$x \ln 1.08 = \ln 2$

$x = \dfrac{\ln 2}{\ln 1.08} \approx 9.0065$ (2-6)

41. $5^{2x-3} = 7.08$

$\ln(5^{2x-3}) = \ln 7.08$

$(2x - 3)\ln 5 = \ln 7.08$

$2x \ln 5 - 3 \ln 5 = \ln 7.08$

$x = \dfrac{\ln 7.08 + 3\ln 5}{2\ln 5}$

$x \approx 2.1081$ (2-6)

42. (A) $x^2 - x - 6 = 0$ at $x = -2, 3$
Domain: all real numbers except $x = -2, 3$

(B) $5 - x > 0$ for $x < 5$
Domain: $x < 5$ or $(-\infty, 5)$ (2-1)

43. $f(x) = 4x^2 + 4x - 3 = 4(x^2 + x) - 3$

$= 4\left(x^2 + x + \frac{1}{4}\right) - 3 - 1$

$= 4\left(x + \frac{1}{2}\right)^2 - 4$ (vertex form)

Intercepts:

y intercept: $f(0) = 4(0)^2 + 4(0) - 3 = -3$

x intercepts: $f(x) = 0$

$4\left(x + \frac{1}{2}\right)^2 - 4 = 0$

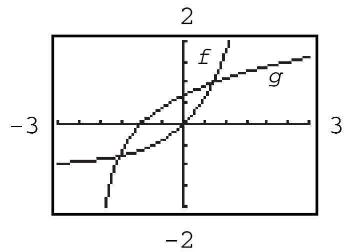
$\left(x + \frac{1}{2}\right)^2 = 1$

$x + \frac{1}{2} = \pm 1$

$x = -\frac{1}{2} \pm 1 = -\frac{3}{2}, \frac{1}{2}$

Vertex: $\left(-\frac{1}{2}, -4\right)$; minimum: -4; range: $y \geq -4$ or $[-4, \infty)$ (2-3)

44. $f(x) = e^x - 1,\ g(x) = \ln(x + 2)$

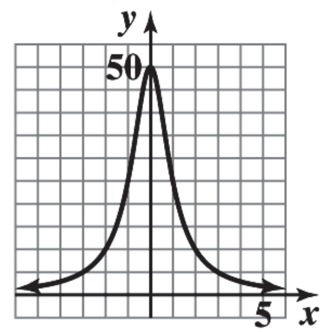


Points of intersection:
(–1.54, –0.79), (0.69, 0.99)

(2-5, 2-6)

45. $f(x) = \dfrac{50}{x^2 + 1}$:

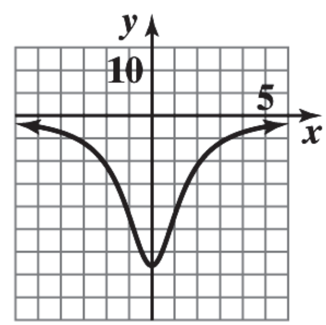
x	−3	−2	−1	0	1	2	3
$f(x)$	5	10	25	50	25	10	5



(2-1)

46. $f(x) = \dfrac{-66}{2 + x^2}$:

x	−3	−2	−1	0	1	2	3
$f(x)$	−6	−11	−22	−66	−22	−11	−6



(2-1)

For Problems 47–50, $f(x) = 5x + 1$.

47. $f(f(0)) = f(5(0) + 1) = f(1) = 5(1) + 1 = 6$ (2-1)

48. $f(f(-1)) = f(5(-1) + 1) = f(-4) = 5(-4) + 1 = -19$ (2-1)

49. $f(2x - 1) = 5(2x - 1) + 1 = 10x - 4$ (2-1)

50. $f(4 - x) = 5(4 - x) + 1 = 20 - 5x + 1 = 21 - 5x$ (2-1)

51. $f(x) = 3 - 2x$

(A) $f(2) = 3 - 2(2) = 3 - 4 = -1$

(B) $f(2 + h) = 3 - 2(2 + h) = 3 - 4 - 2h = -1 - 2h$

(C) $f(2 + h) - f(2) = -1 - 2h - (-1) = -2h$

(D) $\dfrac{f(2+h) - f(2)}{h} = -\dfrac{2h}{h} = -2$ (2-1)

52. $f(x) = x^2 - 3x + 1$

(A) $f(a) = a^2 - 3a + 1$

(B) $f(a+h) = (a+h)^2 - 3(a+h) + 1 = a^2 + 2ah + h^2 - 3a - 3h + 1$

(C) $f(a+h) - f(a) = a^2 + 2ah + h^2 - 3a - 3h + 1 - (a^2 - 3a + 1)$
$= 2ah + h^2 - 3h$

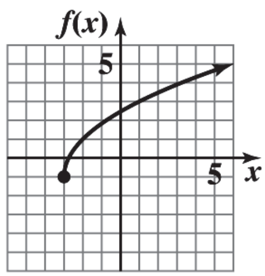
(D) $\dfrac{f(a+h)-f(a)}{h} = \dfrac{2ah+h^2-3h}{h} = \dfrac{h(2a+h-3)}{h} = 2a + h - 3$ (2-1)

53. The graph of m is the graph of $y = |x|$ reflected in the x axis and shifted 4 units to the right. (2-2)

54. The graph of g is the graph of $y = x^3$ vertically contracted by a factor of 0.3 and shifted up 3 units. (2-2)

55. The graph of $y = x^2$ is vertically expanded by a factor of 2, reflected in the x axis and shifted to the left 3 units.
Equation: $y = -2(x+3)^2$ (2-2)

56. Equation: $f(x) = 2\sqrt{x+3} - 1$



(2-2)

57. $f(x) = \dfrac{n(x)}{d(x)} = \dfrac{5x+4}{x^2-3x+1}$. Since degree $n(x) = 1 < 2 =$ degree $d(x)$, $y = 0$ is the horizontal asymptote. (2-4)

58. $f(x) = \dfrac{n(x)}{d(x)} = \dfrac{3x^2+2x-1}{4x^2-5x+3}$. Since degree $n(x) = 2 =$ degree $d(x)$, $y = \dfrac{3}{4}$ is the horizontal asymptote. (2-4)

59. $f(x) = \dfrac{n(x)}{d(x)} = \dfrac{x^2+4}{100x+1}$. Since degree $n(x) = 2 > 1 =$ degree $d(x)$, there is no horizontal asymptote. (2-4)

60. $f(x) = \dfrac{n(x)}{d(x)} = \dfrac{x^2+100}{x^2-100} = \dfrac{x^2+100}{(x-10)(x+10)}$. Since $n(x) = x^2 + 100$ has no real zeros and $d(10) = d(-10) = 0$, $x = 10$ and $x = -10$ are the vertical asymptotes of the graph of f. (2-4)

61. $f(x) = \dfrac{n(x)}{d(x)} = \dfrac{x^2+3x}{x^2+2x} = \dfrac{x(x+3)}{x(x+2)} = \dfrac{x+3}{x+2}$, $x \neq 0$. $x = -2$ is a vertical asymptote of the graph of f. (2-4)

62. True; $p(x) = \dfrac{p(x)}{1}$ is a rational function for every polynomial p. (2-4)

63. False; $f(x) = \frac{1}{x} = x^{-1}$ is not a polynomial function. (2-4)

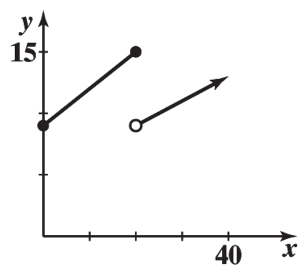
64. False; $f(x) = \frac{1}{x^2+1}$ has no vertical asymptotes. (2-4)

65. True: let $f(x) = b^x$, $(b > 0, b \neq 1)$, then the positive x-axis is a horizontal asymptote if $0 < b < 1$, and the negative x-axis is a horizontal asymptote if $b > 1$. (2-5)

66. True: let $f(x) = \log_b x$ $(b > 0, b \neq 1)$. If $0 < b < 1$, then the positive y-axis is a vertical asymptote; if $b > 1$, then the negative y-axis is a vertical asymptote. (2-6)

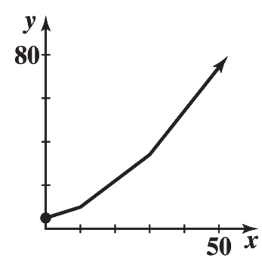
67. True; $f(x) = \frac{x}{x-1}$ has vertical asymptote $x = 1$ and horizontal asymptote $y = 1$. (2-4)

68.



(2-2)

69.



(2-2)

70. $y = -(x-4)^2 + 3$ (2-2, 2-3)

71. $f(x) = -0.4x^2 + 3.2x + 1.2 = -0.4(x^2 - 8x + 16) + 7.6$

$= -0.4(x-4)^2 + 7.6$

(A) y intercept: 1.2

x intercepts: $-0.4(x-4)^2 + 7.6 = 0$

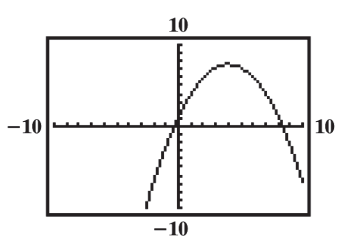
$(x-4)^2 = 19$

$x = 4 + \sqrt{19} \approx 8.4,\ 4 - \sqrt{19} \approx -0.4$

(B) Vertex: (4.0, 7.6) (C) Maximum: 7.6 (D) Range: $y \leq 7.6$ or $(-\infty, 7.6]$

(2-3)

72.



(A) y intercept: 1.2

x intercepts: –0.4, 8.4

(B) Vertex: (4.0, 7.6)

(C) Maximum: 7.6

(D) Range: $y \leq 7.6$ or $(-\infty, 7.6]$ (2-3)

73. $\log 10^{\pi} = \pi \log 10 = \pi$

$10^{\log\sqrt{2}} = y$ is equivalent to $\log y = \log\sqrt{2}$

which implies $y = \sqrt{2}$

Similarly, $\ln e^{\pi} = \pi \ln e = \pi$ (Section 2-5, 4.b & g) and $e^{\ln\sqrt{2}} = y$ implies $\ln y = \ln\sqrt{2}$ and $y = \sqrt{2}$. (2-6)

74. $\log x - \log 3 = \log 4 - \log(x+4)$

$$\log\frac{x}{3} = \log\frac{4}{x+4}$$
$$\frac{x}{3} = \frac{4}{x+4}$$
$$x(x+4) = 12$$
$$x^2 + 4x - 12 = 0$$
$$(x+6)(x-2) = 0$$
$$x = -6,\ 2$$

Since log(−6) is not defined, −6 is not a solution. Therefore, the solution is $x = 2$. (2-6)

75. $\ln(2x-2) - \ln(x-1) = \ln x$

$$\ln\left(\frac{2x-2}{x-1}\right) = \ln x$$
$$\ln\left[\frac{2(x-1)}{x-1}\right] = \ln x$$
$$\ln 2 = \ln x$$
$$x = 2 \qquad (2\text{-}6)$$

76. $\ln(x+3) - \ln x = 2\ln 2$

$$\ln\left(\frac{x+3}{x}\right) = \ln(2^2)$$
$$\frac{x+3}{x} = 4$$
$$x + 3 = 4x$$
$$3x = 3$$
$$x = 1 \qquad (2\text{-}6)$$

77. $\log 3x^2 = 2 + \log 9x$

$$\log 3x^2 - \log 9x = 2$$
$$\log\left(\frac{3x^2}{9x}\right) = 2$$
$$\log\left(\frac{x}{3}\right) = 2$$
$$\frac{x}{3} = 10^2 = 100$$
$$x = 300 \qquad (2\text{-}6)$$

78. $\ln y = -5t + \ln c$

$$\ln y - \ln c = -5t$$
$$\ln\frac{y}{c} = -5t$$
$$\frac{y}{c} = e^{-5t}$$
$$y = ce^{-5t} \qquad (2\text{-}6)$$

79. Let x be *any* positive real number and suppose $\log_1 x = y$. Then $1^y = x$.

But, $1^y = 1$, so $x = 1$, i.e., $x = 1$ for all positive real numbers x.
This is clearly impossible. (2-6)

80. The graph of $y = \sqrt[3]{x}$ is vertically expanded by a factor of 2, reflected in the x axis, shifted 1 unit to the left and 1 unit down.
Equation: $y = -2\sqrt[3]{x+1} - 1$ (2-2)

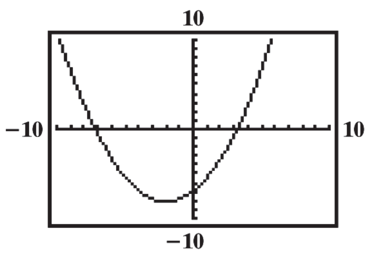
81. $G(x) = 0.3x^2 + 1.2x - 6.9 = 0.3(x^2 + 4x + 4) - 8.1$
$= 0.3(x + 2)^2 - 8.1$

(A) y intercept: -6.9
x intercepts: $0.3(x + 2)^2 - 8.1 = 0$
$(x + 2)^2 = 27$
$x = -2 + \sqrt{27} \approx 3.2, -2 - \sqrt{27} \approx -7.2$

(B) Vertex: $(-2, -8.1)$ (C) Minimum: -8.1 (D) Range: $y \geq -8.1$ or $[-8.1, \infty)$

(2-3)

82.



(A) y intercept: -6.9
x intercept: -7.2, 3.2

(B) Vertex: $(-2, -8.1)$

(C) Minimum: -8.1

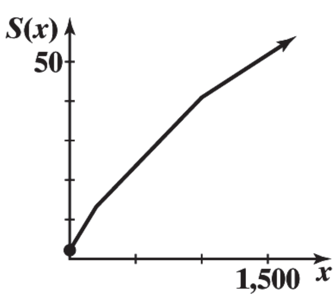
(D) Range: $y \geq -8.1$ or $[-8.1, \infty)$ (2-3)

83. (A) $S(x) = 3$ if $0 \leq x \leq 20$;
$S(x) = 3 + 0.057(x - 20)$
$= 0.057x + 1.86$ if $20 < x \leq 200$;
$S(200) = 13.26$
$S(x) = 13.26 + 0.0346(x - 200)$
$= 0.0346x + 6.34$ if $200 < x \leq 1000$;
$S(1000) = 40.94$

$S(x) = 40.94 + 0.0217(x - 1000)$
$= 0.0217x + 19.24$ if $x > 1000$

$$\text{Therefore, } S(x) = \begin{cases} 3 & \text{if } 0 \leq x \leq 20 \\ 0.057x + 1.86 & \text{if } 20 < x \leq 200 \\ 0.0346x + 6.34 & \text{if } 200 < x \leq 1000 \\ 0.0217x + 19.24 & \text{if } x > 1000 \end{cases}$$

(B)



(2-2)

84. $A = P\left(1 + \frac{r}{m}\right)^{mt}$; $P = 5{,}000$, $r = 0.0125$, $m = 4$, $t = 5$.

$$A = 5000\left(1 + \frac{0.0125}{4}\right)^{4(5)} = 5000\left(1 + \frac{0.0125}{4}\right)^{20} \approx 5321.95$$

After 5 years, the CD will be worth \$5,321.95 (2-5)

85. $A = P\left(1+\frac{r}{m}\right)^{mt}$; $P = 5{,}000,\ r = 0.0105,\ m = 365,\ t = 5$

$$A = 5000\left(1+\frac{0.0105}{365}\right)^{365(5)} = 5000\left(1+\frac{0.0105}{365}\right)^{1825} \approx 5269.51$$

After 5 years, the CD will be worth \$5,269.51. (2-5)

86. $A = P\left(1+\frac{r}{m}\right)^{mt}$, $r = 0.0659$, $m = 12$

Solve $P\left(1+\frac{0.0659}{12}\right)^{12t} = 3P$ or $(1.005492)^{12t} = 3$

for t:

$$12t\ln(1.005492) = \ln 3$$

$$t = \frac{\ln 3}{12\ln(1.005492)} \approx 16.7 \text{ year.}$$ (2-5)

87. $A = Pe^{rt}$, $r = 0.0739$. Solve $2P = Pe^{0.0739t}$ for t.

$$2P = Pe^{0.0739t}$$

$$e^{0.0739t} = 2$$

$$0.0739t = \ln 2$$

$$t = \frac{\ln 2}{0.0739} \approx 9.38 \text{ years.}$$ (2-5)

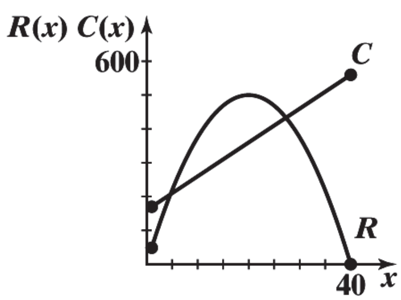
88. $p(x) = 50 - 1.25x$ Price-demand function

$C(x) = 160 + 10x$ Cost function

$R(x) = xp(x)$

$= x(50 - 1.25x)$ Revenue function

(A)



(B) $R = C$

$$x(50 - 1.25x) = 160 + 10x$$

$$-1.25x^2 + 50x = 160 + 10x$$

$$-1.25x^2 + 40x = 160$$

$$-1.25(x^2 - 32x + 256) = 160 - 320$$

$$-1.25(x - 16)^2 = -160$$

$$(x - 16)^2 = 128$$

$$x = 16 + \sqrt{128} \approx 27.314,$$

$$16 - \sqrt{128} \approx 4.686$$

$R = C$ at $x = 4.686$ thousand units (4,686 units) and

$x = 27.314$ thousand units (27,314 units)

$R < C$ for $1 \le x < 4.686$ or $27.314 < x \le 40$

$R > C$ for $4.686 < x < 27.314$

(C) Max Rev: $50x - 1.25x^2 = R$

$-1.25(x^2 - 40x + 400) + 500 = R$

$-1.25(x - 20)^2 + 500 = R$

Vertex at (20, 500)

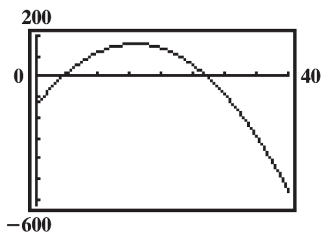
Max. Rev. = 500 thousand ($500,000) occurs when <u>output</u> is 20 thousand (20,000 units)

<u>Wholesale price</u> at this output: $p(x) = 50 - 1.25x$

$p(20) = 50 - 1.25(20) = \$25$ (2-3)

89. (A) $P(x) = R(x) - C(x) = x(50 - 1.25x) - (160 + 10x)$

$= -1.25x^2 + 40x - 160$



(B) $P = 0$ for $x = 4.686$ thousand units (4,686 units) and $x = 27.314$ thousand units (27,314 units)

$P < 0$ for $1 \le x < 4.686$ or $27.314 < x \le 40$

$P > 0$ for $4.686 < x < 27.314$

(C) Maximum profit is 160 thousand dollars ($160,000), and this occurs at $x = 16$ thousand units(16,000 units). The wholesale price at this output is $p(16) = 50 - 1.25(16) = \$30$, which is $5 greater than the $25 found in 88(C). (2-3)

90. (A) The area enclosed by the storage areas is given by

$A = (2y)x$

Now, $3x + 4y = 840$

so $y = 210 - \frac{3}{4}x$

Thus $A(x) = 2\left(210 - \frac{3}{4}x\right)x$

$= 420x - \frac{3}{2}x^2$

(B) Clearly x and y must be nonnegative; the fact that $y \ge 0$ implies

$210 - \frac{3}{4}x \ge 0$

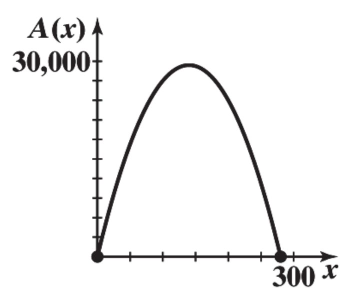
and $210 \ge \frac{3}{4}x$

$840 \ge 3x$

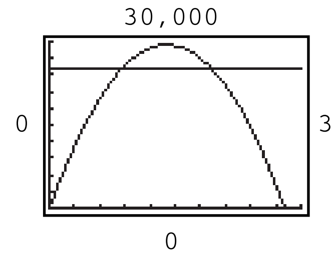
$280 \ge x$

Thus, domain A: $0 \le x \le 280$

(C)



(D) Graph $A(x) = 420x - \frac{3}{2}x^2$ and $y = 25{,}000$ together.



There are two values of x that will produce storage areas with a combined area of 25,000 square feet, one near $x = 90$ and the other near $x = 190$.

(E) $x = 86, x = 194$

(F) $A(x) = 420x - \frac{3}{2}x^2 = -\frac{3}{2}(x^2 - 280x)$

Completing the square, we have

$$A(x) = -\frac{3}{2}(x^2 - 280x + 19{,}600 - 19{,}600)$$
$$= -\frac{3}{2}[(x - 140)^2 - 19{,}600]$$
$$= -\frac{3}{2}(x - 140)^2 + 29{,}400$$

The dimensions that will produce the maximum combined area are:
$x = 140$ ft, $y = 105$ ft. The maximum area is 29,400 sq. ft. (2-3)

91. (A) Quadratic regression model,

Table 1:

```
QuadReg
 y=ax²+bx+c
 a=5.9477212E-6
 b=-.1024018814
 c=422.3467853
```

To estimate the demand at price level of \$180, we solve the equation

$$ax^2 + bx + c = 180$$

for x. The result is $x \approx 2{,}833$ sets.

(B) Linear regression model,

Table 2:

```
LinReg
 y=ax+b
 a=.0387421907
 b=-7.364689544
```

To estimate the supply at a price level of \$180, we solve the equation

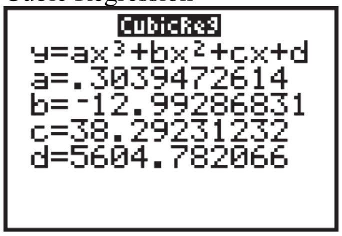
$$ax + b = 180$$

for x. The result is $x \approx 4{,}836$ sets.

(C) The condition is not stable; the price is likely to decrease since the supply at the price level of \$180 exceeds the demand at this level.

(D) Equilibrium price: \$131.59
Equilibrium quantity: 3,587 cookware set. (2-3)

92. (A) Cubic Regression



$y = 0.30395x^3 - 12.993x^2 + 38.292x + 5{,}604.8$

(B) $y = 0.30395(38)^3 - 12.993(38)^2 + 38.292(38) + 5{,}604.8 \approx 4{,}976$

The predicted crime index in 2025 is 4,976.

93. (A)

$$N(0) = 1$$
$$N\left(\frac{1}{2}\right) = 2$$
$$N(1) = 4 = 2^2$$
$$N\left(\frac{3}{2}\right) = 8 = 2^3$$
$$N(2) = 16 = 2^4$$
$$\vdots$$

Thus, we conclude that

$N(t) = 2^{2t}$ or $N = 4^t$

(B) We need to solve:

$$2^{2t} = 10^9$$
$$\log 2^{2t} = \log 10^9 = 9$$
$$2t \log 2 = 9$$
$$t = \frac{9}{2\log 2} \approx 14.95$$

Thus, the mouse will die in 15 days.

(2-6)

94. Given $I = I_0 e^{-kd}$. When $d = 73.6$, $I = \frac{1}{2} I_0$. Thus, we have:

$$\frac{1}{2} I_0 = I_0 e^{-k(73.6)}$$
$$e^{-k(73.6)} = \frac{1}{2}$$
$$-k(73.6) = \ln \frac{1}{2}$$
$$k = \frac{\ln(0.5)}{-73.6} \approx 0.00942$$

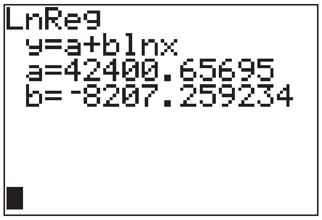
Thus, $k \approx 0.00942$.

To find the depth at which 1% of the surface light remains, we set $I = 0.01 I_0$ and solve

$$0.01 I_0 = I_0 e^{-0.00942d} \quad \text{for } d\text{:}$$
$$0.01 = e^{-0.00942d}$$
$$-0.00942d = \ln 0.01$$
$$d = \frac{\ln 0.01}{-0.00942} \approx 488.87$$

Thus, 1% of the surface light remains at approximately 489 feet. (2-6)

95. (A) Logarithmic regression model:



Year 2023 corresponds to $x = 83$; $y(83) \approx 6{,}134{,}000$ cows.

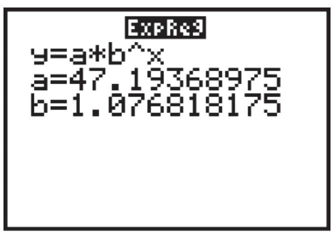
(B) $\ln(0)$ is not defined. (2-6)

96. Using the continuous compounding model, we have:

$$2P_0 = P_0 e^{0.03t}$$
$$2 = e^{0.03t}$$
$$0.03t = \ln 2$$
$$t = \frac{\ln 2}{0.03} \approx 23.1$$

Thus, the model predicts that the population will double in approximately 23.1 years. (2-5)

97. (A)



The exponential regression model is $y = 47.194(1.0768)^x$.

To estimate for the year 2025, let $x = 45 \Rightarrow y = 47.19368975(1.076818175)^{45} \approx 1{,}319.140047$. The estimated annual expenditure for Medicare by the U.S. government, rounded to the nearest billion, is approximately $1,319 billion. (This is $1.319 trillion.)

(B) To find the year, solve $47.194(1.0768)^x = 2{,}000$. Note: Use 2,000 because expenditures are in billions of dollars, and 2 trillion is 2,000 billion.

$$47.194(1.0768)^x = 2{,}000$$
$$1.0768^x = \frac{2{,}000}{47.194}$$
$$\ln(1.0768^x) = \ln\left(\frac{2{,}000}{47.194}\right)$$
$$x\ln 1.0768 = \ln\left(\frac{2{,}000}{47.194}\right)$$
$$x = \frac{\ln\left(\frac{2{,}000}{47.194}\right)}{\ln 1.0768} \approx 50.6 \text{ years}$$

1,980 + 50.63 = 2,030.63 Annual expenditures exceed two trillion dollars in the year 2031.

(2-5)

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Section 2-1 Functions

Goal: To evaluate function values and to determine the domain of functions

Definition: Function
A function is a correspondence between two sets of elements such that to each element in the first set, there corresponds one and only one element in the second set. The first set is called the *domain* and the set of corresponding elements in the second set is called the *range*.

Definition: Function specified by equations
If in an equation in two variables, we get exactly one output (value for the dependent variable) for each input (value for the independent variable), then the equation specifies a function. The graph of such a function is just the graph of the specifying equation.

1. Evaluate the following function at the specified values of the independent variable and simplify the results.

$f(x) = 4x - 5$

a) $f(1) = 4(1) - 5$
$f(1) = 4 - 5$
$f(1) = -1$

b) $f(-3) = 4(-3) - 5$
$f(-3) = -12 - 5$
$f(-3) = -17$

c) $f(x-1) = 4(x-1) - 5$
$f(x-1) = 4x - 4 - 5$
$f(x-1) = 4x - 9$

d) $f\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) - 5$
$f\left(\frac{1}{4}\right) = 1 - 5$
$f\left(\frac{1}{4}\right) = -4$

In Problems 2–10 evaluate the given function for $f(x) = x^2 + 1$ and $g(x) = x - 4$.

2. $(f+g)(5) = f(5) + g(5)$ $\quad f(5) = (5)^2 + 1$ $\quad g(5) = 5 - 4$
$= 26 + 1$ $\quad f(5) = 25 + 1$ $\quad g(5) = 1$
$(f+g)(5) = 27$ $\quad f(5) = 26$

3. $(f-g)(2c) = f(2c) - g(2c)$ $\quad f(2c) = (2c)^2 + 1$ $\quad g(2c) = 2c - 4$
$= 4c^2 + 1 - (2c - 4)$ $\quad f(2c) = 4c^2 + 1$
$(f-g)(2c) = 4c^2 - 2c + 5$

4. $(fg)(-2) = f(-2)g(-2)$ $\quad f(-2) = (-2)^2 + 1$ $\quad g(-2) = -2 - 4$
$= (5)(-6)$ $\quad f(-2) = 4 + 1$ $\quad g(-2) = -6$
$(fg)(-2) = -30$ $\quad f(-2) = 5$

5. $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)}$ $\quad f(0) = (0)^2 + 1$ $\quad g(0) = 0 - 4$
$= \frac{1}{-4}$ $\quad f(0) = 0 + 1$ $\quad g(0) = -4$
$\left(\frac{f}{g}\right)(0) = -\frac{1}{4}$ $\quad f(0) = 1$

6. $4 \cdot g(-3) = 4(-7)$ $\quad g(-3) = -3 - 4$
$4 \cdot g(-3) = -28$ $\quad g(-3) = -7$

7. $3 \cdot f(4) - 2 \cdot g(-1) = 3(17) - 2(-5)$ $\quad f(4) = (4)^2 + 1$ $\quad g(-1) = -1 - 4$
$= 51 + 10$ $\quad f(4) = 16 + 1$ $\quad g(-1) = -5$
$3 \cdot f(4) - 2 \cdot g(-1) = 61$ $\quad f(4) = 17$

8. $\dfrac{f(4)-g(3)}{f(2)}=\dfrac{17-(-1)}{5}$

$\dfrac{f(4)-g(3)}{f(2)}=\dfrac{18}{5}$

$f(4)=(4)^2+1$
$f(4)=16+1$
$f(4)=17$

$f(2)=(2)^2+1$
$f(2)=4+1$
$f(2)=5$

$g(3)=3-4$
$g(3)=-1$

9. $\dfrac{g(-1+h)-g(-1)}{h}=\dfrac{h-5-(-5)}{h}$

$=\dfrac{h}{h}$

$\dfrac{g(-1+h)-g(-1)}{h}=1$

$g(-1+h)=-1+h-4$
$g(-1+h)=h-5$

$g(-1)=-1-4$
$g(-1)=-5$

10. $\dfrac{f(3+h)-f(3)}{h}=\dfrac{h^2+6h+10-(10)}{h}$

$=\dfrac{h^2+6h}{h}$

$\dfrac{f(3+h)-f(3)}{h}=h+6$

$f(3+h)=(3+h)^2+1$
$f(3+h)=h^2+6h+9+1$
$f(3+h)=h^2+6h+10$

$f(3)=(3)^2+1$
$f(3)=9+1$
$f(3)=10$

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Section 2-2 Elementary Functions: Graphs and Transformations

Goal: To determine the domain of a function and to describe the shapes of graphs based on vertical and horizontal shifts and reflections, stretches, and shrinks

The domain of the following functions will be the set of real numbers unless it meets one of the following conditions:

1. The function contains a fraction whose denominator has a variable.
   The domain of such a function is the set of real numbers EXCEPT the values of the variable that make the denominator zero.
2. The function contains an even root (square root $\sqrt{\ }$, fourth root $\sqrt[4]{\ }$, etc.).
   The domain of such a function is limited to values of the variable that make the radicand (the part under the radical) greater than or equal to 0.

Basic Elementary Functions:

$f(x) = x$	Identity function
$h(x) = x^2$	Square function
$m(x) = x^3$	Cube function
$n(x) = \sqrt{x}$	Square root function
$p(x) = \sqrt[3]{x}$	Cube root function
$g(x) = \lvert x \rvert$	Absolute value function

In Problems 1–8 find the domain of each function.

1. $g(x) = \dfrac{5}{x-5}$

The domain is restricted by the denominator. Since it cannot equal zero, the domain is all real numbers except 5.

2. $f(x)=\dfrac{4x}{5x+6}$

The domain is restricted by the denominator. Since it cannot equal zero, the domain is all real numbers except $-\dfrac{6}{5}$.

3. $h(t)=\sqrt[4]{1-5t}$

The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $t\leq\dfrac{1}{5}$.

4. $g(x)=1-2x^2$

There are no restrictions on the domain, therefore, the domain is all real numbers.

5. $f(x)=\sqrt[3]{x+4}$

There are no restrictions on the domain since it has an odd root, therefore, the domain is all real numbers.

6. $h(w)=\sqrt{w-3}$

The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $w\geq 3$.

7. $f(x)=2x^3+5x^2-x+17$

There are no restrictions on the domain, therefore, the domain is all real numbers.

8. $g(x)=\dfrac{3x^4}{4}$

There are no restrictions on the domain since there is no variable in the denominator, therefore, the domain is all real numbers

In Problems 9–22 describe how the graph of each function is related to the graph of one of the six basic functions. State the domain of each function. (Do not use a graphing calculator and do not make a chart.)

9. $g(x) = x^2 - 12$

The graph is the square function that is shifted down 12 units. There are no restrictions on the domain, therefore, the domain is all real numbers.

10. $f(x) = \sqrt{x} + 3$

The graph is the square root function that is shifted up 3 units. The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $x \geq 0$.

11. $f(x) = -\sqrt{x}$

The graph is the square root function that is reflected over the *x*-axis. The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $x \geq 0$.

12. $f(x) = \sqrt[3]{x-4}$

The graph is the cube root function that is shifted 4 units to the right. There are no restrictions on the domain since it has an odd root, therefore, the domain is all real number.

13. $g(x) = (x-6)^2 + 3$

The graph is the square function that is shifted to the right 6 units and up 3 units. There are no restrictions on the domain, therefore, the domain is all real numbers.

14. $f(x) = -x^2 + 1$

The graph is the square function that is reflected over the *x*–axis and shifted up 1 unit. There are no restrictions on the domain, therefore, the domain is all real numbers.

15. $g(x) = 2 - \sqrt{x-4}$

The graph is the square root function that is shifted 4 units to the right, reflected over the *x*-axis, and shifted 2 units up. The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $x \geq 4$.

16. $h(x) = |x+7|$

The graph is the absolute value function that is shifted 7 units to the left. There are no restrictions on the domain, therefore, the domain is all real numbers.

17. $g(x) = \sqrt[3]{x} - 3$

The graph is the cube root function that is shifted 3 units down. There are no restrictions on the domain since it has an odd root, therefore, the domain is all real number.

18. $f(x) = |x+3| - 2$

The graph is the absolute value function that is shifted 3 units to the left and 2 units down. There are no restrictions on the domain, therefore, the domain is all real numbers.

19. $h(x) = -|x-3| + 2$

The graph is the absolute value function that is shifted 3 units to the right, reflected over the x-axis, and shifted 2 units up. There are no restrictions on the domain, therefore, the domain is all real numbers.

20. $f(x) = x^3 + 2$

The graph is the cube function that is shifted 2 units up. There are no restrictions on the domain, therefore, the domain is all real numbers.

21. $f(x) = -(x+5)^3 - 3$

The graph is the cube function that is shifted 5 units to the left, reflected over the x-axis, and then shifted 3 units down. There are no restrictions on the domain, therefore, the domain is all real numbers.

22. $h(x) = 3 - \sqrt[3]{x-4}$

The graph is the cube root function that is shifted 4 units to the right, reflected over the x-axis, and then shifted 3 units up. There are no restrictions on the domain since it has an odd root, therefore, the domain is all real numbers.

In Problems 23–31 write an equation for a function that has a graph with the given characteristics.

23. The shape of $y = x^3$ shifted 8 units right.

$y = (x-8)^3$

24. The shape of $y = \sqrt{x}$ shifted 5 units down.

$y = \sqrt{x} - 5$

25. The shape of $y = |x|$ reflected over the x-axis and shifted 5 units up.

$y = -|x| + 5$

26. The shape of $y = x^2$ shifted 5 units right and 3 units up.

$y = (x-5)^2 + 3$

27. The shape of $y = \sqrt[3]{x}$ reflected over the x-axis and shifted 1 unit up.

$y = 1 - \sqrt[3]{x}$

28. The shape of $y = x^2$ reflected over the x-axis and shifted 3 units down.

$y = -x^2 - 3$

29. The shape of $y = \sqrt{x}$ shifted 4 units left.

$y = \sqrt{x+4}$

30. The shape of $y = x^3$ shifted 6 units right and 2 units down.

$y = (x-6)^3 - 2$

31. The shape of $y = |x|$ shifted 6 units right and 5 units up.

$y = |x-6| + 5$

Name ______________________________ **Date** _____________ **Class** ___________

Section 2-3 Quadratic Functions

Goal: To describe functions that are linear and quadratic in nature

Quadratic Functions:

Standard form of a quadratic: $f(x) = ax^2 + bx + c$, where *a, b, c* are real and $a \neq 0$.

Vertex form of a quadratic: $f(x) = a(x-h)^2 + k$, where $a \neq 0$ and (h, k) is the vertex.

Axis of symmetry: $x = h$

Minimum/Maximum value:

If $a > 0$, then the turning point (or vertex) is a minimum point on the graph and the minimum value would be *k*.
If $a < 0$, then the turning point (or vertex) is a maximum point on the graph and the maximum value would be *k*.

For 1–8 find:
a) the domain
b) the vertex
c) the axis of symmetry
d) the *x*-intercept(s)
e) the *y*-intercept
f) the maximum or minimum value of the function

then:
g) Graph the function.
h) State the range.
i) State the interval over which the function is decreasing.
j) State the interval over which the function is increasing.

1. $f(x)=(x+1)^2-3$

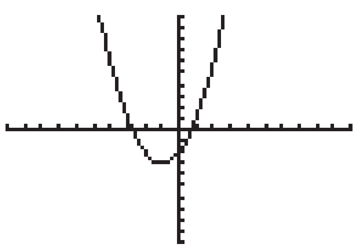
a) The function is a quadratic, therefore, the domain is all real numbers.
b) The function is in vertex form, therefore, the vertex is (–1, – 3).
c) The axis of symmetry is the x-value of the vertex, therefore the axis of symmetry is $x=-1$.
d) The x-intercepts are found by setting $f(x)=0$.

$$f(x)=(x+1)^2-3$$
$$0=x^2+2x+1-3$$
$$0=x^2+2x-2$$
Solve using the quadratic formula
$$x=-1\pm\sqrt{3}$$
Therefore, the x-intercepts are $(-1+\sqrt{3},0)$ and $(-1-\sqrt{3},0)$.

e) The y-intercepts are found by setting $x=0$.

$$f(x)=(x+1)^2-3$$
$$f(0)=(0+1)^2-3$$
$$f(0)=(1)^2-3$$
$$f(0)=-2$$
Therefore, the y-intercept is (0, –2).

f) The graph opens upward, therefore, the graph has a minimum value that is the y-coordinate of the vertex or –3.
g)



h) The graph has a minimum value of –3, therefore, the range is $y\geq-3$.
i) Based on the graph, the function is decreasing over the interval $(-\infty,-1)$.
j) Based on the graph, the function is increasing over the interval $(-1,\infty)$.

2. $f(x)=(x+2)^2+4$

a) The function is a quadratic, therefore, the domain is all real numbers.
b) The function is in vertex form, therefore, the vertex is (–2, 4).
c) The axis of symmetry is the x-value of the vertex, therefore the axis of symmetry is $x=-2$.
d) The x-intercepts are found by setting $f(x)=0$.

$$f(x)=(x+2)^2+4$$
$$0=x^2+4x+4+4$$
$$0=x^2+4x+8$$

Solving the above equation by the quadratic formula will result in complex roots, therefore, no x-intercepts are present.

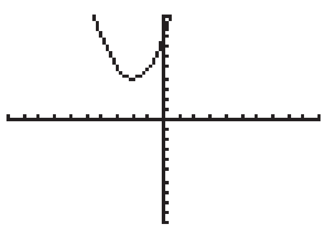
e) The y-intercepts are found by setting $x=0$.

$$f(x)=(x+2)^2+4$$
$$f(0)=(0+2)^2+4$$
$$f(0)=(2)^2+4$$
$$f(0)=8$$

Therefore, the y-intercept is (0, 8).

f) The graph opens upward, therefore the graph has a minimum value that is the y-coordinate of the vertex or 4.

g)



h) The graph has a minimum value of 4, therefore, the range is $y\geq 4$.
i) Based on the graph, the function is decreasing over the interval $(-\infty,-2)$.
j) Based on the graph, the function is increasing over the interval $(-2,\infty)$.

3. $f(x) = -x^2 + 9$

a) The function is a quadratic, therefore, the domain is all real numbers.

b) The function in vertex form is $f(x) = -(x-0)^2 + 9$, therefore, the vertex is (0, 9).

c) The axis of symmetry is the *x*-value of the vertex, therefore, the axis of symmetry is $x = 0$.

d) The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = -x^2 + 9$$

$$0 = -x^2 + 9$$

Solve using the quadratic formula

$$x = \pm 3$$

Therefore, the *x*-intercepts are $(3, 0)$ and $(-3, 0)$.

e) The *y*-intercepts are found by setting $x = 0$.

$$f(x) = -x^2 + 9$$

$$f(0) = -0^2 + 9$$

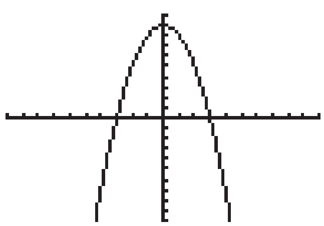
$$f(0) = 0 + 9$$

$$f(0) = 9$$

Therefore, the *y*-intercept is (0, 9).

f) The graph opens downward, therefore, the graph has a maximum value that is the *y*-coordinate of the vertex or 9.

g)



h) The graph has a maximum value of 9, therefore, the range is $y \le 9$.

i) Based on the graph, the function is decreasing over the interval $(0, \infty)$.

j) Based on the graph, the function is increasing over the interval $(-\infty, 0)$.

4. $f(x) = -(x-1)^2 - 1$

a) The function is a quadratic, therefore, the domain is all real numbers.
b) The function is in vertex form, therefore, the vertex is (1, –1).
c) The axis of symmetry is the *x*-value of the vertex, therefore, the axis of symmetry is $x = 1$.
d) The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = -(x-1)^2 - 1$$
$$0 = -x^2 + 2x - 1 - 1$$
$$0 = -x^2 + 2x - 2$$

Solving the above equation by the quadratic formula will result in complex roots, therefore, no *x*-intercepts are present.

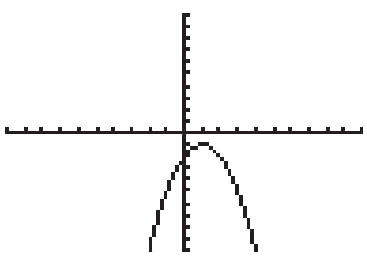
e) The *y*-intercepts are found by setting $x = 0$.

$$f(x) = -(x-1)^2 - 1$$
$$f(0) = -(0-1)^2 - 1$$
$$f(0) = -(-1)^2 - 1$$
$$f(0) = -2$$

Therefore, the *y*-intercept is (0, –2).

f) The graph opens downward, therefore, the graph has a maximum value that is the *y*-coordinate of the vertex or –1.

g)



h) The graph has a maximum value of –1, therefore, the range is $y \le -1$.
i) Based on the graph, the function is decreasing over the interval $(1, \infty)$.
j) Based on the graph, the function is increasing over the interval $(-\infty, 1)$.

5. $f(x) = x^2 - 4x$

a) The function is a quadratic, therefore the domain is all real numbers.
b) The function in vertex form is $f(x) = (x-2)^2 - 4$, therefore, the vertex is (2, – 4).
c) The axis of symmetry is the *x*-value of the vertex, therefore, the axis of symmetry is $x = 2$.
d) The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = x^2 - 4x$$
$$0 = x^2 - 4x$$
$$0 = x(x-4)$$
$$x = 0, 4$$

Therefore, the *x*-intercepts are $(0,0)$ and $(4,0)$.

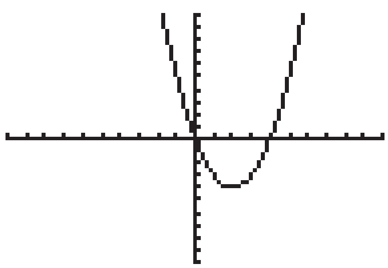
e) The *y*-intercepts are found by setting $x = 0$.

$$f(x) = x^2 - 4x$$
$$f(0) = 0^2 - 4(0)$$
$$f(0) = 0 + 0$$
$$f(0) = 0$$

Therefore, the *y*-intercept is (0, 0).

f) The graph opens upward, therefore, the graph has a minimum value that is the *y*-coordinate of the vertex or – 4.

g)



h) The graph has a minimum value of – 4, therefore, the range is $y \geq -4$.
i) Based on the graph, the function is decreasing over the interval $(-\infty, 2)$.
j) Based on the graph, the function is increasing over the interval $(2, \infty)$.

6. $f(x) = x^2 + 2x - 4$

a) The function is a quadratic, therefore, the domain is all real numbers.

b) The function in vertex form is $f(x) = (x+1)^2 - 5$, therefore, the vertex is (–1, –5).

c) The axis of symmetry is the *x*–value of the vertex, therefore, the axis of symmetry is $x = -1$.

d) The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = (x+1)^2 - 5$$
$$0 = x^2 + 2x + 1 - 5$$
$$0 = x^2 + 2x - 4$$

Solve using the quadratic formula

$$x = -1 \pm \sqrt{5}$$

Therefore, the *x*-intercepts are $(-1+\sqrt{5}, 0)$ and $(-1-\sqrt{5}, 0)$.

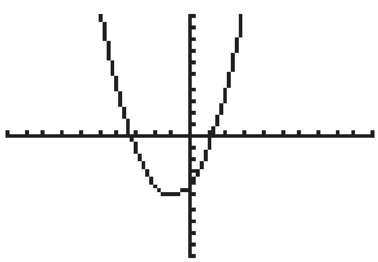
e) The *y*-intercepts are found by setting $x = 0$.

$$f(x) = (x+1)^2 - 5$$
$$f(0) = (0+1)^2 - 5$$
$$f(0) = (1)^2 - 5$$
$$f(0) = -4$$

Therefore, the *y*-intercept is (0, –4).

f) The graph opens upward, therefore, the graph has a minimum value that is the *y*-coordinate of the vertex or –5.

g)



h) The graph has a minimum value of –5, therefore, the range is $y \geq -5$.

i) Based on the graph, the function is decreasing over the interval $(-\infty, -1)$.

j) Based on the graph, the function is increasing over the interval $(-1, \infty)$.

7. $f(x) = x^2 + 2x + 1$

a) The function is a quadratic, therefore, the domain is all real numbers.

b) The function in vertex form is $f(x) = (x+1)^2 + 0,$ therefore, the vertex is (–1, 0).

c) The axis of symmetry is the *x*-value of the vertex, therefore, the axis of symmetry is $x = -1$.

d) The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = x^2 + 2x + 1$$
$$0 = x^2 + 2x + 1$$
$$0 = (x+1)(x+1)$$
$$x = -1$$

Therefore, there is only one *x*-intercept, which is $(-1, 0)$.

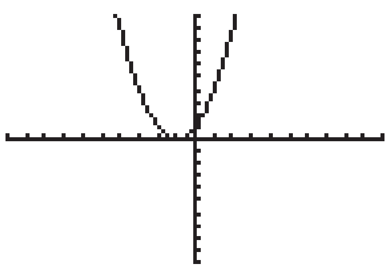
e) The *y*-intercepts are found by setting $x = 0$.

$$f(x) = x^2 + 2x + 1$$
$$f(0) = 0^2 + 2(0) + 1$$
$$f(0) = 0 + 0 + 1$$
$$f(0) = 1$$

Therefore, the *y*-intercept is (0, 1).

f) The graph opens upward, therefore, the graph has a minimum value that is the *y*-coordinate of the vertex or 0.

g)



h) The graph has a minimum value of 0, therefore, the range is $y \geq 0$.

i) Based on the graph, the function is decreasing over the interval $(-\infty, -1)$.

j) Based on the graph, the function is increasing over the interval $(-1, \infty)$.

8. $f(x) = -x^2 + 10x - 19$

a) The function is a quadratic, therefore, the domain is all real numbers.
b) The function in vertex form is $f(x) = -(x-5)^2 + 6$, therefore, the vertex is (5, 6).
c) The axis of symmetry is the x-value of the vertex, therefore, the axis of symmetry is $x = 5$.
d) The x-intercepts are found by setting $f(x) = 0$.

$$f(x) = -x^2 + 10x - 19$$
$$0 = -x^2 + 10x - 19$$
$$0 = x^2 - 10x + 19$$

Solve using the quadratic formula

$$x = 5 \pm \sqrt{6}$$

Therefore, the x-intercepts are $(5+\sqrt{6}, 0)$ and $(5-\sqrt{6}, 0)$.

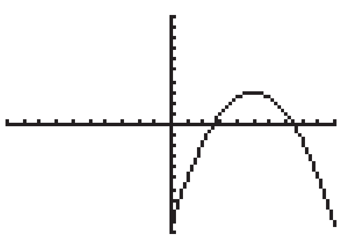
e) The y-intercepts are found by setting $x = 0$.

$$f(x) = -x^2 + 10x - 19$$
$$f(0) = -0^2 + 10(0) - 19$$
$$f(0) = 0 + 0 - 19$$
$$f(0) = -19$$

Therefore, the y-intercept is (0, –19).

f) The graph opens downward, therefore, the graph has a maximum value that is the y-coordinate of the vertex or 6.

g)



h) The graph has a maximum value of 4, therefore, the range is $y \le 6$.
i) Based on the graph, the function is decreasing over the interval $(5, \infty)$.
j) Based on the graph, the function is increasing over the interval $(-\infty, 5)$.

9. The revenue and cost functions for a company that manufactures components for washing machines were determined to be

$$R(x) = x(200-4x) \qquad \text{and} \qquad C(x) = 160 + 20x,$$

where x is the number of components in millions and $R(x)$ and $C(x)$ are in millions of dollars.

a) How many components must be sold in order for the company to break even? (Break-even points are when $R(x) = C(x)$.) (Round answers to nearest million.)

$$R(x) = C(x)$$
$$x(200-4x) = 160 + 20x$$
$$200x - 4x^2 = 160 + 20x$$
$$0 = 4x^2 - 180x + 160$$

Solve the equation by the quadratic formula

$$x \approx 0.9, 44.09$$

The company would need to sell approximately 1 million or 44 million to break even.

b) Find the profit equation. ($P(x) = R(x) - C(x)$)

$$P(x) = R(x) - C(x)$$
$$P(x) = x(200-4x) - (160+20x)$$
$$P(x) = 200x - 4x^2 - 160 - 20x$$
$$P(x) = -4x^2 + 180x - 160$$

c) Determine the maximum profit. How many components must be sold in order to achieve that maximum profit?

The maximum profit occurs at the vertex of the profit function. The x-coordinate is $x = -\frac{b}{2a} = -\frac{180}{2(-4)} = \frac{-180}{-8} = 22.5$. To find the y-coordinate of the vertex, substitute the value into the function as follows:

$$P(x) = -4x^2 + 180x - 160$$
$$P(22.5) = -4(22.5)^2 + 180(22.5) - 160$$
$$P(22.5) = -2025 + 4050 - 160$$
$$P(22.5) = 1865$$

The maximum profit of \$1,865 million is achieved when 22.5 million components are sold.

10. A company keeps records of the total revenue (money taken in) in thousands of dollars from the sale of x units (in thousands) of a product. It determines that total revenue is a function $R(x)$ given by

$$R(x) = 300x - x^2.$$

It also keeps records of the total cost of producing x units of the same product. It determines that the total cost is a function $C(x)$ given by

$$C(x) = 40x + 1600.$$

a) Find the break-even points for this company. (Round answer to nearest 1000.)

$$R(x) = C(x)$$
$$300x - x^2 = 40x + 1600$$
$$0 = x^2 - 260x + 1600$$

Solve the equation by the quadratic formula

$$x \approx 6.307, 253.693$$

The company would need to sell approximately 6,000 or 254,000 to break even.

b) Determine at what point profit is at a maximum. What is the maximum profit? How many units must be sold in order to achieve maximum profit?

The profit equation is

$$P(x) = R(x) - C(x)$$
$$P(x) = 300x - x^2 - (40x + 1600)$$
$$P(x) = -x^2 + 260x - 1600.$$

The maximum profit occurs at the vertex of the profit function. The x-coordinate is $x = -\frac{b}{2a} = -\frac{260}{2(-1)} = \frac{-260}{-2} = 130$. To find the y-coordinate of the vertex, substitute the value into the function as follows:

$$P(x) = -x^2 + 260x - 1600$$
$$P(130) = -(130)^2 + 260(130) - 1600$$
$$P(130) = -16,900 + 33,800 - 1600$$
$$P(130) = 15,300$$

The maximum profit of \$15,300 thousands or \$15,300,000 is achieved when 130,000 units are sold.

11. The cost, $C(x)$, of building a shed is a function of the number of square feet, x, in the shed. If the cost function can be approximated by

$$C(x) = 0.01x^2 - 20x + 25{,}000, \text{ where } 1000 \le x \le 3500$$

a) What would be the cost of building a 1500-square-foot shed?

Substitute the value of 1500 into the cost function:

$$\begin{aligned} C(x) &= 0.01x^2 - 20x + 25{,}000 \\ C(1500) &= 0.01(1500)^2 - 20(1500) + 25{,}000 \\ C(1500) &= 0.01(2{,}250{,}000) - 30{,}000 + 25{,}000 \\ C(1500) &= 17{,}500 \end{aligned}$$

It will cost \$17,500 to build a 1500-square-foot shed.

b) Find the minimum cost to build a shed. How many square-feet would that shed have?

The minimum cost occurs at the vertex of the cost function. The x-coordinate is $x = -\frac{b}{2a} = -\frac{-20}{2(0.01)} = \frac{20}{0.02} = 1000$. To find the y-coordinate of the vertex, substitute the value into the function as follows:

$$\begin{aligned} C(x) &= 0.01x^2 - 20x + 25000 \\ C(1000) &= 0.01(1000)^2 - 20(1000) + 25{,}000 \\ C(1000) &= 10{,}000 - 20{,}000 + 25{,}000 \\ C(1000) &= 15{,}000 \end{aligned}$$

The minimum cost of \$15,000 is achieved when a 1000-square-foot shed is built.

12. The cost of producing computer software is a function of the number of hours worked by the employees. If the cost function can be approximated by

$$C(x) = 0.04x^2 - 20x + 6000, \text{ where } 200 \le x \le 1000$$

a) What would be the cost if the employees worked 800 hours?

Substitute the value of 800 into the cost function:

$$C(x) = 0.04x^2 - 20x + 6000$$
$$C(800) = 0.04(800)^2 - 20(800) + 6000$$
$$C(800) = 0.04(640,000) - 16,000 + 6000$$
$$C(800) = 15,600$$

If the employees work 800 hours, it will cost $15,600 to produce the software.

b) Find the number of hours the employees should work in order to minimize the cost. What would the minimum cost be?
The minimum cost occurs at the vertex of the cost function. The x-coordinate is $x = -\frac{b}{2a} = -\frac{-20}{2(0.04)} = \frac{20}{0.08} = 250.$ To find the y-coordinate of the vertex, substitute the value into the function as follows:

$$C(x) = 0.04x^2 - 20x + 6000$$
$$C(250) = 0.04(250)^2 - 20(250) + 6000$$
$$C(250) = 2500 - 5000 + 6000$$
$$C(250) = 3500$$

The minimum cost of $3500 is achieved when the employees work 250 hours.

Name ______________________________ **Date** _____________ **Class** ___________

Section 2-4 Polynomial and Rational Functions

Goal: To describe and identify functions that are polynomial and rational in nature

Definition: Polynomial function

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ for n a nonnegative integer, called the degree of the polynomial. The coefficients $a_0, a_1, \cdots, a_n$ are real numbers with $a_n \neq 0$. The domain of a polynomial function is the set of all real numbers.

Definition: Rational function

$f(x) = \dfrac{n(x)}{d(x)}$ $d(x) \neq 0$, where $n(x)$ and $d(x)$ are polynomials. The domain is the set of all real numbers such that $d(x) \neq 0$.

Vertical Asymptotes:

Case 1: Suppose $n(x)$ and $d(x)$ have no real zero in common. If c is a real number such that $d(x) = 0$, then the line $x = c$ is a vertical asymptote of the graph.

Case 2: If $n(x)$ and $d(x)$ have one or more real zeros in common, cancel common linear factors and apply Case 1 to the reduced fraction.

Horizontal Asymptotes:

Case 1: If degree $n(x)$ < degree $d(x)$, then $y = 0$ is the horizontal asymptote.

Case 2: If degree $n(x)$ = degree $d(x)$, then $y = a/b$ is the horizontal asymptote, where a is the leading coefficient of $n(x)$ and b is the leading coefficient of $d(x)$.

Case 3: If degree $n(x)$ > degree $d(x)$, there is no horizontal asymptote.

For 1–6, determine each of the following for the polynomial functions:

a) the degree of the polynomial

b) the x-intercept(s) of the graph of the polynomial

c) the y-intercept of the graph of the polynomial

1. $f(x) = x^3 + 2x^2 - 23x - 60 = (x+3)(x-5)(x+4)$

a) The degree of the polynomial is the highest exponent, which is 3.
b) The x-intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form, therefore, if we set the factors equal to zero, the zeros of the polynomial occur at –3, –4, and 5.
c) The y-intercept occurs when the x-value is zero. If the x-value is zero, the only term that will not be zero is the constant term, therefore, the y-intercept is (0, –60).

2. $f(x) = x^3 + 8x^2 - 9x - 72 = (x-3)(x+3)(x+8)$

a) The degree of the polynomial is the highest exponent, which is 3.
b) The x-intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form, therefore, if we set the factors equal to zero, the zeros of the polynomial occur at 3, –3, and –8.
c) The y-intercept occurs when the x-value is zero. If the x-value is zero, the only term that will not be zero is the constant term, therefore, the y-intercept is (0, –72).

3. $f(x) = x^3 - 3x^2 - 10x + 24 = (x+3)(x-2)(x-4)$

a) The degree of the polynomial is the highest exponent, which is 3.
b) The x-intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form, therefore, if we set the factors equal to zero, the zeros of the polynomial occur at –3, 2, and 4.
c) The y-intercept occurs when the x value is zero. If the x value is zero, the only term that will not be zero is the constant term, therefore, the y-intercept is (0, 24).

4. $f(x) = x^3 + 4x^2 - x - 4 = (x+4)(x+1)(x-1)$

a) The degree of the polynomial is the highest exponent, which is 3.
b) The x-intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form, therefore, if we set the factors equal to zero, the zeros of the polynomial occur at –4, –1, and 1.
c) The y-intercept occurs when the x value is zero. If the x value is zero, the only term that will not be zero is the constant term, therefore, the y-intercept is (0, –4).

5. $f(x) = x^4 - 2x^3 + x^2 + 2x + 2 = (x-1)(x+1)(x^2 - 2x - 2)$

a) The degree of the polynomial is the highest exponent, which is 4.
b) The x-intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form. The third factor must be solved using the quadratic formula, therefore, the zeros of the polynomial occur at 1, –1, and $1 \pm \sqrt{3}$.
c) The y-intercept occurs when the x value is zero. If the x value is zero, the only term that will not be zero is the constant term, therefore, the y-intercept is (0, 2).

6. $f(x) = x^5 + 5x^4 - 20x^2 - x + 15 = (x+3)(x-1)(x+1)(x^2 + 2x - 5)$

a) The degree of the polynomial is the highest exponent, which is 5.
b) The x-intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form. The fourth factor must be solved using the quadratic equation, therefore, the zeros of the polynomial occur at –3, –1, 1, and $-1 \pm \sqrt{6}$.
c) The y-intercept occurs when the x-value is zero. If the x-value is zero, the only term that will not be zero is the constant term, therefore, the y-intercept is (0, –15).

For the given rational functions in 7–12,

a) Find the domain.
b) Find any x-intercept(s).
c) Find any y-intercept.
d) Find any vertical asymptote.
e) Find any horizontal asymptote.
f) Sketch a graph of $y = f(x)$ for $-10 \le x \le 10$.

7. $f(x) = \dfrac{1}{2x}$

a) The function is defined everywhere except when the denominator is zero. The domain is, therefore, all real numbers except 0.

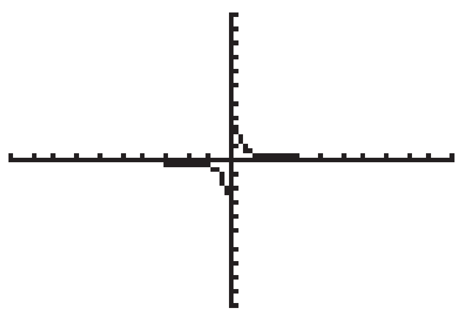
b) The x-intercept(s) are found by setting the function equal to 0. Since the function is a rational expression and the numerator cannot be zero, the function value cannot be zero. Therefore, there is no x-intercept.

c) The y-intercept is found when the value of x is zero. Since $x = 0$ is not in the domain, there is no y-intercept.

d) Vertica asymptotes occur at values where the function is not defined. Since the function is not defined for $x = 0$, the vertical asymptote is the line $x = 0$.

e) Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x) = \dfrac{1}{2x} = \dfrac{\frac{1}{2x}}{\frac{2x}{2x}} = \dfrac{\frac{1}{2x}}{1}$, as x increases or decreases without bound, the denominator is always 1 and the numerator tends to 0; so $f(x)$ tends to 0. The horizontal asymptote is the line $y = 0$.

f)



8. $f(x)=\dfrac{4x}{x-4}$

a) The function is defined everywhere except when the denominator is zero.

$$x-4=0$$
$$x=4$$

The domain is, therefore, all real numbers except 4.

b) The x-intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, the function value is zero when the numerator is zero.

$$4x=0$$
$$x=0$$

Therefore, the x–intercept is (0, 0).

c) The y-intercept is found when the value of x is zero.

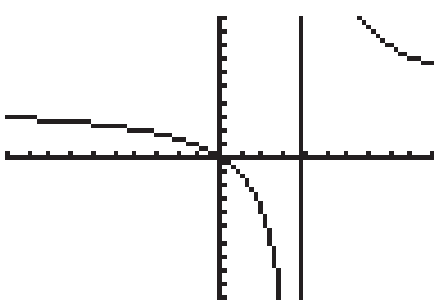
$$f(x)=\frac{4x}{x-4}$$
$$f(0)=\frac{4(0)}{0-4}$$
$$f(0)=\frac{0}{-4}=0$$

Therefore, the y-intercept is (0, 0).

d) Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x = 4$, the vertical asymptote is the line $x = 4$.

e) Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x)=\dfrac{4x}{x-4}=\dfrac{\frac{4x}{x}}{\frac{x}{x}-\frac{4}{x}}=\dfrac{4}{1-\frac{4}{x}}$, as x increases or decreases without bound, the numerator is always 4 and the denominator tends to 1 – 0, or 1; so $f(x)$ tends to 4. The horizontal asymptote is the line $y = 4$.

f)



9. $f(x) = \dfrac{5x}{x-2}$

a) The function is defined everywhere except when the denominator is zero.

$$x - 2 = 0$$
$$x = 2$$

The domain is, therefore, all real numbers except 2.

b) The x-intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, the function value is zero when the numerator is zero.

$$5x = 0$$
$$x = 0$$

Therefore, the x-intercept is (0, 0).

c) The y-intercept is found when the value of x is zero.

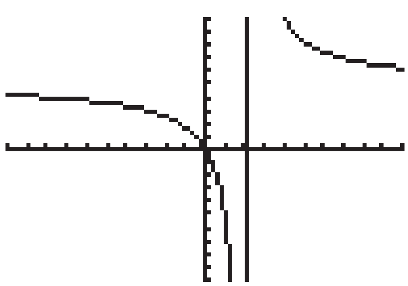
$$f(x) = \frac{5x}{x-2}$$
$$f(0) = \frac{5(0)}{0-2}$$
$$f(0) = \frac{0}{-2} = 0$$

Therefore, the y-intercept is (0, 0).

d) Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x = 2$, the vertical asymptote is the line $x = 2$.

e) Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x) = \dfrac{5x}{x-2} = \dfrac{\frac{5x}{x}}{\frac{x}{x} - \frac{2}{x}} = \dfrac{5}{1 - \frac{2}{x}}$, as x increases or decreases without bound, the numerator is always 5 and the denominator tends to 1 – 0, or 1; so $f(x)$ tends to 5. The horizontal asymptote is the line $y = 5$.

f))



10. $f(x)=\dfrac{2x-4}{x+3}$

a) The function is defined everywhere except when the denominator is zero.

$$x+3=0$$
$$x=-3$$

The domain is, therefore, all real numbers except –3.

b) The x-intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, the function value is zero when the numerator is zero.

$$2x-4=0$$
$$2x=4$$
$$x=2$$

Therefore, the x-intercept is (2, 0).

c) The y-intercept is found when the value of x is zero.

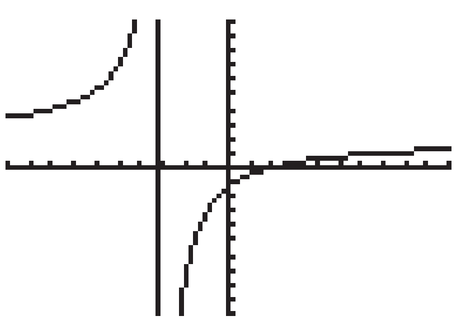
$$f(x)=\frac{2x-4}{x+3}$$
$$f(0)=\frac{2(0)-4}{0+3}$$
$$f(0)=\frac{-4}{3}$$

Therefore, the y-intercept is $(0,-\frac{4}{3})$.

d) Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for x = –3, the vertical asymptote is the line x = –3.

e) Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x)=\dfrac{2x-4}{x+3}=\dfrac{\frac{2x}{x}-\frac{4}{x}}{\frac{x}{x}+\frac{3}{x}}=\dfrac{2-\frac{4}{x}}{1+\frac{3}{x}}$, as x increases or decreases without bound, the numerator tends to 2 – 0, or 2 and the denominator tends to 1 – 0, or 1; so $f(x)$ tends to 2. The horizontal asymptote is the line $y=2$.

f)



11. $f(x) = \dfrac{4+x}{4-x}$

a) The function is defined everywhere except when the denominator is zero.

$$4 - x = 0$$
$$4 = x$$

The domain is, therefore, all real numbers except 4.

b) The x-intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, the function value is zero when the numerator is zero.

$$4 + x = 0$$
$$x = -4$$

Therefore, the x-intercept is (–4, 0).

c) The y-intercept is found when the value of x is zero.

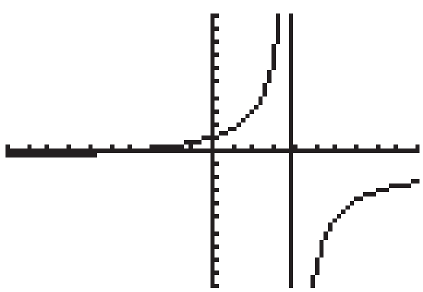
$$f(x) = \frac{4+x}{4-x}$$
$$f(0) = \frac{4+0}{4-0}$$
$$f(0) = \frac{4}{4} = 1$$

Therefore, the y-intercept is $(0,1)$.

d) Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x = 4$, the vertical asymptote is the line $x = 4$.

e) Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x) = \dfrac{4+x}{4-x} = \dfrac{\frac{4}{x}+\frac{x}{x}}{\frac{4}{x}-\frac{x}{x}} = \dfrac{\frac{4}{x}+1}{\frac{4}{x}-1}$, as x increases or decreases without bound, the numerator tends to 0 + 1, or 1 and the denominator tends to 0 – 1, or –1; so $f(x)$ tends to –1. The horizontal asymptote is the line $y = -1$.

f)



12. $f(x)=\dfrac{1-5x}{1+2x}$

a) The function is defined everywhere except when the denominator is zero.

$$1+2x=0$$
$$2x=-1$$
$$x=-\frac{1}{2}$$

The domain is, therefore, all real numbers except $-\frac{1}{2}$.

b) The x-intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, when the numerator is zero, the function value is zero.

$$1-5x=0$$
$$-5x=-1$$
$$x=\frac{1}{5}$$

Therefore, the x-intercept is $(\frac{1}{5},0)$.

c) The y-intercept is found when the value of x is zero.

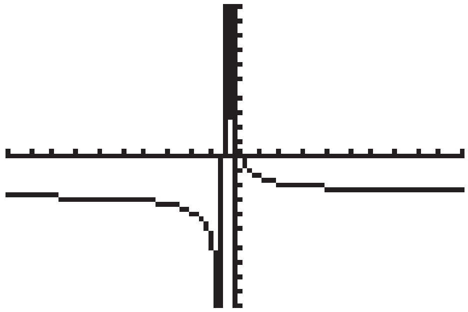
$$f(x)=\frac{1-5x}{1+2x}$$
$$f(0)=\frac{1-5(0)}{1+2(0)}$$
$$f(0)=\frac{1}{1}=1$$

Therefore, the y-intercept is $(0,1)$.

d) Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x=-\frac{1}{2}$, the vertical asymptote is the line $x=-\frac{1}{2}$.

e) Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x)=\dfrac{1-5x}{1+2x}=\dfrac{\frac{1}{x}-\frac{5x}{x}}{\frac{1}{x}+\frac{2x}{x}}=\dfrac{\frac{1}{x}-5}{\frac{1}{x}+2}$, as x increases or decreases without bound, the numerator tends to 0 – 5, or –5 and the denominator tends to 0 + 2, or 2; so $f(x)$ tends to $-\frac{5}{2}$. The horizontal asymptote is the line $y=-\frac{5}{2}$.

f)



13. A video production company is planning to produce a documentary. The producer estimates that it will cost \$104,000 to produce the video and \$40 per video to copy and distribute the tape.

a) Assuming that the total cost to market the video, $C(n)$, is linearly related to the total number, n, of videos produced, write an equation for the cost function.

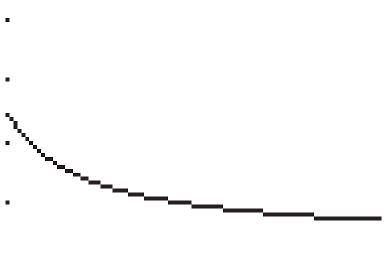
$$C(n) = 40n + 104,000$$

b) The average cost per video for an output of n videos is given by $\overline{C}(n) = \frac{C(n)}{n}$.

Find the average cost function. $\overline{C}(n) = \frac{C(n)}{n} = \frac{40n + 104,000}{n}$

c) Sketch a graph of the average cost function for $1000 \le n \le 6000$.

The x-axis scale shown is from 1000 to 6000. Each tick mark is 1000 units. The y-axis scale shown is from 0 to 400. Each tick mark is 100 units.



d) What does the average cost per video tend to as production increases?

To find the value that the function tends to go towards, you will find the horizontal asymptote of the function. Horizontal asymptotes are found by dividing all terms by the highest power of n. Therefore, $\overline{C}(n) = \frac{\frac{40n}{n} + \frac{104,000}{n}}{\frac{n}{n}} = \frac{40 + \frac{104,000}{n}}{1}$, as n increases without bound, the numerator tends to 40 + 0, or 40 and the denominator is always 1; so $\overline{C}(n)$ tends to 40. This means that the average cost per video tends towards \$40 each.

14. A contractor purchases a piece of equipment for $36,000. The equipment requires an average expenditure of $8.25 per hour for fuel and maintenance, and the operator is paid $13.50 per hour to operate the machinery.

a) Assuming that the total cost per day, $C(h)$, is linearly related to the number of hours, h, that the machine is operated, write an equation for the cost function.

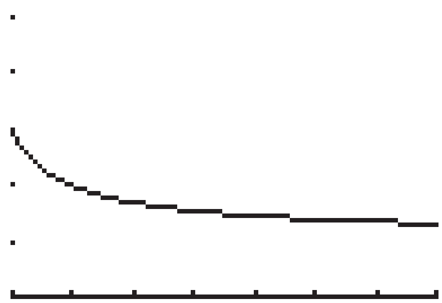
$$C(h) = 21.75h + 36{,}000$$

b) The average cost per hour of operating the machine is given by $\overline{C}(h) = \frac{C(h)}{h}$.

Find the average cost function. $\overline{C}(h) = \frac{C(h)}{h} = \frac{21.75h + 36{,}000}{h}$

c) Sketch a graph of the average cost function for $1000 \le h \le 8000$.

The x-axis scale shown is from 1000 to 8000. Each tick mark is 1000 units. The y-axis scale shown is from 0 to 100. Each tick mark is 20 units.



d) What cost per hour does the average cost per hour tend to as the number of hours of use increases?

To find the value that the function tends to go towards, you will find the horizontal asymptote of the function. Horizontal asymptotes are found by dividing all terms by the highest power of h. Therefore, $\overline{C}(h) = \frac{\frac{21.75n}{h} + \frac{36{,}000}{h}}{\frac{h}{h}} = \frac{21.75 + \frac{36{,}000}{h}}{1}$, as h increases without bound, the numerator tends to 21.75 + 0, or 21.75 and the denominator is always 1; so $\overline{C}(h)$ tends to 21.75. This means that the average cost per hour tends towards $21.75 as the number of hours of use increases.

15. The daily cost function for producing x printers for home computers was determined to be

$$C(x) = x^2 + 8x + 6000,$$

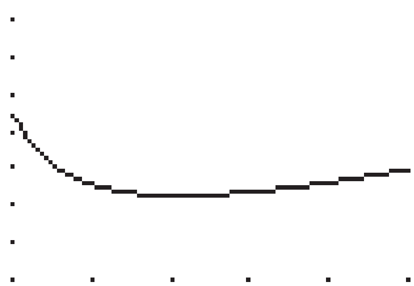
The average cost per printer at a production level of x printers per day is $\bar{C}(x) = \dfrac{C(x)}{x}$.

a) Find the average cost function.

$$\bar{C}(x) = \frac{C(x)}{x} = \frac{x^2 + 8x + 6000}{x}$$

b) Sketch a graph of the average cost function for $25 \le x \le 150$.

The x-axis scale shown is from 25 to 150. Each tick mark is 25 units. The y-axis scale shown is from 50 to 400. Each tick mark is 50 units.



c) At what production level is the daily average cost at a minimum? What is that minimum value?

Based on the graph above, the minimum value occurs around the third tick mark, which has a value of 75. By substituting values into the average value equation, we would have the following:

$$\bar{C}(75) = 163$$
$$\bar{C}(76) = 162.947$$
$$\bar{C}(77) = 162.922$$
$$\bar{C}(78) = 162.923$$
$$\bar{C}(79) = 162.949$$

Therefore, the minimum average cost of 162.92 occurs when 77 printers are produced.

16. The monthly cost function for producing x brake assemblies for a certain type of car is given by

$$C(x) = 3x^2 + 36x + 9000.$$

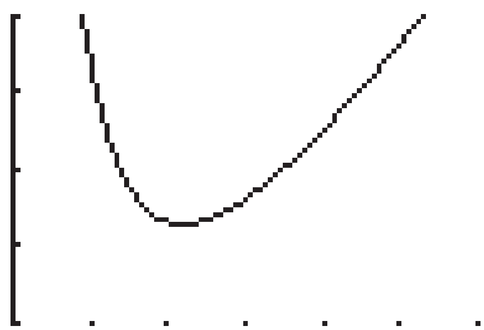
The average cost per brake assembly at a production level of x assemblies per month is $\overline{C}(x) = \frac{C(x)}{x}$.

a) Find the average cost function.

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{3x^2 + 36x + 9000}{x}$$

b) Sketch a graph of the average cost function for $0 \le x \le 150$.

The x-axis scale shown is from 0 to 150. Each tick mark is 25 units. The y-axis scale shown is from 300 to 500. Each tick mark is 50 units.



c) At what production level is the daily average cost at a minimum? What is that minimum value?

Based on the graph above, the minimum value occurs just beyond the third tick mark, which has a value of 50. By substituting values into the average value equation, we would have the following:

$$\overline{C}(53) = 364.811$$
$$\overline{C}(54) = 364.667$$
$$\overline{C}(55) = 364.636$$
$$\overline{C}(56) = 364.714$$

Therefore, the minimum average cost of 364.63 occurs when 55 brake assemblies are produced.

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Section 2-5 Exponential Functions

Goal: To describe and solve functions that are exponential in nature

Rules for Exponents:

$a^m \cdot a^n = a^{m+n}$ Product Rule $a^0 = 1,\ a \neq 0$ Zero Exponent Rule

$\frac{a^m}{a^n} = a^{m-n}$ Quotient Rule $\left(a^m\right)^n = a^{mn}$ Power Rule

In Problems 1–8, describe in words the transformations that can be used to obtain the graph of $g(x)$ from the graph of $f(x)$.

1. $g(x) = 4^{x+5} - 3;\quad f(x) = 4^x$

   The function f is shifted 5 units to the left and 3 units down.

2. $g(x) = -3^x - 7;\quad f(x) = 3^x$

   The function f is reflected over the x-axis and shifted 7 units down.

3. $g(x) = 2^{x-4} - 6;\quad f(x) = 2^x$

   The function f is shifted 4 units to the right and 6 units down.

4. $g(x) = -5^{x-4} + 5;\quad f(x) = 5^x$

   The function f is shifted 4 units to the right, reflected over the x-axis, and shifted 5 units up.

5. $g(x) = 10^{x-2} - 5;\quad f(x) = 10^x$

   The function *f* is shifted 2 units to the right and 5 units down.

6. $g(x) = -10^x - 3;\quad f(x) = 10^x$

   The function *f* is reflected over the *x*-axis and shifted 3 units down.

7. $g(x) = e^{x+1} + 2;\quad f(x) = e^x$

   The function *f* is shifted 1 unit to the left and 2 units up.

8. $g(x) = -e^x + 5;\quad f(x) = e^x$

   The function *f* is reflected over the *x*-axis and shifted 5 units up.

In Problems 9–20, solve each equation for *x*.

9. $10^{7x-4} = 10^{4x+5}$

$$7x - 4 = 4x + 5$$
$$3x = 9$$
$$x = 3$$

10. $10^{x^2} = 10^{2x+8}$

$$x^2 = 2x + 8$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x = -2, 4$$

11. $6^{5x-4} = 6^{x^2}$

$$5x - 4 = x^2$$
$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$
$$x = 1, 4$$

12. $8^{2x^2} = 8^{16x}$

$$2x^2 = 16x$$
$$2x^2 - 16x = 0$$
$$2x(x-8) = 0$$
$$x = 0, 8$$

13. $(x+4)^3 = (3x-10)^3$

$$x + 4 = 3x - 10$$
$$14 = 2x$$
$$7 = x$$

14. $(2x-7)^5 = (x+1)^5$

$$2x - 7 = x + 1$$
$$x = 8$$

15. $(e^2)^3 = e^x$

$2(3) = x$
$6 = x$

16. $(e^x)^5 = e^{x^2}$

$5x = x^2$
$x^2 - 5x = 0$
$x(x-5) = 0$
$x = 0, 5$

17. $(e^{2x})^x = e^{15+x}$

$2x^2 = 15 + x$
$2x^2 - x - 15 = 0$
$(2x+5)(x-3) = 0$
$x = -\frac{5}{2}, 3$

18. $3^x \cdot 3^4 = 3^{3x^2}$

$x + 4 = 3x^2$
$3x^2 - x - 4 = 0$
$(3x-4)(x+1) = 0$
$x = \frac{4}{3}, -1$

19. $2^{x^2} = 2^{12x} \cdot 2^{-32}$

$x^2 = 12x - 32$
$x^2 - 12x + 32 = 0$
$(x-4)(x-8) = 0$
$x = 4, 8$

20. $9^x \cdot 9 = 9^{2x^2}$

$x + 1 = 2x^2$
$2x^2 - x - 1 = 0$
$(2x+1)(x-1) = 0$
$x = -\frac{1}{2}, 1$

Interest Formulas

Simple Interest: $A = P(1 + rt)$

Compound Interest: $A = P\left(1 + \frac{r}{m}\right)^{mt}$

Continuous Compound Interest: $A = Pe^{rt}$

where P is the amount invested (principal), r (expressed as a decimal) is the annual interest rate, t is time invested (in years), m is the number of times a year the interest is compounded, and A is the amount of money in the account after t years (future value).

(Round answers for Problems 21–28 to the nearest dollar)

21. Fred inherited \$50,000 from his uncle. He decides to invest his money for 8 years in order to have the greatest down payment when he buys a house. He can choose from 3 different banks.

Bank A offers 1% compounded monthly.
Bank B offers .5% compounded continuously.
Bank C offers .75% compounded daily.

Which bank offers the best plan so Fred can earn the most money from his investment?

Bank A:

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 50,000\left(1+\frac{0.01}{12}\right)^{12(8)}$$
$$A = 50000(1.000833)^{96}$$
$$A \approx \$54,163$$

Bank B:

$$A = Pe^{rt}$$
$$A = 50,000e^{(0.005)(8)}$$
$$A = 50,000e^{0.04}$$
$$A \approx \$52,041$$

Bank C:

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 50,000\left(1+\frac{0.0075}{365}\right)^{365(8)}$$
$$A = 50,000(1.000021)^{2920}$$
$$A \approx \$53,092$$

Therefore, Bank A is the best option.

22. The day your first child is born, you invest \$20,000 in an account that pays 2.5% interest compounded quarterly. How much will be in the account when the child is 18 years old and ready to start to college?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 20,000\left(1+\frac{0.025}{4}\right)^{4(18)}$$
$$A = 20,000(1.00625)^{72}$$
$$A \approx \$31,322$$

23. When your second child is born, you are able to invest only $10,000 but the account pays 3.2% interest compounded daily. How much will be in the account when this child is 18 years old and ready to start to college?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 10,000\left(1+\frac{0.032}{365}\right)^{365(18)}$$
$$A = 10,000(1.000088)^{6570}$$
$$A \approx \$17,789$$

24. When your third child comes along, money is even tighter and you are able to invest only $5000, but you are able to find a bank that will let you invest the money at 5% compounded continuously. How much will be in the account when this third child is 18 years old and ready to start to college?

$$A = Pe^{rt}$$
$$A = 5000e^{(0.05)(18)}$$
$$A = 5000e^{0.9}$$
$$A \approx \$12,298$$

25. Joe Vader plans to start his own business in ten years. How much money would he need to invest today in order to have $25,000 in ten years if Joe's bank offers a 10-year CD that pays 1.8% interest compounded monthly.

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$25,000 = P\left(1+\frac{0.018}{12}\right)^{12(10)}$$
$$25,000 = P(1.0015)^{120}$$
$$P \approx \$20,885$$

26. Bill and Sue plan to buy a home in 5 years. How much would they need to invest today at 1.2% compounded daily in order to have $30,000 in five years?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$30{,}000 = P\left(1+\frac{0.012}{365}\right)^{365(5)}$$
$$30{,}000 = P(1.000088)^{1825}$$
$$P \approx \$28{,}253$$

27. Suppose you invest $3000 in a four-year certificate of deposit (CD) that pays 1.5% interest compounded monthly the first 3 years and 2.2% compounded daily the last year. What is the value of the CD at the end of the four years?

$$A = P\left(1+\frac{r}{m}\right)^{mt}\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 3000\left(1+\frac{0.015}{12}\right)^{12(3)}\left(1+\frac{0.022}{365}\right)^{365(1)}$$
$$A = 3000(1.00125)^{36}(1.000060)^{365}$$
$$A \approx \$3208$$

28. Suppose you invest $15,000 in a 10-year certificate of deposit (CD) that pays 3.1% interest compounded daily the first 4 years and 3.9% compounded continuously the last six years. What is the value of the CD at the end of the 10 years?

$$A = P\left(1+\frac{r}{m}\right)^{mt} e^{rt}$$
$$A = 15{,}000\left(1+\frac{0.031}{365}\right)^{365(4)} e^{(0.039)(6)}$$
$$A = 15{,}000(1.000085)^{1460} e^{0.234}$$
$$A \approx \$21{,}457$$

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Section 2-6 Logarithmic Functions

Goal: To solve problems that are logarithmic in nature

Properties of Logarithms

$\log_a x + \log_a y = \log_a xy$

$\log_a x - \log_a y = \log_a \frac{x}{y}$

$\log_a x^n = n \log_a x$

Exponential Function: $f(x) = a^x,\ a > 0,\ a \neq 1$

$y = \log_a x$ means $x = a^y$

In Problems 1–20, find the value of *x*. (Evaluate to four decimal places if necessary.)

1. $\log_5 x = 3$

$$x = 5^3$$
$$x = 125$$

2. $\log_4 (x+3) = 3$

$$x + 3 = 4^3$$
$$x + 3 = 64$$
$$x = 61$$

3. $\log_3 3^8 = 7 + 3x$

$$3^{7+3x} = 3^8$$
$$7 + 3x = 8$$
$$3x = 1$$
$$x = \frac{1}{3}$$

4. $\log_2 2^6 = 4 - 3x$

$$2^{4-3x} = 2^6$$
$$4 - 3x = 6$$
$$-3x = 2$$
$$x = -\frac{2}{3}$$

5. $\ln(x+5)=8$

$$e^8 = x+5$$
$$e^8 - 5 = x$$
$$2975.96 \approx x$$

6. $\log_x(x+20)=2$

$$x^2 = x+20$$
$$x^2 - x - 20 = 0$$
$$(x-5)(x+4)=0$$
$$x=5$$
$$x \neq -4$$

The log function cannot have a negative base.

7. $\log_2(4x+3)=\log_2(6x-5)$

$$4x+3=6x-5$$
$$8=2x$$
$$4=x$$

8. $\log_3(8-5x)=\log_3(2x-13)$

$$8-5x=2x-13$$
$$21=7x$$
$$3=x$$

Since the value of 3 creates a negative log, there is no solution.

9. $\ln x+\ln 8=5$

$$\ln 8x=5$$
$$e^5=8x$$
$$\frac{e^5}{8}=x$$
$$18.552 \approx x$$

10. $\ln x-\ln 2=0$

$$\ln \tfrac{x}{2}=0$$
$$e^0=\tfrac{x}{2}$$
$$2(1)=x$$
$$2=x$$

11. $\log(x-1)-\log 4=3$

$$\log \tfrac{x-1}{4}=3$$
$$10^3=\tfrac{x-1}{4}$$
$$(4)10^3=x-1$$
$$4(1000)+1=x$$
$$4001=x$$

12. $\ln(x-1)-\ln 6=2$

$$\ln \tfrac{x-1}{6}=2$$
$$e^2=\tfrac{x-1}{6}$$
$$6e^2=x-1$$
$$6e^2+1=x$$
$$45.3343 \approx x$$

13. $3^{2x} = 14$

$$\begin{aligned} \log 3^{2x} &= \log 14 \\ 2x \log 3 &= \log 14 \\ x &= \frac{\log 14}{2 \log 3} \\ x &\approx 1.2011 \end{aligned}$$

14. $7^{x-2} = 19$

$$\begin{aligned} \log 7^{x-2} &= \log 19 \\ (x-2)\log 7 &= \log 19 \\ x \log 7 - 2 \log 7 &= \log 19 \\ x \log 7 &= \log 19 + 2 \log 7 \\ x &= \frac{\log 19 + 2 \log 7}{\log 7} \\ x &\approx 3.513 \end{aligned}$$

15. $7^{x-1} = 8^x$

$$\begin{aligned} \log 7^{x-1} &= \log 8^x \\ (x-1)\log 7 &= x \log 8 \\ x \log 7 - \log 7 &= x \log 8 \\ x \log 7 - x \log 8 &= \log 7 \\ x(\log 7 - \log 8) &= \log 7 \\ x &= \frac{\log 7}{\log 7 - \log 8} \\ x &\approx -14.5727 \end{aligned}$$

16. $4^{2x+3} = 5^{x-2}$

$$\begin{aligned} \log 4^{2x+3} &= \log 5^{x-2} \\ (2x+3)\log 4 &= (x-2)\log 5 \\ 2x \log 4 + 3 \log 4 &= x \log 5 - 2 \log 5 \\ 2x \log 4 - x \log 5 &= -2 \log 5 - 3 \log 4 \\ x(2 \log 4 - \log 5) &= -2 \log 5 - 3 \log 4 \\ x &= \frac{-2 \log 5 - 3 \log 4}{2 \log 4 - \log 5} \\ x &\approx -6.3429 \end{aligned}$$

17. $7^{x+1} = 10^{2x}$

$$\begin{aligned} \log 7^{x+1} &= \log 10^{2x} \\ (x+1)\log 7 &= 2x \log 10 \\ x \log 7 + \log 7 &= 2x \log 10 \\ x \log 7 - 2x \log 10 &= -\log 7 \\ x(\log 7 - 2 \log 10) &= -\log 7 \\ x &= \frac{-\log 7}{\log 7 - 2 \log 10} \\ x &\approx 0.7317 \end{aligned}$$

18. $e^{x+4} = 14.654$

$$\begin{aligned} \ln e^{x+4} &= \ln(14.654) \\ x + 4 &= \ln(14.654) \\ x &= \ln(14.654) - 4 \\ x &\approx -1.3153 \end{aligned}$$

19. $x + 7 = e^2$

$$\begin{aligned} x &= e^2 - 7 \\ x &\approx 0.3891 \end{aligned}$$

20. $e^{8x} = e^{32}$

$$\begin{aligned} 8x &= 32 \\ x &= 4 \end{aligned}$$

21. You want to accumulate $30,000 by your son's eighteenth birthday. How much do you need to invest on the day he is born in an account that will pay 1.9% interest compounded quarterly? (Round your answer to the nearest dollar.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$30,000 = P\left(1+\frac{0.019}{4}\right)^{4(18)}$$
$$30,000 = P(1.00475)^{72}$$
$$\$21,328 \approx P$$

22. Using the information in Problem 21, how much would you need to invest if you waited until he is 10 years old to start the fund?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$30,000 = P\left(1+\frac{0.019}{4}\right)^{4(8)}$$
$$30,000 = P(1.00475)^{32}$$
$$\$25,779 \approx P$$

23. A bond that sells for $1000 today can be redeemed for $1200 in 10 years. If interest is compounded quarterly, what is the annual interest rate for this investment? (Round your answer to two decimal places when written as a percentage.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$1200 = 1000\left(1+\frac{r}{4}\right)^{4(10)}$$
$$1.2 = \left(1+\frac{r}{4}\right)^{40}$$
$$1.2^{1/40} = 1+\frac{r}{4}$$
$$1.2^{1/40} - 1 = \frac{r}{4}$$
$$4(1.2^{1/40} - 1) = r$$
$$1.83\% \approx r$$

24. A bond that sells for $22,000 today can be redeemed for $25,000 in 4 years. If interest is compounded monthly, what is the annual interest rate for this investment? (Round your answer to two decimal places when written as a percentage.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$25,000 = 22,000\left(1+\frac{r}{12}\right)^{12(6)}$$

$$1.136 = \left(1+\frac{r}{12}\right)^{72}$$

$$1.136^{1/72} = 1+\frac{r}{12}$$

$$1.136^{1/72} - 1 = \frac{r}{12}$$

$$12(1.136^{1/72} - 1) = r$$

$$2.13\% \approx r$$

25. What is the minimum number of months required for an investment of $5,000 to grow to at least $10,000 (double in value) if the investment earns 2.6% annual interest rate compounded monthly? What would be the actual value of the investment after that many months?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$10,000 = 5,000\left(1+\frac{0.026}{12}\right)^{12t}$$

$$2 = (1.00217)^{12t}$$

$$\ln 2 = \ln(1.00217)^{12t}$$

$$\ln 2 = 12t\ln(1.00217)$$

$$\frac{\ln 2}{12\ln 1.00217} = t$$

$$26.7 \approx t$$

It will take about $(26.7)(12) = 320$ months to double.

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 5,000\left(1+\frac{0.026}{12}\right)^{12(26.7)}$$
$$A = 5,000(1.00217)^{320}$$
$$A = 5,000(1.998872)$$
$$A \approx \$9,994$$

26. What is the minimum number of months required for an investment of \$5,000 to grow to at least \$15,000 (triple in value) if the investment earns 2.6% annual interest rate compounded monthly? What would be the actual value of the investment after that many months?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$15,000 = 5,000\left(1+\frac{0.026}{12}\right)^{12t}$$
$$3 = (1.00217)^{12t}$$
$$\ln 3 = \ln(1.00217)^{12t}$$
$$\ln 3 = 12t\ln(1.00217)$$
$$\frac{\ln 3}{12\ln 1.00217} = t$$
$$42.3 \approx t$$

It will take about $(42.3)(12) = 508$ months to triple.

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 5,000\left(1+\frac{0.026}{12}\right)^{12(42.3)}$$
$$A = 5,000(1.00217)^{508}$$
$$A = 5,000(3.002592)$$
$$A \approx \$15,013$$

27. Some years ago, Ms. Martinez invested $7000 at 2% compounded quarterly. The account now contains $10,000. How long ago did she start the account? (Round your answer up to the next year.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$10,000 = 7000\left(1+\frac{0.02}{4}\right)^{4t}$$
$$\tfrac{10}{7} = (1.005)^{4t}$$
$$\ln\tfrac{10}{7} = \ln(1.005)^{4t}$$
$$\ln\tfrac{10}{7} = 4t\ln(1.005)$$
$$\frac{\ln\tfrac{10}{7}}{4\ln 1.005} = t$$
$$17.88 \approx t$$

It took approximately 18 years to have a balance of $10,000.

28. Some years ago, Mr. Tang invested $18,000 at 5% compounded monthly. The account now contains $24,000. How long ago did he start the account? (Round your answer up to the next year.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$24,000 = 18,000\left(1+\frac{0.05}{12}\right)^{12t}$$
$$\tfrac{4}{3} = (1.00417)^{12t}$$
$$\ln\tfrac{4}{3} = \ln(1.00417)^{12t}$$
$$\ln\tfrac{4}{3} = 12t\ln(1.00417)$$
$$\frac{\ln\tfrac{4}{3}}{12\ln 1.00417} = t$$
$$5.77 \approx t$$

It took approximately 6 years to have a balance of $24,000.

29. In a certain country, the number of people above the poverty level is currently 30 million and growing at a rate of 5% annually. Assuming that the population is growing continuously, the population, P (in millions), t years from now, is determined by the formula:

$$P = 30e^{0.05t}$$

In how many years will there be 40 million people above the poverty level? 50 million? (Round your answers to nearest tenth of a year.)

40 million people

$$P = 30e^{0.05t}$$
$$40 = 30e^{0.05t}$$
$$1.33 = e^{0.05t}$$
$$\ln 1.33 = \ln e^{0.05t}$$
$$\ln 1.33 = 0.05t$$
$$\frac{\ln 1.33}{0.05} = t$$
$$5.7 \approx t$$

50 million people

$$P = 30e^{0.05t}$$
$$50 = 30e^{0.05t}$$
$$1.67 = e^{0.05t}$$
$$\ln 1.67 = \ln e^{0.05t}$$
$$\ln 1.67 = 0.05t$$
$$\frac{\ln 1.67}{0.05} = t$$
$$10.3 \approx t$$

It will take approximately 5.7 years to reach 40 million people and 10.3 years to reach 50 million people.

30. The number of bacteria present in a culture at time t is given by the formula $N = 20e^{0.35t}$, where t is in hours. How many bacteria are present initially (that is when $t = 0$)? How many are present after 24 hours? How many hours does it take for the bacteria population to double? (Round your answers to nearest whole number.)

Initially, there are $N = 20e^{0.35(0)} = 20e^{0} = 20$ bacteria present.
After 24 hours there will be $N = 20e^{0.35(24)} = 20e^{8.4} = 20(4447.066748) = 88,941$ bacteria present.

$$
\begin{aligned}
N &= 20e^{0.35t} \\
40 &= 20e^{0.35t} \\
2 &= e^{0.35t} \\
\ln 2 &= \ln e^{0.35t} \\
\ln 2 &= 0.35t \\
\frac{\ln 2}{0.35} &= t \\
1.98 &\approx t
\end{aligned}
$$

The number of bacteria will double after approximately 2 hours.

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Section 2-1 Functions

Goal: To evaluate function values and to determine the domain of functions

Definition: Function
A function is a correspondence between two sets of elements such that to each element in the first set, there corresponds one and only one element in the second set. The first set is called the *domain* and the set of corresponding elements in the second set is called the *range*.

Definition: Function specified by equations
If in an equation in two variables, we get exactly one output (value for the dependent variable) for each input (value for the independent variable), then the equation specifies a function. The graph of such a function is just the graph of the specifying equation.

1. Evaluate the following function at the specified values of the independent variable and simplify the results.

$f(x) = 4x - 5$

a) $f(1) =$

b) $f(-3) =$

c) $f(x-1) =$

d) $f\left(\frac{1}{4}\right) =$

In Problems 2–10 evaluate the given function for $f(x) = x^2 + 1$ and $g(x) = x - 4$.

2. $(f+g)(5) =$

3. $(f-g)(2c) =$

4. $(fg)(-2) =$

5. $\left(\dfrac{f}{g}\right)(0) =$

6. $4 \cdot g(-3) =$

7. $3 \cdot f(4) - 2 \cdot g(-1) =$

8. $\dfrac{f(4)-g(3)}{f(2)} =$

9. $\dfrac{g(-1+h)-g(-1)}{h} =$

10. $\dfrac{f(3+h)-f(3)}{h}$

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Section 2-2 Elementary Functions: Graphs and Transformations

Goal: To describe the shapes of graphs based on vertical and horizontal shifts and reflections, stretches, and shrinks

The domain of the following functions will be the set of real numbers unless it meets one of the following conditions:

1. The function contains a fraction whose denominator has a variable.
   The domain of such a function is the set of real numbers EXCEPT the values of the variable that make the denominator zero.
2. The function contains an even root (square root $\sqrt{\ }$, fourth root $\sqrt[4]{\ }$, etc.).
   The domain of such a function is limited to values of the variable that make the radicand (the part under the radical) greater than or equal to 0.

Basic Elementary Functions:

$f(x) = x$	Identity function
$h(x) = x^2$	Square function
$m(x) = x^3$	Cube function
$n(x) = \sqrt{x}$	Square root function
$p(x) = \sqrt[3]{x}$	Cube root function
$g(x) = \lvert x \rvert$	Absolute value function

In Problems 1–8 find the domain of each function.

1. $g(x) = \dfrac{5}{x-5}$

2. $f(x) = \dfrac{4x}{5x+6}$

3. $h(t) = \sqrt[4]{1-5t}$

4. $g(x) = 1-2x^2$

5. $f(x) = \sqrt[3]{x+4}$

6. $h(w) = \sqrt{w-3}$

7. $f(x) = 2x^3 + 5x^2 - x + 17$

8. $g(x) = \dfrac{3x^4}{4}$

In Problems 9–22 describe how the graph of each function is related to the graph of one of the six basic functions. State the domain of each function. (Do not use a graphing calculator and do not make a chart.)

9. $g(x) = x^2 - 12$

10. $f(x) = \sqrt{x} + 3$

11. $f(x) = -\sqrt{x}$

12. $f(x) = \sqrt[3]{x-4}$

13. $g(x) = (x-6)^2 + 3$

14. $f(x) = -x^2 + 1$

15. $g(x) = 2 - \sqrt{x-4}$

16. $h(x) = |x+7|$

17. $g(x) = \sqrt[3]{x} - 3$

18. $f(x) = |x+3| - 2$

19. $h(x) = -|x-3| + 2$

20. $f(x) = x^3 + 2$

21. $f(x) = -(x+5)^3 - 3$

22. $h(x) = 3 - \sqrt[3]{x-4}$

In Problems 23–31 write an equation for a function that has a graph with the given characteristics.

23. The shape of $y = x^3$ shifted 8 units right.

24. The shape of $y = \sqrt{x}$ shifted 5 units down.

25. The shape of $y = |x|$ reflected over the x-axis and shifted 5 units up.

26. The shape of $y = x^2$ shifted 5 units right and 3 units up.

27. The shape of $y = \sqrt[3]{x}$ reflected over the x-axis and shifted 1 unit up.

28. The shape of $y = x^2$ reflected over the x-axis and shifted 3 units down.

29. The shape of $y = \sqrt{x}$ shifted 4 units left.

30. The shape of $y = x^3$ shifted 6 units right and 2 units down.

31. The shape of $y = |x|$ shifted 6 units right and 5 units up.

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Section 2-3 Quadratic Functions

Goal: To describe functions that are linear and quadratic in nature

Quadratic Functions:

Standard form of a quadratic: $f(x) = ax^2 + bx + c$, where *a, b, c* are real and $a \neq 0$.

Vertex form of a quadratic: $f(x) = a(x-h)^2 + k$, where $a \neq 0$ and (h,k) is the vertex.

Axis of symmetry: $x = h$

Minimum/Maximum value:

If $a > 0$, then the turning point (or vertex) is a minimum point on the graph and the minimum value would be *k*.
If $a < 0$, then the turning point (or vertex) is a maximum point on the graph and the maximum value would be *k*.

For 1–8 find:

a) the domain
b) the vertex
c) the axis of symmetry
d) the *x*-intercept(s)
e) the *y*-intercept
f) the maximum or minimum value of the function

then:

g) Graph the function.
h) State the range.
i) State the interval over which the function is decreasing.
j) State the interval over which the function is increasing.

1. $f(x)=(x+1)^2-3$

2. $f(x) = (x + 2)^2 + 4$

3. $f(x) = -x^2 + 9$

4. $f(x) = -(x-1)^2 - 1$

5. $f(x) = x^2 - 4x$

6. $f(x) = x^2 + 2x - 4$

7. $f(x) = x^2 + 2x + 1$

8. $f(x) = -x^2 + 6x - 5$

9. The revenue and cost functions for a company that manufactures components for washing machines were determined to be

$$R(x) = x(200 - 4x) \qquad \text{and} \qquad C(x) = 160 + 20x,$$

where x is the number of components in millions and $R(x)$ and $C(x)$ are in millions of dollars.

a) How many components must be sold in order for the company to break even? (Break-even points are when $R(x) = C(x)$.) (Round answers to nearest million.)

b) Find the profit equation. ($P(x) = R(x) - C(x)$)

c) Determine the maximum profit. How many components must be sold in order to achieve that maximum profit?

10. A company keeps records of the total revenue (money taken in) in thousands of dollars from the sale of x units (in thousands) of a product. It determines that total revenue is a function $R(x)$ given by

$$R(x) = 300x - x^2.$$

It also keeps records of the total cost of producing x units of the same product. It determines that the total cost is a function $C(x)$ given by

$$C(x) = 40x + 1600.$$

a) Find the break-even points for this company. (Round answer to nearest 1000.)

b) Determine at what point profit is at a maximum. What is the maximum profit? How many units must be sold in order to achieve maximum profit?

11. The cost, $C(x)$, of building a shed is a function of the number of square feet, x, in the shed. If the cost function can be approximated by

$$C(x) = 0.01x^2 - 20x + 25{,}000, \text{ where } 1000 \le x \le 3500$$

a) What would be the cost of building a 1500-square-foot shed?

b) Find the minimum cost to build a shed. How many square-feet would that shed have?

12. The cost of producing computer software is a function of the hourly salary paid (which includes benefits) to employees. If the cost function can be approximated by

$$C(x) = 0.04x^2 - 20x + 6000, \text{where } 200 \le x \le 1000$$

a) What would be the cost if the employees worked 800 hours?

b) Find the number of hours the employees should work in order to minimize the cost. What would the minimum cost be?

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Section 2-4 Polynomial and Rational Functions

Goal: To describe and identify functions that are polynomial and rational in nature

Definition: Polynomial function

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ for n a nonnegative integer, called the degree of the polynomial. The coefficients $a_0, a_1, \cdots, a_n$ are real numbers with $a_n \neq 0$. The domain of a polynomial function is the set of all real numbers.

Definition: Rational function

$f(x) = \dfrac{n(x)}{d(x)}$ $d(x) \neq 0$, where $n(x)$ and $d(x)$ are polynomials. The domain is the set of all real numbers such that $d(x) \neq 0$.

Vertical Asymptotes:

Case 1: Suppose $n(x)$ and $d(x)$have no real zero in common. If c is a real number such that $d(x) = 0$, then the line $x = c$ is a vertical asymptote of the graph.

Case 2: If $n(x)$ and $d(x)$ have one or more real zeros in common, cancel common linear factors and apply Case 1 to the reduced fraction.

Horizontal Asymptotes:

Case 1: If degree $n(x)$ < degree $d(x)$, then $y = 0$ is the horizontal asymptote.
Case 2: If degree $n(x)$ = degree $d(x)$, then $y = a/b$ is the horizontal asymptote, where a is the leading coefficient of $n(x)$ and b is the leading coefficient of $d(x)$.
Case 3: If degree $n(x)$ > degree $d(x)$, there is no horizontal asymptote.

For 1–6, determine each of the following for the polynomial functions:
a) the degree of the polynomial
b) the x-intercept(s) of the graph of the polynomial
c) the y-intercept of the graph of the polynomial

1. $f(x) = x^3 + 2x^2 - 23x - 60 = (x+3)(x-5)(x+4)$

2. $f(x) = x^3 + 8x^2 - 9x - 72 = (x-3)(x+3)(x+8)$

3. $f(x) = x^3 - 3x^2 - 10x + 24 = (x+3)(x-2)(x-4)$

4. $f(x) = x^3 + 4x^2 - x - 4 = (x+4)(x+1)(x-1)$

5. $f(x) = x^4 - 2x^3 - x^2 + 2x + 2 = (x-1)(x+1)(x^2 - 2x - 2)$

6. $f(x) = x^5 + 5x^4 - 20x^2 - x + 15 = (x+3)(x-1)(x+1)(x^2 + 2x - 5)$

For the given rational functions in 7–12,

a) Find the domain.
b) Find any x-intercepts.
c) Find any y-intercept.
d) Find any vertical asymptote.
e) Find any horizontal asymptote.
f) Sketch a graph of $y = f(x)$ for $-10 \le x \le 10$.

7. $f(x) = \dfrac{1}{2x}$

8. $f(x) = \dfrac{4x}{x-4}$

9. $f(x) = \dfrac{5x}{x-2}$

10. $f(x) = \dfrac{2x-4}{x+3}$

11. $f(x)=\dfrac{4+x}{4-x}$

12. $f(x)=\dfrac{1-5x}{1+2x}$

13. A video production company is planning to produce a documentary. The producer estimates that it will cost $104,000 to produce the video and $40 per video to copy and distribute the tape.

a) Assuming that the total cost to market the video, $C(n)$, is linearly related to the total number, n, of videos produced, write an equation for the cost function.

b) The average cost per video for an output of n videos is given by $\bar{C}(n) = \frac{C(n)}{n}$.

Find the average cost function.

c) Sketch a graph of the average cost function for $1000 \le n \le 6000$.

d) What does the average cost per video tend to as production increases?

14. A contractor purchases a piece of equipment for \$36,000. The equipment requires an average expenditure of \$8.25 per hour for fuel and maintenance, and the operator is paid \$13.50 per hour to operate the machinery.

a) Assuming that the total cost per day, $C(h)$, is linearly related to the number of hours, h, that the machine is operated, write an equation for the cost function.

b) The average cost per hour of operating the machine is given by $\bar{C}(h) = \dfrac{C(h)}{h}$.

Find the average cost function.

c) Sketch a graph of the average cost function for $1000 \le h \le 8000$.

d) What cost per hour does the average cost per hour tend to as the number of hours of use increases?

15. The daily cost function for producing x printers for home computers was determined to be

$$C(x) = x^2 + 8x + 6000.$$

The average cost per printer at a production level of x printers per day is $\bar{C}(x) = \frac{C(x)}{x}$.

a) Find the average cost function.

b) Sketch a graph of the average cost function for $25 \le x \le 150$.

c) At what production level is the daily average cost at a minimum? What is that minimum value?

16. The monthly cost function for producing x brake assemblies for a certain type of car is given by

$$C(x) = 3x^2 + 36x + 9000.$$

The average cost per brake assembly at a production level of x assemblies per month is $\bar{C}(x) = \dfrac{C(x)}{x}$.

a) Find the average cost function.

b) Sketch a graph of the average cost function for $5 \le x \le 150$.

c) At what production level is the daily average cost at a minimum? What is that minimum value?

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Section 2-5 Exponential Functions

Goal: To describe and solve functions that are exponential in nature

Rules for Exponents:

$a^m \cdot a^n = a^{m+n}$ Product Rule

$a^0 = 1, \quad a \neq 0$ Zero Exponent Rule

$\dfrac{a^m}{a^n} = a^{m-n}$ Quotient Rule

$\left(a^m\right)^n = a^{mn}$ Power Rule

In Problems 1–8, describe in words the transformations that can be used to obtain the graph of $g(x)$ from the graph of $f(x)$.

1. $g(x) = 4^{x+5} - 4; \quad f(x)=4^x$

2. $g(x) = -3^x - 7; \quad f(x) = 3^x$

3. $g(x) = 2^{x-4} - 6; \quad f(x) = 2^x$

4. $g(x) = -5^{x-4} + 5; \quad f(x) = 5^x$

5. $g(x) = 10^{x-2} - 5;\quad f(x)=10^x$

6. $g(x) = -10^x - 3;\quad f(x) = 10^x$

7. $g(x) = e^{x+1} + 2;\quad f(x) = e^x$

8. $g(x) = -e^x + 5;\quad f(x) = e^x$

In Problems 9–20, solve each equation for x.

9. $10^{7x-4} = 10^{4x+5}$

10. $10^{x^2} = 10^{2x+8}$

11. $6^{5x-4} = 6^{x^2}$

12. $8^{2x^2} = 8^{16x}$

13. $(x+4)^3 = (3x-10)^3$

14. $(2x-7)^5 = (x+1)^5$

15. $(e^2)^3 = e^x$

16. $(e^x)^5 = e^{x^2}$

17. $(e^{2x})^x = e^{15+x}$

18. $3^x \cdot 3^4 = 3^{3x^2}$

19. $2^{x^2} = 2^{12x} \cdot 2^{-32}$

20. $9^x \cdot 9 = 9^{2x^2}$

Interest Formulas

Simple Interest: $A = P(1+rt)$

Compound Interest: $A = P\left(1+\frac{r}{m}\right)^{mt}$

Continuous Compound Interest: $A = Pe^{rt}$

where P is the amount invested (principal), r (expressed as a decimal) is the annual interest rate, t is time invested (in years), m is the number of times a year the interest is compounded, and A is the amount of money in the account after t years (future value).

(Round answers Problems for 21–28 to the nearest dollar)

21. Fred inherited $50,000 from his uncle. He decides to invest his money for 8 years in order to have the greatest down payment when he buys a house. He can choose from 3 different banks.

Bank A offers 1% compounded monthly.
Bank B offers .5% compounded continuously.
Bank C offers .75% compounded daily.

Which bank offers the best plan so Fred can earn the most money from his investment?

22. The day your first child is born, you invest $20,000 in an account that pays 2.5% interest compounded quarterly. How much will be in the account when the child is 18 years old and ready to start to college?

23. When your second child is born, you are able to invest only $10,000 but the account pays 3.2% interest compounded daily. How much will be in the account when this child is 18 years old and ready to start to college?

24. When your third child comes along, money is even tighter and you are able to invest only $5000, but you are able to find a bank that will let you invest the money at 5% compounded continuously. How much will be in the account when this third child is 18 years old and ready to start to college?

25. Joe Vader plans to start his own business in ten years. How much money would he need to invest today in order to have $25,000 in ten years if Joe's bank offers a 10-year CD that pays 1.8% interest compounded monthly.

26. Bill and Sue plan to buy a home in 5 years. How much would they need to invest today at 1.2% compounded daily in order to have $30,000 in five years?

27. Suppose you invest $3000 in a four-year certificate of deposit (CD) that pays 1.5% interest compounded monthly the first 3 years and 2.2% compounded daily the last year. What is the value of the CD at the end of the four years?

28. Suppose you invest $15,000 in a 10-year certificate of deposit (CD) that pays 3.1% interest compounded daily the first 4 years and 3.9% compounded continuously the last six years. What is the value of the CD at the end of the 10 years?

Name ________________________ **Date** ____________ **Class** ___________

Section 2-6 Logarithmic Functions

Goal: To solve problems that are logarithmic in nature

Properties of Logarithms

$\log_a x + \log_a y = \log_a xy$

$\log_a x - \log_a y = \log_a \frac{x}{y}$

$\log_a x^n = n \log_a x$

Exponential Function: $f(x) = a^x,\ a > 0,\ a \neq 1$

$y = \log_a x$ means $x = a^y$

In Problems 1–20, find the value of *x*. (Evaluate to four decimal places if necessary.)

1. $\log_5 x = 3$

2. $\log_4(x+3) = 3$

3. $\log_3 3^8 = 7 + 3x$

4. $\log_2 2^6 = 4 - 3x$

5. $\ln(x+5)=8$

6. $\log_x(x+20)=2$

7. $\log_2(4x+3)=\log_2(6x-5)$

8. $\log_3(8-5x)=\log_3(2x-13)$

9. $\ln x+\ln 8=5$

10. $\ln x-\ln 2=0$

11. $\log(x-1)-\log 4=3$

12. $\ln(x-1)-\ln 6=2$

13. $3^{2x}=14$

14. $7^{x-2}=19$

15. $7^{x-1}=8^{x}$

16. $4^{2x+3}=5^{x-2}$

17. $7^{x+1}=10^{2x}$

18. $e^{x+2}=34.896$

19. $x+7=e^{2}$

20. $e^{8x}=e^{32}$

21. You want to accumulate \$30,000 by your son's eighteenth birthday. How much do you need to invest on the day he is born in an account that will pay 1.9% interest compounded quarterly? (Round your answer to the nearest dollar.)

22. Using the information in Problem 21, how much would you need to invest if you waited until he is 10 years old to start the fund?

23. A bond that sells for \$1000 today can be redeemed for \$1200 in 10 years. If interest is compounded quarterly, what is the annual interest rate for this investment? (Round your answer to two decimal places when written as a percentage.)

24. A bond that sells for $22,000 today can be redeemed for $25,000 in 4 years. If interest is compounded monthly, what is the annual interest rate for this investment? (Round your answer to two decimal places when written as a percentage.)

25. What is the minimum number of months required for an investment of $5,000 to grow to at least $10,000 (double in value) if the investment earns 2.6% annual interest rate compounded monthly? What would be the actual value of the investment after that many months?

26. What is the minimum number of months required for an investment of $5,000 to grow to at least $15,000 (triple in value) if the investment earns 2.6% annual interest rate compounded monthly? What would be the actual value of the investment after that many months?

27. Some years ago, Ms. Martinez invested $7000 at 2% compounded quarterly. The account now contains $10,000. How long ago did she start the account? (Round your answer up to the next year.)

28. Some years ago, Mr. Tang invested $18,000 at 5% compounded monthly. The account now contains $24,000. How long ago did he start the account? (Round your answer up to the next year.)

29. In a certain country, the number of people above the poverty level is currently 30 million and growing at a rate of 5% annually. Assuming that the population is growing continuously, the population, P (in millions), t years from now, is determined by the formula:

$$P = 30e^{0.05t}$$

In how many years will there be 40 million people above the poverty level? 50 million? (Round your answers to nearest tenth of a year.)

30. The number of bacteria present in a culture at time t is given by the formula $N = 20e^{0.35t}$, where t is in hours. How many bacteria are present initially (that is when $t = 0$)? How many are present after 24 hours? How many hours does it take for the bacteria population to double? (Round your answers to nearest whole number.)

NOTES

Chapter 1-2 Graphs and Lines

The Cartesian Coordinate System

The **Cartesian** or **rectangular coordinate system** consists of two real number lines, one horizontal and one vertical, that intersect at their origins.

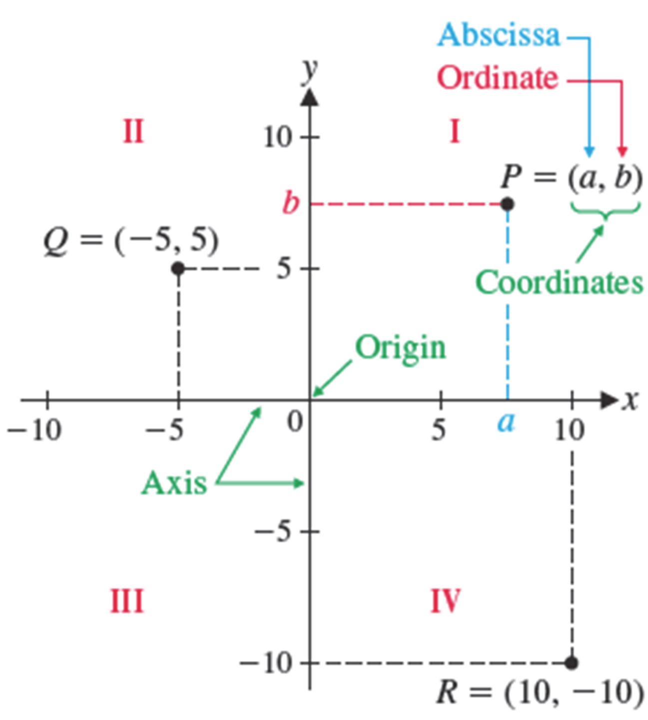
The two number lines are called the **horizontal axis** and the **vertical axis**.

Together, they are referred to as the **coordinate axes**.

The horizontal axis is usually referred to as the ***x* axis** and the vertical axis as the ***y* axis**.

The coordinate axes divide the plane into four parts called **quadrants**, which are numbered counterclockwise using Roman numerals from I to IV, starting with the upper right quadrant.

The figure shows a Cartesian coordinate system with labeled components.



NOTES

The Fundamental Theorem of Analytic Geometry

> There is a one-to-one correspondence between the points in a plane and the elements in the set of all ordered pairs of real numbers.

Write a brief explanation about what this means.

> **DEFINITION Linear Equation in Two Variables**
> A **linear equation in two variables** is an equation that can be written in the **standard form**
>
> $$Ax + By = C$$
>
> where A, B, and C are constants (A and B not both 0), and x and y are variables.

Solutions to Equations in Two Variables

An ordered pair of numbers is a **solution** to an equation in two variables that when substituted into the equation results in a true statement.

Find solutions to the standard form linear equation from the PPT $2x - 3y = 6$ in addition to those outlined in the PPT.

Show that the point (0, –2) is a solution.

Theorem 1 Graph of a Linear Equation in Two Variables

The graph of any equation of the form

$$Ax + By = C \quad (A \text{ and } B \text{ not both } 0)$$

is a line, and any line in a Cartesian coordinate system is the graph of an equation of this form.

Notes and observations about Theorem 1.

Graphing a Linear Equation in Two Variables

To graph a linear function in two variables, plot any two points in the solution set and use a straightedge to draw the line through these two points.

Note how graphing points where $x = 0$ and $y = 0$ can simplify the process of graphing a line.

The point corresponding to $x = 0$ is a point on the ___ axis and is called the __________________.

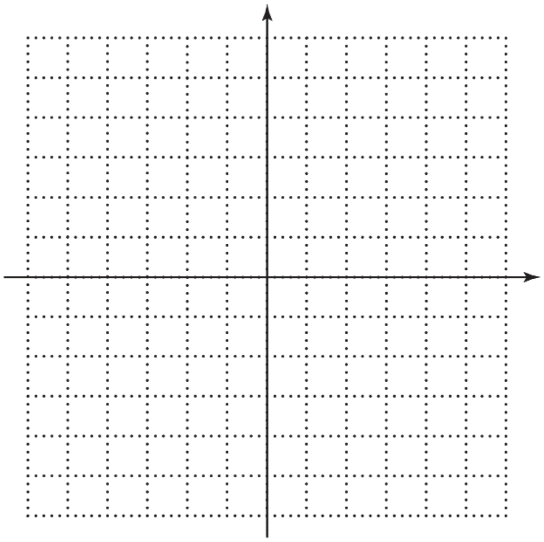
The point corresponding to $y = 0$ is a point on the ____axis and is called the __________________.

Example 1 Using Intercepts to Graph a Line

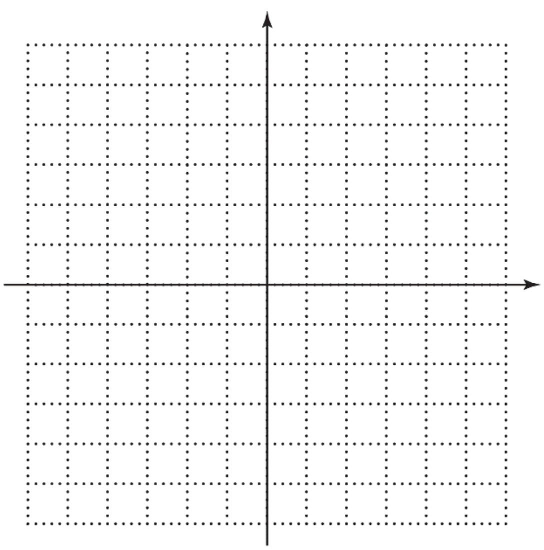
Graph $3x - 4y = 12$

<u>Notes From PPT</u>

NOTES



Matched Problem 1 Graph: $4x - 3y = 12$



NOTES

Example 2 Graph a Line Using a Graphing Calculator

Graph: $3x - 4y = 12$ on a graphing calculator and find the intercepts.

<u>Notes From PPT</u>

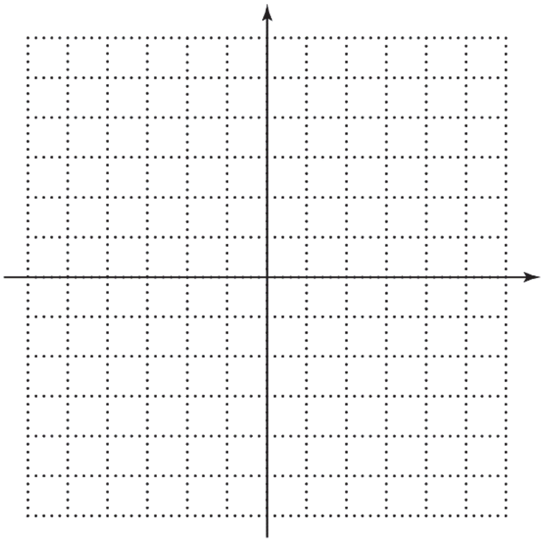
<u>Matched Problem 2</u> Graph $4x - 3y = 12$ on a graphing calculator and find the intercepts.

Example 3 Horizontal and Vertical Lines

(A) Graph $x = -4$ and $y = 6$ simultaneously in the same rectangular coordinate system.

<u>Notes From PPT</u>

NOTES

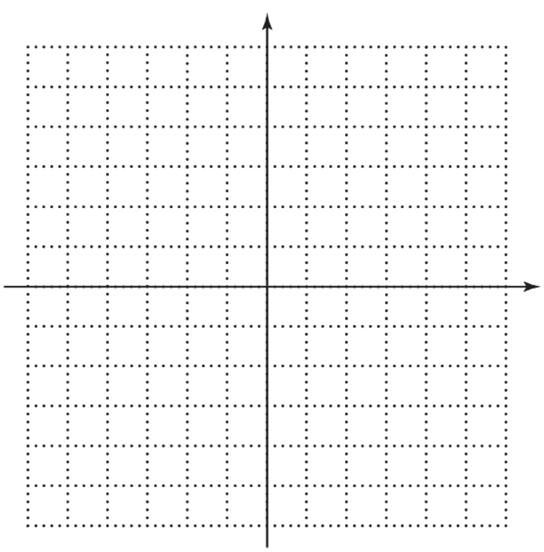


(B) Write the equations of the vertical and horizontal lines that pass through the point (7, –5).

Notes From PPT

Matched Problem 3

(A) Graph $x = 5$ and $y = -3$ simultaneously in the same rectangular coordinate system.



(B) Write the equations of the vertical and horizontal lines that pass through the point (–8, 2)

The Slope of a Line

For two points, (x_1, y_1) and (x_2, y_2), on a line, the ratio of the change in y to the change in x is called the **slope** of the line.

The change in x is often called the ________.

The change in y is often called the ________.

Definition Slope of a Line

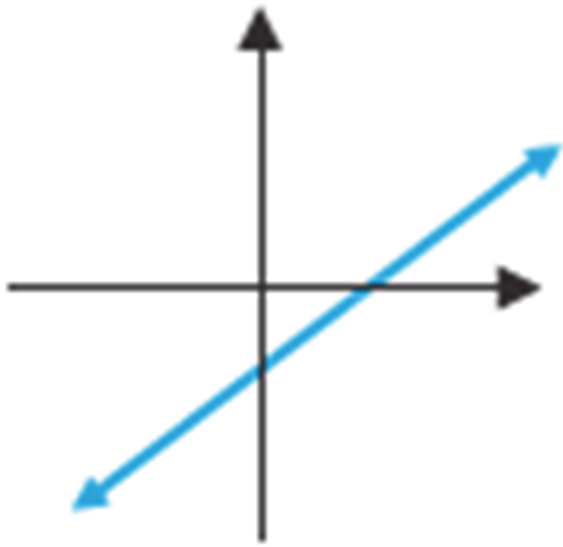
If a line passes through two distinct points, (x_1, y_1) and (x_2, y_2), then its slope is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{for } x_1 \neq x_2$$
$$= \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}}$$

Geometric Interpretation of Positive Slope

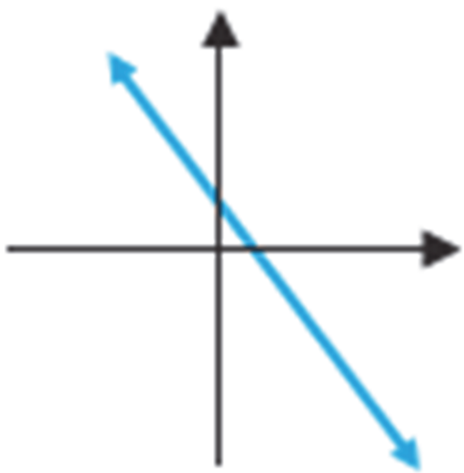
Write a verbal description explaining when a line has a positive slope

NOTES



Geometric Interpretation of Positive Slope

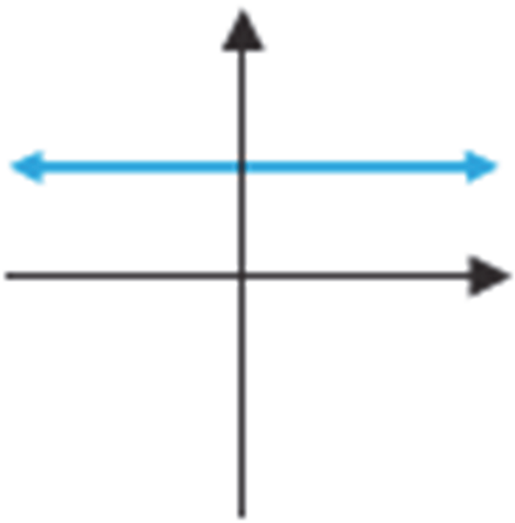
Write a verbal description explaining when a line has a negative slope



Geometric Interpretation of Slope Equal to Zero

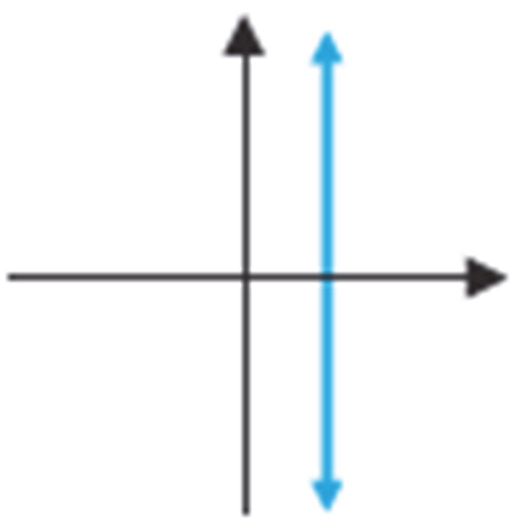
Write a verbal description explaining when a line has a slope equal to zero.

NOTES



Geometric Interpretation of Slope Equal to Zero

Write a verbal description explaining when a line has undefined slope.



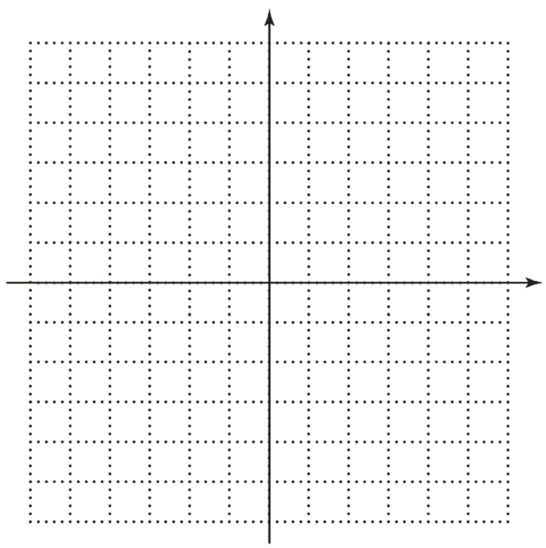
NOTES

Example 4 Finding Slopes

Sketch a line through each pair of points and find the slope of the line.

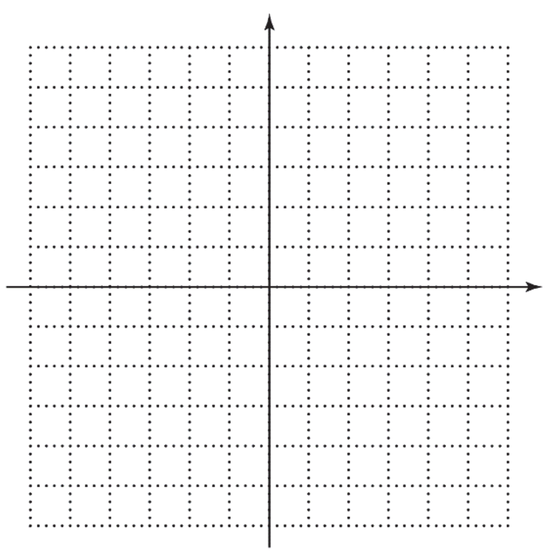
(A) (–3, –2) and (3, 4)

Notes From PPT



(B) (–1, 3) and (2, –3)

Notes From PPT

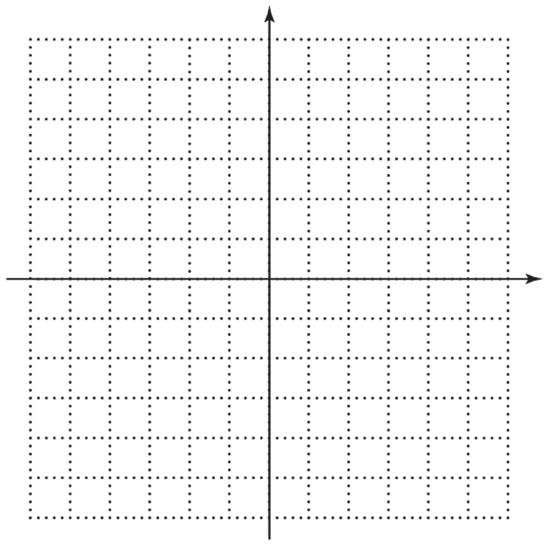


CLICK HERE TO ACCESS THE COMPLETE Solutions

NOTES

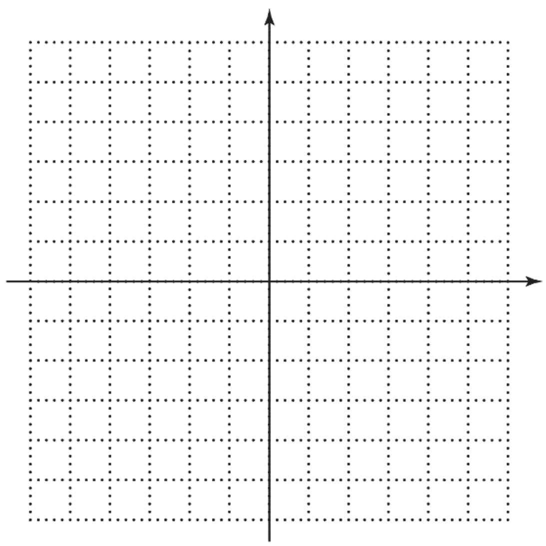
(C) (–2, –3) and (3, –3)

Notes From PPT



(D) (–2, 4) and (–2, –2)

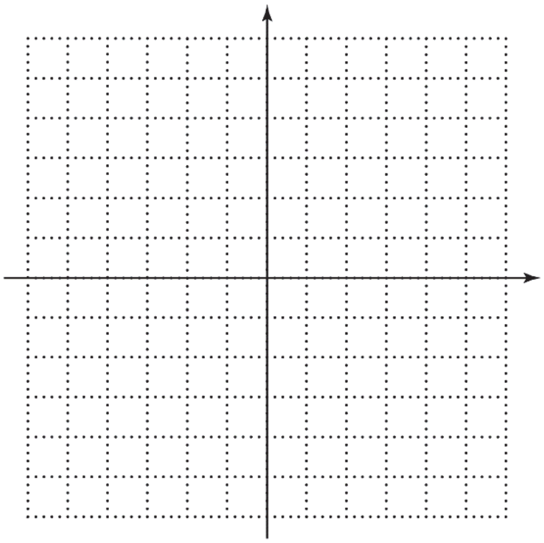
Notes From PPT



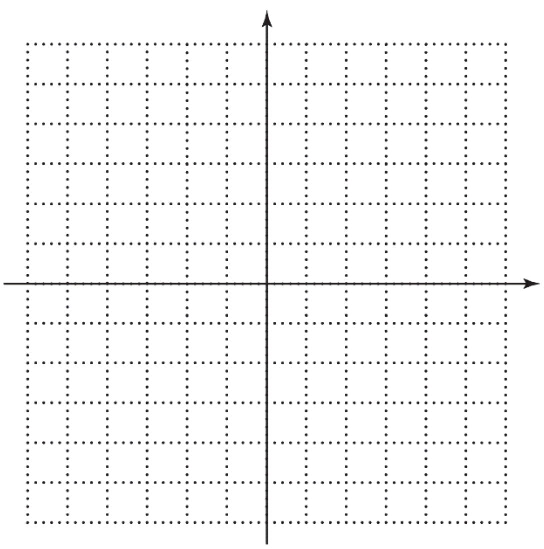
NOTES

Matched Problem 4 Sketch a line through the pair of points and find the slope of the line.

(A) (–2, 4), (3, 4)

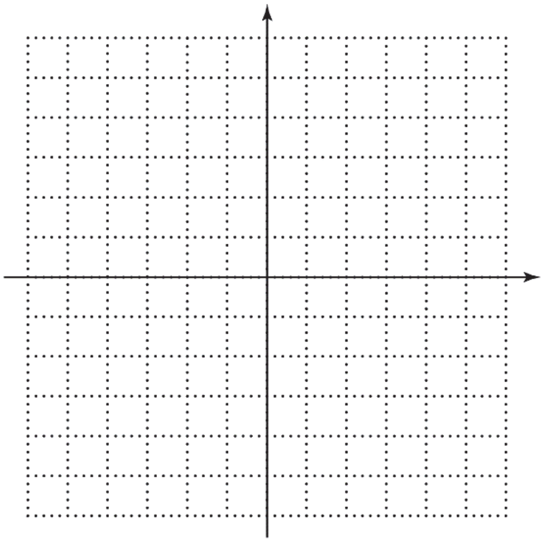


(B) (–2, 4), (0, –4)

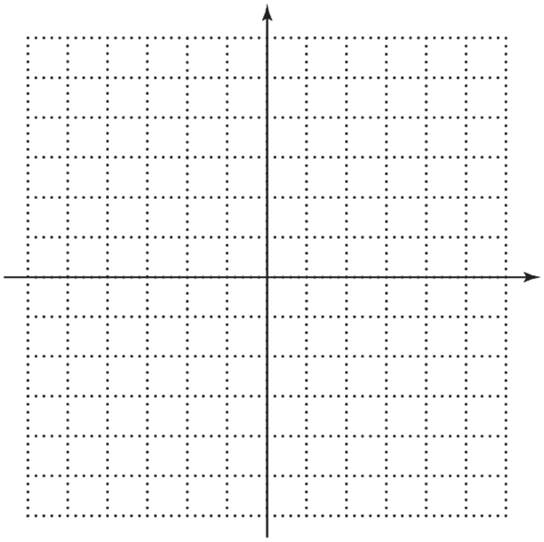


NOTES

(C) (–1, 5), (–1, –2)



(D) (–1, –2), (2, 1)



Equations of Lines: Special Forms

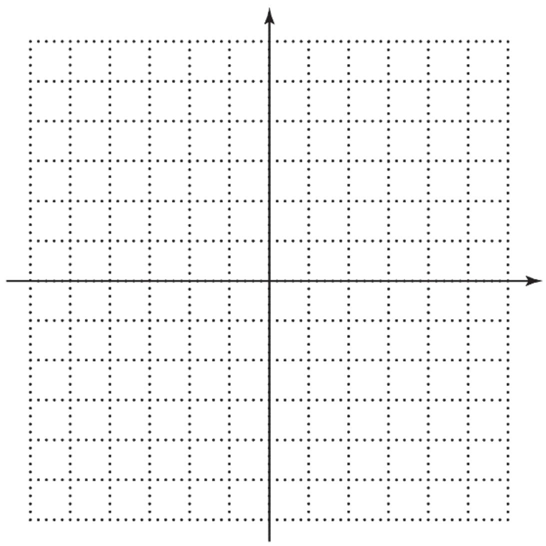
Definition Slope-Intercept Form
The equation $y = mx + b$
(m = slope, b = y intercept)
is called the **slope-intercept form** of an equation of a line.

NOTES

Example 5 Using the Slope-Intercept Form

(A) Find the slope and y intercept, and graph $y = -\frac{2}{3}x - 3$.

Notes From PPT



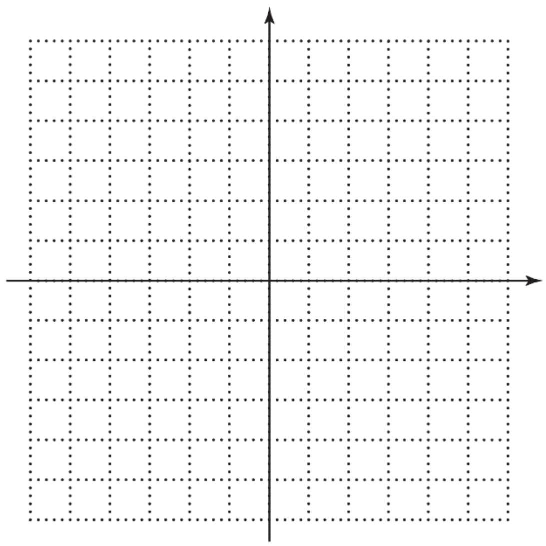
(B) Write the equation of the line with slope $\frac{2}{3}$ and y intercept –2.

Notes From PPT

NOTES

Matched Problem 5 Write the equation of the line with slope $\frac{1}{2}$ and y intercept -1. Graph.

Notes From PPT



Definition Point-Slope Form

An equation of a line with slope m that passes through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$ which is called the **point-slope form** of an equation of a line.

NOTES

Example 6 Using the Point-Slope Form

(A) Find an equation for the line with slope ½ that passes through the point (–4, 3).

Write the final answer in the form $Ax + By = C$.

Notes From PPT

(B) Find an equation for the line that contains the points (–3, 2) and (–4, 5).

Write the final answer in the form $y = mx + b$.

Notes From PPT

Matched Problem 6

(A) Find an equation for the line that has slope $\frac{2}{3}$ and passes through (6, –2).

Write the resulting equation in the form $Ax + By = C$, $A > 0$.

(B) Find an equation for the line that passes through (2, –3) and (4, 3).

Write the resulting equation in the form $y = mx + b$

Summary Table Equations of Lines

Standard form	$Ax + By = C$	A and B not both 0
Slope-intercept form	$y = mx + b$	Slope: m; y intercept: b
Point-slope form	$y - y_1 = m(x - x_1)$	Slope: m; point: (x_1, y_1)
Horizontal line	$y = b$	Slope: 0
Vertical line	$x = a$	Slope: undefined

Example 7 Cost Equation

The management of a company that manufactures skateboards has fixed costs (costs at 0 output) of $300 per day and total costs of $4,300 per day at an output of 100 skateboards per day.

Assume that cost C is linearly related to output x.

(A) Find the slope of the line joining the points associated with outputs of 0 and 100, that is, the line passing through the points (0, 300) and (100, 4,300).

Notes From PPT

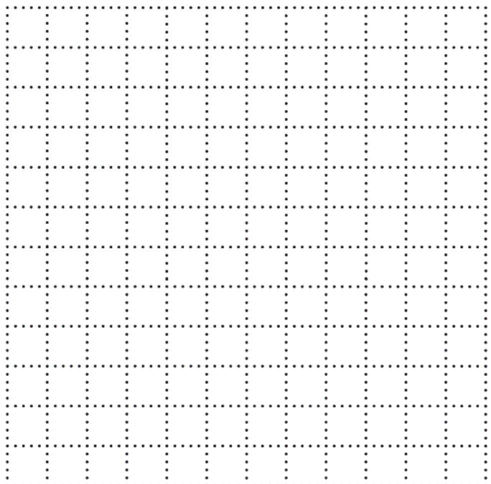
(B) Find an equation of the line relating output to cost. Write the final answer in the form $C = mx + b$.

Notes From PPT

NOTES

(C) Graph the cost equation from part 7(B) for $0 \leq x \leq 200$.

Notes From PPT

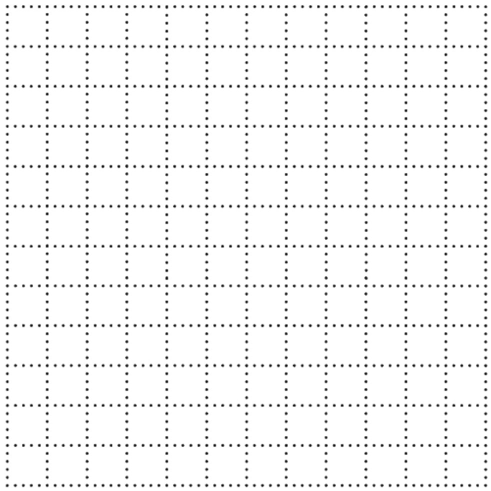


Matched Problem 7 The management of a company that manufactures skateboards has fixed costs (costs at 0 output) of $250 per day and total costs of $3,450 per day at an output of 80 skateboards per day. Assume that cost C is linearly related to output x.

(A) Find the slope of the line joining the points associated with outputs of 0 and 100, that is, the line passing through (0, 250) and (80, 3,450).

(B) Find an equation of the line relating output to cost. Write the final answer in the form $C = mx + b$.

(C) Graph the cost equation from part (B) for $0 \le x \le 200$



Supply and Demand

In a free market, the price of a product is determined by the relationship between supply and demand.

The price tends to stabilize at the point of intersection of the demand and supply equations.

This point of intersection is called the ________________.

The corresponding price is called the _________________.

The common value of supply and demand is called the _________________.

Example 8 Supply and Demand

At a price of \$9.00 per box of oranges, the supply is 320,000 boxes and the demand is 200,000 boxes.

At a price of \$8.50 per box, the supply is 270,000 boxes and the demand is 300,000 boxes.

(A) Find a price-supply equation of the form $p = mx + b$, where p is the price in dollars and x is the corresponding supply in thousands of boxes.

<u>Notes From PPT</u>

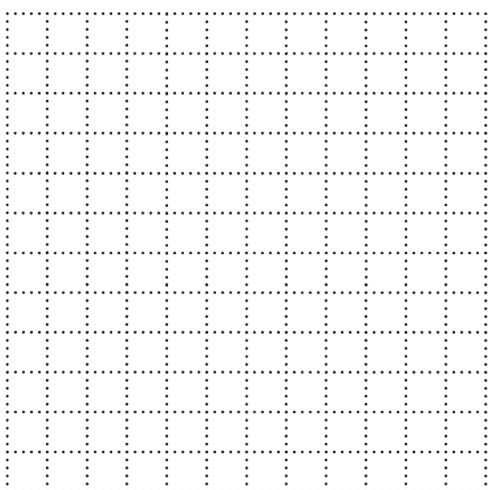
NOTES

(B) Find a price-demand equation of the form $p = mx + b$, where p is the price in dollars and x is the corresponding demand in thousands of boxes.

Notes From PPT

(C) Graph the price-supply and price-demand equations in the same coordinate system and find their intersection.

Notes From PPT



NOTES

Matched Problem 8 At a price of $12.59 per box of grapefruit, the supply is 595,000 boxes and the demand is 650,000 boxes. At a price of $13.19 per box, the supply is 695,000 boxes and the demand is 590,000 boxes. Assume that the relationship between price and supply is linear and that the relationship between price and demand is linear.

(A) Find a price-supply equation of the form $p = mx + b$.

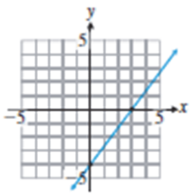
(B) Find a price-demand equation of the form $p = mx + b$.

(C) Find the equilibrium point.

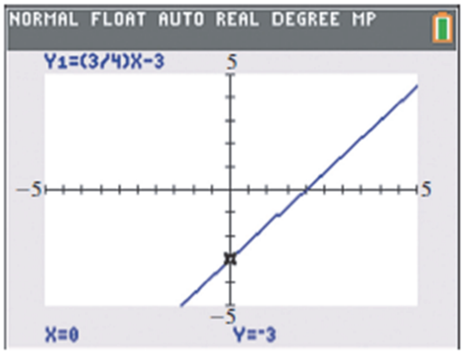
NOTES

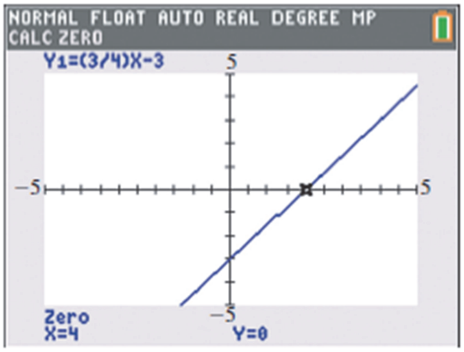
Answers to Matched Problems

1.

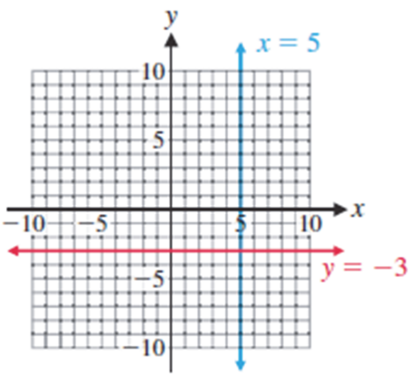


2. y intercept $= -4$, x intercept $= 3$





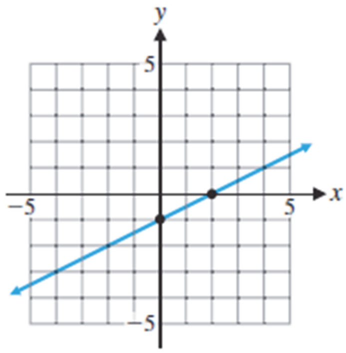
3. (A)



(B) Horizontal line: $y = 2$;
vertical line: $x = -8$

4. (A) 0 **(B)** –4 **(C)** Not defined **(D)** 1

5. $y = \frac{1}{2}x - 1$



6. (A) $2x - 3y = 18$ **(B)** $y = 3x - 9$

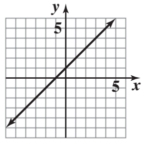
7. (A) $m = 40$ **(C)** $C = 40x + 250$

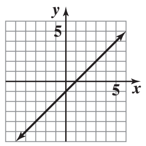
8. (A) $p = 0.006x + 9.02$ **(B)** $p = -0.01x + 19.09$ **(C)** (629, 12.80)

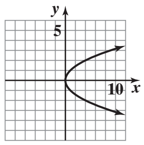
ANSWERS

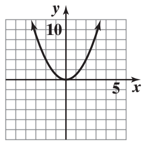
Chapter 2

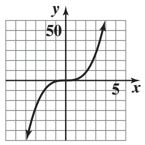
Exercises 2.1

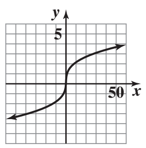
1. 

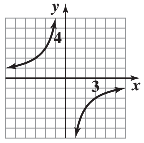
2. 

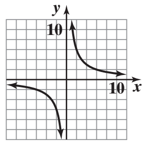
3. 

4. 

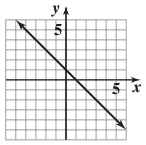
5. 

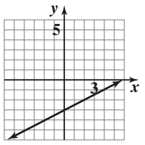
6. 

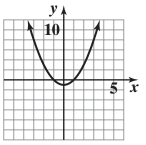
7. 

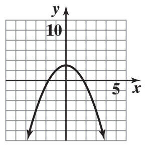
8. 

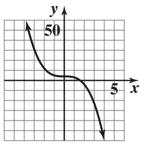
9. A function **10.** A function **11.** Not a function **12.** Not a function **13.** A function **14.** A function **15.** A function **16.** A function **17.** Not a function **18.** Not a function **19.** A function **20.** Not a function **21.** Linear **22.** Neither **23.** Neither **24.** Linear **25.** Linear **26.** Neither **27.** Constant **28.** Constant

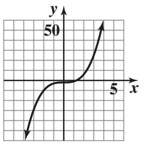
29. 

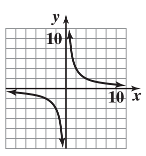
30. 

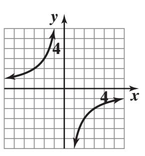
31. 

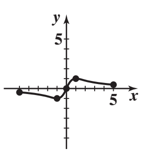
32. 

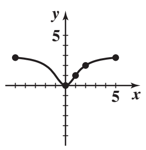
33. 

34. 

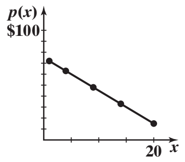
35. 

36. 

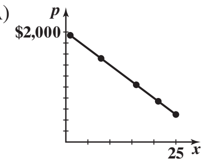
37. 

38. 

39. $y = 0$ **40.** $y = 0$ **41.** $y = 4$ **42.** $y = 3$ **43.** $x = -5$ **44.** $x = 5$ **45.** $x = -6$ **46.** $x = -5, 0, 4$ **47.** All real numbers **48.** All real numbers **49.** All real numbers except -4 **50.** All real numbers except 2 **51.** $x \le 7$ **52.** $x > -5$ **53.** Yes; all real numbers **54.** Yes; all real numbers **55.** No; for example, when $x = 0$, $y = \pm 2$ **56.** Yes; all real numbers except 0 **57.** Yes; all real numbers except 0 **58.** No; when $x = 0$, $y = \pm 3$ **59.** No; when $x = 1$, $y = \pm 1$ **60.** Yes; $x \ge 0$ **61.** $25x^2 - 4$ **62.** $9x^2 - 4$ **63.** $x^2 + 4x$ **64.** $x^2 - 2x - 3$ **65.** $x^4 - 4$ **66.** $x^6 - 4$ **67.** $x - 4$ **68.** $\sqrt{x} - 4$ **69.** $h^2 - 4$ **70.** $h^2 + 1$ **71.** $4h + h^2$ **72.** $5 - 6h + h^2$ **73.** $4h + h^2$ **74.** $-6h + h^2$ **75.** (A) $4x + 4h - 3$ (B) $4h$ (C) 4 **76.** (A) $-3x - 3h + 9$ (B) $-3h$ (C) -3 **77.** (A) $4x^2 + 8xh + 4h^2 - 7x - 7h + 6$ (B) $8xh + 4h^2 - 7h$ (C) $8x + 4h - 7$ **78.** (A) $3x^2 + 6xh + 3h^2 + 5x + 5h - 8$ (B) $6xh + 3h^2 + 5h$ (C) $6x + 3h + 5$ **79.** (A) $20x + 20h - x^2 - 2xh - h^2$ (B) $20h - 2xh - h^2$ (C) $20 - 2x - h$ **80.** (A) $x^2 + 2xh + h^2 + 40x + 40h$ (B) $2xh + h^2 + 40h$ (C) $2x + h + 40$ **81.** $P(w) = 2w + \frac{50}{w}, w > 0$ **82.** $P(l) = 2l + \frac{162}{l}, l > 0$ **83.** $A(l) = l(50 - l), 0 < l < 50$ **84.** $A(w) = (80 - w)w, 0 < w < 80$

85. (A) 

(B) \$54; \$42

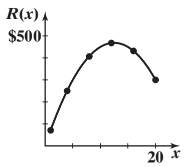
86. (A) 

(B) \$1,340; \$920

87. (A) $R(x) = (75 - 3x)x, 1 \le x \le 20$

(B)

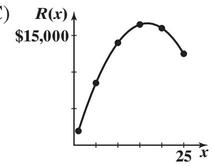
x	$R(x)$
1	72
4	252
8	408
12	468
16	432
20	300

(C) 

88. (A) $R(x) = x(2{,}000 - 60x), 1 \le x \le 25$

(B)

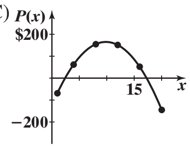
x	$R(x)$
1	1,940
5	8,500
10	14,000
15	16,500
20	16,000
25	12,500

(C) 

89. (A) $P(x) = 59x - 3x^2 - 125, 1 \le x \le 20$

(B)

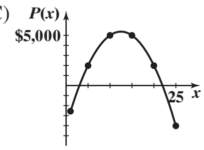
x	$P(x)$
1	-69
4	63
8	155
12	151
16	51
20	-145

(C) 

90. (A) $P(x) = 1{,}500x - 60x^2 - 4{,}000, 1 \le x \le 25$

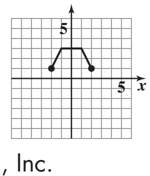
(B)

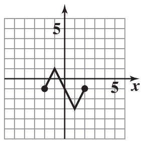
x	$P(x)$
1	$-2{,}560$
5	2,000
10	5,000
15	5,000
20	2,000
25	$-4{,}000$

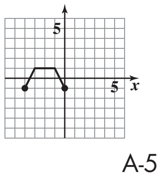
(C) 

91. $v = \frac{75 - w}{15 + w}$; 1.9032 cm/sec **92.** (A) $v = 0.4s + 0.28$; 48.4% (B) $s = 2.5v - 0.7$; 57.5%

Exercises 2.2

1. Domain: all real numbers; range: $[-4, \infty]$ **2.** Domain: $[0, \infty)$; range: $[1, \infty)$ **3.** Domain: all real numbers; range: all real numbers **4.** Domain: all real numbers; range: $[10, \infty)$ **5.** Domain: $[0, \infty)$; range: $(-\infty, 8]$ **6.** Domain: all real numbers; range: all real numbers **7.** Domain: all real numbers; range: all real numbers **8.** Domain: all real numbers; range: $(-\infty, 15]$ **9.** Domain: all real numbers; range: $[9, \infty)$ **10.** Domain: all real numbers; range: all real numbers **11.** 

12. 

13. 

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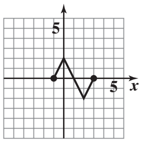

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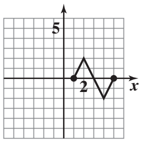

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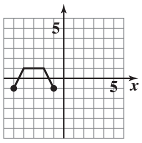

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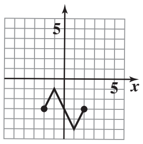

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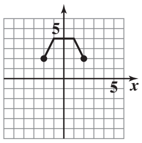

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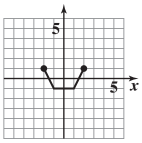

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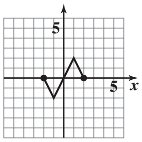

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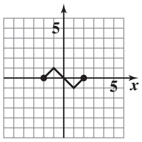

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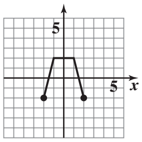

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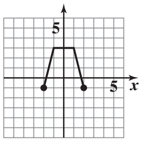

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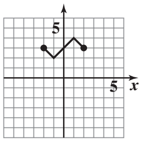

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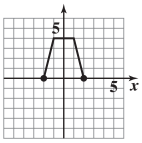

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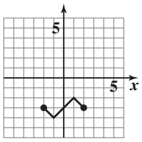
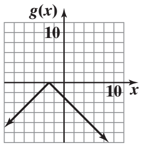
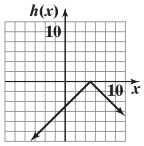
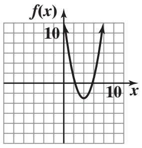
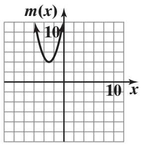
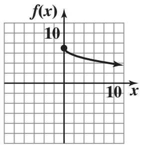


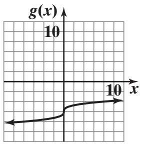
27. The graph of $g(x) = -|x + 3|$ is the graph of $y = |x|$ reflected in the x axis and shifted 3 units to the left.


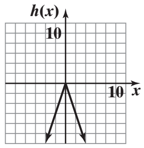
28. The graph of $h(x) = -|x - 5|$ is the graph of $y = |x|$ reflected in the x axis and shifted 5 units to the right.


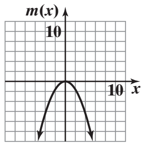
29. The graph of $f(x) = (x - 4)^2 - 3$ is the graph of $y = x^2$ shifted 4 units to the right and 3 units down.


30. The graph of $m(x) = (x + 3)^2 + 4$ is the graph of $y = x^2$ shifted 3 units to the left and 4 units up.


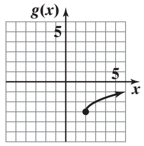
31. The graph of $f(x) = 7 - \sqrt{x}$ is the graph of $y = \sqrt{x}$ reflected in the x axis and shifted 7 units up.


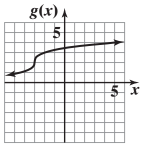
32. The graph of $g(x) = -6 + \sqrt[3]{x}$ is the graph of $y = \sqrt[3]{x}$ shifted 6 units down.


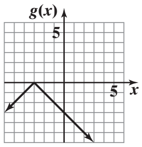
33. The graph of $h(x) = -3|x|$ is the graph of $y = |x|$ reflected in the x axis and vertically stretched by a factor of 3.


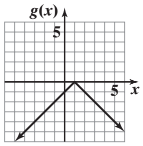
34. The graph of $m(x) = -0.4x^2$ is the graph of $y = x^2$ reflected in the x axis and vertically shrunk by a factor of 0.4.


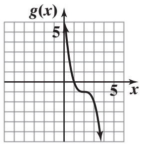
35. The graph of the basic function $y = x^2$ is shifted 2 units to the left and 3 units down. Equation: $y = (x + 2)^2 - 3$. **36.** The graph of the basic function $y = |x|$ is shifted 3 units to the right and 2 units up. Equation: $y = |x - 3| + 2$. **37.** The graph of the basic function $y = x^2$ is reflected in the x axis and shifted 3 units to the right and 2 units up. Equation: $y = 2 - (x - 3)^2$. **38.** The graph of the basic function $y = |x|$ is reflected in the x axis, shifted 2 units to the left and 3 units up. Equation: $y = 3 - |x + 2|$. **39.** The graph of the basic function $y = \sqrt{x}$ is reflected in the x axis and shifted 4 units up. Equation: $y = 4 - \sqrt{x}$. **40.** The graph of the basic function $\sqrt[3]{x}$ is reflected in the x axis and shifted 2 units up. Equation: $y = 2 - \sqrt[3]{x}$. **41.** The graph of the basic function $y = x^3$ is shifted 2 units to the left and 1 unit down. Equation: $y = (x + 2)^3 - 1$. **42.** The graph of the basic function $y = x^3$ is reflected in the x axis, shifted 3 units to the right and 1 unit up. Equation: $y = 1 - (x - 3)^3$.

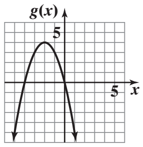
43. $g(x) = \sqrt{x - 2} - 3$


44. $g(x) = \sqrt[3]{x + 3} + 2$


45. $g(x) = -|x + 3|$


46. $g(x) = -|x - 1|$


47. $g(x) = -(x - 2)^3 - 1$


48. $g(x) = 4 - (x + 2)^2$


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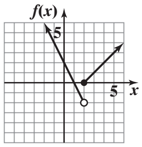

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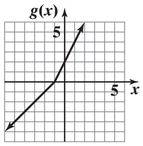

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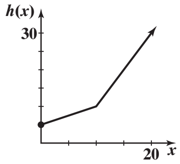

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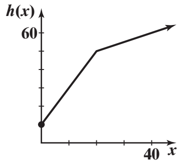

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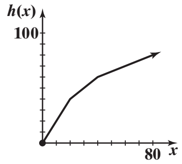
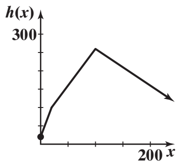
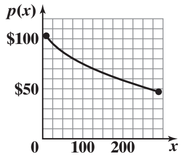


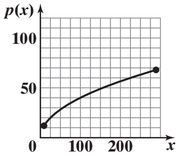
54.


55. The graph of the basic function $y = |x|$ is reflected in the x axis and vertically shrunk by a factor of 0.5. Equation: $y = -0.5|x|$.

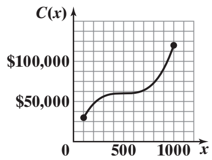
56. The graph of the basic function $y = x$ is reflected in the x axis and vertically stretched by a factor of 2. Equation: $y = -2x$. **57.** The graph of the basic function $y = x^2$ is reflected in the x axis and vertically stretched by a factor of 2. Equation: $y = -2x^2$. **58.** The graph of the basic function $y = |x|$ is vertically stretched by a factor of 4. Equation: $y = 4|x|$. **59.** The graph of the basic function $y = \sqrt[3]{x}$ is reflected in the x axis and vertically stretched by a factor of 3. Equation: $y = -3\sqrt[3]{x}$. **60.** The graph of the basic function $y = x^3$ is vertically shrunk by a factor of 0.25. Equation: $y = 0.25x^3$. **61.** Reversing the order does not change the result. **62.** Reversing the order does not change the result. **63.** Reversing the order can change the result. **64.** Reversing the order can change the result. **65.** Reversing the order can change the result. **66.** Reversing the order does not change the result. **67.** (A) The graph of the basic function $y = \sqrt{x}$ is reflected in the x axis, vertically expanded by a factor of 4, and shifted up 115 units.

(B)


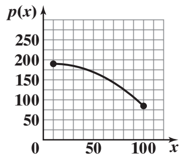
68. (A) The graph of the basic function $y = \sqrt{x}$ is vertically expanded by a factor of 4.

(B)


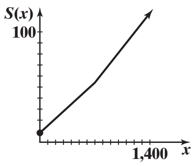
69. (A) The graph of the basic function $y = x^3$ is vertically contracted by a factor of 0.000 48 and shifted right 500 units and up 60,000 units.

(B)


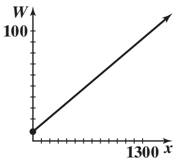
70. (A) The graph of the basic function $y = x^2$ is reflected in the x axis, vertically contracted by a factor of 0.013, and shifted 10 units to the right and 190 units up.

(B)


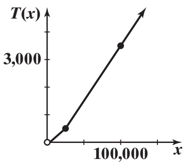
71. (A) $S(x) = \begin{cases} 8.5 + 0.065x & \text{if } 0 \le x \le 700 \\ -9 + 0.09x & \text{if } x > 700 \end{cases}$

(B)


72. (A) $W(x) = \begin{cases} 8.5 + 0.065x & \text{if} 0 \le x \le 700 \\ 16.9 + 0.053x & \text{if} x > 700 \end{cases}$

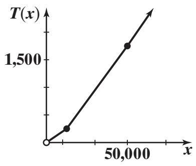
(B)


73. (A) $T(x) = \begin{cases} 0.02x & \text{if } 0 < x \le 25{,}000 \\ 0.04x - 500 & \text{if } 25{,}000 < x \le 100{,}000 \\ 0.06x - 2{,}500 & \text{if } x > 100{,}000 \end{cases}$

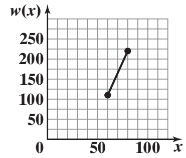
(B)


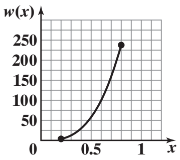
(C) \$1,700; \$4,100

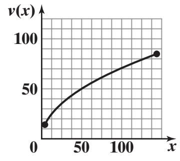
74. (A) $T(x) = \begin{cases} 0.02x & \text{if } 0 \le x < 12{,}500 \\ 0.04x - 250 & \text{if } 12{,}500 < x \le 50{,}000 \\ 0.06x - 1{,}250 & \text{if } x > 50{,}000 \end{cases}$

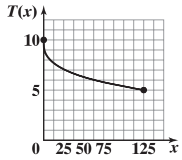
(B)


(C) \$1,030; \$2,590 **75.** (A) The graph of the basic function $y = x$ is vertically stretched by a factor of 5.5 and shifted down 220 units.

(B)


76. (A) The graph of the basic function $y = x^3$ is vertically stretched by a factor of 463. (B)


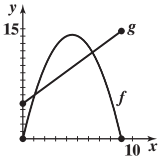
77. (A) The graph of the basic function $y = \sqrt{x}$ is vertically stretched by a factor of 7.08. (B)


78. (A) The graph of the basic function $y = \sqrt[3]{x}$ is reflected in the x axis and shifted up 10 units. (B)


Exercises 2.3

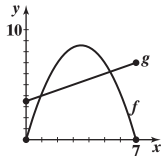
1. $f(x) = (x - 5)^2 - 25$ **2.** $f(x) = (x + 8)^2 - 64$
3. $f(x) = (x + 10)^2 - 50$ **4.** $f(x) = (x - 6)^2 - 44$
5. $f(x) = -2(x - 1)^2 - 3$ **6.** $f(x) = 3(x + 3)^2 - 6$
7. $f(x) = 2\left(x + \frac{1}{2}\right)^2 + \frac{1}{2}$ **8.** $f(x) = -5\left(x - \frac{3}{2}\right)^2 + \frac{1}{4}$
9. The graph of $f(x)$ is the graph of $y = x^2$ shifted right 2 units and down 1 unit. **10.** The graph of $g(x)$ is the graph of $y = x^2$ shifted right 1 unit and down 6 units. **11.** The graph of $m(x)$ is the graph of $y = x^2$ reflected in the x axis, then shifted right 3 units and up 5 units. **12.** The graph of $n(x)$ is the graph of $y = x^2$ reflected in the x axis, then shifted right 4 units and up 7 units. **13.** (A) m (B) g (C) f (D) n **14.** (A) g (B) m (C) n (D) f **15.** (A) x int.: 1, 3; y int.: -3 (B) Vertex: (2, 1) (C) Max.: 1 (D) Range: $y \le 1$ or $(-\infty, 1]$ **16.** (A) x int.: $-5, -1$; y int.: -5 (B) Vertex: $(-3, 4)$ (C) max.: 4 (D) Range: $y \le 4$ or $(-\infty, 4]$ **17.** (A) x int.: $-3, -1$; y int.: 3 (B) Vertex: $(-2, -1)$ (C) Min.: -1 (D) Range: $y \ge -1$ or $[-1, \infty)$ **18.** (A) x int.: 1, 5; y int.: 5 (B) Vertex: $(3, -4)$ (C) min.: -4 (D) Range: $y \ge -4$ or $[-4, \infty)$ **19.** (A) x int.: $3 \pm \sqrt{2}$; y int.: -7 (B) Vertex: $(3, 2)$ (C) Max.: 2 (D) Range: $y \le 2$ or $(-\infty, 2]$ **20.** (A) x int.: $-2 \pm \sqrt{3}$; y int.: -1 (B) Vertex: $(-2, 3)$ (C) max.: 3 (D) Range: $y \le 3$ or $(-\infty, 3]$ **21.** (A) x int.: $-1 \pm \sqrt{2}$; y int.: -1 (B) Vertex: $(-1, -2)$ (C) Min.: -2 (D) Range: $y \ge -2$ or $[-2, \infty)$ **22.** (A) x int.: $4 \pm \sqrt{3}$; y int.: 13 (B) Vertex: $(4, -3)$ (C) min.: -3 (D) Range: $y \ge -3$ or $[-3, \infty)$ **23.** $y = -[x - (-2)]^2 + 5$ or $y = -(x + 2)^2 + 5$ **24.** $y = -(x - 4)^2 + 2$ **25.** $y = (x - 1)^2 - 3$ **26.** $y = [x - (-3)]^2 + 1$ or $y = (x + 3)^2 + 1$ **27.** Vertex form: $(x - 4)^2 - 4$ (A) x int.: 2, 6;

y int.: 12 (B) Vertex: $(4, -4)$ (C) Min.: -4 (D) Range: $y \geq -4$ or $[-4, \infty)$ **28.** Vertex form: $(x - 3)^2 - 4$ (A) x int.: 1, 5; y int.: 5 (B) Vertex: $(3, -4)$ (C) min.: -4 (D) Range: $y \geq -4$ or $[-4, \infty)$ **29.** Vertex form: $-4(x - 2)^2 + 1$ (A) x int.: 1.5, 2.5; y int.: -15 (B) Vertex: $(2, 1)$ (C) Max.: 1 (D) Range: $y \leq 1$ or $(-\infty, 1]$ **30.** Vertex form: $-4(x + 1)^2 + 1$ (A) x int.: $-\frac{3}{2}, -\frac{1}{2}$; y int.: -3 (B) Vertex: $(-1, 1)$ (C) max.: 1 (D) Range: $y \leq 1$ or $(-\infty, 1]$
31. Vertex form: $0.5(x - 2)^2 + 3$ (A) x int.: none; y int.: 5 (B) Vertex: $(2, 3)$ (C) Min.: 3 (D) Range: $y \geq 3$ or $[3, \infty)$
32. Vertex form: $0.5(x + 4)^2 + 2$ (A) x int.: none; y int.: 10 (B) Vertex: $(-4, 2)$ (C) min.: 2 (D) Range: $y \geq 2$ or $[2, \infty)$
33. (A) $-4.87, 8.21$ (B) $-3.44, 6.78$ (C) No solution
34. (A) $-1.53, 6.53$ (B) 0.36, 4.64 (C) No solution **35.** 651.0417
36. 347.1429 **37.** $g(x) = 0.25(x - 3)^2 - 9.25$ (A) x int.: -3.08, 9.08; y int.: -7 (B) Vertex: $(3, -9.25)$ (C) Min.: -9.25 (D) Range: $y \geq -9.25$ or $[-9.25, \infty)$ **38.** $m(x) = 0.20(x - 4)^2 - 4.2$ (A) x int.: -0.58, 8.58; y int.: -1 (B) Vertex: $(4, -4.2)$ (C) min.: -4.2 (D) Range: $y \geq -4.2$ or $[-4.2, \infty)$ **39.** $f(x) = -0.12(x - 4)^2 + 3.12$ (A) x int.: -1.1, 9.1; y int.: 1.2 (B) Vertex: $(4, 3.12)$ (C) Max.: 3.12 (D) Range: $y \leq 3.12$ or $(-\infty, 3.12]$ **40.** $n(x) = -0.15(x + 3)^2 + 4.65$ (A) x int.: -8.57, 2.57; y int.: 3.30 (B) Vertex: $(-3, 4.65)$ (C) max.: 4.65 (D) Range: $y \leq 4.65$ or $(-\infty, 4.65]$ **41.** $(-\infty, -5) \cup (3, \infty)$
42. $(-6, 3)$ **43.** $[-3, 2]$ **44.** $(-\infty, -4] \cup [-3, \infty)$
45. $x = -5.37, 0.37$ **46.** $x = -1.27, 2.77$ **47.** $-1.37 < x < 2.16$
48. $-0.88 \leq x \leq 3.52$ **49.** $x \leq -0.74$ or $x \geq 4.19$ **50.** $x < -1$ or $x > 2.72$ **51.** Axis: $x = 2$; vertex: (2, 4); range: $y \geq 4$ or $[4, \infty)$; no x int.
52. Axis: $x = -3$; vertex: $(-3, -5)$; range: $y \leq -5$ or $(-\infty, -5]$; no x intercept **53.** (A)



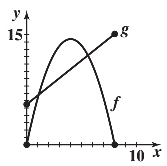
(B) 1.64, 7.61
(C) $1.64 < x < 7.61$
(D) $0 \leq x < 1.64$ or $7.61 < x \leq 10$

54. (A)



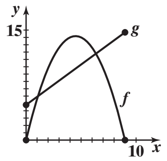
(B) 0.93, 5.35 (C) $0.93 < x < 5.35$
(D) $0 \leq x < 0.93$ or $5.35 < x \leq 7$

55. (A)



(B) 1.10, 5.57 (C) $1.10 < x < 5.57$
(D) $0 \leq x < 1.10$ or $5.57 < x \leq 8$

56. (A)

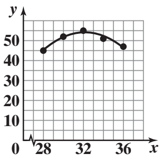


(B) 1.08, 6.35 (C) $1.08 < x < 6.35$
(D) $0 \leq x < 1.08$ or $6.35 < x \leq 9$

65. (A)

x	28	30	32	34	36
Mileage	45	52	55	51	47
$f(x)$	45.3	51.8	54.2	52.4	46.5

(B)

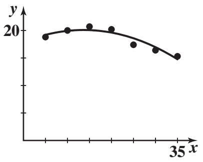


(C) $f(31) = 53.50$ thousand miles;
(D) $f(35) = 49.95$ thousand miles;

66. (A)

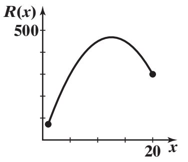
x	5	10	15	20	25	30	35
Market share	18.8	20.0	20.7	20.2	17.4	16.4	15.3
$f(x)$	19.2	19.9	20.1	19.6	18.6	17.0	14.8

(B)



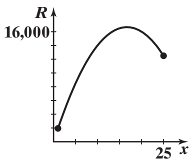
(C) $f(45) = 8.6\%$; $f(48) = 6.3\%$

69. (A)



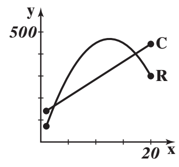
(B) 12.5 (12,500,000 chips); \$468,750,000 (C) \$37.50

70. (A)



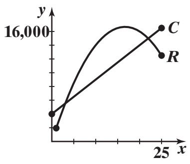
(B) 16.667 (16,667 computers); \$16,667,000 (C) \$1,000

71. (A)



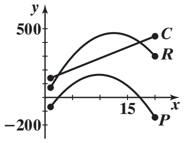
(B) 2,415,000 chips and 17,251,000 chips (C) Loss: $1 \leq x < 2.415$ or $17.251 < x \leq 20$; profit: $2.415 < x < 17.251$

72. (A)

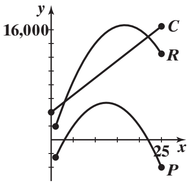


(B) 3,035 computers and 21,965 computers (C) Loss: $1 \leq x < 3.035$ or $21.965 < x \leq 25$; profit: $3.035 < x < 21.965$

73. (A) $P(x) = 59x - 3x^2 - 125$

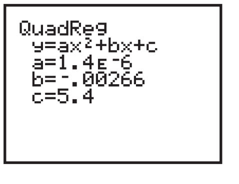


(C) Intercepts and break-even points: 2,415,000 chips and 17,251,000 chips
(D) Maximum profit is \$165,083,000 at a production level of 9,833,000 chips. This is much smaller than the maximum revenue of \$468,750,000.
74. (A) $P(x) = 1{,}500x - 60x^2 - 4{,}000$

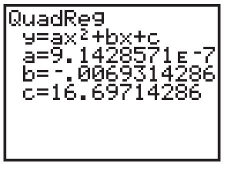


(C) Intercepts and break-even points: 3,035 computers and 21,965 computers
(D) Maximum profit is \$5,375,000 at a production level of 12,500 computers. This is much smaller than the maximum revenue of \$16,666,667.

75. $x = 0.14$ cm **76.** $x = 0.10$ cm **77.** 10.6 mph



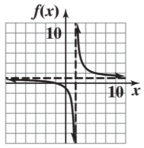
78. 5.6 mpg



Exercises 2.4

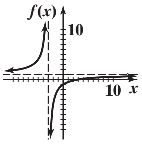
1. (A) 1 (B) −3 (C) 21 **2.** (A) 2 (B) 2, 3 (C) 6 **3.** (A) 2 (B) −5, −4 (C) 20 **4.** (A) 1 (B) 10 (C) 30 **5.** (A) 6 (B) None (C) 15 **6.** (A) 8 (B) None (C) 10 **7.** (A) 5 (B) 0, −6 (C) 0 **8.** (A) 4 (B) −7, 5 (C) 1,225 **9.** (A) 11 (B) −5, −2, 5 (C) −12,800 **10.** (A) 10 (B) $-3, \frac{5}{2}, 3$ (C) 164,025 **11.** (A) 4 (B) Negative **12.** (A) 2 (B) Negative **13.** (A) 5 (B) Negative **14.** (A) 3 (B) Negative **15.** (A) 1 (B) Negative **16.** (A) 4 (B) Positive **17.** (A) 6 (B) Positive **18.** (A) 6 (B) Positive **19.** 10 **20.** 7 **21.** 1 **22.** 0 **23.** (A) x int.: −2; y int.: −1 (B) Domain: all real numbers except 2 (C) Vertical asymptote: $x = 2$; horizontal asymptote: $y = 1$

(D)

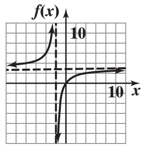


24. (A) x int.: 3; y int.: −1 (B) Domain: all real numbers except −3 (C) Vertical asymptote: $x = -3$; horizontal asymptote: $y = 1$

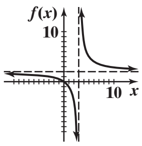
(D)



25. (A) x int.: 0; y int.: 0 (B) Domain: all real numbers except −2 (C) Vertical asymptote: $x = -2$; horizontal asymptote: $y = 3$ (D)

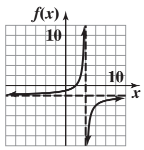


26. (A) x int.: 0; y int.: 0 (B) Domain: all real numbers except 3 (C) Vertical asymptote: $x = 3$; horizontal asymptote: $y = 2$ (D)

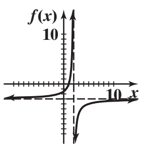


27. (A) x int.: 2; y int.: −1 (B) Domain: all real numbers except 4 (C) Vertical asymptote: $x = 4$; horizontal asymptote: $y = -2$

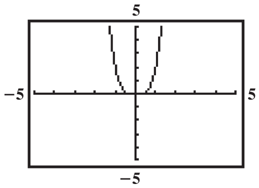
(D)

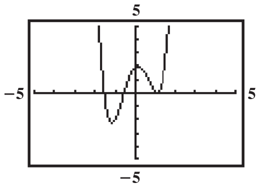


28. (A) x int.: 1; y int.: $-\frac{3}{2}$ (B) Domain: all real numbers except 2 (C) Vertical asymptote: $x = 2$; horizontal asymptote: $y = -3$ (D)

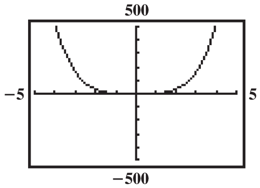


29. (A)

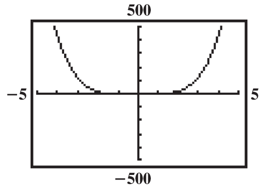




(B)

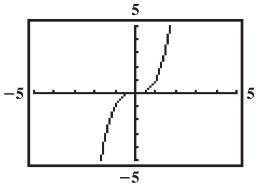


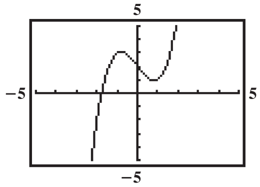
$y = 2x^4$



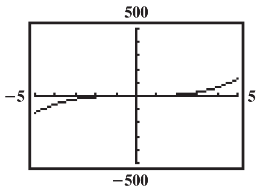
$y = 2x^4 - 5x^2 + x + 2$

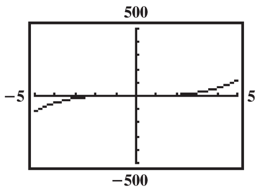
30. (A)



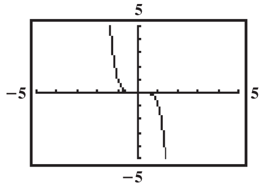


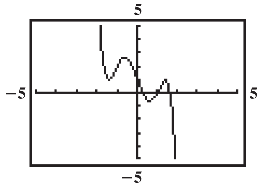
(B)



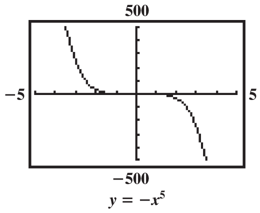


31. (A)

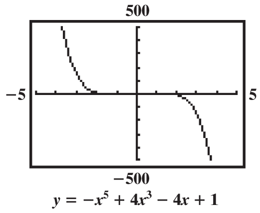




(B)

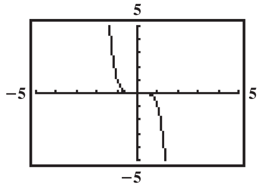


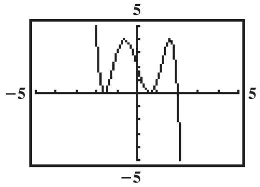
$y = -x^5$



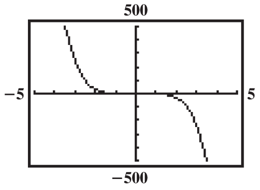
$y = -x^5 + 4x^3 - 4x + 1$

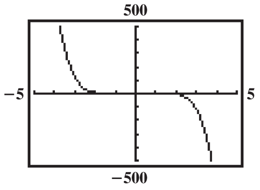
32. (A)





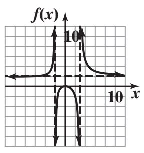
(B)



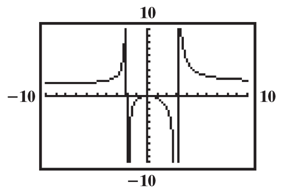


33. $y = \frac{5}{6}$ **34.** $y = \frac{3}{2}$ **35.** $y = \frac{1}{4}$ **36.** $y = -\frac{1}{2}$ **37.** $y = 0$ **38.** $y = 0$ **39.** None **40.** None **41.** $x = -1, x = 1, x = -3, x = 3$ **42.** $x = -2, x = 2, x = -4, x = 4$ **43.** $x = 5$ **44.** $x = -8$ **45.** $x = -6, x = 6$ **46.** $x = 0, x = 2$ **47.** (A) x int.: 0; y int.: 0 (B) Vertical asymptotes: $x = -2, x = 3$; horizontal asymptote: $y = 2$

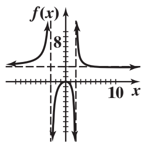
(C)



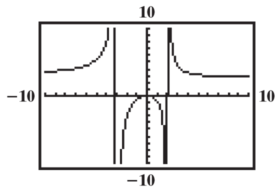
(D)



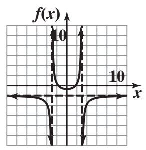
48. (A) x int.: 0; y int.: 0 (B) Vertical asymptotes: $x = -3, x = 2$; horizontal asymptote: $y = 3$ (C)



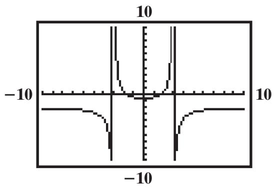
(D)



49. (A) x int.: $\pm\sqrt{3}$; y int.: $-\frac{2}{3}$ (B) Vertical asymptotes: $x = -3, x = 3$; horizontal asymptote: $y = -2$ (C)

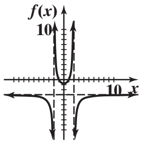


(D)

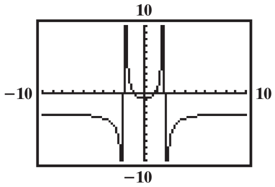


50. (A) x int.: ± 1; y int.: $-\frac{3}{4}$
(B) Vertical asymptote: $x = -2, x = 2$; horizontal asymptote: $y = -3$

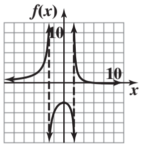
(C)



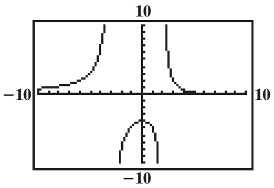
(D)



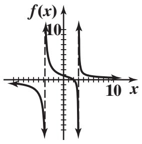
51. (A) x int.: 6; y int.: -4 (B) Vertical asymptotes: $x = -3, x = 2$; horizontal asymptote: $y = 0$ (C)



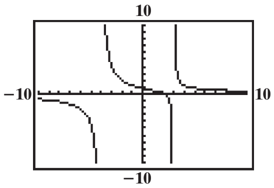
(D)



52. (A) x int.: 2; y int.: $\frac{5}{6}$ (B) Vertical asymptotes: $x = -4, x = 3$; horizontal asymptote: $y = 0$ (C)

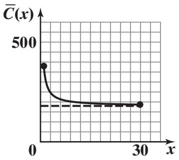


(D)



53. $f(x) = x^2 - x - 2$
54. $f(x) = 2 - x - x^2$
55. $f(x) = 4x - x^3$
56. $f(x) = x^3 - x$
57. (A) $C(x) = 180x + 200$
(B) $\overline{C}(x) = \frac{180x + 200}{x}$

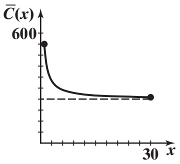
(C)



(D) $180 per board

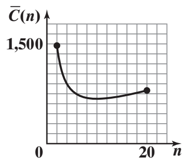
58. (A) $C(x) = 240x + 300$ (B) $\overline{C}(x) = \frac{240x + 300}{x}$

(C)

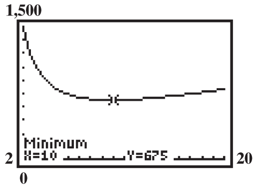


(D) $240 per board

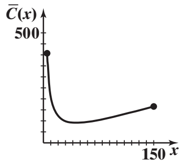
59. (A) $\overline{C}(n) = \frac{2{,}500 + 175n + 25n^2}{n}$ (B)



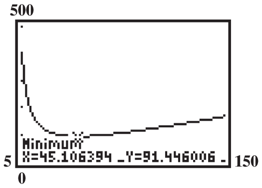
(C) 10 yr; $675.00 per year (D) 10 yr; $675.00 per year



60. (A) $\overline{C}(x) = \frac{x^2 + 2x + 2{,}000}{x}$ (B)

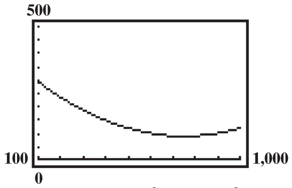


(C) 45 units; $91.44 per player (D) 45 units; $91.44 per player



61. (A) $\overline{C}(x) = \frac{0.00048(x - 500)^3 + 60{,}000}{x}$

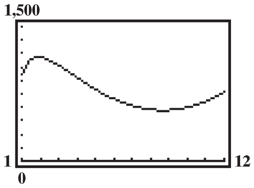
(B)



(C) 750 cases per month; $90 per case

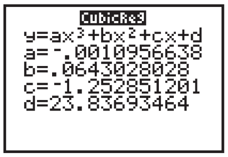
62. (A) $\overline{C}(x) = \frac{20x^3 - 360x^2 + 2{,}300x - 1{,}000}{x}$

(B)



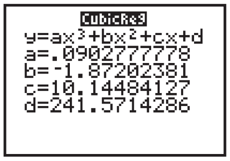
(C) 8.667 thousand cases per month; $567 per case

63. (A)



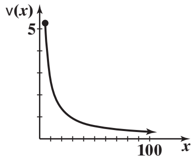
(B) 1.7 lb

64. (A)

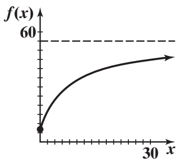


(B) 583 eggs

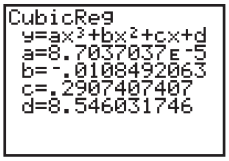
65. (A) 0.06 cm/sec
(B)



66. (A) 55 words per minute
(B)

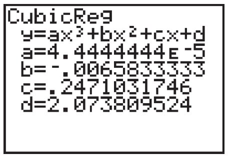


67. (A)



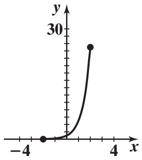
(B) 5.5

68. (A)



(B) 2.5

Exercises 2.5

1. (A) k (B) g (C) h (D) f **2.** (A) g (B) f (C) h (D) k **3.**


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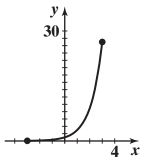

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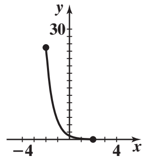

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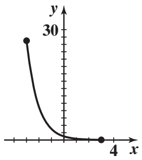
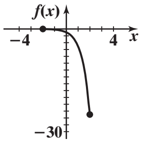


7.


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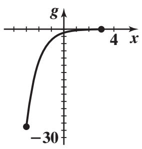

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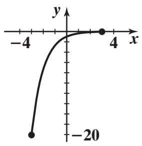
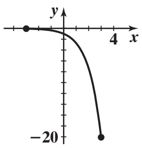
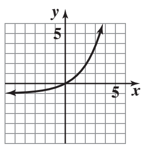
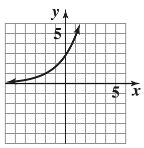
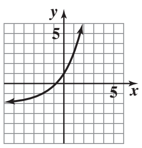


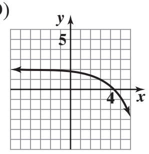
10.


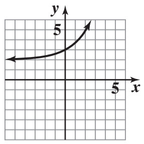
11. The graph of g is the graph of f reflected in the x axis. **12.** The graph of g is the graph of f shifted 2 units to the right. **13.** The graph of g is the graph of f shifted 1 unit to the left. **14.** The graph of g is the graph of f reflected in the x axis. **15.** The graph of g is the graph of f shifted 1 unit up. **16.** The graph of g is the graph of f shifted 2 units down. **17.** The graph of g is the graph of f vertically stretched by a factor of 2 and shifted to the left 2 units. **18.** The graph of g is the graph of f vertically shrunk by a factor of 0.5 and shifted 1 unit to the right.

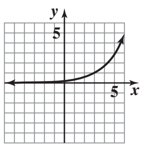
19. (A)


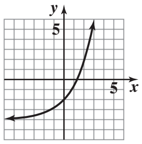
(B)


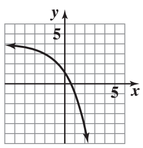
(C)


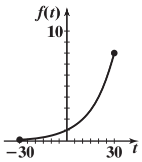
(D)


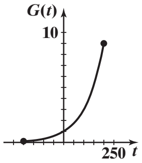
20. (A)


(B)


(C)


(D)


21.


22.


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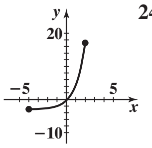

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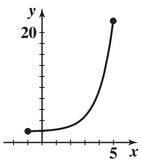
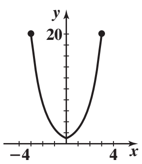
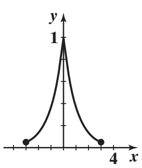
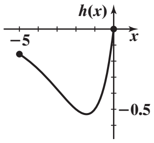
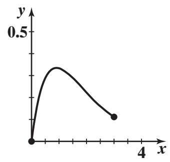


25.


26.


27. $a = 1, -1$ **28.** $a = 2, b = -2$, for example **29.** $x = 48$ **30.** $x = 9$ **31.** $x = -2, 5$ **32.** $x = -6, 7$ **33.** $x = -9$ **34.** $x = 16$ **35.** $x = 3, 19$ **36.** $x = 0, 2$ **37.** $x = -4, -3$ **38.** $x = 1, 11$ **39.** $x = -7$ **40.** $x = 1/2$ **41.** $x = -2, 2$ **42.** $x = -3, 3$ **43.** $x = 1/4$ **44.** No solution **45.** No solution **46.** $x = 2/3$ **47.**


48.


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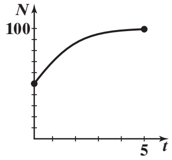

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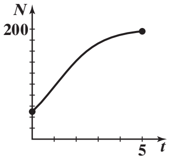
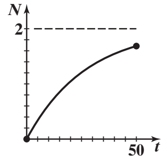
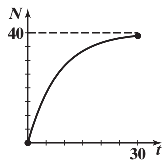


51. \$16,064.07 **52.** \$32,542.72 **53.** (A) \$2,633.56 (B) \$7,079.54 **54.** (A) \$4,121.75 (B) \$7,285.95 **55.** \$10,706 **56.** \$15,705 **57.** (A) \$10,095.41 (B) \$10,080.32 (C) \$10,085.27 **58.** (A) \$10,697.09 (B) \$10,671.21 (C) \$10,644.93 **59.** N approaches 2 as t increases without bound.


60. 40 boards per day


61. (A)

```
ExpReg
y=a*b^x
a=.4832958514
b=1.134278922
```

(B) 9.94 billion

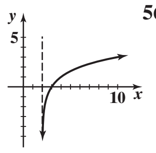
62. (A)

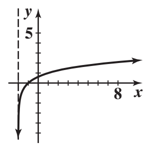
```
ExpReg
y=a*b^x
a=3.996184237
b=1.523286295
```

(B) 268.8 exabytes per month **63.** (A) 10% (B) 1% **64.** (A) 62% (B) 39% **65.** (A) $P = 12e^{0.0402x}$ (B) 17.9 million **66.** (A) $P = 204e^{0.0077x}$ (B) 229 million **67.** (A) $P = 127e^{-0.0016x}$ (B) 124 million **68.** (A) $P = 7.4e^{0.0113x}$ (B) 2025: 8.2 billion; 2033: 9.0 billion

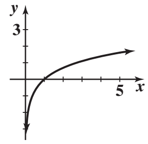
Exercises 2.6

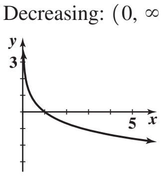
1. $27 = 3^3$ **2.** $32 = 2^5$ **3.** $10^0 = 1$ **4.** $e^0 = 1$ **5.** $8 = 4^{3/2}$ **6.** $27 = 9^{3/2}$ **7.** $\log_7 49 = 2$ **8.** $\log_6 36 = 2$ **9.** $\log_4 8 = \frac{3}{2}$ **10.** $\log_{27} 9 = 2/3$ **11.** $\log_b A = u$ **12.** $\log_b M = x$ **13.** 6 **14.** −3 **15.** −5 **16.** 4 **17.** 7 **18.** −6 **19.** −3 **20.** Not defined **21.** Not defined **22.** −1 **23.** $\log_b P - \log_b Q$ **24.** $\log_b F + \log_b G$ **25.** $5 \log_b L$ **26.** $15 \log_b w$ **27.** q^p **28.** $\log_R P$ **29.** $x = 1/10$ **30.** $x = 10$ **31.** $b = 4$ **32.** $b = 1/5$ **33.** $y = -3$ **34.** $y = 1/2$ **35.** $b = 1/3$ **36.** $b = 100$ **37.** $x = 8$ **38.** $x = 32$ **39.** False **40.** False **41.** True **42.** False **43.** True **44.** False **45.** False **46.** True **47.** $x = 2$ **48.** $x = 12$ **49.** $x = 8$ **50.** $x = 2$ **51.** $x = 7$ **52.** $x = 4$ **53.** No solution **54.** $x = 4$

55.


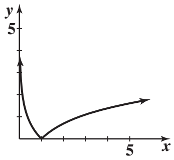
56.


57. The graph of $y = \log_2 (x - 2)$ is the graph of $y = \log_2 x$ shifted to the right 2 units.

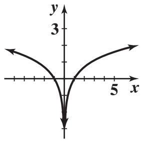
59. Domain: $(-1, \infty)$; range: all real numbers **60.** Domain: $(1, \infty)$; range: all real numbers **61.** (A) 3.547 43 (B) −2.160 32 (C) 5.626 29 (D) −3.197 04 **62.** (A) 1.860 96 (B) −1.48095 (C) 10.603 04 (D) −5.12836 **63.** (A) 13.4431 (B) 0.0089 (C) 16.0595 (D) 0.1514 **64.** (A) 121.1156 (B) 0.0089 (C) 22.9565 (D) 0.0132 **65.** 1.0792 **66.** 2.1847 **67.** 1.4595 **68.** −1.1845 **69.** 18.3559 **70.** 8.7507 **71.** Increasing: $(0, \infty)$


72. Decreasing: $(0, \infty)$


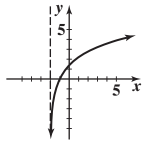
73. Decreasing: $(0, 1]$
Increasing: $[1, \infty)$



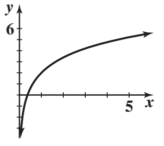
74. Decreasing: $(-\infty, 0)$
Increasing: $(0, \infty)$



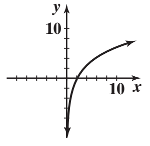
75. Increasing: $(-2, \infty)$



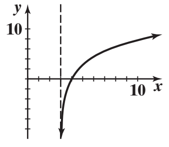
76. Increasing: $(0, \infty)$



77. Increasing: $(0, \infty)$



78. Increasing: $(3, \infty)$



79. Because $b^0 = 1$ for any permissible base $b\,(b > 0, b \neq 1)$.
80. Because the function $y = 1^x$ is not one-to-one **81.** $x > \sqrt{x} > \ln x$ for $1 < x \le 16$ **82.** $\log x < \sqrt[3]{x} < x$ **83.** 4 yr **84.** 8 yr **85.** 9.87 yr; 9.80 yr
86. 5.17 yr; 5.09 yr **87.** 7.51 yr **88.** 29.84 yr
89. (A) 5,373

```
LnReg
 y=a+blnx
 a=256.4659159
 b=-24.03812068
```

(B) 7,220

```
LnReg
 y=a+blnx
 a=-127.8085281
 b=20.01315349
```

90. Equilibrium price: \$46.77; Equilibrium quantity: 6,145
92. (A) 30 (B) 65 (C) 80 (D) 150
93. 168 bushels/acre

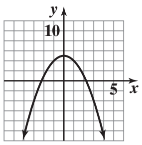
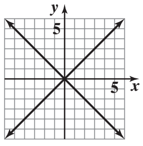
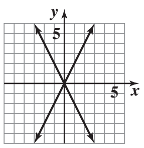
```
LnReg
 y=a+blnx
 a=-551.2132518
 b=149.1505358
```

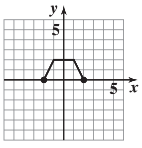
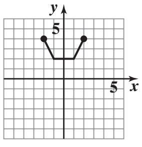
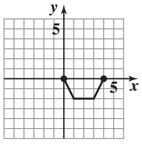
94. 12,628 million bushels

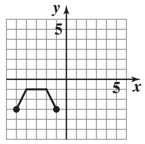
```
LnReg
 y=a+blnx
 a=-45845.97493
 b=12130.89096
```

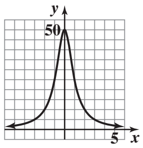
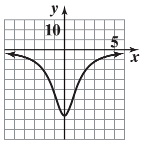
95. 912 yr **96.** 18,569 yr

Chapter 2 Review Exercises

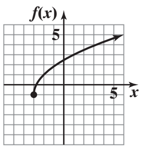
1.  *(2.1)* **2.**  *(2.1)* **3.**  *(2.1)*

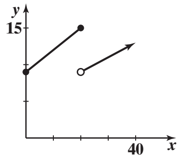
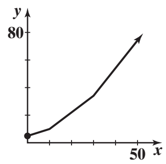
4. (A) Not a function (B) A function (C) A function (D) Not a function *(2.1)* **5.** (A) -2 (B) -8 (C) 0 (D) Not defined *(2.1)*
6. $v = \ln u$ *(2.6)* **7.** $y = \log x$ *(2.6)* **8.** $M = e^N$ *(2.6)* **9.** $u = 10^v$ *(2.6)*
10. $x = 9$ *(2.6)* **11.** $x = 6$ *(2.6)* **12.** $x = 4$ *(2.6)* **13.** $x = 2.157$ *(2.6)*
14. $x = 13.128$ *(2.6)* **15.** $x = 1{,}273.503$ *(2.6)* **16.** $x = 0.318$ *(2.6)*
17. (A) $y = 4$ (B) $x = 0$ (C) $y = 1$ (D) $x = -1$ or 1 (E) $y = -2$ (F) $x = -5$ or 5 *(2.1)*
18. (A)  (B)  (C) 

(D)  *(2.2)* **19.** $f(x) = -(x-2)^2 + 4$. The graph of $f(x)$ is the graph of $y = x^2$ reflected in the x axis, then shifted right 2 units and up 4 units. *(2.2)* **20.** (A) g (B) m (C) n (D) f *(2.2, 2.3)*
21. (A) x intercepts: $-4, 0$; y intercept: 0 (B) Vertex: $(-2, -4)$ (C) Minimum: -4 (D) Range: $y \ge -4$ or $[-4, \infty)$ *(2.3)* **22.** Quadratic *(2.3)*
23. Linear *(2.1)* **24.** None *(2.1, 2.3)* **25.** Constant *(2.1)* **26.** $x = 8$ *(2.6)*
27. $x = 3$ *(2.6)* **28.** $x = 3$ *(2.5)* **29.** $x = -1, 3$ *(2.5)* **30.** $x = 0, \frac{3}{2}$ *(2.5)*
31. $x = -2$ *(2.6)* **32.** $x = \frac{1}{2}$ *(2.6)* **33.** $x = 27$ *(2.6)* **34.** $x = 13.3113$ *(2.6)*
35. $x = 158.7552$ *(2.6)* **36.** $x = 0.0097$ *(2.6)* **37.** $x = 1.4359$ *(2.6)*
38. $x = 1.4650$ *(2.6)* **39.** $x = 92.1034$ *(2.6)* **40.** $x = 9.0065$ *(2.6)*
41. $x = 2.1081$ *(2.6)* **42.** (A) All real numbers except $x = -2$ and 3 (B) $x < 5$ *(2.1)* **43.** Vertex form: $4\left(x + \frac{1}{2}\right)^2 - 4$; x intercepts: $-\frac{3}{2}$ and $\frac{1}{2}$; y intercept: -3; vertex: $(-\frac{1}{2}, -4)$; minimum: -4; range: $y \ge -4$ or $[-4, \infty)$ *(2.3)* **44.** $(-1.54, -0.79)$; $(0.69, 0.99)$ *(2.5, 2.6)*

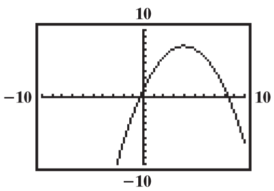
45.  *(2.1)* **46.**  *(2.1)* **47.** 6 *(2.1)*

48. -19 *(2.1)* **49.** $10x - 4$ *(2.1)* **50.** $21 - 5x$ *(2.1)* **51.** (A) -1 (B) $-1 - 2h$ (C) $-2h$ (D) -2 *(2.1)* **52.** (A) $a^2 - 3a + 1$ (B) $a^2 + 2ah + h^2 - 3a - 3h + 1$ (C) $2ah + h^2 - 3h$ (D) $2a + h - 3$ *(2.1)* **53.** The graph of function m is the graph of $y = |x|$ reflected in the x axis and shifted to the right 4 units. *(2.2)* **54.** The graph of function g is the graph of $y = x^3$ vertically contracted by a factor of 0.3 and shifted up 3 units. *(2.2)* **55.** The graph of $y = x^2$ is vertically expanded by a factor of 2, reflected in the x axis, and shifted to the left 3 units. Equation: $y = -2(x+3)^2$. *(2.2)* **56.** $f(x) = 2\sqrt{x+3} - 1$ *(2.2)*



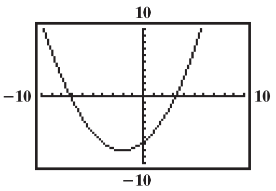
57. $y = 0$ *(2.4)* **58.** $y = \frac{3}{4}$ *(2.4)* **59.** None *(2.4)*
60. $x = -10, x = 10$ *(2.4)* **61.** $x = -2$ *(2.4)*
62. True *(2.3)* **63.** False *(2.3)* **64.** False *(2.3)*
65. True *(2.4)* **66.** True *(2.5)* **67.** True *(2.3)*
68.  *(2.2)* **69.**  *(2.2)*

70. $y = -(x-4)^2 + 3$ *(2.2, 2.3)* **71.** $f(x) = -0.4(x-4)^2 + 7.6$ (A) x intercepts: $-0.4, 8.4$; y intercept: 1.2 (B) Vertex: $(4.0, 7.6)$ (C) Maximum: 7.6 (D) Range: $y \le 7.6$ or $(-\infty, 7.6]$ *(2.3)*
72.



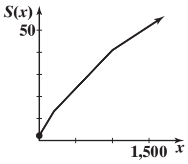
(A) x intercepts: $-.4, 8.4$; y intercept: 1.2 (B) Vertex: (4.0, 7.6) (C) Maximum: 7.6 (D) Range: $y \le 7.6$ or $(-\infty, 7.6]$ *(2.3)*

73. $\log 10^\pi = \pi$ and $10^{\log\sqrt{2}} = \sqrt{2}$; $\ln e^\pi = \pi$ and $e^{\ln\sqrt{2}} = \sqrt{2}$ *(2.6)*
74. $x = 2$ *(2.6)* **75.** $x = 2$ *(2.6)* **76.** $x = 1$ *(2.6)* **77.** $x = 300$ *(2.6)*
78. $y = ce^{-5t}$ *(2.6)* **79.** If $\log_1 x = y$, then $1^y = x$; that is, $1 = x$ for all positive real numbers x, which is not possible. *(2.6)* **80.** The graph of $y = \sqrt[3]{x}$ is vertically expanded by a factor of 2, reflected in the x axis, and shifted 1 unit left and 1 unit down. Equation: $y = -2\sqrt[3]{x+1} - 1$. *(2.2)*
81. $G(x) = 0.3(x+2)^2 - 8.1$ (A) x intercepts: $-7.2, 3.2$; y intercept: -6.9 (B) Vertex: $(-2, -8.1)$ (C) Minimum: -8.1 (D) Range: $y \ge -8.1$ or $[-8.1, \infty)$ *(2.3)*

82. 

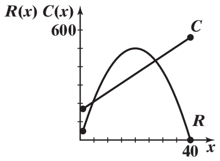
(A) x intercepts: -7.2, 3.2; y intercept: -6.9 (B) Vertex: $(-2, -8.1)$ (C) Minimum: -8.1 (D) Range: $y \geq -8.1$ or $[-8.1, \infty)$ *(2.3)*

83. (A) $S(x) = \begin{cases} 3 & \text{if } 0 \leq x \leq 20 \\ 0.057x + 1.86 & \text{if } 20 < x \leq 200 \\ 0.0346x + 6.34 & \text{if } 200 < x \leq 1{,}000 \\ 0.0217x + 19.24 & \text{if } x > 1{,}000 \end{cases}$

(B) 

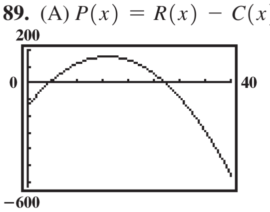
(2.2) **84.** \$5,321.95 *(2.5)* **85.** \$5,269.51 *(2.5)*

86. 201 months ($\approx$16.7 years) *(2.5)* **87.** 9.38 yr *(2.5)*

88. (A) 

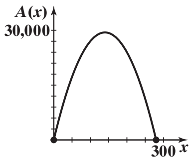
(B) $R = C$ for $x = 4.686$ thousand units (4,686 units) and for $x = 27.314$ thousand units (27,314 units); $R < C$ for $1 \leq x < 4.686$ or $27.314 < x \leq 40$; $R > C$ for $4.686 < x < 27.314$. (C) Maximum revenue is 500 thousand dollars (\$500,000). This occurs at an output of 20 thousand units (20,000 units). At this output, the wholesale price is $p(20) = \$25$. *(2.3)*

89. (A) $P(x) = R(x) - C(x) = x(50 - 1.25x) - (160 + 10x)$

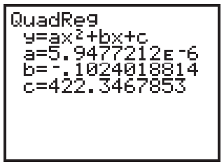


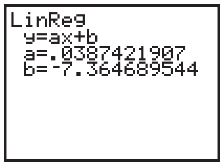
(B) $P = 0$ for $x = 4.686$ thousand units (4,686 units) and for $x = 27.314$ thousand units (27,314 units); $P < 0$ for $1 \leq x < 4.686$ or $27.314 < x \leq 40$; $p > 0$ for $4.686 < x < 27.314$. (C) Maximum profit is 160 thousand dollars (\$160,000). This occurs at an output of 16 thousand units (16,000 units). At this output, the wholesale price is $p(16) = \$30$. *(2.3)*

90. (A) $A(x) = -\frac{3}{2}x^2 + 420x$ (B) Domain: $0 \leq x \leq 280$

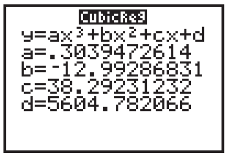
(C) 

(D) There are two solutions to the equation $A(x) = 25{,}000$, one near 90 and another near 190. (E) 86 ft; 194 ft (F) Maximum combined area is 29,400 ft^2. This occurs for $x = 140$ ft and $y = 105$ ft. *(2.3)*

91. (A) 2,833 sets 

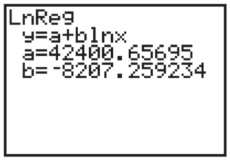
(B) 4,836 

(C) Equilibrium price: \$131.59; equilibrium quantity: 3,587 cookware sets *(2.3)*

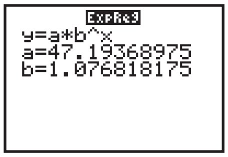
92. (A) 

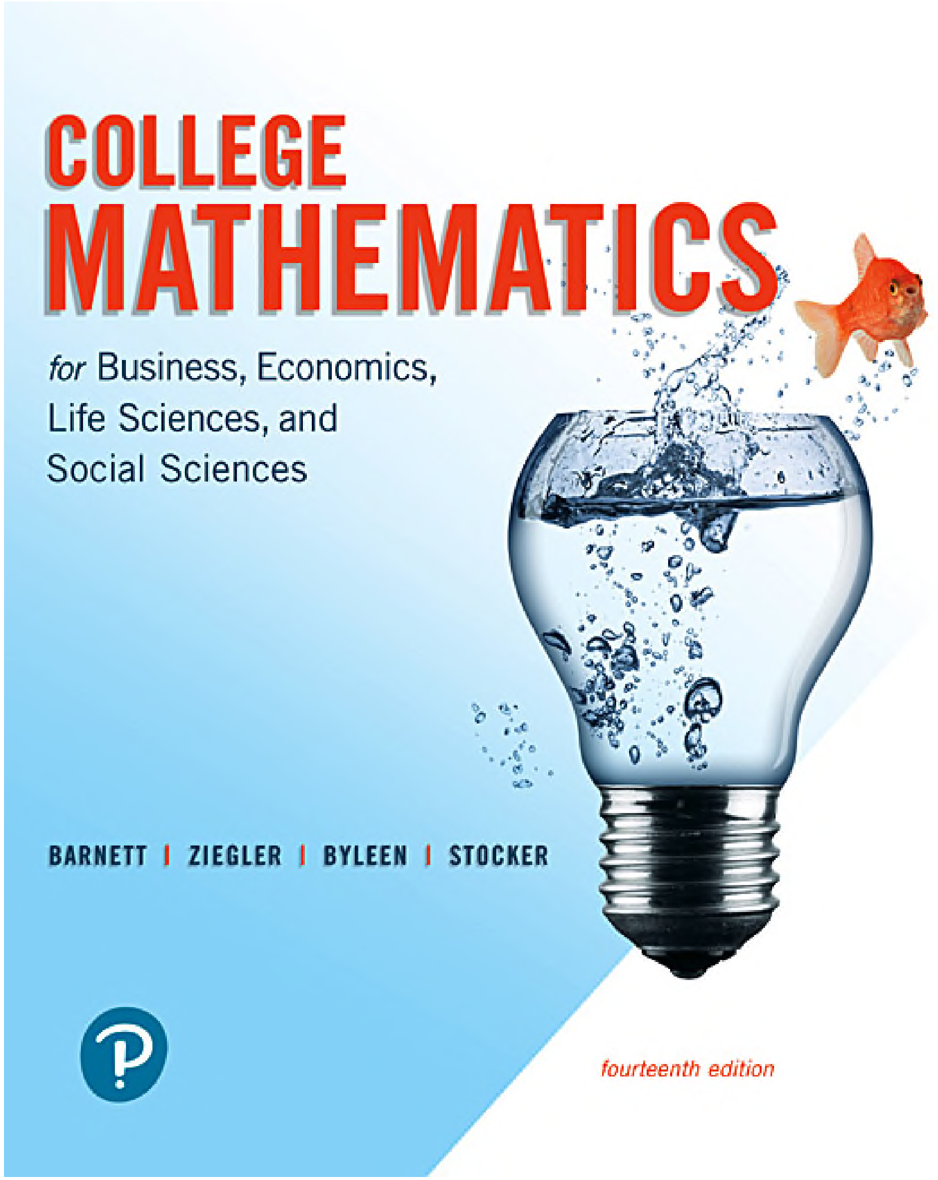
(B) 4976 *(2.4)*

93. (A) $N = 2^{2t}$ or $N = 4^t$ (B) 15 days *(2.5)* **94.** $k = 0.009\,42$; 489 ft *(2.6)* **95.** (A) 6,134,000 *(2.6)*



96. 23.1 yr *(2.5)* **97.** (A) \$1,319 billion (B) 2031 *(2.5)*





Chapter 2

Functions and Graphs

Section 1
Functions

Solutions to Equations in Two Variables

Consider the equation in two variables, $y - x^2 = 5.$

A **solution** to an equation in two variables is an ordered pair of numbers that when substituted into the equation result in a true statement.

The ordered pairs $(x, y) = (2, 9)$ and $(x, y) = (-2, 9)$ are solutions to the equation. When values in these ordered pairs are substituted into the equation, the result is a true statement.

The ordered pair $(x, y) = (0, 2)$ is not a solution to the equation. When values in this ordered pair are substituted into the equation, the resulting statement is not true.

Finding Solutions to Equations in Two Variables

Find a solution to the equation, $y - x^2 = 5,$ when $x = 4.$

One process for finding a solution to an equation in two variables is to choose a value for one variable, substitute that value into the equation, then solve for the other variable.

Substitute $x = 4$ into the equation to obtain the equation, $y - 16 = 5$.

It follows that $y = 21$ and (4, 21) is a solution to the equation.

Point-by-Point Graphing

- To sketch the graph an equation in *x* and *y*, we find ordered pairs that solve the equation and plot the ordered pairs on a grid.
- We must find sufficiently many pairs so that the shape of the graph is apparent.
- This process is called **point-by-point plotting**.
- The points corresponding to the solution are then connected with a smooth curve.
- The resulting graph is a visual representation of the **solution set** for the equation.

Point-by-Point Graphing
Create a Table of Solutions

Create a table of solutions to the equation, $y - x^2 = 5$.

- We select a collection of values for one of the variables in the equation. In this example, we select values for the variable, *x*.
- For this example, we will select the *x*-values, -3, -2, -1, 0, 1, 2, and 3.

Point-by-Point Graphing Create a Table of Solutions

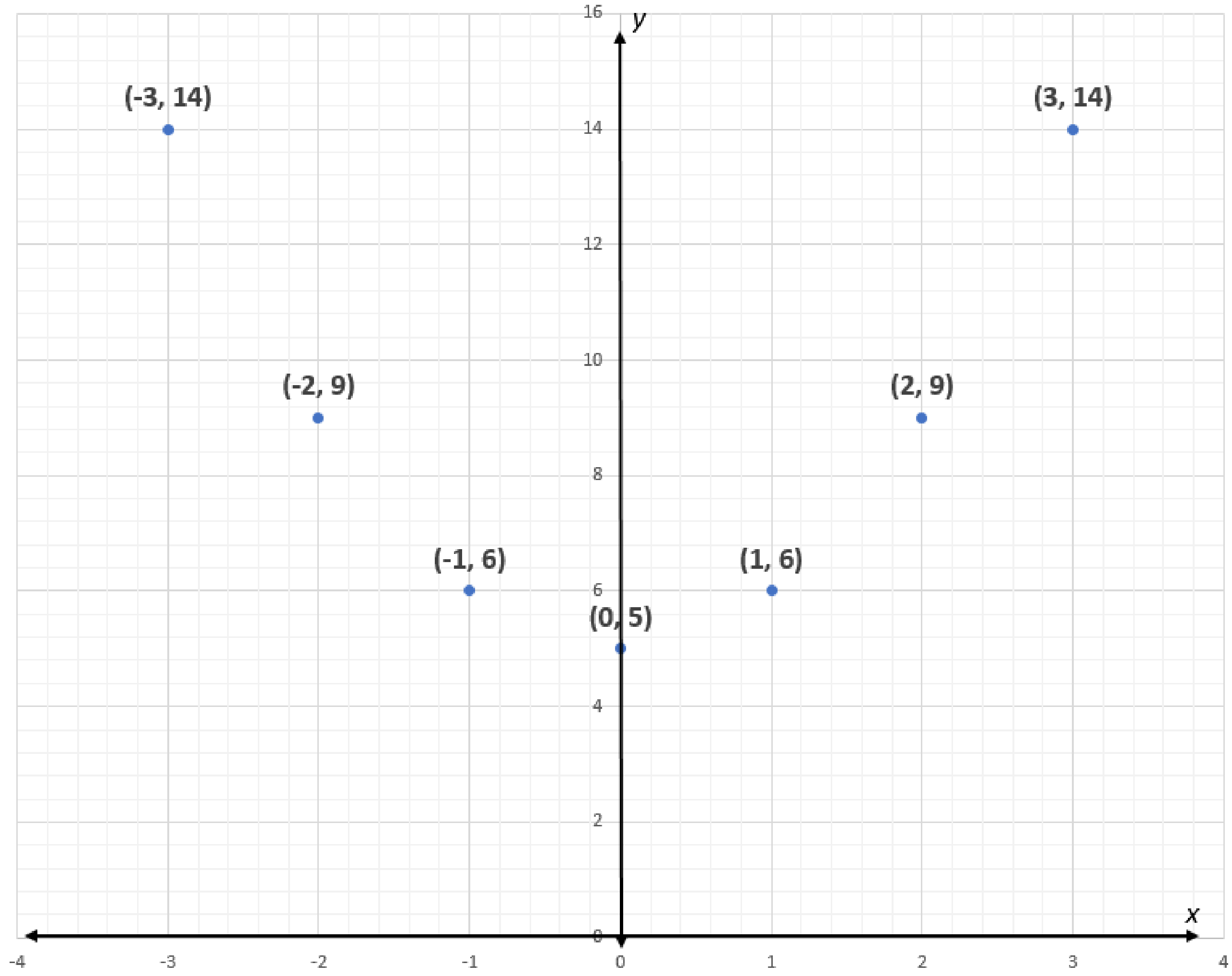
Create a table of solutions to the equation, $y - x^2 = 5$.

- Using the *x*-values, -3, -2, -1, 0, 1, 2, and 3, we substitute and solve for *y*. The results are shown in the table.

x	***y***	***Solution, (x, y)***
-3	$y - (-3)^2 = 5$ gives $y = 9 + 5 = 14$	(-3, 14)
-2	$y - (-2)^2 = 5$ gives $y = 4 + 5 = 9$	(-2, 9)
-1	$y - (-1)^2 = 5$ gives $y = 1 + 5 = 6$	(-1, 6)
0	$y - (0)^2 = 5$ gives $y = 0 + 5 = 5$	(0, 5)
1	$y - (1)^2 = 5$ gives $y = 1 + 5 = 6$	(1, 6)
2	$y - (2)^2 = 5$ gives $y = 4 + 5 = 9$	(2, 9)
3	$y - (3)^2 = 5$ gives $y = 9 + 5 = 14$	(3, 14)

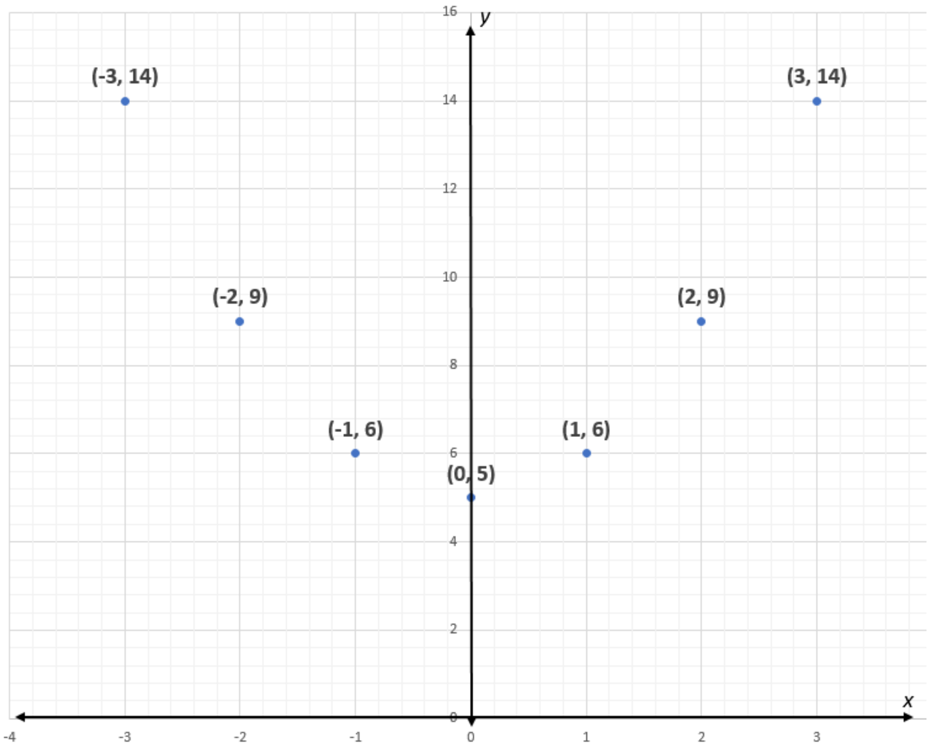
Point-by-Point Graphing
Plotting the Points

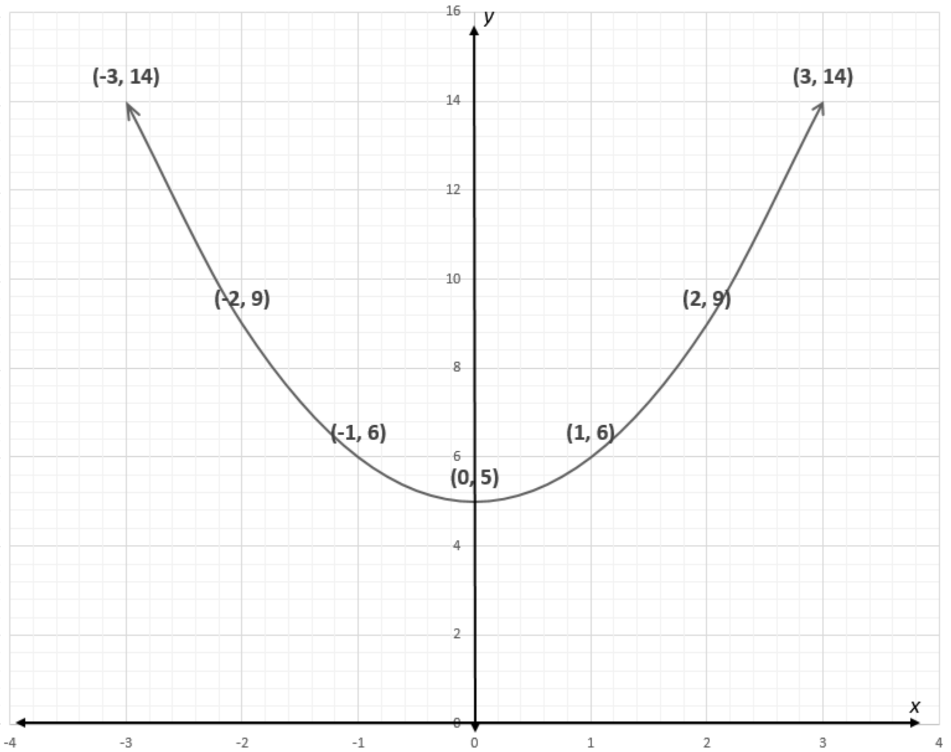
Solution, (x, y)
(-3, 14)
(-2, 9)
(-1, 6)
(0, 5)
(1, 6)
(2, 9)
(3, 14)



Plot the points in the solution.

Point-by-Point Graphing Connecting the Points





Graph the points in the solution.

Connect the points with a smooth curve.

Correspondences

- An important aspect of any science is the establishment of correspondences among various types of phenomena.
- Correspondences are central to the concept of mathematical functions.
- Some familiar correspondences include the following:
  - To each item in a store, there corresponds a price.
  - To each university student, there corresponds their grade-point average.
  - To each circle, there corresponds a circumference.
  - To each number, there corresponds its cube.
  - To each day, there corresponds a maximum temperature.

Mathematical Function

A **function** is a correspondence between two sets of elements such that to each element in the first set, there corresponds one and only one element in the second set.

The first set in such a correspondence is called the **domain** of the function.

The second set in such a correspondence is called the **range** of the function.

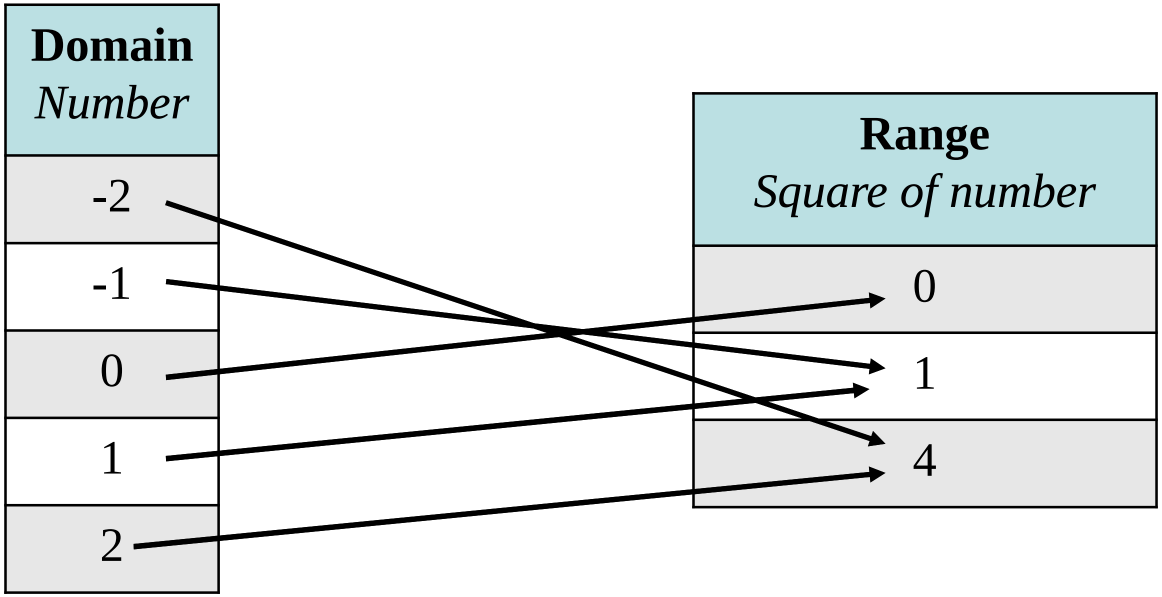
A key concept for functions is the requirement that each domain element corresponds with one and only one range element.

Function or Not?

Domain *Number*		**Range** *Cube of number*
-2	→	-8
-1	→	-1
0	→	0
1	→	1
2	→	8

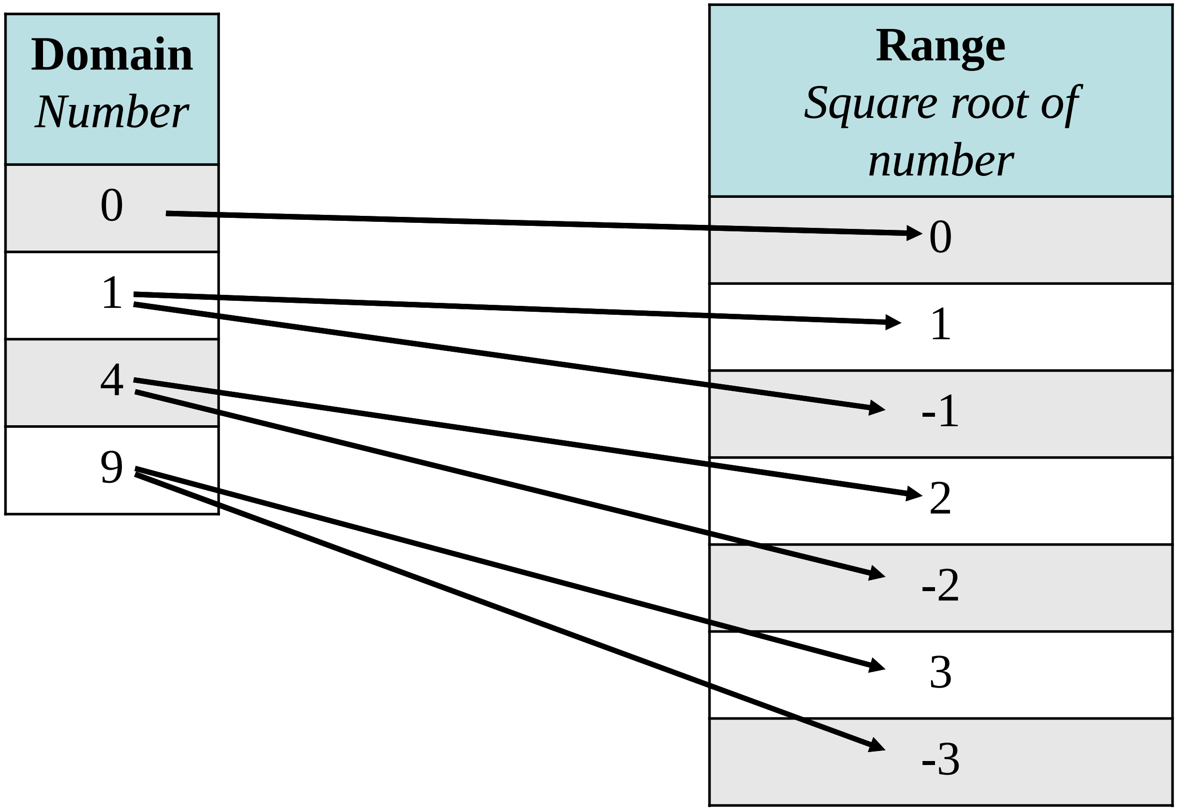
This table represents a function since each domain element corresponds to one and only one range element.

Function or Not?



This table represents a function since each domain element corresponds to one and only one range element.

Function or Not?



This table does not represent a function since there are domain values that correspond to more than one range value (for example, the range values 2, and -2 correspond to the domain value 4).

Functions Specified by Equations

If an equation in two variables has the property that each domain value is associated with one and only one range value, such an equation represents a function.

If an equation in two variables has the property that there is at least one domain value that is associated with more than one range value, such an equation does not represent a function.

Functions

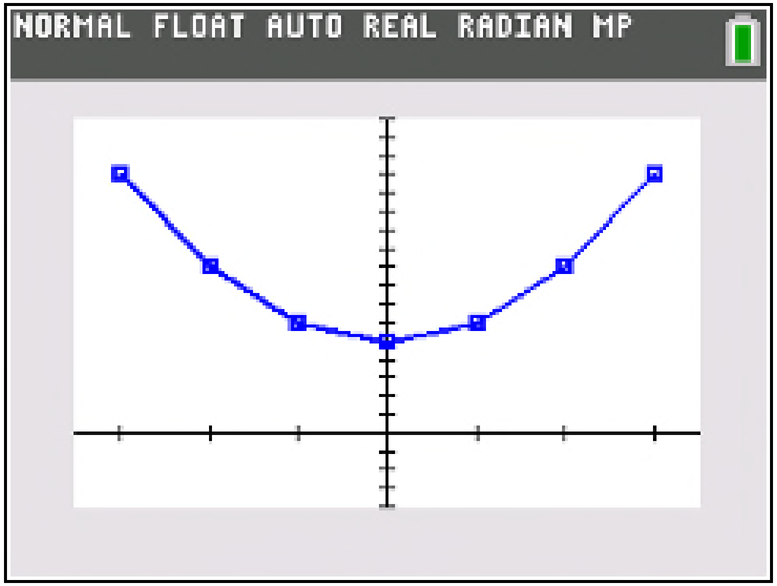
In the equation, $y - x^2 = 5$, y is a function of x.

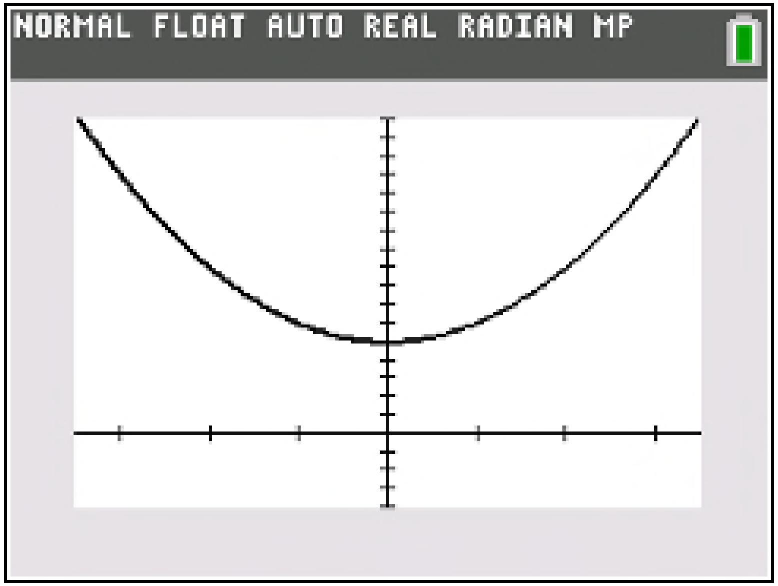
- The collection of x values is the domain of the function.
- The collection of y values associated with each x value is the range of the function.
- Each domain value in this equation corresponds to one and only one range value.

The Graph of a Function

We previously graphed and connected some of the points in the solution set for the equation, $y - x^2 = 5$.

Additional points in the solution set can be graphed to obtain the **graph of the function.**





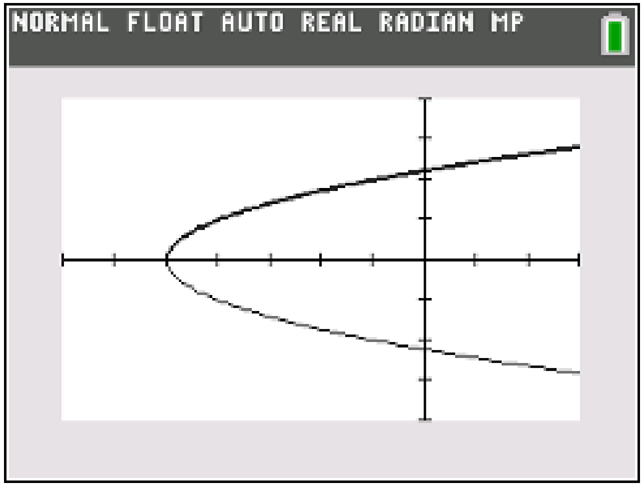
Equations that are Not Functions

In the equation, $y^2 - x = 5$, y is not a function of x.

The domain value, $x = 4$ corresponds to two range values, $y = 3$ and $y = -3$.

When an equation has at least one domain value that corresponds to more than one range value, the equation does not represent a function.

The graph of this specific correspondence is shown below.



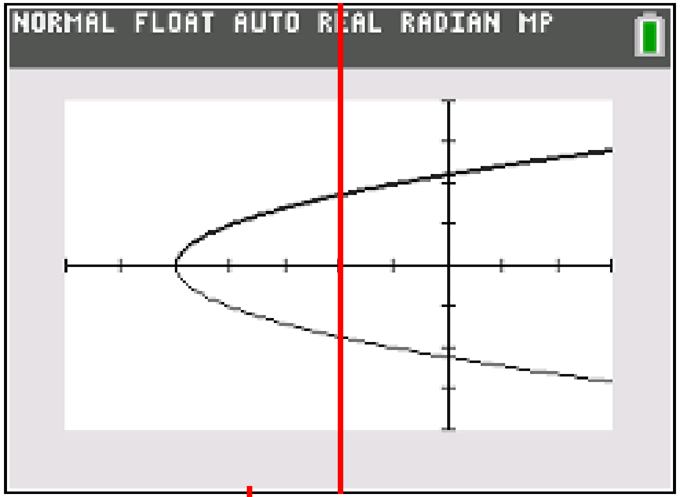
Vertical Line Test for a Function

THEOREM 1:

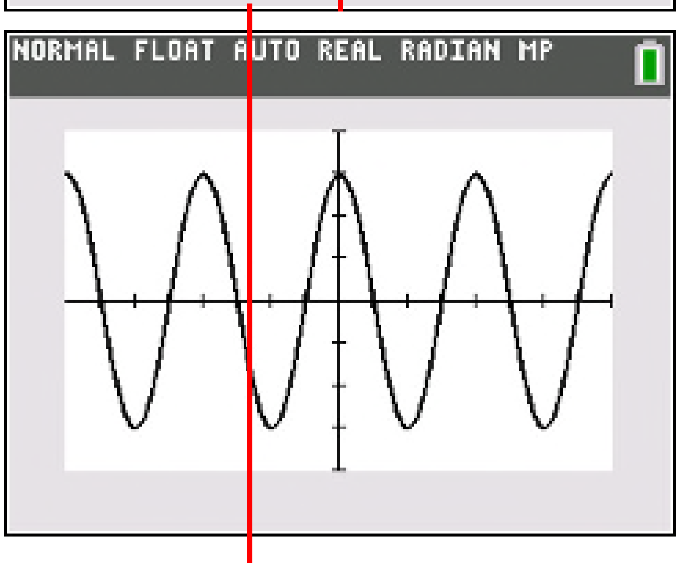
An equation specifies a function if each vertical line in the coordinate system passes through, at most, one point on the graph of the equation.

If any vertical line passes through two or more points on the graph of an equation, then the equation does not specify a function.

Vertical Line Test for a Function



This graph is not the graph of a function because is it possible to draw a vertical line which intersects the graph more than one time.



This graph represents a function because any vertical line will intersect the graph in at most one point.

Independent and Dependent Variables

Input values for functions are domain values and output values for functions are range values.

Function equations often use the variable, x as the domain variable and the variable, y as the range variable.

The input variable is called an **independent variable**.

The output variable is called a **dependent variable**.

If a function is specified by an equation and the domain is not indicated, we assume that the domain is the set of all real-number replacements of the independent variable that produce real values for the dependent variable.

The range of a function is the set of all outputs corresponding to input values.

Function Notation

Functions involve two sets, a domain and a range along with a correspondence that assigns to each element in the domain, exactly one element in the range.

Function notation gives an alternate method for naming functions so that the correspondence between specific input and output values can be easily shown.

Letters can be used as names for functions as well as names for numbers.

The letters, f and g may be used to name the functions specified by the equations $y = 2x + 1$ and $y = x^2 + 2x - 3$ as follows:

$$f : y = 2x + 1$$

$$g : y = x^2 + 2x - 3.$$

Function Notation

Using the functions named, $f: y = 2x + 1$ and $g: y = x^2 + 2x - 3$, if x represents an element in the domain of the function, f, we use the notation $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 3$ to specify these functions.

The symbol *f(x)* is used in place of *y* to designate the number in the range of *f* to which *x* is paired using the equation that is named *f*.

f(x) is read "*f* of *x*," or "the value of *f* at *x*."

Similarly, *g(x)* is read "*g* of *x*," or "the value of *g* at *x*."

Whenever we use function notation such as *f(x)* or *g(x)*, we assume that the variable *x* is an independent variable and both *y* and *f(x)* are dependent variables.

Finding a Domain

Find the domain of the function specified by the equation $y = \sqrt{10 - x}$, assuming that x is the independent variable.

For the dependent variable, y to be real, the expression under the radical sign must be non-negative.

We solve the inequality, $10 - x \geq 0$ to obtain $-x \geq -10$ which gives $x \leq 10$.

The domain in inequality notation form is $x \leq 10$.

The domain in interval notation form is $(-\infty, 10]$.

Function Evaluation

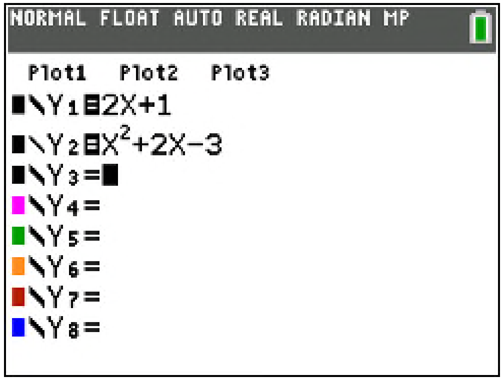
Use the functions $f(x)=2x+1$ and $g(x)=x^2+2x-3$ to evaluate $f(5), g(5), f(-2),$ and $g(-2)$.

- $f(5) = 2\cdot 5 + 1 = 10 + 1 = 11$.
  - The point (5, 11) is a point on the graph of the function, *f*.
- $g(5) = 5^2 + 2\cdot 5 - 3 = 25 + 10 - 3 = 35 - 3 = 32$.
  - The point (5, 32) is a point on the graph of the function, *g*.
- $f(-2) = 2\cdot(-2) + 1 = -4 + 1 = -3$.
  - The point (-2, -3) is a point on the graph of the function, *f*.
- $g(-2) = (-2)^2 + 2\cdot(-2) - 3 = 4 - 4 - 3 = -3$.
  - The point (–2, –3) is a point on the graph of the function, *g*.
- Since the point (–2, –3) is on the graph of each function, it is a point of intersection for the two graphs.

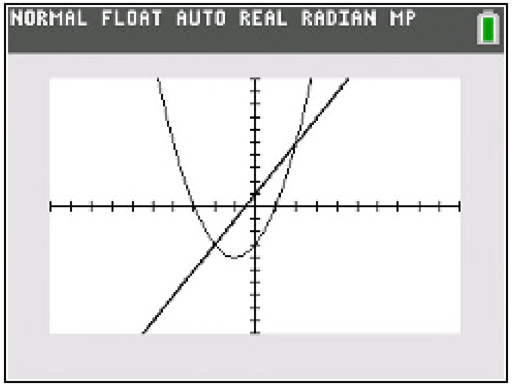
Graphing Functions Using Calculator Technology

Use calculator technology to graph the functions $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 3$ in a standard viewing window.

Input the functions into the graphing calculator as Y1 = *f(x)*, and Y1 = *g(x)*.



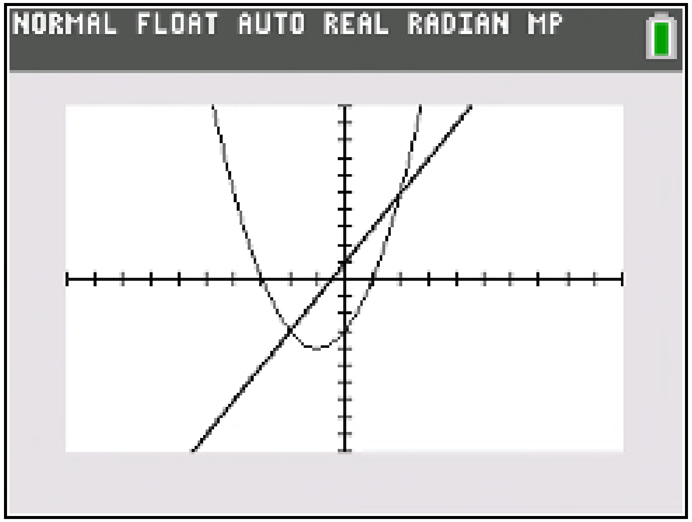
Press *zoom 6:Zstandard* to view the graphs in a standard viewing window.



CLICK HERE TO ACCESS THE COMPLETE Solutions

Function Evaluation

The graphs of the functions $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 3$ are shown in a standard viewing window.



The two functions intersect at two different points. The point (–2, –3) was found to be a point on the graphs of both functions.

What are the coordinates for the other point of intersection?

The Difference Quotient

- In addition to evaluating functions at specific numbers, an important skill is evaluating functions at expressions that involve one or more variables.
- A foundational concept in higher level mathematics areas such as calculus is the **difference quotient**.

The difference quotient for a function $f(x)$ where x and $x+h$ are in the domain of f, with $h \neq 0$ is $\dfrac{f(x+h)-f(x)}{h}$.

- The difference quotient computes the slope of the line between two points on the graph of a function.

The Difference Quotient

The difference quotient for a function $f(x)$ is $\dfrac{f(x+h)-f(x)}{h}$.

Find the difference quotient for $f(x)=2x+1$.

- $f(x+h) = 2(x+h) + 1 = 2x + 2h + 1.$
- $f(x) = 2x + 1.$
- $f(x+h) - f(x) = 2x + 2h + 1 - (2x + 1)$

$$= \cancel{2x} + 2h + \cancel{1} - \cancel{2x} - \cancel{1}$$

$$= 2h.$$

$$\frac{f(x+h)-f(x)}{h} = \frac{2h}{h} = 2.$$

The Difference Quotient

Find the difference quotient $\dfrac{g(x+h)-g(x)}{h}$

for the function $g(x) = x^2 + 2x - 3.$

- $g(x + h) = (x + h)^2 + 2(x + h) - 3$

  $= x^2 + 2xh + h^2 + 2x + 2h - 3$
- $g(x) = x^2 + 2x - 3$
- $g(x + h) - g(x) = x^2 + 2xh + h^2 + 2x + 2h - 3 - (x^2 + 2x - 3)$

  $= x^2 + 2xh + h^2 + 2x + 2h - 3 - x^2 - 2x + 3$

  $= 2xh + h^2 + 2h$

$$\frac{g(x+h)-g(x)}{h} = \frac{2xh+h^2+2h}{h} = 2x+h+2.$$

Profit-loss Analysis

Companies use Profit-loss analysis to support decisions regarding the pricing of products and appropriate levels of production in order to maximize company profit.

A manufacturing company has costs, C, which include **fixed costs** (plant overhead, product design, setup, and promotion) and **variable costs** (costs that depend on the number of items produced.)

The **revenue** (income) for a company, R, is the amount of money the company receives from selling its product.

If $R < C$, the company loses money.

If $R = C$, the company breaks even.

If $R > C$, the company makes a profit.

Profit-loss Analysis

Profit, P, is equal to revenue, R, minus cost, C.

$P = R - C$.

When $P < 0$, the company loses money (cost exceeds revenue).

When $P = 0$, the company breaks even (cost equals revenue).

When $P > 0$, the company makes a profit (revenue exceeds cost).

Price-demand Analysis

Companies use a **price-demand** function, *p(x)*, often determined using historical data or sampling techniques, that specifies the relationship between the demand for a product, *x*, and the price of the product, *p*.

A point (*x*, *p*) is on the graph of the price-demand function if *x* items can be sold at a price of $*p* per item.

Generally, a reduction in price results in an increase in the demand, thus the graph of the price-demand function is expected to go downhill as prices increase from left to right.

The revenue, *R*, is equal to the number of items sold multiplied by the price per item, $R = x \cdot p$.

A Price-demand Example

The price-demand function for a company is given by

$$p(x) = 1000 - 5x, \qquad 0 \le x \le 100$$

where x represents the number of items and $p(x)$ represents the price of the item.

Determine the revenue function and find the revenue generated if 50 items are sold.

Price-demand Solution

Solution:

Revenue = number of items sold times price per item.

$R(x) = x \cdot p(x) = x(1000 - 5x)$

When 50 items are sold $R(50) = 50(1000 - 5 \cdot 50)$

$= 50(1000 - 250)$

$= 50(750) = 37{,}500$

When 50 items are sold, the price is \$750 and the revenue from selling the 50 items is \$37,500.

Break-Even and Profit-Loss Analysis

Cost, revenue, and profit functions are often represented symbolically as *C(x)*, *R(x)*, and *P(x)* where the independent variable, *x*, represents the number of items manufactured and sold.

These functions often have the following forms, where *a*, *b*, *m*, and *n* are positive constants determined from the context of the particular manufacturer.

Cost function, $C(x) = a + b \cdot x$

C = fixed costs + variable costs

Price-demand function, $p(x) = m - n \cdot x$

x is the number of items that can be sold at \$$p$ per item.

Revenue function, $R(x) = x \cdot p = x(m - n \cdot x)$

R = number of items sold times the price per item.

Profit function, $P(x) = R(x) - C(x) = x(m - n \cdot x) - (a + b \cdot x)$.

Be careful not to confuse the price demand function, $p(x)$ with the profit function, $P(x)$. The price function always uses the lower case, p.

Profit-Loss Analysis Example

A company manufactures notebook computers. Its marketing research department has determined that the data is modeled by the price-demand function $p(x) = 2,000 – 60x$, when $1 \leq x \leq 25$, (x is in thousands).

Find the expression representing the company's revenue function and find the domain for this function?

Solution:

Since Revenue = Price · Quantity,

$R(x) = x \cdot p(x) = x \cdot (2000 – 60x) = 2000x – 60x^2$

The domain for this function is the same as the domain of the price-demand function, $1 \leq x \leq 25$, (x is in thousands of items).

Profit Problem

An analyst for the company in the preceding problem found the following cost function that models producing and selling x thousand notebook computers:

$$C(x) = 4{,}000 + 500x,\ 1 \leq x \leq 25.$$

Write a profit function for producing and selling x thousand notebook computers, give the domain of this function, and graph the function in a window that covers the domain of the function.

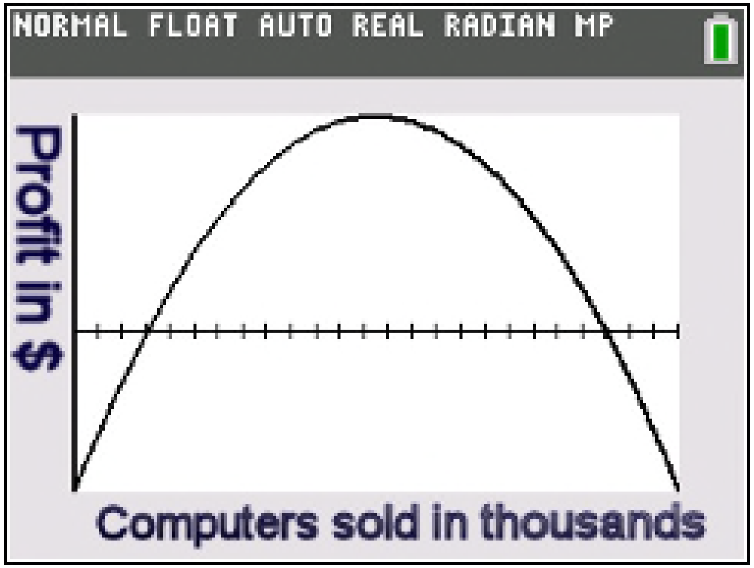
Solution to Profit Problem

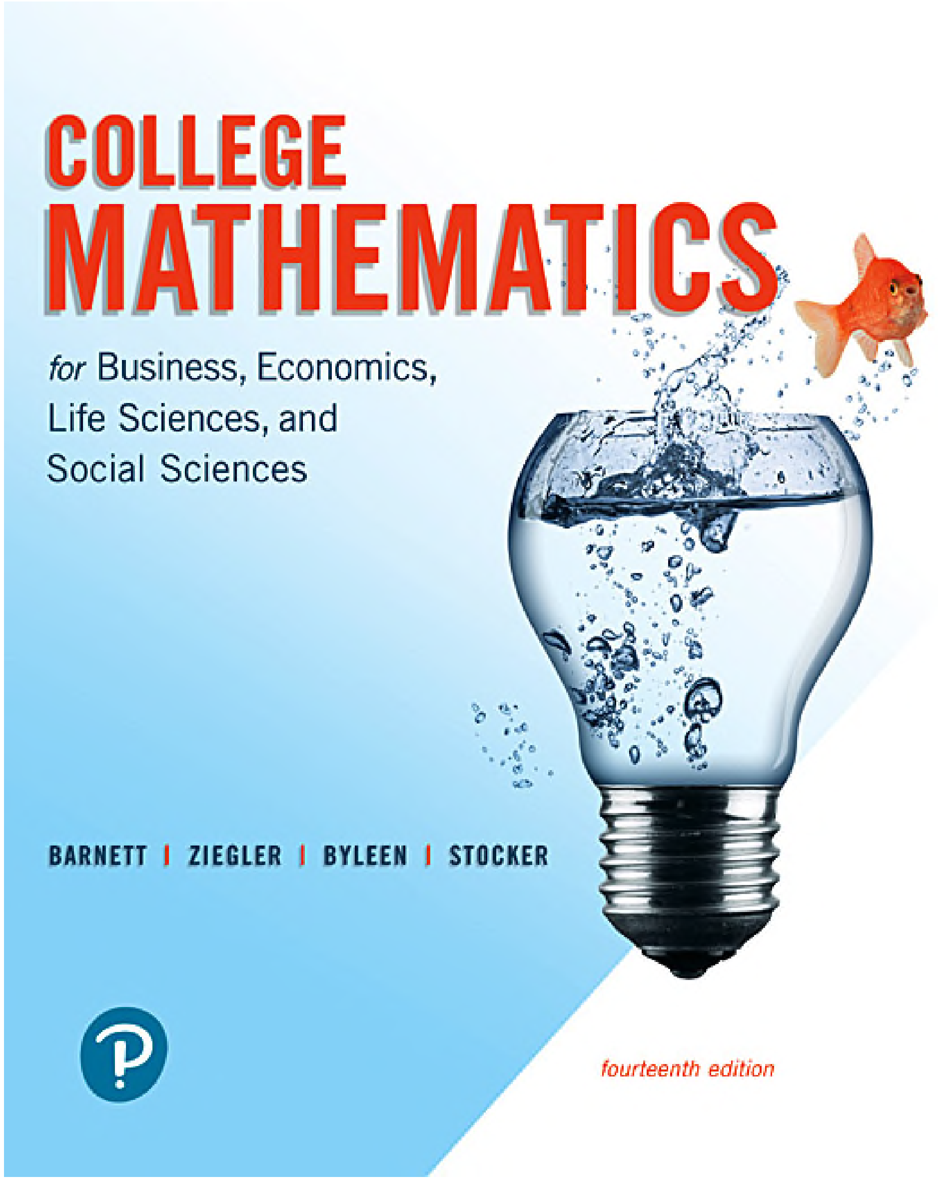
Profit = Revenue – Cost.

The revenue function was found to be $R(x) = 2000x - 60x^2$.

$$P(x) = R(x) - C(x) = 2000x - 60x^2 - (4000 + 500x)$$
$$= -60x^2 + 1500x - 4000.$$

The domain of this function is the same as the domain of the original price-demand function, $1 \le x \le 25$ (where x represents the number of computers produced and sold in thousands.)





Chapter 2

Functions and Graphs

Section 2

Elementary Functions:

Graphs

And

Transformations

Section 2.2 Topics

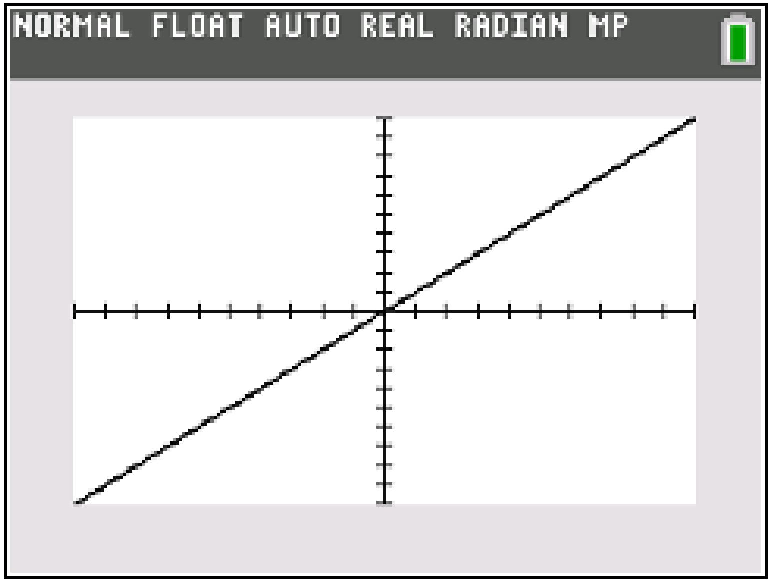
- A beginning library of elementary functions.
- Vertical and horizontal shifts.
- Reflections, stretches, and shrinks.
- Piecewise-defined functions.

A Beginning Library of Elementary Functions

- The basic properties of a small number of elementary functions will become an important addition to your mathematical toolbox.
- We begin with six basic functions.
  - Identity function
  - Square function
  - Cube function
  - Square root function
  - Cube root function
  - Absolute value function

Identity Function

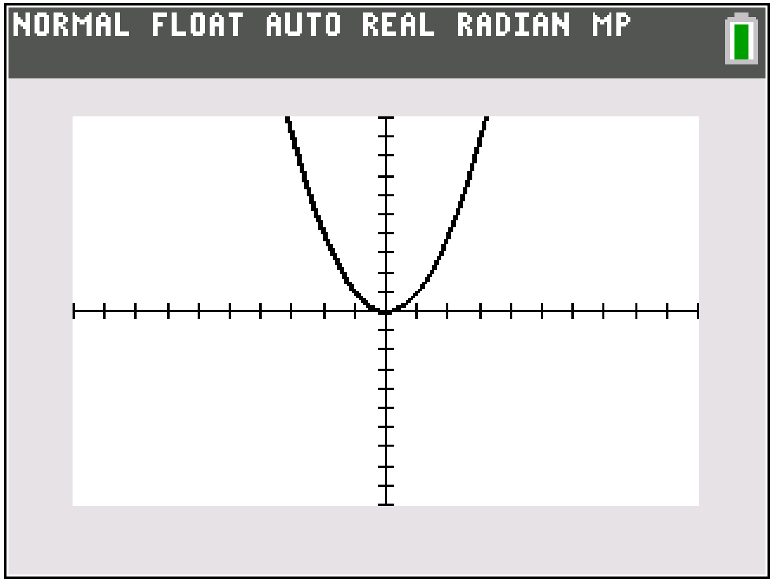
$$f(x) = x.$$



The identity function has the set of real numbers, *R*, as its domain and range.

Square Function

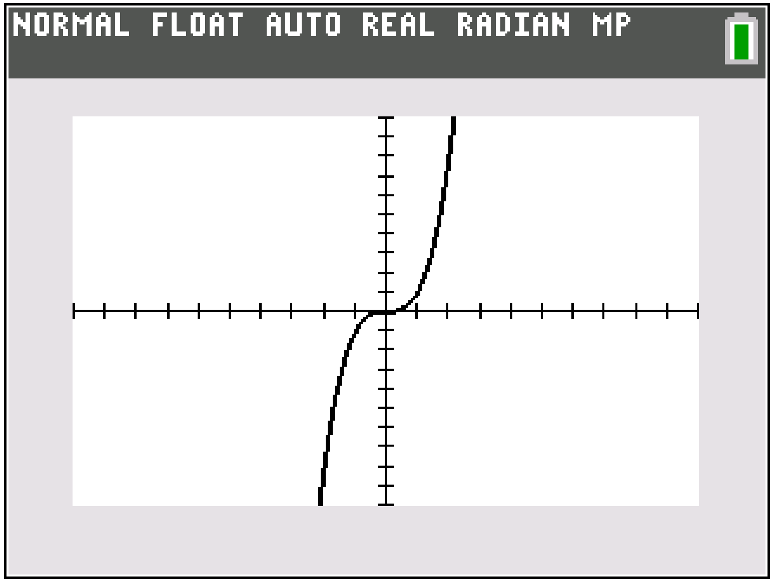
$$h(x) = x^2$$



The square function has domain, R, and range, $[0, \infty)$.

Cube Function

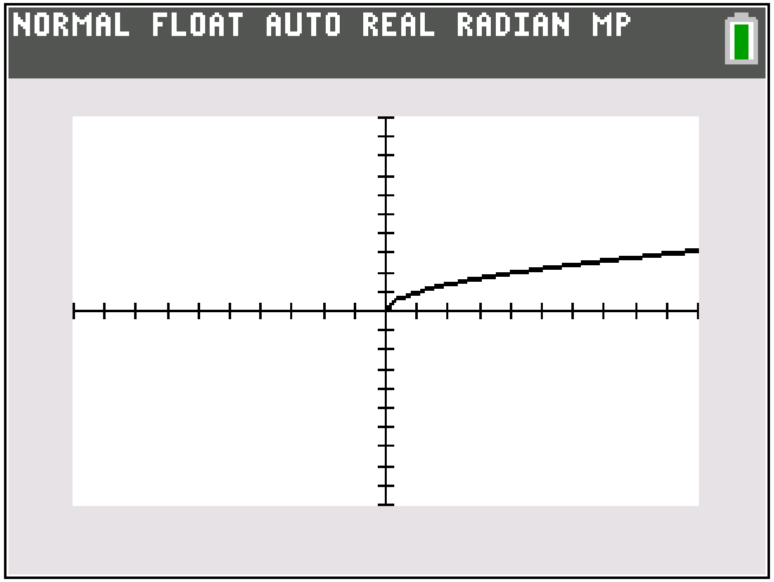
$$m(x) = x^3$$



The cube function has domain, *R*, and range, *R*.

Square Root Function

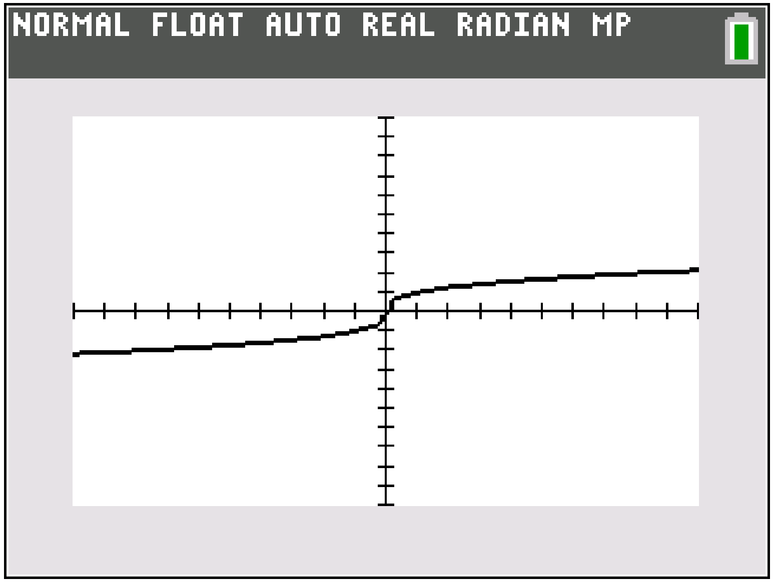
$$n(x) = \sqrt{x}$$



The square root function has domain, $[0, \infty)$, and range, $[0, \infty)$.

Cube Root Function

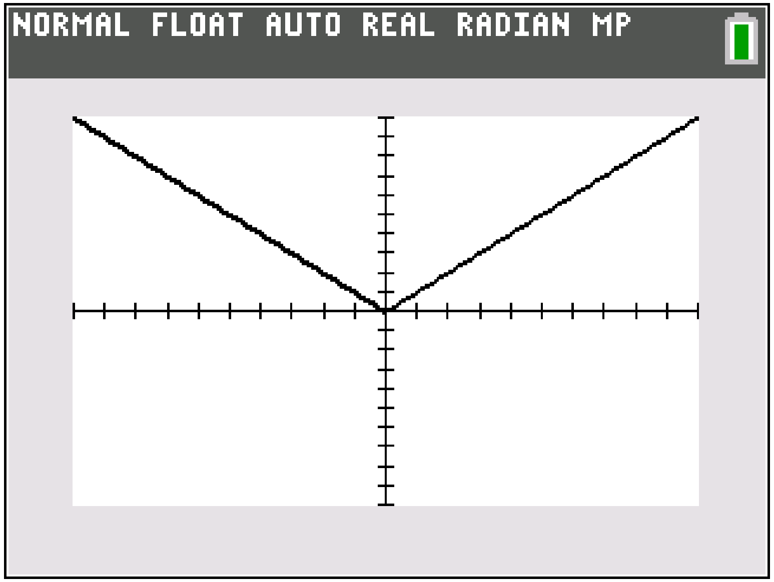
$$p(x) = \sqrt[3]{x}$$



The cube root function has domain, *R*, and range, *R*.

Absolute Value Function

$$g(x) = |x|$$



The absolute value function has domain, R, and range, $[0, \infty)$.

Vertical and Horizontal Shifts

When a new function is formed by performing an operation on a given function, the graph of the new function is called a **transformation** of the graph of the original function.

The graph of $y = f(x) + k$ gives a vertical transformation of the graph of $y = f(x)$.

If k is positive, the vertical transformation is upward by k units.

The graph of $y = f(x + h)$ gives a horizontal transformation of the graph of $y = f(x)$.

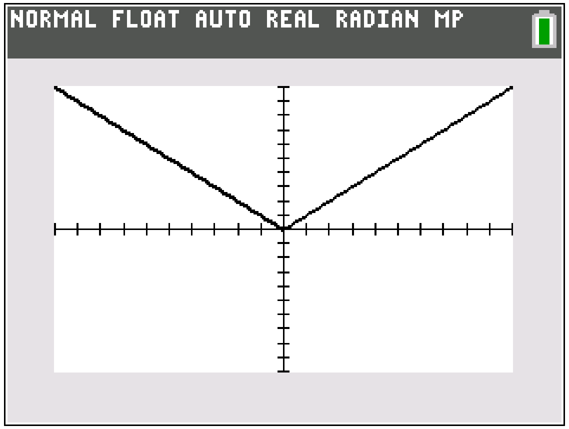
If h is positive, the horizontal transformation is to the left by h units. (Note that this is the opposite of what you might expect.)

Vertical Shift

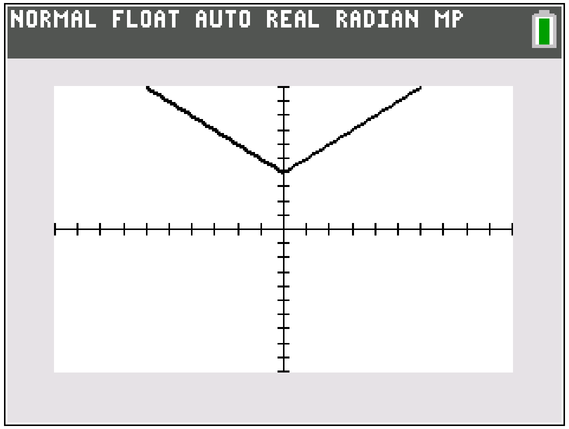
The graph of $y = f(x) + k$ can be obtained from the graph of $y = f(x)$ by **vertically translating** (shifting) the graph of the latter upward k units if k is positive and downward $|k|$ units if k is negative.

Example: Vertical Shift

$$f(x) = |x|$$



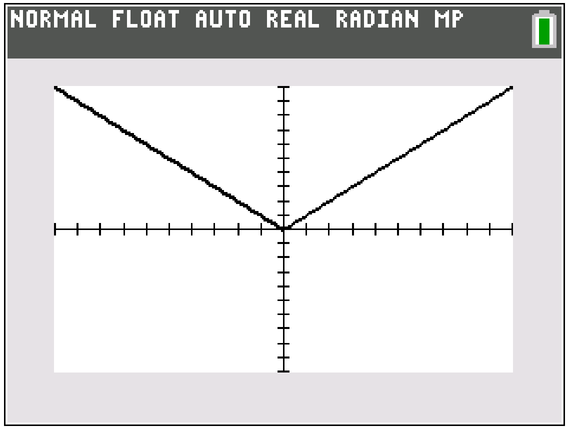
$$g(x) = |x| + 4$$



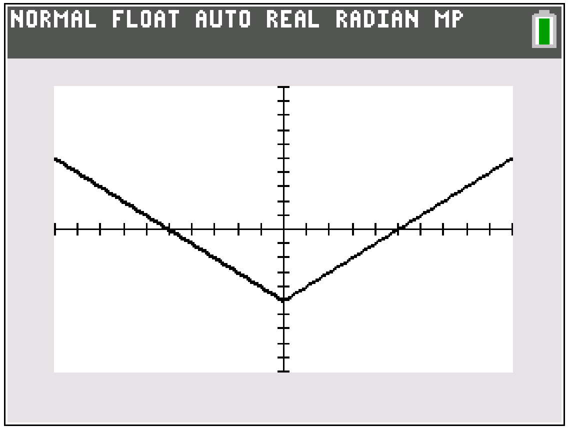
The graph is shifted vertically upward by 4 units.

Example: Vertical Shift

$$f(x) = |x|$$



$$g(x) = |x| - 5$$



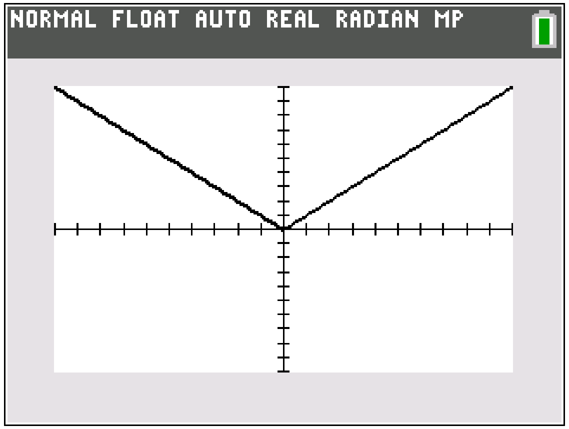
The graph is shifted vertically downward by 5 units.

Horizontal Shift

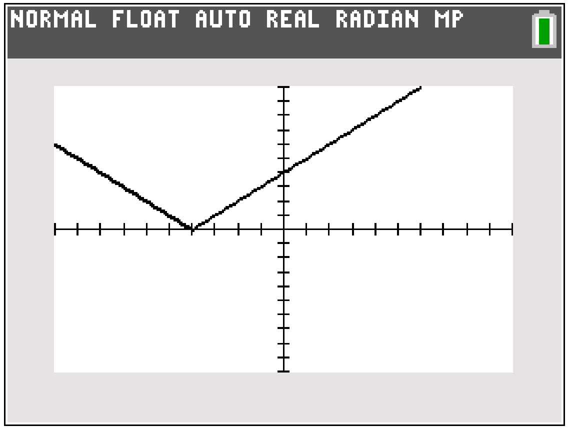
The graph of $y = f(x + h)$ can be obtained from the graph of $y = f(x)$ by **horizontally translating** (shifting) the graph of the latter h units to the left if h is positive and $|h|$ units to the right if h is negative.

Example: Horizontal Shift

$$f(x) = |x|$$



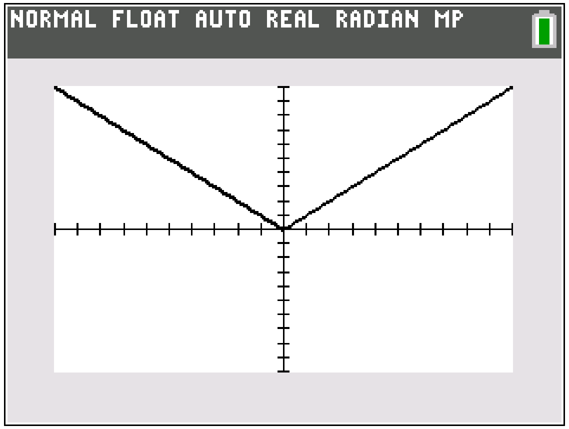
$$g(x) = |x + 4|$$



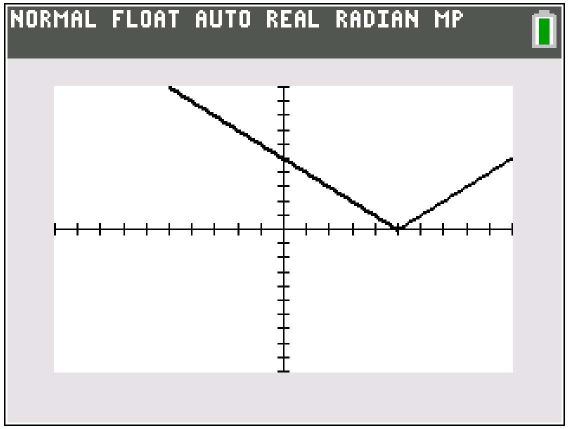
The graph is shifted horizontally left by 4 units.

Example: Horizontal Shift

$$f(x) = |x|$$



$$g(x) = |x - 5|$$



The graph is shifted horizontally right by 5 units.

Graph Translations Summary

Vertical Translation: $y = f(x) + k$

$k > 0$ Shifts the graph of $y = f(x)$ up k units.

$k < 0$ Shifts the graph of $y = f(x)$ down $|k|$ units.

Horizontal Translation: $y = f(x + h)$

$h > 0$ Shifts the graph of $y = f(x)$ left h units.

$h < 0$ Shifts the graph of $y = f(x)$ right $|h|$ units.

Stretches, Shrinks, and Reflections

The graph of $y = A \cdot f(x)$ can be obtained from the graph of $y = f(x)$ by multiplying each ordinate value of the latter by A and graphing the points with these new ordinate values.

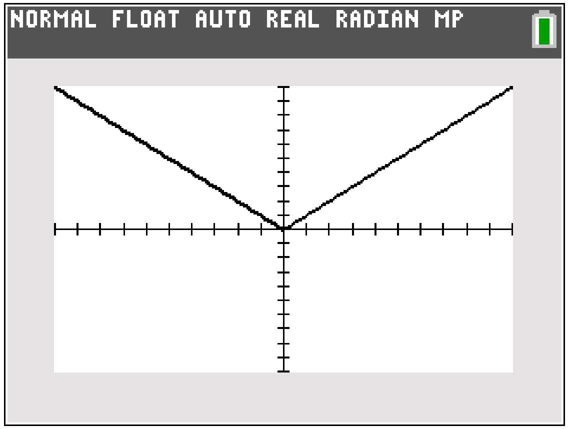
Values where $A > 1$ result in a **vertical stretch** of the graph of $y = f(x)$.

Values where $0 < A < 1$ result in a **vertical shrink** of the graph of $y = f(x)$.

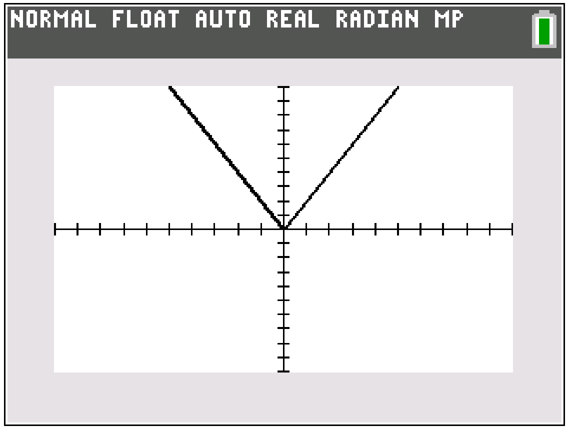
If $A = –1$, the result is a **reflection of the graph in the *x*-axis**.

Example: Vertical Stretch

$$f(x) = |x|$$



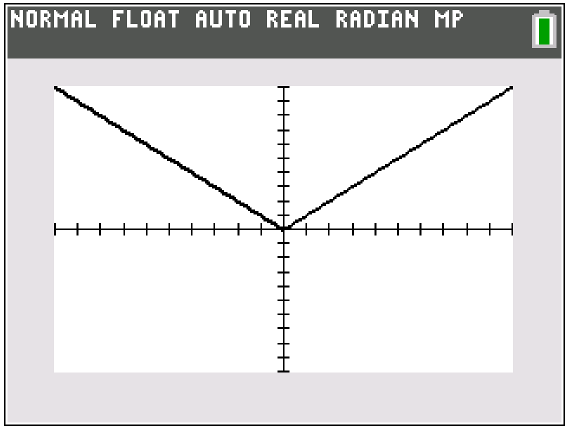
$$g(x) = 2|x|$$



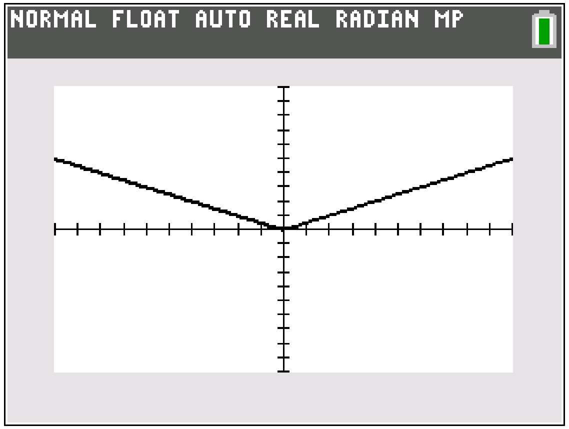
The graph is stretched vertically by a factor of 2.

Example: Vertical Shrink

$$f(x) = |x|$$



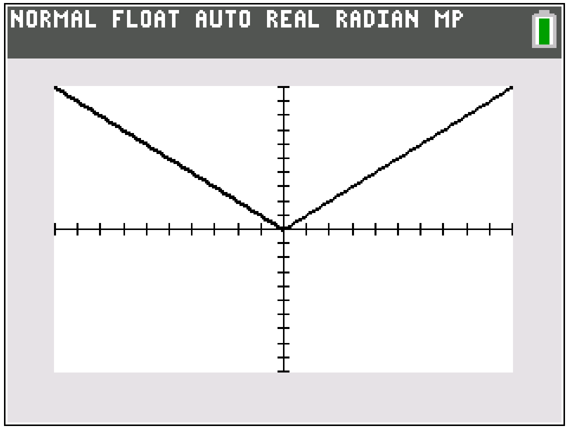
$$g(x) = 0.5|x|$$



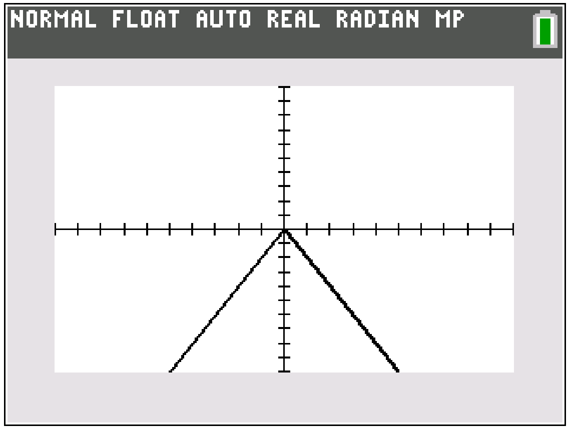
The graph is shrunk vertically by a factor of 0.5.

Example: Vertical Stretch and Reflection

$$f(x) = |x|$$



$$g(x) = -2|x|$$



The graph is stretched vertically by a factor of 2 and reflected in the *x*-axis.

Graph Stretching/Shrinking Summary

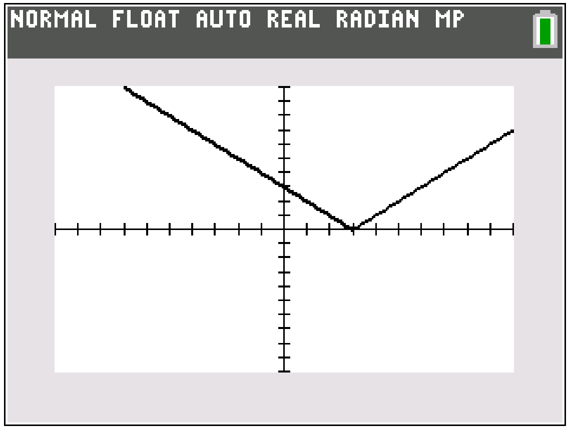
Vertical Stretch and Shrink: $y = A \cdot f(x)$

$A > 1$: Stretches the graph of $y = f(x)$ vertically by multiplying each ordinate value by A.

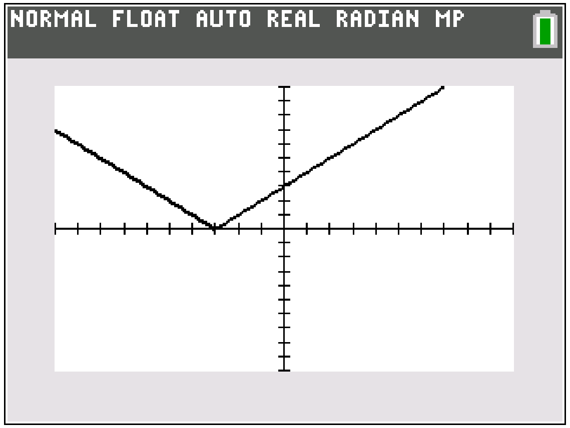
$0 < A < 1$: Shrinks the graph of $y = f(x)$ vertically by multiplying each ordinate value by A.

Example: *y*-axis Reflection

$$f(x) = |x - 3|$$



$$g(x) = |-x - 3|$$



When the input variable, *x* is replaced by –*x*, the graph is reflected in the *y*-axis.

Graph Reflections Summary

Reflection: $y = -f(x)$

Reflects the graph of $y = f(x)$ in the x axis.

Reflection: $y = f(-x)$

Reflects the graph of $y = f(x)$ in the y axis.

Piecewise Defined Functions

Functions whose definitions involve more than one rule are called **piecewise-defined** functions.

Graphing one of these functions involves graphing each rule over the appropriate portion of the domain.

The absolute value function $f(x) = |x|$

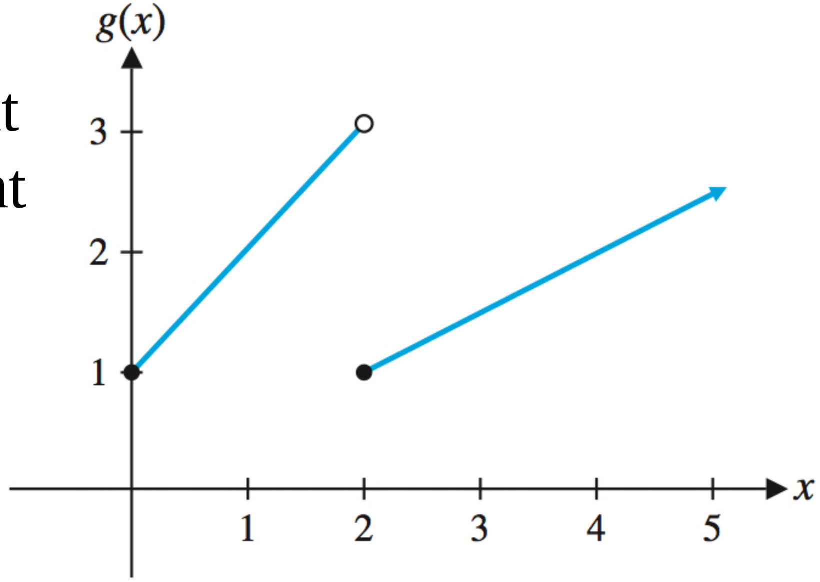
is the piecwise defined function, $f(x) = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

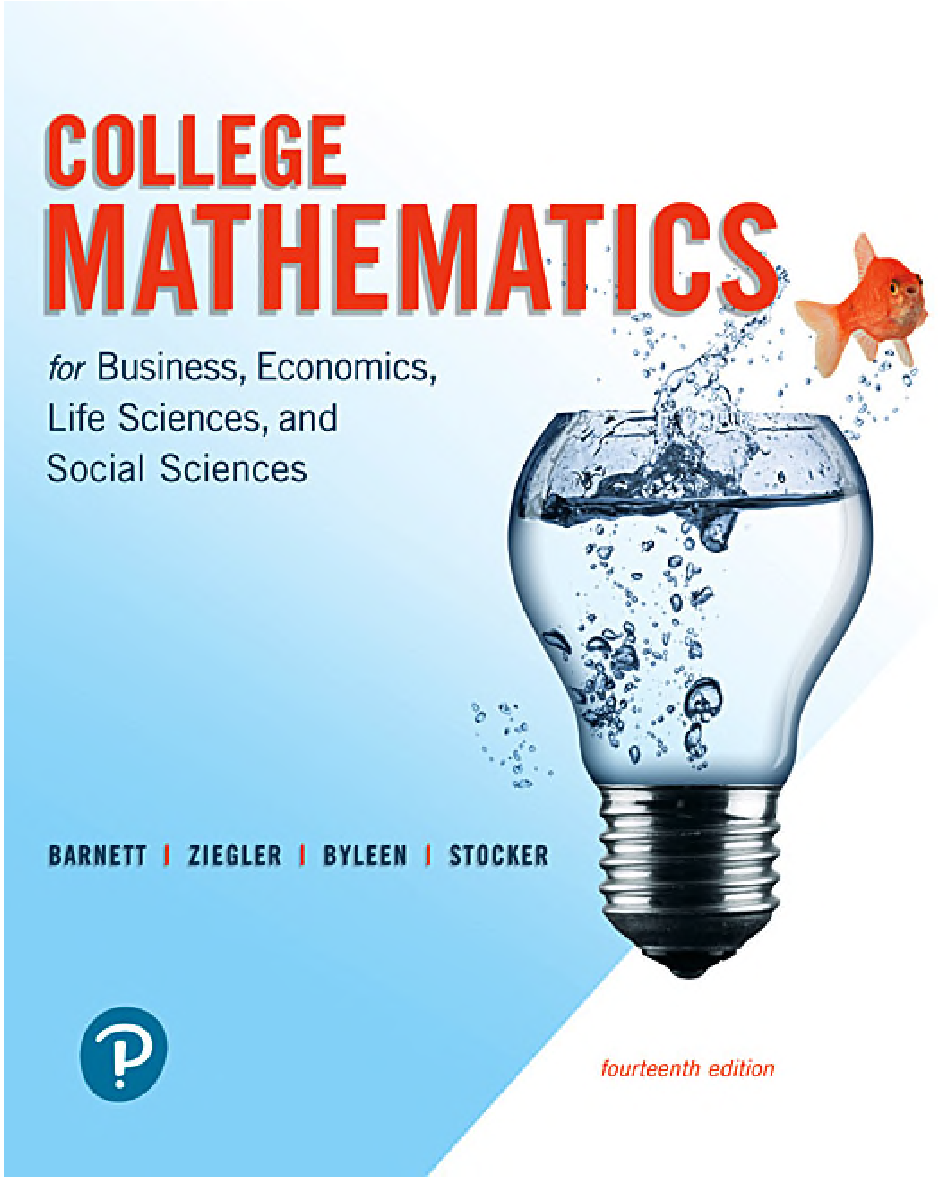
The absolute value function is defined by different rules for different parts of its domain. It is a piece-wise defined function.

Example: Graphing a Piecewise-Defined Function

Graph the function $g(x) = \begin{cases} x+1 & \text{if } 0 \le x < 2 \\ 0.5x & \text{if } x \ge 2 \end{cases}$

The point (2, 1) is a point on the graph but the point (2, 3) is not.





Chapter 2

Functions and Graphs

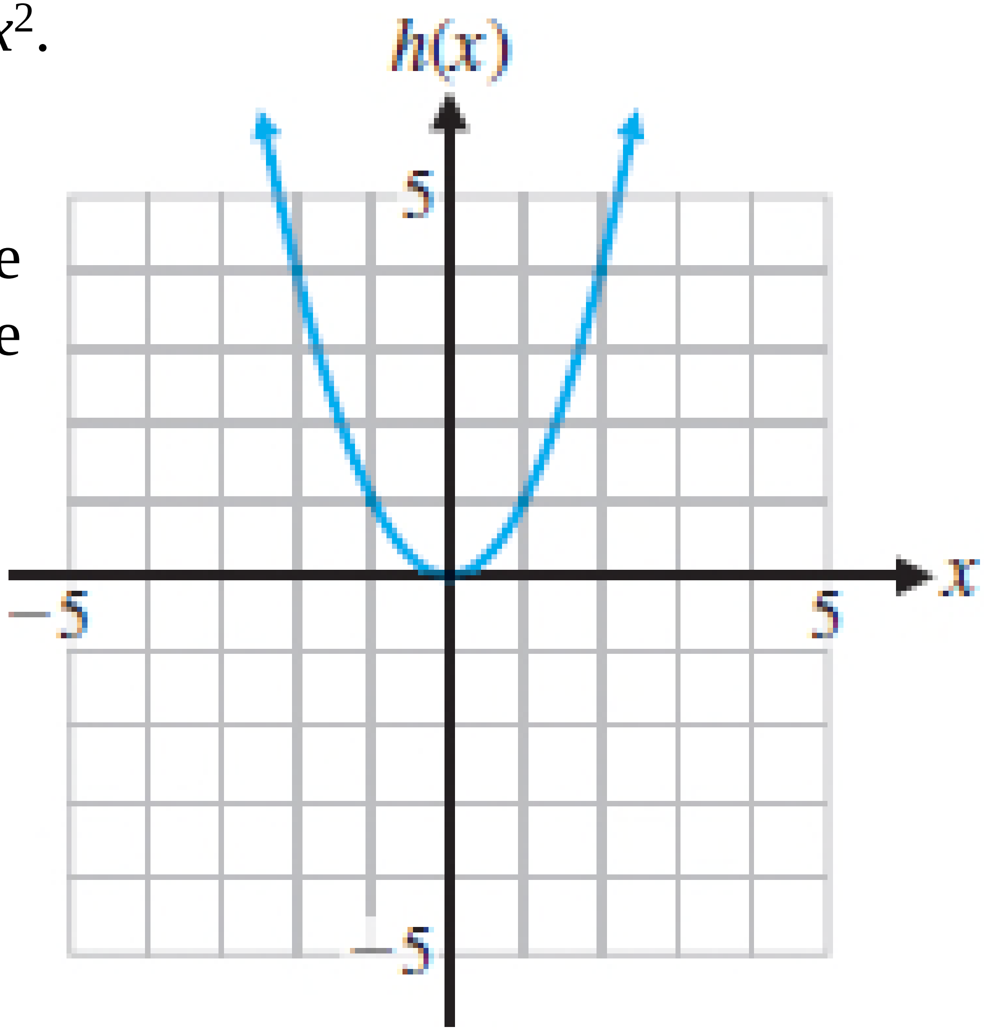
Section 3

Quadratic Functions

The Square Function

One of the basic elementary functions (from Section 2.2) is the square function, $h(x) = x^2$.

The graph of the square function gives a shape called a **parabola**.



What is the Deal With Parabolas

The graphs of every function with equation form $ax^2 + bx + c$ for real numbers *a*, *b*, and *c* are shaped similar to the square function (as long as *a* is not zero).

All such graphs are *parabolas*.

Parabolas appear in many places in our world:

The arc of a basketball shot is a parabola.

Reflecting telescopes use parabolic shaped mirrors.

Automobile headlights use parabolic reflectors.

Satellite dish antennas are parabolic shapes.

Solar furnaces are shaped like parabolas.

Parabolic microphones are often used at sporting events.

Definition: Quadratic Functions

If $a, b,$ and c are real numbers with $a \neq 0,$ then the function $f(x) = ax^2 + bx + c$ is a quadratic function in standard form and its graph is a parabola.

The domain of any quadratic function is the set of all real numbers.

The range of a quadratic function is a proper subset of the set of all real numbers.

Methods for finding the range of a quadratic function will be outlined later in this section.

Quadratic Function Concepts

An x intercept of a function is also called a **zero** of the function.

The x intercepts (if they exist) can be found by solving the quadratic equation $y = ax^2 + bx + c = 0$ for x. Recall that the quadratic equation requires that the leading coefficient a is non-zero.

A common method for solving quadratic equations is the quadratic formula.

If $ax^2 + bx + c = 0, a \neq 0,$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},$

provided $b^2 - 4ac \geq 0.$

Example 1: Intercepts, Equations, and Inequalities

(A) Sketch a graph of $f(x) = -x^2 + 5x + 3$ in a rectangular coordinate system.

Solution: Find the function value for each of the *x* values to create a table of solutions.

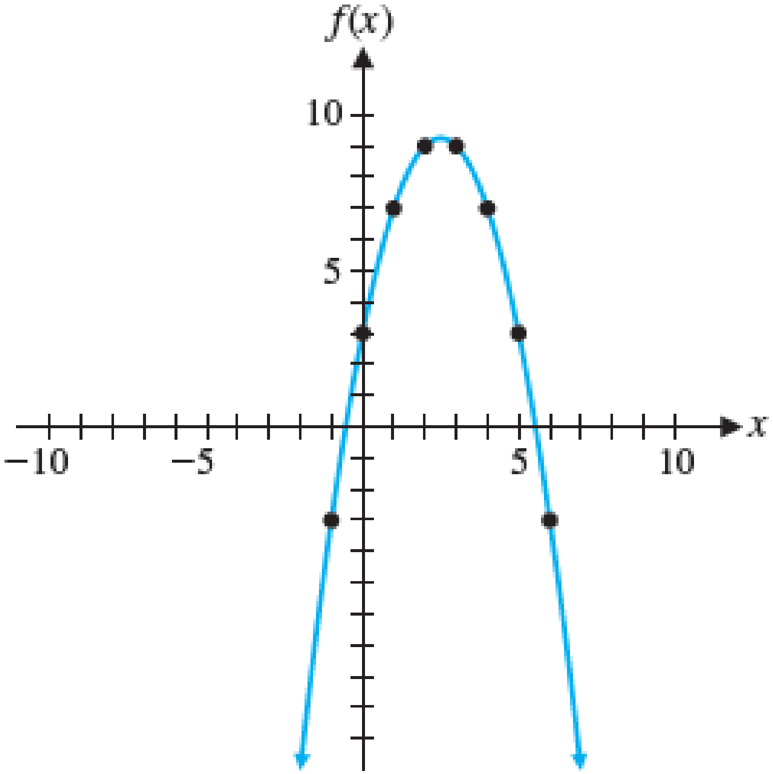
x	y
−1	−3
0	3
1	7
2	9
3	9
4	7
5	3
6	−3

Example 1: Intercepts, Equations, and Inequalities continued

A) Sketch a graph of $f(x) = -x^2 + 5x + 3$ in a rectangular coordinate system.

Solution: Plot the points (x, y) from the table and hand-sketch a graph of f by drawing a smooth curve through the plotted points.

x	y
−1	−3
0	3
1	7
2	9
3	9
4	7
5	3
6	−3



Example 1: Intercepts, Equations, and Inequalities continued

(B) Find x and y intercepts for $f(x) = -x^2 + 5x + 3$ algebraically to four decimal places.

Solution: The y intercept is found by substituting $x = 0$ into $f(x)$ to obtain $f(0) = -(0)^2 + 5(0) + 3 = 3$.

The y intercept is 3.

Use the quadratic formula with a = -1, b = 5, and c = 3 to find the x intercepts.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(-1)(3)}}{2(-1)}$$

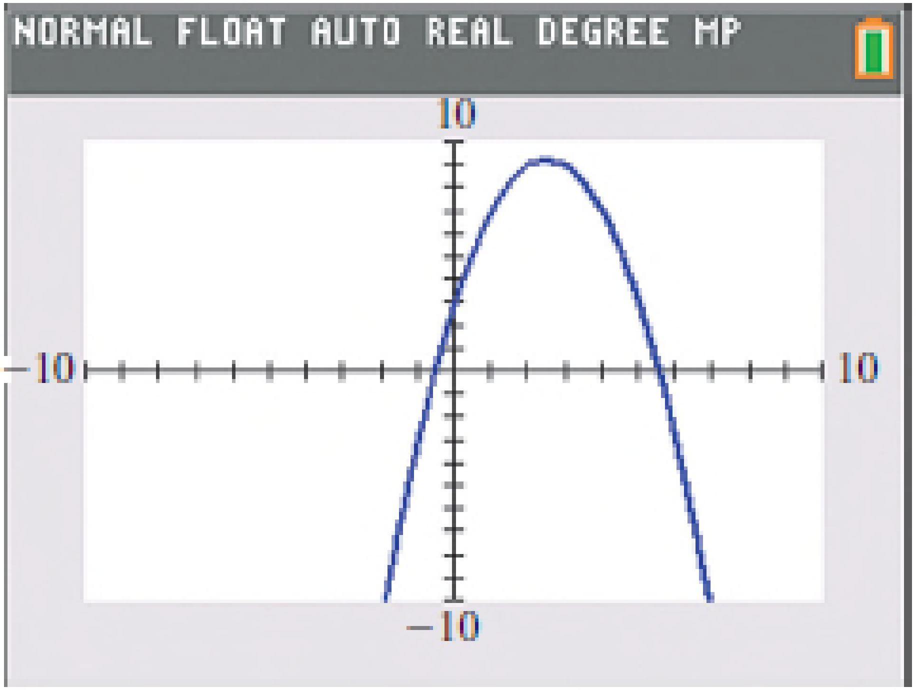
$$x = \frac{-5 \pm \sqrt{37}}{-2}$$

$$x = -0.5414 \text{ or } x = 5.5414$$

Example 1: Intercepts, Equations, and Inequalities continued

(C) Graph $f(x) = -x^2 + 5x + 3$ using a standard viewing window on a graphing calculator.

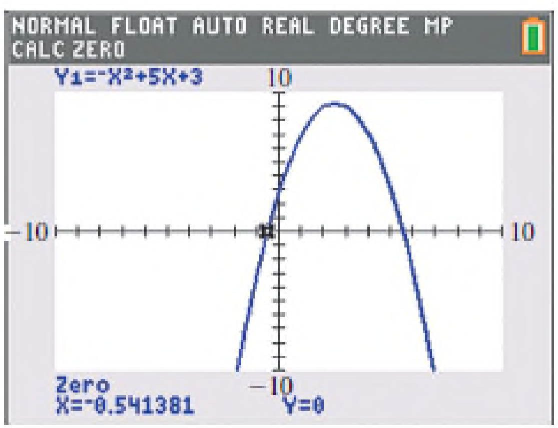
Solution: The function is input into the graphing calculator as $y_1 = -x^2 + 5x + 3$ and graphed using a standard viewing window.



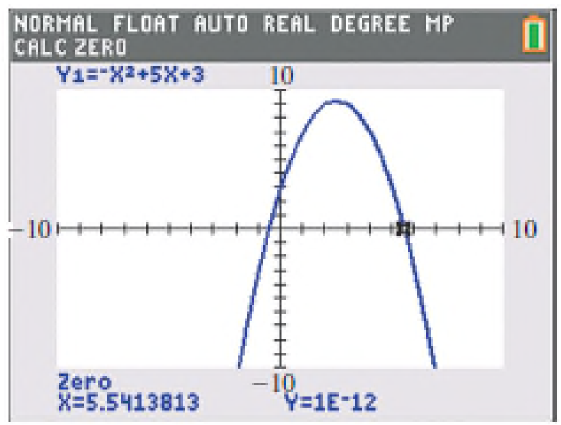
Example 1: Intercepts, Equations, and Inequalities continued

(D) Find the intercepts of $f(x) = -x^2 + 5x + 3$ graphically.

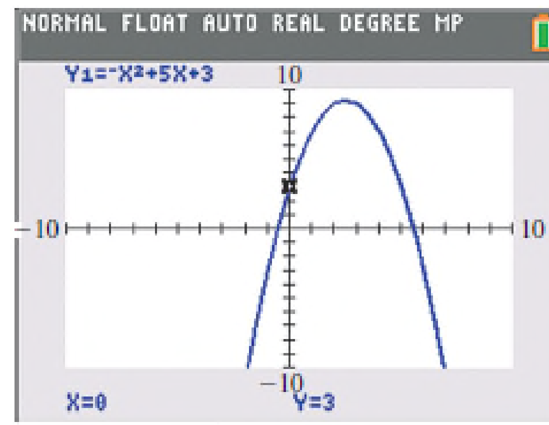
Solution: With the function input into the calculator and graphed, the *x* intercepts are found using the *zero* process. The *y* intercept is found using the *value* process.



(A) *x* intercept: −0.5414



(B) *x* intercept: 5.5414



(C) *y* intercept: 3

Example 1: Intercepts, Equations, and Inequalities continued

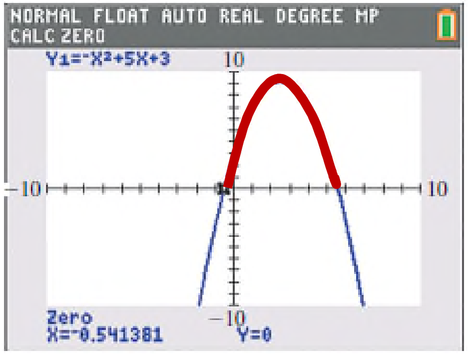
(E) Solve $-x^2 + 5x + 3 \geq 0$ graphically.

Solution: The quadratic inequality $-x^2 + 5x + 3 \geq 0$ is true for those values of x for which the graph of $f(x) = -x^2 + 5x + 3$ is at or above the x axis.

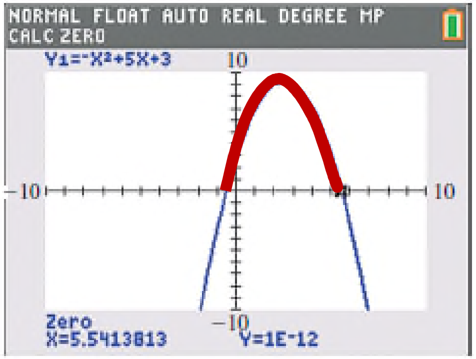
This occurs for x between the two x intercepts previously found.

The solution set is $-0.5414 \leq x \leq 5.5414$.

In interval form, the solution is [-0.5414, 5.5414].



(A) x intercept: -0.5414



(B) x intercept: 5.5414

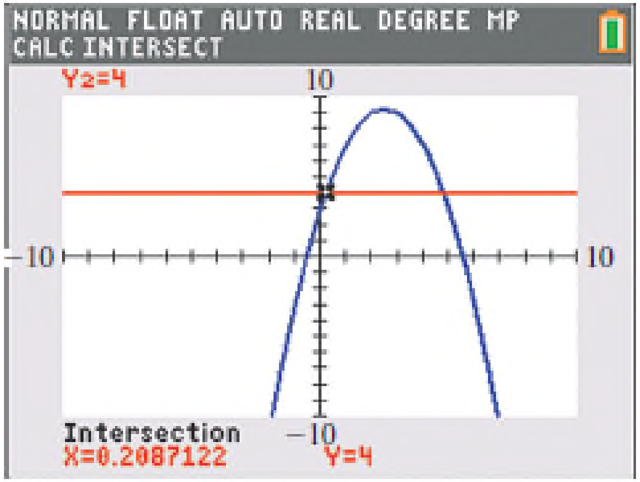
Example 1: Intercepts, Equations, and Inequalities continued

(F) Solve $-x^2 + 5x + 3 = 4$ using a graphing calculator.

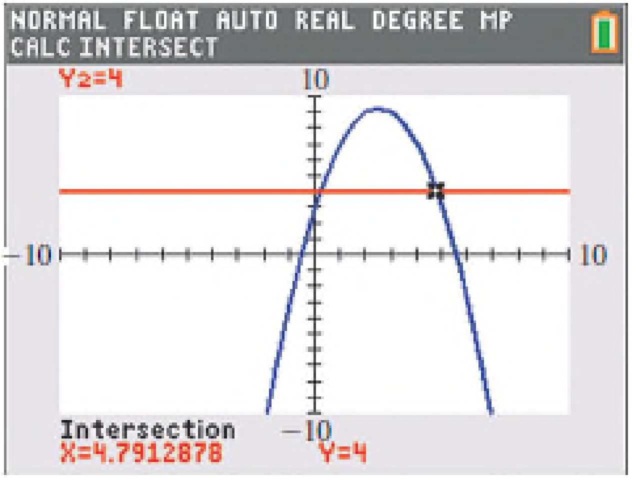
Solution: With the graph of $y_1 = -x^2 + 5x + 3$ in the calculator, input $y_2 = 4$ and graph.

Use the *intersection* process to find the points of intersection of the two graphs.

The solutions are $x = 0.2087$ and $x = 4.7913$.



(A) $-x^2 + 5x + 3 = 4$ at $x = 0.2087$



(B) $-x^2 + 5x + 3 = 4$ at $x = 4.7913$

Vertex Form of the Quadratic Function

The vertex form for a quadratic function gives additional insight into properties of a standard quadratic function.

For $f(x) = ax^2 + bx + c,\ a \neq 0,$ the vertex form is

$f(x) = a(x-h)^2 + k$ where the vertex has coordinates, (h, k).

The vertex can be found using a process called *completing the square.*

The vertex can be found using graphing calculator technology.

Other analytical processes can be used to find the coordinates for the vertex.

Example: Use Completing the Square to Find the Vertex Form of a Quadratic Function

Use the process of completing the square to rewrite the standard form quadratic, $f(x) = -3x^2 + 6x - 1$ in vertex form.

Solution:

$f(x) = -3(x^2 - 2x) - 1$ (What was done in this step? Why?)

$f(x) = -3(x^2 - 2x + 1) - 1 + 3$ (What was done in this step? Why?)

$f(x) = -3(x - 1)^2 + 2$ (What was done in this step? Why?)

The vertex is located at the point (1, 2).
The graph of this quadratic function opens down since the leading coefficient $a = -3$.

Example: Graphing the Vertex Form of a Quadratic Function

The vertex form of the standard form quadratic,

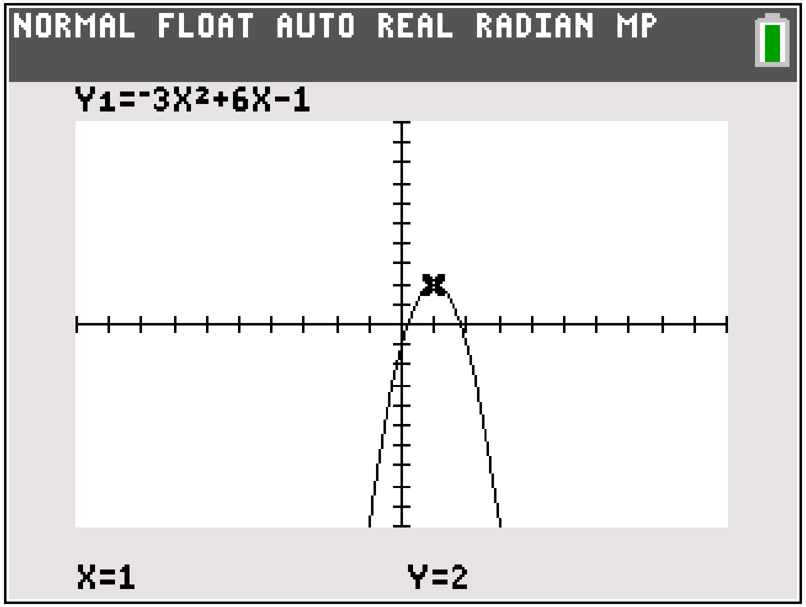
$f(x) = -3x^2 + 6x - 1$ is $f(x) = -3(x-1)^2 + 2.$

Use a graphing calculator to graph $f(x)$.

Solution: Input the function into the calculator and graph using a standard window.

The vertex is located at the point (1, 2) and is shown on the graph.

The parabola opens down since the leading coefficient $a = -3$.



Example: Analyzing the Vertex Form of a Quadratic Function

The process of completing the square was used to rewrite the standard form quadratic, $f(x) = -3x^2 + 6x - 1$ in vertex form as $f(x) = -3(x-1)^2 + 2$.

The vertex of the parabola is at the point (1, 2).
If $x = 1$, then $-3(x - 1)^2 = 0$ and $f(1) = 2$.
For any other value of x, the negative number $-3(x - 1)^2$ is added to 2, making the result smaller than 2.
Therefore, $f(1) = 2$ is the *maximum value* of $f(x)$ for all x.
The function value corresponding to any two x values that are the same distance from 1 are equal.
For example, $x = -1$ and $x = 3$ are each two units from 1 and
$f(-1) = -3(-1 - 1)^2 + 2 = -10$ and

Example: Analyzing the Vertex Form of a Quadratic Function

The standard form quadratic, $f(x) = -3x^2 + 6x - 1$

in vertex form is $f(x) = -3(x-1)^2 + 2$.

The function value corresponding to any two *x* values that are the same distance from 1 are equal.

For example, $x = -1$ and $x = 3$ are each two units from 1 and

$f(-1) = -3(-1-1)^2 + 2 = -3(-2)^2 + 2 = -12 + 2 = -10$ and

$f(-1) = -3(3-1)^2 + 2 \quad = -3(2)^2 + 2 \quad = -12 + 2 = -10$

It follows that the vertical line $x = 1$ is a line of symmetry. This means that if the graph of the function is drawn on a piece of paper and the paper is folded on the line $x = 1$, then the two sides of the parabola will match exactly.

Example: The Line of Symmetry for a Parabola

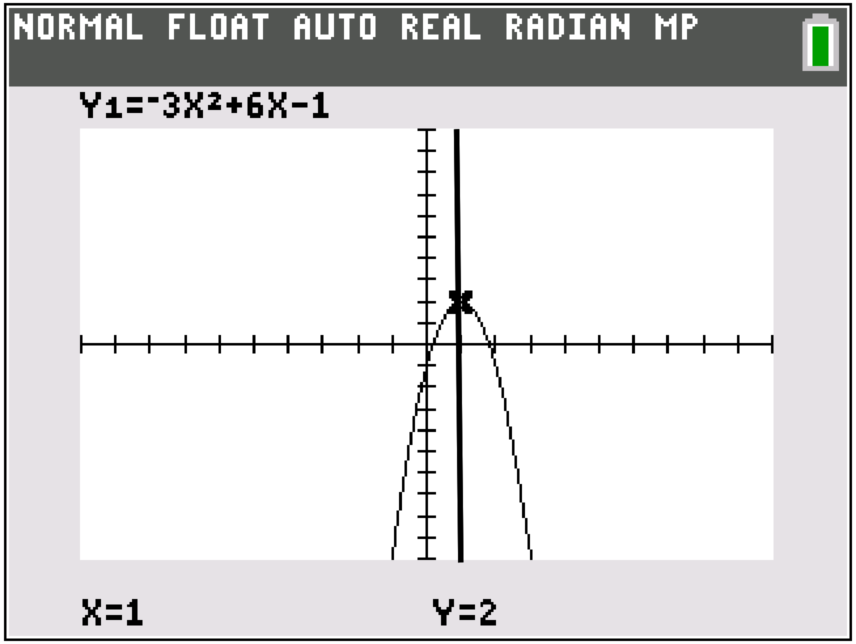
The standard form quadratic, $f(x) = -3x^2 + 6x - 1$

in vertex form is $f(x) = -3(x-1)^2 + 2$.

The line of symmetry for a parabola is the vertical line containing the vertex.

The line of symmetry, $x = 1$ is shown in the graph.

The line of symmetry is called the **axis** of the parabola.



Example: The Range of a Quadratic Function

The standard form quadratic, $f(x) = -3x^2 + 6x - 1$

in vertex form is $f(x) = -3(x-1)^2 + 2$.

This parabola opens downward with a maximum function value of 2.

For all x values, $f(x) \leq 2$.

Therefore, the range of f is $y \leq 2$ or $(-\infty, 2]$.

Transformations of the Square Function

The standard form quadratic, $f(x) = ax^2 + bx + c$

in vertex form is $f(x) = a(x-h)^2 + k$.

The transformation properties outlined in Section 2.2 can be applied to the square function $h(x) = x^2$ to vertically stretch (or compress) by a factor of a), shift horizontally by h and vertically by k and reflect in the x axis if $a < 0$ to obtain the function $f(x) = a(x - h)^2 + k$.

Each unique quadratic function is a result of appropriate horizontal and vertical shifts, vertical stretches (or compressions), and reflection in the x axis for $a < 0$.

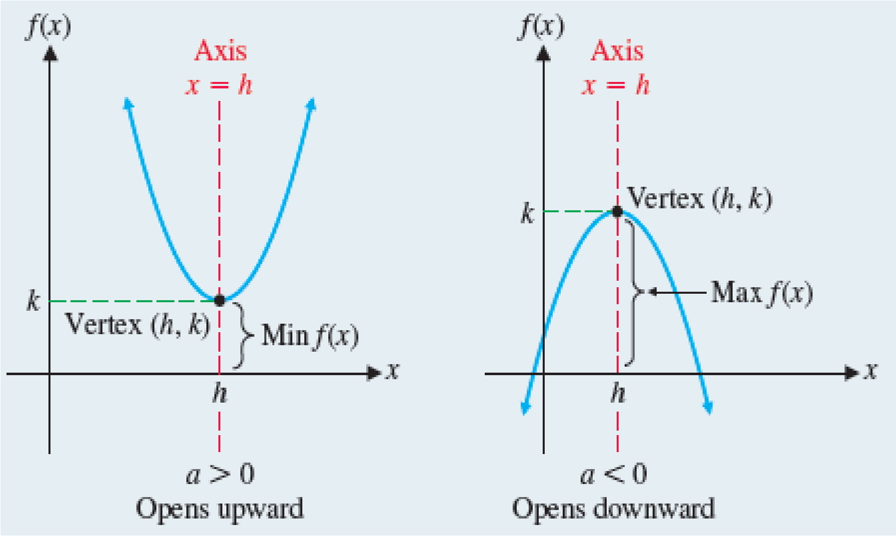
Summary: Properties of a Quadratic Function and Its Graph

The vertex form for a standard form quadratic can be obtained by completing the square

$$f(x) = ax^2 + bx + c \qquad a \neq 0 \qquad \text{Standard form}$$
$$= a(x - h)^2 + k \qquad \text{Vertex form}$$

we summarize its general properties as follows:

1. The graph of f is a parabola that opens upward if $a > 0$, downward if $a < 0$.



Summary: Properties of a Quadratic Function and Its Graph

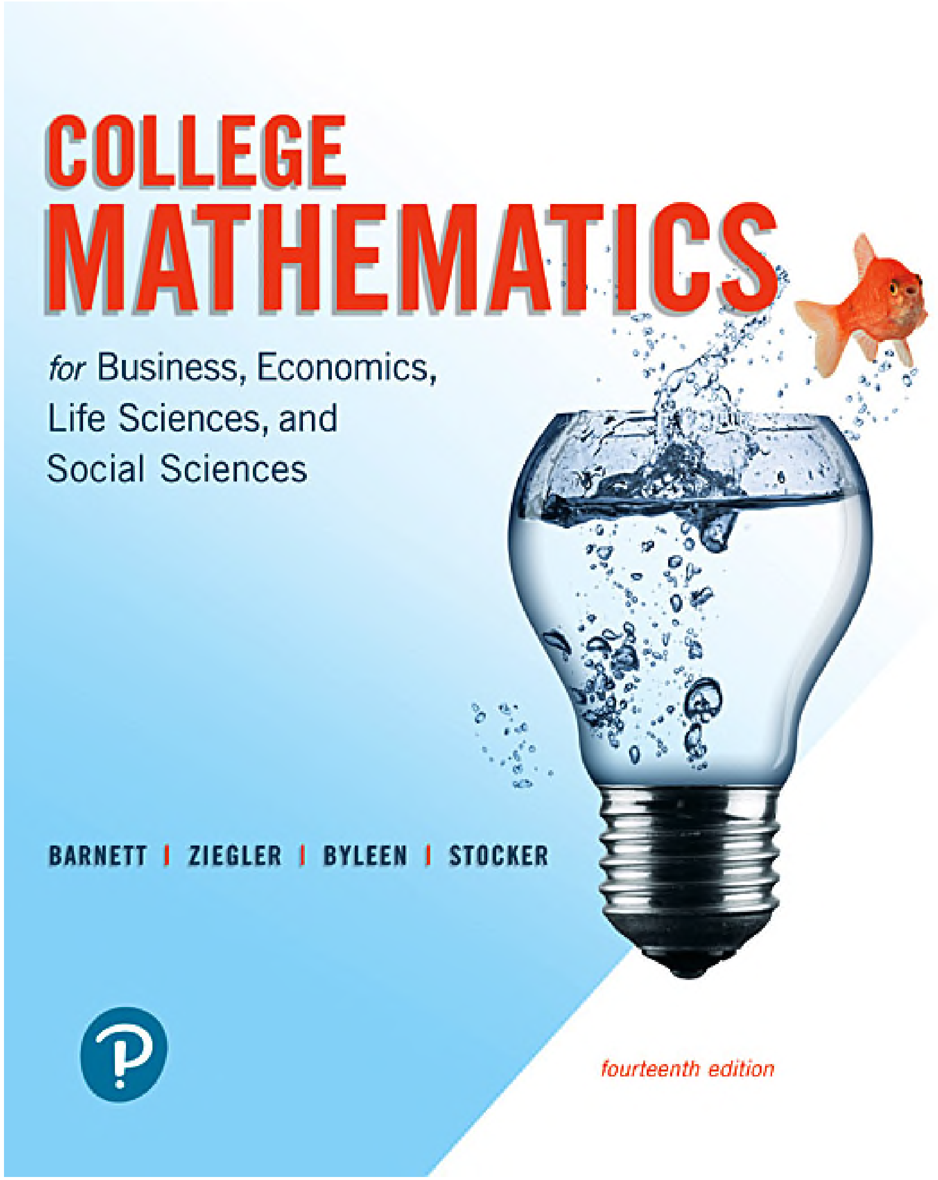
2. Vertex: (h, k) (The parabola increases on one side of the vertex and decreases on the other)

3. Axis (of symmetry): $x = h$ (parallel to the y axis)

4. $f(h) = k$ is the minimum if $a > 0$ and the maximum if $a < 0$

5. Domain: All real numbers. Range: $(-\infty, k]$ if $a < 0$ or $[k, \infty)$ if $a > 0$

6. The graph of f is the graph of $g(x) = ax^2$ translated horizontally h units and vertically k units.



Chapter 2

Functions and Graphs

Section 4
Polynomial
and
Rational Functions

Section 2.4 Polynomial and Rational Functions Topics

Properties of polynomial functions

Polynomial regression using a calculator

Properties of rational functions

Applications of polynomial and rational functions

Definition: Polynomial Function

A **polynomial function** is a function that can be written in the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ for a nonnegative integer *n*, called the **degree** of the polynomial.

The coefficients $a_0, a_1, \ldots, a_n$ are real numbers with $a_n \neq 0$.

The **leading coefficient** is the coefficient of the term of highest degree a_n.

The **domain** of a polynomial function is the set of all real numbers.

A polynomial of degree 0 is a constant function.

A polynomial of degree 1 is a linear function.

A polynomial of degree 2 is a quadratic function.

Shapes of Odd Degree Polynomial Graphs

- Shared properties for **odd degree** polynomials.
  - All odd degree polynomials cross the *x* axis at least one time.
  - If the polynomial starts negative it ends positive.
    - This will happen whenever the **leading coefficient** is positive.
  - If the polynomial starts positive it ends negative.
    - This will happen whenever the **leading coefficient** is negative.

Shapes of Even Degree Polynomial Graphs

- Shared properties for **even degree** polynomials.
  - Some even polynomials never intersect the *x* axis.
  - **ALL** even polynomials that start positive, end positive.
    - This will happen when the leading coefficient is positive.
  - **ALL** even polynomials that start negative, end negative.
    - This will happen when the leading coefficient is negative.
- As you view the graphs of the polynomials on the following slides, look for these characteristic.

Other Characteristics of Polynomial Functions

A polynomial of degree *n* can have, at most, *n* linear factors.

It follows that the graph of a polynomial function of degree *n* can intersect the *x* axis at most *n* times.

The graph of a polynomial function of degree *n* can intersect the *x* axis fewer than *n* times.

An *x* intercept of a function is called a **zero** or **root** of the function.

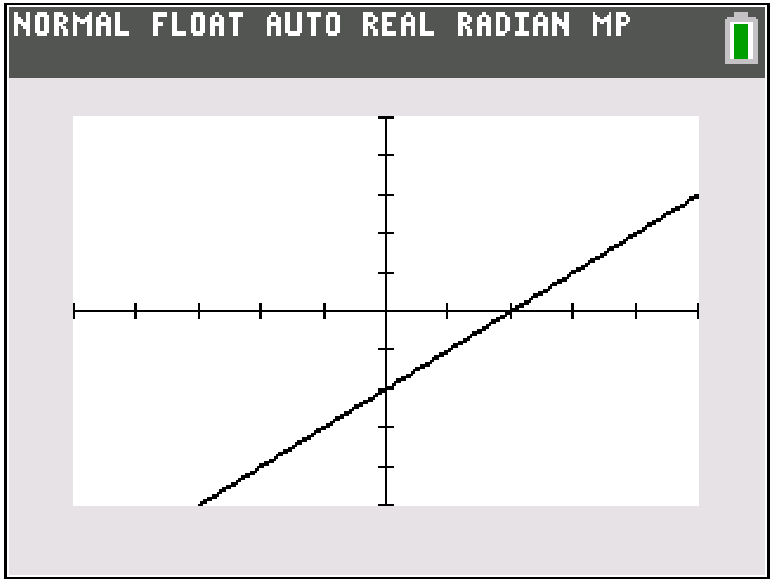
The graph of a polynomial function is **continuous**, with no holes or breaks. This means that the graph can be drawn without ever lifting a pen from the paper.

The graph of a polynomial function is smooth and has no sharp corners.

Example 1: A Graph of a Polynomial of Degree 1

Graph the polynomial function $f(x) = x - 2$.

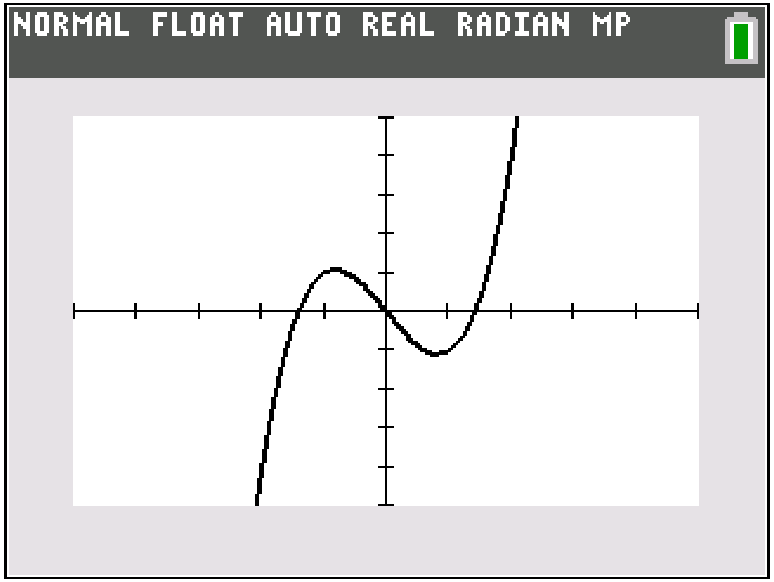
Note that this is an odd degree polynomial.



Example 2: A Graph of a Polynomial of Degree 3

Graph the polynomial function $g(x) = x^3 - 2x$.

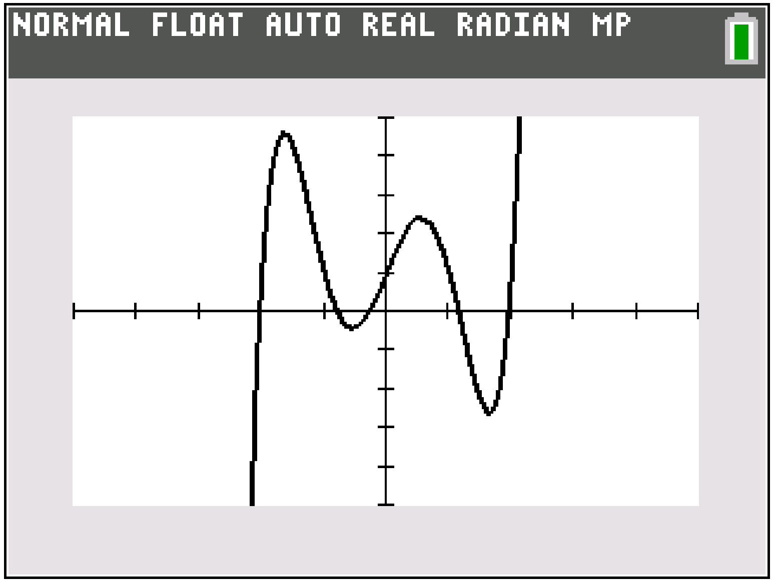
Note that this is an odd degree polynomial.



Example 3: A Graph of a Polynomial of Degree 5

Graph the polynomial function $h(x) = x^5 - 5x^3 + 4x + 1.$

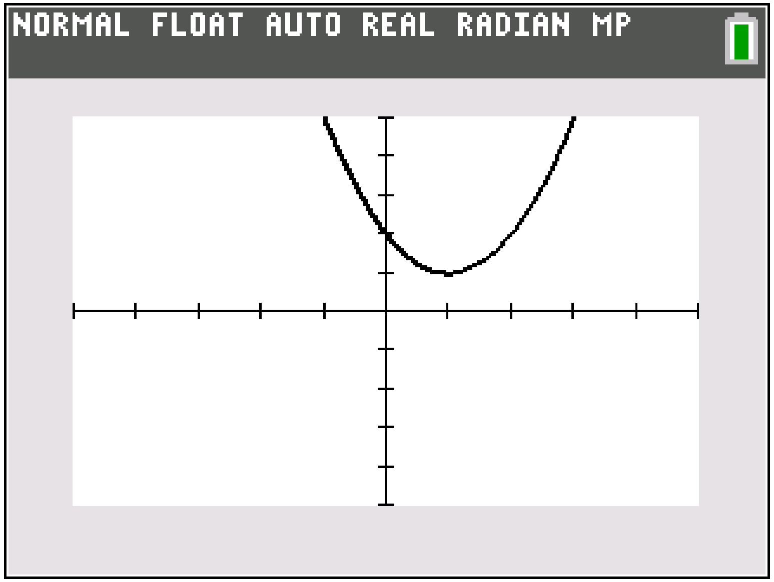
Note that this is an odd degree polynomial.



Example 4: A Graph of a Polynomial of Degree 2

Graph the polynomial $F(x) = x^2 - 2x + 2$.

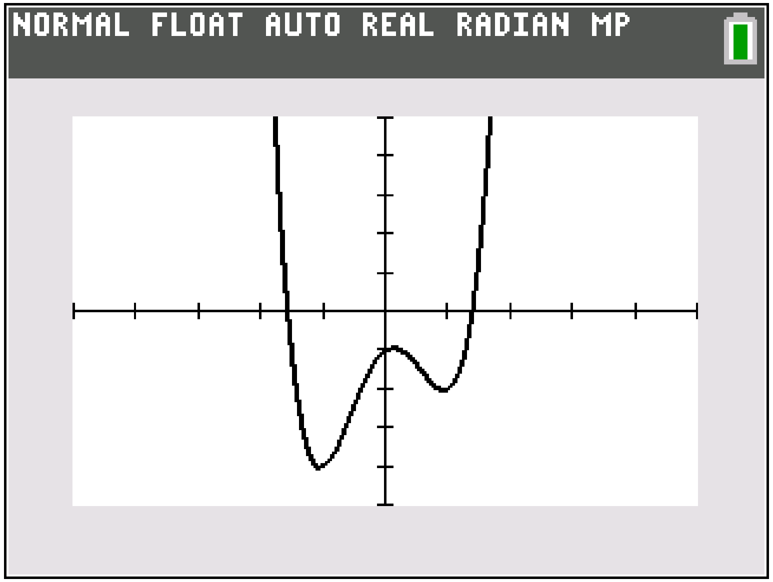
Note that this is an even degree polynomial.



Example 5: A Graph of a Polynomial of Degree 4

Graph the polynomial function $G(x) = 2x^4 - 4x^2 + x - 1.$

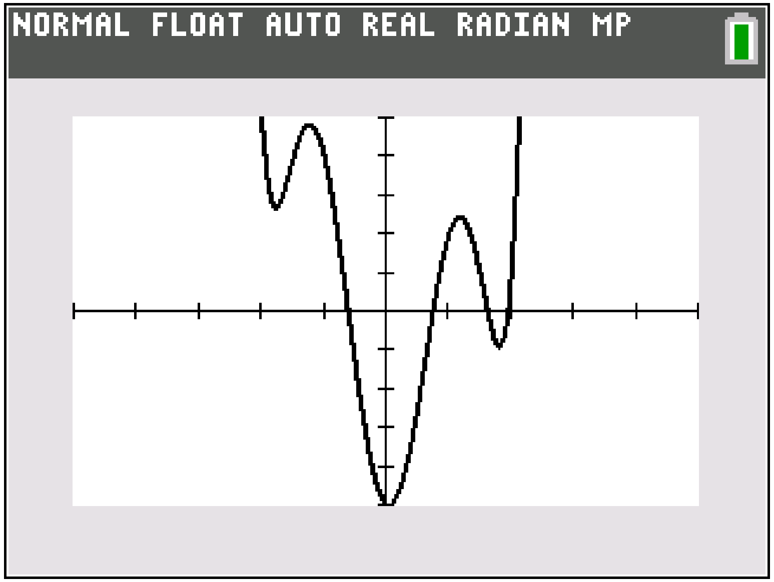
Note that this is an even degree polynomial.



Example 6: A Graph of a Polynomial of Degree 6

Graph the polynomial function $H(x) = x^6 - 7x^4 + 14x^2 - x - 5.$

Note that this is an even degree polynomial.



Observations About the Previous Graphs

In each odd degree polynomial the graph started negative and ended positive.

The ends of each odd degree polynomial moved in opposite directions.

Each odd degree polynomial crossed the *x* axis at least one time.

Observations About the Previous Graphs

In each even degree polynomial, the graph started positive and ended positive.

Some even degree polynomials crossed the *x* axis and some did not.

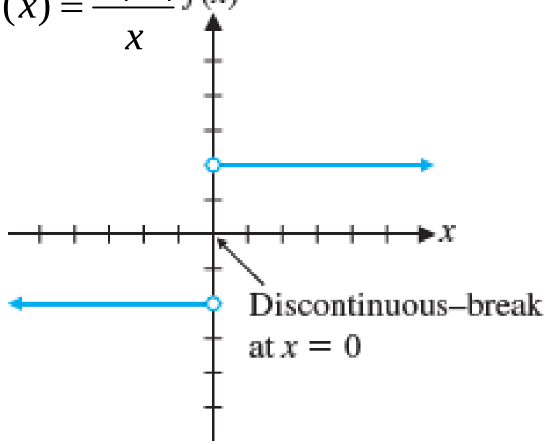
The ends of each even degree polynomial moved in the same direction.

None of the graphs crossed the *x* axis more times than their degree.

Polynomials are Continuous Functions

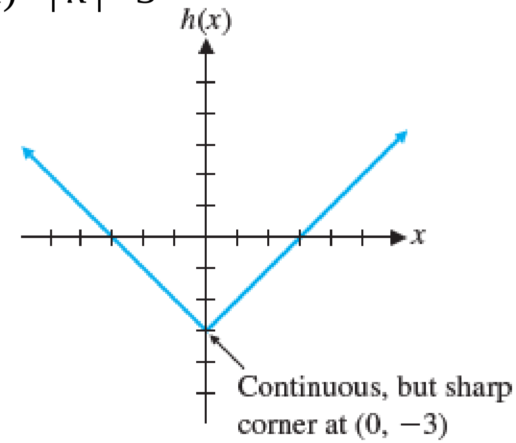
Graphs of polynomials are **continuous** and **smooth.** One can sketch all points on the graph without lifting pen from paper. The graph has no sharp corners.

$f(x) = \frac{2|x|}{x}$



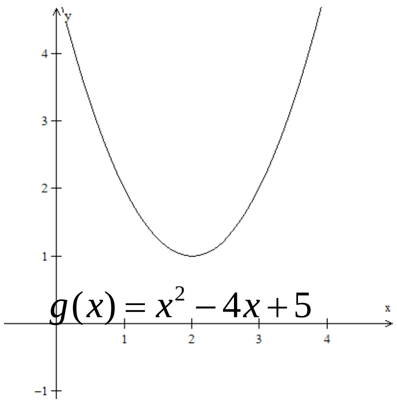
This graph is not continuous.

$h(x) = |x| - 3$



This graph is continuous but not smooth.

$g(x) = x^2 - 4x + 5$



This graph is continuous and smooth.

Regression Polynomials

In Section 2-3 we examined the process of quadratic regression. This is one type of regression polynomial that can be used to model sets of data.

Most graphing calculators have processes for finding other types of regression polynomials as best fit models for sets of data.

Example: Cubic Regression

The table of data gives the marriage and divorce rates per 1,000 population for selected years since 1960.

Let *x* represent the number of years since 1960.

Create a scatter plot for the data.

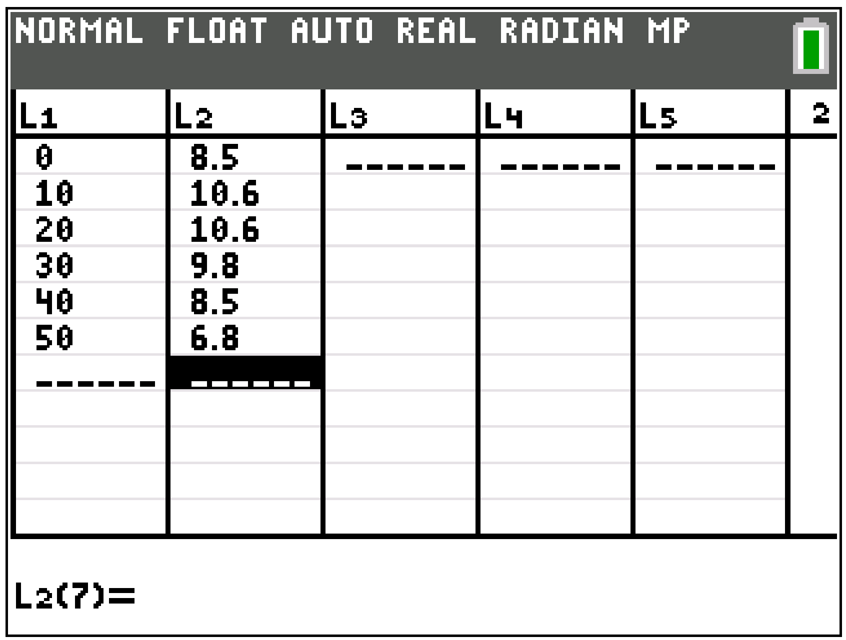
Find a cubic regression polynomial that models the marriage rate.

Marriages and Divorces (per 1000 population)		
Date	**Marriages**	**Divorces**
1960	8.5	2.2
1970	10.6	3.5
1980	10.6	5.2
1990	9.8	4.7
2000	8.5	4.1
2010	6.8	3.6

Example: Create a Scatter Plot

Input the data using the statistics editor.

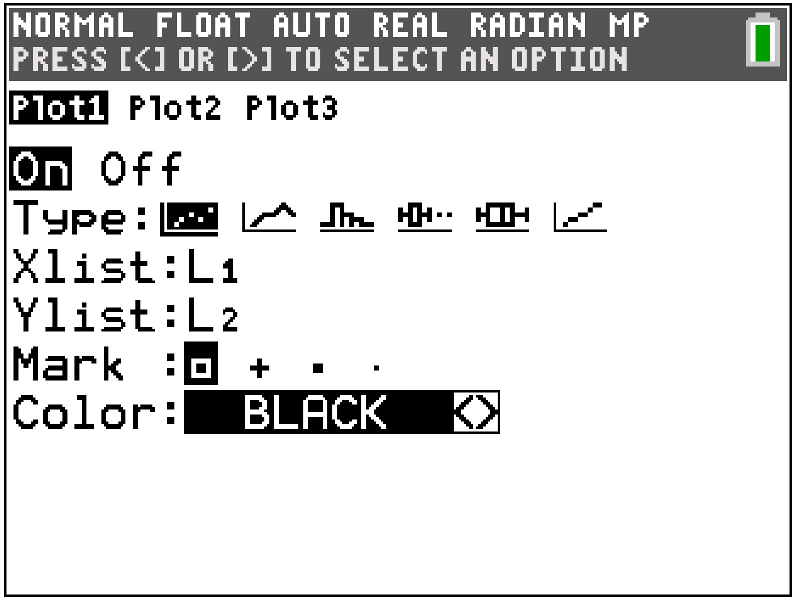
The calculator screen shot shows input values in L1 for the number of years after 1960 with the marriage rate in L2.



Example: Set up the Scatter Plot

The stat plot set up menu is used to select the type of plot desired and to indicate where the data is stored in the calculator.

The screen shot shows the correct settings for this example.

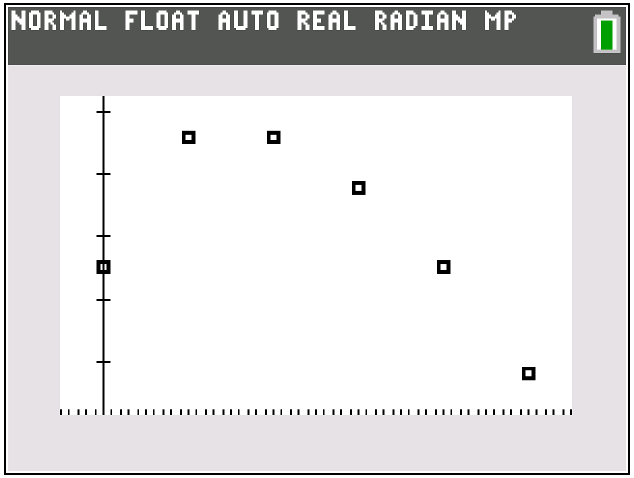


Example: Graphing the Scatter Plot

Equations in the graphing (y =) screen be deleted before viewing the scatter plot.

The calculator zoom option Zoom 9:ZoomStat is used to size the graphing display window and show the graph.

The resulting scatter plot for this example is shown.

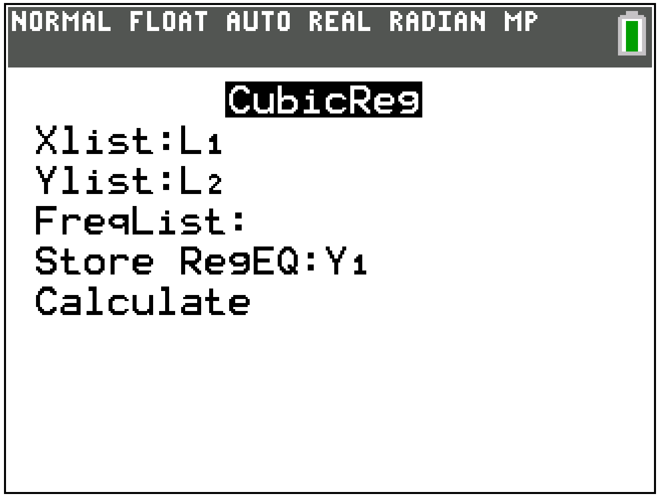


Example: Create a Cubic Regression Model

The cubic regression process is option 6: CubicReg under the Stat Calc menu.

The appropriate settings for this example are shown in the calculator window screen shot below.

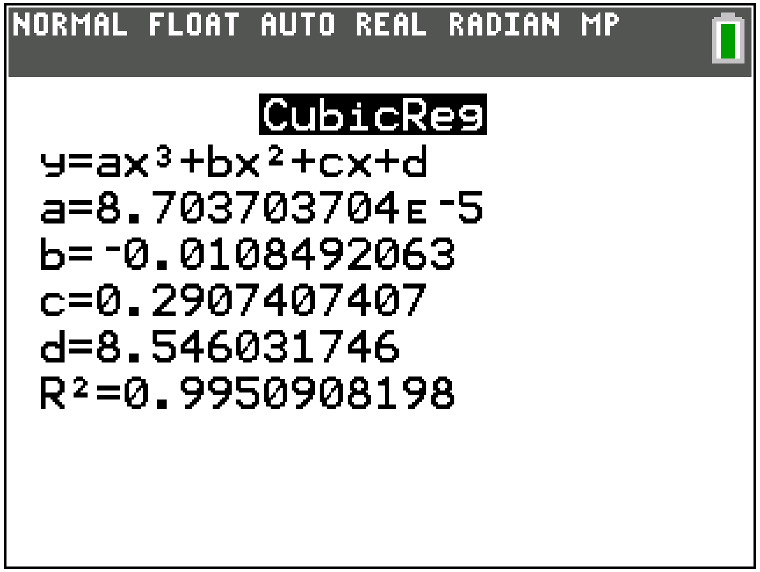
The variable Y1 (indicating where the resulting cubic regression function is to be stored) requires the keystrokes *vars*, *Y-vars*, *Function*, 1:*Y*1.



Example: Create a Cubic Regression Model

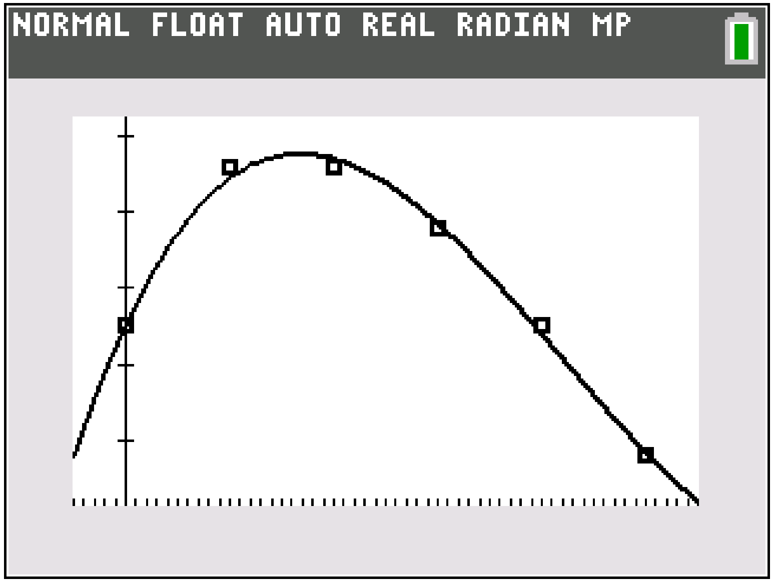
As shown in the calculator screen shot, the best fit cubic regression model for these data is $y = 8.70x^3 - 0.01x^2 + 0.29x + 8.55$ (values rounded to the nearest hundredth).

The R^2 value 0.995 indicates that the accuracy of fit is approximately 99.5%.



Example: Scatter plot compared to the cubic regression function.

The calculator graph shows the scatter plot of the data and the graph of the cubic regression function.



The graph of the cubic regression function closely matches the scatter plot of the data.

Definition: Rational Function

A **rational function** is any function that can be written as

$$f(x) = \frac{n(x)}{d(x)} \qquad d(x) \neq 0$$

where $n(x)$ and $d(x)$ are polynomials.

The **domain** is the set of all real numbers for which $d(x) \neq 0$.

Rational functions are quotients of polynomial functions.

Example: Rational Function

Example: Let $n(x) = x - 3$ and $d(x) = x - 2$.

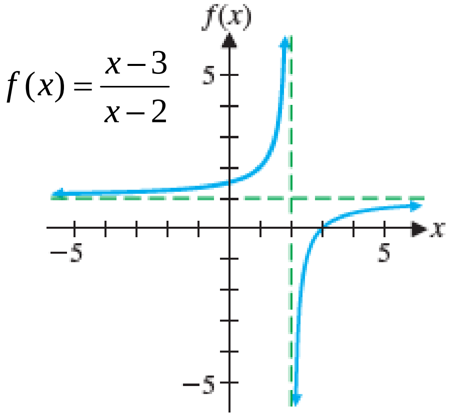
Write the rational function $f(x) = \dfrac{n(x)}{d(x)}$ and give the domain for $f(x)$.

Solution: $f(x) = \dfrac{n(x)}{d(x)} = \dfrac{x-3}{x-2}$

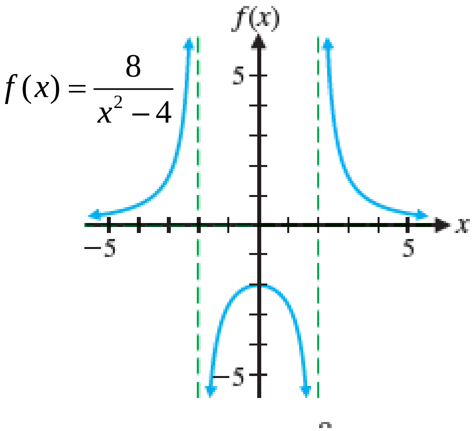
Since $d(2) = 0$, $x = 2$ is not in the domain of the rational function, $f(x)$.
The domain of $f(x)$ is the set of all real numbers $x \neq 2$.

Vertical Asymptotes of Rational Functions

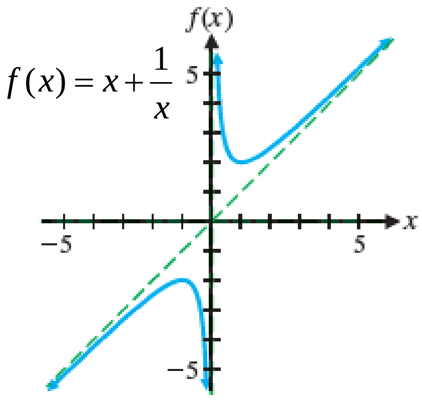
A *vertical asymptote* of a rational function *f*(*x*) is a line of the form *x* = *h* which the graph of the function approaches but does not cross.



f(*x*) has a vertical asymptote at *x* = 2.



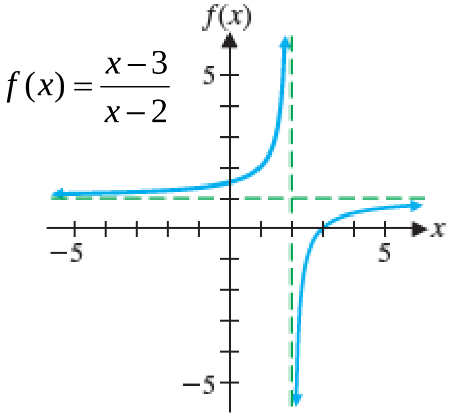
f(*x*) has vertical asymptotes at *x* = –2 and *x* = 2.



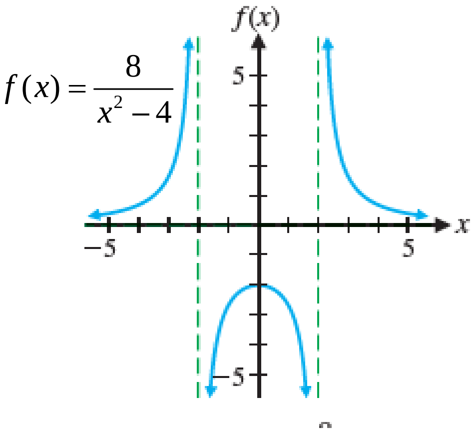
f(*x*) has no vertical asymptotes.

Horizontal Asymptotes of Rational Functions

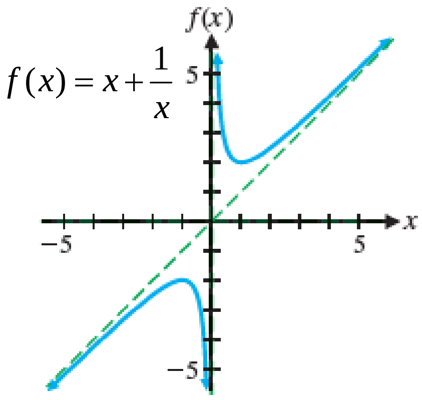
A *horizontal asymptote* of a rational function is a line of the form $y = k$ which the graph of the function approaches but does not cross as both x increases and decreases without bound.



f(*x*) has a horizontal asymptote at $y = 1$.



f(*x*) has a horizontal asymptote at $y = 0$.



f(*x*) has no horizontal asymptote.

Number of Vertical Asymptotes of Rational Functions

If the denominator $d(x)$ of a rational function has degree n, the rational function can have no more than n vertical asymptotes.

If the numerator $n(x)$ and denominator $d(x)$ of the rational function have no common real zeros, if $d(c) = 0$, then $x = c$ is a vertical asymptote of the rational function.

If the numerator $n(x)$ and denominator $d(x)$ of the rational function share common real zeroes, they share linear factors for each shared real zero, and the rational function can be reduced by cancelling these common factors.

Any remaining zeroes for the denominator of the reduced rational function will be vertical asymptotes.

Horizontal Asymptotes of Rational Functions

A rational function with degree of the numerator less than degree of the denominator has horizontal asymptote $y = 0$.

A rational function with degree of the numerator is equal to degree of the denominator has horizontal asymptote $y = a/b$ where a is the leading coefficient of the numerator and b is the leading coefficient of the denominator.

A rational function with degree of the numerator is greater than degree of the denominator has no horizontal asymptote.

Example: Find Asymptotes

Find the vertical and horizontal asymptotes of the rational function.

$$f(x) = \frac{3x^2 + 3x - 6}{2x^2 - 2}$$

Solution: Factor the numerator and denominator to find any common factors.

$$n(x) = 3(x^2 + x - 2) = 3(x - 1)(x + 2)$$

$$d(x) = 2(x^2 - 1) = 2(x - 1)(x + 1)$$

The rational function reduces to $\frac{3(x+2)}{2(x+1)}$.

Example: Find Asymptotes continued

Solution: $f(x) = \dfrac{3x^2 + 3x - 6}{2x^2 - 2}$ is reduced to $\dfrac{3(x+2)}{2(x+1)}$.

Vertical Asymptote: The denominator of the rational function resulting from dividing common linear factors has a zero for $x = -1$. The denominator has no other zeroes. Therefore, $x = -1$ is the only vertical asymptote of $f(x)$.

Horizontal Asymptote: The numerator and denominator of $f(x)$ were each of degree 2. The leading coefficient in the numerator is 3 and the leading coefficient in the denominator is 2. The horizontal asymptote is $y = 3/2$.

Example: Find Asymptotes You Try It!

- Find the vertical and horizontal asymptotes of the rational function

$$f(x)=\frac{x^3-4x}{x^2+5x}.$$

Bounded Functions

A function *f* is **bounded** if the entire graph of *f* lies between two horizontal lines.

The only polynomials that are bounded are the constant functions, but there are many rational functions that are bounded.

Students: Give an example of a bounded, non-constant rational function with the set of all real numbers as its domain.

Example: Application of Rational Functions

A company that manufactures computers has established that, on the average, a new employee can assemble $N(t)$ components per day after t days of on-the-job training, as given by

$$N(t) = \frac{50t}{t+4}, \quad t \geq 0$$

Sketch a graph of N, $0 \leq t \leq 100$, including any vertical or horizontal asymptotes.

What does $N(t)$ approach as t increases without bound?

Solution to Example: Application of Rational Functions

Vertical asymptote: The domain of the function ($t \geq 0$) excludes the value of t for which the denominator is zero so there is no vertical asymptote for the function with the stated domain.

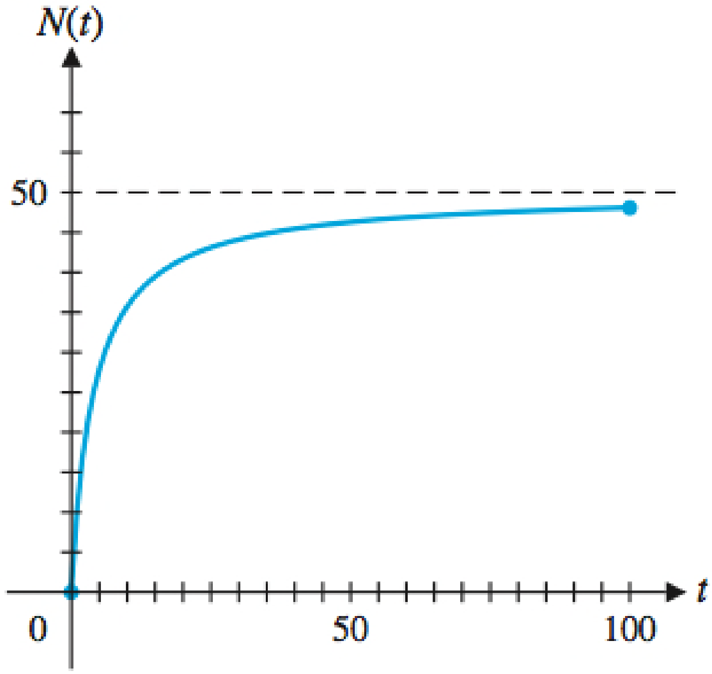
Horizontal asymptote: $$N(t) = \frac{50t}{t+4} = \frac{50}{1+\frac{4}{t}}$$

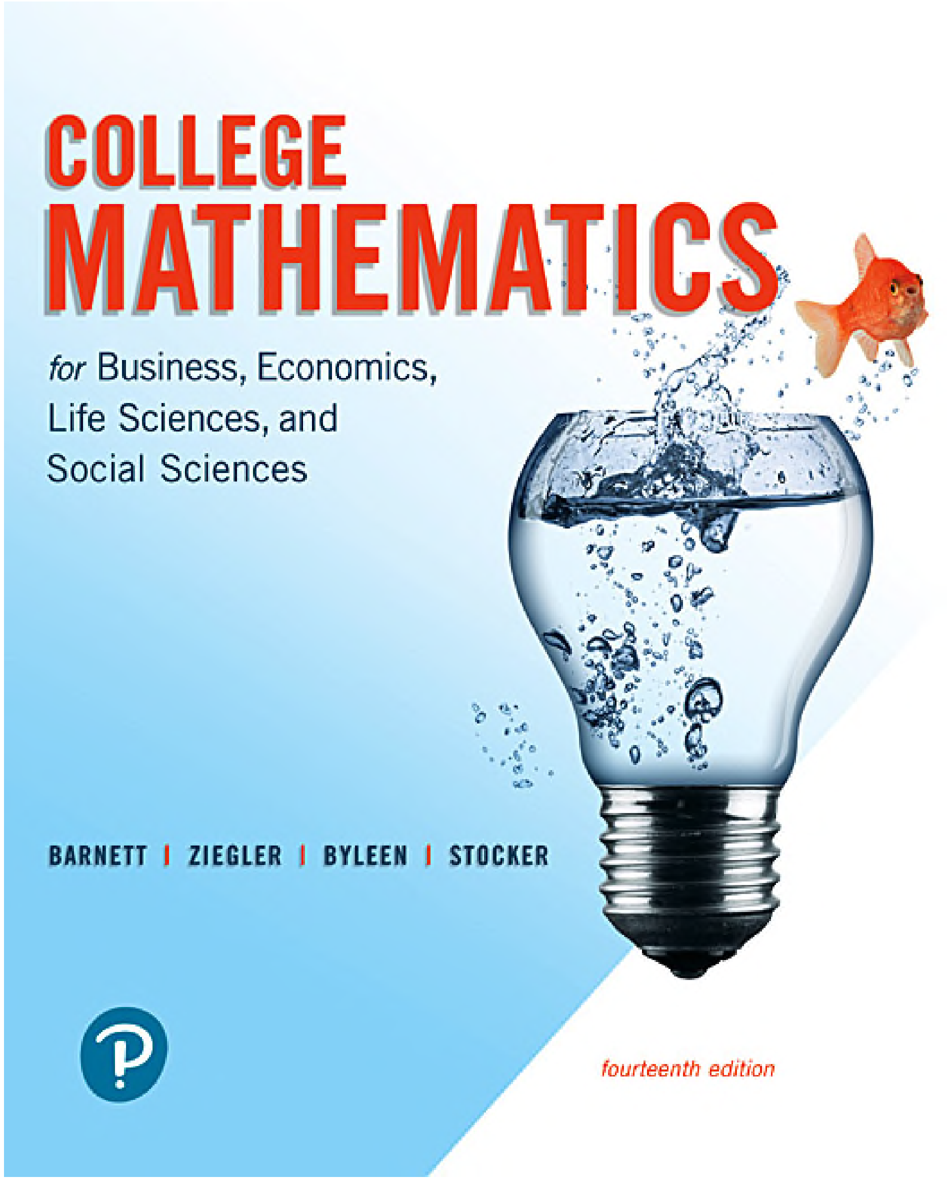
$N(t)$ approaches 50 (the leading coefficient of $50t$ divided by the leading coefficient of (t + 4) as t increases without bound. So $y = 50$ is a horizontal asymptote.

Example: Application of Rational Functions Graph

$N(t) = \dfrac{50t}{t+4}$ approaches 50 as t increases without bound.

As shown in the graph, about 50 components per day would be the upper limit that an employee would be expected to assemble.





Chapter 2

Functions and Graphs

Section 5

Exponential Functions

2.5 Exponential Functions

- This section introduces an important class of functions called *exponential functions*.
- Exponential functions are used extensively in modeling and solving a wide variety of real-world problems.
  - Continuous compounded interest
  - Population growth
  - Radioactive decay
  - And numerous other applications

Same Components Different Functions

- The functions $f(x) = 2^x$ and $g(x) = x^2$ are different.
  - In the case of $g(x) = x^2$, the variable is the base.
  - In the case of $f(x) = 2^x$, the variable is the exponent.

DEFINITION: Exponential Function

The equation

$$f(x) = b^x \quad b > 0, b \neq 1$$

defines an **exponential function** for each different constant b, called the **base**.

The **domain** of f is the set of all real numbers.

The **range** of f is the set of all positive real numbers.

Graphing an Exponential Function

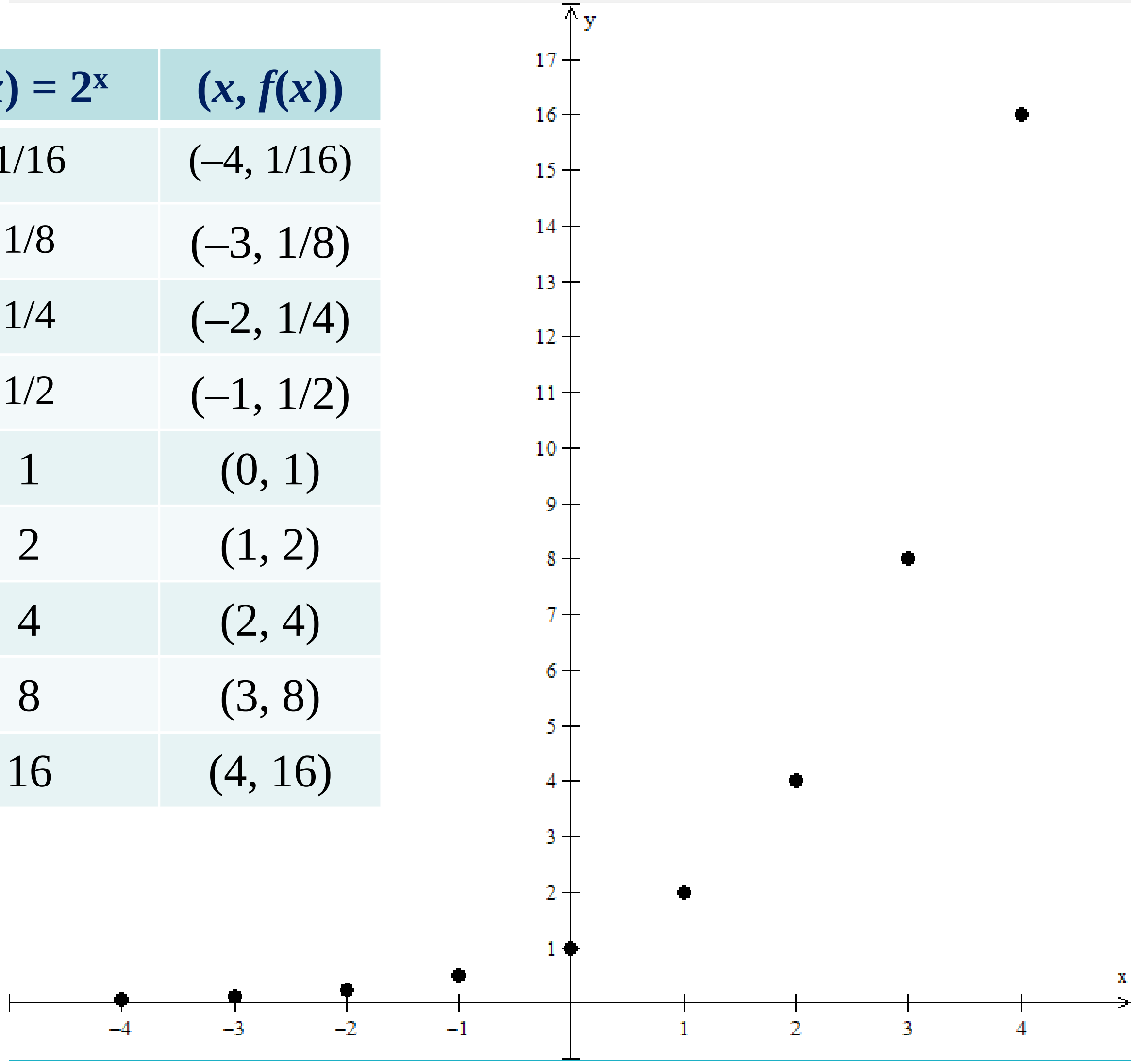
The table gives selected *x* values and the corresponding function values for the function, $f(x) = 2^x$.

x	$f(x) = 2^x$	$(x, f(x))$
–4	1/16	(–4, 1/16)
–3	1/8	(–3, 1/8)
–2	1/4	(–2, 1/4)
–1	1/2	(–1, 1/2)
0	1	(0, 1)
1	2	(1, 2)
2	4	(2, 4)
3	8	(3, 8)
4	16	(4, 16)

Point-by-point Plotting

The graph shows the scatter plot graph of the selected points.

x	$f(x) = 2^x$	$(x, f(x))$
–4	1/16	(–4, 1/16)
–3	1/8	(–3, 1/8)
–2	1/4	(–2, 1/4)
–1	1/2	(–1, 1/2)
0	1	(0, 1)
1	2	(1, 2)
2	4	(2, 4)
3	8	(3, 8)
4	16	(4, 16)

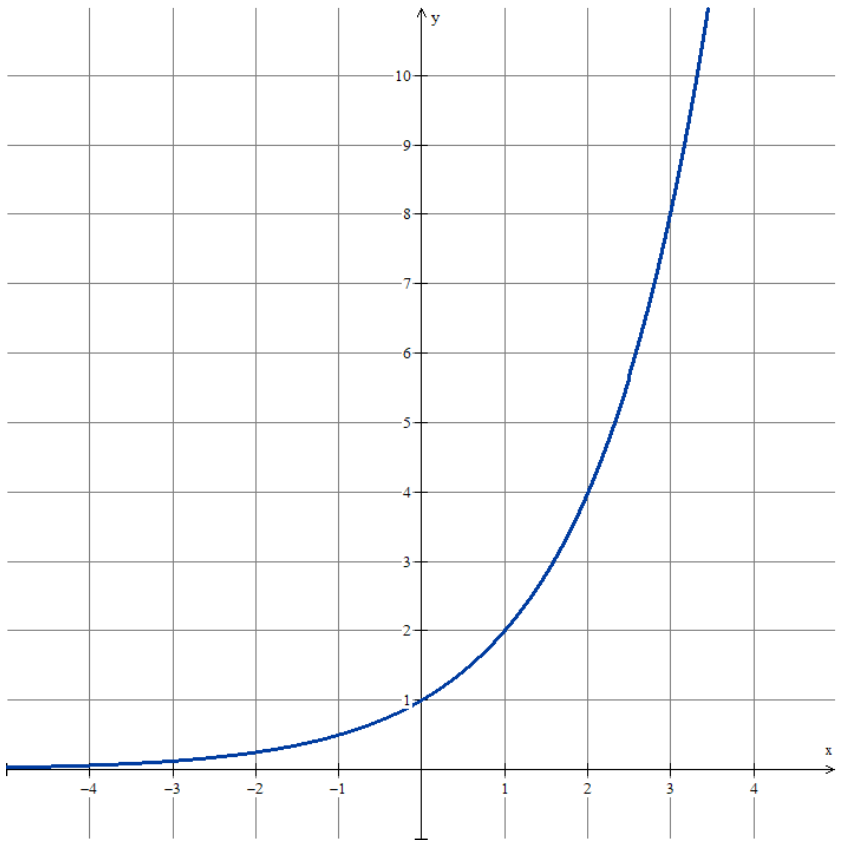


Graph of an Exponential Function

The graph of the exponential function with base 2, $f(x) = 2^x$ is shown.

This graph is drawn by connecting the points on the scatterplot with a smooth curve.

The graph levels off toward the negative x-axis and increases as x increases.



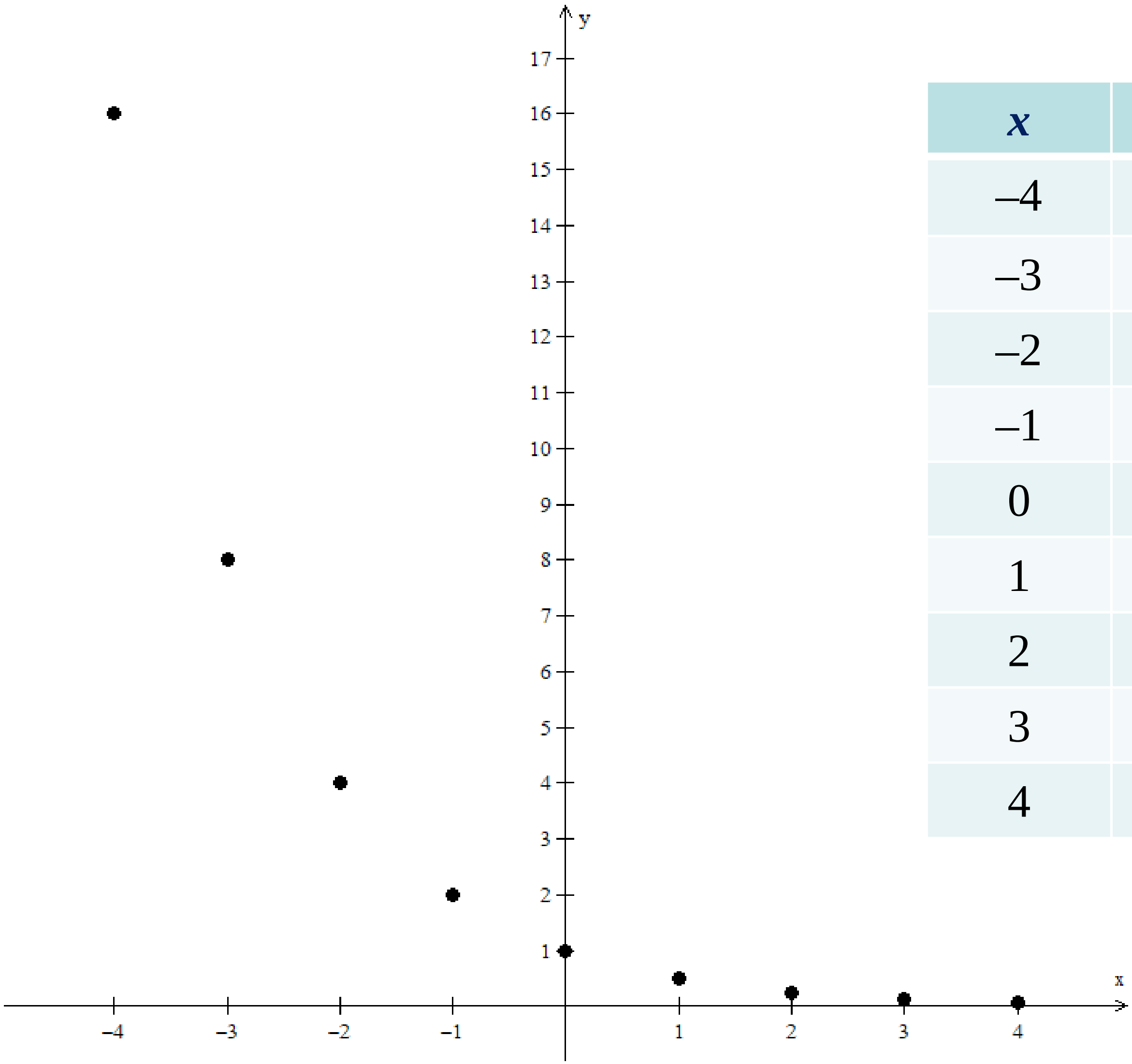
Graphing an Exponential Function

The table gives selected x values and the corresponding function values for the function, $f(x) = 2^{-x}$.

x	$f(x) = 2^{-x}$	$(x, f(x))$
–4	16	(–4, 16)
–3	8	(–3, 8)
–2	4	(–2, 4)
–1	2	(–1, 2)
0	1	(0, 1)
1	1/2	(1, 1/2)
2	1/4	(2, 1/4)
3	1/8	(3, 1/8)
4	1/16	(4, 1/16)

Point-by-point Plotting

The graph shows the scatter plot graph of the selected points.



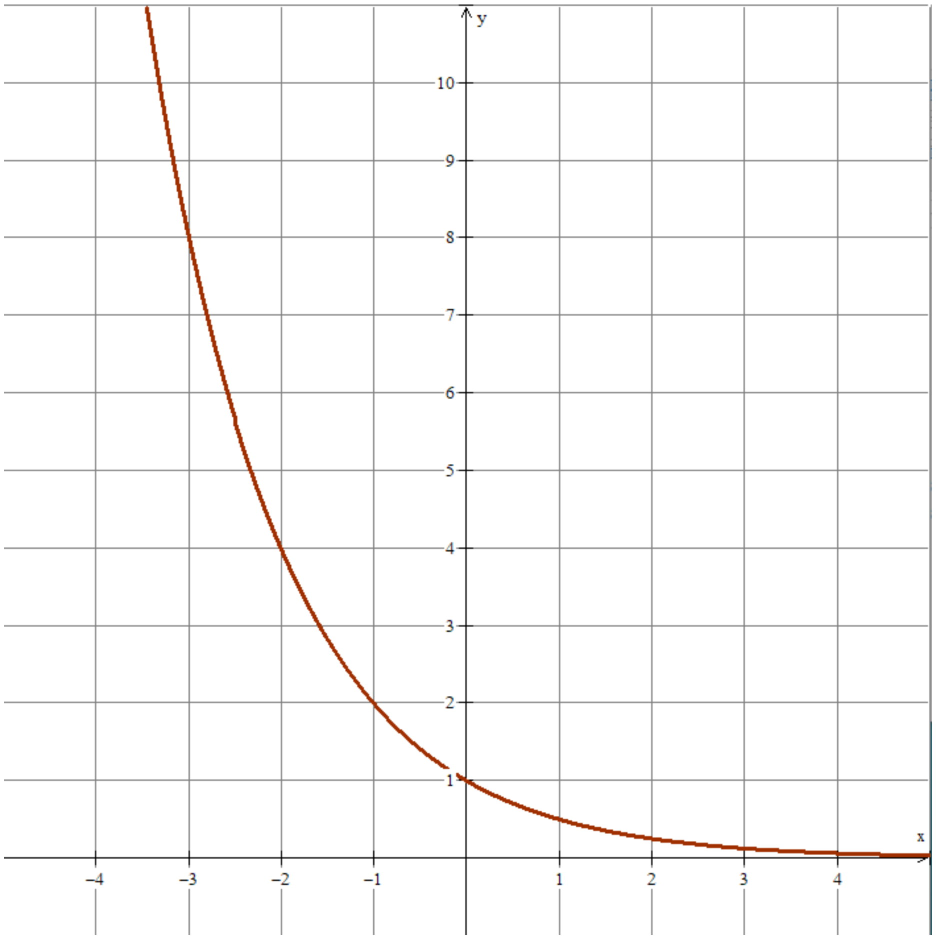
x	$f(x) = 2^{-x}$	$(x, f(x))$
–4	16	(–4, 16)
–3	8	(–3, 8)
–2	4	(–2, 4)
–1	2	(–1, 2)
0	1	(0, 1)
1	1/2	(1, 1/2)
2	1/4	(2, 1/4)
3	1/8	(3, 1/8)
4	1/16	(4, 1/16)

Graph of an Exponential Function

The graph of the exponential function $g(x) = 2^{-x}$ is shown.

This graph is drawn by connecting the points on the scatterplot with a smooth curve.

The graph decreases and levels off toward the positive *x*-axis as *x* increases.



Theorem 1: Basic Properties of the Graph of $f(x) = b^x$, $b > 0$, $b \neq 1$

1. All graphs contain the point (0, 1) since $b^0 = 1$ for any permissible base b.

2. All graphs are continuous curves, with no holes or jumps.

3. The x axis is a horizontal asymptote.

4. If $b > 1$, then b^x increases as x increases.

5. If $0 < b < 1$, then b^x decreases as x increases.

Theorem 2:
Properties of Exponential Functions

For a and b positive, $a \neq 1$, $b \neq 1$, and x and y real,

1. Exponent laws: $a^x a^y = a^{x+y}$ $\quad \dfrac{a^x}{a^y} = a^{x-y}$

$$(a^x)^y = a^{xy} \qquad (ab)^x = a^x b^x \qquad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

2. $a^x = a^y$ if and only if $x = y$

3. For $x \neq 0$, $a^x = b^x$ if and only if $a = b$

Note that $(-2)^2 = 2^2$ does not contradict property 3 of Theorem 2 since Theorem 2 applies only for both a and b are positive.

Base *e* Exponential Function

One of the most useful values for the base of an exponential function is somewhat of a surprise.

Most current calculators have keys for 10^x and e^x.

The value 10 as a base is important because our numbering system is built on base 10.

The value *e* is actually used more often than all other bases.

Certain mathematical processes in calculus and advanced mathematics courses are in simplest form in base *e*.

The function with equation $y = e^x$ is sometimes referred to as *the* exponential function.

Examining the base *e*

The base *e* is an irrational number and, like π it cannot be represented exactly by any finite decimal or fraction.

The base *e* can be approximated by evaluating the expression

$\left(1+\frac{1}{x}\right)^x$ for sufficiently large values of x.

Estimating the Base *e*

x	$\left(1 + \frac{1}{x}\right)^x$
1	2
10	2.593 74…
100	2.704 81…
1,000	2.716 92…
10,000	2.718 14…
100,000	2.718 27…
1,000,000	2.718 28…

The table shows the value of the expression for increasingly large values of *x*.

The irrational number *e* to 12 decimal places is

$$e = 2.718\ 281\ 828\ 459$$

Definition

Exponential Functions with Base *e* and Base 1/*e*

The exponential functions with base e and base $1/e$, respectively, are defined by

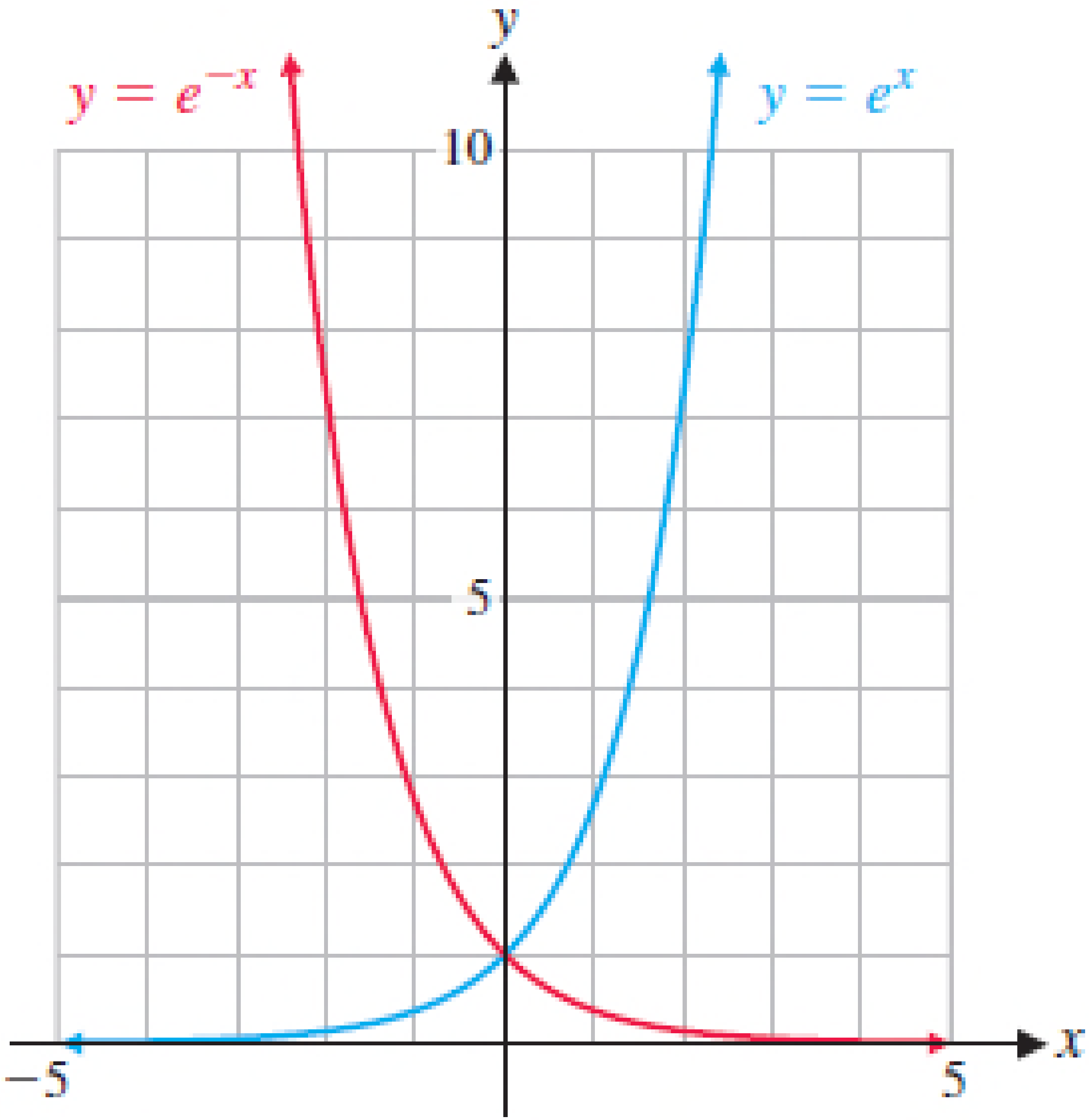
$$y = e^x \qquad \text{and} \qquad y = e^{-x}$$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

The graphs of these two important functions are shown on the next slide.

Graphs of e^x and e^{-x}



Exponential Growth and Decay

- Functions of the form $y = ce^{kt}$, where c and k are constants and the independent variable t represents time are often used to model population growth and radioactive decay.
  - If $t = 0$, then $y = c$.
    - The constant c represents the initial population (or initial amount).
  - The constant k is called the **relative growth rate**.
    - The relative growth rate is a percentage rate per unit of time and is written in decimal form for the model.

Example: Exponential Growth of Cholera Bacteria

Cholera, an intestinal disease, is caused by a cholera bacterium that multiplies exponentially. The number of bacteria grows continuously at relative growth rate 1.386, that is, $N = N_0 e^{1.386t}$ where N is the number of bacteria present after t hours and N_0 is the number of bacteria present at the start.

If there are 25 bacteria at the start, how many bacteria will be present A) In 0.8 hour? B) In 2.5 hours?

Solution: Cholera Bacteria Growth Example

Substitute $N_0 = 25$ into the function $N = N_0 e^{1.386t}$ to obtain

$$N = 25e^{1.386t}$$

A) Solve for N when $t = 0.8$ by substituting and evaluating with a calculator to obtain

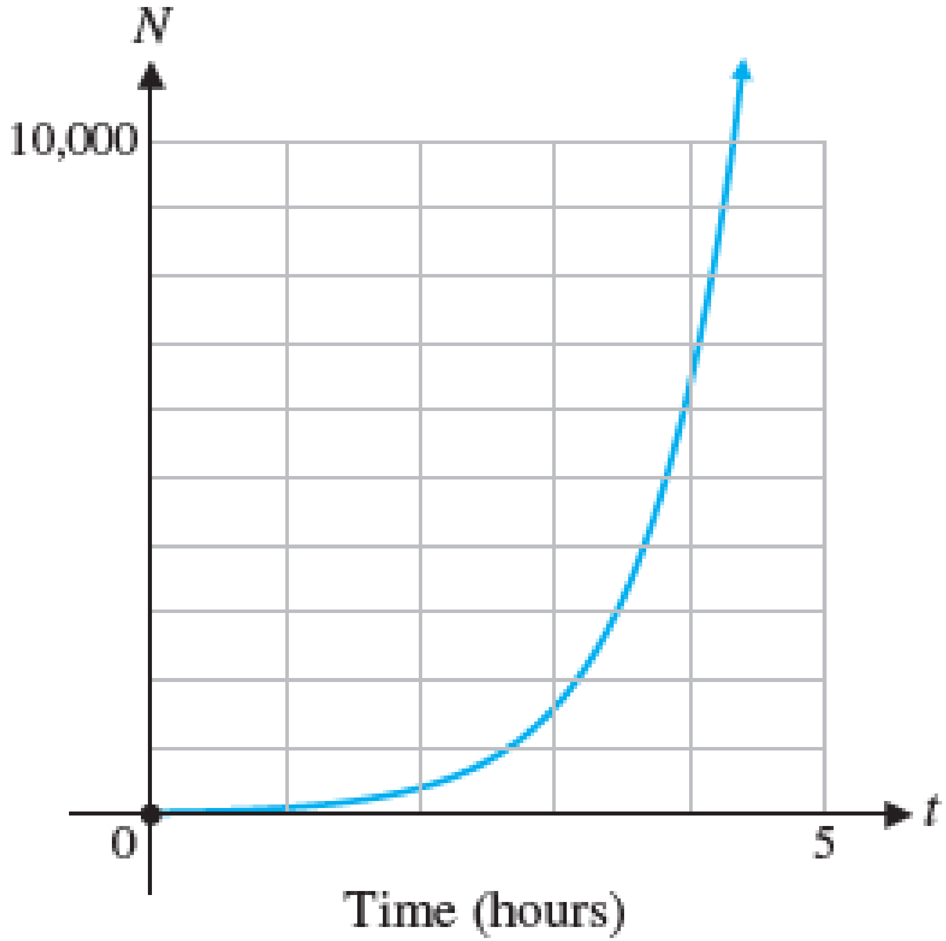
$$N = 25e^{1.386(0.8)} = 76 \text{ bacteria (rounded)}$$

B) Solve for N when $t = 2.5$ by substituting and evaluating with a calculator to obtain

$$N = 25e^{1.386(2.5)} = 799 \text{ bacteria (rounded)}$$

Cholera Bacteria Growth Graph

The graph of the function $N = 25e^{1.386t}$ is shown.
Notice that this graph is shaped like previous examples where the base was greater than 1.



Exponential Decay
Carbon-14

Cosmic-ray bombardment of the atmosphere produces neutrons, which in turn react with nitrogen to produce radioactive carbon-14 (^{14}C).

Radioactive ^{14}C enters all living tissues through carbon dioxide, which is first absorbed by plants.

As long as a plant or animal is alive, ^{14}C is maintained in the living organism at a constant level.

Once the organism dies, however, ^{14}C decays according to the equation $A = A_0e^{-0.000124t}$ where A is the amount present after t years and A_0 is the amount present at time $t = 0$.

Example (A) Carbon-14 Decay

If 500 milligrams of ^{14}C is present in a sample from a skull at the time of death, how many milligrams will be present in the sample in 15,000 years? Round the result to two decimal places.

Solution: Substitute $A_0 = 500$ into the decay equation to obtain $A = 500e^{-0.000124t}$.

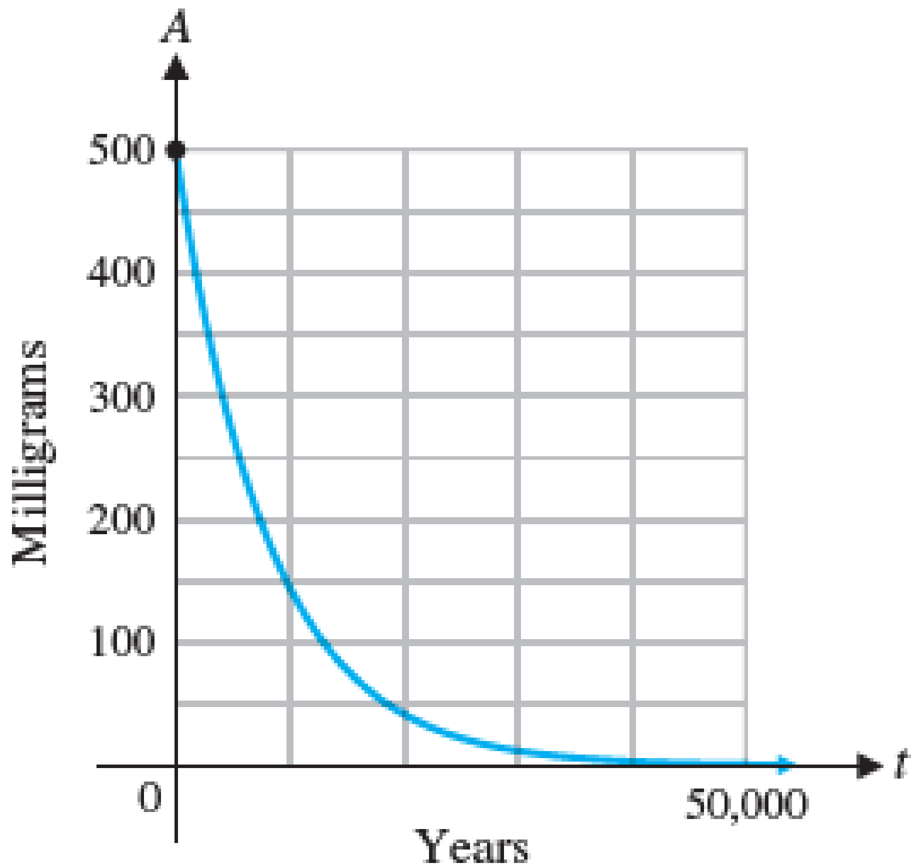
Solve for A when $t = 15{,}000$ by substituting and evaluating using a calculator to obtain

$$A = 500e^{-0.000124(15{,}000)} = 77.84 \text{ milligrams}$$

Half-life Carbon-14

The **half-life** of ^{14}C is the time t at which the amount present is one-half the amount at time $t = 0$.

The graph of the decay function for ^{14}C is shown.



Solution: Example (B) Half-life Carbon-14

Use the graph to estimate the half-life of ^{14}C.

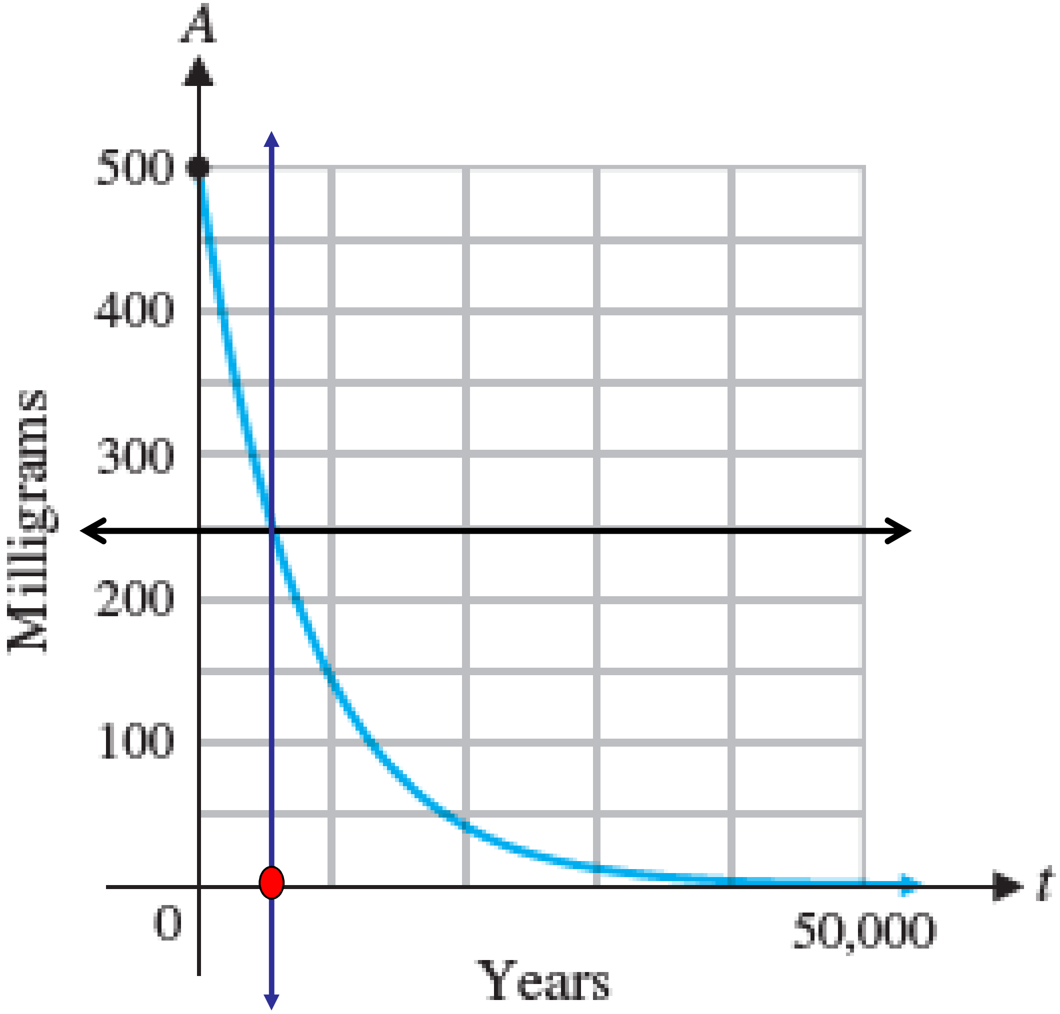
Solution:

Draw a horizontal line through $A = 250$.

Draw a vertical line through the point where the graph and this horizontal line intersect.

Estimate the value of t that corresponds to the vertical line location.

The value of t is about 6,000 years.



Solution to Carbon-14 Example

Use a graphing calculator technique to find a better estimate for the half-life of ^{14}C.

Find the half-life in this example requires solving the equation $250 = 500e^{-0.000124t}$ for t.

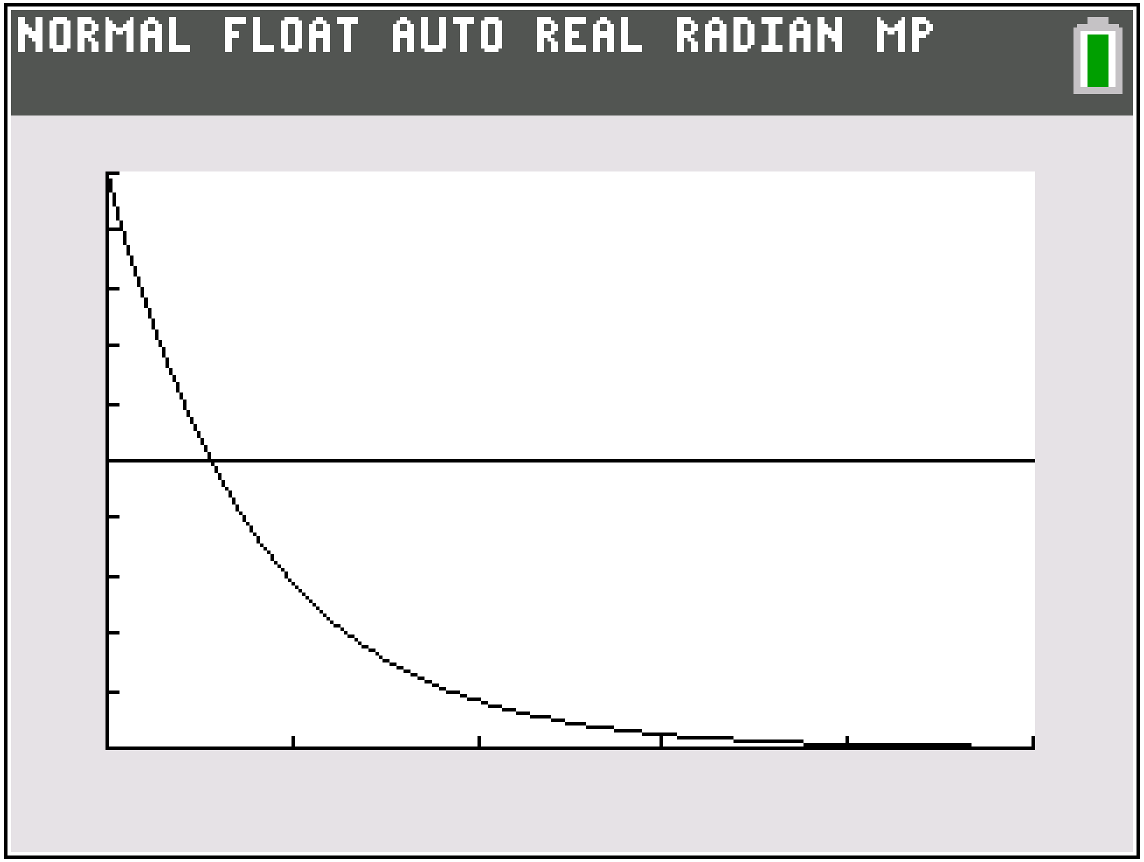
For the purposes of using the graphing calculator, we use the variable x for t.

We find the intersection of $y_1 = 500e^{-0.000124x}$ and $y_2 = 250$ using a graphing calculator process.

The functions y_1 and y_2 are input into the calculator and the graph is obtained using a viewing window [0, 50,000] by [0, 500].

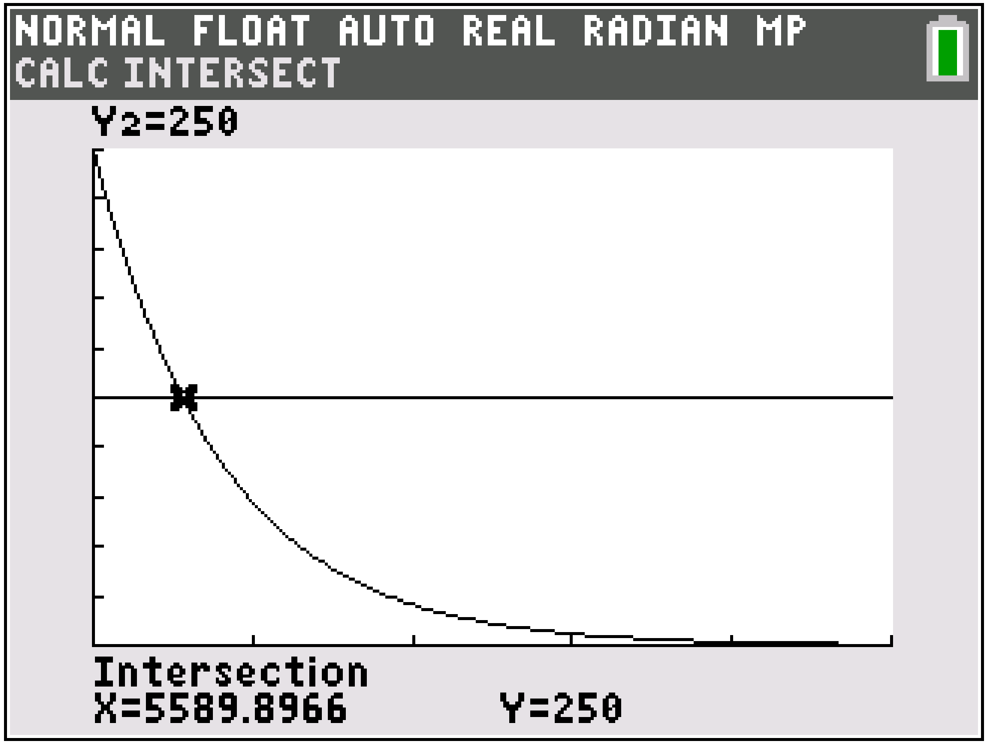
Solution: Finding the Half-life

The graph of the functions $y_1 = 500e^{-0.000124x}$ and $y_2 = 250$ in the window [0, 50,000] by [0, 500] is shown.



Solution: Finding the Half-life

The intersection process is used to find the intersection of the functions $y_1 = 500e^{-0.000124x}$ and $y_2 = 250$.



The solution to the equation is rounded to $x = 5{,}590$.
This gives a better estimate of the half-life.

Exponential Regression

The context of a "real-world" situation sometimes suggests an exponential function model.

Current calculator technology allows analyzing observed data to get a best-fit regression model for the data.

The resulting regression equation has the form, $y = a \cdot b^x$ where the values a and b best fit the data set.

Example: Exponential Regression

The table gives the number of individuals (in billions) worldwide who could access the internet from home for selected years since 2000.

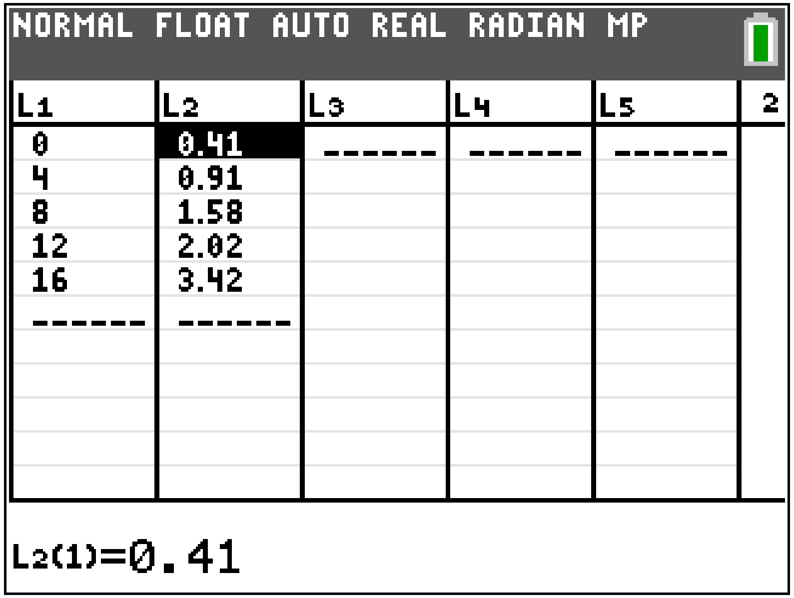
Let x represent the number of years since 2000.
Create a scatterplot for the data to see if exponential modeling is a reasonable choice.

Year	Users
2000	0.41
2004	0.91
2008	1.58
2012	2.02
2016	3.42

Example: Create and View a Scatterplot-Input the Data

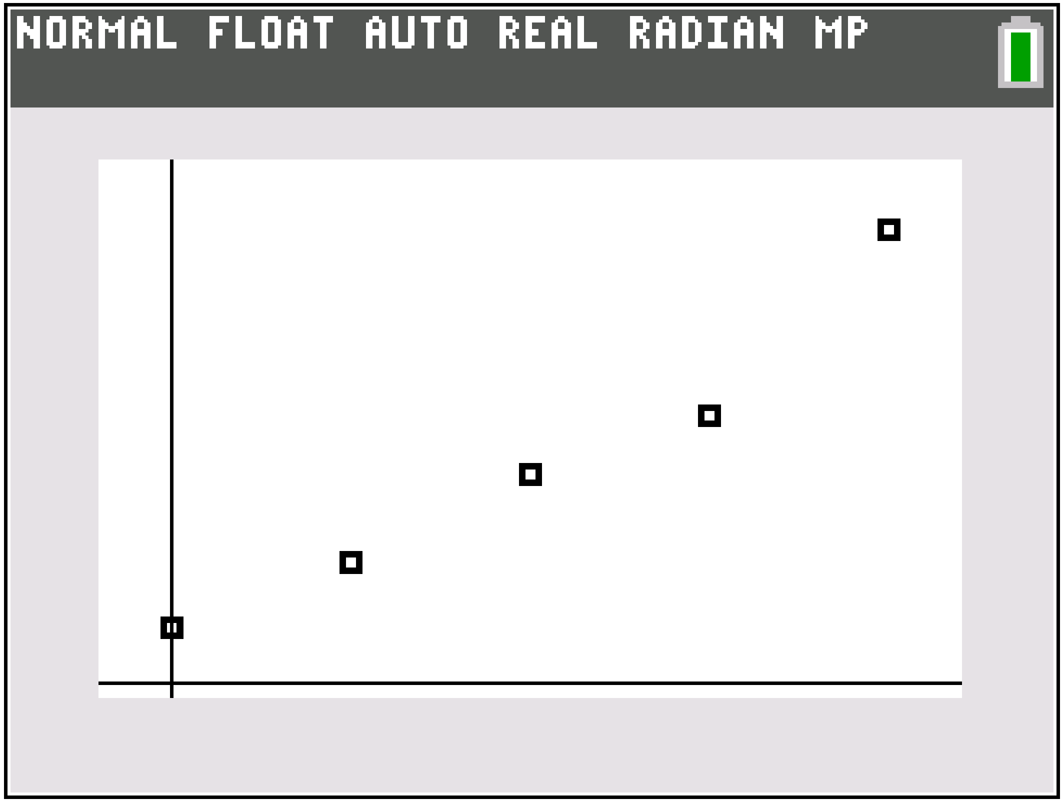
Align values so that x represents the number of years after 2000 and input the data into the calculator.

Year	Users
2000	0.41
2004	0.91
2008	1.58
2012	2.02
2016	3.42



Example: View the Scatterplot

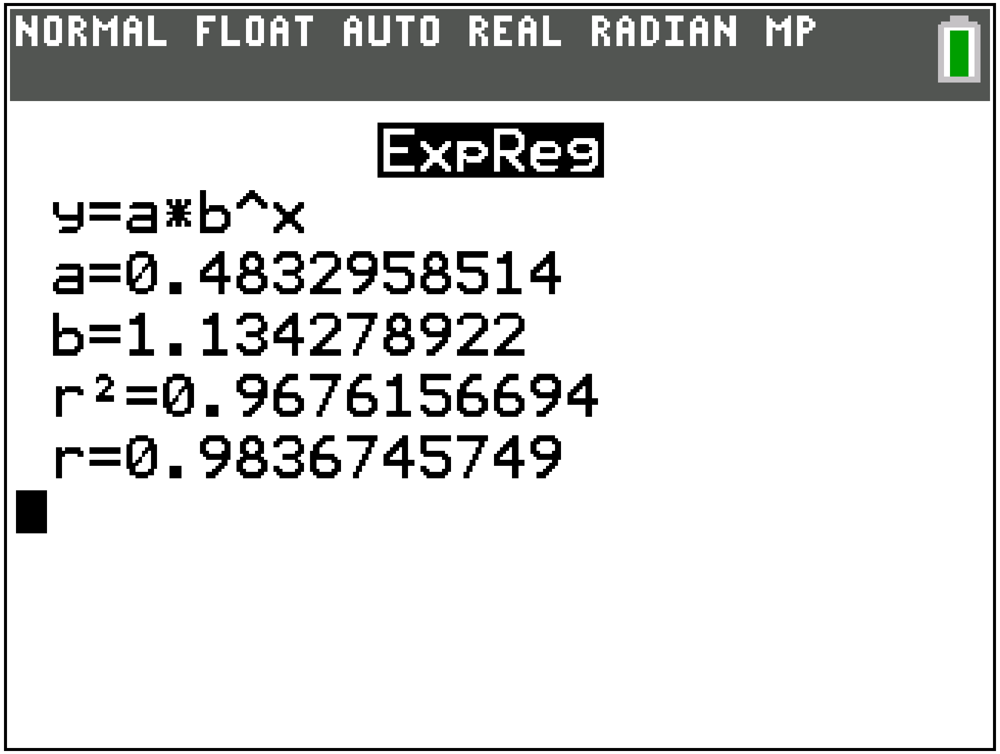
The shape of the scatterplot confirms that exponential regression is a good choice to model the data.



Example: Find the Regression Equation

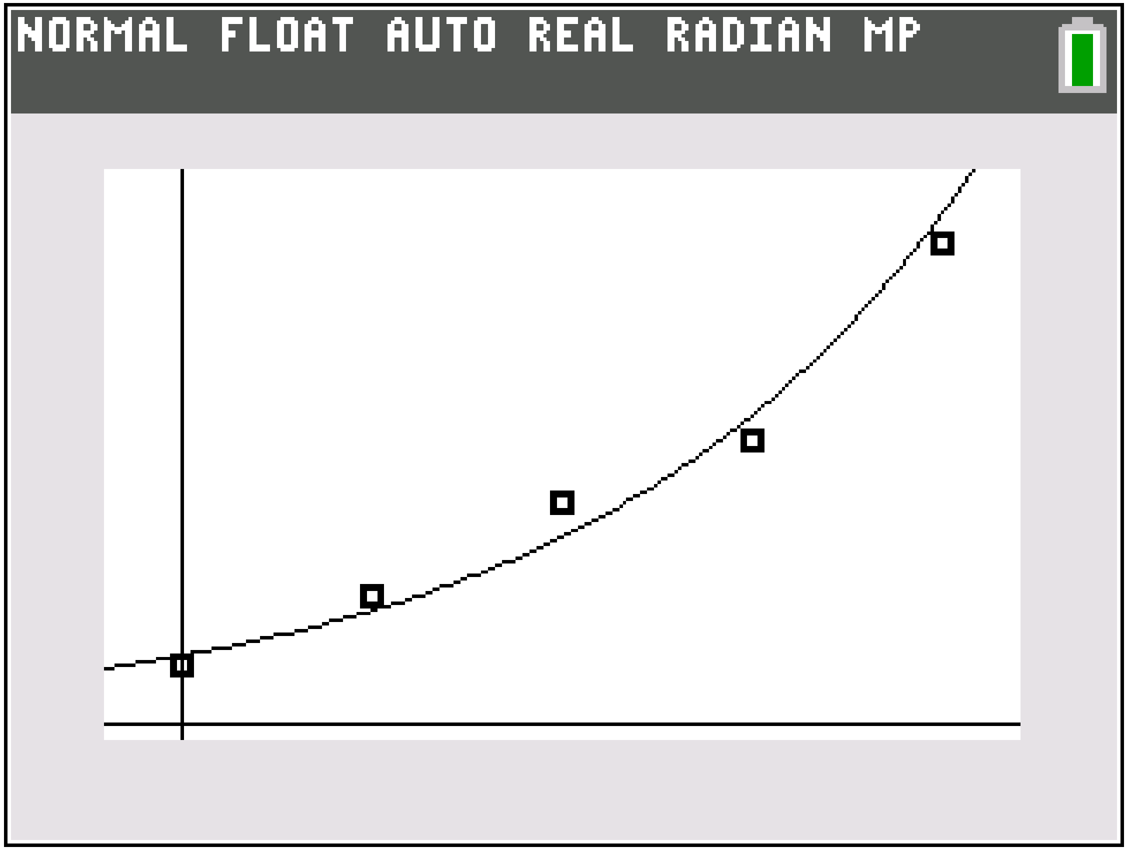
The regression equation is $y = 0.4833 \cdot 1.1343^x$ (values rounded to four decimal places).

The r^2 value indicates that the regression equation is a good fit for the data.



Example: Compare the Scatterplot to the Regression Equation

The graph of the regression equation is a close match to the scatterplot of the data.



Example:
Predict Using the Regression Equation

Use the regression model to estimate the number of internet users in 2024.

Solution: The year 2024 is 24 years past the initial year 2000. Substitute $x = 24$ into the regression equation,

$$y = 0.4833 \cdot 1.1343^x$$

to obtain

$$y = 0.4833 \cdot 1.1343^{24} = 9.94$$

In the year 2024, there will be approximately 9.94 billion internet users.

Compound Interest

The fee paid to use another's money is called **interest**.

Interest is usually computed as a percent (called **interest rate**) of the principal over a given period of time.

If, at the end of a payment period, the interest due is reinvested at the same rate, then the interest paid on interest reinvested is called **compound interest.**

If a **principal *P* (present value)** is invested at an annual **rate *r*** (expressed as a decimal) compounded *m* times a year, then the **amount *A* (future value)** in the account at the end of *t* years is given by

$$A = P\left(1 + \frac{r}{m}\right)^{m \cdot t}$$

Example: Compound Interest

Suppose $10,000 is deposited into an account paying interest at a 7.5% annual rate with interest compounded monthly. How much will be in the account at the end of 20 years?

Solution:

Use the values $P = 10{,}000$, $r = 0.075$, $m = 12$, and $t = 20$ in the compound interest formula.

$$A = P\left(1 + \frac{r}{m}\right)^{m \cdot t}$$

$$A = 10000\left(1 + \frac{0.075}{12}\right)^{12 \cdot 20} = 44{,}608.17$$

The account will have $44,608.17 at the end of 20 years.

Continuous Compound Interest

Continuous compounding occurs when the frequency of compounding is increased infinitely so that at every instant in time, interest is computed and added.

Recall that the base *e* of the exponential function is found using increasingly large value of *x* in

$$\left(1+\frac{1}{x}\right)^{x}$$

When a principal amount *P* is invested at an annual rate, *r*, is compounded continuously, then the amount, *A*, in the account at the end of *t* years is given by

$$A = Pe^{rt}$$

where the constant $e \approx 2.71828$ is the exponential function base.

Example: Continuous Compound Interest

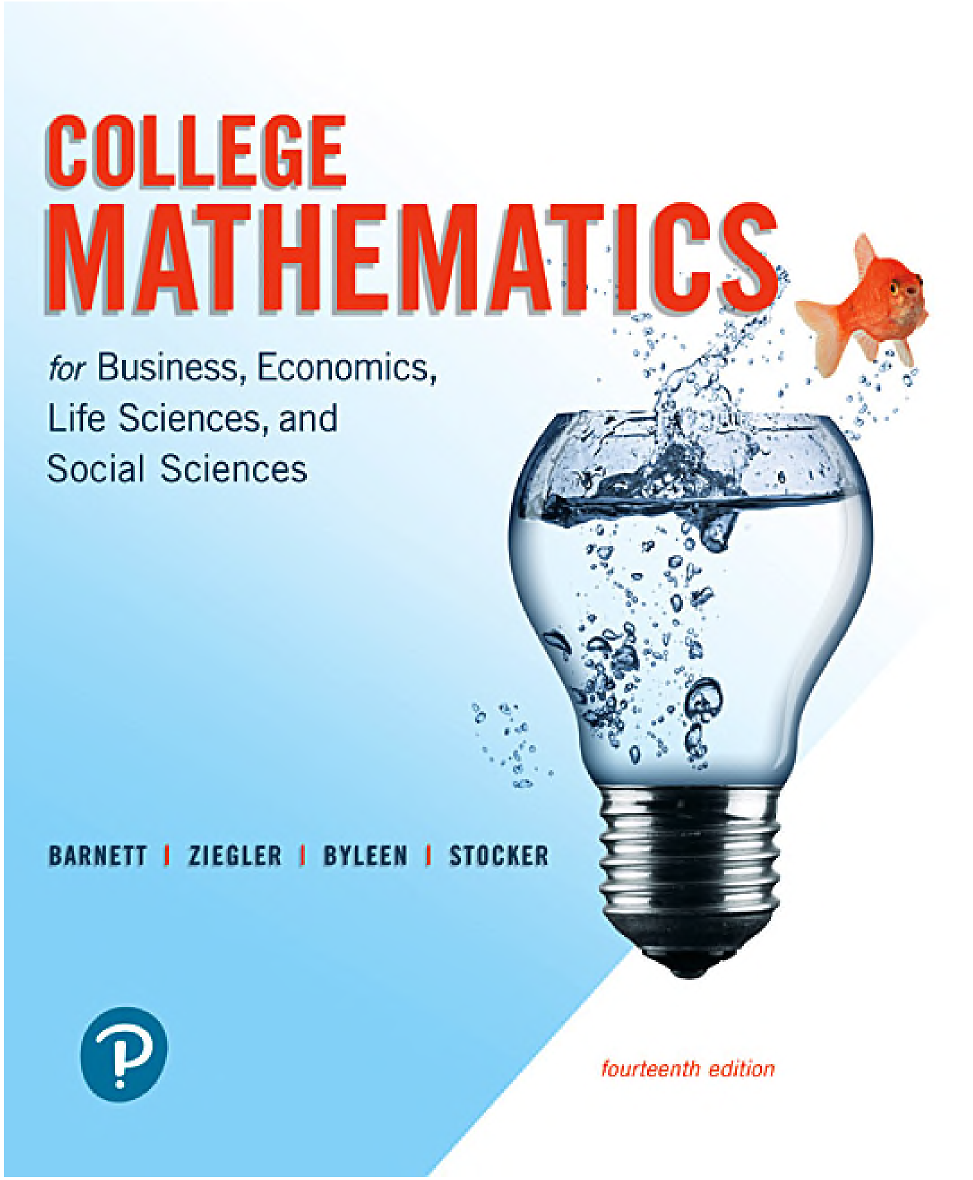
Suppose \$10,000 is invested at 7.5% with interest compounded continuously.

How much will be in the account in 20 years?

Solution:

Substitute the values $P = 10000$, $r = 0.075$, and $t = 20$ into the continuous compounded interest formula $A = Pe^{rt}$.

This gives $A = 10000e^{0.075 \cdot 20} = \$44{,}816.89$.



Chapter 2

Functions and Graphs

Section 6
Logarithmic Functions

2.6 Logarithmic Functions

Logarithmic functions are closely related to exponential functions.

The exponential function and corresponding logarithm function are *inverses* of each other.

We begin this section by examining the concept of inverse functions.

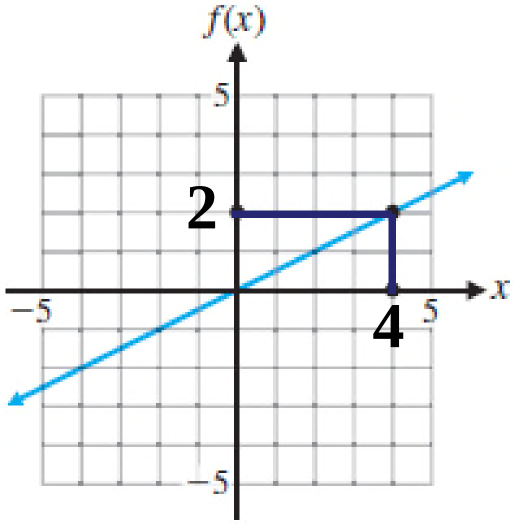
We then define a logarithmic function as the inverse of an exponential function.

Logarithm functions model and facilitate solving many types of problems.

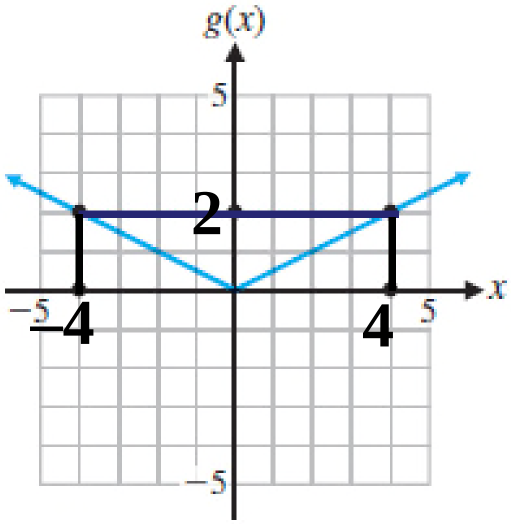
- The Richter scale (measuring the force of earthquakes).
- The decibel scale (measuring sound intensity).
- Finding doubling time and half-life for exponential change.

One-to-One Functions

Consider the graphs of $f(x) = \dfrac{x}{2}$ and $g(x) = \dfrac{|x|}{2}$ shown below.



(A) $f(x) = \dfrac{x}{2}$



(B) $g(x) = \dfrac{|x|}{2}$

In *f* and *g*, each domain value corresponds to exactly one range value.

In *f*, the range value 2 corresponds to the single domain value 4.

In *g*, the range value 2 corresponds to both -4 and 4.

Function *f* is said to be *one-to-one*.

Definition: One-to-One Functions

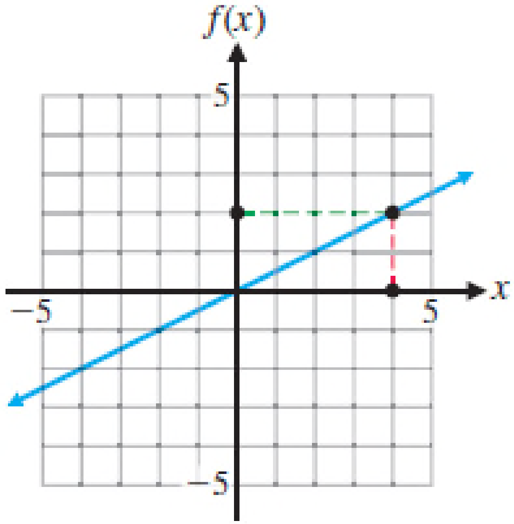
A function *f* is said to be **one-to-one** if each range value corresponds to exactly one domain value.

Any continuous function that is either increasing or decreasing for all domain values is one-to-one.
WHY?

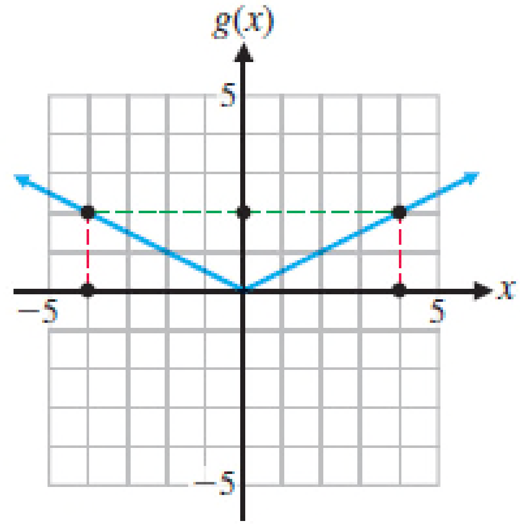
If a continuous function increases for some domain values and decreases for others, then it is not one-to-one.
WHY?

One-to-One Functions

Recall the graphs of $f(x)=\dfrac{x}{2}$ and $g(x)=\dfrac{|x|}{2}$ shown below.



(A) $f(x)=\frac{x}{2}$



(B) $g(x)=\frac{|x|}{2}$

Which graph is increasing for all domain values?
This graph (function) is one-to-one.
Which graph increases for some domain values and decreases for others?
This graph (function) is not one-to-one.

Definition: Inverse of a Function

If f is a one-to-one function, then the **inverse** of f is the function formed by interchanging the independent and dependent variables for f.

Thus, if (a, b) is a point on the graph of f, then (b, a) is a point on the graph of the inverse of f.

Note: If f is not one-to-one, then f **does not have an inverse**.

Logarithmic Functions

The exponential function *f* defined by $y = 2^x$ increases for all of its domain values and is one-to-one.

The inverse of *f* exists and is formed by interchanging the domain and range variables to obtain $x = 2^y$.

We call this inverse the **logarithm function with base 2**, and write $y = \log_2 x$ if and only if $x = 2^y$

The graphs of $y = \log_2 x$ and $x = 2^y$ are equivalent.

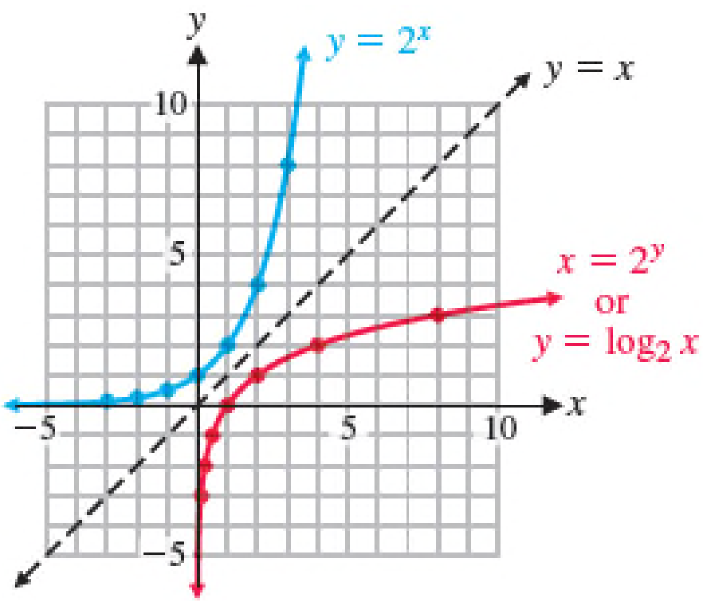
Graphing The Logarithmic Function

Reflecting the graph of the exponential function $y = 2^x$ in the line $y = x$ gives the logarithmic function $y = \log_2 x$.

The table shows coordinates for points on the exponential function and coordinates for points on the logarithmic function formed by interchanging the variables.

The graphs of both functions along with the graph of the line $y = x$ are shown.

Exponential Function		Logarithmic Function	
x	$y = 2^x$	$x = 2^y$	y
−3	$\frac{1}{8}$	$\frac{1}{8}$	−3
−2	$\frac{1}{4}$	$\frac{1}{4}$	−2
−1	$\frac{1}{2}$	$\frac{1}{2}$	−1
0	1	1	0
1	2	2	1
2	4	4	2
3	8	8	3



Definition: Logarithmic Functions

The inverse of an exponential function is called a **logarithmic function**.

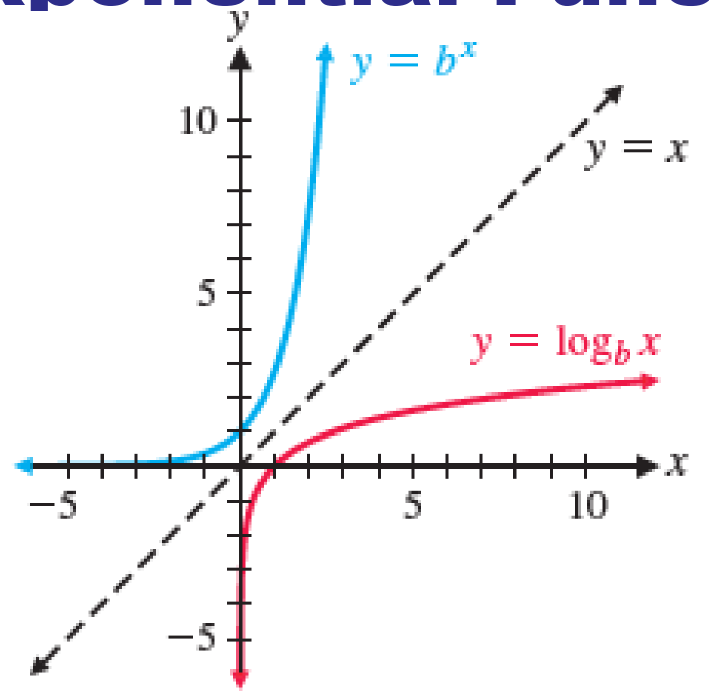
For $b > 0$ and $b \neq 1$, $y = \log_b x$ (the logarithmic form) is equivalent to $x = b^y$ (the exponential form.)

The **log to the base *b* of *x*** is the exponent to which b must be raised to obtain x.

The **domain** of the logarithmic function is the set of all positive real numbers (which is also the range of the corresponding exponential function).

The **range** of the logarithmic function is the set of all real numbers (which is also the domain of the corresponding exponential function).

General Graphs for the Logarithmic and Exponential Functions



The domain of the logarithmic function is the set of positive numbers.

It follows that all points on the logarithmic graph are to the right of the *y* axis.

Logarithmic-Exponential Conversion

Example: Change each logarithmic form to an equivalent exponential form:

$$\text{A) } \log_5 125 = 3 \quad \text{B) } \log_4 2 = \tfrac{1}{2} \quad \text{C) } \log_3(\tfrac{1}{27}) = -3$$

Solution: Converting from logarithmic to exponential form is simplified by noting the base of the logarithm function and using that value as the base for the exponential form.

A) The base for $\log_5 125$ is 5. The logarithm value 3 is the power.

Write the base 5 to the logarithm power 3 to obtain $5^3 = 125$

B) The base for $\log_4 2$ is 4. The logarithm value $\frac{1}{2}$ is the exponent.

Write the base 4 to the power $\frac{1}{2}$ to obtain $4^{\frac{1}{2}} = 2$

C) The base for $\log_3(\frac{1}{27})$ is 3. The logarithm value -3 is the exponent.

Write the base 3 to the power -3 to obtain $3^{-3} = \frac{1}{27}$

You Try It!

Convert each from logarithmic to exponential form.

A) $\log_8 64 = 2$

B) $\log_{½} 16 = -4$

C) $\log_4 4 = 1$

D) $\log_b 1 = 0$

E) $\log_b b = 1$

F) $\log_b b^x = x$

Exponential-Logarithmic Conversion

Example: Change each exponential form to an equivalent logarithmic form:

$$A)\ 3^{-2} = \tfrac{1}{9} \quad B)\ 125^{\frac{2}{3}} = 25 \quad C)\ 2^{8} = 256$$

Solution: Converting from exponential to logarithmic form is simplified by noting the base of the exponential function and using that value as the base for the logarithmic form.

A) The base for 3^{-2} is 3. The exponent -2 is the logarithm value.

Write the base 3 logarithm as $\log_3 \tfrac{1}{9} = -2$.

B) The base for $125^{\frac{2}{3}}$ is 125. The exponent $\tfrac{2}{3}$ is the logarithm value.

Write the base 125 logarithm as $\log_{125} 25 = \tfrac{2}{3}$.

C) The base for 2^8 is 2. The exponent 8 is the logarithm value.

Write the base 2 logarithm as $\log_2 256 = 8$.

You Try It!

Convert each from exponential to logarithmic form.

A) $3^3 = 27$

B) $\sqrt[3]{27} = 3$

C) $10^{-2} = \frac{1}{100}$

D) $b^1 = b$

E) $b^0 = 1$

Solve a Logarithm Equation

Example: Solve $2 = \log_b 100$ for b.

Solution: We write the logarithmic equation in exponential form as $b^2 = 100$.

Since b is the base for a logarithm, it is a positive number.

$b^2 = 100$ and $10^2 = 100$ so the value $b = 10$ is a solution to the equation.

Solve a Logarithm Equation

Example: Solve $\log_6 x = 5$ for x.

Solution: We write the logarithmic equation in exponential form as $6^5 = x$.

Completing the computation gives $x = 7776$.

Theorem 1: Properties of Logarithmic Functions

If b, M, and N are positive real numbers, $b \neq 1$, and p and x are real numbers, then

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b b^x = x$
4. $b^{\log_b x} = x, x > 0$
5. $\log_b MN = \log_b M + \log_b N$
6. $\log_b \frac{M}{N} = \log_b M - \log_b N$
7. $\log_b M^p = p \log_b M$
8. $\log_b M = \log_b N$ if and only if $M = N$.

Example: Use Logarithm Properties to Rewrite Expressions

Write the expression in simpler form: $\log_b \dfrac{xy}{tz}$

Solution: $\log_b \dfrac{xy}{tz} = \log_b xy - \log_b tz$

$$= \log_b x + \log_b y - (\log_b t + \log_b z)$$

$$= \log_b x + \log_b y - \log_b t - \log_b z$$

Example: Use Logarithm Properties to Rewrite Expressions

Write the expression in simpler form: $\log_b (xy)^4$

Solution: $\log_b (xy)^4 = 4\log_b xy$

$$= 4(\log_b x + \log_b y)$$

$$= 4\log_b x + 4\log_b y$$

Example: Use Logarithm Properties to Rewrite Expressions

Write the expression in simpler form: $3^{x \log_3 b}$

Solution: $3^{x \log_3 b} = 3^{\log_3 b^x}$

$$= b^x$$

Example: Solving Logarithmic Equations

Solve for $x: \log_4(x+6) + \log_4(x-6) = 3$

Solution: $\log_4(x+6) + \log_4(x-6) = 3$

$$\log_4\left((x+6)(x-6)\right) = 3$$

$$\log_4(x^2 - 36) = 3$$

$$x^2 - 36 = 4^3$$

$$x^2 - 36 = 64$$

$$x^2 = 100$$

$$x = 10 \text{ or } x = -10$$

$x + 6$ and $x - 6$ are negative when $x = -10$.

Negative values are not in the domain of logarithmic functions so $x = -10$ is not a solution.

The solution to the equation is $x = 10$.

Common and Natural Logarithms

Of all possible logarithmic bases, *e* and 10 are used almost exclusively.

Common logarithms are logarithms with base 10.

In mathematics notation, $\log x$ means $\log_{10} x$.

Natural logarithms are logarithms with base *e*.

In mathematics notation, $\ln x$ means $\log_e x$.

Most current calculators include keys labeled LOG and LN for common and natural logarithms respectively.

You Try It!

$$\textbf{Solve:} \log \pi - \log(10,000\pi) = x$$

Example: Solve a Logarithmic Equation

Solve for x: Give the exact solution in terms of e.

$$\ln(x+1) - \ln x = 1$$

Solution: $\ln(x+1) - \ln x = 1$ gives $\ln\left(\frac{x+1}{x}\right) = 1$

$$\frac{x+1}{x} = e^1 = e$$

$$x + 1 = xe$$

$$xe - x = 1$$

$$x(e-1) = 1$$

$$x = \frac{1}{e-1}$$

Calculators and Logarithms

As previously noted, most current calculators have dedicated keys for common logarithms LOG and natural logarithms LN.

Finding values for common or natural logarithms is easy.On most calculators you press either LOG or LN and enter a number from the domain (positive values) and press the enter key.

Some calculators require entering the domain value first then the LOG or LN key.

Be sure to check which process your calculator requires.

Calculator Evaluation of Logarithms

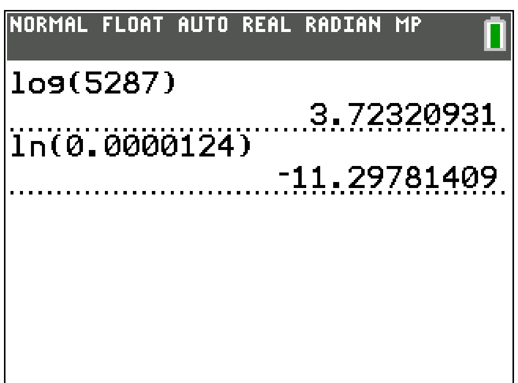
Use a calculator to evaluate each of the following. Give six decimal places in your answer.

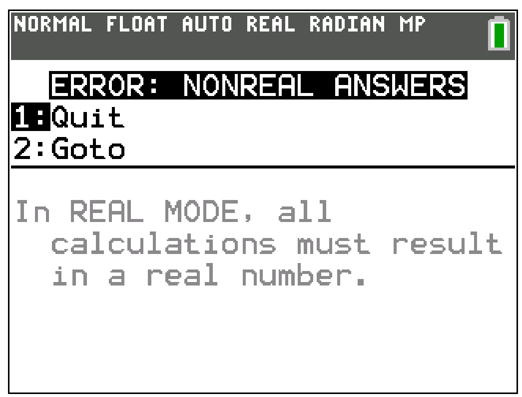
A) log 5,287 B) ln 0.0000124 C) log (-2.88)

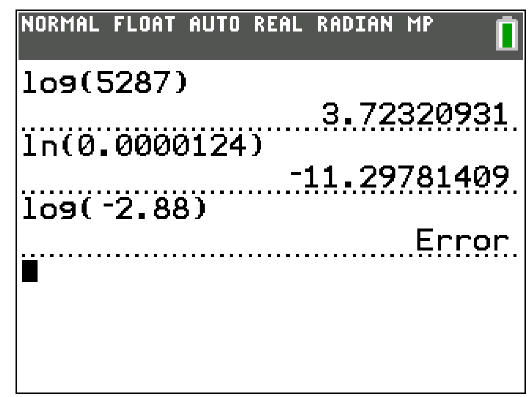
Solution: Input the values for A) and B) into the calculator. The screen shot shows the result. A) log 5,287 = 3.723210

B) ln(0.0000124) = –11.2978141

C) The negative input value is not in the domain giving an error. The calculator indicates the error.







Example: Solving Logarithm Equations Using the Calculator

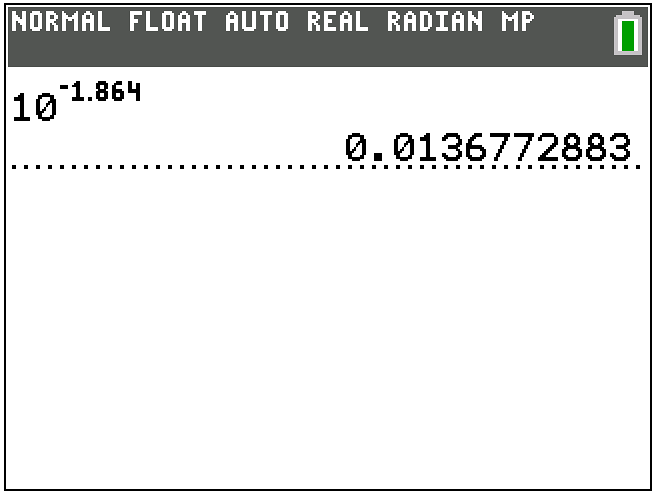
Solve the equation log x = -1.864 for x. Give your answer to four decimal places.

Solution:

Write log x = -1.864 in exponential form.

$x = 10^{-1.864}$ Evaluate with a calculator

$x = 0.0137$ Round the answer to four decimal place



Example: Solving Logarithm Equations Using the Calculator

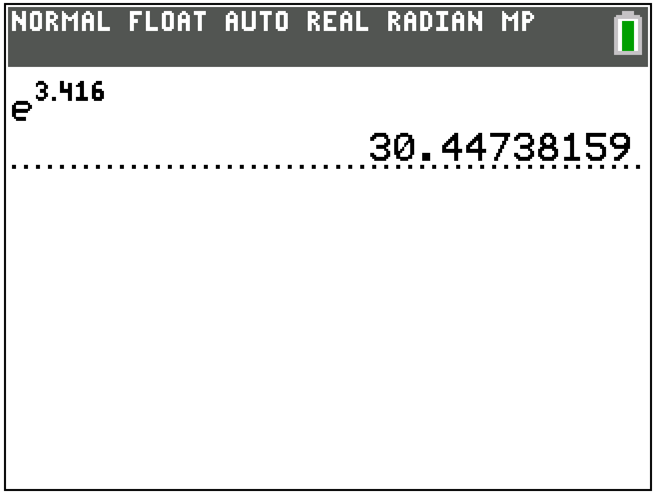
Solve the equation ln $x = 3.416$ for x. Give your answer to four decimal places.

Solution:

Write ln $x = 3.416$ in exponential form.

$x = e^{3.416}$ Evaluate with a calculator

$x = 30.4474$ Round the answer to four decimal places.

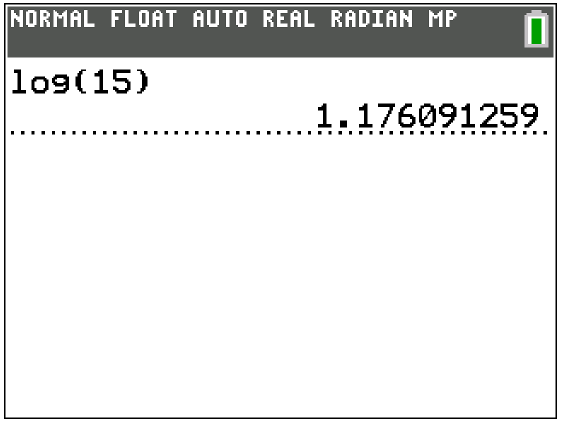


Example: Solving Exponential Equations Using the Calculator

Solve the equation $10^x = 15$ for *x*. Give your answer to four decimal places.

Solution:

$10^x = 15$	Take common logarithm of both sides.
$\log 10^x = \log 15$	Use property 3.
$x = 1.1761$	Evaluate with a calculator. Round to four decimal places.



Example: Solving Exponential Equations Using the Calculator

Solve the equation $5^x = 11$ for x. Give your answer to four decimal places.

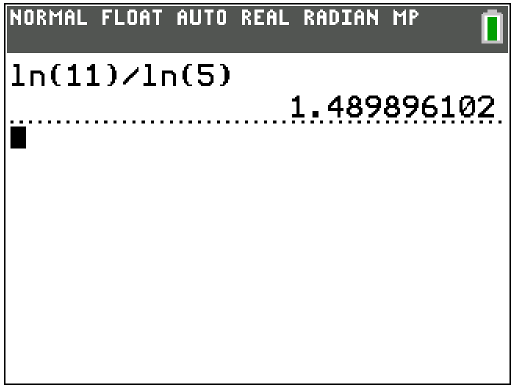
Solution: $5^x = 11$ Take natural logarithm of both sides.

$\ln 5^x = \ln 11$ Use property 7.

$x \ln 5 = \ln 11$ Solve for x.

$$x = \frac{\ln 11}{\ln 5}$$ Use a calculator to evaluate.

$x = 1.4899$ Round the result to four decimal places.



You Try It!

Solve $11^x = 9$ for x to four decimal places.

Example: Simplify to Find a Change of Base Method Using Common Logarithms

Suppose $y = \log_b x$ for some suitable base b.

Then $b^y = x$

$\log b^y = \log x$

$y \log b = \log x$

$$y = \frac{\log x}{\log b}$$

Since $y = \log_b x$, substitute to find $\log_b x = \dfrac{\log x}{\log b}$.

Your Turn: Follow the Example on the Previous Slide to Find a Change of Base Method Using Natural Logarithms

Hint: Let $y = \log_b x$ for some suitable base b.

Write the equation in exponential form, then take the natural logarithm of both sides and solve for y.

An Application of Logarithms

Different investments can be compared by using their **doubling times**.

The doubling time for an exponential growth model is the length of time it takes a given value to double.

Note that exponential growth models are characterized by having a fixed doubling time.

An investment with a shorter doubling time has a higher rate of return.

Logarithms provide a convenient tool for solving doubling time problems.

Example: Find the Doubling Time

How long (to the next whole year) will it take money to double if it is invested at 15% compounded annually?

Solution: We use the compound interest formula (from Section 1.5) in this example.

$$A = P\left(1 + \frac{r}{m}\right)^{m \cdot t}$$

where $r = 0.15$, $m = 1$, and $A = 2P$

$$2P = P(1 + 0.15)^t$$

$$2 = 1.15^t$$

$$\ln 2 = \ln 1.15^t$$

$$\ln 2 = t \ln 1.15$$

$$t = \frac{\ln 2}{\ln 1.15} \approx 4.9595 \approx 5 \text{ years.}$$

Logarithmic Regression

Among increasing functions, the logarithmic functions with bases $b > 1$ increase much more slowly for large values of x than either exponential or polynomial functions.

When a visual inspection of the plot of a data set indicates a slowly increasing function, a logarithmic function often provides a good model.

We use logarithmic regression on a graphing calculator to find the function of the form $y = a + b \cdot \ln x$ that best fits the data.

Example: Logarithmic Regression

A cordless screwdriver is sold through a national chain of discount stores. A marketing company established the following price-demand table, where x is the number of screwdrivers people are willing to buy each month at a price of p dollars per screwdriver.

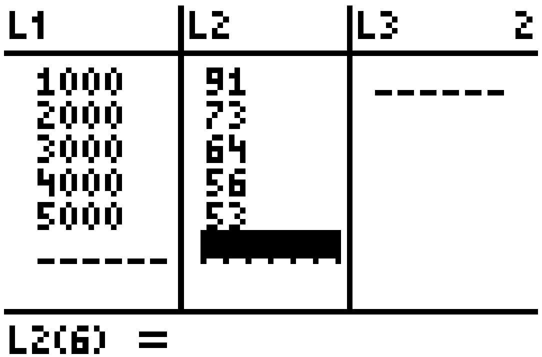
x	$p = D(x)$
1,000	91
2,000	73
3,000	64
4,000	56
5,000	53

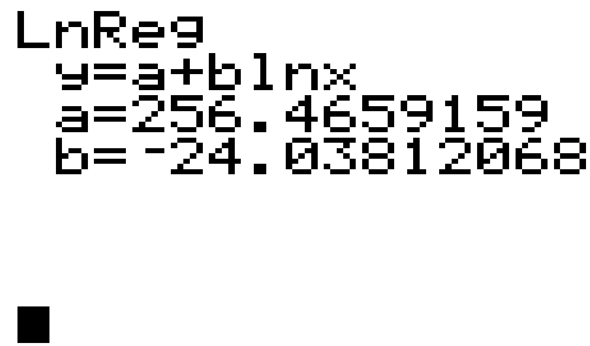
Use technology to find the best fit logarithmic function.

Use the regression equation to predict the price rounded to the nearest dollar required to sell 6,000 screwdrivers each month.

Logarithmic Regression Example continued

To find the logarithmic regression equation, enter the data into your calculator, as shown below.





Use the statistics calculation process for **LnReg** to analyze the data.

The logarithmic regression equation that gives the best fit for these data is $y = 256.4659 - 24.038 \ln x$.

The estimated price required to sell 6,000 screwdrivers is found by evaluating $y = 256.4659 - 24.038 \ln (6000) \approx \$47$.