Solutions for Introductory and Intermediate Algebra 6th Edition by Bittinger

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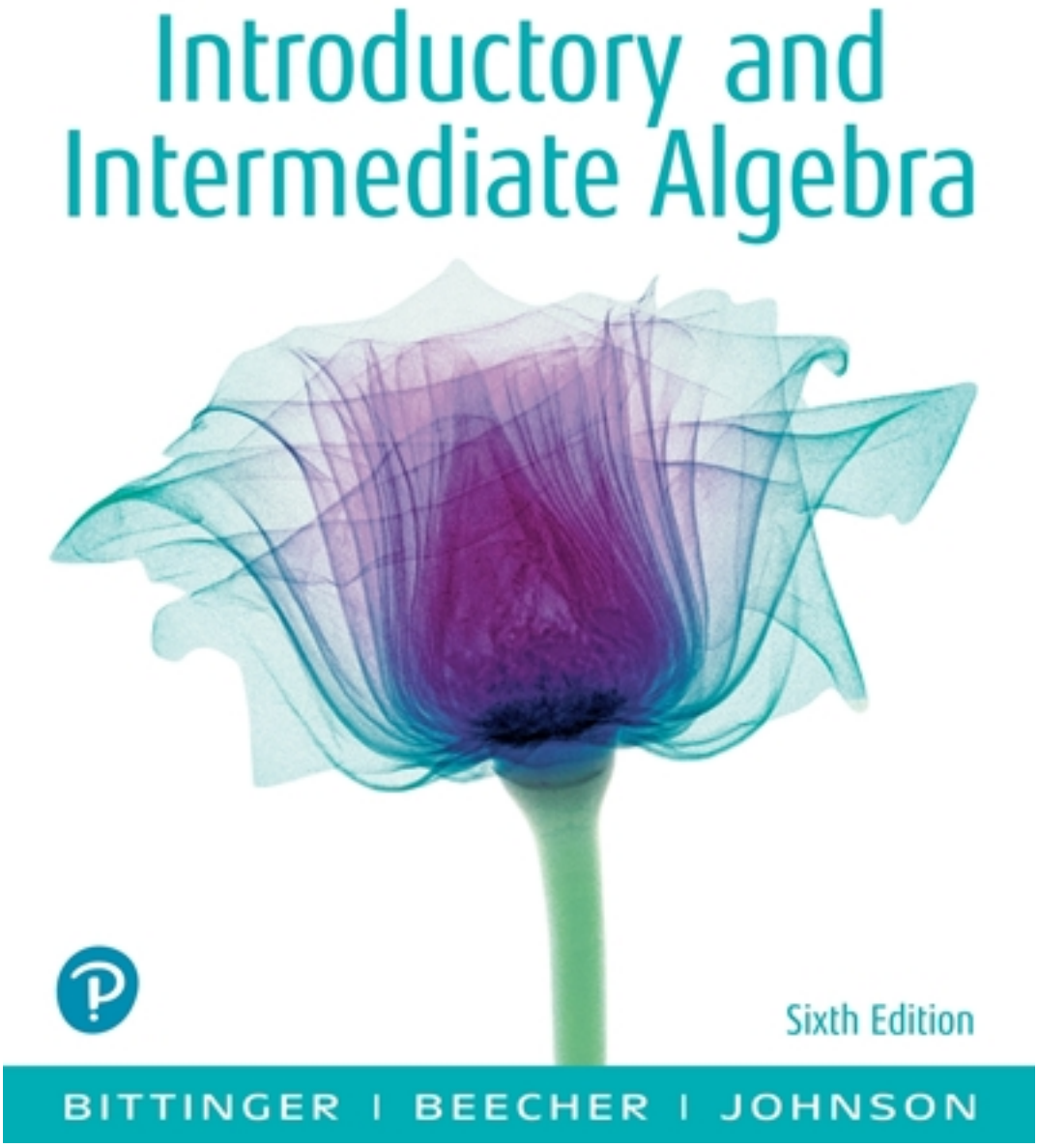


Solutions

Solving Equations: The Addition Principle

Learning Objectives:

a Determine whether a given number is a solution of a given equation.
b Solve equations using the addition principle.

Examples:

1. Determine whether the given number is a solution of the given equation.
   a) Is 0 a solution of $6-3x=6-5x$?
   b) Is 5 a solution of $x+4=10$?
   c) Is 9 a solution of $12=3+x$?
   d) Is 10 a solution of $8(y-3)=54$?

2. Solve using the addition principle.
   a) $x-4=16$ b) $14=x-12$ c) $x+15=18$
   d) $-19=x+16$ e) $21=-16+x$ f) $x-(-6)=18$
   g) $15=x-5$ h) $13+x=13$ i) $-26+x=-32$

3. Solve using the addition principle.
   a) $\frac{1}{4}+x=\frac{3}{4}$ b) $\frac{1}{3}+x=\frac{5}{6}$ c) $x-\frac{1}{5}=0$
   d) $x-\frac{9}{10}=-\frac{3}{5}$ e) $-2.2+x=16$ f) $14\frac{3}{4}=-28+x$

Teaching Notes:

- Encourage students to write all of the addition principle steps and to avoid using shortcuts until they have mastered these types of equations.
- Encourage students to write the steps for solving the equations in a neat and organized manner. This habit will help immensely when the equations become more complex.
- Refer students to the addition principle for equations in the text.

Answers: 1a) Yes, b) no, c) yes, d) no; 2a) 20, b) 26, c) 3, d) -35, e) 37, f) 12, g) 20, h) 0, i) -6; 3a) $\frac{1}{2}$, b) $\frac{1}{2}$, c) $\frac{1}{5}$, d) $\frac{3}{10}$, e) 18.2, f) $42\frac{3}{4}$

Solving Equations: The Multiplication Principle

Learning Objective:

a Solve equations using the multiplication principle.

Examples:

1. By what number is it necessary to multiply each side of each equation in order to obtain x on the left side? Do not solve.

   a) $\frac{1}{5}x = 3$ b) $\frac{x}{6} = -2$ c) $-\frac{3}{4}x = 21$ d) $-x = 41$

2. Tell whether you would use the addition principle or multiplication principle to solve each equation. Do not solve.

   a) $x + 3 = -15$ b) $5x = -25$

3. Solve using the multiplication principle. Be sure to simplify your answers. Check your answers.

   a) $\frac{1}{5}x = 6$ b) $\frac{1}{4}x = -25$ c) $\frac{x}{12} = 5$ d) $-9 = \frac{x}{9}$

4. Solve using the multiplication principle. Be sure to simplify your answers. Check your answers.

   a) $3x = 9$ b) $7x = -56$ c) $-11 = 2x$ d) $1.2x = 88$

   e) $-16 = -x$ f) $-x = 100$ g) $-3.3x = -111$ h) $53 = -8x$

5. Solve using the multiplication principle. Check your answers.

   a) $\frac{1}{6}x = -9$ b) $-43 = 4x$ c) $-9.8x = -210.994$

   d) $-99 = -x$ e) $-8 = \frac{x}{8}$ f) $\frac{x}{15} = 225$

Teaching Notes:

- Some students need to be shown that $\frac{1}{12}x = \frac{1}{12} \cdot \frac{x}{1} = \frac{1 \cdot x}{12 \cdot 1} = \frac{x}{12}$.
- Refer to the multiplication principle for equations in the text.

Answers: 1a) 5, b) 6, c) $-\frac{4}{3}$, d) -1; 2a) Addition principle, b) multiplication principle; 3a) 30, b) -100, c) 60, d) -81; 4a) 3, b) -8, c) $-\frac{11}{2}$, or $-5\frac{1}{2}$, d) $73.\overline{3}$, or $\frac{220}{3}$, e) 16, f) -100, g) $33.\overline{63}$, or $\frac{370}{11}$, h) -6.625, or $-\frac{53}{8}$; 5a) -54, b) -10.75, or $-\frac{43}{4}$, c) 21.53, d) 99, e) -64, f) 3375

Using the Principles Together

Learning Objectives:

a Solve equations using both the addition principle and the multiplication principle.
b Solve equations in which like terms may need to be collected.
c Solve equations by first removing parentheses and collecting like terms; solve equations with an infinite number of solutions and equations with no solutions.

Examples:

1. Determine whether the given number is a solution of the equation.
   a) Is 12 a solution for $5x+4-2x=3x-5+x$?
   b) Is 9 a solution for $5x+4-2x=3x-5+x$?

2. Solve for x. Check your solution.
   a) $10x+7=107$ b) $7x-8=27$ c) $33=6x-3$
   d) $164=15x+14$ e) $\frac{1}{2}x-8=-2$ f) $-\frac{2}{3}x-8=-32$

3. Solve the equation. Check your solution.
   a) $4x=-2x+60$ b) $8x-6=3+9x$ c) $-\frac{3}{4}x-\frac{5}{2}=-\frac{5}{8}+2x$
   d) $0.6y-0.3=0.7-0.3y$ e) $x-12=10-x$ f) $-9x+4+7x=-3x+9$

4. Solve the equation. Check your solution.
   a) $6(2x-1)=30$ b) $-1(x+11)=20$ c) $6(x-8)=6x-48$
   d) $3x+6(x+9)=9x-15$ e) $0.4x-0.2(3-x)=7.6$ f) $7x-3(x-8)=3x+24$

Teaching Notes:

- Encourage students to check their solutions, as in Example 1.
- In Example 3, some students prefer to work to always have the variable on the left, while others prefer to work to always have a positive coefficient of the variable.
- Some students confuse the principles and try to subtract the coefficient from the variable instead of multiplying to obtain a coefficient of 1.
- Some students neglect to collect like terms before trying to solve as in Example 3f.

Answers: 1a) no, b) yes; 2a) 10, b) 5, c) 6, d) 10, e) 12, f) 36; 3a) 10, b) -9, c) $-\frac{15}{22}$, d) $\frac{10}{9}$, or $1\frac{1}{9}$, e) 11, f) 5; 4a) 3, b) -31, c) all real numbers, d) no solution, e) $\frac{41}{3}$, or $13\frac{2}{3}$, f) 0

Formulas

Learning Objectives:

a Evaluate a formula.
b Solve a formula for a specified letter.

Examples:

1. Substitute values into the given formula and solve.
   a) The formula for the perimeter of a rectangle is $P = 2L + 2W$. If the length, L, is 9 m and the width, W, is 5 m, find the perimeter, P, of the rectangle.

   b) The area of a triangle is given by $A = \frac{1}{2}bh$. If the base, b, is 19 in. and the height, h, is 17 in., find the area, A, of the triangle.

2. Solve for the indicated letter.
   a) $d = rt$, for r
   b) $V = \frac{1}{3}Bh$, for h
   c) $P = 2L + 2W$, for W
   d) $F = \frac{9}{5}C + 32$, for C
   e) $y = 3x + 6$, for x
   f) $S = 2\pi rh + 2\pi r^2$, for h

Teaching Notes:

- Many students have difficulty with the problems in Example 2.
- Sometimes it is required that students solve a number of problems using the same formula. It may be advantageous for students to rewrite the formula so that it is solved for the required letter first.
- Encourage students to label answers with the correct units.

Answers: *1a)* $P = 28\ m$, *b)* $A = 161.5\ in^2$; *2a)* $r = \frac{d}{t}$, *b)* $h = \frac{3V}{B}$, *c)* $W = \frac{P - 2L}{2}$, *d)* $C = \frac{5}{9}(F - 32)$, *e)* $x = \frac{y - 6}{3}$, *f)* $h = \frac{S - 2\pi r^2}{2\pi r}$

Applications of Percent

Learning Objective:

a Solve applied problems involving percent.

Examples:

1. Solve.
   a) 7 is 10% of what number?
   b) What is 32% of 224?
   c) Find 190% of 375.
   d) 25 is what percent of 125?
   e) What percent of 80 is 0.8?
   f) 126 is 450% of what number?

2. Solve.
   a) The Smith family paid 22% of the purchase price of a $231,000 home as a down payment. Determine the amount of the down payment.
   b) A waiter received a tip of $7.65 on a meal which cost $42.50. What percent of the cost of the meal was the tip?
   c) Josh controlled 12 cities in a game. This was 75% of all the cities in the game. How many cities were in the game?
   d) The population of Grovestown was 2250 last year. This year it is 2295. What is the percent increase?

Teaching Notes:

- Many students find this section difficult.
- One way to remember which way to move the decimal point when converting between decimal and percent notation is to remember that the decimal is moved right for ***D***ecimal to ***P***ercent, and left for ***P***ercent to ***D***ecimal.

Answers: 1a) 70, b) 71.68, c) 712.5, d) 20%, e) 1%, f) 28; 2a) $50,820, b) 18%, c) 16 cities, d) 2%

Applications and Problem Solving

Learning Objective:

a Solve applied problems by translating to equations.

Examples:

1. Solve.
   a) If 5 is added to a number, and the sum is tripled, the result is 11 more than the number. Find the number.

   b) When six is subtracted from half of a number, the result is -18. What is the original number?

2. In 2014, Pierre's Pizza and Burger Palace together had revenue totaling $470,000. If Burger Palace took in $90,000 less than Pierre's Pizza, how much did each take in as revenue?

3. The perimeter of a triangle is 37 ft. One side of the triangle is 2 ft longer than the second side. The third side is 5 ft longer than the second side. Find the length of each side.

4. Solve.
   a) The sum of two consecutive integers is 59. Find the integers.

   b) When the lesser of two consecutive odd integers is added to twice the greater, the result is 187. Find the integers.

5. A parking lot charges $2.50 for the first hour or part thereof, and $1.25 for each additional hour or part thereof. Determine in terms of an inequality, how many hours, H, Gwen was parked if her total fee was $10.

Teaching Notes:

- Many students find these problems difficult at first.
- Refer students to the five-step problem-solving strategy in the text.
- Encourage students to check whether their final answers are reasonable.

Answers: 1a) -2, b) -24; 2) Pierre's Pizza had revenue of $280,000 and Burger Palace had revenue of $190,000; 3) 12 ft, 10 ft, 15 ft; 4a) 29 and 30, b) 61 and 63; 5) $\{H \mid 6 < H \le 7\}$

Solving Inequalities

Learning Objectives:

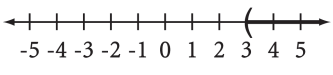
a Determine whether a given number is a solution of an inequality.
b Graph an inequality on the number line.
c Solve inequalities using the addition principle.
d Solve inequalities using the multiplication principle.
e Solve inequalities using the addition principle and the multiplication principle together.

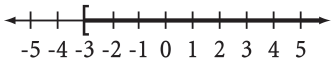
Examples:

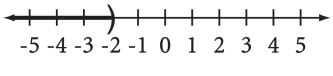
1. Determine whether each number is a solution of $x \le -4$.
   a) -2 b) -5 c) 3 d) -4

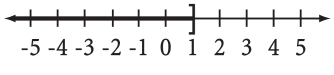
2. Graph each inequality on the number line.
   a) $x \ge 1$ b) $x > -2$ c) $-1 \le x < 2$

3. Translate each graph to an inequality using the variable x.

a) 

b) 

c) 

d) 

4. Solve each inequality.
   a) $x + 3 < -5$ b) $x - \frac{1}{3} > -\frac{2}{9}$ c) $4x \ge -32$ d) $-6x \le 18$

   e) $4x - 10 > 5x + 4$ f) $5x - 2 \le 6x + 4$ g) $-4x - 7 < 5(2x + 7)$ h) $7(x - 3) > 2(5x - 8)$

Teaching Notes:

- Some students are unfamiliar with $<$, $>$, $\le$, and $\ge$ and need to be taught the definitions of each.
- Some students need extra help with Examples 2e.

Answers: *1a) no, b) yes, c) no, d) yes; 2a-e) see graph answer pages; 3a)* $\{x \mid x > 3\}$, *b)* $\{x \mid x \ge -3\}$, *c)* $\{x \mid x < -2\}$, *d)* $\{x \mid x \le 1\}$; *4a)* $\{x \mid x < -8\}$, *b)* $\left\{x \mid x > \frac{1}{9}\right\}$, *c)* $\{x \mid x \ge -8\}$, *d)* $\{x \mid x \ge -3\}$, *e)* $\{x \mid x < -14\}$, *f)* $\{x \mid x \ge -6\}$, *g)* $\{x \mid x > -3\}$, *h)* $\left\{x \mid x < -\frac{5}{3}\right\}$

Applications and Problem Solving with Inequalities

Learning Objectives:

a Translate number sentences to inequalities.
b Solve applied problems using inequalities.

Examples:

1. Translate to an inequality.
   a) A number is greater than -3.

   b) The capacity was at least 200.

   c) Six more than a number is less than 21.

   d) The cost is no more than $500.

2. Solve using an inequality.
   a) A certain car has a weight limit for all passengers and cargo of 1129 lb. The four passengers in the car weigh an average of 165 lb. Find the weight of the cargo that the car can handle.

   b) A store has a fax machine available for use by its customers. The store charges $1.85 to send the first page and $0.45 for each subsequent page. Find the number of pages that can be faxed for $7.25.

   c) An archery set containing a bow and three arrows costs $68. Additional arrows can be purchased for $9 each. Jerry has $230 to spend on the set and additional arrows. Including the arrows in the set, how many arrows can Jerry purchase?

Teaching Notes:

- Review the common English-to-inequality translations.
- Some students benefit from seeing solutions on the number line.

Answers: *1a)* $n > -3$, *b)* $c \geq 200$, *c)* $6 + n < 21$, *d)* $c \leq 500$; *2a)* $\{x | x \leq 469 \text{ lb}\}$, *b)* $\{x | x \leq 13 \text{ pages}\}$, *c)* $\{x | x \leq 21 \text{ arrows}\}$

CHAPTER 2 **NAME**_____

TEST FORM A **CLASS**_____ **SCORE**_____ **GRADE**_____

Solve.

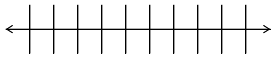
1. $x+8=15$
2. $t-8=16$
3. $4x=-24$
4. $-\frac{3}{4}x=-12$
5. $4t+9=5t-6$
6. $\frac{1}{2}x-\frac{1}{6}=\frac{5}{6}$
7. $14-y=5$
8. $-\frac{2}{3}+x=-\frac{5}{8}$
9. $3(x+4)=36$
10. $-3x-3(x+5)=14$
11. $0.2p-1.8=2.3p-4.5-1.8p$
12. $4(x+5)-9=2(2x+3)+1$
13. $8y+6=-11y-2+19y+8$

Solve. Write set notation for the answers.

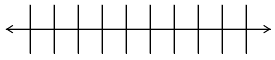
14. $x+5\le 8$
15. $8x+2>9x-6$
16. $4x\ge 24$
17. $-3y\le 15$
18. $-7y\ge -35$
19. $-4x\ge\frac{3}{8}$
20. $3-7x>17$
21. $3-2x\le 21+4x$

Graph on the number line.

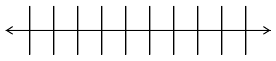
22. $y\le 1$



23. $4x-3>x+6$



24. $-3\le x\le 0$



Solve.

25. What number is 12% of 75?
26. 32.2 is what percent of 92?
27. 24 is 2% of what number?

ANSWERS

1. _____
2. _____
3. _____
4. _____
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9. _____
10. _____
11. _____
12. _____
13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. See graph.
23. See graph.
24. See graph.
25. _____
26. _____
27. _____

CHAPTER 2 **NAME**_____

TEST FORM A

ANSWERS

28. _____
29. _____
30. _____
31. _____
32. _____
33. _____
34. _____
35. _____
36. _____
37. _____
38. _____
39. _____
40. _____

Solve.

28. The population increased from 24,000 to 27,000 over a nine-year period. What was the percent increase?

29. The perimeter of a rectangular flower bed is 56 ft. The length is 4 ft greater than the width. Find the width and the length.

30. Forty-eight percent of the students were girls. There were 12 girls. How many students were there?

31. The sum of the page numbers on the facing pages of a book is 257. What are the page numbers?

32. Money is invested in a savings account with 3% simple interest. After 1 year, there is $1905.50 in the account. How much was originally invested?

33. A 60-in. pipe is cut into two pieces. One piece is 30 in. more than the other. Find the length of each piece.

34. The width of a rectangle is 48 yd. Find all possible lengths so that the perimeter of the rectangle will be at least 200 yd.

35. Greta budgeted an average of $70 a month for clothing. For the first three months of the year she spent $150, $50, and $20. How much can she spend in the fourth month without exceeding her average budget?

36. Solve $F = ma$ for a.

37. Solve $y = 5x - n$ for x.

38. The number of tomato plants Jeff planted went from 4 last year to 12 this year. Find the percent increase.

A. 20% B. $66\frac{2}{3}\%$ C. 150% D. 200%

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39. Solve $k = \dfrac{5}{x-2}$ for x.

40. Solve: $3|x| + 7 = 16$.

CHAPTER 2 **NAME**_____

TEST FORM B **CLASS**_____ **SCORE**_____ **GRADE**_____

Solve.

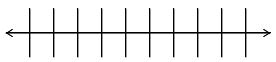
1. $x+9=14$
2. $t-5=9$
3. $3x=-12$
4. $-\frac{3}{4}x=-16$
5. $6t+4=5t-7$
6. $\frac{1}{3}x-\frac{3}{8}=\frac{5}{8}$
7. $10-y=16$
8. $-\frac{3}{8}+x=\frac{5}{6}$
9. $4(x+6)=32$
10. $-2x-2(x-9)=12$
11. $2.4p+4.5=3.6p-1.9+0.4p$
12. $5y-3=-4y+2+9y-6$
13. $8(x-3)+7=6x-17+2x$

Solve. Write set notation for the answers.

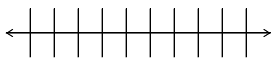
14. $x+2\geq 11$
15. $8x+2<9x-6$
16. $11x\leq 44$
17. $-8y\geq 24$
18. $-2y\leq -28$
19. $-5x\leq \frac{1}{3}$
20. $3-5x<18$
21. $2-7x\geq 26+5x$

Graph on the number line.

22. $y\leq -2$



23. $4x-3<x+9$



24. $-1<x\leq 2$

Solve.

25. What number is 5% of 60?
26. 28.6 is what percent of 52?
27. 18 is 4% of what number?

ANSWERS

1. _____
2. _____
3. _____
4. _____
5. _____
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21. _____
22. See graph.
23. See graph.
24. See graph.
25. _____
26. _____
27. _____

CHAPTER 2 **NAME**_____

TEST FORM B

ANSWERS

28. _____
29. _____
30. _____
31. _____
32. _____
33. _____
34. _____
35. _____
36. _____
37. _____
38. _____
39. _____
40. _____

Solve.

28. Enrollment increased from 136 to 150 over two years. What was the percent increase?

29. The perimeter of a rectangular poster is 108 in. The length is 6 in. greater than the width. Find the width and the length.

30. About 20% of the trees were birches. If there were 38 birch trees, how many trees were there altogether?

31. The sum of the numbers on two consecutive post office boxes is 787. What are the numbers?

32. Danielle paid $122.72 for a dress including 4% tax. How much did the dress itself cost?

33. A 42-in. board is cut into two pieces. One piece is 6 in. longer than the other. Find the length of each piece.

34. The width of a rectangle is 12 in. Find all possible lengths such that the perimeter of the rectangle will be at least 50 in.

35. Joe's Electric made 21 customer calls last week and 18 calls this week. How many calls must be made next week in order to maintain an average of at least 24 calls per week for the three-week period?

36. Solve $d = rt$ for t.

37. Solve $A = 3x - D$ for x.

38. The number of inquiries to customer service went from 16 per day to 24 per day after the release of the new product. Find the percent increase.

A. 200% B. 50% C. 20% D. $33\frac{1}{3}\%$

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39. Solve $2x + 5 = 3x - yx + 2$ for x.

40. Solve: $42 + 3|x| = 258$.

CHAPTER 2 **NAME**_____

TEST FORM C **CLASS**_____ **SCORE**_____ **GRADE**_____

Solve.

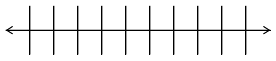
1. $x+4=12$
2. $t-9=21$
3. $8x=-48$
4. $-\frac{2}{3}x=15$
5. $6t+4=7t-8$
6. $\frac{3}{7}x-\frac{1}{8}=\frac{7}{8}$
7. $12-y=17$
8. $-\frac{5}{9}+x=\frac{1}{3}$
9. $6(x+8)=54$
10. $-4x-7(x-2)=8$
11. $0.5p+2.4=1.6p-3.2-0.4p$
12. $-5x+8(x-1)=7+3x+4$
13. $6y-1-9y=4-3y-5$

Solve. Write set notation for the answers.

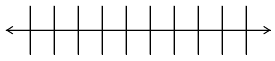
14. $x+7\le 10$
15. $8x+5>7x-9$
16. $8x\ge -24$
17. $-4y\le 44$
18. $-6y\ge -36$
19. $-2x\ge \frac{1}{5}$
20. $8-6x>32$
21. $6+4x<18+7x$

Graph on the number line.

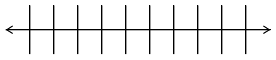
22. $y<1$



23. $11x+5\ge x-15$



24. $-3\le x\le 2$



Solve.

25. What number is 15% of 75?
26. 13.3 is what percent of 70?
27. 18 is 6% of what number?

ANSWERS

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21. _____
22. See graph.
23. See graph.
24. See graph.
25. _____
26. _____
27. _____

CHAPTER 2 **NAME**_____

TEST FORM C

ANSWERS

28. _____
29. _____
30. _____
31. _____
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33. _____
34. _____
35. _____
36. _____
37. _____
38. _____
39. _____
40. _____

Solve.

28. The number of bacteria present decreased from 25,000 to 17,000 in 3 hr. What was the percent decrease?

29. The perimeter of a rectangular tablecloth is 6 m. The length is 1 m greater than the width. Find the width and the length.

30. Five percent of the participants arrived late. If 12 participants arrived late, how many participants were there?

31. The numbers on Tara's three raffle tickets are consecutive whole numbers. The sum of the numbers is 399. What are the numbers?

32. The Andrews are selling their home. They want to keep $150,000 after paying a 7% commission to a realtor. For how much must they sell the house (to the nearest dollar)?

33. A 36-in. board is cut into two pieces. One piece is twice the length of the other. Find the length of each piece.

34. The perimeter of a rectangular playing field is not to exceed 280 ft. The width is to be 20 ft less than the length. What widths will allow these conditions to be met?

35. To be considered eligible for benefits, Marlesa must work an average of at least 20 hr per week. In the first three weeks of one month, she worked 16 hr, 22 hr, and 18 hr. How many hours must she work the next week to have an average of at least 20 hr per week for those 4 weeks?

36. Solve $C = 2\pi r$ for r.

37. Solve $y = 4x + d$ for x.

38. Cassie's weight went from 120 lb to 125 lb. Find the percent increase.

A. 4.2% B. 4% C. 5% D. 4.5%

---

39. Solve $5x - 6 = 3x + xy$ for x.

40. Solve $t = \dfrac{4s}{p - q}$ for q.

CHAPTER 2 **NAME**_____

TEST FORM D **CLASS**_____ **SCORE**_____ **GRADE**_____

Solve.

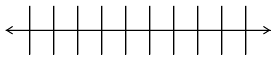
1. $x+7=11$
2. $t-8=22$
3. $6x=-30$
4. $-\frac{2}{5}x=-50$
5. $4t+7=3t-1$
6. $\frac{1}{3}x-\frac{3}{8}=\frac{5}{8}$
7. $6-y=14$
8. $-\frac{2}{3}+x=\frac{5}{8}$
9. $4(x+2)=36$
10. $-4x-5(x-3)=8$
11. $0.2p+1.4=3.5p+6.2-0.8p$
12. $9+8x=4(3+2x)-3$
13. $5x-6-x=4(2+x)-12$

Solve. Write set notation for the answers.

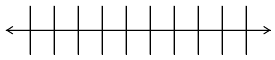
14. $x+9\geq 13$
15. $7x+5<8x-2$
16. $6x\leq 42$
17. $-8y\geq 72$
18. $-3y\leq -24$
19. $-3x\leq \frac{1}{9}$
20. $8-3x\leq 29$
21. $3-5x\geq -6+2x$

Graph on the number line.

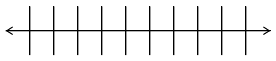
22. $y\geq 1$



23. $3x-2<x+6$



24. $-2\leq x\leq 3$



Solve.

25. What number is 24% of 400?
26. 16.2 is what percent of 80?
27. 13 is 8% of what number?

ANSWERS

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22. See graph.
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CHAPTER 2 **NAME**_____

TEST FORM D

ANSWERS

28. _____
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31. _____
32. _____
33. _____
34. _____
35. _____
36. _____
37. _____
38. _____
39. _____
40. _____

Solve.

28. The number of complaints fell from 40 per month to 24 per month one month after all employees completed the training seminar. What was the percent decrease?

29. The perimeter of a rectangular frame is 48 in. The length is 4 in. greater than the width. Find the width and the length.

30. Twelve percent of the workforce attended the meeting. If 6 workers attended the meeting, how many workers are there altogether?

31. The sum of two consecutive even integers is 770. What are the integers?

32. The price of a wool coat was decreased to a sale price of $124.10. This was a 15% reduction. What was the former price?

33. The second angle of a triangle is three times as large as the first angle. The third angle is half as large as the first angle. How large are the angles?

34. One side of a triangle is 2 cm longer than the base. The other side is 3 cm shorter than the base. What lengths of the base will allow the perimeter to be less than 30 cm?

35. A 15-lb puppy is gaining weight at a rate of 1.5 lb per wk. When will the puppy's weight exceed 40 lb?

36. Solve $A = bh$ for h.

37. Solve $A = \dfrac{2T + Q + H}{4}$ for H.

38. The number of students in Josh's class went from 15 last year to 18 this year. Find the percent increase.

A. 30% B. $16\frac{2}{3}\%$ C. 20% D. 5%

---

39. Solve: $-8|x| + 5 = -11$.

40. Solve: $3 = \dfrac{x}{y - z}$ for z.

CHAPTER 2 **NAME**_____

TEST FORM E **CLASS**_____ **SCORE**_____ **GRADE**_____

1. Determine whether 9 is a solution of $3x-8=19$.

Solve.

2. $t-6=15$

3. $3x=-24$

4. $-\frac{5}{8}x=40$

5. $5t-6=4t+2$

6. $\frac{2}{3}x-\frac{5}{8}=\frac{3}{8}$

7. $9-y=8$

8. $-\frac{3}{5}+x=\frac{2}{3}$

9. $4(x+2)=28$

10. $-5x-3(x-2)=6$

11. $6x-5=2(3x-2)-1$

12. $1.3p-6.2=5.9p+4.6+0.8p$

Solve. Write set notation for the answers.

13. $x+8\geq 5$

14. $2x+4<3x-7$

15. $12x\leq 24$

16. $-3y\geq 27$

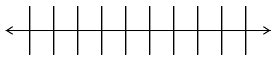
17. $-5y\leq -\frac{1}{2}$

18. $4-2x<8$

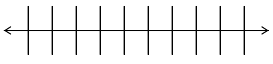
19. $6-5x\geq 3x-32$

Graph on the number line.

20. $y\geq 1$



21. $3x-6<x+2$



ANSWERS

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20. See graph.
21. See graph.

CHAPTER 2 **NAME**_____

TEST FORM E

ANSWERS

22. _____

23. _____

24. _____

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

Solve.

22. If you double a number and then add 8, you get half of the original number. What is the original number?

23. A teacher's salary is $29,355. This is a 3% increase over the previous year's salary. What was the previous salary?

24. What percent of 112 is 22.4?

25. A 100-m rope is cut into two pieces. One piece is 16 m greater than the other. Find the length of each piece.

26. Solve $A = \pi r^2$ for r^2.

27. Translate to an inequality: A number is at least 5.

28. The perimeter of a rectangular field cannot exceed 400 ft. The length is to be 20 ft more than the width. What widths will allow these conditions to be met?

29. Solve: $8|x| - 5 = 43$.

30. Solve $S = 2\pi r^2 + 2\pi rh$ for π.

CHAPTER 2 **NAME**_____

TEST FORM F **CLASS**_____ **SCORE**_____ **GRADE**_____

Solve.

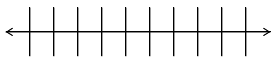
1. $x+5=12$
2. $t-7=12$
3. $-\frac{2}{3}x=36$
4. $6t+5=7t-3$
5. $\frac{1}{2}x-\frac{2}{7}=\frac{5}{7}$
6. $3-y=11$
7. $-\frac{5}{9}+x=\frac{3}{4}$
8. $6(x+2)=42$
9. $-5x-4(x-6)=12$
10. $5x-6=2(3x-1)-x$
11. $1.2p-3.6=0.6p-6.4-2.6p$

Solve. Write set notation for the answers.

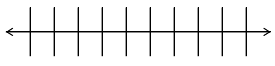
12. $x+3\le 9$
13. $4x+3>5x-8$
14. $8x\ge 48$
15. $-2y\le 16$
16. $-4x\ge\frac{1}{8}$
17. $3-5x>4$
18. $5-7x\le 18+6x$

Graph on the number line.

19. $y\ge -3$



20. $-1\le x<2$



ANSWERS

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19. See graph.
20. See graph.

CHAPTER 2 **NAME**_____

TEST FORM F

ANSWERS

21. _____

22. _____

23. _____

24. _____

25. _____

26. _____

27. a) _____
b) _____
c) _____

28. _____

29. _____

30. _____

Solve.

21. The perimeter of a rectangular picture is 120 in. The length is 12 in. greater than the width. Find the width and the length.

22. The second angle of a triangle is 20° less than the first angle. The third angle is twice the first angle. How large are the angles?

23. What number is 18% of 220?

24. Money is invested in a savings account at 2% simple interest. After 1 year, there is $1606.50 in the account. How much was originally invested?

25. Evaluate $N = n^2 - n$ for $n = 6$ to find the number of games a league of 6 teams will play if each team plays each other team twice.

26. Solve $A = \dfrac{2T + Q + H}{4}$ for T.

27. Determine whether each of the following is a solution of $x \le 6$.
a) -3 b) 10 c) 6

28. Your test grades are 90, 92, 80, and 83. Determine (in terms of an inequality) what scores you can get on the next test and still have an average test score of at least 88.

---

29. Solve: $4|x| + 5 = 3$.

30. Solve: $\dfrac{2}{3}\left(8x - \dfrac{5}{6}\right) - \dfrac{1}{8} = \dfrac{7}{8}$.

CHAPTER 2 **NAME**_____

TEST FORM G **CLASS**_____ **SCORE**_____ **GRADE**_____

1. Solve: $x-7=8$.
   a) 1 b) -1 c) 15 d) -15

2. Solve: $9x=-72$.
   a) 8 b) -8 c) -81 d) -648

3. Solve: $2t+8=3t-11$.
   a) 19 b) $-\frac{19}{5}$ c) 3 d) -19

4. Solve: $-\frac{3}{8}x=24$.
   a) 64 b) -9 c) 9 d) -64

5. Solve: $5-y=12$.
   a) -7 b) 7 c) -17 d) $-\frac{12}{5}$

6. Solve: $8(x+3)=40$.
   a) 8 b) 2 c) 29 d) 317

7. Solve: $4x+3>5x-9$.
   a) $\{x|x<12\}$ b) $\{x|x<6\}$
   c) $\{x|x>12\}$ d) $\{x|x>6\}$

8. Solve: $3-2x<15$.
   a) $\{x|x>6\}$ b) $\{x|x>-6\}$
   c) $\{x|x<-6\}$ d) $\{x|x<6\}$

9. Solve: $3x+4(3+2x)=11x-12$.
   a) All real numbers b) 4
   c) $\frac{19}{6}$ d) No solution

10. 35 is what percent of 560?
    a) 62.5% b) 16% c) 6.25% c) 8.5%

11. Solve: $p=ax+by+c$ for b.
    a) $b=\frac{ax+c-p}{y}$ b) $b=y(p-ax-c)$
    c) $b=\frac{p}{b}-ax-c$ d) $b=\frac{p-ax-c}{y}$

ANSWERS

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CHAPTER 2 **NAME**_____

TEST FORM G

ANSWERS

12. _____
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19. _____
20. _____

12. Which of the following numbers is included in the graph of $3x+8>-2x-16$?
a) -5 b) -6 c) -4 d) -20

13. The perimeter of a rectangular poster is 108 in. The length is 12 in. greater than the width. Find the length.
a) 21 in. b) 42 in. c) 33 in. d) 48 in.

14. If you double a number and then add 8, you get fourteen more than the original number. What is the original number?
a) 11 b) 3 c) 12 d) 6

15. A 12-m board is cut into two pieces. One piece is twice the length of the other. Find the length of the longer piece.
a) 4 m b) 8 m c) 9 m d) 3 m

16. Brenna left a 17% tip on her restaurant tab. If she left a \$4.80 tip, how much was the original tab?
a) \$0.82 b) \$33.04 c) \$32.00 d) \$28.24

17. The height of a triangle is 90 cm. What lengths of the base will make the area at most 500 cm^2?
a) $\{b|b\le 11.\overline{1}\text{ cm}\}$ b) $\{b|b<11.\overline{1}\text{ cm}\}$
c) $\{b|b\ge 5.\overline{5}\text{ cm}\}$ d) $\{b|b\ge 11.\overline{1}\text{ cm}\}$

18. The population increased from 1200 to 1464. What was the percent increase?
a) 18% b) 26% c) 22% d) 5%

19. Solve: $5-|x|=-14$.
a) $-9, 9$ b) $-70, 70$ c) $-\frac{14}{5}, \frac{14}{5}$ d) $-19, 19$

20. Solve: $\frac{2}{3}\left(9x-\frac{3}{8}\right)-\frac{1}{3}=\frac{2}{3}$.
a) $\frac{11}{72}$ b) $\frac{5}{24}$ c) $\frac{25}{216}$ d) $\frac{1}{8}$

CHAPTER 2 **NAME**_____

TEST FORM H **CLASS**_____ **SCORE**_____ **GRADE**_____

1. Solve: $x+8=14$.
   a) 22 b) 6 c) 8 d) -6

2. Solve: $0.2p+2.4=1.6p-1.2+0.6p$.
   a) 1.5 b) 0.6 c) 1.8 d) $0.\overline{5}$

3. Solve: $3-y=11$.
   a) -8 b) 14 c) -14 d) 8

4. Solve: $-\frac{2}{3}+x=-\frac{6}{7}$.
   a) $-\frac{32}{21}$ b) $-\frac{9}{7}$ c) $-\frac{2}{5}$ d) $-\frac{4}{21}$

5. Solve: $-\frac{5}{3}x=15$.
   a) -9 b) -25 c) $16\frac{2}{3}$ d) 9

6. Solve: $-6x-2(x-5)=4$.
   a) $-\frac{11}{7}$ b) $-\frac{9}{8}$ c) $\frac{3}{4}$ d) $-\frac{7}{4}$

7. Solve: $4x-3=9x+6-5x-9$.
   a) No solution b) $\frac{9}{8}$
   c) 1 d) All real numbers

8. Solve: $x+4\le 13$.
   a) $\{x|x\le 11\}$ b) $\{x|x\le 9\}$
   c) $\{x|x\ge -9\}$ d) $\left\{x|x\le \frac{13}{4}\right\}$

9. Solve: $-3y\le 15$.
   a) $\{y|y\ge -5\}$ b) $\{y|y\ge -45\}$
   c) $\{y|y\le 18\}$ d) $\{y|y\le -5\}$

10. Solve: $3-7x\ge -24+2x$.
    a) $\{x|x\le 3\}$ b) $\{x|x\ge 3\}$
    c) $\{x|x\ge -3\}$ d) $\{x|x\le -3\}$

ANSWERS

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CHAPTER 2 **NAME**_____

TEST FORM H

ANSWERS

11. _____
12. _____
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17. _____
18. _____
19. _____
20. _____

11. 21 is 15% of what number?
a) 3.15 b) 315 c) 24.15 d) 140

12. Solve: $y = mx + b$ for m.
a) $m = y - bx$ b) $m = \frac{y-b}{x}$ c) $m = \frac{y+b}{x}$ d) $m = \frac{y}{x} - b$

13. Which of the following numbers is included in the graph of $-1 < x \le 1$?
a) -1 b) -2 c) 1 d) 2

14. The sum of the page numbers on the facing pages of a book is 553. Find the smaller page number.
a) 277 b) 275 c) 226 d) 276

15. Money is invested in a savings account at 3.4% simple interest. After 1 year, there is \$1695.76 in the account. How much was originally invested?
a) \$1753.42 b) \$1638.10 c) \$576.56 d) \$1640.00

16. The amount of apples Blake harvested went from 500 lb in 2009 to 425 lb in 2010. Find the percent decrease.
a) 18% b) 15% c) 5% d) 25%

17. The second angle of a triangle is twice the first angle. The third angle is five more than four times the first angle. Find the measure of the second angle.
a) 25° b) 105° c) 50° d) 30°

18. Find all numbers such that three times the number is greater than eight less than the number.
a) $\{x \mid x > -4\}$ b) $\{x \mid x > -2\}$ c) $\{x \mid x < -4\}$ d) $\{x \mid x > 4\}$

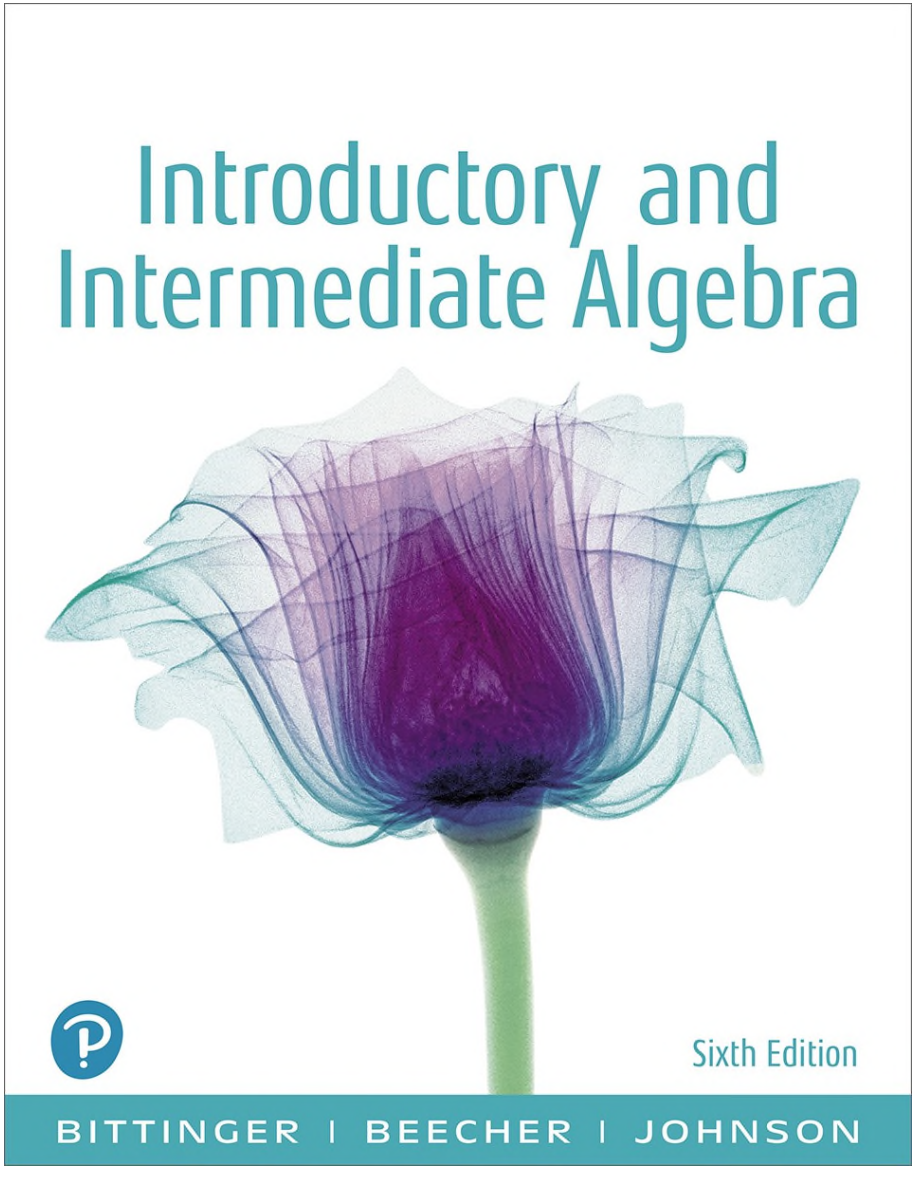
---

19. Solve: $3|x| - 6 = 15$.
a) $-3, 3$ b) $-63, 63$ c) $-27, 27$ d) $-7, 7$

20. Solve: $y = \frac{5}{3-x}$ for x.
a) $x = \frac{3y-5}{y}$ b) $x = -2$ c) $x = 3y - 5$ d) $x = \frac{5}{y} - 3$

Chapter 2

Solving Equations and Inequalities



2.1 SOLVING EQUATIONS: THE ADDITION PRINCIPLE

a. Determine whether a given number is a solution of a given equation.

b. Solve equations using the addition principle.

Equation

An **equation** is a number sentence that says that the expressions on either side of the equals sign, =, represent the same number.

Example

Determine whether the equation is true, false, or neither.

a. $4 + 6 = 10$ True

b. $8 - 3 = 4$ False

c. $x + 9 = 21$ Neither, we do not know what number x represents

Solution of an Equation

Any replacement for the variable that makes an equation true is called a **solution** of the equation. To solve an equation means to find *all* of its solutions.

Example

Determine whether 8 is a solution of $x + 12 = 21$.

Solution

$x + 12 = 21$	Writing the equation
$8 + 12 \mid 21$	Substituting 8 for x
$20 \neq 21$	False

Since the left side and the right side are not the same, 8 is not a solution of the equation.

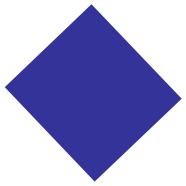
Equivalent Equations

Equations with the same solutions are called **equivalent equations**.

The Addition Principle For Equations

For any real numbers a, b, and c,

$a = b$ is equivalent to $a + c = b + c$.



Example

Solve: $x + 6 = -9$.

Solution

$$x + 6 = -9$$

$$x + 6 + (-6) = -9 + (-6)$$

Using the addition principle: adding –6 to both sides or subtracting 6 on both sides

$$x + 0 = -15$$

Simplifying

$$x = -15$$

Identity property of 0

Check

$$\begin{array}{r|l} x + 6 = & -9 \\ \hline -15 + 6 = & -9 \\ -9 & \text{True} \end{array}$$

Example

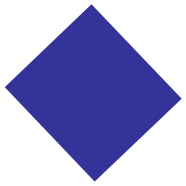
Solve: $x + 8 = 15$.

Solution

$$x + 8 = 15$$
$$x + 8 - 8 = 15 - 8$$
$$x + 0 = 7$$
$$x = 7$$

Check

$$x + 8 = 15$$
$$7 + 8 \mid 15$$
$$15 \qquad \text{True}$$



Example

Solve: $-8.3 = y - 17.9$

$-8.3 + 17.9 = y - 17.9 + 17.9$

$9.6 = y$

Check: $-8.3 = y - 17.9$

$-8.3 \mid 9.6 - 17.9$

$-8.3 = -8.3$ True

The solution is 9.6.

Example

Solve: $-\frac{1}{5}+x=\frac{7}{10}$

$$-\frac{1}{5}+x=\frac{7}{10}$$

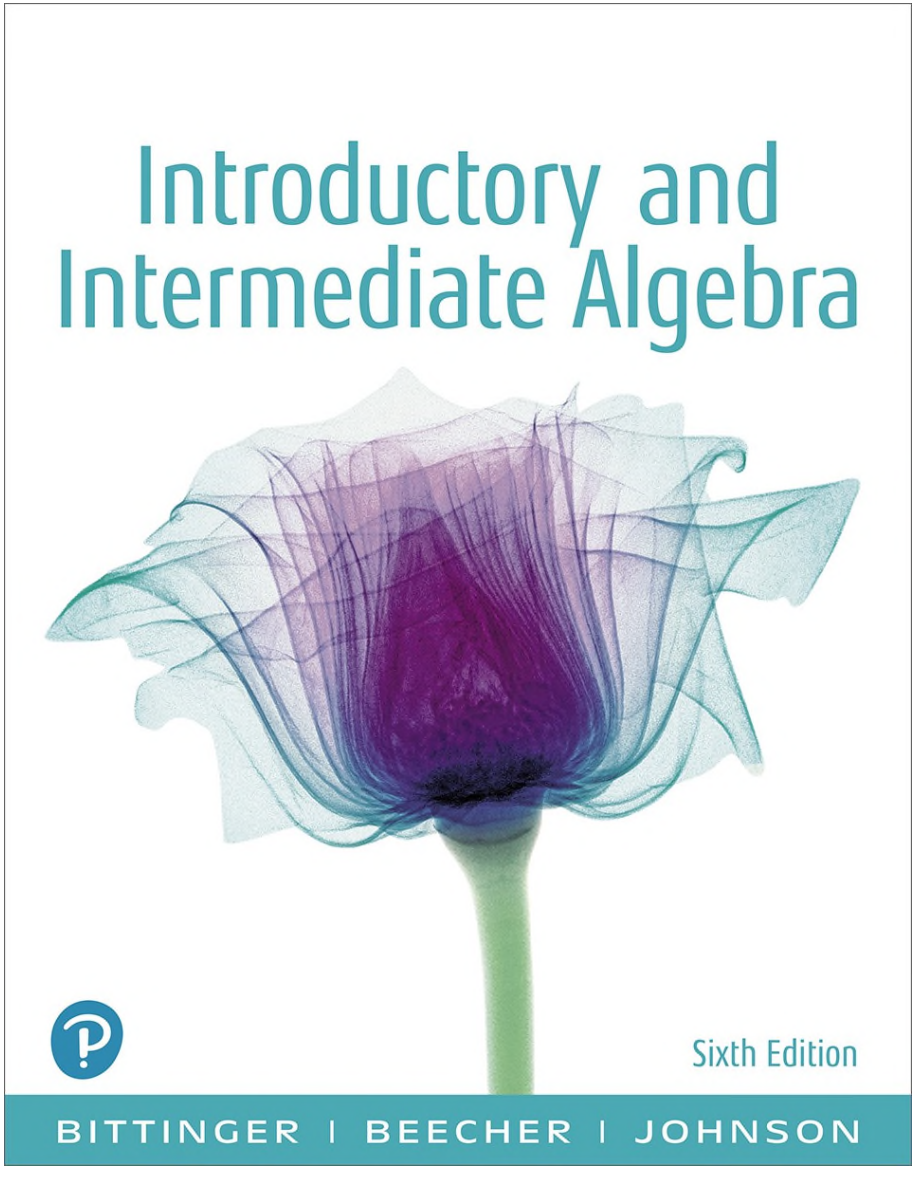
$$\frac{1}{5}-\frac{1}{5}+x=\frac{1}{5}+\frac{7}{10}$$

$$x=\frac{1}{5}\cdot\frac{2}{2}+\frac{7}{10}$$

$$x=\frac{2}{10}+\frac{7}{10}=\frac{9}{10}$$

Chapter 2

Solving Equations and Inequalities



2.2 SOLVING EQUATIONS: THE MULTIPLICATION PRINCIPLE

a. Solve equations using the multiplication principle.

The Multiplication Principle For Equations

For any real numbers a, b, and c with $c \neq 0$,

$a = b$ is equivalent to $a \cdot c = b \cdot c$.

Example

Solve $6x = 96$.

Solution

$$\frac{6x}{6} = \frac{96}{6}$$

$$x = 16$$

Dividing by 6 on both sides

Check:

$$6x = 96$$

$$6(16) = 96$$

$$96 \quad \text{TRUE}$$

The solution is 16.

Example

Solve: $-7x = 84$

$$\frac{-7x}{7} = \frac{84}{-7}$$

Dividing both sides by –7.

$$1 \cdot x = -12$$

$$x = -12$$

Check:

$$-7x = 84$$

$$-7(-12) \mid 84$$

$$84 \mid \text{True}$$

The solution is –12.

Example

Solve $-x = 6$.

Solution

$$-x = 6$$

$$(-1)(-x) = (-1)6$$ Multiplying by –1 on both sides

$$-1(-1)x = -6$$

$$x = -6$$

The solution is –6.

Example

Solve: $\frac{3}{4}x = 15$

Solution

$$\frac{3}{4}x = 15$$

$$\frac{4}{3} \cdot \frac{3}{4}x = 15 \cdot \frac{4}{3}$$

Multiplying by the reciprocal of ¾ on both sides.

$$1x = 20$$

$$x = 20$$

Example

Solve: $3.2y = 9600$

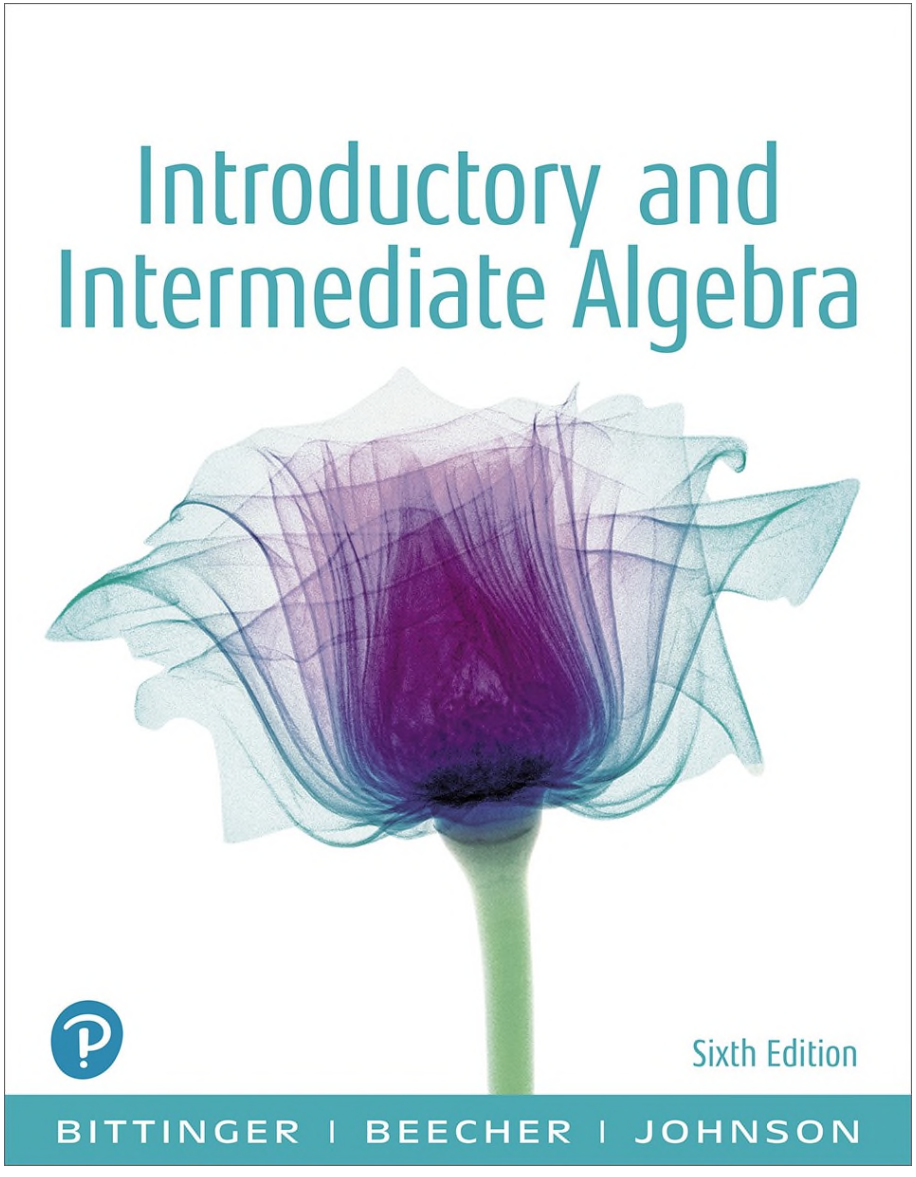
$$3.2y = 9600$$

$$\frac{3.2y}{3.2} = \frac{9600}{3.2}$$

$$y = 3000$$

Chapter 2

Solving Equations and Inequalities



2.3 USING THE PRINCIPLES TOGETHER

a. Solve equations using both the addition principle and the multiplication principle.

b. Solve equations in which like terms may need to be collected.

c. Solve equations by first removing parentheses and collecting like terms; solve equations with an infinite number of solutions and equations with no solutions.

Example

Solve: $9 + 8x = 33$

$9 + 8x - 9 = 33 - 9$ Subtracting 9 from both sides

$9 + (-9) + 8x = 24$

$8x = 24$

$\frac{8x}{8} = \frac{24}{8}$ Dividing both sides by 8

$x = 3$

Check:

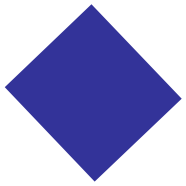
$9 + 8x = 33$

$9 + 8(3) \mid 33$

$9 + 24 \mid$

33 TRUE

The solution is 3.



Example

Solve: $35 - x = 27$.

Solution

$$35 - x = 27$$

$$-35 + 35 - x = -35 + 27 \quad \text{Adding } -35 \text{ on both sides}$$

$$-x = -8$$

$$-1(-x) = -1(-8) \quad \text{Multiplying by } -1 \text{ on both sides}$$

$$x = 8$$

Collecting Like Terms

If there are like terms on one side of the equation, we collect them before using the addition principle or the multiplication principle.

If there are like terms on opposite sides of the equation, we get them on the same side by using the addition principle. Then we collect them. In other words, we get all the terms with a variable on one side of the equation and all the terms without a variable on the other side.

Example

Solve: $5x + 4x = 36$

Collecting like terms

$9x = 36$

Dividing by 9 on both sides

$$\frac{9x}{9} = \frac{36}{9}$$

$x = 4$

Check:

$5x + 4x = 36$

$5(4) + 4(4) \mid 36$

$20 + 16 \mid 36$

36

The solution is 4.

Example

Solve: $4x + 7 - 6x = 10 + 3x + 12$

$-2x + 7 = 22 + 3x$ Collecting like terms

$-2x + 7 - 7 = 22 + 3x - 7$ Subtracting 7 from both sides

$-2x = 15 + 3x$ Simplifying

$-2x - 3x = 15 + 3x - 3x$ Subtracting $3x$ from both sides

$-5x = 15$

$\frac{-5x}{-5} = \frac{15}{-5}$ Dividing both sides by –5

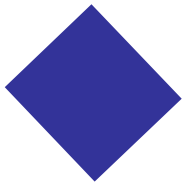
$x = -3$

Clearing Fractions and Decimals

In general, equations are easier to solve if they do not contain fractions or decimals.

The easiest way to clear an equation of fractions is to multiply *every term on both sides* by the **least common multiple of all the denominators**.

To clear an equation of decimals, we count the greatest number of decimals places in any one number and multiply on both sides by that multiple of 10.



Example

Solve $18.4 - 6.2y = 7.24$

Solution

$$18.4 - 6.2y = 7.24$$

$100(18.4 - 6.2y) = 100(7.24)$ Multiplying by 100

$(100)(18.4) - 100(6.2y) = 100(7.24)$ Using the distributive law

$1840 - 620y = 724$ Simplifying

$1840 - 620y - 1840 = 724 - 1840$ Subtracting 1840

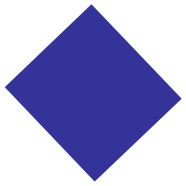
$\frac{-620y}{-620} = \frac{-1116}{-620}$ Collecting like terms

Dividing by – 620

$$y = 1.8$$

Equations Containing Parentheses

To solve certain kinds of equations that contain parentheses, we first use the distributive laws to remove the parentheses. Then we proceed as before.



Example

Solve: $9x = 3(15 - 2x)$

Solution

$$9x = 3(15 - 2x)$$

$$9x = 45 - 6x$$

$$9x + 6x = 45 - 6x + 6x$$

$$\frac{15x}{15} = \frac{45}{15}$$

$$x = 3$$

An Equation-Solving Procedure

1. Multiply on both sides to clear the equation of fractions or decimals. (This is optional, but it can ease computations.)
2. If parentheses occur, multiply to remove them using the *distributive laws*.
3. Collect like terms on each side, if necessary.
4. Get all terms with variables on one side and all numbers (constant terms) on the other side, using the *addition principle*.
5. Collect like terms again, if necessary.
6. Multiply or divide to solve for the variable, using the *multiplication principle*.
7. Check all possible solutions in the original equation.

Example

Solve: $3 - 8(x + 6) = 4(x - 1) - 5.$

$$3 - 8x - 48 = 4x - 4 - 5$$

Using the distributive law to multiply and remove parentheses

$$-45 - 8x = 4x - 9$$

Collecting like terms

$$-8x - 45 + 45 = 4x - 9 + 45$$

Adding 45

$$-8x = 4x + 36$$

Collecting like terms

$$-8x - 4x = 4x + 36 - 4x$$

Subtracting $4x$

$$\frac{-12x}{-12} = \frac{36}{-12}$$

Dividing by -12

$$x = -3$$

Example

Solve: $8x - 18 = 6 + 8(x - 3)$

Solution

$$8x - 18 = 6 + 8(x - 3)$$

Using the distributive law to multiply and remove parentheses

$$8x - 18 = 6 + 8x - 24$$

Collecting like terms

$$8x - 18 = 8x - 18$$

$$8x - 8x - 18 = 8x - 8x - 18$$

Adding $-8x$

$$-18 = -18$$

True for all real numbers.

Every real number is a solution. There are infinitely many solutions.

Example

Solve: $3 - 4(x + 6) = -4(x - 1) - 3$.

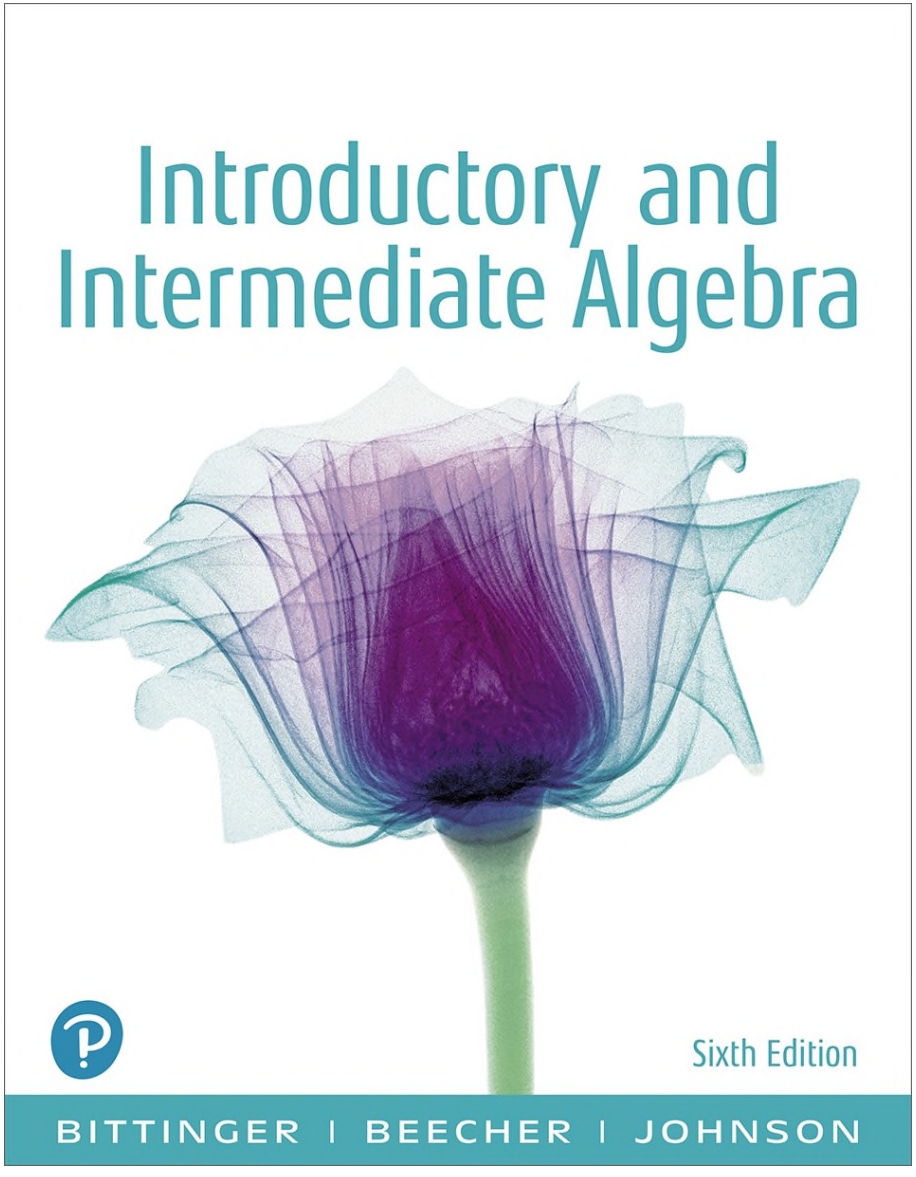
Solution

$$3 - 4(x + 6) = -4(x - 1) - 3$$
$$3 - 4x - 24 = -4x + 4 - 3$$
$$-4x - 21 = -4x + 1$$
$$-4x + 4x - 21 = -4x + 4x + 1$$
$$-21 = 1 \qquad \text{FALSE}$$

There are no solutions.

Chapter 2

Solving Equations and Inequalities



2.4 FORMULAS

a. Evaluate a formula.

b. Solve a formula for a specified variable.

A **formula** is a “recipe” for doing a certain type of calculation. Formulas are often given as equations. When we replace the variables in an equation with numbers and calculate the result, we are **evaluating** the formula.

Example

The formula $M = \frac{1}{5}t$ can be used to determine how far *M*, in miles, you are from lightening when its thunder takes *t* seconds to reach your ears. If it takes 5 seconds for the sound of thunder to reach you after you have seen the lightening, how far away is the storm?

Solution We substitute 5 for *t* and calculate *M*.

$$M = \frac{1}{5}t = \frac{1}{5}(5) = 1$$

The storm is 1 mile away.

Example

The cost C of operating a microwave oven for 1 year is given by the formula

$$C = \frac{W \times h \times 365}{1000} \cdot k$$

where W = the wattage, h = the number of hours used per day, and k = the energy cost per kilowatt-hour. Find the cost of operating a 1200-W microwave oven for 0.25 hr per day if the energy cost is \$0.12 per kilowatt-hour.

Example continued

$$C = \frac{W \times h \times 365}{1000} \cdot k$$

1200-W

0.25 hr per day

cost is \$0.12 per kilowatt-hour

$$C = \frac{1200 \times 0.25 \times 365}{1000} \cdot 0.12 \approx \$13.14$$

Example

Solve for r: $d = rt$.

Solution

$$d = rt$$

We want this letter alone.

$$\frac{d}{t} = \frac{rt}{t}$$

Dividing by t

$$\frac{d}{t} = r \cdot \frac{t}{t}$$

$$\frac{d}{t} = r$$

Simplifying

Example

Solve for x: $y = x + b$.

Solution

$$y = x + b$$ We want this letter alone.

$$y - b = x + b - b$$ Subtracting b

$$y - b = x$$ Simplifying

Example

Solve for y: $wy + x = z$

Solution

$wy + x = z$ We want this letter alone.

$wy + x - x = z - x$ Subtracting x

$wy = z - x$ Simplifying

$\frac{wy}{w} = \frac{z - x}{w}$ Dividing by w

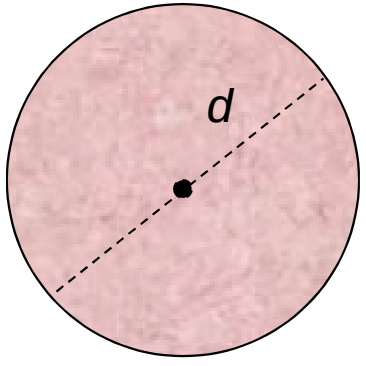
$y = \frac{z - x}{w}$ Simplifying

A Formula-Solving Procedure

1. Multiply on both sides to clear fractions or decimals, if that is needed.
2. Collect like terms on each side, if necessary.
3. Get all terms with the letter to be solved for on one side of the equation and all other terms on the other side.
4. Collect like terms again, if necessary.
5. Solve for the letter in question.

Example

The formula $C = \pi d$ gives the circumference C of a circle with diameter d. Solve for d.



Solution

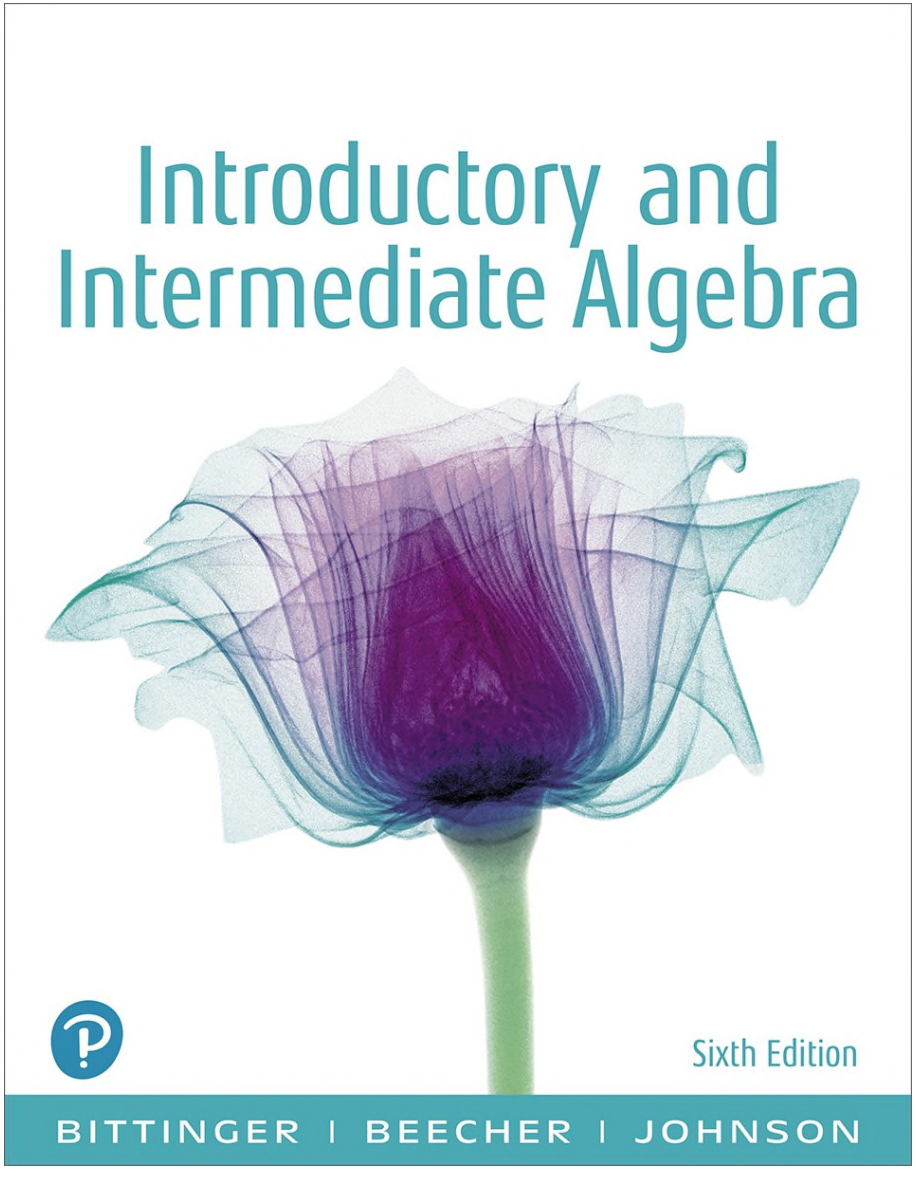
$$C = \pi d$$

$$\frac{C}{\pi} = \frac{\pi d}{\pi}$$

$$\frac{C}{\pi} = d$$

Chapter 2

Solving Equations and Inequalities



2.5 APPLICATIONS OF PERCENT

a. Solve applied problems involving percent.

Key Words in Percent Translations

"**Of**" translates to "•" or "×".

"**Is**" translates to "=".

"**What number**" or "**what percent**" translates to any letter.

% translates to "$\times \frac{1}{100}$" or "$\times 0.01$".

To solve a problem involving percents, it is helpful to translate first to an equation.
For example, “23% of 5 is what?”

23%	of	5	is	what?
↓	↓	↓	↓	↓
0.23	·	5	=	a

Note how the key words are translated.

Example

Translate: What is 19% of 82?

Solution

What	is	19%	of	82?
↓	↓	↓	↓	↓
a	=	0.19	·	82

Example

Translate: 46% of 12 is what number?

Solution

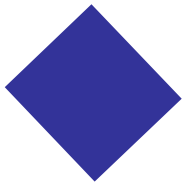
46%	of	12	is	what number?
↓	↓	↓	↓	↓
46%	·	12	=	w

Example

Translate: 7 is 10% of what?

Solution

7	is	10%	of	what?
↓	↓	↓	↓	↓
7	=	0.10	·	b



Example

Translate: 18 is what percent of 38?

Solution

18	is	what percent	of	38
↓	↓	↓	↓	↓
18	=	p	·	38

3 Types of Percent Problems

1. **Finding the *amount*** (the result of taking the percent)

Example: What is 25% of 60?
Translation: $a = 0.25 \cdot 60$

2. **Finding the *base*** (the number you are taking the percent of)

Example: 15 is 25% of what number?
Translation: $15 = 0.25 \cdot b$

3. **Finding the *percent number*** (the percent itself)

Example: 15 is what percent of 60?
Translation: $15 = p \cdot 60$

Example

What is 8% of 34?

Solution

Translate: $a = 0.08 \cdot 34$

Solve: The variable is by itself. To solve the equation, we just multiply.

$$a = 0.08(34)$$

$$\begin{array}{r} 34 \\ \times\ 0.08 \\ \hline 2.72 \end{array}$$

$$a = 2.72$$

Thus, 2.72 is 8% of 34. The answer is 2.72.

Example

15 is 16% of what?

Solution

Translate: 15 is 16% of what?

$$15 = 0.16 \cdot b$$

Solve: To solve we divide both sides of the equation by 0.16:

$$15 = 0.16b$$

$$\frac{15}{0.16} = \frac{0.16b}{0.16}$$

$$93.75 = b$$

$$0.16\overline{)15.00.00} = 93.75$$

144

60

48

120

112

80

80

0

Example

27 is what percent of 36?

Solution

Translate: 27 is what percent of 36?

$$27 = p \cdot 36$$

Solve: To solve we divide both sides by 36 and convert the answer to percent notation:

$$27 = p \cdot 36$$

$$\frac{27}{36} = \frac{27p}{36}$$

$$0.75 = p$$

$$p = 75\%$$

$$\begin{array}{r} .75 \\ 36\overline{)27.00} \\ \underline{252} \\ 180 \\ \underline{180} \\ 0 \end{array}$$

Example

To complete her water safety course instruction, Catherine must complete 45 hours of instruction. If she has completed 75% of her requirement, how many hours has Catherine completed?

Solution

Rewording: What is 75% of 45?

Translating: $a = 0.75 \times 45$

$a = 33.75$

Catherine has completed 33.75 hours of instruction.

Example

A family pays a monthly electric bill of $172.00. With careful monitoring they can reduce their bill to $163.40. What is the percent of decrease?

Solution We find the amount of decrease.

172.00	Original bill
− 163.40	New bill
8.60	Decrease

We rephrase and translate.

8.60 is what percent of 172.00?

$$8.60 = p \cdot 172.00$$

Example continued

Solve. We divide both sides by 172

$$\frac{8.60}{172} = \frac{172p}{172}$$

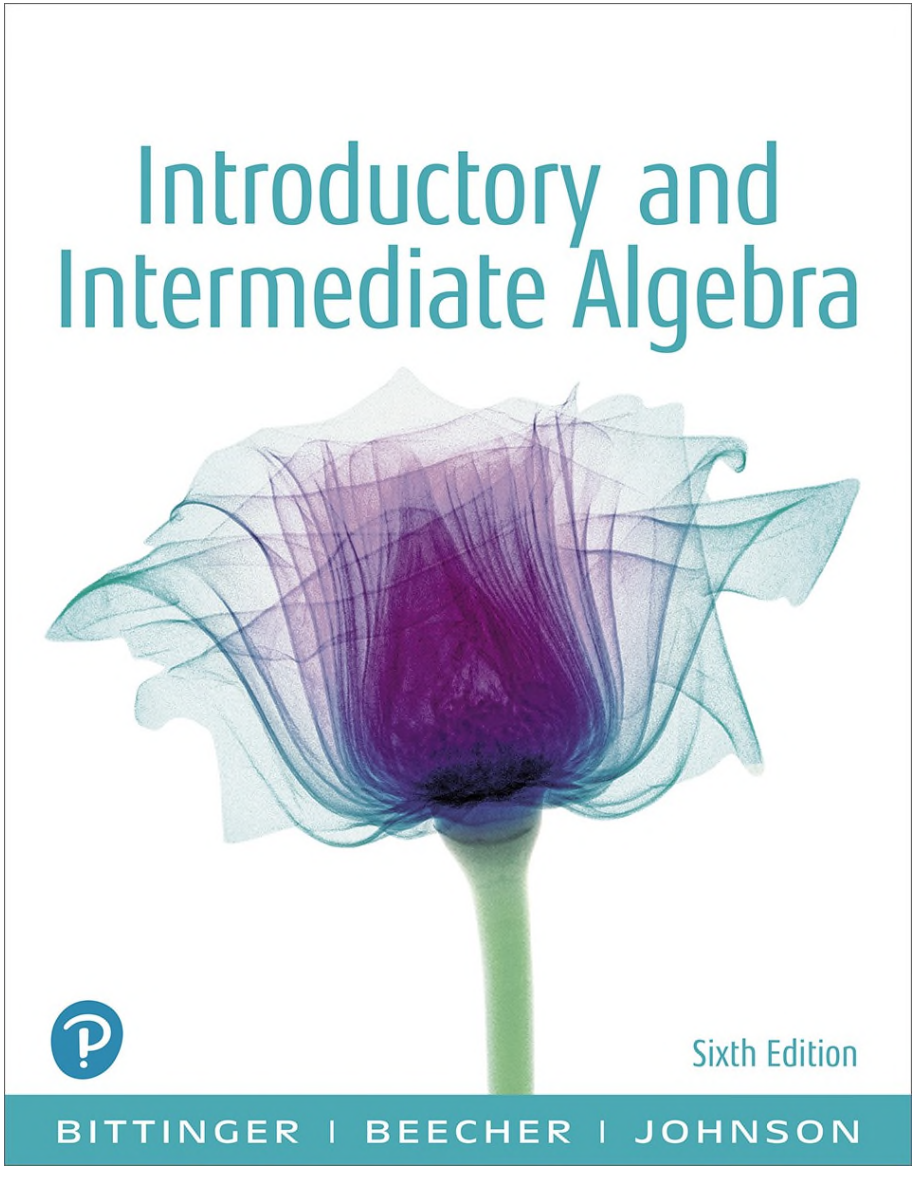
$$0.05 = p$$

$$5\% = p$$

The percent of decrease of the electric bill is 5%.

Chapter 2

Solving Equations and Inequalities



2.6 APPLICATIONS AND PROBLEM SOLVING

a. Solve applied problems by translating to equations.

Five Steps for Problem Solving in Algebra

1. *Familiarize* yourself with the problem situation.
2. *Translate* the problem to an equation.
3. *Solve* the equation.
4. *Check* the answer in the original problem.
5. *State* the answer to the problem clearly.

To Familiarize Yourself with a Problem

- If a problem is given in words, read it carefully. Reread the problem, perhaps aloud. Try to verbalize the problem as if you were explaining it to someone else.
- Choose a variable (or variables) to represent the unknown(s) and clearly state what the variable represents. Be descriptive! For example, let L = length in centimeters, d = distance in miles, and so on.

To Familiarize Yourself with a Problem

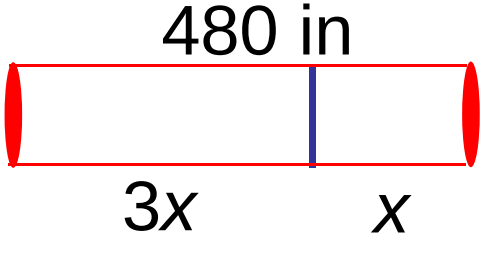
- Make a drawing and label it with known information, using specific units if given. Also, indicate the unknown information.
- Find further information. Look up formulas or definitions with which you are not familiar. Consult the Internet or a reference librarian.
- Create a table that lists all the information you have available. Look for patterns that may help in the translation to an equation.
- Think of a possible answer and check the guess. Note the manner in which the guess is checked.

Example

A 480-in. piece of pipe is cut into two pieces. One piece is three times the length of the other. Find the length of each piece of pipe.

Solution

1. **Familiarize**. Make a drawing. Noting the lengths. Let x = the length of the shorter piece. Then $3x$ = the length of the longer piece.



Example continued

2. **Translate**. From the statement of the problem and drawing, we know the lengths add up to 480 inches.

$$x + 3x = 480$$

3. **Solve**.

$$x + 3x = 480$$

$$\frac{4x}{4} = \frac{480}{4}$$

$$x = 120 \text{ inches}$$

Example continued

4. **Check**. Do we have an answer to the problem?

   No, we need the lengths of both pieces of pipe.

   If $x = 120$ the length of the shorter piece

   $3x$ = the length of the longer piece.

   $3(120) = 360$ inches

   Since $120 + 360 = 480$ our answer checks.

5. **State.** One section of pipe is 120 inches and the other section is 360 inches.

Example

Digicon prints digital photos for $0.12 each plus $3.29 shipping and handling. Your weekly budget for the school yearbook is $22.00. How many prints can you have made if you have $22.00?

Solution

1. **Familarize**. Suppose the yearbook staff takes 220 digital photos. Then the cost to print them would be the shipping charge plus $0.12 times 220. $3.29 + $0.12(220) which is $29.69. Our guess of 220 is too large, but we have familiarized ourselves with the way in which the calculation is made.

Example continued

2. **Translate**.

	Shipping	plus	photo cost	is	$22
Rewording:	↓	↓	↓	↓	↓
Translating:	$3.29	+	$0.12(x)$	=	22

3. **Carry out.** $3.29 + 0.12x = 22$

$$0.12x = 18.71$$

$$x = 155.9 = 155$$

Example continued

4. **Check**. Check in the original problem.
   $3.29 + 155(0.12) = $21.89, which is less than $22.00.

5. **State.** The yearbook staff can have 155 photos printed per week.

Consecutive integers:

16, 17, 18, 19, 20; and so on

x, $x + 1$, $x + 2$; and so on.

Consecutive even integers:

16, 18, 20, 22, and so on

x, $x + 2$, $x + 4$, and so on.

Consecutive odd integers:

21, 23, 25, 27, 29, and so on

x, $x + 2$, $x + 4$, and so on.

Example

The apartments in Wanda's apartment house are consecutively numbered on each floor. The sum of her number and her next door neighbor's number is 723. What are the two numbers?

Solution

1. **Familiarize.** The apartment numbers are consecutive integers.

   Let x = Wanda's apartment

   Let $x + 1$ = neighbor's apartment

Example continued

2. **Translate.**

	First		plus		second			
Rewording:	integer				integer		is	723
	↓		↓		↓		↓	↓
Translating:	x		$+$		$(x+1)$		$=$	723

3. **Carry out.** $x + (x + 1) = 723$

$$2x + 1 = 723$$

$$2x = 722$$

$$x = 361$$

If x is 361, then $x + 1$ is 362.

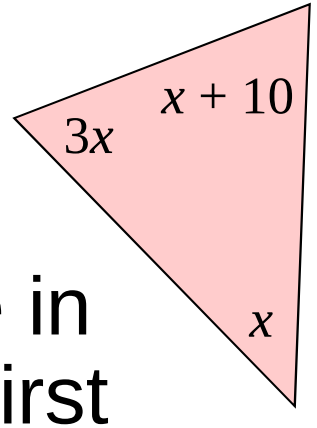
Example continued

4. **Check**. Our possible answers are 361 and 362. These are consecutive integers and the sum is 723.

5. **State.** The apartment numbers are 361 and 362.

Example

You are constructing a triangular kite. The second angle of the kite is three times as large as the first. The third angle is 10 degrees more than the first. Find the measure of each angle.



Solution

1. **Familiarize**. Make a drawing and write in the given information. Let x = measure of first angle, $3x$ = second, $x + 10$ = third.

2. **Translate**. To translate, we need to recall that the sum of the measures of the angles in a triangle is 180 degrees.

Example continued

2. **Translate**.

Measure of first angle	+	measure of second angle	+	measure of third angle	is	180°
↓	↓	↓	↓	↓	↓	↓
x	+	$3x$	+	$(x+10)$	=	180

3. **Solve** $x + 3x + (x + 10) = 180$

$$5x + 10 = 180$$

$$5x = 170$$

$$x = 34$$

Example continued

The measures for the angles appear to be:

first angle: $x = 34$

second angle: $3x = 3(34) = 102$;

third angle: $x + 10 = 34 + 10 = 44$

4. **Check.** Consider 34, 102 and 44 degrees. The sum of these numbers is 180 degrees and the second angle is three times the first angle. The third angle is 10 degrees more than the first. These numbers check.

5. **State.** The measures of the angles are 34, 44 and 102 degrees.

Example

The Hicksons are planning to sell their home. If they want to be left with $142,00 after paying 6% of the selling price to a realtor as a commission, for how much must they sell the house?

Familiarize. Suppose the Hicksons sell the house for $150,000. A 6% commission can be determined by finding 6% of 150,000.

$$6\% \text{ of } 150{,}000 = 0.06(150{,}000) = 9000.$$

Subtracting this commission from $150,000 would leave the Hicksons with 150,000 – 9,000 = 141,000.

Example continued

Translate. Reword the problem and translate.

Selling price less Commission is Amount remaining

$$x \quad - \quad 0.06x \quad = \quad 142{,}000$$

Solve.

$$x - 0.06x = 142{,}000$$

$$1x - 0.06x = 142{,}000$$

$$\frac{0.94x}{0.94} = \frac{142{,}000}{0.94}$$

$$x = 151{,}063.83$$

Example continued

Check.

Find 6% of 151,063.83:

6% of 151,063.83 = (0.06)(151,063.83) = 9063.83

Subtract the commission to find the remaining amount.

151,063.83 – 9063.83 = 142,000.

State.

To be left with $142,000 the Hicksons must sell the house for $151,063.83.

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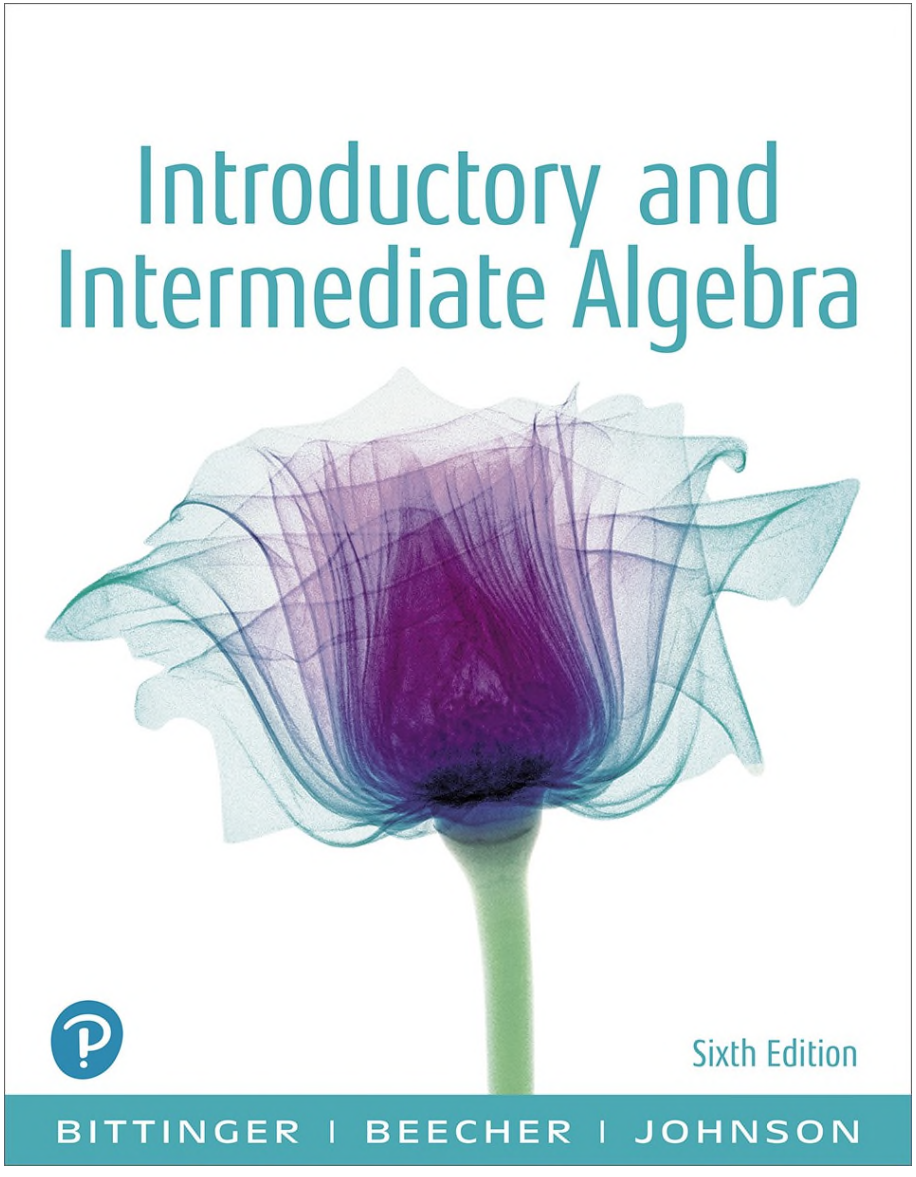
Solving Equations and Inequalities



2.7 SOLVING INEQUALITIES

a. Determine whether a given number is a solution of an inequality.

b. Graph an inequality on a number line.

c. Solve inequalities using the addition principle.

d. Solve inequalities using the multiplication principle.

e. Solve inequalities using the addition and multiplication principles together.

Solutions of Inequalities

An **inequality** is a number sentence with > (is greater than), < (is less than), ≥ (is greater than or equal to), or ≤ (is less than or equal to).

Solution of an Inequality

A replacement that makes an inequality true is called a **solution**. The set of all solutions is called the **solution set**. When we have found the set of all solutions of an inequality, we say that we have **solved** the inequality.

Example

Determine whether the given number is a solution of $x < 5$: a) -4 b) 6

Solution

a) Since $-4 < 5$ is true, -4 is a solution.

b) Since $6 < 5$ is false, 6 is not a solution.

Graphs of Inequalities

Because solutions of inequalities like $x < 4$ are too numerous to list, it is helpful to make a drawing that represents all the solutions.

A **graph** of an inequality is a drawing that represents its solutions. Graphs of inequalities in one variable can be drawn on a number line by shading all the points that are solutions.

Example

Graph each inequality:

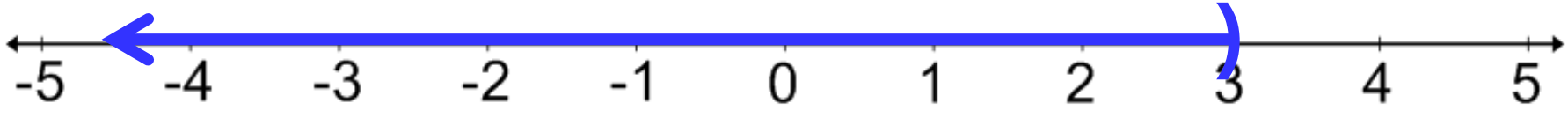
a) $x < 3$, b) $y \geq -4$; c) $-3 < x \leq 5$

Solution

a) The solutions of $x < 3$ are those numbers less than 3.

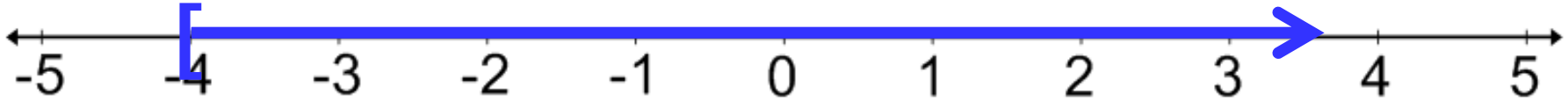
Shade all points to the left of 3.

The parenthesis at 3 and the shading to the left indicate that 3 is *not* part of the graph, but numbers like 1 and –2 are.

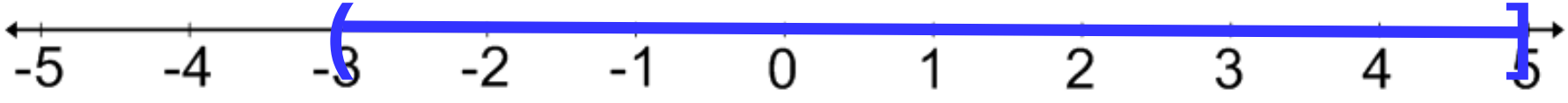


Example continued

b) The solutions of $y \geq -4$ are shown on the number line by shading the point for –4 and all points to the right of –4. The bracket at –4 indicates that –4 is part of the graph.



c) The inequality $-3 < x \leq 5$ is read "–3 is less than *x and x* is less than or equal to 5."



The Addition Principle for Inequalities

For any real numbers a, b, and c:

$a < b$ is equivalent to $a + c < b + c$;

$a \leq b$ is equivalent to $a + c \leq b + c$;

$a > b$ is equivalent to $a + c > b + c$;

$a \geq b$ is equivalent to $a + c \geq b + c$;

When we add or subtract the same number on both sides of an inequality, the direction of the inequality symbol is not changed.

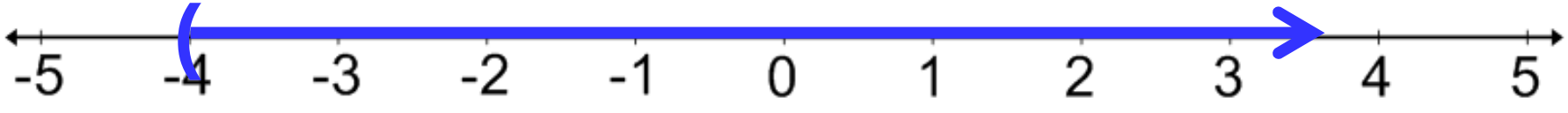
Example

Solve: $x + 6 > 2$ Then graph.

Solution

$$x + 6 > 2$$
$$x + 6 - 6 > 2 - 6$$
$$x > -4$$

Any number greater than –4 makes the statement true.



Example

Solve $4x - 1 \leq 3x - 4$ and then graph the solution.

Solution

$$4x - 1 \leq 3x - 4$$

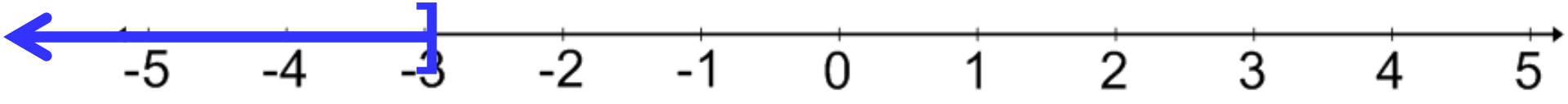
$4x - 1 + 1 \leq 3x - 4 + 1$ Adding 1 to both sides

$4x \leq 3x - 3$ Simplifying

$4x - 3x \leq 3x - 3x - 3$ Subtracting $3x$ from both sides

$x \leq -3$ Simplifying

The solution set is $\{x|x \leq -3\}$.



The Multiplication Principle for Inequalities

For any real numbers a and b, and for any *positive* number c:

$a < b$ is equivalent to $ac < bc$, and

$a > b$ is equivalent to $ac > bc$.

For any real numbers a and b, and for any *negative* number c:

$a < b$ is equivalent to $ac > bc$, and

$a > b$ is equivalent to $ac < bc$.

Similar statements hold for $\leq$ and $\geq$.

The Multiplication Principle for Inequalities (continued)

When we multiply or divide by a positive number on both sides of an inequality, the direction of the inequality symbol stays the same. When we multiply or divide by a negative number on both sides of an inequality, the direction of the inequality symbol is reversed.

Example

Solve and graph each inequality:

a) $7x \leq 35$ b) $-4y < 20$

Solution

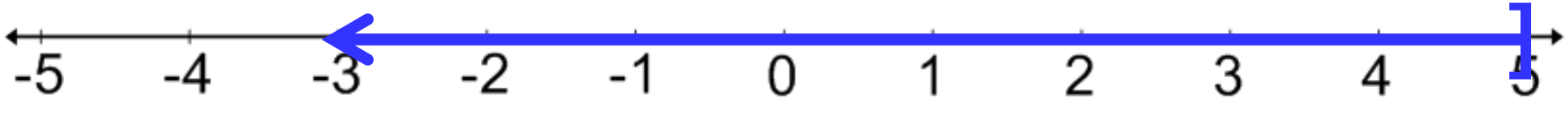
a) $7x \leq 35$

$$\frac{7x}{7} \leq \frac{35}{7}$$

Dividing by 7

The symbol stays the same.

$x \leq 5$ Simplifying



Example continued

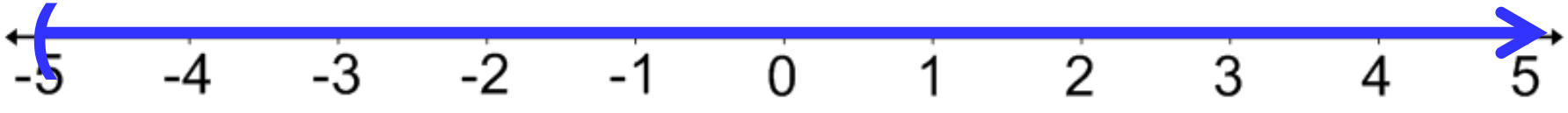
b) $-4y < 20$

$$\frac{-4y}{-4} > \frac{20}{-4}$$ Dividing both sides by -4

The symbol must be reversed!

$$y > -5$$

The solution set is $\{y|y > -5\}$. The graph is shown below.

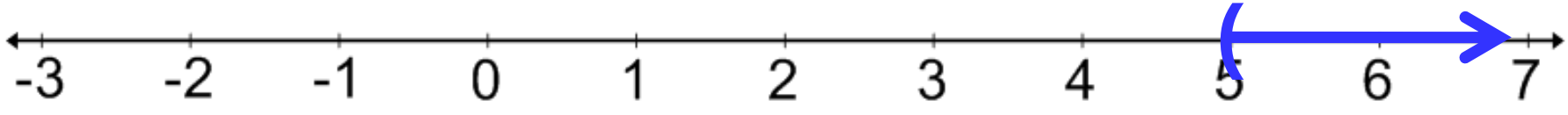


Example

Solve: $3x - 3 > x + 7$

$3x - 3 + 3 > x + 7 + 3$	Adding 3 to both sides
$3x > x + 10$	Simplifying
$3x - x > x - x + 10$	Subtracting x from both sides
$2x > 10$	Simplifying
$\frac{2x}{2} > \frac{10}{2}$	Dividing both sides by 2
$x > 5$	Simplifying

The solution set is $\{x|x > 5\}$.

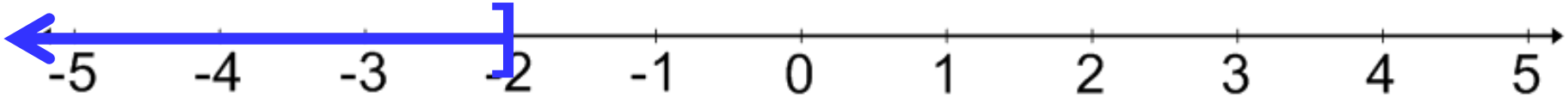


Example

Solve: $5(x - 3) - 7x \geq 4(x - 3) + 9$

$5x - 15 - 7x \geq 4x - 12 + 9$	Using the distributive law to remove parentheses
$-2x - 15 \geq 4x - 3$	Simplifying
$-2x - 15 + 3 \geq 4x - 3 + 3$	Adding 3 to both sides
$-2x - 12 \geq 4x$	Simplifying
$-2x + 2x - 12 \geq 4x + 2x$	Adding $2x$ to both sides
$-12 \geq 6x$	Dividing both sides by 6
$-2 \geq x$	

The solution set is $\{x|x \leq -2\}$.



Example

Solve: $15.4 - 3.2x < -6.76$

$$100(15.4 - 3.2x) < 100(-6.76)$$

$$100(15.4) - 100(3.2x) < 100(-6.76)$$

$$1540 - 320x < -676$$

$$1540 - 320x - 1540 < -676 - 1540$$

$$-320x < -2216$$

$$x > \frac{-2216}{-320}$$

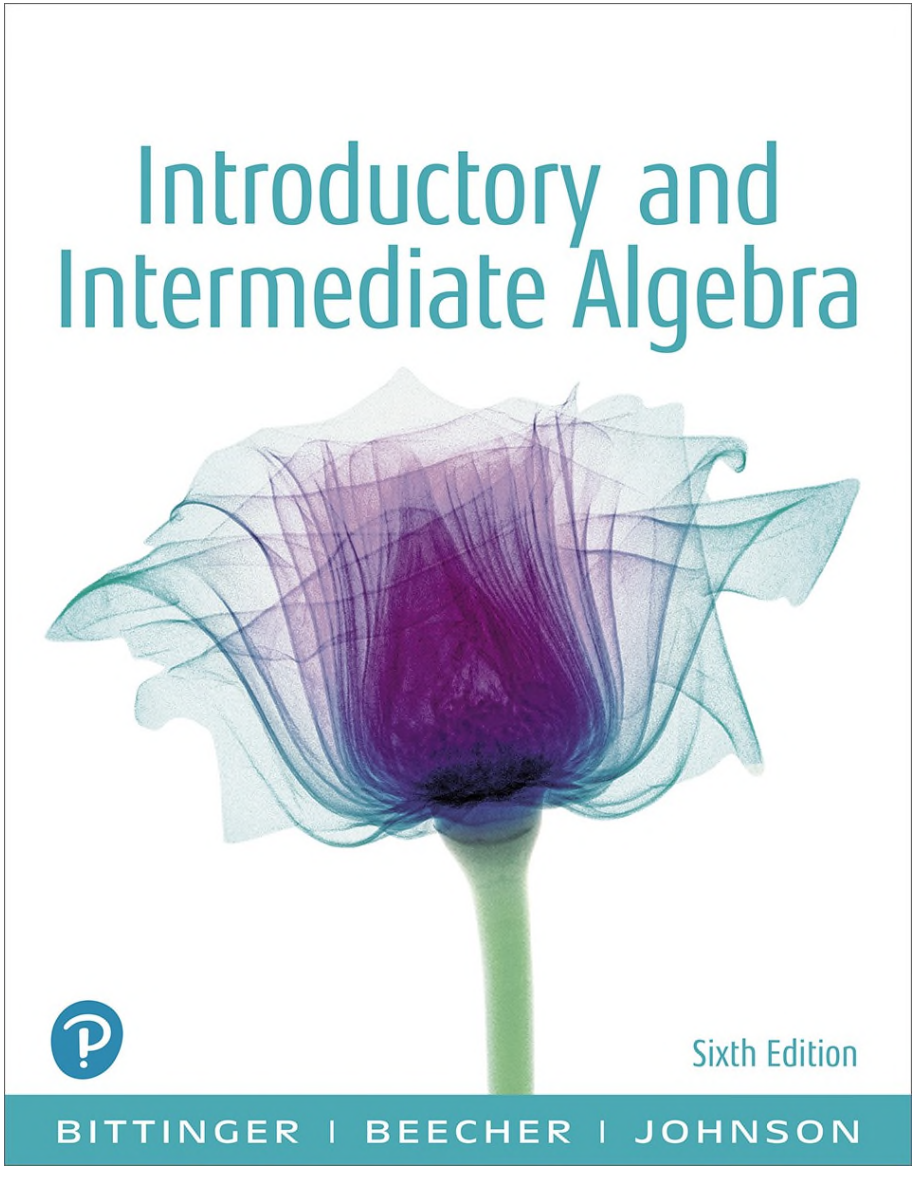
Remember to reverse the symbol!

$$x > 6.925$$

The solution set is $\{x|x > 6.925\}$.

Chapter 2

Solving Equations and Inequalities



2.8 APPLICATIONS AND PROBLEM SOLVING WITH INEQUALITIES

a. Translate number sentences to inequalities.

b. Solve applied problems using inequalities.

Important Words	Sample Sentence	Translation
is at least	Brian is at least 16 years old	$b \geq 16$
is at most	At most 3 students failed the course	$s \leq 3$
cannot exceed	To qualify, earnings cannot exceed $5000	$e \leq \$5000$
must exceed	The speed must exceed 20 mph	$s > 20$
is less than	Nicholas is less than 60 lb.	$n < 60$
is more than	Chicago is more than 300 miles away.	$c > 300$
is between	The movie is between 70 and 120 minutes.	$70 < m < 120$
no more than	The calf weighs no more than 560 lb.	$w \leq 560$
no less than	Carmon scored no less than 9.4.	$c \geq 9.4$

Translating "at least" and "at most"

The quantity x is at least some amount q: $x \geq q$.
(If x is at least q, it cannot be less than q.)

The quantity x is at most some amount q: $x \leq q$.
(If x is at most q, it cannot be more than q.)

Example

Lazer Line charges $65 plus $45 per hour for copier repair. Jonas remembers being billed less than $150. How many hours was Jonas' copier worked on?

Solution

1. **Familarize.** Suppose the copier was worked on for 4 hours. The cost would be $65 + 4($45), or $245. A bill of $150 shows that the copier was worked on for less than 4 hours. Let h = the number of hours.
2. **Translate**.

Rewording:	Initial fee	plus	hours	is	less than	150
	↓	↓	↓	↓		↓
Translating:	65	+	45h	<		150

Example continued

3. **Carry out.**

$$65 + 45h < 150$$

$$45h < 85$$

$$h < \frac{85}{45}$$

$$h < 1\frac{8}{9}$$

Example continued

4. **Check**. Since the time represents hours, we round down to the whole hour. If the copier was worked on for one hour, the cost would be $110, and if worked on for two hours the cost would exceed $150.

5. **State.** Jonas' copier was worked on for less than two hours.

Average or mean

To find the **average** or **mean** of a set of numbers, add the numbers and then divide by the number of addends.

Example

Samantha has test grades of 86, 88, and 78 on her first three math tests. If she wants an average of at least 80 after the fourth test, what possible scores can she earn on the fourth test?

Solution

1. **Familarize.** Suppose she earned an 85 on her fourth test. Her test average would be

$$\frac{86+88+78+85}{4}=84.25$$

This shows she could score an 85. Let's let x represent the fourth test score.

Example continued

2. **Translate.**

Rewording: Average test scores should be at least 80

Translating: $$\frac{86+88+78+x}{4} \geq 80$$

3. **Carry out.**

$$\frac{86+88+78+x}{4} \geq 80$$

$$4\left(\frac{86+88+78+x}{4}\right) \geq 80 \cdot 4$$

$$252 + x \geq 320$$

$$x \geq 68$$

Example continued

4. **Check.**

As a partial check, we show that Samantha can earn a 68 on the fourth test and average 80 for the four tests.

$$\frac{86+88+78+68}{4}=\frac{320}{4}=80.$$

5. **State.**

Samantha's test average will not drop below 80 if she earns at least a 68 on the fourth test.