Solutions for College Algebra Concepts Through Functions 4th Edition by Sullivan

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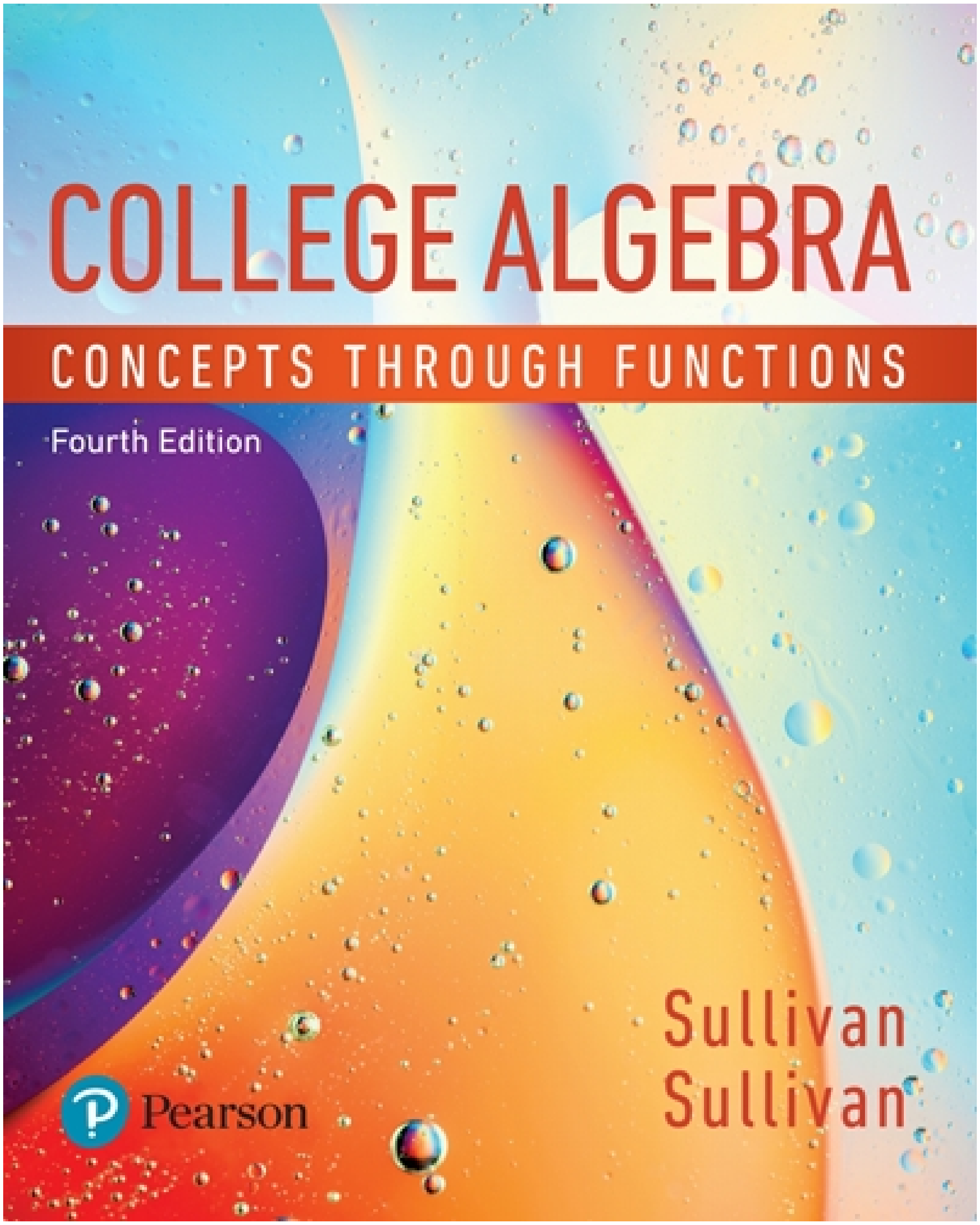
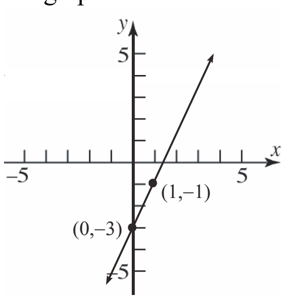


Solutions

Chapter 2
Linear and Quadratic Functions

Section 2.1

1. From the equation $y = 2x - 3$, we see that the y-intercept is -3. Thus, the point $(0, -3)$ is on the graph. We can obtain a second point by choosing a value for x and finding the corresponding value for y. Let $x = 1$, then $y = 2(1) - 3 = -1$. Thus, the point $(1, -1)$ is also on the graph. Plotting the two points and connecting with a line yields the graph below.



2. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-5}{-1-2} = \dfrac{-2}{-3} = \dfrac{2}{3}$

3. $f(2) = 3(2)^2 - 2 = 10$

$f(4) = 3(4)^2 - 2 = 46$

$\dfrac{\Delta y}{\Delta x} = \dfrac{f(4) - f(2)}{4-2} = \dfrac{46-10}{4-2} = \dfrac{36}{2} = 18$

4. $60x - 900 = -15x + 2850$

$75x - 900 = 2850$

$75x = 3750$

$x = 50$

The solution set is $\{50\}$.

5. $f(-2) = (-2)^2 - 4 = 4 - 4 = 0$

6. True

7. slope; y-intercept

8. positive

9. True

10. False. The y-intercept is 8. The average rate of change is 2 (the slope).

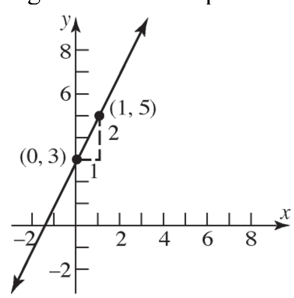
11. a

12. d

13. $f(x) = 2x + 3$

a. Slope = 2; y-intercept = 3

b. Plot the point (0, 3). Use the slope to find an additional point by moving 1 unit to the right and 2 units up.



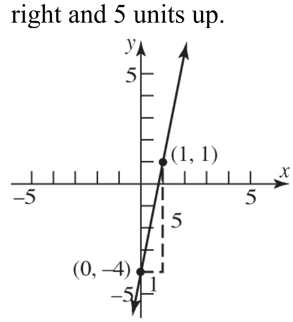
c. average rate of change = 2

d. increasing

14. $g(x) = 5x - 4$

a. Slope = 5; y-intercept = –4

b. Plot the point $(0, -4)$. Use the slope to find an additional point by moving 1 unit to the right and 5 units up.



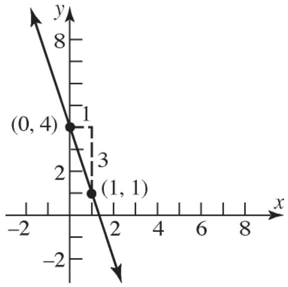
c. average rate of change = 5

d. increasing

15. $h(x) = -3x + 4$

a. Slope = -3; y-intercept = 4

b. Plot the point (0, 4). Use the slope to find an additional point by moving 1 unit to the right and 3 units down.



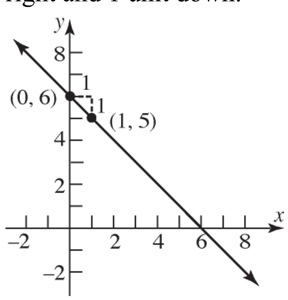
c. average rate of change = -3

d. decreasing

16. $p(x) = -x + 6$

a. Slope = -1; y-intercept = 6

b. Plot the point (0, 6). Use the slope to find an additional point by moving 1 unit to the right and 1 unit down.



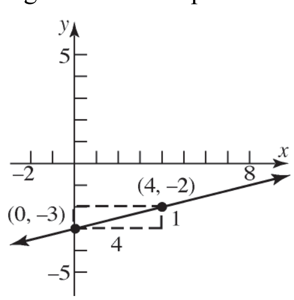
c. average rate of change = -1

d. decreasing

17. $f(x) = \frac{1}{4}x - 3$

a. Slope = $\frac{1}{4}$; y-intercept = -3

b. Plot the point $(0, -3)$. Use the slope to find an additional point by moving 4 units to the right and 1 unit up.



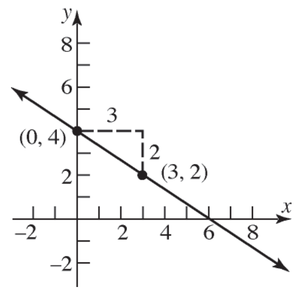
c. average rate of change = $\frac{1}{4}$

d. increasing

18. $h(x) = -\frac{2}{3}x + 4$

a. Slope = $-\frac{2}{3}$; y-intercept = 4

b. Plot the point (0, 4). Use the slope to find an additional point by moving 3 units to the right and 2 units down.



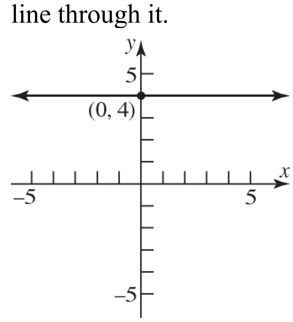
c. average rate of change = $-\frac{2}{3}$

d. decreasing

19. $F(x) = 4$

a. Slope = 0; y-intercept = 4

b. Plot the point (0, 4) and draw a horizontal line through it.



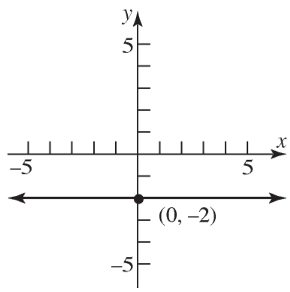
c. average rate of change = 0

d. constant

20. $G(x) = -2$

a. Slope = 0; y-intercept = -2

b. Plot the point $(0, -2)$ and draw a horizontal line through it.



c. average rate of change = 0

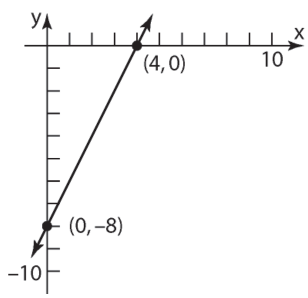
d. constant

21. $g(x) = 2x - 8$

a. zero: $0 = 2x - 8$: y-intercept = -8

$x = 4$

b. Plot the points $(4,0), (0,-8)$.

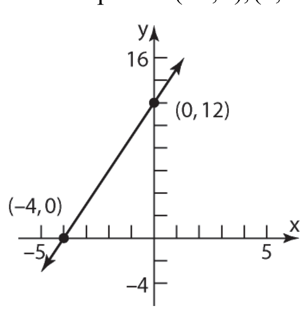


22. $g(x) = 3x + 12$

a. zero: $0 = 3x + 12$: y-intercept = 12

$x = -4$

b. Plot the points $(-4,0), (0,12)$.

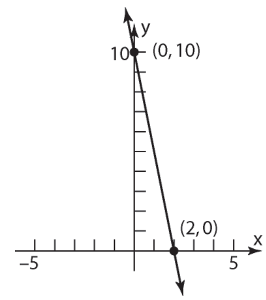


23. $f(x) = -5x + 10$

a. zero: $0 = -5x + 10$: y-intercept = 10

$x = 2$

b. Plot the points 1 unit to the right and 5 units down.

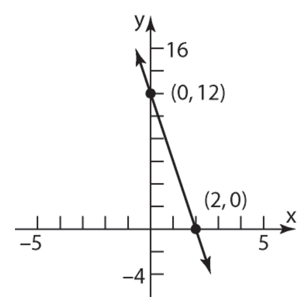


24. $f(x) = -6x + 12$

a. zero: $0 = -6x + 12$: y-intercept = 12

$x = 2$

b. Plot the points $(2,0), (0,12)$.

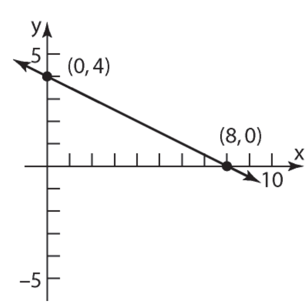


25. $H(x) = -\frac{1}{2}x + 4$

a. zero: $0 = -\frac{1}{2}x + 4$: y-intercept = 4

$x = 8$

b. Plot the points $(8,0), (0,4)$.

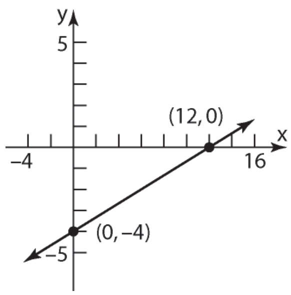


26. $G(x) = \frac{1}{3}x - 4$

a. zero: $0 = \frac{1}{3}x - 4$: y-intercept $= -4$

$x = 12$

b. Plot the points $(12, 0), (0, -4)$.



27.

x	y	Avg. rate of change $= \frac{\Delta y}{\Delta x}$
−2	4	
−1	1	$\frac{1-4}{-1-(-2)} = \frac{-3}{1} = -3$
0	−2	$\frac{-2-1}{0-(-1)} = \frac{-3}{1} = -3$
1	−5	$\frac{-5-(-2)}{1-0} = \frac{-3}{1} = -3$
2	−8	$\frac{-8-(-5)}{2-1} = \frac{-3}{1} = -3$

Since the average rate of change is constant at -3, this is a linear function with slope $= -3$. The y-intercept is $(0, -2)$, so the equation of the line is $y = -3x - 2$.

28.

x	y	Avg. rate of change $= \frac{\Delta y}{\Delta x}$
−2	$\frac{1}{4}$	
−1	$\frac{1}{2}$	$\frac{\left(\frac{1}{2}-\frac{1}{4}\right)}{-1-(-2)} = \frac{\frac{1}{4}}{1} = \frac{1}{4}$
0	1	$\frac{\left(1-\frac{1}{2}\right)}{0-(-1)} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$
1	2	
2	4	

Since the average rate of change is not constant, this is not a linear function.

29.

x	y	Avg. rate of change $= \frac{\Delta y}{\Delta x}$
−2	−8	
−1	−3	$\frac{-3-(-8)}{-1-(-2)} = \frac{5}{1} = 5$
0	0	$\frac{0-(-3)}{0-(-1)} = \frac{3}{1} = 3$
1	1	
2	0	

Since the average rate of change is not constant, this is not a linear function.

30.

x	y	Avg. rate of change $= \frac{\Delta y}{\Delta x}$
−2	−4	
−1	0	$\frac{0-(-4)}{-1-(-2)} = \frac{4}{1} = 4$
0	4	$\frac{4-0}{0-(-1)} = \frac{4}{1} = 4$
1	8	$\frac{8-4}{1-0} = \frac{4}{1} = 4$
2	12	$\frac{12-8}{2-1} = \frac{4}{1} = 4$

Since the average rate of change is constant at 4, this is a linear function with slope = 4. The y-intercept is $(0, 4)$, so the equation of the line is $y = 4x + 4$.

31.

x	y	Avg. rate of change $= \frac{\Delta y}{\Delta x}$
−2	−26	
−1	−4	$\frac{-4-(-26)}{-1-(-2)} = \frac{22}{1} = 22$
0	2	$\frac{2-(-4)}{0-(-1)} = \frac{6}{1} = 6$
1	−2	
2	−10	

Since the average rate of change is not constant, this is not a linear function.

32.

x	y	Avg. rate of change $=\dfrac{\Delta y}{\Delta x}$
-2	-4	
-1	-3.5	$\dfrac{-3.5-(-4)}{-1-(-2)}=\dfrac{0.5}{1}=0.5$
0	-3	$\dfrac{-3-(-3.5)}{0-(-1)}=\dfrac{0.5}{1}=0.5$
1	-2.5	$\dfrac{-2.5-(-3)}{1-0}=\dfrac{0.5}{1}=0.5$
2	-2	$\dfrac{-2-(-2.5)}{2-1}=\dfrac{0.5}{1}=0.5$

Since the average rate of change is constant at 0.5, this is a linear function with slope = 0.5. The y-intercept is $(0,-3)$, so the equation of the line is $y=0.5x-3$.

33.

x	y	Avg. rate of change $=\dfrac{\Delta y}{\Delta x}$
-2	8	
-1	8	$\dfrac{8-8}{-1-(-2)}=\dfrac{0}{1}=0$
0	8	$\dfrac{8-8}{0-(-1)}=\dfrac{0}{1}=0$
1	8	$\dfrac{8-8}{1-0}=\dfrac{0}{1}=0$
2	8	$\dfrac{8-8}{2-1}=\dfrac{0}{1}=0$

Since the average rate of change is constant at 0, this is a linear function with slope = 0. The y-intercept is $(0, 8)$, so the equation of the line is $y=0x+8$ or $y=8$.

34.

x	y	Avg. rate of change $=\dfrac{\Delta y}{\Delta x}$
-2	0	
-1	1	$\dfrac{1-0}{-1-(-2)}=\dfrac{1}{1}=1$
0	4	$\dfrac{4-1}{0-(-1)}=\dfrac{3}{1}=3$
1	9	
2	16	

Since the average rate of change is not constant, this is not a linear function.

35. $f(x)=4x-1;\quad g(x)=-2x+5$

a. $f(x)=0$

$4x-1=0$

$x=\dfrac{1}{4}$

b. $f(x)>0$

$4x-1>0$

$x>\dfrac{1}{4}$

The solution set is $\left\{x \middle| x>\dfrac{1}{4}\right\}$ or $\left(\dfrac{1}{4},\ \infty\right)$.

c. $f(x)=g(x)$

$4x-1=-2x+5$

$6x=6$

$x=1$

d. $f(x)\le g(x)$

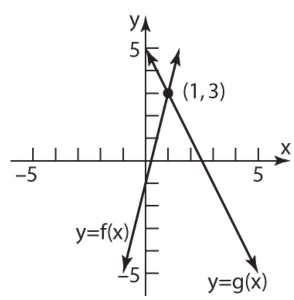
$4x-1\le -2x+5$

$6x\le 6$

$x\le 1$

The solution set is $\{x|x\le 1\}$ or $(-\infty,\ 1]$.

e.



36. $f(x)=3x+5;\quad g(x)=-2x+15$

a. $f(x)=0$

$3x+5=0$

$x=-\dfrac{5}{3}$

b. $f(x)<0$

$3x+5<0$

$x<-\dfrac{5}{3}$

The solution set is $\left\{x \middle| x<-\dfrac{5}{3}\right\}$ or $\left(-\infty,-\dfrac{5}{3}\right)$.

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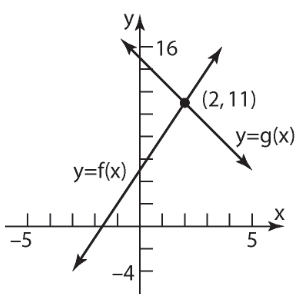
c. $f(x) = g(x)$
$3x + 5 = -2x + 15$
$5x = 10$
$x = 2$

d. $f(x) \geq g(x)$
$3x + 5 \geq -2x + 15$
$5x \geq 10$
$x \geq 2$
The solution set is $\{x | x \geq 2\}$ or $[2, \infty)$.

e.



37. a. The point (40, 50) is on the graph of $y = f(x)$, so the solution to $f(x) = 50$ is $x = 40$.

b. The point (88, 80) is on the graph of $y = f(x)$, so the solution to $f(x) = 80$ is $x = 88$.

c. The point $(-40, 0)$ is on the graph of $y = f(x)$, so the solution to $f(x) = 0$ is $x = -40$.

d. The y-coordinates of the graph of $y = f(x)$ are above 50 when the x-coordinates are larger than 40. Thus, the solution to $f(x) > 50$ is $\{x | x > 40\}$ or $(40, \infty)$.

e. The y-coordinates of the graph of $y = f(x)$ are below 80 when the x-coordinates are smaller than 88. Thus, the solution to $f(x) \leq 80$ is $\{x | x \leq 88\}$ or $(-\infty, 88]$.

f. The y-coordinates of the graph of $y = f(x)$ are between 0 and 80 when the x-coordinates are between -40 and 88. Thus, the solution to $0 < f(x) < 80$ is $\{x | -40 < x < 88\}$ or $(-40, 88)$.

38. a. The point (5, 20) is on the graph of $y = g(x)$, so the solution to $g(x) = 20$ is $x = 5$.

b. The point $(-15, 60)$ is on the graph of $y = g(x)$, so the solution to $g(x) = 60$ is $x = -15$.

c. The point (15, 0) is on the graph of $y = g(x)$, so the solution to $g(x) = 0$ is $x = 15$.

d. The y-coordinates of the graph of $y = g(x)$ are above 20 when the x-coordinates are smaller than 5. Thus, the solution to $g(x) > 20$ is $\{x | x < 5\}$ or $(-\infty, 5)$.

e. The y-coordinates of the graph of $y = f(x)$ are below 60 when the x-coordinates are larger than -15. Thus, the solution to $g(x) \leq 60$ is $\{x | x \geq -15\}$ or $[-15, \infty)$.

f. The y-coordinates of the graph of $y = f(x)$ are between 0 and 60 when the x-coordinates are between -15 and 15. Thus, the solution to $0 < f(x) < 60$ is $\{x | -15 < x < 15\}$ or $(-15, 15)$.

39. a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = -4$.

b. $f(x) \leq g(x)$ when the graph of *f* is above the graph of *g*. Thus, the solution is $\{x |\ x < -4\}$ or $(-\infty, -4)$.

40. a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = 2$.

b. $f(x) \leq g(x)$ when the graph of *f* is below or intersects the graph of *g*. Thus, the solution is $\{x |\ x \leq 2\}$ or $(-\infty, 2]$.

41. a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = -6$.

b. $g(x) \leq f(x) < h(x)$ when the graph of *f* is above or intersects the graph of *g* and below the graph of *h*. Thus, the solution is $\{x | -6 \leq x < 5\}$ or $[-6, 5)$.

42. a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = 7$.

b. $g(x) \le f(x) < h(x)$ when the graph of *f* is above or intersects the graph of *g* and below the graph of *h*. Thus, the solution is $\{x| -4 \le x < 7\}$ or $[-4, 7)$.

43. $C(x) = 2.5x + 85$

a. $C(40) = 2.5(40) + 85 = \$185$.

b. Solve $C(x) = 2.5x + 85 = 245$

$$2.5x + 85 = 245$$
$$2.5x = 100$$
$$x = \frac{160}{2.5} = 64 \text{ miles}$$

c. Solve $C(x) = 0.35x + 45 \le 150$

$$2.5x + 85 \le 150$$
$$2.5x \le 105$$
$$x \le \frac{65}{2.5} = 26 \text{ miles}$$

d. The number of mile towed cannot be negative, so the implied domain for *C* is $\{x \mid x \ge 0\}$ or $[0, \infty)$.

e. The cost of being towed increases \$2.50 for each mile, or there is a charge of \$2.50 per mile towed in addition to a fixed charge of \$85.

f. It costs \$85 for towing 0 miles, or there is a fixed charge of \$85 for towing in addition to a charge that depends on mileage.

44. $C(x) = 0.07x + 24.99$

a. $C(50) = 0.07(50) + 24.99 = \$28.49$.

b. Solve $C(x) = 0.07x + 24.99 = 31.85$

$$0.07x + 24.99 = 31.85$$
$$0.07x = 6.86$$
$$x = \frac{6.86}{0.07} = 98 \text{ minutes}$$

c. Solve $C(x) = 0.07x + 24.99 \le 36$

$$0.07x + 24.99 \le 36$$
$$0.07x \le 11.01$$
$$x \le \frac{11.01}{0.07} \approx 157 \text{ minutes}$$

d. The number of minutes cannot be negative, so $x \ge 0$. If there are 30 days in the month, then the number of minutes can be at most $30 \cdot 24 \cdot 60 = 43,200$. Thus, the implied domain for *C* is $\{x \mid 0 \le x \le 43,200\}$ or $[0, 43200]$.

e. The monthly cost of the plan increases \$0.07 for each minute used, or there is a charge of \$0.07 per minute to use the phone in addition to a fixed charge of \$24.99.

f. It costs \$24.99 per month for the plan if 0 minutes are used, or there is a fixed charge of \$24.99 per month for the plan in addition to a charge that depends on the number of minutes used.

45. $S(p) = -600 + 50p;\ D(p) = 1200 - 25p$

a. Solve $S(p) = D(p)$.

$$-600 + 50p = 1200 - 25p$$
$$75p = 1800$$
$$p = \frac{1800}{75} = 24$$
$$S(24) = -600 + 50(24) = 600$$

Thus, the equilibrium price is \$24, and the equilibrium quantity is 600 T-shirts.

b. Solve $D(p) > S(p)$.

$$1200 - 25p > -600 + 50p$$
$$1800 > 75p$$
$$\frac{1800}{75} > p$$
$$24 > p$$

The demand will exceed supply when the price is less than \$24 (but still greater than \$0). That is, $\$0 < p < \$24$.

c. The price will eventually be increased.

46. $S(p) = -2000 + 3000p;\ D(p) = 10000 - 1000p$

a. Solve $S(p) = D(p)$.

$$-2000 + 3000p = 10000 - 1000p$$
$$4000p = 12000$$
$$p = \frac{12000}{4000} = 3$$
$$S(3) = -2000 + 3000(3) = 7000$$

Thus, the equilibrium price is \$3, and the equilibrium quantity is 7000 hot dogs.

b. Solve $D(p) < S(p)$.

$$10000 - 1000p < -2000 + 3000p$$
$$12000 < 4000p$$
$$\frac{12000}{4000} < p$$
$$3 < p$$

The demand will be less than the supply when the price is greater than \$3.

c. The price will eventually be decreased.

47. a. We are told that the tax function T is for adjusted gross incomes x between \$9,325 and \$37,950, inclusive. Thus, the domain is $\{x \mid 9,325 \le x \le 37,950\}$ or $[9325,\ 37950]$.

b. $T(20,000) = 0.15(20,000 - 9325) + 932.50$
$= 2533.75$

If a single filer's adjusted gross income is \$20,000, then his or her tax bill will be \$2533.75.

c. The independent variable is adjusted gross income, x. The dependent variable is the tax bill, T.

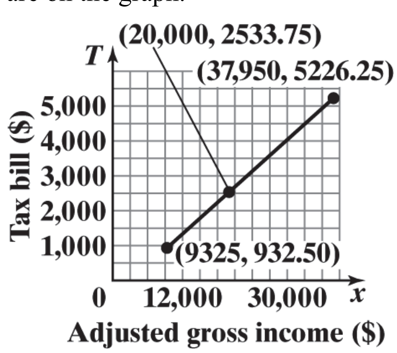
d. Evaluate T at $x = 9325,\ 20000,\ \text{and } 37950$.

$T(9325) = 0.15(9325 - 9325) + 932.50$
$= 932.50$

$T(20,000) = 0.15(20,000 - 9325) + 932.50$
$= 2533.75$

$T(37,950) = 0.15(37950 - 9325) + 932.50$
$= 5226.25$

Thus, the points $(9325, 932.50)$, $(20000, 2533.75)$, and $(37950, 5226.25)$ are on the graph.



e. We must solve $T(x) = 3673.75$.

$$0.15(x - 9325) + 932.50 = 3673.75$$
$$0.15x - 1398.75 + 932.50 = 3673.75$$
$$0.15x - 466.25 = 3673.75$$
$$0.15x = 4140$$
$$x = 27600$$

A single filer with an adjusted gross income of \$27,600 will have a tax bill of \$3673.75.

f. For each additional dollar of taxable income between \$9325 and \$37,950, the tax bill of a single person in 2013 increased by \$0.15.

48. a. The independent variable is payroll, p. The payroll tax only applies if the payroll exceeds \$189 million. Thus, the domain of T is $\{p \mid p > 189\}$ or $(189,\ \infty)$.

b. $T(243.8) = 0.5(243.8 - 189) = 27.4$

The luxury tax for the New York Yankees was \$27.4 million.

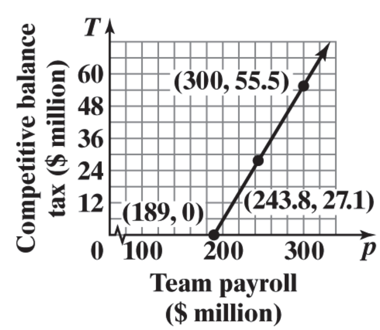
c. Evaluate T at $p = 189$, 243.8, and 300 million.

$T(189) = 0.5(189 - 189) = 0$ million

$T(243.8) = 0.5(243.8 - 189)$
$= 27.4$ million

$T(300) = 0.5(300 - 189) = 55.5$ million

Thus, the points $(189 \text{ million},\ 0 \text{ million})$, $(243.8 \text{ million},\ 27.4 \text{ million})$, and $(300 \text{ million},\ 55.5 \text{ million})$ are on the graph.



d. We must solve $T(p) = 31.8$.

$$0.5(p - 189) = 31.8$$
$$0.5p - 94.5 = 31.8$$
$$0.5p = 126.3$$
$$p = 252.6$$

If the luxury tax is \$31.8 million, then the payroll of the team is \$252.6 million.

e. For each additional million dollars of payroll in excess of \$189 million in 2016, the luxury tax of a team increased by \$0.5 million.

49. $R(x)=8x;\ \ C(x)=4.5x+17{,}500$

a. Solve $R(x)=C(x)$.

$$8x=4.5x+17{,}500$$
$$3.5x=17{,}500$$
$$x=5000$$

The break-even point occurs when the company sells 5000 units.

b. Solve $R(x)>C(x)$

$$8x>4.5x+17{,}500$$
$$3.5x>17{,}500$$
$$x>5000$$

The company makes a profit if it sells more than 5000 units.

50. $R(x)=12x;\ \ C(x)=10x+15{,}000$

a. Solve $R(x)=C(x)$

$$12x=10x+15{,}000$$
$$2x=15{,}000$$
$$x=7500$$

The break-even point occurs when the company sells 7500 units.

b. Solve $R(x)>C(x)$

$$12x>10x+15{,}000$$
$$2x>15{,}000$$
$$x>7500$$

The company makes a profit if it sells more than 7500 units.

51. a. Consider the data points (x,y), where x = the age in years of the computer and y = the value in dollars of the computer. So we have the points $(0,3000)$ and $(3,0)$. The slope formula yields:

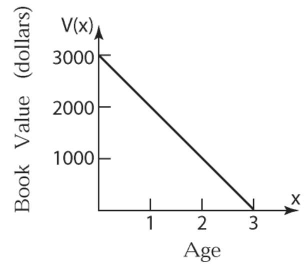
$$m=\frac{\Delta y}{\Delta x}=\frac{0-3000}{3-0}=\frac{-3000}{3}=-1000$$

The y-intercept is $(0,3000)$, so $b=3000$.

Therefore, the linear function is $V(x)=mx+b=-1000x+3000$.

b. The age of the computer cannot be negative, and the book value of the computer will be \$0 after 3 years. Thus, the implied domain for V is $\{x\,|\,0\le x\le 3\}$ or [0, 3].

c. The graph of $V(x)=-1000x+3000$



d. $V(2)=-1000(2)+3000=1000$

The computer's book value after 2 years will be \$1000.

e. Solve $V(x)=2000$

$$-1000x+3000=2000$$
$$-1000x=-1000$$
$$x=1$$

The computer will have a book value of \$2000 after 1 year.

52. a. Consider the data points (x,y), where x = the age in years of the machine and y = the value in dollars of the machine. So we have the points $(0{,}120000)$ and $(10,0)$. The slope formula yields:

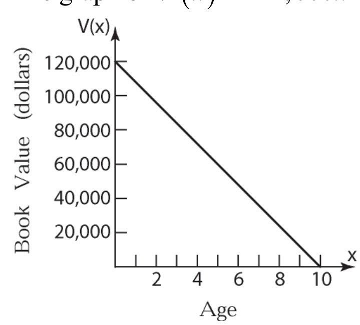
$$m=\frac{\Delta y}{\Delta x}=\frac{0-120000}{10-0}=\frac{-120000}{10}=-12000$$

The y-intercept is $(0{,}120000)$, so $b=120{,}000$.

Therefore, the linear function is $V(x)=mx+b=-12{,}000x+120{,}000$.

b. The age of the machine cannot be negative, and the book value of the machine will be \$0 after 10 years. Thus, the implied domain for V is $\{x\,|\,0\le x\le 10\}$ or [0, 10].

c. The graph of $V(x)=-12{,}000x+120{,}000$



d. $V(4) = -12000(4) + 120000 = 72000$
The machine's value after 4 years is given by \$72,000.

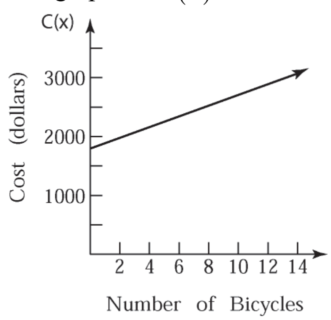
e. Solve $V(x) = 72000$.

$$-12000x + 120000 = 72000$$
$$-12000x = -48000$$
$$x = 4$$

The machine will be worth \$72,000 after 4 years.

53. a. Let x = the number of bicycles manufactured. We can use the cost function $C(x) = mx + b$, with $m = 90$ and $b = 1800$. Therefore $C(x) = 90x + 1800$

b. The graph of $C(x) = 90x + 1800$



c. The cost of manufacturing 14 bicycles is given by $C(14) = 90(14) + 1800 = \$3060$.

d. Solve $C(x) = 90x + 1800 = 3780$

$$90x + 1800 = 3780$$
$$90x = 1980$$
$$x = 22$$

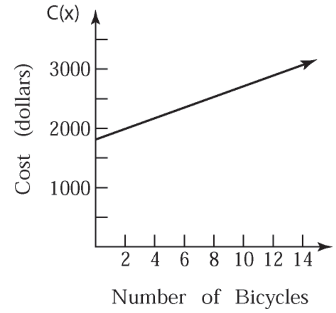
So 22 bicycles could be manufactured for \$3780.

54. a. The new daily fixed cost is

$$1800 + \frac{100}{20} = \$1805$$

b. Let x = the number of bicycles manufactured. We can use the cost function $C(x) = mx + b$, with $m = 90$ and $b = 1805$.
Therefore $C(x) = 90x + 1805$

c. The graph of $C(x) = 90x + 1805$



d. The cost of manufacturing 14 bicycles is given by $C(14) = 90(14) + 1805 = \$3065$.

e. Solve $C(x) = 90x + 1805 = 3780$

$$90x + 1805 = 3780$$
$$90x = 1975$$
$$x \approx 21.94$$

So approximately 21 bicycles could be manufactured for \$3780.

55. a. Let x = number of miles driven, and let C = cost in dollars. Total cost = (cost per mile)(number of miles) + fixed cost
$C(x) = 0.89x + 39.95$

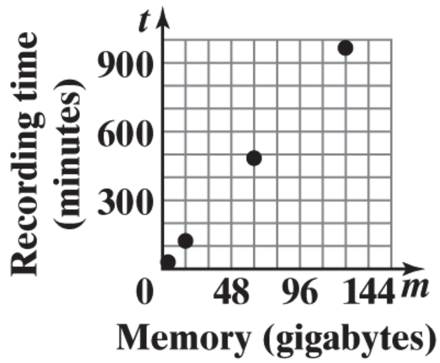
b. $C(110) = (0.89)(110) + 39.95 = \$137.85$
$C(230) = (0.89)(230) + 39.95 = \$244.65$

56. a. Let x = number of megabytes used, and let C = cost in dollars. Total cost = (cost per megabyte)(number of megabytes over 200) + fixed cost:

$$C(x) = 0.25(x - 200) + 40$$
$$= 0.25x - 50 + 40$$
$$= 0.25x - 10, x > 200$$

b. $C(265) = (0.25)(265) - 10 = \$56.25$
$C(300) = (0.25)(300) - 10 = \$65$

57. a.



b.

m	n	Avg. rate of change = $\frac{\Delta n}{\Delta m}$
4	30	
16	120	$\frac{120-30}{16-4} = \frac{90}{12} = \frac{15}{2}$
64	480	$\frac{480-120}{64-16} = \frac{360}{48} = \frac{15}{2}$
128	960	$\frac{960-480}{128-64} = \frac{480}{64} = \frac{15}{2}$

Since each input (memory) corresponds to a single output (recording time), we know that recording time is a function of memory. Also, because the average rate of change is constant at 7.5 minutes per gigabyte, the function is linear.

c. From part (b), we know $slope = 7.5$. Using $(m_1, n_1) = (4, 30)$, we get the equation:

$$t - t_1 = s(m - m_1)$$
$$t - 30 = 7.5(m - 4)$$
$$t - 30 = 7.5m - 30$$
$$t = 7.5m$$

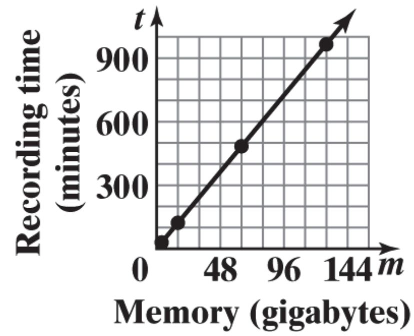
Using function notation, we have $t(m) = 7.5m$.

d. The memory cannot be negative, so $m \geq 0$. Likewise, the time cannot be negative, so, $t(m) \geq 0$.

$$7.5m \geq 0$$
$$m \geq 0$$

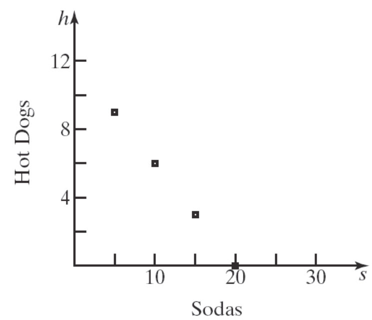
Thus, the implied domain for n(m) is $\{m \mid m \geq 0\}$ or $[0, \infty)$.

e.



f. If memory increases by 1 GB, then the number of songs increases by 218.75.

58. a.



b.

s	h	Avg. rate of change = $\frac{\Delta h}{\Delta s}$
20	0	
15	3	$\frac{3-0}{15-20} = \frac{3}{-5} = -0.6$
10	6	$\frac{6-3}{10-15} = \frac{3}{-5} = -0.6$
5	9	$\frac{9-6}{5-10} = \frac{3}{-5} = -0.6$

Since each input (soda) corresponds to a single output (hot dogs), we know that number of hot dogs purchased is a function of number of sodas purchased. Also, because the average rate of change is constant at -0.6 hot dogs per soda, the function is linear.

c. From part (b), we know $m = -0.6$. Using $(s_1, h_1) = (20, 0)$, we get the equation:

$$h - h_1 = m(s - s_1)$$
$$h - 0 = -0.6(s - 20)$$
$$h = -0.6s + 12$$

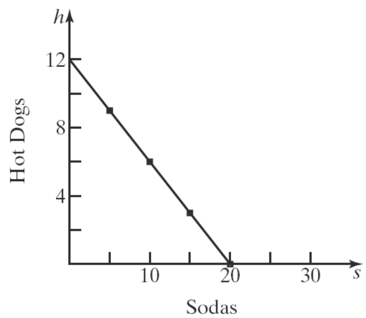
Using function notation, we have $h(s) = -0.6s + 12$.

d. The number of sodas cannot be negative, so $s \geq 0$. Likewise, the number of hot dogs cannot be negative, so, $h(s) \geq 0$.

$$\begin{aligned} -0.6s + 12 &\geq 0 \\ -0.6s &\geq -12 \\ s &\leq 20 \end{aligned}$$

Thus, the implied domain for h(s) is $\{s \mid 0 \leq s \leq 20\}$ or $[0, 20]$.

e.



f. If the number of hot dogs purchased increases by \$1, then the number of sodas purchased decreases by 0.6.

g. s-intercept: If 0 hot dogs are purchased, then 20 sodas can be purchased.
h-intercept: If 0 sodas are purchased, then 12 hot dogs may be purchased.

59. The graph shown has a positive slope and a positive y-intercept. Therefore, the function from (d) and (e) might have the graph shown.

60. The graph shown has a negative slope and a positive y-intercept. Therefore, the function from (b) and (e) might have the graph shown.

61. A linear function $f(x) = mx + b$ will be odd provided $f(-x) = -f(x)$.

That is, provided $m(-x) + b = -(mx + b)$.

$$\begin{aligned} -mx + b &= -mx - b \\ b &= -b \\ 2b &= 0 \\ b &= 0 \end{aligned}$$

So a linear function $f(x) = mx + b$ will be odd provided $b = 0$.

A linear function $f(x) = mx + b$ will be even provided $f(-x) = f(x)$.

That is, provided $m(-x) + b = mx + b$.

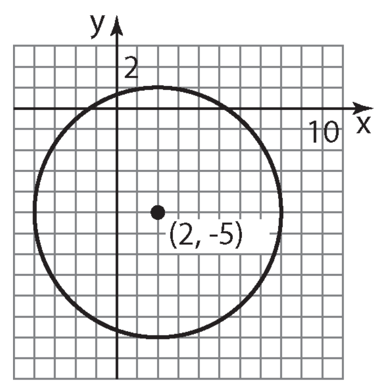
$$\begin{aligned} -mx + b &= mx + b \\ -mxb &= mx \\ 0 &= 2mx \\ m &= 0 \end{aligned}$$

So, yes, a linear function $f(x) = mx + b$ cab be even provided $m = 0$.

62. If you solve the linear function $f(x) = mx + b$ for 0 you are actually finding the x-intercept. Therefore using x-intercept of the graph of $f(x) = mx + b$ would be same x-value as solving $mx + b > 0$ for x. Then the appropriate interval could be determined

63.
$$\begin{aligned} x^2 - 4x + y^2 + 10y - 7 &= 0 \\ (x^2 - 4x + 4) + (y^2 + 10y + 25) &= 7 + 4 + 25 \\ (x-2)^2 + (y+5)^2 &= 6^2 \end{aligned}$$

Center: (2, -5); Radius = 6

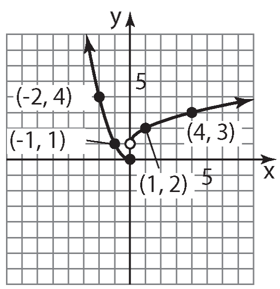


64. $f(x) = \dfrac{2x + B}{x - 3}$

$$\begin{aligned} f(5) = 8 &= \frac{2(5) + B}{5 - 3} \\ 8 &= \frac{10 + B}{2} \\ 16 &= 10 + B \\ B &= 6 \end{aligned}$$

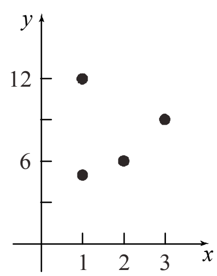
65. $\dfrac{f(3)-f(1)}{3-1}$
$= \dfrac{12-(-2)}{2}$
$= \dfrac{14}{2}$
$= 7$

66.



Section 2.2

1.



No, the relation is not a function because an input, 1, corresponds to two different outputs, 5 and 12.

2. Let $(x_1, y_1) = (1, 4)$ and $(x_2, y_2) = (3, 8)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{3-1} = \frac{4}{2} = 2$$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 2(x-1)$$
$$y - 4 = 2x - 2$$
$$y = 2x + 2$$

3. scatter diagram

4. decrease; 0.008

5. Linear relation, $m > 0$

6. Nonlinear relation

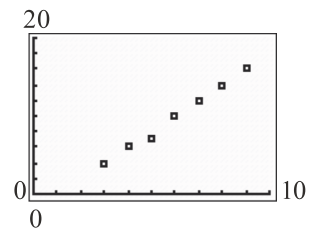
7. Linear relation, $m < 0$

8. Linear relation, $m > 0$

9. Nonlinear relation

10. Nonlinear relation

11. a.



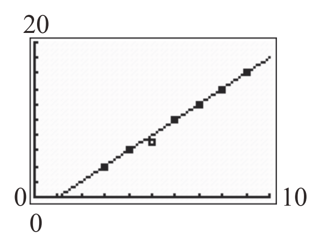
b. Answers will vary. We select (4, 6) and (8, 14). The slope of the line containing these points is:

$$m = \frac{14-6}{8-4} = \frac{8}{4} = 2$$

The equation of the line is:

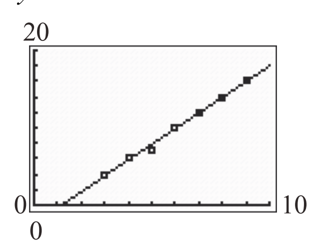
$$y - y_1 = m(x - x_1)$$
$$y - 6 = 2(x-4)$$
$$y - 6 = 2x - 8$$
$$y = 2x - 2$$

c.

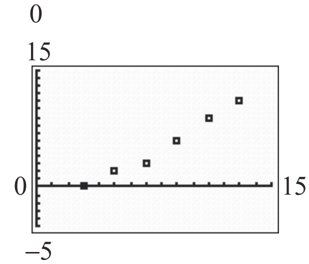


d. Using the LINear REGression program, the line of best fit is:
$y = 2.0357x - 2.3571$

e.



12. a.

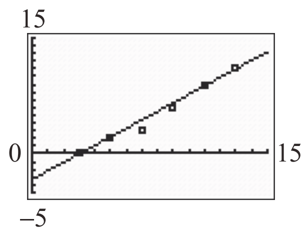


b. Answers will vary. We select (5, 2) and (11, 9). The slope of the line containing these points is: $m = \frac{9-2}{11-5} = \frac{7}{6}$

The equation of the line is:

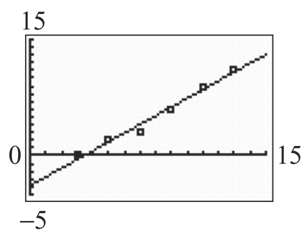
$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{7}{6}(x-5)$$
$$y - 2 = \frac{7}{6}x - \frac{35}{6}$$
$$y = \frac{7}{6}x - \frac{23}{6}$$

c.

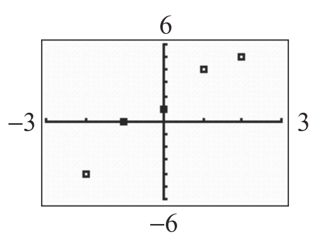


d. Using the LINear REGression program, the line of best fit is:
$y = 1.1286x - 3.8619$

e.



13. a.

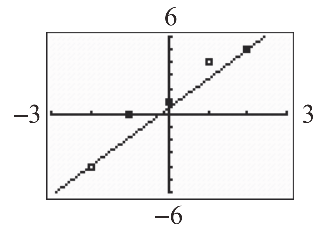


b. Answers will vary. We select (–2,–4) and (2, 5). The slope of the line containing these points is: $m = \frac{5-(-4)}{2-(-2)} = \frac{9}{4}$.

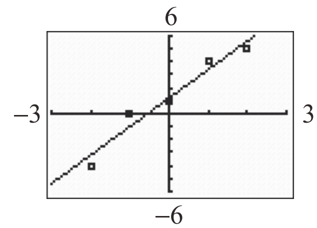
The equation of the line is:

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = \frac{9}{4}(x - (-2))$$
$$y + 4 = \frac{9}{4}x + \frac{9}{2}$$
$$y = \frac{9}{4}x + \frac{1}{2}$$

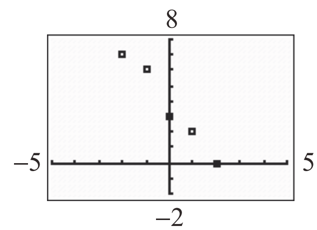
c.



e. Using the LINear REGression program, the line of best fit is:
$y = 2.2x + 1.2$



14. a.

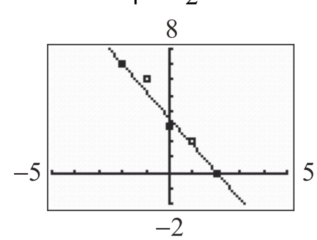


b. Answers will vary. We select (–2, 7) and (2, 0). The slope of the line containing these points is: $m = \frac{0-7}{2-(-2)} = \frac{-7}{4} = -\frac{7}{4}$.

The equation of the line is:

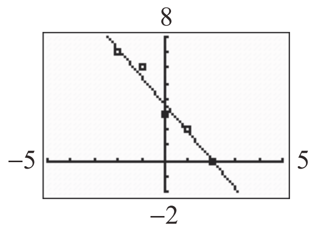
$$y - y_1 = m(x - x_1)$$
$$y - 7 = -\frac{7}{4}(x - (-2))$$
$$y - 7 = -\frac{7}{4}x - \frac{7}{2}$$
$$y = -\frac{7}{4}x + \frac{7}{2}$$

c.

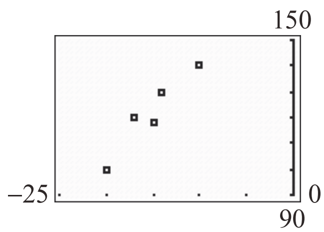


d. Using the LINear REGression program, the line of best fit is:
$y = -1.8x + 3.6$

e.



15. a.



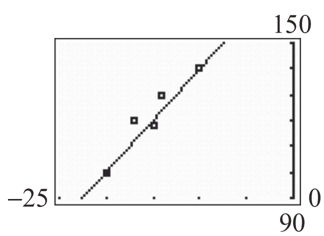
b. Answers will vary. We select (–20,100) and (–10,140). The slope of the line containing these points is:

$$m = \frac{140-100}{-10-(-20)} = \frac{40}{10} = 4$$

The equation of the line is:

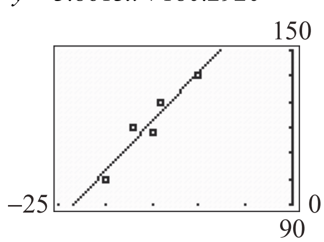
$$y - y_1 = m(x - x_1)$$
$$y - 100 = 4(x - (-20))$$
$$y - 100 = 4x + 80$$
$$y = 4x + 180$$

c.

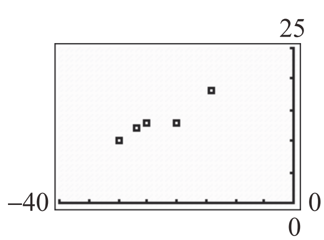


d. Using the LINear REGression program, the line of best fit is:
$y = 3.8613x + 180.2920$

e.



16. a.



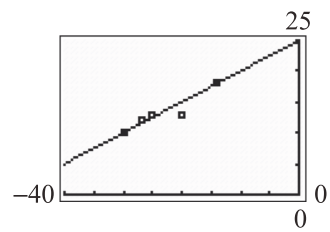
b. Selection of points will vary. We select (–30, 10) and (–14, 18). The slope of the line containing these points is:

$$m = \frac{18-10}{-14-(-30)} = \frac{8}{16} = \frac{1}{2}$$

The equation of the line is:

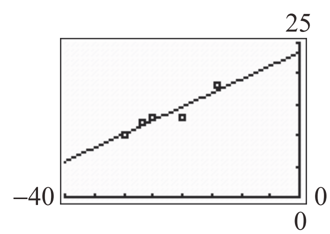
$$y - y_1 = m(x - x_1)$$
$$y - 10 = \frac{1}{2}(x - (-30))$$
$$y - 10 = \frac{1}{2}x + 15$$
$$y = \frac{1}{2}x + 25$$

c.

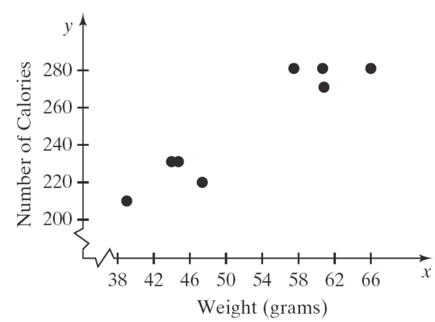


d. Using the LINear REGression program, the line of best fit is:
$y = 0.4421x + 23.4559$

e.



17. a.

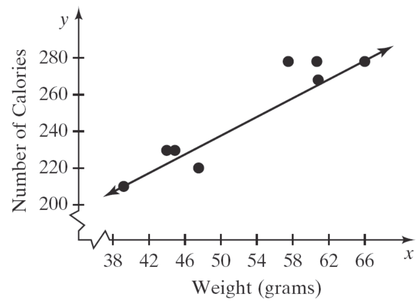


b. Linear.

c. Answers will vary. We will use the points $(39.52,\ 210)$ and $(66.45,\ 280)$.

$$m=\frac{280-210}{66.45-39.52}=\frac{70}{26.93}\approx 2.5993316$$
$$y-210=2.5993316(x-39.52)$$
$$y-210=2.5993316x-102.7255848$$
$$y=2.599x+107.274$$

d.

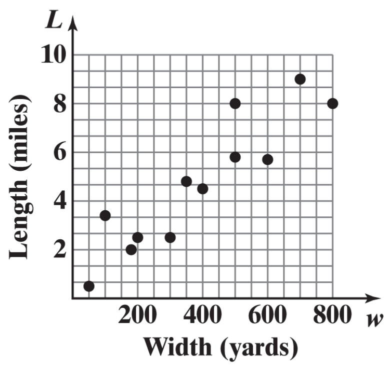


e. $x=62.3$: $y=2.599(62.3)+107.274\approx 269$

We predict that a candy bar weighing 62.3 grams will contain 269 calories.

f. If the weight of a candy bar is increased by one gram, then the number of calories will increase by 2.599.

18. a.

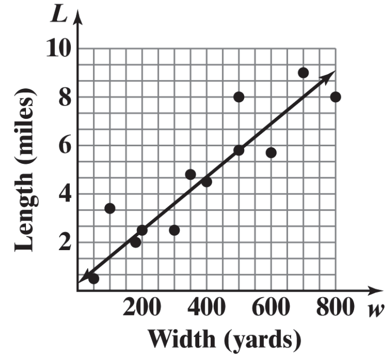


b. Linear with positive slope.

c. Answers will vary. We will use the points $(200,\ 2.5)$ and $(500,\ 5.8)$.

$$m=\frac{5.8-2.5}{500-200}=\frac{3.3}{300}=0.011$$
$$L-L_1=m\left(w-w_1\right)$$
$$L-2.5=0.011\left(w-200\right)$$
$$L-2.5=0.011w-2.2$$
$$L=0.011w+0.3$$

d.



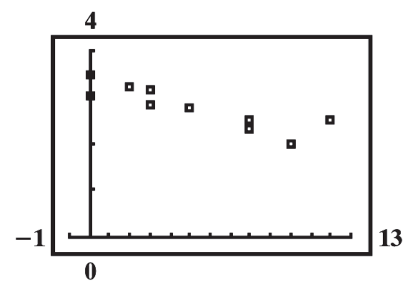
e. $L(450)=0.011(450)+0.3=5.25$

We predict that the approximately length of a 450 yard wide tornado is 5.25 miles.

f. For each 1-yard increase in the width of a tornado, the length of the tornado increases by 0.011 mile, on average.

19. a. The independent variable is the number of hours spent playing video games and cumulative grade-point average is the dependent variable because we are using number of hours playing video games to predict (or explain) cumulative grade-point average.

b.



c. Using the LINear REGression program, the line of best fit is: $G(h)=-0.0942h+3.2763$

d. If the number of hours playing video games in a week increases by 1 hour, the cumulative grade-point average decreases 0.09, on average.

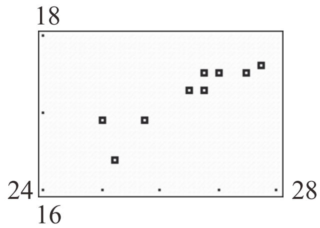
e. $G(8)=-0.0942(8)+3.2763=2.52$

We predict a grade-point average of approximately 2.52 for a student who plays 8 hours of video games each week.

f.
$$2.40=-0.0942(h)+3.2763$$
$$2.40-3.2763=-0.0942h$$
$$-0.8763=-0.0942h$$
$$9.3=h$$

A student who has a grade-point average of 2.40 will have played approximately 9.3 hours of video games.

20. a.



b. Using the LINear REGression program, the line of best fit is:
$w(p) = -1.1857p + 1231.8279$

c. For each 10-millibar increase in the atmospheric pressure, the wind speed of the tropical system decreases by 1.1857 knots, on average.

d. $w(990) = -1.1857(990) + 1231.8279 \approx 58$ knots

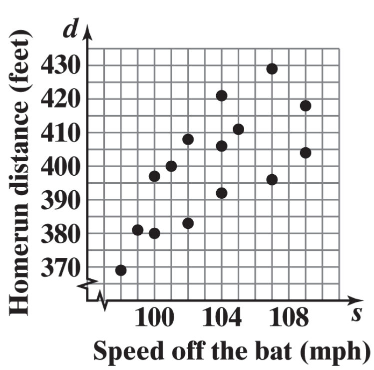
e. To find the pressure, we solve the following equation:

$$85 = -1.1857p + 1231.8279$$
$$-1146.8279 = -1.1857p$$
$$967 \approx p$$

A hurricane with a wind speed of 85 knots would have a pressure of approximately 967 millibars.

21. a. This relation does not represent a function since the values of the input variable s are repeated.

b.



c. Using the LINear REGression program, the line of best fit is: $d = 3.3641s + 51.8233$

d. For each 1-mph increase in the speed off bat, the homerun distance increases by 3.3641 feet, on average.

e. $d(s) = 3.3641s + 51.8233$

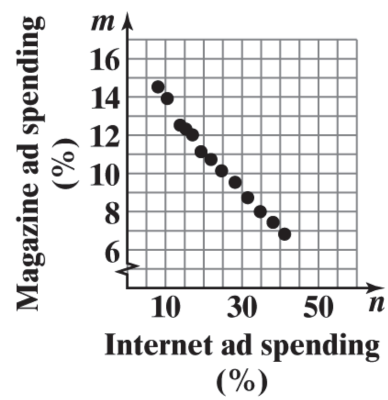
f. Since the speed off bat must be greater than 0 the domain is $\{s \mid s > 0\}$.

g. $d(103) = 3.3641(103) + 51.8233 \approx 398$ ft

A hurricane with a wind speed of 85 knots would have a pressure of approximately 967 millibars.

22. a. The relation is a function because none of the invariables are repeated.

b.



c. Using the LINear REGression program, the line of best fit is: $m = -0.2277n + 15.9370$.

d. If Internet ad spending increases by 1%, magazine ad spending goes down by about 0.2277%, on average.

e. $m(n) = -0.2277n + 15.9370$

f. Domain: $\{n \mid 0 < n \le 70.0\}$

Note that the m-intercept is roughly 15.9 and that the percent of Internet sales cannot be negative.

g. $D(28) = -0.2277(26.0) + 15.9370 \approx 10.0$

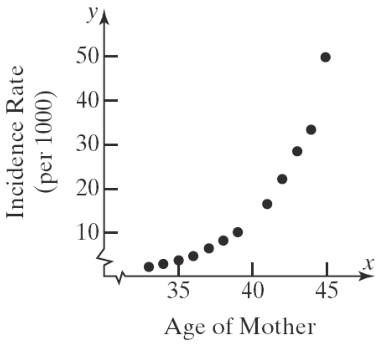
Percent of magazine sales is about 10.0%.

$x^2 - 25 = 0$

$x^2 = 25 \rightarrow x = \pm 5$

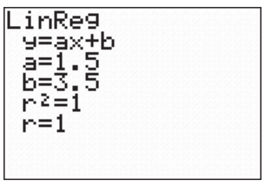
So the domain is: $\{x \mid x \neq 5, -5\}$

23.



The data do not follow a linear pattern so it would not make sense to find the line of best fit.

24. Using the ordered pairs $(1, 5)$ and $(3, 8)$, the line of best fit is $y = 1.5x + 3.5$.



The correlation coefficient is $r = 1$. This is reasonable because two points determine a line.

25. A correlation coefficient of 0 implies that the data do not have a linear relationship.

26. The y-intercept would be the calories of a candy bar with weight 0 which would not be meaningful in this problem.

27. $G(0) = -0.0942(0) + 3.2763 = 3.2763$. The approximate grade-point average of a student who plays 0 hours of video games per week would be 3.28.

28. $m = \dfrac{-3-5}{3-(-1)} = \dfrac{-8}{4} = -2$

$y - y_1 = m(x - x_1)$

$y - 5 = -2(x + 1)$

$y - 5 = -2x - 2$

$y = -2x + 3$ or

$2x + y = 3$

29. The domain would be all real numbers except those that make the denominator zero.

30. $f(x) = 5x - 8$ and $g(x) = x^2 - 3x + 4$

$(g - f)(x) = (x^2 - 3x + 4) - (5x - 8)$

$= x^2 - 3x + 4 - 5x + 8$

$= x^2 - 8x + 12$

31. Since y is shifted to the left 3 units we would use $y = (x+3)^2$. Since y is also shifted down 4 units,we would use $y = (x+3)^2 - 4$.

Section 2.3

1. a. $x^2 - 5x - 6 = (x-6)(x+1)$

b. $2x^2 - x - 3 = (2x-3)(x+1)$

2. $\sqrt{8^2 - 4 \cdot 2 \cdot 3} = \sqrt{64 - 24}$

$= \sqrt{40} = \sqrt{4 \cdot 10} = 2\sqrt{10}$

3. $(x-3)(3x+5) = 0$

$x - 3 = 0$ or $3x + 5 = 0$

$x = 3$ $\quad 3x = -5$

$x = -\dfrac{5}{3}$

The solution set is $\left\{-\dfrac{5}{3}, 3\right\}$.

4. add; $\left(\dfrac{1}{2} \cdot 6\right)^2 = 9$

5. If $f(4) = 10$, then the point (4, 10) is on the graph of *f*.

6. $f(-3) = (-3)^2 + 4(-3) + 3$

$= 9 - 12 + 3 = 0$

-3 is a zero of $f(x)$.

7. repeated; multiplicity 2

8. discriminant; negative

9. A quadratic functions can have either 0, 1 or 2 real zeros.

10. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

11. False; the equation will have only two real solution but not necessarily negatives of one another.

12. b

13.
$$f(x) = 0$$
$$x^2 - 9x = 0$$
$$x(x-9) = 0$$
$$x = 0 \quad \text{or} \quad x - 9 = 0$$
$$x = 9$$

The zeros of $f(x) = x^2 - 9x$ are 0 and 9. The x-intercepts of the graph of f are 0 and 9.

14.
$$f(x) = 0$$
$$x^2 + 4x = 0$$
$$x(x+4) = 0$$
$$x = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = -4$$

The zeros of $f(x) = x^2 + 4x$ are -4 and 0. The x-intercepts of the graph of f are -4 and 0.

15.
$$g(x) = 0$$
$$x^2 - 25 = 0$$
$$(x+5)(x-5) = 0$$
$$x + 5 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = -5 \qquad x = 5$$

The zeros of $g(x) = x^2 - 25$ are -5 and 5. The x-intercepts of the graph of g are -5 and 5.

16.
$$G(x) = 0$$
$$x^2 - 9 = 0$$
$$(x+3)(x-3) = 0$$
$$x + 3 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -3 \qquad x = 3$$

The zeros of $G(x) = x^2 - 9$ are -3 and 3. The x-intercepts of the graph of G are -3 and 3.

17.
$$F(x) = 0$$
$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x + 3 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -3 \qquad x = 2$$

The zeros of $F(x) = x^2 + x - 6$ are -3 and 2. The x-intercepts of the graph of F are -3 and 2.

18.
$$H(x) = 0$$
$$x^2 + 7x + 6 = 0$$
$$(x+6)(x+1) = 0$$
$$x + 6 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = -6 \qquad x = -1$$

The zeros of $H(x) = x^2 + 7x + 6$ are -6 and -1. The x-intercepts of the graph of H are -6 and -1.

19.
$$g(x) = 0$$
$$2x^2 - 5x - 3 = 0$$
$$(2x+1)(x-3) = 0$$
$$2x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -\frac{1}{2} \qquad x = 3$$

The zeros of $g(x) = 2x^2 - 5x - 3$ are $-\frac{1}{2}$ and 3. The x-intercepts of the graph of g are $-\frac{1}{2}$ and 3.

20.
$$f(x) = 0$$
$$3x^2 + 5x + 2 = 0$$
$$(3x+2)(x+1) = 0$$
$$3x + 2 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = -\frac{2}{3} \qquad x = -1$$

The zeros of $f(x) = 3x^2 + 5x + 2$ are -1 and $-\frac{2}{3}$. The x-intercepts of the graph of f are -1 and $-\frac{2}{3}$.

21.
$$P(x)=0$$
$$3x^2-48=0$$
$$3(x^2-16)=0$$
$$3(x+4)(x-4)=0$$
$$t+4=0 \quad \text{or} \quad t-4=0$$
$$t=-4 \qquad t=4$$

The zeros of $P(x)=3x^2-48$ are -4 and 4.
The x-intercepts of the graph of P are -4 and 4.

22.
$$H(x)=0$$
$$2x^2-50=0$$
$$2(x^2-25)=0$$
$$2(x+5)(x-5)=0$$
$$y+5=0 \quad \text{or} \quad y-5=0$$
$$y=-5 \qquad y=5$$

The zeros of $H(x)=2x^2-50$ are -5 and 5.
The x-intercepts of the graph of H are -5 and 5.

23.
$$g(x)=0$$
$$x(x+8)+12=0$$
$$x^2+8x+12=0$$
$$(x+6)(x+2)=0$$
$$x=-6 \quad \text{or} \quad x=-2$$

The zeros of $g(x)=x(x+8)+12$ are -6 and -2.
The x-intercepts of the graph of g are -6 and -2.

24.
$$f(x)=0$$
$$x(x-4)-12=0$$
$$x^2-4x-12=0$$
$$(x-6)(x+2)=0$$
$$x=-2 \quad \text{or} \quad x=6$$

The zeros of $f(x)=x(x-4)-12$ are -2 and 6.
The x-intercepts of the graph of f are -2 and 6.

25.
$$G(x)=0$$
$$4x^2+9-12x=0$$
$$4x^2-12x+9=0$$
$$(2x-3)(2x-3)=0$$
$$2x-3=0 \quad \text{or} \quad 2x-3=0$$
$$x=\frac{3}{2} \qquad x=\frac{3}{2}$$

The only zero of $G(x)=4x^2+9-12x$ is $\frac{3}{2}$.

The only x-intercept of the graph of G is $\frac{3}{2}$.

26.
$$F(x)=0$$
$$25x^2+16-40x=0$$
$$25x^2-40x+16=0$$
$$(5x-4)(5x-4)=0$$
$$5x-4=0 \quad \text{or} \quad 5x-4=0$$
$$x=\frac{4}{5} \qquad x=\frac{4}{5}$$

The only zero of $F(x)=25x^2+16-40x$ is $\frac{4}{5}$.

The only x-intercept of the graph of F is $\frac{4}{5}$.

27.
$$f(x)=0$$
$$x^2-8=0$$
$$x^2=8$$
$$x=\pm\sqrt{8}=\pm 2\sqrt{2}$$

The zeros of $f(x)=x^2-8$ are $-2\sqrt{2}$ and $2\sqrt{2}$.
The x-intercepts of the graph of f are $-2\sqrt{2}$ and $2\sqrt{2}$.

28.
$$g(x)=0$$
$$x^2-18=0$$
$$x^2=18$$
$$x=\pm\sqrt{18}=\pm 3\sqrt{3}$$

The zeros of $g(x)=x^2-18$ are $-3\sqrt{3}$ and $3\sqrt{3}$. The x-intercepts of the graph of g are $-3\sqrt{3}$ and $3\sqrt{3}$.

29.
$$g(x)=0$$
$$(x-1)^2-4=0$$
$$(x-1)^2=4$$
$$x-1=\pm\sqrt{4}$$
$$x-1=\pm 2$$
$$x-1=2 \quad \text{or} \quad x-1=-2$$
$$x=3 \qquad x=-1$$

The zeros of $g(x)=(x-1)^2-4$ are -1 and 3.
The x-intercepts of the graph of g are -1 and 3.

30.
$$G(x) = 0$$
$$(x+2)^2 - 1 = 0$$
$$(x+2)^2 = 1$$
$$x+2 = \pm\sqrt{1}$$
$$x+2 = \pm 1$$
$$x+2 = 1 \quad \text{or} \quad x+2 = -1$$
$$x = -1 \qquad\qquad x = -3$$
The zeros of $G(x) = (x+2)^2 - 1$ are -3 and -1. The x-intercepts of the graph of G are -3 and -1.

31.
$$F(x) = 0$$
$$(2x+3)^2 - 32 = 0$$
$$(2x+3)^2 = 32$$
$$2x+3 = \pm\sqrt{32}$$
$$2x+3 = \pm 4\sqrt{2}$$
$$2x = -3 \pm 4\sqrt{2}$$
$$x = \frac{-3 \pm 4\sqrt{2}}{2}$$
The zeros of $F(x) = (2x+3)^2 - 32$ are $\frac{-3+4\sqrt{2}}{2}$ and $\frac{-3-4\sqrt{2}}{2}$. The x-intercepts of the graph of F are $\frac{-3+4\sqrt{2}}{2}$ and $\frac{-3-4\sqrt{2}}{2}$.

32.
$$F(x) = 0$$
$$(3x-2)^2 - 75 = 0$$
$$(3x-2)^2 = 75$$
$$3x-2 = \pm\sqrt{75}$$
$$3x-2 = \pm 5\sqrt{3}$$
$$3x = 2 \pm 5\sqrt{3}$$
$$x = \frac{2 \pm 5\sqrt{3}}{3}$$
The zeros of $G(x) = (3x-2)^2 - 75$ are $\frac{2+5\sqrt{3}}{3}$ and $\frac{2-5\sqrt{3}}{3}$. The x-intercepts of the graph of G are $\frac{2-5\sqrt{3}}{3}$ and $\frac{2+5\sqrt{3}}{3}$.

33.
$$f(x) = 0$$
$$x^2 + 4x - 8 = 0$$
$$x^2 + 4x = 8$$
$$x^2 + 4x + 4 = 8 + 4$$
$$(x+2)^2 = 12$$
$$x+2 = \pm\sqrt{12}$$
$$x+2 = \pm 2\sqrt{3}$$
$$x = -2 \pm 2\sqrt{3}$$
$$x = -2 + 2\sqrt{3} \quad \text{or} \quad x = -2 - 2\sqrt{3}$$
The zeros of $f(x) = x^2 + 4x - 8$ are $-2+2\sqrt{3}$ and $-2-2\sqrt{3}$. The x-intercepts of the graph of f are $-2+2\sqrt{3}$ and $-2-2\sqrt{3}$.

34.
$$f(x) = 0$$
$$x^2 - 6x - 9 = 0$$
$$x^2 - 6x + 9 = 9 + 9$$
$$(x-3)^2 = 18$$
$$x-3 = \pm\sqrt{18}$$
$$x = 3 \pm 3\sqrt{2}$$
The zeros of $f(x) = x^2 - 6x - 9$ are $3-3\sqrt{2}$ and $3+3\sqrt{2}$. The x-intercepts of the graph of f are $3-3\sqrt{2}$ and $3+3\sqrt{2}$.

35.
$$g(x) = 0$$
$$x^2 - \frac{1}{2}x - \frac{3}{16} = 0$$
$$x^2 - \frac{1}{2}x = \frac{3}{16}$$
$$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{3}{16} + \frac{1}{16}$$
$$\left(x - \frac{1}{4}\right)^2 = \frac{1}{4}$$
$$x - \frac{1}{4} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$
$$x = \frac{1}{4} \pm \frac{1}{2}$$
$$x = \frac{3}{4} \quad \text{or} \quad x = -\frac{1}{4}$$
The zeros of $g(x) = x^2 - \frac{1}{2}x - \frac{3}{16}$ are $-\frac{1}{4}$ and $\frac{3}{4}$.

The x-intercepts of the graph of g are $-\frac{1}{4}$ and $\frac{3}{4}$.

36.
$$g(x)=0$$
$$x^2+\frac{2}{3}x-\frac{1}{3}=0$$
$$x^2+\frac{2}{3}x=\frac{1}{3}$$
$$x^2+\frac{2}{3}x+\frac{1}{9}=\frac{1}{3}+\frac{1}{9}$$
$$\left(x+\frac{1}{3}\right)^2=\frac{4}{9}$$
$$x+\frac{1}{3}=\pm\sqrt{\frac{4}{9}}=\pm\frac{2}{3}$$
$$x=-\frac{1}{3}\pm\frac{2}{3}$$
$$x=\frac{1}{3}\text{ or }x=-1$$

The zeros of $g(x)=x^2+\frac{2}{3}x-\frac{1}{3}$ are -1 and $\frac{1}{3}$.

The x-intercepts of the graph of g are -1 and $\frac{1}{3}$.

37.
$$F(x)=0$$
$$3x^2+x-\frac{1}{2}=0$$
$$x^2+\frac{1}{3}x-\frac{1}{6}=0$$
$$x^2+\frac{1}{3}x=\frac{1}{6}$$
$$x^2+\frac{1}{3}x+\frac{1}{36}=\frac{1}{6}+\frac{1}{36}$$
$$\left(x+\frac{1}{6}\right)^2=\frac{7}{36}$$
$$x+\frac{1}{6}=\pm\sqrt{\frac{7}{36}}=\pm\frac{\sqrt{7}}{6}$$
$$x=\frac{-1\pm\sqrt{7}}{6}$$

The zeros of $F(x)=3x^2+x-\frac{1}{2}$ are $\frac{-1-\sqrt{7}}{6}$ and $\frac{-1+\sqrt{7}}{6}$. The x-intercepts of the graph of F are $\frac{-1-\sqrt{7}}{6}$ and $\frac{-1+\sqrt{7}}{6}$.

38.
$$G(x)=0$$
$$2x^2-3x-1=0$$
$$x^2-\frac{3}{2}x-\frac{1}{2}=0$$
$$x^2-\frac{3}{2}x=\frac{1}{2}$$
$$x^2-\frac{3}{2}x+\frac{9}{16}=\frac{1}{2}+\frac{9}{16}$$
$$\left(x-\frac{3}{4}\right)^2=\frac{17}{16}$$
$$x-\frac{3}{4}=\pm\sqrt{\frac{17}{16}}=\pm\frac{\sqrt{17}}{4}$$
$$x=\frac{3\pm\sqrt{17}}{4}$$

The zeros of $G(x)=2x^2-3x-1$ are $\frac{3-\sqrt{17}}{4}$ and $\frac{3+\sqrt{17}}{4}$. The x-intercepts of the graph of G are $\frac{3-\sqrt{17}}{4}$ and $\frac{3+\sqrt{17}}{4}$.

39.
$$f(x)=0$$
$$x^2-4x+2=0$$
$$a=1,\quad b=-4,\quad c=2$$
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(2)}}{2(1)}=\frac{4\pm\sqrt{16-8}}{2}$$
$$=\frac{4\pm\sqrt{8}}{2}=\frac{4\pm2\sqrt{2}}{2}=2\pm\sqrt{2}$$

The zeros of $f(x)=x^2-4x+2$ are $2-\sqrt{2}$ and $2+\sqrt{2}$. The x-intercepts of the graph of f are $2-\sqrt{2}$ and $2+\sqrt{2}$.

40.
$$f(x)=0$$
$$x^2+4x+2=0$$
$$a=1,\quad b=4,\quad c=2$$
$$x=\frac{-4\pm\sqrt{4^2-4(1)(2)}}{2(1)}=\frac{-4\pm\sqrt{16-8}}{2}$$
$$=\frac{-4\pm\sqrt{8}}{2}=\frac{-4\pm2\sqrt{2}}{2}=-2\pm\sqrt{2}$$

The zeros of $f(x)=x^2+4x+2$ are $-2-\sqrt{2}$ and $-2+\sqrt{2}$. The x-intercepts of the graph of f are $-2-\sqrt{2}$ and $-2+\sqrt{2}$.

41. $g(x)=0$

$x^2-4x-1=0$

$a=1,\quad b=-4,\quad c=-1$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-1)}}{2(1)}=\frac{4\pm\sqrt{16+4}}{2}$$
$$=\frac{4\pm\sqrt{20}}{2}=\frac{4\pm2\sqrt{5}}{2}=2\pm\sqrt{5}$$

The zeros of $g(x)=x^2-4x-1$ are $2-\sqrt{5}$ and $2+\sqrt{5}$. The x-intercepts of the graph of g are $2-\sqrt{5}$ and $2+\sqrt{5}$.

42. $g(x)=0$

$x^2+6x+1=0$

$a=1,\quad b=6,\quad c=1$

$$x=\frac{-6\pm\sqrt{6^2-4(1)(1)}}{2(1)}=\frac{-6\pm\sqrt{36-4}}{2}$$
$$=\frac{-6\pm\sqrt{32}}{2}=\frac{-6\pm4\sqrt{2}}{2}=-3\pm2\sqrt{2}$$

The zeros of $g(x)=x^2+6x+1$ are $-3-2\sqrt{2}$ and $-3+2\sqrt{2}$. The x-intercepts of the graph of g are $-3-2\sqrt{2}$ and $-3+2\sqrt{2}$.

43. $F(x)=0$

$2x^2-5x+3=0$

$a=2,\quad b=-5,\quad c=3$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(3)}}{2(2)}=\frac{5\pm\sqrt{25-24}}{4}$$
$$=\frac{5\pm1}{4}=\frac{3}{2}\text{ or }1$$

The zeros of $F(x)=2x^2-5x+3$ are 1 and $\frac{3}{2}$.

The x-intercepts of the graph of F are 1 and $\frac{3}{2}$.

44. $g(x)=0$

$2x^2+5x+3=0$

$a=2,\quad b=5,\quad c=3$

$$x=\frac{-5\pm\sqrt{5^2-4(2)(3)}}{2(2)}=\frac{-5\pm\sqrt{25-24}}{4}$$
$$=\frac{-5\pm1}{4}=-1\text{ or }-\frac{3}{2}$$

The zeros of $g(x)=2x^2+5x+3$ are $-\frac{3}{2}$ and -1.

The x-intercepts of the graph of g are $-\frac{3}{2}$ and -1.

45. $P(x)=0$

$4x^2-x+2=0$

$a=4,\quad b=-1,\quad c=2$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(4)(2)}}{2(4)}=\frac{1\pm\sqrt{1-32}}{8}$$
$$=\frac{1\pm\sqrt{-31}}{8}=\text{not real}$$

The function $P(x)=4x^2-x+2$ has no real zeros, and the graph of P has no x-intercepts.

46. $H(x)=0$

$4x^2+x+1=0$

$a=4,\quad b=1,\quad c=1$

$$t=\frac{-1\pm\sqrt{1^2-4(4)(1)}}{2(4)}=\frac{-1\pm\sqrt{1-16}}{8}$$
$$=\frac{-1\pm\sqrt{-15}}{8}=\text{not real}$$

The function $H(x)=4x^2+x+1$ has no real zeros, and the graph of H has no x-intercepts.

47. $f(x)=0$

$4x^2-1+2x=0$

$4x^2+2x-1=0$

$a=4,\quad b=2,\quad c=-1$

$$x=\frac{-2\pm\sqrt{2^2-4(4)(-1)}}{2(4)}=\frac{-2\pm\sqrt{4+16}}{8}$$
$$=\frac{-2\pm\sqrt{20}}{8}=\frac{-2\pm2\sqrt{5}}{8}=\frac{-1\pm\sqrt{5}}{4}$$

The zeros of $f(x)=4x^2-1+2x$ are $\frac{-1-\sqrt{5}}{4}$ and $\frac{-1+\sqrt{5}}{4}$. The x-intercepts of the graph of f are $\frac{-1-\sqrt{5}}{4}$ and $\frac{-1+\sqrt{5}}{4}$.

48. $f(x)=0$

$2x^2-1+2x=0$

$2x^2+2x-1=0$

$a=2,\quad b=2,\quad c=-1$

$$x=\frac{-2\pm\sqrt{2^2-4(2)(-1)}}{2(2)}=\frac{-2\pm\sqrt{4+8}}{4}$$
$$=\frac{-2\pm\sqrt{12}}{4}=\frac{-2\pm2\sqrt{3}}{4}=\frac{-1\pm\sqrt{3}}{2}$$

The zeros of $f(x) = 2x^2 - 1 + 2x$ are $\frac{-1-\sqrt{3}}{2}$ and $\frac{-1+\sqrt{3}}{2}$. The x-intercepts of the graph of f are $\frac{-1-\sqrt{3}}{2}$ and $\frac{-1+\sqrt{3}}{2}$.

49.
$$G(x) = 0$$
$$2x(x+2) - 3 = 0$$
$$2x^2 + 4x - 3 = 0$$
$$a = 2, \quad b = 4, \quad c = -3$$
$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(2)(-3)}}{2(2)} = \frac{-4 \pm \sqrt{16+24}}{4}$$
$$= \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}$$
The zeros of $G(x) = 2x(x+2) - 3$ are $\frac{-2+\sqrt{10}}{2}$ and $\frac{-2-\sqrt{10}}{2}$. The x-intercepts of the graph of G are $\frac{-2+\sqrt{10}}{2}$ and $\frac{-2-\sqrt{10}}{2}$.

50.
$$F(x) = 0$$
$$3x(x+2) - 1 = 0 \Rightarrow 3x^2 + 6x - 1 = 0$$
$$a = 3, \quad b = 6, \quad c = -1$$
$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(3)(-1)}}{2(3)} = \frac{-6 \pm \sqrt{36+12}}{6}$$
$$= \frac{-6 \pm \sqrt{48}}{6} = \frac{-6 \pm 4\sqrt{3}}{6} = \frac{-3 \pm 2\sqrt{3}}{3}$$
The zeros of $F(x) = 3x(x+2) - 2$ are $\frac{-3+2\sqrt{3}}{3}$ and $\frac{-3-2\sqrt{3}}{3}$. The x-intercepts of the graph of G are $\frac{-3+2\sqrt{3}}{3}$ and $\frac{-3-2\sqrt{3}}{3}$.

51.
$$p(x) = 0$$
$$9x^2 - 6x + 1 = 0$$
$$a = 9, \quad b = -6, \quad c = 1$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)} = \frac{6 \pm \sqrt{36-36}}{18}$$
$$= \frac{6 \pm 0}{18} = \frac{1}{3}$$
The only real zero of $p(x) = 9x^2 - 6x + 1$ is $\frac{1}{3}$.

The only x-intercept of the graph of g is $\frac{1}{3}$.

52.
$$q(x) = 0$$
$$4x^2 + 20x + 25 = 0$$
$$a = 4, \quad b = 20, \quad c = 25$$
$$x = \frac{-20 \pm \sqrt{(20)^2 - 4(4)(25)}}{2(4)} = \frac{-20 \pm \sqrt{400-400}}{8}$$
$$= \frac{-20 \pm 0}{8} = -\frac{20}{8} = -\frac{5}{2}$$
The only real zero of $q(x) = 4x^2 + 20x + 25$ is $-\frac{5}{2}$. The only x-intercept of the graph of F is $-\frac{5}{2}$.

53.
$$f(x) = g(x)$$
$$x^2 + 6x + 3 = 3$$
$$x^2 + 6x = 0 \Rightarrow x(x+6) = 0$$
$$x = 0 \quad \text{or} \quad x + 6 = 0$$
$$x = -6$$
The x-coordinates of the points of intersection are -6 and 0. The y-coordinates are $g(-6) = 3$ and $g(0) = 3$. The graphs of the f and g intersect at the points $(-6, 3)$ and $(0, 3)$.

54.
$$f(x) = g(x)$$
$$x^2 - 4x + 3 = 3$$
$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$x = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = 4$$
The x-coordinates of the points of intersection are 0 and 4. The y-coordinates are $g(0) = 3$ and $g(4) = 3$. The graphs of the f and g intersect at the points $(0, 3)$ and $(4, 3)$.

55.
$$f(x) = g(x)$$
$$-2x^2 + 1 = 3x + 2$$
$$0 = 2x^2 + 3x + 1$$
$$0 = (2x+1)(x+1)$$
$$2x + 1 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = -\frac{1}{2} \qquad x = -1$$
The x-coordinates of the points of intersection are -1 and $-\frac{1}{2}$. The y-coordinates are $g(-1) = 3(-1) + 2 = -3 + 2 = -1$ and

$g\left(-\frac{1}{2}\right)=3\left(-\frac{1}{2}\right)+2=-\frac{3}{2}+2=\frac{1}{2}$.

The graphs of the *f* and *g* intersect at the points $(-1,-1)$ and $\left(-\frac{1}{2},\frac{1}{2}\right)$.

56.
$$f(x)=g(x)$$
$$3x^2-7=10x+1$$
$$3x^2-10x-8=0$$
$$(3x+2)(x-4)=0$$
$$3x+2=0 \quad \text{or} \quad x-4=0$$
$$x=-\frac{2}{3} \qquad x=4$$

The x-coordinates of the points of intersection are $-\frac{2}{3}$ and 4. The y-coordinates are $g\left(-\frac{2}{3}\right)=10\left(-\frac{2}{3}\right)+1=-\frac{20}{3}+1=-\frac{17}{3}$ and $g(4)=10(4)+1=40+1=41$.

The graphs of the *f* and *g* intersect at the points $\left(-\frac{2}{3},-\frac{17}{3}\right)$ and $(4, 41)$.

57.
$$f(x)=g(x)$$
$$x^2-x+1=2x^2-3x-14$$
$$0=x^2-2x-15$$
$$0=(x+3)(x-5)$$
$$x+3=0 \quad \text{or} \quad x-5=0$$
$$x=-3 \qquad x=5$$

The x-coordinates of the points of intersection are -3 and 5. The y-coordinates are

$f(-3)=(-3)^2-(-3)+1=9+3+1=13$ and $f(5)=5^2-5+1=25-5+1=21$.

The graphs of the *f* and *g* intersect at the points $(-3,13)$ and $(5, 21)$.

58.
$$f(x)=g(x)$$
$$x^2+5x-3=2x^2+7x-27$$
$$0=x^2+2x-24$$
$$0=(x+6)(x-4)$$
$$x+6=0 \quad \text{or} \quad x-4=0$$
$$x=-6 \qquad x=4$$

The x-coordinates of the points of intersection are -6 and 4. The y-coordinates are

$f(-6)=(-6)^2+5(-6)-3=36-30-3=3$ and $f(4)=4^2+5(4)-3=16+20-3=33$.

The graphs of the *f* and *g* intersect at the points $(-6, 3)$ and $(4, 33)$.

59.
$$P(x)=0$$
$$x^4-6x^2-16=0$$
$$(x^2+2)(x^2-8)=0$$
$$x^2+2=0 \quad \text{or} \quad x^2-8=0$$
$$x^2=-2 \qquad x^2=8$$
$$x=\pm\sqrt{-2} \qquad x=\pm\sqrt{8}$$
$$=\text{not real} \qquad =\pm 2\sqrt{2}$$

The zeros of $P(x)=x^4-6x^2-16$ are $-2\sqrt{2}$ and $2\sqrt{2}$. The x-intercepts of the graph of *P* are $-2\sqrt{2}$ and $2\sqrt{2}$.

60.
$$H(x)=0$$
$$x^4-3x^2-4=0$$
$$(x^2+1)(x^2-4)=0$$
$$x^2+1=0 \quad \text{or} \quad x^2-4=0$$
$$x^2=-1 \qquad x^2=4$$
$$x=\pm\sqrt{-1} \qquad x=\pm\sqrt{4}$$
$$=\text{not real} \qquad =\pm 2$$

The zeros of $H(x)=x^4-3x^2-4$ are -2 and 2. The x-intercepts of the graph of *H* are -2 and 2.

61.
$$f(x)=0$$
$$x^4-5x^2+4=0$$
$$(x^2-4)(x^2-1)=0$$
$$x^2-4=0 \quad \text{or} \quad x^2-1=0$$
$$x=\pm 2 \quad \text{or} \quad x=\pm 1$$

The zeros of $f(x)=x^4-5x^2+4$ are -2, -1, 1, and 2. The x-intercepts of the graph of *f* are -2, -1, 1, and 2.

62. $f(x)=0$

$x^4-10x^2+24=0$

$(x^2-4)(x^2-6)=0$

$x^2-4=0$ or $x^2-6=0$

$x^2=4$ $\quad$ $x^2=6$

$x=\pm 2$ $\quad$ $x=\pm\sqrt{6}$

The zeros of $f(x)=x^4-10x^2+24$ are $-\sqrt{6}$, $\sqrt{6}$, 2 and -2. The x-intercepts of the graph of f are $-\sqrt{6}$, $\sqrt{6}$, 2 and -2.

63. $G(x)=0$

$3x^4-2x^2-1=0$

$(3x^2+1)(x^2-1)=0$

$3x^2+1=0$ or $x^2-1=0$

$x^2=-\dfrac{1}{3}$ $\quad$ $x^2=1$

$x=\pm\sqrt{-\dfrac{1}{3}}$ $\quad$ $x=\pm\sqrt{1}=\pm 1$

$x=$ not real

The zeros of $G(x)=3x^4-2x^2-1$ are -1 and 1. The x-intercepts of the graph of G are -1 and 1.

64. $F(x)=0$

$2x^4-5x^2-12=0$

$(2x^2+3)(x^2-4)=0$

$2x^2+3=0$ or $x^2-4=0$

$x^2=-\dfrac{3}{2}$ $\quad$ $x^2=4$

$x=\pm\sqrt{-\dfrac{3}{2}}$ $\quad$ $x=\pm\sqrt{4}=\pm 2$

$=$ not real

The zeros of $F(x)=2x^4-5x^2-12$ are -2 and 2. The x-intercepts of the graph of F are -2 and 2.

65. $g(x)=0$

$x^6+7x^3-8=0$

$(x^3+8)(x^3-1)=0$

$x^3+8=0$ or $x^3-1=0$

$x^3=-8$ $\quad$ $x^3=1$

$x=-2$ $\quad$ $x=1$

The zeros of $g(x)=x^6+7x^3-8$ are -2 and 1. The x-intercepts of the graph of g are -2 and 1.

66. $g(x)=0$

$x^6-7x^3-8=0$

$(x^3-8)(x^3+1)=0$

$x^3-8=0$ or $x^3+1=0$

$x^3=8$ $\quad$ $x^3=-1$

$x=2$ $\quad$ $x=-1$

The zeros of $g(x)=x^6-7x^3-8$ are -1 and 2. The x-intercepts of the graph of g are -1 and 2.

67. $G(x)=0$

$(x+2)^2+7(x+2)+12=0$

Let $u=x+2 \to u^2=(x+2)^2$

$u^2+7u+12=0$

$(u+3)(u+4)=0$

$u+3=0$ or $u+4=0$

$u=-3$ $\quad$ $u=-4$

$x+2=-3$ $\quad$ $x+2=-4$

$x=-5$ $\quad$ $x=-6$

The zeros of $G(x)=(x+2)^2+7(x+2)+12$ are -6 and -5. The x-intercepts of the graph of G are -6 and -5.

68. $f(x)=0$

$(2x+5)^2-(2x+5)-6=0$

Let $u=2x+5 \to u^2=(2x+5)^2$

$u^2-u-6=0$

$(u-3)(u+2)=0$

$u-3=0$ or $u+2=0$

$u=3$ $\quad$ $u=-2$

$2x+5=3$ $\quad$ $2x+5=-2$

$x=-1$ $\quad$ $x=-\dfrac{7}{2}$

The zeros of $f(x)=(2x+5)^2-(2x+5)-6$ are $-\dfrac{7}{2}$ and -1. The x-intercepts of the graph of f are $-\dfrac{7}{2}$ and -1.

69.
$$f(x)=0$$
$$(3x+4)^2-6(3x+4)+9=0$$
Let $u=3x+4 \to u^2=(3x+4)^2$
$$u^2-6u+9=0$$
$$(u-3)^2=0$$
$$u-3=0$$
$$u=3$$
$$3x+4=3$$
$$x=-\frac{1}{3}$$
The only zero of $f(x)=(3x+4)^2-6(3x+4)+9$ is $-\frac{1}{3}$. The x-intercept of the graph of f is $-\frac{1}{3}$.

70.
$$H(x)=0$$
$$(2-x)^2+(2-x)-20=0$$
Let $u=2-x \to u^2=(2-x)^2$
$$u^2+u-20=0$$
$$(u+5)(u-4)=0$$
$u+5=0$ or $u-4=0$
$u=-5$ $u=4$
$2-x=-5$ $2-x=4$
$x=7$ $x=-2$
The zeros of $H(x)=(2-x)^2+(2-x)-20$ are -2 and 7. The x-intercepts of the graph of H are -2 and 7.

71.
$$P(x)=0$$
$$2(x+1)^2-5(x+1)-3=0$$
Let $u=x+1 \to u^2=(x+1)^2$
$$2u^2-5u-3=0$$
$$(2u+1)(u-3)=0$$
$2u+1=0$ or $u-3=0$
$u=-\frac{1}{2}$ $u=3$
$x+1=-\frac{1}{2}$ $x+1=3$
$x=-\frac{3}{2}$ $x=2$
The zeros of $P(x)=2(x+1)^2-5(x+1)-3$ are $-\frac{3}{2}$ and 2. The x-intercepts of the graph of P are $-\frac{3}{2}$ and 2.

72.
$$H(x)=0$$
$$3(1-x)^2+5(1-x)+2=0$$
Let $u=1-x \to u^2=(1-x)^2$
$$3u^2+5u+2=0$$
$$(3u+2)(u+1)=0$$
$3u+2=0$ or $u+1=0$
$u=-\frac{2}{3}$ $u=-1$
$1-x=-\frac{2}{3}$ $1-x=-1$
$x=\frac{5}{3}$ $x=2$
The zeros of $H(x)=3(1-x)^2+5(1-x)+2$ are $\frac{5}{3}$ and 2. The x-intercepts of the graph of H are $\frac{5}{3}$ and 2.

73.
$$G(x)=0$$
$$x-4\sqrt{x}=0$$
Let $u=\sqrt{x} \to u^2=x$
$$u^2-4u=0$$
$$u(u-4)=0$$
$u=0$ or $u-4=0$
$u=4$
$\sqrt{x}=0$ $\sqrt{x}=4$
$x=0^2=0$ $x=4^2=16$

Check:
$G(0)=0-4\sqrt{0}=0$
$G(16)=16-4\sqrt{16}=16-16=0$
The zeros of $G(x)=x-4\sqrt{x}$ are 0 and 16. The x-intercepts of the graph of G are 0 and 16.

74. $f(x)=0$

$x+8\sqrt{x}=0$

Let $u=\sqrt{x}\to u^2=x$

$u^2+8u=0$

$u(u+8)=0$

$u=0$ or $u+8=0$

$u=-8$

$\sqrt{x}=0$ $\quad\sqrt{x}=-8$

$x=0^2=0$ $\quad x=\text{not real}$

Check: $f(0)=0+8\sqrt{0}=0$

The only zero of $f(x)=x+8\sqrt{x}$ is 0. The only x-intercept of the graph of f is 0.

75. $g(x)=0$

$x+\sqrt{x}-20=0$

Let $u=\sqrt{x}\to u^2=x$

$u^2+u-20=0$

$(u+5)(u-4)=0$

$u+5=0$ or $u-4=0$

$u=-5$ $\quad u=4$

$\sqrt{x}=-5$ $\quad\sqrt{x}=4$

$x=\text{not real}$ $\quad x=4^2=16$

Check: $g(16)=16+\sqrt{16}-20=16+4-20=0$

The only zero of $g(x)=x+\sqrt{x}-20$ is 16. The only x-intercept of the graph of g is 16.

76. $f(x)=0$

$x+\sqrt{x}-2=0$

Let $u=\sqrt{x}\to u^2=x$

$u^2+u-2=0$

$(u-1)(u+2)=0$

$u-1=0$ or $u+2=0$

$u=1$ $\quad u=-2$

$\sqrt{x}=1$ $\quad\sqrt{x}=-2$

$x=1^2=1$ $\quad x=\text{not real}$

Check: $f(1)=1+\sqrt{1}-2=1+1-2=0$

The only zero of $f(x)=x+\sqrt{x}-2$ is 1. The only x-intercept of the graph of f is 1.

77. $f(x)=0$

$x^2-50=0$

$x^2=50\Rightarrow x=\pm\sqrt{50}=\pm5\sqrt{2}$

The zeros of $f(x)=x^2-50$ are $-5\sqrt{2}$ and $5\sqrt{2}$. The x-intercepts of the graph of f are $-5\sqrt{2}$ and $5\sqrt{2}$.

78. $f(x)=0$

$x^2-20=0$

$x^2=20\Rightarrow x=\pm\sqrt{20}=\pm2\sqrt{5}$

The zeros of $f(x)=x^2-6$ are $-2\sqrt{5}$ and $2\sqrt{5}$. The x-intercepts of the graph of f are $-2\sqrt{5}$ and $2\sqrt{5}$.

79. $g(x)=0$

$16x^2-8x+1=0$

$(4x-1)^2=0$

$4x-1=0\Rightarrow x=\frac{1}{4}$

The only real zero of $g(x)=16x^2-8x+1$ is $\frac{1}{4}$.

The only x-intercept of the graph of g is $\frac{1}{4}$.

80. $F(x)=0$

$4x^2-12x+9=0$

$(2x-3)^2=0$

$2x-3=0\Rightarrow x=\frac{3}{2}$

The only real zero of $F(x)=4x^2-12x+9$ is $\frac{3}{2}$.

The only x-intercept of the graph of F is $\frac{3}{2}$.

81. $G(x)=0$

$10x^2-19x-15=0$

$(5x+3)(2x-5)=0$

$5x+3=0$ or $2x-5=0$

$x=-\frac{3}{5}$ $\quad x=\frac{5}{2}$

The zeros of $G(x)=10x^2-19x-15$ are $-\frac{3}{5}$ and

$\frac{5}{2}$. The x-intercepts of the graph of G are $-\frac{3}{5}$ and $\frac{5}{2}$.

82.
$$f(x)=0$$
$$6x^2+7x-20=0$$
$$(3x-4)(2x+5)=0$$
$$3x-4=0 \quad \text{or} \quad 2x+5=0$$
$$x=\frac{4}{3} \qquad x=-\frac{5}{2}$$

The zeros of $f(x)=6x^2+7x-20$ are $-\frac{5}{2}$ and $\frac{4}{3}$.

The x-intercepts of the graph of f are $-\frac{5}{2}$ and $\frac{4}{3}$.

83.
$$P(x)=0$$
$$6x^2-x-2=0$$
$$(3x-2)(2x+1)=0$$
$$3x-2=0 \quad \text{or} \quad 2x+1=0$$
$$x=\frac{2}{3} \qquad x=-\frac{1}{2}$$

The zeros of $P(x)=6x^2-x-2$ are $-\frac{1}{2}$ and $\frac{2}{3}$.

The x-intercepts of the graph of P are $-\frac{1}{2}$ and $\frac{2}{3}$.

84.
$$H(x)=0$$
$$6x^2+x-2=0$$
$$(3x+2)(2x-1)=0$$
$$3x+2=0 \quad \text{or} \quad 2x-1=0$$
$$x=-\frac{2}{3} \qquad x=\frac{1}{2}$$

The zeros of $H(x)=6x^2+x-2$ are $-\frac{2}{3}$ and $\frac{1}{2}$.

The x-intercepts of the graph of H are $-\frac{2}{3}$ and $\frac{1}{2}$.

85.
$$G(x)=0$$
$$x^2+\sqrt{2}x-\frac{1}{2}=0$$
$$2\left(x^2+\sqrt{2}x-\frac{1}{2}\right)=(0)(2)$$
$$2x^2+2\sqrt{2}x-1=0$$
$$a=2,\quad b=2\sqrt{2},\quad c=-1$$
$$x=\frac{-(2\sqrt{2})\pm\sqrt{(2\sqrt{2})^2-4(2)(-1)}}{2(2)}$$
$$=\frac{-2\sqrt{2}\pm\sqrt{8+8}}{4}=\frac{-2\sqrt{2}\pm\sqrt{16}}{4}$$
$$=\frac{-2\sqrt{2}\pm 4}{4}=\frac{-\sqrt{2}\pm 2}{2}$$

The zeros of $G(x)=x^2+\sqrt{2}x-\frac{1}{2}$ are $\frac{-\sqrt{2}-2}{2}$ and $\frac{-\sqrt{2}+2}{2}$. The x-intercepts of the graph of G are $\frac{-\sqrt{2}-2}{2}$ and $\frac{-\sqrt{2}+2}{2}$.

86.
$$F(x)=0$$
$$\frac{1}{2}x^2-\sqrt{2}x-1=0$$
$$2\left(\frac{1}{2}x^2-\sqrt{2}x-1\right)=(0)(2)$$
$$x^2-2\sqrt{2}x-2=0$$
$$a=1,\quad b=-2\sqrt{2},\quad c=-2$$
$$x=\frac{-(-2\sqrt{2})\pm\sqrt{(-2\sqrt{2})^2-4(1)(-2)}}{2(1)}$$
$$=\frac{2\sqrt{2}\pm\sqrt{16}}{2}=\frac{2\sqrt{2}\pm 4}{2}=\frac{\sqrt{2}\pm 2}{1}$$

The zeros of $F(x)=\frac{1}{2}x^2-\sqrt{2}x-1$ are $\sqrt{2}-2$ and $\sqrt{2}+2$. The x-intercepts of the graph of F are $\sqrt{2}-2$ and $\sqrt{2}+2$.

87.
$$f(x)=0$$
$$x^2+x-4=0$$
$$a=1,\quad b=1,\quad c=-4$$
$$x=\frac{-(1)\pm\sqrt{(1)^2-4(1)(-4)}}{2(1)}$$
$$=\frac{-1\pm\sqrt{1+16}}{2}=\frac{-1\pm\sqrt{17}}{2}$$

The zeros of $f(x)=x^2+x-4$ are $\frac{-1-\sqrt{17}}{2}$ and $\frac{-1+\sqrt{17}}{2}$. The x-intercepts of the graph of f are $\frac{-1-\sqrt{17}}{2}$ and $\frac{-1+\sqrt{17}}{2}$.

88. $g(x)=0$

$x^2+x-1=0$

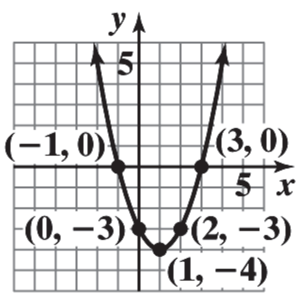
$a=1,\quad b=1,\quad c=-1$

$$x=\frac{-(1)\pm\sqrt{(1)^2-4(1)(-1)}}{2(1)}=\frac{-1\pm\sqrt{5}}{2}$$

The zeros of $g(x)=x^2+x-1$ are $\frac{-1-\sqrt{5}}{2}$ and $\frac{-1+\sqrt{5}}{2}$. The x-intercepts of the graph of g are $\frac{-1-\sqrt{5}}{2}$ and $\frac{-1+\sqrt{5}}{2}$.

89. **a.** $g(x)=(x-1)^2-4$

Using the graph of $y=x^2$, horizontally shift to the right 1 unit, and then vertically shift downward 4 units.



b. $g(x)=0$

$(x-1)^2-4=0$

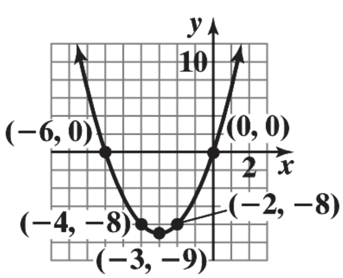
$x^2-2x+1-4=0$

$x^2-2x-3=0$

$(x+1)(x-3)=0\Rightarrow x=-1$ or $x=3$

90. **a.** $F(x)=(x+3)^2-9$

Using the graph of $y=x^2$, horizontally shift to the left 3 units, and then vertically shift downward 9 units.



b. $F(x)=0$

$(x+3)^2-9=0$

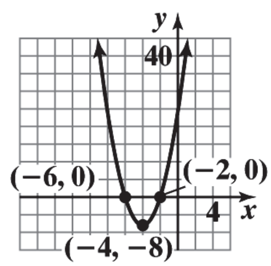
$x^2+6x+9-9=0$

$x^2+6x=0$

$x(x+6)=0\Rightarrow x=0$ or $x=-6$

91. **a.** $f(x)=2(x+4)^2-8$

Using the graph of $y=x^2$, horizontally shift to the left 4 units, vertically stretch by a factor of 2, and then vertically shift downward 8 units.



b. $f(x)=0$

$2(x+4)^2-8=0$

$2(x^2+8x+16)-8=0$

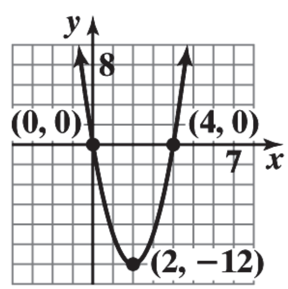
$2x^2+16x+32-8=0$

$2x^2+16x+24=0$

$2(x+2)(x+6)=0\Rightarrow x=-2$ or $x=-6$

92. **a.** $h(x)=3(x-2)^2-12$

Using the graph of $y=x^2$, horizontally shift to the right 2 units, vertically stretch by a factor of 3, and then vertically shift downward 12 units.

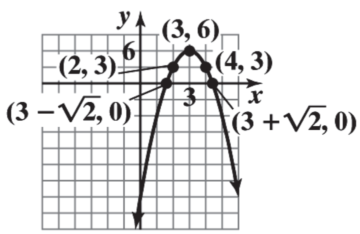


b. $h(x)=0$

$$3(x-2)^2-12=0$$
$$3(x^2-4x+4)-12=0$$
$$3x^2-12x+12-12=0$$
$$3x^2-12x=0$$
$$3x(x-4)=0 \Rightarrow x=0 \text{ or } x=4$$

93. a. $H(x)=-3(x-3)^2+6$

Using the graph of $y=x^2$, horizontally shift to the right 3 units, vertically stretch by a factor of 3, reflect about the x-axis, and then vertically shift upward 6 units.



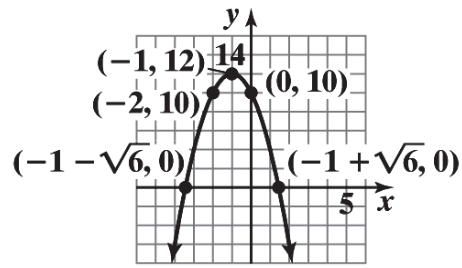
b. $H(x)=0$

$$-3(x-3)^2+6=0$$
$$-3(x^2-6x+9)+6=0$$
$$-3x^2+18x-27+6=0$$
$$-3x^2+18x-21=0$$
$$-3(x^2-6x+7)=0$$
$$a=1,\quad b=-6,\quad c=7$$

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(7)}}{2(1)}=\frac{6\pm\sqrt{36-28}}{2}$$
$$=\frac{6\pm\sqrt{8}}{2}=\frac{6\pm2\sqrt{2}}{2}=3\pm\sqrt{2}$$

94. a. $f(x)=-2(x+1)^2+12$

Using the graph of $y=x^2$, horizontally shift to the left 1 unit, vertically stretch by a factor of 2, reflect about the x-axis, and then vertically shift upward 12 units.



b. $f(x)=0$

$$-2(x+1)^2+12=0$$
$$-2(x^2+2x+1)+12=0$$
$$-2x^2-4x-2+12=0$$
$$-2x^2-4x+10=0$$
$$-2(x^2+2x-5)=0$$
$$a=1,\quad b=2,\quad c=-5$$

$$x=\frac{-(2)\pm\sqrt{(2)^2-4(1)(-5)}}{2(1)}=\frac{-2\pm\sqrt{4+20}}{2}$$
$$=\frac{-2\pm\sqrt{24}}{2}=\frac{-2\pm2\sqrt{6}}{2}=-1\pm\sqrt{6}$$

95. $f(x)=g(x)$

$$5x(x-1)=-7x^2+2$$
$$5x^2-5x=-7x^2+2$$
$$12x^2-5x-2=0$$
$$(3x-2)(4x+1)=0\Rightarrow x=\frac{2}{3} \text{ or } x=-\frac{1}{4}$$
$$f\left(\frac{2}{3}\right)=5\left(\frac{2}{3}\right)\left[\left(\frac{2}{3}\right)-1\right]$$
$$=\left(\frac{10}{3}\right)\left(-\frac{1}{3}\right)=-\frac{10}{9}$$
$$f\left(-\frac{1}{4}\right)=5\left(-\frac{1}{4}\right)\left[\left(-\frac{1}{4}\right)-1\right]$$
$$=\left(-\frac{5}{4}\right)\left(-\frac{5}{4}\right)=\frac{25}{16}$$

The points of intersection are:
$\left(\frac{2}{3},-\frac{10}{9}\right)$ and $\left(-\frac{1}{4},\frac{25}{16}\right)$

96.
$$f(x) = g(x)$$
$$10x(x+2) = -3x+5$$
$$10x^2 + 20x = -3x+5$$
$$10x^2 + 23x - 5 = 0$$
$$(2x+5)(5x-1) = 0 \Rightarrow x = -\frac{5}{2} \text{ or } x = \frac{1}{5}$$
$$f\left(-\frac{5}{2}\right) = 10\left(-\frac{5}{2}\right)\left[\left(-\frac{5}{2}\right)+2\right]$$
$$= (-25)\left(-\frac{1}{2}\right) = \frac{25}{2}$$
$$f\left(\frac{1}{5}\right) = 10\left(\frac{1}{5}\right)\left[\left(\frac{1}{5}\right)+2\right]$$
$$= (2)\left(\frac{11}{5}\right) = \frac{22}{5}$$

The points of intersection are:
$\left(-\frac{5}{2}, \frac{25}{2}\right)$ and $\left(\frac{1}{5}, \frac{22}{5}\right)$

97.
$$f(x) = g(x)$$
$$3(x^2-4) = 3x^2+2x+4$$
$$3x^2 - 12 = 3x^2+2x+4$$
$$-12 = 2x+4$$
$$-16 = 2x \Rightarrow x = -8$$
$$f(-8) = 3\left[(-8)^2 - 4\right]$$
$$= 3[64-4] = 180$$

The point of intersection is: $(-8, 180)$

98.
$$f(x) = g(x)$$
$$4(x^2+1) = 4x^2 - 3x - 8$$
$$4x^2 + 4 = 4x^2 - 3x - 8$$
$$4 = -3x - 8$$
$$12 = -3x \Rightarrow x = -4$$
$$f(-4) = 4\left[(-4)^2 + 1\right]$$
$$= 4[16+1] = 68$$

The point of intersection is: $(-4, 68)$

99.
$$f(x) = g(x)$$
$$\frac{3x}{x+2} - \frac{5}{x+1} = \frac{-5}{x^2+3x+2}$$
$$\frac{3x}{x+2} - \frac{5}{x+1} = \frac{-5}{(x+2)(x+1)}$$
$$3x(x+1) - 5(x+2) = -5$$
$$3x^2 + 3x - 5x - 10 = -5$$
$$3x^2 - 2x - 5 = 0$$
$$(3x-5)(x+1) = 0$$
$$x = \frac{5}{3} \text{ or } \cancel{x = -1}$$
$$f\left(\frac{5}{3}\right) = \frac{3\left(\frac{5}{3}\right)}{\left(\frac{5}{3}\right)+2} - \frac{5}{\left(\frac{5}{3}\right)+1}$$
$$= \frac{(5)}{\left(\frac{11}{3}\right)} - \frac{5}{\left(\frac{8}{3}\right)}$$
$$= \frac{15}{11} - \frac{15}{8}$$
$$= -\frac{45}{88}$$

The point of intersection is: $\left(\frac{5}{3}, -\frac{45}{88}\right)$

100.
$$f(x) = g(x)$$
$$\frac{2x}{x-3} - \frac{3}{x+1} = \frac{2x+18}{x^2-2x-3}$$
$$\frac{2x}{x-3} - \frac{3}{x+1} = \frac{2x+18}{(x-3)(x+1)}$$
$$2x(x+1) - 3(x-3) = 2x+18$$
$$2x^2 + 2x - 3x + 9 = 2x + 18$$
$$2x^2 - 3x - 9 = 0$$
$$(2x+3)(x-3) = 0$$
$$x = -\frac{3}{2} \text{ or } \cancel{x = 3}$$

$$f\left(-\frac{3}{2}\right)=\frac{2\left(-\frac{3}{2}\right)}{\left(-\frac{3}{2}\right)-3}-\frac{3}{\left(-\frac{3}{2}\right)+1}$$
$$=\frac{(-3)}{\left(-\frac{9}{2}\right)}-\frac{3}{\left(-\frac{1}{2}\right)}$$
$$=\frac{6}{9}+6=\frac{2}{3}+6=\frac{20}{3}$$

The point of intersection is: $\left(-\frac{3}{2},\frac{20}{3}\right)$

101. a. $(f+g)(x)=$
$$=x^2+5x-14+x^2+3x-4$$
$$=2x^2+8x-18$$
$$2x^2+8x-18=0$$
$$x^2+4x-9=0$$
$$x=\frac{-(4)\pm\sqrt{(4)^2-4(1)(-9)}}{2(1)}=\frac{-4\pm\sqrt{16+36}}{2}$$
$$=\frac{-4\pm\sqrt{52}}{2}=\frac{-4\pm2\sqrt{13}}{2}=-2\pm\sqrt{13}$$

b. $(f-g)(x)=$
$$=\left(x^2+5x-14\right)-\left(x^2+3x-4\right)$$
$$=x^2+5x-14-x^2-3x+4$$
$$=2x-10$$
$$2x-10=0\Rightarrow x=5$$

c. $(f\cdot g)(x)=$
$$=\left(x^2+5x-14\right)\left(x^2+3x-4\right)$$
$$=(x+7)(x-2)(x+4)(x-1)$$

$$(f\cdot g)(x)=0$$
$$0=(x+7)(x-2)(x+4)(x-1)$$
$$\Rightarrow x=-7 \text{ or } x=2 \text{ or } x=-4 \text{ or } x=1$$

102. a. $(f+g)(x)=$
$$=x^2-3x-18+x^2+2x-3$$
$$=2x^2-x-21$$
$$2x^2-x-21=0$$
$$(2x-7)(x+3)=0\Rightarrow x=\frac{7}{2} \text{ or } x=-3$$

b. $(f-g)(x)=$
$$=\left(x^2-3x-18\right)-\left(x^2+2x-3\right)$$
$$=x^2-3x-18-x^2-2x+3$$
$$=-5x-15$$
$$-5x-15=0\Rightarrow x=-3$$

c. $(f\cdot g)(x)=$
$$=\left(x^2-3x-18\right)\left(x^2+2x-3\right)$$
$$=(x+3)(x-6)(x+3)(x-1)$$
$$(f\cdot g)(x)=0$$
$$0=(x+3)(x-6)(x+3)(x-1)$$
$$\Rightarrow x=-3 \text{ or } x=6 \text{ or } x=1$$

103.
$$A(x)=143$$
$$x(x+2)=143$$
$$x^2+2x-143=0$$
$$(x+13)(x-11)=0$$
~~$x=-13$~~ or $x=11$

Discard the negative solution since width cannot be negative. The width of the rectangular window is 11 feet and the length is 13 feet.

104.
$$A(x)=306$$
$$x(x+1)=306$$
$$x^2+x-306=0$$
$$(x+18)(x-17)=0$$
~~$x=-18$~~ or $x=17$

Discard the negative solution since width cannot be negative. The width of the rectangular window is 17 cm and the length is 18 cm.

105.
$$V(x)=4$$
$$(x-2)^2=4$$
$$x-2=\pm\sqrt{4}$$
$$x-2=\pm2$$
$$x=2\pm2$$
$x=4$ or $x=0$

Discard $x=0$ since that is not a feasible length for the original sheet. Therefore, the original sheet should measure 4 feet on each side.

106. $V(x) = 4$

$(x-2)^2 = 16$

$x - 2 = \pm\sqrt{16}$

$x - 2 = \pm 4$

$x = 2 \pm 4$

$x = 6$ or ~~$x = -2$~~

Discard $x = -2$ since width cannot be negative. Therefore, the original sheet should measure 6 feet on each side.

107. **a.** When the ball strikes the ground, the distance from the ground will be 0. Therefore, we solve

$s = 0$

$96 + 80t - 16t^2 = 0$

$-16t^2 + 80t + 96 = 0$

$t^2 - 5t - 6 = 0$

$(t-6)(t+1) = 0$

$t = 6$ or ~~$t = -1$~~

Discard the negative solution since the time of flight must be positive. The ball will strike the ground after 6 seconds.

b. When the ball passes the top of the building, it will be 96 feet from the ground. Therefore, we solve

$s = 96$

$96 + 80t - 16t^2 = 96$

$-16t^2 + 80t = 0$

$t^2 - 5t = 0$

$t(t-5) = 0$

$t = 0$ or $t = 5$

The ball is at the top of the building at time $t = 0$ seconds when it is thrown. It will pass the top of the building on the way down after 5 seconds.

108. **a.** To find when the object will be 15 meters above the ground, we solve

$s = 15$

$-4.9t^2 + 20t = 15$

$-4.9t^2 + 20t - 15 = 0$

$a = -4.9,\ b = 20,\ c = -15$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-15)}}{2(-4.9)}$$

$$= \frac{-20 \pm \sqrt{106}}{-9.8}$$

$$= \frac{20 \pm \sqrt{106}}{9.8}$$

$t \approx 0.99$ or $t \approx 3.09$

The object will be 15 meters above the ground after about 0.99 seconds (on the way up) and about 3.09 seconds (on the way down).

b. The object will strike the ground when the distance from the ground is 0. Thus, we solve

$s = 0$

$-4.9t^2 + 20t = 0$

$t(-4.9t + 20) = 0$

$t = 0$ or $-4.9t + 20 = 0$

$-4.9t = -20$

$t \approx 4.08$

The object will strike the ground after about 4.08 seconds.

c. $s = 100$

$-4.9t^2 + 20t = 100$

$-4.9t^2 + 20t - 100 = 0$

$a = -4.9,\ b = 20,\ c = -100$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)(-100)}}{2(-4.9)}$$

$$= \frac{-20 \pm \sqrt{-1560}}{-9.8}$$

There is no real solution. The object never reaches a height of 100 meters.

109. For the sum to be 210, we solve

$S(n) = 210$

$\frac{1}{2}n(n+1) = 210$

$n(n+1) = 420$

$n^2 + n - 420 = 0$

$(n-20)(n+21) = 0$

$n - 20 = 0$ or $n + 21 = 0$

$n = 20$ ~~$n = -21$~~

Discard the negative solution since the number of consecutive integers must be positive. For a sum of 210, we must add the 20 consecutive integers, starting at 1.

110. To determine the number of sides when a polygon has 65 diagonals, we solve

$$D(n) = 65$$
$$\frac{1}{2}n(n-3) = 65$$
$$n(n-3) = 130$$
$$n^2 - 3n - 130 = 0$$
$$(n+10)(n-13) = 0$$
$$n+10 = 0 \quad \text{or} \quad n-13 = 0$$
$$\cancel{n = -10} \qquad n = 13$$

Discard the negative solution since the number of sides must be positive. A polygon with 65 diagonals will have 13 sides.

To determine the number of sides if a polygon has 80 diagonals, we solve

$$D(n) = 80$$
$$\frac{1}{2}n(n-3) = 80$$
$$n(n-3) = 160$$
$$n^2 - 3n - 160 = 0$$
$$a = 1,\ b = -3,\ c = -160$$
$$t = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-160)}}{2(1)}$$
$$= \frac{3 \pm \sqrt{649}}{2}$$

Since the solutions are not integers, a polygon with 80 diagonals is not possible.

111. The roots of a quadratic equation are

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a},$$

so the sum of the roots is

$$x_1 + x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} + \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-b - \sqrt{b^2 - 4ac} - b + \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2b}{2a} = -\frac{b}{a}$$

112. The roots of a quadratic equation are

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a},$$

so the product of the roots is

$$x_1 \cdot x_2 = \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)$$
$$= \frac{(-b)^2 - \left(\sqrt{b^2 - 4ac}\right)^2}{(2a)^2} = \frac{b^2 - (b^2 - 4ac)}{4a^2}$$
$$= \frac{b^2 - b^2 + 4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

113. In order to have one repeated real zero, we need the discriminant to be 0.

$$b^2 - 4ac = 0$$
$$1^2 - 4(k)(k) = 0$$
$$1 - 4k^2 = 0$$
$$4k^2 = 1$$
$$k^2 = \frac{1}{4}$$
$$k = \pm\sqrt{\frac{1}{4}}$$
$$k = \frac{1}{2} \quad \text{or} \quad k = -\frac{1}{2}$$

114. In order to have one repeated real zero, we need the discriminant to be 0.

$$b^2 - 4ac = 0$$
$$(-k)^2 - 4(1)(4) = 0$$
$$k^2 - 16 = 0$$
$$(k-4)(k+4) = 0$$
$$k = 4 \quad \text{or} \quad k = -4$$

115. For $f(x) = ax^2 + bx + c = 0$:

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

For $f(x) = ax^2 - bx + c = 0$:

$$x_1^* = \frac{-(-b) - \sqrt{(-b)^2 - 4ac}}{2a}$$
$$= \frac{b - \sqrt{b^2 - 4ac}}{2a} = -\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right) = -x_2$$

and

$$x_2^* = \frac{-(-b)+\sqrt{(-b)^2-4ac}}{2a}$$

$$= \frac{b+\sqrt{b^2-4ac}}{2a} = -\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) = -x_1$$

116. For $f(x) = ax^2+bx+c=0$:

$$x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

For $f(x) = cx^2+bx+a=0$:

$$x_1^* = \frac{-b-\sqrt{b^2-4(c)(a)}}{2c} = \frac{-b-\sqrt{b^2-4ac}}{2c}$$

$$= \frac{-b-\sqrt{b^2-4ac}}{2c}\cdot\frac{-b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}$$

$$= \frac{b^2-(b^2-4ac)}{2c\left(-b+\sqrt{b^2-4ac}\right)} = \frac{4ac}{2c\left(-b+\sqrt{b^2-4ac}\right)}$$

$$= \frac{2a}{-b+\sqrt{b^2-4ac}} = \frac{1}{x_2}$$

and

$$x_2^* = \frac{-b+\sqrt{b^2-4(c)(a)}}{2c} = \frac{-b+\sqrt{b^2-4ac}}{2c}$$

$$= \frac{-b+\sqrt{b^2-4ac}}{2c}\cdot\frac{-b-\sqrt{b^2-4ac}}{-b-\sqrt{b^2-4ac}}$$

$$= \frac{b^2-(b^2-4ac)}{2c\left(-b-\sqrt{b^2-4ac}\right)} = \frac{4ac}{2c\left(-b-\sqrt{b^2-4ac}\right)}$$

$$= \frac{2a}{-b-\sqrt{b^2-4ac}} = \frac{1}{x_1}$$

117. **a.** $x^2=9$ and $x=3$ are not equivalent because they do not have the same solution set. In the first equation we can also have $x=-3$.

b. $x=\sqrt{9}$ and $x=3$ are equivalent because $\sqrt{9}=3$.

c. $(x-1)(x-2)=(x-1)^2$ and $x-2=x-1$ are not equivalent because they do not have the same solution set. The first equation has the solution set $\{1\}$ while the second equation has no solutions.

118. Answers may vary. Methods discussed in this section include factoring, the square root method, completing the square, and the quadratic formula.

119. Answers will vary. Knowing the discriminant allows us to know how many real solutions the equation will have.

120. Answers will vary. One possibility:

Two distinct: $f(x)=x^2-3x-18$

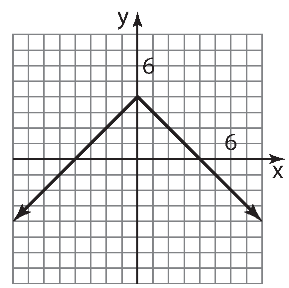
One repeated: $f(x)=x^2-14x+49$

No real: $f(x)=x^2+x+4$

121. Answers will vary.

122. Two quadratic functions can intersect 0, 1, or 2 times.

123. The graph is shifted vertically by 4 units and is reflected about the x-axis.



124. Domain: $\{-3,-1,1,3\}$ Range: $\{2,4\}$

125. $\bar{x} = \frac{-10+2}{2} = \frac{-8}{2} = -4$

$\bar{y} = \frac{4+(-1)}{2} = \frac{3}{2}$

So the midpoint is: $\left(-4, \frac{3}{2}\right)$.

126. If the graph is symmetric with respect to the y-axis then x and –x are on the graph. Thus if $(-1,4)$ is on the graph, then so is $(1,4)$.

Section 2.4

1. $y = x^2 - 9$
   To find the y-intercept, let $x = 0$:
   $y = 0^2 - 9 = -9$.
   To find the x-intercept(s), let $y = 0$:
   $x^2 - 9 = 0$
   $x^2 = 9$
   $x = \pm\sqrt{9} = \pm 3$
   The intercepts are $(0,-9), (-3, 0),$ and $(3, 0)$.

2. $2x^2 + 7x - 4 = 0$
   $(2x-1)(x+4) = 0$
   $2x - 1 = 0$ or $x + 4 = 0$
   $2x = 1$ or $x = -4$
   $x = \frac{1}{2}$ or $x = -4$
   The solution set is $\left\{-4, \frac{1}{2}\right\}$..

3. $\left(\frac{1}{2}\cdot(-5)\right)^2 = \frac{25}{4}$

4. right; 4

5. parabola

6. axis (or axis of symmetry)

7. $-\frac{b}{2a}$

8. True; $a = 2 > 0$.

9. True; $-\frac{b}{2a} = -\frac{4}{2(-1)} = 2$

10. True

11. a

12. d

13. C

14. E

15. F

16. A

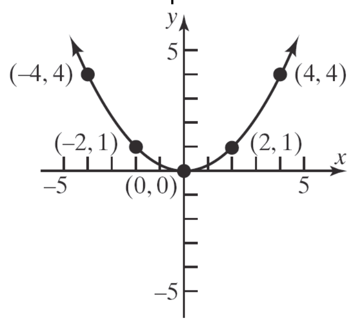
17. G

18. B

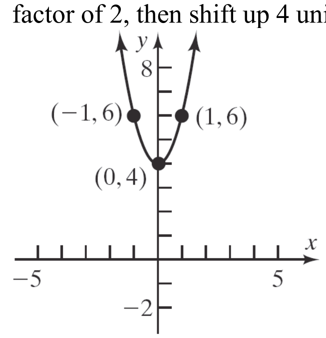
19. H

20. D

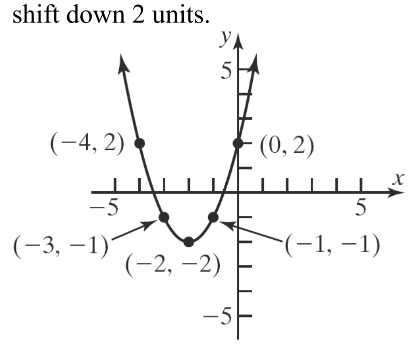
21. $f(x) = \frac{1}{4}x^2$
    Using the graph of $y = x^2$, compress vertically by a factor of $\frac{1}{4}$.



22. $f(x) = 2x^2 + 4$
    Using the graph of $y = x^2$, stretch vertically by a factor of 2, then shift up 4 units.

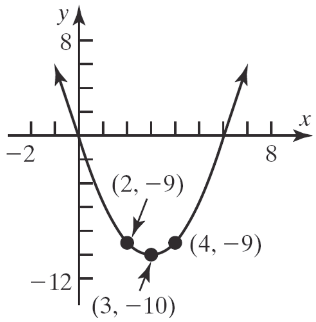


23. $f(x) = (x+2)^2 - 2$
    Using the graph of $y = x^2$, shift left 2 units, then shift down 2 units.



24. $f(x) = (x-3)^2 - 10$

Using the graph of $y = x^2$, shift right 3 units, then shift down 10 units.

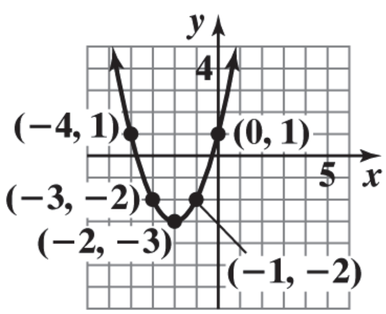


25. $f(x) = x^2 + 4x + 1$

$= (x^2 + 4x + 4) + 1 - 4$

$= (x+2)^2 - 3$

Using the graph of $y = x^2$, shift left 2 units, then shift down 3 units.

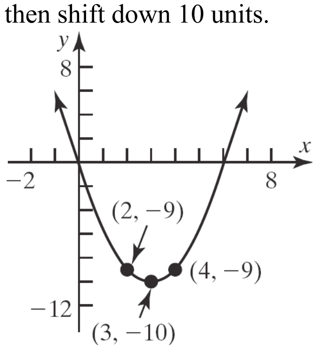


26. $f(x) = x^2 - 6x - 1$

$= (x^2 - 6x + 9) - 1 - 9$

$= (x-3)^2 - 10$

Using the graph of $y = x^2$, shift right 3 units, then shift down 10 units.



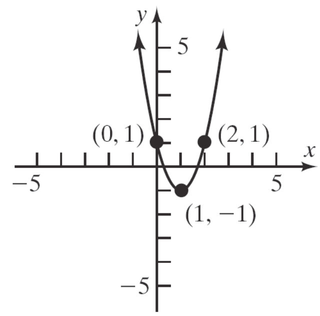
27. $f(x) = 2x^2 - 4x + 1$

$= 2\left(x^2 - 2x\right) + 1$

$= 2(x^2 - 2x + 1) + 1 - 2$

$= 2(x-1)^2 - 1$

Using the graph of $y = x^2$, shift right 1 unit, stretch vertically by a factor of 2, then shift down 1 unit.



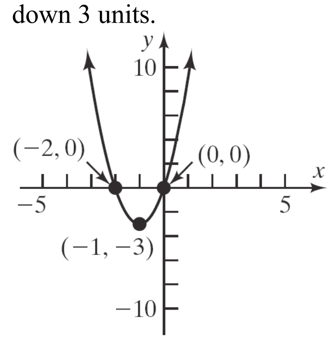
28. $f(x) = 3x^2 + 6x$

$= 3\left(x^2 + 2x\right)$

$= 3(x^2 + 2x + 1) - 3$

$= 3(x+1)^2 - 3$

Using the graph of $y = x^2$, shift left 1 unit, stretch vertically by a factor of 3, then shift down 3 units.



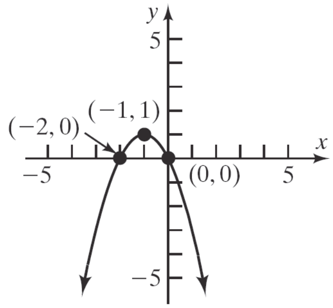
29. $f(x) = -x^2 - 2x$

$= -\left(x^2 + 2x\right)$

$= -(x^2 + 2x + 1) + 1$

$= -(x+1)^2 + 1$

Using the graph of $y = x^2$, shift left 1 unit, reflect across the x-axis, then shift up 1 unit.



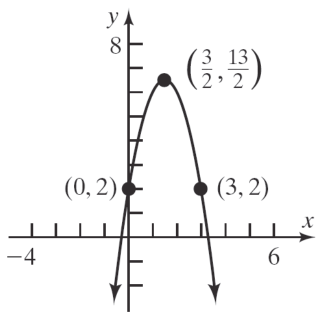
30. $f(x) = -2x^2 + 6x + 2$

$$= -2\left(x^2 - 3x\right) + 2$$

$$= -2\left(x^2 - 3x + \frac{9}{4}\right) + 2 + \frac{9}{2}$$

$$= -2\left(x - \frac{3}{2}\right)^2 + \frac{13}{2}$$

Using the graph of $y = x^2$, shift right $\frac{3}{2}$ units, reflect about the x-axis, stretch vertically by a factor of 2, then shift up $\frac{13}{2}$ units.



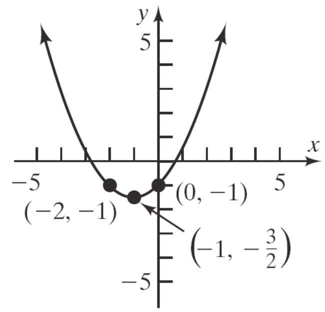
31. $f(x) = \frac{1}{2}x^2 + x - 1$

$$= \frac{1}{2}\left(x^2 + 2x\right) - 1$$

$$= \frac{1}{2}\left(x^2 + 2x + 1\right) - 1 - \frac{1}{2}$$

$$= \frac{1}{2}(x+1)^2 - \frac{3}{2}$$

Using the graph of $y = x^2$, shift left 1 unit, compress vertically by a factor of $\frac{1}{2}$, then shift down $\frac{3}{2}$ units.



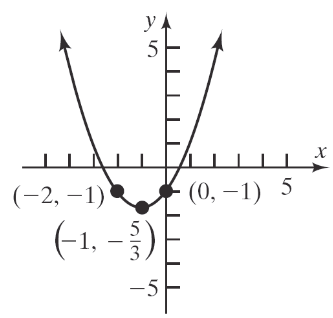
32. $f(x) = \frac{2}{3}x^2 + \frac{4}{3}x - 1$

$$= \frac{2}{3}\left(x^2 + 2x\right) - 1$$

$$= \frac{2}{3}\left(x^2 + 2x + 1\right) - 1 - \frac{2}{3}$$

$$= \frac{2}{3}(x+1)^2 - \frac{5}{3}$$

Using the graph of $y = x^2$, shift left 1 unit, compress vertically by a factor of $\frac{2}{3}$, then shift down $\frac{5}{3}$ unit.



33. **a.** For $f(x) = x^2 + 2x$, $a = 1$, $b = 2$, $c = 0$. Since $a = 1 > 0$, the graph opens up. The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-(2)}{2(1)} = \frac{-2}{2} = -1.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f(-1) = (-1)^2 + 2(-1) = 1 - 2 = -1.$$

Thus, the vertex is $(-1, -1)$.
The axis of symmetry is the line $x = -1$.
The discriminant is
$b^2 - 4ac = (2)^2 - 4(1)(0) = 4 > 0$, so the graph has two x-intercepts.

The x-intercepts are found by solving:

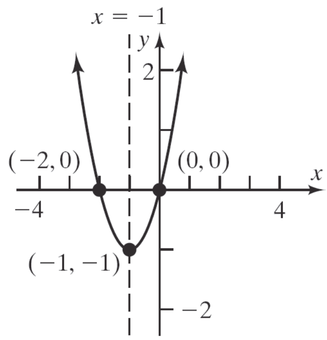
$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0$ or $x = -2$

The x-intercepts are -2 and 0.

The y-intercept is $f(0) = 0$.



b. The domain is $(-\infty, \infty)$.

The range is $[-1, \infty)$.

c. Decreasing on $(-\infty, -1]$.

Increasing on $[-1, \infty)$.

34. a. For $f(x) = x^2 - 4x$, $a = 1$, $b = -4$, $c = 0$.

Since $a = 1 > 0$, the graph opens up.

The x-coordinate of the vertex is

$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$.

The y-coordinate of the vertex is

$f\left(\frac{-b}{2a}\right) = f(2) = (2)^2 - 4(2) = 4 - 8 = -4.$

Thus, the vertex is $(2, -4)$.

The axis of symmetry is the line $x = 2$.

The discriminant is:

$b^2 - 4ac = (-4)^2 - 4(1)(0) = 16 > 0$, so the graph has two x-intercepts.

The x-intercepts are found by solving:

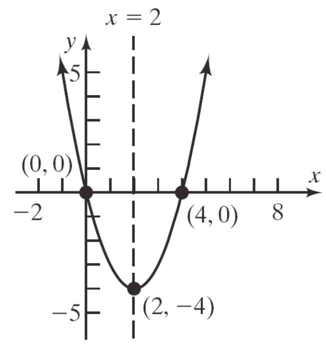
$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ or $x = 4.$

The x-intercepts are 0 and 4.

The y-intercept is $f(0) = 0$.



b. The domain is $(-\infty, \infty)$.

The range is $[-4, \infty)$.

c. Decreasing on $(-\infty, 2]$.

Increasing on $[2, \infty)$.

35. a. For $f(x) = -x^2 - 6x$, $a = -1$, $b = -6$, $c = 0$. Since $a = -1 < 0$, the graph opens down. The x-coordinate of the vertex is

$x = \frac{-b}{2a} = \frac{-(-6)}{2(-1)} = \frac{6}{-2} = -3.$

The y-coordinate of the vertex is

$f\left(\frac{-b}{2a}\right) = f(-3) = -(-3)^2 - 6(-3)$

$= -9 + 18 = 9.$

Thus, the vertex is $(-3, 9)$.

The axis of symmetry is the line $x = -3$.

The discriminant is:

$b^2 - 4ac = (-6)^2 - 4(-1)(0) = 36 > 0$,

so the graph has two x-intercepts.

The x-intercepts are found by solving:

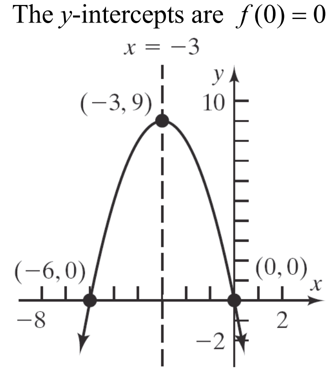
$-x^2 - 6x = 0$

$-x(x+6) = 0$

$x = 0$ or $x = -6.$

The x-intercepts are -6 and 0.

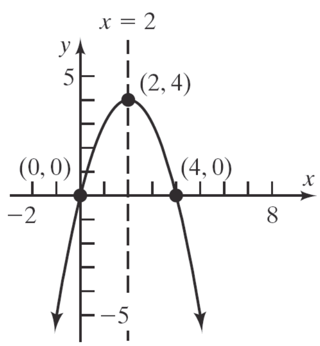
The y-intercepts are $f(0) = 0$.



b. The domain is $(-\infty, \infty)$.
The range is $(-\infty, 9]$.

c. Increasing on $(-\infty, -3]$.
Decreasing on $[-3, \infty)$.

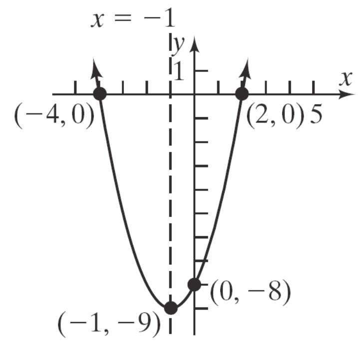
36. a. For $f(x) = -x^2 + 4x$, $a = -1$, $b = 4$, $c = 0$.
Since $a = -1 < 0$, the graph opens down.
The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f(2) = -(2)^2 + 4(2) = 4.$$
Thus, the vertex is $(2, 4)$.
The axis of symmetry is the line $x = 2$.
The discriminant is:
$b^2 - 4ac = 4^2 - 4(-1)(0) = 16 > 0,$
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$-x^2 + 4x = 0$
$-x(x - 4) = 0$
$x = 0$ or $x = 4$.
The x-intercepts are 0 and 4.
The y-intercept is $f(0) = 0$.



b. The domain is $(-\infty, \infty)$.
The range is $(-\infty, 4]$.

c. Increasing on $(-\infty, 2]$.
Decreasing on $[2, \infty)$.

37. a. For $f(x) = x^2 + 2x - 8$, $a = 1$, $b = 2$, $c = -8$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f(-1) = (-1)^2 + 2(-1) - 8 = 1 - 2 - 8 = -9.$$
Thus, the vertex is $(-1, -9)$.
The axis of symmetry is the line $x = -1$.
The discriminant is:
$b^2 - 4ac = 2^2 - 4(1)(-8) = 4 + 32 = 36 > 0$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$x^2 + 2x - 8 = 0$
$(x + 4)(x - 2) = 0$
$x = -4$ or $x = 2$.
The x-intercepts are -4 and 2.
The y-intercept is $f(0) = -8$.



b. The domain is $(-\infty, \infty)$.
The range is $[-9, \infty)$.

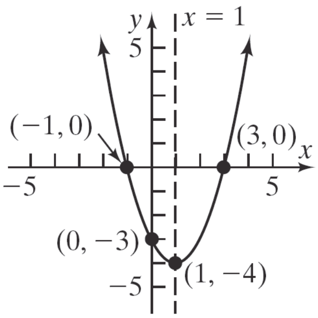
c. Decreasing on $(-\infty, -1]$.
Increasing on $[-1, \infty)$.

38. a. For $f(x) = x^2 - 2x - 3$, $a = 1$, $b = -2$, $c = -3$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f(1) = 1^2 - 2(1) - 3 = -4.$$
Thus, the vertex is $(1, -4)$.
The axis of symmetry is the line $x = 1$.
The discriminant is:
$b^2 - 4ac = (-2)^2 - 4(1)(-3) = 4 + 12 = 16 > 0$,

so the graph has two x-intercepts.
The x-intercepts are found by solving:

$$x^2 - 2x - 3 = 0$$
$$(x+1)(x-3) = 0$$
$$x = -1 \text{ or } x = 3.$$

The x-intercepts are -1 and 3.
The y-intercept is $f(0) = -3$.



b. The domain is $(-\infty, \infty)$. The range is $[-4, \infty)$.

c. Decreasing on $(-\infty, 1]$. Increasing on $[1, \infty)$.

39. a. For $f(x) = x^2 + 2x + 1$, $a = 1$, $b = 2$, $c = 1$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f(-1)$$
$$= (-1)^2 + 2(-1) + 1 = 1 - 2 + 1 = 0.$$

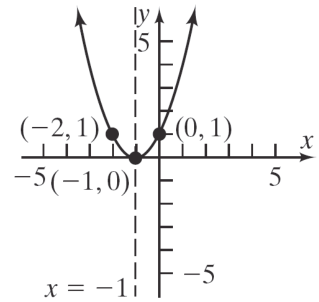
Thus, the vertex is $(-1, 0)$.
The axis of symmetry is the line $x = -1$.
The discriminant is:

$$b^2 - 4ac = 2^2 - 4(1)(1) = 4 - 4 = 0,$$

so the graph has one x-intercept.
The x-intercept is found by solving:

$$x^2 + 2x + 1 = 0$$
$$(x+1)^2 = 0$$
$$x = -1.$$

The x-intercept is -1.
The y-intercept is $f(0) = 1$.



b. The domain is $(-\infty, \infty)$.
The range is $[0, \infty)$.

c. Decreasing on $(-\infty, -1]$.
Increasing on $[-1, \infty)$.

40. a. For $f(x) = x^2 + 6x + 9$, $a = 1$, $b = 6$, $c = 9$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-6}{2(1)} = \frac{-6}{2} = -3.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f(-3)$$
$$= (-3)^2 + 6(-3) + 9 = 9 - 18 + 9 = 0.$$

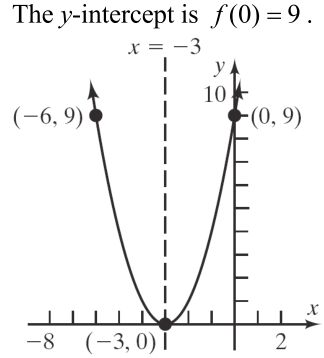
Thus, the vertex is $(-3, 0)$.
The axis of symmetry is the line $x = -3$.
The discriminant is:

$$b^2 - 4ac = 6^2 - 4(1)(9) = 36 - 36 = 0,$$

so the graph has one x-intercept.
The x-intercept is found by solving:

$$x^2 + 6x + 9 = 0$$
$$(x+3)^2 = 0$$
$$x = -3.$$

The x-intercept is -3.
The y-intercept is $f(0) = 9$.



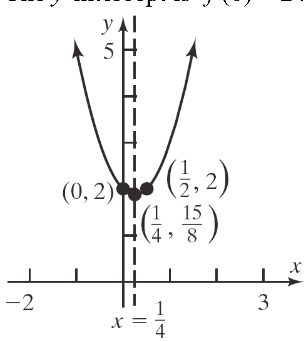
b. The domain is $(-\infty, \infty)$. The range is $[0, \infty)$.

c. Decreasing on $(-\infty, -3]$.
Increasing on $[-3, \infty)$.

41. a. For $f(x) = 2x^2 - x + 2$, $a = 2$, $b = -1$, $c = 2$.
Since $a = 2 > 0$, the graph opens up.
The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-(-1)}{2(2)} = \frac{1}{4}.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{4}\right) = 2\left(\frac{1}{4}\right)^2 - \frac{1}{4} + 2$$
$$= \frac{1}{8} - \frac{1}{4} + 2 = \frac{15}{8}.$$
Thus, the vertex is $\left(\frac{1}{4}, \frac{15}{8}\right)$.

The axis of symmetry is the line $x = \frac{1}{4}$.

The discriminant is:
$b^2 - 4ac = (-1)^2 - 4(2)(2) = 1 - 16 = -15$,
so the graph has no x-intercepts.
The y-intercept is $f(0) = 2$.



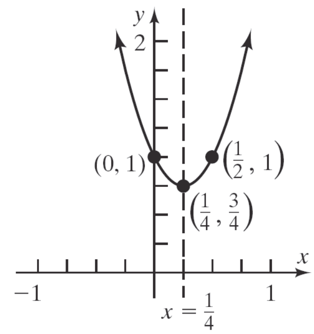
b. The domain is $(-\infty, \infty)$. The range is $\left[\frac{15}{8}, \infty\right)$.

c. Decreasing on $\left(-\infty, \frac{1}{4}\right]$. Increasing on $\left[\frac{1}{4}, \infty\right)$.

42. a. For $f(x) = 4x^2 - 2x + 1$, $a = 4$, $b = -2$, $c = 1$.
Since $a = 4 > 0$, the graph opens up.
The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-(-2)}{2(4)} = \frac{2}{8} = \frac{1}{4}.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right) + 1$$
$$= \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}.$$
Thus, the vertex is $\left(\frac{1}{4}, \frac{3}{4}\right)$.

The axis of symmetry is the line $x = \frac{1}{4}$.

The discriminant is:
$b^2 - 4ac = (-2)^2 - 4(4)(1) = 4 - 16 = -12$,
so the graph has no x-intercepts.
The y-intercept is $f(0) = 1$.



b. The domain is $(-\infty, \infty)$.

The range is $\left[\frac{3}{4}, \infty\right)$.

c. Decreasing on $\left(-\infty, \frac{1}{4}\right]$.

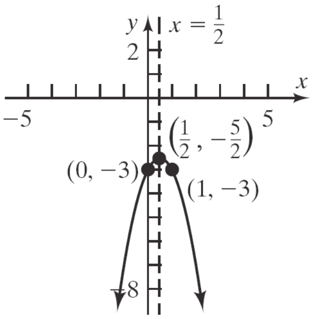
Increasing on $\left[\frac{1}{4}, \infty\right)$.

43. a. For $f(x) = -2x^2 + 2x - 3$, $a = -2$, $b = 2$, $c = -3$. Since $a = -2 < 0$, the graph opens down.
The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-(2)}{2(-2)} = \frac{-2}{-4} = \frac{1}{2}.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{2}\right) = -2\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 3$$
$$= -\frac{1}{2} + 1 - 3 = -\frac{5}{2}.$$
Thus, the vertex is $\left(\frac{1}{2}, -\frac{5}{2}\right)$.

The axis of symmetry is the line $x = \frac{1}{2}$.

The discriminant is:
$b^2 - 4ac = 2^2 - 4(-2)(-3) = 4 - 24 = -20$,
so the graph has no x-intercepts.

The y-intercept is $f(0) = -3$.



b. The domain is $(-\infty, \infty)$.

The range is $\left(-\infty, -\frac{5}{2}\right]$.

c. Increasing on $\left(-\infty, \frac{1}{2}\right]$.

Decreasing on $\left[\frac{1}{2}, \infty\right)$.

44. **a.** For $f(x) = -3x^2 + 3x - 2$, $a = -3$, $b = 3$, $c = -2$. Since $a = -3 < 0$, the graph opens down.
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-3}{2(-3)} = \frac{-3}{-6} = \frac{1}{2}.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{2}\right) = -3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 2$$
$$= -\frac{3}{4} + \frac{3}{2} - 2 = -\frac{5}{4}.$$

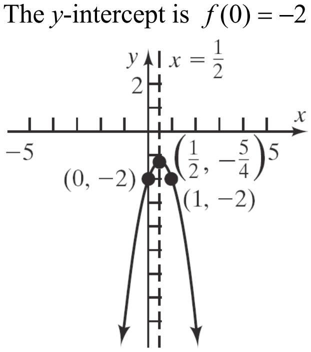
Thus, the vertex is $\left(\frac{1}{2}, -\frac{5}{4}\right)$.

The axis of symmetry is the line $x = \frac{1}{2}$.

The discriminant is:

$b^2 - 4ac = 3^2 - 4(-3)(-2) = 9 - 24 = -15$,

so the graph has no x-intercepts.
The y-intercept is $f(0) = -2$.



b. The domain is $(-\infty, \infty)$.

The range is $\left(-\infty, -\frac{5}{4}\right]$.

c. Increasing on $\left(-\infty, \frac{1}{2}\right]$.

Decreasing on $\left[\frac{1}{2}, \infty\right)$.

45. **a.** For $f(x) = 3x^2 + 6x + 2$, $a = 3$, $b = 6$, $c = 2$. Since $a = 3 > 0$, the graph opens up.
The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-6}{2(3)} = \frac{-6}{6} = -1.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f(-1) = 3(-1)^2 + 6(-1) + 2$$
$$= 3 - 6 + 2 = -1.$$

Thus, the vertex is $(-1, -1)$.
The axis of symmetry is the line $x = -1$.
The discriminant is:

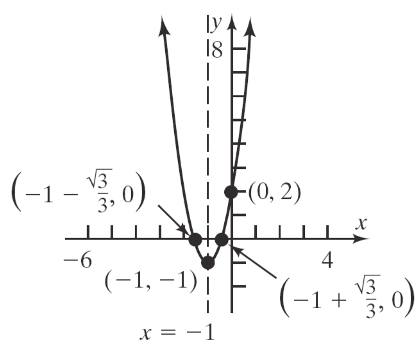
$b^2 - 4ac = 6^2 - 4(3)(2) = 36 - 24 = 12$,

so the graph has two x-intercepts.
The x-intercepts are found by solving:

$$3x^2 + 6x + 2 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-6 \pm \sqrt{12}}{6} = \frac{-6 \pm 2\sqrt{3}}{6} = \frac{-3 \pm \sqrt{3}}{3}$$

The x-intercepts are $-1 - \frac{\sqrt{3}}{3}$ and $-1 + \frac{\sqrt{3}}{3}$.

The y-intercept is $f(0) = 2$.



b. The domain is $(-\infty, \infty)$.

The range is $[-1, \infty)$.

c. Decreasing on $(-\infty, -1]$.

Increasing on $[-1, \infty)$.

46. a. For $f(x)=2x^2+5x+3$, $a=2$, $b=5$, $c=3$. Since $a=2>0$, the graph opens up.
The x-coordinate of the vertex is
$x=\frac{-b}{2a}=\frac{-5}{2(2)}=-\frac{5}{4}$.
The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right)=f\left(-\frac{5}{4}\right)=2\left(-\frac{5}{4}\right)^2+5\left(-\frac{5}{4}\right)+3=\frac{25}{8}-\frac{25}{4}+3=-\frac{1}{8}.$$

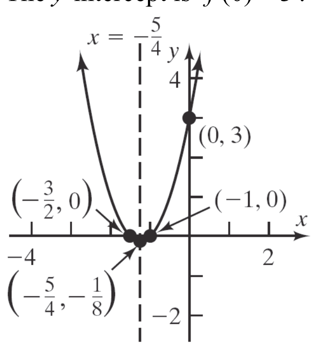
Thus, the vertex is $\left(-\frac{5}{4},-\frac{1}{8}\right)$.

The axis of symmetry is the line $x=-\frac{5}{4}$.

The discriminant is:
$b^2-4ac=5^2-4(2)(3)=25-24=1$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$2x^2+5x+3=0$
$(2x+3)(x+1)=0$
$x=-\frac{3}{2}$ or $x=-1$.

The x-intercepts are $-\frac{3}{2}$ and -1.

The y-intercept is $f(0)=3$.



b. The domain is $(-\infty,\infty)$.

The range is $\left[-\frac{1}{8},\infty\right)$.

c. Decreasing on $\left(-\infty,-\frac{5}{4}\right]$.

Increasing on $\left[-\frac{5}{4},\infty\right)$.

47. a. For $f(x)=-4x^2-6x+2$, $a=-4$, $b=-6$, $c=2$. Since $a=-4<0$, the graph opens down.
The x-coordinate of the vertex is
$x=\frac{-b}{2a}=\frac{-(-6)}{2(-4)}=\frac{6}{-8}=-\frac{3}{4}$.
The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right)=f\left(-\frac{3}{4}\right)=-4\left(-\frac{3}{4}\right)^2-6\left(-\frac{3}{4}\right)+2=-\frac{9}{4}+\frac{9}{2}+2=\frac{17}{4}.$$

Thus, the vertex is $\left(-\frac{3}{4},\frac{17}{4}\right)$.

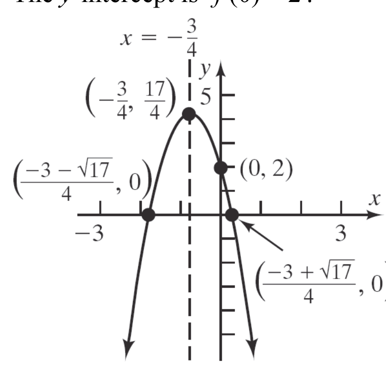
The axis of symmetry is the line $x=-\frac{3}{4}$.

The discriminant is:
$b^2-4ac=(-6)^2-4(-4)(2)=36+32=68$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$-4x^2-6x+2=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-6)\pm\sqrt{68}}{2(-4)}=\frac{6\pm\sqrt{68}}{-8}=\frac{6\pm2\sqrt{17}}{-8}=\frac{3\pm\sqrt{17}}{-4}$$

The x-intercepts are $\frac{-3+\sqrt{17}}{4}$ and $\frac{-3-\sqrt{17}}{4}$.

The y-intercept is $f(0)=2$.



b. The domain is $(-\infty,\infty)$.

The range is $\left(-\infty,\frac{17}{4}\right]$.

c. Decreasing on $\left[-\frac{3}{4}, \infty\right)$.

Increasing on $\left(-\infty, -\frac{3}{4}\right]$.

48. a. For $f(x) = 3x^2 - 8x + 2,\ a = 3,\ b = -8,\ c = 2$.
Since $a = 3 > 0$, the graph opens up.
The x-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-(-8)}{2(3)} = \frac{8}{6} = \frac{4}{3}$.
The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f\left(\frac{4}{3}\right) = 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 2$$
$$= \frac{16}{3} - \frac{32}{3} + 2 = -\frac{10}{3}.$$

Thus, the vertex is $\left(\frac{4}{3}, -\frac{10}{3}\right)$.

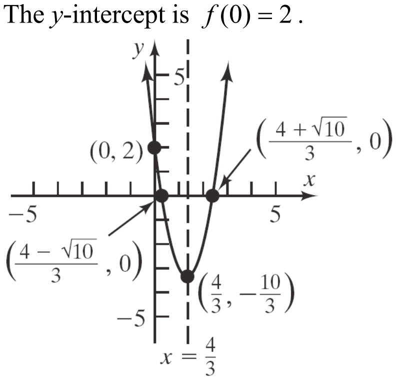
The axis of symmetry is the line $x = \frac{4}{3}$.

The discriminant is:
$b^2 - 4ac = (-8)^2 - 4(3)(2) = 64 - 24 = 40$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$3x^2 - 8x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{40}}{2(3)}$$
$$= \frac{8 \pm \sqrt{40}}{6} = \frac{8 \pm 2\sqrt{10}}{6} = \frac{4 \pm \sqrt{10}}{3}$$

The x-intercepts are $\frac{4+\sqrt{10}}{3}$ and $\frac{4-\sqrt{10}}{3}$.

The y-intercept is $f(0) = 2$.



b. The domain is $(-\infty, \infty)$.

The range is $\left[-\frac{10}{3}, \infty\right)$.

c. Decreasing on $\left(-\infty, \frac{4}{3}\right]$.

Increasing on $\left[\frac{4}{3}, \infty\right)$.

49. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(-1, -2)$ so we have $h = -1$ and $k = -2$. The graph also passes through the point $(x, y) = (0, -1)$. Substituting these values for x, y, h, and k, we can solve for a:

$$-1 = a(0 - (-1))^2 + (-2)$$
$$-1 = a(1)^2 - 2$$
$$-1 = a - 2$$
$$1 = a$$

The quadratic function is
$f(x) = (x+1)^2 - 2 = x^2 + 2x - 1$.

50. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(2, 1)$ so we have $h = 2$ and $k = 1$. The graph also passes through the point $(x, y) = (0, 5)$. Substituting these values for x, y, h, and k, we can solve for a:

$$5 = a(0 - 2)^2 + 1$$
$$5 = a(-2)^2 + 1$$
$$5 = 4a + 1$$
$$4 = 4a$$
$$1 = a$$

The quadratic function is
$f(x) = (x-2)^2 + 1 = x^2 - 4x + 5$.

51. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(-3, 5)$ so we have $h = -3$ and $k = 5$. The graph also passes through the point $(x, y) = (0, -4)$. Substituting these values for x, y, h, and k, we can solve for a:

$$-4 = a(0 - (-3))^2 + 5$$
$$-4 = a(3)^2 + 5$$
$$-4 = 9a + 5$$
$$-9 = 9a$$
$$-1 = a$$

The quadratic function is
$f(x) = -(x+3)^2 + 5 = -x^2 - 6x - 4$.

52. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(2,3)$ so we have $h = 2$ and $k = 3$. The graph also passes through the point $(x, y) = (0, -1)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:
$-1 = a(0-2)^2 + 3$
$-1 = a(-2)^2 + 3$
$-1 = 4a + 3$
$-4 = 4a$
$-1 = a$
The quadratic function is
$f(x) = -(x-2)^2 + 3 = -x^2 + 4x - 1$.

53. Consider the form $y = ax^2 + bx + c$. Substituting the three points from the graph into the general form we have the following three equations.
$5 = a(-1)^2 + b(-1) + c \Rightarrow 5 = a - b + c$
and
$5 = a(3)^2 + b(3) + c \Rightarrow 5 = 9a + 3b + c$
and
$-1 = a(0)^2 + b(0) + c \Rightarrow -1 = c$
Since $-1 = c$, we have the following equations:
$5 = a - b - 1, \quad 5 = 9a + 3b - 1, \quad -1 = c$
Solving the first two simultaneously we have
$$\left.\begin{array}{l} 5 = a - b - 1 \\ 5 = 9a + 3b - 1 \end{array}\right)$$
$$\left.\begin{array}{l} 6 = a - b \\ 6 = 9a + 3b \end{array}\right) \rightarrow \quad a = 2, b = -4$$
The quadratic function is $f(x) = 2x^2 - 4x - 1$.

54. Consider the form $y = ax^2 + bx + c$. Substituting the three points from the graph into the general form we have the following three equations.
$-2 = a(-4)^2 + b(-4) + c \Rightarrow -2 = 16a - 4b + c$
and
$4 = a(-1)^2 + b(-1) + c \Rightarrow 4 = a - b + c$
and
$-2 = a(0)^2 + b(0) + c \Rightarrow -2 = c$
Since $-2 = c$, we have the following equations:
$-2 = 16a - 4b - 2, \quad 4 = a - b - 2, \quad -2 = c$
Solving the first two simultaneously we have
$$\left.\begin{array}{l} -2 = 16a - 4b - 2 \\ 4 = a - b - 2 \end{array}\right)$$
$$\left.\begin{array}{l} 0 = 16a - 4b \\ 6 = a - b \end{array}\right) \rightarrow \quad a = -2, b = -8$$
The quadratic function is $f(x) = -2x^2 - 8x - 2$.

55. For $f(x) = 2x^2 + 12x, \ a = 2, \ b = 12, \ c = 0$.
Since $a = 2 > 0,$ the graph opens up, so the vertex is a minimum point. The minimum occurs at $x = \frac{-b}{2a} = \frac{-12}{2(2)} = \frac{-12}{4} = -3.$
The minimum value is
$f(-3) = 2(-3)^2 + 12(-3) = 18 - 36 = -18$.

56. For $f(x) = -2x^2 + 12x, \ a = -2, \ b = 12, \ c = 0,$.
Since $a = -2 < 0,$ the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \frac{-b}{2a} = \frac{-12}{2(-2)} = \frac{-12}{-4} = 3.$
The maximum value is
$f(3) = -2(3)^2 + 12(3) = -18 + 36 = 18$.

57. For $f(x) = 2x^2 + 12x - 3, \ a = 2, \ b = 12, \ c = -3.$
Since $a = 2 > 0,$ the graph opens up, so the vertex is a minimum point. The minimum occurs at $x = \frac{-b}{2a} = \frac{-12}{2(2)} = \frac{-12}{4} = -3.$ The minimum value is
$f(-3) = 2(-3)^2 + 12(-3) - 3 = 18 - 36 - 3 = -21$.

58. For $f(x) = 4x^2 - 8x + 3, \ a = 4, \ b = -8, \ c = 3.$
Since $a = 4 > 0,$ the graph opens up, so the vertex is a minimum point. The minimum occurs at $x = \frac{-b}{2a} = \frac{-(-8)}{2(4)} = \frac{8}{8} = 1.$ The minimum value is
$f(1) = 4(1)^2 - 8(1) + 3 = 4 - 8 + 3 = -1$.

59. For $f(x) = -x^2 + 10x - 4$, $a = -1$, $b = 10$, $c = -4$. Since $a = -1 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \frac{-b}{2a} = \frac{-10}{2(-1)} = \frac{-10}{-2} = 5$. The maximum value is
$f(5) = -(5)^2 + 10(5) - 4 = -25 + 50 - 4 = 21$.

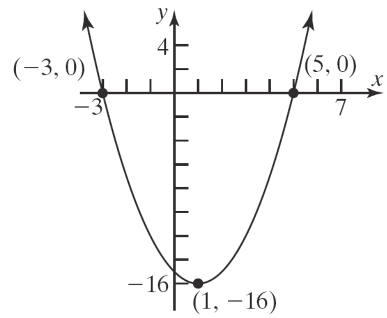
60. For $f(x) = -2x^2 + 8x + 3$, $a = -2$, $b = 8$, $c = 3$. Since $a = -2 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \frac{-b}{2a} = \frac{-8}{2(-2)} = \frac{-8}{-4} = 2$. The maximum value is
$f(2) = -2(2)^2 + 8(2) + 3 = -8 + 16 + 3 = 11$.

61. For $f(x) = -3x^2 + 12x + 1$, $a = -3$, $b = 12$, $c = 1$. Since $a = -3 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \frac{-b}{2a} = \frac{-12}{2(-3)} = \frac{-12}{-6} = 2$. The maximum value is $f(2) = -3(2)^2 + 12(2) + 1 = -12 + 24 + 1 = 13$.

62. For $f(x) = 4x^2 - 4x$, $a = 4$, $b = -4$, $c = 0$. Since $a = 4 > 0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at $x = \frac{-b}{2a} = \frac{-(-4)}{2(4)} = \frac{4}{8} = \frac{1}{2}$. The minimum value is

$$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) = 1 - 2 = -1.$$

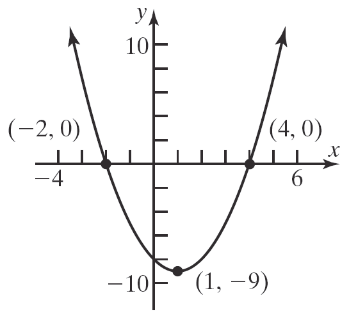
63. a. For $f(x) = x^2 - 2x - 15$, $a = 1$, $b = -2$, $c = -15$. Since $a = 1 > 0$, the graph opens up. The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f(1) = (1)^2 - 2(1) - 15$$
$$= 1 - 2 - 15 = -16.$$
Thus, the vertex is $(1, -16)$.
The discriminant is:
$b^2 - 4ac = (-2)^2 - 4(1)(-15) = 4 + 60 = 64 > 0$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$x^2 - 2x - 15 = 0$
$(x + 3)(x - 5) = 0$
$x = -3$ or $x = 5$
The x-intercepts are -3 and 5.
The y-intercept is $f(0) = -15$.



b. The domain is $(-\infty, \infty)$.
The range is $[-16, \infty)$.

c. Decreasing on $(-\infty, 1]$.
Increasing on $[1, \infty)$.

64. a. For $f(x) = x^2 - 2x - 8$, $a = 1$, $b = -2$, $c = -8$. Since $a = 1 > 0$, the graph opens up. The x-coordinate of the vertex is
$$x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right) = f(1) = (1)^2 - 2(1) - 8 = 1 - 2 - 8 = -9.$$
Thus, the vertex is $(1, -9)$.
The discriminant is:
$b^2 - 4ac = (-2)^2 - 4(1)(-8) = 4 + 32 = 36 > 0$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$x^2 - 2x - 8 = 0$
$(x + 2)(x - 4) = 0$
$x = -2$ or $x = 4$
The x-intercepts are -2 and 4.
The y-intercept is $f(0) = -8$.



b. The domain is $(-\infty, \infty)$. The range is $[-9, \infty)$.

c. Decreasing on $(-\infty, 1]$. Increasing on $[1, \infty)$.

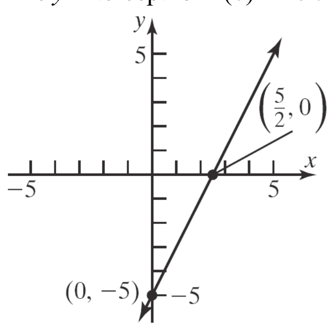
65. a. $F(x) = 2x - 5$ is a linear function.

The x-intercept is found by solving:

$$2x - 5 = 0$$
$$2x = 5$$
$$x = \frac{5}{2}$$

The x-intercept is $\frac{5}{2}$.

The y-intercept is $F(0) = -5$.



b. The domain is $(-\infty, \infty)$.

The range is $(-\infty, \infty)$.

c. Increasing on $(-\infty, \infty)$.

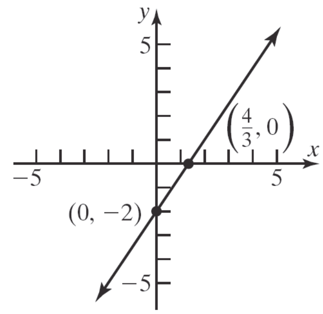
66. a. $f(x) = \frac{3}{2}x - 2$ is a linear function.

The x-intercept is found by solving:

$$\frac{3}{2}x - 2 = 0$$
$$\frac{3}{2}x = 2$$
$$x = 2 \cdot \frac{2}{3} = \frac{4}{3}$$

The x-intercept is $\frac{4}{3}$.

The y-intercept is $f(0) = -2$.



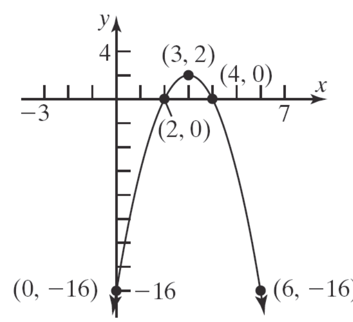
b. The domain is $(-\infty, \infty)$.

The range is $(-\infty, \infty)$.

c. Increasing on $(-\infty, \infty)$.

67. a. $g(x) = -2(x-3)^2 + 2$

Using the graph of $y = x^2$, shift right 3 units, reflect about the x-axis, stretch vertically by a factor of 2, then shift up 2 units.



b. The domain is $(-\infty, \infty)$.

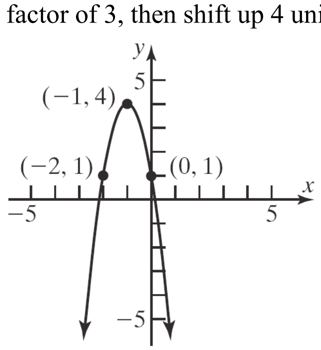
The range is $(-\infty, 2]$.

c. Increasing on $(-\infty, 3]$.

Decreasing on $[3, \infty)$.

68. a. $h(x) = -3(x+1)^2 + 4$

Using the graph of $y = x^2$, shift left 1 unit, reflect about the x-axis, stretch vertically by a factor of 3, then shift up 4 units.



b. The domain is $(-\infty, \infty)$.
The range is $(-\infty, 4]$.

c. Increasing on $(-\infty, -1]$.
Decreasing on $[-1, \infty)$.

69. a. For $f(x) = 2x^2 + x + 1$, $a = 2$, $b = 1$, $c = 1$.
Since $a = 2 > 0$, the graph opens up.
The x-coordinate of the vertex is

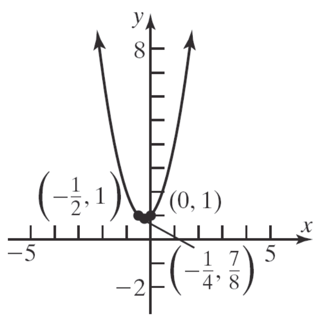
$$x = \frac{-b}{2a} = \frac{-1}{2(2)} = \frac{-1}{4} = -\frac{1}{4}.$$

The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f\left(-\frac{1}{4}\right) = 2\left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right) + 1 = \frac{1}{8} - \frac{1}{4} + 1 = \frac{7}{8}.$$

Thus, the vertex is $\left(-\frac{1}{4}, \frac{7}{8}\right)$.

The discriminant is:
$b^2 - 4ac = 1^2 - 4(2)(1) = 1 - 8 = -7$,
so the graph has no x-intercepts.
The y-intercept is $f(0) = 1$.



b. The domain is $(-\infty, \infty)$.
The range is $\left[\frac{7}{8}, \infty\right)$.

c. Decreasing on $\left(-\infty, -\frac{1}{4}\right]$.
Increasing on $\left[-\frac{1}{4}, \infty\right)$.

70. a. For $G(x) = 3x^2 + 2x + 5$, $a = 3$, $b = 2$, $c = 5$.
Since $a = 3 > 0$, the graph opens up.
The x-coordinate of the vertex is

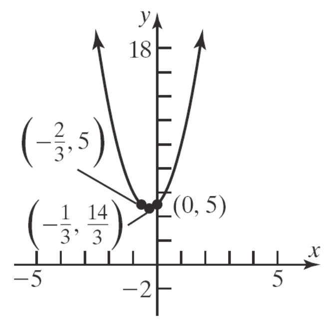
$$x = \frac{-b}{2a} = \frac{-2}{2(3)} = \frac{-2}{6} = -\frac{1}{3}.$$

The y-coordinate of the vertex is

$$G\left(\frac{-b}{2a}\right) = G\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right)^2 + 2\left(-\frac{1}{3}\right) + 5 = \frac{1}{3} - \frac{2}{3} + 5 = \frac{14}{3}.$$

Thus, the vertex is $\left(-\frac{1}{3}, \frac{14}{3}\right)$.

The discriminant is:
$b^2 - 4ac = 2^2 - 4(3)(5) = 4 - 60 = -56$,
so the graph has no x-intercepts.
The y-intercept is $G(0) = 5$.



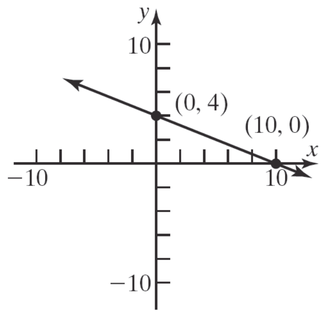
b. The domain is $(-\infty, \infty)$.
The range is $\left[\frac{14}{3}, \infty\right)$.

c. Decreasing on $\left(-\infty, -\frac{1}{3}\right]$.
Increasing on $\left[-\frac{1}{3}, \infty\right)$.

71. a. $h(x) = -\frac{2}{5}x + 4$ is a linear function.
The x-intercept is found by solving:

$$-\frac{2}{5}x + 4 = 0$$
$$-\frac{2}{5}x = -4$$
$$x = -4\left(-\frac{5}{2}\right) = 10$$

The x-intercept is 10.
The y-intercept is $h(0) = 4$.



b. The domain is $(-\infty, \infty)$.

The range is $(-\infty, \infty)$.

c. Decreasing on $(-\infty, \infty)$.

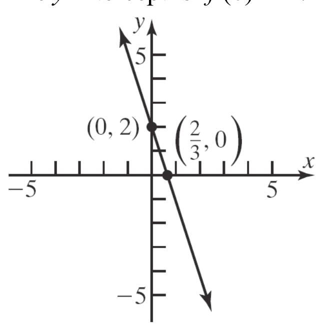
72. a. $f(x) = -3x + 2$ is a linear function.

The x-intercept is found by solving:

$$-3x + 2 = 0$$
$$-3x = -2$$
$$x = \frac{-2}{-3} = \frac{2}{3}$$

The x-intercept is $\frac{2}{3}$.

The y-intercept is $f(0) = 2$.



b. The domain is $(-\infty, \infty)$.

The range is $(-\infty, \infty)$.

c. Decreasing on $(-\infty, \infty)$.

73. a. For $H(x) = -4x^2 - 4x - 1$, $a = -4$, $b = -4$, $c = -1$. Since $a = -4 < 0$, the graph opens down. The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-(-4)}{2(-4)} = \frac{4}{-8} = -\frac{1}{2}.$$

The y-coordinate of the vertex is

$$H\left(\frac{-b}{2a}\right) = H\left(-\frac{1}{2}\right) = -4\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) - 1$$
$$= -1 + 2 - 1 = 0$$

Thus, the vertex is $\left(-\frac{1}{2}, 0\right)$.

The discriminant is:

$b^2 - 4ac = (-4)^2 - 4(-4)(-1) = 16 - 16 = 0$,

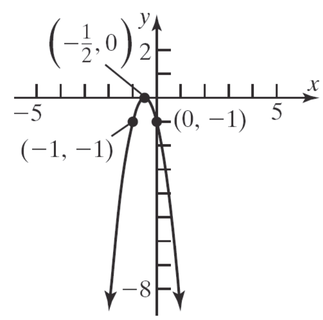
so the graph has one x-intercept.

The x-intercept is found by solving:

$$-4x^2 - 4x - 1 = 0$$
$$4x^2 + 4x + 1 = 0$$
$$(2x + 1)^2 = 0$$
$$2x + 1 = 0$$
$$x = -\frac{1}{2}$$

The x-intercept is $-\frac{1}{2}$.

The y-intercept is $H(0) = -1$.



b. The domain is $(-\infty, \infty)$.

The range is $(-\infty, 0]$.

c. Increasing on $\left(-\infty, -\frac{1}{2}\right]$.

Decreasing on $\left[-\frac{1}{2}, \infty\right)$.

74. a. For $F(x) = -4x^2 + 20x - 25$, $a = -4$, $b = 20$, $c = -25$. Since $a = -4 < 0$, the graph opens down. The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-20}{2(-4)} = \frac{-20}{-8} = \frac{5}{2}.$$

The y-coordinate of the vertex is

$$F\left(\frac{-b}{2a}\right) = F\left(\frac{5}{2}\right) = -4\left(\frac{5}{2}\right)^2 + 20\left(\frac{5}{2}\right) - 25$$
$$= -25 + 50 - 25 = 0$$

Thus, the vertex is $\left(\frac{5}{2}, 0\right)$.

The discriminant is:

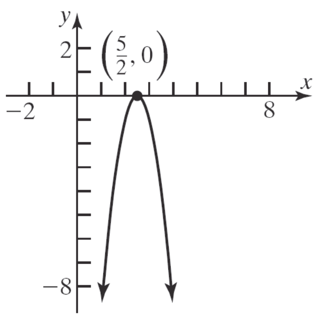
$$b^2 - 4ac = (20)^2 - 4(-4)(-25)$$
$$= 400 - 400 = 0,$$

so the graph has one x-intercept.
The x-intercept is found by solving:

$$-4x^2 + 20x - 25 = 0$$
$$4x^2 - 20x + 25 = 0$$
$$(2x-5)^2 = 0$$
$$2x - 5 = 0$$
$$x = \frac{5}{2}$$

The x-intercept is $\frac{5}{2}$.

The y-intercept is $F(0) = -25$.



b. The domain is $(-\infty, \infty)$.

The range is $(-\infty, 0]$.

c. Increasing on $\left(-\infty, \frac{5}{2}\right]$.

Decreasing on $\left[\frac{5}{2}, \infty\right)$.

75. Use the form $f(x) = a(x-h)^2 + k$.
The vertex is $(0, 2)$, so $h = 0$ and $k = 2$.
$f(x) = a(x-0)^2 + 2 = ax^2 + 2$.
Since the graph passes through $(1, 8)$, $f(1) = 8$.

$$f(x) = ax^2 + 2$$
$$8 = a(1)^2 + 2$$
$$8 = a + 2$$
$$6 = a$$

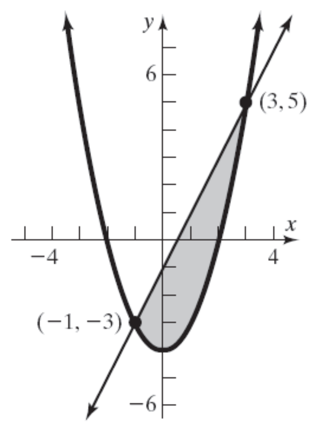
$f(x) = 6x^2 + 2$.
$a = 6, b = 0, c = 2$

76. Use the form $f(x) = a(x-h)^2 + k$.
The vertex is $(1, 4)$, so $h = 1$ and $k = 4$.
$f(x) = a(x-1)^2 + 4$.
Since the graph passes through $(-1, -8)$, $f(-1) = -8$.

$$-8 = a(-1-1)^2 + 4$$
$$-8 = a(-2)^2 + 4$$
$$-8 = 4a + 4$$
$$-12 = 4a$$
$$-3 = a$$
$$f(x) = -3(x-1)^2 + 4$$
$$= -3(x^2 - 2x + 1) + 4$$
$$= -3x^2 + 6x - 3 + 4$$
$$= -3x^2 + 6x + 1$$

$a = -3,\ b = 6,\ c = 1$

77. **a** and **d.**



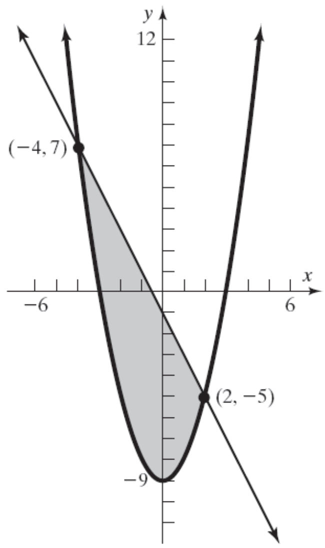
b.
$$f(x) = g(x)$$
$$2x - 1 = x^2 - 4$$
$$0 = x^2 - 2x - 3$$
$$0 = (x+1)(x-3)$$
$$x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -1 \qquad\qquad x = 3$$

The solution set is $\{-1, 3\}$.

c.
$$f(-1) = 2(-1) - 1 = -2 - 1 = -3$$
$$g(-1) = (-1)^2 - 4 = 1 - 4 = -3$$
$$f(3) = 2(3) - 1 = 6 - 1 = 5$$
$$g(3) = (3)^2 - 4 = 9 - 4 = 5$$

Thus, the graphs of f and g intersect at the points $(-1, -3)$ and $(3, 5)$.

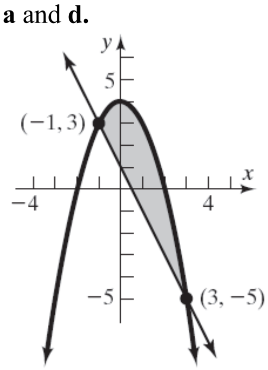
78. **a** and **d.**



b.
$$f(x) = g(x)$$
$$-2x - 1 = x^2 - 9$$
$$0 = x^2 + 2x - 8$$
$$0 = (x+4)(x-2)$$
$$x + 4 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -4 \qquad\qquad x = 2$$
The solution set is $\{-4, 2\}$.

c.
$$f(-4) = -2(-4) - 1 = 8 - 1 = 7$$
$$g(-4) = (-4)^2 - 9 = 16 - 9 = 7$$
$$f(2) = -2(2) - 1 = -4 - 1 = -5$$
$$g(2) = (2)^2 - 9 = 4 - 9 = -5$$
Thus, the graphs of *f* and *g* intersect at the points $(-4,\ 7)$ and $(2, -5)$.

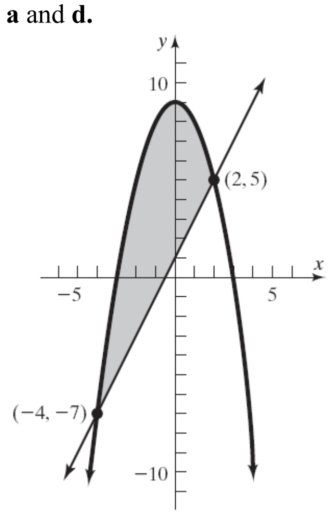
79. **a** and **d.**



b.
$$f(x) = g(x)$$
$$-x^2 + 4 = -2x + 1$$
$$0 = x^2 - 2x - 3$$
$$0 = (x+1)(x-3)$$
$$x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -1 \qquad\qquad x = 3$$
The solution set is $\{-1, 3\}$.

c.
$$f(1) = -(-1)^2 + 4 = -1 + 4 = 3$$
$$g(1) = -2(-1) + 1 = 2 + 1 = 3$$
$$f(3) = -(3)^2 + 4 = -9 + 4 = -5$$
$$g(3) = -2(3) + 1 = -6 + 1 = -5$$
Thus, the graphs of *f* and *g* intersect at the points $(-1,\ 3)$ and $(3, -5)$.

80. **a** and **d.**

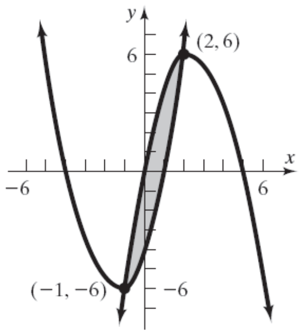


b.
$$f(x) = g(x)$$
$$-x^2 + 9 = 2x + 1$$
$$0 = x^2 + 2x - 8$$
$$0 = (x+4)(x-2)$$
$$x + 4 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -4 \qquad\qquad x = 2$$
The solution set is $\{-4, 2\}$.

c.
$$f(-4) = -(-4)^2 + 9 = -16 + 9 = -7$$
$$g(-4) = 2(-4) + 1 = -8 + 1 = -7$$
$$f(2) = -(2)^2 + 9 = -4 + 9 = 5$$
$$g(2) = 2(2) + 1 = 4 + 1 = 5$$

Thus, the graphs of *f* and *g* intersect at the points $(-4,-7)$ and $(2,\ 5)$.

81. a and **d.**



b.
$$f(x) = g(x)$$
$$-x^2 + 5x = x^2 + 3x - 4$$
$$0 = 2x^2 - 2x - 4$$
$$0 = x^2 - x - 2$$
$$0 = (x+1)(x-2)$$
$$x + 1 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -1 \qquad x = 2$$
The solution set is $\{-1, 2\}$.

c. $f(-1) = -(-1)^2 + 5(-1) = -1 - 5 = -6$

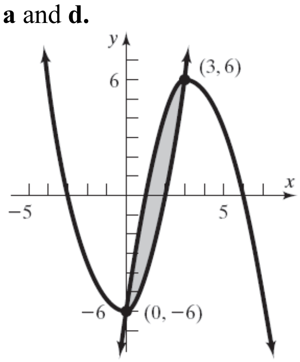
$g(-1) = (-1)^2 + 3(-1) - 4 = 1 - 3 - 4 = -6$

$f(2) = -(2)^2 + 5(2) = -4 + 10 = 6$

$g(2) = 2^2 + 3(2) - 4 = 4 + 6 - 4 = 6$

Thus, the graphs of *f* and *g* intersect at the points $(-1,-6)$ and $(2,\ 6)$.

82. a and **d.**



b.
$$f(x) = g(x)$$
$$-x^2 + 7x - 6 = x^2 + x - 6$$
$$0 = 2x^2 - 6x$$
$$0 = 2x(x-3)$$
$$2x = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = 0 \qquad x = 3$$
The solution set is $\{0, 3\}$.

c. $f(0) = -(0)^2 + 7(0) - 6 = -6$

$g(0) = 0^2 + 0 - 6 = -6$

$f(3) = -(3)^2 + 7(3) - 6 = -9 + 21 - 6 = 6$

$g(3) = 3^2 + 3 - 6 = 9 + 3 - 6 = 6$

Thus, the graphs of *f* and *g* intersect at the points $(0,-6)$ and $(3,\ 6)$.

83. a. For $a = 1$:
$$f(x) = a(x - r_1)(x - r_2)$$
$$= 1(x - (-3))(x - 1)$$
$$= (x+3)(x-1) = x^2 + 2x - 3$$
For $a = 2$:
$$f(x) = 2(x - (-3))(x - 1)$$
$$= 2(x+3)(x-1)$$
$$= 2(x^2 + 2x - 3) = 2x^2 + 4x - 6$$
For $a = -2$:
$$f(x) = -2(x - (-3))(x - 1)$$
$$= -2(x+3)(x-1)$$
$$= -2(x^2 + 2x - 3) = -2x^2 - 4x + 6$$
For $a = 5$:
$$f(x) = 5(x - (-3))(x - 1)$$
$$= 5(x+3)(x-1)$$
$$= 5(x^2 + 2x - 3) = 5x^2 + 10x - 15$$

b. The *x*-intercepts are not affected by the value of *a*. The *y*-intercept is multiplied by the value of a.

c. The axis of symmetry is unaffected by the value of a. For this problem, the axis of symmetry is $x = -1$ for all values of *a*.

d. The *x*-coordinate of the vertex is not affected by the value of *a*. The *y*-coordinate of the vertex is multiplied by the value of a.

e. The *x*-coordinate of the vertex is the mean of the *x*-intercepts.

84. a. For $a=1$:

$f(x)=1(x-(-5))(x-3)$

$=(x+5)(x-3)=x^2+2x-15$

For $a=2$:

$f(x)=2(x-(-5))(x-3)$

$=2(x+5)(x-3)$

$=2(x^2+2x-15)=2x^2+4x-30$

For $a=-2$:

$f(x)=-2(x-(-5))(x-3)$

$=-2(x+5)(x-3)$

$=-2(x^2+2x-15)=-2x^2-4x+30$

For $a=5$:

$f(x)=5(x-(-5))(x-3)$

$=5(x+5)(x-3)$

$=5(x^2+2x-15)=5x^2+10x-75$

b. The x-intercepts are not affected by the value of a. The y-intercept is multiplied by the value of a.

c. The axis of symmetry is unaffected by the value of a. For this problem, the axis of symmetry is $x=-1$ for all values of a.

d. The x-coordinate of the vertex is not affected by the value of a. The y-coordinate of the vertex is multiplied by the value of a.

e. The x-coordinate of the vertex is the mean of the x-intercepts.

85. a. $x=-\dfrac{b}{2a}=-\dfrac{4}{2(1)}=-2$

$y=f(-2)=(-2)^2+4(-2)-21=-25$

The vertex is $(-2,-25)$.

b.

$f(x)=0$

$x^2+4x-21=0$

$(x+7)(x-3)=0$

$x+7=0$ or $x-3=0$

$x=-7$ $\quad$ $x=3$

The x-intercepts of f are $(-7, 0)$ and $(3, 0)$.

c.

$f(x)=-21$

$x^2+4x-21=-21$

$x^2+4x=0$

$x(x+4)=0$

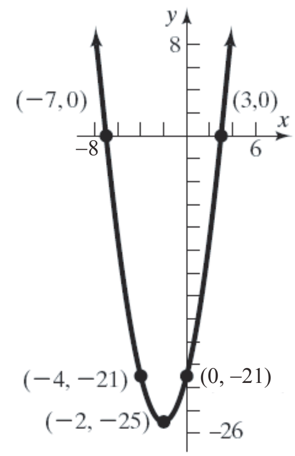
$x=0$ or $x+4=0$

$x=-4$

The solutions $f(x)=-21$ are -4 and 0.

Thus, the points $(-4,-21)$ and $(0,-21)$ are on the graph of f.

d.



86. a. $x=-\dfrac{b}{2a}=-\dfrac{2}{2(1)}=-1$

$y=f(-1)=(-1)^2+2(-1)-8=-9$

The vertex is $(-1,-9)$.

b.

$f(x)=0$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x+4=0$ or $x-2=0$

$x=-4$ $\quad$ $x=2$

The x-intercepts of f are $(-4, 0)$ and $(2, 0)$.

c.

$f(x)=-8$

$x^2+2x-8=-8$

$x^2+2x=0$

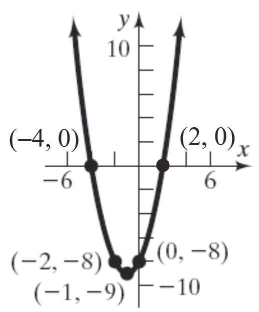
$x(x+2)=0$

$x=0$ or $x+2=0$

$x=-2$

The solutions $f(x)=-8$ are -2 and 0. Thus, the points $(-2,-8)$ and $(0,-8)$ are on the graph of *f*.

d.



87. Let (*x*, *y*) represent a point on the line *y* = *x*. Then the distance from (*x*, *y*) to the point (3, 1) is $d = \sqrt{(x-3)^2 + (y-1)^2}$. Since *y* = *x*, we can replace the *y* variable with *x* so that we have the distance expressed as a function of *x*:

$$d(x) = \sqrt{(x-3)^2 + (x-1)^2}$$
$$= \sqrt{x^2 - 6x + 9 + x^2 - 2x + 1}$$
$$= \sqrt{2x^2 - 8x + 10}$$

Squaring both sides of this function, we obtain $[d(x)]^2 = 2x^2 - 8x + 10$.

Now, the expression on the right is quadratic. Since *a* = 2 > 0, it has a minimum. Finding the *x*-coordinate of the minimum point of $[d(x)]^2$ will also give us the *x*-coordinate of the minimum of $d(x)$: $x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$. So, 2 is the *x*-coordinate of the point on the line *y* = *x* that is closest to the point (3, 1). Since *y* = *x*, the *y*-coordinate is also 2. Thus, the point is (2, 2) is the point on the line *y* = *x* that is closest to (3, 1).

88. Let (*x*, *y*) represent a point on the line *y* = *x* + 1. Then the distance from (*x*, *y*) to the point (4, 1) is $d = \sqrt{(x-4)^2 + (y-1)^2}$. Replacing the *y* variable with *x* + 1, we find the distance expressed as a function of *x*:

$$d(x) = \sqrt{(x-4)^2 + ((x+1)-1)^2}$$
$$= \sqrt{x^2 - 8x + 16 + x^2}$$
$$= \sqrt{2x^2 - 8x + 16}$$

Squaring both sides of this function, we obtain $[d(x)]^2 = 2x^2 - 8x + 16$.

Now, the expression on the right is quadratic. Since *a* = 2 > 0, it has a minimum. Finding the *x*-coordinate of the minimum point of $[d(x)]^2$ will also give us the *x*-coordinate of the minimum of $d(x)$: $x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$. So, 2 is the *x*-coordinate of the point on the line *y* = *x* + 1 that is closest to the point (4, 1). The *y*-coordinate is *y* = 2 + 1 = 3. Thus, the point is (2, 3) is the point on the line *y* = *x* + 1 that is closest to (4, 1).

89. $R(p) = -4p^2 + 4000p$, $a = -4, b = 4000, c = 0.$ Since $a = -4 < 0$ the graph is a parabola that opens down, so the vertex is a maximum point. The maximum occurs at $p = \frac{-b}{2a} = \frac{-4000}{2(-4)} = 500$.

Thus, the unit price should be \$500 for maximum revenue. The maximum revenue is

$$R(500) = -4(500)^2 + 4000(500)$$
$$= -1000000 + 2000000$$
$$= \$1,000,000$$

90. $R(p) = -\frac{1}{2}p^2 + 1900p$, $a = -\frac{1}{2}, b = 1900, c = 0.$

Since $a = -\frac{1}{2} < 0,$ the graph is a parabola that opens down, so the vertex is a maximum point. The maximum occurs at

$p = \frac{-b}{2a} = \frac{-1900}{2(-1/2)} = \frac{-1900}{-1} = 1900$. Thus, the unit price should be \$1900 for maximum revenue. The maximum revenue is

$$R(1900) = -\frac{1}{2}(1900)^2 + 1900(1900)$$
$$= -1805000 + 3610000$$
$$= \$1,805,000$$

91. **a.** $C(x) = x^2 - 140x + 7400$, $a = 1, b = -140, c = 7400.$ Since $a = 1 > 0,$ the graph opens up, so the vertex is a minimum point. The minimum marginal cost occurs at $x = \frac{-b}{2a} = \frac{-(-140)}{2(1)} = \frac{140}{2} = 70$, 70,000 digital music players produced.

b. The minimum marginal cost is

$$f\left(\frac{-b}{2a}\right) = f(70) = (70)^2 - 140(70) + 7400$$
$$= 4900 - 9800 + 7400$$
$$= \$2500$$

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92. a. $C(x) = 5x^2 - 200x + 4000$,
$a = 5, b = -200, c = 4000$. Since $a = 5 > 0$, the graph opens up, so the vertex is a minimum point. The minimum marginal cost occurs at $x = \frac{-b}{2a} = \frac{-(-200)}{2(5)} = \frac{200}{10} = 20$, 20,000 thousand smartphones manufactured.

b. The minimum marginal cost is

$$f\left(\frac{-b}{2a}\right) = f(20) = 5(20)^2 - 200(20) + 4000$$
$$= 2000 - 4000 + 4000$$
$$= \$2000$$

93. a. $R(x) = 75x - 0.2x^2$
$a = -0.2, b = 75, c = 0$
The maximum revenue occurs when

$$x = \frac{-b}{2a} = \frac{-75}{2(-0.2)} = \frac{-75}{-0.4} = 187.5$$

The maximum revenue occurs when $x = 187$ or $x = 188$ watches.
The maximum revenue is:

$$R(187) = 75(187) - 0.2(187)^2 = \$7031.20$$
$$R(188) = 75(188) - 0.2(188)^2 = \$7031.20$$

b.
$$P(x) = R(x) - C(x)$$
$$= 75x - 0.2x^2 - (32x + 1750)$$
$$= -0.2x^2 + 43x - 1750$$

c. $P(x) = -0.2x^2 + 43x - 1750$
$a = -0.2, b = 43, c = -1750$

$$x = \frac{-b}{2a} = \frac{-43}{2(-0.2)} = \frac{-43}{-0.4} = 107.5$$

The maximum profit occurs when $x = 107$ or $x = 108$ watches.
The maximum profit is:

$$P(107) = -0.2(107)^2 + 43(107) - 1750$$
$$= \$561.20$$
$$P(108) = -0.2(108)^2 + 43(108) - 1750$$
$$= \$561.20$$

d. Answers will vary.

94. a. $R(x) = 9.5x - 0.04x^2$
$a = -0.04, b = 9.5, c = 0$
The maximum revenue occurs when

$$x = \frac{-b}{2a} = \frac{-9.5}{2(-0.04)} = \frac{-9.5}{-0.08}$$
$$= 118.75 \approx 119 \text{ boxes of candy}$$

The maximum revenue is:

$$R(119) = 9.5(119) - 0.04(119)^2 = \$564.06$$

b.
$$P(x) = R(x) - C(x)$$
$$= 9.5x - 0.04x^2 - (1.25x + 250)$$
$$= -0.04x^2 + 8.25x - 250$$

c. $P(x) = -0.04x^2 + 8.25x - 250$
$a = -0.04, b = 8.25, c = -250$
The maximum profit occurs when

$$x = \frac{-b}{2a} = \frac{-8.25}{2(-0.04)} = \frac{-8.25}{-0.08}$$
$$= 103.125 \approx 103 \text{ boxes of candy}$$

The maximum profit is:

$$P(103) = -0.04(103)^2 + 8.25(103) - 250$$
$$= \$175.39$$

d. Answers will vary.

95. a.
$$d(v) = 1.1v + 0.06v^2$$
$$d(45) = 1.1(45) + 0.06(45)^2$$
$$= 49.5 + 121.5 = 171 \text{ ft.}$$

b.
$$200 = 1.1v + 0.06v^2$$
$$0 = -200 + 1.1v + 0.06v^2$$
$$x = \frac{-(1.1) \pm \sqrt{(1.1)^2 - 4(0.06)(-200)}}{2(0.06)}$$
$$= \frac{-1.1 \pm \sqrt{49.21}}{0.12}$$
$$\approx \frac{-1.1 \pm 7.015}{0.12}$$

$v \approx 49$ or $v \approx -68$
Disregard the negative value since we are talking about speed. So the maximum speed you can be traveling would be approximately 49 mph.

c. The 1.1v term might represent the reaction time.

96. a.
$$a = \frac{-b}{2a} = \frac{-19.09}{2(-0.34)} = \frac{-19.09}{-0.68} = 28.1 \text{ years old}$$

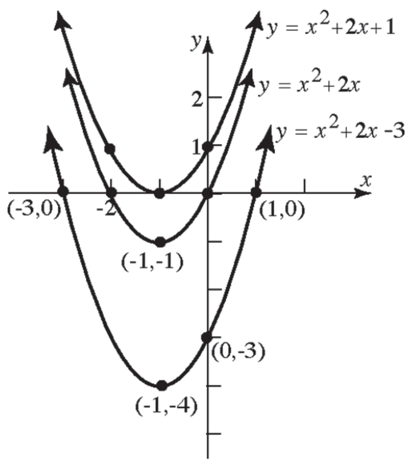
b. $B(28.1) = -0.34(28.1)^2 + 19.09(28.1) - 203.98$
≈ 63.98 births per 1000 unmarried women

c. $B(40) = -0.34(40)^2 + 19.09(40) - 203.98$
$= 15.62$ births/1000 unmarried women over 40

97. If x is even, then ax^2 and bx are even. When two even numbers are added to an odd number the result is odd. Thus, $f(x)$ is odd. If x is odd, then ax^2 and bx are odd. The sum of three odd numbers is an odd number. Thus, $f(x)$ is odd.

98. Answers will vary.

99. $y = x^2 + 2x - 3$; $y = x^2 + 2x + 1$; $y = x^2 + 2x$



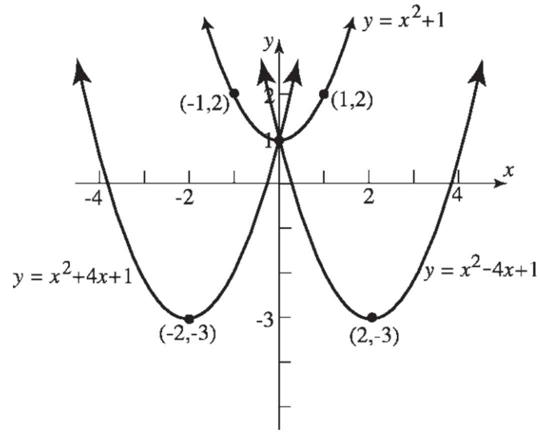
Each member of this family will be a parabola with the following characteristics:

(i) opens upwards since a > 0;

(ii) vertex occurs at $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$;

(iii) There is at least one x-intercept since $b^2 - 4ac \geq 0$.

100. $y = x^2 - 4x + 1$; $y = x^2 + 1$; $y = x^2 + 4x + 1$



Each member of this family will be a parabola with the following characteristics:

(i) opens upwards since $a > 0$

(ii) y-intercept occurs at (0, 1).

101. The graph of the quadratic function $f(x) = ax^2 + bx + c$ will not have any x-intercepts whenever $b^2 - 4ac < 0$.

102. By completing the square on the quadratic function $f(x) = ax^2 + bx + c$ we obtain the equation $y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$. We can then draw the graph by applying transformations to the graph of the basic parabola $y = x^2$, which opens up. When $a > 0$, the basic parabola will either be stretched or compressed vertically. When $a < 0$, the basic parabola will either be stretched or compressed vertically as well as reflected across the x-axis. Therefore, when $a > 0$, the graph of $f(x) = ax^2 + bx + c$ will open up, and when $a < 0$, the graph of $f(x) = ax^2 + bx + c$ will open down.

103. No. We know that the graph of a quadratic function $f(x) = ax^2 + bx + c$ is a parabola with vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. If $a > 0$, then the vertex is a minimum point, so the range is $\left[f\left(-\frac{b}{2a}\right), \infty\right)$. If $a < 0$, then the vertex is a maximum point, so the range is $\left(-\infty, f\left(-\frac{b}{2a}\right)\right]$. Therefore, it is impossible for the range to be $(-\infty, \infty)$.

104. Two quadratic functions can intersect 0, 1, or 2 times.

105. $x^2 + 4y^2 = 16$

To check for symmetry with respect to the x-axis, replace y with –y and see if the equations are equivalent.

$x^2 + 4(-y)^2 = 16$

$x^2 + 4y^2 = 16$

So the graph is symmetric with respect to the x-axis.

To check for symmetry with respect to the y-axis, replace x with –x and see if the equations are equivalent.

$(-x)^2 + 4y^2 = 16$

$x^2 + 4y^2 = 16$

So the graph is symmetric with respect to the y-axis.

To check for symmetry with respect to the origin, replace x with –x and y with –y and see if the equations are equivalent.

$(-x)^2 + 4(-y)^2 = 16$

$x^2 + 4y^2 = 16$

So the graph is symmetric with respect to the origin.

106. The radicand must be non-negative, so

$8 - 2x \ge 0$

$-2x \ge -8$

$x \le 4$

So the solution set is: $(-\infty,\ 4]$ or $\{x \mid x \le 4\}$.

107. $x^2 + y^2 - 10x + 4y + 20 = 0$

$$x^2 - 10x + y^2 + 4y = -20$$
$$(x^2 - 10x + 25) + (y^2 + 4y + 4) = -20 + 25 + 4$$
$$(x-5)^2 + (y+2)^2 = 3^2$$

Center: (5, -2); Radius = 3

108. To reflect a graph about the y-axis, we change $f(x)$ to $f(-x)$ so to reflect $y = \sqrt{x}$ about the y-axis we change it to $y = \sqrt{-x}$.

Section 2.5

1. $-3x - 2 < 7$

$-3x < 9$

$x > -3$

The solution set is $\{x \mid x > -3\}$ or $(-3, \infty)$.

2. $(-2, 7]$ represents the numbers between -2 and 7, including 7 but not including -2. Using inequality notation, this is written as $-2 < x \le 7$.

3. **a.** $f(x) > 0$ when the graph of *f* is above the *x*-axis. Thus, $\{x \mid x < -2 \text{ or } x > 2\}$ or, using interval notation, $(-\infty, -2) \cup (2, \infty)$.

b. $f(x) \le 0$ when the graph of *f* is below or intersects the *x*-axis. Thus, $\{x \mid -2 \le x \le 2\}$ or, using interval notation, $[-2,\ 2]$.

4. **a.** $g(x) < 0$ when the graph of *g* is below the *x*-axis. Thus, $\{x \mid x < -1 \text{ or } x > 4\}$ or, using interval notation, $(-\infty, -1) \cup (4, \infty)$.

b. $g(x) \ge 0$ when the graph of *f* is above or intersects the *x*-axis. Thus, $\{x \mid -1 \le x \le 4\}$ or, using interval notation, $[-1,\ 4]$.

5. **a.** $g(x) \ge f(x)$ when the graph of *g* is above or intersects the graph of *f*. Thus $\{x \mid -2 \le x \le 1\}$ or, using interval notation, $[-2,\ 1]$.

b. $f(x) > g(x)$ when the graph of *f* is above the graph of *g*. Thus, $\{x \mid x < -2 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, -2) \cup (1, \infty)$.

6. **a.** $f(x) < g(x)$ when the graph of *f* is below the graph of *g*. Thus, $\{x \mid x < -3 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, -3) \cup (1, \infty)$.

b. $f(x) \ge g(x)$ when the graph of *f* is above or intersects the graph of *g*. Thus,

$\{x \mid -3 \le x \le 1\}$ or, using interval notation, $[-3, 1]$.

7. $x^2 - 3x - 10 < 0$

We graph the function $f(x) = x^2 - 3x - 10$. The intercepts are

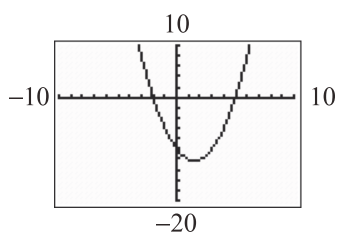
y-intercept: $f(0) = -10$

x-intercepts: $x^2 - 3x - 10 = 0$

$$(x-5)(x+2) = 0$$

$$x = 5,\ x = -2$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-3)}{2(1)} = \frac{3}{2}$. Since $f\left(\frac{3}{2}\right) = -\frac{49}{4}$, the vertex is $\left(\frac{3}{2}, -\frac{49}{4}\right)$.



The graph is below the x-axis for $-2 < x < 5$. Since the inequality is strict, the solution set is $\{x \mid -2 < x < 5\}$ or, using interval notation, $(-2, 5)$.

8. $x^2 + 3x - 10 > 0$

We graph the function $f(x) = x^2 + 3x - 10$. The intercepts are

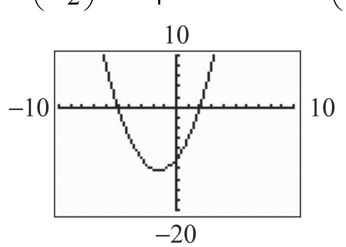
y-intercept: $f(0) = -10$

x-intercepts: $x^2 + 3x - 10 = 0$

$$(x+5)(x-2) = 0$$

$$x = -5,\ x = 2$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(3)}{2(1)} = -\frac{3}{2}$. Since $f\left(-\frac{3}{2}\right) = -\frac{49}{4}$, the vertex is $\left(-\frac{3}{2}, -\frac{49}{4}\right)$.



The graph is above the x-axis when $x < -5$ or $x > 2$. Since the inequality is strict, the solution set is $\{x \mid x < -5 \text{ or } x > 2\}$ or, using interval notation, $(-\infty, -5) \cup (2, \infty)$.

9. $x^2 - 4x > 0$

We graph the function $f(x) = x^2 - 4x$. The intercepts are

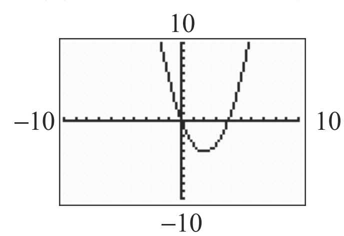
y-intercept: $f(0) = 0$

x-intercepts: $x^2 - 4x = 0$

$$x(x-4) = 0$$

$$x = 0,\ x = 4$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$. Since $f(2) = -4$, the vertex is $(2, -4)$.



The graph is above the x-axis when $x < 0$ or $x > 4$. Since the inequality is strict, the solution set is $\{x \mid x < 0 \text{ or } x > 4\}$ or, using interval notation, $(-\infty, 0) \cup (4, \infty)$.

10. $x^2 + 8x > 0$

We graph the function $f(x) = x^2 + 8x$. The intercepts are

y-intercept: $f(0) = 0$

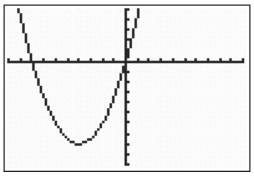
x-intercepts: $x^2 + 8x = 0$

$$x(x+8) = 0$$

$$x = 0,\ x = -8$$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(8)}{2(1)} = \dfrac{-8}{2} = -4$.

Since $f(-4) = -16$, the vertex is $(-4, -16)$.



The graph is above the x-axis when $x < -8$ or $x > 0$. Since the inequality is strict, the solution set is $\{x \mid x < -8 \text{ or } x > 0\}$ or, using interval notation, $(-\infty, -8) \cup (0, \infty)$.

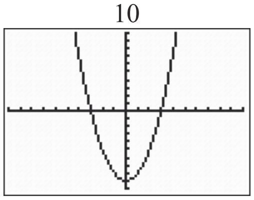
11. $x^2 - 9 < 0$

We graph the function $f(x) = x^2 - 9$. The intercepts are

y-intercept: $f(0) = -9$

x-intercepts:
$$x^2 - 9 = 0$$
$$(x+3)(x-3) = 0$$
$$x = -3, x = 3$$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(0)}{2(1)} = 0$. Since $f(0) = -9$, the vertex is $(0, -9)$.



The graph is below the x-axis when $-3 < x < 3$. Since the inequality is strict, the solution set is $\{x \mid -3 < x < 3\}$ or, using interval notation, $(-3, 3)$.

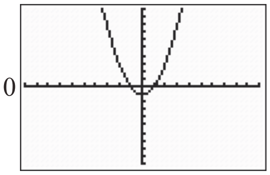
12. $x^2 - 1 < 0$

We graph the function $f(x) = x^2 - 1$. The intercepts are

y-intercept: $f(0) = -1$

x-intercepts:
$$x^2 - 1 = 0$$
$$(x+1)(x-1) = 0$$
$$x = -1, x = 1$$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(0)}{2(1)} = 0$. Since $f(0) = -1$, the vertex is $(0, -1)$.



The graph is below the x-axis when $-1 < x < 1$. Since the inequality is strict, the solution set is $\{x \mid -1 < x < 1\}$ or, using interval notation, $(-1, 1)$.

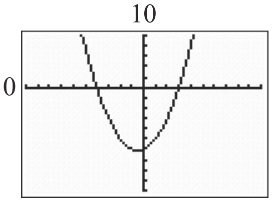
13.
$$x^2 + x > 12$$
$$x^2 + x - 12 > 0$$

We graph the function $f(x) = x^2 + x - 12$.

y-intercept: $f(0) = -12$

x-intercepts:
$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$
$$x = -4, x = 3$$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(1)}{2(1)} = -\dfrac{1}{2}$. Since $f\left(-\dfrac{1}{2}\right) = -\dfrac{49}{4}$, the vertex is $\left(-\dfrac{1}{2}, -\dfrac{49}{4}\right)$.



The graph is above the x-axis when $x < -4$ or $x > 3$. Since the inequality is strict, the solution set is $\{x \mid x < -4 \text{ or } x > 3\}$ or, using interval notation, $(-\infty, -4) \cup (3, \infty)$.

14.
$$x^2 + 7x < -12$$
$$x^2 + 7x + 12 < 0$$

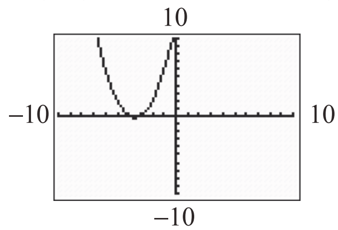
We graph the function $f(x) = x^2 + 7x + 12$.

y-intercept: $f(0) = 12$

x-intercepts:
$$x^2 + 7x + 12 = 0$$
$$(x+4)(x+3) = 0$$
$$x = -4, x = -3$$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(7)}{2(1)} = -\dfrac{7}{2}$. Since

$f\left(-\frac{7}{2}\right)=-\frac{1}{4}$, the vertex is $\left(-\frac{1}{2},-\frac{1}{4}\right)$.



The graph is below the x-axis when $-4<x<-3$. Since the inequality is strict, the solution set is $\{x\,|-4<x<-3\}$ or, using interval notation, $(-4,-3)$.

15. $2x^2<5x+3$

$2x^2-5x-3<0$

We graph the function $f(x)=2x^2-5x-3$. The intercepts are

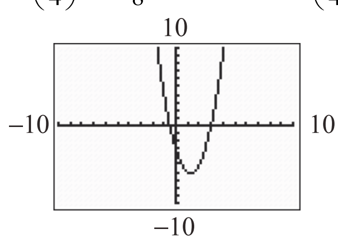
y-intercept: $f(0)=-3$

x-intercepts: $2x^2-5x-3=0$

$(2x+1)(x-3)=0$

$x=-\frac{1}{2},\ x=3$

The vertex is at $x=\frac{-b}{2a}=\frac{-(-5)}{2(2)}=\frac{5}{4}$. Since $f\left(\frac{5}{4}\right)=-\frac{49}{8}$, the vertex is $\left(\frac{5}{4},-\frac{49}{8}\right)$.



The graph is below the x-axis when $-\frac{1}{2}<x<3$. Since the inequality is strict, the solution set is $\left\{x\,\middle|\,-\frac{1}{2}<x<3\right\}$ or, using interval notation, $\left(-\frac{1}{2},\ 3\right)$.

16. $6x^2<6+5x$

$6x^2-5x-6<0$

We graph the function $f(x)=6x^2-5x-6$. The intercepts are

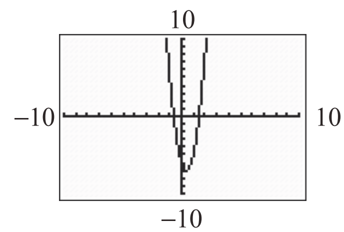
y-intercept: $f(0)=-6$

x-intercepts: $6x^2-5x-6=0$

$(3x+2)(2x-3)=0$

$x=-\frac{2}{3},\ x=\frac{3}{2}$

The vertex is at $x=\frac{-b}{2a}=\frac{-(-5)}{2(6)}=\frac{5}{12}$. Since $f\left(\frac{5}{12}\right)=-\frac{169}{24}$, the vertex is $\left(\frac{5}{12},-\frac{169}{24}\right)$.



The graph is below the x-axis when $-\frac{2}{3}<x<\frac{3}{2}$. Since the inequality is strict, the solution set is $\left\{x\,\middle|\,-\frac{2}{3}<x<\frac{3}{2}\right\}$ or, using interval notation, $\left(-\frac{2}{3},\ \frac{3}{2}\right)$.

17. $x^2-x+1\le 0$

We graph the function $f(x)=x^2-x+1$. The intercepts are

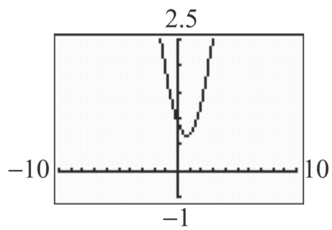
y-intercept: $f(0)=1$

x-intercepts: $x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$

$=\frac{1\pm\sqrt{-3}}{2}$ (not real)

Therefore, f has no x-intercepts.

The vertex is at $x=\frac{-b}{2a}=\frac{-(-1)}{2(1)}=\frac{1}{2}$. Since $f\left(\frac{1}{2}\right)=\frac{3}{4}$, the vertex is $\left(\frac{1}{2},\frac{3}{4}\right)$.



The graph is never below the x-axis. Thus, there is no real solution.

18. $x^2 + 2x + 4 > 0$

We graph the function $f(x) = x^2 + 2x + 4$.

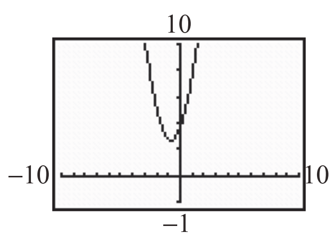
y-intercept: $f(0) = 4$

x-intercepts: $x = \dfrac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$

$= \dfrac{-2 \pm \sqrt{-12}}{2}$ (not real)

Therefore, f has no x-intercepts.

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(2)}{2(1)} = -1$. Since $f(-1) = 3$, the vertex is $(-1, 3)$.



The graph is always above the x-axis. Thus, the solution is all real numbers or using interval notation, $(-\infty,\ \infty)$.

19. $4x^2 + 9 < 6x$

$4x^2 - 6x + 9 < 0$

We graph the function $f(x) = 4x^2 - 6x + 9$.

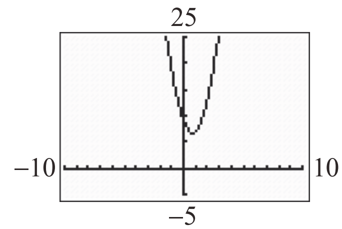
y-intercept: $f(0) = 9$

x-intercepts: $x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$

$= \dfrac{6 \pm \sqrt{-108}}{8}$ (not real)

Therefore, f has no x-intercepts.

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(-6)}{2(4)} = \dfrac{6}{8} = \dfrac{3}{4}$. Since $f\left(\dfrac{3}{4}\right) = \dfrac{27}{4}$, the vertex is $\left(\dfrac{3}{4}, \dfrac{27}{4}\right)$.



The graph is never below the x-axis. Thus, there is no real solution.

20. $25x^2 + 16 < 40x$

$25x^2 - 40x + 16 < 0$

We graph the function $f(x) = 25x^2 - 40x + 16$.

y-intercept: $f(0) = 16$

x-intercepts: $25x^2 - 40x + 16 = 0$

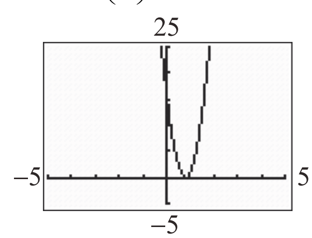
$(5x - 4)^2 = 0$

$5x - 4 = 0$

$x = \dfrac{4}{5}$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(-40)}{2(25)} = \dfrac{40}{50} = \dfrac{4}{5}$.

Since $f\left(\dfrac{4}{5}\right) = 0$, the vertex is $\left(\dfrac{4}{5}, 0\right)$.



The graph is never below the x-axis. Thus, there is no real solution.

21. $6\left(x^2 - 1\right) > 5x$

$6x^2 - 6 > 5x$

$6x^2 - 5x - 6 > 0$

We graph the function $f(x) = 6x^2 - 5x - 6$.

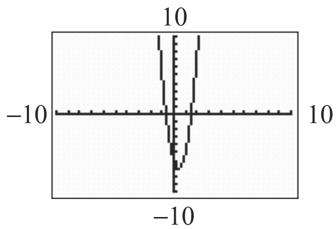
y-intercept: $f(0) = -6$

x-intercepts: $6x^2 - 5x - 6 = 0$

$(3x + 2)(2x - 3) = 0$

$x = -\dfrac{2}{3},\ x = \dfrac{3}{2}$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(-5)}{2(6)} = \dfrac{5}{12}$. Since $f\left(\dfrac{5}{12}\right) = -\dfrac{169}{24}$, the vertex is $\left(\dfrac{5}{12}, -\dfrac{169}{24}\right)$.



The graph is above the x-axis when $x < -\frac{2}{3}$ or $x > \frac{3}{2}$. Since the inequality is strict, solution set is $\left\{ x \,\middle|\, x < -\frac{2}{3} \text{ or } x > \frac{3}{2} \right\}$ or, using interval notation, $\left(-\infty, -\frac{2}{3}\right) \cup \left(\frac{3}{2}, \infty\right)$.

22. $2\left(2x^2 - 3x\right) > -9$

$4x^2 - 6x > -9$

$4x^2 - 6x + 9 > 0$

We graph the function $f(x) = 4x^2 - 6x + 9$.

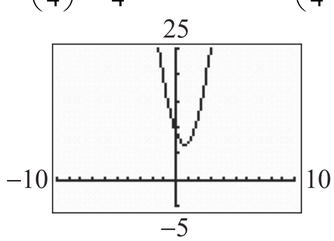
y-intercept: $f(0) = 9$

x-intercepts: $x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$

$= \dfrac{6 \pm \sqrt{-108}}{8}$ (not real)

Therefore, f has no x-intercepts.

The vertex is at $x = \frac{-b}{2a} = \frac{-(-6)}{2(4)} = \frac{6}{8} = \frac{3}{4}$. Since $f\left(\frac{3}{4}\right) = \frac{27}{4}$, the vertex is $\left(\frac{3}{4}, \frac{27}{4}\right)$.



The graph is always above the x-axis. Thus, the solution set is all real numbers or, using interval notation, $(-\infty, \infty)$.

23. The domain of the expression $f(x) = \sqrt{x^2 - 16}$ includes all values for which $x^2 - 16 \ge 0$.

We graph the function $p(x) = x^2 - 16$. The intercepts of p are

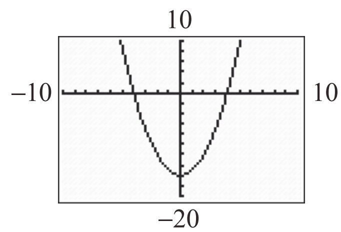
y-intercept: $p(0) = -6$

x-intercepts: $x^2 - 16 = 0$

$(x + 4)(x - 4) = 0$

$x = -4, x = 4$

The vertex of p is at $x = \frac{-b}{2a} = \frac{-(0)}{2(1)} = 0$. Since $p(0) = -16$, the vertex is $(0, -16)$.



The graph of p is above the x-axis when $x < -4$ or $x > 4$. Since the inequality is not strict, the solution set of $x^2 - 16 \ge 0$ is $\{x \mid x \le -4 \text{ or } x \ge 4\}$. Thus, the domain of f is also $\{x \mid x \le -4 \text{ or } x \ge 4\}$ or, using interval notation, $(-\infty, -4] \cup [4, \infty)$.

24. The domain of the expression $f(x) = \sqrt{x - 3x^2}$ includes all values for which $x - 3x^2 \ge 0$.

We graph the function $p(x) = x - 3x^2$. The intercepts of p are

y-intercept: $p(0) = -6$

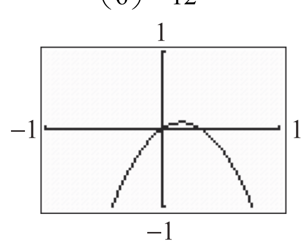
x-intercepts: $x - 3x^2 = 0$

$x(1 - 3x) = 0$

$x = 0, x = \frac{1}{3}$.

The vertex of p is at $x = \frac{-b}{2a} = \frac{-(1)}{2(-3)} = \frac{-1}{-6} = \frac{1}{6}$.

Since $p\left(\frac{1}{6}\right) = \frac{1}{12}$, the vertex is $\left(\frac{1}{6}, \frac{1}{12}\right)$.



The graph of p is above the x-axis when $0 < x < \frac{1}{3}$. Since the inequality is not strict, the solution set of $x - 3x^2 \geq 0$ is $\left\{x \middle| 0 \leq x \leq \frac{1}{3}\right\}$.

Thus, the domain of f is also $\left\{x \middle| 0 \leq x \leq \frac{1}{3}\right\}$ or, using interval notation, $\left[0, \frac{1}{3}\right]$.

25. $f(x) = x^2 - 1$; $g(x) = 3x + 3$

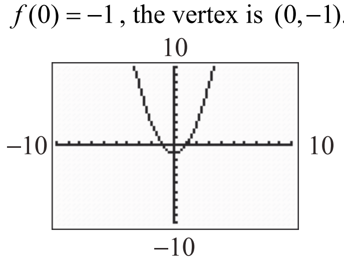
a. $f(x) = 0$
$x^2 - 1 = 0$
$(x-1)(x+1) = 0$
$x = 1; x = -1$
Solution set: $\{-1, 1\}$.

b. $g(x) = 0$
$3x + 3 = 0$
$3x = -3$
$x = -1$
Solution set: $\{-1\}$.

c. $f(x) = g(x)$
$x^2 - 1 = 3x + 3$
$x^2 - 3x - 4 = 0$
$(x-4)(x+1) = 0$
$x = 4; x = -1$
Solution set: $\{-1, 4\}$.

d. $f(x) > 0$
We graph the function $f(x) = x^2 - 1$.
y-intercept: $f(0) = -1$
x-intercepts: $x^2 - 1 = 0$
$(x+1)(x-1) = 0$
$x = -1, x = 1$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(1)} = 0$. Since $f(0) = -1$, the vertex is $(0, -1)$.

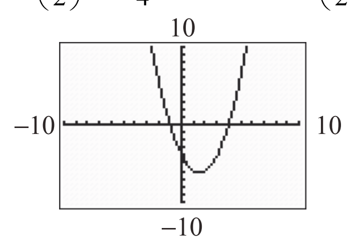


The graph is above the x-axis when $x < -1$ or $x > 1$. Since the inequality is strict, the solution set is $\{x | x < -1 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, -1) \cup (1, \infty)$.

e. $g(x) \leq 0$
$3x + 3 \leq 0$
$3x \leq -3$
$x \leq -1$
The solution set is $\{x | x \leq -1\}$ or, using interval notation, $(-\infty, -1]$.

f. $f(x) > g(x)$
$x^2 - 1 > 3x + 3$
$x^2 - 3x - 4 > 0$
We graph the function $p(x) = x^2 - 3x - 4$.
The intercepts of p are
y-intercept: $p(0) = -4$
x-intercepts: $x^2 - 3x - 4 = 0$
$(x-4)(x+1) = 0$
$x = 4, x = -1$

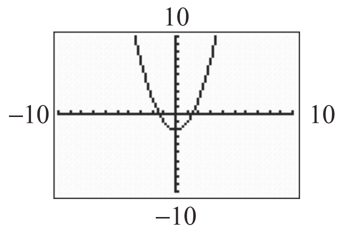
The vertex is at $x = \frac{-b}{2a} = \frac{-(-3)}{2(1)} = \frac{3}{2}$. Since $p\left(\frac{3}{2}\right) = -\frac{25}{4}$, the vertex is $\left(\frac{3}{2}, -\frac{25}{4}\right)$.



The graph of p is above the x-axis when $x < -1$ or $x > 4$. Since the inequality is strict, the solution set is $\{x | x < -1 \text{ or } x > 4\}$ or, using interval notation, $(-\infty, -1) \cup (4, \infty)$.

g. $f(x) \geq 1$
$x^2 - 1 \geq 1$
$x^2 - 2 \geq 0$
We graph the function $p(x) = x^2 - 2$. The intercepts of p are
y-intercept: $p(0) = -2$
x-intercepts: $x^2 - 2 = 0$
$x^2 = 2$
$x = \pm\sqrt{2}$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(1)} = 0$. Since $p(0) = -2$, the vertex is $(0, -2)$.



The graph of p is above the x-axis when $x < -\sqrt{2}$ or $x > \sqrt{2}$. Since the inequality is not strict, the solution set is $\left\{ x \middle| x \le -\sqrt{2} \text{ or } x \ge \sqrt{2} \right\}$ or, using interval notation, $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$.

26. $f(x) = -x^2 + 3; \quad g(x) = -3x + 3$

a.
$$f(x) = 0$$
$$-x^2 + 3 = 0$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$
Solution set: $\left\{-\sqrt{3}, \sqrt{3}\right\}$.

b.
$$g(x) = 0$$
$$-3x + 3 = 0$$
$$-3x = -3$$
$$x = 1$$
Solution set: $\{1\}$.

c.
$$f(x) = g(x)$$
$$-x^2 + 3 = -3x + 3$$
$$0 = x^2 - 3x$$
$$0 = x(x - 3)$$
$$x = 0; x = 3$$
Solution set: $\{0, 3\}$.

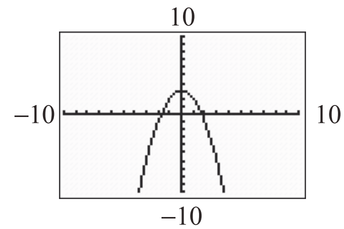
d. $f(x) > 0$

We graph the function $f(x) = -x^2 + 3$.

y-intercept: $f(0) = 3$

x-intercepts: $-x^2 + 3 = 0$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $f(0) = 3$, the vertex is $(0, 3)$.



The graph is above the x-axis when $-\sqrt{3} < x < \sqrt{3}$. Since the inequality is strict, the solution set is $\left\{ x \middle| -\sqrt{3} < x < \sqrt{3} \right\}$ or, using interval notation, $\left(-\sqrt{3}, \sqrt{3}\right)$.

e.
$$g(x) \le 0$$
$$-3x + 3 \le 0$$
$$-3x \le -3$$
$$x \ge 1$$
The solution set is $\{ x | x \ge 1\}$ or, using interval notation, $[1, \infty)$.

f.
$$f(x) > g(x)$$
$$-x^2 + 3 > -3x + 3$$
$$-x^2 + 3x > 0$$
We graph the function $p(x) = -x^2 + 3x$.

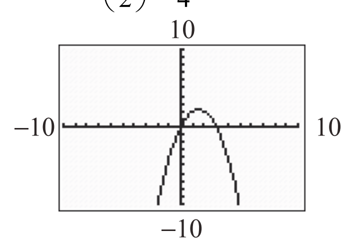
The intercepts of p are

y-intercept: $p(0) = 0$

x-intercepts: $-x^2 + 3x = 0$
$$-x(x - 3) = 0$$
$$x = 0; \; x = 3$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(3)}{2(-1)} = \frac{-3}{-2} = \frac{3}{2}$.

Since $p\left(\frac{3}{2}\right) = \frac{9}{4}$, the vertex is $\left(\frac{3}{2}, \frac{9}{4}\right)$.



The graph of p is above the x-axis when $0 < x < 3$. Since the inequality is strict, the solution set is $\{ x | 0 < x < 3 \}$ or, using interval notation, $(0, 3)$.

g. $f(x) \ge 1$

$-x^2 + 3 \ge 1$

$-x^2 + 2 \ge 0$

We graph the function $p(x) = -x^2 + 2$. The intercepts of *p* are

y-intercept: $p(0) = 2$

x-intercepts: $-x^2 + 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $p(0) = 2$, the vertex is $(0, 2)$.

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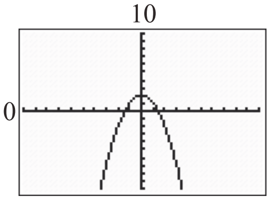
−10  10

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The graph of *p* is above the *x*-axis when $-\sqrt{2} < x < \sqrt{2}$. Since the inequality is not strict, the solution set is $\left\{ x \middle| -\sqrt{2} \le x \le \sqrt{2} \right\}$ or, using interval notation, $\left[-\sqrt{2}, \sqrt{2}\right]$.

27. $f(x) = -x^2 + 1; \quad g(x) = 4x + 1$

a. $f(x) = 0$

$-x^2 + 1 = 0$

$1 - x^2 = 0$

$(1-x)(1+x) = 0$

$x = 1;\ x = -1$

Solution set: $\{-1, 1\}$.

b. $g(x) = 0$

$4x + 1 = 0$

$4x = -1$

$x = -\frac{1}{4}$

Solution set: $\left\{-\frac{1}{4}\right\}$.

c. $f(x) = g(x)$

$-x^2 + 1 = 4x + 1$

$0 = x^2 + 4x$

$0 = x(x+4)$

$x = 0;\ x - 4$

Solution set: $\{-4, 0\}$.

d. $f(x) > 0$

We graph the function $f(x) = -x^2 + 1$.

y-intercept: $f(0) = 1$

x-intercepts: $-x^2 + 1 = 0$

$x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x = -1; x = 1$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $f(0) = 1$, the vertex is $(0, 1)$.

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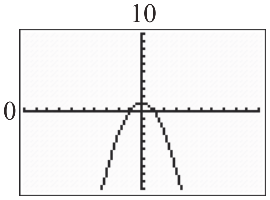
−10  10

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The graph is above the *x*-axis when $-1 < x < 1$. Since the inequality is strict, the solution set is $\{ x \mid -1 < x < 1 \}$ or, using interval notation, $(-1, 1)$.

e. $g(x) \le 0$

$4x + 1 \le 0$

$4x \le -1$

$x \le -\frac{1}{4}$

The solution set is $\left\{ x \middle| x \le -\frac{1}{4} \right\}$ or, using interval notation, $\left(-\infty, -\frac{1}{4}\right]$.

f. $f(x) > g(x)$

$-x^2 + 1 > 4x + 1$

$-x^2 - 4x > 0$

We graph the function $p(x) = -x^2 - 4x$.

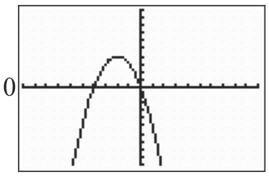
The intercepts of *p* are

y-intercept: $p(0) = 0$

x-intercepts: $-x^2 - 4x = 0$
$-x(x+4) = 0$
$x = 0;\ x = -4$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-4)}{2(-1)} = \frac{4}{-2} = -2$.

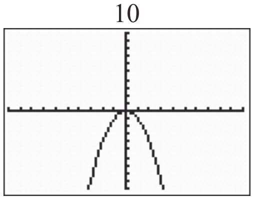
Since $p(-2) = 4$, the vertex is $(-2, 4)$.



The graph of p is above the x-axis when $-4 < x < 0$. Since the inequality is strict, the solution set is $\{x \mid -4 < x < 0\}$ or, using interval notation, $(-4, 0)$.

g. $f(x) \ge 1$
$-x^2 + 1 \ge 1$
$-x^2 \ge 0$

We graph the function $p(x) = -x^2$. The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $p(0) = 0$, the vertex is $(0, 0)$. Since $a = -1 < 0$, the parabola opens downward.



The graph of p is never above the x-axis, but it does touch the x-axis at $x = 0$. Since the inequality is not strict, the solution set is $\{0\}$.

28. $f(x) = -x^2 + 4;\quad g(x) = -x - 2$

a. $f(x) = 0$
$-x^2 + 4 = 0$
$x^2 - 4 = 0$
$(x+2)(x-2) = 0$
$x = -2; x = 2$

Solution set: $\{-2, 2\}$.

b. $g(x) = 0$
$-x - 2 = 0$
$-2 = x$

Solution set: $\{-2\}$.

c. $f(x) = g(x)$
$-x^2 + 4 = -x - 2$
$0 = x^2 - x - 6$
$0 = (x-3)(x+2)$
$x = 3; x = -2$

Solution set: $\{-2, 3\}$.

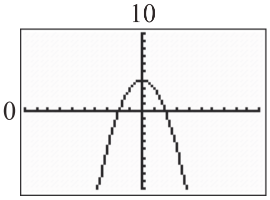
d. $f(x) > 0$
$-x^2 + 4 > 0$

We graph the function $f(x) = -x^2 + 4$.

y-intercept: $f(0) = 4$

x-intercepts: $-x^2 + 4 = 0$
$x^2 - 4 = 0$
$(x+2)(x-2) = 0$
$x = -2;\ x = 2$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $f(0) = 4$, the vertex is $(0, 4)$.



The graph is above the x-axis when $-2 < x < 2$. Since the inequality is strict, the solution set is $\{x \mid -2 < x < 2\}$ or, using interval notation, $(-2, 2)$.

e. $g(x) \le 0$
$-x - 2 \le 0$
$-x \le 2$
$x \ge -2$

The solution set is $\{x \mid x \ge -2\}$ or, using interval notation, $[-2, \infty)$.

f. $f(x) > g(x)$
$-x^2 + 4 > -x - 2$
$-x^2 + x + 6 > 0$

We graph the function $p(x) = -x^2 + x + 6$. The intercepts of p are

y-intercept: $p(0) = 6$

x-intercepts:
$$-x^2 + x + 6 = 0$$
$$x^2 - x - 6 = 0$$
$$(x+2)(x-3) = 0$$
$$x = -2;\ x = 3$$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(1)}{2(-1)} = \dfrac{-1}{-2} = \dfrac{1}{2}$.

Since $p\left(\dfrac{1}{2}\right) = \dfrac{25}{4}$, the vertex is $\left(\dfrac{1}{2}, \dfrac{25}{4}\right)$.

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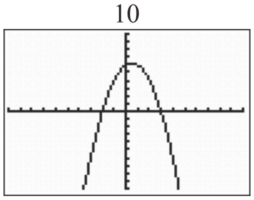
−10  10

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The graph of p is above the x-axis when $-2 < x < 3$. Since the inequality is strict, the solution set is $\{x \mid -2 < x < 3\}$ or, using interval notation, $(-2, 3)$.

g.
$$f(x) \ge 1$$
$$-x^2 + 4 > 1$$
$$-x^2 + 3 > 0$$

We graph the function $p(x) = -x^2 + 3$. The intercepts of p are

y-intercept: $p(0) = 3$

x-intercepts:
$$-x^2 + 3 = 0$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(0)}{2(-1)} = 0$. Since $p(0) = 3$, the vertex is $(0, 3)$.

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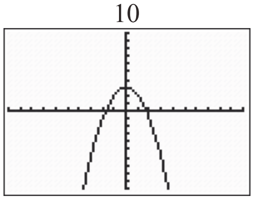
−10  10

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The graph of p is above the x-axis when $-\sqrt{3} < x < \sqrt{3}$. Since the inequality is not strict, the solution set is $\left\{x \mid -\sqrt{3} \le x \le \sqrt{3}\right\}$ or, using interval notation, $\left[-\sqrt{3}, \sqrt{3}\right]$.

29. $f(x) = x^2 - 4;\quad g(x) = -x^2 + 4$

a.
$$f(x) = 0$$
$$x^2 - 4 = 0$$
$$(x-2)(x+2) = 0$$
$$x = 2;\ x = -2$$

Solution set: $\{-2, 2\}$.

b.
$$g(x) = 0$$
$$-x^2 + 4 = 0$$
$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x = -2;\ x = 2$$

Solution set: $\{-2, 2\}$.

c.
$$f(x) = g(x)$$
$$x^2 - 4 = -x^2 + 4$$
$$2x^2 - 8 = 0$$
$$2(x-2)(x+2) = 0$$
$$x = 2;\ x = -2$$

Solution set: $\{-2, 2\}$.

d.
$$f(x) > 0$$
$$x^2 - 4 > 0$$

We graph the function $f(x) = x^2 - 4$.

y-intercept: $f(0) = -4$

x-intercepts:
$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x = -2;\ x = 2$$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(0)}{2(-1)} = 0$. Since $f(0) = -4$, the vertex is $(0, -4)$.

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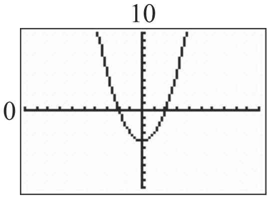
−10  10

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The graph is above the x-axis when $x < -2$ or $x > 2$. Since the inequality is strict, the solution set is $\{x \mid x < -2 \text{ or } x > 2\}$ or, using interval notation, $(-\infty, -2) \cup (2, \infty)$.

e. $g(x) \le 0$

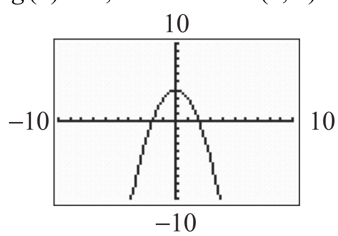
$-x^2 + 4 \le 0$

We graph the function $g(x) = -x^2 + 4$.

y-intercept: $g(0) = 4$

x-intercepts:
$$-x^2 + 4 = 0$$
$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x = -2;\ x = 2$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $g(0) = 4$, the vertex is $(0, 4)$.



The graph is below the x-axis when $x < -2$ or $x > 2$. Since the inequality is not strict, the solution set is $\{x \mid x \le -2 \text{ or } x \ge 2\}$ or, using interval notation, $(-\infty, -2] \cup [2, \infty)$.

f. $f(x) > g(x)$

$x^2 - 4 > -x^2 + 4$

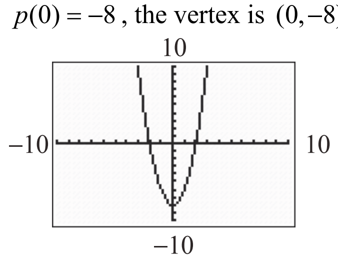
$2x^2 - 8 > 0$

We graph the function $p(x) = 2x^2 - 8$.

y-intercept: $p(0) = -8$

x-intercepts:
$$2x^2 - 8 = 0$$
$$2(x+2)(x-2) = 0$$
$$x = -2;\ x = 2$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(2)} = 0$. Since $p(0) = -8$, the vertex is $(0, -8)$.



The graph is above the x-axis when $x < -2$ or $x > 2$. Since the inequality is strict, the solution set is $\{x \mid x < -2 \text{ or } x > 2\}$ or, using interval notation, $(-\infty, -2) \cup (2, \infty)$.

g. $f(x) \ge 1$

$x^2 - 4 \ge 1$

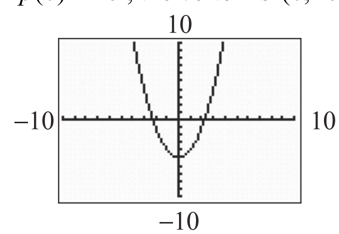
$x^2 - 5 \ge 0$

We graph the function $p(x) = x^2 - 5$.

y-intercept: $p(0) = -5$

x-intercepts:
$$x^2 - 5 = 0$$
$$x^2 = 5$$
$$x = \pm\sqrt{5}$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(1)} = 0$. Since $p(0) = -5$, the vertex is $(0, -5)$.



The graph of p is above the x-axis when $x < -\sqrt{5}$ or $x > \sqrt{5}$. Since the inequality is not strict, the solution set is $\left\{x \middle| x \le -\sqrt{5} \text{ or } x \ge \sqrt{5}\right\}$ or, using interval notation, $\left(-\infty, -\sqrt{5}\right] \cup \left[\sqrt{5}, \infty\right)$.

30. $f(x) = x^2 - 2x + 1;\ g(x) = -x^2 + 1$

a. $f(x) = 0$
$$x^2 - 2x + 1 = 0$$
$$(x-1)^2 = 0$$
$$x - 1 = 0$$
$$x = 1$$

Solution set: $\{1\}$.

b. $g(x) = 0$
$$-x^2 + 1 = 0$$
$$x^2 - 1 = 0$$
$$(x+1)(x-1) = 0$$
$$x = -1; x = 1$$

Solution set: $\{-1, 1\}$.

c. $f(x) = g(x)$
$$x^2 - 2x + 1 = -x^2 + 1$$
$$2x^2 - 2x = 0$$
$$2x(x-1) = 0$$
$$x = 0, x = 1$$

Solution set: $\{0, 1\}$.

d. $f(x) > 0$

$x^2 - 2x + 1 > 0$

We graph the function $f(x) = x^2 - 2x + 1$.

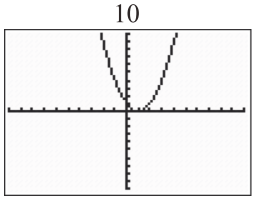
y-intercept: $f(0) = 1$

x-intercepts: $x^2 - 2x + 1 = 0$

$$(x-1)^2 = 0$$
$$x - 1 = 0$$
$$x = 1$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$.

Since $f(1) = 0$, the vertex is $(1, 0)$.



The graph is above the x-axis when $x < 1$ or $x > 1$. Since the inequality is strict, the solution set is $\{x \mid x < 1 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, 1) \cup (1, \infty)$.

e. $g(x) \le 0$

$-x^2 + 1 \le 0$

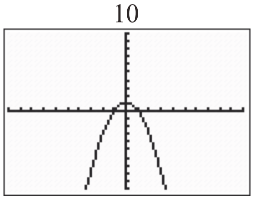
We graph the function $g(x) = -x^2 + 1$.

y-intercept: $g(0) = 1$

x-intercepts: $-x^2 + 1 = 0$

$$x^2 - 1 = 0$$
$$(x+1)(x-1) = 0$$
$$x = -1;\ x = 1$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $g(0) = 1$, the vertex is $(0, 1)$.



The graph is below the x-axis when $x < -1$ or $x > 1$. Since the inequality is not strict, the solution set is $\{x \mid x \le -1 \text{ or } x \ge 1\}$ or, using interval notation, $(-\infty, -1] \cup [1, \infty)$.

f. $f(x) > g(x)$

$x^2 - 2x + 1 > -x^2 + 1$

$2x^2 - 2x > 0$

We graph the function $p(x) = 2x^2 - 2x$.

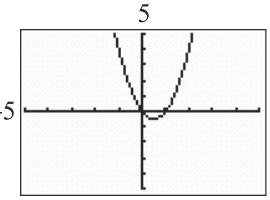
y-intercept: $p(0) = 0$

x-intercepts: $2x^2 - 2x = 0$

$$2x(x-1) = 0$$
$$x = 0;\ x = 1$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-2)}{2(2)} = \frac{2}{4} = \frac{1}{2}$.

Since $p\left(\frac{1}{2}\right) = \frac{1}{2}$, the vertex is $\left(\frac{1}{2}, \frac{1}{2}\right)$.



The graph is above the x-axis when $x < 0$ or $x > 1$. Since the inequality is strict, the solution set is $\{x \mid x < 0 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, 0) \cup (1, \infty)$.

g. $f(x) \ge 1$

$x^2 - 2x + 1 \ge 1$

$x^2 - 2x \ge 0$

We graph the function $p(x) = x^2 - 2x$.

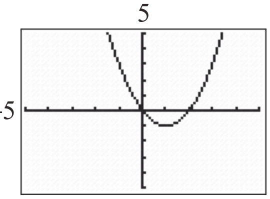
y-intercept: $p(0) = 0$

x-intercepts: $x^2 - 2x = 0$

$$x(x-2) = 0$$
$$x = 0;\ x = 2$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$.

Since $p(1) = -1$, the vertex is $(1, -1)$.



The graph of p is above the x-axis when $x < 0$ or $x > 2$. Since the inequality is not strict, the solution set is $\{x \mid x \le 0 \text{ or } x \ge 2\}$

or, using interval notation, $(-\infty, 0] \cup [2, \infty)$.

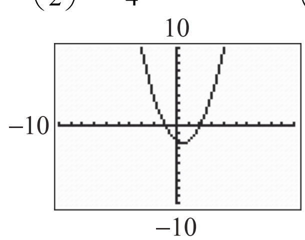
31. $f(x) = x^2 - x - 2$; $g(x) = x^2 + x - 2$

a. $f(x) = 0$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2, x = -1$
Solution set: $\{-1, 2\}$.

b. $g(x) = 0$
$x^2 + x - 2 = 0$
$(x+2)(x-1) = 0$
$x = -2; x = 1$
Solution set: $\{-2, 1\}$.

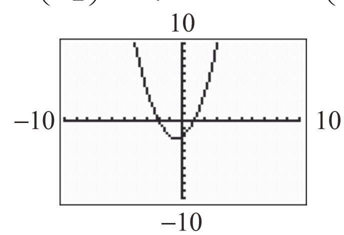
c. $f(x) = g(x)$
$x^2 - x - 2 = x^2 + x - 2$
$-2x = 0$
$x = 0$
Solution set: $\{0\}$.

d. $f(x) > 0$
$x^2 - x - 2 > 0$
We graph the function $f(x) = x^2 - x - 2$.
y-intercept: $f(0) = -2$
x-intercepts: $x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2; x = -1$
The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$. Since $f\left(\frac{1}{2}\right) = -\frac{9}{4}$, the vertex is $\left(\frac{1}{2}, -\frac{9}{4}\right)$.



The graph is above the x-axis when $x < -1$ or $x > 2$. Since the inequality is strict, the solution set is $\{x \mid x < -1 \text{ or } x > 2\}$ or, using interval notation, $(-\infty, -1) \cup (2, \infty)$.

e. $g(x) \le 0$
$x^2 + x - 2 \le 0$
We graph the function $g(x) = x^2 + x - 2$.
y-intercept: $g(0) = -2$
x-intercepts: $x^2 + x - 2 = 0$
$(x+2)(x-1) = 0$
$x = -2; x = 1$
The vertex is at $x = \frac{-b}{2a} = \frac{-(1)}{2(1)} = -\frac{1}{2}$. Since $f\left(-\frac{1}{2}\right) = -\frac{7}{4}$, the vertex is $\left(-\frac{1}{2}, -\frac{7}{4}\right)$.



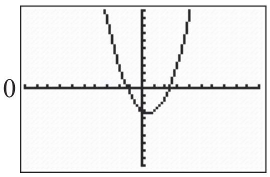
The graph is below the x-axis when $-2 < x < 1$. Since the inequality is not strict, the solution set is $\{x \mid -2 \le x \le 1\}$ or, using interval notation, $[-2, 1]$.

f. $f(x) > g(x)$
$x^2 - x - 2 > x^2 + x - 2$
$-2x > 0$
$x < 0$
The solution set is $\{x \mid x < 0\}$ or, using interval notation, $(-\infty, 0)$.

g. $f(x) \ge 1$
$x^2 - x - 2 \ge 1$
$x^2 - x - 3 \ge 0$
We graph the function $p(x) = x^2 - x - 3$.
y-intercept: $p(0) = -3$
x-intercepts: $x^2 - x - 3 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$$
$$= \frac{1 \pm \sqrt{1+12}}{2} = \frac{1 \pm \sqrt{13}}{2}$$
$x \approx -1.30$ or $x \approx 2.30$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$. Since $p\left(\frac{1}{2}\right) = -\frac{13}{4}$, the vertex is $\left(\frac{1}{2}, -\frac{13}{4}\right)$.



The graph of p is above the x-axis when $x < \frac{1-\sqrt{13}}{2}$ or $x > \frac{1+\sqrt{13}}{2}$. Since the inequality is not strict, the solution set is $\left\{x \,\middle|\, x \le \frac{1-\sqrt{13}}{2} \text{ or } x \ge \frac{1+\sqrt{13}}{2}\right\}$ or, using interval notation, $\left(-\infty, \frac{1-\sqrt{13}}{2}\right] \cup \left[\frac{1+\sqrt{13}}{2}, \infty\right)$.

32. $f(x) = -x^2 - x + 1; \quad g(x) = -x^2 + x + 6$

a.
$$f(x) = 0$$
$$-x^2 - x + 1 = 0$$
$$x^2 + x - 1 = 0$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

Solution set: $\left\{\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right\}$.

b.
$$g(x) = 0$$
$$-x^2 + x + 6 = 0$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3; x = -2$$

Solution set: $\{-2, 3\}$.

c.
$$f(x) = g(x)$$
$$-x^2 - x + 1 = -x^2 + x + 6$$
$$-2x - 5 = 0$$
$$-2x = 5$$
$$x = -\frac{5}{2}$$

Solution set: $\left\{-\frac{5}{2}\right\}$.

d.
$$f(x) > 0$$
$$-x^2 - x + 1 > 0$$

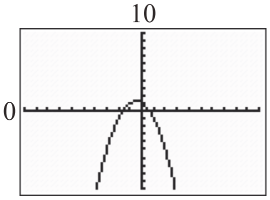
We graph the function $f(x) = -x^2 - x + 1$.

y-intercept: $f(0) = -1$

x-intercepts: $-x^2 - x + 2 = 0$
$$x^2 + x - 2 = 0$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$
$$x \approx -1.62 \text{ or } x \approx 0.62$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(-1)} = \frac{1}{-2} = -\frac{1}{2}$.

Since $f\left(-\frac{1}{2}\right) = \frac{5}{4}$, the vertex is $\left(-\frac{1}{2}, \frac{5}{4}\right)$.



The graph is above the x-axis when $\frac{-1-\sqrt{5}}{2} < x < \frac{-1+\sqrt{5}}{2}$. Since the inequality is strict, the solution set is $\left\{x \,\middle|\, \frac{-1-\sqrt{5}}{2} < x < \frac{-1+\sqrt{5}}{2}\right\}$ or, using interval notation, $\left(\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right)$.

e.
$$g(x) \le 0$$
$$-x^2 + x + 6 \le 0$$

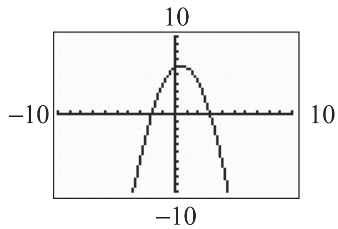
We graph the function $g(x) = -x^2 + x + 6$.

y-intercept: $g(0) = 6$

x-intercepts: $-x^2 + x + 6 = 0$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3; x = -2$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(1)}{2(-1)} = \frac{-1}{-2} = \frac{1}{2}$.

Since $f\left(\frac{1}{2}\right) = \frac{25}{4}$, the vertex is $\left(\frac{1}{2}, \frac{25}{4}\right)$.

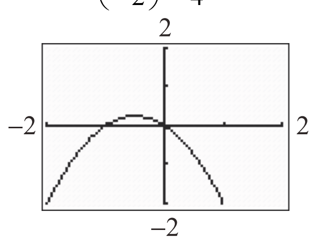


The graph is below the x-axis when $x < -2$ or $x > 3$. Since the inequality is not strict, the solution set is $\{x \mid x \le -2 \text{ or } x \ge 3\}$ or, using interval notation, $(-\infty, 2] \cup [3, \infty)$.

f.
$$f(x) > g(x)$$
$$-x^2 - x + 1 > -x^2 + x + 6$$
$$-2x > 5$$
$$x < -\frac{5}{2}$$
The solution set is $\{x \mid x < -\frac{5}{2}\}$ or, using interval notation, $(-\infty, -\frac{5}{2})$.

g.
$$f(x) \ge 1$$
$$-x^2 - x + 1 \ge 1$$
$$-x^2 - x \ge 0$$
We graph the function $p(x) = -x^2 - x$.
y-intercept: $p(0) = 0$
x-intercepts: $-x^2 - x = 0$
$$-x(x+1) = 0$$
$$x = 0; x = -1$$
The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(-1)} = \frac{1}{-2} = -\frac{1}{2}$.

Since $p\left(-\frac{1}{2}\right) = \frac{1}{4}$, the vertex is $\left(-\frac{1}{2}, \frac{1}{4}\right)$.

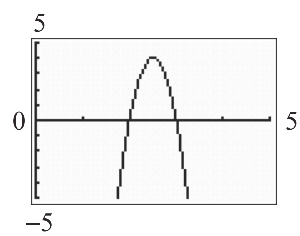


The graph of p is above the x-axis when $-1 < x < 0$. Since the inequality is not strict, the solution set is $\{x \mid -1 \le x \le 0\}$ or, using interval notation, $[-1, 0]$.

33. a. The ball strikes the ground when $s(t) = 80t - 16t^2 = 0$.
$$80t - 16t^2 = 0$$
$$16t(5 - t) = 0$$
$$t = 0, t = 5$$
The ball strikes the ground after 5 seconds.

b. Find the values of t for which
$$80t - 16t^2 > 96$$
$$-16t^2 + 80t - 96 > 0$$
We graph the function $f(t) = -16t^2 + 80t - 96$. The intercepts are
y-intercept: $f(0) = -96$
t-intercepts: $-16t^2 + 80t - 96 = 0$
$$-16(t^2 - 5t + 6) = 0$$
$$16(t-2)(t-3) = 0$$
$$t = 2, t = 3$$
The vertex is at $t = \frac{-b}{2a} = \frac{-(80)}{2(-16)} = 2.5$.

Since $f(2.5) = 4$, the vertex is $(2.5, 4)$.



The graph of f is above the t-axis when $2 < t < 3$. Since the inequality is strict, the solution set is $\{t \mid 2 < t < 3\}$ or, using interval notation, $(2, 3)$. The ball is more than 96 feet above the ground for times between 2 and 3 seconds.

34. a. The ball strikes the ground when $s(t) = 96t - 16t^2 = 0$.
$$96t - 16t^2 = 0$$
$$16t(6 - t) = 0$$
$$t = 0, t = 6$$
The ball strikes the ground after 6 seconds.

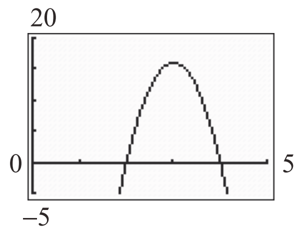
b. Find the values of t for which
$$96t - 16t^2 > 128$$
$$-16t^2 + 96t - 128 > 0$$
We graph $f(t) = -16t^2 + 96t - 128$. The intercepts are

y-intercept: $f(0) = -128$

t-intercepts: $-16t^2 + 96t - 128 = 0$

$$16(t^2 - 6t + 8) = 0$$
$$-16(t-4)(t-2) = 0$$
$$t = 4,\ t = 2$$

The vertex is at $t = \frac{-b}{2a} = \frac{-(96)}{2(-16)} = 3$. Since $f(3) = 16$, the vertex is $(3, 16)$.



The graph of f is above the t-axis when $2 < t < 4$. Since the inequality is strict, the solution set is $\{t \mid 2 < t < 4\}$ or, using interval notation, $(2, 4)$. The ball is more than 128 feet above the ground for times between 2 and 4 seconds.

35. a. $R(p) = -4p^2 + 4000p = 0$

$-4p(p - 1000) = 0$

$p = 0, p = 1000$

Thus, the revenue equals zero when the price is \$0 or \$1000.

b. Find the values of p for which

$$-4p^2 + 4000p > 800,000$$
$$-4p^2 + 4000p - 800,000 > 0$$

We graph $f(p) = -4p^2 + 4000p - 800,000$.

The intercepts are

y-intercept: $f(0) = -800,000$

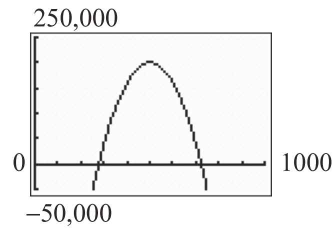
p-intercepts:

$$-4p^2 + 4000p - 800000 = 0$$
$$p^2 - 1000p + 200000 = 0$$
$$p = \frac{-(-1000) \pm \sqrt{(-1000)^2 - 4(1)(200000)}}{2(1)}$$
$$= \frac{1000 \pm \sqrt{200000}}{2}$$
$$= \frac{1000 \pm 200\sqrt{5}}{2}$$
$$= 500 \pm 100\sqrt{5}$$

$p \approx 276.39;\ p \approx 723.61$.

The vertex is at $p = \frac{-b}{2a} = \frac{-(4000)}{2(-4)} = 500$.

Since $f(500) = 200,000$, the vertex is $(500, 200000)$.



The graph of f is above the p-axis when $276.39 < p < 723.61$. Since the inequality is strict, the solution set is $\{p \mid 276.39 < p < 723.61\}$ or, using interval notation, $(276.39, 723.61)$. The revenue is more than \$800,000 for prices between \$276.39 and \$723.61.

36. a. $R(p) = -\frac{1}{2}p^2 + 1900p = 0$

$-\frac{1}{2}p(p - 3800) = 0$

$p = 0, p = 3800$

Thus, the revenue equals zero when the price is \$0 or \$3800.

b. Find the values of p for which

$$-\frac{1}{2}p^2 + 1900p > 1200000$$
$$-\frac{1}{2}p^2 + 1900p - 1200000 > 0$$

We graph $f(p) = -\frac{1}{2}p^2 + 1900p - 1200000$.

The intercepts are

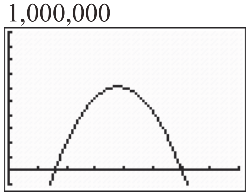
y-intercept: $f(0) = -1,200,000$

p-intercepts: $-\frac{1}{2}p^2 + 1900p - 1200000 = 0$

$$p^2 - 3800p + 2400000 = 0$$
$$(p - 800)(p - 3000) = 0$$
$$p = 800;\ p = 3000$$

The vertex is at $p = \frac{-b}{2a} = \frac{-(-1900)}{2(1/2)} = 1900$.

Since $f(1900) = 605,000$, the vertex is $(1900, 605000)$.



The graph of f is above the p-axis when $800 < p < 3000$. Since the inequality is strict, the solution set is $\{p \mid 800 < p < 3000\}$ or, using interval notation, $(800, 3000)$. The revenue is more than \$1,200,000 for prices between \$800 and \$3000.

37. $y = cx - (1 + c^2)\left(\frac{g}{2}\right)\left(\frac{x}{v}\right)^2$

a. Since the round must clear a hill 200 meters high, this mean $y > 200$.
Now $x = 2000$, $v = 897$, and $g = 9.81$.

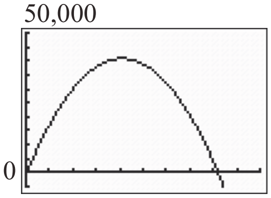
$$c(2000) - (1 + c^2)\left(\frac{9.81}{2}\right)\left(\frac{2000}{897}\right)^2 > 200$$
$$2000c - 24.3845(1 + c^2) > 200$$
$$2000c - 24.3845 - 24.3845c^2 > 200$$
$$-24.3845c^2 + 2000c - 224.3845 > 0$$

We graph $f(c) = -24.3845c^2 + 2000c - 224.3845$.
The intercepts are
y-intercept: $f(0) = -224.3845$
c-intercepts:
$-24.3845c^2 + 2000c - 224.3845 = 0$

$$c = \frac{-2000 \pm \sqrt{(2000)^2 - 4(-24.3845)(-224.3845)}}{2(-24.3845)}$$
$$= \frac{-2000 \pm \sqrt{3,978,113.985}}{-48.769}$$

$c \approx 0.112$ or $c \approx 81.907$

The vertex is at $c = \frac{-b}{2a} = \frac{-(2000)}{2(-24.3845)} = 41.010$. Since $f(41.010) \approx 40,785.273$, the vertex is $(41.010, 40785.273)$.



The graph of f is above the c-axis when $0.112 < c < 81.907$. Since the inequality is strict, the solution set is $\{c \mid 0.112 < c < 81.907\}$ or, using interval notation, $(0.112, 81.907)$.

b. Since the round is to be on the ground $y = 0$. Note, 75 km = 75,000 m. So, $x = 75,000$, $v = 897$, and $g = 9.81$.

$$c(75,000) - (1 + c^2)\left(\frac{9.81}{2}\right)\left(\frac{75,000}{897}\right)^2 = 0$$
$$75,000c - 34,290.724(1 + c^2) = 0$$
$$75,000c - 34,290.724 - 34,290.724c^2 = 0$$
$$-34,290.724c^2 + 75,000c - 34,290.724 = 0$$

We graph $f(c) = -34,290.724c^2 + 75,000c - 34,290.724$.
The intercepts are
y-intercept: $f(0) = -34,290.724$
c-intercepts:
$-34,290.724c^2 + 75,000c - 34,290.724 = 0$

$$c = \frac{-(75,000) \pm \sqrt{(75,000)^2 - 4(-34,290.724)(-34,290.724)}}{2(-34,290.724)}$$
$$= \frac{-75,000 \pm \sqrt{921,584,990.2}}{-68,581.448}$$

$c \approx 0.651$ or $c \approx 1.536$

It is possible to hit the target 75 kilometers away so long as $c \approx 0.651$ or $c \approx 1.536$.

38. $W = \frac{1}{2}kx^2$; $\tilde{W} = \frac{w}{2g}v^2$; $x \ge 0$

Note $v = 25$ mph $= \frac{110}{3}$ ft/sec. For $k = 9450$, $w = 4000$, $g = 32.2$, and $v = \frac{110}{3}$, we solve

$$W > \tilde{W}$$

$$\frac{1}{2}(9450)x^2 > \frac{4000}{2(32.2)}\left(\frac{110}{3}\right)^2$$

$$4725x^2 > 83{,}505.866$$

$$x^2 > 17.6732$$

$$x^2 - 17.6732 > 0$$

We graph $f(x) = x^2 - 17.6732$. The intercepts are

y-intercept: $f(0) = -17.6732$

x-intercepts: $x^2 - 17.6732 = 0$

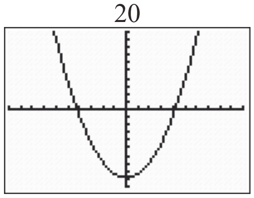
$$x^2 = 17.6732$$

$$x = \pm\sqrt{17.6732}$$

$$x \approx \pm 4.2$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(1)} = 0$. Since $f(0) = -17.6732$, the vertex is $(0, -17.6732)$.

20

−10  10

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The graph of f is above the x-axis when $x < -4.2$ or $x > 4.2$. Since we are restricted to $x \ge 0$, we disregard $x < -4.2$, so the solution is $x > 4.2$. Therefore, the spring must be able to compress at least 4.3 feet in order to stop the car safely.

39. $(x-4)^2 \le 0$

We graph the function $f(x) = (x-4)^2$.

y-intercept: $f(0) = 16$

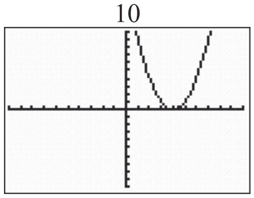
x-intercepts: $(x-4)^2 = 0$

$$x - 4 = 0$$

$$x = 4$$

The vertex is the vertex is $(4, 0)$.

10

−10  10

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The graph is never below the x-axis. Since the inequality is not strict, the only solution comes from the x-intercept. Therefore, the given inequality has exactly one real solution, namely $x = 4$.

40. $(x-2)^2 > 0$

We graph the function $f(x) = (x-2)^2$.

y-intercept: $f(0) = 4$

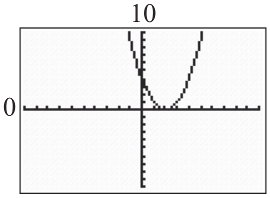
x-intercepts: $(x-2)^2 = 0$

$$x - 2 = 0$$

$$x = 2$$

The vertex is the vertex is $(2, 0)$.

10

−10  10

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The graph is above the x-axis when $x < 2$ or $x > 2$. Since the inequality is strict, the solution set is $\{x \mid x < 2 \text{ or } x > 2\}$. Therefore, the given inequality has exactly one real number that is not a solution, namely $x \ne 2$.

41. Solving $x^2 + x + 1 > 0$

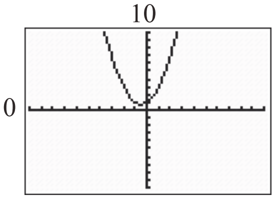
We graph the function $f(x) = x^2 + x + 1$.

y-intercept: $f(0) = 1$

x-intercepts: $b^2 - 4ac = 1^2 - 4(1)(1) = -3$, so f has no x-intercepts.

The vertex is at $x = \frac{-b}{2a} = \frac{-(1)}{2(1)} = -\frac{1}{2}$. Since $f\left(-\frac{1}{2}\right) = \frac{3}{4}$, the vertex is $\left(-\frac{1}{2}, \frac{3}{4}\right)$.

10

−10  10

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The graph is always above the x-axis. Thus, the solution is the set of all real numbers or, using interval notation, $(-\infty, \infty)$.

42. Solving $x^2 - x + 1 < 0$

We graph the function $f(x) = x^2 - x + 1$.

y-intercept: $f(0) = 1$

x-intercepts: $b^2 - 4ac = (-1)^2 - 4(1)(1) = -3$, so f has no x-intercepts.

The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$. Since $f\left(-\frac{1}{2}\right) = \frac{3}{4}$, the vertex is $\left(-\frac{1}{2}, \frac{3}{4}\right)$.

The graph is never below the x-axis. Thus, the inequality has no solution. That is, the solution set is { } or $\varnothing$.

43. The x-intercepts are included when the original inequality is not strict (when it contains an equal sign with the inequality).

44. Since the radical cannot be negative we determine what makes the radicand a nonnegative number.

$$10 - 2x \ge 0$$
$$-2x \ge -10$$
$$x \le 5$$

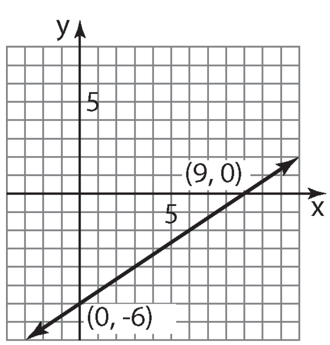
So the domain is: $\{x \mid x \le 5\}$.

45. a. $0 = \frac{2}{3}x - 6$

$$6 = \frac{2}{3}x$$
$$x = 9$$
$$y = \frac{2}{3}(0) - 6$$
$$= -6$$

The intercepts are: $(9,0),(0,-6)$

b.



46. $f(-x) = \frac{-(-x)}{(-x)^2 + 9}$

$$= -\frac{-x}{x^2 + 9} = -f(x)$$

Since $f(-x) = -f(x)$ then the function is odd.

47. $6x - 3y = 10$ $\quad$ $2x + y = -8$

$-3y = -6x + 10$ $\quad$ $y = -2x - 8$

$y = 2x - \frac{10}{3}$

Since the slopes are not equal and are not opposite reciprocals, the graphs are neither.

Section 2.6

1. $R = 3x$

2. Use LIN REGression to get $y = 1.7826x + 4.0652$

3. a. $R(x) = x\left(-\frac{1}{6}x + 100\right) = -\frac{1}{6}x^2 + 100x$

b. The quantity sold price cannot be negative, so $x \ge 0$. Similarly, the price should be positive, so $p > 0$.

$$-\frac{1}{6}x + 100 > 0$$
$$-\frac{1}{6}x > -100$$
$$x < 600$$

Thus, the implied domain for R is $\{x \mid 0 \le x < 600\}$ or $[0, 600)$.

c. $R(200) = -\frac{1}{6}(200)^2 + 100(200)$

$$= \frac{-20000}{3} + 20000$$
$$= \frac{40000}{3} \approx \$13,333.33$$

d. $x = \frac{-b}{2a} = \frac{-100}{2\left(-\frac{1}{6}\right)} = \frac{-100}{\left(-\frac{1}{3}\right)} = \frac{300}{1} = 300$

The maximum revenue is

$$R(300) = -\frac{1}{6}(300)^2 + 100(300)$$
$$= -15000 + 30000$$
$$= \$15,000$$

e. $p = -\frac{1}{6}(300) + 100 = -50 + 100 = \$50$

4. a. $R(x) = x\left(-\frac{1}{3}x + 100\right) = -\frac{1}{3}x^2 + 100x$

b. The quantity sold price cannot be negative, so $x \ge 0$. Similarly, the price should be positive, so $p > 0$.

$$-\frac{1}{3}x + 100 > 0$$
$$-\frac{1}{3}x > -100$$
$$x < 300$$

Thus, the implied domain for R is $\{x \mid 0 \le x < 300\}$ or $[0, 300)$.

c. $R(100) = -\frac{1}{3}(100)^2 + 100(100)$

$$= \frac{-10000}{3} + 10000$$
$$= \frac{20000}{3} \approx \$6,666.67$$

d. $x = \frac{-b}{2a} = \frac{-100}{2\left(-\frac{1}{3}\right)} = \frac{-100}{\left(-\frac{2}{3}\right)} = \frac{300}{2} = 150$

The maximum revenue is

$$R(150) = -\frac{1}{3}(150)^2 + 100(150)$$
$$= -7500 + 15000 = \$7,500$$

e. $p = -\frac{1}{3}(150) + 100 = -50 + 100 = \$50$

5. a. $R(x) = p\left(-5p + 100\right) = -5p^2 + 100p$

b. $R(15) = -5(17)^2 + 100(17)$

$$= -1445 + 1700 = \$255$$

c. $p = \frac{-b}{2a} = \frac{-100}{2(-5)} = \frac{-100}{(-10)} = 10$

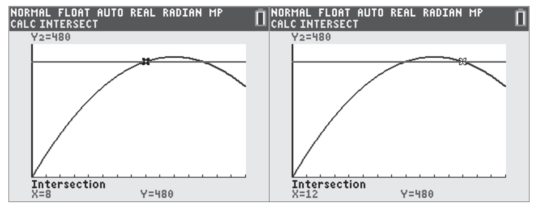
The maximum revenue is

$$R(50) = -5(10)^2 + 100(50)$$
$$= -500 + 1000 = \$500$$

d. $x = -5(10) + 100 = 50$

e. Graph $R = -5p^2 + 100p$ and $R = 480$.

Find where the graphs intersect by solving $480 = -5p^2 + 100x$.



$$5p^2 - 100p + 480 = 0$$
$$p^2 - 20p + 96 = 0$$
$$(p-8)(p-12) = 0$$
$$p = 8, p = 12$$

The company should charge between \$8 and \$12.

6. a. $R(x) = p\left(-20p + 500\right) = -20p^2 + 500p$

b. $R(24) = -20(24)^2 + 500(24)$

$$= -11528 + 12000 = \$480$$

c. $p = \frac{-b}{2a} = \frac{-500}{2(-20)} = \frac{-500}{(-40)} = \$12.50$.

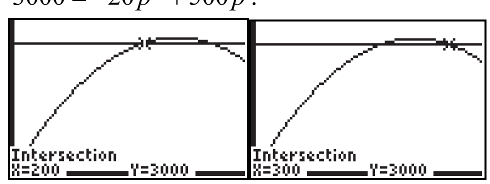
The maximum revenue is

$$R(12.5) = -20(12.5)^2 + 500(12.5)$$
$$= -3125 + 6250 = \$3125$$

d. $x = -20(12.50) + 500 = 250$

e. Graph $R = -20p^2 + 500p$ and $R = 3000$.

Find where the graphs intersect by solving $3000 = -20p^2 + 500p$.



$20p^2 - 500p + 3000 = 0$

$p^2 - 25p + 150 = 0$

$(p-10)(p-15) = 0$

$p = 10, p = 15$

The company should charge between \$10 and \$15.

7. a. Let $w =$ width and $l =$ length of the rectangular area.
Solving $P = 2w + 2l = 400$ for l:

$l = \dfrac{400 - 2w}{2} = 200 - w$.

Then $A(w) = (200 - w)w = 200w - w^2$
$= -w^2 + 200w$

b. $w = \dfrac{-b}{2a} = \dfrac{-200}{2(-1)} = \dfrac{-200}{-2} = 100$ yards

c. $A(100) = -100^2 + 200(100)$
$= -10000 + 20000$
$= 10{,}000 \text{ yd}^2$

8. a. Let $x =$ width and $y =$ width of the rectangle.
Solving $P = 2x + 2y = 3000$ for y:

$y = \dfrac{3000 - 2x}{2} = 1500 - x.$

Then $A(x) = (1500 - x)x$
$= 1500x - x^2$
$= -x^2 + 1500x.$

b. $x = \dfrac{-b}{2a} = \dfrac{-1500}{2(-1)} = \dfrac{-1500}{-2} = 750$ feet

c. $A(750) = -750^2 + 1500(750)$
$= -562500 + 1125000$
$= 562{,}500 \text{ ft}^2$

9. Let $x =$ width and $y =$ length of the rectangle.
Solving $P = 2x + y = 4000$ for y:

$y = 4000 - 2x$.

Then $A(x) = (4000 - 2x)x$
$= 4000x - 2x^2$
$= -2x^2 + 4000x$

$x = \dfrac{-b}{2a} = \dfrac{-4000}{2(-2)} = \dfrac{-4000}{-4} = 1000$ meters

maximizes area.

$A(1000) = -2(1000)^2 + 4000(1000)$.
$= -2000000 + 4000000$
$= 2{,}000{,}000$

The largest area that can be enclosed is 2,000,000 square meters.

10. Let $x =$ width and $y =$ length of the rectangle.

$2x + y = 2000$

$y = 2000 - 2x$

Then $A(x) = (2000 - 2x)x$
$= 2000x - 2x^2$
$= -2x^2 + 2000x$

$x = \dfrac{-b}{2a} = \dfrac{-2000}{2(-2)} = \dfrac{-2000}{-4} = 500$ meters

maximizes area.

$A(500) = -2(500)^2 + 2000(500)$
$= -500{,}000 + 1{,}000{,}000$
$= 500{,}000$

The largest area that can be enclosed is 500,000 square meters.

11. $h(x) = \dfrac{-32x^2}{(50)^2} + x + 200 = -\dfrac{8}{625}x^2 + x + 200$

a. $a = -\dfrac{8}{625}, b = 1, c = 200.$

The maximum height occurs when

$x = \dfrac{-b}{2a} = \dfrac{-1}{2(-8/625)} = \dfrac{625}{16} \approx 39$ feet from base of the cliff.

b. The maximum height is

$h\left(\dfrac{625}{16}\right) = \dfrac{-8}{625}\left(\dfrac{625}{16}\right)^2 + \dfrac{625}{16} + 200$
$= \dfrac{7025}{32} \approx 219.5$ feet.

c. Solving when $h(x) = 0$:

$-\dfrac{8}{625}x^2 + x + 200 = 0$

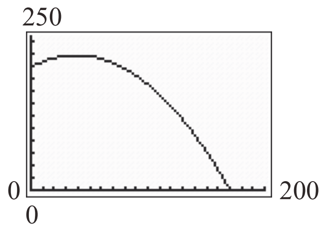
$x = \dfrac{-1 \pm \sqrt{1^2 - 4(-8/625)(200)}}{2(-8/625)}$

$x \approx \dfrac{-1 \pm \sqrt{11.24}}{-0.0256}$

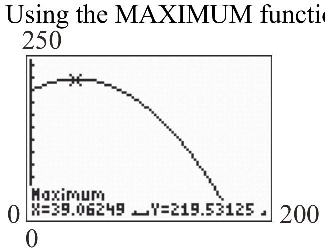
$x \approx -91.90$ or $x \approx 170$

Since the distance cannot be negative, the projectile strikes the water approximately 170 feet from the base of the cliff.

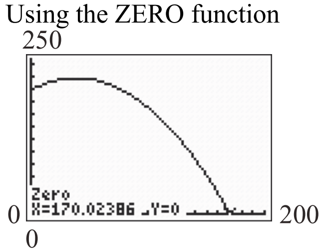
d.



e. Using the MAXIMUM function



Using the ZERO function



f. $-\frac{8}{625}x^2 + x + 200 = 100$

$-\frac{8}{625}x^2 + x + 100 = 0$

$x = \frac{\sqrt{1^2 - 4(-8/625)(100)}}{2(-8/625)} = \frac{-1 \pm \sqrt{6.12}}{-0.0256}$

$x \approx -57.57$ or $x \approx 135.70$

Since the distance cannot be negative, the projectile is 100 feet above the water when it is approximately 135.7 feet from the base of the cliff.

12. a. $h(x) = \frac{-32x^2}{(100)^2} + x = -\frac{2}{625}x^2 + x$

$a = -\frac{2}{625}, b = 1, c = 0.$

The maximum height occurs when

$x = \frac{-b}{2a} = \frac{-1}{2(-2/625)} = \frac{625}{4} = 156.25$ feet

b. The maximum height is

$h\left(\frac{625}{4}\right) = \frac{-2}{625}\left(\frac{625}{4}\right)^2 + \frac{625}{4}$

$= \frac{625}{8} = 78.125$ feet

c. Solving when $h(x) = 0$:

$-\frac{2}{625}x^2 + x = 0$

$x\left(-\frac{2}{625}x + 1\right) = 0$

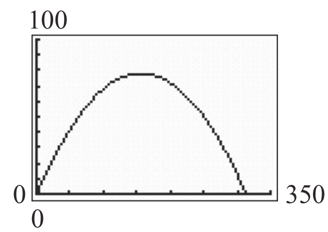
$x = 0$ or $-\frac{2}{625}x + 1 = 0$

$x = 0$ or $1 = \frac{2}{625}x$

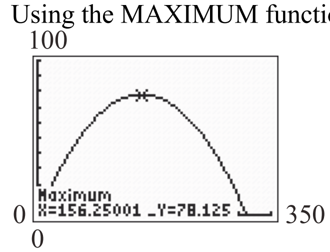
$x = 0$ or $x = \frac{625}{2} = 312.5$

Since the distance cannot be zero, the projectile lands 312.5 feet from where it was fired.

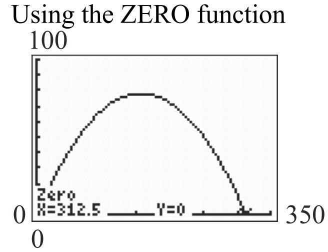
d.



e. Using the MAXIMUM function



Using the ZERO function



f. Solving when $h(x) = 50$:

$-\frac{2}{625}x^2 + x = 50$

$-\frac{2}{625}x^2 + x - 50 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(-2/625)(-50)}}{2(-2/625)}$

$= \frac{-1 \pm \sqrt{0.36}}{-0.0064} \approx \frac{-1 \pm 0.6}{-0.0064}$

$x = 62.5$ or $x = 250$

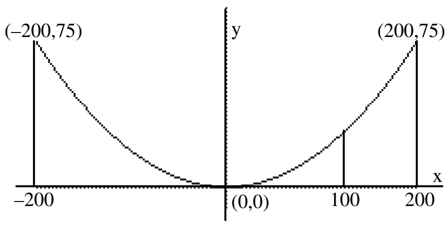
The projectile is 50 feet above the ground 62.5 feet and 250 feet from where it was fired.

13. Locate the origin at the point where the cable touches the road. Then the equation of the parabola is of the form: $y = ax^2$, where $a > 0$. Since the point (200, 75) is on the parabola, we can find the constant a:

Since $75 = a(200)^2$, then $a = \frac{75}{200^2} = 0.001875$.

When $x = 100$, we have:

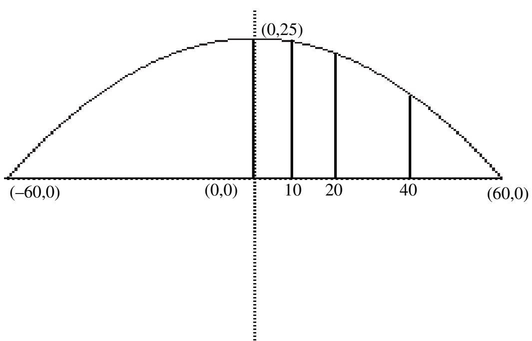
$y = 0.001875(100)^2 = 18.75$ meters.



14. Locate the origin at the point directly under the highest point of the arch. Then the equation of the parabola is of the form: $y = -ax^2 + k$, where $a > 0$. Since the maximum height is 25 feet, when $x = 0$, $y = k = 25$. Since the point (60, 0) is on the parabola, we can find the constant a: Since $0 = -a(60)^2 + 25$ then

$a = \frac{25}{60^2}$. The equation of the parabola is:

$h(x) = -\frac{25}{60^2}x^2 + 25$.



At $x = 10$:

$h(10) = -\frac{25}{60^2}(10)^2 + 25 = -\frac{25}{36} + 25 \approx 24.3$ ft.

At $x = 20$:

$h(20) = -\frac{25}{60^2}(20)^2 + 25 = -\frac{25}{9} + 25 \approx 22.2$ ft.

At $x = 40$:

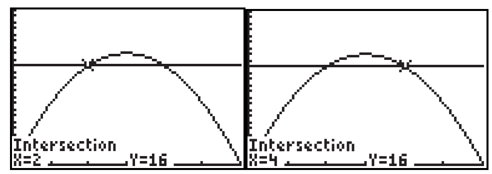
$h(40) = -\frac{25}{60^2}(40)^2 + 25 = -\frac{100}{9} + 25 \approx 13.9$ ft.

15. a. Let x = the depth of the gutter and y the width of the gutter. Then $A = xy$ is the cross-sectional area of the gutter. Since the aluminum sheets for the gutter are 12 inches wide, we have $2x + y = 12$. Solving for y: $y = 12 - 2x$. The area is to be maximized, so: $A = xy = x(12 - 2x) = -2x^2 + 12x$. This equation is a parabola opening down; thus, it has a maximum

when $x = \frac{-b}{2a} = \frac{-12}{2(-2)} = \frac{-12}{-4} = 3$.

Thus, a depth of 3 inches produces a maximum cross-sectional area.

b. Graph $A = -2x^2 + 12x$ and $A = 16$. Find where the graphs intersect by solving $16 = -2x^2 + 12x$.



$$2x^2 - 12x + 16 = 0$$
$$x^2 - 6x + 8 = 0$$
$$(x-4)(x-2) = 0$$
$$x = 4, x = 2$$

The graph of $A = -2x^2 + 12x$ is above the graph of $A = 16$ where the depth is between 2 and 4 inches.

16. Let $x =$ width of the window and $y =$ height of the rectangular part of the window. The perimeter of the window is: $x + 2y + \frac{\pi x}{2} = 20$.

Solving for y: $y = \frac{40 - 2x - \pi x}{4}$.

The area of the window is:

$$A(x) = x\left(\frac{40 - 2x - \pi x}{4}\right) + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$
$$= 10x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8}$$
$$= \left(-\frac{1}{2} - \frac{\pi}{8}\right)x^2 + 10x.$$

This equation is a parabola opening down; thus, it has a maximum when

$$x=\frac{-b}{2a}=\frac{-10}{2\left(-\frac{1}{2}-\frac{\pi}{8}\right)}=\frac{10}{\left(1+\frac{\pi}{4}\right)}\approx 5.6 \text{ feet}$$

$$y=\frac{40-2(5.60)-\pi(5.60)}{4}\approx 2.8 \text{ feet}$$

The width of the window is about 5.6 feet and the height of the rectangular part is approximately 2.8 feet. The radius of the semicircle is roughly 2.8 feet, so the total height is about 5.6 feet.

17. Let $x=$ the width of the rectangle or the diameter of the semicircle and let $y=$ the length of the rectangle. The perimeter of each semicircle is $\frac{\pi x}{2}$.

The perimeter of the track is given by: $\frac{\pi x}{2}+\frac{\pi x}{2}+y+y=1500$.

Solving for x:

$$\pi x+2y=1500$$
$$\pi x=1500-2y$$
$$x=\frac{1500-2y}{\pi}$$

The area of the rectangle is:

$$A=xy=\left(\frac{1500-2y}{\pi}\right)y=\frac{-2}{\pi}y^2+\frac{1500}{\pi}y.$$

This equation is a parabola opening down; thus, it has a maximum when

$$y=\frac{-b}{2a}=\frac{-\frac{1500}{\pi}}{2\left(\frac{-2}{\pi}\right)}=\frac{-1500}{-4}=375.$$

Thus, $x=\frac{1500-2(375)}{\pi}=\frac{750}{\pi}\approx 238.73$

The dimensions for the rectangle with maximum area are $\frac{750}{\pi}\approx 238.73$ meters by 375 meters.

18. Let $x=$ width of the window and $y=$ height of the rectangular part of the window. The perimeter of the window is:

$$3x+2y=16$$
$$y=\frac{16-3x}{2}$$

The area of the window is

$$A(x)=x\left(\frac{16-3x}{2}\right)+\frac{\sqrt{3}}{4}x^2=8x-\frac{3}{2}x^2+\frac{\sqrt{3}}{4}x^2$$
$$=\left(-\frac{3}{2}+\frac{\sqrt{3}}{4}\right)x^2+8x$$

This equation is a parabola opening down; thus, it has a maximum when

$$x=\frac{-b}{2a}=\frac{-8}{2\left(-\frac{3}{2}+\frac{\sqrt{3}}{4}\right)}=\frac{-8}{-3+\frac{\sqrt{3}}{2}}=\frac{-16}{-6+\sqrt{3}}\approx 3.75 \text{ ft.}$$

The window is approximately 3.75 feet wide.

$$y=\frac{16-3\left(\frac{-16}{-6+\sqrt{3}}\right)}{2}=\frac{16+\frac{48}{-6+\sqrt{3}}}{2}=8+\frac{24}{-6+\sqrt{3}}$$

The height of the equilateral triangle is $\frac{\sqrt{3}}{2}\left(\frac{-16}{-6+\sqrt{3}}\right)=\frac{-8\sqrt{3}}{-6+\sqrt{3}}$ feet, so the total height is $8+\frac{24}{-6+\sqrt{3}}+\frac{-8\sqrt{3}}{-6+\sqrt{3}}\approx 5.62$ feet.

19. We are given: $V(x)=kx(a-x)=-kx^2+akx$.

The reaction rate is a maximum when:

$$x=\frac{-b}{2a}=\frac{-ak}{2(-k)}=\frac{ak}{2k}=\frac{a}{2}.$$

20. We have:

$$a(-h)^2+b(-h)+c=ah^2-bh+c=y_0$$
$$a(0)^2+b(0)+c=c=y_1$$
$$a(h)^2+b(h)+c=ah^2+bh+c=y_2$$

Equating the two equations for the area, we have:

$$y_0+4y_1+y_2=ah^2-bh+c+4c+ah^2+bh+c$$
$$=2ah^2+6c.$$

Therefore,

$$\text{Area}=\frac{h}{3}\left(2ah^2+6c\right)=\frac{h}{3}\left(y_0+4y_1+y_2\right) \text{ sq. units.}$$

21. $f(x)=-5x^2+8,\ h=1$

$$\text{Area}=\frac{h}{3}\left(2ah^2+6c\right)=\frac{1}{3}\left(2(-5)(1)^2+6(8)\right)$$
$$=\frac{1}{3}(-10+48)=\frac{38}{3} \text{ sq. units}$$

22. $f(x)=2x^2+8,\quad h=2$

$$\text{Area}=\frac{h}{3}(2ah^2+6c)=\frac{2}{3}\left(2(2)(2)^2+6(8)\right)$$
$$=\frac{2}{3}(16+48)=\frac{2}{3}(64)=\frac{128}{3} \text{ sq. units}$$

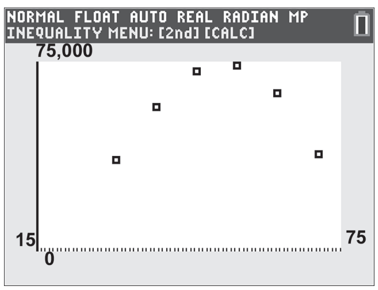
23. $f(x) = x^2 + 3x + 5, \ h = 4$

$$\text{Area} = \frac{h}{3}\left(2ah^2 + 6c\right) = \frac{4}{3}\left(2(1)(4)^2 + 6(5)\right)$$
$$= \frac{4}{3}(32 + 30) = \frac{248}{3} \text{ sq. units}$$

24. $f(x) = -x^2 + x + 4, \quad h = 1$

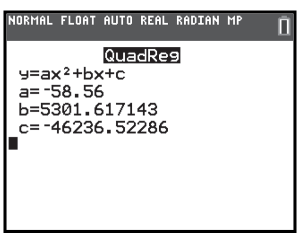
$$\text{Area} = \frac{h}{3}(2ah^2 + 6c) = \frac{1}{3}\left(2(-1)(1)^2 + 6(4)\right)$$
$$= \frac{1}{3}(-2 + 24) = \frac{1}{3}(22) = \frac{22}{3} \text{ sq. units}$$

25. a.



From the graph, the data appear to follow a quadratic relation with $a < 0$.

b. Using the QUADratic REGression program



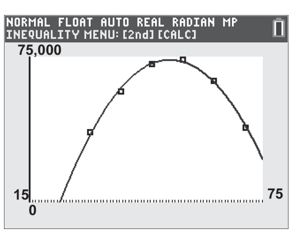
$I(x) = -58.56x^2 + 5301.617x - 46,236.523$

c. $x = \frac{-b}{2a} = \frac{-5301.617}{2(-58.56)} \approx 45.3$

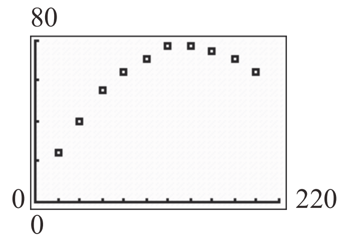
An individual will earn the most income at about 45.3 years of age.

d. The maximum income will be: $I(48.0) =$
$-58.56(45.3)^2 + 5301.617(45.3) - 46,236.523$
$\approx \$73,756$

e.

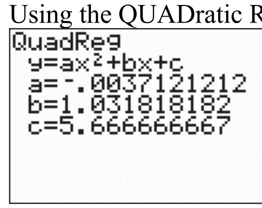


26. a.



From the graph, the data appear to follow a quadratic relation with $a < 0$.

b. Using the QUADratic REGression program



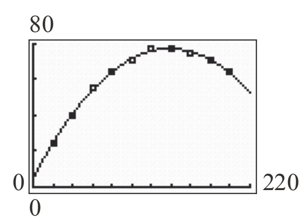
$h(x) = -0.0037x^2 + 1.0318x + 5.6667$

c. $x = \frac{-b}{2a} = \frac{-1.0318}{2(-0.0037)} \approx 139.4$

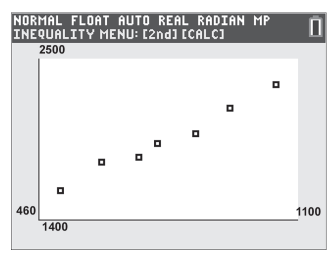
The ball will travel about 139.4 feet before it reaches its maximum height.

d. The maximum height will be: $h(139.4) =$
$-0.0037(139.4)^2 + 1.0318(139.4) + 5.6667$
≈ 77.6 feet

e.

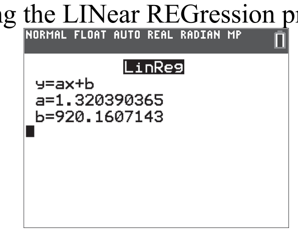


27. a.



From the graph, the data appear to be linearly related with $m > 0$.

b. Using the LINear REGression program

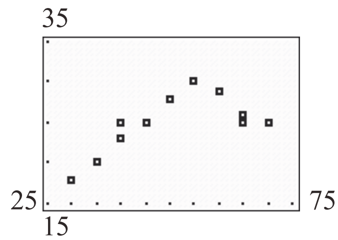


$R(x) = 1.321x + 920.161$

c. $R(875) = 1.321(875) + 920.161 \approx 2076$

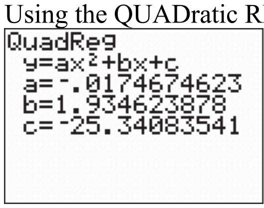
The rent for an 875 square-foot apartment in San Diego will be about \$2076 per month.

28. a.



From the graph, the data appear to follow a quadratic relation with $a < 0$.

b. Using the QUADratic REGression program



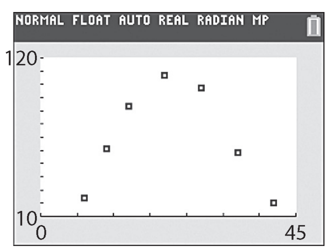
$M(s) = -0.017s^2 + 1.935s - 25.341$

c. $M(63) = -0.017(63)^2 + 1.935(63) - 25.341$

≈ 29.1

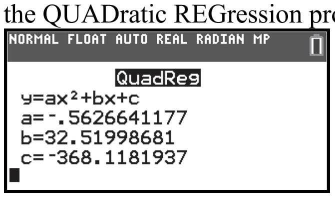
A Camry traveling 63 miles per hour will get about 29.1 miles per gallon.

29. a.



From the graph, the data appear to follow a quadratic relation with $a < 0$.

b. Using the QUADratic REGression program



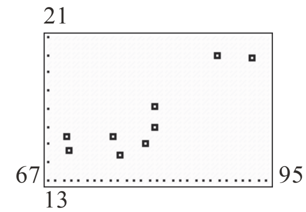
$B(a) = -0.563a^2 + 32.520a - 368.118$

c. $B(35) = -0.563(35)^2 + 32.520(35) - 368.118$

≈ 80.4

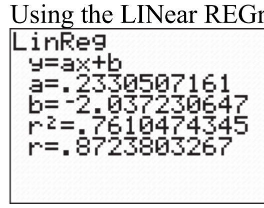
The birthrate of 35-year-old women is about 80.4 per 1000.

30. a.



From the graph, the data appear to be linearly related with $m > 0$.

b. Using the LINear REGression program



$C(x) = 0.233x - 2.037$

c. $C(80) = 0.233(80) - 2.037 \approx 16.6$

When the temperature is $80°\text{F}$, there will be about 16.6 chirps per second.

31. Answers will vary. One possibility follows: If the price is \$140, no one will buy the calculators, thus making the revenue \$0.

32. $m = \dfrac{2-(-2)}{-5-1} = \dfrac{4}{-6} = -\dfrac{2}{3}$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{2}{3}(x - 1)$$
$$y + 2 = -\frac{2}{3}x + \frac{2}{3}$$
$$y = -\frac{2}{3}x - \frac{4}{3}$$

or

$$3y = -2x - 4$$
$$2x + 3y = -4$$

33. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{((-1) - 4)^2 + (5 - (-7))^2}$$
$$= \sqrt{(-5)^2 + (12)^2}$$
$$= \sqrt{25 + 144} = \sqrt{169} = 13$$

34. $(x - h)^2 + (y - k)^2 = r^2$

$$(x - (-6))^2 + (y - 0)^2 = (\sqrt{7})^2$$
$$(x + 6)^2 + y^2 = 7$$

35. $3(0)^2 - 4y = 48$

$-4y = 48$

$y = -12$

The y intercept is $(0, -12)$

$3x^2 - 4(0) = 48$

$3x^2 = 48$

$x^2 = 16$

$x = \pm 4$

The x intercepts are: $(4, 0), (-4, 0)$

Section 2.7

1. Integers: $\{-3, 0\}$

Rationals: $\left\{-3, 0, \frac{6}{5}\right\}$

2. True; the set of real numbers consists of all rational and irrational numbers.

3. $10 - 5i$

4. $2 - 5i$

5. True

6. $9i$

7. $2 + 3i$

8. True

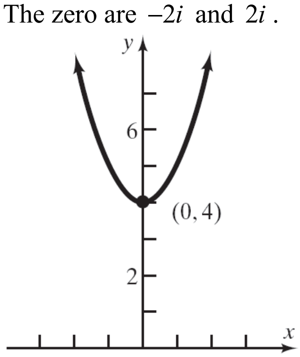
9. $f(x) = 0$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4} = \pm 2i$

The zero are $-2i$ and $2i$.



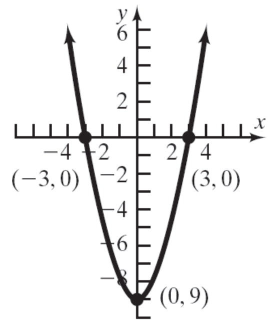
10. $f(x) = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm\sqrt{9} = \pm 3$

The zeros are -3 and 3.



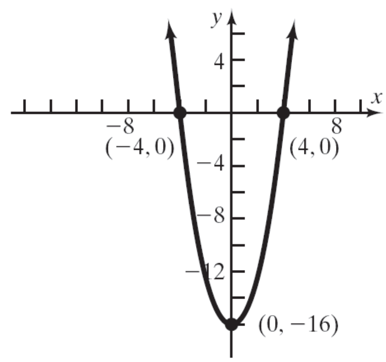
11. $f(x) = 0$

$x^2 - 16 = 0$

$x^2 = 16$

$x = \pm\sqrt{16} = \pm 4$

The zeros are -4 and 4.



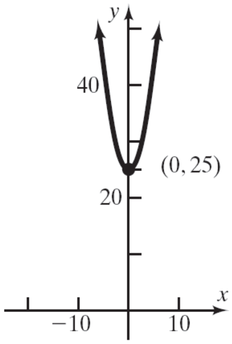
12. $f(x)=0$

$x^2+25=0$

$x^2=-25$

$x=\pm\sqrt{-25}=\pm 5i$

The zeros are $-5i$ and $5i$.



13. $f(x)=0$

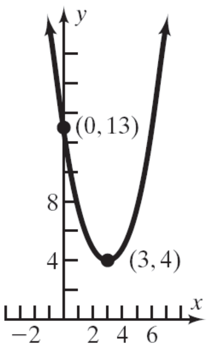
$x^2-6x+13=0$

$a=1, b=-6, c=13,$

$b^2-4ac=(-6)^2-4(1)(13)=36-52=-16$

$$x=\frac{-(-6)\pm\sqrt{-16}}{2(1)}=\frac{6\pm 4i}{2}=3\pm 2i$$

The zeros are $3-2i$ and $3+2i$.



14. $f(x)=0$

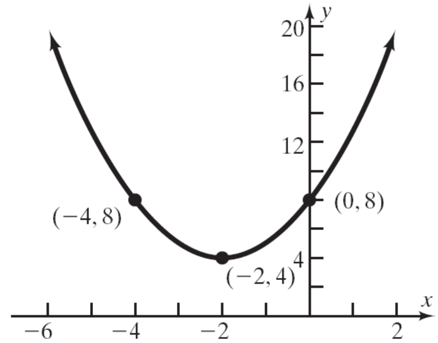
$x^2+4x+8=0$

$a=1, b=4, c=8$

$b^2-4ac=4^2-4(1)(8)=16-32=-16$

$$x=\frac{-4\pm\sqrt{-16}}{2(1)}=\frac{-4\pm 4i}{2}=-2\pm 2i$$

The zeros are $-2-2i$ and $-2+2i$.



15. $f(x)=0$

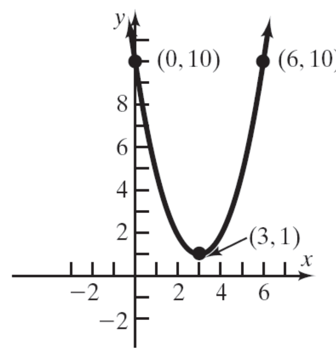
$x^2-6x+10=0$

$a=1, b=-6, c=10$

$b^2-4ac=(-6)^2-4(1)(10)=36-40=-4$

$$x=\frac{-(-6)\pm\sqrt{-4}}{2(1)}=\frac{6\pm 2i}{2}=3\pm i$$

The zeros are $3-i$ and $3+i$.



16. $f(x)=0$

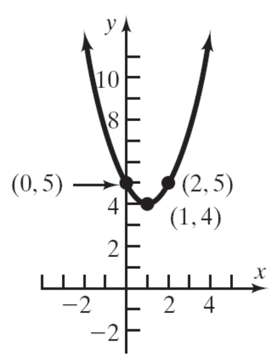
$x^2-2x+5=0$

$a=1, b=-2, c=5$

$b^2-4ac=(-2)^2-4(1)(5)=4-20=-16$

$$x=\frac{-(-2)\pm\sqrt{-16}}{2(1)}=\frac{2\pm 4i}{2}=1\pm 2i$$

The zeros are $1-2i$ and $1+2i$.



17. $f(x)=0$

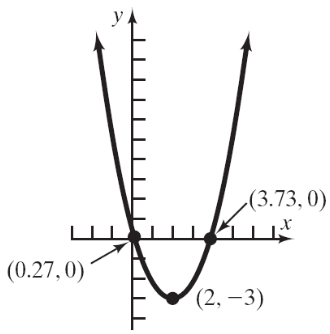
$x^2-4x+1=0$

$a=1, b=-4, c=1$

$b^2-4ac=(-4)^2-4(1)(1)=16-4=12$

$x=\dfrac{-(-4)\pm\sqrt{12}}{2(1)}=\dfrac{4\pm 2\sqrt{3}}{2}=2\pm\sqrt{3}$

The zeros are $2-\sqrt{3}$ and $2+\sqrt{3}$, or approximately 0.27 and 3.73.



18. $f(x)=0$

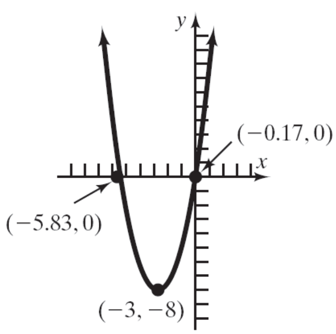
$x^2+6x+1=0$

$a=1, b=6, c=1$

$b^2-4ac=6^2-4(1)(1)=36-4=32$

$x=\dfrac{-6\pm\sqrt{32}}{2(1)}=\dfrac{-6\pm 4\sqrt{2}}{2}=-3\pm 2\sqrt{2}$

The zeros are $-3-2\sqrt{2}$ and $-3+2\sqrt{2}$, or approximately -5.83 and -0.17.



19. $f(x)=0$

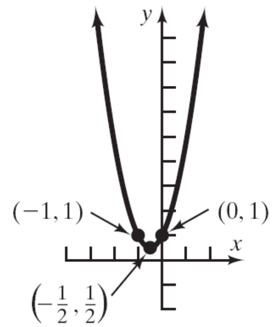
$2x^2+2x+1=0$

$a=2, b=2, c=1$

$b^2-4ac=(2)^2-4(2)(1)=4-8=-4$

$x=\dfrac{-2\pm\sqrt{-4}}{2(2)}=\dfrac{-2\pm 2i}{4}=-\dfrac{1}{2}\pm\dfrac{1}{2}i$

The zeros are $-\dfrac{1}{2}-\dfrac{1}{2}i$ and $-\dfrac{1}{2}+\dfrac{1}{2}i$.



20. $f(x)=0$

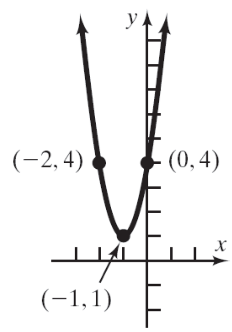
$3x^2+6x+4=0$

$a=3, b=6, c=4$

$b^2-4ac=(6)^2-4(3)(4)=36-48=-12$

$x=\dfrac{-6\pm\sqrt{-12}}{2(3)}=\dfrac{-6\pm 2\sqrt{3}i}{6}=-1\pm\dfrac{\sqrt{3}}{3}i$

The zeros are $-1-\dfrac{\sqrt{3}}{3}i$ and $-1+\dfrac{\sqrt{3}}{3}i$.



21. $f(x)=0$

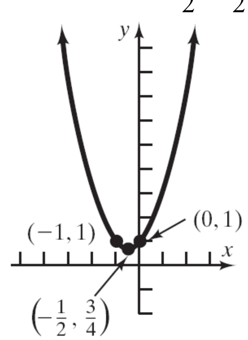
$x^2+x+1=0$

$a=1, b=1, c=1,$

$b^2-4ac=1^2-4(1)(1)=1-4=-3$

$x=\dfrac{-1\pm\sqrt{-3}}{2(1)}=\dfrac{-1\pm\sqrt{3}\,i}{2}=-\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$

The zeros are $-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$ and $-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$.



22. $f(x)=0$

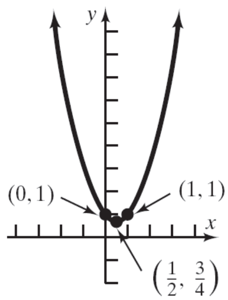
$x^2-x+1=0$

$a=1, b=-1, c=1$

$b^2-4ac=(-1)^2-4(1)(1)=1-4=-3$

$x=\dfrac{-(-1)\pm\sqrt{-3}}{2(1)}=\dfrac{1\pm\sqrt{3}\,i}{2}=\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$

The zeros are $\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$ and $\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$.



23. $f(x)=0$

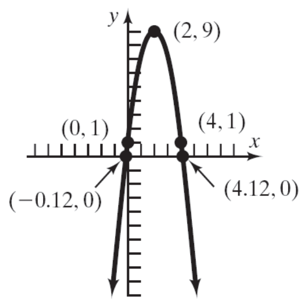
$-2x^2+8x+1=0$

$a=-2, b=8, c=1$

$b^2-4ac=8^2-4(-2)(1)=64+8=72$

$x=\dfrac{-8\pm\sqrt{72}}{2(-2)}=\dfrac{-8\pm 6\sqrt{2}}{-4}=\dfrac{4\pm 3\sqrt{2}}{2}=2\pm\dfrac{3\sqrt{2}}{2}$

The zeros are $\dfrac{4-3\sqrt{2}}{2}$ and $\dfrac{4+3\sqrt{2}}{2}$, or approximately -0.12 and 4.12.



24. $f(x)=0$

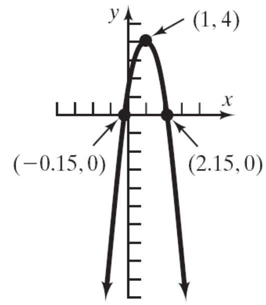
$-3x^2+6x+1=0$

$a=-3, b=6, c=1$

$b^2-4ac=6^2-4(-3)(1)=36+12=48$

$x=\dfrac{-6\pm\sqrt{48}}{2(-3)}=\dfrac{-6\pm 4\sqrt{3}}{-6}=\dfrac{3\pm 2\sqrt{3}}{3}=1\pm\dfrac{2\sqrt{3}}{3}$

The zeros are $\dfrac{3-2\sqrt{3}}{3}$ and $\dfrac{3+2\sqrt{3}}{3}$, or approximately -0.15 and 2.15.



25. $3x^2-3x+4=0$

$a=3, b=-3, c=4$

$b^2-4ac=(-3)^2-4(3)(4)=9-48=-39$

The equation has two complex solutions that are conjugates of each other.

26. $2x^2-4x+1=0$

$a=2, b=-4, c=1$

$b^2-4ac=(-4)^2-4(2)(1)=16-8=8$

The equation has two unequal real number solutions.

27. $2x^2+3x-4=0$

$a=2, b=3, c=-4$

$b^2-4ac=3^2-4(2)(-4)=9+32=41$

The equation has two unequal real solutions.

28. $x^2+2x+6=0$

$a=1, b=2, c=6$

$b^2-4ac=(2)^2-4(1)(6)=4-24=-20$

The equation has two complex solutions that are conjugates of each other.

29. $9x^2-12x+4=0$

$a=9, b=-12, c=4$

$b^2-4ac=(-12)^2-4(9)(4)=144-144=0$

The equation has a repeated real solution.

30. $4x^2+12x+9=0$

$a=4, b=12, c=9$

$b^2-4ac=12^2-4(4)(9)=144-144=0$

The equation has a repeated real solution.

31. $t^4-16=0$

$(t^2-4)(t^2+4)=0$

$t^2=4 \quad t^2=-4$

$t=\pm 2 \quad t=\pm 2i$

32. $y^4 - 81 = 0$

$(y^2 - 9)(y^2 + 9) = 0$

$y^2 = 9 \quad y^2 = -9$

$y = \pm 3 \quad y = \pm 3i$

33. $F(x) = x^6 - 9x^3 + 8 = 0$

$(x^3 - 8)(x^3 - 1) = 0$

$(x-2)(x^2 + 2x + 4)(x-1)(x^2 + x + 1) = 0$

$x^2 + 2x + 4 = 0 \rightarrow a = 1, b = 2, c = 4$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(4)}}{2(1)} = \dfrac{-2 \pm \sqrt{-12}}{2} = \dfrac{-2 \pm 2i\sqrt{3}}{2}$

$= -1 \pm \sqrt{3}i$

$x^2 + x + 1 = 0 \rightarrow a = 1, b = 1, c = 1$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)}}{2(1)} = \dfrac{-1 \pm \sqrt{-3}}{2} = \dfrac{-1 \pm i\sqrt{3}}{2}$

$= -\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}i$

The solution set is $\left\{-1 \pm i\sqrt{3}, -\dfrac{1}{2} \pm i\dfrac{\sqrt{3}}{2}, 2, 1\right\}$

34. $P(z) = z^6 + 28z^3 + 27 = 0$

$(z^3 + 27)(z^3 + 1) = 0$

$(z+3)(z^2 - 3z + 9)(z+1)(z^2 - z + 1) = 0$

$z^2 - 3z + 9 = 0$

$a = 1, b = -3, c = 9$

$z = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(9)}}{2(1)} = \dfrac{3 \pm \sqrt{-27}}{2}$

$= \dfrac{3 \pm 3i\sqrt{3}}{2} = \dfrac{3}{2} \pm \dfrac{3\sqrt{3}}{2}i$

$z^2 - z + 1 = 0 \rightarrow a = 1, b = -1, c = 1$

$z = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)}}{2(1)} = \dfrac{1 \pm \sqrt{-3}}{2} = \dfrac{1 \pm i\sqrt{3}}{2}$

$= \dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}i$

The solution set is $\left\{\dfrac{3}{2} \pm \dfrac{3\sqrt{3}}{2}i, \dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}i, -3, -1\right\}$

35. $f(x) = \dfrac{x}{x+1} \quad g(x) = \dfrac{x+2}{x}$

$(g - f)(x) = \dfrac{x+2}{x} - \dfrac{x}{x+1}$

$= \dfrac{(x+2)(x+1)}{x(x+1)} - \dfrac{x(x)}{x(x+1)}$

$= \dfrac{x^2 + 3x + 2}{x(x+1)} - \dfrac{x^2}{x(x+1)}$

$= \dfrac{x^2 + 3x + 2 - x^2}{x(x+1)}$

$= \dfrac{3x + 2}{x(x+1)}$

Domain: $\{x \mid x \neq -1, x \neq 0\}$

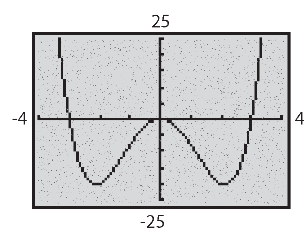
36. a. Domain: $[-3, 3]$ Range: $[-2, 2]$

b. Intercepts: $(-3, 0), (0, 0), (3, 0)$

c. Symmetric with respect to the orgin.

d. The relation is a function. It passes the vertical line test.

37.



Local maximum: (0,0)
Local Minima: (-2.12,-20.25), (2.12,-20.25)
Increasing: (-2.12,0), (2.12,4)
Decreasing: (-4, -2.12), (0,2.12)

38. $y = \dfrac{k}{x^2}$

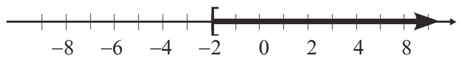
$24 = \dfrac{k}{5^2} = \dfrac{k}{25}$

$k = 600$

$y = \dfrac{600}{x^2}$

Section 2.8

1. $x \geq -2$



2. The distance on a number line from the origin to a is $|a|$ for any real number a.

3. $4x - 3 = 9$

$4x = 12$

$x = 3$

The solution set is $\{3\}$.

4. $3x - 2 > 7$

$3x > 9$

$x > 3$

The solution set is $\{x \mid x > 3\}$ or, using interval notation, $(3, \infty)$.

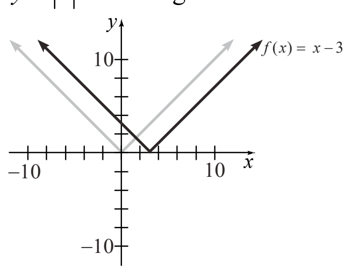
5. $-1 < 2x + 5 < 13$

$-6 < 2x < 8$

$-3 < x < 4$

The solution set is $\{x \mid -3 < x < 4\}$ or, using interval notation, $(-3, 4)$.

6. To graph $f(x) = |x - 3|$, shift the graph of $y = |x|$ to the right 3 units.



7. $-a$; a

8. $-a < u < a$

9. $\leq$

10. True

11. False. Any real number will be a solution of $|x| > -2$ since the absolute value of any real number is positive.

12. False. $|u| > a$ is equivalent to $u < -a$ or $u > a$.

13. a. Since the graphs of *f* and *g* intersect at the points $(-9, 6)$ and $(3, 6)$, the solution set of $f(x) = g(x)$ is $\{-9, 3\}$.

b. Since the graph of *f* is below the graph of *g* when *x* is between -9 and 3, the solution set of $f(x) \leq g(x)$ is $\{x \mid -9 \leq x \leq 3\}$ or, using interval notation, $[-9, 3]$.

c. Since the graph of *f* is above the graph of *g* to the left of $x = -9$ and to the right of $x = 3$, the solution set of $f(x) > g(x)$ is $\{x \mid x < -9 \text{ or } x > 3\}$ or, using interval notation, $(-\infty, -9) \cup (3, \infty)$.

14. a. Since the graphs of *f* and *g* intersect at the points $(0, 2)$ and $(4, 2)$, the solution set of $f(x) = g(x)$ is $\{0, 4\}$.

b. Since the graph of *f* is below the graph of *g* when *x* is between 0 and 4, the solution set of $f(x) \leq g(x)$ is $\{x \mid 0 \leq x \leq 4\}$ or, using interval notation, $[0, 4]$.

c. Since the graph of *f* is above the graph of *g* to the left of $x = 0$ and to the right of $x = 4$, the solution set of $f(x) > g(x)$ is $\{x \mid x < 0 \text{ or } x > 4\}$ or, using interval notation, $(-\infty, 0) \cup (4, \infty)$.

15. a. Since the graphs of *f* and *g* intersect at the points $(-2, 5)$ and $(3, 5)$, the solution set of $f(x) = g(x)$ is $\{-2, 3\}$.

b. Since the graph of *f* is above the graph of *g* to the left of $x = -2$ and to the right of $x = 3$, the solution set of $f(x) \geq g(x)$ is $\{x \mid x \leq -2 \text{ or } x \geq 3\}$ or, using interval notation, $(-\infty, -2] \cup [3, \infty)$.

c. Since the graph of *f* is below the graph of *g* when *x* is between -2 and 3, the solution set of $f(x) < g(x)$ is $\{x \mid -2 < x < 3\}$ or, using interval notation, $(-2, 3)$.

16. a. Since the graphs of *f* and *g* intersect at the points $(-4, 7)$ and $(3, 7)$, the solution set of $f(x) = g(x)$ is $\{-4, 3\}$.

b. Since the graph of *f* is above the graph of *g* to the left of $x = -4$ and to the right of $x = 3$, the solution set of $f(x) \geq g(x)$ is $\{x \mid x \leq -4 \text{ or } x \geq 3\}$ or, using interval notation, $(-\infty, -4] \cup [3, \infty)$.

c. Since the graph of f is below the graph of g when x is between -4 and 3, the solution set of $f(x) < g(x)$ is $\{x \mid -4 < x < 3\}$ or, using interval notation, $(-4, 3)$.

17. $|x| = 6$

$x = 6$ or $x = -6$

The solution set is $\{-6, 6\}$.

18. $|x| = 12$

$x = 12$ or $x = -12$

The solution set is $\{-12, 12\}$.

19. $|2x+3| = 5$

$2x+3 = 5$ or $2x+3 = -5$

$2x = 2$ or $2x = -8$

$x = 1$ or $x = -4$

The solution set is $\{-4, 1\}$.

20. $|3x-1| = 2$

$3x-1 = 2$ or $3x-1 = -2$

$3x = 3$ or $3x = -1$

$x = 1$ or $x = -\frac{1}{3}$

The solution set is $\left\{-\frac{1}{3}, 1\right\}$.

21. $|1-4t| + 8 = 13$

$|1-4t| = 5$

$1-4t = 5$ or $1-4t = -5$

$-4t = 4$ or $-4t = -6$

$t = -1$ or $t = \frac{3}{2}$

The solution set is $\left\{-1, \frac{3}{2}\right\}$.

22. $|1-2z| + 6 = 9$

$|1-2z| = 3$

$1-2z = 3$ or $1-2z = -3$

$-2z = 2$ or $-2z = -4$

$z = -1$ or $z = 2$

The solution set is $\{-1, 2\}$.

23. $|-2x| = 8$

$-2x = 8$ or $-2x = -8$

$x = -4$ or $x = 4$

The solution set is $\{-4, 4\}$.

24. $|-x| = 1$

$-x = 1$ or $-x = -1$

The solution set is $\{-1, 1\}$.

25. $4 - |2x| = 3$

$-|2x| = -1$

$|2x| = 1$

$2x = 1$ or $2x = -1$

$x = \frac{1}{2}$ or $x = -\frac{1}{2}$

The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$.

26. $5 - \left|\frac{1}{2}x\right| = 3$

$-\left|\frac{1}{2}x\right| = -2$

$\left|\frac{1}{2}x\right| = 2$

$\frac{1}{2}x = 2$ or $\frac{1}{2}x = -2$

$x = 4$ or $x = -4$

The solution set is $\{-4, 4\}$.

27. $\frac{2}{3}|x| = 9$

$|x| = \frac{27}{2}$

$x = \frac{27}{2}$ or $x = -\frac{27}{2}$

The solution set is $\left\{-\frac{27}{2}, \frac{27}{2}\right\}$.

28. $\frac{3}{4}|x| = 9$

$|x| = 12$

$x = 12$ or $x = -12$

The solution set is $\{-12, 12\}$.

29. $\left|\frac{x}{3}+\frac{2}{5}\right|=2$

$\frac{x}{3}+\frac{2}{5}=2$ or $\frac{x}{3}+\frac{2}{5}=-2$

$5x+6=30$ or $5x+6=-30$

$5x=24$ or $5x=-36$

$x=\frac{24}{5}$ or $x=-\frac{36}{5}$

The solution set is $\left\{-\frac{36}{5},\frac{24}{5}\right\}$.

30. $\left|\frac{x}{2}-\frac{1}{3}\right|=1$

$\frac{x}{2}-\frac{1}{3}=1$ or $\frac{x}{2}-\frac{1}{3}=-1$

$3x-2=6$ or $3x-2=-6$

$3x=8$ or $3x=-4$

$x=\frac{8}{3}$ or $x=-\frac{4}{3}$

The solution set is $\left\{-\frac{4}{3},\frac{8}{3}\right\}$.

31. $|u-2|=-\frac{1}{2}$

No solution, since absolute value always yields a non-negative number.

32. $|2-v|=-1$

No solution, since absolute value always yields a non-negative number.

33. $|x^2-9|=0$

$x^2-9=0$

$x^2=9$

$x=\pm 3$

The solution set is $\{-3, 3\}$.

34. $|x^2-16|=0$

$x^2-16=0$

$x^2=16$

$x=\pm 4$

The solution set is $\{-4, 4\}$.

35. $|x^2-2x|=3$

$x^2-2x=3$ or $x^2-2x=-3$

$x^2-2x-3=0$ or $x^2-2x+3=0$

$(x-3)(x+1)=0$ or $x^2-2x+3=0$

$x=\frac{2\pm\sqrt{4-12}}{2}$

$=\frac{2\pm\sqrt{-8}}{2}=1\pm\sqrt{2}i$

$x=3$ or $x=-1$

The solution set is $\left\{-1,3,1-\sqrt{2}i,1+\sqrt{2}i\right\}$.

36. $|x^2+x|=12$

$x^2+x=12$ or $x^2+x=-12$

$x^2+x-12=0$ or $x^2+x+12=0$

$(x-3)(x+4)=0$ or $x^2+x+3=0$

$x=\frac{-1\pm\sqrt{1-48}}{2}$

$\frac{-1\pm\sqrt{-47}}{2}=-\frac{1}{2}\pm\frac{\sqrt{47}}{2}i$

$x=3$ or $x=-4$

The solution set is

$\left\{-4,3,-\frac{1}{2}-\frac{\sqrt{47}}{2}i,-\frac{1}{2}+\frac{\sqrt{47}}{2}i\right\}$.

37. $|x^2+x-1|=1$

$x^2+x-1=1$ or $x^2+x-1=-1$

$x^2+x-2=0$ or $x^2+x=0$

$(x-1)(x+2)=0$ or $x(x+1)=0$

$x=1, x=-2$ or $x=0, x=-1$

The solution set is $\{-2,-1,0,1\}$.

38. $|x^2+3x-2|=2$

$x^2+3x-2=2$ or $x^2+3x-2=-2$

$x^2+3x=4$ or $x^2+3x=0$

$x^2+3x-4=0$ or $x(x+3)=0$

$(x+4)(x-1)=0$ or $x=0, x=-3$

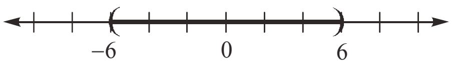
$x=-4, x=1$

The solution set is $\{-4,-3,0,1\}$.

39. $|x| < 6$

$-6 < x < 6$

$\{x \mid -6 < x < 6\}$ or $(-6, 6)$



40. $|x| < 9$

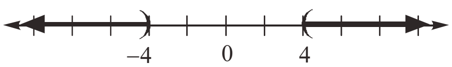
$-9 < x < 9$

$\{x \mid -9 < x < 9\}$ or $(-9, 9)$

41. $|x| > 4$

$x < -4$ or $x > 4$

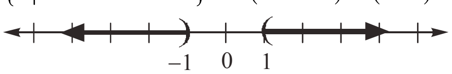
$\{x \mid x < -4 \text{ or } x > 4\}$ or $(-\infty, -4) \cup (4, \infty)$



42. $|x| > 1$

$x < -1$ or $x > 1$

$\{x \mid x < -1 \text{ or } x > 1\}$ or $(-\infty, -1) \cup (1, \infty)$

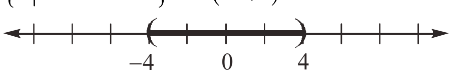


43. $|2x| < 8$

$-8 < 2x < 8$

$-4 < x < 4$

$\{x \mid -4 < x < 4\}$ or $(-4, 4)$

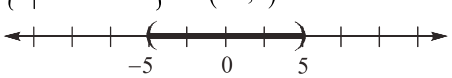


44. $|3x| < 15$

$-15 < 3x < 15$

$-5 < x < 5$

$\{x \mid -5 < x < 5\}$ or $(-5, 5)$

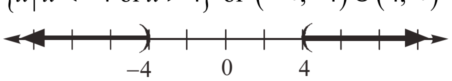


45. $|3x| > 12$

$3x < -12$ or $3x > 12$

$x < -4$ or $x > 4$

$\{x \mid x < -4 \text{ or } x > 4\}$ or $(-\infty, -4) \cup (4, \infty)$

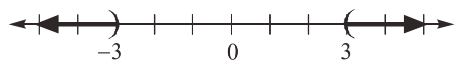


46. $|2x| > 6$

$2x < -6$ or $2x > 6$

$x < -3$ or $x > 3$

$\{x \mid x < -3 \text{ or } x > 3\}$ or $(-\infty, -3) \cup (3, \infty)$



47. $|x-2| + 2 < 3$

$|x-2| < 1$

$-1 < x - 2 < 1$

$1 < x < 3$

$\{x \mid 1 < x < 3\}$ or $(1, 3)$

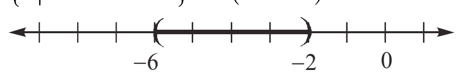
48. $|x+4| + 3 < 5$

$|x+4| < 2$

$-2 < x + 4 < 2$

$-6 < x < -2$

$\{x \mid -6 < x < -2\}$ or $(-6, -2)$



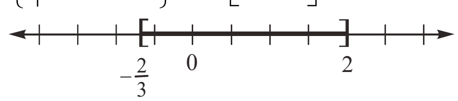
49. $|3t - 2| \le 4$

$-4 \le 3t - 2 \le 4$

$-2 \le 3t \le 6$

$-\frac{2}{3} \le t \le 2$

$\left\{t \,\middle|\, -\frac{2}{3} \le t \le 2\right\}$ or $\left[-\frac{2}{3}, 2\right]$



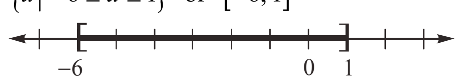
50. $|2u + 5| \le 7$

$-7 \le 2u + 5 \le 7$

$-12 \le 2u \le 2$

$-6 \le u \le 1$

$\{u \mid -6 \le u \le 1\}$ or $[-6, 1]$

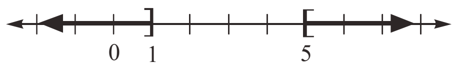


51. $|x-3| \ge 2$

$x-3 \le -2$ or $x-3 \ge 2$

$x \le 1$ or $x \ge 5$

$\{x \mid x \le 1 \text{ or } x \ge 5\}$ or $(-\infty, 1] \cup [5, \infty)$

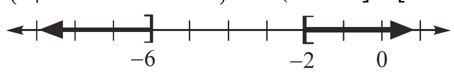


52. $|x+4| \ge 2$

$x+4 \le -2$ or $x+4 \ge 2$

$x \le -6$ or $x \ge -2$

$\{x \mid x \le -6 \text{ or } x \ge -2\}$ or $(-\infty, -6] \cup [-2, \infty)$



53. $|1-4x| - 7 < -2$

$|1-4x| < 5$

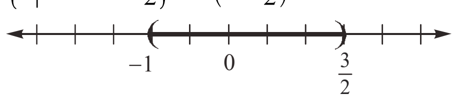
$-5 < 1-4x < 5$

$-6 < -4x < 4$

$\frac{-6}{-4} > x > \frac{4}{-4}$

$\frac{3}{2} > x > -1$ or $-1 < x < \frac{3}{2}$

$\left\{x \mid -1 < x < \frac{3}{2}\right\}$ or $\left(-1, \frac{3}{2}\right)$



54. $|1-2x| - 4 < -1$

$|1-2x| < 3$

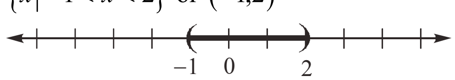
$-3 < 1-2x < 3$

$-4 < -2x < 2$

$\frac{-4}{-2} > x > \frac{2}{-2}$

$2 > x > -1$ or $-1 < x < 2$

$\{x \mid -1 < x < 2\}$ or $(-1, 2)$



55. $|1-2x| > |-3|$

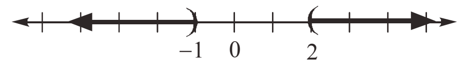
$|1-2x| > 3$

$1-2x < -3$ or $1-2x > 3$

$-2x < -4$ or $-2x > 2$

$x > 2$ or $x < -1$

$\{x \mid x < -1 \text{ or } x > 2\}$ or $(-\infty, -1) \cup (2, \infty)$



56. $|2-3x| > |-1|$

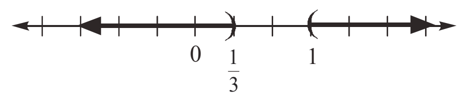
$|2-3x| > 1$

$2-3x < -1$ or $2-3x > 1$

$-3x < -3$ or $-3x > -1$

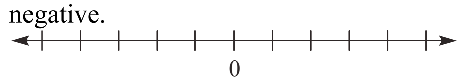
$x > 1$ or $x < \frac{1}{3}$

$\left\{x \,\middle|\, x < \frac{1}{3} \text{ or } x > 1\right\}$ or $\left(-\infty, \frac{1}{3}\right) \cup (1, \infty)$



57. $|2x+1| < -1$

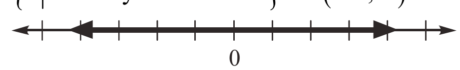
No solution since absolute value is always non-negative.



58. $|3x-4| \ge 0$

All real numbers since absolute value is always non-negative.

$\{x \mid x \text{ is any real number}\}$ or $(-\infty, \infty)$



59. $\left|(3x-2)-7\right| < \frac{1}{2}$

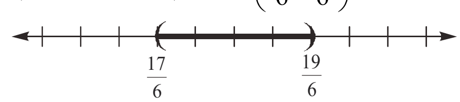
$|3x-9| < \frac{1}{2}$

$-\frac{1}{2} < 3x-9 < \frac{1}{2}$

$\frac{17}{2} < 3x < \frac{19}{2}$

$\frac{17}{6} < x < \frac{19}{6}$

$\left\{x \mid \frac{17}{6} < x < \frac{19}{6}\right\}$ or $\left(\frac{17}{6}, \frac{19}{6}\right)$



60. $\left|(4x-1)-11\right| < \frac{1}{4}$

$\left|4x-12\right| < \frac{1}{4}$

$-\frac{1}{4} < 4x-12 < \frac{1}{4}$

$\frac{47}{4} < 4x < \frac{49}{4}$

$\frac{47}{16} < x < \frac{49}{16}$

$\left\{x \mid \frac{47}{16} < x < \frac{49}{16}\right\}$ or $\left(\frac{47}{16}, \frac{49}{16}\right)$

61. $5-\left|x-1\right| > 2$

$-\left|x-1\right| > -3$

$\left|x-1\right| < 3$

$-3 < x-1 < 3$

$-2 < x < 4$

$\{x \mid -2 < x < 4\}$ or $(-2, 4)$

62. $6-\left|x+3\right| \ge 2$

$-\left|x+3\right| \ge -4$

$\left|x+3\right| \le 4$

$-4 \le x+3 \le 4$

$-7 \le x \le 1$

$\{x \mid -7 \le x \le 1\}$ or $[-7, 1]$

63. a. $f(x) = g(x)$

$-3\left|5x-2\right| = -9$

$\left|5x-2\right| = 3$

$5x-2 = 3$ or $5x-2 = -3$

$5x = 5$ or $5x = -1$

$x = 1$ or $x = -\frac{1}{5}$

b. $f(x) > g(x)$

$-3\left|5x-2\right| > -9$

$\left|5x-2\right| < 3$

$-3 < 5x-2 < 3$

$-1 < 5x < 5$

$-\frac{1}{5} < x < 1$

$\left\{x \mid -\frac{1}{5} < x < 1\right\}$ or $\left(-\frac{1}{5}, 1\right)$

c. $f(x) \le g(x)$

$-3\left|5x-2\right| \le -9$

$\left|5x-2\right| \ge 3$

$5x-2 \ge 3$ or $5x-2 \le -3$

$5x \ge 5$ or $5x \le -1$

$x \ge 1$ or $x \le -\frac{1}{5}$

$\left\{x \mid x \le -\frac{1}{5} \text{ or } x \ge 1\right\}$ or $\left(-\infty, -\frac{1}{5}\right] \cup [1, \infty)$

64. a. $f(x) = g(x)$

$-2\left|2x-3\right| = -12$

$\left|2x-3\right| = 6$

$2x-3 = 6$ or $2x-3 = -6$

$2x = 9$ or $2x = -3$

$x = \frac{9}{2}$ or $x = -\frac{3}{2}$

b. $f(x) = g(x)$

$-2\left|2x-3\right| \ge -12$

$\left|2x-3\right| \le 6$

$-6 \le 2x-3 \le 6$

$-3 \le 2x \le 9$

$-\frac{3}{2} \le x \le \frac{9}{2}$

$\left\{x \mid -\frac{3}{2} \le x \le \frac{9}{2}\right\}$ or $\left[-\frac{3}{2}, \frac{9}{2}\right]$

c. $f(x) = g(x)$

$-2\left|2x-3\right| < -12$

$\left|2x-3\right| > 6$

$2x-3>6$ or $2x-3<-6$

$2x>9$ or $2x<-3$

$x>\frac{9}{2}$ or $x<-\frac{3}{2}$

$\left\{x \mid x<-\frac{3}{2} \text{ or } x>\frac{9}{2}\right\}$ or $\left(-\infty,-\frac{3}{2}\right)\cup\left(\frac{9}{2},\infty\right)$

65. a. $f(x)=g(x)$

$|-3x+2|=x+10$

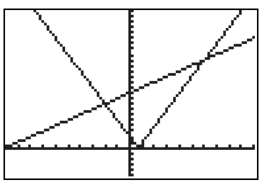
$-3x+2=x+10$ or $-3x+2=-(x+10)$

$-4x=8$ or $-3x+2=-x-10$

$x=-2$ or $-2x=-12$

or $x=6$

b.



Look at the graph of $f(x)$ and $g(x)$ and see where the graph of $f(x)\geq g(x)$. We see that this occurs where $x\leq -2$ or $x\geq 6$. So the solution set is: $\{x \mid x\leq -2 \text{ or } x\geq 6\}$ or $(-\infty,-2]\cup[6,\infty)$.

c. Look at the graph of $f(x)$ and $g(x)$ and see where the graph of $f(x)<g(x)$. We see that this occurs where x is between -2 and 6. So the solution set is: $\{x \mid -2<x<6\}$ or $(-2,6)$.

66. a. $f(x)=g(x)$

$|4x-3|=x+2$

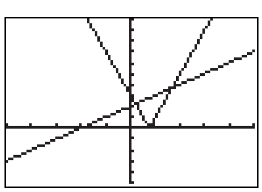
$4x-3=x+2$ or $4x-3=-(x+2)$

$3x=5$ or $4x-3=-x-2$

$x=\frac{5}{3}$ or $5x=1$

or $x=\frac{1}{5}$

b.



Look at the graph of $f(x)$ and $g(x)$ and see where the graph of $f(x)>g(x)$. We see that this occurs where $x<\frac{1}{5}$ or $x>\frac{5}{3}$.

So the solution set is: $\left\{x \mid x<\frac{1}{5} \text{ or } x>\frac{5}{3}\right\}$ or $\left(-\infty,\frac{1}{5}\right)\cup\left(\frac{5}{3},\infty\right)$

c. Look at the graph of $f(x)$ and $g(x)$ and see where the graph of $f(x)\leq g(x)$. We see that this occurs where x is between $\frac{1}{5}$ and $\frac{5}{3}$. So the solution set is: $\left\{x \mid \frac{1}{5}\leq x\leq \frac{5}{3}\right\}$ or $\left[\frac{1}{5},\frac{5}{3}\right]$.

67. $|x-10|<2$

$-2<x-10<2$

$8<x<12$

Solution set: $\{x \mid 8<x<12\}$ or $(8, 12)$

68. $|x-(-6)|<3$

$|x+6|<3$

$-3<x+6<3$

$-9<x<-3$

Solution set: $\{x \mid -9<x<-3\}$ or $(-9,-3)$

69. $|2x-(-1)|>5$

$|2x+1|>5$

$2x+1<-5$ or $2x+1>5$

$2x<-6$ or $2x>4$

$x<-3$ or $x>2$

Solution set: $\{x \mid x<-3 \text{ or } x>2\}$ or $(-\infty,-3)\cup(2,\infty)$

70. $|2x-3|>1$

$2x-3<-1$ or $2x-3>1$

$2x<2$ or $2x>4$

$x<1$ or $x>2$

Solution set: $\{x \mid x<1 \text{ or } x>2\}$ or $(-\infty,1)\cup(2,\infty)$

71. $|x-5.7| \le 0.0005$

$-0.0005 < x-5.7 < 0.0005$

$5.6995 < x < 5.7005$

The acceptable lengths of the rod is from 5.6995 inches to 5.7005 inches.

72. $|x-6.125| \le 0.0005$

$-0.0005 < x-6.125 < 0.0005$

$6.1245 < x < 6.1255$

The acceptable lengths of the rod is from 6.1245 inches to 6.1255 inches.

73. $\left|\dfrac{x-100}{15}\right| > 1.96$

$\dfrac{x-100}{15} < -1.96$ or $\dfrac{x-100}{15} > 1.96$

$x-100 < -29.4$ or $x-100 > 29.4$

$x < 70.6$ or $x > 129.4$

Since IQ scores are whole numbers, any IQ less than 71 or greater than 129 would be considered unusual.

74. $\left|\dfrac{x-266}{16}\right| > 1.96$

$\dfrac{x-266}{16} < -1.96$ or $\dfrac{x-266}{16} > 1.96$

$x-266 < -31.36$ or $x-266 > 31.36$

$x < 234.64$ or $x > 297.36$

Pregnancies less than 235 days long or greater than 297 days long would be considered unusual.

75. $|5x+1|+7=5$

$|5x+1|=-2$

No matter what real number is substituted for *x*, the absolute value expression on the left side of the equation must always be zero or larger. Thus, it can never equal -2.

76. $|2x+5|+3>1 \Rightarrow |2x+5|>-2$

No matter what real number is substituted for *x*, the absolute value expression on the left side of the equation must always be zero or larger. Thus, it will always be larger than -2. Thus, the solution is the set of all real numbers.

77. $|2x-1| \le 0$

No matter what real number is substituted for *x*, the absolute value expression on the left side of the equation must always be zero or larger. Thus, the only solution to the inequality above will be when the absolute value expression equals 0:

$|2x-1|=0$

$2x-1=0$

$2x=1$

$x=\dfrac{1}{2}$

Thus, the solution set is $\left\{\dfrac{1}{2}\right\}$.

78. $f(x)=|2x-7|$

$f(-4)=|2(-4)-7|$

$=|-8-7|=|-15|=15$

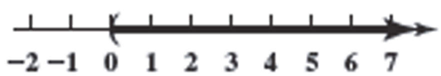
79. $2(x+4)+x<4(x+2)$

$2x+8+x<4x+8$

$3x+8<4x+8$

$-x<0$

$x>0$



80. $(5-i)(3+2i)=$

$15+10i-3i-2i^2=$

$15+7i+2=17+7i$

81. a. Intercepts: (0,0), (4,0)

b. Domain: $[-2,5]$, Range: $[-2,4]$

c. Increasing: $[3,5]$:Decreasing: $[-2,1]$

Constant: $[1,3]$

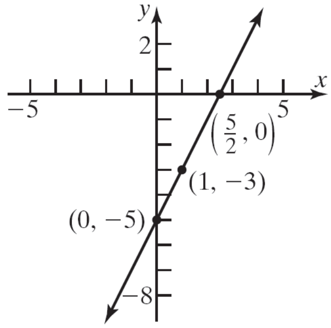
d. Neither

Chapter 2 Review Exercises

1. $f(x)=2x-5$

a. Slope = 2; *y*-intercept = -5

b. Plot the point $(0,-5)$. Use the slope to find an additional point by moving 1 unit to the right and 2 units up.



c. Domain and Range: $(-\infty, \infty)$

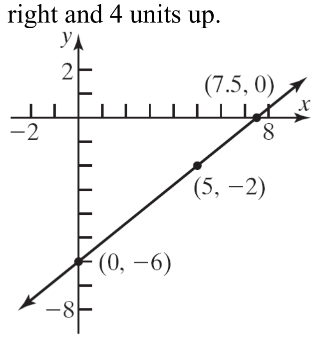
d. Average rate of change = slope = 2

e. Increasing

2. $h(x) = \frac{4}{5}x - 6$

a. Slope = $\frac{4}{5}$; y-intercept = −6

b. Plot the point $(0, -6)$. Use the slope to find an additional point by moving 5 units to the right and 4 units up.



c. Domain and Range: $(-\infty, \infty)$

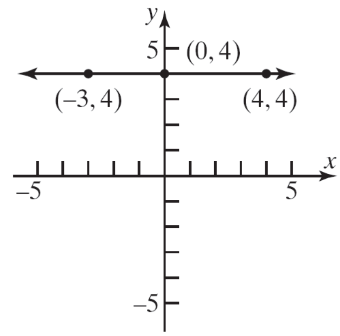
d. Average rate of change = slope = $\frac{4}{5}$

e. Increasing

3. $G(x) = 4$

a. Slope = 0; y-intercept = 4

b. Plot the point (0, 4) and draw a horizontal line through it.



c. Domain: $(-\infty, \infty)$

Range: $\{y \mid y = 4\}$

d. Average rate of change = slope = 0

e. Constant

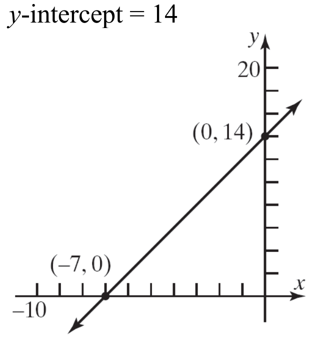
4. $f(x) = 2x + 14$

zero: $f(x) = 2x + 14 = 0$

$2x = -14$

$x = -7$

y-intercept = 14



5.

x	$y = f(x)$	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
−2	−7	
0	3	$\frac{3-(-7)}{0-(-2)} = \frac{10}{2} = 5$
1	8	$\frac{8-3}{1-0} = \frac{5}{1} = 5$
3	18	$\frac{18-8}{3-1} = \frac{10}{2} = 5$
6	33	$\frac{33-18}{6-3} = \frac{15}{3} = 5$

This is a linear function with slope = 5, since the average rate of change is constant at 5. To find the equation of the line, we use the point-slope formula and one of the points.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 5(x - 0)$$
$$y = 5x + 3$$

6.

x	$y = f(x)$	Avg. rate of change $= \dfrac{\Delta y}{\Delta x}$
–1	–3	
0	4	$\dfrac{4-(-3)}{0-(-1)} = \dfrac{7}{1} = 7$
1	7	$\dfrac{7-4}{1-0} = \dfrac{3}{1} = 3$
2	6	
3	1	

This is not a linear function, since the average rate of change is not constant.

7.
$$f(x) = 0$$
$$x^2 + x - 72 = 0$$
$$(x+9)(x-8) = 0$$
$$x + 9 = 0 \quad \text{or} \quad x - 8 = 0$$
$$x = -9 \qquad x = 8$$

The zeros of $f(x) = x^2 + x - 72$ are -9 and 8. The x-intercepts of the graph of f are -9 and 8.

8.
$$P(t) = 0$$
$$6t^2 - 13t - 5 = 0$$
$$(3t+1)(2t-5) = 0$$
$$3t + 1 = 0 \quad \text{or} \quad 2t - 5 = 0$$
$$t = -\frac{1}{3} \qquad t = \frac{5}{2}$$

The zeros of $P(t) = 6t^2 - 13t - 5$ are $-\frac{1}{3}$ and $\frac{5}{2}$.

The t-intercepts of the graph of P are $-\frac{1}{3}$ and $\frac{5}{2}$.

9.
$$g(x) = 0$$
$$(x-3)^2 - 4 = 0$$
$$(x-3)^2 = 4$$
$$x - 3 = \pm\sqrt{4}$$
$$x - 3 = \pm 2$$
$$x = 3 \pm 2$$
$$x = 3 - 2 = 1 \quad \text{or} \quad x = 3 + 2 = 5$$

The zeros of $g(x) = (x-3)^2 - 4$ are 1 and 5. The x-intercepts of the graph of g are 1 and 5.

10.
$$h(x) = 0$$
$$9x^2 + 6x + 1 = 0$$
$$(3x+1)(3x+1) = 0$$
$$3x + 1 = 0 \quad \text{or} \quad 3x + 1 = 0$$
$$x = -\frac{1}{3} \qquad x = -\frac{1}{3}$$

The only zero of $h(x) = 9x^2 + 6x + 1$ is $-\frac{1}{3}$.

The only x-intercept of the graph of h is $-\frac{1}{3}$.

11.
$$G(x) = 0$$
$$2x^2 - 4x - 1 = 0$$
$$x^2 - 2x - \frac{1}{2} = 0$$
$$x^2 - 2x = \frac{1}{2}$$
$$x^2 - 2x + 1 = \frac{1}{2} + 1$$
$$(x-1)^2 = \frac{3}{2}$$
$$x - 1 = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{6}}{2}$$
$$x = 1 \pm \frac{\sqrt{6}}{2} = \frac{2 \pm \sqrt{6}}{2}$$

The zeros of $G(x) = 2x^2 - 4x - 1$ are $\frac{2-\sqrt{6}}{2}$ and $\frac{2+\sqrt{6}}{2}$. The x-intercepts of the graph of G are $\frac{2-\sqrt{6}}{2}$ and $\frac{2+\sqrt{6}}{2}$.

12. $f(x) = 0$

$-2x^2 + x + 1 = 0$

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$2x+1 = 0$ or $x - 1 = 0$

$x = -\frac{1}{2}$ $\quad x = 1$

The zeros of $f(x) = -2x^2 + x + 1$ are $-\frac{1}{2}$ and 1.

The x-intercepts of the graph of f are $-\frac{1}{2}$ and 1.

13. $f(x) = g(x)$

$(x-3)^2 = 16$

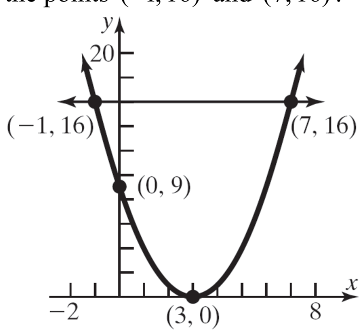
$x - 3 = \pm\sqrt{16} = \pm 4$

$x = 3 \pm 4$

$x = 3 - 4 = -1$ or $x = 3 + 4 = 7$

The solution set is $\{-1, 7\}$.

The x-coordinates of the points of intersection are -1 and 7. The y-coordinates are $g(-1) = 16$ and $g(7) = 16$. The graphs of the f and g intersect at the points $(-1, 16)$ and $(7, 16)$.



14. $f(x) = g(x)$

$x^2 + 4x - 5 = 4x - 1$

$x^2 - 4 = 0$

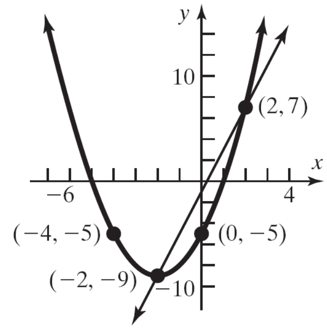
$(x+2)(x-2) = 0$

$x + 2 = 0$ or $x - 2 = 0$

$x = -2$ $\quad x = 2$

The solution set is $\{-2, 2\}$.

The x-coordinates of the points of intersection are -2 and 2. The y-coordinates are $g(-2) = 4(-2) - 1 = -8 - 1 = -9$ and $g(2) = 4(2) - 1 = 8 - 1 = 7$. The graphs of the f and g intersect at the points $(-2, -9)$ and $(2, 7)$.



15. $f(x) = 0$

$x^4 - 5x^2 + 4 = 0$

$(x^2 - 4)(x^2 - 1) = 0$

$x^2 - 4 = 0$ or $x^2 - 1 = 0$

$x = \pm 2$ or $x = \pm 1$

The zeros of $f(x) = x^4 - 5x^2 + 4$ are -2, -1, 1, and 2. The x-intercepts of the graph of f are -2, -1, 1, and 2.

16. $F(x) = 0$

$(x-3)^2 - 2(x-3) - 48 = 0$

Let $u = x - 3 \rightarrow u^2 = (x-3)^2$

$u^2 - 2u - 48 = 0$

$(u+6)(u-8) = 0$

$u + 6 = 0$ or $u - 8 = 0$

$u = -6$ $\quad u = 8$

$x - 3 = -6$ $\quad x - 3 = 8$

$x = -3$ $\quad x = 11$

The zeros of $F(x) = (x-3)^2 - 2(x-3) - 48$ are -3 and 11. The x-intercepts of the graph of F are -3 and 11.

17. $h(x) = 0$

$3x - 13\sqrt{x} - 10 = 0$

Let $u = \sqrt{x} \rightarrow u^2 = x$

$3u^2 - 13u - 10 = 0$

$(3u+2)(u-5) = 0$

$3u + 2 = 0$ or $u - 5 = 0$

$u = -\frac{2}{3}$ $\quad u = 5$

$\sqrt{x} = -\frac{2}{3}$ $\quad \sqrt{x} = 5$

$x = $ not real $\quad x = 5^2 = 25$

Check: $h(25) = 3(25) - 13\sqrt{25} - 10$
$= 3(25) - 13(5) - 10$
$= 75 - 65 - 10 = 0$

The only zero of $h(x) = 3x - 13\sqrt{x} - 10$ is 25. The only x-intercept of the graph of h is 25.

18. $f(x) = 0$

$\left(\frac{1}{x}\right)^2 - 4\left(\frac{1}{x}\right) - 12 = 0$

Let $u = \frac{1}{x} \to u^2 = \left(\frac{1}{x}\right)^2$

$u^2 - 4u - 12 = 0$

$(u+2)(u-6) = 0$

$u + 2 = 0$ or $u - 6 = 0$

$u = -2$ $\quad u = 6$

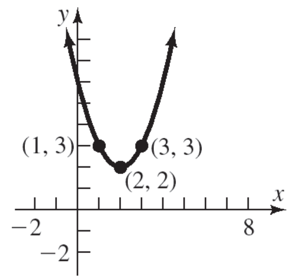
$\frac{1}{x} = -2$ $\quad \frac{1}{x} = 6$

$x = -\frac{1}{2}$ $\quad x = \frac{1}{6}$

The zeros of $f(x) = \left(\frac{1}{x}\right)^2 - 4\left(\frac{1}{x}\right) - 12$ are $-\frac{1}{2}$ and $\frac{1}{6}$. The x-intercepts of the graph of f are $-\frac{1}{2}$ and $\frac{1}{6}$.

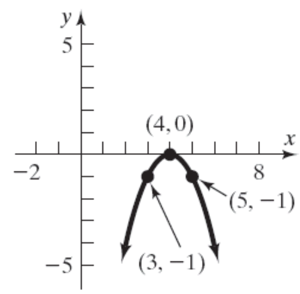
19. $f(x) = (x-2)^2 + 2$

Using the graph of $y = x^2$, shift right 2 units, then shift up 2 units.



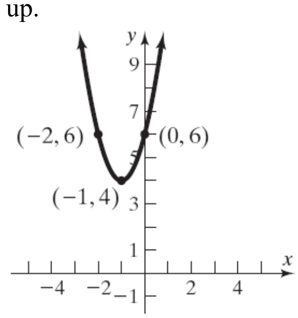
20. $f(x) = -(x-4)^2$

Using the graph of $y = x^2$, shift the graph 4 units right, then reflect about the x-axis.



21. $f(x) = 2(x+1)^2 + 4$

Using the graph of $y = x^2$, stretch vertically by a factor of 2, then shift 1 unit left, then shift 4 units up.



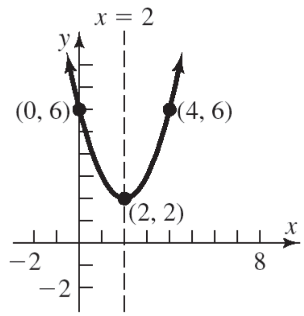
22. a. $f(x) = (x-2)^2 + 2$
$= x^2 - 4x + 4 + 2$
$= x^2 - 4x + 6$

$a = 1, b = -4, c = 6$. Since $a = 1 > 0$, the graph opens up. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{-4}{2(1)} = \frac{4}{2} = 2$.

The y-coordinate of the vertex is

$f\left(-\frac{b}{2a}\right) = f(2) = (2)^2 - 4(2) + 6 = 2$.

Thus, the vertex is (2, 2).
The axis of symmetry is the line $x = 2$.
The discriminant is:
$b^2 - 4ac = (-4)^2 - 4(1)(6) = -8 < 0$, so the graph has no x-intercepts.
The y-intercept is $f(0) = 6$.



b. Domain: $(-\infty, \infty)$. Range: $[2, \infty)$.

c. Decreasing on $(-\infty, 2]$; increasing on $(2, \infty]$.

23. a. $f(x) = \frac{1}{4}x^2 - 16$

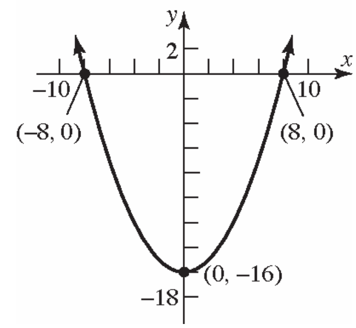
$a = \frac{1}{4}, b = 0, c = -16$. Since $a = \frac{1}{4} > 0$, the graph opens up. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{-0}{2\left(\frac{1}{4}\right)} = -\frac{0}{\frac{1}{2}} = 0$.

The y-coordinate of the vertex is
$f\left(-\frac{b}{2a}\right) = f(0) = \frac{1}{4}(0)^2 - 16 = -16$.
Thus, the vertex is (0, –16).
The axis of symmetry is the line $x = 0$.
The discriminant is:
$b^2 - 4ac = (0)^2 - 4\left(\frac{1}{4}\right)(-16) = 16 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:

$$\frac{1}{4}x^2 - 16 = 0$$
$$x^2 - 64 = 0$$
$$x^2 = 64$$
$$x = 8 \text{ or } x = -8$$

The x-intercepts are –8 and 8.

The y-intercept is $f(0) = -16$.



b. Domain: $(-\infty, \infty)$. Range: $[-16, \infty)$.

c. Decreasing on $(-\infty, 0]$; increasing on $[0, \infty)$.

24. a. $f(x) = -4x^2 + 4x$

$a = -4, b = 4, c = 0$. Since $a = -4 < 0$, the graph opens down. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{4}{2(-4)} = -\frac{4}{-8} = \frac{1}{2}$.

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{2}\right) = -4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)$$
$$= -1 + 2 = 1$$

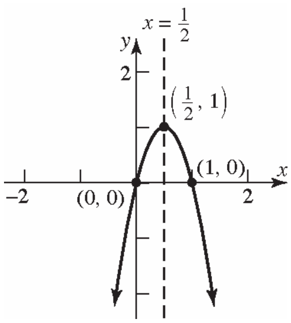
Thus, the vertex is $\left(\frac{1}{2}, 1\right)$.

The axis of symmetry is the line $x = \frac{1}{2}$.

The discriminant is:
$b^2 - 4ac = 4^2 - 4(-4)(0) = 16 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:

$$-4x^2 + 4x = 0$$
$$-4x(x - 1) = 0$$
$$x = 0 \text{ or } x = 1$$

The x-intercepts are 0 and 1.
The y-intercept is $f(0) = -4(0)^2 + 4(0) = 0$.



b. Domain: $(-\infty, \infty)$. Range: $(-\infty, 1]$.

c. Increasing on $\left(-\infty, \frac{1}{2}\right]$; decreasing on $\left[\frac{1}{2}, \infty\right)$.

25. a. $f(x) = \frac{9}{2}x^2 + 3x + 1$

$a = \frac{9}{2}, b = 3, c = 1$. Since $a = \frac{9}{2} > 0$, the graph opens up. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{3}{2\left(\frac{9}{2}\right)} = -\frac{3}{9} = -\frac{1}{3}$.

The y-coordinate of the vertex is

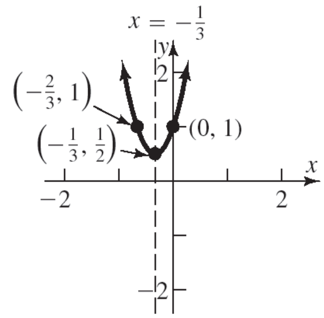
$$f\left(-\frac{b}{2a}\right) = f\left(-\frac{1}{3}\right) = \frac{9}{2}\left(-\frac{1}{3}\right)^2 + 3\left(-\frac{1}{3}\right) + 1$$
$$= \frac{1}{2} - 1 + 1 = \frac{1}{2}$$

Thus, the vertex is $\left(-\frac{1}{3}, \frac{1}{2}\right)$.

The axis of symmetry is the line $x = -\frac{1}{3}$.

The discriminant is:

$b^2 - 4ac = 3^2 - 4\left(\frac{9}{2}\right)(1) = 9 - 18 = -9 < 0$, so the graph has no x-intercepts. The y-intercept is $f(0) = \frac{9}{2}(0)^2 + 3(0) + 1 = 1$.



b. Domain: $(-\infty, \infty)$. Range: $\left[\frac{1}{2}, \infty\right)$.

c. Decreasing on $\left(-\infty, -\frac{1}{3}\right]$; increasing on $\left[-\frac{1}{3}, \infty\right)$.

26. a. $f(x) = 3x^2 + 4x - 1$

$a = 3, b = 4, c = -1$. Since $a = 3 > 0$, the graph opens up. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{4}{2(3)} = -\frac{4}{6} = -\frac{2}{3}$.

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right)^2 + 4\left(-\frac{2}{3}\right) - 1$$
$$= \frac{4}{3} - \frac{8}{3} - 1 = -\frac{7}{3}$$

Thus, the vertex is $\left(-\frac{2}{3}, -\frac{7}{3}\right)$.

The axis of symmetry is the line $x = -\frac{2}{3}$.

The discriminant is:

$b^2 - 4ac = (4)^2 - 4(3)(-1) = 28 > 0$, so the graph has two x-intercepts.

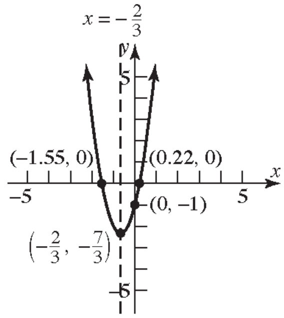
The x-intercepts are found by solving: $3x^2 + 4x - 1 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{28}}{2(3)}$$
$$= \frac{-4 \pm 2\sqrt{7}}{6} = \frac{-2 \pm \sqrt{7}}{3}$$

The x-intercepts are $\frac{-2 - \sqrt{7}}{3} \approx -1.55$ and

$\frac{-2+\sqrt{7}}{3} \approx 0.22$.

The y-intercept is $f(0) = 3(0)^2 + 4(0) - 1 = -1$.



b. Domain: $(-\infty, \infty)$. Range: $\left[-\frac{7}{3}, \infty\right)$.

c. Decreasing on $\left(-\infty, -\frac{2}{3}\right]$; increasing on $\left[-\frac{2}{3}, \infty\right)$.

27. $f(x) = 3x^2 - 6x + 4$

$a = 3, b = -6, c = 4$. Since $a = 3 > 0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at

$$x = -\frac{b}{2a} = -\frac{-6}{2(3)} = \frac{6}{6} = 1.$$

The minimum value is

$$f\left(-\frac{b}{2a}\right) = f(1) = 3(1)^2 - 6(1) + 4 = 3 - 6 + 4 = 1$$

28. $f(x) = -x^2 + 8x - 4$

$a = -1, b = 8, c = -4$. Since $a = -1 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at

$$x = -\frac{b}{2a} = -\frac{8}{2(-1)} = -\frac{8}{-2} = 4.$$

The maximum value is

$$f\left(-\frac{b}{2a}\right) = f(4) = -(4)^2 + 8(4) - 4 = -16 + 32 - 4 = 12$$

29. $f(x) = -3x^2 + 12x + 4$

$a = -3, b = 12, c = 4$. Since $a = -3 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at

$$x = -\frac{b}{2a} = -\frac{12}{2(-3)} = -\frac{12}{-6} = 2.$$

The maximum value is

$$f\left(-\frac{b}{2a}\right) = f(2) = -3(2)^2 + 12(2) + 4 = -12 + 24 + 4 = 16$$

30. Consider the form $y = a(x-h)^2 + k$. The vertex is $(2, -4)$ so we have $h = 2$ and $k = -4$. The function also contains the point $(0, -16)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:

$$-16 = a(0-(2))^2 + (-4)$$
$$-16 = a(-2)^2 - 4$$
$$-16 = 4a - 4$$
$$-12 = 4a$$
$$a = -3$$

The quadratic function is

$f(x) = -3(x-2)^2 - 4 = -3x^2 + 12x - 16$.

31. Use the form $f(x) = a(x-h)^2 + k$.

The vertex is $(-1, 2)$, so $h = -1$ and $k = 2$.

$f(x) = a(x+1)^2 + 2$.

Since the graph passes through $(1, 6)$, $f(1) = 6$.

$$6 = a(1+1)^2 + 2$$
$$6 = a(2)^2 + 2$$
$$6 = 4a + 2$$
$$4 = 4a$$
$$1 = a$$
$$f(x) = (x+1)^2 + 2 = (x^2 + 2x + 1) + 2 = x^2 + 2x + 3$$

32. $x^2 + 6x - 16 < 0$

$f(x) = x^2 + 6x - 16$

$x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

$x = -8, x = 2$ are the zeros of f.

Interval	$(-\infty, -8)$	$(-8, 2)$	$(2, \infty)$
Test Number	-9	0	3
Value of f	11	-16	11
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid -8 < x < 2\}$ or, using interval notation, $(-8, 2)$.

33. $3x^2 \ge 14x + 5$

$3x^2 - 14x - 5 \ge 0$

$f(x) = 3x^2 - 14x - 5$

$3x^2 - 14x - 5 = 0$

$(3x+1)(x-5) = 0$

$x = -\dfrac{1}{3},\ x = 5$ are the zeros of f.

Interval	$\left(-\infty, -\frac{1}{3}\right)$	$\left(-\frac{1}{3}, 5\right)$	$(5, \infty)$
Test Number	-1	0	2
Value of f	12	-5	19
Conclusion	Positive	Negative	Positive

The solution set is $\left\{x \,\middle|\, x \le -\dfrac{1}{3} \text{ or } x \ge 5\right\}$ or, using interval notation, $\left(-\infty, -\dfrac{1}{3}\right] \cup [5, \infty)$.

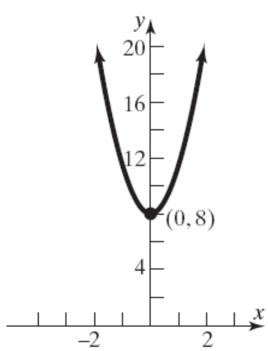
34. $f(x) = 0$

$x^2 + 8 = 0$

$x^2 = -8$

$x = \pm\sqrt{-8} = \pm 2\sqrt{2}\, i$

The zero are $-2\sqrt{2}\, i$ and $2\sqrt{2}\, i$.



35. $g(x) = 0$

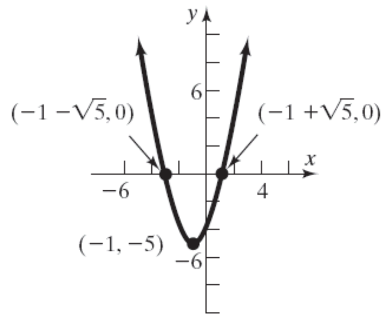
$x^2 + 2x - 4 = 0$

$a = 1, b = 2, c = -4$

$b^2 - 4ac = 2^2 - 4(1)(-4) = 4 + 16 = 20$

$$x = \frac{-2 \pm \sqrt{20}}{2(1)} = \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$$

The zeros are $-1 - \sqrt{5}$ and $-1 + \sqrt{5}$.



36. $p(x) = 0$

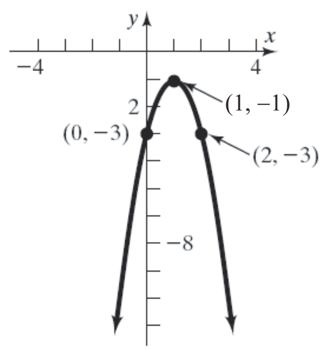
$-2x^2 + 4x - 3 = 0$

$a = -2, b = 4, c = -3$

$b^2 - 4ac = 4^2 - 4(-2)(-3) = 16 - 24 = -8$

$$x = \frac{-4 \pm \sqrt{-8}}{2(-2)} = \frac{-4 \pm 2\sqrt{2}\, i}{-4} = 1 \pm \frac{\sqrt{2}}{2} i$$

The zeros are $1 - \dfrac{\sqrt{2}}{2} i$ and $1 + \dfrac{\sqrt{2}}{2} i$.



37. $f(x) = 0$

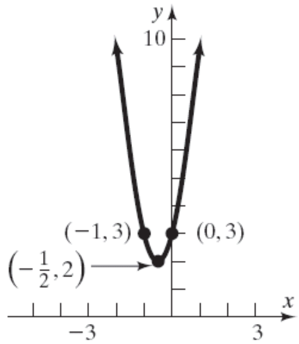
$4x^2 + 4x + 3 = 0$

$a = 4, b = 4, c = 3$

$b^2 - 4ac = 4^2 - 4(4)(3) = 16 - 48 = -32$

$$x = \frac{-4 \pm \sqrt{-32}}{2(4)} = \frac{-4 \pm 4\sqrt{2}\, i}{8} = -\frac{1}{2} \pm \frac{\sqrt{2}}{2} i$$

The zeros are $-\frac{1}{2}-\frac{\sqrt{2}}{2}i$ and $-\frac{1}{2}+\frac{\sqrt{2}}{2}i$.



38. $|2x+3|=7$

$2x+3=7$ or $2x+3=-7$

$2x=4$ or $2x=-10$

$x=2$ or $x=-5$

The solution set is $\{-5, 2\}$.

39. $|2-3x|+2=9$

$|2-3x|=7$

$2-3x=7$ or $2-3x=-7$

$-3x=5$ or $-3x=-9$

$x=-\frac{5}{3}$ or $x=3$

The solution set is $\left\{-\frac{5}{3}, 3\right\}$.

40. $|3x+4|<\frac{1}{2}$

$-\frac{1}{2}<3x+4<\frac{1}{2}$

$-\frac{9}{2}<3x<-\frac{7}{2}$

$-\frac{3}{2}<x<-\frac{7}{6}$

$\left\{x \middle| -\frac{3}{2}<x<-\frac{7}{6}\right\}$ or $\left(-\frac{3}{2}, -\frac{7}{6}\right)$

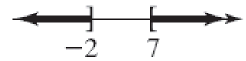
41. $|2x-5|\ge 9$

$2x-5\le -9$ or $2x-5\ge 9$

$2x\le -4$ or $2x\ge 14$

$x\le -2$ or $x\ge 7$

$\{x \mid x\le -2 \text{ or } x\ge 7\}$ or $(-\infty, -2]\cup[7, \infty)$



42. $2+|2-3x|\le 4$

$|2-3x|\le 2$

$-2\le 2-3x\le 2$

$-4\le -3x\le 0$

$\frac{4}{3}\ge x\ge 0$

$\left\{x \middle| 0\le x\le \frac{4}{3}\right\}$ or $\left[0, \frac{4}{3}\right]$

43. $1-|2-3x|<-4$

$-|2-3x|<-5$

$|2-3x|>5$

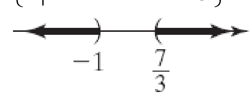
$2-3x<-5$ or $2-3x>5$

$7<3x$ or $-3>3x$

$\frac{7}{3}<x$ or $-1>x$

$x<-1$ or $x>\frac{7}{3}$

$\left\{x \middle| x<-1 \text{ or } x>\frac{7}{3}\right\}$ or $(-\infty, -1)\cup\left(\frac{7}{3}, \infty\right)$



44. **a.** $S(x)=0.01x+25{,}000$

b. $S(1{,}000{,}000)=0.01(1{,}000{,}000)+25{,}000$

$=10{,}000+25{,}000=35{,}000$

Bill's salary would be \$35,000.

c. $0.01x+25{,}000=100{,}000$

$0.01x=75{,}000$

$x=7{,}500{,}000$

Bill's sales would have to be \$7,500,000 in order to earn \$100,000.

d. $0.01x+25{,}000>150{,}000$

$0.01x>125{,}000$

$x>12{,}500{,}000$

Bill's sales would have to be more than

$12,500,000 in order for his salary to exceed $150,000.

45. **a.** If $x = 1500 - 10p$, then $p = \dfrac{1500 - x}{10}$.

$R(p) = px = p(1500 - 10p) = -10p^2 + 1500p$

b. Domain: $\{p \mid 0 < p \le 150\}$

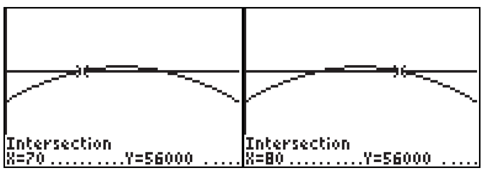
c. $p = \dfrac{-b}{2a} = \dfrac{-1500}{2(-10)} = \dfrac{-1500}{-20} = \$75$

d. The maximum revenue is

$R(75) = -10(75)^2 + 1500(75)$
$= -56250 + 112500 = \$56,250$

e. $x = 1500 - 10(75) = 1500 - 750 = 750$

f. Graph $R = -10p^2 + 1500p$ and $R = 56000$.

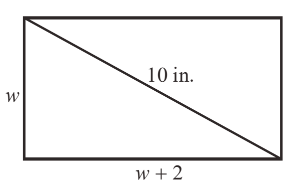


Find where the graphs intersect by solving $56000 = -10p^2 + 1500p$.

$10p^2 - 1500p + 56000 = 0$
$p^2 - 150p + 5600 = 0$
$(p - 70)(p - 80) = 0$
$p = 70, p = 80$

The company should charge between $70 and $80.

46. Let w = the width. Then $w + 2$ = the length.



By the Pythagorean Theorem we have:

$w^2 + (w+2)^2 = (10)^2$
$w^2 + w^2 + 4w + 4 = 100$
$2w^2 + 4w - 96 = 0$
$w^2 + 2w - 48 = 0$
$(w+8)(w-6) = 0$
$w = -8$ or $w = 6$

Disregard the negative answer because the width of a rectangle must be positive. Thus, the width is 6 inches, and the length is 8 inches

47. $C(x) = 4.9x^2 - 617.4x + 19,600$;
$a = 4.9, b = -617.4, c = 19,600$. Since $a = 4.9 > 0$, the graph opens up, so the vertex is a minimum point.

a. The minimum marginal cost occurs at

$x = -\dfrac{b}{2a} = -\dfrac{-617.40}{2(4.9)} = \dfrac{617.40}{9.8} = 63$.

Thus, 63 golf clubs should be manufactured in order to minimize the marginal cost.

b. The minimum marginal cost is

$C\left(-\dfrac{b}{2a}\right) = C(63)$
$= 4.9(63)^2 - (617.40)(63) + 19600$
$= \$151.90$

48. Since there are 200 feet of border, we know that $2x + 2y = 200$. The area is to be maximized, so $A = x \cdot y$. Solving the perimeter formula for y:

$2x + 2y = 200$
$2y = 200 - 2x$
$y = 100 - x$

The area function is:
$A(x) = x(100 - x) = -x^2 + 100x$

The maximum value occurs at the vertex:
$x = -\dfrac{b}{2a} = -\dfrac{100}{2(-1)} = -\dfrac{100}{-2} = 50$

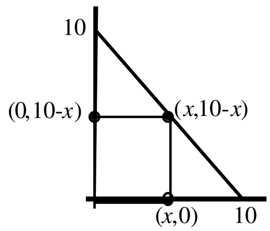
The pond should be 50 feet by 50 feet for maximum area.

49. The area function is:
$A(x) = x(10 - x) = -x^2 + 10x$

The maximum value occurs at the vertex:
$x = -\dfrac{b}{2a} = -\dfrac{10}{2(-1)} = -\dfrac{10}{-2} = 5$

The maximum area is:

$$A(5) = -(5)^2 + 10(5)$$
$$= -25 + 50 = 25 \text{ square units}$$



50. Locate the origin at the point directly under the highest point of the arch. Then the equation is in the form: $y = -ax^2 + k$, where $a > 0$. Since the maximum height is 10 feet, when $x = 0$, $y = k = 10$. Since the point (10, 0) is on the parabola, we can find the constant:

$$0 = -a(10)^2 + 10$$
$$a = \frac{10}{10^2} = \frac{1}{10} = 0.10$$

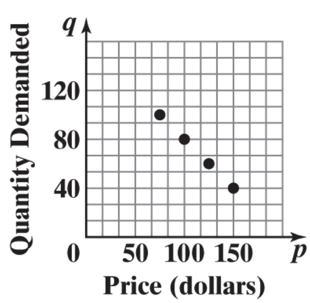
The equation of the parabola is:

$$y = -\frac{1}{10}x^2 + 10$$

At $x = 8$:

$$y = -\frac{1}{10}(8)^2 + 10 = -6.4 + 10 = 3.6 \text{ feet}$$

51. a.



b.

p	q	Avg. rate of change = $\frac{\Delta q}{\Delta p}$
75	100	
100	80	$\frac{80-100}{100-75} = \frac{-20}{25} = -0.8$
125	60	$\frac{60-80}{125-100} = \frac{-20}{25} = -0.8$
150	40	$\frac{40-60}{150-125} = \frac{-20}{25} = -0.8$

Since each input (price) corresponds to a single output (quantity demanded), we know that the quantity demanded is a function of price. Also, because the average rate of change is constant at $-\$0.8$ per LCD monitor, the function is linear.

c. From part (b), we know $m = -0.8$. Using $(p_1, q_1) = (75, 100)$, we get the equation:

$$q - q_1 = m(p - p_1)$$
$$q - 100 = -0.8(p - 75)$$
$$q - 100 = -0.8p + 60$$
$$q = -0.8p + 160$$

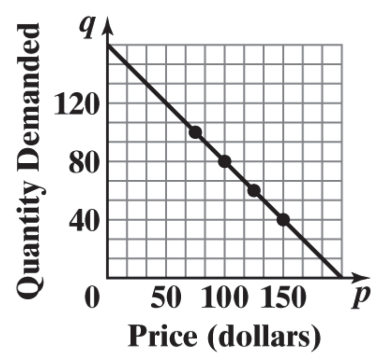
Using function notation, we have $q(p) = -0.8p + 160$.

d. The price cannot be negative, so $p \geq 0$. Likewise, the quantity cannot be negative, so, $q(p) \geq 0$.

$$-0.8p + 160 \geq 0$$
$$-0.8p \geq -160$$
$$p \leq 200$$

Thus, the implied domain for $q(p)$ is $\{p \mid 0 \leq p \leq 200\}$ or $[0, 200]$.

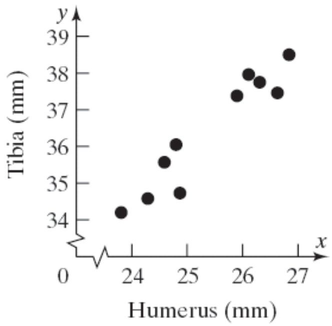
e.



f. If the price increases by \$1, then the quantity demanded of LCD monitors decreases by 0.8 monitor.

g. p-intercept: If the price is \$0, then 160 LCD monitors will be demanded.
q-intercept: There will be 0 LCD monitors demanded when the price is \$200.

52. a.



b. Yes, the two variables appear to have a linear relationship.

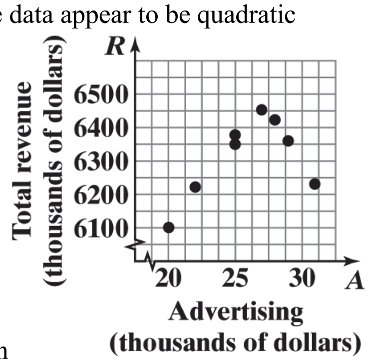
c. Using the LINear REGression program, the line of best fit is:
$y = 1.390171918x + 1.113952697$

```
LinReg
 y=ax+b
 a=1.390171918
 b=1.113952697
 r²=.9050023758
 r=.9513161282
```

d. $y = 1.390171918(26.5) + 1.113952697$
≈ 38.0 mm

53. a.

The data appear to be quadratic



with $a < 0$.

b. Using the QUADratic REGression program, the quadratic function of best fit is:
$y = -7.76x^2 + 411.88x + 942.72$.

```
QuadReg
 y=ax²+bx+c
 a=-7.759570754
 b=411.8750353
 c=942.721091
 R²=.9327651562
```

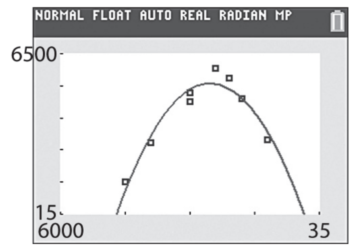
The maximum revenue occurs at

$$A = \frac{-b}{2a} = \frac{-(411.88)}{2(-7.76)}$$
$$= \frac{-411.88}{-15.52} \approx \$26.5 \text{ thousand}$$

c. The maximum revenue is

$$R\left(\frac{-b}{2a}\right) = R(26.53866)$$
$$= -7.76(26.5)^2 + (411.88)(26.5) + 942.72$$
$$\approx \$6408 \text{ thousand}$$

d.



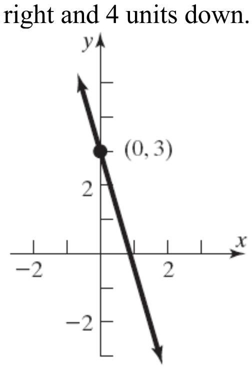
Chapter 2 Test

1. $f(x) = -4x + 3$

a. The slope *f* is -4.

b. The slope is negative, so the graph is decreasing.

c. Plot the point $(0, 3)$. Use the slope to find an additional point by moving 1 unit to the right and 4 units down.



2.

x	y	Avg. rate of change $= \dfrac{\Delta y}{\Delta x}$
-2	12	
-1	7	$\dfrac{7-12}{-1-(-2)} = \dfrac{-5}{1} = -5$
0	2	$\dfrac{2-7}{0-(-1)} = \dfrac{-5}{1} = -5$
1	-3	$\dfrac{-3-2}{1-0} = \dfrac{-5}{1} = -5$
2	-8	$\dfrac{-8-(-3)}{2-1} = \dfrac{-5}{1} = -5$

Since the average rate of change is constant at -5, this is a linear function with slope $= -5$. To find the equation of the line, we use the point-slope formula and one of the points.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -5(x - 0)$$
$$y = -5x + 2$$

3.
$$f(x) = 0$$
$$3x^2 - 2x - 8 = 0$$
$$(3x+4)(x-2) = 0$$
$$3x + 4 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -\frac{4}{3} \qquad x = 2$$

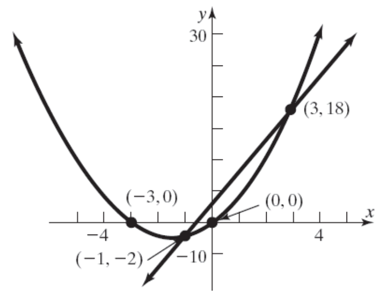
The zeros of f are $-\frac{4}{3}$ and 2.

4.
$$G(x) = 0$$
$$-2x^2 + 4x + 1 = 0$$
$$a = -2, b = 4, c = 1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(-2)(1)}}{2(-2)}$$
$$= \frac{-4 \pm \sqrt{24}}{-4} = \frac{-4 \pm 2\sqrt{6}}{-4} = \frac{2 \pm \sqrt{6}}{2}$$

The zeros of G are $\frac{2-\sqrt{6}}{2}$ and $\frac{2+\sqrt{6}}{2}$.

5.
$$f(x) = g(x)$$
$$x^2 + 3x = 5x + 3$$
$$x^2 - 2x - 3 = 0$$
$$(x+1)(x-3) = 0$$
$$x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -1 \qquad x = 3$$

The solution set is $\{-1, 3\}$.



6.
$$f(x) = 0$$
$$(x-1)^2 + 5(x-1) + 4 = 0$$

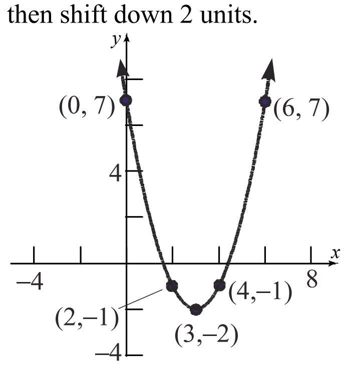
Let $u = x - 1 \rightarrow u^2 = (x-1)^2$

$$u^2 + 5u + 4 = 0$$
$$(u+4)(u+1) = 0$$
$$u + 4 = 0 \quad \text{or} \quad u + 1 = 0$$
$$u = -4 \qquad u = -1$$
$$x - 1 = -4 \qquad x - 1 = -1$$
$$x = -3 \qquad x = 0$$

The zeros of G are -3 and 0.

7. $f(x) = (x-3)^2 - 2$

Using the graph of $y = x^2$, shift right 3 units, then shift down 2 units.



8. a. $f(x) = 3x^2 - 12x + 4$
$a = 3, b = -12, c = 4.$ Since $a = 3 > 0,$ the graph opens up.

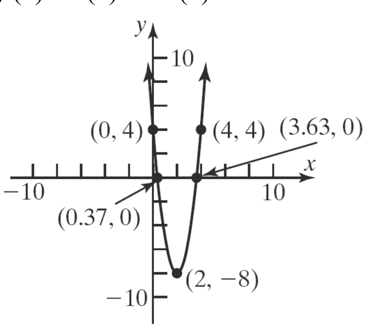
b. The x-coordinate of the vertex is
$x = -\frac{b}{2a} = -\frac{-12}{2(3)} = -\frac{-12}{6} = 2$.

The y-coordinate of the vertex is
$$f\left(-\frac{b}{2a}\right) = f(2) = 3(2)^2 - 12(2) + 4$$
$$= 12 - 24 + 4 = -8$$
Thus, the vertex is $(2, -8)$.

c. The axis of symmetry is the line $x = 2$.

d. The discriminant is:
$b^2 - 4ac = (-12)^2 - 4(3)(4) = 96 > 0$, so the graph has two x-intercepts. The x-intercepts are found by solving: $3x^2 - 12x + 4 = 0$.
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-12) \pm \sqrt{96}}{2(3)}$$
$$= \frac{12 \pm 4\sqrt{6}}{6} = \frac{6 \pm 2\sqrt{6}}{3}$$
The x-intercepts are $\frac{6 - 2\sqrt{6}}{3} \approx 0.37$ and $\frac{6 \pm 2\sqrt{6}}{3} \approx 3.63$. The y-intercept is $f(0) = 3(0)^2 - 12(0) + 4 = 4$.

e.



f. The domain is $(-\infty, \infty)$.
The range is $[-8, \infty)$.

g. Decreasing on $(-\infty, 2]$.
Increasing on $[2, \infty)$.

9. a. $g(x) = -2x^2 + 4x - 5$
$a = -2, b = 4, c = -5.$ Since $a = -2 < 0,$ the graph opens down.

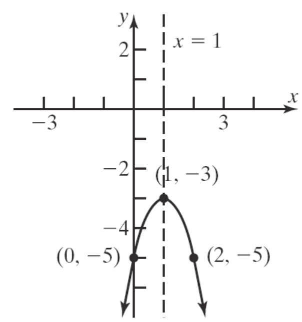
b. The x-coordinate of the vertex is
$x = -\frac{b}{2a} = -\frac{4}{2(-2)} = -\frac{4}{-4} = 1$.

The y-coordinate of the vertex is
$$g\left(-\frac{b}{2a}\right) = g(1) = -2(1)^2 + 4(1) - 5$$
$$= -2 + 4 - 5 = -3$$
Thus, the vertex is $(1, -3)$.

c. The axis of symmetry is the line $x = 1$.

d. The discriminant is:
$b^2 - 4ac = (4)^2 - 4(-2)(-5) = -24 < 0$, so the graph has no x-intercepts. The y-intercept is $g(0) = -2(0)^2 + 4(0) - 5 = -5$.

e.



f. The domain is $(-\infty, \infty)$.
The range is $(-\infty, -3]$.

g. Increasing on $(-\infty, 1]$.
Decreasing on $[1, \infty)$.

10. Consider the form $y = a(x - h)^2 + k$. From the graph we know that the vertex is $(1, -32)$ so we have $h = 1$ and $k = -32$. The graph also passes through the point $(x, y) = (0, -30)$. Substituting these values for x, y, h, and k, we can solve for a:
$-30 = a(0 - 1)^2 + (-32)$ The quadratic function is
$$-30 = a(-1)^2 - 32$$
$$-30 = a - 32$$
$$2 = a$$
$$f(x) = 2(x - 1)^2 - 32 = 2x^2 - 4x - 30.$$

11. $f(x) = -2x^2 + 12x + 3$

$a = -2, b = 12, c = 3$. Since $a = -2 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at

$$x = -\frac{b}{2a} = -\frac{12}{2(-2)} = -\frac{12}{-4} = 3.$$

The maximum value is

$$f(3) = -2(3)^2 + 12(3) + 3 = -18 + 36 + 3 = 21.$$

12. $x^2 - 10x + 24 \geq 0$

$f(x) = x^2 - 10x + 24$

$x^2 - 10x + 24 = 0$

$(x-4)(x-6) = 0$

$x = 4,\ x = 6$ are the zeros of f.

Interval	$(-\infty, 4)$	$(4, 6)$	$(6, \infty)$
Test Number	0	5	7
Value of f	24	−1	3
Conclusion	Positive	Negative	Positive

The solution set is $\{x \mid x \leq 4 \text{ or } x \geq 6\}$ or, using interval notation, $(-\infty, 4] \cup [6, \infty)$.

13. $f(x) = 0$

$2x^2 + 4x + 5 = 0$

$a = 2, b = 4, c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{-4 \pm \sqrt{-24}}{4} = \frac{-4 \pm 2\sqrt{6}\,i}{4} = -1 \pm \frac{\sqrt{6}}{2}i$$

The complex zeros of f are $-1 - \frac{\sqrt{6}}{2}i$ and $-1 + \frac{\sqrt{6}}{2}i$.

14. $|3x + 1| = 8$

$3x + 1 = 8$ or $3x + 1 = -8$

$3x = 7$ or $3x = -9$

$x = \frac{7}{3}$ or $x = -3$

The solution set is $\left\{-3, \frac{7}{3}\right\}$.

15. $\left|\frac{x+3}{4}\right| < 2$

$-2 < \frac{x+3}{4} < 2$

$-8 < x + 3 < 8$

$-11 < x < 5$

$\{x \mid -11 < x < 5\}$ or $(-11, 5)$

16. $|2x + 3| - 4 \geq 3$

$|2x + 3| \geq 7$

$2x + 3 \leq -7$ or $2x + 3 \geq 7$

$2x \leq -10$ or $2x \geq 4$

$x \leq -5$ or $x \geq 2$

$\{x \mid x \leq -5 \text{ or } x \geq 2\}$ or $(-\infty, -5] \cup [2, \infty)$

17. a. $C(m) = 0.15m + 129.50$

b. $C(860) = 0.15(860) + 129.50$

$= 129 + 129.50 = 258.50$

If 860 miles are driven, the rental cost is \$258.50.

c. $C(m) = 213.80$

$0.15m + 129.50 = 213.80$

$0.15m = 84.30$

$m = 562$

The rental cost is \$213.80 if 562 miles were driven.

18. a. $R(x) = x\left(-\frac{1}{10}x + 1000\right) = -\frac{1}{10}x^2 + 1000x$

b. $R(400) = -\frac{1}{10}(400)^2 + 1000(400)$

$= -16{,}000 + 400{,}000$

$= \$384{,}000$

c. $x = \frac{-b}{2a} = \frac{-1000}{2\left(-\frac{1}{10}\right)} = \frac{-1000}{\left(-\frac{1}{5}\right)} = 5000$

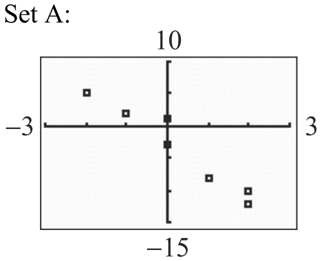
The maximum revenue is

$$R(5000) = -\frac{1}{10}(5000)^2 + 1000(5000)$$
$$= -250{,}000 + 5{,}000{,}000$$
$$= \$2{,}500{,}000$$

Thus, 5000 units maximizes revenue at \$2,500,000.

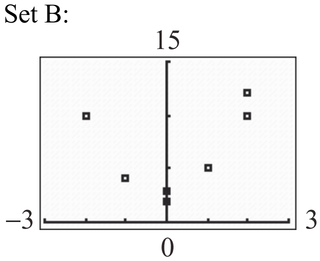
d. $p = -\frac{1}{10}(5000) + 1000$
$= -500 + 1000$
$= \$500$

19. a. Set A:



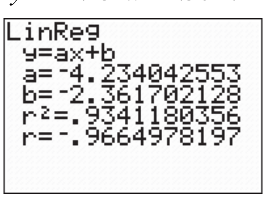
The data appear to be linear with a negative slope.

Set B:

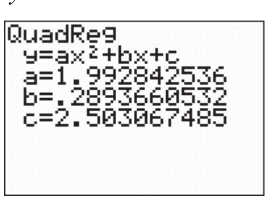


The data appear to be quadratic and opens up.

b. Using the LINear REGression program, the linear function of best fit is: $y = -4.234x - 2.362$.



c. Using the QUADratic REGression program, the quadratic function of best fit is: $y = 1.993x^2 + 0.289x + 2.503$.



Chapter 2 Cumulative Review

1. $P = (-1,3); Q = (4,-2)$

Distance between P and Q:

$$d(P,Q) = \sqrt{(4-(-1))^2 + (-2-3)^2}$$
$$= \sqrt{(5)^2 + (5)^2}$$
$$= \sqrt{25+25}$$
$$= \sqrt{50} = 5\sqrt{2}$$

Midpoint between P and Q:

$$\left(\frac{-1+4}{2}, \frac{3-2}{2}\right) = \left(\frac{3}{2}, \frac{1}{2}\right) = (1.5, 0.5)$$

2. $y = x^3 - 3x + 1$

a. $(-2,-1)$: $-1 = (-2)^3 - 3(-2) + 1$
$-1 = -8 + 6 + 1$
$-1 = -1$

Yes, $(-2,-1)$ is on the graph.

b. $(2,3)$: $3 = (2)^3 - 3(2) + 1$
$3 = 8 - 6 + 1$
$3 = 3$

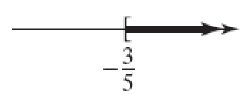
Yes, $(2,3)$ is on the graph.

c. $(3,1)$: $1 = (3)^3 - 3(3) + 1$
$1 = -27 - 9 + 1$
$1 \neq -35$

No, $(3,1)$ is not on the graph.

3. $5x + 3 \geq 0$
$5x \geq -3$
$x \geq -\frac{3}{5}$

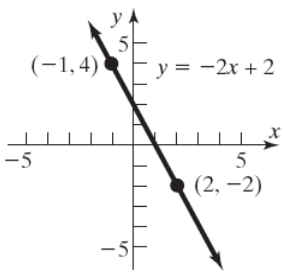
The solution set is $\left\{x \,\middle|\, x \geq -\frac{3}{5}\right\}$ or $\left[-\frac{3}{5}, +\infty\right)$.



4. (–1,4) and (2,–2) are points on the line.

$$\text{Slope} = \frac{-2-4}{2-(-1)} = \frac{-6}{3} = -2$$

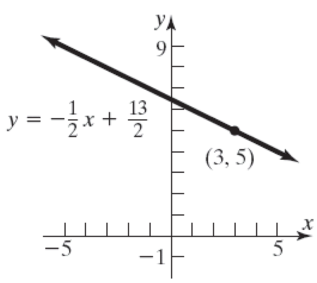
$$y - y_1 = m(x - x_1)$$
$$y - 4 = -2(x - (-1))$$
$$y - 4 = -2(x + 1)$$
$$y - 4 = -2x - 2$$
$$y = -2x + 2$$



5. Perpendicular to $y = 2x + 1$;
Containing (3,5)

$$\text{Slope of perpendicular} = -\frac{1}{2}$$

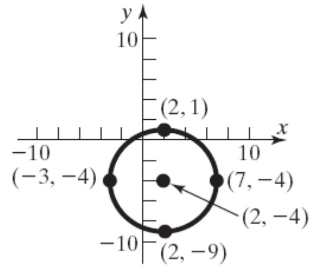
$$y - y_1 = m(x - x_1)$$
$$y - 5 = -\frac{1}{2}(x - 3)$$
$$y - 5 = -\frac{1}{2}x + \frac{3}{2}$$
$$y = -\frac{1}{2}x + \frac{13}{2}$$



6. $x^2 + y^2 - 4x + 8y - 5 = 0$

$$x^2 - 4x + y^2 + 8y = 5$$
$$(x^2 - 4x + 4) + (y^2 + 8y + 16) = 5 + 4 + 16$$
$$(x-2)^2 + (y+4)^2 = 25$$
$$(x-2)^2 + (y+4)^2 = 5^2$$

Center: (2,–4) Radius = 5



7. Yes, this is a function since each x-value is paired with exactly one y-value.

8. $f(x) = x^2 - 4x + 1$

a. $f(2) = 2^2 - 4(2) + 1 = 4 - 8 + 1 = -3$

b. $f(x) + f(2) = x^2 - 4x + 1 + (-3)$
$= x^2 - 4x - 2$

c. $f(-x) = (-x)^2 - 4(-x) + 1 = x^2 + 4x + 1$

d. $-f(x) = -(x^2 - 4x + 1) = -x^2 + 4x - 1$

e. $f(x+2) = (x+2)^2 - 4(x+2) + 1$
$= x^2 + 4x + 4 - 4x - 8 + 1$
$= x^2 - 3$

f.
$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{(x+h)^2 - 4(x+h) + 1 - (x^2 - 4x + 1)}{h}$$
$$= \frac{x^2 + 2xh + h^2 - 4x - 4h + 1 - x^2 + 4x - 1}{h}$$
$$= \frac{2xh + h^2 - 4h}{h}$$
$$= \frac{h(2x + h - 4)}{h} = 2x + h - 4$$

9. $h(z) = \dfrac{3z-1}{6z-7}$

The denominator cannot be zero:

$$6z - 7 \neq 0$$
$$6z \neq 7$$
$$z \neq \frac{7}{6}$$

Domain: $\left\{ z \,\middle|\, z \neq \frac{7}{6} \right\}$

10. Yes, the graph represents a function since it passes the Vertical Line Test.

11. $f(x)=\dfrac{x}{x+4}$

a. $f(1)=\dfrac{1}{1+4}=\dfrac{1}{5}\neq\dfrac{1}{4}$, so $\left(1,\dfrac{1}{4}\right)$ is not on the graph of *f*.

b. $f(-2)=\dfrac{-2}{-2+4}=\dfrac{-2}{2}=-1$, so $(-2,-1)$ is a point on the graph of *f*.

c. Solve for *x*:

$$2=\frac{x}{x+4}$$
$$2x+8=x$$
$$x=-8$$

So, $(-8, 2)$ is a point on the graph of *f*.

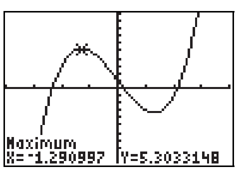
12. $f(x)=\dfrac{x^2}{2x+1}$

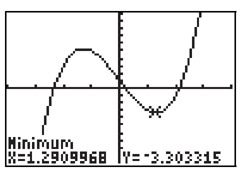
$$f(-x)=\frac{(-x)^2}{2(-x)+1}=\frac{x^2}{-2x+1}\neq f(x)\text{ or }-f(x)$$

Therefore, *f* is neither even nor odd.

13. $f(x)=x^3-5x+4$ on the interval $(-4,4)$

Use MAXIMUM and MINIMUM on the graph of $y_1=x^3-5x+4$.





Local maximum is 5.30 and occurs at $x\approx-1.29$;
Local minimum is –3.30 and occurs at $x\approx1.29$;
f is increasing on $[-4,-1.29]$ or $[1.29,4]$;
f is decreasing on $[-1.29,1.29]$.

14. $f(x)=3x+5;\quad g(x)=2x+1$

a.
$$f(x)=g(x)$$
$$3x+5=2x+1$$
$$3x+5=2x+1$$
$$x=-4$$

b.
$$f(x)>g(x)$$
$$3x+5>2x+1$$
$$3x+5>2x+1$$
$$x>-4$$

The solution set is $\{x|x>-4\}$ or $(-4,\infty)$.

15. a. Domain: $\{x\,|-4\le x\le4\}$ or $[-4,4]$

Range: $\{y\,|-1\le y\le3\}$ or $[-1,3]$

b. Intercepts: $(-1,0)$, $(0,-1)$, $(1,0)$

x-intercepts: $-1, 1$

y-intercept: -1

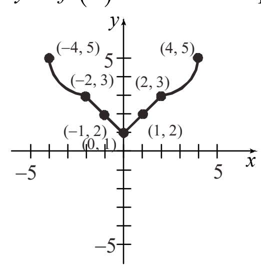
c. The graph is symmetric with respect to the *y*-axis.

d. When $x=2$, the function takes on a value of 1. Therefore, $f(2)=1$.

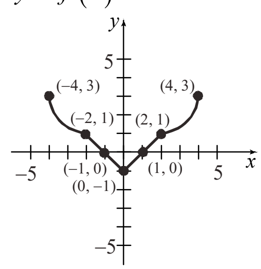
e. The function takes on the value 3 at $x=-4$ and $x=4$.

f. $f(x)<0$ means that the graph lies below the *x*-axis. This happens for *x* values between -1 and 1. Thus, the solution set is $\{x\,|-1<x<1\}$ or $(-1,1)$.

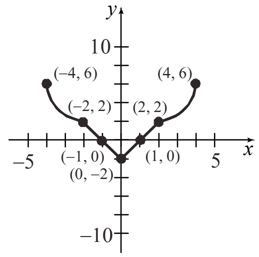
g. The graph of $y=f(x)+2$ is the graph of $y=f(x)$ but shifted up 2 units.



h. The graph of $y=f(-x)$ is the graph of $y=f(x)$ but reflected about the *y*-axis.



i. The graph of $y=2f(x)$ is the graph of $y=f(x)$ but stretched vertically by a factor of 2. That is, the coordinate of each point is multiplied by 2.



j. Since the graph is symmetric about the *y*-axis, the function is even.

k. The function is increasing on the open interval $(0,4)$.

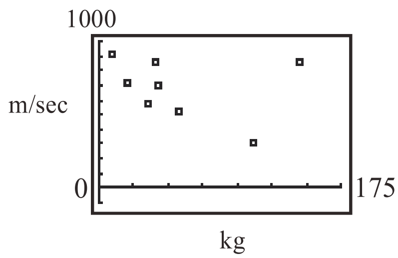
Chapter 2 Projects

Project I – Internet-based Project

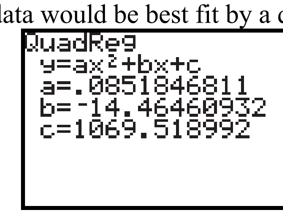
Answers will vary.

Project II

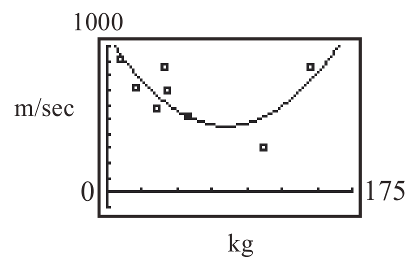
a.



b. The data would be best fit by a quadratic function.



$y = 0.085x^2 - 14.46x + 1069.52$



These results seem reasonable since the function fits the data well.

c. $s_0 = 0\text{m}$

Type	Weight kg	Velocity m/sec	Equation in the form: $s(t) = -4.9t^2 + \frac{\sqrt{2}}{2}v_0 t + s_0$
MG 17	10.2	905	$s(t) = -4.9t^2 + 639.93t$ Best. (It goes the highest)
MG 131	19.7	710	$s(t) = -4.9t^2 + 502.05t$
MG 151	41.5	850	$s(t) = -4.9t^2 + 601.04t$
MG 151/20	42.3	695	$s(t) = -4.9t^2 + 491.44t$
MG/FF	35.7	575	$s(t) = -4.9t^2 + 406.59t$
MK 103	145	860	$s(t) = -4.9t^2 + 608.11t$
MK 108	58	520	$s(t) = -4.9t^2 + 367.70t$
WGr 21	111	315	$s(t) = -4.9t^2 + 222.74t$

$s_0 = 200\text{m}$

Type	Weight kg	Velocity m/sec	Equation in the form: $s(t) = -4.9t^2 + \frac{\sqrt{2}}{2}v_0 t + s_0$
MG 17	10.2	905	$s(t) = -4.9t^2 + 639.93t + 200$ Best. (It goes the highest)
MG 131	19.7	710	$s(t) = -4.9t^2 + 502.05t + 200$
MG 151	41.5	850	$s(t) = -4.9t^2 + 601.04t + 200$
MG 151/20	42.3	695	$s(t) = -4.9t^2 + 491.44t + 200$
MG/FF	35.7	575	$s(t) = -4.9t^2 + 406.59t + 200$
MK 103	145	860	$s(t) = -4.9t^2 + 608.11t + 200$
MK 108	58	520	$s(t) = -4.9t^2 + 367.70t + 200$
WGr 21	111	315	$s(t) = -4.9t^2 + 222.74t + 200$

$s_0 = 30\text{m}$

Type	Weight kg	Velocity m/sec	Equation in the form: $s(t) = -4.9t^2 + \frac{\sqrt{2}}{2}v_0 t + s_0$
MG 17	10.2	905	$s(t) = -4.9t^2 + 639.93t + 30$ Best. (It goes the highest)
MG 131	19.7	710	$s(t) = -4.9t^2 + 502.05t + 30$
MG 151	41.5	850	$s(t) = -4.9t^2 + 601.04t + 30$
MG 151/20	42.3	695	$s(t) = -4.9t^2 + 491.44t + 30$
MG/FF	35.7	575	$s(t) = -4.9t^2 + 406.59t + 30$
MK 103	145	860	$s(t) = -4.9t^2 + 608.11t + 30$
MK 108	58	520	$s(t) = -4.9t^2 + 367.70t + 30$
WGr 21	111	315	$s(t) = -4.9t^2 + 222.74t + 30$

Notice that the gun is what makes the difference, not how high it is mounted necessarily. The only way to change the true maximum height that the projectile can go is to change the angle at which it fires.

Project III

a.

x	1	2	3	4	5
$y=-2x+5$	3	1	-1	-3	-5

b. $\dfrac{\Delta y}{\Delta x}=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{1-3}{1}=-2$

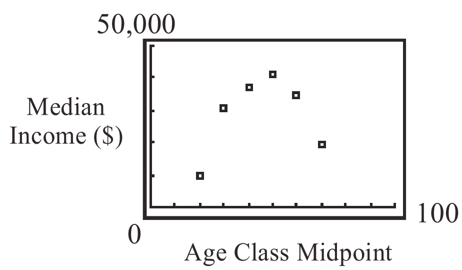
$\dfrac{\Delta y}{\Delta x}=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-1-1}{1}=-2$

$\dfrac{\Delta y}{\Delta x}=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-3-(-1)}{1}=-2$

$\dfrac{\Delta y}{\Delta x}=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-5-(-3)}{1}=-2$

All of the values of $\dfrac{\Delta y}{\Delta x}$ are the same.

c.



d. $\dfrac{\Delta I}{\Delta x}=\dfrac{30633-9548}{10}=2108.50$

$\dfrac{\Delta I}{\Delta x}=\dfrac{37088-30633}{10}=645.50$

$\dfrac{\Delta I}{\Delta x}=\dfrac{41072-37088}{10}=398.40$

$\dfrac{\Delta I}{\Delta x}=\dfrac{34414-41072}{10}=-665.80$

$\dfrac{\Delta I}{\Delta x}=\dfrac{19167-34414}{10}=-1524.70$

These $\dfrac{\Delta I}{\Delta x}$ values are not all equal. The data are not linearly related.

e.

x	-2	-1	0	1	2	3	4
y	23	9	3	5	15	33	59
$\dfrac{\Delta y}{\Delta x}$		-14	-6	2	10	18	26

As x increases, $\dfrac{\Delta y}{\Delta x}$ increases. This makes sense because the parabola is increasing (going up) steeply as x increases.

f.

x	-2	-1	0	1	2	3	4
y	23	9	3	5	15	33	59
$\frac{\Delta^2 y}{\Delta x^2}$			8	8	8	8	8

The second differences are all the same.

g. The paragraph should mention at least two observations:
1. The first differences for a linear function are all the same.
2. The second differences for a quadratic function are the same.

<u>Project IV</u>

a. – i. Answers will vary , depending on where the CBL is located above the bouncing ball.

j. The ratio of the heights between bounces will be the same.

Appendix B
Graphing Utilities

Section B.1

1. $(-1,4)$; Quadrant II

2. (3, 4); Quadrant I

3. (3, 1); Quadrant I

4. $(-6,-4)$; Quadrant III

5. $X\text{ min} = -6$
$X\text{ max} = 6$
$X\text{ scl} = 2$
$Y\text{ min} = -4$
$Y\text{ max} = 4$
$Y\text{ scl} = 2$

6. $X\text{ min} = -3$
$X\text{ max} = 3$
$X\text{ scl} = 1$
$Y\text{ min} = -2$
$Y\text{ max} = 2$
$Y\text{ scl} = 1$

7. $X\text{ min} = -6$
$X\text{ max} = 6$
$X\text{ scl} = 2$
$Y\text{ min} = -1$
$Y\text{ max} = 3$
$Y\text{ scl} = 1$

8. $X\text{ min} = -9$
$X\text{ max} = 9$
$X\text{ scl} = 3$
$Y\text{ min} = -12$
$Y\text{ max} = 4$
$Y\text{ scl} = 4$

9. $X\text{ min} = 3$
$X\text{ max} = 9$
$X\text{ scl} = 1$
$Y\text{ min} = 2$
$Y\text{ max} = 10$
$Y\text{ scl} = 2$

10. $X\text{ min} = -22$
$X\text{ max} = -10$
$X\text{ scl} = 2$
$Y\text{ min} = 4$
$Y\text{ max} = 8$
$Y\text{ scl} = 1$

11. $X\text{ min} = -11$
$X\text{ max} = 5$
$X\text{ scl} = 1$
$Y\text{ min} = -3$
$Y\text{ max} = 6$
$Y\text{ scl} = 1$

12. $X\text{ min} = -3$
$X\text{ max} = 7$
$X\text{ scl} = 1$
$Y\text{ min} = -4$
$Y\text{ max} = 9$
$Y\text{ scl} = 1$

13. $X\text{ min} = -30$
$X\text{ max} = 50$
$X\text{ scl} = 10$
$Y\text{ min} = -90$
$Y\text{ max} = 50$
$Y\text{ scl} = 10$

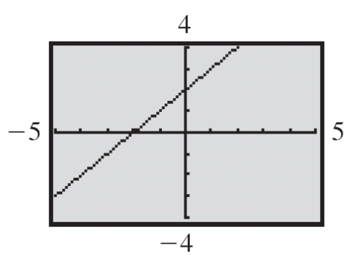
14. $X\min = -90$
$X\max = 30$
$X\text{ scl} = 10$
$Y\min = -50$
$Y\max = 70$
$Y\text{ scl} = 10$

15. $X\min = -10$
$X\max = 110$
$X\text{ scl} = 10$
$Y\min = -10$
$Y\max = 160$
$Y\text{ scl} = 10$

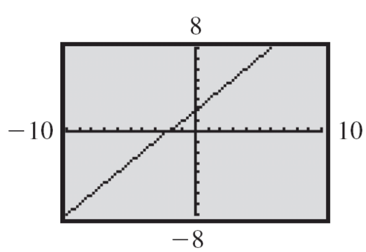
16. $X\min = -20$
$X\max = 110$
$X\text{ scl} = 10$
$Y\min = -10$
$Y\max = 60$
$Y\text{ scl} = 10$

Section B.2

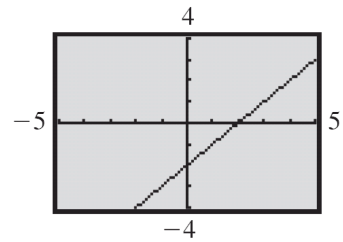
1. **a.**



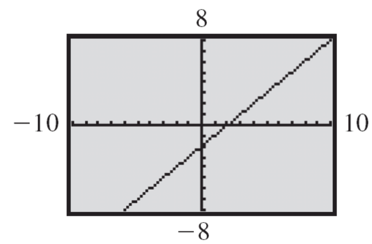
b.



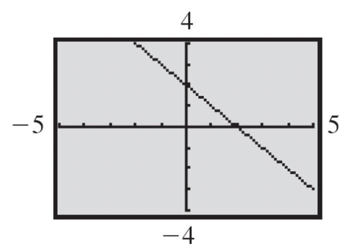
2. **a.**



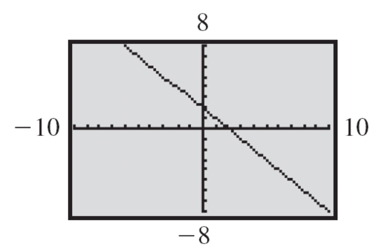
b.



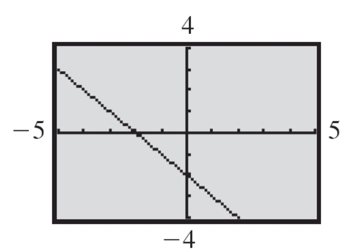
3. **a.**



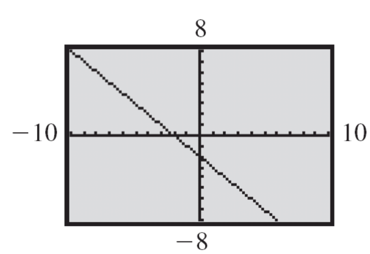
b.



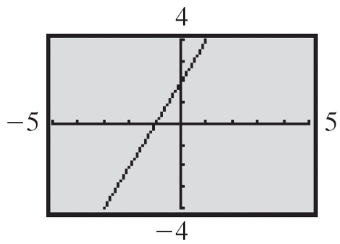
4. **a.**



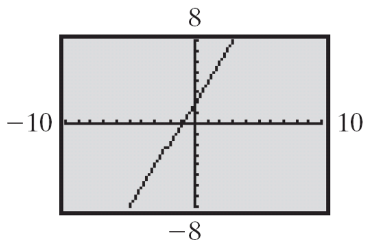
b.



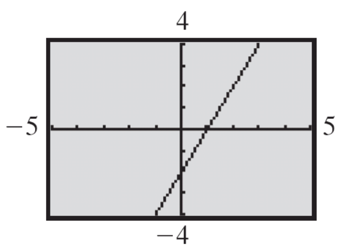
5. **a.**



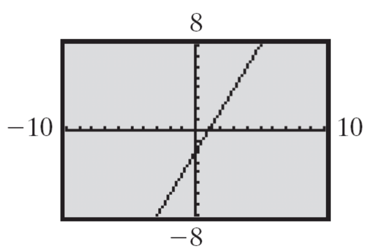
b.



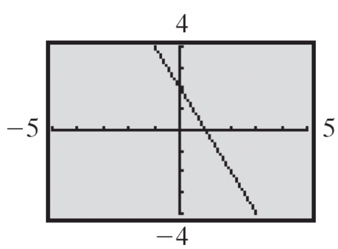
6. **a.**



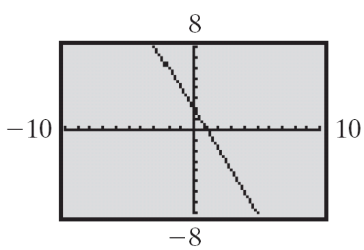
b.



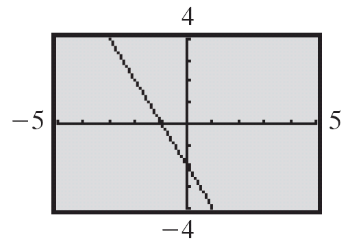
7. **a.**



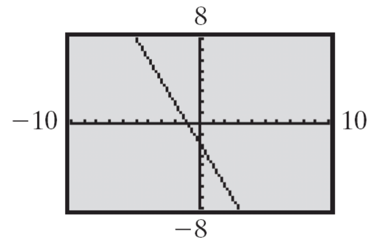
b.



8. **a.**



b.



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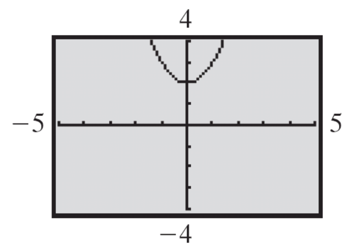

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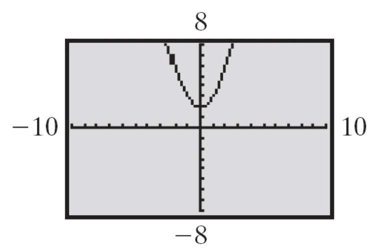

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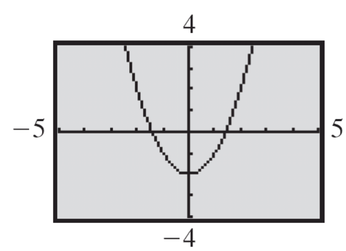

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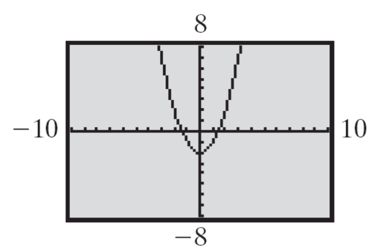

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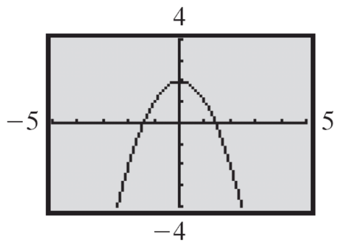

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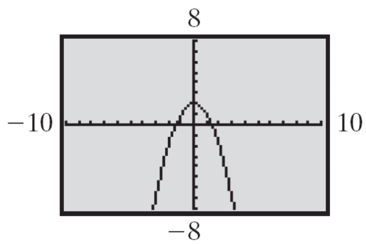
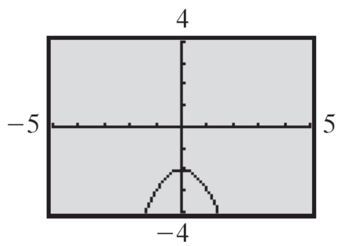


12. **a.**



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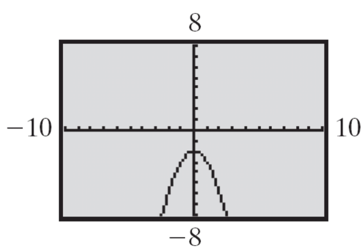

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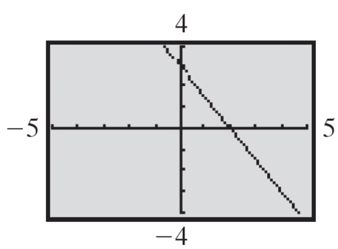

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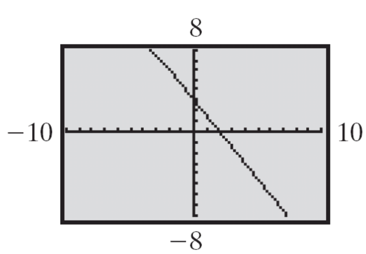

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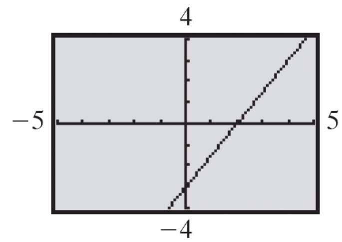

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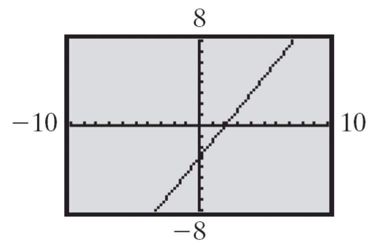
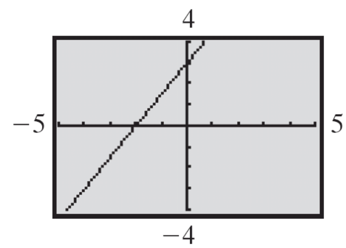


15. **a.**



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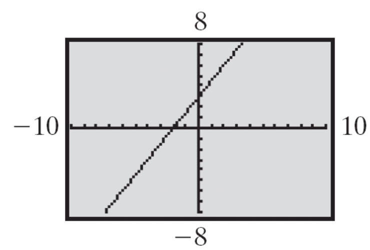
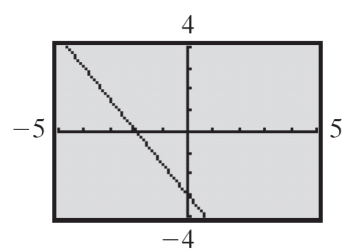


16. **a.**



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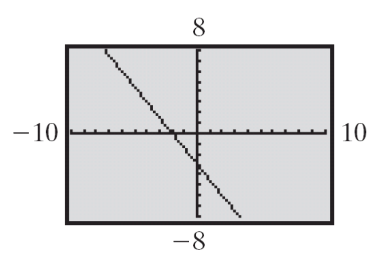


17.

X	Y1
-3	-1
-2	0
-1	1
0	2
1	3
2	4
3	5

$Y_1 = X+2$

Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,-1)$, $(-2,0)$, and $(-1,1)$.

18.

X	Y1
-3	-5
-2	-4
-1	-3
0	-2
1	-1
2	0
3	1

$Y_1 = X-2$

Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,-5)$, $(-2,-4)$, and $(-1,-3)$.

19.

X	Y1
-3	5
-2	4
-1	3
0	2
1	1
2	0
3	-1

$Y_1 = -X+2$

Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,5)$, $(-2,4)$, and $(-1,3)$.

20.

X	Y1
-3	1
-2	0
-1	-1
0	-2
1	-3
2	-4
3	-5

$Y_1 = -X-2$

Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,1)$, $(-2,0)$, and $(0,-2)$.

21.

X	Y1
-3	-4
-2	-2
-1	0
0	2
1	4
2	6
3	8

$Y_1 = 2X+2$

Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,-4)$, $(-2,-2)$, and $(-1,0)$.

22.

X	Y1
-3	-8
-2	-6
-1	-4
0	-2
1	0
2	2
3	4

$Y_1 = 2X-2$

Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,-8)$, $(-2,-6)$, and $(-1,-4)$.

23.

X	Y1
-3	8
-2	6
-1	4
0	2
1	0
2	-2
3	-4

$Y_1 = -2X+2$

Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,8)$, $(-2,6)$, and $(-1,4)$.

24.

X	Y1
-3	4
-2	2
-1	0
0	-2
1	-4
2	-6
3	-8

$Y_1 = -2X-2$

Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,4)$, $(-2,2)$, and $(-1,0)$.

25.

X	Y1
-3	11
-2	6
-1	3
0	2
1	3
2	6
3	11

$Y_1 = X^2+2$

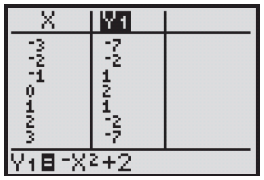
Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,11)$, $(-2,6)$, and $(-1,3)$.

26.

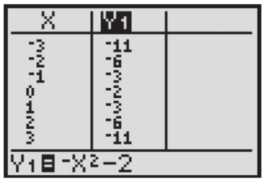
X	Y1
-3	7
-2	2
-1	-1
0	-2
1	-1
2	2
3	7

$Y_1 = X^2-2$

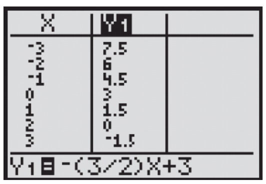
Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,7)$, $(-2,2)$, and $(-1,-1)$.

27. 

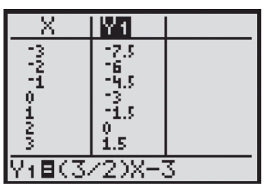
Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,-7)$, $(-2,-2)$, and $(-1,1)$.

28. 

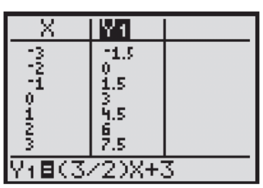
Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,-11)$, $(-2,-6)$, and $(-1,-3)$.

29. 

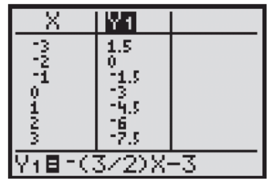
Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,7.5)$, $(-2,6)$, and $(-1,4.5)$.

30. 

Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,-7.5)$, $(-2,-6)$, and $(-1,-4.5)$.

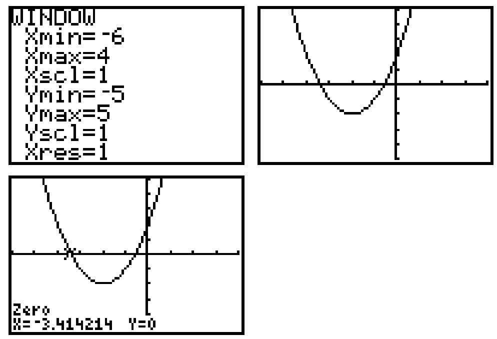
31. 

Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,-1.5)$, $(-2,0)$, and $(-1,1.5)$.

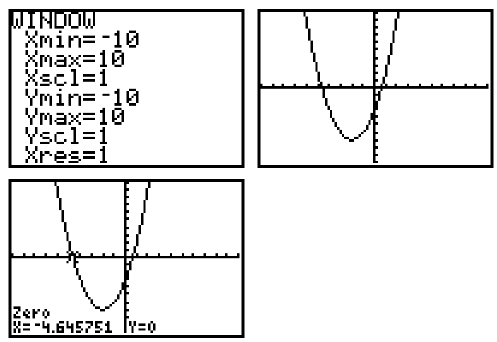
32. 

Each ordered pair from the table corresponds to a point on the graph. Three points on the graph are $(-3,1.5)$, $(-2,0)$, and $(-1,-1.5)$.

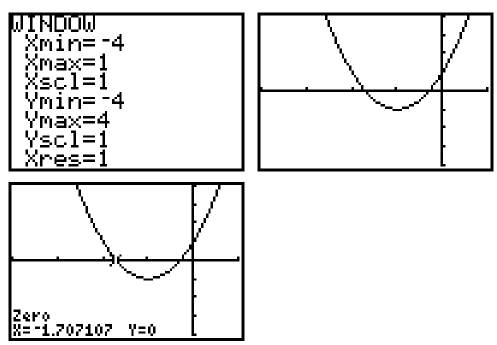
Section B.3

1. 

The smaller x-intercept is roughly -3.41.

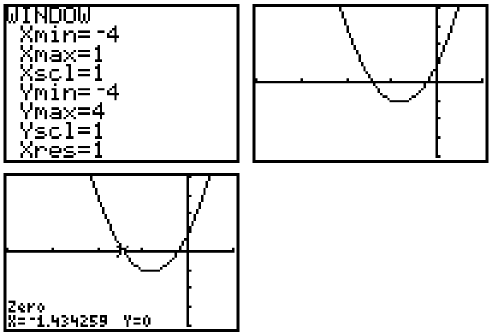
2. 

The smaller x-intercept is roughly -4.65.

3. 

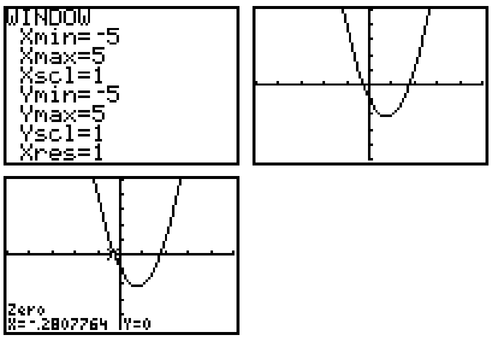
The smaller x-intercept is roughly -1.71.

4.



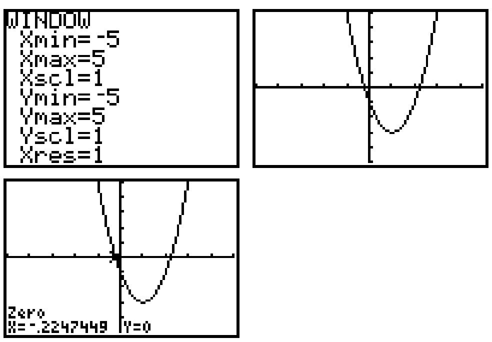
The smaller x-intercept is roughly -1.43 .

5.



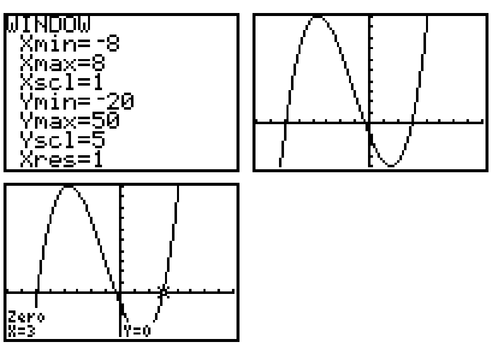
The smaller x-intercept is roughly -0.28 .

6.



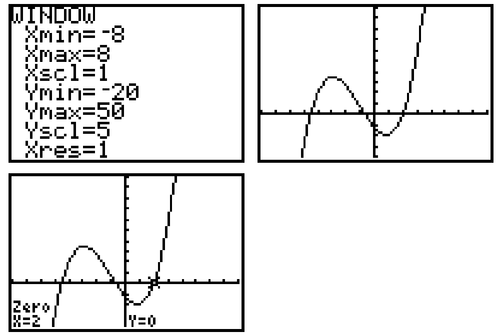
The smaller x-intercept is roughly -0.22 .

7.



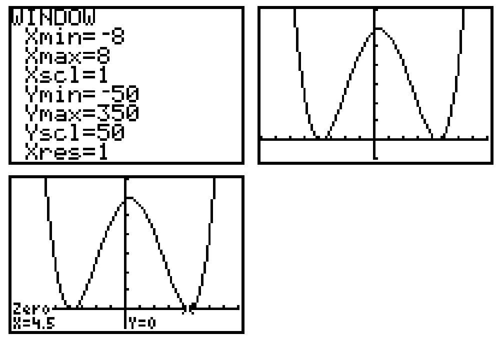
The positive x-intercept is 3.00.

8.



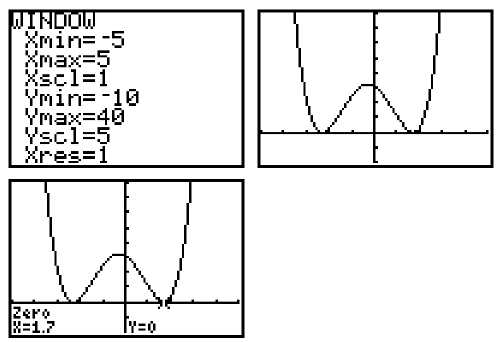
The positive x-intercept is 2.00.

9.



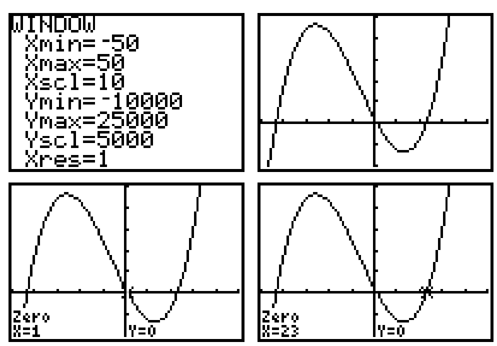
The positive x-intercept is 4.50.

10.



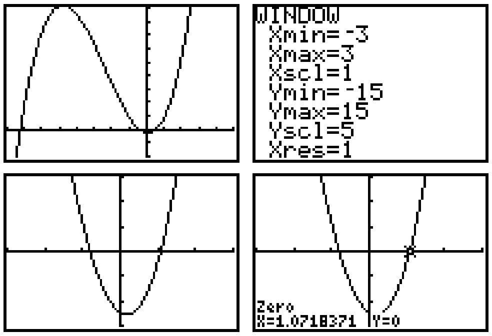
The positive x-intercept is 1.70.

11.



The positive x-intercepts are 1.00 and 23.00.

12.



The positive *x*-intercept is 1.07

Section B.5

Problems 1-8 assume that a ratio of 3:2 is required for a square screen, as with a TI-84 Plus.

1. $\dfrac{X_{\max}-X_{\min}}{Y_{\max}-Y_{\min}}=\dfrac{8-(-8)}{5-(-5)}=\dfrac{16}{10}=\dfrac{8}{5}$
for a ratio of 8:5, resulting in a square screen.

2. $\dfrac{X_{\max}-X_{\min}}{Y_{\max}-Y_{\min}}=\dfrac{5-(-5)}{4-(-4)}=\dfrac{10}{8}=\dfrac{5}{4}$
for a ratio of 5:4, resulting in a screen that is not square.

3. $\dfrac{X_{\max}-X_{\min}}{Y_{\max}-Y_{\min}}=\dfrac{16-0}{8-(-2)}=\dfrac{16}{10}=\dfrac{8}{5}$
for a ratio of 8:5, resulting in a square screen.

4. $\dfrac{X_{\max}-X_{\min}}{Y_{\max}-Y_{\min}}=\dfrac{16-(-16)}{10-(-10)}=\dfrac{32}{20}=\dfrac{8}{5}$
for a ratio of 8:5, resulting in a square screen.

5. $\dfrac{X_{\max}-X_{\min}}{Y_{\max}-Y_{\min}}=\dfrac{6-(-6)}{2-(-2)}=\dfrac{12}{4}=3$
for a ratio of 3:1, resulting in a screen that is not square.

6. $\dfrac{X_{\max}-X_{\min}}{Y_{\max}-Y_{\min}}=\dfrac{8-(-8)}{5-(-5)}=\dfrac{16}{10}=\dfrac{8}{5}$
for a ratio of 8:5, resulting in a square screen.

7. $\dfrac{X_{\max}-X_{\min}}{Y_{\max}-Y_{\min}}=\dfrac{5-(-3)}{3-(-2)}=\dfrac{8}{5}$
for a ratio of 8:5, resulting in a square screen.

8. $\dfrac{X_{\max}-X_{\min}}{Y_{\max}-Y_{\min}}=\dfrac{14-(-10)}{8-(-7)}=\dfrac{24}{15}=\dfrac{8}{5}$
for a ratio of 8:5, resulting in a square screen.

9. Answers will vary.
$X_{\max}-X_{\min}=12-(-4)=16$
We want a ratio of 8:5, so the difference between $Y_{\max}$ and $Y_{\min}$ should be 10. In order to see the point $(4,8)$, the $Y_{\max}$ value must be greater than 8. We might choose $Y_{\max}=10$, which means $10-Y_{\min}=10$, or $Y_{\min}=0$. Since we are on the order of 10, we would use a scale of 1. Thus, $Y_{\min}=0$, $Y_{\max}=10$, and $Y_{\text{scl}}=1$ will make the point $(4,8)$ visible and have a square screen.

10. Answers will vary.
$X_{\max}-X_{\min}=10-(-6)=16$
We want a ratio of 8:5, so the difference between $Y_{\max}$ and $Y_{\min}$ should be 10. In order to see the point $(4,8)$, the $Y_{\max}$ value must be greater than 8. We might choose $Y_{\max}=10$, which means $10-Y_{\min}=10$, or $Y_{\min}=0$. Since our range of *Y* values is more than 10, we might consider using a scale of 2. Thus, $Y_{\min}=0$, $Y_{\max}=10$, and $Y_{\text{scl}}=1$ will make the point $(4,8)$ visible and have a square screen.

Mini-Lecture 2.1
Properties of Linear Functions and Linear Models

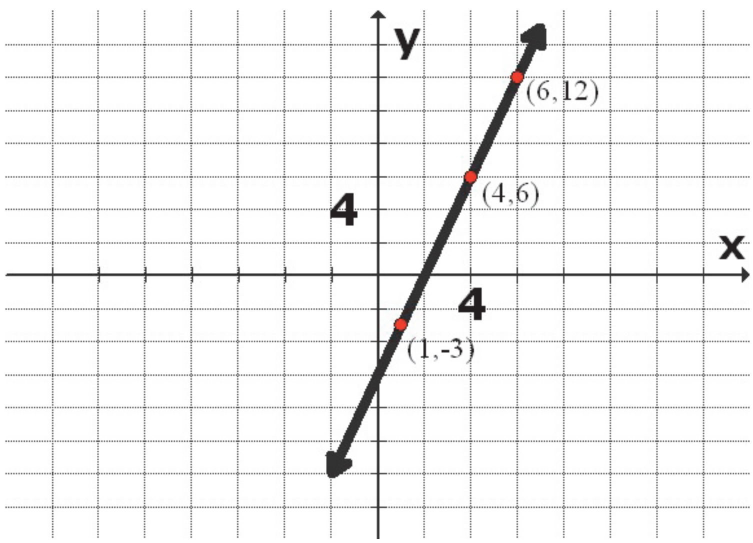
Learning Objectives:

1. Graph Linear Functions
2. Use Average Rate of Change to Identify Linear Functions
3. Determine Whether a Linear function Is Increasing, Decreasing, or Constant
4. Find the Zero of a Linear Function
4. Build Linear Models from Verbal Descriptions

Examples:

1. Suppose that $f(x) = 5x - 9$ and $g(x) = -3x + 7$. Solve $f(x) = g(x)$. Then graph $y = f(x)$ and $y = g(x)$ and label the point that represents the solution to the equation $f(x) = g(x)$.

2. In parts a) and b) using the following figure,



a) Solve $f(x) = 12$.

b) Solve $0 < f(x) < 12$.

3. The monthly cost C, in dollars, for renting a full-size car for a day from a particular agency is modeled by the function $C(x) = 0.12x + 40$, where x is the number of miles driven. Suppose that your budget for renting a car is $\$100$. What is the maximum number of miles that you can drive in one day?

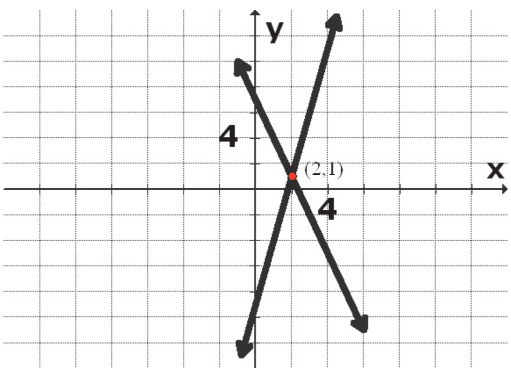
4. Find a firm's break-even point if $R(x) = 10x$ and $C(x) = 7x + 6000$.

Teaching Notes:

- Review the slope-intercept form of the equation of a line.
- Emphasize the theorem for Average Rate of Change of a Linear Function in the book.
- Emphasize the need to express the answer to a verbal problem in terms of the units given in the problem.
- Discuss depreciation, supply and demand, and break-even analysis in more depth before working either the examples in the book or the examples above.

Answers:

1. $x = 2$,



2. a) $x = 6$, b) $2 < x < 6$

3. 500 miles

4. 2000 units

Mini-Lecture 2.2
Building Linear Models from Data

Learning Objectives:

1. Draw and Interpret Scatter Diagrams
2. Distinguish between Linear and Nonlinear Relations
3. Use a Graphing Utility to Find the Line of Best Fit

Examples:

x	3	7	8	9	11	15
y	2	4	7	8	6	10

1. Draw a scatter diagram. Select two points from the scatter diagram and find the equation of the line containing the two points.

2. Graph the line on the scatter diagram.

The marketing manager for a toy company wishes to find a function that relates the demand D for a doll and p the price of the doll. The following data were obtained based on a price history of the doll. The demand is given in thousands of dolls sold per day.

Price	9.00	10.50	11.00	12.00	12.50	13
Demand	12	11	9	10	9.5	8

3. Use a graphing utility to draw a scatter diagram. Then, find and draw the line of best fit.

4. How many dolls will be demanded if the price is \$11.50?

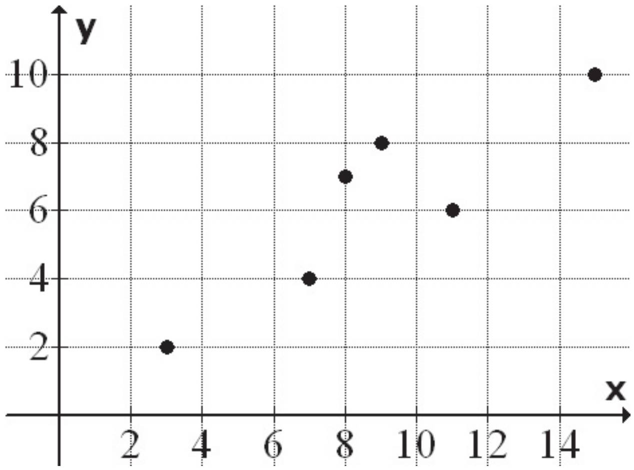
Teaching Notes:

- Many students have trouble deciding what scale to use on the x- and y-axes of their scatter plots. Remind them that the scale does not have to be the same on both axes and that the axes may show a break between zero and the first labeled tick mark.
- Use a set of data and ask different students to pick two points and find the equation of the line through the points. Use the graphing utility to draw a scatter diagram and plot each student's line on the scatter diagram. Then find the line of best fit using the graphing utility and graph it on the scatter diagram. This exercise will give students a better understanding of the line of best fit.
- To help students understand of the correlation coefficient show scatter diagrams for different data sets. Show the students data sets with correlations coefficients close to 1, -1, and 0.

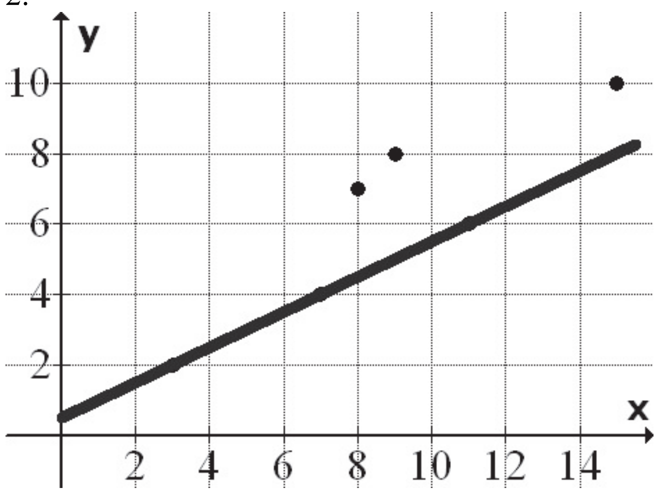
CLICK HERE TO ACCESS THE COMPLETE Solutions

Answers: 1. $y=\frac{1}{2}x+\frac{1}{2}$ for points $(3,2)$ and $(7,4)$ 4. $D=9.77$ thousand dolls

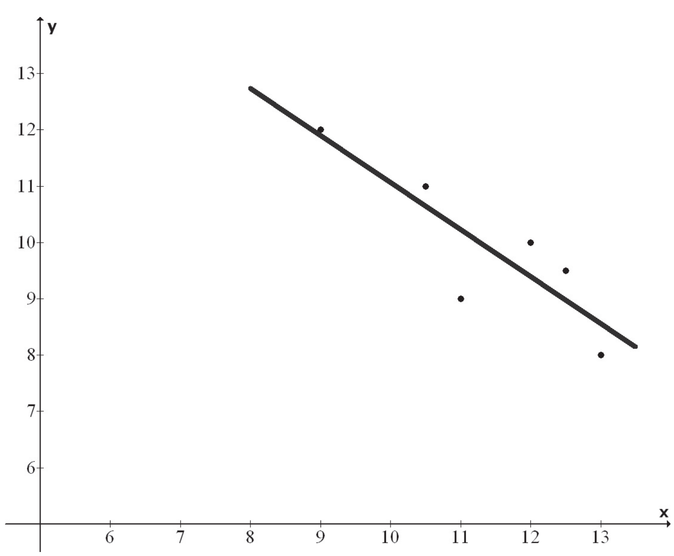
1.



2.



3.



Mini-Lecture 2.3
Quadratic Functions and Their Zeros

Learning Objectives:

1. Find the Zeros of a Quadratic Function by Factoring
2. Find the Zeros of a Quadratic Function Using the Square Root Method
3. Find the Zeros of a Quadratic Function by Completing the Square
4. Find the Zeros of a Quadratic Function Using the Quadratic Formula
5. Find the Point of Intersection of Two Functions
6. Solve Equations That are Quadratic in Form

Examples:

1. Find the zeros by factoring: $f(x) = 3x^2 + 4x - 4$

2. Find the zeros by using the square root method: $f(x) = (4x-1)^2 - 16$

3. Find the zeros by completing the square: $f(x) = x^2 + 4x - 10$

4. Find the zeros by using the quadratic formula: $f(x) = 3x^2 - 5x - 7$

5. Find the real zeros of: $f(x) = x^4 - 11x^2 + 18$

6. Find the points of intersection: $f(x) = x^2 - 4$ & $g(x) = 3 - x^2$

Teaching Notes:

- Students that do not have good skills will struggle with this section. Most students can factor pretty well, but they will commit many types of algebraic mistakes when using the other methods.
- When students use the quadratic formula, they will have trouble simplifying the rational expression. For example, reducing like this $\frac{10 \pm 5\sqrt{10}}{10} = 1 \pm 5\sqrt{10}$ is a common error.
- Completing the square will shine a light on the difficulties that students have with fractions.

Answers:

1. $\frac{2}{3}; -2$

2. $\frac{5}{4}; -\frac{3}{4}$

3. $-2 \pm \sqrt{14}$

4. $\frac{5 \pm \sqrt{109}}{6}$

5. $x = \pm\sqrt{2};\ \pm 3$

6. $\left(\pm\frac{\sqrt{14}}{2}, -\frac{1}{2}\right)$

Mini-Lecture 2.4
Properties of Quadratic Functions

<u>Learning Objectives:</u>

1. Graph a Quadratic Function Using Transformations
2. Identify the Vertex and Axis of Symmetry of a Quadratic Function
3. Graph a Quadratic Function Using Its Vertex, Axis, and Intercepts
4. Find a Quadratic Function Given Its Vertex and One Other Point
5. Find the Maximum and Minimum Value of a Quadratic Function

<u>Examples:</u>

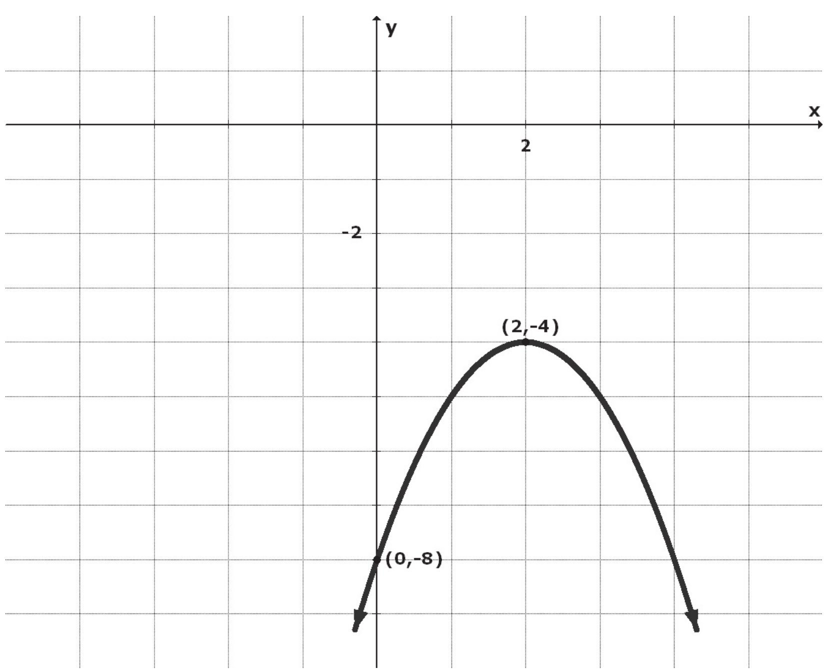
1. Find the coordinates of the vertex for the parabola defined by the given quadratic function. $f(x) = -3x^2 + 5x - 4$

2. Sketch the graph of the quadratic function by determining whether it opens up or down and by finding its vertex, axis of symmetry, y-intercepts, and x-intercepts, if any. $f(x) = 6 - 5x + x^2$

3. For the quadratic function, $f(x) = 4x^2 - 8x$,

   a) determine, without graphing, whether the function has a minimum value or a maximum value,
   b) find the minimum or maximum value.

4. Determine the quadratic function whose graph is given.



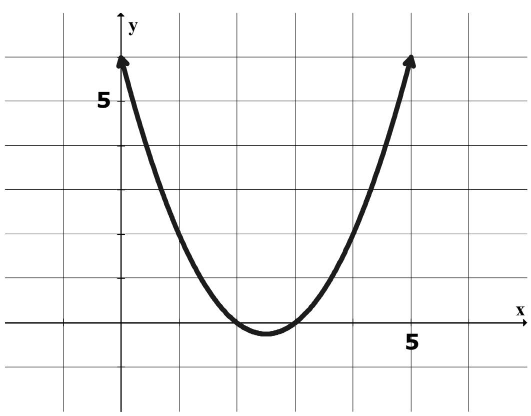
Teaching Notes:

- Remind students to review transformations of graphs before beginning to graph quadratic functions.
- Emphasize from the book "Steps for Graphing a Quadratic Function $f(x) = ax^2 + bx + c$, $a \neq 0$."
- Stress the use of a from the standard form to determine the direction the parabola is opening before beginning to graph it. Students need to recognize early on the benefits of knowing as much about a graph as possible before beginning to draw it.
- In addition to the intercepts, encourage students to use symmetry to find additional points on the graph of a quadratic function.
- Many students will give the x-value found with $x = -\frac{b}{2a}$ as the maximum or minimum value of the quadratic function. Emphasize that finding the maximum or minimum is a two-step process. First, find where it occurs (x), then find what it is (y).

Answers:

1. $\left(\frac{5}{6}, -\frac{23}{12}\right)$

2. opens up, vertex $\left(\frac{5}{2}, -\frac{1}{4}\right)$, axis $x = \frac{5}{2}$, x-intercepts 2 and 3, y-intercept 6



3. a) minimum b) minimum of -4

4. $f(x) = -x^2 + 4x - 8$

Mini-Lecture 2.5

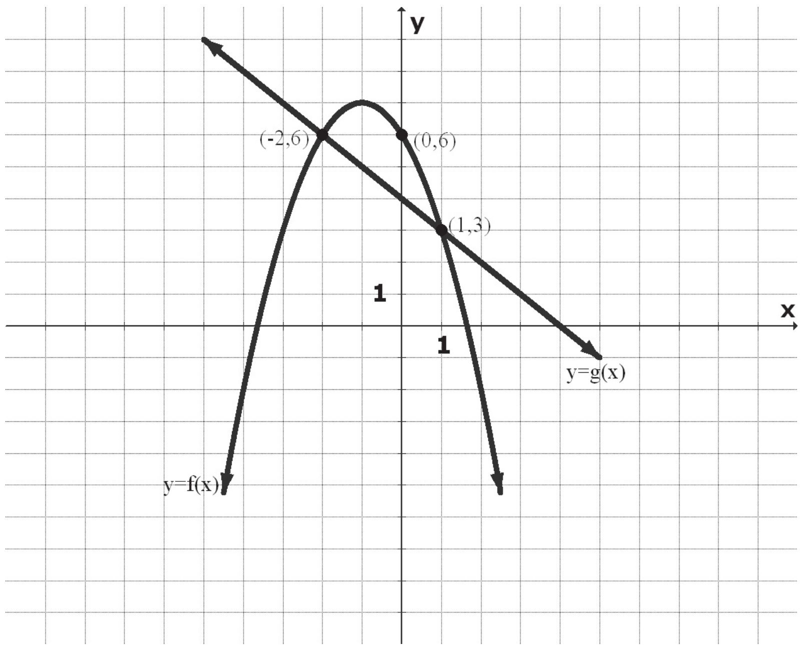
Inequalities Involving Quadratic Functions

Learning Objectives:

1. Solve Inequalities Involving a Quadratic Function

Examples:

1. Use the figure to solve the inequality $f(x) \geq g(x)$.



2. Solve and express the solution in interval notation.

$$9x^2 - 6x + 1 < 0$$

3. Solve the inequality.

$$x(x+2) > 15$$

4. Solve.

$$f(x) > g(x).\quad f(x) = -x^2 + 2x + 3;\ g(x) = -x^2 - 2x + 8$$

Teaching Notes:

- Suggest that students review interval notation. Caution them to check for the correct use of brackets or parentheses in solutions written in interval notation.
- Suggest that students review factoring a trinomial.
- Be sure to show Option I and Option II in Example 2 in the book.

Answers:

1. $[-2,1]$

2. $\varnothing$

3. $(-\infty,-5)\cup(3,\infty)$

4. $\left(\frac{5}{4},\infty\right)$

Mini-Lecture 2.6
Building Quadratic Models from Verbal Descriptions and from Data

Learning Objectives:

1. Build Quadratic Models from Verbal Descriptions
2. Build Quadratic Models from Data

Examples:

1. Among all pairs of numbers whose sum is 50, find a pair whose product is as large as possible. What is the maximum product?

2. A person standing close to the edge of the top of a 180-foot tower throws a ball vertically upward. The quadratic function $s(t) = -16t^2 + 64t + 180$ models the ball's height above ground, $s(t)$, in feet, t seconds after it was thrown. After how many seconds does the ball reach its maximum height? What is the maximum height?

3. The price p (in dollars) and the quantity x sold of a certain product obey the demand equation $p = -\frac{1}{4}x + 120$. Find the model that expresses the revenue R as a function of x. What quantity x maximizes revenue? What is the maximum revenue?

4. The following data represent the percentage of the population in a certain country aged 40 or older whose age is x who do not have a college degree of some type.

Age, x	40	45	50	55	60	65
No college	25.4	23.2	21.8	24.5	26.1	29.8

Find a quadratic model that describes the relationship between age and percentage of the population that do not have a college degree. Use the model to predict the percentage of 53-year-olds that do not have a college degree.

Teaching Notes:

- Show students how to use MAXIMUM and MINIMUM on the graphing utility.
- Show students how to use the QUADratic REGression program on the graphing utility.
- Encourage students to review the discriminant.

Answers:

1. $(25, 25)$, 625

2. 2 sec., 244 ft

3. $R(x) = -\frac{1}{4}x^2 + 120x$, $x = 240$, $R(240) = \$14,400$

4. $P(x) = .0296x^2 - 2.9216x + 94.6550$, 23.0%

Mini-Lecture 2.7
Complex Zeros of a Quadratic Function

Learning Objectives:

1. Find the Complex Zeros of a Quadratic Function

Examples:

1. Find the complex zeros; graph the function; label the intercepts.

a) $f(x) = x^2 + 5$ b) $f(x) = x^2 + 2x + 7$
c) $f(x) = 2x^2 - 4x - 5$ d) $f(x) = x^2 - 2x + 5$

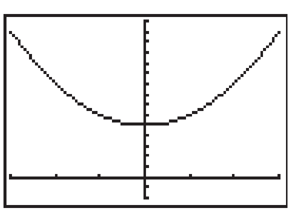
Teaching Notes:

- Have students review the quadratic formula.
- Students will need to know how to reduce radical and rational expressions.
- Remind the students that i is outside the radical when simplifying a radical expression.

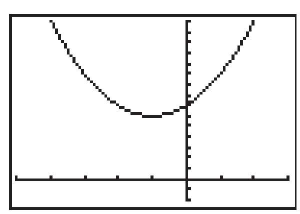
Answers:

1. a) Complex zeros $x = \pm i\sqrt{5}$, y-intercept $= 5$, no x-intercepts.
   b) Complex zeros $x = -1 \pm i\sqrt{6}$, y-intercept $= 7$, no x-intercepts
   c) Complex zeros $x = \dfrac{2 \pm \sqrt{14}}{2}$, y-intercept $= -5$, x-intercepts $x = \dfrac{2 \pm \sqrt{14}}{2}$
   d) Complex zeros $x = 1 \pm 2i$, y-intercept $= 5$, no x-intercepts

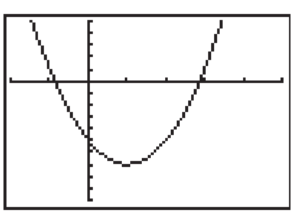
a)



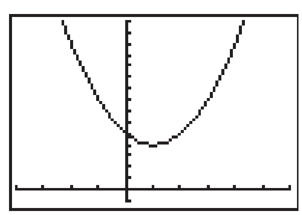
b)



c)



d)



Mini-Lecture 2.8
Equations and Inequalities Involving the Absolute Value Function

Learning Objectives:

1. Solve Absolute Value Equations
2. Solve Absolute Value Inequalities

Examples:

1. Solve each equation.

   a) $|5x-10|=15$ b) $\left|\frac{2}{3}x+6\right|=12$ c) $|4-3x|-4=1$ d) $|3-x|=-7$

2. Solve each absolute value inequality.

   a) $|3x|\le 21$ b) $|4x-3|\ge 9$ c) $|2-6x|-5<1$

Teaching Notes:

- When solving absolute value equations, students will sometimes forget that there are two solutions.
- Students will often not isolate the absolute value expression before trying to solve, such as examples 1 c) and 2 c) above.
- Some students try to combine two intervals that cannot be combined, such as $-3<x>2$.

Answers:

1. a) $x=5, x=-1$ b) $x=9, x=-27$ c) $x=-\frac{1}{3}, x=3$ d) No solution

2. a) $-7\le x\le 7$ b) $x\le -\frac{3}{2}$ *or* $x\ge 3$ c) $-\frac{2}{3}<x<\frac{4}{3}$

4. Cannons The velocity of a projectile depends upon many factors, in particular, the weight of the ammunition.

(a) Plot a scatter diagram of the data in the table below. Let x be the weight in kilograms and let y be the velocity in meters per second.

Type	Weight (kg)	Initial Velocity (m/sec)
MG 17	10.2	905
MG 131	19.7	710
MG 151	41.5	850
MG 151/20	42.3	695
MG/FF	35.7	575
MK 103	145	860
MK 108	58	520
WGr 21	111	315

(Data and information taken from "Flugzeug-Handbuch, Ausgabe Dezember 1996: Guns and Cannons of the Jagdwaffe" at www.xs4all.nl/~rhorta/jgguns.htm*)*

(b) Determine which type of function would fit this data the best: linear or quadratic. Use a graphing utility to find the function of best fit. Are the results reasonable?

(c) Based on velocity, we can determine how high a projectile will travel before it begins to come back down. If a cannon is fired at an angle of 45° to the horizontal, then the function for the height of the projectile is given by $s(t) = -16t^2 + \frac{\sqrt{2}}{2}v_0 t + s_0$, where v_0 is the velocity at which the shell leaves the cannon (initial velocity), and s_0 is the initial height of the nose of the cannon (because cannons are not very long, we may assume that the nose and the firing pin at the back are at the same height for simplicity). Graph the function $s = s(t)$ for each of the guns described in the table. Which gun would be the best for anti-aircraft if the gun were sitting on the ground? Which would be the best to have mounted on a hilltop or on the top of a tall building? If the guns were on the turret of a ship, which would be the most effective?

3. Suppose $f(x) = \sin x$.

(a) Build a table of values for $f(x)$ where $x = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}, 2\pi$. Use exact values.

(b) Find the **first differences** for each consecutive pair of values in part (a). That is, evaluate $g(x_i) = \frac{\Delta f(x_i)}{\Delta x_i} = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$, where $x_1 = 0$, $x_2 = \frac{\pi}{6}, \ldots,$ $x_{17} = 2\pi$. Use your calculator to approximate each value rounded to three decimal places.

(c) Plot the points $(x_i, g(x_i))$ for $i = 1, \ldots, 16$ on a scatter diagram. What shape does the set of points give? What function does this resemble? Fit a sine curve of best fit to the points. How does that relate to your guess?

(d) Find the first differences for each consecutive pair of values in part (b). That is, evaluate $h(x_i) = \frac{\Delta g(x_i)}{\Delta x_i} = \frac{g(x_{i+1}) - g(x_i)}{x_{i+1} - x_i}$ where $x_1 = 0$, $x_2 = \frac{\pi}{6}, \ldots, x_{16} = \frac{11\pi}{6}$. This is the set of **second differences** of $f(x)$. Use your calculator to approximate each value rounded to three decimal places. Plot the points $(x_i, h(x_i))$ for $i = 1, \ldots, 15$ on a scatter diagram. What shape does the set of points give? What function does this resemble? Fit a sine curve of best fit to the points. How does that relate to your guess?

(e) Find the first differences for each consecutive pair of values in part (d). That is, evaluate $k(x_i) = \dfrac{\Delta h(x_i)}{\Delta x_i} = \dfrac{h(x_{i+1}) - h(x_i)}{x_{i+1} - x_i}$, where $x_1 = 0,\ x_2 = \dfrac{\pi}{6}, \ldots, x_{15} = \dfrac{7\pi}{4}$. This is the set of **third differences** of $f(x)$. Use your calculator to approximate each value rounded to three decimal places. Plot the points $(x_i, k(x_i))$ for $i = 1, \ldots, 14$ on a scatter diagram. What shape does the set of points give? What function does this resemble? Fit a sine curve of best fit to the points. How does that relate to your guess?

(f) Find the first differences for each consecutive pair of values in part (e). That is, evaluate $m(x_i) = \dfrac{\Delta k(x_i)}{\Delta x_i} = \dfrac{k(x_{i+1}) - k(x_i)}{x_{i+1} - x_i}$, where $x_1 = 0,\ x_2 = \dfrac{\pi}{6}, \ldots, x_{14} = \dfrac{5\pi}{3}$. This is the set of **fourth differences** of $f(x)$.

Use your calculator to approximate each value rounded to three decimal places. Plot the points $(x_i, m(x_i))$ for $i = 1, \ldots, 13$ on a scatter diagram. What shape does the set of points give? What function does this resemble? Fit a sine curve of best fit to the points. How does that relate to your guess?

(g) What pattern do you notice about the curves that you found? What happened in part (f)? Can you make a generalization about what happened as you computed the differences? Explain your answers.

7. CBL Experiment Locate the motion detector on a Calculator Based Laboratory (CBL) or a Calculator Based Ranger (CBR) above a bouncing ball.

(a) Plot the data collected in a scatter diagram with time as the independent variable.

(b) Find the quadratic function of best fit for the second bounce.

(c) Find the quadratic function of best fit for the third bounce.

(d) Find the quadratic function of best fit for the fourth bounce.

(e) Compute the maximum height for the second bounce.

(f) Compute the maximum height for the third bounce.

(g) Compute the maximum height for the fourth bounce.

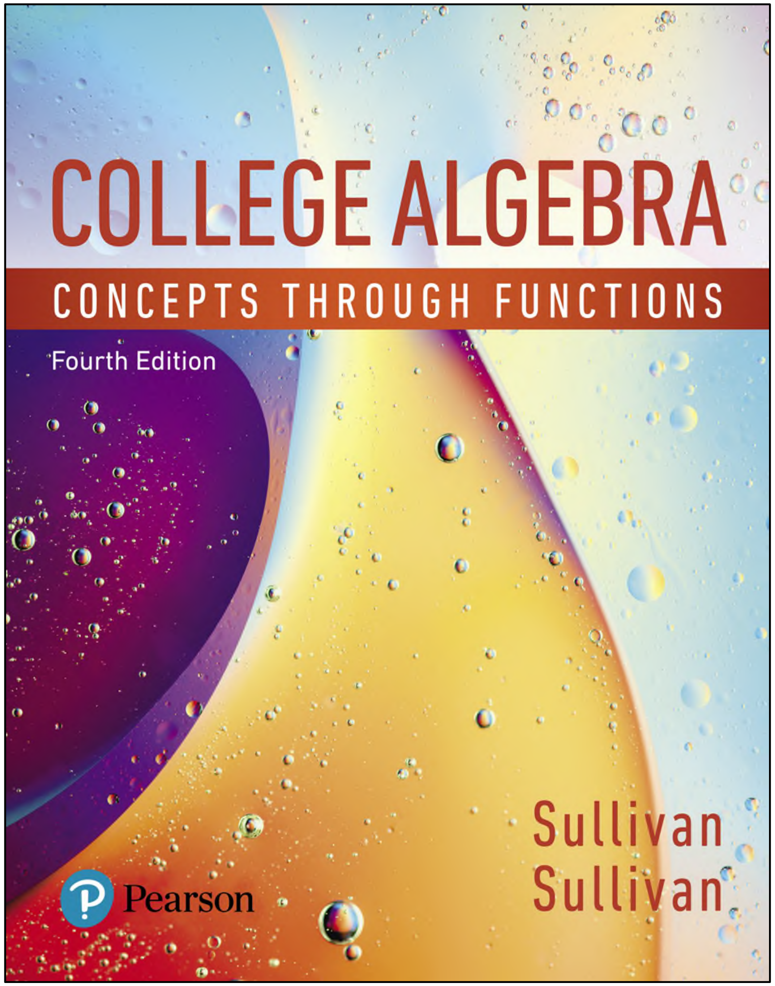
(h) Compute the ratio of the maximum height of the third bounce to the maximum height of the second bounce.

(i) Compute the ratio of the maximum height of the fourth bounce to the maximum height of the third bounce.

(j) Compare the results from parts (h) and (i). What do you conclude?

College Algebra: Concepts Through Functions

Fourth Edition



Chapter 2

Linear and Quadratic Functions

Section 2.1 Properties of Linear Functions and Linear Models

Objectives

1. Graph Linear Functions
2. Use Average Rate of Change to Identify Linear Functions
3. Determine Whether a Linear Function Is Increasing, Decreasing, or Constant
4. Find the Zero of a Linear Function
5. Build Linear Models from Verbal Descriptions

Objective 1 Graph Linear Functions

Definition

A **linear function** is a function of the form

$$f(x) = mx + b$$

The graph of a linear function is a line with slope *m* and *y*-intercept *b*. Its domain is the set of all real numbers.

Functions that are not linear are said to be **nonlinear**.

Example 1: Graphing a Linear Function (1 of 2)

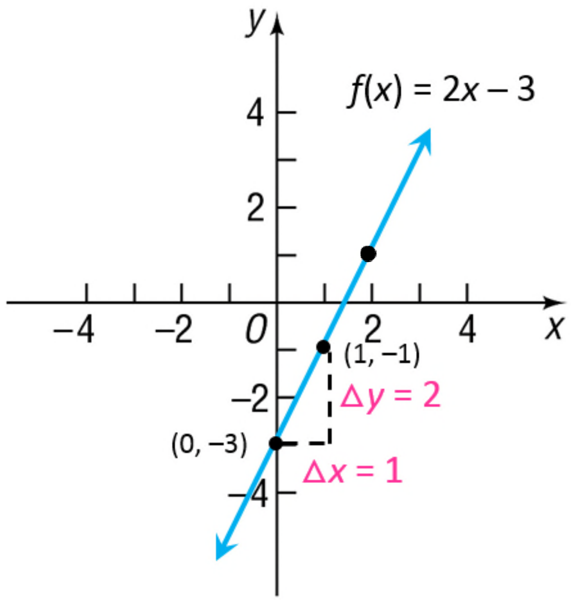
Graph the linear function $f(x) = 2x - 3.$ What are the domain and the range of *f* ?

Solution:

This is a linear function with slope *m* = 2 and *y*-intercept *b* = −3. To graph this function, plot the point (0, −3), the *y*-intercept, and use the slope to find an additional point by moving right 1 unit and up 2 units.

Example 1: Graphing a Linear Function (2 of 2)

The domain and range of *f* are each the set of all real numbers.



Alternatively, an additional point could have been found by evaluating the function at some $x \neq 0$. For $x = 2,\ f(2) = 2(2) - 3 = 1$ and the point (2, 1) lies on the graph.

Objective 2 Use Average Rate of Change to Identify Linear Functions

Theorem 1

Average Rate of Change of a Linear Function

Linear functions have a constant average rate of change. That is, the average rate of change of a linear function $f(x) = mx + b$ is

$$\frac{\Delta y}{\Delta x} = m$$

Example 2: Using the Average Rate of Change to Identify Linear Functions (1 of 4)

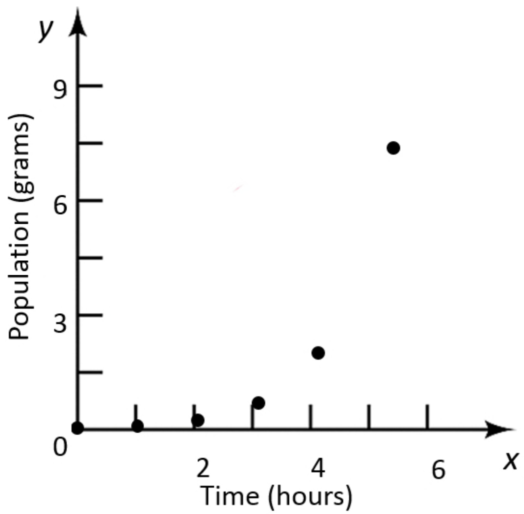
A certain bacteria is placed into a Petri dish at $30°C$ and allowed to grow. The data collected is shown below. The population is measured in grams and the time in hours.

Time (hours), *x*	Population (grams), *y*	(*x*, *y*)
0	0.01	(0, 0.01)
1	0.04	(1, 0.04)
2	0.15	(2, 0.15)
3	0.55	(3, 0.55)
4	2.13	(4, 2.13)
5	7.82	(5, 7.82)

Example 2: Using the Average Rate of Change to Identify Linear Functions (2 of 4)

Solution:

Plot the ordered pairs (x, y) in the Cartesian plane, and use the average rate of change to determine whether the function is linear.



Compute the average rate of change of the function. If the average rate of change is constant, the function is linear. If the average rate of change is not constant, the function is nonlinear.

Example 2: Using the Average Rate of Change to Identify Linear Functions (3 of 4)

The table displays the average rate of change in population.

Time (hours), x	Population (grams), y	Average Rate of Change = $\frac{\Delta y}{\Delta x}$
0	0.01	
		$\frac{0.44 - 0.01}{1 - 0} = 0.03$
1	0.04	
		0.11
2	0.15	
		0.40
3	0.55	
		1.58
4	2.13	
		5.69
5	7.82	

Example 2: Using the Average Rate of Change to Identify Linear Functions (4 of 4)

Because the average rate of change is not constant, the function is not linear. In fact, because the average rate of change is increasing as the value of the independent variable increases, the function is increasing at an increasing rate. So not only is the population increasing over time, but it is also growing more rapidly as time passes.

Objective 3 Determine Whether a Linear Function Is Increasing, Decreasing, or Constant

Theorem 2

Increasing, Decreasing, and Constant Linear Functions

A linear function $f(x) = mx + b$ is increasing over its domain if its slope, *m*, is positive. It is decreasing over its domain if its slope, *m*, is negative. It is constant over its domain if its slope, *m*, is zero.

Example 3: Determining Whether a Linear Function Is Increasing, Decreasing, or Constant

Determine whether the following linear functions are increasing, decreasing, or constant.

(a) $f(x) = 3x - 5$ (b) $g(x) = -6x + 1$

Solution:

(a) For the linear function $f(x) = 3x - 5$, the slope is 3, which is positive. The function *f* is increasing on the interval $(-\infty, \infty)$.

(b) For the linear function $g(x) = -6x + 1$, the slope is −6, which is negative. The function *g* is decreasing on the interval $(-\infty, \infty)$.

Objective 4 Find the Zero of a Linear Function

Example 4: Find the Zero of a Linear Function (1 of 4)

(a) Does $f(x)=4x-8$ have a zero?

(b) If it does, find the zero.

(c) Use the zero along with the *y*-intercept to graph *f*.

(d) Solve $f(x)>0$.

Solution:

(a) The linear function *f* is increasing, so it has one zero.

Example 4: Find the Zero of a Linear Function (2 of 4)

Solution:

(b) Solve the equation $f(x) = 0$ to find the zero.

$$f(x) = 0$$

$$4x - 8 = 0 \qquad f(x) = 4x - 8$$

$$4x = 8 \qquad \text{Add 8 to both sides of the equation.}$$

$$x = 2 \qquad \text{Divide both sides by 4.}$$

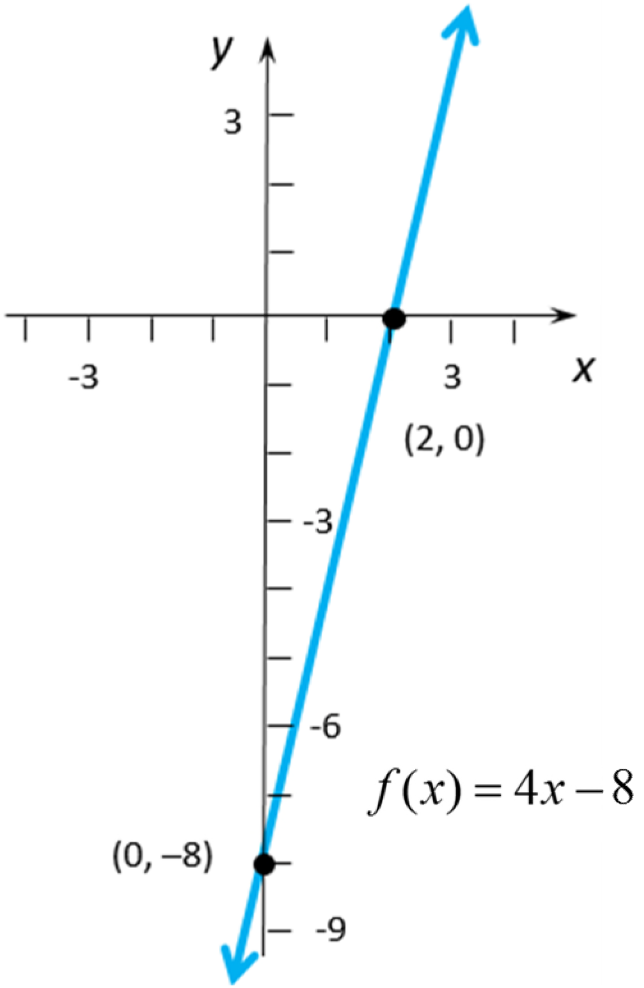
Check: $f(2) = 4(2) - 8 = 0$

The zero of *f* is 2.

Example 4: Find the Zero of a Linear Function (3 of 4)

Solution:

(c) The *y*-intercept is −8, so the point (0, −8) is on the graph of the function. The zero of *f* is 2, so the *x*-intercept is 2 and the point (2, 0) is on the graph.



Example 4: Find the Zero of a Linear Function (4 of 4)

Solution:

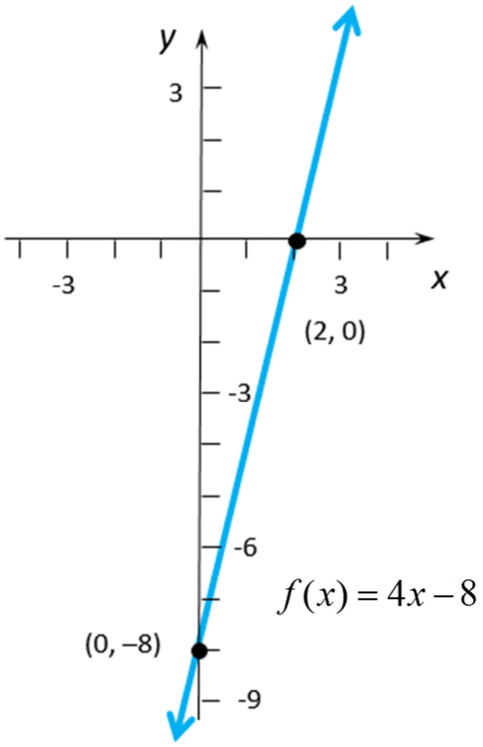
(d) Notice that $f(x) > 0$ when $x > 2$.

$$f(x) > 0$$

$$4x - 8 > 0 \qquad f(x) = 4x - 8$$

$$4x > 8 \qquad \text{Add 8 to both sides of the equation.}$$

$$x > 2 \qquad \text{Divide both sides by 4.}$$



Objective 5 Build Linear Models from Verbal Descriptions

Modeling with a Linear Function

If the average rate of change of a function is a constant *m*, a linear function *f* can be used to model the relation between the two variables as follows:

$$f(x) = mx + b$$

where *b* is the value of *f* at 0; that is, $b = f(0)$.

Example 5: Straight-Line Depreciation (1 of 6)

Book value is the value of an asset that a company uses to create its balance sheet. Some companies depreciate their assets using straight-line depreciation so that the value of the asset declines by a fixed amount each year. The amount of the decline depends on the useful life that the company assigns to the asset. Suppose that your electric company just purchased a fleet of new trucks for its work force at a cost of $35,000 per truck. Your company chooses to depreciate each vehicle using the straight-line method over 7 years. This means that each truck will depreciate by

$$\frac{\$35,000}{7} = \$5,000 \text{ per year.}$$

Example 5: Straight-Line Depreciation (2 of 6)

(a) Write a linear function that expresses the book value *V* of each truck as a function of its age, *x*.

(b) Graph the linear function.

(c) What is the book value of each truck after 4 years?

(d) Interpret the slope.

(e) When will the book value of each truck be $10,000?

Example 5: Straight-Line Depreciation (3 of 6)

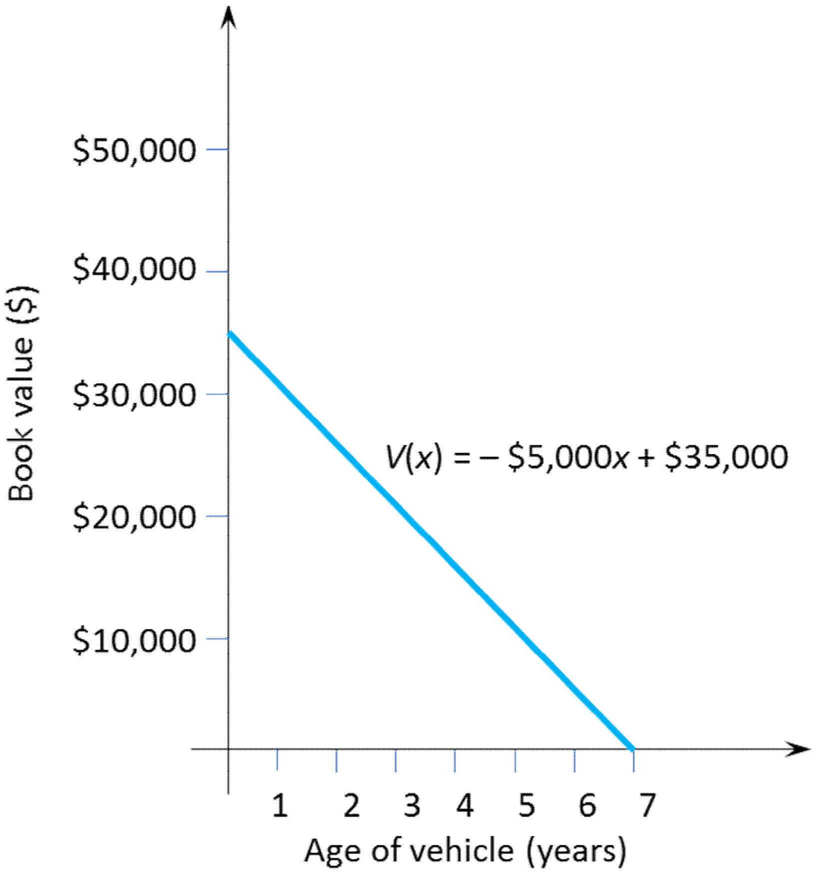
Solution:

(a) If we let $V(x)$ represent the value of each truck after *x* years, then $V(0)$ represents the original value of each truck, so $V(0) = \$35,000$. The *y*-intercept of the linear function is \$35,000. Because each truck depreciates by \$5,000 per year, the slope of the linear function is −5,000. The linear function that represents the book value *V* of each truck after *x* years is

$$V(x) = -5000x + 35,000$$

Example 5: Straight-Line Depreciation (4 of 6)

(b) Graph the function.



Example 5: Straight-Line Depreciation (5 of 6)

(c) The book value of each truck after 4 years is

$$V(4) = -5,000(4) + 35,000$$
$$= \$15,000$$

(d) Since the slope of $V(x) = -5,000x + 35,000$ is −5,000, the average rate of change of the book value is −$5,000/year. So for each additional year that passes, the book value of the truck decreases by $5,000.

Example 5: Straight-Line Depreciation (6 of 6)

(e) To find when the book value will be $10,000, solve the equation

$$V(x) = 10,000$$

$$-5,000x + 35,000 = 10,000$$

$$-5,000x = -25,000$$

Subtract 35,000 from each side.

$$x = \frac{-25,000}{-5,000} = 5$$

Divide by 5,000.

Each truck will have a book value of $10,000 when it is 5 years old.

Example 6: Supply and Demand (1 of 5)

The quantity supplied of a good is the amount of a product that a company is willing to make available for sale at a given price. The quantity demanded of a good is the amount of a product that consumers are willing to purchase at a given price. Suppose that the quantity supplied, *S*, and quantity demanded, *D*, of cell phones each month are given by the following functions:

$$S(p) = 25p - 680$$
$$D(p) = -7p + 3000$$

where *p* is the price (in dollars) of the cell phone.

Example 6: Supply and Demand (2 of 5)

(a) The equilibrium price of a product is defined as the price at which quantity supplied equals quantity demanded. That is, the equilibrium price is the price at which $S(p) = D(p)$. Find the equilibrium price of cell phones. What is the equilibrium quantity, the amount demanded (or supplied) at the equilibrium price?

(b) Determine the prices for which quantity supplied is greater than quantity demanded. That is, solve the inequality $S(p) > D(p)$.

(c) Graph $S = S(p), D = D(p)$ and label the equilibrium point.

Example 6: Supply and Demand (3 of 5)

Solution:

(a) To find the equilibrium price, solve the equation $S(p) = D(p).$

$$25p - 680 = -7p + 3000$$
$$25p = -7p + 3680$$
$$32p = 3680$$
$$p = 115$$

The equilibrium price is \$115 per cell phone. To find the equilibrium quantity, evaluate either $S(p)$ or $D(p)$ at $p = 115.$

$$S(115) = 25(115) - 680 = 2195$$

The equilibrium quantity is 2195 cell phones.

Example 6: Supply and Demand (4 of 5)

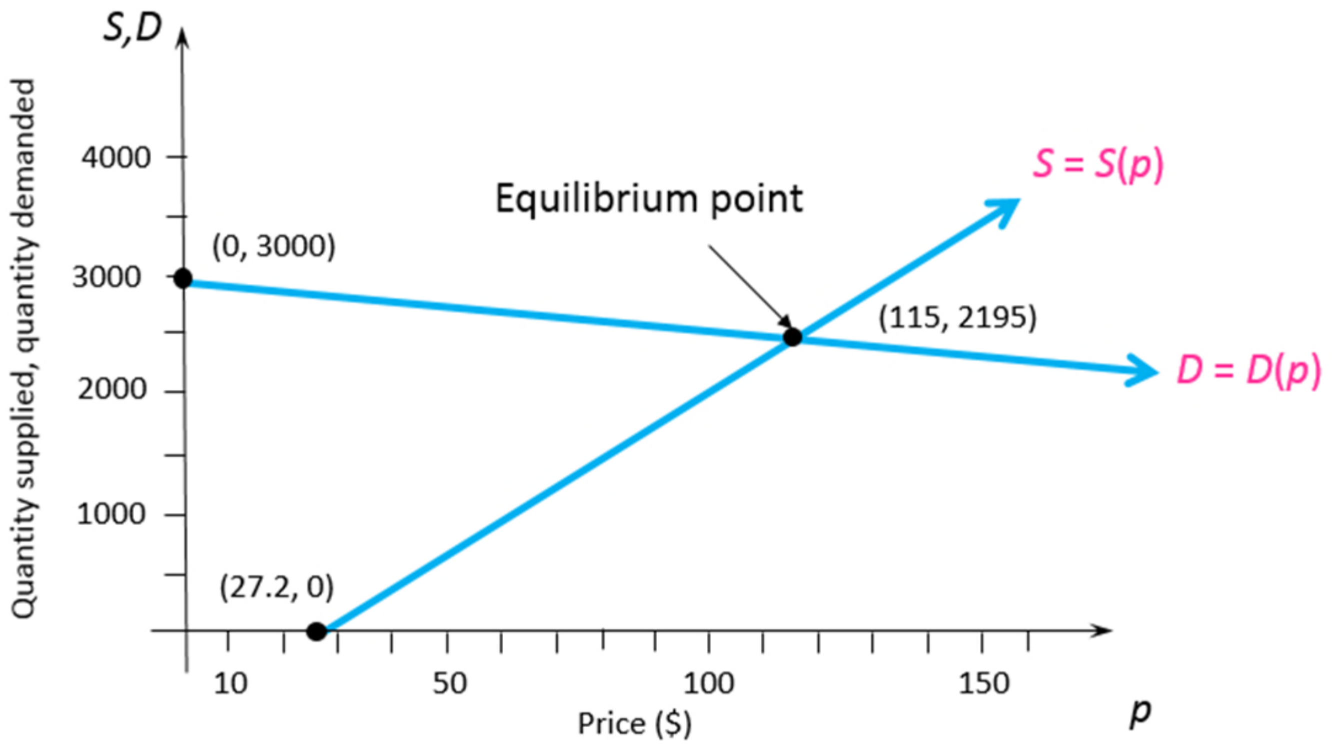
(b) The inequality $S(p) > D(p)$ is

$$25p - 680 > -7p + 3000$$
$$25p > -7p + 3680$$
$$32p > 3680$$
$$p > 115$$

If the company charges more than $115 per phone, quantity supplied will exceed quantity demanded. In this case the company will have excess phones in inventory.

Example 6: Supply and Demand (5 of 5)

(c) The graph shows $S = S(p)$ and $D = D(p)$ with the equilibrium points labeled.



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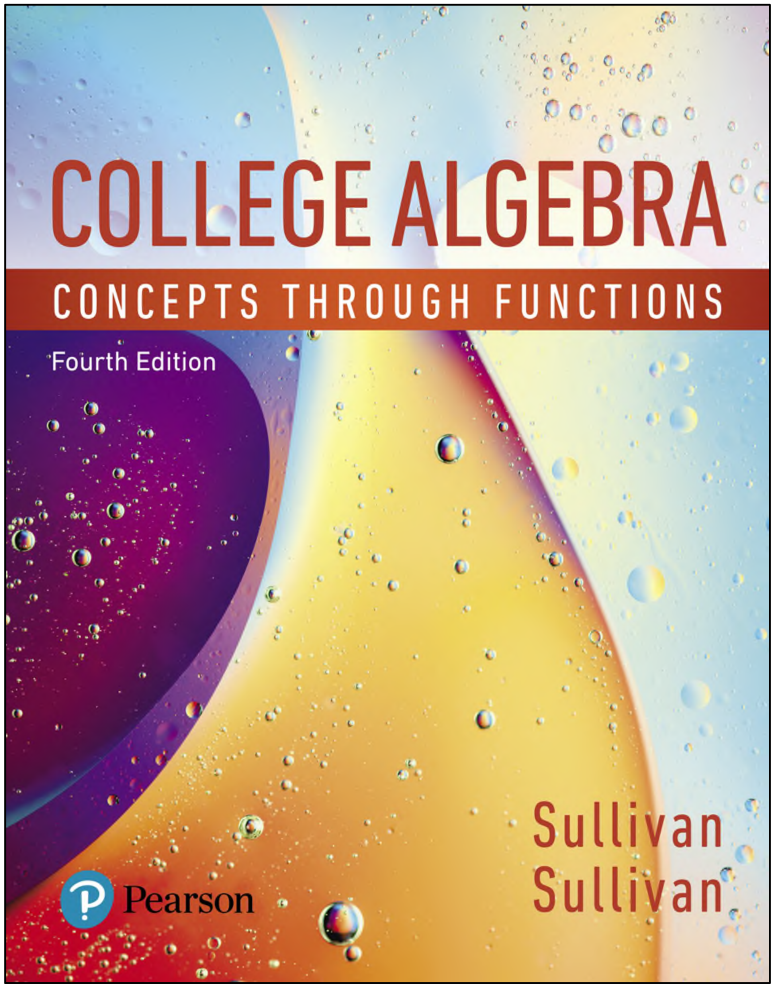


Chapter 2

Linear and Quadratic Functions

Pearson

Section 2.2 Building Linear Models from Data

Objectives

1. Draw and Interpret Scatter Diagrams
2. Distinguish between Linear and Nonlinear Relations
3. Use a Graphing Utility to Find the Line of Best Fit

Objective 1 Draw and Interpret Scatter Diagrams

Example 1: Drawing and Interpreting a Scatter Diagram (1 of 5)

In baseball, the on-base percentage for a team represents the percentage of time that the players safely reach base. The data given represent the number of runs scored *y* and the on-base percentage *x* for teams in the National League during the 2014 baseball season.

Example 1: Drawing and Interpreting a Scatter Diagram (2 of 5)

Team	On-base Percentage, *x*	Runs Scored, *y*	(*x*, *y*)
Arizona	30.2	615	(30.2, 615)
Atlanta	30.5	573	(30.5, 573)
Chicago Cubs	30.0	614	(30.0, 614)
Cincinnati	29.6	595	(29.6, 595)
Colorado	32.7	755	(32.7, 755)
LA Dodgers	33.3	718	(33.3, 718)
Miami	31.7	645	(31.7, 645)
Milwaukee	31.1	650	(31.1, 650)
NY Mets	30.8	629	(30.8, 629)
Philadelphia	30.2	619	(30.2, 619)
Pittsburgh	33.0	682	(33.0, 682)
San Diego	29.2	535	(29.2, 535)
San Francisco	31.1	665	(31.1, 665)
St. Louis	32.0	619	(32.0, 619)
Washington	32.1	686	(32.1, 686)

Source: espn.go.com

Example 1: Drawing and Interpreting a Scatter Diagram (3 of 5)

(a) Draw a scatter diagram of the data, treating on-base percentage as the independent variable.

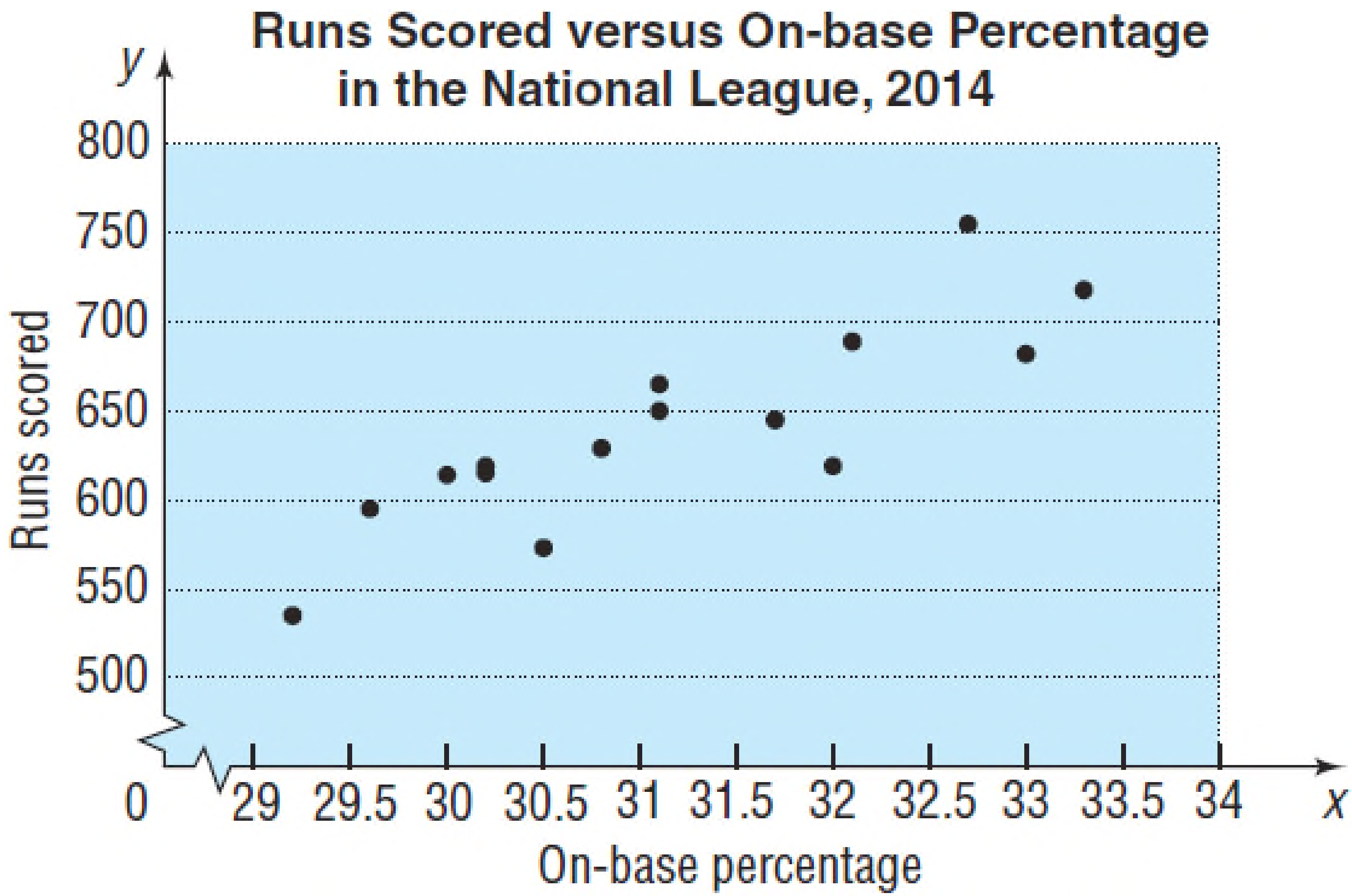
(b) Use a graphing utility to draw a scatter diagram.

(c) Describe what happens to runs scored as the on-base percentage increases.

Solution:

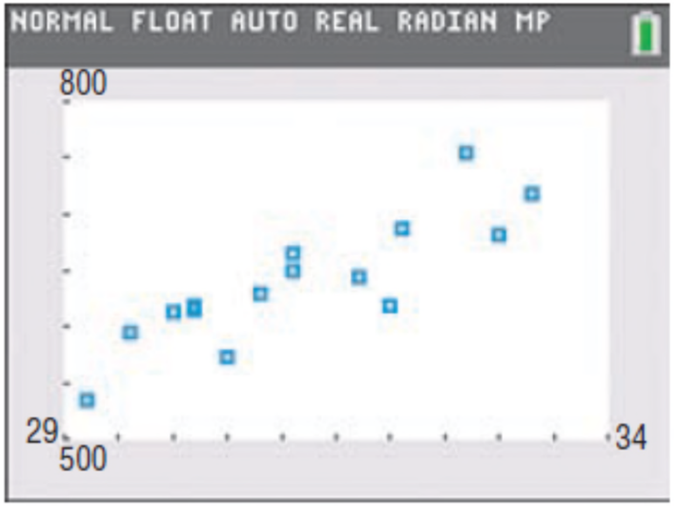
(a) To draw a scatter diagram, plot the ordered pairs, with the on-base percentage as the *x*-coordinate and the runs scored as the *y*-coordinate. Notice that the points in the scatter diagram are not connected.

Example 1: Drawing and Interpreting a Scatter Diagram (4 of 5)



Example 1: Drawing and Interpreting a Scatter Diagram (5 of 5)

(b) The scatter diagram using a TI-84 Plus C graphing calculator.



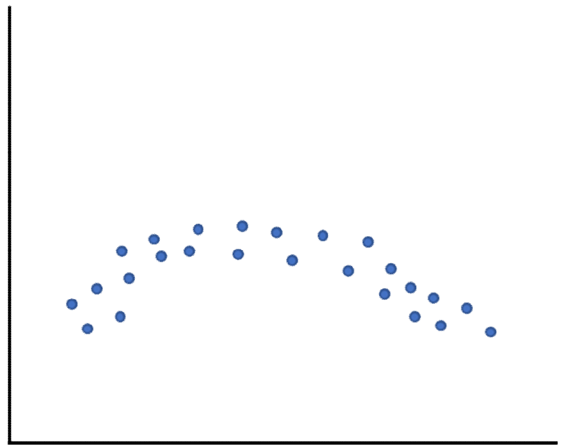
(c) The scatter diagram shows that as the on-base percentage increases, the number of runs scored also increases.

Objective 2 Distinguish Between Linear and Nonlinear Relations

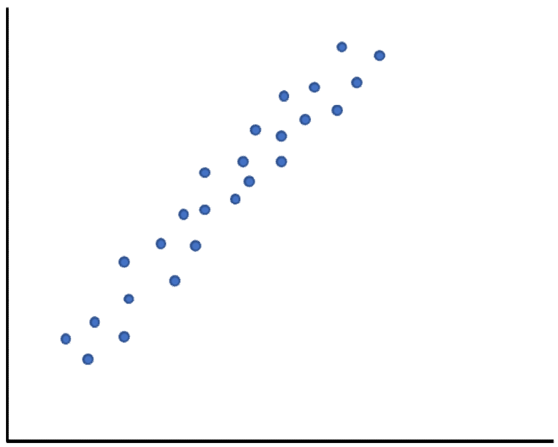
Example 2: Distinguishing Between Linear and Nonlinear Relations

Determine whether the relation between the two variables in each scatter diagram is linear or nonlinear.

(a)



(b)



Solution:

(a) Nonlinear

(b) Linear

Example 3: Finding a Model for Linearly Related Data (1 of 5)

Use the data from a previous example (which is shown on the next slide again).

(a) Select two points and find an equation of the line containing the points.

(b) Graph the line on the scatter diagram obtained in the previous example.

Example 3: Finding a Model for Linearly Related Data (2 of 5)

Team	**On-base Percentage, *x***	**Runs Scored, *y***	**(*x*, *y*)**
Arizona	30.2	615	(30.2, 615)
Atlanta	30.5	573	(30.5, 573)
Chicago Cubs	30.0	614	(30.0, 614)
Cincinnati	29.6	595	(29.6, 595)
Colorado	32.7	755	(32.7, 755)
LA Dodgers	33.3	718	(33.3, 718)
Miami	31.7	645	(31.7, 645)
Milwaukee	31.1	650	(31.1, 650)
NY Mets	30.8	629	(30.8, 629)
Philadelphia	30.2	619	(30.2, 619)
Pittsburgh	33.0	682	(33.0, 682)
San Diego	29.2	535	(29.2, 535)
San Francisco	31.1	665	(31.1, 665)
St. Louis	32.0	619	(32.0, 619)
Washington	32.1	686	(32.1, 686)

Source: espn.go.com

Example 3: Finding a Model for Linearly Related Data (3 of 5)

Solution:

(a) Select two points, say (30.8, 629) and (32.1, 686). The slope of the line joining the points (30.8, 629) and (32.1, 686) is

$$m = \frac{686 - 629}{32.1 - 30.8} = \frac{57}{1.3} \approx 43.85$$

Example 3: Finding a Model for Linearly Related Data (4 of 5)

(a) The equation of the line with slope 43.85 and passing through (30.8, 629) is found using the point–slope form with $m = 43.85$, $x_1 = 30.8$, and $y_1 = 629$.

$$y - y_1 = m(x - x_1)$$

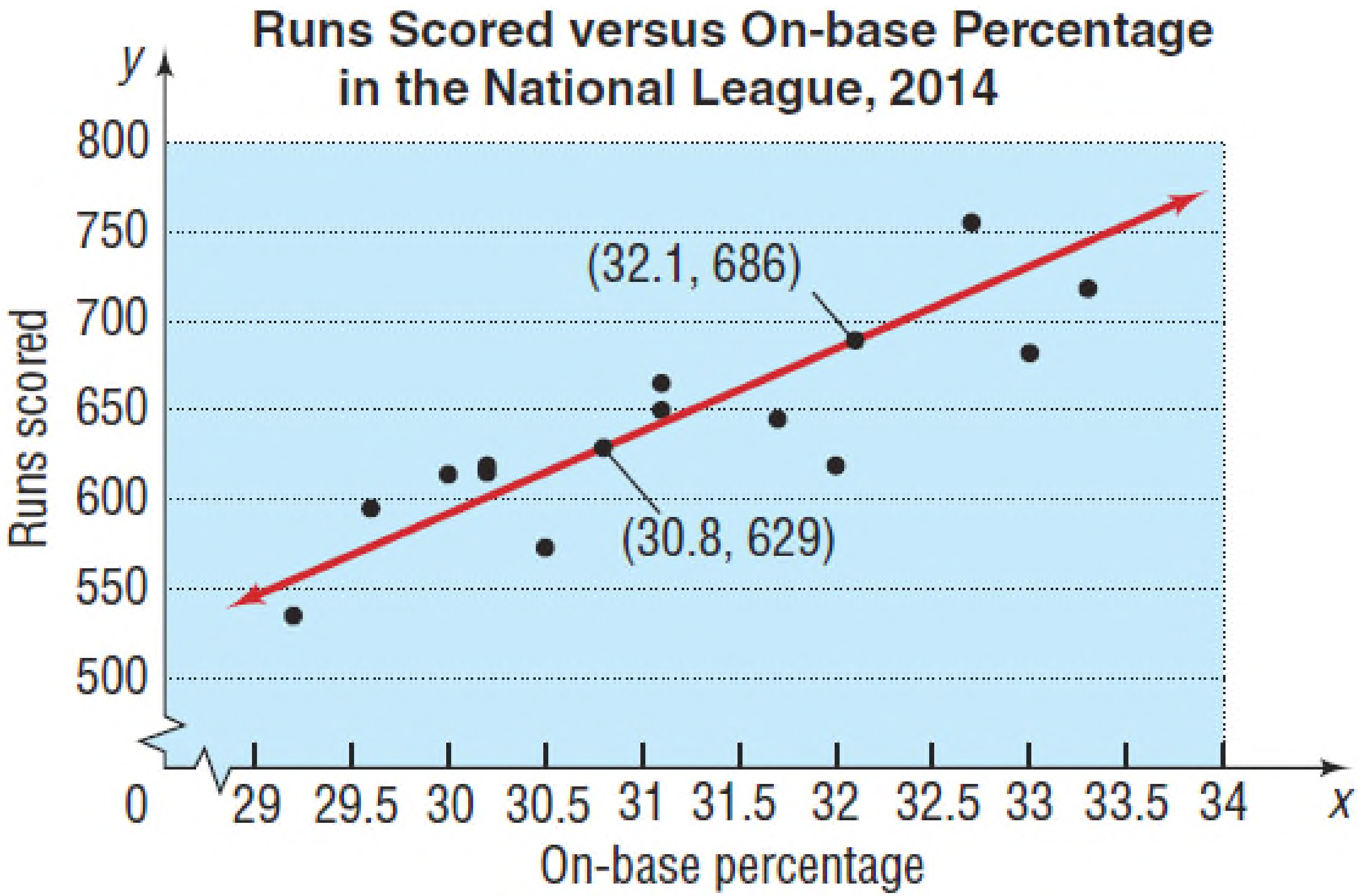
$$y - 629 = 43.85(x - 30.8)$$

$$y - 629 = 43.85x - 1350.58$$

$$y = 43.85x - 721.58$$

Example 3: Finding a Model for Linearly Related Data (5 of 5)

(b)



Objective 3 Use a Graphing Utility to Find the Line of Best Fit

Example 4: Finding a Model for Linearly Related Data (1 of 6)

Team	On-base Percentage, *x*	Runs Scored, *y*	(*x*, *y*)
Arizona	30.2	615	(30.2, 615)
Atlanta	30.5	573	(30.5, 573)
Chicago Cubs	30.0	614	(30.0, 614)
Cincinnati	29.6	595	(29.6, 595)
Colorado	32.7	755	(32.7, 755)
LA Dodgers	33.3	718	(33.3, 718)
Miami	31.7	645	(31.7, 645)
Milwaukee	31.1	650	(31.1, 650)
NY Mets	30.8	629	(30.8, 629)
Philadelphia	30.2	619	(30.2, 619)
Pittsburgh	33.0	682	(33.0, 682)
San Diego	29.2	535	(29.2, 535)
San Francisco	31.1	665	(31.1, 665)
St. Louis	32.0	619	(32.0, 619)
Washington	32.1	686	(32.1, 686)

Source: espn.go.com

Example 4: Finding a Model for Linearly Related Data (2 of 6)

Use the data from the previous example, on the previous slide.

(a) Use a graphing utility to find the line of best fit that models the relation between on-base percentage and runs scored.

(b) Graph the line of best fit on the scatter diagram obtained in the first example.

(c) Interpret the slope.

(d) Use the line of best fit to predict the number of runs a team will score if their on-base percentage is 31.5.

Example 4: Finding a Model for Linearly Related Data (3 of 6)

Solution:

(a) Graphing utilities contain built-in programs that find the line of best fit for a collection of points in a scatter diagram. Executing the LINear REGression program provides the results shown on the next slide. This output shows the equation $y = ax + b,$ where *a* is the slope of the line and *b* is the *y*-intercept. The line of best fit that relates on-base percentage to runs scored may be expressed as the line

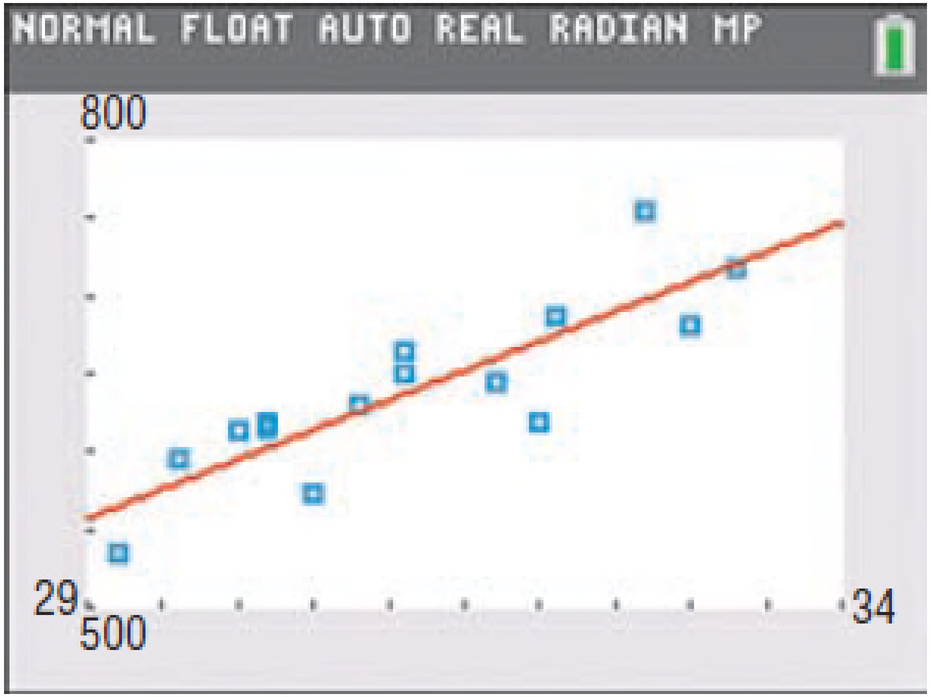
$$y = 38.02x - 544.86$$

Example 4: Finding a Model for Linearly Related Data (4 of 6)

(a) $y = 38.02x - 544.86$

Example 4: Finding a Model for Linearly Related Data (5 of 6)

(b) The graph of the line of best fit, along with the scatter diagram.



Example 4: Finding a Model for Linearly Related Data (6 of 6)

(c) The slope of the line of best fit is 38.02, which means that for every 1 percent increase in the on-base percentage, runs scored increase 38.02, on average.

(d) Letting $x = 31.5$ in the equation of the line of best fit, we obtain $y = 38.02(31.5) - 544.86 \approx 653$ runs.

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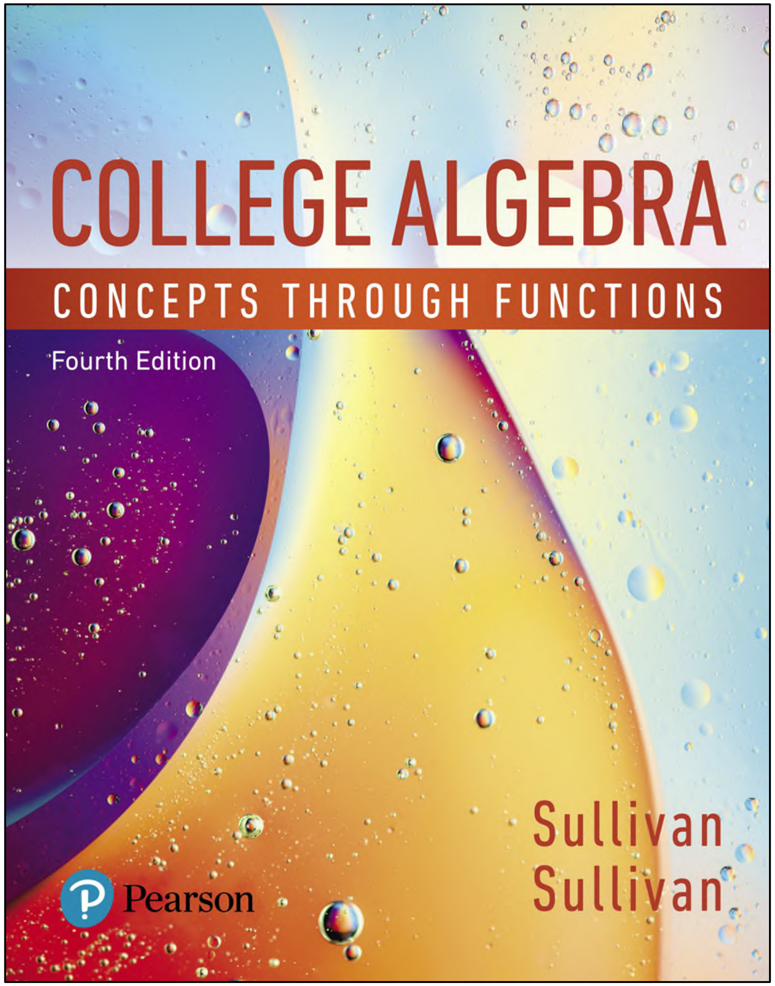


Chapter 2

Linear and Quadratic Functions

Section 2.3 Quadratic Functions and Their Zeros

Objectives (1 of 2)

1. Find the Zeros of a Quadratic Function by Factoring
2. Find the Zeros of a Quadratic Function Using the Square Root Method
3. Find the Zeros of a Quadratic Function by Completing the Square
4. Find the Zeros of a Quadratic Function Using the Quadratic Formula

Objectives (2 of 2)

5. Find the Point of Intersection of Two Functions
6. Solve Equations That Are Quadratic in Form

Quadratic Function

A **quadratic function** is a function of the form

$$f(x) = ax^2 + bx + c$$

where *a*, *b*, and *c* are real numbers and $a \neq 0$. The domain of a quadratic function consists of all real numbers.

Finding the zeros, or *x*-intercepts of the graph, of a quadratic function $f(x) = ax^2 + bx + c$ requires solving the equation $ax^2 + bx + c = 0$. These types of equations are called **quadratic equations.**

Quadratic Equation (1 of 2)

A **quadratic equation** is an equation equivalent to one of the form

$$ax^2 + bx + c = 0 \qquad \textbf{(1)}$$

where *a*, *b*, and *c* are real numbers and $a \neq 0$.

A quadratic equation written in the form $ax^2 + bx + c = 0$ is in **standard form.** Sometimes a quadratic equation is called a **second-degree equation** because, when it is in standard form, the left side is a polynomial of degree 2.

Quadratic Equation (2 of 2)

A **quadratic equation** is an equation equivalent to one of the form

$$ax^2 + bx + c = 0 \qquad \textbf{(1)}$$

where *a*, *b*, and *c* are real number and $a \neq 0$.

Four methods of solving quadratic equations:

- factoring
- the square root method
- completing the square
- the quadratic formula

Objective 1 Find the Zeros of a Quadratic Function by Factoring

Example 1: Finding the Zeros of a Quadratic Function by Factoring (1 of 2)

Find the zeros of $f(x) = 10x^2 + x - 3.$ List any *x*-intercepts of the graph of *f*.

Solution:

To find the zeros of a function *f*, solve the equation $f(x) = 0.$

$$f(x) = 0$$

$$10x^2 + x - 3 = 0$$

$$(2x-1)(5x+3) = 0$$

$$2x - 1 = 0 \quad \text{or} \quad 5x + 3 = 0$$

$$2x = 1 \quad \text{or} \quad 5x = -3$$

$$x = \frac{1}{2} \quad \text{or} \quad x = -\frac{3}{5}$$

Example 1: Finding the Zeros of a Quadratic Function by Factoring (2 of 2)

Solution:

Check: $f(x) = 10x^2 + x - 3.$

$$x = \frac{1}{2}: f\left(\frac{1}{2}\right) = 10\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) - 3$$

$$= \frac{10}{4} + \frac{1}{2} - 3$$

$$= 0$$

$$x = -\frac{3}{5}: f\left(-\frac{3}{5}\right) = 10\left(-\frac{3}{5}\right)^2 + \left(-\frac{3}{5}\right) - 3$$

$$= \frac{90}{25} - \frac{3}{5} - 3$$

$$= 0$$

The zeros of $f(x) = 10x^2 + x - 3$ are $-\frac{3}{5}$ and $\frac{1}{2}$.

The x-intercepts of the graph of $f(x) = 10x^2 + x - 3$ are $-\frac{3}{5}$ and $\frac{1}{2}$.

Multiplicity

Sometimes, the quadratic expression factors into two linear equations with the same solution. When this happens, the quadratic equation is said to have a **repeated solution.** This solution is called a **root of multiplicity 2,** or a **double root.**

Example 2: Finding the Zeros of a Quadratic Function by Factoring (1 of 2)

Find the zeros of $f(x) = 4x^2 + 12x + 9$. List any *x*-intercepts of the graph of *f*.

Solution:

To find the zeros of a function *f*, solve the equation $f(x) = 0$.

$$f(x) = 0$$

$$4x^2 + 12x + 9 = 0$$

$$(2x+3)(2x+3) = 0$$

$$2x + 3 = 0 \quad \text{or} \quad 2x + 3 = 0$$

$$2x = 3 \quad \text{or} \quad 2x = 3$$

$$x = \frac{2}{3} \quad \text{or} \quad x = \frac{2}{3}$$

Example 2: Finding the Zeros of a Quadratic Function by Factoring (2 of 2)

Solution:

The only zero of $f(x) = 4x^2 + 12x + 9$ is $\frac{2}{3}$.

The only *x*-intercept of the graph of

$f(x) = 4x^2 + 12x + 9$ is $\frac{2}{3}$.

Objective 2 Find the Zeros of a Quadratic Function Using the Square Root Method

The Square Root Method

If $x^2 = p$ and $p \geq 0$, then $x = \sqrt{p}$ or $x = -\sqrt{p}$. **(3)**

Using statement (3) to solve a quadratic equation is called the **Square Root Method.**

Note that if $p > 0$, the equation $x^2 = p$ has two solutions, $x = \sqrt{p}$ and $x = -\sqrt{p}$.

Usually these solutions are abbreviated as $x = \pm\sqrt{p}$

Example 3: Finding the Zeros of a Quadratic Function Using the Square Root Method (1 of 3)

Find the zeros of each function.

(a) $f(x) = x^2 - 20$ (b) $f(x) = (x + 3)^2 - 25$

List any *x*-intercepts of the graph.

Solution:

(a) To find the zeros of a function *f*, solve the equation $f(x) = 0.$

$$f(x) = 0$$
$$x^2 - 20 = 0$$
$$x^2 = 20$$
$$x = \pm\sqrt{20}$$
$$x = \pm 2\sqrt{5}$$
$$x = 2\sqrt{5} \quad \text{or} \quad x = -2\sqrt{5}$$

Example 3: Finding the Zeros of a Quadratic Function Using the Square Root Method (2 of 3)

Solution:

(a) The zeros of $f(x) = x^2 - 20$ are $-2\sqrt{5}$ and $2\sqrt{5}.$ The *x*-intercepts of the graph of $f(x) = x^2 - 20$ are $-2\sqrt{5}$ and $2\sqrt{5}.$

Example 3: Finding the Zeros of a Quadratic Function Using the Square Root Method (3 of 3)

Find the zeros of each function.

(b) $f(x) = (x+3)^2 - 25$

Solution:

(b) To find the zeros of a function *f*, solve the equation $f(x) = 0$.

$$f(x) = 0$$
$$(x+3)^2 - 25 = 0$$
$$(x+3)^2 = 25$$
$$x+3 = \pm\sqrt{25}$$
$$x+3 = \pm 5$$
$$x+3 = 5 \text{ or } x+3 = -5$$
$$x = 2 \text{ or } x = -8$$

The zeros of $f(x) = (x+3)^2 - 25$ are −8 and 2.

The *x*-intercepts of the graph of $f(x) = (x+3)^2 - 25$ are −8 and 2.

Objective 3 Find the Zeros of a Quadratic Function by Completing the Square

Example 4: Finding the Zeros of a Quadratic Function by Completing the Square (1 of 2)

Find the zeros of $f(x) = x^2 - 6x + 8$ by completing the square. List any *x*-intercepts of the graph of *f*.

Solution:

To find the zeros of a function *f*, solve the equation $f(x) = 0$.

$$f(x) = 0$$

$$x^2 - 6x + 8 = 0$$

$$x^2 - 6x = -8 \quad \left(\frac{1}{2}\cdot 6\right)^2 = 9$$

$$x^2 - 6x + 9 = -8 + 9$$

$$(x-3)^2 = 1$$

$$x - 3 = \pm\sqrt{1}$$

$$x - 3 = \pm 1$$

$$x - 3 = -1 \text{ or } x - 3 = 1$$

$$x = 2 \text{ or } x = 4$$

Example 4: Finding the Zeros of a Quadratic Function by Completing the Square (2 of 2)

Solution:

The zeros of $f(x) = x^2 - 6x + 8$ are 2 and 4.

The *x*-intercepts of the graph of $f(x) = x^2 - 6x + 8$ are 2 and 4.

Objective 4 Find the Zeros of a Quadratic Function Using the Quadratic Formula

Quadratic Formula

Consider the quadratic equation

$$ax^2 + bx + c = 0, \quad a \neq 0$$

If $b^2 - 4ac < 0$, this equation has no real solution.

If $b^2 - 4ac \geq 0$, the real solution(s) of this equation is (are) given by the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant of a Quadratic Equation

The quantity $b^2 - 4ac$ is called the **discriminant** of the quadratic equation.

For a quadratic equation $ax^2 + bx + c = 0$:

1. If $b^2 - 4ac > 0$, there are two unequal real solutions. Therefore, the graph of $f(x) = ax^2 + bx + c$ will have two distinct *x*-intercepts.
2. If $b^2 - 4ac = 0$, there is a repeated real solution, a root of multiplicity 2. Therefore, the graph of $f(x) = ax^2 + bx + c$ will have one *x*-intercept, at which the graph touches the *x*-axis.
3. If $b^2 - 4ac < 0$, there is no real solution. Therefore, the graph of $f(x) = ax^2 + bx + c$ will have no *x*-intercept.

Example 5: Finding the Zeros of a Quadratic Function Using the Quadratic Formula (1 of 2)

Find the real zeros, if any, of the function $f(x) = 2x^2 - 3x - 4$. List any *x*-intercepts of the graph of *f*.

Solution:

To find the real zeros, solve the equation $f(x) = 0$.

Compare $f(x) = 2x^2 - 3x - 4$ to $f(x) = ax^2 + bx + c$ to find *a*, *b*, and *c*.

$$a = 2,\ b = -3,\ c = -4$$

With *a* = 2, *b* = −3, *c* = −4, evaluate the discriminant $b^2 - 4ac$.

$$b^2 - 4ac = (-3)^2 - 4(2)(-4) = 9 + 32 = 41$$

Since $b^2 - 4ac > 0$, there are two unequal real solutions.

Example 5: Finding the Zeros of a Quadratic Function Using the Quadratic Formula (2 of 2)

Use the quadratic formula with *a* = 2, *b* = −3, *c* = −4, and $b^2 - 4ac = 41.$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{41}}{2(2)} = \frac{3 \pm \sqrt{41}}{4}$$

The real zeros of $f(x) = 2x^2 - 3x - 4$ are $\frac{3+\sqrt{41}}{4}$ and $\frac{3-\sqrt{41}}{4}$.

The *x*-intercepts of the graph of $f(x) = 2x^2 - 3x - 4$ are $\frac{3+\sqrt{41}}{4}$ and $\frac{3-\sqrt{41}}{4}$.

Example 6: Finding the Zeros of a Quadratic Function Using the Quadratic Formula (1 of 2)

Find the real zeros, if any, of the function $f(x) = 2x^2 - 3x + 4.$ List any *x*-intercepts of the graph of *f*.

Solution:

To find the real zeros, solve the equation $f(x) = 0.$

Compare $f(x) = 2x^2 - 3x + 4$ to $f(x) = ax^2 + bx + c$ to find *a*, *b*, and *c*.

$$a = 2,\ b = -3,\ c = 4$$

With $a = 2$, $b = -3$, $c = 4$, evaluate the discriminant $b^2 - 4ac.$

$$b^2 - 4ac = (-3)^2 - 4(2)(4) = 9 - 32 = -23$$

Since $b^2 - 4ac < 0,$ there are no real solutions.

Example 6: Finding the Zeros of a Quadratic Function Using the Quadratic Formula (2 of 2)

Solution:

Therefore the function $f(x) = 2x^2 - 3x + 4$ has no real zeros.

The graph of $f(x) = 2x^2 - 3x + 4$ has no *x*-intercept.

Summary

Finding the Zeros, or *x*-intercepts of the Graph, of a Quadratic Function

Step 1: Given that $f(x) = ax^2 + bx + c$, write the equation $ax^2 + bx + c = 0$.

Step 2: Identify *a*, *b*, and *c*.

Step 3: Evaluate the discriminant, $b^2 - 4ac$.

Step 4: (a) If the discriminant is negative, the equation has no real solution, so the function has no real zeros. The graph of the function has no *x*-intercepts.

(b) If the discriminant is nonnegative (greater than or equal to zero), determine whether the left side can be factored. If you can easily spot factors, use the factoring method to solve the equation. Otherwise, use the quadratic formula or the method of completing the square. The solution(s) to the equation is(are) the real zero(s) of the function. The solution(s) also represent(s) the *x*-intercept(s) of the graph of the function.

Objective 5 Find the Point of Intersection of Two Functions

The Point of Intersection

If *f* and *g* are two functions, each solution to the equation $f(x) = g(x)$ has a geometric interpretation—it represents the *x*-coordinate of each point where the graphs of $y = f(x)$ and $y = g(x)$ intersect.

Example 7: Finding the Points of Intersection of the Graphs of Two Functions (1 of 3)

If $f(x) = x^2 + 7x + 1$ and $g(x) = 4x + 11$, solve $f(x) = g(x)$. At what point(s) do the graphs of the two functions intersect?

Solution:

$$f(x) = g(x)$$

$$x^2 + 7x + 1 = 4x + 11$$

$$x^2 + 3x - 10 = 0$$

$$(x+5)(x-2) = 0$$

$$x + 5 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -5 \quad \text{or} \quad x = 2$$

Example 7: Finding the Points of Intersection of the Graphs of Two Functions (2 of 3)

Solution:

The *x*-coordinates of the points of intersection are −5 and 2. To find the *y*-coordinates of the points of intersection, evaluate either *f* or *g* at *x* = −5 and *x* = 2.

$$g(x) = 4x + 11$$

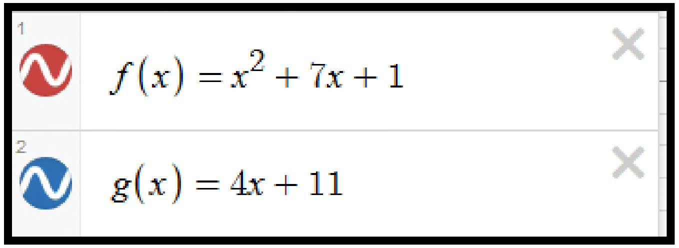
$$x = -5:\ g(-5) = 4(-5) + 11 \qquad x = 2:\ g(2) = 4(2) + 11$$
$$= -9 \qquad\qquad\qquad\qquad = 19$$

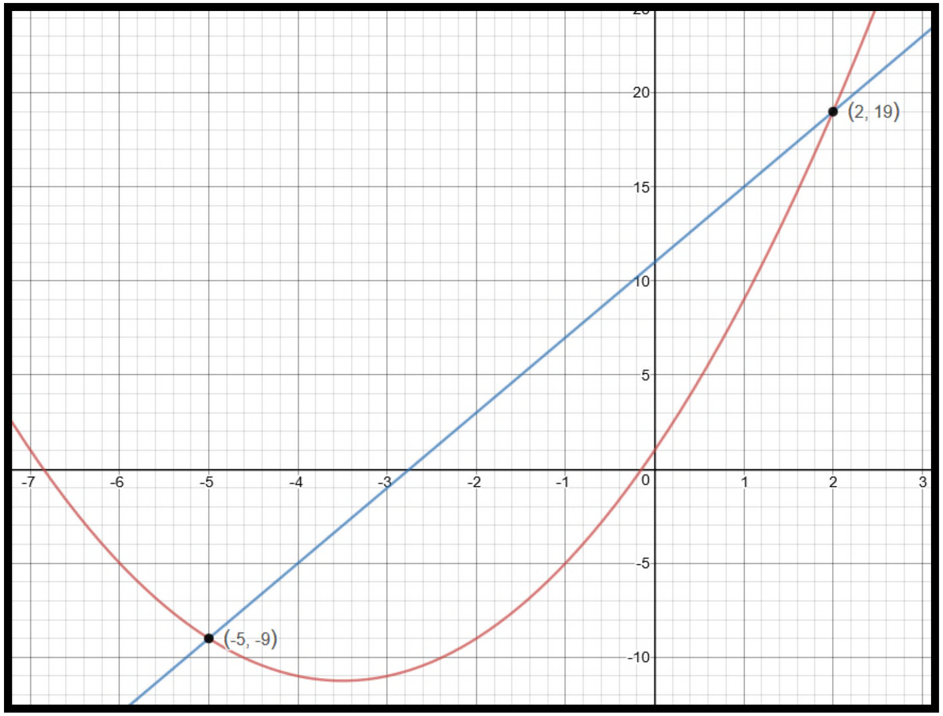
The graphs of the two functions intersect at the points (−5, −9) and (2, 19).

Example 7: Finding the Points of Intersection of the Graphs of Two Functions (3 of 3)

Solution:

Check using Desmos. Each intersection point has been selected, verifying the points (−5, −9) and (2, 19).





Objective 6 Solve Equations That Are Quadratic in Form

Equations in Quadratic Form

In general, if an appropriate substitution *u* transforms an equation into one of the form

$$au^2 + bu + c = 0, \quad a \neq 0$$

the original equation is called an **equation of the quadratic type** or an **equation quadratic in form.**

Example 8: Finding the Real Zeros of a Function (1 of 2)

Find the real zeros of the function

$$f(x) = (x-3)^2 - 7(x-3) + 6.$$

Find any *x*-intercepts of the graph of *f*.

Solution:

To find the real zeros, solve the equation $f(x) = 0.$

$$f(x) = 0$$

$$(x-3)^2 - 7(x-3) + 6 = 0$$

For this equation, let $u = x - 3$. Then $u^2 = (x-3)^2.$

Then $(x-3)^2 - 7(x-3) + 6 = 0$ becomes $u^2 - 7u + 6 = 0.$

Example 8: Finding the Real Zeros of a Function (2 of 2)

Solution:

$$u^2 - 7u + 6 = 0$$

$$(u-6)(u-1) = 0$$

$$u - 6 = 0 \quad \text{or} \quad u - 1 = 0$$

$$u = 6 \quad \text{or} \quad u = 1$$

To solve for *x*, use *u* = *x* − 3, to obtain

$$x - 3 = 6 \quad \text{or} \quad x - 3 = 1$$

$$x = 9 \quad \text{or} \quad x = 4$$

The real zeros of *f* are 4 and 9. The *x*-intercepts of the graph of *f* are 4 and 9.

Example 9: Finding the Real Zeros of a Function (1 of 2)

Find the real zeros of the function $f(x) = x - 5\sqrt{x} - 6.$ Find any *x*-intercepts of the graph of *f*.

Solution:

To find the real zeros, solve the equation $f(x) = 0.$

$$f(x) = 0$$

$$x - 5\sqrt{x} - 6 = 0$$

For this equation, let $u = \sqrt{x}.$ Then $u^2 = x.$

Then $x - 5\sqrt{x} - 6 = 0$ becomes $u^2 - 5u - 6 = 0.$

Example 9: Finding the Real Zeros of a Function (2 of 2)

Solution:

$$u^2 - 5u - 6 = 0$$

$$(u-6)(u+1) = 0$$

$u - 6 = 0$ or $u + 1 = 0$

$u = 6$ or $u = -1$

To solve for *x*, use $u = \sqrt{x}$, to obtain

$\sqrt{x} = 6$ or $\sqrt{x} = -1$

$x = 36$ $\quad \sqrt{x} = -1$ has no real solution.

Check $f(x) = x - 5\sqrt{x} - 6.$

$$f(36) = 36 - 5\sqrt{36} - 6 = 36 - 30 - 6 = 0.$$

The only zero of *f* is 36. The *x*-intercept of the graph of *f* is 36.

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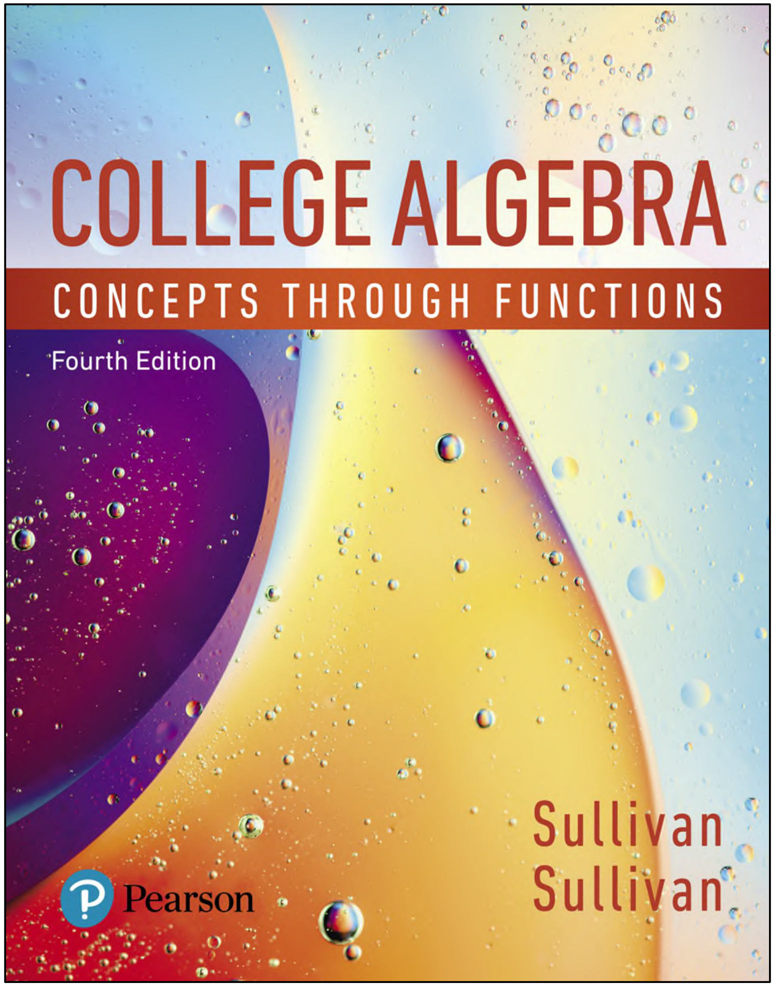


Chapter 2

Linear and Quadratic Functions

Section 2.4 Properties of Quadratic Functions

Objectives

1. Graph a Quadratic Function Using Transformations
2. Identify the Vertex and Axis of Symmetry of a Quadratic Function
3. Graph a Quadratic Function Using Its Vertex, Axis, and Intercepts
4. Find a Quadratic Function Given Its Vertex and One Other Point
5. Find the Maximum or Minimum Value of a Quadratic Function

Quadratic Functions

The revenue *R* is a quadratic function of the price *p*. The figure illustrates the graph of this revenue function.

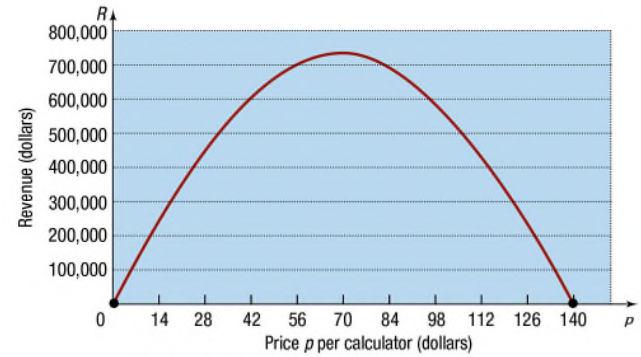
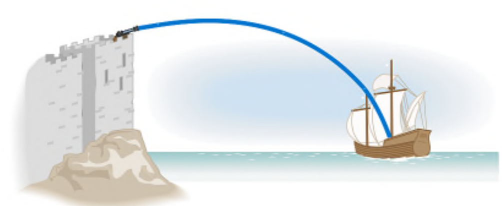


Figure 14 $R(p) = -150p^2 + 21{,}000p$

A second situation in which a quadratic function appears involves the motion of a projectile. Based on Newton's second law of motion (force equals mass times acceleration, $F = ma$), it can be shown that, ignoring air resistance, the path of a projectile propelled upward at an inclination to the horizontal is the graph of a quadratic function.



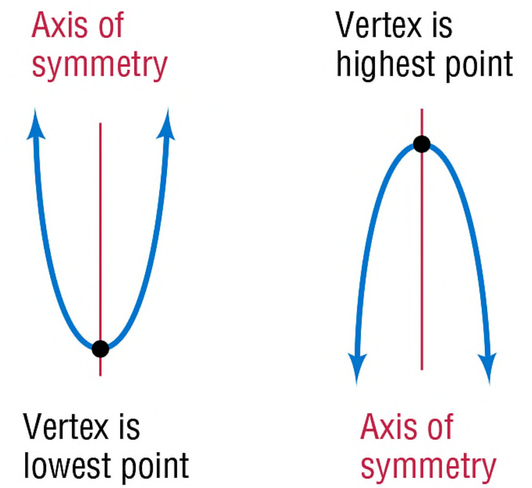
Objective 1 Graph a Quadratic Function Using Transformations

Parabolas (1 of 2)

The graphs in the figures below are typical of the graphs of all quadratic functions, which are called **parabolas**. The parabola on the left **opens up** and has a lowest point; the one on the right **opens down** and has a highest point.

The lowest or highest point of a parabola is called the **vertex**.

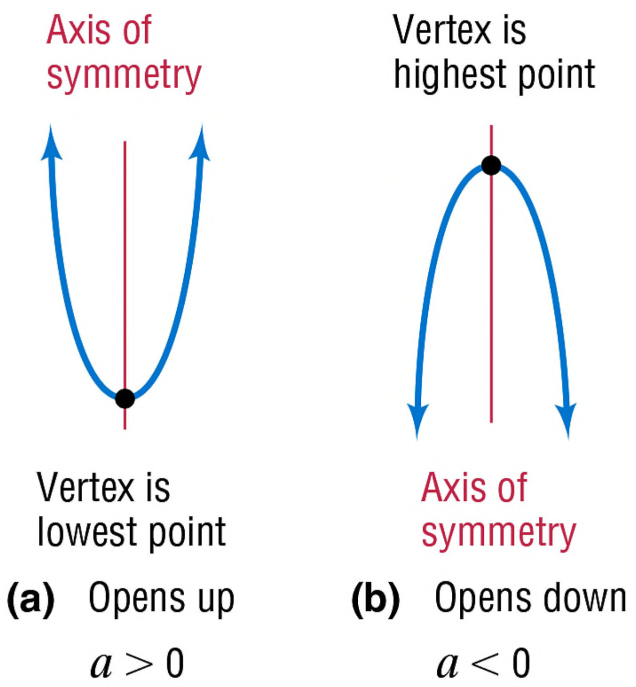
$$f(x) = ax^2 + bx + c,\ a \neq 0$$



(a) Opens up $a > 0$ **(b)** Opens down $a < 0$

Parabolas (2 of 2)

The vertical line passing through the vertex in each parabola is called the **axis of symmetry** (usually abbreviated to **axis**) of the parabola. Because the parabola is symmetric about its axis, the axis of symmetry of a parabola can be used to find additional points on the parabola.



$$f(x) = ax^2 + bx + c,\ a \neq 0$$

Example 1: Graphing a Quadratic Function Using Transformations (1 of 3)

Graph the function $f(x) = 3x^2 - 6x + 5$. Find the vertex and axis of symmetry.

Solution:

$$f(x) = 3x^2 - 6x + 5$$

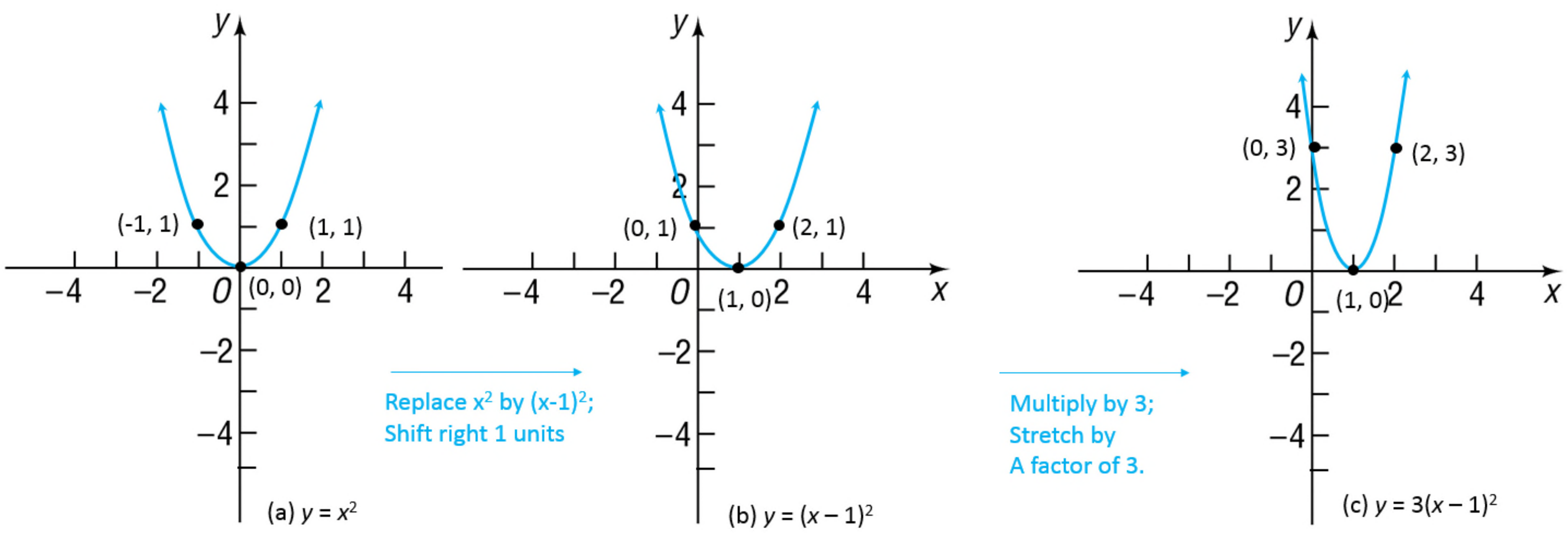
$$= 3(x^2 - 2x) + 5$$

$$= 3(x^2 - 2x + 1) + 5 - 3$$

$$= 3(x - 1)^2 + 2$$

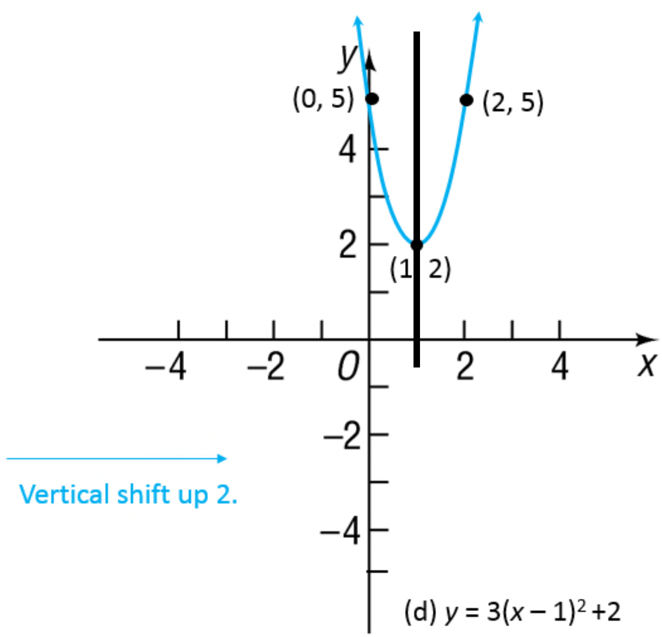
Example 1: Graphing a Quadratic Function Using Transformations (2 of 3)

The graph of *f* can be obtained from the graph of $y = x^2$ in three stages.



Example 1: Graphing a Quadratic Function Using Transformations (3 of 3)

The graph of $f(x) = 3x^2 - 6x + 5$ is a parabola that opens up and has its vertex (lowest point) at (1, 2). Its axis of symmetry is the line $x = 1$.



Axis of Symmetry
$x = 1$

Quadratic Equation

If $h = -\frac{b}{2a}$ and $k = \frac{4ac - b^2}{4a}$, then

$$f(x) = ax^2 + bx + c = a(x-h)^2 + k \qquad \textbf{(1)}$$

The graph of $f(x) = a(x-h)^2 + k$ is the parabola $y = ax^2$ shifted horizontally *h* units (replace *x* by *x* − *h*) and vertically *k* units (add *k*). As a result, the vertex is at (*h*, *k*), and the graph opens up if *a* > 0 and down if *a* < 0. The axis of symmetry is the vertical line *x* = *h*.

Objective 2 Identify the Vertex and Axis of Symmetry of a Quadratic Function

Properties of the Graph of a Quadratic Function

$$f(x) = ax^2 + bx + c \qquad a \neq 0$$

Vertex $= \left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a} \right) \right)$

Axis of symmetry: the vertical line $x = -\dfrac{b}{2a}$

Parabola opens up if $a > 0$; the vertex is a minimum point.

Parabola opens down if $a < 0$; the vertex is a maximum point.

Example 2: Locating the Vertex Without Graphing (1 of 2)

Without graphing, locate the vertex and axis of symmetry of the parabola defined by $f(x) = -2x^2 + 8x - 3$. Does it open up or down?

Solution:

For this quadratic function, *a* = −2, *b* = 8, and *c* = −3. The *x*-coordinate of the vertex is

$$h = -\frac{b}{2a} = -\frac{8}{2(-2)} = 2$$

Example 2: Locating the Vertex Without Graphing (2 of 2)

The *y*-coordinate of the vertex is

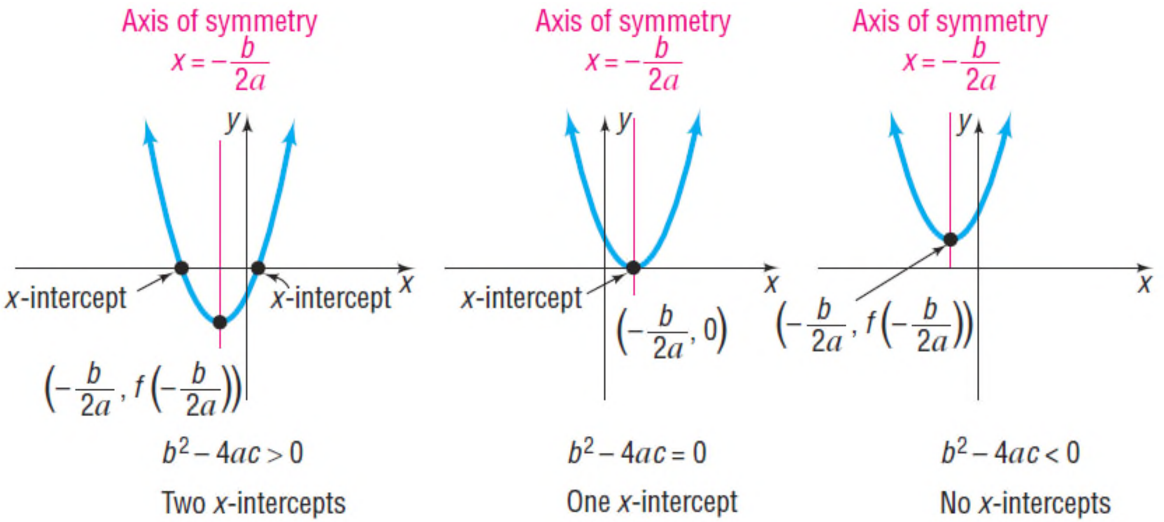
$$k = f\left(-\frac{b}{2a}\right) = f(2) = -2(2)^2 + 8(2) - 3 = 5$$

The vertex is located at the point (2, 5). The axis of symmetry is the line $x = 2$. Because $a = -2 < 0$, the parabola opens down.

Objective 3 Graph a Quadratic Function Using Its Vertex, Axis, and Intercepts

The *x*-Intercepts of a Quadratic Function

1. If the discriminant $b^2 - 4ac > 0$, the graph of $f(x) = ax^2 + bx + c$ has two distinct *x*-intercepts so it crosses the *x*-axis in two places.
2. If the discriminant $b^2 - 4ac = 0$, the graph of $f(x) = ax^2 + bx + c$ has one *x*-intercept so it touches the *x*-axis at its vertex.
3. If the discriminant $b^2 - 4ac < 0$, the graph of $f(x) = ax^2 + bx + c$ has no *x*-intercepts so it does not cross or touch the *x*-axis.



Example 3: How to Graph a Quadratic Function Using Its Properties (1 of 5)

Graph $f(x) = -2x^2 + 4x + 2$ using its properties. Determine the domain and the range of *f*. Determine where *f* is increasing and where it is decreasing.

Solution:

Step 1: Determine whether the graph of *f* opens up or down. The graph of $f(x) = -2x^2 + 4x + 2$ opens down because $a = -2 < 0$.

Example 3: How to Graph a Quadratic Function Using Its Properties (2 of 5)

Solution:

Step 2: Determine the vertex and axis of symmetry of the graph of *f*.

$$h = -\frac{b}{2a} = -\frac{4}{2(-2)} = 1$$

$$k = f(1) = -2(1)^2 + 4(1) + 2 = -2 + 4 + 2 = 4$$

The vertex was found to be at the point whose coordinates are (1, 4). The axis of symmetry is the line $x = 1$.

Example 3: How to Graph a Quadratic Function Using Its Properties (3 of 5)

Step 3: Determine the intercepts of the graph of *f*. The *y*-intercept is found by letting *x* = 0. The *y*-intercept is $f(0) = 2$. The *x*-intercepts are found by solving the equation $f(x) = 0$.

$$f(x) = 0.$$

$$-2x^2 + 4x + 2 = 0 \quad a = -2,\ b = 4,\ c = 2$$

The discriminant $b^2 - 4ac = (4)^2 - 4(-2)(2) = 16 + 16 = 32 > 0,$ so the equation has two real solutions and the graph has two *x*-intercepts.

Example 3: How to Graph a Quadratic Function Using Its Properties (4 of 5)

Use the quadratic formula to find that

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-4 + \sqrt{(4)^2 - 4(-2)(2)}}{2(-2)}$$

$$= \frac{-4 + \sqrt{32}}{-4} = \frac{-4 + 4\sqrt{2}}{-4} = 1 - \sqrt{2} \approx -0.41$$

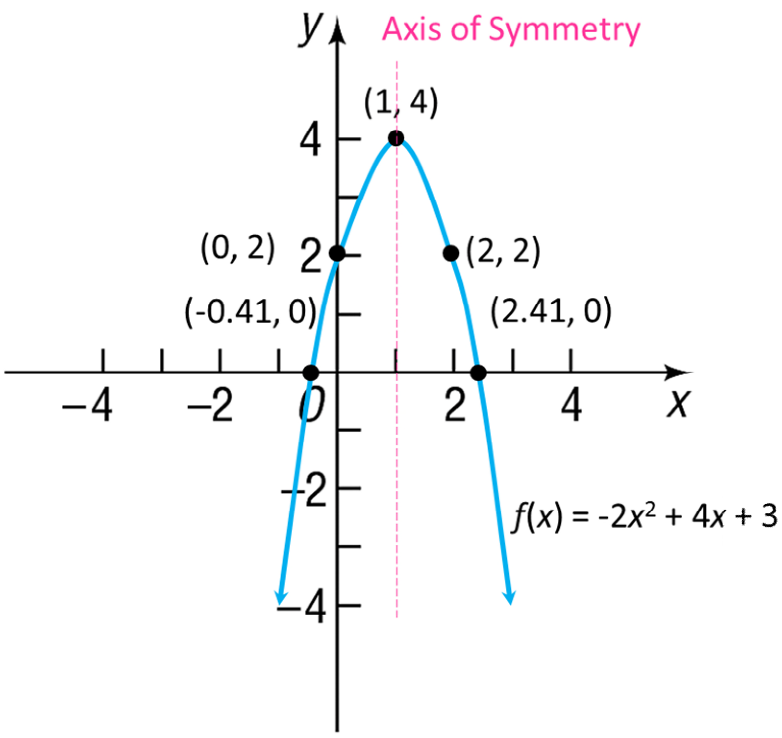
$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-4 - \sqrt{(4)^2 - 4(-2)(2)}}{2(-2)}$$

$$= \frac{-4 - \sqrt{32}}{-4} = \frac{-4 - 4\sqrt{2}}{-4} = 1 + \sqrt{2} \approx 2.41$$

The *x*-intercepts are approximately −0.41 and 2.41.

Example 3: How to Graph a Quadratic Function Using Its Properties (5 of 5)

The graph is illustrated below.



The domain of *f* is the set of all real numbers. Based on the graph, the range of *f* is the interval $(-\infty, 4]$.

The function *f* is increasing on the interval $(-\infty, 1]$ and decreasing on the interval $[1, \infty)$.

Example 4: Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts (1 of 4)

(a) Graph $f(x) = x^2 - 4x + 4$ by determining whether the graph opens up or down and by finding its vertex, axis of symmetry, *y*-intercept, and *x*-intercepts, if any.

(b) Determine the domain and the range of *f.*

(c) Determine where *f* is increasing and where it is decreasing.

Example 4: Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts (2 of 4)

Solution:

(a) **Step 1:** For $f(x) = x^2 - 4x + 4$, note that *a* = 1, *b* = −4, and *c* = 4. Because $a = 1 > 0$, the parabola opens up.

Step 2: The *x*-coordinate of the vertex is

$$h = -\frac{b}{2a} = -\frac{-4}{2(1)} = 2$$

The *y*-coordinate of the vertex is

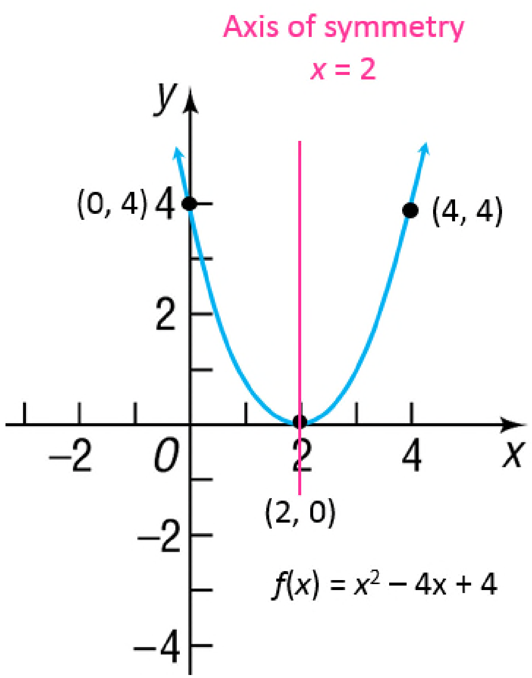
$$k = f(2) = (2)^2 - 4(2) + 4 = 0$$

The vertex is at (2, 0). The axis of symmetry is the line *x* = 2.

Example 4: Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts (3 of 4)

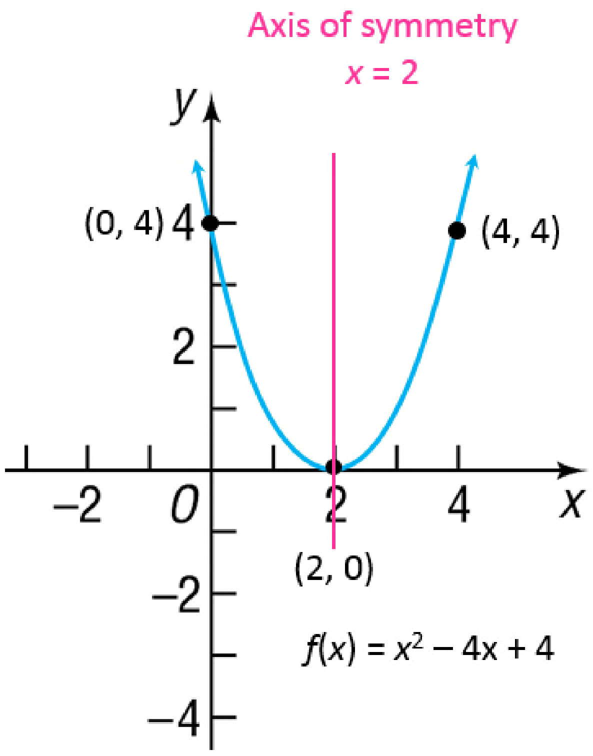
Step 3: The *y*-intercept is $f(0) = 4$. Since the vertex (2, 0) lies on the *x*-axis, the graph touches the *x*-axis at the *x*-intercept.

Step 4: By using the axis of symmetry and the *y*-intercept at (0, 4), we can locate the additional point (4, 4) on the graph.



Example 4: Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts (4 of 4)

(b) The domain of *f* is the set of all real numbers. Based on the graph, the range of *f* is the interval $[0, \infty)$.



(c) The function *f* is decreasing on the interval $(-\infty, 2]$ and increasing on the interval $[2, \infty)$.

Example 5: Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts (1 of 5)

(a) Graph $f(x) = 3x^2 + x + 2$ by determining whether the graph opens up or down and by finding its vertex, axis of symmetry, *y*-intercept, and *x*-intercepts, if any.

(b) Determine the domain and the range of *f*.

(c) Determine where *f* is increasing and where it is decreasing.

Solution:

(a) **Step 1:**

For $f(x) = 3x^2 + x + 2$, we have *a* = 3, *b* = 1, and *c* = 2.

Because $a = 3 > 0$, the parabola opens up.

Example 5: Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts (2 of 5)

Step 2:

The *x*-coordinate of the vertex is

$$h = -\frac{b}{2a} = -\frac{1}{6}$$

The *y*-coordinate of the vertex is

$$k = f\left(-\frac{1}{6}\right) = 3\left(-\frac{1}{6}\right)^2 + \left(-\frac{1}{6}\right) + 2 = \frac{3}{36} - \frac{1}{6} + 2 = \frac{23}{12}$$

The vertex is at $\left(-\frac{1}{6}, \frac{23}{12}\right)$.

The axis of symmetry is the line at $x = -\frac{1}{6}$.

Example 5: Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts (3 of 5)

Step 3:

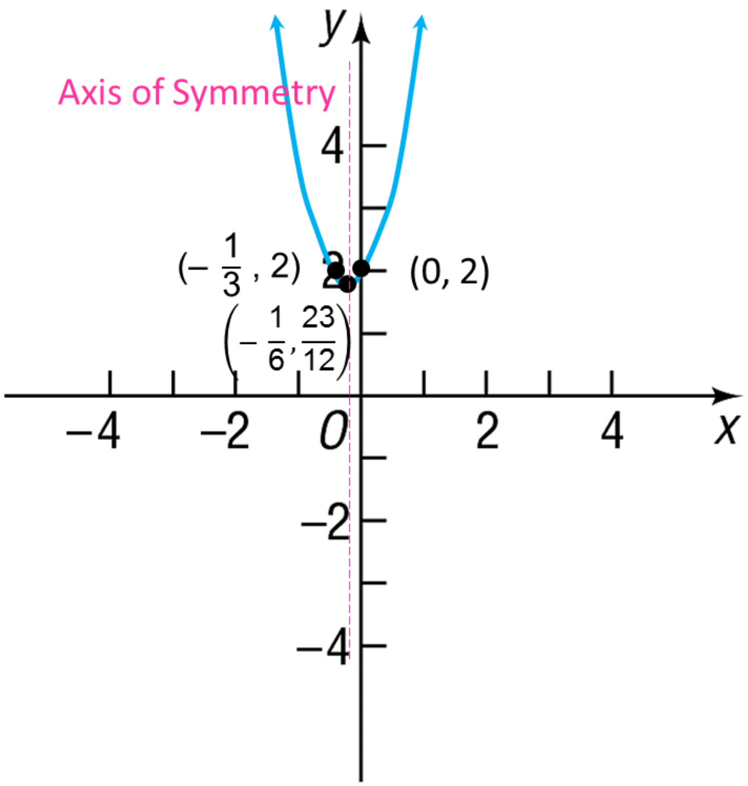
The *y*-intercept is $f(0)=2$. The *x*-intercept(s), if any, obey the equation $3x^2+x+2=0$.

The discriminant $b^2-4ac=(1)^2-4(3)(2)=-23<0$. This equation has no real solutions, which means the graph has no *x*-intercepts.

Example 5: Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts (4 of 5)

Step 4:

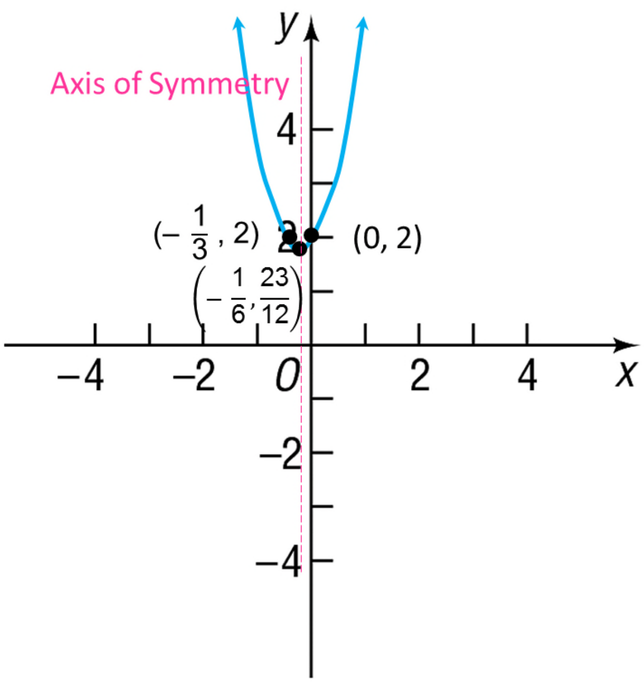
Use the point (0, 2) and the axis of symmetry $x = -\frac{1}{6}$ to locate the additional point $\left(-\frac{1}{3}, 2\right)$ on the graph.



Example 5: Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts (5 of 5)

(b) The domain of *f* is the set of all real numbers. Based on the graph, the range of *f* is the interval $\left[\frac{23}{12}, \infty\right)$.

(c) The function *f* is decreasing on the interval $\left(-\infty, -\frac{1}{6}\right]$ and is increasing on the interval $\left[-\frac{1}{6}, \infty\right)$.



Objective 4 Find a Quadratic Function Given Its Vertex and One Other Point

If the vertex (h, k) and one additional point on the graph of a quadratic function $f(x) = ax^2 + bx + c,\ a \neq 0,$ are known, then

$$f(x) = a(x-h)^2 + k$$

can be used to obtain the quadratic function.

Example 6: Finding the Quadratic Function Given Its Vertex and One Other Point (1 of 2)

Determine the quadratic function whose vertex is (−3, −7) and whose *y*-intercept is 2.

Solution:

The vertex is (−3, −7), so $h = -3$ and $k = -7$.

$$f(x) = a(x-h)^2 + k$$

$$f(x) = a(x+3)^2 - 7$$

Example 6: Finding the Quadratic Function Given Its Vertex and One Other Point (2 of 2)

To determine the value of *a*, use the fact that $f(0)=2$ (the *y*-intercept).

$$f(x)=a(x+3)^2-7$$

$$2=a(0+3)^2-7$$

$$2=9a-7$$

$$9=9a$$

$$a=1$$

Then,

$$f(x)=a(x-h)^2+k$$

$$f(x)=1(x+3)^2-7$$

$$f(x)=x^2+6x+2$$

Objective 5 Find the Maximum or Minimum Value of a Quadratic Function

Example 7: Finding the Maximum or Minimum Value of a Quadratic Function (1 of 2)

Determine whether the quadratic function $f(x) = x^2 - 6x - 3$ has a maximum or a minimum value. Then find the maximum or minimum value.

Solution:

Compare $f(x) = x^2 - 6x - 3$ to $f(x) = ax^2 + bx + c$. Then *a* = 1, *b* = −6, and *c* = −3. Because *a* > 0, the graph of *f* opens up, which means the vertex is a minimum point. The minimum value occurs at

$$x = -\frac{b}{2a} = -\frac{-6}{2(1)} = \frac{6}{2} = 3$$

Example 7: Finding the Maximum or Minimum Value of a Quadratic Function (2 of 2)

The minimum value is

$$f\left(-\frac{b}{2a}\right) = f(3) = 3^2 - 6(3) - 3 = -12$$

Summary

Steps for Graphing a Quadratic Function $\boldsymbol{f(x)=ax^2+bx+c, a\neq 0}$

Option 1

Step 1: Complete the square in *x* to write the quadratic function in the from $f(x)=a(x-h)^2+k.$

Step 2: Graph the function in stages using transformations.

Option 2

Step 1: Determine whether the parabola opens up (*a* > 0) or down *(a* < 0).

Step 2: Determine the vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$

Step 3: Determine the axis of symmetry, $x=-\frac{b}{2a}.$

Step 4: Determine the *y*-intercept, $f(0),$ and the *x*-intercepts, if any.

(a) If $b^2-4ac>0,$ the graph of the quadratic function has two *x*-intercepts, which are found by solving the equation $ax^2+bx+c=0.$

(b) If $b^2-4ac=0,$ the vertex is the *x*-intercept.

(c) If $b^2-4ac<0,$ there are no *x*-intercepts.

Step 5: Determine an additional point by using the *y*-intercept and the axis of symmetry.

Step 6: Plot the points and draw the graph.

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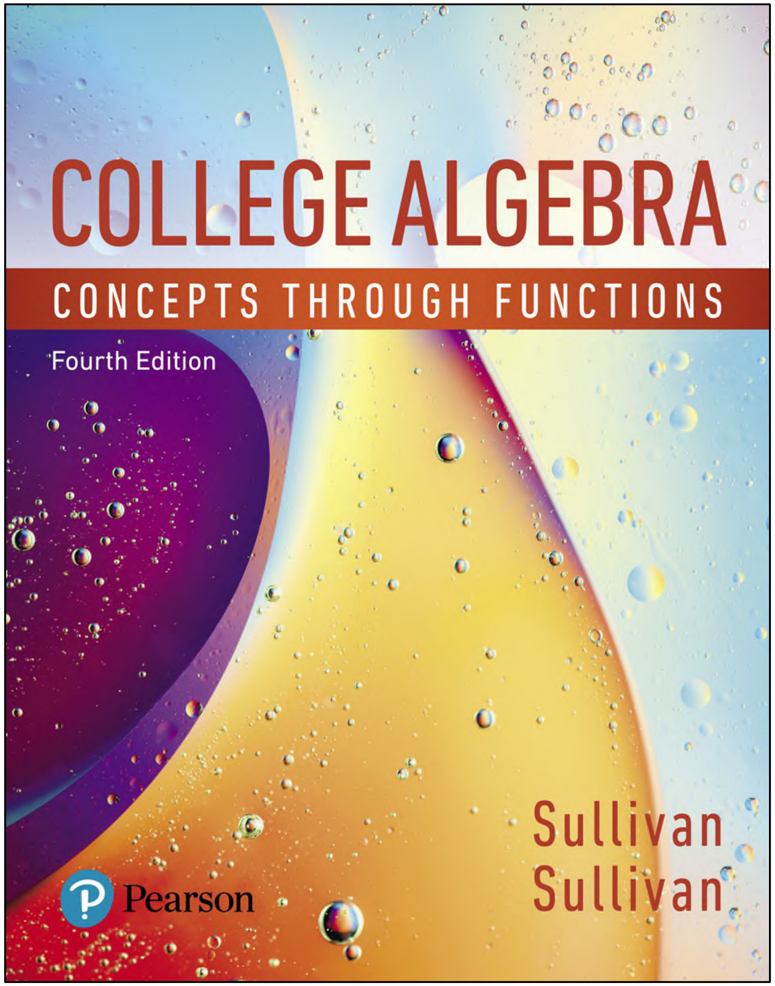


Chapter 2

Linear and Quadratic Functions

Section 2.6 Build Quadratic Models from Verbal Descriptions and from Data

Objectives

1. Build Quadratic Models from Verbal Descriptions
2. Build Quadratic Models from Data

Objective 1 Build Quadratic Models from Verbal Descriptions

Example 1: Maximizing Revenue (1 of 6)

An electronics company has found that when certain calculators are sold at a price of *p* dollars per unit, the number *x* of calculators sold is given by the demand equation

$$x = 18,750 - 125p$$

(a) Find a model that expresses the revenue *R* as a function of the price *p*.

(b) What is the domain of *R*?

(c) What unit price should be used to maximize revenue?

(d) If this price is charged, what is the maximum revenue?

(e) How many units are sold at this price?

Example 1: Maximizing Revenue (2 of 6)

(f) Graph *R*.

(g) What price should the company charge for the product to collect at least $590,625 in revenue?

Solution:

(a) The revenue *R* is $R = xp$, where $x = 18,750 - 125p$.

$$R = xp = (18,750 - 125p)p = -125p^2 + 18,750p$$

(b) *x* represents the number of products sold. We have $x \geq 0$, so $18,750 - 125p \geq 0$. Solving this linear inequality gives $p \leq 150$. In addition, the company will charge only a positive price for the calculator, so $p > 0$. Combining these inequalities gives the domain of *R*, which is $\{p \mid 0 < p \leq 150\}$.

Example 1: Maximizing Revenue (3 of 6)

(c) The function *R* is a quadratic function with $a = -125$, $b = 18{,}750$, and $c = 0$. Because $a < 0$, the vertex is the highest point on the parabola. The revenue *R* is a maximum when the price *p* is

$$p = -\frac{b}{2a} = -\frac{18{,}750}{2(-125)} = \$75.00 \quad a = -125,\ b = 18{,}750$$

(d) The maximum revenue *R* is

$$R(75) = -125(75)^2 + 18{,}750(75) = \$703{,}125$$

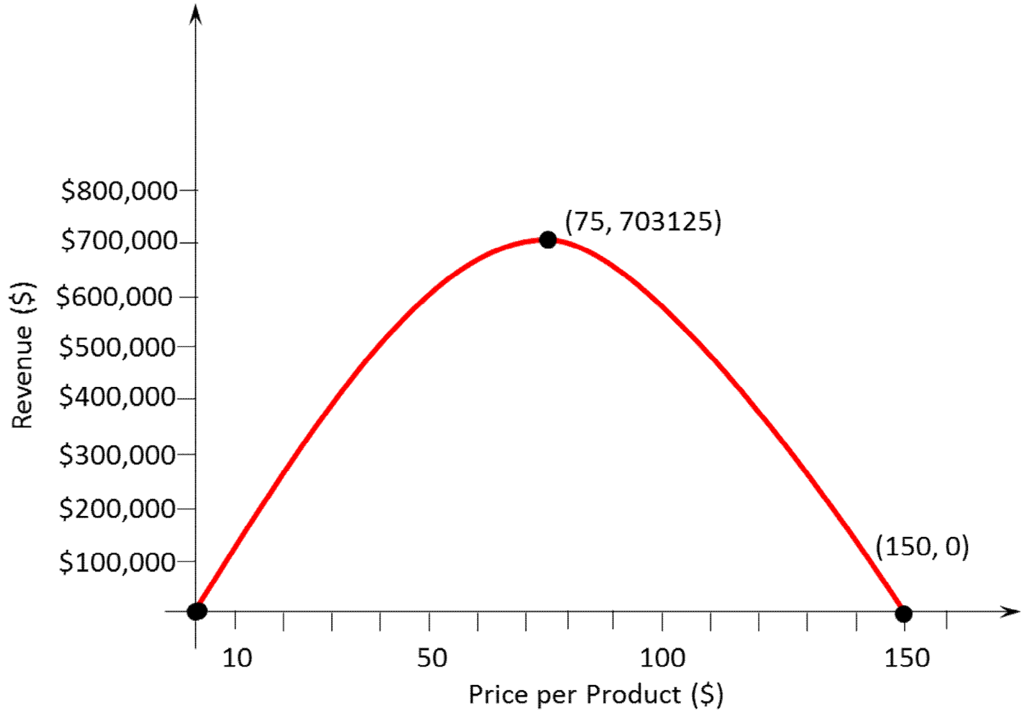
(e) The number of calculators sold is given by the demand equation

$x = 18{,}750 - 125p$. At a price of $p = \$75$,

$x = 18{,}750 - 125(75) = 9{,}375$ calculators are sold.

Example 1: Maximizing Revenue (4 of 6)

(f) To graph *R*, plot the intercept (150, 0) and the vertex (75, 703,125). See the graph below.



Example 1: Maximizing Revenue (5 of 6)

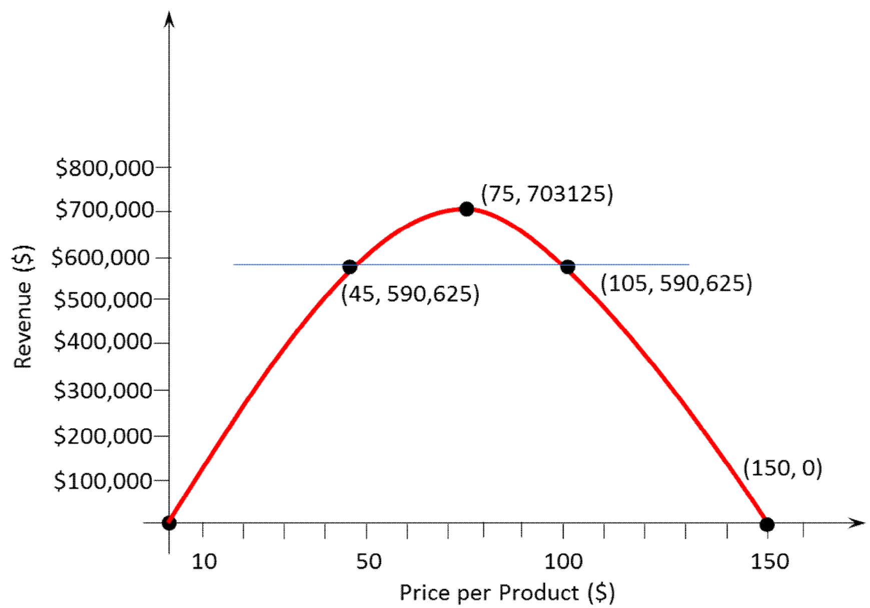
(g) Graph R = 590,625 and $R(p) = -125p^2 + 18,750p$ on the same Cartesian plane. See next slide. We find where the graphs intersect by solving

$$590,625 = -125p^2 + 18,750p$$

$125p^2 - 18,750p + 590,625 = 0$	Add $125p^2 - 18,750p$ to both sides.
$p^2 - 150p + 4,725 = 0$	Divide both sides by 125.
$(p-45)(p-105) = 0$	Factor.
$p = 45$ or $p = 105$	Use the Zero-Product Property.

Example 1: Maximizing Revenue (6 of 6)

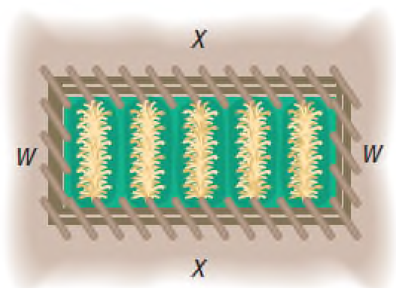
(g) The graphs intersect at (45, 590,625) and (105, 590,625). Based on the graph, the company should charge between $45 and $105 to earn at least $590,625 in revenue.



Example 2: Maximizing the Area Enclosed by a Fence (1 of 4)

A farmer has 2500 yards of fence to enclose a rectangular field. What are the dimensions of the rectangle that encloses the most area?

Solution:



The available fence represents the perimeter of the rectangle. If *x* is the length and *w* is the width, then

$$2x + 2w = 2500$$

The area *A* of the rectangle is $A = xw$

Example 2: Maximizing the Area Enclosed by a Fence (2 of 4)

To express *A* in terms of a single variable, solve the first equation for *w* and substitute the result in $A = xw$.

Then *A* involves only the variable *x*.

$$2x + 2w = 2500$$

$$2w = 2500 - 2x$$

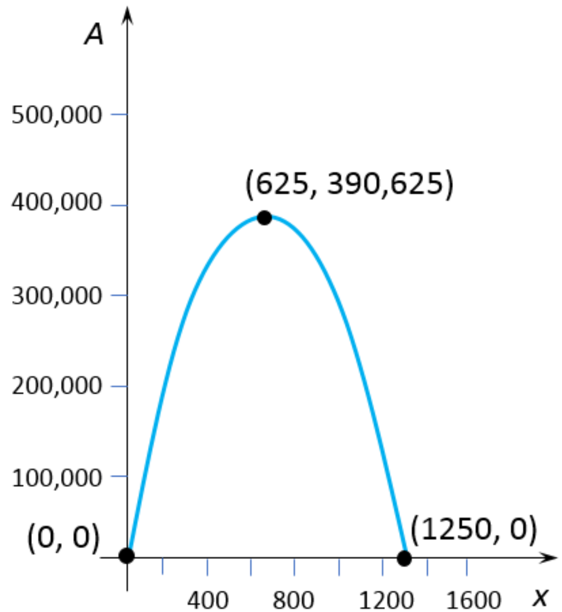
$$w = \frac{2500 - 2x}{2} = 1250 - x$$

Then the area *A* is $A = xw = x(1250 - x) = -x^2 + 1250x$

Now, *A* is a quadratic function of *x.*

$$A(x) = -x^2 + 1250x$$

Example 2: Maximizing the Area Enclosed by a Fence (3 of 4)



$$A(x) = -x^2 + 1250x$$

Because *a* < 0, the vertex is a maximum point on the graph of *A*. The maximum value occurs at

$$x = -\frac{b}{2a} = -\frac{1250}{2(-1)} = 625$$

Example 2: Maximizing the Area Enclosed by a Fence (4 of 4)

The maximum value of *A* is

$$A\left(-\frac{b}{2a}\right) = A(625) = -625^2 + 1250(625)$$

$$= -390{,}625 + 781{,}250 = 390{,}625$$

The largest rectangle that can be enclosed by 2500 yards of fence has an area of 390,625 square yards. Its dimensions are 625 yards by 625 yards.

Example 3: Analyzing the Motion of a Projectile (1 of 4)

A water balloon is launched from a stadium 100 feet above the ground at an inclination of 45° to the horizontal, with an initial velocity of 100 feet per second. From physics, the height *h* of the projectile above the ground can be modeled by

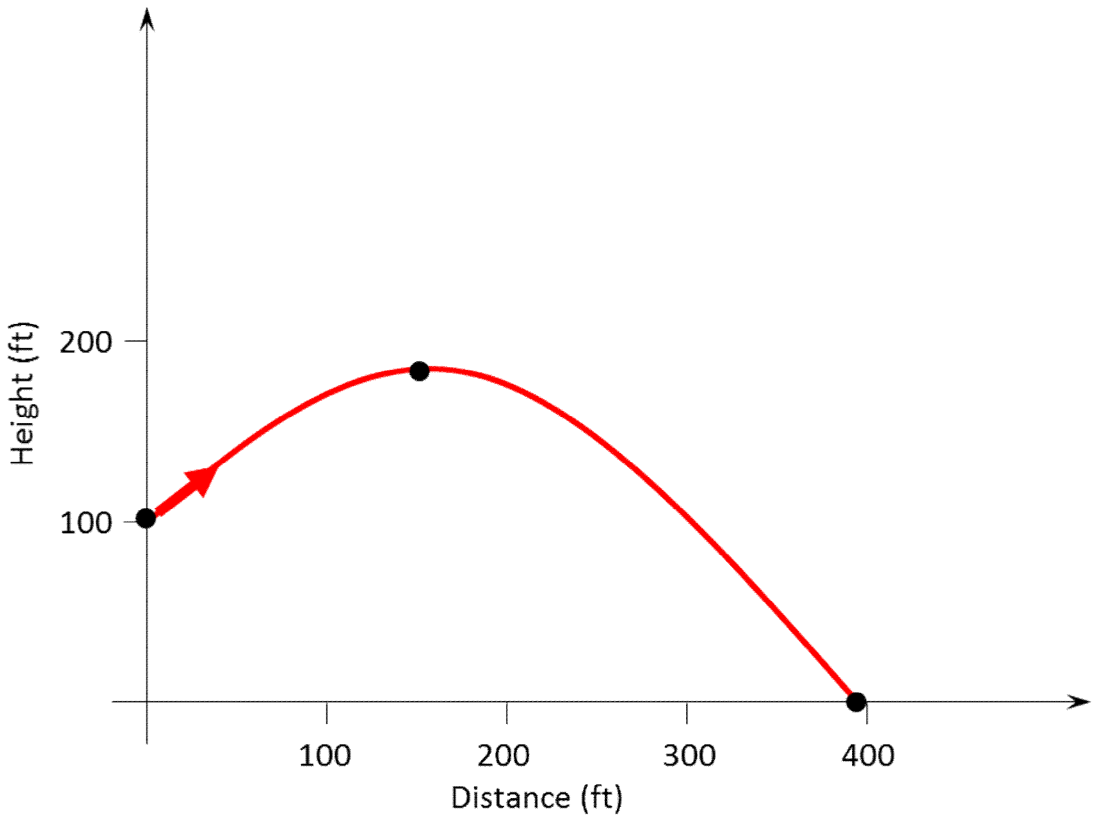
$$h(x) = \frac{-32x^2}{(100)^2} + x + 100$$

where *x* is the horizontal distance of the projectile from the base of the stadium. See the figure on the next slide.

(a) Find the maximum height of the projectile.

(b) How far from the base of the stadium will the projectile hit the ground?

Example 3: Analyzing the Motion of a Projectile (2 of 4)



Example 3: Analyzing the Motion of a Projectile (3 of 4)

Solution:

(a) The height of the projectile is given by a quadratic function.

$$h(x) = \frac{-32x^2}{(100)^2} + x + 100 = -0.0032x^2 + x + 100$$

We are looking for the maximum value of *h*. Because *a* < 0, the maximum value occurs at the vertex, whose *x*-coordinate is

$$x = \frac{-b}{2a} = \frac{-1}{2(-0.0032)} = 156.25$$

The maximum height of the projectile is

$$h(156.25) = -0.0032(156.25)^2 + 156.25 + 100 = 178.125 \text{ ft.}$$

Example 3: Analyzing the Motion of a Projectile (4 of 4)

(b) The projectile will strike the ground when the height is zero. To find the distance *x* traveled, solve the equation

$$h(x) = -0.0032x^2 + x + 100 = 0.$$

The discriminant of this quadratic equation is

$$b^2 - 4ac = 1^2 - 4(-0.0032)(100) = 2.28.$$

Then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-1 \pm \sqrt{2.28}}{2(-0.0032)} \approx -79.68$ or 392.18 ft.

The water balloon will strike the ground at a distance of approximately 392 feet from the stadium.

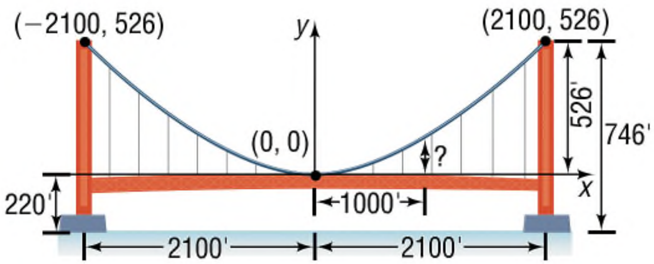
Example 4: The Golden Gate Bridge (1 of 5)

The Golden Gate Bridge, a suspension bridge, spans the entrance to San Francisco Bay. Its 746-foot-tall towers are 4200 feet apart. The bridge is suspended from two huge cables more than 3 feet in diameter; the 90-foot-wide roadway is 220 feet above the water. The cables are parabolic in shape and touch the road surface at the center of the bridge. Find the height of the cable above the road at a distance of 1500 feet from the center.

Example 4: The Golden Gate Bridge (2 of 5)

Solution:

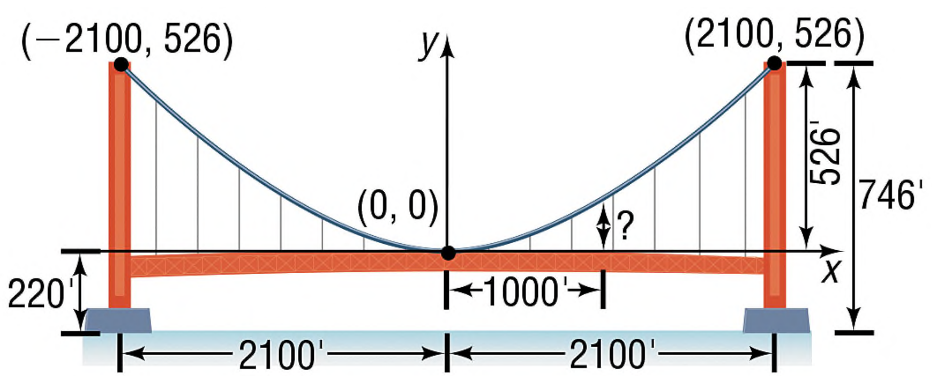
Begin by choosing the placement of the coordinate axes so that the *x*-axis coincides with the road surface and the origin coincides with the center of the bridge. As a result, the twin towers will be vertical (height 746 − 220 = 526 feet above the road) and located 2100 feet from the center.



Example 4: The Golden Gate Bridge (3 of 5)

Solution:

Also, the cable, which has the shape of a parabola, will extend from the towers, open up, and have its vertex at (0, 0). This choice of placement of the axes enables the equation of the parabola to have the form $y = ax^2,\ a > 0.$ Note that the points (−2100, 526) and (2100, 526) are on the graph.



Example 4: The Golden Gate Bridge (4 of 5)

Solution:

Use these facts to find the value of *a* in $y = ax^2$.

$$y = ax^2$$

$$526 = a(2100)^2 \qquad x = 2100,\ y = 526$$

$$a = \frac{526}{(2100)^2}$$

The equation of the parabola is $y = \dfrac{526}{(2100)^2}x^2$

When *x* = 1500, the height of the cable is

$$y = \frac{526}{(2100)^2}(1500)^2 = 268.4\ \text{feet}$$

Example 4: The Golden Gate Bridge (5 of 5)

The cable is 268.4 feet above the road at a distance of 1500 feet from the center of the bridge.

Objective 2 Build Quadratic Models from Data

Example 5: Fitting a Quadratic Function to Data (1 of 7)

The data in the table represent the percentage *D* of the population that is divorced for various ages *x*.

Age, *x*	**Percentage Divorced, *D***
22	0.9
27	3.6
32	7.4
37	10.4
42	12.7
50	15.7
60	16.2
70	13.1
80	6.5

Source: United States Statistical Abstract, 2012

Example 5: Fitting a Quadratic Function to Data (2 of 7)

(a) Draw a scatter diagram of the data treating age as the independent variable. Comment on the type of relation that may exist between age and percentage of the population divorced.

(b) Use a graphing utility to find the quadratic function of best fit that models the relation between age and percentage of the population divorced.

(c) Use the model found in part (b) to approximate the age at which the percentage of the population divorced is greatest.

Example 5: Fitting a Quadratic Function to Data (3 of 7)

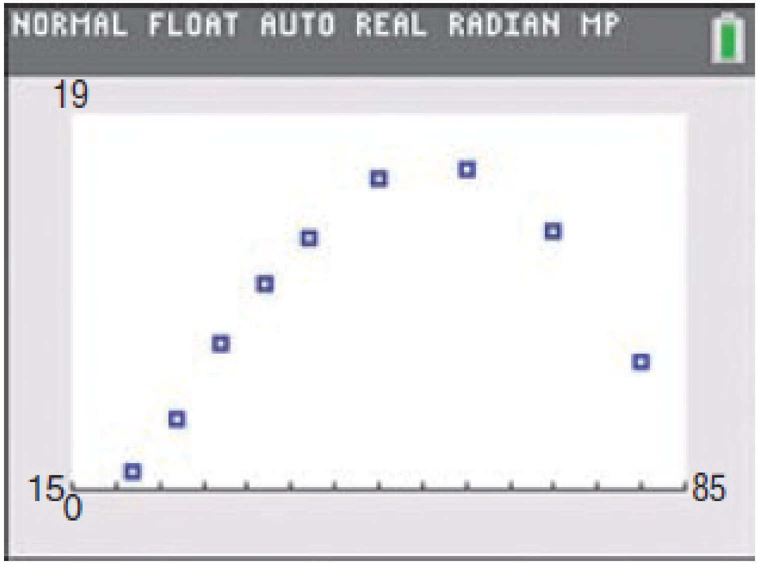
(d) Use the model found in part (b) to approximate the highest percentage of the population that is divorced.

(e) Use a graphing utility to draw the quadratic function of best fit on the scatter diagram.

Example 5: Fitting a Quadratic Function to Data (4 of 7)

Solution:

(a) The figure shows the scatter diagram, from which it appears the data follow a quadratic relation, with $a < 0$.



Example 5: Fitting a Quadratic Function to Data (5 of 7)

(b) Execute the QUADratic REGression program to obtain the results.

The output shows the equation $y = ax^2 + bx + c$. The quadratic function of best fit that models the relation between age and percentage divorced is

$$D(x) = -0.0143x^2 + 1.5861x - 28.1886$$

where *x* represents age and *D* represents the percentage divorced.

Example 5: Fitting a Quadratic Function to Data (6 of 7)

Solution:

(c) Based on the quadratic function of best fit, the age with the greatest percentage divorced is.

$$-\frac{b}{2a} = -\frac{1.5861}{2(-0.0143)} \approx 55 \text{ years}$$

(d) Evaluate the function $D(x)$ at $x = 55$.

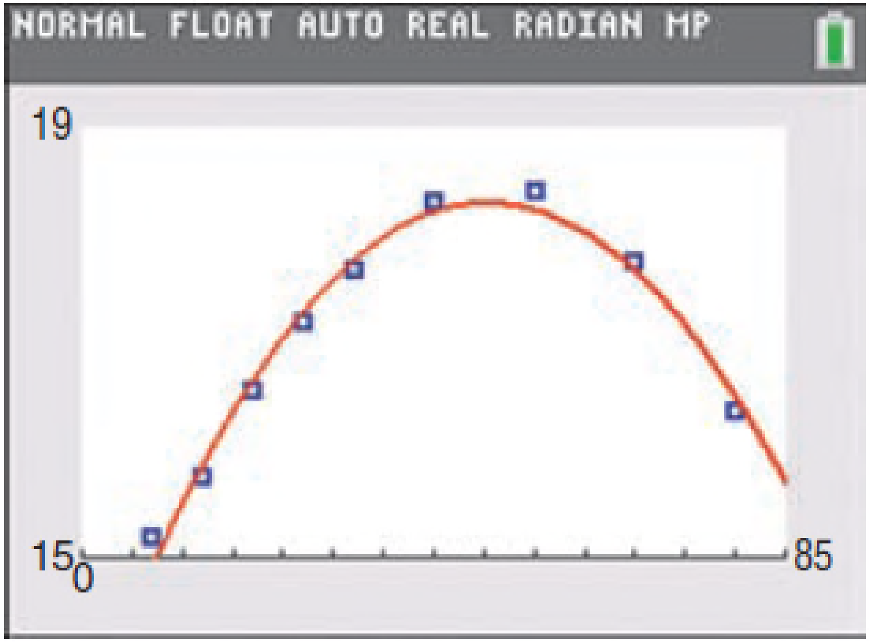
$$D(55) = -0.0143(55)^2 + 1.5861(55) - 28.1886 \approx 15.8 \text{ percent}$$

According to the model, 55-year-olds have the highest percentage divorced at 15.8 percent.

Example 5: Fitting a Quadratic Function to Data (7 of 7)

Solution:

(e) The graph of the quadratic function found in part (b) is drawn on the scatter diagram.



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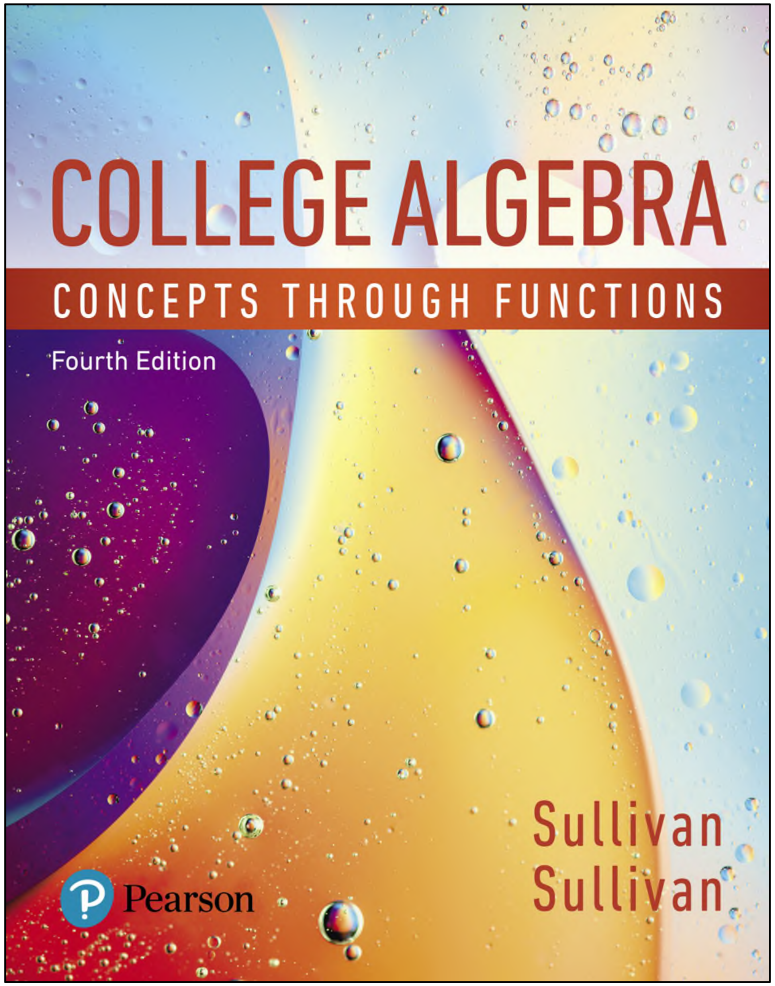


Chapter 2

Linear and Quadratic Functions

Section 2.7 Complex Zeros of a Quadratic Function

Objective

1. Find the Complex Zeros of a Quadratic Function

Complex Zeros

For a function *f*, the solutions of the equation $f(x)=0$ are called the zeros of *f*. When we are working in the real number system, these zeros are called real zeros of *f*. When we are working in the complex number system, these zeros are called **complex zeros of *f*.**

Example 1: Finding Complex Zeros of a Quadratic Function (1 of 4)

Find the complex zeros of each of the following quadratic functions. Graph each function and label the intercepts.

(a) $f(x) = x^2 - 1$ (b) $g(x) = x^2 + 4$

Solution:

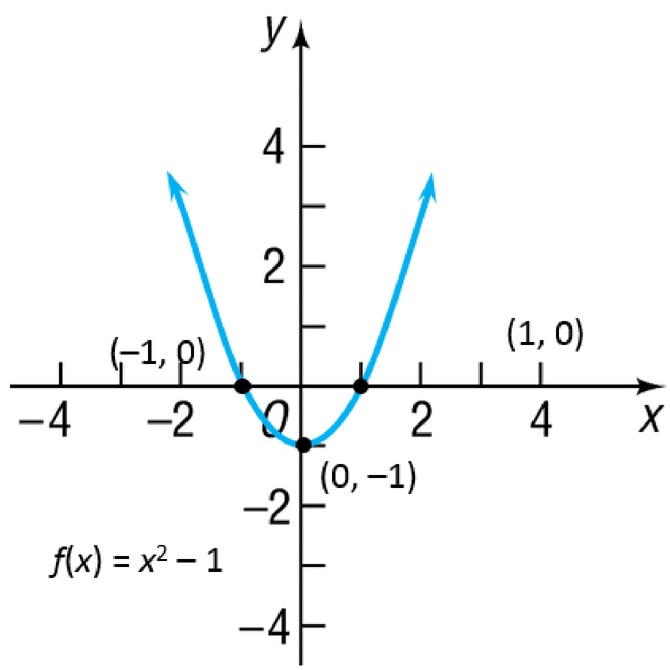
(a) To find the complex zeros of $f(x) = x^2 - 1$, solve the equation $f(x) = 0$.

$$\begin{aligned} f(x) &= 0 \\ x^2 - 1 &= 0 \\ x^2 &= 1 \\ x &= \pm\sqrt{1} \\ x &= \pm 1 \end{aligned}$$

Example 1: Finding Complex Zeros of a Quadratic Function (2 of 4)

Solution:

(a) The complex zeros of *f* are −1 and 1. Because the zeros are real numbers, the *x*-intercepts of the graph of *f* are −1 and 1. The figure shows the graph of $f(x) = x^2 - 1$ with the intercepts labeled.



Example 1: Finding Complex Zeros of a Quadratic Function (3 of 4)

Solution: (b) $g(x) = x^2 + 4$

(b) To find the complex zeros of $g(x) = x^2 + 4,$ solve the equation $g(x) = 0.$

$$g(x) = 0$$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

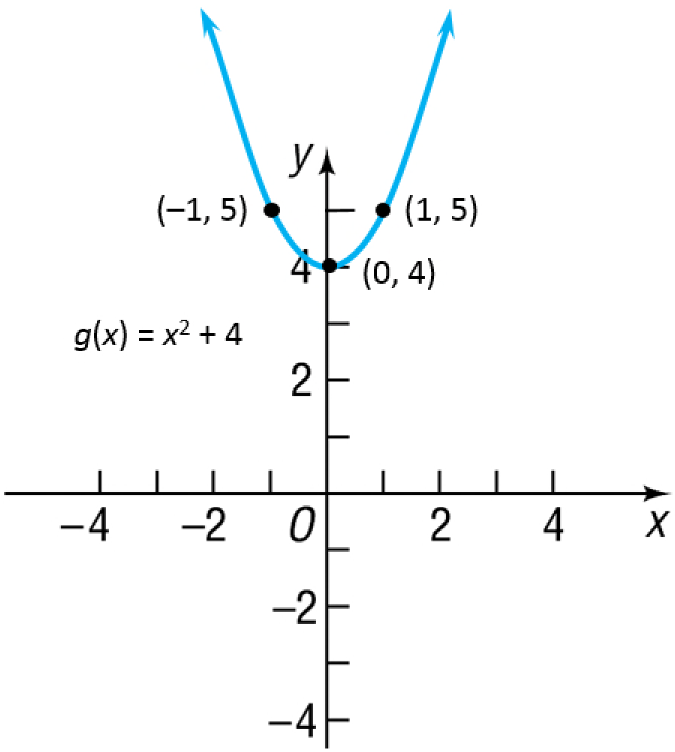
$$x = \pm\sqrt{-4}$$

$$x = \pm 2i$$

Example 1: Finding Complex Zeros of a Quadratic Function (4 of 4)

Solution:

(b) The complex zeros of *g* are $-2i$ and $2i$. Because neither of the zeros is a real number, there are no *x*-intercepts for the graph of *g*. The figure shows the graph of $g(x) = x^2 + 4$.



Theorem

In the complex number system, the solutions of the quadratic equation $ax^2 + bx + c = 0$, where *a*, *b*, and *c* are real numbers and $a \neq 0$, are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \textbf{(1)}$$

Example 2: Finding Complex Zeros of a Quadratic Function (1 of 4)

Find the complex zeros of each of the following quadratic functions. Graph each function and label the intercepts.

(a) $f(x) = x^2 - x - 3$ (b) $g(x) = x^2 + x + 2$

Solution:

(a) To find the complex zeros of $f(x) = x^2 - x - 3,$ solve the equation $f(x) = 0.$

$$f(x) = 0 \qquad a = 1,\ b = -1,\ c = -3$$

$$x^2 - x - 3 = 0 \qquad b^2 - 4ac = (-1)^2 - 4(1)(-3) = 13$$

$$x = \frac{-(-1) \pm \sqrt{13}}{2(1)} = \frac{1 \pm \sqrt{13}}{2} = \frac{1}{2} \pm \frac{\sqrt{13}}{2}$$

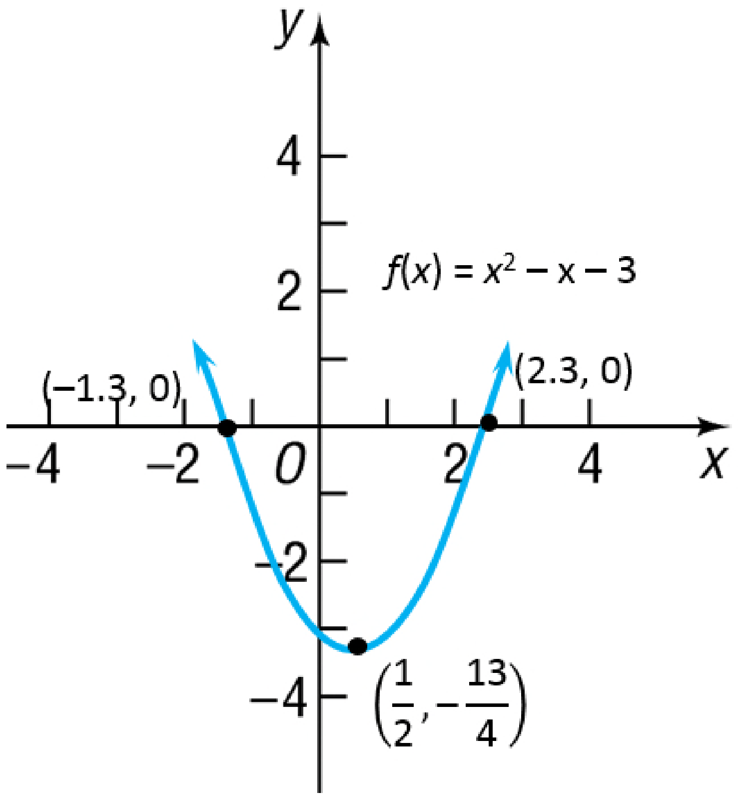
Example 2: Finding Complex Zeros of a Quadratic Function (2 of 4)

Solution:

(a) The complex zeros of *f* are

$$\frac{1}{2}+\frac{\sqrt{13}}{2}\approx 2.3 \text{ and } \frac{1}{2}-\frac{\sqrt{13}}{2}\approx -1.3.$$

Because the zeros are real numbers, the *x*-intercepts of the graph of *f* are the same. The figure shows the graph of $f(x)=x^2-x-3$ with the intercepts labeled.



Example 2: Finding Complex Zeros of a Quadratic Function (3 of 4)

Solution: (b) $g(x) = x^2 + x + 2$

(b) To find the complex zeros of $g(x) = x^2 + x + 2,$ solve the equation $g(x) = 0.$

$$g(x) = 0 \qquad a = 1,\ b = 1,\ c = 2$$

$$x^2 + x + 2 = 0 \qquad b^2 - 4ac = (1)^2 - 4(1)(2) = -7$$

$$x = \frac{-(1) \pm \sqrt{-7}}{2(1)} = \frac{-1 \pm \sqrt{7}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$$

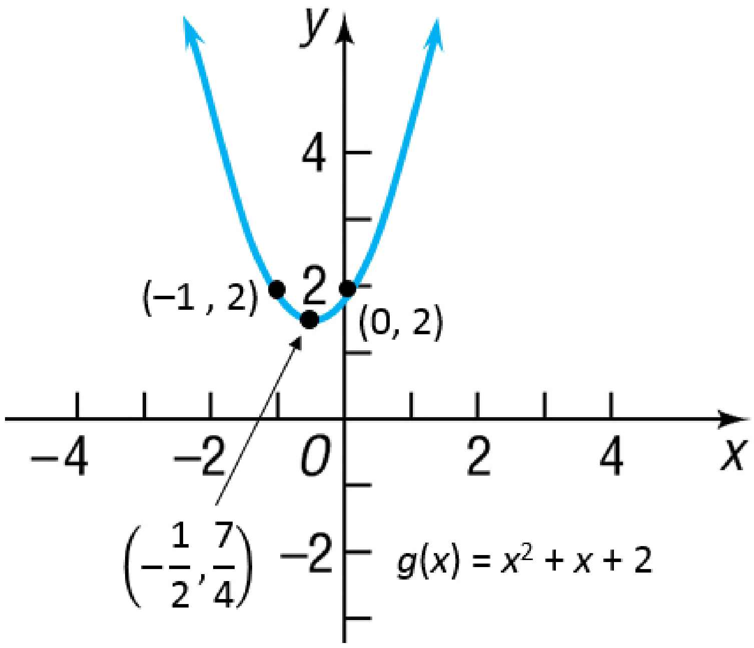
Example 2: Finding Complex Zeros of a Quadratic Function (4 of 4)

Solution:

(b) The complex zeros of *g* are

$$-\frac{1}{2}+\frac{\sqrt{7}}{2}i \text{ and } -\frac{1}{2}-\frac{\sqrt{7}}{2}i.$$

Because neither of the zeros is a real number, there are no *x*-intercepts for the graph of *g*. The figure shows the graph of $g(x)=x^2+x+2.$



Character of the Solutions of a Quadratic Equation

In the complex number system, consider a quadratic equation $ax^2 + bx + c = 0$ with real coefficients.

1. If $b^2 - 4ac > 0$, the equation has two unequal real solutions.
2. If $b^2 - 4ac = 0$, the equation has a repeated real solution, a double root.
3. If $b^2 - 4ac < 0$, the equation has two complex solutions that are not real.

   The complex solutions are conjugates of each other.

Example 3: Determining the Character of the Solutions of a Quadratic Equation

Without solving, determine the character of the solutions of each equation.

(a) $x^2 - 3x + 4 = 0$ (b) $x^2 - 3x + 1 = 0$ (c) $x^2 + 4x + 4 = 0$

Solution:

(a) Here $a = 1,\ b = -3,\ c = 4,$ so $b^2 - 4ac = 9 - 16 = -7.$
The solutions are two complex numbers that are not real and are conjugates of each other.

(b) Here $a = 1,\ b = -3,\ c = 1,$ so $b^2 - 4ac = 9 - 4 = 5.$
The solutions are two unequal real numbers.

(c) Here $a = 1,\ b = 4,\ c = 4,$ so $b^2 - 4ac = 16 - 16 = 0.$
The solution is a repeated real number.

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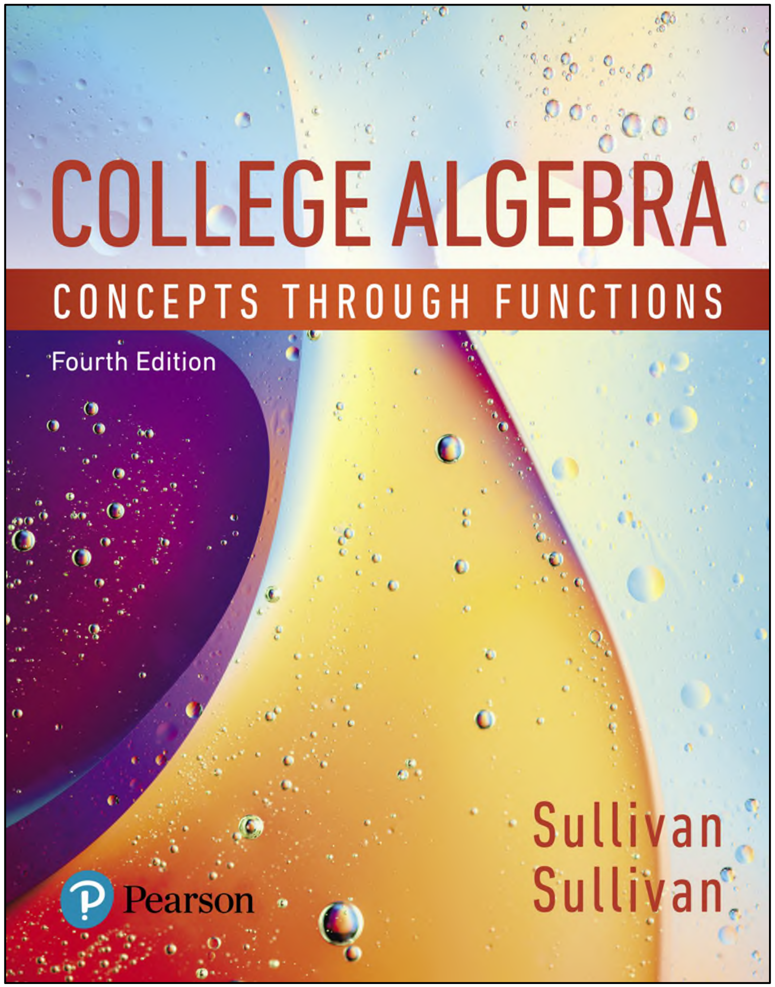


Chapter 2

Linear and Quadratic Functions

Section 2.8 Equations and Inequalities Involving the Absolute Value Function

Objectives

1. Solve Absolute Value Equations
2. Solve Absolute Value Inequalities

Objective 1 Solve Absolute Value Equations

Equations Involving Absolute Value

If *a* is a positive real number and if *u* is any algebraic expression, then

$|u| = a$ is equivalent to $u = a$ or $u = -a$ **(1)**

Example 1: Solving an Equation That Involves Absolute Value

Solve the equation: $|x-3|=2$

Solution:

There are two possibilities:

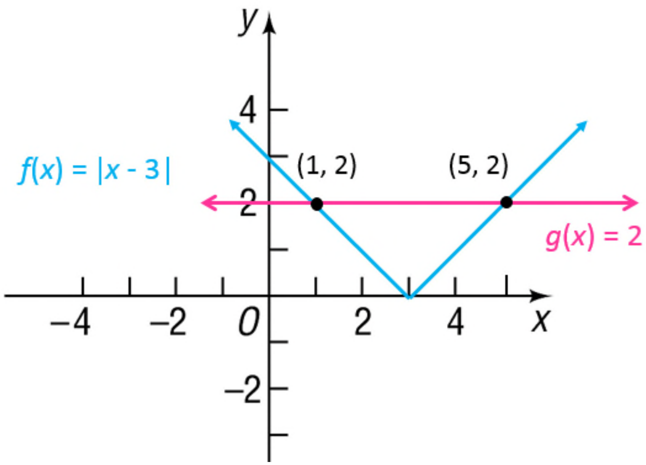
$$x-3=2 \quad \text{or} \quad x-3=-2$$

$$x=5 \quad \text{or} \quad x=1$$

The solution set is $\{1, 5\}$.

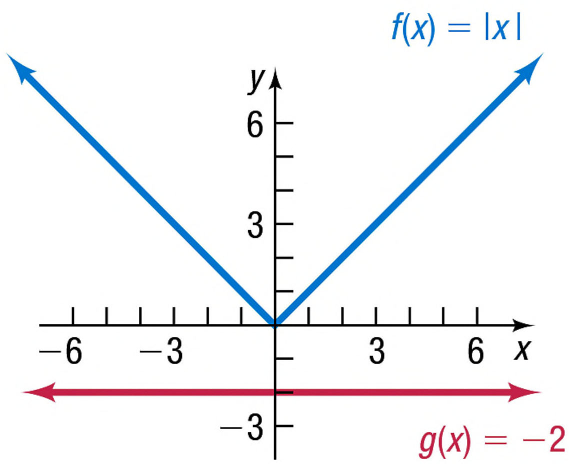
$f(x)=|x-3|;\ g(x)=2.$

The x-coordinates of the points of intersection are 1 and 5, the solutions of the equation $f(x)=g(x)$.



Other Solutions

Equation (1) requires that *a* be a positive number. If *a* = 0, equation (1) becomes $|u| = 0$, which is equivalent to *u* = 0. If *a* is less than zero, the equation has no real solution. The solution set is the empty set, $\varnothing$ or $\{\ \}$.



For example $|x| = -2$ has no solution.

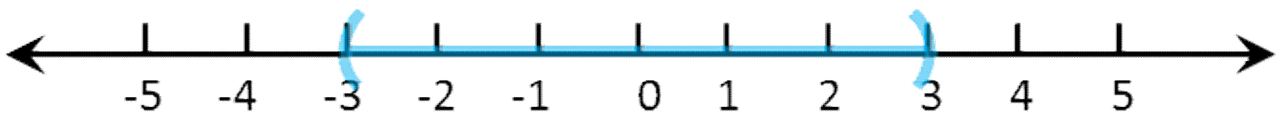
Objective 2 Solve Absolute Value Inequalities

Example 2: Solving an Inequality That Involves Absolute Value (1 of 2)

Solve the inequality: $|x| < 3$

Solution:

The solution is the set of all points whose coordinate *x* is a distance less than 3 units from the origin.



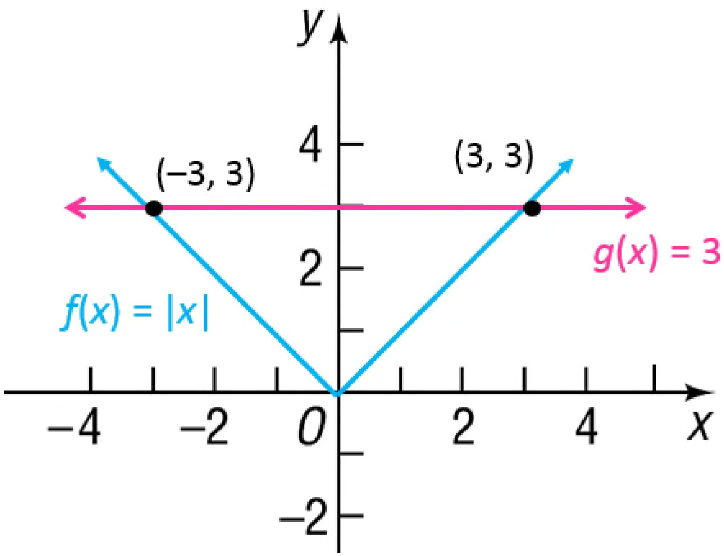
Because any *x* between −3 and 3 satisfies the condition $|x| < 3$, the solution set consists of all numbers *x* for which $-3 < x < 3$, that is, all *x* in the interval (−3, 3).

Example 2: Solving an Inequality That Involves Absolute Value (2 of 2)

Solution:

To visualize the results, graph $f(x) = |x|$ and $g(x) = 3.$ When solving $f(x) < g(x)$, look for all x-coordinates such that the graph of $f(x)$ is below the graph of $g(x)$.

The graph $f(x)$ is below $g(x)$ for all x between −3 and 3. Therefore, we want all x such that $-3 < x < 3$, that is, all x in the interval (−3, 3).



Inequalities Involving Absolute Value (1 of 2)

If *a* is any positive number and if *u* is any algebraic expression, then

$|u| < a$ is equivalent to $-a < u < a$ **(2)**

$|u| \le a$ is equivalent to $-a \le u \le a$ **(3)**

In other words, $|u| < a$ is equivalent to $-a < u$ and $u < a$.

Example 3: Solving an Inequality Involving Absolute Value (1 of 2)

Solve the inequality $|3x-4| \leq 3$, and graph the solution set.

Solution:

$|3x-4| \leq 3$ This follows the form of statement (3); the expression $u = 3x-4$ is inside the absolute value bars.

$-3 \leq 3x-4 \leq 3$ Apply statement (3).

$-3+4 \leq 3x-4+4 \leq 3+4$ Add 4 to each part.

$1 \leq 3x \leq 7$ Simplify.

Example 3: Solving an Inequality Involving Absolute Value (2 of 2)

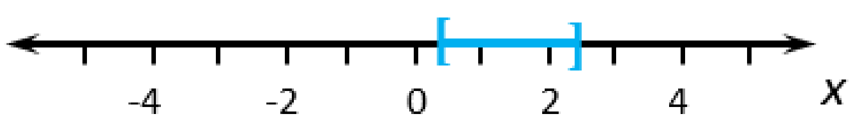
Solution:

$$\frac{1}{3} \leq \frac{3x}{3} \leq \frac{7}{3}$$ Divide each part by 3.

$$\frac{1}{3} \leq x \leq \frac{7}{3}$$ Simplify.

The solution set is $\left\{ x \,\middle|\, \frac{1}{3} \leq x \leq \frac{7}{3} \right\}$.

Using interval notation, the solution is $\left[\frac{1}{3}, \frac{7}{3}\right]$.



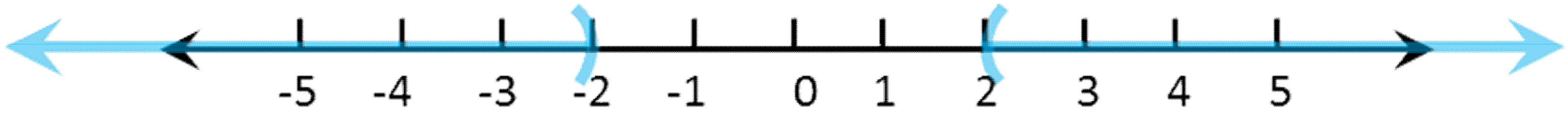
$$|3x - 4| \leq 3$$

Example 4: Solving an Inequality Involving Absolute Value (1 of 2)

Solve the inequality $|x| > 2.$

Solution:

The solution set is the set of all points whose coordinate *x* is a distance greater than 2 units from the origin. The figure illustrates the situation.



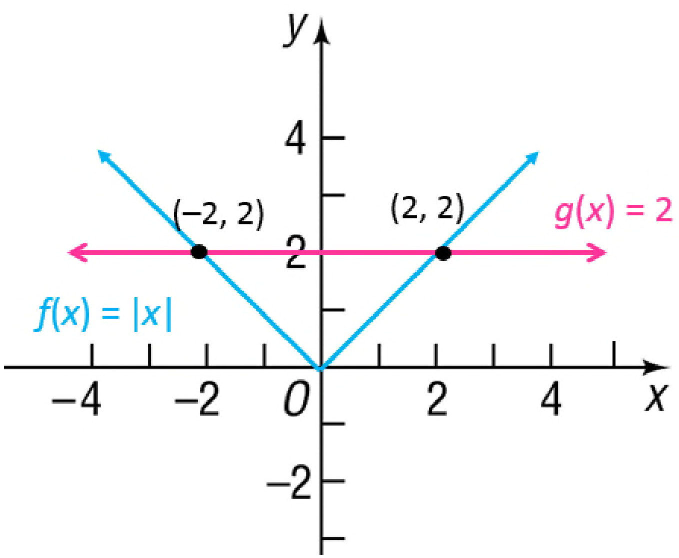
Any *x* less than −2 or greater than 2 satisfies the condition $|x| > 2.$ Consequently, the solution set is $\{x | x < -2 \text{ or } x > 2\}.$ Using interval notation, the solution is $(-\infty, -2) \cup (2, \infty).$

Example 4: Solving an Inequality Involving Absolute Value (2 of 2)

Solution:

To visualize the results, graph $f(x) = |x|$ and $g(x) = 2.$ When solving $f(x) > g(x)$, look for all *x*-coordinates such that the graph of $f(x)$ is above the graph of $g(x)$.

The graph $f(x)$ is above $g(x)$ for all *x* less than −2 or all *x* greater than 2. Therefore, we want all *x* such that $x < -2$ or $x > 2$, that is, all *x* in the interval $(-\infty, -2) \cup (-2, \infty)$.



Inequalities Involving Absolute Value (2 of 2)

If *a* is any positive number and if *u* is any algebraic expression, then

$|u| > a$ is equivalent to $u < -a$ or $u > a$ **(4)**

$|u| \geq a$ is equivalent to $u \leq -a$ or $u \geq a$ **(5)**

Example 5: Solving an Inequality Involving Absolute Value (1 of 2)

Solve the inequality and graph the solution set: $|3x+2| > 5$

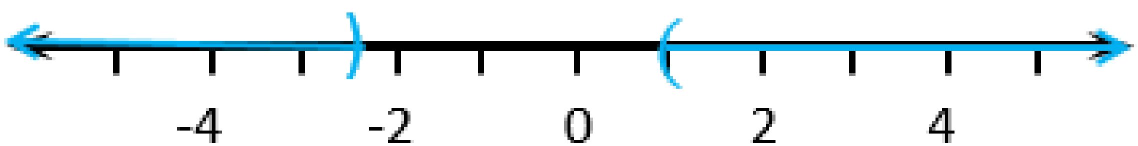
Solution:

$3x+2<-5$ or $3x+2>5$	This follows the form of statement (4).
$3x+2-2<-5-2$ or $3x+2-2>5-2$	Apply statement (4).
$3x<-7$ or $3x>3$	Subtract 2 from each part.
$x<-\dfrac{7}{3}$ or $x>1$	Simplify.

Example 5: Solving an Inequality Involving Absolute Value (2 of 2)

Solution continued:

The solution set is $\{x \mid x < -\frac{7}{3} \text{ or } x > 1\}$. In interval notation, the solution is $\left(-\infty, -\frac{7}{3}\right) \cup (1, \infty)$.



$$|3x + 2| > 5$$