Solutions for Basic College Mathematics 13th Edition by Bittinger

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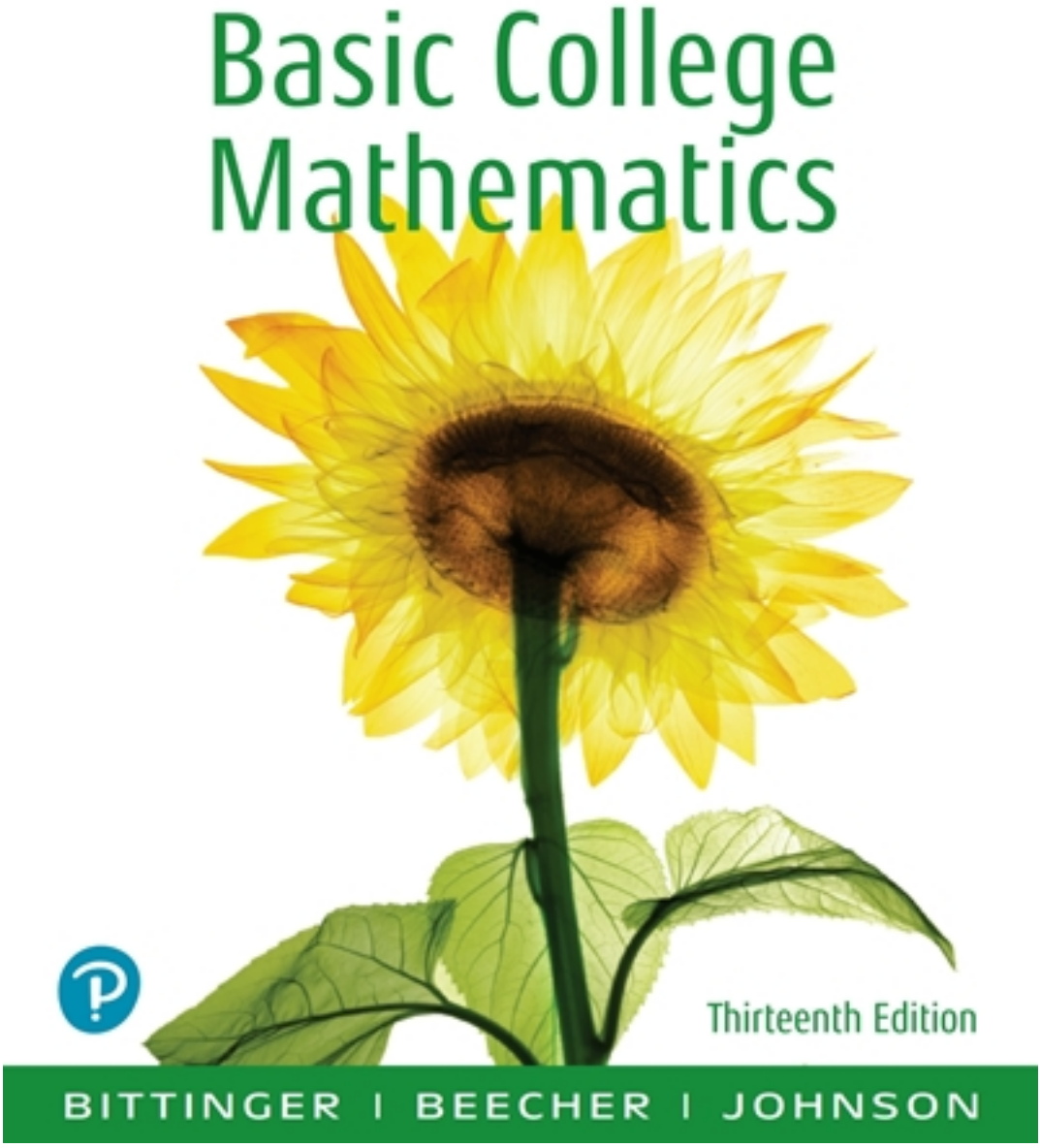


Solutions

Factorizations

Learning Objectives:

a Determine whether one number is a factor of another, and find the factors of a number.
b Find some multiples of a number, and determine whether a number is divisible by another.
c Given a number from 1 to 100, tell whether it is prime, composite, or neither.
d Find the prime factorization of a composite number.

Examples:

1. Determine whether the second number is a factor of the first.

   a) 53; 4 b) 63; 7

2. List all the factors of each number.

   a) 50 b) 109 c) 44

3. Multiply by 1, 2, 3, and so on, to find ten multiples of each number.

   a) 6 b) 15

4. Determine whether the first number is divisible by the second number.

   a) 3892; 7 b) 201; 3

5. Determine whether each number is prime, composite, or neither.

   a) 17 b) 1 c) 33 d) 29

   e) 41 f) 90 g) 59 h) 999

6. Find the prime factorization of each number.

   a) 6 b) 20 c) 30 d) 84

Teaching Notes:

- A common mistake in finding the prime factorization is to not factor completely.
  For example: $60 = 4 \cdot 3 \cdot 5$ or $2 \cdot 2 \cdot 15$, instead of $2 \cdot 2 \cdot 3 \cdot 5$.
- Remind students of the difference between finding the prime factorization and finding all the factors of a number.

Answers: 1a) No, b) Yes; 2a) 1, 2, 5, 10, 25, 50, b) 1, 109, c) 1, 2, 4, 11, 22, 44; 3a) 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, b) 15, 30, 45, 60, 75, 90, 105, 120, 135, 150; 4a) yes, b) yes; 5a) prime, b) neither, c) composite, d) prime, e) prime, f) composite, g) prime, h) composite; 6a) $2 \cdot 3$, b) $2 \cdot 2 \cdot 5$, c) $2 \cdot 3 \cdot 5$, d) $2 \cdot 2 \cdot 3 \cdot 7$

Divisibility

Learning Objective:

a Determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

Example:

1. Consider the following numbers.

468	29	9000	621
84	500	411	14,769
54,555	37,822	95	8271

a) Which of the above are divisible by 2?

b) Which of the above is divisible by 3?

c) Which of the above are divisible by 4?

d) Which of the above is divisible by 5?

e) Which of the above are divisible by 6?

f) Which of the above is divisible by 8?

g) Which of the above are divisible by 9?

h) Which of the above is divisible by 10?

Teaching Note:

- Refer students to the ***Tests for Divisibility*** boxes in the textbook.

Answers: 1a) 468, 84, 500, 37,822, 9000, b) 468, 84, 54,555, 9000, 411, 621, 14,769, 8271, c) 468, 84, 500, 9000, d) 54,555, 500, 9000, 95, e) 468, 84, 9000, f) 9000, g) 468, 9000, 621, 14,769, 8271, h) 500, 9000

Fractions and Fraction Notation

Learning Objectives:

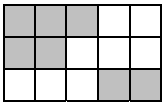
a Identify the numerator and the denominator of a fraction and write fraction notation for part of an object or part of a set of objects and as a ratio.
b Simplify fraction notation like n/n to 1, $0/n$ to 0, and $n/1$ to n.

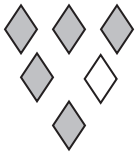
Examples:

1. Identify the numerator and denominator.

   a) $\frac{1}{5}$ b) $\frac{2}{3}$ c) $\frac{17}{15}$ d) $\frac{1}{9}$

2. Use a fraction to represent the shaded part of the object or set of objects.

   a) 

   b) 

3. For a set of five forks and seven spoons, what is the ratio of:

   a) forks to spoons?
   b) spoons to forks?
   c) forks to the total number of elements?
   d) total number of elements to forks?

4. Charlie drove 43 minutes to go to a Celtics game. Seventeen minutes of his trip were spent in bumper-to-bumper traffic. What fractional part of his time was spent in bumper-to-bumper traffic?

5. Simplify.

   a) $\frac{0}{12}$ b) $\frac{17}{17}$ c) $\frac{9-1}{10-9}$ d) $\frac{41}{41}$

   e) $\frac{372}{0}$ f) $\frac{7}{14-14}$ g) $\frac{9}{1}$ h) $\frac{3-3}{74}$

Teaching Notes:

- Some students forget which part of the fraction is the numerator and which is the denominator. An easy way for them to remember which is which is that the ***d***enominator is ***d***own at the bottom.
- Emphasize that the denominator signifies the number of equivalent parts in the whole, whereas the numerator signifies how many parts are being considered.
- Some students forget the difference between a fraction that is zero and a fraction that is not defined.

Answers: 1a) numerator = 1, denominator = 5, b) numerator = 2, denominator = 3, c) numerator = 17, denominator = 15, d) numerator = 1, denominator = 9; 2a) $\frac{7}{15}$, b) $\frac{5}{6}$; 3a) $\frac{5}{7}$, b) $\frac{7}{5}$, c) $\frac{5}{12}$, d) $\frac{12}{5}$; 4) $\frac{17}{43}$; 5a) 0, b) 1, c) 8, d) 1, e) not defined, f) not defined, g) 9, h) 0

Multiplication and Applications

Learning Objectives:

a Multiply a fraction by a fraction, and multiply a fraction by a whole number.
b Solve applied problems involving multiplication of fractions.

Examples:

1. To multiply two fractions, we multiply the _____ of the first fraction by the _____ of the second fraction and then multiply the _____ of the first fraction by the _____ of the second fraction.

2. Multiply.

   a) $\frac{4}{9}\cdot\frac{2}{3}$ b) $\frac{5}{6}\cdot\frac{5}{7}$ c) $\frac{1}{9}\times\frac{2}{5}$

   d) $2\times\frac{1}{9}$ e) $\frac{3}{14}\cdot 5$ f) $\frac{1}{11}\cdot 1$

3. A recipe calls for $\frac{2}{3}$ cup of cheese. How much cheese is needed to make $\frac{1}{3}$ of the recipe?

4. Part of a room is being carpeted. An area $\frac{5}{9}$ of the length and $\frac{7}{11}$ of the width is covered. What fraction of the room is being carpeted?

Teaching Notes:

- Using a visual reference will help students understand the concept of multiplying fractions.
- Remind students to label answers with the appropriate units.

Answers: 1) numerator, numerator, denominator, denominator, or denominator, denominator, numerator, numerator; 2a) $\frac{8}{27}$*, b)* $\frac{25}{42}$*, c)* $\frac{2}{45}$*, d)* $\frac{2}{9}$*, e)* $\frac{15}{14}$*, f)* $\frac{1}{11}$*; 3)* $\frac{2}{9}$ *cup; 4)* $\frac{35}{99}$

Simplifying

Learning Objectives:

a Multiply a number by 1 to find fraction notation with a specified denominator.
b Simplify fraction notation.
c Use the test for equality to determine whether two fractions name the same number.

Examples:

1. Find another name for the given number, but with the denominator indicated. Use multiplying by 1.

   a) $\frac{1}{5}=\frac{?}{20}$ b) $\frac{3}{7}=\frac{?}{21}$ c) $\frac{5}{8}=\frac{?}{40}$ d) $\frac{13}{15}=\frac{?}{45}$

2. Simplify.

   a) $\frac{5}{10}$ b) $\frac{16}{64}$ c) $\frac{42}{77}$

   d) $\frac{88}{90}$ e) $\frac{66}{77}$ f) $\frac{1100}{1200}$

3. Use = or ≠ for $\square$ to write a true sentence.

   a) $\frac{1}{3}\square\frac{2}{6}$ b) $\frac{21}{33}\square\frac{7}{13}$ c) $\frac{4}{10}\square\frac{24}{60}$ d) $\frac{4}{30}\square\frac{16}{90}$

Teaching Notes:

- Emphasize the use of removing factors of 1.
- Warn students of the incorrect ways of canceling.
- The concept of equivalent fractions will be an important skill in later sections.
- Remind students to find the cross products as a test for equality.

Answers: 1a) $\frac{4}{20}$, b) $\frac{9}{21}$, c) $\frac{25}{40}$, d) $\frac{39}{45}$; 2a) $\frac{1}{2}$, b) $\frac{1}{4}$, c) $\frac{6}{11}$, d) $\frac{44}{45}$, e) $\frac{6}{7}$, f) $\frac{11}{12}$; 3a) =, b) ≠, c) =, d) ≠

Multiplying, Simplifying, and Applications

Learning Objectives:

a Multiply and simplify using fraction notation.
b Solve applied problems involving multiplication of fractions.

Examples:

1. Multiply and simplify.

a) $\frac{1}{2}\cdot\frac{3}{4}$ b) $\frac{5}{9}\cdot\frac{3}{15}$ c) $\frac{5}{6}\cdot\frac{9}{2}$

d) $\frac{5}{24}\cdot\frac{36}{25}$ e) $7\cdot\frac{3}{5}$ f) $\frac{4}{9}\cdot 18$

g) $36\cdot\frac{1}{9}$ h) $\frac{4}{5}\cdot\frac{35}{16}$ i) $\frac{12}{37}\cdot\frac{37}{132}$

2. Solve.

a) Of students in a class, $\frac{7}{8}$ are right-handed. Of these students, $\frac{1}{7}$ have declared majors. What fraction of the students are right-handed and have declared majors?

b) A house worth \$210,000 is assessed for $\frac{7}{8}$ of its value. What is the assessed value of the house?

c) On a map, 1 in. represents 80 mi. What distance does $\frac{1}{2}$ in. represent?

Teaching Notes:

- Most students do better if they remove factors of 1 before multiplying.
- Emphasize the importance of factoring before canceling.
- Remind students to label answers with the appropriate units.

Answers: 1a) $\frac{3}{8}$, b) $\frac{1}{9}$, c) $\frac{15}{4}$, d) $\frac{3}{10}$; e) $\frac{21}{5}$, f) 8, g) $\frac{1}{4}$, h) $\frac{7}{4}$, i) $\frac{1}{11}$; 2a) $\frac{1}{8}$, b) \$183,750, c) 40 mi

Division and Applications

Learning Objectives:

a Find the reciprocal of a number.
b Divide and simplify using fraction notation.
c Solve equations of the type $a \cdot x = b$ and $x \cdot a = b$, where a and b may be fractions.
d Solve applied problems involving division of fractions.

Examples:

1. To divide fractions, you multiply the dividend by the _____ of the divisor.

2. Find the reciprocal of each number;

   a) 3 b) $\frac{1}{2}$ c) $\frac{9}{5}$ d) 60

3. Divide and simplify.

   a) $\frac{1}{2} \div \frac{3}{4}$ b) $\frac{5}{9} \div \frac{3}{15}$ c) $\frac{5}{6} \div \frac{9}{2}$ d) $\frac{5}{24} \div \frac{36}{24}$

   e) $7 \div \frac{3}{5}$ f) $\frac{4}{9} \div 4$ g) $2 \div \frac{18}{36}$ h) $\frac{4}{5} \div 28$

4. Solve.

   a) $\frac{5}{8} \cdot x = 25$ b) $\frac{9}{4} \cdot p = 144$ c) $y \cdot \frac{2}{9} = 4$ d) $n \cdot \frac{11}{3} = 77$

5. Solve.

   a) Jody is using a recipe that calls for $\frac{1}{4}$ cup of milk per batch. If she has $\frac{23}{4}$ cups of milk available, how many batches can she make?

   b) A tank had 12 gal of gasoline in it when it was $\frac{3}{5}$ full. How much could it hold when full?

Teaching Notes:

- Every number, except zero, has a reciprocal.
- Remind students that $n/0$ is not defined.
- Emphasize that dividing by a fraction is the same as multiplying by the reciprocal of that fraction.
- Some students incorrectly cancel before taking the reciprocal of the second fraction.
- Many students have trouble determining the divisor in word problems.

Answers: 1) reciprocal; 2a) $\frac{1}{3}$, b) 2, c) $\frac{5}{9}$, d) $\frac{1}{60}$, 3a) $\frac{2}{3}$, b) $\frac{25}{9}$, c) $\frac{5}{27}$, d) $\frac{5}{36}$, e) $\frac{35}{3}$, f) $\frac{1}{9}$, g) 4, h) $\frac{1}{35}$; 4a) 40, b) 64, c) 18, d) 21; 5a) 23 batches, b) 20 gal

CHAPTER 2 **NAME**_____

TEST FORM A **CLASS**_____ **SCORE**_____ **GRADE**_____

1. Find all the factors of 200.

Determine whether each number is prime, composite, or neither.

2. 29

3. 49

Find the prime factorization of the number.

4. 42

5. 60

6. Determine whether 2341 is divisible by 5.

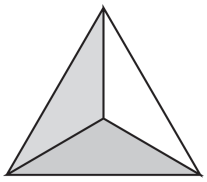
7. Determine whether 1888 is divisible by 6.

8. Determine whether 3186 is divisible by 9.

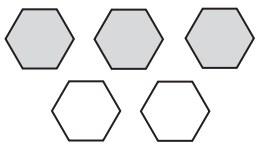
9. Determine whether 522 is divisible by 2.

10. Identify the numerator and the denominator of $\frac{6}{17}$.

11. What part is shaded?



12. What part of the set is shaded?



ANSWERS

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____

CHAPTER 2 **NAME**_____

TEST FORM A

ANSWERS

13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____

13. In 2014, the United States listed 720 plant species as endangered. Of these, 688 species were flowering plants. What was the ratio of endangered flowering plant species to total number of endangered plant species in the U.S.? What was the ratio of the endangered non-flowering plant species to total number of endangered plant species in the U.S.?

Simplify.

14. $\frac{5}{5}$

15. $\frac{12}{1}$

16. $\frac{4}{6-6}$

17. $\frac{35}{7}$

18. $\frac{4}{24}$

19. $\frac{0}{6}$

20. $\frac{8}{0}$

21. $\frac{12}{15}$

Use = or $\neq$ for $\square$ to write a true sentence.

22. $\frac{5}{6} \square \frac{8}{10}$

23. $\frac{24}{66} \square \frac{6}{11}$

CHAPTER 2 **NAME**_____

TEST FORM A

Multiply and simplify.

24. $\frac{3}{4} \cdot 20$

25. $6 \cdot \frac{5}{12}$

26. $\frac{3}{7} \cdot \frac{28}{15}$

27. $\frac{8}{11} \cdot \frac{1}{2}$

Find the reciprocal.

28. $\frac{2}{3}$

29. $\frac{1}{8}$

30. 14

Divide and simplify.

31. $\frac{1}{4} \div \frac{1}{9}$

32. $20 \div \frac{2}{5}$

33. $\frac{4}{9} \div \frac{2}{3}$

Solve.

34. $\frac{2}{7} \cdot x = 56$

35. $t \cdot \frac{2}{5} = \frac{4}{7}$

36. At Northern Spring College there are 576 students, and $\frac{3}{8}$ of them are enrolled in an external degree program. How many are external degree students?

ANSWERS

24. _____

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

31. _____

32. _____

33. _____

34. _____

35. _____

36. _____

CHAPTER 2 **NAME**_____

TEST FORM A

ANSWERS

37. _____

38. _____

39. _____

40. _____

41. _____

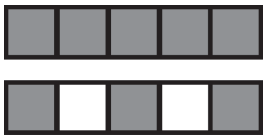
42. _____

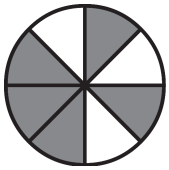
37. A loaf of bread $\frac{1}{2}$ m long is cut into 12 equal pieces. How long is each piece?

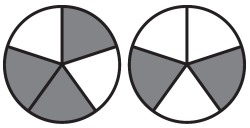
38. A gas tank held 4 gal when it was $\frac{3}{5}$ full. How much gas could it hold when full?

39. A book is $\frac{9}{16}$ in. wide. How wide is a set of 20 books?

40. In which figure does the shaded part represent $\frac{5}{8}$ of the figure?

A. 

B. 

C. 

D. 

41. Jason's house sits on a $\frac{3}{4}$-acre lot and $\frac{5}{8}$ of the lot is lawn. One day Jason mowed $\frac{1}{2}$ of the lawn on the lot. How much did he mow?

42. Simplify: $\left(\frac{3}{4} \div \frac{5}{6} \div \frac{15}{2}\right)^2$.

CHAPTER 2 **NAME**_____

TEST FORM B **CLASS**_____ **SCORE**_____ **GRADE**_____

1. Find all the factors of 210.

Determine whether each number is prime, composite, or neither.

2. 15

3. 31

Find the prime factorization of the number.

4. 66

5. 45

6. Determine whether 2842 is divisible by 6.

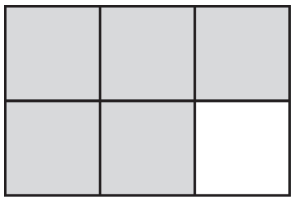
7. Determine whether 4933 is divisible by 9.

8. Determine whether 8762 is divisible by 3.

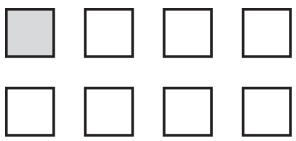
9. Determine whether 14,020 is divisible by 5.

10. Identify the numerator and the denominator of $\frac{9}{10}$.

11. What part is shaded?



12. What part of the set is shaded?



ANSWERS

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____

CHAPTER 2 **NAME**_____

TEST FORM B

ANSWERS

13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____

13. In 2014, there were 326 mammals considered to be endangered. Of these, 70 were listed in the U.S. What was the ratio of endangered U.S. mammals to total endangered mammals. What was the ratio of endangered mammals not listed in the U.S. to total endangered mammals?

Simplify.

14. $\frac{12}{12}$

15. $\frac{11}{1}$

16. $\frac{5}{10-10}$

17. $\frac{20}{4}$

18. $\frac{8}{48}$

19. $\frac{0}{17}$

20. $\frac{31}{0}$

21. $\frac{20}{24}$

Use = or ≠ for ☐ to write a true sentence.

22. $\frac{12}{18} \square \frac{10}{15}$

23. $\frac{4}{6} \square \frac{9}{12}$

CHAPTER 2 **NAME**_____

TEST FORM B

Multiply and simplify.

24. $\frac{4}{7} \cdot 28$

25. $3 \cdot \frac{7}{12}$

26. $\frac{5}{6} \cdot \frac{3}{20}$

27. $\frac{1}{3} \cdot \frac{9}{10}$

Find the reciprocal.

28. $\frac{3}{8}$

29. $\frac{1}{5}$

30. 20

Divide and simplify.

31. $\frac{1}{3} \div \frac{1}{6}$

32. $6 \div \frac{2}{3}$

33. $\frac{3}{5} \div \frac{6}{25}$

Solve.

34. $\frac{2}{3} \cdot x = 18$

35. $t \cdot \frac{6}{11} = \frac{22}{5}$

36. At Northern Spring College there are 576 students, and $\frac{1}{9}$ of them are registered for December graduation. How many are registered to graduate in December?

ANSWERS

24. _____

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

31. _____

32. _____

33. _____

34. _____

35. _____

36. _____

CHAPTER 2 **NAME**_____

TEST FORM B

ANSWERS

37. _____

38. _____

39. _____

40. _____

41. _____

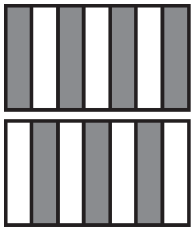
42. _____

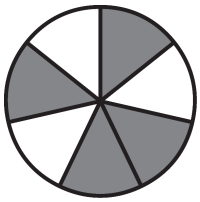
37. A piece of ribbon $\frac{3}{5}$ m long is cut into 4 equal pieces. How long is each piece?

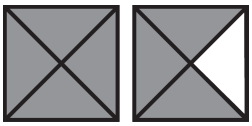
38. A gas tank held 24 gal when it was $\frac{4}{5}$ full. How much gas could it hold when full?

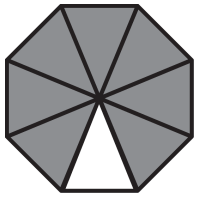
39. A booklet is $\frac{3}{8}$ in. wide. How wide is a set of 10 booklets?

40. In which figure does the shaded part represent $\frac{7}{4}$ of the figure?

A. 

B. 

C. 

D. 

41. Beth's house sits on a $\frac{4}{5}$-acre lot and $\frac{2}{3}$ of the lot is lawn. One day Beth mowed $\frac{3}{4}$ of the lawn on her lot. How much did she mow?

42. Simplify: $\left(\frac{3}{4}\right)^2 \div \frac{1}{9} \div \frac{2}{5}$.

CHAPTER 2 **NAME**_____

TEST FORM C **CLASS**_____**SCORE**_____**GRADE**_____

1. Find all the factors of 240.

Determine whether each number is prime, composite, or neither.

2. 39

3. 1

Find the prime factorization of the number.

4. 18

5. 40

6. Determine whether 4350 is divisible by 3.

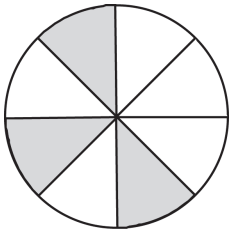
7. Determine whether 3735 is divisible by 5.

8. Determine whether 4264 is divisible by 8.

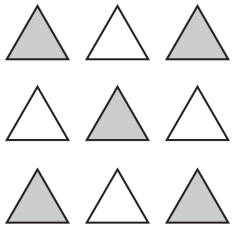
9. Determine whether 13,825 is divisible by 10.

10. Identify the numerator and the denominator of $\frac{7}{8}$.

11. What part is shaded?



12. What part of the set is shaded?



ANSWERS

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CHAPTER 2 **NAME**_____

TEST FORM C

ANSWERS

13. _____
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16. _____
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18. _____
19. _____
20. _____
21. _____
22. _____
23. _____

13. In 2014, there were 26 amphibian species considered to be endangered. Of these, 18 species were listed in the U.S. What was the ratio of endangered U.S. amphibian species to total endangered amphibian species? What was the ratio of endangered amphibian species not listed in the U.S. to total endangered amphibian species?

Simplify.

14. $\frac{4}{4}$

15. $\frac{15}{1}$

16. $\frac{8}{4-4}$

17. $\frac{36}{9}$

18. $\frac{6}{36}$

19. $\frac{0}{12}$

20. $\frac{7}{0}$

21. $\frac{18}{24}$

Use = or ≠ for $\square$ to write a true sentence.

22. $\frac{7}{8} \square \frac{42}{48}$

23. $\frac{12}{15} \square \frac{8}{10}$

CHAPTER 2 **NAME**_____

TEST FORM C

Multiply and simplify.

24. $\frac{3}{5} \cdot 25$

25. $8 \cdot \frac{5}{16}$

26. $\frac{4}{5} \cdot \frac{1}{12}$

27. $\frac{1}{8} \cdot \frac{2}{5}$

Find the reciprocal.

28. 16

29. $\frac{1}{6}$

30. $\frac{9}{10}$

Divide and simplify.

31. $\frac{1}{6} \div \frac{1}{5}$

32. $20 \div \frac{3}{4}$

33. $\frac{4}{5} \div \frac{3}{10}$

Solve.

34. $\frac{2}{5} \cdot x = 10$

35. $t \cdot \frac{4}{9} = \frac{7}{36}$

36. At Northern Spring College there are 576 students, and $\frac{7}{12}$ of them live on campus. How many live on campus?

ANSWERS

24. _____

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

31. _____

32. _____

33. _____

34. _____

35. _____

36. _____

CHAPTER 2 **NAME**_____

TEST FORM C

ANSWERS

37. _____

38. _____

39. _____

40. _____

41. _____

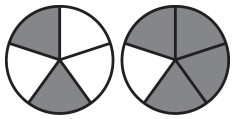
42. _____

37. A piece of metallic thread $\frac{3}{4}$ m long is cut into 6 equal pieces. What is the length of each piece?

38. A gas tank held 8 gal when it was $\frac{4}{5}$ full. How much gas could it hold when full?

39. A booklet is $\frac{1}{4}$ in. wide. How wide is a set of 80 booklets?

40. In which figure does the shaded part represent $\frac{6}{5}$ of the figure?

A. 

B. 

C. 

D. 

41. Charlie's house sits on a $\frac{5}{8}$-acre lot and $\frac{1}{2}$ of the lot is lawn. One day Charlie mowed $\frac{2}{3}$ of the lawn on his lot. How much did he mow?

42. Simplify: $\frac{4}{5} \div \left(\frac{1}{10}\right)^2 \div 20$.

CHAPTER 2 **NAME**_____

TEST FORM D **CLASS**_____ **SCORE**_____ **GRADE**_____

1. Find all the factors of 180.

Determine whether each number is prime, composite, or neither.

2. 18

3. 7

Find the prime factorization of the number.

4. 32

5. 90

6. Determine whether 1500 is divisible by 5.

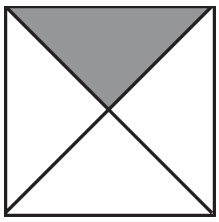
7. Determine whether 1279 is divisible by 9.

8. Determine whether 13,041 is divisible by 3.

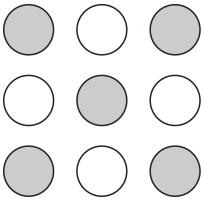
9. Determine whether 17,640 is divisible by 6.

10. Identify the numerator and the denominator of $\frac{2}{3}$.

11. What part is shaded?



12. What part of the set is shaded?



ANSWERS

1. _____
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3. _____
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7. _____
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9. _____
10. _____
11. _____
12. _____

CHAPTER 2 **NAME**_____

TEST FORM D

ANSWERS

13. _____
14. _____
15. _____
16. _____
17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____

13. In 2014, there were 292 species of birds considered endangered. Of these, 80 species were listed in the U.S. What was the ratio of U.S. endangered bird species to total endangered bird species? What was the ratio of endangered bird species not listed in the U.S. to total endangered bird species?

Simplify.

14. $\frac{5}{5}$

15. $\frac{20}{1}$

16. $\frac{12}{3-3}$

17. $\frac{25}{5}$

18. $\frac{8}{48}$

19. $\frac{0}{10}$

20. $\frac{6}{0}$

21. $\frac{16}{56}$

Use = or ≠ for ☐ to write a true sentence.

22. $\frac{3}{8} \square \frac{12}{32}$

23. $\frac{35}{45} \square \frac{14}{16}$

CHAPTER 2 **NAME**_____

TEST FORM D

Multiply and simplify.

24. $\frac{3}{5} \cdot 25$

25. $5 \cdot \frac{3}{20}$

26. $\frac{2}{3} \cdot \frac{9}{10}$

27. $\frac{7}{8} \cdot \frac{2}{3}$

Find the reciprocal.

28. $\frac{1}{8}$

29. 15

30. $\frac{4}{9}$

Divide and simplify.

31. $\frac{1}{5} \div \frac{1}{6}$

32. $15 \div \frac{3}{5}$

33. $\frac{4}{5} \div \frac{8}{9}$

Solve.

34. $\frac{3}{4} \cdot x = 12$

35. $t \cdot \frac{4}{5} = \frac{8}{9}$

36. At Northern Spring College there are 576 students, and $\frac{2}{3}$ of them have declared a major. How many have declared a major?

ANSWERS

24. _____

25. _____

26. _____

27. _____

28. _____

29. _____

30. _____

31. _____

32. _____

33. _____

34. _____

35. _____

36. _____

CHAPTER 2 **NAME**_____

TEST FORM D

ANSWERS

37. _____

38. _____

39. _____

40. _____

41. _____

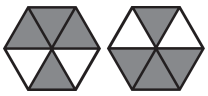
42. _____

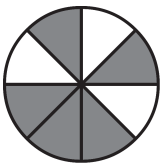
37. A piece of tubing $\frac{4}{5}$ m long is cut into 12 equal pieces. How long is each piece?

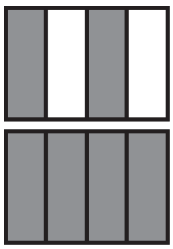
38. A gas tank held 12 gal when it was $\frac{3}{4}$ full. How much gas could it hold when full?

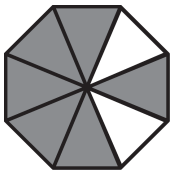
39. A CD case is $\frac{5}{8}$ in. high. How high would a stack of 6 CD cases be?

40. In which figure does the shaded part represent $\frac{6}{8}$ of the figure?

A. 

B. 

C. 

D. 

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41. Theresa's house sits on a $\frac{7}{8}$-acre lot and $\frac{1}{3}$ of the lot is lawn. One day Theresa mowed $\frac{3}{4}$ of the lawn on her lot. How much did she mow?

42. Simplify: $\left(\frac{5}{8}\right)^2 \div \frac{3}{4} \div 2$.

CHAPTER 2 **NAME**_____

TEST FORM E **CLASS**_____**SCORE**_____**GRADE**_____

1. Find all the factors of 200.

2. Determine whether 1 is prime, composite, or neither.

Find the prime factorization of the number.

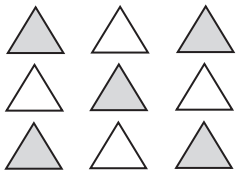
3. 24

4. 125

5. Determine whether 9052 is divisible by 4.

6. Identify the numerator and the denominator of $\frac{3}{4}$.

7. What part of the set is shaded?



Simplify.

8. $\frac{5}{30}$

9. $\frac{4}{1}$

10. $\frac{16}{24}$

11. $\frac{11}{0}$

12. $\frac{0}{15}$

13. $\frac{6}{8-8}$

Use = or ≠ for $\square$ to write a true sentence.

14. $\frac{2}{3} \square \frac{7}{8}$

15. $\frac{15}{25} \square \frac{9}{12}$

Multiply and simplify.

16. $\frac{2}{3} \cdot 12$

17. $7 \cdot \frac{5}{14}$

18. $\frac{2}{3} \cdot \frac{6}{25}$

19. $\frac{3}{5} \cdot \frac{10}{33}$

ANSWERS

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CHAPTER 2 **NAME**_____

TEST FORM E

ANSWERS

20. _____
21. _____
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27. _____
28. _____
29. _____
30. _____
31. _____

Find the reciprocal.

20. $\frac{1}{3}$ 21. $\frac{4}{5}$

Divide and simplify.

22. $\frac{4}{5} \div \frac{11}{10}$ 23. $\frac{1}{4} \div \frac{1}{12}$ 24. $20 \div \frac{4}{25}$

Solve.

25. $\frac{3}{8} \cdot x = 48$ 26. $t \cdot \frac{3}{4} = \frac{20}{11}$

27. In 2014, there were 84 species of reptiles considered endangered. Of these, 14 species were listed in the U.S. What was the ratio of U.S. endangered reptile species to total endangered reptile species? What was the ratio of endangered reptile species not listed in the U.S. to total endangered reptile species?

28. A child's dress requires $\frac{3}{4}$ yd fabric. How many dresses can be made with 12 yd fabric?

29. After Taylor drove 60 mi, she noted that she had driven $\frac{5}{12}$ of her trip. How far was her trip?

30. Simplify: $\frac{7}{8} \cdot \frac{4}{5} \div \left(\frac{1}{2}\right)^3$.

31. Simplify: $\frac{1820}{5148}$.

CHAPTER 2 **NAME**_____

TEST FORM F **CLASS**_____**SCORE**_____**GRADE**_____

1. Find all the factors of 120.

2. Determine whether 15 is prime, composite, or neither.

Find the prime factorization of the number.

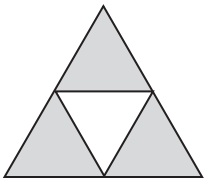
3. 72

4. 98

5. Determine whether 1826 is divisible by 9.

6. Identify the numerator and the denominator of $\frac{1}{8}$.

7. What part is shaded?



Simplify.

8. $\frac{9}{3}$

9. $\frac{6}{18}$

10. $\frac{2}{1-1}$

11. $\frac{15}{0}$

12. $\frac{11}{11}$

13. $\frac{24}{40}$

Use = or ≠ for $\square$ to write a true sentence.

14. $\frac{5}{8} \square \frac{20}{32}$

15. $\frac{8}{12} \square \frac{12}{16}$

Multiply and simplify.

16. $\frac{3}{4} \cdot 24$

17. $20 \cdot \frac{4}{5}$

18. $\frac{4}{9} \cdot \frac{12}{5}$

19. $\frac{5}{8} \cdot \frac{3}{10}$

ANSWERS

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CHAPTER 2 **NAME**_____

TEST FORM F

ANSWERS

20. _____
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23. _____
24. _____
25. _____
26. _____
27. _____
28. _____
29. _____
30. _____
31. _____

Find the reciprocal.

20. 30 21. $\frac{6}{7}$

Divide and simplify.

22. $\frac{3}{8} \div \frac{5}{6}$ 23. $\frac{1}{2} \div \frac{1}{9}$ 24. $14 \div \frac{14}{15}$

Solve.

25. $\frac{3}{4} \cdot x = 18$ 26. $t \cdot \frac{4}{5} = \frac{16}{3}$

27. In 2014, 64 species of insects were considered to be endangered. Of these 60 were listed in the U.S. What was the ratio of endangered U.S. insect species to total endangered insect species? What was the ratio of endangered insect species not listed in the U.S. to total endangered insect species?

28. A loop requires $\frac{5}{8}$ in. wire. How many loops can be made with 40 in. of wire?

29. Thorn's water cooler holds 6 gal of water when it is $\frac{3}{5}$ full. How much does it hold when full?

30. Simplify: $\left(\frac{3}{4} \div \frac{5}{8} \div 2\right)^2$.

31. Simplify: $\frac{4500}{5520}$.

CHAPTER 2 **NAME**_____

TEST FORM G **CLASS**_____**SCORE**_____**GRADE**_____

1. Which of the following numbers is composite?

   a) 1 b) 22 c) 13 d) 2

2. Find the prime factorization of 72.

   a) $8 \cdot 9$ b) $2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

   c) $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$ d) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

3. Determine which of the following is divisible by 6.

   a) 1336 b) 1350 c) 2036 d) 2043

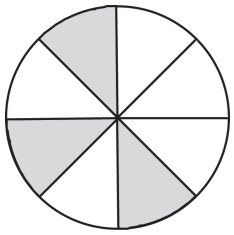
4. Determine which of the following is divisible by 3.

   a) 1313 b) 3176 c) 5302 d) 5082

5. Identify the denominator of $\frac{7}{8}$.

   a) 8 b) 56 c) 7 d) $\frac{8}{7}$

6. What part is shaded?

   a) $\frac{8}{3}$ b) $\frac{5}{8}$

   c) $\frac{3}{5}$ d) $\frac{3}{8}$

7. Simplify: $\frac{6}{18}$.

   a) 3 b) 6 c) $\frac{1}{3}$ d) $\frac{1}{6}$

ANSWERS

1. _____
2. _____
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CHAPTER 2 **NAME**_____

TEST FORM G

ANSWERS

8. _____

9. _____

10. _____

11. _____

12. _____

8. Simplify: $\frac{0}{12}$.

a) 0 b) 1 c) 12 d) Not defined

9. Simplify: $\frac{48}{6}$.

a) $\frac{1}{8}$ b) 7 c) $\frac{28}{3}$ d) 8

10. Multiply and simplify: $\frac{5}{6}\cdot\frac{12}{17}$.

a) $\frac{17}{10}$ b) $\frac{5}{34}$ c) $\frac{85}{72}$ d) $\frac{10}{17}$

11. Multiply and simplify: $20\cdot\frac{15}{16}$.

a) $\frac{3}{4}$ b) $\frac{64}{3}$ c) $\frac{75}{4}$ d) $\frac{335}{16}$

12. By 6 P.M., Julia had typed $\frac{3}{4}$ of her 12-page paper. How many pages had she typed?

a) 8 pages b) 9 pages c) 16 pages d) 3 pages

CHAPTER 2 **NAME**_____

TEST FORM G

13. Find the reciprocal of $\frac{3}{5}$.

a) $\frac{5}{3}$ b) $\frac{9}{25}$ c) $\frac{3}{5}$ d) $\frac{1}{15}$

14. Divide and simplify: $36 \div \frac{4}{9}$.

a) 16 b) 1 c) 81 d) 1296

15. Divide and simplify: $\frac{4}{9} \div \frac{8}{15}$.

a) $\frac{5}{6}$ b) $\frac{6}{5}$ c) $\frac{2}{15}$ d) $\frac{15}{2}$

16. Solve: $\frac{4}{5} \cdot x = \frac{6}{11}$.

a) $\frac{24}{55}$ b) $\frac{15}{22}$ c) $\frac{22}{5}$ d) $\frac{55}{24}$

17. Solve: $\frac{5}{9} \cdot x = 135$.

a) 75 b) 3 c) 6075 d) 243

ANSWERS

13. _____

14. _____

15. _____

16. _____

17. _____

ANSWERS

18. _____

19. _____

20. _____

21. _____

18. A piece of wire $\frac{7}{10}$ m long is cut into 4 equal pieces. What is the length of each piece?

a) $\frac{14}{5}$ m b) $\frac{5}{14}$ m c) $\frac{7}{40}$ m d) $\frac{40}{7}$ m

19. After driving 180 mi, Larry notes that he has completed $\frac{3}{5}$ of his trip. How far is his trip?

a) 300 mi b) 108 mi c) 360 mi d) 288 mi

20. If $\frac{1}{6}$ of a number is $\frac{1}{5}$, how much is $\frac{1}{4}$ of the number?

a) $\frac{1}{120}$ b) $\frac{3}{10}$ c) $\frac{5}{24}$ d) $\frac{10}{3}$

21. Simplify: $\left(\frac{2}{3} \div \frac{5}{8} \div \frac{4}{3}\right)^2$.

a) $\frac{21}{256}$ b) $\frac{4}{5}$ c) $\frac{16}{25}$ d) $\frac{25}{16}$

CHAPTER 2 **NAME**_____

TEST FORM H **CLASS**_____**SCORE**_____**GRADE**_____

1. Which of the following numbers is prime?

   a) 1 b) 15 c) 28 d) 11

2. Find the prime factorization of 60.

   a) $6 \cdot 10$ b) $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

   c) $2 \cdot 3 \cdot 3 \cdot 5$ d) $2 \cdot 2 \cdot 3 \cdot 5$

3. Determine which of the following is divisible by 3.

   a) 2383 b) 9128 c) 1427 d) 1374

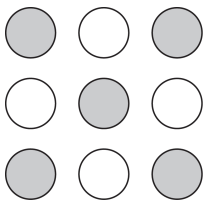
4. Determine which of the following is divisible by 6.

   a) 1332 b) 3676 c) 4838 d) 2451

5. Identify the denominator of $\frac{3}{5}$.

   a) 3 b) $\frac{5}{3}$ c) 15 d) 5

6. What part is shaded?



   a) $\frac{5}{4}$ b) $\frac{4}{9}$

   c) $\frac{5}{9}$ d) $\frac{9}{5}$

7. Simplify: $\frac{20}{24}$.

   a) $\frac{5}{6}$ b) $\frac{5}{8}$ c) $\frac{4}{5}$ d) $\frac{5}{7}$

ANSWERS

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____

CHAPTER 2 **NAME**_____

TEST FORM H

ANSWERS

8. _____

9. _____

10. _____

11. _____

12. _____

8. Simplify: $\frac{6}{6}$.

a) 0 b) 36 c) 1 d) Not defined

9. Simplify: $45 \cdot \frac{9}{5}$.

a) 81 b) $\frac{1}{25}$ c) 25 d) 126

10. Multiply and simplify: $\frac{8}{9} \cdot 24$.

a) $\frac{224}{9}$ b) $\frac{64}{3}$ c) 16 d) $\frac{1}{27}$

11. Multiply and simplify: $\frac{4}{7} \cdot \frac{21}{32}$.

a) $\frac{8}{3}$ b) $\frac{128}{147}$ c) $\frac{3}{8}$ d) $\frac{3}{7}$

12. Kaleb's tuition was \$12,600. He borrowed $\frac{5}{6}$ of the tuition. How much money did Kaleb borrow?

a) \$15,120 b) \$2100 c) \$10,080 d) \$10,500

CHAPTER 2 **NAME**_____

TEST FORM H

13. Find the reciprocal of $\frac{1}{3}$.

a) 0 b) 3 c) 1 d) $\frac{1}{9}$

14. Divide and simplify: $\frac{4}{9} \div \frac{2}{3}$.

a) $\frac{3}{2}$ b) $\frac{10}{9}$ c) $\frac{2}{3}$ d) $\frac{2}{9}$

15. Divide and simplify: $\frac{4}{5} \div \frac{1}{3}$.

a) $\frac{12}{5}$ b) $\frac{4}{15}$ c) $\frac{5}{12}$ d) $\frac{20}{3}$

16. Solve: $t \cdot \frac{3}{8} = 42$.

a) $\frac{63}{4}$ b) 112 c) 1008 d) $\frac{7}{4}$

17. Solve: $t \cdot \frac{3}{4} = 12$.

a) 8 b) 144 c) 9 d) 16

ANSWERS

13. _____

14. _____

15. _____

16. _____

17. _____

CHAPTER 2 **NAME**_____

TEST FORM H

ANSWERS

18. _____

19. _____

20. _____

21. _____

18. A piece of ribbon $\frac{3}{4}$ m long is cut into 5 equal pieces. What is the length of each piece?

a) $\frac{15}{4}$ m b) $\frac{3}{20}$ m c) $\frac{4}{15}$ m d) $\frac{20}{3}$ m

19. A gas tank held 6 gal when it was $\frac{3}{8}$ full. How much gas could it hold when full?

a) $\frac{33}{4}$ gal b) 12 gal c) 16 gal d) 15 gal

20. If $\frac{1}{5}$ of a number is $\frac{1}{8}$, how much is $\frac{1}{2}$ of the number?

a) $\frac{5}{16}$ b) $\frac{1}{8}$ c) $\frac{4}{5}$ d) $\frac{1}{20}$

21. Simplify: $\left(\frac{3}{8}\right)^2 \div \frac{1}{2} \div \frac{6}{7}$.

a) $\frac{21}{256}$ b) $\frac{27}{112}$ c) $\frac{27}{448}$ d) $\frac{21}{64}$

Name Section Date

Activity 2.1 Find all the prime numbers less than 100, using the Sieve of Eratosthenes.

Focus	Prime and composite numbers
Time	10–15 minutes
Group size	2
Materials	Colored pencils (optional)
Background	One of the methods for finding prime numbers was developed around 200 BC by a mathematician named Eratosthenes. He used the process of elimination to "sift" out the composite numbers, leaving only prime numbers. His method became known as the Sieve of Eratosthenes.

1. In Section 2.1 of your textbook, a prime number is defined as a natural number that has exactly two different factors, itself and 1. For example, the number 7 is prime because it has only the factors 1 and 7. The number 14, on the other hand, is not prime because 7 is a factor of 14. Looking at the definition from another point of view, any number that is a multiple of another number will not be prime. In the example above, 14 is a multiple of 7, and so 14 is not prime.

   In this activity, you will cross off all multiples of prime numbers from a grid of numbers. When you are done, the remaining numbers will be prime.

2. Look at the grid on the next page. The number 1 has already been crossed off, as it is not a prime number. The smallest number that is not crossed off is 2. Begin by circling the number 2 on the grid. Then, list the first 10 multiples of 2 in the space below:

   Now, cross off these numbers from the grid. You may want to use a colored pencil to cross off the numbers. Continue crossing off multiples of 2 until you reach the end of the grid.

3. Next, look for the smallest number that is not crossed off and circle it. This is the next prime number. List the first 10 multiples of this number in the space below:

   Cross off these numbers from the grid. Continue, as before, crossing off multiples of the number until you reach the end of the grid.

~~1~~	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

4. Repeat step 3 until all multiples are crossed off. The circled numbers are the prime numbers less than 100. Write the list of circled numbers in the space below:

5. Compare this list with the table of primes given in section 2.1 of your textbook. Are there any differences between the lists? If there are, check your grid to see if you crossed off all multiples. Check also that you did not accidentally cross off a number that is not a multiple.

Conclusion	The Sieve of Eratosthenes can be used anytime you need to list the first few prime numbers. For example, if you need all the prime numbers up to 50, make a list of the numbers from 1 to 50, and start crossing out the multiples of 2, 3, 5, etc.

Name Section Date

Activity 2.2 Use the divisibility rules and properties of numbers to discover an unknown number.

Focus	Rules for divisibility, place value
Time	20–30 minutes
Group size	2
Background	The rules for divisibility given in Section 2.2 of your textbook provide you with fast ways of determining whether numbers are divisible by 2, 3, 4, 5, 6, 8, 9, and 10. This activity will provide practice with these rules, as well as experience in problem solving.
Instructor notes	In step 4, show Puzzles A and B, revealing clues one at a time. You can find more puzzles in the book *Logic Number Problems*, available from Dale Seymour Publications.

For your convenience, the divisibility rules from Section 2.2 are repeated here.

2	A number is divisible by 2 (is even) if it has a ones digit of 0, 2, 4, 6, or 8
3	A number is divisible by 3 if the sum of its digits is divisible by 3
4	A number is divisible by 4 if the number named by its last two digits is divisible by 4
5	A number is divisible by 5 if its ones digit is 0 or 5
6	A number is divisible by 6 if its ones digit is 0, 2, 4, 6, or 8 (is even) and the sum of its digits is divisible by 3
8	A number is divisible by 8 if the number named by its last three digits is divisible by 8
9	A number is divisible by 9 if the sum of its digits is divisible by 9
10	A number is divisible by 10 if its ones digit is 0

1. Each puzzle in this activity gives you clues to the value of an unknown number. The objective is to determine the unknown number by using the fewest number of clues. The clues will be given to you one at a time.

2. First, practice on the following set of clues. Read the clues one at a time, using a sheet of paper to cover up the clues further down.

Clue		Possible solution(s)	Reasoning
1	It is a 3-digit number	___ ___ ___	Write one blank for each digit
2	It is divisible by 5	___ ___ 0 ___ ___ 5	To be divisible by 5, the last digit must be 0 or 5
3	It is an even number	___ ___ 0	The last digit must be 0, 2, 4, 6, or 8
4	It is less than 400	3 ___ 0 2 ___ 0 1 ___ 0	The hundreds digit must be less than 4
5	Each digit is different	3 2 0	No digit can be repeated
6	Its tens digit is greater than its ones digit	3 1 0 2 1 0	The tens digit must be 1 or higher, and the hundreds digit must be 2 or higher.
7	Its hundreds digit is greater than its tens digit		
8	It is divisible by 3	2 1 0	The sum of its digits must be divisible by 3.
9	It has only one odd digit		These clues confirm that the number is 210. They are actually not needed to solve the puzzle.
10	Its tens digit is 1		

Notice that some clues must be considered together (clues 5, 6, and 7), and that only the first 8 clues are needed to solve this puzzle.

3. Here's another puzzle to practice on. One group member writes down the possible solutions, as was done in the example on the previous page. Use complete sentences when writing the reasons for each possible solution. Read the clues one at a time, using a sheet of paper to cover up the clues further down.

Clue		Possible solution(s)	Reasoning
1	It is a 3-digit number		
2	It is an odd number		
3	One of the digits is 7		
4	It is divisible by 5		
5	It is less than 700		
6	It has no even digits		
7	It is divisible by 3		
8	It is greater than 200		
9	Each digit is different		
10	It is a multiple of 25		

When you are done, compare your group's result with the results of the other groups in your class. How many clues did your group need to solve this puzzle? Could you have determined the unknown number with fewer clues? Did you use the remaining clues (if any) to check your answer?

4. Now, let's add a little competition to the problem-solving process. Each group will work as a team to solve a puzzle. Take turns, so each group member has a chance to do the writing. Your instructor will reveal the clues one at a time. The goal is to be the first group to correctly deduce the unknown number by using the fewest number of clues. Your instructor will discuss the scoring scheme; alternatively, the class can propose a scheme that is acceptable to all. The scoring scheme should take into account the correctness of the number, the penalty for a wrong number, the number of clues used, and the penalty for using more clues than needed.

Conclusion	This activity should help you gain experience in applying the divisibility rules. As a side benefit, the problem solving techniques used in solving the puzzles will be useful in solving the applications in your textbook.

Puzzle A

1	It is a 3-digit number
2	It is divisible by 5
3	It is an odd number
4	Each of its digits is different
5	Its tens digit is less than its ones digit
6	Its hundreds digit is less than its tens digit
7	It is greater than 200
8	It is divisible by 3
9	It has two odd digits
10	Its tens digit is 4

Puzzle B

1	It is a 3-digit number
2	It is divisible by 5
3	Its hundreds digit is 8
4	It is divisible by 3
5	Its tens digit is less than its ones digit
6	None of its digits are repeated
7	The sum of two of its digits is 10
8	It has only one odd digit
9	It is divisible by 11
10	Its tens digit is 2

Name Section Date

Activity 2.5 Use fraction bars to represent equivalent fractions.

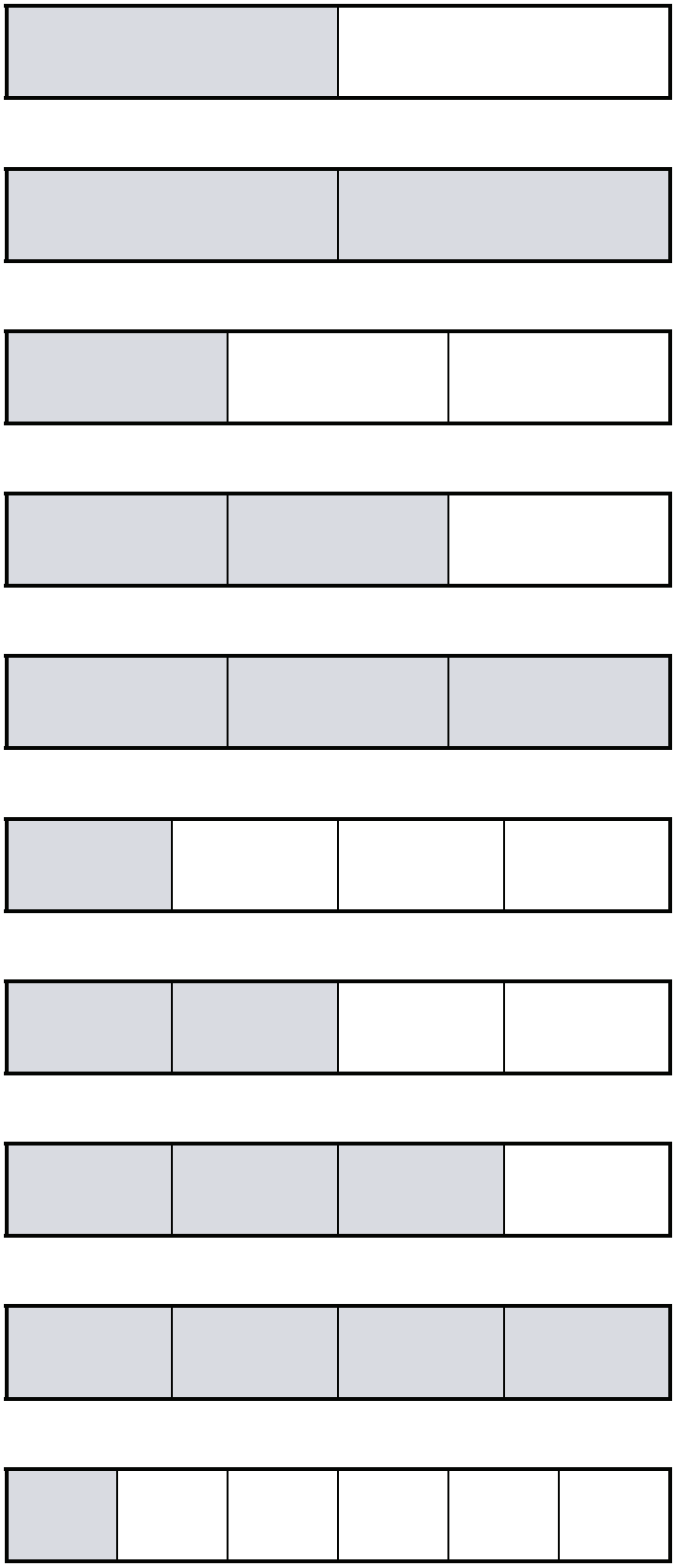
Focus	Equivalent fractions
Time	10–15 minutes
Group size	3
Background	Equivalent fractions are used extensively when adding, subtracting, and simplifying fractions. In Section 2.5 of your textbook, the process of multiplying by one is used to find equivalent fractions, and to simplify fractions. This activity will give you a better understanding of these concepts.
Instructor notes	Each group will need one set of bars from the next three pages. Copy the pages on card stock, and cut along the heavy outline of each bar; do not cut within each fraction bar. You can also purchase fraction bars.

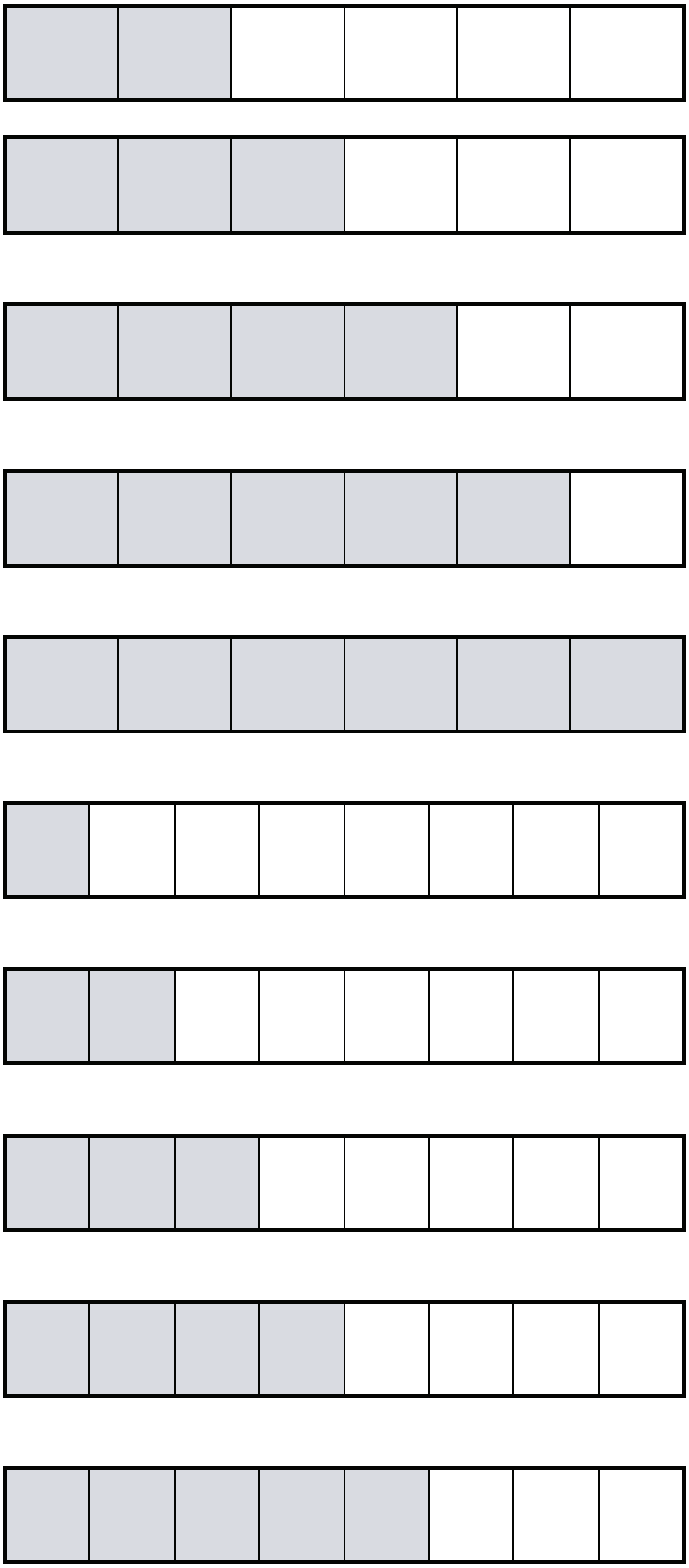
1. Two fractions are equivalent if they represent the same number. For example, 2/3 and 4/6 are two names for the same number. We will use fraction bars to show how equivalent fractions can be represented visually. Take your group's set of fraction bars and mix them up. One group member selects the bar that represents 2/3, and places it in the box below.

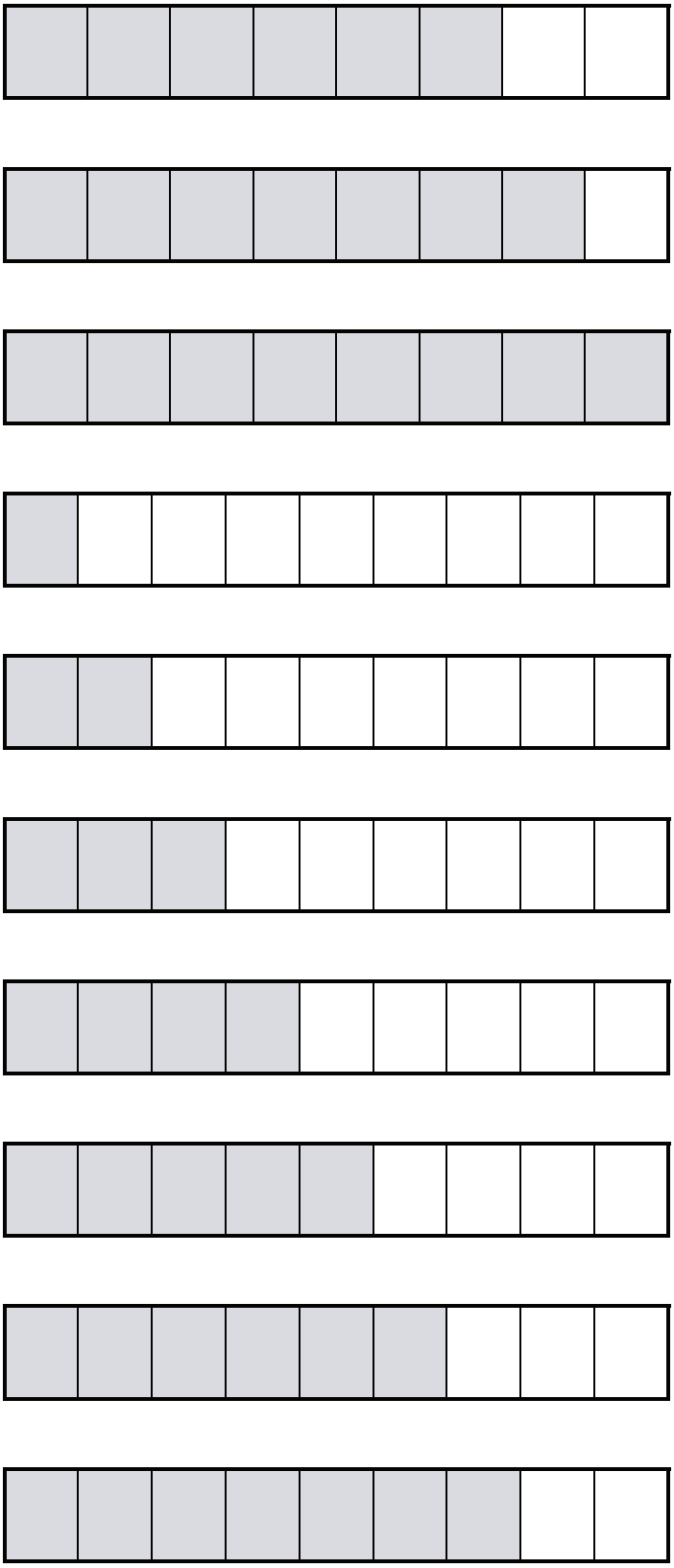
$\frac{2}{3}$

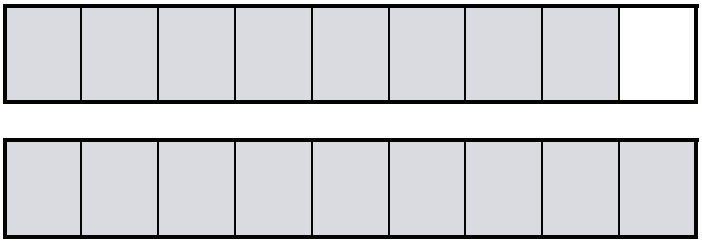
$\frac{4}{6}$

Taking turns, the next group member selects the fraction bar that represents 4/6, and places it in the appropriate box above. The entire group should then examine the two bars. Are the fractions equivalent? Compare the shaded areas in each bar. Are the areas the same? Pick another bar that has the same shaded area as 2/3 and 4/6 and place it in the last box. Write the name for the fraction to the right of the box. Are the fractions equivalent?

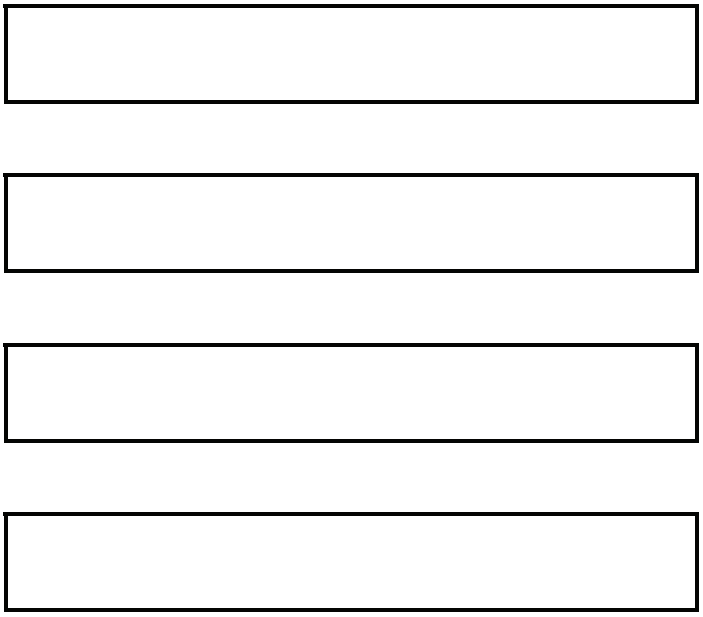








2. Next, one group member picks the bar that represents 1/2 and place it in the appropriate box below.



$\frac{1}{2}$

Taking turns, choose three other bars that are equivalent to 1/2 and place them in the boxes above. Write their fraction names to the right of the appropriate box.

Study the fraction names for these equivalent fractions. How are the numerators and denominators related? Use complete sentences to write your answer.

Write fractions equivalent to 1/2, but with denominators specified. Check your answers with the other group members, and discuss any answers that do not match. If necessary, consult with another group until an agreement is reached on the correct answers.

$\frac{}{10}$ $\frac{}{18}$ $\frac{}{24}$ $\frac{}{100}$

3. Next, consider the fraction 3/9. In turn, each group member should look through the pile of fraction bars, and find the bar that represents 3/9, and two other bars that have the same shaded area as 3/9. Place them in the boxes below, and write the fraction names. Which fraction is simplest? Why?

$\frac{3}{9}$

4. Finally, one group member should draw the bar that represents 9/12.

Another group member then simplifies 9/12 using the process of multiplying by one, as shown in Section 2.5 of your textbook. Write the simplified fraction below, and the third group member should find the fraction bar that represents it.

Compare this fraction bar with the one drawn for 9/12. Are the shaded areas the same? Why or why not? Be sure to use complete sentences for your answer.

Conclusion	Equivalent fractions are represented by the same shaded area on a fraction bar. Therefore, to find equivalent fractions, you multiply by a form of 1, as described in Section 2.5 of your textbook. Conversely, to write a fraction in simplest form, you would reverse the process, and remove a factor of 1.

Name Section Date

Activity 3.1 Find the least common multiple of two or more numbers using shaped markers.

Focus	Least Common Multiples
Time	20–30 minutes
Group size	2
Background	The textbook describes two methods for finding the least common multiple (LCM) of a set of numbers: using multiples and using factorizations. To find the LCM using factorizations, first, find the prime factorization of each number; then, create a product of factors, using each factor the greatest number of times it occurs in any one factorization. The second part of the factorization method requires the creation of a product of factors. We will see how these factors are chosen by using shaped markers to represent the factorizations. This visualization should give you a clearer picture of the process.
Instructor notes	Copy the next page on card stock, and cut out the markers. Each group will need one set of markers. You can also purchase sets of markers (also called pattern blocks).

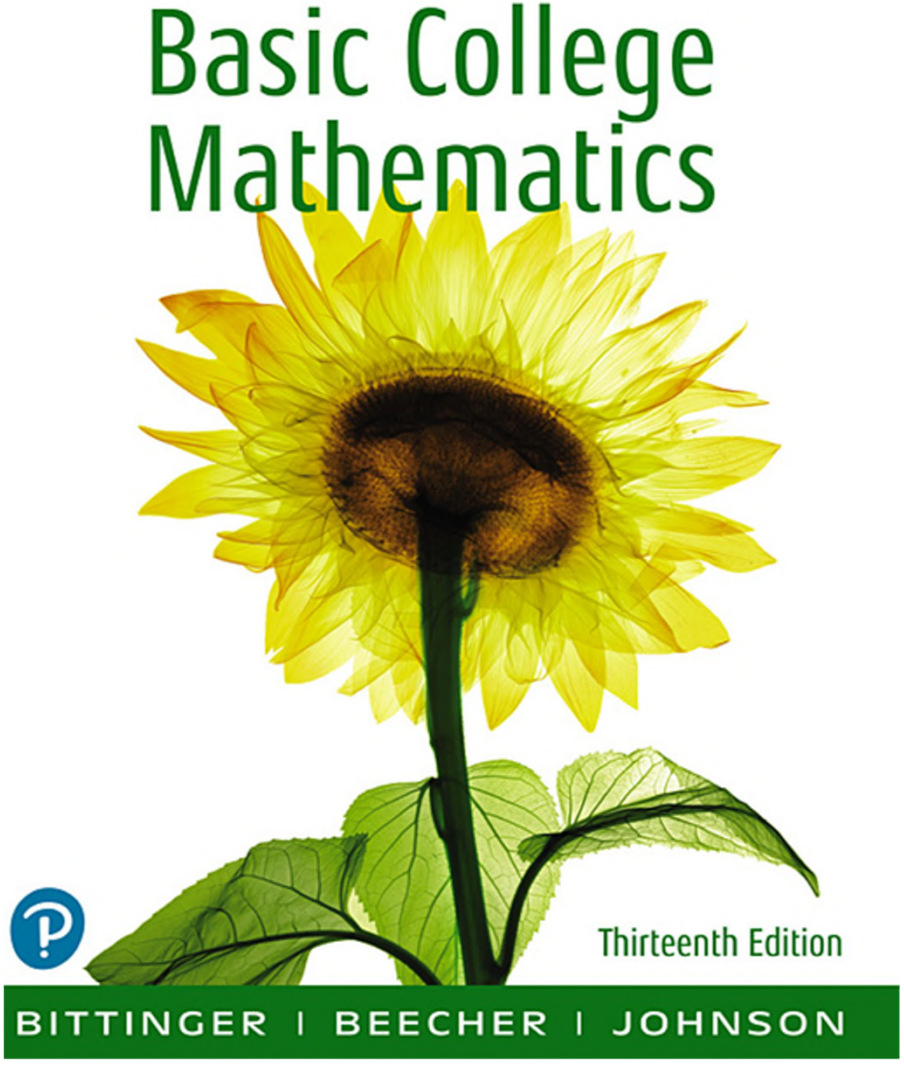
1. Study the set of markers; notice that each type of marker represents a different prime number, as follows.

Marker	Prime number
Circle	2
Triangle	3
Square	5
Hexagon	7

For the first part of this activity, use a restricted subset of markers consisting of 3 circles, 3 triangles, and 3 squares.

Chapter 2

Fraction Notation: Multiplication and Division



2.1 Factorization

a. Determine whether one number is a factor of another, and find the factors of a number.

b. Find some multiples of a number, and determine whether a number is divisible by another.

c. Given a number from 1 to 100, tell whether it is prime, composite, or neither.

d. Find the prime factorization of a composite number.

Factor

In the product $a \cdot b$, a and b are **factors**.

If we divide Q by d and get a remainder of 0, then the divisor d is a **factor** of the dividend Q.

Example

Determine by long division whether 12 is a factor of 3915.

Solution

$$
\begin{array}{r}
326 \\
12\overline{)3915} \\
\underline{36} \\
31 \\
\underline{24} \\
75 \\
\underline{72} \\
3
\end{array}
$$

← Not 0

The remainder is *not* 0, so 12 is not a factor of 3915.

Example

List all the factors of 72.

Solution

Check sequentially the numbers 1, 2, 3, and so on, to see if we can form any factorizations.

$1 \cdot 72$

$2 \cdot 36$

$3 \cdot 24$

$4 \cdot 18$

$6 \cdot 12$

$8 \cdot 9$

Multiples

A **multiple** of a natural number is a product of that number and some natural number.

We find multiples of 2 by counting by twos:

2, 4, 6, 8, and so on. We can find multiples of 3 by counting by threes: 3, 6, 9, 12, and so on.

Example

Multiply by 1, 2, 3,… and so on, to find 6 multiples of seven.

Solution

$1 \cdot 7 = 7$

$2 \cdot 7 = 14$

$3 \cdot 7 = 21$

$4 \cdot 7 = 28$

$5 \cdot 7 = 35$

$6 \cdot 7 = 42$

Divisibility

The number a is divisible by another number b if there exists a number c such that $a = b \cdot c$. The statements "a is divisible by b," "a is a **multiple** of b," and "b is a **factor** of a" all have the same meaning.

Thus,

15 is divisible by 5 because 15 is a multiple of 5 ($15 = 3 \cdot 5$)

40 is divisible by 4 because 40 is a multiple of 4 ($40 = 10 \cdot 4$)

Example

Determine whether 102 is divisible by 4.

Solution

$$
\begin{array}{r}
25 \\
4\overline{)102} \\
\underline{-8} \\
22 \\
\underline{20} \\
2 \longleftarrow \text{Not } 0
\end{array}
$$

Since the remainder is not 0 we know that 102 is not divisible by 4.

Prime and Composite Numbers

- A natural number that has exactly two *different* factors is called a **prime number**.
- The number 1 is *not* prime
- A natural number, other than 1, that is not prime is **composite.**

Example

Determine whether the numbers listed below are prime, composite, or neither.

8 Has factors of 1, 2, 4 and 8, composite

13 Has only two factors 1 and itself, prime

24 Has factors 1, 2, 3, 4, 6, 8, 12, 24, composite

33 Has factors 1, 3, 11, 33, composite

85 Has 5 as a factor, composite

97 Has only two factors 1 and itself, prime

A Table of Primes from 2 to 157

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157

Example

Find the prime factorization of 50.

Solution

a) Since 50 is even, it must have 2 as a factor. $2\overline{)50}$ (25)

b) Since 25 ends in 5, we know 5 is a factor.
Because 5 is prime, we can factor no further. $5\overline{)25}$ (5)

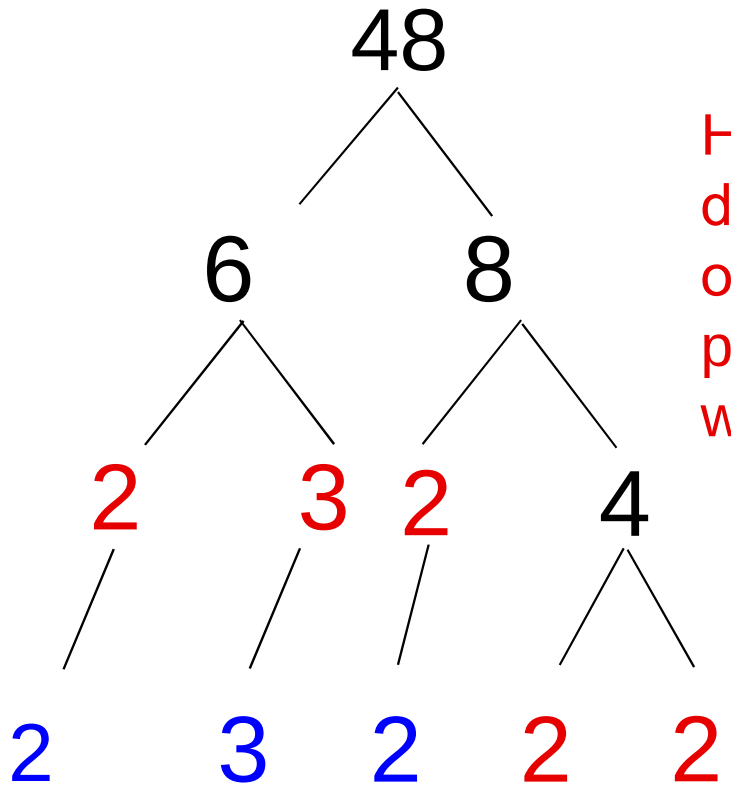
The prime factorization can be written as $2 \cdot 5 \cdot 5$ or $2 \cdot 5^2$.

Every number has just one (unique) prime factorization.

Example

Find the prime factorization of 48 using a factor tree.

Solution

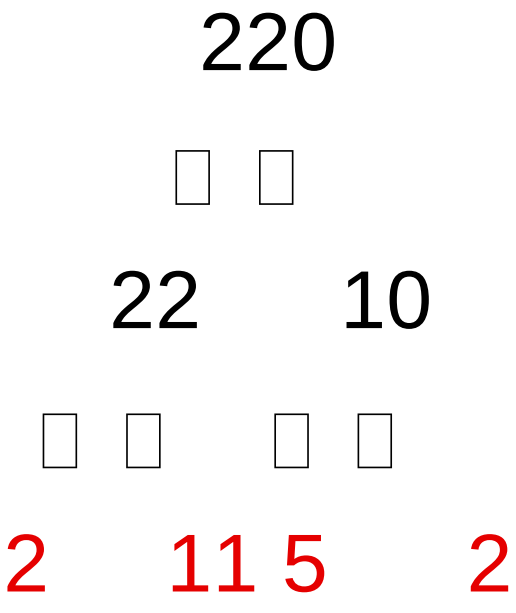


Had we begun with different factors (2 · 24, or 4 · 12), the same prime factorization would result.

Example

Find the prime factorization of 220.

Solution



$$220 = 2 \cdot 2 \cdot 5 \cdot 11$$

Example

Find the prime factorization of 1424.

Solution We use a string of successive divisions.

$$2\overline{)178}\quad 89$$

$$2\overline{)356}\quad 178$$

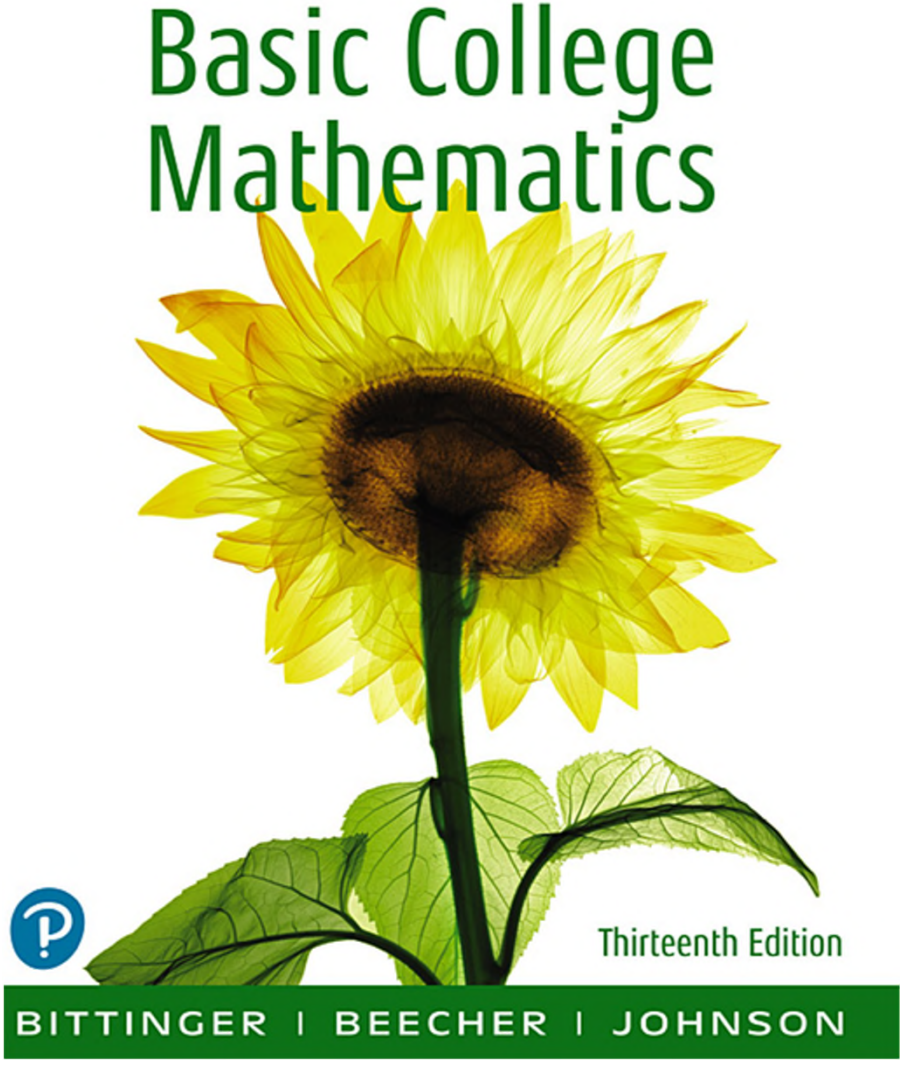
$$2\overline{)712}\quad 356$$

$$2\overline{)1424}\quad 712$$

$$1424 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 89$$

Chapter 2

Fraction Notation: Multiplication and Division



2.2 Divisibility

a. Determine whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

Divisibility by 2

A number is **divisible by 2** (is *even*) if it has a ones digit of 0, 2, 4, 6, or 8 (that is, it has an even ones digit).

Example

Determine whether each of the following numbers is divisible by 2.

1. 457 2. 3488 2. 3200 4. 7893

Solution

1. 457 is *not* divisible by 2; 7 is not even.

2. 3488 is divisible by 2; 8 is even.

2. 3200 is divisible by 2; 0 is even.

4. 7893 is *not* divisible by 2; 3 is not even.

Divisibility by 3

A number is **divisible by 3** if the sum of its digits is divisible by 3.

Example

Determine whether the number is divisible by 3.

1. 12 2. 96 3. 303 4. 374

Solution

1. 12 $1 + 2 = 3$

2. 96 $9 + 6 = 15$

3. 303 $3 + 0 + 3 = 6$

Each is divisible by 3 because the sum of its digits is divisible by 3.

4. 374 $3 + 7 + 4 = 14$

The sum of the digits, 14, is *not* divisible by 3, so 374 is *not* divisible by 3.

Divisibility by 6

A number is **divisible by 6** if its ones digit is 0, 2, 4, 6, or 8 (is even) and the sum of its digits is divisible by3.

Example

Determine whether the number is divisible by 6.

1. 840 2. 90 3. 83

Solution

1. 840 is even, divisible by 2. Also 8 + 4 + 0 = 12, so 840 is divisible by 3, 840 is divisible by 6.

2. 90 is even, divisible by 2. Also 9 + 0 = 9, so 90 is divisible by 3, 90 is divisible by 6.

3. 83 is *not* divisible by 6 because it is *not* even.

Divisibility by 9

A number is **divisible by 9**, if the sum of its digits is divisible by 9.

Example

Determine whether the number is divisible by 9.

1. 4824
2. 524

Solution

1. 4824 — 4 + 8 + 2 + 4 = 18 and 18 is divisible by 9, so 4824 is divisible by 9.

2. 524 — 5 + 2 + 4 = 11 and 11 is *not* divisible by 9, 524 is *not* divisible by 9.

Divisibility by 10

A number is **divisible by 10**, if its ones digit is 0.

Example

Determine whether the number is divisible by 10.

1. 4810
2. 1524

Solution

1. 4810 is divisible by 10 because the ones digit is 0.

2. 1524 is *not* divisible by 10 because the ones digit is not 0.

Divisibility by 5

A number is **divisible by 5** if its ones digit is 0 or 5.

Example

Determine whether each of the following numbers is divisible by 5.

1. 340
2. 885
2. 6721

Solution

1. 340 is divisible by 5; because its one digit is 0.

2. 885 is divisible by 5; because its one digit is 5.

3. 6721 is *not* divisible by 5; because its one digit is neither 0 nor 5.

Divisibility by 4

A number is **divisible by 4** if the number named by its last *two* digits is divisible by 4.

Example

Determine whether each of the following numbers is divisible by 4.

1. 7732
2. 8453

Solution

1. 77**32** is divisible by 4 because 32 is divisible by 4.

2. 84**53** is *not* divisible by 4 because 53 is *not* divisible by 4.

Divisibility by 8

A number is **divisible by 8** if the number named by its last *three* digits is divisible by 8.

Example

Determine whether each of the following numbers is divisible by 8.

1. 1264
2. 43,911

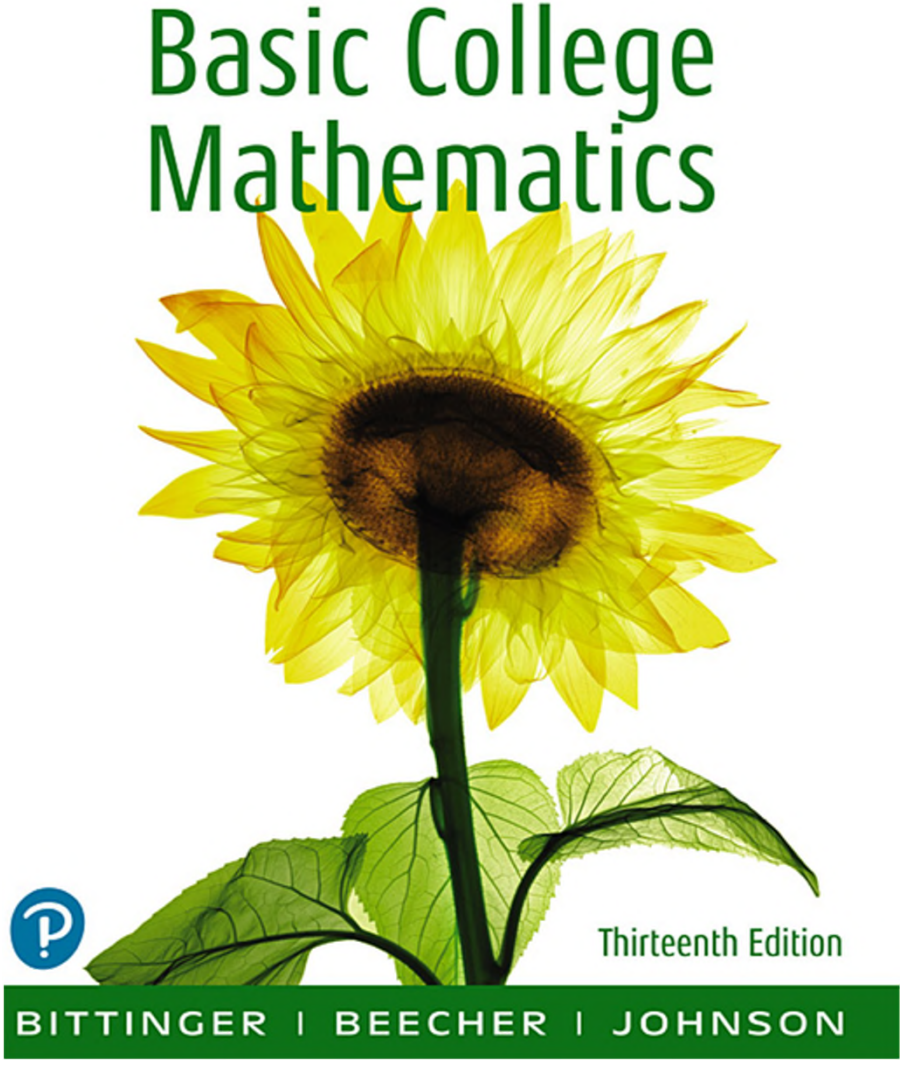
Solution

1. 1264 is divisible by 8 because 264 is divisible by 8.

2. 43,911 is *not* divisible by 8 because 911 is *not* divisible by 8.

Chapter 2

Fraction Notation: Multiplication and Division



2.3 Fractions and Fraction Notation

a. Identify the numerator and the denominator of a fraction, and write fraction notation for part of an object or part of a set of objects and as a ratio.

b. Simplify fraction notation like n/n to 1, $0/n$ to 0, and $n/1$ to n.

The following are some examples of fractions:

$$\frac{3}{4}, \quad \frac{13}{21}, \quad \frac{-7}{6}, \quad \frac{a}{b}, \quad \frac{3a}{4b}$$

This way of writing number names is called **fraction notation.** The top number is called the **numerator** and the bottom number is called the **denominator**.

Example

Identify the numerator and denominator.

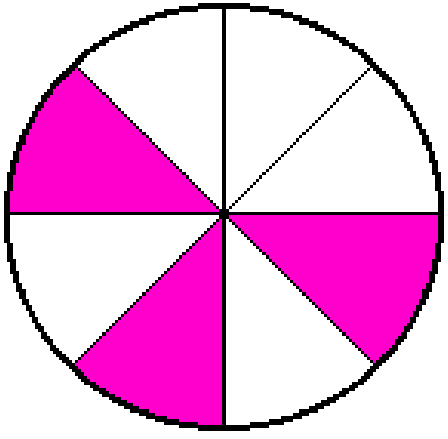
$$\frac{9}{16}$$

← Numerator

← Denominator

Example

What part of the circle is shaded?



Solution

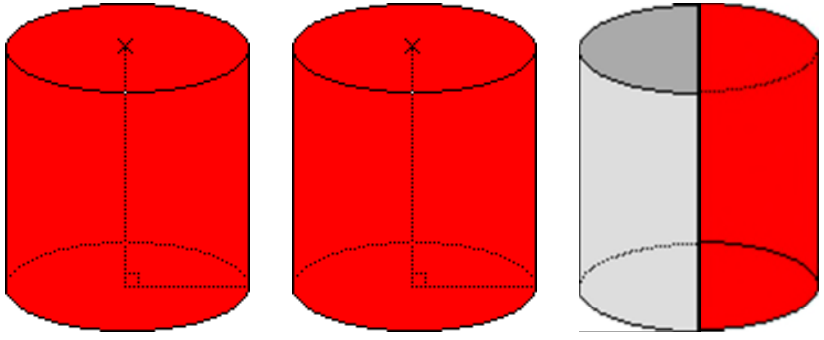
The object is divided into 8 equal parts.

3 of the parts are shaded

$\frac{3}{8}$ of the circle is shaded.

Example

What part is shaded?



Solution

Each cylinder is divided into 2 equal parts.

The unit is ½.

The *denominator* is 2.

We have 5 of the units shaded.

This tells us the *numerator* is 5. $\frac{5}{2}$ is shaded

Fractions as Ratios

A **ratio** is a quotient of two quantities. We can express a ratio with fraction notation.

Example

What part of this set, or collection of toys are balls and clowns?

Solution

There are 10 toys in the set.

3 balls are in the set.

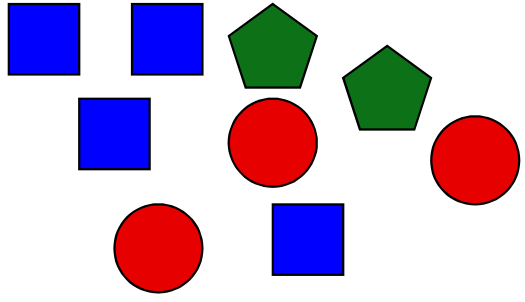
4 clowns are in the set.

$$\text{Balls} = \frac{3}{10} \qquad \text{Clowns} = \frac{4}{10}$$

Example

For the following set of shapes, what is the ratio of:

1. squares to circles? $\frac{4}{3}$



2. circles to the total number of shapes? $\frac{3}{9}$

3. pentagons to squares? $\frac{2}{4}$

4. total number of shapes to pentagons. $\frac{9}{2}$

The Number 1 in Fraction Notation

$\frac{n}{n} = 1$, for any whole number n that is not 0.

Example

Simplify:

1. $\frac{7}{7}$

2. $\frac{12}{12}$

3. $\frac{15x}{15x}$

Solution

1. $\frac{7}{7} = 1$

2. $\frac{12}{12} = 1$

3. $\frac{15x}{15x} = 1$

The Number 0 in Fraction Notation

$\frac{0}{n} = 0,$ for any whole number n that is not 0.

Example

Simplify:

1. $\frac{0}{12}$

2. $\frac{0}{22}$

3. $\frac{0}{6x}$

Solution

1. $\frac{0}{12} = 0$

2. $\frac{0}{22} = 0$

3. $\frac{0}{6x} = 0$

A Denominator of 0

$\frac{n}{0}$ is not defined for any whole number n.

(When asked to simplify $\frac{n}{0}$, we write *undefined.)*

Any Whole Number In Fraction Notation

Any whole number divided by 1 is the whole number.

$$\frac{n}{1} = n, \text{ for any whole number } n.$$

Example

Simplify:

1. $\frac{8}{1}$

2. $\frac{52}{1}$

3. $\frac{4x}{1}$

Solution

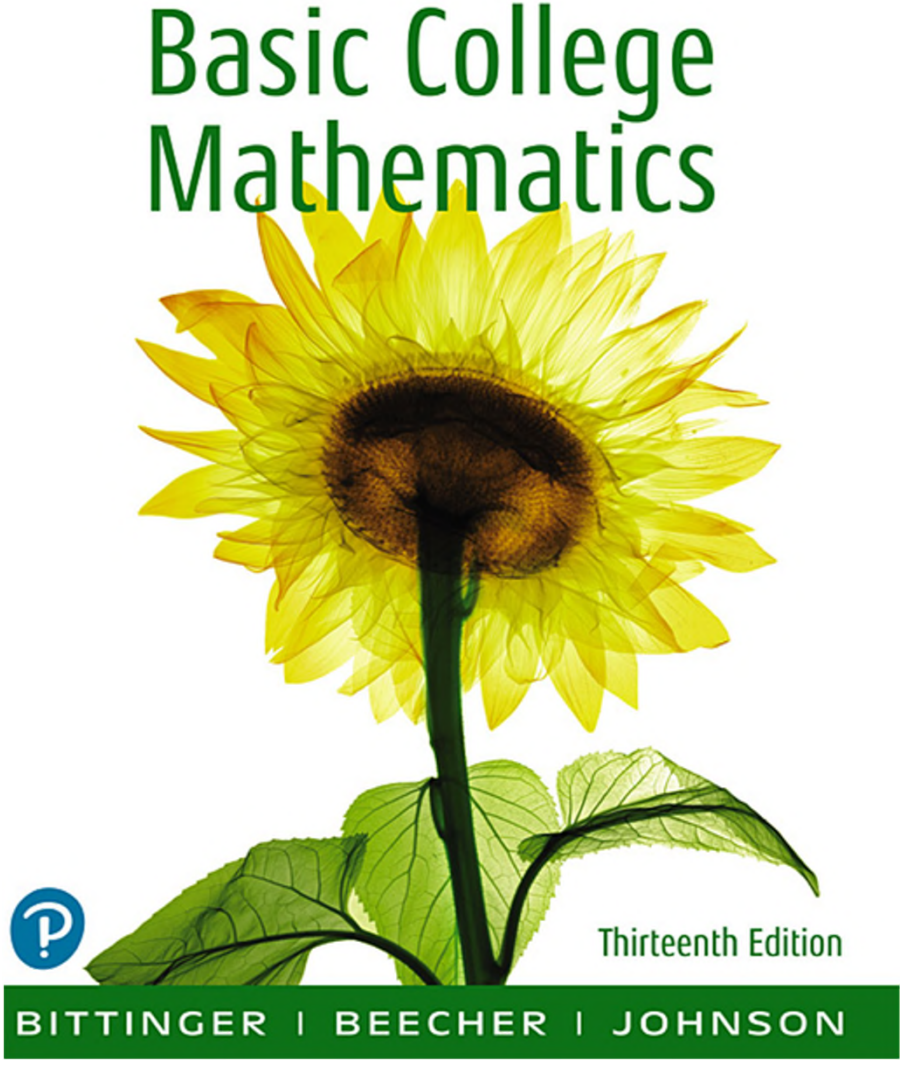
1. $\frac{8}{1} = 8$

2. $\frac{52}{1} = 52$

3. $\frac{4x}{1} = 4x$

Chapter 2

Fraction Notation: Multiplication and Division



2.4 Multiplication and Applications

a. Multiply a fraction by a fraction, and multiply a fraction by a whole number.

b. Solve applied problems involving multiplication of fractions.

To multiply a fraction by a fraction,

a) multiply the numerators to get the new numerator, and

b) multiply the denominators to get the new denominator.

$$\frac{9}{7} \cdot \frac{3}{4} = \frac{9 \cdot 3}{7 \cdot 4} = \frac{27}{28}$$

Example

Multiply.

1. $\frac{4}{9} \cdot \frac{4}{5}$

2. $\frac{3}{4} \cdot \frac{2}{7}$

Solution

1. $\frac{4}{9} \cdot \frac{4}{5} = \frac{4 \cdot 4}{9 \cdot 5} = \frac{16}{45}$

Skip this step when you feel comfortable doing so.

2. $\frac{3}{4} \cdot \frac{2}{7} = \frac{3 \cdot 2}{4 \cdot 7} = \frac{6}{28}$

To multiply a fraction by a whole number, first express the whole number in fraction notation, then multiply the fraction by the fraction.

$$6 \cdot \frac{4}{5} = \frac{6}{1} \cdot \frac{4}{5} = \frac{6 \cdot 4}{1 \cdot 5} = \frac{24}{5}$$

Example

Multiply.

1. $3 \cdot \frac{4}{5}$

2. $\frac{3}{4} \cdot 7$

3. $12 \cdot \frac{1}{9}$

Solution

1. $3 \cdot \frac{4}{5} = \frac{3}{1} \cdot \frac{4}{5} = \frac{3 \cdot 4}{1 \cdot 5} = \frac{12}{5}$

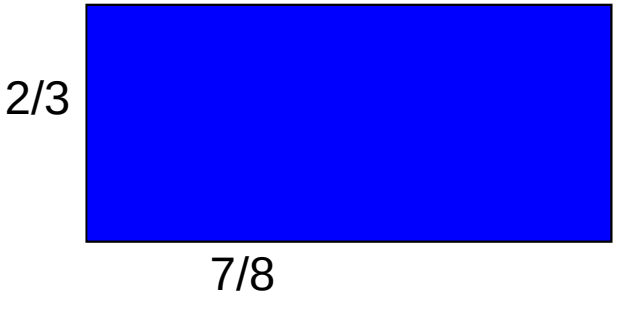
3. $12 \cdot \frac{1}{9} = \frac{12}{1} \cdot \frac{1}{9} = \frac{12}{9}$

2. $\frac{3}{4} \cdot 7 = \frac{3}{4} \cdot \frac{7}{1} = \frac{3 \cdot 7}{4 \cdot 1} = \frac{21}{4}$

Example

The length of a plot of land is 7/8 of a mile and the width is 2/3 of a mile. What is the area of the land?

1. **Familiarize**. Recall that area is the length times the width. We make a drawing and let A = the area of the land.



2. **Translate**.

Area is Length times Width

$$A = \frac{7}{8} \times \frac{2}{3}$$

3. **Solve.**

$$\frac{7}{8} \cdot \frac{2}{3} = \frac{7 \cdot 2}{8 \cdot 3} = \frac{14}{24}.$$

Example continued

4. **Check**. We check by repeating the calculation.

5. **State.**

The area is $\frac{14}{24}$ mi^2.

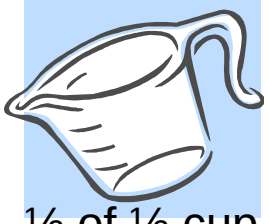
Example

A recipe calls for 1/2 cup of milk. The baker is making ½ of the recipe. How much milk should the baker use?

1. **Familiarize**. Make a drawing or visualize the situation. We let n = amount of milk the baker should use.

2. **Translate**.

   The sentence $\frac{1}{2} \cdot \frac{1}{2} = n$ corresponds to the situation.



½ of ½ cup

2. **Solve.** $\frac{1}{2} \cdot \frac{1}{2} = \frac{1 \cdot 1}{2 \cdot 2} = \frac{1}{4}$

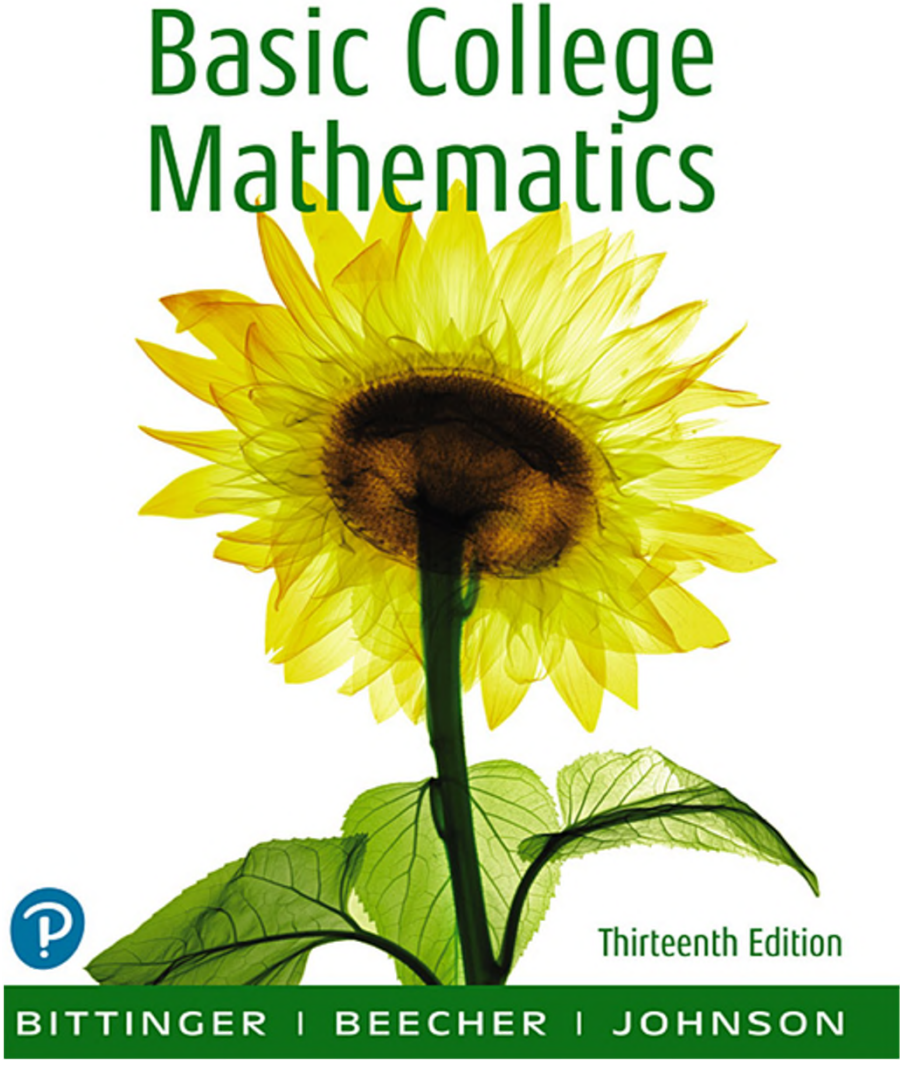
Example continued

4. **Check**. We check by repeating the calculation.

5. **State**. The baker should use $\frac{1}{4}$ cup of milk.

Chapter 2

Fraction Notation: Multiplication and Division



2.5 Simplifying

a. Multiply a number by 1 to find fraction notation with a specified denominator.

b. Simplify fraction notation.

c. Use the test for equality to determine whether two fractions name the same number.

Multiplicative Identity for Fractions

When we multiply a number by 1, we get the same number:

$$a = a \cdot 1 = a \cdot \frac{n}{n} = a.$$

Example

Find a number equivalent to $\frac{3}{4}$ with a denominator of 36.

Solution

Since $36 \div 4 = 9$, we multiply by 1, using $\frac{9}{9}$:

$$\frac{3}{4} = \frac{3}{4} \cdot \frac{9}{9} = \frac{3 \cdot 9}{4 \cdot 9} = \frac{27}{36}$$

Example

Find a number equivalent to $\frac{11}{12}$ with a denominator. of 48.

Solution

Since $48 \div 12 = 4$, we multiply by 1, using $\frac{4}{4}$:

$$\frac{11}{12} \cdot \frac{4}{4} = \frac{44}{48}$$

A fraction is in **simplest form** when it has the smallest numerator and the smallest denominator.

That is, the numerator and denominator have no common factor other than 1.

Example

Simplify.

1. $\frac{28}{35}$ 2. $\frac{3}{24}$ 3. $\frac{8}{32}$

Solution

1. $\frac{28}{35} = \frac{4 \cdot 7}{5 \cdot 7} = \frac{4}{5} \cdot \frac{7}{7} = \frac{4}{5}$

Removing a factor equal to 1: 7/7 = 1

2. $\frac{3}{24} = \frac{1 \cdot 3}{8 \cdot 3} = \frac{1}{8} \cdot \frac{3}{3} = \frac{1}{8}$

Writing 1 allows for pairing of factors in the numerator and the denominator.

3. $\frac{8}{32} = \frac{1 \cdot 8}{4 \cdot 8} = \frac{1}{4} \cdot \frac{8}{8} = \frac{1}{4}$

Example

Simplify: $\frac{210}{245}$.

Solution

$$\frac{210}{245} = \frac{42 \cdot 5}{49 \cdot 5} = \frac{42}{49} \cdot \frac{5}{5} = \frac{42}{49}$$

To find the 42, we divided 210 by 5.

To find the 49, we divided 245 by 5.

The fraction is not simplified since common factors remain in the numerator and denominator.

$$\frac{210}{245} = \frac{42}{49} = \frac{6 \cdot 7}{7 \cdot 7} = \frac{6}{7} \cdot \frac{7}{7} = \frac{6}{7}$$

To find the 6, we divided 42 by 7.

To find the 7, we divided 49 by 7.

Canceling

Canceling is a shortcut that you may have used for removing a factor that equals 1 when working with fraction notation.

Canceling may be done only when removing common factors in numerators and denominators.

Canceling must be done with care and understanding.

Caution!

The difficulty with canceling is that it is often applied incorrectly in situations like the following:

$$\frac{\cancel{2}+3}{\cancel{2}} = 3; \qquad \frac{\cancel{4}+1}{\cancel{4}+2} = \frac{1}{2}; \qquad \frac{15}{\cancel{5}4} = \frac{1}{4}$$

The correct answers are:

$$\frac{2+3}{2} = \frac{5}{2}; \qquad \frac{4+1}{4+2} = \frac{5}{6}; \qquad \frac{15}{54} = \frac{5}{18}$$

In each of the incorrect cancellations, the numbers canceled did not form a factor equal to 1. Factors are parts of products, but in 2 + 3, the numbers 2 and 3 are terms. You cannot cancel terms.

If you cannot factor, you cannot cancel! If in doubt, do not cancel!

A Test for Equality

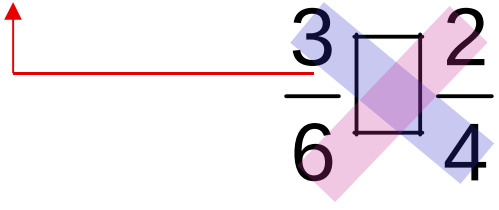
When denominators are the same, we say that the fractions have a **common denominator**. One way to compare fractions is to find a common denominator and compare numerators.

A Test for Equality

Two fractions are equal if their cross products are equal.

We multiply these two numbers: $3 \cdot 4$

We multiply these two numbers $6 \cdot 2$.



We call $3 \cdot 4$ and $6 \cdot 2$ **cross products**. Since the cross products are the same $3 \cdot 4 = 6 \cdot 2$ we know that

$$\frac{3}{6} = \frac{2}{4}$$

Example

Use = or ≠ for □ to write a true sentence: $\frac{7}{8} \square \frac{8}{9}$

Solution

We multiply these two numbers: $7 \cdot 9 = 63$.

We multiply these two numbers $8 \cdot 8 = 64$.

$$\frac{7}{8} = \frac{8}{9}$$

Because $63 \neq 64$ (read "63 is not equal to 64"),

$$\frac{7}{8} \neq \frac{8}{9}$$

Example

Use = or ≠ for □ to write a true sentence: $\frac{3}{8} \square \frac{27}{72}$

Solution

$3 \cdot 72 = 216$ $\qquad$ $27 \cdot 8 = 216$

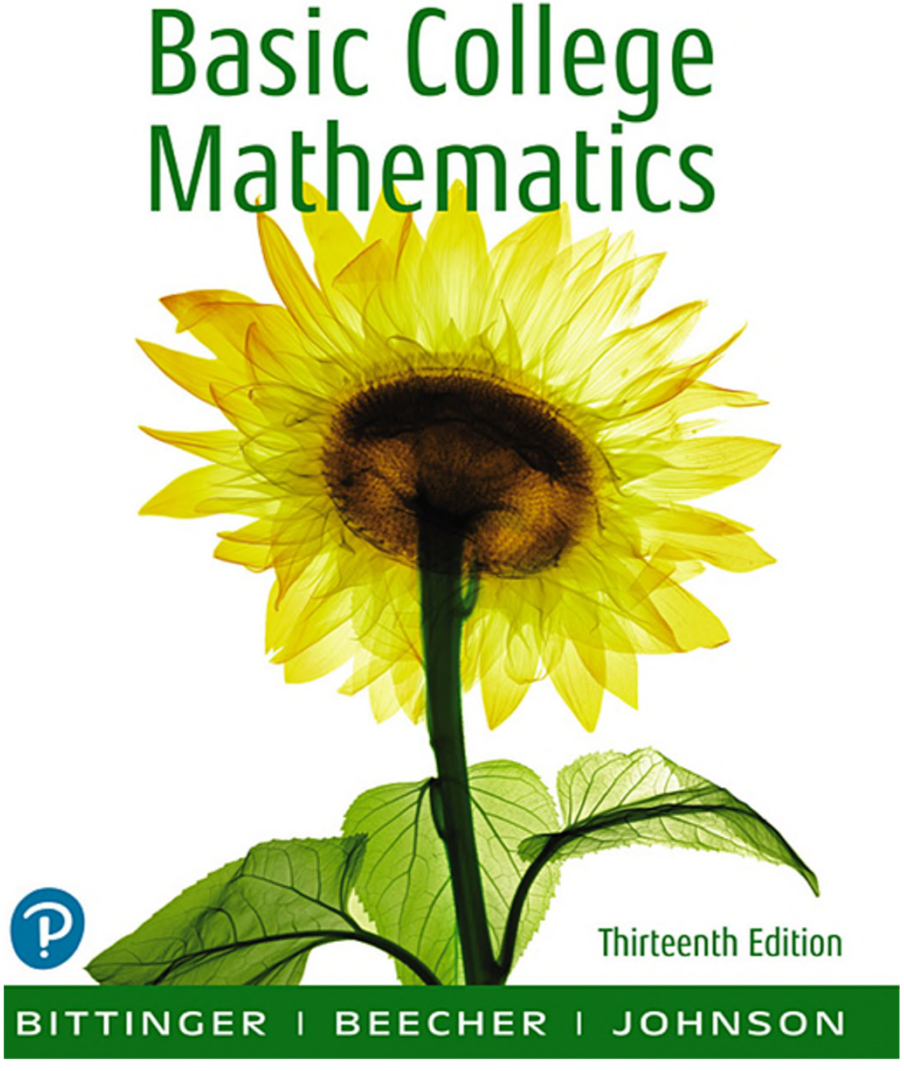
$$\frac{3}{8} = \frac{27}{72}$$

Because the cross products are the same,

$$\frac{3}{8} = \frac{27}{72}$$

Chapter 2

Fraction Notation: Multiplication and Division



2.6 Multiplying, Simplifying, and Applications

a. Multiply and simplify using fraction notation.

b. Solve applied problems involving multiplication of fractions.

To multiply and simplify:

a) Write the products in the numerator and the denominator, but do not carry out the products.
b) Factor the numerator and the denominator.
c) Factor the fraction to remove a factor of 1, if possible.
d) Carry out the remaining products.

Example

Multiply and simplify. $\frac{4}{5} \cdot \frac{7}{8}$

Solution

$$\frac{4}{5} \cdot \frac{7}{8} = \frac{4 \cdot 7}{5 \cdot 8}$$

Note that 4 is a common factor of 4 and 8.

$$= \frac{4 \cdot 7}{5 \cdot 4 \cdot 2}$$

Try to go directly to this step.

$$= \frac{4}{4} \cdot \frac{7}{5 \cdot 2}$$

$$= 1 \cdot \frac{7}{10} = \frac{7}{10}$$

Removing a factor equal to 1

Example

Multiply and simplify. $\frac{12}{28} \cdot \frac{14}{21}$

Solution

$$\frac{12}{28} \cdot \frac{14}{21} = \frac{4 \cdot 3 \cdot 2 \cdot 7}{4 \cdot 7 \cdot 3 \cdot 7}$$

$$= \frac{4 \cdot 7 \cdot 3}{4 \cdot 7 \cdot 3} \cdot \frac{2}{7}$$

$$= \frac{2}{7}$$

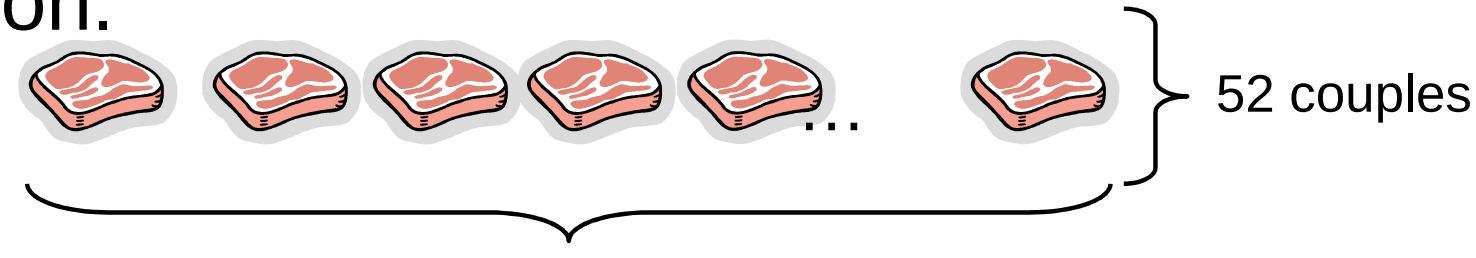
Removing a factor equal to 1

Example

Lisa's Gourmet anticipates 52 couples eating prime rib at a year-end banquet. Allowing ¾ pound of prime rib per couple, how many pounds of prime rib should be prepared?

Solution

1. **Familiarize**. Make a drawing or visualize the situation.



Example continued

2. **Translate**.

number of pounds of prime rib is number of couples times number of pounds per couple

$$n = 52 \cdot \frac{3}{4}$$

$$n = 52 \cdot \frac{3}{4} = \frac{52 \cdot 3}{4}$$

2. **Solve**.

$$= \frac{4 \cdot 13 \cdot 3}{4 \cdot 1} = \frac{4}{4} \cdot \frac{13 \cdot 3}{1} = 39$$

Example continued

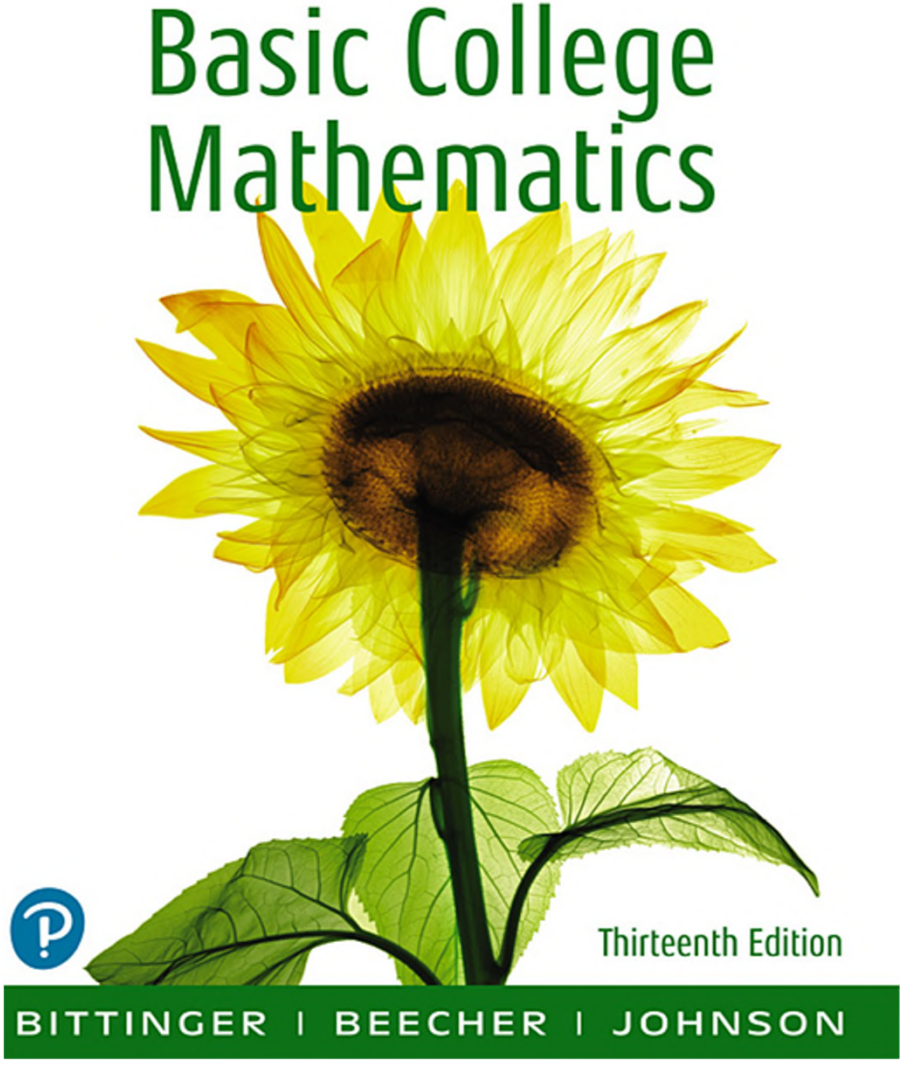
4. **Check.** We could repeat the calculation. We can also think about the reasonableness of the answer.

   Since each couple requires less than 1 pound, it makes sense that 52 couples require fewer than 52 pounds. This provides a partial check of the answer.

5. **State.** Lisa's Gourmet should prepare 39 pounds of prime rib.

Chapter 2

Fraction Notation: Multiplication and Division



2.7 Division and Applications

a. Find the reciprocal of a number.

b. Divide and simplify using fraction notation.

c. Solve equations of the type $a \cdot x = b$ and $x \cdot a = b$, where a and b may be fractions.

d. Solve applied problems involving division of fractions.

RECIPROCALS

If the product of two numbers is 1, we say that they are **reciprocals** of each other. To find the reciprocal of a fraction, interchange the numerator and the denominator.

$$\text{Number: } \frac{3}{4} \longrightarrow \text{Reciprocal: } \frac{4}{3}$$

Example

Find the reciprocal.

1. $\frac{6}{7}$ 2. $\frac{5}{9}$ 3. $\frac{1}{4}$ 4. $\frac{8}{9}$

Solution

1. The reciprocal of $\frac{6}{7}$ is $\frac{7}{6}$.

2. The reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$.

Example continued

Find the reciprocal.

1. $\frac{6}{7}$ 2. $\frac{5}{9}$ 3. $\frac{1}{4}$ 4. $\frac{8}{9}$

Solution

3. The reciprocal of $\frac{1}{4}$ is 4.

4. The reciprocal of $\frac{8}{9}$ is $\frac{9}{8}$.

0 Has No Reciprocal

The number 0, or $\frac{0}{n}$ has no reciprocal.

(Recall that $\frac{n}{0}$ is not defined.)

Division of Fractions

To divide fractions, multiply the dividend by the reciprocal of the divisor.

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \cdot \frac{4}{3} = \frac{2 \cdot 4}{5 \cdot 3} = \frac{8}{15}$$

Multiply by the reciprocal of the divisor.

Example

Divide and simplify. $\frac{3}{4} \div \frac{5}{16}$

Solution

$$\frac{3}{4} \div \frac{5}{16} = \frac{3}{4} \cdot \frac{16}{5}$$

Multiply by the reciprocal of the divisor

$$= \frac{3 \cdot 4 \cdot 4}{4 \cdot 5}$$

Factoring and identifying a common factor

$$= \frac{4}{4} \cdot \frac{3 \cdot 4}{5}$$

Removing a factor equal to 1

$$= \frac{12}{5}$$

Example

Divide and simplify. $\frac{9}{10} \div \frac{13}{15}$

Solution

$$\frac{9}{10} \div \frac{13}{15} = \frac{9}{10} \cdot \frac{15}{13}$$

Multiply by the reciprocal of the divisor

$$= \frac{3 \cdot 3 \cdot 3 \cdot 5}{2 \cdot 5 \cdot 13}$$

Factoring and identifying a common factor

$$= \frac{5}{5} \cdot \frac{3 \cdot 3 \cdot 3}{2 \cdot 13} = \frac{27}{26}$$

Removing a factor equal to 1

Example

Solve: $\frac{4}{5} \cdot x = \frac{14}{5}$.

Solution

$$\frac{4}{5} \cdot x = \frac{14}{5}$$

$$\frac{\frac{4}{5}x}{\frac{4}{5}} = \frac{\frac{14}{5}x}{\frac{4}{5}}$$

Dividing by 4/5 on both sides

$$x = \frac{14}{5} \cdot \frac{5}{4}$$

Multiplying by the reciprocal

$$x = \frac{2 \cdot 7 \cdot 5}{5 \cdot 2 \cdot 2} = \frac{2}{2} \cdot \frac{5}{5} \cdot \frac{7}{2} = \frac{7}{2}$$

Example

Solve: $\frac{4}{5} \cdot x = 20.$

Solution $\frac{4}{5} \cdot x = 20$

$$x = 20 \div \frac{4}{5}$$

$$= 20 \cdot \frac{5}{4}$$

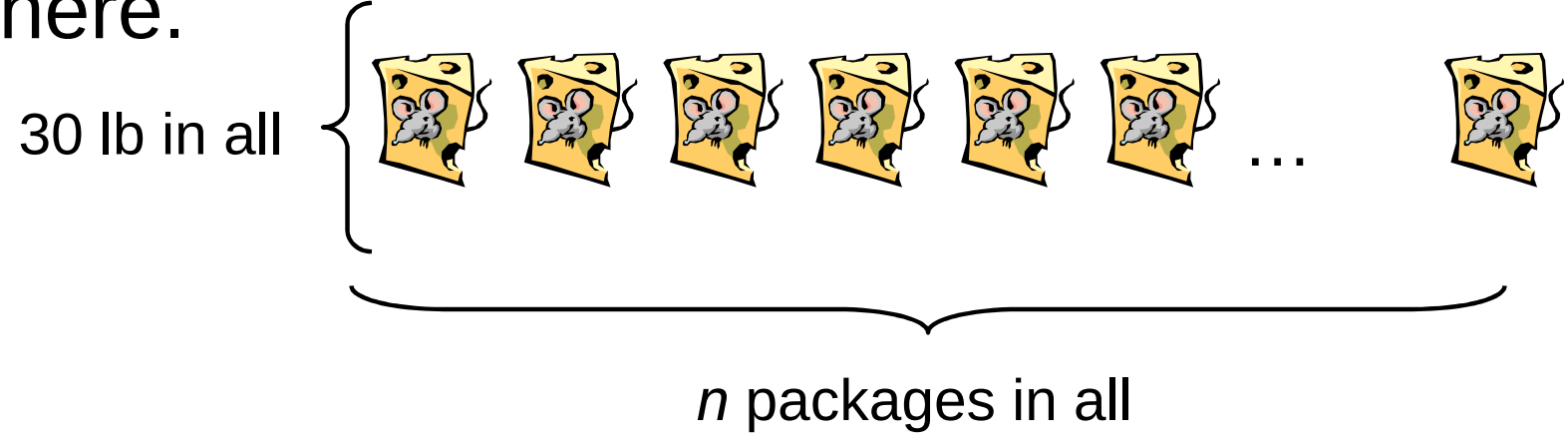
$$= \frac{5 \cdot 20}{4}$$

$$= \frac{5 \cdot 4 \cdot 5}{4} = \frac{4}{4} \cdot \frac{5 \cdot 5}{1} = \frac{5 \cdot 5}{1} = 25$$

Example

Sofo's Market pre-packages mozzarella cheese in ¾ pound packages. How many packages can be made from a 30-lb block of cheese?

Familarize. Make a drawing or at least visualize the situation. Repeated subtraction or division, will work here.



Example continued

Translate. The problem can be translated to the following equation.

$$n = 30 \div \frac{3}{4}$$

Solve.

$$n = 30 \div \frac{3}{4}$$

$$n = 30 \cdot \frac{4}{3} = \frac{30}{1} \cdot \frac{4}{3}$$

$$n = \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5}{3} = \frac{3}{3} \cdot \frac{2 \cdot 2 \cdot 2 \cdot 5}{1} = 40$$

Example continued

Check. If each of the 40 packages took ¾ pound of cheese, we would know that

$$\frac{3}{4} \cdot 40 = \frac{3 \cdot 4 \cdot 10}{4} = \frac{3 \cdot \cancel{4} \cdot 10}{\cancel{4}} = 30$$

Our answer checks.

State. Sofo's Market can fill 40 packages with cheese.