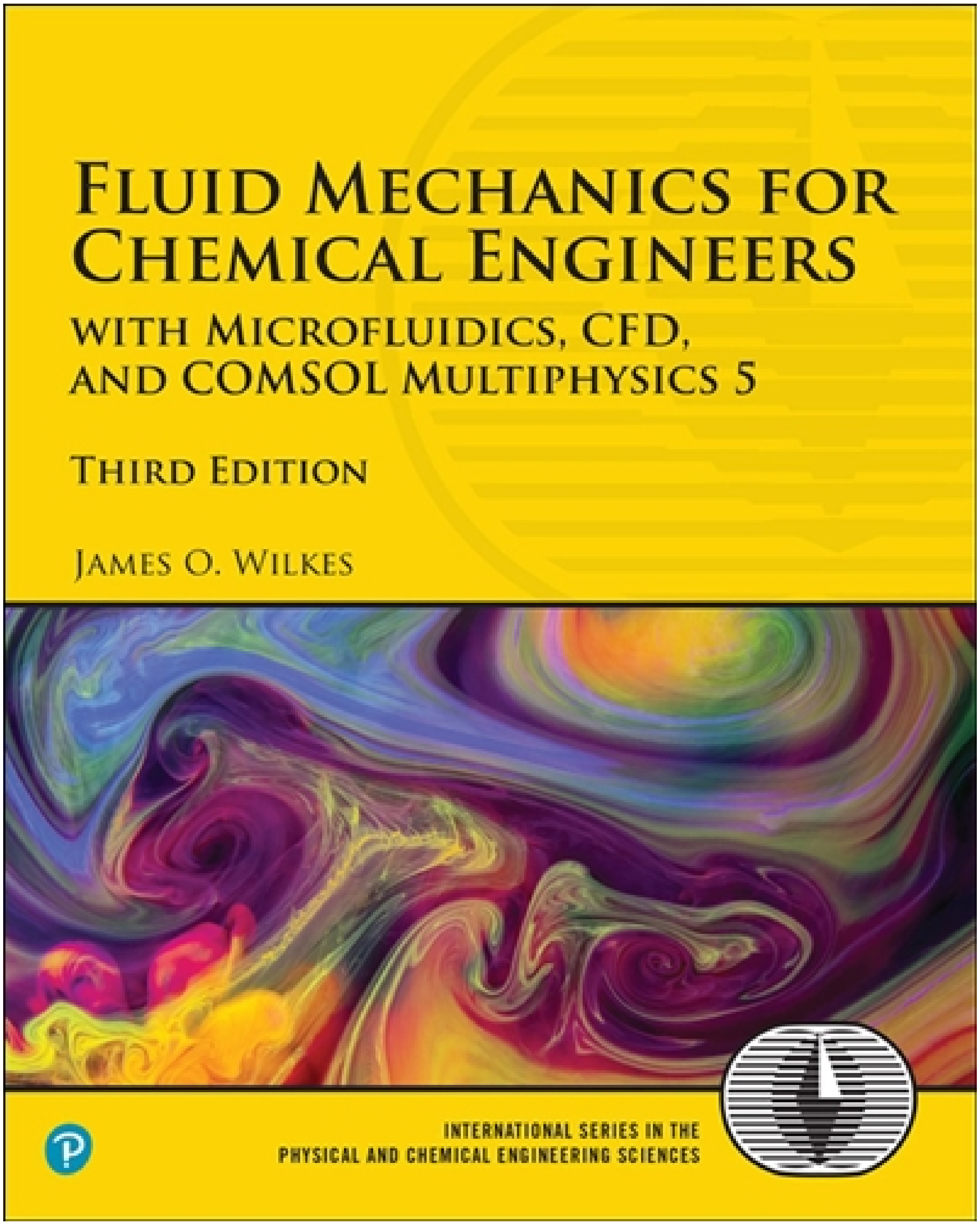
Solutions for Fluid Mechanics for Chemical Engineers 8th Edition by Wilkes



Solutions

Solutions Manual for

Fluid Mechanics for Chemical Engineers

Third Edition

with Microfluidics, CFD, and COMSOL Multiphysics 5

James O. Wilkes

PRENTICE HALL

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1.1

Units Conversion

(a) $\dfrac{1}{640} \times 5280^2 \qquad = \underline{\underline{4.36 \times 10^4 \ ft^3}}$

$acre\ ft \quad \dfrac{mile^2}{acre} \quad \dfrac{ft^2}{miles^2}$

(b) $4.36 \times 10^4 \times 7.48 \qquad = \underline{\underline{3.26 \times 10^5 \ gal}}$

$ft^3 \quad \dfrac{gal}{ft^3}$

(c) $4.36 \times 10^4 \times \dfrac{1}{3.281^3} \qquad = \underline{\underline{1,233 \quad m^3}}$

$ft^3 \quad \dfrac{m^3}{ft^3}$

(d) $M = \rho V$

$$= \underset{m^3}{\overset{V}{1,233}} \times \underset{\frac{kg}{m^3}}{\overset{\rho}{1,000}} = 1.233 \times 10^6 \ kg$$

$= \underline{\underline{1,233 \quad tonnes}} \quad (t)$

1.2

Units Conversion

Viscosity

$$\mu = 10 \text{ centipoise} = 10 \times 0.01 \times \underbrace{10^{-1}\,\frac{kg}{m\,s}}_{1\text{ poise}} = \underline{\underline{0.01\,\frac{kg}{m\,s}}}$$

$$= 0.01 \times \frac{0.3048}{0.4536} = \underline{\underline{6.72 \times 10^{-3}\,\frac{lbm}{ft\,s}}}$$

$$\frac{kg}{m\,s} \qquad \frac{lbm}{kg} \qquad \frac{m}{ft}$$

(Useful conversion factors: $1 \text{ cp} = 0.000672 \text{ lbm/ft s}$

$= 2.42 \text{ lbm/(ft hr)}$)

Density

$$\rho = 0.8 \times 1 \times \frac{(100)^3}{1000} = \underline{\underline{800\,\frac{kg}{m^3}}}$$

$$\frac{g}{cm^3} \qquad \frac{kg}{g} \qquad \left(\frac{cm}{m}\right)^3$$

$$= 0.8 \times 62.4 = \underline{\underline{49.9\,\frac{lbm}{ft^3}}}$$

1.3

Units Conversion

Gravitational Acceleration

$$g = \frac{981}{100} \frac{cm}{s^2} \frac{m}{cm} = 9.81 \frac{m}{s^2}$$

Pressure

$$p = \frac{14.7 \times 32.2 \times 144 \times 3.281}{2.205}$$

$$\frac{lbf}{in^2} \frac{lbm\, ft}{lbf\, s^2} \frac{in^2}{ft^2} \frac{kg}{lbm} \frac{ft}{m}$$

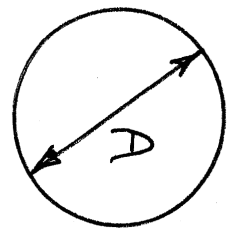
$$= 1.01 \times 10^5 \frac{kg}{m\, s^2} = \frac{N}{m^2} = Pa$$

Since $1\ bar = 10^5\ Pa$

$$p = 1.01\ bar$$

1.4

Meteorite Density



Mass $\quad M = \dfrac{\pi D^3}{6}\rho$

$$\rho = \frac{6M}{\pi D^3} = \frac{6 \times 10^6 \times 1000}{\pi \times 60^3}$$

$$\rho = 8{,}842 \ \frac{kg}{m^3}$$

$$s = \frac{8{,}842}{1{,}000} = 8.84$$

Specific gravities of some elements are

Fe	7.86	Co	8.9	Ag	10.5
Ni	8.9	Pb	11.3	Au	19.3
				U	18.5

Most likely candidate is iron, the deviation being due to the "ball park" figures in the article.

Kinetic Energy

$$\frac{1}{2} M u^2 = \frac{1}{2} 10^9 \times (15000)^2 = 1.125 \times 10^{17} \ J$$

$$\text{TNT equivalent} = \frac{1.125 \times 10^{17}}{5 \times 10^9} = 2.25 \times 10^7 \ \text{tonnes}$$

1.5

Reynolds Number

Cross-sectional area

$$A = \frac{\pi D^2}{4} = \frac{\pi \left(\frac{1.05}{12}\right)^2}{4} = 0.00601 \ ft^2$$

Volumetric flow rate

$$Q = \frac{35}{7.48 \times 60} = 0.0780 \ \frac{ft^3}{s}$$

Mean velocity

$$u_m = \frac{Q}{A} = \frac{0.0780}{0.00601} = 12.98 \ \frac{ft}{s}$$

Reynolds number

$$Re = \frac{\rho u_m D}{\mu} = \frac{62.3 \times 12.98 \times 1.05/12}{1.2 \times 0.000672}$$

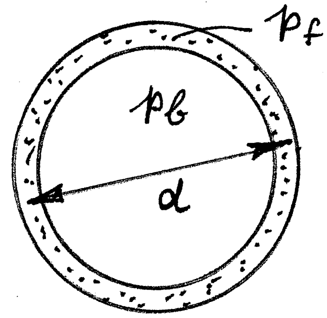
$$\left.\frac{lbm}{ft^3} \ \frac{ft}{s} \ ft \qquad \frac{cP}{cP \ \ lbm/ft\,s}\right\} \text{All units cancel}$$

$$\doteq \underline{\underline{87,740}} \quad (\text{dimensionless})$$

1.6

Pressure in Bubble

Atmosphere p_a



Radius $a = \dfrac{d}{2}$

From notes, the increase in pressure as we go inwards across a convex surface is

$$\left.\begin{aligned} p_f - p_a &= \frac{2\sigma}{a} \\ p_b - p_f &= \frac{2\sigma}{a} \end{aligned}\right\} \text{ Two surfaces are involved}$$

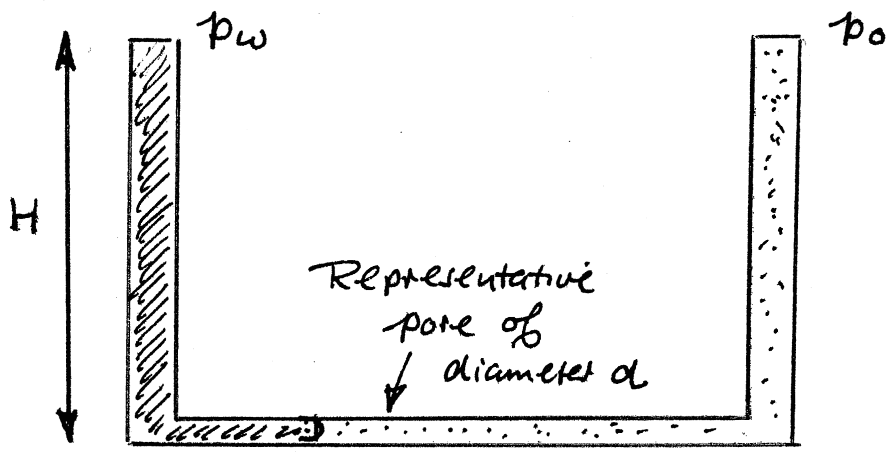
Hence, by addition,

$$p_b - p_a = \frac{4\sigma}{a} = \frac{8\sigma}{d}$$

$$p_b = p_a + \frac{8\sigma}{d}$$

1.7

Reservoir Water-Flooding



$$p_o + \rho_o g H + \underbrace{\frac{4\sigma}{d}}_{\text{Increase in pressure from oil into water}} = p_w + \rho_w g H$$

Hence required water inlet pressure is

$$p_w = p_o - \underbrace{(\rho_w - \rho_o)}_{\text{Positive}} g H + \frac{4\sigma}{d}$$

1.8 – 1

Barometer Reading

(2) • House $\quad z_2 = 950 \text{ ft} \quad p_2 = ?$

$H_2 = ?$

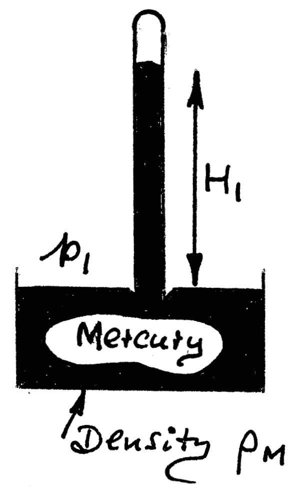
(1) • Weather Station $\quad z_1 = 700 \text{ ft} \quad p_1 = 0.966 \text{ bar}$

$H_1 = ?$

At the weather station

The atmospheric pressure p_1 is balanced by a column of mercury of height H_1:

$$p_1 = \rho_M g H_1$$



Hence

$$H_1 = \frac{p_1}{\rho_M g} = \frac{0.966 \times 10^5 \times 3.281 \times 12}{13.57 \times 1000 \times 9.81}$$

$$\frac{kg}{m^2}\frac{m}{s^2}\quad\frac{m^3}{kg}\frac{s^2}{m}\frac{in}{m} = in$$

$$= \underline{\underline{28.57 \text{ in mercury}}}$$

(Note: $g = 9.81 \text{ m/s}^2$, $3.281 \text{ ft} = 1 \text{ m}$,
$\rho_W = 1000 \frac{kg}{m^3}$, $1 \text{ bar} = 10^5 \text{ pascal} = 10^5 \frac{kg\frac{m}{s^2}}{m^2}$)

Correction for elevation increase. Since $z_2 - z_1$ is "small" for air, we can take ρ_A as essentially constant between ① and ②. Now pressure at weather station is

$$0.966 \text{ bar} \times \frac{14.7 \text{ psia}}{1.01 \text{ bar}} \left(\begin{matrix}\text{see} \\ \text{Problem} \\ 3\end{matrix}\right) = 14.06 \text{ psia}$$

Hence the appropriate mean pressure between ① and ② for purposes of estimating the density can be taken as 14.06 or (as done here, with a trifling change in the answer) slightly less — say 14.0 psia.

$$\rho_A = \frac{M_A p}{R T} = \frac{28.8 \times 14.0}{10.73 \times (460 + \underset{\uparrow}{25})} = 0.0775 \frac{\text{lbm}}{\text{ft}^3}$$

Average for January

Change in Pressure. $p_2 - p_1 = -\rho_A g (z_2 - z_1)$

Change in Barometer Reading $H_2 - H_1 = \dfrac{p_2 - p_1}{\rho_M g} = -\dfrac{\rho_A}{\rho_M}(z_2 - z_1)$

$$H_2 - H_1 = -\frac{0.0775\,(950 - 700) \times 12}{13.57 \times 62.4} = -0.27 \text{ in}$$

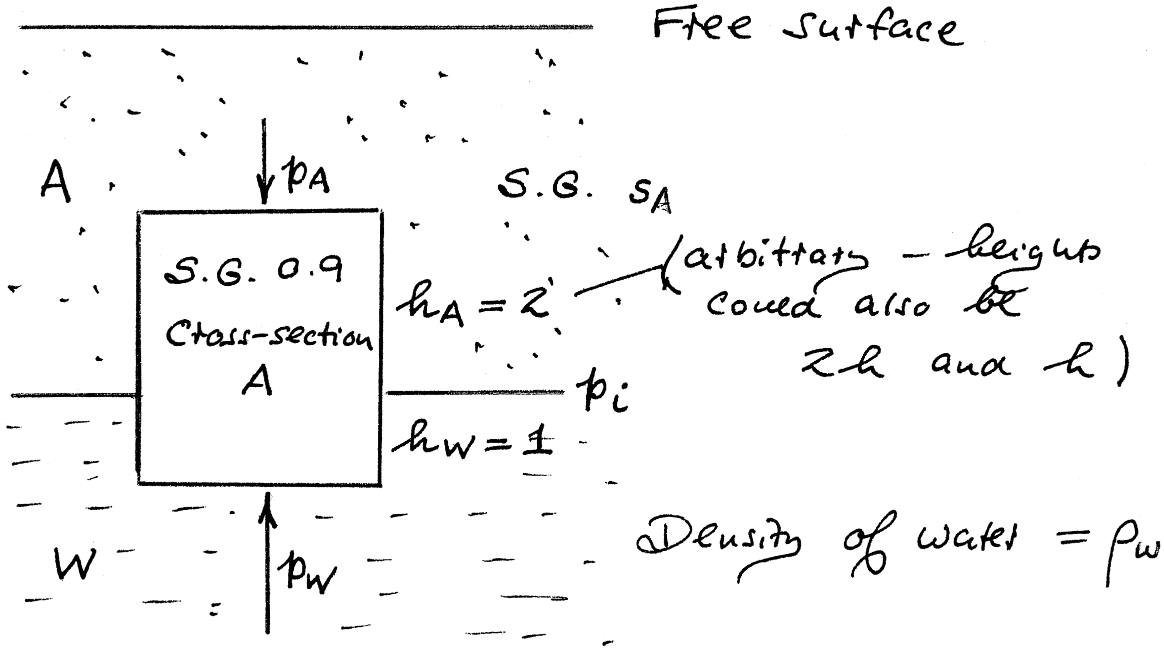
Hence $H_2 = H_1 - 0.27 = 28.57 - 0.27 = \underline{\underline{28.3 \text{ in Hg}}}$

Pressure $p_2 = \rho_M g H_2 = \dfrac{13.57 \times 62.4 \times 32.2 \times 28.3}{32.2 \times 144 \times 12}$

$\underline{\underline{p_2 = 13.87 \text{ psia}}}$ $\qquad \dfrac{\text{lbm}}{\text{ft}^3} \; \dfrac{\text{ft}}{\text{sec}^2} \; \text{in} \; \dfrac{\text{lbf sec}^2}{\text{lbm ft}} \; \dfrac{\text{ft}^2}{\text{in}^2} \; \dfrac{\text{ft}}{\text{in}}$

1.9

Two-Layer Buoyancy



Method 1

Weight displaced (upwards buoyant force) = weight of cylinder downwards

$$\rho_w A g\,(1 + 2 s_A) = 0.9\,\rho_w A\, 3\, g$$

$$\underline{\underline{s_A = 0.85}}$$

Method 2

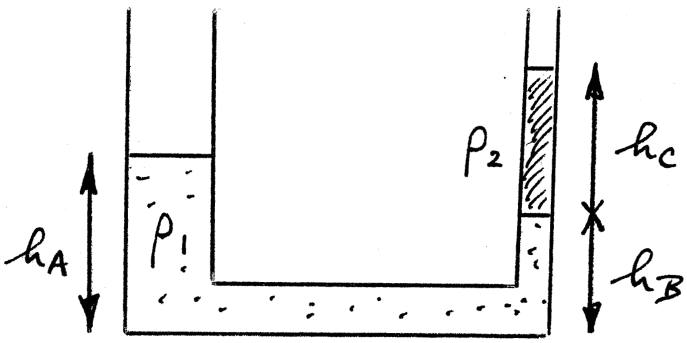
Force balance on cylinder ↓

$$p_A A + \underbrace{0.9 \rho_w A\, 3 g}_{\text{weight}} - p_W A = 0$$

$$p_i - 2 s_A \rho_w g + 2.7 \rho_w g - (p_i + \rho_w g) = 0$$

$$\underline{\underline{s_A = 0.85}}$$

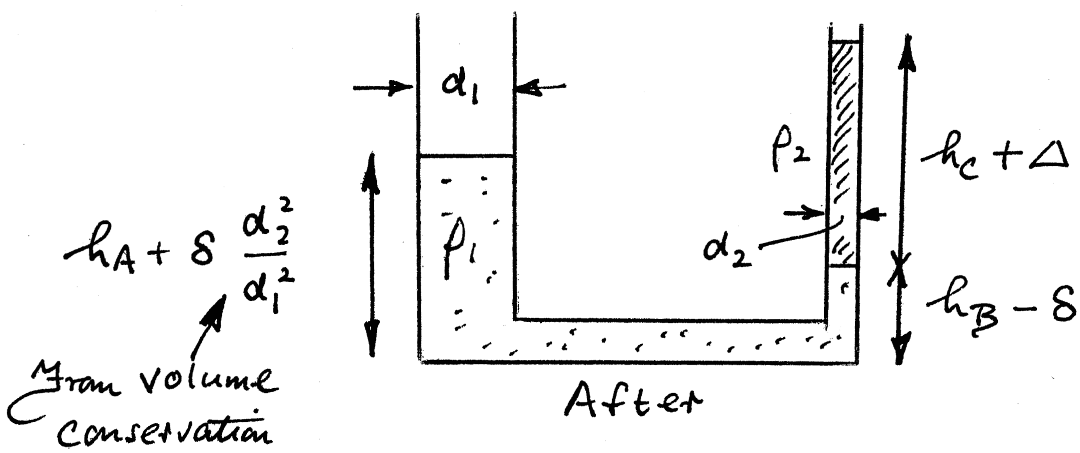
Differential Manometer



Before

$$\rho_1 (h_A - h_B) = \rho_2 h_C \qquad (1)$$

Now add Δ to h_C and let h_B decline by δ



After

$$\rho_1 \left(h_A + \delta \frac{d_2^2}{d_1^2} - (h_B - \delta)\right) = \rho_2 (h_C + \Delta) \qquad (2)$$

Subtract (1) from (2)

$$\rho_1 \delta \left(\frac{d_2^2}{d_1^2} + 1\right) = \rho_2 \Delta = \rho_2 \frac{v_2}{\frac{\pi}{4} d_2^2} \qquad (3)$$

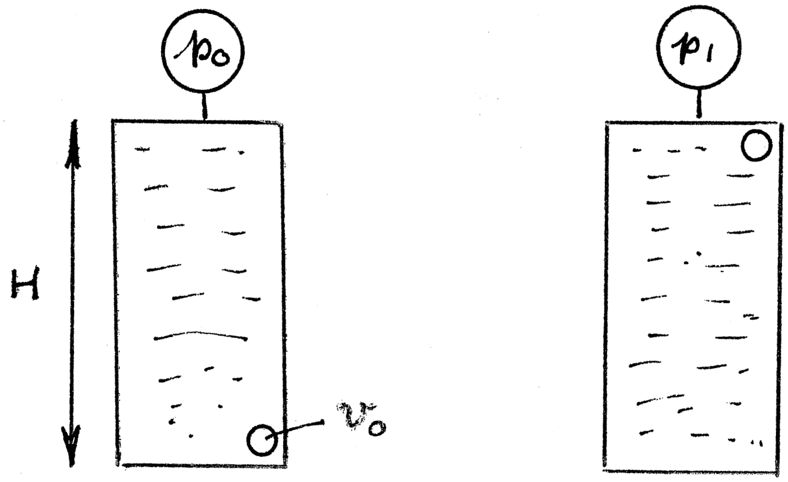
Equation (3) gives δ as a function of v_2, and also enables ρ_2 to be found by observing the value of δ.

1.10-2

Solution for δ gives:

$$\delta = \frac{\rho_2}{\rho_1} \frac{4 v_2}{\pi d_2^2} \left(\frac{d_1^2}{d_1^2 + d_2^2} \right)$$

Ascending Bubble



Since the cylinder and oil volumes don't change, the bubble volume must remain constant at v_o.

But

$$pV_o = nRT$$

Therefore, since T is constant, p within the bubble does not change. Hence

$$p = \underbrace{p_o + \rho g H}_{\text{Before}} = \underset{\text{After}}{p_1}$$

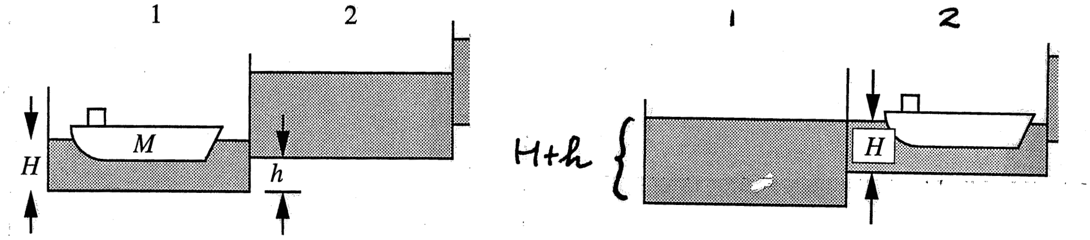
Thus $\quad p_1 = p_o + \rho g H$

1.12

Ship Passing Through Locks

Uphill The ship must increase its elevation by an amount h as it passes from lock 1 to lock 2. Consider the water in lock 1 before and after;

Before **After**



Mass of water in lock $= \rho A H - M$

Mass of water in lock $= \rho A (H+h)$

From Archimedes law, the ship displaces a mass M of water

Hence mass of water to be supplied to lock 1 is

$$\rho A(H+h) - (\rho A H - M) = \rho A h + M$$

Downhill A similar analysis gives the water loss from a lock as

$$\underbrace{\rho A H - M}_{\text{mass at start}} - \underbrace{\rho A (H-h)}_{\text{mass at end}} = \rho A h - M$$

mass at end (note that final depth of water in lock is $H-h$)

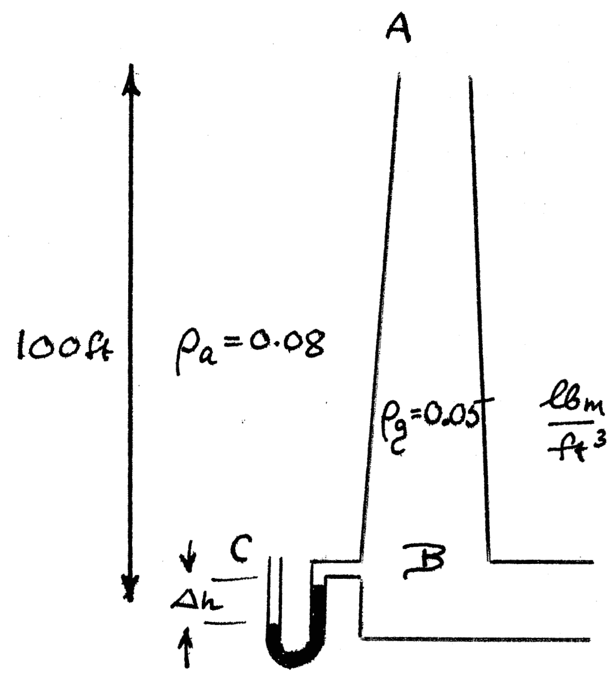
Total water supply

(i) Uphill only: $\rho A h + M$ (depends on M)

(ii) Up and down: $\rho A h + M + \rho A h - M = 2\rho A h$ (independent of M)

1.13

Furnace Stack



Start from point A and consider hydro-static increase of pressure in both cases:

$$p_B = p_A + \rho_g g H$$

$$p_C = p_A + \rho_a g H$$

Hence $p_C = p_B + \underbrace{(\rho_a - \rho_g)}_{\text{positive}} g H$

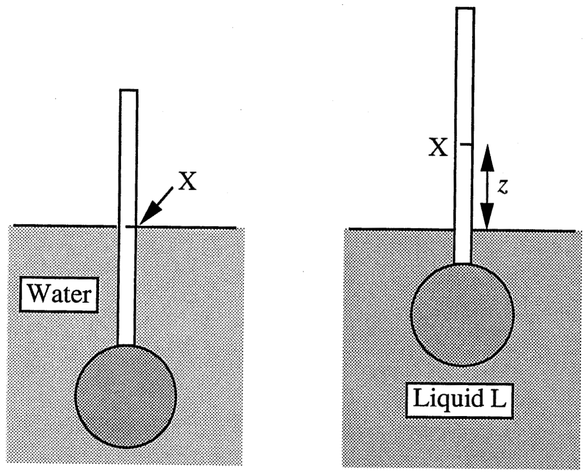
Hence the water moves <u>up</u> in the right-hand leg by Δh given by

$$\rho_w g \, \Delta h = (\rho_a - \rho_g) g H$$

$$\Delta h = \frac{\rho_a - \rho_g}{\rho_w} H = \frac{(0.08 - 0.05) \times 100 \times 12}{62.4} = \underline{\underline{0.58 \text{ in.}}}$$

1.14

Hydrometer



Since the same weight (that of the hydrometer) is supported by the displaced liquid in both cases:

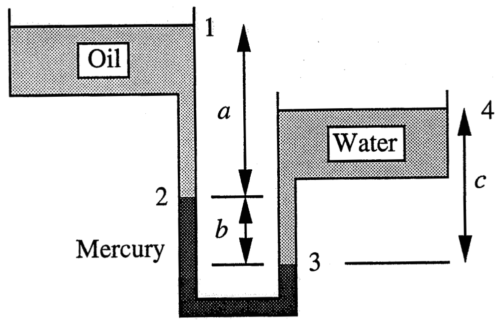
$$\underbrace{Mg}_{\text{mass of hydrometer}} = V \underbrace{\rho_w}_{\text{Density of water}} g = (V - Az)\,\rho_w s g$$

Cancellation of $\rho_w g$ and solution for s gives

$$s = \frac{1}{1 - \frac{Az}{V}}$$

1.15

Three-Liquid Manometer



From hydrostatics:

$$p_4 = p_1 + \rho_o a g + \rho_M b g - \rho_w c g = p_1$$

Cancel p_1 and divide by $\rho_w g$, with $s = \frac{\rho}{\rho_w}$

$$s_o a + s_M b = c$$

$$b = \frac{c - s_o a}{s_M} = \frac{48 - 0.8 \times 72}{13.6} = -0.706$$

Thus the diagram is incorrect as drawn, and the mercury rises on the right by 0.706 m.

1.16

Pressure on Mt. Erebus

From lecture notes, $\frac{dp}{dz} = -\rho g = -\frac{Mp}{RT} g$

Separating variables $\int_{p_0}^{p_H} \frac{dp}{p} = -\frac{Mg}{RT} \int_0^H dz$

$$\ln \frac{p_H}{p_0} = -\frac{MgH}{RT} = \frac{28.8 \times 32.2 \times 13500}{10.73 \times (460-5) \times 32.2 \times 144}$$

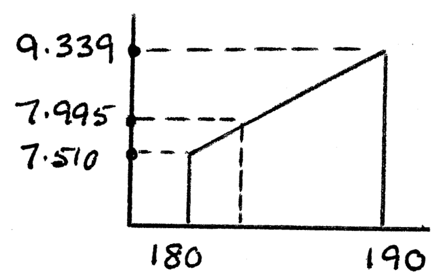
$$\frac{lb_m}{mole} \ \frac{ft}{sec^2} \ ft \ \frac{mole \ ^\circ R}{psia \ ft^3} \ \frac{}{^\circ R} \ \frac{lb_f \ sec^2}{lb_m \ ft} \ \frac{ft^2}{in^2}$$

$$= -0.553 \text{ (dimensionless)}$$

$$p_H = p_0 \, e^{-0.553} = 13.9 \times 0.575 = \underline{\underline{7.995 \text{ psia}}}$$

From Perry's <u>Chemical Engineers Handbook</u>

T (°F)	Vapor Pressure of Water (psia)
180	7.510
190	9.339

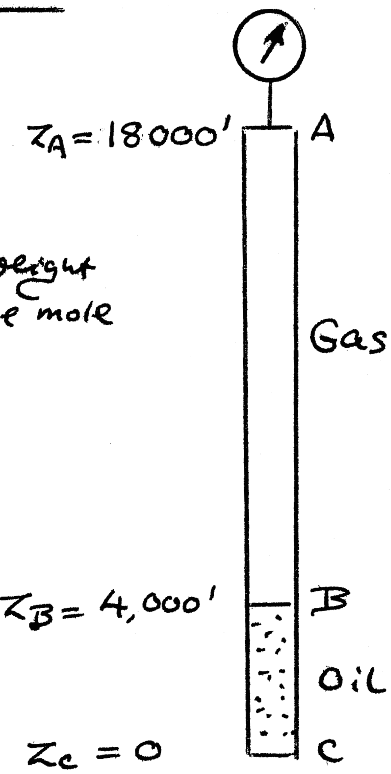


Using linear interpolation

$$T \doteq 180 + \frac{7.995 - 7.510}{9.339 - 7.510} \times 10 \doteq \underline{\underline{182.7 \ ^\circ F}}$$

1.17

Oil and Gas Well Pressures



Hydrostatic variation of pressure

$$\frac{dp}{dz} = -\rho g$$

Imperfect Gas Equation

Molecular weight = mass of one mole

$$pV = zRT \quad \text{or} \quad \rho = \frac{M}{V} = \frac{Mp}{zRT}$$

per mole

For Gas Column

$$\frac{dp}{dz} = -\frac{Mp}{zRT}g \quad \text{or} \quad \int_{p_B}^{p_A} \frac{dp}{p} = -\frac{Mg}{zRT}\int_{z_B}^{z_c} dz$$

$$\ln\frac{p_A}{p_B} = \frac{-0.65 \times 28.8 \times 32.2 \times (18{,}000 - 4{,}000)}{0.8 \times 10.73 \times 520 \times 144 \times 32.2} = -0.408$$

$$p_B = p_A e^{0.408} = 1.503\, p_A$$

$$= 1.503\,(2000 + 14.7) = 3028.9 \text{ psia}$$

$$\underline{\underline{p_B = 3014.2 \text{ psig}}}$$

For Oil Column

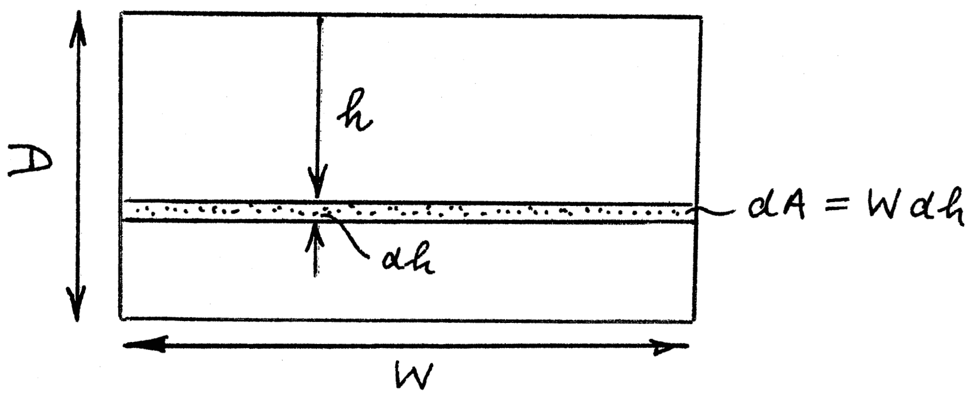
$$p_c = p_B + \rho g (z_B - z_c)$$

$$= 3014.2 + \frac{0.70 \times 62.4 \times 32.2 \times 4{,}000}{32.2 \times 144}$$

$$= 3014.2 + 1{,}213.3 = \underline{\underline{4{,}227.5 \text{ psig}}}$$

1.18

Thrust on Dam



Pressure at depth h is $p = \rho g h$

Force on differential strip

$$dF = p\,dA = \rho g h\,W\,dh$$

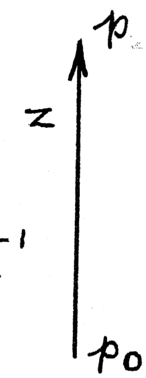
Total thrust is

$$F = \int_0^F dF = \int_0^D \rho g h\,W\,dh$$

$$= \frac{1}{2}\rho g W D^2$$

1.19

Pressure Variations in Air



Accurately $\quad p = p_0\, e^{-(M_w g z / RT)}$

For air at 40°F = 500°R

$$\frac{M_w g}{RT} = \frac{28.8 \times 32.2 \times 5{,}280}{10.73 \times 500 \times 32.2 \times 144} = 0.197 \text{ mile}^{-1}$$

$$\frac{\text{lbm}}{\text{lb mole}}\ \frac{\text{ft}}{\text{s}^2}\ \frac{\text{ft}}{\text{mile}}\ \frac{\text{in}^2}{\text{lbf}}\ \frac{\text{lb mole °R}}{\text{ft}^3\ \text{°R}}\ \frac{\text{lbf s}^2}{\text{lbm ft}}\ \frac{\text{ft}^2}{\text{in}^2} = \frac{1}{\text{mile}}$$

Approximately, $\quad p^a = p_0 \left(1 - \frac{M_w g}{RT} z\right)$

For a 1% error, assuming $p > p^a$ (to be checked)

$$\frac{p - p^a}{p} = 1 - \frac{p^a}{p} = 1 - \frac{1 - 0.197z}{e^{-0.197z}} = 0.01$$

That is, $\quad 0.99\, e^{-0.197z} + 0.197z - 1 = 0$

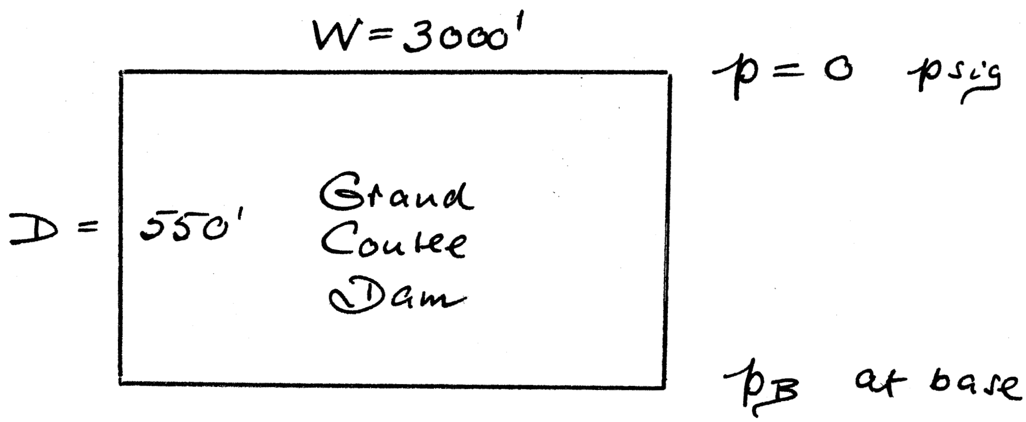
From Excel Solver, $\quad z = 0.686$

That is, the approximation is good within 1% for elevations up to 0.686 miles

Note, for $f(z) = 0$, Solver was asked to find z that minimized $|f(z)|$.

1.20

Grand Coulee Dam



(a) $p_B = \rho g D = \dfrac{62.4 \times 32.2 \times 550}{32.2 \times 144}$

$= \underline{\underline{238.3 \text{ psig}}}$

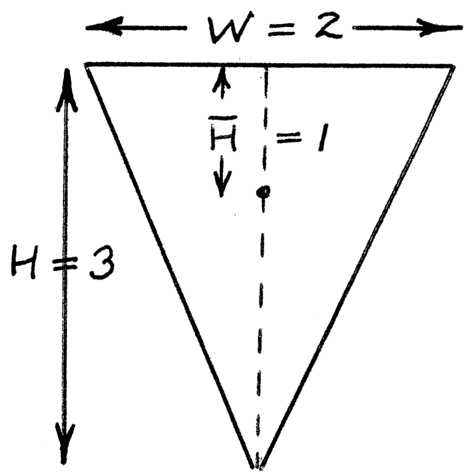
(b) From lecture notes, horizontal force is

$$F = \underbrace{\frac{1}{2} p_B}_{\bar{p}} W D$$

$$= \frac{238.3 \times 3000 \times 550 \times 144}{2}$$

$$= \underline{\underline{2.83 \times 10^{10} \text{ lb}_f}}$$

Force on V-Shaped Dam



$$A = \frac{2 \times 3}{2} = 3 \ m^2$$

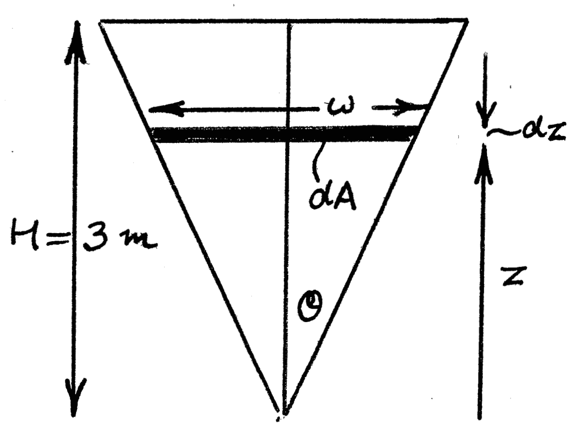
Pressure at centroid

$$\bar{p} = \rho g \bar{H}$$

$$= 10^3 \times 9.81 \times 1$$

$$= 9.81 \times 10^3 \ N/m^2 \text{ or } Pa$$

$$F = \bar{p} A = \underline{\underline{2.943 \times 10^4 \ N}}$$



$$w = 2 z \tan\theta$$

$$F = \int_{z=0}^{H} \underbrace{\rho g (H-z)}_{p} \underbrace{w \, dz}_{dA}$$

where $w = 2 z \tan\theta$

$$F = 2 \rho g \tan\theta \int_0^H (H-z) z \, dz$$

$$= 2 \rho g \tan\theta \left[\frac{Hz^2}{2} - \frac{z^3}{3} \right]_0^H = \frac{1}{3} \rho g H^3 \tan\theta$$

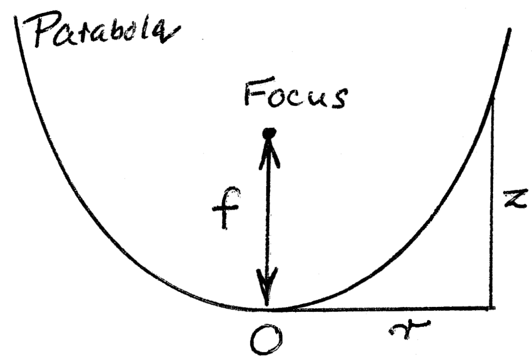
$$= \frac{1}{3} \times 10^3 \times 9.81 \times 3^3 \times \frac{1}{3} = \underline{\underline{29{,}430 \ N}}$$

1.22

Rotating Mercury Mirror

Equation of parabola with focal length f is

$$r^2 = 4fz \qquad (1)$$



But from fluid mechanics, we know that

$$z = \frac{\omega^2 r^2}{2g} \qquad (2)$$

Angular velocity is $\omega = 2\pi N = 2\pi \times \frac{1}{6} = 1.05 \ \frac{rad}{s}$

Focal length, from (1) and (2), is

$$f = \frac{g}{2\omega^2} = \frac{9.81}{2 \times 1.05^2} = \underline{\underline{4.45 \ m}}$$

Depression at center is also elevation z when $r = \frac{D}{2}$

$$\Delta z = \frac{\omega^2 \left(\frac{D}{2}\right)^2}{2g} = \frac{1.05^2 \left(\frac{40}{12 \times 3.281 \times 2}\right)^2}{2 \times 9.81} \quad (\text{since } z = 0 \text{ at } r = 0)$$

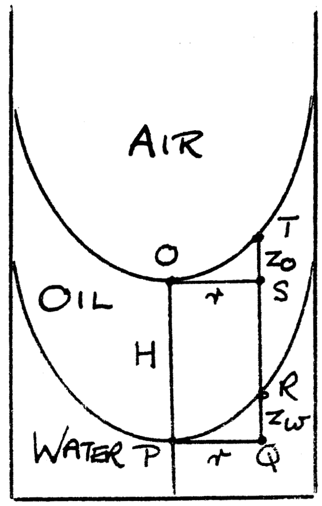
$$= \underline{\underline{0.0145 \ m = 1.45 \ cm}}$$

For new $D = 30$ m mirror (with f unchanged)

$$\Delta z = \frac{1.05^2 \times \left(\frac{30}{2}\right)^2}{2 \times 9.81} = \underline{\underline{12.64 \ m}} \quad (\text{a very large increase})$$

1.23

Oil & Water in Rotating Container



Pressure in water at Q

$$\underset{OP}{\rho_o H g} + \underset{PQ}{\rho_w \frac{\omega^2 r^2}{2}} = \underset{TS}{\rho_o z_o g} + \underset{SR}{\rho_o (H - z_w) g} + \underset{RQ}{\rho_w z_w g}$$

$$\rho_w \frac{\omega^2 r^2}{2} = \rho_o z_o g + (\rho_w - \rho_o) z_w g \qquad (1)$$

Pressure in Oil at S

$$\rho_o \frac{\omega^2 r^2}{2} = \rho_o z_o g \qquad (2)$$

Subtract (2) from (1) $\quad (\rho_w - \rho_o) \dfrac{\omega^2 r^2}{2} = (\rho_w - \rho_o) z_w g$

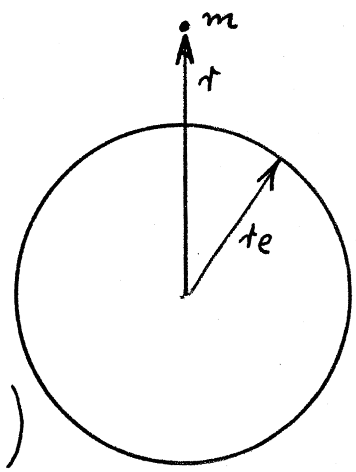
Hence $\quad z_w = \dfrac{\omega^2 r^2}{2g}$

and $\quad z_o = \dfrac{\omega^2 r^2}{2g}$

Hence shapes of both interfaces are identical.

1.24

Energy to Place Satellite in Orbit



Work done in pushing the satellite through a distance dr is

$$dw = F\,dr = G\,m\,M_e\,\frac{dr}{r^2}$$

Total energy expended is

$$w = \int_0^w dw = \int_{r_e}^{r_s} G\,m\,M_e\,\frac{dr}{r^2} = G\,m\,M_e\left(\frac{1}{r_e} - \frac{1}{r_s}\right)$$

For a mass m at the earth's surface,

$$F = m g_e = \frac{G\,m\,M_e}{r_e^2} \quad \text{or} \quad G M_e = g_e r_e^2$$

Thus, $$w = m g_e r_e\left(1 - \frac{r_e}{r_s}\right)$$

Numerical values $\quad m = 6{,}700 \times 0.4536 = 3{,}039\ \text{kg}$

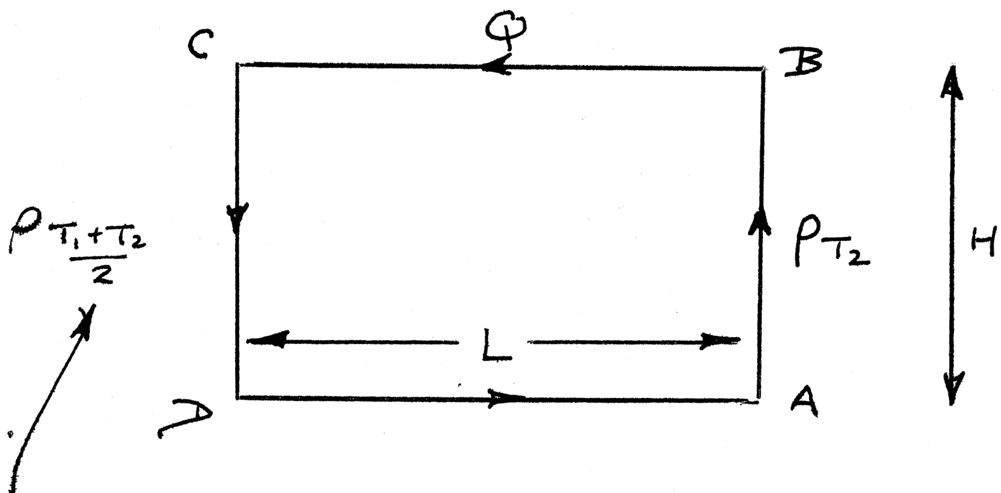
$$r_s - r_e = 360 \times 5{,}280 \times 0.3048 = 5.79 \times 10^5\ \text{m}$$

$$r_s = 5.79 \times 10^5 + 6.37 \times 10^6 = 6.95 \times 10^6\ \text{m}$$

$$w = 3{,}039 \times 9.81 \times 6.37 \times 10^6\left(1 - \frac{6.37 \times 10^6}{6.95 \times 10^6}\right) = \underline{\underline{1.585 \times 10^{10}\ \text{J}}}$$

$$\text{Average power} = \frac{1.585 \times 10^{10}}{78 \times 60} = 3.39 \times 10^6\ \text{W} = \underline{\underline{3.39\ \text{MW}}}$$

Central-Heating Loop



Note that the midpoint temperature along CD is $\frac{1}{2}(T_1+T_2)$

Going round the loop ABCD, there is no net change in the pressure:

$$\underbrace{\underset{\text{Loss in AB}}{-\rho_{T_2} g H} + \underset{\text{Gain in CD}}{\rho_{\frac{T_1+T_2}{2}} g H}}_{\text{Hydrostatic changes}} - \underbrace{\frac{c u^2}{D}(2L+2H)}_{\text{Total frictional loss}} = 0$$

But $\rho = \bar{\rho}\left[1 - \alpha(T - \bar{T})\right]$

$$\rho_{\frac{T_1+T_2}{2}} - \rho_{T_2} = \alpha\bar{\rho}\left(T_2 - \frac{T_1+T_2}{2}\right) = \frac{1}{2}\alpha\bar{\rho}(T_2 - T_1)$$

Hence

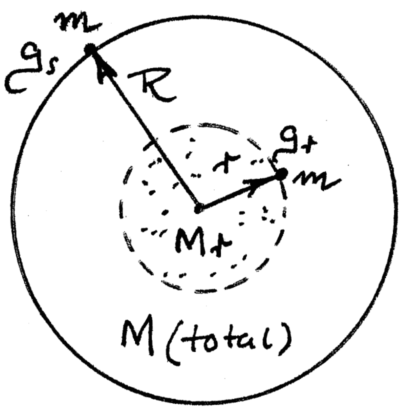
$$\frac{1}{2} g H \alpha \bar{\rho}(T_2 - T_1) = \frac{2cu^2}{D}(L+H)$$

But $Q = \frac{\pi D^2}{4} u$ or $u^2 = \frac{16 Q^2}{\pi^2 D^4}$

Hence $Q = \sqrt{\dfrac{\pi^2 g H \alpha \bar{\rho} D^5 (T_2 - T_1)}{64 c (L+H)}}$

Pressure at Center of Earth

Assume earth is liquid with uniform density ρ.



For mass m at radii r and R, the gravitational attractions are

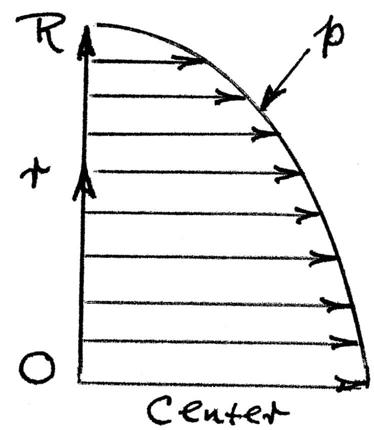
$$\left.\begin{aligned} m g_r &= \frac{G m M_r}{r^2} \\ m g_s &= \frac{G m M}{R^2} \end{aligned}\right\} \quad \frac{g_r}{g_s} = \frac{M_r R^2}{M r^2}$$

Masses:
$$\left.\begin{aligned} M_r &= \tfrac{4}{3}\pi \rho r^3 \\ M &= \tfrac{4}{3}\pi\rho R^3 \end{aligned}\right\} \quad \frac{M_r}{M} = \frac{r^3}{R^3} \qquad \boxed{\frac{g_r}{g_s} = \frac{r}{R}}$$

Hydrostatic Variations

$$\frac{dp}{dr} = -\rho g_r = -\frac{\rho g_s}{R} r \qquad \int_{p_r}^{0} dp = -\frac{\rho g_s}{R}\int_r^R r\,dr$$

Hence $p_r = \dfrac{\rho g_s R}{2}\left(1 - \dfrac{r^2}{R^2}\right)$



Thus, there is a parabolic variation of pressure.

$$p_c = \frac{\rho g_s R}{2}$$

$$p_c = \frac{g_s R}{2}\left(\frac{3M}{4\pi R^3}\right) = \frac{3 M g_s}{8\pi R^2}$$

Numerical Values

$$M = \frac{4}{3}\pi R^3 \rho$$

Hence

$$p_c = \frac{3 M g_s}{8\pi R^2} = \frac{3\left(\frac{4}{3}\pi R^3\right)\rho g_s}{8\pi R^2}$$

$$= \frac{R \rho g_s}{2}$$

$$= \frac{6.37\times10^6 \times 5{,}500 \times 9.81}{2} \quad \underbrace{m \frac{kg}{m^3} \frac{m}{s^2}}_{\frac{kg}{m s^2} = Pa}$$

$$\underline{\underline{p_c = 1.72\times10^{11} \ Pa}}$$

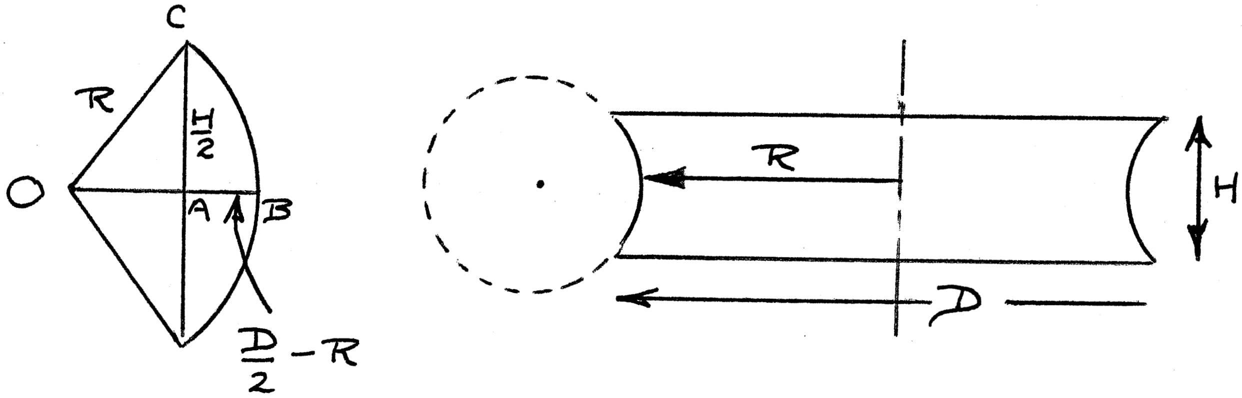
But 1 atm = 14.7 psi = 1.0133×10^5 Pa

Hence

$$p_a = \frac{1.72\times10^{11} \times 14.7}{1.0133\times10^5}$$

$$= \underline{\underline{2.49\times10^7 \ psi}}$$

Soap Film on Wire Rings



For curved surface, $\Delta p = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

Since there is zero pressure change across the film, the two radii of curvature must be equal, the one positive and the other negative.

Geometry gives $\quad OA = R - \left(\frac{D}{2} - R\right) = 2R - \frac{D}{2}$

Pythagoras gives $\quad R^2 = \left(2R - \frac{D}{2}\right)^2 + \left(\frac{H}{2}\right)^2$

$$= 4R^2 - 2RD + \frac{D^2}{4} + \frac{H^2}{4}$$

$$12R^2 - 8RD + (D^2 + H^2) = 0$$

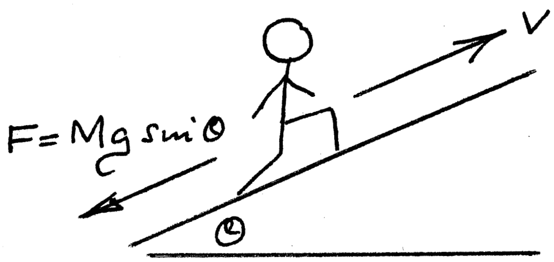
$$R = \frac{8D + \sqrt{64D^2 - 48(D^2+H^2)}}{24} = \frac{1}{6}\left(2D + \sqrt{D^2 - 3H^2}\right)$$

For $D < \sqrt{3}H$ this solution is clearly invalid.

In practice, as the rings are pulled further apart, the film would minimize its total area by forming two separate circles on the rings, with no film in between them.

1.28

Treadmill Stress Test



grade = tan θ

10%: θ = 5.71°, sin θ = 0.100

18%: θ = 10.20°, sin θ = 0.177

Power $P = Fv = Mg \sin\theta \, v$

$$= \frac{163 \times 32.2 \times 0.177 \times 5 \times 5280}{550 \times 32.2 \times 3600} \; \frac{lb_m\, ft}{s^2} \frac{mile}{hr} \frac{ft}{mile} \frac{s\, HP}{ft\, lb_f} \frac{lb_f\, s^2}{lb_m\, ft} \frac{hr}{s}$$

$$P = 0.385 \text{ HP}$$ (at end of test)

Power at start of test

$$= \frac{163 \times 32.2 \times 0.100 \times 1.7 \times 5280}{550 \times 32.2 \times 3600} = 0.0739 \text{ HP}$$

Mean power $\bar{P} \doteq \frac{1}{2}(0.0739 + 0.385) = 0.229$ HP

Energy expended $E = \bar{P} t$

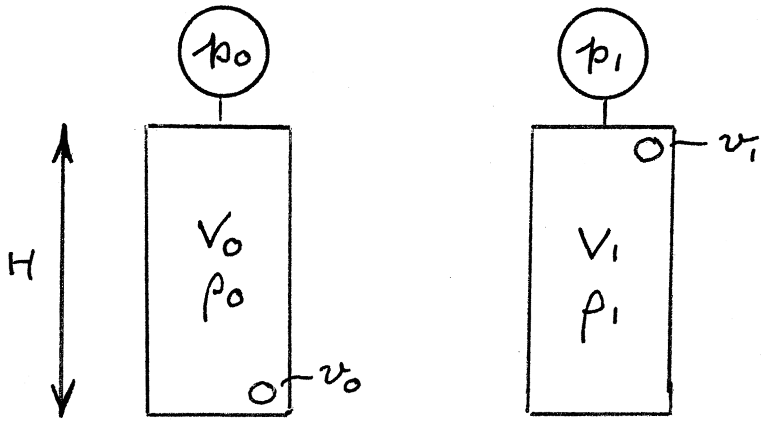
$$= \frac{0.229 \times 13.3 \times 550 \times 60 \times 0.3048}{0.2248} = 1.36 \times 10^5 \text{ J}$$

$$HP\ min\ \frac{ft\, lb_f}{s\, HP} \frac{s}{min} \frac{m}{ft} \frac{N}{lb_f} = N\,m = J$$

Bubble Rising in Compressible Liquid

Isothermal compressibility $\quad \beta = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \doteq -\frac{1}{V}\frac{\Delta V}{\Delta p}$

Hence $\quad \Delta V \doteq -\beta V \Delta p \quad (1)$



Rigid container $\quad V_0 + v_0 = V_1 + v_1 \quad (2)$

Ideal gas $\quad (p_0 + \rho_0 g H) v_0 = p_1 v_1 \quad (3)$

Constant oil mass $\quad m = \rho_0 V_0 = \rho_1 V_1 \quad$ or $\quad \rho_1 = \rho_0 \dfrac{V_0}{V_1} \quad (4)$

From isothermal compressibility (1)

$$\frac{V_1 - V_0}{V_0} = -\beta\Big[\underbrace{p_1 + \tfrac{1}{2}\rho_1 g H - (p_0 + \tfrac{1}{2}\rho_0 g H)}_{\text{Take mean pressures at mid-height}}\Big]$$

$$= -\beta\left[(p_1 - p_0) + \tfrac{1}{2} g H (\rho_1 - \rho_0)\right]$$

From (2) and (3) and (4):

$$\frac{v_0 - v_1}{V_0} = -\beta\left[(p_0 + \rho_0 g H)\frac{v_0}{v_1} - p_0 + \frac{1}{2} g H \rho_0 \left(\frac{V_0}{V_1} - 1\right)\right]$$

$$= -\beta\left[p_0\left(\frac{v_0 - v_1}{v_1}\right) + \rho_0 g H\left(\underset{A}{\frac{v_0}{v_1}} + \frac{1}{2}\underset{B}{\frac{v_1 - v_0}{V_1}}\right)\right]$$

Since $V_1 >> v_1$ or v_0, $B << A$, so that:

$$(v_0 - v_1)\left[\underset{C}{\frac{1}{V_0}} + \underset{D}{\frac{\beta p_0}{v_1}}\right] = -\beta \rho_0 g H \frac{v_0}{v_1}$$

Since $\beta p_0 V_0 >> v_1$, $D >> C$, so β cancels and:

$$(v_0 - v_1)\frac{p_0}{v_1} = -\rho_0 g H \frac{v_0}{v_1}$$

or $$\frac{v_1}{v_0} = 1 + \frac{\rho_0 g H}{p_0} = 1 + \frac{\rho g H}{p_0}$$

Thus, the bubble expands upon rising.

Numerical Values

$$\frac{\rho g H}{p_0} = \frac{800 \times 9.81 \times 1}{10^5} = 0.0785$$

(1 bar = 10^5 Pa) $\quad \frac{kg}{m^3} \quad \frac{m}{s^2} \quad m \quad \frac{m\,s^2}{kg}$

Hence $\frac{v_1}{v_0} = 1.0785$

Check on assumption that $\beta p_0 V_0 \gg v_1$

$$\underset{\frac{m^2}{N}}{5.5 \times 10^{-10}} \times \underset{\frac{N}{m^2}}{10^5} \times \underset{m^3}{0.1} \overset{?}{\gg} \underset{m^3}{10^{-8}}$$

$$5.5 \times 10^{-6} \overset{?}{\gg} 10^{-8} \quad \text{OK}$$

1.30

Pressures in Oil and Gas Well

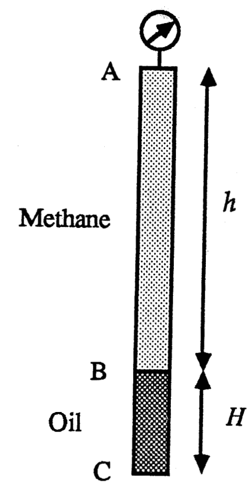


Fig. 1 Well containing oil and methane.

Variation of pressure with elevation for an ideal gas:

$$\frac{dp}{dz} = -\rho g = -\frac{Mp}{RT} g$$

Separate variables and integrate:

$$\int_{p_A}^{p_B} \frac{dp}{p} = -\frac{Mg}{RT} \int_h^0 dz$$

$$\ln \frac{p_B}{p_A} = +\frac{Mgh}{RT} = \frac{16 \times 32.2 \times 10{,}000}{10.73 \times 560 \times 32.2 \times 144} = 0.185$$

$$[=] \quad \frac{\text{lbm}}{\text{mole}} \; \frac{\text{ft}}{\text{s}^2} \; \text{ft} \; \frac{\text{in}^2 \ \text{mole} \ {}^\circ\text{R}}{\text{lb}_f \ \text{ft}^3} \quad \frac{1}{{}^\circ\text{R}} \; \frac{\text{lb}_f \ \text{s}^2}{\text{lb}_m \ \text{ft}} \; \frac{\text{ft}^2}{\text{in}^2} \quad \text{(all units cancel)}$$

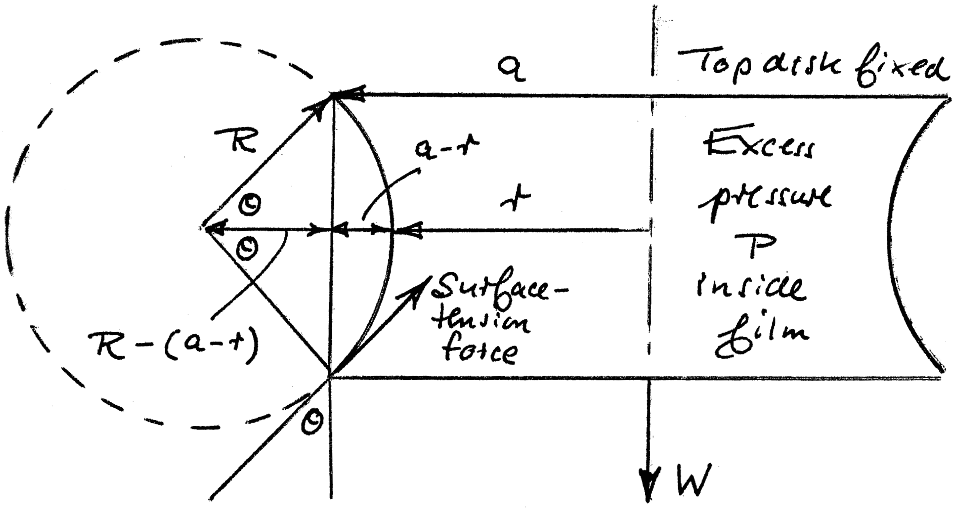
$$p_B = p_A e^{0.185} = 1014.7 \times 1.203 = \underline{\underline{1221 \text{ psia}}}$$

Pressure increase in the oil section

$$p_C - p_B = \rho_o g H = \frac{0.75 \times 62.4 \times 32.2 \times 2000}{32.2 \times 144}$$

$$= \underline{\underline{650 \text{ psi}}}$$

Soap Film Between Two Disks



Force balance ↓ on bottom disk

$$W + P\pi a^2 = 2\pi a(2\sigma)\cos\Theta$$

Film has two sides

Hence $\Theta = \cos^{-1}\left(\dfrac{W + P\pi a^2}{4\pi a\sigma}\right)$

Excess pressure $P = 2\sigma\left(\dfrac{1}{r} - \dfrac{1}{R}\right)$

or $\dfrac{1}{R} = \dfrac{1}{r} - \dfrac{P}{2\sigma}$

From geometry, $\cos\Theta = \dfrac{R-(a-r)}{R} = 1 + \dfrac{r-a}{R}$

Hence $\dfrac{W + P\pi a^2}{4\pi a\sigma} = 1 + (r-a)\left(\dfrac{1}{r} - \dfrac{P}{2\sigma}\right)$

1.32

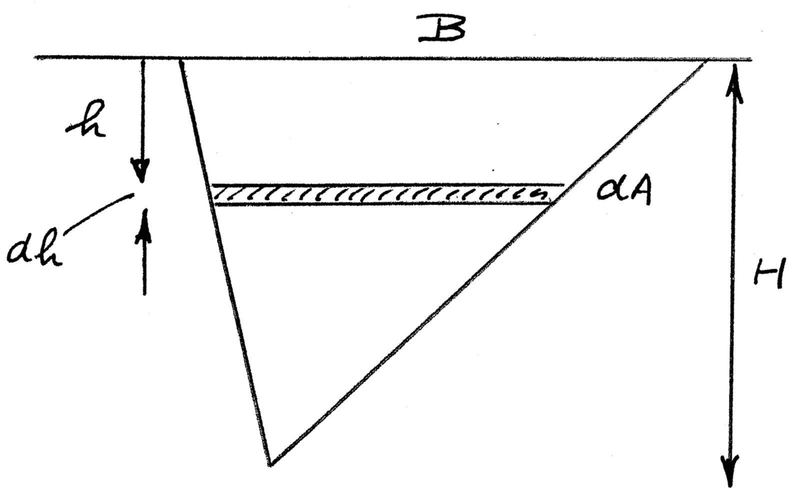
Newspaper Statements About the Erg

The February 10 letter probably said:

"Concerning the definition of the erg, the January 18 statement is incorrect and the January 30 letter is misleading at best. For example, if the mass were on a virtually frictionless horizontal table, it would require virtually no energy to move it through a (horizontal) distance of one centimeter.

The erg is more correctly defined as the amount of work done (equivalent to energy expended) in pushing back a resisting force of one dyne through a distance of one centimeter. One dyne is the force needed, when applied to a mass of one gram, to accelerate it at one centimeter per second per second."

Centroid of Triangle



From the text $h_c \equiv \frac{\int_A h\,dA}{A} = \frac{\int_A h\,dA}{\int_A dA}$

Now the width of the triangle is proportional to $(H-h)$ at any depth h, so that

$$dA = c\,(H-h)\,dh, \text{ where } c \text{ is a constant.}$$

Thus $$h_c = \frac{\int_0^H h\,c\,(H-h)\,dh}{\int_0^H c\,(H-h)\,dh} = \frac{\frac{H^3}{6}}{\frac{H^2}{2}} = \frac{H}{3}$$

Horizontal force = pressure at centroid depth × area

$$F = \left(\rho g \frac{H}{3}\right)\left(\frac{1}{2} B H\right) = \underline{\underline{\frac{1}{6}\rho g B H^2}}$$

1.34

Blake–Kozeny Equation

$$\frac{p_1 - p_2}{L} = 150 \, \frac{\mu v_0}{D_p^2} \, \frac{(1-\varepsilon)^2}{\varepsilon^3}$$

Check dimensions

$$\frac{M}{LT^2} \, \frac{1}{L} \qquad \frac{M}{LT} \, \frac{L}{T} \, \frac{1}{L^2} = \frac{M}{L^2 T^2}$$

Same, hence ε is dimensionless. Also, examination of $(1-\varepsilon)$ gives same conclusion assuming "1" is dimensionless.

Numerical values

$$\frac{\varepsilon^3}{(1-\varepsilon)^2} = \frac{150 \mu v_0 L}{(p_1 - p_2) D_p^2} = \frac{150 \times \overbrace{0.22}^{\frac{g}{cm\,s}} \times 0.1 \times \overset{\frac{ft}{s}}{6} \times \overset{\frac{cm}{ft^2}}{30.48}}{\underset{\frac{g}{lbm}}{453.6} \times \underset{\frac{lbf}{in^2}}{75} \times \underset{in^2}{0.1^2} \times \underset{\frac{lbm\,ft}{lbf\,s^2}}{32.2}}$$

$$= 0.05509$$

Rearrange to $\varepsilon = f(\varepsilon) = \left[0.05509 \times (1-\varepsilon)^2\right]^{1/3}$

Successive substitution

i	ε_i	$f(\varepsilon_i)$
1	0.5000	0.2397
2	0.2397	0.3170
3	0.3170	0.3014
4	0.3014	0.2996
5	0.2996	0.3001
6	0.3001	0.3000

Porosity (void fraction)

$$\boxed{\varepsilon = 0.300}$$

1.35

Shear Stresses for Air and Water

Viscosity of air

$$\mu_a = \mu_0\left(\frac{T}{T_0}\right)^n = 0.0171\left(\frac{293}{273}\right)^{0.768} = 0.0181 \text{ cP}$$

Viscosity of water

$$\mu_w = e^{a + b \ln T} = e^{29.76 - 5.24 \ln 293} = 1.00 \quad \text{cP}$$

Stresses

Air $\quad \tau_a = \mu_a \dfrac{V}{h} = 0.01 \times 0.0181 \times \dfrac{1}{0.1} = 0.00181 \ \dfrac{\text{dynes}}{\text{cm}^2}$

Water $\quad \tau_w = \mu_w \dfrac{V}{h} = 0.01 \times 1.0 \times \dfrac{1}{0.1} = 0.1 \ \dfrac{\text{dynes}}{\text{cm}^2}$

Ratio of stresses

$$\frac{\tau_w}{\tau_a} = \frac{0.1}{0.00181} = 55.2 \quad \left(= \frac{\mu_w}{\mu_a}\right)$$

At higher temperature

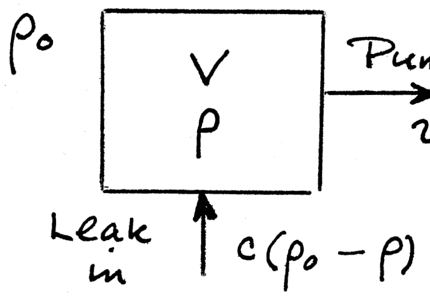
Air: μ_a increases, and so does τ_a

Water: μ_w and hence τ_w decrease

The shear stress τ is uniform across the gap because the velocity profile is linear (a situation itself due to zero applied pressure gradient).

2.1

Evacuation of Leaking Tank



Transient Mass Balance

$$-vp + c(p_0 - p) = \frac{d}{dt} Vp = V\frac{dp}{dt}$$

Separate variables and integrate:

$$\int_0^t dt = V\int_{p_0}^{p} \frac{dp}{c\,p_0 - (c+v)p} \quad \text{or} \quad t = \frac{-V}{c+v} \ln[c p_0 - (c+v)p]\Big|_{p_0}^{p}$$

$$t = \frac{V}{c+v} \ln \frac{-v p_0}{c p_0 - (c+v)p} = \frac{V}{c+v} \ln \frac{v p_0}{(c+v)p - c p_0}$$

$$= \frac{V}{c+v} \ln \frac{1}{\left(1+\frac{c}{v}\right)\frac{p}{p_0} - \frac{c}{v}}$$

Since $\rho = \dfrac{Mp}{RT}$ and T is constant

Lowest attainable pressure

$c = 10^{-5}\ m^3/s$

$v = 10^{-3}\ m^3/s$

As $t \to \infty$ $\quad \underbrace{\left(1+\frac{c}{v}\right)}_{1.01} \frac{p^*}{p_0} = \underbrace{\frac{c}{v}}_{0.01}$

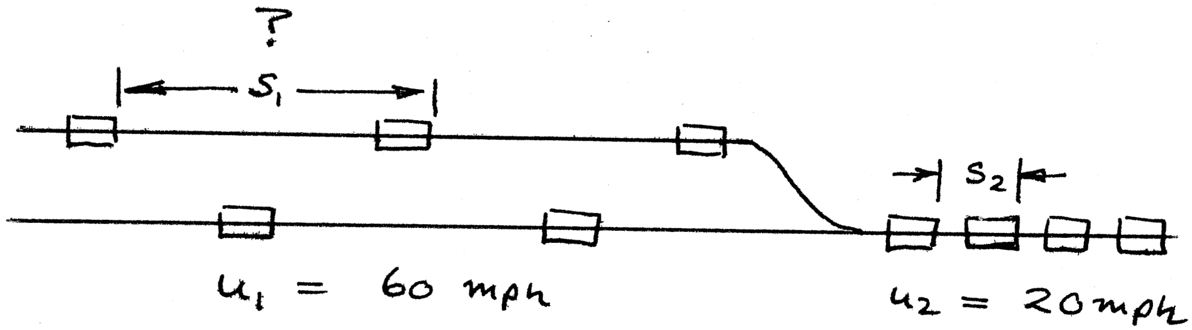
$$p^* = \frac{0.01}{1.01} \times 1 = \underline{\underline{0.00990\ bar}}$$

Time to fall half way to p^*, to $p = \dfrac{1+0.00990}{2} = 0.50495$ bar

$$t = \frac{1}{0.00101} \ln \frac{1}{1.01 \times 0.50495 - 0.01}$$

$$= \underline{\underline{686.3\ s}}$$

2.2
Slowed Traffic



Continuity of Cars

$$\text{"}m\text{"} = 2u_1\rho_1 = u_2\rho_2$$

where $\rho_2 = \frac{1}{S_2} = \frac{1}{25} = 0.04 \ \frac{\text{cars}}{\text{ft}}$

Hence
$$\rho_1 = \frac{1}{2}\frac{u_2}{u_1}\rho_2 = \frac{1}{2}\frac{20}{60} \times 0.04 = 0.00667$$

$$S_1 = \frac{1}{\rho_1} = \frac{1}{0.00667} = \underline{\underline{150 \text{ ft}}}$$

"Flow rate" of cars

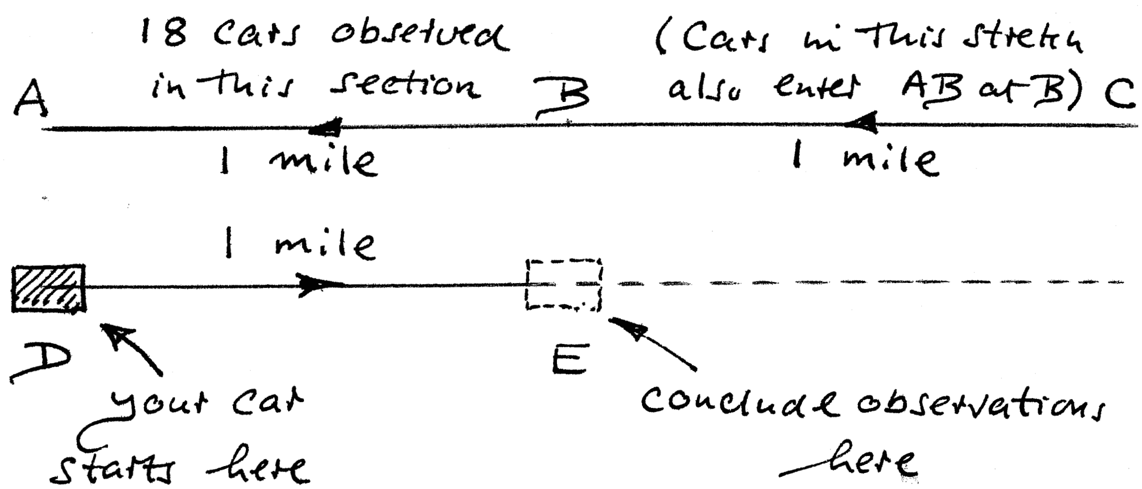
$$m = u_2\rho_2 = 20 \times 0.04 \times 5280 \ \frac{\text{mile}}{\text{hr}}\ \frac{\text{cars}}{\text{ft}}\ \frac{\text{ft}}{\text{mile}}$$

$$= 4,224 \ \frac{\text{cars}}{\text{hr}}$$

$$= \underline{\underline{1.173 \ \frac{\text{cars}}{\text{sec}}}}$$

2.3

"Density" of Cars



The cars you see in the mile between AB includes those in AB when you started at D <u>plus</u> those crossing into AB from BC.

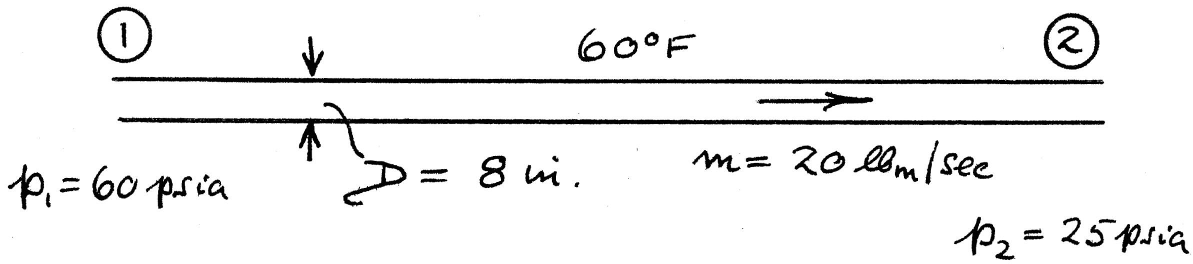
Assume average speed for opposite lane is also 65 mph. Hence BC is also one mile, and a car starting at C reaches B just as you reach E.

Thus the 18 cars you observe originate from <u>two</u> miles of highway (AB + BC)

Hence car density $= \frac{18}{2} = \underline{\underline{9 \text{ per mile}}}$

2.4

Ethylene Pipeline



Ethylene C_2H_4 $MW = 28$

Cross-Sectional Area $A = \frac{\pi}{4}\left(\frac{8}{12}\right)^2 = 0.349\ ft^2$

Densities

$$\rho = \frac{Mp}{RT} \quad \begin{cases} \rho_1 = \dfrac{28 \times 60}{10.73 \times 520} = 0.301\ lb_m/ft^3 \\[2ex] \rho_2 = \dfrac{25}{60} \times 0.301 = 0.125 \end{cases}$$

Flow rate $m = \rho \underbrace{uA}_{Q}$

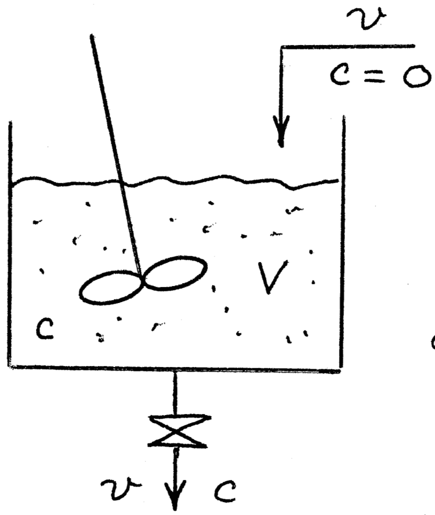
Velocities $u_1 = \dfrac{m}{\rho_1 A} = \dfrac{20}{0.301 \times 0.349} = \underline{\underline{190.4\ ft/sec}}$

$$u_2 = \frac{m}{\rho_2 A} = \frac{20}{0.125 \times 0.349} = \underline{\underline{458.4\ ft/sec}}$$

Pressure falls because of pipe friction (for subsonic flow, at any rate).

2.5

Transient Behavior of a Stirred Tank



$V = 2\,m^3$ (constant)

$v = 0.01\,\frac{m^3}{s}$ (constant)

Unsteady-state mass balance for sodium chloride on the tank as system

Inlet − Exit

$v \times 0 - vc$

$= $ Rate of accumulation

$\frac{d}{dt}(Vc) = V\frac{dc}{dt}$

Separate variables and integrate

$$\int_{c_0}^{c} \frac{dc}{c} = -\frac{v}{V}\int_0^t dt \quad \text{or} \quad \ln\frac{c}{c_0} = -\frac{vt}{V}$$

$$c = c_0\, e^{-vt/V}$$

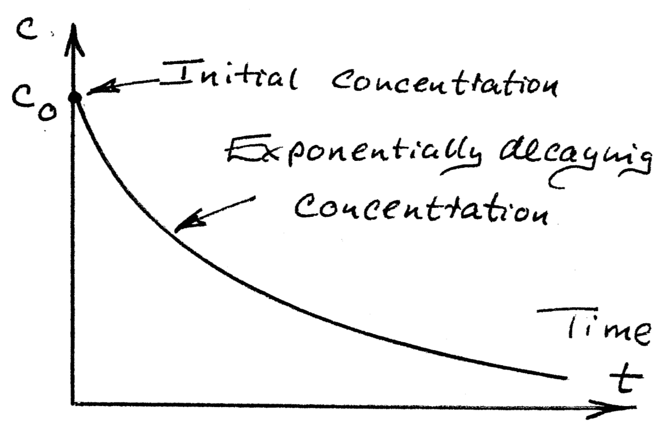
Required time

$c_0 = 1\ kg/m^3$

$c_f = 0.0001\ kg/m^3$

$$t_f = -\frac{V}{v}\ln\frac{c_f}{c_0}$$

$$= -\frac{2}{0.01}\ln\frac{0.0001}{1}$$

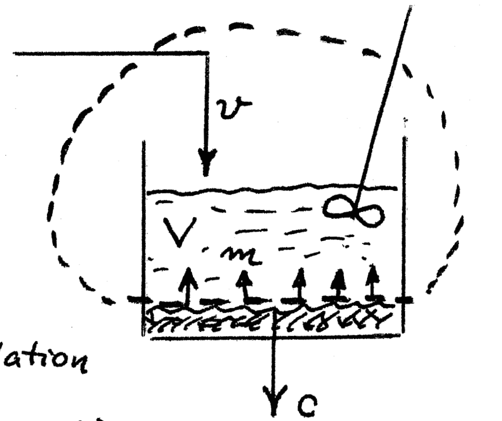


$t_f = 1{,}842\ s = \underline{\underline{30\,m\ \ 42\,s}}$

2.6

Stirred Tank with Crystal Dissolution

Choose the system to exclude the solid salt in the tank. Then perform a rate balance for mass of salt



Entering − leaving = accumulation

$$m - vc = \frac{d}{dt}(Vc) = V\frac{dc}{dt}$$

since V is constant.

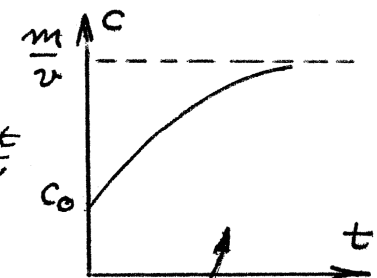
Separate variables and integrate

$$\int_{c_o}^{c} \frac{dc}{\frac{m}{v} - c} = \frac{v}{V}\int_0^t dt$$

$$-\ln\left(\frac{m}{v} - c\right)\Big|_{c_o}^{c} = \frac{vt}{V} \quad \text{or} \quad \ln\frac{m - vc}{m - vc_o} = -\frac{vt}{V}$$

$$m - vc = (m - vc_o)\, e^{-\frac{vt}{V}}$$

$$c = \frac{m}{v} - \left(\frac{m}{v} - c_o\right) e^{-\frac{vt}{V}}$$



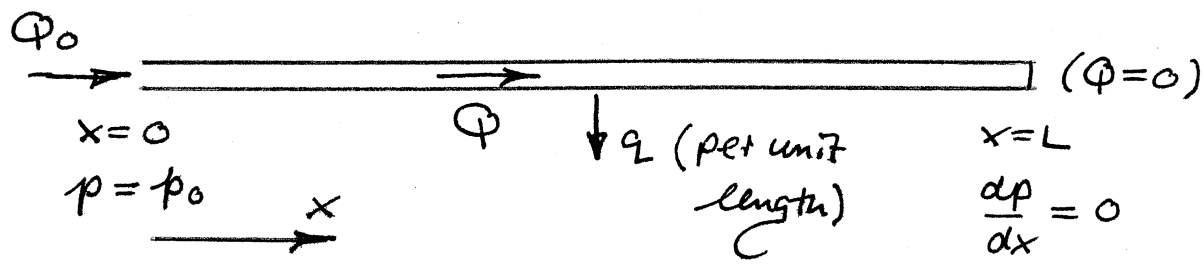
c is its steady-state value m/v, less a deviation that decreases exponentially with time.

$m = 0.02$ kg/s
$v = 0.01$ m³/s
$V = 2$ m³
$c_o = 1$ kg/m³

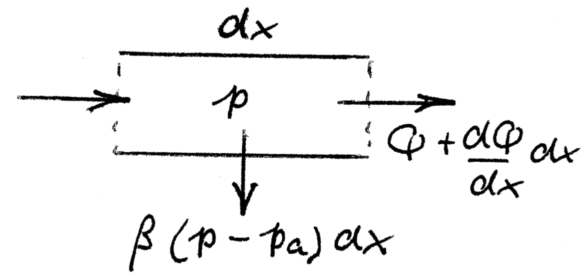
t (sec)	c (kg/m³)
0	1.000
10	1.049
100	1.393
∞	2.000

2.7–1

Soaker Garden Hose



Perform mass balance on element dx



$$Q = Q + \frac{dQ}{dx}dx + \beta(p-p_a)\,dx$$

$$-\frac{dQ}{dx} = \alpha\frac{d^2p}{dx^2} = \beta(p-p_a)$$

Substitute $P = p - p_a$, $\frac{\beta}{\alpha} = \gamma^2$ gives $\frac{d^2P}{dx^2} = \gamma^2 P$

Try $P = A\sinh\gamma x + B\cosh\gamma x$

$$\frac{dP}{dx} = \gamma A\cosh\gamma x + \gamma B\sinh\gamma x$$

$$\frac{d^2P}{dx^2} = \gamma^2 A\sinh\gamma x + \gamma^2 B\cosh\gamma x = \gamma^2 P \quad \underline{\text{OK}}$$

Boundary Conditions

$x=0 \quad P = p_o - p_a = B \qquad (\sinh 0 = 0,\ \cosh 0 = 1)$

$x=L \quad \frac{dP}{dx} = \gamma A\cosh\gamma L + \gamma\underbrace{(p_o-p_a)}_{B}\sinh\gamma L = 0$

Hence $A = -(p_o-p_a)\dfrac{\sinh\gamma L}{\cosh\gamma L}$

2.7-2

The pressure distribution as a function of x is then:

$$\frac{P}{p_o - p_a} = \frac{p - p_a}{p_o - p_a} = -\frac{\sinh \gamma L}{\cosh \gamma L} \sinh \gamma x + \cosh \gamma x$$

$$= \frac{\cosh \gamma (L - x)}{\cosh \gamma L}$$

Rate of loss of water can be obtained by integration

$$Q_{loss} = \int_0^L \beta (p - p_a)\, dx$$

since $p - p_a$ is known now as a function of x.

A somewhat easier approach is to note that $Q_{loss} = Q_o$, the water rate entering the hose:

$$Q_o = -\alpha \left(\frac{dp}{dx}\right)_{x=0} = -\alpha \gamma \left(A \underbrace{\cosh \gamma x}_{1} + B \underbrace{\sinh \gamma x}_{0}\right)_{x=0}$$

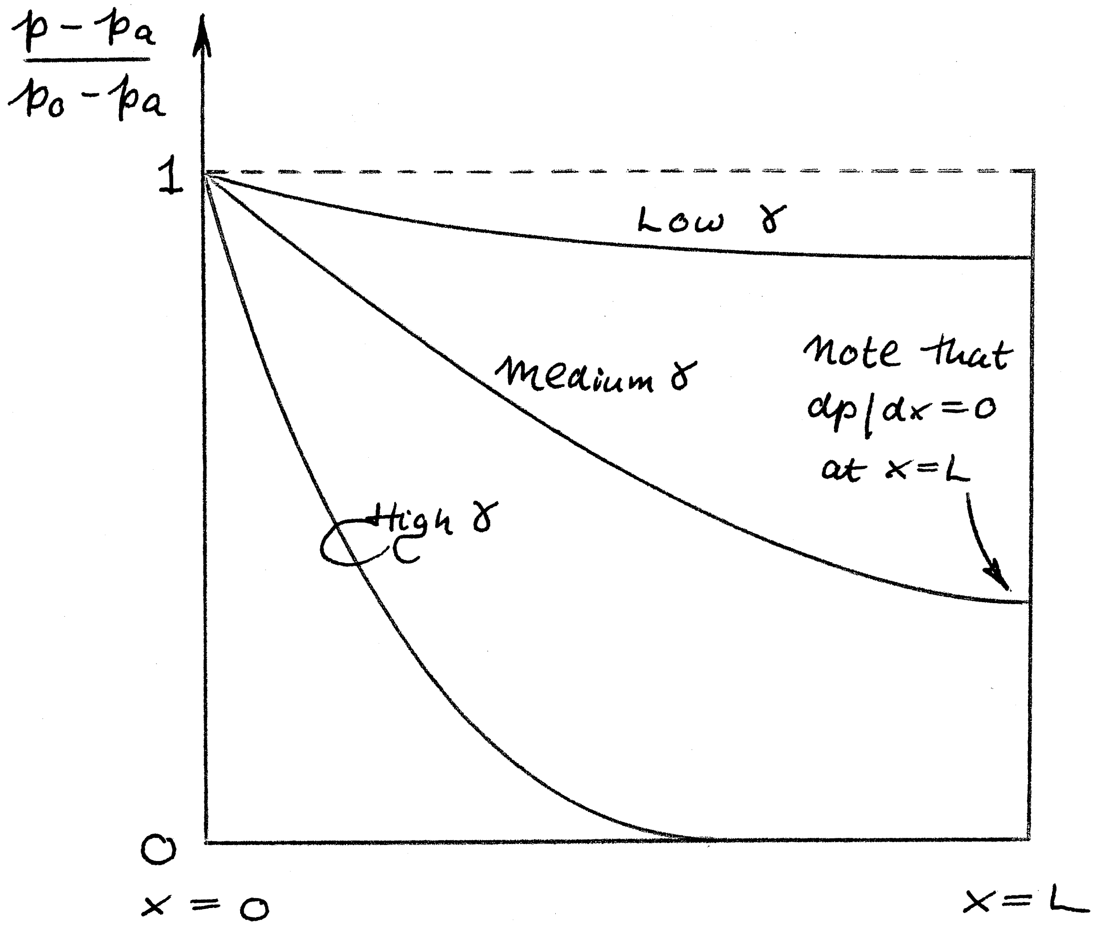
$$= -\alpha \gamma (-)(p_o - p_a) \frac{\sinh \gamma L}{\cosh \gamma L}$$

$$Q_o = \alpha \gamma (p_o - p_a) \tanh \gamma L = Q_{loss}.$$

2.7-3

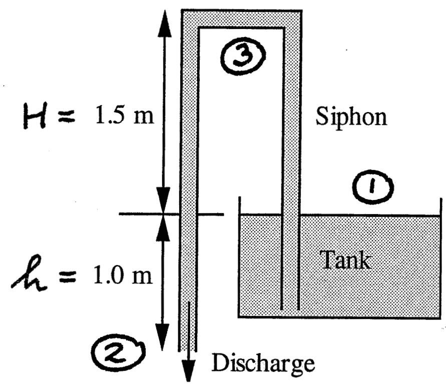
Sketch of pressure distribution

Note, $\gamma = \sqrt{\beta/\alpha}$, so a low value of γ means a relatively low leakage rate, and a high value of γ means a relatively high leakage rate



2·8-1

Performance of a Siphon



Bernoulli 1 → 2 (ignoring friction)

$$\frac{u_1^2}{2} + \frac{p_1}{\rho} + g z_1 = \frac{u_2^2}{2} + \frac{p_2}{\rho} + g z_2$$

zero zero zero zero $-h$

$$u_2 = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 1.00} = \underline{\underline{4.43 \text{ m/s}}}$$

Lowest pressure occurs at point 3 at the top

Bernoulli 1 → 3

$$\frac{u_1^2}{2} + \frac{p_1}{\rho} + g z_1 = 0 = \frac{u_3^2}{2} + \frac{p_3}{\rho} + g H$$

Continuity 3 → 2

$$A u_3 = A u_2$$

$$u_3 = u_2$$

2.8-2

Hence the lowest pressure is:

$$p_3 = -\rho\left(\frac{u_2^2}{2} + gH\right)$$

$$= -10^3\left(\frac{4.43^2}{2} + 9.81 \times 1.5\right)$$

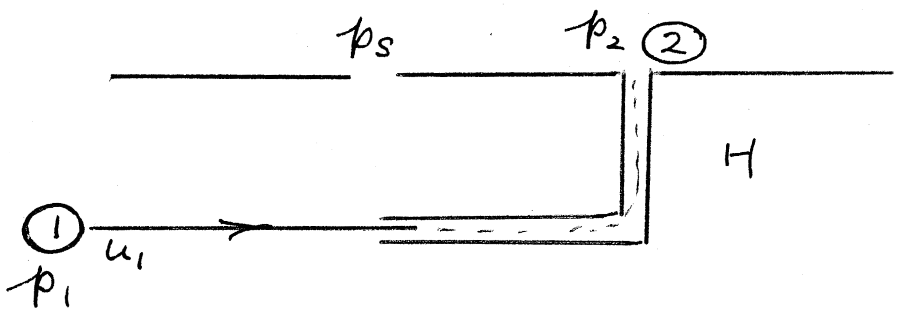
$$= -2.45 \times 10^4 \text{ Pa}$$

$$= \underline{\underline{-0.245 \text{ bar}}} \text{ (gauge)}$$

Yes, the answers are reasonable. The velocity of 4.43 m/s is typical for pipe flow, although in practice it would be reduced somewhat because of friction. Also, $p_3 \doteq 11.0$ psia, which is well above the vapor pressure of water at room temperature ($p_v = 0.363$ psia at 70°F), so vapor-lock will not occur.

The indicated draining time of $t = V/vA$ would only be true for constant velocity v. However, as the level in the tank falls, so does v, so $t = V/vA$ is not true.

2.9
Pitot Tube



Bernoulli
$$\frac{u_1^2}{2} + \frac{p_1}{\rho} = \frac{p_2}{\rho} + gH$$

But $p_1 = p_s + \rho g H$

$$\frac{u_1^2}{2} + \frac{p_s + \rho g H}{\rho} = \frac{p_2}{\rho} + gH$$

Hence
$$u_1 = \sqrt{\frac{2(p_2 - p_s)}{\rho}}$$

Sea Water

(a) $u_1 = \sqrt{\dfrac{2 \times 2.5 \times 144 \times 32.2}{64}} = 19.03$ ft/sec

$= \underline{\underline{12.98 \text{ mph}}}$

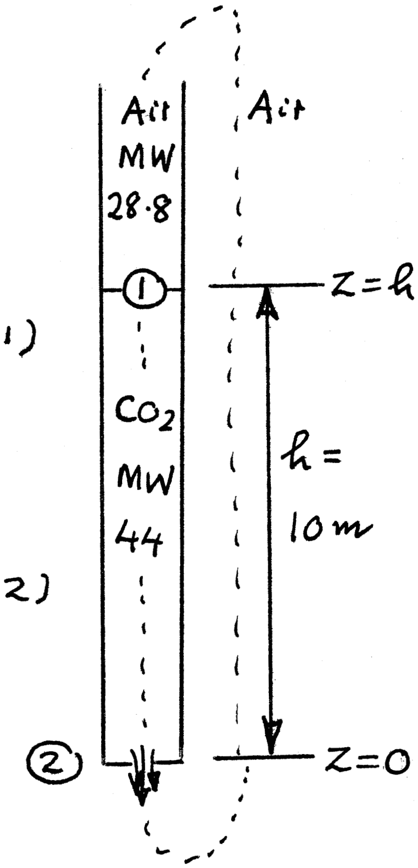
Fresh Water

(b) $u_f = \sqrt{\dfrac{2 \times 2.5 \times 144 \times 32.2}{62.4}} = 19.28$ ft/sec

$= \underline{\underline{13.14 \text{ mph}}}$

2.10

Leaking Carbon Dioxide



Apply Bernoulli 1 → 2

for two different paths

Carbon dioxide

$$\frac{u_1^2}{2} + \frac{p_1}{\rho_{CO_2}} + gh = \frac{u_2^2}{2} + \frac{p_2}{\rho_{CO_2}} \qquad (1)$$

Air

$$\frac{u_1^2}{2} + \frac{p_1}{\rho_A} + gh = 0 + \frac{p_2}{\rho_A} + 0 \qquad (2)$$

Neglect $u_1^2 \ll u_2^2$

From (1) $\quad \dfrac{u_2^2}{2} = \dfrac{1}{\rho_{CO_2}}(p_1 - p_2) + gh$

From (2) $\quad (p_1 - p_2) = -\rho_A g h$

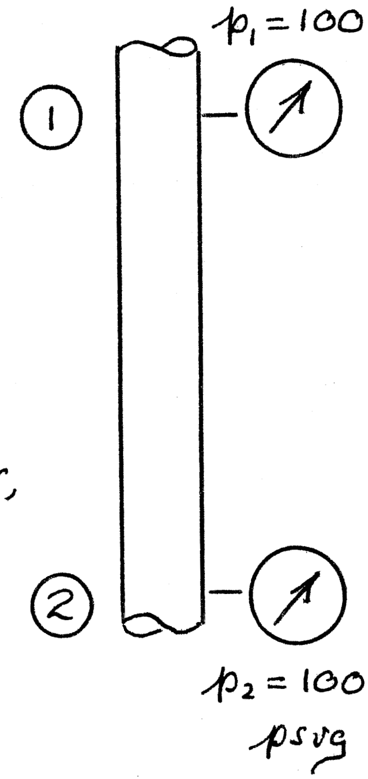
Hence $\quad \dfrac{u_2^2}{2} = gh\left(1 - \dfrac{\rho_A}{\rho_{CO_2}}\right)$

$$u_2 = \sqrt{2 \times 9.81 \times 10\left(1 - \frac{28.8}{44}\right)}$$

$$= \underline{\underline{8.23 \text{ m/s.}}}$$

2.11

Two Pressure Gauges



(a) If the water were not flowing, p_2 would be much higher than p_1, because of the hydrostatic effect.

However, this is not the case, therefore, the water must be flowing.

(b) If the water were flowing upwards, say with $p_2 = 100$, then p_1 would be lower on two counts:

(i) Hydrostatic effect
(ii) Pipe friction.

However, $p_1 \neq p_2$. Therefore, the water cannot be flowing upwards, and must be flowing downwards.

(c) Check on conclusion of (b) by using overall energy balance from (1) to (2) with $w = 0$ and $\Delta(u^2/2) = 0$:

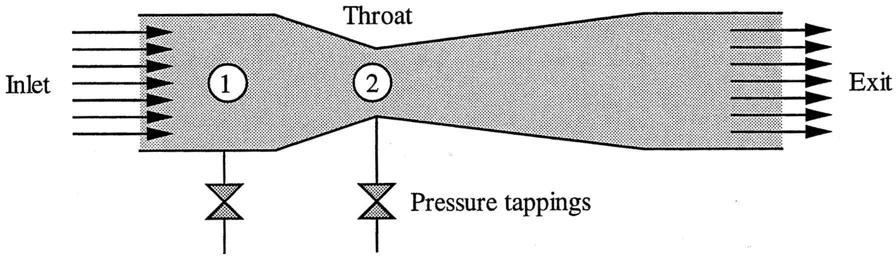
$$g(z_2 - z_1) + \frac{p_2 - p_1}{\rho} + \mathcal{F} = 0$$

Since $p_1 = p_2$, $\quad \mathcal{F} = g(z_1 - z_2)$

Both sides are positive and equal. That is, the hydrostatic or potential energy is exactly consumed by pipe friction.

2.12

Venturi "Meter"



Bernoulli ① → ② $$\frac{u_1^2}{2} + \frac{p_1}{\rho} = \frac{u_2^2}{2} + \frac{p_2}{\rho}$$

Continuity ① → ② $$A u_1 = a u_2$$

Eliminate u_2 gives $$\frac{p_1 - p_2}{\rho} = \frac{u_1^2}{2}\left(\frac{A^2}{a^2} - 1\right)$$

Flow rate $$Q = A u_1 = C_D A \sqrt{\frac{2(p_1 - p_2)}{\rho\left(\frac{A^2}{a^2} - 1\right)}} = C_D A \sqrt{\frac{2(\rho_M - \rho_I) g \Delta h}{\rho_I\left(\frac{A^2}{a^2} - 1\right)}}$$

M = mercury I = isopentane

Substituting numbers, $A = \frac{\pi}{4}\left(\frac{6}{12}\right)^2 = 0.196\ ft^2$, $C_D = 0.98$

$$Q = 0.98 \times 0.196 \sqrt{\frac{2\underbrace{(13.57 \times 62.4 - 38.75) \times 32.2 \times \left(\frac{20}{12}\right)}_{\alpha}}{38.75\left(\frac{6^4}{3^4} - 1\right)}}$$

$$= 0.98 \times 0.196 \times 12.22 = \underline{\underline{2.35\ ft^3/sec}}$$

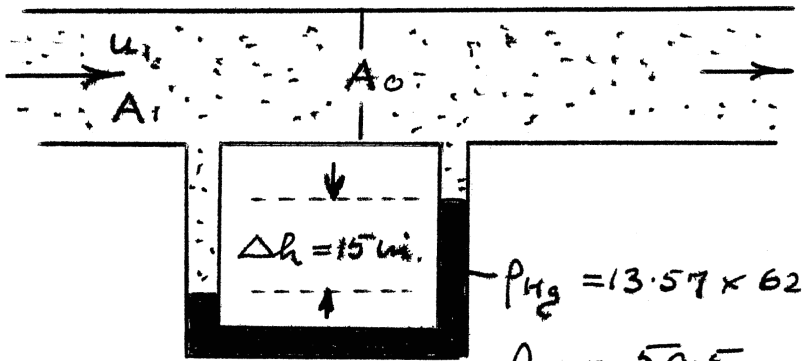
$$= \underline{\underline{1053\ gpm}}$$

$$p_1 - p_2 = \frac{\alpha}{32.2 \times 144} = 9.35\ psi$$

2.13

Orifice Plate

Use the orifice-plate equation $\quad u_1 = C_D \sqrt{\dfrac{2(p_1 - p_2)}{\rho\left(\dfrac{A_1^2}{A_0^2} - 1\right)}}$



$\rho_{Hg} = 13.57 \times 62.4 = 847$

$\rho_K = 50.5$

$A_1 = \dfrac{\pi}{4}\left(\dfrac{2}{12}\right)^2 = 0.0218\ ft^2 \qquad u_1 = \dfrac{560}{60 \times 50.5 \times 0.0218} = 8.48\ \dfrac{ft}{sec}$

For most orifice plates, $C_D \doteq 0.62$, which can be used as first trial.

$$8.48 = 0.62 \sqrt{\dfrac{2 \times \frac{15}{12}(847 - 50.5) \times 32.2}{50.5\left(\dfrac{A_1^2}{A_0^2} - 1\right)}}$$

whence

$\dfrac{A_1}{A_0} = \dfrac{D_1^2}{D_0^2} = 2.79 \qquad \dfrac{D_1}{D_0} = 1.67 \qquad \underline{\underline{D_0 = 1.2 \text{ in dia.}}}$

Check on value of C_D

$$Re_0 = \frac{D_1}{D_0} Re_1 = \frac{\overset{u_1}{1.67 \times 8.48} \times \overset{\rho}{50.5} \times \overset{D_1}{\frac{2}{12}}}{\underset{D_1/D_0}{1} \quad \underset{\mu}{3.18 \times \frac{1}{3600}}} = 1.35 \times 10^5$$

From chart, for $\dfrac{D_0}{D_1} = 0.60$, $Re = 1.35 \times 10^5$

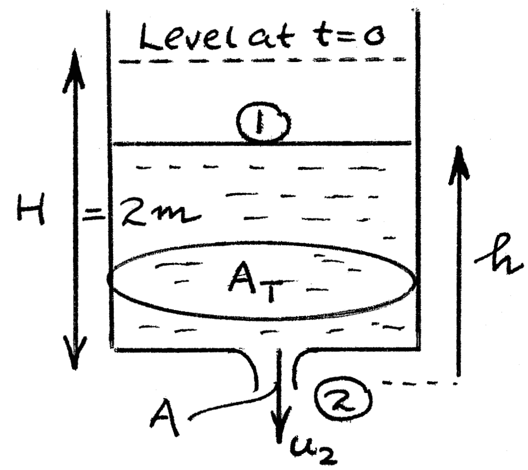
page 56 $\qquad \underline{\underline{C_D = 0.62 \quad \text{Assumption was OK}}}$

2.14

Tank Draining

From lecture notes

$$u_2 = \sqrt{2gh}$$



Mass Balance on Acetone in the Tank

$$-\sqrt{2gh}\,A = \frac{d}{dt}(h A_T)$$

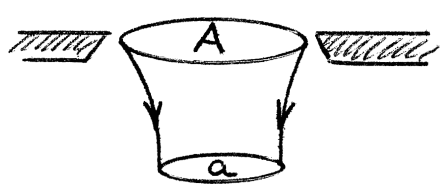
$$= A_T \frac{dh}{dt} \quad \text{or} \quad \int_0^t dt = -\frac{A_T}{A}\frac{1}{\sqrt{2g}}\int_H^0 \frac{dh}{\sqrt{h}}$$

Draining Time

$$t = +\frac{A_T}{A}\frac{1}{\sqrt{2g}}\,2\sqrt{h}\Big|_0^H = \frac{D_T^2}{D_0^2}\sqrt{\frac{2H}{g}}$$

$$t = \left(\frac{1}{0.02}\right)^2\sqrt{\frac{2\times 2}{9.81}} = 1{,}596\ s = \underline{\underline{26m\ 36s}}$$

For sharp-edged orifice, nothing changes except the area ratio, which is now

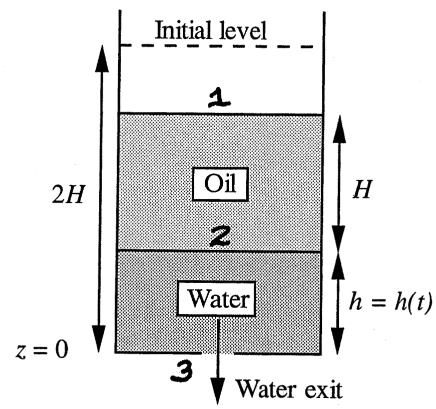


$$\frac{A_T}{a} = \frac{1^2}{0.02^2\times 0.63},$$ so the time increases by a factor of $\frac{1}{0.63} = 1.59$

$$\underline{\underline{t = 2{,}533\ s = 42m\ 13s}}$$

2.15

Draining Immiscible Liquids from a Tank



From hydrostatics,

$$p_2 = \rho_o g H$$

Bernoulli ② → ③

$$\frac{p_2}{\rho_w} + gh = \frac{u_3^2}{2}$$

Hence

$$u_3 = \sqrt{2g\left(\frac{\rho_o}{\rho_w}H + h\right)}$$

Unsteady-State Mass Balance on Tank (Rate balance)

$$-0.62\,a\,u_3\,\rho_w = -0.62\,a\,\rho_w\sqrt{2g\left(\frac{\rho_o}{\rho_w}H + h\right)} = A\rho_w\frac{dh}{dt}$$

Leaving through orifice

Rate of accumulation of water in tank (will be negative, but don't anticipate this with a minus sign)

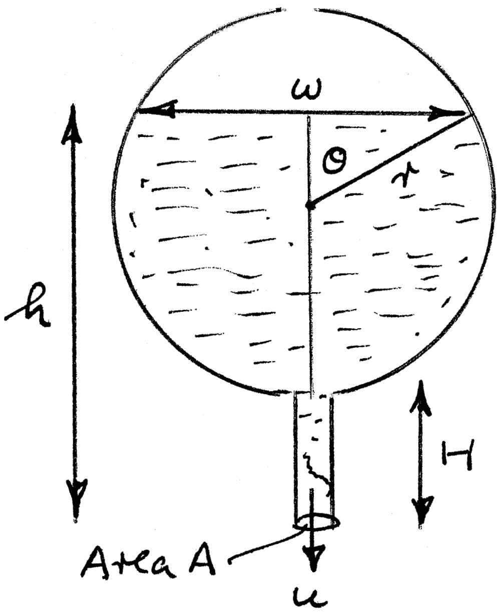
$$\int_0^t dt = -\frac{A}{0.62a\sqrt{2g}}\int_H^0 \frac{dh}{\sqrt{\frac{\rho_o}{\rho_w}H + h}}$$

$b = 1$

$$t = \frac{2A}{\sqrt{2g}\,0.62a}\left[\sqrt{\frac{\rho_o}{\rho_w}H + h}\right]_0^H$$

$$t = \frac{2A}{0.62a\sqrt{2g}}\left[\sqrt{H\left(\frac{\rho_o}{\rho_w}+1\right)} - \sqrt{\frac{\rho_o}{\rho_w}H}\right]$$

Draining a Horizontal Cylindrical Tank



From geometry

$$h = H + r(1+\cos\theta) \qquad (1)$$

$$w = 2r\sin\theta \qquad (2)$$

From Bernoulli

$$u = \sqrt{2gh} \qquad (3)$$

Volume balance

In time dt, a volume dV leaves the tank

$$dV = uA\,dt = -wL\,dh \qquad (4)$$

From (1), $dh = -r\sin\theta\, d\theta \qquad (5)$

Hence, from (2), (3), (4), and (5):

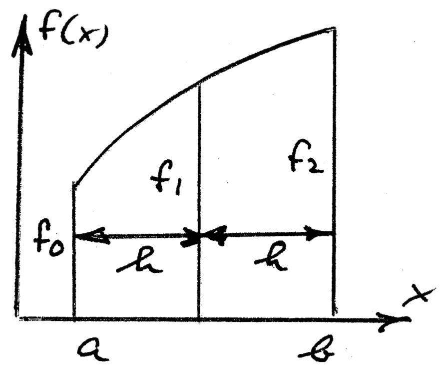
$$\sqrt{2g\,[H + r(1+\cos\theta)]}\; A\, dt = -2r\sin\theta\, L\,(-r\sin\theta\, d\theta)$$

Separate variables and integrate

$$\int_0^t dt = t = \frac{2r^2L}{A\sqrt{2g}} \underbrace{\int_0^{\pi} \frac{\sin^2\theta\, d\theta}{\sqrt{H + r(1+\cos\theta)}}}_{I}$$

Simpson's Rule

$$\int_a^b f(x)dx \doteq \frac{h}{3}(f_0 + 4f_1 + f_2)$$



Approximate I by two applications of Simpson's rule, $h = \pi/4$

$$I \doteq \frac{\pi/4}{3}\left[0 + 4\frac{\sin^2 \pi/4}{\sqrt{2+\cos \pi/4}} + 2\frac{\sin^2 \pi/2}{\sqrt{2+\cos \pi/2}} + 4\frac{\sin^2 3\pi/4}{\sqrt{2+\cos 3\pi/4}} + 0\right]$$

$$= \frac{\pi}{12}(1.216 + 1.414 + 1.759) = 1.149 \ m^{-1/2}$$

(One application of Simpson's rule gives $I \doteq 1.48$)

Required area is

$$A = \frac{2r^2 L\, I}{t\sqrt{2g}} = \frac{2\times 1^2 \times 5 \times 1.149}{3600 \times \sqrt{2\times 9.81}} = \underline{\underline{0.000721\ m^2}}$$

Diameter of tail pipe $A = \frac{\pi d^2}{4}$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4\times 0.000721}{\pi}} = 0.0303\ m$$

$$= \underline{\underline{3.03\ cm}}$$

Is this answer reasonable?

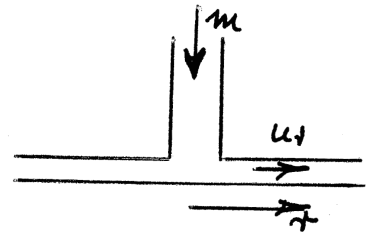
When $\theta = \pi/2$, $h = 2$, $u = \sqrt{2\times 9.81 \times 2} = 6.26\ m/s$

At this rate for 1 hr, $V = 6.26 \times 0.000721 \times 3600$

$$\left.\begin{array}{l} = 16.3\ m^3 \\ \text{Tank volume} = \pi\left(\frac{2r}{4}\right)^2 L = \pi \times \left(\frac{2\times 1}{4}\right)^2 \times 5 = 15.7\ m^3 \end{array}\right\}\ \text{OK}$$

2.17

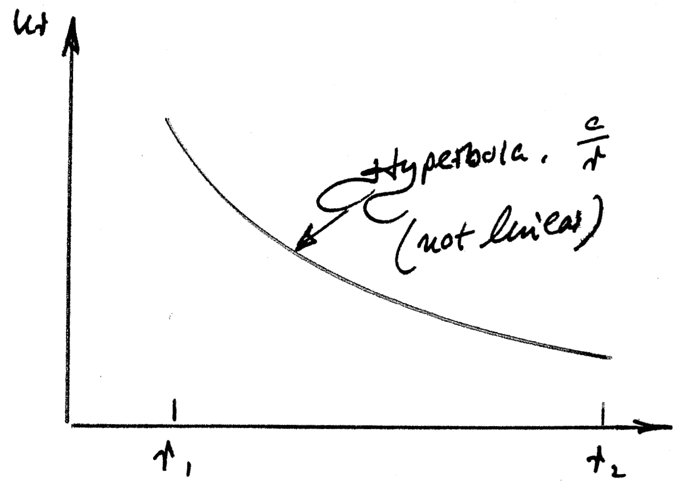
Lifting Silicon Wafers



Mass Balance

$$m = 2\pi r H \rho u_r$$

$$u_r = \frac{m}{2\pi r H \rho}$$

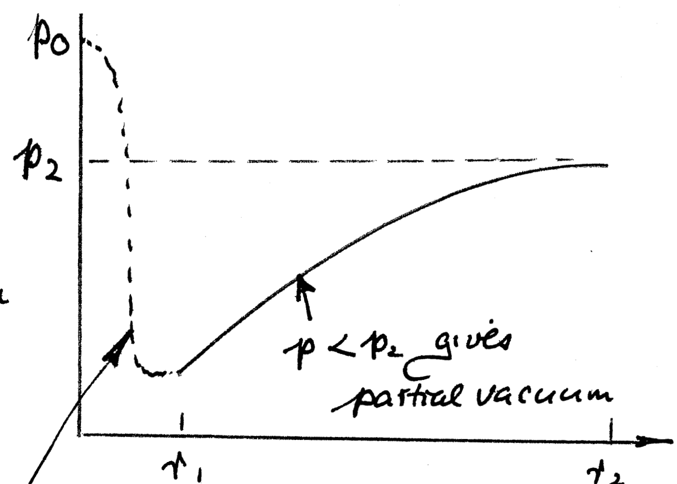


Bernoulli's Equation

$$\frac{u_r^2}{2} + \frac{p}{\rho} = \frac{u_2^2}{2} + \frac{p_2}{\rho}$$

$$p = p_2 - \frac{\rho}{2}\underbrace{\left(u_r^2 - u_2^2\right)}_{\text{positive}}$$

Hence $p < p_2$ and there is a partial vacuum between the flange and wafer, which is therefore picked up.



Reduction in pressure from p_0 as air accelerates into gap.

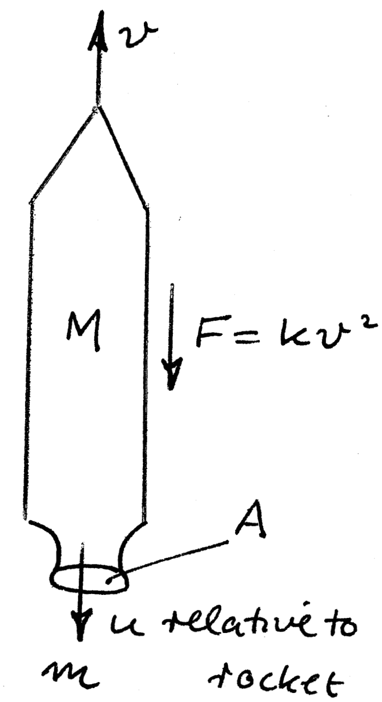
Total Force upwards on Wafer

$$F = \int_0^{r_2} (p_2 - p)\, 2\pi r\, dr.$$

Will be positive, so the device is OK for picking up the wafer

2.18

Rocket Performance



(a) Absolute downwards velocity of propellant

$= u - v$

(b) Momentum flux

$= m(u - v)$

(c) Mass M is decreasing:

$$\frac{dM}{dt} = -m \quad \text{or} \quad M = M_0 - mt$$

(M_0 is initial mass, and this equation holds until all the propellant has been ejected)

(d) Upwards momentum balance on moving rocket

$$\underbrace{-Mg}_{\text{gravity}} \quad \underbrace{-kv^2}_{\text{drag}} \quad \underbrace{-}_{\text{loss}} \quad \underbrace{-}_{\substack{\text{negative}\\ \text{direction}}} \; m(u - v) = \frac{d}{dt}(Mv)$$

$$= M\frac{dv}{dt} + v\frac{dM}{dt}$$

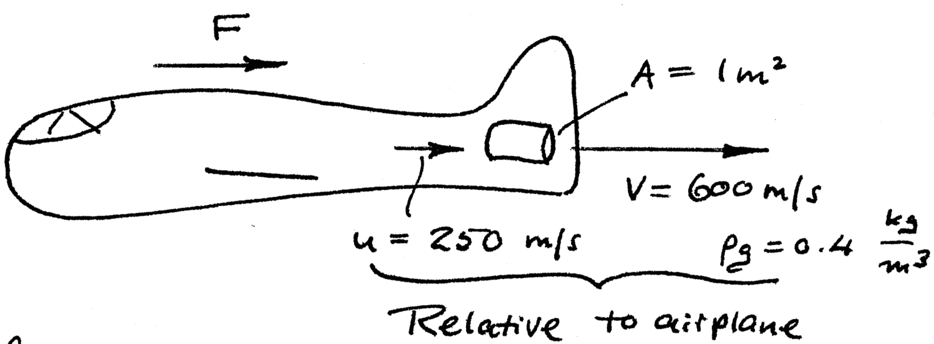
$$= M\frac{dv}{dt} - mv$$

Hence

$$\frac{dv}{dt} = -g + \frac{1}{M_0 - mt}(mu - kv^2)$$

2.19

Jet Airplane



(a) mass flow rate

$$\dot{m} = \rho_g V A = 0.4 \times 600 \times 1 = \underline{\underline{240 \ kg/s}}$$

(b) $\overrightarrow{\text{momentum}}$ balance (as seen by the pilot — this corresponds to a "stationary airplane with air flowing past it)

$$\underbrace{\dot{m}u}_{\text{flow in}} - \underbrace{\dot{m}v}_{\text{flow out}} + \underbrace{F}_{\text{force}} = 0 \quad \left(\begin{matrix}\text{Steady}\\\text{state}\end{matrix}\right)$$

$$F = \dot{m}(v-u) = 240(600-250) = \underline{\underline{8.4 \times 10^4 \ N}}$$

(c) Power may be obtained by different methods, the easiest of which is to note that the engine serves to increase the kinetic energy of the air flowing through it:

$$P = \frac{\dot{m}}{2}(v^2 - u^2) = \frac{240}{2}(600^2 - 250^2) = \underline{\underline{3.57 \times 10^7 \ W}}$$

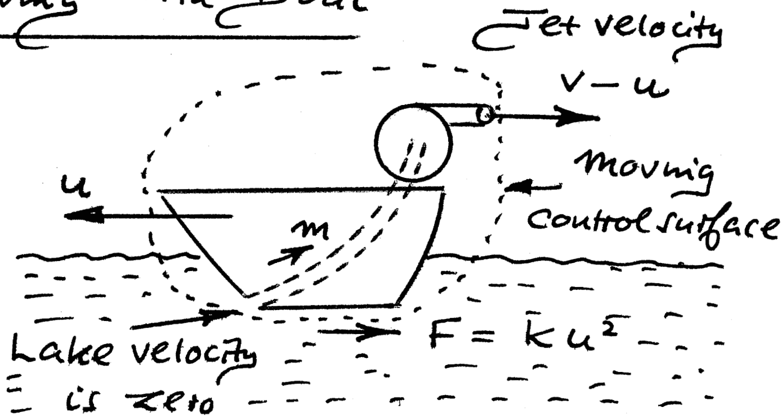
Jet-Propelled Boat

Control Surface Moving with Boat

Momentum balance ←

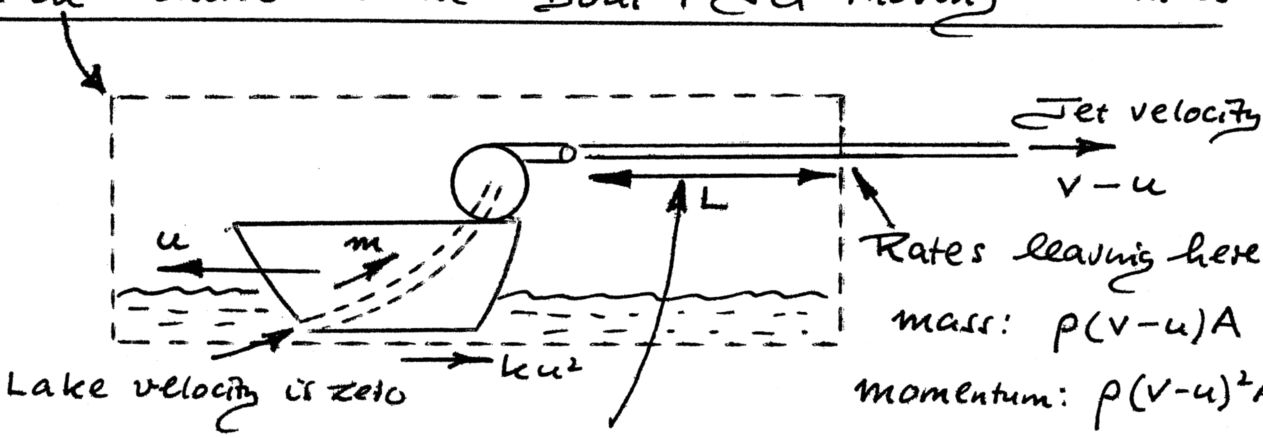
$m \times 0 \quad - \quad - m(v-u)$

Input from lake — Loss — negative direction — Jet



$$-ku^2 = \frac{d}{dt}(Mu) = Ma \quad \text{or} \quad a = \frac{1}{M}\left[m(v-u) - ku^2\right]$$

Fixed Control Surface – Boat + Jet Moving Within it



Rates leaving here:

mass: $\rho(v-u)A$

momentum: $\rho(v-u)^2 A$

Momentum Balance ←

Momentum of this portion of jet $= \rho A L(v-u) \rightarrow$

$$m \times 0 \quad - \quad - \rho(v-u)^2 A = \frac{d}{dt}Mu + \frac{d}{dt}\left[-\rho A L(v-u)\right]$$

From lake — Leaving with jet

$-ku^2$ (friction)

Also note $m = \rho A v$

$$\rho(v-u)^2 A - ku^2 = Ma - \rho A(v-u)\frac{dL}{dt}$$

This equals u

$$a = \frac{1}{M}\left[\rho A(v-u)(v-u+u) - ku^2\right] = \frac{1}{M}\left[m(v-u) - ku^2\right]$$

Numerical Values

At maximum velocity, there is no acceleration, so

$$m(v-u) = Ku^2$$

or $K = \dfrac{m(30-20)}{20^2} = \dfrac{m}{40}$

When $u = 10$ ft/sec,

$$a = \frac{1}{M}\left[m(v-u) - Ku^2\right] = \frac{m}{M}\left[(v-u) - \frac{u^2}{40}\right]$$

$$= \frac{374}{1000}\left[(30-10) - \frac{10^2}{40}\right] = 6.55 \text{ ft/sec}$$

Pumping Power for Propelling Boat

Observer on the boat sees a mass flowrate m with kinetic energy per unit mass of $\frac{u^2}{2}$ coming in and $\frac{v^2}{2}$ leaving. Hence

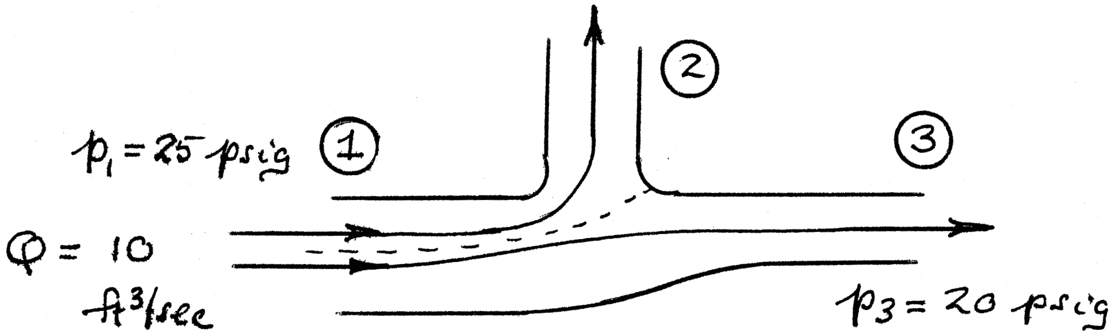
$$P = \frac{m}{2}(v^2 - u^2)$$

[The same result is obtained from the viewpoint of an observer on the shore, who sees power needed to overcome the drag force (Fu), to accelerate water from rest to $(v-u)$ ft/sec ($m\{v-u\}^2/2$), and to increase the kinetic energy of the boat ($\frac{d}{dt} M \frac{u^2}{2} = Mua$)]

Power decreases as u increases because the boat is accelerating less rapidly.

2-21

Branch Pipe



Areas $A_1 = 0.1364$, $A_2 = 0.0873$, $A_3 = 0.0491$ ft²

$$u_1 = \frac{10}{0.1364} = 73.3 \text{ ft/sec}, \quad \rho = 0.8 \times 62.4 = 49.9 \ \frac{\text{lbm}}{\text{ft}^3}$$

Bernoulli 1 → 3 $\quad \dfrac{u_1^2}{2} + \dfrac{p_1}{\rho} = \dfrac{u_3^2}{2} + \dfrac{p_3}{\rho}$

$$\frac{73.3^2}{2} + \frac{25 \times 32.2 \times 144}{49.9} = \frac{u_3^2}{2} + \frac{20 \times 32.2 \times 144}{49.9}$$

Hence $u_3 = 79.4$ ft/sec

Continuity (mass Balance) (ρ constant)

$$10 = 0.0873\, u_2 + 0.0491 \times 79.4$$

$$u_2 = 69.9 \text{ ft/sec}$$

Bernoulli 1 → 2

$$\frac{73.3^2}{2} + \frac{25 \times 32.2 \times 144}{49.9} = \frac{69.9^2}{2} + \frac{p_2 \times 32.2 \times 144}{49.9}$$

Hence $p_2 = 27.6$ psig.

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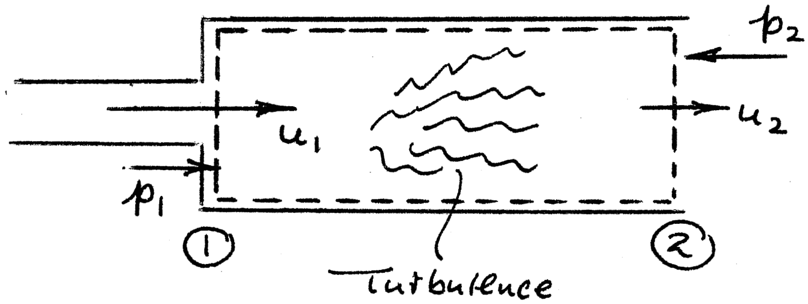
2.22

Sudden Expansion in a Pipe



Bernoulli cannot be applied between 1 and 2 because of the turbulence and energy losses that are characteristic of an expanding jet.

Continuity ① → ② $A_1 u_1 = A_2 u_2$ (1)

Momentum ① → ②

$$p_1 A_2 - p_2 A_2 + \rho A_1 u_1^2 - \rho A_2 u_2^2 = 0 \qquad (2)$$

note

Substitute $u_1 = u_2 \dfrac{A_2}{A_1}$ from (1) into (2)

$$p_2 - p_1 = \rho u_2^2 \left(\frac{A_2}{A_1} - 1 \right), \qquad (3)$$

which is always positive, so pressure increases. Increase in pressure energy is at the expense of kinetic energy

Overall Energy Balance

$$\Delta\left(\frac{u^2}{2}\right) + \frac{\Delta p}{\rho} + \cancel{g\Delta z}^{\text{zero}} + \cancel{w}^{\text{zero}} + \mathcal{J} = 0$$

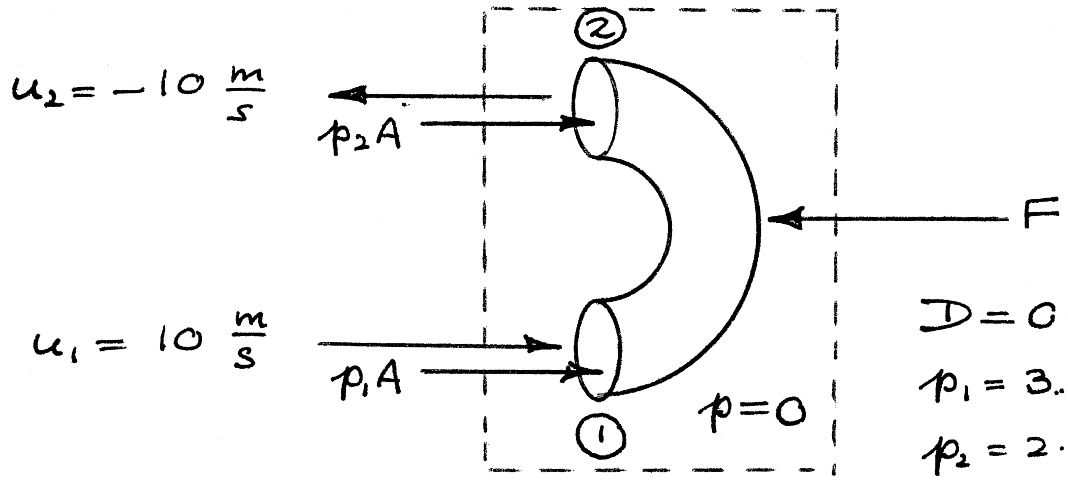
$$\mathcal{J} = \frac{u_1^2}{2} - \frac{u_2^2}{2} + \frac{p_1 - p_2}{\rho} = \frac{u_1^2}{2} - \frac{u_2^2}{2} + \rho u_2^2\left(1 - \frac{A_2}{A_1}\right)$$

$$= \frac{u_2^2}{2}\left(\frac{A_2^2}{A_1^2} - 1 + 2 - 2\frac{A_2}{A_1}\right) = \frac{u_2^2}{2}\left(\frac{A_2}{A_1} - 1\right)^2,$$

which is always positive.

2.23

Force on Return Elbow



$D = 0.1\ m$

$p_1 = 3.0$ bar
$p_2 = 2.5$ (gauge)

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.1)^2}{4} = 0.00785\ m^2$$

$$m = \rho A u_1 = \underset{\frac{kg}{m^3}}{1{,}000} \times \underset{m^2}{0.00785} \times \underset{\frac{m}{s}}{10} = 78.54\ \frac{kg}{s}$$

Momentum Balance → on Return Elbow

$$\underbrace{+\ m u_1 \quad - m u_2}_{\text{By flow}} + \underbrace{p_1 A + p_2 A}_{} - F = 0$$

Addition, Loss — By flow

Both p_1 and p_2 tend to push elbow to right

Elbow stays put.

$$F = (p_1 + p_2) A + 2 m u_1$$

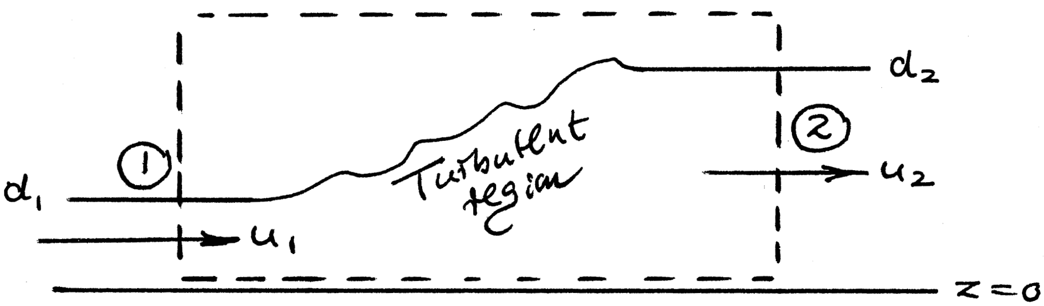
$$= (3.0 + 2.5) \times \underset{\frac{N/m^2}{bar}}{10^5} \times 0.00785 + 2 \times 78.54 \times 10$$

$$= 4{,}317 + 1{,}571 = \underline{\underline{5{,}888\ N}}$$

2.24-1

Hydraulic Jump

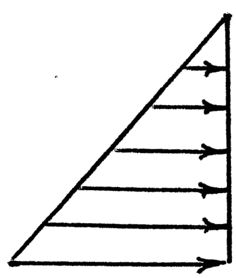
Abrupt, irreversible transition from shallow, rapid stream to deep, tranquil stream.



Ignore shear stress exerted by bottom on liquid

Take unit distance normal to the plane of the diagram

Consider pressure at ① acting on control surface



From earlier note, total force acting to right is

mean pressure × depth

$$= \frac{0 + \rho g d_1}{2} \times d_1 = \frac{\rho g d_1^2}{2}$$

Continuity $\quad u_1 d_1 = u_2 d_2 = \frac{m}{\rho} \qquad (1)$

$\overrightarrow{\text{Momentum}}$ Balance

$$m u_1 + \frac{\rho g d_1^2}{2} - m u_2 - \frac{\rho g d_2^2}{2} = 0 \qquad (2)$$

Eliminating m and u_2

$$\rho u_1^2 d_1 \left(1 - \frac{d_1}{d_2}\right) + \frac{\rho g}{2}\left(d_1^2 - d_2^2\right) = 0$$

Either $d_1 = d_2$ (no change) or

$$-\frac{u_1^2 d_1}{d_2}\cancel{(d_1 - d_2)} + \frac{g}{2}\cancel{(d_1 - d_2)}(d_1 + d_2) = 0$$

$$d_2^2 + d_1 d_2 - \frac{2u_1^2 d_1}{g} = 0 \qquad (3)$$

$$d_2 = \frac{-d_1 \overset{+}{(-)} \sqrt{d_1^2 + \frac{8u_1^2 d_1}{g}}}{2} \qquad (4)$$

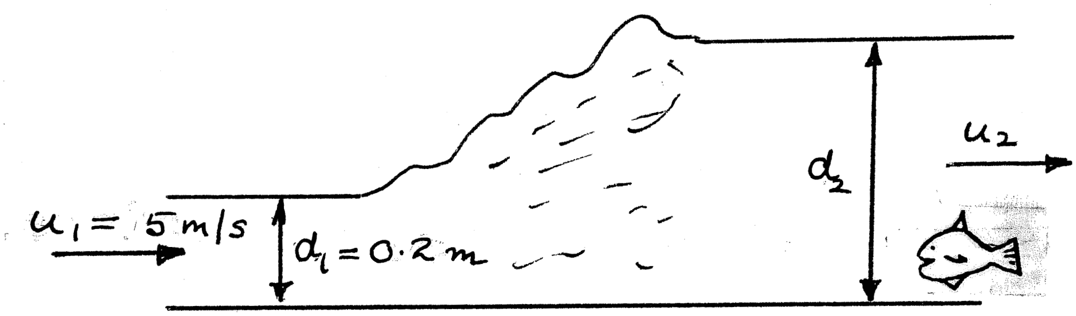
The above equations are satisfied for flow either from L to R or R to L.

It may be determined and is positive for rapid shallow flow becoming deep tranquil flow only, and not vice versa.

Hydraulic jump is similar to a shock wave in a flowing compressible gas (supersonic flow suddenly becomes subsonic, with sudden increase in pressure).

2.24-3

Numbers — Hydraulic Jump



From text: $d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2u_1^2 d_1}{g}}$

$$d_2 = -\frac{0.2}{2} + \sqrt{\frac{0.2^2}{4} + \frac{2 \times 5^2 \times 0.2}{9.81}} = \underline{\underline{0.915 \text{ m}}}$$

Continuity

$$u_1 d_1 = u_2 d_2$$

$$u_2 = u_1 \frac{d_1}{d_2} = 5 \times \frac{0.2}{0.915}$$

$$= \underline{\underline{1.09 \text{ m/s}}}$$

Demonstration that $\mathcal{F}$ is positive for $d_2 > d_1$

Overall Energy Balance

$$\underbrace{\frac{g d_2}{2} - \frac{g d_1}{2}}_{\frac{\Delta \bar{p}}{\rho}} + \frac{u_2^2}{2} - \frac{u_1^2}{2} + \underbrace{\frac{g d_2}{2} - \frac{g d_1}{2}}_{g \Delta \bar{z}} + \mathcal{F} = 0$$

Hence
$$\mathcal{F} = g(d_1 - d_2) + \tfrac{1}{2}(u_1^2 - u_2^2)$$
$$= g(d_1 - d_2) + \frac{u_1^2}{2}\left(1 - \frac{d_1^2}{d_2^2}\right)$$

But from equation (3),

$$\frac{u_1^2}{2} = \frac{g}{4}\,\frac{d_2}{d_1}(d_2 + d_1)$$

Eliminating u_1^2 and dividing by g,

$$\frac{\mathcal{F}}{g} = (d_1 - d_2) + \frac{d_2}{4 d_1}(d_2 + d_1)\frac{(d_2^2 - d_1^2)}{d_2^2}$$

$$= (d_1 - d_2) + \frac{1}{4 d_1 d_2}(d_1 + d_2)^2 (d_2 - d_1)$$

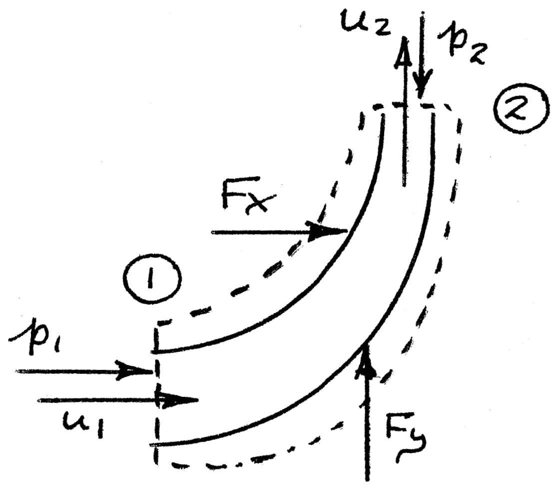
$$= (d_2 - d_1)\left[\frac{(d_1 + d_2)^2}{4 d_1 d_2} - 1\right]$$

$$= \frac{(d_2 - d_1)}{4 d_1 d_2}\left(d_1^2 + 2 d_1 d_2 + d_2^2 - 4 d_1 d_2\right) = \frac{(d_2 - d_1)^3}{4 d_1 d_2}$$

Thus, $\mathcal{F}/g$ is always positive provided that d_2 is greater than d_1 (and <u>not</u> vice versa)

2.25

90° Reducing Elbow



Continuity (ρ constant)

$$A_1 u_1 = A_2 u_2 \qquad (1)$$

Energy (Bernoulli)

$$\frac{u_1^2}{2} + \frac{p_1}{\rho} = \frac{u_2^2}{2} + \frac{p_2}{\rho} \qquad (2)$$

X – Momentum

$$\rho A_1 u_1^2 + p_1 A_1 + F_x = 0 \qquad (3)$$

y – Momentum

$$-\rho A_2 u_2^2 - p_2 A_2 + F_y = 0 \qquad (4)$$

Hence

$$F_x = -(\rho u_1^2 + p_1) A_1$$

$$F_y = (\rho u_2^2 + p_2) A_2 = \left(\rho u_1^2 \frac{A_1^2}{A_2^2} + p_2\right) A_2$$

But $\quad p_2 = p_1 + \frac{\rho u_1^2}{2}\left(1 - \frac{A_1^2}{A_2^2}\right)$

so $\quad F_y = A_2\left[p_1 + \frac{\rho u_1^2}{2}\left(1 + \frac{A_1^2}{A_2^2}\right)\right]$

Numerically, with $p_1 = 1.5 \times 10^5\ N/m^2$, $u_1 = 5.0\ m/s$

$$A_1 = \frac{\pi}{4}(0.20)^2 = 0.03142\ m^2$$

$$A_2 = \frac{\pi}{4}(0.15)^2 = 0.01767\ m^2$$

$$\frac{A_1^2}{A_2^2} = 3.160 \qquad \rho = 1{,}000\ \frac{kg}{m^3}$$

$$\boxed{F_x = -5{,}498\ N}$$

$$\boxed{F_y = 3{,}569\ N}$$