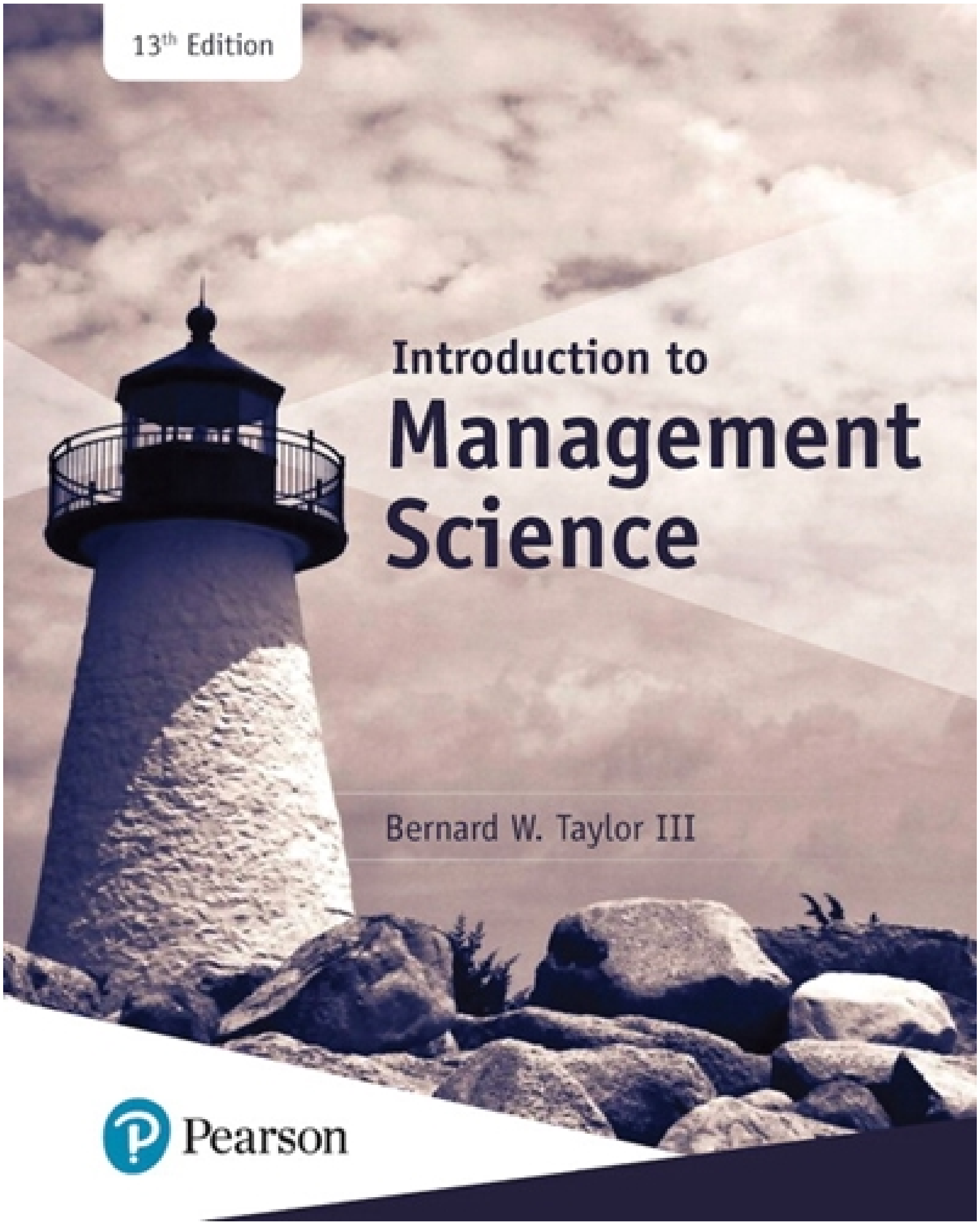
Solutions for Introduction to Management Science 13th Edition by Taylor



Solutions

Chapter Two: Linear Programming: Model Formulation and Graphical Solution

PROBLEM SUMMARY

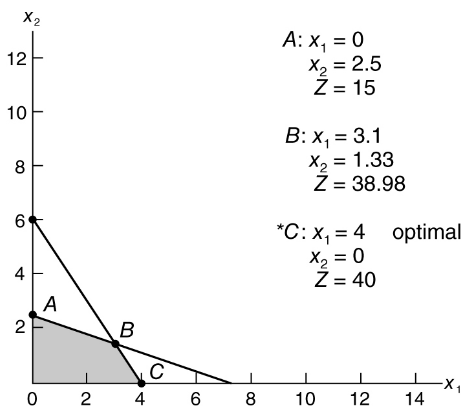
1. Maximization (1–40 continuation), graphical solution
2. Minimization, graphical solution
3. Sensitivity analysis (2–2)
4. Minimization, graphical solution
5. Maximization, graphical solution
6. Slack analysis (2–5), sensitivity analysis
7. Maximization, graphical solution
8. Slack analysis (2–7)
9. Maximization, graphical solution
10. Minimization, graphical solution
11. Maximization, graphical solution
12. Sensitivity analysis (2–11)
13. Sensitivity analysis (2–11)
14. Maximization, graphical solution
15. Sensitivity analysis (2–14)
16. Maximization, graphical solution
17. Sensitivity analysis (2–16)
18. Maximization, graphical solution
19. Standard form (2–18)
20. Maximization, graphical solution
21. Constraint analysis (2–20)
22. Minimization, graphical solution
23. Sensitivity analysis (2–22)
24. Minimization, graphical solution
25. Minimization, graphical solution
26. Sensitivity analysis (2–25)
27. Minimization, graphical solution
28. Maximization, graphical solution
29. Minimization, graphical solution
30. Maximization, graphical solution
31. Sensitivity analysis (2–30)
32. Minimization, graphical solution
33. Maximization, graphical solution
34. Maximization, graphical solution
35. Sensitivity analysis (2–34)
36. Maximization, graphical solution
37. Sensitivity analysis (2–36)
38. Maximization, graphical solution
39. Sensitivity analysis (2–38)
40. Minimization, graphical solution
41. Sensitivity analysis (2–40)
42. Maximization, graphical solution
43. Sensitivity analysis (2–42)
44. Maximization, graphical solution
45. Sensitivity analysis (2–44)
46. Maximization, graphical solution
47. Minimization, graphical solution
48. Sensitivity analysis (2–47)
49. Minimization, graphical solution
50. Sensitivity analysis (2–49)
51. Maximization, graphical solution
52. Minimization, graphical solution
53. Sensitivity analysis (2–52)
54. Maximization, graphical solution
55. Sensitivity analysis (2–54)
56. Maximization, graphical solution
57. Sensitivity analysis (2–56)
58. Multiple optimal solutions
59. Infeasible problem
60. Unbounded problem

PROBLEM SOLUTIONS

1. a) x_1 = # cakes
x_2 = # loaves of bread
maximize $Z = \$10x_1 + 6x_2$
subject to

$$3x_1 + 8x_2 \leq 20 \text{ cups of flour}$$
$$45x_1 + 30x_2 \leq 180 \text{ minutes}$$
$$x_1, x_2 \geq 0$$

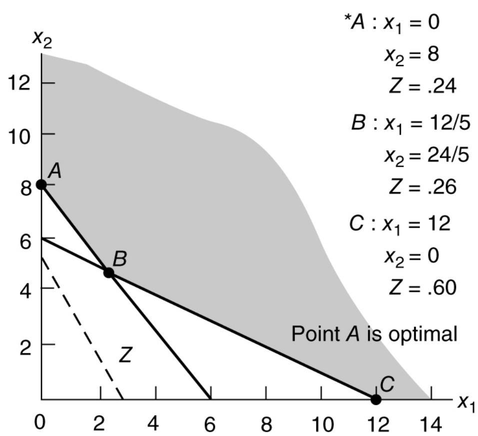
b)



2. a) Minimize $Z = .05x_1 + .03x_2$ (cost, \$)
subject to

$$8x_1 + 6x_2 \geq 48 \text{ (vitamin A, mg)}$$
$$x_1 + 2x_2 \geq 12 \text{ (vitamin B, mg)}$$
$$x_1, x_2 \geq 0$$

b)



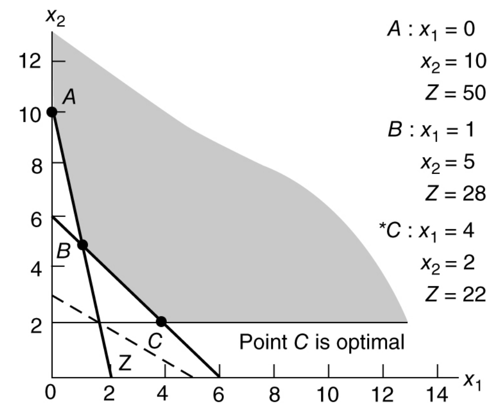
3. The optimal solution point would change from point A to point B, thus resulting in the optimal solution

$x_1 = 12/5 \quad x_2 = 24/5 \quad Z = .408$

4. a) Minimize $Z = 3x_1 + 5x_2$ (cost, \$)
subject to

$$10x_1 + 2x_2 \geq 20 \text{ (nitrogen, oz)}$$
$$6x_1 + 6x_2 \geq 36 \text{ (phosphate, oz)}$$
$$x_2 \geq 2 \text{ (potassium, oz)}$$
$$x_1, x_2 \geq 0$$

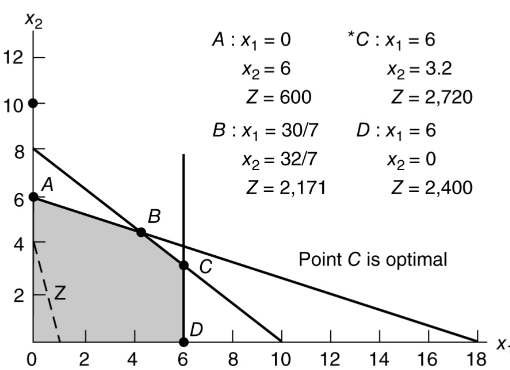
b)



5. a) Maximize $Z = 400x_1 + 100x_2$ (profit, \$)
subject to

$$8x_1 + 10x_2 \leq 80 \text{ (labor, hr)}$$
$$2x_1 + 6x_2 \leq 36 \text{ (wood)}$$
$$x_1 \leq 6 \text{ (demand, chairs)}$$
$$x_1, x_2 \geq 0$$

b)



6. a) In order to solve this problem, you must substitute the optimal solution into the resource constraint for wood and the resource constraint for labor and determine how much of each resource is left over.

Labor

$$8x_1 + 10x_2 \leq 80 \text{ hr}$$
$$8(6) + 10(3.2) \leq 80$$
$$48 + 32 \leq 80$$
$$80 \leq 80$$

There is no labor left unused.

Wood

$$2x_1 + 6x_2 \le 36$$
$$2(6) + 6(3.2) \le 36$$
$$12 + 19.2 \le 36$$
$$31.2 \le 36$$
$$36 - 31.2 = 4.8$$

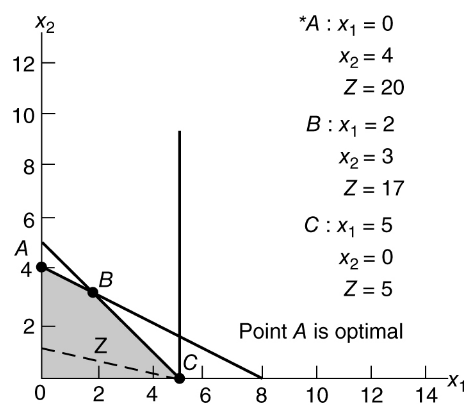
There is 4.8 lb of wood left unused.

b) The new objective function, $Z = 400x_1 + 500x_2$, is parallel to the constraint for labor, which results in multiple optimal solutions. Points B ($x_1 = 30/7$, $x_2 = 32/7$) and C ($x_1 = 6$, $x_2 = 3.2$) are the alternate optimal solutions, each with a profit of $4,000.

7. a) Maximize $Z = x_1 + 5x_2$ (profit, $)
subject to

$5x_1 + 5x_2 \le 25$ (flour, lb)
$2x_1 + 4x_2 \le 16$ (sugar, lb)
$x_1 \le 5$ (demand for cakes)
$x_1, x_2 \ge 0$

b)



8. In order to solve this problem, you must substitute the optimal solution into the resource constraints for flour and sugar and determine how much of each resource is left over.

Flour

$$5x_1 + 5x_2 \le 25 \text{ lb}$$
$$5(0) + 5(4) \le 25$$
$$20 \le 25$$
$$25 - 20 = 5$$

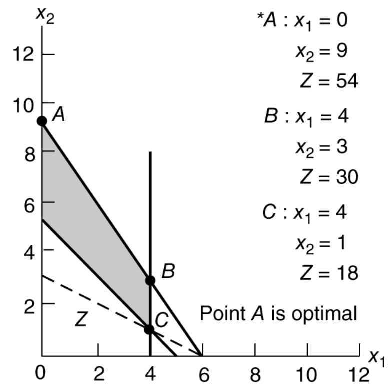
There are 5 lb of flour left unused.

Sugar

$$2x_1 + 4x_2 \le 16$$
$$2(0) + 4(4) \le 16$$
$$16 \le 16$$

There is no sugar left unused.

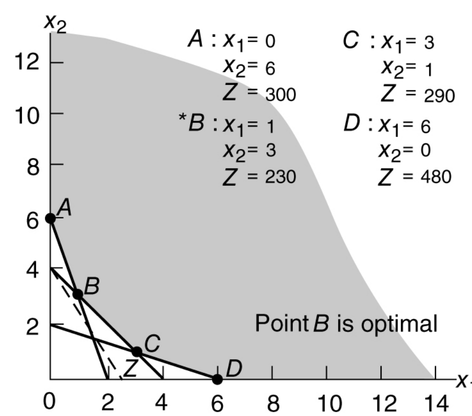
9.



10. a) Minimize $Z = 80x_1 + 50x_2$ (cost, $)
subject to

$3x_1 + x_2 \ge 6$ (antibiotic 1, units)
$x_1 + x_2 \ge 4$ (antibiotic 2, units)
$2x_1 + 6x_2 \ge 12$ (antibiotic 3, units)
$x_1, x_2 \ge 0$

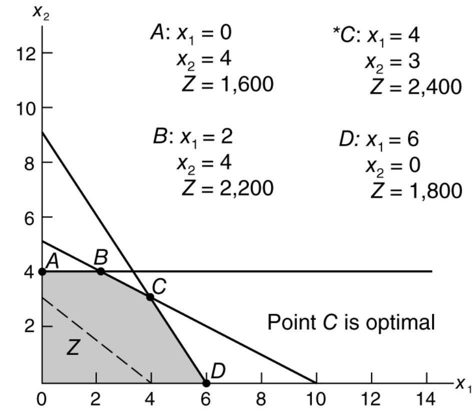
b)



11. a) Maximize $Z = 300x_1 + 400x_2$ (profit, $)
subject to

$3x_1 + 2x_2 \le 18$ (gold, oz)
$2x_1 + 4x_2 \le 20$ (platinum, oz)
$x_2 \le 4$ (demand, bracelets)
$x_1, x_2 \ge 0$

b)



12. The new objective function, $Z = 300x_1 + 600x_2$, is parallel to the constraint line for platinum, which results in multiple optimal solutions. Points B ($x_1 = 2$, $x_2 = 4$) and C ($x_1 = 4$, $x_2 = 3$) are the alternate optimal solutions, each with a profit of \$3,000.

The feasible solution space will change. The new constraint line, $3x_1 + 4x_2 = 20$, is parallel to the existing objective function. Thus, multiple optimal solutions will also be present in this scenario. The alternate optimal solutions are at $x_1 = 1.33$, $x_2 = 4$ and $x_1 = 2.4$, $x_2 = 3.2$, each with a profit of \$2,000.

13. a) Optimal solution: $x_1 = 4$ necklaces, $x_2 = 3$ bracelets. The maximum demand is not achieved by the amount of one bracelet.

b) The solution point on the graph which corresponds to no bracelets being produced must be on the x_1 axis where $x_2 = 0$. This is point D on the graph. In order for point D to be optimal, the objective function "slope" must change such that it is equal to or greater than the slope of the constraint line, $3x_1 + 2x_2 = 18$. Transforming this constraint into the form $y = a + bx$ enables us to compute the slope:

$$2x_2 = 18 - 3x_1$$
$$x_2 = 9 - 3/2x_1$$

From this equation the slope is −3/2. Thus, the slope of the objective function must be at least −3/2. Presently, the slope of the objective function is −3/4:

$$400x_2 = Z - 300x_1$$
$$x_2 = Z/400 - 3/4x_1$$

The profit for a necklace would have to increase to *\$600* to result in a slope of −3/2:

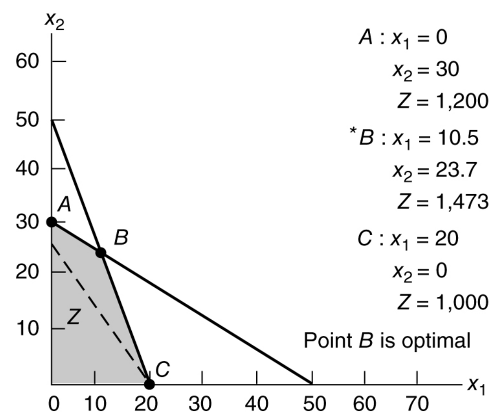
$$400x_2 = Z - 600x_1$$
$$x_2 = Z/400 - 3/2x_1$$

However, this creates a situation where both points C and D are optimal, ie., multiple optimal solutions, as are all points on the line segment between C and D.

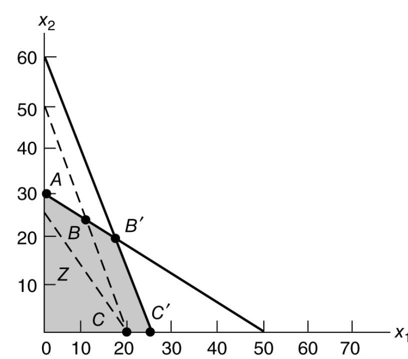
14. a) Maximize $Z = 50x_1 + 40x_2$ (profit, \$) subject to

$$3x_1 + 5x_2 \leq 150 \text{ (wool, yd}^2\text{)}$$
$$10x_1 + 4x_2 \leq 200 \text{ (labor, hr)}$$
$$x_1, x_2 \geq 0$$

b)



15. The feasible solution space changes from the area *0ABC* to *0AB′C′*, as shown on the following graph.



The extreme points to evaluate are now A, B', and C'.

A: $x_1 = 0$
$x_2 = 30$
$Z = 1,200$

B': $x_1 = 15.8$
$x_2 = 20.5$
$Z = 1,610$

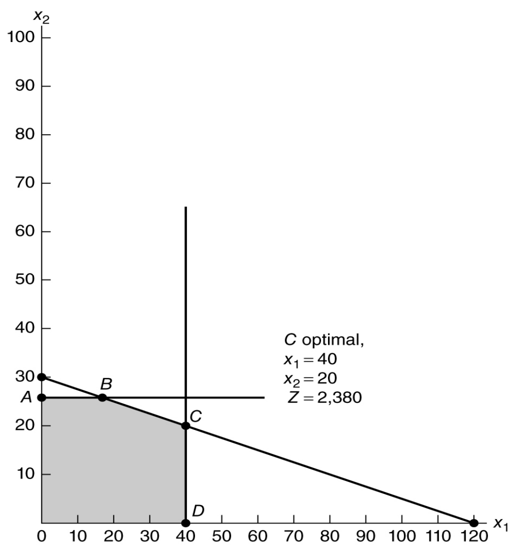
C′: $x_1 = 24$
$x_2 = 0$
$Z = 1{,}200$

Point *B′* is optimal

16. a) Maximize $Z = 23x_1 + 73x_2$
subject to

$$x_1 \le 40$$
$$x_2 \le 25$$
$$x_1 + 4x_2 \le 120$$
$$x_1, x_2 \ge 0$$

b)



17. a) No, not this winter, but they might after they recover equipment costs, which should be after the 2nd winter.

b) $x_1 = 55$
$x_2 = 16.25$
$Z = 1{,}851$

No, profit will go down

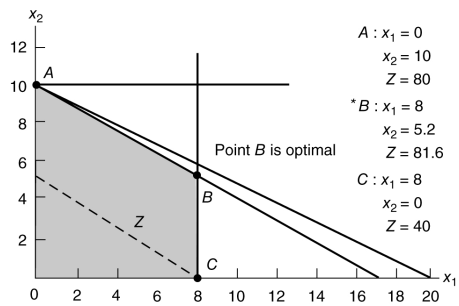
c) $x_1 = 40$
$x_2 = 25$
$Z = 2{,}435$

Profit will increase slightly

d) $x_1 = 55$
$x_2 = 27.72$
$Z = \$2{,}073$

Profit will go down from (c)

18.

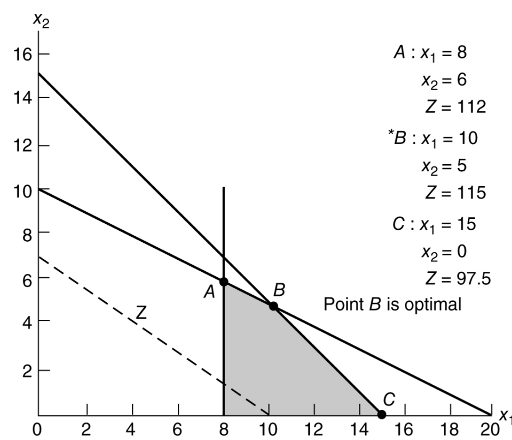


19. Maximize $Z = 5x_1 + 8x_2 + 0s_1 + 0s_3 + 0s_4$
subject to

$$3x_1 + 5x_2 + s_1 = 50$$
$$2x_1 + 4x_2 + s_2 = 40$$
$$x_1 + s_3 = 8$$
$$x_2 + s_4 = 10$$
$$x_1, x_2 \ge 0$$

A: $s_1 = 0,\ s_2 = 0,\ s_3 = 8,\ s_4 = 0$
B: $s_1 = 0,\ s_2 = 3.2,\ s_3 = 0,\ s_4 = 4.8$
C: $s_1 = 26,\ s_2 = 24,\ s_3 = 0,\ s_4 = 10$

20.

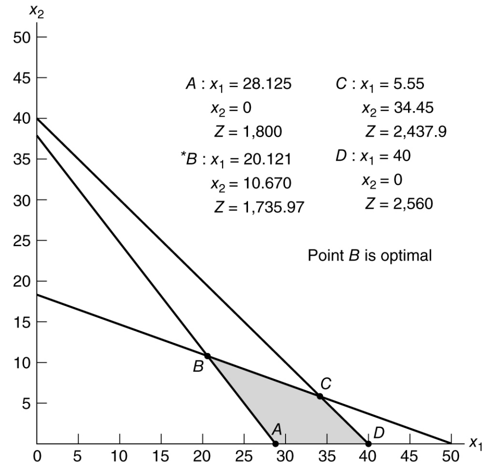


21. It changes the optimal solution to point *A* ($x_1 = 8$, $x_2 = 6$, $Z = 112$), and the constraint, $x_1 + x_2 \le 15$, is no longer part of the solution space boundary.

22. a) Minimize $Z = 64x_1 + 42x_2$ (labor cost, \$)
subject to

$$16x_1 + 12x_2 \ge 450 \text{ (claims)}$$
$$x_1 + x_2 \le 40 \text{ (workstations)}$$
$$0.5x_1 + 1.4x_2 \le 25 \text{ (defective claims)}$$
$$x_1, x_2 \ge 0$$

b)



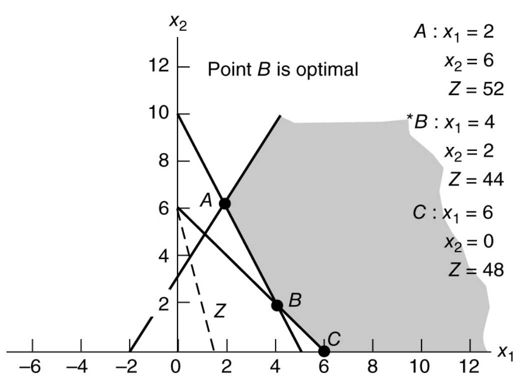
23. a) Changing the pay for a full-time claims solution to point *A* in the graphical solution where $x_1 = 28.125$ and $x_2 = 0$, i.e., there will be no part-time operators.

b) Changing the pay for a part-time operator from $42 to $36 has no effect on the number of full-time and part-time operators hired, although the total cost will be reduced to $1,671.95.

c) Eliminating the constraint for defective claims would result in a new solution, $x_1 = 0$ and $x_2 = 37.5$, where only part-time operators would be hired.

d) The solution becomes infeasible; there are not enough workstations to handle the increase in the volume of claims.

24.



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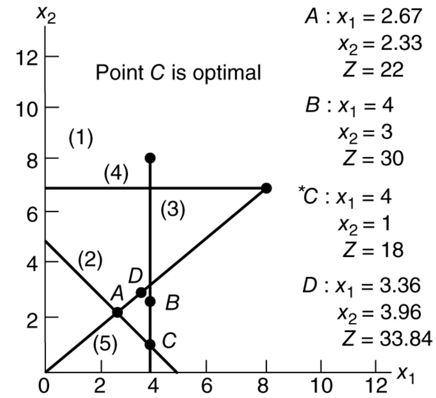
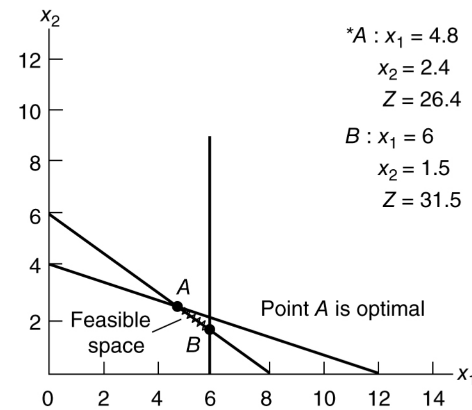
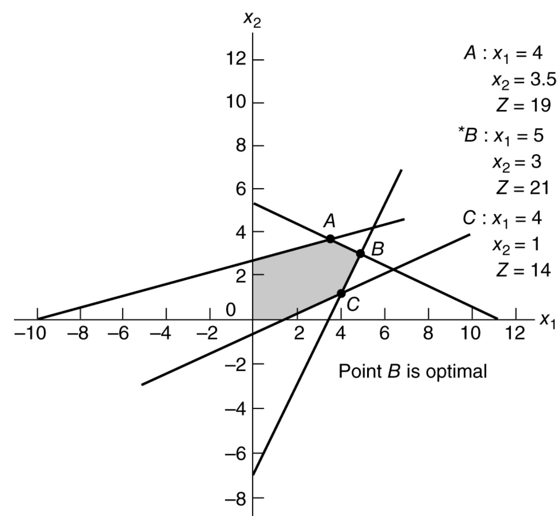


26. The problem becomes infeasible.

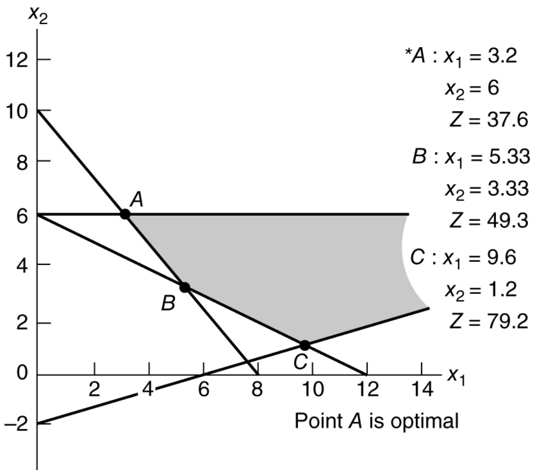
27.



28.



29.



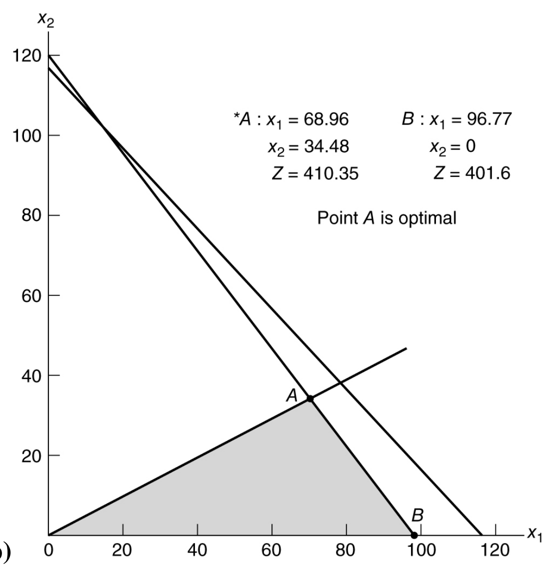
30. a) Maximize $Z = \$4.15x_1 + 3.60x_2$ (profit, \$) subject to

$x_1 + x_2 \leq 115$ (freezer space, gals.)

$0.93x_1 + 0.75x_2 \leq 90$ (budget, \$)

$\frac{x_1}{x_2} \geq \frac{2}{1}$ or $x_1 - 2x_2 \geq 0$ (demand)

$x_1, x_2 \geq 0$



b)

31. No additional profit, freezer space is not a binding constraint.

32. a) Minimize $Z = 200x_1 + 160x_2$ (cost, \$) subject to

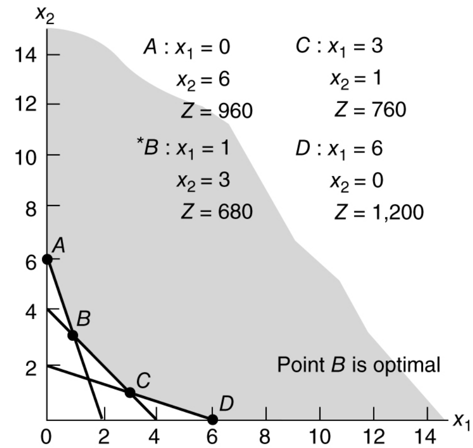
$6x_1 + 2x_2 \geq 12$ (high-grade ore, tons)

$2x_1 + 2x_2 \geq 8$ (medium-grade ore, tons)

$4x_1 + 12x_2 \geq 24$ (low-grade ore, tons)

$x_1, x_2 \geq 0$

b)



33. a) Maximize $Z = 800x_1 + 900x_2$ (profit, \$) subject to

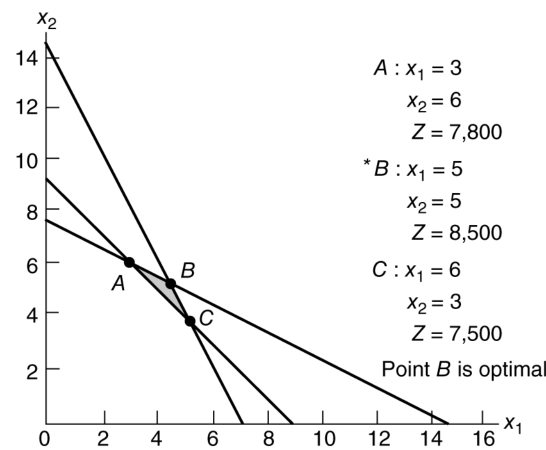
$2x_1 + 4x_2 \leq 30$ (stamping, days)

$4x_1 + 2x_2 \leq 30$ (coating, days)

$x_1 + x_2 \geq 9$ (lots)

$x_1, x_2 \geq 0$

b)



34. a) Maximize $Z = 30x_1 + 70x_2$ (profit, \$) subject to

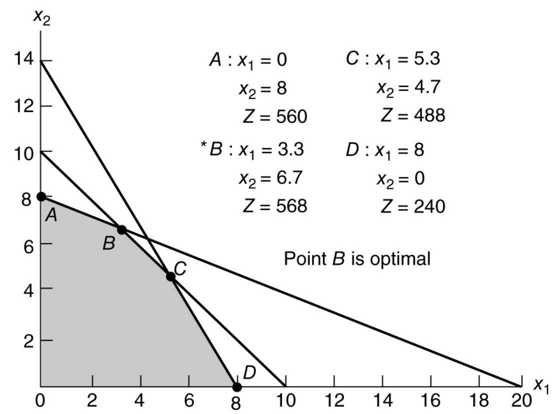
$4x_1 + 10x_2 \leq 80$ (assembly, hr)

$14x_1 + 8x_2 \leq 112$ (finishing, hr)

$x_1 + x_2 \leq 10$ (inventory, units)

$x_1, x_2 \geq 0$

b)



35. The slope of the original objective function is computed as follows:

$$Z = 30x_1 + 70x_2$$
$$70x_2 = Z - 30x_1$$
$$x_2 = Z/70 - 3/7x_1$$
$$\text{slope} = -3/7$$

The slope of the new objective function is computed as follows:

$$Z = 90x_1 + 70x_2$$
$$70x_2 = Z - 90x_1$$
$$x_2 = Z/70 - 9/7x_1$$
$$\text{slope} = -9/7$$

The change in the objective function not only changes the Z values but also results in a new solution point, C. The slope of the new objective function is steeper and thus changes the solution point.

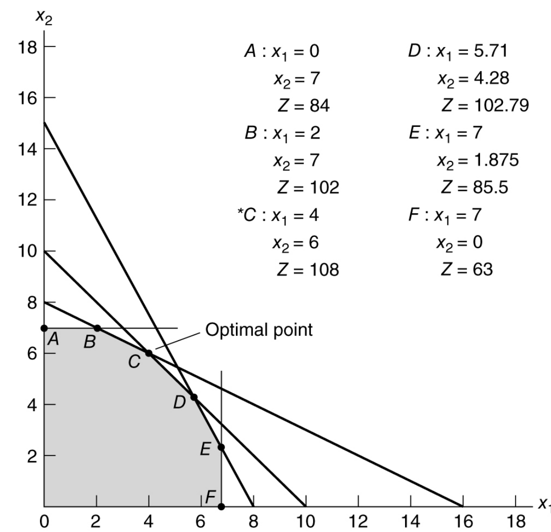
A: $x_1 = 0$, $x_2 = 8$, $Z = 560$ $\quad$ C: $x_1 = 5.3$, $x_2 = 4.7$, $Z = 806$

B: $x_1 = 3.3$, $x_2 = 6.7$, $Z = 766$ $\quad$ D: $x_1 = 8$, $x_2 = 0$, $Z = 720$

36. a) Maximize $Z = 9x_1 + 12x_2$ (profit, \$1,000s) subject to

$4x_1 + 8x_2 \leq 64$ (grapes, tons)
$5x_1 + 5x_2 \leq 50$ (storage space, yd^3)
$15x_1 + 8x_2 \leq 120$ (processing time, hr)
$x_1 \leq 7$ (demand, Nectar)
$x_2 \leq 7$ (demand, Red)
$x_1, x_2 \geq 0$

b)



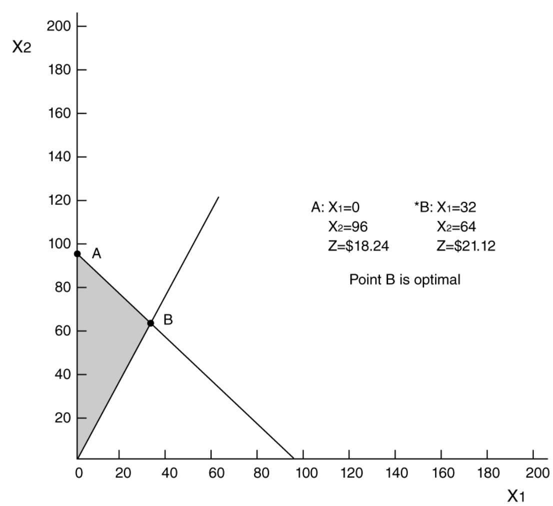
37. a) $15(4) + 8(6) \leq 120$ hr
$60 + 48 \leq 120$
$108 \leq 120$
$120 - 108 = 12$ hr left unused

b) Points C and D would be eliminated and a new optimal solution point at $x_1 = 5.09$, $x_2 = 5.45$, and $Z = 111.27$ would result.

38. a) Maximize $Z = .28x_1 + .19x_2$

$$x_1 + x_2 \leq 96 \text{ cans}$$
$$\frac{x_2}{x_1} \geq 2$$
$$x_1, x_2 \geq 0$$

b)



39. The model formulation would become,
maximize $Z = \$0.23x_1 + 0.19x_2$
subject to

$$x_1 + x_2 \le 96$$
$$-1.5x_1 + x_2 \ge 0$$
$$x_1, x_2 \ge 0$$

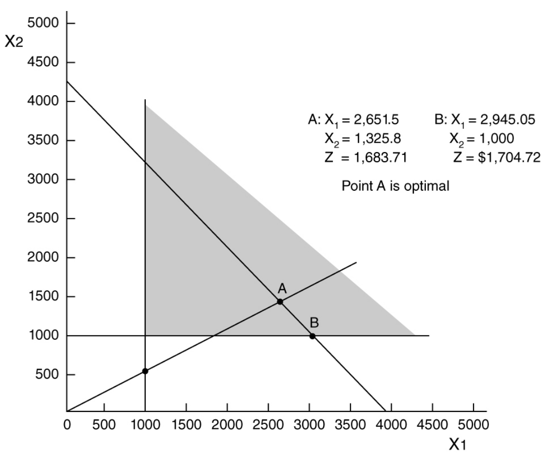
The solution is $x_1 = 38.4$, $x_2 = 57.6$, and $Z = \$19.78$

The discount would reduce profit.

40. a) Minimize $Z = \$0.46x_1 + 0.35x_2$
subject to

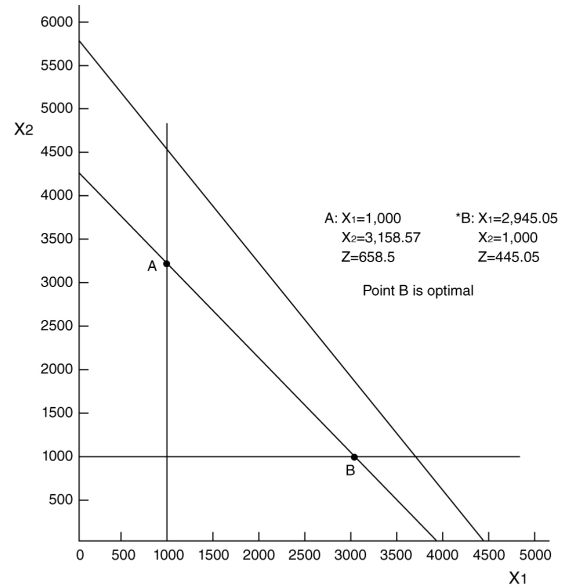
$$.91x_1 + .82x_2 = 3{,}500$$
$$x_1 \ge 1{,}000$$
$$x_2 \ge 1{,}000$$
$$.03x_1 - .06x_2 \ge 0$$
$$x_1, x_2 \ge 0$$

b)



41. a) Minimize $Z = .09x_1 + .18x_2$
subject to

$$.46x_1 + .35x_2 \le 2{,}000$$
$$x_1 \ge 1{,}000$$
$$x_2 \ge 1{,}000$$
$$.91x_1 - .82x_2 = 3{,}500$$
$$x_1, x_2 \ge 0$$

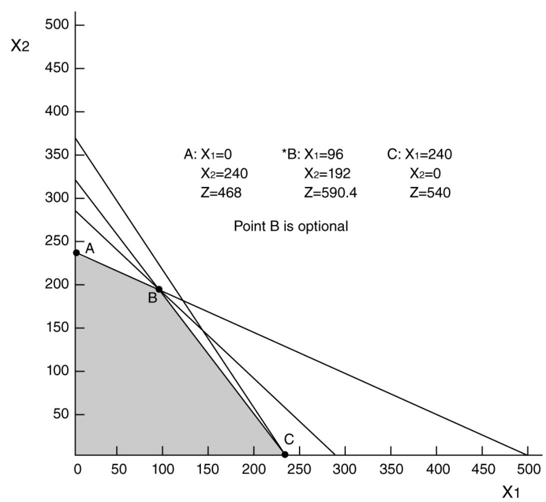


b) $477 - 445 = 32$ fewer defective items

42. a) Maximize $Z = \$2.25x_1 + 1.95x_2$
subject to

$$8x_1 + 6x_2 \le 1{,}920$$
$$3x_1 + 6x_2 \le 1{,}440$$
$$3x_1 + 2x_2 \le 720$$
$$x_1 + x_2 \le 288$$
$$x_1, x_2 \ge 0$$

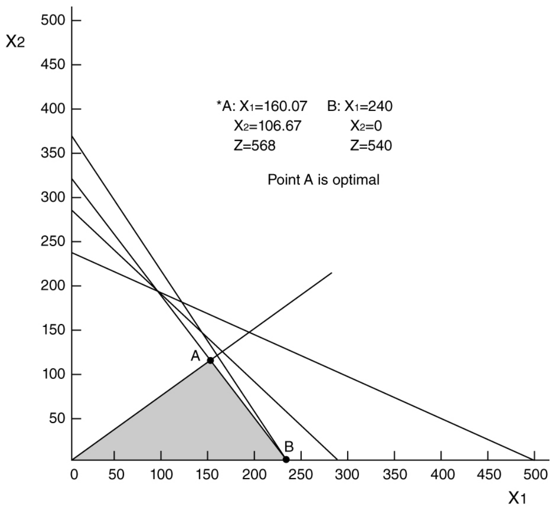
b)



43. A new constraint is added to the model in

$$\frac{x_1}{x_2} \geq 1.5$$

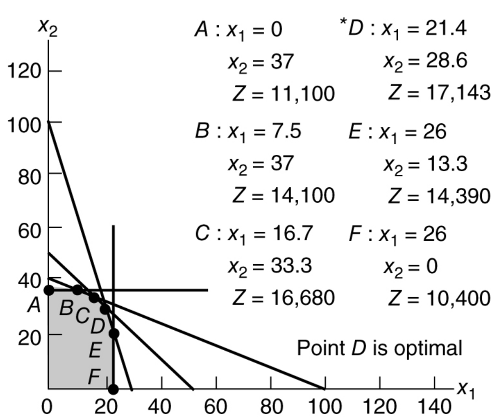
The solution is $x_1 = 160$, $x_2 = 106.67$, $Z = \$568$



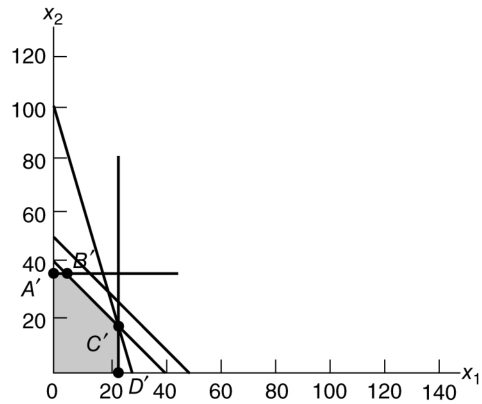
44. a) Maximize $Z = 400x_1 + 300x_2$ (profit, \$) subject to

$$x_1 + x_2 \leq 50 \text{ (available land, acres)}$$
$$10x_1 + 3x_2 \leq 300 \text{ (labor, hr)}$$
$$8x_1 + 20x_2 \leq 800 \text{ (fertilizer, tons)}$$
$$x_1 \leq 26 \text{ (shipping space, acres)}$$
$$x_2 \leq 37 \text{ (shipping space, acres)}$$
$$x_1, x_2 \geq 0$$

b)



45. The feasible solution space changes if the fertilizer constraint changes to $20x_1 + 20x_2 \leq 800$ tons. The new solution space is $A'B'C'D'$. Two of the constraints now have no effect.



The new optimal solution is point C':

A': $x_1 = 0$, $x_2 = 37$, $Z = 11,100$

$*C'$: $x_1 = 25.71$, $x_2 = 14.29$, $Z = 14,571$

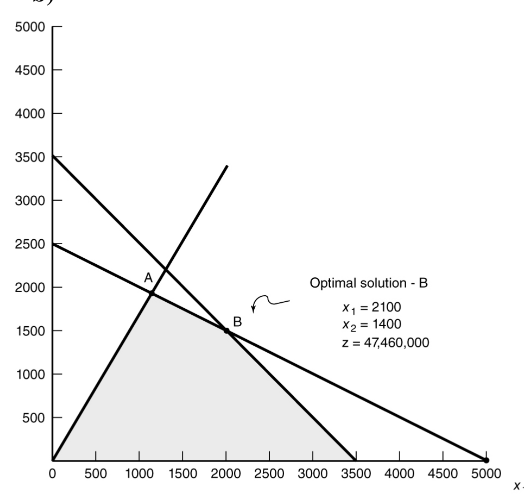
B': $x_1 = 3$, $x_2 = 37$, $Z = 12,300$

D': $x_1 = 26$, $x_2 = 0$, $Z = 10,400$

46. a) Maximize $Z = \$7,600x_1 + 22,500x_2$ subject to

$$x_1 + x_2 \leq 3,500$$
$$x_2/(x_1 + x_2) \leq .40$$
$$.12x_1 + .24x_2 \leq 600$$
$$x_1, x_2 \geq 0$$

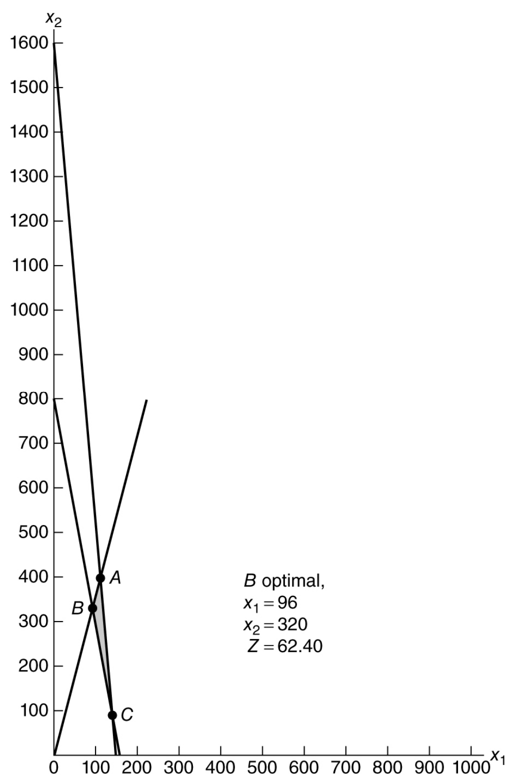
b)



47. a) Minimize $Z = \$(.05)(8)x_1 + (.10)(.75)x_2$
subject to

$$5x_1 + x_2 \geq 800$$
$$\frac{5x_1}{x_2} = 1.5$$
$$8x_1 + .75x_2 \leq 1{,}200$$
$$x_1, x_2 \geq 0$$
$$x_1 = 96$$
$$x_2 = 320$$
$$Z = \$62.40$$

b)



48. The new solution is

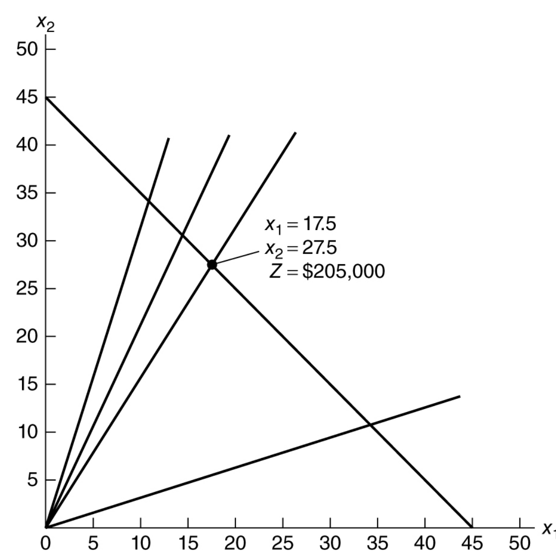
$$x_1 = 106.67$$
$$x_2 = 266.67$$
$$Z = \$62.67$$

If twice as many guests prefer wine to beer, then the Robinsons would be approximately 10 bottles of wine short and they would have approximately 53 more bottles of beer than they need. The waste is more difficult to compute. The model in problem 53 assumes that the Robinsons are ordering more wine and beer than they need, i.e., a buffer, and thus there logically would be some waste, i.e., 5% of the wine and 10% of the beer. However, if twice as many guests prefer wine, then there would logically be no waste for wine but only for beer. This amount "logically" would be the waste from 266.67 bottles, or \$20, and the amount from the additional 53 bottles, \$3.98, for a total of \$23.98.

49. a) Minimize $Z = 3700x_1 + 5100x_2$
subject to

$$x_1 + x_2 = 45$$
$$(32x_1 + 14x_2) / (x_1 + x_2) \leq 21$$
$$.10x_1 + .04x_2 \leq 6$$
$$\frac{x_1}{(x_1 + x_2)} \geq .25$$
$$\frac{x_2}{(x_1 + x_2)} \geq .25$$
$$x_1, x_2 \geq 0$$

b)



50. a) No, the solution would not change

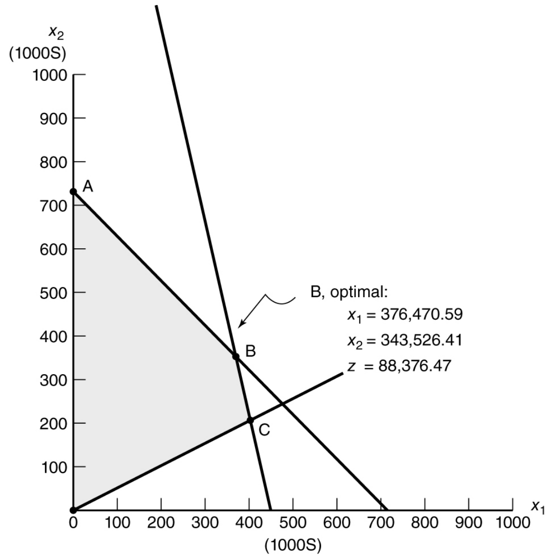
b) No, the solution would not change

c) Yes, the solution would change to China (x_1) = 22.5, Brazil (x_2) = 22.5, and $Z = \$198{,}000$.

51. a) x_1 = \$ invested in stocks
x_2 = \$ invested in bonds
maximize $Z = \$0.18x_1 + 0.06x_2$ (average annual return)
subject to

$x_1 + x_2 \leq \$720{,}000$ (available funds)
$x_1/(x_1 + x_2) \leq .65$ (% of stocks)
$.22x_1 + .05x_2 \leq 100{,}000$ (total possible loss)
$x_1, x_2 \geq 0$

b)



52. x_1 = exams assigned to Brad

x_2 = exams assigned to Sarah

minimize $Z = .10x_1 + .06x_2$

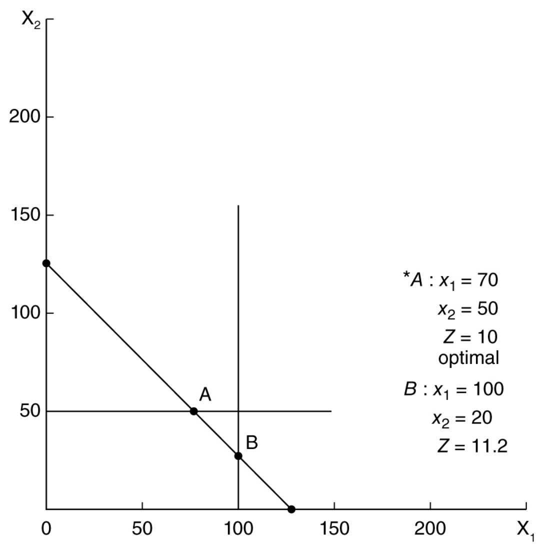
subject to

$x_1 + x_2 = 120$

$x_1 \le (720/7.2)$ or 100

$x_2 \le 50(600/12)$

$x_1, x_2 \ge 0$



53. If the constraint for Sarah's time became $x_2 \le 55$ with an additional hour then the solution point at A would move to $x_1 = 65$, $x_2 = 55$ and $Z = 9.8$. If the constraint for Brad's time became $x_1 \le 108.33$ with an additional hour then the solution point (A) would not change. All of Brad's time is not being used anyway so assigning him more time would not have an effect.

One more hour of Sarah's time would reduce the number of regraded exams from 10 to 9.8, whereas increasing Brad by one hour would have no effect on the solution. This is actually the marginal (or dual) value of one additional hour of labor, for Sarah, which is 0.20 fewer regraded exams, whereas the marginal value of Brad's is zero.

54. a) x_1 = # cups of Pomona

x_2 = # cups of Coastal

Maximize $Z = \$2.05x_1 + 1.85x_2$

subject to

$16x_1 + 16x_2 \le 3{,}840$ oz or (30 gal. × 128 oz)

$(.20)(.0625)x_1 + (.60)(.0625)x_2 \le 6$ lbs. Colombian

$(.35)(.0625)x_1 + (.10)(.0625)x_2 \le 6$ lbs. Kenyan

$(.45)(.0625)x_1 + (.30)(.0625)x_2 \le 6$ lbs. Indonesian

$x_2/x_1 = 3/2$

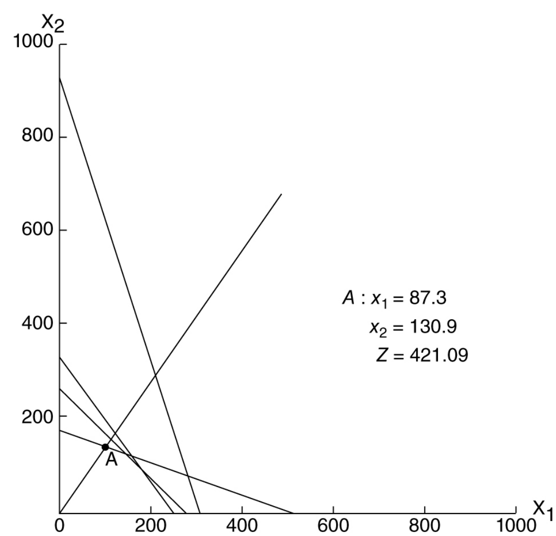
$x_1, x_2 \ge 0$

b) Solution:

$x_1 = 87.3$ cups

$x_2 = 130.9$ cups

$Z = \$421.09$



55. a) The only binding constraint is for Colombian; the constraints for Kenyan and Indonesian are nonbinding and there are already extra, or slack, pounds of these coffees available. Thus, only getting more Colombian would affect the solution.

One more pound of Colombian would increase sales from \$421.09 to \$463.20.

Increasing the brewing capacity to 40 gallons would have no effect since there is already unused brewing capacity with the optimal solution.

b) If the shop increased the demand ratio of Pomona to Coastal from 1.5 to 1 to 2 to 1 it would increase daily sales to \$460.00, so the shop should spend extra on advertising to achieve this result.

56. a) x_1 = 16 in. pizzas

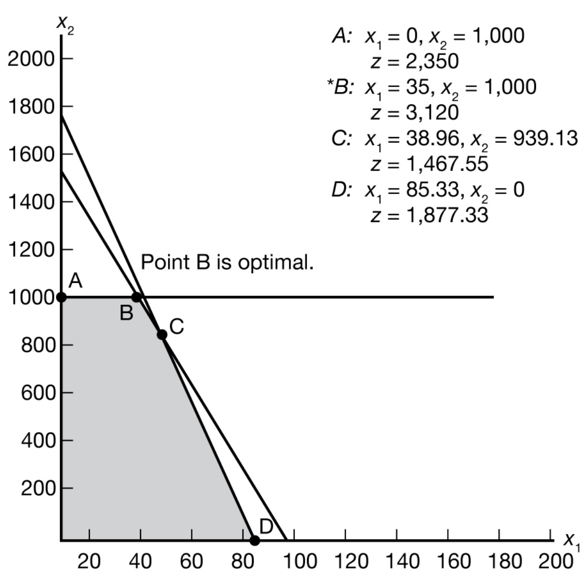
x_2 = hot dogs

Maximize $Z = \$22x_1 + 2.35x_2$

Subject to

$$\$10x_1 + 0.65x_2 \le \$1{,}000$$
$$324 \text{ in}^2 x_1 + 16 \text{ in}^2 x_2 \le 27{,}648 \text{ in}^2$$
$$x_2 \le 1{,}000$$
$$x_1, x_2 \ge 0$$

b)



57. a) $x_1 = 35$, $x_2 = 1{,}000$, $Z = \$3{,}120$

Profit would remain the same (\$3,120) so the increase in the oven cost would decrease the season's profit from \$10,120 to \$8,120.

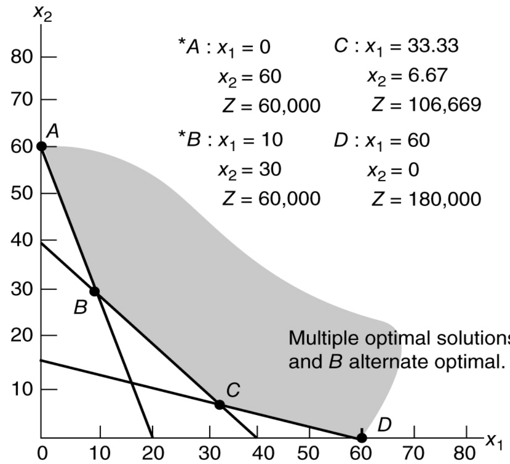
b) $x_1 = 35.95$, $x_2 = 1{,}000$, $Z = \$3{,}140$

Profit would increase slightly from \$10,120 to \$10, 245.46.

c) $x_1 = 55.7$, $x_2 = 600$, $Z = \$3{,}235.48$

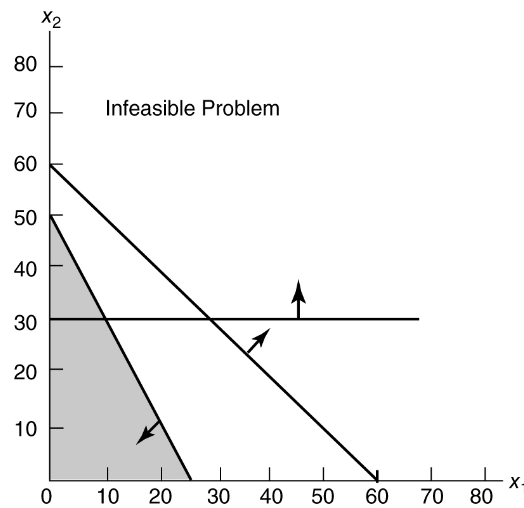
Profit per game would increase slightly.

58.

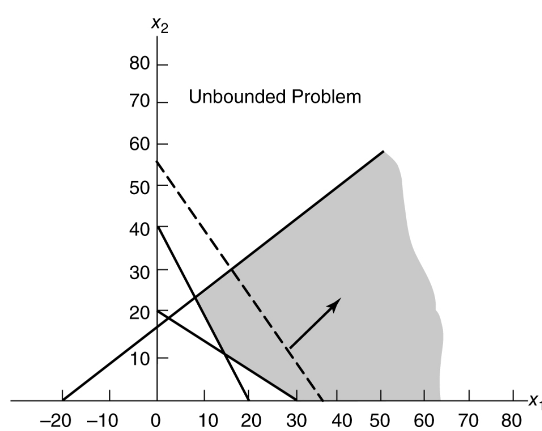


Multiple optimal solutions; *A* and *B* alternate optimal

59.



60.



CASE SOLUTION: METROPOLITAN POLICE PATROL

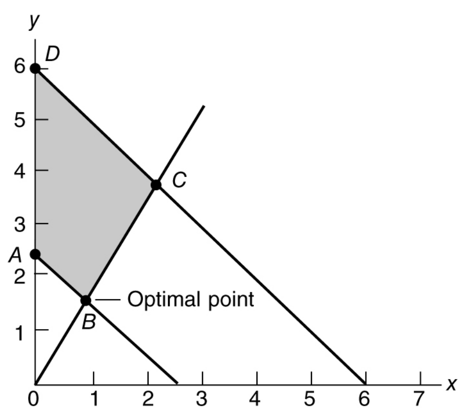
The linear programming model for this case problem is

Minimize $Z = x/60 + y/45$
subject to

$$2x + 2y \geq 5$$
$$2x + 2y \leq 12$$
$$y \geq 1.5x$$
$$x, y \geq 0$$

The objective function coefficients are determined by dividing the distance traveled, i.e., $x/3$, by the travel speed, i.e., 20 mph. Thus, the x coefficient is $x/3 \div 20$, or $x/60$. In the first two constraints, $2x + 2y$ represents the formula for the perimeter of a rectangle.

The graphical solution is displayed as follows.



The optimal solution is $x = 1$, $y = 1.5$, and $Z = 0.05$. This means that a patrol sector is 1.5 miles by 1 mile and the response time is 0.05 hr, or 3 min.

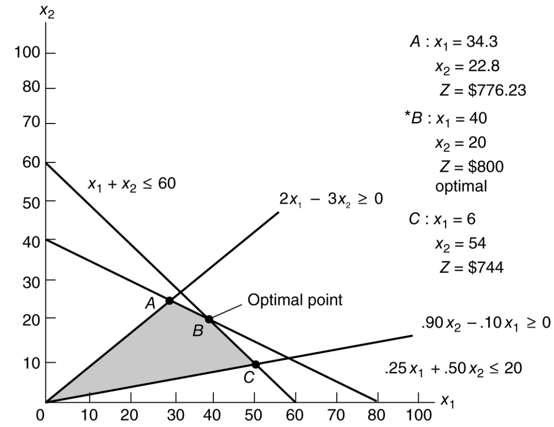
CASE SOLUTION: "THE POSSIBILITY" RESTAURANT

The linear programming model formulation is

Maximize $= Z = \$12x_1 + 16x_2$
subject to

$$x_1 + x_2 \leq 60$$
$$.25x_1 + .50x_2 \leq 20$$
$$x_1/x_2 \geq 3/2 \text{ or } 2x_1 - 3x_2 \geq 0$$
$$x_2/(x_1 + x_2) \geq .10 \text{ or } .90x_2 - .10x_1 \geq 0$$
$$x_1 x_2 \geq 0$$

The graphical solution is shown as follows.



Changing the objective function to $Z = \$16x_1 + 16x_2$ would result in multiple optimal solutions, the end points being B and C. The profit in each case would be \$960.

Changing the constraint from $.90x_2 - .10x_1 \geq 0$ to $.80x_2 - .20x_1 \geq 0$ has no effect on the solution.

CASE SOLUTION: ANNABELLE INVESTS IN THE MARKET

x_1 = no. of shares of index fund
x_2 = no. of shares of internet stock fund

Maximize $Z = (.17)(175)x_1 + (.28)(208)x_2$
$= 29.75x_1 + 58.24x_2$
subject to

$$175x_1 + 208x_2 = \$120{,}000$$
$$\frac{x_1}{x_2} \geq .33$$
$$\frac{x_2}{x_1} \leq 2$$
$$x_1, x_2 > 0$$

$x_1 = 203$
$x_2 = 406$
$Z = \$29{,}691.37$

Eliminating the constraint $\frac{x_2}{x_1} \geq .33$ will have no effect on the solution.

Eliminating the constraint $\frac{x_1}{x_2} \leq 2$ will change the solution to $x_1 = 149$, $x_2 = 451.55$, $Z = \$30{,}731.52$.

Increasing the amount available to invest (i.e., $120,000 to $120,001) will increase profit from Z = $29,691.37 to Z = $29,691.62 or approximately $0.25. Increasing by another dollar will increase profit by another $0.25, and increasing the amount available by one more dollar will again increase profit by $0.25. This indicates that for each extra dollar invested a return of $0.25 might be expected with this investment strategy.

Thus, the *marginal value* of an extra dollar to invest is $0.25, which is also referred to as the “shadow” or “dual” price as described in Chapter 3.

Chapter 2 – Linear Programming: Model Formulation and Graphical Solution

Linear Programming Overview

http://en.wikipedia.org/wiki/Linear_programming

http://www.netmba.com/operations/lp/

YouTube Videos for Linear Programming Graphical Solution

http://www.youtube.com/watch?v=M4K6HYLHREQ

http://www.youtube.com/watch?v=jcdiroeksHE&feature=related

http://www.youtube.com/watch?v=__wAxkYmhvY

http://www.youtube.com/watch?v=pzgnUCFNN7Q

http://www.youtube.com/watch?v=XEA1pOtyrfo

Tomato LP Problem

http://www.youtube.com/watch?v=f0PS8OwXqcw&feature=related

http://www.youtube.com/watch?v=LqWq2qNpGyI&feature=related

http://www.youtube.com/watch?v=M_0jQJ-Ey6c&feature=related

Model Formulation and Graphical Solution

http://www.zweigmedia.com/RealWorld/Summary4.html

http://www2.isye.gatech.edu/~spyros/LP/LP.html

http://homepages.stmartin.edu/fac_staff/dstout/MBA605/Balakrishnan 2e PPT/Chapter 02.ppt

Pioneers in Linear Programming

George Dantzig

http://en.wikipedia.org/wiki/George_Dantzig

http://www.stanford.edu/group/SOL/dantzig.html

http://www-groups.dcs.st-andrews.ac.uk/history/Biographies/Dantzig_George.html

http://www-history.mcs.st-and.ac.uk/Mathematicians/Dantzig_George.html

https://www.informs.org/Explore/History-of-O.R.-Excellence/Oral-Histories/George-Dantzig

MS Application Companies and Organizations

Indian Railways

http://en.wikipedia.org/wiki/Indian_Railways

GE Energy

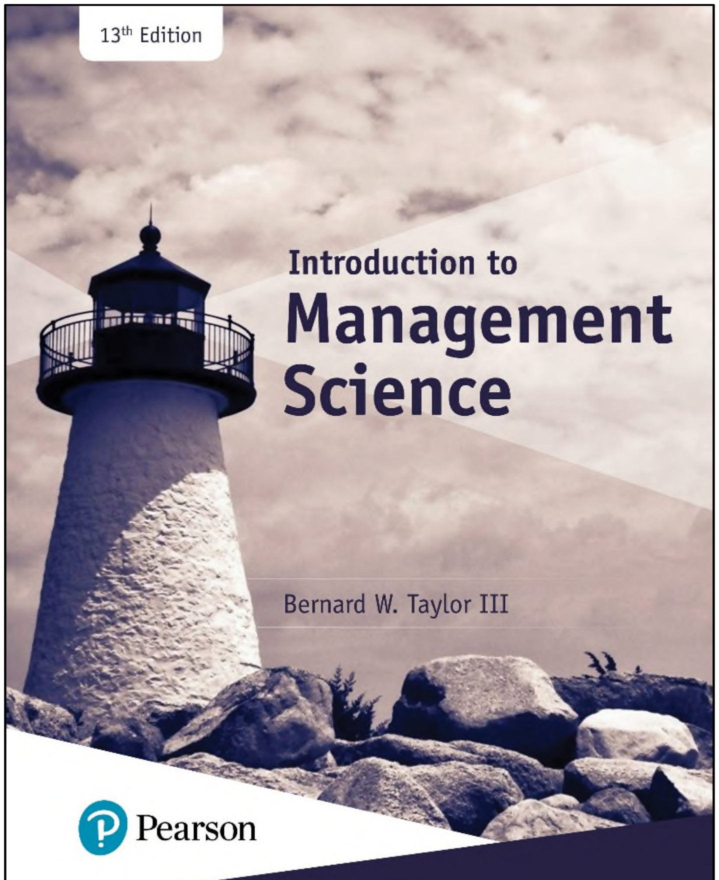
http://www.ge-energy.com/about/index.jsp

Soquimich (S.A.)

http://interfaces.journal.informs.org/cgi/content/abstract/33/4/41

Introduction to Management Science

Thirteenth Edition



Chapter 2

Linear Programming: Model Formulation and Graphical Solution

Pearson

Learning Objectives

2.1 Model Formulation

2.2 A Maximization Model Example

2.3 Graphical Solutions of Linear Programming Models

2.4 A Minimization Model Example

2.5 Irregular Types of Linear Programming Problems

2.6 Characteristics of Linear Programming Problems

Linear Programming: An Overview

- Objectives of business decisions frequently involve **maximizing profit** or **minimizing costs.**
- Linear programming uses **linear algebraic relationships** to represent a firm's decisions, given a business **objective**, and resource **constraints**.
- Steps in application:
  1. Identify problem as solvable by linear programming.
  2. Formulate a mathematical model of the unstructured problem.
  3. Solve the model.

Learning Objective 2.1

- Model Formulation

Model Components

- **Decision variables** - mathematical symbols representing levels of activity by the firm.
- **Objective function** - a linear mathematical relationship describing an objective of the firm, in terms of decision variables - this function is to be maximized or minimized.
- **Constraints** - requirements or restrictions placed on the firm by the operating environment, stated in linear relationships of the decision variables.
- **Parameters** - numerical coefficients and constants used in the objective function and constraints.

Summary of Model Formulation Steps

Step 1: Define the decision variables

How many bowls and mugs to produce?

Step 2: Define the objective function

Maximize profit

Step 3: Define the constraints

The resources (clay and labor) available

Learning Objective 2.2

- A Maximization Model Example

LP Model Formulation 1 (1 of 3)

Resource Requirements

product	Labor (Hr./Unit)	Clay (Lb./Unit)	Profit ($/Unit)
Bowl	1	4	40
Mug	2	3	50

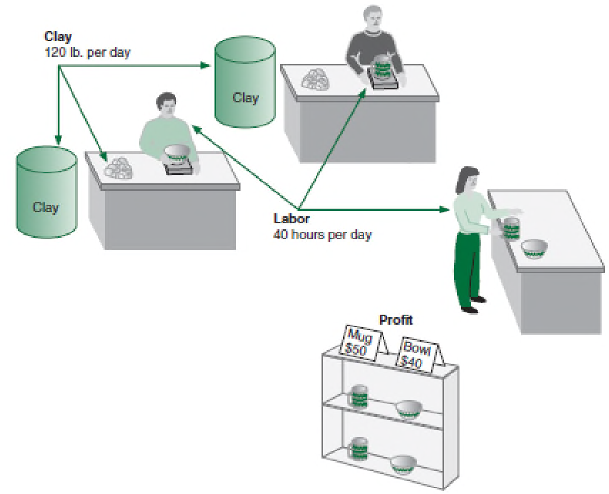


Figure 2.1 Beaver Creek Pottery Company

- Product mix problem - Beaver Creek Pottery Company
- How many bowls and mugs should be produced to maximize profits given labor and materials constraints?
- Product resource requirements and unit profit:

LP Model Formulation 1 (2 of 3)

Resource Availability:

40 hrs of labor per day

120 lbs of clay

Decision Variables:

x_1 = number of bowls to produce per day

x_2 = number of mugs to produce per day

Objective Function:

Maximize $Z = \$40x_1 + \$50x_2$

Where Z = profit per day

Resource Constraints:

$1x_1 + 2x_2 \leq 40$ hours of labor

$4x_1 + 3x_2 \leq 120$ pounds of clay

Non-Negativity Constraints:

$x_1 \geq 0; x_2 \geq 0$

LP Model Formulation 1 (3 of 3)

Complete Linear Programming Model:

Maximize $Z = \$40x_1 + \$50x_2$

subject to: $1x_1 + 2x_2 \leq 40$

$4x_2 + 3x_2 \leq 120$

$x_1, x_2 \geq 0$

Feasible Solutions

A **feasible solution** does not violate **any** of the constraints:

Example: $x_1 = 5$ bowls

$x_2 = 10$ mugs

$Z = \$40x_1 + \$50x_2 = \$700$

Labor constraint check: $1(5) + 2(10) = 25 \leq 40$ hours

Clay constraint check: $4(5) + 3(10) = 70 \leq 120$ pounds

Infeasible Solutions

An **infeasible solution** violates **at least one** of the constraints:

Example:

$$x_1 = 10 \text{ bowls}$$

$$x_2 = 20 \text{ mugs}$$

$$Z = \$40x_1 + \$50x_2 = \$1400$$

Labor constraint check: $1(10) + 2(20) = 50 > 40$ hours

Learning Objective 2.3

- Graphical Solutions of Linear Programming Models

Graphical Solution of LP Models

- Graphical solution is limited to linear programming models containing **only two decision variables** (can be used with three variables but only with great difficulty).
- Graphical methods provide **a picture of how** a solution for a linear programming problem is obtained.

Coordinate Axes

Maximize $Z = \$40x_1 + \$50x_2$

subject to: $1x_1 + 2x_2 \leq 40$

$4x_2 + 3x_2 \leq 120$

$x_1, x_2 \geq 0$

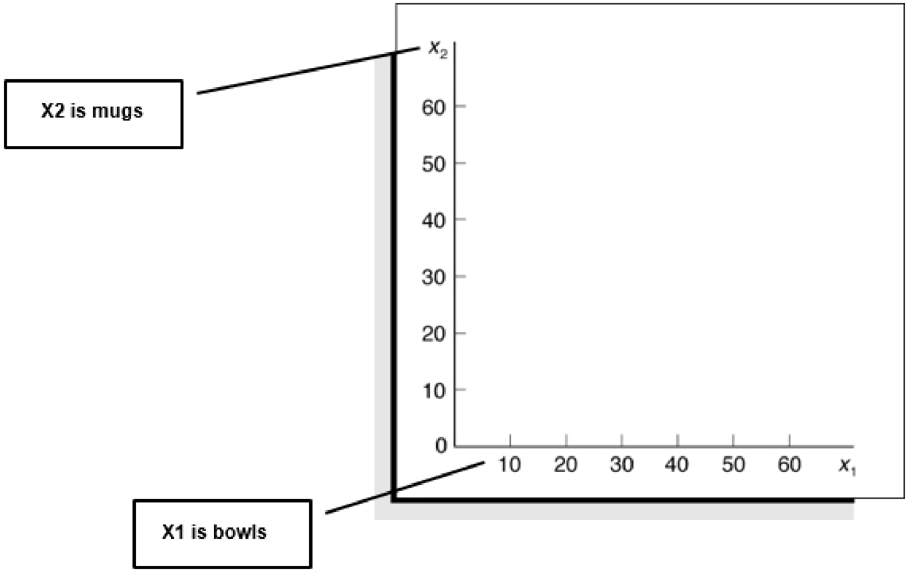
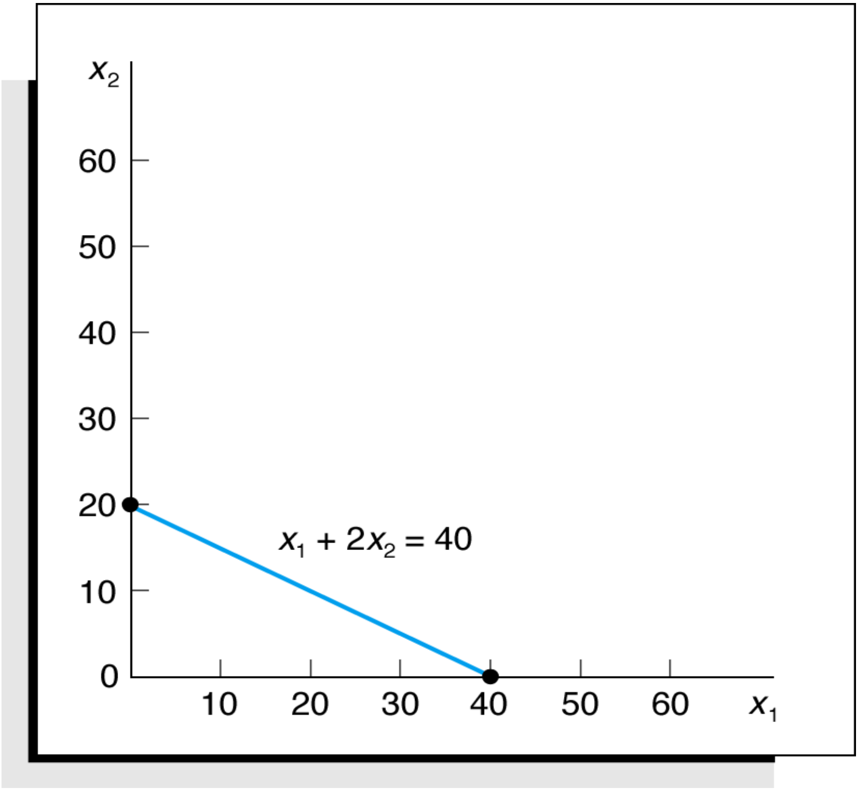


Figure 2.2 Coordinates for graphical analysis

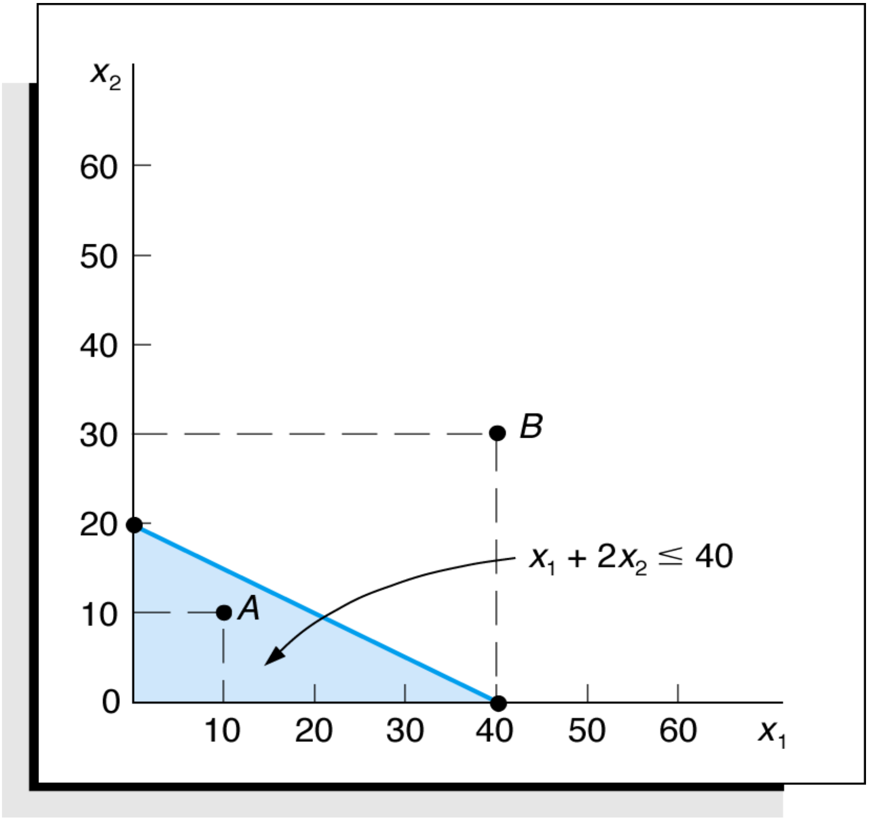
Labor Constraint

Figure 2.3 Graph of labor constraint



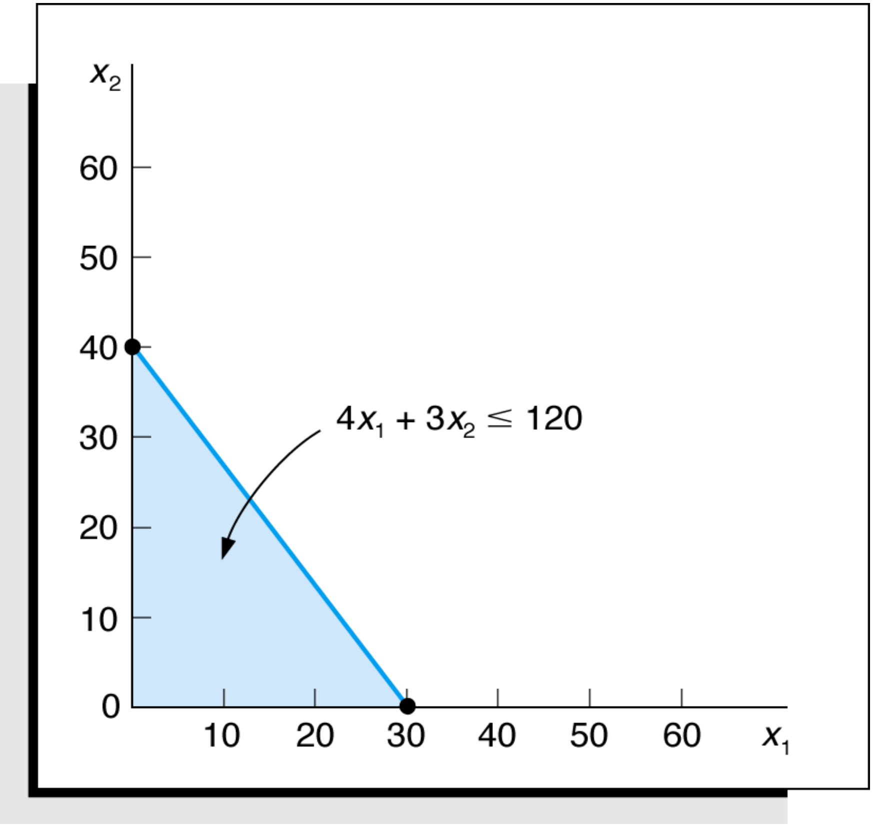
Labor Constraint Area

Figure 2.4 Labor constraint area



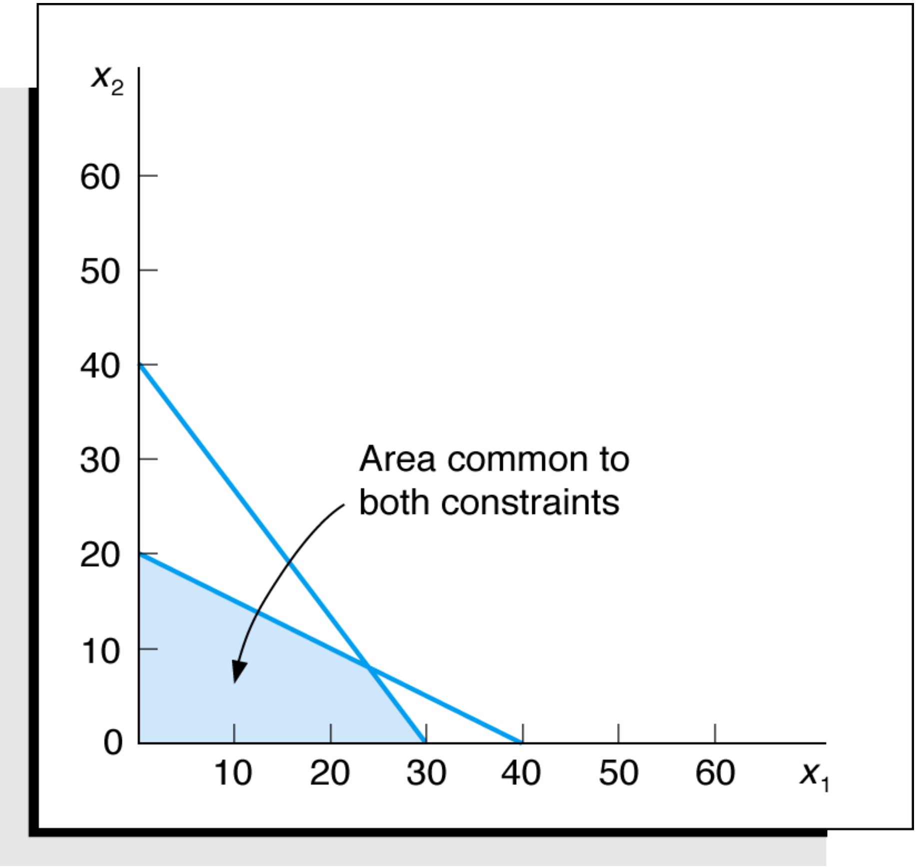
Clay Constraint Area

Figure 2.5 The constraint area for clay



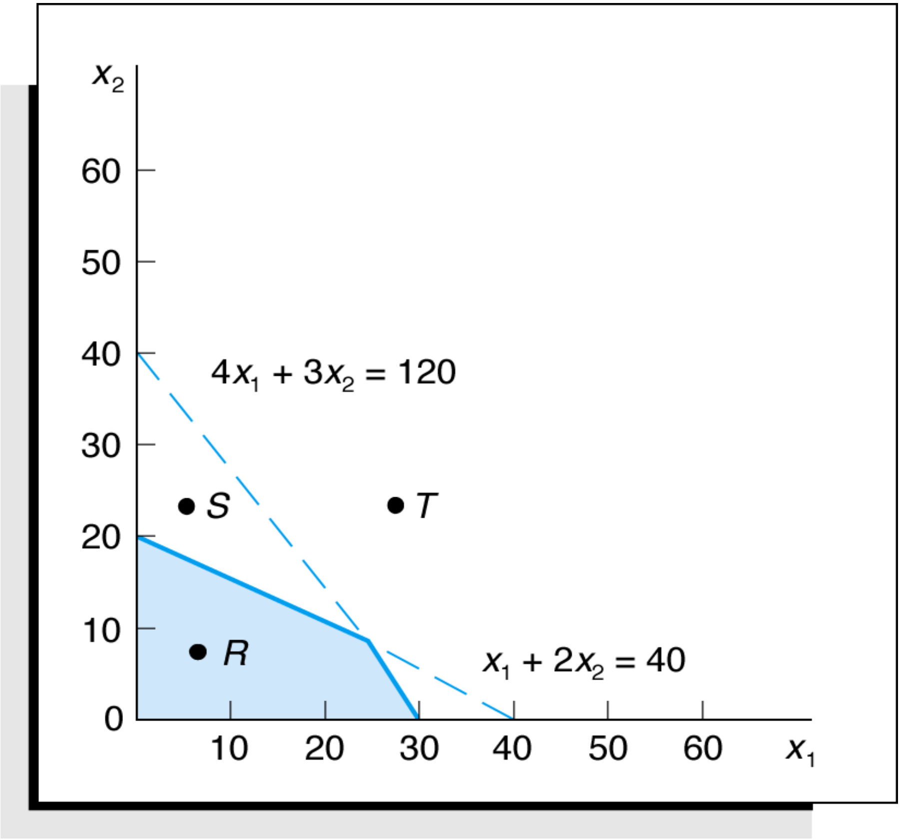
Both Constraints

Figure 2.6 Graph of both model constraints



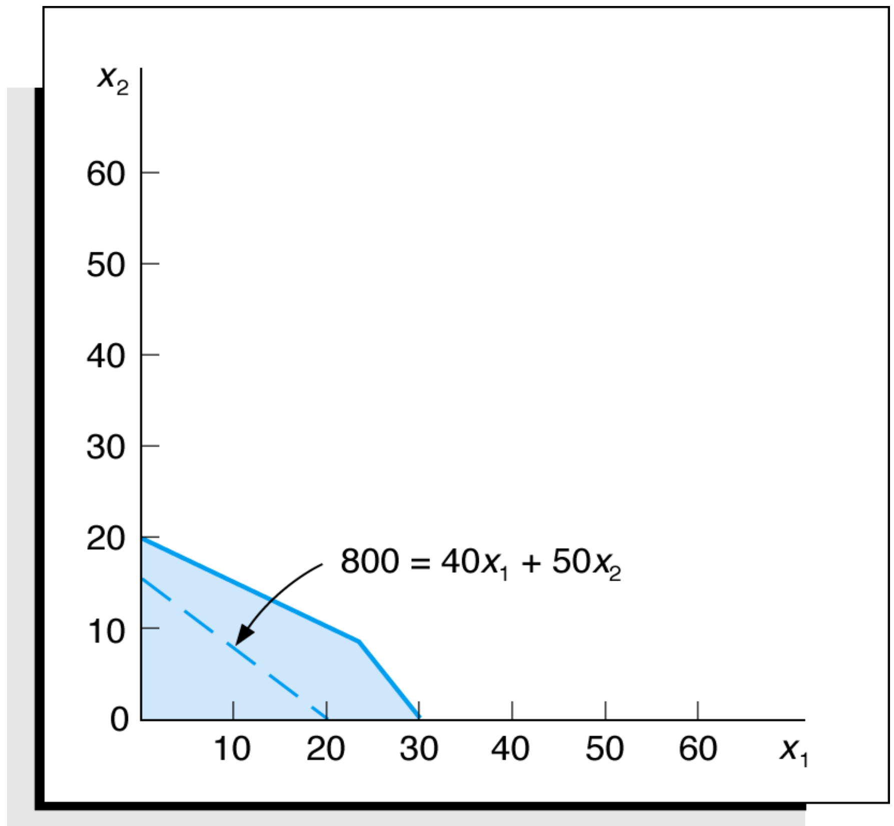
Feasible Solution Area

Figure 2.7 The feasible solution area constraints



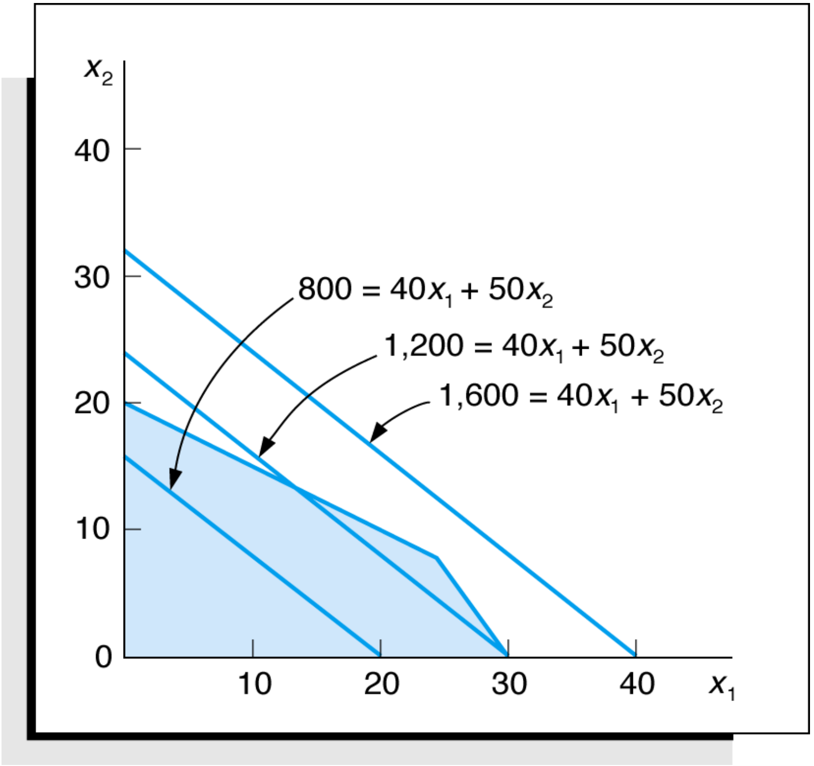
Objective Function Solution Z= \$800

Figure 2.8 Objective function line for Z = \$800



Alternative Objective Function Solution Lines

Figure 2.9 Alternative objective function lines for profits, *Z*, of $800, $1,200, and $1,600



Optimal Solution

The optimal solution point is the last point the objective function touches as it leaves the feasible solution area.

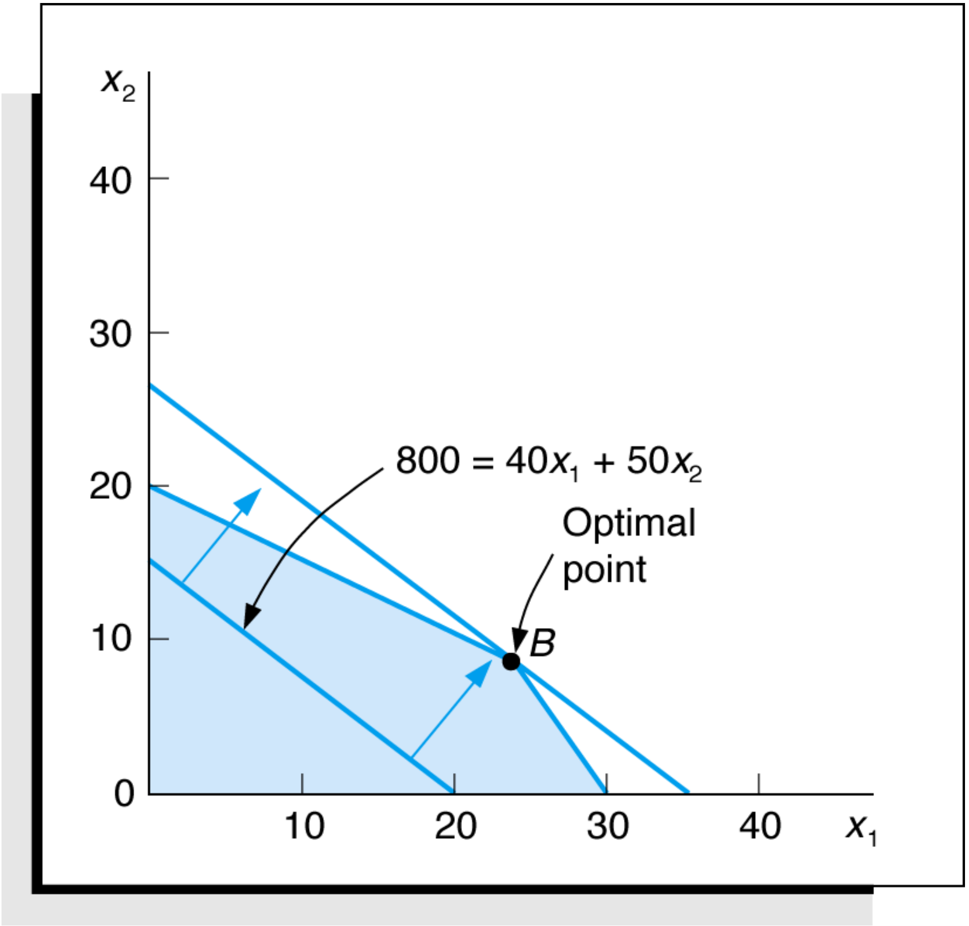
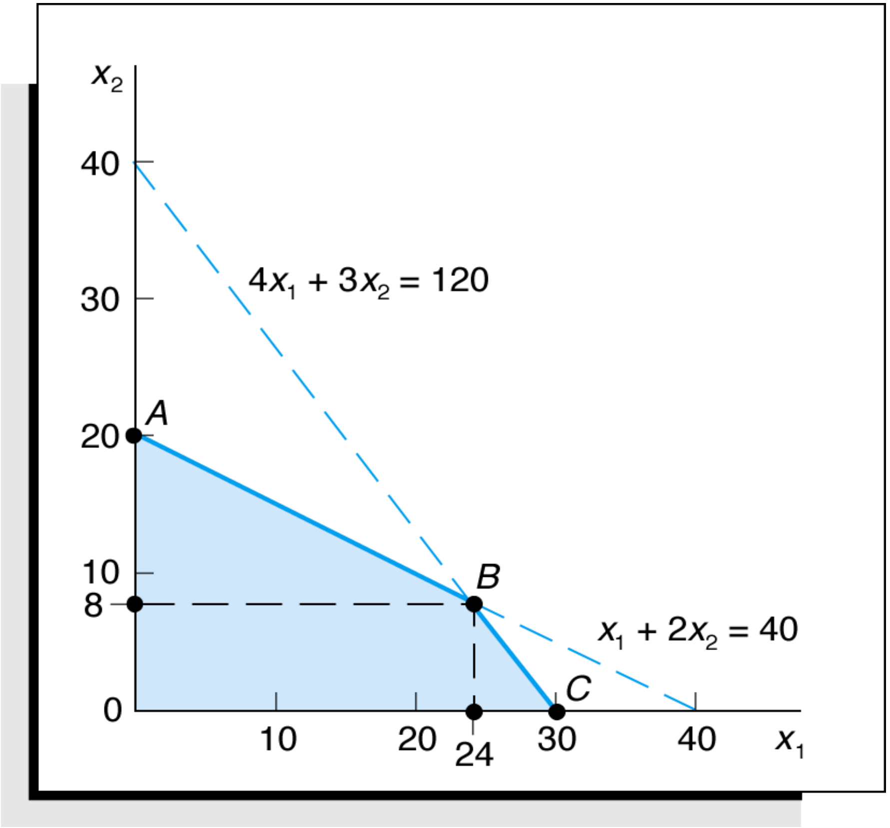


Figure 2.10 Identification of optimal solution point

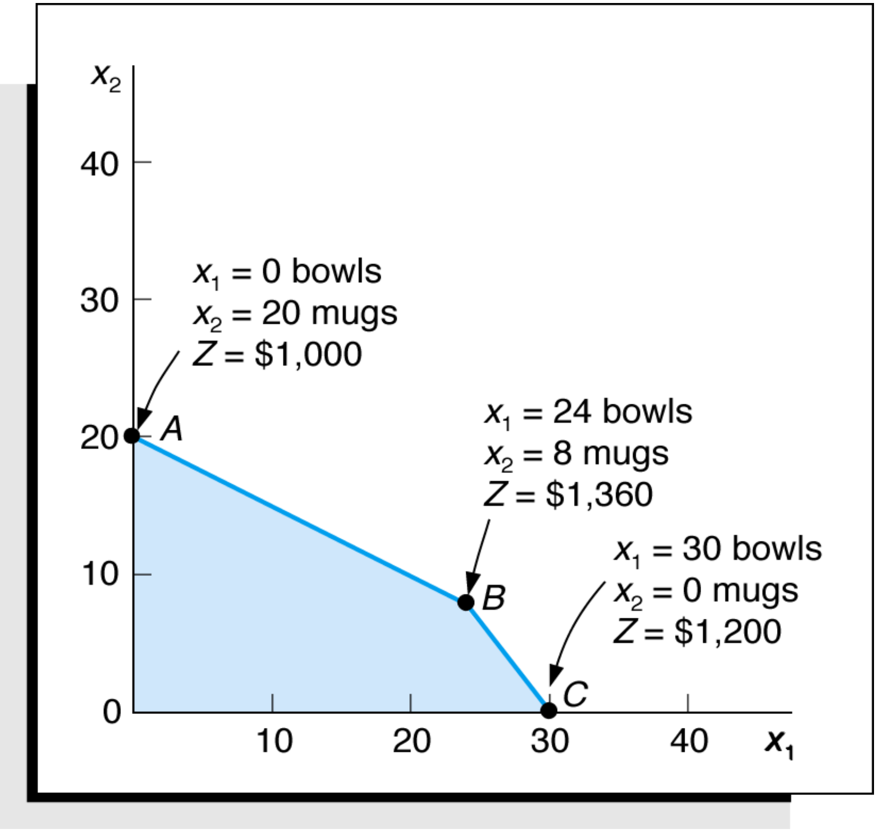
Optimal Solution Coordinates

Figure 2.11 Optimal solution coordinates



Extreme (Corner) Point Solutions

Figure 2.12 Solutions at all corner points



Optimal Solution for New Objective Function

Maximize $Z = \$70x_1 + \$20x_2$

subject to: $1x_1 + 2x_2 \le 40$

$4x_2 + 3x_2 \le 120$

$x_1, x_2 \ge 0$

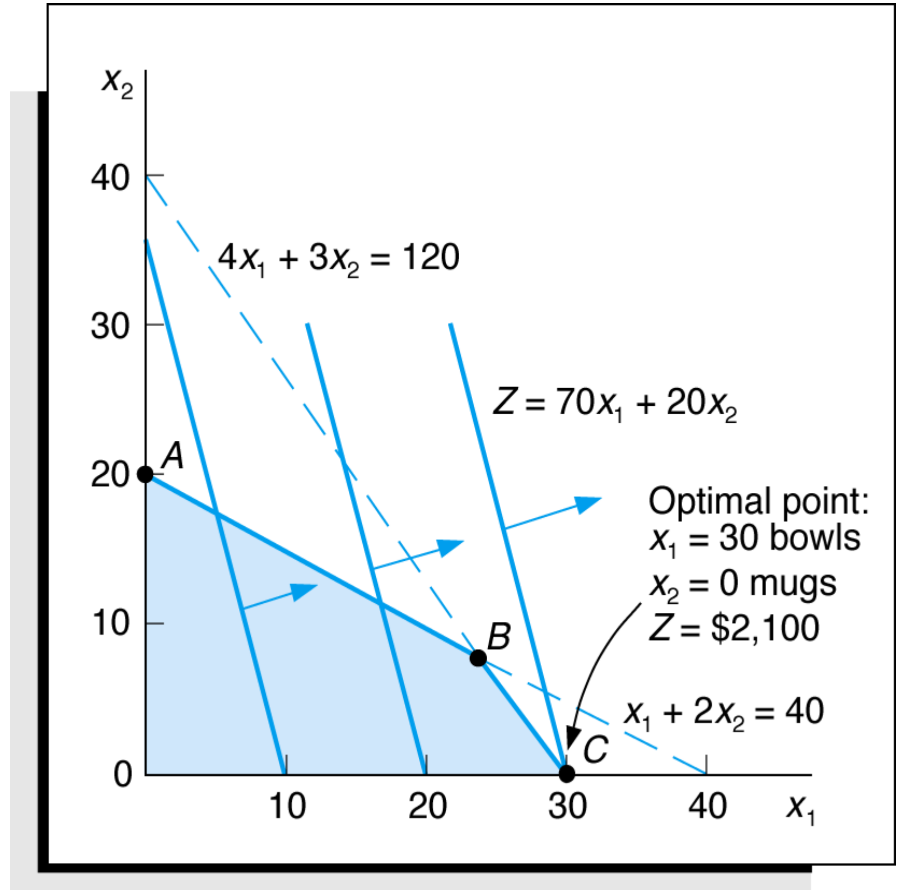


Figure 2.13 Optimal solution with $Z = 70x_1 + 20x_2$

Slack Variables

- Standard form requires that all constraints be in the form of equations (equalities).
- A slack variable is **added to a ≤ constraint** (weak inequality) to convert it to an equation (=).
- A slack variable typically represents an **unused resource**.
- A slack variable **contributes nothing** to the objective function value.

Linear Programming Model: Standard Form

$$\text{Max} \quad Z = 40x_1 + 50x_2 + s_1 + s_2$$

$$\text{subject to:} \quad 1x_1 + 2x_2 + s_1 = 40$$

$$4x_2 + 3x_2 + s_2 = 120$$

$$x_1,\ x_2,\ s_1,\ s_2 \geq 0$$

Where:

x_1 = number of bowls

x_2 = number of mugs

s_1, s_2 are slack variables

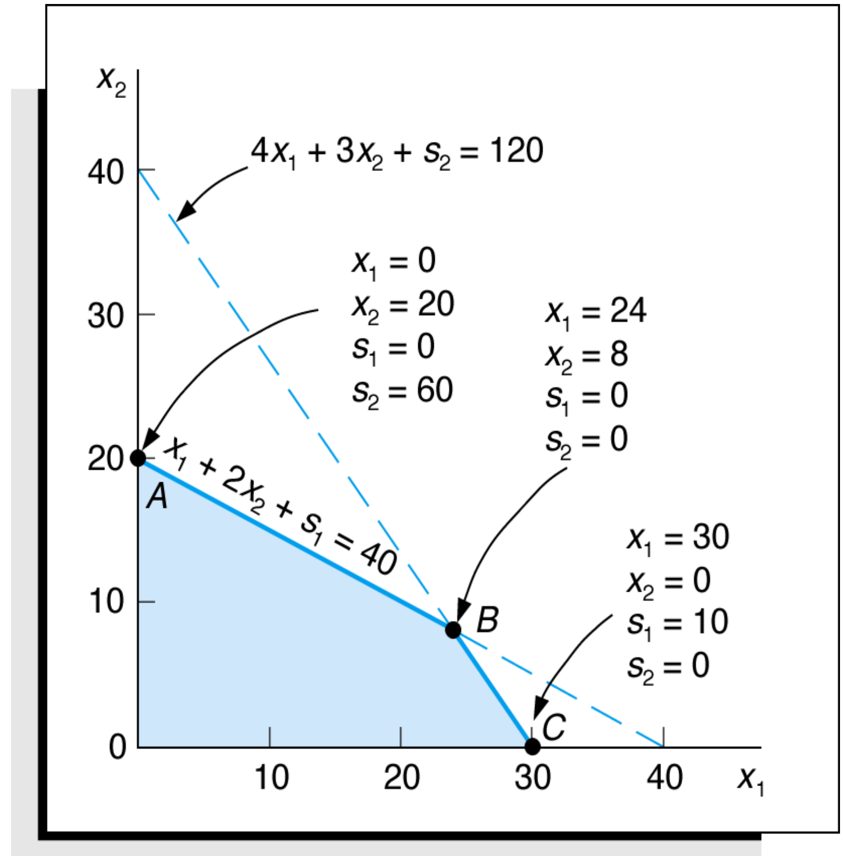


Figure 2.14 Solutions at points *A*, *B*, and *C* with slack

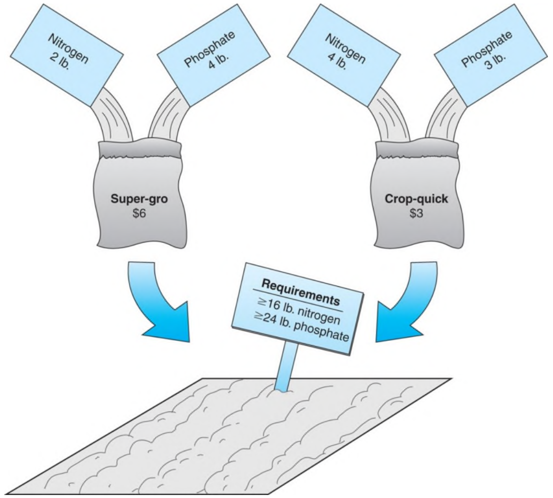
Learning Objective 2.4

- A Minimization Model Example

LP Model Formulation 2 (1 of 2)

- Two brands of fertilizer available – Super-gro, Crop-quick.
- Field requires at least 16 pounds of nitrogen and 24 pounds of phosphate.
- Super-gro costs $6 per bag, Crop-quick $3 per bag.
- Problem: How much of each brand to purchase to minimize total cost of fertilizer given following data ?

Figure 2.15 Fertilizing farmer's field



Chemical Contribution

Brand	Nitrogen (lb./bag)	Phosphate (lb./bag)
Super-gro	2	4
Crop-quick	4	3

LP Model Formulation 2

Decision Variables:

x_1 = bags of Super-gro

x_2 = bags of Crop-quick

The Objective Function:

Minimize $Z = \$6x_1 + 3x_2$

Where: $\$6x_1$ = cost of bags of Super-Gro

$\$3x_2$ = cost of bags of Crop-Quick

Model Constraints:

$$2x_1 + 4x_2 \geq 16 \text{ lb} \;\; (\text{nitrogen constraint})$$

$$4x_1 + 3x_2 \geq 24 \text{ lb} \;\; (\text{phosphate constraint})$$

$$x_1,\; x_2 \geq 0 \;\; (\text{non-negativity constraint})$$

Constraint Graph

Minimize $Z = \$6x_1 + \$3x_2$

subject to: $2x_1 + 4x_2 \geq 16$

$4x_2 + 3x_2 \geq 24$

$x_1, x_2 \geq 0$

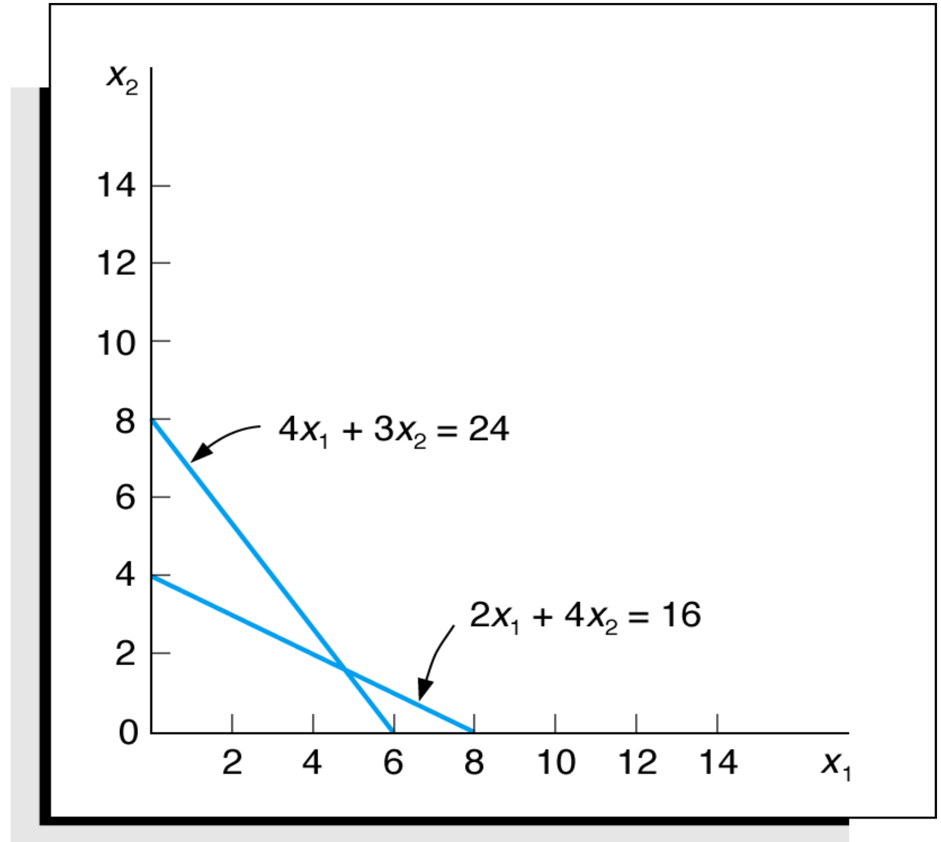
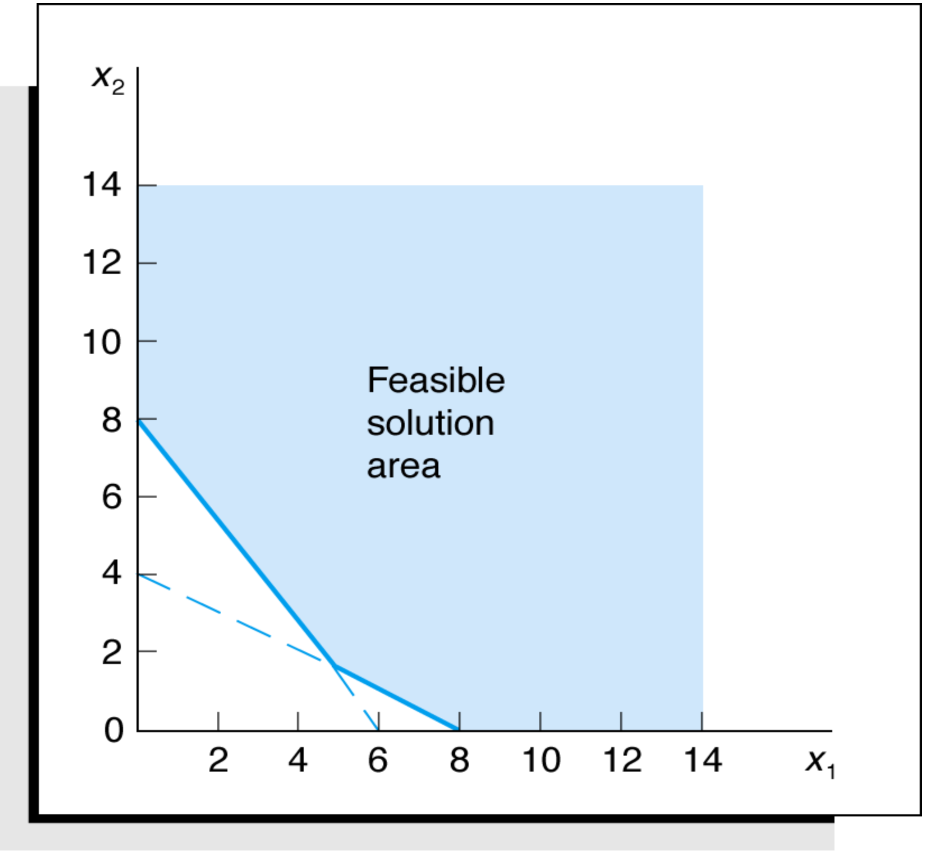


Figure 2.16 Constraint lines for fertilizer model

Feasible Region

Figure 2.17 Feasible solution area



Optimal Solution Point

The optimal solution of a minimization problem is at the extreme point closest to the origin.

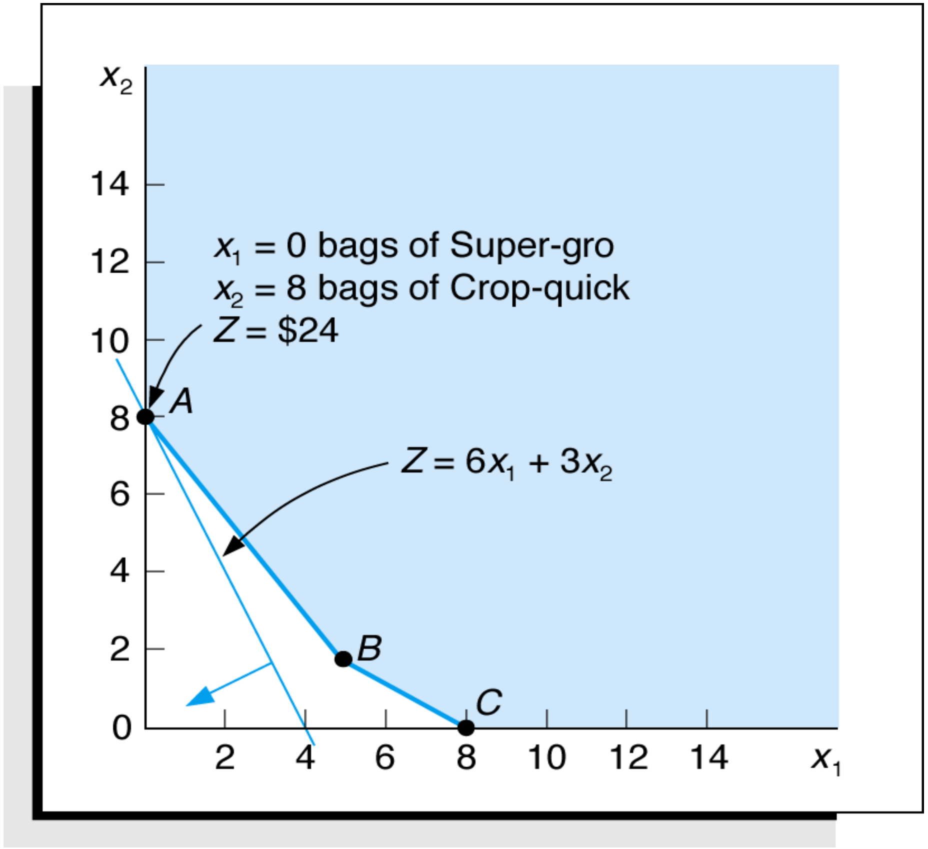


Figure 2.18 The optimal solution point

Surplus Variables

- A surplus variable is **subtracted from a ≥ constraint** to convert it to an equation (=).
- A surplus variable **represents an excess** above a constraint requirement level.
- A surplus variable **contributes nothing** to the calculated value of the objective function.
- Subtracting surplus variables in the farmer problem constraints:

$$2x_1 + 4x_2 - s_1 = 16 \ \text{(nitrogen)}$$

$$4x_1 + 3x_2 - s_2 = 24 \ \text{(phosphate)}$$

Graphical Solutions

Minimize $Z = \$6x_1 + \$3x_2 + 0s_1 + 0s_2$

subject to:

$$2x_1 + 4x_2 - s_1 = 16$$
$$4x_2 + 3x_2 - s_2 = 24$$
$$x_1,\ x_2,\ s_1,\ s_2 \geq 0$$

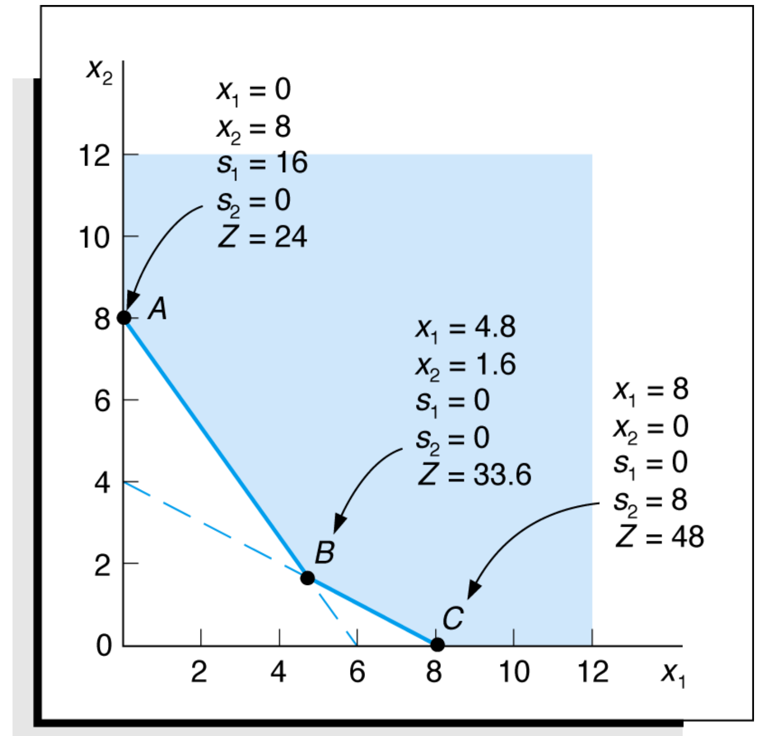


Figure 2.19 Graph of the fertilizer example

Learning Objective 2.5

- Irregular Types of Linear Programming Problems

Irregular Types of Linear Programming Problems

For some linear programming models, the general rules do not apply.

Special types of problems include those with:

- Multiple optimal solutions
- Infeasible solutions
- Unbounded solutions

Multiple Optimal Solutions Beaver Creek Pottery

The objective function is **parallel** to a constraint line.

Maximize $Z = \$40x_1 + 30x_2$

subject to: $1x_1 + 2x_2 \leq 40$

$4x_2 + 3x_2 \leq 120$

$x_1, \ x_2 \geq 0$

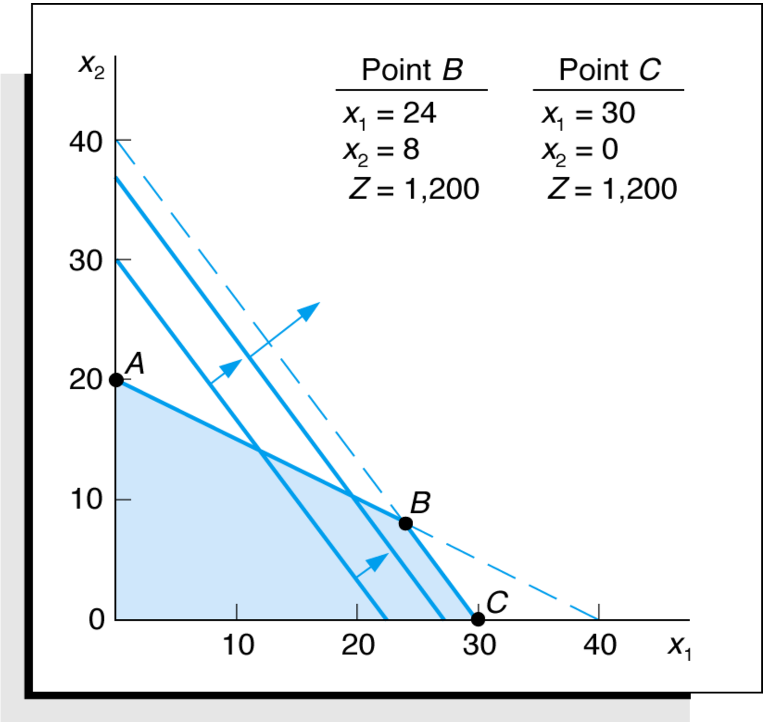


Figure 2.20 Graph of the Beaver Creek Pottery example with multiple optimal solutions

Where:

x_1 = number of bowls

x_2 = number of mugs

An Infeasible Problem

Every possible solution **violates** at least one constraint:

Maximize $Z = 5x_1 + 3x_2$

subject to: $4x_1 + 2x_2 \le 8$

$x_1 \ge 4$

$x_2 \ge 6$

$x_1, x_2 \ge 0$

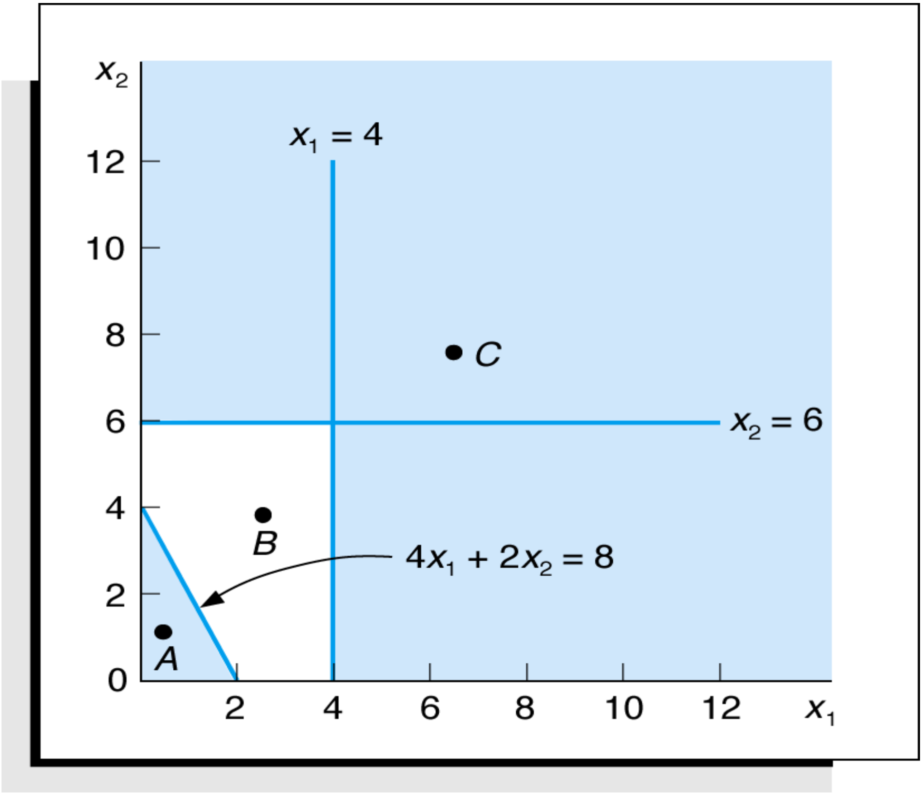


Figure 2.21 Graph of an infeasible problem

An Unbounded Problem

Value of the objective function increases indefinitely:

Maximize $Z = 4x_1 + 2x_2$

subject to: $x_1 \geq 4$

$x_2 \leq 2$

$x_1, x_2 \geq 0$

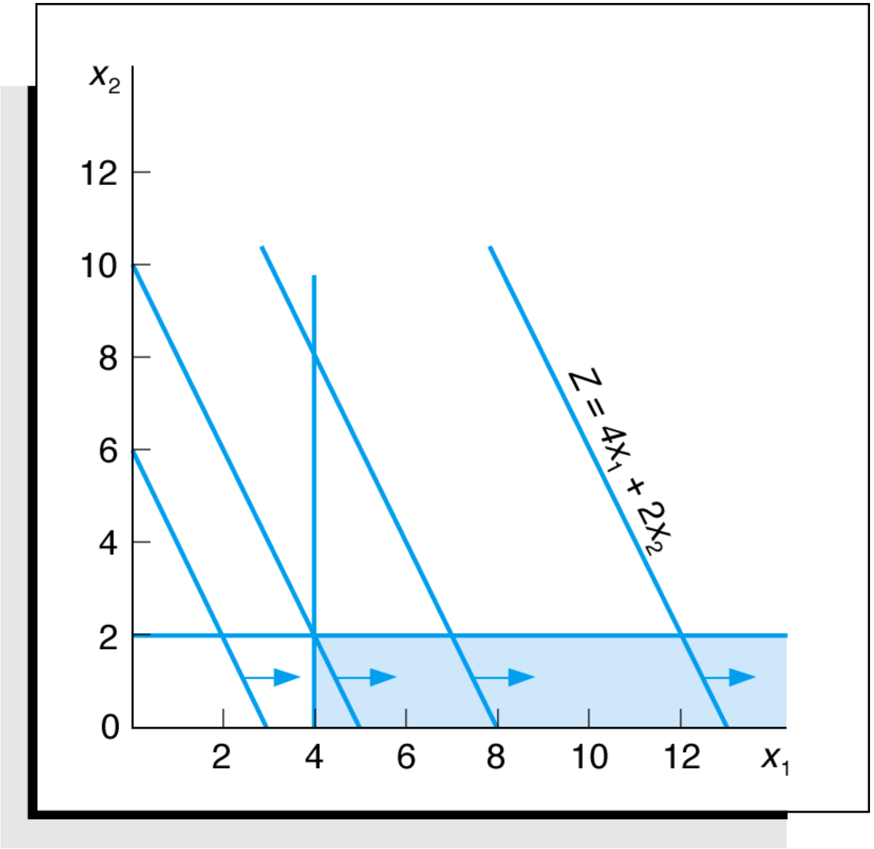


Figure 2.22 Graph of an unbounded problem

Learning Objective 2.6

- Characteristics of Linear Programming Problems

Characteristics of Linear Programming Problems

- A decision among alternative courses of action is required.
- The decision is represented in the model by **decision variables**.
- The problem encompasses a goal, expressed as an **objective function**, that the decision maker wants to achieve.
- Restrictions (represented by **constraints)** exist that limit the extent of achievement of the objective.
- The objective and constraints must be definable by **linear** mathematical functional relationships.

Properties of Linear Programming Models

- **Proportionality** - The rate of change (slope) of the objective function and constraint equations is constant.
- **Additivity** - Terms in the objective function and constraint equations must be additive.
- **Divisibility** - Decision variables can take on any fractional value and are therefore continuous as opposed to integer in nature.
- **Certainty** - Values of all the model parameters are assumed to be known with certainty (non-probabilistic).

Problem Statement: Example Problem No. 1

- Hot dog mixture in 1000-pound batches.
- Two ingredients, chicken ($3/lb) and beef ($5/lb).
- Recipe requirements:

  at least 500 pounds of “chicken”

  at least 200 pounds of “beef”

- Ratio of chicken to beef must be at least 2 to 1.
- Determine optimal mixture of ingredients that will minimize costs.

Solution: Example Problem No. 1 (1 of 2)

Step 1:

Identify decision variables.

x_1 = lb of chicken in mixture

x_2 = lb of beef in mixture

Step 2:

Formulate the objective function.

Minimize $Z = \$3x_1 + \$5x_2$

where Z = cost per 1,000-lb batch

$\$3x_1$ = cost of chicken

$\$5x_2$ = cost of beef

Solution: Example Problem No. 1 (2 of 2)

Step 3: Establish Model Constraints

$$x_1 + x_2 = 1{,}000 \text{ lb}$$
$$x_1 \geq 500 \text{ lb of chicken}$$
$$x_2 \geq 200 \text{ lb of beef}$$
$$\frac{x_1}{x_2} \geq \frac{2}{1} \text{ or } x_1 - 2x_2 \geq 0$$
$$x_1, x_2 \geq 0$$

The Model: Minimize $Z = \$3x_1 + 5x_2$

subject to: $x_1 + x_2 = 1{,}000 \text{ lb}$

$$x_1 \geq 50$$
$$x_2 \geq 200$$
$$x_1 - 2x_2 \geq 0$$
$$x_1, x_2 \geq 0$$

Example Problem No. 2 (1 of 3)

Solve the following model graphically:

$$\text{Maximize } Z = 4x_1 + 5x_2$$

$$\text{subject to: } x_1 + 2x_2 \le 10$$

$$6x_1 + 6x_2 \le 36$$

$$x_1 \le 4$$

$$x_1,\ x_2 \ge 0$$

Step 1: Plot the constraints as equations

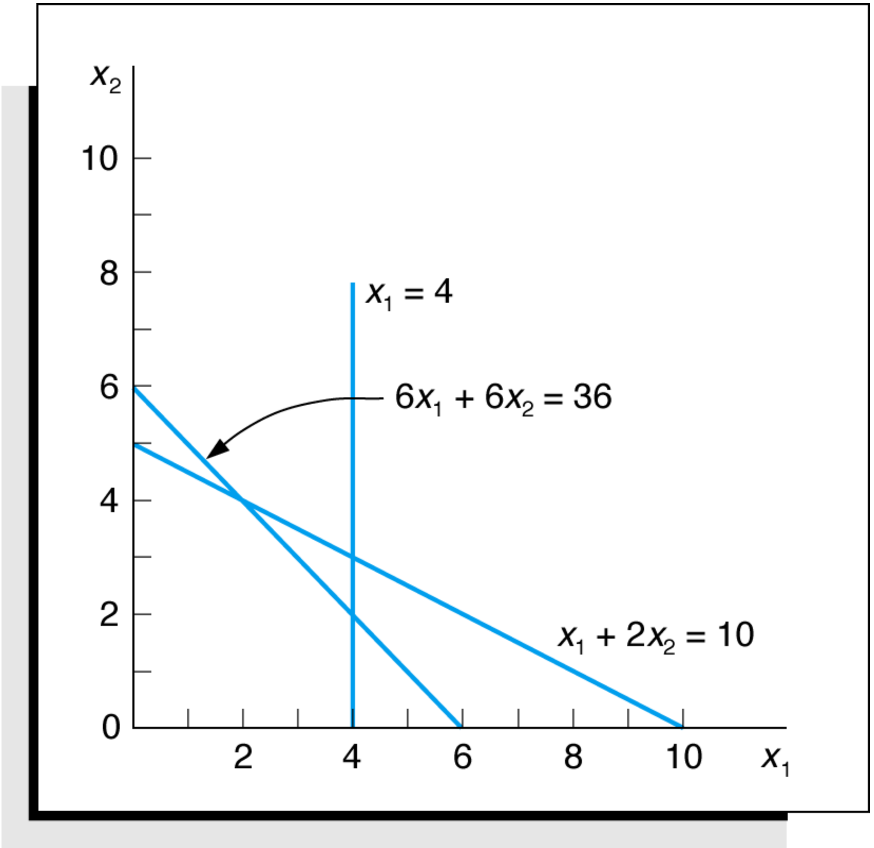


Figure 2.23 Constraint equations

Example Problem No. 2 (2 of 3)

Step 2: Determine the feasible solution space

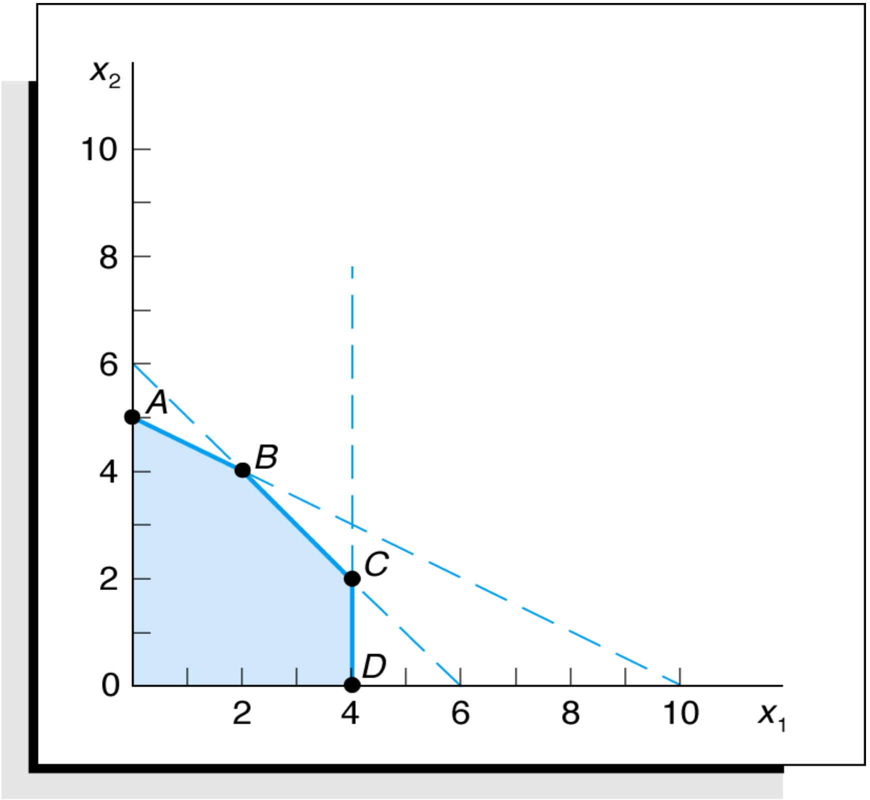


Figure 2.24 Feasible solution space and extreme points

Example Problem No. 2 (3 of 3)

Step 3 and 4: Determine the solution points and optimal solution

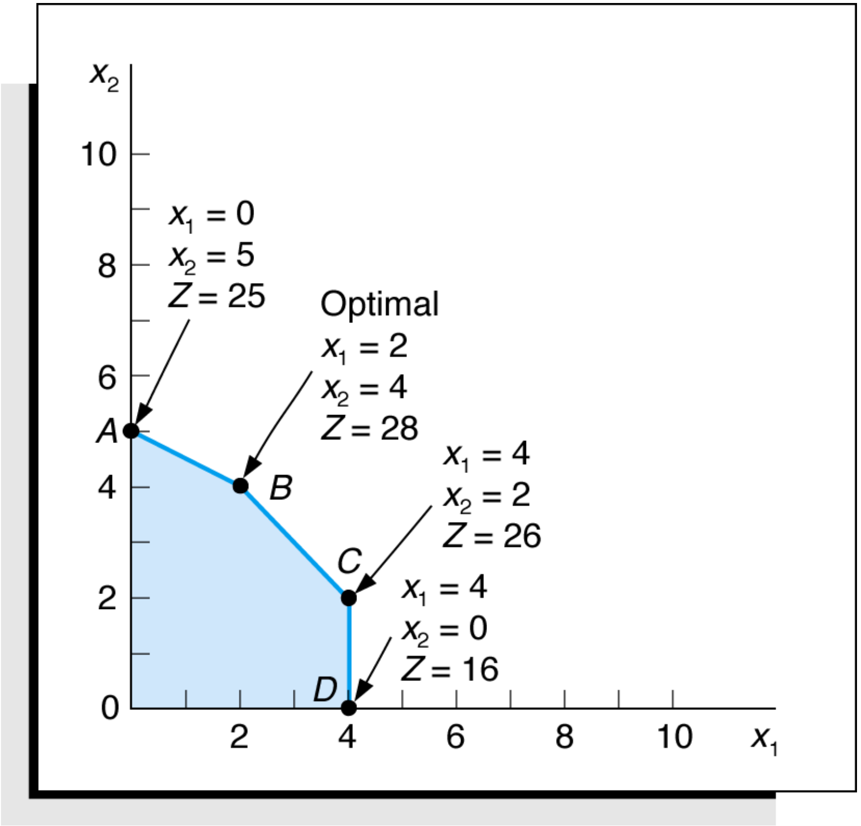


Figure 2.25 Optimal solution point

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