Solutions for Elementary and Intermediate Algebra Functions and Authentic Applications 3rd Edition by Lehmann

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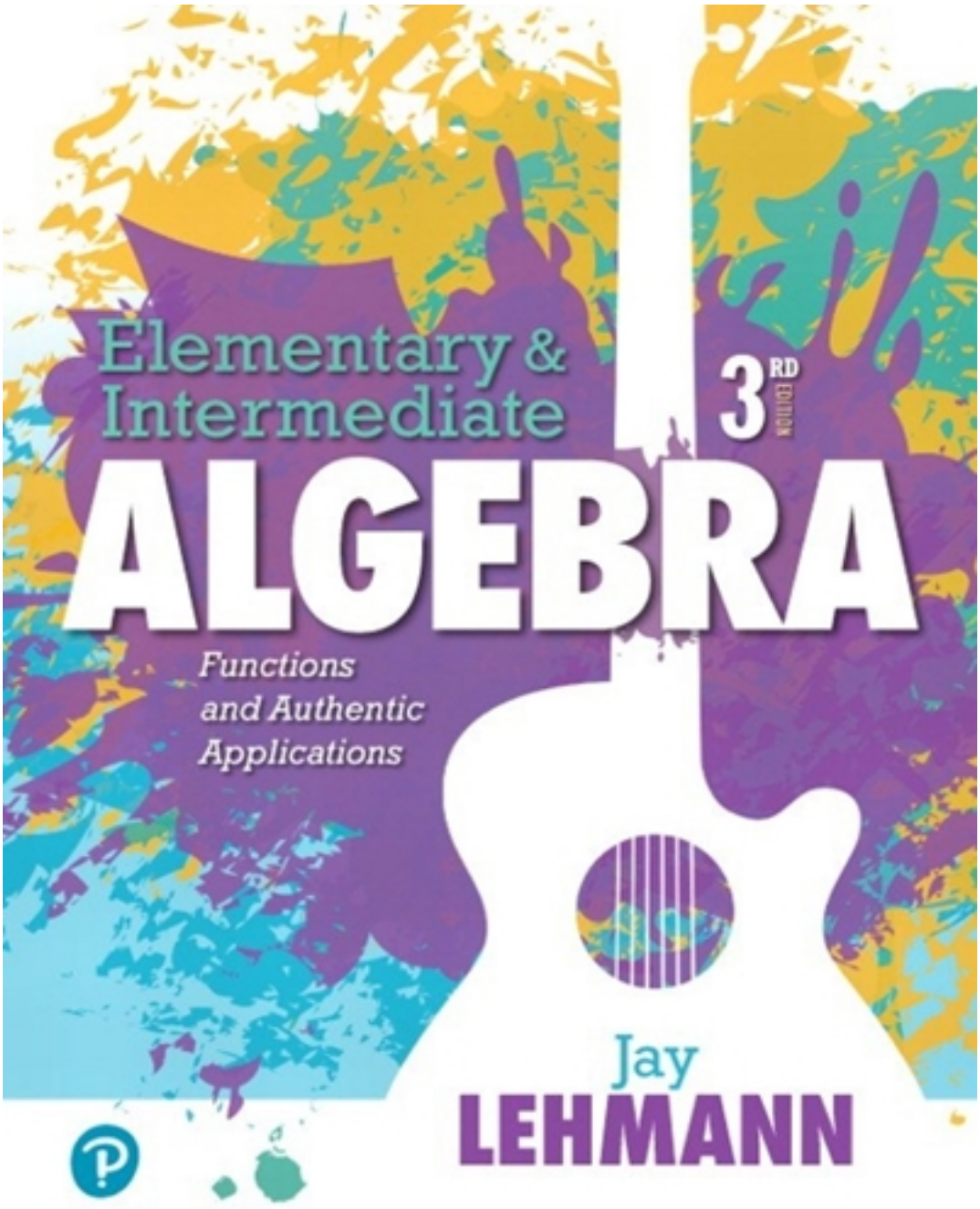

Solutions

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Chapter 2
Operations and Expressions

Homework 2.1

2. Substitute 6 for x in $5+x$: $5+(6)=11$

4. Substitute 6 for x in $x-4$: $(6)-4=2$

6. Substitute 6 for x in $x(9)$: $(6)(9)=54$

8. Substitute 6 for x in $30\div x$: $30\div 6=5$

10. Substitute 6 for x in $x-x$: $(6)-(6)=0$

12. Substitute 6 for x in $x\div x$: $(6)\div(6)=1$

14. Substitute 440 for T in $T\div 5$: $440\div 5=88$
The student's average test score was 88.

16. Substitute 60.9 for F in $F+17.3$:
$60.9+17.3=78.2$
So, the number of General Motors employees in 2016 was 78.2 thousand.

18. a.

Speed Limit (miles per hour)	Driving Speed (miles per hour)
35	$35+5$
40	$40+5$
45	$45+5$
50	$50+5$
s	$s+5$

The expression $s+5$ represents the driving speed if the speed limit is s miles per hour.

b. Substitute 65 for s in $s+5$:
$65+5=70$
So, if the speed limit is 65 miles per hour, the person will be driving 70 miles per hour.

20. a.

Number of Pairs	Total Cost (dollars)
1	$4.05\cdot 1$
2	$4.05\cdot 2$
3	$4.05\cdot 3$
4	$4.05\cdot 4$
n	$4.05\cdot n$

b. Substitute 9 for n in $4.05\cdot n$:
$4.05\cdot 9=36.45$
So, the total cost of 9 pairs Nike Men's Elite Basketball Crew socks is $36.45.

22. a.

Width (feet)	Area (square feet)
1	$20\cdot 1$
2	$20\cdot 2$
3	$20\cdot 3$
4	$20\cdot 4$
w	$20w$

The expression $20w$ represents the area of the garden (in square feet) if the width is w feet.

b. Substitute 10 for w in $20w$: $20(10)=200$
So, the area of the garden will be 200 square feet if the width is 10 feet.

24. $8-x$
Substitute 8 for x in $8-x$: $8-(8)=0$

26. $6+x$
Substitute 8 for x in $6+x$: $6+(8)=14$

28. $x+15$
Substitute 8 for x in $x+15$: $(8)+15=23$

30. $x-7$
Substitute 8 for x in $x-7$: $(8)-7=1$

32. $5x$
Substitute 8 for x in $5x$: $5(8)=40$

34. The quotient of 6 and the number

36. Two less than the number

38. The sum of 4 and the number

40. The product of the number and 5

42. The sum of the number and 3

44. The quotient of the number and 5

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46. Substitute 6 for x and 3 for y in the expression $y + x$: $(3) + (6) = 9$

48. Substitute 6 for x and 3 for y in the expression xy: $(6)(3) = 18$

50. Substitute 6 for x and 3 for y in the expression $x \div y$: $6 \div 3 = 2$

52. $x + y$

Substitute 9 for x and 3 for y in the expression $x + y$: $(9) + (3) = 12$

54. $x \div y$

Substitute 9 for x and 3 for y in the expression $x \div y$: $(9) \div (3) = 3$

56. Substitute 3758 for b and 994 for d in the expression $b - d$: $(3758) - (994) = 2764$

So, the new balance is \$2764 after a check is written for \$994.

58. Substitute 9000 for T and 20 for n in the expression $T \div n$: $9000 \div 20 = 450$

So, if 20 students go on a ski trip that costs \$9000, the cost per student is \$450.

60. Substitute 501 for E and 17 for t (2009 is 17 years after 1992) in the expression $E + t$: $501 + 17 = 518$

So, if the average verbal score was 501 points in 2009, the average math score was 518.

62. a.

n	$3n$
1	$3 \cdot 1 = 3$
2	$3 \cdot 2 = 6$
3	$3 \cdot 3 = 9$
4	$3 \cdot 4 = 12$

So, the cost of 1, 2, 3, or 4 loaves is \$3, \$6, \$9, or \$12, respectively.

b. Each loaf costs \$3. The price per loaf is a constant while the number of loaves is a variable. In the expression $3n$, the constant is 3 and the variable is n.

c. Answers may vary. Example:
For each additional loaf purchased, the total cost increases by \$3.

64. a.

t	$2t$
1	$2 \cdot 1 = 2$
2	$2 \cdot 2 = 4$
3	$2 \cdot 3 = 6$
4	$2 \cdot 4 = 8$

So, the elevator rises 2, 4, 6, or 8 yards after 1, 2, 3, or 4 seconds, respectively.

b. The elevator is rising at a rate of 2 yards per second. The speed is a constant while the number of seconds rising is a variable. In the expression, $2t$, the constant is 2 and the variable is t.

c. Answers may vary. Example:
For each additional second the elevator rises, the total distance risen increases by 2 yards.

66. Answers may vary.

68. Answers may vary.

70. Answers may vary.

Homework 2.2

2. The numerator of $\frac{2}{5}$ is 2.

4. $18 = 2 \cdot 9 = 2 \cdot (3 \cdot 3) = 2 \cdot 3 \cdot 3$

6. $24 = 4 \cdot 6 = (2 \cdot 2) \cdot (2 \cdot 3) = 2 \cdot 2 \cdot 2 \cdot 3$

8. $27 = 3 \cdot 9 = 3 \cdot (3 \cdot 3) = 3 \cdot 3 \cdot 3$

10. $105 = 5 \cdot 21 = 5 \cdot (3 \cdot 7) = 3 \cdot 5 \cdot 7$

12. $\frac{10}{14} = \frac{2 \cdot 5}{2 \cdot 7} = \frac{2}{2} \cdot \frac{5}{7} = \frac{5}{7}$

14. $\frac{7}{28} = \frac{7 \cdot 1}{7 \cdot 4} = \frac{7}{7} \cdot \frac{1}{4} = \frac{1}{4}$

16. $\frac{27}{54} = \frac{3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 2} = \frac{3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3} \cdot \frac{1}{2} = \frac{1}{2}$

18. $\frac{49}{63} = \frac{7 \cdot 7}{7 \cdot 3 \cdot 3} = \frac{7}{7} \cdot \frac{7}{3 \cdot 3} = \frac{7}{3 \cdot 3} = \frac{7}{9}$

20. $\frac{9}{81} = \frac{3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{3 \cdot 3}{3 \cdot 3} \cdot \frac{1}{3 \cdot 3} = \frac{1}{3 \cdot 3} = \frac{1}{9}$

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22. $\frac{15}{18}=\frac{3\cdot 5}{3\cdot 3\cdot 2}=\frac{3}{3}\cdot\frac{5}{3\cdot 2}=\frac{5}{3\cdot 2}=\frac{5}{6}$

24. $\frac{6}{7}\cdot\frac{4}{9}=\frac{6\cdot 4}{7\cdot 9}=\frac{2\cdot 3\cdot 2\cdot 2}{7\cdot 3\cdot 3}=\frac{2\cdot 2\cdot 2}{7\cdot 3}=\frac{8}{21}$

26. $\frac{2}{3}\cdot\frac{5}{6}=\frac{2\cdot 5}{3\cdot 6}=\frac{2\cdot 5}{3\cdot 2\cdot 3}=\frac{5}{3\cdot 3}=\frac{5}{9}$

28. $\frac{5}{12}\cdot 2=\frac{5}{12}\cdot\frac{2}{1}=\frac{5\cdot 2}{2\cdot 2\cdot 3}=\frac{5}{2\cdot 3}=\frac{5}{6}$

30. $\frac{7}{12}\div\frac{2}{3}=\frac{7}{12}\cdot\frac{3}{2}=\frac{7\cdot 3}{12\cdot 2}=\frac{7\cdot 3}{2\cdot 2\cdot 3\cdot 2}=\frac{7}{2\cdot 2\cdot 2}=\frac{7}{8}$

32. $\frac{4}{7}\div\frac{8}{3}=\frac{4}{7}\cdot\frac{3}{8}=\frac{4\cdot 3}{7\cdot 8}=\frac{2\cdot 2\cdot 3}{7\cdot 2\cdot 2\cdot 2}=\frac{3}{7\cdot 2}=\frac{3}{14}$

34. $\frac{4}{9}\div 2=\frac{4}{9}\cdot\frac{1}{2}=\frac{4\cdot 1}{9\cdot 2}=\frac{2\cdot 2}{3\cdot 3\cdot 2}=\frac{2}{3\cdot 3}=\frac{2}{9}$

36. $\frac{5}{9}+\frac{2}{9}=\frac{5+2}{9}=\frac{7}{9}$

38. $\frac{2}{15}+\frac{8}{15}=\frac{2+8}{15}=\frac{10}{15}=\frac{2\cdot 5}{3\cdot 5}=\frac{2}{3}$

40. $\frac{5}{7}-\frac{2}{7}=\frac{5-2}{7}=\frac{3}{7}$

42. $\frac{13}{18}-\frac{9}{18}=\frac{13-9}{18}=\frac{4}{18}=\frac{2\cdot 2}{2\cdot 3\cdot 3}=\frac{2}{3\cdot 3}=\frac{2}{9}$

44. The LCD is 9:

$\frac{1}{3}+\frac{5}{9}=\frac{1}{3}\cdot\frac{3}{3}+\frac{5}{9}=\frac{3}{9}+\frac{5}{9}=\frac{8}{9}$

46. The LCD is 24:

$\frac{3}{8}+\frac{1}{6}=\frac{3}{8}\cdot\frac{3}{3}+\frac{1}{6}\cdot\frac{4}{4}=\frac{9}{24}+\frac{4}{24}=\frac{13}{24}$

48. The LCD is 7:

$2+\frac{3}{7}=\frac{2}{1}\cdot\frac{7}{7}+\frac{3}{7}=\frac{14}{7}+\frac{3}{7}=\frac{17}{7}$

50. The LCD is 4:

$\frac{3}{4}-\frac{1}{2}=\frac{3}{4}-\frac{1}{2}\cdot\frac{2}{2}=\frac{3}{4}-\frac{2}{4}=\frac{1}{4}$

52. The LCD is 42:

$\frac{5}{6}-\frac{4}{7}=\frac{5}{6}\cdot\frac{7}{7}-\frac{4}{7}\cdot\frac{6}{6}=\frac{35}{42}-\frac{24}{42}=\frac{11}{42}$

54. The LCD is 7:

$1-\frac{9}{7}=\frac{1}{1}\cdot\frac{7}{7}-\frac{9}{7}=\frac{7}{7}-\frac{9}{7}=\frac{-2}{7}=-\frac{2}{7}$

56. $\frac{62}{62}=1$

58. $\frac{215}{1}=215$

60. $\frac{713}{0}$ is undefined since division by 0 is not defined.

62. $\frac{0}{798}=0$

64. $\frac{173}{190}\cdot\frac{190}{173}=\frac{173\cdot 190}{173\cdot 190}=1$

66. $\frac{345}{917}-\frac{345}{917}=\frac{345-345}{917}=\frac{0}{917}=0$

68. Substitute 3 for x and 12 for z in the expression $\frac{z}{x}$:

$\frac{12}{3}=\frac{3\cdot 2\cdot 2}{3\cdot 1}=\frac{2\cdot 2}{1}=\frac{4}{1}=4$

70. Substitute 4 for w, 3 for x, 5 for y, and 12 for z in the expression $\frac{y}{z}\cdot\frac{w}{x}$:

$\frac{5}{12}\cdot\frac{4}{3}=\frac{5\cdot 2\cdot 2}{2\cdot 2\cdot 3\cdot 3}=\frac{5}{3\cdot 3}=\frac{5}{9}$

72. Substitute 3 for x, 5 for y, and 12 for z in the expression $\frac{y}{x}+\frac{y}{z}$: $\frac{5}{3}+\frac{5}{12}$

The LCD is 12:

$\frac{5}{3}+\frac{5}{12}=\frac{5}{3}\cdot\frac{4}{4}+\frac{5}{12}=\frac{20}{12}+\frac{5}{12}=\frac{25}{12}$

74.

```
(67/71)*(381/399
)
        .9010907551
```

$\frac{67}{71}\cdot\frac{381}{399}\approx 0.901$

76.

```
(149/215)/(31/52
)
        1.162490623
```

$\frac{149}{215}\div\frac{31}{52}\approx 1.162$

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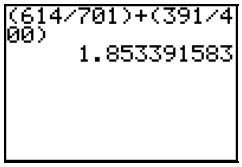
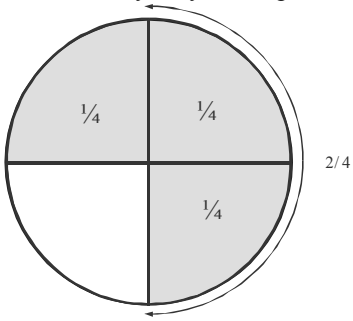
78.



$$\frac{614}{701}+\frac{391}{400}\approx 1.853$$

80. Answers may vary. Example:



82. The perimeter of a rectangle is given by the expression $2L+2W$. Substitute $\frac{2}{3}$ for W and $\frac{3}{4}$ for L in the expression.

$$2\cdot\frac{3}{4}+2\cdot\frac{2}{3}=\frac{2}{1}\cdot\frac{3}{4}+\frac{2}{1}\cdot\frac{2}{3}$$
$$=\frac{2\cdot 3}{2\cdot 2}+\frac{2\cdot 2}{3}$$
$$=\frac{3}{2}+\frac{4}{3}$$

The LCD is 6:

$$\frac{3}{2}\cdot\frac{3}{3}+\frac{4}{3}\cdot\frac{2}{2}=\frac{9}{6}+\frac{8}{6}=\frac{17}{6}$$

The perimeter of the picture is $\frac{17}{6}$ feet.

84. Let m be the fraction of income for mortgage and f be the fraction of income for food. The fraction remaining is given by the expression $1-m-f$. Substitute $\frac{1}{3}$ for m and $\frac{1}{6}$ for f in the expression.

$$1-m-f=1-\frac{1}{3}-\frac{1}{6}$$
$$=\frac{1}{1}\cdot\frac{6}{6}-\frac{1}{3}\cdot\frac{2}{2}-\frac{1}{6}$$
$$=\frac{6}{6}-\frac{2}{6}-\frac{1}{6}$$
$$=\frac{6-2-1}{6}$$
$$=\frac{3}{6}$$
$$=\frac{1}{2}$$

So, $\frac{1}{2}$ of the income remains.

86. The quotient of 5 and the number.

88.

Total Time (hours)	Cost per Hour (dollars per hour)
2	$45\div 2$
3	$45\div 3$
4	$45\div 4$
5	$45\div 5$
t	$45\div t$

So, if a session lasts for t hours, the total cost per hour will be $45\div t$ dollars.

90. a. i. $\frac{2}{3}\cdot\frac{3}{2}=\frac{2\cdot 3}{3\cdot 2}=\frac{6}{6}=1$

ii. $\frac{4}{7}\cdot\frac{7}{4}=\frac{4\cdot 7}{7\cdot 4}=\frac{28}{28}=1$

iii. $\frac{1}{6}\cdot\frac{6}{1}=\frac{1\cdot 6}{6\cdot 1}=\frac{6}{6}=1$

b. Answers may vary. Example:
The product of a fraction and its reciprocal equals 1.

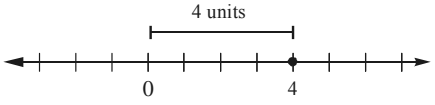
92. Answers may vary. Example:
For addition of fractions, one cannot simply add the numerators and denominators. First get a common denominator, then add numerators, keeping the common denominator.

$$\frac{2}{3}+\frac{5}{6}=\frac{2}{3}\cdot\frac{2}{2}+\frac{5}{6}=\frac{4}{6}+\frac{5}{6}=\frac{4+5}{6}=\frac{9}{6}=\frac{3}{2}$$

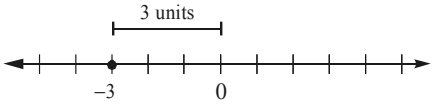
94. Answers may vary. Example:
The student should have multiplied the numerator by 3 instead of the denominator. Rewrite 3 as $\frac{3}{1}$ and then multiply across.

$$3\cdot\frac{7}{2}=\frac{3}{1}\cdot\frac{7}{2}=\frac{3\cdot 7}{1\cdot 2}=\frac{21}{2}$$

96. a. i. The distance between 0 and 4 is 4 units.



ii. The distance between 0 and -3 is 3 units.



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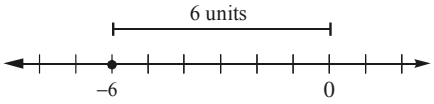
iii. The distance between 0 and -6 is 6 units.



b. The distance between 0 and a number on a number line is equal to the absolute value of the number.

98. a. i. $5+(-2)=3$

ii. $7+(-1)=6$

iii. $8+(-3)=5$

b. When adding two numbers with different signs, if the positive number is farther from 0 on a number line, the result will be positive.

c. i. $2+(-5)=-3$

ii. $1+(-7)=-6$

iii. $3+(-8)=-5$

d. When adding two numbers with different signs, if the positive number is closer to 0 on a number line, the result will be negative.

e. i. $4+(-4)=0$

ii. $7+(-7)=0$

iii. $9+(-9)=0$

f. When adding two numbers with different signs, if the two numbers are the same distance from 0 on a number line, the result will be 0.

g. i. $6+(-4)=2$

ii. $3+(-7)=-4$

iii. $6+(-6)=0$

h. Answers may vary. Example:
When adding two numbers with different signs, take the absolute value of the two numbers and subtract the smaller absolute value from the larger. The sign of the result is the same as the sign of the number with the larger absolute value.

100. Answers may vary. Example:
The denominator of a fraction is the name of the things it represents. The numerator of a fraction is the number of those things it represents. When we add two fractions with the same denominator, we keep the same denominator, or name, and add the two numerators, or number of things.

Homework 2.3

2. $-(-9)=9$

4. $-(-(-2))=-(2)=-2$

6. $|6|=6$ because 6 is a distance of 6 units from 0 on a number line.

8. $|-1|=1$ because -1 is a distance of 1 unit from 0 on a number line.

10. $-|5|=-(5)=-5$

12. $-|-9|=-(9)=-9$

14. The numbers have different signs so subtract the smaller absolute value from the larger.
$|5|-|-3|=5-3=2$
Since $|5|$ is greater than $|-3|$, the sum is positive.
$5+(-3)=2$

16. The numbers have the same sign so add the absolute values.
$|-3|+|-2|=3+2=5$
The numbers are negative, so the sum is negative.
$-3+(-2)=-5$

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18. The numbers have different signs so subtract the smaller absolute value from the larger.
$|-9|-|6|=9-6=3$
Since $|-9|$ is greater than $|6|$, the sum is negative.
$6+(-9)=-3$

20. The numbers have different signs so subtract the smaller absolute value from the larger.
$|4|-|-3|=4-3=1$
Since $|4|$ is greater than $|-3|$, the sum is positive.
$-3+4=1$

22. The numbers have the same sign so add the absolute values.
$|-9|+|-5|=9+5=14$
The numbers are negative, so the sum is negative.
$-9+(-5)=-14$

24. The numbers have different signs so subtract the smaller absolute value from the larger.
$|8|-|-2|=8-2=6$
Since $|8|$ is greater than $|-2|$, the sum is positive.
$8+(-2)=6$

26. $8+(-8)=0$ because the numbers are opposites and the sum of opposites is 0.

28. $-7+7=0$ because the numbers are opposites and the sum of opposites is 0.

30. The numbers have different signs so subtract the smaller absolute value from the larger.
$|17|-|-14|=17-14=3$
Since $|17|$ is greater than $|-14|$, the sum is positive.
$17+(-14)=3$

32. The numbers have different signs so subtract the smaller absolute value from the larger.
$|-89|-|57|=89-57=32$
Since $|-89|$ is greater than $|57|$, the sum is negative.
$-89+57=-32$

34. The numbers have the same sign so add the absolute values.
$|-347|+|-594|=347+594=941$
The numbers are negative, so the sum is negative.
$-347+(-594)=-941$

36. $127,512+(-127,512)=0$ because the numbers are opposites and the sum of opposites is 0.

38. The numbers have the same sign so add the absolute values.
$|-3.7|+|-9.9|=3.7+9.9=13.6$
The numbers are negative, so the sum is negative.
$-3.7+(-9.9)=-13.6$

40. The numbers have different signs so subtract the smaller absolute value from the larger.
$|7|-|-0.3|=7-0.3=6.7$
Since $|7|$ is greater than $|-0.3|$, the sum is positive.
$-0.3+7=6.7$

42. The numbers have different signs so subtract the smaller absolute value from the larger.
$|37.05|-|-19.26|=37.05-19.26=17.79$
Since $|37.05|$ is greater than $|-19.26|$, the sum is positive.
$37.05+(-19.26)=17.79$

44. The numbers have different signs so subtract the smaller absolute value from the larger.
$\left|\frac{2}{5}\right|-\left|-\frac{1}{5}\right|=\frac{2}{5}-\frac{1}{5}=\frac{1}{5}$
Since $\left|\frac{2}{5}\right|$ is greater than $\left|-\frac{1}{5}\right|$, the sum is positive.
$\frac{2}{5}+\left(-\frac{1}{5}\right)=\frac{1}{5}$

46. The numbers have different signs so subtract the smaller absolute value from the larger.
$\left|-\frac{5}{6}\right|-\left|\frac{1}{6}\right|=\frac{5}{6}-\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$
Since $\left|-\frac{5}{6}\right|$ is greater than $\left|\frac{1}{6}\right|$, the sum is negative.
$-\frac{5}{6}+\frac{1}{6}=-\frac{2}{3}$

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48. The numbers have the same sign so add the absolute values.

$$\left|-\frac{2}{3}\right|+\left|-\frac{5}{6}\right|=\frac{2}{3}+\frac{5}{6}=\frac{2}{3}\cdot\frac{2}{2}+\frac{5}{6}=\frac{4}{6}+\frac{5}{6}=\frac{9}{6}=\frac{3}{2}$$

The numbers are negative, so the sum is negative.

$$-\frac{2}{3}+\left(-\frac{5}{6}\right)=-\frac{3}{2}$$

50. The numbers have different signs so subtract the smaller absolute value from the larger.

$$\left|-\frac{3}{4}\right|-\left|\frac{2}{3}\right|=\frac{3}{4}-\frac{2}{3}=\frac{3}{4}\cdot\frac{3}{3}-\frac{2}{3}\cdot\frac{4}{4}=\frac{9}{12}-\frac{8}{12}=\frac{1}{12}$$

Since $\left|-\frac{3}{4}\right|$ is greater than $\left|\frac{2}{3}\right|$, the sum is negative.

$$\frac{2}{3}+\left(-\frac{3}{4}\right)=-\frac{1}{12}$$

52.

```
-7498.34+6435.28
          -1063.06
```

$-7498.34+6435.28=-1063.06$

54.

```
-38487.26+(-8320
5.87)
        -121693.13
```

$-38,487.26+(-83,205.87)=-121,693.13$

56.

```
(-37/642)+(-25/9
83)
     -.0830647487
```

$$-\frac{37}{642}+\left(-\frac{25}{983}\right)\approx-0.08$$

58. Substitute -4 for a and 3 for b in the expression $b+a$ and then find the sum:

$(3)+(-4)=-4+3=-1$

60. Substitute 3 for b and -2 for c in the expression $b+c$ and then find the sum:

$(3)+(-2)=-2+3=1$

62. $x+3$

Substitute -6 for x in the expression and then find the sum: $(-6)+3=-3$

64. $x+(-8)$

Substitute -6 for x in the expression and then find the sum: $(-6)+(-8)=-14$

66. The balance is $-112.50+170$ dollars.

The numbers have different signs so subtract the smaller absolute value from the larger.

$|170|-|-112.50|=170-112.50=57.50$

Since $|170|$ is greater than $|-112.50|$, the sum is positive: $-112.50+170=57.50$

So, the balance is \$57.50.

68. We can find the final balance by finding the balance after each transaction.

Transaction	Balance
Paycheck	$-135.00+549.00=414.00$
FedEx Office	$414.00-10.74=403.26$
ATM	$403.26-21.50=381.76$
Barnes & Noble	$381.76-17.19=364.57$

So, the final balance is \$364.57.

70. The new balance is $-2739+530$.

The numbers have different signs so subtract the smaller absolute value from the larger.

$|-2739|-|530|=2739-530=2209$

Since $|-2739|$ is greater than $|530|$, the sum is negative.

$-2739+530=-2209$

So, the new balance is -2209 dollars.

72. The balance after sending the check is $-873+500=-373$.

The balance after buying the racquet is $-373+(-249)=-622$.

The balance after buying the outfit is $-622+(-87)=-709$.

So, the final balance is -709 dollars.

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74. The current temperature is $-12+8$.
The numbers have different signs so subtract the smaller absolute value from the larger.
$|-12|-|8|=12-8=4$
Since $|-12|$ is greater than $|8|$, the sum is negative.
$-12+8=-4$
So, the current temperature is $-4°$ F.

76. a.

Retail Price (dollars)	Sale Price (dollars)
350	$350+(-35)$
400	$400+(-35)$
450	$450+(-35)$
500	$500+(-35)$
r	$r+(-35)$

From the last row of the table, we see that the expression $r+(-35)$ represents the sale price (in dollars).

b. Evaluate $r+(-35)$ for $r=470$.
$470+(-35)=435$
So, if the retail price is \$470, the sale price is \$435.

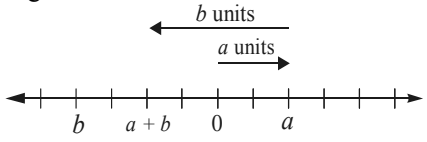
78. a.

Decrease in Temp (°F)	Current Temp (°F)
1	$-2+(-1)$
2	$-2+(-2)$
3	$-2+(-3)$
4	$-2+(-4)$
x	$-2+(-x)$

From the last row of the table, we see that the expression $-2+(-x)$ represents the current temperature (in °F).

b. Evaluate $-2+(-x)$ for $x=7$.
$-2+(-7)=-9$
The current temperature is -9 °F.

80. If a is positive and b is negative (but with a larger absolute value), the sum $a+b$ will be negative.



82. If $a+b$ is positive, then both numbers are positive, or the numbers have opposite signs but the number with the larger absolute value is positive.

84. a. Substitute -2 for a and 5 for b:
$a+b=(-2)+5=3$

b. Substitute -2 for a and 5 for b:
$b+a=5+(-2)=3$

c. The results are the same.

d. Substitute -4 for a and -9 for b:
$a+b=-4+(-9)=-13$
$b+a=-9+(-4)=-13$
The results are the same.

e. Answers may vary.

f. Yes; when adding two quantities, the order of the addition does not matter.

Homework 2.4

2. $3-7=3+(-7)=-4$

4. $-3-9=-3+(-9)=-12$

6. $5-(-1)=5+1=6$

8. $-7-(-3)=-7+3=-4$

10. $-4-7=-4+(-7)=-11$

12. $-4-(-7)=-4+7=3$

14. $-7-7=-7+(-7)=-14$

16. $-100-257=-100+(-257)=-357$

18. $-1939-(-352)=-1939+352=-1587$

20. $5.8-3.7=5.8+(-3.7)=2.1$

22. $-1.7-7.4=-1.7+(-7.4)=-9.1$

24. $3.1-(-3.1)=3.1+3.1=6.2$

26. $-159.24-(-7.8)=-159.24+7.8=-151.44$

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28. $-\frac{1}{5}-\frac{4}{5}=-\frac{1}{5}+\left(-\frac{4}{5}\right)=-\frac{5}{5}=-1$

30. $-\frac{4}{9}-\left(-\frac{7}{9}\right)=-\frac{4}{9}+\frac{7}{9}=\frac{3}{9}=\frac{1}{3}$

32. $\frac{5}{12}-\left(-\frac{1}{6}\right)=\frac{5}{12}+\frac{1}{6}=\frac{5}{12}+\frac{1}{6}\cdot\frac{2}{2}=\frac{5}{12}+\frac{2}{12}=\frac{7}{12}$

34.
$$\begin{aligned}-\frac{2}{3}-\frac{2}{5}&=-\frac{2}{3}+\left(-\frac{2}{5}\right)\\&=-\frac{2}{3}\cdot\frac{5}{5}+\left(-\frac{2}{5}\cdot\frac{3}{3}\right)\\&=-\frac{10}{15}+\left(-\frac{6}{15}\right)\\&=-\frac{16}{15}\end{aligned}$$

36. $-3+9=6$

38. $-4-(-3)=-4+3=-1$

40. $-\frac{5}{6}+\frac{1}{6}=-\frac{4}{6}=-\frac{2}{3}$

42. $-6.4+3.5=-2.9$

44. $-5+(-8)=-13$

46. $5-9=5+(-9)=-4$

48.
```
-6178.39-52.387
          -6230.777
```
$-6178.39-52.387\approx-6230.78$

50.
```
83451.6-(-408.54
9)
          83860.149
```
$83,451.6-(-408.549)\approx 83,860.15$

52.
```
(-49/56)-(85/97)
        -1.75128866
```
$-\frac{49}{56}-\frac{85}{97}\approx-1.75$

54. $-12+18=6$
So, the current temperature is $6°\,\text{F}$.

56. $-13-(-2)=-13+2=-11$
The change in temperature is $-11°\,\text{F}$.

58. a. $9-(-6)=9+6=15$
The change in temperature is $15°\,\text{F}$.

b. To estimate the change in temperature over the past hour, we divide the change over three hours by 3.
$\frac{15}{3}=5$
The estimated change in temperature over the past hour is $5°\,\text{F}$.

c. Answers may vary. Example:
The change in temperature is affected by the time of day in addition to the weather conditions. Thus, temperature change need not be uniform.

60. $29,035-(-1312)=29,035+1312=30,347$
The change in elevation is 30,347 feet.

62. a.

Year	Population	Change in Population
2008	124	–
2009	96	$96-124=-28$
2010	97	$97-96=1$
2011	98	$98-97=1$
2012	83	$83-98=-15$
2013	95	$95-83=12$
2014	104	$104-95=9$

b. The population increased the most from 2012 to 2013 with an increase of 12 wolves.

c. The population decreased the most from 2008 to 2009 with a decrease of 28 wolves.

d. No. The change in population is the difference between births and deaths. An increase of 9 wolves means there were 9 more births than deaths.

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64. a. To find the number of groups in 2015, we add the changes in the number of groups from 2009 to 2015:
$512+312+450+86+(-264)+(-222)+124$
$=998$
So, there were 998 Patriot Groups in 2015.

b. Increasing numbers of groups are indicated by positive changes. Therefore, the number of Patriot Groups was increasing from 2009 to 2012 and from 2014 to 2015.

c. Decreasing numbers of groups are indicated by negative changes. Therefore, the number of Patriot Groups was decreasing from 2012 to 2014.

66. a.

Current Value (dollars)	Change in Value (dollars)
30	$30-35$
35	$35-35$
40	$40-35$
45	$45-35$
x	$x-35$

From the last row of the table, we see that the expression $x-35$ represents the change in value (in dollars) of the stock.

b. Evaluate $x-35$ for $x=44$:
$44-35=9$
So, if the current value is \$44, the stock has increased in value by \$9 from last year.

68. a.

Change in Population	Current Population
10	$10+820$
20	$20+820$
30	$30+820$
40	$40+820$
c	$c+820$

From the last row of the table, we see that the expression $c+820$ represents the current population.

b. Evaluate $c+820$ for $c=-25$:
$-25+820=795$
So, if the change in population is -25, the current population would be 795 due to the decrease in population of 25 deer in the past year.

70. Evaluate $a+c$ for $a=-5$ and $c=-7$:
$(-5)+(-7)=-12$

72. Evaluate $c-a$ for $a=-5$ and $c=-7$:
$(-7)-(-5)=-7+5=-2$

74. Evaluate $b-a$ for $a=-5$ and $b=2$:
$(2)-(-5)=2+5=7$

76. $x-4$
Evaluate the expression for $x=-5$:
$-5-4=-5+(-4)=-9$

78. $x-5$
Evaluate the expression for $x=-5$:
$(-5)-5=-5+(-5)=-10$

80. $x-(-6)$
Evaluate the expression for $x=-5$:
$(-5)-(-6)=-5+6=1$

82. The student changed the order of subtraction without changing the sign of the result.
$2-6=2+(-6)=-4$

84. a. i. $2-8=2+(-8)=-6$

ii. $3-9=3+(-9)=-6$

iii. $1-5=1+(-5)=-4$

b. Answers may vary. Example:
Since the quantity decreased, the final number is smaller than the beginning number. When finding the change in quantity, we subtract the beginning number from the final number. Since the final number is smaller, the result will be negative.

86. a. i. $-2(-5)=10$

ii. $-4(-6)=24$

iii. $-7(-9)=63$

b. Answers may vary. Example:
The results are all positive.

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c. $-3(-7) = 3(7) = 21$

d. Answers may vary. Example:
To multiply two negative numbers, multiply their absolute values.

88. Answers may vary. Example:
In the first situation, the values were on the same side of 0 on a number line, so we subtract to find the distance between the two values on the number line. In the second situation, the values are on opposite sides of 0 on a number line. In this case, we add the distance each number is from 0 to get the overall distance between the numbers.

90. Answers may vary. Example:
It is impossible to find the sign. If $|x|$ is greater than $|y|$, then $x - y$ is negative. If $|x|$ is less than $|y|$, then $x - y$ is positive.

Homework 2.5

2. $91\% = 91.0\% = 0.91$

4. $4\% = 4.0\% = 0.04$

6. $0.01 = 1\%$

8. $3.8\% = 0.038$

10. $0.089 = 8.9\%$

12. To find 67% of 4500 computers, we multiply 0.67 by 4500: $0.67(4500) = 3015$. So 67% of 4500 computers is 3015 computers.

14. To find the 3% grocery tax on the purchase of \$125.35 worth of groceries, we multiply 0.03 by 125.35: $0.03(125.35) = 3.7605$. So the sales tax is about \$3.76.

16. To find the number of undergraduates who lived on campus, we multiply 0.26 by 23,357: $0.26(23{,}357) = 6072.82$. So, the number of undergraduates who lived on campus in the spring semester 2017 at the University of Iowa was about 6073.

18. Since the numbers have different signs, the product is negative: $-5(4) = -20$

20. Since the numbers have the same sign, the product is positive: $-8(-9) = 72$

22. Since the numbers have different signs, the product is negative: $5(-2) = -10$

24. Since the numbers have different signs, the quotient is negative: $-63 \div 7 = -9$

26. Since the numbers have different signs, the quotient is negative: $24 \div (-3) = -8$

28. Since the numbers have the same sign, the quotient is positive: $-1 \div (-1) = 1$

30. Since the numbers have the same sign, the product is positive: $-124(-29) = 3596$

32. Since the numbers have different signs, the quotient is negative: $1008 \div (-21) = -48$

34. Since the numbers have the same sign, the product is positive: $-0.3(-0.3) = 0.09$

36. Since the numbers have different signs, the product is negative: $3.7(-5.24) = -19.388$

38. Since the numbers have different signs, the quotient is negative: $-0.12 \div 0.3 = -0.4$

40. Since the numbers have different signs, the quotient is negative: $\dfrac{9}{-3} = 9 \div (-3) = -3$

42. Since the numbers have the same sign, the quotient is positive: $\dfrac{-72}{-8} = -72 \div (-8) = 9$

44. Since the numbers have different signs, the product is negative: $\dfrac{1}{3}\left(-\dfrac{7}{5}\right) = -\dfrac{7}{15}$

46. Since the numbers have the same sign, the product is positive: $\left(-\dfrac{7}{25}\right)\left(-\dfrac{5}{21}\right) = \dfrac{35}{525} = \dfrac{1}{15}$

48. Since the numbers have different signs, the quotient is negative:
$-\dfrac{5}{7} \div \dfrac{15}{8} = -\dfrac{5}{7} \cdot \dfrac{8}{15} = -\dfrac{40}{105} = -\dfrac{8}{21}$

50. Since the numbers have the same sign, the quotient is positive:
$-\dfrac{3}{8} \div \left(-\dfrac{9}{20}\right) = \dfrac{3}{8} \cdot \dfrac{20}{9} = \dfrac{60}{72} = \dfrac{5}{6}$

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52. $-9+(-4)=-13$

54. $-49\div(7)=-7$

56. $-2-7=-2+(-7)=-9$

58. $(-5)(-9)=45$

60.
$$\begin{aligned}-\frac{8}{3}+\left(-\frac{5}{9}\right)&=-\frac{8}{3}\cdot\frac{3}{3}+\left(-\frac{5}{9}\right)\\&=-\frac{24}{9}+\left(-\frac{5}{9}\right)\\&=\frac{-24+(-5)}{9}\\&=-\frac{29}{9}\end{aligned}$$

62. $\frac{9}{2}\left(-\frac{4}{21}\right)=-\frac{36}{42}=-\frac{6}{7}$

64.
$$\begin{aligned}-\frac{3}{8}-\left(-\frac{1}{10}\right)&=-\frac{3}{8}+\frac{1}{10}\\&=-\frac{3}{8}\cdot\frac{5}{5}+\frac{1}{10}\cdot\frac{4}{4}\\&=-\frac{15}{40}+\frac{4}{40}\\&=\frac{-15+4}{40}\\&=-\frac{11}{40}\end{aligned}$$

66.
$$\begin{aligned}-\frac{22}{9}\div\left(-\frac{33}{18}\right)&=-\frac{22}{9}\cdot\left(-\frac{18}{33}\right)\\&=\frac{2\cdot11\cdot2\cdot9}{9\cdot3\cdot11}\\&=\frac{2\cdot2}{3}\\&=\frac{4}{3}\end{aligned}$$

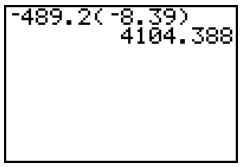
68. $\frac{-15}{35}=-\frac{3\cdot5}{7\cdot5}=-\frac{3}{7}$

70. $\frac{-35}{-21}=\frac{7\cdot5}{7\cdot3}=\frac{5}{3}$

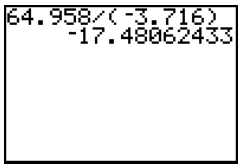
72. $\frac{5}{-6}+\frac{1}{6}=\frac{-5}{6}+\frac{1}{6}=\frac{-5+1}{6}=\frac{-4}{6}=-\frac{2}{3}$

74. $\frac{2}{3}-\left(\frac{1}{-3}\right)=\frac{2}{3}+\frac{1}{3}=\frac{2+1}{3}=\frac{3}{3}=1$

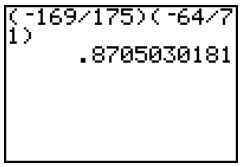
76.
$$\begin{aligned}\frac{1}{4}+\frac{5}{-6}&=\frac{1}{4}\cdot\frac{3}{3}+\frac{-5}{6}\cdot\frac{2}{2}\\&=\frac{3}{12}+\frac{-10}{12}\\&=\frac{3+(-10)}{12}\\&=-\frac{7}{12}\end{aligned}$$

78.


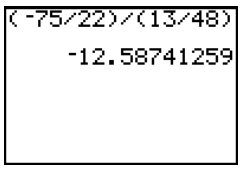
$-489.2(-8.39)\approx 4104.39$

80.


$64.958\div(-3.716)\approx -17.48$

82.


$-\frac{169}{175}\left(-\frac{64}{71}\right)\approx 0.87$

84.


$-\frac{75}{22}\div\frac{13}{48}\approx -12.59$

86. Evaluate ac for $a=-6$ and $c=-8$:
$(-6)(-8)=48$

88. Evaluate $\frac{b}{a}$ for $a=-6$ and $b=4$:
$\frac{4}{-6}=-\frac{2}{3}$

90. Evaluate $-bc$ for $b=4$ and $c=-8$:
$-(4)(-8)=-(-32)=32$

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92. Evaluate $-\frac{a}{c}$ for $a=-6$ and $c=-8$:

$$-\frac{(-6)}{(-8)}=-\frac{3}{4}$$

94. $\frac{w}{4}$; Evaluate the expression for $w=-8$:

$$\frac{(-8)}{4}=-2$$

96. $-2w$; Evaluate the expression for $w=-8$:

$$-2(-8)=16$$

98. $\frac{9}{15}=\frac{3}{5}$

100. $\frac{1032}{45}\approx\frac{22.93}{1}$

Since 1945, the United States has conducted about 22.93 times the number of nuclear bomb tests as has China.

102. $\frac{4.79}{3.77}\approx\frac{1.27}{1}$

On average, there were about 1.27 million more viewers of the *Today* show than of the show *CBS This Morning* in December 2016.

104. a. $\frac{2 \text{ c mushrooms}}{4 \text{ c cooked noodles}}=\frac{0.5 \text{ c mushrooms}}{1 \text{ c cooked noodles}}$

For each cup of cooked noodles, a half cup of sliced mushrooms is required.

b. $\frac{4 \text{ c cooked noodles}}{2 \text{ c mushrooms}}=\frac{2 \text{ c cooked noodles}}{1 \text{ c mushrooms}}$

For each cup of sliced mushrooms, 2 cups of cooked noodles are required.

106. a. $\frac{19,795,791}{8,958,013}\approx\frac{2.21}{1}$

The population of New York is about 2.21 times larger than that of New Jersey.

b. $\frac{571,951}{155,959}\approx\frac{3.67}{1}$

The land area of Alaska is about 3.67 times larger than that of California.

c. Alaska: $\frac{738,432}{571,951}\approx\frac{1.29}{1}$

California: $\frac{39,144,818}{155,959}\approx\frac{250.99}{1}$

Michigan: $\frac{9,922,576}{56,804}\approx\frac{174.68}{1}$

New Jersey: $\frac{8,958,013}{7417}\approx\frac{1207.77}{1}$

New York: $\frac{19,795,791}{47,214}=\frac{419.28}{1}$

d. The state with the greatest population density is New Jersey. The state with the least dense population is Alaska.

e. The person is not correct.
Answers may vary. Example:
Although Michigan has a larger population than New Jersey, it also has a larger land area which serves to lower its population density.

108. a. $\frac{-6810 \text{ dollars}}{-2950 \text{ dollars}}=\frac{681}{295}\approx\frac{2.31}{1}$

b. For each \$1 he pays towards his Sears account, he should pay about \$2.31 towards his Visa account.

110. $0.35(1590)=556.50$

$-1590+556.5=-1033.50$

The new balance would be -1033.50 dollars.

112. $3(89.50)=268.50$

$0-268.50=-268.50$

The new balance is $-\$268.50$.

114. a. $-2+3=1$

b. $-2(3)=-6$

c. The first statement is clearer because it indicates an operation as well. From parts (a) and (b), we see that just having one negative number and one positive number is not enough to guarantee that the result is negative.

d. Answers may vary.

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116. a. Negative; the quotient of two numbers with opposite signs is negative.

b. Negative; the quotient of two numbers with opposite signs is negative.

c. No; the variables a and b can take on positive or negative values, so the sign of the result is not clear without knowing the signs of a and b.

118. Answers may vary. Example:
$$(-4)(5) = (-4)+(-4)+(-4)+(-4)+(-4)$$
$$= -20$$

120. If ab is positive, we can say that the two numbers have the same sign. We cannot say if the two numbers are positive or negative, but we do know they have the same sign.

122. If $\frac{a}{b}$ is negative, we can say that the two numbers have different signs. We cannot say which is positive and which is negative, but we do know they have different signs.

124. a.
$$(-1)(-1) = 1$$
$$(-1)(-1)(-1) = -1$$
$$(-1)(-1)(-1)(-1) = 1$$
$$(-1)(-1)(-1)(-1)(-1) = -1$$

b. Multiplying -1 by itself an even number of times yields 1 as the result, while doing so an odd number of times results in -1.

c. Since the number of -1 's is even, the result will be 1.

d. Since the number of -1 's is odd, the result will be -1.

126. a. Answers may vary. Example:
$2,\ 3,\ 4$

b. Answers may vary. Example:
$0.1,\ 0.2,\ 0.3$

c. For $\frac{2}{x}$ to equal 2, x must be 1.

Homework 2.6

2. $3^4 = 3\cdot 3\cdot 3\cdot 3 = 9\cdot 3\cdot 3 = 27\cdot 3 = 81$

4. $5^3 = 5\cdot 5\cdot 5 = 25\cdot 5 = 125$

6. $-7^2 = -(7\cdot 7) = -49$

8. $(-7)^2 = (-7)(-7) = 49$

10. $\left(\frac{3}{5}\right)^3 = \left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right) = \frac{27}{125}$

12. $8\cdot(2-6) = 8(-4) = -32$

14. $(2+8)(3-8) = (10)(-5) = -50$

16.
$$-6-(4-7)+5 = -6-(-3)+5$$
$$= -6+3+5$$
$$= -3+5$$
$$= 2$$

18. $\frac{3-9}{2-4} = \frac{-6}{-2} = 3$

20. $\frac{4-7}{-3-(-1)} = \frac{-3}{-3+1} = \frac{-3}{-2} = \frac{3}{2}$

22. $\frac{1-9}{2-(-4)} = \frac{-8}{2+4} = \frac{-8}{6} = -\frac{4}{3}$

24. $2-3\cdot 5 = 2-15 = -13$

26. $1+9\cdot(-4) = 1+(-36) = -35$

28. $-16\div(-4)\cdot 2 = 4\cdot 2 = 8$

30. $3-7+1 = -4+1 = -3$

32.
$$2-4(9-6) = 2-4(3)$$
$$= 2-12$$
$$= -10$$

34.
$$2(2-5)+10\div 5 = 2(-3)+10\div 5$$
$$= -6+10\div 5$$
$$= -6+2$$
$$= -4$$

36. $6(2+3)-5\cdot 7 = 6(5)-5\cdot 7 = 30-35 = -5$

38.
$$-3-\left[6+2(4-8)\right] = -3-\left[6+2(-4)\right]$$
$$= -3-[6-8]$$
$$= -3-[-2]$$
$$= -1$$

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40. $-2\left[4(3-8)+1\right]=-2\left[4(-5)+1\right]$
$=-2[-20+1]$
$=-2[-19]$
$=38$

42. $\frac{5}{6}+\frac{2}{3}\div\frac{2}{5}=\frac{5}{6}+\frac{2}{3}\cdot\frac{5}{2}=\frac{5}{6}+\frac{10}{6}=\frac{15}{6}=\frac{5}{2}$

44. $8-3^2=8-9=-1$

46. $8(-2)^3=8(-8)=-64$

48. $\frac{5^2}{2^5}=\frac{5\cdot5}{2\cdot2\cdot2\cdot2\cdot2}=\frac{25}{32}$

50. $5^2+2^5=25+32=57$

52. $-20\div2^2=-20\div4=-5$

54. $4^3-(-4)^3=64-(-64)=64+64=128$

56. $4(-2)^2-3=4(4)-3=16-3=13$

58. $2(-4)^2+3(-4)-7=2(16)+3(-4)-7$
$=32+(-12)-7$
$=20-7$
$=13$

60. $\frac{10-(-2)^2}{3^3}=\frac{10-(4)}{27}=\frac{6}{27}=\frac{2}{9}$

62. $4+(3-6)^2-2=4+(-3)^2-2$
$=4+9-2$
$=13-2$
$=11$

64. $(9-7)^2\cdot(-3)-2^4=(2)^2\cdot(-3)-2^4$
$=4(-3)-16$
$=-12-16$
$=-12+(-16)$
$=-28$

66.
```
8.53/5.26+24.91-
78.3(45.3)
    -3520.458327
```
$8.53\div5.26+24.91-78.3(45.3)\approx-3520.46$

68.
```
1.98+8.22^5*5.29
      198526.7766
```
$1.98+8.22^5\cdot5.29\approx198{,}526.78$

70.
```
(53.25+99.83)/(3
1.28-6.31*89.11)
     -.2882840264
```
$\frac{53.25+99.83}{31.28-(6.31)(89.11)}\approx-0.29$

72. Evaluate $ac-b$ for $a=-2$, $b=-4$, and $c=3$: $(-2)(3)-(-4)=-6+4=-2$

74. Evaluate $c\div a+abc$ for $a=-2$, $b=-4$, and $c=3$:
$3\div(-2)+(-2)(-4)(3)=-\frac{3}{2}+24$
$=-\frac{3}{2}+\frac{48}{2}$
$=\frac{45}{2}$

76. Evaluate c^2-b^2 for $b=-4$ and $c=3$:
$(3)^2-(-4)^2=9-16=-7$

78. Evaluate $-cb^2+a^2$ for $a=-2$, $b=-4$, and $c=3$:
$-(3)(-4)^2+(-2)^2=-3(16)+(-2)^2$
$=-48+4$
$=-44$

80. Evaluate $\frac{a^2-b}{c^2-b}$ for $a=-2$, $b=-4$, and $c=3$:
$\frac{(-2)^2-(-4)}{(3)^2-(-4)}=\frac{4-(-4)}{9-(-4)}=\frac{4+4}{9+4}=\frac{8}{13}$

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82. Substitute $a=5$, $b=-1$, $c=-4$, and $d=7$ in the expression $\dfrac{a-b}{c-d}$:

$$\frac{(5)-(-1)}{(-4)-(7)}=\frac{5+1}{-4+(-7)}=\frac{6}{-11}=-\frac{6}{11}$$

84. Substitute $a=-2$, $b=6$, $c=5$, and $d=-1$ in the expression $\dfrac{a-b}{c-d}$:

$$\frac{(-2)-(6)}{(5)-(-1)}=\frac{-2+(-6)}{5+1}=\frac{-8}{6}=-\frac{4}{3}$$

86. Substitute $a=-3$, $b=-5$, $c=-8$, and $d=-3$ in the expression $\dfrac{a-b}{c-d}$:

$$\frac{(-3)-(-5)}{(-8)-(-3)}=\frac{-3+5}{-8+3}=\frac{2}{-5}=-\frac{2}{5}$$

88. Evaluate $5x^2$ for $x=-3$:

$$5(-3)^2=5(9)=45$$

90. Evaluate $-4x^2+4$ for $x=-3$:

$$\begin{aligned}-4(-3)^2+4&=-4(9)+4\\&=-36+4\\&=-32\end{aligned}$$

92. Evaluate $4x^2+x-2$ for $x=-3$:

$$\begin{aligned}4(-3)^2+(-3)-2&=4(9)+(-3)-2\\&=36+(-3)-2\\&=33-2\\&=31\end{aligned}$$

94. $-3-\dfrac{8}{x}$

Evaluate the expression for $x=-4$:

$$\begin{aligned}-3-\frac{8}{-4}&=-3-(-2)\\&=-3+2\\&=-1\end{aligned}$$

96. $x+x(-5)$

Evaluate the expression for $x=-4$:

$$-4+(-4)(-5)=-4+20=16$$

98. a.

Years since 2014	Number of Countries
0	$70\cdot 0+8.8$
1	$70\cdot 1+8.8$
2	$70\cdot 2+8.8$
3	$70\cdot 3+8.8$
4	$70\cdot 4+8.8$
t	$70t+8.8$

b. Substitute 8 for t in $70t+8.8$:

$$70(8)+8.8=560+8.8=568.8$$

So, in 2022 (8 years after 2014) the number of countries requiring picture warnings on cigarette packages was about 569.

100. a.

Years Since 1990	Percent
0	$-1.1\cdot 0+85$
1	$-1.1\cdot 1+85$
2	$-1.1\cdot 2+85$
3	$-1.1\cdot 3+85$
4	$-1.1\cdot 4+85$
t	$-1.1t+85$

From the last row of the table, we see that the expression $-1.1t+85$ represents the percentage of companies offering traditional benefit plans t years since 1990.

b. Substitute 31 for t in $-1.1t+85$:

$$-1.1(31)+85=-34.1+85=50.9$$

So, in 2021 (31 years after 1990) the percentage of companies offering traditional benefit plans will be about 50.9%.

102. To find the digital ad revenue in 2015, we first need to find 20.4% of \$49.5 billion. To do this multiply: $0.204(49.5)=10.098$.

Now add 10.098 to the 2014 revenue, which is $10.098+49.5=59.598$. The digital ad revenue in 2015 was about \$59.6 billion.

104. To find Nike's North American revenue in 2016, we first need to find 8% of \$13.7 billion. To do this, multiply: $0.8(13.7)=1.096$.

Now add 1.096 to the 2015 revenue of \$13.7 billion, which is $13.7+1.096=14.796$. Nike's North American revenue in 2016 was about \$14.8 billion.

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106. To find the number of Kmart stores in 2016, we first need to find 59.2% of 2165 stores. To do this, multiply: $0.592(2165) = 1281.68$.

Now subtract 1281.68 from the number of Kmart stores in 2000, which is $2165 - 1281.68 = 883.32$. The number of Kmart stores in 2016 was about 883.

108. To find the number of people who participated in the Supplemental Nutrition Assistance Program in 2016, we first need to find 2.3% of 46.5 million. To do this, multiply $0.023(46.5) = 1.0695$ or about 1.07 million.

Now subtract 1.07 million people from the number of people who participated in the program in 2015, which is $46.5 - 1.07 = 45.43$ million. The number of people who participated in the Supplemental Nutrition Assistance Program in 2016 was about 45.43 million people.

110. Substitute 3 for r in the expression $\frac{4}{3}\pi r^3$:

$$\frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi$$

So, the volume is $36\pi \approx 113.10$ cubic inches.

112. In the first line, the student only squared 3 instead of -3. Should be:

$$(-3)^2 + 4(-3) + 5 = 9 + (-12) + 5 = -3 + 5 = 2$$

114. The student did not perform multiplication and division in the correct order (from left to right). $16 \div 2 \cdot 4 = 8 \cdot 4 = 32$.

116. a. $(12 \div 3) \cdot 2 = 4 \cdot 2 = 8$

b. $12 \div (3 \cdot 2) = 12 \div 6 = 2$

c. Yes, the order makes a difference:
L-R: $12 \div 3 \cdot 2 = 4 \cdot 2 = 8$ (part (a))
R-L: $12 \div 3 \cdot 2 = 12 \div 6 = 2$ (part (b))

d. Order of operations dictates that we multiply and divide from left to right in the order that they appear. Therefore, $12 \div 3 \cdot 2 = 4 \cdot 2 = 8$

118. a. $(2+3)+4 = 5+4 = 9$

b. $2+(3+4) = 2+7 = 9$

c. The results are the same.

d. $(4+(-2))+5 = 2+5 = 7$
$4+(-2+5) = 4+3 = 7$
The results are the same.

e. Answers may vary. Example:
In each case, the results should be the same.

f. Yes; this is the associative property of addition.

g. Answers may vary. Example:
For a series of additions, the way in which the additions are grouped does not matter.

120. a. i. $(-1)^2 = (-1)(-1) = 1$

ii. $(-1)^3 = (-1)^2(-1) = 1(-1) = -1$

iii. $(-1)^4 = (-1)^3(-1) = (-1)(-1) = 1$

iv. $(-1)^5 = (-1)^4(-1) = 1(-1) = -1$

v. $(-1)^{87} = (-1)^{86}(-1) = 1(-1) = -1$

vi. $(-1)^{596} = 1$

b. $(-1)^n = 1$ when n is an even counting number. Answers may vary. Example:
The product of an even number of negative numbers is positive.

c. $(-1)^n = -1$ when n is an odd counting number. Answers may vary. Example:
The product of an odd number of negative numbers is negative.

122. Answers may vary.

Chapter 2 Review Exercises

1. $8 + (-2) = 6$

2. $(-5) + (-7) = -12$

3. $6 - 9 = 6 + (-9) = -3$

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4. $8-(-2)=8+2=10$

5. Since the numbers have different signs, the product will be negative: $8(-2)=-16$

6. Since the numbers have different signs, the quotient will be negative: $8\div(-2)=-4$

7. $-24\div(10-2)=-24\div(8)=-3$

8. $(2-6)(5-8)=(-4)(-3)=12$

9. $\dfrac{7-2}{2-7}=\dfrac{5}{-5}=-\dfrac{5}{5}=-1$

10. $\dfrac{2-8}{3-(-1)}=\dfrac{2-8}{3+1}=\dfrac{-6}{4}=-\dfrac{6}{4}=-\dfrac{3}{2}$

11.
$$\begin{aligned}\frac{3-5(-6)}{-2-1}&=\frac{3-(-30)}{-2-1}\\&=\frac{3+30}{-2+(-1)}\\&=\frac{33}{-3}\\&=-\frac{33}{3}\\&=-11\end{aligned}$$

12. $3(-5)+2=-15+2=-13$

13. $-4+2(-6)=-4+(-12)=-16$

14. $2-12\div 2=2-6=-4$

15. $8\div(-2)\cdot 5=(-4)\cdot 5=-20$

16. $4-6(7-2)=4-6(5)=4-30=-26$

17.
$$\begin{aligned}2(4-7)-(8-2)&=2(-3)-(6)\\&=(-6)-(6)\\&=(-6)+(-6)\\&=-12\end{aligned}$$

18.
$$\begin{aligned}-2(3-6)+18\div(-9)&=-2(-3)+18\div(-9)\\&=6+18\div(-9)\\&=6+(-2)\\&=4\end{aligned}$$

19.
$$\begin{aligned}-14\div(-7)-3(1-5)&=-14\div(-7)-3(-4)\\&=2-3(-4)\\&=2-(-12)\\&=2+12\\&=14\end{aligned}$$

20.
$$\begin{aligned}-5-\left[3+2(1-7)\right]&=-5-\left[3+2(-6)\right]\\&=-5-[3-12]\\&=-5-[-9]\\&=-5+9\\&=4\end{aligned}$$

21. $4.2-(-6.7)=4.2+6.7=10.9$

22. $\dfrac{4}{9}\left(-\dfrac{3}{10}\right)=-\dfrac{12}{90}=-\dfrac{2}{15}$

23.
$$\begin{aligned}\left(-\frac{8}{15}\right)\div\left(-\frac{16}{25}\right)&=\left(-\frac{8}{15}\right)\left(-\frac{25}{16}\right)\\&=\frac{8}{15}\cdot\frac{25}{16}\\&=\frac{200}{240}\\&=\frac{5}{6}\end{aligned}$$

24. $\dfrac{5}{9}-\left(-\dfrac{2}{9}\right)=\dfrac{5}{9}+\dfrac{2}{9}=\dfrac{5+2}{9}=\dfrac{7}{9}$

25.
$$\begin{aligned}-\frac{5}{6}+\frac{7}{8}&=-\frac{5}{6}\cdot\frac{4}{4}+\frac{7}{8}\cdot\frac{3}{3}\\&=\frac{-20}{24}+\frac{21}{24}\\&=\frac{-20+21}{24}\\&=\frac{1}{24}\end{aligned}$$

26.
$$\begin{aligned}\frac{-5}{2}-\frac{7}{-3}&=\frac{-5}{2}+\frac{7}{3}\\&=\frac{-5}{2}\cdot\frac{3}{3}+\frac{7}{3}\cdot\frac{2}{2}\\&=\frac{-15}{6}+\frac{14}{6}\\&=\frac{-15+14}{6}\\&=-\frac{1}{6}\end{aligned}$$

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27. $(-8)^2 = (-8)(-8) = 64$

28. $-8^2 = -(8 \cdot 8) = -64$

29. $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

30. $\left(\frac{3}{4}\right)^3 = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{3 \cdot 3 \cdot 3}{4 \cdot 4 \cdot 4} = \frac{27}{64}$

31. $-6(3)^2 = -6(3 \cdot 3) = -6(9) = -54$

32. $24 \div 2^3 = 24 \div (2 \cdot 2 \cdot 2) = 24 \div 8 = 3$

33.
$$\begin{aligned}(-2)^3 - 4(-2) &= (-2)(-2)(-2) - 4(-2)\\ &= -8 - 4(-2)\\ &= -8 - (-8)\\ &= -8 + 8\\ &= 0\end{aligned}$$

34. $\frac{2^3}{3+3^2} = \frac{2 \cdot 2 \cdot 2}{3+(3 \cdot 3)} = \frac{8}{3+9} = \frac{8}{12} = \frac{2}{3}$

35.
$$\begin{aligned}\frac{17-(-3)^2}{5-4^2} &= \frac{17-(-3)(-3)}{5-(4 \cdot 4)}\\ &= \frac{17-9}{5-16}\\ &= \frac{8}{-11}\\ &= -\frac{8}{11}\end{aligned}$$

36.
$$\begin{aligned}-3(2)^2 - 4(2) + 1 &= -3(2 \cdot 2) - 4(2) + 1\\ &= -3(4) - 4(2) + 1\\ &= -12 - 8 + 1\\ &= -20 + 1\\ &= -19\end{aligned}$$

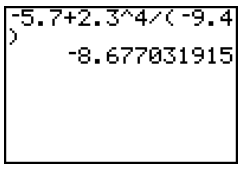
37.
$$\begin{aligned}24 \div (3-5)^3 &= 24 \div (-2)^3\\ &= 24 \div [(-2)(-2)(-2)]\\ &= 24 \div (-8)\\ &= -3\end{aligned}$$

38.
$$\begin{aligned}&7^2 - 3(2-5)^2 \div (-3)\\ &= 7^2 - 3(-3)^2 \div (-3)\\ &= (7 \cdot 7) - 3[(-3)(-3)] \div (-3)\\ &= 49 - 27 \div (-3)\\ &= 49 - (-9)\\ &= 49 + 9\\ &= 58\end{aligned}$$

39. $\frac{-18}{-24} = \frac{18}{24} = \frac{2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3} = \frac{3}{2 \cdot 2} = \frac{3}{4}$

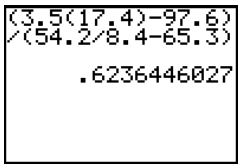
40. $\frac{-28}{35} = -\frac{28}{35} = -\frac{2 \cdot 2 \cdot 7}{5 \cdot 7} = -\frac{2 \cdot 2}{5} = -\frac{4}{5}$

41.



$-5.7 + 2.3^4 \div (-9.4) \approx -8.68$

42.



$\frac{3.5(17.4) - 97.6}{54.2 \div 8.4 - 65.3} \approx 0.62$

43. Substitute $\frac{1}{4}$ for W and $\frac{5}{6}$ for L in the expression $2L + 2W$:

$$\begin{aligned}2\left(\frac{5}{6}\right) + 2\left(\frac{1}{4}\right) &= \frac{2}{1} \cdot \frac{5}{6} + \frac{2}{1} \cdot \frac{1}{4}\\ &= \frac{10}{6} + \frac{2}{4}\\ &= \frac{10}{6} + \frac{1}{2}\\ &= \frac{10}{6} + \frac{1}{2} \cdot \frac{3}{3}\\ &= \frac{10}{6} + \frac{3}{6}\\ &= \frac{13}{6}\end{aligned}$$

So, the perimeter of the rectangle is $\frac{13}{6} = 2\frac{1}{6}$ yards.

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44. $-4789+800-(102.99+3.50)$
$=-4789+800-106.49$
$=-3989-106.49$
$=-4095.49$
The student now owes the credit card company $4095.49.

45. $27,800-32,500=-4700$
The plane had a change in altitude of -4700 feet.

46. a. $-8-4=-12$
The change in temperature is $-12°\,\text{F}$.

b. Divide the change for the past three hours by 3 to estimate the change over 1 hour.
$\frac{-12}{3}=-4$
The estimated change for the past hour is $-4°\,\text{F}$.

c. Answers may vary. Example: Temperature need not change uniformly.

47. a. The change in the annual contributions to the Democratic nominee from 2000 to 2004 can be found by subtracting 631 from 824, which is $824-631=193$.
So, the change in annual contributions to the Democratic nominee from 2000 to 2004 is an increase of $193 million.

b. The change in the annual contribution to the Republican nominee from 2004 to 2008 can be found by subtracting 981 from 920, which is $920-981=-61$.
So, the change in annual contribution to the Republican nominee from 2004 to 2008 is a decrease of $61 million.

c.

Years	Change (millions of dollars)
2000 – 2004	$824-631=193$
2004 – 2008	$962-824=138$
2008 – 2012	$1070-962=108$

The greatest change in annual contribution to the Democratic nominee occurred over the period 2000-2004 with an increase of $193 million.

d.

Years	Change (millions of dollars)
2000 – 2004	$981-923=58$
2004 – 2008	$920-981=-61$
2008 – 2012	$1022-920=102$

The greatest change in annual contribution to the Republican nominee occurred over the period 2008-2012 with an increase of $102 million.

48. To find the unit ratio of the average daily number of texts sent or received by female teens to the average daily number of texts sent or received by male teens, divide:
$\frac{79}{56}\approx\frac{1.41}{1}$.
On average, the number of texts sent or received by female teens is about 1.41 larger than the number sent or received by male teens.

49. $75\%=75.0\%=0.75$

50. $2.9\%=0.029$

51. To find the number of those surveyed who said Five Guys has the best hamburger, find 15% of 1.2 million Americans. To do this, multiply: $0.15(1.2)=0.18$.
So, about 0.18 million Americans (or 180 thousand) said Five Guys has the best hamburger.

52. To find the number of those aged at least 12 years who said they used marijuana in the past month in a survey, find 7.3% of 68,300. To do this, multiply: $0.073(68,300)=4985.9$.
So, according to the survey, about 4985.9 people at least 12 years of age used marijuana in the past month.

53. $-5493+0.2(5493)=-5493+1098.6$
$=-4394.4$
The new balance is -4394.4 dollars.

54. Substitute 2 for a and -4 for c in the expression $ac+c\div a$:
$(2)(-4)+(-4)\div(2)=-8+(-4)\div(2)$
$=-8+(-2)$
$=-10$

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55. Substitute 2 for a, -5 for b, and -4 for c in the expression b^2-4ac:

$$\begin{aligned}(-5)^2-4(2)(-4)&=(-5)(-5)-4(2)(-4)\\&=25-(-32)\\&=25+32\\&=57\end{aligned}$$

56. Substitute 2 for a, -5 for b, and -4 for c in the expression $a(b-c)$:

$$(2)\big((-5)-(-4)\big)=2(-5+4)=2(-1)=-2$$

57. Substitute 2 for a, -5 for b, and -4 for c in the expression $\dfrac{-b-c^2}{2a}$:

$$\begin{aligned}\frac{-(-5)-(-4)^2}{2(2)}&=\frac{-(-5)-(-4)(-4)}{2(2)}\\&=\frac{5-16}{4}\\&=\frac{-11}{4}\\&=-\frac{11}{4}\end{aligned}$$

58. Substitute -4 for c in the expression $2c^2-5c+3$:

$$\begin{aligned}2(-4)^2-5(-4)+3&=2(-4)(-4)-5(-4)+3\\&=32-(-20)+3\\&=32+20+3\\&=55\end{aligned}$$

59. Substitute 2 for a, -5 for b, -4 for c, and 10 for d in the expression $\dfrac{a-b}{c-d}$:

$$\frac{(2)-(-5)}{(-4)-(10)}=\frac{2+5}{-4+(-10)}=\frac{7}{-14}=-\frac{7}{14}=-\frac{1}{2}$$

60. $x+5$

Evaluate the expression for $x=-3$:

$$(-3)+5=2$$

61. $-7-x$

Evaluate the expression for $x=-3$:

$$-7-(-3)=-7+3=-4$$

62. $2-x(4)$

Evaluate the expression for $x=-3$:

$$2-(-3)(4)=2-(-12)=2+12=14$$

63. $1+\dfrac{-24}{x}$

Evaluate the expression for $x=-3$:

$$1+\frac{-24}{-3}=1+8=9$$

64. Substitute 650 for T and 13 for n.

$$\frac{650}{13}=50$$

Each player must pay \$50 for the team to join the softball league.

65. a.

Time (hours)	Volume of Water (cubic feet)
0	$-50\cdot 0+400$
1	$-50\cdot 1+400$
2	$-50\cdot 2+400$
3	$-50\cdot 3+400$
4	$-50\cdot 4+400$
t	$-50\cdot t+400$

From the last row of the table, we see that the expression $-50t+400$ represents the volume of water (in cubic feet) remaining in the basement after water has been pumped out for t hours.

b. Substitute 7 for t in $-50t+400$:

$$-50(7)+400=-350+400=50$$

After 7 hours of pumping, there will be 50 cubic feet of water remaining in the basement.

66. To find the number of Americans who flew to Europe in 2014, we first need to find 10.2% of 10.8 million. To do this, multiply: $0.102(10.8)\approx 1.10$.

Now add 1.10 million people to 10.8 million people, which is: $1.10+10.8=11.9$ million people. So, the number of Americans who flew to Europe in 2014 was about 11.9 million people.

Chapter 2 Test

1. $-8-5=-8+(-5)=-13$

2. Since the two numbers have the same sign, the product will be positive: $-7(-9)=63$

3. $-3+9\div(-3)=-3+(-3)=-6$

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4. $(4-2)(3-7)=(2)(-4)=-8$

5. $\dfrac{4-7}{-1-5}=\dfrac{-3}{-6}=\dfrac{1}{2}$

6. $5-(2-10)\div(-4)=5-(-8)\div(-4)=5-2=3$

7. $-20\div5-(2-9)(-3)=-20\div5-(-7)(-3)$
$=-4-21$
$=-25$

8. Since the two numbers have different signs, the product will be negative:
$0.4(-0.2)=-0.08$

9. $-\dfrac{27}{10}\div\dfrac{18}{75}=-\dfrac{27}{10}\cdot\dfrac{75}{18}$
$=-\dfrac{3\cdot3\cdot3\cdot5\cdot5}{2\cdot5\cdot2\cdot3\cdot3}$
$=-\dfrac{3\cdot3\cdot5}{2\cdot2}$
$=-\dfrac{45}{4}$

10. $-\dfrac{3}{10}+\dfrac{5}{8}=-\dfrac{3}{10}\cdot\dfrac{4}{4}+\dfrac{5}{8}\cdot\dfrac{5}{5}$
$=\dfrac{-12}{40}+\dfrac{25}{40}$
$=\dfrac{-12+25}{40}$
$=\dfrac{13}{40}$

11. $3^4=3\cdot3\cdot3\cdot3=81$

12. $-4^2=-(4\cdot4)=-16$

13. $7+2^3-3^2=7+(2\cdot2\cdot2)-(3\cdot3)$
$=7+8-9$
$=15-9$
$=6$

14. $1-(3-7)^2+10\div(-5)$
$=1-(-4)^2+10\div(-5)$
$=1-(-4)(-4)+10\div(-5)$
$=1-16+(-2)$
$=-15+(-2)$
$=-17$

15. $\dfrac{84}{-16}=-\dfrac{84}{16}=-\dfrac{2\cdot2\cdot3\cdot7}{2\cdot2\cdot2\cdot2}=-\dfrac{3\cdot7}{2\cdot2}=-\dfrac{21}{4}$

16. $5-9=5+(-9)=-4$
The current temperature is $-4°\,\text{F}$.

17. a. The change in the tax audit rate from 2003 to 2005 can be found by subtracting: $9.7-6.5=3.2$.
So, from 2003 to 2005, the tax audit rate increased by 3.2 audits per 1000 tax returns.

b. The change in the tax audit rate from 2013 to 2015 can be found by subtracting: $8.4-9.6=-1.2$.
So, from 2013 to 2015, the tax audit rate decreased by 1.2 audits per 1000 tax returns.

c.

Years	Change
2003 – 2005	$9.7-6.5=3.2$
2005 – 2007	$10.3-9.7=0.6$
2007 – 2009	$10.3-10.3=0.0$
2009 – 2011	$11.1-10.3=0.8$
2011 – 2013	$9.6-11.1=-1.5$
2013 – 2015	$8.4-9.6=-1.2$

The 2-year period for which the change in tax audit rate was the most was from 2003 to 2005 with an increase of 3.2 audits for every 1000 tax returns filed.

18. To find the unit ratio of the number of background checks conducted for gun purchases in 2015 to the number in 2000, divide: $\dfrac{23\text{ million}}{8.4\text{ million}}\approx\dfrac{2.74}{1}$.
The number of background checks conducted for gun purchases in 2015 is about 2.74 million times the number conducted in 2000.

19. Substitute -6 for a, -2 for b, and 5 for c in the expression $ac-\dfrac{a}{b}$:
$(-6)(5)-\dfrac{(-6)}{(-2)}=-30-\dfrac{(-6)}{(-2)}$
$=-30-3$
$=-30+(-3)$
$=-33$

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20. Substitute -6 for a, -2 for b, 5 for c, and -1 for d in the expression $\dfrac{a-b}{c-d}$:

$$\frac{(-6)-(-2)}{(5)-(-1)}=\frac{-6+2}{5+1}=\frac{-4}{6}=-\frac{4}{6}=-\frac{2}{3}$$

21. Substitute -6 for a, -2 for b, and 5 for c in the expression $a+b^3+c^2$:

$$\begin{aligned}(-6)+(-2)^3+(5)^2&=(-6)+(-2)(-2)(-2)+(5\cdot 5)\\&=-6+(-8)+25\\&=-14+25\\&=11\end{aligned}$$

22. Substitute -6 for a, -2 for b, and 5 for c in the expression b^2-4ac:

$$\begin{aligned}(-2)^2-4(-6)(5)&=(-2)(-2)-4(-6)(5)\\&=4-(-120)\\&=4+120\\&=124\end{aligned}$$

23. $2x-3x$

Evaluate the expression for $x=-5$:

$$2(-5)-3(-5)=-10-(-15)=-10+15=5$$

24. $\dfrac{-10}{x}-6$

Evaluate the expression for $x=-5$:

$$\frac{-10}{-5}-6=2-6=-4$$

25. a.

Years since 2014	First-Class Mail Volume (billions of pieces)
0	$-2.1\cdot 0+63.8$
1	$-2.1\cdot 1+63.8$
2	$-2.1\cdot 2+63.8$
3	$-2.1\cdot 3+63.8$
4	$-2.1\cdot 4+63.8$
t	$-2.1t+63.8$

b. Substitute 7 for t in $-2.1t+63.8$:

$$-2.1(7)+63.8=-14.7+63.8=49.1$$

So, in 2021 (7 years after 2014) the U.S. Postal Service first-class mail volume was 49.1 billion pieces.

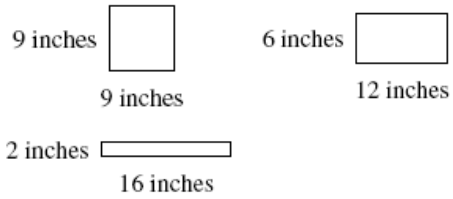
26. To find the number of bats broken in MLB games in 2014, we first need to find 63% of 2400 bats. To do this, multiply:

$0.63(2400)=1512$.

Now subtract 1512 bats from 2400 bats, which is: $2400-1512=888$ bats. So, the number of bats broken at MLB games in 2014 was 888.

Cumulative Review of Chapters 1 and 2

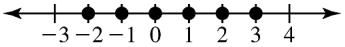
1. a. Answers may vary. Example:



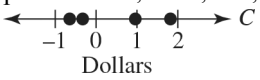
b. W and L are variables because their values are not fixed.

c. P is a constant because the perimeter is fixed at 36 inches.

2.



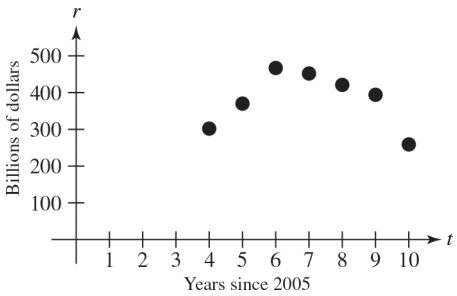
3. Decreases indicate negative values and increases indicate positive values. Graph the points $-0.67, -0.36, 0.96,$ and 1.82.



4. The x-coordinate is -5.

5. Independent variable: t
Dependent variable: V

6. a.



b. According to the table, the year in which the revenue was the most was 2011. The revenue in that year was \$467 billion.

c. According to the table, the year in which the revenue was the least was 2015. The revenue in that year was \$259 billion.

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d.

Years	Change
2009 – 2010	370 – 302 = 68
2010 – 2011	467 – 370 = 97
2011 – 2012	452 – 467 = –15
2012 – 2013	421 – 452 = –31
2013 – 2014	394 – 421 = –27
2014 – 2015	259 – 394 = –135

The annual revenue decreased the most from 2014 to 2015. The decrease in revenue during that period was \$135 billion.

e. The annual revenue increased the most from 2010 to 2011. The increase in revenue during that period was \$97 billion.

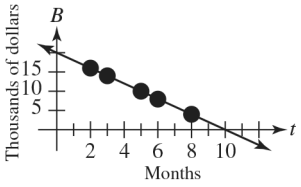
7. The input $x = -4$ leads to the output $y = -3$, so $y = -3$ when $x = -4$.

8. The output $y = 1$ originates from the input $x = 4$, so $y = 1$ when $x = 4$.

9. The line and the y-axis intersect at the point $(0, -1)$ so the y-intercept is $(0, -1)$.

10. The line and the x-axis intersect at the point $(2, 0)$, so the x-intercept is $(2, 0)$.

11. a.



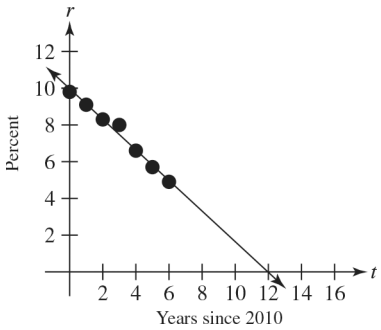
b. The balance in the checking account 4 months after the person was laid off was \$12 thousand.

c. The balance in the checking account was \$6 thousand 7 months after the person was laid off.

d. The B-intercept is (0, 20). This means when the person was laid off, her checking account balance was \$20 thousand.

e. The t-intercept is (10, 0). This means that there was no money in the checking account 10 months after she was laid off.

12. a.



b. According to the model, the unemployment rate in 2013 ($t = 3$) is about 7.5%. According to the table, the unemployment rate in 2013 is 8.0%. The error is -0.5%.

c. The r-intercept of the model is (0, 10). This means in 2010 the unemployment rate was 10%.

d. The t-intercept of the model is (12, 0). This means in 2022 the unemployment rate will be 0%. Since it is unlikely that there will ever be an unemployment rate of 0%, model breakdown has occurred.

13. $$\frac{3(-8)+15}{2-7(2)} = \frac{-24+15}{2-14} = \frac{-9}{-12} = \frac{3}{4}$$

14.
$$\begin{aligned} -4(3)+6-20\div(-10) &= -12+6-20\div(-10) \\ &= -12+6-(-2) \\ &= -12+6+2 \\ &= -6+2 \\ &= -4 \end{aligned}$$

15.
$$\begin{aligned} \left(-\frac{14}{15}\right)\div\left(-\frac{35}{27}\right) &= \left(-\frac{14}{15}\right)\left(-\frac{27}{35}\right) \\ &= \frac{14}{15}\cdot\frac{27}{35} \\ &= \frac{2\cdot 7\cdot 3\cdot 3\cdot 3}{3\cdot 5\cdot 5\cdot 7} \\ &= \frac{2\cdot 3\cdot 3}{5\cdot 5} \\ &= \frac{18}{25} \end{aligned}$$

16. $$\frac{3}{8}-\frac{5}{6} = \frac{3}{8}\cdot\frac{3}{3}-\frac{5}{6}\cdot\frac{4}{4} = \frac{9}{24}-\frac{20}{24} = \frac{9-20}{24} = -\frac{11}{24}$$

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17. $4-(7-9)^4+20\div(-4)$
$=4-(-2)^4+20\div(-4)$
$=4-(-2)(-2)(-2)(-2)+20\div(-4)$
$=4-16+20\div(-4)$
$=4-16+(-5)$
$=-12+(-5)$
$=-17$

18. $\dfrac{5-3^2}{4^2+2}=\dfrac{5-(3\cdot 3)}{(4\cdot 4)+2}=\dfrac{5-9}{16+2}=\dfrac{-4}{18}=-\dfrac{4}{18}=-\dfrac{2}{9}$

19. To find the change in low temperature, find the difference. Subtract: $-5-20=-25$.
The change in low temperature was $-25°\text{F}$.
(This means the temperature dropped $-25°\text{F}$ from $20°\text{F}$ on December 17 to $-5°\text{F}$ on December 18.)

20. $-2692+850-23=-1842-23=-1865$
The student will now owe the credit card company \$1865.

21. Substitute 1 for a, -4 for b, -3 for c, and 7 for d in the expression $\dfrac{a-b}{c-d}$:
$\dfrac{(1)-(-4)}{(-3)-(7)}=\dfrac{1+4}{-3+(-7)}=\dfrac{5}{-10}=-\dfrac{5}{10}=-\dfrac{1}{2}$

22. Substitute 2 for a, -3 for b, and -5 for c in the expression b^2-4ac:
$(-3)^2-4(2)(-5)=(-3)(-3)-4(2)(-5)$
$=9-(-40)$
$=9+40$
$=49$

23. $x-\dfrac{(-12)}{x}$
Evaluate the expression for $x=-4$:
$(-4)-\dfrac{(-12)}{(-4)}=(-4)-3=(-4)+(-3)=-7$

24. $-2x+7$
Evaluate the expression for $x=-4$:
$-2(-4)+7=8+7=15$

25. Evaluate $\dfrac{100(v-42)}{42}$ for $v=45$:
$\dfrac{100(45-42)}{42}=\dfrac{100(3)}{42}=\dfrac{300}{42}=\dfrac{50}{7}\approx 7.14$
A stock value today of \$45 represents about a 7.14% growth of the investment.

26. a.

Years since 2013	Enrollment (thousands of students)
0	$-11.3\cdot 0+67$
1	$-11.3\cdot 1+67$
2	$-11.3\cdot 2+67$
3	$-11.3\cdot 3+67$
4	$-11.3\cdot 4+67$
t	$-11.3t+67$

b. Substitute 5 for t in $-11.3t+67$:
$-11.3(5)+67=-56.5+67=10.5$
So, in 2018 (5 years after 2013) will be about 10.5 thousand students.

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Chapter 2

Lecture Notes, Detailed Comments, and Additional Explorations

This chapter includes chapter overviews, section-by-section lecture notes, comments about the lecture notes, and additional explorations. The lecture notes include short and medium homework assignments.

The comments about the lecture notes describe typical student difficulties and what an instructor can do to help students overcome these obstacles. This chapter contains a few explorations that do not appear in the textbook. For each exploration in this manual and the textbook, there is a discussion of what to do before, during, and after it.

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CHAPTER 1 OVERVIEW

In Section 1.1, students learn the meaning of *variable*, *constant*, and various types of numbers; students also graph data. In Section 1.2, students construct scatterplots of data and read bar graphs. In Section 1.3, students use a linear model to describe an exact linear relationship; students use such a model to make estimates and predictions and find the intercepts of the model. In Section 1.4, students perform similar tasks with a linear model that describes an approximate linear relationship.

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SECTION 1.1 LECTURE NOTES

Objectives

1. Describe the meaning of *variable* and *constant*.
2. Describe the meaning of *counting numbers*, *integers*, *rational numbers*, *irrational numbers*, *real numbers*, *positive numbers*, and *negative numbers*.
3. Use a number line to describe numbers.
4. Graph data.
5. Find the average (or mean) of a group of numbers.
6. Describe a concept or procedure.

Main point: Know the meaning of *variable* and *constant*.

OBJECTIVE 1

Definition *Variable*

A **variable** is a symbol that represents a quantity that can vary.

1. Let p be the price (in dollars) to see a Modest Mouse concert. What is the meaning of $p = 60$?
2. Let t be the number of years since 2015. What is the meaning of $t = 7$? of $t = -5$?
3. Choose a symbol to represent the number of students in a class. Explain why the symbol is a variable. Give two numbers that the variable can represent and two numbers that it cannot represent.

Definition *Constant*

A **constant** is a symbol that represents a specific number (a quantity that does *not* vary).

4. A rectangle has an area of 16 square feet. Let W be the width (in feet), L be the length (in feet), and A be the area (in square feet).
   - **a.** Sketch three possible rectangles of area 16 square feet.
   - **b.** Which of the symbols W, L, and A are variables? Explain.
   - **c.** Which of the symbols W, L, and A are constants? Explain.

OBJECTIVE 2

- The **counting numbers**, or **natural numbers**, are the numbers 1, 2, 3, 4, 5, ...
- The **integers** are the numbers ..., -3, -2, -1, 0, 1, 2, 3, ...
- The **positive integers** are the numbers 1, 2, 3, ...

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- The **negative integers** are the numbers $-1, -2, -3, \ldots$.
- The number 0 is neither positive nor negative.
- The **rational numbers** are the numbers that can be written in the form $\frac{n}{d}$, where n and d are integers and d is nonzero.
- The numbers represented on the number line that are *not* rational are called **irrational numbers**.
- The **real numbers** are all the numbers represented on the number line.

OBJECTIVE 3

5. Graph the integers between -4 and 2, inclusive, on one number line.

6. Graph the integers between -4 and 2 on one number line.

7. Graph all the numbers 3, -5, $\frac{7}{2}$, 0, -2.7, and -1.3 on one number line.

OBJECTIVE 4

8. The number of homes sold by a real estate agent for various months are 2, 3, 0, 7, 5. Let n be the number of homes sold by the agent in a month. Graph the agent's sales on a number line.

OBJECTIVE 5

Definition *Average, mean*

To find the **average** (or **mean**) of a group of numbers, we divide the sum of the numbers by the number of numbers in the group.

9. Use the data values in Problem 8 to find the mean sales of the agent and indicate it on the number line you sketched in Problem 8.

10. The average smartphone screen size (in inches) for the years 2007, 2008, 2009, 2010, 2011, 2012, 2013, and 2014 are 2.59, 2.67, 3.00, 3.27, 3.53, 4.03, 4.38, 4.86, respectively (Source: *PhoneArena.com*). Let A be the average smartphone screen size (in inches) in a given year.

a. Graph the data. Find the mean of the data values and indicate it on the graph.

b. Did the annual average screen size of a smartphone increase, decrease, stay approximately constant, or none of these from 2007 to 2014, inclusive? Explain.

c. Did the annual *increases* in the average size of a smartphone generally increase, decrease, stay approximately constant, or none of these from 2007 to 2014, inclusive? Explain.

When we write numbers on a number line, they should increase by a fixed amount and be equally spaced.

OBJECTIVE 2 (revisited)

- The **negative numbers** are the real numbers less than 0.
- The **positive numbers** are the real numbers greater than 0.

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11. Let T be the temperature (in degrees Fahrenheit). What value of T represents the temperature 20 degrees Fahrenheit below 0? Graph the number on a number line.

OBJECTIVE 6

Discuss the "Guidelines on Writing a Good Response" on page 9 of the textbook.

12. Describe how to graph data values. Describe how to compute the mean of the data values. Describe the meaning of the mean.

SHORT HW 5, 13, 21, 29, 35, 37, 39, 47, 55, 61, 67, 69, 79

MEDIUM HW 5, 9, 11, 13, 21, 23, 29, 31, 35, 37, 39, 45, 47, 55, 59, 61, 67, 69, 73, 75, 79

SECTION 1.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn the meaning of *variable* and *constant*, which lay the foundation for many definitions to come. Students graph numbers on a number line; this skill will be needed for drawing scatterplots in Section 1.2. The section includes definitions of various types of numbers such as rational numbers; the textbook will occasionally use these terminologies. Students calculate the mean of a group of numbers; this terminology will be used throughout the textbook to describe some variables for authentic situations. Student will rarely calculate means in subsequent sections. Finally, students learn some guidelines on how to describe concepts or procedures; most Homework sections contain exercises that instruct students to describe such things.

OBJECTIVE 1 COMMENTS

Problem 2 in the lecture notes is good preparation for working with the explanatory variable for some time-series data. Most students think that time cannot be negative and are surprised that $t = -5$ represents the year 2010. Students will sometimes get a negative value when using a model to make an estimate of the response variable of some time-series data.

For Problem 3 in the lecture notes, discussing unreasonable values of the variable is a nice primer for the concept of model breakdown, which is discussed in Section 1.4.

After completing Problems 1–3 in the lecture notes, I quickly mention several more examples of variables, pointing out that variables "are all around us." I suggest that throughout the rest of the day, my students notice quantities that can be represented by variables. I tell my students that a variable is one of three main concepts in algebra (the other two being expressions and equations).

When discussing constants, I give π and a number such as 5 as examples.

Before doing Problem 4 in the lecture notes, I discuss the meaning of the area of a flat surface. Most students do not know that the area (in square inches) is the number of square inches that it takes to cover the surface. I remind my students that the area of a rectangle is equal to the rectangle's length times its width and explain this by breaking up a rectangle into unit squares.

The perimeter of a rectangle is defined in Exercises 23 and 24 of Homework 1.1. Some students have difficulty with these two exercises (as well as Exercises 25–28) because they are not exactly like Problem 4 in the lecture notes.

OBJECTIVE 2 COMMENTS

When I define counting numbers and integers, I make sure that students understand the meaning of the ellipsis. I quickly define the various types of numbers. I give especially light treatment to defining rational and irrational numbers because even though these terminologies are used in the textbook, a superficial understanding of these terminologies will suffice.

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OBJECTIVE 3 COMMENTS

Most students do not understand how Problems 5 and 6 in the lecture notes differ. The word "inclusive" is used many times in the textbook. I make sure I discuss how to graph decimal numbers because this skill will come in handy throughout the course when drawing scatterplots of data (see Problem 7 in the lecture notes).

OBJECTIVE 4 COMMENTS

For Problem 8 in the lecture notes, I emphasize that the variable "n" and the units of n should be included to the right of the number line.

Textbook and Workbook Exploration

The 15-minute "Reasonable values of a variable" exploration is a good primer for the concept of model breakdown, which students will study in Section 1.4.

OBJECTIVE 5 COMMENTS

I emphasize that the mean of a group of data estimates the center of the numbers graphed on a number line. This is a good primer for students who will take statistics. Means are used a number of times in this course to describe quantities, but students will rarely need to compute means (aside from in this section's Homework).

For Problem 10 in the lecture notes, I discuss the importance of using uniform scaling. This is a good primer for constructing scatterplots in Section 1.2, where some students will mistakenly use nonuniform scaling if not warned against this practice.

Workbook Exploration

In the following 20-minute exploration, groups can have a visceral sense that the mean measures the center of a group of numbers, and it can be used to compare two groups of data.

EXPLORATION *Interpreting the average of some numbers*

1. Plot the given numbers on a number line. Find the average of the given numbers. Indicate it on your number line. How does the position of the average on the number line relate to the numbers you plotted?

a. 1, 3, 5 **b.** 2, 5, 7, 10 **c.** 1, 2, 5, 9, 12, 13

2. In general, what does the average of some numbers tell you about the numbers?

3. The prices (in dollars) of hot dogs at each of the stadiums for the baseball teams in the National League Central (NLC) and the National League West (NLW) are shown in the following table.

NLC Team	Hot Dog Price (dollars)	NLW Team	Hot Dog Price (dollars)
Chicago Cubs	5.50	Arizona Diamondbacks	2.75
Cincinnati Reds	1.00	Colorado Rockies	4.75
Milwaukee Brewers	3.50	Los Angeles Dodgers	5.50
Pittsburgh Pirates	3.25	San Diego Padres	4.00
St. Louis Cardinals	4.25	San Francisco Giants	5.25

Source: *Team Marketing Report*

a. Find the average hot dog price for the NLC.

b. Find the average hot dog price for the NLW.

c. Which tends to have cheaper hot dogs, stadiums in the NLC or the NLW? Explain.

OBJECTIVE 2 (REVISITED) COMMENTS

I emphasize that 0 is neither negative nor positive.

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OBJECTIVE 6 COMMENTS

If I do not have time to do Problem 12 in the lecture notes, I tell my students to read Example 10 in the textbook and the preceding "Guidelines on Writing a Good Response" at home. Students will be asked to explain concepts or procedures in most Homework sections.

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SECTION 1.2 LECTURE NOTES

Objectives

1. Describe the meaning of *ordered pair*, *coordinate*, and *coordinate system*.
2. Construct scatterplots.
3. Describe the meaning of *explanatory variable* and *response variable*.
4. Read bar graphs.
5. Plot points on a coordinate system.

Main point: Construct scatterplots.

OBJECTIVE 1

1. A football team scores the following number of points (listed in chronological order) in five football games: 3, 7, 7, 14, 17. Let s be the team's score (in points) in a game. Use a number line to graph the team's scores.

There are two limitations with using one number line for Problem 1:

- The graph does not show that there were two scores of 7 points each.
- The graph does not show which was the first score, the second score, and so on.

2. Let s be the football team's score (in points) in the nth game.

a. Discuss the meaning: $s = 14$ when $n = 4$.

b. Use a table to describe the values of n and s.

- The *ordered pair* $(4, 14)$ means that $n = 4$ and $s = 14$.
- For $(4, 14)$, the *n-coordinate* is 4 and the *s-coordinate* is 14.
- Draw a coordinate system and indicate the axes, quadrants, and origin.

OBJECTIVE 2

3. Plot the ordered pairs described by the table you constructed in part (b) of Problem 2.

A graph of plotted ordered pairs is called a **scatterplot**.

OBJECTIVE 3

Definition *explanatory and response variables*

Assume that an authentic situation can be described by using the variables t and p, and assume that t affects (explains) p.

- We call t the **explanatory variable** (or **independent variable**).
- We call p the **response variable** (or **dependent variable**).

For each situation, identify the explanatory variable and the response variable.

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4. Let n be the number of words in a document and let t be the number of minutes it takes to type the document.

5. Let L be the length (in inches) of a person's bangs at t weeks since her hair was last cut.

6. Let P be the percentage of American adults who own at least three cars and I be the annual income (in thousands of dollars) of an American adult.

For an ordered pair (a, b), we write the value of the explanatory variable in the first (left) position and the value of the response variable in the second (right) position.

7. Let n be the number of words in a document and let t be the number of minutes it takes to type the document. What does the ordered pair $(400, 8)$ represent?

8. Let L be the length (in inches) of a candle at t minutes since the candle was lit. What does the ordered pair $(30, 6)$ represent?

OBJECTIVE 2 (revisited)

Columns of Tables and Axes of Coordinate Systems

Assume that an authentic situation can be described by using two variables. Then

- For tables, the values of the explanatory variable are listed in the first column and the values of the response variable are listed in the second column. [Construct a table.]
- For coordinate systems, the values of the explanatory variable are described by the horizontal axis and the values of the response variable are described by the vertical axis. [Draw a figure.]

9. The average numbers of Internet searches per day using Google are shown in the following table for various years.

Year	Average Number of Searches Per Day (in billions)
2008	1.7
2009	2.6
2010	3.6
2011	4.7
2012	5.1
2013	5.9
2014	5.7
2015	7.8
2016	9.0

Source: *Google*

Let n be the average number of searches per day (in billions) at t years since 2005.

a. Construct a scatterplot of the data.

b. Did the average number of searches per day increase, decrease, stay approximately constant, or none of these? Explain.

c. Did the annual *increase* in the average number of searches per day increase, decrease, stay approximately constant, or none of these? Explain.

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10. The fatality rates from automobile crashes are shown in the following table for various speeds.

Speed Group (mph)	Speed Used to Represent Speed Group (mph)	Fatality Rate (deaths per 1000 crashes)
0–30	15	2.5
30–40	35	3.5
40–50	45	6.1
50–60	55	15.3
over 60	70	16.9

Source: *National Highway Transportation Safety Administration*

Let r be the fatality rate (deaths per 1000 crashes) for a speed of s mph. Construct a scatterplot of the data.

OBJECTIVE 4

A **bar graph** is a diagram with two axes that we can use to compare measurements of two or more items (see Fig. 2.1).

11. The annual numbers of visitors to ski resorts are illustrated in the bar graph in Fig. 2.1 for various regions in the 2015/2016 season (Source: *National Ski Areas Association*).

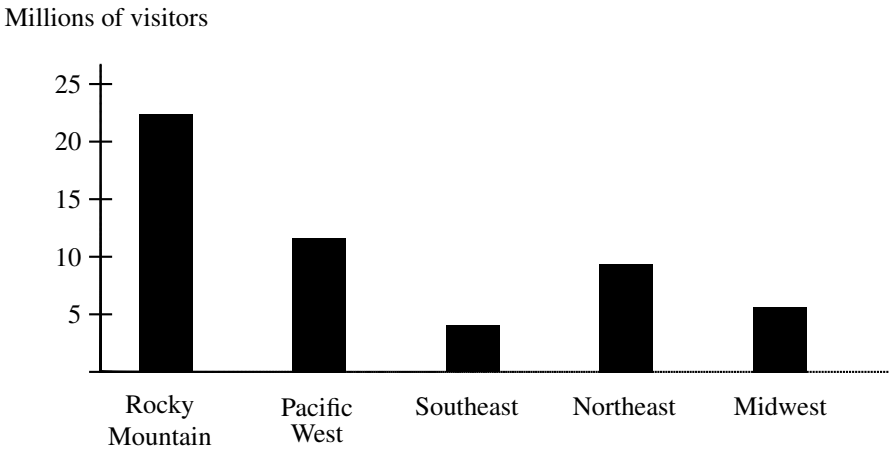

Figure 2.1: Visitors to ski resorts by region

a. For which region is the number of visitors the greatest? What is that number?

b. For which region is the number of visitors the least? What is that number?

c. Estimate the number of visitors to ski resorts in the Northeast.

OBJECTIVE 5

- When we plot points that are not being used to describe authentic situations, we call the horizontal axis the *x-axis* and the vertical axis the *y-axis*.
- In this case, x is the explanatory variable and y is the response variable.
- The ordered pair $(2, 5)$ means that $x = 2$ and $y = 5$.

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12. Plot the points $(6, 2)$, $(-3, 1)$, $(4, -5)$, and $(-2, -4)$ on a coordinate system.

SHORT HW 5, 9, 17, 21, 29, 37, 39, 41, 45, 53, 61

MEDIUM HW 1, 5, 7, 9, 13, 17, 19, 21, 23, 29, 33, 35, 37, 39, 43, 45, 49, 53, 55, 61

SECTION 1.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students construct scatterplots, which is an important modeling skill of the course. Students learn the meaning of explanatory and response variables. They also read bar graphs, although this skill is not required for the rest of the textbook, except for the Stocks Lab in Chapter 2 (see pages 116 and 117 of the textbook).

OBJECTIVE 1 COMMENTS

Although most students have worked with coordinate systems, they have not considered the benefits of such a system (see Problem 1 and the comments that follow it).

When discussing the meaning of the ordered pair $(4, 14)$ within the context of the football team scores, I do not explain why we choose to let the first coordinate represent values of n and the second coordinate represent values of s. I first want students to have a sense of the "big picture" before discussing explanatory and response variables.

OBJECTIVE 2 COMMENTS

For Problem 3, I point out that there is a general upward trend. Sometimes I sketch a line that comes close to the points to suggest what's to come in Section 1.3; if I do this, I make sure that my students understand that the line is not part of the scatterplot.

Workbook Exploration

The following 20-minute exploration addresses an error students make more than any other error when it comes to constructing scatterplots.

Group Exploration

Constructing a scatterplot

The prices of two-pack EpiPens are shown in the following table for various years.

Year	Price (dollars)
2006	108
2008	110
2010	151
2012	209
2014	355
2016	609

Source: *Elsevier*

Let p be the price (in dollars) at t years since 2005.

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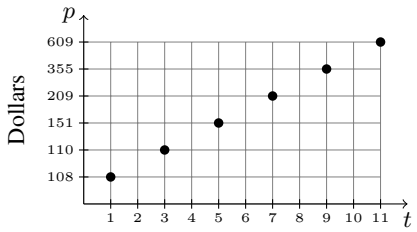
A student constructs the following scatterplot.



1. The student concludes that the price increased by the same amount every two years. By referring to the data table, do you agree with the student? Explain.
2. Describe all errors in the student's scatterplot. Construct a correct scatterplot.
3. Did the *increases* in the price increase, decrease, stay approximately constant, or none of these? Explain.
4. What is the main point of this exploration?

Workbook Exploration

In the following 15-minute exploration, groups can have well-needed practice with constructing a scatterplot. Many groups will likely forget to indicate the units and variables on the axes and use non-uniform scaling.

Group Exploration

Constructing and analyzing a scatterplot

Steam is a popular videogame-download store. The number of new games added to Steam are shown in the following table for various years.

Year	Number of New Games Added (thousands)
2013	0.5
2014	1.3
2015	2.7
2016	4.7
2017	7.0

Source: *SteamSpy*

1. Construct a scatterplot of the data.
2. Did the number of new games added to Steam increase, decrease, stay approximately constant, or none of these? Explain.
3. Did the *increases* in new games added to Steam increase, decrease, stay approximately constant or none of these?
4. In the first six months of 2018, 4.0 thousand new games were added to Steam. A student says that this means $2(4.0) = 8.0$ thousand new games will be added for the entire year. What would you tell the student?

Textbook and Workbook Exploration

In the 20-minute "Analyzing points below, above, and on a line containing points with equal coordinates," groups

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can practice interpreting order pairs in which the data is not time-series data. Students are much more challenged by performing such interpretations with non-time-series data than with time-series data.

OBJECTIVE 3 COMMENTS

Many students have a tough time learning this concept. However, even weaker students will eventually learn this concept because it's addressed so often in the course.

OBJECTIVE 2 (REVISITED) COMMENTS

When constructing the scatterplots for Problems 9 and 10, I emphasize that the scaling should be uniform; it's a common error to use nonuniform scaling. For example, for Problem 10, some students will use 15, 35, 45, 55, 70 (the speeds used to represent the speed groups) as the scaling for the s-axis.

I emphasize that each axis should be labeled with the appropriate variable and units. I warn my students that care should be taken with using zigzag lines for a portion of an axis as they can lead to incorrect results about such things as finding the intercepts of a linear model (in Sections 1.3 and 1.4).

Problem 10 helps students get accustomed to working with grouped data sets. Students will frequently work with such data sets in this course.

OBJECTIVE 4 COMMENTS

If you are tight on time, the topic of bar graphs can be skipped because it is included only in this section's Homework and in the Stocks Lab (in Chapter 2).

OBJECTIVE 5 COMMENTS

For Problem 12, I plot the points by first looking left or right and *then* looking up or down. This is consistent with the later-needed concept of inputs leading to outputs.

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SECTION 1.3 LECTURE NOTES

Objectives

1. Describe the meaning of *linearly related*, *model*, *linear model*, *input*, and *output.*
2. Use a linear model to make estimates and predictions.
3. Use a scatterplot to help decide whether to model a situation with a linear model.
4. Find *intercepts* of a line.
5. Find intercepts of a linear model.

Main point: Use a line to describe an exact linear relationship.

OBJECTIVE 1

1. A basement is flooded with water. Let w be the amount (in hundreds of cubic feet) of water that remains in the basement after t hours of pumping water out of the basement. Values of t and w are listed in the following table.

t (hours)	w (hundreds of cubic feet)
0	9
2	7
4	5
6	3
8	1

a. Construct a scatterplot of the data.

b. Draw the line that contains the points of the scatterplot.

Definition *Linearly related*

If two quantities of an authentic situation are described accurately by a line, then the quantities (and the variables representing those quantities) are **linearly related**.

Definition *Model*

A **model** is a mathematical description of an authentic situation. We say the description *models* the situation.

Definition *Linear model*

A **linear model** is a nonvertical line that describes the relationship between two quantities in an authentic situation.

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OBJECTIVE 2

2. Use the water model to estimate the amount of water in the basement after 3 hours of pumping.

3. Use the model to estimate when there is 2 hundred cubic feet of water in the basement.

Definition *Input, output*

An **input** is a permitted value of the *explanatory* variable that leads to at least one **output,** which is a permitted value of the *response* variable.

For Problems 4–7, refer to the water model you sketched in part (b) of Problem 1.

4. Find an output that comes from the input $t = 5$.

5. Find an input that leads to the output $w = 8$.

6. Is $t = -2$ an input? Explain.

7. Is $w = -1$ an output? Explain.

OBJECTIVE 3

8. Consider the scatterplots of data for situations 1, 2, and 3 shown in Figs. 2.2, 2.3, and 2.4, respectively. For each situation, determine whether a linear model would describe the situation well.

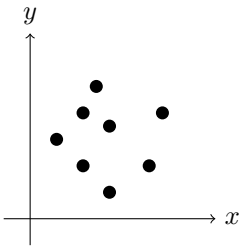

Figure 2.2: Scatterplot for situation 1

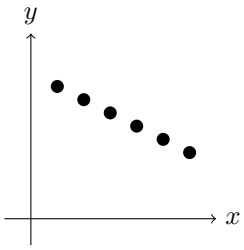

Figure 2.3: Scatterplot for situation 2

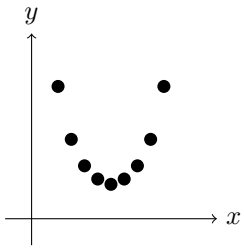

Figure 2.4: Scatterplot for situation 3

We construct a scatterplot of data to determine whether the data points lie on a line. If the points lie on a line, then we draw the line and use it to make estimates and predictions.

OBJECTIVE 4

Definition *Intercepts of a line*

An **intercept** of a line is any point where the line and an axis (or axes) of a coordinate system intersect. There are two types of intercepts of a line sketched on a coordinate system with an x-axis and a y-axis:

- An ***x*-intercept** of a line is a point where the line and the x-axis intersect. [Sketch a figure.] The y-coordinate of an x-intercept is 0.
- A ***y*-intercept** of a line is a point where the line and the y-axis intersect. [Sketch a figure.] The x-coordinate of a y-intercept is 0.

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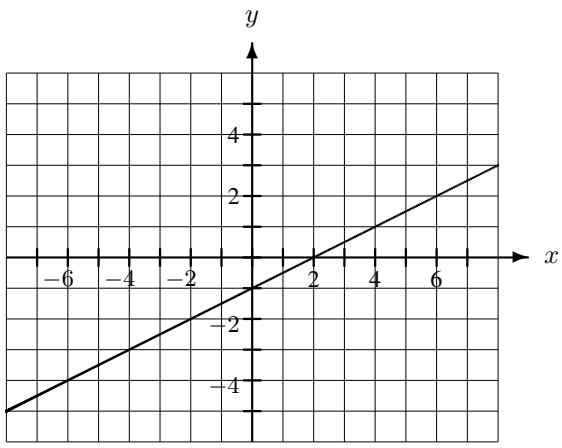
9. Refer to the following figure.

a. Find the x-intercept of the line.

b. Find the y-intercept of the line.

c. Find y when $x = -6$.

d. Find the x when $y = -3$.



OBJECTIVE 5

10. Find the w-intercept of the water model. What does it mean in this situation?

11. Find the t-intercept of the water model. What does it mean in this situation?

OBJECTIVES 1–5 (revisited)

12. Let v be the value (in thousands of dollars) of a car when it is t years old. Some pairs of values of t and v are listed in the following table.

t (years)	v (thousands of dollars)
1	21
2	18
4	12
6	6
7	3

a. Construct a scatterplot of the data. Then draw a linear model.

b. Estimate the value of the car when it is 3 years old.

c. Estimate the age of the car when it is worth $9 thousand.

d. What is the v-intercept of the model? What does it mean in this situation?

e. What is the t-intercept of the model? What does it mean in this situation?

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SHORT HW 1, 3, 5, 11, 13, 15, 19, 21, 27, 31, 43, 47

MEDIUM HW 1–21 odd, 25, 27, 31, 37, 39, 41, 43, 47

SECTION 1.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use a linear model to make estimates and predictions. They find intercepts of lines (nonmodels) and linear models. This section and Section 1.4 provide the "big picture" of Chapters 1–5.

OBJECTIVE 1 COMMENTS

For Problem 1, I remind my students about the concepts of explanatory and response variables and about which type of variable is represented by which axis. I also remind students to use uniform scaling and to label each axis with the appropriate variable and units.

When discussing the meaning of a model and, in particular, a linear model, I refer to the water model and give intuitive examples such as the textbook's discussion about airplane models (see the paragraph in the middle of page 27).

OBJECTIVE 2 COMMENTS

For Problems 2 and 3, I use arrows to set the stage for discussing inputs and outputs. (For an example of using arrows, see Fig. 34 on page 27 of the textbook.) I require my students to use such arrows so that I can better evaluate their work on quizzes and exams. It's difficult to award any partial credit if no arrows are included.

I explain to my students why their results of homework exercises in this section may not match exactly with the answers in the back of the text. Although I want my students to carefully draw scatterplots and linear models, I don't want them to be anxious about getting the exact results.

Problems 6 and 7 are good primers for the concept of model breakdown, which will be discussed in Section 1.4.

Textbook and Workbook Exploration

Having groups work on the 15-minute "Linear modeling" exploration is a great way to introduce linear modeling.

OBJECTIVE 3 COMMENTS

Problem 8 helps students see that it's not a good idea to use a linear model for every situation.

OBJECTIVE 4 COMMENTS

I emphasize that students should use an ordered pair, rather than a single number, to describe an intercept. Some students have difficulties with parts (c) and (d) of Problem 9.

OBJECTIVE 5 COMMENTS

For Problems 10 and 11, some students are not sure which coordinates of the intercepts are 0. It sometimes helps to remind students that the first coordinate is a value of the explanatory variable and the second coordinate is a value of the response variable. Some students have trouble determining the meaning of the intercepts. It can help to write "t" and "w" just below the coordinates of the ordered pairs corresponding to the intercepts.

Having groups do Problem 12 serves as a great summary of the section.

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SECTION 1.4 LECTURE NOTES

Objectives

1. Describe the meaning of *approximately linearly related*.
2. Use a linear model to make estimates and predictions.
3. Find errors in estimations.
4. Describe the meaning of *interpolate*, *extrapolate*, and *model breakdown*.
5. Modify a model.

Main point: Use a linear model to describe an approximate linear relationship.

OBJECTIVE 1

1. The annual revenues of IKEA are shown in the following table for various years.

Year	IKEA Revenue (billons of euros)
2009	21.8
2010	23.5
2011	25.2
2012	27.6
2013	28.5
2014	29.3
2015	32.7

Source: *IKEA*

Let r be the annual revenue (in billions of euros) of IKEA at t years since 2005.

a. Construct a scatterplot of the data.

b. Draw a line that comes close to the points in your scatterplot.

If the points in a scatterplot of data lie close to (or on) a line, we say that the relevant variables are **approximately linearly related.**

OBJECTIVE 2

2. Use the IKEA model to predict the revenue in 2020. Convert your result to U.S. dollars, assuming 1 euro will be worth 1.18 U.S. dollars.

3. Use the IKEA model to predict when the revenue will be $44 billion euros.

- We construct a scatterplot of data to determine whether the relevant variables are approximately linearly related.
- If so, we draw a line that comes close to the data points and use the line to make estimates and predictions.

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OBJECTIVE 3

4. Use the IKEA model to estimate the revenue in 2015. Is your result an underestimate or an overestimate? Calculate the error in the estimate.

5. Use the IKEA model to estimate the revenue in 2014. Is your result an underestimate or an overestimate? Calculate the error in the estimate.

Underestimates and Overestimates

Suppose that an explanatory variable t and a response variable p are approximately linearly related. For a given data point $(a,\ b)$,

- If a linear model is below the data point, the model underestimates the value of p when $t = a$. [Draw a figure.]
- If a linear model is above the data point (a, b), the model overestimates the value of p when $t = a$. [Draw a figure.]

OBJECTIVE 4

6. The percentages of Apple's annual revenue that is from the iPad are shown in the following table for various years.

Year	Percent
2012	19.8
2013	18.7
2014	16.6
2015	9.9
2016	9.6
2017	7.1

Source: *Apple*

Let p be the percentage of Apple's annual revenue that is from the iPad at t years since 2010.

a. Construct a scatterplot of the data. Draw a line that comes close to the points in your scatterplot.

b. Use the model to estimate the percentage of Apple's revenue that is from the iPad in 2015. Is your result an underestimate or an overestimate?

c. Use the model to estimate when 4% of Apple's annual revenue will be from the iPad.

d. What is the n-intercept of the model? What does it mean in this situation?

e. What is the t-intercept of the model? What does it mean in this situation? [might be model breakdown]

f. Use the model to predict the percentage of Apple's revenue that will be from the iPad in 2022. [model breakdown]

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Interpolation and Extrapolation

For a situation that can be described by a model whose explanatory variable is t,

- We preform **interpolation** when we use a part of the model whose t-coordinates are between the t-coordinates of two data points
- We preform **extrapolation** when we use a part of the model whose t-coordinates are not between the t-coordinates of any two data points

Definition *Model breakdown*

When a model yields a prediction that does not make sense or an estimate that is not a good approximation, we say **model breakdown** has occurred.

OBJECTIVE 5

7. In light of your responses to parts (e) and (f) of Problem 6, modify your model accordingly.

SHORT HW 3, 5, 7, 11, 15, 17, 19, 21, 27

MEDIUM HW 1, 3, 5, 7, 11, 15, 17, 19, 21, 23, 27

SECTION 1.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn to use a linear model when two variables are approximately linearly related to make estimates and predictions. Students will calculate errors in estimates and determine when model breakdown has occurred. This section flows nicely from Section 1.3. Upon completion of this section, students will have the "big picture" of Chapters 1–5.

OBJECTIVE 1 COMMENTS

Before showing students the data in Problem 1, I ask them whether they think IKEA's revenue is increasing, decreasing, or neither. I play this "game" with students before revealing each data set in the course because it's a great way to engage students intellectually and sometimes generates interesting discussions.

Some students need yet more reminders about the following: explanatory and response variables, using uniform scaling, including variables and their units on the axes, and the meaning of phrases such as "at t years since 2005."

For Problem 1, I ask my students whether a line contains *all* the data points. Then I ask whether a line comes close to all the points. Then I draw various lines and discuss which ones are acceptable linear models. I emphasize that it's better for a model to come close to *all* the data points, rather than contain a *few* points and not come close to the other points. I compare a reasonable linear model that comes close to the data points, but does not contain any data points, with the highly inaccurate linear model that contains the data points $(4,\ 29.3)$ and $(5,\ 32.7)$.

Problem 1 is a good spring board for defining the terminology *approximately linearly related*.

Textbook and Workbook Exploration

If time permits, having groups of students work on the 15-minute "Approximately linearly related variables" exploration is a great way to begin this section. Without the hint to draw a *straight* line, many groups would

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sketch a "zigzag" curve rather than a line for Problem 2 of the exploration. Make sure to discuss this issue because the homework exercises won't include the word "straight."

OBJECTIVE 2 COMMENTS

For Problems 2 and 3, I continue the practice of using arrows to make estimates and predictions. I remind my students of the meaning of *input* and *output*. I also remind students that their answers may not match exactly with the answers in the back of the textbook; there are now three reasons: we can't draw what we intend to draw perfectly, there are many reasonable linear models when the variables are approximately linearly related, and we must estimate the values of inputs and outputs.

Workbook Exploration

The following 20-minute exploration could be assigned at the start of class. Depending on how well groups plot the data points, some groups may draw zigzag lines, which is actually a reasonable thing to do because they don't know of the advantages of using a line. Because Section 1.3 consists of a graphical approach, most groups will likely do the same for this exploration, but one or two groups may use a numerical approach. For groups that come up with simplistic approaches, challenge them to develop more sophisticated ones. For example, if a group uses the increase in Snapchat users only from 2017 to 2018 to predict the number of users in 2022, you could nudge them toward using the average of the increases from one year to the next. It may seem that a numerical approach is too far askew from the main point of this section, but this open-ended exploration prevents mimicry, supports the powerful learning process of productive struggle, and has students consider the key concept of trend. During the debrief, comparing and contrasting various groups methods can lay a great foundation for the entire chapter.

Group Exploration

Section Opener: Making a prediction

The number of daily active Snapchat users are shown in the following table for the first quarter of various years. Predict the number of daily active Snapchat users in the first quarter of 2022.

Year	Number of Daily Active Snapchat Users (millions)
2014	46
2015	80
2016	122
2017	166
2018	191

Source: *Snap Inc.*

OBJECTIVE 3 COMMENTS

For many students, the distinction between a model and reality is foggy. For example, many students would think that the answers to Problems 4 and 5 are the actual revenues \$32.7 billion and \$29.3 billion, respectively, even if the linear model does not contain the data points $(5, 32.7)$ and $(4, 29.3)$. Making such a distinction should pay dividends both in the short and long run. For an intuitive explanation, I tell my students that a model is fiction, which we hope is very close to the truth.

After completing Problems 4 and 5, I ask my students how we could tell whether an estimate is an underestimate or an overestimate by observing whether the linear model is below or above a given data point.

I point out that for some non-time-series data, several data points might all have the same explanatory-variable coordinate; in such cases, we must specify for which data point we have calculated the error.

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Textbook and Workbook Exploration

The 20-minute "Identifying types of modeling errors" takes a closer look at the various types of errors. Although not absolutely essential to the course, this activity will drive home the point why modeling does not yield exact values, especially when it's done graphically by hand.

OBJECTIVE 4 COMMENTS

Just like when I present Problem 1, before showing students the data in Problem 6, I ask students whether they think the percentage of Apple's annual revenue that is from the iPad is increasing, decreasing, or neither. The answer to this question is not obvious, but students might be more sensitive to what's hot in tech sales than many of us. I follow up this query by asking why this information might be useful to Apple and its competitors. Such questions can attract the attention of students who may be weak in mathematics but strong in other areas, such as business.

Problem 6 serves as a nice summary of the section. Even though finding intercepts of a linear model was discussed in Section 1.3, most students will benefit from reviewing this [in parts (d) and (e)]. I remind students to include both coordinates of an intercept. Students sometimes need help determining which variable is represented by which coordinate.

When discussing interpolation and extrapolation, I refer to parts (b) and (c), respectively, of Problem 6. Discussing the dangers of extrapolation lays a nice foundation for discussing model breakdown. Although extrapolation is highly risky, the textbook has students perform extrapolation, especially with time-series data because students tend to find it to be more interesting to make predictions than to make estimates.

Parts (e) and (f) of Problem 6 serve as a good springboard to defining model breakdown. This concept is important because most models do "break down." I tell students that when model breakdown occurs, they must say so and explain why when completing homework, quizzes, and tests. I reassure them that they do not need to be an expert in all subjects to do so. For example, I tell them whether or not they decide that model breakdown occurs in part (e) of Problem 6, I will reward them all points, provided they state the meaning of the t-intercept, point out that it's a striking result, and give a careful explanation of whether they think model breakdown has occurred.

OBJECTIVE 5 COMMENTS

A discussion about parts (e) and (f) of Problem 6 leads elegantly to a discussion about modifying the model by sketching (nonlinear) curves that might be better models. In general, attempts at such modifications can generate fun, interesting discussions.

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CHAPTER 2 OVERVIEW

In Section 2.1, students use expressions to describe authentic quantities and how to evaluate such expressions. In Section 2.2, students perform operations with fractions. Students add real numbers in Section 2.3 and subtract real numbers in Section 2.4. They also find the change in a quantity in Section 2.4. In Section 2.5, students work with ratios and percents. They also multiply and divide real numbers in Section 2.5. In Section 2.6, students work with exponents and use the rules for order of operations to perform computations and evaluate expressions.

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SECTION 2.1 LECTURE NOTES

Objectives

1. Describe the meaning of *expression* and of *evaluate an expression*.
2. Use expressions to describe authentic quantities.
3. Evaluate expressions.
4. Translate English phrases into and from mathematical expressions.
5. Describe two roles of a variable.
6. Evaluate expressions with more than one variable.

Main point: Describe the meaning of *expression* and how to evaluate one.

OBJECTIVES 1–3

1. A person is driving 3 miles per hour over the speed limit. For each speed limit shown, find the diving speed.

a. 55 mph **b.** 70 mph **c.** s mph

We call $s + 3$ is an *expression*.

Definition *Expression*

An **expression** is a constant, a variable, or combination of constants, variables, operation symbols, and grouping symbols, such as parentheses.

Here are some examples of expressions: $x + 9$, 5, $x - 7$, π, $\dfrac{20}{x}$, xy

2. Substitute 65 for s in the expression $s + 3$ and discuss the meaning of the result (see Problem 1).

We say we have *evaluated the expression* $s + 3$ at $s = 65$.

Definition *Evaluate an expression*

We **evaluate an expression** by substituting a number for each variable in the expression and then calculating the result. If a variable appears more than once in the expression, the same number is substituted for that variable each time.

3. A certain type of pen costs \$3.

a. Complete the following table to help find an expression that describes the total cost (in dollars) of n pens. Show the arithmetic to help see the pattern.

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Number of Pens	Total Cost (dollars)
5	
6	
7	
8	
n	

b. Evaluate your result for $n = 10$. What does it mean in this situation?

- We avoid using $\times$ for the multiplication operation.
- Each of the following expressions describes multiplying 3 by n: $3n$, $3 \cdot n$, $3(n)$, $(3)n$, and $(3)(n)$.

OBJECTIVE 4

Mathematics is a language.

Definition *Product, factor, and quotient*

Let a and b be numbers. Then

- The **product** of a and b is ab. We call a and b **factors** of ab.
- The **quotient** of a and b is $a \div b$, where b is not zero.

4. Use phrases such as "2 plus x" and sentences such as "Add 2 and x." to complete the second column of the following table.

Mathematical Expression	English Phrase or Sentence
$2 + x$	
$2 - x$	
$2 \cdot x$	
$2 \div x$	

Warning: Subtracting 2 from 7 is $7 - 2$, not $2 - 7$.

5. Let x be a number. Have students translate the English phrase into a mathematical expression or vice versa, as appropriate:

a. The difference of the number and 9

b. The product of 5 and the number

c. $7 + x$

d. $x \div 4$

6. Let x be a number. Translate the sentence "Subtract the number from 10." into a mathematical expression. Evaluate the expression at $x = 7$.

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OBJECTIVE 5

Roles of a Variable

Here are two roles of a variable:

- A variable can represent a quantity that can vary.
- In an expression, a variable is a placeholder for a number.

OBJECTIVE 6

Recall that the area of a rectangle is equal to the length times the width of the rectangle.

7. Let W be the width (in feet) and L be the length (in feet) of a rectangle. Evaluate the expression LW for $L = 7$ and $W = 5$. What does your result mean in this situation?

SHORT HW 3, 5, 13, 17, 19, 25, 29, 33, 35, 49, 53, 57, 61, 65

MEDIUM HW 3, 5, 9, 13, 15, 17, 19, 21, 25, 27, 29, 33, 35, 39, 49, 51, 53, 55, 57, 61, 63, 65

SECTION 2.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students evaluate expressions. They use expressions to describe authentic quantities. Students translate English phrases to and from mathematical expressions. They also learn two roles of a variable. These topics will be needed throughout the course.

OBJECTIVES 1–3 COMMENTS

For part (a) of Problem 3, I emphasize that we want to show the arithmetic to help see the pattern. Insisting that students do this now will form good habits so that students will be able to find more challenging expressions of the form $mx + b$ in Section 2.6. The form of students' answers will vary for part (a), and students will be reassured to know that $3n = n(3)$. (The commutative law for multiplication will be discussed in Section 4.1.)

Textbook and Workbook Exploration

The 10-minute"Expressions" exploration is a nice way for students to discover how to use expressions to describe authentic quantities.

Textbook and Workbook Exploration

In the 10-minute "Expressions used to describe a quantity" exploration, students work backwards and find an authentic quantity that can be described by the given expression.

OBJECTIVE 4 COMMENTS

I emphasize that mathematics is a language. Most students have never thought of mathematics in this way.

Most students do not know the meanings of the words "product," "quotient," and "ratio." These words will be used extensively in the textbook.

I emphasize that "Subtracting 2 from 7" is $7 - 2$, not $2 - 7$. I point out that if you take 2 CDs *from* 7 CDs, then 5 CDs remain. This may help students see why we subtract the numbers "out of order."

OBJECTIVE 5 COMMENTS

I tell my students that sometimes a variable serves both roles. (A third role of a variable is discussed in Section 4.3:

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In an equation, a variable represents any number that is a solution of the equation.)

OBJECTIVE 6 COMMENTS

Students have an easy time with this objective.

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SECTION 2.2 LECTURE NOTES

Objectives

1. Describe the meaning of a fraction.
2. Explain why division by zero is undefined.
3. Describe the rules for $a \cdot 1$, $\frac{a}{1}$, and $\frac{a}{a}$.
4. Perform operations with fractions.
5. Find the prime factorization of a number.
6. Simplify fractions.

Main point: Perform operations with fractions.

OBJECTIVE 1

1. Shade a portion of a circle to illustrate the meaning of $\frac{5}{8}$ of a pizza.
2. Use two pizzas with 4 slices each to show that $\frac{8}{4} = 8 \div 4 = 2$.

The fraction $\frac{a}{b}$ means $a \div b$.

OBJECTIVE 2

Use the concept that division is repeated subtraction to show why division by 0 is undefined (see the middle of page 67 of the textbook).

Division by Zero

The fraction $\frac{a}{b}$ is undefined if $b = 0$. Division by 0 is undefined.

OBJECTIVE 3

- $a \cdot 1 = a$
- $\frac{a}{1} = a$
- $\frac{a}{a} = 1$, where a is nonzero

- When we write statements such as $a \cdot 1 = a$, we mean that if we evaluate $a \cdot 1$ and a for *any* value of a in both expressions, the results will be equal.
- We say the expressions $a \cdot 1$ and a are **equivalent expressions.**

OBJECTIVE 4

Multiplying Fractions

If b and d are nonzero, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

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3. $\frac{3}{5} \cdot \frac{7}{2}$

4. $\frac{4}{9} \cdot \frac{5}{7}$

OBJECTIVE 5

Definition *Prime number*

A **prime number** is any counting number larger than 1 whose only positive factors are itself and 1.

5. Prime factor 24.

6. Prime factor 36.

OBJECTIVE 6

Use a drawing of a pizza to show that $\frac{6}{8} = \frac{3}{4}$.

Simplifying a Fraction

To simplify a fraction.

1. Find the prime factorizations of the numerator and the denominator.
2. Find an equal fraction in which the numerator and the denominator do not have common positive factors other than 1 by using the property

$$\frac{ab}{ac} = \frac{a}{a} \cdot \frac{b}{c} = 1 \cdot \frac{b}{c} = \frac{b}{c}$$

where a and c are nonzero.

7. Simplify $\frac{6}{8}$.

8. Simplify $\frac{24}{18}$.

9. Simplify $\frac{7}{49}$.

OBJECTIVE 4 (revisited)

10. $\frac{4}{25} \cdot \frac{35}{8}$

11. $\frac{14}{8} \cdot \frac{10}{21}$

Dividing Fractions

If b, c, and d are nonzero, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.

12. $\frac{9}{2} \div \frac{3}{4}$

13. $\frac{45}{16} \div \frac{35}{8}$

When you use a calculator to check work with fractions, enclose each fraction in parentheses.

14. Use a drawing of a pizza to show that $\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$.

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Adding Fractions with the Same Denominator

If b is nonzero, then $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.

15. $\frac{7}{12} + \frac{2}{12}$ **16.** $\frac{5}{6} + \frac{7}{2}$ **17.** $\frac{4}{5} + \frac{2}{3}$ **18.** $\frac{5}{6} + \frac{3}{8}$

Use a drawing of a pizza to show that $\frac{7}{9} - \frac{2}{9} = \frac{5}{9}$.

Subtracting Fractions with the Same Denominator

If b is nonzero, then $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$.

19. $\frac{5}{6} - \frac{1}{6}$ **20.** $\frac{7}{6} - \frac{2}{3}$ **21.** $\frac{5}{8} - \frac{3}{10}$

Adding (or Subtracting) Fractions with Different Denominators

To add (or subtract) two fractions with different denominators, use the fact that $\frac{a}{a} = 1$, where a is nonzero, to write an equal sum (or difference) of fractions for which each denominator is the LCD.

Throughout the course, if a result is a fraction, it must be simplified.

SHORT HW 7, 21, 25, 31, 37, 45, 49, 69, 75, 79, 81, 87, 89, 97

MEDIUM HW 1, 7, 15, 21, 25, 27, 31, 33, 37, 45, 47, 49, 51, 63, 69, 75, 79, 81, 83, 85, 87, 89, 91, 95, 97

SECTION 2.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students review the meaning of a fraction and that a division by zero is undefined. Students perform operations with fractions and simplify fractions.

Because some students have difficulty with this material, this is a good time to remind them of your availability during office hours and other forms of college support such as math center tutoring. Students' greatest difficulty is usually with adding and subtracting fractions with different denominators. So, it is wise to go quickly through the rest of the material so you have adequate time to focus on this more difficult skill.

OBJECTIVE 1 COMMENTS

Some students will need a reminder about the meaning of a fraction (see Problem 1).

OBJECTIVE 2 COMMENTS

I use repeated subtraction to show that division by zero is undefined. Most students really appreciate this explanation because they have never been told *why* division by zero is undefined.

OBJECTIVE 3 COMMENTS

Although these rules are very basic, they are the foundation for performing operations with fractions.

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OBJECTIVE 4 COMMENTS

After we have discussed how to simplify fractions (Objective 6), we will discuss how to multiply fractions which can be simplified.

OBJECTIVE 5 COMMENTS

In arithmetic, students learn a variety of ways to prime factor a number. Although I allow students to use any method, I encourage them to use the method shown in Example 2 on page 69 of the textbook because this method clearly portrays that the factorization is equal to the original number.

OBJECTIVE 6 COMMENTS

Most students can successfully simplify a fraction, but most are not aware of the concepts involved. I make sure they understand that we use the property $\frac{a}{a} = 1$, where $a \neq 0$, to simplify a fraction. The textbook avoids using the terminology "reduce" because many professors think that this terminology suggests that reducing a fraction makes it smaller. The textbook also avoids using the technique of "canceling" because students misapply this technique in a myriad of ways.

OBJECTIVE 4 (REVISITED) COMMENTS

Once my students know how to simplify fractions, I discuss how to multiply fractions where the result needs to be simplified.

Using a drawing of a pizza to show that $\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$ helps students understand why it doesn't make sense to add the denominators.

When adding fractions with different denominators, I list multiples of the denominators to find the LCD. A different (and unique) technique will be discussed in Section 12.3, but listing multiples will suffice for now. When adding fractions such as $\frac{5}{6}$ and $\frac{3}{8}$ (see Problem 18), I show at least once that we are multiplying fractions by 1 to get each denominator to be the LCD:

$$\begin{aligned}\frac{5}{6} + \frac{3}{8} &= \frac{5}{6} \cdot 1 + \frac{3}{8} \cdot 1 \\ &= \frac{5}{6} \cdot \frac{4}{4} + \frac{3}{8} \cdot \frac{3}{3} \\ &= \frac{20}{24} + \frac{9}{24} \\ &= \frac{29}{24}\end{aligned}$$

Textbook and Workbook Exploration

The 10-minute "Illustrations of simplifying fractions and operations with fractions" exploration is a nice way for students to learn to visualize their work with fractions.

Workbook Exploration

In the following 15-minute exploration, groups reflect on typical errors (especially Problem 4) and inefficient methods (Problem 2). As an extension (either for all groups or those that finish ahead of the rest), you could have groups draw sketches of pizza slices to illustrate why the work in Problems 1, 3, and 4 is incorrect.

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Group Exploration

Performing operations with fractions

For each of Problems 1–4, a student tries to perform an operation of two fractions and then simplify the result. If the result is correct, decide whether there is a more efficient way to do the problem. If the result is incorrect, describe any errors and do the problem correctly.

1. $$\begin{aligned}\frac{3}{4}+\frac{5}{6} &= \frac{3+5}{4+6}\\ &= \frac{8}{10}\\ &= \frac{4}{5}\end{aligned}$$

2. $$\begin{aligned}\frac{1}{3}\cdot\frac{1}{4} &= \left(\frac{1}{3}\cdot\frac{4}{4}\right)\cdot\left(\frac{1}{4}\cdot\frac{3}{3}\right)\\ &= \frac{4}{12}\cdot\frac{3}{12}\\ &= \frac{12}{144}\\ &= \frac{1}{12}\end{aligned}$$

3. $$\begin{aligned}4\cdot\frac{2}{3} &= \frac{4\cdot 2}{4\cdot 3}\\ &= \frac{8}{12}\\ &= \frac{2}{3}\end{aligned}$$

4. $$\begin{aligned}\frac{2}{3}\div\frac{4}{5} &= \frac{3}{2}\cdot\frac{4}{5}\\ &= \frac{3\cdot 4}{2\cdot 5}\\ &= \frac{12}{10}\\ &= \frac{6}{5}\end{aligned}$$

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SECTION 2.3 LECTURE NOTES

Objectives

1. Find the opposite of a number.
2. Find the opposite of the opposite of a number.
3. Find the absolute value of a number.
4. Add real numbers by thinking in terms of money, the number line, and absolute value.
5. Add real numbers pertaining to authentic situations.
6. Find an expression to model an authentic quantity.

Main point: Add real numbers.

OBJECTIVE 1

- Two number are called **opposites** of each other if they are the same distance from 0 on the number line, but are on opposite sides of 0.
- To find the opposite of a number, we reflect the number across 0 on the number line.
- The opposite of 5 is -5. The opposite of -5 is 5. [Draw a figure.]

OBJECTIVE 2

Use reflections with a number line to show that $-(-a) = a$.

1. $-(-6)$

2. $-(-(-5))$

We use parentheses to separate two opposite symbols or an operation symbol and an opposite symbol.

OBJECTIVE 3

Definition *Absolute value*

The **absolute value** of a number is the distance that the number is from 0 on the number line. [Draw a figure.]

3. $|4|$

4. $|-7|$

5. $-|8|$

6. $-|-2|$

OBJECTIVES 4 and 5

7. **a.** A person has a credit card balance with a 0 dollar balance. If she uses her credit card to make two purchases for \$3 and \$4, what is the new balance?

b. Write a sum that is related to the computation in part (a).

c. Use a number line to illustrate the sum found in part (b).

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Adding Two Numbers with the Same Sign

To add two numbers with the same sign:

1. Add the absolute values of the numbers.
2. The sum of the original numbers has the same sign as the sign of the original numbers.

8. $-3+(-8)$ **9.** $-2.57+(-6.84)$ **10.** $-\frac{5}{8}+\left(-\frac{1}{8}\right)$

11. **a.** A brother owes his sister \$5. If he then pays her back \$2, how much does he still owe her?
b. Write a sum that is related to your work in part (a).
c. Use a number line to illustrate the sum you found in part (b).

Adding Two Numbers with Different Signs

To add two numbers with different signs:

1. Find the absolute values of the numbers. Then subtract the smaller absolute value from the larger absolute value.
2. The sum of the original numbers has the same sign as the original number with the larger absolute value.

12. $-9+2$ **13.** $7+(-4)$ **14.** $-\frac{7}{6}+\frac{5}{6}$

15. A person bounces several checks and is charged service fees such that the balance of the checking account is -78. The person then deposits \$250. Find the balance.

16. $-7+(-4)$ **17.** $-5.31+1.98$ **18.** $329+(-838)$ **19.** $\frac{3}{10}+\left(-\frac{5}{6}\right)$

20. Let x be a number. Translate the phrase "the number plus -5" into a mathematical expression. Then evaluate the expression for $x=-9$.

OBJECTIVE 6

21. A hardware store is offering a weekend sale of \$5 off the retail price of any of their power tools.

a. Complete the following table to help find an expression that describes the sale price (in dollars) if the retail price is r dollars. Show the arithmetic to help you see a pattern.

Retail Price (dollars)	Sale Price (dollars)
50	
75	
100	
125	
r	

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b. Evaluate the expression you found in part (a) for $r = 80$. What does your result mean in this situation?

SHORT HW 1, 7, 9, 13, 21, 31, 37, 49, 53, 57, 61, 67, 75, 83

MEDIUM HW 1, 7, 9, 13, 19, 21, 27, 31, 37, 43, 47, 49, 53, 55, 57, 61, 63, 65, 67, 73, 75, 77, 83

SECTION 2.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students add real numbers and find expressions to model authentic quantities. They will use these skills throughout the course.

OBJECTIVE 1 COMMENTS

Although finding the opposite of a number is an easy topic, I discuss reflecting a number across 0 on the number line to set the stage for Objective 2.

OBJECTIVE 2 COMMENTS

This concept will help students subtract numbers in Section 2.4: $a - (-b) = a + [-(-b)] = a + b$.

OBJECTIVE 3 COMMENTS

Although absolute value is an easy topic, defining it in terms of the distance from 0 will lay a good foundation for students to solve absolute value equations and inequalities in subsequent courses.

OBJECTIVES 4 and 5 COMMENTS

Most of my students prefer to think of adding real numbers in terms of money (as opposed to the number line or absolute value). I quickly go over the rules and spend more time having my students do mixed sets of problems such as Problems 16–19.

Workbook Exploration

My students tend to be quick to pick up on how to add real numbers and why the rules make sense, so I usually skip the following 20-minute exploration. But for a weaker class, facilitating the exploration without lecturing is a great way to have students think graphically or intuitively without blindly memorizing the rules.

EXPLORATION *Adding real numbers*

1. Describe how to find the sum by thinking in terms of money. Also show how to find the sum by using a number line.

   a. $-2 + (-3)$ **b.** $-1 + (-8)$ **c.** $-3 + (-5)$

2. What is the sign of the sum of two negative numbers? Explain.

3. Describe how to find the given sum by thinking in terms of money. Also show how to find the sum by using a number line.

   a. $6 + (-2)$ **b.** $7 + (-4)$ **c.** $-3 + 5$

4. If two numbers have different signs, what is the sign of the sum of the numbers if the absolute value of the positive number is larger than the absolute value of the negative number? Explain.

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5. Describe how to find the given sum by thinking in terms of money. Also show how to find the sum by using a number line.

a. $3 + (-5)$ **b.** $1 + (-4)$ **c.** $-6 + 2$

6. If two numbers have different signs, what is the sign of the sum of the numbers if the absolute value of the positive number is smaller than the absolute value of the negative number? Explain.

Textbook and Workbook Exploration

The 10-minute "Adding a number and its opposite" exploration is a nice way for students to learn that $a + (-a) = 0$.

OBJECTIVE 6 COMMENTS

For Problem 21, I emphasize that we want to show the arithmetic. For part (a), the result can be written as $r - 5$ or $r + (-5)$. This suggests that $a - b = a + (-b)$. This could be a nice segue to Section 2.4.

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SECTION 2.4 LECTURE NOTES

Objectives

1. Find the change in a quantity.
2. Subtract real numbers.
3. Find a change in elevation.
4. Determine the sign of the change for an increasing or decreasing quantity.

Main point: Find the change in a quantity and subtract real numbers.

OBJECTIVE 1

1. **a.** If the price of a basket of strawberries increases from \$2 to \$5, find the change in the price.

b. Write a difference that is related to the computation in part (a).

Change in a Quantity

The change in a quantity is the ending amount minus the beginning amount:

$$\text{Change in the quantity} = \text{Ending amount} - \text{Beginning amount}$$

2. The revenues of Facebook are shown in the following table for various years.

Year	Facebook Revenue (billions of dollars)
2011	3.71
2012	5.09
2013	7.87
2014	12.5
2015	17.9
2016	27.6
2017	40.6

Source: *Facebook*

a. For the period 2011–2017, find each of the changes in annual revenue from one year to the next.

b. From which year to the next did the annual revenue increase the most?

c. Did the changes in annual revenue from one year to the next increase, decrease, stay approximately constant, or none of these from 2011 to 2017?

OBJECTIVE 2

3. **a.** The volume of gasoline in a car's gasoline tank decreases from 6 gallons to 2 gallons. What is the change in the volume of gasoline in the tank?

b. Write a difference that is related to the computation in part (a).

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c. From part (b), we found that $2 - 6 = -4$. Find $2 + (-6)$. Explain why we can conclude that $2 - 6 = 2 + (-6)$.

Subtracting a Real Number

$$a - b = a + (-b)$$

4. $3 - 7$ **5.** $-2 - 8$ **6.** $\frac{2}{9} - \frac{8}{9}$

7. **a.** The temperature increases from $-3°$F to $2°$F. Find the change in temperature.

b. Write a difference that is related to the work in part (a).

c. Find the difference obtained in part (b) by using the rule $a - b = a + (-b)$.

8. $5 - (-2)$ **9.** $-6 - (-1)$ **10.** $\frac{2}{7} - \left(-\frac{3}{7}\right)$

11. Translate the sentence "Subtract the number from 5." into a mathematical expression. Then evaluate the expression for $x = -3$.

12. $-5 + (-4)$ **13.** $4 - 11$ **14.** $-9 - 7$ **15.** $-\frac{2}{5} - \left(-\frac{7}{10}\right)$ **16.** $2.5 + (-7.1)$ **17.** $62 - (-95)$ **18.** $\frac{7}{12} - \frac{11}{15}$ **19.** $46 + (-11)$

OBJECTIVE 3

- An object that has a *positive* elevation of 500 feet is 500 feet *above* sea level.
- An object that has a *negative* elevation of -500 feet is 500 feet *below* sea level.

20. The Golden Gate Bridge has two towers that support two main cables of the bridge. The top of each tower is at an elevation of 746 feet, and the foot of each tower is at an elevation of -136 feet. Find the height of each tower.

OBJECTIVE 4

Refer to Problem 2 to motivate the following properties.

Changes of Increasing and Decreasing Quantities

- An increasing quantity has a positive change.
- A decreasing quantity has a negative change.

21. For airplanes, *available seat miles* is the number of seats available times the number of miles flown. The levels of total available seat miles for United Airlines are shown in the following table for various years.

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Year	United's Total Available Seat Miles (billions of seat miles)
2011	219
2012	216
2013	213
2014	214
2015	220
2016	225
2017	235

Source: *United Airlines*

a. Find the change in United's total available seat miles from 2011 to 2012. What does it mean in this situation?

b. Find the change in United's total available seat miles from 2016 to 2017. What does it mean in this situation?

SHORT HW 1, 5, 15, 27, 31, 35, 37, 49, 53, 57, 61, 65, 73, 79, 81

MEDIUM HW 1, 5, 7, 15, 25, 27, 31, 33, 35, 37, 39, 43, 49, 53, 57, 61, 63, 65, 73, 75, 79, 81, 85, 87

SECTION 2.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students subtract real numbers, a skill that they will use throughout the course. In this section, students find the change in a quantity. This important concept is sadly overlooked in most arithmetic and algebra textbooks. This is a central concept needed to understand that in going from a point (x_1, y_1) to a point (x_2, y_2), $y_2 - y_1$ is the vertical change and $x_2 - x_1$ is the horizontal change. This gives meaning to the slope formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. The last piece of the puzzle about slope will be to discuss the meaning of ratios in Section 2.5. Understanding how to find the change in a quantity will also help students make sense of the techniques we use to subtract real numbers.

OBJECTIVE 1 COMMENTS

Problem 1 nicely motivates the change in a quantity property. Note that the results of part (a) of Problem 2 are average rates of change for various years.

Workbook Exploration

Students can complete the following 15-minute exploration with few if any hints. In particular, they are able to determine the signs of the results because they are working with authentic quantities.

Group Exploration

Section Opener: Finding the change in a quantity

1. The sales of craft soda were \$520 million in 2015 and \$541 million in 2016 (Source: *Beverage Marketing Corp*). Is the change in revenue positive or negative? Explain. Also, find the change in the revenue.

2. The enrollment at Eastern Illinois University was about 10 thousand students in 2013 and about 7 thousand students in 2016 (Source: *Eastern Illinois University*). Is the change in enrollment positive or negative? Explain. Also, find the change in enrollment.

3. The viewership of the Academy Awards was 33 million people in 2017 and 27 million people in 2018 (Source: *Nielsen*). Is the change in viewership positive or negative? Also, find the change in viewership.

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4. The revenue for the top 100 North American concert tours was $2.1 billion in 2010 and $3.6 billion in 2017 (Source: *Pollster*). Is the change in revenue positive or negative? Explain. Also, find the change in the revenue.

5. In New York City, the average speed of taxis was 8.2 miles per hour in 2015 and 6.9 miles per hour in 2016 (Source: *Fix NYC Advisory Panel Report*). Is the change in average speed positive or negative? Explain. Also, find the change in the average speed.

6. In general, when is the change in a quantity positive? negative?

7. Describe how to compute the change in a quantity.

OBJECTIVE 2 COMMENTS

Problem 3 suggests the property $a - b = a + (-b)$. Problem 7 suggests how to subtract a negative number. Problems 12–19 serve as a summary of Sections 2.3 and 2.4.

Textbook and Workbook Exploration

The 10-minute "Subtracting numbers" exploration is a nice way for groups to discover that $a - b = a + (-b)$. Some groups will need help with Problems 4 and 5 of this exploration.

OBJECTIVE 3 COMMENTS

Although this is just a special case of computing the change in a quantity, it is nonetheless great practice for later finding the rise in going from one point to another point when calculating the slope of a line.

OBJECTIVE 4 COMMENTS

Although I address the concepts about the significance of signs of change in Objectives 1 and 2, it is nice to reinforce these important concepts at the end of this section.

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SECTION 2.5 LECTURE NOTES

Objectives

1. Find the ratio of two quantities.
2. Describe the meaning of *percent*.
3. Convert percentages to and from decimal numbers.
4. Find the percentage of a quantity.
5. Multiply and divide real numbers.
6. Determine which fractions with negative signs are equal to each other.

Main point: Work with ratios, percents, and multiplying and dividing real numbers.

OBJECTIVE 1

- Suppose that there are 8 women and 4 men on a softball team. The ratio of women to men is

$$\frac{8 \text{ women}}{4 \text{ men}} = \frac{2 \text{ women}}{1 \text{ men}}$$

- This means that there are 2 women to 1 man. This ratio is called a *unit ratio*.
- A **unit ratio** is a ratio written as $\frac{a}{b}$ or as $a : b$ with $b = 1$.

1. The blink of an eye takes 400 milliseconds. One beat of a hummingbird's wings takes 20 milliseconds. Find the unit ratio of the time it takes for the blink of an eye to the time it takes for one beat of a hummingbird's wings. What does your result mean in this situation?

2. The median sales prices of existing homes and the median incomes are shown in the following table for four regions of the United States.

Region	Median Sales Price of Existing Homes (dollars)	Median Income (dollars)
Northeast	402,800	59,210
Midwest	269,700	54,267
South	257,700	49,655
West	333,900	57,688

Sources: *U.S. Census; National Association of Realtors®*

a. Find the unit ratio of the median sales price of existing homes in the Northeast to the median income in the Northeast. What does the result mean?

b. For each of the four regions, find the unit ratio of the median sales price of existing homes to the median income. Taking into account the median income of each region, list the regions in order of affordability of existing homes, from greatest to least.

c. A person believes that existing homes in the South are more affordable than in the Midwest because the median price of existing homes is lower in the South than in the Midwest. What would you tell that person?

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OBJECTIVE 2

- If 57 of 100 songs are hip hop, then 57% of the songs are hip hop.
- **Percent** means "for each hundred": $a\% = \dfrac{a}{100}$.
- A percent is a ratio.

OBJECTIVE 3

Converting Percentages to and from Decimal Numbers

- To write a percentage as a decimal number, remove the percent symbol and divide the number by 100 (move the decimal point two places to the left).
- To write a decimal number as a percentage, multiply the number by 100 (move the decimal point two places to the right) and insert a percent symbol.

3. Write 8% as a decimal number.

4. Write 0.145 as a percent.

Warning: 8% is equal to 0.08, not 0.8.

OBJECTIVE 4

Finding the Percentage of a Quantity

To find the percentage of a quantity, multiply the decimal form of the percentage and the quantity.

5. Find 5% of 80 students.

6. Find 6.3% of $2000.

One Hundred Percent of a Quantity

One hundred percent of a quantity is *all* of the quantity.

OBJECTIVE 5

Use repeated addition to show that $4(-3) = -12$.

Multiplying Two Numbers with Different Signs

The product of two number that have different signs is negative.

7. **a.** Find the products $3(-4) = -12$, $2(-4) = -8$, $1(-4) = -4$, $0(-4) = 0$.

b. Explain why the results of part (a) suggest that $-1(-4) = 4$, $-2(-4) = 8$, and $-3(-4) = 12$.

c. Form a theory about the sign of the product of two numbers that have the same sign.

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Multiplying Two Numbers with the Same Sign

The product of two numbers that have the same sign is positive.

8. $3(-8)$ **9.** $-5(-9)$ **10.** $-0.2(-0.1)$ **11.** $-\frac{4}{9} \cdot \frac{3}{2}$

The rules for dividing numbers are similar to those for multiplying numbers because to divide by a number, we multiply by the reciprocal of the number.

Multiplying or Dividing Real Numbers

The product or quotient of two numbers that have different signs is negative. The product or quotient of two numbers that have the same sign is positive.

12. $-12 \div (-2)$ **13.** $-35 \div 7$ **14.** $475 \div -25$ **15.** $-\frac{5}{49} \div \left(-\frac{25}{21}\right)$

16. A person has credit card balances of -3580 dollars on a Discover® account and -1590 dollars on a MasterCard® account.

a. Find the unit ratio of the Discover balance to the MasterCard balance.

b. If the person wishes to gradually pay off both accounts in the same amount of time, describe how the result in part (a) can help guide the person in making his next payment.

17. $-4 - (-7)$

18. $8 \div (-2)$

19. $-15 + 11$

20. $(-7)(-10)$

21. $-5.8 + 2.9$

22. $-0.2(-0.3)$

23. $-\frac{27}{20} \div \frac{9}{8}$

24. $-\frac{7}{4} - \frac{3}{2}$

OBJECTIVE 6

Compare the results of $\frac{-6}{3}$, $\frac{6}{-3}$, and $-\frac{6}{3}$ to motivate the following property.

Equal Fractions with Negative Signs

If $b \neq 0$, then

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

25. $\frac{-32}{20}$ **26.** $\frac{7}{9} + \frac{-4}{9}$ **27.** $\frac{7}{6} - \left(\frac{5}{-4}\right)$

SHORT HW 3, 15, 17, 23, 27, 49, 51, 55, 67, 73, 91, 93, 99, 107, 115

MEDIUM HW 3, 15, 17, 19, 23, 27, 35, 41, 49, 51, 53, 55, 61, 67, 73, 83, 91, 93, 99, 105, 107, 113, 115, 121, 123, 125

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SECTION 2.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students work with ratios, percents, and multiplying and dividing real numbers.

It may be tempting to skip or deemphasize ratios, but this concept, together with the concept of change, will be very helpful in introducing the slope formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ (in Section 3.3). It is also instrumental in making the connection that slope is a rate of change (in Section 3.5). Students will also work with ratios in Section 12.7, which has strong ties with Section 10.8. Most students have little experience working with ratios.

OBJECTIVE 1 COMMENTS

Most students have an easy time computing ratios but have a harder time thinking about the significance of comparing ratios such as in Problem 2, which emphasizes the important concept that comparing ratios is sometimes more meaningful than comparing quantities. This is a key concept of computing the slope of a line: it is not enough to compare the rises in gong from one point on a line to another point.

Workbook Exploration

After a quick definition of unit ratio, you could have groups work on the following 15-minute exploration, which helps groups see that sometimes comparing ratios is more meaningful than comparing quantities.

Group Exploration

Interpreting ratios

A total of 1824 adults were asked the following question: "Do you favor or oppose the death penalty for persons convicted of murder?" The following table compares the adults' responses with their ethnicities.

	African American	Caucasian	Other	Total
Favor	128	953	108	1189
Oppose	140	414	81	635
Total	268	1367	189	1824

Source: *General Social Survey*

1. Find the number of Caucasians in the survey who oppose the death penalty.
2. Find the number of African Americans in the survey who oppose the death penalty.
3. For the Caucasians in the study, find the unit ratio of the number who oppose the death penalty to the number who are in favor.
4. For the African Americans in the study, find the unit ratio of the number who oppose the death penalty to the number who are in favor.
5. A student says that Caucasians in the study are more likely to oppose the death penalty than African Americans in the study because more of the Caucasians oppose the death penalty than the African Americans. What would you tell the student?

OBJECTIVE 2 COMMENTS

Most students have a weak understanding of percents. Discussing percents now will lay the foundation for the interest problems and mixture problems in Section 6.5.

OBJECTIVE 3 COMMENTS

I use the definition of percent to suggest how to convert percentages to and from decimal numbers.

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For Problem 3, some students think that the result is 0.8.

OBJECTIVE 4 COMMENTS

I first use the definition of percent to find the percentage of a quantity and then streamline the process.

For Problems 5 and 6, I remind students to include units in their results.

I make sure students know the meaning of 100% as this will come in handy for various curve-fitting applications later in the text.

OBJECTIVE 5 COMMENTS

Students have an easy time with these skills.

Writing $4(-3) = (-3) + (-3) + (-3) + (-3) = -12$ suggests the rule for multiplying two numbers with different signs. Problem 7 suggests the rule for multiplying two numbers with the same sign.

For Problem 10, many students think that $-0.2(-0.1) = 0.2$. For Problem 11, some students need to be reminded to simplify their result.

The textbook includes a detailed explanation about why the sign rules for division are the same as for multiplication (see page 101). Due to time constraints, I usually just say the rules for dividing numbers are similar to those for multiplying numbers because to divide by a number, we multiply by the reciprocal of the number.

Problem 16 ties ratios in nicely with dividing two negative numbers. Many students who have credit card debt could directly benefit from learning how to do such work with credit card balances!

I budget plenty of time for students to do Problems 17–24, which serve as a good summary of Sections 2.2–2.5.

Textbook and Workbook Exploration

Groups can discover how to multiply two numbers with different signs by completing the 10-minute "Finding the product of a positive number and a negative number" exploration.

OBJECTIVE 6 COMMENTS

I emphasize the property $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$, where $b \neq 0$ because it will be useful when working with slope. It seems that no amount of emphasis is too much because some students completely forget this property by the time we need to use it for slope (in Sections 3.3 and 3.4).

I tell my students to write their results which are negative fractions in the form $-\frac{a}{b}$ rather than $\frac{-a}{b}$ or $\frac{a}{-b}$.

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SECTION 2.6 LECTURE NOTES

Objectives

1. Describe the meaning of *exponent*.
2. Use the rules for order of operations to perform computations and evaluate expressions.
3. Use the rules for order of operations to make predictions.

Main point: Use the rules for order of operations to perform computations.

OBJECTIVE 1

- $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$
- $4^3 = 4 \cdot 4 \cdot 4 = 64$

Definition *Exponent*

For any counting number n,

$$x^n = \underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}_{n \text{ factors of } x}$$

We refer to x^n as the **power**, the **nth power of x**, or **x raised to the nth power**. We call x the **base** and n the **exponent**.

1. 3^4 **2.** $(-2)^5$ **3.** $(-5)^2$ **4.** -5^2

For an expression of the of the form $-a^n$, we calculate a^n before taking the opposite.

OBJECTIVE 2

- We do operations that lie within grouping symbols before we perform other operations.
- The order of operations does matter:

$$(4+2) \cdot 5 = 6 \cdot 5 = 30$$
$$4 + (2 \cdot 5) = 4 + 10 = 14$$

- For a fraction such as $\dfrac{2+5}{9-4}$, the following use of parentheses is assumed:

$$\frac{2+5}{9-4} = \frac{(2+5)}{(9-4)} = \frac{7}{5}$$

5. $(4-8)(9-2)$ **6.** $\dfrac{2-8}{3-7}$

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Order of Operations

We perform operations in the following order:

1. First, perform operations within parentheses or other grouping symbols, starting with the innermost group.
2. Then perform exponentiations.
3. Next, perform multiplications and divisions, going from left to right.
4. Last, perform additions and subtractions, going from left to right.

7. $3 - 8 \div 4$

8. $4 + (-2)^3$

9. $2(5 - 8) - (4 - 9)$

10. $3 - [6 - 2(3 + 4)]$

11. $3^3 - 2(3 - 5)^2 \div (-4)$

12. $\dfrac{9}{10} - \dfrac{3}{5} \div \dfrac{2}{7}$

13. $\dfrac{4 - (-2)^3}{2 + 4^2}$

Operation	**Computation with 10s**
Exponentiation	$10^{10} = 10,000,000,000$
Multiplication	$10 \cdot 10 = 100$
Addition	$10 + 10 = 20$

Order of Operations and the Strengths of Operations

After we have performed operations in parentheses, the order of operations goes from the most powerful operation, exponentiation, to the next most powerful operations, multiplication and division, to the weakest operations, addition and subtraction.

14. Evaluate $\dfrac{a - b}{c - d}$ for $a = -3$, $b = 5$, $c = 4$, and $d = -6$.

15. Evaluate $3x^2$ for $x = -4$.

16. Evaluate $b^2 - 4ac$ for $a = -2$, $b = -3$, and $c = 5$.

17. Let x be a number. Translate the sentence "Subtract 5 from the product of the number and -3." Then evaluate the expression for $x = -2$.

OBJECTIVE 3

18. The numbers of deaths from opioid drug poisoning in Colorado was 472 deaths in 2015 and has increased by about 32 deaths per year since then (Source: *Colorado Department of Public Health and Environment*). Complete the following table to help find an expression that stands for the number of such deaths at t years since 2015. Show the arithmetic to help see a pattern.

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Years since 2015	Number of Deaths from Opioid Poisoning in Colorado
0	
1	
2	
3	
4	
t	

Evaluate your result for $t = 6$ and discuss what the result means in this situation.

19. The Academy Award viewership was 32.9 million people in 2017, and it decreased by 19.5% in 2018 (Source: *Nielsen*). What was the viewership in 2018?

SHORT HW 5, 7, 15, 21, 35, 45, 63, 67, 77, 81, 91, 95, 97, 107, 113, 117

MEDIUM HW 5, 7, 15, 21, 27, 33, 41, 45, 49, 57, 63, 67, 77, 79, 81, 85, 91, 95, 97, 99, 107, 111, 113, 117, 119

SECTION 2.6 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students work with exponents and use the rules for order of operations to perform computations and evaluate expressions. Students will use this material throughout the course.

OBJECTIVE 1 COMMENTS

It will be especially helpful for students to know the meaning of *exponent*, *base*, and *power* when using the properties of exponents in Sections 7.4, 10.1, and 10.2. Many students think that *power* means the same thing as *exponent*. Explain that x^n is a power of x and that n is the exponent.

For Problem 4, most students are surprised that -5^2 is not equal to 25. I tell students that for an expression of the of the form $-a^n$, we calculate a^n before taking the opposite. It also helps to point out that the base of $(-5)^2$ is -5 and that the base of -5^2 is 5.

OBJECTIVE 2 COMMENTS

When discussing the order of operations, I make sure that my students understand that multiplication and division are on the same level of the hierarchy; some students think that "going from left to right" means that all multiplications should be done before all divisions because the word "multiplications" appears to the left of the word "divisions" in the instructions for the order of operations. The same goes for addition and subtraction.

It is usually best to ignore suggestions from students to use the acronym "PEMDAS" because it lacks the detail needed for students to use it correctly. (Even with commas, the acronym "P,E,MD,AS" lacks necessary details.)

Even though order of operations is a review topic from arithmetic, some students are challenged by Problems 7–13.

Problem 14 is good preparation for using the slope formula in Section 3.3. Problem 16 is good preparation for using the quadratic formula in Section 9.6.

Finally, note that Exercises 81–86 of Homework 2.6 are good practice for using the slope formula. Exercise 77 is a nice primer for the quadratic formula. Exercises 87–92 are good preparation for graphing quadratic equations in two variables in Section 9.2. Exercises 117 and 118 can serve as explorations of the associative laws for multiplication and addition, respectively.

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Textbook and Workbook Exploration

In the 10-minute "Order of operations" exploration, groups discover that it matters in which order we add and multiply.

OBJECTIVE 3 COMMENTS

Having students continue their work with building expressions by recognizing patterns in their arithmetic (see Problem 18) is excellent preparation for Section 3.2, where students will find and graph linear models.

Workbook Exploration

You could have students work on the following 10-exploration without first giving an example because they have done similar (simpler) work in Section 2.1. You will likely have to remind several groups to avoid performing arithmetic so they can more easily see patterns.

Group Exploration

Using an expression to model a situation

The percentage of Americans who have ever listened to a podcast was 33% in 2015 and has increased by about 3.25 percentage points per year since then (Source: *Edison Research and Triton Digital*).

1. Complete the following table to help find an expression that stands for the percentage of Americans who have ever listened to a podcast at t years since 2015. Show arithmetic to help you see a pattern.

Years since 2015	Percent
0	
1	
2	
3	
4	
t	

2. Evaluate the expression you found in Problem 1 for $t = 7$. What does your result mean in this situation?

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CHAPTER 3 OVERVIEW

In Section 3.1, students graph linear equations in two variables and learn the Rule of Four for equations. In Section 3.2, students graph a linear model and perform a unit analysis of it. They also learn the Rule of Four for linear models. In Section 3.3, students calculate the slope of a line and learn the graphical significance of slope. They use slope and y-intercept to graph linear equations in two variables in Section 3.4. In Section 3.5, students find the rate of change of a quantity and use such a rate of change to help find a linear model.

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SECTION 3.1 LECTURE NOTES

Objectives

1. For an equation in two variables, describe the meaning of *solution*, *satisfy*, and *solution set*.
2. Describe the meaning of the *graph* of an equation.
3. Graph equations of the form $y = mx + b$.
4. Describe the meaning of b in equations of the form $y = mx + b$.
5. Graph equations of the form $y = b$ and $x = a$.
6. Describe the Rule of Four for solutions of equations.

Main point: Graph equations of the form $y = mx + b$.

OBJECTIVE 1

1. Show that the equation $y = x + 3$ becomes a true statement when 2 is substituted for x and 5 is substituted for y.

We say $(2, 5)$ *satisfies* $y = x + 3$ and the $(2, 5)$ is a *solution* of $y = x + 3$.

Definition *Solution, satisfy,* and *solution set* of a equation in two variables

An ordered pair (a, b) is a **solution** of an equation in terms of x and y if the equation becomes a true statement when a is substituted for x and b is substituted for y. We say (a, b) **satisfies** the equation. The **solution set** of an equation is the set of all solutions of the equation.

2. Is $(2, 7)$ a solution of $y = 4x - 1$?

3. Is $(3, 5)$ a solution of $y = 4x - 1$?

4. **a.** Find five solutions of $y = 2x - 3$ and plot them in the same coordinate system.

b. Do the five points lie on a line? If so, sketch the line.

c. Select another point on the line and show that the corresponding ordered pair satisfies $y = 2x - 3$.

d. Select a point that doesn't lie on the line and show that the corresponding ordered pair does not satisfy $y = 2x - 3$.

OBJECTIVE 2

We call the line that we sketched in Problem 4 the *graph* of $y = 2x - 3$.

Definition *Graph*

The **graph** of an equation in two variables is the set of points that correspond to all solutions of the equation.

Every point on the graph of an equation represents a solution of the equation. Every point *not* on the graph represents an ordered pair that is *not* a solution.

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OBJECTIVE 3

The equation $y = 2x - 3$ is of the form $y = mx + b$.

Graph of $y = mx + b$

The graph of an equation of the form $y = mx + b$, where m and b are constants, is a line.

Here are some equations whose graphs are lines:

$$y = 3x + 7 \qquad y = -6x + 2 \qquad y = 2x \qquad y = x - 4 \qquad y = 1$$

5. Graph the equation by hand. Also, find the y-intercept.

a. $y = 3x - 5$

b. $y = -2x + 4$

c. $y = -3x$

d. $y = \frac{2}{3}x + 1$

OBJECTIVE 4

Substitute 0 for x into the equation $y = mx + b$:

$$y = m(0) + b = 0 + b = b$$

The y-intercept of the graph of $y = mx + b$ is $(0, b)$.

y-Intercept of the Graph of $y = mx + b$

The graph of an equation of the form $y = mx + b$ has y-intercept $(0, b)$.

For $y = 3x - 5$, the y-intercept is $(0, -5)$ and for $y = -2x + 4$, the y-intercept is $(0, 4)$. See parts (a) and (b) of Problem 5.

OBJECTIVE 5

Find five solutions of the given equation. Then graph the equation by hand.

6. $x = 1$

7. $y = 3$

Equations for Horizontal and Vertical Lines

If a and b are constants, then

- The graph of $y = b$ is a horizontal line. [Show the graph.]
- The graph of $x = a$ is a vertical line. [Show the graph.]

Graph the equation by hand.

8. $x = 4$

9. $y = -2$

10. $x = 0$

11. $y = 0$

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Equations Whose Graphs Are Lines

If an equation can be put into the form $y = mx + b$ or $x = a$, where m, a, and b are constants, then the graph of the equation is a line. We call such an equation a **linear equation in two variables**.

Here are some examples of linear equations in two variables:

$$y = -5x + 9 \qquad y = 3x - 8 \qquad y = 4 \qquad x = -2$$

OBJECTIVE 6

Rule of Four for Solutions of an Equation

We can describe some or all of the solutions of an equation in two variables with:

1. an equation,
2. a table,
3. a graph, or
4. words.

These four ways to describe solutions are known as the **Rule of Four**.

12. **a.** List some solutions of $y = 4x - 3$ by using a table.
b. Describe the solutions of $y = 4x - 3$ by using a graph.
c. Describe the solutions of $y = 4x - 3$ by using words.

13. Consider the equation $y = x + 2$.
a. Substitute 3 for x and show that the input $x = 3$ leads to the output $x = 5$.
b. Graph $y = x + 2$. Then use arrows to show that the input $x = 3$ leads to the output $x = 5$.

SHORT HW 1, 5, 11, 19, 25, 27, 35, 41, 43, 49, 53, 57, 67, 69, 89

MEDIUM HW 1, 5, 11, 13, 15, 19, 25, 27, 31, 35, 41, 43, 45, 47, 49, 51, 53, 57, 61, 65, 67, 69, 71, 81

SECTION 3.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students graph equations of the form $y = mx+b$ and learn the Rule of Four for equations. Students will graph linear equations in two variables in the rest of Chapter 3 as well as in Section 5.1.

OBJECTIVE 1 COMMENTS

I emphasize these terminologies because I use these words so much throughout the course.

Problem 4 is important because it helps students understand the meaning of a graph. Without doing such a problem, most students will not know a graph's meaning.

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OBJECTIVE 2 COMMENTS

I emphasize that every point on the graph represents a solution of the equation and that every point not on the graph represents an ordered pair that is not a solution. This important concept is used when finding equations of lines in Sections 5.4 and 5.5 as well as when solving systems of linear equations by graphing in Section 6.1.

OBJECTIVE 3 COMMENTS

For Problem 5, I suggest that students use three points because the third point can serve as a check. Most students are challenged by part (d). Students will have an easier time graphing such an equation once they have learned about slope.

Textbook and Workbook Exploration

In the 15-minute "Evaluating an expression and plotting points" exploration, groups can be introduced to graphing linear equations. Students have an easy time with this exploration.

Textbook and Workbook Exploration

Although the 15-minute "Solutions of an equation" exploration addresses the meaning of a graph (Objective 2), it is better to use this exploration once students are comfortable graphing linear equations (Objective 3). That way, groups will be able to efficiently do Problem 1 of the exploration and get to the main point of the exploration.

OBJECTIVE 4 COMMENTS

Students will need to be able to determine the y-intercept of an equation of the form $y = mx + b$ when they perform graphing in Section 3.4.

OBJECTIVE 5 COMMENTS

For Problems 6 and 7, many students are thrown for a loop because only one variable appears in these equations. I've included two ways to address this issue below.

One way to address this issue is to begin with an application. Let n be the number of states in the United States at t years since 1960. Next, construct a table of nonnegative values of t and n. Then determine that the model is $n = 50$ and graph the model. Finally, discuss for which years model breakdown occurs. (Hawaii became the 50th state on August 21, 1959.)

Another way to address the issue, when finding solutions of the equation $x = 1$ (Problem 6), is to use the following scenario:

> Suppose that a younger sibling must return home by 1 A.M. but an older sibling can return home at any time. Assuming that the younger sibling always returns home at 1 A.M., discuss possible times when the siblings will return home.

This story helps students see that the equation $x = 1$ is restricting the value of x to be 1, but the value of y can be any real number. (I'd pick an earlier curfew, but I'd rather not graph an equation such as $x = 10$ for an introductory example; plus a curfew of 10 P.M. is a setup for having to contend with the issue that 1 follows 12 with how we keep time.

For students who confuse the graphs of equations of the form $x = a$ and $y = b$, I suggest that they build a table of (at least two) solutions to tip them off whether they should graph a horizontal or vertical line.

Workbook Exploration

Not only does the following 10-minute exploration get across the idea of how to graph equations of the forms $x = a$ and $y = b$, it also encourages students to reflect on the nature of the coordinates of points on a horizontal or vertical line.

Group Exploration

Section Opener: Equations of horizontal and vertical lines

1. Sketch a horizontal line on the given coordinate system.

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2. List the ordered pairs of five points that lie on the horizontal line you sketched in Problem 1.

3. What do the coordinates of the ordered pairs you listed in Problem 2 have in common?

4. Translate the response your wrote in Problem 3 into an equation by filling in the blank: $y =$ _____.

5. Graph $y = -2$.

6. Graph $x = 4$.

Workbook Exploration

Students who have seen a bit of graphing in a previous course will take for granted that graphs of equations of the form $y = mx + b$ are lines because they were told so. In the following 25-minute exploration, students see that graphs of some linear equations are lines *and* that the graphs of some nonlinear equations are not lines.

If you are worried about getting bogged down in Chapter 3, this is a good exploration to skip and just lecture about the concepts because you can communicate the concepts in much less time than it takes groups to do the exploration.

One advantage with having students do the exploration is that they get a preview of the types of curves with which they will be working in subsequent chapters. They'll also see a fuller range of what their graphing calculator can do. And they'll get hands-on experience of the concepts addressed in this exploration.

Group Exploration

Section Opener: Forms of equations whose graphs are lines

1. Use a graphing calculator to determine which of the equations that follow have graphs that are lines. (To view these graphs, use ZStandard followed by ZSquare.)

a. $y = x$

b. $y = x^2$ (For "x^2": Press [X,T,Θ,n] [$\wedge$] **2** or [X,T,Θ,n] [x^2].)

c. $y = 2^x$ (For "2^x": Press **2** [$\wedge$] [X,T,Θ,n].)

d. $y = \dfrac{2}{x}$

e. $y = 3x$

f. $y = 2x + 4$

g. $y = -2x - 8$ (For "$-2x - 8$": Press [(−)] **2** [X,T,Θ,n] [−] **8**.)

h. $y = 4$

2. Make up your own example of an equation whose graph is a line. Use a graphing calculator to verify that you are right.

3. Here are some examples of equations whose graphs are lines.

$$y = 3x + 5 \qquad y = 2x - 8 \qquad y = -4x + 7 \qquad y = -7x - 6 \qquad y = 5x \qquad y = -3$$

Form a theory about how to recognize whether an equation has a graph that is a line without graphing the equation. Test your theory. You can do this by creating equations you think have graphs that are lines and then using a graphing calculator to graph each equation to see if you are correct.

4. Is the graph of $3x^2 + y = 3x^2 + 2x + 8$ a line? Explain. Modify your theory in Problem 3, if necessary.

OBJECTIVE 6 COMMENTS

Students will be asked to demonstrate the Rule of Four in several Homework sections.

Problem 13 is important because it helps students see the connection between finding inputs and outputs symbolically and graphically.

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SECTION 3.2 LECTURE NOTES

Objectives

1. Find an equation of a linear model and make predictions.
2. Describe the Rule of Four for linear models.
3. Perform a unit analysis of a linear model.

Main point: Find an equation of a linear model and describe the Rule of Four for linear models.

OBJECTIVE 1

- Recall that a linear model is a line that describes the relationship between two quantities in an authentic situation.
- An equation of such a line is also called a *linear model*.

1. The profit of a company is \$20 million in 2010. Each year, the profit increases by \$3 million. Let p the annual profit (in millions of dollars) at t years since 2010.

a. Use a table to help find an equation for t and p.

b. Substitute 5 for t into the equation. What does it mean in this situation?

c. Graph the equation.

d. What is the p-intercept? What does it mean in this situation?

e. When will the annual profit be \$41 million? Explain by using arrows on your graph in part (c).

All the equations

$$p = 3t + 20 \qquad p = 20 + 3t \qquad p = t(3) + 20 \qquad p = 20 + t(3)$$

describe the same relationship between t and p and have the same graph.

OBJECTIVE 2

Recall that the Rule of Four of equations states that we can use equations, tables, graphs, and words to describe solutions of equations.

Rule of Four for Linear Models

We can describe a linear model with

1. an equation,
2. a table,
3. a graph, or
4. words.

Refer to Problem 1 to demonstrate the Rule of Four for linear models.

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OBJECTIVE 3

Here is a *unit analysis* of our model $p = 3t + 20$:

$$\underbrace{p}_{\text{millions of dollars}} = \underbrace{3}_{\frac{\text{millions of dollars}}{\text{year}}} \cdot \underbrace{t}_{\text{years}} + \underbrace{20}_{\text{millions of dollars}}$$

We can use the fact that $\frac{\text{years}}{\text{year}} = 1$ to show that the units of the expressions on both sides of the equation are millions of dollars, which suggests that our equation is correct.

Definition *Unit analysis*

We perform a **unit analysis** of a model's equation by determining the units of the expressions on both sides of the equation. The units of the expressions on both sides of the equation should be the same.

We can perform a unit analysis of a model's equation to help verify the equation.

2. The balance of a person's checking account in 2010 is \$15 thousand. Each year the balance decreases by \$2 thousand. Let B be the balance (in thousands of dollars) at t years since 2010.

- **a.** Use a table to help find an equation for t and B.
- **b.** Perform a unit analysis of the equation you found in part (a).
- **c.** Graph the equation.
- **d.** When will the balance be \$3 thousand? Explain by using arrows on your graph in part (c).

SHORT HW 5, 7, 13, 17, 23, 25, 29, 31

MEDIUM HW 1, 3, 5, 7, 11, 13, 17, 21, 23, 25, 27, 29, 31

SECTION 3.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students find an equation of a linear model and make predictions. They perform a unit analysis of such a model and learn the Rule of Four for linear models. This work lays the foundation for working with rates of change in Section 3.5. Students will also graph equations of the form $x = a$ and $y = b$ in this section.

OBJECTIVE 1 COMMENTS

For part (a) of Problem 1, students' work with finding expressions in Chapter 2 should help them find the equation $p = 3t + 20$. I highlight the shift from working with expressions such as $3t + 20$ in Chapter 2 with working with equations such as $p = 3t + 20$ in Chapter 3. In the lecture notes, the profit model is written in four ways. It should be apparent to students that these four equations are equivalent from the various ways that the arithmetic used to construct the table in part (a) could be arranged. (The commutative and associative laws will be discussed in Section 4.1.)

When finding the p-intercept [part (d) of Problem 1], students may have some trouble seeing that the p-intercept is $(0, 20)$ by inspecting the equation. It will help to refer to the table you found in part (a) or to the graph you found in part (c). However, it is important that students learn to find the vertical intercept from the equation alone. Remind students that the y-intercept of the graph of an equation of the form $y = mx + b$ is the point $(0, b)$ as was discussed in Section 3.1.

While doing parts (b) and (e) of Problem 1, I use the terminology of input and output.

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Textbook and Workbook Exploration
You could begin class with the 15-minute "Finding an equation of a linear model" exploration because students have already learned all the necessary concepts in Sections 1.3, 2.6, and 3.1.

OBJECTIVE 2 COMMENTS
I remind students about the Rule of Four for equations in two variables and use this as a segue to discuss the Rule of Four for linear models.

Workbook Exploration
The following 15-minute exploration is a nice summary of a lot of key concepts in Sections 3.1 and 3.2.

Group Exploration
Rule of four for linear models

The number of bankrupt borrowers who asked a judge to cancel their student loans was 667 borrowers in 2015 and has decreased by about 44 borrowers per year since then (Source: *Administrative Office of the U.S. Court documents*). Let n be the number of bankrupt borrowers who asked a judge to cancel their student loans at t years since 2015. Recall that we can describe a linear model using an equation, a table, a graph, or words.

1. Use a table of values of t and n to describe the situation.
2. Use an equation to describe the situation.
3. Use a graph to describe the situation.
4. Use your table, your equation, and your graph to estimate the number of borrowers who asked a judge to cancel their student loans in 2018. Compare your results.

OBJECTIVE 3 COMMENTS
I emphasize that a unit analysis of a model can be used to verify the equation of the model. This work will help students solve value problems in Section 6.5. (The number of items in a group times the value per item is equal to the total value of the group of items.) Students will also convert units of quantities in Section 12.2.

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SECTION 3.3 LECTURE NOTES

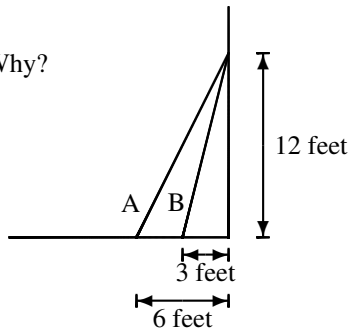
Objectives

1. Use a ratio to compare the steepness of two objects.
2. Describe the meaning of, and how to calculate, the *slope* of a nonvertical line.
3. Determine the sign of the slope of an increasing line and of a decreasing line.
4. Explain why the slope of a horizontal line is zero and that the slope of a vertical line is undefined.

Main point: Describe the meaning of, and how to calculate, the *slope* of a line.

OBJECTIVE 1

1. Ladder B is steeper than ladder A. Why?



Compare $\dfrac{\text{vertical distance}}{\text{horizontal distance}}$ for ladders A and B.

Comparing the Steepness of Two Objects

To compare the steepness of two objects, compute the unit ratio

$$\frac{\text{vertical distance}}{\text{horizontal distance}}$$

for each object. The object with the larger ratio is the steeper object.

2. While taking off, airplane A climbs steadily for 2100 feet over a horizontal distance of 7000 feet. Airplane B climbs steadily for 2850 feet over a horizontal distance of 9900 feet. Which plane is climbing at a greater incline? Explain.

OBJECTIVE 2

Definition *Slope of a nonvertical line*

$$m = \text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$

3. Find the slope of the line that contains the points $(1, 5)$ and $(4, 3)$.

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Calculating Slope

Let (x_1, y_1) and (x_2, y_2) be two distinct points of a nonvertical line. The slope of the line is

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$$

When working with negative coordinates, it can help to first write

$$\frac{(\ ) - (\ )}{(\ ) - (\ )}$$

and then insert the coordinates into the appropriate parentheses.

Find the slope of the line that contains the given points.

4. $(1, 3)$ and $(4, 9)$

5. $(-4, 1)$ and $(2, -3)$

6. $(-5, -1)$ and $(4, 2)$

7. $(-6, -3)$ and $(-2, -5)$

8. Find the approximate slope of the line that contains the points $(-2.8, 5.9)$ and $(-1.1, -3.7)$. Round the result to the second decimal place.

OBJECTIVE 3

For Problems 4 and 6, the *increasing* lines have *positive* slopes. For Problems 5 and 7, the *decreasing* lines have *negative* slopes.

Slopes of Increasing or Decreasing Lines

- An increasing line has positive slope.
- A decreasing line has negative slope.

- Explain this property in general by discussing the signs of rises and runs for an increasing line and for a decreasing line.
- Compare the steepness of two lines with slopes 2 and 3.
- Compare the steepness of two lines with slopes -3 and 2.

Measuring the Steepness of a Line

The absolute value of the slope of a line measures the steepness of the line. The steeper the line, the larger the absolute value of its slope will be.

OBJECTIVE 4

- Draw a horizontal line and find its slope $\left(\frac{0}{\text{run}} = 0\right)$.

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- Draw a vertical line and explain why its slope, $\frac{\text{rise}}{0}$, is undefined.

Slopes of Horizontal and Vertical Lines

- A horizontal line has slope equal to zero. [Draw a figure.]
- A vertical line has undefined slope. [Draw a figure.]

For Problems 9–12, find the slope of the line that contains the given points. Determine whether the line is increasing, decreasing, horizontal, or vertical.

9. $(2, -3)$ and $(2, 1)$

10. $(-3, -2)$ and $(2, 8)$

11. $(-1, -4)$ and $(3, -4)$

12. $(-5, -2)$ and $(-1, -4)$

13. Sketch an increasing line, a decreasing line, a horizontal line, and a vertical line. For each line, determine whether the slope is positive, negative, zero, or undefined (Exercise 51 of Homework 3.3).

SHORT HW 1, 9, 13, 21, 27, 35, 37, 39, 49, 51, 53, 59, 61, 65, 75, 81

MEDIUM HW 1, 5, 9, 13, 15, 21, 25, 27, 33, 35, 37, 39, 43, 49, 51, 53, 57, 59, 61, 63, 65, 73, 75, 81

SECTION 3.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students compare the steepness of two objects and find the slope of a line. They learn about the graphical significance of slope.

OBJECTIVE 1 COMMENTS

When discussing slope, I explain why it is defined in terms of both run and rise of two points on a line and not just the rise of the two points. Students appreciate it when I discuss Problem 2 in the lecture notes. Otherwise, they would have trouble with Exercises 1–4 in Homework 3.3.

The grade of a road is a good motivator of slope (see Example 1 and the following paragraph on page 142 of the textbook.)

Workbook Exploration

The following 20-minute exploration emphasizes that when measuring the steepness of an object, it does not suffice to consider the rise of the object. Problem 2 is a challenging open-ended question. Resist the urge to help students too much. If a group's measure is not valid, point out why that's the case and challenge them to improve their measure or come up with a different one. Remember, even if a group's measure is not valid, putting thought into the concept of steepness will mentally prepare them to then learn about slope and help them appreciate how slope is defined.

Group Exploration

Section Opener: Steepness

Two ladders lean against a building. The foot of ladder A is 3 feet from the building and the top reaches a point at height 12 feet on the building. The foot of ladder B is 6 feet from the building and it reaches a point at height 18 feet on the building.

1. Draw a figure of the situation.

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2. Which ladder is steeper? Explain by using calculations as well as words.

3. A student says ladder B is steeper than ladder A because ladder B reaches a point on the building higher than ladder A does. What would you tell the student?

4. For ladder A, find the unit ratio of the vertical distance to the horizontal distance. Find the unit ratio for ladder B, too. What do the unit ratios mean in this situation? Compare the two ladders' unit ratios. What does your comparison mean in this situation?

5. A student says it would be better to find the unit ratio of the horizontal distance to the vertical distance for each ladder. What would you tell the student?

6. A portion of road A climbs steadily for 105 feet over a horizontal distance of 2950 feet. A portion of road B climbs steadily for 130 feet over a horizontal distance of 4325 feet. Which road is steeper?

OBJECTIVE 2 COMMENTS

To segue from $m = \dfrac{\text{rise}}{\text{run}}$ to $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, I remind my students that the change in a quantity is the final amount minus the beginning amount (Section 2.4).

For Problem 3, I find the slope by plotting the points and using the graph to determine the rise and the run. I continue this practice for at least one of the Problems 4–7 (as well as using the slope formula).

For Problem 4, I also show my students that when we use the slope formula with two points on a line, it doesn't matter which point we choose to be (x_1, y_1) and which we choose to be (x_2, y_2).

I discuss the contents of the warning near the top of page 144 in the textbook.

Problem 8 is good preparation for finding an equation of a linear model in Section 5.5.

When some or all coordinates are negative (see Problems 5–8), a common student error is to omit at least one subtraction symbol from the slope formula or negative sign from the coordinates. The suggestion in the lecture notes to first write the parentheses addresses this type of error.

Textbook and Workbook Exploration

For the 15-minute "For a line, rise over run is constant" exploration, students discover that for a line, the ratio of the rise to the run is constant for any distinct pair of points on the line. Students are often surprised that the value of the slope is independent of which two points are chosen. A good extension of the exploration is to have groups explain why this makes sense.

OBJECTIVES 3 and 4 COMMENTS

Performing a sign analysis of an increasing line and a decreasing line will lead to the meanings of the signs of slope (see Figs. 45 and 46 on pages 144 and 145 of the textbook). Discussing the meanings of the signs of slope will help students understand the meanings of the signs of rates of change in Section 3.5.

A common error when computing the slope is for the sign to be incorrect. To check the sign, a student can quickly plot the points and determine whether the line is increasing or decreasing.

Textbook and Workbook Exploration

The 15-minute "Graphical significance of m for $y = mx$" exploration assists students in discovering the graphical significance of m for an equation of the form $y = mx$, which is a key concept of the course. After students complete the exploration, I have a short classroom discussion in which students tell me their responses for Problem 4 of the exploration. There is a lot of information in this exploration, and students have trouble seeing how it all fits together.

Logically, this exploration belongs in Section 3.4 because it's not until then that it's established that for an equation of the form $y = mx + b$, the slope of the line is m. But aside from this issue, the exploration wonderfully addresses the fundamental idea of Section 3.3.

Workbook Exploration

The following 15-minute exploration might seem redundant with other explorations that have to do with this

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section, but interpreting slope as a measure of steepness is a key concept of the course and one that students tend to forget without a lot of reinforcement.

EXPLORATION *Interpreting slope of a line*

1. Sketch four increasing nonparallel lines. Label your lines l_1, l_2, l_3, and l_4.
2. List your lines from the least steepest line to the steepest line.
3. Compute the slope of each of your four lines. List your lines from the line with the least slope to the one with the greatest slope.
4. Compare the lists you made in Problems 2 and 3. What does this tell you about the slope of a line?
5. Repeat Problems 1–4, but this time for four decreasing nonparallel lines.
6. Summarize what you have learned about slope in this exploration.

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SECTION 3.4 LECTURE NOTES

Objectives

1. Use the slope and the y-intercept of a line to sketch the line.
2. Describe the meaning of m for an equation of the form $y = mx + b$.
3. Graph an equation of the form $y = mx + b$ by using the line's slope and y-intercept.
4. Find an equation of a line from its graph.
5. Graph an equation of a linear model by using the model's slope and y-intercept.
6. Describe the relationship between slopes of parallel lines.
7. Describe the relationship between slopes of perpendicular lines.

Main point: Use the slope and the y-intercept to graph a linear equation.

OBJECTIVE 1

Sketch the line that has the given slope and y-intercept.

1. $m = \frac{2}{5}, (0, -3)$ **2.** $m = -\frac{3}{2}, (0, 4)$ **3.** $m = -2, (0, -1)$

OBJECTIVE 2

4. **a.** Use the method discussed in Section 3.1 to graph $y = 2x + 1$.

b. What is the slope of the line $y = 2x + 1$?

c. Compare the slope with the number multiplied times x in the equation $y = 2x + 1$.

Finding the Slope and y-Intercept from a Linear Equation

For a linear equation of the form $y = mx + b$,

- the slope of the line is m and
- the y-intercept of the line is $(0, b)$.

We say this equation is in **slope-intercept form**.

The graph of the equation $y = -3x + 8$ is a line with slope -3 and y-intercept $(0, 8)$.

OBJECTIVE 3

Sketch the graph of the equation by hand.

5. $y = \frac{3}{5}x - 1$ **6.** $y = -\frac{4}{3}x + 5$ **7.** $y = 3x - 4$ **8.** $y = -2x - 3$

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Graphing an Equation in Slope-Intercept Form

To graph an equation of the form $y = mx + b$,

1. Plot the y-intercept $(0, b)$.
2. Use $m = \dfrac{\text{rise}}{\text{run}}$ to plot a second point. For example, if $m = \dfrac{2}{3}$, then count 3 units to the right (from the y-intercept) and 2 units up to plot another point.
3. Sketch the line that passes through the two plotted points.

OBJECTIVE 4

9. Graph $y = -\dfrac{2}{5}x - 1$ but do not reveal the equation to your students. Find an equation of the line.

Finding an Equation of a Line from a Graph

To find an equation of a line from a graph:

1. Determine the slope m and the y-intercept $(0, b)$ from the graph.

2. Substitute your values for m and b into the equation $y = mx + b$.

OBJECTIVE 5

10. The percentages of Americans who listened to online radio in the past week are shown in the following table for various years.

Year	Percent
2011	22
2012	29
2013	33
2014	36
2015	44
2016	50

Source: *Edison Research*

Let p be the percentage of Americans who listened to online radio in the past week at t years since 2010. A reasonable model is $p = 5.4t + 17$.

a. Graph the model by hand.

b. Use your graph to predict when all Americans will have listened to online radio in the past week. Do you have much faith in your prediction?

OBJECTIVE 6

Sketch two parallel lines and compare their slopes.

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Slopes of Parallel Lines

If lines l_1 and l_2 are parallel nonvertical lines, then the slopes of the lines are equal:

$$m_1 = m_2$$

Also, if two distinct lines have equal slope, then the lines are parallel.

OBJECTIVE 7

Sketch two perpendicular lines and compare their slopes.

Slopes of Perpendicular Lines

If lines l_1 and l_2 are perpendicular nonvertical lines, then the slope of one line is the opposite of the reciprocal of the slope of the other line:

$$m_2 = -\frac{1}{m_1}$$

Also, if the slope of one line is the opposite of the reciprocal of another line's slope, then the lines are perpendicular.

Determine whether the pair of lines is parallel, perpendicular, or neither.

11. $y = -2x + 3$ and $y = 2x - 5$

12. $y = \frac{2}{7}x - 3$ and $y = -\frac{7}{2}x + 4$

SHORT HW 3, 17, 19, 29, 37, 43, 47, 51, 55, 65, 67, 71, 79, 91, 99

MEDIUM HW 1, 3, 9, 13, 15, 17, 19, 27, 29, 37, 43, 47, 49, 51, 55, 61, 65, 67, 71, 79, 81, 87, 91, 95, 99

SECTION 3.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use the slope and the y-intercept to graph equations in slope-intercept form. After students learn how to solve equations in Chapter 4, they will graph linear equations not in slope-intercept form in Section 5.1. Students tend to do better with this organization, rather than having to contend with graphing linear equations of all forms in just one section.

OBJECTIVE 1 COMMENTS

Problems 1–3 provide the "big picture" of the section.

For Problem 2, some students are unsure what to do with the negative sign of the slope. Students want to know whether they should work with the slope $\frac{-3}{2}$ or $\frac{3}{-2}$. I demonstrate that both ways give the same result; I emphasize that our main goal is to graph the line, not find two points on the line. A common error is to write $-\frac{3}{2} = \frac{-3}{-2}$.

For Problem 3, some students need a reminder that $a = \frac{a}{1}$.

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OBJECTIVE 2 COMMENTS

You can address this objective by doing Problem 4 or having groups work on the following exploration.

Textbook and Workbook Exploration

The 15-minute "The meaning of m in the equation $y = mx + b$" exploration in is a great way for groups to discover that for an equation of the form $y = mx + b$, the slope of the equation is m. Many groups will need help with part (e) of Problem 4 of the exploration.

Textbook and Workbook Exploration

Students really enjoy the "Drawing lines with various slopes" exploration, which encourages students to reflect on the graphical significance of m and b of an equation of the form $y = mx + b$. Most students restrict their choices of slopes to positive integers at first and then realize that they need to use negative integers, too. Most students need hints to get them thinking about using slopes between -1 and 1.

OBJECTIVE 3 COMMENTS

This objective follows nicely from objective 1. I use a graphing calculator to verify at least one graph.

OBJECTIVE 4 COMMENTS

Before doing Problem 9, I say a line has slope $\frac{3}{7}$ and y-intercept -4 and ask my students to find the equation of the line. Then I have my students do Problem 10, which they find to be an easy task. I point out that this task is the reverse of graphing. I remind my students about the Rule of Four for equations. I say we can go from any one of these four ways to another. (Students will learn how to go from tables to equations in Section 3.5.)

Textbook and Workbook Exploration

The 10-minute "Finding an equation of a line from a graph" exploration is a fun way for students to practice finding an equation of a line from its graph.

OBJECTIVE 5 COMMENTS

For Problem 10, I tell my students that they will learn to find linear models such as $p = 5.4t + 17$ in Section 5.5. I demonstrate how to use the method in this section to sketch the model $p = 5.4t + 17$, but I also point out that it is often useful to find a point far off to the right such as $(6,\ 49.4)$ by substituting 6 for t. Finding such a third point can help students sketch an accurate line, especially when the run of their first two chosen points is small compared to the largest value of the scaling on the horizontal axis.

OBJECTIVE 6 COMMENTS

When I ask students what we can say about the slopes of two parallel lines, they are quick to say the slopes are equal.

OBJECTIVE 7 COMMENTS

When discussing the slopes of two perpendicular lines, most students will follow the verbal description that the slope of one line is the opposite reciprocal of the slope of the other line. It does not mean much to students to say the product of the slopes is -1 (as most textbooks say).

If you are tight on time, this objective can be skipped because it will not be needed for the rest of the course.

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SECTION 3.5 LECTURE NOTES

Objectives

1. Calculate the rate of change of a quantity.
2. Explain why slope is a rate of change.
3. Use rate of change to help find a linear model.
4. Describe the slope addition property.

Main point: Slope is a rate of change.

OBJECTIVE 1

1. If the temperature increased by $12°$F over 3 hours, estimate how much the temperature increased per hour.
2. If temperature increased from $60°$F at 7 A.M. to $68°$F at 11 A.M., estimate how much the temperature increased per hour.

Formula for Rate of Change

Suppose that a quantity y changes steadily from y_1 to y_2 as a quantity x changes steadily from x_1 to x_2. Then the **rate of change** of y with respect to x is the ratio of the change in y to the change in x:

$$\frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

3. The sales of cigarettes in the United States deceased approximately linearly from 400 billion cigarettes in 2003 to 249 billion cigarettes in 2017 (Source: *Euromonitor International; Centers of Disease Control and Prevention*). Find the approximate rate of change of cigarettes sales.
4. A person of height 132.5 centimeters should use a ski pole of length 90 centimeters. A person of height 153.0 centimeters should use a ski pole of length 110 centimeters. Find the rate of change of ski pole length with respect to a person's height.

Refer to Problems 3 and 4 to motivate the following facts.

Signs of Rate of Change

Suppose that a quantity t affects (explains) a quantity p. Then

- If p increases steadily as t increases steadily, then the rate of change of p with respect to t is positive.
- If p decreases steadily as t increases steadily, then the rate of change of p with respect to t is negative.

OBJECTIVE 2

We use the formula $\frac{y_2 - y_1}{x_2 - x_1}$ to find rate of change and also to find slope of a line. So, slope is a rate of change.

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5. Let d be the distance (in miles) that a student can drive in t hours (see the table).

t (hours)	d (miles)
0	0
1	60
2	120
3	180
4	240
5	300

a. Construct a scatterplot. Then draw a linear model.

b. Find the slope of the linear model.

c. Find the rate of change of the distance traveled in each given period. Compare each result with the slope of the linear model.

i. From $t = 3$ to $t = 4$

ii. From $t = 0$ to $t = 5$

Slope Is a Rate of Change

If there is a linear relationship between quantities t and p, and if t affects (explains) p, then the slope of the linear model is equal to the rate of change of p with respect to t.

Constant Rate of Change

Suppose that a quantity t affects (explains) a quantity p. Then

- If there is a linear relationship between t and p, then the rate of change of p with respect to t is constant.
- If the rate of change of p with respect to t is constant, then there is a linear relationship between t and p.

OBJECTIVE 3

6. A person earns a starting salary of \$30 thousand at a company. Each year, she receives a \$2 thousand raise. Let s be the person's salary (in thousands of dollars) after she has worked at the company for t years.

a. What is the slope of the linear model that describes this situation? What does it mean in this situation?

b. What is the s-intercept of the model? What does it mean in this situation?

c. Find an equation of the model.

d. Perform a unit analysis of the equation you found in part (c).

7. Let T be the total one-semester cost (in dollars) of tuition plus parking fee for u units (credits or hours) of classes at your college. [You could replace the parking fee with any one-time charge such as a student-services fee at your college.]

a. Find an equation of a linear model.

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b. What is the total one-semester cost of tuition plus parking fee for 15 units of classes?

8. The number of multidrug-resistant tuberculosis cases in Estonia was 43 in 2015 and has decreased by about 9 cases per year (Source: *World Health Organization*).

a. Find an equation of a linear model to describe the situation. Explain what your variables represent.

b. Estimate the number of multidrug-resistant tuberculosis cases in 2018.

9. The Gallup U.S. standard of living index measures the level at which Americans are satisfied with their standard of living. The indexes are shown in the following table for various years.

Year	U.S. Standard of Living Index
2008	23
2009	26
2010	31
2011	31
2012	34
2013	38
2014	45
2015	49
2016	50
2017	54

Source: *Gallup*

Let s be the U.S. standard of living index at t years since 2000. A reasonable model is $s = 3.53t - 6.07$.

a. Use a graphing calculator to draw a scatterplot and the model in the same viewing window. Check whether the line comes close to the data points.

b. What is the slope? What does it mean in this situation?

c. Find the rates of change in the U.S. standard of living index from one year to the next. Compare the rates of change with your result in Part (b).

d. Predict the U.S. standard of living index in 2022.

OBJECTIVE 4

Use tables of solutions of $y = 2x + 3$ and $y = -3x + 20$ to motivate the slope addition property.

Slope Addition Property

For a linear equation of the form $y = mx + b$, if the value of x increases by 1, then the value of y changes by the slope m. In other words, if an input increases by 1, the output changes by the slope.

10. Four sets of points are described in the following table. For each set, decide whether there is a line that passes through every point. If so, find the slope of that line. If not, decide whether there is a line that comes close to every point.

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Set 1		Set 2		Set 3		Set 4	
x	y	x	y	x	y	x	y
0	2	0	95	2	11	5	4
1	5	1	91	3	23	6	4
2	8	2	87	4	35	7	4
3	11	3	83	5	47	8	4
4	14	4	79	6	59	9	4

11. Some values of four linear equations are provided in the following table. Complete the table.

Equation 1		Equation 2		Equation 3		Equation 4	
x	y	x	y	x	y	x	y
0	3	0	79	21	25	43	83
1	8	1	72	22		44	
2		2		23		45	
3		3		24	31	46	
4		4		25		47	71

SHORT HW 3, 9, 17, 21, 23, 25, 33, 35, 39, 43, 45, 47, 51, 55, 59

MEDIUM HW 1, 3, 9, 13, 17, 21, 23, 25, 27, 29, 33, 35, 39, 41, 43, 45, 47, 53, 55, 59 Note: Exercises 27–30 ask students to use a graphing calculator to draw a scatterplot.

SECTION 3.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students find the rate of change of a quantity and learn that slope is a rate of change. Students then use rate of change to find a linear model. Students also learn the slope addition property. These concepts will be helpful for work in Sections 5.5, 5.6, 5.7, and 6.4.

OBJECTIVE 1 COMMENTS

If you discussed the change of a quantity in Section 2.4 and unit ratios in Section 2.5, your students should be well primed for this objective. If you didn't discuss these concepts then, spend some time discussing them now.

I tell students that 50 miles per hour is an example of rate of change. I say a rate of change is a description of how quickly a quantity changes in relation to another quantity changing. I say we often use the word "per" to describe a rate of change. I give some more examples of rates of change such as

- A student earns $8 per hour.
- The altitude of an airplane decreases by 2200 feet per minute.
- A college charges $80 per unit. (You could use the charge per unit (hours or credits) at your college.)

Then I discuss the material in the lecture notes. When doing Problems 1–4, I emphasize we should include units when computing changes in the response variable and changes in the explanatory variable. Including units throughout the calculations will help students write the correct units for their result. For Problem 4, I emphasize which change goes in the numerator and which change goes in the denominator.

Workbook Exploration

You could start class by having students work on the following 15-minute exploration. This is a great way

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to motivate the concept rate of change. Some groups may use a numerical approach and other groups may use a graphical approach. Some groups will likely compute changes in the response variable but not take into account changes in the explanatory variable; in fact, I selected the data set so that this approach will yield the wrong conclusion. During the debrief, comparing and contrasting groups' results should generate a productive discussion.

Group Exploration
Section Opener: Rate of change

For which type of car, domestic or imported, has fuel efficiency improved the fastest? Explain.

	Domestic		Imported
Year	(miles per gallon)	Year	(miles per gallon)
2010	33.1	2007	32.2
2011	32.7	2009	33.8
2012	34.8	2011	33.7
2013	36.0	2013	36.6
2014	36.7	2014	36.0

Source: *U.S. Department of Transportation*

OBJECTIVE 2 COMMENTS
I point to the difference quotients used to find results for Problems 2–4 and ask students whether the difference quotients remind them of something. If no one is reminded of slope, I refer to the boxed formula

$$\frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

I then conclude that slope is a rate of change.

If you would like to do this section in one day, you will likely have to skip Problem 5, although it is a nice way to emphasize that slope is a rate of change. Due to time constraints, I usually do not discuss the boxed property with the heading "Constant Rate of Change."

Workbook Exploration
The following exploration is an excellent way for students to discover that for a linear equation in two variables, slope is a rate of change. Students will likely gain a deeper understanding of this concept by completing the exploration than by watching a lecture (which is probably true of most of the explorations, but especially this one.) This concept might be the most important one of the course for those students who will take calculus.

Students have an easy time with this exploration. To save time, I usually assign only Problems 1 and 2. Once most groups are done, I emphasize that slope is a rate of change. I remind my students of the graphical interpretations of slope we've discussed so far.

Group Exploration
Section Opener: Significance of the slope and the response variable's intercept of a model

1. A small airplane is traveling at a constant speed of 100 miles per hour. Let d be the distance (in miles) the airplane can travel in t hours.

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a. Complete the following table.

Time (hours) t	Distance (miles) d
0	
1	
2	
3	
4	

b. Find an equation of a linear model.

c. Compare the slope of your model with the speed of the airplane.

d. What is the d-intercept? What does it mean in this situation?

2. In 2010, a company was worth \$10 million. Each year, its value increases by \$2 million. Let V be the company's value (in millions of dollars) at t years since 2010.

a. Complete the following table.

Year since 2010 t	Value (millions of dollars) V
0	
1	
2	
3	
4	

b. Find an equation of a linear model.

c. Compare the slope of your model with the rate at which the company's value is increasing.

d. What is the V-intercept? What does it mean in this situation?

3. A person is in a hot-air balloon at an altitude of 1600 feet. The person begins gradually letting air out of the balloon, and the balloon descends at a rate of 200 feet per minute. Let H be the balloon's altitude (in feet) after air has been released for t minutes.

a. Complete the following table.

Time (minutes) t	Altitude (feet) H
0	
1	
2	
3	
4	

b. Find an equation of a linear model.

c. Compare the slope of your model with the rate at which the balloon's altitude is changing.

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d. What is the H-intercept? What does it mean in this situation?

4. In general, what is the meaning of the slope in terms of an authentic situation? What is the meaning of the response variable's intercept?

Workbook Exploration

The following 30-minute exploration is similar to the exploration with the same name in the comments about Objective 1, but this exploration provides more structure and has students consider how rate of change and slope of a linear model are connected.

EXPLORATION *Rate of change*

1. World record times for the 200-meter run are listed in the following table for various years.

Women		Men	
Year	Record Time (seconds)	Year	Record Time (seconds)
1973	22.38	1951	20.6
1974	22.21	1963	20.3
1978	22.06	1967	20.14
1984	21.71	1979	19.72
1988	21.34	1996	19.32
		2009	19.19

Source: *IAAF Statistics Handbook*

1. Which gender's record times are decreasing the most per year? Use calculations and words to explain.

2. Construct scatterplots by hand for the women's data and the men's data on the same coordinate system. Make it clear which data points are for which gender.

3. On the scatterplots you constructed in Problem 2, sketch a linear model for each gender.

4. Calculate the slope of each of your linear models. What do your results mean in this situation? Do your results support the claim you made in Problem 1? Explain.

5. A student says the men's record times are decreasing the most per year because the decrease in their record times from 1951 to 2009 is greater than the decrease in the women's record times from 1973 to 1988. What would you tell the student?

6. A student uses the 1974 and 1978 data points to find a linear model for women's record times and the 1963 and 1967 data points to find a linear model for the men's record times. She finds that both gender's record times decreased by about 0.04 second per year. The student concludes that the record times for each gender are decreasing at the same rate. What would you tell the student?

Workbook Exploration

In the following 20-minute exploration, students explore the connection between rate of change and computing the slope of a linear model where the change in the explanatory variable is different than 1. A few groups may need a reminder what "ratio" means. Once groups are over this hurdle, the rest of the exploration flows nicely.

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Group Exploration
Section Opener: Slope is a ratio

Here you will work with the women's 400-meter run model $r = -0.27t + 70.45$, where r is the record time (in seconds) at t years since 1900.

1. What is the slope of the graph of $r = -0.27t + 70.45$? What does the slope mean in this situation? [**Hint:** Consider the slope addition property.]
2. According to the model, by how much should the record time change each year?
3. By how much should the record time change in two years? What is the ratio of the change in the record time to the change in calendar time?
4. By how much does the 400-meter record change in three years? What is the ratio of the change in the record time to the change in calendar time?
5. For any period, what is the ratio of the change in the record time to the corresponding change in calendar time? Explain.

OBJECTIVE 3 COMMENTS
Although students have found equations of models by recognizing patterns of arithmetic, they will initially have trouble with finding such an equation using the concepts in this section.

For Problem 6, I say that because the rate of change of salary per year is a constant, we can model the situation using a linear model. Then I write "$y = mx + b$" on the board, but point out that we must use the variables t and s. I discuss why s is the response variable and t is the explanatory variable. Then I replace y with s and x with t to get $s = mt + b$. Most students are quick to understand that m is 2, but have some trouble with seeing why b is 30. It helps to remind my students that $(0, b)$ is the y-intercept of the graph of an equation of the form $y = mx + b$. It also helps to construct a table for t and s and put the number 0 beneath t and have my students tell me the value of s. But the most effective way might be to sketch a linear model on a coordinate system, which shows that the vertical intercept is 30 and, hence, the value of b is 30.

In part (a) of Problem 9, you are supposed to use a graphing calculator to draw a scatterplot. Because this section is so long, I delay discussing graphing calculator scatterplot instructions until Section 5.5, which is when students will use this skill a great deal. For part (a), I construct a scatterplot by hand, using every other point, and then graph the model.

For Exercises 27–30 of Homework 3.5, I tell my students they can skip part (a), which instructs students to use a graphing calculator to draw a scatterplot. There are also two exercises in Section 4.4 and two exercises in Section 4.6 that require the use of a graphing calculator to draw scatterplots.

Textbook and Workbook Exploration
The 20-minute "Averaging rates of change" exploration has students compare the average of some rates of change with the slope of a linear model. (It is only if the data years are consecutive that we know for sure that the two quantities are equal.)

OBJECTIVE 4 COMMENTS
Students have an easy time with the slope addition property.

Workbook Exploration
The following 15-minute exploration can serve as a good exploration of the slope addition property. You could show students how to use graphing calculator tables to complete the table for Problem 1. Although students should certainly be able to complete the table without technology, a small arithmetic error might prevent a group from discovering the concept.

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EXPLORATION *Slope addition property*

1. Complete the following table.

$y = 2x + 1$		$y = 3x - 5$		$y = -2x + 6$	
x	y	x	y	x	y
0		0		0	
1		1		1	
2		2		2	
3		3		3	
4		4		4	

2. In the table, the x-coordinates increase by 1 each time. For each equation, what do you notice about the y-coordinates? Compare what you notice with the coefficient of x in each equation.

3. Describe what the pattern from Problem 2 would be in general for any equation of the form $y = mx + b$.

4. Create an equation of the form $y = mx + b$, and check whether it behaves as you described in Problem 3.

5. Substitute 1 for x in the equation $y = mx + b$. Then substitute 2 for x. Then substitute 3. Explain why these results suggest that your description in Problem 3 is correct.

Textbook and Workbook Exploration

Groups have an easy time with the 20-minute "Determining whether a line contains some points" exploration. A nice extension is to have groups identify the slope of the line for each of Problems 1 and 2 and discuss the slope addition property in this context. You could also have groups find the equation of the line for each of Problems 1, 2, and 4.

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CHAPTER 4 OVERVIEW

In Section 4.1, students use the commutative, associate, and distributive laws to simplify linear expressions. In Section 4.2, students combine like terms to help simplify more linear expressions. They use symbolic methods, graphing, and tables to solve a linear equation in one variable in Sections 4.3 and 4.4. In Section 4.5, students compare simplifying expressions with solving equations. In Section 4.6, students solve a formula for a variable.

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SECTION 4.1 LECTURE NOTES

Objectives

1. Describe the commutative, associative, and distributive laws.
2. Compare the distributive and associative laws for multiplication.
3. Describe the meaning of *equivalent expressions*.
4. *Simplify* expressions.
5. Subtract two expressions.
6. Show that a statement is false.

Main point: Simplify expressions.

OBJECTIVE 1

The equations $3 + 7 = 7 + 3$ and $3 \cdot 7 = 7 \cdot 3$ suggest the following laws.

Commutative Laws for Addition and Multiplication

Commutative law for addition: $a + b = b + a$
Commutative law for multiplication: $ab = ba$

Use a commutative law to write the expression in another form.

1. $5 + x$

2. $x(2)$

- A **term** is a constant, variable, or a product of a constant and one or more variables raised to powers.
- Here are some terms: $6x$, 5, w, $-3pt$, $\frac{4y}{w}$.
- **Variable terms** are terms that contain variables.
- **Constant terms** are terms that do no contain variables.
- We usually write a sum of both variable and constant terms with the variable terms to the left of the constant terms.

3. Write $7 - 3x$ so that the variable term is to the left of the constant term.

The equations $(3 + 2) + 5 = 3 + (2 + 5)$ and $(3 \cdot 2) \cdot 5 = 3 \cdot (2 \cdot 5)$ suggest the following laws.

Associative Laws for Addition and Multiplication

Associative law for addition: $a + (b + c) = (a + b) + c$
Associative law for multiplication: $a(bc) = (ab)c$

Use an associative law to write the expression in another form.

4. $(w + 4) + 6$

5. $8(4x)$

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6. Rearrange the terms of $7 - 5x + 3 - 9$ so that the numbers can be added.

The equation $3(2 + 4) = 3 \cdot 2 + 3 \cdot 4$ suggests the following law.

Distributive Law

$$a(b + c) = ab + ac$$

Find the product.

7. $2(x + 3)$ **9.** $4(2x + 8)$ **11.** $3(4x - 7y)$ **13.** $(x - 9)(2)$

8. $3(w - 5)$ **10.** $6(5t - 3)$ **12.** $-4(7 - x)$ **14.** $3(2p - 4t + 6)$

OBJECTIVE 2

Compare and contrast:

- The associative laws change the order of *operations*.
- The commutative laws change the order of *terms* or even *expressions*.

OBJECTIVE 3

15. Evaluate both of the expressions $2(x + 3)$ and $2x + 6$ for the values $x = 0$, $x = 1$, $x = 2$, $x = 3$, $x = 4$, and $x = 5$. (After doing a couple of evaluations by hand, use a graphing calculator table.)

Definition *Equivalent expressions*

Two or more expressions are **equivalent expressions** if, when each variable is evaluated for *any* real number (for which all the expressions are defined), the expressions all give equal results.

OBJECTIVE 4

- We **simplify** an expression by using the laws to remove any parentheses and to rearrange terms so that we can add any constant terms.
- The result of simplifying an expression is a **simplified expression**, which is equivalent to the original expression.

Simplify.

16. $2(x + 3) - 8$ **17.** $-4(2w - 5) + 3$ **18.** $7 - 5(x + 4)$ **19.** $6 - 3(4x - 9)$

20. Let x be a number. Translate the phrase "3, minus 2 times the difference of twice the number and 5" to an expression. Then simplify the expression.

21. Use the fact $-a = -1a$ to simplify $-(x + 5)$.

OBJECTIVE 5

Use the fact $a - b = a - 1b$ to simplify the expression.

22. $5 - (2x + 3)$ **23.** $-7 - (8t - 1)$

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OBJECTIVE 6
Show that the statement is false.

24. $a - b = b - a$

25. $a \div b = b \div a$

OBJECTIVE 6
Compare $a(b + c) = ab + ac$ with $a(bc) = (ab)c$.

SHORT HW 3, 9, 15, 25, 35, 49, 55, 61, 77, 79, 85, 91, 99, 103, 113

MEDIUM HW 3, 9, 15, 25, 29, 31, 35, 37, 43, 49, 55, 61, 71, 77, 79, 85, 87, 91, 95, 99, 103, 107, 109, 113

SECTION 4.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text
In this section, students use the commutative, associative, and distributive laws to simplify expressions. These concepts will be used throughout the course. Most students will have an easy time with this section. In fact, many students will recognize this material and think, "Now *this* is algebra!"

OBJECTIVES 1 and 2 COMMENTS
When I am tight on time, I give light treatment to the commutative and associative laws and emphasize the distributive law. Either from past experiences with algebra or by way of intuition, my students seem to know how to apply the commutative and associative laws to simplify expressions. The main thing that students need to know in relation to the commutative and associative laws is that they can rearrange terms.

Textbook and Workbook Exploration
The 20-minute "Laws of operations" exploration has groups discover the three laws of operations discussed in this section. Groups have an easy time with this exploration.

OBJECTIVE 3 COMMENTS
When simplifying an expression, most students think that an equality symbol means, "here's the next step," instead of that the expressions on both sides of the equality symbol are equivalent. To spend time on Problem 15 is well worth it. In fact, this fundamental concept bears repeating multiple times throughout the course. The objective of locating errors in simplifying expressions in Section 4.2 will reinforce this concept (see pages 201 and 202 of the textbook).

Textbook and Workbook Exploration
In the 10-minute "Equivalent expressions" exploration, groups take a closer look at what it means for two expressions to be equivalent.

OBJECTIVE 4 COMMENTS
I discuss how to use the property $-a = -1a$ to simplify expressions such as $-(x+5)$: $-(x+5) = -1(x+5) = -1x + (-1)5 = -x - 5$ (see Problem 21). (The distributive law says that we can distribute a number such as -1, not an opposite symbol.) This will set the stage for Objective 4.

OBJECTIVE 5 COMMENTS
For problems such as Problems 22 and 23, most textbooks suggest to change the signs of the terms in the parentheses, but students don't seem to relate to this technique. Plus, it is not clear to students how that technique is tied to the laws of operations, whereas students can see that the technique that uses the fact $a - b = a - 1b$ is tied

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to the distributive law.

OBJECTIVE 6 COMMENTS

For Problem 24, showing that the statement $a - b = b - a$ is false accomplishes two things: Students are warned that there is no commutative law for subtraction, and they learn how to show that a statement is false.

Many students think that a should be distributed to both b and c in the expression $a(bc)$. In other words, students tend to confuse the distributive law and the associate law for multiplication. Addressing this issue now can help later when students simplify more complicated expressions such as $2(3\sqrt{5})$ in Section 13.2

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SECTION 4.2 LECTURE NOTES

Objectives

1. Combine like terms.
2. Simplify expressions.
3. Locate errors in simplifying expressions by evaluating expressions.

Main point: Use combining like terms to simplify more expressions.

OBJECTIVE 1

- The **coefficient** of a variable term is the constant factor of the term.
- For example, the coefficient of $6x$ is 6, the coefficient of x is 1, and the coefficient of 8 is 8.
- **Like terms** are either constant terms or variable terms that contain the same variable(s) raised to exactly the same power(s).
- For example, $-8x$ and $3x$ are like terms. Also, 2 and 5 are like terms.
- If terms are not like terms, we say they are **unlike terms**.
- For example, $4x$ and $7y$ are unlike terms.
- We use the distributive law to *combine like terms*:

$$3x + 5x = (3 + 5)x = 8x$$

Combine like terms.

1. $3x + 5x$ **2.** $8k - 2k$ **3.** $4x + x$ **4.** $\frac{2}{7}x - \frac{5}{7}x$

Combining Like Terms

To combine like terms, add the coefficients of the terms and keep the same variable factors.

OBJECTIVE 2

We simplify an expression by removing parentheses and combining like terms.

Simplify.

5. $3x - 4y + 2x + 5 - 6y$

6. $2(x + 3) - 4(5x - 2)$

7. $7a - 2(3a - 5b) + 4a$

8. $2(3x - 5y) - 4(2x + 6y + 3)$

9. $3(2t - 5) - (t + 1)$

10. $5(4x - 2y) - (8x - 3y + 1)$

For Problems 11 and 12, let x be a number. Translate the English phrase into a mathematical expression. Then simplify the expression.

11. 7 times the sum of 2 and the number

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12. Twice the number, minus -3 times the sum of the number and 7

For Problems 13 and 14, let x be a number. Translate the expression into an English phrase. Then simplify the expression.

13. $6(x+4)$

14. $x-4(x-2)$

OBJECTIVE 3

Consider the following incorrect work:

$$\begin{aligned} 3(x+4)-2x+5 &= 3x+4-2x+5 \\ &= 3x-2x+4+5 \\ &= x+9 \end{aligned}$$

1. Use a graphing calculator table to show that the work is incorrect.

2. Pinpoint where the error was made by evaluating the expression in each step for $x=2$.

SHORT HW 1, 7, 17, 21, 29, 37, 41, 45, 55, 63, 67, 71, 75, 87, 89

MEDIUM HW 1, 3, 7, 11, 17, 21, 23, 29, 33, 37, 41, 45, 49, 55, 59, 63, 67, 71, 73, 75, 77, 79, 85, 87, 89

SECTION 4.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students combine like terms and use the laws of operations to simplify linear expressions. These skills will be used throughout the course.

OBJECTIVE 1 COMMENTS

Students have an easy time with this skill.

Workbook Exploration

In the following 10-minute exploration, some groups may skip using the distributive law, which is the main point of the exploration.

Group Exploration

Section Opener: Combining like terms

We can write the sum $2x+5x$ as one term by using the distributive law:

$$\begin{aligned} 2x+5x &= (2+5)x && \text{Distributive law: } ac+bc=(a+b)c \\ &= 7x && \text{Add.} \end{aligned}$$

1. Simplify. Your work should include the distributive law.

a. $3x+6x$ **b.** $7x-2x$ **c.** $4x+x$

d. $6x+3x-5x$ **e.** $4x-2y+3-7x-y+6$ **f.** $-3.8y+9.2x+6.3-1.1x+2.5y$

2. A student tries to write $2x+3y$ as one term:

$$2x+3y=5xy$$

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Is the work correct? If yes, use the distributive to explain. If no, substitute numbers for x and y on both sides of the equation to show that the result is false.

OBJECTIVE 2 COMMENTS

For Problem 6, I remind my students how to use a graphing calculator table to verify work in simplifying an expression.

Textbook and Workbook Exploration

The 5-minute "Laws of operations" exploration essentially warns groups of three types of common student errors.

OBJECTIVE 3 COMMENTS

It is unlikely that students will use this technique unless there are points attached to it on an assignment or exam. However, even if you do not wish to attach such points, a demonstration of this technique to students during lecture is a nice way to reinforce their understanding of equivalent expressions.

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SECTION 4.3 LECTURE NOTES

Objectives

1. Describe a *linear equation in one variable*.
2. For an equation in one variable, describe *satisfy*, *solution*, *solution set*, and *solve*.
3. Describe three roles of variables.
4. Describe *equivalent equations*.
5. Apply the addition property of equality.
6. Apply the multiplication property of equality.
7. Solve a linear equation in one variable.
8. Solve a percentage problem.
9. Use a graph or a table to solve a linear equation in one variable.

Main point: Solve linear equations in one variable.

OBJECTIVE 1

Definition *Linear equation in one variable*

A **linear equation in one variable** is an equation that can be put into the form

$$mx + b = 0$$

where m and b are constants and $m \neq 0$.

OBJECTIVE 2

1. Show that the equation $x + 3 = 7$ becomes a true statement if 4 is substituted for x.

Definition *Solution, satisfy, solution set,* and *solve* for an equation in one variable

A number is a **solution** of an equation in one variable if the equation becomes a true statement when the number is substituted for the variable. We say the number **satisfies** the equation. The set of all solutions of the equation is called the **solution set** of the equation. We **solve** the equation by finding its solution set.

2. Is 2 a solution of the equation $2 - 4x = 3(x - 4)$?
3. Is 5 a solution of the equation $2 - 4x = 3(x - 4)$?

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OBJECTIVE 3

Roles of a Variable

Here are three roles of a variable:

1. A variable represents a quantity that can vary.
2. In an expression, a variable is a placeholder for a number.
3. In an equation, a variable represents any number that is a solution of the equation.

OBJECTIVE 4

4. Show that the following equations all have the same solution set: $x = 6$, $x + 2 = 6 + 2$, and $x + 2 = 8$.

Equivalent equations are equations that have the same solution set.

OBJECTIVE 5

Addition Property of Equality

If A and B are expressions and c is a number, then the equations $A = B$ and $A + c = B + c$ are equivalent.

Solve.

5. $x - 4 = 2$ **6.** $x - 1.5 = 8.2$ **7.** $x + 3 = 7$ **8.** $b + 5 = 0$

9. Show that the following equations all have the same solution set: $x = 4$, $3 \cdot x = 3 \cdot 4$, and $3x = 12$.

OBJECTIVE 6

Multiplication Property of Equality

If A and B are expressions and c is a nonzero number, then the equations $A = B$ and $Ac = Bc$ are equivalent.

OBJECTIVE 7

Solve.

10. $\frac{5}{2}x = 3$ **11.** $-15 = -5x$ **12.** $\frac{5}{2}w = \frac{3}{4}$ **13.** $-t = 7$

OBJECTIVE 8

14. A person tips a waiter 15% of the bill. If the tip is \$5.10, how much is the bill?

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OBJECTIVE 9

Using Graphing to Solve an Equation in One Variable

To use graphing to solve an equation $A = B$ in one variable, x, where A and B are expressions:

1. Graph the equations $y = A$ and $y = B$ on the same coordinate system. (For example, if the original equation is $5x - 9 = 3x + 7$, then we would graph the equations $y = 5x - 9$ and $y = 3x + 7$.)
2. Find all intersection points.
3. The x-coordinates of those intersection points are the solutions of the equation $A = B$.

For Problems 15 and 16, graph $y = \frac{3}{5}x + 1$ by hand. Use the graph to solve the given equation.

15. $\frac{3}{5}x + 1 = 4$

16. $\frac{3}{5}x + 1 = -2$

17. Use "intersect" to solve $\frac{2}{3}x + \frac{5}{6} = \frac{3}{2}x - \frac{5}{3}$.

18. Use a table of values of $y = 4x - 3$ to solve $4x - 3 = 5$.

Discuss solving an equation in terms of undoing operations.

SHORT HW 1, 7, 9, 19, 29, 39, 49, 57, 59, 67, 73, 79, 81, 91, 95

MEDIUM HW 1, 7, 9, 17, 19, 29, 31, 35, 39, 41, 47, 49, 57, 59, 67, 73, 75, 79, 81, 83, 91, 95, 97, 99

SECTION 4.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use symbolic methods, graphing, and tables to solve linear equations in one variable. They will use this material in many sections in the textbook.

OBJECTIVE 1 COMMENTS

I remind my students that in Chapter 3 we graphed equations in two variables. I say we will now work with linear equations in *one* variable. I provide a few examples and then define *linear equation in one variable*.

Another option is to introduce the definition after you have solved a few equations so that students know what "put into the form" means.

OBJECTIVE 2 COMMENTS

For Problem 1, I write $x + 3 = 7$ on the board and ask students what x can be. Students are quick to say x is equal to 3. I say 3 is called a *solution* and that it *satisfies* the equation. Then I define the words in the boxed statement and observe that these words mean about the same thing when applied to equations in two variables.

OBJECTIVE 3 COMMENTS

I remind students that we have discussed two roles of a variable so far and say there is a third role.

OBJECTIVE 4 COMMENTS

This objective is setting the stage for Objective 5.

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OBJECTIVES 5 and 6 COMMENTS

For the addition property of equality, most students think that the entire point is that "we can add a number to both sides of an equation." Most students do not know that a solution of an equation will satisfy *all* the equations that lead to finding the solution. At various times in the course, I return to this topic to deepen students' understanding. Similar comments apply to the multiplication property of equality.

Workbook Exploration

The following 15-minute exploration could be thought of as the most important exploration in the sense that students solve so many equations throughout the course yet tend not to know that when solving an equation, all the equations are equivalent.

EXPLORATION *Addition and multiplication properties of equality*

1. The addition property of equality states that if A and B are expressions and c is a number, then the equations $A = B$ and $A + c = B + c$ are equivalent.

a. To show the addition property of equality is true when we add 5 to both sides of $x = 3$, explain why the following three equations are all equivalent.

$$\begin{aligned} x &= 3 \\ x + 5 &= 3 + 5 \\ x + 5 &= 8 \end{aligned}$$

b. Here we use the addition property of equality to solve the equation $x - 4 = 6$ by adding 4 to both sides:

$$\begin{aligned} x - 4 &= 6 \\ x - 4 + 4 &= 6 + 4 \\ x + 0 &= 10 \\ x &= 10 \end{aligned}$$

Show that all four equations are equivalent.

2. The multiplication property of equality states that if A and B are expressions and c is a nonzero number, then the equations $A = B$ and $Ac = Bc$ are equivalent.

a. To show the multiplication property of equality is true when we multiply both sides of $x = 2$ by 3, explain why the following three equations are all equivalent.

$$\begin{aligned} x &= 2 \\ 3 \cdot x &= 3 \cdot 2 \\ 3x &= 6 \end{aligned}$$

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b. Here we use the multiplication property of equality to solve the equation $\frac{3}{5}x = 6$ by multiplying both sides by $\frac{5}{3}$:

$$\begin{aligned} \frac{3}{5}x &= 6 \\ \frac{5}{3} \cdot \frac{3}{5}x &= \frac{5}{3} \cdot 6 \\ 1x &= 10 \\ x &= 10 \end{aligned}$$

Show that all four equations are equivalent.

OBJECTIVE 7 COMMENTS

For Problem 5, I say our objective in solving an equation is to isolate x on one side of the equation.

For Problem 7, I point out that we can still use the addition property of equality because subtracting 3 from both sides of the equation is the same as adding -3 to both sides.

For Problem 10, I first show that the product of a fraction and its reciprocal is equal to 1. Then I use the multiplication property of equality to solve the equation $\frac{5}{2}x = 3$.

For Problem 11, I point out that we can still use the multiplication property of equality because dividing both sides by -5 is the same as multiplying both sides by $-\frac{1}{5}$.

In solving $-t = 7$ (Problem 13), I show students how to solve the equation by dividing both sides of the equation by -1 as well as by multiplying both sides by -1. Almost all my students prefer dividing by -1 rather than multiplying by -1.

Textbook and Workbook Exploration

The 10-minute "Locating an error in solving an equation" exploration is a good reminder to students about the meaning of equivalent equations. This method of locating errors will be formally discussed in Section 4.4.

OBJECTIVE 8 COMMENTS

Perhaps the hardest part of Problem 14 is realizing that it helps to define a variable (for the amount of the bill). Such percentage problems are good preparation for harder ones in Section 4.4.

OBJECTIVE 9 COMMENTS

A discussion about inputs and outputs can be used to explain how to use graphing to solve an equation in one variable. Most students will be challenged by such an explanation. It will be easier to get across why this method works once students learn how to solve systems by using substitution in Section 6.2.

For Problem 17, I point out the advantages of using technology to solve such an equation. It is important that students learn symbolic methods of the course, but it is also important that students learn appropriate uses of technology.

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SECTION 4.4 LECTURE NOTES

Objectives

1. Solve a linear equation in one variable.
2. Make estimates and predictions by solving equations.
3. Translate English sentences into and from mathematical equations.
4. Use a graph or tables to solve a linear equation in one variable.
5. Describe the meaning of *conditional equation*, *inconsistent equation*, and *identity* and how to solve these types of equations.
6. Locate errors in solving linear equations.

Main point: Solve more linear equations in one variable.

OBJECTIVE 1

Solve.

1. $7x + 2 = 23$

2. $3x - 5x + 2 = 6$

3. $2w - 9 = 6w + 23$

4. $5 - 4(2x - 3) = 3(4 - x)$

5. $2(3x + 4) - (4x - 1) = 3(x + 2)$

6. $\frac{5}{6}k - \frac{3}{2} = \frac{7}{3}$

7. $\frac{5(x-4)}{3} = -2x$

8. $\frac{4t-7}{3} = \frac{2t+1}{4}$

- It is wise to check that a result (or the results) of solving an equation does indeed satisfy the equation.
- A key step in solving an equation that contains fractions is to multiply both sides of the equation by the LCD so that there are no fractions on either side of the equation.

9. Solve $2.4(1.7t - 3.58) = 17.94$. Round the result to the second decimal place.

OBJECTIVE 2

10. The number of Bank of America branches was 4542 branches in 2016 and has decreased by about 208 branches per year (Source: *Federal Deposit Insurance Corp*). Let n be the number of Bank of America branches at t years since 2016.

- **a.** Find a model of the situation.
- **b.** Predict the number of Bank of America branches in 2022.
- **c.** Predict when there will be 3500 branches.

11. In 2016, the number of charges alleging sexual harassment was 6758, down 14.9% from 2010 (Source: *U.S. Equal Employment Opportunity Commission*). How many charges alleging sexual harassment were there in 2010?

OBJECTIVE 3

Here are some sentences that have the same meaning as $x = 5$:

- The number is 5.
- The number is equal to 5.
- The number is the same as 5.

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For Problems 12 and 13, find the number.

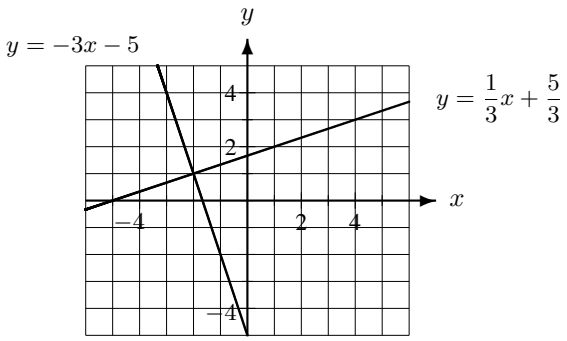
12. Seven times the difference of a number and 3 is -35.

13. Eight, minus 2 times the sum of a number and 5 is 12.

14. Let x be a number. Translate the equation $5x + 2 = 8x - 10$ into an English sentence. Then solve the equation.

OBJECTIVE 4

For Problems 15 and 16, solve the given equation by referring to the graphs of $y = \frac{1}{3}x + \frac{5}{3}$ and $y = -3x - 5$ shown below.



15. $\frac{1}{3}x + \frac{5}{3} = -3x - 5$

16. $\frac{1}{3}x + \frac{5}{3} = 2$

17. Use tables to solve the equation $3x - 2 = 6 - x$. [The solution is 2.]

OBJECTIVE 5

- A **conditional equation** is sometimes true and sometimes false, depending on which values are substituted for the variable(s).
- If an equation does not have any solutions, we call the equation **inconsistent** and say the solution set is the **empty set**.
- An equation that is true for all permissible values of the variable(s) it contains is called an **identity**.

Solve the equation. State whether the equation is a conditional equation, an inconsistent equation, or an identity.

18. $3x + 2 - 7x = 5x - 9x + 8$ **19.** $4(2x - 3) = 5 - x$ **20.** $2(x - 5) - x = x - 10$

OBJECTIVE 6

21. A student incorrectly tries to solve $2(x - 4) = 8$:

$$\begin{aligned} 2(x-4) &= 8 \\ 2x - 4 &= 8 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

Substitute 6 for x in each of the four equations, and explain how the results help you pinpoint the error.

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Locating an Error in Solving a Linear Equation

If the result of an attempt to solve a linear equation in one variable does not satisfy the equation, an error was made. To pinpoint the error, substitute the result into all the equations in the work and find which pair of consecutive equations gives a false statement followed by a true statement.

SHORT HW 13, 17, 23, 29, 39, 43, 49, 55, 65, 75, 81, 85, 93, 95, 117, 119

MEDIUM HW 1, 9, 13, 17, 23, 25, 29, 39, 43, 49, 51, 55, 65, 71, 81, 85, 93, 95, 97, 99, 109, 113, 117, 119, 121, 127

SECTION 4.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use symbolic methods, graphing, and tables to solve linear equations in one variable. Students use models to make predictions. They also learn the meaning of and how to solve conditional equations, inconsistent equations, and identities.

OBJECTIVE 1 COMMENTS

Most students are challenged by Problems 6–8. Some students try to write all the fractions with a common denominator, which is not incorrect but is inefficient, and usually such students make other errors along the way. Some students have trouble finding the LCD. Other students have trouble simplifying both sides of the equation after multiplying both sides of the equation by the LCD.

Textbook and Workbook Exploration

You could start class by having groups work on the 10-minute "Solving linear equations."

Workbook Exploration

The following 15-minute exploration is a more demanding alternative to the exploration just described. Because all the required concepts have already been discussed, you could begin class by facilitating it.

Group Exploration

Section Opener: Solving linear equations

Solve. Check that your result satisfies the original equation.

1. $x + 3 = 11$ **2.** $2x = 8$ **3.** $2x + 3 = 11$

4. $6x - 4x + 3 = 11$ **5.** $2(x + 4) + 3x = 23$ **6.** $3(2x - 5) - (4x + 1) = 8$

OBJECTIVE 2 COMMENTS

For part (a) of Problem 10, some of my students need a reminder about how to find such a linear model (Section 3.5). For Problem 11, students tend to think that what's required is to calculate 14.9% of the number of charges in 2016 (6758). Encourage your students to take the necessary time to understand the situation before trying to perform calculations or solve equations.

OBJECTIVE 3 COMMENTS

Most students have an easy time with this objective.

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OBJECTIVE 4 COMMENTS

For Problems 15 and 16, you could verify the work using "intersect."

OBJECTIVE 5 COMMENTS

This material is not essential for the course. Although Section 4.5 will refer to identities and Chapter 6 includes inconsistent systems, students can pick up on the relevant concepts at those times.

Workbook Exploration

In the following 20-minute exploration, some groups will likely need help seeing how parts (a) and (b) of Problem 3 connect with Problems 2 and 1, respectively.

EXPLORATION *Conditional equations, inconsistent equations, and identities*

1. **a.** Can a number be 1 more than itself? How many solutions does the equation $x = x + 1$ have?

b. If you subtract x from both sides of $x = x + 1$, is the result a true or false statement? Explain why this makes sense by referring to the results you found in part (a).

2. **a.** Can a number be equal to itself? What numbers are solutions of the equation $x = x$?

b. If you subtract x from both sides of $x = x$, is the result a true or false statement? Explain why this makes sense by referring to the results you found in part (a).

3. Solve. Make sure that you clearly state the solutions.

a. $2x + 6x = 8x$ **b.** $4x + 3 = 4x + 7$ **c.** $3x - 1 = 8$

d. $3(2x - 4) + x = 7x - 10$ **e.** $8 - 2(5x - 3) = 6x + 5$ **f.** $10x - 4 - 3x = 2x - 4 + 5x$

Textbook and Workbook Exploration

The 15-minute "Any linear equation in one variable has exactly one solution" exploration not only has groups determine the number of solutions of a linear equation but also has groups work with a formula. Formulas will be discussed in Section 4.6.

OBJECTIVE 6 COMMENTS

Students will not use this strategy on their own to identify errors. If you want them to use this approach, you'll have to attach points on an exam accordingly. Even if you don't attach such points, Problem 21 does reinforce the meaning of equivalent equations.

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SECTION 4.5 LECTURE NOTES

Objectives

1. Compare the meanings of *simplifying expressions* and *solving equations*.
2. Translating English phrases and sentences into mathematical expressions and equations.

Main point: Compare the meanings of *simplifying expressions* and *solving equations*.

OBJECTIVE 1

Definition *Linear expression in one variable*

A **linear expression in one variable** is an expression that can be put into the form

$$mx + b$$

where m and b are constants and $m \neq 0$.

Recall: A *linear equation in one variable* is an equation that can be put into the form $mx + b = 0$, where m and b are constants and $m \neq 0$.

For Problems 1 and 2, is the following a linear expression or a linear equation?

1. $6(2x - 1) + 5 = 35$

2. $6(2x - 1) + 5$

3. Solve $4(x - 5) + 3 = 2x - 9$.

4. Simplify $4(x - 5) + 3$.

5. Use a graphing calculator table to check your work in Problem 3.

6. Use a graphing calculator table to check your work in Problem 4.

Results of Simplifying an Expression and Solving an Equation

The result of simplifying an expression is an expression. The result of solving a linear equation in one variable is a number.

Comparing Simplifying an Expression with Solving an Equation

- If an expression A in one variable is simplified to an expression B, then every real number (for which both expressions are defined) is a solution of the equation $A = B$. In other words, the equation $A = B$ is an identity.
- Exactly one number is a solution of a linear equation in one variable.

7. The simplified form of $2(4x - 7) - 3x$ is $5x - 14$. What does this mean?

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8. The solution of $2(4x - 7) - 3x = -4$ is 2. What does this mean?

Simplify the expression or solve the equation, as appropriate.

9. $2(x - 5) - (6x + 1) - 3(2x + 3)$

10. $2(x - 5) - (6x + 1) = 3(2x + 3)$

11. $\frac{3}{8}x + \frac{7}{6} - \frac{7}{4}x$

12. $\frac{3}{8}x + \frac{7}{6} = \frac{7}{4}x$

13. $\frac{5}{2}x - \frac{3}{4} = \frac{7}{8}x + \frac{1}{4}$

14. $\frac{5}{2}x - \frac{3}{4} - \frac{7}{8}x + \frac{1}{4}$

Multiplying Expressions and Both Sides of Equations by Numbers

When simplifying an expression, the only number that we can multiply the expression by or part of it by is 1. When solving an equation, we can multiply both sides of the equation by *any* number except 0.

OBJECTIVE 2

Let x be a number. Translate the following to an expression or an equation, as appropriate. Then simplify the expression or solve the equation.

15. Five, minus 3 times the sum of the number and 1

16. The sum of 4 and twice the number is 10.

17. The quotient of the number and 3 is equal to the number subtracted from 5.

18. Three, plus 5 times the difference of the number and 2

SHORT HW 5, 7, 21, 23, 29, 31, 37, 39, 41, 43, 51, 55, 59, 61, 63, 71

MEDIUM HW 5, 7, 13, 15, 21, 23, 29, 31, 37, 39, 41, 43, 51, 55, 59, 61, 63, 65, 66, 67, 71, 73

SECTION 4.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students compare the meanings of *simplifying expressions* and *solving equations*. This is excellent preparation for working with polynomial expressions and equations and working with rational expressions and equations; it is with these more complex expressions and equations that students have trouble keeping straight the meanings and methods.

Students also translate English phrases and sentences to mathematical expressions and equations in this section.

OBJECTIVE 1 COMMENTS

This objective has two purposes. Its primary function is to have students compare simplifying expressions and solving equations. A secondary function is to give weaker students another shot at mastering working with linear expressions and equations.

Most students have an easy time with Problems 1–4, 9, and 10. This means that while working with these problems, I can focus on the similarities and differences of simplifying expression and solving equations. Although I will address the same concepts when working with more challenging types of expressions and equations later in the course, it is now that the ideas can really sink in because students are not distracted by the techniques.

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For Problems 11–14, some students struggle with the techniques. Other students are ready to contemplate when they should be multiplying by 1 and when they should be multiplying by the LCD.

Textbook and Workbook Exploration

The 15-minute "Comparing expressions and equations" exploration is a nice way to lead into this section. For Problems 3 and 6 of this exploration, some groups may not be sure how to use substitutions to check their work when simplifying expressions.

Textbook and Workbook Exploration

The 10-minute "Simplifying versus solving" exploration has groups reflect on two common types of student errors.

OBJECTIVE 2 COMMENTS

By this stage of the game, most students have an easy time with Problems 15–18.

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SECTION 4.6 LECTURE NOTES

Objectives

1. Determine area and perimeter formulas of a rectangle and a total-value formula.
2. Use formulas to solve various types of problems.
3. Translate an English sentence into a formula.
4. Solve a formula for a variable.

Main point: Use formulas to solve various types of problems.

OBJECTIVE 1

- Recall: A **formula** is an equation that contains two or more variables (Section 3.3).
- The **perimeter** of a polygon, which is a geometrical object such as a triangle, rectangle, or trapezoid, is the total distance around the object.
- The perimeter P of a rectangle with width W and length L is given by $P = W + L + W + L = 2L + 2W$.

Area and Perimeter of a Rectangle

For a rectangle with length L, width W, area A, and perimeter P:

- $A = LW$
- $P = 2L + 2W$

OBJECTIVE 2

1. A rectangular floor has an area of 192 square feet and a length of 16 feet. Find the width of the room.

To find a single value of a variable in a formula, we often substitute numbers for all the other variables and then solve for that one variable.

2. A photographer plans to use 76 inches of wood to make a rectangular frame for a photo. If the width of the frame is to be 14 inches, what will its length be?

For Problems 3 and 4, substitute the given values for the variables and then solve the equation for the remaining variable.

3. $A = P + Prt$; $A = 1025$, $P = 500$, $t = 15$ (simple interest)
4. $S = 2WL + 2WH + 2LH$; $S = 72$, $W = 2$, $H = 3$ (surface area of a rectangular box)

- Four dimes are worth $10 \cdot 4 = 40$ cents.
- We found the total value of the dimes by multiplying the value of one dime (10 cents) by the number of quarters (4).

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Total-Value Formula

If n objects each have value v, then their total value T is given by $T = vn$.

5. Tickets for a Man Man concert cost $25 each.

a. Find a formula of the total cost T (in dollars) of n of these tickets.

b. Perform a unit analysis of the formula you found in part (a).

c. Substitute 425 for T in the formula from part (a) and solve for n. What does your result mean in this situation?

OBJECTIVE 3

6. Let a, b, and c be the lengths (in inches) of the sides of a triangle. The average length (in inches) A of a side of a triangle is equal to the sum of the lengths of the three sides divided by 3.

a. Write a formula of the average length of a side of a triangle.

b. If the average length of a side of a triangle is 10 inches, and the lengths of two of the sides are 5 inches and 13 inches, find the length of the third side.

OBJECTIVE 4

For Problems 7–10, solve the equation for the specified variable.

7. $W = Fd$, for d

8. $P = 2L + 2W$, for W [Example 6]

9. $5x - 8y = 24$, for y

10. $A = P(1 + rt)$, for t [very similar to Exercise 52 of Homework 4.6]

11. The number of Apple stores was 463 stores in 2015 and has increased by about 26 stores per year since then (Source: *Apple*). Let n be the number of Apple stores at t years since 2015.

a. Find an equation of a linear model to describe the situation.

b. Solve the equation that you found in part (a) for t.

c. Estimate when there will be 600 stores.

d. Use a graphing calculator table to predict in which years the numbers of stores will be 650, 700, 750, 800, and 850.

- For Problem 11, to find numbers of Apple stores, it is convenient to use $n = 26t + 463$. To find years, it is convenient to use $t = \dfrac{n - 463}{26}$.
- Solving for a variable in a formula will not change the relationship between the variables in the formula.
- To find several values of a variable in a formula, we usually solve the formula for that variable before we make any substitutions.

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SHORT HW 3, 7, 13, 17, 21, 31, 35, 41, 45, 51, 63, 67, 77, 81

MEDIUM HW 3, 5, 7, 13, 15, 17, 19, 21, 25, 31, 33, 35, 37, 41, 45, 51, 57, 63, 67, 69, 75, 77, 81

SECTION 4.6 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn the area and perimeter formulas of a rectangle and a total-value formula. Students solve various types of problems. They also solve a formula for a variable.

OBJECTIVE 1 COMMENTS

Some students confuse area with perimeter, so I spend a little time comparing these two concepts.

If time permits, I sketch a polygon such as those in Exercises 1–6 of Homework 4.6 and have my students find a formula of the perimeter of the polygon.

OBJECTIVE 2 COMMENTS

I emphasize the arithmetic-algebra connection for the total value formula $T = vn$. The example about the four dimes in the lecture notes accomplishes this.

OBJECTIVE 3 COMMENTS

Recall that *average* was defined in Section 1.1. So, the first two sentences of Problem 6 could be replaced with "Let a, b, and c be the lengths (in inches) of the three sides of a triangle and let A be the average length (in inches) of a side."

OBJECTIVE 4 COMMENTS

Most students have a tough time with Problems 7–11. It can help to solve each formula alongside a similar equation in one variable. For example, see the solutions of Examples 5 and 6 on pages 240 and 241 of the textbook.

Problem 9 is excellent preparation for Section 5.1. I make sure that I do this problem (and assign several of Exercises 59–66 of Homework 4.6) so that my students have two sections to practice this skill.

Note that part (b) of Problem 11 is essentially having students find the inverse function of the linear model $n = 26t + 463$. When doing part (d), I point out that it is advantageous to first solve for t before substituting the five values for n; if we substituted before solving, we'd have five equations to solve. I say that first solving for t would become a huge advantage if we wanted to make hundreds of substitutions for n.

Most textbooks seem to suggest that we always want to first solve a formula for a variable before making substitutions; this hardly makes sense if we want to find only one value of a quantity.

Textbook and Workbook Exploration

The 10-minute "Solving a formula for one of its variables" is probably the best way to introduce how solve a formula for one of its variables. Groups benefit greatly from hands-on experience with the connection between solving an equation in one variable and solving a formula.

Workbook Exploration

The following 25-minute exploration requires a good chunk of time, but it highlights a key concept of the section and also gives groups practice with solving a formula (Problem 2 in the exploration) that is well-supported by solving equations (Problem 1 in the exploration).

Group Exploration

Determining when to substitute a value for a variable in a formula

Let F be the temperature (in Fahrenheit degrees) and C be the equivalent Celsius reading. Recall that the rela-

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tionship between C and F can be described by the formula $C = \frac{5}{9}(F - 32)$.

1. Suppose the high temperature readings (in Celsius) on June 16 in five European cities are 21, 23, 24, 27, and 28. Convert the temperature readings to Fahrenheit readings by substituting each data value for C in the formula $C = \frac{5}{9}(F - 32)$ and solving for F.

2. Solve the formula $C = \frac{5}{9}(F - 32)$ for F.

3. Perform the same conversions as you did in Problem 1, but this time use a graphing calculator table and the formula you found in Problem 2.

4. In Problem 1 you converted five Celsius readings. In Problem 2 followed by Problem 3 you converted the same five readings.

   a. Which method would you use to convert 100 Celsius readings? Explain.

   b. Which method would you use to convert a single Celsius reading? Explain.

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CHAPTER 5 OVERVIEW

In Section 5.1, students graph linear equations in two variables. In Section 5.2, students learn concepts fundamental to functions, and in Section 5.3 they use function notation to evaluate functions and to find the values of explanatory variables. Students use two points to find an equation of a line in Section 5.4. In Section 5.5, students use technology to draw scatterplots and then find equations of linear models by hand; they also learn how to perform linear regression, although finding linear models by hand (using two "good" points) is emphasized. In Section 5.6, students use linear models to make estimates and predictions. Students solve linear inequalities in Section 5.7.

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SECTION 5.1 LECTURE NOTES

Objectives

1. Graph a linear equation by solving for y.
2. Graph a linear equation by finding the intercepts of its graph.
3. Find coordinates of solutions of a linear equation.
4. Describe the difference between equations in one variable and equations in two variables.

Main point: Graph linear equations in two variables.

OBJECTIVE 1

Determine the slope and the y-intercept. Use the slope and the y-intercept to graph the equation by hand.

1. $2y = 3x$

2. $0 = x + 3y$

3. $3x + 5y = 10$

4. $2(x - 2y) + 1 = 9$

5. $5 - 2(3x - y) = 1 - 3(x - 2y)$

- Before we can use the y-intercept and the slope to graph a linear equation, we must solve for $\boldsymbol{y}$ to put it in the form $y = mx + b$.
- Warning: The slope of the line $3x + 5y = 10$ is *not* 3 (see Problem 3).

OBJECTIVE 2

Intercepts of the Graph of an Equation

For an equation containing the variables x and y,

- To find the x-coordinate of each x-intercept, substitute 0 for y and solve for x.
- To find the y-coordinate of each y-intercept, substitute 0 for x and solve for y.

For Problems 7–9, find the intercepts. Then graph the equation by hand.

6. $3x - 6y = 12$

7. $y = -2x + 5$

8. $\dfrac{x}{2} + \dfrac{y}{4} = 1$

Using Intercepts to Graph an Equation

To graph a linear equation whose graph has exactly two intercepts,

1. Find the intercepts.
2. Plot the intercepts and a third point on the line, and graph the line that contains the three points.

Find the approximate intercepts. Round coordinates to the second decimal place.

9. $3.9x - 5.8y = 23.6$

10. $y = 2.46x - 17.51$

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11. Find the x-intercept and the y-intercept of the graph of $\frac{x}{a} - \frac{y}{b} = 1$.

OBJECTIVE 3

12. Complete the following steps to graph $3x - 5y = 20$.

- **a.** Find y when $x = 5$.
- **b.** Find y when $x = -5$.
- **c.** Find x when $y = -2$.
- **d.** Graph $3x - 5y = 20$ by hand.

Using Three Solutions to Graph a Linear Equation

To graph any linear equation,

1. Find three solutions of the equation.
2. Plot the three solutions and graph the line that contains them.

OBJECTIVE 4

13. Describe the solution(s) of the equation in two variables $y = 2x - 3$.

14. Describe the solution(s) of the equation in one variable $5 = 2x - 3$.

- There is a connection between the equations $y = 2x - 3$ and $5 = 2x - 3$.
- If we substitute 5 for y in the equation $y = 2x - 3$, the result is the equation $5 = 2x - 3$, whose solution is $x = 4$.
- So, $(4, 5)$ is one of an infinite number of solutions of $y = 2x - 3$. [Draw a sketch.]

Comparing Linear Equations in One and Two Variables

A linear equation in two variables has an infinite number of (ordered pair) solutions. A linear equation in one variable has exactly one (real number) solution.

SHORT HW 3, 9, 13, 19, 35, 39, 45, 53, 61, 63, 75, 79, 89, 91, 97

MEDIUM HW 3, 9, 13, 19, 29, 35, 39, 45, 49, 53, 61, 63, 75, 77, 79, 83, 89, 91, 95, 97, 101, 105, 107, 109

SECTION 5.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use three methods to graph linear equations in two variables. Students will use this material in Section 6.1.

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OBJECTIVE 1 COMMENTS

For Problems 1–5, students combine the skill of solving a formula for a variable (Section 4.6) with the skill of graphing a linear equation in slope-intercept form (Section 3.4). The warning about 3 not being the slope of the line $3x + 5y = 10$ is worthwhile because students have been working only with slope-intercept form until now.

Textbook and Workbook Exploration

The 15-minute "Graphing linear equations" exploration is a great introduction to Objective 1.

OBJECTIVE 2 COMMENTS

For Problems 6–11, I emphasize that an intercept is a point and has two coordinates (not one). I tell my students that after they find the intercepts of a line, that they should indicate which is the x-intercept and which is the y-intercept (assuming they are distinct).

When graphing an equation that can be put into the form $y = mx$, I warn my students that the origin is the only intercept. So, using the slope and the origin is a good way to graph the equation.

Textbook and Workbook Exploration

The 20-minute "Comparing two graphing techniques" exploration has groups compare graphing by using the slope and the y-intercept with graphing by finding the intercepts.

OBJECTIVE 3 COMMENTS

This objective has been included in the textbook to open students' horizons to the concept that we can find non-intercepts of a line. I do not recommend that students use this approach to graph linear equations.

OBJECTIVE 4 COMMENTS

Because we describe the solutions of a linear equation in two variables and the solution of a linear equation in one variable in such different formats (graphs versus a statement of the form "$x = a$"), many students lose sight that in both cases, we are finding the solutions of the equations. That is, we are finding all the ordered pairs or all the numbers that satisfy the equations. The point of Problems 13 and 14 is to help students see this commonality and the connection between the two types of equations.

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SECTION 5.2 LECTURE NOTES

Objectives

1. Describe the meanings of *relation*, *domain*, *range*, and *function*.
2. Identify functions by using the *vertical line test*.
3. Describe the definition of a *linear function*.
4. Describe the Rule of Four for functions.
5. Use the graph of a function to find the function's domain and range.

Main point: Describe the meaning of *function* and identify functions.

OBJECTIVE 1

For the ordered pairs $(1, 4)$, $(2, 3)$, $(2, 5)$, and $(3, 1)$:

- Construct a table and a scatterplot.
- The set of ordered pairs are called a *relation*.
- Describe the domain and range of the previous relation.
- A **relation** is a set of ordered pairs. The **domain** of a relation is the set of all values of the explanatory variable, and the **range** of the relation is the set of all values of the response variable.
- Discuss inputs and outputs for the previous relation.
- For the previous relation, observe that the input $x = 2$ has *two* outputs: $y = 3$ and $y = 5$.

Definition *Function*

A **function** is a relation in which each input leads to exactly one output.

Determine whether the given relation is a function.

1. $y = x + 5$ **2.** $y = 2x$ **3.** $y = \pm x$

4. Some ordered pairs of four relations are listed in the following table. Which of these relations could be functions? Explain.

Relation 1		Relation 2		Relation 3		Relation 4	
x	y	x	y	x	y	x	y
0	13	2	20	5	3	7	1
1	18	3	18	6	3	7	2
2	23	3	16	7	3	7	3
3	28	4	14	8	3	7	4
4	33	5	12	9	3	7	5

OBJECTIVE 2

Use arrows to show that there is an input that has two outputs for the relation graphed below. Conclude that the relation is not a function.

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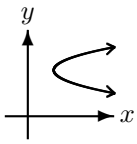
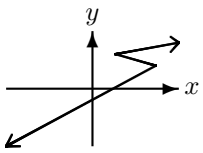
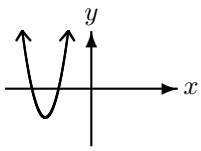


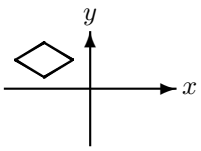
Vertical Line Test

A relation is a function if and only if every vertical line intersects the graph of the relation at no more than one point. We call the is requirement the **vertical line test**.

Determine whether the following graph represents a function. Explain.







OBJECTIVE 3

Use the vertical line test to deduce that any nonvertical line is the graph of a function.

Definition *Linear Function*

A **linear function** is a relation whose equation can be put into the form

$$y = mx + b$$

where m and b are constants.

Determine whether the given relation is a function.

5. $6x + 9y = 27$

6. $x = 2$

7. $y = x^2$

8. $x = y^2$

OBJECTIVE 4

Refer to your previous work to motivate the Rule of Four for functions.

Rule of Four for Functions

We can describe some or all of the input-output pairs of a function by means of

1. an equation

2. a graph

3. a table, or

4. words.

OBJECTIVE 5

Describe the meaning of the notations $\leq$ and $\geq$.

Describe the given inequality in words.

9. $x \leq 7$

10. $x \geq 4$

11. $5 \leq x$

12. $1 \leq x \leq 9$

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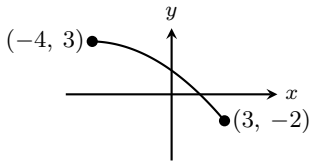
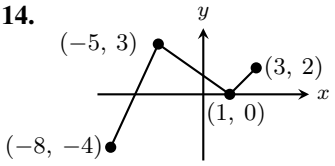
Use the graph of the function to determine the function's domain and range.

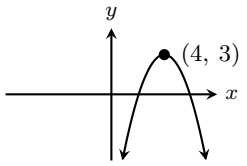
13.



14.



15.



SHORT HW 1, 3, 5, 11, 13, 17, 21, 29, 31, 35, 39, 45, 47, 51, 57

MEDIUM HW 1, 3, 5, 7, 9, 11, 13, 17, 21, 25, 29, 31, 35, 39, 45, 47, 49, 51, 57, 59, 61, 63, 65

SECTION 5.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students learn about the concepts of relation and function in this section. Concepts related to functions will be needed throughout the rest of the textbook.

Functional notation will be introduced in Section 5.3. Delaying this topic until then frees up Section 5.2 to emphasize the meaning of a function.

OBJECTIVE 1 COMMENTS

To get across the definition of a function, I like to draw lots of "function machines" such as the ones in Figs. 11 and 12 on pages 263 and 264, respectively, of the textbook as well as tables such as Tables 3 and 4 on pages 263 and 264, respectively, of the textbook. I make sure that students understand that a function may have two inputs that share the same output.

Workbook Exploration

In the following 20-minute exploration, students practice determining whether a relation is a function. It will be more challenging and beneficial to students if you simply define a relation and function and then have groups work on the exploration, rather than you first giving examples.

Group Exploration

Section Opener: Identifying functions

1. If we think of Americans as inputs to a machine where the outputs are the current ages of the Americans, then each input has exactly one output because each American has exactly one current age. For each situation, determine whether each input has exactly one output.

   a. Inputs: Americans; Outputs: current heights (in inches) of Americans

   b. Inputs: Americans; Outputs: names of siblings of Americans

   c. Inputs: phones; Outputs: names of apps on phones

   d. Inputs: homes in the United States; outputs: numbers of bedrooms in homes

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2. For each table, think of the values of x as inputs and the values of y as outputs. Does each input have exactly one output?

a.

x	y
0	6
1	2
2	5
3	3
4	4

b.

x	y
0	6
1	2
2	5
2	3
3	4

c.

x	y
0	3
1	3
2	3
3	3
4	3

3. For each equation, think of the values of x as inputs and the values of y as outputs. Does each input have exactly one output?

a. $y = 3x$
b. $y = \pm x$
c. $y = x^2$
d. $x = y^2$ [**Hint:** Substitute 9 for x and solve for y.]

OBJECTIVE 2 COMMENTS

Before discussing the vertical line test, I first use arrows similar to those in Fig. 14 on page 265 of the textbook to get students accustomed to thinking visually of inputs being sent to outputs. I find that even calculus students lack this essential perspective. I also use arrows with tables and arrows with graphs to help students see the connection between tables and graphs in terms of inputs being sent to outputs.

Once students see how to draw such arrows with graphs, students are quick to identify graphs of functions. They can do this without having been told about the vertical line test. I prefer having students draw arrows because the vertical line test strips away the key idea of a function. Once students have mastered using arrows to identify graphs of functions, I'll sometimes briefly discuss the vertical line test. Most students persist in drawing arrows on quizzes and exams, which I take to be a good sign.

Textbook and Workbook Exploration

The vertical line test is nicely motivated by the 15-minute "Vertical line test" exploration. By completing the exploration, students will see not only how to use the vertical line test, but also *why* it works.

OBJECTIVE 3 COMMENTS

Students have an easy time with this objective.

OBJECTIVE 4 COMMENTS

There are exercises scattered throughout the textbook that refer to the Rule of Four for functions. For example, see Exercises 29 and 30 of Homework 5.2.

OBJECTIVE 5 COMMENTS

To solve Problem 13, draw arrows indicating around eight inputs being sent to corresponding outputs. Students will then be quick to pick up on the domain and range of the function. Apply a similar approach to Problems 14 and 15. For Problem 15, students will wonder whether the domain is really the set of all real numbers. It will help to explain the meaning of the two displayed arrows.

Students will be quick to pick up on how to use inequality notation to describe the domain and range of a function. If you would rather postpone discussion about inequalities until they are fully developed in Section 5.7, you could have students describe the domain and range of a function using words. Because the answers to such exercises in Homework 5.2 use inequality notation, you should supply verbal answers for such exercises if you want to go that route.

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SECTION 5.3 LECTURE NOTES

Objectives

1. Use *function notation*.
2. Find inputs and outputs of a function.
3. Use function notation with models.

Main point: Use function notation and make estimates and predictions about authentic situations.

OBJECTIVE 1

To use "f" as the name of the function $y = 3x - 5$, we use "$f(x)$" to represent y: $y = f(x)$. We can substitute $f(x)$ for y in the equation $y = 3x - 5$: $f(x) = 3x - 5$. Warning: "$f(x)$" does not mean f times x.

OBJECTIVE 2

Evaluate $f(x) = 3x - 5$ at the given values of x.

1. $f(4)$ **2.** $f\left(-\frac{1}{3}\right)$ **3.** $f(a-3)$ **4.** $f(a+k)$

5. Evaluate $g(x) = 3x^2 - 4x$ at $x = -2$.

6. Evaluate $h(x) = \dfrac{5x-4}{3x+2}$ at $x = -6$.

Let $h(x) = -2x + 1$.

7. Find $h(4)$.

8. Find x when $h(x) = 4$.

Let $f(x) = \frac{3}{5}x + 2$.

9. Find $f(10)$.

10. Find x when $f(x) = 10$.

11. Let $f(x) = -8.73x - 256$. Find x when $f(x) = 96.15$. Round any results to the second decimal place.

Some input-output pairs of a function f are shown in the following table.

12. Find $f(3)$.

13. Find x when $f(x) = 3$.

x	y
1	4
2	3
3	2
4	3
5	4

14. Find the intercepts of the graph of $f(x) = 2x - 7$.

15. Recall that we can describe some or all of the input-output pairs of a function by means of an equation, a graph, a table, or words. Let $f(x) = -3x + 5$.

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a. Describe five input-output pairs of f by using a table.

b. Describe the input-output pairs of f by using a graph.

c. Describe the input-output pairs of f by using words.

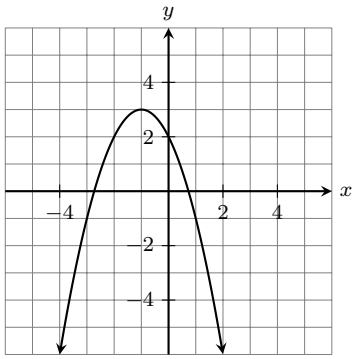
16. A function f is graphed in the figure below.

a. Find $f(-1)$.

b. Find x when $f(x) = -1$.

c. Find the domain of f.

d. Find the range of f.



OBJECTIVE 3

Definition *Function notation*

The response variable of a function f can be represented by the expression formed by writing the explanatory variable name within the parentheses of $f(\ )$:

$$\text{response variable} = f(\text{explanatory variable})$$

We call this representation **function notation**.

17. The percentage of Americans who say reducing the budget deficit should be a top priority for the President and Congress was 65.0% in 2015 and has decreased by 5.7 percentage points per year since then (Source: *Pew Research Center*). Let p be the percentage of Americans who say reducing the budget deficit should be a top priority for the President and Congress, and let t be the number of years since 2015.

a. Find an equation of a model that describes this situation.

b. Rewrite your equation using the function name f.

SHORT HW 5, 9, 19, 27, 31, 37, 41, 45, 51, 57, 69, 71, 75, 81, 83

MEDIUM HW 1, 5, 9, 13, 19, 23, 27, 31, 37, 41, 45, 51, 57, 69, 71, 73, 75, 79, 81, 83, 85, 87, 89, 91

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SECTION 5.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn about functional notation. They will use this notation both with models and functions that are not models throughout the rest of the textbook.

OBJECTIVE 1 COMMENTS

Many students wonder why we would want to represent "y" by the more complicated notation "$f(x)$." I emphasize that functional notation allows us to conveniently name a function. I explain that being able to name a function is especially helpful when working with two models, such as the model described in Problem 17 and the model that describes the percentage of Americans who say at t years since 2015 that climate change should be a top priority for the President and Congress.

Many of my students are reassured when I tell them that $f(x)$ simply stands for y. Later (when discussing Objective 3) I say that, in general, response variable $= f$(explanatory variable).

OBJECTIVE 2 COMMENTS

I spend a good chunk of time discussing how to find values of x or $f(x)$, where a function f is described by a table, a graph, or an equation. I also tie in function notation with function-machine figures such as the one in Fig. 24 on page 273 of the textbook.

Some students have difficulty with Problems 3 and 4. I compare the work for these problems with the work for Problems 1 and 2. I also emphasize that when substituting an expression for x, parentheses must be used.

Many students think that the notation $f(4)$ means that y is 4 or that the equation $f(x) = 4$ means that x is 4. Comparing the work for Problem 7 with the work for Problem 8 and comparing the work for Problem 9 with the work for Problem 10 should help address these issues.

Workbook Exploration

The following 15-minute exploration is a great way for students to think about the numerical aspect of a function. Symbolic work with functions can distract students from the main point, which has to do with inputs and outputs. Problem 2 has students work with two functions, which is good preparation for linear systems in Sections 6.1–6.4, systems of linear inequalities in Section 6.6, and algebra of functions in Sections 7.1, 7.2, and 12.1.

EXPLORATION *Using a table to evaluate a function*

1. Input–output pairs of a function f are shown in the following table.

x	$f(x)$
0	5
1	2
2	4
3	2
4	3
5	0

a. Find $f(4)$.

b. Find $f(0)$.

c. Find x when $f(x) = 4$.

d. Find x when $f(x) = 2$.

2. All Input–output pairs of functions f and g are shown in the following table.

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x	$f(x)$	$g(x)$
0	3	1
1	0	4
2	4	3
3	5	5
4	2	0
5	1	2

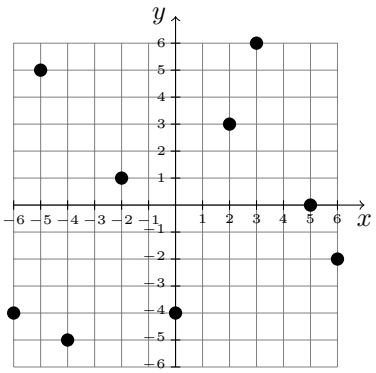
a. Find $f(1)$.
b. Find $g(2)$.
c. Find x when $f(x) = 2$.
d. Find x when $g(x) = 1$.
e. Which is larger, $f(2)$ or $g(2)$? Explain.
f. Which is larger, $f(5)$ or $g(5)$? Explain.
g. Find $f(0) - g(0)$.
h. Find $f(1) - g(1)$.
i. Find x when $f(x) = g(x)$.

Workbook Exploration

The following 30-minute exploration has students consider the graphical aspect of a function, which is what they tend to be weakest in doing. Problem 3 has students work with two functions, which is good preparation for linear systems in Sections 6.1–6.4, systems of linear inequalities in Section 6.6, and algebra of functions in Sections 7.1, 7.2, and 12.1.

EXPLORATION *Using a graph to evaluate a function*

1. The graph of a function f is displayed below.



a. Find $f(-2)$.
b. Find $f(5)$.
c. Find $f(0)$.
d. Find x when $f(x) = 6$.
e. Find x when $f(x) = -4$.
f. Find x when $f(x) = 0$

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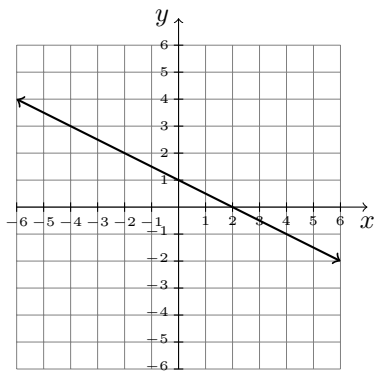
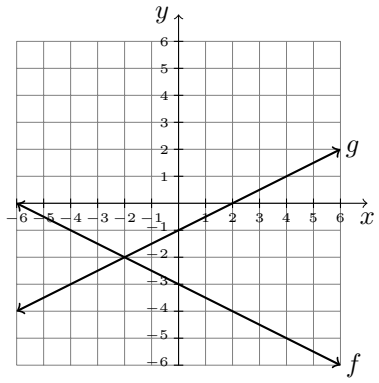
2. The graph of a function f is displayed below.



a. Find $f(-4)$. **b.** Find $f(0)$. **c.** Estimate $f(-4.7)$.
d. Find x when $f(x) = 3$. **e.** Find x when $f(x) = 0$. **f.** Estimate x when $f(x) = -1.2$

3. The graphs of functions f and g are displayed below.



a. Find $f(-4)$. **b.** Find $g(2)$. **c.** Find $f(0)$.

d. Find x when $f(x) = -4$. **e.** Find x when $g(x) = -1$. **f.** Find x when $g(x) = 0$.

g. Which is larger, $f(6)$ or $g(6)$? Explain. **h.** Which is larger, $f(-5)$ or $g(-5)$? Explain.

i. Find $f(4) - g(4)$. **j.** Find $f(-4) - g(-4)$. **k.** Find x when $f(x) = g(x)$.

Textbook and Workbook Exploration

The 10-minute "Formula for slope" exploration is good to assign if a sizable number of your students will eventually take calculus.

OBJECTIVE 3 COMMENTS

Most students will have an easy time understanding Problem 17b, but it is helpful if you address this concept now, before students have to grapple with the symbolic complexity of using function notation with linear models to make estimates and predictions in Section 5.6.

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SECTION 5.4 LECTURE NOTES

Objectives

1. Find an equation of a line by using the slope-intercept form of a linear equation.
2. Find an equation of a line by using the point-slope form of a linear equation.

Main point: Find an equation of a line.

OBJECTIVE 1

Recall that an equation of a line can be put in slope-intercept form $y = mx + b$ (Section 3.4).

Use the slope-intercept form to find an equation of the line that has the given slope and contains the given point.

1. $m = 3$, $(-4, 5)$

2. $m = -2$, $(-3, -6)$

3. $m = \frac{2}{5}$; $(-3, 4)$

4. $m = -\frac{4}{3}$; $(5, -2)$

5. $m = 0$, $(-3, 2)$

6. m is undefined, $(-5, 1)$

Finding an Equation of a Line by Using the Slope, a Point, and the Slope-Intercept Form

To find an equation of a line by using the slope and a point,

1. Substitute the given value of the slope m into the equation $y = mx + b$.
2. Substitute the coordinates of the given point into the equation you found in step 1 and solve for b.
3. Substitute the value of b you found in step 2 into the equation you found in step 1.
4. Check that the graph of your equation contains the given point.

Use the slope-intercept form to find an equation of the line that contains the two given points.

7. $(2, 3)$ and $(4, 7)$ [slope is an integer]

8. $(-3, 4)$ and $(2, -6)$ [slope is an integer]

9. $(-5, -3)$ and $(2, 1)$ [slope is a fraction]

10. $(-6, -2)$ and $(-3, -7)$ [slope is a fraction]

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Finding an Equation of a Line by Using Two Points and the Slope-Intercept Form

To find an equation of the line that passes through two given points whose x-coordinates are different,

1. Use the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ find the slope of the line containing the two points.
2. Substitute the m value you found in step 1 into the equation $y = mx + b$.
3. Substitute the coordinates of one of the given points into the equation you found in step 2 and solve for b.
4. Substitute the b value you found in step 3 into the equation you found in step 2.
5. Check that the graph of your equation contains the two given points.

11. Find an approximate equation of the line that contains the points $(-3.25, 8.11)$ and $(2.67, -1.39)$. Round the slope and the constant term to two decimal places.

12. Find an equation of the line that contains the given points.

a. $(4, -3)$ and $(4, 2)$ [slope is undefined]

b. $(-5, -2)$ and $(3, -2)$ [slope is 0]

13. Find an equation of the line that contains $(-4, 8)$ and is parallel to the line $y = -3x - 2$.

14. Find an equation of the line that contains $(5, -2)$ and is perpendicular to the line $3x + 5y = 10$.

OBJECTIVE 2 (optional)

Let (x_1, y_1) and (x, y) be two distinct points on a line with slope m. Use the true statement $\dfrac{y - y_1}{x - x_1} = m$ to derive the point-slope form.

Point-Slope Form

If a nonvertical line has slope m and contains the point (x_1, y_1), then an equation of the line is

$$y - y_1 = m(x - x_1)$$

Repeat any of Problems 1–5, 7–11, 12b, 13, and 14. Compare your results with your earlier results.

SHORT HW 5, 9, 13, 15, 23, 27, 35, 41, 49, 63, 69, 71, 75, 77, 85

MEDIUM HW 1, 5, 9, 13, 15, 23, 27, 35, 41, 47, 49, 57, 63, 69, 71, 75, 77, 81, 83, 85, 87, 91, 93, 95

SECTION 5.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

This section discusses how to find equations of lines, both by using the slope-intercept form and the point-slope form. In subsequent sections, only the slope-intercept form will be used to find such equations. So, the point-slope

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form is optional. However, if some of your students will eventually take calculus, you should spend some time on the point-slope form because this concept will likely be used in that course.

I have chosen to emphasize finding linear equations using the slope-intercept form for several reasons:

- My students have better success with this method than with using the point-slope form, especially when the slope is a noninteger.
- The notion of substituting a point to find the parameter b of $y = mx + b$ generalizes to finding parameters for quadratic, exponential, rational, and radical functions.
- Students have a better sense of why this method "works" than with using the point-slope form.
- Using two forms for linear equations in Chapters 1–5 confuses the issue for many students.

Students will find equations of linear models in the rest of Chapter 5 as well as in Chapter 6. In particular, this skill is the primary practice of Section 5.5.

The skills required for Problems 13 and 14 will not be needed in the rest of the textbook. However, the concepts about parallel lines addressed in Section 3.4 will be needed in Sections 6.1–6.3.

OBJECTIVE 1 COMMENTS

For Problem 1, when I substitute the ordered pair $(-4, 5)$ into the equation $y = 3x + b$ to find b, I remind students that an ordered pair that corresponds to a point on a graph satisfies an equation of the graph. So, making such a substitution will give a true statement, one that will allow us to find the value of b.

For Problems 3, 4, 9, 10, and 14, I make sure that students understand that they should use fractions, not decimals, when finding linear equations. Some students will have trouble working with fractions. Such work is a good primer for work with rational expressions and equations in Chapter 12. When solving an equation with fractions (to find b of $y = mx + b$), I prefer to have students multiply both sides of the equation by the LCD because that is how students will solve rational equations in Chapter 12; also, a majority of my students prefer this approach.

For Problems 7–10, I show students how to use their graphing calculators to verify the equations.

Problem 11 is a good primer for Section 5.5, where students will find equations of linear models.

Students will be challenged by Problems 13 and 14. One of students' difficulties is trying to track what is true for one line and what is true for the other. When doing Problem 13, it can help to use one color for all words and symbols in the problem statement and solution that describe one line and to use another color for such things that describe the other line. Similar color coding can be used for Problem 14.

Workbook Exploration

You could start class with the following 10-minute exploration, which serves as a good bridge from Section 3.5 to this section. In this exploration, students use the slope addition property to find equations. Although this skill is not necessary for this section, it does have groups reflect on how to go from tables to equations for linear functions. After completing this section, students should be able to go in all six directions between equations, graphs, and tables for linear functions.

Group Exploration

Finding an equation of a line

Some solutions of four linear equations are provided in the following table. Find an equation of each of the four lines.

Equation 1		Equation 2		Equation 3		Equation 4	
x	y	x	y	x	y	x	y
0	3	3	40	2	3	1	22
1	7	4	37	4	15	3	18
2	11	5	34	6	27	5	14
3	15	6	31	8	39	7	10

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Workbook Exploration

The skill of finding an equation using two given points is introduced in the following 15-minute exploration. This exploration is more intuitive than the exploration that directly follows this one because it makes use of a graph. A few groups will have trouble with Problem 2 of the exploration. In particular, some groups will ignore Problem 2 and write $1 = m(2) + b$ in Problem 3.

Group Exploration

Section Opener: Finding linear equations

1. Find the slope of the line that contains the points $(4,\ 5)$ and $(6,\ 8)$.
2. Plot the points $(4,\ 5)$ and $(6,\ 8)$ on the same coordinate system. Then draw the line that contains the points. Finally, find the y-intercept of the line.
3. Recall that a line of the form $y = mx + b$ has slope m and y-intercept $(0,\ b)$. Use the results you found in Problems 1 and 2 to find an equation of the line that contains the points $(4,\ 5)$ and $(6,\ 8)$.
4. Substitute the coordinates of the point $(4,\ 5)$ into the equation $y = \frac{3}{2}x + b$ and solve for b. Then substitute the value for b that you found into the equation $y = \frac{3}{2}x + b$.
5. Explain why it makes sense that the equations you found in Problems 3 and 4 are the same.

Workbook Exploration

The skill of finding an equation using two given points is introduced in the following 15-minute exploration.

EXPLORATION *Finding an equation of a line*

In this exploration, you will use pencil and paper to find an equation of the line passing through the points $(2, 1)$ and $(5, 7)$.

1. Find the slope of the line containing the points $(2, 1)$ and $(5, 7)$.
2. Recall that the equation of a nonvertical line can be put into the form $y = mx + b$. Substitute your value of m from Problem 1 into $y = mx + b$. (If $m = 5$, then $y = 5x + b$, for example.)
3. To find b, recall that any point that lies on the graph of an equation must satisfy that equation. In particular, the point $(2, 1)$ should satisfy the equation from Problem 2. That is—substitute 2 for x and 1 for y, and then solve for b.
4. Write the equation of the line. (If $m = 5$ and $b = 3$, for example, then $y = 5x + 3$.)
5. Use a graphing calculator to verify that the graph of your equation passes through the points $(2, 1)$ and $(5, 7)$.

A few groups will have trouble with Problem 2 of the exploration. In particular, some groups will ignore Problem 2 and write $1 = m(2) + b$ in Problem 3.

Textbook and Workbook Exploration

The 15-minute "Deciding which points to use to find an equation of a line" exploration responds to the frequent student query, "Does it matter which ordered pair is used to find b?" and "Does it matter which two distinct points I choose from the graph of a line to find its equation?" I make sure that students understand the directions for Problem 2 of the exploration, and that they imagine a line that is *not* parallel to either axis. This is an easy exploration for most students, and it could be assigned as homework.

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Textbook and Workbook Exploration

The 15-minute "Finding equations of lines" exploration is a fun critical-thinking activity. Groups can complete this exploration in a timely fashion if they divide up the work. Or, this exploration is suitable to be assigned as homework.

OBJECTIVE 2 COMMENTS

This concept is optional. See my comments made earlier under "Main points."

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SECTION 5.5 LECTURE NOTES

Objectives

1. Find an equation of a linear model by using data described in words.
2. Find an equation of a linear model by using data displayed in a table.

Main point: Find an equation of a linear model.

OBJECTIVE 1

Recall from Section 5.2 that a linear function can be put in the form $y = mx + b$.

1. Let n be the average number of times a person at age a years visits a library per year. The variables a and n are approximately linearly related. The average number of times 29-year-old adults visit libraries per year is 11 times. The average number of times 59-year-old adults visit libraries per year is 9 times (Source: *KRC Research for the American Library Association*). Find an equation of a linear model to describe the data.

2. Faculty Regalia® sells 16 graduation sets (cap, gown, and tassel) for \$31 per set and sells 75 sets for \$24 per set (Source: *Faculty Regalia*). Let p be the price (in dollars) per set for purchasing n sets. The variables n and p are approximately linearly related. Find an equation of a linear model to describe the data.

OBJECTIVE 2

3. The total U.S. consumptions (in billions of pounds) of avocados are shown in the following table for various years.

Table 2.1: Total U.S. Consumption of Avocados

Year	Consumption (billions of pounds)
2009	1.15
2010	1.32
2011	1.24
2012	1.59
2013	1.79
2014	1.97
2015	2.28
2016	2.39

Source: *USDA*

Let A be the annual total U.S. consumption (in billions of pounds) of avocados at t years since 2000. Find an equation of a linear model to describe the data. Use a graphing calculator to verify your equation.

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Finding an Equation of a Linear Model

To find an equation of a linear model, given some data:

1. Construct a scatterplot of the data.
2. Determine whether there is a line that comes close to the data points. If so, choose two points (not necessarily data points) that you can use to find an equation of a linear model.
3. Find an equation of the line.
4. Use a graphing calculator to verify the graph of your equation contains the two chosen points and comes close to all the points of the scatterplot.

4. Annual U.S. new-car and light-truck sales are shown in the following table for various years.

Table 2.2: U.S. New-Car and Light-Truck Sales

Year	Sales (millions)
2009	10.5
2010	11.6
2011	12.8
2012	14.7
2013	15.6
2014	16.7
2015	17.5

Source: *Automotive News*

Let s be the U.S. annual new-car and light-truck sales (in millions) at t years since 2005. Find an equation of a linear model to describe the data. Use a graphing calculator to verify your equation.

SHORT HW 1, 3, 9, 13, 15, 21, 23, 27, 29, 31, 33, 37

MEDIUM HW 1, 3, 7, 9, 13, 15, 17, 18, 21, 23, 24, 25, 27, 29, 31, 33, 37, 39, 41, 43

SECTION 5.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students learn to use technology to draw a scatterplot of data in this section. Then they use two "good points" to find an equation of a linear model. They will use these skills in Sections 5.6, 5.7, and Chapter 6.

OBJECTIVE 1 COMMENTS

For Problems 1 and 2, I remind my students once more of the roles of the explanatory variable and the response variable. It is helpful to organize the information in a table such as Table 16 on page 289 of the textbook. As I've done in previous sections, I write the equation $y = mx + b$ and then replace the variables x and y with the appropriate variables.

I tell my students that when modeling, we round the values of m and b to the second decimal place, unless values are close to 0, such as $m = 0.003$.

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Finally, I use a graphing calculator to verify an equation that we have found. The most efficient way to do this is to use a graphing calculator table in "Ask" mode. However, it is important that students can visualize what they have accomplished, so it's probably best to use TRACE. If you want to avoid discussing window settings, you can enter the two data points in the STAT editor and then use ZoomStat.

OBJECTIVE 2 COMMENTS

I begin solving Problem 3 by showing my students how to use their graphing calculators (or other technology) to draw a scatterplot of the avocado data. It is from this point on that students will have graphing-calculator invalid-dimension errors crop up. I describe the meaning of the message "ERR:INVALID DIM" to my students so that they know what to do when the message is displayed (see the third bulleted comment in Appendix A.26). When my students accidentally delete a list in the STAT editor, I explain how to use "SetUpEditor" to get back the list. (See the first margin note in Section A.8.)

I then number the data points (or use letters) and ask my students which pairs of data points lie on a line that comes close to the other data points and which pairs do not.

Once the class has determined two "good points," I derive an equation of the line that contains the two points. I emphasize that it's okay to use decimals; many students think they need to use fractions because they were required to do so when finding equations of (non-model) lines in Section 5.4. I explain that it's okay to round when finding a model because there are already various types of errors built into the process, anyway (see the exploration "Identifying types of modeling errors" in Section 1.4 of the textbook).

When verifying a model's equation, I explain that the line should contain the two chosen points (and come close to the other data points). I encourage my students to take the time to distinguish between their chosen two points and the other data points. (By the way, most students do not realize that a point that lies close to, but not on a line, may appear to lie *on* the line on a graphing calculator screen.) For quizzes and exams, I tell my students to copy their graphing calculator screen which displays how well their model fits the data; if I don't require this (by making it worth points), some students will resist ever learning how to use a graphing calculator to draw scatterplots and to verify their models.

Students could use two nondata points to find a linear model, but I don't bring up this option for four reasons. First, because using data points works just fine. Second, it is easier to use coordinates of data points than to estimate coordinates of nondata points. Third, skipping this option saves me a bit of class time. Fourth, by not discussing how to use nondata points, I reduce the amount of time I spend grading exams because there is less variation in my students' derived equations for models.

Although the skill of using a graphing calculator to obtain a linear regression equation is described in Section 5.5 of the textbook, discussion of this topic can be postponed until a later point or it can be avoided entirely. Skipping this topic would mean that you would have to be careful to avoid assigning a small number of exercises throughout the textbook that ask students to use regression. The only such exercise in Chapter 5 is Exercise 26 of Homework 5.5.

I have included regression equations as answers to exercises simply because I can't anticipate which two "good" points students will choose. So, when assigning this section's homework, I warn my students that their answers will vary from those in the textbook. I remind them that they can verify their work by using a graphing calculator.

I require my students to use pencil and paper to derive the equations of all types of models in the course. Such skills are foundational to algebra, and students need to do lots of symbolic work in order to prepare for subsequent courses. However, I find regression to be a useful student tool for the four "Topic of Your Choice" labs (see pages 326, 604, 669–70, and 739 of the textbook) in which students may need to find equations of many models while searching through data sets.

Workbook Exploration

The following 15-minute exploration about electric vehicles could be assigned at the start of class. After all, students have already learned how to find an equation of a line using two points in Section 5.4. Most, if not all, groups will forgo constructing a scatterplot and simply choose two data pairs in the table to use to find an equation. This creates the opportunity to point out the danger in doing so during the debrief. In fact, if a group uses the data pairs for 2016 and 2017, you can point to how poorly the model fits all the data points. The workbook exploration

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"Choosing 'good points' to find a model" (included later in these comments) also attends to this.

Most, if not all, groups will ignore that t is defined to be years since 2010 and will in effect, define t to be the year. During the debrief, comparing and contrasting the equations and graphs of the models for the two definitions of t can be very instructive.

Group Exploration
Section Opener: Finding an equation of a linear model

The numbers of electric vehicle models available to consumers in North America are shown in the following table for various years.

Year	Number of Electric Vehicle Models in North America
2011	10
2012	17
2013	24
2014	29
2015	39
2016	44
2017	54

Source: *Bloomberg New Energy Finance*

Let n be the number of electric vehicle models available to consumers in North America at t year since 2010. Find an equation of a linear model.

Workbook Exploration
The following 20-minute exploration about airline passengers provides more structure than the one about electric vehicles, so it's probably less valuable. But it does outline the desired approach. So, having students work on it directly after the electric-vehicle exploration can work well.

Group Exploration
Section Opener: Finding an equation of a linear model

The number of airline passengers worldwide are shown in the following table for various years.

Year	Number of Airline Passengers Worldwide (millions)
2009	2.5
2010	2.7
2011	2.9
2012	3.0
2013	3.2
2014	3.3
2015	3.6
2016	3.8

Source: *ICAO; IATA*

Let n be the number (in millions) of airline passengers worldwide in the year that is t years since 2000.

1. Use a graphing calculator to construct a scatterplot of the data. Copy the screen.
2. Imagine a line that comes close to the data points. Estimate the coordinates of two points that lie on the line. Use the coordinates to find the slope of the line.

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3. Substitute the slope you found in Problem 2 for m in the equation $n = mt + b$.

4. Substitute the coordinates of one of the points you identified in Problem 2 into the equation you found in Problem 3 and solve for b.

5. Substitute the value of b you found in Problem 4 into the equation you found in Problem 3. We call such an equation a *linear model*.

6. Predict the number of airline passengers worldwide in 2018 by substituting 18 for t in your linear model.

7. International Air Transport Association predicts that there will be 4.3 million airline passengers worldwide in 2018. Is this prediction greater than, equal to, or less than the prediction you made in Problem 6?

Workbook Exploration

When finding a linear model, groups can discover the impact of their selection of a pair of points by completing the following 15-minute exploration. It is best, however, to first demonstrate how to derive an equation of a linear model before assigning this exploration, so students can focus on the main point of this exploration, which is the selection of a pair of points, not finding the equation.

You may want your students to skip Problem 7 in the exploration to save class time and/or to save time when you grade exams (as I explained before).

Group Exploration

Choosing "good points" to find a model

The revenues of IKEA are shown in the following table for various years. The table includes a first column that indicates a name for each data point. For example, point D refers to the point $(9,\ 21.8)$.

Name of Point	Years since 2000	Revenue (billions of dollars)
A	6	17.5
B	7	20.0
C	8	21.5
D	9	21.8
E	10	23.5
F	11	25.2
G	12	27.6
H	13	28.5
I	14	29.3
J	15	32.7

Source: *IKEA*

1. Let r be the annual revenue (in billions of dollars) at t years since 2000. Use a graphing calculator to draw a scatterplot of the data. Copy the screen.

2. Find an equation of the line that contains the points A and B.

3. Use a graphing calculator to verify that the graph of your equation passes through both points A and B. Copy the screen. Does the line come close to the other data points?

4. If you had used points A and H, you would have found the equation $r = 1.57t + 8.07$. Compare its graph with the graph you drew in Problem 3. Copy the screen. Explain why the graphs look so different.

5. List five pairs of points that yield equations you think would be good linear models. (You do not have to find the equations.)

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6. Several pairs of points from a scatterplot yield equations that could serve as models of the data. Discuss how to choose two such data points to find an equation that comes reasonably close to all the data points.

7. It is not necessary to use data points to find an equation of a linear model. While viewing the IKEA scatterplot, use the arrow keys on a graphing calculator to identify two nondata points you feel would yield an equation of a line that is close to the data points. Find an equation of the line that contains these two points. Then use a graphing calculator to verify that the graph of your equation comes close to the data points. Copy the screen.

Textbook and Workbook Exploration

Students will have an easy time with the 15-minute "Adjusting the fit of a model" exploration, which is a fun reminder of the graphical significance of slope and the vertical intercept. More generally, the connection between a function's equation and graph is reinforced.

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SECTION 5.6 LECTURE NOTES

Objectives

1. Use function notation to makes estimates and predictions.
2. Find intercepts of a model.
3. Use data described in words to make estimates and predictions.
4. Describe the meaning of *domain* and *range* of a model.

Main point: Use function notation to make estimates and predictions about authentic situations.

OBJECTIVES 1 and 2

1. In Problem 4 of the lecture notes of Section 5.5, we found an equation close to $s = 1.21t + 5.70$, where s id the U.S. annual new-car and light-truck sales (in millions) at t years since 2005. [Use the non-regression equation that you found in class.]

a. Rewrite the equation $s = 1.21t + 5.70$ with the function name f.

b. Find $f(16)$. What does it mean in this situation?

c. Find t when $f(t) = 26$. What does it mean in this situation?

d. Find the s-intercept. What does it mean in this situation?

e. Find the t-intercept. What does it mean in this situation?

Using an Equation of a Linear Model to Find Intercepts

If a function of the form $p = mt + b$, where $m \neq 0$, is used to model a situation, then

- The p-intercept is $(0, b)$.
- To find the t-coordinate of the t-intercept, substitute 0 for p in the model's equation and solve for t.

Four-Step Modeling Process

To find a linear model and make estimates and predictions,

1. Construct a scatterplot of the data to determine whether there is a nonvertical line that comes close to the data points. If so, choose two points (not necessarily data points) that you can use to find an equation of a linear model.
2. Find an equation of your model.
3. Verify your equation by checking that the graph of your model contains the two chosen points and comes close to all the data points.
4. Use the equation of your model to make estimates, make predictions, and draw conclusions.

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OBJECTIVE 3

2. U.S. sales of refrigerated ready-to-drink coffee increased approximately linearly from \$116 million in 2012 to \$249 million in 2016 (Source: *IRI*). Predict when the annual sales will reach \$400 million.

OBJECTIVE 4

Recall from Section 5.2 that the domain of a function is the set of all inputs and the range is the set of all outputs. For the **domain** and **range** of a model, we consider input-output pairs only when both the input and the output make sense in the situation.

3. An airplane at an altitude of 36 thousand feet begins to descend steadily. Ten minutes later, the altitude of the airplane is 12 thousand feet. Let $g(t)$ be the altitude (in thousands of feet) of the airplane after t minutes of descending.

a. Find an equation of g.

b. Graph g by hand.

c. What are the domain and range of g?

SHORT HW 3, 7, 9, 13, 15, 19, 25, 27, 29, 33, 35

MEDIUM HW 1, 3, 7, 9, 11, 13, 15, 19, 23, 25, 27, 29, 33, 35, 37, 39

SECTION 5.6 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use function notation to make estimates and predictions. They will use this skill in modeling sections throughout the rest of the textbook.

OBJECTIVES 1 and 2 COMMENTS

Inform your students to what extent they can use regression for homework exercises, as most problems do not specify how students should find a model's equation. I usually do not tell my students about regression until I know that they are proficient at finding equations for linear, quadratic, and exponential models because I believe these skills are important ones for students to learn. A good time to introduce regression is when you assign projects. If you do choose to introduce regression now, you can prevent your students from becoming too dependent on this tool by requiring them to show their symbolic work when they find equations of models on exams.

In Problem 1, compare part (b) with part (c). For parts (d) and (e), I remind my students that we begin to find an intercept by substituting 0 for the *other* variable into the model's equation.

Background Information for the Exploration That Follows

In the 30-minute exploration that follows, students perform the four-step modeling process. Problems 1–3 of the exploration review finding an equation of a linear model by choosing two points. Many students need to revisit this skill (from Section 5.5) before they become proficient at it. Problems 4 and 5 have students perform symbolic work as well as help them get used to using function notation. After the exploration, I compare the work for Problem 4 with the work for Problem 5 and compare the work for Problem 6 with the work for Problem 7.

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EXPLORATION *Using function notation; making predictions*

The number (in millions) of households with VCRs are shown in the following table for various years.

Year	Number of Households (millions)
2008	84
2009	79
2010	74
2011	69
2012	65

Source: *Nielsen*

Let $n = f(t)$ be the number (in millions) of households with VCRs at t years since 2000.

1. Use a graphing calculator to draw a scatterplot of the data.
2. Find an equation of f.
3. Draw the graph of f and the scatterplot in the same viewing window. Verify that the graph of f contains the two points you chose in finding the equation of f and that it comes close to all the data points.
4. Find $f(17)$. What does it mean in this situation?
5. Find t when $f(t) = 35$. What does it mean in this situation?
6. Find the n-intercept of the model. What does it mean in this situation?
7. Find the t-intercept of the model. What does it mean in this situation?

Workbook Exploration

By completing the following 30-minute exploration, students can see one of the benefits of function notation: naming function is a convenient way to refer to them. The exploration nicely motivates linear systems, which are discussed in Sections 6.1–6.4.

Group Exploration

Comparing linear models

Average U.S. per-person consumptions of chicken and red meat are described in the following table for various years.

	Average Consumption (pounds per person)	
Year	Chicken	Red Meat
1970	40.3	145.8
1980	48.0	136.8
1990	61.5	120.0
2000	78.0	120.7
2010	83.7	108.7
2015	90.0	104.8

Source: *U.S. Department of Agriculture*

Let $C(t)$ be the average annual per-person consumption of chicken and $R(t)$ be the average annual per-person consumption of red meat, both in pounds per person, in the year that is t years since 1970.

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1. Find equations of C and R. Use a graphing calculator to verify that your models fit the data points well. Copy the screen.
2. Compare the values of $C(0)$ and $R(0)$. What does your comparison mean in this situation?
3. Compare the slopes of the two models. What does your comparison mean in this situation?
4. The results you found in Problems 2 and 3 should suggest an event that will happen in the future. Describe that event.
5. Use "intersect" on a graphing calculator to find the point where the graphs of the models intersect. Copy the screen. In terms of consumption, what does it mean that the models intersect at this point? State the year and consumption for this event.
6. For the period 1970–2015, has the *total* average consumption of chicken and red meat generally increased, decreased, or neither? Use calculations and words to explain.

Textbook and Workbook Exploration

The 20-minute "Using function notation to make estimates and predictions" exploration has groups practice interpreting function notation and review interpreting the slope of a linear model.

OBJECTIVE 3 COMMENTS

I tell my students that for exercises similar to Problem 2, it is left to them to define the variables.

OBJECTIVE 4 COMMENTS

On pages 302 and 303 of the textbook, there is a discussion about the domain and range of a linear model. For Example 4 on pages 302 and 303 of the textbook, I point out that both the domain and range of the function $f(t) = 10t$ is the set of (all) real numbers if f is not a model. Such comparisons help students learn about the domain and range of functions not being used as models as well as the domain and range of models.

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SECTION 5.7 LECTURE NOTES

Objectives

1. Describe the meaning of *inequality symbols* and *inequality*.
2. Graph an inequality.
3. Apply the properties of inequalities.
4. Describe the meaning of *satisfy*, *solution*, and *solution set* for a *linear inequality in one variable*.
5. Solve a linear inequality in one variable, and graph the solution set.
6. Solve a *compound inequality in one variable*, and graph the solution set.
7. Use linear inequalities to make predictions about authentic situations.

Main point: Solve linear inequalities in one variable.

OBJECTIVE 1

Here are the meanings of the **inequality symbols** $<$, $\leq$, $>$, and $\geq$ and some examples of *inequalities:*

Symbol	**Meaning**	**Examples of Inequalities**
$<$	less than	$3 < 8,\ \ 0 < 3,\ \ -5 < -1$
$\leq$	less than or equal to	$3 \leq 4,\ \ 5 \leq 5,\ \ -4 \leq 0$
$>$	greater than	$6 > 3,\ \ -5 > -9,\ \ 3 > 0$
$\geq$	greater than or equal to	$5 \geq 1,\ \ 4 \geq 4,\ \ -3 \geq -7$

- An **inequality** contains one of the symbols $<$, $\leq$, $>$, and $\geq$ with expressions on both sides.
- Some examples: $x < 9$, $2x - 7 \leq 5$, $3x - 5 > 4x + 1$, $9 \geq 6$

Decide whether the inequality is true or false.

1. $3 < 7$ **2.** $-6 \geq -2$ **3.** $8 > 8$ **4.** $6 \leq 6$

OBJECTIVE 2

Describe the inequality using words, a graph, and interval notation.

5. $x < 2$ **6.** $x \leq 2$ **7.** $x > 2$ **8.** $x \geq 2$

OBJECTIVE 3

Show what happens when we add a number to both sides of $5 < 7$.

Addition Property of Inequalities

If $a < b$, then $a + c < b + c$.

Similar properties hold for $\leq$, $>$, and $\geq$.

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- Illustrate the addition property of inequalities by using a number line.
- Show what happens when we multiply both sides of the inequality $5 < 7$ by a number.

Multiplication Property of Inequalities

- For a *positive* number c, if $a < b$, then $ac < bc$.
- For a *negative* number c, if $a < b$, then $ac > bc$.

Similar properties hold for $\leq$, $>$, and $\geq$.

Illustrate that if $a < b$, then $-a > -b$ by using a number line.

OBJECTIVE 4

Definition *Linear inequality in one variable*

A **linear inequality in one variable** is an inequality that can be put into one of the forms

$$mx + b < 0, \qquad mx + b \leq 0, \qquad mx + b > 0, \qquad mx + b \geq 0$$

where m and b are constants and $m \neq 0$.

We say a number **satisfies** an inequality in one variable if the inequality becomes a true statement after we have substituted the number for the variable.

9. Does the number 2 satisfy the inequality $4x - 3 < 8$?

10. Does the number 5 satisfy the inequality $4x - 3 < 8$?

Definition *Solution, solution set,* and *solve* for an inequality in one variable

We say a number is a **solution** of an inequality in one variable if it satisfies the inequality. The **solution set** of an inequality is the set of all solutions of the inequality. We **solve** an inequality by finding its solution set.

OBJECTIVE 5

Solve the inequality. Describe the solution set as an inequality, in interval notation, and in a graph.

11. $-4x < 24$

12. $k + 3 \geq 5$

13. $-3x + 1 < -11$

14. $4m + 3 > 2(3m - 4)$

15. $\frac{2}{9} - \frac{5}{3}x \leq \frac{7}{3}$

16. $\frac{2p - 1}{3} - \frac{7p + 3}{4} \geq \frac{5}{6}$

OBJECTIVE 6

The inequality $2 \leq x \leq 5$ is a *compound inequality*.

17. Describe the meaning of $2 \leq x \leq 5$ and graph the solutions.

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Solve the inequality. Describe the solution set as an inequality, in interval notation, and in a graph.

18. $-1 \le 3x - 7 \le 5$ **19.** $2 < 8 - 2x < 10$ **20.** $\frac{1}{2} < 3 - \frac{5}{4}x \le \frac{7}{2}$

OBJECTIVE 7

21. The total annual box office grosses in the United States and Canada are shown in the following table for various years.

Year	Total Box Office Grosses in the United States and Canada (billions of dollars)
1990	5.02
1995	5.27
2000	7.51
2005	8.82
2010	10.58
2015	11.12
2016	11.37

Sources: *AC Nielsen EDI, Rentrak Corporation*

Let B be the total annual box office grosses (in billions of dollars) in the United States and Canada at t years since 1990.

a. Find a linear model to describe the data.

b. In which years will the annual box office gross be more than $13 billion?

SHORT HW 1, 7, 13, 17, 27, 39, 47, 57, 63, 69, 75, 77, 81, 85, 87

MEDIUM HW 1, 7, 13, 17. 19, 27, 33, 39, 47, 57, 63, 69, 75, 77, 81, 83, 85, 87, 91, 95, 97, 101, 103, 105

SECTION 5.7 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students solve linear inequalities in this section. Students will graph linear inequalities in two variables and solve systems of linear inequalities in Section 6.6.

OBJECTIVE 1 COMMENTS

For Problem 4, some students think that $6 \le 6$ is a false statement because 6 is not less than itself.

OBJECTIVE 2 COMMENTS

For Problems 5–8, I try to clear up as many questions as I can so that students can focus on the symbolic work when we reach Problems 11–16.

OBJECTIVE 3 COMMENTS

I emphasize that when we multiply or divide both sides of an inequality by a negative number, we reverse the inequality symbol. Students need several reminders about this.

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Textbook and Workbook Exploration
In the 20-minute "Properties of exponents" exploration, groups can discover properties of inequalities.

OBJECTIVE 4 COMMENTS
I observe with my students that the meanings of the words *satisfy*, *solution*, and *solution set* are similar for equations and inequalities.

OBJECTIVE 5 COMMENTS
I point out that solving a linear inequality is similar to solving a linear equation. However, I continue to heavily emphasize reversing the inequality symbol, when appropriate. When saying something like, "Here we *divide* both sides by *negative* 5," I'll raise my voice dramatically when I say "divide" and "negative." Although this is cheesy, my students get the point that they need to be vigilant about reversing the inequality symbol, when appropriate.

Many students do not understand the significance of the solution set of an inequality in one variable. For example, consider the inequality $-3x + 1 < -11$, whose solution is $x > 4$ (see Problem 13). It is instructive for students to see that numbers greater than 4 such as 4.1, 5, and 6 satisfy the inequality and numbers less than or equal to 4 such as 2, 3, and 4 do not satisfy the inequality. Exercises 87 and 88 of Homework 5.7 address this issue.

Textbook and Workbook Exploration
Another way for students to learn the meaning of the solution set of an inequality is for them to complete the 10-minute "Meaning of the solution set of an inequality" exploration.

OBJECTIVE 6 COMMENTS
Students will have an easy time solving compound inequalities. However, students will need some coaching to see that an inequality such as $a > x > b$ is equivalent to $b < x < a$. This issue naturally arises in Problem 19, where the inequality $2 < 8 - 2x < 10$ has solution $3 > x > -1$, which is equivalent to $-1 < x < 3$.

OBJECTIVE 7 COMMENTS
To solve part (b) of Problem 21, I first translate the phrase "annual box office gross be more than \$13 billion:"

$$B > 13$$

Then I substitute $0.27t + 4.69$ for B:

$$0.27t + 4.69 > 13$$

Many students mistakenly use equality symbols instead of inequality symbols; this is partly due to lack of reading comprehension and partly due to sloppiness in notation. Even more students have trouble interpreting the meaning of an inequality in terms of the situation. For Problem 17, students would tend to say the inequality $t > 30.78$ means that the annual box office gross will be more than \$13 billion *in* 2021, rather than *after* 2021. I warn my students repeatedly about this issue. I also sketch a graph of the model and discuss the situation in detail. Unfortunately, many students still have trouble with these issues on exams.

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CHAPTER 6 OVERVIEW

In Section 6.1, students use graphing to solve systems of linear equations, which are sometimes linear models. They do this by hand and by using technology. In Sections 6.2 and 6.3, students solve such systems using substitution and elimination, respectively. In Section 6.4, students use substitution or elimination to solve systems of two models to make an estimate or prediction. In Section 6.5, they solve perimeter, value, interest, and mixture problems. In Section 6.6, students graph linear inequalities in two variables and solve systems of linear inequalities in two variables. Techniques in this chapter will be useful in many other parts of the text including Sections 9.7—9.9, 10.4, 10.5, 13.6, and 15.5.

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SECTION 6.1 LECTURE NOTES

Objectives

1. Describe the meaning of *solution* and *solution set* of a *system of linear equations in two variables*.
2. Use a graphical approach to solve a systems of linear equations.
3. Use graphing to make predictions about situations that can be modeled by using two linear models.
4. Describe the three types of linear systems of two equations.
5. Find the solution of a system of linear equations from tables of solutions of such equations.

Main point: Solve *systems of linear equations*.

OBJECTIVE 1

A **system of linear equations in two variables** or a **linear system** for short, consists of two or more linear equations in two variables. Here is an example:

$$y = -2x + 5$$
$$y = x - 1$$

1. Find all ordered pairs that satisfy both of the equations

$$y = -2x + 5$$
$$y = x - 1$$

Definition *Solution, solution set,* and *solve* for a system

We say an ordered pair (a, b) is a **solution** of a system of two equations in two variables if it satisfies both equations. The **solution set** of a system is the set of all solutions of the system. We **solve** a system by finding its solution set.

OBJECTIVE 2

Solve the system. Verify your work by using "intersect" on a graphing calculator or by checking that the result satisfies the system.

2.
$$y = -2x + 1$$
$$y = 3x + 6$$

3.
$$-x + 2y = 2$$
$$3x + 4y = -16$$

4.
$$3(y - 1) + 5x = 4x$$
$$3y = 2(6 + x)$$

5.
$$\frac{1}{6}x - \frac{1}{2}y = \frac{3}{2}$$
$$y = x - 5$$

6. Use “intersect” on a graphing calculator to solve the system

$$y = -0.57x - 6.28$$
$$y = 1.45x + 1.31$$

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OBJECTIVE 3

7. The annual U.S. volume sales of soft drinks and bottled water are shown in the following table for various years.

	U.S. Volume Sales (billions of gallons)	
Year	Soft Drinks	Bottled Water
2010	13.78	8.76
2011	13.54	9.11
2012	13.30	9.67
2013	12.87	10.13
2014	12.75	10.87

Source: *Forbes*

Annual U.S. volume sales (in billions of gallons) $f(t)$ and $g(t)$ of soft drinks and bottled water, respectively, are modeled by the system

$$r = f(t) = -0.27t + 13.79$$
$$r = g(t) = 0.52t + 8.66$$

where t is the number of years since 2010.

a. Compare the r-intercepts of the two models. What does your comparison tell you about this situation?

b. Compare the slopes of the two models. What does your comparison tell you about this situation?

c. Explain why your work in parts (a) and (b) suggest there may be a time when the annual U.S. volume sales of soft drinks will equal the annual U.S. volume sales of bottled water.

d. Use "intersect" on a graphing calculator to predict when the annual U.S. volume sales of soft drinks will equal the annual U.S. volume sales of bottled water. What will be that annual volume sales?

Intersection Point of the Graphs of Two Models

If the explanatory variable of two models represents time, then an intersection point of the graphs of the two models indicates when the quantities represented by the response variables were or will be equal.

OBJECTIVE 4

8. Solve the system.

$$y = \frac{1}{3}x + 1$$
$$y = \frac{1}{3}x + 2$$

A linear system whose solution set is the empty set is called an **inconsistent system**.

9. Solve the system.

$$y = 2x - 3$$
$$y = 2x - 3$$

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- A linear system that has an infinite number of solutions is called a **dependent system**.
- A linear system that has one solution is called a *one-solution system*.

Solve the system. If the system is inconsistent or dependent, say so.

10.

$$4x - 10y = 30$$
$$6x - 15y = 60$$

11.

$$y = 3x - 2$$
$$6x - 2y = 4$$

OBJECTIVE 5

12. Use the following table to solve the system

$$y = -4x + 21$$
$$y = 5x + 3$$

x	0	1	2	3	4
$y = -4x + 21$	21	17	13	9	5
$y = 5x + 3$	3	8	13	18	23

SHORT HW 3, 5, 7, 15, 19, 23, 25, 27, 35, 37, 45, 47, 57, 59, 67

MEDIUM HW 3, 5, 7, 15, 19, 23, 25, 27, 31, 35, 37, 41, 45, 47, 49, 53, 55, 57, 59, 67, 71, 75, 77, 79

SECTION 6.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students use graphing and tables to solve systems of linear equations in this section. They use pencil and paper to solve some systems and technology to solve others. In particular, technology is used to solve systems of two linear models. The concepts in this section are relevant to the rest of Chapter 6.

OBJECTIVE 1 COMMENTS

When doing Problem 1, I remind students of the meaning of a graph. This will lead to finding the intersection point of the two lines. I drive this point home by selecting a point that lies on both lines, one line, or neither line and checking whether the corresponding ordered pair satisfies both equations, one equation, or neither equation, respectively.

OBJECTIVE 2 COMMENTS

When some students attempt to solve a system, they sketch the appropriate graphs but do not indicate the solution. I tell students that they must write a sentence describing the solution. Students will have an easy time with Problem 2. For Problem 3, I emphasize that it helps to first solve both of the equations for y. For Problem 5, I remind students that when an equation contains fractions, we multiply both sides of the equation by the LCD to clear the equation of fractions.

For Problem 6, students get a kick out of learning how to use "intersect" on a graphing calculator to solve a linear system.

Background Information for the Exploration That Follows

In the following 10-minute exploration, students discover why the intersection point of two nonparallel lines

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corresponds to the solution of the system of equations of the lines. To prepare students for this exploration, I show them how to use "intersect" on a graphing calculator. Groups' responses to Problem 3 in the exploration will reveal whether or not they remember the meaning of a graph. If they have forgotten, I emphasize this concept.

EXPLORATION *Intersection points satisfy both equations*

1. Use "intersect" on a graphing calculator to find the intersection point of the graphs of the following equations:

$$y = 3x - 7, \text{ and}$$
$$y = -2x + 8$$

   (See Appendix B.18 for calculator instructions.)

2. Show that the ordered pair corresponding to the intersection point satisfies both equations. Explain why this makes sense.

3. Are there points other than the intersection point whose ordered pairs satisfy both equations? If so, find them. If not, explain why not.

Workbook Exploration

In the following 10-minute exploration, students discover why the intersection point of two nonparallel lines corresponds to the solution of the system of equations of the lines. Groups' responses to Problems 2–5 in the exploration will reveal whether or not they remember the meaning of a graph. If they have forgotten, I emphasize this during the debrief.

Group Exploration

Section Opener: Using graphs to solve systems

1. Graph the equations $y = 2x - 1$ and $y = -x + 5$.

2. Find three ordered pairs that satisfy $y = 2x - 1$ but do not satisfy $y = -x + 5$.

3. Find three ordered pairs that satisfy $y = -x + 5$ but do not satisfy $y = 2x - 1$.

4. Find three ordered pairs that satisfy neither of the equations $y = 2x - 1$ and $y = -x + 5$.

5. Find all ordered pairs that satisfy both of the equations $y = 2x - 1$ and $y = -x + 5$.

OBJECTIVE 3 COMMENTS

Problem 7 is a great way to review the meaning of the parameters of a linear model as well as to motivate finding the intersection point of two lines. Most students need help with finding appropriate WINDOW settings. One way to find such settings is to use ZStandard, and then increase the WINDOW setting for Xmax to 10. Another way is to use ZoomStat to draw the two scatterplots of the data and then Zoom Out once. Some students need to be reminded to find the volume sales as well as the year.

Workbook Exploration

You could start class with the following 20-minute exploration, which is an excellent way to begin this section. Groups may come up with a wide variety of approaches: numerical, graphical, and symbolic. Comparing and contrasting groups' approaches can lay a great foundation for the entire chapter.

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Group Exploration

Section Opener: Estimating when quantities were equal

The total worldwide revenues from television, radio, and multimedia, and the worldwide revenues from telecommunication devices are shown in the following table for various years. Estimate when the total revenue from television, radio, and multimedia was equal to the revenue from telecommunication devices. What was that revenue?

	Revenue (billions of dollars)	
Year	Television, Radio, and Multimedia	Telecommunication Devices
2011	56.3	65.5
2012	52.0	69.8
2013	47.5	73.6
2014	45.5	77.2
2015	44.0	79.9

Source: *Statista*

Textbook and Workbook Exploration

The 20-minute "Using a system of equations to model a system" exploration offers more structure than the "Estimating when quantities were equal" exploration, so unless you facilitate both, I would advise using the letter because it will lead to a richer debrief.

Groups are already accustomed to working with linear models so they should easily progress through this exploration. Before groups start working on this exploration, I show them how to use the "intersect" key on their calculator.

OBJECTIVE 4 COMMENTS

When solving Problems 8 and 9, I remind students of the meaning of a graph and the meaning of a solution of a system.

To ensure that my students become proficient at solving inconsistent and dependent systems in this section and Sections 6.2 and 6.3, I carefully and repeatedly discuss the relevant concepts. Students have trouble keeping straight the solutions and the names of these two types of systems.

Textbook and Workbook Exploration

The 15-minute "Comparing the three types of systems" exploration encourages students to reflect on solutions of one-solution systems, inconsistent systems, and dependent systems. Many groups do well with this exploration, including Problem 3.

OBJECTIVE 5 COMMENTS

Students will pick up on this concept quickly. Doing Problem 11 will set the stage for students to solve the more challenging Exercises 49 and 50 of Homework 6.1, Exercises 45 and 46 of Homework 6.2, and Exercises 65 and 66 of Homework 6.3. If you are tight on time, you could skip this method because it is much more effective to use graphing, substitution, or elimination to solve linear systems.

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SECTION 6.2 LECTURE NOTES

Objectives

1. Use substitution to solve a system of two linear equations.
2. Isolate a variable in an equation to help solve a system by substitution.
3. Solve inconsistent and dependent systems by substitution.

Main point: Use substitution to solve a system of two linear equations.

OBJECTIVE 1

Use substitution to solve the system.

1.
$$-3x + 2y = 9$$
$$y = 4x + 7$$

2.
$$x = 4y + 1$$
$$-5x - 2y = 17$$

3.
$$-4x + 3y = -6$$
$$x = 5y - 7$$

4.
$$y = 1 - 3x$$
$$4x + 3y + 2 = 0$$

Using Substitution to Solve a Linear System

To use **substitution** to solve a system of two linear equations,

1. Isolate a variable to one side of either equation.
2. Substitute the expression for the variable found in step 1 into the other equation.
3. Solve the equation in one variable found in step 2.
4. Substitute the solution found in step 3 into one of the original equations, and solve for the other variable.

A system of equations can be solved by substitution as well as by graphing. The methods give the same result. Use substitution to solve the system, with coordinates of solutions rounded to the second decimal place.

5.
$$y = 1.25x - 7.49$$
$$y = -3.18x + 8.66$$

6.
$$y = -1.42x - 6.13$$
$$y = 2.68x - 2.75$$

OBJECTIVE 2

Use substitution to solve the system.

7.
$$x - 3y = 7$$
$$-4x + 5y = -14$$

8.
$$4x - 3y = 3$$
$$-2x + y = -5$$

9.
$$2x - y = 1$$
$$5x - 3y = 5$$

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OBJECTIVE 3

10. What happens when you solve the following inconsistent system by substitution?

$$y = 2x + 3$$
$$y = 2x + 4$$

Solving Inconsistent Systems by Substitution

If the result of applying substitution to a system of equations is a false statement, then the system is inconsistent; that is, the solution set is the empty set.

11. What happens when we solve the following dependent system by substitution? [We solved this system by graphing when we did Problem 11 in the lecture notes of Section 6.1.]

$$y = 3x - 2$$
$$6x - 2y = 4$$

Solving Dependent Systems by Substitution

If the result of applying substitution to a linear system of two equations is a true statement that can be put into the form $a = a$, then the system is dependent; that is, the solution set is the set of ordered pairs represented by every point on the (same) line.

Use substitution to solve the system.

12. [inconsistent]
$$y = 4 - 3x$$
$$15x + 5y = 17$$

13. [dependent]
$$-5x + 20y = 25$$
$$x = 4y - 5$$

14. [dependent]
$$x = 2y - 1$$
$$4x - 8y = -4$$

15. [inconsistent]
$$y = 2x - 5$$
$$-4x + 2y = -6$$

SHORT HW 3, 5, 11, 13, 19, 21, 27, 33, 35, 43, 45, 47, 49, 53, 55

MEDIUM HW 1, 3, 5, 9, 11, 13, 15, 19, 21, 27, 31, 33, 35, 37, 43, 45, 47, 49, 51, 53, 55, 61, 63, 65

SECTION 6.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use substitution to solve systems of linear equations. They will use these skills in Sections 6.3, 6.4, 6.5, and 15.5.

OBJECTIVE 1 COMMENTS

For Problem 1, I say that for any solution of the system, the value of y is equal to the value of $4x + 7$. So, we substitute $4x + 7$ for y in the equation $-3x + 2y = 9$.

It is a common student error to find only one coordinate of a solution of a system. I remind my students that we found both coordinates of solutions of systems in Section 6.1.

Problems 4 and 6 are excellent preparation for Section 6.4, where students will solve systems of linear models.

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Workbook Exploration

The following 15-minute exploration is an effective way to introduce substitution because students take to this method quickly.

Group Exploration

Section Opener: Using substitution to solve systems

1. Use graphing to solve the system

$$\begin{aligned} y &= 3x - 5 \\ 2x + y &= 5 \end{aligned}$$

2. Substitute $3x - 5$ for y in the equation $2x + y = 5$ and then solve for x. How is your result similar to the result you found in Problem 1?

3. Substitute the result you found for x in Problem 2 into the equation $y = 3x - 5$ and solve for y. Also substitute it for x in the equation $2x + y = 5$ and solve for y. Explain why it makes sense that your two results are equal. How do your equal results relate to the system in Problem 1? Explain why this makes sense.

Textbook and Workbook Exploration

The 25-minute "Comparing techniques for solving systems" exploration has groups compare using substitution with using graphing to solve systems. If you are tight on time, you could skip this exploration to save time for assigning the exploration in Section 6.3, which has groups compare three techniques (elimination, substitution, and graphing) to solve systems.

OBJECTIVE 2 COMMENTS

Students have an easy time with Problems 7–9. Exercises 33 and 34 of Homework 6.2 are challenging because using substitution will require working with fractions. However, these exercises have not been included in the recommended homework assignments because it's probably better to use elimination than substitution for these types of systems.

OBJECTIVE 3 COMMENTS

Students have a tough time with this objective. I spend a good amount of time discussing why we arrive at false statements when we use substitution to solve inconsistent systems and arrive at true statements of the form $a = a$ when we use substitution to solve dependent systems. I point out that if a student forgets what it means to get either of these types of statements, the student can sort out what is going on by solving both of the original equations for y.

I carefully distinguish between the types of systems (e.g. inconsistent) and the solution sets of systems (e.g. empty set).

Background Information for the Exploration That Follows

The following 15-minute exploration has students discover what happens when we use substitution to solve an inconsistent system or a dependent system. Although groups may give correct responses to the question "What happens?" in parts 1(b) and 2(b), they may still not get the entire point of this exploration. For example, if a group's result is $0 = 0$ for part 2(b), they may not leap to the generalization that dependent systems will result in true statements of the form $a = a$. Parts 1(c) and 2(c) are meant to have students reflect on their responses to parts 1(b) and 2(b) with a wider scope, but groups may not think of these types of generalizations, especially in part 2(c).

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EXPLORATION *Solving dependent, inconsistent, and one-point solution systems*

1. **a.** Is the following system a dependent system, an inconsistent system, or a one-solution system? Explain.

$$y = 3x - 4$$
$$y = 3x - 5$$

b. Use substitution to solve the system. What happens?

c. Will this always happen with inconsistent systems? Test your theory.

d. What happens when you use "intersect" on a graphing calculator to try to solve the system?

2. **a.** Is the following system a dependent system, an inconsistent system, or a one-solution system? Explain.

$$y = 2x + 1$$
$$y = 2x + 1$$

b. Use substitution to solve the system. What happens?

c. Will this always happen with dependent systems? Test your theory.

d. What happens when you use "intersect" on a graphing calculator to try to solve the system?

3. Summarize your findings. What will happen when you use substitution to solve an inconsistent system? a dependent system? a one-solution system?

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SECTION 6.3 LECTURE NOTES

Objectives

1. Use elimination to solve a system of two linear equations.
2. Solve inconsistent systems and dependent systems by elimination.
3. Compare three ways to solve a system.

Main point: Use elimination to solve systems of two linear equations.

OBJECTIVE 1

If we add the left sides and add the right sides of the equations $2 = 2$ and $4 = 4$, we obtain the true statement $2 + 4 = 2 + 4$.

Adding Left Sides and Right Sides of Two Equations

If $a = b$ and $c = d$, then

$$a + c = b + d$$

Solve the system by elimination.

1.

$$\begin{aligned} 5x - 2y &= -16 \\ -5x + 4y &= 22 \end{aligned}$$

2.

$$\begin{aligned} 4x - 3y &= 5 \\ 3x + y &= 7 \end{aligned}$$

3.

$$\begin{aligned} 4x - 3y &= 13 \\ 7x + 5y &= -8 \end{aligned}$$

4.

$$\begin{aligned} 2x - 3y &= 7 \\ 5x - 7y &= 17 \end{aligned}$$

5.

$$\begin{aligned} \frac{1}{3}x - \frac{1}{2}y &= \frac{5}{6} \\ 4x - 7y &= 11 \end{aligned}$$

6.

$$\begin{aligned} 2(4x - 3) - 3(2y - 1) &= -25 \\ 3(2x + 5) - 4(3y + 2) &= -17 \end{aligned}$$

Using Elimination to Solve a Linear System

To use **elimination** to solve a system of two linear equations,

1. Use the multiplication property of equality (Section 4.3) to get the coefficients of one variable to be equal in absolute value and opposite in sign.
2. Add the left sides and add the right sides of the equations to eliminate one of the variables.
3. Solve the equation in one variable found in step 2.
4. Substitute the solution found in step 3 into one of the original equations, and solve for the other variable.

Use elimination to solve the system, with coordinates of solutions rounded to the second decimal place.

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7.

$$y = 2.18x - 9.66$$
$$y = -4.57x + 8.05$$

8.

$$y = -1.72x + 5.34$$
$$y = -2.39x + 8.76$$

Solve the system by elimination.

9.

$$\frac{1}{5}x - \frac{1}{2}y = \frac{2}{5}$$
$$4x - 3y = -6$$

10.

$$\frac{1}{5}x + \frac{3}{2}y = 7$$
$$\frac{2}{5}x - \frac{9}{2}y = -16$$

OBJECTIVE 2

Solving Inconsistent Systems and Dependent Systems by Elimination

If the result of applying elimination to a linear system of two equations is

- a false statement, the system is inconsistent; that is, the solution set is the empty set.
- a true statement that can be put into the form $a = a$, then the system is dependent; that is, the solution set is the set of ordered pairs represented by every point on the (same) line.

Solve the system by elimination. If the system is inconsistent or dependent, say so.

11. [dependent]

$$-4x + 6y = -10$$
$$6x - 9y = 15$$

12. [inconsistent]

$$6x + 2y = 6$$
$$-15x - 5y = -10$$

13. [inconsistent]

$$4x - 2y = 8$$
$$10x - 5y = 17$$

14. [dependent]

$$-4x + 12y = -8$$
$$5x - 15y = 10$$

OBJECTIVE 3

15. Solve the system of equations three times, once by each of the three methods—graphing by hand, substitution, and elimination. Decide which method you prefer for this system. Explain.

$$4x - y = 8$$
$$5x - 3y = 3$$

SHORT HW 5, 11, 21, 23, 29, 35, 37, 41, 45, 53, 59, 61, 65, 67, 71

MEDIUM HW 1, 5, 11, 19, 21, 23, 29, 31, 35, 37, 41, 45, 53, 57, 59, 61, 65, 67, 69, 71, 73, 79, 81, 83

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SECTION 6.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use elimination to solve systems of linear equations. They will use this skill in Sections 6.4 and 6.5.

OBJECTIVE 1 COMMENTS

While solving the system in Problem 1, I emphasize that things worked out so nicely because the coefficients of x in the original system are equal in absolute value and opposite in sign.

For Problem 2, I ask my students how we can make the coefficients of y to be equal in absolute value and opposite in sign.

I compare solving the system in Problem 3 by eliminating y with finding the sum $\frac{1}{3}+\frac{1}{5}$ (both involve finding the LCM of 3 and 5).

For Problems 7 and 8, some students are surprised that they can use elimination to solve the systems. These problems can be good preparation for working with systems in Section 6.4; however, most students will prefer to use substitution in Section 6.4.

Most students have difficulties solving the systems in Problems 9 and 10.

Background Information for the Exploration That Follows

In the following 20-minute exploration, students practice using elimination and reflect on issues related to using elimination and substitution.

EXPLORATION *Solving systems using elimination*

1. Consider the system

$$x-3y=2$$
$$2x+5y=15$$

- **a.** Solve this system using elimination by first getting the coefficients of x to be equal in absolute value and opposite in sign.
- **b.** Verify that your solution to the system is correct by checking that it satisfies both equations in the system.
- **c.** Explain why it is more efficient to eliminate x rather than y in solving this system.
- **d.** Discuss whether it is more convenient to use elimination or substitution to solve this system.

2. Consider the system

$$25x-4y=1$$
$$37x-6y=1$$

- **a.** Solve this system using elimination by first getting the coefficients of y to be equal in absolute value and opposite in sign.
- **b.** Verify that your solution is correct by checking that it satisfies both equations in the system.
- **c.** Discuss which variable would be easiest to eliminate, x or y.
- **d.** Discuss whether it is more efficient to use elimination or substitution to solve this system.

Background Information for the Exploration That Follows

In the following 20-minute exploration, students practice using elimination and reflect on issues related to using elimination and substitution.

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EXPLORATION *Solving systems using elimination*

1. Consider the system

$$x - 3y = 2$$
$$2x + 5y = 15$$

a. Solve this system using elimination by first getting the coefficients of x to be equal in absolute value and opposite in sign.

b. Verify that your solution to the system is correct by checking that it satisfies both equations in the system.

c. Explain why it is more efficient to eliminate x rather than y in solving this system.

d. Discuss whether it is more convenient to use elimination or substitution to solve this system.

2. Consider the system

$$25x - 4y = 1$$
$$37x - 6y = 1$$

a. Solve this system using elimination by first getting the coefficients of y to be equal in absolute value and opposite in sign.

b. Verify that your solution is correct by checking that it satisfies both equations in the system.

c. Discuss which variable would be easiest to eliminate, x or y.

d. Discuss whether it is more efficient to use elimination or substitution to solve this system.

OBJECTIVE 2 COMMENTS

Most students continue to be greatly challenged by dependent systems and inconsistent systems such as those in Problems 11–14. Most students arrive at a true or false statement and are unsure what this means.

Workbook Exploration

The following 15-minute exploration has students discover what happens when using substitution or elimination to solve an inconsistent system or a dependent system.

EXPLORATION *Solving dependent, inconsistent, and one-point solution systems*

1. Is the following system a dependent system, an inconsistent system, or a one-solution system? Explain.

$$y = 2x + 1$$
$$y = 2x - 4$$

a. Solve the system using either elimination or substitution. What happens?

b. Will this always happen with inconsistent systems? Test your theory.

c. What happens when you use "intersect" on a graphing calculator to try to solve the system?

2. Is the following system a dependent system, an inconsistent system, or a one-solution system? Explain.

$$y = 2x - 3$$
$$y = 2x - 3$$

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a. Solve the system using either elimination or substitution. What happens?

b. Will this always happen with dependent systems? Test your theory.

c. What happens when you use "intersect" on a graphing calculator to try to solve the system?

3. Summarize your findings. What will happen when you solve an inconsistent system? a dependent system? a one-solution system?

OBJECTIVE 3 COMMENTS

I emphasize that any linear system of two equations can be solved by graphing, substitution, or elimination, and that all three methods will give the same result.

If I do not have time to use each of the three methods to solve the system in Problem 15, I quickly discuss what's involved with each method and then ask my students which method might be the best to use.

Textbook and Workbook Exploration

The 15-minute "Comparing techniques of solving systems" exploration has students compare the techniques of solving systems. By completing this exploration, students not only reflect on which technique is best suited to solve a given system, but they also see that each technique gives the same result.

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SECTION 6.4 LECTURE NOTES

Objectives

1. Use substitution and elimination to make predictions about situations described by a table of data.
2. Use substitution and elimination to make predictions about situations described by rates of change.

Main point: Use substitution and elimination to make predictions about authentic situations.

OBJECTIVE 1

- In Section 6.1 we used graphing to find the intersection point of two linear models.
- We can also find an intersection point of the graphs of two linear models by substitution or elimination.
- We will solve modeling problems in this section by using substitution or elimination. Then we will use "intersect" on a graphing calculator to verify our work.

1. When solving Problem 7 of the lecture notes of Section 6.1, we worked with the system

$$r = f(t) = -0.27t + 13.79$$
$$r = g(t) = 0.52t + 8.66$$

where $f(t)$ and $g(t)$ are the annual U.S. volume sales (in billions of gallons) of soft drinks and bottled water, respectively, both at t years since 2010 (see the following table).

	U.S. Volume Sales (billions of gallons)	
Year	Soft Drinks	Bottled Water
2010	13.78	8.76
2011	13.54	9.11
2012	13.30	9.67
2013	12.87	10.13
2014	12.75	10.87

Source: *Forbes*

Use substitution or elimination to predict when the annual U.S. volume sales of soft drinks will equal the annual U.S. volume sales of bottled water. What will be that annual volume sales?

2. The numbers of subscribers to Netflix and to major cable providers are shown in the following table for various years.

	Millions of subscribers	
Year	Netflix	Major Cable Providers
2012	23.4	52.6
2013	29.2	51.0
2014	35.7	50.4
2015	41.4	49.2
2016	47.0	49.1

Source: *Netflix, TDG*

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The number (in millions) of subscribers $f(t)$ and $g(t)$ to Netflix and to major cable providers, respectively, are modeled by the system

$$n = f(t) = 5.94t + 11.58$$
$$n = g(t) = -0.88t + 53.98$$

where t is the number of years since 2010. Use substitution or elimination to predict when the number of subscribers to Netflix will equal the number of subscribers to major cable providers. What is that number of subscribers?

OBJECTIVE 2

Recall from Section 3.5 that the slope of a linear model is a rate of change.

3. The number of freshman applications received at Boston University was 41.7 thousand in 2015 and has increased by about 7.7 thousand applications per year (Source: *Boston University*). The number of freshman applications received at University of Michigan Ann Arbor was 51.8 thousand students in 2015 and has increased by 3.0 thousand applications per year (Source: *MLive Media Group*). Estimate when the number of freshman applications received at Boston University was equal to the number of freshman applications received at University of Michigan Ann Arbor. What was that number of applications?

4. The revenue of Fox News Channel increased approximately linearly from \$0.6 billion in 2005 to \$2.0 billion in 2014. The revenue of CNN/HLN increased approximately linearly from \$0.8 billion in 2005 to \$1.2 billion in 2014 (Source: *SNL Kagan*).

a. Let $F(t)$ be the annual revenue of Fox News Channel and $C(t)$ be the annual revenue of CNN/HLN, both in billions of dollars, at t years since 2000. Find equations of F and C.

b. Use substitution or elimination to estimate when the annual revenue of Fox News Channel was equal to the annual revenue of CNN/HLN. What was that revenue?

c. Use a graphing calculator table or graph to verify your work in part (b).

SHORT HW 1, 3, 5, 11, 15, 19, 21, 25

MEDIUM HW 1, 3, 5, 7, 9, 11, 13, 15, 19, 21, 23, 25, 27, 29

SECTION 6.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use substitution or elimination to solve a system of two models to make an estimate or prediction.

OBJECTIVE 1 COMMENTS

Most students prefer to use substitution to solve modeling problems in this section.

For Problem 1, I emphasize that students are to find both the year and the annual volume sales; without such a warning, many students forget to find the annual volume sales.

After showing my students how to do Problem 1, they have an easy time doing Problem 2. In fact, most students can probably do Problem 1 without much help because students have learned all the relevant skills and concepts in previous parts of the text.

If I have time, I have students find the equations of the models. I remind students to use graphing calculator tables or graphs to verify their work.

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Workbook Exploration

You could start class by facilitating the following 20-minute exploration because students already know how to find equations of models and how to solve linear systems using substitution or elimination. Because groups might use different definitions for the explanatory variable, select different pairs of response variables to analyze, and use different methods to solve their systems, the debrief could be an interesting and instructive discussion.

Group Exploration

Section Opener: Forming a question that can be answered by using a system

The percentages of children living with two, one, or no parents are shown in the following table for various years.

	Percentage of Children Who Live with the Following Number of Parents		
Year	Two	One	None
1960	87	9	4
1970	81	14	4
1980	77	19	4
1990	76	20	4
2000	73	22	5
2010	70	25	5
2014	69	26	5

Source: *Pew Research Center*

Form a question that can be solved by solving a system of linear equations. Then respond to the question by finding the system and solving it using substitution or elimination.

Workbook Exploration

The following 25-minute exploration is similar to the "Using a system of equations to model a situation" exploration in Section 6.1, but now students are asked to solve the new linear system by substitution or elimination. And Problems 6 and 7 are a first step toward working with sum functions in Section 7.1.

Group Exploration

Section Opener: Using systems to model data

Per-person daily consumptions of television and the Internet are shown in the following table for various years.

	Per-Person Daily Consumption (minutes)	
Year	Television	Internet
2009	188	48
2011	182	73
2013	176	102
2015	175	128
2017	167	149

Source: *Zenith via Recode*

Let $f(t)$ and $g(t)$ be the per-person daily consumption (in minutes) of television and the Internet, respectively, both at t years since 2000.

1. Find equations of f and g.
2. Estimate the per-person daily consumptions of television and the Internet in 2010.
3. Compare the slopes of your two models. What does your comparison tell you about this situation?

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4. Explain why your work in Problems 2 and 3 suggest there may be a time when the per-person daily consumptions of television and the Internet will be equal.

5. Use substitution or elimination to predict when the per-person daily consumptions of television and the Internet will be equal.

6. Predict the total per-person daily consumption of television and the Internet in 2020.

7. Find the rate of change of the total per-person daily consumption of television and the Internet.

OBJECTIVE 2 COMMENTS

For Problem 3, I remind students that slope is a rate of change. Even though this concept was the main point in Section 3.5, some students will likely have forgotten this concept and how to use it to find an equation of a linear model.

For Problem 4, if you don't have time to find one or both of the models in class, here are the models:

$$F(t) = 0.16t - 0.18$$
$$C(t) = 0.04t + 0.58$$

Textbook and Workbook Exploration

In the 15-minute "Using a difference to make a prediction" exploration, groups discover how to use a difference to solve a system. Even more important, the exploration has students reflect on how a difference of two quantities yields how much larger or smaller one quantity is than the other. This is good preparation for Section 7.1, which uses differences of models to make predictions about authentic situations.

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SECTION 6.5 LECTURE NOTES

Objectives

1. Describe a five-step problem-solving method.
2. Use a system of two linear equations or a linear function to solve perimeter, value, interest, and mixture problems.

Main point: Solve perimeter, value, interest, and mixture problems.

OBJECTIVE 1

Five-Step Problem-Solving Method

To solve some problems in which we want to find two quantities, it is useful to perform the following five steps:

- ***Step 1: Define each variable.*** For each quantity that we are trying to find, we usually define a variable to be that unknown quantity.
- ***Step 2: Write a system of two equations.*** We find a system of two equations by using the variables from step 1. We can usually write each equation either by translating the information stated in the problem into mathematics or by making a substitution into a formula.
- ***Step 3: Solve the system.*** We solve the system of equations from step 2.
- ***Step 4: Describe each result.*** We use a complete sentence to describe the quantities we found.
- ***Step 5: Check.*** We reread the problem and check that the quantities we found agree with the given information.

OBJECTIVE 2

Recall that the formula of the perimeter P of a rectangle with length L and width W is $P = 2L+2W$ (Section 4.6).

1. For a golden rectangle, the length is equal to about 1.62 times the width. If an architect wants to design the base of a building to be a golden rectangle, what are the dimensions of the base if the perimeter is to be 800 feet?

2. A landscaper plans to dig a rectangular garden for which the length is to be 3 feet less than twice the width. If the landscaper has 66 feet of fencing to enclose the garden, what should be the dimensions of the garden?

Why are 7 nickels worth 35 cents? We find the total value of the nickels by multiplying the value of one nickel times the number of nickels.

Total-Value Formula

If n objects each have value v, then their total value T is given by

$$T = vn$$

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3. A 9000-seat amphitheater will sell tickets at \$15 and \$22 for a Jack Johnson concert. How many tickets should be sold at each price for a sellout performance to generate a total revenue of \$156,000?

4. An auditorium has 900 balcony seats and 3500 main-level seats. If tickets for balcony seats will cost \$20 less than tickets for main-level seats, what should the prices be for each type of ticket so that the total revenue from a sell-out performance will be \$149,200?

5. A 6000-seat amphitheater will sell tickets at \$20 and \$35 for a PJ Harvey concert. Let x and y be the number of tickets that will sell for \$20 and \$35, respectively. Assume that the show will sell out.

a. Let $R = f(x)$ be the total revenue (in dollars) from selling the \$20 and \$35 tickets. Find an equation of f.

b. Use a graphing calculator to draw a graph of f for $0 \le x \le 6000$. What is the slope? What does it mean in this situation?

c. How many of each ticket must be sold for the revenue to be \$135,000?

- Money deposited in an account is called the **principal**.
- A person invests money in hopes of later getting back the principal plus additional money called the **interest**.
- The **annual simple interest rate** is the percentage of the principal that equals the interest earned per year.

6. A person plans to invest a total of \$7500. She will invest in both an account at 5% annual interest and an account at 12% annual interest. How much should she invest in each account so that the total interest in one year will be \$515?

7. A person plans to invest twice as much money in an account at 3% annual interest as in an account at 7% annual interest. How much should he invest in each account to earn a total of \$416 in one year?

8. A person plans to invest a total of \$8000 in an account at 2% and an account at 5%. Let x and y be the money (in dollars) invested in the account at 2% and the account at 5%, respectively.

a. Let $I = f(x)$ be the total interest (in dollars) earned from investing the \$8000 for one year. Find an equation of f.

b. Use a graphing calculator to find the values $f(0)$, $f(2000)$, $f(4000)$, $f(6000)$, and $f(8000)$. What do they mean in this situation?

c. Describe the various possible total interest earnings from investing the \$8000 in the accounts for one year.

d. How much should be invested in each account to earn a total of \$220 in one year?

Discuss the meaning of a 30% lime-juice solution.

9. How many quarts each of a 16% antifreeze solution and a 24% antifreeze solution must be mixed to make 8 quarts of a 18% antifreeze solution?

10. How many quarts each of a 8% acid solution and a 20% acid solution must be mixed to make 9 quarts of a 12% acid solution?

11. A chemist needs 6 ounces of a 20% alcohol solution but has only a 24% alcohol solution. How many ounces each of the 24% solution and water should she mix to make the desired 6 ounces of 20% alcohol solution?

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SHORT HW 1, 7, 9, 13, 17, 27, 31, 35, 45, 49, 53, 57, 61

MEDIUM HW 1, 7, 9, 11, 13, 17, 21, 23, 25, 27, 31, 35, 39, 43, 45, 49, 51, 53, 55, 57, 61, 65, 67

SECTION 6.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use a five-step problem-solving method to solve perimeter, value, interest, and mixture problems. Although students will not solve these types of problems in subsequent sections, they will use the five-step problem-solving method to solve area and revenue problems in Sections 8.6, 9.2, and 9.9.

OBJECTIVE 1 COMMENTS

Before solving problems, I quickly go over the five-step problem-solving method so that my students see the big picture. I tell students that they should number their work with the numbers 1 through 5 accordingly. This makes it a lot easier for me to grade their work.

OBJECTIVE 2 COMMENTS

There are two types of value and interest problems, those that use a nonfunction approach (see Problems 3, 4, 6, and 7) and those that use a function approach (see Problems 5 and 8). Due to time constraints, you will likely have to choose between these two approaches. (All perimeter and mixture problems can be solved using a nonfunction approach.) The function approach is more challenging than the nonfunction approach because it has students reflect on the meaning of parameters such as slope. The function approach is more realistic than the nonfunction approach because students are much more likely to use spreadsheets in their careers to consider various outcomes than to solve a system to find a single ordered-pair result. The nonfunction approach may better prepare students to solve similar problems in subsequent courses.

For Problem 1, I tell my students a bit about golden rectangles (see the introduction to Example 1 on page 368 of the textbook).

When solving most traditional word problems, it is extremely helpful to see the underlying arithmetic. For example, discussing the total value of 7 nickels is a great way to introduce the total-value formula.

For Problem 3, it helps to warm up by finding the total value of, say, 3 \$15 tickets and 5 \$22 tickets. This work will suggest the equation

$$15x + 22y = 156,000$$

where x and y are the number of tickets that will sell for \$15 and \$22, respectively. Performing a unit analysis of the equation can increase students' understanding of the concepts and serve as a check. When finding the system

$$\begin{aligned} 15x + 22y &= 156,000 \quad \text{Equation (1)} \\ x + y &= 9000 \quad \text{Equation (2)} \end{aligned}$$

I emphasize that equation (1) describes the value of tickets and equation (2) describes the number of tickets.

For part (b) of Problem 5, students have trouble understanding the meaning of the slope. In particular, they are baffled that the total revenue would decline as the number of \$20 tickets sold increases. It is instructive for students to understand *why* the slope is equal to the difference of the values of a \$20 ticket and a \$35 ticket ($m = 20 - 35 = -15$).

For interest problems, I explain why investors often diversify by investing both in lower-risk and higher-risk accounts whose interest rates are lower and higher, respectively.

For Problem 6, it helps to warm up by finding the total interest from investing, say, \$4000 in the 5% account and investing \$3500 in the 12% account for one year. This work will suggest the equation

$$0.05x + 0.12y = 515$$

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where x and y are the principals of the accounts at 5% and 12%, respectively. When finding the system

$$\begin{aligned} 0.05x + 0.12y &= 515 \quad \text{Equation (1)} \\ x + y &= 7500 \quad \text{Equation (2)} \end{aligned}$$

I emphasize that equation (1) describes the interest amounts and equation (2) describes the principals.

For mixture problems, most students do not know what it means to have a 30% lime-juice solution. I explain that a 100-ounce 30% lime-juice solution is composed of 30 ounces of lime juice and 70 ounces of water.

I explain why equal amounts of a 30% lime-juice solution and a 70% lime-juice solution mixed together would yield a 50% lime-juice solution (and not a 100% lime-juice solution).

Students are usually challenged by mixture problems that involve pure water, so I take my time when going over Problem 11.

Workbook Exploration

The following 15-minute exploration tries to have students see the connection between arithmetic and algebra for value problems.

Group Exploration

Section Opener: Solving a value problem

1. Write an expression for the total revenue from selling x tickets at \$60.
2. Write an expression for the total revenue from selling x tickets at \$60 and y tickets at \$85.
3. Write an expression for the total *number* of tickets sold from selling x tickets at \$65 and y tickets at \$85.
4. A 12,000-seat amphitheater will sell tickets at \$60 and at \$85 for a Thundercat concert. Let x and y be the number of tickets sold for \$60 and \$85, respectively. Assuming the total revenue is \$795,000 for a sellout performance, fill in the following blanks:

$$\begin{aligned} x + y &= _____ \\ 60x + 85y &= _____ \end{aligned}$$

5. Solve the system you found in Problem 4.
6. What does the solution you found in Problem 5 mean in this situation?
7. Perform calculations to check that your response in Problem 6 is correct.

Textbook and Workbook Exploration

The 30-minute "Describing an authentic situation for a given system" exploration is challenging for many groups because they are not used to creating a context.

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SECTION 6.6 LECTURE NOTES

Objectives

1. Graph a *linear inequality in two variables*.
2. Solve a *system of linear inequalities in two variables*.
3. Use a system of linear inequalities in two variables to make estimates.

Main point: Graph *linear inequalities in two variables* and solve *systems of linear inequalities in two variables*.

OBJECTIVE 1

A **linear inequality in two variables** is an inequality of the form $y < mx + b$ or $x < a$ (or with $<$ replaced with $\leq$, $>$, or $\geq$).

Definition *Satisfy, solution, solution set, and solve*

If an inequality in the two variables x and y becomes a true statement when a is substituted for x and b is substituted for y, we say the ordered pair (a, b) **satisfies** the inequality and call (a, b) a **solution** of the inequality. The **solution set** of an inequality is the set of all solutions of the inequality. We **solve** the inequality by finding its solution set.

Graph the inequality by hand.

1. $y \geq \frac{1}{3}x - 2$
2. $y < -2x + 3$
3. $y \leq -\frac{3}{5}x$
4. $2x - 5y > 10$
5. $2(2x + 1) - 3y \geq 5$
6. $y \leq 2$
7. $x > -3$

Graph of an Inequality in Two Variables

- The graph of an inequality of the form $y > mx + b$ is the region above the line $y = mx + b$. The graph of an inequality of the form $y < mx + b$ is the region below the line $y = mx + b$. For either inequality, we use a dashed line to show that $y = mx + b$ is not part of the graph.
- The graph of an inequality of the form $y \geq mx + b$ is the line $y = mx + b$ and the region above that line. The graph of an inequality of the form $y \leq mx + b$ is the line $y = mx + b$ and the region below that line.

Warning: The graph of $2x - 5y > 10$ is not the region *above* the line $2x - 5y = 10$ (see Problem 4).

OBJECTIVE 2

A **system of linear inequalities in two variables** consists of two or more linear inequalities in two variables.

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Here is an example:

$$y > 2x - 5$$
$$y \le \frac{3}{4}x + 1$$

Definition *Solution, solution set, and solve for a system of inequalities in two variables*

We say an ordered pair (a, b) is a **solution** of a system of inequalities in two variables if it satisfies all the inequalities in the system. The **solution set** of a system is the set of all solutions of the system. We **solve** a system by finding its solution set.

We can find the solution set of a system of inequalities in two variables by locating the intersection of the graphs of all the inequalities.

Graph the solution set of the system by hand.

8.

$$y < \frac{2}{3}x + 2$$
$$y \ge -\frac{3}{5}x + 3$$

9.

$$y \ge -2x + 4$$
$$y \ge x - 5$$
$$x \ge 0$$
$$y \ge 0$$

10.

$$y \le 4$$
$$y \ge -2$$

11.

$$4x - 3y > 9$$
$$2x + 5y < 20$$

OBJECTIVE 3

12. The life expectancies of U.S. females at birth and at age 20 yeas are shown in the following table for various calendar years.

	Life Expectancy (years)	
Year	At Birth	At Age 20 Years
1980	77.6	59.0
1985	78.2	59.3
1990	78.3	59.8
1995	78.9	59.9
2000	79.7	60.5
2005	80.4	61.2
2010	81.1	61.7
2015	81.6	62.2

Source: *U.S. National Center for Health Statistics*

Let $L = B(t)$ be the life expectancy (in years) at birth and $L = T(t)$ be the life expectancy (in years) at age 20 years, both of U.S. females at t years since 1980.

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a. Find equations of B and T.

b. Find a system of inequalities that describes the life expectancies of U.S. females from 0 years through 20 years old from 1985 to 2025.

c. Graph the solution set of the system of inequalities you found in part (b).

d. Predict the life expectancies of U.S. females from 0 years through 20 years old in 2020.

SHORT HW 3, 7, 9, 23, 29, 31, 37, 45, 53, 55, 59, 65, 69, 73, 83

MEDIUM HW 3, 7, 9, 13, 23, 29, 31, 37, 45, 53, 55, 59, 63, 65, 67, 69, 71, 73, 81, 83, 91, 93, 97, 99

SECTION 6.6 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students graph linear inequalities in two variables and solve systems of linear inequalities in two variables.

OBJECTIVE 1 COMMENTS

For an inequality such as $2x - 5y > 10$ (see Problem 4), the text uses the approach of first isolating y and then determining whether the points corresponding to the solution set lie above or below the line $y = \frac{2}{5}x - 2$. By this approach, students must think about the meaning of an inequality and the meaning of a graph.

The text does not use the approach of using a test point to determine which half plane describes a solution. This approach does little to reinforce the meaning of an inequality or a graph. However, the text does encourage students to substitute the coordinates of a point to *verify* that the correct half plane has been shaded.

For Problem 4, some students need a reminder that when we multiply or divide both sides of an inequality by a negative number, we reverse the inequality symbol.

Textbook and Workbook Exploration

By completing the 20-minute "Graphing a linear inequality in two variables" exploration, students will have a much sharper understanding of the logic behind graphing linear inequalities in two variables because they will investigate so closely with coordinates of points.

OBJECTIVE 2 COMMENTS

Students tend to memorize (without understanding) that they are looking for the intersection of half planes when solving a system of linear inequalities in two variables. I remind students that when they solved systems of equations in Sections 6.1 through 6.4, they found ordered pairs that satisfied *all* the equations in the system, which are the points that lie on *all* the graphs of the equations. Likewise, an ordered pair is a solution of a system of inequalities if it lies on *all* the graphs of the inequalities.

Workbook Exploration

Most students can successfully follow a procedure to solve a system of linear equalities, but they tend to lose sight of the meaning of the region they shade. The following 20-minute exploration emphasizes that the shaded region indicates the ordered pairs that satisfy all the inequalities in a system.

EXPLORATION *Meaning of a solution of a system of linear inequalities*

1. Graph the inequalities $y \geq \frac{1}{2}x + 3$ and $y < -\frac{5}{2}x + 1$ on the following coordinate system. Also plot the points $(2,\ 5)$, $(4,\ 5)$, $(4,\ 1)$, $(2,\ -4)$, $(-2,\ -3)$, $(-4,\ 1)$, $(-4,\ 5)$, and $(-2,\ 6)$.

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2. Which of the points that you plotted in Problem 1 satisfy $y \geq \frac{1}{2}x+3$? Give two explanations: by performing calculations and by referring to the graph.

3. Which of the points that you plotted in Problem 1 satisfy $y < -\frac{5}{2}x + 1$? Give two explanations: by performing calculations and by referring to the graph.

4. Which of the points that you plotted in Problem 1 satisfy both $y \geq \frac{1}{2}x + 3$ and $y < -\frac{5}{2}x + 1$?

5. Create a system of two linear inequalities such that the ordered pair $(2, -2)$ is a solution of the system but the ordered pairs $(2, 2)$, $(-2, 2)$, and $(-2, -2)$ are not solutions of the system. Also, graph the linear inequalities.

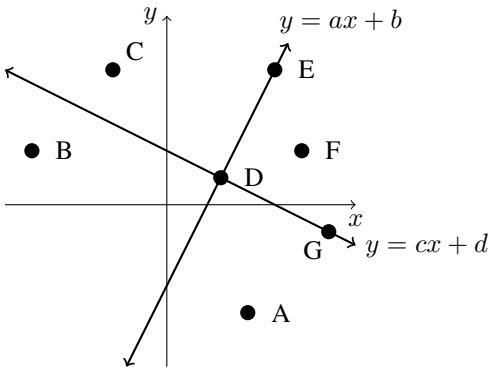
Workbook Exploration

The following 10-minute exploration has students reflect on the connection between the meaning of a graph of an inequality, the meaning of a solution of an inequality, and the meaning of a system of inequalities.

Group Exploration

Meaning of solution of a system of linear inequalities in two variables

The graphs of $y = ax + b$ and $y = cx + d$ are sketched below.



1. For each part, decide which one or more of the points A, B, C, D, E, F, and G represent ordered pairs that

   a. satisfy the inequality $y < ax + b$

   b. satisfy the inequality $y \geq cx + d$

   c. are solutions of the system of inequalities

   $y < ax + b$
   $y \geq cx + d$

2. Write a system of inequalities in terms of a, b, c, d, x, and y such that points A, D, and G are solutions and points B, C, E, and F are not solutions.

3. Write a system of inequalities in terms of a, b, c, d, x, and y such that points A and G are solutions and points B, C, D, E, and F are not solutions.

OBJECTIVE 3 COMMENTS

For part (a) of Problem 12, I find the linear regression equations of f and g to save time.

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CHAPTER 7 OVERVIEW

Students add and subtract polynomials and use sum and difference functions to model authentic situations in Section 7.1. In Section 7.2, students multiply polynomials and use a product functions to model authentic situations. In Section 7.3, students simplify the square of a binomial and find the product of two binomial conjugates. They use properties of exponents to simplify power expressions in Section 7.4, and they divide polynomials in Section 7.5.

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SECTION 7.1 LECTURE NOTES

Objectives

1. Describe the meaning of *term*, *monomial*, *polynomial*, *degree*, *coefficient*, *like terms*, *polynomial function*, *quadratic function*, *parabola*, *vertex*, and *cubic function*.
2. Combine like terms.
3. Add and subtract polynomials.
4. Evaluate polynomial functions.
5. Identify typical graphs of quadratic functions and cubic functions.
6. Find *sum functions* and *difference functions*.
7. Use sum functions and difference functions to describe authentic situations.

Main point: Add and subtract polynomial expressions and functions.

OBJECTIVE 1

- A **term** is a constant, a variable, or a product of a constant and one or more variables raised to powers.
- A **monomial** is a constant, a variable, or a product of a constant and one or more variables raised to *counting-number* powers.
- A **polynomial**, or **polynomial expression**, is a monomial or a sum of monomials.
- We usually write polynomials in one variable so that the exponents of the terms decrease from left to right, which is called **descending order**.
- The **degree of a term** in one variable is the exponent on the variable. The degree of a term in two or more variables is the sum of the exponents on the variables.
- The **degree of a polynomial** is the highest degree of any nonzero term of the polynomial.

Use words such as *linear*, *quadratic*, *cubic*, *polynomial*, *degree*, *one variable*, and *two variables* to describe the expression.

1. $4x^3 - x^2 + 5$ **2.** $-9x^2 + 4x - 7$ **3.** $3p^4q + 7p^2q^2 - 8q^3$

- The **coefficient** of a term is the constant factor of the term.
- The **leading coefficient** of a polynomial is the coefficient of the term with the largest degree.
- **Like terms** are either constant terms or variables terms that contain the same variable(s) raised to exactly the same power(s).
- **Unlike terms** are terms that are not like terms.

OBJECTIVE 2

Using the distributive law to combine like terms: $3x^4 + 5x^4 = (3 + 5)x^4$.

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Combining Like Terms

To combine like terms, add the coefficients of the terms.

Combine like terms.

4. $7x^3 - 8x^2 + 2x^3 + 5x^2$

5. $4a^3b - 7a^2b^2 - 2a^2b^2 - a^3b$

OBJECTIVE 3

Adding Polynomials

To add polynomials, combine like terms.

6. Find the sum $(3p^2 - 5pq + 7q^2) + (-8p^2 + 5pq - 2q^2)$.

We can use the fact $a - b = a - 1b$ to help us subtract two polynomials.

Subtracting Polynomials

To subtract polynomials, first distribute -1 and then combine like terms.

7. Find the difference $(6x^2 - x + 5) - (2x^2 - 7x + 3)$.

OBJECTIVES 4 and 5

A **polynomial function** is a function whose equation can be put into the form $f(x) = P$, where P is a polynomial in terms of the variable x.

Definition *Quadratic function*

A **quadratic function** is a function whose equation can be put into the form

$$f(x) = ax^2 + bx + c$$

where $a \neq 0$. This form is called the **standard form**.

8. For $f(x) = -3x^2 - 2x + 6$, find $f(-2)$.

9. Graph $f(x) = x^2$ by hand.

Definition *Cubic function*

A **cubic function** is a function whose equation can be put into the form

$$f(x) = ax^3 + bx^2 + cx + d$$

where $a \neq 0$.

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10. Graph $f(x) = x^3$ by hand.

OBJECTIVE 6

Definition *Sum function, difference function*

If f and g are functions and x is in the domain of both functions, then we can form the following functions:

- **Sum function $f + g$**, where $(f + g)(x) = f(x) + g(x)$
- **Difference function $f - g$**, where $(f - g)(x) = f(x) - g(x)$

Let $f(x) = 3x^2 - 7x + 2$ and $g(x) = -6x^2 + x - 5$.

11. Find an equation of $f + g$.

12. Find $(f + g)(2)$.

13. Find an equation of $f - g$.

14. Find $(f - g)(2)$.

OBJECTIVE 7

The Meaning of the Sign of a Difference

If a difference $A - B$ is positive, then A is more than B. If a difference $A - B$ is negative, then A is less than B.

15. In Problem 4 of the lecture notes of Section 6.4, we modeled the annual revenues (in billions of dollars) $f(t)$ and $g(t)$ of Fox News Channel and CNN/HLN, by the system

$$F(t) = 0.16t - 0.18$$
$$C(t) = 0.04t + 0.58$$

where t is the number of years since 2000.

a. Find an equation of the sum function $F + C$.

b. Perform a unit analysis of the expression $F(t) + C(t)$.

c. Find $(F + C)(21)$. What does it mean in this situation?

d. Find an equation of the difference function $F - C$.

e. Find $(F - C)(21)$. What does it mean in this situation?

f. Find $(C - F)(21)$. What does it mean in this situation?

SHORT HW 5, 23, 27, 35, 39, 47, 55, 57, 63, 69, 81, 83, 85, 89, 95

MEDIUM HW 5, 15, 23, 27, 31, 35, 39, 43, 47, 55, 57, 63, 69, 81, 83, 85, 87, 89, 95, 97, 99, 103, 105, 107

SECTION 7.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students are introduced to polynomial functions. Students learn to add and subtract polynomials

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and how to find sum and difference functions. Students will use these skills throughout most of the rest of the textbook.

OBJECTIVE 1 COMMENTS

I usually "define" all the polynomial terminology by way of examples. Knowing the definitions will help students communicate with you and other students, and it will help complete Exercises 1–6 of Homework 7.1 as well as many Expressions, Equations, Functions, and Graphs exercises of Chapters 7–15.

OBJECTIVE 2 COMMENTS

Students have an easy time with combining like terms.

Workbook Exploration

Although students can easily combine like terms, they forget that the distributive law is what allows them to do so. The following 10-minute exploration is a good reminder of this.

Group Exploration

Section Opener: Combining like terms

Recall that $2x + 6x$ can be written as one monomial by using the distributive law: $2x + 6x = (2 + 6)x = 8x$.

1. Can the given polynomial be written as one monomial by using the distributive law? If yes, do so. If no, explain why not. [**Hint:** When possible, make sure to show the step that uses the distributive law.]

   a. $3x^4 + 5x^4$

   b. $7x^3y^2 - 5x^3y^2$

   c. $4x^5 + 2x^3$

   d. $5x^4y^2 - 9x^2y^4$

   e. $2x^3y^7 + x^3y^7 + 3x^3y^7$

2. When the sum or difference of two or more monomials can be written as one monomial, we say they are *like terms*. When are monomials like terms? When are they unlike terms?

OBJECTIVE 3 COMMENTS

Although many texts suggest subtracting two polynomials by changing the signs, I find that my students do better by writing a difference of polynomials $P - Q$ as $P - 1Q$ and then distributing the -1.

Textbook and Workbook Exploration

Groups have an easy time with the 15-minute "The degree of a sum of polynomials" exploration. This is a good critical-thinking activity but the content is not essential to the course.

OBJECTIVES 4 and 5 COMMENTS

I make sure to do Problems 9 and 10 as well as to show students typical graphs of quadratic and cubic functions so that students are primed to use their graphing calculators to help them analyze the number of solutions of quadratic and cubic equations in one variable in Section 8.5.

Textbook and Workbook Exploration

Graphing simple quadratic equations in two variables can be introduced by facilitating the 15-minute "Graphing quadratic equations in two variables" exploration. For Problem 4, it's a good idea to remind students that we perform exponentiation before multiplying.

OBJECTIVE 6 COMMENTS

Although my students have an easy time performing the mechanics of finding sum and difference functions, most of them do not realize that they are forming "new" functions from "old" functions.

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Textbook and Workbook Exploration

The 20-minute exploration "Using a difference function to solve a system" exploration draws a connection between a difference function and a system of two equations. This exploration is basically a restatement of the "Using a difference to make a prediction" exploration in Section 6.4. This material will not be directly used in subsequent sections of the course.

OBJECTIVE 7 COMMENTS

For part (e) of Problem 15, most of my students do not know the meaning of $(F-C)(21)$. When comparing parts (e) and (f), it will likely be useful to point out that for any two real numbers a and b, $a-b$ and $b-a$ are opposites: $a-b=-(b-a)$.

Workbook Exploration

The material in this section could be motivated by using the following 10-minute exploration at the start of class. You could extend the exploration having groups predict when there will be the same number of part-time faculty as full-time faculty, which would be a nice reminder of Section 6.4 material.

Group Exploration

Section Opener: Combining polynomials that represent authentic quantities

The numbers of part-time and full-time college instructors are shown in the following table for various years.

	Number of Instructors (thousands)	
Year	Part-Time	Full-Time
1997	421	569
2001	495	618
2005	615	676
2009	710	729
2013	754	791

Source: *U.S. National Center for Education Statistics*

Let n be the number of college instructors (in thousands) at t years since 1990. Reasonable models for part-time instructors and full-time instructors are

$$n = 22.03t + 268.63 \quad \text{Part-time instructors}$$
$$n = 13.88t + 468.48 \quad \text{Full-time instructors}$$

1. Find the sum of the expressions $22.03t + 268.63$ and $13.88t + 468.48$. What does your result represent?
2. Evaluate the result you found in Problem 1 for $t = 31$. What does your result mean in this situation?
3. Find the difference of the expressions $22.03t+268.63$ and $13.88t+468.48$. What does your result represent?
4. Evaluate the result you found in Problem 3 for $t = 31$. What does your result mean in this situation? [**Hint:** The difference $7 - 2 = 5$ means that 7 is 5 more than 2.]

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SECTION 7.2 LECTURE NOTES

Objectives

1. Describe the meaning of *binomial* and *trinomial*.
2. Apply the *product property for exponents*.
3. Multiply monomials.
4. Find the product of a monomial and a polynomial.
5. Multiply polynomials.
6. Find *product functions*.
7. Use product functions to model authentic situations.

Main point: Multiply polynomials and use a product of two polynomials to model an authentic situation.

OBJECTIVE 1

We refer to a polynomial as a **monomial**, a **binomial**, or a **trinomial** depending on whether it has one, two, or three nonzero terms, respectively:

Name	Examples	Meaning
monomial	$5x^3, \quad x^2, \quad -3x^3y, \quad 79$	one nonzero term
binomial	$4x^3+6x, \quad -5x^3-7x, \quad 7y^2-2$	two nonzero terms
trinomial	$-x^3-5x+1, \quad 3x^2-2x+8$	three nonzero terms

OBJECTIVE 2

Show that $x^2 \cdot x^4 = x^{2+4}$:

$$\begin{aligned} x^2 \cdot x^4 &= (x \cdot x)(x \cdot x \cdot x \cdot x) \\ &= x \cdot x \cdot x \cdot x \cdot x \cdot x \\ &= x^6 \end{aligned}$$

Product Property for Exponents

If n and m are counting numbers, then

$$x^m x^n = x^{m+n}$$

For example, $x^3x^5 = x^{3+5} = x^8$. Also, $x^4x = x^4x^1 = x^{4+1} = x^5$.

OBJECTIVE 3

Find the product.

1. $3x(7x^4)$
2. $4x^5(-6x^3)$
3. $-5p^3q(-3p^2q^5)$

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4. $2x^3(7x^5)$

5. $5a^4b^2(-6a^3b^5)$

Warning: $5a^4b^2(-6a^3b^5)$ is a product, not a difference (see Problem 5).

OBJECTIVE 4

Find the product.

6. $-8x(3x-5)$

7. $3wy^2(4w^2-9wy+2y^2)$

OBJECTIVE 5

We can use the distributive law to find $(a+b)(c+d)$:

$$\begin{aligned}(a+b)(c+d) &= a(c+d)+b(c+d)\\ &= ac+ad+bc+bd\end{aligned}$$

Multiplying Two Polynomials

To multiply two polynomials, multiply each term in the first polynomial by each term in the second polynomial. Then combine like terms if possible.

Find the product.

8. $(x+6)(x+2)$

9. $(3m-5n)(7m+2n)$

10. $(4a^2-b^2)(3a^2-2b^2)$

11. $2x(3x^2+5)(4x-3)$

12. $(3x-y)(2x^2+4xy-5)$

13. $(w^2-2w+4)(w^2-5w-2)$

OBJECTIVE 6

Definition *Product function*

If f and g are functions and x is in the domain of both functions, then we can form the **product function** $\boldsymbol{f\cdot g}$:

$$(f\cdot g)=f(x)\cdot g(x)$$

Let $f(x)=2x-3$ and $g(x)=4x-7$.

14. Find an equation of the product function $f\cdot g$.

15. Find $(f\cdot g)(3)$.

OBJECTIVE 7

16. The numbers of F-35 airplanes delivered and their prices are shown in the following table for various years.

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Year	Number of F-35 Airplanes Delivered	Price (millions of dollars)
2012	28	119
2013	34	116
2014	35	111
2015	44	108
2016	45	102
2017	65	96

Source *Lockheed Martin; Pentagon; RBC Capital Markets*

Let $n(t)$ be the number of F-35 airplanes delivered and $p(t)$ be their price (in millions of dollars), both in the year that is t years since 2010. Reasonable models are $n(t) = 1.25t^2 - 4.8t + 34$ and $p(t) = -0.34t^2 - 1.5t + 123$.

a. Find an equation of the product function $n \cdot p$.

b. Perform a unit analysis of the expression $n(t) \cdot p(t)$.

c. Find $(n \cdot p)(11)$. What does it mean in this situation?

SHORT HW 5, 13, 25, 31, 39, 51, 57, 61, 71, 73, 79, 89, 95, 97, 103

MEDIUM HW 1, 5, 13, 25, 31, 39, 43, 51, 57, 61, 71, 73, 79, 89, 93, 95, 97, 99, 103, 105, 107, 113, 115, 117

SECTION 7.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students multiply polynomials and use a product of two polynomials to model an authentic situation. Students will find products of polynomials at many times in the rest of the course.

OBJECTIVE 1 COMMENTS

I point out that we know that the prefixes mono, bi, and tri are used to mean one, two, and three, respectively, for words such as monorail, bicycle, and tricycle.

OBJECTIVE 2 COMMENTS

When discussing the product rule for exponents $x^m x^n = x^{m+n}$, I emphasize that even though we are adding the exponents m and n, we are nonetheless multiplying the powers x^m and x^n. It helps to show the multiplication operation: $x^m \cdot x^n = x^{m+n}$. Many students are unclear on which operation is involved where.

Textbook and Workbook Exploration

Groups have an easy time with the 15-minute "Product property for exponents" exploration. However, this is a useful activity because most students do not understand why exponential properties are true.

OBJECTIVE 3 COMMENTS

The warning about $5a^4b^2(-6a^3b^5)$ just after Problem 5 in the lecture notes is useful because some students do, in fact, think that the expression is a difference.

OBJECTIVES 4 and 5 COMMENTS

When finding products of polynomials, I remind students of the meaning of equivalent expressions. Many students treat the equality symbol as "Here's the next step." To help students understand the meaning of the equality symbol, I choose an expression such as $(x+2)(x+3)$ and show that this expression and the product x^2+5x+6

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both give the same result when evaluated for some real number a. Then I use a graphing calculator table to show that $P(k) = Q(k)$, where $P(x) = (x + 2)(x + 3)$ and $Q(x) = x^2 + 5x + 6$, for various values of k. Finally, I show that the graphs of P and Q are the same and point out that this makes sense because the tables are the same.

I emphasize that students can verify their work by comparing graphing calculator tables or graphs. My students are often reluctant to verify their work, but it can improve their performance on exams and it is worth encouraging them to take advantage of this. To alter my students' perspective on verifying their work on exams, I often ask, "If I gave you an answer key during a test, would you refer to it?"

Workbook Exploration

Unless you are going to do an extremely large number of explorations, the following 20-minute exploration probably should be skipped because the concepts are not needed for the course.

Group Exploration

The degree of a product of polynomials

In this exploration, you will explore the degree of the product of two polynomials.

1. Find the degrees of the given polynomials. Next, multiply the polynomials. Then determine the degree of the product.
   - **a.** $5x + 4$ and $2x - 3$
   - **b.** $2x - 3$ and $x^2 + 2x - 1$
   - **c.** $x^2 + x + 3$ and $x^2 - 2x + 1$
2. What is the degree of the product of a polynomial of degree 1 and a polynomial of degree 1? Explain.
3. What is the degree of the product of a polynomial of degree 1 and a polynomial of degree 2? Explain.
4. What is the degree of the product of a polynomial of degree 2 and a polynomial of degree 2? Explain.
5. Without multiplying the polynomials $2x^8 - 5x^6 + 7x^3 + 8$ and $7x^5 + 2x^4 - 6x + 4$, find the degree of the product of the polynomials. Explain.
6. What is the degree of the product of a polynomial of degree m and a polynomial of degree n? Explain.

OBJECTIVE 6 COMMENTS

When finding a product function, I point out that we are finding a function, not just multiplying two polynomials.

OBJECTIVE 7 COMMENTS

Completing part (b) of Problem 16 should help students determine the meaning of $(n \cdot p)(11)$ in part (c). Students should eventually learn to determine the meaning of an expression such as $(n \cdot p)(11)$ without having a prompt to perform a unit analysis of the expression $n(t) \cdot p(t)$.

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SECTION 7.3 LECTURE NOTES

Objectives

1. Raise a product to a power.
2. Find the power of a monomial.
3. Simplify the square of a binomial.
4. Evaluate quadratic functions, and write them in standard form.
5. Find the product of two binomial conjugates.

Main point: Simplify powers of polynomials and find products of binomial conjugates.

OBJECTIVE 1

We can write $(xy)^4$ as a product of powers:

$$\begin{aligned} (xy)^4 &= (xy)(xy)(xy)(xy) && \text{Write power without exponents.} \\ &= (x \cdot x \cdot x \cdot x)(y \cdot y \cdot y \cdot y) && \text{Rearrange factors.} \\ &= x^4y^4 && \text{Simplify.} \end{aligned}$$

Raising a Product to a Power

If n is a counting number, then

$$(xy)^n = x^n y^n$$

For example, $(xy)^7 = x^7y^7$

OBJECTIVE 2

Perform the indicated operation.

1. $(xy)^5$ **2.** $(2x)^4$ **3.** $(-4p)^3$

Warning: The expression $(3x)^2$ is equivalent to $9x^2$, not $3x^2$.

OBJECTIVE 3

Simplify $(A + B)^2$:

$$\begin{aligned} (A+B)^2 &= (A+B)(A+B) \\ &= A^2 + AB + BA + B^2 \\ &= A^2 + 2AB + B^2 \end{aligned}$$

Squaring a Binomial

$$\begin{aligned} (A+B)^2 &= A^2 + 2AB + B^2 && \text{Square of a sum} \\ (A-B)^2 &= A^2 - 2AB + B^2 && \text{Square of a difference} \end{aligned}$$

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Simplify.

4. $(x+5)^2$
5. $(x-3)^2$
6. $(2k+5)^2$
7. $(3x-6)^2$
8. $(2t+4w)^2$
9. $(2p^2-5q^2)^2$

- A **perfect-square trinomial** is a trinomial equivalent to the square of a binomial (see Problems 4–9).
- Compare $(AB)^2 = A^2B^2$ and $(A+B)^2 = A^2+2AB+B^2$.
- Warning: When simplifying $(A+B)^2$, remember the middle term $2AB$ of $A^2+2AB+B^2$.

OBJECTIVE 4

For $f(x) = x^2 - 4x$, find the following.

10. Find $f(a-5)$
11. Find $f(a+3) - f(a)$

Write the equation in standard form.

12. $y = (x-5)^2 + 3$
13. $y = -2(x+3)^2 - 4$

OBJECTIVE 5

- We say the sum of two terms and the difference of the same two terms are **binomial conjugates**.
- Simplify $(A+B)(A-B)$:

$$\begin{aligned}(A+B)(A-B) &= A^2 - AB + AB - B^2\\ &= A^2 - B^2\end{aligned}$$

Product of Binomial Conjugates

$$(A+B)(A-B) = A^2 - B^2$$

Find the product.

14. $(x+4)(x-4)$
15. $(5x-3)(5x+3)$
16. $(8a^2+6b^2)(8a^2-6b^2)$
17. $(5m-3n)(5m+3n)(25m^2+9n^2)$

Perform the indicated operation.

18. $(3a^2-6ab+2b^2)-(5a^2+3ab-7b^2)$
19. $(2x-5)(3x^2+4x-2)$
20. $(c+7)^2$
21. $(x+5)(x-9)$
22. $5ab^2(-3a^2b)$
23. $(3t^2-5w^2)^2$
24. $(4m^2-2m+8)+(-7m^2-m+3)$
25. $-5xy^2(3x^2-7xy+9y^2)$
26. $(4pw+8q)(4pw-8q)$
27. $(5x^2-4y^2)(3x^2-6y^2)$

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SHORT HW 15, 17, 29, 31, 37, 41, 47, 55, 61, 65, 67, 71, 89, 91, 97

MEDIUM HW 5, 15, 17, 27, 31, 33, 37, 41, 47, 55, 61, 65, 67, 71, 89, 91, 97, 99, 101, 103, 107, 109, 111, 113

SECTION 7.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students simplify powers of polynomials and find the product of two binomial conjugates. Students will use these skills throughout most of the rest of the textbook.

OBJECTIVE 1 COMMENTS

Students have an easy time following the explanation of why $(xy)^4$ is equivalent to x^4y^4.

OBJECTIVE 2 COMMENTS

The warning about $(3x)^2$ is important because many students forget to square the coefficient 3 of the base. This type of error is especially prevalent in more complicated power expressions.

OBJECTIVE 3 COMMENTS

When simplifying polynomials of the form $(A + B)^2$, most of my students omit the middle term $2AB$ of $A^2 + 2AB + B^2$. When simplifying polynomials of the form $(A - B)^2$, many students write $A^2 - B^2$ or $A^2 + B^2$. Because so many students make these types of errors, I do all the following to counteract this behavior:

- For each of the equations $(A + B)^2 = A^2 + B^2$, $(A - B)^2 = A^2 - B^2$, and $(A - B)^2 = A^2 + B^2$, I evaluate both sides of the equation to show that the statement is incorrect.
- I assign many or all of Exercises 89–96 and 99 of Homework 7.3.
- Throughout the rest of the course, when I find the square of a binomial, I underline the middle term several times and remind students to include it.

If I employ these tactics, my students rarely make these types of errors on exams.

For Problems 4–9, I continue to use arrows as shown in the solutions of Examples 2 and 3 on pages 425 and 426 of the textbook until students get the hang of this skill.

Workbook Exploration

On the basis of how frequently students tend to make errors when squaring a binomial, it's a good idea to have groups complete the following 15-minute exploration.

EXPLORATION *Simplifying squares of binomials*

1. Write $(x+3)^2$ without parentheses by first writing it as $(x+3)(x+3)$ and then multiplying pairs of terms. We say you have *simplified* $(x + 3)^2$.
2. Simplify $(A + B)^2$.
3. In Problem 1 you simplified $(x + 3)^2$ by first writing it as $(x + 3)(x + 3)$. Now simplify it by using the formula that you found in Problem 2. Compare your result with the result you found in Problem 1.
4. Use the formula that you found in Problem 2 to simplify $(3x + 4y)^2$.
5. Use the formula that you found in Problem 5 to simplify $(5p - 2w)^2$.

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OBJECTIVE 4 COMMENTS

Just before doing Problem 10, I find $f(2)$ for $f(x) = x^2 - 4x$. This helps students see that to find $f(a-5)$, we must substitute $a-5$ for x in the expression $x^2 - 4x$.

For Problem 13, some of my students distribute the coefficient -2 before simplifying $(x+3)^2$. When simplifying $(x+3)^2$ some students omit the middle term. After simplifying $(x+3)^2$ and then distributing the coefficient -2, some students forget to subtract the 4, the second term of $-2(x+3)^2 - 4$.

Workbook Exploration

In the following 20-minute exploration, students multiply pairs of binomials, investigate whether the equation $(A+B)^2 = A^2 + B^2$ is a true statement, and put quadratic equations of the form $y = a(x-h)^2 + k$ into standard form $y = ax^2 + bx + c$.

EXPLORATION *Working with quadratic expressions and equations*

1. Find the product. Use graphing calculator tables or graphs to verify your result.

a. $(x+2)(x+5)$

b. $(x-3)(x+6)$

2. Two students both attempt to simplify $(x+3)^2$. Did one, both, or neither of these students simplify the expression correctly? If any errors were made, describe them and where they occurred. Also verify your decision by comparing graphing calculator tables or graphs for $y = (x+3)^2$, $y = x^2 + 9$, and $y = x^2 + 6x + 9$.

student #1's work

$$\begin{aligned}(x+3)^2 &= x^2 + 3^2 \\ &= x^2 + 9\end{aligned}$$

student #2's work

$$\begin{aligned}(x+3)^2 &= (x+3)(x+3) \\ &= x^2 + 3x + 3x + 3^2 \\ &= x^2 + 6x + 9\end{aligned}$$

3. Write the equation in standard form $y = ax^2 + bx + c$.

a. $y = (x+3)^2$

b. $y = (x-5)^2$

c. $y = (x-7)^2 + 1$

d. $y = 2(x+4)^2 + 5$

OBJECTIVE 5 COMMENTS

My students do well with this skill. Nonetheless, it doesn't hurt to use arrows as shown in the solution of Example 6 on page 428 of the textbook.

Problems 18-27 address the contents of Sections 7.1–7.3. These types of mixed sets of problems are good practice for exams.

Textbook and Workbook Exploration

The 15-minute "Product of binomial conjugates" exploration helps groups discover how to multiply binomial conjugates. Groups usually have an easy time with this exploration.

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SECTION 7.4 LECTURE NOTES

Objectives

1. Apply properties of exponents.
2. Recognize whether a power expression is simplified.
3. Describe the meaning of the exponent zero.
4. Use combinations of properties of exponents to simplify power expressions.

Main point: Apply the properties of exponents to simplify power expressions.

OBJECTIVES 1 and 2

Recall: If n and m are counting numbers, then

$x^m x^n = x^{m+n}$ Product property for exponents (Section 7.2)

$(xy)^n = x^n y^n$ Raising a product to a power (Section 7.3)

Perform the indicated operations.

1. $x^4 x^8$
2. $(xy)^6$
3. $(3x^3)(-4x^6)$
4. $(5a^4b^2)(3a^5b^4)$
5. $(-3tw)^4$

- A **power expression** is an expression that contains one or more powers.
- Some examples: $4x^6y^2(3x^2y)^2$, $\dfrac{25x^6y^3}{35x^3y}$, $\left(\dfrac{3x^2}{5y}\right)^3$.

Simplifying a Power Expression

A power expression is simplified if

1. It includes no parentheses.
2. In any monomial, each variable or constant appears as a base at most once. For example, for nonzero x, we write x^3x^5 as x^8.
3. Each numerical expression (such as 5^2) has been calculated, and each numerical fraction has been simplified.

$$\frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x}$$ Write quotient without exponents.

$$= \frac{x \cdot x \cdot x}{x \cdot x \cdot x} \cdot \frac{x \cdot x}{1}$$ $\dfrac{ac}{bd} = \dfrac{a}{b} \cdot \dfrac{c}{d}$

$$= 1 \cdot x \cdot x$$ Simplify: $\dfrac{x \cdot x \cdot x}{x \cdot x \cdot x} = 1$.

$$= x^2$$ Simplify.

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This suggests we can subtract the exponents: $\frac{x^5}{x^3} = x^{5-3} = x^2$.

Quotient Property for Exponents

If m and n are counting numbers and x is nonzero, then

$$\frac{x^m}{x^n} = x^{m-n}$$

Simplify.

6. $\frac{6x^7}{9x^4}$ **7.** $\frac{4x^9y^5}{6x^7y^2}$ **8.** $\frac{20a^8b^5}{15a^6b}$

OBJECTIVE 3

Show that $2^4 = 16$, $2^3 = 8$, $2^2 = 4$, $2^1 = 2$ suggests that $2^0 = 1$.

Definition *Zero exponent*

For nonzero x, $x^0 = 1$

So, $7^0 = 1$, $(3xy^2)^0 = 1$, and $\left(\frac{7x}{y}\right)^0 = 1$.

OBJECTIVE 4

$$\begin{aligned}\left(\frac{x}{y}\right)^3 &= \frac{x}{y}\cdot\frac{x}{y}\cdot\frac{x}{y} && \text{Write power expression without exponents.}\\ &= \frac{x\cdot x\cdot x}{y\cdot y\cdot y} && \text{Multiply numerators; multiply denominators.}\\ &= \frac{x^3}{y^3} && \text{Simplify.}\end{aligned}$$

This suggests that we can distribute the exponent 3: $\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$

Raising a Quotient to a Power

If n is a counting number and y is nonzero, then

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

Simplify.

9. $\left(\frac{x}{y}\right)^5$ **10.** $\left(\frac{x}{4}\right)^3$

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$$\begin{aligned}(x^2)^4 &= x^2 \cdot x^2 \cdot x^2 \cdot x^2 && \text{Write expression without exponent 4.}\\ &= x^{2+2+2+2} && \text{Add exponents: } x^m x^n = x^{m+n}.\\ &= x^8 && \text{Simplify.}\end{aligned}$$

This suggests we can multiply the exponents: $(x^2)^4 = x^{2\cdot 4} = x^8$.

Raising a Power to a Power

If m and n are counting numbers, then

$$(x^m)^n = x^{mn}$$

Simplify.

11. $(x^3)^5$

12. $(b^4)^7$

Sometimes we use more than one property of exponents to simplify an expression.

Simplify.

13. $(2x^2)^3$

14. $(3x^5y^2)^3$

15. $5x^6y(3x^3y^5)^2$

16. $(b^5c^2)^3(b^4c^8)^5$

17. $\dfrac{12x^3x^5}{8x^2}$

18. $\left(\dfrac{x^5}{2y}\right)^4$

19. $\left(\dfrac{4x^7}{3y^5w}\right)^3$

20. $\dfrac{(2ab^5)^4}{8a^3b^9}$

SHORT HW 7, 17, 25, 29, 37, 43, 49, 53, 61, 63, 71, 73, 83, 85, 87

MEDIUM HW 7, 17, 25, 29, 33, 37, 43, 49, 53, 55, 61, 63, 65, 71, 73, 79, 81, 83, 85, 87, 97, 99, 101, 103

SECTION 7.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn some properties of exponents and simplify power expressions. They will use these skills many times in the rest of the course.

OBJECTIVES 1 and 2 COMMENTS

For Problem 3, I discuss why we multiply the coefficients but add the appropriate exponents. For Problem 5, I emphasize that we distribute the exponent 4 to the coefficient -3 as well as to the other factors x and y.

When discussing the boxed statement about "Simplifying a Power Expression," I refer to our work on Problems 1–5.

For Problem 6, to make sure students see how to apply the quotient property of exponents, I write $\dfrac{6x^7}{9x^4}$ as the

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product of two fractions:

$$\begin{aligned}\frac{6x^7}{9x^4} &= \frac{6}{9}\cdot\frac{x^7}{x^4}\\ &= \frac{2}{3}\cdot x^{7-4}\\ &= \frac{2}{3}\cdot x^3\\ &= \frac{2x^3}{3}\end{aligned}$$

Then I discuss how we can simplify $\dfrac{6x^7}{9x^4}$ without writing it as the product of two fractions.

OBJECTIVE 3 COMMENTS

Many students are surprised to learn that it makes sense to define x^0 to be 1 (for nonzero values of x).

Workbook Exploration

The following 5-minute exploration is so quick, it's probably worth doing.

EXPLORATION *Zero exponent definition*

1. Complete the following table with ordered-pair solutions of $y = 2^x$.

x	2^x Before Simplifying	2^x After Simplifying
5	2^5	32
4		
3		
2		
1		

2. Look for a pattern in your completed table that suggests a value for 2^0. Use a calculator to verify your result.

OBJECTIVE 4 COMMENTS

I emphasize that for $x^m x^n$, we add the exponents and that for $(x^m)^n$, we multiply the exponents; students often confuse the product property and the property of raising a power to a power.

Many students are challenged by Problems 13–20. Typical errors include

- applying the properties in the wrong order, sometimes because students do not understand or pay attention to the roles of the parentheses.
- not completely simplifying expressions.
- not distributing exponents to coefficients.
- confusing the product property and the property of raising a power to a power.

Because of these difficulties, I go as quickly as possible over Problems 1–12 and the related concepts so that I have plenty of time to discuss Problems 13–20.

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Textbook and Workbook Exploration

The 20-minute "Section Opener: Properties of exponents" exploration helps groups discover the property of raising a quotient to a power, the property of raising a power to a power, and the quotient property.

Textbook and Workbook Exploration

The 15-minute "Properties of exponents" exploration addresses common student errors.

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SECTION 7.5 LECTURE NOTES

Objectives

1. Divide a polynomial by a monomial.
2. Use long division to divide a polynomial by a binomial.
3. Perform synthetic division.

Main point: Divide a polynomial by a monomial or a binomial and perform synthetic division.

OBJECTIVE 1

Dividing by a Monomial

If A, B, and C are monomials and B is nonzero, then

$$\frac{A+C}{B} = \frac{A}{B} + \frac{C}{B}$$

Warning: When dividing a polynomial by a monomial, make sure you divide *every* term of the polynomial by the monomial.

Divide.

1. $\dfrac{15x^3 - 10x^2 + 20x}{5x}$

2. $\dfrac{6p^4 + 7p^3 - 9p}{3p^2}$

3. $\dfrac{20x^5y^3 - 8x^3y^4 + 6xy^6}{-4x^2y^3}$

OBJECTIVE 2

Begin by reminding students how to perform long division with numbers. Also, review how to check the work.

Perform long division.

4. $\dfrac{3x^2 + 10x + 11}{x + 2}$

5. $\dfrac{6x^3 - 17x^2 + 9x - 13}{2x - 5}$

6. $\dfrac{8p^3 - 27}{2p + 3}$

7. $\dfrac{4x^2 - 24 + 3x^3 - 17x}{x^2 - 5}$

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OBJECTIVE 3
Perform synthetic division.

8. $\dfrac{2x^2 - 11x + 17}{x - 3}$

Warning: To perform synthetic division, the divisor must be of the form $x - a$. Off to the left, we write a for the divisor $x - a$.

9. $\dfrac{x^2 + 2 + 3x^3 - 8x}{x + 2}$

10. $\dfrac{2x^3 + 5x^2 + 7}{x + 3}$

SHORT HW 3, 7, 9, 17, 19, 29, 31, 37, 41, 45, 53, 57, 59, 63, 69

MEDIUM HW 3, 7, 9, 17, 19, 27, 29, 31, 35, 37, 41, 45, 51, 53, 57, 59, 61, 63, 69, 71, 73, 75, 79, 83

SECTION 7.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text
Students divide a polynomial by a monomial or a binomial. They also perform synthetic division. The content of this section will not be needed in subsequent sections, except for the Related Review exercises of Section 12.1, where long division is compared to the technique of factoring the numerator and denominator and then using the property $\dfrac{AB}{AC} = \dfrac{B}{C}$, where $A \neq 0$.

OBJECTIVE 1 COMMENTS
Begin by reminding students that we add fractions with a common denominator by using the property $\dfrac{A}{B} + \dfrac{C}{B} = \dfrac{A + C}{B}$. So, it follows that we can divide a polynomial by a monomial by using the property $\dfrac{A + C}{B} = \dfrac{A}{B} + \dfrac{C}{B}$.

Textbook and Workbook Exploration
Although a few groups may have a bit of trouble with signs and/or exponents in Problems 3 and 4, most groups will have an easy time with the 10-minute exploration "Dividing by a monomial" exploration.

OBJECTIVE 2 COMMENTS
If you haven't taught this course before, you'll be surprised by how many of your students don't recall how to perform long division with numbers. Nevertheless, one quick example sets the stage well for performing long division with polynomials. Students often forget to change the signs before adding terms, so make sure to emphasize this. Although it isn't absolutely necessary to use expressions such as $0x^2$ for missing terms, it helps students line up like terms very well.

Many students tend to think that the answer is just the remainder, so I insist that they write their answer separate from their work.

OBJECTIVE 3 COMMENTS
Point out that synthetic division can only be used when we divide by a binomial of the form $x - a$. In particular, emphasize that the coefficient of x must be 1.

For Problem 8, explain that off to the left side, we write 3 (rather than -3) for the divisor $x - 3$, so that we can then add without having to change signs first. This is a subtle point, so take some time with this.

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CHAPTER 8 OVERVIEW

In Section 8.1, students factor trinomials of the form $x^2 + bx + c$ and factor differences of two squares. In Section 8.2, students factor out the GCF of polynomials and factor polynomials by grouping. They factor trinomials of the form $ax^2 + bx + c$ in Section 8.3. In Section 8.4, students factor sums and differences of cubes and learn a factoring strategy. In Section 8.5, students solve polynomial equations. They use quadratic models to make estimates and predictions in Section 8.6.

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SECTION 8.1 LECTURE NOTES

Objectives

1. Explain why multiplying and *factoring* are reverse processes.
2. Factor a trinomial of the form $x^2 + bx + c$.
3. Describe the meaning of a *prime* polynomial.
4. Factor a difference of two squares.

Main point: Factor polynomials.

OBJECTIVE 1

Find the product $(x + 2)(x + 4)$.

- Writing $x^2 + 6x + 8$ as $(x + 2)(x + 4)$ is called *factoring*.
- We **factor** a polynomial by writing it as a product.
- We say $(x + 2)(x + 4)$ is a **factored polynomial** and that both $(x + 2)$ and $(x + 4)$ are factors of the polynomial.
- Multiplying and factoring are reverse processes.

OBJECTIVE 2

Find the product $(x + p)(x + q)$:

$$\begin{aligned}(x + p)(x + q) &= x^2 + qx + px + pq \\ &= x^2 + px + qx + pq \\ &= x^2 + (p + q)x + pq\end{aligned}$$

In the result, the coefficient of x is the sum of the last terms p and q and the constant term is the product of the last terms p and q.

1. Factor $x^2 + 9x + 20$.

Factoring $x^2 + bx + c$

To factor $x^2 + bx + c$, look for two integers p and q whose product is c and whose sum is b. That is, $pq = c$ and $p + q = b$. If such integers exist, the factored polynomial is

$$(x + p)(x + q)$$

Factor.

2. $x^2 + 11x + 28$

3. $x^2 - 7x + 12$

4. $b^2 - 6b + 9$

5. $x^2 - 5x - 14$

6. $x^2 + 3x - 18$

7. $w^2 - 5w - 24$

8. $p^2 + 7pq + 10q^2$

9. $x^2 - 4xy - 32y^2$

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Factoring $x^2 + bx + c$ with c Positive

- To factor a trinomial of the form $x^2 + bx + c$ with a positive constant term c,
  - * If b is positive, look for *two positive* integers whose product is c and whose sum is b.
  - * If b is negative, look for *two negative* integers whose product is c and whose sum is b.

Factoring $x^2 + bx + c$ with c Negative

To factor a trinomial of the form $x^2 + bx + c$ with a negative constant term c, look for two integers with *different* signs whose product is c and whose sum is b.

OBJECTIVE 3

10. Factor $x^2 + 9x + 21$.

A polynomial that cannot be factored is called **prime**.

OBJECTIVE 4

Difference of Two Squares

$$A^2 - B^2 = (A + B)(A - B)$$

Factor.

11. $x^2 - 9$

12. $x^2 - 81$

13. $4r^2 - 25$

14. $36k^2 - 49$

15. $49x^2 - 64y^2$

16. $81k^4 - 16$

A polynomial of the form $x^2 + k^2$, where $k \neq 0$, is prime. For example, $x^2 + 9$ is prime.

SHORT HW 1, 9, 15, 21, 27, 33, 35, 39, 49, 51, 65, 67, 77, 85, 91, 93

MEDIUM HW 1, 9, 15, 21, 27, 33, 35, 39, 49, 51, 55, 65, 67, 75, 77, 81, 85, 87, 91, 93, 95, 97, 103, 105, 107

SECTION 8.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students factor trinomials of the form $x^2 + bx + c$ and factor differences of squares in this section. They will use these skills many times in the course.

OBJECTIVE 1 COMMENTS

Most students tend to think of factoring as a procedure. Some students do not realize that we factor a polynomial by writing it as a product.

OBJECTIVE 2 COMMENTS

To factor a polynomial of the form $x^2 + bx + c$, I usually do not break this process into cases of signs of b and c.

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It works fine to leave it at the general strategy of trying to find two numbers whose product is c and whose sum is b.

For Problem 8, I first factor $p^2 + 7p + 10$ and then insert the variable q in the appropriate places. I use the same tactic with Problem 9.

Some of my students need yet another reminder that they can verify their work using graphing calculator tables or graphs.

Workbook Exploration

Groups can discover the key idea behind factoring polynomials of the form $x^2 + bx + c$ by completing the following 15-minute exploration. Problem 4 gets across why the patterns discovered in this exploration only hold if the coefficient of x^2 is 1.

Group Exploration

Section Opener: Factoring trinomials

The product $(x + 4)(x + 8)$ is equivalent to $x^2 + 12x + 32$ (try it). Working backward, we write

$$x^2 + 12x + 32 = (x + 4)(x + 8)$$

and we call the process factoring. We say $(x+4)(x+8)$ is a factored polynomial. In general, a factored polynomial is a product of two or more polynomials.

1. Complete the following table.

Factored Polynomial	Last Terms	Product of Factored Polynomial	Coefficient of x	Constant Term
$(x + 3)(x + 4)$	3 and 4	$x^2 + 7x + 12$	7	12
$(x + 5)(x - 3)$	5 and -3	$x^2 + 2x - 15$	2	-15
$(x - 2)(x + 6)$				
$(x - 4)(x - 3)$				

2. For each row of the table you completed, what connection do you notice between the last terms of the factored polynomial and the coefficient of x of the product of the factored polynomial? Explain why this happens.

3. For each row of the table you completed, what connection do you notice between the last terms of the factored polynomial and the constant term of the product of the factored polynomial? Explain why this happens.

4. **a.** Do the observations you have made in this exploration apply to the polynomial $(2x + 3)(x + 4)$? If yes, show that they do. If no, explain why not in terms of how you find the product $(2x + 3)(x + 4)$.

b. Do the observations you have made in this exploration apply to the polynomial $(x + 5)(3x + 4)$? If yes, show that they do. If no, explain why not in terms of how you find the product $(x + 5)(3x + 4)$.

c. Discuss, in general, when your observations apply and when they do not.

Textbook and Workbook Exploration

The 15-minute "Factors of a polynomial and x-intercepts of the graph of a function" exploration can lay a foundation that could be further developed in Section 8.5, which discusses how to solve polynomial equations.

OBJECTIVE 3 COMMENTS

For Problem 10, I ask my students to factor $x^2 + 9x + 21$ without any preamble about prime polynomials. Once

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they determine that we cannot factor the polynomial, I tell them that the polynomial is called *prime*. I then compare the meaning of a prime polynomial with the meaning of a prime number.

OBJECTIVE 4 COMMENTS

Some of my students think that a sum of squares can be factored. To dismantle that belief, I write $x^2 + 9$ on the board and ask how to factor it. Someone usually suggests $(x + 3)(x + 3)$ and we observed that the product is $x^2 + 6x + 9$, not $x^2 + 9$. Someone else usually suggests $(x + 3)(x - 3)$ and we observe that the product is $x^2 - 9$, not $x^2 + 9$. It is about this time that the class concludes that $x^2 + 9$ is prime.

Textbook and Workbook Exploration

Teams can discover how to factor differences of squares by completing the 20-minute "Factoring the difference of two squares" exploration.

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SECTION 8.2 LECTURE NOTES

Objectives

1. Factor out the *greatest common factor (GCF)* of a polynomial.
2. Factor polynomials completely.
3. Factor out the opposite of the GCF of a polynomial.
4. Factor a polynomial by grouping.

Main point: Factor the *greatest common factor (GCF)* of a polynomial and factor a polynomial *by grouping*.

OBJECTIVE 1

Factor.

1. $5x + 10$ **2.** $12x^2 - 20x$ **3.** $15w^3 + 12w^2$

Definition *Greatest common factor*

The **greatest common factor (GCF)** of two or more terms is the monomial with the largest coefficient and the largest degree that is a factor of all the terms.

Factor.

4. $25p^3 + 15p^2$

5. $2x^3 - 6x^2 + 14x$

6. $8a^2b^2 - 12ab^3$

7. $6p^4q + 12p^3q - 9pq$

OBJECTIVE 2

If a result cannot be factored further, it is said to be **factored completely**.

8. $3x^2 - 12$

9. $5t^2 + 15t - 50$

10. $36x^3 - 16x$

11. $-12x^3 + 4x^4 - 40x^2$

12. $2a^4b - 2a^3b^2 - 12a^2b^3$

When factoring a polynomial, always factor it *completely*.

OBJECTIVE 3

How to Factor when the Leading Coefficient Is Negative

When the leading coefficient of a polynomial is negative, we first factor out the opposite of the GCF.

Factor.

13. $-5x^3 + 45x$

14. $-5x^2y + 15xy^2 - 10y^3$

15. $-x^2 + 6x - 9$

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OBJECTIVE 4

- First, factor $ax + 5x$: $ax + 5x = (a + 5)x$.
- Next, use the above work to suggest how to factor $a(b + 3) + 5(b + 3)$:

$$a(b + 3) + 5(b + 3) = (a + 5)(b + 3)$$

- Finally, factor $ab + 3a + 5b + 15$:

$$ab + 3a + 5b + 15 = a(b + 3) + 5(b + 3) = (a + 5)(b + 3)$$

16. Factor $5x^3 - 15x^2 + 2x - 6$.

Factoring by Grouping

For a polynomial with four terms, we **factor by grouping** (if it can be done) by

1. Factoring the first two terms and the last two terms.
2. Factoring out the binomial GCF.

Factor.

17. $12x^3 + 9x^2 - 8x - 6$

18. $20x^3 + 8x^2 - 45x - 18$

19. $3a - 3b - ax + xb$

20. $12x^3 - 21x^2 + 4x - 7$

SHORT HW 5, 15, 17, 23, 33, 43, 49, 53, 59, 69, 77, 81, 85, 87, 93

MEDIUM HW 1, 5, 13, 15, 17, 23, 33, 43, 49, 53, 59, 61, 69, 77, 81, 85, 87, 89, 93, 95, 97, 103, 105, 107

SECTION 8.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students factor out the GCF of a polynomial and factor a polynomial by grouping. Students will use this material many times in the rest of the course.

OBJECTIVE 1 COMMENTS

For Problem 2, I point out that if we only factor out 2, 4, or x, one of the factors of the result can still be (and should be) factored. This motivates factoring out the GCF $4x$, because by doing so, the result is factored completely.

Textbook and Workbook Exploration

In the 15-minute "Factoring out the greatest common factor" exploration, teams discover how to factor out the GCF of a polynomial.

OBJECTIVE 2 COMMENTS

I emphasize that when we factor a polynomial, we always factor it *completely*. Many of my students either do not know or forget to factor a polynomial completely.

OBJECTIVE 3 COMMENTS

When the leading coefficient of a polynomial is negative, many students fail to first factor out the opposite of the

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GCF. Sorting this out with students now will pay dividends not only with the rest of Chapter 8, but also when working with rational expressions in Chapter 12.

For Problem 15, some students do not see that they can factor out -1.

OBJECTIVE 4 COMMENTS

For Problem 18, I remind my students that for the expression $-45x - 18$, we factor out the opposite of the GCF to get

$$4x^2(5x + 2) - 9(5x + 2)$$

before factoring out the common factor $(5x + 2)$. Some students think that the expression

$$4x^2(5x + 2) - 9(5x + 2)$$

is factored completely. I remind these students that we factor a polynomial by writing it as a *product*. Finally, some students forget to consider whether the expression $(4x^2 - 9)(5x + 2)$ is factored completely.

For Problem 20, it does not occur to many of my students to factor out a 1 from the polynomial $4x - 7$ to get $3x^2(4x - 7) + 1(4x - 7)$ before then factoring out the common factor $(4x - 7)$.

Workbook Exploration

Problems 1–3 of the following 15-minute exploration illustrate errors most students tend to make. It's well worth the time to have students work on this.

Group Exploration

Factoring out the GCF and factoring by grouping

For each of Problems 1–4, a student tries to factor a polynomial. If the result is correct, decide whether there is a more efficient way to factor the polynomial. If the result is incorrect, describe any errors and factor the polynomial correctly.

1. $3m^3 - 12m^2n - 36mn^2 = 3m(m^2 - 4mn - 12n^2)$

2. $4x^3y^2 + 8x^2y^3 - 12xy^4 = 2xy^2(2x^2 + 4xy - 6y^2)$

3. $8x^3 - 20x^2 - 18x + 45 = 4x^2(2x - 5) - 9(2x - 5)$

4.
$$\begin{aligned} 4x^2 - 16 &= (2x + 4)(2x - 4) \\ &= 2(x + 2)2(x - 2) \\ &= 4(x + 2)(x - 2) \end{aligned}$$

Textbook and Workbook Exploration

The 10-15 minute "Equivalent expressions and locating errors" exploration may be too tedious for students with all of the calculator-entry work. You could avoid the calculator work and simply ask teams to identify the error in Problem 2 and then do the factoring correctly.

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SECTION 8.3 LECTURE NOTES

Objectives

1. Factor a trinomial by trial and error.
2. Rule out possibilities when factoring by trial and error.
3. Factor a trinomial by grouping.

Main point: Factor trinomials of the form $ax^2 + bx + c$, where $a \neq 1$.

OBJECTIVE 1

When factoring by trial and error, it is extremely helpful to find the product of two binomials in one step.

Factor by trial and error.

1. $3x^2 + 17x + 10$

2. $2x^2 - 5x - 12$

3. $4w^2 - 16w + 15$

4. $9p^2 + 12pq + 4q^2$

Factoring $ax^2 + bx + c$ by Trial and Error

To **factor a trinomial** of the form $ax^2 + bx + c$ **by trial and error,** if the trinomial can be factored as a product of two binomials, then the product of the coefficients of the first terms of the binomials is equal to a and the product of the last terms of the binomials is equal to c. To find the correct factored expression, multiply the possible products and identify the one for which the coefficient of x is b.

OBJECTIVE 2

Recall:

- When factoring a polynomial, if the GCF is not 1, then we first factor out the GCF (or its opposite) and continue factoring if possible.
- Always factor a polynomial *completely*.

Factor.

5. $4x^2 + 18x - 36$

6. $60x^4 - 110x^3 - 350x^2$

Factor by trial and error. Rule out possible factorizations to help speed up the process.

7. $6x^2 - 5x - 4$

8. $8a^3b - 49a^2b^2 + 6ab^3$

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OBJECTIVE 3

Factoring $ax^2 + bx + c$ by Grouping

To **factor a trinomial** of the form $ax^2 + bx + c$ **by grouping** (if it can be done),

1. Find pairs of numbers whose product is ac.
2. Determine which of the pairs of numbers from step 1 has the sum b. Call this pair of numbers m and n.
3. Write the bx term as $mx + nx$:

$$ax^2 + bx + c = ax^2 + mx + nx + c$$

4. Factor $ax^2 + mx + nx + c$ by grouping.

Another name for this technique is the **ac method.**

Factor the trinomial by grouping.

9. $3x^2 + 17x + 10$

10. $2x^2 + 11x + 15$

11. $2x^2 - 5x - 12$

12. $4w^2 - 16w + 15$

13. $3x^2 - 10xy - 8y^2$

14. $9p^2 + 12pq + 4q^2$

15. $12k^3 - 28k^2 + 8k$

16. $20t^3w - 38t^2w + 12tw$

SHORT HW 1, 7, 13, 19, 21, 23, 29, 33, 47, 51, 61, 67, 73, 85, 87, 91

MEDIUM HW 1, 7, 13, 15, 19, 21, 23, 29, 33, 39, 47, 51, 57, 61, 67, 73, 85, 87, 91, 95 101, 103, 105, 107, 109

SECTION 8.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students factor trinomials of the form ax^2+bx+c, where $a \neq 1$, by trial and error and by grouping. Students will use these skills throughout most of the rest of the text.

OBJECTIVE 1 COMMENTS

I remind my students that for trinomials of the form $ax^2 + bx + c$, where $a = 1$, we can use the method discussed in Section 8.1. I emphasize that for trinomials of the form $ax^2 + bx + c$, where $a \neq 1$, we must use one of the two methods discussed in this section.

Due to time constraints, when factoring trinomials of the form $ax^2 + bx + c$, you will likely have to choose between showing students how to factor by trial and error or to factor by grouping. One advantage of using trial and error is that it requires students to find lots of products, and so students will become proficient at finding products. Another advantage is that the procedure naturally reinforces the concept that factoring is the reverse process of expanding, and vice versa. However, most students seem to be more successful at using grouping than using trial and error.

Some students' greatest challenge with using trial and error is that they lack the organizational skills to methodically exhaust all possible factorizations. Students tend to try a few possibilities and then stop or retry possibilities, sometimes without realizing that they're revisiting a possible factorization. So, any type of organizational

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structure you can offer to students will likely greatly improve their success in using this method.

Textbook and Workbook Exploration

The 15-minute "Factoring polynomials" exploration addresses some typical student errors. It's a good idea to have groups work on this exploration because students often make these errors–especially the ones addressed in Problems 2 and 3 of the exploration.

OBJECTIVE 2 COMMENTS

The two most prevalent issues with factoring are that students forget to factor out the GCF first and they forget to factor completely. Probably the best way to convince students to factor out the GCF first is to have them factor a polynomial such as $6x^2 + 30x + 36$, which is much easier to factor if the GCF is factored out first.

I don't spend much time discussing how to rule out possibilities while factoring by trial and error, because I suspect that few of my students will remember to use such a strategy in the midst of hunting down a factorization. However, a brief mention probably helps my stronger students.

OBJECTIVE 3 COMMENTS

For factoring trinomials by grouping, I do not try to explain why the four-step algorithm works, because I do not think many of my students would follow such an explanation. Nonetheless, students are quite successful at using this approach, provided that the coefficients of the polynomial are relatively small.

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SECTION 8.4 LECTURE NOTES

Objectives

1. Factor a sum or difference of two cubes.
2. Apply a factoring strategy.

Main point: Factor a sum or difference of two cubes and learn a factoring strategy.

OBJECTIVE 1

Show that $(A+B)(A^2-AB+B^2)=A^3+B^3$.

Sum or Difference of Two Cubes

$A^3+B^3=(A+B)(A^2-AB+B^2)$	Sum of two cubes.
$A^3-B^3=(A-B)(A^2+AB+B^2)$	Difference of two cubes.

Remind students of the following cubes:

$$2^3=8 \qquad 3^3=27 \qquad 4^3=64 \qquad 5^3=125 \qquad 10^3=1000$$

Factor.

1. x^3+27
2. x^3-8
3. $8w^3+125y^3$
4. $2x^5y-54x^2y^4$
5. $64x^6-1$
6. n^6-p^6

OBJECTIVE 2

Five-Step Factoring Strategy

The five steps that follow can be used to factor many polynomials Steps 2–4 can be applied to the entire polynomial or to a factor of the polynomial.

1. If the leading coefficient is positive and the GCF is not 1, factor out the GCF. If the leading coefficient is negative, factor out the opposite of the GCF.
2. For a binomial, try using one of the properties for the difference of two squares, the sum of two cubes, or the difference of two cubes.
3. For a trinomial of the form ax^2+bx+c,
   a. If $a=1$, try to find two integers whose product is c and whose sum is b.
   b. If $a\neq 1$, try to factor by trial and error or by grouping.
4. For an expression with four terms, try factoring by grouping.
5. Continue applying steps 2–4 until the polynomial is factored completely.

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Factor.

7. $3w^4 - 27w^3 + 60w^2$

8. $x^3 - 27$

9. $4m^2 - 16m + 15$

10. $80x^2 - 45$

11. $3x^3 - 2x^2 - 27x + 18$

12. $40 - x^2 - 3x$

13. $6a^2b - 32ab + 10b$

14. $x^2 - 2x - 48$

15. $25p^2 + 9$

16. $8a^3 + 125b^3$

SHORT HW 1, 11, 17, 21, 25, 33, 45, 49, 65, 67, 75, 83, 89, 93, 95

MEDIUM HW 1, 11, 17, 21, 25, 33, 37, 45, 49, 59, 65, 67, 75, 79, 83, 86, 87, 89, 93, 95, 97, 99, 105, 107, 109

SECTION 8.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students factor sums of cubes and differences of cubes in this section. Students also learn a factoring strategy. They will use these skills throughout most of the rest of the textbook.

OBJECTIVE 1 COMMENTS

It will greatly benefit students to memorize the cubes of 2, 3, 4, 5, and 10. These are the only cubes that students will need to know to complete the exercises of Homework 8.4.

For Problem 1, I write the formula $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$ above the equation $x^3 + 27 = (x + 3)(x^2 - x \cdot 3 + 3^2)$ and draw arrows between the terms (see Example 1 on page 479 of the textbook for a similar use of arrows). I use a similar approach for Problems 2–6 until my students get the idea.

When factoring the sum of two cubes, many of my students want to write the incorrect statement $A^3 + B^3 = (A + B)(A^2 - 2AB + B^2)$. They also tend to write a similar incorrect statement for the difference of two cubes. Some students also have trouble with the signs of the coefficients.

Workbook Exploration

Students have such trouble with factoring sums and differences of cubes, the following exploration might help them keep the big picture in mind. Keep close tabs on groups for Problem 1; if their result is wrong, the point of Problem 2 might be lost on them.

Group Exploration

Section Opener: Factoring a sum and a difference of two cubes

1. Find the product $(A + B)(A^2 - AB + B^2)$.

2. Refer to the result you found in Problem 1 to help you factor $x^3 + y^3$.

3. What is the cube root of 8? Use your result and the result you found in Problem 1 to help you factor $w^3 + 8$.

4. Find the product $(A - B)(A^2 + AB + B^2)$.

5. Refer to the result you found in Problem 4 to help you factor $x^3 - y^3$.

6. What is the cube root of 27? Use your result and the result you found in Problem 4 to help you factor $p^3 - 27$.

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OBJECTIVE 2 COMMENTS

Once my students begin to get comfortable with all the factoring techniques discussed in Chapter 8, they still have trouble recognizing which technique is appropriate for a particular polynomial, and they will often fail to factor the polynomial completely. Spending some time discussing the five-step factoring strategy is a good idea. For each of the Problems 7–16, I walk students through the strategy, asking them lots of questions as I do so. Or, I have groups of students work on the problems and then go over a few of them, referring to the strategy as I do so.

Textbook and Workbook Exploration

Students can reflect on the various factoring strategies by completing the 10-minute "Developing a factoring strategy" exploration.

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SECTION 8.5 LECTURE NOTES

Objectives

1. Apply the *zero factor property*.
2. Use factoring to solve *quadratic equations in one variable*.
3. Find the x-intercept(s) of the graph of a polynomial function.
4. Describe a connection between the x-intercepts of the graph of a function and the solutions of a related equation in one variable.
5. Use factoring to solve *cubic equations in one variable*.
6. Use graphing to solve a polynomial equation in one variable.
7. Compare solving polynomial equations with factoring polynomials.

Main point: Use factoring to solve polynomial equations.

OBJECTIVE 1

A **quadratic equation in one variable** is an equation that can be put into the form $ax^2 + bx + c = 0$, where a, b, and c are constants and $a \neq 0$.

Zero Factor Property

Let A and B be real numbers,

If $AB = 0$, then $A = 0$ or $B = 0$.

OBJECTIVE 2

Solve.

1. $(x - 3)(x + 5) = 0$
2. $x^2 + 3x - 28 = 0$
3. $18x^2 - 8 = 0$
4. $9x^2 = 12x - 4$
5. $5w^2 = 30w + 80$
6. $x^2 - 6x = 35 - 4x$
7. $(p - 4)(p - 3) = 2$
8. $\frac{1}{6}x^2 - \frac{1}{3}x = \frac{4}{3}$

OBJECTIVE 3

Find all x-intercepts.

9. $y = x^2 - 10x + 25$
10. $y = x^2 + 4x - 45$

OBJECTIVE 4

Point out that the x-coordinates of the x-intercepts of the graph of $f(x) = x^2 + 4x - 45$ are the solutions of the equation $f(x) = 0$ (see Problem 10).

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Connection Between x-Intercepts and Solutions

Let f be a function. If k is a real-number solution of the equation $f(x) = 0$, then $(k, 0)$ is an x-intercept of the graph of the function f. Also, if $(k, 0)$ is an x-intercept of the graph of f, then k is a solution of $f(x) = 0$.

Explain why it follows that the solution set of a quadratic equation may contain two real numbers, one real number, or no real numbers.

OBJECTIVE 5

A **cubic equation in one variable** is an equation that can be put into the form

$$ax^3 + bx^2 + cx + d = 0$$

where a, b, c, and d are constants and $a \neq 0$.

Solve.

11. $5x^3 - 30x^2 + 40x = 0$

12. $k^3 + 2k^2 = 4k + 8$

Solving Quadratic or Cubic Equations by Factoring

If an equation can be solved by factoring, we solve it by the following steps:

1. Write the equation so that one side of the equation is equal to zero.
2. Factor the nonzero side of the equation.
3. Apply the zero factor property.
4. Solve each equation that results from using the zero factor property.

13. Find the x-intercepts of the graph of $f(x) = x^3 + 2x^2 - 9x - 18$.

The solution set of a cubic equation in one variable may contain one, two, or three real numbers.

OBJECTIVE 6

The graphs of $y = x^2 - 10x + 28$, $y = 7$, $y = 3$, and $y = 1$ are shown below. Use these graphs to solve the given equation.

14. $x^2 - 10x + 28 = 7$

15. $x^2 - 10x + 28 = 3$

16. $x^2 - 10x + 28 = 1$

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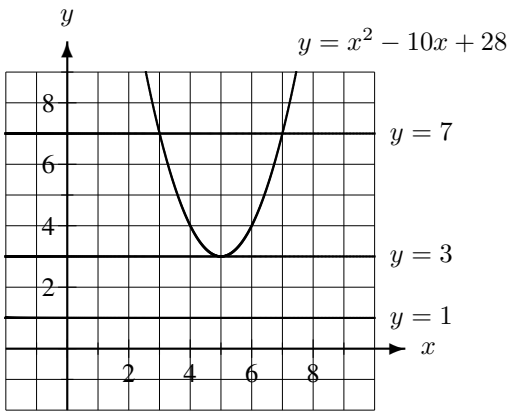

17. Use "intersect" to solve $x^2 - 5x + 1 = x - 3$. Round the solution(s) to the second decimal place.

OBJECTIVE 7

When we solve any equation, our objective is to find any *numbers* that satisfy the equation. When we factor a polynomial, our objective is to write the polynomial as a product of polynomials, which is an *expression.*

18. Solve $x^2 + 5x + 6 = 0$.

19. Factor $x^2 + 5x + 6$.

SHORT HW 5, 13, 19, 21, 33, 41, 47, 51, 57, 63, 69, 77, 79, 83, 87, 93, 99

MEDIUM HW 5, 13, 19, 21, 27, 33, 39, 41, 47, 51, 55, 57, 63, 65, 67, 69, 75, 77, 79, 83, 87, 89, 93, 99, 101, 103, 111, 113, 115

SECTION 8.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students use factoring to solve polynomial equations in this section. They will use this skill for most of the rest of the course. Students also find x-intercepts of the graphs of quadratic and cubic functions and use graphing to solve quadratic and cubic equations in one variable.

OBJECTIVE 1 COMMENTS

I ask my students, "If $AB = 0$, what do we know about A or B?" Students always know the answer.

OBJECTIVE 2 COMMENTS

Most students who are proficient at factoring do well with Problems 1–8.

For Problems 4–8, I say that in order to apply the zero factor property, one side of the equation must be 0.

For Problem 5, some students are baffled when they arrive at the equation $5(w - 8)(w + 2) = 0$. I ask my students "If $5AB = 0$, then what do we know about A or B?" Students then see that either $w - 8$ or $w + 2$ must be 0.

For Problem 7, I remind my students that one side of the equation must be zero in order to apply the zero factor property. Some students think that the solutions are 3 and 4.

For Problem 8, some students need to be reminded to clear each equation of fractions by multiplying by the LCD.

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Textbook and Workbook Exploration
The 15-minute "Zero factor property" exploration is a nice way for groups to discover how to solve quadratic equations.

OBJECTIVE 3 COMMENTS
Some students may need to be reminded that to find x-intercepts, we substitute 0 for y (the other variable) and solve for x.

Workbook Exploration
The following 20-minute exploration is a great way to review finding intercepts (and solving related equations) for all the functions discussed in the course so far. At various times in the semester, it's good practice to pick a theme such as graphing and review that theme for all the functions discussed in the course so far.

Group Exploration
Finding intercepts

Find all intercepts of the graph of the function.

1. $f(x) = 4(2)^x$

2. $f(x) = x^2 - 5x - 36$

3. $f(x) = -3x + 7$

4. $f(x) = 3x^2 + 4x - 4$

5. $f(x) = \log_3(x)$

6. $f(x) = 4x^3 - 12x^2 - 9x + 27$

OBJECTIVE 4 COMMENTS
Referring to problems 9 and 10 will help get across this concept.

OBJECTIVE 5 COMMENTS
I point out that if $ABC = 0$, then $A = 0$, $B = 0$, or $C = 0$.

OBJECTIVE 6 COMMENTS
Although this concept was addressed in Sections 4.3 and 4.4, students will need a refresher. For Problem 17, I point out that we could not use factoring to solve $x^2 - 5x + 1 = x - 3$.

Textbook and Workbook Exploration
In the 15-minute "Finding equations of quadratic functions" exploration, students take another look at the connection between factors of the expression $a(x - b)(x - c)$ and x-intercepts of the graph of the function $y = a(x - b)(x - c)$. The concepts in this exploration have been addressed in preceding explorations and exercises, but this exploration is worth assigning if you want to emphasize these concepts.

OBJECTIVE 7 COMMENTS
Some students confuse solving polynomial equations with factoring polynomials on exams. For example, students will swap the instructions for Problems 18 and 19. Warning students about this issue now will improve their work on polynomial expressions and equations in Chapter 8 as well as their work on rational expressions and equations in Chapter 12.

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SECTION 8.6 LECTURE NOTES

Objectives

1. Use an equation of a quadratic model to make estimates and predictions.
2. Model projectile motion and the area of rectangular objects.

Main point: Use a quadratic model to make estimates and predictions about authentic situations.

OBJECTIVE 1

1. The global average smartphone prices are shown in the following table for various years.

Year	Global Average Smartphone Price (dollars)
2012	378
2013	332
2014	309
2015	302
2016	306
2017	324

Source: *GfK*

Let $f(t)$ be the global average smartphone price (in dollars) at t years since 2012.

a. Use a graphing calculator to draw a scatterplot of the data. Can the data be described better by a linear model or a quadratic model? Explain.

b. Use a graphing calculator to draw the graph of the model $f(t) = 8t^2 - 48t + 370$ and, in the same viewing window, the scatterplot of the data. Does the model fit the data well?

c. Predict the global average smartphone price in 2022.

d. Estimate when the global average smartphone price will be $586.

2. The annual shipments of U.S. craft beers are shown in the following table for various years.

Year	Shipment of U.S. Craft Beers (millions of barrels)
2006	7.0
2007	8.0
2008	8.5
2009	9.0
2010	10.2
2011	11.6
2012	13.0
2013	15.5

Source: *Brewers Association*

Let $s = f(t)$ be the annual shipment (in millions of barrels) of U.S. craft beers at t years since 2005. A reasonable model of the situation is the function $f(t) = \frac{1}{10}t^2 + \frac{1}{5}t + \frac{34}{5}$.

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a. Does the model fit the data well?

b. What is the s-intercept? What does it mean in this situation?

c. Predict the annual shipment in 2022.

d. Predict when the annual shipment will be 18.8 million barrels.

3. A company's profit can be modeled by $g(t) = t^2 - 8t + 20$, where $g(t)$ is the profit (in millions of dollars) for the year that is t years since 2010. Predict when the profit was or will be \$8 million.

OBJECTIVE 2

4. A batter hits a baseball into the air. The height $f(t)$ (in feet) of the baseball after t seconds is given by

$$f(t) = -16t^2 + 80t + 4$$

a. When is the baseball at a height of 68 feet? Explain why there are two such times.

b. When is the baseball at a height of 104 feet? Explain why there is one such time.

c. When is the baseball at a height of 110 feet?

5. A rectangular hallway floor has an area of 64 square feet. If the length is 12 feet more than the width, find the dimensions of the floor.

6. A person has a rectangular garden with a width of 8 feet and a length of 10 feet. He plans to place mulch outside of the garden to form a border of uniform width. If he has just enough mulch to cover 88 square feet of land, determine the width of the border.

SHORT HW 1, 3, 7, 11, 15, 19, 23, 27

MEDIUM HW 1–19 odds, 23, 25, 27, 29, 33, 35, 39

SECTION 8.6 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use quadratic models to make estimates and predictions. They also solve area problems.

OBJECTIVE 1 COMMENTS

Although using factoring to make estimates or predictions when working with a quadratic model is contrived, Problems 1 and 2 and similar exercises of Homework 8.6 are a good way to get students thinking about aspects of quadratic modeling such as constructing scatterplots, evaluating the fit of a model, and deciding for which variable to substitute a value.

For part (d) of Problem 2, sometimes my students need to be reminded to clear the equation

$$18.8 = \frac{1}{10}t^2 + \frac{1}{5}t + \frac{34}{5}$$

of decimals and fractions (by multiplying both sides of the equation by 10).

My students have trouble with some of Exercises 1–10 of Homework 8.6, because the coefficients are fractions or fairly large values.

Textbook and Workbook Exploration

The 25-minute "Modeling with a quadratic function" exploration can give groups much-needed practice with quadratic modeling. You'll have to show them how to use "maximum" on a graphing calculator in Problem 3.

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OBJECTIVE 2 COMMENTS

Problem 4 is a nice opportunity for students to reflect on the connection between the results of the symbolic work and the graph of the model. For part (c), we can deduce that the baseball never reaches a height of 110 feet, because the maximum height is 104 feet [see part (b)].

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CHAPTER 9 OVERVIEW

In Sections 9.1 and 9.2, students graph quadratic functions in vertex form and standard from, respectively. In Section 9.2, they find the maximum or minimum value of a quantity. Students learn to simplify radical expressions in Section 9.3. In Sections 9.4, 9.5, and 9.6, students learn how to solve quadratic equations by extracting roots, completing the square, and using the quadratic formula, respectively. In Section 9.7, students learn how to solve systems of equations in three variables and how to find equations of parabolas. In Section 9.8, they find equations of quadratic models. In Section 9.9, students use a quadratic model to make estimates and predictions. They also solve a system of quadratic models.

Some instructors will chose to include Section 15.2 after Section 9.4. Care must be taken in doing so because some prerequisite concepts are not fully developed until Section 13.3. For example, finding the quotient of two complex numbers follows nicely after having discussed rationalizing the denominator of a radical expression in Section 13.3.

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SECTION 9.1 LECTURE NOTES

Objectives

1. Describe the *vertex form* of a quadratic function.
2. Graph a quadratic function in vertex form.
3. Find the domain and range of a quadratic function.
4. Find a quadratic model in vertex form.

Main point: Graph quadratic functions in vertex form.

OBJECTIVE 1

First remind students of the standard form of a quadratic function and then introduce the vertex form.

Vertex Form of a Quadratic Function

Let $f(t) = a(x-h)^2 + k$, where $a \neq 0$. Then f is a quadratic function, and its graph is a parabola. We say that the equation is in **vertex form**.

Examples: $f(x) = 3(x-7)^2 + 4$ $\quad g(x) = -2(x+4)^2 - 9$ $\quad h(x) = 3x^2 - 5$ $\quad k(x) = x^2$

OBJECTIVE 2

1. Compare the graphs of $f(x) = x^2$, $g(x) = 2x^2$, and $h(x) = 4x^2$.
2. Compare the graph of $f(x) = \frac{1}{2}x^2$ with the graph of $g(x) = -\frac{1}{2}x^2$.

Graphs of Quadratic Functions of the Form $f(x) = ax^2$

For a quadratic function of the form $f(x) = ab^2$,

- The graph is a parabola with vertex $(0, 0)$.
- If $|a|$ is a large number, then the parabola is steep. (It is narrow.)
- If a is near zero, then the parabola is not steep. (It is wide.)
- If $a > 0$, then the parabola opens upward.
- If $a < 0$, then the parabola opens downward.
- The graph of $y = -ax^2$ is the reflection of the graph of $f(x) = ax^2$ across the x-axis.

3. Compare the graphs of $f(x) = x^2$, $g(x) = x^2 + 2$, and $h(x) = x^2 + 4$.
4. Compare the graphs of $f(x) = x^2$, $g(x) = x^2 - 2$, and $h(x) = x^2 - 4$.
5. Compare the graphs of $f(x) = x^2$, $g(x) = (x+2)^2$, and $h(x) = (x+4)^2$.
6. Compare the graphs of $f(x) = x^2$, $g(x) = (x-2)^2$, and $h(x) = (x-4)^2$.

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Three-Step Method of Graphing a Quadratic Function in Vertex Form

To sketch the graph of $f(x) = a(x - h)^2 + k$, where $a \neq 0$.

1. Sketch the graph of $y = ax^2$.
2. Translate the graph from step 1 to the right by h units if $h > 0$ or to the left by $|h|$ units if $h < 0$.
3. Translate the graph from step 2 up by k units if $k > 0$ or down by $|k|$ units if $k < 0$.

Graph the function by hand.

7. $f(x) = -(x + 3)^2$

8. $f(x) = (x - 4)^2 - 3$

9. $f(x) = 3(x + 2) - 6$

10. $f(x) = -2(x - 5) + 3$

Refer to Problems 8–10 to motivate the following property.

Vertex of the Graph of a Quadratic Function

The vertex of the graph of a quadratic function in vertex form, $f(x) = a(x - h)^2 + k$, is the point (h, k).

Find the vertex of the graph of the function.

11. $f(x) = 3(x - 7)^2 + 5$

12. $f(x) = -4(x + 1)^2 - 6$

OBJECTIVE 3

Graph the function by hand, and find the domain and range of the function.

13. $f(x) = 2(x - 2)^2 + 1$

14. $f(x) = -3(x + 4)^2 + 7$

OBJECTIVE 4

15. The total annual revenues of the airline industry are shown in the following table for various years.

Year	Airline Industry Total Revenue (billions of dollars)
2009	476
2010	564
2011	642
2012	706
2013	720
2014	767
2015	721
2016	709

Source: *ICAO; IATA*

Let $f(t)$ be the total annual revenue (in billions of dollars) of the airline industry at t years since 2000.

a. Find an equation of f.

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b. What is the vertex of the graph of f? What does it mean in this situation?

c. Find $f(17)$. What does it mean in this situation?

Finding a Quadratic Model in Vertex Form

To find a quadratic model in vertex form, given some data,

1. Construct a scatterplot of the data.
2. Imagine a parabola that comes close to (or contains) the data points, and select a point (h, k) to be the vertex. Although it is not necessary to select a data point, it is often convenient and satisfactory to do so.
3. Select a nonvertex point (not necessarily a data point) of the parabola, substitute the point's coordinates into the equation $f(t) = a(t - h)^2 + k$, and solve for a.
4. Substitute the result you found for a in step 3 into $f(t) = a(t - h)^2 + k$.

SHORT HW 1, 5, 11, 15, 17, 21, 27, 31, 33, 37, 41, 49, 55, 59, 65

MEDIUM HW 1, 5, 11, 15, 17, 21, 27, 31, 33, 35, 37, 39, 41, 49, 51, 55, 59, 63, 65, 67, 69, 73, 75, 77

SECTION 9.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn how to sketch the graph of a quadratic function in vertex form.

OBJECTIVE 1 COMMENTS

I tell students that both vertex form and standard form have their advantages. For example, it is a little easier to graph quadratic functions in vertex form, whereas it is a little easier to use standard form to find an equation of parabola that contains three given points.

Workbook Exploration

Although the following 15-minute exploration doesn't fall strictly under any of the section's objectives, I've included it here because it's a nice reminder of the meaning of a graph. It also suggests a reasonable way to investigate the graph of a simple, yet unfamiliar, function: find and plot some ordered-pair solutions.

EXPLORATION *Sketching graphs of quadratic functions*

Here you will explore the equation $y = x^2 + 1$.

1. Complete the following table of ordered-pair solutions of the equation $y = x^2 + 1$.

x	y
-2	5
-1	
0	
1	
2	

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2. Use a graphing calculator to draw a scatterplot of the five points in the table. What do you notice about the placement of these points? Find one more solution of the equation $y = x^2 + 1$. Does the graph of this point fit the pattern as well? Does it suggest another solution?

3. Use a graphing calculator to draw the graph of $y = x^2 + 1$ and the scatterplot in the same viewing window. What do you observe?

4. Use a graphing calculator to pick a point that lies on the graph of $y = x^2 + 1$ but is different from the points plotted in the scatterplot. Verify that the ordered pair for this point satisfies the equation $y = x^2 + 1$ and is indeed a solution of the equation.

5. Use a graphing calculator to pick a point that does not lie on the graph of $y = x^2 + 1$. Does the ordered pair for this point satisfy the equation $y = x^2 + 1$?

6. What is the relationship between the equation $y = x^2 + 1$ and its graph? Verify that your theory is true.

OBJECTIVE 2 COMMENTS

Students are quick to pick up on the effect the parameter a has on the graph of $f(x) = ax^2$, but they are satisfied to simply memorize the pattern. I tell them that if they reflect on the reason for the pattern, they will do well on such problems on the final exam, even if they forget the pattern by then.

Of course, students are surprised by what they observe in Problems 5 and 6. At the intermediate algebra level, it is very difficult to explain to students why this happens. For some students, it works to consider the x-intercepts of the functions. For example, in Problem 6, we can tell by inspection that the x-intercepts of the graphs of $f(x) = x^2$, $g(x) = (x - 2)^2$, and $h(x) = (x - 4)^2$, are $(0, 0)$, $(2, 0)$, and $(4, 0)$, which suggests that we translate the graph of f to the right to get the graphs of g and h.

To graph functions of the form $f(x) = a(x-h)^2+k$, students can either carefully sketch the graph of $y = ax^2$ and then translate the graph, or they can they can plot the vertex of f and determine the "width" of the parabola by evaluating the function for a couple of values of x. By considering the symmetry of a parabola, students get two points for the price of one evaluation of f. I require students to find five points of a parabola (including the vertex).

Thinking in terms of symmetry has another benefit. Section 9.2 describes two ways to graph quadratic functions in standard form. One of these methods relies heavily on considering the symmetry of a parabola.

There are several options of how to test students on their ability to graph parabolas. One option is to have students complete the first section of an exam without a graphing calculator. When a student finishes, the student turns in that section and receives the other part of the exam, which allows the use of graphing calculators. A second option is to allow graphing calculators but require students to show work that demonstrates that they can graph parabolas without using a graphing calculator. A third option is to create problems which evaluate students' understanding of concepts related to graphing parabolas, but where a graphing calculator will not be of much help. For example, you could include a problem similar to Exercise 63 of Homework 9.1. A fourth option is to ask to students to graph a function such as $f(x) = 2.8(x - 271)^2 - 349$; this problem evaluates students' understanding of parameters and use of technology.

Textbook and Workbook Exploration

The 25-minute "Significance of a, h, and k for $y = a(x - h)^2 + k$" exploration is a great way to get across the idea of the graphical significance of a, h, and k for quadratic functions in vertex form $y = a(x - h)^2 + k$.

This exploration is similar to the "Graphical significance of m for $y = mx$" exploration in Section 3.3 (see page 148) for linear functions, respectively, so students should already have a sense of what's in store. There is one subtle difference, however. The effects of h and k are to be viewed as translations of the graph of $y = ax^2$, so there is perceived movement involved. In contrast, the role of b for linear functions was most likely viewed by students as the y-intercept, rather than as translating the graph of $y = mx$.

This type of exploration is well-suited for directed-discovery learning. Less directive is given in Problem 4, as groups have had lots of experience with seeing the effects of parameters by now.

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Textbook and Workbook Exploration

Students have fun with the 15-minute "Drawing families of parabolas" exploration which addresses many key concepts of this section. Make sure that students use values of a with absolute value between 0 and 1. Most students tend to use only integer values.

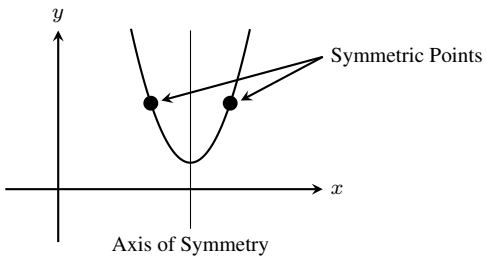
Workbook Exploration

The following 15-minute exploration facilitates students in sketching the graph of a quadratic function in vertex form. Some students may need a reminder about order of operations in order to evaluate the quadratic function.

EXPLORATION *Sketching graphs of quadratic functions in vertex form*

In this exploration, you will use pencil and paper to sketch a graph of the function $f(x) = 2(x + 3)^2 + 1$.

1. What is the vertex of the parabola? Sketch a coordinate system by hand then plot this point.
2. Find another point that lies on the parabola. This point will be referred to as point A. Plot point A.
3. Use the fact that the part of the parabola that lies to the left of the axis of symmetry is the mirror reflection of the part of the parabola that lies to the right of the axis of symmetry to find another point—the reflection of point A—that also lies on the parabola (see the following figure). This point is called the **symmetric point** to point A. Plot this point.



4. Find a fourth point of the parabola. Plot this point and the point symmetric to it.
5. Sketch the parabola that passes through the five points you have plotted.
6. Explain how the sign of the constant a for your equation [thinking of the form $f(x) = a(x - h)^2 + k$] serves as a partial check that you have sketched your parabola correctly.
7. Use a graphing calculator to draw the parabola and verify that your graph is accurate.

OBJECTIVE 3 COMMENTS

Most students easily pick up on finding the domain and range of a quadratic function due to their previous work with domains and ranges of linear functions in Section 5.2. After doing Problems 13 and 14, I have students compare the domain and range of $f(x) = (x - 4)^2 + 1$ with the domains and ranges of $g(x) = 2(x - 4)^2 + 1$ and $h(x) = 3(x - 4)^2 + 1$. Next, I ask students to compare the domain and range of $y = (x - 4)^2 + 1$ with the domain and range of $y = -(x - 4)^2 + 1$.

OBJECTIVE 4 COMMENTS

The method shown in Example 10 of the textbook (see pages 516 and 517) is fairly easy to use, provided that it's convenient to estimate the vertex of a reasonable model by inspecting a scatterplot of the data. This method is also used in Section 9.4. Then in Section 9.8, students will learn how to find quadratic models in standard form using three given points.

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Workbook Exploration

The following 20-minute exploration guides students to apply their understanding of the graphical significance of parameters to help them find an equation of a quadratic model. This is an ideal exploration for instructors who enjoy using an application-driven approach. Students usually find Problem 2 to be challenging. In this problem, most groups will use $h = 1790$ and $k = 3.9$, which actually will yield a good fit, once an appropriate value for a is found through trial and error.

Group Exploration

Finding an equation of a quadratic model

The number of lower skilled worker visa requests are shown in the following table for various years.

Year	Number of Lower Skilled Worker Visa Requests (thousands)
2010	113
2011	97
2012	95
2013	98
2014	108
2015	126
2016	143
2017	165

Source: *U.S. Labor Department*

Let $f(t)$ be the number (in thousands) of lower skilled worker visa requests in the year that is t years since 2010.

1. Use a graphing calculator to draw a scatterplot of the data. Copy the screen.
2. Recall that for the graph of a quadratic function $f(t) = a(t - h)^2 + k$, the vertex is (h, k). Imagine a parabola that comes close to the data points. Estimate the coordinates of the vertex (h, k). Substitute the values of h and k into the equation $f(t) = a(t - h)^2 + k$.
3. Estimate the coordinates of another point that lies on the parabola. Substitute the coordinates into the equation you found in Problem 2 and solve for a.
4. Substitute the value of a you found in Problem 3 into the equation you found in Problem 2. We call the equation a *quadratic model*.
5. Draw your quadratic model and the scatterplot in the same viewing window. Copy the model and the scatterplot. Does the model come close to all the data points? If no, use trial and error to find a better equation.
6. Use your quadratic model to predict the number of lower skilled worker visa requests in 2021.

Workbook Exploration

If you have students work on the following 15-minute exploration at the start of class, groups would likely either use a numerical approach and guess a minimum value or construct a scatterplot and possibly sketch a crude parabola and estimate a minimum value. Groups would be quick to agree that they are not that confident in their estimates; this would motivate the material in this section.

If you have students work on the exploration after most or all the section's material has been discussed, then groups might use a trial and error approach to estimate the coordinates of the vertex of a quadratic model.

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Group Exploration
Estimating a minimum value

The percentages of U.S. consumer debt in non-mortgage borrowing are shown in the following table for various years. For the data in the table, the lowest percent is 21.9%. Estimate the lowest percent for any year between 2003 and 2015, inclusive.

Year	Percent
2003	28.4
2004	25.6
2005	23.8
2006	22.7
2011	21.9
2012	23.1
2013	24.5
2014	25.4
2015	26.8

Source: *New York Federal Reserve*

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SECTION 9.2 LECTURE NOTES

Objectives

1. Graph a quadratic function in standard form by using two symmetric points to find the vertex.
2. Graph a quadratic function in standard form by using the *vertex formula* to find the vertex.
3. Find the *minimum value* or *maximum value* of a quantity.

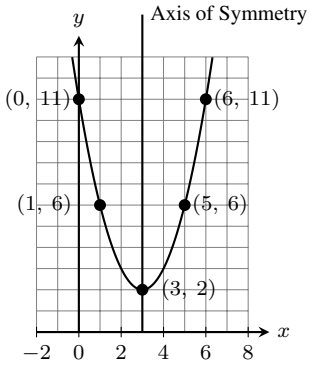
Main point: Graph quadratic functions in standard form.

OBJECTIVE 1

Sketch the following parabola. Then show that the average of the x-coordinates of the given pair of symmetric points is equal to the x-coordinate of the vertex.

1. $(1, 6)$ and $(5, 6)$

2. $(0, 11)$ and $(6, 11)$



Using Two Symmetric Points to Find the Vertex

To find the vertex of a parabola in which (p, s) and (q, s) are symmetric points,

1. Find the x-coordinate by using the formula $x = \dfrac{p+q}{2}$.
2. Find the y-coordinate by evaluating f at the value found in step 1. That is, find $f\left(\dfrac{p+q}{2}\right)$.

In short, the vertex is $\left(\dfrac{p+q}{2}, f\left(\dfrac{p+q}{2}\right)\right)$.

Graph the function by hand.

3. $y = 2x^2 - 8x + 5$ **4.** $y = -x^2 - 6x - 4$ **5.** $y = 1.5x^2 + 5.6x - 1.8$

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Using Symmetric Points to Graph a Quadratic Function in Standard Form (Method 1)

To sketch a graph of a quadratic function $f(x) = ax^2 + bx + c$, where $b \neq 0$,

1. Find the y-intercept.
2. Find the y-intercept's symmetric point.
3. Average the x-coordinates of the two symmetric points to find the x-coordinate of the vertex.
4. Find the y-coordinate of the vertex.
5. Depending on how accurate your sketch is to be, find additional points on the parabola, as needed.
6. Sketch a parabola that contains the points found.

OBJECTIVE 2

Apply Method 1 to the standard form $y = ax^2 + bx + c$ to derive the vertex formula.

Vertex Formula

To find the vertex of the graph of a quadratic function $f(x) = ax^2 + bx + c$,

1. Find the x-coordinate of the vertex by using the **vertex formula** $x = -\dfrac{b}{2a}$.
2. Find the y-coordinate of the vertex by evaluating f at the value found in step 1. That is, find $f\left(-\dfrac{b}{2a}\right)$.

In short, the vertex is $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$.

Use the vertex formula to help graph the function by hand.

6. $y = 2x^2 - 8x + 5$ **7.** $y = -x^2 - 6x - 4$ **8.** $y = 1.5x^2 + 5.6x - 1.8$

OBJECTIVE 3

Maximum or Minimum Value of a Function

For a quadratic function $f(x) = ax^2 + bx + c$ whose graph has vertex (h, k),

- If $a < 0$, then the parabola opens downward and the maximum value of the function is k. [Sketch such a parabola.]
- If $a > 0$, then the parabola opens upward and the minimum value of the function is k. [Sketch such a parabola.]

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9. The percentage of households that have credit cards are shown in the following table for various income groups.

Annual Household Income Group (thousands of dollars)	Income Used to Represent Income Group (thousands of dollars)	Percent of Households That Have Credit Cards
0–25	12.5	41.6
25–49	37	68.4
50–74	62	77.9
75–99	87	93.9
100–124	112	99.1
125–199	162	94.0

Source: *Consumer Payments Research Center*

Let $f(t)$ be the percentage of households that have credit cards for households with income D thousand dollars.

a. Does the model $f(t) = -0.0044D^2 + 1.11D + 29.63$ fit the data well?

b. Use f to estimate at what income is a household most likely to have credit cards. What percentage of these households have credit cards?

10. A person plans to use 100 feet of fencing to enclose a rectangular garden. What dimensions of the rectangle would give the maximum area? What is that area?

SHORT HW 1, 3, 9, 11, 19, 25, 31, 37, 41, 49, 55, 59, 65, 67, 75

MEDIUM HW 1, 3, 9, 11, 19, 23, 25, 31, 37, 41, 45, 49, 51, 55, 59, 65, 67, 75, 77, 79, 81, 83, 85, 87

SECTION 9.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students graph quadratic functions in standard from $y = ax^2 + bx + c$ by hand. The key step to graph a quadratic function in standard form is to find the vertex. This section describes two ways to find the vertex: using symmetry (Method 1) and using the vertex formula (Method 2). Students will graph quadratic functions in Sections 9.7—9.9, but with the aid of technology. Students will also find extrema values for quadratic models in this section. They will use this skill again in Section 9.9.

OBJECTIVE 1 COMMENTS

Students have more trouble with Method 1 than Method 2. However, Method 1 reinforces important concepts such symmetry, solving quadratic equations, and locating the y-intercept. Such concepts are important even in calculus, whereas Method 2 has no importance for calculus students.

Many of my students need a quick review on how to find the average of two numbers. They also need a reminder about the geometrical significance of the average.

To make it easier to grade students' work on exams, I require that students number the first five of six steps when they use Method 1.

It is interesting that the y-intercept's symmetric point can be found for any quadratic function $f(x) = ax^2 + bx + c$, where $b \neq 0$, by solving the simple equation $ax^2 + bx = 0$. Even so, I tend to focus on simpler functions such as those in Problems 3 and 4 and avoid more complicated functions such as the one in Problem 5.

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Workbook Exploration

Students can discover how to find the x-coordinate of the vertex by completing the following 15-minute exploration.

EXPLORATION *Sketching graphs of equations in standard form*

1. First, you will explore a connection between the x-coordinates of two symmetric points and the x-coordinate of the vertex for $f(x) = x^2 - 8x + 12$. Study the following figure and discover the connection. If you see the connection, describe it. If not, complete Table 2.3 so you can see the connection.

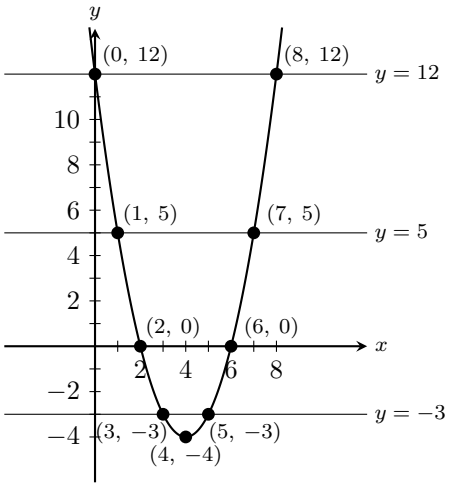

Table 2.3: x-coordinates of Symmetric Points of $f(x) = x^2 - 8x + 12$

Symmetric Points	x-coordinates of Symmetric Points	x-coordinate of Vertex
$(0, 12)$ and $(8, 12)$	0 and 8	4
		4
		4
		4

2. What is the x-coordinate of the vertex of the parabola graphed in Fig. 2.5?

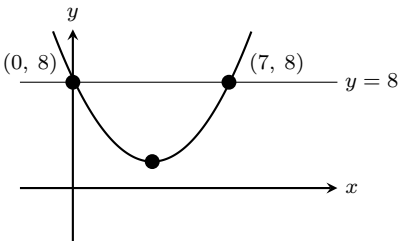

Figure 2.5: Problem 2

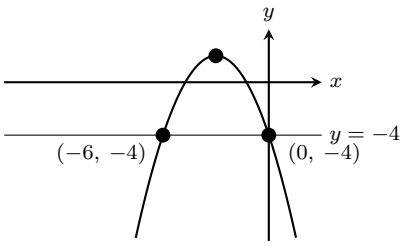

Figure 2.6: Problem 3

3. What is the x-coordinate of the vertex of the parabola graphed in Fig. 2.6?

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Workbook Exploration

The following 15-minute exploration walks students through the process of sketching the graph of a quadratic equation in standard form. It is probably best to show students a couple of examples before they attempt this exploration.

If you do assign this exploration before providing examples, most groups will want help with most of it. For Problem 2, I ask groups to tell me the y-coordinate of the y-intercept and this often helps them focus on what's relevant. For Problem 3, I ask, "How do we *always* find an x-coordinate if we know the y-coordinate?" For Problem 5, I ask, "How do we *always* find a y-coordinate if we know the x-coordinate?"

EXPLORATION *Sketching the graph of a quadratic equation in standard form*

In this exploration, you will sketch by hand the graph of $g(x) = x^2 - 4x + 7$. You will do this by finding the y-intercept, its symmetric point, and then the vertex.

1. Find the y-intercept. Plot this point on a coordinate system.

2. Find the y-coordinate of the symmetric point to the y-intercept. [**Hint**: Symmetric points have the same y-coordinate.]

3. Use your y-coordinate of the symmetric point to find the x-coordinate. [**Hint**: Substitute the y-coordinate of the symmetric point in for y in $g(x) = x^2 - 4x + 7$ and solve for x.] Then plot the symmetric point.

4. Use the connection between the x-coordinates of symmetric points and the x-coordinate of the vertex to find the x-coordinate of the vertex.

5. Use the x-coordinate of the vertex to find the y-coordinate of the vertex. [**Hint**: Substitute the x-coordinate of the vertex for x in $g(x) = x^2 - 4x + 7$.] Then plot the vertex.

6. You now have three points that lie on the parabola. Find one more point that lies on the parabola and its symmetric point. Plot this pair of symmetric points. [**Hint**: Now that you know the vertex, it is easy to find symmetric points.]

7. You now have five points that lie on the parabola. Use these five points to sketch the parabola.

8. Use a graphing calculator to verify your sketch.

OBJECTIVE 2 COMMENTS

After doing Problems 3–5, students have an easy time following the derivation of the vertex formula (see pages 525 and 526 of the textbook). I then do Problem 6 and compare the graph to the graph found in Problem 3. I usually require students to use Method 1 to graph equations such as those in Problems 3 and 4, but allow students to use the vertex formula when finding maximum values and minimum values of quadratic models.

Textbook and Workbook Exploration

The 35-minute "Comparing methods of graphing quadratic functions" exploration is a great review of all graphing techniques included in Sections 9.1 and 9.2. During the debrief, make sure to compare and contrast the techniques. If can't afford 35 minutes, having students complete Problem 1 would still make for a good review.

Workbook Exploration

In the following 10-minute exploration, groups have practice identifying linear and quadratic functions. Weaker students will tend to use two points of a parabola to mistakenly sketch a line.

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Group Exploration
Identifying linear and quadratic functions

Determine whether the graph of the equation is a line or a parabola.

1. $y = 4x^2 + 7$ **2.** $y = 4x + 7$ **3.** $y = 4x^2 - 7x$

4. $y = 4(x + 7)^2$ **5.** $y = 2x^2 - 8x + 3.$ **6.** $y = -2x^2 + 12x - 13.$

OBJECTIVE 3 COMMENTS
For Problem 9, I emphasize that the function has a maximum value because $a < 0$. Students do pretty well with optimization problems in the short run, but often forget what to do by the time the exam rolls around. It helps to remind students to consider finding the vertex when they see words such as "maximum," "largest," "minimum," and "smallest"

Area optimization problems such as Problem 10 will help students prepare for revenue optimization problems in Section 9.9. Both area and revenue optimization problems will be especially good preparation for calculus-bound students.

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SECTION 9.3 LECTURE NOTES

Objectives

1. Describe the meaning of *square root* and *principal square root*.
2. Approximate a principal square root.
3. Apply the *product property for square roots*.
4. Use the product property for square roots to simplify radicals.
5. Apply the *quotient property for square roots*.
6. Simplify a radical expression.
7. *Rationalize the denominator* of a fraction.
8. Simplify a radical expression completely.

Main point: Use properties of square roots to simplify radicals.

OBJECTIVE 1

- A **square root** of a number a is the number we square to get a.
- The square roots of 25 are -5 and 5 because $(-5)^2 = 25$ and $5^2 = 25$.

Definition *Principal square root*

If a is a nonnegative number, then $\sqrt{a}$ is the nonnegative number we square to get a. We call $\sqrt{a}$ the **principal square root** of a.

- For example $\sqrt{49} = 7$ because $7^2 = 49$.
- The symbol "$\sqrt{}$" is called a **radical sign**.
- An expression under a radical sign is called a **radicand**.
- For $\sqrt{4x-7}$, the radicand is $4x - 7$.
- A radical sign together with a radicand is called a **radical**.
- Here are some more radicals: $\sqrt{3}$, $\sqrt{x}$, $\sqrt{2x+7}$.
- An expression that contains a radical is called a **radical expression**.
- Here are some radical expressions: $\sqrt{7}$, $\sqrt{x}$, $\sqrt{3x+1}$, $2\sqrt{x+2} - 9x$, $(4\sqrt{x}+3)(\sqrt{x}-5)$
- A square root of a negative number is not a real number.

Find the square root.

1. $\sqrt{64}$ **2.** $\sqrt{-64}$ **3.** $-\sqrt{64}$ **4.** $-\sqrt{-64}$

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- A **perfect square** is a number whose principal square root is rational.
- Students should memorize the integer perfect squares from 0 to 225.

OBJECTIVE 2

State whether the square root is rational or irrational. If the square root is rational, find the (exact) value. If the square root is irrational, estimate its value by rounding to the second decimal place.

5. $\sqrt{23}$ **6.** $\sqrt{196}$

OBJECTIVE 3

Show students that $\sqrt{4 \cdot 25} = \sqrt{4} \cdot \sqrt{25}$. This equation suggests the following property.

Product Property for Square Roots

For nonnegative numbers a and b,

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

For example, $\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6$.

OBJECTIVE 4

A square root is **simplified** when the radicand does not have any perfect-square factors other than 1.

Simplify.

7. $\sqrt{18}$ **8.** $\sqrt{20}$ **9.** $4\sqrt{50}$ **10.** $3\sqrt{28}$

Simplifying a Square Root with a Small Radicand

To simplify a square root in which it is easy to determine the largest perfect-square factor of the radicand,

1. Write the radicand as the product of the *largest* perfect-square factor and another number.
2. Apply the product property for square roots.

Simplify.

11. $\sqrt{150}$ **12.** $\sqrt{360}$

Simplifying a Square Root with a Large Radicand

To simplify a square root in which it is difficult to determine the largest perfect-square factor of the radicand,

1. Find the prime factorization of the radicand.
2. Look for pairs of identical factors of the radicand and rearrange the factors to highlight them.
3. Use the product property for square roots.

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OBJECTIVE 5

Here we compute the square root of a quotient and a quotient of square roots:

$$\text{Square root of a quotient: } \sqrt{\frac{9}{25}} = \frac{3}{5} \qquad \text{because } \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

$$\text{Quotient of square roots: } \frac{\sqrt{9}}{\sqrt{25}} = \frac{4}{5}$$

Because the two results are equal, we can write

$$\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}}$$

This equation suggests the **quotient property for square roots**.

Quotient Property for Square Roots

For a nonnegative number a and a positive number b,

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

OBJECTIVE 6

If a radical has a fractional radicand, we **simplify the radical** by writing it as an expression whose radicand is not a fraction.

Simplify. Assume that all variables are positive.

13. $\sqrt{\frac{25}{81}}$ **14.** $-\sqrt{\frac{7}{9}}$ **15.** $\sqrt{\frac{12}{25}}$

OBJECTIVE 7

We simplify an expression of the form $\frac{p}{\sqrt{q}}$ by leaving no denominator as a radical expression. We call this process **rationalizing the denominator**.

16. Simplify $\frac{2}{\sqrt{5}}$.

To rationalize the denominator of a fraction of the form $\frac{p}{\sqrt{q}}$, where q is positive, we multiply the fraction by 1 in the form $\frac{\sqrt{q}}{\sqrt{q}}$.

Simplify.

17. $\frac{7}{\sqrt{12}}$ **18.** $\sqrt{\frac{2}{5}}$ **19.** $\sqrt{\frac{5}{18}}$ **20.** $\sqrt{\frac{7}{20}}$

OBJECTIVE 8

Simplify the *radical quotient*.

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21. $\dfrac{8+6\sqrt{7}}{10}$

22. $\dfrac{35-15\sqrt{3}}{45}$

23. $\dfrac{14-\sqrt{12}}{8}$

24. $\dfrac{6+\sqrt{18}}{12}$

Simplifying a Radical Expression

To simplify a radical expression,

1. Use the quotient property for square roots so that no radicand is a fraction.
2. Use the product property for square roots so that no radicands have perfect-square factors other than 1.
3. Rationalize denominators so that no denominator is a radical expression.
4. Use the property $\dfrac{a}{a}=1$, where a is nonzero, to simplify.
5. Continue applying steps 1–4 until the radical expression is simplified completely .

SHORT HW 1, 11, 13, 19, 33, 35, 47, 51, 57, 61, 63, 73, 75, 79, 91, 95

MEDIUM HW 1, 11, 13, 19, 29, 33, 35, 47, 51, 57, 61, 63, 67, 75, 79, 85, 87, 91, 95, 97, 99, 101, 103, 105

SECTION 9.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use the product property for square roots, the quotient property for square roots, and the technique of rationalizing denominators to simplify radicals. Students will use this material in Chapters 13 and 15.

OBJECTIVE 1 COMMENTS

Most students are surprised to learn that a positive number has two square roots.

I tell my students that it is important to know the meanings of *radical*, *radicand*, and *radical sign* because I will use these terminologies frequently in Chapter 13. I also make a big deal about memorizing the integer perfect squares from 0 to 225.

OBJECTIVE 2 COMMENTS

Students have an easy time with this objective.

Workbook Exploration

Students tend to not understand that a square root such as $\sqrt{13}$ is simply a (real) number. The following 15-minute exploration may help such students.

Group Exploration

Section Opener: Estimating square roots

If a is a nonnegative number, then $\sqrt{a}$ is the nonnegative number we square to get a. We call $\sqrt{a}$ the principal square root of a. So, $\sqrt{9}=3$ because $3^2=9$.

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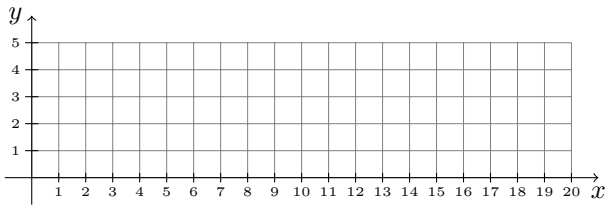
1. Complete the following table.

x	$\sqrt{x}$
0	
1	
4	
9	
16	

2. Graph $y = \sqrt{x}$.



3. Draw arrows on the coordinate system in Problem 2 to show how to find $\sqrt{4}$.

4. Draw arrows on the coordinate system in Problem 2 to help find the two consecutive integers nearest $\sqrt{7}$.

5. Draw arrows on the coordinate system in Problem 2 to help you estimate $\sqrt{13}$. Round your result to the first decimal place.

OBJECTIVE 3 COMMENTS

I tell my students that we will use the product property for square roots frequently in Chapter 13.

Workbook Exploration

The following 10-minute "Product property for square roots" exploration leads groups to discover the product property for square roots and to observe that the statement $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ is not true in general. This is a good exploration to assign, because so many students think that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ is true.

Group Exploration

Section Opener: Product property for square roots

1. If a is a nonnegative number, then $\sqrt{a}$ is the nonnegative number we square to get a. We call $\sqrt{a}$ the principal square root of a. So, $\sqrt{9} = 3$ because $3^2 = 9$. Find the principal square root or the product of principal square roots.

a. $\sqrt{4}$

b. $\sqrt{2 \cdot 8}$

c. $\sqrt{25} \cdot \sqrt{49}$

d. $\sqrt{15}$ [**Hint:** Use a calculator.]

2. Find the principal square root $\sqrt{4 \cdot 9}$ and the product $\sqrt{4} \cdot \sqrt{9}$. Then compare your results.

3. Find the principal square root $\sqrt{4 \cdot 25}$ and the product $\sqrt{4} \cdot \sqrt{25}$. Then compare your results.

4. Write $\sqrt{ab}$ as a product of two principal square roots. [**Hint:** See Problems 2 and 3.] This is called the *product property for square roots*.

5. Use a calculator to estimate $\sqrt{4+9}$. Then find the sum $\sqrt{4} + \sqrt{9}$. Finally, compare your results.

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6. Use a calculator to estimate $\sqrt{16+25}$. Then find the sum $\sqrt{16}+\sqrt{25}$. Finally, compare your results.

7. Is the statement $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$ true for all real numbers a and b? Explain.

OBJECTIVE 4 COMMENTS

For Problem 7, some students try to simplify the radical by writing 18 as $3 \cdot 6$. I point out that nothing is gained by doing so because 3 and 6 are not perfect squares.

OBJECTIVE 5 COMMENTS

I compare the quotient property for square roots with the product property for square roots.

Workbook Exploration

The following 15-minute "Quotient property for square roots" exploration helps groups discover the quotient property and how to use it to simplify radicals. For part (b) of Problem 5 of the exploration, some groups may need to be reminded that $\sqrt{x^2}=x$, where x is nonnegative.

Group Exploration

Section Opener: Quotient property for square roots

1. Find $\sqrt{\frac{4}{9}}$ and $\frac{\sqrt{4}}{\sqrt{9}}$. Then compare your results.

2. Find $\sqrt{\frac{16}{25}}$ and $\frac{\sqrt{16}}{\sqrt{25}}$. Then compare your results.

3. Find $\sqrt{\frac{49}{81}}$ and $\frac{\sqrt{49}}{\sqrt{81}}$. Then compare your results.

4. Write $\sqrt{\frac{a}{b}}$ as a quotient of two square roots. [**Hint:** See Problems 1, 2, and 3.]

5. Write each expression as a radical expression in which no radicand is a fraction. Assume that x is positive.

a. $\sqrt{\frac{5}{16}}$ **b.** $\sqrt{\frac{20}{x^2}}$

OBJECTIVE 6 COMMENTS

For Problem 15, after writing $\sqrt{\frac{12}{25}}=\frac{\sqrt{12}}{\sqrt{25}}$, some students need to be reminded to simplify $\sqrt{12}$.

OBJECTIVE 7 COMMENTS

When rationalizing the denominator of an expression, I emphasize that we can only multiply the expression by a strategic form $\frac{a}{a}$, where $a \neq 0$, of the number 1.

For Problem 19, after applying the quotient property, many students multiply the expression $\frac{\sqrt{5}}{\sqrt{18}}$ by $\frac{\sqrt{18}}{\sqrt{18}}$, when it is probably better to simplify $\frac{\sqrt{5}}{\sqrt{18}}$ to $\frac{\sqrt{5}}{3\sqrt{2}}$ and then multiply this result by $\frac{\sqrt{2}}{\sqrt{2}}$.

OBJECTIVE 8 COMMENTS

I go over Problems 1–20 and related concepts as quickly as possible so I have plenty of time to discuss Problems 21–24 because these problems involve an important skill when using the quadratic formula.

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For Problem 21, I first remind my students that we can simplify $\frac{8}{10}$ because we can write

$$\begin{aligned}\frac{8}{10} &= \frac{2 \cdot 4}{2 \cdot 5} \\ &= \frac{2}{2} \cdot \frac{4}{5} \\ &= 1 \cdot \frac{4}{5} \\ &= \frac{4}{5}\end{aligned}$$

Next, I compare factoring $8 + 6\sqrt{7}$ with factoring $8 + 6x$. Then, I simplify $\frac{8 + 6\sqrt{7}}{10}$:

$$\begin{aligned}\frac{8 + 6\sqrt{7}}{10} &= \frac{2(4 + 3\sqrt{7})}{2(5)} \\ &= \frac{2}{2} \cdot \frac{4 + 3\sqrt{7}}{5} \\ &= 1 \cdot \frac{4 + 3\sqrt{7}}{5} \\ &= \frac{4 + 3\sqrt{7}}{5}\end{aligned}$$

Finally, I discuss how to simplify the radical quotient in fewer steps.

Textbook and Workbook Exploration

The 15-minute "Estimating the value of a radical expression" exploration is not essential to the course. However, it does promote some nice critical thinking and number sense. In fact, it may be the best way to show students that each of the radical quotients in Problems 21–24 is a number. Such a radical quotient looks so strange to some students that they don't realize that it's a number.

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SECTION 9.4 LECTURE NOTES

Objectives

1. Use the *square root property* to solve quadratic equations.
2. Make predictions with a quadratic function in vertex form.
3. Describe the meaning of *pure imaginary number*, *complex number*, and *imaginary number*.

Main point: Use the square root property to solve quadratic equations.

OBJECTIVE 1

Use factoring to show that the solutions of $x^2 = 25$ are ± 5, which can be written as $\pm\sqrt{25}$.

Square Root Property

Let k be a nonnegative constant. Then $x^2 = k$ is equivalent to

$$x = \pm\sqrt{k}$$

Solve.

1. $x^2 = 36$
2. $x^2 = 18$
3. $5t^2 - 3 = 4$
4. $(x - 6)^2 = 3$
5. $(4x + 7)^2 = 12$
6. $\left(p - \frac{7}{3}\right)^2 = \frac{5}{9}$

OBJECTIVE 2

7. The average debts of Americans are shown in the following table for various age groups.

Age Group (years)	Age Used to Represent Age Group (years)	Average Debt (thousands of dollars)
18–29	23.5	8.3
30–39	34.5	16.4
40–49	44.5	19.5
50–59	54.5	21.3
60–69	64.5	16.8
over 69	75.0	7.1

Source: *Experian*

Let $f(t)$ be the average debt (in thousands of dollars) of an American at age t years.

a. Find an equation of f.

b. Estimate the age of Americans whose average debt is \$20 thousand.

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OBJECTIVE 3

- The **imaginary unit**, written i, is the number whose square is -1. That is, $i^2 = -1$ and $i = \sqrt{-1}$.
- If p is a positive number, then $\sqrt{-p} = i\sqrt{p}$.
- If b is a nonzero real number, then we call the expression of the form bi a **pure imaginary number**.

Write the number in bi form, where b is a real number. Simplify the result.

8. $\sqrt{-9}$ **9.** $\sqrt{-50}$ **10.** $-\sqrt{-27}$ **11.** $\sqrt{-\frac{5}{7}}$

- A **complex number** is a number of the form $a + bi$, where a and b are real numbers.
- An **imaginary number** is a number $a + bi$, where a and b are real numbers and $b \neq 0$.

Solve.

12. $x^2 = -49$ **13.** $r^2 = -32$ **14.** $(x+5)^2 = -20$ **15.** $\left(x - \frac{5}{2}\right)^2 = -\frac{3}{4}$

SHORT HW 1, 9, 19, 29, 33, 37, 43, 49, 59, 65, 71, 81, 83, 87, 91

MEDIUM HW 1, 9, 19, 21, 29, 33, 37, 43, 49, 59, 65, 67, 71, 81, 83, 87, 89, 91, 93, 95, 97, 107, 109, 111

SECTION 9.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn how to use the square root property to solve quadratic equations. They will need this skill when they solve quadratic equations by completing the square. Students also learn about complex numbers and imaginary numbers.

OBJECTIVE 1 COMMENTS

Students have an easy time with solving equations of the form $x^2 = k$, but some students have some trouble with solving equations of the form $(mx+b)^2 = k$ such as $(x-6)^2 = 3$ (see Problem 4). To solve $(x-6)^2 = 3$, I write the equation on the board. Then I cover the expression $x - 6$ with my hand and say the equation is now "hand" squared is equal to 3. Next, I ask students what the value of "hand" must be. Many students see that "hand" must be $\pm\sqrt{3}$. Therefore, $x - 6 = \pm\sqrt{3}$.

Many of my students forget to write "$\pm$." I remind my students several times to include this notation while discussing this section and Section 9.5.

Problem 6 will lay the foundation to solve quadratic equations of the form $ax^2 + bx + c = 0$, where $a \neq 1$, by completing the square. After solving the equation, I show my students how to store values in their calculator so that they can check the solutions more easily. Or, students can use graphing calculator tables or graphs to verify their work.

Workbook Exploration

The following 15-minute exploration can be used to introduce using the square root property to solve quadratic equations.

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Group Exploration

Section Opener: Using the square root property to solve an equation

1. Use factoring to solve $x^2 = 9$.
2. Describe how to find the solutions of $x^2 = 9$ in one step.
3. Use factoring to solve $x^2 = 25$.
4. Describe how to find the solutions of $x^2 = 25$ in one step.
5. Find the solutions of $x^2 = 7$ in one step.
6. Find the solutions of $x^2 = k$ in one step, where k is a nonnegative constant. This is called the **square root property**.
7. Use the square root property to solve $2x^2 = 10$.

Textbook and Workbook Exploration

The 15-minute "Deriving a formula for solving quadratic equations in $a(x - h)^2 + k = p$ form" exploration is fairly easy and serves as a nice primer for deriving the quadratic formula, which you could have groups do in the "Deriving a formula for solving quadratic equations of the form $ax^2 + bx + c = 0$" exploration in Section 9.6.

OBJECTIVE 2 COMMENTS

For part (a) of Problem 7, I use the method discussed in Section 9.1 to find an equation of f.

OBJECTIVE 3 COMMENTS

All exercises throughout the textbook that have imaginary solutions will have the instruction "Find all complex-number solutions." This was done to make it easy for professors who do not want to discuss complex numbers to be able to avoid such exercises, which are included in Homework sections 9.4–9.6, 12.5, and 15.2.

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SECTION 9.5 LECTURE NOTES

Objectives

1. Describe the relationship between b and c in a perfect-square trinomial of the form $x^2 + bx + c$.
2. Solve quadratic equations by *completing the square*.
3. Explain why any quadratic equation can be solved by completing the square.

Main point: Solve quadratic equations by completing the square.

OBJECTIVE 1

For the right-hand side of each of the following equations, show that if we divide the coefficient of x by 2 and square the result, we get the constant term.

- $(x + 5)^2 = x^2 + 10x + 25$
- $(x - 3)^2 = x^2 - 6x + 9$
- $(x + k)^2 = x^2 + (2k)x + k^2$

Perfect-Square Trinomial Property

For a perfect-square trinomial of the form $x^2 + bx + c$, dividing b by 2 and squaring the result gives c:

$$x^2 + bx + c$$
$$\left(\frac{b}{2}\right)^2 = c$$

Find the value of c such that the expression is a perfect-square trinomial. Then factor the perfect-square trinomial.

1. $x^2 + 8x + c$ **2.** $x^2 - 18x + c$ **3.** $x^2 - 5x + c$ **4.** $x^2 + \frac{3}{7}x + c$

OBJECTIVE 2

Solve.

5. $x^2 + 10x = -3$ **6.** $x^2 - 14x + 41 = 0$ **7.** $x^2 - 3x - 2 = 0$ **8.** $x^2 - \frac{3}{5}x = \frac{1}{5}$

- The perfect-square trinomial property given for $ax^2 + bx + c$, where $a = 1$, does not extend to trinomials with $a \neq 1$.
- For example, consider simplifying $(2x + 3)^2$:

$$(2x + 3)^2 = 4x^2 + 12x + 9$$

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Dividing $b = 12$ by 2 and squaring the result does *not* give 9:

$$\left(\frac{12}{2}\right)^2 = 6^2 = 36 \neq 9$$

When solving a quadratic equation of the form $ax^2 + bx + c = 0$ with $a \neq 1$, we can first divide both sides by a to obtain an equation involving "$1x^2$"–an equation to which we *can* apply the property.

Solve.

9. $3x^2 - 18x = -5$

10. $2x^2 + 5x - 1 = 0$

11. $5x^2 - 3x = 4$

12. $4x^2 - 32x + 80 = 0$ [imaginary-number solutions]

Solving a Quadratic Equation by Completing the Square

To solve a quadratic equation $ax^2 + bx + c = 0$ by **completing the square,**

1. If $a \neq 1$, divide both sides of the equation by a.
2. Write the equation in the form $x^2 + dx = k$, where d and k are constants.
3. Complete the square for the polynomial on the left side of the equation.
4. Solve the equation by using the square root property.

OBJECTIVE 3

Any quadratic equation can be solved by completing the square.

SHORT HW 1, 7, 11, 13, 27, 41, 51, 57, 67, 71, 75, 79, 81, 85, 89

MEDIUM HW 1, 7, 11, 13, 27, 35, 41, 45, 51, 57, 61, 67, 71, 75, 79, 81, 83, 85, 89, 91, 93, 95, 97, 99

SECTION 9.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students solve quadratic equations by completing the square. The main reason this section is included in the text is so that students understand the derivation of the quadratic formula in Section 9.6. Some instructors may choose to do light treatment of this section because factoring and the quadratic formula will be the main tools for solving quadratic equations for the rest of the course.

OBJECTIVE 1 COMMENTS

For Problem 4, I write $a \div 2 = a \cdot \frac{1}{2}$ to show that dividing by 2 is the same as multiplying by $\frac{1}{2}$. So, to find c, I multiply $\frac{3}{7}$ by $\frac{1}{2}$ and then square the result. Students have a tough time remembering this tactic.

Workbook Exploration

The following 15-minute exploration includes enough detail that groups have a reasonable chance of learning the main point of completing the square; it is doubtful that many students would pick this up from a lecture.

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Group Exploration

Section Opener: The connection between b and c of a perfect square trinomial $x^2 + bx + c$

1. Simplify $(x + 5)^2$.
2. For $(x + 5)^2 = x^2 + 10x + 25$, explain why the coefficient of x, 10, turns out to be twice the second term of $x + 5$, 5.
3. For $(x + 5)^2 = x^2 + 10x + 25$, explain why the constant term, 25, turns out to be the square of the second term of $x + 5$, 5.
4. For the perfect square trinomial $x^2 + 10x + 25$, show that if you divide the coefficient of x, 10, by 2 and then square the result, you will get the constant term, 25. Why does this make sense? [**Hint:** See your explanations in Problems 2 and 3.]
5. Simplify $(x + k)^2$.
6. Let k be a nonzero constant. For $(x + k)^2 = x^2 + (2k)x + k^2$, explain why the coefficient of x, $2k$, turns out to be twice the second term of $x + k$, k.
7. Let k be a nonzero constant. For $(x + k)^2 = x^2 + (2k)x + k^2$, explain why the constant term, k^2, turns out to be the square of the second term of $x + k$, k.
8. For the perfect square trinomial $x^2 + (2k)x + k^2$, show that if you divide the coefficient of x, $2k$, by 2 and then square the result, you will get the constant term, k^2. Why does this make sense? [**Hint:** See your explanations in Problems 6 and 7.]
9. Find the value of c for which the polynomial $x^2 + 8x + c$ is a perfect-square trinomial. Then factor the perfect-square trinomial.

OBJECTIVE 2 COMMENTS

For Problem 5, I remind students that if we add 25 to the left side of the equation, we must add 25 to the right side as well (addition property of equality). Some students forget to do this. For the step where the expression $x^2 + 10x + 25$ is factored as $(x + 5)^2$, many students wonder "where did the x (from the $10x$) go?" It helps to show students that $(x + 5)^2 = x^2 + 10x + 25$ by multiplying pairs of terms (and combining like terms).

I remind students to write "$\pm$," as appropriate. I also remind students that they can store values in their calculators to help them verify their solutions. Or, students can use graphing calculator tables or graphs to verify their work.

Most students need several reminders that the perfect square trinomial property given for $ax^2 + bx + c$, where $a = 1$, does not extend to trinomials with $a \neq 1$.

Students may need a reminder that when a radicand of a solution is negative, they should find the imaginary solution, rather than simply say, "There is no real-number solution."

OBJECTIVE 3 COMMENTS

I tell students that although any quadratic equation can be solved by completing the square, we will discuss an easier method in Section 9.6 that can also be used to solve any quadratic equation. I say that this method involves the *quadratic formula*, which we will derive by using completing the square.

Textbook and Workbook Exploration

The 15-minute "Identifying errors in solving by completing the square" exploration is a great activity because it singles out two errors that are frequently made by students.

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SECTION 9.6 LECTURE NOTES

Objectives

1. Solve quadratic equations by using the *quadratic formula*.
2. Use the *discriminant* to determine the number and type of solutions of a quadratic equation.
3. Decide which method to use to solve a quadratic equation.
4. Use the quadratic formula to make predictions with a quadratic model.

Main point: Use the quadratic formula to solve quadratic equations.

OBJECTIVE 1

Solve the general quadratic equation $ax^2 + bx + c = 0$ by completing the square.

Quadratic Formula

The solutions of a quadratic equation $ax^2 + bx + c = 0$ are given by the **quadratic formula:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Warning: For the fraction in the quadratic formula, note that $-b$ is part of the numerator.

1. Solve $x^2 + 3x - 10$ by using the quadratic formula and then by factoring. Compare your results.

Solve.

2. $3x^2 - 5x - 7 = 0$ [Radicand does not need simplifying.]

3. $5x^2 - 4x = 2$ [Radicand requires simplifying.]

4. $\frac{2}{3}x^2 = \frac{5}{6} - \frac{1}{2}x$ [Radicand does not need simplifying.]

5. $(2x - 3)(x + 5) = -26$ [Imaginary-number solutions]

6. Find approximate solutions of the equation $1.7x(2.8x - 3.1) = 2.1x + 4.9$. Round any results to the second decimal place.

OBJECTIVE 2

Determining the Number and Type of Solutions

For the quadratic equation $ax^2 + bx + c = 0$, the **discriminant** is $b^2 - 4ac$. Also,

- If $b^2 - 4ac > 0$, there are two real-number solutions.
- If $b^2 - 4ac = 0$, there is one real-number solution.
- If $b^2 - 4ac < 0$, there are two imaginary-number solutions (and no real-number solutions).

Determine the number and type of solutions of the equation.

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7. $5x^2 - 3x + 2 = 0$ [two imaginary-number solutions]

8. $x^2 + 6x + 9 = 0$ [one real-number solutions]

9. $3x^2 - x - 1 = 0$ [two real-number solutions]

Use graphs to verify your results in Problems 7, 8, and 9.

OBJECTIVE 3

Method:	**When to Use:**
Factoring	For equations that can easily be put into the form $ax^2 + bx + c = 0$ and where $ax^2 + bx + c$ can easily be factored.
Square root property	For equations that can easily be put into the form $x^2 = k$ or $(x + p)^2 = k$.
Completing the square	Use when the directions require it.
Quadratic formula	For all equations except those that can easily be solved by factoring or by using the square root property.

Solve.

10. $3(t - 5)^2 - 1 = 6$ [Square root property.]

11. $(x - 2)(x - 3) = 2(x - 5) + 24$ [Factor.]

12. $2x(x - 2) = 5$ [Quadratic formula.]

OBJECTIVE 4

13. The total numbers of employees at U.S. breweries are shown in the following table for various years.

Year	Total Number of of Employees at U.S. Breweries (thousand)
2011	26.7
2012	29.3
2013	34.5
2014	40.1
2015	48.4
2016	58.6
2017	69.4

Source: *The Bureau of Labor Statistics*

Let $f(t)$ be the number (in thousands) of employees at U.S. breweries at t years since 2010. A reasonable model is $f(t) = 0.85t^2 + 0.36t + 25.4$.

a. Verify that f models the data well.

b. Predict the number of employees at U.S. breweries in 2020.

c. Predict the average number of employees at U.S. breweries per state in 2020. Do you think your result will be a reasonable estimate for every state? Explain.

d. Predict when the number of employees at U.S. breweries will reach 150 thousand.

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SHORT HW 1, 7, 17, 19, 25, 31, 33, 37, 43, 47, 63, 71, 77, 81, 91, 93, 97

MEDIUM HW 1, 7, 13, 15, 17, 19, 25, 27, 31, 33, 37, 43, 47, 57, 63, 65, 71, 73, 77, 79, 81, 87, 89, 91, 93, 95, 97, 103, 105, 107, 109, 111

SECTION 9.6 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use the quadratic formula to solve quadratic equations. They will use this skill in Sections 9.9, 12.1, 12.5, 12.6, 13.5, and 15.2.

OBJECTIVE 1 COMMENTS

Before deriving the quadratic formula, I tell students that the derivation is in their textbooks and I encourage students to watch carefully and not take notes. That way, they can focus on the concepts and I can get through the derivation more quickly because I won't have to wait for them to copy it from the board.

Although I am not in favor of students memorizing proofs or derivations, I sometimes tell students that I will ask them to derive the quadratic formula on the next exam. The derivation is difficult to memorize without understanding it because it is so long. Also, knowing the derivation may increase the student success rate in solving specific equations by completing the square.

For Problem 3, a few students may think that $a = 5$, $b = -4$, and $c = 2$ (when really, $c = -2$). I emphasize that students must write the equation so that one side is 0. Also for Problem 3, many students do not simplify the radicand of their results. I revisit this topic a couple of times before the next exam.

Problem 6 is good preparation for part (d) of Problem 13. After I have applied the quadratic formula, I show students how to use their calculators to compute the two approximate solutions. First, I have students calculate the square root of the discriminant. Next, I have them calculate the numerators (both the sum and the difference). Then, I have them calculate the entire fractions. Students do pretty well by this stepwise calculator-entry approach. I warn them that they had better practice such problems until they can do one in about five minutes. Without such a warning, some students can take as long as 20 minutes for one problem!

Textbook and Workbook Exploration

The 20-minute "Deriving a formula for solving quadratic equations of the form $ax^2 + bx + c = 0$" exploration is a challenging group activity that asks students to derive the quadratic formula.

Workbook Exploration

The following 20-minute exploration has students find intercepts by using numerical, graphical, and symbolic methods. Students need lots of practice with making such connections.

EXPLORATION *Finding intercepts by tables, graphs, and the quadratic formula*

1. Use a graphing calculator to help you complete the following table for the function $f(x) = 2x^2 - 5x - 2$. Then use the table to estimate the x-intercepts of the graph of f.

x	$f(x)$	x	$f(x)$
-4		1	
-3		2	
-2		3	
-1		4	
0		5	

2. Use a graphing calculator to graph $f(x) = 2x^2 - 5x - 2$. Copy the screen. Use your graph to estimate the x-intercepts of the graph of $f(x) = 2x^2 - 5x - 2$.

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3. Use the quadratic formula to find the x-intercepts of the graph of $f(x) = 2x^2 - 5x - 2$. Round your results to the second decimal place.

4. Are the results you found in Problems 1, 2, and 3 equal? Are they approximately equal? Which method is easiest to use? Which method gives the most precise results?

5. Would your responses to the questions in Problem 4 be the same for all quadratic functions? [**Hint:** Carefully consider all sorts of quadratic functions.]

OBJECTIVE 2 COMMENTS

After discussing how to use the discriminant to determine the number of solutions, I sometimes ask students to create three equations that have two, one, and no real number solutions. If time permits, I demonstrate the graphical significance of this concept by providing an example similar to Example 9 on pp. 550–51.

OBJECTIVE 3 COMMENTS

Determining which method to use to solve a quadratic equation is a great challenge for most students. Without much discussion, most students will rarely think to solve *any* quadratic equations by using the square root property. Some students will use the quadratic formula to solve an equation which can easily be solved by factoring. Some students will do a poor job of solving an equation by completing the square when they could have used the quadratic formula (and hopefully have done a better job).

I try to train my students to first inspect an equation and see if the square root property can easily be used. If not, I suggest that students check whether the equation can easily be solved by factoring. If not, then I strongly recommend that students use the quadratic formula. I say flat out that they should only use completing the square when the directions require it. And I promise that there will be such directions for one problem on the upcoming exam.

Textbook and Workbook Exploration

The 20-minute "Comparing methods of solving quadratic equations" exploration encourages groups to consider the big picture of solving quadratic equations. It sets the stage nicely for the upcoming exam where students will have to grapple with which technique is most appropriate to use in solving an equation.

OBJECTIVE 4 COMMENTS

For part (d) of Problem 13, see my calculator keystroke suggestions for Problem 6 included in my objective 1 comments.

Workbook Exploration

The following 15-minute exploration is straightforward, but students need lots of practice with using their calculators after they've applied the quadratic formula (see Problem 4 of the exploration).

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Group Exploration

Using a quadratic model to make predictions

The numbers of undocumented immigrants removed from the interior of the United States are shown in the following table for various years.

Year	Removal of Undocumented Immigrants from Interior of the United States (thousands)
2012	181
2013	134
2014	102
2015	69
2016	65
2017	82

Source: *U.S. Customs and Border Protection*

Let $f(t)$ be the number (in thousands) of undocumented immigrants removed from the interior of the United States in the year that is t years since 2010.

1. Use a graphing calculator to draw a scatterplot of the data. Copy the screen. Can the data be better described by a linear model or a quadratic model? Explain.
2. Use a graphing calculator to draw the graph of the model $f(t) = 7.71t^2 - 90.43t + 333.71$ and, in the same viewing window, the scatterplot of the data. Copy the screen. Does the model fit the data well?
3. Predict the number of undocumented immigrants who will be removed from the interior of the United States in 2019.
4. Predict in which year 200 million undocumented immigrants will be removed from the interior of the United States.

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SECTION 9.7 LECTURE NOTES

Objectives

1. Describe the meaning of *linear equation in three variables*.
2. Solve a *system of linear equations in three variables*.
3. Find a quadratic equation, in standard form, of a parabola that contains three given points.

Main point: Find an equation of a parabola.

OBJECTIVE 1

Definition *Linear equation in three variables*

A **linear equation in three variables** is an equation that can be put into the form $Ax + By + Cz = D$, where A, B, C, and D are constants and A, B, and C are not all zero.

Here is an example of a linear equation in three variables: $2x + 7y - 4x = 9$

- An **ordered triple** (x, y, z) represents values of x, y, and z.
- For example, the ordered triple $(2, -4, 7)$ represents the values $x = 2$, $y = -4$, and $z = 7$.
- An ordered triple is a **solution** of an equation in terms of x, y, and z if the equation becomes a true statement when a, b, and c are substituted for x, y, and z, respectively.
- We say a solution **satisfies** the equation.

Decide whether the given ordered triple is a solution of the equation $2x - 7x + 4z = -5$.

1. $(-3, 1, 2)$

2. $(7, 0, -4)$

OBJECTIVE 2

A **system of linear equations in three variables** consists of two or more linear equations in three variables. Here is an example:

$$\begin{aligned} 3x - 5y + z &= -2 \\ 7x + 4y - 8z &= -46 \\ x - 3y + 4z &= 14 \end{aligned}$$

The **solution** of a system of linear equations in three variables is an ordered triple that satisfies *all* the equations.

3. Solve the system.

$$\begin{aligned} x + 4y + 2z &= 3 \\ 2x - 3y + 5z &= -7 \\ 3x + 5y - 4z &= 22 \end{aligned}$$

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4. Solve the system.

$$\begin{aligned} 4x + y + 3z &= 9 \\ 3x - y - 2z &= 4 \\ -2x + 2y - 6z &= 10 \end{aligned}$$

5. Solve the system.

$$\begin{aligned} 2x \quad + 3z &= 6 \\ 2y \quad - z &= -2 \\ 4x - y \quad &= -13 \end{aligned}$$

Solving a System of Three Linear Equations in Three Variables

To solve a system of three linear equations in three variables,

1. Select a pair of equations and eliminate a variable.
2. Select any other pair of equations and eliminate the *same variable* as in step 1.
3. The equations you found in steps 1 and 2 form a system of linear equations in *two* variables. Use elimination or substitution to solve this system.
4. Substitute the values of the two variables you found in step 3 into one of the original equations that contains the third variable. Solve for the third variable.
5. Write your solution as an ordered triple.

Discuss the graphical significance of solving a system of linear equations in three variables (see pages 575 of the textbook).

OBJECTIVE 3

Find an equation of the parabola that contains the given points.

6. $(1, 4), (2, 7), (4, 25)$

7. $(2, 7), (4, 1), (5, -8)$

Finding an Equation of a Parabola

To find an equation of a parabola that contains three given points,

1. Obtain a system of three linear equations in three variables by substituting the coordinates of each of the three given points into the general equation $y = ax^2 + bx + c$.
2. Solve the system you found in step 1.
3. Substitute the values of a, b, and c you found in step 2 into the equation $y = ax^2 + bx + c$.

Find an equation of the parabola that contains the given points.

8. $(0, 2), (2, 12), (3, 26)$

9. $(0, -4), (4, 8), (5, 26)$

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SHORT HW 1, 9, 13, 17, 23, 29, 37, 45, 49, 53

MEDIUM HW 1, 3, 9, 13, 17, 23, 29, 31, 37, 41, 45, 49, 51, 53, 55, 57, 59

SECTION 9.7 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students solve systems of linear equations in three variables in this section. Students also find a quadratic equation, in standard form, of a parabola that contains three given points. This skill will help students find an equation of a quadratic model in Sections 9.8 and 9.9. If you are tight on time, you could skip solving general systems and focus on finding equations of parabolas.

OBJECTIVE 1 COMMENTS

I point out that the meaning of an ordered triple is similar to the meaning of an ordered pair and that we are using the terminologies *solution* and *satisfies* in ways similar to what we've done in the past.

OBJECTIVE 2 COMMENTS

Once students get accustomed to the length of solving systems of three equations, most of my students do well. I emphasize that the process involves performing elimination three times, and that the same variable should be eliminated for the first two eliminations.

After solving Problem 3, I describe what is going on geometrically. I tell my students that the graph of each equation is a plane and that our result is the intersection of the three planes. For an example, I point to the ceiling and two adjacent walls, and ask students to point to the intersection point. Sometimes, I discuss all four cases as shown in Fig. 86 on page 575 of the textbook, although this goes beyond what students will do symbolically; all homework exercises have one-point solutions.

My students are sometimes thrown off when variables are missing, like in Problem 5. I point out that because variables are missing, we need only perform elimination once to then have two equations in (the same) two variables.

OBJECTIVE 3 COMMENTS

To begin, I tell my students that we will find an equation of a parabola that contains three given points. Then I construct a scatterplot with no scaling on the axes and demonstrate how in Section 9.8, we will choose three points to use to find a quadratic model, which we will use to make predictions in Section 9.9. This serves as good motivation for this objective and helps students hold the big picture for Sections 9.7–9.9.

For Problem 8, after substituting the coordinates of the given points in the general quadratic equation $y = ax^2 + bx + c$, we have the system

$$\begin{aligned} 2 &= c && \text{Equation (1)} \\ 12 &= 4a + 2b + c && \text{Equation (2)} \\ 26 &= 9a + 3b + c && \text{Equation (3)} \end{aligned}$$

Some of my students do not realize that they can substitute 2 for c into equations (2) and (3):

$$\begin{aligned} 12 &= 4a + 2b + 2 && \text{Equation (4)} \\ 26 &= 9a + 3b + 2 && \text{Equation (5)} \end{aligned}$$

Almost all my students do not realize that they can now subtract 2 from both sides. Instead, they try to eliminate b, but many students get confused due to there being so many constants present. After discussing these two pitfalls, I emphasize that when one of the given points is the y-intercept, we need only use elimination once.

Textbook and Workbook Exploration

The 20-minute "Finding an equation of a parabola" exploration directs students how to find an equation of a

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parabola that contains three given points. When many instructors first see this exploration, they think that intermediate algebra students would not be able to handle it. I have used this exploration many times and each time groups have done the activity successfully. Of course, some groups need a few hints along the way.

If groups have questions with Problem 1 of the exploration, I remind them that a point that lies on a graph of an equation satisfies the equation. For Problem 2 of the exploration, I remind groups that they used elimination in Chapter 6.

Textbook and Workbook Exploration

The 25-minute "For any three points, is there a quadratic function that contains them?" exploration is not important content-wise, but is a good critical thinking exercise. If stronger groups finish early, I ask a similar question about linear functions. The issue addressed in Problem 1 of the exploration is also touched on in Exercises 51 and 52 of Homework 9.7.

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SECTION 9.8 LECTURE NOTES

Objectives

1. Find an equation of a quadratic model in standard form.
2. Determine whether to model a situation by using a linear function, a quadratic function, or neither.

Main point: Find an equation of a quadratic model in standard form.

OBJECTIVES 1 and 2

1. The numbers of football bowls are shown in the following table for various years.

Year	Number of Bowls
1920	2
1930	1
1940	5
1050	8
1960	9
1970	11
1980	15
1990	19
2000	25
2010	35
2017	40

Source: *College Football Data Warehouse*

Let $f(t)$ be the number of football bowls at t years since 1900.

a. Find a linear equation and a quadratic equation of f. Compare how well the models fit the data.

b. Use each model to estimate the number of bowls in 1901. The actual number was 1 bowl (the Rose Bowl, which was originally called "Tournament East-West football game"). Which model gives the better estimate?

c. Which function better models the situation for years before 1924? Explain.

Finding an Equation of a Quadratic Model

To find an equation of a quadratic model, given some data,

1. Construct a scatterplot of the data.
2. Imagine a parabola that comes close to the data points, and choose three points (not necessarily data points) that lie on or close to the parabola.
3. Use the three points to find an equation of the parabola.
4. Use a graphing calculator to verify that the graph of the equation comes close to the points of the scatterplot.

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2. The total costs for companies to respond to cyberattacks are shown in the following table for various years.

Year	Total Cost for Companies to Respond to Cyberattacks (billions of dollars)
2013	12
2014	13
2015	15
2016	17
2017	21

Source: *Ponemon Institute LLC; Accenture*

Let $f(t)$ be the total annual cost (in billions of dollars) for companies to respond to cyberattacks at t years since 2010.

a. Find a linear equation and a quadratic equation of f. Compare how well the models fit the data.

b. For the years before 2013, which of the two models likely better describes the situation? Explain.

Selecting a Model

When performing step 1 of the modeling process, we must decide whether a linear function, a quadratic function, or neither of these is suitable for modeling the situation. Here are the criteria for selecting a model:

- The graph of the model should fit the points well.
- The model should make sense within the context of the authentic situation.

Four-Step Modeling Process

To find a model and make estimates and predictions:

1. Construct a scatterplot of the data. Decide whether a line, a parabola, or neither of these comes close to the points.
2. Find an equation of your model.
3. Verify that your equation has a graph that comes close to the points in the scatterplot. If it doesn't, check for calculation errors or use different points to find the equation. An alternative is to reconsider your choice of model in step 1.
4. Use your equation of the model to draw conclusions, make estimates, and/or make predictions.

SHORT HW 1, 3, 11, 13, 15, 17, 21, 23

MEDIUM HW 1, 3, 9–31 odds

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SECTION 9.8 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students find an equation of a quadratic model in this section. They can learn to do this by hand and by using regression.

OBJECTIVE 1 COMMENTS

If I'm short on time, I simply show students how to find quadratic regression equations. This shifts the emphasis of the section almost entirely to objective 2.

If I do have students find quadratic models by hand, students have two challenges. First, they tend to feel overwhelmed with all of the decimals from working with real data. Second, they are not sure which three points to use to generate the quadratic equation.

It is interesting to note that many students tend to choose the highest data point if the parabola opens downward and the lowest data point if the parabola opens upward. Further, they are surprised to find out that this type of data point usually does not turn out to be the maximum or minimum point of the model.

Textbook and Workbook Exploration

You could begin class with the 30-minute "Choosing three 'good' points to find a quadratic model" exploration. It might be nice if you did an example first to get students acclimated to the idea that working with decimals is not *that* hard, but keep in mind that these problems take time! Or, you can greatly reduce the time spent on this exploration by allowing students to use regression for Problem 2 of the exploration.

If you do not assign the exploration just described, you may want to assign one or both of Exercises 11 and 12 of Homework 9.8, which address the same issue.

Workbook Exploration

In the following 25-minute exploration, groups practice finding an equation of a quadratic model.

Group Exploration

Section Opener: Finding a quadratic model

The numbers of international visitors to the United States are shown in the following table for the first quarters of various years.

Year	Number of International Visitors (in millions)
2011	12.9
2012	14.1
2013	15.0
2014	15.9
2015	16.5
2016	16.5
2017	15.8

Source: *DOC; NTTO, ITA*

Let $f(t)$ be the number (in millions) of international visitors to the United States in the first quarter of the year that is t years since 2010.

1. Use a graphing calculator to draw a scatterplot of the data. Copy the screen.

2. Imagine a parabola that comes close to the data points in your scatterplot. Use three points on your parabola to find an equation of the parabola. We call such an equation a *quadratic model*.

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3. Draw your quadratic model and the scatterplot in the same viewing window. Verify that the model passes through the three points you chose in finding the equation in Problem 2 and that it comes close to all the data points.

4. Use a graphing calculator to find a regression equation of f.

5. Use a graphing calculator to graph the equations you found in Problems 2 and 4 in the same viewing window. Copy the screen. Compare the graphs.

OBJECTIVE 2 COMMENTS

Because students' tool bag of functions consists only of linear and quadratic ones, it is easy for them to determine which type of function to use to model some data. If the data points "bend," students will use a quadratic function. If the points lie near a straight line, students will use a linear function. Nonetheless, a discussion about the context of a situation will lay a foundation for when students reach Section 10.5, where they will have to choose between the less obvious choices of quadratic and exponential functions for data points that "bend."

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SECTION 9.9 LECTURE NOTES

Objectives

1. Use a quadratic model to make estimates and predictions.
2. Estimate the maximum or minimum value of a quantity.
3. Use a system of a quadratic model and another model to make estimates and predictions.
4. Find the maximum revenue of a business venture.

Main point: Use a quadratic model to make estimates and predictions.

OBJECTIVES 1 and 2

1. The numbers of models of plug-in hybrid electric vehicles in the United States are shown in the following table for various years.

Year	Number of Models of Plug-In Hybrid Electric Vehicles
2011	1
2012	2
2013	4
2014	9
2015	13
2016	14
2017	23

Source: *University of Michigan Transportation*

Let $f(t)$ be the number of models of plug-in electric vehicles at t years since 2010.

a. Find a linear equation and a quadratic equation of f. Compare how well the graphs of the functions fit the data.

b. Use both functions to predict the number of plug-in hybrid electric models in 2021. Explain why your results are so different.

c. Predict when there will be 60 plug-in hybrid electric models.

d. Find the t-intercepts of the graph of the quadratic model. What do they mean in this situation?

e. For what years is there model breakdown for the quadratic model for certain? Explain.

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2. The fatality rates of passenger vehicle occupants are shown in the following table for various age groups.

Age Group (years)	Age Used to Represent Age Group	Fatality Rate (number of deaths per 100,000 people)
25–29	27	14.3
30–34	32	11.1
35–39	37	10.0
40–44	42	9.4
45–49	47	9.2
50–54	52	8.9
55–59	57	8.7
60–64	62	9.9
65–69	67	10.6
70–74	72	12.3
75–79	77	14.9
80–84	82	17.3

Source: *Insurance Institute for Highway Safety*

Let $f(t)$ be the fatality rate of passenger vehicle occupants (number of deaths per 100,000 people) for passengers who are t years old.

a. Find an equation of f.

b. Estimate the age of occupants with a fatality rate equal to 10.3 deaths per 100,000 people.

c. Estimate the age of occupants with the lowest fatality rate.

d. Estimate the fatality rate of 22 year-old occupants. The actual fatality rate is 22.1 deaths per 100,000 people. Is your result an underestimate or an overestimate?

OBJECTIVE 3

3. The revenues from digital videogame products and physical videogame products are shown in the following table for various years.

	Revenue (billions of dollars)	
Year	Digital	Physical
2012	7.5	15.8
2013	11.7	14.6
2014	15.8	13.8
2015	20.0	13.3
2016	22.8	13.1

Source: *SuperData Research, Inc.*

Let $d(t)$ and $p(t)$ be the annual revenue (in billions of dollars) from digital videogame products and physical videogame products, both at t years since 2010.

a. Find quadratic regression equations of d and p.

b. Estimate when the annual revenue of digital videogame products was equal to the annual revenue of physical videogame products.

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OBJECTIVE 4

4. A company charters a party boat that normally costs $36 per person. A group discount reduces the fare by $0.20 for each ticket sold; the more tickets sold, the lower the per-person fare. The maximum capacity of the boat is 100 people, including the crew of 6 people. What size of group would maximize the boat owner's revenue?

5. Some students plan to reserve a room in a restaurant where they will have a buffet dinner as part of a graduation party. The buffet dinner normally costs $20 per person. A group discount reduces the price by $0.10 for each student who attends; the more dinners sold, the lower the per-person price. The maximum capacity of the buffet room is 120 people. What size party would maximize the restaurant's revenue?

SHORT HW 1, 5, 7, 11, 13, 15, 19, 21. 25

MEDIUM HW 1–35 odds

SECTION 9.9 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use quadratic models to make predictions, locate extrema, and identify where there is model breakdown. Students solve maximum revenue problems and work with systems of quadratic equations to make predictions.

OBJECTIVES 1 and 2 COMMENTS

For part (a) of Problem 1, I use a scatterplot to illustrate why it makes more sense to find a quadratic model than a linear model.

For part (a) of Problem 2, I point out that inspection of the table of data suggests that no line comes close to the data points, but that we should view a scatterplot of the data to check whether a quadratic model might be acceptable. I have students use regression (via a calculator) due to time constraints.

Although students have used the quadratic formula to make predictions for the explanatory variable in Section 9.6, they will still need to practice this skill in this section. For example, in part (b) of Problem 2, some students successfully substitute 10.3 for $f(t)$ in the regression model $f(t) = 0.009t^2 - 0.92t + 31.95$, but then use $c = 31.95$ rather than first subtracting 10.3 from both sides to use the correct value $c = 21.65$.

I remind my students that locating the vertex of a quadratic model is sometimes helpful for determining the maximum or minimum value of a quantity (see part (c) of Problem 2) and is sometimes helpful for determining where there may be model breakdown. Parts (d) and (e) of Problem 1 illustrate that sometimes the horizontal intercepts can help determine where model breakdown occurs for certain.

Although my students could use technology to quickly find a vertex, I prefer to have them continue to use the vertex formula to locate it. I want my students to continue to improve in their symbolic work on such problems. Students can then use technology to verify the work.

Background Information for the Exploration That Follows

In the following 25-minute exploration, students find the vertex to help them determine where there may be model breakdown.

EXPLORATION *Making predictions using quadratic models*

Organic food sales (in billions of dollars) in the U.S. are shown in the following table for various years.

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Year	Organic Food Sales (billions of dollars)
2009	22.5
2010	24.1
2011	26.3
2012	29.0
2013	32.3
2014	36.0
2015	39.8
2016	43.0

Source: *Organic Trade Association*

Let $f(t)$ be the organic food sales (in billions of dollars) in the year that is t years since 2000. A possible equation of f is

1. Find a quadratic equation of f.
2. Predict the organic food sales in 2021.
3. Predict in which year organic food sales will be \$74 billion.
4. Find the vertex of the graph of f. What does it mean in this situation?
5. For what years is model breakdown likely? Explain.

OBJECTIVE 3 COMMENTS

My students quickly see how to solve part (b) of Problem 3 because they have already used substitution in a similar manner when working with pairs of linear models in Section 6.4.

Textbook and Workbook Exploration

In the 25-minute "Modeling differences of quantities" exploration, students compare the results of two different ways to find a difference function that describes an authentic situation.

OBJECTIVE 4 COMMENTS

The maximum revenue problems are nice preparation for students who will eventually take calculus. Some of my students have trouble with these problems, probably because these problems are so different than the rest of the section. I point out to my students that we are performing the same three steps that we used to find the maximum area of a rectangle in Section 9.2.

In Section 6.5, we used the equation $T = vn$, where T is the total value of n objects that each have value v. I point out to my students that in Problem 4, this leads to the equation $R = pn$, where R is the revenue (in dollars) from selling n tickets at p dollars per ticket. It can also help to show students a unit analysis of $R = pn$.

Some of my students are surprised to find out that increasing the price of the tickets may not increase the revenue. I point out that as the price rises, the demand falls. As an extreme example, no one would purchase a music CD that sold for a billion dollars and so this high price gives zero revenue. To address this concept in Problem 4, I use *one* graphing calculator table to display the values of n, $p = -0.2n+36$, and $R = -0.2n^2+36n$, where n is equal to 88, 89, 90, 91, and 92. I also explore the connection between a table of values of $R = -0.2n^2 + 36n$ and the graph of $R = -0.2n^2 + 36n$.

Workbook Exploration

You can also address the concept of optimization by assigning the following 15-minute exploration.

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Group Exploration

Section Opener: Elasticity of demand

A company sells digital televisions. Consider the model $n = -30p + 22,500$, where n is the number of digital televisions sold per month at price p dollars.

1. Let $R = f(p)$ be the total monthly revenue (in dollars) from selling the televisions at price p dollars per television. Find an equation of f.
2. Use a graphing calculator to draw a graph of f. Copy the screen.
3. Use "maximum" on a graphing calculator to find the maximum point of the graph of f. What does it mean in this situation?
4. Suppose that the current price of the television is less than \$375. If the price is increased (but not beyond \$375), how will that impact the revenue? Explain. [**Hint:** Think in terms of the graph of f.]
5. Now suppose that the current price of the television is more than \$375. If the price is increased, how will that impact the revenue? Explain.

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CHAPTER 10 OVERVIEW

In Sections 10.1 and 10.2, students use exponential definitions and properties to simplify expressions involving exponents. They also study scientific notation in Section 10.1. In Section 10.3, they learn to sketch graphs of exponential functions. In Section 10.4, students use exponential properties to help them find an exponential equation whose graph contains two given points. They also solve equations of the form $ab^n = k$ for the base b. In Section 10.5, they use skills learned in Section 10.4 to help them find exponential models and make estimates and predictions. They also review model breakdown in this section. Students will use concepts from this chapter in Chapter 11 and Sections 14.2, and 14.4.

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SECTION 10.1 LECTURE NOTES

Objectives

1. Describe the meaning of a negative-integer exponent.
2. Simplify expressions involving negative-integer exponents.
3. Describe an *exponential function*.
4. Work with models whose equations involve negative-integer exponents.
5. Use *scientific notation*.

Main point: Describe the meaning of a negative exponent and simplify power expressions.

OBJECTIVE 1

Here we write $\frac{b^2}{b^5}$ in two different forms:

$$\frac{b^2}{b^5} = b^{2-5} = b^{-3}$$
$$\frac{b^2}{b^5} = \frac{b \cdot b}{b \cdot b \cdot b \cdot b \cdot b} = \frac{1}{b^3}$$

Equating the two results, we have

$$b^{-3} = \frac{1}{b^3}$$

Definition *Negative-integer exponent*

If n is a counting number and $b \neq 0$, then

$$b^{-n} = \frac{1}{b^n}$$

OBJECTIVE 2

Part of simplifying a power expression includes writing it so that each exponent is positive.

Simplify.

1. 7^{-2}

2. b^{-4}

$$\begin{aligned}
\frac{1}{b^{-n}} &= 1 \div b^{-n} && \frac{a}{b} = a \div b \\
&= 1 \div \frac{1}{b^n} && \text{Write power so exponent is positive: } b^{-n} = \frac{1}{b^n} \\
&= 1 \cdot \frac{b^n}{1} && \text{Multiply by reciprocal of } \frac{1}{b^n}\text{, which is } \frac{b^n}{1}. \\
&= b^n && \text{Simplify.}
\end{aligned}$$

So, $\frac{1}{b^{-n}} = b^n$.

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Negative-Integer Exponent in a Denominator

If n is a counting number and $b \neq 0$, then

$$\frac{1}{b^{-n}} = b^n$$

Simplify.

3. $\dfrac{1}{4^{-3}}$ **4.** $\dfrac{1}{b^{-9}}$

Properties of Exponents

If m and n are integers, $b \neq 0$, and $c \neq 0$, then

- $b^m b^n = b^{m+n}$ Product property for exponents
- $\dfrac{b^m}{b^n} = b^{m-n}$ Quotient property for exponents
- $(bc)^n = b^n c^n$ Raising a product to a power
- $\left(\dfrac{b}{c}\right)^n = \dfrac{b^n}{c^n}$ Raising a quotient to a power
- $(b^m)^n = b^{mn}$ Raising a power to a power

Simplify.

5. $(3b^4c^7)^2$ **6.** $(2b^5c^3)(-4b^7c^6)$ **7.** $\dfrac{14b^8c^5}{21b^2c^3}$ **8.** $\dfrac{(2b^3c^3)^5}{b^4c}$

Simplifying Expressions Involving Exponents

A expression involving exponents is simplified if

1. It includes no parentheses.
2. Each variable or constant appears as a base as few times as possible.
3. Each numerical expression (such as 7^2) has been calculated, and each numerical fraction has been simplified.
4. Each exponent is positive.

Simplify.

9. $(6b^{-9}c^4)(-2b^5c^{-3})$ **10.** $\dfrac{b^{-7}}{b^{-2}}$ **11.** $\dfrac{18b^{-5}c^{-4}}{27b^3c^{-6}}$ **12.** $\left(\dfrac{2b^{-4}c^4}{3b^2c^{-5}}\right)^{-3}$

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OBJECTIVE 3

Definition *Exponential function*

An **exponential function** is a function whose equation can be put into the form

$$f(x) = ab^x$$

where $a \neq 0$, $b > 0$, and $b \neq 1$. The constant b is called the **base**.

For $f(x) = 3(4)^x$ and $g(x) = 2^x$, find the following.

13. $f(2)$ **14.** $f(-2)$ **15.** $g(a+3)$ **16.** $g(8a)$

17. For $E(x) = 2^x$ and $L(x) = 2x$, compare $E(5)$ with $L(5)$.

OBJECTIVE 4

18. The weight of an astronaut w (in pounds) when d thousand miles from the center of Earth is described by the equation $w = 3060d^{-2}$.

a. Simplify the right-hand side of the equation.

b. When at sea level, the astronaut is about 4 thousand miles from Earth's center. How much does the astronaut weigh at sea level?

c. Substitute 6 for d in the equation found in part (a), and solve for w. What does the result mean in this situation?

OBJECTIVE 5

Definition *Scientific notation*

A number is written in **scientific notation** if it has the form $N \times 10^k$, where k is an integer and the absolute value of N is between 1 and 10 or is equal to 1.

Examples: 6.74×10^7, 9.2×10^{-6}

Write the number in standard decimal notation.

19. 5.36×10^3 **20.** 8.192×10^6 **21.** 4.6×10^{-3} **22.** 2.99×10^{-5}

Converting from Scientific Notation to Standard Decimal Notation

To write the scientific notation $N \times 10^k$ in standard decimal notation, we move the decimal point of the number N as follows:

- If k is *positive*, we multiply N by 10 k times and hence move the decimal point k places to the *right*.
- If k is *negative*, we divide N by 10 k times and hence move the decimal point k places to the *left*.

Write the number in scientific notation.

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23. The first evidence of life on Earth dates back to 3.6×10^9 years ago.

24. The wavelength of violet light is about 0.00000047 meter.

Converting from Standard Decimal Notation to Scientific Notation

To write a number in scientific notation, count the number k of places that the decimal point must be moved so that the absolute value of the new number N is between 1 and 10 or equal to 1.

- If the decimal point is moved to the left, then the scientific notation is written as $N \times 10^k$.
- If the decimal point is moved to the right, then the scientific notation is written as $N \times 10^{-k}$.

- Most calculators represent 7.29×10^{26} as 7.29 E 26.
- Most calculators represent 5.96×10^{-23} as 5.96 E -23.

SHORT HW 5, 17, 23, 27, 35, 43, 51, 55, 59, 67, 73, 81, 89, 93, 95, 103, 109, 113, 119

MEDIUM HW 5, 9, 17, 23, 27, 35, 43, 47, 51, 55, 59, 63, 67, 69, 73, 81, 89, 93, 95, 103, 109, 113, 115, 119, 121, 123, 125, 127, 129, 131

SECTION 10.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students learn many definitions and properties of exponents in this section and Section 10.2. They will use these concepts in many sections of Chapters 10–15. In Sections 10.1 and 10.2, students also learn to simplify expressions involving exponents, a skill that is especially helpful for Section 11.6.

OBJECTIVE 1 COMMENTS

Most students have difficulty with negative exponents. The work showing why $x^{-3} = \dfrac{1}{x^3}$ doesn't help students' intuition much. The statement $x^{-n} = \dfrac{1}{x^n}$ seems mysterious to most students.

Textbook and Workbook Exploration

In the 15-minute "Negative-integer exponents" exploration, groups discover the meaning of a negative-integer exponent. This may be the best way to help students gain some confidence with negative-integer exponents.

Workbook Exploration

The following 15-minute exploration is another option for groups to discover the meaning of a negative exponent.

EXPLORATION *Negative-integer exponent definition*

1. Complete the following table with ordered-pair solutions of $y = 2^x$. For results that are less than 1, use fractions, not decimal numbers.

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x	2^x Before Simplifying	2^x After Simplifying
5	2^5	32
4		
3		
2		
1		
0		
-1		
-2		
-3		
-4		
-5		

2. Look for a pattern in your completed table that suggests a value for 2^{-6}. Write your result as a fraction. We say we have *simplified* 2^{-6}.

3. Simplify. Write your result as a fraction. If you have difficulty doing this, construct a table similar to the one you constructed in Problem 1.

a. 5^{-2} **b.** 3^{-4} **c.** 4^{-3} **d.** 7^{-1}

4. Simplify.

a. x^{-2} **b.** b^{-4} **c.** y^{-3} **d.** t^{-1}

5. Simplify b^{-n}, where n is a positive integer.

OBJECTIVE 2 COMMENTS

Although students may know the properties of exponents discussed in this section from elementary algebra, they probably do not know why they are true. Students often have trouble remembering the definition of negative exponents even after it has been reviewed in this course.

When simplifying expressions involving exponents, students struggle with knowing the order in which to apply the properties and definitions.

When simplifying $(3b^4c^7)^2$, I emphasize that the exponent 2 should be distributed to all three factors of the base (see Problem 5). Students often omit distributing the exponent 2 to the factor 3.

There are many ways to simplify quotients of powers of the form $\frac{b^m}{b^n}$. One way is to use the property $\frac{b^m}{b^n} = b^{m-n}$ and then simplify so that there are no negative exponents. Another way is to "move" the power with the smaller exponent across the fraction bar (and change the sign of the exponent). Showing more than one method usually confuses students. It is best to select one method and use it whenever appropriate. Students tend to have less trouble with signs of exponents when they use the property $\frac{b^m}{b^n} = b^{m-n}$.

You may prefer to show students that $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$ and then use this fact to do Problem 12. I have not included this fact in the textbook in an attempt to avoid overloading students with too many options.

Textbook and Workbook Exploration

The 15-minute "Properties of exponents" exploration addresses common student errors.

Background Information for the Exploration That Follows

In the following 20-minute exploration, students can practice simplifying expressions involving exponents. Most

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groups will do pretty well on this exploration, yet students perform poorly on this material when working individually. If I have time, I have groups do additional problems. Or, during the next class meeting, I have groups do additional problems, perhaps after doing a couple more examples.

EXPLORATION *Simplifying expressions involving exponents*

Use pencil and paper to simplify the following expressions.

1. $\dfrac{2^{5004}}{2^{5001}}$ (If your calculator displays an error message, it may indicate "OVERFLOW." This means that the numerator and/or the denominator are too large for the calculator to handle.)
2. $(2b^2c^3)^5$
3. $\dfrac{35b^{-2}c^4}{15b^5c^{-3}}$
4. $\left(\dfrac{413^{10}b^{-3}c^{-7}}{26^9b^{-5}c^{-3}}\right)^0$
5. $(2^4b^{-4}c^6)(2^{-2}bc^{-3})$
6. $\dfrac{(2b^5c^{-5})^3}{(3b^3c^{-2})^4}$
7. $\left(\dfrac{3b^4c^{-7}}{4d^{-3}}\right)^{-2}$

OBJECTIVE 3 COMMENTS

After completing Problem 17, consider comparing input-output tables of E and L to help students begin to distinguish exponential functions from linear functions. Or, you could assign Exercise 77 of Homework 10.1 and have such a discussion the next class meeting.

OBJECTIVE 4 COMMENTS

Most students know that an astronaut is weightless in outer space, but few students know that an astronaut's weight declines *continuously* as the astronaut's distance from the center of Earth increases.

OBJECTIVE 5 COMMENTS

Deemphasizing scientific notation is one way to handle getting through this long section. However, make sure that students know how to interpret their graphing calculator's way of expressing scientific notation because students will run into this notation in Chapters 10 and 11.

One way to motivate scientific notation is to ask students to use their graphing calculators to compute 2^{50} or 2^{-50}. Students will naturally run into such computations in Chapters 10 and 11 from time to time.

For converting scientific notation to and from standard decimal notation, students are not sure whether to move the decimal point to the right or left. I encourage students to reflect on what makes sense, rather than to memorize rules. For example, for 3.56×10^{-8}, 3.56 is divided by 10^8 (a product of eight 10's), so we need to move the decimal point 8 places to the left.

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SECTION 10.2 LECTURE NOTES

Objectives

1. Explain the meaning of *rational exponents*.
2. Simplify expressions that have rational exponents.

Main point: Know definitions of rational exponents.

OBJECTIVE 1

Argue that

$$\left(16^{1/2}\right)^2 = 16^{\frac{1}{2}\cdot 2} = 16^1 = 16$$

suggests that $16^{1/2} = 4$. (Note that $16^{1/2}$ is *not* equal to 8, which is half of 16.)

Argue that

$$\left(8^{1/3}\right)^3 = 8^{\frac{1}{3}\cdot 3} = 8^1 = 8$$

suggests that $8^{1/3} = 2$.

Definition $\boldsymbol{b^{1/n}}$

For the counting n, where $n \neq 1$,

- If n is odd, then $\boldsymbol{b^{1/n}}$ is the number whose nth power is b, and we call $b^{1/n}$ the **nth root of b**.
- If n is even and $b \geq 0$, then $\boldsymbol{b^{1/n}}$ is the nonnegative number whose nth power is b, and we call $b^{1/n}$ the **principal nth root of b**.
- If n is even and $b < 0$, then $b^{1/n}$ is not a real number.

$\boldsymbol{b^{1/n}}$ may be represented by $\sqrt[n]{b}$.

Simplify.

1. $49^{1/2}$ **3.** $(-27)^{1/3}$ **5.** $-81^{1/4}$ **7.** $64^{-1/2}$

2. $27^{1/3}$ **4.** $32^{1/5}$ **6.** $(-81)^{1/4}$ **8.** $(-64)^{-1/3}$

Show that $16^{3/4} = 16^{\frac{1}{4}\cdot 3} = \left(16^{1/4}\right)^3 = 2^3 = 8$

Definition *Rational exponent*

For the counting numbers m and n, where $n \neq 1$, and b is any real number for which $b^{1/n}$ is a real number,

- $\boldsymbol{b^{m/n}} = \left(b^{1/n}\right)^m = (b^m)^{1/n}$
- $\boldsymbol{b^{-m/n}} = \dfrac{1}{b^{m/n}}, b \neq 0$

A power of the form $b^{m/n}$ or $b^{-m/n}$ is said to have a **rational exponent**.

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Simplify.

9. $25^{3/2}$ **10.** $8^{5/3}$ **11.** $(-27)^{4/3}$ **12.** $(-81)^{3/4}$ **13.** $9^{-3/2}$ **14.** $(-32)^{-2/5}$

For $f(x) = 2(27)^x$, find the following.

15. $f\left(\frac{2}{3}\right)$ **16.** $f\left(-\frac{1}{3}\right)$

OBJECTIVE 2

The properties of exponents that we discussed in Section 10.1 are valid for *rational exponents*.

Properties of Rational Exponents

If m and n are rational numbers and b and c are any real numbers for which b^m, b^n, and c^n are real numbers, then

- $b^m b^n = b^{m+n}$ Product property for exponents
- $\frac{b^m}{b^n} = b^{m-n}, b \neq 0$ Quotient property for exponents
- $(bc)^n = b^n c^n$ Raising a product to a power
- $\left(\frac{b}{c}\right)^n = \frac{b^n}{c^n}, c \neq 0$ Raising a quotient to a power
- $(b^m)^n = b^{mn}$ Raising a power to a power

Simplify. Assume that b and c are positive.

17. $\left(8b^6\right)^{4/3}$

18. $\frac{b^{-5/9}}{b^{-2/9}}$

19. $b^{3/5}b^{7/8}$

20. $(7b^2c^5)^{4/9}(7b^2c^5)^{5/9}$

21. $\left[\left(2b^3\right)^2\left(4b^2c^6\right)\right]^{1/4}$

22. $\frac{(8b^{-6}c^3)^{1/3}}{(16b^4c^{-12})^{1/4}}$

23. $\left(\frac{49b^5c^{-8}}{64b^{-3}c^{-2}}\right)^{-1/2}$

24. $\frac{\left(81b^8c^4\right)^{3/4}}{\left(64bc^7\right)^{1/3}}$

SHORT HW 1, 13, 23, 25, 37, 43, 47, 53, 57, 63, 71, 73, 77, 79, 83, 89

MEDIUM HW 1, 7, 13, 17, 25, 29, 33, 37, 39, 43, 47, 53, 57, 63, 65, 71, 73, 77, 79, 83, 89, 91, 93, 99, 101, 103

SECTION 10.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn the meaning of rational exponents. They will use this concept throughout Chapters 10, 11, and 13. Students also simplify more expressions involving exponents in this section.

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OBJECTIVE 1 COMMENTS

When introducing rational exponents, I write $16^{1/2}$ on the board and say we will find its value, which I will call "hand." Next, I write $\left(16^{1/2}\right)^2$ on the board and point out that this expression is "hand squared," and I place my hand over just $16^{1/2}$ to illustrate. Then I remove my hand and perform the following calculation:

$$\left(16^{1/2}\right)^2 = 16^{\frac{1}{2}\cdot 2} = 16^1 = 16$$

Then I cover $16^{1/2}$ (on the farthest left side of the equation) and observe that "hand squared is equal to 16." I ask my students, "So, what is 'hand' equal to?" Someone always sees that "hand" is equal to 4 (no one ever mentions -4, which we won't include in our definition, anyway). Finally, we discuss that $16^{1/2}$ means to find the square root of 16.

Then I use "hand" again to find the value of $8^{1/3}$. After that, students are ready for me to discuss the meaning of $b^{1/n}$ in general.

For powers with positive rational exponents, I encourage my students to practice computing them mentally, without writing all the steps. This will keep the amount of writing to a more reasonable level when they are simplifying complicated expressions (see Objective 2).

Most students have difficulty computing powers with negative rational exponents, especially after some time has passed (see Problems 7, 8, 13, and 14).

I delay discussing how to use a calculator to compute powers with rational exponents until I'm confident that students can compute such powers without one. When using a calculator to find a power such as $\left(\frac{9}{2}\right)^{3/4}$, I emphasize that two sets of parentheses must be used.

Textbook and Workbook Exploration

Another way to introduce rational exponents is to assign the 20-minute "Definition of $b^{1/n}$" exploration. Students may get more out of this exploration than from a lecture because they are more involved with each step of investigating the meaning of rational exponents.

OBJECTIVE 2 COMMENTS

Most students have a tough time simplifying expressions with rational exponents. They will continue to be challenged by determining which order to apply the properties and definitions of exponents. Further, some students may need reminders on how to perform operations with fractions (e.g. add fractions with different denominators). Sometimes students get psyched out when trying to simplify such expressions. So as not to overwhelm them, I point out that we are still performing the same types of steps as in Section 10.1; it is mostly performing operations of fractions that extends the time to simplify expressions involving rational exponents.

Problems 22–24 are more difficult than what's included in most intermediate algebra textbooks. If you are tight on time, you could omit discussing these problems and assigning these types of exercises.

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SECTION 10.3 LECTURE NOTES

Objectives

1. Graph an exponential function.
2. Describe the *base multiplier property* and the *increasing or decreasing property*.
3. Find the y-intercept of the graph of an exponential function.
4. Describe the *reflection property for exponential functions*.

Main point: Graph exponential functions.

OBJECTIVE 1

1. Create a table of solutions of $f(x) = 2^x$ and graph f by hand.
2. Create a table of solutions of $g(x) = 8\left(\frac{1}{2}\right)^x$ and graph g by hand.

Use a graphing calculator to verify your graphs in Problems 1 and 2.

OBJECTIVE 2

Refer to your tables of solutions of $f(x) = 2^x$ and $g(x) = 8\left(\frac{1}{2}\right)^x$ to motivate the base multiplier property.

Base Multiplier Property

For an exponential function of the form $y = ab^x$, if the value of x increases by 1, then the value of y is multiplied by b. In other words, if the input increases by 1, then the output is multiplied by b.

Describe the base multiplier property as applied to the given function.

3. $f(x) = 5(4)^x$

4. $g(x) = 7\left(\frac{2}{3}\right)^x$

Use the base multiplier property to explain the following connections:

- $f(x) = 2^x$ is an increasing function and $b = 2 > 1$ (see Problem 1).
- $g(x) = 8\left(\frac{1}{2}\right)^x$ is a decreasing function and $b = \frac{1}{2} < 1$ (see Problem 2).

Increasing or Decreasing Property

Let $f(x) = ab^x$, where $a > 0$. Then

- If $b > 1$, then the function f is increasing. We say the function **grows exponentially**.
- If $0 < b < 1$, then the function f is decreasing. We say the function **decays exponentially**.

Draw graphs to illustrate this property.

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OBJECTIVE 3

Substitute 0 for x in the general equation $y = ab^x$ to show the following property.

y-Intercept of an Exponential Function

For the graph of an exponential function of the form

$$y = ab^x$$

the y-intercept is $(0, a)$. [Draw a sketch.]

Find the y-intercept of the graph of the function.

5. $y = 8(5)^x$

6. $y = -4\left(\frac{2}{7}\right)^x$

Warning: The y-intercept of the graph of a function of the form $y = b^x$ is $(0,\ 1)$, *not* $(0,\ b)$.

Find the y-intercept of the graph of the given function. Then use the base multiplier property to graph the function by hand.

7. $f(x) = 2(3)^x$

8. $g(x) = 12\left(\frac{1}{3}\right)^x$

OBJECTIVE 4

9. Sketch and compare the graphs of $f(x) = -4(2)^x$ and $g(x) = 4(2)^x$.

Reflection Property for Exponential Functions

The graphs of $f(x) = -ab^x$ and $g(x) = ab^x$ are reflections of each other across the x-axis.

Graph the function by hand.

10. $f(x) = -3(4)^x$

11. $g(x) = -16\left(\frac{1}{4}\right)^x$

Discuss the domain and range of an exponential function of the form $f(x) = ab^x$ where $a > 0$ and do the same where $a < 0$.

Find the domain and range of the function.

12. $f(x) = 3(2)^x$

13. $g(x) = -10\left(\frac{1}{2}\right)^x$

SHORT HW 5, 11, 15, 21, 27, 29, 33, 37, 41, 45, 47, 63, 71, 77, 85

MEDIUM HW 1, 5, 11, 15, 17, 21, 23, 27, 29, 33, 37, 41, 45, 47, 61, 63, 71, 75, 77, 85, 87, 99, 101, 103

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SECTION 10.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students learn to sketch graphs of exponential functions in this section. Concepts related to graphing exponential functions will be very helpful throughout the rest of this chapter as well as Chapter 11. They will also learn the important base multiplier property. This property will help them find equations of exponential curves in Sections 10.4, 10.5, and 11.5 and help them see the connection between geometric sequences and exponential functions in Section 14.2.

OBJECTIVE 1 COMMENTS

For Problem 1, I evaluate f at -2, -1, 0, 1, 2, 3, and 4 to find points to plot. Students can observe that the graph of f is not a line. Then I point out the base multiplier property without yet calling it as such to find other points (without evaluating f) and sketch the exponential curve. For example, to plot a point with x-coordinate -3, I plot a point above $(-3, 0)$ that is half as high above the x-axis as the point $(-2, \frac{1}{4})$. By thinking in terms of halving distances visually each time we move to the left one unit, students can see that the curve is getting closer and closer to the x-axis, but never reaching it. I then introduce the terminology of a horizontal asymptote.

For Problem 2, I evaluate g at 0, 1, 2, 3, and 4 to find points to plot. Students can see that the graph of g is not a line. We can again use the base multiplier property (without naming it yet) to find other points (without evaluating g) and sketch the exponential curve.

Workbook Exploration

In the following 10-minute exploration, students sketch the graph of an exponential function. Problems 6 and 7 are nice reminders about the meaning of a graph. Most groups have an easy time with this exploration.

Group Exploration

Section Opener: Sketching the graph of an exponential function

1. Complete the following table of ordered-pair solutions for $f(x) = 2^x$.

x	$f(x)$
-2	
-1	
0	
1	
2	
3	
4	

2. Use a graphing calculator table to verify that you have completed the table correctly.

3. Construct a scatterplot by hand of the ordered pairs you found in Problem 1.

4. Sketch a curve that passes through the points of the scatterplot.

5. Use a graphing calculator to verify your graph. Copy the screen.

6. Choose two points that are on the graph of f. Show that the ordered pairs that correspond to these points are solutions of $f(x) = 2^x$.

7. Choose two points that are not on the graph of f. Show that the ordered pairs that correspond to these points are not solutions of $f(x) = 2^x$.

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OBJECTIVE 2 COMMENTS

I refer to Problems 1 and 2 to introduce the base multiplier property. For Problems 3 and 4, I do not graph the functions; I wait to do more graphing until I've discussed finding the y-intercept of the graph of an exponential function.

I show students how the increasing or decreasing property follows directly from the base multiplier property. The concepts of exponential growth and exponential decay will come in handy when performing modeling in Sections 10.5 and 11.5.

Textbook and Workbook Exploration

Groups can discover the base multiplier property by completing the 15-minute "Numerical significance of a and b for $f(x) = ab^x$" exploration. After groups complete the exploration, remind them of the slope addition property for linear functions and compare it to the base multiplier property.

OBJECTIVE 3 COMMENTS

For a function of the form $f(x) = b^x$, most students think that the y-intercept of the graph of f is $(0, b)$. By writing $f(x) = 1(b^x)$, students see that the y-intercept is $(0, 1)$. It is usually necessary to remind my students of this at least a couple of times.

After discussing the y-intercept, I show students how to efficiently graph an exponential function. First, I plot the y-intercept. The I use the base multiplier property to determine other points to plot. By this method, I never have to evaluate the function. Students tend to resist this method; they initially prefer evaluating the function. However, with some encouragement, most students will switch to using the base multiplier method. I prefer this method, not only because it is more efficient, but because it reinforces the base multiplier property and has students think more geometrically than the other method.

Workbook Exploration

The following 20-minute exploration is a great way to have students reflect on the base multiplier property, the y-intercept of the graph of an exponential function, and systems of equations. Because of the rich interplay between these three concepts, this is one of my favorite explorations.

EXPLORATION *Creating nonlinear systems*

1. Solve the system of two exponential equations in two variables.

$$y = 5(3)^x$$
$$y = 5\left(\frac{1}{2}\right)^x$$

2. Create a system of two exponential equations in two variables that has the given solution set.

a. $(0,\ 12)$ **b.** $(1,\ 12)$ **c.** $(2,\ 12)$

3. Create a system of an exponential equation and a linear equation, both in two variables, that has the given solution set.

a. $(0,\ 3), (1,\ 6)$ **b.** $(1,\ 12), (3,\ 3)$

c. $(0,\ 4)$ [**Hint:** Make sure the curves intersect exactly once.]

d. Is your line in part (c) decreasing? If yes, then find an increasing line that works. [**Hint:** Make sure the curves intersect exactly once. It may help to use trial and error.]

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OBJECTIVE 4 COMMENTS

The base multiplier method can also be used to graph the functions in Problem 9, which leads to discussing the reflection property. To get across the idea of reflection, I say that if a mirror was placed along the x-axis, the graph of $g(x) = 4(2)^x$ would see the graph of $f(x) = -4(2)^x$ as its reflection in the mirror. I also say that if we painted the graph of g on graph paper and folded the paper along the x-axis, the blotted image would be the graph of f.

Due to students' work with domain and range in Section 5.2, most students have an easy time finding the domain and range of an exponential function. A reminder that the graph of an exponential function $f(x) = ab^x$ does not intersect the x-axis helps students find the range of such functions.

Textbook and Workbook Exploration

The 20-minute "Graphical significance of a and b for $y = ab^x$" exploration is a great way to address the key concepts of this section; in fact, you could begin class by having groups work on it. If you facilitated the similar exploration for graphing linear functions, you will find that your students are more efficient in completing this exploration than with the linear one. Once groups are done, you can have a classroom discussion about Problems 4 and 5 in the exploration to get closure.

Functions of the form $y = ab^x + c$ will not be needed for modeling in the text, so this exploration omits investigation of vertical translation. If you want your students to explore the graphical significance of c, you could ask groups to explore its significance at the end of the exploration. A good follow-up question is to ask groups to describe the y-intercept of the graph of the function $y = ab^x + c$.

Textbook and Workbook Exploration

The 15-minute "Drawing families of exponential curves" exploration is a fun way for students to reflect on the graphical significance of parameters a and b for an exponential function of the form $y = ab^x$. Even if students have explored the graphical significance of the base b, groups may still need a hint to use values of b between 0 and 1 for Problems 1 and 2.

Textbook and Workbook Exploration

If you prefer an application-driven course, consider assigning the 15-minute "Using trial and error to find a model" exploration In Section 10.5 in *this* section. This exploration has students reflect on curve fitting and on the graphical significance of the parameters of an exponential function.

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SECTION 10.4 LECTURE NOTES

Objectives

1. Use the base multiplier property to find an exponential equation.
2. Solve an equation of the form $ab^n = k$ for the base b.
3. Use two points to find an exponential equation.

Main point: Find an equation of an exponential function.

OBJECTIVE 1

1. Use the base multiplier property to find an equation of an exponential function whose graph contains the points listed in the following table.

x	$f(x)$
0	2
1	6
2	18
3	54
4	162

2. Some values of functions f, g, h, and k are provided in the following table. Find a possible equation of each function. Verify your results with a graphing calculator table.

x	$f(x)$	$g(x)$	$h(x)$	$k(x)$
0	3	3	96	96
1	6	6	48	48
2	9	12	24	0
3	12	24	12	-48
4	15	48	6	-96

OBJECTIVE 2

Find all real-number solutions.

3. $b^2 = 9$

4. $b^3 = 27$

5. $3b^4 = 48$

6. $5b^5 = 95$

7. $b^4 = -67$

Discuss how Problems 3–7 suggest how to solve equations of the form $b^n = k$ for b.

Solving Equations of the Form $b^n = k$ for b

To solve an equation of the form $b^n = k$ for b, where n is a counting number and $n \neq 1$,

1. If n is odd, the real-number solution is $k^{1/n}$.
2. If n is even and $k \geq 0$, the real-number solutions are $\pm k^{1/n}$.
3. If n is even and $k < 0$, there is no real-number solution.

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Find all real-number solutions. Round all results to the second decimal place.

8. $2.7b^4 - 5.3 = 351.9$

9. $\frac{5}{6}b^4 - \frac{3}{8} = \frac{7}{12}$

10. $\frac{b^7}{b^2} = \frac{83}{4}$

11. Solve $b^n - \frac{k}{a} = p$ for b, where n is odd and $a \neq 0$.

OBJECTIVE 3

Find an approximate equation $y = ab^x$ of the exponential curve that contains the given pair of points. Round the value of b to two decimal places. Verify your result with a graphing calculator.

12. $(0, 7)$ and $(6, 273)$

13. $(0, 95)$ and $(3, 17)$

Show that if we divide the left sides and divide the right sides of the equations $5 = 5$ and $7 = 7$, we obtain the true statement $\frac{5}{7} = \frac{5}{7}$.

Dividing Left Sides and Right Sides of Two Equations

If $a = b$, $c = d$, $c \neq 0$, and $d \neq 0$, then

$$\frac{a}{c} = \frac{b}{d}$$

Find an approximate equation $y = ab^x$ of the exponential curve that contains the given pair of points. Round the values of a and b to two decimal places. Use a graphing calculator to verify your result.

14. $(2, 5)$ and $(6, 77)$

15. $(4, 47)$ and $(9, 6)$

SHORT HW 3, 9, 15, 17, 21, 25, 29, 35, 41, 47, 53, 55, 57, 59, 63

MEDIUM HW 1, 3, 7, 9, 15, 17, 21, 25, 29, 31, 35, 41, 45, 47, 53, 55, 57, 59, 63, 73, 77, 79, 81, 83

SECTION 10.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn three methods to find an equation of an exponential curve that contains two given points. They will use these methods to perform modeling in Sections 10.5 and 11.5.

OBJECTIVE 1 COMMENTS

Students have an easy time with exercises similar to Problem 1. For Problem 2, I remind students about the slope addition property.

Workbook Exploration

Students can discover how to apply the base multiplier property to find an equation of an exponential function in the following 10-minute exploration.

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Group Exploration

Section Opener: Finding an equation of an exponential function

1. Some input–output pairs of an exponential function f are listed in the following table.

x	$f(x)$
0	5
1	10
2	20
3	40
4	80

a. What is the y-intercept?

b. State the base multiplier property in terms of the function f.

c. What is an equation of f?

2. Some input–output pairs of an exponential function g are listed in the following table.

x	$g(x)$
0	162
1	54
2	18
3	6
4	2

a. What is the y-intercept?

b. State the base multiplier property in terms of the function g.

c. What is an equation of g?

Workbook Exploration

The following 15-minute exploration builds nicely on the previous exploration. If groups try to find an exponential function for data that is linearly related, suggest that they construct a scatterplot.

EXPLORATION *Finding equations of exponential functions*

1. Some values of functions f, g, h, and k are provided in the table below. Find a possible equation of each function. Verify your results with a graphing calculator table.

x	$f(x)$	x	$g(x)$	x	$h(x)$	x	$k(x)$
0	2	1	48	2	3	3	16
1	6	2	24	3	15	4	13
2	18	3	12	4	75	5	10
3	54	4	6	5	375	6	7
4	162	5	3	6	1875	7	4

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2. Some values of functions f, g, h, and k are provided in the table below. Find a possible equation of each function. Verify your results with a graphing calculator table.

x	$f(x)$	x	$g(x)$	x	$h(x)$	x	$k(x)$
2	512	3	6	1	5	59	1
4	128	5	24	3	15	60	-1
6	32	8	192	5	45	61	1
8	8	10	768	7	135	62	-1
10	2	13	6144	9	405	63	1

OBJECTIVE 2 COMMENTS

Problems 3–7 relate nicely to previous work done in Section 10.2 on rational exponents. Students tend to forget that there can be two solutions, so I remind them about this several times. For Problems 5, 6, 8, 9, and 11, I emphasize that we need to isolate the power b^n to one side of the equation. Problem 10 is excellent preparation for the work required to find an exponential equation that contains two given points, where neither point is the y-intercept.

Workbook Exploration

Students can learn how to solve equations of the form $b^n = c$ for b by completing the following 15-minute exploration.

Group Exploration

Section Opener: Solving equations of the form $b^n = k$ for b

1. Solve. [**Hint:** Consider negative solutions.]

a. $b^2 = 25$ **b.** $b^4 = 16$ **c.** $b^6 = 1$

2. To solve the equation $b^n = k$ for b, where n is even and $k > 0$, how many solutions are there?

3. Solve. [**Hint:** Consider negative solutions.]

a. $b^2 = -9$ **b.** $b^4 = -16$ **c.** $b^6 = -1$

4. To solve the equation $b^n = k$ for b, where n is even and $k < 0$, how many solutions are there?

5. Solve. [**Hint:** Consider negative solutions.]

a. $b^3 = 27$ **b.** $b^5 = 32$ **c.** $b^7 = 1$

d. $b^3 = -8$ **e.** $b^5 = -32$ **f.** $b^7 = -1$

6. To solve the equation $b^n = k$ for b, where n is odd, how many solutions are there?

7. Summarize what you have learned from this exploration.

OBJECTIVE 3 COMMENTS

Students have an easy time finding an exponential equation if one of the two given points is the y-intercept.

Although it will take some practice, students can become very successful at finding equations when neither of the given points is the y-intercept. I like this method because students have the rare opportunity to witness a meaningful application of the quotient property for exponents.

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For Problem 14, I tell students to write the system

$$77 = ab^6 \quad \text{Equation 1}$$
$$5 = ab^2 \quad \text{Equation 2}$$

so that the equation with the larger exponent, $77 = ab^6$, is on top. That way, when students later subtract the exponents, the difference of the exponents will be positive. When many students simplify the sides of the equation $\frac{77}{5} = \frac{ab^6}{ab^2}$, they want to perform the same operation with the 77 and 5 as with the exponents 6 and 2. So, they either divide 77 by 5 and divide 6 by 2, or they subtract 5 from 77 and subtract 2 from 6.

There is a way to avoid these pitfalls. Instead of dividing the left sides and dividing the right sides of equations (1) and (2), you can solve equation (2) for a: $a = \frac{5}{b^2}$. Next, substitute $\frac{5}{b^2}$ for a in equation (1): $77 = \frac{5}{b^2} \cdot b^6$ and solve for b. Note that solving equation (2) for a is preferable because if we had solved equation (1) for a, the substitution would've led to the equation $5 = b^{-4}$, which contains a negative exponent.

Students usually need a reminder on how to use technology to verify their work.

It is a good idea to compare and contrast equations of the form $ab^n = c$ (e.g. $5b^7 = 95$) and exponential functions (e.g. $y = 5(7)^x$).

Workbook Exploration

In the following 15-minute exploration, students learn how to find an equation of the exponential curve that contains two given points, where one of the points is the y-intercept.

Group Exploration

Section Opener: Finding an equation of an exponential curve

In this exploration, you will find an equation of the exponential curve that passes through the points $(0, 3)$ and $(4, 70)$.

1. Recall that an equation of an exponential curve can be put into the form $y = ab^x$. Find the constant a. [**Hint**: Recall that the y-intercept is $(0, a)$.]
2. Substitute the value of a into $y = ab^x$. (If $a = 5$, then $y = 5b^x$, for example.)
3. To find the constant b, use the fact that $(4, 70)$ should satisfy the equation you found in Problem 2. [**Hint:** First substitute for x and y, then solve for b. Also recall that the base of an exponential function is positive.]
4. Write an equation of the exponential curve. (If $a = 7$ and $b = 6$, then $y = 7(6)^x$, for example.)
5. Use a graphing calculator to graph your equation and verify that the graph passes through the points $(0, 3)$ and $(4, 70)$. Copy the screen.

After groups complete the exploration, discuss the subtle point that the equation $3b^4 = 70$ has two solutions, but b must be positive because it is the base of an exponential function.

Textbook and Workbook Exploration

The 20-minute "Comparing three ways to find exponential equations" exploration enables students to compare three methods for finding an equation of an exponential function.

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SECTION 10.5 LECTURE NOTES

Objectives

1. Find an equation of an *exponential model* by using the base multiplier property.
2. Model a *half-life* situation.
3. Find an equation of an exponential model by using data described in words.
4. Find an equation of an exponential model by using data described in a table.
5. For a model $f(t) = ab^t$, know the meaning of the coefficient a and the base b in terms of the situation being modeled.
6. Make estimates and predictions by using an exponential model.
7. Determine whether to model a situation by using a linear function, a quadratic function, an exponential function, or neither of these.

Main: Use an exponential model to describe an authentic situation.

OBJECTIVES 1, 5, and 6

An **exponential model** is an exponential function, or its graph, that describes the relationship between two quantities for an authentic situation. If all the data points for a situation lie on an exponential curve, then we say the explanatory and response variables are **exponentially related**. If no exponential curve contains all the data points but an exponential curve comes close to all the data points (and perhaps contains some of them), then we say the variables are **approximately exponentially related**.

1. Suppose a rumor is spreading at your school that classes will be free next semester. Assume that 5 people have heard the rumor as of today and that each day the total number of people who have heard it doubles. Let $f(t)$ be the total number of people who have heard the rumor at t days since today.
   - **a.** Find an equation of f.
   - **b.** How many people will have heard the rumor 7 days from now?
   - **c.** How many people will have heard the rumor 30 days from now?

If $y = ab^t$ is an exponential model where y is a quantity at time t, then the coefficient a is the value of that quantity present at time $t = 0$.

The term r **percent interest compounded annually** means that the interest earned each year equals r percent of the principal plus *any interest earned in previous years* (all of which becomes part of the investment).

2. A person invests \$7500 in an account at 8% interest compounded annually. Let $f(t)$ be the value (in dollars) of the account at t years after he has invested the \$7500.
   - **a.** Find an equation of f.
   - **b.** What will be the account's value in 20 years?
3. Oregon's Death with Dignity Act (DWDA) allows terminally-ill adult Oregon residents to use prescriptions from their doctors for self-administered, lethal doses of medications. The number of DWDA deaths in 2014 was 105 deaths and has grown by about 10% per year since then (Source: *Oregon Health Authority*). That is, each year the number of DWDA deaths is about 1.1 times the previous year's number of DWDA deaths.
   - **a.** Let $d = g(t)$ be the number of DWDA deaths in the year that is t years since 2014. Find an equation of g.

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b. What is the d-intercept of the model? What does it mean in this situation?

c. What is the base b of $g(t) = ab^t$? What does it mean in this situation?

d. Predict the number of DWDA deaths in 2021.

e. A student predicts that there will be 210 DWDA deaths in 2024 because $10(10\%) = 100\%$. What would you tell the student?

OBJECTIVES 2, 5, and 6

If a quantity decays exponentially, the **half-life** is the amount of time it takes for that quantity to be reduced to half. (Draw a graph similar to Fig. 51 on page 654 of the textbook.)

4. A storage tank contains cobalt-60, a radioactive element with a half-life of 5.3 years. Let $h(t)$ be the percent of the cobalt-60 that remains at t years since the element was placed in the tank.

a. Find an equation of h.

b. What percentage of the cobalt-60 will remain after 20 years?

- Because $1.08 - 1 = 0.08$, the base 1.08 of the compound interest model indicates that the value grows exponentially by 8% per year (see Problem 2).
- Because $1 - 0.877 = 0.123$, the base 0.877 of the cobalt-60 model indicates that the cobalt-60 decays exponentially by 12.3% per year (see Problem 4).

Meaning of the Base of an Exponential Model

If $f(t) = ab^t$, where $a > 0$, models a quantity at time t, then the percent rate of change is constant. In particular,

- If $b > 1$, then the quantity grows exponentially at a rate of $b - 1$ percent (in decimal form) per unit of time.
- If $0 < b < 1$, then the quantity decays exponentially at a rate of $1 - b$ percent (in decimal form) per unit of time.

OBJECTIVES 3 and 6

5. The number of osteopathic doctors grew approximately exponentially from 30.9 thousand doctors in 1990 to 91.6 thousand doctors in 2015 (Source: *American Osteopathic Association*). Predict the number of osteopathic doctors in 2022.

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OBJECTIVES 4–7

6. The numbers of public school teachers are shown in the following table for various years of experience.

Years of Experience	Number of Teachers (thousands)
5	185
10	145
15	90
20	63
25	54
30	50
35	38
40	6

Source: *R. Ingersoll and E. Merrill, University of Pennsylvania, original analyses for NCTAF of the Schools and Staffing Survey*

Let $n = f(t)$ be the number (in thousands) of public school teachers who have t years of experience.

a. Find a linear equation, a quadratic equation, and an exponential equation of f. Compare how well the models fit the data.

b. Which of the three models probably best models the situation for levels of experience greater than 40 years? Explain.

c. What is the base b of the exponential model $f(t) = ab^t$? What does it mean in this situation?

d. What is the n-intercept of the exponential model $n = ab^t$? What does it mean in this situation?

e. Use the exponential model to estimate the number of teachers who have 18 years of experience.

7. The numbers of Amazon employees are shown in the following table for various years.

Year	Number of Employees (thousands)
2007	17
2008	21
2009	24
2010	34
2011	56
2012	88
2013	117
2014	154
2015	231
2016	341

Source: *Amazon*

Let $f(t)$ be the number of Amazon employees at t years since 2000.

a. Find a linear equation, a quadratic equation, and an exponential equation of f. Compare how well the models fit the data.

b. Which of the three models probably best models the situation for years before 2007? Explain.

c. What is the percentage growth rate of the number of Amazon employees?

d. Predict the number of Amazon employees in 2022.

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SHORT HW 1, 5, 9, 13, 15, 17, 21, 23, 27, 29, 35, 39, 41, 43, 47

MEDIUM HW 1, 3, 5, 7, 9, 13, 15, 17, 19, 21, 23, 27, 29, 31, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 57

SECTION 10.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students work with the following applications: using the base multiplier property, compound interest, $\frac{1}{2}$-life applications, and working with tables of data. Students use concepts in Section 10.4 to help them find equations of exponential models. They then use these models to make estimates and predictions. They will use these techniques in Section 10.5 and 11.5.

OBJECTIVES 1, 5, and 6 COMMENTS

Students will quickly pick up on how to use the base multiplier property to find an equation of an exponential model.

Sometimes I repeat Problem 1, but assume that each day the total number of people who have heard the rumor increases by 5 people. By comparing the results of this version with the original version, students can see a nice comparison between linear and exponential growth. It is good to compare that in the original version, the growth could be caused by each person who hears the rumor continues to tell exactly one new person the rumor each day, whereas in the linear version, each person who hears the rumor tells exactly one person the rumor on the following day and then doesn't tell anyone else the rumor from then on.

I take care when discussing compound-interest accounts because most students have a weak understanding of percents and possibly no familiarity with the concept of compounding. It is instructive to compare compound interest with simple interest (see Exercise 49 of Homework 10.5).

For Problem 3, I point out that the percentage of 10% is to the base 1.10 of $g(t) = 105(1.10)^t$ in much the same way as in Problem 2, the percentage 8% is to the base 1.08 of $f(t) = 7500(1.08)^t$.

OBJECTIVES 2, 5 and 6 COMMENTS

Students have a tough time with half-life applications.

In Problem 1 of Example 3 in the textbook (page 654), two methods are shown to find an equation of a model for a half-life application. Method 1 is difficult for students to follow, but yields an exact model, assuming that the given half-life is exact. Method 2 is familiar to students (Section 10.4), but the model will not give as accurate estimates and predictions as the first method. It's also easier to determine the percentage rate of decay using Method 2 than Method 1. I prefer using Method 2 because it draws from Section 10.4 and because it is can be used to find models for other types of situations in Homework 10.5.

For Problem 4 in the lecture notes, I encourage students to first create a table of values of h:

t	$h(t)$
0	100
5.3	50

I find that students who follow this directive tend to do much better than students who don't.

Discussing or assigning Exercises 41 and 42 of Homework 10.5 can help students better grasp the concept of half-life.

Most students have a much easier time determining an exponential growth rate than an exponential decay rate. This is likely due to students' weak understanding of percents. In particular, most students lose sight that the "1" in the expression $1 - b$ is the decimal form of 100%.

Workbook Exploration

The following 20-minute exploration emphasizes percent change, which is at the heart and soul of exponential modeling. For Problem 4 in the exploration, groups could compare typical percent changes for each gender. A

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more sophisticated approach would be to find exponential models for each gender and compare the bases. If you don't nudge groups in this direction during the exploration, you could discuss it with them during the debrief.

Group Exploration

Section Opener: Percent rate of change

New York Life offers a $250,000 life insurance policy. Quarterly rates for women and men are shown in the following table for various ages.

Age (years)	Quarterly Rate (dollars)	
	Women	Men
35	25.00	28.75
40	33.75	35.75
45	51.25	57.50
50	70.00	87.50
55	104.50	145.00
60	145.75	230.75

Source: *New York Life*

The percent change of a quantity can be found by dividing the change in the quantity by the beginning value of the quantity and then converting the decimal result into percent form (by multiplying by 100). For the women's quarterly rate from age 35 years to age 40 years, the change in quarterly rate is its ending value minus its beginning value: $33.75 - 25.00 = 8.75$ dollars. Here we find the percent change in value:

$$\begin{aligned}\text{Percent change} &= \frac{\text{Change in value}}{\text{Beginning value}} \cdot 100 \\ &= \frac{33.75 - 25.00}{25.00} \cdot 100 \\ &= \frac{8.75}{25.00} \cdot 100 \\ &= 35\end{aligned}$$

So, the percent change is 35%.

1. Complete the following table.

Age	Quarterly Rate Women	Percent Change	Quarterly Rate Men	Percent Change
35	25.00	35	28.75	
40	33.75		35.75	
45	51.25		57.50	
50	70.00		87.50	
55	104.50		145.00	
60	145.75		230.75	

2. Compare the percentage rates of change of women's quarterly rates with each other.

3. Compare the percentage rates of change of men's quarterly rates with each other.

4. Are the women's percent changes in quarterly rates greater than, approximately equal to, or less than men's percent changes in quarterly rates? Is this fair? Explain.

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OBJECTIVES 3 and 6 COMMENTS

For Problem 5, I remind students that in such problems we must define the variables including units. I point out that it will be easier to derive an equation if we define t to be the number of years since 1990 rather than some other year. That way, one of the data points is $(0, 30.9)$ and the model will be of the form $f(t) = 30.9b^t$.

OBJECTIVES 4–7 COMMENTS

Some students will want to skip constructing a scatterplot and simply use the first and last data points. There are two problems with this strategy:

- The data may not be approximately exponentially related.
- If the data are approximately exponentially related, the first and last data points may not be the best pair of points to use to find an equation of a model.

For the teacher data (see Problem 6), both the quadratic regression model and the exponential regression model fit the data well, although the quadratic model might fit the data ever so slightly better. But the exponential model most likely describes the situation better for years of experience after 40 because the quadratic model estimates that the number of teachers will increase as the years of experience increase. I emphasize that students must consider both how well a function fits the data as well as the context of the situation in order to determine which type of function to select as a model.

In some cases, it can be difficult to determine which pair of points will generate the best exponential model. Sometimes students express frustration that the pair of points they thought would work well generates a model that doesn't fit the data very well. I reassure such students that as long as they make a reasonable guess at a "good" pair of points and derive the model correctly, they will earn full credit on quizzes and exams.

When preparing a lecture, I often "cheat" by determining a "good" pair of points by graphing the regression exponential model and selecting two data points that lie closest (or on) the curve. However, as I've mentioned before, I require my students to derive (nonregression) equations of models by hand because I want students to know those fundamental skills of the course.

Textbook and Workbook Exploration

The 15-minute "Using trial and error to find a model" exploration has students reflect on curve fitting and on the graphical significance of the parameters of an exponential function.

Workbook Exploration

You can emphasize the importance of constructing a scatterplot by assigning the following 20-minute exploration. After the exploration, I emphasize the importance of choosing two "good" points to find an equation of an exponential model as addressed in Problems 5 and 6 of the exploration.

Group Exploration

Finding the equation of an exponential model

The average monthly smartphone data consumptions are shown in the following table for various years.

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Year	Average Monthly Smartphone Data Consumption (gigabytes)
2010	0.3
2011	0.4
2012	0.6
2013	1.1
2014	1.3
2015	2.8
2016	3.9

Source: *CITA*

Let $f(t)$ be the average monthly smartphone data consumption (in gigabytes) at t years since 2010.

1. Find a linear equation, a quadratic equation, and an exponential equation of f. Compare how well the models fit the data.
2. Which of the three models probably best models the situation for years before 2010? Explain.
3. If you had used the data points for 2013 and 2014, you would have found the exponential model $f(t) = 0.67(1.18)^t$. If you had used the data points for 2014 and 2015, you would have found the model $f(t) = 0.06(2.15)^t$. Compare how these models and the exponential model you found in Problem 1 fit the data. Which is the best model? Explain.
4. Explain how you can determine two "good" points to use to find an equation of an exponential model.

Workbook Exploration

The following 30-minute exploration is a great review of exponential modeling. During the debrief, I highlight how Problems 2–4 of the exploration are related to the base multiplier property, and how we observed a similar connection between data that is approximately linearly related and the slope addition property.

Group Exploration

Using an exponential model to make predictions

The number of U.S. bike-sharing programs are shown in the following table for various years.

Year	Number of Bike-Sharing Programs	Ratio of Number of Programs (current to previous)
2011	22	leave blank
2012	30	1.36
2013	48	
2014	62	
2015	95	
2016	134	
2017	255	
2018	285	

Source: *bikesharingmap.com*

Let $n = f(t)$ be the number of U.S. bike-sharing programs at t years since 2010.

1. Find an exponential equation of f.

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2. What is the base b of your model $f(t) = ab^t$? What does it mean in this situation?

3. Complete the third column of the table of data. The first entry is 1.36 because

$$\frac{\text{2012 number of programs}}{\text{2011 number of programs}} = \frac{30}{22} \approx 1.36$$

4. Find the average of the ratios in the third column of the table. Compare your result to the base b of your model $f(t) = ab^t$. Explain why your comparison makes sense.

5. Predict the number of bike-sharing programs in 2021.

6. What is the coefficient a of your model $f(t) = ab^t$? What does it mean in this situation?

7. There are 3035 towns or cities with populations of at least 10,000 people. Use "intersect" on a graphing calculator to predict when all 3035 of those towns and cities will have ride-sharing programs, assuming towns and cities with populations less than 10,000 people will not have ride-sharing programs. Copy the screen. [**Hint:** Graph $n = 3035$.]

Textbook and Workbook Exploration

The 20-minute "Comparing a linear model with an exponential model" exploration is a nice activity for students to compare the growth predicted by linear and exponential models. The exploration also sends a subtle message (which you could emphasize) that it is foolish to extrapolate from just a few data points, especially just *two* data points!

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CHAPTER 11 OVERVIEW

In Section 11.1, students learn about composite functions and use this knowledge to check their work when finding inverses of invertible functions in Section 11.2. In Section 11.3, they apply their knowledge about inverse functions to learn about logarithms. In Section 11.4, they solve exponential equations and then apply this skill in Section 11.5 to use exponential models to make predictions for the explanatory variable. Students learn more properties of logarithms in Section 11.6 and learn about the natural logarithm in Section 11.7. Students will apply logarithm concepts in Sections 14.2 and 14.4.

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SECTION 11.1 LECTURE NOTES

Objectives

1. Describe the meaning of *composite function*.
2. Use tables to evaluate a composite function.
3. Use equations to evaluate a composite function.
4. Find an equation of a composite function.
5. Use graphs to evaluate a composite function.
6. Use a composite function to model an authentic situation.

Main point: Describe the meaning of *composite function*.

OBJECTIVE 1

Give an everyday example of composition.

Definition *Composite function*

If f and g are functions, x is in the domain of g, and $g(x)$ is in the domain of f, then we can form the **composite function $f \circ g$:**

$$(f \circ g)(x) = f(g(x))$$

We say $f \circ g$ is the **composition of f and g**.

OBJECTIVE 2

1. all the input-output pairs of functions g and f are shown in the following tables.

x	g(x)
0	8
1	5
2	9
3	6
4	7

x	f(x)
5	30
6	27
7	24
8	21
9	18

a. Find $(f \circ g)(0)$.

b. Use a table to describe the five input-output pairs of $f \circ g$.

OBJECTIVE 3

Let $f(x) = 3x + 5$ and $g(x) = 4x - 7$.

2. Find $(f \circ g)(2)$.

3. Find $(g \circ f)(2)$.

In general, $(f \circ g)(a)$ and $(g \circ f)(a)$ may or may not be the same function.

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OBJECTIVE 4

Let $f(x) = 2x$ and $g(x) = x + 4$.

4. Find an equation of $f \circ g$.

5. Find an equation of $g \circ f$.

Let $f(x) = x - 8$ and $g(x) = 5^x$.

6. Find an equation of $f \circ g$.

7. Find an equation of $g \circ f$.

Let $f(x) = 2x - 6$ and $g(x) = 3x - 5$.

8. Find $(f \circ g)(4)$.

9. Find an equation of $f \circ g$.

10. Use the equation of $f \circ g$ to find $(f \circ g)(4)$.

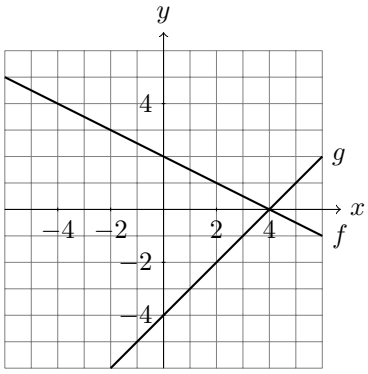
Compare the answers for Problems 8 and 10.

11. If $h(x) = (5x - 9)^4$, find equations of f and g such that $h(x) = (f \circ g)(x)$.

12. If $h(x) = \dfrac{1}{4x - 8}$, find equations of f and g such that $h(x) = (f \circ g)(x)$.

OBJECTIVE 5

13. Refer to the graphs in the following figure to find $(f \circ g)(2)$.



OBJECTIVE 6

14. Let $f(p)$ be the sales (in thousands of albums) of The Queens of the Stone Age album ... *Like Clockwork* at price p dollars. Let $g(n)$ be the revenue (in millions of dollars) from selling n thousand albums. What does $(g \circ f)(9.99) = 11.2$ mean in this situation?

15. Let $f(N)$ be the value (in cents) of N nickels, and let $g(D)$ be the number of nickels worth D dollars.

- **a.** Find equations of f and g.
- **b.** Find an equation of $f \circ g$.
- **c.** Find $(f \circ g)(4)$. What does it mean in this situation?

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16. In Problem 3 of the lecture notes of Section 5.5, we found an equation close to $f(t) = 0.19t - 0.65$, where $f(t)$ is the annual U.S. consumption (in billions of pounds) of avocados at t years since 2000.

a. Let $g(w)$ be the number (in billions) of avocados that weigh w billion pounds. Assuming that the average weight of an avocado is 0.38 pound, find an equation of g.

b. Let $h(t)$ be the number (in billions) of avocados consumed in the year that is t years since 2000. Determine which of the following is true: $h(t) = (f \circ g)(t)$ or $h(t) = (g \circ f)(t)$. Explain.

c. Find an equation of h.

d. Find $h(21)$. What does it mean in this situation?

SHORT HW 1, 7, 21, 23, 29, 31, 33, 51, 55, 63, 65, 69, 71, 77, 81

MEDIUM HW 1, 7, 15, 21, 23, 25, 29, 31, 33, 39, 51, 55, 63, 65, 69, 71, 75, 77, 81, 83, 85, 91, 93, 95

SECTION 11.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn to use equations, tables, and graphs to compose two functions. Students will use composition to check their work after finding an equation of the inverse of an invertible function in Section 11.2.

OBJECTIVE 1 COMMENTS

The notation for composing two functions is cumbersome, so it is important that you use an everyday example such as the combining of a change maker and old-fashioned vending machine to make a modern vending machine as described on pages 675 and 676 of the textbook. Drawing function diagrams like the ones in Figs. 3 and 4 of the textbook on page 676 is also helpful.

Workbook Exploration

In the following 10-minute exploration, groups can practice interpreting compositions of functions without having to get bogged down with the functions' equations. Once they understand this fundamental idea, they will be ready to work with tables and then equations.

EXPLORATION *Composing two models*

1. Let $f(t)$ be the sales price (in dollars) of a television at t years since 2015. Let $g(p)$ be the sales tax (in dollars) of a television whose price is p dollars. Which composition makes sense, $f(g(p))$ or $g(f(t))$? Describe its meaning.

2. Let $f(L)$ be the percentage of Drake fans whose ears ring the day after a concert with sound level L decibels. Let $g(n)$ be the sound level (in decibels) if n speakers are used at the concert. Which composition makes sense, $f(g(n))$ or $g(f(L))$? Describe its meaning.

3. Let $f(t)$ be the number of cars a salesperson sells by working t hours per week. Let $g(n)$ be the amount of money (in thousands of dollars) the salesperson earns from selling n cars. What does $g(f(52)) = 2$ mean in this situation?

4. Let $f(n)$ be the total cost of purchasing n gallons of gasoline. Let $g(d)$ be the number of gallons of gasoline consumed by driving d miles. What does $f(g(200)) = 18$ mean in this situation?

OBJECTIVE 2 COMMENTS

Rather than starting with graphs or equations, it is best to first evaluate a composite functions using tables because

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the action of stringing along two functions is most apparent. You can even draw arrows from table entries to other entries similar to the arrows shown in the function diagrams in Figs. 3 and 4 on page 676 of the textbook.

OBJECTIVE 3 COMMENTS

After completing Problems 2 and 3, emphasize that the results are different.

OBJECTIVE 4 COMMENTS

When working with equations to compose functions, students are often confused with what substitutions should be made. For Problem 4, it helps to prime the pump by first demonstrating how to evaluate $f \circ g$ at a specific number, such as 3. When you follow up by finding the equation of $f \circ g$, point out that the required substitutions are similar to the ones you made when finding $(f \circ g)(3)$.

When comparing the results for Problems 8 and 10, point out that if you want to evaluate a composition once, then using the method in Problem 8 is more efficient, but if you want to evaluate a composition many times, then using the method illustrated in Problems 9 and 10 is more efficient.

Workbook Exploration

In the following 20-minute exploration, students can practice identifying functions in a composition, which is a key skill for applying the chain rule in calculus.

EXPLORATION *Identifying functions in a composition*

1. Find equations of f and g such that $h(x) = f(g(x))$.

a. $h(x) = (x+5)^2$
b. $h(x) = 2^{x-7}$
c. $h(x) = x^2 + 4$
d. $h(x) = \dfrac{1}{x-2}$
e. $h(x) = 5^x - 1$
f. $h(x) = 4(x+9)$
g. $h(x) = \dfrac{1}{x} - 8.$
h. $h(x) = 7x - 2$

2. Find equations of f, g, and h such that $p(x) = f(g(h(x)))$.

a. $p(x) = 4^{x-5} + 1$
b. $p(x) = (x+9)^2 - 3$
c. $p(x) = \dfrac{1}{x+9} - 4$
d. $p(x) = 6^{2x-8}$

OBJECTIVE 5 COMMENTS

Students have a tough time with using graphs to evaluate compositions. Drawing arrows like in Fig. 6 on page 680 of the textbook can help. Remind students that the output of g becomes the input of f.

Textbook and Workbook Exploration

In the 15-minute "Composition of two linear functions" exploration, students discover that the composition of two linear functions is a linear function. This offers the payoff of having to find and plot only two points in order to graph the composition of two linear functions. In fact, only one point need be found directly because we can find a second point using the slope, which is the product of the slopes of the graphs of the two linear functions.

OBJECTIVE 6 COMMENTS

Modeling with composite functions may be the most challenging topic in the course. Students are often confused with which function should be the inner function and which should be the outer function. Add that difficulty to the complexity of finding equations of two models plus composing the functions leads to quite a challenge! In sum, think carefully if this challenge is worth the effort. Students won't have to compose models in subsequent sections. Although I seem to be making a case to skip this material, I actually discuss it but keep realistic expectations about how well students will do on this material. Basically, you have to devote quite a bit of time to this topic for students to do well.

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For Problem 15, perhaps the hardest part is simply finding the equations of f and g. For part (a) of Problem 16, students will have an easier time thinking of the appropriate conversion if you tell them that 2.63 avocados weigh 1 pound, rather than that an avocado weighs 0.38 pound, but I chose to provide the latter information because that seems like the more natural way to describe how much avocados weigh. After completing part (a), you could jump to finding an equation of h, but I prefer to slow things down by having students focus on the key intermediate step illustrated in part (b) because it is so challenging for students.

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SECTION 11.2 LECTURE NOTES

Objectives

1. Describe the meaning of *inverse of a function*.
2. Graph the inverse of a function.
3. Find an equation of the inverse of a model.
4. Find an equation of the inverse of a function that is not a model.
5. Compose an invertible function with its inverse.

Main point: Know the meaning of *inverse of a function*.

OBJECTIVE 1

An employee is paid \$10 per hour. Let $I = f(t)$ be the employee's income (in dollars) from working t hours.

- The function f sends values of t (input values) to values of I (output values). For example, f sends 8 to 80 (for 8 hours of work, the employee earns \$80).
- The *inverse* of f is a function that sends outputs of f to inputs of f. For example, the inverse of f sends 80 to 8 (to earn \$80, the employee must work 8 hours).
- Show input-output diagrams similar to Figs. 12 and 13 in the textbook (page 686) for f and the inverse of f.

The **inverse of a function f** is a relation that sends b to a if $f(a) = b$. If the inverse of a function f is also a function, we say that f is **invertible** and use "f^{-1}" as a name for the inverse of f. We say that f^{-1} is the **inverse function of f**.

Property of an Inverse Function

For an invertible function f, the following statements are equivalent: $f(a) = b$ and $f^{-1}(b) = a$.

1. Let f be an invertible function where $f(5) = 7$. Find $f^{-1}(7)$.

Some values of an invertible function f are shown in the following table.

x	$f(x)$
2	3
3	6
4	12
5	24
6	48

2. Find $f(3)$.

3. Find $f^{-1}(3)$.

4. Create a table of values of f^{-1}.

Refer to the employee model and its inverse to motivate the following property.

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f and f^{-1} are Inverses of Each Other

If f is an invertible function, then

- f^{-1} is invertible, and
- f and f^{-1} are inverses of each other.

OBJECTIVE 2

5. Graph by hand the equations $f(x) = 2^x$, f^{-1}, and $y = x$ on the same set of axes.

Reflection Property of Inverse Functions

For an invertible function f, the graph of f^{-1} is the reflection of the graph of f across the line $y = x$.

Note that in Problem 5, if a point $(a,\ b)$ is on the graph of f, then the point $(b,\ a)$ is on the graph of the inverse of f.

Graphing an Inverse Function

For an inverse function f, we sketch the graph of f^{-1} by the following steps:

1. Sketch the graph of f.
2. Choose several points that lie on the graph of f.
3. For each point (a, b) chosen in step 2, plot the point (b, a).
4. Sketch the curve that contains the points plotted in step 3.

6. Let $f(x) = \frac{1}{2}x + 1$. Graph by hand the equations of f, f^{-1}, and $y = x$ on the same set of axes.

OBJECTIVE 3

7. In Problem 4 of the lecture notes of Section 5.5, we found an equation close to $f(t) = 1.21t - 0.37$, where $s = f(t)$ is the U.S. annual new-car and light-truck sales (in millions) at t years since 2000. [Use the non-regression equation that you found in class.]

a. Find an equation of f^{-1}.

b. Find $f(23)$. What does it mean in this situation?

c. Find $f^{-1}(23)$. What does it mean in this situation?

Three-Step Process for Finding the Inverse of a Model

To find the inverse of an invertible *model* f, where $p = f(t)$,

1. Replace $f(t)$ with p.
2. Solve for t.
3. Replace t with $f^{-1}(p)$.

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OBJECTIVE 4

Find the inverse of the given function.

8. $f(x) = 5x + 3$ **9.** $f(x) = 2(3 - 2x) + x$ **10.** $f(x) = \dfrac{2x - 7}{3}$

Four-Step Process for Finding the Inverse of a Function That is Not a Model

Let f be an invertible function that is *not* a model. To find the inverse of f, where $y = f(x)$,

1. Replace $f(x)$ with y.
2. Solve for x.
3. Replace x with $f^{-1}(y)$.
4. Write the equation of f^{-1} in terms of x.

OBJECTIVE 5

Composing an Invertible Function with Its Inverse

If f is an invertible function, then

- $(f^{-1} \circ f)(x) = x$, where x is in the domain of f, and
- $(f \circ f^{-1})(x) = x$, where x is in the domain of f^{-1}.

In Problem 8, we found that if $f(x) = 5x + 3$, then $f^{-1}(x) = \dfrac{1}{5}x - \dfrac{3}{5}$. Show that the following is true.

11. $(f^{-1} \circ f)(x) = x$

12. $(f \circ f^{-1})(x) = x$

SHORT HW 1, 5, 7, 9, 27, 33, 43, 55, 65, 73, 83, 89, 91, 93, 100

MEDIUM HW 1, 3, 5, 7, 9, 21, 27, 31, 33, 37, 41, 43, 53, 55, 65, 73, 83, 85, 89, 91, 93, 100, 101, 103, 105, 107

SECTION 11.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students learn to use graphs, tables, and equations to describe the inverse of an invertible function. This will be a fundamental concept when discussing logarithms throughout the rest of Chapter 11.

You will probably not have time to do everything in this section. Finding an equation of an inverse function is of secondary importance to describing inverse functions with tables and graphs because it is the work with tables and graphs that will be needed in Section 11.3. Also, the four-step algorithm to find an equation of an inverse function has diminished importance because it cannot be applied to find the inverse of any invertible function. It may suffice for you to use just graphs and tables to describe inverse functions.

OBJECTIVE 1 COMMENTS

When first discussing inverse functions, I use tables to illustrate concepts. My students gain a much better grasp of the meaning of an inverse function by considering tables rather than graphs or equations.

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When illustrating the idea that f^{-1} "undoes" f, I make analogies of everyday events, such as how taking off shoes undoes putting on shoes. I also discuss how subtracting 2 undoes adding 2 and how dividing by 5 undoes multiplying by 5.

A discussion about one-to-one functions is probably not necessary in this course, so I did not include a prompt for this topic in the lecture notes. If you would like to discuss this concept, a presentation such as the one on page 693 of the textbook will likely suffice.

Workbook Exploration

In the following 10-minute exploration, students work with the inverse of a function that is not a model.

Group Exploration

Section Opener: Numerical introduction to an inverse function

Some values of a function f are given in the following table.

x	$f(x)$
0	2
1	4
2	3
3	1
4	0

Note that f sends the input 0 to the output 2. There is a function, f^{-1}, called the **inverse of** f that "undoes" f. For example, f^{-1} sends 2 to 0. In general, f^{-1} sends the *outputs* of f to the corresponding *inputs* of f.

1. Fill in the blank:

a. f^{-1} sends 4 to _____.

b. f^{-1} sends 0 to _____.

2. Complete the following table.

x	$f^{-1}(x)$
0	
1	
2	0
3	
4	

3. Because f^{-1} sends 2 to 0, we can write $f^{-1}(2) = 0$.

a. Find $f^{-1}(0)$.

b. Find $f^{-1}(1)$.

4. We can summarize what we've found using *one* table (see the following table on the left). Now, complete the following table on the right.

x	$f(x)$	x	$f^{-1}(x)$
0	2	0	4
1	4	1	3
2	3	2	0
3	1	3	2
4	0	4	1

x	$g(x)$	x	$g^{-1}(x)$
0	4		
1	0		
2	3		
3	1		
4	2		

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5. Describe the meaning of f^{-1} in terms of inputs and outputs of the function f.

OBJECTIVE 2 COMMENTS

Students have a better chance of understanding the general four steps to graph an inverse function if I first do Problem 5. Of the four steps, Step 3 is the crucial step. I explained it by reminding students that for the inverse function, outputs of f are sent to inputs of f.

After completing Problem 5, I have my students fold their paper along the line $y = x$ so they can see that the graphs of f and f^{-1} match up. My students have a relatively easy time graphing the inverse of an increasing exponential function. They have more trouble graphing the inverse of an invertible linear function f probably because the graph of f intersects the line $y = x$. My students have even more trouble graphing the inverse of a decreasing exponential function. I point out that the reflection property suggests that if a point lies on the intersection of the graphs of f and $y = x$, it will also lie on the graph of f^{-1}.

Workbook Exploration

In the following 15-minute exploration, groups can discover how to graph an inverse function, including having a visceral sense of why the graph of f^{-1} is the reflection of the graph of f across the line $y = x$.

Group Exploration

Section Opener: Graphing inverse functions

1. On a single coordinate system, plot each of the pair of given points. Also plot the point between them.

a. $(1, 5)$ and $(5, 1)$
b. $(0, 2)$ and $(2, 0)$
c. $(-4, 2)$ and $(2, -4)$
d. $(-3, -1)$ and $(-1, -3)$

2. Describe any patterns from your work in Problem 1. If you do not see any patterns, on the same coordinate system you worked with in Problem 1, graph other pairs of points of the forms (a, b) and (b, a).

3. Graph the function $y = 2^x$.

4. List seven points that lie on the graph of $y = 2^x$, where at least two of the points do not lie in the first quadrant. Then switch the coordinates of the seven points and plot the new points on the same coordinate system you worked with in Problem 3.

5. On the same coordinate system you constructed in Problem 3, sketch a curve that contains all the points obtained by switching the coordinates of all the points that lie on the curve $y = 2^x$.

OBJECTIVE 3 COMMENTS

I introduce the three-step method to find the inverse of a model by going over Problem 7. In Step 2, I obtain the equation $t = 0.83s + 0.31$. Before going on to Step 3, I show students that this equation is helpful for finding values of t for given values of s (the main idea of the inverse function).

The steps make more sense to students within the context of a situation. After completing Problem 7, I then list the steps in general.

Most of my students have a tough time discriminating between the work required for parts (b) and (c) of Problem 7. It helps to show students function diagrams for f and f^{-1}. It would help even more if students would draw such figures on exams, but it is very difficult to convince them to go to this length (even though it hardly takes any time).

Textbook and Workbook Exploration

Consider assigning the 15-minute "Inverse function" exploration as a way to introduce and motivate the inverse of a linear model.

OBJECTIVE 4 COMMENTS

For Problem 8, after applying the first three of four steps, we have $f^{-1}(y) = \frac{1}{5}y - \frac{3}{5}$. To apply step 4 and write

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$f^{-1}(x) = \frac{1}{5}x - \frac{3}{5}$ seems mysterious to students. To convince students that there's no harm in replacing y with x, I show them that $g(x) = 3x$, $g(y) = 3y$, and $g(w) = 3w$ all have the same input-output pairs. I also say that sometimes we like to consider an inverse function in its own right (such as $\sin^{-1}$) and when we do so, we usually use the variable x for the explanatory variable (of the inverse function).

OBJECTIVE 5 COMMENTS

Problems 11 and 12 are challenging for students. They also loose sight of the main point. I keep reminding them that f^{-1} undoes f, and vice versa. I say simple things, like, "If I turn on the lights and then turn off the lights, I'm right back with the lights off." It also helps to sketch function diagrams like the ones in Figs. 26 and 27 on page 692 of the textbook.

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SECTION 11.3 LECTURE NOTES

Objectives

1. Describe the meaning of *logarithm*.
2. Find logarithms.
3. Apply properties of logarithmic functions.
4. Graph logarithmic functions.
5. Use logarithms to model authentic situations.

Main point: Know the meaning of logarithms and logarithmic functions.

OBJECTIVE 1

Create tables of input-output pairs for $f(x) = 2^x$ and f^{-1} (see Tables 2.4 and 2.5).

Table 2.4: Values of f

x	$f(x)$
0	2^0
1	2^1
2	2^2
3	2^3
4	2^4

Table 2.5: Values of f^{-1}

x	$f^{-1}(x)$
2^0	0
2^1	1
2^2	2
2^3	3
2^4	4

- So, $f^{-1}(2^3) = 3$ and $f^{-1}(2^4) = 4$.
- Conclude that outputs of f^{-1} are exponents of powers of 2.
- Explain that we write $f^{-1}(8)$ as $\log_2(8)$ and that we write $f^{-1}(x) = \log_2(x)$.

Definition *Logarithm*

For $b > 0$, $b \neq 1$, and $a > 0$, the **logarithm $\log_b(a)$** is the number k such that $b^k = a$.

In short, a logarithm is an exponent.

OBJECTIVE 2

Find the logarithm.

1. $\log_2(32)$ **2.** $\log_5(25)$ **3.** $\log_3(81)$ **4.** $\log_4(64)$

Definition *Common logarithm*

A **common logarithm** is a logarithm with base 10. We write $\log(a)$ to represent $\log_{10}(a)$.

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Find the logarithm.

5. $\log(10,000)$

6. $\log(0.0001)$

7. $\log_6(6)$

8. $\log_8(1)$

9. $\log_4\left(\frac{1}{16}\right)$

10. $\log_3\left(\sqrt{3}\right)$

OBJECTIVE 3

Properties of Logarithms

For $b > 0$ and $b \neq 1$,

- $\log_b(b) = 1$
- $\log_b(1) = 0$

Definition *Logarithmic function*

A **logarithm function, base *b***, is a function that can be put into the form

$$f(x) = \log_b(x)$$

where $b > 0$ and $b \neq 1$.

- Discuss why $\log_3(0)$ and $\log_3(-9)$ are undefined.
- The domain of a logarithmic function $\log_b$ is the set of all positive real numbers.

Refer to Tables 2.4 and 2.5 in these notes to explain the following property.

Logarithmic and Exponential Functions Are Inverses of Each Other

- For an exponential function $f(x) = b^x$, $f^{-1}(x) = \log_b(x)$.
- For a logarithmic function $g(x) = \log_b(x)$, $g^{-1}(x) = b^x$.

Find the inverse of the function.

11. $f(x) = 5^x$

12. $g(x) = 10^x$

13. $h(x) = \log_7(x)$

14. $k(x) = \log(x)$

Let $f(x) = 4^x$.

15. Find $f(4)$.

16. Find $f^{-1}(4)$.

17. Find $f(3)$.

18. Find $f^{-1}(16)$.

Composing Logarithmic and Exponential Functions with the Same Base

For $b > 0$ and $b \neq 1$,

- $\log_b\left(b^x\right) = x$,
- $b^{\log_b(x)} = x$, where $x > 0$

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Simplify.

19. $\log_7\left(7^3\right)$ **20.** $2^{\log_2(6)}$ **21.** $10^{\log(4)}$ **22.** $\log\left(10^5\right)$

OBJECTIVE 4

Because a logarithmic function is the inverse of an exponential function, we can use the four-step graphing method of Section 11.2 for graphing inverse functions to help us graph a logarithmic function.

Graph by hand.

23. $y = \log_3(x)$ **24.** $y = \log_{\frac{1}{4}}(x)$

OBJECTIVE 5

The energy released by an earthquake is sometimes measured on the *Richter scale*. The **Richter number**, R, of an earthquake is given by

$$R = \log\left(\frac{A}{A_0}\right)$$

where A is the amplitude (maximum value) of a seismic wave and A_0, called the **reference amplitude**, is the amplitude of the smallest seismic wave that a seismograph can detect.

25. In 1906, an earthquake in San Francisco had an amplitude of 2×10^8 times the reference amplitude A_0. In 1989, an earthquake in San Francisco had an amplitude of 8×10^6 times A_0.

a. Find the Richter number of both earthquakes.

b. Find the ratio of the amplitudes of the 1906 and 1989 earthquakes.

SHORT HW 1, 9, 13, 25, 35, 37, 41, 43, 49, 61, 63, 65, 71, 77, 81, 85, 91

MEDIUM HW 1, 9, 13, 17, 23, 25, 29, 35, 37, 41, 43, 49, 55, 57, 61, 63, 65, 71, 77, 81, 83, 85, 87, 91, 93, 95, 99, 101, 103

SECTION 11.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students apply what they learned about inverse functions in Section 11.2 to now learn about logarithms. Students will work with logarithms throughout the rest of Chapter 11.

OBJECTIVE 1 COMMENTS

There are two paths that I have used to introduce logarithms. Each path has its advantages and disadvantages.

One path, which is in the lecture notes, introduces a logarithm as being the inverse of an exponential function. This is directly in sync with the foundation laid in Section 11.2. This path affords students the best chance of seeing why a logarithm is the inverse of an exponential function. However, it is a little more difficult to see that a logarithm is an exponent. So, it is also a little more difficult to see how to evaluate a logarithm.

For the other path, logarithms are introduced by way of solving exponential equations by trial and error (see the description of "A logarithm is an exponent" exploration below). By this path, students develop a firm understanding that a logarithm is an exponent. As a result, they quickly see how to evaluate a logarithm. However, by this path, students do not tend to view a logarithm as a function and they do not tend to understand why a logarithm is the inverse of an exponential function.

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Using a combination of these two paths is probably the best tact. Unfortunately, no matter what approach I've tried, it seems that the concept that a logarithmic function is the inverse of an exponential function fades from students minds quite quickly!

Textbook and Workbook Exploration

The 15-minute "A logarithm is an exponent" exploration is an excellent introduction to logarithms. In addition, the process of finding solutions by trial and error is a worthwhile experience in its own right.

OBJECTIVE 2 COMMENTS

When finding logarithms such as $\log_4\left(\frac{1}{16}\right)$ and $\log_3(\sqrt{3})$, I remind my students of the exponential properties discussed in Sections 10.1 and 10.2 (see Problems 9 and 10). I emphasize the connection between logarithmic properties and exponential properties throughout Chapter 11.

Background Information for the Exploration That Follows

In the following 10-minute exploration, students have practice evaluating logarithms.

EXPLORATION *Evaluating logarithmic functions*

Find the logarithm.

1. $\log_6(36)$

2. $\log_4(64)$

3. $\log_5(125)$

4. $\log_{10}(100,000)$

5. $\log_2\left(\frac{1}{2}\right)$

6. $\log_3\left(\frac{1}{9}\right)$

7. $\log_7(\sqrt{7})$ [**Hint**: Recall that $\sqrt{7} = 7^{1/2}$.]

OBJECTIVE 3 COMMENTS

My students often lose sight that $\log_b$ is a name of a function and they need to be reminded of this. The concept that $f(x) = b^x$ and $\log_b(x)$ are inverse functions of each other is probably best shown through comparing tables for both types of functions.

After I do Problems 11–14, my students can certainly do similar problems just after that. However, without more reminders later on, few of my students can do such problems on the next exam. This is not surprising because the notation is challenging for students and most students forget that exponential functions and logarithmic functions are inverse functions of each other.

Workbook Exploration

In the following 5-minute exploration, students discover that $\log_b(b) = 1$ and $\log_b(1) = 0$. Groups will have an easy time with this exploration. After the exploration, I point out that there is no need for students to memorize the properties in this exploration because they can always evaluate the logarithms directly.

You may find that you can skip this exploration as your students may be able to find the values of $\log_b(b)$ and $\log_b(1)$ without first doing part (a) of Problem 1 and part (a) of Problem 2.

EXPLORATION *Properties of logarithms*

1. **a.** Find the logarithm.

i. $\log_3(3)$ **ii.** $\log_7(7)$ **iii.** $\log_{12}(12)$

b. What is the value of $\log_b(b)$? Explain.

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2. **a.** Find the logarithm.

i. $\log_5(1)$ **ii.** $\log_6(1)$ **iii.** $\log_{14}(1)$

b. What is the value of $\log_b(1)$? Explain.

OBJECTIVE 4 COMMENTS

There are two ways to do Problem 23. One way is to first graph $y = 3^x$ and then reflect the graph across the line $y = x$. For this method, most students need a reminder that $\log_3$ is the inverse function of $y = 3^x$. Another way to graph $y = \log_3(x)$ is to build a table of values using powers of 3 as inputs, plot the solutions, and sketch a curve that contains the points. Even though the latter method is likely easier for students, I prefer the former method because it reinforces key concepts that we've discussed in Section 11.2.

Workbook Exploration

In the following 20-minute exploration, students will graph $\log_2$. The exploration supports a thorough investigation of the function. In fact, you could have groups work on it at the start of class. Presenting a logarithm as a function provides the big picture, which learning theory supports. And introducing it via a graph could work well for students because they tend to be such visual learners.

Group Exploration

Section Opener: Analyzing the graph of a logarithmic function

1. Graph the inverse of the function $f(x) = 2^x$.

2. Is the graph you sketched in Problem 1 a function? Explain.

3. The function you graphed in Problem 1 is called "$\log_2$". Use the graph to find the given output.

a. $\log_2(2)$ **b.** $\log_2(4)$ **c.** $\log_2(8)$ **d.** $\log_2(1)$ **e.** $\log_2\left(\frac{1}{2}\right)$

4. Find the domain of the function $\log_2$. Explain.

5. Find the range of the function $\log_2$. Explain.

6. Is $\log_2$ an increasing function, a decreasing function, or neither? Explain.

7. For $x \geq 1$, which grows the fastest, $f(x) = 2^x$, $y = x$, or $f^{-1}(x) = \log_2(x)$? Which grows the next fastest? Explain.

OBJECTIVE 5 COMMENTS

Students will have an easy time with the applications in this section.

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SECTION 11.4 LECTURE NOTES

Objectives

1. Convert equations in *logarithmic form* to *exponential form* and vice versa.
2. Solve logarithmic equations.
3. Solve exponential equations.
4. Solve equations in one variable by graphing.

Main point: Solve exponential and logarithmic equations.

OBJECTIVE 1

- Remind students that $\log_2(16) = 4$ because $2^4 = 16$.
- $\log_2(16) = 4$ is in *logarithmic form*.
- $2^4 = 16$ is in *exponential form*.

Write the equation in exponential form. Assume that all constants are positive and not equal to 1.

1. $\log_5(125) = 3$

2. $\log_k(t) = w$

Write the equation in logarithmic form. Assume that all constants are positive and not equal to 1.

3. $3^4 = 81$

4. $p^r = q$

Exponential/Logarithmic Forms Property

For $a > 0$, $b > 0$, and $b \neq 1$, the equations $\log_b(a) = c$ and $b^c = a$ are equivalent.

The equation $\log_b(a) = c$ is in **logarithmic form** and the equation $b^c = a$ is in **exponential form**. Either form can replace the other when you solve a problem.

OBJECTIVE 2

A logarithmic equation in one variable is an equation in one variable that contains one or more logarithms.

Solve. Round approximate solutions to the fourth decimal place.

5. $\log_5(x) = 4$

6. $\log(x) = 3$

7. $\log(4x + 976) = 3$

8. $3\log_{27}(x) + 5 = 7$

9. $2\log_2(5x - 3) = 10$

10. $\log_2(\log_2(x)) = 3$

11. $\log_b(64) = 3$

12. $\log_b(95) = 7$

OBJECTIVE 3

Power Property for Logarithms

For $x > 0$, $b > 0$, and $b \neq 1$, $\log_b(x^p) = p\log_b(x)$.

For example, $\log_3(x^7) = 7\log_3(x)$ for $x > 0$.

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Proof of the power property: Let $k = \log_b(x)$. The exponential form of this equation is $b^k = x$. Taking the exponent p on both sides and simplifying gives

$$\begin{aligned} (b^k)^p &= x^p && \text{Raise both sides to the power } p. \\ b^{kp} &= x^p && \text{Multiply exponents: } (b^m)^n = b^{mn} \end{aligned}$$

The logarithmic form of this equation is

$$\begin{aligned} \log_b(x^p) &= kp \\ &= pk && \text{Commutative law.} \\ &= p\log_b(x) && \text{Substitute } \log_b(x) \text{ for } k. \end{aligned}$$

Therefore

$$\log_b(x^p) = p\log_b(x)$$

This statement is what we set out to prove.

Logarithm Property of Equality

For positive real numbers a, b, and c, where $b \neq 1$, the equations $a = c$ and $\log_b(a) = \log_b(c)$ are equivalent.

An **exponential equation in one variable** is an equation in one variable in which an exponent contains a variable.

Solve. Round the solution to the fourth decimal place.

13. $5^x = 97$ [Warning: Watch parentheses when computing $\dfrac{\log(97)}{\log(5)}$.]

14. $2(6)^x = 253$ [Warning: $2(6)^x \neq 12^x$; Divide both sides by 2 before taking log of both sides.]

15. $3^{4x+2} = 341$

16. $8(2^x) + 3 = 5(2^x) + 81$

17. $4^x = -15$

Warning: $\log_b(ax^p) \neq p\log_b(ax)$. See Problems 14 and 16.

18. Solve the equation $ab^{p-x} = c$ for x, where $b > 0$ and $b \neq 1$, and the constants have values for which the equation has exactly one real-number solution.

19. Compare and contrast the methods required to solve the equations $\log_5(x) = 2$, $\log_b(5) = 2$, and $5^x = 2$.

OBJECTIVE 4

20. Use graphing to solve $3^x = 5x$. Round the solutions to the fourth decimal place.

It is impossible to solve $3^x = 5x$ by performing operations on both sides of the equation.

SHORT HW 5, 15, 23, 29, 31, 37, 41, 45, 51, 53, 61, 63, 67, 71, 81, 89, 91, 101, 105

MEDIUM HW 1, 5, 9, 15, 19, 23, 29, 31, 37, 41, 45, 49, 51, 53, 57, 61, 63, 67, 71, 81, 89, 91, 95, 101, 105, 107, 111, 113, 115, 117

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SECTION 11.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn some properties of logarithms and how to apply these properties to solve logarithmic and exponential equations. They will use these properties throughout the rest of Chapter 11 and Sections 14.2 and 14.4. In particular, they will use the power property to solve for the explanatory variable of exponential models in Section 11.5.

OBJECTIVE 1 COMMENTS

Although the exponential/logarithmic forms property is typically how a logarithm is defined in most texts, I have found that my students are confused by this type of introduction of logarithms. However, when presented as a property following student work in Section 11.3, most of my students pick up on the point of this property quickly.

OBJECTIVE 2 COMMENTS

When solving the equations in Problems 5–12, I ask students, "What is the base?" and "What is the exponent?" I use their responses to convert the equations from logarithmic to exponential form. At some point, we switch roles and I have the students ask these two questions. I do this so that students learn to ask *themselves* these two questions when solving such equations in the homework and on exams.

OBJECTIVE 3 COMMENTS

In Section 10.5, at least one of my students asks whether there is a symbolic way to solve equations of the form $b^x = c$ for x. I motivate the power property by telling groups that they will now learn a property that will allow them to solve such equations.

At some point while discussing Chapter 11, I start to notice students "bringing down the exponent" for an expression that does not contain a logarithm. For example, $2^3 = 2 \cdot 3 = 6$. A quick reminder that $2^3 = 2 \cdot 2 \cdot 2$ sets students' thinking straight.

When "taking log of both sides" of an equation, most of my students think that they are multiplying both sides by log; they treat "log" as if it were a variable. To help my students understand what is really going on, I have them consider solving the linear equation $x + 2 = 5$. I show them that we can solve for x by "taking" f of both sides, where $f(x) = x - 2$.

For Problem 14, many of my students mistakenly write $2(6)^x$ as 12^x. Other students take the log of both sides and then write $\log[2(6)^x]$ as $x \log(2 \cdot 6)$. It is time well spent to show students that these steps are incorrect.

Textbook and Workbook Exploration

The 10-minute "Power property for logarithms" exploration is a nice introduction to the power property. The exploration is probably more convincing to students that the power property is true than a formal proof, although if time permits, I like to demonstrate the proof to my students as well.

Textbook and Workbook Exploration

After discussing the error $\log(ab^x) = x \log(ab)$ and solving at least one equation of the form $ab^x = c$ for x, groups will be ready to do the 15-minute "Comparing the power property with other statements" exploration. Many groups will have a hard time with Problem 1.

OBJECTIVE 4 COMMENTS

For Problem 20, I emphasize that solving the equation $3^x = 5x$ by graphing has really paid off because it is impossible to solve the equation by performing operations on both sides of the equation.

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SECTION 11.5 LECTURE NOTES

Objectives

1. Use the power property for logarithms with exponential models to make predictions.
2. Use the half-life of carbon-14 to date archaeological artifacts and fossils.

Main point: Use the power property with exponential models to make predictions.

Now that we can solve exponential equations, we can make predictions for the explanatory variable of an exponential model.

OBJECTIVE 1

1. A person invests $15,000 in a bank account with a yearly interest rate of 7% compounded annually. When will the balance be $22,000?

2. Suppose a rumor is spreading in the United States that fasting one day per week will add ten years to your life. Assume 10 people have heard the rumor as of today and each day the number of people (both past and present) who will have heard the rumor triples. Predict when all Americans (both past and present) will have heard the rumor. Assume the U.S. population is 328 million.

3. CD album sales have decayed approximately exponentially from 943 million CDs in 2000 to 99 million CDs in 2016 (Source: *RIAA*). Predict in which year the sales will be 43 million CDs.

4. The buying powers of U.S. Latinos are shown in the following table for various years.

Year	Buying Power of U.S. Latinos (trillions of dollars)
1990	0.2
1995	0.3
2000	0.5
2005	0.7
2010	1.0
2015	1.5

Source *The Selig Center for Economic Growth; Hispanic Retail 360; University of Georgia*

Let $f(t)$ be the buying power (in trillions of dollars) of U.S. Latinos at t years since 1990.

a. Use a graphing calculator to draw a scatterplot of the data. Is it better to model the data by using a linear or an exponential function? Explain.

b. Find an equation of f.

c. What is the coefficient a of your model $f(t) = ab^t$? What does it mean in this situation?

d. What is the base b of your model $f(t) = ab^t$? What does it mean in this situation?

e. Find t when $f(t) = 2.5$. What does it mean in this situation?

f. Find $f^{-1}(2.5)$. What does it mean in this situation?

g. Find $f(32)$. What does it mean in this situation?

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OBJECTIVE 2

5. The oldest human footprints on the American continents were found in Managua, Nicaragua. If 48.39% of the carbon-14 remains in the petrified footprints, estimate the age of the footprints. The half-life of carbon-14 is 5730 years.

6. An archaeologist discovers a spear made of wood. If 15% of the wood's carbon-14 remains, how old is the wood? The half-life of carbon-14 is 5730 years.

SHORT HW 1, 5, 7, 13, 17, 21, 27, 29, 35, 37, 39,

MEDIUM HW 1, 5, 7, 9, 13, 17, 21, 23, 27, 29, 33, 35, 37, 39, 41, 43, 45, 47

SECTION 11.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students work with the same applications as in Section 10.5: using the base multiplier property, compound interest, $\frac{1}{2}$-life applications, and working with tables of data. Students use the power property to make predictions for the explanatory variable of exponential models. There are no applications in Section 11.6 and just a few in Section 11.7, so this is the main application section in Chapter 11.

OBJECTIVE 1 COMMENTS

For Problem 1, I remind students why the base of the exponential model is $b = 1.07$ (and not $b = 0.07$). For part (a) of Problem 4, some of my students need a reminder on how to use two points to find an exponential model, especially if the data point $(0,\ 0.2)$ is not used.

For part (f) of Problem 4, I emphasize that $f^{-1}(2.5)$ can be found by solving the exponential equation $2.5 = 0.21(1.083)^t$ (if the model is $f(t) = 0.21(1.083)^t$). I emphasize that parts (e) and (f) should have the same results. This is a good opportunity to sketch some function diagrams like those in Figs. 49 and 50 on page 715 of the textbook. I also compare the meaning of expressions such as $f(2)$ and $f^{-1}(2)$.

This may be a good time to compare how we find $f^{-1}(a)$ when f is a linear model with when f is an exponential model.

Workbook Exploration

In the following 20-minute exploration, groups can participate in what is generally a well-needed review of finding an equation of an exponential model.

Group Exploration

Section Opener: Using the power property to make predictions

The numbers of U.S. lawyers are shown in the following table for various years.

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Year	Numbers of Lawyers (thousands)
1880	64
1900	114
1920	123
1940	181
1960	286
1980	575
2000	1022
2017	1336

Source: *American Bar Association*

Let $n = f(t)$ be the number (in thousands) of lawyers at t years since 1880.

1. Find an equation of f.
2. Find the n-intercept. What does it mean in this situation?
3. Find the exponential growth rate per year of the number of lawyers.
4. Predict the number of lawyers in 2022.
5. Predict when there will be 1.5 million lawyers. [**Hint:** The units of n are *thousands* of lawyers.]

Background Information for the Exploration That Follows

The following 10-minute exploration addresses the true-to-life significance of notations such as $f(c)$ and $f^{-1}(c)$.

EXPLORATION *Inverse of an exponential model*

Assume that there were 1 million bacteria on an apple at noon on Monday and that a bacterium divides into two bacteria every hour, on average. Let $M = f(t)$ be the number of bacteria (in millions) on an apple at t hours after Monday noon. An equation of f is $f(t) = 2^t$.

1. Find $f(3)$. What does it mean in this situation?
2. Find $f^{-1}(8)$. What does it mean in this situation? [**Hint**: Recall that f^{-1} sends outputs of f to inputs of f.]
3. Find $f(4)$. What does it mean in this situation?
4. Find $f^{-1}(16)$. What does it mean in this situation?
5. Find $f^{-1}(1000)$. What does it mean in this situation?
6. Explain why $f^{-1}(M) = \log_2(M)$.

Textbook and Workbook Exploration

The 15-minute "Finding an equation of the inverse of an exponential model" exploration assists students in discovering how to first find an equation of an inverse function and then evaluate the inverse function to make predictions. Problem 5 in the exploration is meant to show that finding the equation of the inverse function is helpful when many predictions for the explanatory variable need to be found.

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OBJECTIVE 2 COMMENTS

For Problem 5, I define $f(t)$ to be the percentage of carbon-14 that remains t years after the petrified footprint formed. I remind students that it helps to create a table of values of f to organize the information:

t	$f(t)$
0	100
5730	50

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SECTION 11.6 LECTURE NOTES

Objectives

1. Apply the *product property* for logarithms.
2. Apply the *quotient property* for logarithms.
3. Solve logarithmic equations.
4. Apply the *change-of-base property*.
5. Compare the quotient property for logarithms with the change-of-base property.

Main point: Know the *product*, *quotient*, and *change of base properties* for logarithms.

OBJECTIVE 1

Product Property for Logarithms

For $x > 0$, $y > 0$, $b > 0$, and $b \neq 1$,

$$\log_b(x) + \log_b(y) = \log_b(xy)$$

Show that the product property works: $\log_2(8) + \log_2(4) = \log_2(8 \cdot 4)$.
A proof of the product property for logarithms follows.
Let $m = \log_b(x)$ and $n = \log_b(y)$. Writing both equations in exponential form, we have

$$x = b^m$$
$$y = b^n$$

Multiplying the left sides and multiplying the right sides yields

$$\begin{aligned} xy &= (b^m)(b^n) \\ &= b^{m+n} \end{aligned}$$

Writing $xy = b^{m+n}$ in logarithmic form gives

$$m + n = \log_b(xy)$$

Substituting $\log_b(x)$ for m and $\log_b(y)$ for n yields:

$$\log_b(x) + \log_b(y) = \log_b(xy)$$

This statement is what we set out to prove.
Simplify. Write the result as a single logarithm with a coefficient of 1.

1. $\log_b(5x) + \log_b(x)$
2. $3\log_b(x) + \log_b(8x)$
3. $4\log_b(x^3) + 3\log_b(2x)$
4. $2\log_b(5x) + 5\log_b(x^3)$

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Warning: To apply the product property for logarithms, the coefficient of each logarithm must be 1.

OBJECTIVE 2

Quotient Property for Logarithms

For $x > 0$, $y > 0$, $b > 0$, and $b \neq 1$,

$$\log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$$

- Show that the quotient property works: $\log_2(32) - \log_2(8) = \log_2\left(\frac{32}{8}\right)$.
- Students can prove the quotient property in Exercise 50 of Homework 11.6.

Simplify. Write the result as a single logarithm with a coefficient of 1.

5. $\log_b(10x) - \log_b(5)$

6. $\log_b(6t^3) - 5\log_b(t)$

7. $4\log_b(2t) - 3\log_b(t^4)$

8. $3\log_b(x^2) + \log_b(7x) - 2\log_b(4x)$

OBJECTIVE 3

Solve. Round any solutions to the fourth decimal place.

9. $\log_2(4x) + \log_2(x) = 5$

10. $\log_7(12x) - \log_7(6) = 2$

11. $2\log_3(x^2) + 3\log_3(2x) = 4$

12. $4\log(2x) - 3\log(x^2) = 3$

OBJECTIVE 4

Change-of-Base Property

For $a > 0$, $b > 0$, $a \neq 1$, $b \neq 1$, and $x > 0$,

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Evaluate. Round your result to the fourth decimal place.

13. $\log_5(31)$

14. $\log_9\left(\frac{1}{75}\right)$

Write the expression as a single logarithm.

15. $\dfrac{\log_4(x)}{\log_4(8)}$

16. $\dfrac{\log_b(w)}{\log_b(k)}$

17. Use a graphing calculator to graph $y = \log_4(x)$.

Solve by using "intersect" on a graphing calculator. Round any solutions to the fourth decimal place.

18. $\log(x+2) + \log(x+7) = 2^x - 3$

19. $\log_2(x+5) + \log_3(x+3) = -2x + 8$

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OBJECTIVE 5
In general,

- $\log_b(x) - \log_b(y) \neq \dfrac{\log_b(x)}{\log_b(y)}$
- $\log_b\left(\dfrac{x}{y}\right) \neq \dfrac{\log_b(x)}{\log_b(y)}$

SHORT HW 1, 3, 7, 11, 13, 15, 19, 23, 25, 35, 37, 41, 45, 47, 52

MEDIUM HW 1, 3, 7, 9, 11, 13, 15, 19, 21, 23, 25, 35, 37, 41, 45, 47, 52, 57, 59, 65, 67, 69, 71, 73

SECTION 11.6 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text
Students study the product property, quotient property, and the change of base property for logarithms in this section. Students use these properties to simplify expressions and to solve equations that contain a sum or difference of two logarithms. Homework 12.5 and 13.5 both contain one or two Related Review exercises where students will use a new skill such as solving rational equations to help solve equations with a sum or difference of two logarithms. All the logarithm properties in Chapter 11 are revisited in Section 11.7.

OBJECTIVE 1 COMMENTS
When discussing the product and quotient properties for logarithms, I emphasize the connection between these properties and the corresponding product and quotient properties of exponents, respectively. Students will see this connection at a greater depth if you prove the product property for logarithms. Also, proving the property gives students a good idea of what a proof is. The proof is challenging for students to follow, but it's within students' grasp because they have been converting expressions from logarithmic form to exponential form (or vice versa) in preceding sections.

For Problems 2–4, I emphasize that the coefficients of the logs must be 1 before we can apply the product property for logarithms. Many students fail to use the power property to get the 1 coefficients unless I warn them several times.

For Problem 4, after I write $2\log_b(5x)$ as $\log_b(5x)^2$, I remind students that we distribute the exponent 2 to *both* of the factors 5 and x of the base. Students often omit squaring the factor 5.

OBJECTIVE 2 COMMENTS
Students are asked to prove the quotient property in Exercise 50 of Homework 11.6.

Textbook and Workbook Exploration
Students can discover the idea behind the product property and the quotient property in the 15-minute "Product and quotient properties for logarithms" exploration.

OBJECTIVE 3 COMMENTS
For Problem 9, once I have written the equation as $\log_2(4x^2) = 5$, I remind students to ask the questions "What is the base?" and "What is the exponent?" to assist in writing the equation in exponential form. Students usually have an easy time with Problems 10–12.

Workbook Exploration
The following 20-minute exploration could be used before demonstrating how to solve any logarithmic equations because students already know how to combine the sum or difference of two logarithms and how to solve an equation with just one logarithm.

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Group Exploration
Section Opener: Solving logarithmic equations

For Problems 1–4, round any solutions to the fourth decimal place.

1. Solve $\log_4(5x^2) = 3$.
2. Solve $\log_3(2x) + \log_3(x) = 4$. [**Hint:** Write the left-hand side of the equation as a single logarithm.]
3. Solve $3\log_5(2x) + 2\log_5\left(x^3\right) = 2$.
4. Solve $4\log_2\left(3x^3\right) - 5\log_2(2x) = 4$.
5. Solve $a\log_b(mx) + c\log_b\left(x^p\right) = k$ for x. Assume $b > 0$, $b \neq 1$, and the constants have values for which the equation has exactly one real-number solution.

OBJECTIVE 4 COMMENTS
For the change of base property, I emphasize that students can now use their calculators to compute logarithms of bases other than 10.

One pitfall of teaching students the change of base property is that they can then use their calculators to find logarithms such as $\log_2(8)$ and avoid thinking about logarithms in terms of exponents. One way to counteract this is to give students problems such as $\log_b(b^3)$ so they won't rely solely on the change of base property. Another way to counteract this is to discuss this property after the exam on logarithms.

Workbook Exploration
In the following 15-minute exploration, students discover the change of base property. Although Problem 1 of the exploration is fairly involved, you will find that many groups can do it.

EXPLORATION *Change of base property for logarithms*

1. First, you will compute $\log_2(7)$. To begin, let $k = \log_2(7)$. Your task is to find the value of k. To do this, write the equation $k = \log_2(7)$ in exponential form. Then solve this exponential equation for k. Your result will be in terms of $\log(2)$ and $\log(7)$. Do not compute these logarithms so that you can notice a pattern in Problem 3.
2. Now do the same process described in Problem 1 for $\log_3(5)$.
3. Describe any pattern suggested by your results for $\log_2(7)$ in Problem 1 and for $\log_3(5)$ in Problem 2.
4. Compute $\log_5(17)$ according to the pattern you described in Problem 3.
5. Write $\log_b(x)$ in terms of $\log(x)$ and $\log(b)$. Explain how you can use this result to find $\log_2(7)$ quickly.

OBJECTIVE 5 COMMENTS
Students tend to confuse the quotient property and the change of base property as shown in the lecture notes.

Textbook and Workbook Exploration
As I mentioned before, many students treat the notation "log" as if it were a variable name, not the name of a function. For example, they believe that the statement $\log(x + y) = \log(x) + \log(y)$ is true because they can distribute "log." The 15-minute "Function of a sum" exploration attends to this issue.

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SECTION 11.7 LECTURE NOTES

Objectives

1. Find *natural logarithms*.
2. Use properties of natural logarithms to simplify expressions and solve equations.
3. Use exponential models with base e to make estimates and predictions.

Main point: Know the meaning of a *natural logarithm*.

OBJECTIVE 1

To the nearest ten-thousandth, $e = 2.7183$.

Definition *Natural Logarithm*

A **natural logarithm** is a logarithm with base e. We write $\ln(a)$ to represent $\log_e(a)$.

For $a > 0$, $\ln(a)$ is the exponent on the base e that gives a.

Exponential/Natural Logarithmic Forms Property

For $a > 0$, the equations $\ln(a) = c$ and $e^c = a$ are equivalent.

1. Use a calculator to find $\ln(40)$.

Find the natural logarithm. Use a calculator to verify your result.

2. $\ln(e^4)$

3. $\ln\left(\dfrac{1}{e^3}\right)$

4. $\ln(e)$ [Exercise 7 of Homework 11.7]

5. $\ln(1)$ [Exercise 8 of Homework 11.7]

For *any* logarithmic function, the domain is the set of all positive real numbers (Section 11.3).

OBJECTIVE 2

Solve. Round any solutions to the fourth decimal place.

6. $\ln(x + 7) = 4$

7. $6\ln(5x) - 3 = 9$

8. $9e^x = 45$

9. $2e^{t+4} = 76$

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For the following properties, assume that $x > 0$, $y > 0$, $b > 0$, and $b \neq 1$ unless stated otherwise:

Properties of Logarithms	**Properties of Natural Logarithms**	
$\log_b(1) = 0$	$\ln(1) = 0$	The natural logarithm of 1 is 0.
$\log_b(b) = 1$	$\ln(e) = 1$	The natural logarithm of e is 1.
$\log_b(b^x) = x$, for real number x	$\ln(e^x) = x$, for real number x	Composing $y = \ln(x)$ with $y = e^x$
$b^{\log_b(x)} = x$	$e^{\ln x} = x$	Composing $y = e^x$ with $y = \ln(x)$
$\log_b(x^p) = p\log_b(x)$	$\ln(x^p) = p\ln(x)$	Power Property
$\log_b(x) + \log_b(y) = \log_b(xy)$	$\ln(x) + \ln(y) = \ln(xy)$	Product Property
$\log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$	$\ln(x) - \ln(y) = \ln\left(\frac{x}{y}\right)$	Quotient Property

We can use the power property for natural logarithms to solve exponential equations.
Solve. Round the solution to the fourth decimal place.

10. $3(4)^w - 6 = 91$

11. $6(2)^x + 17 = 250$

Write the expression as a single logarithm with a coefficient of 1.

12. $3\ln(4x) + 4\ln(x^2)$

13. $4\ln(r^3) - 2\ln(7r)$

Solve. Round any solutions to the fourth decimal place.

14. $2\ln(x^4) + \ln(3x) = 9$

15. $5\ln(2p) - 2\ln(p^2) = 3$

OBJECTIVE 3

16. The following table shows that people with more education are less likely to be unemployed.

Grade-Level Completion or Degree	Full-Time Equivalent Years in School	Unemployment Rate Percent
9th to 12th Grade, no diploma	10	7.3
High school	12	4.2
Some college	13	3.5
Associate	14	2.9
Bachelor's	16	2.5
Master's	18	2.1
Doctoral	20	1.1

Source: *U.S. Census Bureau*

Let $f(t)$ be the unemployment rate for people with a full-time equivalent of t years of education. A model for the situation is

$$f(t) = 32.66e^{-0.16t}$$

a. Verify that f models the situation well.

b. Use f to estimate the unemployment rate for people whose highest degree is a bachelor's. Is this estimate an underestimate or an overestimate?

c. Estimate the full-time equivalent number of years of education for people whose unemployment rate is 1.5%.

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SHORT HW 1, 11, 19, 21, 31, 37, 41, 49, 55, 61, 63, 67, 69, 73, 76

MEDIUM HW 1, 11, 13, 19, 21, 25, 31, 37, 41, 43, 49, 55, 59, 61, 63, 67, 69, 73, 75, 76, 83, 85, 87, 89, 91, 95

SECTION 11.7 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students learn about a natural logarithm in this section. Most of the concepts have been discussed in previous sections. This section is not needed for the rest of the text, so it can be skipped.

OBJECTIVE 1 COMMENTS

If your students understand the concepts in the previous sections of Chapter 11, they should easily pick up on the concepts in this section. However, students wonder why the number e is so important. Any motivation for e in an algebra course is largely artificial because we can rewrite any exponential expression or logarithmic expression in terms of any positive base not equal to one (for example, $e^x = 2^{x \log_2(e)}$).

The text motivates e with a brief reference to a bell curve. With this motivation students have a nice opportunity to use their graphing calculators to draw a graph that would be too challenging for them to do by hand. Further, students can reflect on why such a curve might be appropriate for describing heights of women (or men), IQs of people, diameters of trunks of redwood trees, and so on. For most colleges, a large portion of intermediate algebra students will take statistics. So, this is a worthwhile seed to plant.

When discussing natural logarithms, I emphasize that all the concepts in this section (aside from the value of e) have been addressed in previous sections.

For Problem 2, I emphasize that $\ln$ is simply a logarithm with a specific base. It can help to ask students the value of $\log_b(b^4)$ before finding $\ln(e^4)$.

OBJECTIVE 2 COMMENTS

My students usually have an easy time simplifying expressions and equations that contain natural logarithms. I continue to emphasize that $\ln$ is a logarithm with a specific base. I conclude that all previous properties of logarithms discussed in Sections 11.3 through 11.6 are true for the natural logarithm as well.

OBJECTIVE 3 COMMENTS

Although there is no need to use e as the base of the exponential model in Problem 16, doing so gives students practice working with such models before they see them again in calculus.

Textbook and Workbook Exploration

The 20-minute "Newton's law of cooling" exploration directs groups to find an equation of a model written in terms of e. This exploration serves as good preparation for Exercises 65 and 66 of Homework 11.7.

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CHAPTER 12 OVERVIEW

In Section 12.1, students find the domains of rational functions, work with quotient functions, and simplify rational expressions. They perform operations with rational expressions in Section 12.2 and 12.3. They simplify complex rational expressions in Section 12.4 and solve rational equations in Section 12.5. They use what they have learned in Section 12.5 to help them make estimates and predictions with rational models in Section 12.6 and solve proportions in Section 12.7. In Section 12.8, they use curve fitting in conjunction with direct and inverse variation concepts to perform more modeling. Skills related to multiplying rational expressions will be needed in Section 13.3 and quotient functions will be revisited in Section 13.4.

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SECTION 12.1 LECTURE NOTES

Objectives

1. Describe the meaning of *rational function* and *vertical asymptote*.
2. Find the domain of a rational function.
3. Simplify a *rational expression*.
4. Describe the connection between *vertical asymptotes* of graphs of rational functions and domains of rational functions.
5. Form *quotient functions*.
6. Use a *rational model* to describe the percentage of a quantity.

Main point: Find domains of rational functions and simplify rational expressions.

OBJECTIVE 1

If P and Q are polynomials, where Q is nonzero, we call the ratio $\dfrac{P}{Q}$ a **rational expression.**

Definition *Rational function*

A **rational function** is a function whose equation can be put into the form

$$f(x) = \frac{P(x)}{Q(x)}$$

where $P(x)$ and $Q(x)$ are polynomials and $Q(x)$ is nonzero.

Evaluate $f(x) = \dfrac{3x+1}{x^2-2x+5}$ at the indicated value.

1. $f(3)$

2. $f(-2)$

Warning: For calculator entry, enclose both the numerator and the denominator in parentheses.

OBJECTIVE 2

Find the domain of the function.

3. $f(x) = \dfrac{7}{x-4}$

4. $f(x) = \dfrac{x-2}{x^2-9}$

Domain of a Rational Function

The domain of a rational function $f(x) = \dfrac{P(x)}{Q(x)}$ is the set of all real numbers except for those numbers that, when substituted for x, give $Q(x) = 0$.

Find the domain of the function.

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5. $f(x) = \dfrac{x+5}{x^2 - x - 20}$ **6.** $f(x) = \dfrac{2}{3x^2 + x - 5}$ **7.** $f(x) = \dfrac{2x}{x^2 + 16}$

Definition *Excluded value*

A number is an **excluded value** of a rational expression if substituting the number into the expression leads to a division by 0.

OBJECTIVE 3

- A rational expression is in **lowest terms** if the numerator and the denominator have no common factors other than 1 or -1.
- We **simplify a rational expression** by writing it in lowest terms.

Simplifying a Rational Expression

To simplify a rational expression,

1. Factor the numerator and the denominator.
2. Use the property
$$\frac{AB}{AC} = \frac{A}{A} \cdot \frac{B}{C} = 1 \cdot \frac{B}{C} = \frac{B}{C}$$
where A and C are nonzero, so that the expression is in lowest terms.

Simplify the right-hand side of the equation or simply the expression, whichever is appropriate.

8. $f(x) = \dfrac{21x^2}{14x^5}$ **10.** $f(x) = \dfrac{3 - x}{2x^2 - 3x - 9}$ **12.** $\dfrac{x^2 + 5xy + 6y^2}{x^2 + 6xy + 9y^2}$

9. $f(x) = \dfrac{x^2 - 25}{x^2 - 2x - 15}$ **11.** $f(x) = \dfrac{x^3 - 8}{x^3 - 2x^2 - 9x + 18}$

OBJECTIVE 4

Vertical Asymptotes and Domains of Rational Functions

If the graph of a rational function f has a vertical asymptote $x = k$, then k is not in the domain of f. If k is not in the domain of a rational function g, then $x = k$ may or may not be a vertical asymptote of the graph of g.

To discuss this property, refer to Problems 8–11.

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OBJECTIVE 5

Definition *Quotient Function*

If f and g are functions, x is in the domain of both functions, and $g(x)$ is nonzero, then we can form the **quotient function** $\frac{f}{g}$: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$.

13. Let $f(x) = x^2 - 5x - 14$ and $g(x) = x^2 - 49$.

a. Find an equation of $\frac{f}{g}$.

b. Find $\left(\frac{f}{g}\right)(3)$.

OBJECTIVE 6

A **rational model** is a rational function, or its graph, that describes an authentic situation.

Percentage Formula

If m items out of n items have a certain attribute, then the percentage p (written $p\%$) of the n items that have the attribute is

$$p = \frac{m}{n} \cdot 100$$

We call this equation the **percentage formula**.

14. The numbers of people in the United States who are foreign-born are shown in the following table for various years.

Year	People in the United States Who are Foreign-Born (millions)
1970	9.6
1980	14.1
1990	19.8
2000	31.1
2010	40.0
2016	43.7

Source: *Census Bureau*

Let $F(t)$ be the number (in millions) of people in the United States who are foreign-born and $U(t)$ be the U.S. population (in millions), both at t years since 1970. Reasonable models are

$$F(t) = 0.79t + 7.27$$
$$U(t) = 0.0068t^2 + 2.31t + 202.80$$

For U.S. population data, see Table 51 in Exercise 11 of Homework 9.8.

a. Let $P(t)$ be the percentage of people who live in the United States who are foreign-born at t years since 1970. Find an equation of P.

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b. Use P to estimate the percentage of people who lived in the United States who were foreign-born in 2010. Compare your result to the actual percentage. The number of people living in the United States in 2010 was 308.7 million.

c. Find $P(52)$. What does it mean in this situation?

SHORT HW 1, 9, 13, 19, 25, 33, 45, 55, 59, 65, 73, 77, 81, 87, 91

MEDIUM HW 1, 9, 13, 19, 21, 25, 33, 45, 49, 55, 59, 65, 73, 77, 79, 81, 87, 89, 91, 95, 97, 99, 101, 103

SECTION 12.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students find domains of rational functions and simplify rational expressions. They form quotient functions and use them to describe the percentage of a quantity. Students will use these skills in Sections 12.2–12.6.

OBJECTIVE 1 COMMENTS

I point out that it makes sense that the word "rational" contains the word "ratio" because a rational expression is the ratio of two polynomials.

OBJECTIVE 2 COMMENTS

To begin, I remind my students that division by 0 is undefined. Then we discuss the domain of $f(x) = \dfrac{1}{x}$. For Problem 3, students can determine the domain by inspection. For Problem 4, I set the denominator equal to zero to find the domain. Problem 6 requires the use of the quadratic formula.

I briefly discuss excluded values so that students are prepared to find them when they solve rational equations in Sections 12.5 and 12.6.

OBJECTIVE 3 COMMENTS

I remind my students several times throughout this chapter that the property

$$\frac{AB}{AC} = \frac{B}{C}, \quad \text{where } A \text{ and } C \text{ are nonzero}$$

can be used only if A is a factor both of the (entire) numerator and of the (entire) denominator. Many students have difficulty with this concept. For example, it is a common student error to think that $\dfrac{3x^2 - 5}{2x} = \dfrac{3x - 5}{2}$.

Some of my students have difficulties with Problem 8 because they have forgotten the properties of exponents discussed in Section 10.1.

Problem 10 is challenging for some of my students. To factor the numerator, I write it in descending order as $-x + 3$ and then factor out -1, which is the opposite of the GCF.

Some of my students have trouble with Problem 11 because they've forgotten how to factor the numerator, which is a difference of two cubes, or how to factor the denominator by grouping.

I encourage my students to verify their work with graphing calculator tables and graphs. I discuss how to use parentheses correctly for their equation entries. One way to avoid using as many parentheses is to build functions using Y_n references as described in Appendix B.25.

Textbook and Workbook Exploration

Students can discover how to simplify rational expressions by completing the 15-minute "Simplifying rational expressions" exploration. If your students have seen how to simplify rational functions before, they will probably remember how to do it, but will not know why it is correct to do so.

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OBJECTIVE 4 COMMENTS

The connection between the domain and the vertical asymptotes of a rational function is misunderstood by most students. My students tend to think that if k is not in the domain, then there is a vertical asymptote at $x = k$. Exploring domains and graphs of functions such as $f(x) = \dfrac{x-2}{x-2}$ and $g(x) = \dfrac{x^2-9}{x+3}$ helps.

Textbook and Workbook Exploration

After discussing some examples of the connection between domains and vertical asymptotes, consider assigning the 20-minute "Connection between the domain and vertical asymptotes" exploration as a class activity. Showing examples before assigning this exploration will not spoil the activity. Students will need all the hands-on experience they can get with this concept.

OBJECTIVE 5 COMMENTS

Students have an easy time with this concept.

OBJECTIVE 6 COMMENTS

Many students have trouble with this concept because they have a weak foundation with percents. I first discuss a scenario in which a student earns 4 out of 5 points on a quiz. After calculating the percentage of points that the student earned, I introduce the formula $p = \dfrac{m}{n} \cdot 100$.

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SECTION 12.2 LECTURE NOTES

Objectives

1. Multiply rational expressions.
2. Divide rational expressions.
3. Convert units of quantities

Main point: Multiply and divide rational expressions.

OBJECTIVE 1

Recall how to multiply two fractions such as $\frac{4}{7}$ and $\frac{3}{5}$:

$$\frac{4}{7} \cdot \frac{3}{5} = \frac{4 \cdot 3}{7 \cdot 5} = \frac{12}{35}$$

Multiplying Rational Expressions

If $\frac{A}{B}$ and $\frac{C}{D}$ are rational expressions and B and D are nonzero, then

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$$

Find the product.

1. $\dfrac{3x+5}{x^2} \cdot \dfrac{x^6}{2x-1}$

2. $\dfrac{x^2-2x-3}{3x^2+13x+4} \cdot \dfrac{x^2-16}{x^2-8x+15}$

How to Multiply Rational Expressions

To multiply two rational expressions,

1. Factor the numerators and the denominators.
2. Multiply by using the property $\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$, where B and D are nonzero.
3. Simplify the result.

Find the product.

3. $\dfrac{-2x^2+12x}{25-x^2} \cdot \dfrac{3x-15}{x^2-2x-24}$

4. $\dfrac{a^2-7ab+10b^2}{a^2+6ab+9b^2} \cdot \dfrac{-ab^2-3b^3}{a^2-25b^2}$

5. Let $f(x) = \dfrac{x^2+2x-8}{x^2-7x+12}$ and $g(x) = \dfrac{2x-6}{3x-6}$.

a. Find an equation of $f \cdot g$.

b. Find $(f \cdot g)(3)$.

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OBJECTIVE 2

Recall how to divide two fractions such as $\frac{2}{9}$ and $\frac{5}{7}$:

$$\frac{2}{9} \div \frac{5}{7} = \frac{2}{9} \cdot \frac{7}{5} = \frac{14}{45}$$

Dividing Rational Expressions

If $\frac{A}{B}$ and $\frac{C}{D}$ are rational expressions and B, C, and D are nonzero, then

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C}$$

Find the quotient.

6. $\dfrac{2x^4}{x^2+2x+1} \div \dfrac{8x^6}{x^2-1}$

7. $\dfrac{x^2-5x}{x^2+2x-8} \div \dfrac{x^2+x}{x^2-4}$

How to Divide Rational Expressions

To divide two rational expressions,

1. Write the quotient as a product by using the property $\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C}$, where B, C, and D are nonzero.
2. Find the product.
3. Simplify.

For Problems 8 and 9, find the quotient.

8. $\dfrac{x^3-27}{x^3-x^2-4x+4} \div \dfrac{2x^2+6x+18}{2-x}$.

9. $\dfrac{8a^3-b^3}{4a^2b+2ab^2+b^3} \div \dfrac{4a^2-b^2}{2a^2+ab}$

10. Let $f(x) = \dfrac{x^2+12x+36}{x^2-x}$ and $g(x) = \dfrac{x^2-36}{2x-2}$.

a. Find an equation of $\frac{f}{g}$.

b. Find $\left(\frac{f}{g}\right)(2)$.

11. Perform the indicated operations and simplify the result:

$$\frac{6x^2}{x^2-36} \div \left(\frac{4x^3}{5x-x^2} \cdot \frac{3x-15}{x^2+3x-18}\right)$$

OBJECTIVE 3

12. A couch is 8 feet long. What is its length in inches?

13. A jug contains 5 gallons of water. How many cups of water does it contain?

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14. A car travels at a speed of 65 miles per hour. What is the car's speed in feet per second?

Converting Units

To convert the units of a quantity.

1. Write the quantity in the original units.
2. Multiply by fractions equal to 1 so the units you want to eliminate appear in one numerator and one denominator.

SHORT HW 11, 19, 27, 29, 43, 53, 55, 61, 63, 67, 73, 79, 85, 91, 95, 98

MEDIUM HW 11, 19, 27, 29, 39, 43, 53, 55, 61, 63, 65, 67, 73, 79, 85, 89, 91, 95, 98, 99, 103, 105, 107, 109

SECTION 12.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students multiply and divide rational expressions. They combine two rational functions to form product functions and quotient functions. These skills will be needed in Sections 12.3–12.6.

OBJECTIVES 1 and 2 COMMENTS

Most students have an easy time with this objective.

For Problems 3, 4, 8 and 11, I remind students that if the leading coefficient of a polynomial is negative, then we factor out the opposite of the GCF.

For Problems 6–9, some of my students forget to take the reciprocal of the appropriate rational expression. Some of my students make mistakes when simplifying, usually because they try to simplify before taking the reciprocal.

For Problem 11, I remind students that we first perform operations within parentheses.

To use a graphing calculator table or graph to verify results, I use Y_n references (see Appendix B.25).

Textbook and Workbook Exploration

Students can discover how to multiply and divide rational expressions in the 20-minute "Multiplying and dividing rational expressions" exploration. If they have not divided fractions in a long time, they may need to be reminded that $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, where b, c, and d are nonzero.

OBJECTIVE 3 COMMENTS

Most students have an easy time with this objective. I emphasize that we perform conversions by multiplying by fractions equal to 1.

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SECTION 12.3 LECTURE NOTES

Objectives

1. Add rational expressions with a common denominator.
2. Add rational expressions with different denominators.
3. Subtract rational expressions with a common denominator.
4. Subtract rational expressions with different denominators.

Main point: Add and subtract rational expressions.

OBJECTIVE 1

Recall how to add two fractions with a common denominator such as $\frac{2}{9}$ and $\frac{5}{9}$: $\frac{2}{9}+\frac{5}{9}=\frac{7}{9}$.

Adding Rational Expressions That Have a Common Denominator

If $\frac{A}{B}$ and $\frac{C}{B}$ are rational expressions, where B is nonzero, then $\frac{A}{B}+\frac{C}{B}=\frac{A+C}{B}$.

1. Find the sum $\frac{x^2}{x^2-2x-8}+\frac{3x-28}{x^2-2x-8}$.

OBJECTIVE 2

Suppose that two brothers, John and Paul, own the following numbers and types of musical instruments:

John	Paul
3 guitars	1 guitar
1 bass guitar	2 bass guitars
1 sitar	1 sitar

- The brothers will not give their instruments to each other, but each brother wants to own the same numbers and types of musical instruments that the other brother has. Which instruments do the brothers want?
- Compare this situation with finding the LCD for $\frac{3}{40}+\frac{7}{50}$.

Find the sum.

2. $\frac{5}{8x^2}+\frac{7}{6x^3}$

3. $\frac{5}{x-2}+\frac{3}{x+4}$

4. $\frac{3a}{a^2+5ab+6b^2}+\frac{5b}{a^2-3ab-10b^2}$

5. $\frac{x-2}{x^3+4x^2+3x}+\frac{4}{3x^3+11x^2+6x}$

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How to Add Two Rational Expressions That Have Different Denominators

To add two rational expressions that have different denominators,

1. Factor the denominators of the expressions if possible. Determine which factors are missing.
2. Use the property $\frac{A}{A} = 1$, where A is nonzero, to introduce missing factors.
3. Add the expressions by using the property $\frac{A}{B} + \frac{C}{B} = \frac{A+C}{B}$, where B is nonzero.
4. Simplify.

6. Let $f(x) = \frac{x+2}{x+3}$ and $g(x) = \frac{x+4}{x+5}$.

a. Find an equation of $f + g$.

b. Find $(f + g)(-2)$.

OBJECTIVE 3

Recall how to subtract two fractions with a common denominator such as $\frac{6}{7}$ and $\frac{4}{7}$: $\frac{6}{7} - \frac{4}{7} = \frac{2}{7}$.

Subtracting Rational Expressions That Have a Common Denominator

If $\frac{A}{B}$ and $\frac{C}{B}$ are rational expressions, where B is nonzero, then $\frac{A}{B} - \frac{C}{B} = \frac{A-C}{B}$.

7. Find the difference $\frac{x^2}{x^2-25} - \frac{3x+40}{x^2-25}$.

Warning: When subtracting rational expressions, be sure to subtract the *entire* numerator.

OBJECTIVE 4

Find the difference.

8. $\frac{3}{4ab^3} - \frac{1}{6a^2b^2}$

9. $\frac{5x}{x^2-x-12} - \frac{2x-1}{x^2-3x-4}$

10. $\frac{-3x}{5-3x} - \frac{x+2}{9x^2-25}$

11. $\frac{4p}{p^3-q^3} - \frac{3q}{2p^3+2p^2q+2pq^2}$

How to Subtract Rational Expressions That Have Different Denominators

To subtract two rational expressions that have different denominators,

1. Factor the denominators of the expressions if possible. Determine which factors are missing.
2. Use the property $\frac{A}{A} = 1$, where A is nonzero, to introduce missing factors.
3. Subtract the expressions by using the property $\frac{A}{B} - \frac{C}{B} = \frac{A-C}{B}$, where B is nonzero.
4. Simplify.

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12. Perform the indicated operations.

$$\frac{4}{5x-20} - \left(\frac{3x+7}{x^2-16} + \frac{2}{x+4}\right)$$

13. Let $f(x) = \dfrac{2x}{x^2+7x+10}$ and $g(x) = \dfrac{x+1}{3x+6}$.

a. Find an equation of $f - g$.

b. Find $(f-g)(3)$.

SHORT HW 7, 13, 25, 37, 45, 57, 63, 73, 77, 79, 83, 89, 93, 95, 99

MEDIUM HW 7, 13, 25, 33, 37, 45, 51, 57, 63, 69, 73, 77, 79, 83, 89, 91, 93, 95, 99, 101, 107, 109, 111, 113

SECTION 12.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students add and subtract rational expressions and combine two rational expressions to form sum and difference functions. They will use these concepts in Sections 12.4–12.6.

OBJECTIVES 1 and 2 COMMENTS

I find that the John-Paul analogy (see the lecture notes or the more detailed explanation in the textbook) is extremely effective in getting across the main idea of finding the LCD. The analogy has students focus on the key element of the procedure, which is to determine which factors are missing in each denominator.

I remind my students that they should simplify their results.

Workbook Exploration

Students can discover how to add rational expressions by completing the following 20-minute exploration.

Group Exploration

Section Opener: Adding rational expressions

We can perform the addition for $\dfrac{5}{2} + \dfrac{3}{7}$ by the following steps:

$$\begin{aligned}\frac{5}{2}+\frac{3}{7} &= \frac{5}{2}\cdot 1 + \frac{3}{7}\cdot 1\\ &= \frac{5}{2}\cdot\frac{7}{7} + \frac{3}{7}\cdot\frac{2}{2}\\ &= \frac{35}{14} + \frac{6}{14}\\ &= \frac{35+6}{14}\\ &= \frac{41}{14}\end{aligned}$$

1. Find the sum.

a. $\dfrac{3}{5} + \dfrac{7}{2}$

b. $\dfrac{b}{2} + \dfrac{b}{3}$

c. $\dfrac{x-2}{4x} + \dfrac{x+3}{6x}$

d. $\dfrac{5}{x-3} + \dfrac{2}{x+4}$

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e. $\dfrac{w}{(w+3)(w-5)} + \dfrac{4}{(w-2)(w-5)}$

f. $\dfrac{3}{x^2+3x+2} + \dfrac{2}{x^2+7x+6}$

g. $\dfrac{x+1}{x^2-16} + \dfrac{x-2}{2x-8}$

h. $\dfrac{a+1}{10a^2+13a-3} + \dfrac{a-2}{2a^2-5a-12}$

OBJECTIVE 3 COMMENTS

For Problem 7, most of my students fail to subtract the *entire* numerator $3x + 40$. I revisit this topic many times before we reach the next exam.

OBJECTIVE 4 COMMENTS

For Problem 10, some of my students need to be reminded to factor out the opposite of the GCF of the denominator $5 - 3x$.

Many of my students struggle with Problem 11 because the factoring is fairly challenging.

Textbook and Workbook Exploration

The 15-minute "Adding and subtracting rational expressions" exploration (in the middle of page 781 of the textbook) serves as a nice introduction to this section. Students often have trouble with Problem 3.

Textbook and Workbook Exploration

For the 10-minute "Adding and subtracting rational expressions" exploration (at the bottom of page 781 and continues on page 782 of the textbook) students consider errors or inefficient algorithms typically performed by students. If I assign this exploration and spend ample time discussing these pitfalls, I find that my students rarely make these type of errors on exams.

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SECTION 12.4 LECTURE NOTES

Objective

1. Simplify *complex rational expressions*.

Main point: Simplify complex rational expressions.

OBJECTIVE 1

- A **complex rational expression** is a rational expression whose numerator or denominator (or both) is a rational expression.

- Here are some examples:

$$\frac{\dfrac{x}{x+5}}{\dfrac{3}{x^2-25}} \qquad \frac{\dfrac{1}{a}+\dfrac{2}{a^2}}{\dfrac{3}{4}-\dfrac{1}{a}} \qquad \frac{\dfrac{x-3}{x-2}}{x^2-5x+6}$$

- Here we find the values of two numerical complex rational expressions:

$$\frac{\dfrac{3}{3}}{3}=\frac{3}{1}=3 \qquad \frac{\dfrac{3}{3}}{3}=\frac{1}{3}$$

- From these two examples, we see that it is important to keep track of the main fraction bar (the longest one) of the complex fraction.

- We **simplify a complex rational expression** by writing it as a rational expression $\dfrac{P}{Q}$, with $\dfrac{P}{Q}$ in lowest terms.

Method 1: An expression of the form $\dfrac{R}{S}$ can be written in the form $R \div S$. Use this fact to simplify the following complex rational expression:

$$\frac{\dfrac{2}{9}}{\dfrac{5}{7}}$$

Simplify by method 1.

1. $\dfrac{\dfrac{8}{x^3}}{\dfrac{6}{x^2}}$ **2.** $\dfrac{\dfrac{3x+6}{x^2+7x+12}}{\dfrac{5x+10}{x^2-16}}$ **3.** $\dfrac{\dfrac{5}{ab}-\dfrac{2}{b^2}}{\dfrac{3}{a^3}+\dfrac{4}{ab}}$

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Using Method 1 to Simplify a Complex Rational Expression

To simplify a complex rational expression by method 1,

1. Write both the numerator and the denominator as fractions.
2. Write the complex rational expression as the quotient of two rational expressions:

$$\frac{\frac{A}{B}}{\frac{C}{D}} = \frac{A}{B} \div \frac{C}{D}, \text{ where } B, C, \text{ and } D \text{ are nonzero}$$

3. Divide the rational expressions.

Method 2: First find the LCD of all the fractions in the numerator and the denominator. Then multiply by 1 in the form $\frac{\text{LCD}}{\text{LCD}}$.

Simplify by method 2.

4. $\dfrac{\frac{9}{x^4}}{\frac{12}{x^7}}$

5. $\dfrac{\frac{4x^2-10x}{x^2-11x+18}}{\frac{3x^2-15x}{x^2-81}}$

6. $\dfrac{\frac{7}{3x^2}-\frac{4}{x}}{\frac{9}{2x^3}-\frac{5}{x^2}}$

Using Method 2 to Simplify a Complex Rational Expression

To simplify a rational expression by method 2,

1. Find the LCD of all the fractions appearing in the numerator and denominator.
2. Multiply by 1 in the form $\frac{\text{LCD}}{\text{LCD}}$.
3. Simplify the numerator and the denominator to polynomials.
4. Simplify the rational expression.

For Problems 7 and 8, use the method of your choice.

7. $\dfrac{\frac{5}{x^2-x-6}-\frac{2}{x-3}}{\frac{7}{x^2+3x+2}-\frac{4}{x+1}}$

8. Let $f(x) = \dfrac{5}{x-2} + 3$ and $g(x) = 2 - \dfrac{4}{x+5}$. Find an equation of $\dfrac{f}{g}$.

SHORT HW 5, 11, 13, 19, 23, 33, 37, 41, 43, 45, 49, 51, 55, 59, 63, 69

MEDIUM HW 5, 11, 13, 19, 23, 33, 37, 41, 43, 45, 49, 51, 53, 55, 59, 63, 67, 69, 73, 77, 79, 81, 83, 85

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SECTION 12.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students simplify complex rational expressions using one of two methods called "method 1" (writing such expressions as a quotient of two rational expressions) and "method 2" (multiplying such an expression by $\frac{\text{LCD}}{\text{LCD}}$). For instructors who would like to decrease the amount of symbol manipulation in the course, this may be a good section to skip. Perhaps the most compelling reason to do this section is to prepare students for simplifying difference quotients for rational functions in calculus.

OBJECTIVE 1 COMMENTS

Many reviewers of the textbook swear by method 2. Most of my students prefer method 1. The section has been written so that students can compare method 1 and method 2 and decide for themselves which method they think is best for a particular expression. I discuss method 1 and leave it up to my students to read about and use method 2, if they so choose.

When using method 1, some students use pairs of equality symbols aligned vertically. They seem to treat a complex rational expression as if it were *two* separate expressions. I am not sure what such students are thinking, but the error of using two equality symbols is definitely worth discussing!

When I use method 2 (for a higher level course), I emphasize that the expression

$$\frac{\frac{\text{LCD}}{1}}{\frac{\text{LCD}}{1}}$$

is equal to 1. I remind my students that multiplying an expression by 1 in this form will give an equivalent expression.

For Problem 4, it is a common student error to multiply the original expression by $\frac{\frac{x^4}{1}}{\frac{x^7}{1}}$. Many students make similar mistakes for Problems 5 and 6.

For Problem 3, many of my students perform the following incorrect work:

$$\frac{\frac{5}{ab}-\frac{2}{b^2}}{\frac{3}{a^3}+\frac{4}{ab}} = \left(\frac{5}{ab}-\frac{2}{b^2}\right)\cdot\left(\frac{a^3}{3}+\frac{ab}{4}\right)$$

To make matters worse, some of my students omit using any parentheses.

Textbook and Workbook Exploration

The 15-minute "Simplifying complex rational expressions" exploration guides groups in using method 1 to simplify complex rational expressions.

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SECTION 12.5 LECTURE NOTES

Objectives

1. Solve *rational equations in one variable*.
2. Solve formulas involving rational expressions.
3. Compare solving rational equations with simplifying rational expressions.
4. Use a rational model to makes estimates and predictions for the explanatory variable.

Main point: Solve rational equations.

OBJECTIVE 1

- A **rational equation in one variable** is an equation in one variable in which both sides can be written as rational expressions.
- With rational equations, it is possible to take the usual steps for solving equations, yet arrive at x values that are excluded values for one or more of the fractions in the equation. These values of x are *not* solutions. We call these values **extraneous solutions**.

Solve. Any solution is a real number.

1. $\dfrac{2}{x} - 3 = \dfrac{9}{x} + 4$

2. $\dfrac{3x+2}{x-3} = \dfrac{x+8}{x-3}$ [extraneous solution]

3. $\dfrac{3}{x-4} - \dfrac{x}{x+2} = \dfrac{2x+10}{x^2-2x-8}$ [one extraneous solution, one solution]

4. $\dfrac{2x}{x^2-4x-5} - \dfrac{x+3}{x^2-1} = \dfrac{10}{x^2-6x+5}$ [quadratic formula]

Solving a Rational Equation in One Variable

To solve a rational equation in one variable,

1. Factor the denominator(s) if possible.
2. Identify any excluded values.
3. Find the LCD of all the fractions.
4. Multiply both sides of the equation by the LCD, which gives a simpler equation to solve.
5. Solve the simpler equation.
6. Discard any proposed solutions that are excluded values.

5. Find all complex-number solutions of $\dfrac{x+2}{2x-4} - \dfrac{2x}{x-2} = \dfrac{1}{x^2-3x+2}$.

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OBJECTIVE 2

6. Solve $P = \dfrac{nrT}{V}$ for T.

7. Solve $T = \dfrac{d_1}{r_1} + \dfrac{d_2}{r_2}$ for r_2.

OBJECTIVE 3

Solving a Rational Equation Versus Simplifying a Rational Expression

To solve a rational equation, clear the fractions in it by multiplying both sides of the equation by the LCD. To simplify a rational expression, do *not* multiply it by the LCD—the only multiplication permissible is multiplication by 1, usually in the form $\dfrac{A}{A}$, where A is a nonzero polynomial.

Solve or simplify, as appropriate. For the equations, any solution is a real number.

8. $\dfrac{7}{x} + \dfrac{2}{x-5} - \dfrac{4}{x}$

9. $\dfrac{7}{x} + \dfrac{2}{x-5} = \dfrac{4}{x}$

10. $\dfrac{3x-1}{x^2-9} - \dfrac{5}{x+3} = \dfrac{7}{x-3}$

11. $\dfrac{3x-1}{x^2-9} - \dfrac{5}{x+3} + \dfrac{7}{x-3}$

The result of solving a rational equation is the empty set or a set of one or more numbers. The result of simplifying a rational expression is an expression.

OBJECTIVE 4

12. In Problem 14 of the lecture notes of Section 12.1, we found the model

$$P(t) = \frac{79t + 727}{0.0068t^2 + 2.31t + 202.80}$$

where $P(t)$ is the percentage of people who live in the United States who are foreign-born at t years since 1970 (see the following table and Table 51 in Exercise 11 of Homework 9.8).

Year	People in the United States Who are Foreign-Born (millions)
1970	9.6
1980	14.1
1990	19.8
2000	31.1
2010	40.0
2016	43.7

Source: *Census Bureau*

Predict when 14% of people who live in the United States will be foreign-born.

SHORT HW 1, 7, 21, 29, 35, 43, 47, 51, 55, 63, 69, 71, 77, 83, 87, 89

MEDIUM HW 1, 7, 21, 29, 35, 41, 43, 47, 51, 55, 61, 63, 69, 71, 77, 83, 85, 87, 89, 95, 97, 105, 107, 109

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SECTION 12.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students solve rational equations in this section. They use a rational model to make predictions for the explanatory variable.

OBJECTIVE 1 COMMENTS

After solving Problem 2, I provide an explanation similar to the one just after Example 3 on page 795 of the textbook to show where we introduced the extraneous solution, 3. Almost all students fail to check for extraneous solutions. I tell my students to write "extraneous solutions" at the top of their exam to remind them to check for extraneous solutions.

For Problem 3, after multiplying both sides of the original equation by $(x-4)(x+2)$, some of my students make the following error on the left-hand side:

$$(x-4)(x+2)\left(\frac{3}{x-4}-\frac{x}{x+2}\right)=3-x$$

I remind my students that they cannot "cancel" across a subtraction symbol. Many of my students also have difficulties with the signs due to the difference

$$\frac{3}{x-4}-\frac{x}{x+2}$$

on the left-hand side of the original equation.

Textbook and Workbook Exploration

One way to begin this section is to assign the 15-minute "Solving rational equations" exploration.

Workbook Exploration

Another way to begin this section is to assign the following 25-minute exploration as a class activity. You may want to go over the example in the exploration or a similar example on the board, rather than have students read the example. If you go over another example, make sure it has an extraneous solution(s) so you can discuss this concept. Problems 1 and 3 of this exploration have extraneous solutions.

If students have seen this material before, they should make good progress with the problems in this exploration.

EXPLORATION *Solving rational equations*

The steps for solving the rational equation $\frac{x}{x+2}+\frac{7}{x-5}=\frac{14}{x^2-3x-10}$ are shown below:

$$\begin{aligned}\frac{x}{x+2}+\frac{7}{x-5} &= \frac{14}{x^2-3x-10}\\ \frac{x}{x+2}+\frac{7}{x-5} &= \frac{14}{(x-5)(x+2)}\end{aligned}$$

The excluded values are 5 and -2.

We can get rid of the denominators that appear in this equation by multiplying both sides of the equation by the LCD, $(x+2)(x-5)$:

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$$\begin{aligned}
(x+2)(x-5)\cdot\left(\frac{x}{x+2}+\frac{7}{x-5}\right) &= (x+2)(x-5)\cdot\frac{14}{(x-5)(x+2)}\\
(x+2)(x-5)\cdot\frac{x}{x+2}+(x+2)(x-5)\cdot\frac{7}{x-5} &= 14\\
(x-5)\cdot x+(x+2)\cdot 7 &= 14\\
x^2-5x+7x+14 &= 14\\
x^2+2x+14 &= 14\\
x^2+2x &= 0\\
x(x+2) &= 0\\
x=0 \quad \text{or} \quad x+2&=0\\
x=0 \quad \text{or} \quad x&=-2
\end{aligned}$$

It *appears* that the solutions are 0 and -2. However, recall that -2 is an excluded value, so the only solution is 0.

Solve:

1. $\dfrac{x}{x-3}=4+\dfrac{3}{x-3}$

2. $\dfrac{5}{x+1}=\dfrac{2}{x-5}-\dfrac{1}{x+1}$

3. $\dfrac{x}{x-3}-\dfrac{7}{x+5}=\dfrac{24}{x^2+2x-15}$ [**Hint**: Begin by factoring $x^2+2x-15$.]

4. $\dfrac{6}{(x+2)(x-4)}=\dfrac{4}{(x+2)(x+1)}-\dfrac{3}{(x-4)(x+1)}$

OBJECTIVE 2 COMMENTS

If students are troubled by Problem 7, I first solve a similar equation in one variable such as $7=\dfrac{2}{3}+\dfrac{5}{r_2}$ to help students see the big picture.

OBJECTIVE 3 COMMENTS

My students often confuse simplifying expressions with solving equations. This issue has been addressed in various Related Review exercises. However, most of my students still need this issue to be discussed in this section. Without frequent warnings, many students introduce multiplications of rational expressions by expressions not equivalent to 1. Many students multiply both sides of an equation by an expression equivalent to 1, rather than simply multiplying both sides by the LCD.

Textbook and Workbook Exploration

The 15-minute "Simplifying versus solving" exploration addresses some student misconceptions about simplifying rational expressions and solving rational equations.

OBJECTIVE 4 COMMENTS

Although Problem 12 first seems challenging to my students, most of them can solve such a problem on an exam, provided that they are proficient in using the quadratic formula.

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SECTION 12.6 LECTURE NOTES

Objectives

1. Use a rational function to model the *mean of a quantity*.
2. Model the percentage of a quantity.
3. Use a rational function to model the time it takes to travel a given distance at a constant speed.

Main point: Use rational functions to model authentic situations.

OBJECTIVE 1

Suppose that four students spend a total of \$40 to buy refreshments for a party. Consider the following possibilities:

- Each person pays \$10 for refreshments.
- The amounts contributed for refreshments are \$9, \$9, \$11, and \$11.
- One student pays \$40 and the other three students do not contribute for refreshments.

Compute the mean amount of money each student spent for refreshments. Discuss how well the mean estimates the per-person contribution.

Computing the Mean

If a quantity Q is divided into n parts, the **mean** amount M of the quantity per part is given by $M = \dfrac{Q}{n}$.

1. The underground band Melted Zipper wants to make and sell a CD of its original songs. It costs about \$1000 to record the music onto a digital audiotape (DAT), \$100 to rearrange the music and improve the sound quality, \$350 for artwork for the cover and inside leaflet, and \$350 to set up production. In addition, it will cost \$2.50 for each CD manufactured.

   a. What is the total cost of making 300 CDs?

   b. Let $C(n)$ be the total cost (in dollars) of making n CDs. Find an equation of C.

   c. Let $P(n)$ be the price (in dollars) the band should set for each CD so it breaks even by making and selling n CDs. Find an equation of P.

   d. Find $P(300)$. What does it mean in this situation?

   e. Find n when $P(n) = 10$. What does it mean in this situation?

   f. Describe the values of $P(n)$ for large values of n.

2. In Problem 3 of the lecture notes of Section 5.5, we modeled the total U.S. consumption of avocados. A reasonable model is $A(t) = 189t - 645$, where $A(t)$ is the annual total U.S. consumption (in *millions* of pounds) at t years since 2000 (see the following table). The function $U(t) = 0.0068t^2 + 2.72t + 278.3$ models the U.S. population (in millions) at t years since 2000 (see Table 51 in Exercise 11 of Homework 9.8).

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Year	Consumption of Avocados (millions of pounds)
2009	1150
2010	1320
2011	1240
2012	1590
2013	1790
2014	1970
2015	2280
2016	2390

Source: *USDA*

a. Let $M(t)$ be the mean annual per-person consumption (in pounds) of avocados at t years since 2000. Find an equation of M.

b. Find $M(10)$. What does it mean in this situation?

c. Find t when $M(t) = 10$. What does it mean in this situation?

OBJECTIVE 2

3. In Problem 3 of the lecture notes of Section 9.9, we modeled the annual revenues (in billions of dollars) from digital videogame products and physical videogame products, $f(t)$ and $g(t)$, respectively, by the system

$$f(t) = -0.19t^2 + 5.43t - 2.7$$
$$g(t) = 0.16t^2 - 1.98t + 19.1$$

where t is the number of years since 2010 (see the following table).

	Revenue (billions of dollars)	
Year	Digital	Physical
2012	7.5	15.8
2013	11.7	14.6
2014	15.8	13.8
2015	20.0	13.3
2016	22.8	13.1

Source: *SuperData Research, Inc.*

a. Find an equation of the sum function $f + g$. What do the inputs and the outputs of $f + g$ mean in this situation?

b. Let $P(t)$ be the percentage of revenue from videogame products that are from digital products at t years since 2010. Find an equation of P.

c. Estimate when two-thirds of the revenue from videogame products will be from digital products.

OBJECTIVE 3

Distance-Speed-Time Relationship

If an object is moving at a constant speed s for an amount of time t, then the distance d traveled is given by $d = st$ and the time is given by $t = \frac{d}{s}$.

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4. A student at Seattle Central Community College plans to drive from Seattle, Washington, to Eugene, Oregon. The speed limit is 70 mph in Washington and 65 mph in Oregon. She will drive 164 miles in Washington, then 121 miles in Oregon.

a. If the student drives steadily at the speed limits, compute the driving time.

b. If the student exceeds the speed limits, let $T(a)$ be the driving time (in hours) at a mph above the speed limits. Find an equation of T.

c. Find $T(0)$. Compare this speed with the result in part (a).

d. If the student drives 5 mph over the speed limits, compute the driving time.

e. If the student wants the driving time to be 4 hours, how much over the speed limits would she have to drive?

SHORT HW 1, 5, 7, 9, 13, 19, 23, 25

MEDIUM HW 1, 5, 7, 9, 11, 13, 15, 17, 19, 23, 25, 27, 29, 31

SECTION 12.6 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students make estimates and predictions using rational models.

OBJECTIVE 1 COMMENTS

Problem 1 is the same as Example 1 of the textbook on pages 806 and 807. I've included this application in both places because most students find this situation interesting.

For part (a) of Problem 2, some students will not be sure whether $M(t) = \dfrac{A(t)}{U(t)}$ or $M(t) = \dfrac{U(t)}{A(t)}$. Performing a unit analysis of each equation should help.

In Problem 3 of the lecture notes of Section 5.5, we modeled the annual consumption of avocados where the units of annual consumption were in *billions* of pounds. As an extra challenge, you could continue working with the model using those units: $A(t) = 0.189t - 0.645$; then the equation of M would be given by

$$M(t) = \frac{1000A(t)}{U(t)}$$

Textbook and Workbook Exploration

In the 20-minute "Modeling the mean of a quantity" exploration, many groups will likely need a hint for part (b) of Problem 2. The hint could be as simple as having groups reflect on the computations they performed in Problem 1 of the exploration.

OBJECTIVE 2 COMMENTS

For Problem 3, although finding percentage models was already discussed in Sections 12.1 and 12.5, this problem requires students to first form the sum function $f + g$ before finding an equation of P. This is a challenging problem for students mostly because of its length.

Workbook Exploration

In the following 20-minute exploration, groups can practice finding a percentage model.

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Group Exploration

Section Opener: Using a rational model to make predictions

The numbers of households that stream Netflix are shown in the following table for various years.

Year	Households That Stream Netflix (millions)	Total Number of Households (millions)
2012	23.4	121.1
2013	29.2	122.5
2014	35.7	123.2
2015	41.4	124.6
2016	47.0	125.8

Source: *Netflix; U.S. Census Bureau*

1. Let $f(t)$ be the number (in millions) of households that stream Netflix at t years since 2010. Find an equation of f.
2. Let $g(t)$ be the number (in millions) of households at t years since 2010. Find an equation of g.
3. Let $p(t)$ be the percentage of households that stream Netflix at t years since 2010. Find an equation of p.
4. Predict the percentage of household that will stream Netflix in 2022.
5. Predict when 62% of households will stream Netflix.

OBJECTIVE 3 COMMENTS

I emphasize that the model $d = st$ can only be applied for constant-rate situations.

For part (b) of Problem 4, I refer to the expression $\frac{164}{70}+\frac{121}{65}$ to suggest the expression $\frac{164}{a+70}+\frac{121}{a+65}$. Note that Problem 4 is a more realistic motion problem than the motion problems included in most algebra textbooks.

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SECTION 12.7 LECTURE NOTES

Objectives

1. Use *proportions* to make estimates.
2. Find the length of a side of a *similar triangle*.

Main point: Use proportions to make estimates. Work with similar triangles.

OBJECTIVE 1

- A **proportion** is a statement of the equality of two ratios.
- Here is a proportion: $\frac{6}{8} = \frac{3}{4}$.

If the ratio of two related quantities is constant, we say that the two quantities are **proportional**.

1. A person pays \$297 for 6 months' use of cable service. How much will the person pay for 8 months' use of cable service?

2. The weight of an object on Earth and the weight of the same object on the Moon are proportional. An astronaut who weighs 160 pounds on Earth weighs 26.5 pounds on the Moon. What is the weight of a person on Earth if she weighs 24.0 pounds on the Moon?

3. A family spends \$416 on groceries in a 4-month period.

a. Estimate the family's total money for groceries during a 6-month period.

b. Discuss any assumptions that you made in part (a).

4. In a poll of 1000 adults, 770 adults said that engineers are largely responsible for Americans having a high standard of living (Source: *Harris Interactive, Inc.*).

a. Estimate how many of 12,500 students at a college would say that engineers are largely responsible for Americans having a high standard of living.

b. Discuss any assumptions that you made in part (a). Describe a possible scenario in which the result in part (a) is an underestimate.

OBJECTIVE 2

- Two triangles are called **similar triangles** if their corresponding angles are equal in measure.
- The two triangles in Fig. 2.7 are similar triangles because the measures of angles A and X are equal, the measures of angles B and Y are equal, and the measures of angles C and Z are equal.

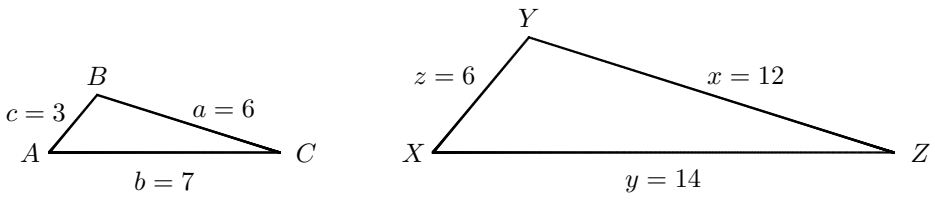

Figure 2.7: Two similar triangles

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- Here we find the ratio of the lengths of corresponding sides of the two triangles in Fig. 2.7:

$$\frac{c}{z} = \frac{3}{6} = \frac{1}{2}, \qquad \frac{a}{x} = \frac{6}{12} = \frac{1}{2}, \qquad \frac{b}{y} = \frac{7}{14} = \frac{1}{2}$$

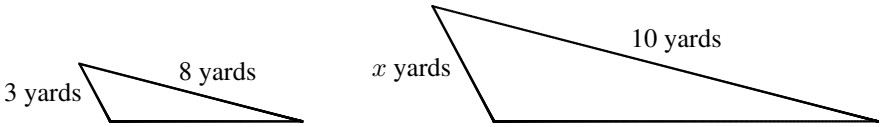
  Notice that all three ratios are equal.

Side Lengths of Similar Triangles

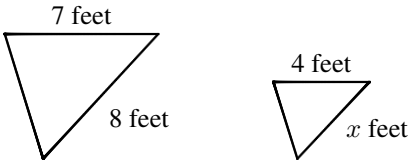
The lengths of the corresponding sides of two similar triangles are proportional.

Similar triangles have the same shape but not necessarily the same size.

5. The two triangles shown in the following figure are similar. Find the length of the side labeled "x yards" on the larger triangle. Round your result to the second decimal place.



6. The two triangles shown in the following figure are similar. Find the length of the side labeled "x feet" on the smaller triangle. Round your result to the second decimal place.



SHORT HW 1, 7, 9, 11, 17, 19, 25, 29

MEDIUM HW 1, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 29, 33, 35, 37, 39

SECTION 12.7 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students use proportions to make estimates about authentic situations and find the length of a side of a similar triangle.

OBJECTIVE 1 COMMENTS

Students have an easy time working with proportions. To begin, I quickly remind students of the meaning of a ratio. When setting up proportions, I emphasize that it's important to be consistent in describing quantities in the numerators and the denominators. I remind students that they should describe their result using units and a complete sentence.

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Part (b) of Problem 3 and part (b) of Problem 4 are great opportunities for critical thinking. Most algebra textbooks do not address the dangers of generalizing from a sample. This is unfortunate, because without such a discussion, work with proportions can be misleading.

Textbook and Workbook Exploration

The 15-minute "Proportions" exploration is a nice way to introduce the concept of proportion.

OBJECTIVE 2 COMMENTS

Again, I emphasize the importance of being consistent. This time, students must be consistent in describing the lengths of the sides of the triangles in the numerators and the denominators of a proportion.

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SECTION 12.8 LECTURE NOTES

Objectives

1. Describe the meaning of *direct variation* and *inverse variation*.
2. In direct variation and inverse variation, describe how a change in the explanatory variable affects the value of the response variable.
3. Use a single point to find a *direct variation equation* or an *inverse variation equation*.
4. Use direct variation models and inverse variation models to make estimates.
5. Use ratios to find a direct variation constant, and use products to find an inverse variation constant.

Main point: Work with *direct variation equations* and *inverse variation equations*.

OBJECTIVE 1

1. A person drives d miles at 50 mph for t hours. Find an equation for t and d.

Definition *Direct Variation*

If $y = kx$ for some nonzero constant k, we say $\boldsymbol{y}$ **varies directly as** $\boldsymbol{x}$ or that $\boldsymbol{y}$ **is proportional to** $\boldsymbol{x}$. We call k the **variation constant** or the **constant of proportionality**. The equation $y = kx$ is called a **direct variation equation**.

OBJECTIVES 2 and 3

Graph $y = \frac{1}{2}x$, $y = x$, and $y = 2x$ to motivate the following property.

Changes in Values of Variables for Direct Variation

Assume that y varies directly as x with some positive variation constant k:

- If the value of x increases, then the value of y increases.
- If the value of x decreases, then the value of y decreases.

2. The variable y varies directly as x with positive variation constant k.

a. What happens to the value of y as the value of x increases?

b. If $y = 7$ when $x = 5$, find an equation for x and y.

3. The variable p varies directly as r. If $p = 5$ when $r = 2$, find the value of r when $p = 7$.

OBJECTIVE 4

4. For a resident of Jackson County, Michigan, the total cost of tuition at Jackson Community College varies directly as the number of billing contact hours (units or credits) a student takes. For the academic year 2016–2017, the total cost of 9 billion contact hours is \$1215. Find the total cost of 12 billing contact hours.

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Finding and Using a Direct Variation Model

Assume a quantity p varies directly as a quantity t. To make estimates about an authentic situation:

1. Substitute the values of a data point into the equation $p = kt$; then solve for k.
2. Substitute the value of k you found in step 1 into the equation $p = kt$.
3. Use the equation you found in step 2 to make estimates of quantity t or p.

Give examples of functions such as $y = kx^2$ in which the explanatory variable varies directly as an *expression*.

OBJECTIVE 1 (revisited)

Definition *Inverse Variation*

If $y = \dfrac{k}{x}$ for some nonzero constant k, we say $\boldsymbol{y}$ **varies inversely as** $\boldsymbol{x}$ or that $\boldsymbol{y}$ **is inversely proportional to** $\boldsymbol{x}$. We call k the **variation constant** or the **constant of proportionality**. The equation $y = \dfrac{k}{x}$ is called an **inverse variation equation**.

OBJECTIVES 2 and 3 (revisited)

Graph $y = \dfrac{1}{x}$, $y = \dfrac{2}{x}$, and $y = \dfrac{4}{x}$ to motivate the following property.

Changes in Values of Variables for Inverse Variation

Assume y varies inversely as x for some positive variation constant k. For positive values of x:

- If the value of x increases, then the value of y decreases.
- If the value of x decreases, then the value of y increases.

5. The variable y varies inversely as x with positive variation constant k.

a. What happens to the value of y as the value of x increases?

b. If $y = 5$ when $x = 3$, find an equation for x and y.

6. The variable w varies inversely as t. If $w = 4$ when $t = 3$, find the value of t when $w = 6$.

OBJECTIVE 4 (revisited)

7. When a stone is tied to a string and whirled in a circle at constant speed, the tension in the string varies inversely as the radius of the circle. If the radius is 55 centimeters, the tension is 80 newtons. Find the radius if the tension is 90 newtons.

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Finding and Using an Inverse Variation Model

Assume a quantity p varies inversely as a quantity t. To make estimates about an authentic situation:

1. Substitute the values of a data point into the equation $p = \frac{k}{t}$; then solve for k.
2. Substitute k value you found in step 1 into the equation $p = \frac{k}{t}$.
3. Use the equation you found in step 2 to make estimates of quantity t or p.

Give examples of functions such as $y = \frac{5}{x^2}$ in which the explanatory variable varies inversely as an *expression*.

8. A person plays an electric guitar outside. The sound level (in decibels) varies inversely as the square of the distance (in feet) from the amplifier. If the sound level is 112 decibels at 35 feet from the amplifier, what is the sound level at 45 feet?

OBJECTIVE 5

Go over or refer to Examples 9 and 10 in Section 12.8 in the text (pages 831 and 832 of Homework 12.8).

SHORT HW 1, 5, 11, 13, 19, 23, 25, 31, 33, 37, 41, 43, 45, 53, 59, 65

MEDIUM HW 1, 5, 11, 13, 15, 19, 23, 25, 27, 31, 33, 37, 41, 43, 45, 49, 53, 57, 59, 65, 69, 71, 75, 77

SECTION 12.8 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students learn about direct and inverse variation in this section. They use variation models to make estimates.

OBJECTIVE 1 COMMENTS

When discussing direct variation, I emphasize that an equation such as $y = kx$ describes a linear function. Hence, most of the concepts associated with direct variation are review material. I point out that if one quantity y varies directly as another quantity x, linear regression may generate a model of the form $y = kx + b$ where b is near zero and explain that b does not turn out to be zero due to experimental error.

I make sure that my students understand the difference between direct variation and an increasing function. Most people (for example, in the media) state that two quantities vary directly when what they mean is that the quantities can be modeled by an increasing function. Exercise 57 of Homework 12.8 addresses this confusion.

Textbook and Workbook Exploration

The 15-minute "Comparing methods of finding a direction variation equation" exploration ties together what students learned about linear models in Chapters 1–5 with the current material.

OBJECTIVES 2 and 3 COMMENTS

For the equation $y = kx$, I emphasize how the value of y changes as the value of x increases or decreases.

OBJECTIVE 4 COMMENTS

I spend some time discussing the significance of k for a model of the form $y = kx$. For example, when modeling the bounce height y of a ball for a drop height x, the textbook discusses that k takes into account how "bouncy" the ball is (see the bottom of page 826 of the textbook).

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And for Problem 4 in the lecture notes, I remind students that the slope of the graph of the tuition model, 104.46, means that the cost per credit is $104.46. Discussing the domain and range of the tuition model can serve as a nice review, although domain and range won't be addressed in Homework 12.8.

OBJECTIVE 1 (REVISITED) COMMENTS

I spend more time discussing inverse variation than direct variation because students are less familiar with graphs and tables of equations of the form $y = \frac{k}{x^p}$, where k and p are constants.

OBJECTIVES 2 and 3 (REVISITED) COMMENTS

For the equation $y = \frac{k}{x}$, I emphasize how the value of y changes as the value of x increases or decreases.

OBJECTIVE 4 (REVISITED) COMMENTS

For Problem 7, I emphasize that the phrase "varies inversely" is important. Many students who do not read the problem carefully treat such a problem as a direct variation application.

For Problem 8, I emphasize the word "square." Many of my students who do not read the problem carefully write the equation $L = \frac{k}{d}$, where L is the sound level (in decibels) and d is the distance (in feet).

Textbook and Workbook Exploration

The 15-minute "Inverse variation" exploration has groups consider asymptotic behavior for a model of the form $y = \frac{k}{x}$.

OBJECTIVE 5 COMMENTS

Examples 9 and 10 of Section 12.8 in the textbook demonstrate far more realistic applications than what's included in most textbooks. However, it is difficult to discuss this with students due to time constraints.

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CHAPTER 13 OVERVIEW

In section 13.1, students simplify radical expressions. In Sections 13.2 and 13.3, they perform operations with radical expressions. They sketch graphs of square root functions and continue work with algebra of functions in section 13.4. In section 13.5, students solve square root equations. In section 13.6, students perform curve fitting with square root models.

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SECTION 13.1 LECTURE NOTES

Objectives

1. Evaluate radicals.
2. Convert expressions in radical form to exponential form and vice versa.
3. Simplify radical expressions.
4. Describe the meaning of *radical function*, *square root function*, *radical model*, and *square root model*.
5. Use a radical model to make an estimate or prediction.

Main point: Simplify radical expressions.

OBJECTIVE 1

- Recall that we can represent $x^{1/n}$ as $\sqrt[n]{x}$, where n is a counting number greater than 1.
- We say n is the **index** and the symbol $\sqrt{\ }$ is the **radical sign**.
- The notation $\sqrt{x}$ is shorthand for $\sqrt[2]{x}$.
- An expression under the radical sign is called a **radicand**.
- A radical sign together with an index and a radicand is called a **radical**.

Evaluate the radical.

1. $\sqrt[4]{16}$ **2.** $\sqrt[3]{-64}$

OBJECTIVE 2

- An expression that contains a radical is called a **radical expression**.
- A number that has an nth root that is rational is called a **perfect nth power**.
- For example, 8 is a perfect 3rd power, or **perfect cube**.

Recall from Section 10.2 that if $a^{1/n}$ is defined, then

$$a^{m/n} = \left(a^{1/n}\right)^m = (a^m)^{1/n}$$

or we can write

$$a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

We say the expressions $\left(\sqrt[n]{a}\right)^m$ and $\sqrt[n]{a^m}$ are in *radical form* and that the expression $x^{m/n}$ is in *exponential form*.

If the expression is in exponential form, write it in radical form. If it is in radical form, write it in exponential form.

3. $x^{2/5}$ **4.** $\sqrt[7]{x^4}$ **5.** $(4x+2)^{3/8}$ **6.** $\sqrt{7x+3}$

OBJECTIVE 3

Write the expression in exponential form; then simplify the result. Assume x is nonnegative.

7. $\sqrt[4]{x^4}$ **8.** $\sqrt[4]{x^{12}}$ **9.** $\sqrt[4]{x^{28}}$

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Product Property for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are defined, then $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$.

Here is the proof:

$$\sqrt[n]{ab} = (ab)^{1/n} = a^{1/n}b^{1/n} = \sqrt[n]{a}\,\sqrt[n]{b}$$

A radical with index n is **simplified** when the radicand does not have any factors that are perfect nth powers (other than 1 or -1) and the index is as small as possible.

Simplify. Assume x is nonnegative.

10. $\sqrt{98}$ **11.** $\sqrt{x^9}$ **12.** $\sqrt{18x^{13}}$

When simplifying a radical,

- If the radicand has more than one numerical factor that is a perfect nth power, select the largest one.
- If the radicand has more than one factor that is a perfect nth power with the same variable base, select the one with the largest exponent.

Simplify. Assume all variables are nonnegative.

13. $\sqrt{45x^{10}y^7}$ **15.** $\sqrt{(2x+7)^5}$ **17.** $\sqrt[4]{x^{31}}$ **19.** $\sqrt[5]{64x^{30}y^{23}}$

14. $\sqrt{50x^3y^{12}}$ **16.** $\sqrt[3]{x^{17}}$ **18.** $\sqrt[3]{24x^{22}}$ **20.** $\sqrt[4]{(6x+1)^{22}}$

- Show how to write $\sqrt[9]{x^6}$ with a smaller index.
- If $\sqrt[n]{x}$ is defined and the fraction $\dfrac{m}{n}$ can be simplified, then we can decrease the index of $\sqrt[n]{x^m}$ by writing $\sqrt[n]{x^m} = x^{m/n}$ and simplifying the exponent $\dfrac{m}{n}$.

Simplify. Assume all variables are nonnegative.

21. $\sqrt[6]{x^4}$ **22.** $\sqrt[10]{(2x+9)^5}$ **23.** $\sqrt[8]{16}$ **24.** $\sqrt[6]{64x^8}$ **25.** $\sqrt[3]{\sqrt{x}}$

Simplifying a Radical Expression

To simplify a radical expression with index n,

1. Find perfect nth-power factors of the radicand.
2. Apply the product property for radicals.
3. Find the nth root of each perfect nth power.
4. Write the radical with as small an index as possible.

Give examples to motivate the following property.

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Power Property for Radicals

Let n be a counting number greater than 1:

- If n is even, then $\sqrt[n]{x^n} = |x|$.
- If n is odd, then $\sqrt[n]{x^n} = x$.

OBJECTIVE 4

A **radical function** is a function whose equation contains a radical with a variable in the radicand.

For $f(x) = 2\sqrt{x} - 5$ and $g(x) = \sqrt[4]{8x+1}$, find the following.

26. $f(36)$

27. $g(10)$

A **square root function** is a radical function in which any radicals are square root radicals.

28. Graph $f(x) = \sqrt{x}$ by hand.

OBJECTIVE 5

- A **radical model** is a radical function, or its graph, that describes an authentic situation.
- A **square root model** is a square root function, or its graph, that describes an authentic situation.

29. The percentages of U.S. households that are one-person households are shown in the following table for various years.

Year	Percentage of U.S. Households That Are One-Person Households
1960	13.3
1970	17.6
1980	22.7
1990	24.6
2000	25.8
2010	26.7
2017	27.9

Source: *Census Bureau*

Let $f(t)$ be the percentage of U.S. households that are one-person households at t years since 1960. A reasonable model is $f(t) = 1.95\sqrt{t} + 13.3$.

a. Does f fit the data well?

b. Predict the percentage of U.S. households that will be one-person households in 2022.

c. Estimate the *number* of U.S. households that were one-person households in 2009. There were 117.2 million households in 2009.

SHORT HW 1, 9, 17, 25, 29, 39, 43, 53, 63, 69, 71, 81, 85, 87, 92

MEDIUM HW 1, 9, 17, 25, 29, 37, 39, 43, 53, 55, 63, 69, 71, 79, 81, 83, 85, 87, 91, 92, 93, 95, 97, 99

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SECTION 13.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn about properties of radicals and how to apply these properties to simplify radical expressions.

OBJECTIVES 1 and 2 COMMENTS

Students will need a reminder that $x^{1/n}$ can be written as $\sqrt[n]{x}$, where n is a counting number greater than 1 (Section 10.2). I emphasize the vocabulary in this section because students are not very familiar with it and I use this terminology frequently in Chapter 13.

Textbook and Workbook Exploration

Groups can learn how to simplify radical expressions by writing them in exponential form by completing the 10-minute "Simplifying radical expressions" exploration.

OBJECTIVES 2 and 3 COMMENTS

For Problems 11 and 12, it can help to list the perfect squares x^2, x^4, x^6, x^8, x^{10}, and x^{12}. For Problem 12, some students do not immediately see why it is better to write x^{13} as $x^{12} \cdot x$, rather than some other product such as $x^6 \cdot x^7$.

For Problem 16, some of my students are surprised that writing x^{17} as $x^{16} \cdot x$ is not beneficial. I point out that we now need to factor out perfect *cubes*, not squares, and list the perfect cubes x^3, x^6, x^9, x^{12} and x^{15}.

The power property is a difficult concept for most students and probably should be discussed toward the end of class after students have first had some experience applying it to radical expressions where all variables are positive. This property can be skipped because we will assume all variables are nonnegative when indexes are even.

Workbook Exploration

The following 15-minute exploration is a great opportunity for students to learn how to prove a property. Students should be able to prove the index property in Problem 2 of the exploration after having seen you prove the product property and after they have completed Problem 1 of the exploration.

Group Exploration

Section Opener: Index property for radicals

Assume $x \geq 0$.

1. Write the expression with as small an index as possible. [**Hint:** First write the expression in exponential form.]

a. $\sqrt[8]{x^6}$ **b.** $\sqrt[6]{x^4}$ **c.** $\sqrt[16]{b^{10}}$

d. $\sqrt[9]{x^3}$ **e.** $\sqrt[22]{w^4}$ **f.** $\sqrt[30]{x^{25}}$

2. Let k, m, and n be counting numbers, where m and n have no common factors. Write the expression $\sqrt[kn]{x^{km}}$ with as small an index as possible.

3. Describe what your result from Problem 2 tells you about simplifying a radical expression. Use this observation to simplify $\sqrt[20]{x^{16}}$ in one step.

OBJECTIVE 4 COMMENTS

Problem 28 lays a foundation for section 13.4.

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OBJECTIVE 5 COMMENTS

It is a good idea to do Problem 29 because it is with radical equations that students are most likely to wonder how this material could possibly be used to describe authentic situations.

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SECTION 13.2 LECTURE NOTES

Objectives

1. Add and subtract radical expressions.
2. Multiply radical expressions.
3. Apply another version of the power property for radicals.
4. Simplify the square of a radical expression that has two terms.

Main point: Add, subtract, and multiply radical expressions.

OBJECTIVE 1

- Radicals that have the same index and the same radicands are called **like radicals**. For example, $2\sqrt[3]{5x}$ and $6\sqrt[3]{5x}$ are like radicals.
- To add or subtract like radicals, we use the distributive property. Here we add two like radicals:

$$2\sqrt{x} + 6\sqrt{x} = (2+6)\sqrt{x} = 8\sqrt{x}$$

- When we add or subtract like radicals, we say we *combine like radicals*.

Combine like radicals.

1. $4\sqrt{x} + 2\sqrt{x}$

2. $3\sqrt[4]{2x^3y} - 8\sqrt[4]{2x^3y}$

3. $6\sqrt[5]{x} + 4\sqrt[3]{x}$

4. $5\sqrt[3]{x+2} - 3\sqrt[3]{x}$

Perform the indicated operations.

5. $2\sqrt[3]{x} - 4\sqrt{x} - 6\sqrt[3]{x} + 3\sqrt{x}$

6. $4(3\sqrt[4]{x+5} - 5) + 2\sqrt[4]{x+5}$

7. $\sqrt{12x} + \sqrt{75x}$

8. $3a\sqrt{8ab^3} - 4b\sqrt{18a^3b}$

9. $5x\sqrt[3]{24x^2} - 2\sqrt[3]{3x^5}$

OBJECTIVE 2

Find the product.

10. $6\sqrt{5x} \cdot 4\sqrt{2}$

11. $5\sqrt{6x}\left(-2\sqrt{3x}\right)$

12. $2\sqrt{7x}(4\sqrt{x} - 3\sqrt{7})$

OBJECTIVE 3

Another Version of the Power Property for Radicals

If $\sqrt[n]{x}$ is defined, then $\left(\sqrt[n]{x}\right)^n = x$.

In particular, we have $\left(\sqrt{x}\right)^2 = x$ if $x \geq 0$.

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Product of Two Radical Expressions, Each with Two Terms

To find the product of two radical expressions in which each factor has two terms,

1. Multiply each term in the first radical expression by each term in the second radical expression.
2. Combine like radicals.

Simplify.

13. $(4\sqrt{x}+5)(2\sqrt{x}-7)$

14. $(3\sqrt{5x}-2)(4\sqrt{5x}-6)$

Simplifying the Square of a Radical Expression

To simplify the square of a radical expression with two terms,

- Use the square-of-a-sum property $(A+B)^2 = A^2+2AB+B^2$ or the square-of-a difference property $(A-B)^2 = A^2-2AB+B^2$,

or

- Use $C^2 = CC$ to write the square as a product of two identical expressions, and then multiply each term in the first radical expression by each term in the second radical expression.

OBJECTIVE 4

Simplify.

15. $(x+\sqrt{7})^2$

16. $(\sqrt{x}-\sqrt{3})^2$

17. $(5\sqrt{a}+2\sqrt{b})^2$

18. $(\sqrt{6x+3}-4)^2$

OBJECTIVES 2 and 3 (revisited)

Simplify.

19. $6\sqrt[3]{x^2}\cdot 3\sqrt[3]{x^2}$

20. $(\sqrt[4]{x^3}+2)(\sqrt[4]{x^3}+3)$

Multiplying Two Radicals That Have Different Indexes but the Same Radicand

To multiply two radicals that have different indexes but the same radicand,

1. Write the radicals in exponential form.
2. Use exponential properties to simplify the expression involving exponents.
3. Write the simplified expression in radical form.

Perform the operations.

21. $\sqrt[4]{x}\sqrt[3]{x}$

22. $5\sqrt[3]{x}(4\sqrt[6]{x}+2)$

23. $(\sqrt{x}-5\sqrt[3]{x})^2$

24. $(5\sqrt[3]{x}-2)(3\sqrt[4]{x}-1)$

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Simplifying a Radical Expression

To simplify a radical expression,

1. Perform any indicated multiplications.
2. Combine like radicals.
3. For any radical with index n, write the radical as a product of one or more perfect nth powers and another expression that has no factors that are perfect nth powers. Then apply the product property for radicals.
4. Write any radicals with as small an index as possible.

SHORT HW 9, 23, 25, 35, 43, 47, 61, 67, 73, 77, 81, 85, 87, 91, 99, 101

MEDIUM HW 9, 17, 23, 25, 35, 43, 47, 59, 61, 67, 73, 77, 81, 85, 87, 91, 97, 99, 101, 103, 105, 107, 109, 111

SECTION 13.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students add, subtract, and multiply radical expressions. They simplify the square of a radical expression that has two terms. Students will use these skills throughout the rest of Chapter 13.

OBJECTIVE 1 COMMENTS

Students have difficulty with Problems 7–9 in which they have to factor out nth powers before combining like radicals.

Textbook and Workbook Exploration

Groups can learn how to combine like radicals by completing the 15-minute "Combining like radicals" exploration. If students have seen this material before, they will probably remember how to do the symbolic work, but they likely will not know why their work is correct. Problem 1 can help groups see why it is correct to combine like radicals. Problems 2 and 3 has groups further reflect on when they can and when they cannot combine radical expressions.

OBJECTIVES 2 and 3 COMMENTS

Most students have an easy time with problems similar to Problems 13 and 14.

Workbook Exploration

In the following 20-minute exploration, students discover the version of the power property described on page 860 of the textbook. Tell groups that this exploration will address some concepts that will be helpful in solving radical equations, which will in turn be helpful in making estimations and predictions using square root models.

Despite the instructions in Problem 1, some groups may graph the two functions in the same viewing window and, thus, fail to notice that the graphs differ. You can rectify this by showing them how to use thicker lines or other calculator features to distinguish between the two graphs.

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Group Exploration
Section Opener: A property of radical expressions

1. **a.** Simplify $(\sqrt{x})^2$. [**Hint:** Write $\sqrt{x}$ in the form x^k and then use an exponential property.]
   b. What is the domain of $f(x) = \sqrt{x}$?
   c. Use a graphing calculator to draw the graph of $y = (\sqrt{x})^2$. Do not simplify the right-hand side of the equation when entering it in your calculator. Then draw the graph of $y = x$. Draw the graphs in different viewing windows so you can clearly see how they compare. Copy the screens. Describe your observations and why they make sense.
   d. What is the relationship between $(\sqrt{x})^2$ and x? State it carefully.

2. **a.** Simplify $(\sqrt[3]{x})^3$.
   b. What is the domain of $g(x) = \sqrt[3]{x}$? [**Hint:** What is the cube root of -8? Can you find cube roots of negative numbers?]
   c. Use a graphing calculator to draw the graph of $y = (\sqrt[3]{x})^3$. (Press [(] **3**, then MATH, then choose "$\sqrt[x]{}$". Then press [X,T,Θ,n] [)] [∧] **3**.) Do not simplify the right-hand side of this equation when entering it into your calculator. Then draw the graph of $y = x$. Draw the graphs in different viewing windows so you can clearly see how they compare. Copy the screens. Describe your observations and why they make sense.

3. Simplify $(\sqrt[n]{x})^n$, where n is a counting number. For what values of n is it necessary that x be greater than or equal to zero? Explain why this makes sense. [**Hint:** Experiment with graphs of $y = \sqrt[4]{x}$, $y = \sqrt[5]{x}$, $y = \sqrt[6]{x}$, ...]

OBJECTIVE 4 COMMENTS
For Problems 15–18, I remind students to remember the middle term $2AB$ of the right-hand side of $(A + B)^2 = A^2 + 2AB + B^2$. I make a similar comment about $(A - B)^2 = A^2 - 2AB + B^2$. Students will need to keep these ideas in mind when solving square root equations in section 13.5.

For Problem 15, aside from the most frequent error $(x + \sqrt{7})^2 = x^2 + 7$, a common error is to write

$$(x + \sqrt{7})^2 = x^2 + x\sqrt{14} + 7$$

I find that weaker students have better success writing $(x+\sqrt{7})^2$ as $(x+\sqrt{7})(x+\sqrt{7})$ and then multiplying pairs of terms than by using the property $(A + B)^2 = A^2 + 2AB + B^2$.

Workbook Exploration
In the following 10-minute exploration, students discover how to multiply two radical expressions and how to simplify the square of a radical expression with two terms.

Group Exploration
Section Opener: Multiplying two radical expressions

Recall the product property for square roots:

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad \text{where } a \text{ and } b \text{ are nonnegative numbers}$$

We can use this property to multiply two radical expressions:

$$\sqrt{2}\sqrt{7} = \sqrt{2 \cdot 7} = \sqrt{14}$$

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Perform the indicated operation.

1. $\sqrt{3} \cdot \sqrt{5}$ **2.** $6\sqrt{5} \cdot \sqrt{2}$ **3.** $\sqrt{3}\left(\sqrt{2} - \sqrt{5}\right)$ **4.** $\left(x + \sqrt{5}\right)\left(x + \sqrt{7}\right)$

5. $\left(x - \sqrt{2}\right)\left(x + \sqrt{3}\right)$ **6.** $\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$ **7.** $\left(x - \sqrt{5}\right)^2$

OBJECTIVES 2 and 3 (REVISITED) COMMENTS

For Problem 20, after students multiply pairs of terms, most students do not simplify $\sqrt[4]{x^6}$.

For Problems 22–24, students have difficulty with eventually having to find a product of radicals with different indexes.

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SECTION 13.3 LECTURE NOTES

Objectives

1. Rationalize the denominator of a radical expression.
2. Apply the *quotient property for radicals*.
3. Use the quotient property to simplify radical expressions.
4. Use a *radical conjugate* to rationalize the denominator of a radical expression.

Main point: Simplify quotients of radical expressions.

OBJECTIVE 1

Recall that in Section 9.3, we discussed how to rationalize the denominator of an expression of the form $\frac{p}{\sqrt{q}}$.

Simplify.

1. $\frac{7}{\sqrt{3x}}$ **2.** $\frac{8}{\sqrt{20x}}$ **3.** $\frac{5}{\sqrt[3]{2}}$ **4.** $\frac{1}{\sqrt[5]{8x^2}}$

To rationalize the denominator of a radical expression of the form $\frac{A}{\sqrt[n]{x^m}}$, we multiply the expression by a fraction of the form $\frac{\sqrt[n]{x^k}}{\sqrt[n]{x^k}}$ so the radical in the denominator has a perfect nth-power radicand.

OBJECTIVES 2 and 3

Quotient Property for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are defined and b is nonzero, then

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Students can prove the quotient property in Exercise 74 of Homework 13.3.
Simplify.

5. $\sqrt{\frac{5}{7}}$ **6.** $\sqrt[3]{\frac{5}{9x}}$ **7.** $\sqrt[5]{\frac{7}{4x^2}}$ **8.** $\sqrt[4]{\frac{2w}{8xy^2}}$

OBJECTIVE 4

- We say the sum of two radicals and the difference of the same two radicals are **radical conjugates** of each other.
- Remind students that $(A+B)(A-B) = A^2 - B^2$.

Find the product.

9. $(\sqrt{3}+\sqrt{7})(\sqrt{3}-\sqrt{7})$ **10.** $(4+\sqrt{x})(4-\sqrt{x})$

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The product of two radical conjugates contain no radicals (see the results of Problems 9 and 10).

Simplify.

11. $\dfrac{2}{3+\sqrt{5}}$

12. $\dfrac{5}{\sqrt{x}-\sqrt{3}}$

13. $\dfrac{6\sqrt{x}}{5\sqrt{x}-2}$

14. $\dfrac{5\sqrt{x}-2\sqrt{3}}{3\sqrt{x}+4\sqrt{2}}$

15. $\dfrac{\sqrt{a}-2\sqrt{b}}{\sqrt{a}+2\sqrt{b}}$

16. $\dfrac{3}{\sqrt{x+5}-\sqrt{x}}$

Rationalizing a Denominator by Using a Radical Conjugate

To rationalize the denominator of a square root expression if the denominator is a sum or difference involving radicals,

1. Determine the radical conjugate of the denominator.
2. Multiply the original fraction by the fraction $\dfrac{\text{conjugate}}{\text{conjugate}}$.
3. Find the product of the denominator by using $(A+B)(A-B)=A^2-B^2$.

SHORT HW 5, 15, 17, 21, 29, 37, 39, 47, 53, 59, 61, 65, 67, 73, 77

MEDIUM HW 5, 15, 17, 21, 27, 29, 37, 39, 47, 53, 57, 59, 61, 63, 65, 67, 71, 73, 74, 77, 79, 81, 83, 85

SECTION 13.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students rationalize denominators of radical expressions in this section. For instructors who want to reduce the amount of symbol manipulation, this section can be skipped. Perhaps the best reason to do this section is to prepare students to simplify difference quotients in calculus.

OBJECTIVE 1 COMMENTS

When discussing how to rationalize denominators, I emphasize that we are multiplying an expression by 1 and, thus, are finding an equivalent expression.

For Problem 1, I remind students that we can't simplify $\dfrac{7}{\sqrt{3x}}$ by squaring it or by multiplying it by $\dfrac{1}{\sqrt{3x}}$.

For Problem 3, a common student error is to multiply $\dfrac{5}{\sqrt[3]{2}}$ by $\dfrac{\sqrt{2}}{\sqrt{2}}$. It is by looking at examples such as this that students learn to focus on making the denominator the nth root of the nth power of an expression, rather than just multiplying by an expression of the form $\dfrac{\sqrt{A}}{\sqrt{A}}$.

For Problem 4, some students prefer to write $\dfrac{1}{\sqrt[5]{8x^2}}$ as $\dfrac{1}{\sqrt[5]{2^3x^2}}$, which suggests that we need to multiply it by $\dfrac{\sqrt[5]{2^2x^3}}{\sqrt[5]{2^2x^3}}$. More students prefer to leave the expression as $\dfrac{1}{\sqrt[5]{8x^2}}$ and multiply it by $\dfrac{\sqrt[5]{4x^3}}{\sqrt[5]{4x^3}}$.

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Textbook and Workbook Exploration

The 15-minute "Rationalizing the denominator" exploration is a great way to introduce the skill of rationalizing denominators.

OBJECTIVES 2 and 3 COMMENTS

Although students find it easy to apply the quotient property, they continue to struggle with then rationalizing the denominator for indexes other than 2.

OBJECTIVE 4 COMMENTS

For Problem 11, I demonstrate that multiplying $\dfrac{2}{3+\sqrt{5}}$ by $\dfrac{\sqrt{5}}{\sqrt{5}}$ does not work. I then explain why multiplying $\dfrac{2}{3+\sqrt{5}}$ by the expression $\dfrac{3-\sqrt{5}}{3-\sqrt{5}}$ works in terms of the product of binomial conjugates property $(A+B)(A-B)=A^2-B^2$.

For Problems 12–16, I continue to suggest that students think in terms of the product of binomial conjugates property.

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SECTION 13.4 LECTURE NOTES

Objectives

1. Describe the graphical significance of a, h, and k for a square root function of the form $y = a\sqrt{x-h}+k$.
2. Sketch graphs of square root functions.
3. Find the domain and range of square root functions.
4. Describe the meaning of b in equations of the form $y = a\sqrt{x}+b$.
5. Find the sum function, the difference function, the product function, and the quotient function of two square root functions.

Main point: Graph square root functions and perform operations with square root functions.

OBJECTIVE 1

Compare the graph of the given function with the graph of $f(x) = \sqrt{x}$.

1. $g(x) = \sqrt{x-2}$

2. $g(x) = \sqrt{x+2}$

3. $g(x) = \sqrt{x}-2$

4. $g(x) = \sqrt{x}+2$

The values of a, h, and k have similar effects on a square root function $g(x) = a\sqrt{x-h}+k$, where $a \neq 0$, as on a quadratic function $Q(x) = a(x-h)^2+k$.

Function	To graph the function $g(x) = \sqrt{x-h}+k$,
$g(x) = \sqrt{x-4}$	translate the graph of $y = \sqrt{x}$ to the right by 4 units.
$g(x) = \sqrt{x+4}$	translate the graph of $y = \sqrt{x}$ to the left by 4 units.
$g(x) = \sqrt{x}-4$	translate the graph of $y = \sqrt{x}$ down by 4 units.
$g(x) = \sqrt{x}+4$	translate the graph of $y = \sqrt{x}$ up by 4 units.

Compare the graph of the given function with the graph of $f(x) = \sqrt{x}$.

5. $g(x) = 2\sqrt{x}$ **6.** $g(x) = -2\sqrt{x}$ **7.** $g(x) = \frac{1}{2}\sqrt{x}$ **8.** $g(x) = -\frac{1}{2}\sqrt{x}$

- The graphs of $f(x) = -a\sqrt{x}$ and $g(x) = a\sqrt{x}$ are reflections of each other across the x-axis.
- If $a > 0$, then $g(x) = a\sqrt{x-h}+k$ is an increasing function and (h, k) is the minimum point.
- If $a < 0$, then g is a decreasing function and (h, k) is the maximum point.
- If $a > 1$, then the graph of $g(x) = a\sqrt{x-h}+k$ rises more quickly than the graph of $y = \sqrt{x-h}+k$.
- If $0 < a < 1$, then the graph of $g(x) = a\sqrt{x-h}+k$ rises more slowly than the graph of $y = \sqrt{x-h}+k$.

[Draw sketches for the preceding concepts.]

OBJECTIVES 2 and 3

Graph the function by hand. Also, find the domain and range of the function.

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9. $y = 3\sqrt{x}$

10. $y = -\sqrt{x+5}$

11. $f(x) = 2\sqrt{x-3} - 1$

12. $f(x) = -3\sqrt{x+1} + 4$

The graph of a square root function is called a **square root curve.**

Three-Step Method for Graphing a Square Root Function

To sketch the graph of $f(x) = a\sqrt{x-h} + k$, where $a \neq 0$,

1. Sketch the graph of $y = a\sqrt{x}$.
2. Translate the graph sketched in step 1 to the right by h units if $h > 0$ and to the left by $|h|$ units if $h < 0$.
3. Translate the graph sketched in step 2 up by k units if $k > 0$ and down by $|k|$ units if $k < 0$.

OBJECTIVE 4

By substituting 0 for x in $y = a\sqrt{x} + b$, we see that the y-intercept is $(0, b)$: $y = a\sqrt{0} + b = b$

y-Intercept of the Graph of $y = a\sqrt{x} + b$

The graph of an equation of the form $y = a\sqrt{x} + b$ has y-intercept $(0,\ b)$. [Sketch a figure.]

Find the y-intercept.

13. $y = -3\sqrt{x} + 7$

14. $y = 5\sqrt{x} - 2$

OBJECTIVE 5

For $f(x) = 3\sqrt{x} - 5$ and $g(x) = 4\sqrt{x} - 2$, find an equation of the given function.

15. $f + g$

16. $f - g$

17. $f \cdot g$

18. $\dfrac{f}{g}$

SHORT HW 1, 9, 15, 19, 25, 29, 37, 41, 47, 57, 59, 61, 69, 71, 73, 81

MEDIUM HW 1, 9, 15, 19, 21, 25, 29, 35, 37, 41, 47, 57, 59, 61, 69, 71, 73, 77, 81, 83, 87, 91, 93, 95, 97

SECTION 13.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students sketch the graphs of square root functions in this section.

OBJECTIVES 1 and 2 COMMENTS

I emphasize that the graphical significance of the parameters of $y = a\sqrt{x-h} + k$ are the same as of $y = a(x-h)^2 + k$. In fact, it may be instructive for students to compare the graphs of $y = 2\sqrt{x+5} - 3$ and $y = 2(x+5)^2 - 3$ and other such pairs of functions.

Textbook and Workbook Exploration

The 15-minute "Sketching graphs of square root functions" exploration is a great way to address the key concepts

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of this section. Students should be able to traverse this exploration fairly quickly if they have done the similar explorations for linear, quadratic, and exponential functions.

Textbook and Workbook Exploration

If you assign the 15-minute "Translating and reflecting the absolute value function" exploration, your stronger students will be ready for a discussion about the graphical significance of parameters of the general equation $y = af(x - h) + k$, where f is *any* function and a, h, and k are constants.

OBJECTIVE 3 COMMENTS

For Problem 10, I show my students how to find the domain by both solving the inequality $x + 5 \geq 0$ and by viewing the graph of $y = -\sqrt{x + 5}$.

OBJECTIVE 4 COMMENTS

This concept is not that important in this section because students can graph square root functions by thinking in terms of translations. But this concept will come in handy when finding equations of square root functions and models in section 13.6.

OBJECTIVE 5 COMMENTS

Algebra of square root functions is included in this section to reinforce what students have learned about algebra of functions in Chapters 7 and 12.

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SECTION 13.5 LECTURE NOTES

Objectives

1. Solve square root equations in one variable.
2. Find any x-intercepts of the graph of a square root function.
3. Solve radical equations in one variable.
4. Use a radical model to make predictions about the explanatory variable.

Main point: Solve radical equations.

OBJECTIVE 1

A **radical equation in one variable** is an equation in one variable that contains a radical with the variable in the radicand. Here are some examples:

$$\sqrt{x} = 7 \qquad \sqrt[3]{3x-4} = 5 \qquad \sqrt{x+3} = x-1 \qquad \sqrt{x-2} + \sqrt{3x-4} = 3$$

1. Solve $\sqrt{x} = 5$ by squaring both sides of the equation.

Squaring Property of Equality

If A and B are expressions, then all solutions of the equation $A = B$ are *among* the solutions of the equation $A^2 = B^2$. That is, the solutions of an equation are among the solutions of the equation obtained by squaring both sides.

- If a proposed solution of any type of equation is *not* a solution, we call it an **extraneous solution.**
- Show that squaring both sides of the equation $x = 3$ introduces the extraneous solution -3.

Checking Proposed Solutions

Because squaring both sides of a square root equation may introduce extraneous solutions, it is essential to check that each proposed solution satisfies the original equation.

Solve.

2. $4\sqrt{x} + 3 = 15$ [eventually linear: one solution]

To solve a square root equation, we isolate a square root term to one side of the equation before we square both sides.

3. $\sqrt{3x-5} - 1 = 3$ [eventually linear: one solution]

4. $3\sqrt{x-4} - \sqrt{4x-6} = 0$ [eventually linear: one solution]

5. $\sqrt{5x-9} = x - 3$ [factoring; one extraneous solution, one solution]

- Warning: To solve $\sqrt{5x-9} = x - 3$, it is incorrect to square each term: $\left(\sqrt{5x-9}\right)^2 = x^2 - 3^2$.

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- It is correct to square both sides of $\sqrt{5x-9} = x-3$. When simplifying $(x-3)^2$, don't forget the middle term $-6x$ of $x^2 - 6x + 9$.
- If an equation contains two or more square root terms, we may need to use the squaring property of equality twice.

6. $\sqrt{x} - \sqrt{3x} = -2$ [quadratic formula; one extraneous solution, one solution]

7. $\sqrt{3x+4} - \sqrt{2x-4} = 2$ [factoring; two solutions]

Solving a Square Root Equation in One Variable

To solve a square root equation in one variable,

1. Isolate a square root term on one side of the equation.
2. Square both sides.
3. Repeat steps 1 and 2 until no square root terms remain.
4. Solve the new equation.
5. Check that each proposed solution satisfies the original equation.

8. Solve the equation $V = \sqrt{\dfrac{2GM}{R}}$ for M. Assume the constants have values for which the equation has exactly one real-number solution.

9. Use "intersect" on a graphing calculator to solve $\sqrt{x+5} = x^2 - 3x - 5$.

OBJECTIVE 2

Find all x-intercepts.

10. $f(x) = 2\sqrt{x+2} - 4$

11. $g(x) = \sqrt{2x+4} - \sqrt{3x+1}$

Power Property of Equality

If A and B are expressions and n is a counting number greater than 1, then all solutions of the equation $A = B$ are *among* the solutions of the equation $A^n = B^n$. That is, the solutions of an equation are among the solutions of the equation obtained by raising both sides to the nth power.

OBJECTIVE 3

Solve.

12. $\sqrt[3]{5x-3} + 6 = 9$

13. $\sqrt[5]{7x-6} = \sqrt[5]{4x+3}$

OBJECTIVE 4

14. In Problem 29 of the lecture notes of section 13.1, we worked with the model $f(t) = 1.95\sqrt{t} + 13.3$, where $p = f(t)$ is the percentage of U.S. households that are one-person households at t years since 1960 (see the following table).

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Year	Percentage of U.S. Households That Are One-Person Households
1960	13.3
1970	17.6
1980	22.7
1990	24.6
2000	25.8
2010	26.7
2017	27.9

Source: *Census Bureau*

a. Find the p-intercept. What does it mean in this situation?

b. Find $f(29)$. What does it mean in this situation?

c. Find t when $f(t) = 29$. What does it mean in this situation?

SHORT HW 7, 15, 21, 31, 39, 43, 45, 53, 57, 59, 65, 75, 79, 83, 86

MEDIUM HW 7, 11, 15, 21, 25, 31, 39, 43, 45, 53, 57, 59, 65, 75, 77, 79, 83, 86, 95, 99, 101, 103, 105, 107

SECTION 13.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students solve square root equations.

OBJECTIVE 1 COMMENTS

The key concept in this section is the power property $(\sqrt[n]{x})^n = x$, where $\sqrt[n]{x}$ is defined. I remind students why this property is true.

I remind students several times that $(A+B)^2$ is not equivalent to A^2+B^2. For example, to solve the equation $\sqrt{3x+4} = x+3$, some students will correctly write

$$(\sqrt{3x+4})^2 = (x+3)^2$$

as their first step and then incorrectly write

$$(\sqrt{3x+4})^2 = x^2 + 3^2$$

Other students will not show the above first step, but will square each term of the right side of the equation:

$$(\sqrt{3x+4})^2 = x^2 + 3^2$$

For the first type of error, I remind students how to simplify $(A+B)^2$. For the second type of error, I demonstrate that squaring each term of the true statement $3 = 1 + 2$ yields $3^2 = 1^2 + 2^2$, a false statement.

For Problems 2 and 3, I emphasize that we want to isolate the radical to one side of the equation. For Problems 6 and 7, I emphasize that we want to isolate one of the radicals to one side of the equation. I point out that for these two problems, each time we square both sides, we reduce the number of radicals in the equation by one radical.

I emphasize that we must check for extraneous solutions.

Textbook and Workbook Exploration

The 15-minute "Solving square root equations" exploration addresses errors that students frequently make.

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Workbook Exploration

Although the following 30-minute exploration takes a lot of time for groups to complete, they will cover quite a lot of ground.

Group Exploration

Section Opener: Solving radical equations

1. Solve $\sqrt{x} = 4$ by finding a number whose square root is 4.

2. Solve $\sqrt{x} = 4$ by squaring both sides of the equation. Compare your result to the result you found in Problem 1.

3. Solve $\sqrt{x-3} = 2$ by squaring both sides of the equation.

4. **a.** A student tries to solve the equation $\sqrt{x} = -2$:

$$\begin{aligned} \sqrt{x} &= -2 \\ (\sqrt{x})^2 &= (-2)^2 \\ x &= 4 \end{aligned}$$

Check whether 4 satisfies the equation $\sqrt{x} = -2$. (As was the case with rational equations, the result $x = 4$ is called an *extraneous solution*.) What is the solution of the equation $\sqrt{x} = -2$?

b. Do you think checking your results using substitution is an optional or "mandatory" step when solving radical equations? Explain.

c. Solve the equation $\sqrt{x} = -3$.

5. Three students tried to solve $\sqrt{x} + 1 = 3$. Which students, if any, solved the equation correctly? Describe any errors and where they occurred.

Student 1's Work

$$\begin{aligned} \sqrt{x} + 1 &= 3 \\ (\sqrt{x} + 1)^2 &= 3^2 \\ (\sqrt{x})^2 + 1^2 &= 9 \\ x + 1 &= 9 \\ x &= 8 \end{aligned}$$

Student 2's Work

$$\begin{aligned} \sqrt{x} + 1 &= 3 \\ \sqrt{x} &= 3 - 1 \\ \sqrt{x} &= 2 \\ (\sqrt{x})^2 &= 2^2 \\ x &= 4 \end{aligned}$$

Student 3's Work

$$\begin{aligned} \sqrt{x} + 1 &= 3 \\ (\sqrt{x} + 1)^2 &= 3^2 \\ (\sqrt{x})^2 + 2\sqrt{x} + 1^2 &= 9 \\ x + 2\sqrt{x} + 1 &= 9 \\ 2\sqrt{x} &= 8 - x \\ (2\sqrt{x})^2 &= (8 - x)^2 \\ 4x &= 64 - 16x + x^2 \\ 0 &= x^2 - 20x + 64 \\ 0 &= (x - 4)(x - 16) \\ x = 4 \quad &\text{or} \quad x = 16 \end{aligned}$$

6. Solve.

a. $2\sqrt{x} + 9 = 1$ **b.** $\sqrt{x+5} - 2 = 4$ **c.** $\sqrt{x+3} - x = 3$

Textbook and Workbook Exploration

Students can investigate how extraneous solutions are introduced in the 15-minute "Extraneous solutions" exploration.

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OBJECTIVE 2 COMMENTS

By this point in the course, most students will remember to substitute 0 for $f(x)$ to find the x-intercepts.

OBJECTIVE 3 COMMENTS

I tell students that extraneous solutions do not get introduced by raising both sides of an equation to an odd power, but I advise them to still check that any results satisfy the original equation to make sure the work is correct.

OBJECTIVE 4 COMMENTS

Students have an easy time with Problem 14.

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SECTION 13.6 LECTURE NOTES

Objectives

1. Find an equation of a square root curve that contains two given points.
2. Find an equation of a square root model.
3. Use a square root model to make estimates and predictions.

Main point: Find equations of square root curves and use such equations to model authentic situations.

OBJECTIVE 1

Find an equation of a square root curve that contains the given points.

1. $(3, 2)$ and $(11, 7)$

2. $(5, 6)$ and $(37, 29)$

Finding an Equation of a Square Root Curve

To find an equation of a square root curve that contains two given points.

1. Obtain a system of two linear equations in two variables by substituting the coordinates of both points into the general equation $y = a\sqrt{x} + b$.
2. Solve the system that you found in step 1.
3. Substitute the values of a and b you found in step 2 into the equation $y = a\sqrt{x} + b$.

Find an equation of a square root curve that contains the given points.

3. $(0, 3)$ and $(9, 5)$

4. $(0, 13)$ and $(15, 6)$

OBJECTIVE 2

5. The worldwide sales of General Motors (GM) vehicles are shown in the following table for various years.

Year	Sales (millions of vehicles)
2009	7.3
2010	8.3
2011	8.9
2012	9.3
2013	9.7
2014	9.9
2015	9.8
2016	10.0

Source: *GM*

Let $n = S(t)$ be the worldwide annual sales (in millions) of GM vehicles at t years since 2009.

a. Find a square root equation of S.

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b. A quadratic model of the situation is given by $Q(t) = -0.073t^2 + 0.86t + 7.4$.

i. Describe how well both of the functions S and Q fit the data points in the scatterplot.

ii. Which of the two models does GM hope turns out to give better predictions? Explain.

Finding an Equation of a Square Root Model

To find an equation of a square root model, given some data,

1. Construct a scatterplot of the data.
2. Imagine a square root curve that comes close to the data points, and choose two points (not necessarily data points) that lie on or close to the square root curve.
3. Use the two data points to find an equation of the square root curve.
4. Use a graphing calculator to verify that the graph of the equation comes close to the points of the scatterplot.

OBJECTIVE 3

6. Use the model S you found in Problem 5 to respond to the following questions.

a. Find the n-intercept of the graph of S. What does it mean in this situation?

b. Predict the worldwide sales of GM vehicles in 2022.

c. Predict when the worldwide annual sales of GM vehicles will be 11 million vehicles.

Four-Step Modeling Process

To find a model and make estimates and predictions,

1. Construct a scatterplot of the data. Decide whether a line, an exponential curve, a parabola, a square root curve, or none of these comes close to the points.
2. Find an equation of your model.
3. Verify that your equation has a graph that comes close to the points in the scatterplot. If it doesn't, check for calculation errors or use different points to find the equation. An alternative is to reconsider your choice of model in step 1.
4. Use your equation of the model to draw conclusions, make estimates, and/or make predictions.

SHORT HW 1, 13, 19, 21, 23, 31, 33, 35, 37

MEDIUM HW 1, 7, 13, 19, 21, 23, 25, 27, 31 33, 35, 37, 39, 41, 43, 45, 47

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SECTION 13.6 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students perform curve fitting with square root functions. If you are tight for time, this section can be skipped because students will have the idea of curve fitting by now. However, if you've got the time, this is a relatively easy section that may make a heavily-symbolic chapter more meaningful and interesting to students.

OBJECTIVE 1 COMMENTS

For Problems 1–4, I use decimal approximations throughout the derivations of the equations. If you would like this section to tie in more closely with Sections 13.1—13.3, you can require that students use the exact radicals rather than decimal approximations.

Textbook and Workbook Exploration

In the 15-minute "Finding an equation of a square root curve" exploration, students can discover how to derive the equation of a square root curve that contains two given points, where one of the points is the y-intercept.

OBJECTIVE 2 COMMENTS

Rather than use the more flexible form $y = a\sqrt{x-h}+k$, students will use the form $y = a\sqrt{x}+k$ to model situations so they won't have to solve 3 by 3 systems at this late stage of the course. This is not too much of a restriction because the data in this chapter can be modeled fairly well by functions of the form $y = a\sqrt{x}+k$.

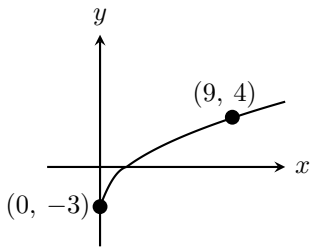
For Problem 5, I remind my students to use a graphing calculator to draw a scatterplot so that two "good points" can be chosen.

Workbook Exploration

The following 15-minute exploration could be a great way to start class.

EXPLORATION *Finding an equation using trial and error*

1. In this problem, you will use your graphing calculator to draw a radical curve like the one in the following figure. Use a function of the form $f(x) = a\sqrt{x}+b$, where a and b are real number constants. What is the equation of f that works? You should be able to determine the value of b by inspecting the graph. You can find the value of a through trial-and-error graphing.



2. The total revenues of car rental companies are shown in the following table for various years.

Year	Revenue (billions of dollars)
2012	16.8
2013	17.7
2014	18.4
2015	18.9
2016	19.1

Source: *Phocuswright*

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Let $f(t)$ be the total annual revenue (in billions of dollars) of car rental companies at t years since 2012. Find an equation of the form $f(t) = a\sqrt{t} + b$ that models the situation well. Find the equation through trial-and-error graphing.

OBJECTIVE 3 COMMENTS

Students have an easy time with Problem 6.

Workbook Exploration

The following 15-minute exploration directs students to find a square root model and then use the model to make estimates.

Group Exploration

Using a square root model to make predictions

The number of shopping malls are shown in the following table for various years.

Year	Number of Malls
1977	576
1987	874
1997	1043
2007	1165
2017	1211

Source: *CoStar Group*

Let $n = f(t)$ be the number of shopping malls at t years since 1977.

1. Find a square root equation of f.
2. What is the n-intercept? What does it mean in this situation?
3. Predict the number of malls in 2020.
4. Predict the average number of malls per state in 2022.
5. Predict when there will be 1275 malls.
6. As t increases, does the graph of f get steeper, have the same steepness, or get less steep? What does that mean in this situation?

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CHAPTER 14 OVERVIEW

In Sections 14.1 and 14.2, students learn about arithmetic sequences and geometric sequences, respectively. In Sections 14.3 and 14.4, students study arithmetic series and geometric series, respectively.

Students should see the connection between a linear function and an arithmetic sequence as well as the connection between an exponential function and a geometric sequence. To foster this connection, some instructors choose to use Section 14.1 in Chapter 5 and use Section 14.2 in Chapter 11.

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SECTION 14.1 LECTURE NOTES

Objectives

1. Describe the meaning of *arithmetic sequence*.
2. Find a formula of an arithmetic sequence.
3. Find a term or term number of an arithmetic sequence.
4. Use an arithmetic sequence to make estimates and predictions.
5. Compare a linear function and an arithmetic sequence.

Main point: Work with *arithmetic sequences*.

OBJECTIVE 1

1. A bicycle rental shop charges a flat fee of \$10 plus \$4 per hour to rent a bicycle. List the total charges (in dollars) for 1 through 10 hours in order.

Definition *Sequence*

Any ordered list of numbers is called a **sequence**. Each number is a **term** of the sequence.

- A sequence that has a last term, such as 50 in the bicycle sequence, is called a **finite sequence**.
- A sequence that does not have a last term is called an **infinite sequence**.
- Show that the common difference of the bicycle sequence is 4. This concept will motivate the following definition.

Definition *Arithmetic Sequence*

If the difference between any term of a sequence and the preceding term is a constant d for every such pair of terms, then the sequence is an **arithmetic sequence.** We call the constant d the **common difference.**

Determine whether the sequence is arithmetic. If it is, find the common difference d.

2. 3, 8, 13, 18, 23, ...

3. 80, 73, 66, 59, 52, ...

4. 32, 16, 8, 4, 2, ...

5. 5, 55, 555, 5555, 55555, ...

- We use the notation $a_1, a_2, a_3, \ldots$ to denote the terms of a sequence.
- We say a_n is the $\boldsymbol{n}$**th term** of the sequence and that its **term number** is n.
- For the bicycle sequence we write
$$a_1 = 14, a_2 = 18, a_3 = 22, \ldots, a_{10} = 50$$
where a_n is the charge (in dollars) for renting a bicycle for n hours.

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OBJECTIVE 2

Find a formula that describes the nth term of an arithmetic sequence:

$a_1 = a_1$	
$a_2 = a_1 + d$	Add d once to the first term to get the second term.
$a_3 = a_1 + 2d$	Add d twice to the first term to get the third term.
$a_4 = a_1 + 3d$	Add d three times to the first term to get the fourth term.
$a_n = a_1 + (n-1)d$	Add d a total of $(n-1)$ times to the first term to get the nth term.

Formula That Describes the nth Term of an Arithmetic Sequence

If an arithmetic sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ has the common difference d, then

$$a_n = a_1 + (n-1)d$$

Find a formula of the sequence.

6. 8, 11, 14, 17, 20, ...

7. 46, 41, 36, 31, 26, ...

8. **a.** Find a formula that describes the bicycle sequence.

b. Use a graphing calculator table to verify the formula.

OBJECTIVE 3

Because the formula $a_n = a_1 + (n-1)d$ is valid for arithmetic sequences only, we must first check that a sequence is arithmetic before we use the formula.

9. Find the 27th term of the sequence 7, 10, 13, 16, 19, ...

10. Find the term a_{54} of the sequence 99, 93, 87, 81, 75, ...

11. Find the term number n of the last term of the sequence 5, 12, 19, 26, 33, ..., 1377.

12. Find the term number n of the last term of the sequence 75, 71, 67, 63, 59, ..., -781.

OBJECTIVE 4

13. A person's salary is \$29,000 in the first year. It will increase by \$1160 each year. Let a_n be the salary (in dollars) in the nth year.

a. Find a formula that describes a_n.

b. What will be the salary in the 25th year?

c. In what year will the salary be \$62,640?

OBJECTIVE 5

- The bicycle model formula $a_n = 4n + 10$ describes a linear function whose graph has a slope of 4, which is the common difference of the sequence.

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- If we let $f(n)$ be the charge (in dollars) for renting a bicycle for n hours, then $f(n) = 4n + 10$ and the sequence 14, 18, 22, 26, 30, . . . , 50 is given by

$$f(1), f(2), f(3), \ldots, f(10)$$

Connection Between a Linear Function and an Arithmetic Sequence

If f is a linear function of the form $f(x) = mx + b$, then

$$f(1), f(2), f(3), \ldots$$

is an arithmetic sequence with common difference equal to the slope m.

14. For $f(x) = 5x + 7$, is the sequence $f(1), f(2), f(3), \ldots$ arithmetic?

SHORT HW 1, 3, 9, 13, 17, 19, 21, 25, 31, 37, 39, 43, 45, 51, 55

MEDIUM HW 1, 3, 9, 13, 17, 19, 21, 25, 31, 35, 37, 39, 43, 45, 51, 55, 57, 59, 61, 63

SECTION 14.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students learn about arithmetic sequences in this section. Many concepts in this section will be needed in Section 14.3.

OBJECTIVES 1 and 2 COMMENTS

Many students confuse the meaning of *term* with *term number*. I point out that for the bicycle sequence, the third term is 22 and the term number of 22 is 3.

Background Information for the Exploration That Follows

You could start class with the following 20-minute exploration.

Group Exploration

Arithmetic Sequences

A math tutor charges \$25 per hour to tutor one student, plus \$8 per hour for each additional student.

1. Let $f(n)$ be the amount of money (in dollars) the tutor will charge per hour to work with n students. Find an equation of f.

2. Evaluate f at each given value of n. Explain what each result means in this situation.

a. $f(3.8)$ **b.** $f(-2)$ **c.** $f(0)$

3. On the basis of your results in Problem 2, determine a domain of the model f. [**Hint:** Which inputs make sense in this situation?]

4. On the basis of your domain of f, find the range of f. List the values of the range of f in this order: $f(1)$, $f(2)$, $f(3)$, $f(4)$, ... What do you notice about these numbers?

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5. Sketch a graph of f, but plot only points whose n-coordinates are in the domain. [**Hint:** Your graph will look like a scatterplot.]

OBJECTIVE 3 COMMENTS

I emphasize that the formula $a_n = a_1 + (n-1)d$ only "works" for arithmetic sequences. This point will be driven home in Section 14.2, where students must determine whether a sequence is arithmetic, geometric, or neither before applying an appropriate formula.

OBJECTIVE 4 COMMENTS

For part (a) of Problem 13, students tend think the formula that describes a_n is $a_n = 1160n + 29,000$.

It may be instructive to compare and contrast finding the equation of the model using the formula for arithmetic sequences and using the method in Section 5.5 with the ordered pairs $(1,\ 29,000)$ and $(2,\ 30,160)$.

OBJECTIVE 5 COMMENTS

It is important that students see the connection between an arithmetic sequence and a linear function. When discussing this connection, I remind students of the slope addition property. Many of the concepts addressed in this section follow logically from concepts learned in Chapters 1–5.

Textbook and Workbook Exploration

In the 15-minute "Connection between combining linear functions and forming new sequences" exploration, students revisit algebra of functions. They will consider the connection between arithmetic sequences and linear functions from a different perspective.

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SECTION 14.2 LECTURE NOTES

Objectives

1. Describe the meaning of *geometric sequence*.
2. Find a formula of a geometric sequence.
3. Find a term or term number of a geometric sequence.
4. Use a geometric sequence to make estimates and predictions.
5. Compare an exponential function and a geometric sequence.

Main point: Work with geometric sequences.

OBJECTIVE 1

Show that the common ratio of the following sequence is 2:

$$3, 6, 12, 24, 48, \ldots$$

Definition *Geometric sequence*

If the ratio of any term of a sequence to the preceding term is a constant r for every such pair of terms, then the sequence is a **geometric sequence.** We call the constant r the **common ratio**.

Determine whether the sequence is geometric. If so, find the common ratio.

1. 4, 12, 36, 108, 324, ...

2. 80, 40, 20, 10, 5, ...

3. 6, 12, 36, 72, 216, ...

4. 0.03, 0.3, 3, 30, 300, ...

Find a formula that describes the nth term of a geometric sequence:

$a_1 = a_1$	
$a_2 = a_1 r$	Multiply a_1 by r once to get the second term.
$a_3 = a_1 r^2$	Multiply a_1 by r twice to get the third term.
$a_4 = a_1 r^3$	Multiply a_1 by r three times to get the fourth term.
$a_n = a_1 r^{n-1}$	Multiply a_1 by r a total of $(n-1)$ times to get the nth term.

OBJECTIVE 2

Formula That Describes the nth Term of a Geometric Sequence

If a geometric sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ has the common ratio r, then

$$a_n = a_1 r^{n-1}$$

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Find a formula of the sequence.

5. 4, 8, 16, 32, 64, ...

6. 405, 135, 45, 15, 5, ...

- In working with a sequence, we must first determine whether it is arithmetic, geometric, or neither.
- An arithmetic sequence has a common difference, whereas a geometric sequence has a common ratio.

OBJECTIVE 3

Find the indicated term of the sequence. Write the result in scientific notation $N \times 10^k$ with N rounded to the fourth decimal place.

7. Find the 15th term of the sequence 3, 12, 48, 192, 768, ...

8. Find the 14th term of the sequence 112, 56, 28, 14, 7, ...

Find the term number n of the last term of the sequence.

9. 5, 20, 80, 320, 1280, ..., 1,310,720

10. 1875, 375, 75, 15, 3, ..., 0.0000384

OBJECTIVE 4

11. A person's salary is \$29,000 in the first year. It will increase by 4% each year. Let a_n be the salary (in dollars) in the nth year.

a. Find a formula that describes a_n.

b. What will be the salary in the 25th year?

c. In what year will the salary first exceed \$62,640?

d. Compare the results from parts (b) and (c) with the results from parts (b) and (c) of Problem 13 in the lecture notes of Section 14.1 [answer to 13(b): \$56,840; answer to Problem 13(c): 30th year]

Write the salary sequence $a_n = 29,000(1.04)^{n-1}$ in the form $a_n = ab^n$: $a_n = \dfrac{29,000}{1.04}(1.04)^n$.

OBJECTIVE 5

- Observe that the formula
$$a_n = \frac{29,000}{1.04}(1.04)^n$$
describes an exponential function with base 1.04, which is the common ratio of the salary sequence.
- If we let $f(n)$ be the salary (in dollars) in the nth year, then $f(n) = \dfrac{29,000}{1.04}(1.04)^n$ and the salary sequence is given by
$$f(1), f(2), f(3), \ldots$$

Connection Between an Exponential Function and a Geometric Sequence

If f is an exponential function of the form $f(x) = ab^x$, then
$$f(1), f(2), f(3), \ldots$$
is a geometric sequence with common ratio equal to the base b of f.

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12. For $f(x) = 3(6)^x$ is the sequence $f(1)$, $f(2)$, $f(3)$, ... geometric?

SHORT HW 1, 9, 11, 13, 19, 23, 27, 33, 35, 39, 43, 47, 49, 51, 59

MEDIUM HW 1, 3, 5, 9, 11, 13, 17, 19, 23, 27, 31, 35, 37, 39, 41, 43, 35, 47, 49, 51, 59, 63, 65, 67

SECTION 14.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students learn about geometric sequences in this section. Most of the concepts in this section will be useful in Section 14.4.

OBJECTIVE 1 COMMENTS

For Problem 1, I ask my students why the sequence is not arithmetic with common difference $d = 8$.

Workbook Exploration

You could begin class with the following 10-minute exploration.

EXPLORATION *Introduction to geometric sequences*

A legal secretary earns a salary of $25,000 during his first year of work. He will receive a 3% raise each year. Let $f(t)$ be the salary (in dollars) of the legal secretary in his nth year of work.

1. Find an equation of f.
2. Use a graphing calculator table to verify your equation.
3. Predict the legal secretary's salary in his 20th year of work.
4. Find the value of each term of the sequence $f(1)$, $f(2)$, $f(3)$, $f(4)$, $f(5)$. What pattern do you notice? What do your results mean in this situation?

OBJECTIVE 2 COMMENTS

After introducing the general formula $a_n = a_1 r^{n-1}$, I emphasize that it only can be used for geometric sequences.

OBJECTIVE 3 COMMENTS

I tell students that they must first determine whether a sequence is arithmetic, geometric, or neither so that they can determine what formula, if any, they can use.

For Problems 9 and 10, I remind my students of the power property for logarithms.

OBJECTIVE 4 COMMENTS

For part (a) of Problem 11, I remind my students that a salary increase of 4% means that the salary in a certain year is 104% of the salary in the preceding year. So, a_n is a geometric sequence and $r = 1.04$.

OBJECTIVE 5 COMMENTS

It is enlightening for students to see a comparison between an arithmetic sequence and a geometric sequence (see Problem 11). I remind students of similar comparisons made earlier in the course between linear and exponential functions.

When discussing the connection between an exponential function and a geometric sequence, I remind students of the base multiplier property.

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Workbook Exploration

By completing the following 15-minute exploration, many students will see the connection between geometric sequences, exponential functions, and modeling.

Group Exploration

Using two data points to find an equation

The following table lists worldwide wind-generating capacities in thousands of megawatts (MW) for various years.

Year	Worldwide Wind-Generating Capacity (thousand MW)
2001	24.3
2004	47.7
2007	93.9
2010	196.9
2013	318.9
2016	486.7

Source: *World Wind Energy Association*

Let $f(t)$ be the wind-generating capacity (in thousand MW) at t years since 1990.

1. Use a graphing calculator to construct a scatterplot of the data. Copy the screen. What type of function seems best as a model for these data? Explain.
2. Find an equation of f.
3. Verify your equation by using a graphing calculator to draw a graph of f and your scatterplot in the same viewing window. Copy the screen.
4. Use a graphing calculator table to find the sequence $f(1)$, $f(2)$, $f(3)$, $f(4)$, $f(5)$. Is this sequence a geometric sequence? Explain.
5. Use your equation to predict the worldwide wind-generating capacity in 2021.

Textbook and Workbook Exploration

In the 15-minute "Connection between combining exponential functions and forming new sequences" exploration, students use algebra of functions to construct new sequences. Students also reflect on the connection between an exponential function and a geometric sequence.

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SECTION 14.3 LECTURE NOTES

Objectives

1. Describe the meaning of *arithmetic series*.
2. Evaluate the sum of an arithmetic series.
3. Use arithmetic series to make estimates and predictions.

Main point: Work with arithmetic series.

OBJECTIVE 1

- A person's salary is \$25,000 in the first year and increases by \$3000 each year.
- The following sequence describes the salaries (in thousands of dollars) for the first 20 years:

$$25, 28, 31, \ldots 76, 79, 82$$

- The following sum describes the *total* earnings (in thousand of dollars) during the first 20 years:

$$25 + 28 + 31 + \cdots + 76 + 79 + 82$$

Definition *Arithmetic series*

If the sequence $a_1, a_2, a_3, \ldots, a_n$ is an arithmetic sequence, then the sum $a_1 + a_2 + a_3 + \cdots + a_n$ is an **arithmetic series.** We say a_i is the ***i*th term** of the series and that its **term number** is i.

We use S_n to represent the sum of the first n terms of an arithmetic sequence:

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

OBJECTIVE 2

Next, we find the total earnings (in thousands of dollars) for 20 years, S_{20}, where

$$S_{20} = 25 + 28 + 31 + \cdots + 76 + 79 + 82$$

We find the sum in a way that suggests a general formula for S_n for any arithmetic series.

To find the sum, we write the equation for S_{20} twice, the second time with the terms in reverse order. Then we add the left-hand sides and add the right-hand sides of the two equations:

$$\begin{array}{rrcl} & S_{20} & = & 25 + 28 + 31 + \cdots + 76 + 79 + 82 \\ + & S_{20} & = & 82 + 79 + 76 + \cdots + 31 + 28 + 25 \\ \hline & 2S_{20} & = & 107 + 107 + 107 + \cdots + 107 + 107 + 107 \end{array}$$

On the right-hand side of the last equation, the number 107 appears 20 times, so the sum equals $20(107)$:

$$\begin{array}{ll} 2S_{20} = 20(107) & \text{Equation (1)} \\ S_{20} = \dfrac{20(107)}{2} & \text{Divide both sides by 2: Equation (2)} \\ S_{20} = 1070 & \text{Equation (3)} \end{array}$$

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The total earnings for 20 years will be $1,070,000.

Note that 107 is the sum of the first and last terms of the series:

$$107 = 25 + 82$$

So, we can write equation (2) as

$$S_{20} = \frac{20(25 + 82)}{2}$$

Our process and result suggest that we can find S_n for any arithmetic series by first multiplying the sum of the first and last terms $a_1 + a_n$ by the number of terms n and dividing the product by 2.

Formula for Sum of an Arithmetic Series

If $S_n = a_1 + a_2 + a_3 + \cdots + a_n$ is an arithmetic series, then

$$S_n = \frac{n(a_1 + a_n)}{2}$$

To evaluate the sum $S_n = a_1 + a_2 + a_3 + \cdots a_n$, we must first check that the sequence $a_1, a_2, a_3, \ldots, a_n$ is arithmetic before we use the formula $S_n = \frac{n(a_1 + a_n)}{2}$.

Evaluate the sum of the arithmetic series with the given values of a_1, a_n, and n.

1. $a_1 = 6$, $a_n = 1154$, and $n = 165$

2. $a_1 = 95$, $a_n = -265$, and $n = 46$

Evaluate the sum of the series.

3. $S_{83} = 9 + 15 + 21 + 27 + 33 + \cdots + 501$

4. $S_{135} = 82 + 79 + 76 + 73 + 70 + \cdots + (-320)$

In evaluating S_n for an arithmetic series, we sometimes must use the formula $a_n = a_1 + (n - 1)d$ to find n, a_1, or a_n before we can use the formula $s_n = \frac{n(a_1 + a_n)}{2}$.

Evaluate the sum of the series.

5. $S_{49} = 23 + 32 + 41 + 50 + 59 + \cdots + a_{49}$

6. $S_{99} = 801 + 799 + 797 + 795 + 793 + \cdots + a_{99}$

7. $15 + 27 + 39 + 51 + 63 + \cdots + 543$

8. $1000 + 990 + 980 + 970 + 960 + \cdots + (-40)$

OBJECTIVE 3

9. A person's salary is $32,000 in the first year and increases by $1600 each year. Find the person's total earnings for the first 30 years.

Derive the formula $S_n = \frac{n(a_1 + a_n)}{2}$. See pages 925 and 926 of the textbook.

SHORT HW 1, 7, 9, 13, 19, 21, 23, 27, 29, 31, 35, 41, 45, 47

MEDIUM HW 1, 5, 7, 9, 13, 17, 19, 21, 23, 27, 29, 31, 33, 35, 41, 45, 47, 49, 51, 53

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SECTION 14.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students work with arithmetic series.

OBJECTIVE 1 COMMENTS

I emphasize the connection between an arithmetic sequence a_1, a_2, a_3, …, a_n and the related arithmetic series $a_1 + a_2 + a_3 + \cdots + a_n$. This connection is crucial for many problems in this section.

OBJECTIVE 2 COMMENTS

A fun way to begin this section is to demonstrate how to find the sum $1 + 2 + 3 + \cdots + 1000$.

Most students have a tough time with Problems 7 and 8 because these problems require the use of both the arithmetic sequence formula and the arithmetic series formula. Many students lose sight of the connection between a series and its corresponding sequence. As a result, many students are unclear which formula is needed. They also have trouble assessing which variables have known values and which do not. I often tell my students that if an equation contains, say, four variables and I know values of three of the four variables, then I can solve for the value of the other variable.

Workbook Exploration

In the following 20-minute exploration, students discover the basic idea behind finding the sum of an arithmetic series.

EXPLORATION *Introduction to arithmetic series*

Suppose that a person's salary at a company is \$23,000 during the first year, and that it will increase by \$2,000 each year. Also assume this person will hold the job for 32 years.

1. What will the salary be during the 32nd year?

2. Let S_{32} be the total income (in thousands of dollars) that the person earns while working at the company. Explain why it makes sense that $S_{32} = 23 + 25 + 27 + 29 + 31 + \cdots + 81 + 83 + 85$. (This sum is called a *series*.)

3. One way to find S_{32} is to add the 32 salaries twice in a special way and then divide this result by 2.

a. Add both sides of the equations shown here, as suggested.

$$\begin{array}{rrl} & S_{32} &= 23 + 25 + 27 + \cdots + 81 + 83 + 85 \\ + & S_{32} &= 85 + 83 + 81 + \cdots + 27 + 25 + 23 \\ \hline & 2S_{32} &= 108 + \ ? \ + \ ? \ + \cdots + \ ? \ + \ ? \ + \ ? \end{array}$$

b. Simplify the right-hand side of the equation you obtained in part (a). [**Hint**: There is a quick way to do this.]

c. You should now have an equation of the form $2S_{32} = k$, where k is some number. Now solve this equation for S_{32}. [**Hint**: Solving this equation is like solving the equation $2x = 7$ for x.] What does your result mean in this situation?

4. Is the sequence 23, 25, 27, 29, 31, …, 81, 83, 85 an arithmetic sequence, a geometric sequence, or neither? Explain. (Your answer should suggest why the series $23 + 25 + 27 + 29 + 31 + \cdots + 81 + 83 + 85$ is called an *arithmetic series*.)

OBJECTIVE 3 COMMENTS

Students have an easy time with Problem 9.

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Textbook and Workbook Exploration

The formula $S_n = \dfrac{n(a_1 + a_n)}{2}$ will likely make more sense to groups after completing the 15-minute exploration, "Other ways to evaluate the sum of an arithmetic series."

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SECTION 14.4 LECTURE NOTES

Objectives

1. Describe the meaning of *geometric series*.
2. Evaluate the sum of a geometric series.
3. Use geometric series to make estimates and predictions.

Main point: Work with geometric series.

OBJECTIVE 1

Consider the geometric sequence 2, 6, 18, 54, 162, …, 39,366. We call the sum

$$2 + 6 + 18 + 54 + 162 + \cdots + 39,366$$

a geometric series.

Definition *Geometric series*

If the sequence a_1, a_2, a_3, …, a_n is a geometric sequence, then the sum $a_1 + a_2 + a_3 + \cdots + a_n$ is a **geometric series**. We say a_i is the ***i*th term** of the series and that its **term number** is i.

OBJECTIVE 2

We can derive a general formula that describes the sum of a geometric series $S_n = a_1 + a_2 + a_3 + \cdots + a_n$. We can express the series as

$$S_n = a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-3} + a_1r^{n-2} + a_1r^{n-1} \qquad \text{Equation (1)}$$

We multiply both sides of this equation by r to obtain

$$rS_n = a_1r + a_1r^2 + a_1r^3 + \cdots + a_1r^{n-2} + a_1r^{n-1} + a_1r^n \qquad \text{Equation (2)}$$

Next, we multiply both sides of equation (2) by -1, add the left-hand sides, and add the right-hand sides of this new equation and equation (1):

$$\begin{array}{rcl} S_n & = a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-2} + a_1r^{n-1} & \\ -rS_n & = -a_1r - a_1r^2 - \cdots - a_1r^{n-2} - a_1r^{n-1} - a_1r^n & \\ \hline S_n - rS_n & = a_1 + 0 + 0 + \cdots + 0 + 0 - a_1r^n & \end{array}$$

We can simplify the right-hand side of the last equation:

$$S_n - rS_n = a_1 - a_1r^n$$

We now factor out S_n from the left-hand side and a_1 from the right-hand side:

$$\begin{aligned} S_n(1 - r) &= a_1(1 - r^n) \\ S_n &= \frac{a_1(1 - r^n)}{1 - r}, \quad r \neq 1 \end{aligned} \qquad \text{Divide both sides by } 1 - r.$$

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Formula for the Sum of a Geometric Series

If $S_n = a_1 + a_2 + a_3 + \cdots + a_n$ is a geometric series with common ratio $r \neq 1$, then

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Evaluate the sum of the geometric series with the indicated values of a_1, r, and n. Round any approximate results to the fourth decimal place.

1. $a_1 = 7$, $r = 2$, and $n = 15$

2. $a_1 = 62$, $r = 0.7$, and $n = 11$

To find the sum $S_n = a_1 + a_2 + a_3 + \cdots + a_n$, we must first determine whether the series is arithmetic, geometric, or neither. If the series is arithmetic, we use $S_n = \dfrac{n(a_1 + a_n)}{2}$. If the series is geometric with common ratio $r \neq 1$, we use $S_n = \dfrac{a_1(1 - r^n)}{1 - r}$.

Find the sum of the series. Round any approximate results to the fourth decimal place.

3. $6 + 24 + 96 + 384 + 1536 + \cdots + a_{10}$

4. $12 + 8 + \dfrac{16}{3} + \dfrac{32}{9} + \dfrac{64}{27} + \cdots + a_{12}$

5. $2 + 6 + 18 + 54 + 162 + \cdots + 118,098$

6. $5120 + 2560 + 1280 + 640 + 320 + \cdots + 0.078125$

OBJECTIVE 3

7. A person's salary is \$32,000 in the first year and increases by 5% at the end of each year.

a. Calculate the person's total earnings for the first 30 years.

b. Compare the result from part (a) with the result we found in Problem 9 when discussing Section 14.3, where we assumed the person's salary increases by a constant \$1600 each year.

SHORT HW 1, 7, 9, 13, 17, 19, 21, 25, 27, 33, 37, 41, 43

MEDIUM HW 1, 7, 9, 13, 17, 19, 21, 25, 27, 33, 35, 37, 41, 43, 45, 47, 49, 51, 53

SECTION 14.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students work with geometric series in this section.

OBJECTIVE 1 COMMENTS

I emphasize the connection between a geometric sequence a_1, a_2, a_3, …, a_n and the related geometric series $a_1 + a_2 + a_3 + \cdots + a_n$.

OBJECTIVE 2 COMMENTS

Most students are challenged by Problems 5 and 6 because they must use both the geometric sequence formula and the geometric series formula.

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My students appreciate it when I review the two formulas for arithmetic and geometric sequences and the formula for an arithmetic series.

Workbook Exploration

You can have groups derive the formula for the sum of a geometric series by assigning the following 20-minute exploration.

EXPLORATION *Introduction to geometric series*

Suppose a person's salary is \$23,000 for the first year, and that at the end of each year there is a raise equal to 5% of the salary for that year. Assume the person stays with this company for 32 years.

1. What will the salary be for the 2nd year?

2. What will the salary be for the 32nd year?

3. Let S_{32} be the total amount of money (in thousands of dollars) the person will earn while working at the company. Explain why the following equation makes sense.

$$S_{32} = 23 + 23(1.05) + 23(1.05)^2 + 23(1.05)^3 + \cdots + 23(1.05)^{29} + 23(1.05)^{30} + 23(1.05)^{31}$$

4. If both sides of the equation in Problem 3 are multiplied by 1.05, the result is the equation

$$1.05S_{32} = 23(1.05) + 23(1.05)^2 + 23(1.05)^3 + \cdots + 23(1.05)^{30} + 23(1.05)^{31} + 23(1.05)^{32}$$

Explain why this makes sense.

5. The equations in Problems 3 and 4 are written below. **Subtract** on both sides of the equations.

$$\begin{array}{rcccccccccc}
S_{32} & = & 23 + 23(1.05) + 23(1.05)^2 + \cdots + 23(1.05)^{30} + 23(1.05)^{31} & & & & & & & & \\
1.05S_{32} & = & & 23(1.05) + 23(1.05)^2 + \cdots + 23(1.05)^{30} + 23(1.05)^{31} + 23(1.05)^{32} & & & & & & & \\
\hline
S_{32} - 1.05S_{32} & = & 23 + & 0 & + & ? & + \cdots + & ? & + & ? & + & ?
\end{array}$$

6. Explain why the equation below follows from the equation that you obtained in Problem 5.

$$(1 - 1.05)S_{32} = 23(1 - 1.05^{32})$$

7. Now solve the equation in Problem 6 for S_{32}. [**Hint**: Solving this equation is like solving the equation $3x = 5$ for x.] What does S_{32} mean in this situation?

8. Is the sequence

$$23,\ 23(1.05),\ 23(1.05)^2,\ 23(1.05)^3,\ \ldots,\ 23(1.05)^{29},\ 23(1.05)^{30},\ 23(1.05)^{31}$$

an arithmetic sequence, a geometric sequence, or neither? Explain. (Your answer should suggest why the series

$$23 + 23(1.05) + 23(1.05)^2 + 23(1.05)^3 + \cdots + 23(1.05)^{29} + 23(1.05)^{30} + 23(1.05)^{31}$$

is called a *geometric series*.)

OBJECTIVE 3 COMMENTS

In part (b) of Problem 7, many students are surprised by how much more total money can be earned by salary raises of 5% per year than by salary raises of \$1600 per year.

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Textbook and Workbook Exploration

In the 15-minute "Stacks of pennies on a chessboard" exploration, students solve the problem about the grains of rice on a chessboard, but use pennies instead of rice. The pedagogical benefit with pennies is that students can then compute the height of the stack of pennies on the 64th square, assuming they do not fall over. (It's hard to stack even just a few grains of rice!) This way, students consider both a geometric sequence and a geometric series in the same problem. They also get a physical sense of the number of pennies on the chessboard.

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CHAPTER 15 OVERVIEW

In my opinion, most of the topics in Chapter 15 are not essential for an intermediate algebra course. These topics are also not necessary for curve fitting, so I have placed them at the end of the text so that Chapters 1-14 can be more unified and compact.

Chapter 15 is meant to serve professors and students in two ways. One option is to insert sections of Chapter 15 into previous chapters. Another option is to discuss selected sections of Chapter 15 toward the end of the course, which is my preference.

Here are some ways to insert sections of Chapter 15 into earlier chapters:

- Use Section 15.1 just after Section 5.7.
- Use Section 15.2 just after Section 13.3. Section 15.2 can be used just after Section 9.6 but you will not be able to draw parallels between radical expressions and complex numbers that way.
- Use Section 15.3 just after Section 9.4. However, the equation of a circle is derived by using the distance formula and then squaring both sides, which is not discussed until Section 13.5.
- Use Sections 15.4 and 15.5 anytime after Section 13.5 (but probably not in the middle of a chapter). Sections 15.3–15.5 make for a nice module.

The section quizzes at the end of each homework section can be used to form a chapter review. There is no chapter review (or chapter test) for Chapter 15, because some professors will insert sections of Chapter 15 into previous chapters. The section quizzes give professors more flexibility. For a section inserted into an earlier chapter, the quiz can be appended to that chapter's review exercises.

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SECTION 15.1 LECTURE NOTES

Objectives

1. Describe the meaning of *absolute value function*, *absolute value equation in one variable*, and *absolute value inequality in one variable*.
2. Solve absolute value equations in one variable.
3. Solve absolute value inequalities in one variable.

Main point: Solve equations and inequalities that have absolute values.

OBJECTIVE 1

Definition *Absolute value*

The **absolute value** of a number a, written $|a|$, is the distance from a to 0 on the number line.

- For example, $|3| = 3$ and $|-5| = 5$.
- The absolute value of any number is nonnegative.

An **absolute value function** is a function whose equation contains the absolute value of a variable expression. Here are some examples:

$$f(x) = |x| \qquad g(x) = 2\,|4x - 8| + 3 \qquad h(x) = \frac{|x| + 3}{7}$$

1. Graph $y = |x|$ by hand.

OBJECTIVE 2

- An **absolute value equation in one variable** is an equation in one variable that contains the absolute value of a variable expression.
- Here are some examples:

$$|x| = 3 \qquad |4x - 7| = 5 \qquad |3x + 8| = |6x - 1|$$

2. Use the definition of absolute value to find the solutions of $|x| = 2$.

Absolute Value Property for Equations

For an expression A and a positive constant k, the equation $|A| = k$ is equivalent to

$$A = -k \text{ or } A = k$$

Solve.

3. $|2x - 5| = 11$

4. $5\,|x| + 4 = 24$

5. $|3x - 21| = 0$

6. $|9x + 4| = -6$

7. $\left|\frac{2}{3}x - \frac{1}{6}\right| = \frac{5}{2}$

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Use the definition of absolute value to explain the following property.

Solving an Equation of the Form $|A| = |B|$

For expressions A and B, the equation $|A| = |B|$ is equivalent to

$$A = -B \text{ or } A = B$$

8. Solve $|3x - 8| = |5x + 2|$.

9. Use graphing to solve $7 - |x + 2| = 2|x + 3| - 5$.

OBJECTIVE 3

- An **absolute value inequality in one variable** is an inequality in one variable that contains the absolute value of a variable expression.
- Here are some examples:

$$|x| < 4 \qquad 4|x| + 3 \le 9 \qquad |x - 5| > 2 \qquad \left|\frac{4x + 7}{2}\right| \ge 1$$

If A and B are sets, then the **union of A and B**, denoted $A \cup B$, is the set of all members of A together with all members of B.

Absolute Value Property for Inequalities

For an expression A and a positive constant k,

- The inequality $|A| < k$ is equivalent to $-k < A < k$.
- The inequality $|A| > k$ is equivalent to $A < -k$ or $A > k$.

Solve. Describe the solution set as an inequality, in a graph, and in interval notation.

10. $|x| < 4$

11. $|x| > 3$

12. $|2x + 5| \le 7$

13. $|3x - 1| \ge 14$

14. $|x| + 6 < 9$

15. $7 - |x + 2| > 4$

16. $\left|\frac{x - 4}{5}\right| \le 3$

17. $|8x + 3| < -4$

18. $|5x - 2| \ge -3$

SHORT HW 5, 11, 23, 27, 31, 37, 41, 51, 63, 67, 73, 79, 83, 85, 91

MEDIUM HW 1, 5, 11, 17, 23, 27, 31, 37, 41, 47, 49, 51, 53, 61, 63, 67, 73, 79, 81, 83, 85, 91, 99, 101, 103

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SECTION 15.1 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn to solve absolute value equations and inequalities. This section could be inserted after Section 5.7, because students will have just worked with inequalities in Section 5.7.

OBJECTIVE 1 COMMENTS

I emphasize that the absolute value of a number is a measure of *distance*. It is this interpretation that will help students to solve absolute equations and inequalities.

OBJECTIVE 2 COMMENTS

To solve $|x| = 2$, I use the analogy of a person living 2 miles along a road from a bank (where the road is the number line, and the bank is at 0). Note that solving $|x| = k$ suggests how to solve equations such as $|mx + b| = k$ in much the same way that solving $x^2 = k$ suggests how to solve equations such as $(mx + b)^2 = k$.

For Problem 3, some students mistakenly write the equation $|2x - 5| = 11$ as $2x + 5 = 11$. Other students simply drop the absolute value symbols and write $2x - 5 = 11$. These types of errors tend to crop up by exam time, unless I make several warnings.

For Problem 4, I emphasize that we isolate $|x|$ to one side of the equation.

For Problem 6, I point out that it is essential that we inspect the equation to determine whether the solution set is the empty set. The usual algorithm can't be successfully applied.

Workbook Exploration

In the following 10-minute exploration, groups have practice thinking of absolute value numerically, graphically, and symbolically, and how these three ways of describing functions relate.

EXPLORATION *Number of solutions of an absolute value equation*

1. Complete the table of input–output pairs of $f(x) = |x|$. Use your completed table to solve $|x| = 3$.

x	$f(x)$	x	$f(x)$
-4		0.5	
-3		1	
-2		2	
-1		3	
-0.5		4	
0		5	

2. Graph $f(x) = |x|$. Use your graph to solve $|x| = 3$.

3. Use the definition of absolute value to solve $|x| = 3$. Compare your result to the results you found in Problems 1 and 2.

4. Use the table you completed in Problem 1, the graph you sketched in Problem 2, and the definition of absolute value to solve the given equation.

a. $|x| = 4$ **b.** $|x| = 1$ **c.** $|x| = 0$ **d.** $|x| = -2$ **e.** $|x| = -3$

5. For what values of k does the equation $|x| = k$ have two solutions? One solution? No solutions? Explain by referring to the table you completed in Problem 1, the graph you sketched in Problem 2, and the definition of absolute value.

OBJECTIVE 3 COMMENTS

Most students have a hard time solving absolute value inequalities. Here is where interpreting absolute value as

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a measure of distance is extremely helpful. I spend a good chunk of time making sure students see the graphical significance of statements such as $|x| < 4$ and $|x| > 3$ (see Problems 10 and 11).

To solve $|x| < 4$, I use the analogy of a person living less than 4 miles along a road from a bank (where the road is the number line and the bank is at 0). To solve $|x| > 3$, I use the analogy of a person living more than 3 miles from the bank.

Most students are unsure of the mathematical meaning of "and" as well as "or." To get across the idea of these terminologies, I say that the set of students who are tall and female is the set of tall, female students. The set of students who are tall or female is the set of all tall students as well as female students.

Then I solve the inequality $|x| < 4$ and point out that the statement $-4 < x < 4$ means that x is *both* greater than -4 *and* less than 4. I explain that "and" is used to state that both requirements must be true. Including a graph will help students follow the discussion.

A similar demonstration of solving $|x| < 4$ can be performed to discuss the meaning of "or."

Workbook Exploration

Once students know how to solve simple absolute value inequalities, you could have them complete the following 10-minute exploration. You may want to bring some pens with different colored ink, although if students share their own writing implements with each other, the class should have a large enough bank of colors.

EXPLORATION *Comparing solutions of absolute value inequalities*

1. Solve $|x| = 3$.
2. Solve $|x| < 3$.
3. Solve $|x| > 3$.
4. Graph the solutions in Problems 1, 2, and 3 on the same number line. Use three colors to identify the different solutions. Make some observations about the solutions. Explain these observations.

Textbook and Workbook Exploration

In the 20-minute "Graphical meaning of $|a - b|$" exploration, students will learn that $|a - b|$ gives the distance between the numbers a and b on the number line. This interpretation is especially helpful for students who will later study $\epsilon - \delta$ proofs in calculus.

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SECTION 15.2 LECTURE NOTES

Objective

1. Perform operations with complex numbers.

Main point: Perform operation with complex numbers.

OBJECTIVE 1

- Recall from Section 9.4 that $i^2 = -1$, $i = \sqrt{-1}$, and $\sqrt{-p} = i\sqrt{p}$.
- Because $i = \sqrt{-1}$, we perform operations with complex numbers in much the same way as we do with radical expressions.
- Compare finding the following sums.
  - $(5 + 4\sqrt{3}) + (2 + 6\sqrt{3}) = 7 + 10\sqrt{3}$
  - $(5 + 4i) + (2 + 6i) = 7 + 10i$
- If a radicand is a negative real number, we first write the radical in terms of i before performing any operations:

$$\sqrt{-1}\sqrt{-1} = i \cdot i = i^2 = -1 \qquad \text{Correct}$$
$$\sqrt{-1}\sqrt{-1} = \sqrt{-1 \cdot -1} = \sqrt{1} = 1 \qquad \text{Incorrect}$$

- So, there is no product property $\sqrt{a}\sqrt{b} = \sqrt{ab}$ when a and b are both negative.
- We write results in the form $a + bi$, where a and b are in lowest terms.

Perform the indicated operations. Simplify the result.

1. $(6 - 5i) + (3 + 2i)$

2. $(2 - 8i) - (7 - 5i)$

3. $3i \cdot 7i$

4. $\sqrt{-5}\sqrt{-7}$

5. $2 - 4i(3 - 9i)$

6. $(5 + 2i)(6 - 3i)$

7. $(6 + 2i)^2$

8. $(4 - 7i)^2$

Definition *Complex conjugate*

The complex numbers $a + bi$ and $a - bi$ are called **complex conjugates** of each other.

Find the product.

9. $(5 + 3i)(5 - 3i)$

10. $(2 - 9i)(2 + 9i)$

The product of *any* two complex conjugates is a real number.

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- We **simplify** a quotient of two complex numbers by removing i from the denominator of the quotient.
- To simplify the quotient of two complex numbers, we multiply the quotient by

$$\frac{\text{complex conjugate of the denominator}}{\text{complex conjugate of the denominator}}$$

Simplify.

11. $\dfrac{7}{8+3i}$

12. $\dfrac{2i}{4-i}$

13. $\dfrac{2+5i}{4-3i}$

14. $\dfrac{2-8i}{2+8i}$

When the denominator of a fraction is a pure imaginary number, the easiest way to remove i from the denominator is to multiply the fraction by $\dfrac{i}{i}$.

Simplify.

15. $\dfrac{4+7i}{2i}$

16. $\dfrac{3}{4i}$

SHORT HW 5, 7, 13, 17, 27, 29, 39, 43, 47, 53, 55, 62, 63, 71, 75

MEDIUM HW 5, 7, 11, 13, 17, 27, 29, 39, 43, 47, 53, 55, 57, 60, 62, 63, 71, 75, 77, 79, 81

SECTION 15.2 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students learn to perform operations with complex numbers. This section assumes that students have already completed Sections 13.1—13.3. Much of the symbolic work with radical expressions suggests how to perform symbolic work with complex numbers.

However, with care, you could use this section just after Section 9.6, because symbolic work with polynomials also supports working with complex numbers. If you follow this route, you will have to give lots of examples on techniques such as simplifying the quotient of two complex numbers.

OBJECTIVE 1 COMMENTS

Most students have an easy time with this section. I emphasize that we must first write the square root of a negative number in terms of i before we perform any operations. Showing students the example about $\sqrt{-1}\sqrt{-1}$ in the lecture notes helps students understand why we do this.

Workbook Exploration

In the following 15-minute exploration, some groups may have trouble with signs. They also may forget to write i^2 as -1.

Group Exploration

Section Opener: Performing operations with complex numbers

Recall that $i^2 = -1$ and $i = \sqrt{-1}$. Because $i = \sqrt{-1}$, we perform operations with complex numbers in much the same way as we do with radical expressions. For example,

$$(4+2\sqrt{5})+(3+6\sqrt{5}) = 7+8\sqrt{5} \qquad \text{Add two radical expressions.}$$
$$(4+2i)+(3+6i) = 7+8i \qquad \text{Add two complex numbers.}$$

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Perform the indicated operation.

1. $(5-2i)+(4+9i)$ **2.** $(4-7i)-(3-5i)$ **3.** $3i \cdot 8i$ **4.** $(5i)^2$
5. $4i(6i-3i)$ **6.** $(3-6i)(4-2i)$ **7.** $(2+3i)^2$ **8.** $(6-4i)^2$

Textbook and Workbook Exploration

The 10-minute "Finding powers of i" exploration is a nice opportunity for students to discover a pattern. The experience of discovering the pattern is just as important, if not more important, than the facts learned about powers of i.

For Problem 11, I point out that simplifying $\dfrac{7}{8+3i}$ is similar to simplifying $\dfrac{7}{8+3\sqrt{x}}$.

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SECTION 15.3 LECTURE NOTES

Objectives

1. Use the Pythagorean theorem to find the length of a side of a right triangle.
2. Use the Pythagorean theorem to make estimates about authentic situations.
3. Use the distance formula to find the distance between two points.
4. Find or graph the equation of a circle.

Main point: Work with the *Pythagorean theorem*, *distance formula*, and circles.

OBJECTIVE 1

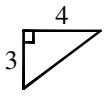
- An angle of 90° is called a *right angle*.
- If one angle of a triangle measures 90°, the triangle is a **right triangle**.
- The side opposite the right angle is the triangle's longest side. We call that side the **hypotenuse**, and we call the two shorter sides the **legs**. [Draw such a triangle.]

Pythagorean Theorem

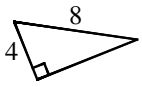
If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2+b^2 = c^2$. [Draw a sketch.]

The lengths of two sides of a right triangle are given. Find the length of the third side.

1.



2.



OBJECTIVE 2

3. The bottom of a ladder is 4 feet from the base of the building against which the ladder leans. How long must the ladder be so that the top will reach a window that is 12 feet above the ground?

4. The size of a rectangular television screen is usually described as the length of a diagonal. If a 19-inch screen has a height of 11 inches, what is the screen's width?

OBJECTIVE 3

Refer to the figure on the next page to show that

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 \qquad \text{Distance formula}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{Square root property; } d \geq 0$$

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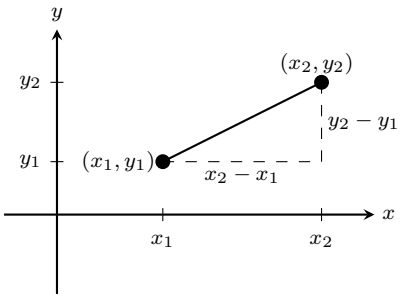

Distance Formula

The distance d between points (x_1, y_1) and (x_2, y_2) is given by the **distance formula:**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the distance between the two given points.

5. $(3, 2)$ and $(5, 6)$

6. $(-3, -1)$ and $(2, -6)$

Find the distance between the two given points. Round your result to the second decimal place.

7. $(-5.72, 2.11)$ and $(3.99, -8.06)$

8. $(-3.94, -6.25)$ and $(-1.28, -9.47)$

OBJECTIVE 4

Definition *Circle*

A **circle** with **center** point C and **radius** r, where $r > 0$, is the set of all points in a plane that are r units from point C in that plane. [Draw a sketch.]

Sketch a circle with center (h, k) and radius r. Explain the following:

$$\sqrt{(x - h)^2 + (y - k)^2} = r \qquad \text{Distance formula}$$
$$(x - h)^2 + (y - k)^2 = r^2 \qquad \text{Square both sides.}$$

Equation of a Circle

If a circle has center (h, k) and radius r, then an equation of the circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

The graph of an equation of this form with $r > 0$ is a circle with center $(h,\ k)$ and radius r.

Find an equation of the circle with the given center and radius.

9. Center $(0, 0)$, radius 5

10. Center $(0, 0)$, radius 3.9

11. Center $(5, 2)$, radius 3

12. Center $(-4, 3)$, radius $\sqrt{7}$

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Equation of a Circle Centered at the Origin

If a circle has center $(0, 0)$ and radius r, then an equation of the circle is

$$x^2 + y^2 = r^2$$

Find the center and radius of the circle. Graph the equation by hand.

13. $x^2 + y^2 = 9$

14. $x^2 + y^2 = 15$

15. $(x - 5)^2 + (y - 3)^2 = 4$

16. $(x + 4)^2 + (y - 2)^2 = 25$

SHORT HW 5, 11, 13, 15, 21, 29, 33, 35, 39, 55, 57, 63, 67, 72

MEDIUM HW 1, 5, 11, 13, 15, 17, 19, 21, 25, 29, 33, 35, 39, 43, 45, 49, 51, 55, 57, 59, 63, 65, 67, 72, 77, 79, 81, 83

SECTION 15.3 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students learn about the Pythagorean theorem, the distance formula, and circles in this section. This section can be used just after Section 9.4. However, the equation of a circle is derived by using the distance formula and then squaring both sides, which is not discussed until Section 13.5. Sections 15.3–15.5 can be taught as a module anytime after Section 13.5. Teaching the material in these sections at the end of the course serves as a first step in reviewing for the final exam. Note that these sections draw from a wide variety of concepts already addressed in the course including solving systems; sketching graphs of linear, quadratic, and square root functions; and solving quadratic and radical equations.

OBJECTIVE 1 COMMENTS

For the Pythagorean theorem, I emphasize that c in the equation $a^2 + b^2 = c^2$ is the length of the hypotenuse.

Textbook and Workbook Exploration

You could begin this section by providing a brief introduction to the Pythagorean theorem and then assigning the 20-minute "Pythagorean theorem and its converse" exploration as a class activity. I emphasize that the value of c represents the length of the hypotenuse.

OBJECTIVE 2 COMMENTS

For Problems 3 and 4, I encourage students to draw figures.

OBJECTIVE 3 COMMENTS

For Problems 5 and 6, I remind students that we must factor out the largest perfect square factor of the radicand.

Workbook Exploration

The following 20-minute exploration guides groups to derive the distance formula.

EXPLORATION *Finding the distance between two points*

1. Plot the points $(2,\ 1)$ and $(5,\ 5)$. Find the distance between the points. [**Hint:** Sketch a right triangle whose hypotenuse is the line segment from $(2,\ 1)$ to $(5,\ 5)$. Then use the Pythagorean theorem.]

2. Plot the points $(-3,\ -2)$ and $(2,\ 1)$ to help you find the distance between the points.

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3. Plot the points $(-4,\ 3)$ and $(3,\ -2)$ to help you find the distance between the points.

4. Find the distance between $(x_1,\ x_2)$ and $(y_1,\ y_2)$. [**Hint:** Write the slope formula for a line that contains the two points. Then refer to the formula to determine the run and the rise. Next, sketch a right triangle and write your expressions for the run and the rise next to the appropriate sides.]

OBJECTIVE 4 COMMENTS

For Problem 15, some students have trouble drawing a graph that resembles a circle. It can help to first plot the four points $(3, 3)$, $(5, 1)$, $(5, 5)$, and $(7, 3)$ and then draw the circle that contains these points. I point out that the graph of $(x-5)^2+(y-3)^2=4$ is the graph of $x^2+y^2=4$ translated 5 units to the right and 3 units up. I make similar observations for Problem 16.

Workbook Exploration

Once students understand how to use the distance formula, you could assign the following 15-minute exploration, which serves as a bridge from the distance formula to the general equation of a circle. It is helpful to supply groups with a compass or some other tool for drawing circles. Some groups have trouble with Problem 5 and many groups are challenged by Problem 6.

EXPLORATION *General equation of a circle*

For this exploration, you will need graph paper. It is also helpful to have a tool for drawing circles.

1. Sketch a circle that is not centered at the origin. Estimate the coordinates of the center.

2. Choose a point on the circle and estimate the coordinates of the point. Then find the distance between the point and the center of the circle.

3. Choose two more points on the circle and find the distance from each point to the center of the circle.

4. Compare the distances you calculated in Problems 2 and 3.

5. Let (x, y) be a point on the circle. Use the distance formula to write the radius r of your circle in terms of x and y. Then square both sides of your equation.

6. If a circle has center (h, k) with radius r, what is an equation of the circle?

Workbook Exploration

In the following 25-minute exploration, groups can possibly make connections between the Pythagorean theorem, the distance formula, and circles. Problem 3 is a nice extension that goes beyond the scope of the textbook.

EXPLORATION *Points inside, on, or outside a circle*

1. Consider the circle with center at the origin and radius 10. Does the point lie inside, on, or outside the circle?

   a. $(7,\ 6)$ **b.** $(-5,\ 9)$ **c.** $(-8,\ -6)$ **d.** $(\sqrt{15},\ -\sqrt{85})$

2. Consider the circle with center at the origin and radius 4. For the given point, what can you say about a^2+b^2?

   a. The point $(a,\ b)$ lies on the circle.

   b. The point $(a,\ b)$ lies inside the circle.

   c. The point $(a,\ b)$ lies outside the circle.

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3. Graph the inequality.

a. $x^2 + y^2 < 16$

b. $x^2 + y^2 > 16$

c. $x^2 + y^2 \geq 9$

d. $x^2 + y^2 \leq 25$

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SECTION 15.4 LECTURE NOTES

Objectives

1. Graph equations of ellipses.
2. Graph equations of hyperbolas.

Main point: Graph equations of ellipses and hyperbolas.

OBJECTIVE 1

1. Consider the equation $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$.

- **a.** Find the x-intercepts.
- **b.** Find the y-intercepts.
- **c.** Find the domain of the relation.
- **d.** Find points on the graph where x is $-3, -2, -1, \ldots, 3$.
- **e.** Sketch a curve that contains the points you have found.

Equation of an Ellipse

An equation that can be put into the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad \text{where } a > 0 \text{ and } b > 0$$

has an **ellipse** as its graph.

Find the intercepts of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

Intercepts of an Ellipse

The ellipse described by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

has x-intercepts $(-a, 0)$ and $(a, 0)$ and y-intercepts $(0, -b)$ and $(0, b)$. [Sketch such an ellipse.]

Graph the equation by hand.

2. $\dfrac{x^2}{4} + \dfrac{y^2}{36} = 1$

3. $\dfrac{x^2}{49} + \dfrac{y^2}{9} = 1$

4. $4x^2 + 9y^2 = 36$

5. $25x^2 + 7y^2 = 175$

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OBJECTIVE 2

Equations of a Hyperbola

An equation that can be put into one of the following forms has a **hyperbola** as its graph:

- An equation that can be put into the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \qquad \text{where } a > 0 \text{ and } b > 0$$

  is a hyperbola with x-intercepts $(-a, 0)$ and $(a, 0)$. [Sketch such a hyperbola.] There are no y-intercepts.

- An equation that can be put into the form

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1, \qquad \text{where } a > 0 \text{ and } b > 0$$

  is a hyperbola with y-intercepts $(0, -b)$ and $(0, b)$. [Sketch such a hyperbola.] There are no x-intercepts.

- The dashed lines that go through opposite corners of the dashed rectangle are *inclined asymptotes*.
- The dashed rectangle and the inclined asymptotes are not parts of the hyperbola.

Graphing Hyperbolas

To sketch a hyperbola on the basis of its equation,

1. Sketch a dashed rectangle whose sides are parallel to the axes and contain the points $(-a, 0)$, $(a, 0)$, $(0, -b)$, and $(0, b)$.
2. Sketch two dashed lines (the inclined asymptotes) that contain the diagonals of the rectangle.
3. Plot the intercepts of the hyperbola.
4. Sketch the branches to contain the intercepts and get closer to the asymptotes as $|x|$ gets large.

Graph the equation by hand.

6. $\dfrac{x^2}{25} - \dfrac{y^2}{4} = 1$

7. $\dfrac{y^2}{49} - \dfrac{x^2}{16} = 1$

8. $9y^2 - 25x^2 = 225$

9. $x^2 - 9y^2 = 9$

SHORT HW 1, 7, 13, 17, 19, 25, 29, 35, 43, 47, 53, 55, 57, 64, 65

MEDIUM HW 1, 7, 13, 17, 19, 25, 29, 35, 43, 47, 53, 55, 57, 64, 65, 69, 73, 75, 77

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SECTION 15.4 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

In this section, students graph ellipses and hyperbolas. This section can be used directly after Section 15.3.

OBJECTIVE 1 COMMENTS

For graphing equations of ellipses, I encourage my students to find the intercepts by substituting zero for variables rather than memorizing that an equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has x-intercepts $(-a, 0)$ and $(a, 0)$ and y-intercepts $(0, -b)$ and $(0, b)$.

Workbook Exploration

You could start class by warming groups up to ellipses by having them find intercepts in the following 10-minute exploration.

Group Exploration

Section Opener: Finding intercepts of ellipses

Find the intercepts of the graph of the given equation.

1. $\frac{x^2}{4} + \frac{y^2}{9} = 1$ **2.** $\frac{x^2}{36} + \frac{y^2}{16} = 1$ **3.** $\frac{x^2}{25} + \frac{y^2}{49} = 1$ **4.** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

OBJECTIVE 2 COMMENTS

For graphing equations of hyperbolas, I also encourage students to find the intercepts to then determine what type of hyperbola they need to sketch rather than memorizing the forms of the graphs of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$.

Textbook and Workbook Exploration

In the 15-minute "Graphical significance of a and b for ellipses and hyperbolas" exploration, students reflect on the graphical significance of a and b for ellipses and hyperbolas by "going backwards" from graphs to equations.

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SECTION 15.5 LECTURE NOTES

Objectives

1. Describe the meaning of a *nonlinear system*.
2. Solve nonlinear systems by graphing, substitution, or elimination.

Main point: Solve *nonlinear systems of equations*.

OBJECTIVE 1

A **nonlinear system of equations** is a system of equations in which *at least* one of the equations is not linear. Here is an example:

$$\begin{aligned} 4x^2 - 9y^2 &= 1 \\ y &= x^2 - 7 \end{aligned}$$

- A **solution** of a nonlinear system is an ordered pair that satisfies *all* the equations in the system.
- The **solution set** of a nonlinear system is the set of all solutions of the system.
- When solving nonlinear systems in this course, we find only solutions that have real-number coordinates.
- The solution set of a system of nonlinear equations can be found by locating the intersection points of the graphs of *all* the equations.

OBJECTIVE 2

Solve the system by graphing by hand. Also, solve the system by substitution.

1.

$$\begin{aligned} y &= x^2 - 3 \\ 4x - y &= 6 \end{aligned}$$

2.

$$\begin{aligned} y &= x + 1 \\ x^2 - y^2 &= 1 \end{aligned}$$

Solve the system by graphing by hand. Also, solve the system by elimination.

3.

$$\begin{aligned} x^2 + y^2 &= 25 \\ x^2 - y^2 &= 25 \end{aligned}$$

4.

$$\begin{aligned} y^2 - 9x^2 &= 9 \\ 9x^2 + y^2 &= 9 \end{aligned}$$

If it is reasonable to do so, we first graph the equations of a nonlinear system to determine the number of solutions and to find approximate coordinates of the solutions. Then we solve the system by substitution or elimination.

5. Solve the system by substitution or elimination. Round the coordinates of your solution(s) to the second decimal place.

$$\begin{aligned} x^2 - y^2 &= 9 \\ 4x^2 + 25y^2 &= 100 \end{aligned}$$

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SHORT HW 1, 3, 7, 9, 11, 15, 19, 23, 27, 29, 33, 35, 37, 41

MEDIUM HW 1, 3, 7, 9, 11, 15, 19, 23, 27, 29, 33, 35, 37, 39, 41, 43, 45, 49, 51, 53

SECTION 15.5 DETAILED COMMENTS AND EXPLORATIONS

Main points/connection to other parts of the text

Students solve nonlinear systems of equations in this section, which can be used directly after Section 15.4. This section serves as an excellent review of solving systems; graphing equations of lines, parabolas, circles, ellipses, and hyperbolas; solving quadratic equations; and even a little bit on solving radical equations (see Exercises 19 and 20 of Homework 15.5).

OBJECTIVES 1 and 2 COMMENTS

When solving nonlinear systems, I emphasize the connection between solving these systems symbolically and verifying the work graphically. It may be instructive to compare and contrast solving one of the systems in Problems 1–4 by all three methods (graphing, substitution, and elimination).

Workbook Exploration

You could start class with the following 15-minute exploration.

Group Exploration

Section Opener: Using graphing to solve nonlinear systems of equations

Solve the system by graphing.

1. $y = x^2$
$y = x + 2$

2. $x^2 + y^2 = 25$
$y = x + 5$

3. $x^2 + y^2 = 4$
$4x^2 + y^2 = 16$

4. $y^2 - 9x^2 = 9$
$9x^2 + y^2 = 9$

Textbook and Workbook Exploration

Note that the 15-minute "Using graphs to find the number of solutions" exploration ties in nicely with the exploration in Section 15.4, although this exploration can be assigned even if you did not assign the one in Section 15.4.

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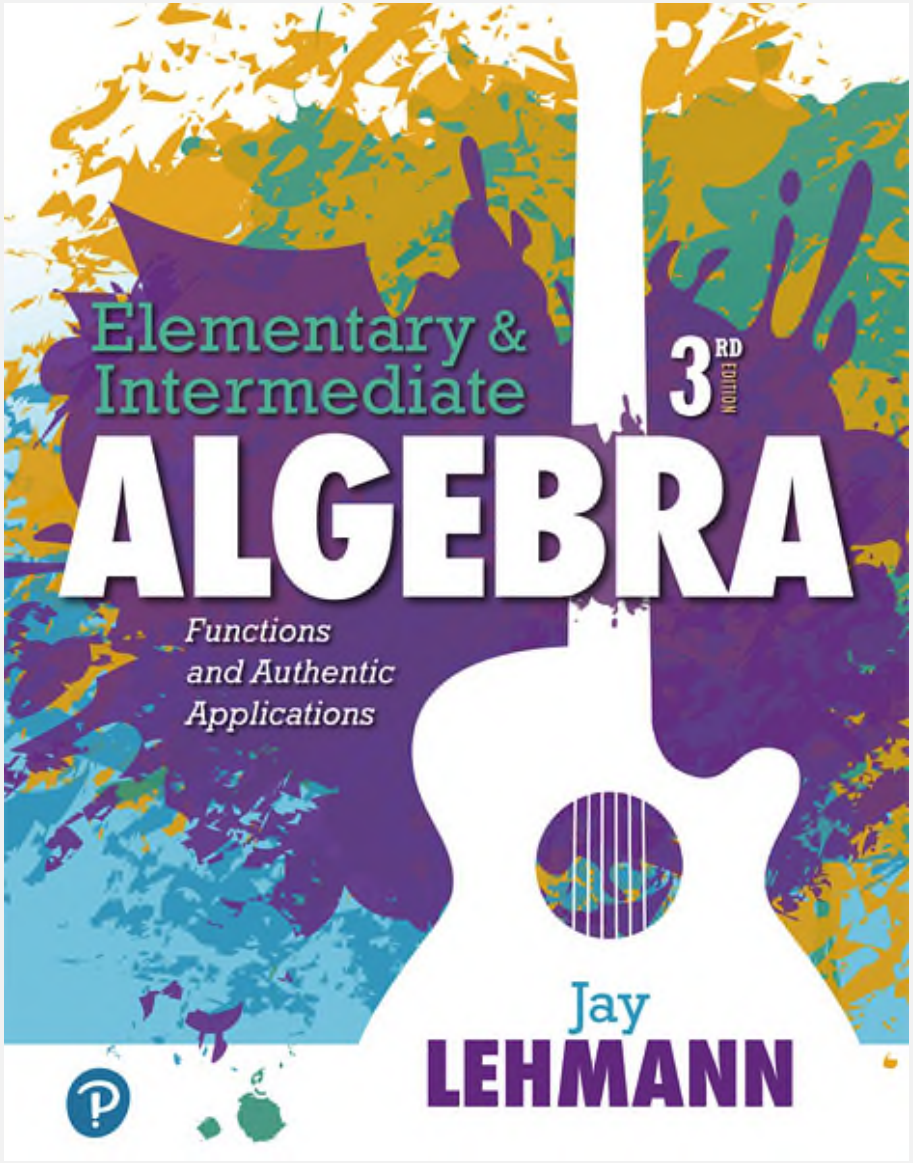
Chapter 10

Exponential Functions



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10.2 Rational Exponents

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$b^{1/n}$

Definition

For the counting number n, where $n \neq 1$,

- If n is odd, then $\boldsymbol{b^{1/n}}$ is the number whose nth power is b, and we call $b^{1/n}$ the ***n*th root of *b*.**
- If n is even and $b \geq 0$, then $\boldsymbol{b^{1/n}}$ is the nonnegative number whose nth power is b, and we call $b^{1/n}$ the **principal *n*th root of *b***.
- If n is even and $b < 0$, then $b^{1/n}$ is not a real number.

$b^{1/n}$ may be represented by $\sqrt[n]{b}$.

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Example: Simplifying Expressions Involving Rational Exponents (1 of 3)

Simplify.

1. $25^{1/2} = 5,$ since $5^2 = 25.$

2. $64^{1/3} = 4,$ since $4^3 = 64.$

3. $(-64)^{1/3} = -4,$ since $(-4)^3 = -64.$

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Example: Simplifying Expressions Involving Rational Exponents (2 of 3)

Simplify.

4. $16^{1/4} = 2,$ since $2^4 = 16.$

5. $-16^{1/4} = -\left(16^{1/4}\right) = -2.$

6. $\left(-16\right)^{1/4}$

is not a real number, since the fourth power of any real number is nonnegative.

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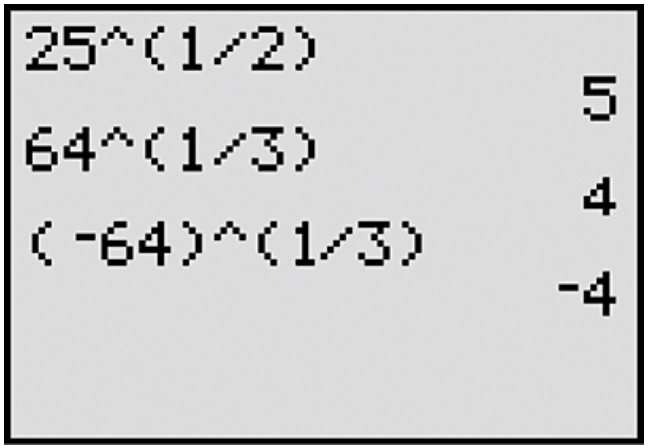
Example: Simplifying Expressions Involving Rational Exponents (3 of 3)

Graphing calculator checks for Problems 1, 2, and 3 are shown below. If an exponent involves an operation, you must use parentheses.



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Rational Exponent

Definition

For the counting numbers m and n, where $n \neq 1$ and b is any real number for which $b^{1/n}$ is a real number,

- $b^{m/n} = \left(b^{1/n}\right)^m = \left(b^m\right)^{1/n}$
- $b^{-m/n} = \dfrac{1}{b^{m/n}}, b \neq 0$

A power of the form $b^{m/n}$ or $b^{-m/n}$ is said to have a **rational exponent.**

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Example: Evaluating an Exponential Function

For $f(x) = 64^x$, $g(x) = 3(16)^x$, and $h(x) = -5(9)^x$, find the following.

1. $f\left(\frac{2}{3}\right) = f\left(\frac{2}{3}\right) = 64^{2/3} = \left(64^{1/3}\right)^2 = 4^2 = 16$

2. $g\left(\frac{3}{4}\right) = g\left(\frac{3}{4}\right) = 3(16)^{3/4} = 3\left(16^{1/4}\right)^3 = 3(2)^3 = 3 \cdot 8 = 24$

3. $h\left(-\frac{1}{2}\right) = h\left(-\frac{1}{2}\right) = -5(9)^{-1/2} = \frac{-5}{9^{1/2}} = -\frac{5}{3}$

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Properties of Rational Exponents

If m and n are rational numbers and b and c are any real numbers for which b^m, b^n, and c^n are real numbers, then

- $b^m b^n = b^{m+n}$ Product property for exponents
- $\frac{b^m}{b^n} = b^{m-n}, b \neq 0$ Quotient property for exponents
- $(bc)^n = b^n c^n$ Raising a product to a power
- $\left(\frac{b}{c}\right)^n = \frac{b^n}{c^n}$ Raising a quotient to a power
- $(b^m)^n = b^{mn}$ Raising a power to a power

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Example: Simplifying Expressions Involving Rational Exponents (1 of 2)

Simplify. Assume b is positive.

1. $b^{2/3}b^{1/2} = b^{\frac{2}{3}+\frac{1}{2}}$

$= b^{\frac{4}{6}+\frac{3}{6}}$

$= b^{7/6}$

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Example: Simplifying Expressions Involving Rational Exponents (2 of 2)

Simplify. Assume b is positive.

2. $\left(\frac{32b^2}{b^{12}}\right)^{2/5} = \left(32b^{2-12}\right)^{2/5}$

$= \left(32b^{-10}\right)^{2/5}$

$= \left(\frac{32}{b^{10}}\right)^{2/5}$

$= \frac{32^{2/5}}{\left(b^{10}\right)^{2/5}}$

$= \frac{\left(32^{1/5}\right)^2}{b^{10\cdot\frac{2}{5}}}$

$= \frac{2^2}{b^4}$

$= \frac{4}{b^4}$

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Example: Simplifying an Expression Involving Rational Exponents

Simplify $\dfrac{\left(81b^6c^{20}\right)^{1/2}}{\left(27b^{12}c^9\right)^{2/3}}$. Assume b and c are positive.

$$\frac{\left(81b^6c^{20}\right)^{1/2}}{\left(27b^{12}c^9\right)^{2/3}} = \frac{81^{1/2}\left(b^6\right)^{1/2}\left(c^{20}\right)^{1/2}}{27^{2/3}\left(b^{12}\right)^{2/3}\left(c^9\right)^{2/3}}$$

$$= \frac{9b^{6\cdot\frac{1}{2}}c^{20\cdot\frac{1}{2}}}{\left(27^{1/3}\right)^2 b^{12\cdot\frac{2}{3}}c^{9\cdot\frac{2}{3}}} = \frac{9b^3c^{10}}{3^2b^8c^6}$$

$$= \frac{9b^{-5}c^4}{9} = \frac{c^4}{b^5}$$

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