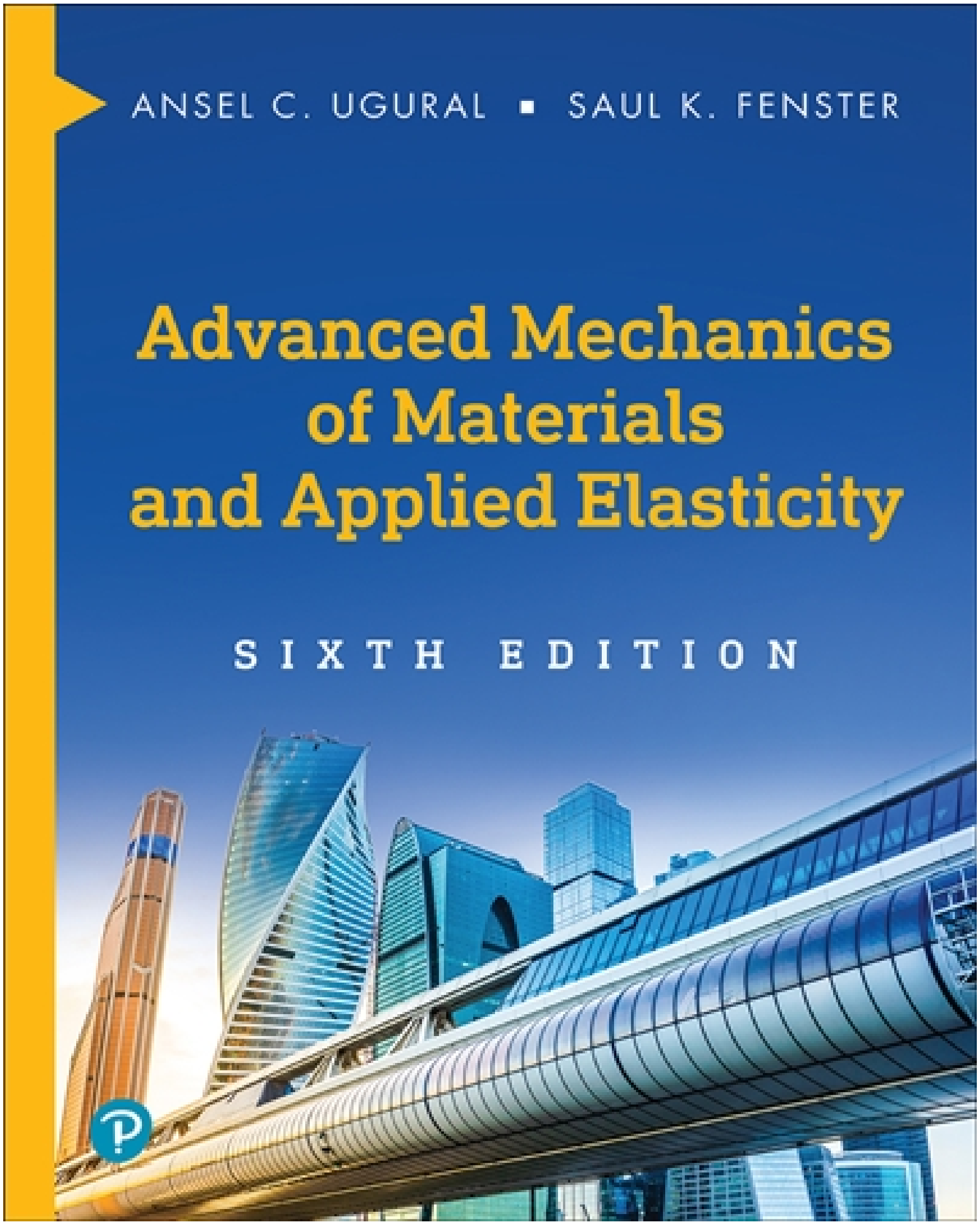
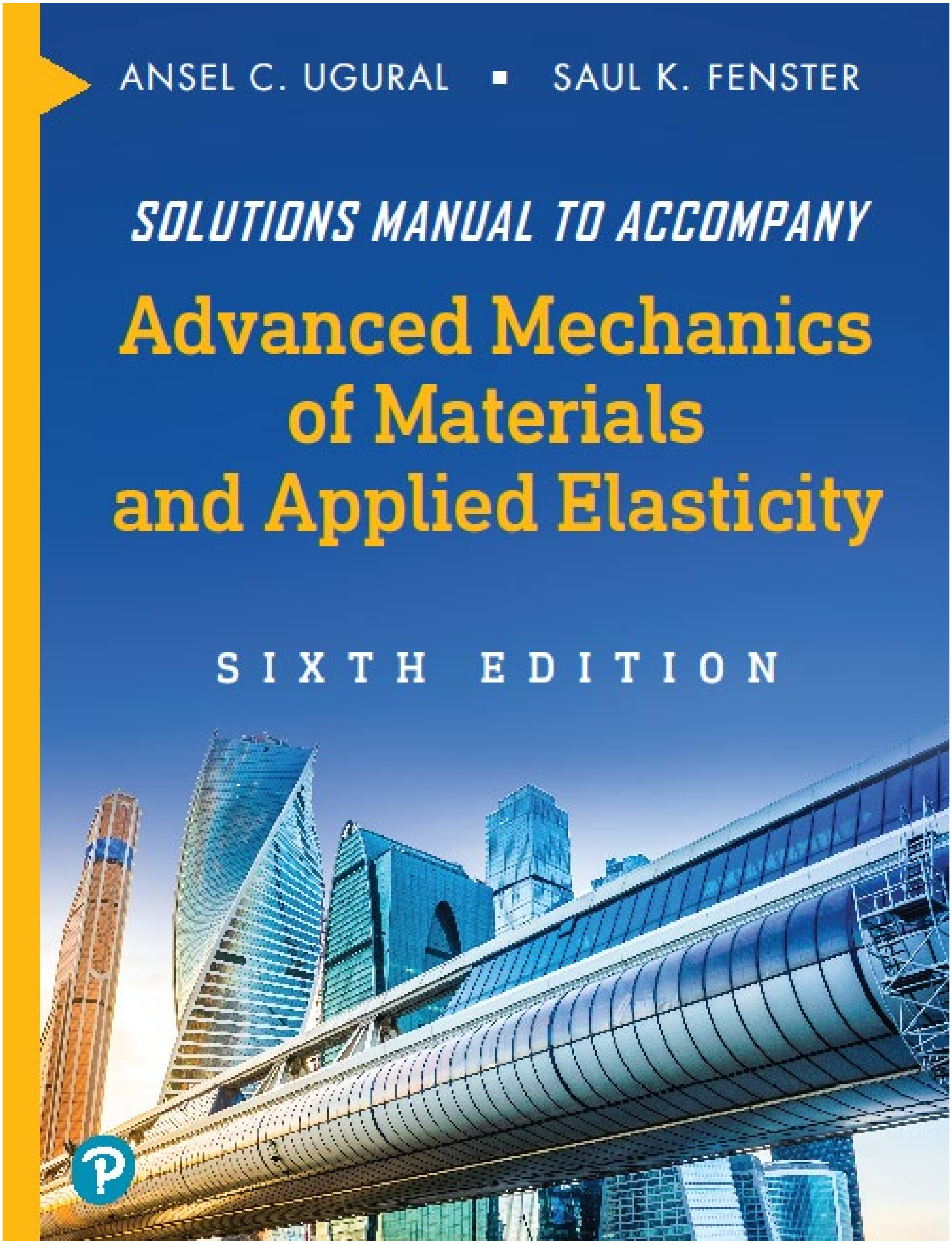
Solutions for Advanced Mechanics of Materials and Applied Elasticity 6th Edition by Ugural



Solutions


ANSEL C. UGURAL ▪ SAUL K. FENSTER
SOLUTIONS MANUAL TO ACCOMPANY
Advanced Mechanics of Materials and Applied Elasticity
SIXTH EDITION

CONTENTS

NOTES TO THE INSTRUCTOR

The **Solutions Manual** to accompany the text *Advanced Mechanics of Materials and Applied Elasticity* supplements the study of stress and deformation analyses developed in the book. The main objective of the manual is to provide efficient solutions for problems dealing with variously loaded members. This manual can also serve to guide the instructor in the assignments of problems, in grading these problems, and in preparing lecture materials as well as examination questions. Every effort has been made to have a solutions manual that can cut through the clutter and is self - explanatory as possible thus reducing the work on the instructor. It is written and class tested by the author, Ansel Ugural.

As indicated in its *preface*, the text is designed for the senior and/or first year graduate level courses in stress analysis. In order to accommodate courses of varying emphasis, considerably more material has been presented in the book than can be covered effectively in a single three-credit course. The instructor has the choice of assigning a variety of problems in each chapter. Answers to selected problems are given at the end of the text. A description of the topics covered is given in the *introduction* of each chapter throughout the text. It is hoped that the foregoing materials will help instructor in organizing his or her course to best fit the needs of his or her students.

Ansel C. Ugural

Holmdel, NJ

CHAPTER 1

SOLUTION (1.1)

We have

$$A = 50 \times 75 = 3.75(10^{-3})\ \text{m}^2, \theta = 50^o, \text{ and } \sigma_x = P/A.$$

Equations (1.11), with $\theta = 50^o$:

$$\sigma_{x'} = 700(10^3) = \sigma_x \cos^2 50^o = 0.413\sigma_x = 110.18P$$

or $P = 6.35\ \text{kN}$

and

$$\left|\tau_{x'y'}\right| = 560(10^3) = \sigma_x \sin 50^o \cos 50^o = 0.492\sigma_x = 131.2P$$

Solving

$$P = 4.27\ \text{kN} = P_{all}$$ ▲

SOLUTION (1.2)

Normal stress is

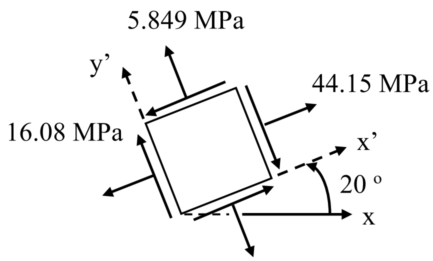
$$\sigma_x = \frac{P}{A} = \frac{125(10^3)}{0.05 \times 0.05} = 50\ \text{MPa}$$

(a) Equations (1.11), with $\theta = 20^o$:

$$\sigma_{x'} = 50\cos^2 20^o = 44.15\ \text{MPa}$$

$$\tau_{x'y'} = -50\sin 20^o \cos 20^o = -16.08\ \text{MPa}$$ ▲

$$\sigma_{y'} = 50\cos^2(20^o + 90^o) = 5.849\ \text{MPa}$$

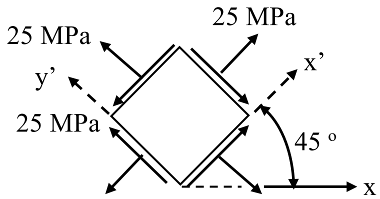


(b) Equations (1.11), with $\theta = 45^o$:

$$\sigma_{x'} = 50\cos^2 45^o = 25\ \text{MPa}$$

$$\tau_{x'y'} = -50\sin 45^o \cos 45^o = -25\ \text{MPa}$$ ◄

$$\sigma_{y'} = 50\cos^2(45^o + 90^o) = 25\ \text{MPa}$$



SOLUTION (1.3)

From Eq. (1.11a),

$$\sigma_x = \frac{\sigma_{x'}}{\cos^2\theta} = \frac{-75}{\cos^2 30^o} = -100 \text{ MPa}$$

For $\theta = 50^o$, Eqs. (1.11) give then

$$\sigma_{x'} = -100\cos^2 50^o = -41.32 \text{ MPa} \quad \blacktriangle$$

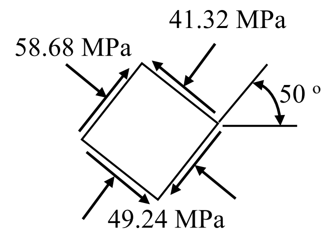
$$\tau_{x'y'} = -(-100)\sin 50^o \cos 50^o$$

$$= 49.24 \text{ MPa}$$

Similarly, for $\theta = 140^o$:

$$\sigma_{x'} = -100\cos^2 140^o = -58.68 \text{ MPa} \quad \blacktriangle$$

$$\tau_{x'y'} = -49.24 \text{ MPa}$$



SOLUTION (1.4)

Refer to Fig. 1.6c. Equations (1.11) by substituting the double angle-trigonometric relations, or Eqs. (1.18) with $\sigma_y = 0$ and $\tau_{xy} = 0$, become

$$\sigma_{x'} = \tfrac{1}{2}\sigma_x + \tfrac{1}{2}\sigma_x \cos 2\theta \qquad \text{and} \qquad |\tau_{x'y'}| = \tfrac{1}{2}\sigma_x \sin 2\theta$$

or

$$20 = \tfrac{P}{2A}(1+\cos 2\theta) \qquad \text{and} \qquad 10 = \tfrac{P}{2A}\sin 2\theta$$

The foregoing lead to

$$2\sin 2\theta - \cos 2\theta = 1 \qquad \text{(a)}$$

By introducing trigonometric identities, Eq. (a) becomes

$4\sin\theta\cos\theta - 2\cos^2\theta = 0$ or $\tan\theta = 1/2$. Hence

$$\theta = 26.56^o \quad \blacktriangle$$

Thus,

$$20 = \tfrac{P}{2(1300)}(1+0.6)$$

gives

$$P = 32.5 \text{ kN} \quad \blacktriangle$$

It can be shown that use of Mohr's circle yields readily the same result.

SOLUTION (1.5)

Equations (1.12):

$$\sigma_1 = \frac{P}{A} = \frac{-150(10^3)}{\frac{\pi}{4}(50)^2} = -76.4 \text{ MPa} \quad \blacktriangle$$

$$\tau_{max} = \frac{P}{2A} = 38.2 \text{ MPa} \quad \blacktriangle$$

SOLUTION (1.6)

Shaded transverse area:

$$A = 2at = 2(10)(75) = 1.5(10^3)\ \text{mm}^2$$

Metal is capable of supporting the load

$$P = \sigma A = 90(10^6)(1.5\times 10^{-3}) = 135\ \text{kN}$$

Apply Eqs. (1.11):

$$\sigma_{x'} = 25(10^6) = \frac{P}{1.5(10^{-3})}(\cos^2 55^o), \quad P = 114\ \text{kN}$$

$$\tau_{x'y'} = 12(10^6) = -\frac{P}{1.5(10^{-3})}\sin 55^o \cos 55^o, \quad P = 38.3\ \text{kN}$$

Thus,

$$P_{all} = 38.3\ \text{kN}$$ ▲

SOLUTION (1.7)

Use Eqs. (1.11):

$$\sigma_{x'} = 20(10^6) = \frac{P}{1.5(10^{-3})}(\cos^2 40^o), \quad P = 51.1\ \text{kN}$$

$$\tau_{x'y'} = 8(10^6) = -\frac{P}{1.5(10^{-3})}\sin 40^o \cos 40^o, \quad P = 24.4\ \text{kN}$$

Thus,

$$P_{all} = 24.4\ \text{kN}$$ ▲

SOLUTION (1.8)

$$A = 15\times 30 = 450\ \text{mm}^2$$

Apply Eqs. (1.11):

$$\sigma_{x'} = \frac{120(10^3)}{450\times 10^{-6}}(\cos^2 40^o) = 156\ \text{MPa}$$

$$\tau_{x'y'} = -\frac{120(10^3)}{450\times 10^{-6}}\sin 40^o \cos 40^o = -131\ \text{MPa}$$ ◀

SOLUTION (1.9)

We have $A = 450(10^{-6})\ \text{m}^2$. Use Eqs. (1.11):

$$\sigma_{x'} = \frac{-100(10^3)}{450\times 10^{-6}}(\cos^2 60^o) = -55.6\ \text{MPa}$$

$$\tau_{x'y'} = -\frac{-100(10^3)}{450\times 10^{-6}}\sin 60^o \cos 60^o = 96.2\ \text{MPa}$$ ◀

SOLUTION (1.10)

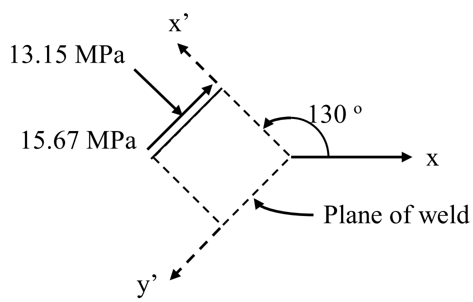
$$\theta = 40^o + 90^o = 130^o$$

$$\sigma_x = \frac{P}{A} = -\frac{150(10^3)}{\pi(0.08^2 - 0.07^2)} = -31.83 \text{ MPa}$$

Equations (1.11):

$$\sigma_{x'} = -31.83\cos^2 130^o = -13.15 \text{ MPa}$$ ◀

$$\tau_{x'y'} = 31.83\sin 130^o \cos 130^o = -15.67 \text{ MPa}$$ ◀



SOLUTION (1.11)

Use Eqs. (1.14),

$$(2x) + (-2xy) + (x) + F_x = 0$$

$$(-y^2) + (-2yz + x) + (0) + F_y = 0$$

$$(z - 4xy) + (0) + (-2z) + F_z = 0$$

Solving, we have (in MN/m^3):

$$F_x = -3x + 2xy \qquad F_y = -x + y^2 + 2yz \qquad F_z = 4xy + z \quad \text{(a)}$$ ◀

Substituting x=-0.01 m, y=0.03 m, and z=0.06 m, Eqs. (a) yield the following values

$$F_x = 29.4 \text{ kN/m}^3 \qquad F_y = 14.5 \text{ kN/m}^3 \qquad F_z = 58.8 \text{ kN/m}^3$$

Resultant body force is thus

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = 67.32 \text{ kN/m}^3$$ ◀

SOLUTION (1.12)

Equations (1.14):

$$-2c_1y - 2c_1y + 0 + 0 = 0, \qquad 4c_1y \neq 0$$

$$0 + c_3z + 0 + 0 = 0, \qquad c_3z \neq 0$$

$$0 + 0 + 0 + 0 = 0$$

No. Eqs. (1.14) are not satisfied. ◀

SOLUTION (1.13)

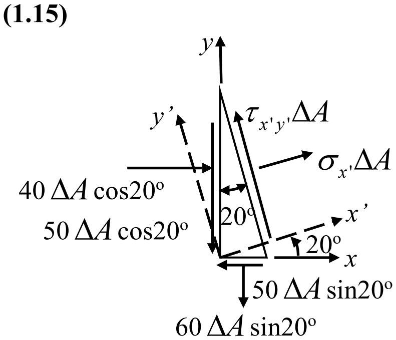
(a) No. Eqs. (1.14) are not satisfied. ▶
(b) Yes. Eqs. (1.14) are satisfied. ▶

SOLUTION (1.14)

Eqs. (1.14) for the given stress field yield: ▶

$$F_x = F_y = F_z = 0$$

SOLUTION (1.15)



$$\sum F_{x'} = 0: \quad \sigma_{x'}\Delta A + 40\cos^2 20^o - 60\Delta A \sin^2 20^o$$

$$-2(50\Delta A \sin 20^o \cos 20^o) = 0$$

$$\sigma_{x'} = -35.32 + 7.02 + 32.14 = 3.8 \text{ MPa}$$ ▶

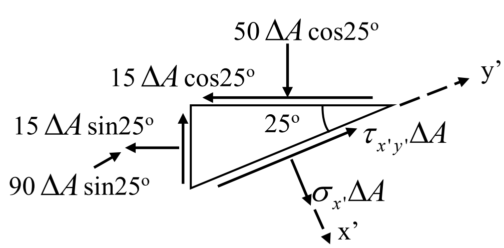
$$\sum F_{y'} = 0: \quad \tau_{x'y'}\Delta A - 40\Delta A \sin 20^o \cos 20^o$$

$$-60\Delta A \sin 20^o \cos 20^o - 50\Delta A \cos^2 20^o$$

$$+50\Delta A \sin^2 20^o = 0$$

$$\tau_{x'y'} = 12.86 + 19.28 + 44.15 - 5.85 = 70.4 \text{ MPa}$$ ▶

SOLUTION (1.16)



$$\sum F_{x'} = 0: \quad \sigma_{x'}\Delta A + 50\Delta A \cos^2 25^o$$

$$-90\Delta A \sin^2 25^o - 2(15\Delta A \sin 25^o \cos 25^o) = 0$$

$$\sigma_{x'} = -41.07 + 16.07 + 11.49 = -13.5 \text{ MPa}$$ ▶

(CONT.)

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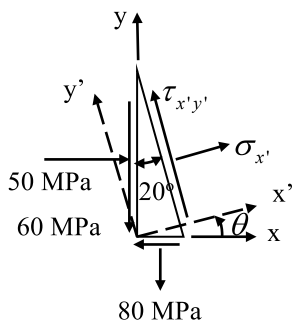
1.16 (CONT.)

$$\sum F_{y'} = 0: \quad \tau_{x'y'}\Delta A - 50\Delta A \sin 25^o \cos 25^o$$
$$-90\Delta A \sin 25^o \cos 25^o - 15\Delta A \cos^2 25^o + 15\Delta A \sin^2 25^o = 0$$
$$\tau_{x'y'} = 19.15 + 34.47 + 12.32 - 2.68 = 63.3 \text{ MPa}$$

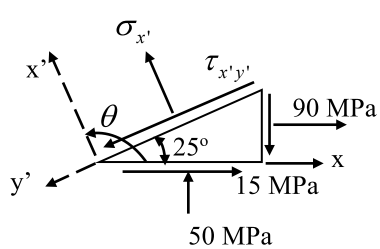
SOLUTION (1.17)



θ=20°

$$\sigma_{x'} = \frac{1}{2}(-40+60) + \frac{1}{2}(-40-60)\cos 40^o + 50\sin 40^o$$
$$= 10 - 38.3 + 32.1 = 3.8 \text{ MPa}$$
$$\tau_{x'y'} = -\frac{1}{2}(-40-60)\sin 40^o + 50\cos 40^o$$
$$= 32.14 + 38.3 = 70.4 \text{ MPa}$$

SOLUTION (1.18)



θ=115°

$$\sigma_{x'} = \frac{1}{2}(90-50) + \frac{1}{2}(90+50)\cos 230^o - 15\sin 230^o$$
$$= 20 - 45 + 11.5 = -13.5 \text{ MPa}$$
$$\tau_{x'y'} = -\frac{1}{2}(90+50)\sin 230^o - 15\cos 230^o$$
$$= 53.62 + 9.64 = 63.3 \text{ MPa}$$

SOLUTION (1.19)

Transform from $\theta = 40^o$ to $\theta = 0$. For convenience in computations, Let

$$\sigma_x = -160 \text{ MPa}, \quad \sigma_y = -80 \text{ MPa}, \quad \tau_{xy} = 40 \text{ MPa} \text{ and } \theta = -40^o$$

Then

$$\sigma_{x'} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{1}{2}(-160 - 80) + \frac{1}{2}(-160 + 80)\cos(-80^o) + 40\sin(-80^o)$$

$$= -166.3 \text{ MPa} \quad \blacktriangle$$

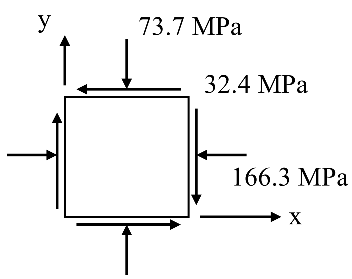
$$\tau_{x'y'} = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{1}{2}(-160 + 80)\sin(-80^o) + 40\cos(-80^o)$$

$$= -32.4 \text{ MPa} \quad \blacktriangle$$

So $\quad \sigma_{y'} = \sigma_x + \sigma_y - \sigma_{x'} = -160 - 80 + 166.3 = -73.7 \text{ MPa}$ ▲

For $\theta = 0^o$:



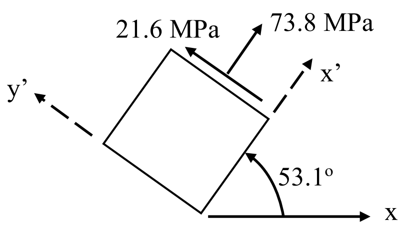
SOLUTION (1.20)

$$\theta = \tan^{-1}\frac{4}{3} = 53.1^o$$

$$\sigma_{x'} = \frac{45 + 90}{2} + \frac{45 - 90}{2}\cos 106.2^o$$

$$= 67.5 + 6.28 = 73.8 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -\frac{45 - 90}{2}\sin 106.2^o = 21.6 \text{ MPa}$$



SOLUTION (1.21)

$\tau_{xy} = 0 \qquad \theta = 70^o$

(a) $\tau_{x'y'} = -30 = -\dfrac{\sigma - 60}{2}\sin 140^o \qquad \sigma = 153.3 \text{ MPa}$ ◄

(b) $\sigma_{x'} = 80 = \dfrac{\sigma + 60}{2} + \dfrac{\sigma - 60}{2}\cos 140^o \qquad \sigma = 231 \text{ MPa}$ ◄

SOLUTION (1.22)

Equations(1.18) with $\theta = 60^o$, $\sigma_x = 110$ MPa, $\sigma_y = 0$, $\tau_{xy} = 50$ MPa give

$$\sigma_{x'} = \tfrac{1}{2}(110) + \tfrac{1}{2}(110)\cos 120^o + 50\sin 120^o = 70.8 \text{ MPa}$$
$$\tau_{x'y'} = -\tfrac{1}{2}(110)\sin 120^o + 50\cos 120^o = -72.6 \text{ MPa}$$
$$\sigma_{y'} = \tfrac{1}{2}(110) - \tfrac{1}{2}(110)\cos 120^o - 50\sin 120^o = 39.2 \text{ MPa}$$

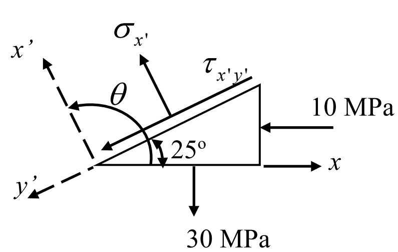
SOLUTION (1.23)

Equations(1.18) with $\theta = 30^o$, $\sigma_x = 110$ MPa, $\sigma_y = 0$, $\tau_{xy} = 50$ MPa result in

$$\sigma_{x'} = \tfrac{1}{2}(110) + 55\cos 60^o + 50\sin 60^o = 125.8 \text{ MPa}$$
$$\tau_{x'y'} = -\tfrac{1}{2}(110)\sin 60^o + 50\cos 60^o = -22.6 \text{ MPa}$$
$$\sigma_{y'} = \tfrac{1}{2}(110) - 55\cos 60^o - 50\sin 60^o = -15.8 \text{ MPa}$$

SOLUTION (1.24)

We have



$\theta = 25 + 90 = 115^o$

$\sigma_x = -10$ MPa

$\sigma_y = 30$ MPa

$\tau_{xy} = 0$

(a) $\sigma_{x'} = \dfrac{1}{2}(\sigma_x + \sigma_y) + \dfrac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta$

$= \dfrac{1}{2}(-10 + 30) + \dfrac{1}{2}(-10 - 30)\cos 230^o = 22.86 \text{ MPa}$ ◄

Thus,

$$\sigma_w = \sigma_{x'} = 22.86 \text{ MPa}$$

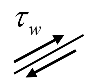
(CONT.)

(b) $\tau_{x'y'} = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$

$$= -\frac{1}{2}(-10-30)\sin 230^o = -15.32 \text{ MPa}$$

So

$$\tau_w = \tau_{x'y'} = -15.32 \text{ MPa}$$



▲

SOLUTION (1.25)

(a) $\sigma_1 = 80 = \frac{0+50}{2} + \sqrt{(\frac{0-50}{2})^2 + \tau^2}$

$\tau = 49 \text{ MPa}$ ▲

(b) $\tau_{\max} = \sqrt{(\frac{-50}{2})^2 + 49^2} = 55 \text{ MPa}$

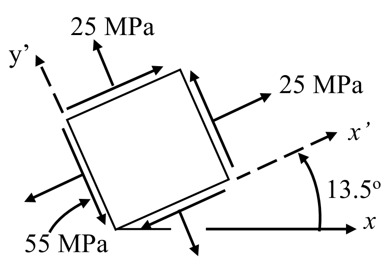
$\sigma' = \frac{50}{2} = 25 \text{ MPa}$ ▲

$2\theta_s = \tan^{-1}[-\frac{0-50}{2(49)}] = 27^o$

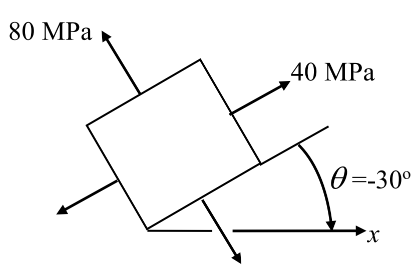
$$\tau_{x'y'} = \frac{50}{2}\sin 27^o + 49\cos 27^o = 55 \text{ MPa}$$

Thus,

$$\theta_s' = 13.5^o$$ ▲



SOLUTION (1.26)



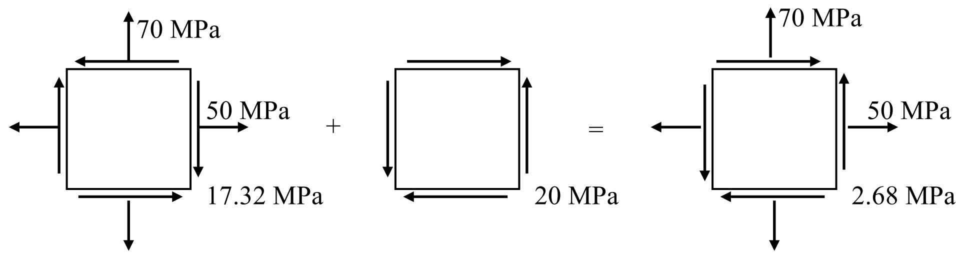
(CONT.)

1.26 (CONT.)

$$\sigma_x = \frac{40+80}{2} + \frac{40-80}{2}\cos(-60^o) = 60-10 = 50 \text{ MPa}$$

$$\sigma_y = 60+10 = 70 \text{ MPa}$$

$$\tau_{xy} = -\frac{40-80}{2}\sin(-60^o) = -17.32 \text{ MPa}$$



$$\sigma_{1,2} = \frac{50+70}{2} \pm \sqrt{(\frac{50-70}{2})^2 + 2.68^2} = 60 \pm 10.35$$

$$\sigma_1 = 70.35 \text{ MPa} \qquad \sigma_2 = 49.65 \text{ MPa}$$ ▲

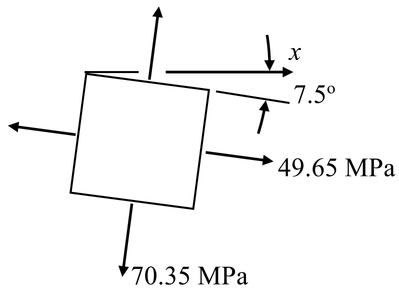
$$2\theta_p = \tan^{-1}[\frac{2(2.68)}{50-70}] = -15^o$$

$$\sigma_{x'} = 60 + \frac{50-70}{2}\cos(-15^o) + 2.68\sin(-15^o)$$

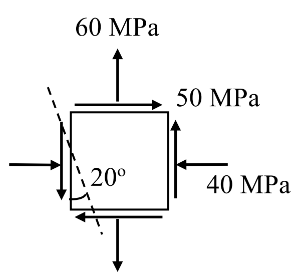
$$= 60 - 9.66 - 0.694 = 49.65 \text{ MPa}$$

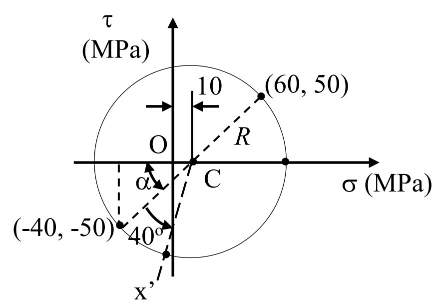
Thus,

$$\theta_p'' = -7.5^o$$ ▲



SOLUTION (1.27)





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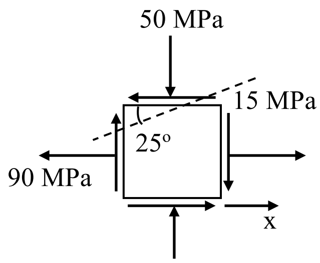
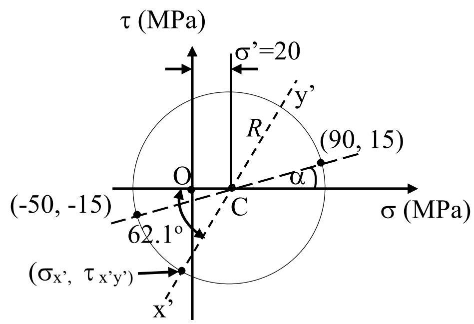
1.27 (CONT.)

$$\alpha = \tan^{-1}\frac{50}{50} = 45^o$$
$$R = (50^2 + 50^2)^{\frac{1}{2}} = 70.7$$
$$\tau_{x'y'} = \sin 85^o (70.7) = 70.4 \text{ MPa}$$
$$\sigma_{x'} = 10 - \cos 85^o (70.7) = 3.84 \text{ MPa}$$

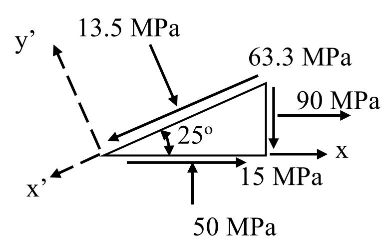
▲

SOLUTION (1.28)





$$\alpha = \tan^{-1}\frac{15}{70} = 12.1^o$$
$$R = (15^2 + 70^2)^{\frac{1}{2}} = 71.6$$
$$\tau_{x'y'} = 71.6 \sin 62.1^o = 63.3 \text{ MPa}$$
$$\sigma_{x'} = -71.6 \cos 62.1^o + 20$$
$$= -13.5 \text{ MPa}$$



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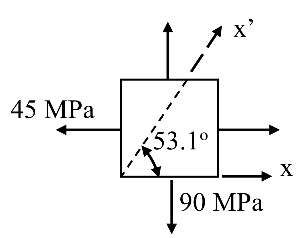
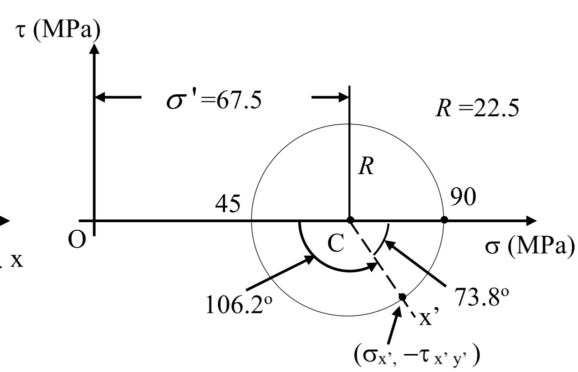
SOLUTION (1.29)





(CONT.)

1.29 (CONT.)

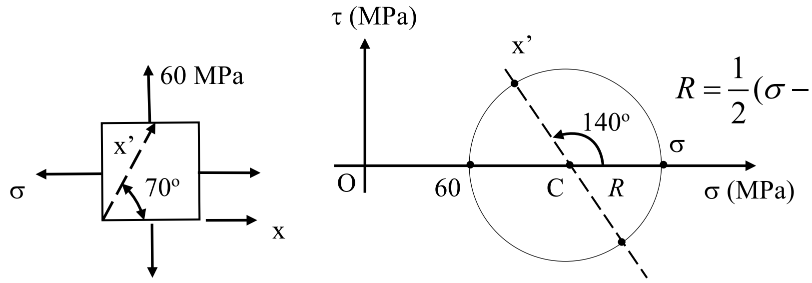
$$\tau_{x'y'} = 22.5\sin 73.8^o = 21.6 \text{ MPa}$$

$$\sigma_{x'} = 67.5 + 22.5\cos 73.8^o = 73.8 \text{ MPa}$$

Sketch of results is as shown in solution of Prob. 1.20.

SOLUTION (1.30)

(a)



$$R = \frac{1}{2}(\sigma - 60)$$

$$\tau_{x'y'} = -30 = \frac{\sigma - 60}{2}\sin(-40^o); \qquad \sigma = 153.3 \text{ MPa}$$

(b) $$\sigma_{x'} = 80 = 60 + \frac{\sigma - 60}{2}[1 - \cos(-40^o)]$$

$$\sigma = 231 \text{ MPa}$$

SOLUTION (1.31)

(a) From Mohr's circle, Fig. (a):

$$\sigma_1 = 121 \text{ MPa} \qquad \sigma_2 = -71 \text{ MPa} \qquad \tau_{max} = 96 \text{ MPa}$$

$$\theta_p' = -19.3^o \qquad \theta_s' = 25.7^o$$

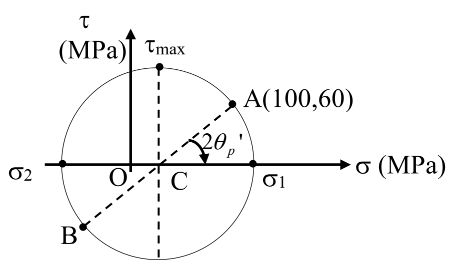


Figure (a)

By applying Eq. (1.20):

$$\sigma_{1,2} = \tfrac{50}{2} \pm \left[\tfrac{22,500}{4} + 3600\right]^{\frac{1}{2}} = 25 \pm 96$$

or $\sigma_1 = 121 \text{ MPa} \qquad \sigma_2 = -71 \text{ MPa}$

Using Eq. (1.19):

$$\tan 2\theta_p = -\tfrac{12}{15} = -0.8$$

$$\theta_p' = -19.3^o \qquad \theta_s' = 25.7^o$$

(CONT.)

1.31 (CONT.)

(b) From Mohr's circle, Fig. (b):

$$\sigma_1 = 200 \text{ MPa} \qquad \sigma_2 = -50 \text{ MPa} \qquad \tau_{max} = 125 \text{ MPa}$$

$$\theta_p' = 26.55^o \qquad \theta_s' = 71.55^o$$

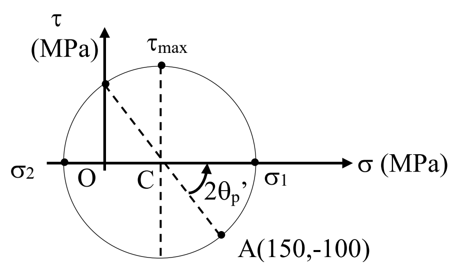


Figure (b)

Through the use of Eq. (1.20),

$$\sigma_{1,2} = 75 \pm \left[\tfrac{22,500}{4} + 10,000\right]^{\frac{1}{2}} = 75 \pm 125$$

or

$$\sigma_1 = 200 \text{ MPa} \qquad \sigma_2 = -50 \text{ MPa}$$

Using Eq. (1.19), $\tan 2\theta_p = 4/3$:

$$\theta_p' = 26.57^o \qquad \theta_s' = 71.57^o$$

SOLUTION (1.32)

Referring to Mohr's circle, Fig. 1.15:

$$\sigma_{x'} = \tfrac{\sigma_1+\sigma_2}{2} + \tfrac{\sigma_1-\sigma_2}{2}\cos 2\theta \qquad \text{(a)}$$

$$\sigma_{y'} = \tfrac{\sigma_1+\sigma_2}{2} - \tfrac{\sigma_1-\sigma_2}{2}\cos 2\theta$$

$$\tau_{x'y'} = \tfrac{\sigma_1-\sigma_2}{2}\sin 2\theta \qquad \text{(b)}$$

From Eqs. (a),

$$\sigma_{x'} + \sigma_{y'} = \sigma_1 + \sigma_2$$

By using $\cos^2 2\theta + \sin^2 2\theta = 1$, and Eqs. (a) and (b), we have

$$\sigma_{x'} \cdot \sigma_{y'} - \tau_{x'y'}^2 = \sigma_1 \cdot \sigma_2 = const.$$

SOLUTION (1.33)

We have

$$\tan 2\theta_p = \tfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \tfrac{2(-70)}{50-(-190)} = -0.583$$

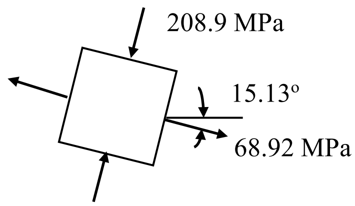
$$2\theta_p = -30.24^o \qquad \text{and } \theta_p = -15.12^o$$

(CONT.)

1.33 (CONT.)

Equations (1.18):

$$\sigma_{x'} = \tfrac{50-190}{2} + \tfrac{50+190}{2}\cos(-30.26^o) - 70\sin(-30.26^o)$$
$$= -70 + 103.65 + 35.275 = 68.93 \text{ MPa} = \sigma_1$$
$$\sigma_{y'} = \sigma_x + \sigma_y - \sigma_{x'} = -208.9 \text{ MPa} = \sigma_2$$



SOLUTION (1.34)

$$\tau_{\max} = \sqrt{(\tfrac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$$

Substituting the given values

$$140^2 = \left(\tfrac{60+100}{2}\right)^2 + \tau_{xy}^2$$

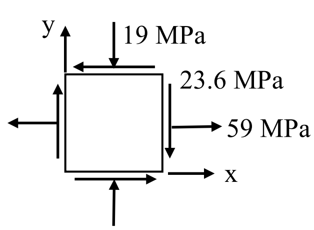
or

$$\tau_{xy,\max} = 114.89 \text{ MPa}$$

SOLUTION (1.35)

Transform from $\theta = 60^o$ to $\theta = 0^o$ with $\sigma_{x'} = -20$ MPa, $\sigma_{y'} = 60$ MPa, $\tau_{x'y'} = -22$ MPa, and $\theta = -60^o$. Use Eqs. (1.18):

$$\sigma_x = \tfrac{-20+60}{2} + \tfrac{-20-60}{2}\cos 2(-60^o) - 22\sin 2(-60^o) = 59 \text{ MPa}$$
$$\sigma_y = \sigma_{x'} + \sigma_{y'} - \sigma_x = -19 \text{ MPa}$$
$$\tau_{xy} = -23.6 \text{ MPa}$$



SOLUTION (1.36)

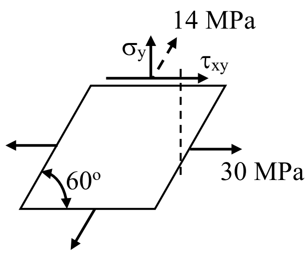


Figure (a)

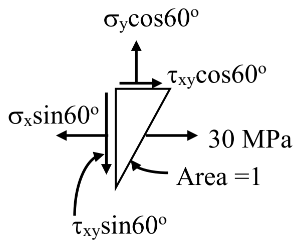


Figure (b)

(a) Figure (a):

$$\sigma_y = 14\sin 60^o = 12.12 \text{ MPa}$$ ▲

$$\tau_{xy} = 14\cos 60^o = 7 \text{ MPa}$$

Figure (b):

$$\sum F_y = 12.12\cos 60^o - \tau_{xy}\sin 60^o = 0$$

or

$$\tau_{xy} = 7 \text{ MPa} \text{ (as before)}$$

$$\sum F_x = -\sigma_x \sin 60^o + 30 + 7\cos 60^o = 0$$

or

$$\sigma_x = 38.68 \text{ MPa}$$ ▲

(b) Equation (1.20) is therefore:

$$\sigma_{1,2} = \tfrac{38.68+12.12}{2} \pm \left[\left(\tfrac{38.68-12.12}{2}\right)^2 + 7^2\right]^{\frac{1}{2}}$$ ▲

or $\sigma_1 = 40.41 \text{ MPa}, \qquad \sigma_2 = 10.39 \text{ MPa}$

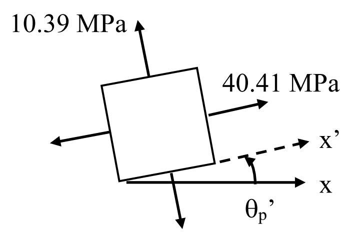
Also,

$$\theta_p = \tfrac{1}{2}\tan^{-1}\frac{2(7)}{38.68-12.12} = 13.9^o$$

Note: Eq. (1.18a) gives, $\sigma_{x'} = 40.41 \text{ MPa}$.

Thus,

$$\theta_p{}' = 13.9^o$$ ▲



SOLUTION (1.37)

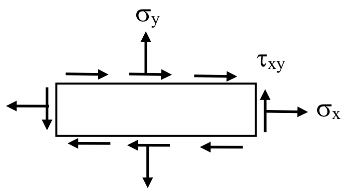


Figure (a)

Figure (a):

$$\sigma_x = 100\cos 45^o = 70.7 \text{ MPa}$$
$$\sigma_y = 100\sin 45^o = 70.7 \text{ MPa}$$
$$\tau_{xy} = 100\cos 45^o = 70.7 \text{ MPa}$$

Now, Eqs. (1.18) give (Fig. b):

$$\sigma_{x'} = 70.7 + 0 + 70.7\sin 240^o = 9.47 \text{ MPa}$$
$$\tau_{x'y'} = -0 + 70.7\cos 240^o = -35.35 \text{ MPa}$$ ▲
$$\sigma_{y'} = 70.7 - 0 - 70.7\sin 240^o = 131.9 \text{ MPa}$$

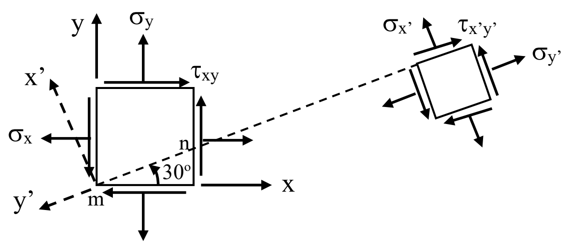


Figure (b)

SOLUTION (1.38)

$$\sigma_y = -70\sin 30^o = -35 \text{ MPa}$$ ◄
$$\tau_{xy} = 70\cos 30^o = 60.6 \text{ MPa}$$

(a) Figure (a):

$$\sum F_x = -150 + 0.5\sigma_x + 60.6(0.866) = 0$$

or $\sigma_x = 195 \text{ MPa}$ ◄

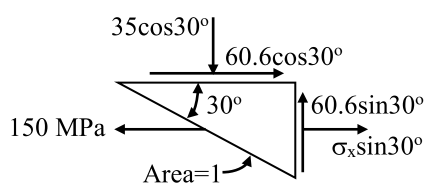


Figure (a)

(CONT.)

1.38 (CONT.)

(b) Equation (1.20):

$$\sigma_{1,2} = \tfrac{195-35}{2} \pm \left[\left(\tfrac{195+35}{2}\right)^2 + 60.6^2 \right]^{\frac{1}{2}}$$

or $\sigma_1 = 210$ MPa $\quad \sigma_2 = -50$ MPa

Also,

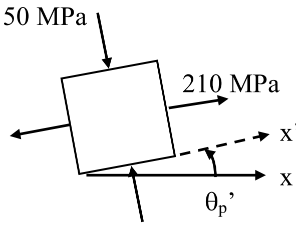
$$\theta_p = \tfrac{1}{2}\tan^{-1}\tfrac{2(60.6)}{195+35} = 13.89^o$$

Equation (1.18a):

$$\sigma_{x'} = 80 + 115\cos 2(13.89^o) + 60.6\sin 2(13.89^o) = 210 \text{ MPa}$$

Thus,

$$\theta_p' = 13.89^o$$



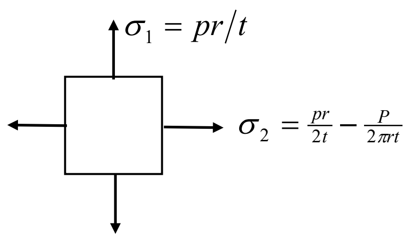
SOLUTION (1.39)

For pure shear, $\sigma_1 = -\sigma_2$:

$$\tfrac{pr}{t} = -\tfrac{pr}{2t} + \tfrac{P}{2\pi rt}$$

from which

$$P = 3\pi pr^2$$



SOLUTION (1.40)

Table D.4:

$$A = 2\pi rt$$
$$J = 2\pi r^3 t$$

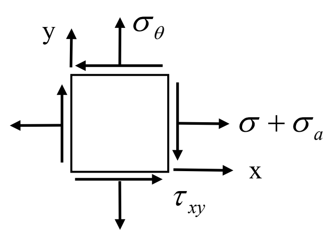


Figure (a)

Stresses are (Fig. a):

$$\sigma = \tfrac{-P}{A} = \frac{-30(10^3)\pi}{2\pi(0.12)(0.005)} - 25 \text{ MPa}$$

$$\sigma_a = \tfrac{pr}{2t} = \frac{4(10^6)120}{2(5)} = 48 \text{ MPa}$$

$$\sigma_\theta = 2\sigma_a = 96 \text{ MPa}$$

$$\tau_{xy} = \tfrac{-Tr}{J} = \frac{-10\pi(10^3)}{2\pi(0.12^2)(0.005)} = -69.4 \text{ MPa}$$

(CONT.)

1.40 (CONT.)

Hence,

$$\sigma_x = 48 - 25 = 23 \text{ MPa} \qquad \sigma_y = 96 \text{ MPa}$$

Therefore, we have

$$\tau_{\max} = \pm\left[\left(\tfrac{23-96}{2}\right)^2 + 69.4^2\right]^{\frac{1}{2}} = \pm 78.4 \text{ MPa}$$

Also

$$\sigma' = \tfrac{1}{2}(23+96) = 59.5 \text{ MPa}$$

and

$$\theta_s = -\tfrac{1}{2}\tan^{-1}\tfrac{23-96}{2(-69.4)} = -13.87^o$$

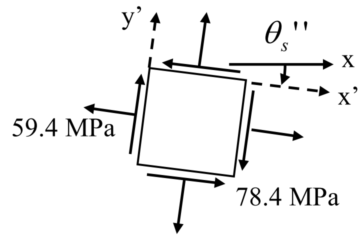


Figure (b)

Equation (1.18b) with $\theta_s = -13.87^o$:

$$\tau_{x'y'} = -16.99 - 61.42 = -78.4 \text{ MPa}$$

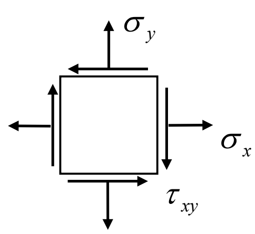
Thus,

$$\theta_s'' = 13.87^o$$

SOLUTION (1.41)

$$A = 2\pi rt = 2\pi(60)(4) = 1508 \text{ mm}^2$$
$$J = 2\pi r^3 t = 2\pi(60)^3(4) = 5.429\times10^6 \text{ mm}^4$$

$$\sigma_y = \tfrac{pr}{t} = \tfrac{5(60)}{4} = 75 \text{ MPa}$$
$$\tau_{xy} = -\tfrac{Tr}{J} = -\tfrac{600(0.05)}{5.429(10^{-6})}$$
$$= -5.526 \text{ MPa}$$
$$\sigma_x = \tfrac{pr}{2t} + \tfrac{P}{A} = 37.5 + \tfrac{P}{1508}$$
(P in newtons)



Thus

$$\sigma_1 = \tfrac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\tfrac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substituting the numerical values gives

$$80 = 56.3 + 331.6\times10^{-6}P + \left[(-18.75 + 331.6\times10^{-6}P)^2 + (-5.526)^2\right]^{\frac{1}{2}}$$

Solving,

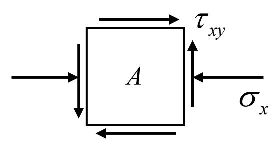
$$P = 64.01 \text{ kN}$$

SOLUTION (1.42)

At point A, $T = 8\,\text{kN}$ and $P = 400$ kN

$$\sigma_x = -\frac{P}{A} = -\frac{4(400\times10^3)}{\pi(0.1)^2} = -50.9 \text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(8\times10^3)}{\pi(0.1)^3} = 40.7 \text{ MPa}$$

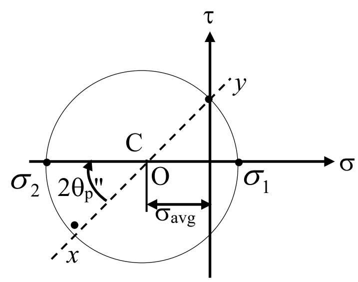


Hence

$$\tau_{max} = \sqrt{(\frac{\sigma_x}{2})^2 + \tau_{xy}^2}$$
$$= \sqrt{(-25.45)^2 + (40.7)^2}$$
$$= 48 \text{ MPa} = R$$

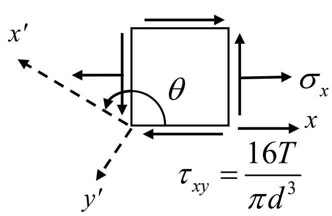
$$\sigma_{avg} = \frac{\sigma_x}{2} = -25.45 \text{ MPa}$$

$$\tan 2\theta_p" = \frac{40.7}{25.45}, \quad \theta_p" = 29^o$$



SOLUTION (1.43)

$$\theta = \alpha + 90 = 50 + 90 = 140^o$$



$$\sigma_x = \frac{P}{A} \qquad \tau_{xy} = \frac{16T}{\pi d^3}$$

$$\sigma_x = \frac{120\times10^3}{\frac{\pi}{4}(0.04)^2} = 95.5 \text{ MPa}$$

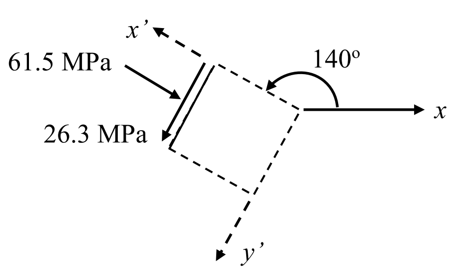
$$\tau = \frac{16(1.5\times10^{-3})}{\pi(0.04)^3} = 119.4 \text{ MPa}$$

Equations(1.18):

$$\sigma_w = \sigma_{x'} = \frac{95.5}{2} + \frac{95.5}{2}\cos 280^o + 119.4\sin 280^o = -61.5 \text{ MPa}$$

and

$$\tau_w = \tau_{x'y'} = -\frac{95.5}{2}\sin 280^o - 119.4\cos 280^o = 47.02 - 20.73 = 26.3 \text{ MPa}$$



SOLUTION (1.44)

$$A = \tfrac{\pi}{4}(180^2 - 120^2) = 14.14 \times 10^3 \text{ mm}^2$$
$$J = \tfrac{\pi}{32}(180^4 - 120^4) = 82.70 \times 10^6 \text{ mm}^4$$

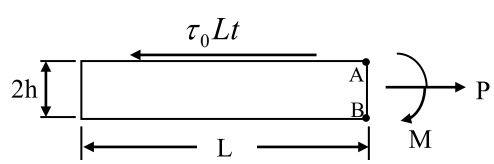
$$\sigma_x = -\tfrac{P}{A} = -\tfrac{700}{14.14} = -49.5 \text{ MPa}, \qquad \sigma_y = 0$$
$$\tau_{xy} = \tfrac{Tr}{J} = \tfrac{20(90)}{82.70 \times 10^{-6}} = 21.77 \text{ MPa}$$

Equation (1.20) is therefore

$$\sigma_{\max} = \sigma_1 = -\tfrac{49.5}{2} + \sqrt{(\tfrac{49.5}{2})^2 + (21.76)^2}$$
$$= 8.205 \text{ MPa} \qquad \blacktriangle$$

SOLUTION (**1.45**)

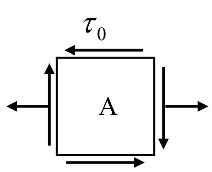
$$P = \tau_0 L t$$
$$M = \tau_0 L t h$$
$$A = 2ht$$
$$I = \tfrac{1}{12} t (2h)^3 = \tfrac{2}{3} t h^3$$



Axial stress: $\sigma_a = \frac{P}{A} = \frac{\tau_0 L}{2h}$

Bending stress: $\sigma_b = \frac{Mc}{I} = \frac{3\tau_0 L}{2h}$

Point A

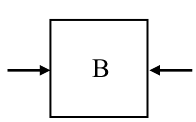


$\sigma_a + \sigma_b = \frac{\tau_0 L}{2h}$

From Eqs. (1.20) and (1.22), we obtain

$$\sigma_{1,2} = \tfrac{\tau_0 L}{h} \pm \tau_0 \sqrt{(\tfrac{L}{h})^2 + 1} \qquad \blacktriangle$$
$$\tau_{\max} = \tau_0 \sqrt{(\tfrac{L}{h})^2 + 1}$$

Point B

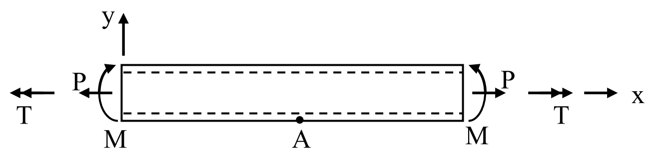


$\sigma_b - \sigma_a = \frac{\tau_0 L}{h}$

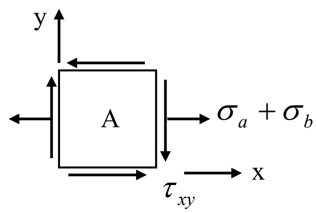
Hence

$$\sigma_1 = \tfrac{\tau_0 L}{h} \qquad \sigma_2 = 0 \qquad \tau_{\max} = \tfrac{\tau_0}{2h} \qquad \blacktriangle$$

SOLUTION (1.46)



$$A=\pi(30^2-15^2)=2.121(10^{-3})\ \text{m}^2$$
$$I=\tfrac{\pi}{4}(30^4-15^4)=0.596(10^{-6})\ \text{m}^4 \qquad J=2I$$



We have

$$\sigma_a=\tfrac{P}{A}=\frac{50(10^3)}{2.12(10^{-3})}=23.58\ \text{MPa}$$
$$\sigma_b=\tfrac{Mr}{I}=\frac{200(0.03)}{0.596(10^{-6})}=10.07\ \text{MPa}$$
$$\tau_{xy}=\tfrac{-Tr}{J}=\frac{-500(0.03)}{1.192(10^{-6})}=-12.58\ \text{MPa}$$

Thus,

$$\sigma_x=23.58+10.07=33.65\ \text{MPa} \qquad \sigma'=16.83\ \text{MPa}$$

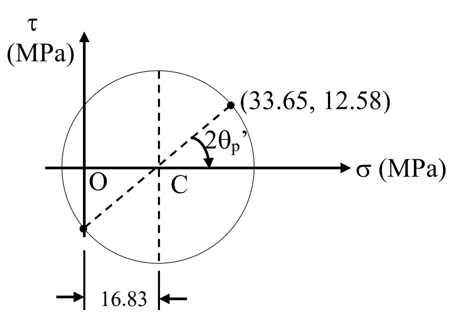


Figure (a)

From Mohr's circle (Fig. a):

$$r=\sqrt{12.58^2+16.83^2}=21.01\ \text{MPa} \qquad \theta_p'=\tfrac{1}{2}\tan^{-1}\tfrac{12.58}{16.83}=18.39^o$$
$$\sigma_1=16.83+21.01=37.84\ \text{MPa}$$
$$\sigma_2=-4.18\ \text{MPa}$$

Results are shown in Fig. (b).

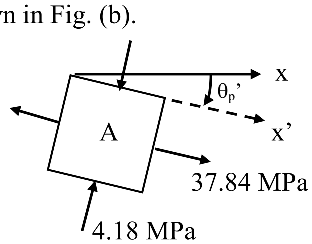


Figure (b)

SOLUTION (1.47)

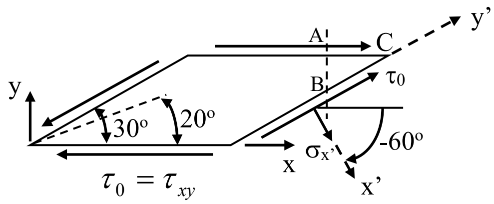


Figure (a)

(a) At $\theta = -60^o$ (Fig. a):

$$0 = \tfrac{\sigma_x + \sigma_y}{2} + \tfrac{\sigma_x - \sigma_y}{2}\cos 2(-60^o) + \tau_0 \sin 2(-60^o)$$

or

$$0 = 0.5\sigma_x + 1.5\sigma_y - 1.732\tau_0 \qquad \text{(a)}$$

We also have

$$\tau_0 = -\tfrac{\sigma_x - \sigma_y}{2}\sin 2(-60^o) + \tau_o \cos 2(-60^o)$$

or

$$\sigma_x = 3.464\tau_0 + \sigma_y \qquad \text{(b)}$$

Substituting Eq. (b) into (a), we obtain $\sigma_y = 0$. Results are shown in Fig. b.

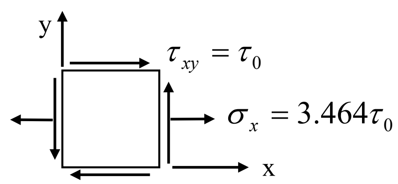


Figure (b)

Alternatively, using an element ABC (Fig. c):

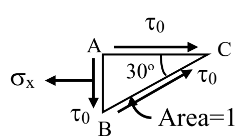


Figure (c)

$$\sum F_x = 0.5\sigma_x - 0.866\tau_0 - 0.866\tau_0 = 0$$

or $\sigma_x = 3.464\tau_0$, as before.

Stresses on planes at 20^o, taking $\theta = -70^o$ (Fig. b):

$$\sigma_{20^o} = [\tfrac{3.464}{2} + \tfrac{3.464}{2}\cos(-140^o) + \sin(-140^o)]\tau_0 = -0.237\tau_0 \quad ◀$$

$$\tau_{20^o} = [-\tfrac{3.464}{2}\sin(-140^o) + \cos(-140^o)]\tau_0 = 0.347\tau_0 \quad ◀$$

(CONT.)

1.47 (CONT.)

(b) Principal stresses:

$$\sigma_{1,2} = \tfrac{3.464\tau_o}{2} \pm \left[(\tfrac{3.464\tau_o}{2})^2 + \tau_0^2 \right]^{\frac{1}{2}}$$

$$\sigma_1 = 3.732\tau_0 \qquad \sigma_2 = -0.268\tau_0$$ ▲

The maximum principal stress is on plane inclined at

$$\theta_p' = \tfrac{1}{2}\tan^{-1}\tfrac{\tau_0}{1.732\tau_0} = 15^o$$ ▲

SOLUTION (1.48)

At a critical point on the shaft surface, the state of stress of stress is as shown in Fig. (a).

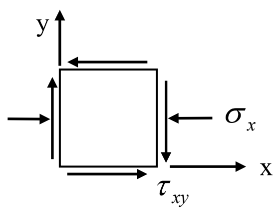


Figure (a)

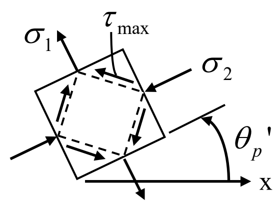


Figure (b)

We have

$$\sigma_x = -\tfrac{P}{A} - \tfrac{Mr}{I}$$

$$= -\frac{81(10^3)}{\pi(0.075)^2} - \frac{13(10^3)(0.075)}{\pi(0.075)^4/4} = -43.818 \text{ MPa}$$

$$\tau_{xy} = -\tfrac{Tr}{J} = -\frac{(15.6\times10^3)0.075}{\pi(0.075)^4/2} = -23.54 \text{ MPa}$$

Therefore,

$$\sigma_{1,2} = \tfrac{-43.818}{2} \pm \left[(\tfrac{43.818}{2})^2 + (-23.54)^2 \right]^{\frac{1}{2}}$$

or

$$\sigma_1 = 10.248 \text{ MPa}, \qquad \sigma_2 = -54.066 \text{ MPa}$$ ▲

$$\tau_{max} = \tfrac{1}{2}(\sigma_1 - \sigma_2) = 32.157 \text{ MPa}$$

and

$$\theta_p'' = \tfrac{1}{2}\tan^{-1}\tfrac{2(23.54)}{43.818} = 23.53^o$$ ▲

Results are shown in Fig. (b).

SOLUTION (1.49)

Apply Eqs. (1.20) to Fig. P1.49b, for $\theta = -30^o$:

$$\sigma_{xb} = -40\sin 2(-30^o) = 20\sqrt{3} \text{ MPa}$$

$$\sigma_{yb} = -20\sqrt{3} \text{ MPa} \qquad \text{(b)}$$

$$\tau_{xyb} = -40\cos 2(-30^o) = -20 \text{ MPa}$$

(CONT.)

Now apply Eqs. (1.18) to Fig. P1.49c, for $\theta = -60^o$:

$$\sigma_{xc} = 10\sin 2(-60^o) = -5\sqrt{3} \text{ MPa}$$
$$\sigma_{yc} = 5\sqrt{3} \text{ MPa} \qquad \text{(c)}$$
$$\tau_{xyc} = 10\cos 2(-60^o) = -5 \text{ MPa}$$

Superposing stresses in Eqs. (b) and (c) and those in Fig. P1.49a, we obtain Fig. (a).

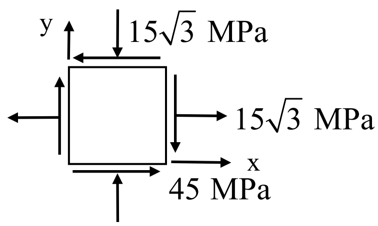


Figure (a)

Referring to Fig. (a):

$$\sigma_{1,2} = 0 \pm \left[(15\sqrt{3})^2 + (-45)^2\right]^{\frac{1}{2}}$$

or

$$\sigma_1 = 51.96 \text{ MPa} \qquad \sigma_2 = -51.96 \text{ MPa}$$

When

$$\theta_p' = \tfrac{1}{2}\tan^{-1}\frac{2(-45)}{2(15\sqrt{3})} = -30^o$$

is substituted into Eq. (1.18a), we have 51.96 MPa (Fig. b).

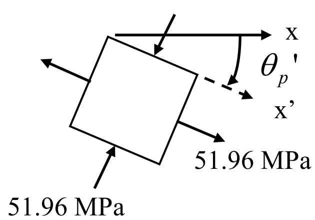


Figure (b)

SOLUTION (1.50)

Apply Eqs. (1.18) to Fig. P1.50a, for $\theta = -15^o$, to obtain stresses in Fig. (a):

$$\sigma_{xa} = -\tfrac{30}{2} - \tfrac{30}{2}\cos 2(-15^o) = -27.99 \text{ MPa}$$
$$\sigma_{ya} = -15 + 15\cos 2(-15^o) = -2.01 \text{ MPa}$$
$$\tau_{xya} = 15\sin 2(-15^o) = -7.5 \text{ MPa}$$

(CONT.)

1.50 (CONT.)

Superposition of stresses in Figs. (a) and P1.50b gives Fig. (b).

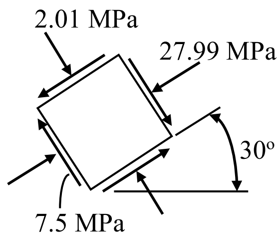


Figure (a)

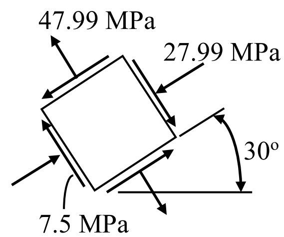


Figure (b)

Apply Eq. (1.20) to Fig. (b):

$$\sigma_{1,2} = \tfrac{-27.99+47.99}{2} \pm \left[\tfrac{1}{4}(-27.99-47.99)^2 + (-7.5)^2\right]^{\frac{1}{2}}$$

or

$$\sigma_1 = 48.72 \text{ MPa}, \qquad \sigma_2 = -28.72 \text{ MPa}$$ ▲

When

$$\theta_p = \tfrac{1}{2}\tan^{-1}\tfrac{2(-7.5)}{-(27.99+47.99)} = 5.58^o$$

is substituted into Eq. (1.18a), we obtain –28.72 MPa (Fig. c).

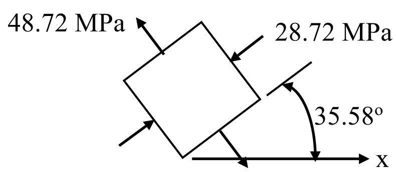


Figure (c)

SOLUTION (1.51)

Equations (1.18) are applied to Fig. P1.51a, for $\theta = -30^o$:

$$\sigma_{xa} = \tfrac{20+30}{2} + \tfrac{20-30}{2}\cos 2(-30^o) = 22.5 \text{ MPa}$$

$$\sigma_{ya} = 25 - (-5)\cos 2(-30^o) = 27.5 \text{ MPa}$$

$$\tau_{xya} = -(-5)\sin 2(-30^o) = -4.33 \text{ MPa}$$

These stresses and that of Fig. P1.51b are superimposed to yield Fig. (a).

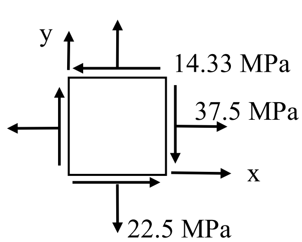


Figure (a)

(CONT.)

1.51 (CONT.)

Principal stresses are thus

$$\sigma_{1,2} = \tfrac{37.5+22.5}{2} \pm \left[\left(\tfrac{37.5-22.5}{2}\right)^2 + 14.33^2\right]^{\frac{1}{2}}$$

or

$$\sigma_1 = 46.17 \text{ MPa} \qquad \sigma_2 = 13.83 \text{ MPa}$$

Hence

$$\tau_{max} = \tfrac{1}{2}(\sigma_1 - \sigma_2) = 16.17 \text{ MPa}$$

We have

$$\theta_p = \tfrac{1}{2}\tan^{-1}\tfrac{2(-14.33)}{37.5-22.5} = -31.2^o$$

Equation (1.18a) results in

$$\sigma_{x'} = \tfrac{37.5+22.5}{2} + \tfrac{37.5-22.5}{2}\cos(-62.4^o) - 14.33\sin(-62.4^o) = 46.17 \text{ MPa}$$

Therefore

$$\theta_p' = 31.2^o$$

Results are shown in a properly oriented element in Fig. (b).

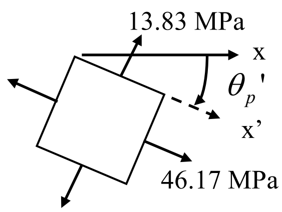


Figure (b)

SOLUTION (1.52)

State of stress is represented by Mohr's circle in Fig. (a).

From this circle, we determine

$$\sigma_x = -40 \text{ MPa}$$
$$\sigma_y = 20 \text{ MPa}$$
$$\theta_p' = \tfrac{1}{2}\tan^{-1}\tfrac{4}{3} = 26.57^o$$

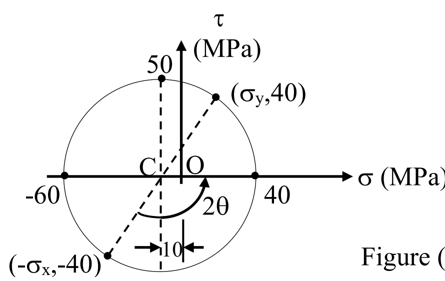


Figure (a)

Results are shown in Fig. (b).

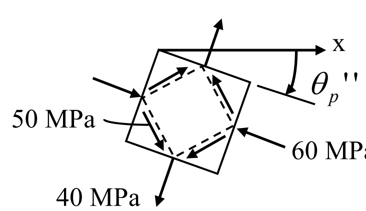
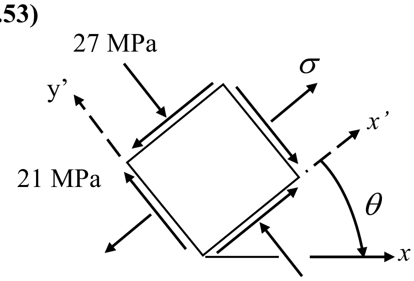


Figure (b)

SOLUTION (1.53)



$$\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$$

$$45 - 30 = -27 + \sigma; \qquad \sigma = 42 \text{ MPa}$$ ▲

$$\sigma_{x'} = 42 = \frac{45-30}{2} + \frac{45+30}{2}\cos 2\theta + 15\sin 2\theta$$

or

$$34.5 = 37.5\cos 2\theta + 15\sin 2\theta \qquad (1)$$

$$\tau_{x'y'} = -21 = -37.5\sin 2\theta + 15\cos 2\theta$$

Multiply this by -2.5:

$$52.5 = 93.75\sin 2\theta - 37.5\cos 2\theta \qquad (2)$$

Add Eqs. (1) and (2),

$$87 = 108.75\sin 2\theta, \quad 2\theta = 53.13^o$$

or

$$\theta = 26.6^o$$ ▲

SOLUTION (1.54)

State of stress is represented by Mohr's circle in Fig. (a).

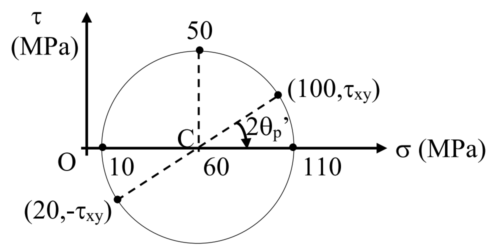


Figure (a)

Referring to this circle, we obtain the results (Fig. b). ▲

$$\theta_p' = \tfrac{1}{2}\tan^{-1}\tfrac{3}{4} = 18.43^o$$

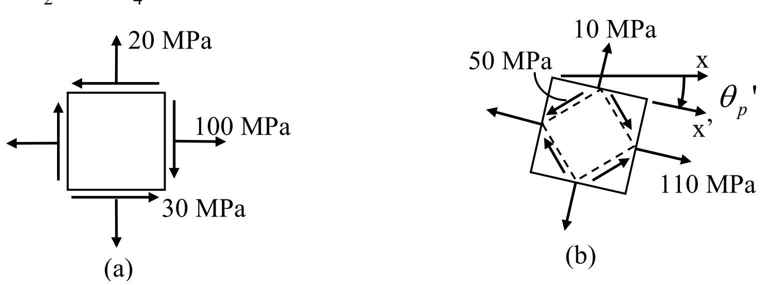


Figure (b)

SOLUTION (1.55)

$$\sigma_x = 60 \text{ MPa} \qquad \sigma_y = -18 \text{ MPa} \qquad \tau_{xy} = -15 \text{ MPa} \qquad \sigma_{x'} = 30 \text{ MPa}$$

From Equation (1.18a):

$$30 = \tfrac{60-18}{2} + \tfrac{60+18}{2}\cos 2\theta_1 - 15\sin 2\theta_1$$

or

$$13\cos 2\theta_1 - 5\sin 2\theta_1 - 3 = 0$$

Solving

$$2\theta_1 = 56.52^o \qquad \theta_1 = 28.26^o$$ ▲

We have

$$\sigma_{y'} = \sigma_x + \sigma_y - \sigma_{x'} = 12 \text{ MPa}$$ ▲

Equation (1.18b) gives

$$\tau_{x'y'} = -\tfrac{60+18}{2}\sin 56.52^o - 15\cos 56.52^o = -40.8 \text{ MPa}$$ ▲

SOLUTION (1.56)

We have

$$\sigma = \tfrac{4M}{\pi r^3} = \frac{4(21\pi)10^3}{\pi(0.1)^3} = 84 \text{ MPa}$$

State of stress is represented by Mohr's circle in Fig. (a).

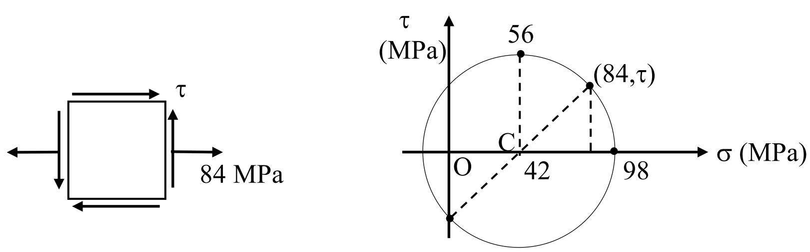


Figure (a)

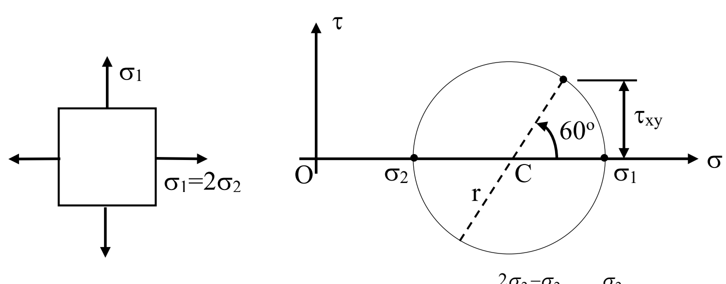
$$\tau = (56^2 - 42^2)^{\frac{1}{2}} = 37.04 \text{ MPa}$$

Thus $T = \tfrac{\tau J}{r} = \frac{(37.04)(10^6)\pi(0.1^3)}{2} = 58.18 \text{ kN}\cdot\text{m}$ ▲

Hence

$$P = 2\pi f T = 2\pi(20)58.18 = 7311 \text{ kW}$$ ▲

SOLUTION (1.57)



$$r = \frac{2\sigma_2 - \sigma_2}{2} = \frac{\sigma_2}{2}$$

(CONT.)

From Mohr's circle,

$$\tau_{x'y'} = (\tfrac{\sigma_2}{2})\sin 60^o = 0.433\sigma_2$$

Therefore $\quad 30 = 0.433\sigma_2 \qquad \sigma_2 = 69.28 \text{ MPa}$

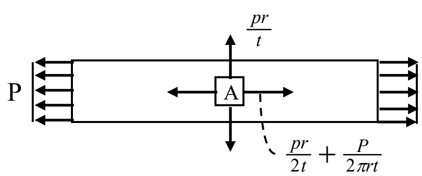
We have

$$\sigma_2 = \tfrac{pr}{2t} = 69.28 \qquad p(\tfrac{250}{2\times 5}) = 69.28$$

Solving

$$p = 2.771 \text{ MPa}$$

SOLUTION (1.58)



Mohr's circle representing stress at point A is shown in Fig. (a).

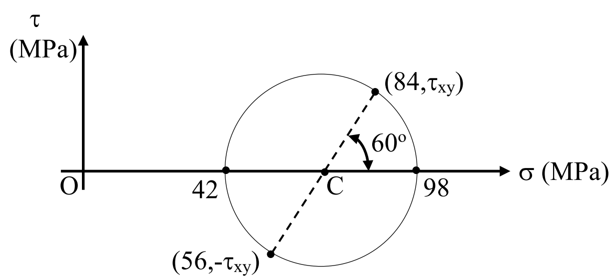


Figure (a)

From this circle:

$$42 = \tfrac{p(0.45)}{0.005} = 90p \qquad \text{or} \qquad p = 467 \text{ kPa}$$

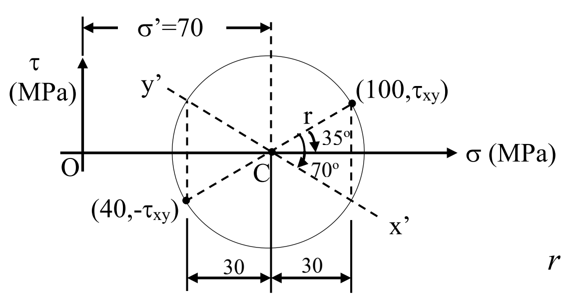
Then

$$98(10^3) = \tfrac{467(0.45)}{2(0.005)} + \tfrac{P}{2\pi(0.45)(0.005)}$$

gives

$$P = 1088 \text{ kN}$$

SOLUTION (1.59)



$$r = \tfrac{30}{\cos 35^o} = 36.62 \text{ MPa}$$

(CONT.)

1.59 (CONT.)

(a) $\tau_{xy} = -36.62 \sin 35^o = -21 \text{ MPa}$ ▶

(b) Because of symmetry:

$$\tau_{x'y'} = -\tau_{xy} = 21 \text{ MPa}$$ ▶

and

$$\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'} = 140 \text{ MPa}$$

gives

$$\sigma_{y'} = 40 \text{ MPa}$$ ▶

SOLUTION (1.60)

State of stress is represented by Mohr's circle in Fig. (a).

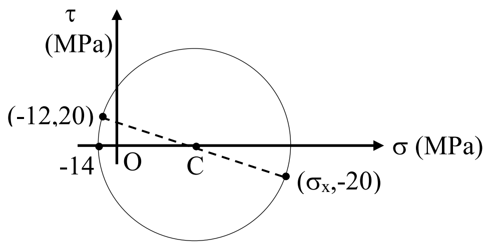


Figure (a)

(a) Using this circle, we write

$$\tau_{\max} = \left[\left(\tfrac{\sigma_x + 12}{2}\right)^2 + 20^2\right]^{\frac{1}{2}}$$

and

$$\tau_{\max} = 14 + OC = 14 + \tfrac{1}{2}(\sigma_x - 12)$$

Solving,

$$\sigma_x = 186 \text{ MPa}$$ ◀

Note that, alternately,

$$-14 = \tfrac{\sigma_x - 12}{2} - \left[\left(\tfrac{\sigma_x + 12}{2}\right)^2 + 20^2\right]^{\frac{1}{2}}$$

yields $\sigma_x = 186 \text{ MPa}$, as before.

(b) We have

$$\sigma_{1,2} = \tfrac{186-12}{2} \pm \left[\left(\tfrac{186+12}{2}\right)^2 + 20^2\right]^{\frac{1}{2}}$$

or

$$\sigma_1 = 188 \text{ MPa} \qquad \sigma_2 = -14 \text{ MPa}$$ ◀

and

$$\tau_{\max} = \tfrac{1}{2}(\sigma_1 - \sigma_2) = 101 \text{ MPa}$$ ◀

Also

$$\theta_p' = \tfrac{1}{2} \tan^{-1} \tfrac{2(20)}{186+12} = 5.71^o$$ ◀

(CONT.)

1.60 (CONT.)

Results are shown in Fig. (b).

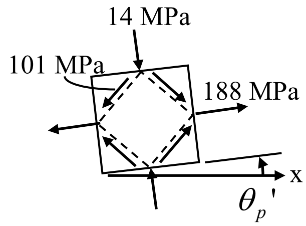


Figure (b)

SOLUTION (1.61)

(a) $\sigma_1 = 96.05\text{ MPa} \qquad \sigma_2 = 23.95\text{ MPa} \qquad \sigma_3 = 0$

(b)
$$(\tau_{12})_{\max} = \tfrac{1}{2}(\sigma_1 - \sigma_2) = 36.05\text{ MPa}$$
$$(\tau_{13})_{\max} = \tfrac{1}{2}(\sigma_1 - \sigma_3) = 48.03\text{ MPa}$$
$$(\tau_{23})_{\max} = \tfrac{1}{2}(\sigma_2 - \sigma_3) = 11.98\text{ MPa}$$

▲

Plane of $(\tau_{12})_{\max}$ is shown in Fig. (a). Other maximum shear planes are sketched similarly.

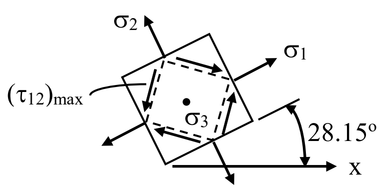
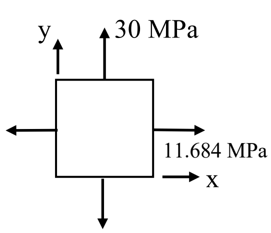
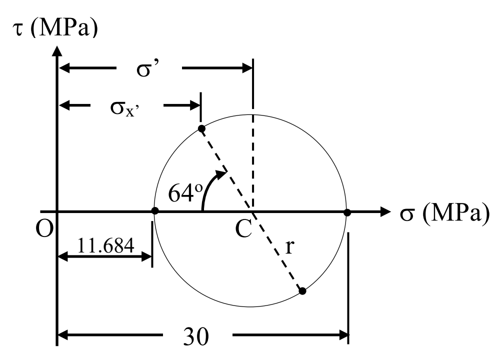


Figure (a)

SOLUTION (1.62)

$$A = 2\pi rt = 2\pi(60)(4) = 1508\text{ mm}^2$$
$$\sigma_y = \tfrac{pr}{t} = \tfrac{2(60)}{4} = 30\text{ MPa}$$
$$\sigma_x = -\tfrac{P}{A} + \tfrac{pr}{2t} = -\frac{5(10^3)}{1508(10^{-6})} + 15 = 11.684\text{ MPa}$$





(CONT.)

1.62 (CONT.)

$$\sigma' = \tfrac{1}{2}(30 + 11.684) = 20.84 \text{ MPa}$$
$$r = \tfrac{1}{2}(30 - 11.684) = 9.158 \text{ MPa}$$

(a) $\sigma_{x'} = \sigma' - r\cos 64^o = 16.82 \text{ MPa}$

(b) $\tau_{x'y'} = r\sin 64^o = 8.231 \text{ MPa}$

SOLUTION (1.63)

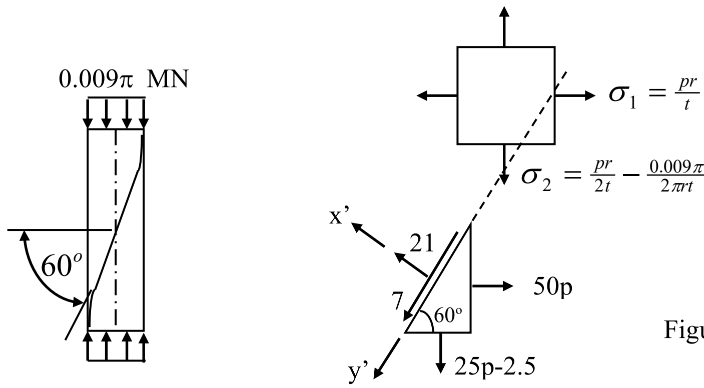


Figure (a)

Equilibrium of x' and y' directed forces results in (Fig. a):

$$21 - 50p(\tfrac{\sqrt{3}}{2})^2 - (25p - 2.5)(\tfrac{1}{2})^2 = 0$$

or

$$p_{all} = 494 \text{ kPa}$$

and

$$7 + (25p - 2.5)(\tfrac{\sqrt{3}}{4}) - 50p(\tfrac{\sqrt{3}}{4}) = 0$$

from which

$$p = 547 \text{ kPa}$$

SOLUTION (1.64)

Direction cosines are:

$$l_1 = \sqrt{3}/2 \quad m_1 = 1/2 \quad n_1 = 0$$
$$l_2 = -1/2 \quad m_2 = \sqrt{3}/2 \quad n_2 = 0$$
$$l_3 = 0 \quad m_3 = 0 \quad n_3 = 1$$

Equation (1.28a) is thus

$$\sigma_{x'} = 20(\tfrac{3}{4}) + 0 + 0 + 2(12)(\tfrac{\sqrt{3}}{2})(\tfrac{1}{2}) + 0 + 0 = 25.392 \text{ MPa}$$

Similarly, applying Eqs. (1.28b) through (1.28e), we obtain $[\tau_{i'j'}]$:

(CONT.)

1.64 (CONT.)

$$\begin{bmatrix} 25.392 & -2.66 & -7.99 \\ -2.66 & -5.392 & 16.16 \\ -7.99 & 16.16 & 6 \end{bmatrix} \text{MPa}$$

Then, Eqs. (1.34) result in

$$I_1 = I_1' = 26 \text{ MPa} \qquad I_2 = I_2' = -349 \text{ (MPa)}^2$$
$$I_3 = I_3' = -6464 \text{ (MPa)}^3$$

SOLUTION (1.65)

Direction cosines are:

$$l_1 = \sqrt{3}/2 \quad m_1 = 1/2 \quad n_1 = 0$$
$$l_2 = -1/2 \quad m_2 = \sqrt{3}/2 \quad n_2 = 0$$
$$l_3 = 0 \quad m_3 = 0 \quad n_3 = 1$$

Equation (1.28a) is therefore

$$\sigma_{x'} = 60(\tfrac{3}{4}) + 0 + 0 + 2(40)(\tfrac{\sqrt{3}}{2})(\tfrac{1}{2}) + 0 + 0$$
$$= 20[(\tfrac{9}{4}) + \sqrt{3}] = 79.64 \text{ MPa}$$

Similarly, applying Eqs. (1.28b) through (1.28e), we obtain $[\tau_{i'j'}]$:

$$\begin{bmatrix} 79.64 & -5.98 & -44.64 \\ -5.98 & -19.64 & 2.68 \\ -44.64 & 2.68 & 20 \end{bmatrix} \text{MPa}$$

Then, Eqs. (1.34) lead to

$$I_1 = I_1' = 80 \text{ MPa} \qquad I_2 = I_2' = -2400 \text{ (MPa)}^2$$
$$I_3 = I_3' = 8000 \text{ (MPa)}^3$$

SOLUTION (1.66)

Referring to Appendix B:

$$\sigma_1 = 13.212 \text{ MPa} \qquad \sigma_2 = 5.684 \text{ MPa} \qquad \sigma_3 = -8.896 \text{ MPa}$$

and

$$l_1 = 0.9556 \qquad m_1 = 0.1688 \qquad n_1 = 0.2416$$

Thus,

$$\tau_{max} = \tfrac{1}{2}(\sigma_1 - \sigma_3) = 11.054 \text{ MPa}$$

SOLUTION (1.67)

Referring to Appendix B:

$$\sigma_1 = 66.016 \text{ MPa} \qquad \sigma_2 = 28.418 \text{ MPa} \qquad \sigma_3 = -44.479 \text{ MPa}$$

and

$$l_1 = 0.9556 \qquad m_1 = 0.1688 \qquad n_1 = 0.2416$$

SOLUTION (1.68)

Referring to Appendix B:

$$\sigma_1 = 30.493 \text{ MPa} \qquad \sigma_2 = 12.485 \text{ MPa} \qquad \sigma_3 = -16.979 \text{ MPa}$$

Thus,

$$\tau_{\max} = \tfrac{1}{2}(\sigma_1 - \sigma_3) = 23.736 \text{ MPa}$$

SOLUTION (1.69)

Referring to Appendix B:

$$\sigma_1 = 24.747 \text{ MPa} \qquad \sigma_2 = 8.480 \text{ MPa} \qquad \sigma_3 = 2.773 \text{ MPa}$$

and

$$l_1 = 0.6467 \qquad m_1 = 0.3958 \qquad n_1 = 0.6421$$

SOLUTION (1.70)

(a) Equation (1.32) becomes

$$\begin{vmatrix} (30-\sigma_p) & 0 & 20 \\ 0 & -\sigma_p & 0 \\ 20 & 0 & -\sigma_p \end{vmatrix} = 0$$

Expanding,

$$-\sigma_p[\sigma_p(\sigma_p - 30) - 400] = 0$$

or

$$\sigma_p = 0, \qquad \sigma_p = -10, \qquad \sigma_p = 40$$

Thus

$$\sigma_1 = 40 \text{ MPa}, \qquad \sigma_2 = 0, \qquad \sigma_3 = -10 \text{ MPa}$$

(b) For $\sigma_1 = 40 \text{ MPa}$:

$$(30-40)l_1 + (0)m_1 + 20n_1 = 0, \qquad l_1 = 2n_1$$

$$(0)m_1 = 0, \qquad m_1 = 0$$

The condition $l_1^2 + 0 + n_1^2 = 1$ gives

$$(2n_1)^2 + n_1^2 = 1, \qquad n_1 = \tfrac{1}{\sqrt{5}}$$

Thus

$$l_1 = \tfrac{2}{\sqrt{5}}, \qquad m_1 = 0, \qquad n_1 = \tfrac{1}{\sqrt{5}}$$

SOLUTION (1.71)

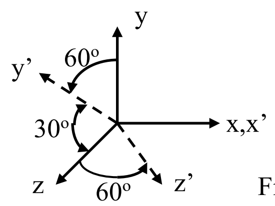


Figure (a)

(a) At point (3,1,5) with respect to xyz axis, we have $[\tau_{ij}]$:

$$\begin{bmatrix} 10 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 8 \end{bmatrix} \text{MPa} \qquad (a)$$

Then, Eqs. (1.34) result in

$$I_1 = 14 \text{ MPa} \qquad I_2 = 8 \text{ (MPa)}^2 \qquad I_3 = -320 \text{ (MPa)}^3 \qquad \blacktriangle$$

Direction cosines of x' y' z', referring to Fig. (a) are

$$l_1 = 1 \quad m_1 = 0 \quad n_1 = 0$$
$$l_2 = 0 \quad m_2 = 1/2 \quad n_2 = \sqrt{3}/2$$
$$l_3 = 0 \quad m_3 = -\sqrt{3}/2 \quad n_3 = 1/2$$

Now Eqs. (1.28) and (a) give $[\tau_{i'j'}]$:

$$\begin{bmatrix} 10 & 0 & 0 \\ 0 & 5 & 3\sqrt{3} \\ 0 & 3\sqrt{3} & -1 \end{bmatrix} \text{MPa} \qquad \blacktriangle$$

Thus, Eqs. (1.34) yield

$$I_1' = 14 \text{ MPa} \qquad I_2' = 8 \text{ (MPa)}^2 \qquad I_3' = -320 \text{ (MPa)}^3 \qquad \blacktriangle$$

as before.

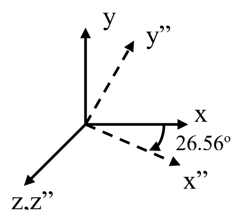


Figure (b)

(b) Direction cosines are (Fig. b):

$$l_1 = 2/\sqrt{5} \quad m_1 = -1/\sqrt{5} \quad n_1 = 0$$
$$l_2 = 1/\sqrt{5} \quad m_2 = 2/\sqrt{5} \quad n_2 = 0$$
$$l_3 = 0 \quad m_3 = 0 \quad n_3 = 1$$

(CONT.)

1.71 (CONT.)

With these and Eq. (a), Eqs. (1.28) yield $[\tau_{i'j'}]$:

$$\begin{bmatrix} 7.2 & 5.6 & 0 \\ 5.6 & -1.2 & 0 \\ 0 & 0 & 8 \end{bmatrix} \text{MPa}$$ ◄

Thus, Eqs. (1.34) result in

$$I_1'' = 14 \text{ MPa} \qquad I_2'' = 8 \text{ (MPa)}^2 \qquad I_3'' = -320 \text{ (MPa)}^3$$ ◄

The I's are thus invariants.

SOLUTION (1.72)

Introducing the given data into Eq. (1.28a), we obtain

$$\sigma_{x'} = 12(\tfrac{1}{2})^2 + 10(\tfrac{\sqrt{3}}{2})^2 + 14(0) + 2[6(\tfrac{1}{2})(\tfrac{\sqrt{3}}{2})] + 0 + 0$$
$$= 15.696 \text{ MPa}$$

Remaining stress components are determined in a like manner. The result, $[\tau_{i'j'}]$, is

$$\begin{bmatrix} 15.696 & -3.866 & 7.089 \\ -3.866 & 6.304 & -6.294 \\ 7.089 & -6.294 & 14. \end{bmatrix} \text{MPa}$$ ◄

SOLUTION (1.73)

Equations (1.34) become

$$I_1 = \sigma_x + \sigma_y \qquad I_2 = \sigma_x \cdot \sigma_y - \tau_{xy}^2 \qquad I_3 = 0$$

Equation (1.33) is then

$$\sigma_p^3 - (\sigma_x + \sigma_y)\sigma_p^2 + (\sigma_x\sigma_y - \tau_{xy}^2)\sigma_p = 0$$

or $$\sigma_p^2 - (\sigma_x + \sigma_y)\sigma_p + (\sigma_x\sigma_y - \tau_{xy}^2) = 0$$

Solution of this quadratic equation is

$$\sigma_{1,2} = \tfrac{\sigma_x + \sigma_y}{2} \pm \tfrac{1}{2}[\sigma_x^2 + 2\sigma_x\sigma_y + \sigma_y^2 - 4(\sigma_x + \sigma_y - \tau_{xy}^2)]^{\frac{1}{2}}$$
$$= \tfrac{\sigma_x + \sigma_y}{2} \pm [(\tfrac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2]^{\frac{1}{2}}$$ ◄

SOLUTION (1.74)

Referring to Appendix B, we obtain the following values.

(a) $\sigma_1 = 12.049$ MPa $\qquad \sigma_2 = -1.521$ MPa $\qquad \sigma_3 = -4.528$ MPa ◄

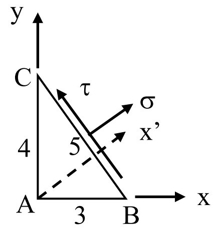
and $l_1 = 0.6184 \qquad m_1 = 0.5333 \qquad n_1 = 0.5772$

(b) $\sigma_1 = 19.238$ MPa $\qquad \sigma_2 = 13.704$ MPa $\qquad \sigma_3 = 4.648$ MPa ◄

and $l_1 = 0.3339 \qquad m_1 = 0.3862 \qquad n_1 = 0.8599$

SOLUTION (1.75)

(a) Direction cosines are:



$l = 4/5 = 0.8$
$m = 3/5 = 0.6$
$n = 0$

Equation (1.40) is thus

$$\sigma = 100(0.8)^2 + 60(0.6)^2 + 2(40)(0.8)(0.6) = 124 \text{ MPa}$$ ▲

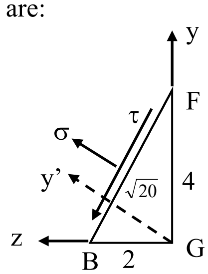
Equations (1.26) yield

$$p_x = 100(0.8) + 40(0.6) = 104 \text{ MPa}$$
$$p_y = 40(0.8) + 60(0.6) = 68 \text{ MPa}$$
$$p_z = 80(0.6) = 48 \text{ MPa}$$

Equation (1.41) is then

$$\tau = [104^2 + 68^2 + 48^2 - 124^2]^{\frac{1}{2}} = 48.66 \text{ MPa}$$ ▲

(b) Direction cosines are:



$l = 0$
$m = 2/\sqrt{20} = 0.447$
$n = 4/\sqrt{20} = 0.894$

Equation (1.40) results in

$$\sigma = 60(0.447)^2 + 20(0.894)^2 + 2(80)(0.447)(0.894) = 91.912 \text{ MPa}$$ ◄

Equations (1.26) yield

$$p_x = 40(0.447) = 17.88 \text{ MPa}$$
$$p_y = 60(0.447) + 80(0.894) = 98.34 \text{ MPa}$$
$$p_z = 80(0.447) + 20(0.894) = 53.64 \text{ MPa}$$

Equation (1.41) leads to

$$\tau = [17.88^2 + 98.34^2 + 53.64^2 - 91.912^2]^{\frac{1}{2}} = 66.482 \text{ MPa}$$ ◄

(c) Direction cosines are:

$l = 0.512$ $\quad m = 0.384$ $\quad n = 0.768$

Equation (1.40) is therefore

(CONT.)