Solutions for Algebra for College Students 9th Edition by Lial

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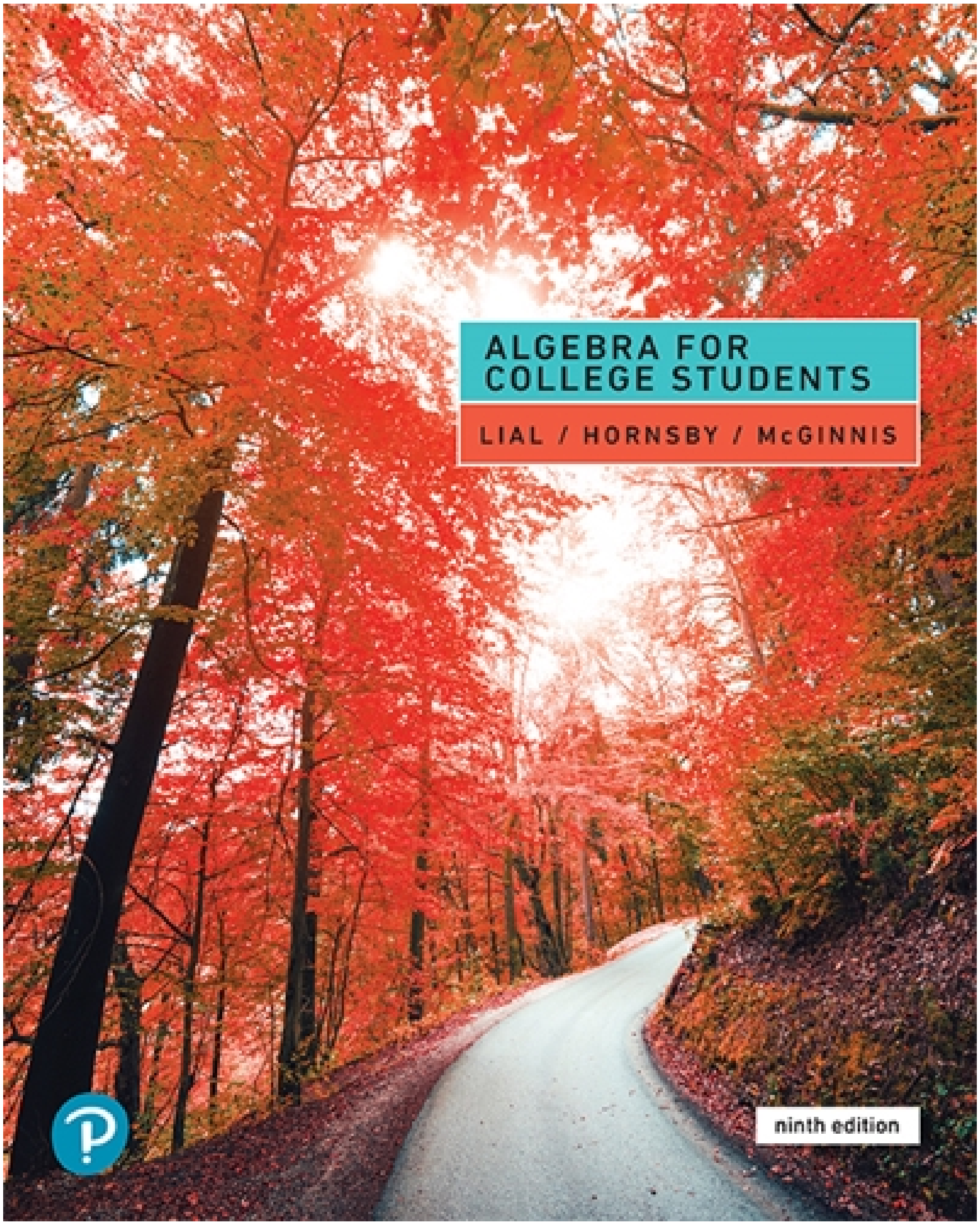


Solutions

Chapter R
Review of the Real Number System

R.1 Fractions, Decimals, and Percents

Classroom Examples, Now Try Exercises

1. **(a)** $\frac{12}{20}=\frac{3\cdot4}{5\cdot4}=\frac{3}{5}\cdot\frac{4}{4}=\frac{3}{5}\cdot1=\frac{3}{5}$

(b) $\frac{8}{48}=\frac{8}{6\cdot8}=\frac{1}{6\cdot1}=\frac{1}{6}$

(c) $\frac{90}{162}=\frac{5\cdot18}{9\cdot18}=\frac{5}{9}\cdot1=\frac{5}{9}$

N1. **(a)** $\frac{30}{42}=\frac{5\cdot6}{7\cdot6}=\frac{5}{7}\cdot\frac{6}{6}=\frac{5}{7}\cdot1=\frac{5}{7}$

(b) $\frac{10}{70}=\frac{10}{7\cdot10}=\frac{1}{7\cdot1}\cdot=\frac{1}{7}$

(c) $\frac{72}{120}=\frac{3\cdot24}{5\cdot24}=\frac{3}{5}\cdot1=\frac{3}{5}$

2. The fraction bar represents division. Divide the numerator of the improper fraction by the denominator.

$$\begin{array}{r} 3 \\ 10\overline{)37} \\ \underline{30} \\ 7 \end{array}$$

Thus, $\frac{37}{10}=3\frac{7}{10}$.

N2. The fraction bar represents division. Divide the numerator of the improper fraction by the denominator.

$$\begin{array}{r} 18 \\ 5\overline{)92} \\ \underline{5} \\ 42 \\ \underline{40} \\ 2 \end{array}$$

Thus, $\frac{92}{5}=18\frac{2}{5}$.

3. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$5\cdot3=15$ and $15+4=19$
The denominator of the improper fraction is the same as the denominator in the mixed number.

Thus, $3\frac{4}{5}=\frac{19}{5}$.

N3. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$3\cdot11=33$ and $33+2=35$
The denominator of the improper fraction is the same as the denominator in the mixed number.

Thus, $11\frac{2}{3}=\frac{35}{3}$.

4. To multiply two fractions, multiply their numerators and then multiply their denominators. Then simplify and write the answer in lowest terms.

$$\begin{aligned}\frac{5}{9}\cdot\frac{18}{25}&=\frac{5\cdot18}{9\cdot25}\\&=\frac{90}{225}\\&=\frac{2\cdot45}{5\cdot45}\\&=\frac{2}{5}\end{aligned}$$

N4. To multiply two fractions, multiply their numerators and then multiply their denominators. Then simplify and write the answer in lowest terms.

$$\begin{aligned}\frac{4}{7}\cdot\frac{5}{8}&=\frac{4\cdot5}{7\cdot8}\\&=\frac{20}{56}\\&=\frac{5\cdot4}{14\cdot4}\\&=\frac{5}{14}\end{aligned}$$

5. **(a)** To divide fractions, multiply by the reciprocal of the divisor.

$$\begin{aligned}\frac{9}{10}\div\frac{3}{5}&=\frac{9}{10}\cdot\frac{5}{3}\\&=\frac{3\cdot3\cdot5}{2\cdot5\cdot3}\\&=\frac{3}{2},\text{ or }1\frac{1}{2}\end{aligned}$$

(b) Change both mixed numbers to improper fractions. Then multiply by the reciprocal of the second fraction.

$$2\frac{3}{4}\div 3\frac{1}{3}=\frac{11}{4}\div\frac{10}{3}$$
$$=\frac{11}{4}\cdot\frac{3}{10}$$
$$=\frac{33}{40}$$

N5. **(a)** To divide fractions, multiply by the reciprocal of the divisor.

$$\frac{2}{7}\div\frac{8}{9}=\frac{2}{7}\cdot\frac{9}{8}$$
$$=\frac{2\cdot 3\cdot 3}{7\cdot 2\cdot 4}$$
$$=\frac{9}{28}$$

(b) To divide fractions, multiply by the reciprocal of the divisor.

$$3\frac{3}{4}\div 4\frac{2}{7}=\frac{15}{4}\div\frac{30}{7}$$
$$=\frac{15}{4}\cdot\frac{7}{30}$$
$$=\frac{15\cdot 7}{4\cdot 2\cdot 15}$$
$$=\frac{7}{8}$$

6. **(a)** To find the sum of two fractions having the same denominator, add the numerators and keep the same denominator.

$$\frac{1}{9}+\frac{5}{9}=\frac{1+5}{9}$$
$$=\frac{6}{9}$$
$$=\frac{2\cdot 3}{3\cdot 3}$$
$$=\frac{2}{3}$$

(b) Since $30=2\cdot 3\cdot 5$ and $45=3\cdot 3\cdot 5$, the least common denominator must have one factor of 2 (from 30), two factors of 3 (from 45), and one factor of 5 (from either 30 or 45), so it is $2\cdot 3\cdot 3\cdot 5=90$.
Write each fraction with a denominator of 90.

$$\frac{7}{30}=\frac{7}{30}\cdot\frac{3}{3}=\frac{21}{90}\text{ and }\frac{2}{45}=\frac{2}{45}\cdot\frac{2}{2}=\frac{4}{90}$$

Now add.

$$\frac{7}{30}+\frac{2}{45}=\frac{21}{90}+\frac{4}{90}=\frac{21+4}{90}=\frac{25}{90}$$

Write $\frac{25}{90}$ in lowest terms.

$$\frac{25}{90}=\frac{5\cdot 5}{18\cdot 5}=\frac{5}{18}$$

(c) Since $10=2\cdot 5$ and $4=2\cdot 2$, the least common denominator is $2\cdot 2\cdot 5=20$. Write each fraction with a denominator of 20.

$$\frac{3}{10}=\frac{3}{10}\cdot\frac{2}{2}=\frac{6}{20}\text{ and }\frac{1}{4}=\frac{1}{4}\cdot\frac{5}{5}=\frac{5}{20}$$

Now subtract.

$$\frac{3}{10}-\frac{1}{4}=\frac{6}{20}-\frac{5}{20}=\frac{1}{20}$$

(d) Write each mixed number as an improper fraction.

$$3\frac{3}{8}-1\frac{1}{2}=\frac{27}{8}-\frac{3}{2}$$

The least common denominator is 8. Write each fraction with a denominator of 8. $\frac{27}{8}$ remains unchanged, and $\frac{3}{2}=\frac{3}{2}\cdot\frac{4}{4}=\frac{12}{8}$.

Now subtract.

$$\frac{27}{8}-\frac{3}{2}=\frac{27}{8}-\frac{12}{8}=\frac{27-12}{8}=\frac{15}{8},\text{ or }1\frac{7}{8}$$

N6. **(a)** To find the sum of two fractions having the same denominator, add the numerators and keep the same denominator.

$$\frac{1}{8}+\frac{3}{8}=\frac{1+3}{8}$$
$$=\frac{4}{8}$$
$$=\frac{1\cdot 4}{2\cdot 4}$$
$$=\frac{1}{2}$$

(b) Since $12=2\cdot 2\cdot 3$ and $8=2\cdot 2\cdot 2$, the least common denominator must have three factors of 2 (from 8) and one factor of 3 (from 12), so it is $2\cdot 2\cdot 2\cdot 3=24$.
Write each fraction with a denominator of 24.

$$\frac{5}{12}=\frac{5}{12}\cdot\frac{2}{2}=\frac{10}{24}\text{ and }\frac{3}{8}=\frac{3}{8}\cdot\frac{3}{3}=\frac{9}{24}$$

Now add.

$$\frac{5}{12}+\frac{3}{8}=\frac{10}{24}+\frac{9}{24}=\frac{10+9}{24}=\frac{19}{24}$$

(c) Since $11 = 11$ and $9 = 3 \cdot 3$, the least common denominator is $3 \cdot 3 \cdot 11 = 99$. Write each fraction with a denominator of 99.

$$\frac{5}{11} = \frac{5}{11} \cdot \frac{9}{9} = \frac{45}{99} \text{ and } \frac{2}{9} = \frac{2}{9} \cdot \frac{11}{11} = \frac{22}{99}$$

Now subtract.

$$\frac{5}{11} - \frac{2}{9} = \frac{45}{99} - \frac{22}{99} = \frac{23}{99}$$

(d) Write each mixed number as an improper fraction.

$$4\frac{1}{3} - 2\frac{5}{6} = \frac{13}{3} - \frac{17}{6}$$

The least common denominator is 6. Write each fraction with a denominator of 6. $\frac{17}{6}$ remains unchanged, and $\frac{13}{3} = \frac{13}{3} \cdot \frac{2}{2} = \frac{26}{6}$.

Now subtract.

$$\frac{13}{3} - \frac{17}{6} = \frac{26}{6} - \frac{17}{6} = \frac{26-17}{6} = \frac{9}{6}$$

Now reduce.

$$\frac{9}{6} = \frac{3 \cdot 3}{2 \cdot 3} = \frac{3}{2}, \text{ or } 1\frac{1}{2}$$

7. **(a)** $0.15 = \frac{15}{100}$

(b) $0.009 = \frac{9}{1000}$

(c) $2.5 = 2\frac{5}{10} = \frac{25}{10}$

N7. **(a)** $0.8 = \frac{8}{10}$

(b) $0.431 = \frac{431}{1000}$

(c) $2.58 = 2\frac{58}{100} = \frac{258}{100}$

8. **(a)**

$$\begin{array}{r} 42.830 \\ 71.000 \\ +\ 3.074 \\ \hline 116.904 \end{array}$$

(b)

$$\begin{array}{r} 32.50 \\ -21.72 \\ \hline 10.78 \end{array}$$

N8. **(a)**

$$\begin{array}{r} 68.900 \\ 42.720 \\ +\ 8.973 \\ \hline 120.593 \end{array}$$

(b)

$$\begin{array}{r} 351.800 \\ -\quad 2.706 \\ \hline 349.094 \end{array}$$

9. **(a)**

$$\begin{array}{r r} 30.2 & \text{1 decimal place} \\ \times 0.052 & \text{3 decimal places} \\ \hline 604 & \downarrow \\ 1510 & 1+3=4 \\ \hline 1.5704 & \text{4 decimal places} \end{array}$$

(b)

$$\begin{array}{r r} 0.06 & \text{2 decimal places} \\ \times 0.12 & \text{2 decimal places} \\ \hline 12 & \downarrow \\ 6 & 2+2=4 \\ \hline 0.0072 & \text{4 decimal places} \end{array}$$

(c) To change the divisor 0.37 into a whole number, move each decimal point two places to the right. Move the decimal point straight up and divide as with whole numbers.

$$\begin{array}{r} 14.8 \\ 37\overline{)547.6} \\ \underline{37} \\ 177 \\ \underline{148} \\ 296 \\ \underline{296} \\ 0 \end{array}$$

Therefore, $5.476 \div 0.37 = 14.8$.

(d) To change the divisor 3.1 into a whole number, move each decimal point one place to the right. Move the decimal point straight up and divide as with whole numbers.

$$\begin{array}{r} 1.21 \\ 31\overline{)37.60} \\ \underline{31} \\ 66 \\ \underline{62} \\ 40 \\ \underline{31} \\ 9 \end{array}$$

We carried out the division to 2 decimal places so that we could round to 1 decimal place. Therefore, $3.76 \div 3.1 \approx 1.2$.

N9. **(a)**

$$\begin{array}{rr} 9.32 & \text{2 decimal places} \\ \underline{\times\ 1.4} & \text{1 decimal place} \\ 3728 & \downarrow \\ \underline{932} & 2+1=3 \\ 13.048 & \text{3 decimal places} \end{array}$$

(b)

$$\begin{array}{rr} 0.6 & \text{1 decimal place} \\ \underline{\times 0.004} & \text{3 decimal places} \\ \underline{24} & 1+3=4 \\ 0.0024 & \text{4 decimal places} \end{array}$$

(c) To change the divisor 1.3 into a whole number, move each decimal point one place to the right. Move the decimal point straight up and divide as with whole numbers.

$$\begin{array}{r} 5.641 \\ 13\overline{)73.340} \\ \underline{65} \\ 83 \\ \underline{78} \\ 54 \\ \underline{52} \\ 20 \\ \underline{13} \\ 7 \end{array}$$

We carried out the division to 3 decimal places so that we could round to 2 decimal places. Therefore, $7.334 \div 1.3 \approx 5.64$.

10. **(a)** Move the decimal point three places to the right.
$19.5 \times 1000 = 19{,}500$

(b) Move the decimal point one place to the left.
$960.1 \div 10 = 96.01$

N10. **(a)** Move the decimal point one place to the right.
$294.72 \times 10 = 2947.2$

(b) Move the decimal point two places to the left. Insert a 0 in front of the 4 to do this.
$4.793 \div 100 = 0.04793$

11. **(a)** Divide 3 by 50. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.06 \\ 50\overline{)3.00} \\ \underline{3\ 00} \\ 0 \end{array}$$

Therefore, $\frac{3}{50} = 0.06$.

(b) Divide 11 by 1. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.090909... \\ 11\overline{)1.000000...} \\ \underline{99} \\ 100 \\ \underline{99} \\ 100 \\ \underline{99} \\ 1 \end{array}$$

Note that the pattern repeats. Therefore, $\frac{1}{11} = 0.\overline{09}$, or about 0.091.

N11. **(a)** Divide 20 by 17. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.85 \\ 20\overline{)17.00} \\ \underline{160} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

Therefore, $\frac{17}{20} = 0.85$.

(b) Divide 2 by 9. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.222... \\ 9\overline{)2.000...} \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

Note that the pattern repeats. Therefore, $\frac{2}{9} = 0.\overline{2}$, or 0.222.

12. **(a)** $85\% = 0.85$

(b) $110\% = 1.10$, or 1.1

(c) $0.30 = 30\%$

(d) $0.165 = 16.5\%$

N12. **(a)** $52\% = 0.52$

(b) $2\% = 02\% = 0.02$

(c) $0.45 = 45\%$

(d) $3.5 = 3.50 = 350\%$

13. **(a)** $65\% = \frac{65}{100}$

In lowest terms,

$$\frac{65}{100} = \frac{13 \cdot 5}{20 \cdot 5} = \frac{13}{20}$$

(b) $1.5\% = \frac{1.5}{100} = \frac{1.5}{100} \cdot \frac{10}{10} = \frac{15}{1000} = \frac{3}{200}$

N13. **(a)** $20\% = \frac{20}{100}$

In lowest terms,

$$\frac{20}{100} = \frac{1 \cdot 20}{5 \cdot 20} = \frac{1}{5}$$

(b) $160\% = \frac{160}{100}$

In lowest terms,

$$\frac{160}{100} = \frac{8 \cdot 20}{5 \cdot 20} = \frac{8}{5}, \text{ or } 1\frac{3}{5}$$

14. **(a)**
$$\begin{aligned} \frac{3}{50} &= \frac{3}{50} \cdot 100\% \\ &= \frac{3}{50} \cdot \frac{100}{1}\% \\ &= \frac{3 \cdot 50 \cdot 2}{50}\% \\ &= 6\% \end{aligned}$$

(b)
$$\begin{aligned} \frac{1}{3} &= \frac{1}{3} \cdot 100\% \\ &= \frac{1}{3} \cdot \frac{100}{1}\% \\ &= \frac{100}{3}\% \\ &= 33\frac{1}{3}\%, \text{ or } 33.\overline{3}\% \end{aligned}$$

N14. **(a)**
$$\begin{aligned} \frac{6}{25} &= \frac{6}{25} \cdot 100\% \\ &= \frac{6}{25} \cdot \frac{100}{1}\% \\ &= \frac{6 \cdot 25 \cdot 4}{25}\% \\ &= 24\% \end{aligned}$$

(b)
$$\begin{aligned} \frac{7}{9} &= \frac{7}{9} \cdot 100\% \\ &= \frac{7}{9} \cdot \frac{100}{1}\% \\ &= \frac{700}{9}\% \\ &= 77\frac{7}{9}\%, \text{ or } 77.\overline{7}\% \end{aligned}$$

Exercises

1. True; the number above the fraction bar is called the numerator and the number below the fraction bar is called the denominator.

2. True; 5 divides the 31 six times with a remainder of one, so $\frac{31}{5} = 6\frac{1}{5}$.

3. False; this is an improper fraction. Its value is 1.

4. False; the reciprocal of $\frac{6}{2} = 3$ is $\frac{2}{6} = \frac{1}{3}$.

5. $\frac{16}{24} = \frac{2 \cdot 8}{3 \cdot 8} = \frac{2}{3}$

Therefore, C is correct.

6. Simplify each fraction to find which are equal to $\frac{5}{9}$.

$$\frac{15}{27} = \frac{3 \cdot 5}{3 \cdot 9} = \frac{5}{9}$$
$$\frac{30}{54} = \frac{6 \cdot 5}{6 \cdot 9} = \frac{5}{9}$$
$$\frac{40}{74} = \frac{2 \cdot 20}{2 \cdot 37} = \frac{20}{37}$$
$$\frac{55}{99} = \frac{11 \cdot 5}{11 \cdot 9} = \frac{5}{9}$$

Therefore, C is correct.

7. $\frac{8}{16} = \frac{1 \cdot 8}{2 \cdot 8} = \frac{1}{2} \cdot \frac{8}{8} = \frac{1}{2} \cdot 1 = \frac{1}{2}$

8. $\frac{4}{12}=\frac{1\cdot 4}{3\cdot 4}=\frac{1}{3}\cdot\frac{4}{4}=\frac{1}{3}\cdot 1=\frac{1}{3}$

9. $\frac{15}{18}=\frac{3\cdot 5}{3\cdot 6}=\frac{3}{3}\cdot\frac{5}{6}=1\cdot\frac{5}{6}=\frac{5}{6}$

10. $\frac{16}{20}=\frac{4\cdot 4}{5\cdot 4}=\frac{4}{5}\cdot\frac{4}{4}=\frac{4}{5}\cdot 1=\frac{4}{5}$

11. $\frac{90}{150}=\frac{3\cdot 30}{5\cdot 30}=\frac{3}{5}\cdot\frac{30}{30}=\frac{3}{5}\cdot 1=\frac{3}{5}$

12. $\frac{100}{140}=\frac{5\cdot 20}{7\cdot 20}=\frac{5}{7}\cdot\frac{20}{20}=\frac{5}{7}\cdot 1=\frac{5}{7}$

13. $\frac{18}{90}=\frac{1\cdot 18}{5\cdot 18}=\frac{1}{5}\cdot\frac{18}{18}=\frac{1}{5}\cdot 1=\frac{1}{5}$

14. $\frac{16}{64}=\frac{1\cdot 16}{4\cdot 16}=\frac{1}{4}\cdot\frac{16}{16}=\frac{1}{4}\cdot 1=\frac{1}{4}$

15. $\frac{144}{120}=\frac{6\cdot 24}{5\cdot 24}=\frac{6}{5}\cdot\frac{24}{24}=\frac{6}{5}\cdot 1=\frac{6}{5}$

16. $\frac{132}{77}=\frac{12\cdot 11}{7\cdot 11}=\frac{12}{7}\cdot\frac{11}{11}=\frac{12}{7}\cdot 1=\frac{12}{7}$

17.
$$\begin{array}{r} 1 \\ 7\overline{)12} \\ \underline{7} \\ 5 \end{array}$$

Therefore, $\frac{12}{7}=1\frac{5}{7}$.

18.
$$\begin{array}{r} 1 \\ 9\overline{)16} \\ \underline{9} \\ 7 \end{array}$$

Therefore, $\frac{16}{9}=1\frac{7}{9}$.

19.
$$\begin{array}{r} 6 \\ 12\overline{)77} \\ \underline{72} \\ 5 \end{array}$$

Therefore, $\frac{77}{12}=6\frac{5}{12}$.

20.
$$\begin{array}{r} 5 \\ 13\overline{)67} \\ \underline{65} \\ 2 \end{array}$$

Therefore, $\frac{67}{13}=5\frac{2}{13}$.

21. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$5\cdot 2=10$ and $10+3=13$
The denominator of the improper fraction is the same as the denominator in the mixed number.
Thus, $2\frac{3}{5}=\frac{13}{5}$.

22. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$7\cdot 5=35$ and $35+6=41$
The denominator of the improper fraction is the same as the denominator in the mixed number.
Thus, $5\frac{6}{7}=\frac{41}{7}$.

23. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$3\cdot 12=36$ and $36+2=38$
The denominator of the improper fraction is the same as the denominator in the mixed number.
Thus, $12\frac{2}{3}=\frac{38}{3}$.

24. Multiply the denominator of the fraction by the natural number and then add the numerator to obtain the numerator of the improper fraction.
$5\cdot 10=50$ and $50+1=51$
The denominator of the improper fraction is the same as the denominator in the mixed number.
Thus, $10\frac{1}{5}=\frac{51}{5}$.

25. $\frac{4}{5}\cdot\frac{6}{7}=\frac{4\cdot 6}{5\cdot 7}=\frac{24}{35}$

26. $\frac{5}{9}\cdot\frac{2}{7}=\frac{5\cdot 2}{9\cdot 7}=\frac{10}{63}$

27. $\frac{2}{15}\cdot\frac{3}{8}=\frac{2\cdot 3}{15\cdot 8}=\frac{6}{120}=\frac{1\cdot 6}{20\cdot 6}=\frac{1}{20}$

28. $\frac{3}{20}\cdot\frac{5}{21}=\frac{3\cdot 5}{20\cdot 21}=\frac{15}{420}=\frac{1\cdot 15}{28\cdot 15}=\frac{1}{28}$

29. $\frac{1}{10}\cdot\frac{12}{5}=\frac{1\cdot 12}{10\cdot 5}=\frac{1\cdot 2\cdot 6}{2\cdot 5\cdot 5}=\frac{6}{25}$

30. $\frac{1}{8}\cdot\frac{10}{7}=\frac{1\cdot 10}{8\cdot 7}=\frac{1\cdot 2\cdot 5}{2\cdot 4\cdot 7}=\frac{5}{28}$

31. $$\begin{aligned}\frac{15}{4}\cdot\frac{8}{25}&=\frac{15\cdot 8}{4\cdot 25}\\&=\frac{3\cdot 5\cdot 4\cdot 2}{4\cdot 5\cdot 5}\\&=\frac{3\cdot 2}{5}\\&=\frac{6}{5},\text{ or } 1\frac{1}{5}\end{aligned}$$

32. $$\begin{aligned}\frac{21}{8}\cdot\frac{4}{7}&=\frac{21\cdot 4}{8\cdot 7}\\&=\frac{3\cdot 7\cdot 4}{4\cdot 2\cdot 7}\\&=\frac{3}{2},\text{ or } 1\frac{1}{2}\end{aligned}$$

33. $$\begin{aligned}21\cdot\frac{3}{7}&=\frac{21}{1}\cdot\frac{3}{7}\\&=\frac{21\cdot 3}{1\cdot 7}\\&=\frac{3\cdot 7\cdot 3}{1\cdot 7}\\&=\frac{3\cdot 3}{1}=9\end{aligned}$$

34. $$\begin{aligned}36\cdot\frac{4}{9}&=\frac{36}{1}\cdot\frac{4}{9}\\&=\frac{36\cdot 4}{1\cdot 9}\\&=\frac{4\cdot 9\cdot 4}{1\cdot 9}\\&=\frac{4\cdot 4}{1}=16\end{aligned}$$

35. Change both mixed numbers to improper fractions.

$$\begin{aligned}3\frac{1}{4}\cdot 1\frac{2}{3}&=\frac{13}{4}\cdot\frac{5}{3}\\&=\frac{13\cdot 5}{4\cdot 3}\\&=\frac{65}{12},\text{ or } 5\frac{5}{12}\end{aligned}$$

36. Change both mixed numbers to improper fractions.

$$\begin{aligned}2\frac{2}{3}\cdot 1\frac{3}{5}&=\frac{8}{3}\cdot\frac{8}{5}\\&=\frac{8\cdot 8}{3\cdot 5}\\&=\frac{64}{15},\text{ or } 4\frac{4}{15}\end{aligned}$$

37. To divide fractions, multiply by the reciprocal of the divisor.

$$\begin{aligned}\frac{7}{9}\div\frac{3}{2}&=\frac{7}{9}\cdot\frac{2}{3}\\&=\frac{7\cdot 2}{9\cdot 3}\\&=\frac{14}{27}\end{aligned}$$

38. To divide fractions, multiply by the reciprocal of the divisor.

$$\begin{aligned}\frac{6}{11}\div\frac{5}{4}&=\frac{6}{11}\cdot\frac{4}{5}\\&=\frac{6\cdot 4}{11\cdot 5}\\&=\frac{24}{55}\end{aligned}$$

39. To divide fractions, multiply by the reciprocal of the divisor.

$$\begin{aligned}\frac{5}{4}\div\frac{3}{8}&=\frac{5}{4}\cdot\frac{8}{3}\\&=\frac{5\cdot 8}{4\cdot 3}\\&=\frac{5\cdot 4\cdot 2}{4\cdot 3}\\&=\frac{5\cdot 2}{3}\\&=\frac{10}{3},\text{ or } 3\frac{1}{3}\end{aligned}$$

40. To divide fractions, multiply by the reciprocal of the divisor.

$$\begin{aligned}\frac{7}{5}\div\frac{3}{10}&=\frac{7}{5}\cdot\frac{10}{3}\\&=\frac{7\cdot 10}{5\cdot 3}\\&=\frac{7\cdot 2\cdot 5}{5\cdot 3}\\&=\frac{14}{3},\text{ or } 4\frac{2}{3}\end{aligned}$$

41. To divide fractions, multiply by the reciprocal of the divisor.

$$\begin{aligned}\frac{32}{5}\div\frac{8}{15}&=\frac{32}{5}\cdot\frac{15}{8}\\&=\frac{32\cdot 15}{5\cdot 8}\\&=\frac{8\cdot 4\cdot 3\cdot 5}{1\cdot 5\cdot 8}\\&=\frac{4\cdot 3}{1}=12\end{aligned}$$

42. To divide fractions, multiply by the reciprocal of the divisor.

$$\begin{aligned}\frac{24}{7}\div\frac{6}{21}&=\frac{24}{7}\cdot\frac{21}{6}\\&=\frac{24\cdot 21}{7\cdot 6}\\&=\frac{4\cdot 6\cdot 3\cdot 7}{1\cdot 7\cdot 6}\\&=\frac{4\cdot 3}{1}=12\end{aligned}$$

43. To divide fractions, multiply by the reciprocal of the divisor.

$$\begin{aligned}\frac{3}{4}\div 12&=\frac{3}{4}\cdot\frac{1}{12}\\&=\frac{3\cdot 1}{4\cdot 12}\\&=\frac{3\cdot 1}{4\cdot 3\cdot 4}\\&=\frac{1}{4\cdot 4}=\frac{1}{16}\end{aligned}$$

44. To divide fractions, multiply by the reciprocal of the divisor.

$$\begin{aligned}\frac{2}{5}\div 30&=\frac{2}{5}\cdot\frac{1}{30}\\&=\frac{2\cdot 1}{5\cdot 30}\\&=\frac{2\cdot 1}{5\cdot 2\cdot 15}\\&=\frac{1}{5\cdot 15}=\frac{1}{75}\end{aligned}$$

45. To divide fractions, multiply by the reciprocal of the divisor.

$$\begin{aligned}6\div\frac{3}{5}&=\frac{6}{1}\cdot\frac{5}{3}\\&=\frac{6\cdot 5}{1\cdot 3}\\&=\frac{2\cdot 3\cdot 5}{1\cdot 3}\\&=\frac{2\cdot 5}{1}=10\end{aligned}$$

46. To divide fractions, multiply by the reciprocal of the divisor.

$$\begin{aligned}8\div\frac{4}{9}&=\frac{8}{1}\cdot\frac{9}{4}\\&=\frac{8\cdot 9}{1\cdot 4}\\&=\frac{2\cdot 4\cdot 9}{1\cdot 4}\\&=\frac{2\cdot 9}{1}=18\end{aligned}$$

47. Change the first number to an improper fraction, and then multiply by the reciprocal of the divisor.

$$\begin{aligned}6\frac{3}{4}\div\frac{3}{8}&=\frac{27}{4}\div\frac{3}{8}\\&=\frac{27}{4}\cdot\frac{8}{3}\\&=\frac{27\cdot 8}{4\cdot 3}\\&=\frac{3\cdot 9\cdot 2\cdot 4}{4\cdot 3}\\&=\frac{9\cdot 2}{1}=18\end{aligned}$$

48. Change the first number to an improper fraction, and then multiply by the reciprocal of the divisor.

$$\begin{aligned}5\frac{3}{5}\div\frac{7}{10}&=\frac{28}{5}\div\frac{7}{10}\\&=\frac{28}{5}\cdot\frac{10}{7}\\&=\frac{28\cdot 10}{5\cdot 7}\\&=\frac{4\cdot 7\cdot 2\cdot 5}{5\cdot 7}\\&=\frac{4\cdot 2}{1}=8\end{aligned}$$

49. Change both mixed numbers to improper fractions, and then multiply by the reciprocal of the divisor.

$$\begin{aligned}2\frac{1}{2}\div 1\frac{5}{7}&=\frac{5}{2}\div\frac{12}{7}\\&=\frac{5}{2}\cdot\frac{7}{12}\\&=\frac{5\cdot 7}{2\cdot 12}\\&=\frac{35}{24},\text{ or }1\frac{11}{24}\end{aligned}$$

50. Change both mixed numbers to improper fractions, and then multiply by the reciprocal of the divisor.

$$\begin{aligned}2\frac{2}{9}\div 1\frac{2}{5}&=\frac{20}{9}\div\frac{7}{5}\\&=\frac{20}{9}\cdot\frac{5}{7}\\&=\frac{20\cdot 5}{9\cdot 7}\\&=\frac{100}{63},\text{ or }1\frac{37}{63}\end{aligned}$$

51. Change both mixed numbers to improper fractions, and then multiply by the reciprocal of the divisor.

$$\begin{aligned}2\frac{5}{8}\div 1\frac{15}{32}&=\frac{21}{8}\div\frac{47}{32}\\&=\frac{21}{8}\cdot\frac{32}{47}\\&=\frac{21\cdot 32}{8\cdot 47}\\&=\frac{21\cdot 8\cdot 4}{8\cdot 47}\\&=\frac{21\cdot 4}{47}\\&=\frac{84}{47},\text{ or }1\frac{37}{47}\end{aligned}$$

52. Change both mixed numbers to improper fractions, and then multiply by the reciprocal of the divisor.

$$\begin{aligned}2\frac{3}{10}\div 1\frac{4}{5}&=\frac{23}{10}\div\frac{9}{5}\\&=\frac{23}{10}\cdot\frac{5}{9}\\&=\frac{23\cdot 5}{2\cdot 5\cdot 9}\\&=\frac{23}{18},\text{ or }1\frac{5}{8}\end{aligned}$$

53. $\frac{7}{15}+\frac{4}{15}=\frac{7+4}{15}=\frac{11}{15}$

54. $\frac{2}{9}+\frac{5}{9}=\frac{2+5}{9}=\frac{7}{9}$

55.

$$\begin{aligned}\frac{7}{12}+\frac{1}{12}&=\frac{7+1}{12}\\&=\frac{8}{12}\\&=\frac{2\cdot 4}{3\cdot 4}\\&=\frac{2}{3}\end{aligned}$$

56. $\frac{3}{16}+\frac{5}{16}=\frac{3+5}{16}=\frac{8}{16}=\frac{1}{2}$

57. Since $9=3\cdot 3$, and 3 is prime, the LCD (least common denominator) is $3\cdot 3=9$.

$$\frac{1}{3}=\frac{1}{3}\cdot\frac{3}{3}=\frac{3}{9}$$

Now add the two fractions with the same denominator.

$$\frac{5}{9}+\frac{1}{3}=\frac{5}{9}+\frac{3}{9}=\frac{8}{9}$$

58. To add $\frac{4}{15}$ and $\frac{1}{5}$, first find the LCD. Since $15=3\cdot 5$ and 5 is prime, the LCD is 15.

$$\begin{aligned}\frac{4}{15}+\frac{1}{5}&=\frac{4}{15}+\frac{1}{5}\cdot\frac{3}{3}\\&=\frac{4}{15}+\frac{3}{15}\\&=\frac{4+3}{15}\\&=\frac{7}{15}\end{aligned}$$

59. Since $8=2\cdot 2\cdot 2$ and $6=2\cdot 3$, the LCD is $2\cdot 2\cdot 2\cdot 3=24$.

$$\frac{3}{8}=\frac{3}{8}\cdot\frac{3}{3}=\frac{9}{24}\text{ and }\frac{5}{6}=\frac{5}{6}\cdot\frac{4}{4}=\frac{20}{24}$$

Now add fractions with the same denominator.

$$\frac{3}{8}+\frac{5}{6}=\frac{9}{24}+\frac{20}{24}=\frac{29}{24},\text{ or }1\frac{5}{24}$$

60. Since $6=2\cdot 3$ and $9=3\cdot 3$, the LCD is $2\cdot 3\cdot 3=18$.

$$\frac{5}{6}=\frac{5}{6}\cdot\frac{3}{3}=\frac{15}{18}\text{ and }\frac{2}{9}=\frac{2}{9}\cdot\frac{2}{2}=\frac{4}{18}$$

Now add fractions with the same denominator.

$$\frac{5}{6}+\frac{2}{9}=\frac{15}{18}+\frac{4}{18}=\frac{19}{18},\text{ or }1\frac{1}{18}$$

61. Since $9=3\cdot 3$ and $16=4\cdot 4$, the LCD is $3\cdot 3\cdot 4\cdot 4=144.$

$$\frac{5}{9}=\frac{5}{9}\cdot\frac{16}{16}=\frac{80}{144}\text{ and }\frac{3}{16}=\frac{3}{16}\cdot\frac{9}{9}=\frac{27}{144}$$

Now add fractions with the same denominator.

$$\frac{5}{9}+\frac{3}{16}=\frac{80}{144}+\frac{27}{144}=\frac{107}{144}$$

62. Since $4=2\cdot 2$ and $25=5\cdot 5$, the LCD is $2\cdot 2\cdot 5\cdot 5=100.$

$$\frac{3}{4}=\frac{3}{4}\cdot\frac{25}{25}=\frac{75}{100}\text{ and }\frac{6}{25}=\frac{6}{25}\cdot\frac{4}{4}=\frac{24}{100}$$

Now add fractions with the same denominator.

$$\frac{3}{4}+\frac{6}{25}=\frac{75}{100}+\frac{24}{100}=\frac{99}{100}$$

63.

$$3\frac{1}{8}=3+\frac{1}{8}=\frac{24}{8}+\frac{1}{8}=\frac{25}{8}$$

$$2\frac{1}{4}=2+\frac{1}{4}=\frac{8}{4}+\frac{1}{4}=\frac{9}{4}$$

$$3\frac{1}{8}+2\frac{1}{4}=\frac{25}{8}+\frac{9}{4}$$

Since $8=2\cdot 2\cdot 2$ and $4=2\cdot 2$, the LCD is $2\cdot 2\cdot 2$ or 8.

$$\begin{aligned}3\frac{1}{8}+2\frac{1}{4}&=\frac{25}{8}+\frac{9}{4}\cdot\frac{2}{2}\\&=\frac{25}{8}+\frac{18}{8}\\&=\frac{43}{8},\text{ or }5\frac{3}{8}\end{aligned}$$

64.

$$4\frac{2}{3}=4+\frac{2}{3}=\frac{12}{3}+\frac{2}{3}=\frac{14}{3}$$

$$2\frac{1}{6}=2+\frac{1}{6}=\frac{12}{6}+\frac{1}{6}=\frac{13}{6}$$

Since $6=2\cdot 3$, the LCD is 6.

$$\begin{aligned}4\frac{2}{3}+2\frac{1}{6}&=\frac{14}{3}\cdot\frac{2}{2}+\frac{13}{6}\\&=\frac{28}{6}+\frac{13}{6}\\&=\frac{41}{6},\text{ or }6\frac{5}{6}\end{aligned}$$

65. $$\frac{7}{9}-\frac{2}{9}=\frac{7-2}{9}=\frac{5}{9}$$

66. $$\frac{8}{11}-\frac{3}{11}=\frac{8-3}{11}=\frac{5}{11}$$

67. $$\begin{aligned}\frac{13}{15}-\frac{3}{15}&=\frac{13-3}{15}\\&=\frac{10}{15}\\&=\frac{2\cdot 5}{3\cdot 5}=\frac{2}{3}\end{aligned}$$

68. $$\begin{aligned}\frac{11}{12}-\frac{3}{12}&=\frac{11-3}{12}\\&=\frac{8}{12}\\&=\frac{2\cdot 4}{3\cdot 4}=\frac{2}{3}\end{aligned}$$

69. Since $12=4\cdot 3$ (12 is a multiple of 3), the LCD is 12.

$$\frac{1}{3}\cdot\frac{4}{4}=\frac{4}{12}$$

Now subtract fractions with the same denominator.

$$\frac{7}{12}-\frac{1}{3}=\frac{7}{12}-\frac{4}{12}=\frac{3}{12}=\frac{1\cdot 3}{4\cdot 3}=\frac{1}{4}$$

70. Since $6=3\cdot 2$ (6 is a multiple of 2), the LCD is 6.

$$\frac{1}{2}\cdot\frac{3}{3}=\frac{3}{6}$$

Now subtract fractions with the same denominator.

$$\frac{5}{6}-\frac{1}{2}=\frac{5}{6}-\frac{3}{6}=\frac{2}{6}=\frac{1\cdot 2}{3\cdot 2}=\frac{1}{3}$$

71. Since $12=2\cdot 2\cdot 3$ and $9=3\cdot 3$, the LCD is $2\cdot 2\cdot 3\cdot 3=36.$

$$\frac{7}{12}=\frac{7}{12}\cdot\frac{3}{3}=\frac{21}{36}\text{ and }\frac{1}{9}\cdot\frac{4}{4}=\frac{4}{36}$$

Now subtract fractions with the same denominator.

$$\frac{7}{12}-\frac{1}{9}=\frac{21}{36}-\frac{4}{36}=\frac{17}{36}$$

72. $$\begin{aligned}\frac{11}{16}-\frac{1}{12}&=\frac{11}{16}\cdot\frac{3}{3}-\frac{1}{12}\cdot\frac{4}{4}\\&=\frac{33}{48}-\frac{4}{48}\\&=\frac{29}{48}\end{aligned}$$

The LCD of 12 and 16 is 48.

73. $4\frac{3}{4} = 4 + \frac{3}{4} = \frac{16}{4} + \frac{3}{4} = \frac{19}{4}$

$1\frac{2}{5} = 1 + \frac{2}{5} = \frac{5}{5} + \frac{2}{5} = \frac{7}{5}$

Since $4 = 2 \cdot 2$, and 5 is prime, the LCD is $2 \cdot 2 \cdot 5 = 20$.

$$\begin{aligned} 4\frac{3}{4} - 1\frac{2}{5} &= \frac{19}{4} \cdot \frac{5}{5} - \frac{7}{5} \cdot \frac{4}{4} \\ &= \frac{95}{20} - \frac{28}{20} \\ &= \frac{67}{20}, \text{ or } 3\frac{7}{20} \end{aligned}$$

74. Change both numbers to improper fractions then add, using 45 as the common denominator.

$$\begin{aligned} 3\frac{4}{5} - 1\frac{4}{9} &= \frac{19}{5} - \frac{13}{9} \\ &= \frac{19}{5} \cdot \frac{9}{9} - \frac{13}{9} \cdot \frac{5}{5} \\ &= \frac{171}{45} - \frac{65}{45} \\ &= \frac{106}{45}, \text{ or } 2\frac{16}{45} \end{aligned}$$

75. $8\frac{2}{9} = 8 + \frac{2}{9} = \frac{72}{9} + \frac{2}{9} = \frac{74}{9}$

$4\frac{2}{3} = 4 + \frac{2}{3} = \frac{12}{3} + \frac{2}{3} = \frac{14}{3}$

Since $9 = 3 \cdot 3$, and 3 is prime, the LCD is $3 \cdot 3 = 9$.

$$\begin{aligned} 8\frac{2}{9} - 4\frac{2}{3} &= \frac{74}{9} - \frac{14}{3} \cdot \frac{3}{3} \\ &= \frac{74}{9} - \frac{42}{9} \\ &= \frac{32}{9}, \text{ or } 3\frac{5}{9} \end{aligned}$$

76. $7\frac{5}{12} = 7 + \frac{5}{12} = \frac{84}{12} + \frac{5}{12} = \frac{89}{12}$

$4\frac{5}{6} = 4 + \frac{5}{6} = \frac{24}{6} + \frac{5}{6} = \frac{29}{6}$

Since $12 = 2 \cdot 2 \cdot 3$ and $6 = 2 \cdot 3$, the LCD is $2 \cdot 2 \cdot 3 = 12$.

$$\begin{aligned} 7\frac{5}{12} - 4\frac{5}{6} &= \frac{89}{12} - \frac{29}{6} \cdot \frac{2}{2} \\ &= \frac{89}{12} - \frac{58}{12} \\ &= \frac{31}{12}, \text{ or } 2\frac{7}{12} \end{aligned}$$

77. Observe that there are 24 dots in the entire figure, 6 dots in the triangle, 12 dots in the rectangle, and 2 dots in the overlapping region.

(a) $\frac{12}{24} = \frac{1}{2}$ of all the dots are in the rectangle.

(b) $\frac{6}{24} = \frac{1}{4}$ of all the dots are in the triangle.

(c) $\frac{2}{6} = \frac{1}{3}$ of the dots in the triangle are in the overlapping region.

(d) $\frac{2}{12} = \frac{1}{6}$ of the dots in the rectangle are in the overlapping region.

78. **(a)** 12 is $\frac{1}{3}$ of 36, so Maureen got a hit in exactly $\frac{1}{3}$ of her at-bats.

(b) 5 is a little less than $\frac{1}{2}$ of 11, so Chase got a hit in just less than $\frac{1}{2}$ of his at-bats.

(c) 9 is a little less than $\frac{1}{4}$ of 40, so Christine got a hit in just less than $\frac{1}{4}$ of her at-bats.

(d) 8 is $\frac{1}{2}$ of 16, and 10 is $\frac{1}{2}$ of 20, so Joe and Greg each got hits $\frac{1}{2}$ of the time they were at bat.

79. 367.9412

(a) Tens: 6

(b) Tenths: 9

(c) Thousandths: 1

(d) Ones: 7

(e) Hundredths: 4

80. Answers will vary. One example is 5243.0164.

81. 46.249

(a) 46.25

(b) 46.2

(c) 46

(d) 50

82. **(a)** 0.889

(b) 0.444

(c) 0.976

(d) 0.865

83. $0.4 = \frac{4}{10}$

84. $0.6 = \frac{6}{10}$

85. $0.64 = \frac{64}{100}$

86. $0.82 = \frac{82}{100}$

87. $0.138 = \frac{138}{1000}$

88. $0.104 = \frac{104}{1000}$

89. $0.043 = \frac{43}{1000}$

90. $0.087 = \frac{87}{1000}$

91. $3.805 = 3\frac{805}{1000} = \frac{3805}{1000}$

92. $5.166 = 5\frac{166}{1000} = \frac{5166}{1000}$

93.
$$\begin{array}{r} 25.320 \\ 109.200 \\ +\ \ 8.574 \\ \hline 143.094 \end{array}$$

94.
$$\begin{array}{r} 90.527 \\ 32.430 \\ +589.800 \\ \hline 712.757 \end{array}$$

95.
$$\begin{array}{r} 28.73 \\ -\ \ 3.12 \\ \hline 25.61 \end{array}$$

96.
$$\begin{array}{r} 46.88 \\ -13.45 \\ \hline 33.43 \end{array}$$

97.
$$\begin{array}{r} 43.50 \\ -28.17 \\ \hline 15.33 \end{array}$$

98.
$$\begin{array}{r} 345.10 \\ -\ \ 56.31 \\ \hline 288.79 \end{array}$$

99.
$$\begin{array}{r} 3.87 \\ 15.00 \\ +\ \ 2.90 \\ \hline 21.77 \end{array}$$

100.
$$\begin{array}{r} 8.20 \\ 1.09 \\ +12.00 \\ \hline 21.29 \end{array}$$

101.
$$\begin{array}{r} 32.560 \\ 47.356 \\ +\ \ 1.800 \\ \hline 81.716 \end{array}$$

102.
$$\begin{array}{r} 75.200 \\ 123.960 \\ +\ \ \ 3.897 \\ \hline 203.057 \end{array}$$

103.
$$\begin{array}{r} 18.000 \\ -\ \ 2.789 \\ \hline 15.211 \end{array}$$

104.
$$\begin{array}{r} 29.000 \\ -\ \ 8.582 \\ \hline 20.418 \end{array}$$

105.
$$\begin{array}{rr} 12.8 & 1\text{ decimal place} \\ \times\ 9.1 & 1\text{ decimal place} \\ \hline 128 & \downarrow \\ 1152 & 1+1=2 \\ \hline 116.48 & 2\text{ decimal places} \end{array}$$

106.
$$\begin{array}{rr} 34.04 & \text{2 decimal places} \\ \times\ 0.56 & \text{2 decimal places} \\ \hline 20424 & \downarrow \\ 17020 & 2+2=4 \\ \hline 19.0624 & \text{4 decimal places} \end{array}$$

107.
$$\begin{array}{rr} 22.41 & \text{2 decimal places} \\ \times\ 33 & \text{0 decimal places} \\ \hline 6723 & \downarrow \\ 6723 & 2+0=2 \\ \hline 739.53 & \text{2 decimal places} \end{array}$$

108.
$$\begin{array}{rr} 55.76 & \text{2 decimal places} \\ \times\ 72 & \text{0 decimal places} \\ \hline 11152 & \downarrow \\ 39032 & 2+0=2 \\ \hline 4014.72 & \text{2 decimal places} \end{array}$$

109.
$$\begin{array}{rr} 0.2 & \text{1 decimal place} \\ \times 0.03 & \text{2 decimal places} \\ \hline 6 & 1+2=3 \\ \hline 0.006 & \text{3 decimal places} \end{array}$$

110.
$$\begin{array}{rr} 0.07 & \text{2 decimal places} \\ \times 0.004 & \text{3 decimal places} \\ \hline 28 & 2+3=5 \\ \hline 0.00028 & \text{5 decimal places} \end{array}$$

111.
$$\begin{array}{r} 7.15 \\ 11\overline{)78.65} \\ \underline{77} \\ 16 \\ \underline{11} \\ 55 \\ \underline{55} \\ 0 \end{array}$$

112.
$$\begin{array}{r} 5.24 \\ 14\overline{)73.36} \\ \underline{70} \\ 33 \\ \underline{28} \\ 56 \\ \underline{56} \\ 0 \end{array}$$

113. To change the divisor 11.6 into a whole number, move each decimal point one place to the right. Move the decimal point straight up and divide as with whole numbers.

$$\begin{array}{r} 2.8 \\ 116\overline{)324.8} \\ \underline{232} \\ 928 \\ \underline{928} \\ 0 \end{array}$$

Therefore, $32.48 \div 11.6 = 2.8$.

114. To change the divisor 17.4 into a whole number, move each decimal point one place to the right. Move the decimal point straight up and divide as with whole numbers.

$$\begin{array}{r} 4.9 \\ 174\overline{)852.6} \\ \underline{696} \\ 1566 \\ \underline{1566} \\ 0 \end{array}$$

Therefore, $85.26 \div 17.4 = 4.9$.

115. To change the divisor 9.74 into a whole number, move each decimal point two places to the right. Move the decimal point straight up and divide as with whole numbers.

$$\begin{array}{r} 2.05 \\ 974\overline{)1996.70} \\ \underline{1948} \\ 4870 \\ \underline{4870} \\ 0 \end{array}$$

Therefore, $19.967 \div 9.74 = 2.05$.

116. To change the divisor 5.27 into a whole number, move each decimal point two places to the right. Move the decimal point straight up and divide as with whole numbers.

$$\begin{array}{r} 8.44 \\ 527\overline{)4447.88} \\ \underline{4216} \\ 2318 \\ \underline{2108} \\ 2108 \\ \underline{2108} \\ 0 \end{array}$$

Therefore, $44.4788 \div 5.27 = 8.44$.

117. Move the decimal point one place to the right.
$123.26 \times 10 = 1232.6$

118. Move the decimal point one place to the right.
$785.91 \times 10 = 7859.1$

119. Move the decimal point two places to the right.
$57.116 \times 100 = 5711.6$

120. Move the decimal point two places to the right.
$82.053 \times 100 = 8205.3$

121. Move the decimal point three places to the right.
$0.094 \times 1000 = 94$

122. Move the decimal point three places to the right.
$0.025 \times 1000 = 25$

123. Move the decimal point one place to the left.
$1.62 \div 10 = 0.162$

124. Move the decimal point one place to the left.
$8.04 \div 10 = 0.804$

125. Move the decimal point two places to the left.
$124.03 \div 100 = 1.2403$

126. Move the decimal point two places to the left.
$490.35 \div 100 = 4.9035$

127. Move the decimal point three places to the left.
$23.29 \div 1000 = 023.29 \div 1000 = 0.02329$

128. Move the decimal point three places to the left.
$59.8 \div 1000 = 059.8 \div 1000 = 0.0598$

129. Convert from a decimal to a percent.
$0.01 = 0.01 \cdot 100\% = 1\%$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{100}$	0.01	1%

130. Convert from a percent to a decimal.

$$2\% = \frac{2}{100} = 0.02$$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{50}$	0.02	2%

131. Convert from a percent to a fraction.

$$5\% = \frac{5}{100}$$

In lowest terms,

$$\frac{5}{100} = \frac{1 \cdot 5}{20 \cdot 5} = \frac{1}{20}$$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{20}$	0.05	5%

132. Convert to a decimal first. Divide 1 by 10. Move the decimal point one place to the left.
$1 \div 10 = 0.1$
Convert the decimal to a percent.
$0.1 = 0.1 \cdot 100\% = 10\%$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{10}$	0.1	10%

133. Convert the decimal to a percent.
$0.125 = 0.125 \cdot 100\% = 12.5\%$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{8}$	0.125	12.5%

134. Convert the percent to a decimal first.
$20\% = 0.20$, or 0.2
Convert from a percent to a fraction.

$$20\% = \frac{20}{100}$$

In lowest terms,

$$\frac{20}{100} = \frac{1 \cdot 20}{5 \cdot 20} = \frac{1}{5}$$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{5}$	0.2	20%

135. Convert to a decimal first. Divide 1 by 4. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.25 \\ 4\overline{)1.00} \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Convert the decimal to a percent.

$0.25 = 0.25 \cdot 100\% = 25\%$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{4}$	0.25	25%

136. Convert to a decimal first. Divide 1 by 3. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.33\ldots \\ 3\overline{)1.00\ldots} \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

Note that the pattern repeats. Therefore,

$\frac{1}{3} = 0.\overline{3}.$

Convert the decimal to a percent.

$0.33\overline{3} = 0.33\overline{3} \cdot 100\% = 33.\overline{3}\%,$ or $33\frac{1}{3}\%$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{3}$	$0.\overline{3}$	$33.\overline{3}\%$ or $33\frac{1}{3}\%$

137. Convert the percent to a decimal first.

$50\% = 0.50,$ or 0.5

Convert from a percent to a fraction.

$50\% = \frac{50}{100}$

In lowest terms,

$\frac{50}{100} = \frac{1 \cdot 50}{2 \cdot 50} = \frac{1}{2}$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{2}$	0.5	50%

138. Divide 2 by 3. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r} 0.66\ldots \\ 3\overline{)2.00\ldots} \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

Note that the pattern repeats. Therefore,

$\frac{2}{3} = 0.\overline{6}.$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{2}{3}$	$0.\overline{6}$	$66.\overline{6}\%$ or $66\frac{2}{3}\%$

139. Convert the decimal to a percent first.
$0.75 = 0.75 \cdot 100\% = 75\%$
Convert from a percent to a fraction.
$75\% = \frac{75}{100}$
In lowest terms,
$\frac{75}{100} = \frac{3 \cdot 25}{4 \cdot 25} = \frac{3}{4}$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{3}{4}$	0.75	75%

140. Convert the decimal to a percent.
$1.0 = 1.0 \cdot 100\% = 100\%$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
1	1.0	100%

141. Divide 21 by 5. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r}
4.2 \\
5\overline{)21.0} \\
\underline{20} \\
1\,0 \\
\underline{1\,0} \\
0
\end{array}$$

142. Divide 9 by 5. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r}
1.8 \\
5\overline{)9.0} \\
\underline{5} \\
4\,0 \\
\underline{4\,0} \\
0
\end{array}$$

143. Divide 9 by 4. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r}
2.25 \\
4\overline{)9.00} \\
\underline{8} \\
1\,0 \\
\underline{8} \\
20 \\
\underline{20} \\
0
\end{array}$$

144. Divide 15 by 4. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r}
3.75 \\
4\overline{)15.00} \\
\underline{12} \\
3\,0 \\
\underline{2\,8} \\
20 \\
\underline{20} \\
0
\end{array}$$

145. Divide 3 by 8. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r}
0.375 \\
8\overline{)3.000} \\
\underline{2\,4} \\
60 \\
\underline{56} \\
40 \\
\underline{40} \\
0
\end{array}$$

146. Divide 7 by 8. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r}
0.875 \\
8\overline{)7.000} \\
\underline{6\,4} \\
60 \\
\underline{56} \\
40 \\
\underline{40} \\
0
\end{array}$$

147. Divide 5 by 9. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r@{}l} & \;0.555... \\ 9\overline{)} & 5.000... \\ & \underline{45} \\ & \;\;50 \\ & \;\;\underline{45} \\ & \;\;\;\;50 \\ & \;\;\;\;\underline{45} \\ & \;\;\;\;\;\;5 \end{array}$$

Note that the pattern repeats. Therefore, $\frac{5}{9} = 0.\overline{5}$, or about 0.556.

148. Divide 8 by 9. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r@{}l} & \;0.888... \\ 9\overline{)} & 8.000... \\ & \underline{72} \\ & \;\;80 \\ & \;\;\underline{72} \\ & \;\;\;\;80 \\ & \;\;\;\;\underline{72} \\ & \;\;\;\;\;\;8 \end{array}$$

Note that the pattern repeats. Therefore, $\frac{8}{9} = 0.\overline{8}$, or about 0.889.

149. Divide 1 by 6. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r@{}l} & \;0.166... \\ 6\overline{)} & 1.000... \\ & \underline{\;\;6} \\ & \;\;40 \\ & \;\;\underline{36} \\ & \;\;\;\;40 \\ & \;\;\;\;\underline{36} \\ & \;\;\;\;\;\;4 \end{array}$$

Note that the pattern repeats. Therefore, $\frac{1}{6} = 0.1\overline{6}$, or about 0.167.

150. Divide 5 by 6. Add a decimal point and as many 0s as necessary.

$$\begin{array}{r@{}l} & \;0.833... \\ 6\overline{)} & 5.000... \\ & \underline{48} \\ & \;\;20 \\ & \;\;\underline{18} \\ & \;\;\;\;20 \\ & \;\;\;\;\underline{18} \\ & \;\;\;\;\;\;2 \end{array}$$

Note that the pattern repeats. Therefore, $\frac{5}{6} = 0.8\overline{3}$, or about 0.833.

151. $54\% = 0.54$

152. $39\% = 0.39$

153. $7\% = 07\% = 0.07$

154. $4\% = 04\% = 0.04$

155. $117\% = 1.17$

156. $189\% = 1.89$

157. $2.4\% = 02.4\% = 0.024$

158. $3.1\% = 03.1\% = 0.031$

159. $6\frac{1}{4}\% = 6.25\% = 06.25\% = 0.0625$

160. $5\frac{1}{2}\% = 5.5\% = 05.5\% = 0.055$

161. $0.79 = 79\%$

162. $0.83 = 83\%$

163. $0.02 = 2\%$

164. $0.08 = 8\%$

165. $0.004 = 0.4\%$

166. $0.005 = 0.5\%$

167. $1.28 = 128\%$

168. $2.35 = 235\%$

169. $0.40 = 40\%$

170. $0.6 = 0.60 = 60\%$

171. $51\% = \frac{51}{100}$

172. $47\% = \frac{47}{100}$

173. $15\% = \frac{15}{100}$

In lowest terms,

$$\frac{15}{100} = \frac{3 \cdot 5}{20 \cdot 5} = \frac{3}{20}$$

174. $35\% = \frac{35}{100}$

In lowest terms,

$$\frac{35}{100} = \frac{7 \cdot 5}{20 \cdot 5} = \frac{7}{20}$$

175. $2\% = \frac{2}{100}$

In lowest terms,

$$\frac{2}{100} = \frac{1 \cdot 2}{50 \cdot 2} = \frac{1}{50}$$

176. $8\% = \frac{8}{100}$

In lowest terms,

$$\frac{8}{100} = \frac{2 \cdot 4}{25 \cdot 4} = \frac{2}{25}$$

177. $140\% = \frac{140}{100}$

In lowest terms,

$$\frac{140}{100} = \frac{7 \cdot 20}{5 \cdot 20} = \frac{7}{5}, \text{ or } 1\frac{2}{5}$$

178. $180\% = \frac{180}{100}$

In lowest terms,

$$\frac{180}{100} = \frac{9 \cdot 20}{5 \cdot 20} = \frac{9}{5}, \text{ or } 1\frac{4}{5}$$

179. $7.5\% = \frac{7.5}{100} = \frac{7.5}{100} \cdot \frac{10}{10} = \frac{75}{1000}$

In lowest terms,

$$\frac{75}{1000} = \frac{3 \cdot 25}{40 \cdot 25} = \frac{3}{40}$$

180. $2.5\% = \frac{2.5}{100} = \frac{2.5}{100} \cdot \frac{10}{10} = \frac{25}{1000}$

In lowest terms,

$$\frac{25}{1000} = \frac{1 \cdot 25}{40 \cdot 25} = \frac{1}{40}$$

181. $\frac{4}{5} = \frac{4}{5} \cdot 100\% = \frac{4}{5} \cdot \frac{100}{1}\% = \frac{4 \cdot 5 \cdot 20}{5}\% = 80\%$

182.
$$\begin{aligned}\frac{3}{25} &= \frac{3}{25} \cdot 100\% \\ &= \frac{3}{25} \cdot \frac{100}{1}\% \\ &= \frac{3 \cdot 4 \cdot 25}{25}\% \\ &= 12\%\end{aligned}$$

183.
$$\begin{aligned}\frac{7}{50} &= \frac{7}{50} \cdot 100\% \\ &= \frac{7}{50} \cdot \frac{100}{1}\% \\ &= \frac{7 \cdot 2 \cdot 50}{50}\% \\ &= 14\%\end{aligned}$$

184.
$$\begin{aligned}\frac{9}{20} &= \frac{9}{20} \cdot 100\% \\ &= \frac{9}{20} \cdot \frac{100}{1}\% \\ &= \frac{9 \cdot 5 \cdot 20}{20}\% \\ &= 45\%\end{aligned}$$

185.
$$\begin{aligned}\frac{2}{11} &= \frac{2}{11} \cdot 100\% \\ &= \frac{2}{11} \cdot \frac{100}{1}\% \\ &= \frac{200}{11}\% \\ &= 18.\overline{18}\%\end{aligned}$$

186. $\frac{4}{9} = \frac{4}{9} \cdot 100\% = \frac{4}{9} \cdot \frac{100}{1}\% = \frac{400}{9}\% = 44.\overline{4}\%$

187. $\frac{9}{4} = \frac{9}{4} \cdot 100\% = \frac{9}{4} \cdot \frac{100}{1}\% = \frac{9 \cdot 4 \cdot 25}{4}\% = 225\%$

188. $\frac{8}{5} = \frac{8}{5} \cdot 100\% = \frac{8}{5} \cdot \frac{100}{1}\% = \frac{8 \cdot 5 \cdot 20}{5}\% = 160\%$

189.
$$\begin{aligned}\frac{13}{6} &= \frac{13}{6} \cdot 100\% \\ &= \frac{13}{6} \cdot \frac{100}{1}\% \\ &= \frac{13 \cdot 2 \cdot 50}{2 \cdot 3}\% \\ &= 216.\overline{6}\%\end{aligned}$$

190. $$\begin{aligned}\frac{31}{9}&=\frac{31}{9}\cdot 100\%\\&=\frac{31}{9}\cdot\frac{100}{1}\%\\&=\frac{3100}{9}\%\\&=344.\overline{4}\%\end{aligned}$$

R.2 Basic Concepts from Algebra

Classroom Examples, Now Try Exercises

1. $\{x \mid x$ is a natural number greater than $12\}$

$\{13, 14, 15, \ldots\}$ is the set of natural numbers greater than 12.

N1. $\{p \mid p$ is a natural number less than $6\}$

$\{1, 2, 3, 4, 5\}$ is the set of natural numbers less than 6.

2. One answer is $\{x \mid x$ is a whole number less than $6\}$.

N2. One answer is $\{x \mid x$ is a natural number between 8 and $13\}$.

3. (a) -7 is an integer, a rational number $\left(\text{since } -7=\frac{-7}{1}\right)$, and a real number.

(b) 3.14, a terminating decimal, is a rational number and a real number.

(c) $\sqrt{4}=2$, so it is a whole number, an integer, a rational number, and a real number.

N3. (a) The whole numbers in the given set are in the set $\{0, 5\}$.

(b) The rational numbers in the given set are in the set $\left\{-2.4,\ -\sqrt{1},\ -\frac{1}{2},\ 0,\ 0.\overline{3},\ 5\right\}$.

4. (a) The statement "Some integers are whole numbers" is *true* since 0 and the positive integers are whole numbers. The negative integers are not whole numbers.

(b) The statement "Every real number is irrational" is *false*. Some real numbers such as $\frac{4}{9}$, $0.\overline{3}$, and $\sqrt{16}$ are rational. Others such as $\sqrt{2}$ and π are irrational.

N4. (a) The statement "All integers are irrational numbers" is *false*. In fact, all integers are rational.

(b) The statement "Every whole number is an integer" is *true* since the integers include the whole numbers, the negatives of the whole numbers, and 0.

5. (a) $|3|=3$

(b) $|-3|=-(-3)=3$

(c) Evaluate the absolute value. Then find the additive inverse.
$-|3|=-(3)=-3$

(d) $-|-3|=-(3)=-3$

(e) Evaluate each absolute value, and then add.
$|8|+|-1|=8+1=9$

(f) $|8-1|=|7|=7$

N5. (a) $|-7|=-(-7)=7$

(b) Evaluate the absolute value. Then find the additive inverse.
$-|7|=-(7)=-7$

(c) Evaluate the absolute value. Then find the additive inverse.
$-|-7|=-(7)=-7$

(d) Evaluate each absolute value, and then subtract.
$|4|-|-4|=4-4=0$

(e) $|4-4|=|0|=0$

6. Physicians assistants: $|37.4|=37.4$

Locomotive firers: $|-78.6|=78.6$

Since $78.6>37.4$, locomotive firers will show the greater change.

N6. Home health aides: $|46.7|=46.7$

Parking enforcement workers: $|-35.3|=35.3$

Locomotive firers: $|-78.6|=78.6$

Since 35.3 is less than 46.7 and 78.6, parking enforcement workers will show the least change.

7. (a) $-8>-4$ is *false*, since -8 is to the left of -4 on a number line, -8 is less than, not greater than, -4.

(b) $-2<5$ is *true*, since -2 is to the left of 5 on a number line.

N7. **(a)** $-5 > -1$ is *false*, since -5 is to the left of -1 on a number line, -5 is less than, not greater than, -1.

(b) $-7 < -6$ is *true*, since -7 is to the left of -6 on a number line.

8. **(a)** $-2 \le -3$ is *false*, since -2 is to the right of -3 on a number line, it must be greater.

(b) $-1 \ge -9$ reads "-1 is greater than or equal to -9". This is *true*, since -1 is to the right of -9 on a number line.

(c) $8 \le 8$ is *true*. The statement is "8 is less than *or* equal to 8". Since 8 is equal to 8, the statement is true.

(d) Since $3(4) = 12$ and $2 \cdot 6 = 12$, the statement becomes $12 > 12$, which is *false*.

N8. **(a)** Since -6 is to the left of -5 on a number line, -6 is less than, not greater than -5. The statement is *false*.

(b) Since -10 is to the left of -2 on a number line, the statement is *true*.

(c) Since the statement is "0.5 is less than *or* equal to 0.5." and 0.5 is equal to 0.5, the statement is *true*.

(d) Since $10 \cdot 6 = 60$ and $8(7) = 56$, the statement becomes $60 > 56$, which is *true*.

Exercises

1. Yes, the choice of variables is not important. The sets have the same description so they represent the same set.

2. **(a)** The only integer between 6.75 and 7.75 is 7.

(b) There are an infinite number of rational numbers between $\frac{1}{4}$ and $\frac{3}{4}$; one example is $\frac{1}{2}$.

(c) The number 0 is the only whole number that is not a natural number.

(d) The opposite of any natural number is an integer that is not a whole number; one example is -1.

(e) There are an infinite number of irrational numbers between $\sqrt{4}$ and $\sqrt{9}$; one example is $\sqrt{5}$.

(f) One example is $-\pi$. Other answers are possible.

3. The set of natural numbers is $\{1, 2, 3, \ldots\}$, so the set of natural numbers less than 6 is $\{1, 2, 3, 4, 5\}$.

4. The set is $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

5. The set of integers is $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$, so the set of integers greater than 4 is $\{5, 6, 7, 8, \ldots\}$.

6. The set is $\{9, 10, 11, 12, \ldots\}$.

7. The set of integers is $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$, so the set of integers less than or equal to 4 is $\{\ldots, -1, 0, 1, 2, 3, 4\}$.

8. The set is $\{\ldots, -2, -1, 0, 1, 2\}$.

9. The set of even integers is $\{\ldots, -2, 0, 2, 4, \ldots\}$, so the set of even integers greater than 8 is $\{10, 12, 14, 16, \ldots\}$.

10. The set is $\{\ldots, -7, -5, -3, -1\}$.

11. This is the set of numbers that lie a distance of 4 units from 0 on the number line. Thus, the set of numbers whose absolute value is 4 is $\{-4, 4\}$.

12. The set is $\{-7, 7\}$.

13. $\{x \mid x$ is an irrational number that is also rational$\}$
Irrational numbers cannot also be rational numbers. The set of irrational numbers that are also rational is $\varnothing$.

14. The set is $\varnothing$, the empty set.

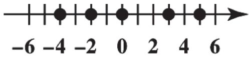
15. $\{2, 4, 6, 8\}$ can be described by $\{x \mid x$ is an even natural number less than or equal to $8\}$.

16. $\{11, 12, 13, 14\}$ can be described by $\{x \mid x$ is an integer between 10 and $15\}$.

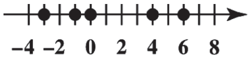
17. $\{4, 8, 12, 16, \ldots\}$ can be described by $\{x \mid x$ is a multiple of 4 greater than $0\}$.

18. $\{\ldots, -6, -3, 0, 3, 6, \ldots\}$ can be described by $\{x \mid x$ is an integer multiple of $3\}$.

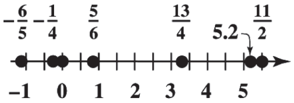
19. Place dots for $-4, -2, 0, 3,$ and 5 on a number line.



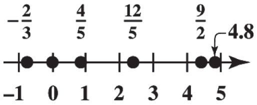
20. Place dots for $-3, -1, 0, 4,$ and 6 on a number line.



21. Place dots for $-\frac{6}{5} = -1.2, -\frac{1}{4} = -0.25, 0,$ $\frac{5}{6} = 0.8\overline{3}, \frac{13}{4} = 3.25, 5.2,$ and $\frac{11}{2} = 5.5,$ on a number line.



22. Place dots to indicate the points $-\frac{2}{3} = -0.\overline{6}, 0, \frac{4}{5} = 0.8, \frac{12}{5} = 2.4, \frac{9}{2} = 4.5,$ and 4.8 on a number line.



23. **(a)** The elements 8, 13, and $\frac{75}{5}$ (or 15) are natural numbers.

(b) The elements 0, 8, 13, and $\frac{75}{5}$ are whole numbers.

(c) The elements $-9,$ 0, 8, 13, and $\frac{75}{5}$ are integers.

(d) The elements $-9, -0.7, 0, \frac{6}{7}, 4.\overline{6},$ $8, \frac{21}{2}, 13,$ and $\frac{75}{5}$ are rational numbers.

(e) The elements $-\sqrt{6}$ and $\sqrt{7}$ are irrational numbers.

(f) All the elements are real numbers.

24. **(a)** The elements 5, 17, and $\frac{40}{2}$ (or 20) are natural numbers.

(b) The elements 0, 5, 17, and $\frac{40}{2}$ are whole numbers.

(c) The elements $-8,$ 0, 5, 17, and $\frac{40}{2}$ are integers.

(d) The elements $-8, -0.6, 0, \frac{3}{4}, 5, \frac{13}{2},$ 17, and $\frac{40}{2}$ are rational numbers.

(e) The elements $-\sqrt{5}, \sqrt{3},$ and π are irrational numbers.

(f) All the elements are real numbers.

25. The statement “Every integer is a whole number” is *false.* Some integers are whole numbers, but the negative integers are not whole numbers.

26. The statement “Every natural number is an integer” is *true.* The integers include the natural numbers, their negatives, and zero.

27. The statement “Every irrational number is an integer” is *false.* Irrational numbers have decimal representations that neither terminate nor repeat, so no irrational numbers are integers.

28. The statement “Every integer is a rational number” is *true.* Every integer q can be written as $\frac{q}{1}.$

29. The statement “Every natural number is a whole number” is *true.* Whole numbers consist of the natural numbers and zero.

30. The statement “Some rational numbers are irrational” is *false.* No rational numbers are irrational numbers.

31. The statement “Some rational numbers are whole numbers” is *true.* Every whole number is rational.

32. The statement “Some real numbers are integers” is *true.* Every integer is a real number.

33. The statement “The absolute value of any number is the same as the absolute value of its additive inverse” is *true.* The distance on a number line from 0 to a number is the same as the distance from 0 to its additive inverse.

34. The statement “The absolute value of any nonzero number is positive” is *true.* Every nonzero number is a positive distance from 0.

35. **(a)** $-(-4) = 4$ (Choice A)

(b) $|-4| = 4$ (Choice A)

(c) $-|-4| = -(4) = -4$ (Choice B)

(d) $-|-(-4)| = -|4| = -4$ (Choice B)

36. In general $|x| = a$ (with $a \neq 0$), is true for two values of a; namely, a and $-a.$ In this case, $|x| = 4$ is true for $x = 4$ and $x = -4.$

37. **(a)** The additive inverse of 6 is -6.

(b) $6 > 0$, so $|6| = 6$.

38. **(a)** The additive inverse of 9 is -9.

(b) $9 > 0$, so $|9| = 9$.

39. **(a)** The additive inverse of -12 is $-(-12) = 12$.

(b) $-12 < 0$, so $|-12| = -(-12) = 12$.

40. **(a)** The additive inverse of -14 is $-(-14) = 14$.

(b) $-14 < 0$, so $|-14| = -(-14) = 14$.

41. **(a)** The additive inverse of $\frac{6}{5}$ is $-\frac{6}{5}$.

(b) $\frac{6}{5} > 0$, so $\left|\frac{6}{5}\right| = \frac{6}{5}$.

42. **(a)** The additive inverse of 0.16 is -0.16.

(b) $0.16 > 0$, so $|0.16| = 0.16$.

43. Use the definition of absolute value. $-8 < 0$, so $|-8| = -(-8) = 8$.

44. $|-19| = -(-19) = 19$

45. $\frac{3}{2} > 0$, so $\left|\frac{3}{2}\right| = \frac{3}{2}$.

46. $\left|\frac{3}{4}\right| = \frac{3}{4}$

47. $-|5| = -(5) = -5$

48. $-|12| = -(12) = -12$

49. $-|-2| = -[-(-2)] = -(2) = -2$

50. $-|-6| = -[-(-6)] = -(6) = -6$

51. $-|4.5| = -(4.5) = -4.5$

52. $-|12.4| = -(12.4) = -12.4$

53. $|-2| + |3| = 2 + 3 = 5$

54. $|-16| + |14| = 16 + 14 = 30$

55. $-|10 - 9| = -(1) = -1$

56. $-|12 - 6| = -(6) = -6$

57. $|-9| - |-3| = 9 - 3 = 6$

58. $|-10| - |-7| = 10 - 7 = 3$

59.
$$\begin{aligned} |-1| + |-2| - |-3| &= 1 + 2 - 3 \\ &= 3 - 3 \\ &= 0 \end{aligned}$$

60. $|-7| + |-3| - |-10| = 7 + 3 - 10 = 0$

61. **(a)** The greatest absolute value is $|13.9| = 13.9$. Therefore, New Orleans had the greatest change in population. The population increased by 13.9%.

(b) The least absolute value is $|-3.0| = 3.0$. Therefore, Toledo had the least change in population. The population decreased by 3.0%.

62. **(a)** The greatest absolute value is $|-35,230| = 35,230$. For China, imports exceeded exports by $35,230 million.

(b) The least absolute value is $|230| = 230$. For New Zealand, exports exceeded imports by $230 million.

63. Compare the depths of the bodies of water. The deepest, that is, the body of water whose depth has the greatest absolute value, is the Pacific Ocean $(|-14,040| = 14,040)$ followed by the Indian Ocean $(|-12,800| = 12,800)$, the Caribbean Sea $(|-8448| = 8448)$, the South China Sea $(|-4802| = 4802)$, and the Gulf of California $(|-2375| = 2375)$.

64. Compare the heights of the mountains. The shortest, that is, the smallest value, is Point Success $(|14,164| = 14,164)$ followed by Rainier $(|14,410| = 14,410)$, Matlalcueyetl $(|14,636| = 14,636)$, Steele $(|16,624| = 16,624)$, and Denali $(|20,310| = 20,310)$.

65. *True*; the absolute value of the depth of the Pacific Ocean is $|-14,040| = 14,040$ which is greater than the absolute value of the depth of the Indian Ocean, $|-12,800| = 12,800$.

66. *False*; the absolute value of the depth of the Gulf of California is $|-2375| = 2375$ which is less than the absolute value of the depth of the Caribbean Sea, $|-8448| = 8448$.

67. *True*; since -6 is to the left of -1 on a number line, -6 is less than -1.

68. *True*; since -4 is to the left of -2 on a number line, -4 is less than -2.

69. *False*; since -4 is to the left of -3 on a number line, -4 is *less* than -3, not greater.

70. *False*; since -3 is to the left of -1 on a number line, -3 is *less* than -1, not greater.

71. *True*; since 3 is to the right of -2 on a number line, 3 is greater than -2.

72. *True*; since 6 is to the right of -3 on a number line, 6 is greater than -3.

73. *False*; since $-3 = -3, -3$ is *not* greater than -3.

74. *False*; since $-5 = -5, -5$ is *not* less than -5.

75. The inequality $6 > 2$ can also be written $2 < 6$. In each case, the inequality symbol points toward the smaller number so both inequalities are true.

76. $5 > 1$ is equivalent to $1 < 5$.

77. $-9 < 4$ is equivalent to $4 > -9$.

78. $-6 < 1$ is equivalent to $1 > -6$.

79. $-5 > -10$ is equivalent to $-10 < -5$.

80. $-7 > -12$ is equivalent to $-12 < -7$.

81. "7 is greater than -1" can be written as $7 > -1$.

82. "-4 is less than 10" can be written as $-4 < 10$.

83. "5 is greater than or equal to 5" can be written as $5 \geq 5$.

84. "-6 is less than or equal to -6" can be written as $-6 \leq -6$.

85. "$13-3$ is less than or equal to 10" can be written as $13-3 \leq 10$.

86. "$5+14$ is greater than or equal to 19" can be written as $5+14 \geq 19$.

87. "$5+0$ is not equal to 0" can be written as $5+0 \neq 0$.

88. "$6+7$ is not equal to -13" can be written as $6+7 \neq -13$.

89. $0 \leq -5$ False

90. $-11 \geq 0$ False

91. $7 \leq 7$ True

92. $10 \geq 10$ True

93. $-6 \overset{?}{<} 7+3$

$-6 < 10$ True

The last statement is true since -6 is to the left of 10 on a number line.

94. $-7 \overset{?}{<} 4+1$

$-7 < 5$ True

95. $-2 \cdot 5 \overset{?}{\geq} 4+6$

$10 \geq 10$ True

96. $-8+7 \overset{?}{\leq} 3 \cdot 5$

$15 \geq 15$ True

97. $-|-3| \overset{?}{\geq} -3$

$-3 \geq -3$ True

98. $-|-4| \overset{?}{\leq} -4$

$-4 \leq -4$ True

99. $-8 \overset{?}{>} -|-6|$

$-8 > -6$ False

100. $-10 \overset{?}{>} -|-4|$

$-10 > -4$ False

101. 2016: IA $= 13{,}608$, OH $= 9546$, PA $= 8212$

In 2016, Iowa (IA), Ohio (OH), and Pennsylvania (PA) had production greater than 6000 million eggs.

102. OH: $9546 < 9708$; FL: $2364 < 2463$

For Ohio (OH) and Florida (FL), egg production was less in 2016 than in 2015.

103. 2016: TX $= x = 5572$, OH $= y = 9546$

Since $5572 < 9546$, $x < y$ is true.

104. 2015: IA $= x = 12{,}471$, PA $= y = 7778$

Two inequalities that compare the production in these two states are $x > y$ and $y < x$.

R.3 Operations on Real Numbers

Classroom Examples, Now Try Exercises

1. **(a)** $-6+(-15)=-(6+15)=-21$

(b) $-1.1+(-1.2)=-(1.1+1.2)=-2.3$

(c)
$$\begin{aligned} -\frac{3}{4}+\left(-\frac{5}{8}\right) &= -\left(\frac{3}{4}+\frac{5}{8}\right) \\ &= -\left(\frac{6}{8}+\frac{5}{8}\right) \\ &= -\frac{11}{8} \end{aligned}$$

N1. **(a)** $-4+(-9)=-(4+9)=-13$

(b)
$$\begin{aligned} -7.25+(-3.57) &= -(7.25+3.57) \\ &= -10.82 \end{aligned}$$

(c)
$$\begin{aligned} -\frac{2}{5}+\left(-\frac{3}{10}\right) &= -\left(\frac{2}{5}+\frac{3}{10}\right) \\ &= -\left(\frac{4}{10}+\frac{3}{10}\right) \\ &= -\frac{7}{10} \end{aligned}$$

2. **(a)** Subtract the lesser absolute value from the greater absolute value $(7-3)$, and take the sign of the larger.
$3+(-7)=-(7-3)=-4$

(b) $-3+7=7-3=4$

(c) $-3.8+4.6=4.6-3.8=0.8$

(d)
$$\begin{aligned} -\frac{3}{8}+\frac{1}{4} &= -\frac{3}{8}+\frac{1\cdot 2}{4\cdot 2}=-\frac{3}{8}+\frac{2}{8} \\ &= -\left(\frac{3}{8}-\frac{2}{8}\right)=-\frac{1}{8} \end{aligned}$$

N2. **(a)** Subtract the lesser absolute value from the greater absolute value $(15-7)$, and take the sign of the larger.
$-15+7=-(15-7)=-8$

(b) $-5+12=12-5=7$

(c) $4.6+(-2.8)=4.6-2.8=1.8$

(d)
$$\begin{aligned} -\frac{5}{9}+\frac{2}{7} &= \frac{5\cdot 7}{9\cdot 7}+\frac{2\cdot 9}{7\cdot 9}=-\frac{35}{63}+\frac{18}{63} \\ &= -\left(\frac{35}{63}-\frac{18}{63}\right)=-\frac{17}{63} \end{aligned}$$

3. **(a)** $5-12=5+(-12)=-7$

(b) $12-(-5)=12+5=17$

(c)
$$\begin{aligned} -11.5-(-6.3) &= -11.5+6.3 \\ &= -(11.5-6.3) \\ &= -5.2 \end{aligned}$$

(d)
$$\begin{aligned} \frac{3}{4}-\left(-\frac{2}{3}\right) &= \frac{3}{4}+\frac{2}{3}=\frac{3\cdot 3}{4\cdot 3}+\frac{2\cdot 4}{3\cdot 4} \\ &= \frac{9}{12}+\frac{8}{12}=\frac{17}{12} \end{aligned}$$

N3. **(a)** $9-15=9+(-15)=-6$

(b) $-4-11=-4+(-11)=-15$

(c)
$$\begin{aligned} -5.67-(-2.34) &= -5.67+2.34 \\ &= -(5.67-2.34) \\ &= -3.33 \end{aligned}$$

(d)
$$\begin{aligned} \frac{4}{9}-\frac{3}{5} &= \frac{4}{9}-\frac{3}{5}=\frac{4\cdot 5}{9\cdot 5}+\frac{3\cdot 9}{5\cdot 9} \\ &= \frac{20}{45}-\frac{27}{45}=-\frac{7}{45} \end{aligned}$$

4. **(a)**
$$\begin{aligned} -6-(-2)-8-1 &= -6+2-8-1 \\ &= -4+(-8)-1 \\ &= -12-1 \\ &= -13 \end{aligned}$$

(b)
$$\begin{aligned} -3-[(-7)+15]-6 &= -3-8-6 \\ &= -11-6 \\ &= -17 \end{aligned}$$

N4.
$$\begin{aligned} -4-(-2-7)-12 &= -4-(-9)-12 \\ &= -4+9-12 \\ &= 5-12 \\ &= -7 \end{aligned}$$

5. $|-1-(-8)|=|-1+8|=|7|=7$

N5. $|10-(-7)|=|10+7|=|17|=17$

6. **(a)** $7(-2)=-14$
The numbers have different signs, so the product is negative.

(b) $-0.9(-15)=13.5$

(c) $-\frac{5}{8}(16)=-\frac{5\cdot 2\cdot 8}{8}=-10$

(d) $0(-3) = 0 \cdot (-3) = 0$

The multiplication property of 0 states that any product of any real number and 0 is 0.

N6. **(a)** $-3(-10) = 30$

The numbers have the same sign, so the product is positive.

(b) $0.7(-1.2) = -0.84$

The numbers have different signs, so the product is negative.

(c) $-\frac{8}{11}(33) = \frac{8 \cdot 3 \cdot 11}{11} = -24$

(d) $14(0) = 14 \cdot 0 = 0$

The multiplication property of 0 states that any product of any real number and 0 is 0.

7. **(a)** $\frac{-15}{-3} = -15\left(-\frac{1}{3}\right) = 5$

(b) $\frac{2.7}{-0.3} = -9$

(c) $\frac{10}{0}$ is undefined.

(d)

$$\frac{-\frac{3}{8}}{\frac{11}{16}} = -\frac{3}{8} \div \frac{11}{16} = -\frac{3}{8} \cdot \frac{16}{11}$$

$$= -\frac{3 \cdot 2 \cdot 8}{8 \cdot 11} = -\frac{6}{11}$$

(e)

$$\frac{3}{4} \div \left(-\frac{7}{16}\right) = \frac{3}{4} \cdot \left(-\frac{16}{7}\right) = -\frac{3 \cdot 4 \cdot 4}{4 \cdot 7}$$

$$= -\frac{3 \cdot 4}{7} = -\frac{12}{7}$$

N7. **(a)** $\frac{-10}{-5} = -10\left(-\frac{1}{5}\right) = 2$

(b) $\frac{-1.5}{0.3} = -5$

(c) $\frac{0}{-4} = 0$

(d)

$$\frac{-\frac{10}{3}}{\frac{3}{8}} = -\frac{10}{3} \div \frac{3}{8} = -\frac{10}{3} \cdot \frac{8}{3}$$

$$= -\frac{10 \cdot 8}{3 \cdot 3} = -\frac{80}{9}$$

(e)

$$-\frac{6}{5} \div \left(-\frac{3}{7}\right) = -\frac{6}{5} \cdot \left(-\frac{7}{3}\right) = \frac{2 \cdot 3 \cdot 7}{5 \cdot 3}$$

$$= \frac{2 \cdot 7}{5} = \frac{14}{5}$$

Exercises

1. The sum of two positive numbers is a positive number. For example, $18 + 6 = 24$.

2. The sum of two negative numbers is a negative number. For example, $-7 + (-21) = -28$.

3. The sum of a positive number and a negative number is negative if the negative number has the greater absolute value. For example, $-14 + 9 = -5$.

4. The sum of a positive number and a negative number is positive if the positive number has the greater absolute value. For example, $15 + (-2) = 13$.

5. The difference of two positive numbers is negative if the number with the greater absolute value is subtracted from the one with the lesser absolute value. For example, $5 - 12 = -7$.

6. The difference of two negative numbers is negative if the number with lesser absolute value is subtracted from the one with greater absolute value. For example, $-15 - (-3) = -12$.

7. The product of two numbers with the same sign is positive. For example, $(-2)(-8) = 16$ and $2 \cdot 8 = 16$.

8. The product of two numbers with different signs is negative. For example, $-5(15) = -75$.

9. The quotient formed by any nonzero number divided by 0 is undefined, and the quotient formed by 0 divided by any nonzero number is 0. For example, $\frac{-17}{0}$ is undefined and $\frac{0}{42}$ is equal to 0.

10. The sum of a positive number and a negative number is 0 if the numbers are additive inverses. For example, $4 + (-4) = 0$.

11. $-6 + (-13) = -(6 + 13) = -19$

12. $-8 + (-16) = -(8 + 16) = -24$

13. $-15 + 6 = -(15 - 6) = -9$

14. $-17 + 9 = -(17 - 9) = -8$

15. $13+(-4)=13-4=9$

16. $19+(-13)=19-13=6$

17. $-17+22=22-17=5$

18. $-12+16=16-12=4$

19. $-\frac{7}{3}+\frac{3}{4}=-\frac{28}{12}+\frac{9}{12}=-\frac{19}{12}$

20. $-\frac{5}{6}+\frac{4}{9}=-\frac{15}{18}+\frac{8}{18}=-\frac{7}{18}$

21. The difference between 4.5 and 2.8 is 1.7. The number with the greater absolute value, 4.5 is positive, so the answer is positive. Thus, $-2.8+4.5=1.7$.

22.
$$\begin{aligned}-3.8+6.2&=6.2-3.8\\&=2.4\end{aligned}$$

23. $4-9=4+(-9)=-5$

24. $3-7=3+(-7)=-4$

25. $-6-5=-6+(-5)=-(6+5)=-11$

26. $-8-17=-8+(-17)=-(8+17)=-25$

27. $8-(-13)=8+13=21$

28. $12-(-22)=12+22=34$

29. $-16-(-3)=-16+3=-13$

30. $-21-(-6)=-21+6=-15$

31.
$$\begin{aligned}-12.31-(-2.13)&=-12.31+2.13\\&=-(12.31-2.13)\\&=-10.18\end{aligned}$$

32.
$$\begin{aligned}-15.88-(-9.42)&=-15.88+9.42\\&=-(15.88-9.42)\\&=-6.46\end{aligned}$$

33. $\frac{9}{10}-\left(-\frac{4}{3}\right)=\frac{9}{10}+\frac{4}{3}=\frac{27}{30}+\frac{40}{30}=\frac{67}{30}$

34. $\frac{3}{14}-\left(-\frac{3}{4}\right)=\frac{3}{14}+\frac{3}{4}=\frac{6}{28}+\frac{21}{28}=\frac{27}{28}$

35. $|-8-6|=|-14|=-(-14)=14$

36. $|-7-15|=|-22|=-(-22)=22$

37. $-|-4+9|=-|5|=-5$

38. $-|-5+6|=-|1|=-1$

39. $-2-|-4|=-2-4=-2+(-4)=-6$

40. $16-|-13|=16-13=16+(-13)=3$

41. $-7+5-9=(-7+5)-9=-2-9=-11$

42. $-12+14-18=2-18=-16$

43. $6-(-2)-8=6+2-8=8-8=0$

44. $7-(-4)-11=7+4-11=11-11=0$

45.
$$\begin{aligned}8-(-12)-2-6&=8+12-2-6\\&=20-2-6\\&=18-6\\&=12\end{aligned}$$

46.
$$\begin{aligned}3-(-14)-6-4&=3+14-6-4\\&=17-6-4\\&=11-4\\&=7\end{aligned}$$

47.
$$\begin{aligned}-9-4-(-3)+6&=(-9-4)+3+6\\&=-13+3+6\\&=-10+6\\&=-4\end{aligned}$$

48.
$$\begin{aligned}-10-6-(-12)+9&=-10+(-6)+12+9\\&=-16+12+9\\&=-4+9\\&=5\end{aligned}$$

49.
$$\begin{aligned}-0.38+4-0.62&=3.62-0.62\\&=3\end{aligned}$$

50.
$$\begin{aligned}-2.95+8-0.05&=5.05-0.05\\&=5\end{aligned}$$

51.
$$\begin{aligned}\left(-\frac{5}{4}-\frac{2}{3}\right)+\frac{1}{6}&=\left(-\frac{15}{12}-\frac{8}{12}\right)+\left(\frac{2}{12}\right)\\&=-\frac{23}{12}+\frac{2}{12}\\&=-\frac{21}{12},\text{ or }-\frac{7}{4}\end{aligned}$$

52. $\left(-\frac{5}{9}-\frac{1}{6}\right)+\frac{1}{2}=\left(-\frac{10}{18}-\frac{3}{18}\right)+\frac{1}{2}$
$=-\frac{13}{18}+\frac{9}{18}$
$=-\frac{4}{18}$, or $-\frac{2}{9}$

53. $-\frac{3}{4}-\left(\frac{1}{2}-\frac{3}{5}\right)=-\frac{6}{8}-\left(\frac{4}{8}-\frac{3}{8}\right)$
$=-\frac{6}{8}-\frac{1}{8}$
$=-\frac{7}{8}$

54. $-\frac{1}{2}-\left(\frac{5}{6}-\frac{5}{12}\right)=-\frac{1}{2}-\left(\frac{10}{12}-\frac{5}{12}\right)$
$=-\frac{1}{2}-\left(\frac{5}{12}\right)$
$=-\frac{6}{12}-\frac{5}{12}$
$=-\frac{11}{12}$

55. $-4-[-3-(-6)]+9$
$=-4-[-3+6]+9$
$=-4-3+9$
$=-7+9$
$=2$

56. $-10-[-2-(-5)]+16$
$=-10-[-2+5]+16$
$=-10-3+16$
$=-13+16$
$=3$

57. $|-11|-|-5|-|7|+|-2|$
$=11-5-7+2$
$=6-7+2$
$=-1+2$
$=1$

58. $|-6|+|-3|-|4|-|-8|$
$=6+3-4-8$
$=9-4-8$
$=5-8$
$=-3$

59. $A=-4$ and $B=2$. The distance between A and B is the absolute value of their difference.
$|-4-2|=|-6|=6$

60. $A=-4$ and $C=5$. The distance between A and C is the absolute value of their difference.
$|-4-5|=|-9|=9$

61. $D=-6$ and $F=\frac{1}{2}$. The distance between D and F is the absolute value of their difference.
$\left|-6-\frac{1}{2}\right|=\left|-\frac{12}{2}-\frac{1}{2}\right|$
$=\left|-\frac{13}{2}\right|$
$=\frac{13}{2}$, or $6\frac{1}{2}$

62. $E=-\frac{3}{2}$ and $C=5$. The distance between E and C is the absolute value of their difference.
$\left|-\frac{3}{2}-5\right|=\left|-\frac{3}{2}-\frac{10}{2}\right|$
$=\left|-\frac{13}{2}\right|$
$=\frac{13}{2}$, or $6\frac{1}{2}$

63. It is true for multiplication (and division). It is false for addition and subtraction when the number to be subtracted has the lesser absolute value. A more precise statement is, "The product or quotient of two negative numbers is positive."

64. Because the reciprocal of a number is the quotient of 1 (a positive number) and the number, the reciprocal always has the same sign as the number.

65. The product of two numbers with the same sign is positive, so $-8(-5)=40$.

66. The product of two numbers with the same sign is positive, so $-20(-4)=80$.

67. The product of two numbers with different signs is negative, so $5(-7)=-35$.

68. The product of two numbers with different signs is negative, so $6(-9)=-54$.

69. $4(0)=4\cdot 0=0$
The multiplication property of 0 states that any product of any real number and 0 is 0.

70. $0(-8) = 0 \cdot -8 = 0$
The multiplication property of 0 states that any product of any real number and 0 is 0.

71. $\frac{1}{2}(0) = \frac{1}{2} \cdot 0 = 0$
The multiplication property of 0 states that any product of any real number and 0 is 0.

72. $0(-4.5) = 0 \cdot -4.5 = 0$
The multiplication property of 0 states that any product of any real number and 0 is 0.

73. The product of two numbers with different signs is negative, so $\frac{3}{4}(-16) = -\frac{3}{4} \cdot 4 \cdot 4 = -12.$

74. The product of two numbers with different signs is negative, so $\frac{3}{5}(-35) = -\frac{3}{5} \cdot 5 \cdot 7 = -21.$

75. The product of two numbers with the same sign is positive, so $-10\left(-\frac{1}{5}\right) = 2 \cdot 5 \cdot \left(\frac{1}{5}\right) = 2.$

76. The product of two numbers with the same sign is positive, so $-\frac{1}{2}(-18) = \frac{1}{2} \cdot 2 \cdot 9 = 9.$

77. The product of two numbers with the same sign is positive, so $-\frac{5}{2}\left(-\frac{12}{25}\right) = \frac{5 \cdot 2 \cdot 6}{2 \cdot 5 \cdot 5} = \frac{6}{5}.$

78. The product of two numbers with the same sign is positive, so $-\frac{9}{7}\left(-\frac{21}{36}\right) = \frac{9 \cdot 3 \cdot 7}{7 \cdot 4 \cdot 9} = \frac{3}{4}.$

79. The product of two numbers with the same sign is positive, so $-\frac{3}{8}\left(-\frac{24}{9}\right) = \frac{3 \cdot 3 \cdot 8}{8 \cdot 9} = 1.$

80. The product of two numbers with the same sign is positive, so $-\frac{2}{11}\left(-\frac{22}{4}\right) = \frac{2 \cdot 2 \cdot 11}{11 \cdot 2 \cdot 2} = 1.$

81. The product of two numbers with the same sign is positive, so $-0.8(-0.5) = 0.4.$

82. The product of two numbers with the same sign is positive, so $-0.5(-0.6) = 0.3.$

83. The product of two numbers with different signs is negative, so $-0.06(0.4) = -0.024.$

84. The product of two numbers with different signs is negative, so $-0.08(0.7) = -0.056.$

85. The quotient of two nonzero real numbers with different signs is negative, so
$\frac{-14}{2} = -14 \cdot \frac{1}{2} = -2 \cdot 7 \cdot \frac{1}{2} = -7.$

86. The quotient of two nonzero real numbers with different signs is negative, so
$\frac{-39}{13} = -39 \cdot \frac{1}{13} = -3 \cdot 13 \cdot \frac{1}{13} = -3.$

87. The quotient of two nonzero real numbers with the same sign is positive, so
$\frac{-24}{-4} = 24 \cdot \frac{1}{4} = 4 \cdot 6 \cdot \frac{1}{4} = 6.$

88. The quotient of two nonzero real numbers with the same sign is positive, so
$\frac{-45}{-9} = 45 \cdot \frac{1}{9} = 5 \cdot 9 \cdot \frac{1}{9} = 5.$

89. The quotient of two nonzero real numbers with different signs is negative, so
$\frac{100}{-25} = -100 \cdot \frac{1}{25} = -4 \cdot 25 \cdot \frac{1}{25} = -4.$

90. The quotient of two nonzero real numbers with different signs is negative, so
$\frac{150}{-30} = -150 \cdot \frac{1}{30} = -5 \cdot 30 \cdot \frac{1}{30} = -5.$

91. $\frac{0}{-8} = 0 \cdot \left(-\frac{1}{8}\right) = 0$

92. $\frac{0}{-14} = 0 \cdot \left(-\frac{1}{14}\right) = 0$

93. Division by 0 is undefined, so $\frac{5}{0}$ is undefined.

94. Division by 0 is undefined, so $\frac{13}{0}$ is undefined.

95. The quotient of two nonzero real numbers with the same sign is positive, so
$-\frac{10}{17} \div \left(-\frac{12}{5}\right) = \frac{10}{17} \cdot \frac{5}{12} = \frac{2 \cdot 5 \cdot 5}{17 \cdot 2 \cdot 6} = \frac{25}{102}.$

96. $-\frac{22}{23} \div \left(-\frac{33}{5}\right) = \frac{22}{23} \cdot \frac{5}{33} = \frac{2 \cdot 11 \cdot 5}{23 \cdot 3 \cdot 11} = \frac{10}{69}$

97. $\frac{\frac{12}{13}}{-\frac{4}{3}} = \frac{12}{13} \div \left(-\frac{4}{3}\right) = \frac{12}{13}\left(-\frac{3}{4}\right)$
$= -\frac{3 \cdot 4 \cdot 3}{13 \cdot 4} = -\frac{9}{13}$

98. $\frac{\frac{7}{6}}{-\frac{2}{3}} = \frac{7}{6} \div \left(-\frac{2}{3}\right) = \frac{7}{6}\left(-\frac{3}{2}\right)$

$= -\frac{7 \cdot 3}{2 \cdot 3 \cdot 2} = -\frac{7}{4}$

99. $\frac{-7.2}{0.8} = -9$

100. $\frac{-4.5}{0.9} = -5$

101. $\frac{-1.28}{-0.4} = 3.2$

102. $\frac{-1.82}{-0.2} = 9.1$

103. $\frac{1}{6} - \left(-\frac{7}{9}\right) = \frac{1}{6} + \frac{7}{9} = \frac{3}{18} + \frac{14}{18} = \frac{17}{18}$

104. $\frac{7}{10} - \left(-\frac{1}{6}\right) = \frac{7}{10} + \frac{1}{6} = \frac{21}{30} + \frac{5}{30} = \frac{26}{30} = \frac{13}{15}$

105. $-\frac{1}{9} + \frac{7}{12} = -\frac{4}{36} + \frac{21}{36} = \frac{17}{36}$

106. $-\frac{1}{12} + \frac{13}{16} = -\frac{4}{48} + \frac{39}{48} = \frac{35}{48}$

107. $-\frac{3}{8} - \frac{5}{12} = -\frac{9}{24} - \frac{10}{24} = -\frac{19}{24}$

108. $-\frac{11}{15} - \frac{4}{9} = -\frac{33}{45} - \frac{20}{45} = -\frac{53}{45}$

109. $-\frac{7}{30} + \frac{2}{45} - \frac{3}{10} = -\frac{21}{90} + \frac{4}{90} - \frac{27}{90} = -\frac{44}{90} = -\frac{22}{45}$

110. $-\frac{8}{15} + \frac{7}{6} - \frac{3}{20} = -\frac{32}{60} + \frac{70}{60} - \frac{9}{60} = \frac{29}{60}$

111. $\frac{8}{25}\left(-\frac{5}{12}\right) = -\frac{2 \cdot 4 \cdot 5}{5 \cdot 5 \cdot 3 \cdot 4} = -\frac{2}{5 \cdot 3} = -\frac{2}{15}$

112. $\frac{9}{20}\left(-\frac{7}{15}\right) = -\frac{3 \cdot 3 \cdot 7}{4 \cdot 5 \cdot 3 \cdot 5} = -\frac{3 \cdot 7}{4 \cdot 5 \cdot 5} = -\frac{21}{100}$

113. $\frac{5}{6}\left(-\frac{9}{10}\right)\left(-\frac{4}{5}\right) = \frac{5 \cdot 3 \cdot 3 \cdot 2 \cdot 2}{2 \cdot 3 \cdot 2 \cdot 5 \cdot 5} = \frac{3}{5}$

114. $\frac{4}{3}\left(-\frac{9}{20}\right)\left(-\frac{5}{12}\right) = \frac{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}{3 \cdot 2 \cdot 2 \cdot 5 \cdot 4 \cdot 3} = \frac{1}{4}$

115. $\frac{8}{3} \div \left(-\frac{14}{15}\right) = \frac{8}{3} \cdot \left(-\frac{15}{14}\right)$

$= -\frac{2 \cdot 4 \cdot 3 \cdot 5}{3 \cdot 2 \cdot 7}$

$= -\frac{4 \cdot 5}{7}$

$= -\frac{20}{7}$

116. $\frac{12}{5} \div \left(-\frac{18}{25}\right) = \frac{12}{5} \cdot \left(-\frac{25}{18}\right)$

$= -\frac{2 \cdot 2 \cdot 3 \cdot 5 \cdot 5}{5 \cdot 2 \cdot 3 \cdot 3}$

$= -\frac{2 \cdot 5}{3}$

$= -\frac{10}{3}$, or $-3\frac{1}{3}$

117. $\frac{\frac{2}{3}}{-2} = \frac{2}{3} \div \frac{-2}{1} = \frac{2}{3} \cdot \frac{1}{-2} = -\frac{2 \cdot 1}{3 \cdot 2} = -\frac{2}{6} = -\frac{1}{3}$

118. $\frac{\frac{3}{4}}{-6} = \frac{3}{4} \div \frac{-6}{1} = \frac{3}{4} \cdot \frac{1}{-6} = -\frac{3 \cdot 1}{4 \cdot 6} = -\frac{3}{24} = -\frac{1}{8}$

119. $\frac{-\frac{8}{9}}{\frac{2}{3}} = -\frac{8}{9} \div \frac{2}{3} = -\frac{8}{9} \cdot \frac{3}{2} = -\frac{8 \cdot 3}{9 \cdot 2} = -\frac{24}{18} = -\frac{4}{3}$

120. $\frac{-\frac{15}{16}}{\frac{3}{8}} = -\frac{15}{16} \div \frac{3}{8} = -\frac{15}{16} \cdot \frac{8}{3}$

$= -\frac{15 \cdot 8}{16 \cdot 3} = -\frac{120}{48} = -\frac{5}{2}$

121. $-8.6 - 23.751 = -(8.6 + 23.751) = -32.351$

122. $-37.8 - 13.582 = -(37.8 + 13.582) = -51.382$

123. $-2.5(0.8)(1.5) = (-2)(1.5) = -3$

124. $-1.6(0.5)(2.5) = (-0.8)(2.5)$

$= -2$

125. $-24.84 \div 6 = -4.14$

126. $-32.84 \div 8 = -4.105$

127. $-2496 \div (-0.52) = 4800$

128. $-1875 \div (-0.25) = 7500$

129. $$\frac{-100}{-0.01} = \frac{100}{\frac{1}{100}} = 100 \div \frac{1}{100} = 100 \cdot 100$$
$$= 10,000$$

130. $$\frac{-60}{-0.06} = \frac{60}{\frac{6}{100}} = 60 \div \frac{6}{100} = 60 \cdot \frac{100}{6}$$
$$= 10 \cdot 100 = 1000$$

131. $-14.2 + 9.81 = -4.39$

132. $-89.41 + 21.325 = -68.085$

133. To find the difference between these two temperatures, subtract the lowest temperature from the highest temperature.
$$100° - (-80°) = 100° + 80°$$
$$= 180°$$
The difference is 180°F.

134. $$120° - (-29°) = 120° + 29°$$
$$= 149°$$
The difference is 149°F.

135. $$48.35 - 35.99 - 20.00 - 28.50 + 66.27$$
$$= 12.36 - 20.00 - 28.50 + 66.27$$
$$= -7.64 - 28.50 + 66.27$$
$$= -36.14 + 66.27$$
$$= 30.13$$
His balance is \$30.13.

136. $$37.60 - 25.99 - 19.34 - 25.00 + 58.66$$
$$= 11.61 - 19.34 - 25.00 + 58.66$$
$$= -7.73 - 25.00 + 58.66$$
$$= -32.73 + 58.66$$
$$= 25.93$$
Her balance is \$25.93.

137. $$-382.45 + 25.10 + 34.50 - 45.00 - 98.17$$
$$= -466.02$$
His balance is $-\$466.02$.

(a) To pay off the balance, his payment should be \$466.02.

(b) $-466.02 + 300 - 24.66 = -190.68$
His balance is $-\$190.68$.

138. $-237.59 + 47.25 - 12.39 - 20.00 = -222.73$
Her balance is $-\$222.73$.

(a) To pay off her balance, her payment should be \$222.73.

(b) $-222.73 + 75.00 - 32.06 = -179.79$
Her balance is $-\$179.79$.

139. **(a)** $4.93 + 16.27 + 8.41 + (-28.46) + 30.18$
$= 31.33$
The sum for 2014–2018 was 31.33%.

(b) $-28.46 - (8.41) = -36.87$
The difference between the percents from 2008 and 2007 was -36.87%.

(c) $30.18 - (-28.46) = 58.64$
The difference between the percents from 2009 and 2008 was 58.64%.

140. **(a)** $-142 - 225 - 185 + 77 = -475$
The total is $-\$475$ thousand, which represents a loss of \$475 thousand.

(b) $77 - (-185) = 77 + 185$
$= 262$
The difference was \$262 thousand dollars.

(c) $-225 - (-142) = -225 + 142$
$= -83$
The difference was $-\$83$ thousand dollars.

141.

Year	**Difference (in billions)**
2001	$\$1991 - \$1863 = \$128$
2006	$\$2407 - \$2655 = -\$248$
2011	$\$2303 - \$3603 = -\$1300$
2016	$\$3268 - \$3853 = -\$585$

142. There was a surplus in the year 2001. There were defecits in the years 2006, 2011, and 2016. A positive difference between receipts and outlays indicates a surplus, while a negative difference indicates a deficit.

R.4 Exponents, Roots, and Order of Operations

Classroom Examples, Now Try Exercises

1. **(a)** $\frac{2}{7} \cdot \frac{2}{7} \cdot \frac{2}{7} \cdot \frac{2}{7} = \left(\frac{2}{7}\right)^4$

(b) $(-10)(-10)(-10) = (-10)^3$

(c) $y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y = y^8$

N1. **(a)** $(-3)(-3)(-3) = (-3)^3$

(b) $(0.5)(0.5) = (0.5)^2$

(c) $t \cdot t \cdot t \cdot t \cdot t = t^5$

2. **(a)** $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

(b) $(0.5)^2 = (0.5)(0.5) = 0.25$

(c) $(-4)^3 = (-4)(-4)(-4) = -64$

(d) $(-4)^4 = (-4)(-4)(-4)(-4) = 256$

N2. **(a)** $5^3 = 5 \cdot 5 \cdot 5 = 125$

(b) $\left(\frac{2}{5}\right)^4 = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} = \frac{16}{625}$

(c) $(-3)^2 = (-3)(-3) = 9$

(d) $(-3)^3 = (-3)(-3)(-3) = -27$

3. **(a)**
$$\begin{aligned} 3^4 &= 3 \cdot 3 \cdot 3 \cdot 3 \\ &= 9 \cdot 3 \cdot 3 \\ &= 27 \cdot 3 = 81 \end{aligned}$$

(b) $(-3)^4 = (-3)(-3)(-3)(-3) = 81$

(c) $-3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$

N3. **(a)** $7^2 = 7 \cdot 7 = 49$

(b) $(-7)^2 = (-7)(-7) = 49$

(c) $-7^2 = -(7 \cdot 7) = -49$

4. **(a)** $\sqrt{49} = 7$ because 7 is positive and $7^2 = 49$.

(b) $\sqrt{\frac{121}{81}} = \frac{11}{9}$ because $\left(\frac{11}{9}\right)^2 = \frac{121}{81}$, so $-\sqrt{\frac{121}{81}} = -\frac{11}{9}$.

(c) $-\sqrt{49} = -7$ because the negative sign is outside the radical symbol.

(d) $\sqrt{-49}$ is not a real number.

N4. **(a)** $\sqrt{121} = 11$ because $11^2 = 121$.

(b) $\sqrt{\frac{100}{9}} = \frac{10}{3}$ because $\left(\frac{10}{3}\right)^2 = \frac{100}{9}$.

(c) $-\sqrt{121} = -11$ because the negative sign is outside the radical symbol.

(d) $\sqrt{-121}$ is not a real number.

5. **(a)**
$$\begin{aligned} 5 \cdot 9 + 2 \cdot 4 &= 45 + 8 && \text{Multiply.} \\ &= 53 && \text{Add.} \end{aligned}$$

(b)
$$\begin{aligned} 4 - 12 \div 4 \cdot 2 &= 4 - 3 \cdot 2 && \text{Divide.} \\ &= 4 - 6 && \text{Multiply.} \\ &= -2 && \text{Subtract.} \end{aligned}$$

N5.
$$\begin{aligned} 15 - 3 \cdot 4 + 2 &= 15 - 12 + 2 && \text{Multiply.} \\ &= 3 + 2 && \text{Subtract.} \\ &= 5 && \text{Add.} \end{aligned}$$

6. **(a)** Add and subtract inside parentheses.

$(4+2) - 4^2 - (8-3)$

$= 6 - 4^2 - 5$

Evaluate the power.

$= 6 - 16 - 5$

$= -10 - 5$ Subtract from left to right.

$= -15$

(b) $6 + \frac{5}{4}(-12) - \frac{2}{3} \cdot 18$

$= 6 + \frac{5}{4} \cdot \frac{-12}{1} - \frac{2}{3} \cdot \frac{18}{1}$

$= 6 - \frac{60}{4} - \frac{36}{3}$ Multiply.

Subtract left from right.

$= 6 - 15 - 12$

$= -21$

N6. **(a)** Work inside the absolute value bars.

$-5^2 + 10 \div 5 - |3 - 7|$

Subtract inside the absolute value bars.

$$\begin{aligned} &= -5^2 + 10 \div 5 - |-4| \\ &= -5^2 + 10 \div 5 - 4 && \text{Take absolute value.} \\ &= -25 + 10 \div 5 - 4 && \text{Evaluate the power.} \\ &= -25 + 2 - 4 && \text{Divide.} \\ &= -23 - 4 && \text{Add.} \\ &= -27 && \text{Subtract.} \end{aligned}$$

(b) $6+\frac{2}{3}(-9)-\frac{5}{8}\cdot 16$

$$\begin{aligned}&=6+\frac{2}{3}\cdot\frac{-9}{1}-\frac{5}{8}\cdot\frac{16}{1}\\&=6-\frac{18}{3}-\frac{80}{8}&&\text{Multiply}\\&=6-6-10&&\text{Subtract from left}\\&=-10&&\text{to right.}\end{aligned}$$

7. $\dfrac{\frac{1}{2}\cdot 10-6+\sqrt{9}}{\frac{5}{6}\cdot 12-3^2-1}$

$$\begin{aligned}&=\frac{\frac{1}{2}\cdot 10-6+3}{\frac{5}{6}\cdot 12-9-1}&&\text{Evaluate powers and roots.}\\&=\frac{5-6+3}{10-9-1}&&\text{Multiply.}\\&=\frac{-1+3}{1-1}&&\text{Add and subtract.}\\&=\frac{2}{0}&&\text{Undefined.}\end{aligned}$$

N7. $\dfrac{\sqrt{36}-4\cdot 3^2}{-2^2-8\cdot 3+28}$

$$\begin{aligned}&=\frac{6-4\cdot 9}{-4-8\cdot 3+28}&&\text{Evaluate powers and roots.}\\&=\frac{6-36}{-4-24+28}&&\text{Multiply.}\\&=\frac{-30}{-28+28}&&\text{Add and subtract.}\\&=\frac{-30}{0}&&\text{Undefined.}\end{aligned}$$

8. (a) $\dfrac{5x+z\sqrt{y}}{x-1}=\dfrac{5(-12)+(-3)\sqrt{64}}{-12-1}$

$$\begin{aligned}&=\frac{5(-12)-3(8)}{-12-1}\\&=\frac{-60-24}{-12-1}\\&=\frac{-84}{-13}=\frac{84}{13}\end{aligned}$$

(b) $w^2+2z^3=4^2+2(-3)^3$

$$\begin{aligned}&=16+2(-27)\\&=16+(-54)\\&=-38\end{aligned}$$

N8. (a) $3y-2x=3(7)-2(-4)$

$$\begin{aligned}&=21-(-8)\\&=21+8\\&=29\end{aligned}$$

(b) $\dfrac{x^2-\sqrt{z}}{-3xy}=\dfrac{(-4)^2-\sqrt{36}}{-3(-4)(7)}$

$$\begin{aligned}&=\frac{16-6}{-3(-4)(7)}\\&=\frac{16-6}{12(7)}\\&=\frac{16-6}{84}\\&=\frac{10}{84}=\frac{5}{42}\end{aligned}$$

Exercises

1. $-7^6=-(7\cdot 7\cdot 7\cdot 7\cdot 7\cdot 7)=-117{,}649,$
$(-7)^6=(-7)(-7)(-7)(-7)(-7)(-7)$
$=117{,}649$
Thus, the original statement, $-7^6=(-7)^6$, is *false*. Note that the statement $-7^6=-(7^6)$ is *true*.

2. $-5^7=(-5)^7$ is *true*. $(-5)^7=-5^7$ since $(-5)^7$ has an odd number of negative factors.

3. The statement "$\sqrt{25}$ is a positive number" is *true*. The symbol $\sqrt{}$ always gives a positive square root provided the radicand is positive.

4. $3+5\cdot 8=3+(5\cdot 8)$ is *true*. The order of operations says that multiplication should be done before addition.

5. The statement "$(-6)^7$ is a negative number" is *true*. $(-6)^7$ gives an odd number of negative factors, so the product is negative.

6. The statement "$(-6)^8$ is a positive number" is *true*. $(-6)^8$ gives an even number of negative factors, so the product is positive.

7. The statement "The product of 10 positive factors and 10 negative factors is positive" is *true*. The product of an even number of negative factors is positive.

8. The statement "The product of 5 positive factors and 5 negative factors is positive" is *false.* The product of 5 positive factors is positive and the product of 5 negative factors is negative, so the product of these two products is negative.

9. The statement "In the exponential expression -2^5, the base is -2" is *false.* The base is 2, not -2. If the problem were written $(-2)^5$, then -2 would be the base.

10. The statement "$\sqrt{a}$ is positive for all positive numbers *a*" is *true.*

11. $10 \cdot 10 \cdot 10 \cdot 10 = 10^4$

12. $8 \cdot 8 \cdot 8 = 8^3$

13. $\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \left(\frac{3}{4}\right)^5$

14. $\frac{1}{2} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^2$

15. $(-9)(-9)(-9) = (-9)^3$

16. $(-4)(-4)(-4)(-4) = (-4)^4$

17. $(0.8)(0.8) = (0.8)^2$

18. $(0.1)(0.1)(0.1)(0.1)(0.1)(0.1) = (0.1)^6$

19. $z \cdot z \cdot z \cdot z \cdot z \cdot z \cdot z = z^7$

20. $a \cdot a \cdot a \cdot a \cdot a = a^5$

21. **(a)** $8^2 = 64$

(b) $-8^2 = -(8 \cdot 8) = -64$

(c) $(-8)^2 = (-8)(-8) = 64$

(d) $-(-8)^2 = -(64) = -64$

22. **(a)** $4^3 = 64$

(b) $-4^3 = -(4 \cdot 4 \cdot 4) = -64$

(c) $(-4)^3 = (-4)(-4)(-4) = -64$

(d) $-(-4)^3 = -(-64) = 64$

23. $4^2 = 4 \cdot 4 = 16$

24. $6^2 = 6 \cdot 6 = 36$

25. $(0.3)^3 = (0.3)(0.3)(0.3) = 0.027$

26. $(0.1)^3 = (0.1)(0.1)(0.1) = 0.001$

27. $\left(\frac{1}{5}\right)^3 = \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{125}$

28.
$$\left(\frac{1}{6}\right)^4 = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{6 \cdot 6 \cdot 6 \cdot 6} = \frac{1}{1296}$$

29.
$$\left(\frac{4}{5}\right)^4 = \left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right) = \frac{4 \cdot 4 \cdot 4 \cdot 4}{5 \cdot 5 \cdot 5 \cdot 5} = \frac{256}{625}$$

30. $\left(\frac{7}{10}\right)^3 = \left(\frac{7}{10}\right)\left(\frac{7}{10}\right)\left(\frac{7}{10}\right) = \frac{343}{1000}$

31. $(-5)^3 = (-5)(-5)(-5) = -125$

32. $(-3)^5 = (-3)(-3)(-3)(-3)(-3) = -243$

33.
$$(-2)^8 = (-2)(-2)(-2)(-2)(-2)(-2)(-2)(-2) = 256$$

34. $(-3)^6 = (-3)(-3)(-3)(-3)(-3)(-3) = 729$

35. $-3^6 = -(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) = -729$

36. $-4^6 = (4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4) = -4096$

37. $-8^4 = -(8 \cdot 8 \cdot 8 \cdot 8) = -4096$

38. $-10^3 = -(10 \cdot 10 \cdot 10) = -1000$

39. **(a)** -7^2 is negative because the negative sign is not part of the base.

(b) $(-7)^2$ is positive because the negative sign is part of the base, and the base is squared, so there are an even number of factors.

(c) -7^3 is negative because the negative sign is not part of the base.

(d) $(-7)^3$ is negative because the negative sign is part of the base, and the base is cubed, so there are an odd number of factors.

(e) -7^4 is negative because the negative sign is not part of the base.

(f) $(-7)^4$ is positive because the negative sign is part of the base, and the base is raised to the fourth power, so there are an even number of factors.

40. **(a)** $\sqrt{144}=12,$ which is choice B.

(b) $\sqrt{-144}$ is not a real number, which is choice C.

(c) $-\sqrt{144}=-12,$ which is choice A.

41. $\sqrt{81}=9$ since 9 is positive and $9^2=81$.

42. $\sqrt{64}=8$ since 8 is positive and $8^2=64$.

43. $\sqrt{169}=13$ since 13 is positive and $13^2=169$.

44. $\sqrt{225}=15$ since 15 is positive and $15^2=225$.

45. $-\sqrt{400}=-\left(\sqrt{400}\right)=-(20)=-20$

46. $-\sqrt{900}=-\left(\sqrt{900}\right)=-(30)=-30$

47. $\sqrt{\dfrac{100}{121}}=\dfrac{10}{11}$ since $\dfrac{10}{11}$ is positive and $\left(\dfrac{10}{11}\right)^2=\dfrac{100}{121}.$

48. $\sqrt{\dfrac{225}{169}}=\dfrac{15}{13}$ since $\dfrac{15}{13}$ is positive and $\left(\dfrac{15}{13}\right)^2=\dfrac{225}{169}.$

49. $-\sqrt{0.49}=-\sqrt{(0.7)^2}=-(0.7)=-0.7$

50. $-\sqrt{0.64}=-\sqrt{(0.8)^2}=-(0.8)=-0.8$

51. There is no real number whose square is negative, so $\sqrt{-36}$ is not a real number.

52. There is no real number whose square is negative, so $\sqrt{-121}$ is not a real number.

53. If a is a positive number, then $-a$ is a negative number. Therefore, $\sqrt{-a}$ is not a real number, and $-\sqrt{-a}$ *is not a real number.*

54. If a is a positive number, then $\sqrt{a}$ is also a positive number, and $-\sqrt{a}$ is a *negative* number.

55. **(a)** Since $9+15\div3=9+5=14,$ the grandson's answer was correct.

(b) The reasoning was incorrect. The operation of division must be done first, and then the addition follows. The grandson's "Order of Process rule" is not correct. It just happens coincidentally in this problem that he obtained the correct answer the wrong way.

56.
$$\begin{aligned}7+7\div7+7\times7-7&=7+1+49-7\\&=8+49-7\\&=57-7\\&=50\end{aligned}$$

57.
$$\begin{aligned}12+3\cdot4&=12+12 &&\text{Multiply.}\\&=24 &&\text{Add.}\end{aligned}$$

58.
$$\begin{aligned}15+5\cdot2&=15+10 &&\text{Multiply.}\\&=25 &&\text{Add.}\end{aligned}$$

59.
$$\begin{aligned}6\cdot3-12\div4&=18-12\div4 &&\text{Multiply.}\\&=18-3 &&\text{Divide.}\\&=15 &&\text{Subtract.}\end{aligned}$$

60.
$$\begin{aligned}9\cdot4-8\div2&=36-8\div2 &&\text{Multiply.}\\&=36-4 &&\text{Divide.}\\&=32 &&\text{Subtract.}\end{aligned}$$

61.
$$\begin{aligned}10+30\div2\cdot3&=10+15\cdot3 &&\text{Divide.}\\&=10+45 &&\text{Multiply.}\\&=55 &&\text{Add.}\end{aligned}$$

62.
$$\begin{aligned}12+24\div3\cdot2&=12+8\cdot2 &&\text{Divide.}\\&=12+16 &&\text{Multiply.}\\&=28 &&\text{Add.}\end{aligned}$$

63.
$$\begin{aligned}&-3(5)^2-(-2)(-8)\\&=-3(25)-(-2)(-8) &&\text{Evaluate power.}\\&=-75-16 &&\text{Multiply.}\\&=-91 &&\text{Subtract.}\end{aligned}$$

64.
$$\begin{aligned}&-9(2)^2-(-3)(-2)\\&=-9(4)-(-3)(-2) &&\text{Evaluate power.}\\&=-36-6 &&\text{Multiply.}\\&=-42 &&\text{Subtract.}\end{aligned}$$

65.
$$\begin{aligned}&5-7\cdot3-(-2)^3\\&=5-7\cdot3-(-8) &&\text{Evaluate power.}\\&=5-21+8 &&\text{Multiply, change sign.}\\&=-16+8 &&\text{Subtract.}\\&=-8 &&\text{Add.}\end{aligned}$$

66. $-4-3\cdot 5-(-3)^3$

$=-4-3\cdot 5-(-27)$ Evaluate power.

$=-4-15+27$ Multiply.

$=-19+27$ Subtract.

$=8$ Add.

67. $-7\left(\sqrt{36}\right)-(-2)(-3)$

$=-7(6)-(-2)(-3)$ Evaluate root.

$=-42-6$ Multiply.

$=-48$ Subtract.

68. $-8\left(\sqrt{64}\right)-(-3)(-7)$

$=-8(8)-(-3)(-7)$ Evaluate root.

$=-64-21$ Multiply.

$=-85$ Subtract.

69. Simplify within absolute value bars.

$6|4-5|-24\div 3$

$=6|-1|-24\div 3$

$=6(1)-24\div 3$ Take absolute value.

$=6-8$ Multiply and divide.

$=-2$ Subtract.

70. Simplify within absolute value bars.

$-4|2-4|+8\cdot 2$

$=-4|-2|+8\cdot 2$

$=-4(2)+8\cdot 2$ Take absolute value.

$=-8+16$ Multiply.

$=8$ Add.

71. Simplify within absolute value bars.

$|-6-5|(-8)-3^2$

$=|-11|(-8)-3^2$

$=11(-8)-3^2$ Take absolute value.

$=11(-8)-9$ Evaluate power.

$=-88-9$ Multiply.

$=-97$ Subtract.

72. Simplify within absolute value bars.

$|-2-3|(-9)-4^2$

$=|-5|(-9)-4^2$

$=5(-9)-4^2$ Take absolute value.

$=5(-9)-16$ Evaluate power.

$=-45-16$ Multiply.

$=-61$ Subtract.

73. Simplify within parentheses.

$18-4^2+5-(3-7)$

$=18-4^2+5-(-4)$

$=18-4^2+5+4$ Simplify parentheses.

$=18-16+5+4$ Evaluate power.

$=2+5+4$ Subtract.

$=7+4$ Add.

$=11$ Add.

74. Simplify within parentheses.

$10-2^2+9-(1-8)$

$=10-2^2+9-(-7)$

$=10-2^2+9+7$ Simplify parentheses.

$=10-4+9+7$ Evaluate power.

$=6+9+7$ Subtract.

$=15+7$ Add.

$=22$ Add.

75. $6+\frac{2}{3}(-9)-\frac{5}{8}\cdot 16=6+(-6)-10$ Multiply.

$=0-10$ Add.

$=-10$ Subtract.

76. $7-\frac{3}{4}(-8)+12\cdot\frac{5}{6}=7+6+10$ Multiply.

$=13+10$ Add.

$=23$ Add.

77. $-14\left(-\frac{2}{7}\right)\div(2\cdot 6-10)$

$=4\div(12-10)$ Multiply.

$=4\div 2$ Work inside parentheses.

$=2$ Divide.

78. $-12\left(-\frac{3}{4}\right)-(6\cdot 5\div 3)$

$=9-(30\div 3)$ Multiply.

$=9-10$ Work inside parentheses.

$=-1$ Subtract.

79. Evaluate root and power in numerator, and subtract in the denominator.

$$\begin{aligned} &\frac{(-5+\sqrt{4})-2^2}{-5-2}\\ &=\frac{(-5+2)-4}{-7}\\ &=\frac{(-3)-4}{-7} && \text{Work inside parentheses.}\\ &=\frac{-7}{-7} && \text{Subtract.}\\ &=1 && \text{Divide.}\end{aligned}$$

80. Evaluate root and power in numerator, and subtract in the denominator.

$$\begin{aligned} &\frac{(-9+\sqrt{16})-3^2}{-6-1}\\ &=\frac{(-9+4)-9}{-7}\\ &=\frac{(-5)-9}{-7} && \text{Work inside parentheses.}\\ &=\frac{-14}{-7} && \text{Subtract.}\\ &=2 && \text{Divide.}\end{aligned}$$

81. $\dfrac{2(-5)+(-3)(-2)}{-8+3^2-1}$

$$\begin{aligned} &=\frac{-10+6}{-8+9-1} && \text{Evaluate power, multiply.}\\ &=\frac{-4}{1-1} && \text{Add.}\\ &=\frac{-4}{0} && \text{Subtract.}\end{aligned}$$

Since division by 0 is undefined, the given expression is *undefined.*

82. $\dfrac{3(-4)+(-5)(-8)}{2^3-2-6}$

$$\begin{aligned} &=\frac{-12+40}{8-2-6} && \text{Evaluate power, multiply.}\\ &=\frac{28}{6-6} && \text{Add, Subtract.}\\ &=\frac{28}{0} && \text{Subtract.}\end{aligned}$$

Since division by 0 is undefined, the given expression is *undefined.*

83.
$$\begin{aligned} 3a+\sqrt{b}&=3(-3)+\sqrt{64}\\ &=3(-3)+8\\ &=-9+8\\ &=-1\end{aligned}$$

84.
$$\begin{aligned} -2a-\sqrt{b}&=-2(-3)-\sqrt{64}\\ &=6-8\\ &=-2\end{aligned}$$

85.
$$\begin{aligned} \sqrt{b}+c-a&=\sqrt{64}+6-(-3)\\ &=8+6+3\\ &=14+3=17\end{aligned}$$

86.
$$\begin{aligned} \sqrt{b}-c+a&=\sqrt{64}-6+(-3)\\ &=8-6+(-3)\\ &=2+(-3)=-1\end{aligned}$$

87.
$$\begin{aligned} 4a^3+2c&=4(-3)^3+2(6)\\ &=4(-27)+12\\ &=-108+12=-96\end{aligned}$$

88.
$$\begin{aligned} -3a^4-3c&=-3(-3)^4-3(6)\\ &=-3(81)-18\\ &=-243-18=-261\end{aligned}$$

89.
$$\begin{aligned} 2(a-c)^2-ac&=2(-3-6)^2-(-3)(6)\\ &=2(-9)^2-(-18)\\ &=2(81)+18\\ &=162+18\\ &=180\end{aligned}$$

90.
$$\begin{aligned} -4ac+(c-a)^2&=-4(-3)(6)+(6-(-3))^2\\ &=-4(-3)(6)+(6+3)^2\\ &=-4(-3)(6)+(9)^2\\ &=-4(-3)(6)+81\\ &=72+81\\ &=153\end{aligned}$$

91.
$$\begin{aligned} \frac{\sqrt{b}-4a}{c^2}&=\frac{\sqrt{64}-4(-3)}{(6)^2}\\ &=\frac{8-4(-3)}{36}\\ &=\frac{8+12}{36}\\ &=\frac{20}{36}\\ &=\frac{5}{9}\end{aligned}$$

92. $$\begin{aligned}\frac{a^3+2c}{-\sqrt{b}} &= \frac{(-3)^3+2(6)}{-\sqrt{64}}\\ &= \frac{-27+2(6)}{-8}\\ &= \frac{-27+12}{-8}\\ &= \frac{-15}{-8}\\ &= \frac{15}{8}\end{aligned}$$

93. $$\begin{aligned}\frac{2c+a^3}{4b+6a} &= \frac{2(6)+(-3)^3}{4(64)+6(-3)}\\ &= \frac{12+(-27)}{256+(-18)}\\ &= \frac{-15}{238} = -\frac{15}{238}\end{aligned}$$

94. $$\begin{aligned}\frac{3c+a^2}{2b-6a} &= \frac{3(6)+(-3)^2}{2(64)-6(6)}\\ &= \frac{18+9}{128-36}\\ &= \frac{27}{92}\end{aligned}$$

95. $$\begin{aligned}wy-8x &= 4\left(\frac{1}{2}\right)-8\left(-\frac{3}{4}\right)\\ &= 2+6\\ &= 8\end{aligned}$$

96. $$\begin{aligned}wz-12y &= 4(1.25)-12\left(\frac{1}{2}\right)\\ &= 5-6\\ &= -1\end{aligned}$$

97. $$\begin{aligned}xy+y^4 &= -\frac{3}{4}\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^4\\ &= -\frac{3}{8}+\frac{1}{16}\\ &= -\frac{6}{16}+\frac{1}{16} = -\frac{5}{16}\end{aligned}$$

98. $$\begin{aligned}xy-x^2 &= -\frac{3}{4}\left(\frac{1}{2}\right)-\left(-\frac{3}{4}\right)^2\\ &= -\frac{3}{8}-\frac{9}{16}\\ &= -\frac{6}{16}-\frac{9}{16} = -\frac{15}{16}\end{aligned}$$

99. $$\begin{aligned}&-w+2x+3y+z\\ &= -4+2\left(-\frac{3}{4}\right)+3\left(\frac{1}{2}\right)+1.25\\ &= -4-\frac{3}{2}+\frac{3}{2}+1.25\\ &= -4+1.25\\ &= -2.75\end{aligned}$$

100. $$\begin{aligned}&w-6x+5y-3z\\ &= 4-6\left(-\frac{3}{4}\right)+5\left(\frac{1}{2}\right)-3(1.25)\\ &= 4+4.5+2.5-3.75\\ &= 7.25\end{aligned}$$

101. $$\begin{aligned}&(v\times 0.5485-4850)\div 1000\times 31.44\\ &= (150{,}000\times 0.5485-4850)\div 1000\times 31.44\\ &= (82{,}275-4850)\div 1000\times 31.44\\ &= 77{,}425\div 1000\times 31.44\\ &= 77.425\times 31.44\\ &= 2434.242\approx 2434\end{aligned}$$

The owner would pay $2434 in property taxes.

102. $$\begin{aligned}&(v\times 0.5485-4850)\div 1000\times 31.44\\ &= (200{,}000\times 0.5485-4850)\div 1000\times 31.44\\ &= (109{,}700-4850)\div 1000\times 31.44\\ &= 104{,}850\div 1000\times 31.44\\ &= 104.85\times 31.44\\ &= 3296.484\approx 3296\end{aligned}$$

The owner would pay $3296 in property taxes.

103. **(a)** $$\begin{aligned}&\text{number of oz}\times\%\text{ alcohol}\times 0.075\div\text{body}\\ &\text{weight in lb}-\text{hr of drinking}\times 0.015\\ &= 48\times 3.2\times 0.075\div 190-2\times 0.015\end{aligned}$$

(b) $$\begin{aligned}&48\times 3.2\times 0.075\div 190-2\times 0.015\\ &= 153.6\times 0.075\div 190-2\times 0.015\\ &= 11.52\div 190-2\times 0.015\\ &\approx 0.061-2\times 0.015\\ &= 0.061-0.03\\ &= 0.031\end{aligned}$$

104. number of oz × % alcohol × 0.075 ÷ body weight in lb − hr of drinking × 0.015

$= 36 \times 4.0 \times 0.075 \div 135 - 3 \times 0.015$

$= 144 \times 0.075 \div 135 - 3 \times 0.015$

$= 10.8 \div 135 - 3 \times 0.015$

$= 0.08 - 3 \times 0.015$

$= 0.08 - 0.015$

$= 0.035$

105. **(a)** BAC for the man

$= 48 \times 3.2 \times 0.075 \div (190 + 25) - 2 \times 0.015$

≈ 0.024

BAC for the woman

$= 36 \times 4.0 \times 0.075 \div (135 + 25) - 3 \times 0.015$

≈ 0.023

Increased weight results in lower BACs.

(b) Decreased weight will result in higher BACs.

BAC for the man

$= 48 \times 3.2 \times 0.075 \div (190 - 25) - 2 \times 0.015$

≈ 0.040

BAC for the woman

$= 36 \times 4.0 \times 0.075 \div (135 - 25) - 3 \times 0.015$

≈ 0.053

106. BAC for the man

$= 48 \times 3.2 \times 0.075 \div 190 - 1 \times 0.015$

≈ 0.046

BAC for the woman

$= 36 \times 4.0 \times 0.075 \div 135 - 2 \times 0.015$

$= 0.05$

Decreased time results in higher BACs.

107. $2.493x - 4962$

(a) $2.493(2003) - 4962 \approx \$31.5$ billion

(b) $2.493(2010) - 4962 \approx \$48.9$ billion

(c) $2.493(2016) - 4962 \approx \$63.9$ billion

(d) The amount spent on pets roughly doubled from 2003 to 2016.

108. **(a)**

Year	Average Price (in dollars)
2000	$0.2050(2000) - 404.6 = 5.40$
2005	6.43
2010	$0.2050(2010) - 404.6 = 7.45$
2016	$0.2050(2000) - 404.6 = 8.68$

(b) The average price of a movie ticket in the United States increased by $\$8.68 - \$5.40 =$ $3.28 from 2000 to 2016.

R.5 Properties of Real Numbers

Classroom Examples, Now Try Exercises

1. **(a)** $-4(p-5) = -4[p + (-5)]$

$= -4(p) + (-4)(-5)$

$= -4p + 20$

(b) Use the second form of the distributive property.

$-6m + 2m = (-6 + 2)m$

$= -4m$

(c) Since there is no common number or variable here, the distributive property cannot be used.

(d) $5(4p - 2q + r) = 5(4p) + 5(-2q) + 5(r)$

$= 20p - 10q + 5r$

N1. **(a)** $-2(3x - y) = -2[3x + (-y)]$

$= -2(3x) + (-2)(-y)$

$= -6x + 2y$

(b) Use the second form of the distributive property.

$4k - 12k = [4 + (-12)]k$

$= -8k$

(c) Since there is no common number or variable here, the distributive property cannot be used.

2. **(a)** $x - 3x = 1x - 3x$ Identity property

$= (1-3)x$ Distributive property

$= -2x$

(b) $-(3 + 4p)$

$= -1(3 + 4p)$ Identity property

$= -1(3) + (-1)(4p)$ Distributive property

$= -3 + (-4p)$

$= -3 - 4p$

N2. **(a)** $7x + x = 7x - 1x$ Identity property

$= (7+1)x$ Distributive property

$= 8x$

(b)
$$\begin{aligned}&-(5p-3q)\\&=-1(5p-3q) && \text{Identity prop.}\\&=-1(5p)+(-1)(-3q) && \text{Distributive prop.}\\&=-5p+3q\end{aligned}$$

3. (See Example 3 for more detailed steps.)
$$\begin{aligned}&12b-9+4b-7b+1\\&=12b+4b-7b-9+1\\&=(12+4-7)b-9+1\\&=9b-8\end{aligned}$$

N3. (See Example 3 for more detailed steps.)
$$\begin{aligned}&-7x+10-3x-4+x\\&=-7x-3x+1x+10-4\\&=(-7-3+1)x+10-4\\&=-9x+6\end{aligned}$$

4. (a)
$$\begin{aligned}&12b-9b+5b-7b\\&=(12-9+5-7)b\\&=1b\\&=b\end{aligned}$$

(b)
$$\begin{aligned}&6-(2x+7)-3\\&=6-2x-7-3 && \text{Distributive property}\\&=-2x+6-7-3 && \text{Commutative prop.}\\&=-2x-4 && \text{Combine like terms.}\end{aligned}$$

(c)
$$\begin{aligned}&4m(2n)\\&=[4m(2)]n && \text{Associative property}\\&=[4(m\cdot 2)]n && \text{Associative property}\\&=[4(2m)]n && \text{Commutative property}\\&=[(4\cdot 2)m]n && \text{Associative property}\\&=(8m)n && \text{Multiply.}\\&=(8mn) && \text{Associative property}\\&=8mn\end{aligned}$$

(d)
$$\begin{aligned}&2(x+3)-5(2x-1)\\&=2x+6-10x+5 && \text{Distributive property}\\&=2x-10x+6+5 && \text{Commutative prop.}\\&=-8x+11 && \text{Combine like terms.}\end{aligned}$$

N4. (a)
$$\begin{aligned}&-3(t-4)-t+15\\&=-3t+12-t+15 && \text{Distributive prop.}\\&=-3t-t+12+15 && \text{Commutative prop.}\\&=-4t+27 && \text{Combine terms.}\end{aligned}$$

(b)
$$\begin{aligned}&7x-(4x-2)\\&=7x-1(4x-2) && \text{Identity property}\\&=7x-4x+2 && \text{Distributive property}\\&=3x+2 && \text{Subtract.}\end{aligned}$$

(c)
$$\begin{aligned}&5x(6y)\\&=[5x(6)]y && \text{Associative property}\\&=[5(x\cdot 6)]y && \text{Associative property}\\&=[5(6x)]y && \text{Commutative property}\\&=[(5\cdot 6)x]y && \text{Associative property}\\&=(30x)y && \text{Multiply.}\\&=30(xy) && \text{Associative property}\\&=30xy\end{aligned}$$

(d)
$$\begin{aligned}&3(5x-7)-8(x+4)\\&=15x-21-8x-32 && \text{Distributive property}\\&=15x-8x-21-32 && \text{Commutative prop.}\\&=7x-53 && \text{Combine like terms.}\end{aligned}$$

Exercises

1. The identity element for addition is 0 since, for any real number a, $a+0=0+a=a$. Choice B is correct.

2. The identity element for multiplication is 1 since, for any real number a, $a\cdot 1=1\cdot a=a$. Choice C is correct.

3. The additive inverse of a is $-a$ since, for any real number a, $a+(-a)=0$ and $-a+a=0$. Choice A is correct.

4. The multiplicative inverse of a is $\frac{1}{a}$ since, for any nonzero real number a, $a\cdot\frac{1}{a}=1$ and $\frac{1}{a}\cdot a=1$. Choice D is correct.

5. The distributive property provides a way to rewrite a product such as $a(b+c)$ as the sum $\underline{ab+ac}$.

6. The commutative property is used to change the order of two terms or factors.

7. The associative property is used to change the grouping of three terms or factors.

8. Like terms are terms with the same variables raised to the same powers.

9. When simplifying an expression, only like terms can be combined.

10. The numerical coefficient in the term $-7yz^2$ is -7.

11. Using the distributive property, $2(m+p)=2m+2p$.

12. $3(a+b)=3a+3b$

13. Using the distributive property,

$$\begin{aligned}-12(x-y)&=-12[x+(-y)]\\&=-12(x)+(-12)(-y)\\&=-12x+12y.\end{aligned}$$

14.

$$\begin{aligned}-10(p-q)&=-10[p+(-q)]\\&=-10p+10q\end{aligned}$$

15. Use the second form of the distributive property.

$$\begin{aligned}5k+3k&=(5+3)k\\&=8k\end{aligned}$$

16.

$$\begin{aligned}6a+5a&=(6+5)a\\&=11a\end{aligned}$$

17.

$$\begin{aligned}7r-9r&=7r+(-9r)\\&=[7+(-9)]r\\&=-2r\end{aligned}$$

18.

$$\begin{aligned}4n-6n&=4n+(-6n)\\&=[4+(-6)]n\\&=-2n\end{aligned}$$

19. Since there is no common variable factor here, we cannot use the distributive property to simplify the expression.

20. Since there is no common variable factor here, we cannot use the distributive property to simplify the expression.

21. Use the identity property, then the distributive property.

$$\begin{aligned}a+7a&=1a+7a\\&=(1+7)a\\&=8a\end{aligned}$$

22.

$$\begin{aligned}s+9s&=1\cdot s+9s \quad \text{Identity property}\\&=(1+9)s\\&=10s\end{aligned}$$

23.

$$\begin{aligned}x+x&=1x+1x\\&=(1+1)x\\&=2x\end{aligned}$$

24.

$$\begin{aligned}a+a&=1a+1a\\&=(1+1)a\\&=2a\end{aligned}$$

25.

$$\begin{aligned}-(2d-f)&=-1(2d-f)\\&=-1(2d)+(-1)(-f)\\&=-2d+f\end{aligned}$$

26.

$$\begin{aligned}-(3m-n)&=-1(3m-n)\\&=-1(3m)+(-1)(-n)\\&=-3m+n\end{aligned}$$

27.

$$\begin{aligned}-(-x-y)&=-1(-x-y)\\&=-1(-x)+(-1)(-y)\\&=x+y\end{aligned}$$

28.

$$\begin{aligned}-(-3x-4y)&=-1(-3x-4y)\\&=-1(-3x)+(-1)(-4y)\\&=3x+4y\end{aligned}$$

29.

$$\begin{aligned}2(x-3y+2z)&=2x+2(-3y)+2(2z)\\&=2x-6y+4z\end{aligned}$$

30.

$$\begin{aligned}8(3x+y-5z)&=8(3x)+8y+8(-5z)\\&=24x+8y-40z\end{aligned}$$

31.

$$\begin{aligned}&-12y+4y+3y+2y\\&=(-12+4+3+2)y\\&=-3y\end{aligned}$$

32.

$$\begin{aligned}&-5r-9r+8r-5r\\&=(-5-9+8-5)r\\&=-11r\end{aligned}$$

33.

$$\begin{aligned}&-6p+5-4p+6+11p\\&=-6p-4p+11p+5+6\\&=(-6-4+11)p+11\\&=1p+11 \text{ or } p+11\end{aligned}$$

34.

$$\begin{aligned}&-8x-12+3x-5x+9\\&=-8x+3x-5x-12+9\\&=(-8+3-5)x-3\\&=-10x-3\end{aligned}$$

35. $3(k+2)-5k+6+3$
$=3k+6-5k+6+3$
$=3k-5k+6+6+3$
$=(3-5)k+6+6+3$
$=-2k+15$

36. $5(r-3)+6r-2r+4$
$=5r-15+6r-2r+4$
$=5r+6r-2r-15+4$
$=(5+6-2)r-11$
$=9r-11$

37. $10-(4y+8)$
$=10-1(4y+8)$
$=10-4y-8$
$=-4y+2$

38. $6-(9y+5)$
$=6-1(9y+5)$
$=6-9y-5$
$=-9y+1$

39. $10x(3)(y)=[10x(3)]y$
$=[30x]y$
$=30xy$

40. $8x(6)(y)=[8x(6)]y$
$=[48x]y$
$=48xy$

41. $-\frac{2}{3}(12w)(7z)=\left[-\frac{2}{3}(12w)\right](7z)$
$=[-8w](7z)$
$=-56wz$

42. $-\frac{5}{6}(18w)(5z)=\left[-\frac{5}{6}(18w)\right](5z)$
$=[-15w](5z)$
$=-75wz$

43. $3(m-4)-2(m+1)$
$=3m-12-2m-2$
$=3m-2m-12-2$
$=m-14$

44. $6(a-5)-4(a+6)$
$=6a-30-4a-24$
$=6a-4a-30-24$
$=2a-54$

45. $0.25(8+4p)-0.5(6+2p)$
$=0.25(8)+0.25(4p)+(-0.5)(6)+(-0.5)(2p)$
$=2+p-3-p$
$=p-p+2-3$
$=(1-1)p+2-3$
$=0p-1$
$=-1$

46. $0.4(10-5x)-0.8(5+10x)$
$=0.4(10)+0.4(-5x)+(-0.8)(5)+(-0.8)(10x)$
$=4-2x-4-8x$
$=-10x$

47. $-(2p+5)+3(2p+4)-2p$
$=-2p-5+6p+12-2p$
$=(-2+6-2)p+(-5+12)$
$=2p+7$

48. $-(7m-12)+2(4m+7)-6m$
$=-1(7m-12)+2(4m+7)-6m$
$=-7m+12+8m+14-6m$
$=-5m+26$

49. $2+3(2z-5)-3(4z+6)-8$
$=2+6z-15-12z-18-8$
$=6z-12z+2-15-18-8$
$=(6-12)z-13-18-8$
$=-6z-31-8$
$=-6z-39$

50. $-4+4(4k-3)-6(2k+8)+7$
$=-4+16k-12-12k-48+7$
$=16k-12k-4-12-48+7$
$=4k-57$

51. $5x+8x=(5+8)x=13x$ Distributive prop.

52. $9y-6y=(9-6)y=3y$ Distributive property

53. $5(9r)=(5\cdot 9)r=45r$ Associative prop.

54. $-4+(12+8)=(-4+12)+8$ Assoc. prop.
$=8+8=16$

55. $5x+9y=9y+5x$ Commutative prop.

56. $-5 \cdot 7 = 7 \cdot (-5) = -35$ Commutative prop.

57. $1 \cdot 7 = 7$ Identity property

58. $-12x + 0 = -12x$ Identity property

59. $-\frac{1}{4}ty + \frac{1}{4}ty = 0$ Inverse prop.
A number plus its opposite equals 0.

60. $-\frac{9}{8}\left(-\frac{8}{9}\right) = 1$ Inverse prop.

61. $8(-4 + x) = 8(-4) + 8x$ Distributive prop.
$= -32 + 8x$

62. $3(x - y + z) = 3x - 3y + 3z$ Distributive prop.

63. Use the multiplication property of 0.
$0(0.875x + 9y) = 0$
Zero times any quantity equals 0.

64. Use the multiplication property of 0.
$0(0.35t + 12u) = 0$

65. Answers will vary. One example of commutativity is washing your face and brushing your teeth. The activities can be carried out in either order. An example of non-commutativity is putting on your socks and putting on your shoes.

66. Answers will vary. One example is waking up and going to sleep. Another example is tying and untying your shoes. These activities are inverses.

67.
$$\begin{aligned} 96 \cdot 19 + 4 \cdot 19 &= (96 + 4)19 \\ &= (100)19 \\ &= 1900 \end{aligned}$$

68.
$$\begin{aligned} 27 \cdot 60 + 27 \cdot 40 &= 27(60 + 40) \\ &= 27(100) \\ &= 2700 \end{aligned}$$

69.
$$\begin{aligned} 58 \cdot \frac{3}{2} - 8 \cdot \frac{3}{2} &= (58 - 8)\frac{3}{2} \\ &= (50)\frac{3}{2} = \frac{50}{1} \cdot \frac{3}{2} \\ &= \frac{150}{2} = 75 \end{aligned}$$

70.
$$\begin{aligned} \frac{8}{5}(17) + \frac{8}{5}(13) &= \frac{8}{5}(17 + 13) \\ &= \frac{8}{5}(30) = 8 \cdot \frac{30}{5} \\ &= 8 \cdot 6 = 48 \end{aligned}$$

71.
$$\begin{aligned} 8.75(15) - 8.75(5) &= 8.75(15 - 5) \\ &= 8.75(10) \\ &= 87.5 \end{aligned}$$

72.
$$\begin{aligned} 4.31(69) + 4.31(31) &= 4.31(69 + 31) \\ &= 4.31(100) \\ &= 431 \end{aligned}$$

73. The terms have been grouped using the associative property of addition.

74. The terms have been regrouped using the associative property of addition.

75. The order of the terms inside the parentheses has been changed using the commutative property of addition.

76. The terms have been regrouped using the associative property of addition.

77. The common factor, x, has been factored out using the distributive property.

78. The numbers in parentheses have been added using arithmetic facts to simplify the expression.

Chapter R Test

1.
$$\begin{aligned} \frac{3}{4} + \frac{1}{6} &= \frac{3}{4} \cdot \frac{3}{3} + \frac{1}{6} \cdot \frac{2}{2} \\ &= \frac{9}{12} + \frac{2}{12} \\ &= \frac{9 + 2}{12} \\ &= \frac{11}{12} \end{aligned}$$

2.
$$\begin{aligned} \frac{3}{7} \div \frac{9}{14} &= \frac{3}{7} \cdot \frac{14}{9} \\ &= \frac{3 \cdot 14}{7 \cdot 9} \\ &= \frac{3 \cdot 2 \cdot 7}{7 \cdot 3 \cdot 3} \\ &= \frac{2}{3} \end{aligned}$$

3.
$$\begin{array}{r} 13.250 \\ -\ \ 6.417 \\ \hline 6.833 \end{array}$$

4.
$$\begin{array}{rr} 0.7 & 1 \text{ decimal place} \\ \times 0.04 & 2 \text{ decimal places} \\ \hline 28 & 1+2=3 \\ \hline 0.028 & 3 \text{ decimal places} \end{array}$$

5. Convert the percent to a decimal first.
$4\% = 0.04$
Convert from a decimal to a fraction.
$0.04 = \frac{4}{100} = \frac{1}{25}$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{1}{25}$	0.04	4%

6. Convert the fraction to a decimal first.
$\frac{5}{6} = 0.8\overline{3}$
Convert from a decimal to a percent.
$0.8\overline{3} = 0.8\overline{3} \cdot 100\% = 83.\overline{3}\%$

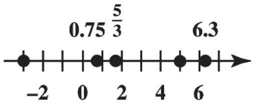
Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{5}{6}$	$0.8\overline{3}$	$83.\overline{3}\%$

7. Convert the decimal to a percent first.
$1.5 = 1.5 \cdot 100\% = 150\%$
Convert from a decimal to a fraction.
$1.5 = 1\frac{1}{2} = \frac{3}{2}$

Fraction in Lowest Terms (or Whole Number)	Decimal	Percent
$\frac{3}{2}$	1.5	150%

8. $\left\{-3, 0.75, \frac{5}{3}, 5, 6.3\right\}$

Place dots at $-3, 0.75, \frac{5}{3} = 1.\overline{6}, 5,$ and 6.3.



9. The elements $0, 3, \sqrt{25}$ (or 5), and $\frac{24}{2}$ (or 12) are whole numbers.

10. The elements $-1, 0, 3, \sqrt{25}$ (or 5), and $\frac{24}{2}$ (or 12) are integers.

11. The elements $-1, -0.5, 0, 3, \sqrt{25}$ (or 5), 7.5, and $\frac{24}{2}$ (or 12) are rational numbers.

12. All the elements in the set are real numbers except $\sqrt{-4}$.

13.
$$\begin{aligned} &-6+14+(-11)-(-3) \\ &= 8+(-11)+3 \\ &= -3+3 = 0 \end{aligned}$$

14.
$$\begin{aligned} -\frac{5}{7}-\left(-\frac{10}{9}+\frac{2}{3}\right) &= -\frac{5}{7}+\frac{10}{9}-\frac{2}{3} \\ &= -\frac{5}{7}\cdot\frac{9}{9}+\frac{10}{9}\cdot\frac{7}{7}-\frac{2}{3}\cdot\frac{21}{21} \\ &= -\frac{45}{63}+\frac{70}{63}-\frac{42}{63} \\ &= \frac{-45+70-42}{63} \\ &= -\frac{17}{63} \end{aligned}$$

15.
$$\begin{aligned} &10-4\cdot 3+6(-4) \\ &= 10-12+(-24) \\ &= -2+(-24) = -26 \end{aligned}$$

16.
$$\begin{aligned} &7-4^2+2(6)+(-4)^2 \\ &= 7-16+12+16 \\ &= 19 \end{aligned}$$

17. $$\frac{-2[3-(-1-2)+2]}{\sqrt{9}(-3)-(-2)}$$
$$=\frac{-2[3-(-3)+2]}{3(-3)-(-2)}$$
$$=\frac{-2[8]}{-9-(-2)}$$
$$=\frac{-16}{-7}=\frac{16}{7}$$

18. $$\frac{8\cdot 4-3^2\cdot 5-2(-1)}{-3\cdot 2^3+24}$$
$$=\frac{8\cdot 4-9\cdot 5-2(-1)}{-3\cdot 8+24}$$
$$=\frac{32-45+2}{-24+24}$$
$$=\frac{-11}{0}=\text{Undefined}$$

19. $\sqrt{196}=14,$ because 14 is positive and $14^2=196.$

20. $-\sqrt{225}=-(\sqrt{225})=-(15)=-15$

21. Since there is no real number whose square is $-16,$ $\sqrt{-16}$ is not a real number.

22. Let $k=-3,$ $m=-3,$ and $r=25$
$$\frac{8k+2m^2}{r-2}$$
$$=\frac{8(-3)+2(-3)^2}{25-2}$$
$$=\frac{8(-3)+2(9)}{23}$$
$$=\frac{-24+18}{23}$$
$$=\frac{-6}{23}\text{ or }-\frac{6}{23}$$

23. $-3(2k-4)+4(3k-5)-2+4k$
$$=-3(2k)+(-3)(-4)+4(3k)$$
$$+4(-5)-2+4k$$
$$=-6k+12+12k-20-2+4k$$
$$=-6k+12k+4k+12-20-2$$
$$=10k-10$$

24. When simplifying $(3r+8)-(-4r+6),$ the subtraction sign in front of $(-4r+6)$ changes the sign of the terms $-4r$ and 6.
$$(3r+8)-(-4r+6)$$
$$=3r+8-(-4r)-6$$
$$=3r+8+4r-6$$
$$=3r+4r+8-6$$
$$=7r+2$$

25. **(a)** The answer is B, Inverse property. The sum of 6 and its inverse, $-6,$ equals zero.

(b) The answer is D, Associative property. The order of the terms is the same, but the grouping has changed.

(c) The answer is A, Distributive property. This is the second form of the distributive property.

(d) The answer is F, Multiplication property of 0. Multiplication by 0 always equals 0.

(e) The answer is C, Identity property. The addition of 0 to any number does not change the number.

(f) The answer is C, Identity property. Multiplication of any number by 1 does not change the number.

(g) The answer is E, Commutative property. The order of the terms *a* and *b* is reversed.

Chapter 1
Linear Equations, Inequalities, and Applications

1.1 Linear Equations in One Variable

Classroom Examples, Now Try Exercises

1. **(a)** $9x+10=0$ is an *equation* because it contains an equality symbol.

(b) $9x+10$ is an *expression* because it does not contain an equality symbol.

N1. **(a)** $2x+17-3x$ is an *expression* because it does not contain an equality symbol.

(b) $2x+17=3x$ is an *equation* because it contains an equality symbol.

2.

$$\begin{aligned} 4x+8x &= -9+17x-1 & & \\ 12x &= 17x-10 & & \text{Combine terms.} \\ 12x-17x &= 17x-10-17x & & \text{Subtract } 17x. \\ -5x &= -10 & & \text{Combine terms.} \\ \frac{-5x}{-5} &= \frac{-10}{-5} & & \text{Divide by } -5. \\ x &= 2 & & \end{aligned}$$

Check by substituting 2 for x in the original equation.

$$\begin{aligned} 4x+8x &= -9+17x-1 & & \\ 4(2)+8(2) &\overset{?}{=} -9+17(2)-1 & & \text{Let } x=2. \\ 8+16 &\overset{?}{=} -9+34-1 & & \\ 24 &\overset{?}{=} 25-1 & & \\ 24 &= 24 & & \text{True} \end{aligned}$$

The solution set is $\{2\}$.

N2.

$$\begin{aligned} 5x+11 &= 2x-13-3x & & \\ 5x+11 &= -x-13 & & \text{Combine terms.} \\ 5x+11+x &= -x-13+x & & \text{Add } x. \\ 6x+11 &= -13 & & \text{Combine terms.} \\ 6x+11-11 &= -13-11 & & \text{Subtract } 11. \\ 6x &= -24 & & \text{Combine terms.} \\ \frac{6x}{6} &= \frac{-24}{6} & & \text{Divide by } 6. \\ x &= -4 & & \end{aligned}$$

Check by substituting -4 for x in the original equation.

$$\begin{aligned} 5x+11 &= 2x-13-3x & & \\ 5(-4)+11 &\overset{?}{=} 2(-4)-13-3(-4) & & \text{Let } x=-4. \\ -20+11 &\overset{?}{=} -8-13+12 & & \\ -9 &\overset{?}{=} -21+12 & & \\ -9 &= -9 & & \text{True} \end{aligned}$$

The solution set is $\{-4\}$.

3.

$$\begin{aligned} 3(4+x)-x &= 5x-3 & & \\ 12+3x-x &= 5x-3 & & \text{Distributive prop.} \\ 12+2x &= 5x-3 & & \text{Combine terms.} \\ 12+2x-2x &= 5x-3-2x & & \text{Subtract } 2x. \\ 12 &= 3x-3 & & \text{Combine terms.} \\ 12+3 &= 3x-3+3 & & \text{Add } 3. \\ 15 &= 3x & & \text{Combine terms.} \\ \frac{15}{3} &= \frac{3x}{3} & & \text{Divide by } 3. \\ 5 &= x & & \end{aligned}$$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

Check $x=5$: $27-5=25-3$ True

The solution set is $\{5\}$.

N3.

$$\begin{aligned} 5(x-4)-12 &= 3-2x & & \\ 5x-20-12 &= 3-2x & & \text{Distributive prop.} \\ 5x-32 &= 3-2x & & \text{Combine terms.} \\ 5x-32+2x &= 3-2x+2x & & \text{Add } 2x. \\ 7x-32 &= 3 & & \text{Combine terms.} \\ 7x-32+32 &= 3+32 & & \text{Add } 32. \\ 7x &= 35 & & \text{Combine terms.} \\ \frac{7x}{7} &= \frac{35}{7} & & \text{Divide by } 7. \\ x &= 5 & & \end{aligned}$$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

Check $x=5$: $5-12=3-10$ True

The solution set is $\{5\}$.

4. $$\begin{aligned} 6-(4+x)&=8x-2(3x+5) \\ 6-4-x&=8x-6x-10 && \text{Distributive prop.} \\ 2-x&=2x-10 && \text{Combine terms.} \\ 2-x+x&=2x-10+x && \text{Add } x. \\ 2&=3x-10 && \text{Combine terms.} \\ 2+10&=3x-10+10 && \text{Add 10.} \\ 12&=3x && \text{Combine terms.} \\ \frac{12}{3}&=\frac{3x}{3} && \text{Divide by 3.} \\ 4&=x \end{aligned}$$

Check $x=4$: $-2=32-34$ True

The solution set is $\{4\}$.

N4. $$\begin{aligned} 2-3(2+6x)&=4(x+1)+36 \\ 2-6-18x&=4x+4+36 && \text{Dist. prop.} \\ -4-18x&=4x+40 && \text{Combine terms.} \\ -4-18x+18x&=4x+40+18x && \text{Add } 18x. \\ -4&=22x+40 && \text{Combine terms.} \\ -4-40&=22x+40-40 && \text{Subtract 40.} \\ -44&=22x && \text{Combine terms.} \\ \frac{-44}{22}&=\frac{22x}{22} && \text{Divide by 22.} \\ -2&=x \end{aligned}$$

Check $x=-2$: $2+30=-4+36$ True

The solution set is $\{-2\}$.

5. Multiply each side by the LCD, 4, and use the distributive property

$$\begin{aligned} \frac{x+1}{2}+\frac{x+3}{4}&=\frac{1}{2} \\ 4\left(\frac{x+1}{2}\right)+4\left(\frac{x+3}{4}\right)&=4\left(\frac{1}{2}\right) \\ 2(x+1)+1(x+3)&=2 \\ 2x+2+x+3&=2 \\ 3x+5&=2 \\ 3x&=-3 && \text{Subtract 5.} \\ x&=-1 && \text{Divide by 3.} \end{aligned}$$

Check $x=-1$: $0+\frac{2}{4}=\frac{1}{2}$ True

The solution set is $\{-1\}$.

N5. Multiply each side by the LCD, 8, and use the distributive property.

$$\begin{aligned} \frac{x-4}{4}+\frac{2x+4}{8}&=5 \\ 8\left(\frac{x-4}{4}\right)+8\left(\frac{2x+4}{8}\right)&=8(5) \\ 2(x-4)+1(2x+4)&=40 \\ 2x-8+2x+4&=40 \\ 4x-4&=40 \\ 4x&=44 && \text{Add 4.} \\ x&=11 && \text{Divide by 4.} \end{aligned}$$

Check $x=11$: $\frac{7}{4}+\frac{13}{4}=5$ True

The solution set is $\{11\}$.

6. Multiply each term by 100.

$$\begin{aligned} 0.02(60)+0.04x&=0.03(50+x) \\ 2(60)+4x&=3(50+x) \\ 120+4x&=150+3x \\ 4x&=30+3x \\ x&=30 \end{aligned}$$

Check $x=30$: $1.2+1.2=2.4$ True

The solution set is $\{30\}$.

N6. Multiply each term by 100.

$$\begin{aligned} 0.08x-0.12(x-4)&=0.03(x-5) \\ 8x-12(x-4)&=3(x-5) \\ 8x-12x+48&=3x-15 \\ -4x+48&=3x-15 \\ -4x+63&=3x \\ 63&=7x \\ 9&=x \end{aligned}$$

Check $x=9$: $0.72-0.60=0.12$ True

The solution set is $\{9\}$.

7. **(a)** $$\begin{aligned} 5(x+2)-2(x+1)&=3x+1 \\ 5x+10-2x-2&=3x+1 \\ 3x+8&=3x+1 \\ 3x+8-3x&=3x+1-3x \\ 8&=1 && \text{False} \end{aligned}$$

Since the result, $8=1$, is *false*, the equation has no solution and is called a *contradiction*.
The solution set is $\varnothing$.

(b) Multiply each side by the LCD, 3, and use the distributive property.

$$\frac{x+1}{3}+\frac{2x}{3}=x+\frac{1}{3}$$
$$3\left(\frac{x+1}{3}\right)+3\left(\frac{2x}{3}\right)=3\left(x+\frac{1}{3}\right)$$
$$x+1+2x=3x+1$$
$$3x+1=3x+1$$

This is an *identity*. Any real number will make the equation true.
The solution set is {all real numbers}.

(c) $5(3x+1)=x+5$

$$\begin{aligned} 15x+5&=x+5 \\ 14x+5&=5 && \text{Subtract } x. \\ 14x&=0 && \text{Subtract 5.} \\ x&=0 && \text{Divide by 14.} \end{aligned}$$

This is a *conditional equation.*
Check $x=0$: $5(1)=0+5$ True
The solution set is $\{0\}$.

N7. **(a)** $9x-3(x+4)=6(x-2)$

$$\begin{aligned} 9x-3x-12&=6x-12 \\ 6x-12&=6x-12 \end{aligned}$$

This is an *identity*. Any real number will make the equation true.
The solution set is $\{\text{all real numbers}\}$.

(b) $-3(2x-1)-2x=3+x$

$$\begin{aligned} -6x+3-2x&=3+x \\ -8x+3&=3+x \\ -9x+3&=3 && \text{Subtract } x. \\ -9x&=0 && \text{Subtract 3.} \\ x&=0 && \text{Divide by} -9. \end{aligned}$$

This is a *conditional equation.*
Check $x=0$: $-3(-1)=3$ True
The solution set is $\{0\}$.

(c)

$$\begin{aligned} 10x-21&=2(x-5)+8x \\ 10x-21&=2x-10+8x \\ 10x-21&=10x-10 \\ 10x-21-10x&=10x-10-10x \\ -21&=-10 \quad \text{False} \end{aligned}$$

Since the result, $-21=-10$, is *false*, the equation has no solution and is called a *contradiction.*
The solution set is $\varnothing$.

Exercises

1. A collection of numbers, variables, operation symbols, and grouping symbols, such as $2(8x-15)$, is an algebraic expression. While an equation *does* include an equality symbol, there *is not* an equality symbol in an algebraic expression.

2. A linear equation in one variable (here x) can be written in the form $ax+b=0$, with $a\neq 0$. Another name for a linear equation is a first-degree equation, because the greatest power on the variable is *one*.

3. If we let $x=2$ in the linear equation $2x+5=9$, a *true* statement results. The number 2 is a solution of the equation, and $\{2\}$ is the solution set.

4. A linear equation with one solution in its solution set, such as $2x+5=9$, is a conditional equation.

5. A linear equation with an infinite number of solutions is an identity. Its solution set is $\{\text{all real numbers}\}$.

6. A linear equation with no solution is a contradiction. Its solution set is the empty set $\varnothing$.

7. $3x+x-2=0$ can be written as $4x=2$, so it is linear.
$9x-4=9$ is in linear form.
Choices A and C are linear.

8. $12=x^2$ in choice B is nonlinear because the variable is squared.
$3x+2y=6$ in choice D is nonlinear because there are two variables.
Choices B and D are nonlinear.

9. $-3x+2-4=x$ is an *equation* because it contains an equality symbol.

10. $-3x+2-4-x=4$ is an *equation* because it contains an equality symbol.

11. $4(x+3)-2(x+1)-10$ is an *expression* because it does not contain an equality symbol.

12. $4(x+3)-2(x+1)+10$ is an *expression* because it does not contain an equality symbol.

13. $-10x+12-4x=-3$ is an *equation* because it contains an equality symbol.

14. $-10x+12-4x+3=0$ is an *equation* because it contains an equality symbol.

15. A sign error was made when the distributive property was applied. The left side of the second line should be $8x-4x+6.$

$$\begin{aligned} 8x-2(2x-3)&=3x+7 && \\ 8x-4x+6&=3x+7 && \text{Distributive property} \\ 4x+6&=3x+7 && \text{Combine like terms.} \\ x&=1 && \text{Subtract } 3x \text{ and } 6. \end{aligned}$$

The correct solution is 1.

16. The first step should have been to distribute -2 to $(3x+1)$.

$$\begin{aligned} 12-2(3x+1)&=11 && \\ 12-6x-2&=11 && \text{Distributive property} \\ -6x+10&=11 && \text{Combine terms.} \\ -6x+10-10&=11-10 && \text{Subtract 10.} \\ -6x&=1 && \\ x&=-\frac{1}{6} && \text{Divide by } -6. \end{aligned}$$

The correct solution is $-\frac{1}{6}$.

17. **(a)** The true statement 7 = 7 indicates that the solution set is {all real numbers}, choice B.

(b) The final line is $x = 0$, which is the form of solution for a conditional equation. The solution set is {0}, choice A.

(c) The false statement 7 = 0 indicates that the equation has no solution. The solution set is the empty set $\varnothing$, choice C.

18. An equation will have the solution set {all real numbers} if it is an identity. The only equation listed here that is **not** an identity is choice C:

$$\begin{aligned} 4x&=3x \\ 4x-3x&=0 \\ x&=0 \end{aligned}$$

19.

$$\begin{aligned} 7x+8&=1 && \\ 7x+8-8&=1-8 && \text{Subtract 8.} \\ 7x&=-7 && \\ \frac{7x}{7}&=\frac{-7}{7} && \text{Divide by 7.} \\ x&=-1 && \end{aligned}$$

We will use the following notation to indicate the value of each side of the original equation after we have substituted the proposed solution and simplified.

Check $x=-1$: $-7+8=1$ True

The solution set is $\{-1\}$.

20.

$$\begin{aligned} 5x-4&=21 && \\ 5x-4+4&=21+4 && \text{Add 4.} \\ 5x&=25 && \\ \frac{5x}{5}&=\frac{25}{5} && \text{Divide by 5.} \\ x&=5 && \end{aligned}$$

Check $x=5$: $25-4=21$ True

The solution set is $\{5\}$.

21.

$$\begin{aligned} 5x+2&=3x-6 && \\ 5x+2-3x&=3x-6-3x && \text{Subtract } 3x. \\ 2x+2&=-6 && \\ 2x+2-2&=-6-2 && \text{Subtract 2.} \\ 2x&=-8 && \\ \frac{2x}{2}&=\frac{-8}{2} && \text{Divide by 2.} \\ x&=-4 && \end{aligned}$$

Check $x=-4$: $-20+2=-12-6$ True

The solution set is $\{-4\}$.

22.

$$\begin{aligned} 9x+1&=7x-9 && \\ 9x+1-7x&=7x-9-7x && \text{Subtract } 7x. \\ 2x+1&=-9 && \\ 2x+1-1&=-9-1 && \text{Subtract 1.} \\ 2x&=-10 && \\ \frac{2x}{2}&=\frac{-10}{2} && \text{Divide by 2.} \\ x&=-5 && \end{aligned}$$

Check $x=-5$: $-45+1=-35-9$ True

The solution set is $\{-5\}$.

23.

$$\begin{aligned} 7x-5x+15&=x+8 && \\ 2x+15&=x+8 && \text{Combine terms.} \\ 2x+15-x&=x+8-x && \text{Subtract } x. \\ x+15&=8 && \text{Combine terms.} \\ x+15-15&=8-15 && \text{Subtract 15.} \\ x&=-7 && \text{Combine terms.} \end{aligned}$$

Check $x=-7$: $-49+35+15=-7+8$ True

The solution set is $\{-7\}$.

24. $2x+4-x=4x-5$

$x+4=4x-5$ Combine terms.

$-3x+4=-5$ Subtract $4x$.

$-3x=-9$ Subtract 4.

$x=3$ Divide by -3.

Check $x=3:$ $6+4-3=12-5$ True

The solution set is $\{3\}$.

25. $12w+15w-9+5=-3w+5-9$

$27w-4=-3w-4$ Combine terms.

$30w-4=-4.$ Add $3w$.

$30w=0$ Add 4.

$w=0$ Divide by 30.

Check $w=0:$ $-9+5=5-9$ True

The solution set is $\{0\}$.

26. $-4x+5x-8+4=6x-4$

$x-4=6x-4$ Combine terms.

$-5x-4=-4$ Subtract $6x$.

$-5x=0$ Add 4.

$x=0$ Divide by -5.

Check $x=0:$ $-8+4=-4$ True

The solution set is $\{0\}$.

27. $3(2t-4)=20-2t$

$6t-12=20-2t$ Distributive property

$8t-12=20$ Add $2t$.

$8t=32$ Add 12.

$t=4$ Divide by 8.

Check $t=4:$ $3(4)=20-8$ True

The solution set is $\{4\}$.

28. $2(3-2x)=x-4$

$6-4x=x-4$ Distributive property

$6-5x=-4$ Subtract x.

$-5x=-10$ Subtract 6.

$x=2$ Divide by -5.

Check $x=2:$ $2(-1)=2-4$ True

The solution set is $\{2\}$.

29. $-5(x+1)+3x+2=6x+4$

$-5x-5+3x+2=6x+4$ Distributive prop.

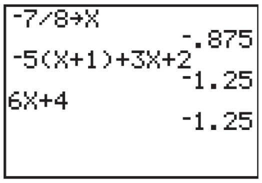
$-2x-3=6x+4$ Combine terms.

$-3=8x+4$ Add $2x$.

$-7=8x$ Subtract 4.

$-\frac{7}{8}=x$ Divide by 8.

Check: Substitute $-\frac{7}{8}$ for x and show that both sides equal -1.25. The screen shows a typical Check on a calculator.



```
-7/8→X
                 -.875
-5(X+1)+3X+2
                 -1.25
6X+4
                 -1.25
```

The solution set is $\left\{-\frac{7}{8}\right\}$.

30. $5(x+3)+4x-5=4-2x$

$5x+15+4x-5=4-2x$ Distributive prop.

$9x+10=4-2x$ Combine terms.

$11x+10=4$ Add $2x$.

$11x=-6$ Subtract 10.

$x=-\frac{6}{11}$ Divide by 11.

Check

$x=-\frac{6}{11}:$

$\frac{135}{11}-\frac{24}{11}-\frac{55}{11}=\frac{44}{11}+\frac{12}{11}$ True

The solution set is $\left\{-\frac{6}{11}\right\}$.

31. $-2x+5x-9=3(x-4)-5$

$3x-9=3x-12-5$

$3x-9=3x-17$

$-9=-17$ False

The equation is a *contradiction*.
The solution set is $\varnothing$.

32. $-6x+2x-11=-2(2x-3)+4$

$-4x-11=-4x+6+4$

$-4x-11=-4x+10$

$-11=10$ False

The equation is a *contradiction*.
The solution set is $\varnothing$.

33.
$$\begin{aligned} -2(x+3) &= -6(x+7) \\ -2x-6 &= -6x-42 \\ 4x-6 &= -42 \\ 4x &= -36 \\ x &= -9 \end{aligned}$$

Check $x=-9$: $-2(-6)=-6(-2)$ True

The solution set is $\{-9\}$.

34.
$$\begin{aligned} -4(x-9) &= -8(x+3) \\ -4x+36 &= -8x-24 \\ 4x+36 &= -24 \\ 4x &= -60 \\ x &= -15 \end{aligned}$$

Check $x=-15$: $-4(-24)=-8(-12)$ True

The solution set is $\{-15\}$.

35.
$$\begin{aligned} 3(2x+1)-2(x-2) &= 5 \\ 6x+3-2x+4 &= 5 \\ 4x+7 &= 5 \\ 4x &= -2 \\ x &= \frac{-2}{4} = -\frac{1}{2} \end{aligned}$$

Check $x=-\frac{1}{2}$: $3(0)-2\left(-\frac{5}{2}\right)=5$ True

The solution set is $\left\{-\frac{1}{2}\right\}$.

36.
$$\begin{aligned} 4(x-2)+2(x+3) &= 6 \\ 4x-8+2x+6 &= 6 \\ 6x-2 &= 6 \\ 6x &= 8 \\ x &= \frac{8}{6} = \frac{4}{3} \end{aligned}$$

Check $x=\frac{4}{3}$: $4\left(-\frac{2}{3}\right)+2\left(\frac{13}{3}\right)=6$ True

The solution set is $\left\{\frac{4}{3}\right\}$.

37.
$$\begin{aligned} 2x+3(x-4) &= 2(x-3) \\ 2x+3x-12 &= 2x-6 \\ 5x-12 &= 2x-6 \\ 3x &= 6 \\ x &= \frac{6}{3} = 2 \end{aligned}$$

Check $x=2$: $4+3(-2)=2(-1)$ True

The solution set is $\{2\}$.

38.
$$\begin{aligned} 6x-3(5x+2) &= 4(1-x) \\ 6x-15x-6 &= 4-4x \\ -9x-6 &= 4-4x \\ -5x &= 10 \\ x &= \frac{10}{-5} = -2 \end{aligned}$$

Check $x=-2$: $-12-3(-8)=4(3)$ True

The solution set is $\{-2\}$.

39.
$$\begin{aligned} 6x-4(3-2x) &= 5(x-4)-10 \\ 6x-12+8x &= 5x-20-10 \\ 14x-12 &= 5x-30 \\ 9x &= -18 \\ x &= -2 \end{aligned}$$

Check $x=-2$: $-12-4(7)=5(-6)-10$ True

The solution set is $\{-2\}$.

40.
$$\begin{aligned} -2x-3(4-2x) &= 2(x-3)+2 \\ -2x-12+6x &= 2x-6+2 \\ 4x-12 &= 2x-4 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

Check $x=4$: $-8-3(-4)=2(1)+2$ True

The solution set is $\{4\}$.

41.
$$\begin{aligned} -2(x+3)-x-4 &= -3(x+4)+2 \\ -2x-6-x-4 &= -3x-12+2 \\ -3x-10 &= -3x-10 \end{aligned}$$

The equation is an *identity.*
The solution set is {all real numbers}.

42.
$$\begin{aligned} 4(2x+7) &= 2x+25+3(2x+1) \\ 8x+28 &= 2x+25+6x+3 \\ 8x+28 &= 8x+28 \end{aligned}$$

The equation is an *identity.*
The solution set is {all real numbers}.

43.
$$\begin{aligned} 2[x-(2x+4)+3] &= 2(x+1) \\ 2[x-2x-4+3] &= 2(x+1) \\ 2[-x-1] &= 2(x+1) \\ -x-1 &= x+1 && \text{Divide by 2.} \\ -1 &= 2x+1 && \text{Add } x. \\ -2 &= 2x && \text{Subtract 1.} \\ -1 &= x && \text{Divide by 2.} \end{aligned}$$

Check $x=-1$: $2[-1-2+3]=0$ True

The solution set is $\{-1\}$.