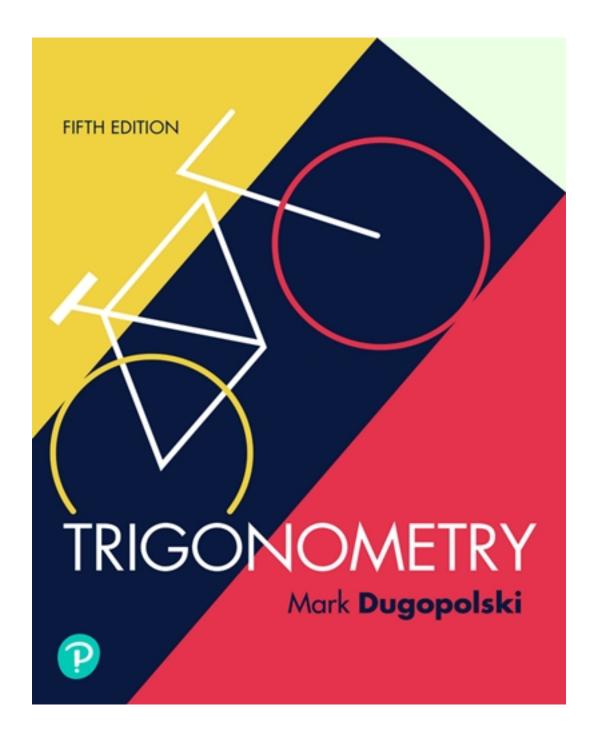
Solutions for Trigonometry 5th Edition by Dugopolski

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Solutions

INSTRUCTOR'S SOLUTIONS MANUAL

EDGAR REYES

Southeastern Louisiana University

TRIGONOMETRY FIFTH EDITION

Mark Dugopolski

Southeastern Louisiana University



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For Thought

- 1. False, the point (2, -3) is in Quadrant IV.
- **2.** False, the point (4,0) does not belong to any quadrant.
- **3.** False, since the distance is $\sqrt{(a-c)^2+(b-d)^2}$.
- **4.** False, since Ax + By = C is a linear equation.
- 5. True
- **6.** False, since $\sqrt{7^2 + 9^2} = \sqrt{130} \approx 11.4$
- 7. True
- 8. True
- 9. True
- 10. False, since the radius is 3.

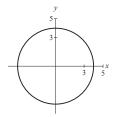
P.1 Exercises

- 1. ordered
- 2. Cartesian
- **3.** *x*-axis
- 4. origin
- **5.** Pythagorean theorem
- 6. circle
- 7. linear equation
- 8. y-intercept
- **9.** (4, 1), Quadrant I
- **10.** (-3,2), Quadrant II
- **11.** (1,0), x-axis
- **12.** (-1, -5), Quadrant III
- **13.** (5, -1), Quadrant IV
- **14.** (0, -3), y-axis
- **15.** (-4, -2), Quadrant III
- **16.** (-2,0), x-axis
- 17. (-2,4), Quadrant II

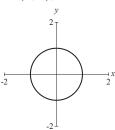
- **18.** (1, 5), Quadrant I
- **19.** $c = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$
- **20.** Since $a^2 + a^2 = \sqrt{2}^2$, we get $2a^2 = 2$ or $a^2 = 1$. Then a = 1.
- **21.** Since $b^2 + 2^2 = 3^2$, we get $b^2 + 4 = 9$ or $b^2 = 5$. Then $b = \sqrt{5}$.
- **22.** Since $b^2 + \left(\frac{1}{2}\right)^2 = 1^2$, we get $b^2 + \frac{1}{4} = 1$ or $b^2 = \frac{3}{4}$. Thus, $b = \frac{\sqrt{3}}{2}$.
- **23.** Since $a^2 + 3^2 = 5^2$, we get $a^2 + 9 = 25$ or $a^2 = 16$. Then a = 4.
- **24.** $c = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$
- **25.** $\sqrt{4 \cdot 7} = 2\sqrt{7}$
- **26.** $\sqrt{25 \cdot 2} = 5\sqrt{2}$
- **27.** $\frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$
- **28.** $\frac{\sqrt{3}}{\sqrt{16}} = \frac{\sqrt{3}}{4}$
- **29.** $\frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$
- **30.** $\frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$
- **31.** $\frac{2\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{15}}{5}$
- **32.** $\frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$
- **33.** $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
- **34.** $\frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$
- **35.** $\frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$
- **36.** $\frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{2}$
- **37.** Distance is $\sqrt{(4-1)^2 + (7-3)^2} = \sqrt{9+16} = \sqrt{25} = 5$, midpoint is (2.5, 5)
- **38.** Distance is $\sqrt{144 + 25} = 13$, midpoint is (3, 0.5)
- **39.** Distance is $\sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$, midpoint is (0,-1)

- **40.** Distance is $\sqrt{4+4} = 2\sqrt{2}$, midpoint is (0,1)
- **41.** Distance is $\sqrt{\left(\frac{\sqrt{2}}{2} 0\right)^2 + \left(\frac{\sqrt{2}}{2} 0\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{1} = 1$, midpoint is $\left(\frac{\sqrt{2}/2 + 0}{2}, \frac{\sqrt{2}/2 + 0}{2}\right) = \left(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\right)$
- **42.** Distance is $\sqrt{\left(\sqrt{3} 0\right)^2 + (1 0)^2} = \sqrt{3 + 1} = 2$, midpoint is $\left(\frac{\sqrt{3} + 0}{2}, \frac{1 + 0}{2}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
- **43.** Distance is $\sqrt{\left(\sqrt{18} \sqrt{8}\right)^2 + \left(\sqrt{12} \sqrt{27}\right)^2} = \sqrt{(3\sqrt{2} 2\sqrt{2})^2 + (2\sqrt{3} 3\sqrt{3})^2} = \sqrt{(\sqrt{2})^2 + (-\sqrt{3})^2} = \sqrt{5},$ midpoint is $\left(\frac{\sqrt{18} + \sqrt{8}}{2}, \frac{\sqrt{12} + \sqrt{27}}{2}\right) = \left(\frac{3\sqrt{2} + 2\sqrt{2}}{2}, \frac{2\sqrt{3} + 3\sqrt{3}}{2}\right) = \left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{3}}{2}\right)$
- **44.** Distance is $\sqrt{\left(\sqrt{72} \sqrt{50}\right)^2 + \left(\sqrt{45} \sqrt{20}\right)^2} = \sqrt{(6\sqrt{2} 5\sqrt{2})^2 + (3\sqrt{5} 2\sqrt{5})^2} = \sqrt{(\sqrt{2})^2 + (\sqrt{5})^2} = \sqrt{7},$ midpoint is $\left(\frac{\sqrt{72} + \sqrt{50}}{2}, \frac{\sqrt{45} + \sqrt{20}}{2}\right) = \left(\frac{6\sqrt{2} + 5\sqrt{2}}{2}, \frac{3\sqrt{5} + 2\sqrt{5}}{2}\right) = \left(\frac{11\sqrt{2}}{2}, \frac{5\sqrt{5}}{2}\right)$
- **45.** Distance is $\sqrt{(1.2+3.8)^2 + (4.4+2.2)^2} = \sqrt{25+49} = \sqrt{74}$, midpoint is (-1.3, 1.3)
- **46.** Distance is $\sqrt{49 + 81} = \sqrt{130}$, midpoint is (1.2, -3)
- **47.** Distance is $\frac{\sqrt{\pi^2+4}}{2}$, midpoint is $\left(\frac{3\pi}{4},\frac{1}{2}\right)$

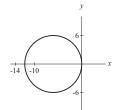
- **48.** Distance is $\frac{\sqrt{\pi^2+4}}{2}$, midpoint is $\left(\frac{\pi}{4},\frac{1}{2}\right)$
- **49.** Distance is $\sqrt{(2\pi \pi)^2 + (0 0)^2} = \sqrt{\pi^2} = \pi$, midpoint is $\left(\frac{2\pi + \pi}{2}, \frac{0 + 0}{2}\right) = \left(\frac{3\pi}{2}, 0\right)$
- **50.** Distance is $\sqrt{\left(\pi \frac{\pi}{2}\right)^2 + (1 1)^2} = \sqrt{\frac{\pi^2}{4}} = \frac{\pi}{2}$, midpoint is $\left(\frac{\pi + \pi/2}{2}, \frac{1 + 1}{2}\right) = \left(\frac{3\pi}{4}, 1\right)$
- **51.** Distance is $\sqrt{\left(\frac{\pi}{2} \frac{\pi}{3}\right)^2 + \left(-\frac{1}{3} \frac{1}{2}\right)^2} = \sqrt{\frac{\pi^2}{36} + \frac{25}{36}} = \frac{\sqrt{\pi^2 + 25}}{6},$ midpoint is $\left(\frac{\pi}{3} + \frac{\pi}{2}, \frac{1}{2} \frac{1}{3}\right) = \left(\frac{5\pi}{12}, \frac{1}{12}\right)$
- **52.** Distance is $\sqrt{\left(\pi \frac{2\pi}{3}\right)^2 + \left(-1 + \frac{1}{2}\right)^2} = \sqrt{\frac{\pi^2}{9} + \frac{1}{4}} = \frac{\sqrt{4\pi^2 + 9}}{6},$ midpoint is $\left(\frac{2\pi}{3} + \pi, \frac{-\frac{1}{2} 1}{2}\right) = \left(\frac{5\pi}{6}, -\frac{3}{4}\right)$
- **53.** Center(0,0), radius 4



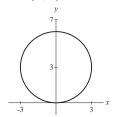
54. Center (0,0), radius 1



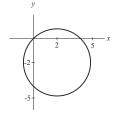
55. Center (-6,0), radius 6



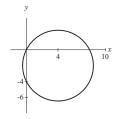
56. Center (0,3), radius 3



57. Center (2, -2), radius $2\sqrt{2}$



58. Center (4, -2), radius $2\sqrt{5}$



59.
$$x^2 + y^2 = 7$$

60.
$$x^2 + y^2 = 12$$
 since $(2\sqrt{3})^2 = 12$

61.
$$(x+2)^2 + (y-5)^2 = 1/4$$

62.
$$(x+1)^2 + (y+6)^2 = 1/9$$

- **63.** The distance between (3,5) and the origin is $\sqrt{34}$ which is the radius. The standard equation is $(x-3)^2 + (y-5)^2 = 34$.
- **64.** The distance between (-3,9) and the origin is $\sqrt{90}$ which is the radius. The standard equation is $(x+3)^2 + (y-9)^2 = 90$.
- **65.** Note, the distance between $(\sqrt{2}/2, \sqrt{2}/2)$ and the origin is 1. Thus, the radius is 1. The standard equation is $x^2 + y^2 = 1$.

- **66.** Note, the distance between $(\sqrt{3}/2, 1/2)$ and the origin is 1. Thus, the radius is 1. The standard equation is $x^2 + y^2 = 1$.
- **67.** The radius is $\sqrt{(-1-0)^2 + (2-0)^2} = \sqrt{5}$. The standard equation is $(x+1)^2 + (y-2)^2 = 5$.
- **68.** Since the center is (0,0) and the radius is 2, the standard equation is $x^2 + y^2 = 4$.
- **69.** Note, the center is (1,3) and the radius is 2. The standard equation is $(x-1)^2 + (y-3)^2 = 4$.
- **70.** The radius is $\sqrt{(2-0)^2 + (2-0)^2} = \sqrt{8}$. The standard equation is $(x-2)^2 + (y-2)^2 = 8$.
- **71.** We solve for a.

$$a^{2} + \left(\frac{3}{5}\right)^{2} = 1$$

$$a^{2} = 1 - \frac{9}{25}$$

$$a^{2} = \frac{16}{25}$$

$$a = \pm \frac{4}{5}$$

72. We solve for a.

$$a^{2} + \left(-\frac{1}{2}\right)^{2} = 1$$

$$a^{2} = 1 - \frac{1}{4}$$

$$a^{2} = \frac{3}{4}$$

$$a = \pm \frac{\sqrt{3}}{2}$$

73. We solve for a.

$$\left(-\frac{2}{5}\right)^2 + a^2 = 1$$

$$a^2 = 1 - \frac{4}{25}$$

$$a^2 = \frac{21}{25}$$

$$a = \pm \frac{\sqrt{21}}{5}$$

74. Solve for *a*:

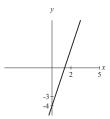
$$\left(\frac{2}{3}\right)^2 + a^2 = 1$$

$$a^2 = 1 - \frac{4}{9}$$

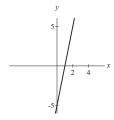
$$a^2 = \frac{5}{9}$$

$$a = \pm \frac{\sqrt{5}}{3}$$

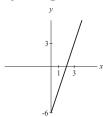
75. y = 3x - 4 goes through $(0, -4), (\frac{4}{3}, 0)$.



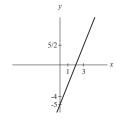
76. y = 5x - 5 goes through (0, -5), (1, 0).



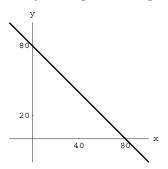
77. 3x - y = 6 goes through (0, -6), (2, 0).



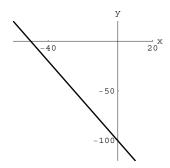
78. 5x - 2y = 10 goes through (0, -5), (2, 0).



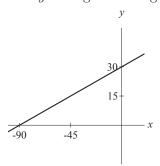
79. x + y = 80 goes through (0, 80), (80, 0).



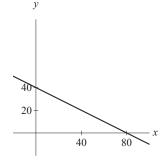
80. 2x + y = -100 goes through (0, -100), (-50, 0).



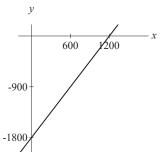
81. x = 3y - 90 goes through (0, 30), (-90, 0).



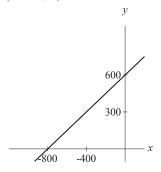
82. x = 80 - 2y goes through (0, 40), (80, 0).



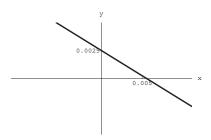
83. $\frac{1}{2}x - \frac{1}{3}y = 600$ goes through (0, -1800), (1200, 0).



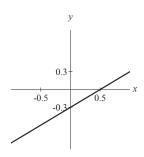
84. $\frac{2}{3}y - \frac{1}{2}x = 400$ goes through (0, 600), (-800, 0).



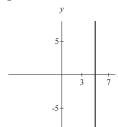
85. Intercepts are (0, 0.0025), (0.005, 0).



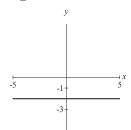
86. Intercepts are (0, -0.3), (0.5, 0).



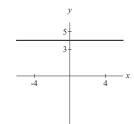
87. x = 5



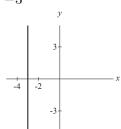
88. y = -2



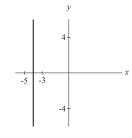
89. y = 4



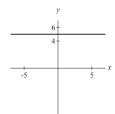
90. x = -3



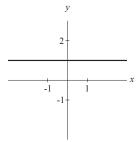
91. x = -4



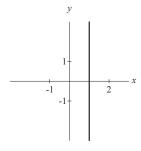
92.
$$y = 5$$



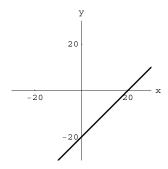
93. Solving for y, we have y = 1.



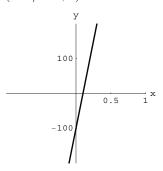
94. Solving for x, we get x = 1.



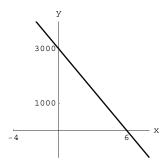
95. y = x - 20 goes through (0, -20), (20, 0).



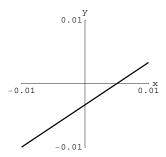
96. y = 999x - 100 goes through (0, -100), (100/999, 0).



97. y = 3000 - 500x goes through (0, 3000), (6, 0).



98. $y = \frac{1}{300}(200x - 1)$ goes through (0, -1/300), (1/200, 0).



- **99.** The hypotenuse is $\sqrt{6^2 + 8^2} = \sqrt{100} = 10$.
- **100.** The other leg is $\sqrt{10^2 4^2} = \sqrt{84} = 2\sqrt{21}$ ft.
- 101. a) Let r be the radius of the smaller circle. Consider the right triangle with vertices at the origin, another vertex at the center of a smaller circle, and a third vertex at the center of the circle of radius 1. By the Pythagorean Theorem, we obtain

$$1 + (2 - r)^2 = (1 + r)^2$$
$$5 - 4r = 1 + 2r$$

$$\begin{array}{rcl}
4 & = & 6r \\
r & = & \frac{2}{3}.
\end{array}$$

The diameter of the smaller circle is $2r = \frac{4}{3}$.

b) The smallest circles are centered at $(\pm r, 0)$ or $(\pm 4/3, 0)$. The equations of the circles are

$$\left(x - \frac{4}{3}\right)^2 + y^2 = \frac{4}{9}$$

and

$$\left(x + \frac{4}{3}\right)^2 + y^2 = \frac{4}{9}$$

- 102. Draw a right triangle with vertices at the centers of the circles, and another vertex at a point of intersection of the two circles. The legs of the right triangle are 5 and 12. By the Pythagorean theorem, the hypotenuse is $\sqrt{5^2 + 12^2} = 13$.
- 103. Let C(h, k) and r be the center and radius of the smallest circle, respectively. Then k = -r. We consider two right triangles each of which has a vertex at C.

The right triangles have sides that are perpendicular to the coordinate axes. Also, one side of each right triangle passes through the center of a larger circle.

Applying the Pythagorean Theorem, we list a system of equations

$$(r+1)^2 = h^2 + (1-r)^2$$

$$(2-r)^2 = h^2 + r^2.$$

The solutions are r = 1/2, $h = \sqrt{2}$, and k = -r = -1/2.

The equation of the smallest circle is

$$(x - \sqrt{2})^2 + (y + 1/2)^2 = 1/4.$$

104. We apply symmetry to the centers of the remaining three circles. From the answer or equation in Exercise 103, the equations of the remaining circles are

$$(x - \sqrt{2})^2 + (y - 1/2)^2 = 1/4$$

$$(x+\sqrt{2})^2 + (y-1/2)^2 = 1/4$$

$$(x+\sqrt{2})^2 + (y+1/2)^2 = 1/4.$$

105. The midpoint of (0, 20.8) and (48, 27.4) is

$$\left(\frac{0+48}{2}, \frac{20.8+27.4}{2}\right) = (24, 24.1).$$

In 1994 (= 1970 + 24), the median age at first marriage was 24.1 years.

106. a) If h = 0, then 0 = 0.229n + 5.203. Then $n = -5.203/0.229 \approx -22.72$. The *n*-intercept is

$$(-22.72,0).$$

There were no unmarried couples in 1977 ($\approx 2000 - 22.7$). Nonsense.

- b) If n = 0, then h = 0.229(0) + 5.203 = 5.203. The *h*-intercept is (0, 5.203). In 2000, there where 5,203,000 unmarried-couple households.
- 107. The distance between (10,0) and (0,0) is 10. The distance between (1,3) and the origin is $\sqrt{10}$. If two points have integer coordinates, then the distance between them is of the form $\sqrt{s^2+t^2}$ where $s^2,t^2\in\{0,1,2^2,3^2,4^2,\ldots\}=\{0,1,4,9,16,\ldots\}$.

Note, there are no numbers s^2 and t^2 in $\{0, 1, 4, 9, 16, ...\}$ satisfying $s^2 + t^2 = 19$. Thus, one cannot find two points with integer coordinates whose distance between them is $\sqrt{19}$.

108. One can assume the vertices of the right triangle are A(0,0), $B(1,\sqrt{3})$, and C(1,0).

The midpoint of the hypotenuse AB is

 $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. The distance between the midpoint

and
$$C$$
 is $\sqrt{\left(\frac{1}{2}-1\right)^2+\left(\frac{\sqrt{3}}{2}-0\right)^2}=1$, which

is also the distance from the midpoint to A, and the distance from the midpoint to B.

111. On day 1, break off a 1-dollar piece and pay the gardener.

On day 2, break of a 2-dollar and pay the gardener. The gardener will give you back your change which is a 1-dollar piece.

On day 3, you pay the gardener with the 1-dollar piece you received as change from the previous day.

On day 4, pay the gardener with the 4-dollar bar. The gardener will give you back your change which will consist of a 1-dollar piece and a 2-dollar piece.

On day 5, you pay the gardener with the 1-dollar piece you received as change from the previous day.

On day 6, pay the gardener with the 2-dollar piece you received as change from day 4. The gardener will give you back your change which is a 1-dollar piece.

On day 7, pay the gardener with the 1-dollar piece you received as change from day 6.

112. Let $\triangle ABC$ be a right triangle with vertices at A(2,7), B(0,-3), and C(6,1). Notice, the midpoints of the sides of $\triangle ABC$ are (3,-1), (4,4) and (1,2). The area of $\triangle ABC$ is

$$\frac{1}{2}\overline{AC} \times \overline{BC} = \frac{1}{2}\sqrt{4^2 + 6^2}\sqrt{6^2 + 4^2}$$
$$= \frac{1}{2}(52) = 26.$$

P.1 Pop Quiz

- 1. The distance is $\sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$.
- **2.** Center (3, -5), radius 9
- **3.** Completing the square, we find

$$(x^2 + 4x + 4) + (y^2 - 10y + 25) = -28 + 4 + 25$$

 $(x + 2)^2 + (y - 5)^2 = 1.$

The center is (-2,5) and the radius is 1.

- **4.** The distance between (3,4) and the origin is 5, which is the radius. The circle is given by $(x-3)^2 + (y-4)^2 = 25$.
- **5.** By setting x = 0 and y = 0 in 2x 3y = 12 we find -3y = 12 and 2x = 12, respectively. Since y = -4 and x = 6 are the solutions of the two equations, the intercepts are (0, -4) and (6,0).
- 6. (5,-1)

For Thought

- 1 True, since the number of gallons purchased is 20 divided by the price per gallon.
- 2. False, since a student's exam grade is a function of the student's preparation. If two classmates had the same IQ and only one prepared then the one who prepared will most likely achieve a higher grade.
- **3.** False, since $\{(1,2),(1,3)\}$ is not a function.
- 4. True
- 5. True
- 6. True
- 7. False, the domain is the set of all real numbers.
- 8. True
- **9.** True, since $f(0) = \frac{0-2}{0+2} = -1$.
- **10.** True, since if a 5 = 0 then a = 5.

P.2 Exercises

- 1. function
- 2. independent, dependent
- 3. domain, range
- 4. parabola
- 5. function
- **6.** function
- 7. Note, $b = 2\pi a$ is equivalent to $a = \frac{b}{2\pi}$. Then a is a function of b, and b is a function of a.
- 8. Note, b = 2(5 + a) is equivalent to $a = \frac{b 10}{2}$. Then a is a function of b, and b is a function of a.
- **9.** *a* is a function of *b* since a given denomination has a unique length. Since a dollar bill and a five-dollar bill have the same length, then *b* is not a function of *a*.

P.2 Functions

- 10. Since different U.S. coins have different diameters, then a is a function of b, and b is a function of a.
- **11.** Since an item has only one price, b is a function of a. Since two items may have the same price, a is not a function of b.
- 12. a is not a function of b since there may be two students with the same semester grades but different final exams scores. b is not a function of a since there may be identical final exam scores with different semester grades.
- **13.** *a* is not a function of *b* since it is possible that two different students can obtain the same final exam score but the times spent on studying are different.

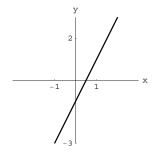
b is not a function of a since it is possible that two different students can spend the same time studying but obtain different final exam scores.

14. a is not a function of b since it is possible that two adult males can have the same shoe size but have different ages.

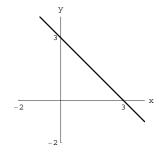
b is not a function of a since it is possible for two adults with the same age to have different shoe sizes.

- **15.** Since 1 in ≈ 2.54 cm, a is a function of b and b is a function of a.
- 16. Since there is only one cost for mailing a first class letter, then a is a function of b. Since two letters with different weights each under 1/2-ounce cost 47 cents to mail first class, b is not a function of a.
- 17. Since $b = a^3$ and $a = \sqrt[3]{b}$, we get that b is a function of a, and a is a function of b.
- **18.** Since $b = a^4$ and $a = \pm \sqrt[4]{b}$, we get that b is a function of a, but a is not a function of b.
- **19.** Since b = |a|, we get b is a function of a. Since $a = \pm b$, we find a is not a function of b.
- **20.** Note, $b = \sqrt{a}$ since $a \ge 0$, and $a = b^2$. Thus, b is a function of a, and a is a function of b.
- **21.** $A = s^2$ **22.** $s = \sqrt{A}$ **23.** $s = \frac{\sqrt{2}d}{2}$

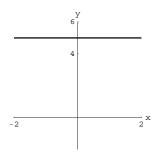
- **24.** $d = s\sqrt{2}$ **25.** P = 4s **26.** s = P/4
- **27.** $A = P^2/16$ **28.** $d = \sqrt{2A}$
- **29.** y = 2x 1 has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$, some points are (0, -1) and (1, 1)



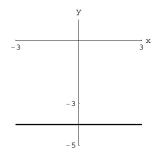
30. y = -x + 3 has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$, some points are (0, 3) and (3, 0)



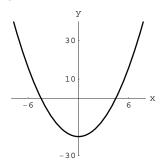
31. y = 5 has domain $(-\infty, \infty)$ and range $\{5\}$, some points are (0, 5) and (1, 5)



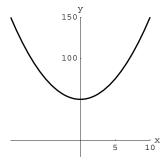
32. y = -4 has domain $(-\infty, \infty)$ and range $\{-4\}$, some points are (0, -4) and (1, -4)



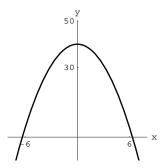
33. $y = x^2 - 20$ has domain $(-\infty, \infty)$ and range $[-20, \infty)$, some points are (0, -20) and (6, 16)



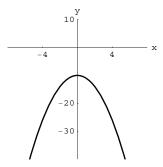
34. $y = x^2 + 50$ has domain $(-\infty, \infty)$ and range $[50, \infty)$, some points are (0, 50) and (5, 75)



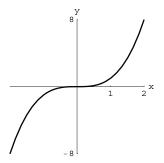
35. $y = 40 - x^2$ has domain $(-\infty, \infty)$ and range $(-\infty, 40]$, some points are (0, 40) and (6, 4)



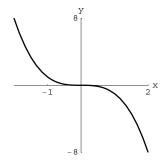
36. $y=-10-x^2$ has domain $(-\infty,\infty)$ and range $(-\infty,-10]$, some points are (0,-10) and (4,-26)



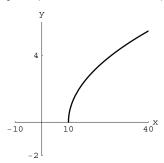
37. $y = x^3$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$, some points are (0,0) and (2,8)



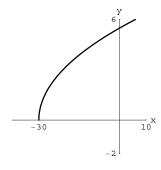
38. $y = -x^3$ has domain $(-\infty, \infty)$ and range $(-\infty, \infty)$, some points are (0,0) and (0,-8)



39. $y = \sqrt{x-10}$ has domain $[10, \infty)$ and range $[0, \infty)$, some points are (10, 0) and (14, 2)

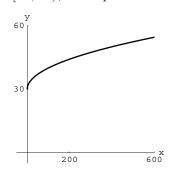


40. $y = \sqrt{x+30}$ has domain $[-30, \infty)$ and range $[0, \infty)$, some points are (-30, 0) and (-26, 2)

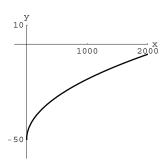


P.2 Functions

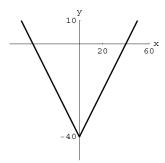
41. $y = \sqrt{x} + 30$ has domain $[0, \infty)$ and range $[30, \infty)$, some points are (0, 30) and (400, 50)



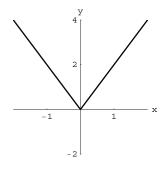
42. $y = \sqrt{x} - 50$ has domain $[0, \infty)$ and range $[-50, \infty)$, some points are (0, -50) and (900, -20)



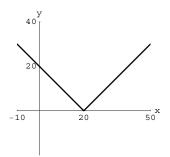
43. y = |x| - 40 has domain $(-\infty, \infty)$ and range $[-40, \infty)$, some points are (0, -40) and (40, 0)



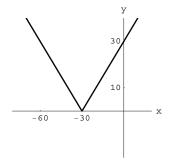
44. y = 2|x| has domain $(-\infty, \infty)$ and range $[0, \infty)$, some points are (0, 0) and (1, 2)



45. y = |x - 20| has domain $(-\infty, \infty)$ and range $[0, \infty)$, some points are (0, 20) and (20, 0)



46. y = |x + 30| has domain $(-\infty, \infty)$ and range $[0, \infty)$, some points are (0, 30) and (-30, 0)



47. $3 \cdot 4 - 2 = 10$ **48.** 3(16) + 4 = 52

49. -4-2=-6 **50.** -8-2=-10

51. |8| = 8 **52.** |-1| = 1

53. 4 + (-6) = -2 **54.** $24 \cdot 6 = 144$

55. 80 - 2 = 78 **56.** 2/2 = 1

57. $3a^2 - a$ **58.** 4b - 2

59. $f(-x) = 3(-x)^2 - (-x) = 3x^2 + x$

60. g(-x) = 4(-x) - 2 = -4x - 2

61. Factoring, we get x(3x-1)+0. So x=0,1/3.

62. Since 4x - 2 = 3, we get x = 5/4.

63. Since |a + 3| = 4 is equivalent to a + 3 = 4 or a + 3 = -4, we have a = 1, -7.

64. Since $3t^2 - t - 10 = (t - 2)(3t + 5) = 0$, we find t = 2, -5/3.

65. C = 353n

66. P = 580n

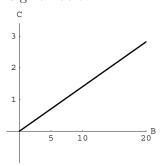
67.
$$C = 35n + 50$$

68.
$$C = 2.50 + 0.50n$$

69. We find

$$C = \frac{4B}{\sqrt[3]{D}} = \frac{4(12 + 11/12)}{\sqrt[3]{22,800}} \approx 1.822$$

and a sketch of the graph of $C = \frac{4B}{\sqrt[3]{22,800}}$ is given below.



70. Solving for B, we get

$$\frac{4B}{\sqrt[3]{22,800}} < 2$$

$$B < \frac{\sqrt[3]{22,800}}{2}$$

$$B < 14 \text{ ft, 2 in.}$$

Then the maximum displacement is 14 ft, 2 in. With D fixed, we get that C becomes larger as the beam B becomes larger. Thus, a boat is more likely to capsize as the beam gets larger.

71. Let $N=2,\,B=3.498,$ and S=4.250. Then $D\ =\ \frac{\pi}{4}B^2\cdot S\cdot N$

$$= 81.686 \text{ in.}^3$$

Then $D \approx 81.7$ in.³.

72. Let N = 2, B = 3.518, and S = 4.250. Then

$$D = \frac{\pi}{4}B^2 \cdot S \cdot N$$
$$= 82.622 \text{ in.}^3$$

Using the unrounded answer to Exercise 71, the difference in the displacement is

$$82.622 - 81.686 \approx 0.94 \text{ in.}^3$$
.

73. Solving for B,

$$D = \frac{\pi}{4}B^2 \cdot S \cdot N$$

$$\frac{4D}{\pi S \cdot N} = B^2$$

$$B = 2\sqrt{\frac{D}{\pi S \cdot N}}.$$

74. Solving for V,

$$CR = 1 + \frac{\pi B^2 \cdot S}{4V}$$

$$CR - 1 = \frac{\pi B^2 \cdot S}{4V}$$

$$\frac{1}{CR - 1} = \frac{4V}{\pi B^2 \cdot S}$$

$$V = \frac{\pi B^2 \cdot S}{4(CR - 1)}$$

- **75.** Pythagorean, legs, hypotenuse
- **76.** circle, radius, center

77 .
$$\sqrt{(2+3)^2 + (-4+6)^2} = \sqrt{29}$$

78.
$$\left(\frac{4-6}{2}, \frac{-8+16}{2}\right) = (-1, 4)$$

79. If we replace x = 0 in 4x - 6y = 40, then -6y = 40 or y = -20/3. The *y*-intercept is $(0, -\frac{20}{3})$.

If we replace y = 0 in 4x - 6y = 40, then 4x = 40 or x = 10. The x-intercept is (10,0).

- **80.** The diagonal is $\sqrt{3^2 + 7^2} = \sqrt{58}$ ft.
- **81.** Rewriting the equation, we find

$$\frac{1}{3^{3}} \cdot 3^{100} \cdot \frac{1}{3^{4}} \cdot 3^{2x} = \frac{1}{3} \cdot 3^{x}$$

$$\frac{1}{3^{3}} \cdot 3^{100} \cdot \frac{1}{3^{4}} \cdot 3^{2x} = 3^{x-1}$$

$$3^{2x+93} = 3^{x-1}$$

$$2x+93 = x-1$$

$$x = -94.$$

82. First, $9 = (a+b)^2 = (a^2+b^2) + 2ab = 89 + 2ab$. Then ab = -40. Thus,

$$a^{3} + b^{3} = (a+b)(a^{2} + b^{2} - ab)$$

= 3(89 + 40)
= 387.

P.2 Pop Quiz

- 1. Yes, since $r \ge 0$ and $r = \sqrt{A/\pi}$
- **2.** Since $A = s^2$ and $s \ge 0$, we obtain $s = \sqrt{A}$.
- **3.** No, since $b = \pm a$.
- **4.** $[1, \infty)$
- **5.** $[2, \infty)$
- **6.** f(3) = 3(3) + 6 = 15
- 7. We find

$$2a - 4 = 10$$
$$2a = 14$$
$$a = 7.$$

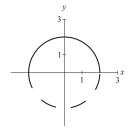
For Thought

- 1. False, it is a reflection in the y-axis.
- 2. False, the graph of $y = x^2 4$ is shifted down 4 units from the graph of $y = x^2$.
- **3.** False, rather it is a left translation.
- 4. True
- 5. True
- **6.** False, the down shift should come after the reflection.
- 7. True
- 8. False, since the domains are different.
- 9. True
- 10. True, since f(-x) = -f(x) where $f(x) = x^3$.

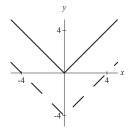
P.3 Exercises

- 1. rigid
- 2. nonrigid
- 3. reflection
- 4. upward translation, downward translation
- 5. right, left
- 6. stretching, shrinking
- **7.** odd
- 8. even
- 9. transformation
- **10.** family
- **11.** $f(x) = \sqrt{x}, g(x) = -\sqrt{x}$
- 12. $f(x) = x^2 + 1$, $g(x) = -x^2 1$
- **13.** y = x, y = -x

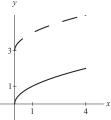
14.
$$y = \sqrt{4 - x^2}$$
, $y = -\sqrt{4 - x^2}$



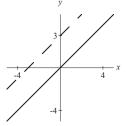
15.
$$f(x) = |x|, g(x) = |x| - 4$$



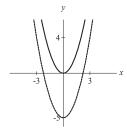
16.
$$f(x) = \sqrt{x}$$
, $g(x) = \sqrt{x} + 3$



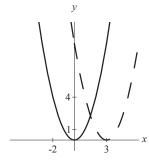
17.
$$f(x) = x$$
, $g(x) = x + 3$



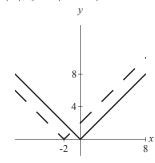
18.
$$f(x) = x^2$$
, $g(x) = x^2 - 5$



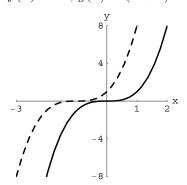
19.
$$y = x^2$$
, $y = (x - 3)^2$



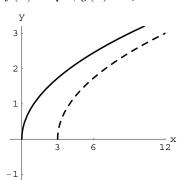
20.
$$y = |x|, y = |x + 2|$$



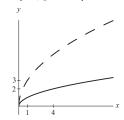
21.
$$f(x) = x^3$$
, $g(x) = (x+1)^3$



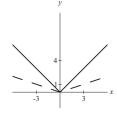
22.
$$f(x) = \sqrt{x}, g(x) = \sqrt{x-3}$$



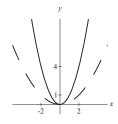
23.
$$y = \sqrt{x}, y = 3\sqrt{x}$$



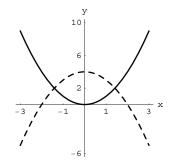
24.
$$y = |x|, y = \frac{1}{3}|x|$$



25.
$$y = x^2, y = \frac{1}{4}x^2$$



26.
$$y = x^2$$
, $y = 4 - x^2$



35.
$$y = (x - 10)^2 + 4$$

36.
$$y = \sqrt{x+5} - 12$$

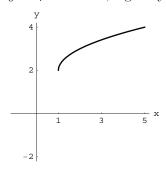
37.
$$y = -3|x - 7| + 9$$

38.
$$y = -2(x+6) - 8$$
 or $y = -2x - 20$

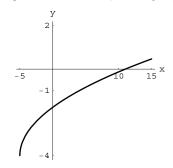
39.
$$y = -(3\sqrt{x} + 5)$$
 or $y = -3\sqrt{x} - 5$

40.
$$y = -((x-13)^2 - 6)$$
 or $y = -(x-13)^2 + 6$

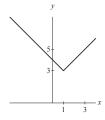
41. $y = \sqrt{x-1} + 2$; right by 1, up by 2



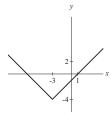
42. $y = \sqrt{x+5} - 4$; left by 5, down by 4



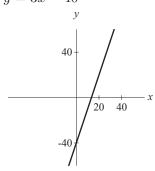
43. y = |x - 1| + 3; right by 1, up by 3

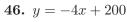


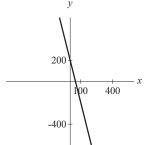
44. y = |x + 3| - 4; left by 3, down by 4



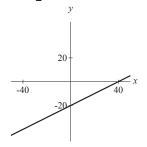
45. y = 3x - 40



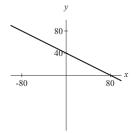




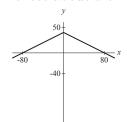
47.
$$y = \frac{1}{2}x - 20$$



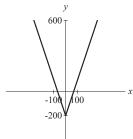
48.
$$y = -\frac{1}{2}x + 40$$



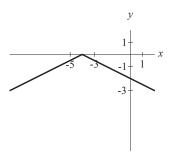
49. $y = -\frac{1}{2}|x| + 40$, shrink by 1/2, reflect about *x*-axis, up by 40



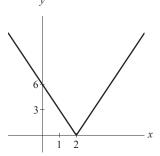
50. y = 3|x| - 200, stretch by 3, down by 200



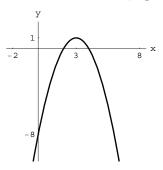
51. $y = -\frac{1}{2}|x+4|$, left by 4, reflect about x-axis, shrink by 1/2



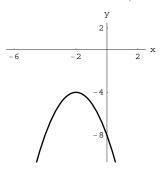
52. y = 3|x - 2|, right by 2, stretch by 3



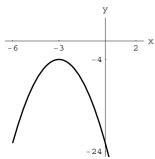
53. $y = -(x-3)^2 + 1$; right by 3, reflect about *x*-axis, up by 1



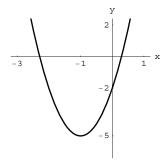
54. $y = -(x+2)^2 - 4$; left by 2, reflect about *x*-axis, down by 4



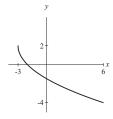
55. $y = -2(x+3)^2 - 4$; left by 3, stretch by 2, reflect about x-axis, down by 4



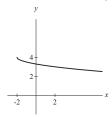
56. $y = 3(x+1)^2 - 5$; left by 1, stretch by 3, down by 5



57. $y = -2\sqrt{x+3} + 2$, left by 3, stretch by 2, reflect about x-axis, up by 2



58. $y = -\frac{1}{2}\sqrt{x+2} + 4$, left by 2, shrink by 1/2, reflect about x-axis, up by 4



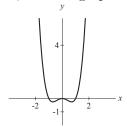
- **59.** Symmetric about y-axis, even function since f(-x) = f(x)
- **60.** Symmetric about y-axis, even function since f(-x) = f(x)
- **61.** No symmetry, neither even nor odd since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

- **62.** Symmetric about the origin, odd function since f(-x) = -f(x)
- **63.** Neither symmetry, neither even nor odd since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$
- **64.** Neither symmetry, neither even nor odd since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$
- **65.** No symmetry, not an even or odd function since f(-x) = -f(x) and $f(-x) \neq -f(x)$
- **66.** Symmetric about the y-axis, even function since f(-x) = f(x)
- **67.** Symmetric about the origin, odd function since f(-x) = -f(x)
- **68.** Symmetric about the origin, odd function since f(-x) = -f(x)
- **69.** No symmetry, not an even or odd function since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$
- **70.** No symmetry, not an even or odd function since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$
- **71.** No symmetry, not an even or odd function since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$
- **72.** Symmetric about the y-axis, even function since f(-x) = f(x)
- **73.** Neither symmetry, not an even or odd function since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$
- **74.** Symmetric about the y-axis, even function since f(-x) = f(x)
- **75.** Symmetric about the y-axis, even function since f(-x) = f(x)
- **76.** Symmetric about the y-axis, even function since f(-x) = f(x) since f(-x) = f(x) since f(-x) = f(x)
- **77.** e **78.** a **79.** g **80.** h
- **81.** b **82.** d **83.** c **84.** f
- **85.** N(x) = x + 2000

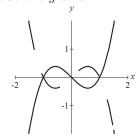
- **86.** N(x) = 1.05x + 3000. Yes, if the merit increase is followed by the cost of living raise then the new salary becomes higher and is N'(x) = 1.05(x + 3000) = 1.05x + 3150.
- 87. If inflation rate is less than 50%, then $1-\sqrt{x}<\frac{1}{2}.$ This simplifies to $\frac{1}{2}<\sqrt{x}.$ After squaring we have $\frac{1}{4}< x$ and so x>25%.
- 88. If production is at least 28 windows, then $1.75\sqrt{x} \ge 28$. They need at least $x = \left(\frac{28}{1.75}\right)^2 = 256$ hrs.

89.

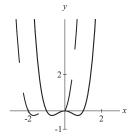
(a) Both functions are symmetric about the y-axis, and the graphs are identical.



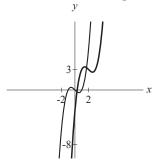
(b) One graph is a reflection of the other about the y-axis, and both are symmetric about the y-axis.



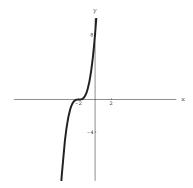
(c) The second graph is obtained by translating the first one to the left by 1 unit.



(d) The second graph is obtained by translating the first one to the right by 2 units and 3 units up.



90. The graph of $y = x^3 + 6x^2 + 12x + 8$ or equivalently $y = (x+2)^3$ can be obtained by shifting the graph of $y = x^3$ to the left by 2 units.



- **91.** $x^2 + y^2 = 1$
- **92.** y = 5
- **93.** x = 4
- **94.** If A is the area and s is the length of a side, then $A = s^2$.

95.
$$f(6) = 2(36) - 3(6) = 72 - 18 = 54$$

 $f(-x) = 2(-x)^2 - 3(-x) = 2x^2 + 3x$

96. Solve for *a*:

$$2a^{2} + 1 = 9$$

$$2a^{2} = 8$$

$$a^{2} = 4$$

$$a = \pm 2$$

97. Draw the numbers in $\{1, 2, 3, ..., 999\}$ that can be written in the form

$$(2n+1)\cdot 2^{2m}$$

for $n, m \geq 0$. These means that you will have drawn 666 numbers. Then the 667th number you will draw has the form

$$(2n+1)\cdot 2^{2m+1}$$

which must be twice one of the first 666 numbers.

98. Let $A(x_1, 2)$ and $B(x_2, 2)$ be the other two opposite vertices where $x_1 < x_2$. Since the sides at B are perpendicular, the slopes are negative reciprocals, i.e.,

$$\frac{3}{3-x_2} = -\frac{x_2-1}{3}.$$

Solving, $x_2 = 2 + \sqrt{10}$. Since the distance AB is the same as the distance between (1, -1) and (3, 5), we obtain

$$x_{2} - x_{1} = \sqrt{40} = 2\sqrt{10}$$

$$x_{1} = x_{2} - 2\sqrt{10}$$

$$x_{1} = (2 + \sqrt{10}) - 2\sqrt{10}$$

$$x_{1} = 2 - \sqrt{10}$$

Then the other vertices are $(2 \pm \sqrt{10}, 2)$.

P.3 Pop Quiz

1.
$$y = \sqrt{x} + 8$$

2.
$$y = (x - 9)^2$$

3.
$$y = (-x)^3$$
 or $y = -x^3$

4. Domain $[1, \infty)$, range $(-\infty, 5]$

5.
$$y = -3(x-6)^2 + 4$$

6. Even function

For Thought

- **1** True, since $A = P^2/16$.
- **2.** False, rather $y = (x-1)^2 = x^2 2x + 1$.
- **3.** False, rather $(f \circ g)(x) = \sqrt{x-2}$.
- 4. True
- **5.** False, since $(h \circ g)(x) = x^2 9$.
- **6.** False, rather $f^{-1}(x) = \frac{1}{2}x \frac{1}{2}$.
- 7. True
- **8.** False; g^{-1} does not exist since the graph of g which is a parabola fails the horizontal line test.
- **9.** False, since $y = x^2$ is a function which does not have an inverse function.
- **10.** True

P.4 Exercises

- 1. composition
- 2. one-to-one
- 3. invertible
- 4. inverse
- **5.** switch-and-solve
- 6. symmetric

7.
$$y = 2(3x + 1) - 3 = 6x - 1$$

8.
$$y = -4(-3x - 2) - 1 = 12x + 7$$

9.
$$y = (x^2 + 6x + 9) - 2 = x^2 + 6x + 7$$

10.
$$y = 3(x^2 - 2x + 1) - 3 = 3x^2 - 6x$$

11.
$$y = 3 \cdot \frac{x+1}{3} - 1 = x + 1 - 1 = x$$

12.
$$y = 2\left(\frac{1}{2}x - \frac{5}{2}\right) + 5 = x - 5 + 5 = x$$

13.
$$y = 2 \cdot \frac{x+1}{2} - 1 = x + 1 - 1 = x$$

14.
$$y = \frac{(5x-1)+1}{5} = \frac{5x}{5} = x$$

15.
$$f(2) = 5$$
 16. $g(-4) = 17$

17.
$$f(2) = 5$$
 18. $h(-22) = -7$

19.
$$f(17) = 3(17) - 1 = 50$$

20.
$$g\left(-\frac{8}{3}\right) = \frac{64}{9} + 1 = \frac{73}{9}$$

21.
$$3(x^2+1)-1=3x^2+2$$

22.
$$(3x-1)^2 + 1 = 9x^2 - 6x + 2$$

23.
$$(3x-1)^2+1=9x^2-6x+2$$

24.
$$3(x^2+1)-1=3x^2+2$$
 25. $\frac{x^2+2}{3}$

26.
$$\frac{x^2+2x+1}{9}+1=\frac{1}{9}x^2+\frac{2}{9}x+\frac{10}{9}$$

27.
$$F = g \circ h$$
 28. $G = g \circ f$

29.
$$H = h \circ g$$
 30. $M = f \circ g$

31.
$$N = f \circ h$$
 32. $R = h \circ f$

33. Interchange x and y then solve for y.

$$x = 3y - 7$$

$$\frac{x+7}{3} = y$$

$$\frac{x+7}{3} = f^{-1}(x)$$

34. Interchange x and y then solve for y.

$$x = -2y + 5$$

$$2y = 5 - x$$

$$f^{-1}(x) = \frac{5 - x}{2}$$

$$f^{-1}(x) = \frac{x - 5}{-2}$$

35. Interchange x and y then solve for y.

$$x = 2 + \sqrt{y-3}$$
 for $x \ge 2$
 $(x-2)^2 = y-3$ for $x \ge 2$
 $f^{-1}(x) = (x-2)^2 + 3$ for $x \ge 2$

36. Interchange x and y then solve for y.

$$x = \sqrt{3y-1} \quad \text{for } x \ge 0$$

$$x^2 + 1 = 3y \quad \text{for } x \ge 0$$

$$f^{-1}(x) = \frac{x^2 + 1}{3} \quad \text{for } x \ge 0$$

37. Interchange x and y then solve for y.

$$x = -y - 9$$

$$y = -x - 9$$

$$f^{-1}(x) = -x - 9$$

38. Interchange x and y then solve for y.

$$x = -y+3$$

$$y = -x+3$$

$$f^{-1}(x) = -x+3$$

39. Interchange x and y then solve for y.

$$x = -\frac{1}{y}$$

$$xy = -1$$

$$f^{-1}(x) = -\frac{1}{x}$$

- **40.** Clearly $f^{-1}(x) = x$
- **41.** Interchange x and y then solve for y.

$$x = \sqrt[3]{y-9} + 5$$

$$x-5 = \sqrt[3]{y-9}$$

$$(x-5)^3 = y-9$$

$$f^{-1}(x) = (x-5)^3 + 9$$

42. Interchange x and y then solve for y.

$$x = \sqrt[3]{\frac{y}{2}} + 5$$

$$x - 5 = \sqrt[3]{\frac{y}{2}}$$

$$(x - 5)^3 = \frac{y}{2}$$

$$f^{-1}(x) = 2(x - 5)^3$$

43. Interchange x and y then solve for y.

$$x = (y-2)^2 \quad x \ge 0$$

$$\sqrt{x} = y-2$$

$$f^{-1}(x) = \sqrt{x}+2$$

44. Interchange x and y then solve for y.

$$x = (y+3)^2 \quad x \ge 0$$

$$\sqrt{x} = y+3$$

$$f^{-1}(x) = \sqrt{x}-3$$

45.
$$(f^{-1} \circ f)(x) = \frac{1}{2}(2x - 1) + \frac{1}{2} = x$$
 and $(f \circ f^{-1})(x) = 2\left(\frac{1}{2}x + \frac{1}{2}\right) - 1 = x$

46.
$$(f^{-1} \circ f)(x) = 3\left(\frac{x+1}{3}\right) - 1 = x$$
 and $(f \circ f^{-1})(x) = \frac{(3x-1)+1}{3} = x$

47.
$$(f^{-1} \circ f)(x) = 0.25(4x + 4) - 1 = x$$
 and $(f \circ f^{-1})(x) = 4(0.25x - 1) + 4 = x$

48.
$$(f^{-1} \circ f)(x) = -0.2(20 - 5x) + 4 = x$$
 and $(f \circ f^{-1})(x) = 20 - 5(-0.2x + 4) = x$

49. We obtain

$$(f^{-1} \circ f)(x) = \frac{4 - \left(\sqrt[3]{4 - 3x}\right)^3}{3}$$
$$= \frac{4 - (4 - 3x)}{3}$$
$$= \frac{3x}{3}$$
$$(f^{-1} \circ f)(x) = x$$

and

$$(f \circ f^{-1})(x) = \sqrt[3]{4 - 3\left(\frac{4 - x^3}{3}\right)}$$
$$= \sqrt[3]{4 - (4 - x^3)}$$
$$= \sqrt[3]{x^3}$$
$$(f \circ f^{-1})(x) = x.$$

50. We obtain

$$(f^{-1} \circ f)(x) = \sqrt[3]{(x^3 + 5) - 5}$$

= $\sqrt[3]{x^3}$
 $(f^{-1} \circ f)(x) = x$

and

$$(f \circ f^{-1})(x) = (\sqrt[3]{x-5})^3 + 5$$

= $(x-5)+5$
 $(f \circ f^{-1})(x) = x.$

51. We obtain

$$(f^{-1} \circ f)(x) = \sqrt[5]{\left(\sqrt[3]{x^5 - 1}\right)^3 + 1}$$

$$= \sqrt[5]{x^5 - 1 + 1}$$

$$= \sqrt[5]{x^5}$$

$$(f^{-1} \circ f)(x) = x$$

and

$$(f \circ f^{-1})(x) = \sqrt[3]{\left(\sqrt[5]{x^3 + 1}\right)^5 - 1}$$
$$= \sqrt[3]{x^3 + 1 - 1}$$
$$= \sqrt[3]{x^3}$$
$$(f \circ f^{-1})(x) = x.$$

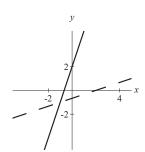
52. We find

$$(f^{-1} \circ f)(x) = (x^{3/5} - 3) + 3)^{5/3}$$
$$= (x^{3/5})^{5/3}$$
$$(f^{-1} \circ f)(x) = x$$

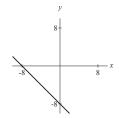
and

$$(f \circ f^{-1})(x) = ((x+3)^{5/3})^{3/5} - 3$$
$$= (x-3) - 3$$
$$(f \circ f^{-1})(x) = x.$$

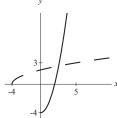
53.
$$f^{-1}(x) = \frac{x-2}{3}$$



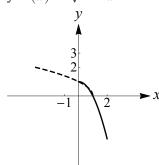
54.
$$f^{-1}(x) = -x - 8$$



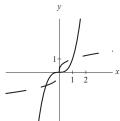
55.
$$f^{-1}(x) = \sqrt{x+4}$$



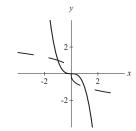
56.
$$f^{-1}(x) = \sqrt{1-x}$$



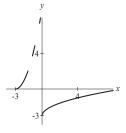
57.
$$f^{-1}(x) = \sqrt[3]{x}$$



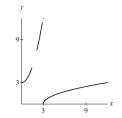
58.
$$f^{-1}(x) = -\sqrt[3]{x}$$



59.
$$f^{-1}(x) = (x+3)^2$$
 for $x \ge -3$



60.
$$f^{-1}(x) = x^2 + 3$$
 for $x \ge 0$



- **61.** The inverse is the composition of subtracting 2 and dividing by 5, i.e., $f^{-1}(x) = \frac{x-2}{5}$
- **62.** The inverse is the composition of adding 1 and dividing by 0.5, i.e., $f^{-1}(x) = \frac{x+1}{0.5} = 2x+2$
- **63.** The inverse is the composition of adding 88 and dividing by 2, i.e.,

$$f^{-1}(x) = \frac{x+88}{2} = \frac{1}{2}x + 44$$

64. The inverse is the composition of subtracting 99 and dividing by 3, i.e.,

$$f^{-1}(x) = \frac{x - 99}{3} = \frac{1}{3}x - 33$$

65. The inverse is the composition of subtracting 4 and dividing by -3, i.e.,

$$f^{-1}(x) = \frac{x-4}{-3} = -\frac{1}{3}x + \frac{4}{3}$$

66. The inverse is the composition of adding 5 and dividing by -2, i.e.,

$$f^{-1}(x) = \frac{x-5}{-2} = -\frac{1}{2}x + \frac{5}{2}$$

67. The inverse is the composition of adding 9 and multiplying by 2, i.e.,

$$f^{-1}(x) = 2(x+9) = 2x+18$$

68. The inverse is the composition of subtracting 6 and multiplying by 3, i.e.,

$$f^{-1}(x) = 3(x - 6) = 3x - 18$$

69. The inverse is the composition of subtracting 3 and taking the multiplicative inverse, i.e.,

$$f^{-1}(x) = \frac{1}{x-3}$$

70. The inverse is the composition of taking the multiplicative inverse and subtracting 3, i.e.,

$$f^{-1}(x) = \frac{1}{x} - 3$$

71. The inverse is the composition of adding 9 and raising an expression to the third power, i.e.,

$$f^{-1}(x) = (x+9)^3.$$

- **72.** The inverse is the composition of raising an expression to the third power and adding 9, i.e., $f^{-1}(x) = x^3 + 9$.
- **73.** The inverse is the composition of adding 7, dividing by 2, and taking the cube root, i.e.,

$$f^{-1}(x) = \sqrt[3]{\frac{x+7}{2}}.$$

- **74.** The inverse is the composition of subtracting by 4, multiplying by -1, and taking the cube root, i.e., $f^{-1}(x) = \sqrt[3]{-(x-4)} = \sqrt[3]{4-x}$
- **75.** $A = d^2/2$ **76.** $P = 4\sqrt{A}$
- **77.** Let x be the number of ice cream bars sold. Then W(x) = (0.20)(0.40x + 200) = 0.08x + 40.
- **78.** Let x be the number of radios manufactured. Then P(x) = (0.80)(0.90x) = 0.72x.
- **79.** C = 1.08P expresses the total cost as a function of the purchase price; and P = C/1.08 is the purchase price as a function of the total cost.
- **80.** $V(x) = x^3, S(x) = \sqrt[3]{x}$

81.
$$r = \sqrt{5.6}$$

$$r = \sqrt{5.625 \times 10^{-5} - \frac{V}{500}}$$

where $0 \le V \le 0.028125$

82. Solving for I in $E = 0.4I + 10{,}000$, we

obtain
$$\frac{E - 10,000}{0.4} = I$$
 or equivalently

$$I = \frac{5}{2}E - 25,000.$$

If E=I, then I=0.4I+10,000 or 0.6I=10,000. Thus, if E=I then $I=\frac{10,000}{0.6}$ or I=\$16,666.67.

83. When V = \$18,000, the depreciation rate is

$$1 - \left(\frac{18,000}{50,000}\right)^{1/5} \approx 0.1848$$

or the depreciation rate is 18.48%. Solving for V, we obtain

$$\left(\frac{V}{50,000}\right)^{1/5} = 1 - r$$

$$\frac{V}{50,000} = (1 - r)^5$$

$$V = 50,000(1 - r)^5.$$

84. When P = \$23,580, the annual growth rate is

$$\left(\frac{23,580}{10,000}\right)^{1/10} - 1 \approx 0.0896$$

or the annual growth rate is 8.96%.

Solving for P, we obtain

$$\left(\frac{P}{10,000}\right)^{1/10} = r+1$$

$$\frac{P}{10,000} = (r+1)^{10}$$

$$P = 10,000(r+1)^{10}.$$

85. Since $g^{-1}(x) = \frac{x+5}{3}$ and $f^{-1}(x) = \frac{x-1}{2}$, we have

$$g^{-1} \circ f^{-1}(x) = \frac{\frac{x-1}{2} + 5}{\frac{3}{2}} = \frac{x+9}{6}.$$

Likewise, since $(f \circ g)(x) = 6x - 9$, we get

$$(f \circ g)^{-1}(x) = \frac{x+9}{6}.$$

Hence, $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

- **86.** Since $(f \circ g \circ g^{-1} \circ f^{-1})(x) = (f \circ (g \circ g^{-1}))(f^{-1}(x)) = f(f^{-1}(x)) = x$ and the range of one function is the domain of the other function, we have $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.
- **87.** $x^2 + (y-1)^2 = 9$

88.
$$\left(\frac{\pi/3 + \pi/2}{2}, \frac{1+1}{2}\right) = \left(\frac{5\pi/6}{2}, 1\right) = \left(\frac{5\pi}{12}, 1\right)$$

- 89. function
- 90. reflection
- **91.** Domain $(-\infty, \infty)$, range $[1, \infty)$
- **92.** Since $x 2 \ge 0$, the domain $[2, \infty)$. Note, the range of $-\sqrt{x}$ is $(-\infty, 0]$. Then the range of $y = -\sqrt{x-2} + 3$ is $(-\infty, 3]$.
- **93.** Note, if a > b > 0 and n > 0, then

$$\frac{a}{n} + \frac{b}{n+1} > \frac{b}{n} + \frac{a}{n+1}.$$

Thus, the arrangement with the largest sum is

$$\frac{2025}{1} + \frac{2024}{2} + \dots + \frac{2}{2024} + \frac{1}{2025}$$
.

94. If f(x) = mx + b, then

$$(f \circ f \circ f)(x) = m^3 x + b(m^2 + m + 1).$$

Since

$$(f \circ f \circ f)(x) = 27x + 26$$

we obtain $m^3 = 27$ or m = 3.

Also, $b(m^2 + m + 1) = 26$. If we substitute m = 3, then 13b = 26 or m = 2.

Thus, f(x) = 3x + 2. The y-intercept is (0, 2).

P.4 Pop Quiz

1.
$$A = \pi r^2 = \pi (d/2)^2$$
 or $A = \frac{\pi d^2}{4}$

2.
$$y = 3(4x - 5) - 1 = 12x - 16$$

3.
$$(f \circ g)(5) = f(3) = 9$$

- **4.** $M = g \circ h \circ f$
- **5.** Interchange x and y then solve for y.

$$2y - 1 = x$$

$$y = \frac{x+1}{2}$$

$$f^{-1}(x) = \frac{x+1}{2}$$

6. Interchange x and y then solve for y.

$$\sqrt[3]{y+1} - 4 = x$$

$$y+1 = (x+4)^3$$

$$y = (x+4)^3 - 1$$

$$g^{-1}(x) = (x+4)^3 - 1$$

Chapter P Review Exercises

1.
$$\sqrt{49 \cdot 2} = 7\sqrt{2}$$

2.
$$\sqrt{100 \cdot 2} = 10\sqrt{2}$$

3.
$$\frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$4. \ \frac{4}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{4\sqrt{13}}{13}$$

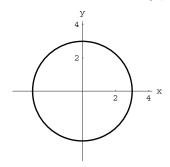
5.
$$\frac{\sqrt{5}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{30}}{6}$$

6.
$$\frac{\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{4}$$

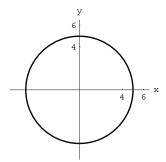
7.
$$\frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

8.
$$\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$$

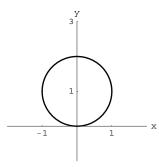
- 9. The distance is $\sqrt{(-3-2)^2 + (5-(-6))^2} = \sqrt{(-5)^2 + 11^2} = \sqrt{25 + 121} = \sqrt{146}$. The midpoint is $\left(\frac{-3+2}{2}, \frac{5-6}{2}\right) = \left(-\frac{1}{2}, -\frac{1}{2}\right)$.
- **10.** The distance is $\sqrt{(-1 (-2))^2 + (1 (-3))^2} = \sqrt{1^2 + 4^2} = \sqrt{17}.$ The midpoint is $\left(\frac{-1 2}{2}, \frac{1 3}{2}\right) = \left(-\frac{3}{2}, -1\right).$
- 11. The distance is $\sqrt{\left(\pi \frac{\pi}{2}\right)^2 + (1-1)^2} = \sqrt{\left(\frac{\pi}{2}\right)^2 + 0} = \frac{\pi}{2}$. The midpoint is $\left(\frac{\pi/2 + \pi}{2}, \frac{1+1}{2}\right) = \left(\frac{3\pi}{4}, 1\right)$.
- **12.** The distance is $\sqrt{\left(\frac{\pi}{2} \frac{\pi}{3}\right)^2 + (2-2)^2} = \sqrt{\left(\frac{\pi}{6}\right)^2 + 0} = \frac{\pi}{6}$. The midpoint is $\left(\frac{\pi/2 + \pi/3}{2}, \frac{2+2}{2}\right) = \left(\frac{5\pi}{12}, 2\right)$.
- 13. Circle with center at (0,0) and radius 3



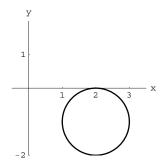
14. Circle with center at (0,0) and radius 5



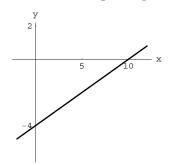
15. Circle with center at (0,1) and radius 1



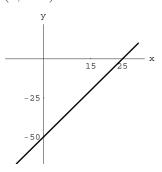
16. Circle with center at (2, -1) and radius 1



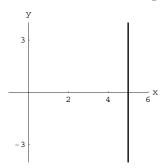
17. The line through the points (10,0) and (0,-4).



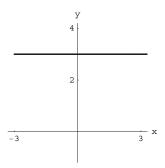
18. The line through the points (25,0) and (0,-50).



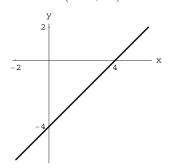
19. The vertical line through the point (5,0).



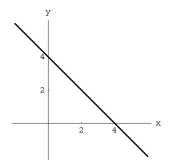
20. The horizontal line through the point (0,3).



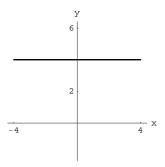
21. Domain $(-\infty, \infty)$ and range $(-\infty, \infty)$



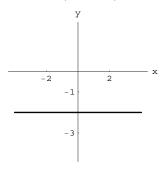
22. Domain $(-\infty, \infty)$ and range $(-\infty, \infty)$



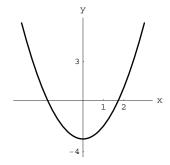
23. Domain $(-\infty, \infty)$ and range $\{4\}$. The horizontal line through the point (0, 4).



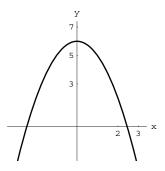
24. Domain $(-\infty, \infty)$ and range $\{-2\}$



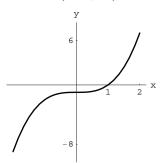
25. Domain $(-\infty, \infty)$ and range $[-3, \infty)$



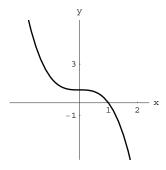
26. Domain $(-\infty, \infty)$ and range $(-\infty, 6]$



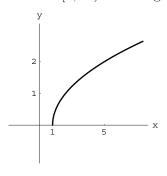
27. Domain $(-\infty, \infty)$ and range $(-\infty, \infty)$



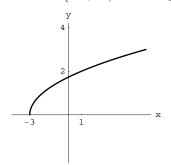
28. Domain $(-\infty, \infty)$ and range $(-\infty, \infty)$



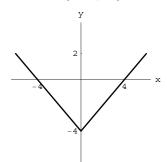
29. Domain $[1, \infty)$ and range $[0, \infty)$



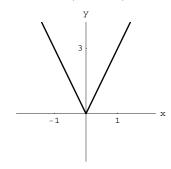
30. Domain $[-3, \infty)$ and range $[0, \infty)$



31. Domain $(-\infty, \infty)$ and range $[-4, \infty)$



32. Domain $(-\infty, \infty)$ and range $[0, \infty)$



33.
$$9+3=12$$
 34. $6-7=-1$

35.
$$24 - 7 = 17$$
 36. $1 + 3 = 4$

37.
$$g(12) = 17$$
 38. $f(-1) = 4$

39.
$$f(-3) = 12$$
 40. $g(7) = 7$

41.

$$f(g(x)) = f(2x-7)$$

$$= (2x-7)^{2} + 3$$

$$= 4x^{2} - 28x + 52$$

42.

$$g(f(x)) = g(x^2 + 3)$$

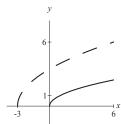
= $2(x^2 + 3) - 7$
= $2x^2 - 1$

43.
$$g\left(\frac{x+7}{2}\right) = (x+7) - 7 = x$$

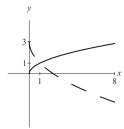
44.
$$f(\sqrt{x-3}) = (x-3) + 3 = x$$

45.
$$g^{-1}(x) = \frac{x+7}{2}$$

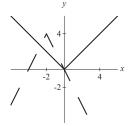
- **46.** From No. 45, $g^{-1}(-3) = \frac{-3+7}{2} = 2$
- **47.** $f(x) = \sqrt{x}$, $g(x) = 2\sqrt{x+3}$; left by 3, stretch by 2



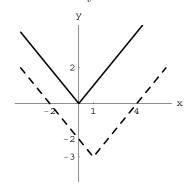
48. $f(x) = \sqrt{x}$, $g(x) = -2\sqrt{x} + 3$; stretch by 2, reflect about x-axis, up by 3



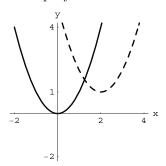
49. f(x) = |x|, g(x) = -2|x+2| + 4; left by 2, stretch by 2, reflect about x-axis, up by 4



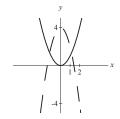
50. f(x) = |x|, g(x) = |x - 1| - 3; right by 1 and down by 3



51. $f(x) = x^2$, $g(x) = (x-2)^2 + 1$; right by 2 and up by 1

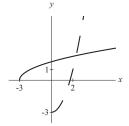


52. $f(x) = x^2$, $g(x) = -2x^2 + 4$; stretch by 2, reflect about x-axis, up by 4



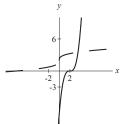
- **53.** $N = h \circ j$ **54.** $P = j \circ h$
- **55.** $R = h \circ g$ **56.** $Q = g \circ h$
- **57.** $F = f \circ g$ **58.** $G = g \circ f$
- **59.** $H = f \circ g \circ j$ **60.** $M = j \circ h \circ g$
- **61.** y = |x| 3, domain is $(-\infty, \infty)$, range is $[-3, \infty)$
- **62.** y = |x 3| 1, domain is $(-\infty, \infty)$, range is $[-1, \infty)$
- **63.** y = -2|x| + 4, domain is $(-\infty, \infty)$, range is $(-\infty, 4]$
- **64.** $y = -\frac{1}{2}|x| + 2$, domain is $(-\infty, \infty)$, range is $(-\infty, 2]$
- **65.** y = |x+2| + 1, domain is $(-\infty, \infty)$, range is $[1, \infty)$
- **66.** y = -|x+1| 2, domain $(-\infty, \infty)$, range $(-\infty, -2]$
- **67.** Symmetry: *y*-axis **68.** Symmetry: *y*-axis
- **69.** Symmetric about the origin
- **70.** Symmetric about the origin

- 71. Neither symmetry 72. Neither symmetry
- **73.** Symmetric about the y-axis
- **74.** Symmetric about the *y*-axis
- **75.** inverse functions, $f(x) = \sqrt{x+3}, g(x) = x^2 3$ for $x \ge 0$



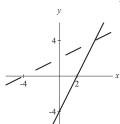
76. inverse functions,

$$f(x) = (x-2)^3, g(x) = \sqrt[3]{x} + 2$$



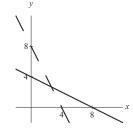
77. inverse functions,

$$f(x) = 2x - 4, g(x) = \frac{1}{2}x + 2$$



78. inverse functions,

$$f(x) = -\frac{1}{2}x + 4, g(x) = -2x + 8$$



79.
$$f^{-1}(x) = \frac{x}{5}$$
 80. $f^{-1}(x) = x - 9$

81.
$$f^{-1}(x) = \frac{x+21}{3}$$
 or $f^{-1}(x) = \frac{1}{3}x + 7$

82.
$$f^{-1}(x) = \frac{x+5}{2}$$
 or $f^{-1}(x) = \frac{1}{2}x + \frac{5}{2}$

83.
$$f^{-1}(x) = \sqrt[3]{x+1}$$

84.
$$f^{-1}(x) = x^3 - 5$$

85.
$$f^{-1}(x) = \frac{1}{x} + 3$$

86.
$$f^{-1}(x) = \frac{1}{x-5}$$

87.
$$f^{-1}(x) = \sqrt{x}$$

88.
$$f^{-1}(x) = x^2 + 3$$
 where $x \ge 0$

89.
$$\sqrt{(\sqrt{5})^2 + 2^2} = \sqrt{9} = 3$$

90.
$$\sqrt{3^2-1^2}=\sqrt{8}=2\sqrt{2}$$

- **91.** From 3x 4(0) = 9, we get that the x-intercept is (3,0). From 3(0) 4y = 9, we find that the y-intercept is $\left(0, -\frac{9}{4}\right)$.
- **92.** From 0 = 2x 3, we get that the x-intercept is $\left(\frac{3}{2}, 0\right)$. From y = 2(0) 3, we find that the y-intercept is (0, -3).

93.
$$(x+3)^2 + (y-5)^2 = 3$$

94. The center is (3, -1) and the radius is 4.

95. Since
$$C = 2\pi r$$
, we get $r = \frac{C}{2\pi}$.

96. Since
$$A = \pi \left(\frac{d}{2}\right)^2$$
, we get $4A = \pi d^2$.
Then $d = \sqrt{\frac{4A}{\pi}}$ or $d = 2\sqrt{\frac{A}{\pi}}$.

97. No, since the product of the numbers from 1 through 9 is

$$1 \cdot 2 \cdot 3 \cdot \cdot \cdot 9 = 362,880$$

and
$$71^3 = 357911$$
.

98. The fourth point of the parallelogram is either Q(15, a-30) or R(25, a+30). Suppose the two unknown points are P(20, a) and Q(15, a-30). The line through the origin and point Q is

$$\frac{a - 30}{15}x - y = 0.$$

We apply a known formula. The shortest distance between point (x_0, y_0) and a line Ax + By + C = 0 is

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}.$$

Then the shortest distance h between (5,30) and the line through the origin and Q is

$$h = \frac{\left|\frac{a-30}{15}(5) - 30\right|}{\sqrt{\left(\frac{a-30}{15}\right)^2 + 1}} = \frac{\left|\frac{a-120}{3}\right|}{\sqrt{\left(\frac{a-30}{15}\right)^2 + 1}}.$$

The distance b between the origin and Q is

$$b = \sqrt{15^2 + (a - 30)^2}.$$

The area of a parallelogram is the product of the base b and height h. Since the area is 580, we obtain through some cancelations that

$$\sqrt{15^2 + (a - 30)^2} \frac{\left|\frac{a - 120}{3}\right|}{\sqrt{\left(\frac{a - 30}{15}\right)^2 + 1}} = 580$$

$$15\left|\frac{a - 120}{3}\right| = 580$$

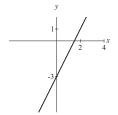
$$|a - 120| = 116.$$

Hence, a = 4 or a = 236.

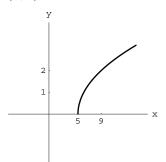
If the fourth point is R(25, a + 30), we still find a = 4 or a = 236. There are four parallelograms but only two values of a.

Chapter P Test

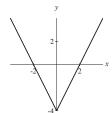
- 1. Domain $(-\infty, \infty)$, range $[0, \infty)$
- **2.** Domain $[9, \infty)$, range $[0, \infty)$
- **3.** Domain $(-\infty, \infty)$, range $(-\infty, 2]$
- **4.** The graph of y = 2x 3 includes the points (3/2, 0), (0, -3)



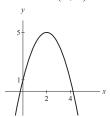
5. The graph of $f(x) = \sqrt{x-5}$ goes through (5,0), (9,2).



6. The graph of y = 2|x| - 4 includes the points $(0, -4), (\pm 3, 2)$



7. The graph of $y = -(x-2)^2 + 5$ is a parabola with vertex (2,5)



- 8. $\sqrt{9} = 3$
- 9. 3(2) 1 = 5

Chapter P Test

10.
$$f(5) = \sqrt{7}$$

11.
$$g^{-1}(x) = \frac{x+1}{3}$$
 or $g^{-1}(x) = \frac{1}{3}x + \frac{1}{3}$

12. Using the answer from Exercise 11, we find $g^{-1}(20) = \frac{20+1}{3} = 7$.

13.
$$\sqrt{72} = 6\sqrt{2}$$

14.
$$\frac{\sqrt{25}}{\sqrt{6}} = \frac{5\sqrt{6}}{6}$$

15.
$$\frac{20}{2\sqrt{2}} = 5\sqrt{2}$$

16.
$$\left(\frac{3+(-3)}{2}, \frac{5+6}{2}\right) = \left(0, \frac{11}{2}\right)$$

17.
$$\sqrt{(-2-3)^2 + (4-(-1)^2)} = \sqrt{25+25} = 5\sqrt{2}$$
.

18.
$$(x+4)^2 + (y-1)^2 = 7$$

19. Center
$$(4, -1)$$
, radius $\sqrt{6}$

20. Solving
$$3a - 9 = 6$$
, we obtain $a = 5$.

21. Symmetric about the *y*-axis since
$$f(-x) = f(x)$$

22. Interchange x and y then solve for y.

$$\begin{array}{rcl}
 x & = & 3 + \sqrt[3]{y - 2} \\
 x - 3 & = & \sqrt[3]{y - 2} \\
 (x - 3)^3 & = & y - 2 \\
 (x - 3)^3 + 2 & = & y
 \end{array}$$

The inverse is $g^{-1}(x) = (x-3)^3 + 2$.

23. Since
$$A = \left(\frac{p}{4}\right)^2$$
, we get $A = \frac{p^2}{16}$.