Solutions for Manufacturing Engineering and Technology 8th Edition by Kalpakjian

Solutions

Solutions Manual

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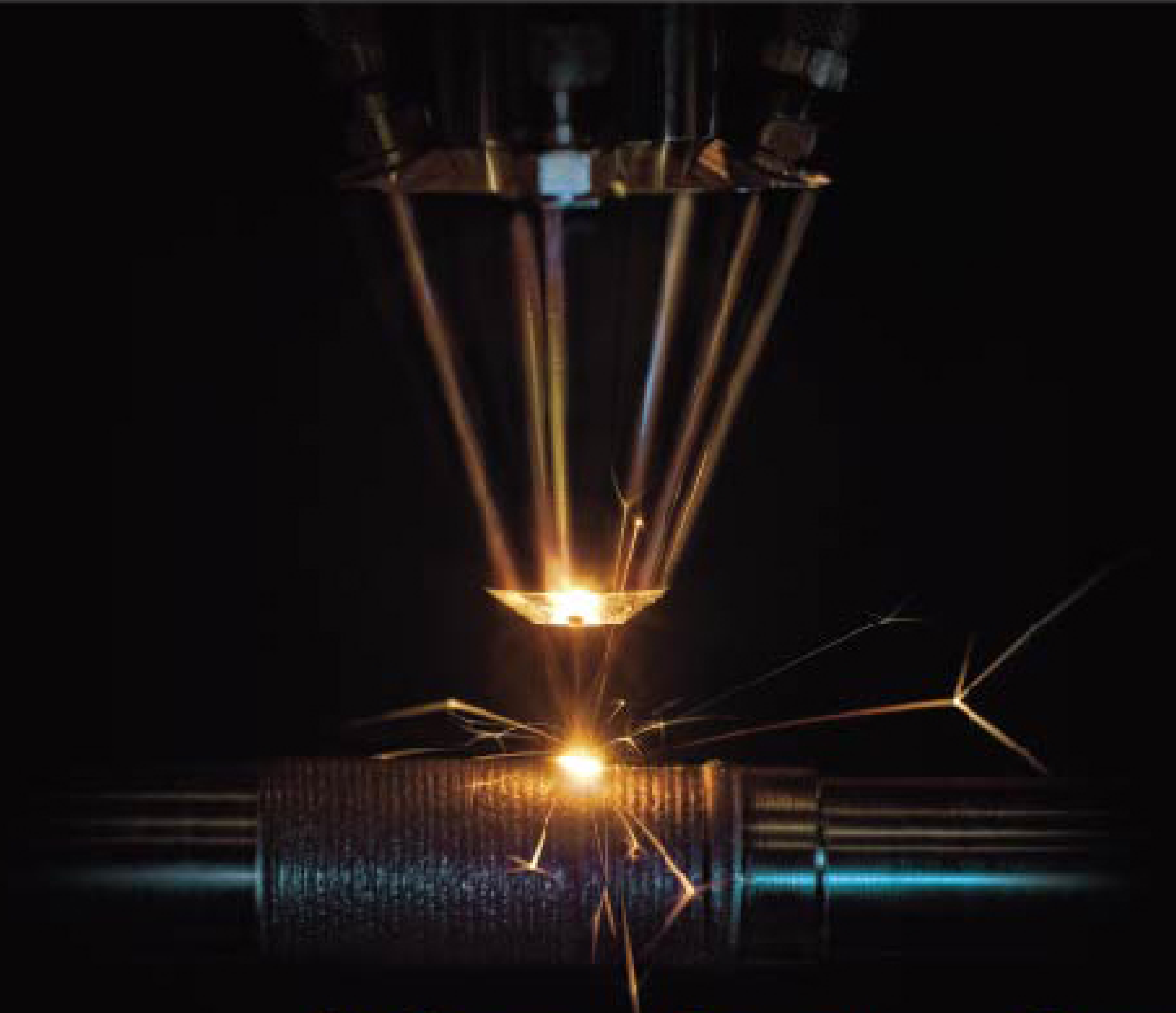

Manufacturing Engineering and Technology

Pearson

Eighth Edition

Preface

This solutions manual is intended to assist instructors in the organization of assignments and discussions associated with courses in manufacturing engineering, and using the textbook *Manufacturing Engineering and Technology, 8th ed.* In addition to these solutions, instructors can find other education resources at the Prentice Hall maintained website, as discussed and connected with links in the e-book.

Manufacturing presents a number of challenges and opportunities to instructors. As a topic of study it is exciting because of its breadth and unending ability to provide fascinating opportunities for research, analysis, and creativity. Literally every discipline and sub-discipline in engineering has strong ties to manufacturing, and a number of universities have used design and manufacturing as the basis of a capstone course that culminates a mechanical engineering bachelor's degree. To students of manufacturing, it is, at first, a field so enormous that any semester or academic year sequence in manufacturing can do nothing but scratch the surface of the subject. This perception is absolutely true: Manufacturing, like so many other areas of specialization within engineering, truly is an area where lifelong learning is necessary.

As educators, we have a responsibility to prepare our students as best we can for a life of continued education. Lifelong learning need not be restricted to formal classroom training, but it should be impressed upon students that they need to continually and systematically examine the physical world in order to achieve continued levels of improvement.

A challenging question, and perhaps one without any one good answer is: how should one teach an effective manufacturing course? We have seen examples of many successful strategies, some based solely on analytical methods, others involving surveys of manufacturing, and again others emphasizing the impact of manufacturing on engineering design. Most instructors develop hybrid approaches that are not restricted to any one area. We have attempted to include problems at the end of every chapter to accommodate each of these approaches, and it is our hope that instructors will find good homework assignments in the book.

Manufacturing is a challenge to instructors. There are a number of courses, such as statics, dynamics, solid and fluid mechanics, etc., where topics for study are broken down into small enough portions and where closed-form, quantitative problems are routinely solved by students and by faculty during lectures. Such problems are important for learning concepts, and they give students a sense of security in that absolute answers can be determined.

In manufacturing practice, such closed-form solutions do exist, but they are relatively rare. Usually, multiple disciplines are blended, and the information available is insufficient to truly optimize a desired outcome. In practice, manufacturing engineers need to apply good judgment after they have researched a problem as best they can given budgetary and time restrictions. These difficult open-ended problems are much more demanding than closed-form solutions, and require a different mindset. Instead of considering a number as valid or invalid (usually by checking against the answer provided in the book or by the instructor), an open-ended problem can be evaluated only with respect to whether or not the result is reasonable and if good scientific methods were used to obtain the result.

This textbook has been intentionally designed with a large number of open-ended problems. Such problems are, in our experience, extremely valuable when teaching manufacturing engineering. However, a solutions manual is well-suited for closed-form solutions, not open-ended problems. We have attempted to describe the pitfalls and methods that will be valuable in solving the open-ended problems, but by their nature, it is difficult to give *a* correct answer to these problems. Often, such problems have as the first sentence in the solution "By the student". We will describe acceptable solutions or approaches to obtaining those solutions, but it should be recognized that many potential answers exist for these problems.

Bloom's taxonomy of learning objectives, illustrated in Fig. 1, suggest that there is a hierarchy of skills that students can acquire. The chapter-ending problems have been designed with Bloom's taxonomy in mind, with Review Questions and Qualitative Problems emphasizing lower-order thinking skills, and

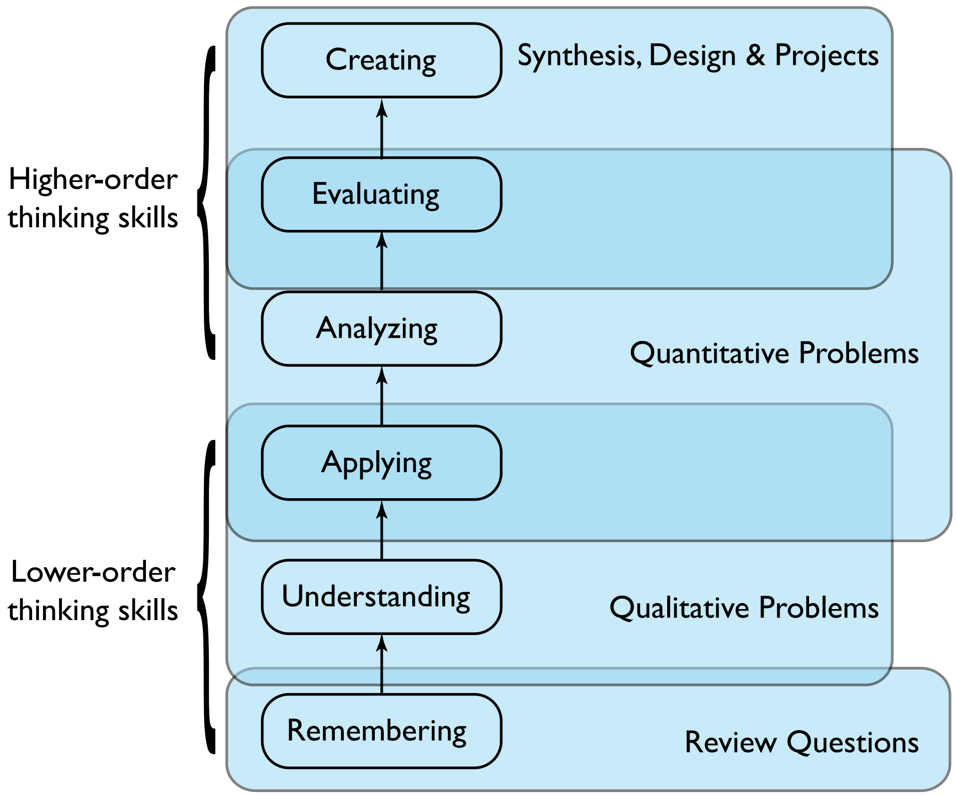

Figure 1: Bloom's taxonomy of learning objectives. *Source:* Anderson, L.W., and Krathwold, D.R., *A taxonomy for learning, teaching and assessing: A revision of Bloom's Taxonomy of educational objectives: Complete Edition*, New York, Longman, 2001.

Quantitative Problems and Synthesis, Design and Projects to exercise higher-order thinking skills. It is recognized that truly effective courses will address all Bloom levels; the chapter-ending problems have been designed to assist instructors in this task.

We encourage faculty to communicate with us and to give us feedback in any of the areas of the book.

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Chapter 1

The Structure of Metals

Qualitative Problems

1.25. Explain your understanding of why the study of the crystal structure of metals is important.

The study of crystal structure is important for a number of reasons. Basically, the crystal structure influences a material's performance from both a design and manufacturing standpoint. For example, the number of slip systems in a crystal has a direct bearing on the ability of a metal to undergo plastic deformation without fracture. Similarly, the crystal structure has a bearing on strength, ductility and corrosion resistance. Metals with face-centered cubic structure, for example, tend to be ductile whereas hexagonal close-packed metals tend to be brittle. The crystal structure and size of atom determines the largest interstitial sites, which has a bearing on the ability of that material to form alloys, and with which materials, as interstitials or substitutionals.

1.26. What is the significance of the fact that some metals undergo allotropism?

Allotropism (also called polymorphism) means that a metal can change from one crystal structure to another. Since properties vary with crystal structures, allotropism is useful and essential in heat treating of metals to achieve desired properties (Chapter 4). A major application is hardening of steel, which involves the change in iron from the fcc structure to the bcc structure (see Fig. 1.3). By heating the steel to the fcc structure and quenching, it develops into martensite, which is a very hard, hence strong, structure.

1.27. Is it possible for two pieces of the same metal to have different recrystallization temperatures? Is it possible for recrystallization to take place in some regions of a part before it does in other regions of the same part? Explain.

Two pieces of the same metal can have different recrystallization temperatures if the pieces have been cold worked to different amounts. The piece that was cold worked to a greater extent (higher strains), will have more internal energy (stored energy) to drive the recrystallization process, hence its recrystallization temperature will be lower. Recrystallization may also occur in some regions of the part before others if it has been unevenly strained (since varying amounts of cold work have different recrystallization temperatures), or if the part has different thicknesses in various sections. The thinner sections will heat up to the recrystallization temperature faster.

1.28. Describe your understanding of why different crystal structures exhibit different strengths and ductilities.

Different crystal structures have different slip systems, which consist of a slip plane (the closest packed plane) and a slip direction (the close-packed direction). The fcc structure has 12 slip systems, bcc has 48, and hcp has 3. The ductility of a metal depends on how many of the slip systems can be operative. In general, fcc and bcc structures possess higher ductility than hcp structures, because they have more slip systems. The shear strength of a metal decreases for decreasing b/a ratio (b is inversely proportional to atomic density in the slip plane and a is the plane spacing), and the b/a ratio depends on the slip system of the chemical structure (see Section 1.4).

1.29. A cold-worked piece of metal has been recrystallized. When tested, it is found to be anisotropic. Explain the probable reason.

The anisotropy of the workpiece is likely due to preferred orientation remaining from the recrystallization process. Copper is an example of a metal that has a very strong preferred orientation after annealing. Also, it has been shown that below a critical amount of plastic deformation, typically 5%, no recrystallization occurs.

1.30. What materials and structures can you think of (other than metals) that exhibit anisotropic behavior?

This is an open-ended problem and the students should be encouraged to develop their own answers. However, some examples of anisotropic materials are wood, polymers that have been cold worked, bone, any woven material (such as cloth) and composite materials.

1.31. Two parts have been made of the same material, but one was formed by cold working and the other by hot working. Explain the differences you might observe between the two.

There are a large number of differences that will be seen between the two materials, including:

1. The cold worked material will have a higher strength than the hot worked material, and this will be more pronounced for materials with high strain hardening exponents.
2. Since hardness (see Section 2.6.2) is related to strength, the cold worked material will also have a higher hardness.
3. The cold worked material will have smaller grains and the grains will be elongated.
4. The hot worked material will probably have fewer dislocations, and they will be more evenly distributed.
5. The cold worked material can have a superior surface finish when in an as-formed condition. Also, it can have better tolerances.
6. A cold worked material will have a lower recrystallization temperature than a hot worked material.

1.32. Explain the importance of homologous temperature.

The homologous temperature is defined as the ratio of a metal's current temperature to its melting temperature on an absolute scale (Kelvin or Rankine, not Celsius or Fahrenheit). This is important for determining whether or not the metal will encounter recrystallization or grain growth. The homologous temperature is more important than actual temperature, because recrystallization occurs at very different temperatures for different metals, but the homologous temperature effects are fairly consistent. The homologous temperature therefore allows one to distinguish between cold, warm, and hot working, as discussed in Table 1.2.

1.33. Do you think it might be important to know whether a raw material to be used in a manufacturing process has anisotropic properties? What about anisotropy in the finished product? Explain.

Anisotropy is important in cold-working processes, especially sheet-metal forming where the material's properties should preferably be uniform in the plane of the sheet and stronger in the thickness direction. As shown in Section 16.7, these characteristics allow for deep drawing of parts (like beverage cans) without earing, tearing, or cracking in the forming operations involved. In a finished part, anisotropy is important so that the strongest direction of the part can be designed to support the largest load in service. Also, the efficiency of transformers can be improved by using a sheet steel with anisotropy that can reduce *magnetic hysteresis* losses. Hysteresis is well known in ferromagnetic materials. When an external magnetic field is applied to a ferromagnet, the ferromagnet absorbs some of the external field. When sheet steel is highly anisotropic, it contains small grains and a crystallographic orientation that is far more uniform than for isotropic materials, and this orientation will reduce magnetic hysteresis losses.

1.34. What is the difference between an interstitial atom and a substitutional atom?

The difference can be seen in Fig. 1.8. A substitutional atom replaces an atom in the repeating lattice without distortion. An interstitial does not fit in the normal lattice; it can fit in the gap between atoms, or else it distorts the lattice. Examples of substitutionally are copper-nickel, gold-silver, and molybdenum-tungsten. Interstitials can be self-interstitials, but common other examples are carbon, lithium, sodium, and nitrogen.

1.35. Explain why the strength of a polycrystalline metal at room temperature decreases as its grain size increases.

Strength increases as more entanglements of dislocations occur with grain boundaries (Section 1.4.2). Metals with larger grains have less grain-boundary area per unit volume, and hence will not be as able to generate as many entanglements at grain boundaries, thus the strength will be lower.

1.36. Describe the technique you would use to reduce the orange-peel effect on the surface of workpieces.

Orange peel is surface roughening induced by plastic strain. There are a number of ways of reducing the orange peel effect, including:

- Performing all forming operations without a lubricant, or else a very thin lubricant film (smaller than the desired roughness) and very smooth tooling. The goal is to have the surface roughness of the tooling imparted onto the workpiece.
- Large grains exacerbate orange peel, so the use of small grained materials would reduce orange peel.
- If deformation processes can be designed so that the surfaces see no deformation, then there would be no orange peel. For example, upsetting beneath flat dies can lead to a reduction in thickness with very little surface strains beneath the platen (see Fig. 14.3).
- Finishing operations can remove orange peel effects.

1.37. What is the significance of the fact that such metals as lead and tin have a recrystallization temperature that is about room temperature?

Recrystallization around room temperature prevents these metals from work hardening when cold worked. This characteristic prevents their strengthening and hardening, thus requiring a recrystallization cycle to restore their ductility. This behavior is also useful in experimental verification of analytical results concerning force and energy requirements in metalworking processes (see Part III of the text).

1.38. It was stated in this chapter that twinning usually occurs in hcp materials, but Fig. 1.6b shows twinning in a rectangular array of atoms. Can you explain the discrepancy?

The hcp unit cell shown in Fig. 1.6a has a hexagon on the top and bottom surfaces. However, an intersecting plane that is vertical in this figure would intersect atoms in a rectangular array as depicted in Fig. 1.6b. Thus, twinning occurs in hcp materials, but not in the hexagonal (close packed) plane such as in the top of the unit cell.

1.39. It has been noted that the more a metal has been cold worked, the less it strain hardens. Explain why.

This phenomenon can be observed in stress-strain curves, such as those shown in Figs. 2.2 and 2.5. Recall that the main effects of cold working are that grains become elongated and that the average grain size becomes smaller (as grains break down) with strain. Strain hardening occurs when dislocations interfere with each other and with grain boundaries. When a metal is annealed, the grains are large, and a small strain results in grains moving relatively easily at first, but they increasingly interfere with each other as strain increases. This explains that there is strain hardening for annealed materials at low strain. To understand why there is less strain hardening at higher levels of cold work, consider the extreme case of a very highly cold-worked material, with very small grains and very many dislocations that already interfere with each other. For this highly cold-worked material, the stress cannot be increased much more with strain, because the dislocations have nowhere else to go - they already interfere with each other and are pinned at grain boundaries.

1.40. Is it possible to cold work a metal at temperatures above the boiling point of water? Explain.

The metallurgical distinction between cold and hot working is associated with the *homologous temperature*. Cold working is associated with plastic deformation of a metal when it is below one-third of its melting temperature on an absolute scale. At the boiling point of water, the temperature is 100°C, or 373 Kelvin. If this value is one-third the melting temperature, then a metal would have to have a melting temperature of 1119K, or 846°C. As can be seen in Table 3.1, there are many such metals.

1.41. Comment on your observations regarding Fig. 1.14.

This is an open-ended problem with many potential answers. Students may choose to address this problem by focusing on the shape of individual curves or their relation to each other. The instructor may wish to focus the students on a curve or two, or ask if the figure would give the same trends for a material that is quickly heated, held at that temperature for a few seconds, and then quenched, or alternatively for one that is maintained at the temperatures for very long times.

1.42. Is it possible for a metal to be completely isotropic? Explain.

This answer can be answered only if isotropy is defined within limits. For example:

- A single crystal of a metal has an inherent an unavoidable anisotropy. Thus, at a length scale that is on the order of a material's grain size, a metal will always be anisotropic.
- A metal with elongated grains will have a lower strength and hardness in one direction than in others, and this is unavoidable.
- However, a metal that contains a large number of small and equiaxed grains will have the first two effects essentially made very small; the metal may be isotropic within measurement limits.
- Annealing can lead to equiaxed grains, and depending on the measurement limits, this can essentially result in an isotropic metal.
- A metal with a very small grain size (i.e., a metal glass) can have no apparent crystal structure or slip systems, and can be essentially isotropic.

1.43. Referring to Fig. 1.1, assume you can make a ball bearing from a single crystal. What advantages and disadvantages would such a bearing have?

This is a challenging problem, and is one that can be reintroduced at the end of Chapters 1-4. Students should be encouraged to provide answers that are based on their understanding and experience; these are intended to start a discussion.

The advantages of a single crystal bearing include:

- Corrosion generally starts at a grain boundary, where loosely packed atoms are more susceptible to chemical attack; therefore, the bearing could be more corrosion resistant.
- Fatigue cracks may be more difficult to form, but with some materials are more difficult to propogate through a grain than along a grain boundary.
- The strength could be higher than a polycrystalline metal.
- Distortion of the bearing at elevated temperatures may not be an issue.

Disadvantages include:

- The ductility of the bearing may be suspect, since plastic deformation associated with dislocation motion are not present.
- Because of the anisotropy of the material, producing a perfect sphere may be very difficult.
- Cost may be a serious concern.

1.44. Referring to Fig. 1.10, explain why edge dislocations cannot cross grain boundaries using appropriate sketches.

Figure 1.10 shows that dislocations can move inside a lattice in close-packed directions. At grain boundaries, it should be recognized that the direction of the lattice changes on each side; there is then no natural direction for a dislocation to travel. This is shown below for a simple cubic arrangement. Note that the grain on the left (blue) and the grain on the right (red) are rotated 15° from each other. The grain boundary in between has only occasional direct contact between atoms, and has a large gap where dislocations would have a difficult time moving and translating across. Note that grain boundaries often have impurities, which are not shown.

Quantitative Problems

1.45. How many atoms are in a single repeating cell of an bcc crystal structure? How many in a repeating cell of an hcp structure?

For an fcc structure, refer to Fig. 1.4a. The atoms at each corner are shared by eight unit cells, and there are eight of these atoms. Therefore, the corners contribute one total atom. The atoms on the faces are each shared by two cells, and there are six of these atoms. Therefore, the atoms on the faces contribute a total of three atoms to the unit cell. Therefore, the total number of atoms in an fcc unit cell is four atoms.

For the hcp, refer to Fig. 1.5. The atoms on the periphery of the top and bottom are each shared by six cells, and there are 12 of these atoms (on top and bottom), for a contribution of two atoms. The atoms in the center of the hexagon are shared by two cells, and there are two of these atoms, for a net contribution of one atom. There are also three atoms fully contained in the unit cell. Therefore, there are six atoms in an hcp unit cell.

1.46. The atomic weight of gold is 196.97, meaning that 6.023×10^{23} atoms weigh 196.97 grams. The density of gold is 19,320 kg/m^3, and pure gold forms fcc crystals. Estimate the diameter of a gold atom.

Consider the face of the fcc unit cell, which consists of a right triangle with side length a and hypoteneuse of $4r$. From the Pythagorean theorem, $a = 4r/\sqrt{2}$. Therefore, the volume of the unit cell is

$$V = a^3 = \left(\frac{4r}{\sqrt{2}}\right)^3 = 22.63r^3$$

Each fcc unit cell has four atoms (see Prob. 1.34), and each atom has a mass of

$$\text{Mass} = \frac{196.97 \text{ g}}{6.023 \times 10^{23}} = 3.27 \times 10^{-22} \text{ g} = 3.27 \times 10^{-25} \text{ kg}$$

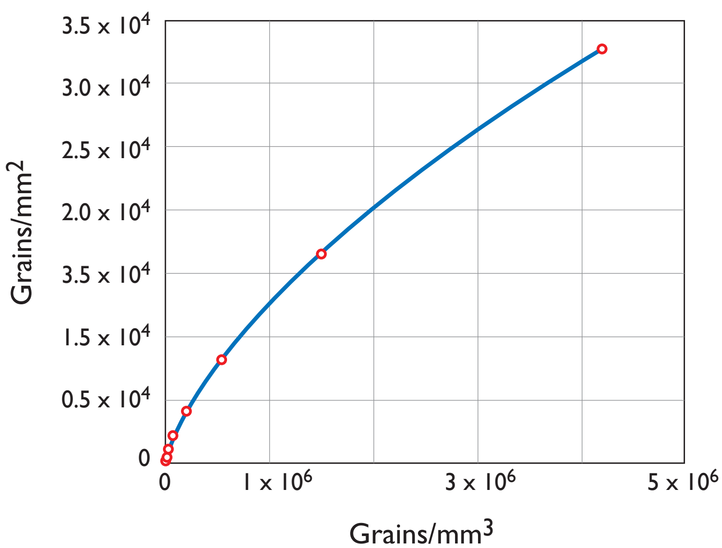
So that the density inside a fcc unit cell is

$$\rho = \frac{\text{Mass}}{V} = 19,320 \text{ kg/m}^3 = \frac{4(3.27 \times 10^{-25}) \text{ kg}}{22.63r^3}$$

Solving for r yields $r = 1.444 \times 10^{-10}$ m, or 1.44 Å. Note that the accepted value is 1.6 Å; the difference is attributable to a number of factors, including impurities in crystal structure and a concentration of mass in the nucleus of the atom.

1.47. Plot the data given in Table 1.1 in terms of grains/mm^2 vs. grains/mm^3, and discuss your observations.

The plot is shown below. It can be seen that the grains per cubic millimeter increases faster than the grains per square millimeter. This relationship is to be expected since the volume of an equiaxed grain depends on the diameter cubed, whereas its area depends on the diameter squared.



1.48. A strip of metal is reduced from 40 mm in thickness to 20 mm by cold working; a similar strip is reduced from 50 mm to 30 mm. Which of these cold-worked strips will recrystallize at a lower temperature? Why?

The metal that is reduced to 20 mm by cold working will recrystallize at a lower temperature. The more the cold work the lower the temperature required for recrystallization. This is because the number of dislocations and energy stored in the material increases with cold work. Thus, when recrystallizing a more highly cold worked material, this energy can be recovered and less energy needs to be imparted to the material.

1.49. The ball of a ballpoint pen is 0.5 mm in diameter and has an ASTM grain size of 12. How many grains are there in the ball?

From Table 1.1, a metal with an ASTM grain size of 10 has about 520,000 grains/mm^3. The volume of the ball is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1)^3 = 4.189 \text{ mm}^3$$

Multiplying the volume by the grains per cubic millimeter gives the number of grains in the paper clip as about 2.18 million.

1.50. How many grains are on the surface of the head of a pin? Assume that the head of a pin is spherical with a 1-mm diameter and has an ASTM grain size of 10.

Note that the surface area of a sphere is given by $A = 4\pi r^2$. Therefore, a 1 mm diameter head has a surface area of

$$A = 4\pi r^2 = 4\pi(0.5)^2 = \pi \text{ mm}^2$$

From Eq. (1.2), the number of grains per area is

$$N = 2^{11} = 2048$$

This is the number of grains per 0.0645 mm^2 of actual area; therefore the number of grains on the surface is

$$N_g = \frac{2048}{0.0645(\pi)} = 99,750 \text{ grains}$$

1.51. The unit cells shown in Figs. 1.3 through 1.5 can be represented by tennis balls arranged in various configurations in a box. In such an arrangement, the *atomic packing factor* (APF) is defined as the ratio of the sum of the volumes of the atoms to the volume of the unit cell. Show that the APF is 0.68 for the bcc structure and 0.74 for the fcc structure.

Note that the bcc unit cell in Fig. 1.3a has 2 atoms inside of it; one inside the unit cell and eight atoms that have one-eighth of their volume inside the unit cell. Therefore the volume of atoms inside the cell is $8\pi r^3/3$, since the volume of a sphere is $4\pi r^3/3$. Note that the diagonal of a face of a unit cell has a length of $a\sqrt{2}$, which can be easily determined from the Pythagorean theorem. Using that diagonal and the height of a results in the determination of the diagonal of the cube as $a\sqrt{3}$. Since there are four radii across that diagonal, it can be deduced that

$$a\sqrt{3} = 4r \quad \rightarrow \quad a = \frac{4r}{\sqrt{3}}$$

The volume of the unit cell is a^3, so

$$V = a^3 = \left(\frac{4r}{\sqrt{3}}\right)^3 = \left(\frac{4}{\sqrt{3}}\right)^3 r^3.$$

Therefore, the atomic packing factor is

$$\text{APF}_{\text{bcc}} = \frac{\frac{8}{3}\pi r^3}{\left(\frac{4}{\sqrt{3}}\right)^3 r^3} = 0.68$$

For the fcc cell, there are four atoms in the cell, so the volume of atoms inside the fcc unit cell is $16\pi r^3/3$. On a face of the fcc cell, it can be shown from the Pythagorean theorem that the hypotenuse is $a\sqrt{2}$. Also, there are four radii across the diameter, so that

$$a\sqrt{2} = 4r \quad \rightarrow \quad a = 2r\sqrt{2}$$

Therefore, the volume of the unit cell is

$$V = a^3 = \left(2r\sqrt{2}\right)^3$$

so that the atomic packing factor is

$$\text{APF}_{\text{fcc}} = \frac{\frac{16}{3}\pi r^3}{\left(2\sqrt{2}\right)^3 r^3} = 0.74$$

1.52. Show that the lattice constant a in Fig. 1.4a is related to the atomic radius by the formula $\alpha = 2\sqrt{2}R$, where R is the radius of the atom as depicted by the tennis-ball model.

For a face centered cubic unit cell as shown in Fig. 1.4a, the Pythagorean theorem yields

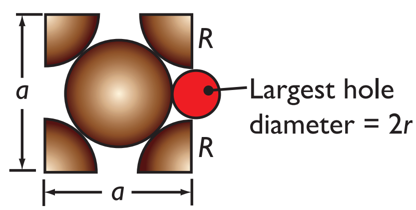
$$a^2 + a^2 = (4r)^2$$

Therefore,

$$2a^2 = 16r^2 \quad \rightarrow \quad a = 2\sqrt{2}r$$

1.53. Show that, for the fcc unit cell, the radius r of the largest hole is given by $r = 0.414R$. Determine the size of the largest hole for the iron atoms in the fcc structure.

The largest hole is shown in the sketch below. Note that this hole occurs in other locations, in fact in three other locations of this sketch.



For a face centered cubic unit cell as shown in Fig. 1.4a, the Pythagorean theorem yields

$$a^2 + a^2 = (4R)^2$$

Therefore,

$$2a^2 = 16R^2 \quad \rightarrow \quad a = 2\sqrt{2}R$$

Also, the side dimension a is

$$a = 2R + 2r$$

Therefore, substituting for a,

$$2\sqrt{2}R = 2R + 2r \quad \rightarrow \quad r = \left(\sqrt{2} - 1\right) R = 0.414R$$

1.54. A technician determines that the grain size of a certain etched specimen is 8. Upon further checking, it is found that the magnification used was $150\times$, instead of the $100\times$ that is required by the ASTM standards. Determine the correct grain size.

If the grain size is 8, then there are 2048 grains per square millimeter (see Table 1.1). However, the magnification was too large, meaning that too small of an area was examined. For a magnification of $100\times$, the area is reduced by a factor of 1/1.82=0.309. Therefore, there really are 632 grains per mm^2, which corresponds to a grain size between 6 and 7.

1.55. If the diameter of the aluminum atom is 0.28 nm, how many atoms are there in a grain of ASTM grain size 10?

If the grain size is 8, there are 65,000 grains per cubic millimeter of aluminum - see Table 1.1. Each grain has a volume of $1/65,000 = 1.538 \times 10^{-5}$ mm^3. Note that for an fcc material there are four atoms per unit cell (see solution to Prob. 1.43), with a total volume of $16\pi R^3/3$, and that the diagonal, a, of the unit cell is given by

$$a = \left(2\sqrt{2}\right) R$$

Hence,

$$\text{APF}_{\text{fcc}} = \frac{\left(16\pi R^3/3\right)}{\left(2R\sqrt{2}\right)^3} = 0.74$$

Note that as long as all the atoms in the unit cell have the same size, the atomic packing factors do not depend on the atomic radius. Therefore, the volume of the grain which is taken up by atoms is $(4.88 \times 10^{-4})(0.74) = 3.61 \times 10^{-4}$ mm^3. (Recall that 1 mm=10^6 nm.) If the diameter of an aluminum atom is 0.5 nm, then its radius is 0.25 nm or 0.25×10^{-6} mm. The volume of an aluminum atom is then

$$V = 4\pi R^3/3 = 4\pi(0.25 \times 10^{-6})^3/3 = 6.54 \times 10^{-20} \text{ mm}^3$$

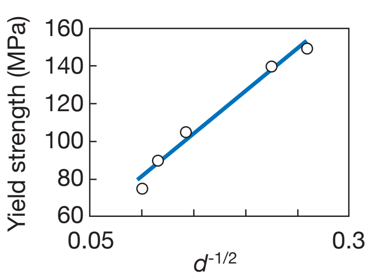
Dividing the volume of aluminum in the grain by the volume of an aluminum atom yields the total number of atoms in the grain as $(1.538 \times 10^{-5})/(6.54 \times 10^{-20}) = 2.35 \times 10^{14}$.

1.56. The following data are obtained in tension tests of brass:

Grain size (μm)	Yield stress (MPa)
30	150
40	140
100	105
150	90
200	75

Does the material follow the Hall-Petch effect? If so, what is the value of k?

First, it is obvious from this table that the material becomes stronger as the grain size decreases, which is the expected result. However, it is not clear whether Eq. (1.1) is applicable. It is possible to plot the yield stress as a function of grain diameter, but it is better to plot it as a function of $d^{-1/2}$, as follows:



The least-squares curve fit for a straight line is

$$Y = 35.22 + 458d^{-1/2}$$

with an R factor of 0.990. This suggests that a linear curve fit is proper, and it can be concluded that the material does follow the Hall-Petch effect, with a value of $k = 458$ MPa-$\sqrt{\mu\text{m}}$.

1.57. Does water have a homologous temperature? What is the highest temperature where water (ice) can get cold worked?

All materials with a melting point have a homologous temperature, defined as the ratio of the working temperature to the melting temperature on an absolute scale. If water had strengthening mechanisms like metals, it could be cold worked up to one-third of its melting temperature. Since water melts at 0°C or 273 K, it could be cold worked up to 273/3=91 K = -182 °C. However, water is a complicated material with many possible crystal structures, and does not in general display cold working like metals.

1.58. The atomic radius of iron is 0.125 nm, while that of a carbon atom is 0.070 nm. Can a carbon atom fit inside a steel bcc structure without distorting the neighboring atoms?

Consider the bcc cell shown in Fig. 1.3a. The diagonal is $4R$. The length of a side of the bcc unit cell is a, so that

$$a^2 + a^2 + a^2 = (4R)^2$$

or $a = 2.31R$. Since there are two radii on an edge, the size of the opening is $0.31R$. For iron, this would be (0.31)(0.125)=0.0387 nm, which is too small for a carbon atom to fit without distortion.

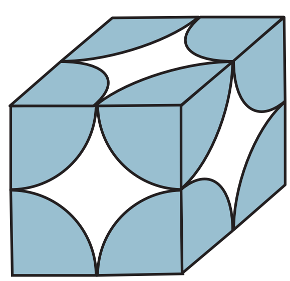
1.59. Estimate the atomic radius for the following materials and data: (a) Aluminum (atomic weight = 26.98 grams per mole, density = 2700 kg/m^3) (b) silver (atomic weight=107.87 grams/mole, density = 10,500 kg/m^3); (c) titanium (atomic weight = 47.87 grams per mole, density = 4506 kg/m^3).

Refer to Problem 1.46. Aluminum and silver are fcc materials; titanium can be fcc or hcp. The radii for aluminum is found as $r = 1.44\mathring{A}$; 1.45 Åfor silver, and 1.46 for Titanium. The following Matlab code can be used to evaluate other materials:

```
M=47.87/(6.023e23)/1000;
rho=4506;
r=(4*M/rho/22.63)^0.3333;
```

1.60. A simple cubic structure involves atoms located at the cube corners that are in contact with each other along the cube edges. Make a sketch of a simple cubic structure, and calculate its atomic packing factor.

The sketch is as follows:



The edge has a length of two times the atom radius, so that the volume of the cube is $8a^3$. There is one atom in the structure, which would take a volume of πa^3. Therefore, the atomic packing factor is

$$\text{APF} = \frac{\pi a^3}{8a^3} = 39\%.$$

1.61. Estimate the ASTM grain size number for a 300 mm (12 in.) silicon wafer used to produce computer chips.

The notion of a grain is not useful for silicon wafers, but it provides an interesting demonstration of the ASTM grain size number. If a cross section of a 300-mm diameter is considered, it would have an area of $A = \pi d^2/4 = \pi(300)^2/4 = 70,680\ \text{mm}^2$. 0.0645 mm would represent 9.12×10^{-7} grains. Therefore,

$$9.12 \times 10^{-7} = 2^{n-1}$$

$$\log 9.12 \times 10^{-7} = (n-1)\log 2$$

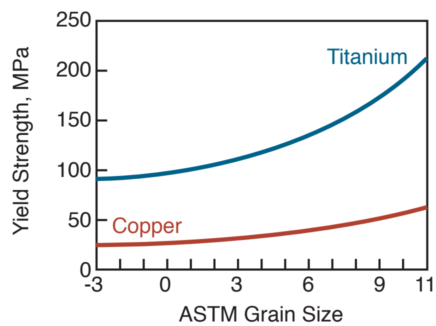
or $n = -19$.

1.62. Pure copper and pure titanium follow the Hall-Petch equation. For copper, $S_{yi} = 24$ MPa and $k = 0.12$ MPa-m$^{1/2}$. For titanium, $S_{yi} = 80$ MPa and $k = 0.40$ MPa-m$^{1/2}$. (a) Plot the yield strength of these metals as a function of grain size for ASTM grain sizes of -3 to 11. (b) Explain which material would see greater strengthening from a reduction in grain size, as in cold working.

Note from Eq. (1.2) that for an ASTM grain size of -3, there would be $N = 2^{-4} = 0.0625$ grains in 0.0645 mm^2. Hence, each grain would have an area of around 1.03 mm^2. Using $A = \pi d^2/4$ would give an estimate of a grain of diameter 1.15 mm. For a grain size of 11, the grain diameter is similarly estimated as $d = 0.008955\ \text{mm} = 8.955 \times 10^{-6}$ m. The Hall-Petch equation is given by

$$S_y = S_{yi} + kd^{-1/2}$$

The desired plot is as follows:



As can be seen, titanium strengthens more with a decrease in grain size (higher ASTM grain size number).

Synthesis, Design, and Projects

1.63. By stretching a thin strip of polished metal, as in a tension-testing machine, demonstrate and comment on what happens to its reflectivity as the strip is being stretched.

The polished surface is initially smooth, which allows light to be reflected uniformly across the surface. As the metal is stretched, the reflective surface of the polished sheet metal will begin to become dull. The slip and twin bands developed at the surface cause roughening (see Fig. 1.7), which tends to scatter the reflected light.

1.64. Draw some analogies to mechanical fibering—for example, layers of thin dough sprinkled with flour or melted butter between each layer.

A wide variety of acceptable answers are possible based on the student's experience and creativity. Some examples of mechanical fibering include: (a) food products such as lasagna, where layers of noodles bound sauce, or pastries with many thin layers, such as baklava; (b) log cabins, where tree trunks are oriented to construct walls and then sealed with a matrix; and (c) straw-reinforced mud.

1.65. Draw some analogies to the phenomenon of hot shortness.

Some analogies to hot shortness include: (a) a brick wall with deteriorating mortar between the bricks, (b) time-released medicine, where a slowly soluble matrix surrounds doses of quickly soluble medicine, and (c) an Oreo cookie at room temperature compared to a frozen cookie.

1.66. Obtain a number of small balls made of plastic, wood, marble, or metal, and arrange them with your hands or glue them together to represent the crystal structures shown in Figs. 1.3–1.5. Comment on your observations.

By the student. There are many possible comments, including the relative densities of the three crystal structures (hcp is clearly densest). Also, the ingenious and simple solid-ball models are striking when performing such demonstrations.

1.67. Take a deck of playing cards, place a rubber band around it, and then slip the cards against each other to represent Figs. 1.6a and 1.7. If you repeat the same experiment with more and more rubber bands around the same deck, what are you accomplishing as far as the behavior of the deck is concerned?

By the student. With an increased number of rubber bands, you are physically increasing the friction force between each card. This is analogous to increasing the magnitude of the shear stress required to cause slip. Furthermore, the greater the number of rubber bands, the higher the shear or elastic modulus of the material (see Section 2.4). This problem can be taken to a very effective extreme by using small C-clamps to highly compress the cards; the result is an object that acts like one solid, with much higher stiffness than the loose cards.

1.68. Give examples in which anisotropy is scale dependent. For example, a wire rope can contain annealed wires that are isotropic on a microscopic scale, but the rope as a whole is anisotropic.

All materials may behave in an anisotropic manner when considered at atomic scales, but when taken as a continuum, many materials are isotropic. Other examples include:

- Clothing, which overall appears to be isotropic, but clearly has anisotropy defined by the direction of the threads in the cloth. This anisotropic behavior can be verified by pulling small patches of the cloth in different directions.
- Wood has directionality (orthotropic) but it can be ignored for many applications.
- Human skin: it appears isotropic at large length scales, but microscopically it consists of cells with varying strengths within the cell.

1.69. The movement of an edge dislocation was described in Section 1.4.1 by means of an analogy involving a hump in a carpet on the floor and how the whole carpet can eventually be moved by moving the hump forward. Recall that the entanglement of dislocations was described in terms of two humps at different angles. Use a piece of cloth placed on a flat table to demonstrate these phenomena.

By the student. This can be clearly demonstrated, especially with a cloth that is compliant (flexible) but has high friction with a flat surface. Two methods of ensuring this is the case are (a) to use a cotton material (as found in T-shirts) and wetting it before conducting the experiments, or (b) spraying the bottom side of the fabric with temporary adhesives, as found in most arts and office supply stores. The experiments (single and two lumps) can then be conducted and observations made.

1.70. If you wanted to strengthen a material, would you wish to have it consist of one grain, or would you want it to have grains that contain the minimum number of atoms? Explain.

Most likely, the desire would be to have grains with the minimum number of atoms, as suggested by Eq. (1.1) (The Hall-Petch equation). This is the rationale for the use of metallic glasses, with no identifiable grain structure.

Chapter 2

Mechanical Behavior, Testing, and Manufacturing Properties of Materials

Qualitative Problems

2.29. On the same scale for stress, the tensile true stress-true strain curve is higher than the engineering stress-engineering strain curve. Explain whether this condition also holds for a compression test.

During a compression test, the cross-sectional area of the specimen increases as the load is increased. Since true stress is defined as load divided by the instantaneous cross-sectional area of the specimen, the true stress in compression will be lower than the engineering stress for a given load, providing that frictional forces (between the platens and the specimen) are negligible.

2.30. Explain why it is difficult to break a sheet of paper in tension, but easy to cut it with scissors.

There are a few reasons for this. First, in tension, the entire cross section of the paper is loaded, whereas in cutting with scissors only a small area (theoretically a zero area line) sees a stress. Also, the design of scissors incorporates natural leverage - the shearing takes place closer to the pivot than the force that is applied.

2.31. What are the similarities and differences between deformation and strain?

The similarities are that they are both a measure of a change in shape; strain is a deformation normalized by initial length, and therefore is dimensionless.

2.32. Can a material have a negative Poisson's ratio? Give a rationale for your answer.

Solid material do not have a negative Poisson's ratio, with the exception of some composite materials (see Chapter 10), where there can be a negative Poisson's ratio in a given direction.

The rationale is harder to express. It should make sense to students that a material, when stretched, should become narrower in the transverse directions. If Poisson's ratio were zero, then there would be no lateral deflection. If the Poisson's ratio were negative, it would expand laterally when stretched longitudinally. Also consider compression - if compressed axially, the material would need to thin laterally. This shouldn't make sense to students.

This can be proven to violate the second law of thermodynamics by calculating all components of strain energy in this case, but this is an advanced proof.

2.33. Referring to Table 2.2, explain why there can be so much variation in the strength and elongation in a class of metal alloys.

Consider aluminum and its alloys, for example, where the yield strength has a range of 35 to 550 MPa, and an elongation of 45 to 4%. A number of reasons can explain this large range, as described in this chapter, including:

- The lowest strength is associated with pure aluminum, the highest with highly alloyed aluminum. This demonstrates the dramatic effect of alloying elements on mechanical properties, and is the most important factor.
- If a material is cold worked, it can strain harden, leading to an increase in mechanical properties.
- Heat treating or annealing can increase or decrease the mechanical properties, respectively.
- Residual stresses or the presence of internal flaws can have a large effect on the strength and elongation of a material.

2.34. Referring to Table 2.2, explain why the stiffness of diamond has so much variation.

As shown in Table 2.2, the stiffness of diamond varies from 820 to 1050 GPa. This can be explained by a number of factors:

- Single crystal diamond will have different stiffnesses in different directions, which is associated with the crystal direction.
- Diamond can be single crystal or polycrystalline. A polycrystalline material will have more isotropic properties, but not the extreme values of stiffness.
- In tension, the presence of internal flaws or cracks can lead to a lower elastic modulus, although this is generally less important than the other factors.

2.35. It has been stated that the higher the value of m, the more diffuse the neck is, and likewise, the lower the value of m, the more localized the neck is. Explain the reason for this behavior.

As discussed in Section 2.2.7, with high m values, the material stretches to a greater length before it fails; this behavior is an indication that necking is delayed with increasing m. When necking is about to begin, the necking region's strength with respect to the rest of the specimen increases, due to strain hardening. However, the strain rate in the necking region is also higher than in the rest of the specimen, because the material is elongating faster there. Since the material in the necked region becomes stronger as it is strained at a higher rate, the region exhibits a greater resistance to necking. The increase in resistance to necking thus depends on the magnitude of m.

2.36. Explain why materials with high m values, such as hot glass and taffy, when stretched slowly, undergo large elongations before failure. Consider events taking place in the necked region of the specimen.

The answer is similar to Answer 2.35 above. The use of glass and taffy as examples, and the availability of videos of deforming glass and taffy on the Internet, clearly demonstrates the importance of strain hardening in achieving large strains before fracture. When necking is about to begin, the necking region's strength with respect to the rest of the specimen increases, due to strain hardening. With enough strain hardening, the deformed section resists further strain, forcing the material to strain elsewhere. This leads to more uniform deformation and a pronounced neck.

2.37. Explain if it is possible for stress-strain curves in tension tests to reach 0% elongation as the gage length is increased further.

Refer to Figure 2.1 to see the gage length, l_o, and the elongation, $(l_e - l_o)$. It would appear that if the gage length becomes infinite, then the engineering strain would approach zero. However, this is not likely from a practical standpoint, since testing equipment generally does not have the capacity for very long pieces, recognizing that the initial specimen size needs to be smaller than the largest displacement by the machine jaws. However, gage length clearly influences the measured elongation.

2.38. With a simple sketch, explain whether it is necessary to use the offset method to determine the yield strength, S_y, of a material that has been highly cold worked.

As can be seen by reviewing Fig. 2.3, a highly cold-worked metal will have a distinct change in slope on its stress-strain curve occurring at the yield point, so that the offset method is not necessary (see also Fig. 2.5).

2.39. Explain why the difference between engineering strain and true strain becomes larger as strain increases. Does this difference occur for both tensile and compressive strains? Explain.

The answer lies in the fact that the definitions of engineering strain and true strain are different, the latter being based on the actual or instantaneous dimensions, as can be seen in Eqs. (2.2) and (2.7), respectively. In both cases of tension and compression, the difference increases as strain increases. This is shown quantitatively in Problem 2.74.

2.40. Consider an elastomer, such as a rubber band. This material can undergo a large elastic deformation before failure, but after fracture it recovers completely to its original shape. Is this material brittle or ductile? Explain.

This is an interesting question and one that can be answered in a number of ways. From a stress analysis standpoint, the large elastic deformations would lead to blunting of stress concentrations, and the material would be considered ductile. However, in manufacturing, ductility implies an ability to achieve a permanent change in shape; in this case, a rubber band is extremely brittle, as there is essentially no permanent deformation when the rubber band fractures in a tension test, for example.

2.41. If a material (such as aluminum) does not have an endurance limit, how then would you estimate its fatigue life?

Materials without endurance limits have their fatigue life defined as a certain number of cycles to failure at a given stress level. For engineering purposes, this definition allows for an estimate of the expected lifetime of a part. The part is then usually taken out of service before its lifetime is reached. An alternative approach is to use nondestructive test techniques (Section 36.10) to periodically measure the accumulated damage in a part, and then use fracture mechanics approaches to estimate the remaining life.

2.42. What role, if any, does friction play in a hardness test? Explain.

The effect of friction has been found to be minimal. In a hardness test, most of the indentation occurs through plastic deformation, and there is very little sliding at the indenter-workpiece interface; see Fig. 2.14.

2.43. Which hardness tests and scales would you use for very thin strips of metal, such as aluminum foil? Explain.

A hardness test that produces small indentations would have to be used; also, since aluminum foil is relatively soft, a very light load would be required. Two scales that satisfy these requirements are the Knoop microhardness (HK) and the Vickers hardness (HV) at very light loads (see Fig. 2.13). An area of current research is the use of atomic force microscopy and nanoindenters to obtain the hardness of very thin materials and coatings. The shape of the indenter used is not exactly the same as in Fig. 2.13, and the loads are in the micro- to milli-Newton range.

2.44. Consider the circumstance where a Vickers hardness test is conducted on a material. Sketch the resulting indentation shape if there is a residual stress on the surface.

There are many possible shapes. Consider the simple sketches below, where the Vickers indentation for a material without residual stresses (the baseline) is shown in red. The green example would be particular to the case where a uniaxial residual stress is aligned with the indenter and is constant through the thickness. The blue example is more typical, and shows a biaxial residual compressive residual stress. If the Vickers inventor is not aligned with the stress, then the shape will be more of a rhomboid than square.

2.45. Which of the two tests, tension or compression, would require a higher capacity of testing machine, and why?

The compression test requires a higher capacity machine since the cross-sectional area of the specimen increases as the test progresses. The increase in area requires a load higher than that for the tension test to achieve the same stress level. Also, there is friction between the flat dies (platens) and the workpiece surfaces in a compression test (see Sections 2.3 and 14.2) which results in higher pressures than in tension; this higher pressure then requires larger forces for the same cross-sectional area. In addition, there is more redundant work in compression testing than in tension testing, so the material will work harden more (unless the test is conducted at elevated temperatures).

2.46. In a Brinell hardness test, the resulting impression is found to be an ellipse. Give possible explanations for this result.

Two possible explanations for an elliptical impression after a Brinell test are: (a) An obvious reason is the possible presence of asymmetric residual stresses in the surface layers of the material before the test. (b) The material itself may be highly anisotropic, such as a fiber-reinforced composite material, or due to severe cold working.

2.47. List and explain briefly the conditions that induce brittle fracture in an otherwise ductile metal.

Brittle fracture can be induced by high deformation rates, lower temperatures (particularly those with bcc structure), the presence of stress concentration (notches and cracks), state of stress, radiation damage, corrosion (including hydrogen embrittlement). In each case, the stress needed to cause yielding is raised above the stress needed to cause failure, or the stress needed for a crack to propagate is below the yield stress of the material (as with stress concentration).

2.48. List the factors that you would consider in selecting a hardness test. Explain why.

Hardness tests mainly have three differences: (a) type of indenter, (b) applied load, and (c) method of indentation measurement, i.e., depth or surface area of indentation, or rebound of indenter. The hardness test selected would depend on the estimated hardness of the workpiece, its size and thickness, and if average hardness or the hardness of individual microstructural components is desired. For instance, the scleroscope, which is portable, is capable of measuring the hardness of large pieces that cannot be used for measurement by other techniques.

The Brinell hardness test leaves a fairly large indentation, thus providing a good measure of average hardness, while the Knoop test leaves a small indentation that allows for determination of the hardness of the individual phases in a two-phase alloy. The small indentation of the Knoop test also allows it to be useful in measuring the hardness of very thin layers or plated layers on parts. Note that the depth of indentation should be small relative to part thickness, and that any change in the appearance of the bottom surface the part will make the test results invalid.

Figure 2.15 is a useful guide for determining which hardness test is valid for a class of material. Note that often numerous hardness tests are suitable for a material. In these cases, the best hardness test is the one that has one or more of the following characteristics:

- The best hardness test is often one that can be performed quickly; thus, it may be desirable to also select a hardness test based on available equipment.
- Hardness tests are often specified by customers as part of a quality control requirement. Whatever form of hardness test is specified by the customer is the appropriate one to use.
- A hardness test that is most commonly used in a plant may be the best choice since technicians will be most familiar with the test protocol and the equipment is most likely to be in good calibration.
- Experimental error can be minimized by selecting a hardness test that gives the largest penetration or indentation size.

2.49. List two situations where a material's toughness is important from a design standpoint.

Toughness is important for applications where fracture is to be avoided or where energy is to be absorbed. For example, metals that are used in fatigue applications are generally tough (although strength is also important). Bumpers on vehicles or used as devices to stop motion are tough, so that the material can deform and absorb energy. Students should be encouraged to produce many more examples.

2.50. On the basis of Fig. 2.5, can you calculate if a metal tension-test specimen is rapidly pulled and broken, where would the temperature be highest, and why?

Since temperature rise is due to work input, it is obvious that the temperature will be highest in the necked region because that is where the strain is highest and, hence, the energy dissipated per unit volume in plastic deformation is highest.

2.51. Comment on the temperature distribution if the specimen in Question 2.50 is pulled very slowly.

Regardless of the speed, there is work put into the specimen. However, this energy likely will not cause an appreciable or even noticeable increase in temperature, since there is more time for heat to be conducted in the specimen.

2.52. Comment on your observations regarding the contents of Table 2.2.

By the student. There are a large number of acceptable answers to this problem. A student may compare the values in the table between different materials or material classes. Alternatively, the students may comment on the size of the range in properties. Students should be encouraged to develop well thought-out answers to this question.

2.53. Will the disk test be applicable to a ductile material? Why or why not?

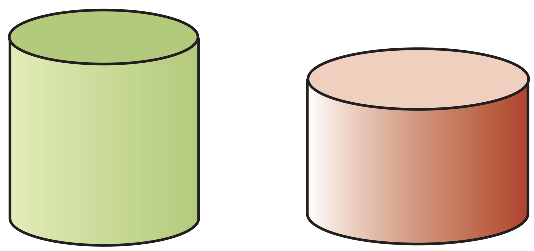
With a ductile material, a point load on a disk results in the circular disk being flattened at the platens and attaining elliptical shape of the originally round specimen. The flattening converts the point load to a distributed load, completely changing the stress state in the piece. Therefore, Eq. (2.10) is not valid, and the usefulness of the test is compromised.

2.54. Refer to Table 2.4, and note the true strain encountered by a material in different manufacturing processes. Explain why some typical strains are large and others are small.

Some of the answers will be clearer as the student progresses through later chapters. However, it can be seen that much larger strains are often encountered in extrusion as compared to forging. The main reason is that with large extrusion ratios, the material can have a pronounced improvement in its ductility and avoidance of chevron cracking. In a different operation such as forging, the exterior of the part will crack as it is not under hydrostatic compression as in extrusion.

2.55. Refer to Table 2.4, and sketch the original and deformed shape of a 25 mm specimen subjected to the largest typical strain for each process. What are your observations regarding strains that can be achieved?

As examples, the left figure shows a cylinder that is upset forged with a total true strain of -0.3 (midrange of the values in Table 2.4). That is, the initial height is $l_o = 25$ mm, the final height is 18.5 mm. The initial diameter is 25 mm, so that the final diameter (to maintain the part volume) is 29.06 mm. The sketch (at scale) is:



As a more extreme case, consider extrusion, where the 1 mm height is extended (due to a mid-range strain of 3.5) to 827 mm. The diameter for volume constancy would change from 25 mm to 4.34 mm. The following sketch is at 50% reduction in size:

This demonstrates that very large strains can be achieved in metal forming, but more moderate strains are common.

2.56. If a tension test on carbon steel is conducted at room temperature, and then with a bath of boiling water, would you expect the strength to be different? Explain.

The strength would not change. The melting point of steel is around 1500°C, or around 1770K. One-third of this temperature is around 860°C. Boiling water is not ha high enough temperature to exceed a homologous temperature of one-third.

2.57. What hardness test is suitable for determining the hardness of a thin ceramic coating on a piece of metal?

For a thin ceramic coating, it is still important that the hardness of the coating and not the substrate be measured. Most ceramics have limited ductility (Section 8.3), so that Knoop or Vickers tests are suitable, although the Mohs test can also be used to obtain a qualitative value. Because of the increasing importance of coatings, special microhardness tests have been developed for their hardness measurement.

2.58. Wire rope consists of many wires that bend and unbend as the rope is run over a sheave. A wire-rope failure is investigated, and it is found that some of the wires, when examined under a scanning electron microscope, display cup-and-cone failure surfaces, while others display transgranular fracture surfaces. Comment on these observations.

There are a large number of potential reasons for this behavior. However, a likely explanation is that when the wire rope was in use, it was run over a sheave or drum repeatedly. As a result, some of the wires in the rope failed due to fatigue, so they display brittle fracture surfaces. At some point, enough wires have failed so that the remaining wires fail due to static overload, and display cup-and-cone failure surfaces as a result.

2.59. A statistical sampling of Rockwell C hardness tests are conducted on a material, and it is determined that the material is defective because of insufficient hardness. The supplier claims that the tests are flawed because the diamond cone indenter was probably dull. Is this a valid claim? Explain.

Refer to Fig. 2.13 and note that if an indenter is blunt, then the penetration, t, under a given load will be smaller than that using a sharp indenter. This then translates into a higher hardness. The explanation is plausible, but in practice, hardness tests are fairly reliable and measurements are consistent if the testing equipment is properly calibrated and routinely serviced.

2.60. In a Brinell hardness test, the resulting impression is found to be elliptical. Give possible explanations for this result.

Two possible explanations for an elliptical impression after a Brinell test are: (a) An obvious reason is the possible presence of asymmetric residual stresses in the surface layers of the material before the test. (b) The material itself may be highly anisotropic, such as a fiber-reinforced composite material, or due to severe cold working.

2.61. In the machining of an extruded aluminum block to produce a smart phone case, it is seen that there is significant warpage after machining. Explain why. What would you do to reduce this warpage?

The warpage is likely due to residual stress in the aluminum. Therefore, strategies should be followed to reduce residual stresses or their effects. These can include:

- The aluminum can be annealed before machining.
- The aluminum can be stretched to a small plastic strain, say 10%, to eliminate residual stresses.
- The stiffness of the phone case can be increased by placing more material at corners or even in walls.
- The residual stress can be measured using a neutron radiation source, and then planning a machining path that allows the part to relax to the desired shape.

Note that the first three approaches are *much* easier than the last one.

2.62. Some coatings are extremely thin – some as thin as a few nanometers. Explain why even the Knoop test is not able to give reliable results for such coatings. Recent investigations have attempted to use highly polished diamonds (with a tip radius around 5 nm) to indent such coatings in atomic force microscopes. What concerns would you have regarding the appropriateness of the test results?

With a coating of thickness of 5 nm, the stressed volume has to be approximately one-tenth this depth, which begins to approach the size of individual atoms. Thus, a knoop indentor would need to have a tip radius that was atomically sharp in order to get results. Even with highly polished diamond tips in atomic force microscopes, this scale problem is unavoidable. However, there are additional concerns in that the diamond indenter may not be symmetric, there are large adhesive forces at the small scales, there are complicated elastic and viscoelastic recovery at small length scales, there may be residual stresses at the surface, and the stressed volume may or may not contain a dislocation (whereas with Knoop tests, there is always a number of dislocations).

2.63. Select an appropriate hardness test for each of the following materials, and justify your answer:

1. Cubic boron nitride
2. Lead
3. Cold-drawn 0.5%C steel
4. Diamond
5. Caramel candy
6. Granite

Figure 2.15 is a useful guide for selecting hardness tests.

1. Cubic boron nitride is very hard, and useful data can be obtained only from the Knoop and Mohs tests. The Mohs scale is qualitative and does not give numerical values for hardness, so the Knoop test is preferable.
2. Lead. As shown in Fig. 2.15, lead is so soft that only the Brinell and Vickers tests yield useful data. Recognizing that lead is very soft, the lightest loads in these tests should be used. Consider the expected results in this test if a typical value of hardness is 4 HB or 4 HV. For the Brinell test, Fig. 2.13 suggests that the expected indentation for a 500 kg load is:

$$\text{HB} = \frac{2P}{(\pi D)\left(D - \sqrt{D^2 - d^2}\right)}$$

   Therefore, solving for d,

$$d = \sqrt{D^2 - \left(D - \frac{2P}{(\pi D)(\text{HB})}\right)^2} = \sqrt{10^2 - \left(10 - \frac{2(500)}{[\pi(10)]\,(4)}\right)^2} = 9.79 \text{ mm}$$

   Note that this dimension is almost the same as the diameter of the indentor, and makes the usefulness of the test highly questionable. For the Vickers test, the expected indentation test, using the lowest allowable load of 1 kg, is:

$$\text{HV} = \frac{1.854P}{L^2} \quad \rightarrow \quad L = \sqrt{\frac{1.854P}{\text{HX}}} = \sqrt{\frac{1.854(1)}{4}} = 0.68 \text{ mm}$$

   This is much more reasonable, suggesting that the Vickers test is the best alternative for lead.
3. Cold-drawn 0.5% steel. From Fig. 2.15, all of the hardness tests are suitable for this material. As discussed in Problem 2.40, the best choice for this material will depend on a number of factors.
4. Diamond. The hardness of diamond is difficult to obtain. The hardness of diamond is really determined by extrapolating the hardness on the Mohs curve to another scale in Fig. 2.15. The hardness of diamond is usually quoted as 8000 to 10,000 HK.
5. Caramel (candy). This would be an interesting experiment to perform, but the result will be that none of the hardness tests can be used for this material because it is far too soft. Also, the hardness of caramel is strongly temperature-dependent and that it creeps, so that hardness measurement may be meaningless.
6. Granite. The hardness of granite varies according to the source, but it is approximately around apatite on the Mohs scale. Thus, various hardness tests can give valuable information on granite. Note, however, that in inspecting granite surfaces, one can see various regions within which there would be hardness variations. The particular hardness test selected will depend on various factors, as discussed in part (c) above.