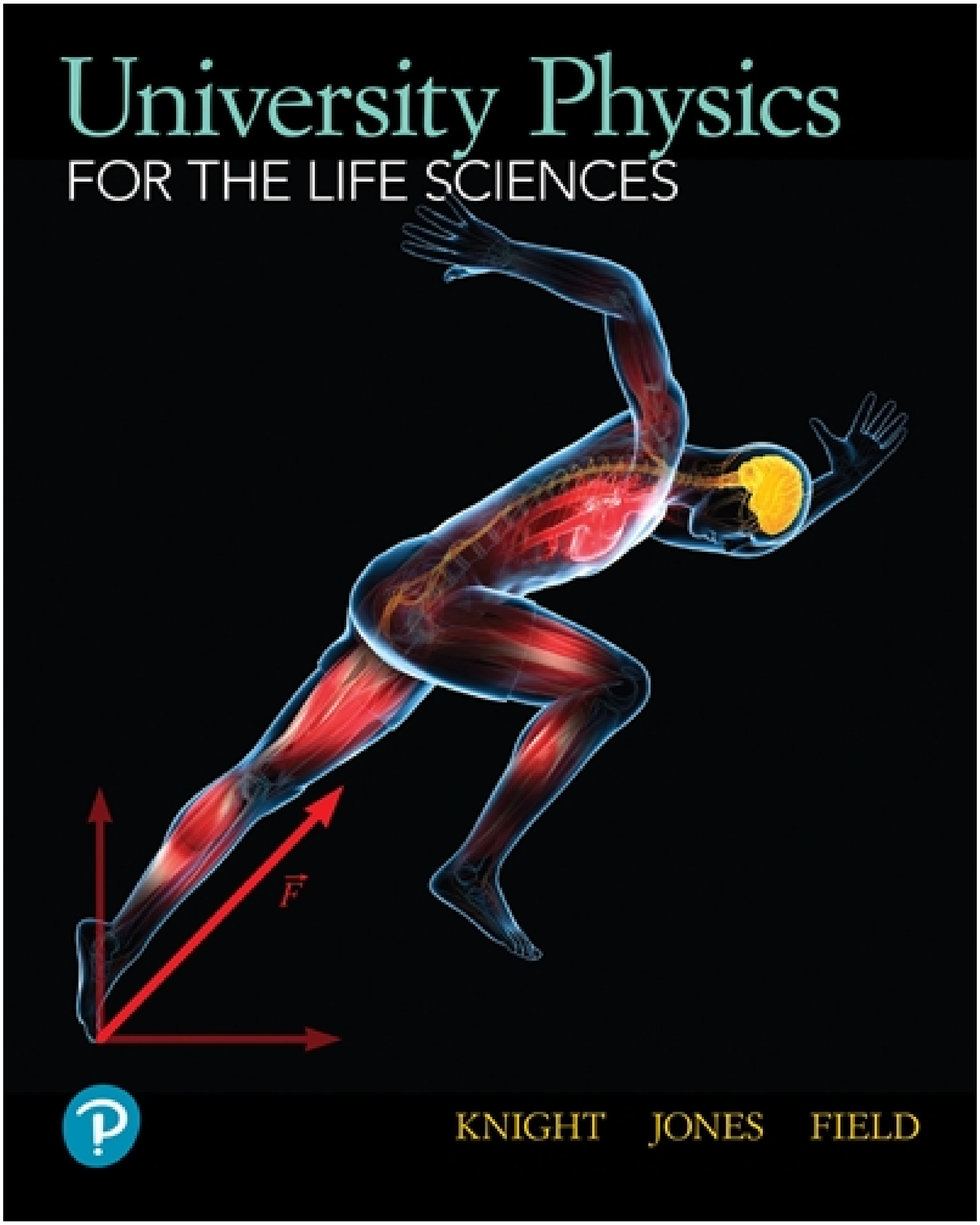
Solutions for University Physics for Life Sciences 1st Edition by Randall



Solutions

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PHYSICS FOR THE LIFE SCIENCES

QUESTIONS

Q1.1 **REASON:** We know from the reading that diffusion distances scale with the square root of time. Specifically, Equation 1.3 tells us $x_{\text{rms}} = \sqrt{2Dt}$. So, if the time were quadrupled, we would expect the diffusion distance to double from d to $2d$.

ASSESS: This is not specific to one dimension. Equation 1.5 for three dimensions reads $r_{\text{rms}} = \sqrt{6Dt}$. It has a different numerical constant, but the dependence on time is the same.

Q1.2 **REASON:** The diffusion constant is given by Equation 1.2, $D = \frac{1}{2}vd$. The fact that Xenon atoms are heavier won't affect the distance between collisions very much, but the fact that they are slower means the diffusion constant will be smaller for Xenon.

ASSESS: It is intuitive that a heavy, slow-moving thing will spread more slowly than a light, fast-moving thing.

Q1.3 **REASON:** We know that higher temperature indicates faster molecular motion on a microscopic scale. So, we expect diffusion to be faster in hot water than in cold water. This means the diffusion constant will be larger for hot water.

ASSESS: This can be seen easily by putting drops of food coloring or other dyes into glasses of water with different temperatures.

Q1.4 **REASON:** One reason might be that the elephant trunk snake is larger. The surface area of skin available for gas exchange scales with $\ell \times r$ (treating the snake as cylindrical). The volume available for lungs scales with $\ell \times r^2$. So as snake size increases, the surface area increases linearly with r , whereas the volume increases quadratically in r , meaning $V \propto r^2$. Thus, as snake size increases, volume increases faster than surface area, and using the volume for lungs becomes more efficient than using the skin for gas exchange.

ASSESS: This is not the only possible reason. A larger snake may also have a thicker skin, such that diffusion of gases through the skin would take longer for the larger snake.

Q1.5 **REASON:** We can use the formula for the volume of a sphere, and check our answer by considering scaling. For a sphere, $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$. So if $V_A = \frac{1}{6}\pi d_A^3$ and $V_B = \frac{1}{6}\pi d_B^3 = \frac{1}{6}\pi(2d_A)^3 = 8\left(\frac{1}{6}\pi d_A^3\right)$, then clearly $V_B = 8V_A$.

ASSESS: We know that the volume of a sphere depends on the diameter cubed. So doubling the diameter should result in the volume increasing by a factor of $2^3 = 8$.

Q1.6 **REASON:** We can express this proportionality as $T = c\sqrt{L}$, where c is some unknown quantity that does not depend on length. If we have two such pendula, we can write $T_A = c\sqrt{L_A}$ and $T_B = c\sqrt{L_B}$, such that

$$\frac{T_B}{T_A} = \frac{c\sqrt{L_B}}{c\sqrt{L_A}} \Rightarrow L_B = \left(\frac{T_B}{T_A}\right)^2 L_A .$$

Inserting numbers, we have

$$L_B = \left(\frac{2\text{ s}}{1\text{ s}}\right)^2 (1\text{ m}) = 4\text{ m} .$$

ASSESS: If the period depended linearly on the length, doubling the period would require doubling the length. We know the period doesn't depend that strongly on the length; it depends on the square root of the length. So, it makes sense that we would need to do more than double the length to double the period.

Q1.7 **REASON:** This is a diffusion problem. We will use the expression $x_{rms} = \sqrt{2Dt}$, where x_{rms} is the root mean squared average distance that the gas diffuses. The decay time is

$$(20\text{ days}) \times \frac{(24\text{ h})}{(1\text{ day})} \times \frac{(3600\text{ s})}{(1\text{ h})} = 1.73\times10^6\text{ s}$$

We can use this in the diffusion equation above, and we find

$$x_{rms} = \sqrt{2Dt} = \sqrt{2\left(1\times10^{-6}\text{ m}^2/\text{s}\right)\left(1.73\times10^6\text{ s}\right)} = 2\text{m}$$

So, the correct answer is C.

ASSESS: This is consistent with the problem statement in that it means radon from deep inside the earth will not reach the atmosphere. It is important to remember that this is the length that the gas will diffuse on average; some radon particles will diffuse farther than that. If we are concerned with trace amounts of radon, we may need to consider a depth of more than 2 m.

Q1.8 **REASON:** Another way of writing $d \propto m^{3/8}$ is $d = cm^{3/8}$, where c is an unknown constant. Then we can write $d_E = cm_E^{3/8}$ and $d_H = cm_H^{3/8}$, from which it follows that

$$\frac{d_E}{d_H} = \frac{cm_E^{3/8}}{cm_H^{3/8}} = \left(\frac{m_E}{m_H}\right)^{3/8} \Rightarrow d_E = d_H\left(\frac{m_E}{m_H}\right)^{3/8} = (0.020\text{ m})\left(\frac{5000\text{ kg}}{70\text{ kg}}\right)^{3/8} = 0.1\text{ m}$$

This is the same as 10 cm. The correct answer is C.

ASSESS: It is certainly reasonable that an elephant's aorta would be larger than a human's.

Q1.9 **REASON:** The line goes up two orders of magnitude every time it goes over one order of magnitude. On the plot, this corresponds to a slope of 2. So, the correct answer is B.

ASSESS: As described in the chapter, the slope of a line on a log-log is equivalent to the exponent in a scaling law; such that $y = x^2$.

Q1.10 **REASON:** The given scaling law can be written as $f \propto m^{-1/2}$ or $f = cm^{-1/2}$, where c is an unknown constant. Call the 200 g mass A, and the other mass B. Then $f_A = cm_A^{-1/2}$ and $f_B = cm_B^{-1/2}$, such that

$$\frac{f_B}{f_A} = \frac{cm_B^{-1/2}}{cm_A^{-1/2}} \Rightarrow f_B = f_A\sqrt{\frac{m_A}{m_B}} = (2.0 \text{ cycles/s})\sqrt{\frac{200 \text{ g}}{400 \text{ g}}} = 1.4 \text{ cycles/s}$$

So, the correct answer is B.

ASSESS: It makes sense that a heavier mass would oscillate more slowly if the spring stiffness is unchanged.

PROBLEMS

P1.1 **PREPARE:** In a random walk in one dimension, the particle moves to the left or right with equal probability.

SOLVE: (a) After one step, half the particles (500) will move to $x = d$ and half to $x = -d$. In the second step, half the particles on $x = d$ will move further to $x = 2d$ and half (250) will move back to $x = 0$. It is similar on the left, such that another 250 move back to $x = 0$. Thus, we expect a total of 500 random walkers at $x = 0$ after 2 steps.

(b) There cannot be any particles at $x = d$ after two steps. They have to move after each time step. Any particles that moved to $x = d$ in the first time step, now must move to either $x = 2d$ or back to $x = 0$ in the second step.

(c) As described in part (a), 500 will initially move to $x = d$, and in the second step 250 of those will move on to $x = 2d$.

ASSESS: The presence of more random walkers in the center than on the edges is consistent with what we know of diffusion.

P1.2 **PREPARE:** We will use Equation 1.1: $x_{rms} = \sqrt{(x^2)_{avg}} = \sqrt{n}\, d$ for part (a). For part (b), we remember that diffusion occurs equivalently in all directions.

SOLVE: (a) Since $x_{rms} = \sqrt{(x^2)_{avg}} = \sqrt{n}\, d$, we can write $(x^2)_{avg} = nd^2$, or equivalently

$$d = \sqrt{(x^2)_{avg} / n} = \sqrt{\left(100\times\left(10^{-6} \text{ m}\right)^2\right)/\left(10^6\right)} = 10^{-8} \text{ m} = 10 \text{ nm}$$

(b) The average position along the x-axis is zero, because diffusion to the left is equivalent to diffusion to the right.

ASSESS: The answer to part (b) is why we often speak of $x_{rms} = \sqrt{\left(x^2\right)_{avg}}$. The average position is zero. But average of the square of the position is not.

P1.3 **PREPARE:** We can start with Equation 1.1: $x_{\text{rms}} = \sqrt{(x^2)_{\text{avg}}} = \sqrt{n}\,d$, and modify it slightly using the fact that $n = \Delta t / \Delta t_{\text{step}}$.

SOLVE: (a) Combing the expressions above yields $x_{\text{rms}} = \sqrt{\Delta t / \Delta t_{\text{step}}}\,d$, and inserting the given numbers yields

$$x_{\text{rms}} = \sqrt{(20\text{s})/(2.0\times10^{-3}\text{ s})}\left(2.0\times10^{-6}\text{ m}\right) = 2.0\times10^{-4}\text{ m } = 0.2\text{ mm}$$

(b) We have $x_{\text{rms}} = \sqrt{\Delta t / \Delta t_{\text{step}}}\,d$. The diffusion distance and step size don't change, so we need the quantity $\sqrt{\Delta t / \Delta t_{\text{step}}}$ to also be unchanged. Thus, if the diffusion time is to be halved to only 10 s, the step time must also be halved to 1.0 ms.

(c) Rearranging our expression from (a) to solve for the step size, we find

$$d = x_{\text{rms}}\sqrt{\Delta t_{\text{step}} / \Delta t} = \left(2.0\times10^{-4}\text{ m}\right)\sqrt{\left(2.0\times10^{-3}\text{ s}\right)/(10\text{ s})} = 2.8\times10^{-6}\text{ m}$$

or $2.8\ \mu\text{m}$.

ASSESS: In order for the diffusion to occur faster, in only 10 s, it makes sense that we would either have to increase the step size or decrease the time for each step.

P1.4 **PREPARE:** We can start with Equation 1.1: $x_{\text{rms}} = \sqrt{(x^2)_{\text{avg}}} = \sqrt{n}\,d$ and modify it slightly using the fact that $n = \Delta t / \Delta t_{\text{step}}$. We can rearrange the resulting expression to solve for time: $x_{\text{rms}} = \sqrt{\Delta t / \Delta t_{\text{step}}}\,d \Rightarrow \Delta t = \Delta t_{\text{step}}\left(x_{\text{rms}}^2 / d^2\right)$.

SOLVE: (a) Inserting the given numbers, we find

$$\Delta t = \Delta t_{\text{step}}\left(x_{\text{rms}}^2 / d^2\right) = (1.0\text{ s})\left(\frac{5.0\text{ m}}{0.50\text{ m}}\right)^2 = 100\text{ s}$$

(b) Inserting the new numbers, we find

$$\Delta t = \Delta t_{\text{step}}\left(x_{\text{rms}}^2 / d^2\right) = (1.0\text{ s})\left(\frac{50\text{ m}}{0.50\text{ m}}\right)^2 = 10^4\text{ s or } 2.8\text{ h}$$

ASSESS: This demonstrates, with familiar times and distances, that diffusing ten times as far takes much more than ten times as long.

P1.5 **PREPARE:** We can start with Equation 1.1: $x_{\text{rms}} = \sqrt{(x^2)_{\text{avg}}} = \sqrt{n}\,d$ and modify it slightly using the fact that $n = \Delta t / \Delta t_{\text{step}}$, to obtain $\Delta t = \Delta t_{\text{step}}\left(x_{\text{rms}}^2 / d^2\right)$.

SOLVE: Clearly, if x_{rms} doubles, Δt will increase by a factor of four. Hence, the time required will be 40 s.

ASSESS: We know that diffusion distance is not linear in time, and that diffusion is slower than that. Thus, it makes sense that for the distance to double, the time would have to increase by more than a factor of two.

P1.6 **PREPARE:** We know from the reading that we can approximate the speed of a small molecule in air can be approximated as 320 m/s. We can use this, and the given information to find the diffusion constant from Equation 1.2: $D = \frac{1}{2}vd$, to answer part (a). For part (b) we will use Equation 1.5: $r_{\text{rms}} = \sqrt{6Dt}$.

SOLVE: (a) Inserting the numerical values, we find

$$D = \frac{1}{2}vd = \frac{1}{2}(320\text{ m/s})(250\times10^{-9}\text{ m}) = 4.0\times10^{-5}\text{ m}^2/\text{s}$$

(b) We know $r_{\text{rms}} = \sqrt{6Dt}$. Rearranging for time and inserting numerical values from the problem statement and from part (a), we have

$$t = \frac{r_{\text{rms}}^2}{6D} = \frac{(3.0\text{ m})^2}{6\left(4.0\times10^{-5}\text{ m}^2/\text{s}\right)} = 3.8\times10^5\text{ s or 10 h}$$

(c) The time required for particles to reach your nose via diffusion is orders of magnitude larger than the time observed.

ASSESS: The phenomenon is actually due to convection currents in the air, which carry the particles faster than they would move through diffusion.

P1.7 **PREPARE:** Both parts of this question are straightforward applications of Equation 1.3: $x_{\text{rms}} = \sqrt{2Dt}$, which we can rearrange and solve for time: $t = x_{\text{rms}}^2/2D$.

SOLVE: (a) $t = x_{\text{rms}}^2/2D = \dfrac{\left(8.0\times10^{-9}\text{ m}\right)^2}{2\left(7\times10^{-10}\text{ m}^2/\text{s}\right)} = 4.6\times10^{-8}\text{ s or 46 ns}$

(b) $t = x_{\text{rms}}^2/2D = \dfrac{\left(1.0\times10^{-3}\text{ m}\right)^2}{2\left(7\times10^{-10}\text{ m}^2/\text{s}\right)} = 710\text{ s or 12 min}$

ASSESS: It is reasonable that cellular transport would happen much more quickly than transport across larger biological systems like a leaf.

P1.8 **PREPARE:** The given scaling law can be written as $T \propto L^{1/2}$ or $T = cL^{1/2}$, where c is an unknown constant. Call the 1.0 m pendulum A and the other pendulum B.

SOLVE: Combining $T_{\text{A}} = cL_{\text{A}}^{1/2}$ and $T_{\text{B}} = cL_{\text{B}}^{1/2}$, and taking the ratio of each side, we find that

$$\frac{T_{\text{B}}}{T_{\text{A}}} = \frac{cL_{\text{B}}^{1/2}}{cL_{\text{A}}^{1/2}} \Rightarrow T_{\text{B}} = T_{\text{A}}\sqrt{\frac{L_{\text{B}}}{L_{\text{A}}}} = (4.0\text{ s})\sqrt{\frac{2.0\text{ m}}{1.0\text{ m}}} = 5.7\text{ s}$$

ASSESS: If the period increases as the length increases, we expect the longer pendulum to have a period greater than 4.0, which it does.

P1.9 **PREPARE:** The scaling law described can be written as $x_{\text{rms}} = c\sqrt{t}$, where c is an unknown quantity that doesn't depend on time. We can write this for the first case described and for the second case, about which we are asked.

SOLVE: We can write $x_{\text{rms},1} = c\sqrt{t_1}$ and $x_{\text{rms},2} = c\sqrt{t_2}$. Taking the ratio of the right-hand sides and left-hand sides, we find

$$\frac{x_{\text{rms},2}}{x_{\text{rms},1}} = \frac{c\sqrt{t_2}}{c\sqrt{t_1}} \Rightarrow x_{\text{rms},2} = x_{\text{rms},1}\sqrt{\frac{t_2}{t_1}} = (10\ \mu\text{m})\sqrt{\frac{20\ \text{ms}}{2\ \text{ms}}} = 32\ \mu\text{m}$$

ASSESS: Because of the distance scaling with the square root of the time, we expect that when the time increases by a factor of 10, the distance should increase but not quite by a factor of 10. Our answer is 3.2 times greater than the diffusion distance given in the problem; this is reasonable.

P1.10 **PREPARE:** Equation 1.2 tells us $D = \frac{1}{2}vd$. We will approximate the distance d between collisions as the same $d \approx 0.05\ \text{nm} = 5\times10^{-11}\ \text{m}$ given in Chapter 1. Also, in the text, we used 320 m/s as an estimate for the average speed of small molecules or atoms. Using the mass information in the problem statement and the scaling $v \propto m^{-1/2}$, we can come up with an estimate of the average speed of the proteins.

SOLVE: (a) First, we can write $v_{\text{small}} = cm_{\text{small}}^{-1/2}$ and $v_{\text{protein}} = cm_{\text{protein}}^{-1/2}$, where c is some constant. Combining, we find

$$\frac{v_{\text{small}}}{v_{\text{protein}}} = \frac{cm_{\text{small}}^{-1/2}}{cm_{\text{protein}}^{-1/2}} \Rightarrow v_{\text{protein}} = v_{\text{small}}\sqrt{\frac{m_{\text{small}}}{m_{\text{protein}}}} \approx (320\ \text{m/s})\sqrt{\frac{(30\ \text{Da})}{(10^5\ \text{Da})}} = 5.54\ \text{m/s}$$

Now, Equation 1.2 yields

$$D = \frac{1}{2}vd = \frac{1}{2}(5.54\ \text{m/s})\left(5\times10^{-11}\ \text{m}\right) = 1.4\times10^{-10}\ \text{m}^2/\text{s}$$

(b) Equation 1.5 relates the diffusion constant to distance and time: $r_{\text{rms}} = \sqrt{6Dt}$. Solving this expression for time, and inserting the value from part (a), we find

$$t = \frac{r_{\text{rms}}^2}{6D} = \frac{\left(10\times10^{-6}\ \text{m}\right)^2}{6\left(1.4\times10^{-10}\ \text{m}^2/\text{s}\right)} = 0.12\ \text{s}$$

ASSESS: We expect very short time scales for cellular processes, so this time is plausible.

P1.11 **PREPARE:** We can rewrite the given scaling relation as $R = cm^{-1/4}$, where c is some constant. We can write this once for each animal: $R_{\text{E}} = cm_{\text{E}}^{-1/4}$ and $R_{\text{M}} = cm_{\text{M}}^{-1/4}$.

SOLVE: Combining the equations above, we have

$$\frac{R_{\text{M}}}{R_{\text{E}}} = \frac{cm_{\text{M}}^{-1/4}}{cm_{\text{E}}^{-1/4}} \Rightarrow R_{\text{M}} = R_{\text{E}}\left(\frac{m_{\text{E}}}{m_{\text{M}}}\right)^{1/4} = (30\ \text{bpm})\left(\frac{(5000\ \text{kg})}{(5.0\ \text{kg})}\right)^{1/4} = 1.7\times10^2\ \text{bpm or 170 bpm}$$

ASSESS: This answer is consistent with the heart rate increasing as the mass decreases. It is also reassuring that a normal human heart rate of around 60–70 bpm falls between these extreme heart rates, since humans' mass falls between the extreme masses of these two animals.

P1.12 **PREPARE:** The scaling law described can be written as $S \propto A^z$ or $S = cA^z$, where c is a constant. Writing this for each island, we have $S_A = cA_A^z$ and $S_B = cA_B^z$.

SOLVE: Combining the equations above, we have

$$\frac{S_B}{S_A} = \frac{cA_B^z}{cA_A^z} \Rightarrow S_B = S_A\left(\frac{A_B}{A_A}\right)^z = (300\text{ species})\left(\frac{1}{10}\right)^{(0.33)} = 140\text{ species}$$

ASSESS: It is certainly reasonable that a smaller island would have fewer species living on it.

P1.13 **PREPARE:** The line goes over by two units (corresponding to two orders of magnitude on this log-log plot) as it rises one unit. The slope is ½.

SOLVE: We know from the reading that the slope of a log-log plot is the exponent z in an expression $y \propto x^z$. Thus, the scaling law implied by Figure P1.13 is $y \propto x^{1/2}$ or $y \propto \sqrt{x}$.

ASSESS: Note that when x changes from 10 to 1000, the line only rises from 10 to 100. In other words, when x changes by a factor of 100, y changes by only a factor of 10.

P1.14 **PREPARE:** The line moves downward by two units each time it moves over by one unit. The slope is –2.

SOLVE: We know from the reading that the slope of a log-log plot is the exponent z in an expression $y \propto x^z$. Thus, the scaling law implied by Figure P1.14 is $y \propto x^{-2}$. In other words, the number of trees scales like the trunk diameter to the power –2.

ASSESS: This result means that small trunk diameters are very common, whereas larger trunks are less common.

P1.15 **PREPARE:** Let us write the scaling law $A \propto m^{3/4}$ as $A = cm^{3/4}$, where c is some constant. We will write out expressions for the initial and final surface areas of the plants.

SOLVE: Initially, there is only one larger tree: $A_i = cm_i^{3/4}$, whereas finally, we have two trees each with $A_f = cm_f^{3/4}$. So, the ratio of the new area to the initial area is

$$\frac{2A_f}{A_i} = \frac{2cm_f^{3/4}}{cm_i^{3/4}} = 2\left(\frac{m_f}{m_i}\right)^{3/4} = 2\left(\frac{1}{2}\right)^{3/4} = 1.2$$

So, the area has been increased by a factor of 1.2.

ASSESS: The area doesn't quite scale linearly with mass. So, halving the mass does not quite halve the area. It is therefore reasonable that two trees each with an area not quite halved would result in more area than before.

P1.16 **PREPARE:** We can rewrite the given scaling law as $d = cm^{2/3}$, where c is some constant. This will hold for the heavy tortoise and the light tortoise: $d_H = cm_H^{2/3}$ and $d_L = cm_L^{2/3}$.

SOLVE: Combining the two equations above, we have

$$\frac{d_{\rm H}}{d_{\rm L}} = \frac{cm_{\rm H}^{2/3}}{cm_{\rm L}^{2/3}} \Rightarrow d_{\rm H} = d_{\rm L}\left(\frac{m_{\rm H}}{m_{\rm L}}\right)^{2/3} = (300\text{ m})\left(\frac{(200\text{ kg})}{(50\text{ kg})}\right)^{2/3} = 7.6\times10^{2}\text{ m}$$

ASSESS: Although tortoises are not known for their speed, 760 m in a day is entirely possible. If the tortoise were only active for 6 hours, that would an average speed of about 3.5 cm/s.

P1.17 **PREPARE:** We can rewrite the given scaling law as $v = cm^{-1/4}$, where c is some constant. This will hold for the heavy goose and the light hawk: $v_{\rm G} = cm_{\rm G}^{2/3}$ and $v_{\rm H} = cm_{\rm H}^{2/3}$.

SOLVE: Combining the two equations above, we have

$$\frac{v_{\rm H}}{v_{\rm G}} = \frac{cm_{\rm H}^{-1/4}}{cm_{\rm G}^{-1/4}} \Rightarrow v_{\rm H} = v_{\rm L}\left(\frac{m_{\rm G}}{m_{\rm H}}\right)^{1/4} = (60\text{ km/h})\left(\frac{(5\text{ kg})}{(1\text{ kg})}\right)^{1/4} = 90\text{ km/h}$$

ASSESS: This is extremely fast, but this is to be expected since a hawk is a quick bird of prey.

P1.18 **PREPARE:** We will use $V_{\rm m} = \pi r^2 \ell$ for the volume of the cylindrical mitochondria, and $V_{\rm y} = \frac{4}{3}\pi R^3$ for the volume of the spherical yeast cell (note the different radii). We want the ratio of the total mitochondria volume to that of the yeast cell.

SOLVE: The ratio desired is

$$\frac{40V_{\rm m}}{V_{\rm y}} = \frac{40\pi r^2\ell}{\frac{4}{3}\pi R^3} = \frac{30r^2\ell}{R^3} = \frac{30\left(0.5\times10^{-6}\text{ m}\right)^2\left(2.0\times10^{-6}\text{ m}\right)}{\left(2.5\times10^{-6}\text{ m}\right)^3} = 0.96$$

So 96% of the volume of yeast is taken up by the mitochondria.

ASSESS: This is almost the entire volume.

P1.19 **PREPARE:** For part (a), we will simply calculate the volume of the roundworm using $V_{\rm r} = \pi R^2 \ell$, and divide it by the number of cells. For part (b), we take the volume of each cell and equate it to $V_{\rm c} = L^3$.

SOLVE: (a) $V_{\rm cell} = \dfrac{V_{\rm r}}{N} = \dfrac{\pi R^2 \ell}{N} = \dfrac{\pi\left(0.04\times10^{-3}\text{ m}\right)^2\left(1.0\times10^{-3}\text{ m}\right)}{959} = 5.2\times10^{-15}\text{ m}^3 \text{ or } 5.2\times10^{3}\ \mu\text{m}^3$

(b) $V_{\rm cell} = L^3 \Rightarrow L = V_{\rm cell}^{1/3} = \left(5.2\times10^{3}\ \mu\text{m}^3\right)^{1/3} = 17\ \mu\text{m}$

ASSESS: This is the right order of magnitude for biological cells.

P1.20 **PREPARE:** Both parts (a) and (b) are essentially unit conversions.

SOLVE: (a) Inserting the given numbers, we have

$$5\text{ L}\times\frac{6\times10^{6}\text{ cells}}{10^{-6}\text{ L}} = 3\times10^{13}\text{ cells}$$

Which we report to one significant digit as 3×10^{13}cells

(b) $\dfrac{150\ \mu\text{g hem}}{1\ \mu\text{L blood}}\times\dfrac{1\ \mu\text{L}}{6\times10^{6}\text{ cells}}\times\dfrac{1\text{ g}}{10^{6}\ \mu\text{g}}\times\dfrac{1\text{ Da}}{1.66\times10^{-24}\text{ g}}\times\dfrac{1\text{ hem}}{64{,}000\text{ Da}} = 2.35\times10^{8}\ \dfrac{\text{hem}}{\text{cells}}$

$\approx 2\times10^{8}$ hemoglobin per red blood cell

ASSESS: Since proteins carry out vital functions all throughout a cell and body, we expect to find many copies of a given protein in a cell. This number still seems astronomical, but a quick search online confirms that this is the right order of magnitude!

P1.21 **PREPARE:** Both parts (a) and (b) use the fact that speed is distance over time: $s = d/t$.

SOLVE: (a) $s = \dfrac{d}{t} = \dfrac{60\text{ bp}}{\text{s}} \times \dfrac{0.34\text{ nm}}{1\text{ bp}} = 20\text{ nm/s}$

(b) $t = d/s = \dfrac{12{,}000\text{ bp}}{60\text{ bp/s}} = 200\text{ s or }3.3\text{ min}$

ASSESS: This is a surprisingly long time for a small-scale biological process.

P1.22 **PREPARE:** We will pretend that the oil will spread until it is a single molecule thick. This isn't true, but it is the estimation method being described. We can determine the volume of the oil from $\rho = m/V$ and let the volume be given by $V = Ad$, where d is the unknown thickness of the layer.

SOLVE: Combining the equations and rearranging, we have $\rho = m/Ad \Rightarrow d = m/A\rho$. Inserting the given numbers, we find

$$d = m/A\rho = \frac{\left(1.0\times10^{-3}\text{ g}\right)}{\left(8500\text{ cm}^2\right)\left(0.92\text{ g/cm}^3\right)} = 1.28\times10^{-7}\text{ cm or about 1.3 nm}$$

ASSESS: As stated in the problem stem, this is slightly larger than the actual size of simple molecules like those in oil. But, it was a good first step.

P1.23 **PREPARE:** We can set the total mass of a human equal to the mass of mammalian cells and bacterial cells combined, and we will require that the number N of mammalian cells be one-third the number of bacterial cells. We will treat both cells as spheres, so that we can find the volume using $V = \frac{4}{3}\pi r^3$ and from there, we can determine the masses of cells using $\rho = m/V$.

SOLVE: (a) The total mass of cells can be expressed as

$$m = N\rho V_\text{m} + 3N\rho V_\text{b} = N\rho\frac{4}{3}\pi r_\text{m}^3 + 3N\rho\frac{4}{3}\pi r_\text{b}^3 = N\rho\frac{4}{3}\pi\left(r_\text{m}^3 + 3r_\text{b}^3\right)$$

This must yield the total mass of the human. We can rearrange this to solve for the number of mammalian cells N:

$$N = \frac{3m}{4\pi\rho\left(r_\text{m}^3 + 3r_\text{b}^3\right)} = \frac{3(70\text{ kg})}{4\pi\left(1000\text{ kg/m}^3\right)\left(\left(6.0\times10^{-6}\text{ m}\right)^3 + 3\left(5.0\times10^{-7}\text{ m}\right)^3\right)} = 7.7\times10^{13}$$

There are about 7.7×10^{13} mammalian cells in such a human.

(b) Here, we simply take the ratios of the mass of the bacterial cells to the total as follows:

$$\frac{m_\text{b}}{m} = \frac{3N\rho V_\text{b}}{m} = \frac{3N\rho}{m}\frac{4}{3}\pi r_b^3 = \frac{4\pi\left(7.72\times10^{13}\text{ cells}\right)\left(1000\text{ kg/m}^3\right)\left(5.0\times10^{-7}\text{m}\right)^3}{(70\text{ kg})} = 0.0017\text{ or }0.17\%$$

ASSESS: The question premise that humans have more bacterial cells than mammalian cells may have seemed counterintuitive. But, part (b) helps us reconcile this with what we know about our bodies: the smaller size of bacterial cells means that the premise can be true, even though not much of our bodies' mass is made up of bacteria.

P1.24 **PREPARE:** Part (a) simply requires converting from base pairs to SI units of length. In part (b), we will model a single nucleotide as a cube such that its volume is $V_n = L^3$. We will also use that $V_{\text{sphere}} = \frac{4}{3}\pi r^3$.

SOLVE: (a) The length is $\ell_{\text{total}} = N\ell_n = (6\times10^9 \text{ nucleotides})(0.3\times10^{-9} \text{ m/nucleotides}) = 1.8 \text{ m} \approx 2 \text{ m}$.

(b) Equating the volume of a tightly packed sphere to that of all the nucleotides, we have

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3 = NV_n = NL^3 \Rightarrow r = \left(\frac{3N}{4}\right)^{1/3} L = \left(\frac{3(6\times10^9)}{4}\right)^{1/3} (0.3\times10^{-9} \text{ m}) = 3.4\times10^{-7} \text{ m}$$

The diameter of such a sphere would be about $0.7\ \mu\text{m}$.

ASSESS: Clearly, there is sufficient space in the cell nucleus to hold all nucleotides, if they are compressed together tightly.

P1.25 **PREPARE:** The surface area of a sphere is $A = 4\pi r^2$.

SOLVE: (a) The density of hairs is the number of hairs divided by the area of the head that is covered with hair.

$$N/A = \frac{N}{(0.25)4\pi r^2} = \frac{(10^5 \text{ hairs})}{\pi(7.5 \text{ cm})^2} = 5.66\times10^2 \text{ hairs/cm}^2 \approx 560 \text{ hairs/cm}^2$$

(b) If 566 hairs were arranged on a square grid 1 cm on a side, there would be $\sqrt{566} = 23.8$ hairs on one side. That means the hairs would have to be separated by approximately 1/24 of a centimeter. Thus, the spacing is approximately $(1 \text{ cm})/(24) = 0.04$ cm or 0.4 mm.

ASSESS: We could have guessed that the spacing between hairs should be on the order of a millimeter, or smaller. So, this answer is very reasonable.

P1.26 **PREPARE:** We will repeatedly use the formulas for the volume and surface area of spheres: $V_{\text{sphere}} = (4/3)\pi r^3$ and $A_{\text{sphere}} = 4\pi r^2$.

SOLVE: (a) The volume of each lung would need to be 500 cm^3. This corresponds to a radius of

$$r = \left(\frac{3V_{\text{sphere}}}{4\pi}\right)^{1/3} = \left(\frac{3(500\,\text{cm}^3)}{4\pi}\right)^{1/3} = 4.9 \text{ cm}$$

The diameter would then be 9.8 cm.

(b) The surface area of the two spherical lungs would then be

$$A = 2A_{\text{sphere}} = 8\pi r^2 = 8\pi(4.92 \text{ cm})^2 = 6.1\times10^2 \text{ cm}^2 \text{ or } 0.061 \text{ m}^2$$

(c) The given surface area of $70\ \text{m}^2$ must come from 300 million times the surface area of a single alveolus. Thus,

$$A_{\text{total}} = N4\pi r^2 \Rightarrow r = \sqrt{\frac{A_{\text{total}}}{N4\pi}} = \sqrt{\frac{(70\ \text{m}^2)}{(3.00\times10^8)4\pi}} = 1.36\times10^{-4}\text{m}$$

This means that the diameter is 0.27 mm or $270\ \mu\text{m}$.

ASSESS: It makes sense that the alveoli must be very small in order for their total surface area to be so much larger than the surface area of the lung when modeled as one large sack.

P1.27 **PREPARE:** We can rewrite the given scaling law as $S = cm^{-3/4}$, where c is an unknown constant. This holds for the larger animals and for the smaller animals: $S_{\text{L}} = cm_{\text{L}}^{-3/4}$ and $S_{\text{S}} = cm_{\text{S}}^{-3/4}$.

SOLVE: Combining the equations above, we have

$$\frac{S_{\text{S}}}{S_{\text{L}}} = \frac{cm_{\text{S}}^{-3/4}}{cm_{\text{L}}^{-3/4}} = \left(\frac{m_{\text{L}}}{m_{\text{S}}}\right)^{3/4} = \left(\frac{5\ \text{kg}}{0.5\ \text{kg}}\right)^{3/4} = 5.6 \approx 6$$

There will be approximately 6 times as many small species as large species.

ASSESS: The original scaling law can be interpreted as saying that the number of species drops off as the mass gets larger, so it is reasonable that we would find more of the lightweight species than of the heavy species.

P1.28 **PREPARE:** This is an estimation problem, so a range of answers may be correct. Let us assume the back-to-back distance from one person to the next is about 0.3 m on average. We will also use $C = 2\pi r$ for the circumference of Earth.

SOLVE: (a) $L = Nd = (8\times10^9)(0.3\ \text{m}) = 2.4\times10^9\ \text{m} \approx 2\times10^9\ \text{m}$.

(b) We simply divide the distance by the circumference of Earth as follows:

$$N = \frac{L}{2\pi r} = \frac{(2.4\times10^9\ \text{m})}{2\pi(6.4\times10^6\ \text{m})} = 60$$

This line of people would stretch around the earth 60 times.

ASSESS: One might note how important dimensionality is here. The people described are right next to each other. Yet, when they spread out away from the equator in a second dimension (latitude), there is plenty of space.

P1.29 **PREPARE:** We can express the mass of the soil and of the bacteria using their densities and volumes, according to $m = \rho V$. We are given the volume of soil, and the volume of a bacterium will be calculated by treating it as a sphere: $V_{\text{sphere}} = \frac{4}{3}\pi r^3$.

SOLVE: We wish to find the ratio of bacterial mass to the total mass:

$$\frac{m_{\text{b}}}{m_{\text{t}}} = \frac{\rho_{\text{b}}V_{\text{b}}}{\rho_{\text{t}}V_{\text{t}}} = \frac{\rho_{\text{b}}N4\pi r^3/3}{\rho_{\text{t}}V_{\text{t}}} = \frac{(1000\ \text{kg/cm}^3)(10^9)4\pi(5.0\times10^{-7}\ \text{m})^3}{3(1300\ \text{kg/m}^3)(0.010\ \text{m})^3} = 4\times10^{-4}$$

ASSESS: This explains why the presence of bacteria does not usually play an important role in the calculations of mass. Although there may be billions of them, they are so small that they hardly affect the overall mass.

P1.30 **PREPARE:** None of the answers below exactly restate anything from the text. We must consider whether any of them are consistent with the text.

SOLVE: The text contains no information on the rate at which microbes are discovered, so it will not be option C. There is no mention of cost to the host animal. There may be a cost; there may be a benefit as with bacteria that aid digestion in the gut. It is not clear whether D might be true or not, but it is likely not true in general. We are left with options A and B. If most microbes lived on the surface of animals, then their number would tend to scale with the surface area of the animal. But, surface area does not scale linearly with mass. Volume scales linearly with mass: $V = m/\rho$. So, the scaling law (which is very close to linear dependence on mass) is most consistent with bacteria occupying the entire volume of the animal, rather than the surface. We therefore discount option A. This leaves us with option B. Although we are not told that most of the bacteria live in the gut, it is true that the volume of the gut should roughly scale with the volume of the animal and the mass of the animal. Although this does not prove that most bacteria live in the gut, that statement is consistent with the scaling law. We choose option B.

ASSESS: One may also simply know from other fields of study that bacteria in the gut play an important role in digestion.

P1.31 **PREPARE:** We can rewrite the given scaling law for bacterial diversity $D \propto m^{0.34}$ as $D_\text{E} = cm_\text{E}^{0.34}$ and $D_\text{C} = cm_\text{C}^{0.34}$, where c is an unknown constant.

SOLVE: Combining the equations above, we have

$$\frac{D_\text{E}}{D_\text{C}} = \frac{cm_\text{E}^{0.34}}{cm_\text{C}^{0.34}} = \left(\frac{m_\text{E}}{m_\text{C}}\right)^{0.34} = \left(\frac{5000\text{ kg}}{500\text{ kg}}\right)^{0.34} = 2.2 \approx 2$$

The answer is A.

ASSESS: If the gut diversity scaled linearly with mass, we would expect the gut diversity of the elephant to be 10 times that of the cow. Since the dependence on mass is weaker than that, it is reasonable that we got a smaller factor.

P1.32 **PREPARE:** We examine both scaling laws: $N \propto m^{1.07}$ and $D \propto m^{0.34}$.

SOLVE: Note that both the number of microbes and the diversity increase as mass increases. The number of microbes increases with mass with a higher exponent than diversity does, so the number of microbes will increase faster than diversity as mass increases. This matches answer option B.

ASSESS: For either of these quantities to decrease, they would need to depend on mass to a negative exponent, such as m^{-1}. Clearly, both quantities must increase as an organism grows larger and more massive.

P1.33 **PREPARE:** We must first recognize that the number of animal cells scales exactly with the mass of the animal: $N_A \propto m^{1.00}$.

SOLVE: We note that the number of microbial cells increases slightly faster than that: $N \propto m^{1.07}$. It is not obvious from the text that all or most of these microbes are in the gut. However, the fact that the number of microbes scales almost linearly with mass, certainly indicates that the microbes live throughout the volume of the body rather than on the surface, or in an isolated area that does not grow. It is a safe assumption that the number of microbes in the gut follows a similar or identical law. Thus, we see that the number of microbes in the gut increases a litte faster than the animal cells. Thus, answer C is correct.

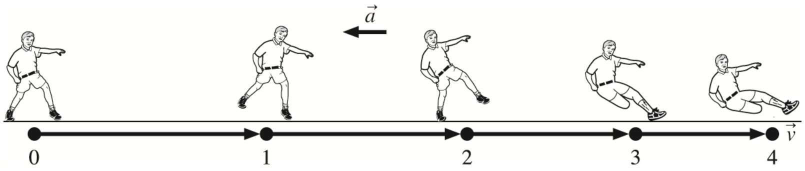
ASSESS: The rate of increase with mass is very similar for the cells and microbes, so this does not represent a huge shift in the makeup of the animal.

2

DESCRIBING MOTION

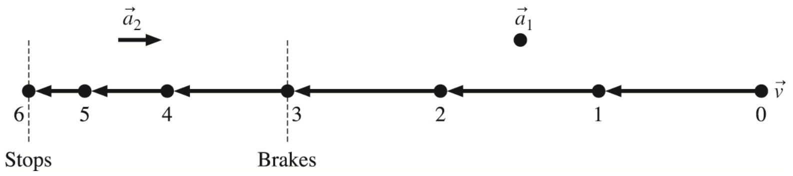
QUESTIONS

Q2.1 **REASON:** The softball player starts with an initial velocity but as he slides, he moves slower and slower until coming to rest at the base. The distance he travels in successive times will become smaller and smaller until he comes to a stop. See the figure below.



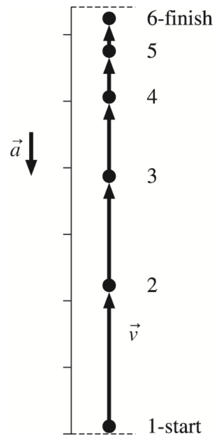
ASSESS: Compare to Figure 2.8 in the text.

Q2.2 **REASON:**



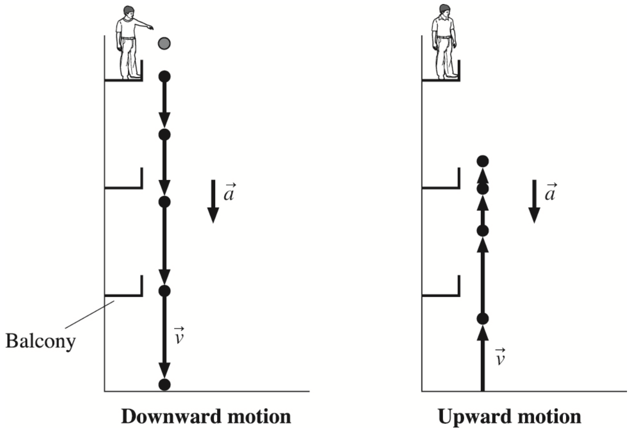
ASSESS: The dots are equally spaced until the brakes are applied to the car. Equidistant dots indicate constant average speed. On braking, the dots get closer as the average speed decreases.

Q2.3 **REASON:**



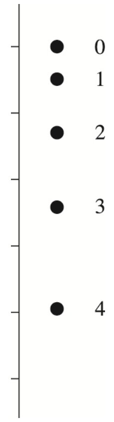
ASSESS: The spacing between dots is initially large, since the initial speed with which the bush baby leaves the ground is large. As the bush baby rises and gravity slows the ascent, the speed decreases, and therefore the spacing between adjacent dots in the motion diagram decreases.

Q2.4 **REASON:** The tennis ball falls freely from the three stories under the pull of gravity. Since gravity is pulling it downward, its speed increases with time. It strikes the ground and very quickly slows down to a stop (while compressing the ball) then bounces back upward (while the ball decompresses). After the bounce, it travels upward while still under the pull of gravity. As it is traveling upward, the pull of gravity decreases its velocity to zero at a height of two stories. The downward and upward motions of the ball are shown in the figure below. The increasing length of the arrows during the downward motion indicates increasing velocity. The decreasing length of the arrows during the upward motion indicates the particle slowing down.



ASSESS: Note that gravity always pulls downward, no matter what the direction of motion of something under its influence. So, the ball will constantly be slowing down during its upward motion.

Q2.5 **REASON:** As the ball drops from the tall building, the ball will go faster and faster the farther it falls under the pull of gravity. The motion diagram should show the displacements for later times to be getting larger and larger. The successive displacements in the diagram given in the text get smaller and smaller. So, the diagram given in the problem is incorrect. The correct diagram is below.



ASSESS: The displacements increase during the fall of the object as we reasoned.

Q2.6 **REASON:** The distance you travel will be recorded on the odometer. As you travel, the distance you travel accumulates, which is recorded by the odometer, and is independent of the direction of travel. Your displacement is the difference between your final position and your initial position. If you travel around a 440 m track and end up where you started, you have traveled 440 m; however, since you ended up where you started, your change in position and hence displacement is zero.

ASSESS: If you watch a track meet, you will observe the 440-m race. As you watch the race, it is obvious that the runners travel a distance of 440 m (assuming they complete the race). Yet, since they end up where they started, their final position is the same as their initial position and hence their displacement is zero.

Q2.7 **REASON:** Since the jogger is running around a track, she returns to her starting point at the end of the lap. Since her final position is the same as her initial position, her displacement is 0 m. Velocity is defined as

$$\text{velocity} = \frac{\text{displacement in a given time interval}}{\text{time interval}}$$

So, her average velocity is 0 m/s.

However, though her displacement is 0 m, the actual distance she traveled is 400 m. Her average speed is not zero, since speed is defined in terms of distance, not displacement.

$$\text{speed} = \frac{\text{distance traveled in a given time interval}}{\text{time interval}}$$

Her average speed is then $\frac{400\text{ m}}{100\text{ s}} = 4\text{ m/s}$.

The second friend is correct.

Since the motion of a runner is not uniform, we can only calculate average velocity and average speed.

ASSESS: This problem illustrates a very important difference between speed and velocity. Speed depends on total distance traveled. Velocity depends on displacement, which only takes into account the starting and ending points of a motion.

Q2.8 **REASON:** Since the velocity of the skateboard is negative during the whole time of its motion, it is moving in the negative direction the entire 5 s. In order to move closer to the origin, which is in the positive direction relative to the starting point of the skateboard, the skateboard must have had a velocity in the positive direction for some time. Since the velocity is always negative, the skateboard must be farther from the origin than initially.

ASSESS: Velocity gives direction of motion since it refers to displacements and not only distance traveled. If velocity is always negative, displacement will be negative also.

Q2.9 **REASON:** If the position of the bicycle is negative, it is to your left. The bicycle's velocity is positive, or to the right, so the bicycle is getting closer to you.

ASSESS: If the initial position had been positive and the velocity positive, the bicycle would be getting farther away from you.

Q2.10 **REASON:** Because the velocity vectors get shorter for each time step, the object must be slowing down as it travels to the left. The acceleration vector must therefore point opposite the velocity. In this case, that means the acceleration vector points to the right. Thus, a_x is positive as per our convention.

ASSESS: We can also see that if we wanted to add a vector to the first velocity vector to attain the size of the next velocity vector, we would need to add a vector pointing to the right.

Q2.11 **REASON:** Because the velocity vectors get shorter for each time step, the object must be slowing down as it travels in the $-y$ direction (downward). The acceleration vector must therefore point in the direction opposite to the velocity; namely, in the $+y$ direction (upward). Thus, a_y is positive as per our convention.

ASSESS: We can also see that the difference between the second and first velocity vector; $\vec{v}_2 - \vec{v}_1$ is an upward vector, so the acceleration vector must point upward.

Q2.12 **REASON:** The particle position is to the left of zero on the *x*-axis, so its position is negative. The particle is moving to the left, so its velocity is negative. The particle's speed is decreasing as it moves to the left, so its acceleration vector points in the direction opposite the velocity vector (i.e., to the right). Thus, the acceleration is positive.

ASSESS: We can also see that the difference between the second and first velocity vector, $\vec{v}_2 - \vec{v}_1$, is a rightward vector, so the acceleration vector must point to the right.

Q2.13 **REASON:** The particle position is below zero on the *y*-axis, so its position is negative. The particle is moving up, so its velocity is positive. The particle's speed is increasing as it moves in the positive direction, so its acceleration vector points in the same direction as its velocity vector (i.e., up). Thus, the acceleration is also positive.

ASSESS: We can also see that the difference between the second and first velocity vector, $\vec{v}_2 - \vec{v}_1$, is an upward vector, so the acceleration vector must point upward.

Q2.14 **REASON:** We do not believe Travis. 55 m is more than 165 ft. It is not reasonable to think that a unicycle could move that quickly.

ASSESS: The evaluation may be easier in different units. Consider converting to miles per hour:

$$\frac{55\text{ m}}{\text{s}}\times\frac{1\text{ mi}}{1610\text{ m}}\times\frac{3600\text{ s}}{1\text{ h}}=120\text{ mph}$$

This is twice the speed limit on many freeways and not reasonable for a unicycle.

Q2.15 **REASON:** Because of the numbering of the dots, we see the object is moving to the left. It is slowing because the dots are getting closer together. The choice that fits this scenario is a cyclist moving to the left and braking to a stop. So, choice B is correct.

ASSESS: If the dots were numbered in reverse order, then choice C would be correct.

Q2.16 **REASON:** Because the dots are getting farther apart to the right (and the numbers are increasing to the right), we know that the object is speeding up. The choice that best fits that is a car pulling away (to the right) from a stop sign. So, the correct choice is C.

ASSESS: An ice skater gliding (choice A) would likely have nearly constant velocity (constant spacing between dots). The motion diagram for a plane braking (choice B) might look like the given diagram with the dots numbered in reverse order. The pool ball reversing direction (choice D) would have dot numbers increasing in one direction at first, but then going the other way.

Q2.17 **REASON:** The speed is the distance divided by the time.

$$\text{speed}=\frac{\text{distance}}{\text{time}}=\frac{0.30\text{ km}}{5\text{ min}}\left(\frac{1{,}000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)=1.0\text{ m/s}$$

So, the correct choice is -C (rearranged in 1st pages.

ASSESS: 1 m/s does seem like a reasonable speed for a seal in water.

Q2.18 **REASON:** Following is a simple unit conversion problem:

$$\frac{400\text{ m}}{51.9\text{ s}}\times\frac{1\text{ mi}}{1610\text{ m}}\times\frac{3600\text{ s}}{1\text{ h}}=17.2\text{ mph}$$

So, the correct choice is C.

ASSESS: This is the speed of a slow-moving car and is reasonable for a very fast-moving human.

Q2.19 **REASON:** Since the rock is above the origin (further up the $+y$ axis), the position is positive; since it is still moving upward the vertical component of velocity is also positive. Hence, the correct answer is A.

ASSESS: After it gets to the top and starts back down, the position will still be positive, but the velocity will be negative.

Q2.20 **REASON:** Since we are using the convention that the $+y$ points vertically upward, we know a downward-moving rock will have a negative velocity. Clearly, gravity accelerates things downward, so the acceleration is also negative. The answer is D.

ASSESS: We know from our reading that objects speed up when acceleration and velocity are in the same direction. We know from experience that a dropped stone gains speed as it falls. So, the fact that our answer has the same sign for both velocity and acceleration is encouraging.

Q2.21 **REASON:** The second rule on significant figures says that when adding two numbers the number of decimal places in the answer should match the *smallest* number of decimal places of any number used in the calculation. There are three decimal places in 0.532 m, but only two in 3.24 m, so our answer must have only two decimal places. Therefore, the correct choice is B: 3.77 m.

ASSESS: On contemplation, we realize that we don't know the 3.24 m number better than to the nearest centimeter and that will also be true of the answer, even though we may know the other number to the nearest millimeter.

Q2.22 **REASON:** When multiplying numbers, the correct number of significant digits is the smaller of the numbers of significant digits in the two numbers. That is, here, we have four significant figures in 109.7 m and three in 48.8 m, and so our product must have three significant digits:

$$(109.7\text{ m}) \times (48.8\text{ m}) = 5.35 \times 10^3\text{ m}^2$$

The correct answer is C.

ASSESS: Our final answer has three digits of precision, since we multiplied by something with only three digits of precision.

Q2.23 **REASON:** This is a straightforward unit conversion question.

$$4.57\times10^9\text{ y} = 4.57\times10^9\text{ y}\left(\frac{365.25\text{ d}}{1\text{ y}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 1.44 \times 10^{17}\text{ s}$$

The correct choice is D.

ASSESS: Notice that even though we have more significant figures in some of the conversion factors (we accounted for leap year, and the other factors are exact and have as many significant figures as we need) that we obey the significant figure rules and report the answer to the same number of significant figures as the number of significant figures, in the least precisely known number in the calculation (the age in years).

Q2.24 **REASON:** We are given an equation for density and are asked to calculate the density of the earth given its mass and volume. However, the units must be converted before the calculation is done since we're given volume in km^3 and the answer must be given in terms of m^3.

$$V = (1.08\times10^{12}\text{ km}^3)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)$$
$$= (1.08\times10^{12}\text{ km}^3)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)^3 = (1.08\times10^{12}\text{ km}^3)\left(\frac{10^9\text{ m}^3}{1\text{ km}^3}\right)$$
$$= 1.08\times10^{21}\text{ m}^3$$

Note carefully that we needed *three* conversion factors for the conversion from kilometer to meter here since we are dealing with cubic kilometers. Three factors are needed to cancel the factor of $\text{km}^3 = \text{km}\cdot\text{km}\cdot\text{km}$.

So, the density is as follows:

$$\rho = \frac{M}{V} = \frac{5.94\times10^{24}\text{ kg}}{1.08\times10^{21}\text{ m}^3} = 5.50\times10^3\text{ kg/m}^3$$

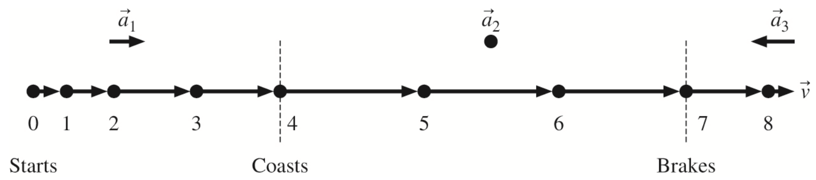
The correct choice is A.

ASSESS: For cubic and square units (or units to any power), you must include the correct number of conversion factors to convert every factor in the original quantity. Since the density of water is $1.0 \times 10^3\ \text{kg/m}^3$, it seems reasonable that the earth would be 5.5 times as dense.

PROBLEMS

P2.1 **PREPARE:** The spacing of dots in a motion diagram corresponds to the distance traveled in constant time intervals. So, when the bike is speeding up from rest, the spacing should begin small and increase. Similarly, when the bike travels at a constant speed, the spacing between dots should be constant, and so on.

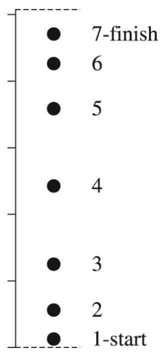
SOLVE: The diagram below reflects the considerations above and also shows a decreasing spacing as the bike stops.



ASSESS: The velocity vectors between dots intuitively show that the speed increases, holds stead, and then decreases.

P2.2 **PREPARE:** As the elevator begins to rise, its speed changes from zero to some other speed. An elevator typically maintains a steady speed for a while, and then slows as it reaches the desired floor. So our diagram should start with very small but increasing spacing between dots (getting started), then have wider spaced dots with constant spacing for some time (constant speed), and finally dots with decreasing spacing (slowing to a stop).

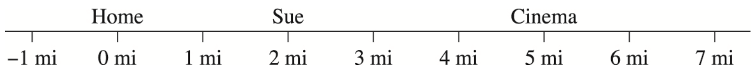
SOLVE:



ASSESS: The speed (and therefore the spacing between dots) increases from 1 to 3, then stays constant from 3 to 5, and decreases from 5 to 7.

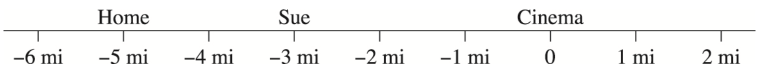
P2.3 **PREPARE:** We are asked to find a position, which depends on the origin. Therefore, we expect to find different answers to parts (a) and (b).

SOLVE: **(a)** Refer to the figure below.



If Sue's home is the origin of the x-axis, she is 2 mi to the right of the origin. This is the positive side of the axis, so her position $x = +2$ mi.

(b) Refer to the figure below.



Taking the cinema as the origin of the $x-$axis, then Sue is 3 mi to the left of the origin. This is the negative portion of the axis, so Sue's position is $x = -3$ mi.

ASSESS: Position along a straight line is a *signed* quantity. It's important to indicate the sign when reporting a displacement relative to a chosen origin.

P2.4 **PREPARE:** The displacement is the difference between two positions. It does not depend on the origin. So, we expect to find the same answer in both cases. To find Sue's displacement, we want to find the difference between her initial position between her home and the cinema, and her position at her home.

SOLVE: It is graphically clear that $\Delta x = x_f - x_i = 2$ mi. This is the answer for both parts (a) and (b).

ASSESS: Sue's position depends on where the origin is. But, the displacement is the difference between two points (initial and final), and that is independent of what point in space we label as our origin.

P2.5 **PREPARE:** Let us assume that the ruler is oriented along the positive x axis. We will use that $\Delta x = x_f - x_i$ and $\Delta t = t_f - t_i$.

SOLVE: Inserting the given values, we have

$$\Delta x = x_f - x_i = (42 \text{ mm}) - (65 \text{ mm}) = -23 \text{ mm}$$

and

$$\Delta t = t_f - t_i = (9:12:05 \text{ AM}) - (9:11:15 \text{ AM}) = 50 \text{ s}$$

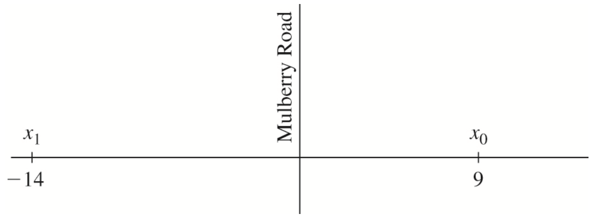
ASSESS: Since the ruler marking decreased, it makes sense that the displacement is negative. The passage of time should certainly be positive.

P2.6 **PREPARE:** Displacement along a straight line is a signed quantity, and is given by $\Delta x = x_f - x_i$. Since this problem asks for displacements relative to Mulberry Road, we will choose Mulberry Road as the origin.

SOLVE: Since Mulberry Road was chosen as the origin and x increases to the east, the final position of the car is $x_f = -14$ mi. We are told that the displacement of the car was $\Delta x = -23$ mi, so during its motion the car traveled 23 mi west. We can use the definition of displacement, $\Delta x = x_f - x_i$, to find the car's initial position.

$$x_i = x_f - \Delta x = (-14 \text{ mi}) - (-23 \text{ mi}) = +9 \text{ mi}$$

The car started 9 mi east of Mulberry Road.



ASSESS: Note the choice of origin for measuring positions and displacements is arbitrary, but in many cases it's convenient to choose a reference point in the problem as the origin. Here, all displacements and positions were mentioned relative to Mulberry Road, which was chosen as the origin and made the calculations direct.

P2.7 **PREPARE:** We are told the guard walks at a steady pace, so there is only one speed in this problem: the average speed. The average speed is defined in Equation 2.1 of the text. Speed is the distance traveled in some time interval divided by the length of the time interval (at least in this case of a steady pace, in which average speed and speed refer to the same thing).

SOLVE: From Equation 2.1,

$$\text{speed } v = \frac{\text{distance traveled}}{\text{time interval spent traveling}} = \frac{110\text{ m}}{240\text{ s}} = 0.46\text{ m/s}$$

ASSESS: Someone walking at a brisk pace will easily travel more than 1 m/s. However, since a guard would travel at more like a stroll, this is a reasonable speed.

P2.8 **PREPARE:** In all cases, objects are moving at a steady pace. So, speed is just the distance divided by the time. We are asked to rank in order three different speeds, so we simply compute each one according to Equation 2.1:

$$\text{speed } v = \frac{\text{distance traveled}}{\text{time interval spent traveling}}$$

SOLVE: (i) Toy $\frac{0.15\text{ m}}{2.5\text{ s}} = 0.060\text{ m/s}$

(ii) Ball $\frac{2.3\text{ m}}{0.55\text{ s}} = 4.2\text{ m/s}$

(iii) Bicycle $\frac{0.60\text{ m}}{0.075\text{ s}} = 8.0\text{ m/s}$

(iv) Cat $\frac{8.0\text{ m}}{2.0\text{ s}} = 4.0\text{ m/s}$

So the order from fastest to slowest is bicycle, ball, cat, and toy car.

ASSESS: We reported all answers to two significant figures, as we should according to the significant figure rules.

The result is probably what we would have guessed before solving the problem, although the cat and ball are close. These numbers all seem reasonable for the respective objects.

P2.9 **PREPARE:** We want to find the highest speed, meaning the highest value of: speed = $\Delta x/\Delta t$. Since we are told the times after intervals of 100 m, the Δx is the same over each interval. Only Δt changes. We need to find the shortest time interval, since that will correspond to the highest speed.

SOLVE: We find the duration of each of the four intervals:

$$\Delta t_1 = 11.20 \text{ s}$$
$$\Delta t_2 = (21.32 \text{ s}) - (11.20 \text{ s}) = 10.12 \text{ s}$$
$$\Delta t_3 = (31.76 \text{ s}) - (21.32 \text{ s}) = 10.44 \text{ s}$$
$$\Delta t_4 = (43.18 \text{ s}) - (31.76 \text{ s}) = 11.42 \text{ s}$$

(a) Clearly, the second 100 m was done in the shortest amount of time. So, the second 100 m was the fastest.

(b) $\text{speed} = \dfrac{\Delta x}{\Delta t} = \dfrac{100 \text{ m}}{10.12 \text{ s}} = 9.88 \text{ m/s}$

ASSESS: This is about 22 mph: extremely fast for a human but still entirely plausible.

P2.10 **PREPARE:** We will use the distance and time to calculate average speed. Because we want the average speed, the elapsed time is not only the time spent in motion but also the delay.

SOLVE: We know $\text{speed} = \dfrac{d}{\Delta t}$. Inserting numbers, we have

$$\text{speed} = \frac{d}{\Delta t} = \frac{(8\times 10^{-9} \text{ m})}{(15\times 10^{-3} \text{ s}) + (50\times 10^{-6} \text{ s})} = 5\times 10^{-7} \text{ m/s°}$$

or 500 nm/s.

ASSESS: Since we are told that the kinesin moves a few nanometers in much less than a second, a speed of several hundred nanometers each second is reasonable.

P2.11 **PREPARE:** We can use Equation 2.3 and the times and positions given in the diagram to calculate the horse's velocity at the different times.

SOLVE: Since the dots are spaced at equal intervals of time, and there is one dot between the time 10 s and 30 s, the spacing between the dots indicate a 10 s time interval. The dot between 10 s and 30 s will mark a time of 20 s. The horse is moving to the left, as time increases to the left, so the rightmost dot must be at 0 s. We will use the definition of the average velocity in Equation 2.3, applied to the horizontal direction: $(v_x)_{\text{avg}} = \Delta x/\Delta t$. Looking at Figure P2.11, it should be noted that you can't determine the distance information to better than 10 m and certainly not to 1 m. As a result, 100 m has two significant figures (you know it is between 90 m and 110 m) and 50 m has one significant figure (you know it is between 40 m and 60 m). For this reason, the answer to part (b) should have only one significant figure.

(a) Referring to the P1.12 in the text, $x_f = 500$ m, $x_i = 600$ m, $t_f = 10$ s, $t_i = 0$ s, so

$$(v_x)_{avg} = \frac{\Delta x}{\Delta t} = \frac{500\text{ m} - 600\text{ m}}{10\text{ s} - 0\text{ s}} = \frac{-100\text{ m}}{10\text{ s}} = -10\text{ m/s}$$

(b) Here, $x_f = 300$ m, $x_i = 350$ m, $t_f = 40$ s, $t_i = 30$ s, so

$$(v_x)_{avg} = \frac{\Delta x}{\Delta t} = \frac{300\text{ m} - 350\text{ m}}{40\text{ s} - 30\text{ s}} = \frac{-50\text{ m}}{10\text{ s}} = -5\text{ m/s}$$

(c) In this case, $x_f = 50$ m, $x_i = 250$ m, $t_f = 70$ s, $t_i = 50$ s, so

$$(v_x)_{avg} = \frac{\Delta x}{\Delta t} = \frac{50\text{ m} - 250\text{ m}}{70\text{ s} - 50\text{ s}} = \frac{-200\text{ m}}{20\text{ s}} = -10\text{ m/s}$$

ASSESS: Displacement and velocities are signed quantities. Since the *x*-axis increases to the right and the horse is traveling to the left, we should expect all the velocities to be negative.

P2.12 **PREPARE:** Average velocity in this one-dimensional problem is defined as the displacement Δx divided by the time interval Δt. We are given the initial and final positions, and the time interval, so we can use Equation 2.3 applied to the horizontal direction.

SOLVE: $x_f = 3$ m, $x_i = -12$ m, $\Delta t = 10$ s, so

$$(v_x)_{avg} = \frac{\Delta x}{\Delta t} = \frac{8\text{ m} - (-12)\text{ m}}{10\text{ s}} = \frac{20\text{ m}}{10\text{ s}} = +2.0\text{ m/s}$$

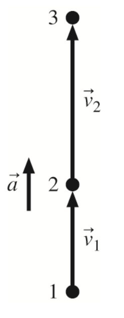
So the average velocity is 2.0 m/s in the $+x$ direction.

ASSESS: Note that it's important to keep track of signs on positions and displacements in equations.

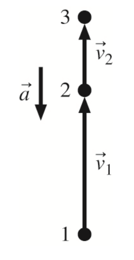
P2.13 **PREPARE:** We know that if the velocity vectors and acceleration point in the same direction, the velocity vectors will get better. If the velocity vectors and acceleration point opposite each other, the velocity vectors will become smaller. We can use this to determine appropriate sizes for the additional velocity vectors.

SOLVE:

(a)



(b)

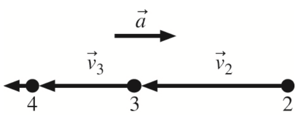


ASSESS: Part (b) in particular is familiar to us. This is a good representation of a ball thrown upward with a speed that is initially large. The ball will slow as it ascends, because of the downward acceleration due to gravity.

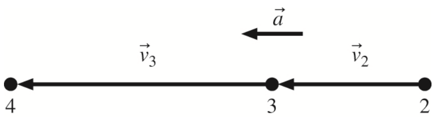
P2.14 **PREPARE:** We know that if the velocity vectors and acceleration point in the same direction, the velocity vectors will get better. If the velocity vectors and acceleration point opposite each other, the velocity vectors will become smaller. We can use this to determine appropriate sizes for the additional velocity vectors.

SOLVE:

(a)



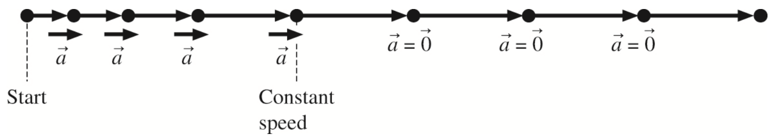
(b)



ASSESS: Part (a) is similar to the case of a car driving to the left, and applying the brakes. The acceleration is opposite the velocity, and so the car slows.

P2.15 **PREPARE:** Model the skater as a particle. The dots must start out close together, with spacing that initially increases. But, after the skater reaches constant speed, the dots are equally spaced.

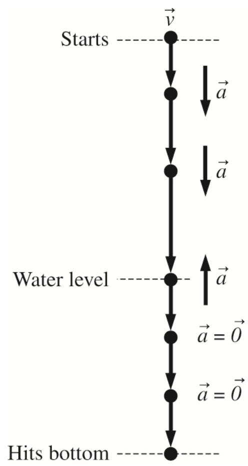
SOLVE:



ASSESS: We note that when the acceleration is in the same direction as the velocity, the speed increases, as it should.

P2.16 **PREPARE:** Represent the tile as a particle. Starting from rest, the tile's velocity increases until it hits the water surface. This part of the motion is represented by dots with increasing separation, indicating increasing average velocity. After the tile enters the water, it settles to the bottom at roughly constant speed, so this part of the motion is represented by equally spaced dots.

SOLVE:



ASSESS: We note that when the acceleration is in the same direction as the velocity, the speed increases, as it should. When the acceleration is briefly directed oppose the velocity, the tile slows.

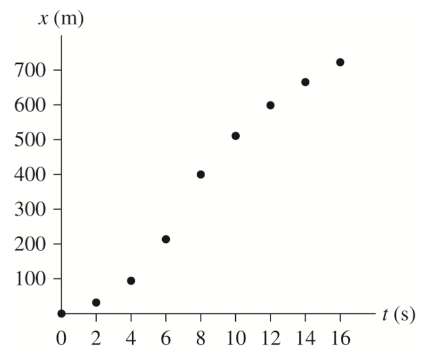
P2.17 **PREPARE:** The time is evenly spaced, with two seconds between each measurement. The position is not evenly spaced, since the lengths of the velocity vectors vary. Therefore, we expect a graph that is not linear.

SOLVE:

(A)

Dot	Time (s)	x (m)
1	0	0
2	2	30
3	4	95
4	6	215
5	8	400
6	10	510
7	12	600
8	14	670
9	16	720

(B)



ASSESS: Note that when the velocity vectors are longest (near the middle of the time interval) the position vs. time plot has the highest slope, as it should.

P2.18 **PREPARE:** All we need is some conditions in which the initial speed is low, but in the $+x$ direction, then the person/object remains still. Then the final speed must be higher than the initial speed, but in the same direction.

SOLVE: A forgetful physics professor is walking from one class to the next. Walking at a constant speed, he covers a distance of 100 m in 200 s. He then stops and chats with a student for 200 s. Suddenly, he realizes he is going to be late for his next class, so he hurries on and covers the remaining 200 m in 200 s to get to the class on time.

ASSESS: Note that the direction of motion does not change, and that the final speed is higher than the initial speed.

P2.19 **PREPARE:** All we need is some conditions in which the person/object is initially moving in the $-x$, then remains still for one hour. Then the final speed must be slightly slower than the initial speed, but in the opposite direction.

SOLVE: Eustace, the truck driver, had a load in a city 120 miles east of El Dorado. He drove west at 60 mph for two hours to El Dorado where he spent an hour unloading the truck and loading up different cargo. He then drove back east at 40 mph for two hours to the final destination 80 miles east of El Dorado.

ASSESS: Note that the change in direction of the truck driver's velocity matches the change in slope of the position versus time plot.

P2.20 **PREPARE:** We see from the Figure P2.20 that $+y$ points upward. The sign of position is taken from the location of the dots relative to the origin, and the sign of the velocity is taken from the direction of the velocity arrows. To determine the sign of the acceleration, we must look at how the velocity vectors are changing.

SOLVE: **(a)** Since the positions are all above the origin (marked "0"), the position is always positive.

(b) The velocity arrows always point downward in the direction of the $-y$ axis. Thus v_y is negative.

(c) The velocity vectors point downward but are getting smaller with time. That means the acceleration must point opposite the velocity: in the $+y$ direction. Thus a_y is positive.

ASSESS: We can see that adding an upward acceleration vector to the first velocity vector would be required to obtain the next velocity vector. This is consistent with our answer that the vertical component of the acceleration is positive.

P2.21 **PREPARE:** These are all simple unit conversions. We want quantities in base units such as meters and seconds, with no scaling prefix. For example, we want to write $1\ \mu\text{s}$ as 10^{-6} s. We will keep all original significant digits, since these metric conversions are exact. We first collect the necessary conversion factors: $1\ \mu\text{s} = 10^{-6}$ s; $1\text{ km} = 10^3$ m; $1\text{ m} = 10^2$ cm; $1\text{ h} = 60$ min; $1\text{ min} = 60$ s.

SOLVE:

(a) $$9.12\ \mu\text{s} = (9.12\ \mu\text{s})\left(\frac{10^{-6}\text{ s}}{1\ \mu\text{s}}\right) = 9.12 \times 10^{-6}\text{ s}$$

(b) $$3.42\text{ km} = (3.42\text{ km})\left(\frac{10^{3}\text{ m}}{1\text{ km}}\right) = 3.42 \times 10^{3}\text{ m}$$

(c) $$44\text{ cm/ms} = 44\left(\frac{\text{cm}}{\text{ms}}\right)\left(\frac{10^{-2}\text{ m}}{1\text{ cm}}\right)\left(\frac{1\text{ ms}}{10^{-3}\text{ s}}\right) = 4.4 \times 10^{2}\text{ m/s}$$

(d) $$80\text{ km/h} = 80\left(\frac{\text{km}}{\text{h}}\right)\left(\frac{10^{3}\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3.6 \times 10^{2}\text{ s}}\right) = 22\text{ m/s}$$

ASSESS: The conversion factors are applied in such a manner that we obtain the desired units. Scientific notation is used and the answer has no more significant figures than the starting number.

P2.22 **PREPARE:** These are all simple unit conversions. SI units refer to the base units such as meters and seconds, with no scaling prefix. In particular, we do not want to use imperial units like feet or inches. We first collect the necessary conversion factors:

$$1\text{ in} = 2.54\text{ cm};\ 1\text{ cm} = 10^{-2}\text{ m};\ 1\text{ ft} = 12\text{ in};\ 39.37\text{ in} = 1\text{ m};\ 1\text{ mi} = 1.609\text{ km};\ 1\text{ km} = 10^{3}\text{ m};\ 1\text{ h} = 3600\text{ s}$$

SOLVE:

(a) $$8.0\text{ in} = 8.0\text{ (in)}\left(\frac{2.54\text{ cm}}{1\text{ in}}\right)\left(\frac{10^{-2}\text{ m}}{1\text{ cm}}\right) = 0.20\text{ m}$$

(b) $$66\text{ ft/s} = 66\left(\frac{\text{ft}}{\text{s}}\right)\left(\frac{12\text{ in}}{1\text{ ft}}\right)\left(\frac{1\text{ m}}{39.37\text{ in}}\right) = 20\text{ m/s}$$

(c) $$60\text{ mph} = 60\left(\frac{\text{mi}}{\text{h}}\right)\left(\frac{1.609\text{ km}}{1\text{ mi}}\right)\left(\frac{10^{3}\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 27\text{ m/s}$$

ASSESS: Each of these appears qualitatively correct in the following sense. We know a meter is much longer than an inch. So finding that 8 in is less than a meter is reasonable. Similar logic holds relating feet and meters, or miles per hour and meters per second.

P2.23 **PREPARE:** The SI unit of time is seconds. So, this is a simple time conversion from various units into seconds. We will use the conversion factors: 1 year = 365.25 days; 1 day = 24 hours; 1 h = 3600 s.

SOLVE: **(a)** $1.0 \text{ hour} = 1.0\text{ (h)}\left(\dfrac{3600 \text{ s}}{1 \text{ h}}\right) = 3600 \text{ s} = 3.6 \times 10^3 \text{ s}$

(b) $1.0 \text{ day} = 1.0\text{ (d)}\left(\dfrac{24 \text{ h}}{1 \text{ d}}\right)\left(\dfrac{3600 \text{ s}}{1 \text{ h}}\right) = 8.6 \times 10^4 \text{ s}$

(c) $1.0 \text{ year} = 1.0\text{ (y)}\left(\dfrac{365.25 \text{ d}}{1 \text{ y}}\right)\left(\dfrac{8.64\times 10^4 \text{ s}}{1 \text{ d}}\right) = 3.2 \times 10^7 \text{ s}$

ASSESS: Since the given information contains only two significant figures, all answers contain only two significant figures.

P2.24 **PREPARE:** Review the rules for significant figures from Tactics Box 2.5. Pay particular attention to any zeros and whether or not they are significant. Note for example that the left-most zero in (c) is a place-holder and is not a significant digit, but the right-most zero in (c) specifies an additional digit of precision and is significant.

SOLVE: **(a)** The number 6.21 has three significant figures.

(b) The number 62.1 has three significant figures.

(c) The number 0.620 has three significant figures.

(d) The number 0.062 has two significant figures.

ASSESS: In part (c), the final zero is significant because it is expressed and in part (d), the second zero locates the decimal point but is not significant.

P2.25 **PREPARE:** Review the rules for significant figures from Tactics Box 2.5. Pay particular attention to any zeros and whether or not they are significant.

SOLVE: **(a)** The number 0.621 has three significant figures.

(b) The number 0.006200 has four significant figures.

(c) The number 1.0621 has five significant figures.

(d) The number 6.21×10^3 has three significant figures.

ASSESS: In part (b), the initial two zeros place the decimal point. The last two zeros do not have to be there, but when they are, they are significant.

P2.26 **PREPARE:** Review the rules for significant figures from Tactics Box 2.5. Pay particular attention to the rules for addition (and subtraction) and multiplication (and division).

SOLVE: **(a)** $33.3 \times 25.4 = 846$

(b) $33.3 - 25.4 = 7.9$

(c) $\sqrt{33.3} = 5.77$

(d) $333.3 \div 25.4 = 13.1$

ASSESS: In part (a), the two numbers multiplied each have three significant figures and the answer has three significant figures. In part (b), even though each number has three significant figures, no information is significant past the tenths column. As a result, the answer is expressed only to the tenths column. In part (c), the number and the answer both have three significant figures. In part (d), the answer is expressed to three significant figures since this is the least number of significant figures in either of the two numbers in the problem.

P2.27 **PREPARE:** Recall the rules for significant figures in multiplying, dividing, adding, and subtracting.

SOLVE: **(a)** $159.31 \times 204.6 = 32590$. This is reported to four significant figures since that is the smallest number of significant figures in the factors.

(b) $5.1125 + 0.67 + 3.2 = 9.0$. This is reported to the tenths digit since that is the least significant digit in 3.2.

(c) $7.662 - 7.425 = 0.237$. This is reported to the thousandths digit since that is the least significant digit in both of the numbers.

(d) $16.5/3.45 = 4.78$. This is reported to three significant figures since that is the smallest number of significant figures in the two numbers.

ASSESS: A quick logical check is that none of the answers have more significant figures than were given in the problem statement.

P2.28 **PREPARE:** We will use the conversion factors given in Table 2.5, and our answers must have the same number of significant digits as were given in the problem statement.

SOLVE:

(a) $30 \text{ cm} \approx (30 \text{ cm})\left(\dfrac{4 \text{ in}}{10 \text{ cm}}\right) \approx 10 \text{ in or } 1\times 10^1 \text{ in}$

(b) $25 \text{ m/s} \approx (25 \text{ m/s})\left(\dfrac{2 \text{ mph}}{1 \text{ m/s}}\right) \approx 50 \text{ mph or } 5\times 10^1 \text{ mph}$

(c) $5 \text{ km} \approx (5 \text{ km})\left(\dfrac{0.6 \text{ mi}}{1 \text{ km}}\right) \approx 3 \text{ mi}$

(d) $0.5 \text{ cm} \approx (0.5 \text{ cm})\left(\dfrac{1/2 \text{ in}}{1 \text{ cm}}\right) \approx 0.3 \text{ in}$

ASSESS: All of these pass common sense checks. For example, knowing that a centimeter is smaller than an inch, it is reasonable that the number of inches equivalent to 30 cm is smaller than 30.

P2.29 **PREPARE:** We will use the conversion factors given in Table 2.5, and our answers must have the same number of significant digits as were given in the problem statement.

Solve: **(a)** $20 \text{ ft} \approx (20 \text{ ft})\left(\dfrac{1 \text{ m}}{3 \text{ ft}}\right) \approx 7 \text{ m}$

(b) $60 \text{ mi} \approx (60 \text{ mi})\left(\dfrac{1 \text{ km}}{0.6 \text{ mi}}\right)\left(\dfrac{1000 \text{ m}}{1 \text{ km}}\right) \approx 100{,}000 \text{ m}$

(c) $60 \text{ mph} \approx (60 \text{ mph})\left(\dfrac{1 \text{ m/s}}{2 \text{ mph}}\right) \approx 30 \text{ m/s}$

(d) $8 \text{ in} \approx (8 \text{ in})\left(\dfrac{1 \text{ cm}}{1/2 \text{ in}}\right)\left(\dfrac{10^{-2} \text{ m}}{1 \text{ cm}}\right) \approx 0.2 \text{ m}$

ASSESS: All of these pass common sense checks. For example, knowing that a meter is longer than a foot, it is reasonable that the number of meters equivalent to 20 ft is smaller than 20.

P2.30 **PREPARE:** The number of significant digits should reflect the certainty of the measurement. A digit that is not certain should not be specified. The uncertainty is nearly a half an inch in either direction; this is about one centimeter in either direction, so we'll express the answer to the nearest centimeter.

SOLVE: Convert the man's height to inches: 6 ft 1 in = 73 in.

$$73 \text{ in}\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right) = 1.85 \text{ m}$$

ASSESS: To round this to two significant figures, 1.9 m, would give the wrong impression of the man's height.

P2.31 **PREPARE:** This is an estimation problem. We will need to assemble estimates from life experience, and many answers are possible. My barber trims about an inch of hair when I visit him every month for a haircut. The rate of hair growth thus is one inch per month. We also need the conversions $1 \text{ inch} = 2.54 \text{ cm}$; $1 \text{ day} = 24 \text{ h}$; $1 \text{ month} = 30 \text{ days}$; $1 \text{ hour} = 3600 \text{ s}$; $1 \text{ cm} = 10^{-2} \text{ m}$.

SOLVE: The rate of hair growth is

$$\left(\frac{1 \text{ in}}{\text{month}}\right)\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)\left(\frac{10^{-2} \text{ m}}{1 \text{ cm}}\right)\left(\frac{1 \text{ month}}{30 \text{ d}}\right)\left(\frac{1 \text{ d}}{24 \text{ h}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 9.8 \times 10^{-9} \text{ m/s} \approx 35\ \mu\text{m/h}$$

ASSESS: Since we expect an extremely small number for the rate at which our hair grows per second, this figure is not unreasonable.

P2.32 **PREPARE:** This is an estimation problem, so a range of answers may be acceptable. We will use the formula for the volume of a cylinder: $V_{\text{cyl}} = \pi r^2 \ell$, and will treat the cells as cubes with side length L and hence volume $V_{\text{cube}} = L^3$.

SOLVE: The volume of each cell is the volume of the total organism, divided by the number of cells:

$$V_{\text{cell}} = \frac{V_{\text{cyl}}}{N} = \frac{\pi r^2 \ell}{N}$$

We are equating this to the volume of a cube of side length $L = V_{\text{cube}}^{1/3}$, so

$$L = V_{\text{cell}}^{1/3} = \left(\frac{\pi r^2 \ell}{N}\right)^{1/3}$$

Using orders of magnitude, we find

$$L=\left(\frac{\pi\left(40\times10^{-6}\ \text{m}\right)^2\left(1\times10^{-3}\ \text{m}\right)}{959}\right)^{1/3}=\left(5.2\times10^{-15}\ \text{m}^3\right)^{1/3}\approx 2\times10^{-5}\ \text{m}$$

ASSESS: Table 2.6 gives the diameter of a mammalian cell as approximately 10^{-5} m. Although various cell types need not be identical, it is reassuring that we obtained the same order of magnitude. Also, doing the more precise calculation yields 1.7×10^{-5} m.

P2.33 **PREPARE:** For part (a), we simply divide the mass of a person by the mass of a mammalian cell, using orders of magnitude only. For part (b), we will use the volume of a cylinder: $V_{\text{cyl}}=\pi r^2\ell$ to determine the volume of a human, and then divide that by the volume of a cell. Again, we will use orders of magnitude only.

SOLVE: **(a)** Clearly, $m_{\text{human}}=Nm_{\text{cell}}\Rightarrow N=m_{\text{human}}/m_{\text{cell}}$. Inserting the number from Table 2.7, we find

$$N=m_{\text{human}}/m_{\text{cell}}=\frac{(70\ \text{kg})}{\left(10^{-12}\ \text{kg}\right)}\approx\frac{\left(10^{2}\ \text{kg}\right)}{\left(10^{-12}\ \text{kg}\right)}=10^{14}\ \text{cells}$$

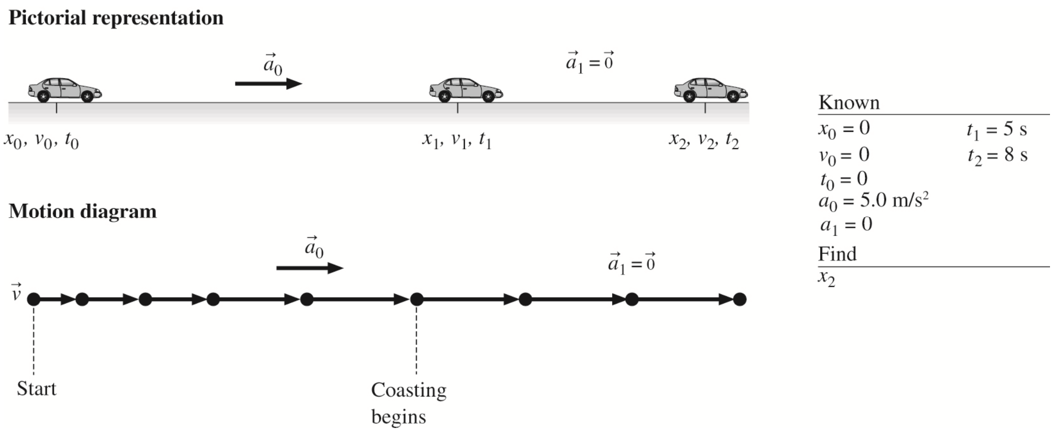
(b) Since $V_{\text{human}}=NV_{\text{cell}}$ we have

$$N=V_{\text{human}}/V_{\text{cell}}=\frac{\pi r^2\ell}{(4\pi/3)(d/2)^3}=\frac{\pi(0.125\ \text{m})^2(1.4\ \text{m})}{(4\pi/3)\left(5\times10^{-6}\ \text{m}\right)^3}\approx\frac{\left(10^{-2}\ \text{m}^2\right)\left(10^{0}\ \text{m}\right)}{\left(10^{-17}\ \text{m}^3\right)}=10^{15}\text{cells}$$

ASSESS: We do not need to be concerned that the obtained slightly different orders. Several estimations went into the calculation, such that an overall difference of a factor of 10 is possible.

P2.34 **PREPARE:** Represent the Porsche as a particle for the motion diagram. Assume the car moves at a constant speed when it coasts, such that dots are evenly spaced. While the car is accelerating from rest, the dots should start out close together, and their spacing should increase.

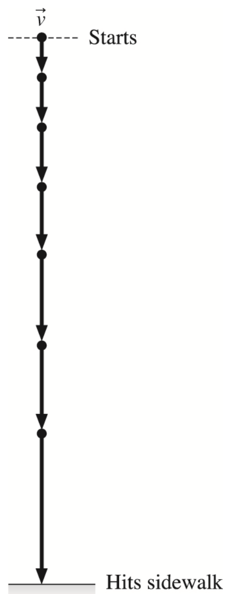
SOLVE:



ASSESS: Note that while the acceleration and velocity vectors are pointing in the same direction, the speed increases, as it should.

P2.35 **PREPARE:** The watermelon, represented as a particle, falls freely and speeds up during its downward motion along the *y*-direction. The velocity vectors should thus be increasing in length.

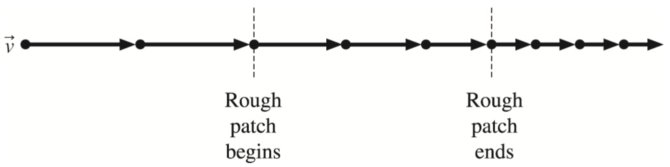
SOLVE:



ASSESS: As a result of the acceleration due to gravity, as the watermelon falls, its velocity and the distance between the position dots increases. This is the case in the figure shown.

P2.36 **PREPARE:** The skater moves along the *x*-axis. She slows down or has decreasing velocity vectors during a patch of rough ice. She has constant velocity vectors before the rough patch begins and after the rough patch ends, that is, velocity vectors are of the same length.

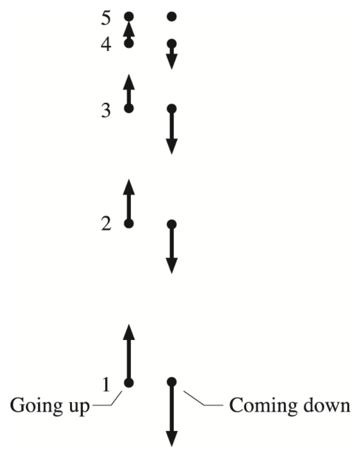
SOLVE:



ASSESS: Before and after the rough ice, the velocity vector is constant in length and the position dots are uniformly spaced. Since the skater is traveling slower after the rough ice than before, the velocity vectors after the rough ice are shorter than they are before the rough ice and the position dots are closer together after the rough ice than they are before the rough ice. During the rough ice section, the velocity vector decreases in length and the dot position gets closer together.

P2.37 **PREPARE:** Since the eland has a positive velocity but is slowing down, the velocity will decrease to zero and the spacing between the position dots will decrease. The velocity vector at each position on the way up has the same magnitude but opposite direction as the velocity at each position on the way down.

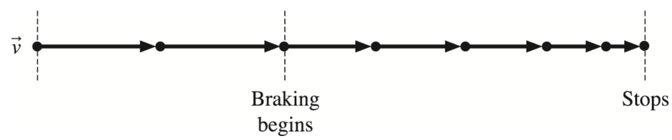
SOLVE:



ASSESS: On the way up, the velocity vector decreases to zero as it should and the spacing between the position dots decreases, as it should. The magnitude of the velocity vector at any position is the same on the way up as it is on the way down. This allows us to conclude that the figure is correct.

P2.38 **PREPARE:** The motorist, represented as a particle, is moving along the *x*-axis. He slows down during braking, at which time the spacing between dots should decrease. As he coasts during his reaction time, his velocity doesn't change, and the dots should be evenly spaced.

SOLVE:



ASSESS: During the reaction time, the velocity vector is constant in length and the position dots are uniformly spaced. During the braking process, the velocity vector decreases in length and the position dots get closer together.

P2.39 **PREPARE:** The dots are initially evenly spaced, and then are closer together. We need a situation where the speed is initially constant, and then decreases until the person/object comes to rest.

SOLVE: Rahul was coasting on interstate highway I-44 from Tulsa to Springfield at 70 mph. Seeing an accident at a distance of 200 feet in front of him, he began to brake. What steady deceleration will bring him to a stop at the accident site?

ASSESS: Since the position dots are initially and equally spaced, the first few velocity vectors have the same length; this is consistent with Rahul initially traveling at a constant velocity. The fact that the dots get closer together and the velocity vectors get shorter is consistent with Rahul's braking. The fact that there is no velocity vector associated with the last dot is consistent with the fact that he braked to a stop.

P2.40 **PREPARE:** The spacing between dots initially decreases, the object stops, and then the spacing increases again. So we need any situation in which the speed initially decreases to rest, then increases again.

SOLVE: Reema passes 3rd street doing 40 mph, slows steadily to the stop sign at 4th street, stops for 1 s, then speeds up and reaches her original speed as she passes 5th street. If the blocks are 50 m long, how long does it take Reema to drive from 3rd street to 5th street?

ASSESS: The statement that Reema slows to a stop in one block and regains her initial velocity in one block is consistent with the symmetry of the position dots and the velocity vectors about the stop position.

P2.41 **PREPARE:** Many answers are possible. As the object moves downward, its speed is increasing. So, we need a situation where the acceleration is downward. This is consistent with the second stage of motion. The object is moving upward, but slowing. A constant downward acceleration might cause us to use gravity.

SOLVE: A bowling ball is at rest at the top of an incline. You nudge the ball giving it an initial velocity and causing it to roll down an incline. At the bottom of the incline, it bounces off a sponge and travels back up the incline until it stops.

ASSESS: The statement that you give the ball an initial velocity is consistent with the fact that the start position dot has a velocity vector. The statement that the ball rolls down the incline is consistent with the fact that the dots are getting farther apart and the velocity vectors are increasing in length. The statement that the ball bounces off a sponge is consistent with the fact that ball does not bounce back to its original position.

P2.42 **PREPARE:** We are asked for the average speed, which is given by $\text{speed} = \dfrac{\text{distance}}{\text{time}}$. It is very important to

realize that $v_{\text{av}} \neq \frac{1}{2}(v_1 + v_2)$, because there is no reason to think that the two speeds will be equally important. We can determine the amount of time required for each leg of the journey, and then combine our information to determine the total distance and total time.

SOLVE: For the first leg of the journey: $v_1 = d_1/\Delta t_1 \Rightarrow \Delta t_1 = d_1/v_1 = (25\text{ mi})/(55\text{ mi/h}) = 0.455\text{ h}$, and for the second leg of the journey, we have $v_2 = d_2/\Delta t_2 \Rightarrow \Delta t_2 = d_2/v_2 = (15\text{ mi})/(70\text{ mi/h}) = 0.214\text{ h}$

Now the average speed for the entire trip is $v_{\text{av}} = d_{\text{total}}/\Delta t_{\text{total}} = \dfrac{(25\text{ mi}) + (15\text{ mi})}{(0.455\text{ h}) + (0.214\text{ h})} = 60\text{ mi/h}$

ASSESS: We do expect the average speed to be somewhere between the two speeds at which Joseph traveled. It makes sense that this is the case.

P2.43 **PREPARE:** We will use the relationship $v_{\text{av}} = \dfrac{d}{\Delta t}$ to determine the new speed required. We also know that the distance between Evan and his grandmother's house does not change. But his speed and the duration of the trip will both be different today than on a normal day. Thus, we will write out $v_{\text{av,norm}} = \dfrac{d}{\Delta t_{\text{norm}}}$ and $v_{\text{av, today}} = \dfrac{d}{\Delta t_{\text{today}}}$ separately, and then relate the two.

SOLVE: From the information about ordinary days, we know

$$d = v_{\text{av,norm}}\Delta t_{\text{norm}} = (55\text{ mi/h})(25\text{ min})\left(\frac{1\text{ h}}{60\text{ min}}\right) = 22.9\text{ mi}$$

Now we can use this distance, and the fact that Evan must make the trip in 5 minutes less than usual to write

$$v_{\text{av,today}} = \frac{d}{\Delta t_{\text{today}}} = \frac{(22.9\text{ mi})}{(20\text{ min})}\left(\frac{60\text{ min}}{1\text{ h}}\right) = 69\text{ mph.}$$

ASSESS: Since Evan has to make the trip in 25% less time than usual, it makes perfect sense that his average speed must be 25% greater than usual.

P2.44 **PREPARE:** Although Gretchen ran at varying speed, we want to determine one average speed that Gretchen could have run the entire time that is equivalent to $v_{\text{av}} = \frac{d_{\text{total}}}{\Delta t_{\text{total}}}$. We can consider the two segments of the race during which Gretchen ran at a constant speed, separately. That way, we can write $v_{\text{av},1} = \frac{d_1}{\Delta t_1}$ and $v_{\text{av},2} = \frac{d_2}{\Delta t_2}$, and then combine information to determine $v_{\text{av,total}} = \frac{d_{\text{total}}}{\Delta t_{\text{total}}} = \frac{d_1 + d_2}{\Delta t_1 + \Delta t_2}$.

SOLVE: We write the expression for the average speed for the whole trip, and then write the two relevant times in terms of the given speeds and distances.

$$v_{\text{av,total}} = \frac{d_{\text{total}}}{\Delta t_{\text{total}}} = \frac{d_1 + d_2}{\Delta t_1 + \Delta t_2} = \frac{d_1 + d_2}{(d_1 / v_{\text{av},1}) + (d_2 / v_{\text{av},2})}$$

$$v_{\text{av,total}} = \frac{(4.0\times10^3\text{ m}) + (1.0\times10^3\text{ m})}{((4.0\times10^3\text{ m})/(5.0\text{ m/s})) + ((1.0\times10^3\text{ m})/(4.0\text{ m/s}))} = 4.8\text{ m/s}$$

ASSESS: It makes sense that the average speed for the entire trip would be between the two speeds Gretchen ran during the race.

P2.45 **PREPARE:** The speed of the glacier has been given in feet per year. Expressing it in m/s is just a matter of unit conversion.

SOLVE: The speed of the glacier in m/s is determined as follows:

$$v = \left(\frac{105\text{ ft}}{\text{year}}\right)\left(\frac{12\text{ in}}{\text{feet}}\right)\left(\frac{2.54\text{ cm}}{1\text{ in}}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right)\left(\frac{1\text{ y}}{3.16\times10^7\text{ s}}\right) = 1.0\times10^{-6}\text{ m/s}$$

ASSESS: All of the unit conversions are correct, after units are canceled, we obtain the desired units (m/s) and we are expecting a very small number.

P2.46 **PREPARE:** This problem is mostly a unit conversion. We will also make use of the given growth rate of the shark. The length of the shark can be converted to centimeters, and then the length can be related to an age using the given growth rate. The increase in length during the shark's lifetime can be 14 ft.

SOLVE: Starting with the increase in length, we have

$$(14\text{ ft})\left(\frac{12\text{ in}}{1\text{ ft}}\right)\left(\frac{2.54\text{ cm}}{1\text{ in}}\right)\left(\frac{1\text{ y}}{1\text{ cm}}\right)=430\text{ y.}$$

ASSESS: Although this is an incredible age, it is consistent with the statement that the Greenland shark is thought to be the longest-living vertebrate on Earth.

P2.47 **PREPARE:** We can use the equation $v_{\text{av}}=\frac{d}{\Delta t}$ over any period when the average speed is given. Note that we cannot use $v_{\text{av}}=\frac{1}{2}(v_1+v_2)$, since the uphill and downhill speeds might not be equally important, as the cyclist rode at those two speeds for different periods of time. We can apply the above expression to the uphill segment, the downhill segment.

$$v_{\text{av,uphill}}=\frac{d_{\text{uphill}}}{\Delta t_{\text{uphill}}},\qquad v_{\text{av,downhill}}=\frac{d_{\text{downhill}}}{\Delta t_{\text{downhill}}},\text{ and we also note }\Delta t_{\text{total}}=\Delta t_{\text{downhill}}+\Delta t_{\text{uphill}}$$

We know both distances, the total time, and the average speed for the uphill leg. We can relate the speeds, distances, and times for each segment and solve for the unknown average speed on the downhill leg.

SOLVE: Starting with $\Delta t_{\text{total}}=\Delta t_{\text{downhill}}+\Delta t_{\text{uphill}}$ and inserting $\Delta t_{\text{uphill}}=\frac{d_{\text{uphill}}}{v_{\text{av,uphill}}}$, we can obtain

$$\Delta t_{\text{downhill}}=\Delta t_{\text{total}}-\frac{d_{\text{uphill}}}{v_{\text{av,uphill}}}=(2{,}845\text{ s})-\frac{(4.6\text{ mi})}{(8.75\text{ mi/h})}\left(\frac{3600\text{ s}}{1\text{ h}}\right)=952\text{ s}$$

Now that we know the time, we simply use

$$v_{\text{av,downhill}}=\frac{d_{\text{downhill}}}{\Delta t_{\text{downhill}}}=\frac{6.9\text{ mi}}{952\text{ s}}\left(\frac{3600\text{ s}}{1\text{ hr}}\right)=26\text{ mph}$$

ASSESS: This is much faster than the uphill leg of the trip, which is to be expected.

P2.48 **PREPARE:** According to her speedometer, Shannon is traveling 70 mph. Knowing that mile markers are 1 mi apart and knowing that she travels the distance between two markers in 54 s, we can determine her speed using Equation 2.1 and compare it to the value from the speedometer to see if her speedometer is accurate.

SOLVE: Her actual speed in mph may be determined by

$$v=\frac{\text{distance}}{\text{time}}=\left(\frac{1\text{ mi}}{54\text{ s}}\right)\left(\frac{3.6\times10^{2}\text{ s}}{\text{h}}\right)=67\text{ mph}$$

Since Shannon's speedometer reads 70 mph and her actual speed is 67 mph, we conclude that her speedometer is not accurate. It shows a value that is too high.

ASSESS: This is a reasonable answer—your speedometer is intended to give you a good estimate to your actual speed. Her speedometer is reading within 5% of the true value and rumor has it that the highway patrol will allow up to 10% on a good day. A well-calibrated speedometer could be this much off by changing the size of the tires.

P2.49 **PREPARE:** This problem involves estimation, such that a range of answers may be correct. Knowing the speed at which the signal travels (approximately 25 m/s) and estimating the distance from your brain to your hand to be about 1.0 m, we can determine the transmission time. Some unit conversion will be required to get the answer in ms.

SOLVE: The transmission time for the signal may be determined by

$$t = \frac{\text{distance}}{\text{speed}} = \left(\frac{1\text{ m}}{25\text{ m/s}}\right)\left(\frac{10^3\text{ ms}}{\text{s}}\right) = 40\text{ ms}$$

ASSESS: When you touch something very hot, it takes a fraction of a second to remove your hand. Keeping in mind, that in this case the signal must make a round trip, the answer, while very small, seems reasonable.

P2.50 **PREPARE:** In Figure P2.50, time is measured in seconds, and position is measured in nanometers. We need only read off the step length for part (a). For part (b), we can simply divide the total time by the number of steps to obtain the average time per step. For the average speed, we can use the average step size and average time from parts (a) and (b). We expect a small range of answers from uncertainty in reading the graph.

SOLVE: (a) A set of two steps exactly reaches the 16-nm mark, so the step length is 8 nm.

(b) The first step starts in its resting period, and then jumps. We count 9 total such steps. The resting phase of a 10th one has begun, but the step has not been completed. The 9th step was completed just past the halfway point between 0.3 s and 0.4 s, which we can estimate as 3.6 s. Thus, the average time interval between steps is

$$\Delta t_{\text{av step}} = \frac{\Delta t_{\text{total}}}{N_{\text{steps}}} = \frac{(3.6\text{ s})}{(9\text{ steps})} = 0.4\text{ s/step}$$

(c) The average speed is given by

$$\text{speed} = \frac{\Delta x_{\text{step}}}{\Delta t_{\text{av step}}} = \frac{(8\text{ nm})}{(0.4\text{ s})} = 20\text{ nm/s}$$

ASSESS: We can check this using the total displacement over the interval shown and divide it by the total time as follows:

$$\text{speed} = \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{(72\text{ nm})}{(3.6\text{ s})} = 20\text{ nm/s}$$

P2.51 **PREPARE:** This question involves unit conversion and the definition of average speed. We will need to convert to SI units throughout the problem.

SOLVE: (a) The length is given as $2\ \mu\text{m} = 2 \times 10^{-6}\text{ m}$ (using the data from Table 2.3)

(b) The diameter is $1\ \mu\text{m} = 1 \times 10^{-6}\text{ m}$ (using the data from Table 2.3)

(c) The mass is given as $1 \times 10^{-12}\text{ g} = (1 \times 10^{-12}\text{ g})(1\text{ kg}/1000\text{ g}) = 1 \times 10^{-15}\text{ kg}$

(d) The length of the bacterium's DNA is given as 700 times longer than the bacterium's length. Therefore, the length of DNA is $(700)(2\times10^{-6}\text{ m}) = 1\times10^{-3}\text{ m}$. We need to convert this to millimeters.

$$1 \times 10^{-3}\text{ m} = 1 \times 10^{-3}\text{ m}\left(\frac{1000\text{ mm}}{1\text{ m}}\right) = 1\text{ mm}$$

(e) The organism travels at $20\ \mu\text{m/s}$. Assume this is given to two significant figures. Converting to m/s, $v = 20\ \mu\text{m/s} = 2.0 \times 10^{-5}\ \text{m/s}$.

A day is

$$1\ \text{day} = (1\ \text{day})\left(\frac{24\ \text{h}}{\text{d}}\right)\left(\frac{60\ \text{min}}{\text{h}}\right)\left(\frac{60\ \text{s}}{\text{min}}\right) = 86{,}400\ \text{s}$$

So, the bacterium travels

$$\Delta x = v\Delta t = \left(2.0\times10^{-5}\,\frac{\text{m}}{\text{s}}\right)(86400\ \text{s}) = 1.7\ \text{m}$$

ASSESS: Use the method of multiplying by one to help keep track of multiple unit conversions.

P2.52 **PREPARE:** This problem involves displacement, which is note the same as the distance travelled. It is the difference between initial and final positions. We also distinguish between speed and velocity. Since velocity has a direction associated with it, in order for two segments to describe the same velocity, they would need to be the same length and the same direction. For them to describe the same speed, they only need to have the same length (since time intervals are fixed). Assume that the bacterium moves along the path to consecutive letters.

SOLVE: **(a)** The displacements in segments AB and CD are the same (five right and one up). No other pairs appear to be the same.

(b) The problem explicitly stated that the bacteria move at a constant speed, so the answer is all of the segments.

(c) Since the displacements in segments AB and CD are the same (and the bacterium had the same speed in both segments, i.e., Δt is the same for both segments), then the velocity is the same in those two segments. Since no other pairs of segments have the same direction, they can't have the same velocity.

ASSESS: Remember that both displacement and velocity are vectors, so the direction matters. Only the length (magnitude) matters with speed since it is a scalar.

P2.53 **PREPARE:** Both parts (a) and (b) are essentially unit conversions.

SOLVE: **(a)** Inserting the given numbers, we have

$$5\ \text{L}\times\frac{5\times10^{6}\ \text{cells}}{10^{-6}\ \text{L}} = 2.5\times10^{13}\text{cells}$$

Which we report to one significant digit as 3×10^{13}cells

(b)

$$\frac{150\ \mu\text{g hem}}{1\ \mu\text{L blood}}\times\frac{1\ \mu\text{L}}{5\times10^{6}\ \text{cells}}\times\frac{1\ \text{kg}}{10^{9}\ \mu\text{g}}\times\frac{1\ \text{Da}}{1.7\times10^{-27}\ \text{kg}}\times\frac{1\ \text{hem}}{64{,}000\ \text{Da}}$$
$$\approx\ 3\times10^{8}\ \text{hemoglobin per red blood cell}$$

ASSESS: If we put the numbers into our calculators, we find an answer of about 2.76×10^{8}, so our estimation process was done correctly. We might expect a high number, although this is astronomically high. A quick search online can confirm that this is the correct order of magnitude.

P2.54 **PREPARE:** We will use the formula for the volume of a sphere: $V=(4\pi/3)r^3=(4\pi/3)(d/2)^3$ for the volume of both the yeast cells and each of the mitochondria. We then simply take the ratio of the total volume of all 40 mitochondria to that of the yeast cells.

SOLVE: We want the ratio $40V_{\text{mit}}/V_{\text{yeast}}$. Inserting the equations above and the given numbers, we find

$$40\frac{V_{\text{mit}}}{V_{\text{yeast}}}=40\frac{(4\pi/3)(d_{\text{mit}}/2)^3}{(4\pi/3)(d_{\text{yeast}}/2)^3}=40\left(\frac{d_{\text{mit}}}{d_{\text{yeast}}}\right)^3=40\left(\frac{(0.75\ \mu\text{m})}{(5.0\ \mu\text{m})}\right)^3=0.135\approx 14\%$$

ASSESS: Since the mitochondria play a vital role inside the cell, it is plausible that they take up an appreciable fraction of the volume.

P2.55 **PREPARE:** We will calculate the volume of the bacterium using $V_{\text{cyl}}=\pi R^2\ell$, and then divide that up between all the proteins. Finally, we will treat the volume per protein as though it is a spherical space, and determine the radius of that spherical space using $V_{\text{sphere}}=(4\pi/3)r^3$. Note the use of R for the bacterium and r for the protein.

SOLVE: **(a)** We set the volume per protein equal to the volume of a sphere, and rearrange:

$$V_{\text{sphere}}=\frac{V_{\text{cyl}}}{N}\Rightarrow\frac{4\pi}{3}r^3=\frac{\pi R^2\ell}{N}$$

$$r=\left(\frac{3R^2\ell}{4N}\right)^{1/3}=\left(\frac{3\left(5\times10^{-7}\text{ m}\right)^2\left(2\times10^{-6}\text{ m}\right)}{4\left(3\times10^6\right)}\right)^{1/3}=5\times10^{-9}\text{ m}$$

The spacing is then 10 nm.

(b) If the center-to-center spacing between proteins is about 10 nm, and each is about 5 nm across, this is a fairly dense fluid. Collisions/interactions will occur constantly.

ASSESS: This is only an estimate, but it still has substantial meaning for working with cytoplasm and the contents of bacterial cells: expect a dense, viscous fluid.

P2.56 **PREPARE:** Since area equals length × width, the smallest area will correspond to the smaller length and the smaller width. Similarly, the largest area will correspond to the larger length and the larger width.

SOLVE: The smallest area is $A_{\text{min}}=(64\text{ m})(100\text{ m})=6.4\times10^3\text{m}^2$. The largest area is $A_{\text{max}}=(75\text{ m})(110\text{ m})=8.3\times10^3\text{m}^2$.

ASSESS: Clearly, these areas are different by about 2,000 sq. mt, which is significant. We can have some confidence in our calculation, though, since the two areas are of the same order of magnitude, as they should be.

P2.57 **PREPARE:** This is simply a unit conversion.

SOLVE: $$9.0\text{ g/L}=\left(9.0\ \frac{\text{g}}{\text{L}}\right)\left(\frac{1\text{ kg}}{1000\text{ g}}\right)\left(\frac{1\text{ L}}{1000\text{ mL}}\right)\left(\frac{1\text{ mL}}{1\text{ cm}^3}\right)\left(\frac{100\text{ cm}}{1\text{ m}}\right)^3=9.0\text{ kg/m}^3$$

ASSESS: Although 9.0 kg seems like a lot of salt, a cubic meter of water is a similarly huge volume of water. Note that both the numerator and denominator in the units increased by a factor of 1,000.

P2.58 **PREPARE:** Use the letter ρ for density. In each case, we will simply take the ratio of the mass to the volume, making sure to use base SI units. Since we are starting in SI units, we won't need conversions involving imperial units like feet or pounds.

SOLVE: **(a)** $\rho = \left(\dfrac{0.0179\ \text{kg}}{215\ \text{cm}^3}\right)\left(\dfrac{100\ \text{cm}}{1\ \text{m}}\right)^3 = 83.3\ \text{kg/m}^3$

(b) $\rho = \left(\dfrac{77\ \text{g}}{95\ \text{cm}^3}\right)\left(\dfrac{1\ \text{kg}}{1000\ \text{g}}\right)\left(\dfrac{100\ \text{cm}}{1\ \text{m}}\right)^3 = 810\ \text{kg/m}^3$

ASSESS: Glancing back at the given numbers, we see the volume in (b) was a little smaller than in (a), and the mass in (b) was a little greater than in (a). Therefore, we expect a larger density in (b) than in (a), which fits with our answers above.

P2.59 **PREPARE:** This is a simple calculation of growth speed, given the change in height and the time interval. Assume that the growth is steady during this interval from the end of year 1 to the end of year 3.
We will subtract any two heights and divide by the corresponding time interval.

SOLVE:

$$v = \frac{30\ \text{ft} - 12\ \text{ft}}{3\ \text{y} - 1\ \text{y}} = \frac{18\ \text{ft}}{2\ \text{y}} = 9\frac{\text{ft}}{\text{y}}$$

So the correct answer is B.

ASSESS: This is an extremely fast-growing tree, but that is what was advertised. You could check to make sure you get a similar answer using different times, such as from year 2 to year 3.

P2.60 **PREPARE:** This is a simple unit conversion problem. We must convert feet/year to m/s using known conversion factors.

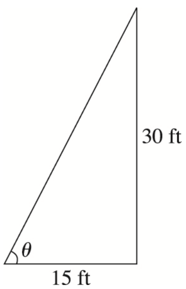
SOLVE: The tree grows at 9 ft/year, so

$$9\frac{\text{ft}}{\text{y}} = \left(\frac{9\ \text{ft}}{\text{y}}\right)\left(\frac{0.305\ \text{m}}{1\ \text{ft}}\right)\left(\frac{1\ \text{y}}{365\ \text{days}}\right)\left(\frac{1\ \text{day}}{24\ \text{h}}\right)\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 9\times10^{-8}\ \text{m/s}$$

The correct answer is A.

ASSESS: We expect a very small answer in these units, so our answer seems reasonable.

P2.61 **PREPARE:** Since we are given the vertical height of the tree and the horizontal distance from the tree, we have a right triangle and can use trigonometry. Draw a diagram and label the sides and the angle you want to know.



SOLVE:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\theta = \tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right) = \tan^{-1}\left(\frac{30\text{ ft}}{15\text{ ft}}\right) = 63°$$

The correct answer is A.

ASSESS: Imagine holding your protractor to your diagram and convince yourself that 63° is in the right ballpark.

3

Motion Along a Line

Questions

Q3.1 **Reason:** Any story will work if it satisfies the following criteria: the initial velocity is 15 mi/h in the +*x*-direction for 20 min, then the object/animal remains still for 20 min, and then it moves in the –*x*-direction at a speed of 15 mi/h for 40 min. For example: An antelope starts out five miles from a research station, and runs for 20 min at a speed of 15 mi/h to the east (which we will call the +*x*-direction). It stops to drink and rests for 20 min. Then the antelope realizes that this area has quite a few predators on the prowl and it runs for 40 min west, still at 15 mi/h, until it arrives at the research station.

Assess: Many stories are possible. Make sure you have specified your directions of motion, and make sure the speeds involved are reasonable for your story.

Q3.2 **Reason:** Many answers could be correct; here is an example. Lola is just learning to drive and starts her car slowly from rest. She is driving in the +*x*-direction, and speeds up to about 10 m/s. At that point, her speed becomes constant.

Assess: The car starts from rest, and this is consistent with the initial slope being zero on the plot.

Q3.3 **Reason:** The speed is the slope of the displacement-versus-time graph. So we need to pay attention to the slopes of the two lines, not the value on the vertical axis.

(a) At $t = 1$ s, the slope of the line for object A is greater than that for object B. Therefore, object A's speed is greater. (Both are positive slopes.)

(b) No, the speeds are never the same. Each has a constant speed (constant slope) and A's speed is always greater.

Assess: This problem is emphasizing the relationship between position and speed plots. Had the question asked if the positions were ever the same, we would have answered "Yes, at $t = 3$ s."

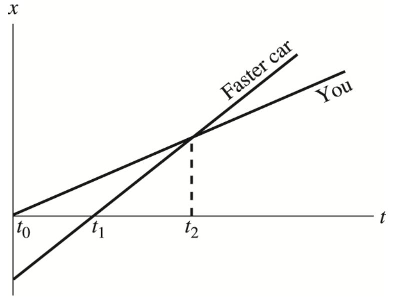
Q3.4 **Reason:** The speed is the slope of the displacement-versus-time graph. So we need to pay attention to the slopes of the two lines, not the value on the vertical axis.

(a) A's speed is greater at $t = 1$ s. The slope of the tangent to B's curve at $t = 1$ s is smaller than the slope of A's line.

(b) A and B have the same speed at just about $t = 3$ s. At that time, the slope of the tangent to the curve representing B's motion is equal to the slope of the line representing A.

ASSESS: This problem is emphasizing the relationship between position and speed plots. Had the question asked if the two objects ever had the same position, we would have answered "No, because the lines never cross."

Q3.5 **REASON:** Let $t_0 = 0$ be when you pass the origin. The other car will pass the origin at a later time t_1 and passes you at time t_2.



ASSESS: The slope of the position graph is the velocity, and the slope for the faster car is steeper.

Q3.6 **REASON: (a)** 4. The steepness of the tangent line is greatest at 4.

(b) 6. Motion to the left is indicated by a decreasing segment on the graph.

(c) 2. The speed corresponds to the steepness of the tangent line, so the question can be recast as "Where is the tangent line getting steeper (either positive or negative slope, but getting steeper)?" The slope at 1 is zero and is greatest at 3, so it must be getting steeper at 2.

(d) 2. The speed corresponds to the steepness of the tangent line, so the question can be recast as "Where is the tangent line getting less steep (either positive or negative slope, but getting less steep)?"

(e) 5. Before 5 the object is moving right, and after 5 it is moving left.

ASSESS: It is amazing that we can get so much information about the velocity (and even about the acceleration) from a position-versus-time graph.

Q3.7 **REASON: (a)** For the velocity to be constant, the velocity-versus-time graph must have zero slope. Looking at the graph, there are three time intervals where the graph has zero slope: segment A, segment D, and segment F.

(b) For an object to be speeding up, the magnitude of the velocity of the object must be increasing. When the slope of the lines on the graph is nonzero, the object is accelerating and therefore changing speed.

Consider segment B. The velocity is positive while the slope of the line is negative. Since the velocity and acceleration are in opposite directions, the object is slowing down. At the start of segment B, we can see the velocity is +2 m/s, while at the end of segment B the velocity is 0 m/s.

During segment E the slope of the line is positive, which indicates positive acceleration, but the velocity is negative. Since the acceleration and velocity are in opposite directions, the object is slowing here also. Looking at the graph at the beginning of segment E the velocity is –2 m/s, which has a magnitude of 2 m/s. At the end of segment E the velocity is 0 m/s, so the object has slowed down.

Consider segment C. Here the slope of the line is negative and the velocity is negative. The velocity and acceleration are in the same direction, so the object is speeding up. The object is gaining velocity in the negative direction. At the beginning of that segment the velocity is 0 m/s, and at the end the velocity is –2 m/s, which has a magnitude of 2 m/s.

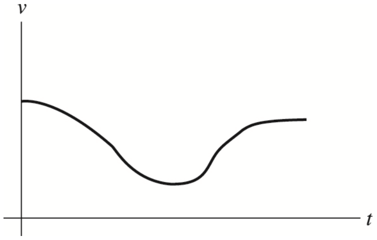
(c) In the analysis for part (b), we found that the object is slowing down during segments B and E.

(d) An object standing still has zero velocity. The only time this is true on the graph is during segment F, where the line has zero slope, and is along $v = 0$ m/s. The velocity is also zero for an instant at time $t = 5$ s between segments B and C.

(e) For an object to be moving to the right, the convention is that the velocity is positive. In terms of the graph, positive values of velocity are above the time axis. The velocity is positive for segments A and B. The velocity must also be greater than zero. Segment F represents a velocity of 0 m/s.

ASSESS: The slope of the velocity graph is the acceleration graph.

Q3.8 **REASON:** Where the rings are far apart, the tree is growing rapidly. It appears that the rings are quite far apart near the center (the origin of the graph), then get closer together, then farther apart again.



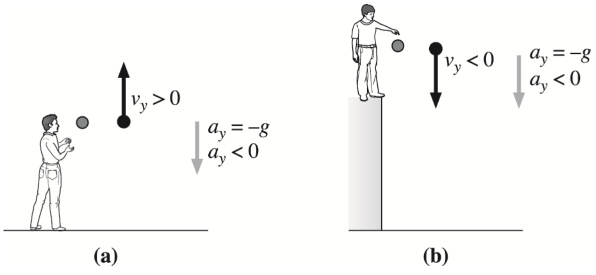
ASSESS: After drawing velocity-versus-time graphs (as well as others), stop and think if it matches the physical situation, especially by checking end points, maximum values, places where the slope is zero, and so on. This one passes those tests.

Q3.9 **REASON:** The figure shows the x-component of velocity as a function of time. This quantity is always positive for both objects, which means they are always moving in the same direction (just with varying speeds, one with increasing speed and one with decreasing speed). Victoria is not correct, since her scenario involves balls initially moving in different directions. In addition, the acceleration due to gravity in her case would have the same sign for both balls, such that the velocity-versus-time plot would have the same slope for both balls. Wes describes two airplanes moving in the same direction, one with increasing speed and one with decreasing speed. This matches the figure. Zach describes cars going in opposite directions. Their accelerations might match the figure; Zach doesn't describe how their speeds are changing, but their initial directions of travel do not match the figure. Wes is correct.

ASSESS: Zach might have been confusing the vertical axis with position. If it were a position-versus-time plot, then it would look like two cars passing each other moving at constant speeds in opposite directions.

Q3.10 **REASON:** **(a)** Positive velocity in vertical motion means an object is moving upward. Negative acceleration means the acceleration of the object is downward. Therefore, the upward velocity of the object is decreasing. An example would be a ball thrown upward, before it starts to fall back down. Since it's moving upward, its velocity is positive. Since gravity is acting on it and the acceleration due to gravity is always downward, its acceleration is negative.

(b) To have a negative vertical velocity means that an object is moving downward. The acceleration due to gravity is always downward, so it is always negative. An example of a motion where both velocity and acceleration are negative would be a ball dropped from a height during its downward motion. Since the acceleration is in the same direction as the velocity, the velocity is increasing.



ASSESS: For vertical displacement, the convention is that upward is positive and downward is negative for both velocity and acceleration.

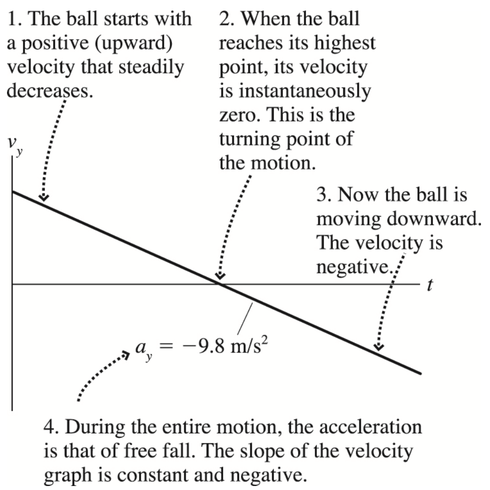
Q3.11 **REASON:** A predator capable of running at a great speed while not being capable of large accelerations could overtake slower prey that were capable of large accelerations, given enough time. However, it may not be as effective as surprising and grabbing prey that are capable of higher acceleration. For example, prey could escape if the safety of a burrow were nearby. If a predator were capable of larger accelerations than its prey, while being slower in speed than the prey, it would have a greater chance of surprising and grabbing the prey, quickly, though the prey might outrun it if given enough warning.

ASSESS: Consider the horse-man race discussed in the text.

Q3.12 **REASON:** Call "up" the positive direction. Further, assume that there is no air resistance. This assumption is probably not true (unless the rock is thrown on the moon), but air resistance is a complication that will be addressed later, and for small, heavy items like rocks no air resistance is a pretty good assumption if the rock isn't going too fast.

To be able to draw this graph without help demonstrates a good level of understanding of these concepts. The velocity graph will not go up and down as the rock does—that would be a graph of the position. Think carefully about the velocity of the rock at various points during the flight.

At the instant the rock leaves the hand, it has a large positive (up) velocity, so the value on the graph at $t = 0$ needs to be a large positive number. The velocity decreases as the rock rises, but the velocity arrow would still point up. So the graph is still above the t axis, but decreasing. At the tippy-top the velocity is zero; that corresponds to a point on the graph where it crosses the t axis. Then, as the rock descends with increasing velocity (in the negative, or down, direction), the graph continues below the t axis. It may not have been totally obvious before, but this graph will be a *straight line* with a negative slope.



ASSESS: Make sure that the graph touches or crosses the t axis whenever the velocity is zero. In this case, that is only when it reaches the top of its trajectory and the velocity vector is changing direction from up to down. It is also worth noting that this graph would be more complicated if we were to include the time at the beginning when the rock is being accelerated by the hand. Think about what that would entail.

Q3.13 **REASON:** We will neglect air resistance, and thus assume that the ball is in free fall.

(a) $-g$ After leaving your hand, the ball is traveling up but slowing, therefore the acceleration is down (i.e., negative).

(b) $-g$ At the very top the velocity is zero, but it had previously been directed up and will consequently be directed down, so it is changing direction (i.e., accelerating) down.

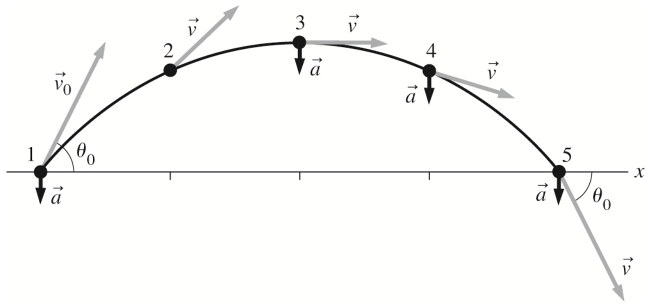
(c) $-g$ Just before hitting the ground it is going down (velocity is down) and getting faster; this also constitutes an acceleration down.

ASSESS: As simple as this question is, it is sure to illuminate a student's understanding of the difference between velocity and acceleration. Students would be wise to dwell on this question until it makes complete sense.

Q3.14 **REASON:** A typical trajectory of a projectile is shown in the figure below. The acceleration due to gravity always points down. The velocity changes direction from the launch angle $\theta = \theta_0$ above the $+x$-axis to zero at the top of the trajectory, to $\theta = \theta_0$ below the $+x$-axis when it hits the ground.

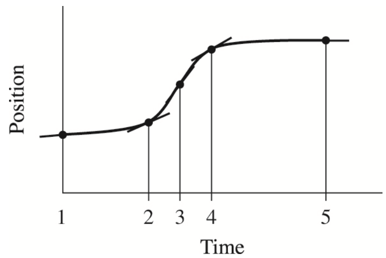
(a) At no time are $\vec{v}$ and $\vec{a}$ parallel if $\dfrac{|v_{1_y}|}{v_{1_x}} = \tan 30°$

(b) At the top of the trajectory $\vec{v}$ and $\vec{a}$ are perpendicular.



ASSESS: Since acceleration always points directly downward, we are really being asked if the velocity is ever vertical or horizontal. Since it is launched at an angle, the velocity is never vertical. And we identified the one instant at which it is horizontal.

Q3.15 **REASON:** This graph shows a curved position-versus-time line. Since the graph is curved, the motion is *not* uniform. The instantaneous velocity, or the velocity at any given instant of time, is the slope of a line tangent to the graph at that point in time. Consider the graph below, where tangents have been drawn at each labeled time.



Comparing the slope of the tangents at each time in the figure above, the speed of the car is greatest at time 3.

ASSESS: Instantaneous velocity is given by the slope of a line tangent to a position-versus-time curve at a given instant of time.

Q3.16 **REASON:** C. Negative, negative; since the slope of the tangent line is negative at both 1 and 2.

ASSESS: The car's position at 2 is at the origin, but it is traveling to the left and therefore has negative velocity in this coordinate system.

Q3.17 **REASON:** The velocity of an object is given by the physical slope of the line on the position-versus-time graph. Since the graph has constant slope, the velocity is constant. We can calculate the slope by using Equation 3.1, choosing any two points on the line since the velocity is constant. In particular, at $t_1 = 0$ s the position is $x_1 = 5$ m. At time $t_2 = 3$ s the position is $x_2 = 15$ m. The points on the line can be read to two significant figures.

The velocity is

$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{15\text{ m} - 5\text{ m}}{3\text{ s} - 0\text{ s}} = \frac{10\text{ m}}{3\text{ s}} = +3.3\text{ m/s}$$

The correct choice is C.

ASSESS: Since the slope is positive, the value of the position is increasing with time, as can be seen from the graph.

Q3.18 **REASON:** The dots from time 0 to 9 seconds indicate a direction of motion to the right. The dots are getting closer and closer. This indicates that the object is moving to the right and slowing down. From 9 to 16 seconds, the object remains at the same position, so it has no velocity. From 16 to 23 seconds, the object is moving to the left. Its velocity is constant since the dots are separated by identical distances.

The velocity-versus-time graph that matches this motion closest is B.

ASSESS: The slope of the line in a velocity-versus-time graph gives an object's acceleration.

Q3.19 **REASON:** We are asked to find the largest of four accelerations, so we compute all four from Equation 3.18:

$$a_x = \frac{\Delta v_x}{\Delta t}$$

A $a_x = \dfrac{10\text{ m/s}}{5.0\text{ s}} = 2.0\text{ m/s}^2$

B $a_x = \dfrac{5.0\text{ m/s}}{2.0\text{ s}} = 2.5\text{ m/s}^2$

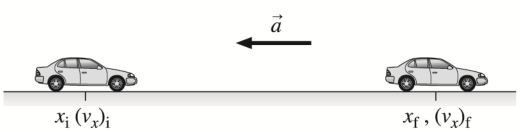
C $a_x = \dfrac{20\text{ m/s}}{7.0\text{ s}} = 2.9\text{ m/s}^2$

D $a_x = \dfrac{3.0\text{ m/s}}{1.0\text{ s}} = 3.0\text{ m/s}^2$

The largest of these is the last, so the correct choice is D.

ASSESS: A large final speed, such as in choices A and C, does not necessarily indicate a large acceleration.

Q3.20 **REASON:** The initial velocity is 20 m/s. Since the car comes to a stop, the final velocity is 0 m/s. We are given the acceleration of the car, and need to find the stopping distance. See the pictorial representation, which includes a list of values below.



Known
$(v_x)_i = 20$ m/s
$(v_x)_f = 0$ m/s
$a_x = -4.0\text{ m/s}^2$

Find
Δx

An equation that relates acceleration, initial velocity, final velocity, and distance is Equation 3.23.

$$(v_x)_f^2 = (v_x)_i^2 + 2a_x\Delta x$$

Solving for Δx,

$$\Delta x = \frac{(v_x)_f{}^2 - (v_x)_i{}^2}{2a_x} = \frac{(0\text{ m/s})^2 - (20\text{ m/s})^2}{2(-4.0\text{ m/s}^2)} = 50\text{ m}$$

The correct choice is D.

ASSESS: We are given initial and final velocities and acceleration. We are asked to find a displacement, so Equation 3.23 is an appropriate equation to use.

Q3.21 **REASON:** All dogs have the same final speed. The dog who has travelled the greatest distance will be the dog who reached higher speeds earlier. This describes Spot. B.

ASSESS: We also know that the area under the velocity-versus-time curve (the integral of the x-component of velocity) will give the displacement along the x-direction. So the dog that has gone the greatest distance will be the one with the largest area under the velocity-versus-time plot. Again, this is clearly Spot B.

Q3.22 **REASON:** Acceleration is the rate of change of velocity. So, the figure with the highest initial slope on the velocity-versus-time plot has the highest initial acceleration. This is Spot. B.

ASSESS: A rapid initial acceleration helps a dog or a runner cover a greater distance in a short amount of time, even if the maximum speed is fixed. They can run at the highest speed for a longer time, that way.

Q3.23 **REASON:** For either animal, the distance traveled over a time t is equal to the area under the velocity-versus-time curve. The speeds are equal at $t = 4$ s, so we want the area under each curve up to that time:

$$\Delta x_L = \frac{1}{2}(12 \text{ m/s})(2 \text{ s}) + (12 \text{ m/s})(2 \text{ s}) = 36 \text{ m}$$

$$\Delta x_G = \frac{1}{2}(12 \text{ m/s})(4 \text{ s}) = 24 \text{ m}$$

So, the lion has run 12 m farther than the gazelle. **ASSESS:** This treatment says the gazelle is eaten. But, note that the quantities calculated above only describe the displacement of each animal (along the x - direction). Claiming that this means the lion eats the gazelle means we are assuming they started at exactly the same position. If they started more than 12 m apart (or with some separation in the y - direction), then the gazelle may still escape.

Q3.24 **REASON:** By definition, the acceleration in the x-direction is the rate of change of the velocity in the x-direction. This means,

$$a_x = \Delta v_x / \Delta t = \frac{(v_x)_f - (v_x)_i}{t_f - t_i} = \frac{(15 \text{ m/s}) - (5 \text{ m/s})}{(4 \text{ s}) - (0 \text{ s})} = 2.5 \text{ m/s}^2$$

The correct answer is B.

ASSESS: Since the slope is positive, we expect a positive acceleration. The magnitude is also reasonable for the components of velocity given.

Q3.25 **REASON:** We can solve this with a straightforward application of Equation 3.23. One can easily obtain the acceleration from the slope, and this was the goal of Question 3.24. The result is $a_x = 2.5 \text{ m/s}^2$; see the solution to Q3.24 for details.

$$(v_x)_f^2 = (v_x)_i^2 + 2a_x\Delta x \Rightarrow \Delta x = \frac{(v_x)_f^2 - (v_x)_i^2}{2a_x} = \frac{(15 \text{ m/s})_f^2 - (5 \text{ m/s})_i^2}{2(2.5 \text{ m/s}^2)} = 40 \text{ m}$$

The correct answer is C.

ASSESS: The speed is never less than 5 m/s, so the distance traveled must be greater than $(5 \text{ m/s})(4 \text{ s}) = 20 \text{ m}$. The speed is never greater than 15 m/s, so the distance traveled must be less than $(15 \text{ m/s})(4 \text{ s}) = 60 \text{ m}$. Our answer lies directly between these limits, and is therefore very reasonable.

Q3.26 **REASON:** The maximum height the ball reaches only depends on the initial velocity it had in the vertical direction. The *y*-component of the velocity of this ball is

$$(v_y)_i = v_i \sin(\theta) = (23.0 \text{ m/s}) \sin(37.0°) = 13.8 \text{ m/s}$$

In order to reach the same height when being thrown vertically upward the ball's initial velocity must be 13.8 m/s. The correct choice is A.

ASSESS: Projectile motion is made up of two independent motions: uniform motion at constant velocity in the horizontal direction and free-fall motion in the vertical direction.

Q3.27 **REASON:** We can apply one kinematic equation to the vertical direction (which we call *y*) to determine how long the balloon is in the air, then apply a kinematic equation to the horizontal direction (which we call *x*) to determine how far it can go in that amount of time. Let us use $\Delta y = (v_y)_i \Delta t + \frac{1}{2} a_y (\Delta t)^2$. Noting that the initial velocity is entirely horizontal such that $(v_y)_i = 0$, we find $\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-10 \text{ m})}{(-9.8 \text{ m/s}^2)}} = 1.43$ s. Now, we can use the same kinematic equation again but for the horizontal direction. This time, we note that there is no horizontal acceleration, such that

$$\Delta x = (v_x)_i \Delta t + \frac{1}{2} a_x (\Delta t)^2 = (v_x)_i \Delta t = (8.2 \text{ m/s})(1.43 \text{ s}) = 12 \text{ m}$$

The correct answer is C.

ASSESS: We have separated the motion into horizontal and vertical components and connected them through time.

PROBLEMS

P3.1 **PREPARE:** Ignoring air resistance, the horizontal component of the velocity should be constant. Although there is motion in the vertical direction, this is separable from motion in the horizontal direction; we will consider only horizontal motion. Since there is no horizontal acceleration, Equation 3.21 becomes $\Delta x = v_{i,x} \Delta t$.

Solve: Rearranging the equation above yields $\Delta t = \Delta x / v_{i,x}$. Inserting the given numbers, and converting units as needed, we have

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{60 \text{ ft}}{95 \frac{\text{mi}}{\text{h}}} \left(\frac{1 \text{ mi}}{5280 \text{ ft}}\right)\left(\frac{60 \text{ min}}{1 \text{ h}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 0.43 \text{ s}$$

ASSESS: Just under a half second is reasonable for a major league pitch.

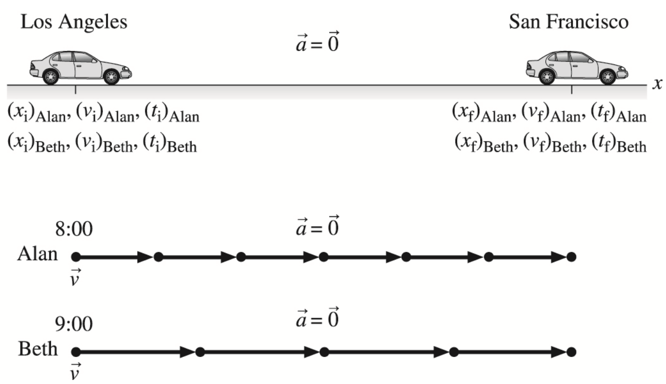
P3.2 **PREPARE:** A visual overview of Alan's and Beth's motion that includes a pictorial representation, a motion diagram, and a list of values is shown below. Our strategy is to calculate and compare Alan's and Beth's time of travel from Los Angeles to San Francisco, assuming constant speed for each.

Known

$(x_i)_{Alan} = (x_i)_{Beth} = 0$
$(x_f)_{Alan} = (x_f)_{Beth} = 400$ miles
$(v_i)_{Alan} = (v_f)_{Alan} = 50$ mph
$(v_i)_{Beth} = (v_f)_{Beth} = 60$ mph
$(t_i)_{Alan} = 8{:}00$ AM
$(t_i)_{Beth} = 9{:}00$ AM

Find

$(t_f)_{Alan}$ $(t_f)_{Beth}$



SOLVE: Beth and Alan are moving at a constant speed, so we can calculate the time of arrival as follows:

$$v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \Rightarrow t_f = t_i + \frac{x_f - x_i}{v}$$

Using the known values identified in the pictorial representation, we find

$$(t_f)_{Alan} = (t_i)_{Alan} + \frac{(x_f)_{Alan} - (x_i)_{Alan}}{v} = 8{:}00\ \text{AM} + \frac{400\ \text{mi}}{50\ \text{mi/h}} = 8{:}00\ \text{AM} + 8\ \text{h} = 4{:}00\ \text{PM}$$

$$(t_f)_{Beth} = (t_i)_{Beth} + \frac{(x_f)_{Beth} - (x_i)_{Beth}}{v} = 9{:}00\ \text{AM} + \frac{400\ \text{mi}}{60\ \text{mi/h}} = 9{:}00\ \text{AM} + 6.67\ \text{h} = 3{:}40\ \text{PM}$$

(a) Beth arrives first.

(b) Beth has to wait 20 min for Alan.

ASSESS: Times of the order of 7 or 8 h are reasonable in the present problem.

P3.3 **PREPARE:** The speed is constant over each of the two intervals, but in between there is a quick acceleration. Thus, we can use kinematic equations to describe each interval separately, but we cannot use a kinematic equation to describe the entire trip, since the acceleration changes.

SOLVE: (a) The time for each segment is $\Delta t_1 = 50\ \text{mi}/40\ \text{mph} = 5/4\ \text{h}$ and $\Delta t_2 = 50\ \text{mi}/60\ \text{mph} = 5/6\ \text{h}$.

The average speed to the house is

$$\frac{100\ \text{mi}}{5/6\ \text{h} + 5/4\ \text{h}} = 48\ \text{mph}$$

(b) Julie drives the distance Δx_1 in time Δt_1 at 40 mph. She then drives the distance Δx_2 in time Δt_2 at 60 mph. She spends the same amount of time at each speed, thus

$$\Delta t_1 = \Delta t_2 \Rightarrow \Delta x_1/40\ \text{mph} = \Delta x_2/60\ \text{mph} \Rightarrow \Delta x_1 = (2/3)\Delta x_2$$

But $\Delta x_1 + \Delta x_2 = 100$ mi, so $(2/3)\Delta x_2 + \Delta x_2 = 100$ mi. This means $\Delta x_2 = 60$ mi and $\Delta x_1 = 40$ mi. Thus, the times spent at each speed are $\Delta t_1 = 40\ \text{mi}/40\ \text{mph} = 1$ h and $\Delta t_2 = 60\ \text{mi}/60\ \text{mph} = 1$ h. The total time for her return trip is $\Delta t_1 + \Delta t_2 = 2$ h. So, her average speed is $100\ \text{mi}/2\ \text{h} = 50\ \text{mph}$.

ASSESS: Comparing parts (a) and (b), it makes sense that the answer to part (a) is a little less than 50 mph. Driving half the distance at 40 mph takes longer than driving half the distance at 60 mph. Since she drove at 40 mph for a longer time, the 40 mph leg of the journey is weighted more heavily in the calculation.

P3.4 **PREPARE:** This problem involves constant-velocity motion. Assume that Richard only speeds on the 125 mi stretch of the interstate. We then need to compute the times that correspond to two different speeds for that given distance. Rearrange Equation 2.1 to produce

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

SOLVE: At the speed limit

$$\text{time}_1 = \frac{125 \text{ mi}}{65 \text{ mi/h}}\left(\frac{60 \text{ min}}{1 \text{ h}}\right) = 115.4 \text{ min}$$

At the faster speed

$$\text{time}_2 = \frac{125 \text{ mi}}{70 \text{ mi/h}}\left(\frac{60 \text{ min}}{1 \text{ h}}\right) = 107.1 \text{ min}$$

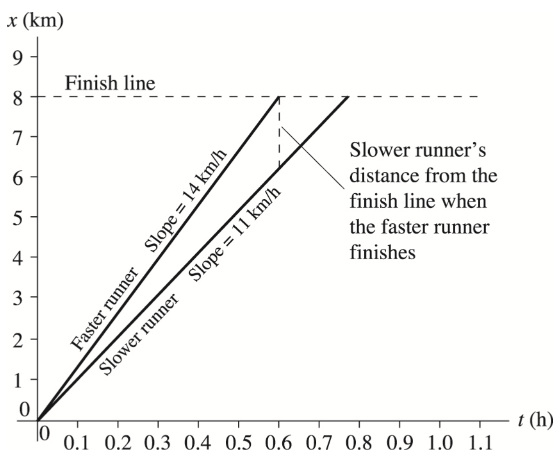
By subtracting, we see that Richard saves 8.3 min.

ASSESS: Breaking the law to save 8.3 min is normally not worth it; Richard's parents can wait 8 min. Notice how the hours (as well as the miles) cancel in the equations.

P3.5 **PREPARE:** This problem involves constant-velocity motion. We'll do this problem in multiple steps. Rearrange Equation 2.1 to produce

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Use this to compute the time the faster runner takes to finish the race; then use $\text{distance} = \text{speed} \times \text{time}$ to see how far the slower runner has gone in that amount of time. Finally, subtract that distance from the 8.00 km length of the race to find out how far the slower runner is from the finish line.



SOLVE: The faster runner finishes in

$$t = \frac{8.00 \text{ km}}{14.0 \text{ km/h}} = 0.571 \text{ h}$$

In that time, the slower runner runs $d = (11.0 \text{ km/h}) \times (0.571 \text{ h}) = 6.29 \text{ km}$.

This leaves the slower runner $8.00\text{ km} - 6.29\text{ km} = 1.71\text{ km}$ from the finish line as the faster runner crosses the line.

ASSESS: The slower runner will not even be in sight of the faster runner when the faster runner crosses the line.

We did not need to convert hours to seconds because the hours canceled out of the last equation. Notice we kept 3 significant figures, as indicated by the original data.

P3.6 **PREPARE:** Since she is running at a steady pace, this is a constant-velocity problem. We will use $v_x = \dfrac{\Delta x}{\Delta t}$, holding the speed constant. This will give us ratios of distances and times to compare.

SOLVE:

(a)

$$\frac{100\text{ m}}{18\text{ s}} = \frac{400\text{ m}}{\Delta t} \Rightarrow \Delta t = 18\text{ s}\left(\frac{400\text{ m}}{100\text{ m}}\right) = 72\text{ s}$$

(b)

$$\frac{100\text{ m}}{18\text{ s}} = \frac{1.0\text{ mi}}{\Delta t} \Rightarrow \Delta t = 18\text{s}\left(\frac{1.0\text{ mi}}{100\text{ m}}\right)\left(\frac{1609\text{ m}}{1.0\text{ mi}}\right) = 290\text{ s} = 4.8\text{ min}$$

ASSESS: This pace does give about the right answer for the time required to run a mile for a good marathoner.

P3.7 **PREPARE:** This is a position-versus-time plot, and we are asked for the top speed. So, we are interested in the maximum slope in the given plot. The slope of the path traced by the dots in the position-versus-time plot is initially somewhat inclined, but then increases after the first two seconds. After that point near the two second mark (which we approximate as 1.9 s), the slope appears fairly constant. We can take this larger, sustained slope as the maximum speed.

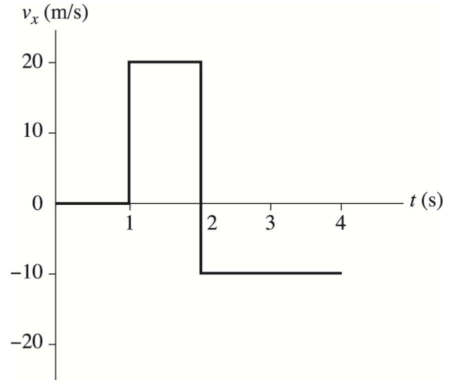
SOLVE: Since the speed after the dot at 1.9 s mark appears roughly constant, we can use the expression for the average speed over that interval:

$$v_{x,\text{av}} = \frac{\Delta x}{\Delta t} = \frac{x_\text{f} - x_\text{i}}{t_\text{f} - t_\text{i}} = \frac{(100\text{ m}) - (10\text{ m})}{(9.5\text{ s}) - (1.9\text{ s})} = 12\text{ m/s}$$

ASSESS: This is extremely fast. One can perform a simple check by noting that the average time for the entire run is $(100\text{ m})/(9.5\text{ s}) = 11\text{ m/s}$. Hence, the fact that we got a slightly higher speed for Usain Bolt's maximum speed is reasonable.

P3.8 **PREPARE:** In this problem, the velocity is changing, but we can determine v_x at a given time by looking at the slope of the x versus t graph. The graph in Figure P3.8 shows distinct slopes in the time intervals: 0–1 s, 1–2 s, and $2-4$ s. Thus, we can obtain the velocity values from this graph using $v = \Delta x/\Delta t$.

SOLVE: (a)



(b) There is only one turning point. At $t = 2$ s, the velocity changes from +20 m/s to −10 m/s, thus reversing the direction of motion. At $t = 1$ s, there is an abrupt change in motion from rest to +20 m/s, but there is no reversal in motion.

ASSESS: As shown above in part (a), a positive slope must give a positive velocity and a negative slope must yield a negative velocity.

P3.9 **PREPARE:** The distance traveled is the area under the v_y graph. Since the graph of v_y vs. t is linear in each region, calculating the area under the line is straightforward: $A = \frac{1}{2}BH$.

SOLVE:

(a) Equating the area with the displacement, we have

$$\Delta y = \text{area} = \frac{1}{2}BH = \frac{1}{2}(0.20\text{ s})(0.75\text{ m/s}) = 7.5\text{ cm}$$

(b) Using the given distance of 30 cm, we find

$$\Delta t = \frac{\Delta y}{v_y} = \frac{30\text{ cm}}{7.5\text{ cm/beat}} = 4.0\text{ beats}$$

ASSESS: Four beats seems reasonable. There is some doubt that we are justified using two significant figures here.

P3.10 **PREPARE:** Displacement is given by the area under the velocity-versus-time graph. In this case, the displacement is equal to the area under the velocity graph between t_i and t_f. We can find the car's final position from its initial position and the area.

SOLVE: (a) Using the equation $x_f = x_i$ + area of the velocity graph between t_i and t_f,

$$x_{2\text{ s}} = 10\text{ m} + \text{area of trapezoid between 0 and 2 s}$$
$$= 10\text{ m} + \frac{1}{2}(12\text{ m/s} + 4\text{ m/s})(2\text{ s}) = 26\text{ m}$$
$$x_{3\text{ s}} = 10\text{ m} + \text{area of triangle between 0 and 3 s}$$
$$= 10\text{ m} + \frac{1}{2}(12\text{ m/s})(3\text{ s}) = 28\text{ m}$$
$$x_{4\text{ s}} = x_{3\text{ s}} + \text{area between 3 and 4 s}$$
$$= 28\text{ m} + \frac{1}{2}(-4\text{ m/s})(1\text{ s}) = 26\text{ m}$$

(b) The car reverses direction at $t = 3$ s, because its velocity becomes negative.

ASSESS: The car starts at $x_i = 10$ m at $t_i = 0$. Its velocity decreases as time increases, it is zero at $t = 3$ s, and then becomes negative. The slope of the velocity-versus-time graph is negative, which means the car's acceleration is negative and a constant. From the acceleration thus obtained and given velocities on the graph, we can also use kinematic equations to find the car's position at various times.

P3.11 **PREPARE:** We know from eq. 3.4 that the $v_x = dx/dt$. We can simply apply this to the given expression and evaluate it at the times specified.

SOLVE: (a) Taking the derivative, we obtain

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left[2t^2 - 8t\right] = 2(2)t - 8$$

Evaluating this at $t = 1.0$ s, we find $v_x = 2(2)(1.0) - 8 = -4$ m/s.

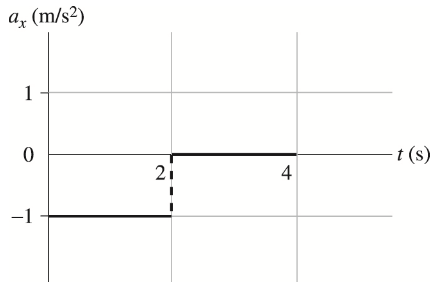
(b) Evaluating the expression from (a) at $t = 2.0$ s, we find $v_x = 2(2)(2.0) - 8 = 0$ m/s.

(c) Evaluating the expression from (a) at $t = 3.0$ s, we find $v_x = 2(2)(3.0) - 8 = 4$ m/s.

ASSESS: From the original expression, it is clear that as time passes, the position increases more rapidly, because of the quadratic dependence on time. It is therefore reasonable that our sequence of answers shows a greater and greater (more positive) number for v_x, as time passes.

P3.12 **PREPARE:** The graph in Figure P3.12 shows the horizontal component of velocity as a function of time. We know the acceleration is the rate of change of the velocity. So we can determine the acceleration using the slope of this graph. Mathematically, $a_x = \Delta v_x/\Delta t$. A linear decrease in velocity from $t = 0$ s to $t = 2$ s implies a constant negative acceleration. On the other hand, a constant velocity between $t = 2$ s and $t = 4$ s means zero acceleration.

SOLVE:



ASSESS: The discontinuous jump in acceleration is expected, since the slope of the velocity-versus-time plot changes discontinuously at the 2-second mark.

P3.13 **PREPARE:** In part (a), we will use Equation 3.16, which describes how acceleration can be found from the slope of a velocity-versus-time plot. Equation 3.12 describes how to determine the position from a velocity-versus-time plot, by integrating. This will be useful in part (b).

SOLVE: (a) We know $a_x = \Delta v_x / \Delta t = (4 \text{ m/s})/(4 \text{ s}) = 1 \text{ m/s}^2$.

(b) The integral of v_x over time is equivalent to the area under the curve. This area is as follows:

$$\Delta x = \frac{1}{2}(-2 \text{ m/s})(2 \text{ s}) + \frac{1}{2}(2 \text{ m/s})(2 \text{ s}) + (2 \text{ m/s})(2 \text{ s}) = 4 \text{ m}$$

This only describes how the position of the train has changed. Since, at $t = 0$, the train was already at $x = 2.0$ m, the new position of the train is at $x = 2 \text{ m} + 4 \text{ m} = 6 \text{ m}$.

ASSESS: Since the train has a positive v_x for most of the time, it is reasonable that the displacement in x would be positive.

P3.14 **PREPARE:** Although this problem does not involve constant acceleration throughout the entire time shown, the acceleration is constant on intervals. We recall that displacement is equal to area under the velocity graph between t_i and t_f, and acceleration is the slope of the velocity-versus-time graph.

SOLVE: (a) Using the equation, $x_f = x_i$ + area under the velocity-versus-time graph between t_i and t_f we have,

x (at $t = 1$ s) = x (at $t = 0$ s) + area between $t = 0$ s and $t = 1$ s = 0.0 m + (4 m/s)(1 s) = 4.0 m.

Reading from the velocity-versus-time graph, v_x (at $t = 1$ s) = 4.0 m/s. Also, a_x = slope = $\Delta v/\Delta t = 0 \text{ m/s}^2$.

(b)

$$\begin{aligned} x(\text{at } t = 3.0 \text{ s}) &= x(\text{at } t = 0 \text{ s}) + \text{area between } t = 0 \text{ s and } t = 3 \text{ s} \\ &= 0.0 \text{ m} + 4 \text{ m/s} \times 2 \text{ s} + 2 \text{ m/s} \times 1 \text{ s} + (1/2) \times 2 \text{ m/s} \times 1 \text{ s} = 11.0 \text{ m} \end{aligned}$$

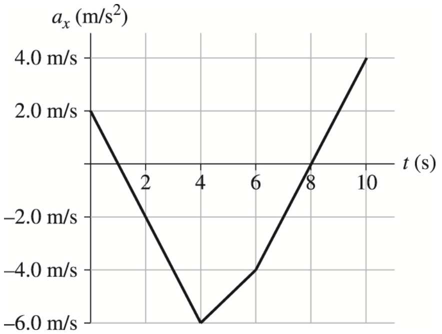
Reading from the graph, v_x ($t = 3$ s) = 2 m/s. The acceleration is

$$a_x(t = 3 \text{ s}) = \text{slope} = \frac{v_x(\text{at } t = 4 \text{ s}) - v_x(\text{at } t = 2 \text{ s})}{2 \text{ s}} = -2.0 \text{ m/s}^2$$

ASSESS: Due to the negative slope of the velocity graph between 2 s and 4 s, a negative acceleration was expected.

P3.15 **PREPARE:** Acceleration is the rate of change of velocity. We must draw a velocity versus time graph in which the slope at a given point is equal to the value of the acceleration in the plot above. We are told the initial speed of the object is 2.0 m/s. We can simply start drawing a line from that point with the appropriate slope, changing slopes at the appropriate times.

SOLVE:



ASSESS: We can check our answer by calculating the velocity after a certain time and seeing if it matches the graph. Let us check the lowest point, which on our graph is –6.0 m/s and occurs at 4 s. Using Equation 3.19, we have $(v_x)_f = (v_x)_i + a_x \Delta t = (2.0\text{ m/s}) + (-2.0\text{ m/s}^2)(4\text{ s}) = -6.0\text{ m/s},$ which is consistent.

P3.16 **PREPARE:** We calculate the acceleration from the slope of each straight line segment.

SOLVE: Speeding up:

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{0.75\text{ m/s}}{0.05\text{ s}} = 15\text{ m/s}^2$$

Slowing down:

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{-0.75\text{ m/s}}{0.15\text{ s}} = -5\text{ m/s}^2$$

ASSESS: Indeed, the slope looks three time steeper in the first segment than in the second. These are pretty large accelerations.

P3.17 **PREPARE:** Since acceleration is given by the rate of change of velocity with respect to time, we will compute the slope of each straight-line segment in the graph.

$$a_x = \frac{(v_x)_f - (v_x)_i}{t_f - t_i}$$

The trickiest part is reading the values off of the graph.

SOLVE: (a)

$$a_x = \frac{5.5\text{ m/s} - 0.0\text{ m/s}}{1.0\text{ s} - 0.0\text{ s}} = 5.5\text{ m/s}^2$$

(b)

$$a_x = \frac{9.0\text{ m/s} - 5.5\text{ m/s}}{2.5\text{ s} - 1.0\text{ s}} = 2.3\text{ m/s}^2$$

(c)

$$a_x = \frac{11\text{ m/s} - 9.0\text{ m/s}}{3.5\text{ s} - 2.5\text{ s}} = 2.0\text{ m/s}^2$$

ASSESS: This graph is difficult to read to more than one significant figure. I did my best to read a second significant figure, but there is some estimation in the second significant figure.

It takes Carl Lewis almost 10 s to run 100 m, so this graph covers only the first third of the race. Were the graph to continue, the slope would continue to decrease until the slope is zero as he reaches his (fastest) cruising speed.

Also, if the graph were continued out to the end of the race, the area under the curve should total 100 m.

P3.18 **PREPARE:** In reality, biological systems rarely move with constant acceleration. But we will assume constant acceleration over this very short time interval. Use the definition of acceleration: $a_y = \Delta v_y/\Delta t$. Also, 60 ms = 0.060 s.

SOLVE:

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{3.7\text{ m/s}}{0.060\text{ s}} = 62\text{ m/s}^2$$

ASSESS: Frogs are quite impressive! Humans can't jump with this kind of acceleration.

P3.19 **PREPARE:** We will assume constant accelerations for both animals. We can calculate acceleration from Equation 3.8.

SOLVE: For the gazelle:

$$(a_x) = \left(\frac{\Delta v_x}{\Delta t}\right) = \frac{13\text{ m/s}}{3.0\text{ s}} = 4.3\text{ m/s}^2$$

For the lion:

$$(a_x) = \left(\frac{\Delta v_x}{\Delta t}\right) = \frac{9.5\text{ m/s}}{1.0\text{ s}} = 9.5\text{ m/s}^2$$

For the trout:

$$(a_x) = \left(\frac{\Delta v_x}{\Delta t}\right) = \frac{2.8\text{ m/s}}{0.12\text{ s}} = 23\text{ m/s}^2$$

The trout is the animal with the largest acceleration.

ASSESS: A lion would have an easier time snatching a gazelle than a trout.

P3.20 **PREPARE:** We will assume constant acceleration, such that we can use kinematic equations. Acceleration is the rate of change of velocity.

$$a_x = \frac{\Delta v_x}{\Delta t}$$

Where $\Delta v_x = 4.0\text{ m/s}$ and $\Delta t = 0.11\text{s}$.

We will then use that acceleration to compute the final position after the strike:

$$x_f = \frac{1}{2}a_x(\Delta t)^2$$

where we are justified in using the special case because $(v_x)_i = 0.0$ m/s and $x_i = 0$ m.

SOLVE: (a)

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{4.0 \text{ m/s}}{0.11 \text{ s}} = 36 \text{ m/s}^2$$

(b)

$$x_f = \frac{1}{2}a_x(\Delta t)^2 = \frac{1}{2}(36 \text{ m/s}^2)(0.11 \text{ s})^2 = 0.22 \text{ m}$$

ASSESS: The answer is remarkable but reasonable. The pike strikes quickly and so is able to move 0.22 m in 0.11 s, even starting from rest. The seconds squared cancel in the last equation.

P3.21 **PREPARE:** Model the air as a particle. We will convert to base SI units, and make use of Equation 3.19 to determine acceleration.

SOLVE:

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{150 \text{ km/h}}{0.50 \text{ s}}\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{60 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 83 \text{ m/s}^2$$

ASSESS: 83 m/s^2 is a remarkable acceleration.

P3.22 **PREPARE:** During the acceleration phase, acceleration is constant, and we can use kinematic equations. Once the acceleration drops to zero, acceleration will once again be constant, and we can apply kinematic equations over the phase of constant velocity. But we cannot apply kinematic equations from the beginning of the dash to the end, since acceleration changes in between.

Use Equation 3.19 to find the acceleration during that first phase.

$$v_x = a_x t_1 \qquad \text{where } v_0 = 0 \text{ and } t_0 = 0$$

$$a_x = \frac{v_x}{t_1} = \frac{11.2 \text{ m/s}}{2.14 \text{ s}} = 5.23 \text{ m/s}^2$$

SOLVE: The distance traveled during the acceleration phase will be

$$\begin{aligned}\Delta x &= \frac{1}{2}a_x(\Delta t)^2 \\ &= \frac{1}{2}(5.23 \text{ m/s}^2)(2.14 \text{ s})^2 \\ &= 12.0 \text{ m}\end{aligned}$$

The distance left to go at constant velocity is $100 \text{ m} - 12.0 \text{ m} = 88.0 \text{ m}$. The time this takes at the top speed of 11.2 m/s is

$$\Delta t = \frac{\Delta x}{v_x} = \frac{88.0 \text{ m}}{11.2 \text{ m/s}} = 7.86 \text{ s}$$

The total time is $2.14 \text{ s} + 7.86 \text{ s} = 10.0 \text{ s}$.

ASSESS: This is indeed about the time it takes a world-class sprinter to run 100 m (the world record is a bit under 9.8 s).

Compare the answer to this problem with the accelerations given in Problem 3.17 for Carl Lewis.

P3.23 **PREPARE:** This problem involves free fall, in which the acceleration is constant. Thus, we can use the kinematic equations. The bill must drop its own length in free fall. We can use Equation 3.21, with an initial downward velocity of zero.

SOLVE:

$$\Delta y = \frac{1}{2}g(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(0.16 \text{ m})}{9.8 \text{ m/s}^2}} = 0.18 \text{ s}$$

ASSESS: This is less than the typical 0.25 s reaction time, so most people miss the bill.

P3.24 **PREPARE:** This problem involves free fall, in which the acceleration is constant. Thus, we can use the kinematic equations. We will assume that, as stated in the chapter, the bill is held at the top, and the other person's fingers are bracketing the bill at the bottom.

We will make use of Equation. 3.21. Call the initial position of the top of the bill the origin, $y_0 = 0.0$ m, and, for convenience, call the down direction positive.

In free fall, the acceleration a_y will be −9.8 m/s². The length of the bill will be Δy, the distance the top of the bill can fall from rest in 0.25 s.

SOLVE:

$$y_f = \frac{1}{2}a_y(\Delta t)^2 = \frac{1}{2}(9.8 \text{ m/s}^2)(0.25 \text{ s})^2 = 0.31 \text{ m}$$

ASSESS: This is about twice as long as real bills are (they are really 15.5 cm long), so if a typical reaction time is 0.25 s, then almost no one would catch one in this manner. To catch a bill as small as real bills, one would need a reaction time of 0.13 s.

P3.25 **PREPARE:** This problem involves free fall, in which the acceleration is constant. Thus, we can use the kinematic equations. Equation 3.23 will be useful in this situation, since we know the acceleration, the final speed, and we are seeking the distance covered.

SOLVE:

$$(v_y)_f^2 = (v_y)_i^2 + 2g\Delta y \Rightarrow \Delta y = \frac{(v_y)_f^2}{2g} = \frac{(32 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 52 \text{ m}$$

ASSESS: A distance of 52 meters seems a reasonable height from which to begin the dive.

P3.26 **PREPARE:** This problem deals with hoverflies in free fall, which is a special case of motion with a constant acceleration. So we can apply kinematic equations as needed. We will use the convention that $-y$ is downward.

SOLVE: (a) We are given the fall time and we know the acceleration due to gravity is $a_y = -9.8 \text{ m/s}^2$.

Applying Equation 3.21, we find

$$\Delta y = v_{i,y}\,\Delta t + \frac{1}{2}a_y(\Delta t)^2 = 0 + \frac{1}{2}\left(-9.8 \text{m/s}^2\right)(0.150 \text{ s})^2 = -0.11 \text{ m}$$

Although this displacement is correct as a negative number, the problem asks how far the hoverflies have fallen; it does not ask for a vector or a component of a vector. We report the distance as 11 cm.

(b) We must first determine the speed the flies have reached at this intermediate stage, and the distance that remains: $(40\text{ cm})-(11\text{ cm})=29$ cm. Let us use the subscript "2" to refer to the instant after 0.150 s have passed, and "3" to refer to the moment the hoverflies reach the bottom of the container. We can use Equation 3.18 to determine the speed at time 2:

$$v_{2,y}=v_{i,y}+a_y\Delta t=0+\left(-9.8\text{ m/s}^2\right)(0.150\text{s})=-1.47\text{ m/s}$$

Now, Equation 3.23 can be used to find the required acceleration to ensure the hoverflies come to rest just as they reach the bottom of the container:

$$v_3^2=v_2^2+2a_y\Delta y\Rightarrow a_y=\frac{v_3^2-v_2^2}{2\Delta y}=\frac{(0\text{ m/s})^2-(-1.47\text{ m/s})^2}{2(-0.29\text{ m})}=3.7\text{ m/s}^2$$

The acceleration could, of course, be larger than this. This is the acceleration that would see the hoverflies just barely come to rest at the bottom of the container.

ASSESS: It is reasonable that we obtained a positive acceleration. It is also reasonable that the acceleration we obtained is smaller than g, since the hoverflies gained their speed through a free fall of only 11 cm, and they have 29 cm in which to stop.

P3.27 **PREPARE:** Once the trout leaves the water, it is subject to acceleration due to gravity only. The acceleration is constant, so we can use kinematic equations. In order to find the maximum height, we can use Equation 3.23. We recognize that at the maximum height the vertical component of the velocity is momentarily zero. Let us call the vertically upward direction $+y$.

SOLVE: To find the maximum height, we write

$$(v_y)_f^2=(v_y)_i^2+2a_y\Delta y\Rightarrow\Delta y=\frac{(v_y)_f^2-(v_y)_i^2}{2a_y}\Rightarrow\Delta y_{\max}=\frac{(0\text{ m/s})^2-(8.0\text{ m/s})^2}{2(-9.8\text{ m/s}^2)}=3.3\text{ m}$$

ASSESS: This a reasonable time of flight for a fish jumping out of the water.

P3.28 **PREPARE:** Once the jumper leaves the ground, he or she is in free fall, in which the acceleration is constant. Thus, we can use the kinematic equations. Specifically, we will use $(v_y)_f^2=(v_y)_i^2+2a_y\Delta y$ where $(v_y)_f^2=0$ at the top of the leap.

We assume $a_y=-9.8\text{ m/s}^2$, and we are given $\Delta y=1.1$ m.

SOLVE:

$$(v_y)_i^2=-2a_y\Delta y\quad\Rightarrow\quad(v_y)_i=\sqrt{-2a_y\Delta y}=\sqrt{-2(-9.8\text{ m/s}^2)(1.1\text{ m})}=4.6\text{ m/s}$$

ASSESS: This is an achievable take off speed for good jumpers. The units also work out correctly and the two minus signs under the square root make the radicand positive.

P3.29 **PREPARE:** Assume the trajectory is symmetric (i.e., the ball leaves the ground), so half of the total time is the upward portion and half downward. Put the origin at the ground. Assume no air resistance, such that the acceleration is due to gravity and is constant. This enables us to use kinematic equations.

SOLVE:

(a) On the way down $(v_y)_i = 0$ m/s, $y_f = 0$ m, and $\Delta t = 2.6$ s. Solve for y_i.

$$0 = y_i + \frac{1}{2}a_y(\Delta t)^2 \quad \Rightarrow \quad y_i = -\frac{1}{2}a_y(\Delta t)^2 = -\frac{1}{2}(-9.8 \text{ m/s}^2)(2.6 \text{ s})^2 = 33.1 \text{ m}$$

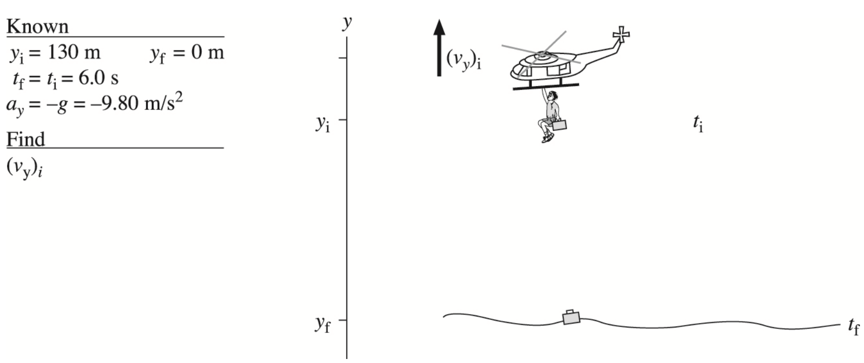
or 33 m to two significant figures.

(b) On the way up $(v_y)_f = 0$ m/s.

$$(v_y)_i^2 = -2a_y\Delta y \quad \Rightarrow \quad (v_y)_i = \sqrt{-2a_y\Delta y} = \sqrt{-2(-9.8 \text{ m/s}^2)(33.1 \text{ m})} = 25 \text{ m/s}$$

ASSESS: When thinking about real football games, this speed seems reasonable.

P3.30 **PREPARE:** Once the briefcase is dropped it is in free fall, during which the acceleration is constant. Thus, we can use kinematic equations. Since the villain is hanging on to the ladder as the helicopter is ascending, he and the briefcase are moving with the same upward velocity as the helicopter. We can calculate the initial velocity of the briefcase, which is equal to the upward velocity of the helicopter. See the following figure.



SOLVE: We can use Equation 3.22 here. We know the time it takes the briefcase to fall, its acceleration, and the distance it falls. Solving for $(v_y)_i\,\Delta t$,

$$(v_y)_i\Delta t = (y_f - y_i) - \frac{1}{2}(a_y)\Delta t^2 = -130 \text{ m} - \left[\frac{1}{2}(-9.80 \text{ m/s}^2)(6.0 \text{ s})^2\right] = 46 \text{ m}$$

Dividing by Δt to solve for $(v_y)_i$,

$$(v_x)_i = \frac{46 \text{ m}}{6.0 \text{ s}} = 7.7 \text{ m/s}$$

ASSESS: Note the placement of negative signs in the calculation. The initial velocity is positive, as expected for a helicopter ascending.

P3.31 **PREPARE:** The jumper is in free fall after leaving the ground, so we can use the kinematic equations. We know $a_y = -9.8$ m/s^2 and we are given $(y_f - y_i) = 1.1$ m. We can use Equation 3.12 to determine the unknown time.

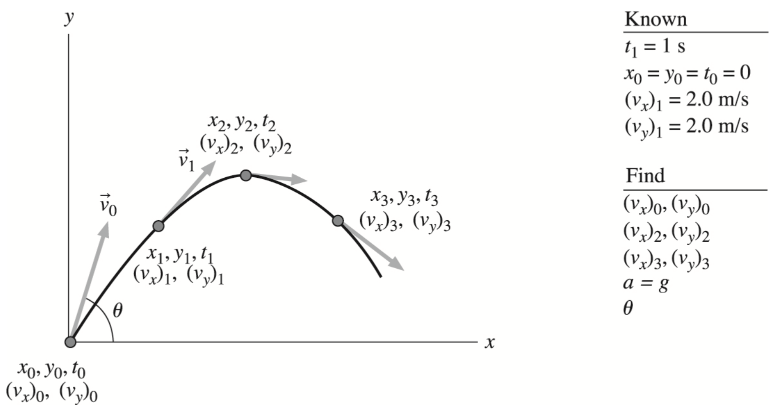
SOLVE: Since the trajectory is symmetric, we'll compute the time it takes to come down from 1.1 m to the floor and then double it.

$$(y_f - y_i) = \frac{1}{2}a_y(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2(y_f - y_i)}{a_y}} = \sqrt{\frac{2(-1.1\text{ m})}{-9.8\text{ m/s}^2}} = 0.47\text{ s}$$

The whole "hang time" will be double this, or 0.95 s.

ASSESS: This is about the time for a big leap. The units also work out correctly and the two minus signs under the square root make the radicand positive.

P3.32 **PREPARE:** The object is in free fall, just on a different planet, such that we don't know the value of the gravitational acceleration. We do know that it will be constant, so that we can use kinematic equations. A visual overview is shown in the following figure.



We can use the change in the vertical component of velocity between 1.0 s and 2.0 s to determine the acceleration due to gravity on this planet.

SOLVE: (a) We know the velocity $\vec{v}_1 = (\vec{v}_x)_1 + (\vec{v}_y)_1$ with $(v_x)_1 = 2.0$ m/s and $(v_y)_1 = 2.0$ m/s at $t = 1$ s. The ball is at its highest point at $t = 2$ s, so $v_y = 0$ m/s. The horizontal velocity is constant in projectile motion, so $v_x = 2.0$ m/s at all times. Thus, $\vec{v}_2 = (\vec{v}_x)_2 + (\vec{v}_y)_2$, with $(v_x)_2 = 2.0$ m/s and $(v_y)_2 = 0$ m/s at $t = 2$ s. We can see that the *y*-component of velocity *changed* by $\Delta v_y = -2.0$ m/s between $t = 1$ s and $t = 2$ s. Because a_y is constant, v_y changes by –2.0 m/s in *any* 1-s interval. At $t = 3$ s, v_y is 2.0 m/s less than its value of 0 at $t = 2$ s. At $t = 0$ s, v_y must have been 2.0 m/s more than its value of 2.0 m/s at $t = 1$ s. Consequently, at $t = 0$ s,

$$\vec{v}_0 = (\vec{v}_x)_0 + (\vec{v}_y)_0, \text{ with } (v_x)_0 = 2.0\text{ m/s and } (v_y)_0 = 4.0\text{ m/s}$$

At $t = 1$ s,

$$\vec{v}_1 = (\vec{v}_x)_1 + (\vec{v}_y)_1, \text{ with } (v_x)_1 = 2.0\text{ m/s and } (v_y)_1 = 2.0\text{ m/s}$$

At $t = 2$ s,

$$\vec{v}_2 = (\vec{v}_x)_2 + (\vec{v}_y)_2, \text{ with } (v_x)_2 = 2.0\text{ m/s and } (v_y)_2 = 0\text{ m/s}$$

At $t = 3$ s,

$$\vec{v}_3 = (\vec{v}_x)_3 + (\vec{v}_y)_3, \text{ with } (v_x)_3 = 2.0 \text{ m/s and } (v_y)_3 = -2.0 \text{ m/s}$$

(b) Because v_y is changing at the rate –2.0 m/s per s, the y-component of acceleration is $a_y = -2.0 \text{ m/s}^2$.

But $a_y = -g$ for projectile motion, so the value of g on Exidor is $g = 2.0 \text{ m/s}^2$.

(c) From part (a) the components of $\vec{v}_0$ are $(v_x)_0 = 2.0$ m/s and $(v_y)_0 = 4.0$ m/s. This means

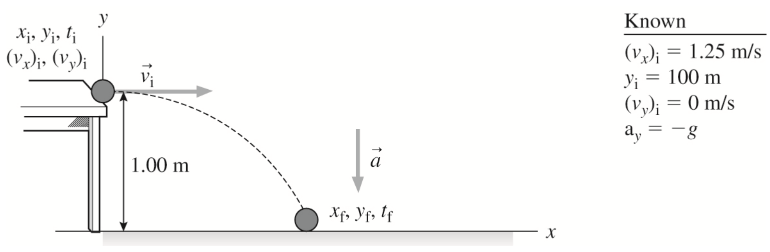
$$\theta = \tan^{-1}\left(\frac{(v_y)_0}{(v_x)_0}\right) = \tan^{-1}\left(\frac{4.0 \text{ m/s}}{2.0 \text{ m/s}}\right) = 63° \text{ above} + x$$

ASSESS: The y-component of the velocity vector decreases from 2.0 m/s at $t = 1$ s to 0 m/s at $t = 2$ s. This gives an acceleration of -2 m/s^2. All the other values obtained above are also reasonable.

P3.33 **PREPARE:** We can use the vertical-position equation to find the time it takes the ball to reach the floor: $y_f = y_i + (v_y)_i \Delta t - \frac{1}{2} g(\Delta t)^2$. The distance the ball travels horizontally is governed by the horizontal-position Equation 3.21:

$$x_f = x_i + (v_x)_i \, \Delta t$$

SOLVE: Refer to the visual overview shown.



The initial vertical velocity is zero. Take the floor as the origin of coordinates. The ball falls from $y_i = 1.00$ m and lands at $y_f = 0$ m.

(a Rearranging to solve for time, and inserting the given values, we have

$$\Delta t = \sqrt{\frac{-2\Delta y}{g}} = \sqrt{\frac{-2(-1.00 \text{ m})}{9.80 \text{ m/s}^2}} = 0.452 \text{ s}$$

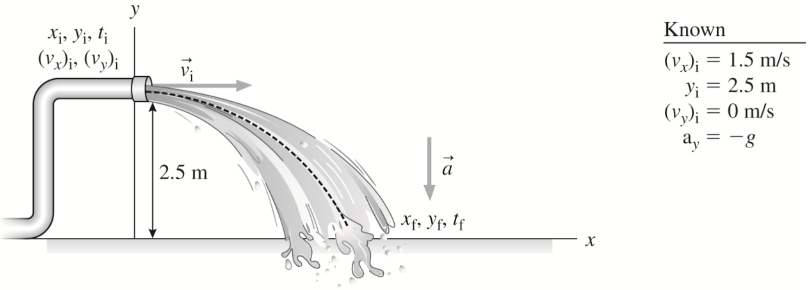
(b) Inserting the result from (a), we have

$$x_f = x_i + (v_x)_i \Delta t = (0.0 \text{ m}) + (1.25 \text{ m/s})(0.452 \text{ s}) = 0.565 \text{ m}$$

ASSESS: This seems reasonable for a ball rolling off a table.

P3.34 **PREPARE:** Assume the water is in free fall. We will also treat the water as though it is made up of particles, each of which roughly obeys the projectile motion we expect for solid objects. We ignore air resistance. We can use the vertical-position equation to find the time it takes the water to reach the floor: $y_f = y_i + (v_y)_i \Delta t - \frac{1}{2} g(\Delta t)^2$. The distance the water travels horizontally is governed by the horizontal-position equation: $x_f = x_i + (v_x)_i \Delta t$.

SOLVE: Refer to the visual overview shown.



The initial vertical velocity is zero. Take the creek as the origin of coordinates. The ball falls from $y_i = 2.5$ m and lands at $y_f = 0$ m. Rearranging the vertical equation to solve for time, we find

$$\Delta t = \sqrt{\frac{-2\Delta y}{g}} = \sqrt{\frac{-2(-2.5\text{ m})}{(9.8\text{ m/s}^2)}} = 0.71\text{ s}.$$

(b) Inserting the result from part (a), we find

$$x_f = x_i + (v_x)_i \Delta t = (0.0\text{ m}) + (1.5\text{ m/s})(0.714\text{ s}) = 1.1\text{ m}$$

ASSESS: This is the right order of magnitude, since the water is only in the air for less than one second.

P3.35 **PREPARE:** This problem involves constant acceleration (as in projectile motion). So, we can use the kinematic equations in the horizontal and vertical directions. If we consider half the motion from the launch to the peak height, then we can use Equation 3.23 to determine the initial velocity in the vertical direction (which we will call *y*). Then we can use Equation 3.19 to determine the time that the mountain lion was in the air. We also know that there is no acceleration in the *x*-direction (ignoring air resistance). So, once we have that hang time, we can easily determine the initial velocity in the horizontal direction using $(v_x)_i = \Delta x/\Delta t$.

SOLVE: (a) Considering only half the trip such that the "final" time is at the peak height, Equation 2.13 yields

$$(v_y)_f^2 = (v_y)_i^2 + 2a_y\,\Delta y \Rightarrow (v_y)_i = \sqrt{-2a_y\,\Delta y} = \sqrt{-2(-9.8\text{ m/s}^2)(3.0\text{ m})} = 7.67\text{ m/s}$$

Now, employing Equation 3.19 to the same time interval, we find

$$(v_y)_f = (v_y)_i + a_y\Delta t \Rightarrow \Delta t = -(v_y)_i/a_y = -(7.67\text{ m/s})/(-9.8\text{ m/s}^2) = 0.782\text{ s}$$

But note that this is the time for only half the projectile motion of the mountain lion. By symmetry, the full time should be twice that: 1.56 s.

Finally, using $(v_x)_i = \Delta x/\Delta t$ (because there is no acceleration in the horizontal direction), we find

$$(v_x)_i = \Delta x/\Delta t = (10\text{ m})/(1.56\text{ s}) = 6.4\text{m/s}$$

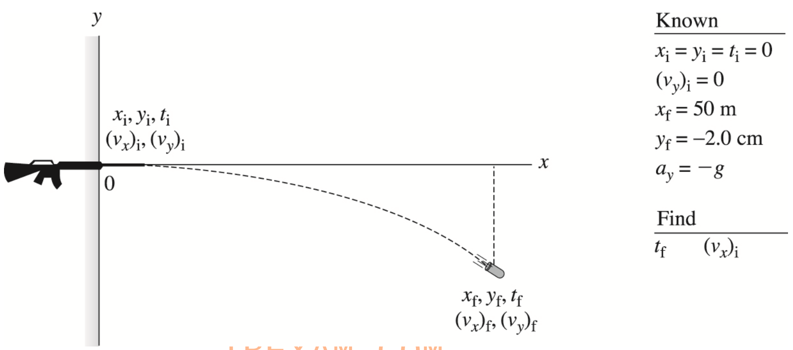
Now that we have the horizontal and vertical components of the velocity, we can find its magnitude (the speed) using the Pythagorean theorem: $v_i = \sqrt{(v_x)_i^2 + (v_y)_i^2} = \sqrt{(6.4\text{ m/s})^2 + (7.67\text{ m/s})^2} = 10\text{ m/s}.$

(b) Now that we know the horizontal and vertical components of the mountain lion's initial velocity, we can determine the angle from horizontal using simple trigonometry: $\tan(\theta) = opp./adj. = (v_y)_i/(v_x)_i$. So

$$\theta = \tan^{-1}\left((v_y)_i/(v_x)_i\right) = \tan^{-1}\left((7.67 \text{ m/s})/(6.40 \text{ m/s})\right) = 50°$$

ASSESS: Components of velocity equal to a few meters per second are reasonable for a mountain lion. The initial jumping angle fits expectations, since it would need to have a significant initial upward component to stay in the air 1.56 s.

P3.36 **PREPARE:** We will apply the constant-acceleration kinematic equations to the horizontal and vertical motions. The effect of air resistance on the motion of the bullet is neglected. We will begin by drawing a diagram of the situation and identifying known quantities and the quantities we want to find.



SOLVE: **(a)** Using $y_f = y_i + (v_y)_i(t_f - t_i) + \frac{1}{2}a_y(t_f - t_i)^2$, we obtain

$$(-2.0 \times 10^{-2} \text{ m}) = 0 \text{ m} + 0 \text{ m} + \tfrac{1}{2}(-9.8 \text{ m/s}^2)(t_f - 0 \text{ s})^2 \Rightarrow t_f = 0.0639 \text{ s}$$

or 0.064 s to two significant figures.

(b) Using $x_f = x_i + (v_x)_i(t_f - t_i) + \frac{1}{2}a_x(t_f - t_i)^2$,

$$(50 \text{ m}) = 0 \text{ m} + (v_x)_i(0.0639 \text{ s} - 0 \text{ s}) + 0 \text{ m} \Rightarrow (v_x)_i = 780 \text{ m/s}$$

ASSESS: The bullet falls 2.0 cm during a horizontal displacement of 50 m. This implies a large initial velocity, and a value of 780 m/s is not surprising.

P3.37 **PREPARE:** We can consider vertical and horizontal motion separately to obtain an expression for the distance in terms of the takeoff speed. We are asked to find the takeoff speed and horizontal speed of the kangaroo given its initial angle, 20°, and its range. Since the horizontal speed is given by $v_x = v\cos\theta$ and the time of flight is given by $\Delta t = 2v\,\sin\theta/g$, the range of the kangaroo is given by the product of these: $\Delta x = 2v\,\sin\theta\cos\theta/g$.

SOLVE: **(a)** We can solve the above formula for v, and then plug in the range and angle to find the takeoff speed:

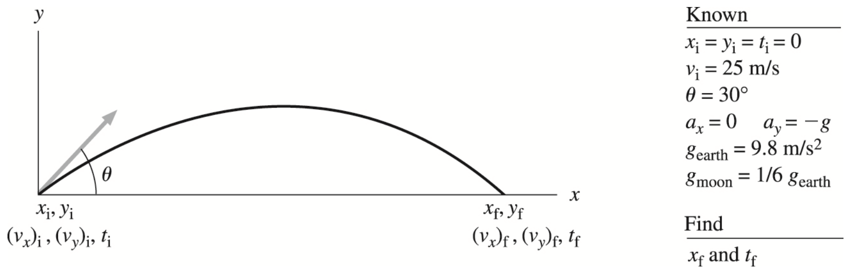
$$v = g\Delta x/(2\,\sin\theta\,\cos\theta) = (9.8 \text{ m/s}^2)(10 \text{ m})/(2\,\sin 20°\cos 20°) = 12.3 \text{ m/s}$$

Its takeoff speed is 12 m/s, to two significant figures.

(b) Its horizontal speed is given by $v_x = v\cos\theta = (12.3 \text{ m/s}) \cos 20° = 11.6 \text{ m/s}$ or 12 m/s to two significant figures.

ASSESS: The reason the horizontal speed and takeoff speed appear the same is that $20°$ is a small angle and the cosine of a small angle is approximately equal to 1.

P3.38 **PREPARE:** We will begin by drawing a diagram of the situation and identifying known and unknown quantities. This is projectile motion, so we know the acceleration and we know that it is constant, such that we can use kinematic equations.



SOLVE: (a) The distance traveled is $x_f = (v_i)_x t_f = v_i \cos\theta \times t_f$. The flight time is found from the y-equation, using the fact that the ball starts and ends at $y = 0$:

$$y_f - y_i = 0 = v_i \sin\theta\, t_f - \tfrac{1}{2} g t_f^2 = (v_i \sin\theta - \tfrac{1}{2} g t_f)\, t_f \Rightarrow t_f = \frac{2v_i \sin\theta}{g}$$

Thus, the distance traveled is

$$x_f = v_i \cos\theta \times \frac{2v_i \sin\theta}{g} = \frac{2v_i^2 \sin\theta \cos\theta}{g}$$

For $\theta = 30°$, the distances are

$$(x_f)_{earth} = \frac{2v_i^2 \sin\theta \cos\theta}{g_{earth}} = \frac{2(25 \text{ m/s})^2 \sin 30° \cos 30°}{9.80 \text{ m/s}^2} = 55.2 \text{ m}$$

$$(x_f)_{moon} = \frac{2v_i^2 \sin\theta \cos\theta}{g_{moon}} = \frac{2v_i^2 \sin\theta \cos\theta}{\frac{1}{6} g_{earth}} = 6 \times \frac{2v_i^2 \sin\theta \cos\theta}{g_{earth}} = 6(x_f)_{earth} = 331.2 \text{ m}$$

The flight times are

$$(t_f)_{earth} = \frac{2v_i \sin\theta}{g_{earth}} = 2.55 \text{ s}$$

$$(t_f)_{moon} = \frac{2v_i \sin\theta}{g_{moon}} = \frac{2v_i \sin\theta}{\frac{1}{6} g_{earth}} = 6\ (t_f)_{earth} = 15.30 \text{ s}$$

or 15 s to two significant figures.

(The ball spends $15.30 - 2.55 \text{ s} = 12.75 \text{ s} = 13 \text{ s}$ longer in flight on the moon.)

(b) From part (a), the distance traveled on the moon is 331 m or 330 m to two significant figures.

(c) From part (a), the golf ball travels $331.2 - 55.2 \text{ m} = 276 \text{ m}$ or 280 m to two significant figures, farther on the moon than on earth.

ASSESS: It is reasonable that a ball would travel farther on the moon than on earth, since the acceleration due to gravity is smaller on the moon.

P3.39 **PREPARE:** This problem involves constant acceleration (as in projectile motion). So, we can use the kinematic equations in the horizontal and vertical directions. We will write expressions for the initial components of velocity in the vertical and horizontal directions, respectively, then relate them to the magnitude of the initial velocity using the given angle.

Solve: First, we note that $(v_x)_\text{i} = v_\text{i}\cos(\theta)$ and $(v_y)_\text{i} = v_\text{i}\sin(\theta)$. Using Equation 3.22 for the *x*- and *y*-directions, respectively, yields

$$\Delta x = v_\text{i}\cos(\theta)\Delta t \Rightarrow \Delta t = \frac{\Delta x}{v_\text{i}\cos(\theta)}$$

$$\Delta y = v_\text{i}\sin(\theta)\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

Inserting the expression for time from the horizontal motion equation into the second equation related to vertical motion, we have

$$\Delta y = v_\text{i}\sin(\theta)\left(\frac{\Delta x}{v_\text{i}\cos(\theta)}\right) + \frac{1}{2}a_y\left(\frac{\Delta x}{v_\text{i}\cos(\theta)}\right)^2 \Rightarrow v_\text{i} = \sqrt{\frac{a_y(\Delta x)^2}{2\cos^2(\theta)(\Delta y - \tan(\theta)\Delta x)}}$$

$$v_\text{i} = \sqrt{\frac{(-9.8\,\text{m/s}^2)(20\,\text{m})^2}{2\cos^2(32°)((2.4\text{ m}) - \tan(32°)(20\,\text{m}))}} = 16\text{ m/s}$$

ASSESS: This is about 37 mph, which is a reasonable speed for a kick of a soccer ball.

P3.40 **PREPARE:** This problem involves motion in two dimensions. We can use kinematic equations because the accelerations in the x- and y-directions are constant (0 m/s^2 and -9.8 m/s^2, respectively). We will use the vertical direction to determine how long the droplet will be in the air, and then use the resulting time to determine the horizontal range.

To determine the initial speed of the launch, we will use Equation 3.24: $v_{s,f}^2 = v_{s,i}^2 + 2a_s\,\Delta s$. Once the water is in free fall, we will use Equation 3.19: $v_{f,s} = v_{i,s} + a_s\,\Delta t$, and 3.21: $\Delta s = v_{i,s}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$. Basic trigonometry tells us that $v_{i,x} = v_i\cos(\theta)$ and $v_{i,y} = v_i\sin(\theta)$.

SOLVE: We must first determine the launch speed of the water droplet:

$$v_{s,f}^2 = v_{s,f}^2 + 2a_s\Delta s = (0)^2 + 2(100)(9.8\text{ m/s}^2)(500\times10^{-6}\text{ m}) \Rightarrow v_{f,s} = 0.99\text{ m/s}$$

Second, we determine the time the droplet is in the air. Neglecting air resistance, the path is symmetric, such that the final downward speed of the droplet will equal its initial upward speed. Thus,

$$v_{f,y} = v_{i,y} + a_y\Delta t \Rightarrow -v_{i,y} = v_{i,y} + a_y\Delta t \Rightarrow \Delta t = \frac{-2v_i\sin(\theta)}{a_y} = \frac{-2(0.99\text{ m/s})\sin(60°)}{(-9.8\text{ m/s}^2)} = 0.175\text{ s}$$

Now, finally, we can use this time in expressions for the horizontal direction to find the range:

$$\Delta x = v_{i,x}\Delta t + \frac{1}{2}a_x(\Delta t)^2 = v_i\cos(\theta)\Delta t + 0 = (0.99\text{ m/s})\cos(60°)(0.175\text{ s}) = 0.087\text{ m}$$

So, the droplet travels a horizontal distance of 8.7 cm.

ASSESS: A few centimeters is a plausible distance for a bug to launch a water droplet. Despite the astronomical acceleration of $100g$, owing to the very short distance over which this acceleration was maintained, the launch speed ended up being a modest 0.99 m/s. This is also reasonable.

P3.41 **PREPARE:** Since we know the initial velocity, finding the final velocity is accomplished by finding the change in velocity. That is found by integrating the acceleration. We know that this integral is the same thing as the area under the acceleration-versus-time curve.

SOLVE: The formula for the particle's velocity is given by

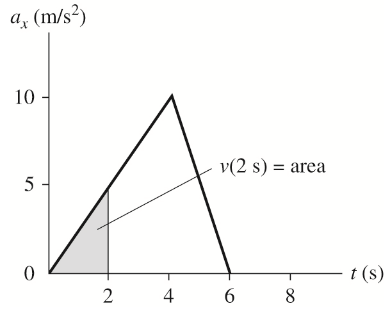
$v_f = v_i +$ area under the acceleration curve between t_i and t_f

For $t = 4$ s, we get

$$v_{4\text{ s}} = 8\text{ m/s} + \frac{1}{2}(4\text{ m/s}^2)\,4\text{ s} = 16\text{ m/s}$$

ASSESS: The acceleration is positive but decreases as a function of time. The initial velocity of 8.0 m/s will therefore increase. A value of 16 m/s is reasonable.

P3.42 **PREPARE:** Since we know the initial velocity, finding the final velocity is accomplished by finding the change in velocity. That is found by integrating the acceleration. We know that this integral is the same thing as the area under the acceleration-versus-time curve.



SOLVE: We will determine the object's velocity using graphical methods first and then using calculus. Graphically, $v(t) = v_0 +$ area under the acceleration curve from 0 to t. In this case, $v_0 = 0$ m/s. The area at the time requested is a triangle.

$$t = 6\text{ s} \quad v(t = 6\text{ s}) = \frac{1}{2}(6\text{ s})(10\text{ m/s}) = 30\text{ m/s}$$

Let us now use calculus. The acceleration function $a(t)$ consists of three pieces and can be written as follows:

$$a(t) = \begin{cases} 2.5t & 0\text{ s} \le t \le 4\text{ s} \\ -5t + 30 & 4\text{ s} \le t \le 6\text{ s} \\ 0 & 6\text{ s} \le t \le 8\text{ s} \end{cases}$$

These were determined by the slope and the y-intercept of each of the segments of the graph. The velocity function is found by integration as follows: For $0 \le t \le 4$ s,

$$v(t) = v(t = 0\ \text{s}) + \int_0^t a(t)dt = 0 + 2.5\frac{t^2}{2}\bigg|_0^t = 1.25t^2$$

This gives

$$t = 4\ \text{s} \quad v(t = 4\ \text{s}) = 20\ \text{m/s}$$

For $4\ \text{s} \le t \le 6$ s,

$$v(t) = v(t = 4\ \text{s}) + \int_4^t a(t)dt = 20\ \text{m/s} + \left[\frac{-5t^2}{2} + 30t\right]_4^t = -2.5t^2 + 30t - 60$$

This gives,

$$t = 6\ \text{s} \quad v(t = 6\ \text{s}) = 30\ \text{m/s}$$

ASSESS: The same velocities are found using calculus and graphs, but the graphical method is easier for simple graphs.

P3.43 **PREPARE:** Part (a) of this problem requires that we integrate the acceleration-versus-time plot to obtain the change in vertical velocity. This is accomplished by finding the area under the curve of the given plot. In part (b), we will use the launch speed found in part (a) and the known acceleration due to gravity to determine the maximum height the fish reaches. In this second part, we will use Equation 3.23: $v_{s,f}^2 = v_{s,i}^2 + 2a_s\Delta s$.

SOLVE: (a) The change in v_x is the area under the curve:

$$\Delta v_x = \frac{1}{2}\left(75\times10^{-3}\ \text{s}\right)\left(45\ \text{m/s}^2\right) = 1.69\ \text{m/s} \approx 1.7\ \text{m/s}.$$

Since the fish starts from rest, this is also the launch speed of the fish: 1.7 m/s.

(b) At its highest point, the archerfish will momentarily come to rest. Using the result of part (a), we have

$$v_{y,f}^2 = v_{y,i}^2 + 2a_y\Delta y \Rightarrow \Delta y = \frac{v_{y,f}^2 - v_{y,i}^2}{2a_y} = \frac{(0\ \text{m/s})^2 - (1.69\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} = 0.145\ \text{m} \approx 0.15\ \text{m}$$

ASSESS: 15 cm is a reasonable jump height for a fish.

P3.44 **PREPARE:** This problem will be solved using calculus. We are given the position as a function of time $x(t)$.

$v_x = \dfrac{dx(t)}{dt}$ and $a_x = \dfrac{dv_x(t)}{dt} = \dfrac{d^2x(t)}{dt^2}$. So we simply evaluate the expression in (a), and take one derivative in (b), and two derivatives in (c).

SOLVE: $x = (2t^3 + 2t + 1)$ m.

(a) The position $t = 2$ s is $x_{2\text{s}} = [2(2)^3 + 2(2) + 1]\ \text{m} = 21\ \text{m}$

(b) The velocity is the derivative $v = dx/dt$ and the velocity at $t = 2$ s is calculated as follows:

$$v = (6t^2 + 2)\ \text{m/s} \Rightarrow v_{2\text{s}} = [6(2^2) + 2]\ \text{m/s} = 26\ \text{m/s}$$

(c) The acceleration is the derivative $a = dv/dt$ and the acceleration at $t = 2$ s is calculated as follows:

$$a = (12t)\ \text{m/s}^2 \Rightarrow a_{2\,\text{s}} = 24\ \text{m/s}^2$$

ASSESS: Note that the position is always increasing. Therefore, we expect the velocity in that direction to always be positive, which we found to be true at a specific time.

P3.45 **PREPARE:** We will solve this problem using calculus. We are given the velocity as a function of time. We know to differentiate this to obtain the following acceleration: $a_x = \dfrac{dv_x(t)}{dt}$, and to integrate to obtain the displacement.

SOLVE: The formula for the particle's position along the *x*-axis is given by

$$x_\text{f} = x_\text{i} + \int_{t_\text{i}}^{t_\text{f}} v_x\,dt$$

Using the expression for v_x, we get

$$x_\text{f} = x_\text{i} + \tfrac{2}{3}[t_\text{f}^3 - t_\text{i}^3] \qquad a_x = \frac{dv_x}{dt} = \frac{d}{dt}(2t^2\ \text{m/s}) = 4t\ \text{m/s}^2$$

(a) The particle's position at $t = 1$ s is $x_{1\,\text{s}} = 1\ \text{m} + \tfrac{2}{3}\ \text{m} = \tfrac{5}{3}$ m.

(b) The particle's speed at $t = 1$ s is $v_{1\,\text{s}} = 2$ m/s.

(c) The particle's acceleration at $t = 1$ s is $a_{1\,\text{s}} = 4\ \text{m/s}^2$.

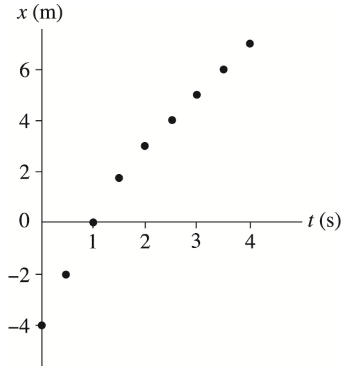
ASSESS: Since the initial position and v_x are always positive for $t > 0$, we expect positive answers for part (a) and (b), which we found. It happens that the acceleration is also always positive for $t > 0$.

P3.46 **PREPARE:** The timing between images in Figure P3.46 is constant. We expect to find a period of constant velocity, followed by a decrease in speed, followed by another period of constant (lower) velocity. We assume that the track, except for the sticky section, is frictionless and aligned along the *x*-axis. Because the motion diagram of Figure P3.46 is made at two frames of film per second, the time interval between consecutive ball positions is 0.5 s.

SOLVE: **(a)**

Times (s)	Position
0	–4.0
0.5	–2.0
1.0	0
1.5	1.8
2.0	3.0
2.5	4.0
3.0	5.0
3.5	6.0
4.0	7.0

(b)



(c) $\Delta x = x\,(\text{at } t = 1\text{ s}) - x\,(\text{at } t = 0\text{ s}) = 0\text{ m} - (-4\text{ m}) = 4\text{ m}$.

(d) $\Delta x = x\,(\text{at } t = 4\text{ s}) - x\,(\text{at } t = 2\text{ s}) = 7\text{ m} - 3\text{ m} = 4\text{ m}$.

(e) From $t = 0$ s to $t = 1$ s, $v_s = \Delta x/\Delta t = 4$ m/s.

(f) From $t = 2$ s to $t = 4$ s, $v_x = \Delta x/\Delta t = 2$ m/s.

(g) The average acceleration is

$$a = \frac{\Delta v}{\Delta t} = \frac{2\text{ m/s} - 4\text{ m/s}}{2\text{ s} - 1\text{ s}} = -2\text{ m/s}^2$$

ASSESS: The sticky section has decreased the ball's speed from 4 m/s to 2 m/s, which is a reasonable magnitude.

P3.47 **PREPARE:** This problem involves two phases, each of which is a constant velocity phase. But in between, the truck speeds up and has nonzero acceleration briefly. Thus, we can apply kinematic equations to either constant-velocity phase of the trip, but not to the trip as a whole.

SOLVE: Since the driver usually takes 8 hours to travel 440 miles, his usual velocity is

$$v_{\text{usual }x} = \frac{\Delta x}{\Delta t_{\text{usual}}} = \frac{440\text{ mi}}{8\text{ h}} = 55\text{ mph}$$

However, during this trip he was driving slower for the first 120 miles. Usually, he would be at the 120-mile point in

$$\Delta t_{\text{usual at 120 mi}} = \frac{\Delta x}{v_{\text{usual at 120 mi }x}} = \frac{120\text{ mi}}{55\text{ mph}} = 2.18\text{ h}$$

He is 15 min, or 0.25 h late. So the time he's taken to get 120 mi is $2.18\text{ h} + 0.25\text{ h} = 2.43\text{ h}$. He wants to complete the entire trip in the usual 8 h, so he only has 8 h – 2.43 h = 5.57 h left to complete 440 mi – 120 mi = 320 mi. So he needs to increase his velocity to

$$v_{\text{to catch up }x} = \frac{\Delta x}{\Delta t_{\text{ to catch up}}} = \frac{320\text{ mi}}{5.57\text{ h}} = 57\text{ mph}$$

where additional significant figures were kept in the intermediate calculations.

ASSESS: This result makes sense. He is only 15 min late.

P3.48 **PREPARE:** This problem involves motion at constant speeds. The position of either runner can be described by $\Delta x = v_x \Delta t$. We will apply this to each runner separately and use subscripts H and K for the respective runners. In order for Hanna to pass Kara, the distance covered by Hanna and Kara must satisfy $\Delta x_{\rm H} = \Delta x_{\rm K} + (400\ \text{m})$.

SOLVE: We have $\Delta x_{\rm H} = v_{{\rm H},x}\Delta t$ and $\Delta x_{\rm K} = v_{{\rm K},x}\Delta t$. Note that no subscript is required on the time. Requiring $\Delta x_{\rm H} = \Delta x_{\rm K} + (400\ \text{m})$, we find

$$v_{{\rm H},x}\Delta t = (400\ \text{m}) + v_{{\rm K},x}\Delta t \Rightarrow \Delta t = \frac{(400\ \text{m})}{(v_{{\rm H},x} - v_{{\rm K},x})}$$

The x-components of the women's velocities can be calculated from the given distances and times for the total run as follows:

$$v_{{\rm H},x} = \frac{(12.5)(400\ \text{m})}{(15.3\ \text{min})}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 5.45\ \text{m/s}$$

$$v_{{\rm K},x} = \frac{(12.5)(400\ \text{m})}{(17.5\ \text{min})}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 4.76\ \text{m/s}$$

Inserting these into the equation above, we find

$$\Delta t = \frac{(400\ \text{m})}{((5.45\ \text{m/s}) - (4.76\ \text{m/s}))} = 580\ \text{s}$$

Thus, when Hanna passes Kara, Hanna has traveled $\Delta x_{\rm H} = v_{{\rm H},x}\, t = (5.45\ \text{m/s})(580\ \text{s}) = 3.16\times10^3\ \text{m}$, or 7.9 laps.

ASSESS: One could check the math by finding how many laps Kara had completed after 580 s. One finds $\Delta x_{\rm H} = v_{{\rm H},x}\, t = (4.76\ \text{m/s})(580\ \text{s}) = 2.76\times10^3\ \text{m}$ or 6.9 laps. This confirms that Hanna passes Kara at that time.

P3.49 **PREPARE:** We will represent the jetliner's motion to be along the x-axis. We will assume constant acceleration, such that we can use the kinematic equations. Specifically, in the first part, we will use the definition of acceleration: $a_x = \Delta v/\Delta t$. In the second part, we can use Equation 3.23, and in the third part we will use Equation 3.21.

SOLVE:

(a) Using $a_x = \Delta v/\Delta t$, we have,

$$a_x(t=0 \text{ to } t=10\ \text{s}) = \frac{23\ \text{m/s} - 0\ \text{m/s}}{10\ \text{s} - 0\ \text{s}} = 2.3\ \text{m/s}^2 \quad a_x(t=20\ \text{s to } t=30\ \text{s}) = \frac{69\ \text{m/s} - 46\ \text{m/s}}{30\ \text{s} - 20\text{s}} = 2.3\ \text{m/s}^2$$

For all time intervals a_x is 2.3 m/s^2. In gs this is (2.3 m/s^2)/(9.8 m/s^2) = 0.23g

(b) Because the jetliner's acceleration is constant, we can use kinematics as follows:

$$(v_x)_{\rm f} = (v_x)_{\rm i} + a_x(t_{\rm f} - t_{\rm i}) \Rightarrow 80\ \text{m/s} = 0\ \text{m/s} + (2.3\ \text{m/s}^2)(t_{\rm f} - 0\ \text{s}) \Rightarrow t_{\rm f} = 34.8\ \text{s}$$

or 35 s to two significant figures.

(c) Using the above values, we calculate the takeoff distance as follows:

$$x_f = x_i + (v_x)_i(t_f - t_i) + \frac{1}{2}a_x(t_f - t_i)^2 = 0\text{ m} + (0\text{ m/s})(34.8\text{ s}) + \frac{1}{2}(2.3\text{ m/s}^2)(34.8\text{ s})^2 = 1390\text{ m}$$

For safety, the runway should be 3×1390 m = 4.2 km.

ASSESS: A few kilometers is a reasonable length for a runway.

P3.50 **PREPARE:** We know the distance that must be crossed by the vane, which can be obtained by integrating the velocity over time. This is equal to the area under the curve of the velocity-versus-time graph given.

SOLVE: $\Delta x = \frac{1}{2}v_{\text{max}}\Delta t \Rightarrow v_{\text{max}} = 2\frac{\Delta x}{\Delta t} = 2\frac{(0.0015\text{ m})}{(0.020\text{ s})} = 0.15\text{ m/s}$

ASSESS: For a small part of a plant, this is a pretty impressive speed.

P3.51 **PREPARE:** This is an estimation problem, so a range of answers may be acceptable. Note that the slope of the velocity-versus-time graph is changing, meaning the acceleration is not constant. We cannot use kinematic equations in solving this problem. Given the velocity-versus-time graph we need to estimate slopes to determine accelerations and then estimate the area under the curve to determine distance traveled.

SOLVE:

(a) At the origin a tangent line looks like it goes through (0 s, 0 m/s) and (2 s, 10 m/s), so the slope is

$$a(0\text{ s}) = \frac{10\text{ m/s}}{2.0\text{ s}} = 5\text{ m/s}^2$$

(b) Compute slopes similarly for $t = 2.0$ s and $t = 4.0$ s.

$$a(2.0\text{ s}) = \frac{8.0\text{ m/s}}{4.0\text{ s}} = 2\text{ m/s}^2 \qquad a(4.0\text{ s}) = \frac{5.0\text{ m/s}}{6.0\text{ s}} = 0.8\text{ m/s}^2$$

(c) We estimate the area under the curve. It looks like the area under the curve but above 10 m/s is a bit larger than the area above the curve but below 10 m/s. If they were equal the area would be $(8\text{ s})(10\text{ m/s}) = 80\text{ m}$, so we estimate a little more than 80 m.

ASSESS: It is very difficult to get a good estimate of slopes and areas from such small graphs, but the answers are reasonable. We do see the acceleration decreasing as we expected.

P3.52 **PREPARE:** This problem will be solved with calculus. We will take one derivative of $x(t)$ to obtain $v_x(t)$, and a second to obtain $a_x(t)$. Also, recall that if the velocity is changing in time and reaches a minimum, right at that minimum the slope will be zero. We will use this to find the minimum speed in part (a).

SOLVE: First take the derivatives. Every quantity is in SI units.

$$x = 2t^3 - 6t^2 + 12$$
$$v = 6t^2 - 12t$$
$$a = 12t - 12$$

(a) To find the minimum *v* we take the derivative of the velocity function and set it equal to zero (this is how we always minimize things). But we have already taken the derivative of *v* as follows:

$\frac{dv}{dt} = 12t - 12 = 0 \Rightarrow t = 1\text{ s}$. This is the first answer. Now we plug that time back into the equation for

v: $v_{\min} = 6(1)^2 - 12(1) = -6\text{ m/s}$.

(b) We have already done this when we minimized *v*. The acceleration is zero at $0 = 12t - 12 \Rightarrow t = 1\text{ s}$.

ASSESS: It is clear that at very large times, $v_x(t)$ will always be positive, because the quadratic term will grow most quickly. So it makes sense that if there is going to be a minimum in $v_x(t)$, it should happen at fairly small t.

P3.53 **PREPARE:** The position is the integral of the velocity, so we must first integrate the given expression, then match the result to the position given.

SOLVE: Integrating the function yields

$$x_1 = x_0 + \int_{t_0}^{t_1} v_x dt = x_0 + \int_0^{t_1} kt^2 dt = x_0 + \tfrac{1}{3}kt^3\Big|_0^{t_1} = x_0 + \tfrac{1}{3}kt_1^3$$

We're given that $x_0 = -9.0\text{ m}$ and that the particle is at $x_1 = 9.0\text{ m}$ at $t_1 = 3.0\text{ s}$. Thus,

$$9.0\text{ m} = (-9.0\text{ m}) + \tfrac{1}{3}k(3.0\text{ s})^3 = (-9.0\text{ m}) + k(9.0\text{ s}^3)$$

Solving for *k* gives $k = 2.0\text{ m/s}^3$.

ASSESS: Since the position along the x-direction is positive, it is reasonable that we obtained a positive number for k. This ensures that v_x is always positive.

P3.54 **PREPARE:** The velocity is the integral of acceleration, so we must integrate once to answer part (a). The position is the integral of the velocity, so we must first integrate the given expression a second time to answer part (b).

Solve: (a) Integrating the expression for acceleration once, we obtain an expression for velocity:

$$v_{1x} = v_{0x} + \int_{t_0}^{t_1} a_x dt = 0\text{ m/s} + \int_0^{t_1} (10 - t)dt = (10t - \tfrac{1}{2}t^2)\Big|_0^{t_1} = 10t_1 - \tfrac{1}{2}t_1^2$$

The velocity is zero when

$$v_{1x} = 0\text{ m/s} = \left(10t_1 - \tfrac{1}{2}t_1^2\right) = (10 - \tfrac{1}{2}t_1) \times t_1 \Rightarrow t_1 = 0\text{ s} \quad \text{or} \quad t_1 = 20\text{ s}$$

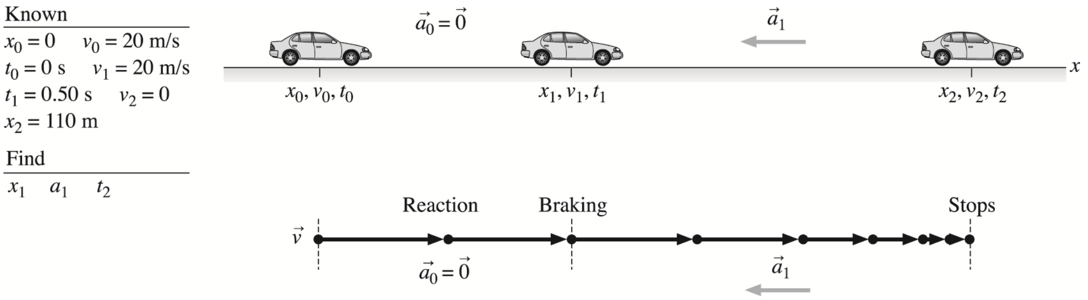
The first solution is the initial condition. Thus, the particle's velocity is again 0 m/s at $t_1 = 20\text{ s}$.

(b) Position is the integral of the velocity. At $t_1 = 20\text{ s}$, and using $x_0 = 0\text{ m}$ at $t_0 = 0\text{ s}$, the position is

$$x_1 = x_0 + \int_{t_0}^{t_1} v_x dt = 0\text{ m} + \int_0^{20} \left(10t - \tfrac{1}{2}t^2\right)\, dt = 5t^2\Big|_0^{20} - \tfrac{1}{6}t^3\Big|_0^{20} = 667\text{ m}$$

ASSESS: This large position is reasonable, since the velocity has been positive the entire time.

P3.55 **PREPARE:** Shown below is a visual overview of your car's motion that includes a pictorial representation, a motion diagram, and a list of values. We label the car's motion along the x-axis. This is a two-part problem. First, we will find the car's displacement during your reaction time when the car's deceleration is zero. This will give us the distance over which you must brake to bring the car to rest. Kinematic equations can then be used to find the required deceleration, since that deceleration is constant.



SOLVE: **(a)** During the reaction time,

$$x_1 = x_0 + v_0(t_1 - t_0) + 1/2\, a_0(t_1 - t_0)^2$$

$$= 0 \text{ m} + (20 \text{ m/s})(0.70 \text{ s} - 0 \text{ s}) + 0 \text{ m} = 14 \text{ m}$$

After reacting, $x_2 - x_1 = 110 \text{ m} - 14 \text{ m} = 96$ m, that is, you are 96 m away from the intersection.

(b) To stop successfully,

$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) \Rightarrow (0 \text{ m/s})^2 = (20 \text{ m/s})^2 + 2a_1(96 \text{ m}) \Rightarrow a_1 = -2.1 \text{ m/s}^2$$

The magnitude of the acceleration is therefore $|a_1| = 2.1 \text{ m/s}^2$.

(c) The time it takes to stop can be obtained as follows:

$$v_2 = v_1 + a_1(t_2 - t_1) \Rightarrow 0 \text{ m/s} = 20 \text{ m/s} + (-2.1 \text{ m/s}^2)(t_2 - 0.70 \text{ s}) \Rightarrow t_2 = 10 \text{ s}$$

ASSESS: 20 m/s is about 45 mph. Braking to a stop could reasonably take 10 s.

P3.56 **PREPARE:** Because the skier slows steadily, her acceleration is a constant during the glide and we can use the kinematic equations. We can use Equation 3.23 to determine the unknown acceleration.

SOLVE: Since we know the skier's initial and final speeds and the width of the patch over which she decelerates, we will use

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

$$\Rightarrow a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(6.0 \text{ m/s})^2 - (8.0 \text{ m/s})^2}{2(5.0 \text{ m})} = -2.8 \text{ m/s}^2$$

The magnitude of this acceleration is 2.8 m/s^2.

ASSESS: A deceleration of 2.8 m/s^2 or 6.3 mph/s is reasonable.

P3.57 **PREPARE:** We will assume constant acceleration of both planes, such that we can use the kinematic equations of motion. The kinematic equation that relates velocity, acceleration, and distance is $(v_x)_f^2 = (v_x)_i^2 + 2a_x\Delta x$. Solve for Δx.

$$\Delta x = \frac{(v_x)_f^2 - (v_x)_i^2}{2a_x}$$

Note that $(v_x)_i^2 = 0$ for both planes.

SOLVE: The accelerations are same, so they cancel.

$$\frac{\Delta x_{\text{jet}}}{\Delta x_{\text{prop}}} = \frac{\left(\frac{(v_x)_f^2}{2a_x}\right)_{\text{jet}}}{\left(\frac{(v_x)_f^2}{2a_x}\right)_{\text{prop}}} = \frac{\left((v_x)_f\right)_{\text{jet}}^2}{\left((v_x)_f\right)_{\text{prop}}^2} = \frac{\left((2v_x)_f\right)_{\text{prop}}^2}{\left((v_x)_f\right)_{\text{prop}}^2} = 4 \Rightarrow \Delta x_{\text{jet}} = 4\Delta x_{\text{prop}} = 4(1/4 \text{ mi}) = 1 \text{ mi}$$

ASSESS: It seems reasonable to need a mile for a passenger jet to take off.

P3.58 **PREPARE:** During the phase of constant acceleration, we can use kinematic equations, and during the subsequent phase of zero acceleration, we can again use kinematic equations. This must be done in two parts, since in between the acceleartion changes. First compute the distance traveled during the acceleration phase and what speed it reaches. Then compute the additional distance traveled at that constant speed. We can use Equations 3.21 and 3.19.

SOLVE: During the acceleration phase, since $(v_x)_i = 0$ and $x_i = 0$,

$$x_f = \frac{1}{2}a_x(\Delta t)^2 = \frac{1}{2}(250 \text{ m/s}^2)(20 \text{ ms})^2 = 0.05 \text{ m} = 5.0 \text{ cm}$$

We also compute the speed it attains.

$$v_x = a_x\Delta t = (250 \text{ m/s}^2)(20 \text{ ms}) = 5.0 \text{ m/s}$$

Now the distance traveled at a constant speed of 5.0 m/s.

$$\Delta x = v_x\Delta t = (5.0 \text{ m/s})(30 \text{ ms}) = 0.15 \text{ m} = 15 \text{ cm}$$

Now add the two distances to get the total.

$$\Delta x_{\text{total}} = 5.0 \text{ cm} + 15 \text{ cm} = 20 \text{ cm}$$

ASSESS: A 20-cm-long tongue is impressive, but possible.

P3.59 **PREPARE:** The total time will be the reaction time Δt_r, the time spent at constant acceleration Δt_{acc} and the time spent at a constant speed Δt_{const}. We will determine each of these separately. During any interval described, the acceleration is constant, so we can use kinematic equations during the period of constant acceleration or constant speed. We cannot apply kinematic equations from the start of the motion to the end, since there is a one discrete change in acceleration in the middle.

Let us call the moment the parent starts running "1," the moment the parent reaches peak speed "2," and the moment the parent reaches the child "3."

SOLVE: The time Δt_{acc} is given, but we need to know the distance covered in that time:

$$\Delta x = v_{1,x}\Delta t_{\text{acc}} + \frac{1}{2}a_x\left(\Delta t_{\text{acc}}\right)^2 = 0 + \frac{1}{2}\left(2.3 \text{ m/s}^2\right)\left(3.0 \text{ s}\right)^2 = 10.35 \text{ m}$$

This means there is a distance of 9.65 m to be covered at the constant speed. The maximum speed is given by

$$v_{2,x} = v_{1,x} + a_x\Delta t = 0 + \left(2.3 \text{ m/s}^2\right)\left(3.0 \text{s}\right) = 6.9 \text{ m/s}$$

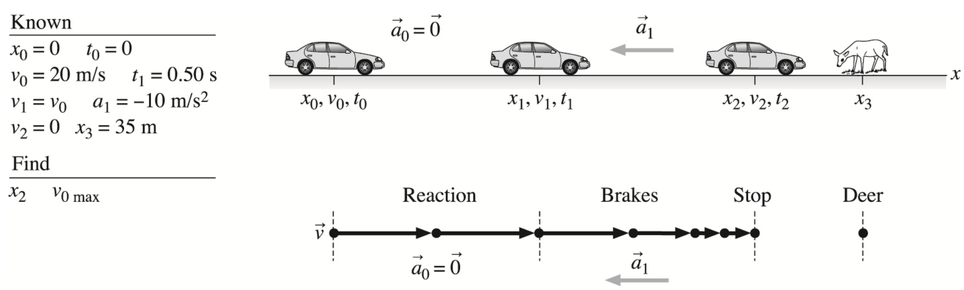
The time required for the portion at constant speed is then

$$\Delta t_{\text{const}} = \frac{\Delta x_{23}}{v_{2,x}} = \frac{9.65 \text{ m}}{6.9 \text{ m/s}} = 1.398 \text{ s}$$

The total time is now $\Delta t_{\text{total}} = \Delta t_{\text{r}} + \Delta t_{\text{acc}} + \Delta t_{\text{const}} = \left(0.25 \text{ s}\right) + \left(3.0 \text{ s}\right) + \left(1.398 \text{ s}\right) = 4.6 \text{ s}.$

ASSESS: 4.6 s is a reasonable time for a parent to run across a short distance in the woods. We see that most of the time was spent getting up to speed.

P3.60 **PREPARE:** We will assume that you achieve the maximum magnitude of acceleration possible, and that this acceleration is constant. A visual overview of your car's motion that includes a pictorial representation, a motion diagram, and a list of values is shown below. We label the car's motion along the x-axis. For maximum (constant) deceleration of your car, kinematic equations hold. This is a two-part problem. We will first find the car's displacement during your reaction time when the car's deceleration is zero. Then we will find the displacement as you bring the car to rest with maximum deceleration.



SOLVE: **(a)** To find x_2, we first need to determine x_1. Using $x_1 = x_0 + v_0(t_1 - t_0)$, we get $x_1 = 0$ m + (20 m/s) (0.50 s – 0 s) = 10 m. Now, with $a_1 = 10 \text{ m/s}^2$, $v_2 = 0$ and $v_1 = 20$ m/s, we can use

$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) \Rightarrow 0 \text{ m}^2/\text{s}^2 = (20 \text{ m/s})^2 + 2(-10 \text{ m/s}^2)(x_2 - 10 \text{ m}) \Rightarrow x_2 = 30 \text{ m}$$

The distance between you and the deer is $(x_3 - x_2)$ or (35 m – 30 m) = 5 m.

(b) Let us find $v_{0\text{ max}}$ such that $v_2 = 0$ m/s at $x_2 = x_3 = 35$ m. Using the following equation,

$$v_2^2 - v_{0\text{ max}}^2 = 2a_1(x_2 - x_1) \Rightarrow 0 \text{ m}^2/\text{s}^2 - v_{0\text{ max}}^2 = 2(-10 \text{ m/s}^2)(35 \text{ m} - x_1)$$

Also, $x_1 = x_0 + v_{0\text{ max}}(t_1 - t_0) = v_{0\text{ max}}(0.50 \text{ s} - 0 \text{ s}) = (0.50 \text{ s})v_{0\text{ max}}$. Substituting this expression for x_1 in the above equation yields

$$-v_{0\text{ max}}^2 = (-20 \text{ m/s}^2)[35 \text{ m} - (0.50 \text{ s})\, v_{0\text{ max}}] \Rightarrow v_{0\text{ max}}^2 + (10 \text{ m/s})v_{0\text{ max}} - 700 \text{ m}^2/\text{s}^2 = 0$$

The solution of this quadratic equation yields $v_{0\text{ max}} = 22$ m/s. (The other root is negative and unphysical for the present situation.)

ASSESS: An increase of speed from 20 m/s to 22 m/s is very reasonable for the car to cover an additional distance of 5 m with a reaction time of 0.50 s and a deceleration of 10 m/s^2.

P3.61 **PREPARE:** We will assume constant acceleration, such that we can use kinematic equations. Call the point where the motorcycle started the origin. We will convert to base SI units. The time required for the motorcycle to get up to speed can be found using Equation 3.19, and the distances covered can be found using Equation 3.21.

SOLVE:

(a)

$$a = \frac{\Delta v}{\Delta t} \Rightarrow \Delta t = \frac{\Delta v}{a} = \frac{80\text{ km/h}}{8.0\text{ m/s}^2}\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right) = 2.78\text{ s} \approx 2.8\text{ s}$$

(b) Compute the distance traveled in 10 s for each vehicle.

For the car: $\Delta x = v\Delta t = (80\text{ km/h})(2.78\text{ s})\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right) = 61.7\text{ m}$

For the motorcycle: $\Delta x = \frac{1}{2}a(\Delta t)^2 = \frac{1}{2}(8.0\text{ m/s}^2)(2.78\text{ s})^2 = 30.7\text{ m}$

The difference is the distance between the motorcycle and the car at that time, 61.7 m – 30.7 m = 31 m.

ASSESS: The motorcycle will never catch up if it never exceeds the speed of the car.

P3.62 **PREPARE:** In this problem, we will use the slope of a velocity-versus-time graph to determine acceleration and other kinematic variables. The acceleration is assumed to be approximately constant for this insect. We can answer part (a) by using Equation 3.19, which is equivalent to determining the slope of a velocity-versus-time graph in the case of a constant acceleration. We can then use Equation 3.21 to determine the distance covered by the insect.

SOLVE: (a) The acceleration is given by

$$a_y = \frac{(v_y)_\text{f} - (v_y)_\text{i}}{\Delta t} = \frac{(0.90\text{ m/s}) - (0\text{ m/s})}{(5.0\times10^{-3}\text{ s})} = 1.8\times10^2\text{ m/s}^2$$

(b) The distance can be determined by

$$\Delta y = (v_y)_\text{i}\Delta t + \frac{1}{2}a_y(\Delta t)^2 = (0\text{ m/s})(5\times10^{-3}\text{ m/s}) + \frac{1}{2}(180\text{ m/s}^2)(5\times10^{-3}\text{ m/s})^2 = 2.3\times10^{-3}\text{ m}$$

ASSESS: The distance covered is very reasonable for an insect in the process of jumping.

P3.63 **PREPARE:** This problem involves some estimation, so a range of answers may be correct. Because the problem assumes constant acceleration, we will make use of kinematic equations.

SOLVE: (a) Mine is approximately 1.5 cm. Anything around that is probably reasonable.

(b) Equation 3.21 can be used to determine the acceleration. The eyelid will start from rest.

$$\Delta y = v_{i,y}\Delta t + \frac{1}{2}a_y(\Delta t)^2 \rightarrow 0 + \frac{1}{2}a_y(\Delta t)^2 \Rightarrow a_y = \frac{2\Delta y}{(\Delta t)^2} = \frac{2(0.015\text{ m})}{(0.024\text{ s})^2} = 52\text{ m/s}^2$$

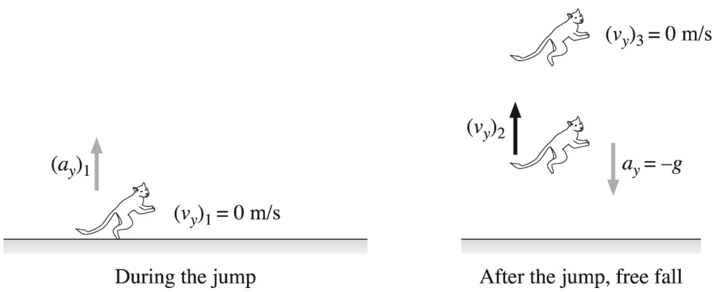
(c) Here, we will make sure of Equation 3.19.

$$v_{f,y} = v_{i,y} + a_y\Delta t = 0 + (52\text{ m/s}^2)(0.024\text{ s}) = 1.3\text{ m/s}$$

ASSESS: The acceleration is amazing; it is many times the acceleration due to gravity! Part (c) makes it clear that such a large acceleration is reasonable over a very short time, such that the speeds involved are of order 1 m/s.

P3.64 **PREPARE:** There are two separate segments of this motion, the jump and the free fall after the jump. Let us assume the acceleration is constant during the jump, such that we can use the kinematic equations. Acceleration is also constant during the free fall, although clearly the acceleration changes between these two segments. We will make use of Equation 3.24.

SOLVE: See the following figure. Before the jump, the velocity of the bush baby is 0 m/s.



We could solve for the acceleration of the bush baby during the jump using Equation 3.23 if we knew the final velocity the bush baby reached at the end of the jump, $(v_y)_2$.

We can find this final velocity from the second part of the motion. During this part of the motion, the bush baby travels with the acceleration of gravity. The initial velocity it has obtained from the jump is $(v_y)_2$.

When it reaches its maximum height, its velocity is $(v_y)_3 = 0$ m/s. It travels 2.3 m during the upward free-fall portion of its motion. The initial velocity it had at the beginning of the free-fall motion can be calculated from

$$(v_y)_2 = \sqrt{-2(a_y)_2 \Delta y_2} = \sqrt{-2(-9.80 \text{ m/s}^2)(2.3 \text{ m})} = 6.714 \text{ m/s}$$

This is the bush baby's final velocity at the end of the jump, just as it leaves the ground, legs straightened. Using this velocity and Equation 3.23, we can calculate the acceleration of the bush baby during the jump.

$$(a_y)_1 = \frac{(v_y)_2^2 - (v_y)_1^2}{2\Delta y_1} = \frac{(6.714 \text{ m/s})^2 - (0 \text{ m/s})_1^2}{2(0.16\text{m})} = 140 \text{ m/s}^2$$

In g's, the acceleration is $\frac{140 \text{ m/s}^2}{9.80 \text{ m/s}^2} = 14g$'s.

ASSESS: This is a very large acceleration, which is not unexpected considering the height of the jump. Note the acceleration during the jump is positive, as expected.

P3.65 **PREPARE:** If we assume the acceleration is constant as the beetle speeds up, and is then (a different) constant after the beetle is in the air, then we can use kinematic equations during each of those two phases, separately. For part (a), we will use Equation 3.23 with $(v_y)_i = 0$ m/s. For part (b), we will use Equation 3.19. Finally, during the free fall, we will again use Equation 3.24, this time with $(v_y)_f = 0$ m/s.

SOLVE: **(a)** It leaves the ground with the final speed of the jumping phase.

$$(v_y)_f^2 = 2a_y \Delta y = 2(400)(9.8 \text{ m/s}^2)(0.0060 \text{ m}) \quad \Rightarrow \quad (v_y)_f = 6.86 \text{ m/s}$$

or 6.9 m/s to two significant figures.

(b)

$$\Delta t = \frac{\Delta v_y}{a_y} = \frac{6.86 \text{ m/s}}{(400)(9.8 \text{ m/s}^2)} = 1.7496 \text{ ms} \approx 1.7 \text{ ms}$$

(c) Now the initial speed for the free-fall phase is the final speed of the jumping phase and $(v_y)_\text{f} = 0$.

$$(v_y)_\text{i}^2 = -2a_y\Delta y \quad \Rightarrow \quad \Delta y = \frac{(v_y)_\text{i}^2}{-2a_y} = \frac{(6.86 \text{ m/s})^2}{-2(-9.8 \text{ m/s}^2)} = 2.4 \text{ m}$$

ASSESS: This is an amazing height for a beetle to jump, but given the large acceleration, this sounds right.

P3.66 **PREPARE:** This problem involves constant acceleration, such that we can use the kinematic equation. In particular, equation 3.24 will be useful in part (a), and also in part (b).

SOLVE: (a) $v_{f,x}^2 = v_{i,x}^2 + 2a_x\Delta x \Rightarrow a_x = \dfrac{v_{f,x}^2 - v_{i,x}^2}{2\Delta x} = \dfrac{(2.7 \text{ m/s})^2 - (0)^2}{2(2.0 \text{ m})} = 1.82 \text{ m/s}^2 \approx 1.8 \text{ m/s}^2$

(b) Using the acceleration from part (a), we have $v_{f,x}^2 = v_{i,x}^2 + 2a_x\Delta x = 0 + 2(1.82 \text{ m/s}^2)(1.0 \text{ m}) = 1.9 \text{ m/s}$.

ASSESS: It makes sense that our answer to (b) is between the initial 0 m/s and the final 2.7 m/s, since the acceleration is constant.

P3.67 **PREPARE:** This problem involves two intervals, and over each interval acceleration is constant. It is not constant between the two intervals, though. So, we can use kinematic equations, but only for the intervals over which acceleration is constant. We will separately determine the distance covered between the start and the time when the maximum speed is reached (call these times "1" and "2"), and the distance covered during the interval of constant speed. Call the end of the 10 s time "3".

SOLVE: First, let us determine the distance covered during the acceleration period and the time required:

$$v_{2,x}^2 = v_{1,x}^2 + 2a_x\Delta x_{12} \Rightarrow \Delta x_{12} = \frac{v_{2,x}^2 - v_{1,x}^2}{2a_x} = \frac{(11 \text{ m/s})^2 - (0 \text{ m/s})^2}{2(4.0 \text{ m/s}^2)} = 15.125 \text{ m}$$

$$v_{2,x} = v_{1,x} + a_x\Delta t_{12} \Rightarrow \Delta t_{12} = \frac{v_{2,x} - v_{1,x}}{a_x} = \frac{(11 \text{ m/s}) - 0}{(4.0 \text{ m/s}^2)} = 2.75 \text{ s}$$

We now know that the total 10 s interval will be made of 2.75 s of acceleration and 7.25 s of motion at constant speed. For this motion at constant speed, we see

$$\Delta x_{23} = v_{2,x}\Delta t_{23} + \frac{1}{2}a_x(\Delta t_{23})^2 = (11 \text{ m/s})(7.25 \text{ s}) + 0 = 79.75 \text{ m}$$

Adding the distances crossed during the two intervals, we have

$\Delta x_\text{total} = \Delta x_{12} + \Delta x_{23} = (15.125 \text{ m}) + (79.75 \text{ m}) = 95 \text{ m}$.

ASSESS: If the acceleration were instant, and the zebra were traveling at 11 m/s the entire time, then the distance covered would be (11 m/s)(10 s) = 110 m. Since the acceleration does take some time, it is reasonable that our answer was slightly less than this.

P3.68 **PREPARE:** This problem involves two intervals, and over each interval acceleration is constant. It is not constant between the two intervals, though. So, we can use kinematic equations, but only for the intervals over which acceleration is constant. We will separately determine the distance covered between the start and the time when the maximum speed is reached (call these times "1" and "2"), and the distance covered during the interval of constant speed. Call the end of the 10 s time "3".

SOLVE: First, let us determine the distance covered during the acceleration period and the time required:

$$v_{2,x}^2 = v_{1,x}^2 + 2a_x\Delta x \Rightarrow \Delta x = \frac{v_{2,x}^2 - v_{1,x}^2}{2a_x} = \frac{(14\text{ m/s})^2 - (0\text{ m/s})^2}{2(5.5\text{ m/s}^2)} = 17.82\text{ m}$$

$$v_{2,x} = v_{1,x} + a_x\Delta t_{12} \Rightarrow \Delta t_{12} = \frac{v_{2,x} - v_{1,x}}{a_x} = \frac{(14\text{ m/s}) - 0}{(5.5\text{ m/s}^2)} = 2.545\text{ s}$$

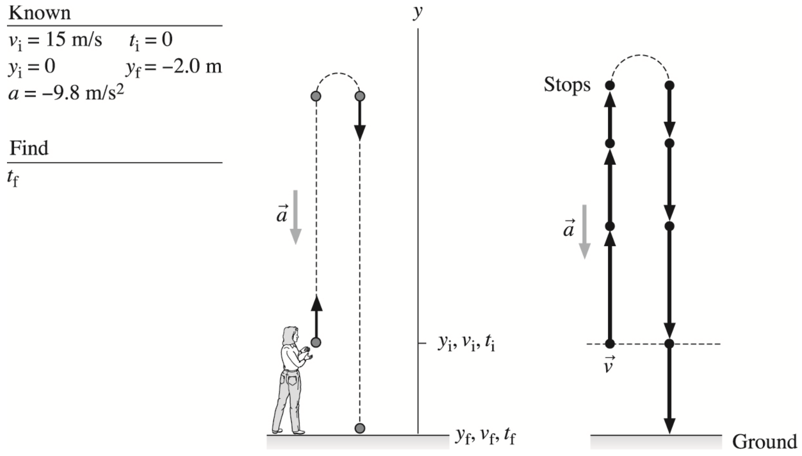
We now know that the total 100 m distance will be partly made up of the 17.8 m during constant acceleration, and the remaining 82.2 m will be covered during a period of motion at constant speed. For this motion at constant speed, we see

$$\Delta x_{23} = v_{2,x}\Delta t_{23} + \frac{1}{2}a_x(\Delta t_{23})^2 \rightarrow \Delta x_{23} = v_{2,x}\Delta t_{23} \Rightarrow \Delta t_{23} = \frac{\Delta x_{23}}{v_{2,x}} = \frac{82.2\text{ m}}{(14\text{ m/s})} = 5.87\text{ s}$$

Adding the times for the two intervals, we have $\Delta t_{\text{total}} = \Delta t_{12} + \Delta t_{23} = (2.545\text{ s}) + (5.870\text{ s}) = 8.4\text{ s}$.

ASSESS: If the acceleration were instant, and the lion were traveling at 14 m/s the entire time, then the time required would be (100 m)/(14 m/s) = 7.1 s. Since the acceleration does take some time, it is reasonable that our answer was slightly more than this.

P3.69 **PREPARE:** A visual overview of the ball's motion that includes a pictorial representation, a motion diagram, and a list of values is shown below. We label the ball's motion along the y-axis. Once the ball leaves the student's hands, its acceleration is equal to the acceleration due to gravity that always acts vertically downward toward the center of the earth. Because it is constant, we can use the kinematic equations. The initial position of the ball is at the origin where $y_i = 0$, but the final position is below the origin at $y_f = -2.0$ m. Recall sign conventions, which tell us that v_i is positive and a is negative.



SOLVE: With all the known information, it is clear that we must use

$$y_f = y_i + v_i \Delta t + \frac{1}{2} a \Delta t^2$$

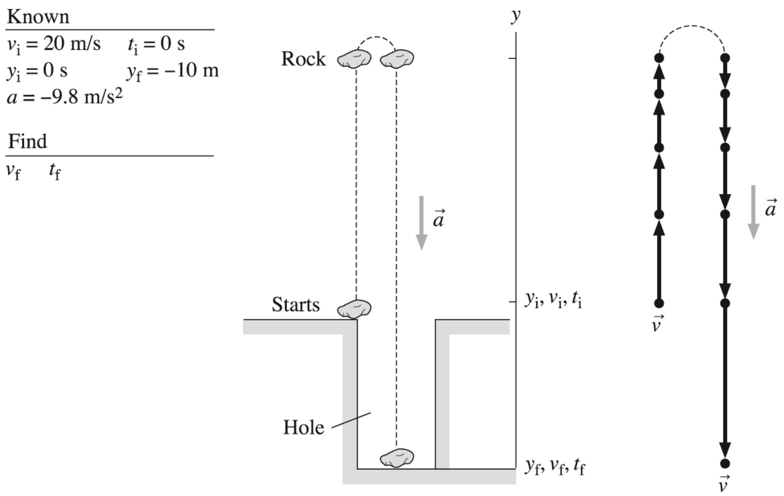
Substituting the known values

$$-2 \text{ m} = 0 \text{ m} + (15 \text{ m/s})t_f + (1/2)(-9.8 \text{ m/s}^2)t_f^2$$

The solution of this quadratic equation gives $t_f = 3.2$ s. The other root of this equation yields a negative value for t_f, which is not physical for this problem.

ASSESS: A time of 3.2 s is reasonable for a ball thrown up in the air.

P3.70 **PREPARE:** A visual overview of the rock's motion that includes a pictorial representation, a motion diagram, and a list of values is shown below. We represent the rock's motion along the y-axis. Once the rock leaves the person's hands, its acceleration is equal to the acceleration due to gravity that always acts vertically downward toward the center of the earth. Because this acceleration is constant, we can use the kinematic equations. The initial position of the rock is at the origin where $y_i = 0$, but the final position is below the origin at $y_f = -10$ m. Recall sign conventions which tell us that v_i is positive and a is negative.



SOLVE: **(a)** Substituting the known values into $y_f = y_i + v_i \Delta t + \frac{1}{2} a \Delta t^2$, we get

$$-10 \text{ m} = 0 \text{ m} + 20 \text{ (m/s)}t_f + \frac{1}{2}(-9.8 \text{ m/s}^2)t_f^2$$

One of the roots of this equation is negative and is not physically relevant. The other root is $t_f = 4.53$ s which is the answer to part (b). Using $v_f = v_i + a\Delta t$, we obtain

$$v_f = 20\text{(m/s)} + (-9.8 \text{ m/s}^2)(4.53 \text{ s}) = -24 \text{ m/s}$$

(b) The time is 4.5 s.

ASSESS: A time of 4.5 s is a reasonable value. The rock's velocity as it hits the bottom of the hole has a negative sign because of its downward direction. The magnitude of 24 m/s compared to 20 m/s when the rock was tossed up is consistent with the fact that the rock travels an additional distance of 10 m into the hole.

P3.71 **PREPARE:** Since constant acceleration is involved, we can use kinematic equations. We are given initial and final speeds, and a displacement, and we are asked for acceleration. Equation 3.23 can be used to answer this problem. All values must first be converted to SI units. Let us call the initial direction of motion the $+x$ direction.

SOLVE: We start by expressing all given quantities in SI units:

$$(75 \text{ mph})\left(\frac{1610 \text{ m}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 33.5 \text{ m/s}$$

$$(55 \text{ mph})\left(\frac{1610 \text{ m}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 24.6 \text{ m/s}$$

$$(0.5 \text{ mi})\left(\frac{1610 \text{ m}}{1 \text{ mi}}\right) = 805 \text{ m}$$

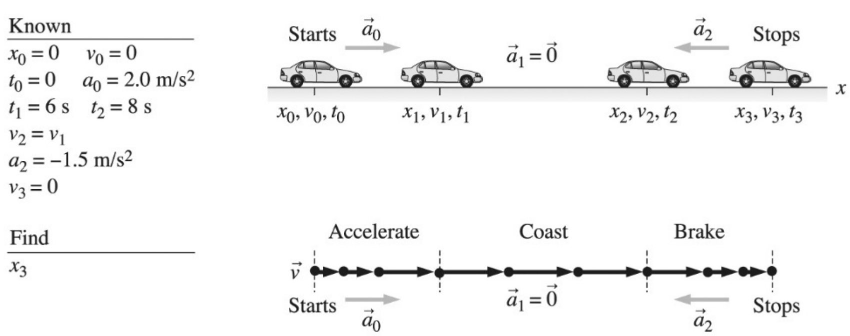
Employing Equation 2.13, we find

$$(v_x)_f^2 = (v_x)_i^2 + 2a_x\Delta x \Rightarrow a_x = \frac{(v_x)_f^2 - (v_x)_i^2}{2\Delta x} = \frac{(24.6 \text{ m/s})^2 - (33.5 \text{ m/s})^2}{2(805 \text{ m})} = -0.32 \text{ m/s}^2$$

So the magnitude of the acceleration is 0.32 m/s^2.

ASSESS: Since the change in speed can occur over such a long distance, we expect a relatively small answer, and since the vehicle is slowing in the x-direction, we expected the answer to be negative as well. This answer is very reasonable.

P3.72 **PREPARE:** A visual overview of car's motion that includes a pictorial representation, a motion diagram, and a list of values is shown below. We label the car's motion along the x-axis. This is a three-part problem. First the car accelerates, then it moves with a constant speed, and then it decelerates. We assume that during each part the acceleration is constant, such that we can use the kinematic equations. The total displacement between the stop signs is equal to the sum of the three displacements, that is, $x_3 - x_0 = (x_3 - x_2) + (x_2 - x_1) + (x_1 - x_0)$.



SOLVE: First, the car accelerates:

$$v_1 = v_0 + a_0(t_1 - t_0) = 0 \text{ m/s} + (2.0 \text{ m/s}^2)(6 \text{ s} - 0 \text{ s}) = 12 \text{ m/s}$$

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = 0 \text{ m} + \frac{1}{2}(2.0 \text{ m/s}^2)(6 \text{ s} - 0 \text{ s})^2 = 36 \text{ m}$$

Second, the car moves at v_1:

$$x_2 - x_1 = v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2 = (12\text{ m/s})(8\text{ s} - 6\text{ s}) + 0\text{ m} = 24\text{ m}$$

Third, the car decelerates:

$$v_3 = v_2 + a_2(t_3 - t_2) \Rightarrow 0\text{ m/s} = 12\text{ m/s} + (-1.5\text{ m/s}^2)(t_3 - t_2) \Rightarrow (t_3 - t_2) = 8\text{ s}$$

$$x_3 = x_2 + v_2(t_3 - t_2) + \frac{1}{2}a_2(t_3 - t_2)^2 \Rightarrow x_3 - x_2 = (12\text{ m/s})(8\text{ s}) + \frac{1}{2}(-1.5\text{ m/s}^2)(8\text{ s})^2 = 48\text{ m}$$

Thus, the total distance between stop signs is

$$x_3 - x_0 = (x_3 - x_2) + (x_2 - x_1) + (x_1 - x_0) = 48\text{ m} + 24\text{ m} + 36\text{ m} = 108\text{ m}$$

or 110 m to two significant figures.

ASSESS: A distance of approximately 360 ft in a time of around 16 s with an acceleration/deceleration is reasonable.

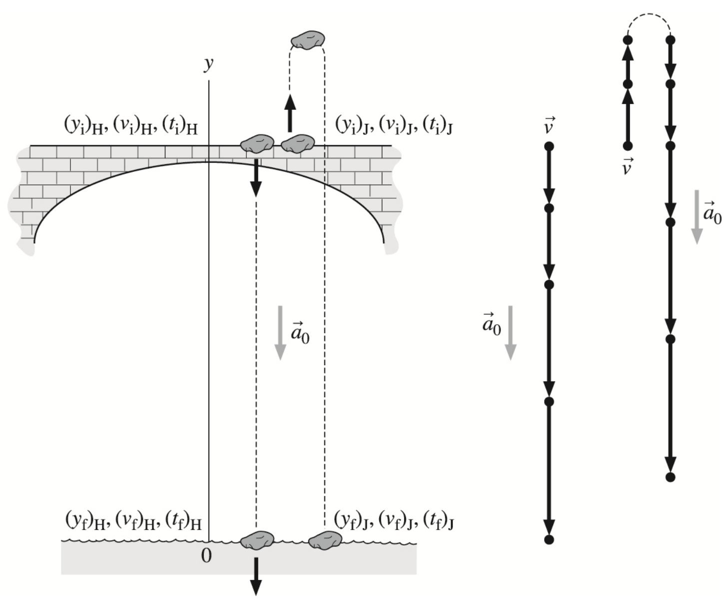
P3.73 **PREPARE:** A visual overview of the motion of the two rocks, one thrown down by Heather and the other thrown up at the same time by Jerry, which includes a pictorial representation, a motion diagram, and a list of values is shown below. We represent the motion of the rocks along the y-axis with origin at the surface of the water. The initial position for both cases is y_i = 50 m and similarly the final position for both cases is at $y_f = 0$. Recall sign conventions, which tell us that $(v_i)_J$ is positive and $(v_i)_H$ is negative. As soon as the rocks are thrown, they fall freely and thus kinematics equations are applicable.

Known

$(y_i)_H = 50$ m $(v_i)_H = -20$ m/s
$(t_i)_H = 0$ $a_0 = -9.8$ m/s^2
$(y_f)_H = 0$ $(y_i)_J = 50$ m
$(v_i)_J = +20$ m/s $(t_i)_J = 0$ s
$a_0 = -9.8$ m/s^2
$(y_f)_J = 0$

Find

$(v_f)_J$ $(v_f)_H$ and $|(t_f)_J - (t_f)_H|$



SOLVE: **(a)** For Heather,

$$(y_f)_H = (y_i)_H + (v_i)_H[(t_f)_H - (t_i)_H] + \frac{1}{2}a_0[(t_f)_H - (t_i)_H)]^2$$

$$\Rightarrow 0\text{ m} = (50\text{ m}) + (-20\text{ m/s})[(t_f)_H - 0\text{ s}] + \frac{1}{2}(-9.8\text{ m/s}^2)[(t_f)_H - 0\text{ s}]^2$$

$$\Rightarrow 4.9\text{ m/s}^2\ (t_f)_H^2\ + 20\text{ m/s}\ (t_f)_H - 50\text{ m} = 0$$

The two mathematical solutions of this equation are −5.83 s and +1.75 s. The first value is not physically acceptable since it represents a rock hitting the water before it was thrown, therefore, $(t_f)_H$ = 1.75 s.

For Jerry,

$$(y_f)_J = (y_i)_J + (v_i)_J[(t_f)_J - (t_i)_J] + \frac{1}{2}a_0[(t_f)_J - (t_i)_J)]^2$$

$$\Rightarrow 0 \text{ m} = (50 \text{ m}) + (+20 \text{ m/s})[(t_f)_J - 0 \text{ s}] + \frac{1}{2}(-9.8 \text{ m/s}^2)[(t_f)_J - 0 \text{ s}]^2$$

Solving this quadratic equation will yield $(t_f)_J$ = −1.75 s and +5.83 s. Again, only the positive root is physically meaningful. The elapsed time between the two splashes is $(t_f)_J - (t_f)_H$ = 5.83 s – 1.75 s = 4.1 s.

(b) Knowing the times, it is easy to find the impact velocities:

$$(v_f)_H = (v_i)_H + a_0[(t_f)_H - (t_i)_H] = (-20 \text{ m/s}) + (-9.8 \text{ m/s})(1.75 \text{ s} - 0 \text{ s}) = -37 \text{ m/s}$$
$$(v_f)_J = (v_i)_J + a_0[(t_f)_J - (t_i)_J] = (+20 \text{ m/s}) + (-9.8 \text{ m/s}^2)(5.83 \text{ s} - 0 \text{ s}) = -37 \text{ m/s}$$

The two rocks hit the water with equal speeds.

ASSESS: The two rocks hit the water with equal speeds because Jerry's rock has the same downward speed as Heather's rock when it reaches Heather's starting position during its downward motion.

P3.74 **PREPARE:** This problem can be broken up into two parts for either creature: a period of acceleration, which we will assume to be constant, and a period of constant speed (zero acceleration). During each segment separately, we can use kinematic equations. We can begin in the phase of constant acceleration with Equation 3.19, with $(v_x)_i = 0$ m/s.

SOLVE:

(a) The gazelle gains speed at a steady rate for the first 6.5 s.

$$(v_x)_f = (v_x)_f + a_x\Delta t = 0 \text{ m/s } + (4.2 \text{ m/s}^2)(6.5 \text{ s}) = 27.3 \text{ m/s} \approx 27 \text{ m/s}$$

(b) Use a different kinematic equation to find the time during the acceleration phase.

$$\Delta t = \sqrt{\frac{2\Delta x}{a_x}} = \sqrt{\frac{2(30 \text{ m})}{4.2 \text{ m/s}^2}} = 3.8 \text{ s}$$

So, indeed, the fast human wins by 0.2 s.

(c) We'll do this in two parts. First, we'll find out how far the gazelle goes during the 6.5 s acceleration phase.

$$\Delta x = \frac{1}{2}a_x(\Delta t)^2 = \frac{1}{2}\left(4.2 \text{ m/s}^2\right)\left(6.5 \text{ s}\right)^2 = 88.725 \text{ m}$$

We subtract this distance from the 200 m total to find out how long it takes the gazelle to do the constant speed phase at 27.3 m/s. 200 m – 88.725 m = 111.275 m.

$$\Delta t = \frac{\Delta x}{v_x} = \frac{111.275 \text{ m}}{27.3 \text{ m/s}} = 4.1 \text{ s}$$

The total time for the gazelle is then 6.5 s + 4.1 s = 10.6 s, which is much less than the human.

ASSESS: We might be surprised that humans can beat gazelles in short races, but we are not surprised that the gazelle wins the 200 m race. The numbers are in the right ballpark.

P3.75 **PREPARE:** Assume the vaulter is in free fall before he hits the pad, during which acceleration is constant. Then we can use the kinematic equations to describe the fall. We will also assume that the acceleration is constant (but different) during the compression of the mat. He falls a distance of 4.2 m – 0.8 m = 3.4 m before hitting the pad.

SOLVE: We will find the impact speed assuming $(v_x)_i = 0$ m/s

$$(v_y)_f^2 = (v_y)_i^2 + 2a_y\Delta y \Rightarrow (v_y)_f = \sqrt{2(9.8 \text{ m/s}^2)(3.4 \text{ m})} = 8.16 \text{ m/s}$$

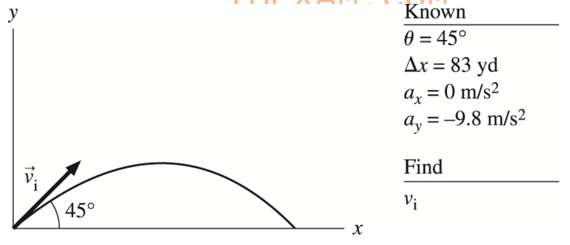
We use the same equation for the pad-compression phase, but now the 8.16 m/s is the initial speed and the final speed is zero. Solve for a_x.

$$(v_y)_f^2 = (v_y)_i^2 + 2a_y\Delta y \Rightarrow a_y = \frac{-(v_y)_i^2}{2\Delta y} = \frac{-(8.16 \text{ m/s})^2}{2(-0.50 \text{ m})} = 67 \text{ m/s}^2$$

ASSESS: This is a large acceleration, but it is not dangerous for such short periods of time. It took a lot longer for the vaulter to gain 8.16 m/s of speed at an acceleration of g than it did to lose 8.16 m/s of speed at a much larger acceleration.

P3.76 **PREPARE:** This problem involves the projectile motion of a football. We convert the 83 yd to meters, and then use Equation 3.28 for the range at a launch angle of $45°$.

$$83 \text{ yards} = 83 \text{ yd}\left(\frac{36 \text{ in}}{1 \text{ yd}}\right)\left(\frac{1 \text{ m}}{39.37 \text{ in}}\right) = 75.9 \text{ m}$$



SOLVE: The record range is 83 yd or 75.9 m. We use an extra significant figure since this datum will be used to get another result. Based on the formula for range, we can say, $v = \sqrt{gR/\sin(2\theta)}$. Using $45°$ for the angle gives the following:

$$v = \sqrt{(9.8 \text{ m/s}^2)(75.9 \text{ m})} = 27 \text{ m/s}$$

The launch speed of this record pass was 27 m/s.

ASSESS: The launch speed is about 60 mi/h This is impressive for a football. We would expect the launch speed to be less than that for a baseball because of a football's greater mass.

P3.77 **PREPARE:** Assume that there is no air resistance $(a_y = -g)$ and that the marble leaves the gun with the same speed (muzzle speed) each time it is fired. We can determine the muzzle speed by applying Equation 3.23 to the vertical case. We can use Equation 3.21 to determine the time to fall 1.5 m in the case of the horizontal launch.

SOLVE: Equation 3.23 tells us

$$(v_y)_f^2 = (v_y)_i^2 + 2a_y\,\Delta y$$

where at the top of the trajectory $(v_y)_f = 0.0$ m/s and $\Delta y = 6.0$ m.

$$(v_y)_i^2 = 2g\Delta y \Rightarrow (v_y)_i = \sqrt{2g\Delta y} = 10.8 \text{ m/s}$$

We also rearrange Equation 3.21 to find the time for an object to fall 1.5 m from rest: $y_f - y_i = -15$ m now instead of the 6.0 m used previously.

$$\Delta y = \frac{1}{2}a_y(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{-g}} = \sqrt{\frac{2(-1.5 \text{ m})}{-9.8 \text{ m/s}^2}} = 0.553 \text{ s}$$

Finally, we combine this information into the equation for constant horizontal velocity.

$$\Delta x = v_x\Delta t = (10.8 \text{ m/s})(0.553 \text{ s}) = 6.0 \text{ m}$$

ASSESS: Is it a coincidence that the marble has a horizontal range of 6.0 m when it can reach a height of 6.0 m when fired straight up, or will those numbers always be the same? Well, the 6.0 m horizontal range depends on the height (1.5 m) from which you fire it, so if that were different the range would be different. This leads us to conclude that it *is* a coincidence. You can go back, though, and do the problem algebraically (with no numbers) and find that g cancels and that the horizontal range is 2 times the square root of the product of the vertical height it can reach and the height from which you fire it horizontally.

P3.78 **PREPARE:** Assume that there is no air resistance and that the plane's velocity is constant. We will use Equation 3.21 once in the vertical direction to determine the time required for the weight to fall, and then again in the horizontal direction to determine the horizontal distance from the target at which the weight should be released.

SOLVE: **(a)** We also use the kinematic equations to find the time for an object to fall 60 m from rest.

$$(y_f - y_i) = \frac{1}{2}a_y\,(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{2(y_f - y_i)}{-g}} = \sqrt{\frac{2(-60 \text{ m})}{-9.8 \text{ m/s}^2}} = 3.5 \text{ s}$$

Now, we combine this information into the equation for constant horizontal velocity.

$$\Delta x = v_x\Delta t = (45 \text{ m/s})(3.5 \text{ s}) = 160 \text{ m}$$

(b) Because they have the same horizontal speed, the plane will be directly over the target when the weight hits the ground.

ASSESS: In real-life, air resistance would alter these results.

P3.79 **PREPARE:** Example 3.15 works out an expression for the horizontal range of a projectile in terms of the launch angle:

$$\Delta x = \frac{2v_i^2\sin(\theta)\cos(\theta)}{g} = \frac{v_i^2\sin(2\theta)}{g}$$

This was obtained by finding the time of flight by looking at the vertical component of motion, and using that time in equations describing the horizontal motion. We can use the result here, with the modification that the initial throw speed is now also a function of θ.

SOLVE: (a) Inserting $v_i = v_{\max}\sqrt{1-\frac{\theta}{95°}}$ into the result of Example 3.15, we have

$$\Delta x = \frac{v_i^2 \sin(2\theta)}{g} = v_{\max}^2\left(1-\frac{\theta}{95°}\right)\frac{\sin(2\theta)}{g}$$

This range will have different values for different input angles. When the maximum range is reached, the slope $d\Delta x/d\theta$ will momentarily be zero. So, maximizing this range corresponds to finding for what value of θ is $d\Delta x/d\theta = 0$:

$$\frac{d\Delta x}{d\theta} = \frac{v_{\max}^2}{g}\left[\left(1-\frac{\theta}{95°}\right)2\cos(2\theta)+\left(-\frac{\theta}{95°}\right)\sin(2\theta)\right]=0$$
$$\Rightarrow \left(1-\frac{\theta}{95°}\right)2\cos(2\theta)=\left(\frac{\theta}{95°}\right)\sin(2\theta)$$
$$\Rightarrow \tan(\theta) = 2\left(\frac{95°}{\theta}-1\right)$$

This is a transcendental expression: one that cannot be solved by algebraic methods. Numerically (such as by plotting the right-hand side and left-hand side on a graphing calculator and looking for the intersection), we can find that this is satisfied for $\theta = 32.7°$.

(b) Inserting this angle back into the original expression we had for the range, we find

$$\Delta x = v_{\max}^2\left(1-\frac{\theta}{95°}\right)\frac{\sin(2\theta)}{g} = (40 \text{ m/s})^2\left(1-\frac{32.6°}{95°}\right)\frac{\sin(2(32.6°))}{g} = 97 \text{ m}$$

(c) We know $v_i = v_{\max}\sqrt{1-\frac{\theta}{95°}}$. Inserting the angle found in part (a), we have

$$\frac{v_i}{v_{\max}} = \sqrt{1-\frac{\theta}{95°}} = \sqrt{1-\frac{32.6°}{95°}} = 0.81$$

or 81%.

ASSESS: Without considering the human body's ability to throw a ball, the maximum range is achieved at $45°$. Since the human body can most easily throw forward, at small angles of elevation, it is reasonable that the combined effect to for the greatest range to be achieved at an angle somewhat smaller than $45°$.

P3.80 PREPARE: In order to model his speed and acceleration, we will use the exponential expressions:

$$v_x(t) = v_{\max,x}\left(1-e^{-t/\tau}\right)$$
$$a_x(t) = a_{\max,x}e^{-t/\tau}$$

We are given the maximum speed and the time constant. Since the rate of change of the velocity must be equal to the acceleration, we can determine $a_{\max,x}$ by differentiating $v_x(t)$ with respect to time:

$$\frac{dv_x(t)}{dt} = v_{\max,x}\frac{d}{dt}\left(1-e^{-t/\tau}\right) = \frac{v_{\max,x}}{\tau}e^{-t/\tau} = a_x(t)$$

Comparing expressions for the acceleration, we see $a_{\max,x} = v_{\max,x}/\tau$. Now we only need to insert the given numbers.

SOLVE: (a) Inserting $t = 0$ s, we have

$$v_x(t) = v_{\max,x}\left(1-e^{-t/\tau}\right) = (11.8\text{ m/s})\left(1-e^{-(0\text{ s})/(1.45\text{ s})}\right) = 0\text{ m/s}$$

$$a_x(t) = a_{\max,x}e^{-t/\tau} = \frac{(11.8\text{ m/s})}{(1.45\text{ s})}e^{-(0\text{ s})/(1.45\text{ s})} = 8.14\text{ m/s}^2$$

(b) Inserting $t = 3.00$ s, we have

$$v_x(t) = v_{\max,x}\left(1-e^{-t/\tau}\right) = (11.8\text{ m/s})\left(1-e^{-(3.00\text{ s})/(1.45\text{ s})}\right) = 10.3\text{ m/s}$$

$$a_x(t) = a_{\max,x}e^{-t/\tau} = \frac{(11.8\text{ m/s})}{(1.45\text{ s})}e^{-(3.00\text{ s})/(1.45\text{ s})} = 1.03\text{ m/s}^2$$

(c) Inserting $t = 6.00$ s, we have

$$v_x(t) = v_{\max,x}\left(1-e^{-t/\tau}\right) = (11.8\text{ m/s})\left(1-e^{-(6.00\text{ s})/(1.45\text{ s})}\right) = 11.6\text{ m/s}$$

$$a_x(t) = a_{\max,x}e^{-t/\tau} = \frac{(11.8\text{ m/s})}{(1.45\text{ s})}e^{-(6.00\text{ s})/(1.45\text{ s})} = 0.130\text{ m/s}^2$$

ASSESS: The speed and accelerations we obtained match our expectations for the exponential functions we used. That is, the speed asymptotically approaches its maximum, and the acceleration starts at a maximum and asymptotically approaches zero.

P3.81 **PREPARE:** This problem involves modeling speeds and accelerations using exponential functions. Specifically,

$$v_x(t) = v_{\max,x}\left(1-e^{-t/\tau}\right)$$
$$a_x(t) = a_{\max,x}e^{-t/\tau}$$

We are given the maximum speed and the maximum acceleration but not the time constant. Since the rate of change of the velocity must be equal to the acceleration, we can determine τ, by differentiating $v_x(t)$ with respect to time:

$$\frac{dv_x(t)}{dt} = v_{\max,x}\frac{d}{dt}\left(1-e^{-t/\tau}\right) = \frac{v_{\max,x}}{\tau}e^{-t/\tau} = a_x(t)$$

Comparing expressions for the acceleration, we see

$$a_{\max,x} = v_{\max,x}/\tau \Rightarrow \tau = v_{\max,x}/a_{\max,x} = (24\text{ m/s})/\left(5.7\text{ m/s}^2\right) = 4.21\text{ s}.$$

Now we have an expression for the velocity in the direction of motion, but we must integrate this expression to find the displacement.

SOLVE: (a) Integrating, we find

$$\Delta x(t_f) = \int_0^{t_f} v_x(t)\,dt = v_{\max,x}\int_0^{t_f}\left(1 - e^{-t/\tau}\right)dt = v_{\max,x}\left(t_f + \tau e^{-t_f/\tau} - \tau\right)$$

Inserting $t = 2.0$ s, we have

$$\Delta x = v_{\max,x}\left(t_f + \tau e^{-t_f/\tau} - \tau\right) = (24\text{ m/s})\left((2.0\text{ s}) + (4.21\text{ s})e^{-(2.0\text{ s})/(4.21\text{ s})} - (4.21\text{ s})\right) = 9.8\text{ m}$$

(b) Inserting $t = 4.0$ s, we have

$$\Delta x = v_{\max,x}\left(t_f + \tau e^{-t_f/\tau} - \tau\right) = (24\text{ m/s})\left((4.0\text{ s}) + (4.21\text{ s})e^{-(4.0\text{ s})/(4.21\text{ s})} - (4.21\text{ s})\right) = 34\text{ m}$$

(c) Inserting $t = 8.0$ s, we have

$$\Delta x = v_{\max,x}\left(t_f + \tau e^{-t_f/\tau} - \tau\right) = (24\text{ m/s})\left((8.0\text{ s}) + (4.21\text{ s})e^{-(8.0\text{ s})/(4.21\text{ s})} - (4.21\text{ s})\right) = 106\text{ m} \approx 110\text{ m}$$

ASSESS: These are reasonable distances for a racehorse to cover in a few seconds. The speed should be increasing over time, and we can see that in the first 4 s, the horse travels 34 m, and in the second 4 s, it covers 72 m. This should give us some confidence in our answer.

P3.82 **PREPARE:** In order to model the speed and acceleration, we will use the exponential expressions:

$$v_x(t) = v_{\max,x}e^{-t/\tau}$$
$$a_x(t) = a_{\max,x}e^{-t/\tau}$$

We have chosen these because we expect both the speed and the acceleration to exponentially decay. We are given the maximum speed and the time constant. Since the rate of change of the velocity must be equal to the acceleration, we can determine $a_{\max,x}$ by differentiating $v_x(t)$ with respect to time:

$$\frac{dv_x(t)}{dt} = v_{\max,x}\frac{d}{dt}e^{-t/\tau} = -\frac{v_{\max,x}}{\tau}e^{-t/\tau} = a_x(t)$$

Comparing expressions for the acceleration, we see $a_{\max,x} = -v_{\max,x}/\tau$. Now, we only need to insert the given numbers.

SOLVE: (a) Inserting $t = 0$ s, we have

$$\text{(i)}\quad v_x(t) = v_{\max,x}e^{-t/\tau} = (200\ \mu\text{m/s})e^{-(0.5\text{ ms})/(0.5\text{ ms})} = 70\ \mu\text{m/s}$$

(b) Inserting $t = 0.5$ ms, we have

$$\text{(i)}\ (v_x(t) = v_{\max,x}e^{-t/\tau} = (200\ \mu\text{m/s})e^{-(0.5\text{ ms})/(0.5\text{ ms})} = 70\ \mu\text{m/s}$$

$$\text{(ii)}\ |a_x(t)| = |a_{\max,x}|e^{-t/\tau} = \frac{(200\ \mu\text{m/s})}{(0.5\times10^{-3}\text{ s})}e^{-(0.5\text{ ms})/(0.5\text{ ms})} = 0.1\text{ m/s}^2$$

(c) Inserting $t = 1.0$ ms, we have

$$\text{(i)}\ v_x(t) = v_{\max,x}e^{-t/\tau} = (200\ \mu\text{m/s})e^{-(1.0\text{ ms})/(0.5\text{ ms})} = 30\ \mu\text{m/s}$$

$$\text{(ii)}\ |a_x(t)| = |a_{\max,x}|e^{-t/\tau} = \frac{(200\ \mu\text{m/s})}{(0.5\times10^{-3}\text{ s})}e^{-(1.0\text{ ms})/(0.5\text{ ms})} = 0.05\text{ m/s}^2$$

ASSESS: The speed and accelerations we obtained match our expectations for the exponential functions we used. That is, the speed is decreasing, and the acceleration is also decreasing. Both appear to be approaching zero.

P3.83 **PREPARE:** Acceleration in free fall due to gravity is a constant in either case. On Earth $a_y = -9.8\,\text{m/s}^2$ (constant), and on the moon $a_y = -1.63\,\text{m/s}^2$ (constant). So we can use kinematic equations. We can calculate the initial velocity obtained by the astronaut on the earth and then use that to calculate the maximum height the astronaut can jump on the moon.

SOLVE: The astronaut can jump a maximum 0.50 m on the earth. The maximum initial velocity his leg muscles can give him can be calculated with Equation 3.23. His velocity at the peak of his jump is zero.

$$(v_y)_\text{i} = \sqrt{-2(a_y)\Delta y} = \sqrt{-2(-9.80\text{ m/s}^2)(0.50\text{ m})} = 3.1\text{ m/s}$$

We can also use Equation 3.23 to find the maximum height the astronaut can jump on the moon. The acceleration due to the moon's gravity is $\frac{9.80\text{ m/s}^2}{6} = 1.63\text{ m/s}^2$. On the moon, given the initial velocity above, the astronaut can jump

$$\Delta y_\text{moon} = \frac{-(v_y)_\text{i}^2}{2(a_y)_\text{moon}} = \frac{-(3.1\text{ m/s})^2}{2(-1.63\text{ m/s}^2)} = 3.0\text{ m}$$

ASSESS: The answer, choice B, makes sense. The astronaut can jump much higher on the moon.

P3.84 **PREPARE:** Acceleration due to gravity is a constant in either case. On Earth $a_y = -9.8\,\text{m/s}^2$ (constant), and on the moon $a_y = -1.63\,\text{m/s}^2$ (constant). So we can use kinematic equations. We assume that the astronaut's safe landing speed on the moon should be the same as the safe landing speed on the earth.

SOLVE: The brute force method is to compute the landing speed on the earth with Equation 3.23, and plug that back into the Equation 3.23 for the moon and see what the Δy could be there. This works, but is unnecessarily complicated and gives information (the landing speed) we don't really need to know.

To be more elegant, set up Equation 3.23 for the earth and moon, with both initial velocities zero, but then set the final velocities (squared) equal to each other.

$$(v_\text{earth})_\text{f}^2 = 2(a_\text{earth})\Delta y_\text{earth} \quad (v_\text{moon})_\text{f}^2 = 2(a_\text{moon})\Delta y_\text{moon}$$
$$2(a_\text{earth})\Delta y_\text{earth} = 2(a_\text{moon})\Delta y_\text{moon}$$

Dividing both sides by $2(a_\text{moon})\Delta y_\text{earth}$ gives

$$\frac{a_\text{earth}}{a_\text{moon}} = \frac{\Delta y_\text{moon}}{\Delta y_\text{earth}}$$

This result could also be accomplished by dividing the first two equations; the left side of the resulting equation would be 1, and then one arrives at our same result.

Since the acceleration on the earth is six times greater than on the moon, then one can safely jump from a height six times greater on the moon and still have the same landing speed.

So the answer is B.

ASSESS: Notice that in the elegant method we employed we did not need to find the landing speed (but for the sake of curiosity, it is 4.4 m/s, which seems reasonable).

P3.85 **PREPARE:** Acceleration in free fall due to gravity is a constant in either case. On Earth $a_y = -9.8\,\text{m/s}^2$ (constant), and on the moon $a_y = -1.63\,\text{m/s}^2$ (constant). So we can use kinematic equations. We can calculate the initial velocity with which the astronaut throws the ball on the earth and then use that to calculate the time the ball is in motion after it is thrown and comes back down on the moon. The initial velocity with which the ball is thrown on the earth can be calculated from Equation 3.21.

SOLVE: Since the ball starts near the ground and lands near the ground, $x_\text{f} = x_\text{i}$. Solving the equation for $(v_y)_\text{i}$,

$$(v_y)_\text{i} = -\frac{1}{2}a_y\Delta t = -\frac{1}{2}(-9.80\ \text{m/s}^2)(3.0\ \text{s}) = 15\ \text{m/s}$$

The acceleration due to the moon's gravity is $\dfrac{9.80\ \text{m/s}^2}{6} = 1.63\ \text{m/s}^2$. We can find the time it takes to return to the lunar surface using the same equation as above, this time solving for Δt. If thrown upward with this initial velocity on the moon,

$$\Delta t = \frac{-2(v_y)_\text{i}}{a_y} = \frac{-2(15\ \text{m/s})}{-1.63\ \text{m/s}^2} = 18\ \text{s}$$

The correct choice is B.

ASSESS: This makes sense. The ball is in motion for a much longer time on the moon.

4

FORCE AND MOTION

QUESTIONS

Q4.1. **REASON:** Even if an object is not moving, forces can be acting on it. However, the *net* force must be zero. As an example, consider a book on a flat table. The forces that act on the book are the weight of the book (a long-ranger force) and the normal force exerted by the table (a contact force). There are two forces acting on the book, but it is not moving because the net force on the book is zero.

ASSESS: The net force, which is the vector sum of the forces acting on an object, governs the acceleration of objects through Equation 4.4.

Q4.2. **REASON:** No. If you know all of the forces, then you know the direction of the acceleration, not the direction of the motion (velocity). For example, a car moving forward could have on it a net force forward if speeding up or backward if slowing down or no net force at all if moving at constant speed.

ASSESS: Consider carefully what Newton's *second* law says, and what it doesn't say. The net force must *always* be in the direction of the acceleration. This is also the direction of the *change* in velocity, although not necessarily in the direction of the velocity itself.

Q4.3. **REASON:** The arrows have been shot horizontally and air resistance is negligible. The only force on the arrows is due to gravity, which always points straight down. There is no component of a force in the horizontal direction. Each arrow has exactly the same magnitude of horizontal force on it, 0 N. No horizontal force is required to keep the arrows moving. A horizontal force is only required if we wish to *change* the horizontal motion of the arrows.

ASSESS: The gravitational force always points vertically downward. Once the arrow has been fired, it is moving with a constant velocity, a force is required to change the velocity but not to maintain the velocity.

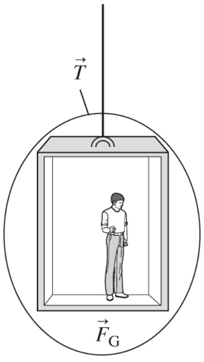
Q4.4. **REASON:** What you feel is a contact force between your back and the seat. That could be explained by something pushing you back into the seat, or by the seat pushing against you. The latter is what is actually happening. The car and the seat are accelerating from rest, and the seat has to exert a force on you in order for you to accelerate with the rest of the car.

ASSESS: There is no force pushing you backward. Your inertia would cause you to remain still while the car accelerates away, except that the seat exerts a forward force on your body.

Q4.5. **REASON:** The inertia of the ketchup will keep it from moving if it isn't too tightly adhered to the sides of the moving bottle.

ASSESS: If you hit the bottle downward (while it is upside down), then the ketchup will end up farther from the opening.

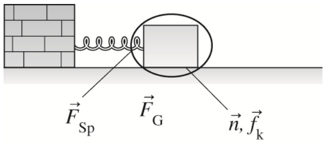
Q4.6. **REASON:** Other than gravity, forces can only be exerted on the elevator by objects in contact with the elevator. Only the cable is in contact with the elevator.



Two forces are present, tension $\vec{T}$ in the cable and gravitational force $\vec{F}_G$ as seen in the figure.

ASSESS: We are told that the elevator is descending at constant speed. But one can see that with these two forces, the net force could be either upward or downward depending on which of the two forces is larger.

Q4.7 **REASON:** We know that only objects in contact with the block can exert forces on it, with the exception of gravity. We have the spring in contact with the block, as well as the rough floor.



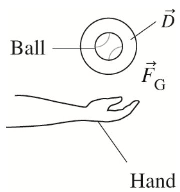
Four forces act on the block: the push of the spring $\vec{F}_{sp}$, gravitational force $\vec{F}_G$, a normal force from the tabletop $\vec{n}$, and a kinetic frictional force due to the rough table surface $\vec{f}_k$.

ASSESS: Gravity and the normal force will cancel each other if the system is not accelerating upward or downward. It is not known whether the spring force is greater than or less than the force of friction; that is, the block could either be speeding up or slowing down.

Q4.8 **REASON:** Two forces act on the ball after it leaves your hand: the long-range gravitational force $\vec{F}_G$ and the contact force of air resistance or drag $\vec{D}$.

(a) $\vec{F}_G$ is a long-range force; the earth and the ball do not need to be in contact for them to be attracted by gravity. $\vec{D}$ is a contact force; air particles must be in contact with the ball in order to exert a force on the ball.

(b) $\vec{F}_G$ is due to an interaction between the ball and earth, so earth is the agent. Drag $\vec{D}$ is due to an interaction between the ball and the air, so the air is the agent. Some students are tempted to list a force due to the hand acting on the ball. As can be seen in the figure, the hand is not touching the ball, and therefore it no longer applies a force to the ball.



ASSESS: A similar diagram during the throwing process, while the ball is still in the hand, would indeed show an upward force exerted on the ball by the hand.

Q4.9. **REASON:** Kinetic friction opposes the motion, but static friction is in the direction to prevent motion.

(a) Examples of motion where the frictional force on the object is directed opposite the motion would be a block sliding across a tabletop or the friction of the road on car tires in a skid.

(b) An example of motion in which the frictional force on the object is in the same direction as the motion would be a crate (not sliding) in the back of a pickup truck that is speeding up. The static frictional force of the truck bed on the crate is in the forward direction (the same direction as the motion), because it *is* the net force on the crate that accelerates it forward.

ASSESS: It is easy to think that the direction of the frictional force is *always* opposite the direction of motion, but static frictional forces are in the direction to prevent relative motion between the surfaces and can be in various directions depending on the situation.

Q4.10. **REASON:** Since there is no source of gravity, you will not be able to feel the weight of the objects. However, Newton's second law is true even in an environment without gravity. Assuming you can exert a reproducible force in throwing both objects, you could throw each and note the acceleration each obtains.

ASSESS: Mass is independent of the force of gravity and exists even in environments with no sources of gravity.

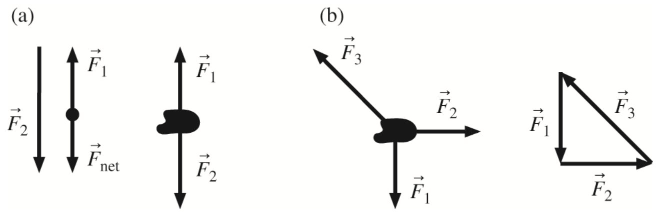
Q4.11. **REASON:** Both objects (Jonathan and his daughter) experience the same acceleration (i.e. the acceleration of the car). However, since the objects do not have the same mass, the forces required to accelerate them will be different. The object with the greater mass (Jonathan) will require the greater force.

ASSESS: This is a straightforward application of Newton's second law.

Q4.12. **REASON:** Newton's first law says that if no force acts on an object, that object will continue to move with the same velocity. Inside the tube, there was a contact force between the walls of the tube and the ball, so the direction of the velocity could change. But as the ball emerges, the wall vanishes and there is no more force acting on it. The ball must continue on with the same velocity it had when it exited the tube. Thus, the ball follows path C.

ASSESS: It is reasonable that once the ball is free of the curved tube, its motion should not be affected by the curvature of the tube.

Q4.13 **REASON:** In order for an object to be in equilibrium, the sum of all forces on the object must be zero. We can draw free-body diagrams for the two balls, and examine the vector sum of all forces.



(a) Basketball A is not in equilibrium because $|\vec{F}_2| > |\vec{F}_1|$, so there is a net downward force on A.

(b) Basketball B is in equilibrium because the vector sum of the three forces is zero: $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$.

ASSESS: This problem emphasizes that equilibrium does not necessarily mean that no force is acting on an object, but simply that the vector sum of all forces is zero.

Q4.14. **REASON:** Since the block glued on to the original block is identical to the original block, the mass of the two together must be twice as large as the mass of the original block. If the force applied is also twice as large, the acceleration will be the same. Explicitly applying Newton's second law to the two blocks glued together gives

$$a_{new} = \frac{2F}{2m} = \frac{F}{m} = a_{old}$$

The correct choice is C.

ASSESS: In Newton's second law, the acceleration is proportional to the net force and inversely proportional to the mass accelerated. As a result, if you double both the mass and the force, the acceleration will remain the same.

Q4.15. **REASON:** We can apply Newton's second law twice to solve this problem, first to determine the force and second to determine the mass.

$$F = ma = (5.0\,\text{kg})(0.20\,\text{m/s}^2) = 1.0\,\text{N}$$

Then

$$m = F/a = 1.0\text{ N}/0.10\text{ m/s}^2 = 10\text{ kg}$$

The correct answer is A.

ASSESS: This problem is a straightforward application of Newton's second law.

Q4.16. **REASON:** Drag points opposite to the direction of motion. As the ball is going up, the drag force acts downward. As the ball comes down, the drag force acts upward. The correct choice is D.

ASSESS: Drag always acts opposite to the direction of motion of an object.

Q4.17. **REASON:** In Newton's second law $a = F/m$ the force is nearly constant, but the mass is decreasing; therefore the acceleration will increase, so the answer is A.

ASSESS: Many objects we study in physics have approximately constant mass, but rockets don't; they get lighter as the fuel burns.

Q4.18. **REASON:** The direction of the kinetic friction force will be opposite the motion, so the friction points down while the box goes up, and the friction points up while the box slides down.

The answer is D.

ASSESS: Drawing a free-body diagram (with tilted axes) and applying Newton's second law will support this conclusion.

Q4.19. **REASON:** Friction is holding the crate in place, but we do not know if the crate is on the verge of slipping down the incline due to gravity or on the verge of slipping up the incline due to Craig's push. The direction of static friction depends on whether or not Craig is overcoming the force of gravity. Since we are not given this information, the correct answer is D.

ASSESS: Consider a very massive crate and a very small push. Friction would be acting up the ramp. Alternatively, a lighter crate and a very large push could result in friction acting down the ramp.

Q4.20. **REASON:** To remain stationary there needs to be a zero net force on the scallop. The downward gravitational force is not quite balanced by the upward buoyant force so the thrust force must also be up. For the thrust force on the scallop to be up, it must eject water in the downward direction.

The answer is C.

ASSESS: Drawing a free-body diagram (with tilted axes) and applying Newton's second law will support this conclusion.

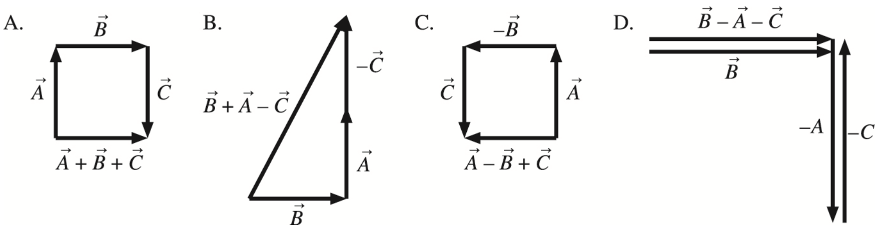
Q4.21. **REASON:** Since block A rides without slipping, it, too, must be accelerating to the right. If it is accelerating to the right there must be a net force to the right, according to Newton's second law. The only object that can exert a force to the right is block B.

This static friction force is to the right to prevent slippage of block A to the left (relative to block B).

The correct choice is B.

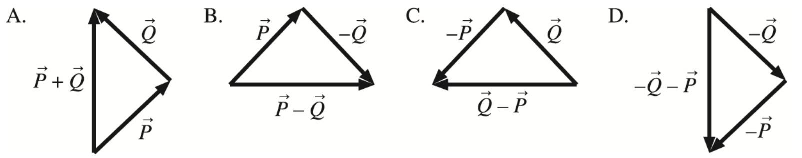
ASSESS: This is one of those cases in which the static friction force can be in the same direction as the motion to prevent slippage the other way. Verify with a free-body diagram of block A.

Q4.22. **REASON:** The longest vector will be obtained by turning $\vec{C}$ around or by turning $\vec{A}$ around so that the two point in the same direction. The choices A, C, and D all have $\vec{A}$ and $\vec{C}$ added together. B is the only choice in which the two vectors are subtracted and not added together. Since $\vec{B}$ is perpendicular to $\vec{A}$ and $\vec{C}$, we will get a vector of the same length whether we add or subtract $\vec{B}$ from the other two. See the figure for the lengths of the different vector combinations. The answer is B.



ASSESS: A longer vector can be created by adding two vectors, which point in the same direction.

Q4.23. **Reason:** To generate a vector that points to the left, we could add two vectors that point left, one pointing up and the other down. In C, $\vec{Q}$, and $-\vec{P}$ fit this description so their sum points to the left. The various vector combinations are shown.



The answer is C.

Assess: If two vectors have equal and opposite components in a certain direction, say the x-direction, then when we add the vectors, the equal and opposite components will cancel and leave us with a vector perpendicular to that direction.

Problems

P4.1. **Prepare:** This problem deals with an object (human head) that has inertia. It will continue with constant velocity or remain at rest until a force acts on it. Note that time progresses to the right in each sequence of pictures. In one case, the head is thrown back, and in the other, forward.

Solve: Using the principle of inertia, the head will tend to continue with the same velocity after the collision that it had before.

In the first series of sketches, the head is lagging behind because the car has been quickly accelerated forward (to the right). This is the result of a rear-end collision.

In the second series of sketches, the head is moving forward relative to the car because the car is slowing down and the head's inertia keeps it moving forward at the same velocity (although external forces do eventually stop the head as well). This is the result of a head-on collision.

Assess: Hopefully you haven't experienced either of these in an injurious way, but you have felt similar milder effects as the car simply speeds up or slows down.

It is for this reason that cars are equipped with headrests, to prevent the whiplash shown in the first series of sketches, because rear-end collisions are so common. The laws of physics tell us how wise it is to have the headrests properly positioned for our own height.

Air bags are now employed to prevent injury in the second scenario.

P4.2. **Prepare:** This problem deals with an object (human head) that has inertia. It will continue with constant velocity or remain at rest until a force acts on it. We can apply Newton's first law to determine the motion of the brain during the collision.

SOLVE: During the collision of the automobile, the passenger and the passenger's brain continue moving at the speed the car was moving just prior to the collision. When the passenger's head is stopped by a part of the inside of the car, such as an airbag or the windshield, the passenger's brain is traveling at the speed of the car before the collision. The brain continues to move forward, and comes to a rest as it compresses against the forward part of the skull. The frontal portion of the brain will be compressed.

ASSESS: The damage is diminished if the compression takes place over a longer interval of time; hence, padded dashboards.

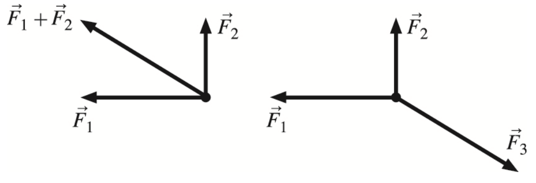
P4.3. **PREPARE:** This problem deals with an object (a human infant) that has inertia. It will continue with constant velocity or remain at rest until a force acts on it. As background, look at Question Q4.6 and Problem P4.1. Also, think about the design and orientation of the seat and how the child rides in the seat. Finally recall Newton's second law.

SOLVE: As the child rides in the seat, his/her head and back rest against the padded back of the seat. If the car is brought to a rapid stop (as in a head-on collision), the child will continue to move forward at the before-crash speed until he/she hits something. The object hit is the back of the seat (supporting the entire back and head), which is padded and as a result the force increases to the maximum value over a time interval. Granted this time interval may be small but that is considerably better than instantaneous. Also, since the head is supported, there will be no whiplash.

ASSESS: The fact the force acting on the child is spread over a time interval is a critical factor. In later chapters, you will learn to call this concept impulse.

P4.4. **PREPARE:** Draw the vector sum $\vec{F}_1+\vec{F}_2$ of the two forces $\vec{F}_1$ and $\vec{F}_2$. Then look for a vector that will "balance" the force vector $\vec{F}_1+\vec{F}_2$.

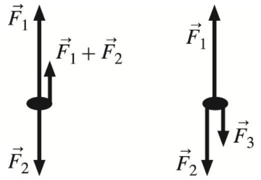
SOLVE: The object will be in equilibrium if $\vec{F}_3$ has the same magnitude as $\vec{F}_1+\vec{F}_2$ but is in the opposite direction so that the sum of all the three forces is zero.



ASSESS: Adding the new force vector $\vec{F}_3$ with length and direction as shown will cause the object to be at rest.

P4.5. **PREPARE:** Draw the vector sum $\vec{F}_1+\vec{F}_2$ of the two forces $\vec{F}_1$ and $\vec{F}_2$. Then look for a vector that will "balance" the force vector $\vec{F}_1+\vec{F}_2$.

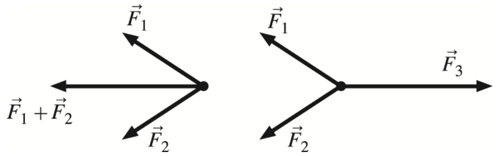
SOLVE: The object will be in equilibrium if $\vec{F}_3$ has the same magnitude as $\vec{F}_1 + \vec{F}_2$ but is in the opposite direction so that the sum of all three forces is zero.



ASSESS: Adding the new force vector $\vec{F}_3$ with length and direction as shown will cause the object to be at rest.

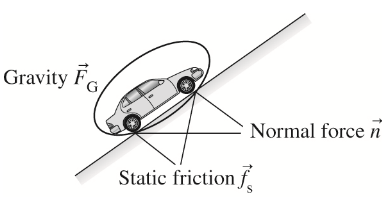
P4.6. **PREPARE:** Draw the vector sum $\vec{F}_1 + \vec{F}_2$ of the two forces $\vec{F}_1$ and $\vec{F}_2$. Then look for a vector that will "balance" the force vector $\vec{F}_1 + \vec{F}_2$.

SOLVE: The object will be in equilibrium if $\vec{F}_3$ has the same magnitude as $\vec{F}_1 + \vec{F}_2$ but is in the opposite direction so that the sum of all three forces is zero.



ASSESS: Adding the new force vector $\vec{F}_3$ with length and direction as shown will cause the object to be at rest.

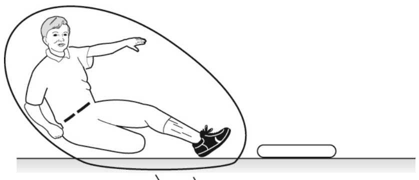
P4.7 **PREPARE:** Gravity can act without contact between objects, but any other force exerted on the car must come from physical contact with an object. Sketch the setup, circle the car, and determine objects in contact with it.
SOLVE: As shown in the figure, only the road is in contact with the car. The road can exert two kinds of forces: a force perpendicular to the surface (normal force $\vec{n}$) and parallel to the surface (frictional force, in this case static $\vec{f}_s$). Gravity is also acting on the car. Thus there are three forces being exerted on the car: $\vec{n}$, $\vec{f}_s$, and $\vec{w}$.



Assess: Since the car is in static equilibrium, the gravitational force must be canceled out both in the direction perpendicular to the road, and parallel to it. The normal force and the force of static friction respectively cancel out these components of gravity.

P4.8 **PREPARE:** Gravity can act without contact between objects, but any other force exerted on the player must come from physical contact with an object. Sketch the setup, circle the player, and determine objects in contact with him/her.

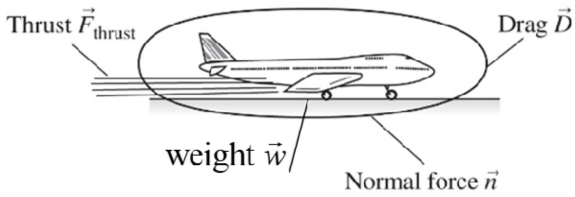
SOLVE: As shown in the figure, only the ground is in contact with the player. The ground can exert two kinds of forces: a force perpendicular to the surface (normal force $\vec{n}$) and parallel to the surface (frictional force, in this case kinetic $\vec{f}_k$). Gravity is also acting on the player. Thus there are three forces being exerted on the player: $\vec{n}$, $\vec{f}_k$, and $\vec{w}$.



ASSESS: Because the player is not accelerating vertically upward or downward, we need a pair of vertical forces that can cancel each other out. We have these in gravity and the normal force. In the horizontal direction, we can have a net force, since we expect the player to be slowing down as he/she slides. This will be the kinetic force of friction.

P4.9 **PREPARE:** Gravity can act without contact between objects, but any other force exerted on the plane must come from physical contact with an object. Sketch the setup, circle the plane, and determine objects in contact with it. In this case, air particles will collectively play the role of an "object."

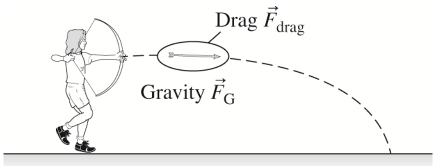
SOLVE: As shown in the figure, the runway is in contact with the plane, as is the air. The runway can exert two kinds of forces: a force perpendicular to the surface (normal force $\vec{n}$) and parallel to the surface (frictional force). In this case, the plane is on wheels meant to minimize friction, and rolling friction is likely negligible compared to other forces in this problem; we will ignore it. Air is in contact with the plane, and this has at least two effects and possibly a third: the engines can push against air and achieve thrust $\vec{F}_{thrust}$, and air particles in front of the plane collide with it, causing a drag force $\vec{D}$. Depending on how the flaps on the wings are oriented, some of the air striking the plane may cause lift $\vec{F}_{lift}$. It is a good bet that during takeoff, the lift would be significant. Gravity is also acting on the plane. Thus there are four forces being exerted on the plane: $\vec{n}$, $\vec{D}$, $\vec{F}_{thrust}$, and $\vec{w}$.



ASSESS: Depending on how the flaps on the wings are oriented, some of the air striking the plane may cause lift $\vec{F}_{lift}$. It is a good bet that during takeoff, the lift would be significant, but we have omitted it from our list since we are not given information about the flaps. The combination of gravity, normal force, and lift will be in equilibrium, until the lift exceeds the force of gravity, at which time the place will begin to accelerate upward.

P4.10 **PREPARE:** Gravity can act without contact between objects, but any other force exerted on the arrow must come from physical contact with an object. Sketch the setup, circle the arrow, and determine objects in contact with it. In this case, air particles will collectively play the role of an "object."

SOLVE: The arrow experiences drag due to air particles striking it, and gravity is also exerted. There are no objects (other than air particles) in contact with the arrow, and so there are no other forces being exerted on it. The two forces are $\vec{w}$ and $\vec{D}$.



ASSESS: Since there is nothing to counteract gravity, the arrow should accelerate toward the ground, which we know, it does.

P4.11. **PREPARE:** Force and acceleration are related through mass by Newton's second law. We will use Equation 4.3 and model the object as a particle.

SOLVE: **(a)** We are told that for an unknown force (call it F_0) acting on an unknown mass (call it m_A), the acceleration of the mass is $5\ \text{m/s}^2$. The accelerations are $3\ \text{m/s}^2$ and $8\ \text{m/s}^2$ for objects B and C. According to Newton's second law,

$$F_0 = m_A(5\ \text{m/s}^2) = m_B(3\ \text{m/s}^2) = m_C(8\ \text{m/s}^2)$$

This means B has the largest mass.

(b) Object C has the smallest mass because it has the largest acceleration.

(c) From the equation in part (a) the ratio of mass A to mass B is 3/5.

ASSESS: Since the force is constant and Newton's second law tells us the product of the mass and acceleration is equal to the force, given the accelerations, finding the mass is relatively straightforward.

P4.12. **PREPARE:** We will use Equation 4.3: $\vec{F} = m\vec{a}$, and assume that the maximum force the road exerts on the car is the same in both cases.

SOLVE: Use primed quantities for the case with the four new passengers and unprimed quantities for the original case with just the driver. $F' = F$. The original mass was 1200 kg and the new mass is 1600 kg.

$$a' = \frac{F'}{m'} = \frac{F}{m'} = \frac{ma}{m'} = \frac{(1200 \text{ kg})(4 \text{ m/s}^2)}{1600 \text{ kg}} = 3.0 \text{ m/s}^2$$

ASSESS: We expected the maximum acceleration to be less with the passengers than with the driver only.

P4.13. **PREPARE:** The problem may be solved by applying Newton's second law to the present and the new situation: $\vec{F} = m\vec{a}$.

SOLVE: **(a)** We are told that for an unknown force (call it F_o) acting on an unknown mass (call it m_o), the acceleration of the mass is 8.0 m/s^2. According to Newton's second law

$$F_o = m_o(8.0 \text{ m/s}^2) \quad \text{or} \quad F_o/m_o = 8.0 \text{ m/s}^2$$

For the new situation, the new force is $F_{new} = 2F_o$, the mass is not changed $(m_{new} = m_o)$ and we may find the acceleration by

$$F_{new} = m_{new}a_{new}$$

or

$$a_{new} = F_{new}/m_{new} = 2F_o/m_o = 2(F_o/m_o) = 2(8 \text{ m/s}^2) = 16 \text{ m/s}^2$$

(b) For the new situation, the force is unchanged $F_{new} = F_o$, the new mass is half the old mass $(m_{new} = m_o/2)$, and we may find the acceleration by

$$F_{new} = m_{new}a_{new}$$

or

$$a_{new} = F_{new}/m_{new} = F_o/2m_o = (F_o/m_o)/2 = (8.0 \text{ m/s}^2)/2 = 4.0 \text{ m/s}^2$$

(c) A similar procedure gives $a = 8.0 \text{ m/s}^2$.

(d) A similar procedure gives $a = 32 \text{ m/s}^2$.

ASSESS: From the algebraic relationship $a = F/m$, we can see that when (a) the force is doubled, the acceleration is doubled; (b) the mass is doubled, the acceleration is halved; (c) both force and mass are doubled, the acceleration doesn't change; and (d) force is doubled and mass is halved, the acceleration will be four times larger.

P4.14. **PREPARE:** We will use Figure P4.14 and Equation 4.3: $\vec{F} = m\vec{a}$. In the direction of acceleration, we see that the ratio $a/F = m$ is the same as the slope in the figure.

SOLVE: Applying Newton's second law to curves 1 at the point F = 3 rubber bands and to curve 2 at the point F = 5 rubber bands gives

$$\left.\begin{aligned} 3F &= m_1(5a_1) \\ 5F &= m_2(4a_1) \end{aligned}\right\} \frac{3}{5} = \frac{5m_1}{4m_2} \quad \Rightarrow \quad \frac{m_1}{m_2} = \frac{12}{25}$$

Assess: The line with the steepest slope should have the smallest mass, so we expect $m_1 < m_2$, which is consistent with our calculation.

P4.15 **Prepare:** We will use Figure P4.15 and Equation 4.3: $\vec{F} = m\vec{a}$. In the direction of acceleration, we see that the ratio $a / F = m$ is the same as the slope in the figure.

Solve: Newton's second law is $F = ma$. Applying this to curves 1 and 2 at the point F = 2 rubber bands gives

$$\left.\begin{aligned}2F &= m_1(5a_1)\\ 2F &= m_2(2a_1)\end{aligned}\right\}1 = \frac{5m_1}{2m_2} \quad \Rightarrow \quad m_1 = \frac{2}{5}m_2 = \frac{2}{5}(0.20\text{ kg}) = 0.080\text{ kg}$$

Repeating the calculation for curves 2 and 3 at the point F = 5 rubber bands gives

$$\left.\begin{aligned}5F &= m_2(5a_1)\\ 5F &= m_3(2a_1)\end{aligned}\right\}1 = \frac{5m_2}{2m_3} \quad \Rightarrow \quad m_3 = \frac{5}{2}m_2 = \frac{5}{2}(0.20\text{ kg}) = 0.50\text{ kg}$$

Assess: The line with the steepest slope should have the smallest mass, so we expect $m_1 < m_2 < m_3$, which is consistent with our calculation.

P4.16. **Prepare:** We can use Newton's second law, Equation 4.3, to find the acceleration of the wagon with the child.

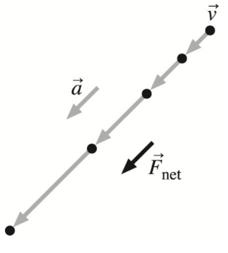
Solve: We do not know the mass of the wagon, but we do know that the same force is applied to the empty wagon as to the wagon with the child in it. We also know that the mass of the child is three times that of the wagon, so the child and wagon together have a mass of four times the mass of the empty wagon. Using Newton's second law for both the empty wagon and the wagon containing the child we have

$$a_{\text{empty wagon}} = \frac{F}{m} = 1.4\text{ m/s}^2$$

$$a_{\text{wagon and child}} = \frac{F}{4\,m} = \frac{1}{4}\frac{F}{m} = \frac{1}{4}(1.4\text{ m/s}^2) = 0.35\text{ m/s}^2$$

Assess: We could have also used the fact that acceleration is inversely proportional to mass as discussed in Section 4.4. Since the mass is four times larger, the acceleration must be a quarter the original acceleration.

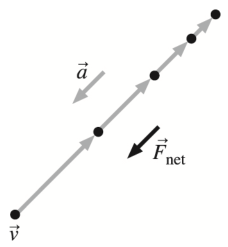
P4.17. **Prepare:** Motion diagrams show changes in velocity vectors, from which we can determine acceleration. The acceleration and the net force must be in the same direction. Redraw the motion diagram as shown.



SOLVE: The velocity vector in the previous figure is shown downward and to the left. So movement is downward and to the left. The velocity vectors get successively longer which means the speed is increasing. Therefore, the acceleration is downward and to the left. By Newton's second law, $\vec{F} = m\vec{a}$, the net force must be in the same direction as the acceleration. Thus, the net force is downward and to the left.

ASSESS: Since the object is speeding up, the acceleration vector must be parallel to the velocity vector. This means the acceleration vector must be pointing along the direction of velocity. Therefore, the net force must also be downward and to the left.

P4.18. **PREPARE:** Motion diagrams show changes in velocity vectors, from which we can determine acceleration. The acceleration and the net force must be in the same direction. Redraw the motion diagram as shown.



SOLVE: The velocity vector shown is upward and to the right. So movement is upward and to the right. The velocity vector gets successively shorter, which means the speed is decreasing. Therefore, the acceleration is downward and to the left. By Newton's second law, $\vec{F} = m\vec{a}$, the net force must be in the same direction as the acceleration. Thus, the net force is downward and to the left.

ASSESS: Since the object is slowing down, the acceleration vector must be antiparallel to the velocity vector. This means the acceleration vector must be pointing in the opposite direction of velocity. Therefore, the net force must also be pointing in the opposite direction of velocity. In other words, the net force must be downward and to the left.

P4.19 **PREPARE:** We will use Newton's second law, $F = ma$, at each point.

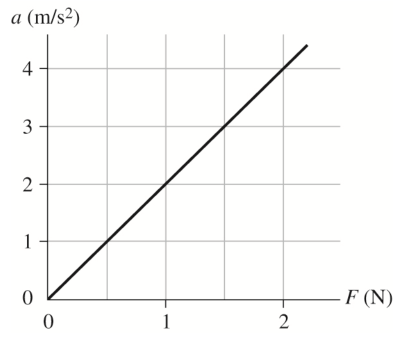
SOLVE: Compute F for each case:

(a) $F = (0.200\ \text{kg})(5\ \text{m/s}^2) = 1\ \text{N}$.

(b) $F = (0.200\ \text{kg})(10\ \text{m/s}^2) = 2\ \text{N}$.

ASSESS: To double the acceleration we must double the force, as expected.

P4.20 **PREPARE:** We will use Newton's second law, $F = ma$, at each point.



SOLVE: **(a)** When $F = 2$ N, we have $2\text{ N} = (0.5\text{ kg})a$, so $a = 4\text{ m/s}^2$.

(b) When $F = 1$ N, we have $1\text{ N} = (0.5\text{ kg})a$, so $a = 2\text{ m/s}^2$.

ASSESS: The fact that doubling the force doubles the acceleration is consistent with expectations.

P4.21 **PREPARE:** Newton's second law is $F = ma$. The graph in Figure P4.21 tells us the acceleration as a function of mass. Knowing the mass and acceleration for any given point, we can find the force.

SOLVE: Let us examine the case $m = 300\text{ g} = 0.30\text{ kg}$, which corresponds to $a = 5.0\text{ m/s}^2$. Newton's second law yields $F = ma = (0.30\text{ kg})(5.0\text{ m/s}^2) = 1.5\text{ N}$

ASSESS: To double-check the result, insert $F = 1.5$ N into Newton's law for $m = 100$ g = 0.10 kg. This gives $a = F/m = (1.5\text{ N})/(0.1\text{ kg}) = 15\text{ m/s}^2$, which is consistent with the graph.

P4.22. **PREPARE:** The graph shows velocity versus time. We get the acceleration from the slope of the graph and then apply Newton's second law: $\vec{F} = m\vec{a}$. The mass of the scallop is 25 g or 0.025 kg.

SOLVE: The slope of the line in the graph is 1 m/s^2; this is the acceleration of the scallop.

$$F = ma = (0.025\text{ kg})(1\text{ m/s}^2) = 0.025\text{ N}$$

ASSESS: This force is 1/10 of the weight of the scallop, which seems reasonable.

P4.23. **PREPARE:** This problem involves forces acting on varying mass to produce different accelerations. We will assume that the car's ability to produce a force does not depend on the mass. Then we can write Newton's second law for two cases: with a passenger and without. We call the force provided by the car F. Then $F = m_{\text{with}}a_{\text{with}}$ and $F = m_{\text{without}}a_{\text{without}}$. We can relate these two and solve for the unknown acceleration.

SOLVE: Equating the two equations for the force provided by the car, we find

$$F = m_{\text{with}}a_{\text{with}} = m_{\text{without}}a_{\text{without}} \Rightarrow a_{\text{with}} = \frac{m_{\text{without}}}{m_{\text{with}}}a_{\text{without}} = \left(\frac{1510\text{ kg}}{1590\text{ kg}}\right)(0.75)g = (0.71)g$$

ASSESS: It is reasonable that an increase in mass should result in a smaller acceleration. But since the increase in mass is small, the decrease in acceleration is also small.

P4.24. **PREPARE:** The average acceleration is $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$. Since we are not given information about a changing acceleration, we use this average. Use Newton's second law in the second part: $\vec{F} = m\vec{a}$.

SOLVE: **(a)** $a_x = \dfrac{\Delta v_x}{\Delta t} = \dfrac{12\ \text{m/s} - 0\ \text{m/s}}{0.002\ \text{s}} = 6000\ \text{m/s}^2$

(b) $F = ma = (0.140\ \text{kg})(6000\ \text{m/s}^2) = 840\ \text{N}$

ASSESS: Even little leaguers hit the ball with quite a bit of force.

P4.25. **PREPARE:** Call the direction of the skater's initial motion the $+x$-direction. We know the skater's mass, initial and final speeds, and the time over which the speeds changed. We treat the acceleration as being constant, such that we can use Newton's second law and the definition of acceleration to write

$$\sum \vec{F} = m\vec{a} = m\left(\frac{\vec{v}_\text{f} - \vec{v}_\text{i}}{\Delta t}\right)$$

SOLVE: The only force in the horizontal direction is the force of kinetic friction between the skates and the ice. Thus, we can write

$$\sum F_x = f_{k,x} = m\left(\frac{(v_x)_\text{f} - (v_x)_\text{i}}{\Delta t}\right) = (55\ \text{kg})\left(\frac{(2.9\ \text{m/s}) - (3.5\ \text{m/s})}{(5.0\ \text{s})}\right) = -6.6\ \text{N}$$

So the magnitude is 6.6 N.

ASSESS: This is a reasonable magnitude for ice skates.

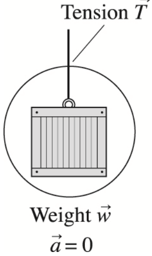
P4.26. **PREPARE:** Assuming the force described in the graph is the only force acting on the head, we can equate $F_\text{ave} = ma_\text{ave}$ and use this to determine the average acceleration required to find the HIC.

SOLVE: The acceleration is $a_\text{ave} = F_\text{ave}/m = (2000\ \text{N})/(4.5\ \text{kg}) = 444\ \text{m/s}^2$. Inserting this into the expression for the HIC and reading the duration from the graph, we have

$$\text{HIC} = (a_\text{avg}/g)^{2.5}\Delta t = \left((444\ \text{m/s}^2)/(9.8\ \text{m/s}^2)\right)^{2.5}(0.080\ \text{s}) = 1.1\times10^3\ \text{N}$$

ASSESS: This collision is likely to cause serious head injury or even death.

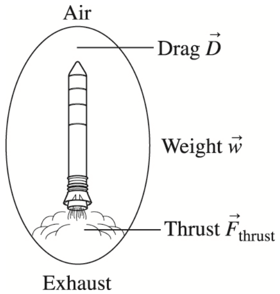
P4.27. **PREPARE:** The free-body diagram shows two equal and opposite forces such that the net force is zero. The force directed down is labeled as a weight, and the force directed up is labeled as a tension. With zero net force the acceleration is zero. Draw a picture of a real object, as shown, with two forces to match the given free-body diagram.



SOLVE: A possible description is: "An object hangs from a rope and is at rest." Or, "An object hanging from a rope is moving up or down with a constant speed."

ASSESS: This problem and the following problem make it clear how important it is to know all forces (and their direction) acting on an object in order to determine the net force acting on the object.

P4.28. PREPARE: The free-body diagram shows three forces with a net force (and therefore net acceleration) upward. There is a force labeled $\vec{w}$ directed down, a force $\vec{F}_{\text{thrust}}$ directed up, and a force $\vec{D}$ directed down. Now, draw a picture of a real object with three forces to match the given free-body diagram.

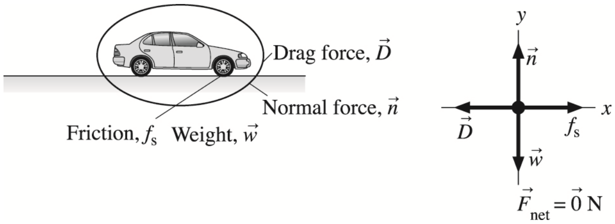


SOLVE: A possible description is, "A rocket accelerates upward."

ASSESS: It is given that the net force is pointing up. Then, $\vec{F}_{\text{net}} = \vec{F}_{\text{thrust}} - \vec{w} - \vec{D}$ must be greater than zero. In other words, $\vec{F}_{\text{thrust}}$ must be larger than $(\vec{w} + \vec{D})$.

P4.29. PREPARE: Refer to Tactics Box 4.2 and Tactics Box 4.3 for identification of forces and for drawing free-body diagrams. We will draw a correct free-body diagram and compare.

SOLVE: Your car is the system. See the following diagram.



There are contact forces where the car touches the road. One of them is the normal force of the road on the car. The other is the force of static friction between the car's tires and the road since the car is moving. In addition to this, there must be a force in the opposite direction to the car's motion, since the car is moving at constant speed. If only the frictional force acted in the horizontal direction, the car would be accelerating! The diagram omits one of the forces. A possible force that acts in this direction is the force of air drag on the car, which is indicated on the diagram.

The only long-range force acting is the weight of the car.

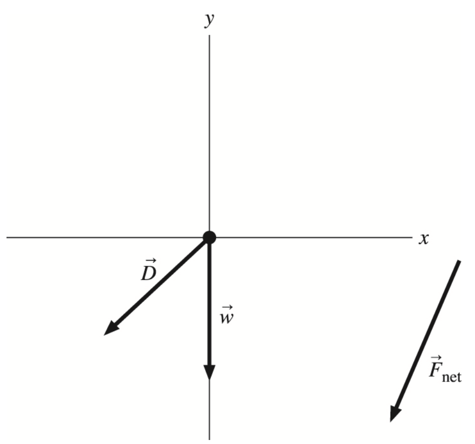
The diagram also identifies the weight of the car and the normal force on the car as an action/reaction pair. This isn't possible, since both these forces act on the same object, while action/reaction pairs always act on *different* objects. The normal force on an object and its weight are *never* action/reaction pairs.

ASSESS: In order for an object to be moving at constant velocity, the net force on it must be zero. Action/reaction pairs always act on two different objects.

P4.30. **PREPARE:** Free-body diagrams shown all force vectors acting on a particular object. The object is treated as a point, and the force vectors are drawn on the same scale. Review Tactics Box 4.3 about drawing free-body diagrams.

SOLVE: One error is that there isn't a force along the direction of motion in this case; $\vec{F}_{\text{motion}}$ should be erased completely. Another error is that the drag force should be opposite the direction of the velocity, not straight left.

A correct free-body diagram would be

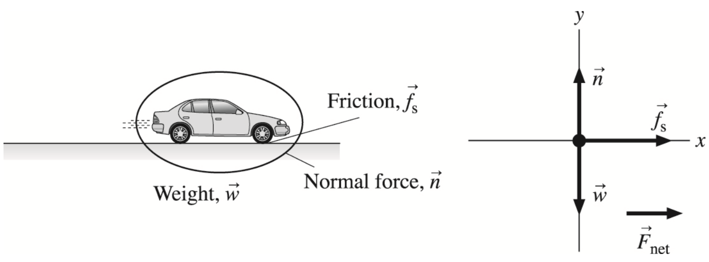


The acceleration (not shown) is in the same direction as $\vec{F}_{\text{net}}$. The velocity (not shown) is up to the right, opposite the drag force.

ASSESS: Motion is not a force. To draw a free-body diagram you must simply consider all of the forces acting *on* the object of interest. Do this by considering which objects are in contact with the object of interest, and which long-range forces act on the object.

P4.31. **PREPARE:** Free-body diagrams show all force vectors acting on a particular object. The object is treated like a single point. We will follow the procedures in Tactics Box 4.2 and Tactics Box 4.3.

SOLVE: Your car is the system. See the following diagram.



There are contact forces where the car touches the road. One of them is the normal force of the road on the car. The other is the force of static friction between the car's tires and the road, since the car is accelerating from a stop. The only long-range force acting is the weight of the car.

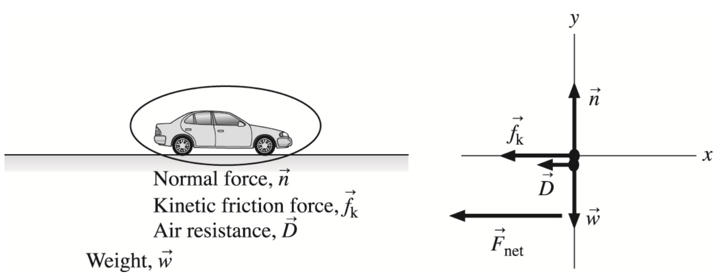
ASSESS: Tactics Box 4.2 and Tactics Box 4.3 give a systematic method for determining all forces on an object and drawing a free-body diagram.

P4.32. PREPARE: Free-body diagrams show all force vectors acting on a particular object. The object is treated like a single point. We follow the steps outlined in Tactics Boxes 4.2 and 4.3. We assume the road is level. We do not neglect air resistance, because at a "high speed" it is significant.

SOLVE: The system is your car.

The objects in contact with your car are the air and the road. The road exerts two forces on the car: the normal force (directed up) and the kinetic friction force (directed horizontally back, parallel to the road). The air exerts a drag force (air resistance) on the car in the same direction as the friction force (i.e., opposite the velocity). The downward pull of the earth's gravitational force is the long-range force.

There is a large friction force between the road and the tires of the car.

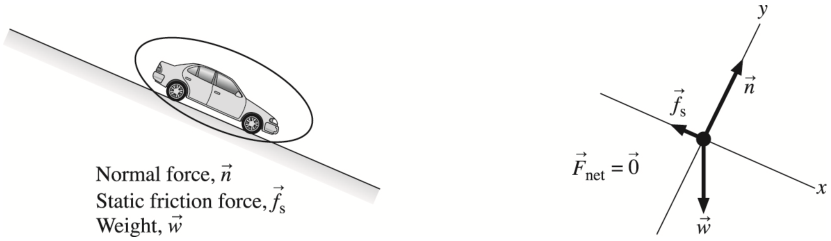


ASSESS: $\vec{F}_{net}$ points to the left, as does the $\vec{a}$ for a car that is moving to the right but slowing down.

P4.33. PREPARE: We will follow the procedures in Tactics Box 4.2 and Tactics Box 4.3. We know the car is not accelerating, so the vector sum of all the forces must be zero.

SOLVE: The only object in contact with the car is the road. The road can exert forces normal to its surface (the normal force), and along its surface (friction). Because this is taking place near a very massive object (Earth), there is also a gravitational force. The forces that are being exerted on the car are gravity, the normal force, and a force of friction. Because the car is not moving relative to the road, the friction is static friction.

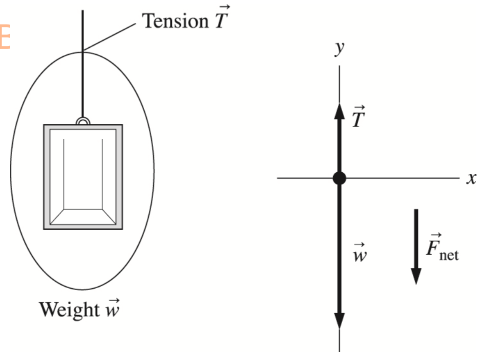
In sketching the free body diagram, it is important to note that the normal force is not "upward", but rather perpendicular to the surface of the road. Note that the components of gravity along the road, and normal to the road must cancel the static friction and normal forces, respectively.



ASSESS: We can see that if the static force of friction were removed, there would be a net force along the surface of the road (downhill). This fits our prior knowledge of the physical world, in that if nothing is holding the car in place (such as if the brakes fail) it would roll or slide down the hill.

P4.34. PREPARE: Free-body diagrams show all force vectors acting on a particular object. The object is treated like a single point. Follow the steps outlined in Tactics Boxes 4.2 and 4.3. Draw a picture of the situation, identify the system, in this case the elevator, and draw a closed curve around it. Name and label all relevant contact forces (the tension) and long-range forces (weight).

SOLVE:



There are two forces acting *on* the elevator due to its interactions with the two agents, earth and the cable. One of the forces *on* the elevator is the long-range weight force *by* the earth. Another force is the tension force exerted *by* the cable due to the contact between the elevator and the cable. Since the elevator is coming to a stop while ascending, it has a negative acceleration, hence a negative net force. As a result, we know that the length of the vector representing the weight of the elevator must be greater than the length of the vector representing the tension in the cable. The free-body diagram is shown in the previous diagram on the right.

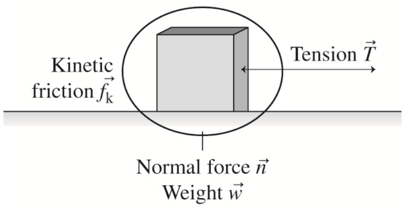
ASSESS: There are only two forces on the elevator. The weight is directed down and the tension in the cable is directed up. Since the elevator is slowing down (has a negative acceleration), the tension must be less than the weight.

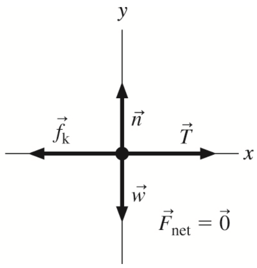
P4.35. **PREPARE:** Free-body diagrams show all force vectors acting on a particular object. The object is treated like a single point. We follow the steps outlined in Tactics Boxes 4.2 and 4.3.

SOLVE: The system is the box.

The objects in contact with the box are the floor and the rope. The floor exerts an upward normal force and a backwards friction force. The rope exerts a tension force.

The important long-range force is the gravitational force of the earth on the box (i.e., the weight).

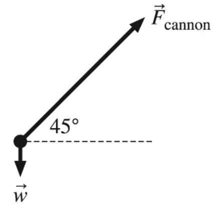




ASSESS: The net force is zero, as it should be for an object that is moving at constant velocity.

P4.36. **PREPARE:** We will identify action/reaction pairs as described in Newton's third law. When a cannon is fired, gas is ignited and the explosion of hot gas propels the cannonball down the muzzle of the cannon. We will refer to this expanding hot gas as the force of the cannon on the cannonball.

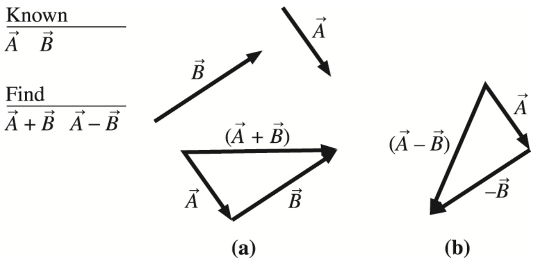
SOLVE: The force of the cannon on the cannonball and the force of the cannonball back on the cannon make up an action/reaction pair. This can be seen when a cannon lurches backward upon being fired. As the cannon is fired, gravity continues to act on the cannonball. Although there is drag/air resistance on a cannonball, we will ignore that force for this instant at which it is fired.



ASSESS: Note that the force from the cannon on the cannonball is much larger than the cannonball's weight.

P4.37. **PREPARE:** This problem involves vector addition. We add vectors "tip to tail." **(a)** To find $\vec{A}+\vec{B}$, we place the tail of vector $\vec{B}$ on the tip of vector $\vec{A}$ and then connect vector $\vec{A}$'s tail with vector $\vec{B}$'s tip. **(b)** To find $\vec{A}-\vec{B}$, we note that $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$. We place the tail of vector $-\vec{B}$ on the tip of vector $\vec{A}$ and then connect vector $\vec{A}$'s tail with the tip of vector $-\vec{B}$.

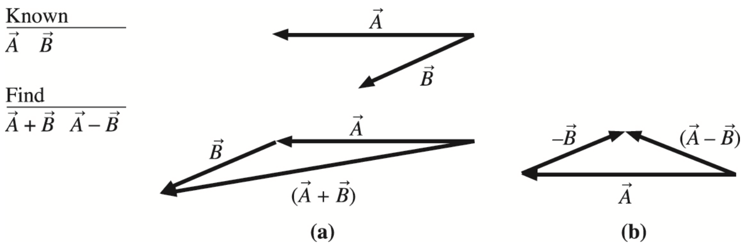
SOLVE:



ASSESS: Since the vectors are perpendicular, or nearly so, it is reasonable that the lengths of their sum and their difference are equal in magnitude.

P4.38. PREPARE: **(a)** To find $\vec{A}+\vec{B}$, we place the tail of vector $\vec{B}$ on the tip of vector $\vec{A}$ and connect the tail of vector $\vec{A}$ with the tip of vector $\vec{B}$. **(b)** Since $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$, we place the tail of the vector $(-\vec{B})$ on the tip of vector $\vec{A}$ and then connect the tail of vector $\vec{A}$ with the tip of vector $(-\vec{B})$.

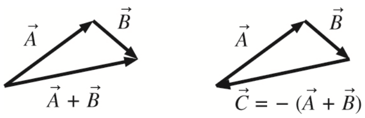
SOLVE:



ASSESS: Since the vectors are pointing in similar directions, it makes sense that their sum would have a greater magnitude than their difference.

P4.39. PREPARE: In order for the sum of three vectors to be zero, adding them tip to tail must bring us back to the point where we started. We can draw $\vec{A}+\vec{B}$, and then sketch in the vector that brings us back to where $\vec{A}$ began.

SOLVE: Figure P4.39 (a) shows the copied vectors, (b) shows the correct tip-to-tail addition $\vec{A}+\vec{B}$, and (c) shows the vector $\vec{C}$ required to make $\vec{A}+\vec{B}+\vec{C}=0$.



ASSESS: It is particularly clear that this sum is zero, if one thinks of the arrows as displacement vectors. The three displacements clearly would return a person to the starting point.

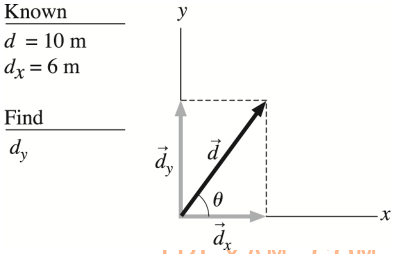
P4.40. **PREPARE:** Diagrammatically we add vectors tip-to-tail. However, since the vectors are orthogonal, we can determine the magnitude of their sum using the Pythagorean Theorem. Let us call $\vec{C}$ the resultant sum of $\vec{A}+\vec{B}$.

SOLVE: For the special case of orthogonal vectors, we can write

$$A^2 + B^2 = C^2 \Rightarrow B = \sqrt{C^2 - A^2} = \sqrt{(2)^2 - (1)^2} = \sqrt{3} \approx 1.7$$

ASSESS: Note that adding a vector of length 1 and of 1.7 could never yield anything smaller than 0.7 and could never yield anything greater than 2.7. Thus, our claim that the sum has magnitude 2 is reasonable.

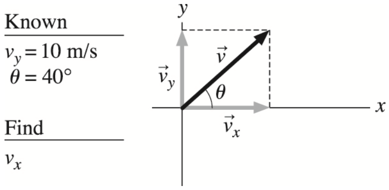
P4.41. **PREPARE:** This problem involves two orthogonal legs of a triangle. In right triangles, we can use trigonometric functions. The position vector $\vec{d}$ whose magnitude d is 10 m has an x-component of 6 m. It makes an angle θ with the $+x$-axis in the first quadrant. We will use trigonometric relations to find the y-component of the position vector.



SOLVE: Using trigonometry, $d_x = d\cos\theta$ or $6 \text{ m} = (10 \text{ m})\cos\theta$. This gives $\theta = 53.1°$. Thus the y-component of the position vector $\vec{d}$ is $d_y = d\sin\theta = (10 \text{ m})\sin 53.1° = 8.0 \text{ m}$.

ASSESS: The y-component is positive since the position vector is in the first quadrant.

P4.42. **PREPARE:** This problem involves two orthogonal legs of a triangle. In right triangles, we can use trigonometric functions. The figure below shows the components v_x and v_y, and the angle θ. We will use Tactics Box 4.4 to find the sign attached to the components of a vector.



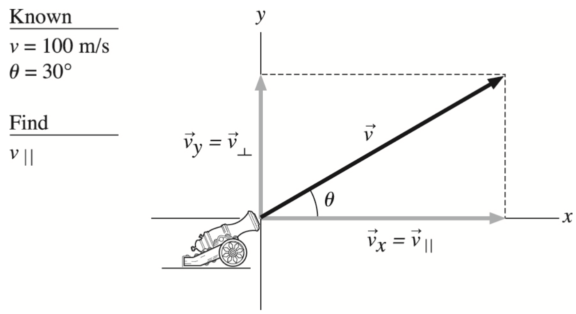
SOLVE: We have,

$$v_y = v\sin 40°, \quad \text{or} \quad 10 \text{ m/s} = v\sin 40°, \text{ or } v = 15.56 \text{ m/s}.$$

Thus, the x-component is $v_x = v\cos 40° = (15.56 \text{ m/s})\cos 40° = 12 \text{ m/s}$.

ASSESS: Note that we had to insert the minus sign manually with v_y since the vector is in the fourth quadrant.

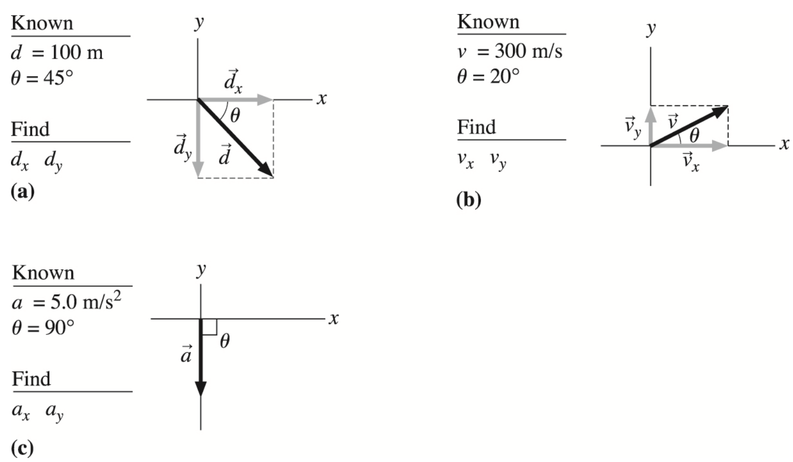
P4.43. **PREPARE:** This problem involves a right triangle. In right triangles, we can use trigonometric functions. The figure below shows the components $v_{\parallel}$ and $v_{\perp}$, and the angle θ. We will use Tactics Box 4.4 to find the sign attached to the components of a vector.



SOLVE: We have $\vec{v} = \vec{v}_x + \vec{v}_y = \vec{v}_{\parallel} + \vec{v}_{\perp}$. Thus, $v_{\parallel} = v \cos\theta = (100 \text{ m/s}) \cos 30° = 87 \text{ m/s}$.

ASSESS: For the small angle of 30°, the obtained value of 87 m/s for the horizontal component is reasonable.

P4.44. **PREPARE:** We will follow rules given in Tactics Box 4.4, using appropriate trigonometric functions to determine components of vectors.



SOLVE: **(a)** Vector $\vec{d}$ points to the right and down, so the components d_x and d_y are positive and negative, respectively:

$$d_x = d \cos\theta = (100 \text{ m}) \cos 45° = 71 \text{ m} \quad d_y = -d \sin\theta = -(100 \text{ m}) \sin 45° = -71 \text{ m}$$

(b) Vector $\vec{v}$ points to the right and up, so the components v_x and v_y are both positive:

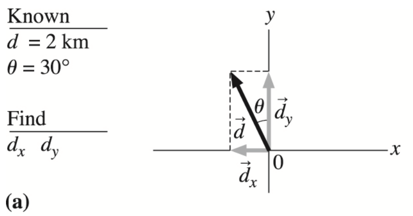
$$v_x = v \cos\theta = (300 \text{ m/s}) \cos 20° = 280 \text{ m/s} \quad v_y = v \sin\theta = (300 \text{ m/s}) \sin 20° = 100 \text{ m/s}$$

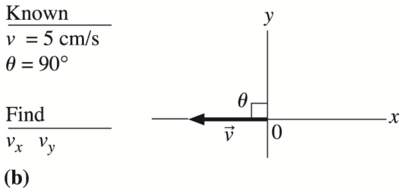
(c) Vector $\vec{a}$ has the following components:

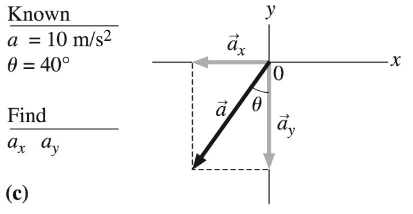
$$a_x = -a \cos\theta = -(5.0 \text{ m/s}^2) \cos 90° = 0.0 \text{ m/s}^2 \quad a_y = -a \sin\theta = -(5.0 \text{ m/s}^2) \sin 90° = -5.0 \text{ m/s}^2$$

ASSESS: The components have the same units as the vectors. Note the minus signs we have manually inserted according to Tactics Box 4.4.

P4.45. **PREPARE:** Start by drawing each of the given vectors. Then use trigonometry to determine the components.







SOLVE: **(a)** $d_x = -(2\text{ km})\sin 30° = -1.0\text{ km}$ $d_y = (2\text{ km})\cos 30° = 1.7\text{ km}$

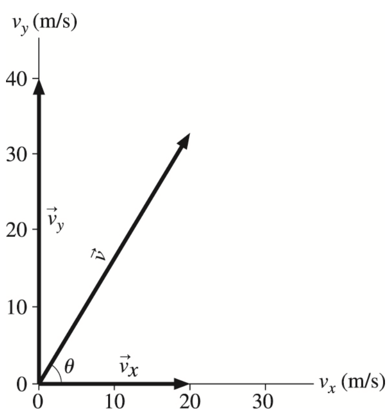
(b) $v_x = -(5\text{ cm/s})\sin 90° = -5.0\text{ cm/s}$ $v_y = (5\text{ cm/s})\cos 90° = 0\text{ cm/s}$

(c) $a_x = -(10\text{ m/s}^2)\sin 40° = -6.4\text{ m/s}^2$ $a_y = -(10\text{ m/s}^2)\cos 40° = -7.7\text{ m/s}^2$

ASSESS: The components have the same units as the vectors. Note the minus signs we have manually inserted according to Tactics Box 4.4.

P4.46. **PREPARE:** We will draw the vectors to scale and label the angles from the positive *x*-axis (positive angles go counterclockwise). We also use Equations 4.9 and 4.10. Make sure your calculator is in degree mode.

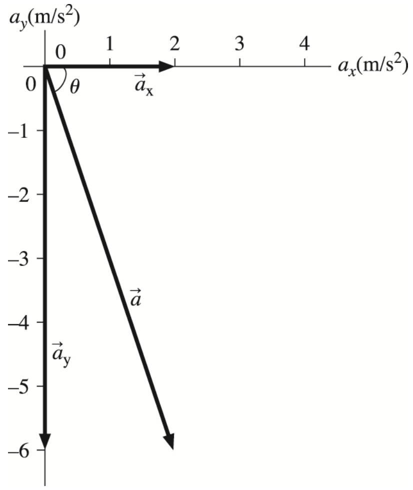
SOLVE: **(a)**



$$v = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(20\text{ m/s})^2 + (40\text{ m/s})^2} = 45\text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{40\text{ m/s}}{20\text{ m/s}}\right) = \tan^{-1}(2) = 63°$$

(b)



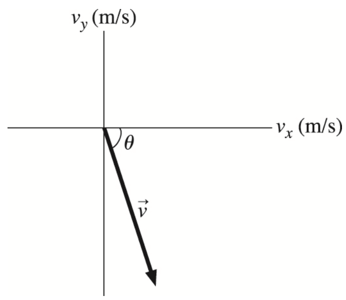
$$a=\sqrt{(a_x)^2+(a_y)^2}=\sqrt{(2.0\text{ m/s}^2)^2+(-6.0\text{ m/s}^2)^2}=6.3\text{ m/s}^2$$

$$\theta=\tan^{-1}\left(\frac{a_y}{a_x}\right)=\tan^{-1}\left(\frac{-6.0\text{ m/s}^2}{2.0\text{ m/s}^2}\right)=\tan^{-1}(-3)=-72°$$

ASSESS: In each case the magnitude is longer than either component, as is required for the hypotenuse of a right triangle. The negative angle in part (b) corresponds to a clockwise direction from the positive *x*-axis.

P4.47. PREPARE: We can use Equations 4.8 and 4.9 to find the magnitude and direction of a vector given its components.

SOLVE: (a) See the following diagram.



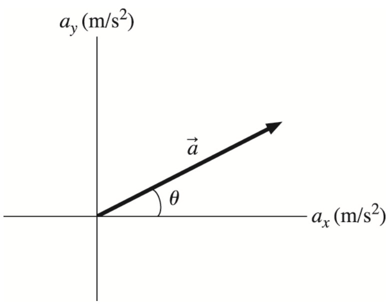
Using Equation 4.8,

$$v=\sqrt{(v_x)^2+(v_y)^2}=\sqrt{(10\text{ m/s})^2+(-30\text{ m/s})^2}=32\text{ m/s}$$

Using Equation 4.9,

$$\theta=\tan^{-1}\left(\frac{|v_y|}{v_x}\right)=\tan^{-1}\left(\frac{30\text{ m/s}}{10\text{ m/s}}\right)=72°$$

(b) See the following diagram.



Using Equation 4.8,

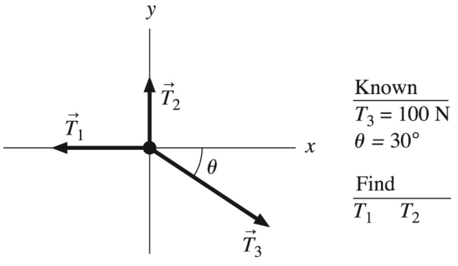
$$a = \sqrt{(a_x)^2 + (a_y)^2} = \sqrt{(20\ \text{m/s}^2)^2 + (10\ \text{m/s}^2)^2} = 22\ \text{m/s}^2$$

Using Equation 4.9,

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{10\ \text{m/s}^2}{20\ \text{m/s}^2}\right) = 27°$$

ASSESS: The vector diagrams are consistent with the calculations.

P4.48. PREPARE: This problem involves forces in two dimensions. We will separate the forces into components and use Newton's second law. The massless ring is in static equilibrium, so all the forces acting on it must cancel to give a zero net force. The forces acting on the ring are shown on a free-body diagram below.



SOLVE: Written in component form, Newton's first law is

$$(F_{\text{net}})_x = \Sigma F_x = T_{1x} + T_{2x} + T_{3x} = 0\ \text{N} \quad (F_{\text{net}})_y = \Sigma F_y = T_{1y} + T_{2y} + T_{3y} = 0\ \text{N}$$

Evaluating the components of the force vectors from the free-body diagram,

$$T_{1x} = -T_1 \quad T_{2x} = 0\ \text{N} \quad T_{3x} = T_3 \cos 30°$$

$$T_{1y} = 0\ \text{N} \quad T_{2y} = T_2 \quad T_{3y} = -T_3 \sin 30°$$

Using Newton's first law,

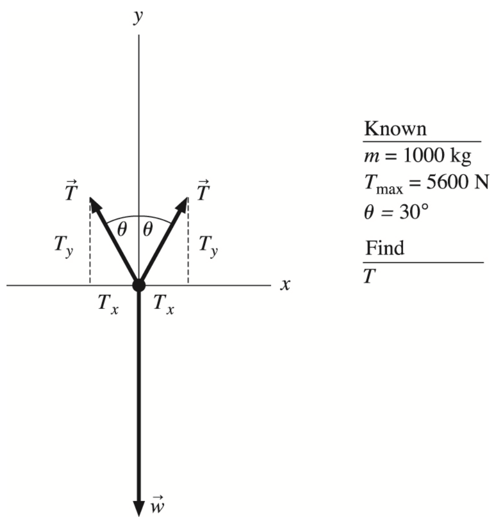
$$-T_1 + T_3 \cos 30° = 0\ \text{N} \quad T_2 - T_3 \sin 30° = 0\ \text{N}$$

Rearranging

$$T_1 = T_3 \cos 30° = (100\ \text{N})(0.8666) = 87\ \text{N} \quad T_2 = T_3 \sin 30° = (100\ \text{N})(0.5) = 50\ \text{N}$$

ASSESS: Since $\vec{T}_3$ acts closer to the x-axis than to the y-axis, it makes sense that $T_1 > T_2$.

P4.49. PREPARE: This problem involves forces in two dimensions. We separate the forces into components and use Newton's second law. The forces acting on the beam are shown on a free-body diagram below. You can model the beam as a particle and assume $\vec{F}_{\text{net}} = \mathbf{0}$ N to calculate the tensions in the suspension ropes.



SOLVE: The beam attached to the ropes will remain in static equilibrium only if $T < T_{\max}$, where $T_{\max}$ is the maximum sustained tension. The equilibrium equations in vector and component form are as follows:

$$\vec{F}_{net} = \vec{T} + \vec{T} + \vec{w} = \vec{0}\ \text{N}$$
$$(F_{\text{net}})_x = T_x + T_x = 0\ \text{N}$$
$$(F_{\text{net}})_y = T_y + T_y + w_y = 0\ \text{N}$$

Using the free-body diagram and $w = mg$ yields

$$2T\cos\theta - w = 0\ \text{N} \Rightarrow T = \frac{mg}{2\cos\theta} = \frac{(1000\ \text{kg})(9.8\ \text{m/s}^2)}{2\cos 30°} = 5660\ \text{N}$$

So, the ropes break.

ASSESS: The above approach and result seem reasonable.

P4.50. PREPARE: This problem involves forces acting vertically, and one of those forces is drag. We remind ourselves that drag always acts opposite the direction of motion. In all three cases (stationary and lowered or raised at a steady rate) the acceleration is zero, so the net force must be zero. In the stationary case,

$$F_{\text{net}} = T + F_{\text{buoy}} - mg = 0 \Rightarrow T = mg - F_{\text{buoy}} = 6000\ \text{N}.$$

SOLVE: (a) When the craft is being lowered, the drag is upward, so it is positive in the equation.

$$F_{\text{net}} = T + F_{\text{buoy}} + D - mg = 0 \Rightarrow T = mg - F_{\text{buoy}} - D = 6000\ \text{N} - 1800\ \text{N} = 4200\ \text{N}$$

(b) When the craft is being raised, the drag is downward, so it is negative in the equation.

$$F_{\text{net}} = T + F_{\text{buoy}} - D - mg = 0 \Rightarrow T = mg - F_{\text{buoy}} + D = 6000\ \text{N} + 1800\ \text{N} = 7800\ \text{N}$$

ASSESS: The buoyant force means the tension in the stationary case is less than the actual weight of the craft, but it contributes the same amount in each of the three cases.

P4.51. **PREPARE:** This is an application of Newton's second law, in two dimensions. We can separate forces into components as needed and use the sum of all forces in the x-direction. The free-body diagram shows three forces acting on an object whose mass is 2.0 kg. The force in the first quadrant has two components: 4 N along the x-axis and 3 N along the y-axis.

Solve: Applying Newton's second law to the diagram on the left

$$a_x = \frac{(F_{\text{net}})_x}{m} = \frac{4\text{ N} - 2\text{ N}}{2\text{ kg}} = 1.0\text{ m/s}^2$$

ASSESS: The object's acceleration is only along the x-axis.

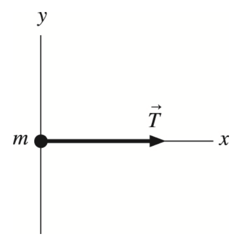
P4.52. **PREPARE:** This is an application of Newton's second law, in two dimensions. There is no need to consider components, since all forces lie along x- or y-axes. The free-body diagram shows five forces acting on an object whose mass is 2.0 kg. We will first find the net force along the x- and the y-axes and then divide these forces by the object's mass to obtain the x- and y-components of the object's acceleration.

SOLVE: Applying Newton's second law,

$$a_x = \frac{(F_{\text{net}})_x}{m} = \frac{4\text{ N} - 2\text{ N}}{2\text{ kg}} = 1.0\text{ m/s}^2 \qquad a_y = \frac{(F_{\text{net}})_y}{m} = \frac{3\text{ N} - 1\text{ N} - 2\text{ N}}{2\text{ kg}} = 0.0\text{ m/s}^2$$

ASSESS: The object's acceleration is only along the x-axis.

P4.53. **PREPARE:** We assume that the box is a particle being pulled in a straight line. Since the ice is frictionless, the tension in the rope is the only horizontal force on the box and is shown below in the free-body diagram. Since we are looking at horizontal motion of the box, we are not interested in the vertical forces in this problem.



SOLVE: (a) Since the box is at rest, $a_x = 0\text{ m/s}^2$, the net force on the box must be zero or the tension in the rope must be zero.

(b) For this situation again, $a_x = 0\text{ m/s}^2$, so $\boldsymbol{F_{\text{net}} = T = 0\text{ N}}$.

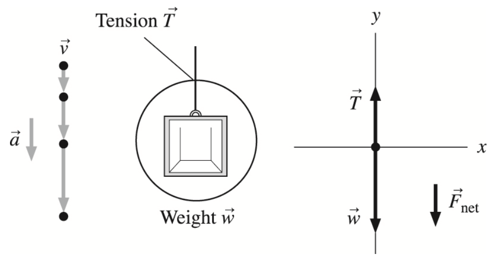
(c) Here, the velocity of the box is irrelevant, since only a *change* in velocity requires a nonzero net force. Since $a_x = 5.0\text{ m/s}^2$,

$$\boldsymbol{F_{\text{net}} = T = ma_x = (50\text{ kg})(5.0\text{ m/s}^2) = 250\text{ N}}$$

ASSESS: For parts (a) and (b), the zero acceleration immediately implies that the rope is exerting no horizontal force on the box. For part (c), the 250 N force (the equivalent of about half the weight of a small person) seems reasonable to accelerate a box of this mass at 5.0 m/s^2.

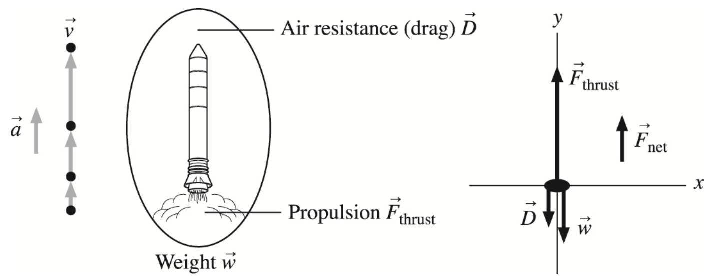
P4.54. PREPARE: There are two forces acting *on* the elevator due to its interactions with the two agents the earth and the cable. One of the forces *on* the elevator is the long-range weight force *by* the earth. Another force is the tension force exerted *by* the cable due to the contact between the elevator and the cable. Since the elevator is speeding up as it descends, its acceleration is pointing downward. Tension is the only contact force. The downward acceleration implies that $w > T$. Therefore, the net force on the elevator must also point downward.

SOLVE: A force-identification diagram, a motion diagram, and a free-body diagram are shown.



ASSESS: Note that if the elevator had been moving downward at a constant speed, the tension would need to be the same magnitude as the weight. Only because the elevator is speeding up as it descends, we know the net force must be downward.

P4.55. PREPARE: There are three forces acting *on* the rocket due to its interactions with the three agents the earth, the air, and the hot gases exhausted to the environment. One force on the rocket is the long-range weight force *by* the earth. The second force is the drag force *by* the air. The third is the thrust force exerted on the rocket *by* the hot gas that is being let out to the environment. Since the rocket is being launched upward, it is being accelerated upward. Therefore, the net force on the rocket must also point upward. Draw a picture of the situation, identify the system, in this case the rocket, and draw a motion diagram. Draw a closed curve around the system, and name and label all relevant contact forces and long-range forces.



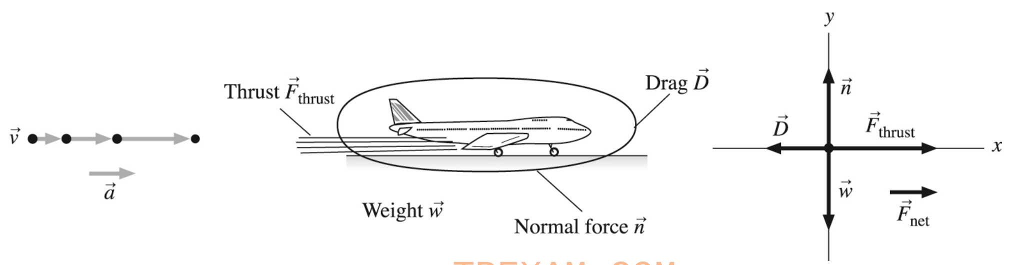
SOLVE: A force-identification diagram, a motion diagram, and a free-body diagram are shown.

ASSESS: It is reasonable that the net force is upward. In the launch of a rocket, the thrust must greatly exceed drag and weight, so that the rocket can accelerate to very high speeds.

P4.56. **PREPARE:** The normal force is perpendicular to the ground. The thrust force is parallel to the ground and in the direction of acceleration. The drag force is opposite to the direction of motion. There are four forces acting *on* the jet plane due to its interactions with the four agents the earth, the air, the ground, and the hot gases exhausted to the environment. One force on the rocket is the long-range weight force *by* the earth. The second force is the drag force *by* the air. Third is the normal force on the rocket *by* the ground. The fourth is the thrust force exerted on the jet plane *by* the hot gas that is being let out to the environment. Since the jet plane is speeding down the runway, its acceleration is pointing to the right. Therefore, the net force on the jet plane must also point to the right.

Now, draw a picture of the situation, identify the system, in this case the jet plane, and draw a motion diagram. Draw a closed curve around the system, and name and label all relevant contact forces and long-range forces.

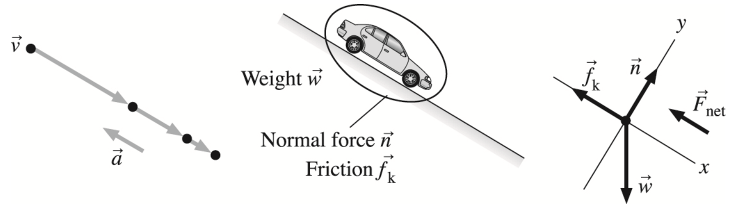
SOLVE: A force-identification diagram, a motion diagram, and a free-body diagram are shown.



ASSESS: Since the plane is still on the runway, the net upward acceleration should be zero. This fits with our free body diagram, since the sum of all forces in the vertical direction is zero.

P4.57. **PREPARE:** The normal force is perpendicular to the hill. The frictional force is parallel to the hill.

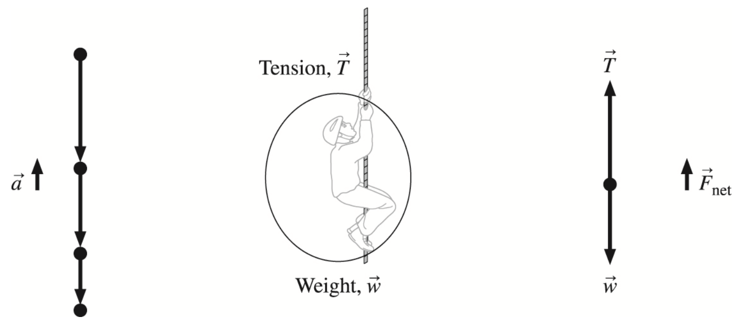
SOLVE: A force-identification diagram, a motion diagram, and a free-body diagram are shown.



ASSESS: You now have three important tools in your "Physics Toolbox," motion diagrams, force diagrams, and free-body diagrams. Careful use of these tools will give you an excellent conceptual understanding of a situation.

P4.58. **PREPARE:** We will draw a motion diagram with downward velocity vectors that decrease in size as the explorer descends. Our force identification diagram should include the force applied by the rope or chain and gravity. These should also be reflected in our free-body diagram.

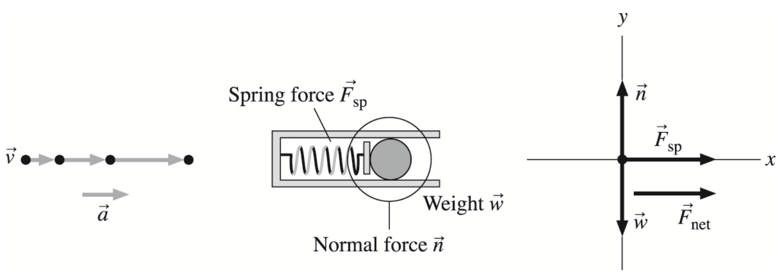
SOLVE:



ASSESS: Note that the upward force from the rope must be greater than the downward weight force in order for the acceleration to be upward.

P4.59. PREPARE: The ball rests on the floor of the barrel because the weight is equal to the normal force. There is a force of the spring to the right, which causes acceleration. Now, draw a picture of the situation, identify the system, in this case the plastic ball, and draw a motion diagram. Draw a closed curve around the system, and name and label all relevant contact forces and long-range forces. Neglect friction.

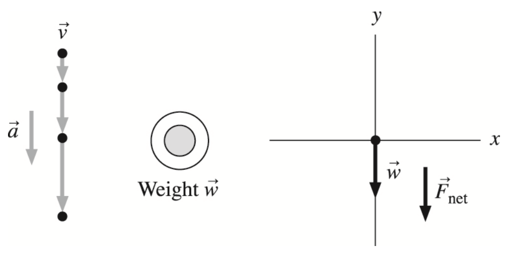
SOLVE: A force-identification diagram, a motion diagram, and a free-body diagram are shown.



ASSESS: Since the normal force acting on the ball and the weight of the ball are equal in magnitude and opposite in direction, the ball experiences no vertical motion.

P4.60. PREPARE: There are no contact forces on the rock. There is only one force acting *on* the rock due to its interaction with one agent, the earth. Since weight force points down, the acceleration must also point downward. Now, draw a picture of the situation, identify the system, in this case the rock, and draw a motion diagram. Draw a closed curve around the system, and name and label all relevant contact forces and long-range forces.

SOLVE: A force-identification diagram, a motion diagram, and a free-body diagram are shown.



ASSESS: If there is only one force acting on an object, that will be the net force.

P4.61. **PREPARE:** This problem involves vector addition/subtraction. We will break both vectors into components in the x and y directions, using trigonometry.

SOLVE: In terms of components, we have $A_x + B_x + C_x = 0 \Rightarrow C_x = -A_x - B_x$. Similarly, $C_y = -A_y - B_y$.

Using trigonometry, we can write these expressions in terms of magnitudes and angles:

$$C_x = -(4.0\text{ m})\cos(60°) - (3.0\text{ m})\cos(20°) = -4.8\text{ m}$$

$$C_y = -(4.0\text{ m})\sin(60°) - (-(3.0\text{ m})\sin(20°)) = -2.4\text{ m}$$

So $\vec{C} = (-4.8\text{ m}, -2.4\text{ m})$.

ASSESS: It is clear that the vector sum of $\vec{A} + \vec{B}$ would be in the upper right quadrant. It is therefore reasonable that a vector intended to cancel out $\vec{A} + \vec{B}$ would need to be in the lower left quadrant, which is what we found.

P4.62. **PREPARE:** This problem involves vector addition/subtraction. We will break both vectors into components in the x - and y -directions, using trigonometry.

SOLVE: In terms of components, we have $D_x = 2A_x + 3B_x$. Similarly, $D_y = A_y + B_y$. Using trigonometry, we can write these expressions in terms of magnitudes and angles:

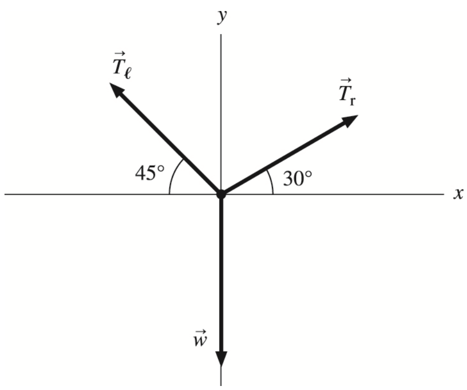
$$D_x = 2(2.0\text{ m})\sin(15°) + 3(4.0\text{ m})\cos(15°) = 12.6\text{ m}$$

$$D_y = 2(2.0\text{ m})\cos(15°) + 3(-4.0\text{ m})\sin(15°) = 0.8\text{ m}$$

So, $\vec{D} = (12.6\text{ m}, 0.8\text{ m})$.

ASSESS: Both vectors point toward the right. So we expect the weighted sum of these vectors to still have a large component to the right, D_x . One has an upward component and one has a downward component. We expect their vertical components to partially cancel such that he final vertical component D_y is small, relative to the horizontal component. This is what we found.

P4.63. PREPARE: Let us call the vertically upward direction $+y$ and the direction horizontally to the right $+x$. We will apply Newton's second law to both of these directions. We will refer to the tension in the rope that makes a $45°$ angle (taken to be the right-most rope) as T_r and that in the left rope as T_l. We will start by drawing a free-body diagram.



SOLVE: Because Bethany is not accelerating, we know that the horizontal and vertical components must sum to zero. We have

$$\sum F_x = T_r\cos(45°) - T_l\cos(30°) = ma_x = 0$$
$$\sum F_y = T_r\sin(45°) + T_l\sin(30°) - w = ma_y = 0$$

The first equation tells us $T_r = T_l\dfrac{\cos(30°)}{\cos(45°)}$, which we then insert into the second equation to obtain

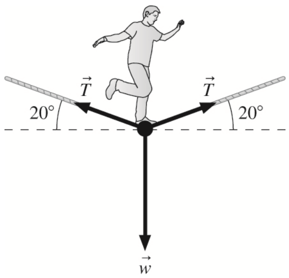
$$T_l\big(\cos(30°)\tan(45°)+\sin(30°)\big) = w$$
$$T_l = \frac{w}{\big(\cos(30°)\tan(45°)+\sin(30°)\big)} = \frac{(560\ \text{N})}{\big(\cos(30°)\tan(45°)+\sin(30°)\big)}$$
$$T_l = 4.1\times10^2\ \text{N}$$

Inserting this value (409.9 N, without intermediate rounding) into our expression above for T_r, we find

$$T_r = (409.9\ \text{N})\frac{\cos(30°)}{\cos(45°)} = 5.0\times10^2\ \text{N}.$$

ASSESS: A cursory review of these magnitudes shows that the sum of the magnitudes is larger than Bethany's weight, which must be the case since only one component of each force supports her against gravity.

P4.64. **PREPARE:** This problem involves forces in two dimensions. We separate the forces into components and use Newton's second law. We remind ourselves that the tension in a rope (or in this case slackline) acts along its length. And in cases such as this where the line is bent by some object in the middle, the tension acts twice, just as though there were two ropes (one on the left and one on the right). The piece of rope under the student's foot is in static equilibrium, so the net force is zero. The sum of the forces in the y-direction must be zero.

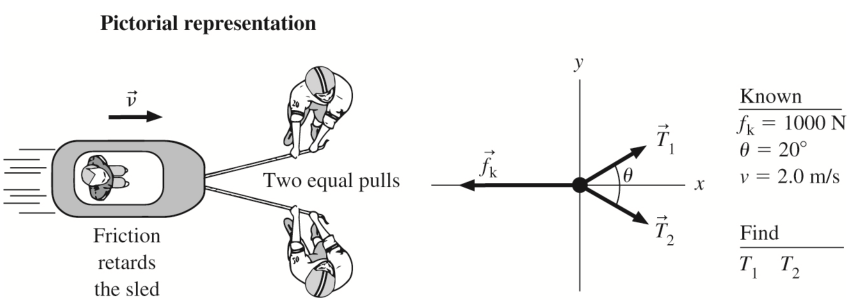


SOLVE: The only forces on the piece of rope under the student's foot are the two tension forces and the downward weight of the student.

$$\Sigma F_y = 2T\sin\theta - mg = 0 \Rightarrow T = \frac{mg}{2\sin\theta} = \frac{(65\text{ kg})(9.80\text{ m/s}^2)}{2\sin 20°} = 930\text{ N}$$

ASSESS: The tension is greater than the student's weight, so the slackline needs to be sturdy.

P4.65 **PREPARE:** We will treat the coach and his sled like a particle being towed at a constant velocity by the two ropes, with friction resisting the pull. We start by making a sketch of the setup and a free-body diagram for the coach/sled combined object.



SOLVE: Since the sled is not accelerating, it is in dynamic equilibrium and Newton's first law applies:

$$(F_{net})_x = \sum F_x = T_{1x} + T_{2x} + f_{kx} = 0\text{ N} \qquad (F_{net})_y = \sum F_y = T_{1y} + T_{2y} + f_{ky} = 0\text{ N}$$

From the free-body diagram,

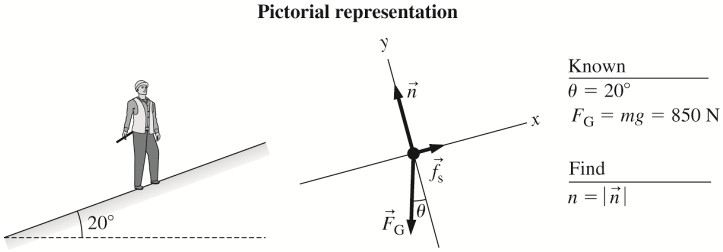
$$T_1\cos\left(\frac{1}{2}\theta\right) + T_2\cos\left(\frac{1}{2}\theta\right) - f_k = 0\text{ N} \qquad T_1\sin\left(\frac{1}{2}\theta\right) - T_2\sin\left(\frac{1}{2}\theta\right) + 0\text{ N} = 0\text{ N}$$

From the second of these equations $T_1 = T_2$. Then from the first:

$$2T_1\cos 10° = 1000\text{ N} \Rightarrow T_1 = \frac{1000\text{ N}}{2\cos 10°} = \frac{1000\text{ N}}{1.970} = 508\text{ N} \approx 510\text{ N}$$

ASSESS: The two tensions are equal, as expected, since the two players are pulling at the same angle. It is reasonable that each force is slightly more than half the 1000 N frictional force. Only the forward (x) component of the pulling forces should add to 1000 N; they should be slightly larger when the y components are taken into account.

P4.66 **PREPARE:** We will model the worker as a particle. In equilibrium the net force is zero both parallel and perpendicular to the roof. Note that there must be a static friction force to keep her from sliding off. We begin by making a sketch of the setup and drawing a careful free-body diagram. We can then use Newton's second law to set the sum of all forces perpendicular to the roof to zero.

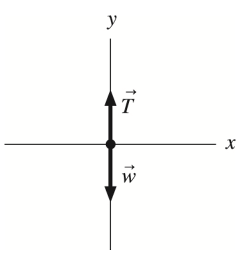


SOLVE: We only need to examine the y-direction.

$$(\Sigma F)_y = n - mg\cos\theta = 0 \Rightarrow n = mg\cos\theta = (850\text{ N})(\cos 20°) = 799\text{ N} \approx 800\text{ N}$$

ASSESS: A good way to assess solutions like this is to consider what happens in the limit as $\theta \to 0$ and as $\theta \to 90°$. In the first case, $n \to mg$, and in the second, $n \to 0$, as expected.

P4.67. **PREPARE:** To solve this problem, we can apply Newton's second law and solve for tension. The box is acted on by two forces: the tension in the rope and the pull of gravity. Both the forces act along the same vertical line, which is taken to be the y-axis. The free-body diagram for the box is shown below.



SOLVE: (a) Since the box is at rest, $a_y = 0\text{ m/s}^2$ and the net force on it must be zero:

$$F_{net} = T - w = 0\text{ N} \Rightarrow T = w = mg = (50\text{ kg})(9.80\text{ m/s}^2) = 490\text{ N}$$

(b) The velocity of the box is irrelevant, since only a *change* in velocity requires a nonzero net force. Since $\boldsymbol{a_y = 5.0\text{ m/s}^2}$,

$$\boldsymbol{F_{net} = T - w = ma_y = (50\text{ kg})(5.0\text{ m/s}^2) = 250\text{ N} \Rightarrow T = 250\text{ N} + w = 250\text{ N} + 490\text{ N} = 740\text{ N}}$$

ASSESS: For part (a) the zero acceleration immediately implies that the box's weight must be exactly balanced by the upward tension in the rope. For part (b) the tension not only has to support the box's weight but must also accelerate it upward, hence, T must be greater than w.

P4.68. PREPARE: We'll first use Newton's second law $\left(F_{x,\text{net}} = ma_x\right)$ to find the acceleration of the car, then the kinematic equations to find the stopping distance. The given values are $m = 1500$ kg, $f_\text{k} = 7200$ N, $v_\text{i} = 20$ m/s, and $v_\text{f} = 0$ m/s.

SOLVE: The acceleration of the car is $a = \dfrac{F}{m} = \dfrac{f_\text{k}}{m} = \dfrac{7200\text{ N}}{1500\text{ kg}} = 4.8\text{ m/s}^2$. The distance required to stop is computed from the kinematic equation: $v_\text{f}^2 - v_\text{i}^2 = 2a\Delta x \Rightarrow \Delta x = \dfrac{v_\text{f}^2 - v_\text{i}^2}{2a} = \dfrac{(20\text{ m/s})^2 - (0\text{ m/s}^2)}{2(4.8\text{ m/s}^2)} = 42$ m.

ASSESS: 42 m is a longer stopping distance than we would want in an emergency, but it seems plausible given the friction from the gravel. If the friction were greater, the acceleration would be greater and the stopping distance less.

P4.69. PREPARE: Ignoring drag, there is only thrust in the horizontal direction. Thus, the given 0.65 N thrust is also the net force in the x-direction. We are given $m = 1.1$ kg, so we can determine the acceleration using Equation 4.3. We are then given $v_{x,\text{i}} = 0.45$ m/s and $\Delta x = 1.0$ m sufficient kinematic information to use Equation 3.23 to determine $v_{x,\text{f}}$.

SOLVE: The acceleration is $a_x = F_{x,\text{net}}/m = (0.65\text{ N})/(1.1\text{ kg}) = 0.591\text{ m/s}^2$. Equation 3.23 then yields

$$v_{x,\text{f}} = \sqrt{v_{x,\text{i}}^2 + 2a_x\Delta x} = \sqrt{(0.45\text{ m/s})^2 + 2\left(0.591\text{ m/s}^2\right)(1.0\text{ m})} = 1.2\text{ m/s}$$

ASSESS: It is reasonable that a fish could swim at a speed of order 1 m/s, and it is reasonable that the fish could double its speed over a fairly short distance.

P4.70. PREPARE: Force can be found from the graph of acceleration using Newton's second law and the given mass of the greyhound. The distance travelled when the dog reaches 4.0 m/s can be found using kinematic equations, since the acceleration is constant over that interval.

SOLVE: (a) The ground is the agent of force. The greyhound exerts a force on the ground and the ground exerts a force back on the greyhound; the latter is the force that propels the greyhound forward.

(b) Assuming the force the ground exerts on the greyhound is the only force in the horizontal direction acting on the greyhound, we can write

$$\sum F_x = F_{\text{ground greyhound}} = ma_x = (32\text{ kg})\left(10\text{ m/s}^2\right) = 320\text{ N}$$

(c) Note that until the greyhound reaches 4.0 m/s, the acceleration is a constant 10m/s^2. Using the definition of average acceleration, we find the time required for this is $\Delta t = \Delta v_x / a_x = (4.0\text{ m/s})/\left(10\text{m/s}^2\right) = 0.40$ s. Combining this with the fact that the greyhounds start from rest, kinematic equations tell us

$$\Delta x = (v_x)_\text{i}\Delta t + \frac{1}{2}a_x(\Delta t)^2 = \frac{1}{2}\left(10\text{m/s}^2\right)(0.40\text{ s})^2 = 0.80\text{ m}$$

ASSESS: This is a reasonable distance for extremely fast racing dogs.

P4.71. PREPARE: We are given sufficient kinematic information to use Equation 3.23 to determine acceleration. Then Equation 4.3 will be used to find the average force.

SOLVE: Rearranging Equation 3.23, we have

$$a_x = \frac{v_{x,\text{f}}^2 - v_{x,\text{i}}^2}{2\Delta x}$$

Such that, using Equation 4.3,

$$F_{x,\text{net}} = ma_x = m\left(\frac{v_{x,\text{f}}^2 - v_{x,\text{i}}^2}{2\Delta x}\right) = (0.150\ \text{kg})\left(\frac{(47\ \text{m/s})^2 - 0}{2(1.0\ \text{m})}\right) = 1.7\times10^2\ \text{N}$$

ASSESS: This is about 20% the weight of a 180 lbs baseball player. It is reasonable for an athlete to exert a force equal to 20% of his/her body weight. What is exceptional is being able to keep the arm and hand moving quickly enough to keep up with the ball, as it accelerates.

P4.72. PREPARE: In part (a), we use the fact that acceleration is the rate of change of the velocity. In part (b) we will determine the force using Equation 4.3 and the results from part (a). Part (c) is a simple comparison with $w = mg$.

SOLVE: (a) The largest acceleration corresponds to the largest slope on the plot. This happens between $t = 40\ \mu\text{s}$ and $t = 60\ \mu\text{s}$ as follows:

$$a_x = \frac{v_{x,\text{f}} - v_{x,\text{i}}}{\Delta t} = \frac{(40\ \text{m/s}) - (5\ \text{m/s})}{(60\ \mu\text{s}) - (40\ \mu\text{s})} = 1.8\times10^6\ \text{m/s}^2$$

(b) In this context, the force we are asked for is the only force exerted in the direction of the mandible's motion. So we can write

$$F_{x,\text{net}} = ma_x = (130\times10^{-9}\ \text{kg})(1.75\times10^6\ \text{m/s}^2) = 0.23\ \text{N}$$

(c) We simply divide

$$\frac{F_{x,\text{net}}}{mg} = \frac{(0.228\ \text{N})}{(14\times10^{-6}\ \text{kg})(9.8\ \text{m/s}^2)} = 1.7\times10^3\ \text{times}$$

ASSESS: This is a staggering force for such a small creature, as indicated in the problem text.

P4.73. PREPARE: We can relate acceleration to force using the mass and Newton's second law. We know the maximum force should be exerted with the maximum acceleration is achieved. The maximum force appears to be about 45 mN.

SOLVE: The sum of all forces in the vertical direction is given by

$$\sum F_y = F_{\text{plate}} - mg = ma_y \Rightarrow a_y = (F_{\text{plate}}/m) - g = ((45\ \text{mN})/(5.0\times10^{-4}\ \text{kg})) - (9.8\ \text{m/s}^2) = 80\ \text{m/s}^2$$

ASSESS: The insect had to push with enough force to cancel out gravity just to remain stationary on the plate. This must be subtracted from the total force to obtain the net force accelerating the locust upward.

P4.74. **Prepare:** Newton's second law relates force to acceleration through mass. In this case, the mass of the tiny section of rope in your hands is very small. Consider what would happen to a small section of rope, if the sum of all forces acting on it were not zero: $a_y = F_{y,\text{net}}/m$ would be huge.

Solve: When we assume the vector sum of the forces on an object is zero, it is usually because we know the acceleration of the object is zero.

The correct choice is C.

Assess: The other choices don't really have much to do with whether the vector sum of the forces on an object is zero.

P4.75. **Prepare:** The direction of a tension force is always along the rope that is tense. Tension force is discussed in Section 4.2. We will use Newton's third law to find the force on the rope.

Solve: In the diagram given in the problem, the force that the left part of the rope exerts on you is mostly in the westerly direction, with a small component to the south. The force that you exert on the left portion of the rope is the reaction force to this force and would be in the exact opposite direction, mostly to the east and a little to the north. The rope is connected directly to the tree. As explained in Section 4.2, the tension force is "transmitted" through the rope by the molecular bonds in the rope. So the force on the tree is directly mostly to the east with a small component to the north. The correct choice is C.

Assess: Tension is "transmitted" along a rope by the molecular bonds in the rope.

P4.76. **Prepare:** This problem involves Newton's second law. If the car is not moving over time, then its acceleration is zero. This tells us about the sum of all forces, and can tell us about the directions of the forces involved.

Solve: If the acceleration on an object is zero, then the sum of the forces on the object is zero. In the horizontal direction, the rope is pulling on the car approximately west, and therefore the mud must exert a force on the car approximately east.

The answer is C.

Assess: If there are only two horizontal forces on a stationary object, they must be in opposite directions.

P4.77. **Prepare:** This is an application of Newton's second law. Consider whether there is a net force on the car, and what this would mean for the acceleration.

Solve: The rope exerts a force on the car and the mud exerts a force on the car in the opposite direction. No other forces act in the horizontal direction. If one of these forces were larger than the other, then there would be a net force in one direction, and the car would accelerate. We are told the car moves at a constant speed. Thus, there is no net force; the two forces must be the same magnitude and in opposite directions, such that they cancel. The correct answer is C.

Assess: It is true that the forces could not always have been perfectly equal, since the car did change from being at rest to moving. But the car could certainly start moving and then proceed with a constant speed.

5

INTERACTING SYSTEMS

QUESTIONS

Q5.1 REASON: For an object to be in equilibrium, the net force (i.e., sum of the forces) must be zero. Assume that the two forces mentioned in the question are the only ones acting on the object.

The question boils down to asking if two forces can sum to zero if they aren't in opposite directions. Mental visualization shows that the answer is no, but so does a careful analysis. Set up a coordinate system with the x-axis along one of the forces. If the other force is not along the negative x-axis, then there will be a y-(or z-) component that cannot be canceled by the first one along the x-axis.

ASSESS: In summary, two forces not in opposite directions cannot sum to zero. Neither can two forces with different magnitudes. However, three can.

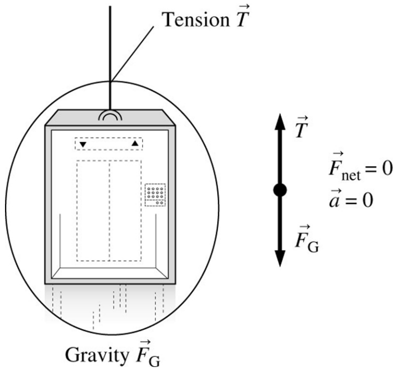
Q5.2 REASON: No. The ball is still changing its speed, and just momentarily has zero velocity.

ASSESS: The force of gravity is certainly always acting on the ball. Even though the velocity is momentarily zero, the acceleration is not, and neither is the sum of all forces.

Q5.3 REASON: Kat is closest to the correct statement, which should read, "Gravity pulls down on it, but the table pushes it up so that the net force on the book is zero."

ASSESS: A very common misunderstanding is that if an object is not accelerating, then no forces can be acting on it. In fact, arbitrarily many forces may be acting on an object in static equilibrium, as long as their vector sum is zero.

Q5.4 REASON:



Equal. The tension in the cable is equal to the force of gravity, since the net force must be zero in order for the elevator to move with constant speed.

ASSESS: If the elevator were accelerating upward from rest, then the tension would need to be greater than the weight.

Q5.5 REASON: If the blocks remain at rest, then clearly the sum of all forces in the horizontal direction must be zero; otherwise, there would be some nonzero acceleration. Thus, in the case of both boxes A and B, the force of static friction is exactly canceling the force applied (along the rope) in the horizontal direction. The frictional force is 30 N to the left on both boxes A and B; they are equal.

ASSESS: It is easy to get confused by applying $(f_s)_{max} = \mu_s n$. But remember that is the expression for the maximum frictional force before the object starts to slip.

Q5.6 REASON: The reading on the moon will be the moon-weight, or the gravitational force of the moon on the astronaut. This would be about 1/6 of the astronaut's earth-weight or the gravitational force of the earth on the astronaut (while standing on the scales on the earth).

ASSESS: The astronaut's *mass* does not change by going to the moon.

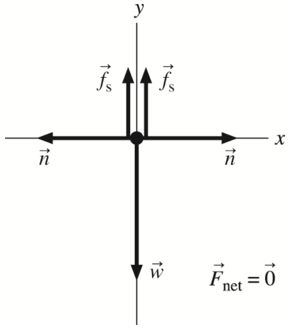
Q5.7 REASON The ball filled with lead is more massive. Since the balls are weightless, the astronaut must measure their inertia (mass) directly. One easy option is to move each ball side to side in turn. More force is required to change the more massive lead-filled ball's direction of motion.

ASSESS: This highlights the difference between weight and mass. Even in deep space where there is no weight force due to gravity, one ball still has more mass, more stuff, and is therefore harder to accelerate.

Q5.8 REASON: During the upward part of the motion, the drag force and weight of the ball both act in the downward direction. During the downward part of the motion, the drag force acts upward while the weight of the ball acts downward. The net force accelerating the ball during the upward motion is greater than during downward motion, so the ball takes a shorter time on the upward part of the trip, so a longer time to fall back down.

ASSESS: The force of air drag always acts opposite the direction of motion of an object.

Q5.9 REASON: Each hand exerts a horizontal normal force on the box, but the two add up to zero. The downward weight force is balanced out by the upward static friction force that each hand exerts.



ASSESS: The net force on the box must be zero since it isn't accelerating.

Q5.10 **REASON:** Use the simple model in Section 5.4 and assume that

$$D \approx \frac{1}{4}\rho A v^2$$

For object 1: $A = 0.20\text{ m} \times 0.30\text{ m} = 0.060\text{ m}^2$; $v^2 = (6\text{ m/s})^2 = 36\text{ m}^2/\text{s}^2$; so $Av^2 = 2.2\text{ m}^4/\text{s}^2$

For object 2: $A = 0.20\text{ m} \times 0.20\text{ m} = 0.040\text{ m}^2$; $v^2 = (6\text{ m/s})^2 = 36\text{ m}^2/\text{s}^2$; so $Av^2 = 1.4\text{ m}^4/\text{s}^2$

For object 3: $A = 0.30\text{ m} \times 0.30\text{ m} = 0.090\text{ m}^2$; $v^2 = (4\text{ m/s})^2 = 16\text{ m}^2/\text{s}^2$; so $Av^2 = 1.4\text{ m}^4/\text{s}^2$

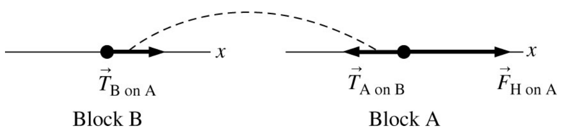
The density of air ρ is the same for all three objects, so it won't affect the ranking.

Therefore, $D_1 > D_2 = D_3$.

Assess: Note that because v is squared, object 3's greater cross-sectional area did not produce the largest drag force.

ASSESS: Note that because v is squared, object 3's greater cross-sectional area did not produce the largest drag force.

Q5.11 **REASON:**



The figure shows the horizontal forces on blocks B and A using the massless-string approximation in the absence of friction. The hand must accelerate both blocks A and B, so more force is required to accelerate the greater mass. Thus, the force of the string on B is smaller than the force of the hand on A.

ASSESS: If there were friction, this would not be as simple. Friction could be slowing the system, such that other forces could be larger than the pushing force from the hand.

Q5.12 **REASON:** Note that Carlos takes the place of the wall, and that the force on the spring is still 200 N. The spring still stretches 20 cm.

ASSESS: This highlights the point that inanimate objects are regularly exerting forces, equivalent to those humans exert. Here, the wall was always exerting a 200 N force on the left side of the spring, to keep it from accelerating away due to Bob's pull.

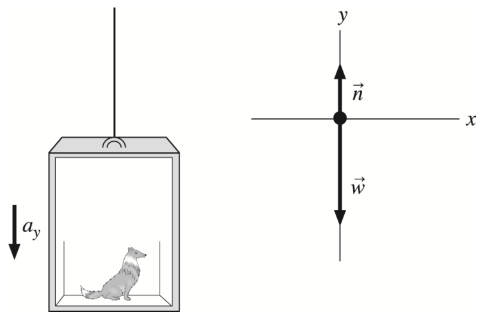
Q5.13 **REASON:** Newton's third law tells us that the forces are equal. They are also clearly equal when Newton's law of gravity is examined: $F_{12} = Gm_1m_2/r^2$ has the same value whether $m_1 = \text{Earth}$ and $m_2 = \text{sun}$ or vice versa.

ASSESS: The effects of the forces can be different in terms of the acceleration they cause, since the earth is so much less massive than the sun.

Q5.14 **REASON:** In this case there is not enough information to tell, because we don't know which way the block would go if the friction were reduced. Think of extreme cases to see this. If Block 1 were much, much more massive than Block 2 it would slide down the ramp if friction were reduced sufficiently; in that case (if the friction weren't reduced) the static would have to be up the ramp to hold Block 1 there. On the other hand, if Block 2 were much more massive than Block 1, then Block 1 would slide up the ramp if friction were reduced sufficiently; in that case (if the friction weren't reduced) the static friction would have to be down the ramp to hold block 1 there. Since we don't know the masses we don't know which extreme case is closer to our situation. So the answer is D.

ASSESS: By examining limiting cases, we get a good feel for the situation. From the figure it *looks* like block 1 is more massive than block 2, but we aren't told, and there isn't enough information to decide which way it would slide if friction were reduced.

Q5.15 REASON: We will use Equation 5.4 since neither the dog nor the floor is in equilibrium.



From the free-body diagram above, we have $n - w = ma_y$.

Solving for the normal force,

$$n = w + ma_y = mg + ma_y = (5.0\text{ kg})(9.80\text{ m/s}^2) + (5.0\text{ kg})(-1.20\text{ m/s}^2) = 43\text{ N}$$

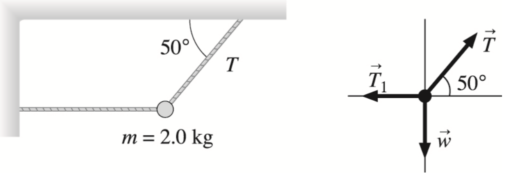
The correct choice is B.

(b) The normal force on the dog is the force of the floor of the elevator on the dog. The force of the dog on the elevator floor is the reaction force to this. The correct choice is D.

Assess: This result makes sense; the normal force will be less than the weight of the dog, which is 49 N.

Q5.16 REASON: The ball is in equilibrium. We will use Equation 5.1.

See the free-body diagram below.



In the vertical direction, we have

$$T\sin(50°) - w = T\sin(50°) - mg = 0$$

Solving for T, we obtain

$$T = \frac{mg}{\sin(50°)} = \frac{(2.0\text{ kg})(9.80\text{ m/s}^2)}{\sin(50°)} = 26\text{ N}$$

The correct choice is D.

ASSESS: Note that we did not need to use the horizontal components of the forces.

Q5.17 **REASON:** This is still a Newton's second law question; the only twist is that the object is not in equilibrium, i.e., the right side of the second law is not zero.

The forces on Eric are the downward gravitational force of the earth on him *w*, and the upward normal force of the scale on him *n* (which we want to know).

We note that $a = -1.7 \text{ m/s}^2$ and $w = mg$.

This is a one-dimensional question in the vertical direction, so the following equations are all in the *y*-direction.

$$F_{y,\text{net}} = ma_y$$
$$n - w = ma_y$$
$$n = ma_y + mg = (62 \text{ kg})(-1.7 \text{ m/s}^2 + 9.8 \text{ m/s}^2) = 502 \text{ N} \approx 500 \text{ N}$$

The correct choice is B.

ASSESS: Because the elevator is accelerating down, we expect the scale to read a bit less than Eric's normal weight. This is the case.

It is important that neither the question nor the answer specify whether the elevator is moving up or down. The elevator can be accelerating down in two ways: It can be moving up and slowing (such as the end of a trip from a low floor to a high floor), or it can be moving down and gaining speed (such as the beginning of a trip from a high floor to a low floor). The answer is the same in both cases.

Q5.18 **REASON:** We will assume a constant direction so that plus the "constant speed" means no acceleration. The sled is in equilibrium and the net force on it must be zero.

In the horizontal direction, there are two forces on the sled: the football player pushing on it, and kinetic friction acting in the opposite direction. These two must have the same magnitude.

Equation 5.6 tells us that $f_k = \mu_k n$, but we don't yet know *n*.

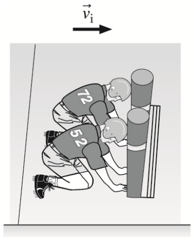
Independently analyzing the vertical direction reveals that the magnitude of $\vec{n}$ is the same as the magnitude of $w = mg = (60 \text{ kg})(9.80 \text{ m/s}^2) = 590 \text{ N}$.

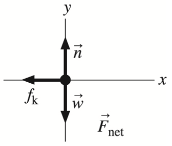
So the kinetic friction force is $f_k = \mu_k n = (0.30)(590 \text{ N}) = 180 \text{ N}$. And that must also be the magnitude of the football player's pushing force.

The correct answer is C.

ASSESS: Choices A and B don't seem very strenuous for a football player, but choice D seems like too much. Choice C is in the right range.

Q5.19 **REASON:** Friction will slow down and stop the sled once the players stop pushing. The only horizontal force on the sled while it is slowing down is the force of kinetic friction. See the diagram below.





In the vertical direction, Equation 5.1 gives $n = w$. The force of kinetic friction is given by Equation 5.6.

$$f_k = \mu_k n = \mu_k mg$$

The net force in the horizontal direction is $\vec{F}_{net} = \vec{f}_k$. We can find the acceleration of the sled using Newton's second law.

$$a_x = \frac{-f_k}{m} = \frac{-\mu_k mg}{m} = -\mu_k g = -(0.30)(9.80\text{ m/s}^2) = -2.94\text{ m/s}^2$$

Additional significant figures have been retained in this intermediate calculation.

We can find how far the sled slides before stopping using kinematic equations. We have the initial velocity of the sled is $v_1 = 2.0$ m/s. The final velocity of the sled is $v_f = 0.0$ m/s. Using Equation 3.23 and solving for Δx,

$$\Delta x = \frac{(v_i)^2}{2a_x} = \frac{(2.0\text{ m/s})^2}{2(2.94\text{ m/s}^2)} = 0.68\text{ m}$$

The correct choice is B.

ASSESS: This result is reasonable. The sled would be expected to stop in a short distance.

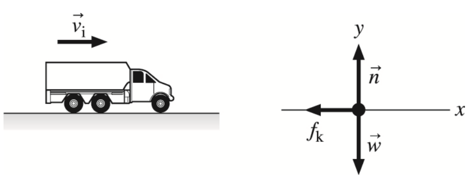
Q5.20 **REASON:** Let us call the direction pointing down the incline the $+x$ direction. Because the car is parked, it is not accelerating along the incline. Thus, we know the sum of all forces along the incline is zero. The only forces acting on the car in this direction are gravity and the force of friction. Thus, we can write

$$\sum F_x = w_x - f_s = ma_x = 0$$
$$mg\sin(\theta) - f_s = 0$$
$$f_s = mg\sin(\theta) = (2{,}000\text{ kg})(9.8\text{ m/s}^2)\sin(30°)$$
$$f_s = 9800\text{ N}$$

So the correct answer is B.

ASSESS: Knowing that the friction must exactly counteract the component of gravity pointing down in the incline is sufficient to determine the magnitude of the frictional force. The coefficient of friction is not needed. However, we know that the maximum force of static friction is given by $(f_s)_{max} = \mu_s n$ which in this case is $(f_s)_{max} = (0.90)(2{,}000\text{ kg})(9.8\text{ m/s}^2)\cos(30°) = 1.5\times10^4\text{N}$, which is greater than the 9,800 N required to hold the car in place. So, we determined first what the frictional force must be, and now we have shown that the materials are such that it can be large.

Q5.21 **REASON:** Friction will slow down and stop the truck once the truck starts to skid. The only horizontal force on the truck while it is skidding is the force of kinetic friction. See the diagram below.



In the vertical direction, Equation 5.1 gives $n = w.$ The force of kinetic friction is given by Equation 5.6.

$$f_k = \mu_k n = \mu_k mg$$

The net force in the horizontal direction is $\vec{F}_{net} = \vec{f}_k.$ We can find the acceleration of the truck using Newton's second law.

$$a_x = \frac{-f_k}{m} = \frac{-\mu_k mg}{m} = -\mu_k g = -(0.20)(9.80\ \text{m/s}^2) = -1.96\ \text{m/s}^2$$

Additional significant figures have been retained in this intermediate calculation for use later.
We can find how far the truck skids before stopping using the kinematic equations. We have the initial velocity of the truck is $v_i = 30\ \text{m/s}.$ The final velocity of the truck is $v_f = 0.0\ \text{m/s}.$ Using Equation 3.23 and solving for $\Delta x,$

$$\Delta x = \frac{(v_i)^2}{2a_x} = \frac{(30\ \text{m/s})^2}{2(1.96\ \text{m/s}^2)} = 230\ \text{m}$$

The correct choice is A.

Assess: A speed of 30 m/s is almost 70 mph. Note that the truck takes nearly a quarter of a kilometer to skid to a stop.

Q5.22 **Reason:** Equation 5.27 gives

$$g_{planet} = \frac{GM}{R^2}$$

If the mass stays the same while the radius doubles, then the new g will be 1/4 of the old one. Since $g \approx 10\ \text{m/s}^2$ now, then one quarter of that is $2.5\ \text{m/s}^2.$

The correct choice is A.

Assess: Especially note that in part (b) the magnitude of the force of the floor on you is not the same as the magnitude of the earth's gravitational force on you, as it would have been if you hadn't been pushing on the ceiling.

Q5.23 **Reason:** We will model the elastic rope like a spring, assuming the restoring force is linear in its stretch: $F_{spring} = -k\Delta x$. The constant k tells us about the stiffness of the "spring," or the force required to stretch the rope. This is the same material property regardless of what mass is hanging from the rope. Thus, we can equate:

$$\frac{F_1}{\Delta y_1} = -k = \frac{F_2}{\Delta y_2}$$

We keep in mind that here Δy is the stretch of the rope from its original, unstretched length (10 m):

$$\Delta y_2 = \frac{F_2}{F_1}\Delta y_1 = \frac{(75\ \text{kg})g}{(60\ \text{kg})g}(0.5\,\text{m}) = 0.625 \approx 0.6\ \text{m}$$

So the new length of the rope will be 10.6 m. The correct answer is A.

Assess: It makes sense that a heavier person will stretch the rope more. It also makes sense that the stretch won't me more than a meter, since the 60 kg person only stretched the rope half a meter.

PROBLEMS

P5.1. **PREPARE:** We will simply use Newton's second law with the appropriate value of g for each astronomical object.

SOLVE: (a) The woman's weight on the earth is

$$w_{\text{earth}} = mg_{\text{earth}} = (55.0\text{ kg})(9.80\text{ m/s}^2) = 539\text{ N}$$

(b) Since mass is a measure of the amount of matter, the woman's mass is the same on the moon as on the earth.

Her weight on the moon is

$$w_{\text{moon}} = mg_{\text{moon}} = (55.0\text{ kg})(1.62\text{ m/s}^2) = 89.1\text{ N}$$

ASSESS: The smaller acceleration due to gravity on the moon reveals that objects are less strongly attracted to the moon than to the earth. Thus, the woman's smaller weight on the moon makes sense.

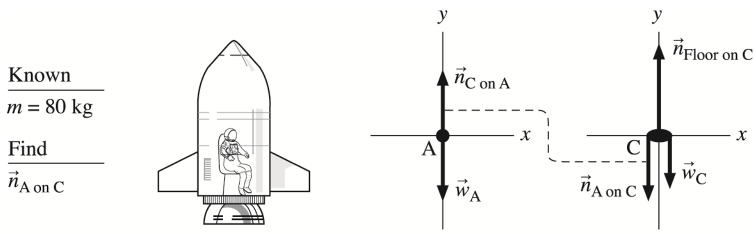
P5.2. **PREPARE:** The apparent weight is the contact force between the floor of the spacecraft and the astronauts' bodies. We will use Newton's second law in the vertical direction (calling vertically upward from the surface of the moon $+y$.

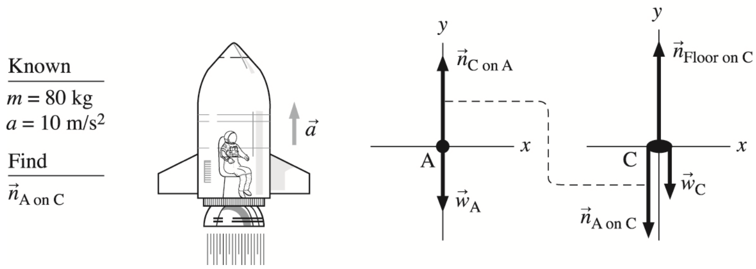
SOLVE: We have, for the sum of all forces on the astronauts $\sum(F_{\text{on astro}})_y = n - mg = ma_y$. Here, g refers to the acceleration due to gravity on the moon. Thus,

$n = m(a_y + g) = (75\text{ kg})\left((3.4\text{m/s}^2) + (1.6\text{m/s}^2)\right) = 3.8\times10^2$ N. This normal force between the floor and the astronaut is the apparent weight.

ASSESS: Note that on Earth the weight of a 75 kg astronaut would be about 740 N. It is reasonable that the apparent weight taking off from the moon is still smaller than this since gravity is so weak there and the acceleration of the spacecraft was also small compared to Earth's 9.8m/s^2 acceleration due to gravity.

P5.3. **PREPARE:** We will use Newton's second law in the vertical direction. The astronaut and the chair will be denoted by A and C, respectively, and they are separate systems. The launch pad is a part of the environment. In the following free-body diagrams for both the astronaut and the chair are shown at rest on the launch pad (top) and while accelerating (bottom).





SOLVE: (a) Newton's second law for the astronaut is

$$\sum(F_{\text{on A}})_y = n_{\text{C on A}} - w_{\text{A}} = m_{\text{A}}a_{\text{A}} = 0 \text{ N} \Rightarrow n_{\text{C on A}} = w_{\text{A}} = m_{\text{A}}g$$

By Newton's third law, the astronaut's force on the chair is

$$n_{\text{A on C}} = n_{\text{C on A}} = m_{\text{A}}g = (80 \text{ kg})(9.80 \text{ m/s}^2) = 780 \text{ N}$$

(b) Newton's second law for the astronaut is

$$\sum(F_{\text{on A}})_y = n_{\text{C on A}} - w_{\text{A}} = m_{\text{A}}a_{\text{A}} \Rightarrow n_{\text{C on A}} = w_{\text{A}} + m_{\text{A}}a_{\text{A}} = m_{\text{A}}(g + a_{\text{A}})$$

By Newton's third law, the astronaut's force on the chair is

$$n_{\text{A on C}} = n_{\text{C on A}} = m_{\text{A}}(g + a_{\text{A}}) = (80 \text{ kg})(9.80 \text{ m/s}^2 + 10 \text{ m/s}^2) = 1600 \text{ N}$$

ASSESS: This is a reasonable value because the astronaut's acceleration is more than g.

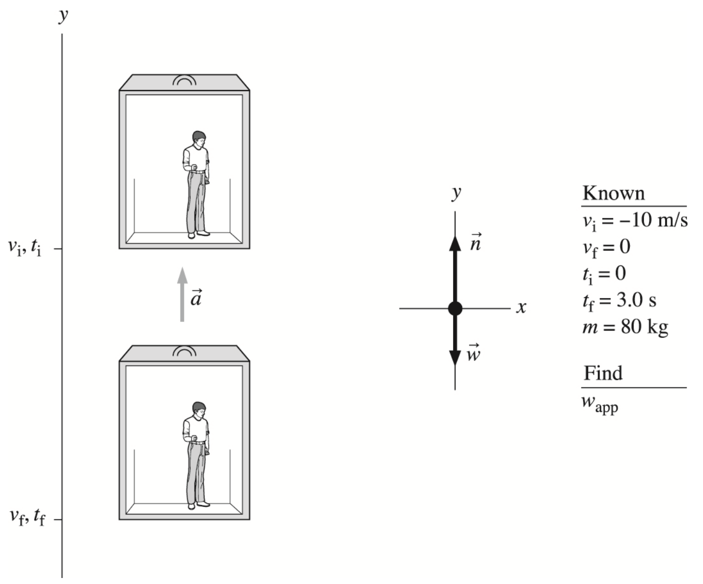
P5.4. PREPARE: We are asked for apparent weight, which is the contact force between the people and the surface supporting them. Thus, we will apply Newton's second law to determine the contact force (which here is a normal force that the seat exerts up on a passenger). Newton's second law can be used in the vertical direction to obtain $w_{\text{app}} = m(g + a_y)$. We need to add the 4$g$ acceleration to the g in the equation.

SOLVE: (a) The rider's apparent weight will be $(60 \text{ kg})(5)(9.80 \text{ m/s}^2) = 2900 \text{ N}$.

(b) The rider's apparent weight will be $(60 \text{ kg})(9.80 \text{ m/s}^2) + (-9.80 \text{ m/s}^2) = 0$.

ASSESS: The apparent weight is always zero for objects in free fall.

P5.5. PREPARE: We are asked for apparent weight, which is the contact force between Zach and the surface supporting him. Thus, we will apply Newton's second law to determine the contact force (which here is a normal force that the floor exerts upward on Zach). We note that since the acceleration is different in parts (a) and (b), we expect the statement of Newton's second law to be different in those cases, meaning we expect to find different apparent weights. We begin by drawing a free-body diagram.



SOLVE: (a) Before the elevator starts braking, Zach is not accelerating. His apparent weight is

$$w_{app} = w\left(1+\frac{a}{g}\right) = w\left(1+\frac{0\text{ m/s}^2}{g}\right) = mg = (80\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$$

or 780 N to two significant figures.

(b) Using the definition of acceleration,

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{0-(-10)\text{ m/s}}{3.0\text{ s}} = 3.33\text{ m/s}^2$$

$$\Rightarrow w_{app} = w\left(1+\frac{a}{g}\right) = (80\text{ kg})(9.80\text{ m/s}^2)\left(1+\frac{3.33\text{ m/s}^2}{9.80\text{ m/s}^2}\right) = (784\text{ N})(1+0.340) = 1100\text{ N}$$

ASSESS: While the elevator is braking, it not only must support Zach's weight but must also push upward on him to decelerate him, so the apparent weight is greater than his normal weight.

P5.6. PREPARE: This problem deals with apparent weight, which is the contact force between the passenger and the surface on which he/she is resting. We will use the sum of all forces in the direction of motion (here the y direction) to determine the unknown contact force. The passenger is acted on by only two vertical forces: the downward pull of gravity and the upward force of the elevator floor. Referring to Figure P5.6, the graph has three segments corresponding to different conditions: (1) increasing velocity, meaning an upward acceleration, (2) a period of constant upward velocity, and (3) decreasing velocity, indicating a period of deceleration (negative acceleration). Given the assumptions of our model, we can calculate the acceleration for each segment of the graph.

SOLVE: The acceleration for the first segment is

$$a_y = \frac{v_f - v_i}{t_f - t_i} = \frac{8\text{ m/s} - 0\text{ m/s}}{2\text{ s} - 0\text{ s}} = 4\text{ m/s}^2 \Rightarrow w_{app} = w\left(1+\frac{a_y}{g}\right) = (mg)\left(1+\frac{4\text{ m/s}^2}{9.80\text{ m/s}^2}\right)$$

$$= (75\text{ kg})(9.80\text{ m/s}^2)\left(1+\frac{4}{9.8}\right) = 1000\text{ N}$$

For the second segment, $a_y = 0\ \text{m/s}^2$ and the apparent weight is

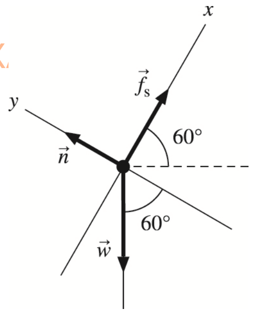
$$w_{\text{app}} = w\left(1 + \frac{0\ \text{m/s}^2}{g}\right) = mg = (75\ \text{kg})(9.80\ \text{m/s}^2) = 740\ \text{N}$$

For the third segment,

$$a_y = \frac{v_3 - v_2}{t_3 - t_2} = \frac{0\ \text{m/s} - 8\ \text{m/s}}{10\ \text{s} - 6\ \text{s}} = -2\ \text{m/s}^2$$
$$\Rightarrow w_{\text{app}} = w\left(1 + \frac{-2\ \text{m/s}^2}{9.80\ \text{m/s}^2}\right) = (75\ \text{kg})(9.80\ \text{m/s}^2)(1 - 0.2) = 590\ \text{N}$$

ASSESS: As expected, the apparent weight is greater than normal when the elevator is accelerating upward and lower than normal when the acceleration is downward. When there is no acceleration the weight is normal. In all three cases, the magnitudes are reasonable, given the mass of the passenger and the accelerations of the elevator.

P5.7. PREPARE: We will write the sum of all forces along the slope and perpendicular to the slope to determine what forces are required to keep the mountain goat from slipping. This process will be aided by the free-body diagram below. Let us call the direction up the incline the $+x$-direction, and the direction somewhat upward but perpendicular to the slope the $+y$-direction. When the mountain goat is not slipping, the acceleration should be zero along both of these directions.



SOLVE: Writing Newton's second law for both directions, we find

$$\sum F_x = -mg\sin(\theta) + f_s = ma_x = 0$$
$$\Rightarrow f_s = mg\sin(\theta) = (900\ \text{N})\sin(60°) = 7.8\times10^2\ \text{N}$$
$$\sum F_y = -mg\cos(\theta) + n = ma_y = 0$$
$$\Rightarrow n = mg\cos(\theta) = (900\ \text{N})\cos(60°) = 4.5\times10^2\ \text{N}$$

ASSESS: Note that each of these forces is smaller than the total weight of the goat, and that they are similar in magnitude to the weight of the goat. This is reasonable.

P5.8 PREPARE: This problem deals with kinetic friction and its relationship to the normal force: $f_k = \mu_k n$. Because the child is not on level ground, we expect that the normal force will not simply be mg. The child is not accelerating perpendicular to the slide; we will use the fact that the sum of all forces normal to the slide must be zero. Call the direction down the incline the $+x$-direction, and call the direction normal to the slide and somewhat upward the $+y$-direction. The only forces acting on the child are gravity, friction, and the normal force.

SOLVE: (a) The component of gravity perpendicular to the incline is $w_y = -mg\cos(\theta)$. The sum of all forces in that direction can be written as

$$\sum F_y = n - mg\cos(\theta) = ma_y = 0$$

Thus

$$n = mg\cos(\theta) = (23\text{ kg})(9.8\text{ m/s}^2)\cos(35°) = 180\text{ N}$$

(b) Kinetic friction is given by

$$f_k = \mu_k n = \mu_k mg\cos(\theta) = (0.25)(23\text{ kg})(9.8\text{ m/s}^2)\cos(35°) = 46\text{ N}$$

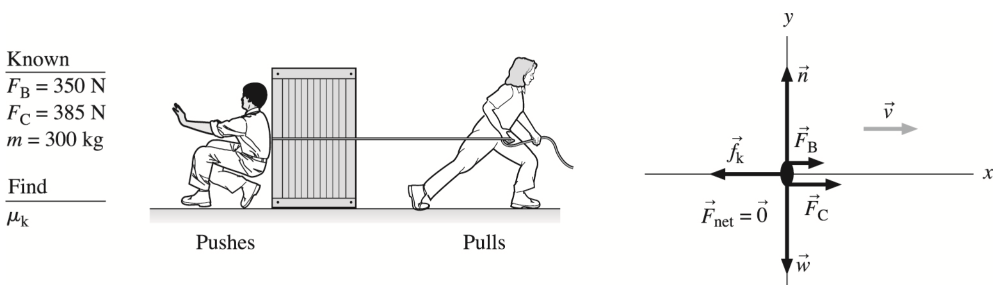
ASSESS: One can check that the component of gravity pulling the child down the incline is 130 N, which is greater than the 46 N of friction. This means the child will accelerate down the slide. This fits our experience.

P5.9. **PREPARE:** The force of friction between the crate and the horizontal floor surface is proportional to the crate's mass. Specifically, $f_{s\,max} = \mu_s n = \mu_s mg = ma_x$. That is, the acceleration as the crate slows down is unchanged. We can now use kinematics equations to find the stopping distance.

SOLVE: (a) The block will slide the same distance *d*. The acceleration is the same as before and the velocity is the same as before, so from Equation 3.23 the distance traveled *d* remains the same.

(b) The block will slide a distance of 4*d*. Because the acceleration is unchanged, but the velocity is doubled, Equation 3.23 yields a stopping distance of 4*d*.

P5.10. **PREPARE:** We assume that the safe is a particle moving only in the *x*-direction. Since it is sliding during the entire problem, the force of kinetic friction opposes the motion by pointing to the left. In the following diagram, we give a pictorial representation and a free-body diagram for the safe. The safe is in dynamic equilibrium, since it's not accelerating. We can determine the unknown force by using Newton's second law.



SOLVE: We apply Newton's first law in the vertical and horizontal directions:

$$(F_{net})_x = \Sigma F_x = F_B + F_C - f_k = 0\text{ N} \Rightarrow f_k = F_B + F_C = 350\text{ N} + 385\text{ N} = 735\text{ N}$$
$$(F_{net})_y = \Sigma F_y = n - w = 0\text{ N} \Rightarrow n = w = mg = (300\text{ kg})(9.80\text{ m/s}^2) = 2940\text{ N}$$

Then, for kinetic friction

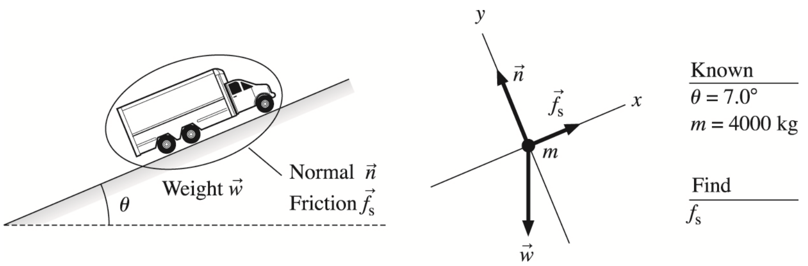
$$f_k = \mu_k n \Rightarrow \mu_k = \frac{f_k}{n} = \frac{735\text{ N}}{2940\text{ N}} = 0.25$$

ASSESS: The value of $\mu_k = 0.25$ is hard to evaluate without knowing the material the floor is made of, but it seems reasonable.

P5.11. PREPARE: The truck is in static equilibrium. We can apply Newton's second law to determine the unknown force of friction. Below we identify the forces acting on the truck and construct a free-body diagram.

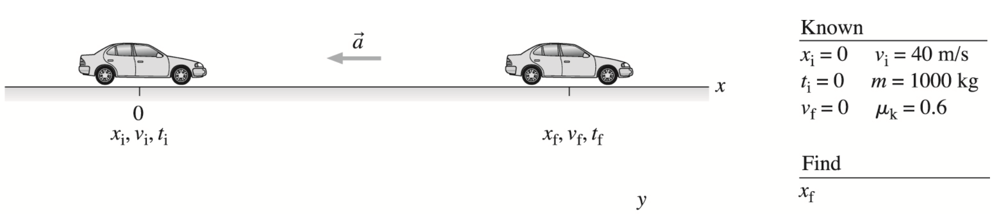
SOLVE: The truck is not accelerating, so it is in equilibrium, and we can apply Newton's first law. The normal force has no component in the x-direction, so we can ignore it here. For the other two forces

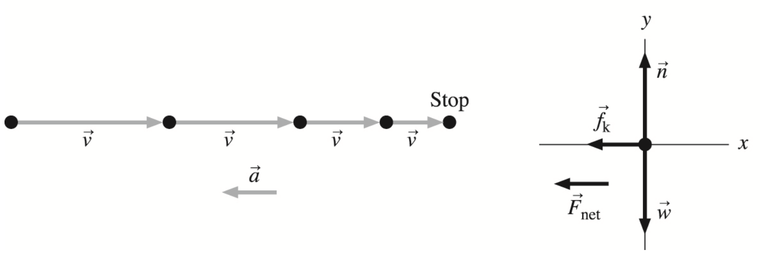
$$(F_{\text{net}})_x = \Sigma F_x = f_s - w_x = 0\ \text{N} \Rightarrow f_s = w_x = mg\sin\theta = (4000\ \text{kg})(9.80\ \text{m/s}^2)(\sin 7.0°) = 4800\,\text{N}$$



ASSESS: The truck's weight (mg) is roughly 40,000 N. A friction force that is $\approx 12\%$ of the truck's weight seems reasonable.

P5.12. PREPARE: The car is undergoing skidding, so it is decelerating and the force of kinetic friction acts to the left. We give below an overview of the pictorial representation, a motion diagram, a free-body diagram, and a list of values. We will first apply Newton's second law to find the deceleration and then use kinematics to obtain the length of the skid marks.





SOLVE: We begin with Newton's second law. Although the motion is one-dimensional, we need to consider forces in both the x- and y-directions. However, we know that $a_y = 0\ \text{m/s}^2$. We have

$$a_x = \frac{(F_{\text{net}})_x}{m} = \frac{-f_k}{m} \qquad a_y = 0\ \text{m/s}^2 = \frac{(F_{\text{net}})_y}{m} = \frac{n-w}{m} = \frac{n-mg}{m}$$

We used $(f_k)_x = -f_k$ because the free-body diagram tells us that $\vec{f}_k$ points to the left. The force of kinetic friction relates $\vec{f}_k$ to $\vec{n}$ with the equation $f_k = \mu_k n$. The y-equation is solved to give $n = mg$. Thus, the kinetic friction force is $f_k = \mu_k mg$.

Substituting this into the x-equation yields

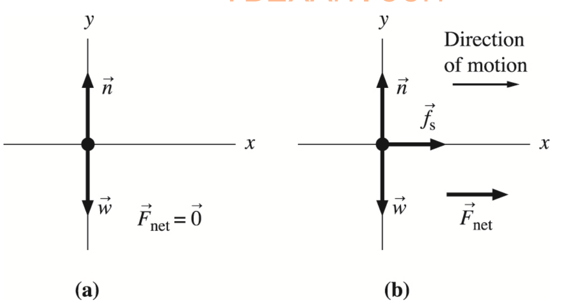
$$a_x = \frac{-\mu_k mg}{m} = -\mu_k g = -(0.6)(9.80 \text{ m/s}^2) = -5.88 \text{ m/s}^2$$

The acceleration is negative because the acceleration vector points to the left as the car slows. Now we have a constant-acceleration kinematics problem. Δt isn't known, so use

$$v_f^2 = 0 \text{ m}^2/\text{s}^2 = v_i^2 + 2a_x\Delta x \Rightarrow \Delta x = -\frac{(40 \text{ m/s})^2}{2(-5.88 \text{ m/s}^2)} = 140 \text{ m}$$

ASSESS: The skid marks are 140 m long. This is ≈ 430 feet, reasonable for a car traveling at ≈ 80 mph. It is worth noting that an algebraic solution led to the *m* canceling out.

P5.13. PREPARE: The force of kinetic friction is always given by $f_k = \mu_k n$, whereas the force of static friction can be anything up to a maximum value of $(f_s)_{max} = \mu_s n$. If the crate does not slip across the surface of the conveyor belt, then we will use static friction; if it does slip we will use kinetic friction. We will write down Newton's second law to help us find the required accelerations. We show below the free-body diagrams of the crate when the conveyer belt runs at constant speed (part (a)) and the belt is speeding up (part (b)).



SOLVE: (a) When the belt runs at constant speed, the crate has an acceleration $\vec{a} = \vec{0}$ m/s^2 and is in dynamic equilibrium. Thus $\vec{F}_{net} = \vec{0}$. It is tempting to think that the belt exerts a friction force on the crate. But if it did, there would be a *net* force because there are no other possible horizontal forces to balance a friction force. Because there is no net force, there cannot be a friction force. The only forces are the upward normal force and the crate's weight. (A friction force would have been needed to get the crate moving initially, but no horizontal force is needed to keep it moving once it is moving with the same constant speed as the belt.)

(b) If the belt accelerates gently, the crate speeds up without slipping on the belt. Because it is accelerating, the crate must have a net horizontal force. So *now* there is a friction force, and the force points in the direction of the crate's motion. Is it static friction or kinetic friction? Although the crate is moving, there is *no* motion of the crate relative to the belt. Thus, it is a *static* friction force that accelerates the crate so that it moves without slipping on the belt.

(c) The static friction force has a maximum possible value $(f_s)_{max} = \mu_2 n$. The maximum possible acceleration of the crate is

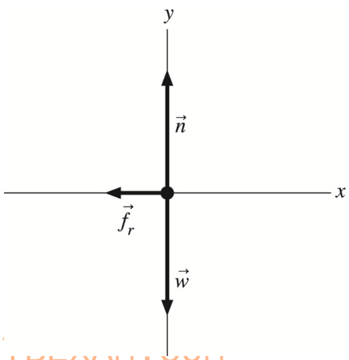
$$a_{max} = \frac{(f_s)_{max}}{m} = \frac{\mu_s n}{m}$$

If the belt accelerates more rapidly than this, the crate will not be able to keep up and will slip. It is clear from the free-body diagram that $n = w = mg$. Thus,

$$a_{max} = \mu_s g = (0.50)(9.80\ \text{m/s}^2) = 4.9\ \text{m/s}^2$$

(d) The acceleration of the crate will be $a = \mu_k g = (0.30)(9.80\ \text{m/s}^2) = 2.9\ \text{m/s}^2$.

P5.14. **PREPARE:** Assume the locomotive is on level ground and the acceleration is constant. Assume rolling friction is the only force acting on the locomotive in the horizontal direction.



SOLVE: Since the locomotive does not accelerate in the vertical direction, the free-body diagram shows that $n = w = mg$. The friction force is $f_r = \mu_r mg$.

$$F_{net} = f_r = \mu_r mg \Rightarrow a = \frac{F_{net}}{m} = \mu_r g$$

The definition of acceleration $a = \Delta v/\Delta t$ gives

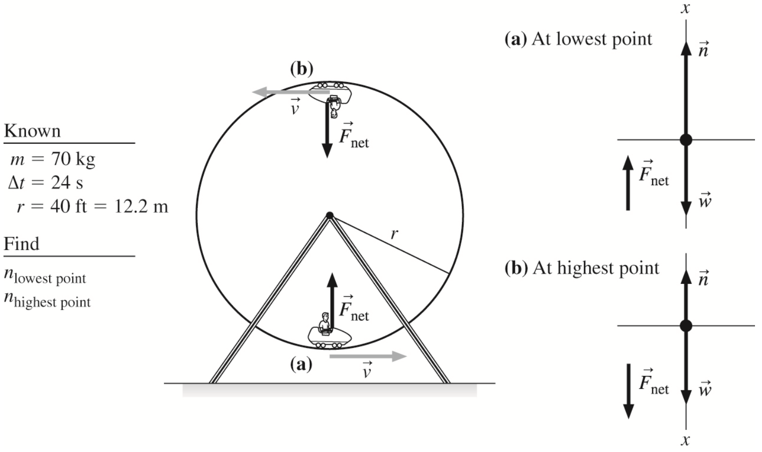
$$\Delta t = \frac{\Delta v}{a} = \frac{\Delta v}{\mu_r g} = \frac{10\ \text{m/s}}{(0.002)(9.80\ \text{m/s}^2)} = 510.2\ \text{s} \approx 500\ \text{s}$$

We now use the kinematic equation to find how far the locomotive will move during this time.

$$\Delta x = \frac{1}{2}a(\Delta t)^2 = \frac{1}{2}(0.002)(9.80\ \text{m/s}^2)(510.2\ \text{s})^2 = 2550\ \text{m} \approx 3000\ \text{m}$$

ASSESS: We are impressed, but not surprised, by the long time it would take the locomotive to coast to a stop without brakes. And it covers almost 3 km in that time. The mass was irrelevant in this problem.

P5.15. **PREPARE:** Model the passenger on the Ferris wheel as a particle in uniform circular motion. We will use Newton's second law to relate the forces acting on the passenger to the passenger's centripetal acceleration. A pictorial representation of the passenger, its free-body diagram, and a list of values is shown below. Note that the normal force $\vec{n}$ of the seat pushing on the passenger is the passenger's apparent weight. Draw the x-axis pointing toward the center of the circle in each case.



A preliminary calculation gives the speed: $v = \dfrac{2\pi r}{\Delta t} = \dfrac{2\pi(12.2 \text{ m})}{24 \text{ s}} = 3.19 \text{ m/s}$

SOLVE: Use Newton's second law in the x-direction.

(a) The net force at the lowest point of the circle is:

$$\sum F = n - w = \frac{mv^2}{r} \Rightarrow n = mg + \frac{mv^2}{r} = m\left(g + \frac{v^2}{r}\right) = 740 \text{ N}$$

(b) The net force at the highest point of the circle is:

$$\sum F = w - n = \frac{mv^2}{r} \Rightarrow n = mg - \frac{mv^2}{r} = m\left(g - \frac{v^2}{r}\right) = 630 \text{ N}$$

ASSESS: It feels right that the apparent weight would be a bit more at the bottom and a bit less at the top.

P5.16 **PREPARE:** The Reynolds number is determine using Equation 5.9: $Re = \dfrac{\rho v L}{\eta}$. We find from Table 5.3 that $\rho_{H_2O} = 1{,}000 \text{ kg/m}^3$, $\rho_{\text{air}}(20° \text{ C}) = 1.2 \text{ kg/m}^3$, $\eta_{H_2O}(20° \text{ C}) = 1.0\times10^{-3}$ Pa, and $\eta_{\text{air}}(20° \text{ C}) = 1.8\times10^{-5}$ Pa. Finally, we note that for seawater (in which dolphins swim) $\rho_{\text{seawater}} = 1{,}030 \text{ kg/m}^3$.

Solve: In all cases, we simply plug in numbers to Equation 5.9.

(a) $Re = \dfrac{\rho v L}{\eta} = \dfrac{(1.2 \text{ kg/m}^3)(15 \text{ m/s})(0.030 \text{ m})}{(1.8\times10^{-5} \text{ Pa}\cdot\text{s})} = 3\times10^4$

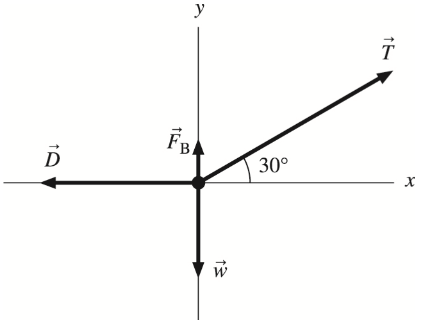
(b) $Re = \dfrac{\rho v L}{\eta} = \dfrac{(1030 \text{ kg/m}^3)(15 \text{ m/s})(0.30 \text{ m})}{(1.0\times10^{-3} \text{ Pa}\cdot\text{s})} = 5\times10^6$

(c) $Re = \dfrac{\rho v L}{\eta} = \dfrac{(1.2 \text{ kg/m}^3)(15 \text{ m/s})(0.001 \text{ m})}{(1.8\times10^{-5} \text{ Pa}\cdot\text{s})} = 1\times10^3$

(d) $Re = \dfrac{\rho v L}{\eta} = \dfrac{(1000 \text{ kg/m}^3)(0.015 \text{ m/s})(20\times10^{-6} \text{ m})}{(1.0\times10^{-3} \text{ Pa}\cdot\text{s})} = 0.3$

ASSESS: We see that the lowest Reynolds number is for the diatom in water. This is reasonable since viscous forces would be expected to matter most for a very small object like a diatom in a relatively viscous fluid (compared to air, anyway) such as water.

P5.17. **PREPARE:** Let us call the direction of horizontal motion of the submersible the $+x$-direction. We can determine the drag force in that direction and relate that to the horizontal component of the tension in the cable. Since the velocity is constant, the acceleration in the horizontal direction is zero. In calculating drag, note that the density of seawater is really somewhat greater than that of freshwater, approximately $1025\ \text{kg/m}^3$. But because this number is variable (depending on salinity) and because it is only slightly different from the density of freshwater, we will use $1000\ \text{kg/m}^3$. The forces acting on the submersible are shown in the free-body diagram.



SOLVE: Newton's second law tells us

$$\sum F_x = T\cos(\theta) - D = ma_x = 0$$

$$T = \frac{D}{\cos(\theta)} = \frac{1}{\cos(\theta)}\frac{1}{2}C_D\rho A v^2 = \frac{1}{\cos(30°)}\frac{1}{2}(1.2)(1000\ \text{kg/m}^3)(1.3\ \text{m}^2)(5.1\ \text{m/s})^2$$

$$T = 2.4\times10^4\ \text{N}$$

ASSESS: This is a tremendous force, which explains why a hefty steel cable must be used.

P5.18. **PREPARE:** The first part is a straightforward calculation of the drag force. The second part involves viscosity, which we look up from Table 5.3. We are given the geometric and kinematic information necessary to calculate the drag force on the dolphin using $D = \frac{1}{2}C_D\rho A v^2$. The Reynolds number is $\rho v L/\eta$. In this problem we will take the density of seawater to be approximately the same as that of freshwater: $1000\ \text{kg/m}^3$, and the viscosity for seawater is approximately the same as for freshwater: $\eta(20°\ \text{C}) = 1.0\times10^{-3}\ \text{Pa}\cdot\text{s}$.

SOLVE: (a) Inserting the given values, we have

$$D = \frac{1}{2}C_D\rho A v^2 = \frac{1}{2}(0.090)(1000\ \text{kg/m}^3)\left(\pi(0.25\ \text{m})^2\right)(7.5\ \text{m/s})^2$$

$$D = 5.0\times10^2\ \text{N}$$

(b) Inserting values into the expression for the Reynolds number, we have

$$\text{Re} = \frac{\rho v L}{\eta} = \frac{(1025\ \text{kg/m}^3)(7.5\ \text{m/s})(0.50\ \text{m})}{(1.0\times10^{-3}\ \text{Pa}\cdot\text{s})} = 3.8\times10^6$$

ASSESS: The drag force is comparable to about 114 lbs of force slowing the dolphin.

P5.19. **PREPARE:** It is likely that the Reynolds number is very small in this case. One can guess this based on Example 5.10, or by doing an order of magnitude estimate. The speed of the particle could not possibly be greater than 1 m/s, and is likely much less. Using that speed, we find a Reynolds number of 0.16, so our suspicion that it is low was correct. Thus, we will use Stokes' law to determine the drag force. The particles will begin to settle and will quickly reach their terminal velocity. We calculate their terminal speed by equating the force of gravity to $D = 6\pi\eta rv$, and solving for the speed. Then we will use the speed to determine the settling time, using $\Delta y \approx (v_{\text{term}})_y \Delta t$. We look up the viscosity of air from Table 5.3, and use half the diameter of the particles as the characteristic radius.

SOLVE: The sum of all forces in the vertical direction yields

$$\sum F_y = D - mg = 6\pi\eta rv - mg = ma_y = 0$$

$$v = \frac{mg}{6\pi\eta r} = \frac{(1.4\times10^{-14}\ \text{kg})(9.8\ \text{m/s}^2)}{6\pi(1.8\times10^{-5}\ \text{Pa}\cdot\text{s})(1.25\times10^{-6}\ \text{m})} = 3.23\times10^{-4}\ \text{m/s}$$

Finally, $\Delta t \approx \Delta y/(v_{\text{term}})_y = (0.15\ \text{m})/(3.23\times10^{-4}\ \text{m/s}) = 4.6\times10^2\ \text{s}$ or about 7.7 min.

ASSESS: It is reasonable that it takes several minutes for such fine grains of pollution to settle.

P5.20. **PREPARE:** We can find the drag force using the equation in the text. From Example 5.9, we'll assume the cross-sectional area of the runner is $0.72\ \text{m}^2$. We'll also assume the skydiver's mass is 75 kg, as in the example.

SOLVE: Follow Example 5.9 with $\rho = (0.043)(1.22\ \text{kg/m}^3)$.

$$v_{\text{term}} \approx \sqrt{\frac{2mg}{C_D\rho A}} = \sqrt{\frac{2(75\ \text{kg})(9.80\ \text{m/s}^2)}{(0.043)(1.1)(1.22\ \text{kg/m}^3)(0.72\ \text{m}^2)}} = 1.9\times10^2\ \text{m/s}$$

This is almost five times faster than the terminal speed at sea level.

ASSESS: We expect the terminal speed to be much higher where the air is so thin.

P5.21. **PREPARE:** From Example 5.9 we'll assume the cross-sectional area of the runner is $0.72\ \text{m}^2$. Table 5.4 gives the drag coefficient of a running person as $C_D = 1.2$. Converting 18 min to SI units gives 1080 s.

SOLVE: Using Equation 5.10, $D = \frac{1}{2}C_D\rho Av^2$ with $\rho = 1.22\ \text{kg/m}^3$. We need to compute the speed of the runner from the data given. $v = \dfrac{5000\ \text{m}}{1080\ \text{s}} = 4.63\ \text{m/s}$.

$$D = \tfrac{1}{2}C_D\rho Av^2 = \tfrac{1}{2}(1.2)(1.22\ \text{kg/m}^3)(0.72\ \text{m}^2)(4.63\ \text{m/s})^2 = 1.1\times10^2\ \text{N}$$

This is a fairly small fraction of the weight of the runner but is nonnegligible:

$D/w = (11\ \text{N})/(590\ \text{N}) = 1.9\%$.

ASSESS: The drag force is fairly small because the speed of the runner is small.

P5.22. **PREPARE:** The skydiver will have a fairly high Reynolds number, so we will use the quadratic drag model, from Equation 5.9. We will write out the sum of all forces in the vertical (y) direction, for a general case. Then we will specify that the velocity is ½ the terminal velocity, and see what we obtain.

SOLVE: In general, the sum of all forces in the vertical direction is $\sum F_y = D - mg = ma_y$. If we write down the expression for the terminal case, there is a trivial modification describing the drag at ½ the terminal speed:

$$\frac{1}{2}C_D \rho A v_{\text{term}}^2 - mg = 0 \Rightarrow \frac{1}{2}C_D \rho A \left(\frac{v_{\text{term}}}{2}\right)^2 = \frac{mg}{4}$$

Finally, writing the sum of all forces in the case where the speed is ½ the eventual terminal speed, we find

$$\sum F_y = D - mg = ma_y \Rightarrow \frac{mg}{4} - mg = ma_y \Rightarrow a_y = -\frac{3}{4}g = -7.4 \text{ m/s}^2$$

We are asked for the magnitude, which is 7.4 m/s^2.

ASSESS: This answer has the right sign and is slightly smaller than the acceleration due to gravity, which makes sense.

P5.23 **PREPARE:** Equation 5.22 relates the stopping distance to the time constant: $\Delta x_{\text{stop}} = v_{\max}\tau$. The time constant is related to other variables, including mass, through Equation 5.21: $\tau = \dfrac{m}{6\pi\eta r}$. We can combine these and solve for mass.

SOLVE: Combining Equations 5.21 and 5.22 yields

$$\Delta x_{\text{stop}} = v_{\max}\frac{m}{6\pi\eta r} \Rightarrow m = \frac{6\pi\eta r \Delta x_{\text{stop}}}{v_{\max}}$$

$$m = \frac{6\pi(8.4\times10^{-2} \text{ Pa}\cdot\text{s})(125\times10^{-6} \text{ m})(11\times10^{-6} \text{ m})}{(0.25 \text{ m/s})}$$

$$= 8.7\times10^{-9} \text{ kg} = 8.7 \ \mu\text{g}$$

ASSESS: This is a reasonable mass for a bead of this size, since it corresponds to a density of

$$\frac{m}{(4/3)\pi r^3} = \frac{(8.7\times10^{-9} \text{ kg})}{(4/3)\pi(125\times10^{-6} \text{ m})^3} \approx 10^3 \text{ kg/m}^3$$

This is around the density of water.

P5.24. **PREPARE:** This problem involves a spring restoring force holding an object in static equilibrium. The spring will stretch and exert a restoring force according to $F_{\text{sp},y} = -k\Delta y$. In static equilibrium, we know $\sum F_y = F_{\text{sp},y} - mg = 0$. We will refer to the vertically upward direction as $+y$.

SOLVE: **(a)** Using the condition for static equilibrium, and inserting the form of the restoring force, we have $-k\Delta y = mg$. We know the displacement of the spring from equilibrium is $\Delta y = -0.024$ m from the given lengths. Inserting this, and solving for the spring constant yields

$$k = -\frac{mg}{\Delta y} = -\frac{(4.0 \text{ kg})(9.8 \text{ m/s}^2)}{(-0.024 \text{ m})} = 1.6\times10^3 \text{ N}.$$

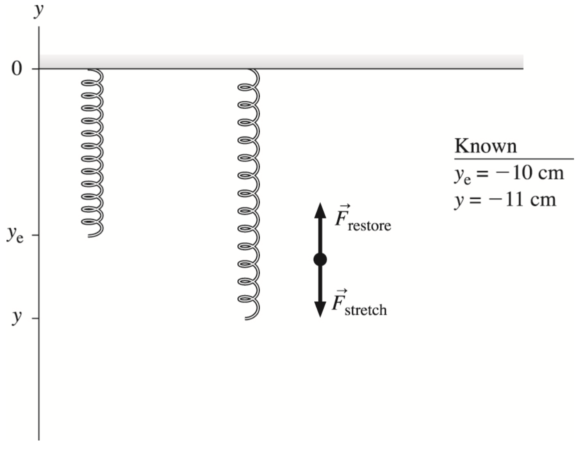
(b) Using the value for the spring constant we found in part (a), we find

$$\Delta y = -\frac{mg}{k} = -\frac{(8.0\ \text{kg})(9.8\ \text{m/s}^2)}{(1.63\times10^3\ \text{N})} = -0.048\ \text{m}$$

The spring has an initial length of 10.0 cm, and is then stretched downward an additional 4.8 cm. So, the final length of the spring is 14.8 cm.

ASSESS: The stretch of the spring doubles from 2.4 to 4.8 cm when the mass of the hanging fish is doubled. This fits our expectations.

P5.25. PREPARE: A visual overview below shows the details, including a free-body diagram, of the problem. We will assume an ideal spring that obeys Hooke's law.



SOLVE: (a) The spring force or the restoring force is $F_{sp} = -k\Delta y$. For $\Delta y = -1.0$ cm and the force in Newtons,

$$F_{sp} = F = -k\Delta y \Rightarrow k = -F/\Delta y = -F/(-0.01\ \text{m}) = 100F\ \text{N/m}$$

Notice that Δy is negative, so F_{sp} is positive.

We can now calculate the new length for a restoring force of $3F$:

$$F_{sp} = 3F = -k\Delta y = (-100\ F)\Delta y \Rightarrow \Delta y = -0.03\ \text{m}$$

From $\Delta y = y - y_e = -0.03$ m or $y = -0.03\ \text{m} + y_e$, or $y = -0.03\ \text{m} + (-0.10\ \text{m}) = -0.13\ \text{m}$, the length of the spring is 0.13 m.

(b) The new compressed length for a restoring force of $2F$ can be calculated as follows:

$$F_{sp} = 2F = -k\Delta y = (-100\ F)\Delta y \Rightarrow \Delta y = -0.02\ \text{m}$$

Using $\Delta y = y_e - y = -0.02$ m, or $y = 0.02\ \text{m} + y_e$, or $y = 0.02\ \text{m} + (-0.10\ \text{m}) = -0.08\ \text{m}$, the length of the compressed spring is 0.08 m.

ASSESS: The stretch Δx is proportional to the applied force, as both parts of this problem demonstrate. Of course, this bet is off if the spring is stretched or compressed far enough to take it out of the linear region.

P5.26. **PREPARE:** Before the passengers get in, the springs are already compressed due to the weight of the train car. But, we can treat this compressed position as a new equilibrium position and measure only the additional compression due to the people. We will use $F_{sp,y} = -k\Delta y$, where we call the vertically upward direction $+y$.

Initially, we have a sum of all forces of the car and all contents: $\sum F_y = -m_{car}g + F_{sp,i,y} = 0$, and after people get on board, we have $\sum F_y = -\left(m_{car} + m_{people}\right)g + F_{sp,f,y} = 0$.

Since the eight springs act identically, we can treat the train as being supporting by one spring system with an effective spring constant of $8k$.

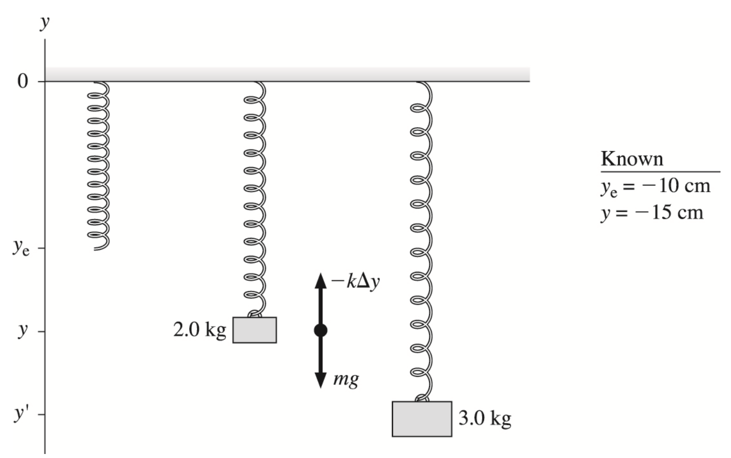
SOLVE: Inserting the form of the spring restoring force, we have $-m_{car}g + -k\Delta y_i = 0$ and $-\left(m_{car} + m_{people}\right)g + -k\Delta y_f = 0$. Combining these, we find that the additional compression $\Delta y_f - \Delta y_i$ is given by

$$k\Delta y_f - k\Delta y_i = -\left(m_{car} + m_{people}\right)g + -m_{car}g$$
$$\Delta y_f - \Delta y_i = -m_{people}g / k = 30\left(80\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right) / \left(8\left(2.8\times10^7\ \text{N/m}\right)\right)$$
$$\Delta y_f - \Delta y_i = -1.1\times10^{-4}\ \text{m}$$

Here, the negative sign indicates that the springs are compressed downward.

ASSESS: The mass of 30 passengers is small compared to a train car. It makes sense that springs designed to support a train car would be only slightly affected by the addition of 30 passengers.

P5.27. **PREPARE:** This problem involves an object held in static equilibrium by forces including the elastic restoring force of a spring. We will assume an ideal spring that obeys Hooke's law. A visual overview below shows the details, including a free-body diagram, of the problem.



SOLVE: (a) The spring force on the 2.0 kg mass is $F_{sp} = -k\Delta y$. Notice that Δy is negative, so F_{sp} is positive. This force is equal to mg, because the 2.0 kg mass is at rest. We have $-k\Delta y = mg$. Solving for k:

$$k = -(mg/\Delta y) = -(2.0\ \text{kg})\ (9.80\ \text{m/s}^2)/(-0.15\ \text{m} - (-0.10\ \text{m})) = 392\ \text{N/m} = 390\ \text{N/m}$$

(b) Again using $-k\Delta y = mg$:

$$\Delta y = -mg/k = -(3.0\text{ kg})(9.80\text{ m/s}^2)/(392\text{ N/m}) = -0.075\text{ m}$$
$$y' - y_e = -0.075\text{ m} \Rightarrow y' = y_e - 0.075\text{ m} = -0.10\text{ m} - 0.075\text{ m} = -0.175\text{ m} = -18\text{ cm}$$

The length of the spring is 18 cm when a mass of 3.0 kg is attached to the spring.

Assess: The *position* of the end of the spring is negative because it is below the origin, but length must be a positive number. We expected the length to be a little more than 15 cm.

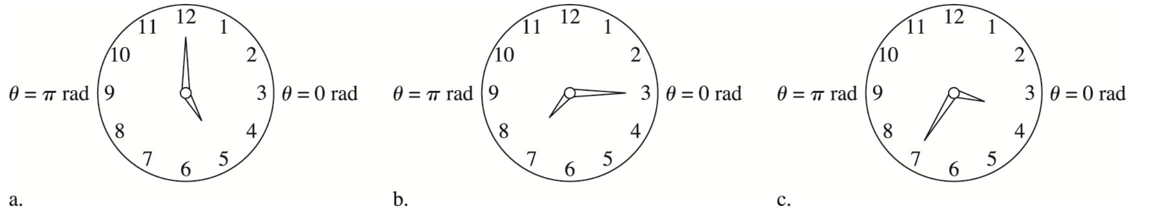
P5.28. Prepare: According to Hooke's law, the spring force acting on a mass (m) attached to the end of a spring is given as $F_{sp} = k\Delta y$, where Δy is the change in length of the spring. If the mass m is at rest, then F_{sp} is also equal to the weight $w = mg$.

Solve: We have $F_{sp} = k\Delta y = mg$. We want a 0.10 kg mass to give $\Delta y = 0.010$ m. This means,

$$k = mg/\Delta y = (0.10\text{ kg})(9.80\text{ m/s}^2)/(0.010\text{ m}) = 98\text{ N/m}$$

Assess: If you double the mass and hence the weight, the displacement of the end of the spring will double as well.

P5.29. Prepare: This problem deals with angular measurements in radians. Recall that 2π rad $= 360°$, and that angle measurements start at the $+x$-axis in standard Cartesian coordinates. The position of the minute hand is determined by the number after the colon. There are 60 min in an hour so the number of minutes after the hour, when divided by 60, gives the fraction of a circle which has been covered by the minute hand. Also, the minute hand starts at $\pi/2$ rad and travels clockwise, thus decreasing the angle. If we get a negative angle, we can make it positive by adding 2π rad.



Solve: **(a)** The angle is calculated as described above. Since the number after the colon is 0, we subtract nothing from $\pi/2$ rad, so $\theta = \pi/2$.

(b) We subtract 15/60 of 2π rad from the starting angle, so we have:

$$\theta = \frac{\pi}{2} - \left(\frac{15}{60}\right)(2\pi) = 0$$

(c) As before, the angle is given by:

$$\theta = \frac{\pi}{2} - \left(\frac{35}{60}\right)(2\pi) = -\frac{2}{3}\pi$$

Since this angle is negative, we can add 2π rad to obtain: $\theta = -2\pi/3 + 2\pi = 4\pi/3$.

Assess: The first two parts make sense from our experience with clocks. In part (a), the minute hand is straight up. In part (b), it points to the right.

P5.30. **PREPARE:** This problem deals with angular displacements in radians. Recall that 2π rad = 360°, and that angle measurements start at the $+x$-axis in standard Cartesian coordinates. We are given a period of rotation and a time interval. If we find the angular velocity, we can use $\Delta\theta = \omega\Delta t$.

SOLVE: Since he or she completes one revolution in 3.0 s, his or her angular velocity is given by: $\omega = \Delta\theta/\Delta t = (2\pi \text{ rad})/(3.0 \text{ s}) = 2.09 \text{ rad/s}$. We include an extra digit beyond the significant two digits since this is an intermediate step. Now we can find his or her angular displacement as described previously

$$\Delta\theta = (2.09 \text{ rad/s})(1.0 \text{ s}) = 2.1 \text{ rad}$$

ASSESS: This makes sense because if he or she completes one revolution in 3.0 s, then he or she completes one third of a revolution in 1.0 s.

P5.31. **PREPARE:** To compute the angular speed ω, we use the equation in the text and convert to rad/s: 2π rad = 360°. The minute hand takes an hour to complete one revolution.

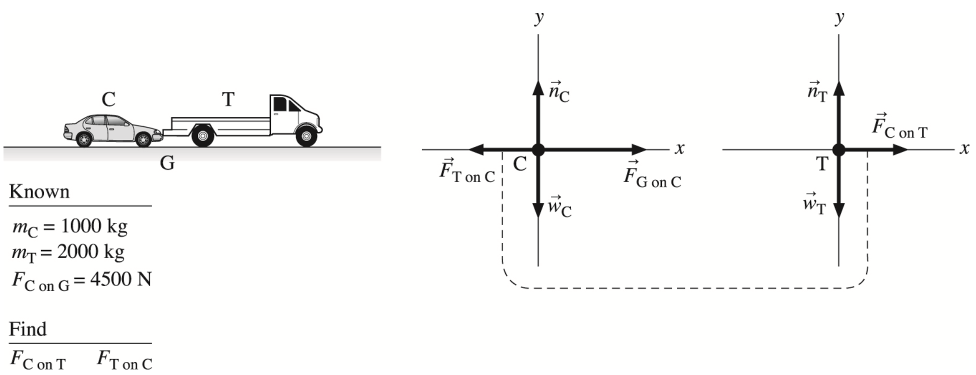
SOLVE:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{1.0 \text{ rev}}{60 \text{ min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 0.0017 \text{ rad/s} = 1.7\times10^{-3} \text{ rad/s}$$

ASSESS: This answer applies not just to the tip, but the whole minute hand. The answer is small, but the minute hand moves quite slowly.

The second hand moves 60 times faster or 0.10 rad/s. This too seems reasonable.

P5.32. **PREPARE:** We will write down Newton's second law for the car and truck separately, and we will use Newton's third law to describe interactions between the car and truck. The car and the truck will be denoted by the symbols C and T, respectively. The ground will be denoted by the symbol G. A visual overview shows a pictorial representation, a list of known and unknown values, and a free-body diagram for both the car and the truck. Since the car and the truck move together in the positive *x*-direction, they have the same acceleration.



SOLVE: **(a)** The *x*-component of Newton's second law for the car is

$$\Sigma(F_{\text{on C}})_x = F_{\text{G on C}} - F_{\text{T on C}} = m_C a_C$$

The *x*-component of Newton's second law for the truck is

$$\Sigma(F_{\text{on T}})_x = F_{\text{C on T}} = m_T a_T$$

Using $a_C = a_T = a$ and $F_{\text{T on C}} = F_{\text{C on T}}$, we get

$$(F_{\text{C on G}} - F_{\text{C on T}})\left(\frac{1}{m_C}\right) = a \quad (F_{\text{C on T}})\left(\frac{1}{m_T}\right) = a$$

Combining these two equations,

$$(F_{\text{C on G}} - F_{\text{C on T}})\left(\frac{1}{m_C}\right) = (F_{\text{C on T}})\left(\frac{1}{m_T}\right) \Rightarrow F_{\text{C on T}}\left(\frac{1}{m_C} + \frac{1}{m_T}\right) = (F_{\text{C on G}})\left(\frac{1}{m_C}\right)$$

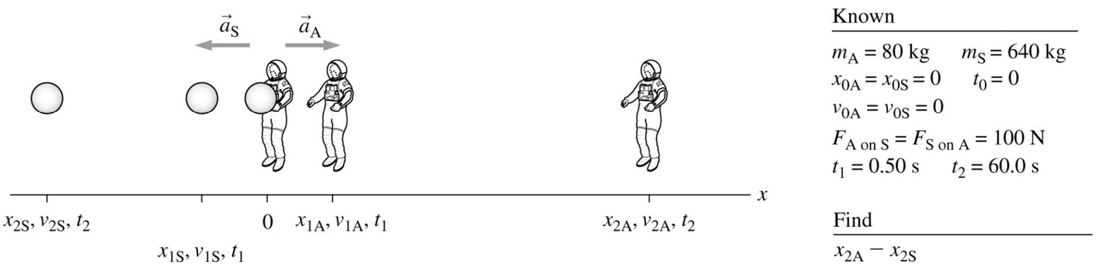
$$\Rightarrow F_{\text{C on T}} = (F_{\text{C on G}})\left(\frac{m_T}{m_C + m_T}\right) = (4500\text{ N})\left(\frac{2000\text{ kg}}{1000\text{ kg} + 2000\text{ kg}}\right) = 3000\text{ N}$$

(b) Due to Newton's third law, $F_{\text{T on C}} = 3000\text{ N}$.

ASSESS: Even though the forces each vehicle exerts on the other are equal, the accelerations they cause are very different for the two vehicles. The lighter car will be accelerated twice as much, making it potentially much more dangerous for the passengers of the car than for those of the truck.

P5.33. PREPARE: The astronaut and the satellite, the two objects in our system, will be treated as particles.

Pictorial representation



SOLVE: The astronaut and the satellite accelerate in opposite directions for 0.50 s. The force on the satellite and the force on the astronaut are an action/reaction pair, so both have a magnitude of 100 N. Newton's second law for the satellite along the x-direction gives

$$\Sigma(F_{\text{on S}})_x = F_{\text{A on S}} = m_S a_S \quad \Rightarrow \quad a_S = \frac{F_{\text{A on S}}}{m_S} = \frac{-(100\text{ N})}{640\text{ kg}} = -0.156\text{ m/s}^2$$

Newton's second law for the astronaut along the x-direction is

$$\Sigma(F_{\text{on A}})_x = F_{\text{S on A}} = m_A a_A \quad \Rightarrow \quad a_A = \frac{F_{\text{S on A}}}{m_A} = \frac{F_{\text{A on S}}}{m_A} = \frac{100\text{ N}}{80\text{ kg}} = 1.25\text{ m/s}^2$$

Let us first calculate the positions and velocities of the astronaut and the satellite at $t_1 = 0.50\text{ s}$ under the accelerations a_A and a_S:

$$x_{1A} = x_{0A} + v_{0A}(t_1 - t_0) + \tfrac{1}{2}a_A(t_1 - t_0)^2 = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(1.25\text{ m/s}^2)(0.50\text{ s} - 0.00\text{ s})^2 = 0.156\text{ m}$$

$$x_{1S} = x_{0S} + v_{0S}(t_1 - t_0) + \tfrac{1}{2}a_S(t_1 - t_0)^2 = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(-0.156\text{ m/s}^2)(0.50\text{ s} - 0.00\text{ s})^2 = -0.020\text{ m}$$

$$v_{1A} = v_{0A} + a_A(t_1 - t_0) = 0\text{ m/s} + (1.25\text{ m/s}^2)(0.50\text{ s} - 0.00\text{ s}) = 0.625\text{ m/s}$$

$$v_{1S} = v_{0S} + a_S(t_1 - t_0) = 0\text{ m/s} + (-0.156\text{ m/s}^2)(0.5\text{ s} - 0.00\text{ s}) = -0.078\text{ m/s}$$

With x_{1A} and x_{1S} as initial positions, v_{1A} and v_{1S} as initial velocities, and zero acceleration, we can now obtain the new positions at $(t_2 - t_1) = 59.5$ s:

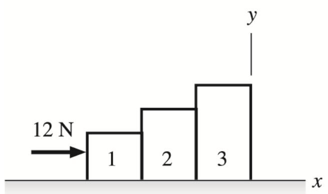
$$x_{2A} = x_{1A} + v_{1A}(t_2 - t_1) = 0.156 \text{ m} + (0.625 \text{ m/s})(59.5 \text{ s}) = 37.34 \text{ m}$$

$$x_{2S} = x_{1S} + v_{1S}(t_2 - t_1) = -0.02 \text{ m} + (-0.078 \text{ m/s})(59.5 \text{ s}) = -4.66 \text{ m}$$

Thus, the astronaut and the satellite are $x_{2A} - x_{2S} = (37.34 \text{ m}) - (-4.66 \text{ m}) = 42 \text{ m}$ apart.

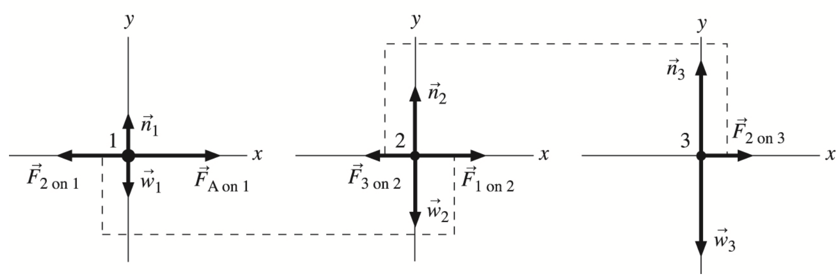
ASSESS: Since the initial push was not that strong, nor exerted for too long, this is a reasonable distance after one minute.

P5.34. PREPARE: We will write down Newton's second law for each object, and relate them by requiring that Newton's third law hold. For example: $(F_{1 \text{ on } 2})_x = -(F_{2 \text{ on } 1})_x$. The blocks are denoted as 1, 2, and 3. The surface is frictionless and along with the earth it is a part of the environment. The three blocks are our three systems of interest. The force applied on block 1 is $F_{\text{A on 1}} = 12$ N. The acceleration for all the blocks is the same and is denoted by a. A visual overview shows a pictorial representation, a list of known and unknown values, and a free-body diagram for the three blocks.



Known
$m_1 = 1.0$ kg
$m_2 = 2.0$ kg
$m_3 = 3.0$ kg
$F_{\text{A on 1}} = 12$ N

Find
$F_{2 \text{ on } 3}$
$F_{2 \text{ on } 1}$



SOLVE: Newton's second law for the three blocks along the x-direction is

$$\Sigma(F_{\text{on 1}})_x = F_{\text{A on 1}} - F_{2 \text{ on } 1} = m_1 a \quad \Sigma(F_{\text{on 2}})_x = F_{1 \text{ on } 2} - F_{3 \text{ on } 2} = m_2 a \quad \Sigma(F_{\text{on 3}})_x = F_{2 \text{ on } 3} = m_3 a$$

Adding these three equations and using Newton's third law ($F_{2 \text{ on } 1} = F_{1 \text{ on } 2}$ and $F_{3 \text{ on } 2} = F_{2 \text{ on } 3}$), we get

$$F_{\text{A on 1}} = (m_1 + m_2 + m_3)a \Rightarrow (12 \text{ N}) = (1.0 \text{ kg} + 2.0 \text{ kg} + 3.0 \text{ kg})a \Rightarrow a = 2.0 \text{ m/s}^2$$

(a) Using this value of a, the force equation on block 3 gives

$$F_{2 \text{ on } 3} = m_3 a = (3.0 \text{ kg})(2.0 \text{ m/s}^2) = 6.0 \text{ N}$$

(b) Substituting into the force equation on block 1,

$$12\text{ N} - F_{2\text{ on }1} = 12\text{ N} - (1.0\text{ kg})(2.0\text{ m/s}^2) \Rightarrow F_{2\text{ on }1} = 10\text{ N}$$

ASSESS: Because all three blocks are pushed forward by a force of 12 N, the value of 10 N for the force that the 2.0 kg block exerts on the 1.0 kg block is reasonable.

P5.35. PREPARE: This problem involves the sum of all torques equaling zero, in which case the rigid object does not experience any angular acceleration. The height, thickness, and mass of the door are all irrelevant for this problem. If the door closer exerts a torque of 5.2 N · m, then you need to also apply a torque of 5.2 N · m in the opposite direction. The way to do that with the least force is to make r as big as possible (the entire width of the door), and make sure the angle $\phi = 90°$.

SOLVE: From $\tau = rF\sin\phi$ solve for F. Then we see that the needed torque is produced with the smallest force by maximizing r and $\sin\phi$.

$$F = \frac{\tau}{r\sin\phi} = \frac{5.2\text{ N}\cdot\text{m}}{(0.91\text{ m})\sin 90°} = 5.7\text{ N}$$

ASSESS: It is good to have problems where more than the required information is given. Part of learning to solve real-world problems is knowing (or learning) which quantities are significant, which are irrelevant, and which are negligible.

The answer of 5.7 N seems like a reasonable amount of force, which might be supplied, say, by a doorstop. If your doorstop is a simple wedge of wood inserted under the door (as they are at my college), you can see that it should be positioned near the outside edge of the door so the friction force will produce enough torque to keep the door open.

P5.36. PREPARE: Call the mass of the star *M.* Write Newton's law of gravitation for each planet.

$$F_1 = \frac{GMm_1}{r_1^2}$$

$$F_2 = \frac{GMm_2}{r_2^2} = \frac{GM(2m_1)}{(2r_1)^2}$$

SOLVE: Divide the two equations to get the ratio desired.

$$\frac{F_2}{F_1} = \frac{\frac{GM(2m_1)}{(2r_1)^2}}{\frac{GMm_1}{r_1^2}} = \frac{1}{2}$$

ASSESS: The answer is expected. Even with twice the mass, because the radius in the denominator is squared, we expect the force on planet 2 to be less than the force on planet 1.

P5.37. PREPARE: We will use Newton's Law of Universal Gravitation to determine the gravitational attraction between spheres. Assume the two lead balls are spherical masses with their centers separated by 10 cm.

SOLVE: (a)

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(10\text{ kg})(0.100\text{ kg})}{(0.10\text{ m})^2} = 6.67\times10^{-9}\text{ N} = 6.7\times10^{-9}\text{ N}$$

(b) The ratio of the above gravitational force to the weight of the 100 g ball is

$$\frac{6.7\times10^{-9}\text{ N}}{(0.100\text{ kg})(9.8\text{ m/s}^2)}=6.8\times10^{-9}$$

ASSESS: The answer in part (b) shows the smallness of the gravitational force between two lead balls separated by 10 cm compared to the weight of the 100 g ball.

P5.38. PREPARE: We can use the equation for free-fall acceleration on the surface of a given planet. Assume the two planets are spherical masses, M_1 and M_2 with radii R_1 and R_2, respectively. $M_2=2M_1$ and $R_2=2R_1$. We will use Newton's Law of Universal Gravitation, pulling out the mass that is being accelerated to obtain an expression for the acceleration due to gravity.

SOLVE: (a) From the equation for free fall

$$g_1=\frac{GM_1}{R_1^2}\quad\text{and}\quad g_2=\frac{GM_2}{R_2^2}$$

So,

$$\frac{g_2}{g_1}=(M_2/M_1)(R_1/R_2)^2=(2M_1/M_1)(R_1/2R_1)^2=0.5\Rightarrow g_2=0.5g_1=0.5(20\text{ m/s}^2)=10\text{ m/s}^2$$

ASSESS: The answer shows clearly the inverse square dependence on distance versus the direct dependence on mass of the acceleration due to gravity on the surface of a planet.

P5.39. PREPARE: We will use Newton's Law of Universal Gravitation and known physical characteristics of Jupiter to determine the free-fall acceleration on this new planet. Look up the data for Jupiter.

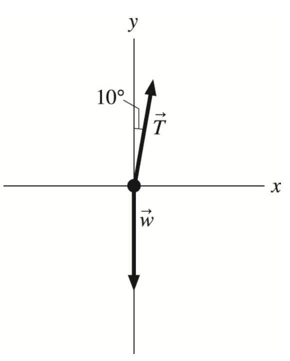
$M_{\text{Jupiter}}=1.90\times10^{27}$ kg, $R_{\text{Jupiter}}=6.99\times10^{7}$ m

SOLVE: From the equation in the text,

$$g=\frac{GM}{R^2}=\frac{G(0.43M_{\text{Jupiter}})}{(1.7R_{\text{Jupiter}})^2}=\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\big((0.43)(1.90\times10^{27}\text{ kg})\big)}{((1.7)(6.99\times10^{7}\text{ m}))^2}=3.9\text{ m/s}^2$$

ASSESS: This is in the range of g for other planets.

P5.40. PREPARE: Note that the medal would hang straight down if the car were going at a constant velocity, so the deviation from vertical only occurs while the car is accelerating. The medal must have the same acceleration as the car. We can use that information, along with geometric information given to us to determine the unknown acceleration. We apply Newton's second law.



SOLVE: (a) Because she accelerates onto the highway, we assume she is accelerating forward so the medal hangs away from the windshield.

(b) Use Newton's law in vertical and horizontal directions. In the horizontal direction, there is only one (component of) force, but there is an acceleration.

$$\Sigma F_x = T \sin\theta = ma_x$$

There is no acceleration in the y-direction.

$$\Sigma F_y = T\cos\theta - mg = 0 \Rightarrow T\cos\theta = mg$$

Divide the first equation by the second and cancel m and T.

$$\frac{\sin\theta}{\cos\theta} = \frac{a_x}{g} \Rightarrow a_x = g\tan\theta = (9.80\,\text{m/s}^2)(\tan 10°) = 1.7\,\text{m/s}^2$$

ASSESS: This is a reasonable acceleration for a car.

P5.41. PREPARE: Because the times where we are asked to find the net force fall on distinct slopes of the velocity-versus-time graph, we can use the constant slopes of the three segments of the graph to calculate the three accelerations.

SOLVE: For t between 0 s and 3 s,

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{12\text{ m/s} - 0\text{ s}}{3\text{ s}} = 4\text{ m/s}^2$$

For t between 3 s and 6 s, $\Delta v_x = 0$ m/s, so $a_x = 0\text{ m/s}^2$. For t between 6 s and 8 s,

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{0\text{ m/s} - 12\text{ m/s}}{2\text{ s}} = -6\text{ m/s}^2$$

From Newton's second law, at $t = 1$ s we have

$$F_{\text{net}} = ma_x = (2.0\text{ kg})(4\text{ m/s}^2) = 8\text{ N}$$

At $t = 4$ s, $a_x = 0\text{ m/s}^2$, so $F_{\text{net}} = 0$ N.

At $t = 7$ s,

$$F_{\text{net}} = ma_x = (2.0\text{ kg})(-6.0\text{ m/s}^2) = -12\text{ N}$$

ASSESS: The magnitudes of the forces look reasonable, given the small mass of the object. The positive and negative signs are appropriate for an object first speeding up, then slowing down.

P5.42. PREPARE: This problem involves the determination of force from kinematic information through the application of Newton's second law. In order to relate the force the forehead exerts on the baseball to the force the baseball exerts on the forehead, we must also employ Newton's third law. We will assume constant acceleration so we can use the kinematic equations. Assume the baseball is initially moving in the positive x-direction.

We list the known quantities:

Known
$m = 0.14\text{ kg}$
$(v_x)_i = 30\text{ m/s}$
$\Delta t = 0.0015\text{ s}$
Find
a
F

SOLVE: (a) With $(v_x)_f = 0\text{ m/s},$ we solve for a from $(v_x)_f = (v_x)_i + a_x\Delta t.$

$$a_x = \frac{-(v_x)_i}{\Delta t} = \frac{-(30\text{ m/s})}{0.0015\text{ s}} = -20{,}000\text{ m/s}^2$$

The magnitude of this is $20{,}000\text{ m/s}^2$ or $2.0\times 10^4\text{ m/s}^2.$

(b) Apply Newton's second law: $\Sigma F_x = ma_x$ where the force of the body on the ball is the only force (and is therefore the net force).

$$\Sigma F_x = ma_x = (0.14\text{ kg})(-20{,}000\text{ m/s}^2) = -2800\text{ N}$$

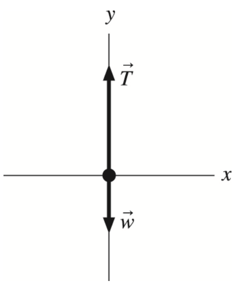
The magnitude of this is $2800\text{ N}.$ This force is exerted by the body the ball hits.

(c) By Newton's third law if the body exerts a force on the ball, then the ball exerts a force equal in magnitude and opposite in direction on the body. Therefore, the ball applies a force of 2800 N to the object it hits.

(d) This force of 2.8 kN is less than 6.0 kN, so the forehead is not in danger (although it would still hurt and maybe raise a lump). This force of 2.8 kN is greater than 1.3 kN, so the cheek is in danger of fracture.

ASSESS: This is a nice real-life problem that employs the definition of acceleration and Newton's second and third laws. The data provided are typical of real baseballs and real pitching speeds, so the conclusion is also true to life. Catchers, whose faces are in the line of fire, wear masks for this reason.

P5.43. PREPARE: The fish is acted on by two forces: the tension in the line and the pull of gravity. Both the forces act along the same vertical line, which is taken to be the y-axis. The free-body diagram for the fish is shown below. The fish could rise at a constant velocity (and the two forces would have equal magnitudes), but the fisherman can get it up more quickly by accelerating it (so the tension is greater than the weight). First, apply Newton's second law and then the kinematic equations.



Known
$m = 5.0\text{ kg}$
$T_{max} = 54\text{ N}$
$\Delta y = 2.0\text{ m}$
Find
Δt_{min}

SOLVE: Since we want the shortest possible time, we'll use the maximum tension, which produces the maximum acceleration.

$$F_{net} = T_{max} - mg = ma_{max} \Rightarrow a_{max} = \frac{T_{max}}{m} - g = \frac{54\text{ N}}{5.0\text{ kg}} - 9.80\text{ m/s}^2 = 1.0\text{ m/s}^2$$

Now use the kinematic equations for constant-acceleration for an object starting from rest.

$$\Delta y = \frac{1}{2}a_{max}(\Delta t_{min})^2 \Rightarrow \Delta t_{min} = \sqrt{\frac{2\Delta y}{a_{max}}} = \sqrt{\frac{2(2.0\text{ m})}{1.0\text{ m/s}^2}} = 2.0\text{ s}$$

Assess: This seems like a reasonable amount of time to raise a fish two meters.

P5.44. Prepare: This problem deals with apparent weight, which is the contact force between a surface and the people resting on it. We can use kinematics to determine the acceleration. From there we can use Newton's second law to relate that acceleration to the forces involved, and specifically to the apparent weight. Use the kinematic equations twice. The first time find out the velocity of the rider at the end of the 2.0 s and then use that as the initial velocity during the second part to compute the acceleration.

Solve: During the free fall phase the initial velocity is zero, so

$$v_f = a\Delta t = (-9.80\text{ m/s}^2)(2.0\text{ s}) = -19.6\text{ m/s}$$

then, as the rider slows,

$$a = \frac{\Delta v}{\Delta t} = \frac{0\text{ m/s} - (-19.6\text{ m/s})}{0.50\text{ s}} = 39.2\text{ m/s}^2$$

This is an upward acceleration. The apparent weight is

$$w_{app} = m(g + a_y) = (65\text{ kg})(9.80\text{ m/s}^2 + 39.2\text{ m/s}^2) = 3200\text{ N}$$

This is 5.0x the rider's actual weight.

Assess: We would say the rider experiences 5g's, which is a significant acceleration, but it lasts for only a short while.

P5.45. Prepare: This problem deals with apparent weight, which is the contact force between the astronaut and the surface on which he/she is resting (the normal force acting on the astronaut). We will write out the sum of all forces in the vertically upward ($+y$) direction, and use Newton's second law to relate this to the acceleration of the astronaut. This will enable us to determine the normal force acting on the astronaut. We can get the acceleration by using $(v_f)_y = (v_i)_y + a_y\Delta t$ and converting all information to SI units: $\frac{160\text{ km}}{1\text{ h}} \times \left(\frac{1\text{ h}}{3600\text{ s}}\right) \times \left(\frac{1000\text{ m}}{1\text{ km}}\right) = 44.4\text{ m/s}.$

Solve: Newton's second law tells us $\sum(F_{on\ astro})_y = n - mg = ma_y$. Inserting the expression for the acceleration and rearranging, we have

$$n = m(a_y + g) = m\left(\frac{(v_f)_y - (v_i)_y}{\Delta t} + g\right) = (72\text{ kg})\left(\frac{(44.4\text{ m/s})_y - (0)}{(8.0\text{ s})} + (9.8\text{ m/s}^2)\right) = 1.1\times10^3\text{ N}$$

Assess: This is about 1.5 times the astronaut's actual weight on Earth, which is reasonable.

P5.46 **PREPARE:** Let us call the direction of the final recoil of the woodpecker's head the $+x$-direction. Part (a) can be solved by using the definition of constant-acceleration: $a_x = \Delta v_x / \Delta t$. Since the force and acceleration are approximately constant, we can use other kinematic equations, specifically $v_{x,f}^2 = v_{x,i}^2 + 2a_x \Delta x$ to solve part (b). Finally, part (c) requires Newton's second law: $\sum F_x = ma_x$.

SOLVE: (a) $a_x = \Delta v_x / \Delta t = \dfrac{(3.6\text{ m/s}) - (-3.6\text{ m/s})}{(2.0\times10^{-3}\text{ s})} = 3600\text{ m/s}^2$

(b) Let the "final" time be the halfway point, where the beak has momentarily come to rest before recoiling. Using the result of part (a), we have

$$\Delta x = \frac{v_{x,f}^2 - v_{x,i}^2}{2a_x} = \frac{(0)^2 - (3.6\text{ m/s})^2}{2(3600\text{ m/s}^2)} = -1.8\text{ mm}$$

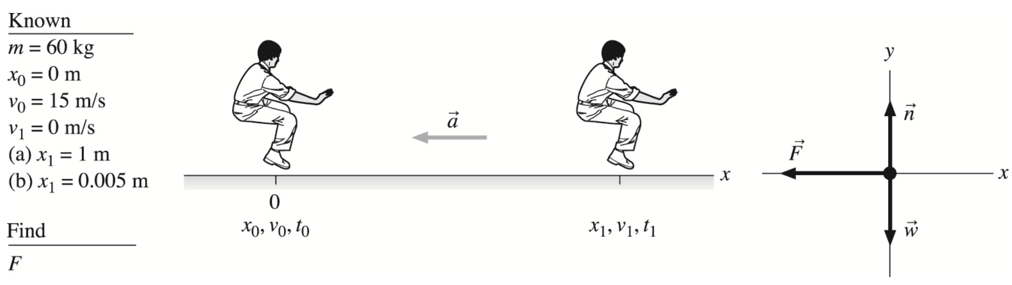
Here the "–" sign simply indicates that the motion is in the direction we chose to call the $-x$ direction. The distance is 1.8 mm.

(c) The only force on the woodpecker's head in the direction of motion is the force from the tree on the woodpecker. Thus

$$\sum F_x = F_{\text{tree}} = ma_x = (9.0\times10^{-3}\text{ kg})(3600\text{ m/s}^2) = 32\text{ N}$$

ASSESS: This is an enormous force for such a tiny head, as is evident from the very large acceleration.

P5.47. **PREPARE:** We can use the stopping distances to determine the acceleration in each case. Then we can relate the acceleration to the forces involved using Newton's second law. Assume the person is moving in a straight line under the influence of the combined decelerating forces of the air bag and seat belt or, in the absence of restraints, the dashboard or windshield. The following is an overview of the situation in a pictorial representation and the occupant's free-body diagram is shown below. Note that the occupant is brought to rest over a distance of 1 m in the former case, but only over 5 mm in the latter.



SOLVE: (a) In order to use Newton's second law for the passenger, we'll need the acceleration. Since we don't have the stopping time,

$$v_f^2 = v_i^2 + 2a(x_f - x_i) \Rightarrow a = \frac{v_f^2 = v_i^2}{2(x_f - x_i)} = \frac{0\text{ m}^2/\text{s}^2 - (15\text{ m/s}^2)}{2(1\text{ m} - 0\text{ m})} = -112.5\text{ m/s}^2$$

$$\Rightarrow F_{\text{net}} = F = ma = (60\text{ kg})(-112.5\text{ m/s}^2) = -6750\text{ N}$$

The net force is 6800 N to the left.

(b) Using the same approach as in part (a),

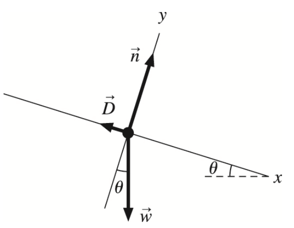
$$F = ma = m = \frac{v_f^2 = v_i^2}{2(x_f - x_i)} = (60\text{ kg})\frac{0\text{ m}^2/\text{s}^2 - (15\text{ m/s}^2)}{2(0.005\text{ m})} = -1{,}350{,}000\text{ N}$$

The net force is 1.4×10^6 N to the left.

(c) The passenger's weight is $mg = (60\text{ kg})(9.80\text{ m/s}^2) = 590\text{ N}.$ The force in part (a) is 11 times the passenger's weight. The force in part (b) is 2300 times the passenger's weight.

ASSESS: An acceleration of 11g is well within the capability of the human body to withstand. A force of 2300 times the passenger's weight, on the other hand, would surely be catastrophic.

P5.48. **PREPARE:** We will consider Newton's second law during the first and second 3.0 s intervals. The apparent weight tells us about the normal force acting on Corey, and we know the force of gravity. So, we will be able to determine Corey's acceleration in the vertical direction. Let us call vertically upward the $+y$-direction, as shown in the free-body diagram below.



SOLVE: During the first three seconds, we have:

$$\sum F_y = n - mg = ma_y \Rightarrow a_y = \frac{n}{m} - g = \left(\frac{830\text{ N}}{95\text{ kg}}\right) - \left(9.8\text{ m/s}^2\right) = -1.06\text{ m/s}^2$$

So, we know there will be 3.0 seconds in which Corey accelerates downward at a rate of 1.06 m/s^2. This means the vertical component of Corey's velocity will be

$(v_f)_y = (v_i)_y + a_y\Delta t = (0) + (-1.06\text{ m/s}^2)(3.0\text{ s}) = -3.2\text{ m/s}.$

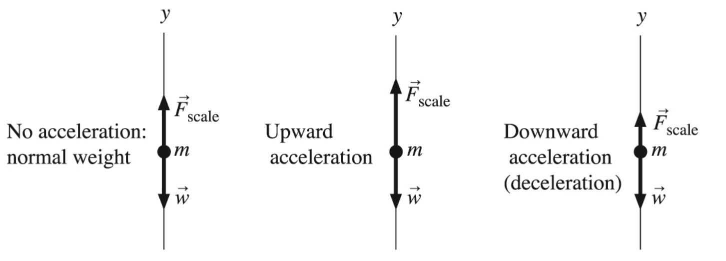
Now for the second 3.0 s period, we have

$$\sum F_y = n - mg = ma_y \Rightarrow a_y = \frac{n}{m} - g = \left(\frac{930\text{ N}}{95\text{ kg}}\right) - \left(9.8\text{ m/s}^2\right) = 0\text{ m/s}^2 \text{ to two significant digits.}$$

Thus, we see that the second 3.0 s period was one of constant downward speed. So, the final velocity after 6.0 s is 3.2 m/s downward.

ASSESS: This is a reasonable speed for an elevator.

P5.49. **PREPARE:** This problem deals with the concept of apparent weight, which in this case is the contact force that the scale exerts upward on your body. We can use this reading as the normal force and apply Newton's second law to determine the unknown accelerations. Your body is moving in a straight line along the y-direction under the influence of two forces: gravity and the support force of the scale. The free-body diagrams for you for the following three cases are shown below: no acceleration, upward acceleration, and downward acceleration. The apparent weight of an object moving in an elevator is $w_{app} = w(1 + \frac{a}{g}) \Rightarrow a = (\frac{w_{app}}{w} - 1)g.$



SOLVE: (a) When accelerating upward, the acceleration is

$$a = \left(\frac{170 \text{ lb}}{150 \text{ lb}} - 1\right)(9.80 \text{ m/s}^2) = 1.3 \text{ m/s}^2$$

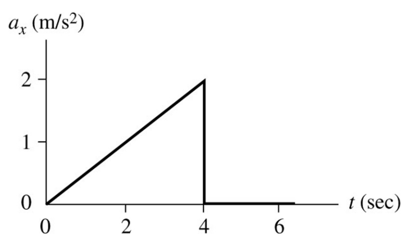
(b) When braking, the acceleration is

$$a = \left(\frac{120 \text{ lb}}{150 \text{ lb}} - 1\right)(9.80 \text{ m/s}^2) = -2.0 \text{ m/s}^2$$

The magnitude of this acceleration is 2.0 m/s^2.

ASSESS: A 10–20% change in apparent weight seems reasonable for a fast elevator, as the one in the Empire State Building must be. Also, note that we did not have to convert the units of the weights from pounds to Newtons because the weights appear as a ratio.

P5.50. PREPARE: The net force in the horizontal direction is related to the acceleration in that direction through Newton's second law. This enables us to sketch an acceleration-versus-time plot.



For example, the peak acceleration was calculated as follows:

$$a_{\text{max}} = \frac{F_{\text{max}}}{m} = \frac{10 \text{ N}}{5 \text{ kg}} = 2 \text{ m/s}^2$$

Integrating the acceleration overtime yields the change in v_x.

SOLVE: The acceleration is not constant, so we cannot use constant-acceleration kinematics. Instead, we use the more general result that

$$v(t) = v_0 + \text{area under the acceleration curve from 0 s to } t$$

The object starts from rest, so $v_0 = 0$ m/s. The area under the acceleration curve between 0 and 6 s is $\frac{1}{2}(4 \text{ s})(2 \text{ m/s}^2) = 4.0$ m/s. We've used the fact that the area between 4 and 6 s is zero. Thus, at $t = 6$ s, $v_x = 4.0$ m/s.

ASSES: If the acceleration had been a constant $2.0\ \text{m/s}^2$, then it would be easy to see that

$$v_{x,\text{f}} = v_{x,\text{i}} + a_x \Delta t = \left(2.0\ \text{m/s}^2\right)\left(4.0\ \text{s}\right) = 8\ \text{m/s}$$

Our answer is less than this, which is reasonable, since $2.0\ \text{m/s}^2$ was the maximum acceleration.

P5.51 **PREPARE:** The force and acceleration are related through Newton's 2nd law: $a_x = \dfrac{\sum F_x}{m}$. The change in v_x is calculated by integrating the acceleration over time.

SOLVE: The change in v_x is

$$\Delta v_x = \int_{t_\text{i}}^{t_\text{f}} a_x dt = \frac{1}{m}\int_{t_\text{i}}^{t_\text{f}} \sum F_x dt = \frac{1}{m}\int_{t_\text{i}}^{t_\text{f}} (2.0)t^2 dt = \frac{(2.0)}{(0.050\,\text{kg})}\frac{1}{3}\left((2.0)^3 - (0)^3\right)$$

$$\Delta v_x = 106.7\ \text{m/s} \approx 110\ \text{m/s}$$

Since the particle started from rest, the change in v_x is also the final value: 110 m/s.

ASSESS: Although the force being applied is on the order of a few Newtons, the mass is very small. So we expect large accelerations and large speeds after even just 2.0 s.

P5.52 **PREPARE:** The force and acceleration are related through Newton's second law: $a_x = \dfrac{\sum F_x}{m}$. The change in v_x is calculated by integrating the acceleration overtime.

SOLVE: The change in v_x is

$$\Delta v_x = \int_{t_\text{i}}^{t_\text{f}} a_x dt = \frac{1}{m}\int_{t_\text{i}}^{t_\text{f}} \sum F_x dt = \frac{1}{m}\int_{t_\text{i}}^{t_\text{f}} (2.0\ \text{N})e^{-t/1.0\ \text{s}} dt = \frac{(2.0\ \text{N})}{(0.050\,\text{kg})}(-1.0\ \text{s})\left(\left(e^{-2.0\ \text{s}/1.0\ \text{s}}\right) - \left(e^0\right)\right)$$

$$\Delta v_x = 34.6\ \text{m/s} \approx 35\ \text{m/s}$$

Since the particle started from rest, the change in v_x is also the final value: 35 m/s.

ASSESS: Although the force being applied is on the order of a newton or two, the mass is very small. So, we expect large accelerations and large speeds after even just 2.0 s.

P5.53. **PREPARE:** This problem involves two stages of acceleration: the first with the impala pushing on the ground, and the second with the impala accelerating only due to gravity. The first stage will involve consideration of Newton's second law, but the free fall can be treated with kinematics. Assume no air resistance. The strategy will be to first find the speed when the impala leaves the ground from the height of the leap, then use that to find the acceleration during the jumping motion.

SOLVE: Solve the kinematic equation in the y-direction to find the initial speed with the final speed at the top being zero.

$$v_\text{f}^2 = v_\text{i}^2 + 2g\Delta y \Rightarrow v_\text{i} = \sqrt{-2g\Delta y} = \sqrt{2(9.80\ \text{m/s}^2)(2.5\ \text{m})} = 7.0\ \text{m/s}$$

Use another kinematic equation to find the acceleration during the jump, with $v_1 = 0$.

$$a = \frac{\Delta v}{\Delta t} = \frac{7.0\ \text{m/s}}{0.21\ \text{s}} = 33.3\ \text{m/s}^2$$

The force of the impala on the ground is the same as the force of the ground on the impala.

$$F_{net} = n - w \Rightarrow n = F_{net} + w = ma + mg = (45\text{ kg})(33.3\text{ m/s}^2 + 9.80\text{ m/s}^2) = 1900\text{ N}$$

That answer is 4.4 times the impala's actual weight.

ASSESS: These seem like reasonable numbers.

P5.54. PREPARE: Let us call the vertically upward direction $+y$. Once we write the sum of all forces in the vertical direction, we can use the acceleration in $(v_f)_y^2 = (v_i)_y^2 + 2a_y\Delta y$. We will apply this kinematic equation once for the process of extending the legs, and then a second time after the athlete has left the ground. We must break the problem apart like this because the acceleration changes in between.

SOLVE: During the jump, the acceleration can be determined as follows:

$$\sum F_y = n - mg = ma_y$$
$$2mg - mg = ma_y \Rightarrow a_y = g$$

Inserting this into the kinematic equation, we find that the launch speed after a 60 cm leg extension is

$$(v_f)_y^2 = (v_i)_y^2 + 2a_y\Delta y = (0) + 2(9.8\text{ m/s}^2)(0.60\text{ m}) \Rightarrow (v_f)_y = 3.43\text{ m/s}$$

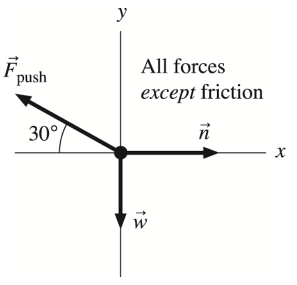
We now apply the same kinematic equation (in reverse) to see what change in height corresponds to zero upward speed:

$$\Delta y = \frac{(v_f)_y^2 - (v_i)_y^2}{2a_y} = \frac{(0) - (3.43\text{ m/s})_y^2}{2(-9.8\text{ m/s}^2)} = 0.60\text{ m}$$

So, the athlete's center will rise 0.60 m off the ground.

ASSESS: This is a reasonable height for a well-trained athlete.

P5.55. PREPARE: We will apply Newton's second law to the block. We must be especially careful with the direction of friction. We show below the free-body diagram of the 1 kg block. The block is initially at rest, so initially the friction force is static friction. If the 12 N pushing force is too strong, the box will begin to move up the wall. If it is too weak, the box will begin to slide down the wall. And, if the pushing force is within the proper range, the box will remain stuck in place.



SOLVE: First, let's evaluate the sum of all the forces *except* friction:

$$\sum F_x = n - F_{push}\cos 30° = 0\text{ N} \Rightarrow n = F_{push}\cos 30°$$

$$\sum F_y = F_{push}\sin 30° - w = F_{push}\sin 30° - mg = (12\text{ N})\sin 30° - (1\text{ kg})(9.80\text{ m/s}^2) = -3.8\text{ N}$$

In the first equation, we have utilized the fact that any motion is parallel to the wall, so $a_x = 0\ \text{m/s}^2$.

The two forces in the second *y*-equation add up to $-3.8\ \text{N}$. This means the static friction force will be able to prevent the box from moving if $f_s = +3.8\ \text{N}$. Using the *x*-equation, we get

$$f_{\text{s max}} = \mu_s n = \mu_s F_{\text{push}} \cos 30° = 5.2\ \text{N}$$

where we used $\mu_s = 0.5$ for wood on wood. The static friction force $\vec{f}_s$ needed to keep the box from moving is *less* than $f_{\text{s max}}$. Thus, the box will stay at rest.

ASSESS: Clearly, the box could be accelerated upward or downward by changing the angle and/or magnitude of the force.

P5.56 **PREPARE:** We will use Newton's second law in both the horizontal and vertical directions: $\sum F_x = ma_x$ and $\sum F_y = ma_y$.

SOLVE: From the sum of all forces in the vertical direction, we have

$$\sum F_y = F_{y,\text{grf}} - mg = (1.7mg)\cos(15°) - mg = ma_y \Rightarrow a_y = g(1.7\cos(15°) - 1)$$
$$a_y = (9.8\ \text{m/s}^2)(1.7\cos(15°) - 1) = 6.3\ \text{m/s}^2$$

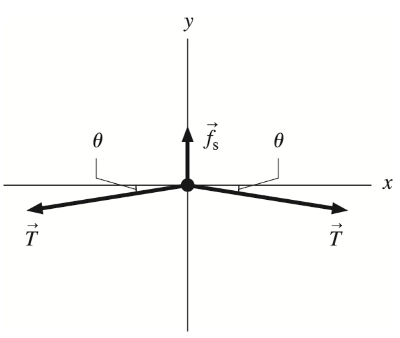
In the horizontal direction, we have

$$\sum F_x = F_{x,\text{grf}} = (1.7mg)\sin(15°) = ma_x \Rightarrow a_x = 1.7g\sin(15°)$$
$$a_y = (1.7)(9.8\ \text{m/s}^2)\sin(15°) = 4.3\ \text{m/s}^2$$

So, the components of acceleration are $a_x = 4.3\ \text{m/s}^2$, $a_y = 6.3\ \text{m/s}^2$.

ASSESS: Note that the horizontal acceleration is almost half the acceleration due to gravity!

P5.57. **PREPARE:** Let the string lie along the y-axis, such that the bow pulls the string in the $+x$-direction. As the string is pulled, some component of its tension will act against the friction from the bow. When this component maxes out the force of static friction, the string will slip. A free-body diagram will help us apply Newton's second law.



SOLVE: Newton's second law tells us

$$\sum F_x = -2T\sin(\theta) + \mu_s n = ma_x = 0$$
$$\sin(\theta) = \frac{\mu_s n}{2T}$$

If the length of the string is called L, then the displacement of the string at the midpoint is $\Delta x = \frac{L}{2}\sin(\theta)$.

Or using the results from the line above: $\Delta x = \frac{L}{2}\frac{\mu_s n}{2T} = \frac{(0.33\text{ m})}{4}\frac{(0.80)(0.75\text{ N})}{(50\text{N})} = 1.0\times10^{-3}\text{ m}$

ASSESS: If you have ever seen a violin string vibrating, you know that an answer on the order of 1 mm is very reasonable.

P5.58. PREPARE: This problem involves two stages of motion: one without friction and one with friction. We must split up the problem and apply Newton's second law to each stage. The length of the hill is $\Delta x = h/\sin\theta$ and the acceleration is $g\sin\theta$.

SOLVE: First use the kinematic equation, with $v_i = 0$ m/s at the top of the hill, to determine the speed at the bottom of the hill.

$$(v_f)_1^2 = (v_i)_1^2 + 2a\Delta x \Rightarrow (v_f)_1^2 = 2(g\sin\theta)(h/\sin\theta) = 2gh$$

Now apply the same kinematic equation to the horizontal patch of snow, only this time we want Δx. To connect the two parts $(v_f)_1 = (v_i)_2$. The final speed is zero: $(v_f)_2 = 0$.

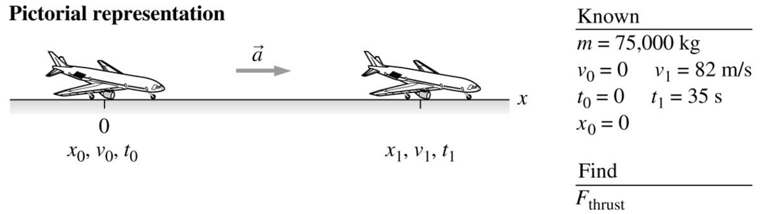
$$(v_f)_2^2 = (v_i)_2^2 + 2a\Delta x = (v_f)_1^2 + 2a\Delta x = 2gh + 2a\Delta x = 0$$

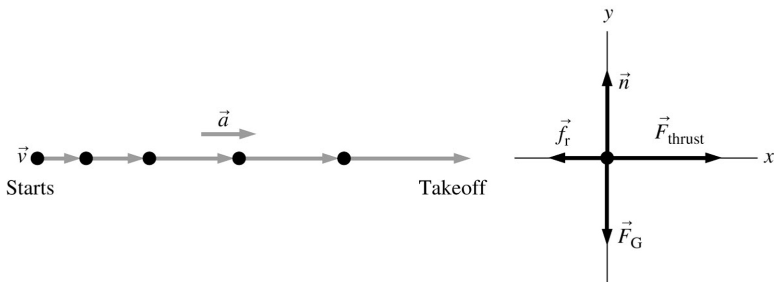
The friction force is the net force, so $a = -f_k/m$. Note $f_k = \mu_k n = \mu_k mg$. Solve for Δx.

$$\Delta x = \frac{-2gh}{2a} = \frac{-gh}{-f_k/m} = \frac{gh}{\mu_k mg/m} = \frac{h}{\mu_k} = \frac{3.0\text{ m}}{0.05} = 60\text{ m}$$

ASSESS: It seems reasonable to glide 60 m with such a low coefficient of friction. It is interesting that we did not need to know the angle of the (frictionless) slope; this will become clear in the chapter on energy. The answer is also independent of Josh's mass.

P5.59. PREPARE: We assume that the plane is a particle accelerating in a straight line under the influence of two forces: the thrust of its engines and the rolling friction of the wheels on the runway. The free-body diagram below will help us apply Newton's second law. We can use one-dimensional kinematics, first, to find the acceleration.





SOLVE: We can use the definition of acceleration to find a, and then apply Newton's second law. We obtain:

$$a = \frac{\Delta v}{\Delta t} = \frac{82 \text{ m/s} - 0 \text{ m/s}}{35 \text{ s}} = 2.34 \text{ m/s}^2$$

$$(F_{\text{net}}) = \sum F_x = F_{\text{thrust}} - f_{\text{r}} = ma \Rightarrow F_{\text{thrust}} = f_{\text{r}} + ma$$

For rubber rolling on concrete, $\mu_r = 0.02$ (Table 5.2), and since the runway is horizontal, $n = F_{\text{G}} = mg$.

Thus,

$$F_{\text{thrust}} = \mu_{\text{r}} F_{\text{G}} + ma = \mu_{\text{r}} mg + ma = m(\mu_{\text{r}} g + a)$$
$$= (75{,}000 \text{ kg})[(0.02)(9.8 \text{ m/s}^2) + 2.34 \text{ m/s}^2] = 190{,}000 \text{ N}$$

ASSESS: It's hard to evaluate such an enormous thrust, but comparison with the plane's mass suggests that 190,000 N is enough to produce the required acceleration.

P5.60 **PREPARE:** We will need to use Newton's second law in two directions: down the incline ($+x$) and normal to the board, somewhat upward ($+y$).

SOLVE: In the vertical direction, we have

$$\sum F_y = n - mg\cos(\theta) = ma_y = 0 \Rightarrow n = mg\cos(\theta)$$

In the horizontal direction, we have

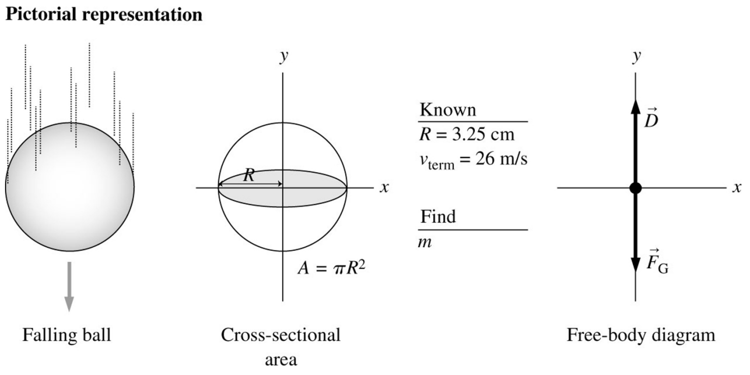
$$\sum F_x = mg\sin(\theta) - f_{\text{s}} = ma_x$$

When the cricket is right on the verge of slipping, the acceleration is approximately zero, and the static frictional force is at its maximum. This allows us to write

$$\sum F_x = mg\sin(\theta) - \mu_{\text{s}} n = mg\sin(\theta) - \mu_{\text{s}} mg\cos(\theta) = ma_x = 0$$
$$\Rightarrow \sin(\theta) = \mu_{\text{s}}\cos(\theta) \Rightarrow \mu_{\text{s}} = \tan(\theta) = \tan(40°) \approx 0.84$$

ASSESS: Since the cricket remains until such a large angle is reached, it is reasonable that the coefficient of static friction would be very high.

P5.61. PREPARE: We will represent the tennis ball as a particle. The drag coefficient is 0.5.



The tennis ball falls straight down toward the earth's surface. The ball is subject to a net force that is the resultant of the gravitational and drag force vectors acting vertically, in the downward and upward directions, respectively. Once the net force acting on the ball becomes zero, the terminal velocity is reached and remains constant for the rest of the motion.

SOLVE: The mathematical equation defining the dynamical equilibrium situation for the falling ball is

$$\vec{F}_{net} = \vec{F}_G + \vec{D} = \vec{0}\ \text{N}$$

Since only the vertical direction matters, one can write:

$$\sum F_y = 0\ \text{N} \Rightarrow \quad F_{net} = D - F_G = 0\ \text{N}$$

When this condition is satisfied, the speed of the ball becomes the constant terminal speed $v = v_{term}$. The magnitudes of the gravitational and drag forces acting on the ball are:

$$F_G = mg = m(9.80\ \text{m/s}^2)$$

$$D \approx \frac{1}{2}(C\rho A v_{term}^2) = 0.5(0.5)(1.2\ \text{kg/m}^3)(\pi R^2)v_{term}^2 = (0.3\pi)(0.0325\ \text{m})^2(26\ \text{m/s})^2 = 0.67\ \text{N}$$

The condition for dynamic equilibrium becomes:

$$(9.80\ \text{m/s}^2)m - 0.67\ \text{N} = 0\ \text{N} \Rightarrow m = \frac{0.67\ \text{N}}{9.80\ \text{m/s}^2} = 69\ \text{g}$$

ASSESS: The value of the mass of the tennis ball obtained above seems reasonable.

P5.62. PREPARE: The Reynolds number for this situation is large, so we can use the quadratic expression for drag: $D = \frac{1}{2}C_D A\rho v^2$. Let us call the direction of motion (down the hill) the $+x$-direction. We recognize that the component of gravity acting along this direction is $mg\sin(\theta)$.

SOLVE: From Newton's second law, we have

$$\sum F_x = mg\sin(\theta) - \frac{1}{2}C_D A\rho v^2 = ma_x = 0$$

$$v = \sqrt{\frac{2mg\sin(\theta)}{C_D A\rho}} = \sqrt{\frac{2(70\ \text{kg})(9.8\ \text{m/s}^2)\sin(3.5°)}{(0.88)(0.32\ \text{m}^2)(1.22\ \text{kg/m}^3)}} = 16\ \text{m/s}$$

ASSESS: This is a reasonable terminal speed given the small slope and the fact that the rider is just coasting.