# Solutions for Modern Control Systems 14th Edition by Dorf

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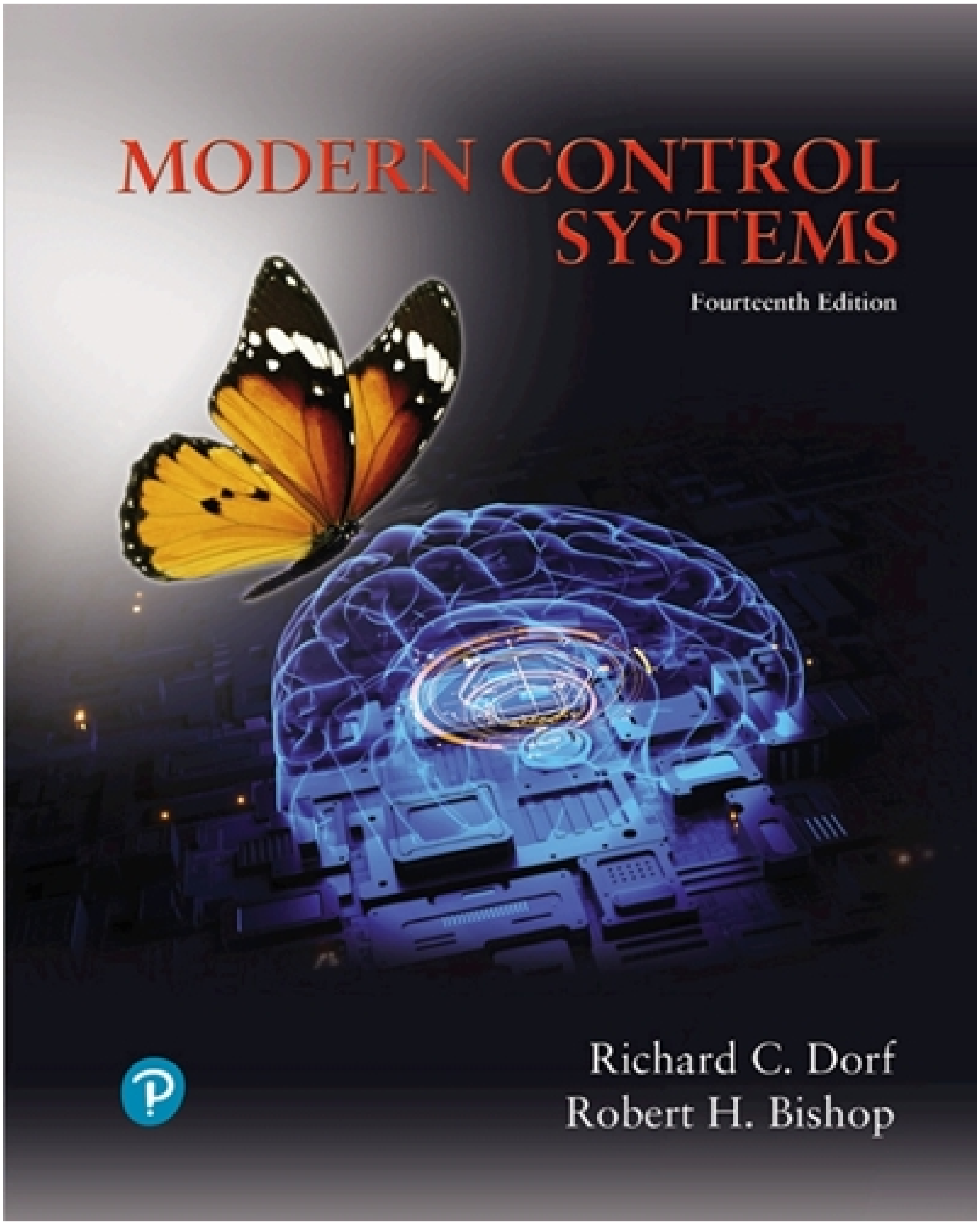


# Solutions

# INSTRUCTOR'S SOLUTIONS MANUAL

# MODERN CONTROL SYSTEMS

## FOURTEENTH EDITION

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Pearson

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ISBN-13: 978-0-13-730729-6
ISBN-10: 0-13-730729-2

# PREFACE

In each chapter, there are five problem types:

- Exercises
- Problems
- Advanced Problems
- Design Problems/Continuous Design Problem
- Computer Problems

In total, there are over 980 problems. The abundance of problems of increasing complexity gives students confidence in their problem-solving ability as they work their way from the exercises to the design and computer-based problems.

It is assumed that instructors (and students) have access to MATLAB and the Control System Toolbox or to LabVIEW and the MathScript RT Module. All of the computer solutions in this *Solution Manual* were developed and tested on an Apple MacBook Pro platform using MATLAB R2020a and the Control System Toolbox Version 9.6 and LabVIEW 2020. It is not possible to verify each solution on all the available computer platforms that are compatible with MATLAB and LabVIEW MathScript RT Module. Please forward any incompatibilities you encounter with the scripts to Prof. Bishop at the email address given below.

The authors and the staff at Pearson Education would like to establish an open line of communication with the instructors using *Modern Control Systems*. We encourage you to contact Pearson with comments and suggestions for this and future editions.

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CHAPTER 1

# Introduction to Control Systems

There are, in general, no unique solutions to the following exercises and problems. Other equally valid block diagrams may be submitted by the student.

## Exercises

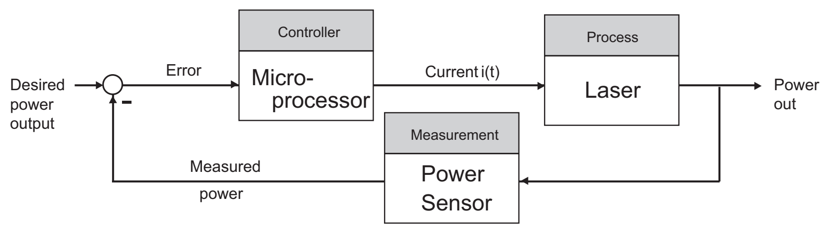
**E1.1** Describe typical sensors that can measure each of the following:

a. Linear position → ultrasonic transducer
b. Velocity (or speed) → Doppler radar
c. Non-gravitational acceleration → inertial measurement unit
d. Rotational position (or angle) → rotary encoder
e. Rotational velocity → gyroscope
f. Temperature → thermocouple
g. Pressure → barometer
h. Liquid (or gas) flow rate → velocimeter
i. Torque → torquemeter
j. Force → load cell
k. Earth's magnetic field → magnetometer
l. Heart rate → electrocardiograph

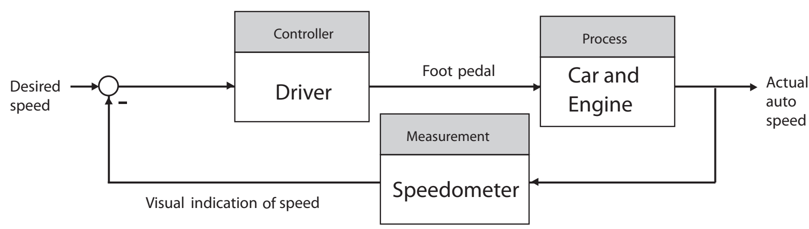
**E1.2** Describe typical actuators that can convert the following:

a. Fluidic energy to mechanical energy → hydraulic cylinder
b. Electrical energy to mechanical energy → electric motor
c. Mechanical deformation to electrical energy → piezoelectric actuator
d. Chemical energy to kinetic energy → automobile engine
e. Heat to electrical energy → thermoelectric generator

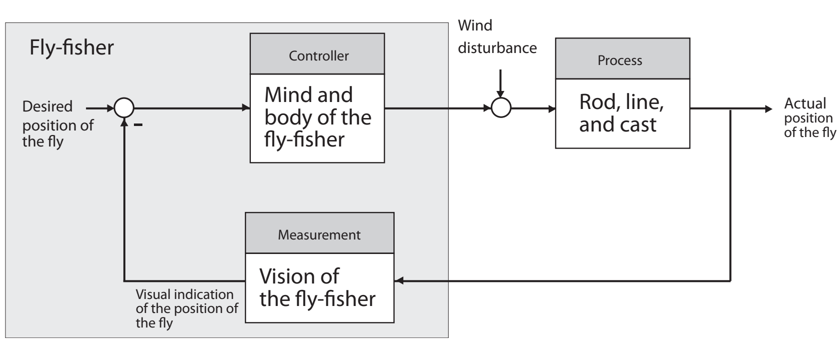
**E1.3** A microprocessor controlled laser system:



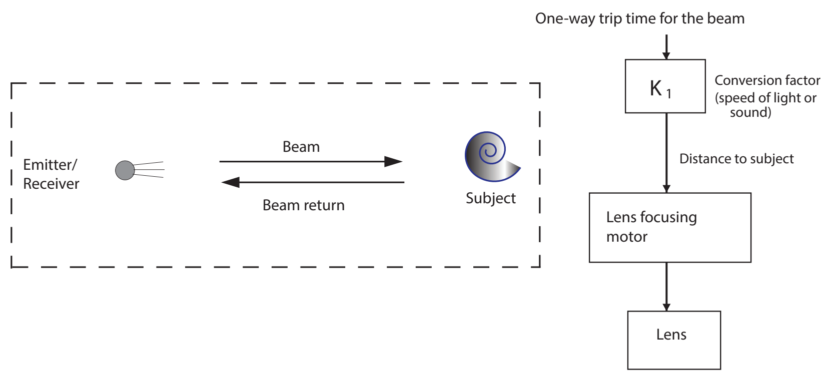

**E1.4** A driver controlled cruise control system:



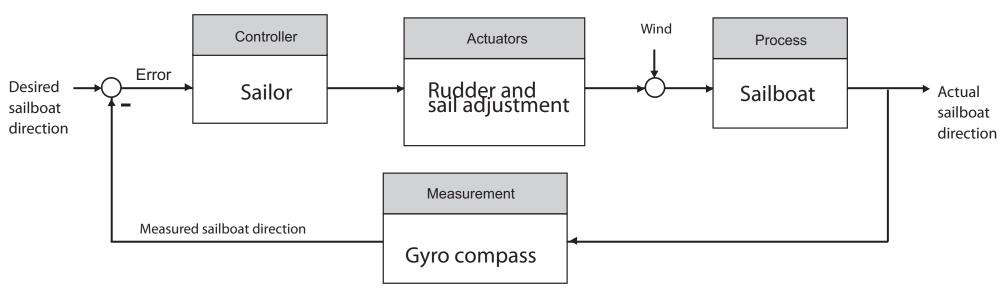

**E1.5** Although the principle of conservation of momentum explains much of the process of fly-casting, there does not exist a comprehensive scientific explanation of how a fly-fisher uses the small backward and forward motion of the fly rod to cast an almost weightless fly lure long distances (the current world-record is 236 ft). The fly lure is attached to a short invisible leader about 15-ft long, which is in turn attached to a longer and thicker Dacron line. The objective is cast the fly lure to a distant spot with dead-eye accuracy so that the thicker part of the line touches the water first and then the fly gently settles on the water just as an insect might.



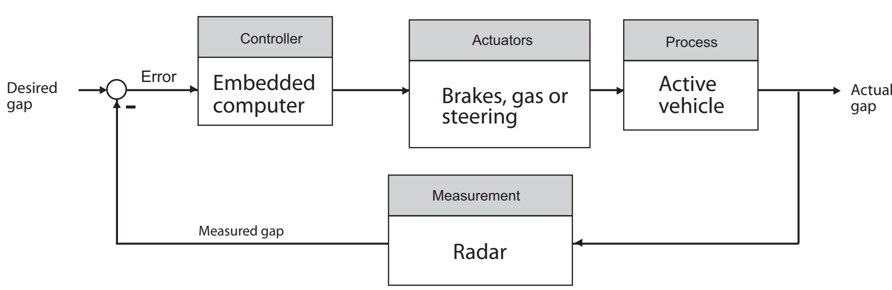
**E1.6** An autofocus camera control system:



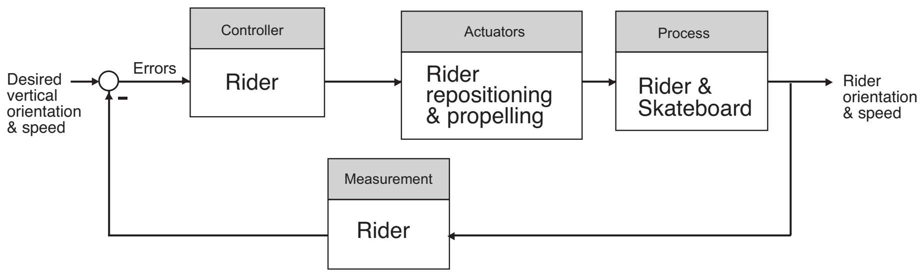

**E1.7** Tacking a sailboat as the wind shifts:



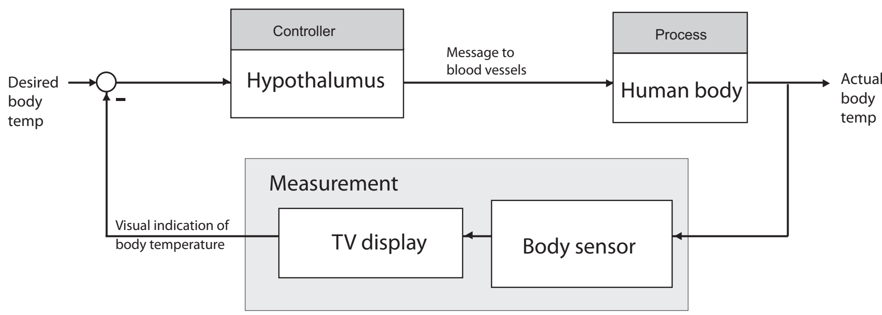

**E1.8** An automated highway control system merging two lanes of traffic:



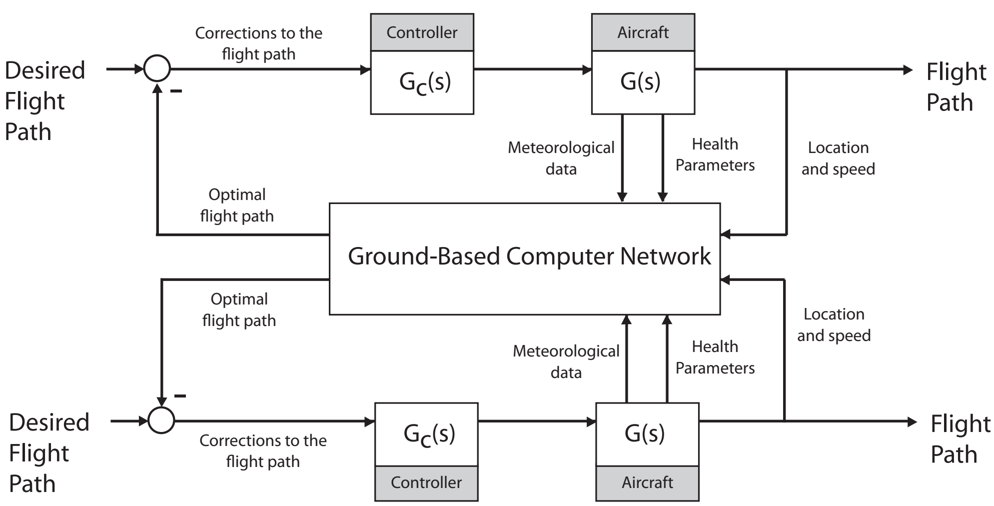
**E1.9** A skateboard rider maintaining vertical orientation and desired speed:



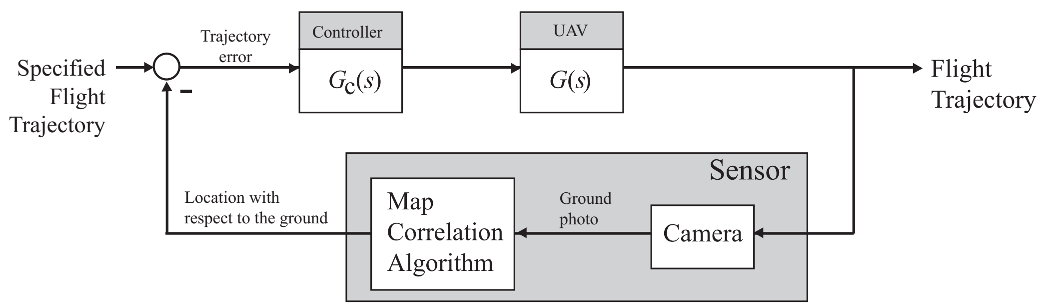

**E1.10** Human biofeedback control system: E1.11



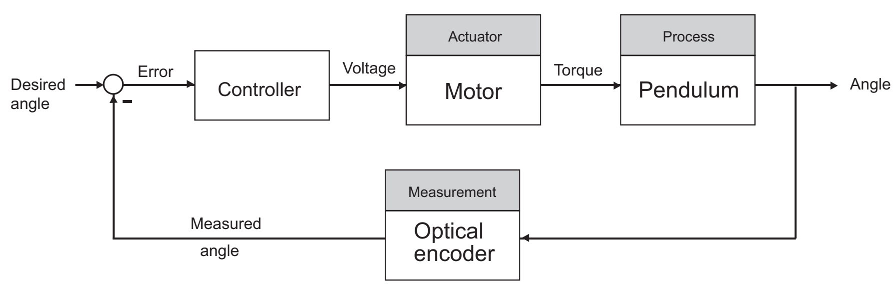

**E1.11** E-enabled aircraft with ground-based flight path control:



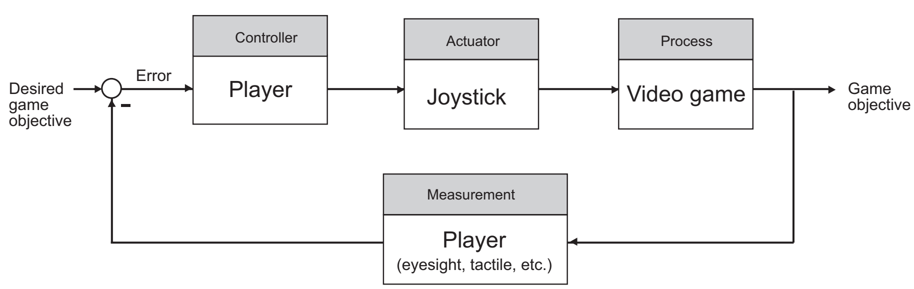
**E1.12** Unmanned aerial vehicle used for crop monitoring in an autonomous mode:



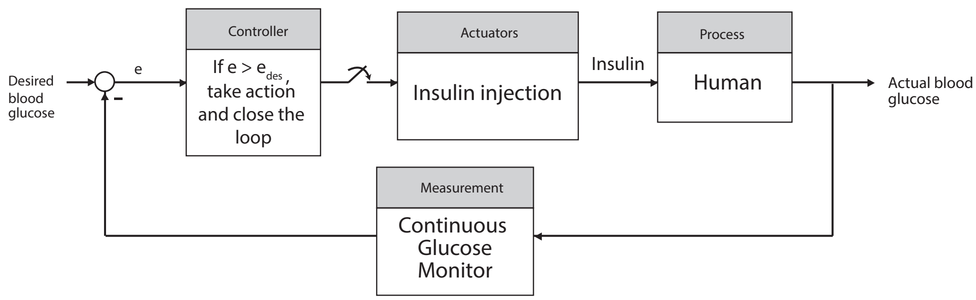

**E1.13** An inverted pendulum control system using an optical encoder to measure the angle of the pendulum and a motor producing a control torque:




**E1.14** In the video game, the player can serve as both the controller and the sensor. The objective of the game might be to drive a car along a prescribed path. The player controls the car trajectory using the joystick using the visual queues from the game displayed on the computer monitor.




**E1.15** A closed-loop blood glucose system with a continuous glucose measurement informing the decision to inject insulin or not:


Desired blood glucose
e
Controller
If e > e_des, take action and close the loop
Actuators
Insulin injection
Insulin
Process
Human
Actual blood glucose
Measurement
Continuous Glucose Monitor

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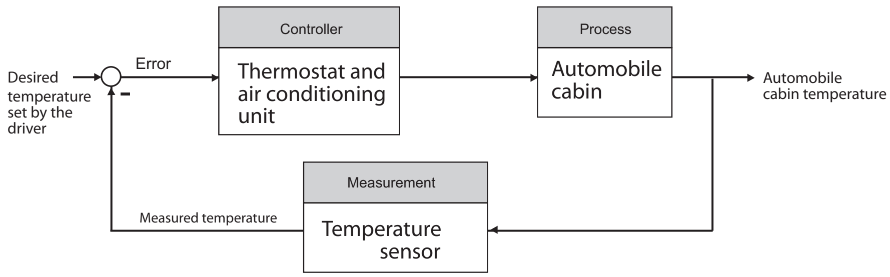
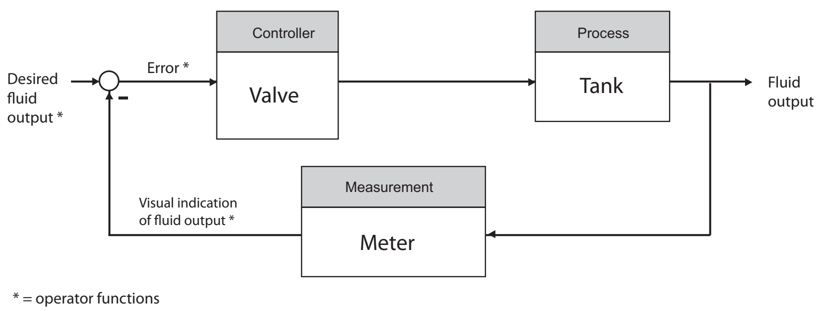
# Problems

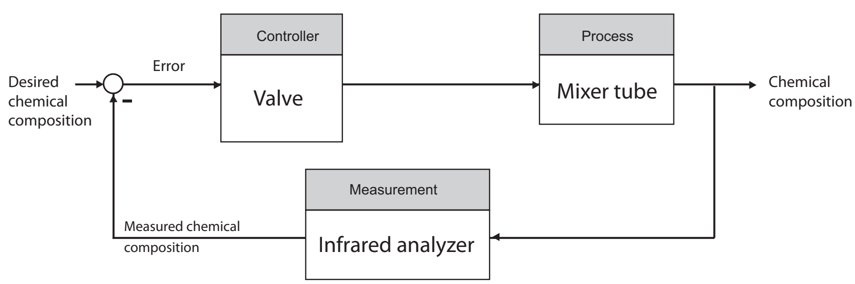
**P1.1** An automobile interior cabin temperature control system block diagram:



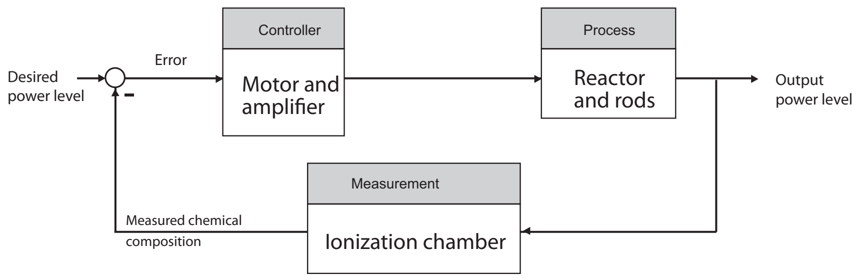

**P1.2** A human operator controlled valve system:



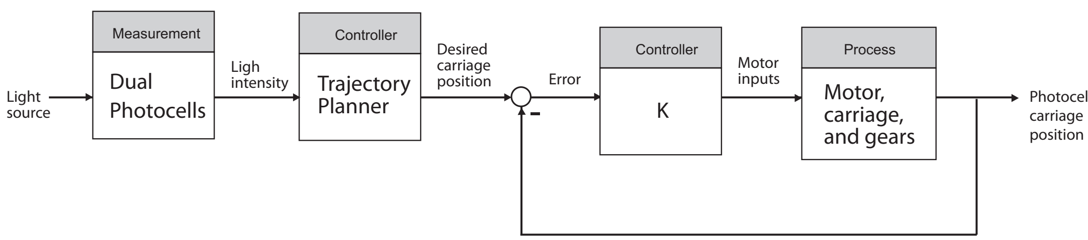

**P1.3** A chemical composition control block diagram:



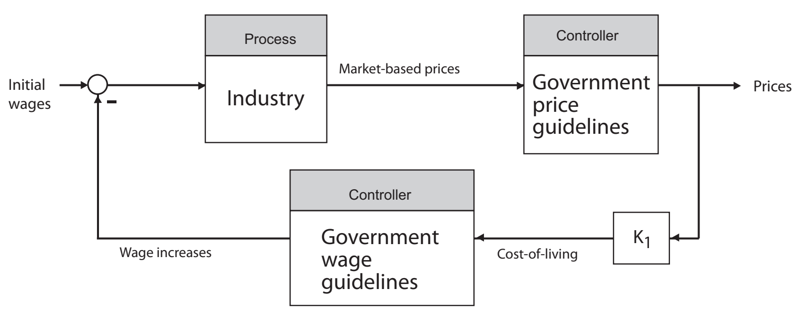
**P1.4** A nuclear reactor control block diagram:




**P1.5** A light seeking control system to track the sun:




**P1.6** If you assume that increasing worker's wages results in increased prices, then by delaying or falsifying cost-of-living data you could reduce or eliminate the pressure to increase worker's wages, thus stabilizing prices. This would work only if there were no other factors forcing the cost-of-living up. Government price and wage economic guidelines would take the place of additional "controllers" in the block diagram, as shown in the block diagram.



**P1.7** Assume that the cannon fires initially at exactly 5:00 p.m.. We have a positive feedback system. Denote by $\Delta t$ the time lost per day, and the net time error by $E_T$. Then the follwoing relationships hold:

$$\Delta t = 4/3 \text{ min.} + 3 \text{ min.} = 13/3 \text{ min.}$$

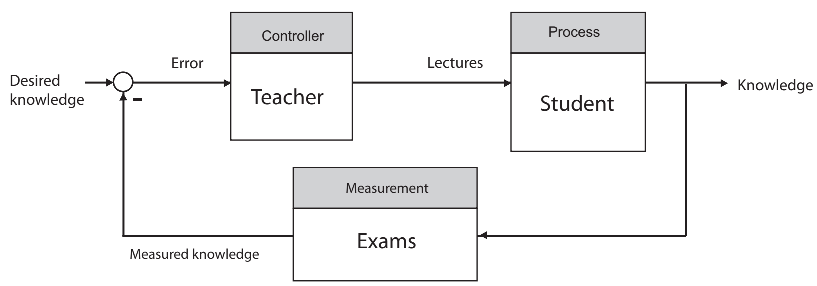
and

$$E_T = 12 \text{ days} \times 13/3 \text{ min./day} .$$

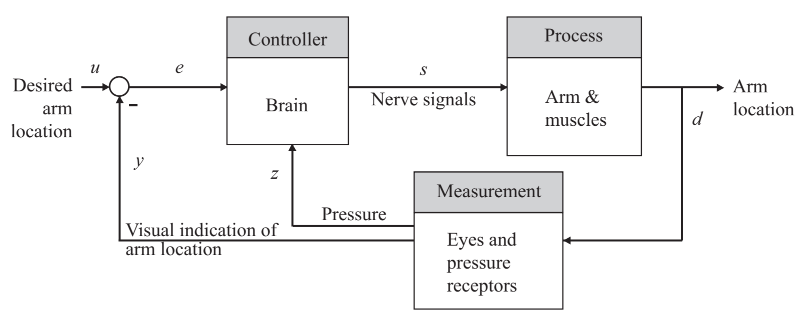
Therefore, the net time error after 15 days is

$$E_T = 52 \text{ min.}$$

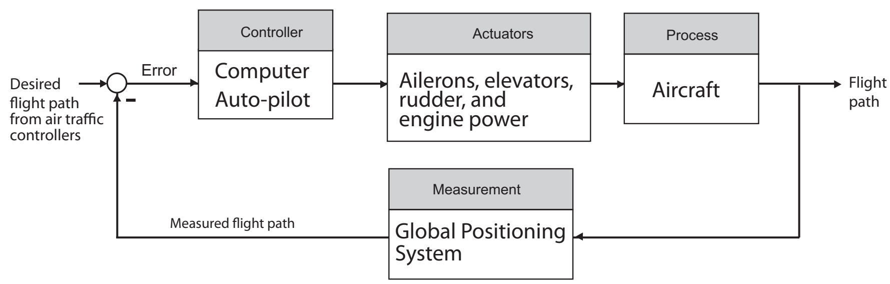
**P1.8** The student-teacher learning process:



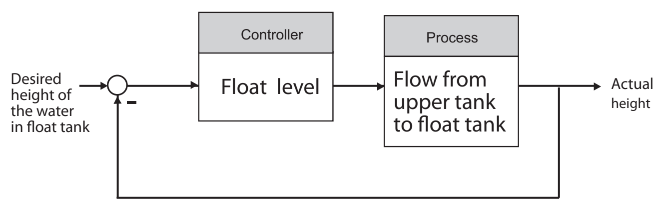

**P1.9** A human arm control system:



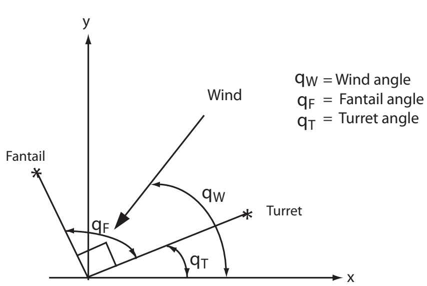
**P1.10** An aircraft flight path control system using GPS:

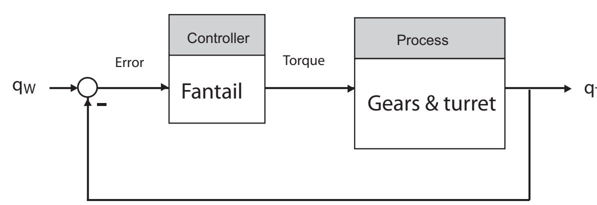



**P1.11** The accuracy of the clock is dependent upon a constant flow from the orifice; the flow is dependent upon the height of the water in the float tank. The height of the water is controlled by the float. The control system controls only the height of the water. Any errors due to enlargement of the orifice or evaporation of the water in the lower tank is not accounted for. The control system can be seen as:

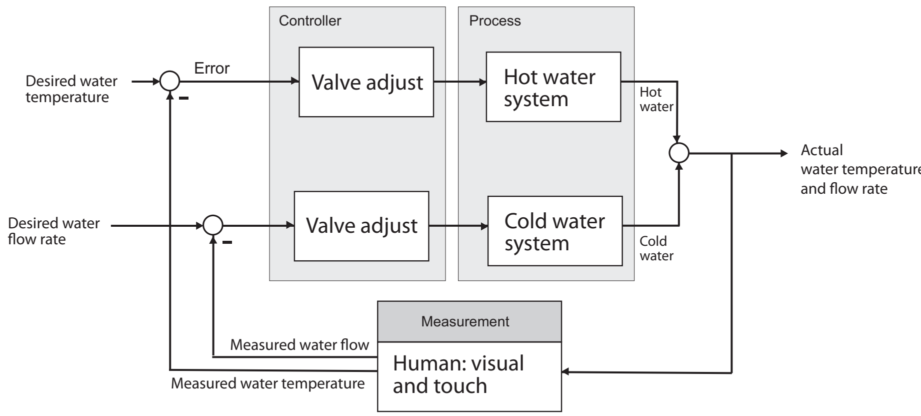



**P1.12** Assume that the turret and fantail are at 90°, if $\theta_w \neq \theta_F$-90°. The fantail operates on the error signal $\theta_w$ - $\theta_T$, and as the fantail turns, it drives the turret to turn.



**P1.13** This scheme assumes the person adjusts the hot water for temperature control, and then adjusts the cold water for flow rate control.

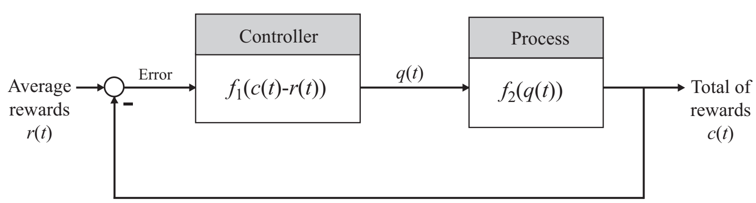



**P1.14** If the rewards in a specific trade is greater than the average reward, there is a positive influx of workers, since

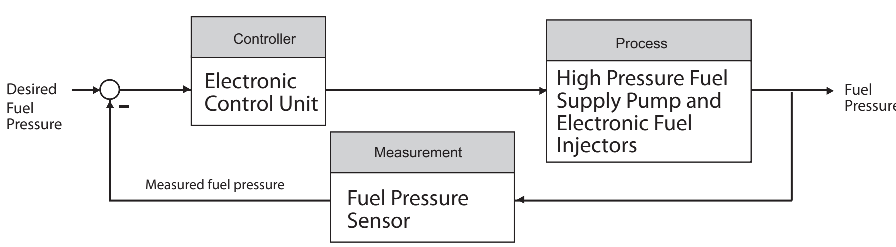
$$q(t) = f_1(c(t) - r(t)).$$

If an influx of workers occurs, then reward in specific trade decreases, since

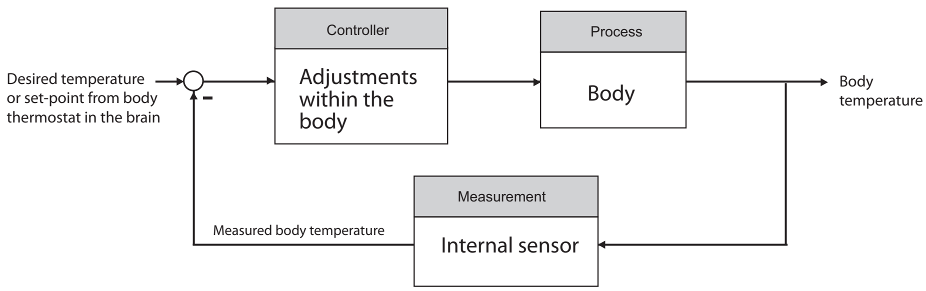
$$c(t) = -f_2(q(t)).$$




**P1.15** A computer controlled fuel injection system:

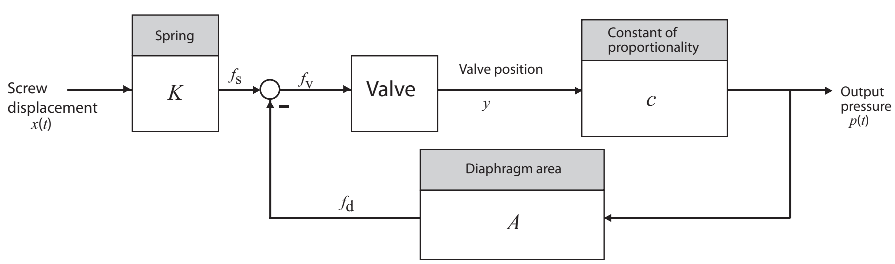


**P1.16** With the onset of a fever, the body thermostat is turned up. The body adjusts by shivering and less blood flows to the skin surface. Aspirin acts to lowers the thermal set-point in the brain.

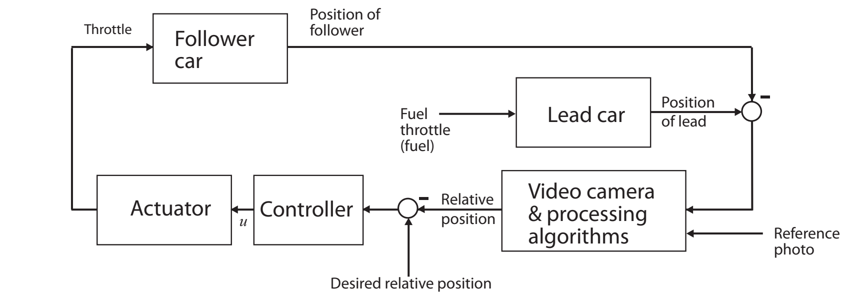



**P1.17** Hitting a baseball is arguably one of the most difficult feats in all of sports. Given that pitchers may throw the ball at speeds of 90 mph (or higher!), batters have only about 0.1 second to make the decision to swing—with bat speeds aproaching 90 mph. The key to hitting a baseball a long distance is to make contact with the ball with a high bat velocity. This is more important than the bat's weight, which is usually around 33 ounces. Since the pitcher can throw a variety of pitches (fast ball, curve ball, slider, etc.), a batter must decide if the ball is going to enter the strike zone and if possible, decide the type of pitch. The batter uses his/her vision as the sensor in the feedback loop. A high degree of eye-hand coordination is key to success—that is, an accurate feedback control system.

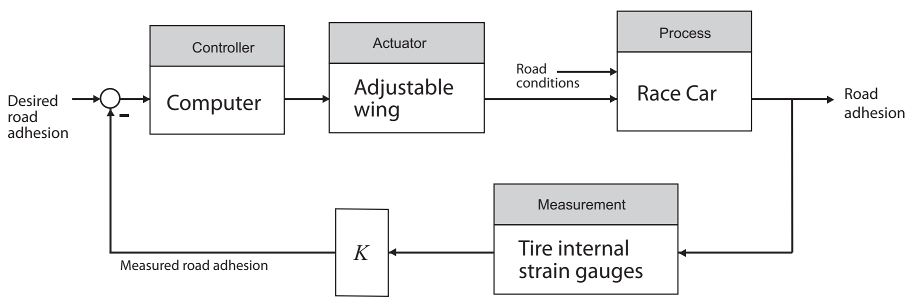
**P1.18** Define the following variables: $p$ = output pressure, $f_s$ = spring force = $Kx$, $f_d$ = diaphragm force = $Ap$, and $f_v$ = valve force = $f_s$ - $f_d$. The motion of the valve is described by $\ddot{y} = f_v/m$ where $m$ is the valve mass. The output pressure is proportional to the valve displacement, thus $p = cy$ , where $c$ is the constant of proportionality.



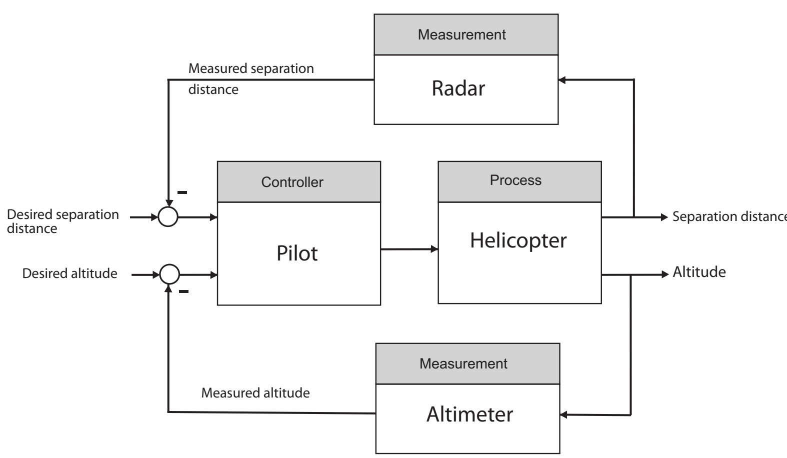
**P1.19** A control system to keep a car at a given relative position offset from a lead car:



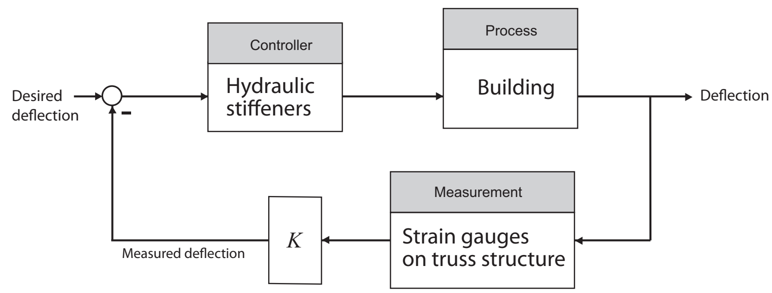

**P1.20** A control system for a high-performance car with an adjustable wing:



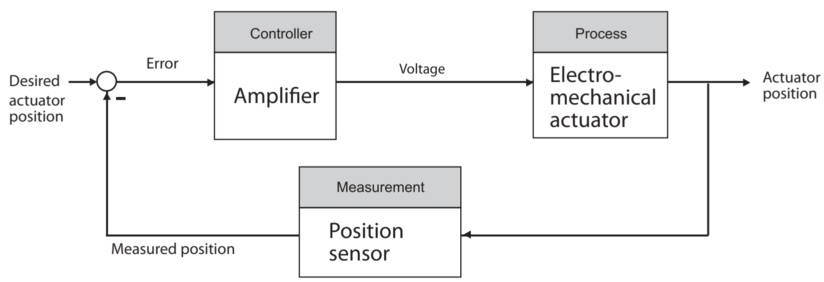

**P1.21** A control system for a twin-lift helicopter system:



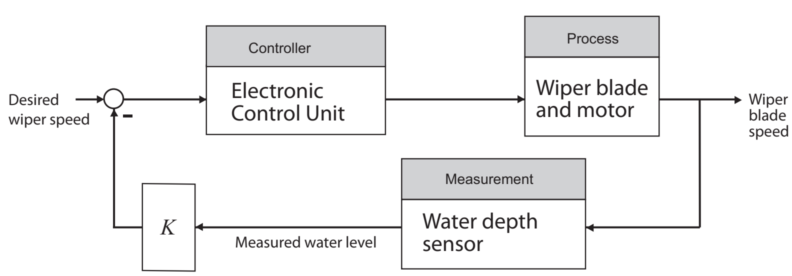
**P1.22** The desired building deflection would not necessarily be zero. Rather it would be prescribed so that the building is allowed moderate movement up to a point, and then active control is applied if the movement is larger than some predetermined amount.



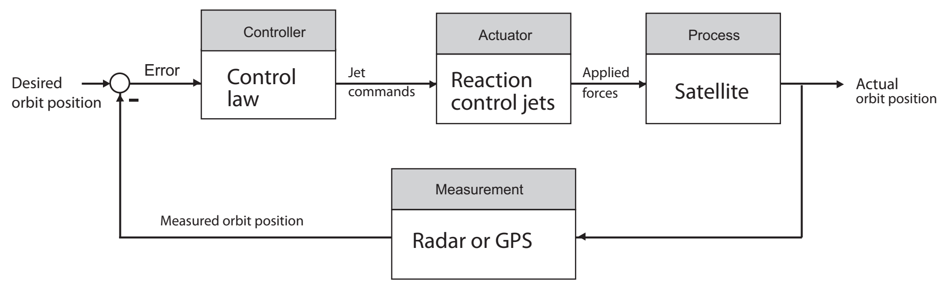

**P1.23** The human-like face of the robot might have micro-actuators placed at strategic points on the interior of the malleable facial structure. Cooperative control of the micro-actuators would then enable the robot to achieve various facial expressions.



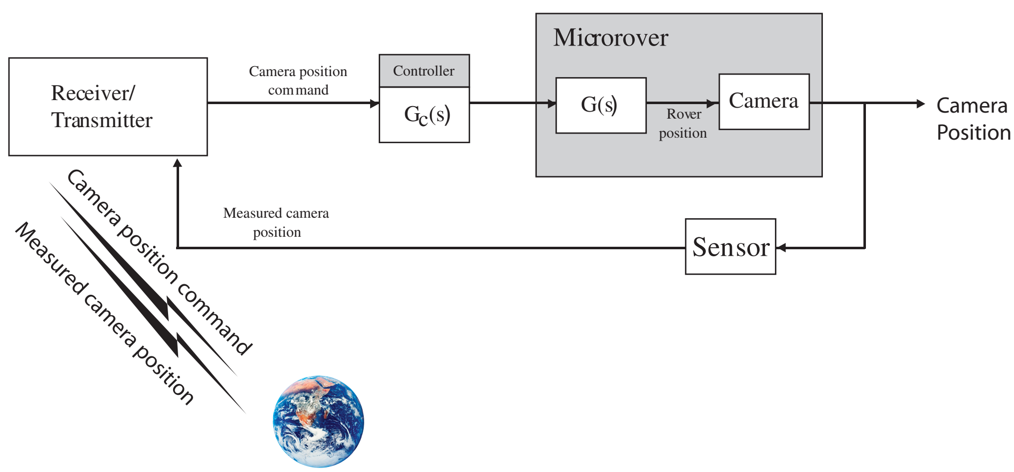

**P1.24** We might envision a sensor embedded in a "gutter" at the base of the windshield which measures water levels—higher water levels corresponds to higher intensity rain. This information would be used to modulate the wiper blade speed.



**P1.25** A feedback control system for the space traffic control:

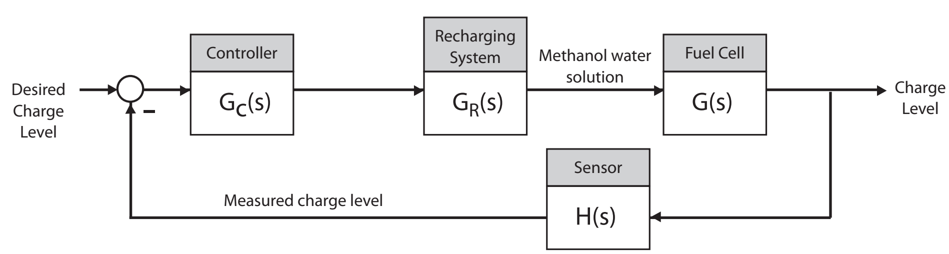



**P1.26** Earth-based control of a microrover to point the camera:



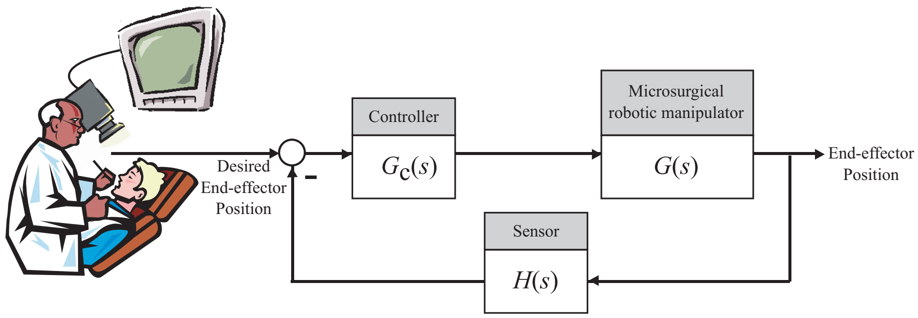

(Dundanim/Shutterstock.)

**P1.27** Control of a methanol fuel cell:

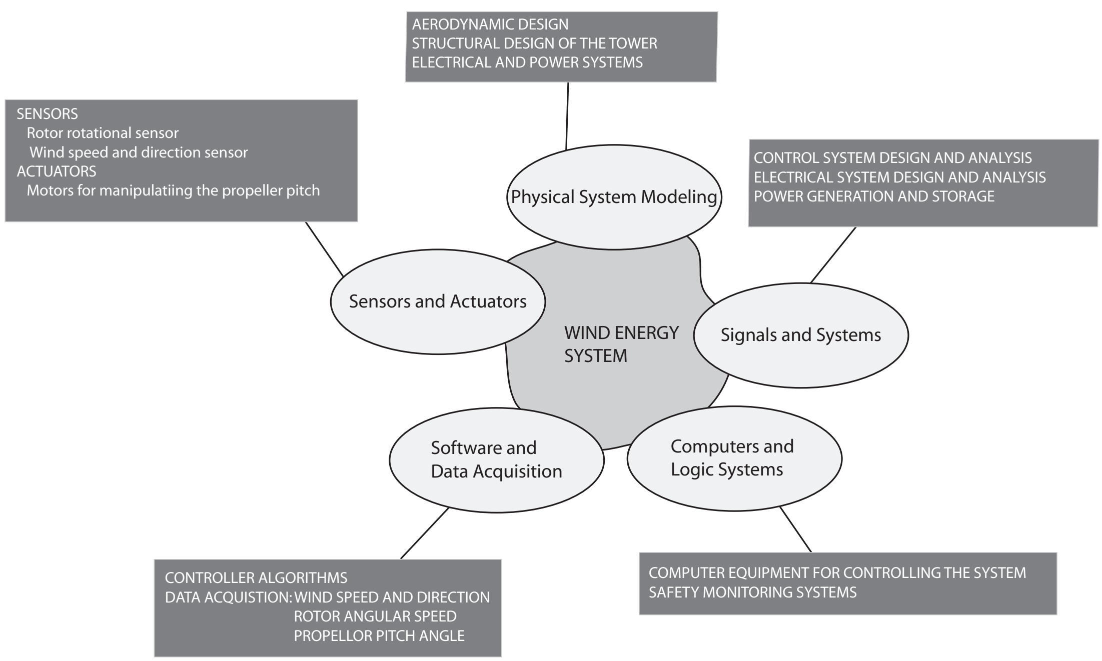


# Advanced Problems

**AP1.1** Control of a robotic microsurgical device:

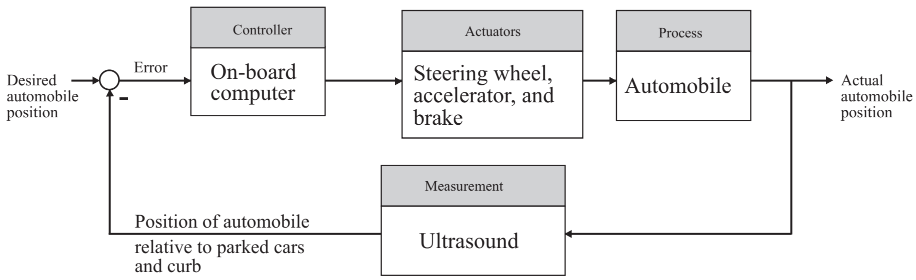



**AP1.2** An advanced wind energy system viewed as a mechatronic system:

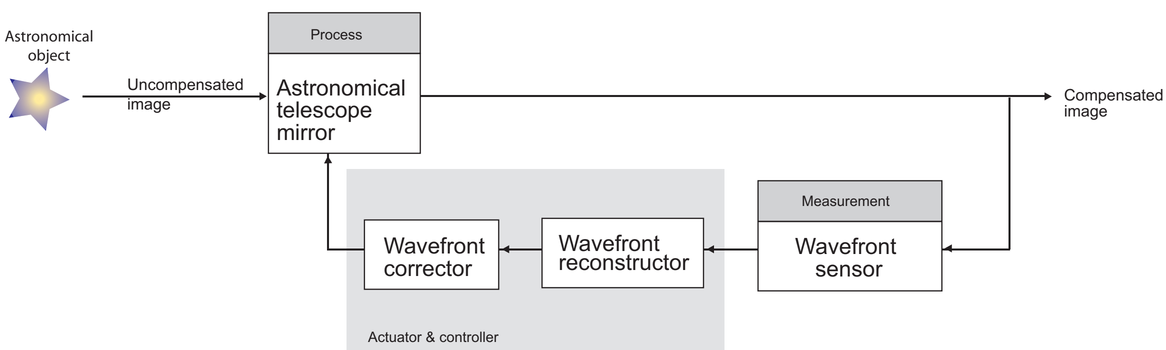



**AP1.3** The automatic parallel parking system might use multiple ultrasound sensors to measure distances to the parked automobiles and the curb. The sensor measurements would be processed by an on-board computer to determine the steering wheel, accelerator, and brake inputs to avoid collision and to properly align the vehicle in the desired space.

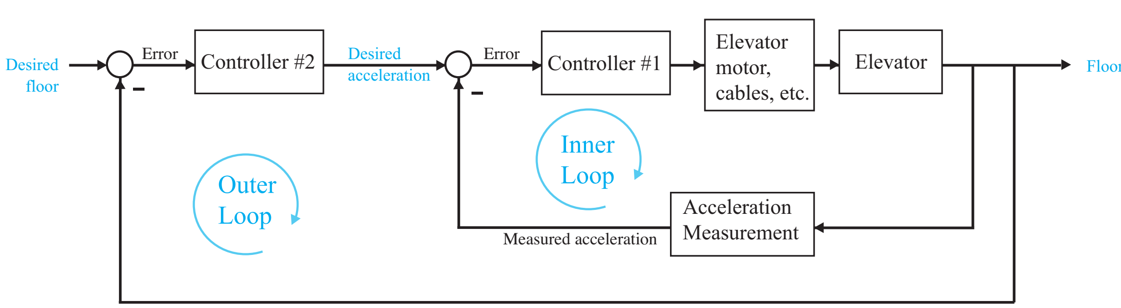
Even though the sensors may accurately measure the distance between the two parked vehicles, there will be a problem if the available space is not big enough to accommodate the parking car.



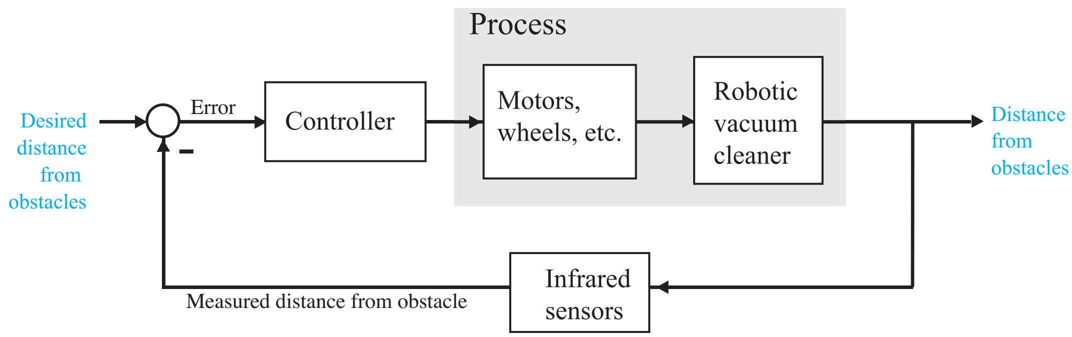

**AP1.4** There are various control methods that can be considered, including placing the controller in the feedforward loop (as in Figure 1.3). The adaptive optics block diagram below shows the controller in the feedback loop, as an alternative control system architecture.



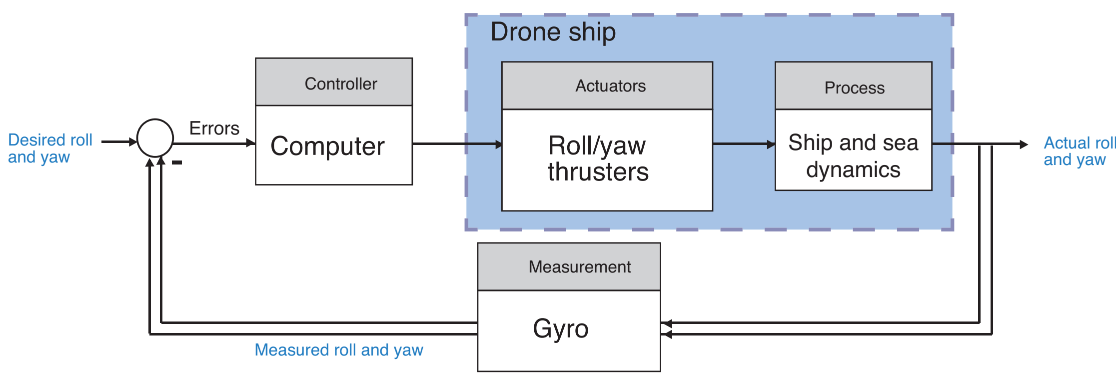

**AP1.5** The control system might have an inner loop for controlling the acceleration and an outer loop to reach the desired floor level precisely.



**AP1.6** An obstacle avoidance control system would keep the robotic vacuum cleaner from colliding with furniture but it would not necessarily put the vacuum cleaner on an optimal path to reach the entire floor. This would require another sensor to measure position in the room, a digital map of the room layout, and a control system in the outer loop.

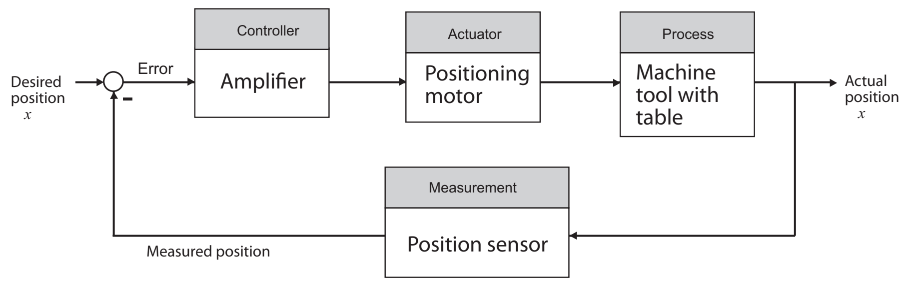



**AP1.7** The attitude control of the drone ship requires measuring the yaw and roll using a gyro. Often the gyro measures attitude rate, therefore, it may be necessary to integrate the gyro output to compute the measured roll and yaw.

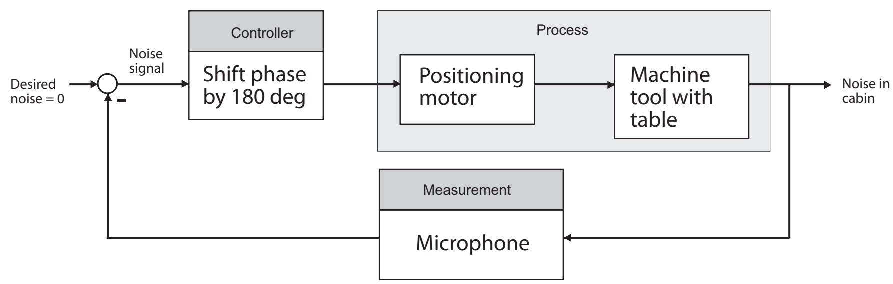


# Design Problems

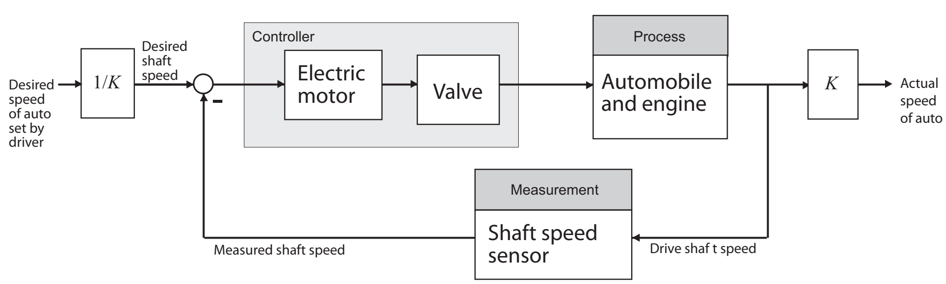
CDP1.1 The machine tool with the movable table in a feedback control configuration:



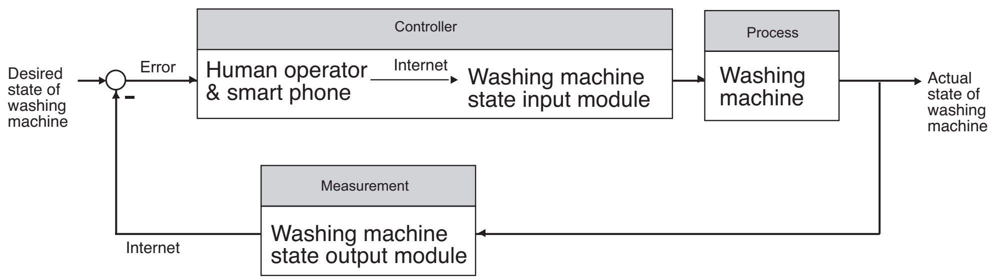

**DP1.1** Use the stereo system and amplifiers to cancel out the noise by emitting signals 180° out of phase with the noise.



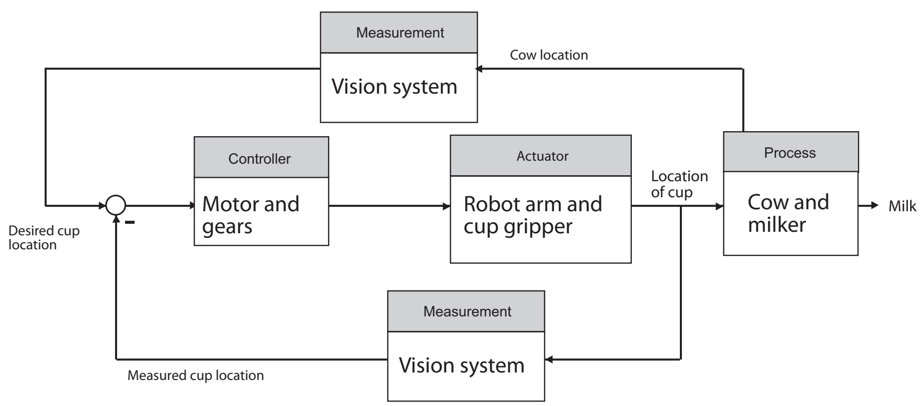

**DP1.2** An automobile cruise control system:



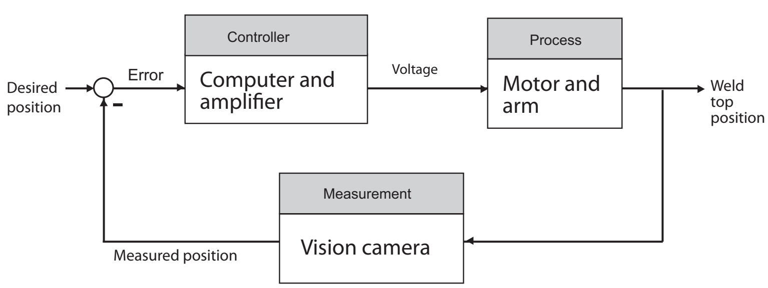
**DP1.3** Utilizing a smart phone to remotely monitor and control a washing machine:



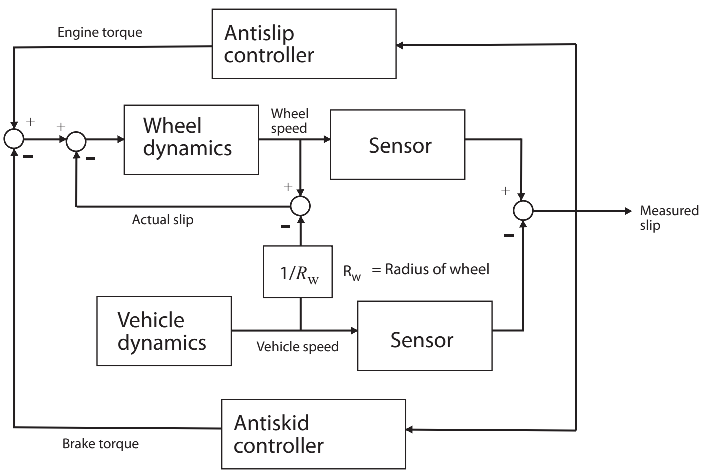

**DP1.4** An automated cow milking system:



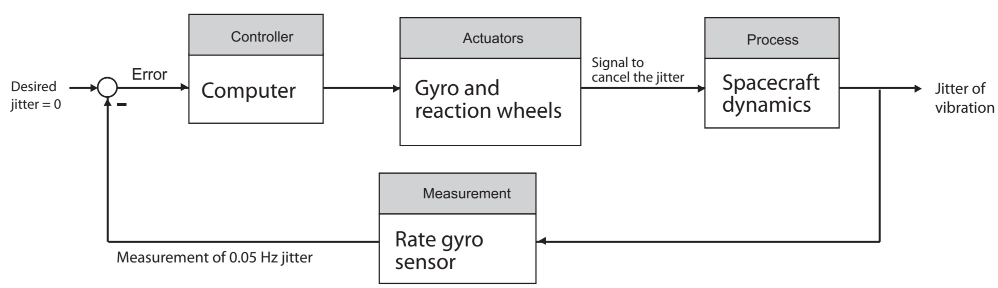

**DP1.5** A feedback control system for a robot welder:



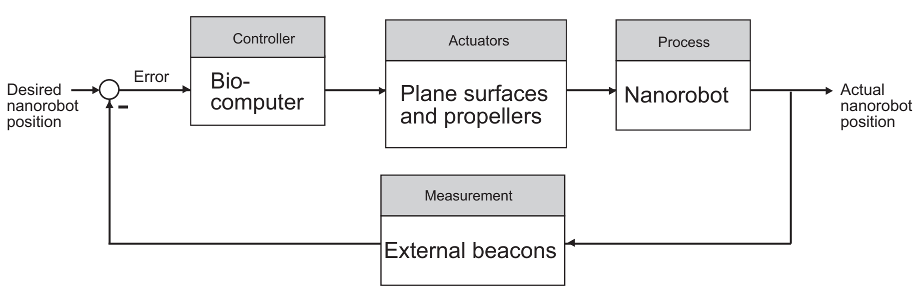
**DP1.6** A control system for one wheel of a traction control system:




**DP1.7** A vibration damping system for the Hubble Space Telescope:

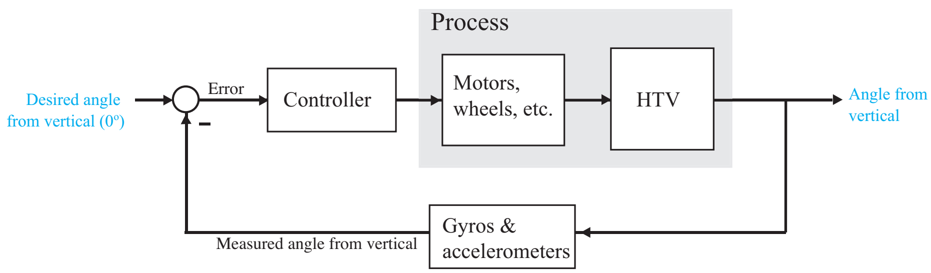



**DP1.8** A control system for a nanorobot:

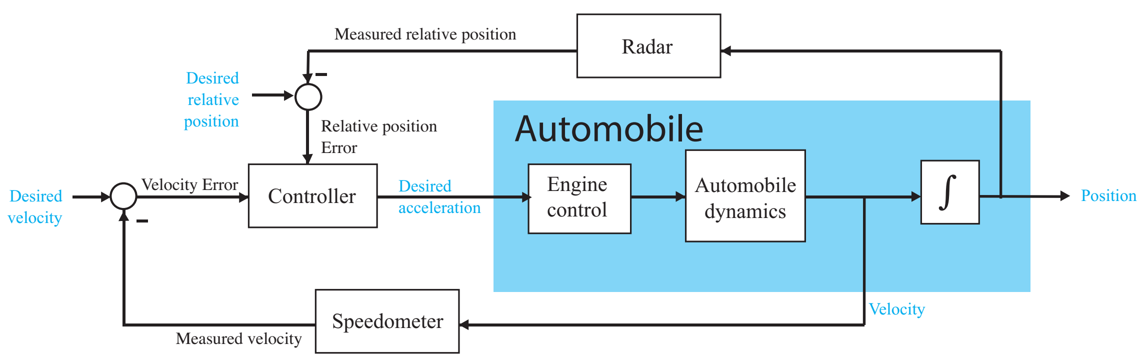



Many concepts from underwater robotics can be applied to nanorobotics within the bloodstream. For example, plane surfaces and propellers can provide the required actuation with screw drives providing the propulsion. The nanorobots can use signals from beacons located outside the skin as sensors to determine their position. The nanorobots use energy from the chemical reaction of oxygen and glucose available in the human body. The control system requires a bio-computer–an innovation that is not yet available.

**DP1.9** The feedback control system might use gyros and/or accelerometers to measure angle change and assuming the HTV was originally in the vertical position, the feedback would retain the vertical position using commands to motors and other actuators that produced torques and could move the HTV forward and backward.




**DP1.10** There are two loops in this control system, one to control the automobile velocity and one to control the relative position of the two vehicles. Since we have no way to measure the velocity of the forward vehicle, we rely on the radar to provide relative positioning. The controller will need to account for both the velocity error and the relative position error in computing the desired acceleration.



CHAPTER 2

# Mathematical Models of Systems

## Exercises

**E2.1** We have for the open-loop

$$y = r^2$$

and for the closed-loop

$$e = r - y \quad \text{and} \quad y = e^2 \ .$$

So, $e = r - e^2$ and $e^2 + e - r = 0$ .

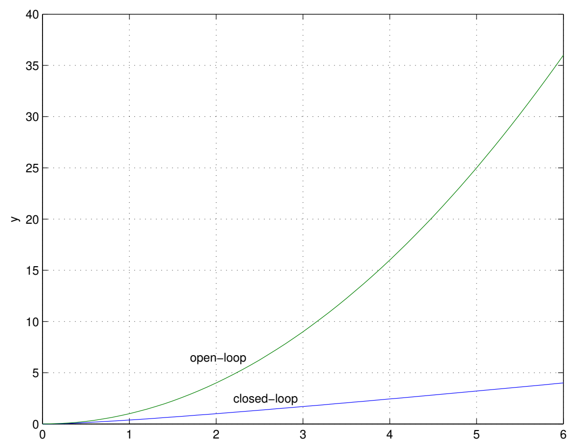



FIGURE E2.1
Plot of open-loop versus closed-loop.

For example, if $r = 1$, then $e^2 + e - 1 = 0$ implies that $e = 0.618$. Thus, $y = 0.382$. A plot $y$ versus $r$ is shown in Figure E2.1.

**E2.2** Define

$$f(T) = R = R_0 e^{-0.1T}$$

and

$$\Delta R = f(T) - f(T_0) \ , \quad \Delta T = T - T_0 \ .$$

Then,

$$\Delta R = f(T) - f(T_0) = \left.\frac{\partial f}{\partial T}\right|_{T=T_0=20^\circ} \Delta T + \cdots$$

where

$$\left.\frac{\partial f}{\partial T}\right|_{T=T_0=20^\circ} = -0.1 R_0 e^{-0.1T_0} = -135,$$

when $R_0 = 10,000\Omega$. Thus, the linear approximation is computed by considering only the first-order terms in the Taylor series expansion, and is given by

$$\Delta R = -135 \Delta T \ .$$

**E2.3** The spring constant for the equilibrium point is found graphically by estimating the slope of a line tangent to the force versus displacement curve at the point $y = 0.5$cm, see Figure E2.3. The slope of the line is $K \approx 1$.

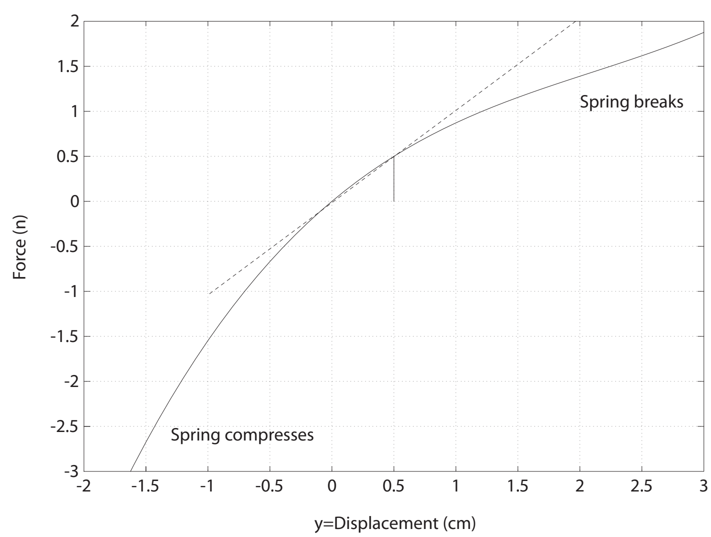



FIGURE E2.3
Spring force as a function of displacement.

**E2.4** Since

$$R(s) = \frac{1}{s}$$

we have

$$Y(s) = \frac{4(s+50)}{s(s+20)(s+10)} .$$

The partial fraction expansion of $Y(s)$ is given by

$$Y(s) = \frac{A_1}{s} + \frac{A_2}{s+20} + \frac{A_3}{s+10}$$

where

$$A_1 = 1 , \quad A_2 = 0.6 \text{ and } \quad A_3 = -1.26 .$$

Using the Laplace transform table, we find that

$$y(t) = 1 + 0.6e^{-20t} - 1.6e^{-10t} .$$

The final value is computed using the final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s \left[ \frac{4(s+50)}{s(s^2+30s+200)} \right] = 1 .$$

**E2.5** The circuit diagram is shown in Figure E2.5.

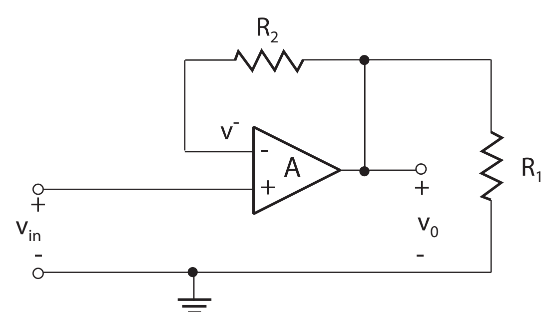



FIGURE E2.5
Noninverting op-amp circuit.

With an ideal op-amp, we have

$$v_o = A(v_{in} - v^-),$$

where $A$ is very large. We have the relationship

$$v^- = \frac{R_1}{R_1 + R_2} v_o.$$

Therefore,

$$v_o = A(v_{in} - \frac{R_1}{R_1 + R_2} v_o),$$

and solving for $v_o$ yields

$$v_o = \frac{A}{1 + \frac{AR_1}{R_1+R_2}} v_{in}.$$

Since $A \gg 1$, it follows that $1 + \frac{AR_1}{R_1+R_2} \approx \frac{AR_1}{R_1+R_2}$. Then the expression for $v_o$ simplifies to

$$v_o = \frac{R_1 + R_2}{R_1} v_{in}.$$

**E2.6** Given

$$y = f(x) = e^x$$

and the operating point $x_o = 1$, we have the linear approximation

$$y = f(x) = f(x_o) + \left.\frac{\partial f}{\partial x}\right|_{x=x_o} (x - x_o) + \cdots$$

where

$$f(x_o) = e, \quad \left.\frac{df}{dx}\right|_{x=x_o=1} = e, \quad \text{and} \quad x - x_o = x - 1.$$

Therefore, we obtain the linear approximation $y = ex$.

**E2.7** The block diagram is shown in Figure E2.7.

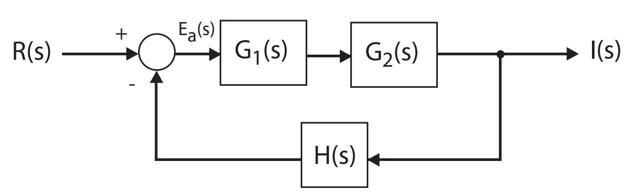



FIGURE E2.7
Block diagram model.

Starting at the output we obtain

$$I(s) = G_1(s)G_2(s)E(s).$$

But $E(s) = R(s) - H(s)I(s)$, so

$$I(s) = G_1(s)G_2(s)\left[R(s) - H(s)I(s)\right].$$

Solving for $I(s)$ yields the closed-loop transfer function

$$\frac{I(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}.$$

**E2.8** The block diagram is shown in Figure E2.8.

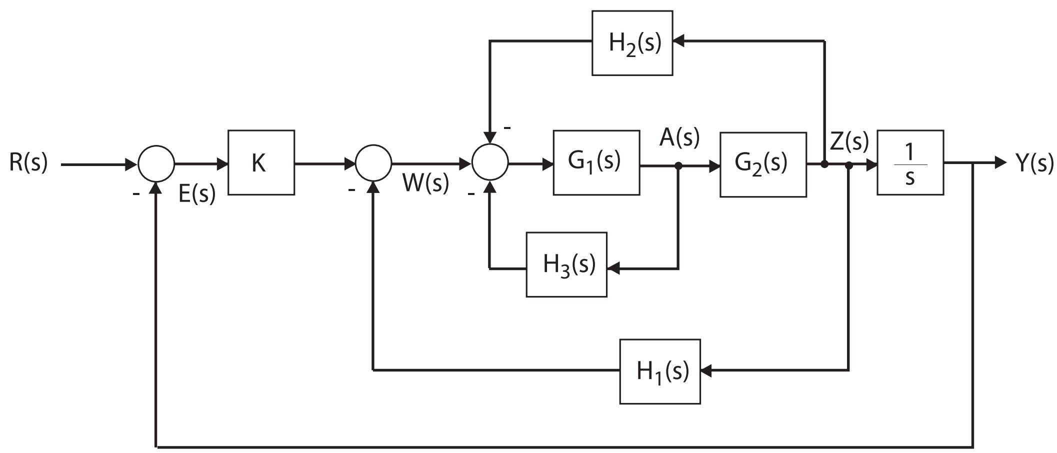



FIGURE E2.8
Block diagram model.

Starting at the output we obtain

$$Y(s) = \frac{1}{s}Z(s) = \frac{1}{s}G_2(s)A(s).$$

But $A(s) = G_1(s)\left[-H_2(s)Z(s) - H_3(s)A(s) + W(s)\right]$ and $Z(s) = sY(s)$, so

$$Y(s) = -G_1(s)G_2(s)H_2(s)Y(s) - G_1(s)H_3(s)Y(s) + \frac{1}{s}G_1(s)G_2(s)W(s).$$

Substituting $W(s) = KE(s) - H_1(s)Z(s)$ into the above equation yields

$$Y(s) = -G_1(s)G_2(s)H_2(s)Y(s) - G_1(s)H_3(s)Y(s) + \frac{1}{s}G_1(s)G_2(s)\left[KE(s) - H_1(s)Z(s)\right]$$

and with $E(s) = R(s) - Y(s)$ and $Z(s) = sY(s)$ this reduces to

$$Y(s) = [-G_1(s)G_2(s)\left(H_2(s) + H_1(s)\right) - G_1(s)H_3(s) \\ - \frac{1}{s}G_1(s)G_2(s)K]Y(s) + \frac{1}{s}G_1(s)G_2(s)KR(s).$$

Solving for $Y(s)$ yields the transfer function

$$Y(s) = T(s)R(s),$$

where

$$T(s) = \frac{KG_1(s)G_2(s)/s}{1 + G_1(s)G_2(s)\left[(H_2(s) + H_1(s)\right] + G_1(s)H_3(s) + KG_1(s)G_2(s)/s}.$$

**E2.9** From Figure E2.9, we observe that

$$F_f(s) = G_2(s)U(s)$$

and

$$F_R(s) = G_3(s)U(s) \ .$$

Then, solving for $U(s)$ yields

$$U(s) = \frac{1}{G_2(s)}F_f(s)$$

and it follows that

$$F_R(s) = \frac{G_3(s)}{G_2(s)}U(s) \ .$$

Again, considering the block diagram in Figure E2.9 we determine

$$F_f(s) = G_1(s)G_2(s)[R(s) - H_2(s)F_f(s) - H_2(s)F_R(s)] \ .$$

But, from the previous result, we substitute for $F_R(s)$ resulting in

$$F_f(s) = G_1(s)G_2(s)R(s) - G_1(s)G_2(s)H_2(s)F_f(s) - G_1(s)H_2(s)G_3(s)F_f(s) \ .$$

Solving for $F_f(s)$ yields

$$F_f(s) = \left[\frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H_2(s) + G_1(s)G_3(s)H_2(s)}\right]R(s) \ .$$

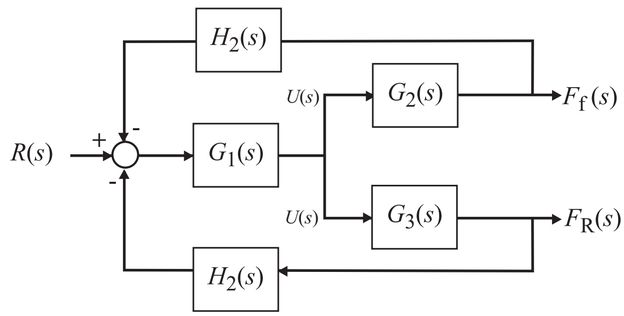



FIGURE E2.9
Block diagram model.

**E2.10** The shock absorber block diagram is shown in Figure E2.10. The closed-loop transfer function model is

$$T(s) = \frac{G_c(s)G_p(s)G(s)}{1 + H(s)G_c(s)G_p(s)G(s)} .$$

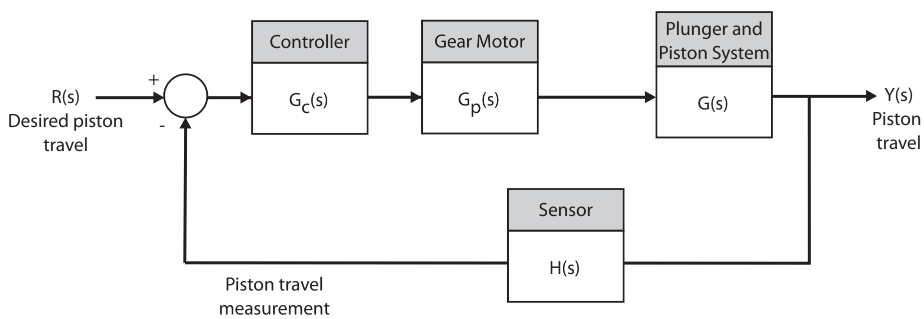



FIGURE E2.10
Shock absorber block diagram.

**E2.11** Let $f$ denote the spring force (n) and $x$ denote the deflection (m). Then

$$K = \frac{\Delta f}{\Delta x} .$$

Computing the slope from the graph yields:

(a) $x_o = -0.14\text{m} \rightarrow K = \Delta f/\Delta x = 10$ n / 0.04 m = 250 n/m

(b) $x_o = 0\text{m} \rightarrow K = \Delta f/\Delta x = 10$ n / 0.05 m = 200 n/m

(c) $x_o = 0.35\text{m} \rightarrow K = \Delta f/\Delta x = 3$n / 0.05 m = 60 n/m

**E2.12** The signal flow graph is shown in Fig. E2.12. Find $Y(s)$ when $R(s) = 0$.

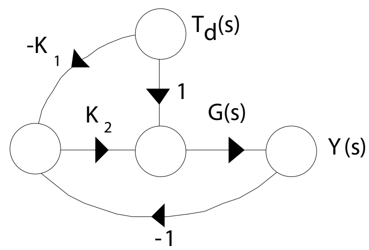



FIGURE E2.12
Signal flow graph.

The transfer function from $T_d(s)$ to $Y(s)$ is

$$Y(s) = \frac{G(s)T_d(s) - K_1K_2G(s)T_d(s)}{1 - (-K_2G(s))} = \frac{G(s)(1 - K_1K_2)T_d(s)}{1 + K_2G(s)} \ .$$

If we set

$$K_1K_2 = 1 \ ,$$

then $Y(s) = 0$ for any $T_d(s)$.

**E2.13** The transfer function from $R(s)$, $T_d(s)$, and $N(s)$ to $Y(s)$ is

$$Y(s) = \left[\frac{K}{s^2 + 25s + K}\right] R(s) + \left[\frac{1}{s^2 + 25s + K}\right] T_d(s) - \left[\frac{K}{s^2 + 25s + K}\right] N(s)$$

Therefore, we find that

$$Y(s)/T_d(s) = \frac{1}{s^2 + 25s + K} \quad \text{and} \quad Y(s)/N(s) = -\frac{K}{s^2 + 25s + K}$$

**E2.14** Since we want to compute the transfer function from $R_2(s)$ to $Y_1(s)$, we can assume that $R_1 = 0$ (application of the principle of superposition). Then, starting at the output $Y_1(s)$ we obtain

$$Y_1(s) = G_3(s)\left[-H_1(s)Y_1(s) + G_2(s)G_8(s)W(s) + G_9(s)W(s)\right],$$

or

$$\left[1 + G_3(s)H_1(s)\right]Y_1(s) = \left[G_3(s)G_2(s)G_8(s)W(s) + G_3(s)G_9(s)\right]W(s).$$

Considering the signal $W(s)$ (see Figure E2.14), we determine that

$$W(s) = G_5(s)\left[G_4(s)R_2(s) - H_2(s)W(s)\right],$$

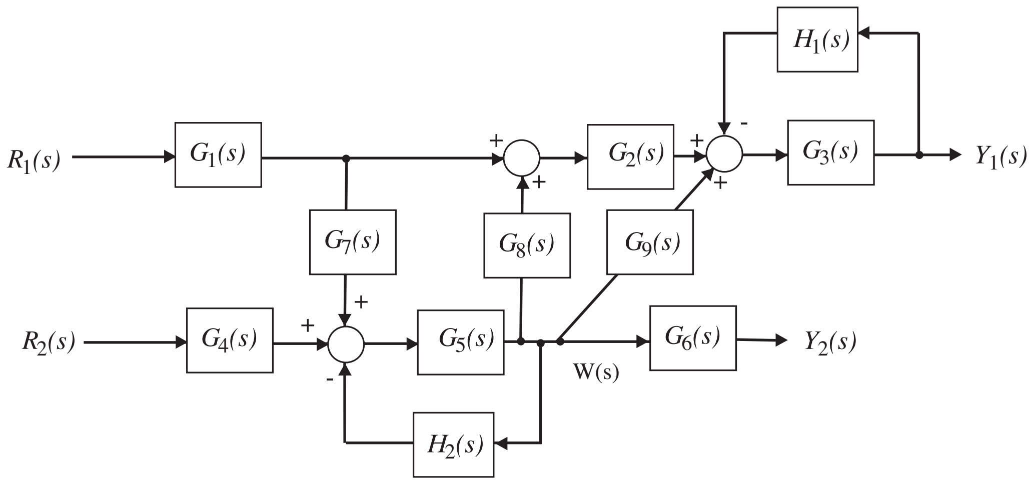



FIGURE E2.14
Block diagram model.

or

$$[1 + G_5(s)H_2(s)]\, W(s) = G_5(s)G_4(s)R_2(s).$$

Substituting the expression for $W(s)$ into the above equation for $Y_1(s)$ yields

$$\frac{Y_1(s)}{R_2(s)} = \frac{G_2(s)G_3(s)G_4(s)G_5(s)G_8(s) + G_3(s)G_4(s)G_5(s)G_9(s)}{1 + G_3(s)H_1(s) + G_5(s)H_2(s) + G_3(s)G_5(s)H_1(s)H_2(s)}.$$

**E2.15** For loop 1, we have

$$R_1 i_1 + L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int (i_1 - i_2)dt + R_2(i_1 - i_2) = v(t)\ .$$

And for loop 2, we have

$$\frac{1}{C_2}\int i_2 dt + L_2 \frac{di_2}{dt} + R_2(i_2 - i_1) + \frac{1}{C_1}\int (i_2 - i_1)dt = 0\ .$$

**E2.16** The transfer function from $R(s)$ to $P(s)$ is

$$\frac{P(s)}{R(s)} = \frac{4.2}{s^3 + 2s^2 + 4s + 4.2}\ .$$

The block diagram is shown in Figure E2.16a. The corresponding signal flow graph is shown in Figure E2.16b for

$$P(s)/R(s) = \frac{4.2}{s^3 + 2s^2 + 4s + 4.2}\ .$$

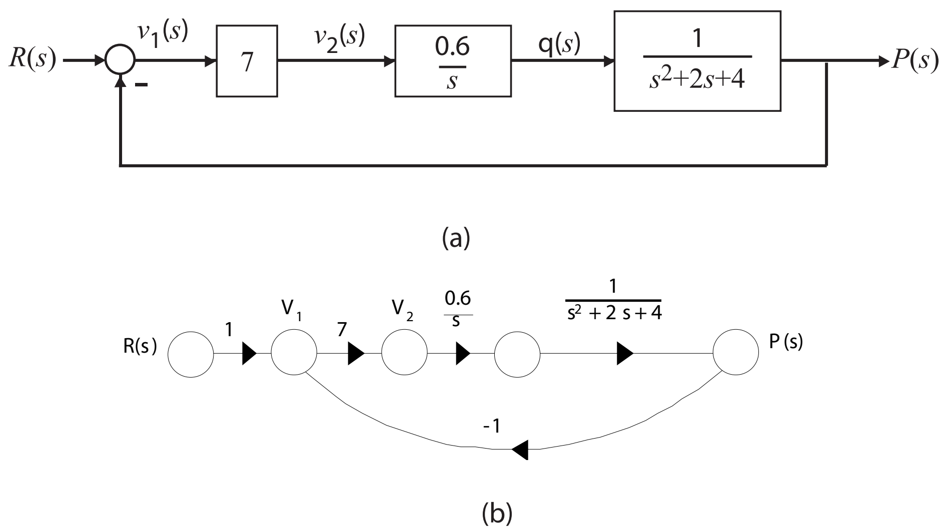



FIGURE E2.16
(a) Block diagram, (b) Signal flow graph.

**E2.17** A linear approximation for $f$ is given by

$$\Delta f = \left.\frac{\partial f}{\partial x}\right|_{x=x_o} \Delta x = 2kx_o\Delta x = k\Delta x$$

where $x_o = 1/2$, $\Delta f = f(x) - f(x_o)$, and $\Delta x = x - x_o$.

**E2.18** The linear approximation is given by

$$\Delta y = m\Delta x$$

where

$$m = \left.\frac{\partial y}{\partial x}\right|_{x=x_o} .$$

(a) When $x_o = 1$, we find that $y_o = 2.4$, and $y_o = 13.2$ when $x_o = 2$.

(b) The slope $m$ is computed as follows:

$$m = \left.\frac{\partial y}{\partial x}\right|_{x=x_o} = 1 + 4.2x_o^2 .$$

Therefore, $m = 5.2$ at $x_o = 1$, and $m = 18.8$ at $x_o = 2$.

**E2.19** The output (with a step input) is

$$Y(s) = \frac{30(s+1)}{s(s+5)(s+6)} .$$

The partial fraction expansion is

$$Y(s) = \frac{5}{s} - \frac{20}{s+3} + \frac{15}{s+2} .$$

Taking the inverse Laplace transform yields

$$y(t) = 5 - 20e^{-3t} + 15e^{-2t} .$$

**E2.20** The input-output relationship is

$$\frac{V_o}{V} = \frac{A(K-1)}{1+AK}$$

where

$$K = \frac{Z_1}{Z_1 + Z_2} .$$

Assume $A \gg 1$. Then,

$$\frac{V_o}{V} = \frac{K-1}{K} = -\frac{Z_2}{Z_1}$$

where

$$Z_1 = \frac{R_1}{R_1C_1s + 1} \quad \text{and} \quad Z_2 = \frac{R_2}{R_2C_2s + 1} .$$

Therefore,

$$\frac{V_o(s)}{V(s)} = -\frac{R_2(R_1C_1s+1)}{R_1(R_2C_2s+1)} = -\frac{2(s+1)}{s+2} .$$

**E2.21** The equation of motion of the mass $m_c$ is

$$m_c\ddot{x}_p + (b_d + b_s)\dot{x}_p + k_dx_p = b_d\dot{x}_{in} + k_dx_{in} .$$

Taking the Laplace transform with zero initial conditions yields

$$[m_cs^2 + (b_d + b_s)s + k_d]X_p(s) = [b_ds + k_d]X_{in}(s) .$$

So, the transfer function is

$$\frac{X_p(s)}{X_{in}(s)} = \frac{b_ds + k_d}{m_cs^2 + (b_d + b_s)s + k_d} = \frac{0.65s + 1.8}{s^2 + 1.55s + 1.8} .$$

**E2.22** The rotational velocity is

$$\omega(s) = \frac{2(s+4)}{(s+5)(s+1)^2}\frac{1}{s} .$$

Expanding in a partial fraction expansion yields

$$\omega(s) = \frac{8}{5}\frac{1}{s} + \frac{1}{40}\frac{1}{s+5} - \frac{3}{2}\frac{1}{(s+1)^2} - \frac{13}{8}\frac{1}{s+1} .$$

Taking the inverse Laplace transform yields

$$\omega(t) = \frac{8}{5} + \frac{1}{40}e^{-5t} - \frac{3}{2}te^{-t} - \frac{13}{8}e^{-t} .$$

**E2.23** The closed-loop transfer function is

$$\frac{Y(s)}{R(s)} = T(s) = \frac{K_1K_2}{s^2 + (K_1 + K_2K_3 + K_1K_2)s + K_1K_2K_3} .$$

**E2.24** Let $x = 0.6$ and $y = 0.8$. Then, with $y = ax^3$, we have

$$0.8 = a(0.6)^3 .$$

Solving for $a$ yields $a = 3.704$. A linear approximation is

$$y - y_o = 3ax_o^2(x - x_o)$$

or $y = 4x - 1.6$, where $y_o = 0.8$ and $x_o = 0.6$.

**E2.25** The closed-loop transfer function is

$$\frac{Y(s)}{R(s)} = T(s) = \frac{10}{s^2 + 21s + 10} .$$

**E2.26** The equations of motion are

$$m_1\ddot{x}_1 + k(x_1 - x_2) = F$$
$$m_2\ddot{x}_2 + k(x_2 - x_1) = 0 .$$

Taking the Laplace transform (with zero initial conditions) and solving for $X_2(s)$ yields

$$X_2(s) = \frac{k}{(m_2s^2 + k)(m_1s^2 + k) - k^2}F(s) .$$

Then, with $m_1 = m_2 = k = 1$, we have

$$X_2(s)/F(s) = \frac{1}{s^2(s^2 + 2)} .$$

**E2.27** The transfer function from $T_d(s)$ to $Y(s)$ is

$$Y(s)/T_d(s) = \frac{G_2(s)}{1 + G_1G_2H(s)} .$$

**E2.28** The transfer function is

$$\frac{V_o(s)}{V(s)} = \frac{R_2R_4C}{R_3}s + \frac{R_2R_4}{R_1R_3} = 46.08s + 344.91 .$$

**E2.29** (a) If

$$G(s) = \frac{1}{s^2 + 15s + 50} \quad \text{and} \quad H(s) = 2s + 15 ,$$

then the closed-loop transfer function of Figure E2.28(a) and (b) (in Dorf & Bishop) are equivalent.

(b) The closed-loop transfer function is

$$T(s) = \frac{1}{s^2 + 17s + 65} .$$

**E2.30** (a) The closed-loop transfer function is

$$T(s) = \frac{G(s)}{1 + G(s)}\frac{1}{s} = \frac{15}{s(s^2 + 5s + 30)} \quad \text{where} \quad G(s) = \frac{15}{s^2 + 5s + 15} .$$

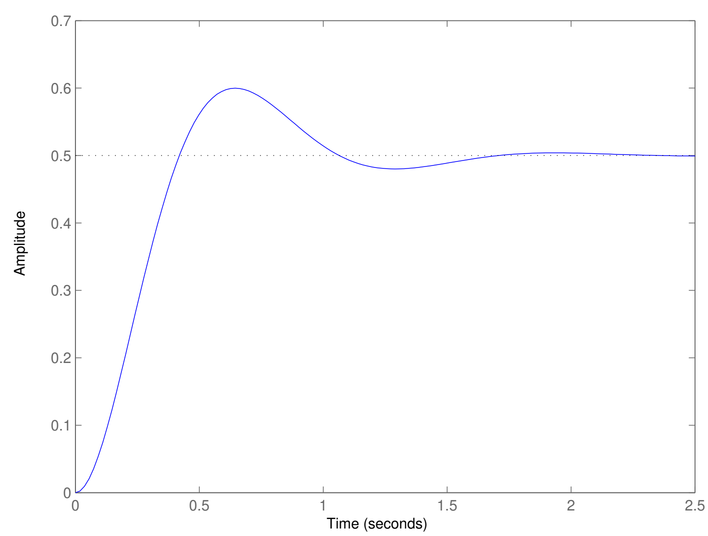



FIGURE E2.30
Step response.

(b) The output $Y(s)$ (when $R(s) = 1/s$) is

$$Y(s) = \frac{0.5}{s} + \frac{-0.25 + 0.1282j}{s + 2.5 - 4.8734} + \frac{-0.25 - 0.1282j}{s + 2.5 + 4.8734j}$$

or

$$Y(s) = \frac{1}{2}\left(\frac{1}{s} - \frac{s+5}{s^2 + 5s + 30}\right)$$

(c) The plot of $y(t)$ is shown in Figure E2.30. The output is given by

$$y(t) = 0.5(1 - 1.1239e^{-2.5t}\sin(4.8734t + 1.0968));$$

**E2.31** The partial fraction expansion is

$$V(s) = \frac{a}{s + p_1} + \frac{b}{s + p_2}$$

where $p_1 = 5 - 8.66j$ and $p_2 = 5 + 8.66j$. Then, the residues are

$$a = -5.77j \quad b = 5.77j \ .$$

The inverse Laplace transform is

$$v(t) = -5.77je^{(-5+8.66j)t} + 5.77je^{(-5-8.66j)t} = 11.55e^{-5t}\sin 8.66t \ .$$