Solutions for Physics for Scientists and Engineers 5th Edition by Knight

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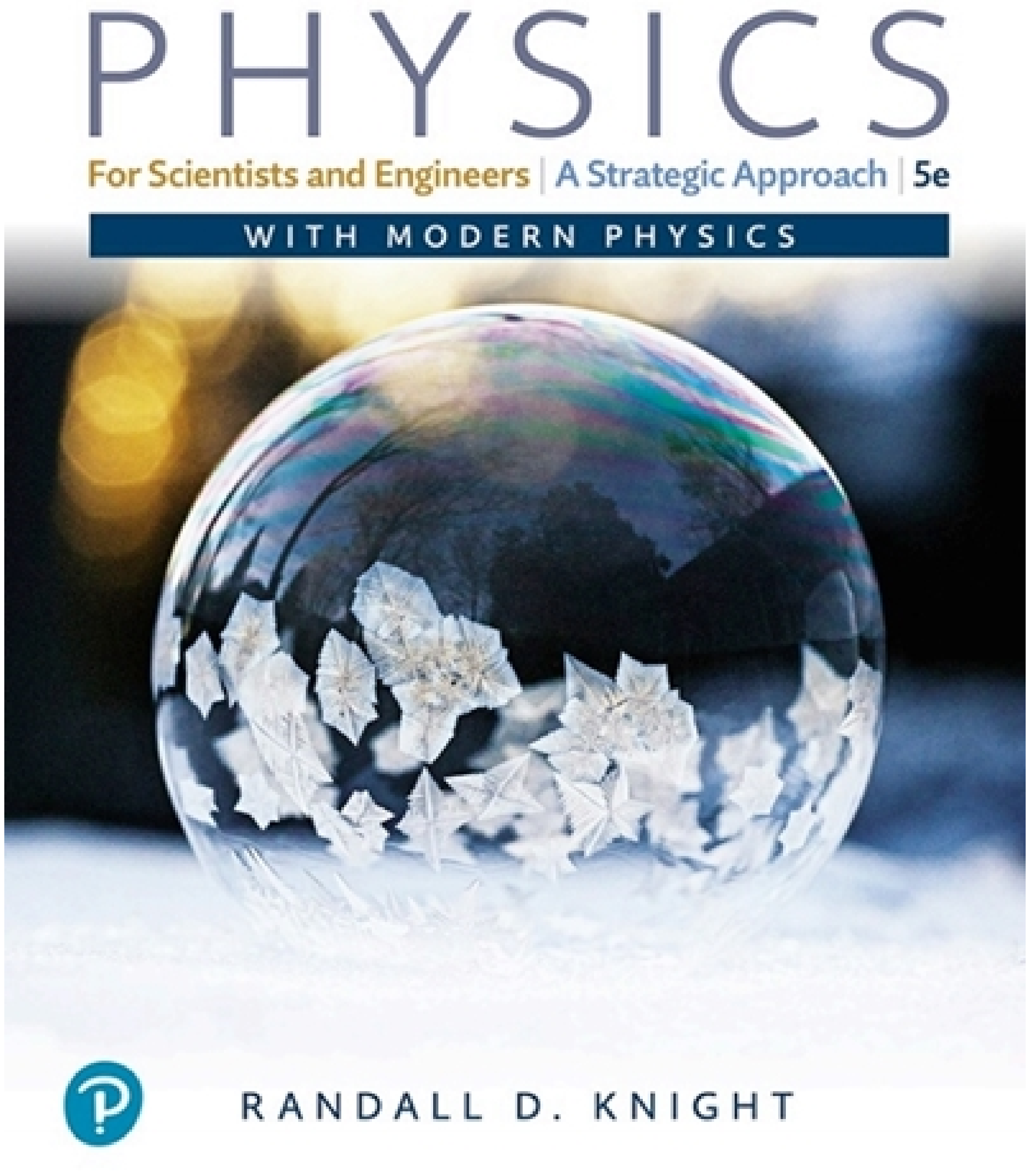


Solutions

2

KINEMATICS IN ONE DIMENSION

CONCEPTUAL QUESTIONS

2.1. I left my friend's house 5 mi east of town and rode my bike east at 4 mi/h for 5 more miles; this took 20 min. I then rested and ate a sandwich for 20 min and then rode all the way back to town at 4 mi/h.

2.2. With a slow start out of the blocks, a super sprinter reached top speed in about 5 s, having gone only 30 m. He was still able to finish his 100 m in only just over 9 s by running a world record pace for the rest of the race.

2.3. A crane picks up a crate from a platform 50 ft above the ground and then lifts it to 100 ft above the ground in 10 s where it is marked before lowering it all the way to the ground at a faster rate in another 10 s.

2.4. **(a)** At $t = 1$ s, the slope of the line for object A is greater than that for object B. Therefore, object A's speed is greater. (Both are positive slopes.)

(b) No, the speeds are never the same. Each has a constant speed (constant slope) and A's speed is always greater.

2.5. **(a)** A's speed is greater at $t = 1$ s. The slope of the tangent to B's curve at $t = 1$ s is smaller than the slope of A's line.

(b) A and B have the same speed at just about $t = 3$ s. At that time, the slope of the tangent to the curve representing B's motion is equal to the slope of the line representing A.

2.6. **(a)** 2. The object is still moving, but the magnitude of the slope of the position-versus-time curve is smaller than at 4.

(b) 4. The slope is greatest at 4.

(c) At points 1, 3, and 5 the slope of the curve is zero, so the object is not moving.

(d) At point B the slope is negative, so the object is moving to the left.

2.7. **(a)** The slope of the position-versus-time graph is greatest at 3, so the object is moving fastest at this point.

(b) The slope is negative at point 6, meaning the object is moving to the left there.

(c) At point 6 the slope is increasing in magnitude (getting more negative), meaning that the object is speeding up to the left.

(d) At point 5 the object is not moving since the slope is zero. Before point 5, the slope is positive, while after 5 it is negative, so the object is turning around at 5.

2.8. **(a)** The positions of the third dots of both motion diagrams are the same, as are the sixth dots of both, so cars A and B are at the same locations at the time corresponding to dot 3 and again at that of dot 6.

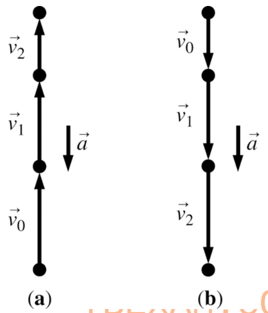
(b) The spacing of dots 4 and 5 in both diagrams is the same, so the cars are traveling at the same speeds between times corresponding to dots 4 and 5.

2.9. No, though you have the same position along the road, his velocity is greater because he is passing you. If his velocity were not greater, then he would remain even with the front of your car.

2.10. Yes. The acceleration vector will point west when the bicycle is slowing down while traveling east.

2.11. **(a)** As a ball tossed upward moves upward, its vertical velocity is positive, while its vertical acceleration is negative, opposite the velocity, causing the ball to slow down.

(b) The same ball on its way down has downward (negative) velocity. The downward negative acceleration is pointing in the same direction as the velocity, causing the speed to increase.



2.12. For all three of these situations the acceleration is equal to g in the downward direction. The magnitude and direction of the velocity of the ball do not matter. Gravity pulls down at constant acceleration. (Air friction is ignored.)

2.13. **(a)** The magnitude of the acceleration while in free fall is equal to g at all times, independent of the initial velocity. The acceleration only tells how the velocity is changing.

(b) The magnitude of the acceleration is still g because the rock is still in free fall. The speed is increasing at the same rate each instant, that is, by the same Δv each second.

2.14. **(a)** The vertical axis of the graph is velocity, not position. The object is speeding up where the velocity is increasing; this is the case at point 3.

(b) The object is slowing down at point 1 because the velocity in the x direction is getting smaller.

(c) The graph of velocity is always above the t axis, so the velocity is always positive, or in the direction to the right. At none of the points 1, 2, or 3 is it moving to the left.

(d) The object is moving to the right at all three points because the velocity is positive at all three points.

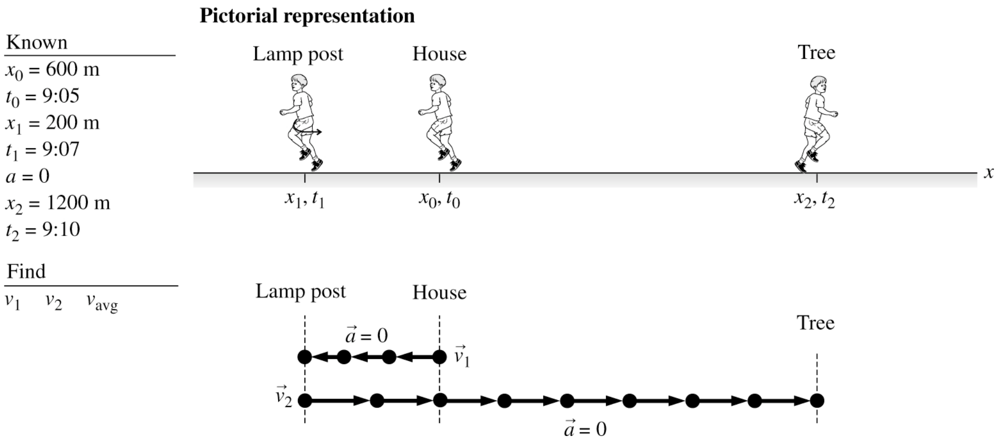
EXERCISES AND PROBLEMS

EXERCISES

Section 2.1 Uniform Motion

2.1. **MODEL:** We will consider Larry to be a particle.

VISUALIZE:



SOLVE: Since Larry's speed is constant, we can use the following equation to calculate the velocities:

$$v_s = \frac{s_f - s_i}{t_f - t_i}$$

(a) For the interval from the house to the lamppost:

$$v_1 = \frac{200\text{ m} - 600\text{ m}}{9:07 - 9:05} = -200\text{ m/min}$$

For the interval from the lamppost to the tree:

$$v_2 = \frac{1200\text{ m} - 200\text{ m}}{9:10 - 9:07} = +333\text{ m/min}$$

(b) For the average velocity for the entire run:

$$v_{avg} = \frac{1200\text{ m} - 600\text{ m}}{9:10 - 9:05} = +120\text{ m/min}$$

2.2. **SOLVE:** **(a)** The time for each segment is $\Delta t_1 = 50\text{ mi}/40\text{ mph} = 5/4\text{ h}$ and $\Delta t_2 = 50\text{ mi}/60\text{ mph} = 5/6\text{h}$. The average speed to the house is

$$\frac{100\text{ mi}}{5/6\text{ h} + 5/4\text{ h}} = 48\text{ mph}$$

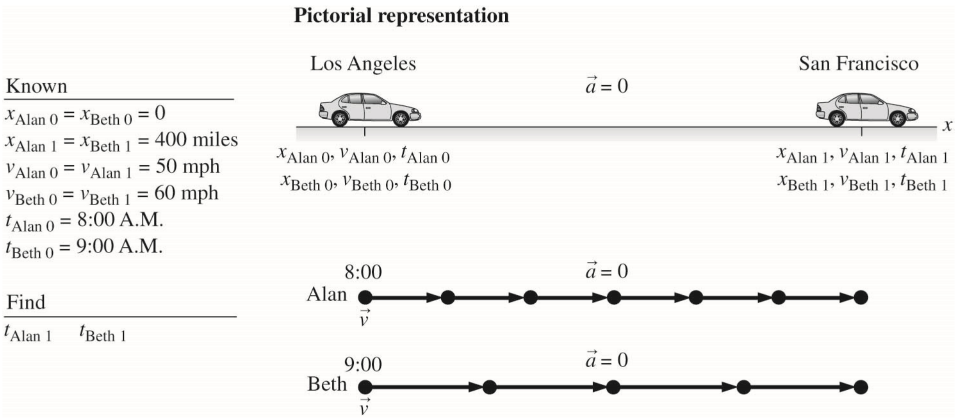
(b) Julie drives the distance Δx_1 in time Δt_1 at 40 mph. She then drives the distance Δx_2 in time Δt_2 at 60 mph. She spends the same amount of time at each speed, thus

$$\Delta t_1 = \Delta t_2 \Rightarrow \Delta x_1/40\text{ mph} = \Delta x_2/60\text{ mph} \Rightarrow \Delta x_1 = (2/3)\Delta x_2$$

But $\Delta x_1 + \Delta x_2 = 100$ mi, so $(2/3)\Delta x_2 + \Delta x_2 = 100$ mi. This means $\Delta x_2 = 60$ mi and $\Delta x_1 = 40$ mi. Thus, the times spent at each speed are $\Delta t_1 = 40$ mi/40 mph $= 1.00$ h and $\Delta t_2 = 60$ mi/60 mph $= 1.00$ h. The total time for her return trip is $\Delta t_1 + \Delta t_2 = 2.00$ h. So, her average speed is 100 mi/2 h $= 50$ mph.

2.3. **MODEL:** Cars will be treated by the particle model.

VISUALIZE:



SOLVE: Beth and Alan are moving at a constant speed, so we can calculate the time of arrival as follows:

$$v = \frac{\Delta x}{\Delta t} = \frac{x_1 - x_0}{t_1 - t_0} \Rightarrow t_1 = t_0 + \frac{x_1 - x_0}{v}$$

Using the known values identified in the pictorial representation, we find:

$$t_{\text{Alan }1} = t_{\text{Alan }0} + \frac{x_{\text{Alan }1} - x_{\text{Alan }0}}{v} = 8{:}00 \text{ A.M.} + \frac{400 \text{ mi}}{50 \text{ mi/h}} = 8{:}00 \text{ A.M.} + 8 \text{ h} = 4{:}00 \text{ P.M.}$$

$$t_{\text{Beth }1} = t_{\text{Beth }0} + \frac{x_{\text{Beth }1} - x_{\text{Beth }0}}{v} = 9{:}00 \text{ A.M.} + \frac{400 \text{ mi}}{60 \text{ mi/h}} = 9{:}00 \text{ A.M.} + 6.67 \text{ h} = 3{:}40 \text{ P.M.}$$

(a) Beth arrives first.

(b) Beth has to wait $t_{\text{Alan }1} - t_{\text{Beth }1} = 20$ min for Alan.

REVIEW: Times of the order of 7 or 8 h are reasonable in the present problem.

2.4. **MODEL:** We will treat the bicycle as a point particle, meaning we won't be concerned with measuring from one end or the other. The velocity is the rate of change of position. Here, the motion is in the x direction, so the velocity we determine will also be along the x direction. We will use $v_x = \Delta x / \Delta t$.

SOLVE: (a) The horizontal component of velocity at $t = 5.0$ s is

$$v_x = \frac{\Delta x}{\Delta t} = \frac{(50 \text{ m}) - (150 \text{ m})}{(10 \text{ s}) - (10 \text{ s})} = -10 \text{ m/s}$$

So, $\vec{v} = -10$ m/s in x.

(b) The horizontal component of velocity at $t = 15$ s is

$$v_x = \frac{\Delta x}{\Delta t} = \frac{(50 \text{ m}) - (50 \text{ m})}{(10 \text{ s}) - (10 \text{ s})} = 0 \text{ m/s}$$

So, $\vec{v} = \vec{0}$ m/s.

(c) The horizontal component of velocity at $t = 30$ s is

$$v_x = \frac{\Delta x}{\Delta t} = \frac{(150\text{ m}) - (50\text{ m})}{(40\text{ s}) - (20\text{ s})} = 5.0\text{ m/s}$$

So, $\vec{v} = 5.0$ m/s in x.

Review: Since the x component of velocity is the slope of the position vs. time graph, it is reasonable that we obtained a speed of zero at the 15 second mark, where the position vs. time graph is flat.

Section 2.2 Instantaneous Velocity

Section 2.3 Finding Position from Velocity

2.5. **Model:** We can model the object as a particle, so that it has one well-defined position. The motion is in the x direction, so the velocity we determine will really be the x component of the velocity $v_x = \Delta x / \Delta t$.

Visualize: Note that a flat line in the position vs. time graph indicates that the position is not changing, and must correspond to a velocity of zero.

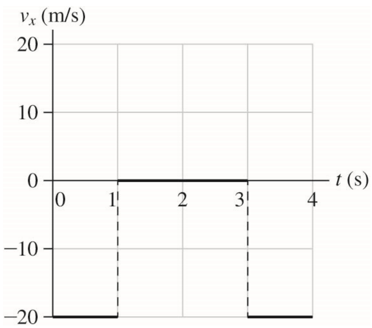
Solve: (a) We can calculate $v_x = \Delta x / \Delta t$ on each time interval, and plot the results:

$$t = 0 \text{ to } t = 1\text{ s: } v_x = \frac{(10\text{ m}) - (20\text{ m})}{(1\text{ s}) - (0\text{ s})} = -10\text{ m/s},$$

$$t = 1 \text{ to } t = 3\text{ s: } v_x = \frac{(10\text{ m}) - (10\text{ m})}{(3\text{ s}) - (1\text{ s})} = 0\text{ m/s},$$

$$t = 3 \text{ to } t = 4\text{ s: } v_x = \frac{(0\text{ m}) - (10\text{ m})}{(4\text{ s}) - (3\text{ s})} = -10\text{ m/s}$$

Plotting this yields



(b) No, the motion is always in the –x direction. The cyclist does stop, but does not turn around.

Review: Our math in part (a) confirms what we expected: a flat line segment on a position vs. time graph indicates a velocity of zero.

2.6. **VISUALIZE:** Please refer to Figure EX2.6 in the text. The particle starts at $x_0 = 10$ m at $t_0 = 0$. Its velocity is initially in the $-x$ direction. The speed decreases as time increases during the first second, is zero at $t = 1$ s, and then increases after the particle reverses direction.

SOLVE: **(a)** The particle reverses direction at $t = 1$ s, when v_x changes sign.

(b) Using the equation $x_\text{f} = x_0 +$ area of the velocity graph between t_1 and t_f,

$$x_{2\text{s}} = 10 \text{ m} - (\text{area of triangle between 0 s and 1 s}) + (\text{area of triangle between 1 s and 2 s})$$
$$= 10 \text{ m} - \frac{1}{2}(4 \text{ m/s})(1 \text{ s}) + \frac{1}{2}(4 \text{ m/s})(1 \text{ s}) = 10 \text{ m}$$
$$x_{4\text{s}} = x_{2\text{s}} + \text{area between 2 s and 4 s}$$
$$= 10 \text{ m} + \frac{1}{2}(4 \text{ m/s} + 12 \text{ m/s})(2 \text{ s}) = 26 \text{ m}$$

REVIEW: It is reasonable for the particle to be back at its starting position after 2 seconds, since the area "under" the curve is zero.

2.7. **SOLVE:** We can calculate the position of the particle at every instant with the equation

$x_\text{f} = x_\text{i} +$ area under the velocity-versus-time graph between t_i and t_f

The particle starts from the origin at $t = 0$ s, so $x_i = 0$ m. Notice that the each square of the grid in Figure EX2.8 has "area" $(5 \text{ m/s}) \times (2 \text{ s}) = 10$ m. We can find the area under the curve, and thus determine x, by counting squares. You can see that x = 35 m at 6s because there are 3.5 squares under the curve. In addition, $x = 35$ m at t = 10 s because the 5 m represented by the half square between 6 s and 8 s is canceled by the –5 m represented by the half square between 8 s and 10 s. Areas beneath the axis are negative areas. The particle passes through $x = 35$ m at $t = 6$ s and again at $t = 10$ s.

REVIEW: It is reasonable that if the particle passes through the 35 m mark while moving to the right, it should pass through this point a second time once it changes direction. Our answers are reasonable.

2.8. **MODEL:** The graph shows the assumption that the blood isn't moving for the first 0.1 s nor at the end of the beat.

VISUALIZE: The graph is a graph of velocity versus time, so the displacement is the area under the graph—that is, the area of the triangle. The velocity of the blood increases quickly and decreases a bit more slowly.

SOLVE: Call the distance traveled Δy. The area of a triangle is $\frac{1}{2}BH$.

$$\Delta y = \frac{1}{2}BH = \frac{1}{2}(0.20 \text{ s})(0.80 \text{ m/s}) = 8.0 \text{ cm}$$

REVIEW: This distance seems reasonable for one beat.

Section 2.4 Motion with Constant Acceleration

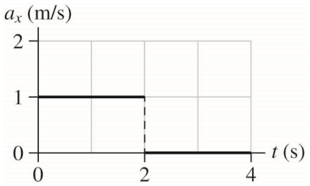
2.9. **MODEL:** We will model the object like a particle. The motion is entirely in the x direction, so the acceleration we find will be a_x. Wherever the acceleration is constant $a_x = \Delta v_x / \Delta t$.

VISUALIZE: Wherever the acceleration is constant (meaning the graph of v_x is a straight line segment with one slope, we can write $a_x = \Delta v_x / \Delta t$.

SOLVE: Up to 2 s, the slope of v_x vs. *t* is constant:

$$a_x = \Delta v_x / \Delta t = \frac{(3 \text{ m/s}) - (1 \text{ m/s})}{(2 \text{ s}) - (0 \text{ s})} = 1 \text{ m/s}^2 \cdot$$

After that, the *x*-component of the velocity is constant, and we have $a_x = \Delta v_x / \Delta t = 0 \text{ m/s}^2$. Plotting these values, we have



REVIEW: Initially, the *x*-components of both the acceleration and velocity are positive. This should cause the particle to speed up in the +*x* direction. This is exactly what happens, since v_x rises from 1 m/s to 3 m/s, initially. This makes sense.

2.10. **SOLVE:** **(a)** At $t = 2.0$ s, the position of the particle is

$$x_2 = 2.0 \text{ m} + \text{area under velocity graph from } t = 0 \text{ s to } t = 2.0 \text{ s}$$
$$= 2.0 \text{ m} + \frac{1}{2}(4.0 \text{ m/s})(2.0 \text{ s}) + 2.0 \text{ m} = 8.0 \text{ m}$$

(b) From the graph itself at $t = 2.0$ s, $v = 2$ m/s.

(c) The acceleration is

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{fx} - v_{ix}}{\Delta t} = \frac{0.0 \text{ m/s} - 6.0 \text{ m/s}}{3 \text{ s}} = -2.0 \text{ m/s}^2$$

2.11. **VISUALIZE:** The graph is a graph of velocity versus time, so the acceleration is the slope of the graph.

SOLVE: When the blood is speeding up the acceleration is

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{0.80 \text{ m/s}}{0.05 \text{ s}} = 16 \text{ m/s}^2$$

When the blood is slowing down the acceleration is

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{-0.80 \text{ m/s}}{0.15 \text{ s}} = -5.3 \text{ m/s}^2$$

REVIEW: 16 m/s^2 is an impressive but reasonable acceleration.

2.12. **SOLVE:** **(a)** Using the equation

$x_f = x_i +$ area under the velocity-versus-time graph between t_i and t_f

we have

$$x(\text{at } t = 1 \text{ s}) = x(\text{at } t = 0 \text{ s}) + \text{area between } t = 0 \text{ s and } t = 1 \text{ s}$$
$$= 2.0 \text{ m} + (4 \text{ m/s})(1 \text{ s}) = 6 \text{ m}$$

Reading from the velocity-versus-time graph, $v_x(\text{at } t = 1 \text{ s}) = 4$ m/s. Also, $a_x = \text{slope} = \Delta v/\Delta t = 0 \text{ m/s}^2$.

(b) $x(\text{at } t = 3.0 \text{ s}) = x(\text{at } t = 0 \text{ s}) +$ area between $t = 0$ s and $t = 3$ s

$$= 2.0 \text{ m} + 4 \text{ m/s} \times 2 \text{ s} + 2 \text{ m/s} \times 1 \text{ s} + (1/2) \times 2 \text{ m/s} \times 1 \text{ s} = 13.0 \text{ m}$$

Reading from the graph, $v_x(t = 3 \text{ s}) = 2$ m/s. The acceleration is

$$a_x(t = 3 \text{ s}) = \text{slope} = \frac{v_x(\text{at } t = 4 \text{ s}) - v_x(\text{at } t = 2 \text{ s})}{2 \text{ s}} = -2 \text{ m/s}^2$$

2.13. **MODEL:** Represent the car as a particle.

SOLVE: **(a)** First, we will convert units:

$$60 \frac{\text{mi}}{\text{h}} \times \frac{1 \text{ h}}{3600 \text{ s}} \times \frac{1610 \text{ m}}{1 \text{ mi}} = 27 \text{ m/s}$$

The motion is constant acceleration, so

$$v_{1x} = v_{0x} + a_x \Delta t \Rightarrow a_x = \frac{v_{1x} - v_{0x}}{\Delta t} = \frac{(27 \text{ m/s} - 0 \text{ m/s})}{10 \text{ s}} = 2.7 \text{ m/s}^2$$

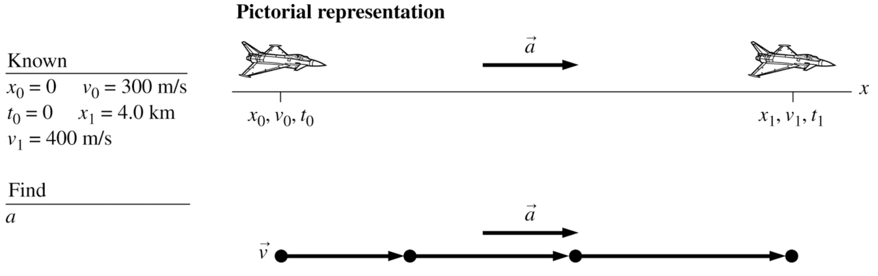
(b) The distance is calculated as follows:

$$x_1 = x_0 + v_{0x}\Delta t + \frac{1}{2}a_x(\Delta t)^2 = \frac{1}{2}a_x(\Delta t)^2 = 1.3 \times 10^2 \text{ m} = 4.3 \times 10^2 \text{ ft}$$

REVIEW: We expect a car driving for 10 s to move many feet. We would expect an answer in the hundreds. Our answer seems reasonable.

2.14. **MODEL:** Represent the jet plane as a particle.

VISUALIZE:



SOLVE: Since we don't know the time of acceleration, we will use

$$v_1^2 = v_0^2 + 2a(x_1 - x_0)$$
$$\Rightarrow a = \frac{v_1^2 - v_0^2}{2x_1} = \frac{(400 \text{ m/s})^2 - (300 \text{ m/s})^2}{2(4000 \text{ m})} = 8.75 \text{ m/s}^2 \approx 8.8 \text{ m/s}^2$$

REVIEW: The acceleration of the jet is not quite equal to *g*, the acceleration due to gravity; this seems reasonable for a jet.

2.15. **MODEL:** Throughout the problem, we will ignore the spatial dimensions of the probe, and treat it like a particle.

VISUALIZE: We will use $v_x = v_{0x} + a_x \Delta t$ as well as $v_x^2 = v_{0x}^2 + 2a_x \Delta x$.

SOLVE: **(a)** Starting from rest, we have $a_x = v_{max} / t_{push}$.

(b) We know $v_x^2 = v_{0x}^2 + 2a_x \Delta x$. Combining this with the result from (a), we have

$$v_{max}^2 = v_{0x}^2 + 2a_x \Delta x = 0 + 2\left(\frac{v_{max}}{t_{push}}\right)\Delta x \Rightarrow \Delta x = \frac{1}{2} v_{max} t_{push}$$

(c) The speed of light is 3.00×10^8 m/s, so the acceleration required is

$$a_x = \frac{(0.10)c}{t_{push}} = \frac{(0.10)(3.00 \times 10^8 \text{ m/s})}{(1.0 \text{ y})(365 \text{ d/y})(24 \text{ h/d})(3600 \text{ s/h})} = 0.95 \text{ m/s}^2.$$

(d) Using the result from part (b), we have

$$\frac{\Delta x}{(1c \cdot \text{y})} = \frac{1}{2(1c \cdot \text{y})} v_{max} t_{push} = \frac{1}{2} \frac{(0.10)c}{(1c \cdot \text{y})}(1.0 \text{ y}) = \frac{0.10}{2} = 0.050$$

Or 5.0% of a light year.

REVIEW: Since the probe never travels faster than 1/10 of the speed of light, we expect the distance it crosses in a year to be much less than a light-year. Our answer, 5.0% of a light year, is reasonable.

2.16. **MODEL:** Model the air as a particle.

VISUALIZE: Use the definition of acceleration and then convert units.

SOLVE:

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{150 \text{ km/h}}{0.50 \text{ s}}\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{60 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 83 \text{ m/s}^2$$

REVIEW: 83 m/s^2 is a remarkable acceleration.

2.17. **MODEL:** We are using the particle model for the skater and the kinematics model of motion under constant acceleration.

SOLVE: Since we don't know the time of acceleration we will use

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

$$\Rightarrow a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(-6.0 \text{ m/s})^2 - (-8.0 \text{ m/s})^2}{2(5.0 \text{ m})} = -2.8 \text{ m/s}^2$$

Her acceleration is positive 2.8 m/s^2 because it points to the right; not because she is speeding up (she isn't).

REVIEW: A deceleration of 2.8 m/s^2 is reasonable.

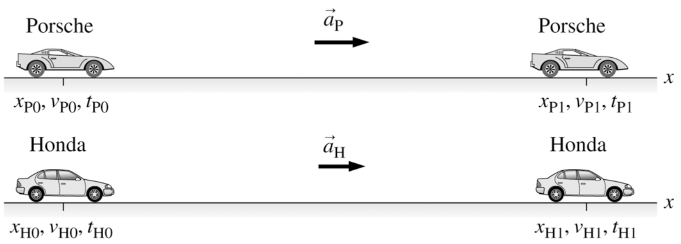
2.18. **MODEL:** We are assuming both cars are particles.

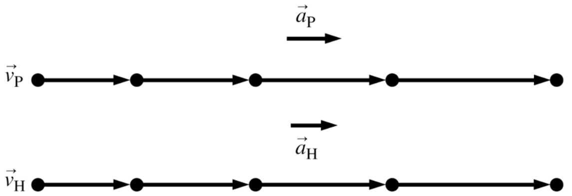
VISUALIZE:

Pictorial representation

Known	
$x_{P0} = 0$	$v_{P0} = 0$
$t_{P0} = 0$	$a_P = 3.5\ \text{m/s}^2$
$x_{P1} = 400\ \text{m}$	$x_{H0} = 0\ \text{m}$
$v_{H0} = 0$	$t_{H0} = -1\ \text{s}$
$a_H = 3.0\ \text{m/s}^2$	
$x_{H1} = 400\ \text{m}$	

Find	
t_{P1}	t_{H1}





SOLVE: The Porsche's time to finish the race is determined from the position equation

$$x_{P1} = x_{P0} + v_{P0}(t_{P1} - t_{P0}) + \frac{1}{2}a_P(t_{P1} - t_{P0})^2$$

$$\Rightarrow 400\ \text{m} = 0\ \text{m} + 0\ \text{m} + \frac{1}{2}(3.5\ \text{m/s}^2)(t_{P1} - 0\ \text{s})^2 \Rightarrow t_{P1} = 15.12\ \text{s}$$

The Honda's time to finish the race is obtained from Honda's position equation as

$$x_{H1} = x_{H0} + v_{H0}(t_{H1} - t_{H0}) + \frac{1}{2}a_{H0}(t_{H1} - t_{H0})^2$$

$$400\ \text{m} = 0\ \text{m} + 0\ \text{m} + \frac{1}{2}(3.0\ \text{m/s}^2)(t_{H1} + 1\ \text{s})^2 \Rightarrow t_{H1} = 15.33\ \text{s}$$

So, the Porsche wins by 0.21 s.

REVIEW: The numbers are contrived for the Porsche to win, but the time to go 400 m seems reasonable.

2.19. **MODEL:** For now, we will treat the Porsche as though it were a particle, ignoring its length and other complications. We can use the kinematic equations, because we are told to assume the acceleration is constant. 60 mph is 26.8 m/s.

Solve: (a) The acceleration is given by $v_x = v_{0x} + a_x\Delta t$. Rearranging, and inserting numbers, we have

$$a_x = \frac{v_x - v_{0x}}{\Delta t} = \frac{(26.8\ \text{m/s}) - (0\ \text{m/s})}{(2.5\ \text{s})} = 10.7\ \text{m/s}^2 \approx 11\ \text{m/s}^2$$

(b) The distance travelled can be obtained from $\Delta x = v_{0x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$. Using the acceleration from part (a), we have

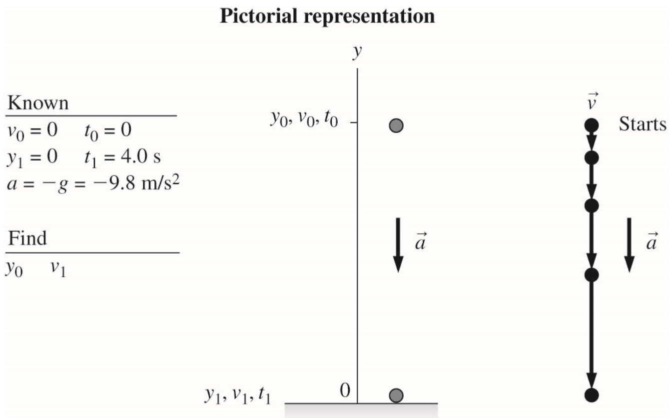
$$\Delta x = v_{0x}\Delta t + \frac{1}{2}a_x(\Delta t)^2 = 0 + \frac{1}{2}(10.7\ \text{m/s}^2)(2.5\ \text{s})^2 = 33.5\ \text{m} \approx 34\ \text{m}$$

REVIEW: Our answer to part (a) is an amazingly high acceleration, comparable to the acceleration due to gravity of an object falling.

Section 2.5 Free Fall

2.20. **MODEL:** Represent the spherical drop of molten metal as a particle.

VISUALIZE:



SOLVE: **(a)** The shot is in free fall, so we can use free-fall kinematics with $a = -g$. The height must be such that the shot takes 4.0 s to fall, so we choose $t_1 = 4.0$ s. Then,

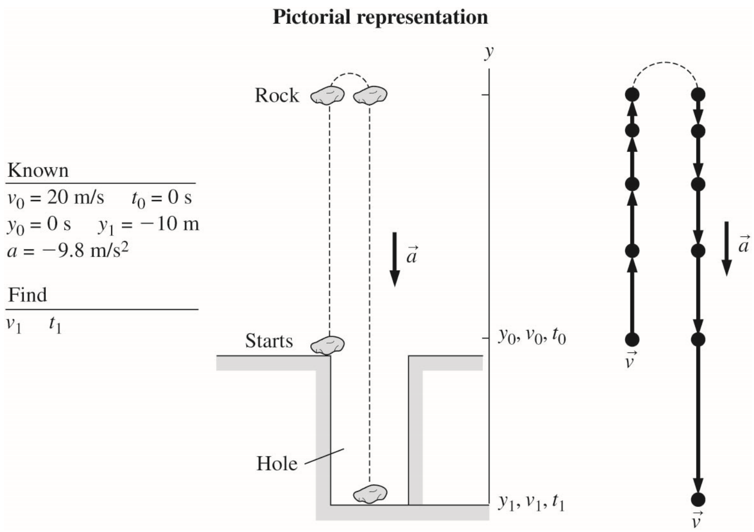
$$y_1 = y_0 + v_0(t_1 - t_0) - \frac{1}{2}g(t_1 - t_0)^2 \Rightarrow y_0 = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.8 \text{ m/s}^2)(4.0 \text{ s})^2 = 78.4 \text{ m}$$

(b) The impact velocity is $v_1 = v_0 - g(t_1 - t_0) = -gt_1 = -39.2$ m/s.

REVIEW: Note the minus sign. The question asked for *velocity*, not speed, and the *y*-component of $\vec{v}$ is negative because the vector points downward.

2.21. **MODEL:** We will use the particle model and the constant-acceleration kinematic equations.

VISUALIZE:



Solve: **(a)** Substituting the known values into $y_1 = y_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2$, we get

$$-10\text{ m} = 0\text{ m} + 20\text{ (m/s)}t_1 + \frac{1}{2}(-9.8\text{ m/s}^2)t_1^2$$

One of the roots of this equation is negative and is not relevant physically. The other root is $t_1 = 4.53$ s, which is the answer to part (b). Using $v_1 = v_0 + a\Delta t$, we obtain

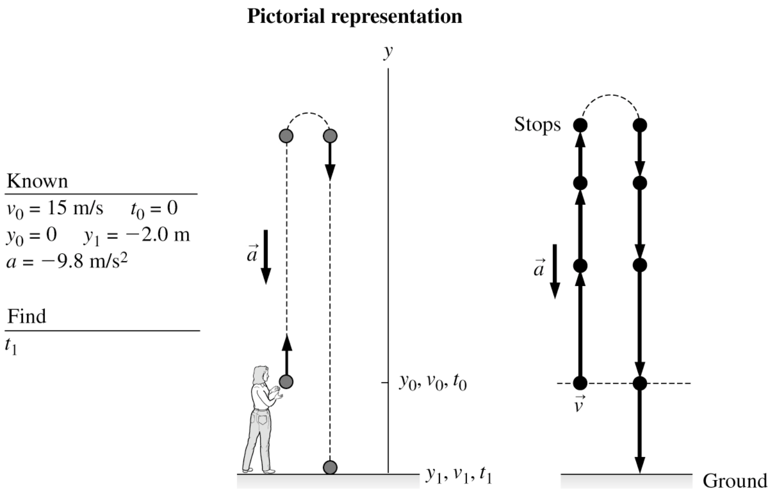
$$v_1 = 20\text{(m/s)} + (-9.8\text{ m/s}^2)(4.53\text{ s}) = -24\text{ m/s}$$

(b) The time is 4.5 s.

Review: A time of 4.5 s is a reasonable value. The rock's velocity as it hits the bottom of the hole has a negative sign because of its downward direction. The magnitude of 24 m/s compared to 20 m/s, when the rock was tossed up, is consistent with the fact that the rock travels an additional distance of 10 m into the hole.

2.22. **Model:** We model the ball as a particle.

Visualize:



Solve: Once the ball leaves the student's hand, the ball is in free fall and its acceleration is equal to the free-fall acceleration g that always acts vertically downward toward the center of the earth. According to the constant-acceleration kinematic equations of motion

$$y_1 = y_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2$$

Substituting the known values

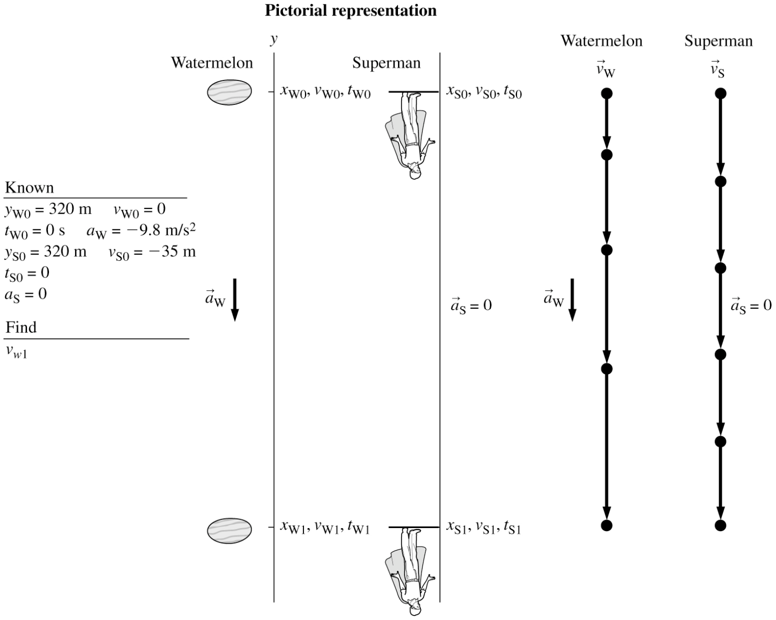
$$-2\text{ m} = 0\text{ m} + (15\text{ m/s})t_1 + (1/2)(-9.8\text{ m/s}^2)t_1^2$$

One solution of this quadratic equation is $t_1 = 3.2$ s. The other root of this equation yields a negative value for t_1, which is not valid for this problem.

Review: A time of 3.2 s is reasonable.

2.23. **MODEL:** The watermelon and Superman will be treated as particles that move according to constant-acceleration kinematic equations.

VISUALIZE:



SOLVE: The watermelon's and Superman's position as they meet each other are

$$y_{W1} = y_{W0} + v_{W0}(t_{W1} - t_{W0}) + \frac{1}{2}a_{W0}(t_{W1} - t_{W0})^2$$

$$y_{S1} = y_{S0} + v_{S0}(t_{S1} - t_{S0}) + \frac{1}{2}a_{S0}(t_{S1} - t_{S0})^2$$

$$\Rightarrow y_{W1} = 320 \text{ m} + 0 \text{ m} + \frac{1}{2}(-9.8 \text{ m/s}^2)(t_{W1} - 0 \text{ s})^2$$

$$\Rightarrow y_{S1} = 320 \text{ m} + (-35 \text{ m/s})(t_{S1} - 0 \text{ s}) + 0 \text{ m}$$

Because $t_{S1} = t_{W1}$,

$$y_{W1} = 320 \text{ m} - (4.9 \text{ m/s}^2)\, t_{W1}^2 \qquad y_{S1} = 320 \text{ m} - (35 \text{ m/s})\, t_{W1}$$

Since $y_{W1} = y_{S1}$,

$$320 \text{ m} - (4.9 \text{ m/s}^2)t_{W1}^2 = 320 \text{ m} - (35 \text{ m/s})t_{W1} \Rightarrow t_{W1} = 0 \text{ s and } 7.1 \text{ s}$$

Indeed, $t_{W1} = 0$ s corresponds to the situation when Superman arrives just as the watermelon is dropped off the Empire State Building. The other value, $t_{W1} = 7.1$ s, is the time when the watermelon will catch up with Superman. The speed of the watermelon as it passes Superman is

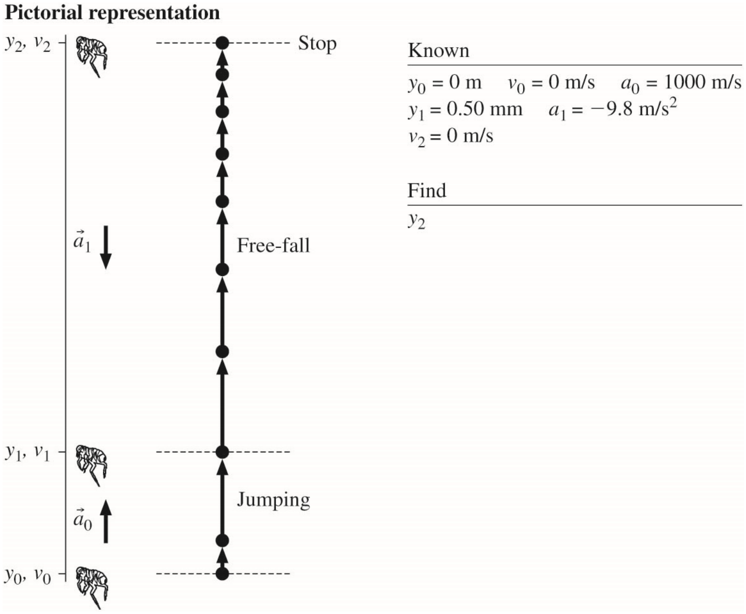
$$v_{W1} = v_{W0} + a_{W0}(t_{W1} - t_{W0}) = 0 \text{ m/s} + (-9.8 \text{ m/s}^2)(7.1 \text{ s} - 0 \text{ s}) = -70 \text{ m/s}$$

Note that the negative sign implies a downward velocity.

REVIEW: A speed of 157 mph for the watermelon is understandable in view of the significant distance (250 m) involved in the free fall.

2.24. **MODEL:** Model the flea as a particle. Both the initial acceleration phase and the free-fall phase have constant acceleration, so use the kinematic equations.

VISUALIZE:



SOLVE: We can apply the kinematic equation $v_f^2 - v_i^2 = 2a\Delta y$ twice, once to find the take-off speed and then again to find the final height. In the first phase the acceleration is up (positive) and $v_0 = 0$.

$$v_1^2 = 2a_0(y_1 - y_0) = 2(1000\ \text{m/s}^2)(0.50\times10^{-3}\ \text{m}) v_1 = 1.0\ \text{m/s}$$

In the free-fall phase, the acceleration is $a_1 = -g$ and $v_1 = 1.0$ m/s and $v_2 = 0$ m/s.

$$y_2 - y_1 = \frac{v_2^2 - v_1^2}{2a_1} = \frac{-v_1^2}{2(-g)} = \frac{-(1.0\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} = 5.1\ \text{cm}$$

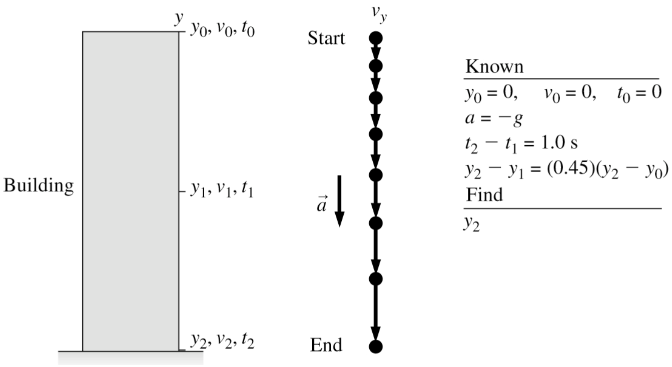
So the final height is $y_2 = 5.1\ \text{cm} + y_1 = 5.1\ \text{cm} + 0.50\ \text{mm} = 5.2\ \text{cm}$.

REVIEW: This is pretty amazing—about 10–20 times the size of a typical flea.

2.25. **MODEL:** Model the rock as a particle and assume it is in free fall.

VISUALIZE: Set up a coordinate system where the origin is at the top of the building, but up is still positive. That is, the location of the bottom of the building is y_2 which is a negative number, and downward velocities are also negative.

Pictorial representation



SOLVE: Use the third kinematic equation for $y_2 - y_1$ and $y_2 - y_0$.

$$y_2 - y_1 = \frac{v_2^2 - v_1^2}{2a} \qquad y_2 - y_0 = \frac{v_2^2 - v_0^2}{2a}$$

Now use $v_0 = 0$ and set up a ratio where the $2a$ cancels.

$$0.45 = \frac{y_2 - y_1}{y_2 - y_1} = \frac{v_2^2 - v_1^2}{v_2^2} = 1 - \frac{v_1^2}{v_2^2}$$

Now the first kinematic equation comes into play.

$$v_2 = v_1 + a(t_2 - t_1)$$

$$1 - \frac{v_1^2}{v_2^2} = 1 - \frac{v_1^2}{(v_1 + a(t_2 - t_1))^2} = 0.45$$

$$1 - 0.45 = \frac{v_1^2}{(v_1 + a(t_2 - t_1))^2} = 0.55$$

Take square roots of both sides and solve for v_1.

$$\frac{v_1}{v_1 + a(t_2 - t_1)} = \sqrt{0.55} \Rightarrow$$

$$v_1 = \frac{a(t_2 - t_1)\sqrt{0.55}}{1 - \sqrt{0.55}} = \frac{(-9.8 \text{ m/s}^2)(1 \text{ s})\sqrt{0.55}}{1 - \sqrt{0.55}} = -28.13 \text{ m/s}$$

Now compute v_2.

$$v_2 = v_1 + a(t_2 - t_1) = -28.13 \text{ m/s} + (-9.8 \text{ m/s}^2)(1 \text{ s}) = -37.93 \text{ m/s}$$

Plug back into the third kinematic equation with $v_0 = 0$ and $y_0 = 0$.

$$y_2 = \frac{v_2^2}{2a} = \frac{(-37.93 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = -73 \text{ m}$$

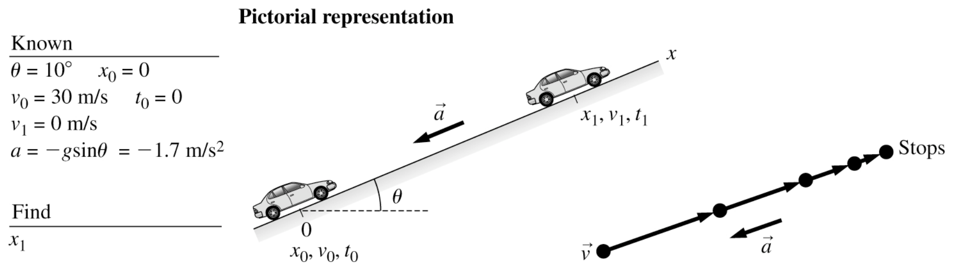
So the building is 73 m tall.

REVIEW: 73 m is a reasonable height of a building. Also, with the information calculated so far we check our work by using kinematic equations to find the other variables: $y_1 = -40$ m, $t_1 = 2.9$ s, $t_2 = 3.9$ s. It is important to keep extra digits in the intermediate calculations and then round to the correct number of significant figures at the end.

Section 2.6 Motion on an Inclined Plane

2.26. **MODEL:** Represent the car as a particle.

VISUALIZE:



SOLVE: Note that the problem "ends" at a turning point, where the car has an instantaneous speed of 0 m/s before rolling back down. The rolling back motion is *not* part of this problem. If we assume the car rolls without friction, then we have motion on a frictionless inclined plane with an acceleration

$a = -g\sin\theta = -g\sin 10° = -1.7\ \text{m/s}^2$. Constant-acceleration kinematics gives

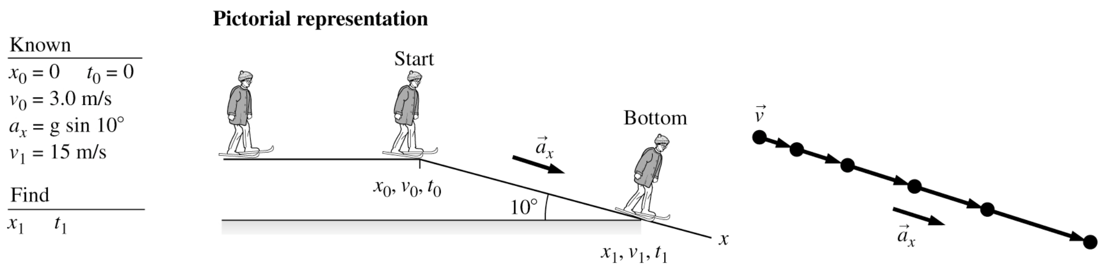
$$v_1^2 = v_0^2 + 2a(x_1 - x_0) \Rightarrow 0\ \text{m}^2/\text{s}^2 = v_0^2 + 2ax_1 \Rightarrow x_1 = -\frac{v_0^2}{2a} = -\frac{(30\ \text{m/s})^2}{2(-1.7\ \text{m/s}^2)} = 265\ \text{m}$$

Notice how the two negatives canceled to give a positive value for x_1.

REVIEW: We must include the minus sign because the $\vec{a}$ vector points *down* the slope, which is in the negative *x*-direction.

2.27. **MODEL:** We will model the skier as a particle.

VISUALIZE:



Note that the skier's motion on the horizontal, frictionless snow is not of any interest to us. Also note that the acceleration parallel to the incline is equal to $g\sin 10°$.

SOLVE: **(a)** Using the following constant-acceleration kinematic equations,

$$v_{fx}^2 = v_{ix}^2 + 2a_x(x_f - x_i)$$

$$\Rightarrow (15\ \text{m/s})^2 = (3.0\ \text{m/s})^2 + 2(9.8\ \text{m/s}^2)\sin 10°(x_1 - 0\ \text{m}) \Rightarrow x_1 = 64\ \text{m}$$

$$v_{fx} = v_{ix} + a_x(t_f - t_i)$$

$$\Rightarrow (15\ \text{m/s}) = (3.0\ \text{m/s}) + (9.8\ \text{m/s}^2)(\sin 10°)t \Rightarrow t = 7.1\ \text{s}$$

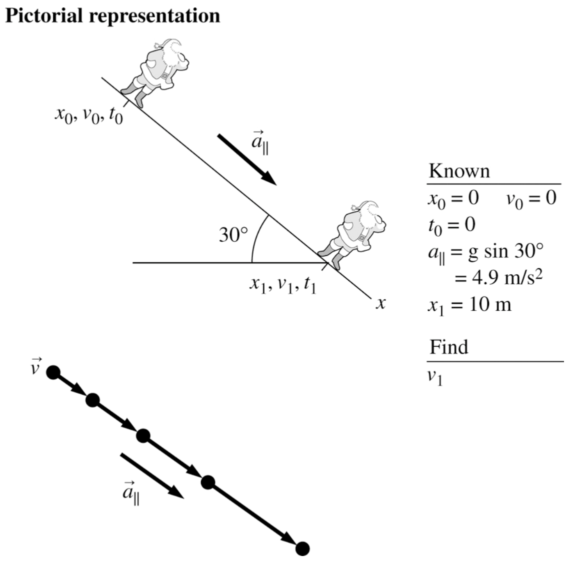
(b) Using a different kinematic equation this time, we find

$$v_{fx} = v_{ix} + a_x(t_f - t_i)$$
$$\Rightarrow (15\text{ m/s}) = (3.0\text{ m/s}) + (9.8\text{ m/s}^2)(\sin 10°)t \Rightarrow t = 7.1\text{ s}$$

REVIEW: A time of 7.1 s to cover 64 m is a reasonable value.

2.28. **MODEL:** Santa is a particle moving under constant-acceleration kinematic equations.

VISUALIZE: Note that our x-axis is positioned along the incline.



SOLVE: Using the following kinematic equation,

$$v_1^2 = v_0^2 + 2a_{||}(x_1 - x_2) = (0\text{ m/s})^2 + 2(4.9\text{ m/s}^2)(10\text{ m} - 0\text{ m}) \Rightarrow v_1 = 9.9\text{ m/s}$$

REVIEW: Santa's speed of 9.9 m/s (or about 20 mph) as he reaches the edge is reasonable.

2.29. **MODEL:** We will model the bicycle as a particle, ignoring its length and height. We will ignore rolling friction. We know an object free to slide on an incline experiences an acceleration of $a_x = -g\sin(\theta)$, where x is assumed to point up the incline.

VISUALIZE: Note that the angle between the incline and the ground is $\theta = \sin^{-1}(1/5) = 11.5°$, and the length is given as 5.0 m.

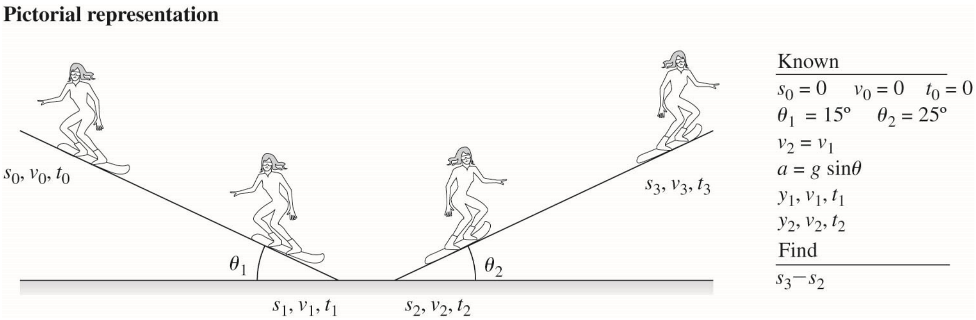
SOLVE: We can use $v_x^2 = v_{0x}^2 + 2a_x\Delta x$, inserting the acceleration from above to obtain

$$v_x = \sqrt{v_{0x}^2 + 2a_x\Delta x} = \sqrt{(8.0\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)(\sin(\theta))(5.0\text{ m})} = 6.7\text{ m/s}$$

REVIEW: We expect the bicycle to slow down, so it is sensible that the answer we obtained is less than the original 8.0 m/s.

2.30. MODEL: The snowboarder is a particle moving under constant-acceleration kinematic equations. The speed does not diminish on the horizontal section.

VISUALIZE: Note that our *s*-axis is positioned along both inclines.



SOLVE: **(a)** Using the following kinematic equation for the downward slope

$$v_1^2 = v_0^2 + 2a_0(s_1 - s_0) = (0\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(\sin 15^\circ)(50\text{ m} - 0\text{ m}) \Rightarrow v_1 = 15.93\text{ m/s}$$

which is reported as 16 m/s to two significant figures.

(b) Use the same equation again on the upward slope (with $v_2 = v_1$).

$$v_3^2 = v_2^2 + 2a_2(s_3 - s_2) = (0\text{ m/s})^2 \Rightarrow$$

$$(s_3 - s_2) = \frac{-v_1^2}{2a_2} = \frac{-(15.93\text{ m/s})^2}{2(-9.8\text{ m/s}^2)(\sin 25^\circ)} = 31\text{ m}$$

REVIEW: Because the upward slope is steeper we did not expect the snowboarder to travel as far up the slope.

Section 2.7 Instantaneous Acceleration

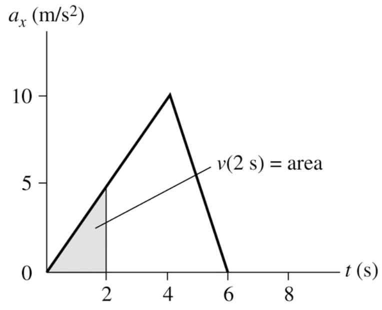
2.31. MODEL: We are treating the object like a particle.

VISUALIZE: The acceleration is the rate of change of the velocity. We can determine the total change in the velocity from $\Delta v_x = \int_{t_i}^{t_f} a_x dt$. This corresponds to the area under the curve of Figure EX2.31.

SOLVE: The area under the curve is $\Delta v_x = \int_{t_i}^{t_f} a_x dt = (4\text{ s})(2\text{ m/s}^2) + \frac{1}{2}(4\text{ s})(2\text{ m/s}^2) = 12\text{ m/s}$. Since the initial velocity was $v_{0x} = 8.0\text{ m/s}$, we have $v_{fx} = v_{0x} + \Delta v_x = (8.0\text{ m/s}) + (12\text{ m/s}) = 20\text{ m/s}$. The velocity is 20 m/s in the x direction.

REVIEW: The initial direction of motion is the positive x direction, and the acceleration is always in the positive x direction. It is therefore reasonable to find a higher speed finally than initially.

2.32. VISUALIZE:



SOLVE: We will determine the object's velocity using graphical methods first and then using calculus. Graphically, $v(t) = v_0 +$ area under the acceleration curve from 0 to t. In this case, $v_0 = 0$ m/s. The area at the time requested is a triangle.

$$t = 6 \text{ s} \qquad v(t = 6 \text{ s}) = \frac{1}{2}(6 \text{ s})(10 \text{ m/s}) = 30 \text{ m/s}$$

Let us now use calculus. The acceleration function $a(t)$ consists of three pieces and can be written:

$$a(t) = \begin{cases} 2.5t & 0 \text{ s} \le t \le 4 \text{ s} \\ -5t + 30 & 4 \text{ s} \le t \le 6 \text{ s} \\ 0 & 6 \text{ s} \le t \le 8 \text{ s} \end{cases}$$

These were determined by the slope and the y-intercept of each of the segments of the graph. The velocity function is found by integration as follows: For $0 \le t \le 4$ s,

$$v(t) = v(t = 0 \text{ s}) + \int_0^t a(t)dt = 0 + 2.5\frac{t^2}{2}\bigg|_0^t = 1.25t^2$$

This gives

$$t = 4 \text{ s} \quad v(t = 4 \text{ s}) = 20 \text{ m/s}$$

For $4 \text{ s} \le t \le 6$ s,

$$v(t) = v(t = 4 \text{ s}) + \int_4^t a(t)dt = 20 \text{ m/s } + \left[\frac{-5t^2}{2} + 30t\right]_4^t = -2.5t^2 + 30t - 60$$

This gives:

$$t = 6 \text{ s} \quad v(t = 6 \text{ s}) = 30 \text{ m/s}$$

REVIEW: The same velocities are found using calculus and graphs, but the graphical method is easier for simple graphs.

2.33. SOLVE: $x = (2t^3 + 2t + 1)$ m

(a) The position $t = 2$ s is $x_{2\text{s}} = [2(2)^3 + 2(2) + 1]$ m $= 21$ m

(b) The velocity is the derivative $v = dx/dt$ and the velocity at $t = 2$ s is calculated as follows:

$$v = (6t^2 + 2) \text{ m/s} \Rightarrow v_{2\text{s}} = [6(2^2) + 2] \text{ m/s} = 26 \text{ m/s}$$

(c) The acceleration is the derivative $a = dv/dt$ and the acceleration at $t = 2$ s is calculated as follows:

$$a = (12t)\ \text{m/s}^2 \Rightarrow a_{2\,\text{s}} = 24\ \text{m/s}^2$$

2.34. SOLVE: The formula for the particle's position along the *x*-axis is given by

$$x_\text{f} = x_\text{i} + \int_{t_\text{i}}^{t_\text{f}} v_x\, dt$$

Using the expression for v_x we get

$$x_\text{f} = x_\text{i} + \tfrac{2}{3}[t_\text{f}^3 - t_\text{i}^3] \qquad a_x = \frac{dv_x}{dt} = \frac{d}{dt}(2t^2\ \text{m/s}) = 4t\ \text{m/s}^2$$

(a) The particle's position at $t = 1$ s is $x_{1\,\text{s}} = 1\ \text{m} + \tfrac{2}{3}\ \text{m} = \tfrac{5}{3}$ m.

(b) The particle's velocity at $t = 1$ s is $v_{1\,\text{s}} = 2$ m/s.

(c) The particle's acceleration at $t = 1$ s is $a_{1\,\text{s}} = 4\ \text{m/s}^2$.

2.35. MODEL: We are modeling the object like a particle moving in the vertical direction only.

VISUALIZE: The given equation is that of a parabola. We expect it to have a turning point. We can find the turning point by realizing that the vertical speed is momentarily zero at that point, such that $dy/dt = 0$.

SOLVE: (a) Differentiating, we find

$$v_y = \frac{dy}{dt} = \frac{d}{dt}\left(t^2 - 4t + 2\right) = 2t - 4$$

Requiring that this be zero yields $t = 2$ s.

(b) Using the time from (a), we find the position is

$$y = \left(t^2 - 4t + 2\right)\ \text{m} = \left((2)^2 - 4(2) + 2\right)\text{m} = -2\ \text{m}$$

REVIEW: One could check that this is indeed the minimum in the position by inserting slightly shorter and longer times. For example, $y(1\ \text{s}) = -1$ m and $y(3\ \text{s}) = -1$ m, both above the position found in (b), as we expected.

2.36. SOLVE: First take the derivatives. Every quantity is in SI units.

$$x = 2t^3 - 6t^2 + 12$$
$$v = 6t^2 - 12t$$
$$a = 12t - 12$$

(a) To find the minimum *v* we take the derivative of the velocity function and set it equal to zero (this is how we always minimize things). But we have already taken the derivative of *v*: $\dfrac{dv}{dt} = 12t - 12 = 0 \Rightarrow t = 1$ s. This is the first answer. Now we plug that time back into the equation for *v*: $v_\text{min} = 6(1)^2 - 12(1) = -6$ m/s.

(b) We have already done this when we minimized *v*. The acceleration is zero at $0 = 12t - 12 \Rightarrow t = 1$ s.

PROBLEMS

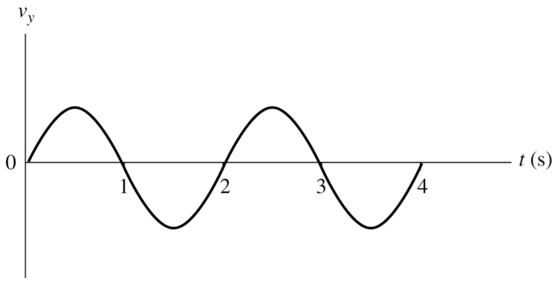
2.37. **SOLVE:** The graph for particle A is a straight line from $t = 2$ s to $t = 8$ s. The slope of this line is -10 m/s, which is the velocity at $t = 7.0$ s. The negative sign indicates motion toward lower values on the *x*-axis. The velocity of particle B at $t = 7.0$ s can be read directly from its graph. It is -20 m/s. The velocity of particle C can be obtained from the equation

$v_f = v_i +$ area under the acceleration curve between t_i and t_f

This area can be calculated by adding up three sections. The area between $t = 0$ s and $t = 2$ s is 40 m/s, the area between $t = 2$ s and $t = 5$ s is 45 m/s, and the area between $t = 5$ s and $t = 7$ s is -20 m/s. We get $(10 \text{ m/s}) + \ (60 \text{ m/s}) + (45 \text{ m/s}) - (20 \text{ m/s}) = 95 \text{ m/s}$.

2.38. **SOLVE:** **(a)** The velocity-versus-time graph is the derivative with respect to time of the distance-versus-time graph. The velocity is zero when the slope of the position-versus-time graph is zero, the velocity is most positive when the slope is most positive, and the velocity is most negative when the slope is most negative. The slope is zero at *t* = 0 s, 1 s, 2 s, 3 s, . . . ; the slope is most positive at *t* = 0.5 s, 2.5 s, . . . ; and the slope is most negative at *t* = 1.5 s, 3.5 s, . . .

(b)



2.39. **SOLVE:** The given function for the velocity is $v_x = \left(t^2 - 7t + 10\right)$ m/s.

(a) The turning points are when the velocity changes sign. Set $v_x = 0$ and check that it actually changes sign at those times. The function factors into the product of two binomials:

$$v_x = (t-2)(t-5) \text{ m/s} \Rightarrow v_x = 0 \text{ m/s when } t = 2 \text{ s and } t = 5 \text{ s}$$

Indeed, the function changes sign at those two times.

(b) The acceleration is given by the derivative of the velocity.

$$a_x = \frac{dv_x}{dt} = \left(2t - 7\right) \text{ m/s}^2$$

Plug in the times from part **(a)**: $a_x(2 \text{ s}) = \left(2(2) - 7\right) \text{ m/s}^2 = -3 \text{ m/s}^2$ and $a_x(5 \text{ s}) = 2(5) - 7 = 3 \text{ m/s}^2$

REVIEW: This problem does not have constant acceleration so the kinematic equations do not apply, but $a = dv/dt$ always applies.

2.40. SOLVE: The position is the integral of the velocity.

$$x_1 = x_0 + \int_{t_0}^{t_1} v_x dt = x_0 + \int_0^{t_1} kt^2 dt = x_0 + \tfrac{1}{3}kt^3\Big|_0^{t_1} = x_0 + \tfrac{1}{3}kt_1^3$$

We're given that $x_0 = -9.0$ m and that the particle is at $x_1 = 9.0$ m at $t_1 = 3.0$ s. Thus

$$9.0 \text{ m} = (-9.0 \text{ m}) + \tfrac{1}{3}k(3.0 \text{ s})^3 = (-9.0 \text{ m}) + k(9.0 \text{ s}^3)$$

Solving for k gives $k = 2.0 \text{ m/s}^3$.

2.41. SOLVE: (a) The velocity is the integral of the acceleration.

$$v_{1x} = v_{0x} + \int_{t_0}^{t_1} a_x dt = 0 \text{ m/s} + \int_0^{t_1} (10 - t)dt = (10t - \tfrac{1}{2}t^2)\Big|_0^{t_1} = 10t_1 - \tfrac{1}{2}t_1^2$$

The velocity is zero when

$$v_{1x} = 0 \text{ m/s} = \left(10t_1 - \tfrac{1}{2}t_1^2\right) = (10 - \tfrac{1}{2}t_1) \times t_1$$
$$\Rightarrow t_1 = 0 \text{ s} \quad \text{or} \quad t_1 = 20 \text{ s}$$

The first solution is the initial condition. Thus the particle's velocity is again 0 m/s at $t_1 = 20$ s.

(b) Position is the integral of the velocity. At $t_1 = 20$ s, and using $x_0 = 0$ m at $t_0 = 0$ s, the position is

$$x_1 = x_0 + \int_{t_0}^{t_1} v_x dt = 0 \text{ m} + \int_0^{20} \left(10t - \tfrac{1}{2}t^2\right) dt = 5t^2\Big|_0^{20} - \tfrac{1}{6}t^3\Big|_0^{20} = 667 \text{ m}$$

2.42. SOLVE: (a) The turning point is when $v = 0$, so set the equation for v equal to zero.

The velocity is zero when

$$v = (20 \text{ m/s})\sin(\pi t) = 0 \Rightarrow t = 0 \text{ s, } 1 \text{ s, } 2 \text{ s, } \ldots$$

The first time after $t = 0$ s is $t = 1$ s.

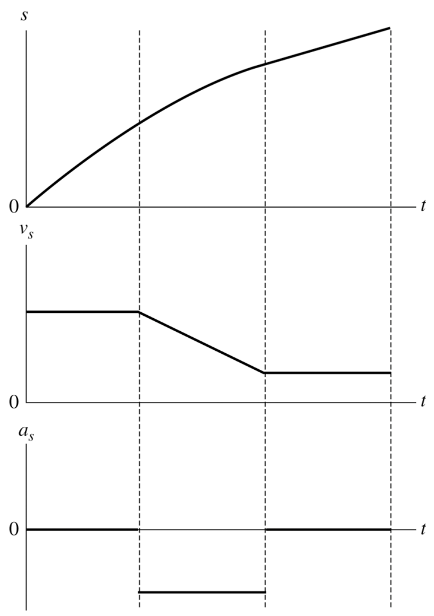
(b) The acceleration at t = 1 s is found by taking the derivative of the velocity equation and inserting t = 1 s.

$$a = \frac{dv}{dt} = (2.0 \text{ m/s})\pi \cdot \cos(\pi t)$$
$$a(t = 1 \text{ s}) = (2.0 \text{ m/s})\pi \cdot \cos(\pi) = -2.0\pi \text{ m/s}^2$$

2.43. MODEL: Represent the ball as a particle.

VISUALIZE: Please refer to the figure in the problem.

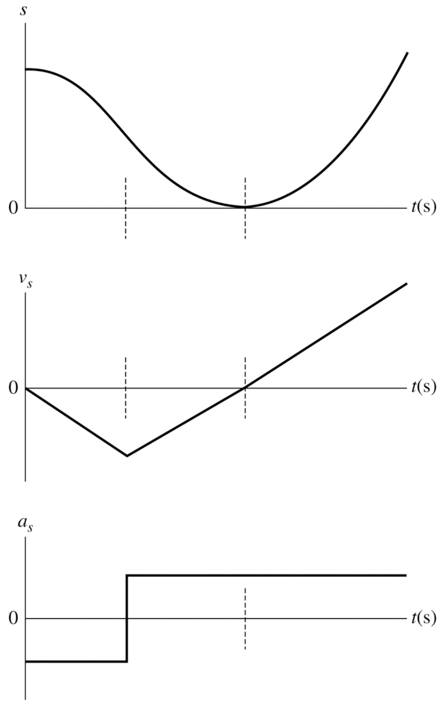
SOLVE: In the first and third segments the acceleration a_s is zero. In the second segment the acceleration is negative and constant. This means the velocity v_s will be constant in the first two segments and will decrease linearly in the third segment. Because the velocity is constant in the first and third segments, the position s will increase linearly. In the second segment, the position will increase parabolically rather than linearly because the velocity decreases linearly with time.



2.44. **Model:** Represent the ball as a particle.

Visualize: Please refer to Figure P2.44. The ball rolls down the first short track, then up the second short track, and then down the long track. s is the distance along the track measured from the left end (where $s = 0$). Label $t = 0$ at the beginning, that is, when the ball starts to roll down the first short track.

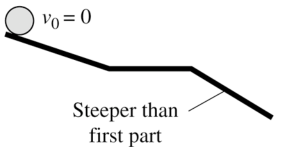
Solve: Because the incline angle is the same, the magnitude of the acceleration is the same on all of the tracks.



REVIEW: Note that the derivative of the *s*-versus-*t* graph yields the v_s-versus-*t* graph. And the derivative of the v_s-versus-*t* graph gives rise to the a_s-versus-*t* graph.

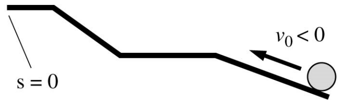
2.45. VISUALIZE: Please refer to Figure P2.45.

SOLVE:



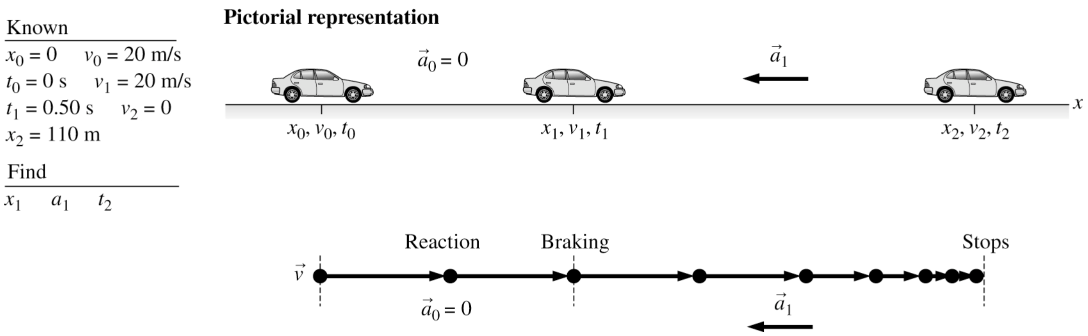
2.46. VISUALIZE: Please refer to Figure P2.46.

SOLVE:



2.47. MODEL: The car is a particle and constant-acceleration kinematic equations hold.

VISUALIZE:



SOLVE: This is a two-part problem. During the reaction time,

$$x_1 = x_0 + v_0(t_1 - t_0) + 1/2a_0(t_1 - t_0)^2$$
$$= 0\text{ m} + (20\text{ m/s})(0.50\text{ s} - 0\text{ s}) + 0\text{ m} = 10\text{ m}$$

After reacting, $x_2 - x_1 = 110\text{ m} - 10\text{ m} = 100\text{ m}$, that is, you are 100 m away from the intersection.

To stop successfully,

$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) \Rightarrow (0\text{ m/s})^2 = (20\text{ m/s})^2 + 2a_1(100\text{ m}) \Rightarrow a_1 = -2\text{ m/s}^2$$

2.48. MODEL: The plane is a particle and the constant-acceleration kinematic equations hold.

SOLVE: (a) Using $a_s = \Delta v/\Delta t$, we have,

$$a_s(t = 0\text{ s to }t = 10\text{ s}) = \frac{23\text{ m/s} - 0\text{ m/s}}{10\text{ s} - 0\text{ s}} = 2.3\text{ m/s}^2 \qquad a_s(t = 20\text{ s to }t = 30\text{ s}) = \frac{69\text{ m/s} - 46\text{ m/s}}{30\text{ s} - 20\text{ s}} = 2.3\text{ m/s}^2$$

Since the change in speed is constant over every one-second time interval, *a* is a constant $\left(2.3\text{ m/s}^2\right)$.

(b) Using kinematics as follows:

$$v_{\text{fs}} = v_{\text{is}} + a(t_{\text{f}} - t_{\text{i}}) \Rightarrow 80 \text{ m/s} = 0 \text{ m/s} + (2.3 \text{ m/s}^2)(t_{\text{f}} - 0 \text{ s}) \Rightarrow t_{\text{f}} = 35 \text{ s}$$

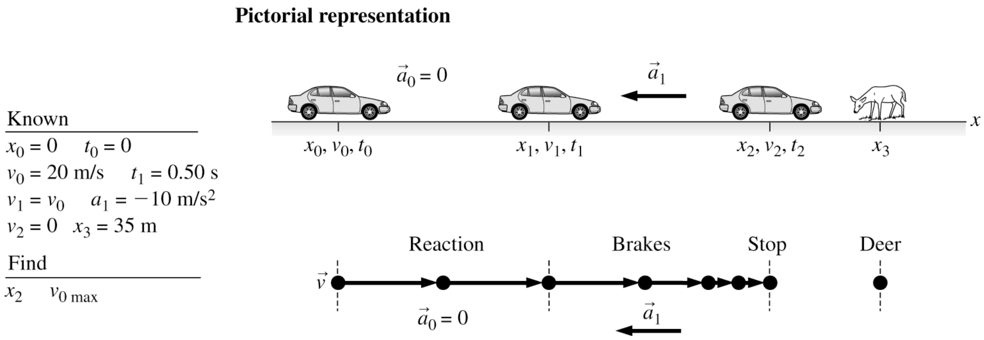
(c) Using the above values, we calculate the take off distance as follows:

$$s_{\text{f}} = s_{\text{i}} + v_{\text{is}}(t_{\text{f}} - t_{\text{i}}) + \frac{1}{2}a_{\text{s}}(t_{\text{f}} - t_{\text{i}})^2 = 0 \text{ m} + (0 \text{ m/s})(35 \text{ s}) + \frac{1}{2}(2.3 \text{ m/s}^2)(35 \text{ s})^2 = 1410 \text{ m}$$

For safety, the runway should be $3 \times 1410 \text{ m} = 4230 \text{ m}$ or 2.6 mi. This is longer than the 2.5-mi-long runway, so the takeoff is not safe.

2.49. MODEL: We will use the particle model and the constant-acceleration kinematic equations.

VISUALIZE:



SOLVE: (a) To find x_2, we first need to determine x_1. Using $x_1 = x_0 + v_0(t_1 - t_0)$, we get $x_1 = 0 \text{ m} + (20 \text{ m/s}) \; (0.50 \text{ s} - 0 \text{ s}) = 10 \text{ m}$. Now,

$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) \Rightarrow 0 \text{ m}^2/\text{s}^2 = (20 \text{ m/s})^2 + 2(-10 \text{ m/s}^2)(x_2 - 10 \text{ m}) \Rightarrow x_2 = 30 \text{ m}$$

The distance between you and the deer is $(x_3 - x_2)$ or $(35 \text{ m} - 30 \text{ m}) = 5 \text{ m}$.

(b) Let us find $v_{0\text{ max}}$ such that $v_2 = 0$ m/s at $x_2 = x_3 = 35$ m. Using the following equation,

$$v_2^2 - v_{0\text{ max}}^2 = 2a_1(x_2 - x_1) \Rightarrow 0 \text{ m}^2/\text{s}^2 - v_{0\text{ max}}^2 = 2(-10 \text{ m/s}^2)(35 \text{ m} - x_1)$$

Also, $x_1 = x_0 + v_{0\text{ max}}(t_1 - t_0) = v_{0\text{ max}}(0.50 \text{ s} - 0 \text{ s}) = (0.50 \text{ s})v_{0\text{ max}}$. Substituting this expression for x_1 in the above equation yields

$$-v_{0\text{ max}}^2 = (-20 \text{ m/s}^2)[35 \text{ m} - (0.50 \text{ s})\, v_{0\text{ max}}] \Rightarrow v_{0\text{ max}}^2 + (10 \text{ m/s})v_{0\text{ max}} - 700 \text{ m}^2/\text{s}^2 = 0$$

The solution of this quadratic equation yields $v_{0\text{ max}} = 22$ m/s. (The other root is negative and unphysical for the present situation.)

REVIEW: An increase of speed from 20 m/s to 22 m/s is very reasonable for the car to cover an additional distance of 5 m with a reaction time of 0.50 s and a deceleration of 10 m/s^2.

2.50. **MODEL:** We will model the pod as a particle. There is a period of speeding up, a period of constant speed, and a period of slowing down.

VISUALIZE: We can use kinematic equations for each segment over which acceleration is constant. Specifically, we will use $v_{fx} = v_{0x} + a_x\Delta t$, and $\Delta x = v_{0x}\Delta t + \frac{1}{2}a_x\Delta t^2$. Call the initial time $t_0 = 0$ s, the instant the constant speed begins t_1, the instant the constant speed ends t_2, and the time of arrival at the destination t_3.

SOLVE: (a) The time required for the initial acceleration is

$$t_1 = \frac{v_{1x} - v_{0x}}{a_x} = \frac{(330 \text{ m/s}) - (0 \text{ m/s})}{(2.5 \text{ m/s}^2)} = 132 \text{ s}\cdot$$

And during this interval, the pod would move a distance

$$x_1 = v_{0x}\Delta t + \frac{1}{2}a_x\Delta t^2 = (0 \text{ m/s})(132 \text{ s}) + \frac{1}{2}(2.5 \text{ m/s}^2)(132 \text{ s})^2 = 21,780 \text{ m}$$

During the braking period, the time required to stop is

$$t_3 - t_2 = \frac{v_{3x} - v_{2x}}{a_x} = \frac{(0 \text{ m/s}) - (330 \text{ m/s})}{(-1.5 \text{ m/s}^2)} = 220 \text{ s}\cdot$$

During this interval, the pod would cover a distance

$$x_3 - x_2 = v_{2x}\Delta t + \frac{1}{2}a_x\Delta t^2 = (330 \text{ m/s})(220 \text{ s}) + \frac{1}{2}(-1.5 \text{ m/s}^2)(220 \text{ s})^2 = 36,300 \text{ m}$$

The remaining distance to be covered is

$$\Delta x_{12} = x_{\text{Total}} - \Delta x_{01} - \Delta x_{23} = 540,000 \text{ m} - 21,780 \text{ m} - 36,300 \text{ m}$$
$$= 481,920 \text{ m}$$

Thus, this constant speed must be maintained for a time

$$\Delta t_{12} = \frac{\Delta x_{12}}{v} = \frac{(481,920 \text{ m})}{(330 \text{ m/s})} = 1460.4 \text{ s}$$

Adding together the times from the period of speeding up, slowing down, and constant speed, we find a total time

$$\Delta t_{\text{total}} = \Delta t_{01} + \Delta t_{12} + \Delta t_{23} = (132 \text{ s}) + (220 \text{ s}) + (1460.4 \text{ s}) = 1812.4 \text{ s} = 30 \text{ min}$$

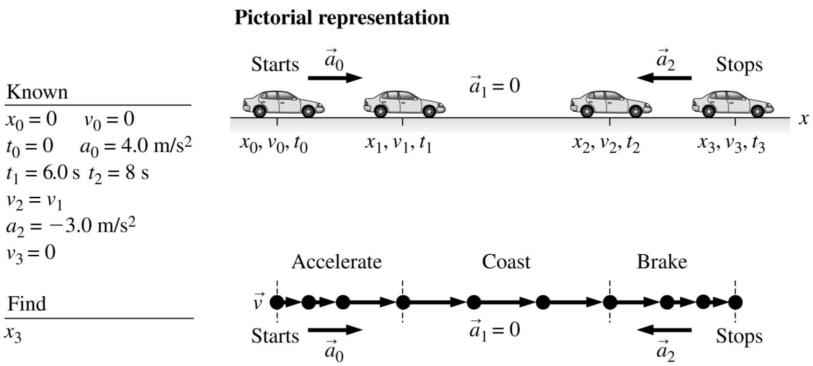
(b) When an initial car reaches the constant speed, it will be moving at 330 m/s. We know it takes the next pod 132 s to reach the constant speed, and it also waits 60 s before launching. So the next pod will reach the constant-speed segment 192 s after the first. In that 192 s, the first pod will move

$$\Delta x = v\Delta t = (330 \text{ m/s})(192 \text{ s}) = 63,360 \text{ m} \approx 63 \text{ km}$$

REVIEW: We obtained a very reasonable travel time of 30 minutes, which makes this plan fairly plausible. Also, the 63 km between pods would seem to give sufficient stopping distance in the event of an emergency.

2.51. **MODEL:** The car is a particle moving under constant-acceleration kinematic equations.

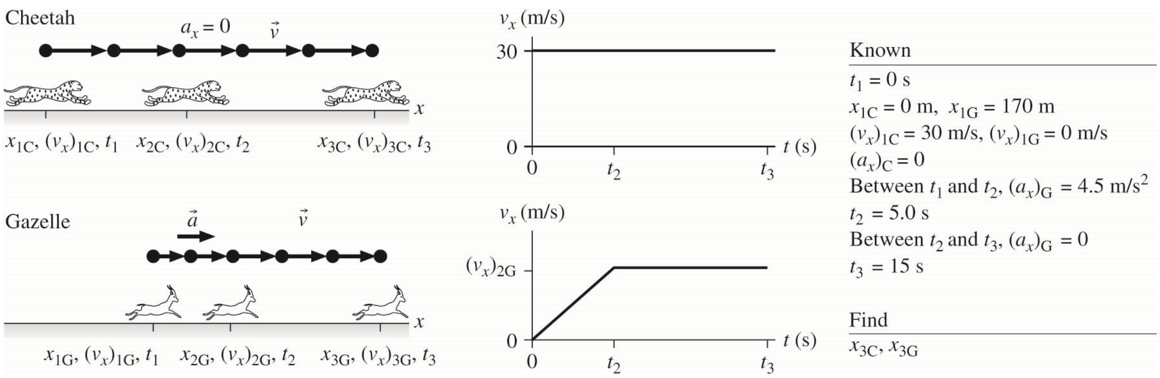
VISUALIZE:



SOLVE: This is a three-part problem. First the car accelerates, then it moves with a constant speed, and then it decelerates.

2.52. **MODEL:** Model each of the animals as a particle and use kinematic equations. Assume that the time it takes the cheetah to accelerate to $30\ \text{m/s}$ is negligible.

VISUALIZE:



SOLVE: The cheetah is in uniform motion for the entire duration of the problem, so we can easily solve for its position at $t_3 = 25\ \text{s}$:

$$x_{3C} = x_{1C} + (v_x)_{1C}\,\Delta t = 0\ \text{m} + (30\ \text{m/s})(15\ \text{s}) = 450\ \text{m}$$

The gazelle's motion has two phases: one of constant acceleration and then one of constant velocity. We can solve for the position and the velocity at t_2, the end of the first phase.

$$(v_x)_{2G} = (v_x)_{1G} + (a_x)_G\,\Delta t = 0\ \text{m/s} + (4.6\ \text{m/s}^2)(5.0\ \text{s}) = 23\ \text{m/s}$$

$$x_{2G} = x_{1G} + (v_x)_{1G}\,\Delta t + \frac{1}{2}(a_x)_G(\Delta t)^2 = 170\ \text{m} + 0\ \text{m} + \frac{1}{2}(4.6\ \text{m/s}^2)(5.0\ \text{s})^2 = 227.5\ \text{m}$$

From t_2 to t_3 the gazelle moves at a constant speed, so we can use the equation for uniform motion to find its final position.

$$x_{3G} = x_{2G} + (v_x)_{2G}\,\Delta t = 227.5\ \text{m} + (23\ \text{m/s})(10.0\ \text{s}) = 457.5\ \text{m} \approx 460\ \text{m}$$

x_{3C} is 450 m; x_{3G} is 460 m. The gazelle is just 10 m ahead of the cheetah when the cheetah has to break off the chase, so the gazelle escapes.

REVIEW: The numbers in the problem statement are realistic, so we expect our results to mirror real life. The speed for the gazelle is close to that of the cheetah, which seems reasonable for two animals known for their speed. And the result is the most common occurrence—the chase is very close, but the gazelle gets away.

2.53. **MODEL:** We will treat the car like a particle with a single, well-defined position.

VISUALIZE: We can use kinematic equations wherever acceleration will be constant. In particular, in part (a) $v_{fx}^2 = v_{0x}^2 + 2a_x\Delta x$ will be useful. But we must note that this is valid only over the period with constant acceleration, such as while braking. Note also that the car will be slowing, which means $\vec{a}_{\text{brake}}$ must be directed opposite the initial direction of motion, meaning $a_x = -|a_{\text{brake}}|$. The absolute value is not necessary, since a_{brake} means the size of $\vec{a}_{\text{brake}}$. We have added it only for clarity. Before the braking begins, the car will move at a constant speed during the reaction time. In part (b), we will apply our result from part (a), inserting numbers.

SOLVE: **(a)** The total distance will be the distance during the reaction and the distance travelled during braking: $d = d_{\text{react}} + d_{\text{brake}}$. We see that $d_{\text{react}} = v_0\Delta t_{\text{react}}$. For the distance traveled during the braking, we know $v_{fx} = 0$. Thus, we have

$$2a_{\text{brake},x}d_{\text{brake}} = -v_0^2 \Rightarrow d_{\text{brake}} = \frac{-v_0^2}{2a_{\text{brake},x}} = \frac{v_0^2}{2|a_{\text{brake}}|}$$

Thus the total distance required to stop is

$$d = d_{\text{react}} + d_{\text{brake}} = d_{\text{react}} = v_0\Delta t_{\text{react}} + \frac{v_0^2}{2|a_{\text{brake}}|} = v_0\Delta t_{\text{react}} + \frac{v_0^2}{2a_{\text{brake}}}$$

As described above, either of the last two expressions is acceptable, since a_{brake} means the size of $\vec{a}_{\text{brake}}$, and the absolute value can be inferred.

(b) The first set of numbers can be used to determine the acceleration, which can then be used in the same expression with the second speed:

$$d = v_0\Delta t_{\text{react}} + \frac{v_0^2}{2a_{\text{brake}}} \Rightarrow a_{\text{brake}} = \frac{v_0^2}{2(d - v_0\Delta t_{\text{react}})}$$

$$a_{\text{brake}} = \frac{(30\text{ m/s})^2}{2((60\text{ m}) - (30\text{ m/s})(0.50\text{ s}))} = 10\text{ m/s}^2$$

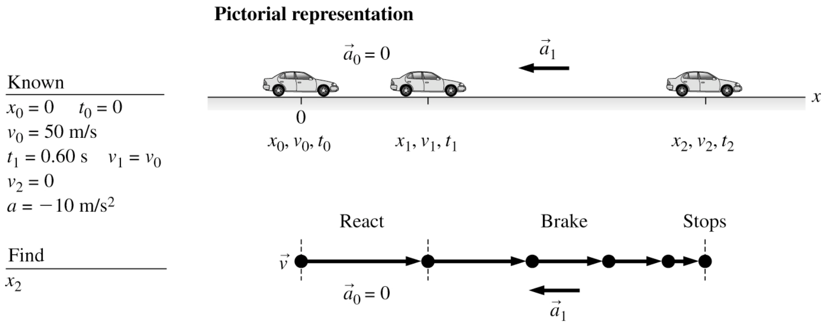
Using this value with the second speed, we find

$$d = v_0\Delta t_{\text{react}} + \frac{v_0^2}{2a_{\text{brake}}} = (40\text{ m/s})(0.50\text{ s}) + \frac{(40\text{ m/s})^2}{2(10\text{ m/s}^2)} = 100\text{ m}$$

REVIEW: We found an extremely high acceleration, almost the same as that due to gravity. This would require excellent tires and a clean, dry road, but might be possible. Note that increasing the speed by 33% increased the stopping distance by 67%.

2.54. MODEL: We will use the particle model and the kinematic equations at constant-acceleration.

VISUALIZE:



SOLVE: To find x_2, let us use the kinematic equation

$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) = (0 \text{ m/s})^2 = (50 \text{ m/s})^2 + 2(-10 \text{ m/s}^2)(x_2 - x_1) \Rightarrow x_2 = x_1 + 125 \text{ m}$$

Since the nail strip is at a distance of 150 m from the origin, we need to determine x_1:

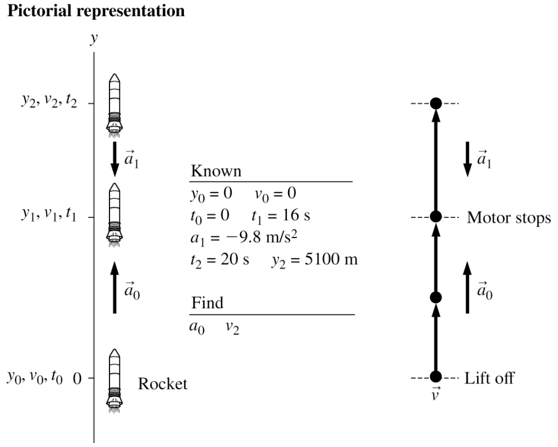
$$x_1 = x_0 + v_0(t_1 - t_0) = 0 \text{ m} + (50 \text{ m/s})(0.60 \text{ s} - 0.0 \text{ s}) = 30 \text{ m}$$

Therefore, we can see that $x_2 = (30 + 125) \text{ m} = 155 \text{ m}$. That is, he can't stop within a distance of 150 m so he stops after the nail strip by 5.0 m. He is in jail.

REVIEW: Bob is driving at approximately 100 mph and the stopping distance is of the correct order of magnitude.

2.55. MODEL: We will model the rocket as a particle. Air resistance will be neglected.

VISUALIZE:



SOLVE: Using the constant-acceleration kinematic equations,

$$v_1 = v_0 + a_0(t_1 - t_0) = 0 \text{ m/s} + a_0(16 \text{ s} - 0 \text{ s}) = a_o(16 \text{ s})$$

$$y_1 = y_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = \frac{1}{2}a_0(16 \text{ s} - 0 \text{ s})^2 = a_o(128 \text{ s}^2)$$

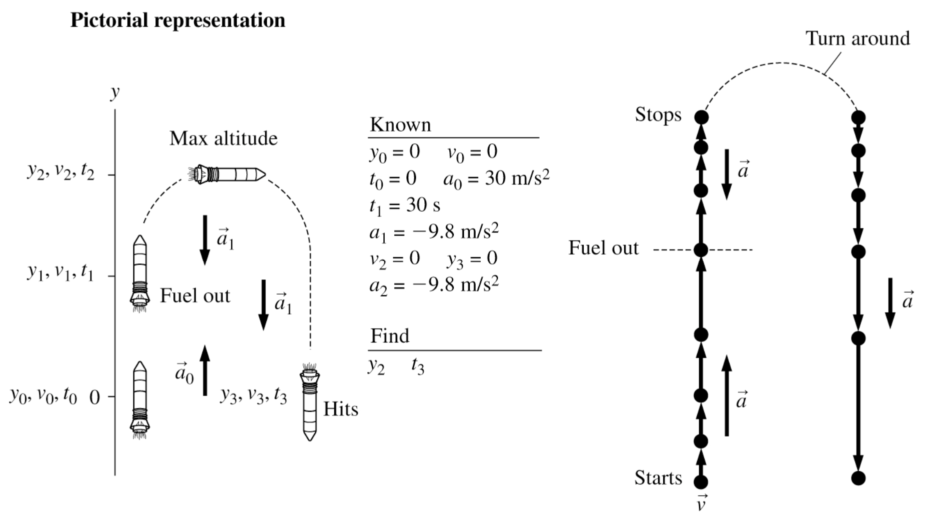
$$y_2 = y_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$$

$$\Rightarrow 5100 \text{ m} = 128 \text{ s}^2\ a_0 + 16 \text{ s}\ a_0(20 \text{ s} - 16 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(20 \text{ s} - 16 \text{ s})^2 \Rightarrow a_0 = 27 \text{ m/s}^2$$

REVIEW: This acceleration would produce a final speed (after 16 s) of 400 m/s ≈ 900 mph, which would be reasonable.

2.56. MODEL: The rocket is represented as a particle.

VISUALIZE:



SOLVE: **(a)** There are three parts to the motion. Both the second and third parts of the motion are free fall, with $a = -g$. The maximum altitude is y_2. In the acceleration phase:

$$y_1 = y_0 + v_0(t_1 - t_0) + \frac{1}{2}a(t_1 - t_0)^2 = \frac{1}{2}at_1^2 = \frac{1}{2}(30 \text{ m/s}^2)(30 \text{ s})^2 = 13{,}500 \text{ m}$$

$$v_1 = v_0 + a(t_1 - t_0) = at_1 = (30 \text{ m/s}^2)(30 \text{ s}) = 900 \text{ m/s}$$

In the coasting phase,

$$v_2^2 = 0 = v_1^2 - 2g(y_2 - y_1) \Rightarrow y_2 = y_1 + \frac{v_1^2}{2g} = 13{,}500 \text{ m} + \frac{(900 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 54{,}800 \text{ m} = 54.8 \text{ km}$$

The maximum altitude is 54.8 km (≈ 33 mi).

(b) The rocket is in the air until time t_3. We already know $t_1 = 30$ s. We can find t_2 as follows:

$$v_2 = 0 \text{ m/s} = v_1 - g(t_2 - t_1) \Rightarrow t_2 = t_1 + \frac{v_1}{g} = 122 \text{ s}$$

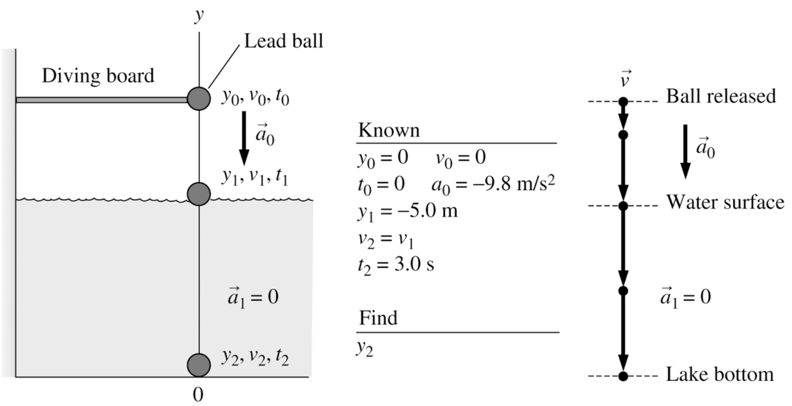
Then t_3 is found by considering the time needed to fall 54,800 m:

$$y_3 = 0 \text{ m} = y_2 + v_2(t_3 - t_2) - \frac{1}{2}g(t_3 - t_2)^2 = y_2 - \frac{1}{2}g(t_3 - t_2)^2 \Rightarrow t_3 = t_2 + \sqrt{\frac{2y_2}{g}} = 228 \text{ s}$$

REVIEW: In reality, friction due to air resistance would prevent the rocket from reaching such high speeds as it falls, and the acceleration upward would not be constant because the mass changes as the fuel is burned, but that is a more complicated problem.

2.57. MODEL: We will model the lead ball as a particle and use the constant-acceleration kinematic equations.

VISUALIZE:



Note that the particle undergoes free fall until it hits the water surface.

SOLVE: The kinematic equation $y_1 = y_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2$ becomes

$$-5.0 \text{ m} = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}(-9.8 \text{ m/s}^2)(t_1 - 0)^2 \Rightarrow t_1 = 1.01 \text{ s}$$

Now, once again,

$$y_2 = y_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$$
$$\Rightarrow y_2 - y_1 = v_1(3.0 \text{ s} - 1.01 \text{ s}) + 0 \text{ m/s} = 1.99\, v_1$$

v_1 is easy to determine since the time t_1 has been found. Using $v_1 = v_0 + a_0(t_1 - t_0)$, we get

$$v_1 = 0 \text{ m/s} - (9.8 \text{ m/s}^2)(1.01 \text{ s} - 0 \text{ s}) = -9.898 \text{ m/s}$$

With this value for v_1, we go back to:

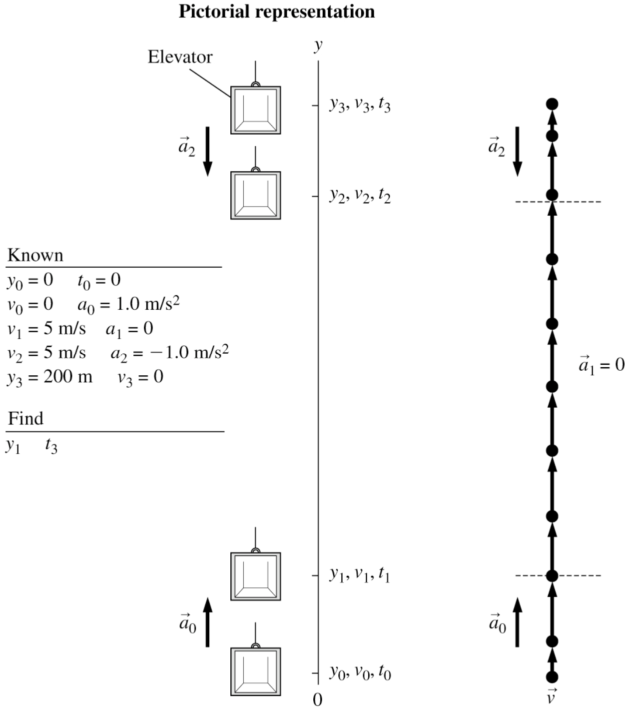
$$y_2 - y_1 = 1.99v_1 = (1.99)(-9.898 \text{ m/s}) = -19.7 \text{ m}$$

$y_2 - y_1$ is the displacement of the lead ball in the lake and thus corresponds to the depth of the lake, which is 19.7 m. The negative sign shows the direction of the displacement vector.

REVIEW: A depth of about 60 ft for a lake is not unusual.

2.58. **MODEL:** The elevator is a particle moving under constant-acceleration kinematic equations.

VISUALIZE:



SOLVE: **(a)** To calculate the distance to accelerate up:

$$(v_1)^2 = v_0^2 + 2a_0(y_0 - y_0) \Rightarrow (5 \text{ m/s})^2 = (0 \text{ m/s})^2 + 2(1 \text{ m/s}^2)(y_1 - 0 \text{ m}) \Rightarrow y_1 = 12.5 \text{ m}$$

(b) To calculate the time to accelerate up:

$$v_1 = v_0 + a_0(t_1 - t_0) \Rightarrow 5 \text{ m/s} = 0 \text{ m/s} + (1 \text{ m/s}^2)(t_1 - 0 \text{ s}) \Rightarrow t_1 = 5 \text{ s}$$

To calculate the distance to decelerate at the top:

$$v_3^2 = v_2^2 + 2a_2(y_3 - y_2) \Rightarrow (0 \text{ m/s})^2 = (5 \text{ m/s})^2 + 2(-1 \text{ m/s}^2)(y_3 - y_2) \Rightarrow y_3 - y_2 = 12.5 \text{ m}$$

To calculate the time to decelerate at the top:

$$v_3 = v_2 + a_2(t_3 - t_2) \Rightarrow 0 \text{ m/s} = 5 \text{ m/s} + (-1 \text{ m/s}^2)(t_3 - t_2) \Rightarrow t_3 - t_2 = 5 \text{ s}$$

The distance moved up at 5 m/s is

$$y_2 - y_1 = (y_3 - y_0) - (y_3 - y_2) - (y_1 - y_0) = 200 \text{ m} - 12.5 \text{ m} - 12.5 \text{ m} = 175 \text{ m}$$

The time to move up 175 m is given by

$$y_2 - y_1 = v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2 \Rightarrow 175 \text{ m} = (5 \text{ m/s})(t_2 - t_1) \Rightarrow (t_2 - t_1) = 35 \text{ s}$$

To total time to move to the top is

$$(t_1 - t_0) + (t_2 - t_1) + (t_3 - t_2) = 5 \text{ s} + 35 \text{ s} + 5 \text{ s} = 45 \text{ s}$$

REVIEW: To cover a distance of 200 m at 5 m/s (ignoring acceleration and deceleration times) will require a time of 40 s. This is comparable to the time of 45 s for the entire trip as obtained above.

2.59. **MODEL:** We will treat the cars like particles, each with a single, well-defined position.

VISUALIZE: We can use kinematic equations wherever acceleration will be constant. In particular $v_{fx}^2 = v_{0x}^2 + 2a_x\Delta x$ will be useful. We can apply this over the entire stopping interval if we ignore reaction time and assume the acceleration is constant throughout the process. We will apply this twice: once to each case, and then take the difference of the two stopping distances. Note that the x components of acceleration are negative, since the cars will be slowing down.

SOLVE: For the case of locking and skidding, we have

$$\Delta x_{\text{skid}} = \frac{v_{fx}^2 - v_{0x}^2}{2a_x} = \frac{(0\text{ m/s})^2 - (30\text{ m/s})^2}{2(-4.8\text{ m/s}^2)} = 93.75\text{ m}.$$

Here, we keep additional digits until the end of the calculation. In the case of anti-lock brakes, we have

$$\Delta x_{\text{anti-lock}} = \frac{v_{fx}^2 - v_{0x}^2}{2a_x} = \frac{(0\text{ m/s})^2 - (30\text{ m/s})^2}{2(-7.0\text{ m/s}^2)} = 64.29\text{ m}$$

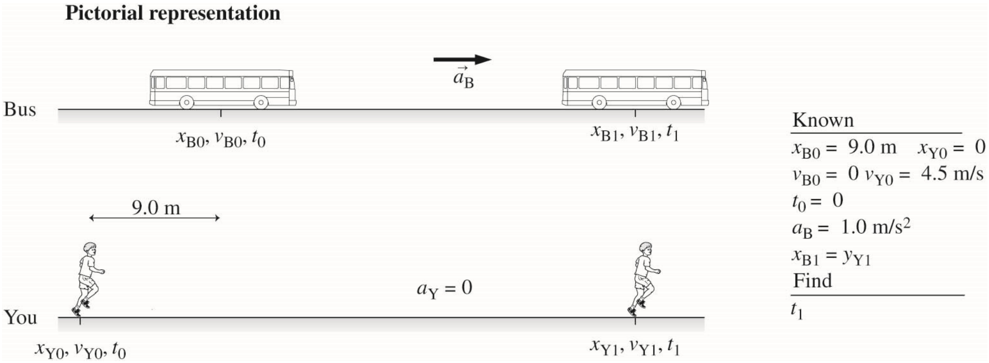
The difference in stopping distances is

$$\Delta x_{\text{skid}} - \Delta x_{\text{anti-lock}} = (93.75\text{ m}) - (64.29\text{ m}) = 29.46\text{ m} \approx 29\text{ m}$$

REVIEW: Clearly, there is a substantial safety advantage to having anti-lock brakes, since 29 m can mean the difference between avoiding a collision, and a serious accident.

2.60. **MODEL:** You and the bus are particles; the bus has a constant acceleration and your acceleration is zero.

VISUALIZE:



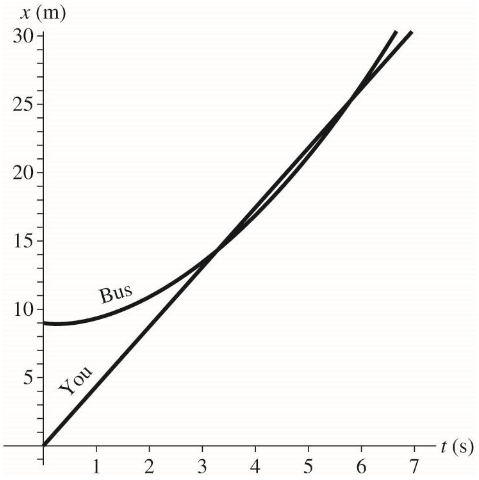
SOLVE: (a) For the bus:

$$x_{B1} = x_{B0} + \frac{1}{2}a_B t_1^2$$

For you:

$$x_{Y1} = v_{Y0}t_1$$

Equations for the bus and you are graphed on the same position-versus-time graph.



When you catch the bus $x_{B1} = x_{Y1}$. Then solve the resulting quadratic equation.

$$\frac{1}{2}a_B t_1^2 - v_{Y0}t_1 + x_{B0} = 0 \Rightarrow$$

$$t_1 = \frac{v_{Y0} \pm \sqrt{v_{Y0}^2 - 2a_B x_{B0}}}{a_B} = \frac{(4.5 \text{ m/s}) \pm \sqrt{(4.5 \text{ m/s})^2 - 2(1.0 \text{ m/s}^2)(9.0 \text{ m})}}{1.0 \text{ m/s}^2}$$

$$= (4.5 \text{ s}) \pm (1.5 \text{ s}) = 3.0 \text{ s}, 6.0 \text{ s}$$

There are two positive answers because if you don't jump on the bus when you first catch up to it at 3.0 s then you would pass it and then later it would catch up to you at 6.0 s. You actually have two chances to jump on the bus in this scenario, but the first one comes at 3.0 s.

(b) The maximum time you could wait before starting to run is best seen in the position-versus-time graph above. The parabola for the bus stays the same, but the straight line for you could move horizontally until it is just tangent to the parabola (touches at one point). The slope of the position graph for the bus is the velocity graph, and we seek the time when its slope is v_{Y0}.

$$v_B = a_B t = v_{Y0}$$

$$(4.5 \text{ m/s}) = (1.0 \text{ m/s}^2)t \Rightarrow t = 4.5 \text{ s}$$

When you catch the bus $x_B = x_Y$.

$$x_B(4.5 \text{ s}) = 9 + \frac{1}{2}(1.0 \text{ m/s}^2)(4.5 \text{ s})^2 = 19.125 \text{ m}$$

Now we find x_{Y0}.

$$x_Y = x_{Y0} + v_{Y0}t$$

$$19.125 \text{ m} = x_{Y0} - (4.5 \text{ m/s})(4.5 \text{ s}) \Rightarrow x_{Y0} = -1.125 \text{ m}$$

Now we can find the time when $x_Y = 0$.

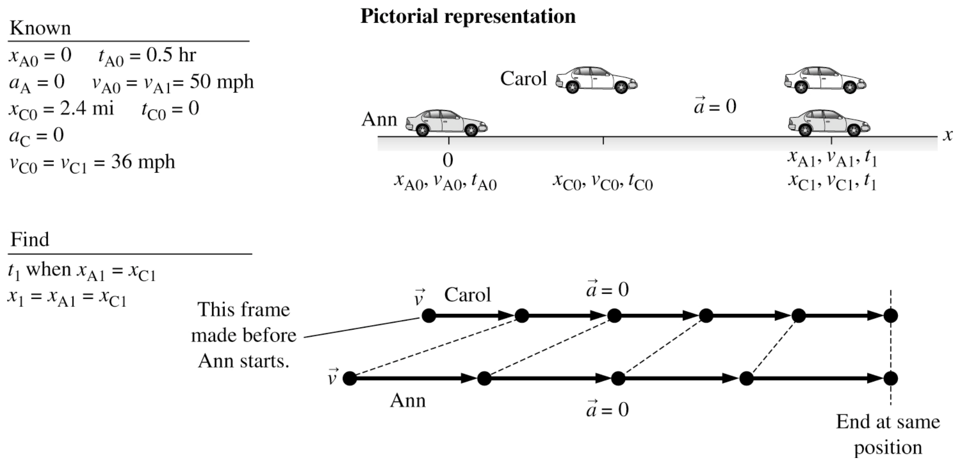
$$0 \text{ m} = -1.125 \text{ m} + (4.5 \text{ m/s})t \Rightarrow t = 0.25 \text{ s}$$

So you could wait ¼ s and still catch the bus by running at the same speed. In this scenario you don't get two chances to catch the bus.

REVIEW: The data and answers all seem reasonable. When you catch the bus it is not going the same speed you are in part **(a)**, but it is in part **(b)**.

2.61. MODEL: The cars are represented as particles.

VISUALIZE:



SOLVE: **(a)** Ann and Carol start from different locations at different times and drive at different speeds. But at time t_1 they have the *same* position. It is important in a problem such as this to express information in terms of *positions* (that is, coordinates) rather than distances. Each drives at a constant velocity, so using constant velocity kinematics gives

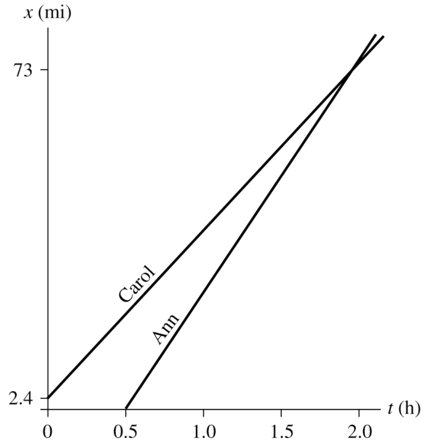
$$x_{A1} = x_{A0} + v_A(t_1 - t_{A0}) = v_A(t_1 - t_{A0}) \qquad x_{C1} = x_{C0} + v_C(t_1 - t_{C0}) = x_{C0} + v_C t_1$$

The critical piece of information is that Ann and Carol have the same position at t_1, so $x_{A1} = x_{C1}$. Equating these two expressions, we can solve for the time t_1 when Ann passes Carol:

$$v_A(t_1 - t_{A0}) = x_{C0} + v_C t_1$$
$$\Rightarrow (v_A - v_C)t_1 = x_{C0} + v_A t_{A0}$$
$$\Rightarrow t_1 = \frac{x_{C0} + v_A t_{A0}}{v_A - v_C} = \frac{2.4 \text{ mi} + (50 \text{ mph})(0.5 \text{ h})}{50 \text{ mph} - 36 \text{ mph}} = 1.96 \text{ h} \approx 2.0 \text{ h}$$

(b) Their position is $x_1 = x_{A1} = x_{C1} = x_{C0} + v_C t_1 = 72.86 \text{ mi} \approx 73 \text{ mi}$

(c) Note that Ann's graph doesn't start until $t = 0.5$ h, but her graph has a steeper slope so it intersects Carol's graph at $t \approx 2.0$ h.



2.62. MODEL: We will treat the ball like a particle. We will assume that interactions between the felt and the ball remain constant such that the acceleration from the initial period of rolling persists until the ball comes to rest or leaves the felt.

VISUALIZE: With the assumption of constant acceleration, we can use kinematic equations. In particular, we will find $v_{fx}^2 = v_{0x}^2 + 2a_x\Delta x$ to be of use. We will use the subscript "0" to refer to the moment the ball rolls onto the felt, the subscript "1" to refer to the moment when it reaches the 20 cm mark, and "2" to refer to the moment the ball would stop, assuming the constant acceleration persists. We can then check whether it would stop in a distance less than or greater than the remaining 10 cm of felt.

SOLVE: Because we are assuming constant acceleration, we can write

$$\frac{v_{1x}^2 - v_{0x}^2}{2\Delta x_{01}} = a_x = \frac{v_{2x}^2 - v_{1x}^2}{2\Delta x_{12}}$$

Because the speed at point 1 has dropped to half the initial speed, we can write $v_{1x} = \frac{1}{2}v_{0x}$. Also, because we are looking for the stopping position, we can require that $v_{2x} = 0$. Rearranging, we have

$$\frac{\left(\frac{1}{2}v_{0x}\right)^2 - v_{0x}^2}{\Delta x_{01}} = \frac{0 - \left(\frac{1}{2}v_{0x}\right)^2}{\Delta x_{12}} \Rightarrow -\frac{3}{4}\frac{v_{0x}^2}{\Delta x_{01}} = -\frac{1}{4}\frac{v_{0x}^2}{\Delta x_{12}}$$

$$3\Delta x_{12} = \Delta x_{01}$$

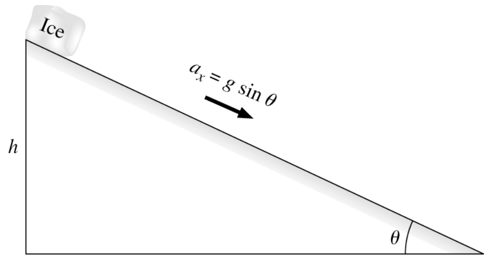
$$\Delta x_{12} = \frac{1}{3}\Delta x_{01} = \frac{1}{3}(0.20\text{ m}) = 0.067\text{ m}$$

Since only 6.7 cm will be required for the ball to stop, and 10 cm of felt remain, the ball will stop on the felt.

REVIEW: This problem calls attention to how changes in speed depend quadratically on distance when there is constant acceleration.

2.63. **MODEL:** Model the ice as a particle and use the kinematic equations for constant acceleration. Model the "very slippery block" and "smooth ramp" as frictionless. Set the *x*-axis parallel to the ramp.

VISUALIZE:



Note that the distance down the ramp is $\Delta x = h/\sin\theta$. Also $a_x = g\sin\theta$ down a frictionless ramp.

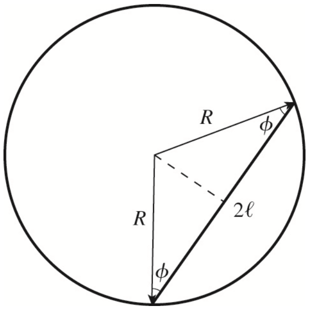
SOLVE: Use $v_f^2 = v_i^2 + 2a_x\Delta x$, where $v_i = 0$.

$$v_f^2 = 2a\Delta x \Rightarrow v_f = \sqrt{2(g\sin\theta)\frac{h}{\sin\theta}} = \sqrt{2gh}$$

REVIEW: We will later learn how to solve this problem in an easier way with energy.

2.64. **MODEL:** We can model the bead as a particle undergoing constant acceleration.

VISUALIZE: Because this bead is effectively on an incline, we can use equation 2.26. Note that the angle used in equation 2.26 is measured from horizontal, such that we will use $a_x = \pm g\sin(90° - \phi) = \pm g\cos(\phi)$. In this case, we will choose our axes such that +*x* points up the incline, and $a_x = -g\cos(\phi)$. We can use kinematic equations, but we will need a total distance traveled. Refer to the diagram below.



Note that $R\cos(\phi) = \ell$, such that the total distance the bead must traverse is $2\ell = 2R\cos(\phi)$. With this distance, we can apply $\Delta x = v_{0x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$.

SOLVE: Combining the above expressions, and using the fact that the bead starts from rest, we have

$$\Delta x = -2R\cos(\phi) = (0)\Delta t + \frac{1}{2}(-g\cos(\phi))(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{4R}{g}} = 2\sqrt{\frac{R}{g}}$$

Review: Our answer turns out not to depend on the angle, which is initially surprising. However, note that as the angle increases, both the acceleration and the distance decrease. It is plausible (though not obvious) that these effects would cancel to make the answer independent of the angle ϕ.

2.65. **MODEL:** We will model the skateboarder as a particle undergoing constant acceleration.

VISUALIZE: Because the skateboarder is on an incline, we can use equation 2.26 to describe his/her acceleration. Note that $\sin(\theta) = h/\Delta x$, where h is the vertical height we seek, and Δx is the distance along the incline. Because we are seeking the highest possible incline, we will consider the case where the speed has dropped almost to zero, and the skateboarder just barely makes it to the top. We can use $v_{0x}^2 = v_{fx}^2 + 2a_x\Delta x$.

Solve: Combining the expressions above, we have

$$v_{fx}^2 = v_{0x}^2 + 2a_x\Delta x = v_{0x}^2 - 2g\sin(\theta)\Delta x = v_{0x}^2 - 2g\frac{h}{\Delta x}\Delta x$$

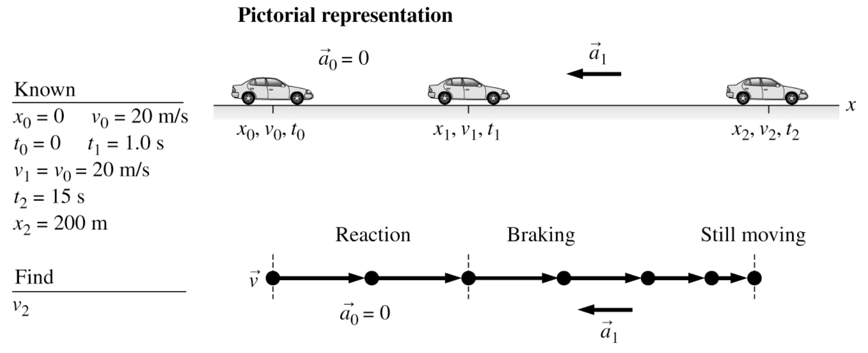
$$v_{fx}^2 = v_{0x}^2 - 2gh \Rightarrow h = \frac{v_{fx}^2 - v_{0x}^2}{-2g}$$

$$h = \frac{(0\text{ m/s})^2 - (4.0\text{ m/s})^2}{-2(9.8\text{ m/s}^2)} = 0.82\text{ m}$$

REVIEW: The initial speed of 4.0 m/s is a plausible speed for a skateboarder. So, we expect an intuitively plausible height for the skateboarder to reach going up a ramp. A height just under 1 m seems reasonable.

2.66. **MODEL:** The car is a particle that moves with constant linear acceleration.

VISUALIZE:



SOLVE: The reaction time is 1.0 s, and the motion during this time is

$$x_1 = x_0 + v_0(t_1 - t_0) = 0\text{ m} + (20\text{ m/s})(1.0\text{ s}) = 20\text{ m}$$

During slowing down,

$$x_2 = x_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2 = 200\text{ m}$$

$$= 20\text{ m} + (20\text{ m/s})(15\text{ s} - 1.0\text{ s}) + \frac{1}{2}a_1(15\text{ s} - 1.0\text{ s})^2 \Rightarrow a_1 = -1.02\text{ m/s}^2$$

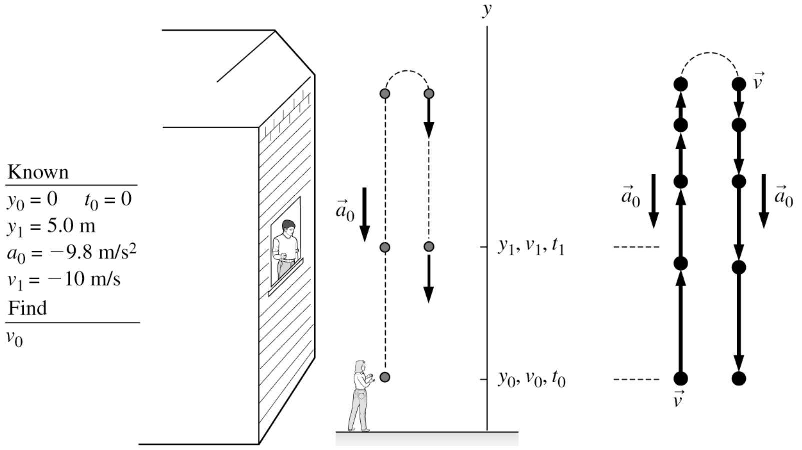
The final speed v_2 can now be obtained as

$$v_2 = v_1 + a_1(t_2 - t_1) = (20\text{ m/s}) + (-1.02\text{ m/s}^2)(15\text{ s} - 1\text{ s}) = 5.7\text{ m/s}$$

2.67. MODEL: The ball is a particle that exhibits freely falling motion according to the constant-acceleration kinematic equations.

VISUALIZE:

Pictorial representation



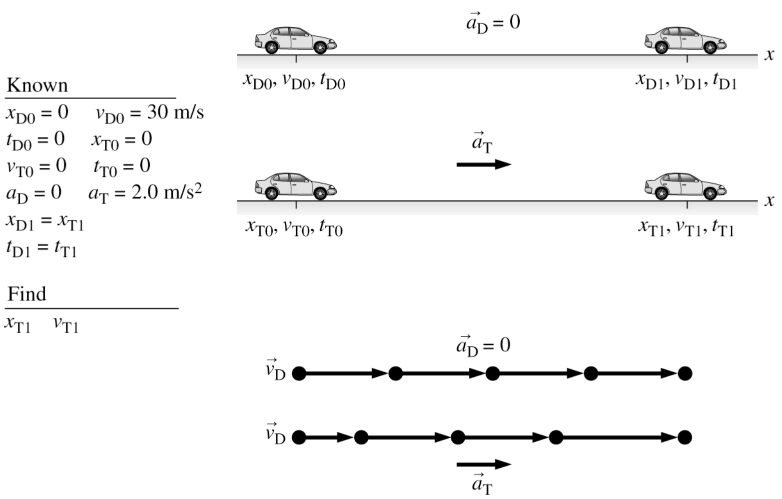
SOLVE: Using the known values, we have

$$v_1^2 = v_0^2 + 2a_0(y_1 - y_0) \Rightarrow (-10 \text{ m/s})^2 = v_0^2 + 2(-9.8 \text{ m/s}^2)(5.0 \text{ m} - 0 \text{ m}) \Rightarrow v_0 = 14 \text{ m/s}$$

2.68. MODEL: Both cars are particles that move according to the constant-acceleration kinematic equations.

VISUALIZE:

Pictorial representation



SOLVE: **(a)** David's and Tina's motions are given by the following equations:

$$x_{D1} = x_{D0} + v_{D0}(t_{D1} - t_{D0}) + \frac{1}{2}a_D(t_{D1} - t_{D0})^2 = v_{D0}t_{D1}$$

$$x_{T1} = x_{T0} + v_{T0}(t_{T1} - t_{T0}) + \frac{1}{2}a_T(t_{T1} - t_{T0})^2 = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}a_T t_{T1}^2$$

When Tina passes David the distances are equal and $t_{D1} = t_{T1}$, so we get

$$x_{D1} = x_{T1} \Rightarrow v_{D0}t_{D1} = \frac{1}{2}a_T t_{T1}^2 \Rightarrow v_{D0} = \frac{1}{2}a_T t_{T1} \Rightarrow t_{T1} = \frac{2v_{D0}}{a_T} = \frac{2(30 \text{ m/s})}{2.0 \text{ m/s}^2} = 30 \text{ s}$$

Using Tina's position equation,

$$x_{T1} = \frac{1}{2}a_T t_{T1}^2 = \frac{1}{2}(2.0 \text{ m/s}^2)(30 \text{ s})^2 = 900 \text{ m}$$

(b) Tina's speed v_{T1} can be obtained from

$$v_{T1} = v_{T0} + a_T(t_{T1} - t_{T0}) = (0 \text{ m/s}) + (2.0 \text{ m/s}^2)(30 \text{ s} - 0 \text{ s}) = 60 \text{ m/s}$$

REVIEW: This is a high speed for Tina (~134 mph) and so is David's velocity (~67 mph). Thus the large distance for Tina to catch up with David (~0.6 mi) is reasonable.

2.69. MODEL: We will model the Tesla as a particle. We will definitely not model the acceleration as constant, since we are given explicit time dependence of the acceleration. This means we cannot use kinematic equations.

VISUALIZE: The change in the x component of the velocity is given by the integral of the acceleration, over time. This is equivalent to the area under the curve of the acceleration vs. time plot. Let us integrate over the entire first time interval, and then see how far into the second time interval we need to integrate to reach 60 mph.

SOLVE: **(a)** 60 mph is 26.8 m/s. In the first time interval, we have

$$\Delta v_x = \int_0^{0.40 \text{ s}} \left(35 \text{ m/s}^3\right) t\,dt = \left(35 \text{ m/s}^3\right)\frac{1}{2}t^2\Big|_0^{0.40 \text{ s}} = \left(35 \text{ m/s}^3\right)\frac{1}{2}\left(0.40 \text{ s}\right)^2 = 2.8 \text{ m/s}$$

This means that in the second time interval, the change in speed must be 26.8 m/s – 2.8 m/s = 24 m/s. We can write

$$\Delta v_x = \int_{0.40 \text{ s}}^{t_f} \left(14.6 \text{ m/s}^2\right) - \left(1.5 \text{ m/s}^3\right) t\,dt = \left(14.6 \text{ m/s}^2\right)\left(t_f - 0.40 \text{ s}\right) - \left(1.5 \text{ m/s}^3\right)\frac{1}{2}\left(t_f^2 - \left(0.40 \text{ s}\right)^2\right) = 24 \text{ m/s}$$

$$\Rightarrow -\left(1.5 \text{ m/s}^3\right)\frac{1}{2}t_f^2 + \left(14.6 \text{ m/s}^2\right)t_f = \left(24 \text{ m/s}\right) + \left(14.6 \text{ m/s}^2\right)\left(0.40 \text{ s}\right) - \left(1.5 \text{ m/s}^3\right)\frac{1}{2}\left(0.40 \text{ s}\right)^2$$

$$\Rightarrow t_f = 2.3 \text{ s}$$

In the last step, we applied the quadratic equation. Note that the quadratic equation yields two solutions, but one is outside the range given in the problem statement for this acceleration behavior.

(b) Now assuming constant acceleration, we can use kinematic equations:

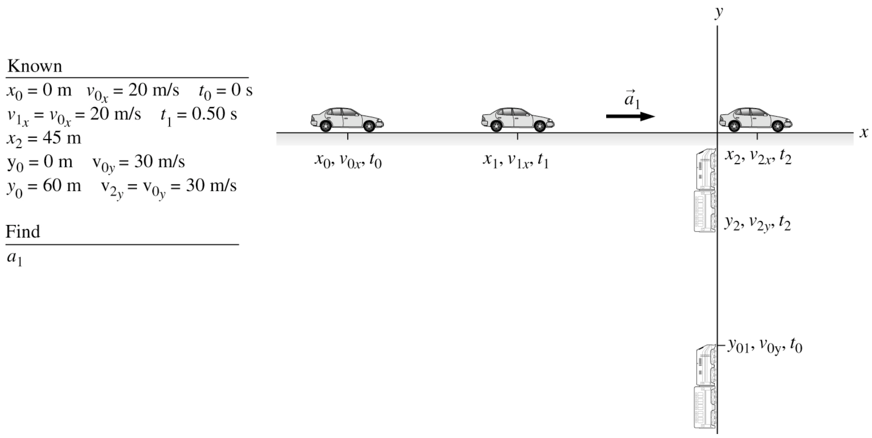
$$v_{fx} = v_{0x} + a_x\Delta t \Rightarrow a_x = \frac{v_{fx} - v_{0x}}{\Delta t} = \frac{\left(26.8 \text{ m/s}\right) - \left(0 \text{ m/s}\right)}{\left(2.3 \text{ s}\right)} = 11.7 \text{ m/s}^2 = 1.2g$$

REVIEW: We see in part (b) what the problem statement meant by "impossibly fast"! If the acceleration were constant, it would need to be greater than the acceleration due to gravity!

2.70. **MODEL:** Treat the car and train in the particle model and use the constant-acceleration kinematic equations.

VISUALIZE:

Pictorial representation



SOLVE: In the particle model the car and train have no physical size, so the car has to reach the crossing at an infinitesimally sooner time than the train. Crossing at the same time corresponds to the minimum a_1 necessary to avoid a collision. So the problem is to find a_1 such that $x_2 = 45$ m when $y_2 = 60$ m.

The time it takes the train to reach the intersection can be found by considering its known constant velocity.

$$v_{0y} = v_{2y} = 30 \text{ m/s} = \frac{y_2 - y_0}{t_2 - t_0} = \frac{60 \text{ m}}{t_2} \Rightarrow t_2 = 2.0 \text{ s}$$

Now find the distance traveled by the car during the reaction time of the driver.

$$x_1 = x_0 + v_{0x}(t_1 - t_0) = 0 + (20 \text{ m/s})(0.50 \text{ s}) = 10 \text{ m}$$

The kinematic equation for the final position at the intersection can be solved for the minimum acceleration a_1.

$$x_2 = 45 \text{ m} = x_1 + v_{1x}(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$$
$$= 10 \text{ m} + (20 \text{ m/s})(1.5 \text{ s}) + \frac{1}{2}a_1(1.5 \text{ s})^2$$
$$\Rightarrow a_1 = 4.4 \text{ m/s}^2$$

REVIEW: The acceleration of $4.4 \text{ m/s}^2 = 2.0$ miles/h/s is reasonable for an automobile to achieve. However, you should not try this yourself! Always pay attention when you drive! Train crossings are dangerous locations, and many people lose their lives at one each year.

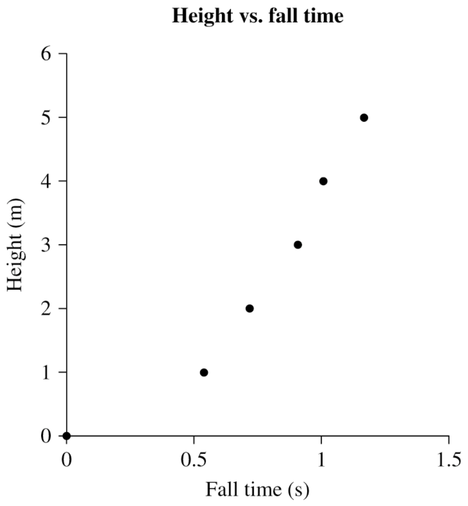
2.71. **MODEL:** Model the ball as a particle. Since the ball is heavy we ignore air resistance.

VISUALIZE: We use the kinematic equation $\Delta y = v_0\Delta t + \frac{1}{2}a(\Delta t)^2$, but we set the origin at the ground so $y_0 = h$ and $y_1 = 0$; this means $\Delta y = y_1 - y_0 = -h$. We release the ball from rest so at $t_0 = 0$ we have $v_0 = 0$ and $\Delta t = t$. We also note that $a = -g$ where g is the free-fall acceleration on Planet X. Making all these substitutions leaves

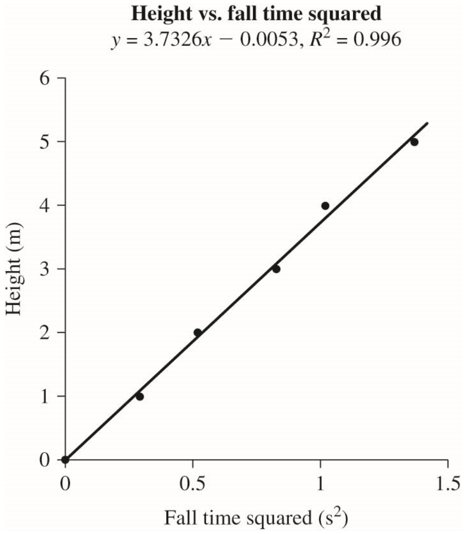
$$h = \left(\frac{1}{2}g\right)t^2$$

So we expect a graph of h versus t^2 to produce a straight line whose slope is $g/2$ and whose intercept is zero. Compare to $y = mx + b$ where $y = h$, $m = g/2$, $x = t^2$, and $b = 0$.

SOLVE: First look at a graph of height versus fall time and notice that it is not linear. It would be difficult to analyze. Even though the point $(0,0)$ is not a measured data point, it is valid to add to the data table and graph because it would take zero time to fall zero distance.



However, the theory has guided us to expect that a graph of height versus fall time **squared** would be linear and the slope would be $g/2$. First we use a spreadsheet to square the fall times and then graph the height versus. fall time squared to see if it looks linear and that the intercept is close to zero.



It looks linear and $R^2 = 0.996$ tells us the linear fit is very good. We also see that the intercept is a very small negative number that is close to zero, so we have confidence in our model. The fit is not perfect and the intercept is not exactly zero probably because of uncertainties in timing the fall.

We now conclude that the slope of the best fit line $m = 3.7326$ is $g/2$ in the proper units, so

$g = 2\times 3.7326\ \text{m/s}^2 = 7.5\ \text{m/s}^2$ on Planet X.

Review: The free-fall acceleration on Planet X is a little bit smaller than on earth, but is reasonable. It is customary to put the independent variable on the horizontal axis and the dependent variable along the vertical axis. Had we done so here we would have graphed t^2 versus h and the slope would have been $2/g$. Our answer to the question would be the same.

2.72. Model: We will model the ball as a particle. We can use kinematic equations, since the ball is in freefall throughout the period of interest.

Visualize: We will label the halfway point with the subscript "1", and the maximum height with "2". We know that the vertical component of velocity is momentarily zero at the highest point. We can use this in conjunction with $v_{2y} = v_{1y} + a_y\Delta t$ to find the speed at the halfway point. This enables us to determine the distance traveled in the last second (and by extension, the maximum height) through $v_{2y}^2 = v_{1y}^2 + 2a_y\Delta y$.

Solve: Using the first equation above, we have

$$v_{1y} = v_{2y} - a_y\Delta t = (0\ \text{m/s}) + (9.8\ \text{m/s}^2)(1.0\ \text{s}) = 9.8\ \text{m/s}$$

Now, using the second kinematic equation from above, we have

$$\Delta y = \frac{v_{2y}^2 - v_{1y}^2}{2a_y} = \frac{(0\ \text{m/s})^2 - (9.8\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} = 4.9\ \text{m}$$

This was the second half of the vertical distance travelled, such that the total height is 9.8 m. The first 4.9 m interval tells us

$$v_{0y} = \sqrt{v_{1y}^2 - 2a_y\Delta y} = \sqrt{(9.8\,\text{m/s})^2 - 2(-9.8\ \text{m/s}^2)(4.9\ \text{m})} = 13.9\,\text{m/s}$$

and requires a time

$$\Delta t = \frac{v_{1y} - v_{0y}}{a_y} = \frac{(9.8\ \text{m/s}^2) - (13.9\ \text{m/s}^2)}{(-9.8\,\text{m/s}^2)} = 0.414\ \text{s}$$

The total time is thus 1.4 s.

Review: Because the ball slows as it rises, we expect the second half of the rise to take longer than the first half. It is thus reasonable that our answer is less than 2.0 s.

2.73. Solve: **(a)** With $v_x = \sqrt{\frac{2P}{m}}t^{1/2}$, we have

$$a_x = \frac{dv_x}{dt} = \sqrt{\frac{2P}{m}} \times \frac{1}{2}t^{-1/2} = \sqrt{\frac{P}{2mt}}$$

(b) The quantity $\frac{2P}{m} = \frac{2(3.6\times10^4 \text{ W})}{1200 \text{ kg}} = 60 \text{ m}^2/\text{s}^3$. Thus $v_x = \sqrt{(60 \text{ m}^2/\text{s}^3)t}$

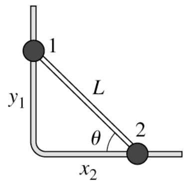
At $t = 2$ s, $v_x = \sqrt{(60 \text{ m}^2/\text{s}^3)(2 \text{ s})} = 11$ m/s (≈ 25 mph).

At $t = 10$ s, $v_x = \sqrt{(60 \text{ m}^2/\text{s}^3)(10 \text{ s})} = 24$ m/s (≈ 50 mph).

(c) At $t = 2$ s, $a_x = \sqrt{\frac{P}{2mt}} = \sqrt{\frac{(3.6\times10^4 \text{ W})}{2(1200 \text{ kg})(2 \text{ s})}} = 2.7 \text{ m/s}^2$. Similarly, at $t = 10$ s, $a_x = 1.2 \text{ m/s}^2$.

2.74. MODEL: The masses are particles.

VISUALIZE:



SOLVE: The rigid rod forms the hypotenuse of a right triangle, which defines a relationship between x_2 and y_1:

$$x_2^2 + y_1^2 = L^2.$$

Taking the time derivative of both sides yields

$$2x_2\frac{dx_2}{dt} + 2y_1\frac{dy_1}{dt} = 0$$

We can now use $v_{2x} = \frac{dx_2}{dt}$ and $v_{1y} = \frac{dy_1}{dt}$ to write $x_2v_{2x} + y_1v_{1y} = 0$.

Thus $v_{2x} = -\left(\frac{y_1}{x_2}\right)v_{1y}$. But from the figure, $\frac{y_1}{x_2} = \tan\theta \Rightarrow v_{2x} = -v_{1y}\tan\theta$.

REVIEW: As x_2 decreases $(v_{2x} < 0)$, y_1 increases $(v_{1y} > 0)$, and vice versa.

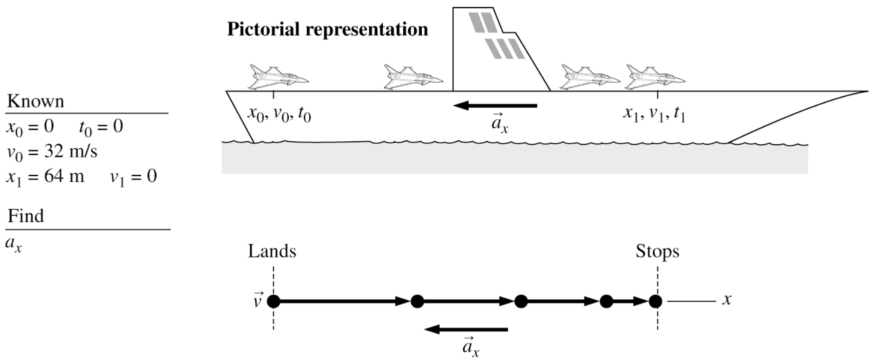
2.75. SOLVE: A comparison of the given equation with the constant-acceleration kinematic equation

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2$$

yields the following information: $x_0 = 0$ m, $x_1 = 64$ m, $t_0 = 0, t_1 = 4$ s, and $v_0 = 32$ m/s.

(a) After landing on the deck of a ship at sea with a velocity of 32 m/s, a fighter plane is observed to come to a complete stop in 4 s over a distance of 64 m. Find the plane's deceleration.

(b)



(c) $64\text{ m} = 0\text{ m} + (32\text{ m/s})(4\text{ s} - 0\text{ s}) + \frac{1}{2}a_x(4\text{ s} - 0\text{ s})^2$ $\quad 64\text{ m} = 128\text{ m} + (8\text{ s}^2)a_x \Rightarrow a_x = -8\text{ m/s}^2.$

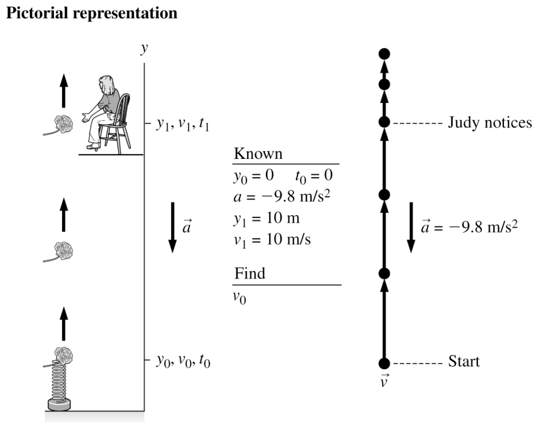
The deceleration is the absolute value of the acceleration, or 8 m/s^2.

2.76. **Solve:** **(a)** A comparison of this equation with the constant-acceleration kinematic equation

$$(v_{1y})^2 = v_{0y}^2 + 2(a_y)(y_1 - y_0)$$

yields the following information: $y_0 = 0$ m, $y_1 = 10$ m, $a_y = -9.8\text{ m/s}^2$, and $v_{1y} = 10$ m/s. It is clearly a problem of free fall. On a romantic Valentine's Day, John decided to surprise his girlfriend, Judy, in a special way. As he reached her apartment building, he found her sitting in the balcony of her second floor apartment 10 m above the first floor. John quietly armed his spring-loaded gun with a rose, and launched it straight up to catch her attention. Judy noticed that the flower flew past her at a speed of 10 m/s. Judy is refusing to kiss John until he tells her the initial speed of the rose as it was released by the spring-loaded gun. Can you help John on this Valentine's Day?

(b)



(c) $(10\text{ m/s})^2 = v_{0y}^2 - 2(9.8\text{ m/s}^2)(10\text{ m} - 0\text{ m}) \Rightarrow v_{0y} = 17.2\text{ m/s}$

Review: The initial velocity of 17.2 m/s, compared to a velocity of 10 m/s at a height of 10 m, is very reasonable.

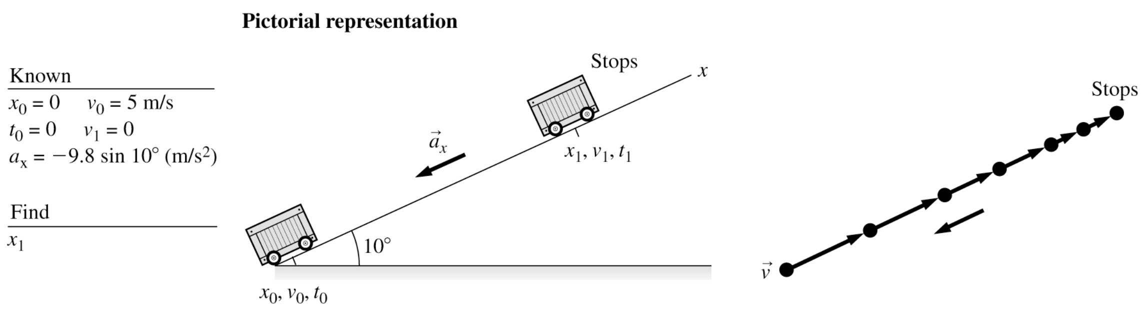
2.77. **SOLVE:** A comparison with the constant-acceleration kinematic equation

$$(v_{1x})^2 = (v_{0x})^2 + 2a_x(x_1 - x_0)$$

yields the following quantities: $x_0 = 0$ m, $v_{0x} = 5$ m/s, $v_{1x} = 0$ m/s, and $a_x = -(9.8 \text{ m/s}^2)\sin 10°$.

(a) A wagon at the bottom of a frictionless 10° incline is moving up at 5 m/s. How far up the incline does it move before reversing direction and then rolling back down?

(b)

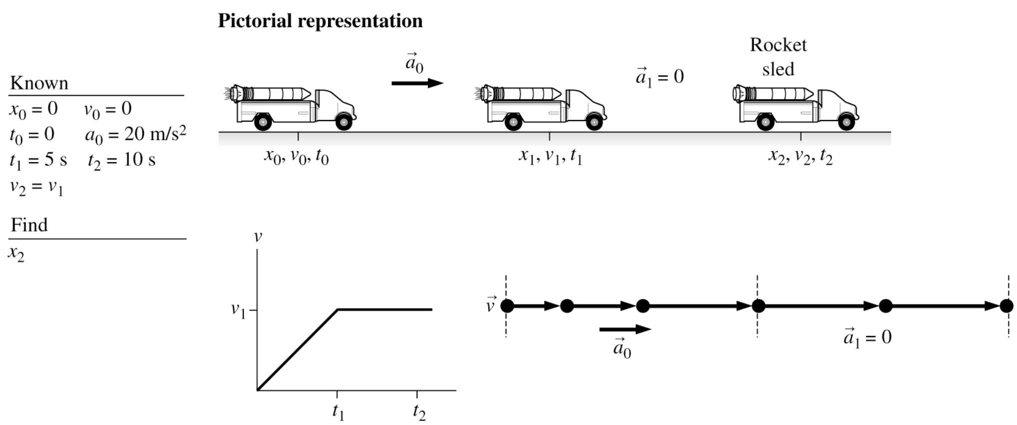


(c) $(0 \text{ m/s})^2 = (5 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)\sin 10°(x_1 - 0 \text{ m})$

$\Rightarrow 25(\text{m/s})^2 = 2(9.8 \text{ m/s}^2)(0.174)x_1 \Rightarrow x_1 = 7.3$ m

2.78. **SOLVE:** **(a)** From the first equation, the particle starts from rest and accelerates for 5 s. The second equation gives a position consistent with the first equation. The third equation gives a subsequent position following the second equation with zero acceleration. A rocket sled accelerates from rest at 20 m/s^2 for 5 s and then coasts at constant speed for an additional 5 s. Draw a graph showing the velocity of the sled as a function of time up to t = 10 s. Also, how far does the sled move in 10 s?

(b)



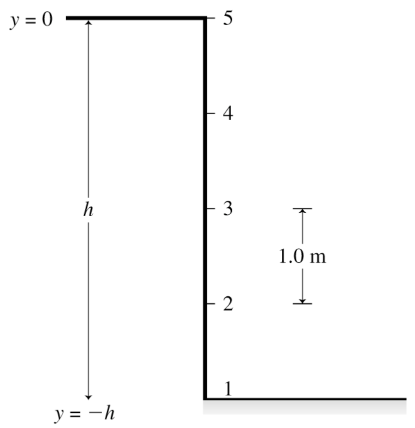
(c) $x_1 = \frac{1}{2}(20 \text{ m/s}^2)(5 \text{ s})^2 = 250$ m $\quad v_1 = 20 \text{ m/s}^2(5 \text{ s}) = 100$ m/s $\quad x_2 = 250 \text{ m} + (100 \text{ m/s})(5 \text{ s}) = 750$ m

Challenge Problems

2.79. **Model:** Assume the water drops are particles in free fall and use the constant-acceleration kinematic equations.

Visualize: Each drop falls from rest so the initial speed is zero. We will also put the origin of the coordinate system at the level of the roof, so the values of the position will be negative. The acceleration for each drop will be $a = -g$.

For each drop $y = \frac{1}{2}(-g)(\Delta t)^2$.



In the figure the labels show where the different drops are at the instant the first drop hits the ground and the fifth drop starts to fall. $y_1 = -h$

Solve: First we want to calculate the time between drops Δt. It is important to note that drop 1 was where drop 2 now is one Δt before. So the first drop has fallen for $4\Delta t$, the second drop has fallen for $3\Delta t$, and the third drop has fallen for $2\Delta t$.

$$y_3 - y_2 = \frac{1}{2}(-g)\left[(2\Delta t)^2 - (3\Delta t)^2\right] = 1.0 \text{ m}$$

$$4(\Delta t)^2 - 9(\Delta t)^2 = \frac{2(1.0 \text{ m})}{-(9.8 \text{ m/s}^2)}$$

$$(-5)(\Delta t)^2 = \frac{2(1.0 \text{ m})}{-(9.8 \text{ m/s}^2)} \Rightarrow \Delta t = \sqrt{\frac{2(1.0 \text{ m})}{(5)(9.8 \text{ m/s}^2)}} = 0.202 \text{ s}$$

Now plug Δt into the y equation for the first drop.

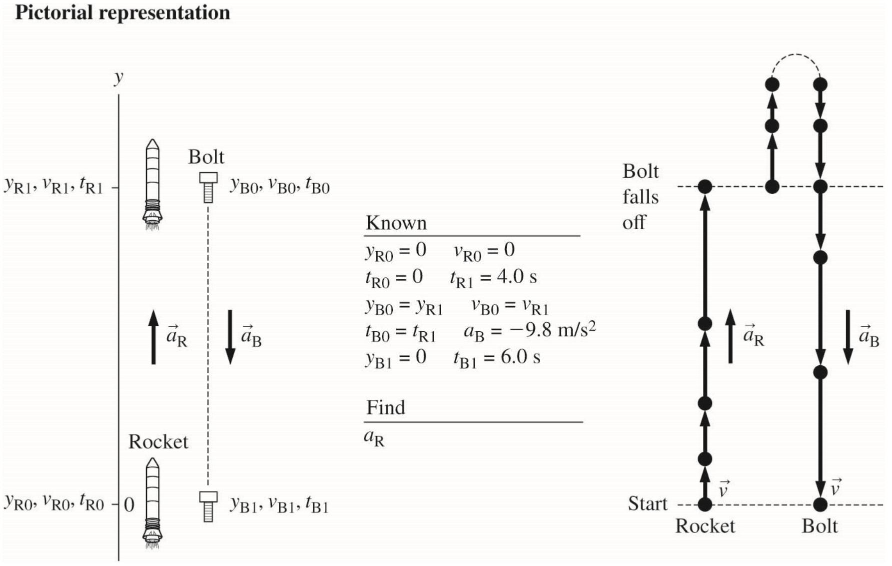
$$y_1 = \frac{1}{2}(-9.8 \text{ m/s}^2)(4(0.202 \text{ s}))^2 = -3.2 \text{ m}$$

So the height of the building is $h = -y_1 = 3.2$ m.

Review: This is a low roof, so this isn't a multistory building. But checking our work gives $y_3 = -1.8$ m, and $y_2 = -0.8$ m, which is the proper spacing.

2.80. MODEL: The rocket and the bolt will be represented as particles to investigate their motion.

VISUALIZE:



The initial velocity of the bolt as it falls off the side of the rocket is the same as that of the rocket, that is, $v_{B0} = v_{R1}$ and it is positive since the rocket is moving upward. The bolt continues to move upward with a deceleration equal to $g = 9.8 \text{ m/s}^2$ before it comes to rest and begins its downward journey.

SOLVE: To find a_R we look first at the motion of the rocket:

$$y_{R1} = y_{R0} + v_{R0}(t_{R1} - t_{R0}) + \frac{1}{2}a_R(t_{R1} - t_{R0})^2$$
$$= 0 \text{ m} + 0 \text{ m/s} + \frac{1}{2}a_R(4.0 \text{ s} - 0 \text{ s})^2 = 8a_R$$

To find a_R we must determine the magnitude of y_{R1} or y_{B0}. Let us now look at the bolt's motion:

$$y_{B1} = y_{B0} + v_{B0}(t_{B1} - t_{B0}) + \frac{1}{2}a_B(t_{B1} - t_{B0})^2$$
$$0 = y_{R1} + v_{R1}(6.0 \text{ s} - 0 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(6.0 \text{ s} - 0 \text{ s})^2$$
$$\Rightarrow y_{R1} = 176.4 \text{ m} - (6.0 \text{ s})\ v_{R1}$$

Since $v_{R1} = v_{R0} + a_R(t_{R1} - t_{R0}) = 0 \text{ m/s} + 4\ a_R = 4\ a_R$ the above equation for y_{R1} yields $y_{R1} = 176.4 - 6.0(4a_R)$.
We know from the first part of the solution that $y_{R1} = 8a_R$. Therefore, $8a_R = 176.4 - 24.0a_R$ and hence $a_R = 5.5 \text{ m/s}^2$.

2.81. MODEL: We will model the car as a particle. The acceleration is not constant, but is a function of velocity. So we cannot use kinematic equations.

VISUALIZE: Note that the maximum speed occurs when the acceleration becomes zero.

SOLVE: (a) We can require $a_x = a_0 - kv_{\max} = 0 \Rightarrow k = a_0 / v_{\max}$.

(b) Rearranging the expression for acceleration, we see

$$a_x = \frac{dv_x}{dt} = a_0 - \frac{a_0}{v_{\max}} v_x$$

We recognize this as a differential equation, and recall that when the derivative of a function depends on the function itself, it is a good idea to try solutions of the form $v_x = Ae^{-bt} + c$. We want the car to start from rest, such that $A = -c$, and we have $v_x = A\left(e^{-bt} - 1\right)$. We also know that after long times $v_x \to v_{\max} \Rightarrow A = -v_{\max}$. This yields

$$\frac{dv_x}{dt} = -b\left(-v_{\max}\right)e^{-bt} = a_0\left(1 - \frac{1}{v_{\max}} v_{\max}\left(1 - e^{-bt}\right)\right) \Rightarrow -b\left(-v_{\max}\right)e^{-bt} = a_0 e^{-bt} \Rightarrow b = a_0 / v_{\max}$$

such that

$$v_x(t) = v_{\max}\left(1 - e^{-ta_0/v_{\max}}\right)$$

(c) We require

$$v_x(t) = 0.95 v_{\max} = v_{\max}\left(1 - e^{-ta_0/v_{\max}}\right) \Rightarrow \left(1 - e^{-ta_0/v_{\max}}\right) = 0.95 \Rightarrow e^{-ta_0/v_{\max}} = 0.05$$

$$t = -\frac{v_{\max}}{a_0}\ln(0.05)$$

Inserting the given numbers, we have

$$t = -\frac{(60\ \text{m/s})}{\left(4.0\ \text{m/s}^2\right)}\ln(0.05) = 45\ \text{s}$$

REVIEW: One way of checking our answers is by examining units. Note that the exponent has no units: $ta_0 / v_{\max}$, and that the speed has units of m/s.

2.82. SOLVE: (a) The acceleration is the time derivative of the velocity.

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}[a(1 - e^{-bt})] = abe^{-bt}$$

With $a = 11.81$ m/s and $b = 0.6887\ \text{s}^{-1}$, $a_x = 8.134e^{-0.6887t}\ \text{m/s}^2$. At the times $t = 0$ s, 2.00 s, and 4.00 s, this has the values 8.134 m/s^2, 2.052 m/s^2, and 0.5175 m/s^2.

(b) Since $v_x = \dfrac{dx}{dt}$, the position x is the integral of the velocity. With $v_x = \dfrac{dx}{dt} = a - ae^{-bt}$ and the initial condition that $x_i = 0$ m at $t_i = 0$ s,

$$\int_o^x dx = \int_o^t dt - \int_o^t ae^{-bt}dt$$

Thus

$$x = at\Big|_o^t + \frac{a}{b}e^{2bt}\Big|_o^t = at + \frac{a}{b}e^{-bt} - \frac{a}{b}$$

This can be written a little more neatly as

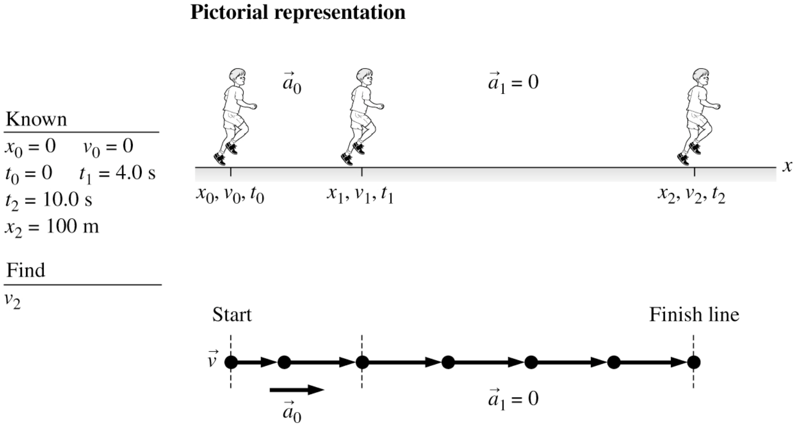
$$x = \frac{a}{b}(bt + e^{-bt} - 1)$$
$$= 17.15(0.6887t + e^{-0.6887t} - 1)\text{ m}$$

(c) By trial and error, t = 9.92 s yields x = 100.0 m.

REVIEW: Lewis's actual time was 9.93 s.

2.83. **MODEL:** We will use the particle-model to represent the sprinter and the equations of kinematics.

VISUALIZE:



SOLVE: Substituting into the constant-acceleration kinematic equations,

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = 0\text{ m} + 0\text{ m} + \frac{1}{2}a_0(4\text{ s} - 0\text{ s})^2 = \frac{1}{2}a_0t_1^2 = \frac{1}{2}a_0(4.0\text{ s})^2$$
$$\Rightarrow x_1 = (8\text{ s}^2)a_0$$
$$v_1 = v_0 + a_0(t_1 - t_0) = 0\text{ m/s} + a_0(4.0\text{ s} - 0\text{ s}) \Rightarrow v_1 = (4.0\text{ s})\ a_0$$

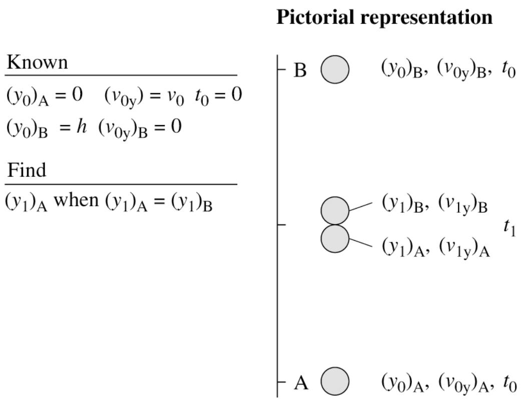
From these two results, we find that $x_1 = (2\text{ s})v_1$. Now,

$$x_2 = x_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$$
$$\Rightarrow 100\text{ m} = (2\text{ s})v_1 + v_1(10\text{ s} - 4\text{ s}) + 0\text{ m} \Rightarrow v_1 = 12.5\text{ m/s}$$

REVIEW: Using the conversion 2.24 mph = 1 m/s, $v_1 = 12.5\text{ m/s} = 28$ mph. This speed as the sprinter reaches the finish line is physically reasonable.

2.84. **MODEL:** The balls are particles undergoing constant acceleration.

VISUALIZE:



SOLVE: **(a)** The positions of each of the balls at t_1 is found from kinematics.

$$(y_1)_A = (y_0)_A + (v_{0y})_A t_1 - \frac{1}{2}gt_1^2 = v_0 t_1 - \frac{1}{2}gt_1^2$$

$$(y_1)_B = (y_0)_B + (v_{0y})_B t_1 - \frac{1}{2}gt_1^2 = h - \frac{1}{2}gt_1^2$$

In the particle model the balls have no physical extent, so they meet when $(y_1)_A = (y_1)_B$. This means

$$v_0 t_1 - \frac{1}{2}gt_1^2 = h - \frac{1}{2}gt_1^2 \Rightarrow t_1 = \frac{h}{v_0}$$

Thus the collision height is $y_{coll} = h - \frac{1}{2}gt_1^2 = h - \frac{gh^2}{2v_0^2}$.

(b) We need the collision to occur while $y_{coll} \geq 0$. Thus

$$h - \frac{gh^2}{2v_0^2} \geq 0 \Rightarrow 1 \geq \frac{gh}{2v_0^2} \Rightarrow h \leq \frac{2v_0^2}{g}$$

So $h_{max} = \frac{2v_0^2}{g}$.

(c) Ball A is at its highest point when its velocity $(v_{1y})_A = 0$.

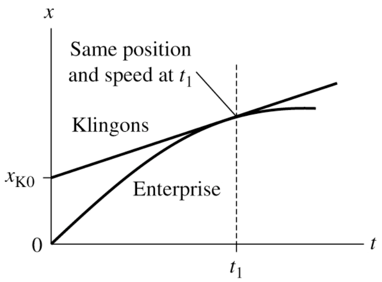
$$(v_{1y})_A = (v_{0y})_A - gt_1 \Rightarrow 0 = v_0 - gt_1 \Rightarrow t_1 = \frac{v_0}{g}$$

In **(a)** we found that the collision occurs at $t_1 = \frac{h}{v_0}$. Equating these, $\frac{h}{v_0} = \frac{v_0}{g} \Rightarrow h = \frac{v_0^2}{g}$.

REVIEW: Interestingly, the height at which a collision occurs while Ball A is at its highest point is exactly half of h_{max}.

2.85. MODEL: The spaceships are represented as particles.

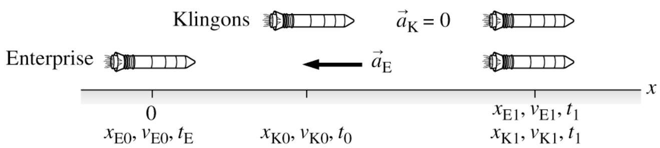
VISUALIZE:

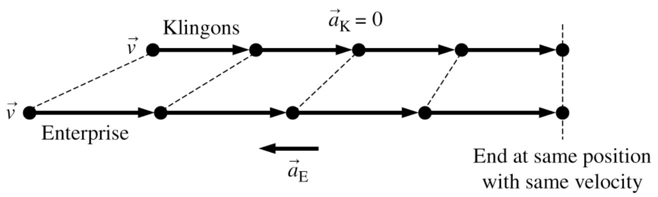


Pictorial representation

Known
$x_{E0} = 0$ $v_{E0} = 50$ km/s
$t_0 = 0$ $x_{K0} = 100$ km
$v_{K0} = 20$ km/s $a_K = 0$
$v_{K1} = v_{K0}$ $x_{E1} = x_{K1}$
$v_{E1} = v_{K1}$

Find
$a_E = a$





SOLVE: The difficulty with this problem is how to describe "barely avoid." The Klingon ship is moving with constant speed, so its position-versus-time graph is a straight line from $x_{K0} = 100$ km. The Enterprise will be decelerating, so its graph is a parabola with decreasing slope. The Enterprise doesn't have to stop; it merely has to slow quickly enough to match the Klingon ship speed at the point where it has caught up with the Klingon ship. (You do the same thing in your car when you are coming up on a slower car; you decelerate to match its speed just as you come up on its rear bumper.) Thus the parabola of the Enterprise will be tangent to the straight line of the Klingon ship, showing that the two ships have the same speed (same slopes) when they are at the same position. Mathematically, we can say that at time t_1 the two ships will have the same position $(x_{E1} = x_{K1})$ *and* the same velocity $(v_{E1} = v_{K1})$. Note that we are using the particle model, so the ships have zero length. At time t_1,

$$x_{K1} = x_{K0} + v_{K0}t_1 \qquad v_{K1} = v_{K0}$$
$$x_{E1} = v_{E0}t_1 + \frac{1}{2}at_1^2 \qquad v_{E1} = v_{E0} + at_1$$

Equating positions and velocities at t_1:

$$x_{K0} + v_{K0}t_1 = v_{E0}t_1 + \frac{1}{2}at_1^2 \qquad v_{K0} = v_{E0} + at_1$$

We have two simultaneous equations in the two unknowns a and t_1. From the velocity equation,

$$t_1 = (v_{K0} - v_{E0})/a$$

Substituting into the position equation gives

$$x_{K0} = -(v_{K0} - v_{E0}) \cdot \frac{(v_{K0} - v_{E0})}{a} + \frac{1}{2} a \cdot \left(\frac{(v_{K0} - v_{E0})}{a} \right)^2 = 2 \frac{(v_{K0} - v_{E0})^2}{2a}$$

$$\Rightarrow a = 2 \frac{(v_{K0} - v_{E0})^2}{2x_{K0}} = -\frac{(20{,}000 \text{ m/s} - 50{,}000 \text{ m/s})^2}{2(100{,}000 \text{ m})} = -4500 \text{ m/s}^2$$

The magnitude of the acceleration is 4500 m/s^2.

REVIEW: The deceleration is 4500 m/s^2, which is a rather extreme $\approx 460g$. Fortunately, the Enterprise has other methods to keep the crew from being killed.