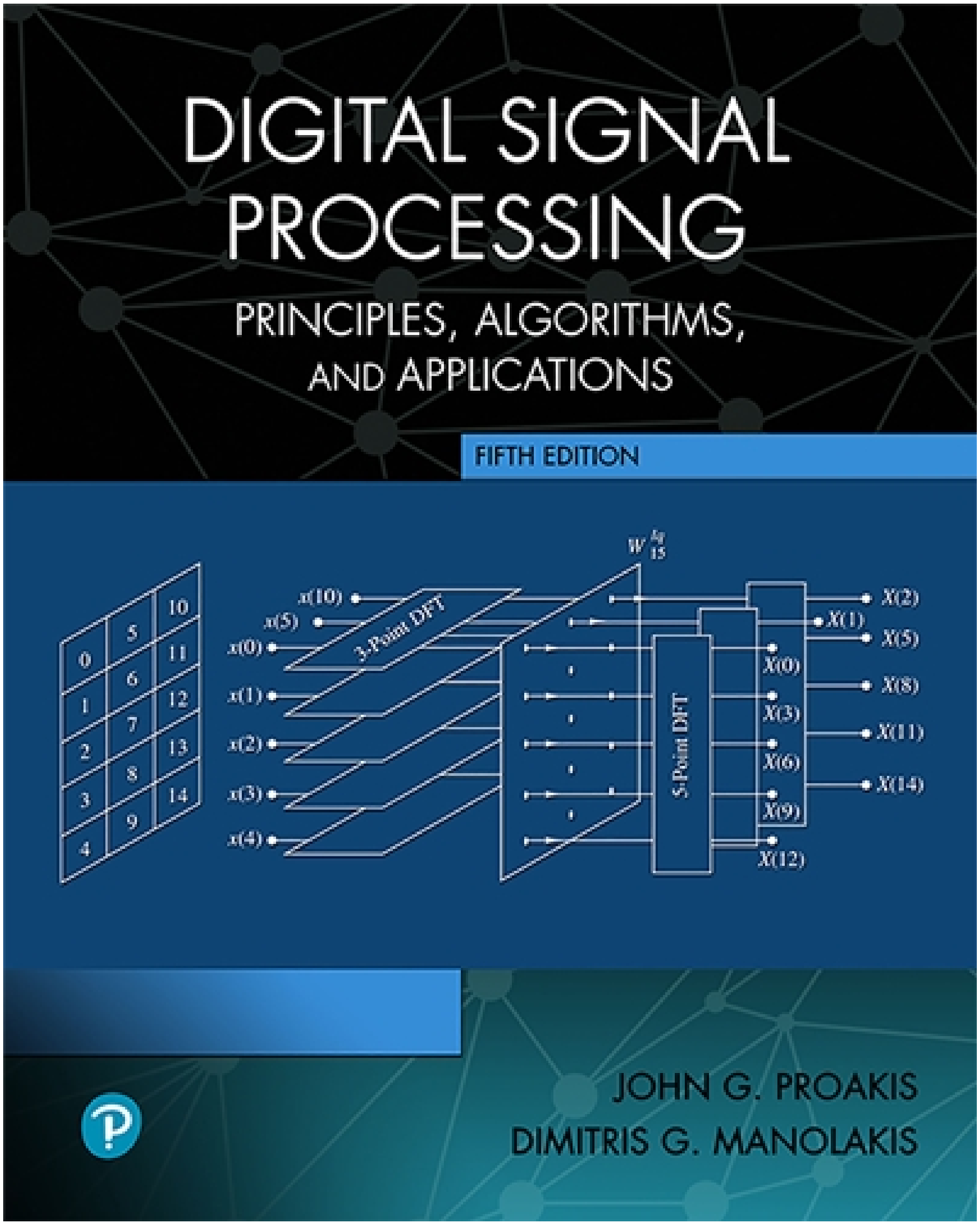
Solutions for Digital Signal Processing 5th Edition by Proakis



Solutions

SOLUTIONS MANUAL

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DIGITAL SIGNAL PROCESSING

Principles, Algorithms, and Applications

5th Edition

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Chapter 1

1.1

(a) One dimensional, multichannel, discrete time, and digital.
(b) Multi dimensional, single channel, continuous-time, analog.
(c) One dimensional, single channel, continuous-time, analog.
(d) One dimensional, single channel, continuous-time, analog.
(e) One dimensional, multichannel, discrete-time, digital.

Chapter 2

2.1

(a)

$$x(n) = \left\{ \ldots 0, \frac{1}{3}, \frac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots \right\}$$

. Refer to fig 2.1-1.
(b) After folding s(n) we have

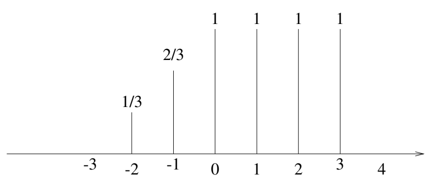

Figure 2.1-1:

$$x(-n) = \left\{ \ldots 0, 1, 1, 1, \underset{\uparrow}{1}, \frac{2}{3}, \frac{1}{3}, 0, \ldots \right\}.$$

After delaying the folded signal by 4 samples, we have

$$x(-n+4) = \left\{ \ldots 0, \underset{\uparrow}{0}, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \ldots \right\}.$$

On the other hand, if we delay x(n) by 4 samples we have

$$x(n-4) = \left\{ \ldots \underset{\uparrow}{0}, 0, \frac{1}{3}, \frac{2}{3}, 1, 1, 1, 1, 0, \ldots \right\}.$$

Now, if we fold $x(n-4)$ we have

$$x(-n-4) = \left\{ \ldots 0, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \underset{\uparrow}{0}, \ldots \right\}$$

(c)

$$x(-n+4) = \left\{\ldots \underset{\uparrow}{0}, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \ldots\right\}$$

(d) To obtain $x(-n+k)$, first we fold $x(n)$. This yields $x(-n)$. Then, we shift $x(-n)$ by k samples to the right if $k > 0$, or k samples to the left if $k < 0$.
(e) Yes.

$$x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n+1) + u(n) - u(n-4)$$

2.2

$$x(n) = \left\{\ldots 0, 1, \underset{\uparrow}{1}, 1, 1, \frac{1}{2}, \frac{1}{2}, 0, \ldots\right\}$$

(a)

$$x(n-2) = \left\{\ldots 0, \underset{\uparrow}{0}, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, 0, \ldots\right\}$$

(b)

$$x(4-n) = \left\{\ldots 0, \underset{\uparrow}{\frac{1}{2}}, \frac{1}{2}, 1, 1, 1, 1, 0, \ldots\right\}$$

(see 2.1(d))
(c)

$$x(n+2) = \left\{\ldots 0, 1, 1, 1, \underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{2}, 0, \ldots\right\}$$

(d)

$$x(n)u(2-n) = \left\{\ldots 0, 1, \underset{\uparrow}{1}, 1, 1, 0, 0, \ldots\right\}$$

(e)

$$x(n-1)\delta(n-3) = \left\{\ldots \underset{\uparrow}{0}, 0, 1, 0, \ldots\right\}$$

(f)

$$\begin{aligned} x(n^2) &= \{\ldots 0, x(4), x(1), x(0), x(1), x(4), 0, \ldots\} \\ &= \left\{\ldots 0, \frac{1}{2}, 1, \underset{\uparrow}{1}, 1, \frac{1}{2}, 0, \ldots\right\} \end{aligned}$$

(g)

$$\begin{aligned} x_e(n) &= \frac{x(n) + x(-n)}{2}, \\ x(-n) &= \left\{\ldots 0, \frac{1}{2}, \frac{1}{2}, 1, 1, \underset{\uparrow}{1}, 1, 0, \ldots\right\} \\ &= \left\{\ldots 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 1, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0, \ldots\right\} \end{aligned}$$

(h)

$$\begin{aligned} x_o(n) &= \frac{x(n) - x(-n)}{2} \\ &= \left\{\ldots 0, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0, \ldots\right\} \end{aligned}$$

2.3

(a)

$$u(n) - u(n-1) = \delta(n) = \begin{cases} 0, & n < 0 \\ 1, & n = 0 \\ 0, & n > 0 \end{cases}$$

(b)

$$\sum_{k=-\infty}^{n} \delta(k) = u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

$$\sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

2.4

Let

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)],$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)].$$

Since

$$x_e(-n) = x_e(n)$$

and

$$x_o(-n) = -x_o(n),$$

it follows that

$$x(n) = x_e(n) + x_o(n).$$

The decomposition is unique. For

$$x(n) = \left\{2, 3, \underset{\uparrow}{4}, 5, 6\right\},$$

we have

$$x_e(n) = \left\{4, 4, \underset{\uparrow}{4}, 4, 4\right\}$$

and

$$x_o(n) = \left\{-2, -1, \underset{\uparrow}{0}, 1, 2\right\}.$$

2.5

First, we prove that

$$\sum_{n=-\infty}^{\infty} x_e(n)x_o(n) = 0$$

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x_e(n)x_o(n) &= \sum_{m=-\infty}^{\infty} x_e(-m)x_o(-m) \\
&= -\sum_{m=-\infty}^{\infty} x_e(m)x_o(m) \\
&= -\sum_{n=-\infty}^{\infty} x_e(n)x_o(n) \\
&= \sum_{n=-\infty}^{\infty} x_e(n)x_o(n) \\
&= 0
\end{aligned}$$

Then,

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x^2(n) &= \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2 \\
&= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + \sum_{n=-\infty}^{\infty} 2x_e(n)x_o(n) \\
&= E_e + E_o
\end{aligned}$$

2.6

(a) No, the system is time variant. Proof: If

$$\begin{aligned}
x(n) \to y(n) &= x(n^2) \\
x(n-k) \to y_1(n) &= x[(n-k)^2] \\
&= x(n^2 + k^2 - 2nk) \\
&\neq y(n-k)
\end{aligned}$$

(b) (1)

$$x(n) = \left\{0, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots\right\}$$

(2)

$$y(n) = x(n^2) = \left\{\ldots, 0, 1, \underset{\uparrow}{1}, 1, 0, \ldots\right\}$$

(3)

$$y(n-2) = \left\{\ldots, 0, \underset{\uparrow}{0}, 1, 1, 1, 0, \ldots\right\}$$

(4)

$$x(n-2) = \left\{\ldots, \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0, \ldots\right\}$$

(5)

$$y_2(n) = \mathcal{T}[x(n-2)] = \left\{\ldots, 0, 1, 0, \underset{\uparrow}{0}, 0, 1, 0, \ldots\right\}$$

(6)

$$y_2(n) \neq y(n-2) \Rightarrow \text{system is time variant.}$$

(c) (1)

$$x(n) = \left\{\underset{\uparrow}{1}, 1, 1, 1\right\}$$

(2)

$$y(n) = \left\{\underset{\uparrow}{1}, 0, 0, 0, 0, -1\right\}$$

(3)

$$y(n-2) = \left\{\underset{\uparrow}{0}, 0, 1, 0, 0, 0, 0, -1\right\}$$

(4)

$$x(n-2) = \left\{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1\right\}$$

(5)

$$y_2(n) = \left\{\underset{\uparrow}{0}, 0, 1, 0, 0, 0, 0, -1\right\}$$

(6)

$$y_2(n) = y(n-2).$$

The system is time invariant, but this example alone does not constitute a proof.

(d) (1)

$$y(n) = nx(n),$$

$$x(n) = \left\{\ldots, 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots\right\}$$

(2)

$$y(n) = \left\{\ldots, \underset{\uparrow}{0}, 1, 2, 3, \ldots\right\}$$

(3)

$$y(n-2) = \left\{\ldots, \underset{\uparrow}{0}, 0, 0, 1, 2, 3, \ldots\right\}$$

(4)

$$x(n-2) = \left\{\ldots, 0, \underset{\uparrow}{0}, 0, 1, 1, 1, 1, \ldots\right\}$$

(5)
$$y_2(n) = \mathcal{T}[x(n-2)] = \{\ldots, 0, 0, 2, 3, 4, 5, \ldots\}$$

(6)
$$y_2(n) \neq y(n-2) \Rightarrow \text{the system is time variant.}$$

2.7

(a) Static, nonlinear, time invariant, causal, stable.
(b) Dynamic, linear, time invariant, noncausal and unstable. The latter is easily proved. For the bounded input $x(k) = u(k)$, the output becomes

$$y(n) = \sum_{k=-\infty}^{n+1} u(k) = \begin{cases} 0, & n < -1 \\ n+2, & n \geq -1 \end{cases}$$

since $y(n) \to \infty$ as $n \to \infty$, the system is unstable.
(c) Static, linear, timevariant, causal, stable.
(d) Dynamic, linear, time invariant, noncausal, stable.
(e) Static, nonlinear, time invariant, causal, stable.
(f) Static, nonlinear, time invariant, causal, stable.

2.8

(a) True. If
$$v_1(n) = \mathcal{T}_1[x_1(n)] \text{ and}$$
$$v_2(n) = \mathcal{T}_1[x_2(n)],$$
then
$$\alpha_1 x_1(n) + \alpha_2 x_2(n)$$
yields
$$\alpha_1 v_1(n) + \alpha_2 v_2(n)$$
by the linearity property of $\mathcal{T}_1$. Similarly, if
$$y_1(n) = \mathcal{T}_2[v_1(n)] \text{ and}$$
$$y_2(n) = \mathcal{T}_2[v_2(n)],$$
then
$$\beta_1 v_1(n) + \beta_2 v_2(n) \to y(n) = \beta_1 y_1(n) + \beta_2 y_2(n)$$
by the linearity property of $\mathcal{T}_2$. Since
$$v_1(n) = \mathcal{T}_1[x_1(n)] \text{ and}$$
$$v_2(n) = \mathcal{T}_2[x_2(n)],$$
it follows that
$$A_1 x_1(n) + A_2 x_2(n)$$
yields the output
$$A_1 \mathcal{T}[x_1(n)] + A_2 \mathcal{T}[x_2(n)],$$

where $\mathcal{T} = \mathcal{T}_1\mathcal{T}_2$. Hence $\mathcal{T}$ is linear.
(b) True. For $\mathcal{T}_1$, if

$$x(n) \to v(n) \text{ and}$$
$$x(n-k) \to v(n-k),$$

For $\mathcal{T}_2$, if

$$v(n) \to y(n)$$
$$\text{and} v(n-k) \to y(n-k).$$

Hence, For $\mathcal{T}_1\mathcal{T}_2$, if

$$x(n) \to y(n) \text{ and}$$
$$x(n-k) \to y(n-k)$$

Therefore, $\mathcal{T} = \mathcal{T}_1\mathcal{T}_2$ is time invariant.
(c) True. $\mathcal{T}_1$ is causal $\Rightarrow v(n)$ depends only on x(k) for $k \leq n$. $\mathcal{T}_2$ is causal $\Rightarrow y(n)$ depends only on $v(k)$ for $k \leq n$. Therefore, $y(n)$ depends only on $x(k)$ for $k \leq n$. Hence, $\mathcal{T}$ is causal.
(d) True. Combine (a) and (b).
(e) True. This follows from $h_1(n) * h_2(n) = h_2(n) * h_1(n)$
(f) False. For example, consider

$$\mathcal{T}_1 : y(n) = nx(n) \text{ and}$$
$$\mathcal{T}_2 : y(n) = nx(n+1).$$

Then,

$$\begin{aligned} \mathcal{T}_2[\mathcal{T}_1[\delta(n)]] &= \mathcal{T}_2(0) = 0. \\ \mathcal{T}_1[\mathcal{T}_2[\delta(n)]] &= \mathcal{T}_1[\delta(n+1)] \\ &= -\delta(n+1) \\ &\neq 0. \end{aligned}$$

(g) False. For example, consider

$$\mathcal{T}_1 : y(n) = x(n) + b \text{ and}$$
$$\mathcal{T}_2 : y(n) = x(n) - b, \text{ where } b \neq 0.$$

Then,

$$\mathcal{T}[x(n)] = \mathcal{T}_2[\mathcal{T}_1[x(n)]] = \mathcal{T}_2[x(n) + b] = x(n).$$

Hence $\mathcal{T}$ is linear.
(h) True.

$$\mathcal{T}_1 \text{ is stable } \Rightarrow v(n) \text{ is bounded if } x(n) \text{ is bounded.}$$
$$\mathcal{T}_2 \text{ is stable } \Rightarrow y(n) \text{ is bounded if } v(n) \text{ is bounded .}$$

Hence, y(n) is bounded if x(n) is bounded $\Rightarrow \mathcal{T} = \mathcal{T}_1\mathcal{T}_2$ is stable.
(i) Inverse of (c). $\mathcal{T}_1$ and for $\mathcal{T}_2$ are noncausal $\Rightarrow \mathcal{T}$ is noncausal. Example:

$$\begin{aligned} \mathcal{T}_1 : y(n) &= x(n+1) \text{ and} \\ \mathcal{T}_2 : y(n) &= x(n-2) \\ \Rightarrow \mathcal{T} : y(n) &= x(n-1), \end{aligned}$$

which is causal. Hence, the inverse of (c) is false.
Inverse of (h): $\mathcal{T}_1$ and/or $\mathcal{T}_2$ is unstable, implies $\mathcal{T}$ is unstable. Example:

$$\mathcal{T}_1 : y(n) = e^{x(n)}, \text{ stable and } \mathcal{T}_2 : y(n) = ln[x(n)], \text{ which is unstable.}$$

But $\mathcal{T} : y(n) = x(n)$, which is stable. Hence, the inverse of (h) is false.

2.9

(a)

$$
\begin{aligned}
y(n) &= \sum_{k=-\infty}^{n} h(k)x(n-k), x(n)=0, n<0 \\
y(n+N) &= \sum_{k=-\infty}^{n+N} h(k)x(n+N-k) = \sum_{k=-\infty}^{n+N} h(k)x(n-k) \\
&= \sum_{k=-\infty}^{n} h(k)x(n-k) + \sum_{k=n+1}^{n+N} h(k)x(n-k) \\
&= y(n) + \sum_{k=n+1}^{n+N} h(k)x(n-k)
\end{aligned}
$$

For a BIBO system, $\lim_{n\to\infty}|h(n)| = 0$. Therefore,

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)x(n-k) = 0 \text{ and}$$

$$\lim_{n\to\infty} y(n+N) = y(N).$$

(b) Let $x(n) = x_o(n) + au(n)$, where a is a constant and

$$x_o(n) \text{ is a bounded signal with } \lim_{n\to\infty} x_o(n) = 0.$$

Then,

$$
\begin{aligned}
y(n) &= a\sum_{k=0}^{\infty} h(k)u(n-k) + \sum_{k=0}^{\infty} h(k)x_o(n-k) \\
&= a\sum_{k=0}^{n} h(k) + y_o(n)
\end{aligned}
$$

clearly, $\sum_n x_o^2(n) < \infty \Rightarrow \sum_n y_o^2(n) < \infty$ (from (c) below) Hence,

$$\lim_{n\to\infty}|y_o(n)| = 0.$$

and, thus, $\lim_{n\to\infty} y(n) = a\sum_{k=0}^{n} h(k) = \text{constant}$.
(c)

$$
\begin{aligned}
y(n) &= \sum_k h(k)x(n-k) \\
\sum_{-\infty}^{\infty} y^2(n) &= \sum_{-\infty}^{\infty} \left[\sum_k h(k)x(n-k)\right]^2 \\
&= \sum_k \sum_l h(k)h(l) \sum_n x(n-k)x(n-l)
\end{aligned}
$$

But

$$\sum_n x(n-k)x(n-l) \le \sum_n x^2(n) = E_x.$$

Therefore,

$$\sum_n y^2(n) \le E_x \sum_k |h(k)| \sum_l |h(l)|.$$

For a BIBO stable system,

$$\sum_k |h(k)| < M.$$

Hence,

$$E_y \le M^2 E_x, \text{ so that}$$
$$E_y < 0 \text{ if } E_x < 0.$$

2.10

The system is nonlinear. This is evident from observation of the pairs

$$x_3(n) \leftrightarrow y_3(n) \text{ and } x_2(n) \leftrightarrow y_2(n).$$

If the system were linear, $y_2(n)$ would be of the form

$$y_2(n) = \{3, 6, 3\}$$

because the system is time-invariant. However, this is not the case.

2.11

since

$$x_1(n) + x_2(n) = \delta(n)$$

and the system is linear, the impulse response of the system is

$$y_1(n) + y_2(n) = \left\{0, \underset{\uparrow}{3}, -1, 2, 1\right\}.$$

If the system were time invariant, the response to $x_3(n)$ would be

$$\left\{\underset{\uparrow}{3}, 2, 1, 3, 1\right\}.$$

But this is not the case.

2.12

(a) Any weighted linear combination of the signals $x_i(n), i = 1, 2, \ldots, N$.
(b) Any $x_i(n-k)$, where k is any integer and $i = 1, 2, \ldots, N$.

2.13

A system is BIBO stable if and only if a bounded input produces a bounded output.

$$\begin{aligned} y(n) &= \sum_k h(k)x(n-k) \\ |y(n)| &\le \sum_k |h(k)||x(n-k)| \\ &\le M_x \sum_k |h(k)| \end{aligned}$$

where $|x(n-k)| \leq M_x$. Therefore, $|y(n)| < \infty$ for all n, if and only if

$$\sum_k |h(k)| < \infty.$$

2.14

(a) A system is causal $\Leftrightarrow$ the output becomes nonzero after the input becomes non-zero. Hence,

$$x(n) = 0 \text{ for } n < n_o \Rightarrow y(n) = 0 \text{ for } n < n_o.$$

(b)

$$y(n) = \sum_{-\infty}^{n} h(k)x(n-k), \text{ where } x(n) = 0 \text{ for } n < 0.$$

If $h(k) = 0$ for $k < 0$, then

$$y(n) = \sum_{0}^{n} h(k)x(n-k), \text{ and hence, } y(n) = 0 \text{ for } n < 0.$$

On the other hand, if $y(n) = 0$ for $n < 0$, then

$$\sum_{-\infty}^{n} h(k)x(n-k) \Rightarrow h(k) = 0, k < 0.$$

2.15

(a)

$$\begin{aligned}
\text{For } a = 1, \sum_{n=M}^{N} a^n &= N - M + 1 \\
\text{for } a \neq 1, \sum_{n=M}^{N} a^n &= a^M + a^{M+1} + \ldots + a^N \\
(1-a)\sum_{n=M}^{N} a^n &= a^M + a^{M+1} - a^{M+1} + \ldots + a^N - a^N - a^{N+1} \\
&= a^M - a^{N+1}
\end{aligned}$$

(b) For $M = 0, |a| < 1$, and $N \to \infty$,

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, |a| < 1.$$

2.16

(a)

$$
\begin{aligned}
y(n) &= \sum_k h(k)x(n-k) \\
\sum_n y(n) &= \sum_n \sum_k h(k)x(n-k) = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k) \\
&= \left(\sum_k h(k)\right)\left(\sum_n x(n)\right)
\end{aligned}
$$

(b) (1)

$$y(n) = h(n) * x(n) = \{1, 3, 7, 7, 7, 6, 4\}$$

$$\sum_n y(n) = 35, \quad \sum_k h(k) = 5, \quad \sum_k x(k) = 7$$

(2)

$$y(n) = \{1, 4, 2, -4, 1\}$$

$$\sum_n y(n) = 4, \quad \sum_k h(k) = 2, \quad \sum_k x(k) = 2$$

(3)

$$y(n) = \left\{0, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -2, 0, -\frac{5}{2}, -2\right\}$$

$$\sum_n y(n) = -5, \quad \sum_n h(n) = 2.5, \quad \sum_n x(n) = -2$$

(4)

$$y(n) = \{1, 2, 3, 4, 5\}$$

$$\sum_n y(n) = 15, \quad \sum_n h(n) = 1, \quad \sum_n x(n) = 15$$

(5)

$$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$$

$$\sum_n y(n) = 8, \quad \sum_n h(n) = 4, \quad \sum_n x(n) = 2$$

2.17

(a)

$$\begin{aligned}
x(n) &= \left\{\underset{\uparrow}{1}, 1, 1, 1\right\} \\
h(n) &= \left\{\underset{\uparrow}{6}, 5, 4, 3, 2, 1\right\} \\
y(n) &= \sum_{k=0}^{n} x(k)h(n-k) \\
y(0) &= x(0)h(0) = 6, \\
y(1) &= x(0)h(1) + x(1)h(0) = 11 \\
y(2) &= x(0)h(2) + x(1)h(1) + x(2)h(0) = 15 \\
y(3) &= x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0) = 18 \\
y(4) &= x(0)h(4) + x(1)h(3) + x(2)h(2) + x(3)h(1) + x(4)h(0) = 14 \\
y(5) &= x(0)h(5) + x(1)h(4) + x(2)h(3) + x(3)h(2) + x(4)h(1) + x(5)h(0) = 10 \\
y(6) &= x(1)h(5) + x(2)h(4) + x(3)h(2) = 6 \\
y(7) &= x(2)h(5) + x(3)h(4) = 3 \\
y(8) &= x(3)h(5) = 1 \\
y(n) &= 0, n \geq 9 \\
y(n) &= \left\{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1\right\}
\end{aligned}$$

(b) By following the same procedure as in (a), we obtain

$$y(n) = \left\{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\right\}$$

(c) By following the same procedure as in (a), we obtain

$$y(n) = \left\{1, \underset{\uparrow}{2}, 2, 2, 1\right\}$$

(d) By following the same procedure as in (a), we obtain

$$y(n) = \left\{\underset{\uparrow}{1}, 2, 2, 2, 1\right\}$$

2.18

(a)

$$\begin{aligned}
x(n) &= \left\{\underset{\uparrow}{0}, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\right\} \\
h(n) &= \left\{1, 1, \underset{\uparrow}{1}, 1, 1\right\} \\
y(n) &= x(n) * h(n) \\
&= \left\{\frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\right\}
\end{aligned}$$

(b)

$$\begin{aligned}
x(n) &= \frac{1}{3}n[u(n) - u(n-7)], \\
h(n) &= u(n+2) - u(n-3) \\
y(n) &= x(n) * h(n) \\
&= \frac{1}{3}n[u(n) - u(n-7)] * [u(n+2) - u(n-3)] \\
&= \frac{1}{3}n[u(n) * u(n+2) - u(n) * u(n-3) - u(n-7) * u(n+2) + u(n-7) * u(n-3)] \\
y(n) &= \frac{1}{3}\delta(n+1) + \delta(n) + 2\delta(n-1) + \frac{10}{3}\delta(n-2) + 5\delta(n-3) + \frac{20}{3}\delta(n-4) + 6\delta(n-5) \\
& +5\delta(n-6) + 5\delta(n-6) + \frac{11}{3}\delta(n-7) + \delta(n-8)
\end{aligned}$$

2.19

$$\begin{aligned}
y(n) &= \sum_{k=0}^{4} h(k)x(n-k), \\
x(n) &= \left\{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \ldots, \alpha^{5}\right\} \\
h(n) &= \left\{\underset{\uparrow}{1}, 1, 1, 1, 1\right\} \\
y(n) &= \sum_{k=0}^{4} x(n-k), -3 \leq n \leq 9 \\
&= 0, \text{ otherwise.}
\end{aligned}$$

Therefore,

$$\begin{aligned}
y(-3) &= \alpha^{-3}, \\
y(-2) &= x(-3) + x(-2) = \alpha^{-3} + \alpha^{-2}, \\
y(-1) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1}, \\
y(0) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 \\
y(1) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha, \\
y(2) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^{2} \\
y(3) &= \alpha^{-1} + 1 + \alpha + \alpha^{2} + \alpha^{3}, \\
y(4) &= \alpha^{4} + \alpha^{3} + \alpha^{2} + \alpha + 1 \\
y(5) &= \alpha + \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5}, \\
y(6) &= \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5} \\
y(7) &= \alpha^{3} + \alpha^{4} + \alpha^{5}, \\
y(8) &= \alpha^{4} + \alpha^{5}, \\
y(9) &= \alpha^{5}
\end{aligned}$$

2.20

(a) 131 x 122 = 15982

(b) $\{1_{\uparrow}, 3, 1\} * \{1_{\uparrow}, 2, 2\} = \{1, 5, 9, 8, 2\}$

(c) $(1 + 3z + z^2)(1 + 2z + 2z^2) = 1 + 5z + 9z^2 + 8z^3 + 2z^4$
(d) 1.31 x 12.2 = 15.982.
(e) These are different ways to perform convolution.

2.21

(a)

$$y(n) = \sum_{k=0}^{n} a^k u(k) b^{n-k} u(n-k) = b^n \sum_{k=0}^{n} (ab^{-1})^k$$

$$y(n) = \begin{cases} \frac{b^{n+1}-a^{n+1}}{b-a} u(n), & a \neq b \\ b^n (n+1) u(n), & a = b \end{cases}$$

(b)

$$\begin{aligned} x(n) &= \left\{1, 2, \underset{\uparrow}{1}, 1\right\} \\ h(n) &= \left\{\underset{\uparrow}{1}, -1, 0, 0, 1, 1\right\} \\ y(n) &= \left\{1, 1, -\underset{\uparrow}{1}, 0, 0, 3, 3, 2, 1\right\} \end{aligned}$$

(c)

$$\begin{aligned} x(n) &= \left\{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\right\}, \\ h(n) &= \left\{1, 2, \underset{\uparrow}{3}, 2, 1\right\} \\ y(n) &= \left\{1, 3, 6, \underset{\uparrow}{8}, 9, 8, 5, 1, -2, -2, -1\right\} \end{aligned}$$

(d)

$$\begin{aligned} x(n) &= \left\{\underset{\uparrow}{1}, 1, 1, 1, 1\right\}, \\ h'(n) &= \left\{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1, 1\right\} \\ h(n) &= h'(n) + h'(n-9), \\ y(n) &= y'(n) + y'(n-9), \text{ where} \\ y'(n) &= \left\{\underset{\uparrow}{0}, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\right\} \end{aligned}$$

2.22

(a)

$$\begin{aligned} y_i(n) &= x(n) * h_i(n) \\ y_1(n) &= x(n) + x(n-1) \\ &= \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 12, 15, 9\}, \text{ similarly} \\ y_2(n) &= \{1, 6, 11, 11, 13, 16, 14, 13, 15, 21, 25, 28, 24, 9\} \\ y_3(n) &= \{0.5, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, 4.5\} \\ y_4(n) &= \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 3.75, 5.25, 6.25, 7, 6, 2.25\} \\ y_5(n) &= \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0, 0.25, -0.75, 1, -3, -2.25\} \end{aligned}$$

(b)

$$\begin{aligned} y_3(n) &= \frac{1}{2} y_1(n), \text{ because} \\ h_3(n) &= \frac{1}{2} h_1(n) \\ y_4(n) &= \frac{1}{4} y_2(n), \text{ because} \\ h_4(n) &= \frac{1}{4} h_2(n) \end{aligned}$$

(c) $y_2(n)$ and $y_4(n)$ are smoother than $y_1(n)$, but $y_4(n)$ will appear even smoother because of the smaller scale factor.

(d) System 4 results in a smoother output. The negative value of $h_5(0)$ is responsible for the non-smooth characteristics of $y_5(n)$

(e)

$$y_6(n) = \left\{\frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{3}{2}, -\frac{9}{2}\right\}$$

$y_2(n)$ is smoother than $y_6(n)$.

2.23

We can express the unit sample in terms of the unit step function as $\delta(n) = u(n) - u(n-1)$. Then,

$$\begin{aligned} h(n) &= h(n) * \delta(n) \\ &= h(n) * (u(n) - u(n-1)) \\ &= h(n) * u(n) - h(n) * u(n-1) \\ &= s(n) - s(n-1) \end{aligned}$$

Using this definition of $h(n)$

$$\begin{aligned} y(n) &= h(n) * x(n) \\ &= (s(n) - s(n-1)) * x(n) \\ &= s(n) * x(n) - s(n-1) * x(n) \end{aligned}$$

2.24

If

$$\begin{aligned} y_1(n) &= ny_1(n-1) + x_1(n) \text{ and} \\ y_2(n) &= ny_2(n-1) + x_2(n) \text{ then} \\ x(n) &= ax_1(n) + bx_2(n) \end{aligned}$$

produces the output

$$y(n) = ny(n-1) + x(n), \text{ where}$$
$$y(n) = ay_1(n) + by_2(n).$$

Hence, the system is linear. If the input is $x(n-1)$, we have

$$\begin{aligned} y(n-1) &= (n-1)y(n-2) + x(n-1). \text{ But} \\ y(n-1) &= ny(n-2) + x(n-1). \end{aligned}$$

Hence, the system is time variant. If $x(n) = u(n)$, then $|x(n)| \le 1$. But for this bounded input, the output is

$$y(0) = 1, \quad y(1) = 1 + 1 = 2, \quad y(2) = 2\text{x}2 + 1 = 5, \ldots$$

which is unbounded. Hence, the system is unstable.

2.25

(a)

$$\begin{aligned} \delta(n) &= \gamma(n) - a\gamma(n-1) \text{ and,} \\ \delta(n-k) &= \gamma(n-k) - a\gamma(n-k-1). \text{ Then,} \\ x(n) &= \sum_{k=-\infty}^{\infty} x(k)\delta(n-k) \\ &= \sum_{k=-\infty}^{\infty} x(k)[\gamma(n-k) - a\gamma(n-k-1)] \\ x(n) &= \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k)\gamma(n-k-1) \\ x(n) &= \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k-1)\gamma(n-k) \\ &= \sum_{k=-\infty}^{\infty} [x(k) - ax(k-1)]\gamma(n-k) \\ \text{Thus, } c_k &= x(k) - ax(k-1) \end{aligned}$$

(b)

$$\begin{aligned} y(n) &= \mathcal{T}[x(n)] \\ &= \mathcal{T}[\sum_{k=-\infty}^{\infty} c_k\gamma(n-k)] \\ &= \sum_{k=-\infty}^{\infty} c_k\mathcal{T}[\gamma(n-k)] \\ &= \sum_{k=-\infty}^{\infty} c_k g(n-k) \end{aligned}$$

(c)

$$\begin{aligned} h(n) &= \mathcal{T}[\delta(n)] \\ &= T[\gamma(n) - a\gamma(n-1)] \\ &= g(n) - ag(n-1) \end{aligned}$$

2.26

With x(n) = 0, we have

$$\begin{aligned} y(n-1) + \frac{4}{3}y(n-1) &= 0 \\ y(-1) &= -\frac{4}{3}y(-2) \\ y(0) &= (-\frac{4}{3})^2 y(-2) \\ y(1) &= (-\frac{4}{3})^3 y(-2) \\ &\vdots \\ y(k) &= (-\frac{4}{3})^{k+2} y(-2) \leftarrow \text{ zero-input response.} \end{aligned}$$

2.27

$$\begin{aligned} h(n) &= h_1(n) * h_2(n) \\ &= \sum_{k=-\infty}^{\infty} a^k [u(k) - u(k-N)][u(n-k) - u(n-k-M)] \\ &= \sum_{k=-\infty}^{\infty} a^k u(k)u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k)u(n-k-M) \\ &\quad - \sum_{k=-\infty}^{\infty} a^k u(k-N)u(n-k) + \sum_{k=-\infty}^{\infty} a^k u(k-N)u(n-k-M) \\ &= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-M} a^k\right) - \left(\sum_{k=N}^{n} a^k - \sum_{k=N}^{n-M} a^k\right) \\ &= 0 \end{aligned}$$

2.28

(a) $L_1 = N_1 + M_1$ and $L_2 = N_2 + M_2$

(b) Partial overlap from left:

low $N_1 + M_1$ high $N_1 + M_2 - 1$

Full overlap: low $N_1 + M_2$ high $N_2 + M_1$

Partial overlap from right:

low $N_2 + M_1 + 1$ high $N_2 + M_2$

(c)

$$
\begin{aligned}
x(n) &= \left\{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\right\} \\
h(n) &= \left\{2, \underset{\uparrow}{2}, 2, 2\right\} \\
N_1 &= -2, \\
N_2 &= 4, \\
M_1 &= -1, \\
M_2 &= 2,
\end{aligned}
$$

$$\text{Partial overlap from left: } n = -3 \qquad n = -1 \qquad L_1 = -3$$

$$\text{Full overlap: } n = 0 \qquad n = 3$$

$$\text{Partial overlap from right:} n = 4 \qquad n = 6 \qquad L_2 = 6$$

2.29

(a)

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n).$$

The characteristic equation is

$$\lambda^2 - 0.6\lambda + 0.08 = 0.$$

$\lambda = 0.2, 0.4$ Hence,

$$y_h(n) = c_1 \frac{1}{5}^n + c_2 \frac{2}{5}^n.$$

With $x(n) = \delta(n)$, the initial conditions are

$$
\begin{aligned}
y(0) &= 1, \\
y(1) - 0.6y(0) &= 0 \Rightarrow y(1) = 0.6. \\
\text{Hence,} c_1 + c_2 &= 1 \text{ and} \\
\frac{1}{5}c_1 + \frac{2}{5} &= 0.6 \Rightarrow c_1 = -1, c_2 = 3. \\
\text{Therefore } h(n) &= \left[-(\frac{1}{5})^n + 2(\frac{2}{5})^n\right] u(n)
\end{aligned}
$$

The step response is

$$
\begin{aligned}
s(n) &= \sum_{k=0}^{n} h(n-k), n \geq 0 \\
&= \sum_{k=0}^{n} \left[2(\frac{2}{5})^{n-k} - (\frac{1}{5})^{n-k}\right] \\
&= \left\{\frac{1}{0.12}\left[(\frac{2}{5}^{n+1} - 1\right] - \frac{1}{0.16}\left[(\frac{1}{5}^{n+1} - 1\right]\right\} u(n)
\end{aligned}
$$

(b)

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2).$$

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0.$$

$\lambda = \frac{1}{2}, \frac{1}{5}$ Hence,

$$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n.$$

With $x(n) = \delta(n)$, we have

$$\begin{aligned}
y(0) &= 2, \\
y(1) - 0.7y(0) &= 0 \Rightarrow y(1) = 1.4. \\
\text{Hence,} c_1 + c_2 &= 2 \text{ and} \\
\frac{1}{2}c_1 + \frac{1}{5} &= 1.4 = \frac{7}{5} \\
\Rightarrow c_1 + \frac{2}{5}c_2 &= \frac{14}{5}.
\end{aligned}$$

These equations yield

$$c_1 = \frac{10}{3}, c_2 = -\frac{4}{3}.$$

$$h(n) = \left[\frac{10}{3}(\frac{1}{2})^n - \frac{4}{3}(\frac{1}{5})^n\right] u(n)$$

The step response is

$$\begin{aligned}
s(n) &= \sum_{k=0}^{n} h(n-k), \\
&= \frac{10}{3}\sum_{k=0}^{n}(\frac{1}{2})^{n-k} - \frac{4}{3}\sum_{k=0}^{n}(\frac{1}{5})^{n-k} \\
&= \frac{10}{3}(\frac{1}{2})^n\sum_{k=0}^{n}2^k - \frac{4}{3}(\frac{1}{5})^n\sum_{k=0}^{n}5^k \\
&= \frac{10}{3}(\frac{1}{2}^n(2^{n+1}-1)u(n) - \frac{1}{3}(\frac{1}{5}^n(5^{n+1}-1)u(n)
\end{aligned}$$

2.30

$$\begin{aligned}
h(n) &= \left\{\underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\right\} \\
y(n) &= \left\{\underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0\right\} \\
x(0)h(0) &= y(0) \Rightarrow x(0) = 1 \\
\frac{1}{2}x(0) + x(1) &= y(1) \Rightarrow x(1) = \frac{3}{2}
\end{aligned}$$

By continuing this process, we obtain

$$x(n) = \left\{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2}, \ldots\right\}$$

2.31

(a) $h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$
(b)

$$\begin{aligned}
h_3(n) * h_4(n) &= (n-1)u(n-2) \\
h_2(n) - h_3(n) * h_4(n) &= 2u(n) - \delta(n) \\
h_1(n) &= \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2) \\
\text{Hence } h(n) &= \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right] * [2u(n) - \delta(n)] \\
&= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)
\end{aligned}$$

(c)

$$\begin{aligned}
x(n) &= \left\{1, 0, \underset{\uparrow}{0}, 3, 0, -4\right\} \\
y(n) &= \left\{\frac{1}{2}, \frac{5}{4}, \underset{\uparrow}{2}, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, 0, \ldots\right\}
\end{aligned}$$

2.32

First, we determine

$$\begin{aligned}
s(n) &= u(n) * h(n) \\
s(n) &= \sum_{k=0}^{\infty} u(k)h(n-k) \\
&= \sum_{k=0}^{n} h(n-k) \\
&= \sum_{k=0}^{\infty} a^{n-k} \\
&= \frac{a^{n+1}-1}{a-1}, n \geq 0
\end{aligned}$$

For $x(n) = u(n+5) - u(n-10)$, we have the response

$$s(n+5) - s(n-10) = \frac{a^{n+6}-1}{a-1}u(n+5) - \frac{a^{n-9}-1}{a-1}u(n-10)$$

From figure P2.33,

$$\begin{aligned}
y(n) &= x(n) * h(n) - x(n) * h(n-2) \\
\text{Hence, } y(n) &= \frac{a^{n+6}-1}{a-1}u(n+5) - \frac{a^{n-9}-1}{a-1}u(n-10) \\
&\quad - \frac{a^{n+4}-1}{a-1}u(n+3) + \frac{a^{n-11}-1}{a-1}u(n-12)
\end{aligned}$$

2.33

$$
\begin{aligned}
h(n) &= \left[u(n) - u(n-M)\right]/M \\
s(n) &= \sum_{k=-\infty}^{\infty} u(k)h(n-k) \\
&= \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{M}, & n < M \\ 1, & n \geq M \end{cases}
\end{aligned}
$$

2.34

$$
\begin{aligned}
\sum_{n=-\infty}^{\infty} |h(n)| &= \sum_{n=0, neven}^{\infty} |a|^n \\
&= \sum_{n=0}^{\infty} |a|^{2n} \\
&= \frac{1}{1-|a|^2}
\end{aligned}
$$

Stable if $|a| < 1$

2.35

$h(n) = a^n u(n)$. The response to $u(n)$ is

$$
\begin{aligned}
y_1(n) &= \sum_{k=0}^{\infty} u(k)h(n-k) \\
&= \sum_{k=0}^{n} a^{n-k} \\
&= a^n \sum_{k=0}^{n} a^{-k} \\
&= \frac{1-a^{n+1}}{1-a} u(n) \\
\text{Then, } y(n) &= y_1(n) - y_1(n-10) \\
&= \frac{1}{1-a}\left[(1-a^{n+1})u(n) - (1-a^{n-9})u(n-10)\right]
\end{aligned}
$$

2.36

We may use the result in problem 2.32 with $a = \frac{1}{2}$. Thus,

$$
y(n) = 2\left[1 - (\frac{1}{2})^{n+1}\right] u(n) - 2\left[1 - (\frac{1}{2})^{n-9}\right] u(n-10)
$$

2.37

(a)

$$\begin{aligned} y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \\ &= \sum_{k=0}^{n} (\frac{1}{2})^k 2^{n-k} \\ &= 2^n \sum_{k=0}^{n} (\frac{1}{4})^k \\ &= 2^n \left[1 - (\frac{1}{4})^{n+1}\right] (\frac{4}{3}) \\ &= \frac{2}{3} \left[2^{n+1} - (\frac{1}{2})^{n+1}\right] u(n) \end{aligned}$$

(b)

$$\begin{aligned} y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \\ &= \sum_{k=0}^{\infty} h(k) \\ &= \sum_{k=0}^{\infty} (\frac{1}{2})^k = 2, n < 0 \\ y(n) &= \sum_{k=n}^{\infty} h(k) \\ &= \sum_{k=n}^{\infty} (\frac{1}{2})^k \\ &= \sum_{k=0}^{\infty} (\frac{1}{2})^k - \sum_{k=0}^{n-1} (\frac{1}{2})^k \\ &= 2 - (\frac{1 - (\frac{1}{2})^n}{\frac{1}{2}}) \\ &= 2(\frac{1}{2})^n, n \geq 0. \end{aligned}$$

2.38

(a)

$$\begin{aligned} h_e(n) &= h_1(n) * h_2(n) * h_3(n) \\ &= [\delta(n) - \delta(n-1)] * u(n) * h(n) \\ &= [u(n) - u(n-1)] * h(n) \\ &= \delta(n) * h(n) \\ &= h(n) \end{aligned}$$

(b) No.

2.39

(a) $x(n)\delta(n-n_0) = x(n_0)$. Thus, only the value of $x(n)$ at $n = n_0$ is of interest.
$x(n) * \delta(n-n_0) = x(n-n_0)$. Thus, we obtain the shifted version of the sequence $x(n)$.
(b)

$$\begin{aligned}
y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \\
&= h(n) * x(n) \\
\text{Linearity:} x_1(n) \to y_1(n) &= h(n) * x_1(n) \\
x_2(n) \to y_2(n) &= h(n) * x_2(n) \\
\text{Then } x(n) &= \alpha x_1(n) + \beta x_2(n) \to y(n) = h(n) * x(n) \\
y(n) &= h(n) * [\alpha x_1(n) + \beta x_2(n)] \\
&= \alpha h(n) * x_1(n) + \beta h(n) * x_2(n) \\
&= \alpha y_1(n) + \beta y_2(n) \\
\text{Time Invariance:} & \\
x(n) \to y(n) &= h(n) * x(n) \\
x(n-n_0) \to y_1(n) &= h(n) * x(n-n_0) \\
&= \sum_k h(k)x(n-n_0-k) \\
&= y(n-n_0)
\end{aligned}$$

(c) $h(n) = \delta(n-n_0)$.

2.40

(a) $s(n) = -a_1 s(n-1) - a_2 s(n-2) - \ldots - a_N s(n-N) + b_0 v(n)$. Refer to fig 2.40-1.
(b) $v(n) = \frac{1}{b_0}\left[s(n) + a_1 s(n-1) + a_2 s(n-2) + \ldots + a_N s(n-N)\right]$. Refer to fig 2.40-2

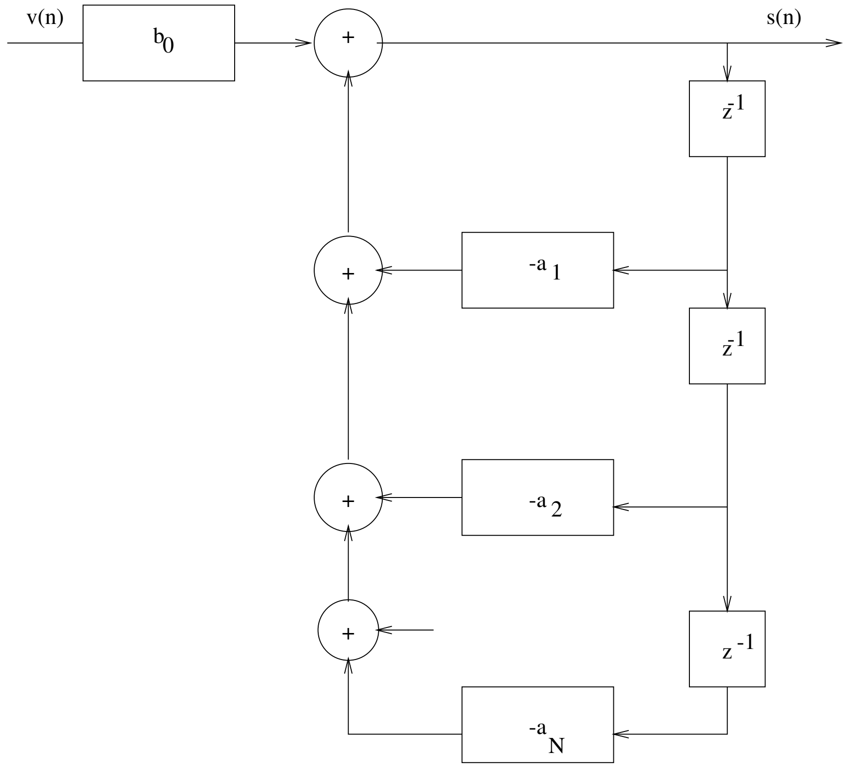

Figure 2.40-1:

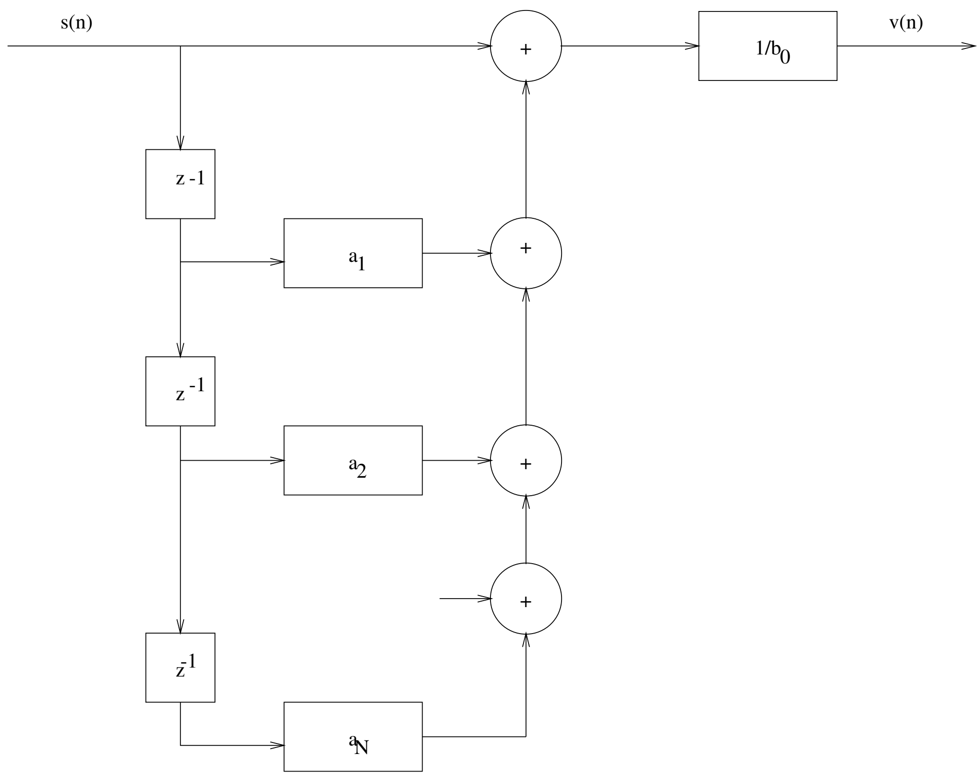

Figure 2.40-2:

2.41

$$
\begin{aligned}
y(n) &= -\frac{1}{2}y(n-1) + x(n) + 2x(n-2) \\
y(-2) &= -\frac{1}{2}y(-3) + x(-2) + 2x(-4) = 1 \\
y(-1) &= -\frac{1}{2}y(-2) + x(-1) + 2x(-3) = \frac{3}{2} \\
y(0) &= -\frac{1}{2}y(-1) + 2x(-2) + x(0) = \frac{17}{4} \\
y(1) &= -\frac{1}{2}y(0) + x(1) + 2x(-1) = \frac{47}{8}, \text{ etc}
\end{aligned}
$$

2.42

(a) Refer to fig 2.42-1
(b) Refer to fig 2.42-2

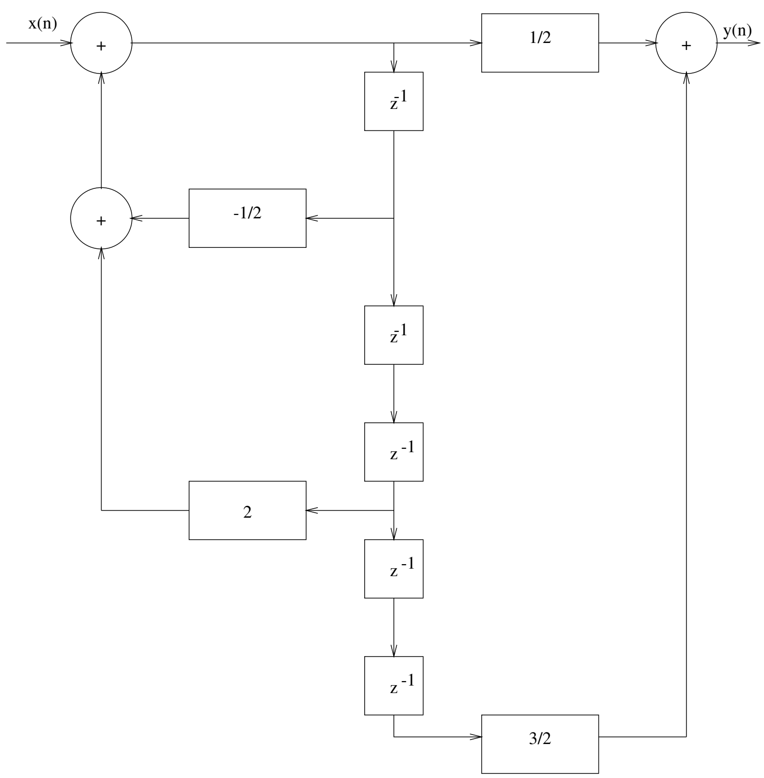

Figure 2.42-1:

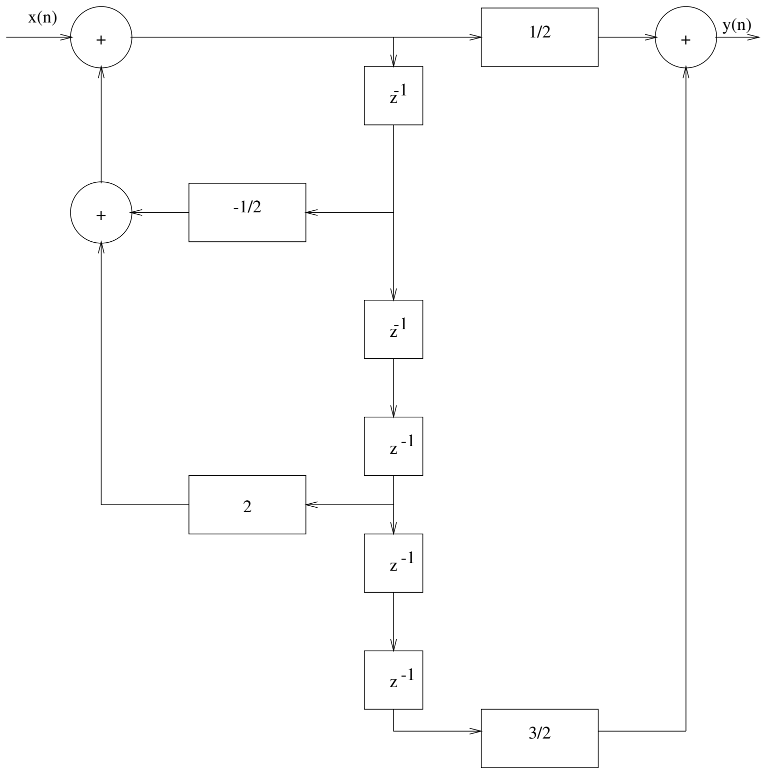

Figure 2.42-2:

2.43

(a)

$$\begin{aligned}
x(n) &= \left\{\underset{\uparrow}{1}, 0, 0, \ldots\right\} \\
y(n) &= \frac{1}{2}y(n-1) + x(n) + x(n-1) \\
y(0) &= x(0) = 1, \\
y(1) &= \frac{1}{2}y(0) + x(1) + x(0) = \frac{3}{2} \\
y(2) &= \frac{1}{2}y(1) + x(2) + x(1) = \frac{3}{4}. \text{ Thus, we obtain} \\
y(n) &= \left\{1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, \ldots\right\}
\end{aligned}$$

(b) $y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$
(c) As in part(a), we obtain

$$y(n) = \left\{1, \frac{5}{2}, \frac{13}{4}, \frac{29}{8}, \frac{61}{16}, \ldots\right\}$$