Solutions for Precalculus A Unit Circle Approach 4th Edition by Ratti

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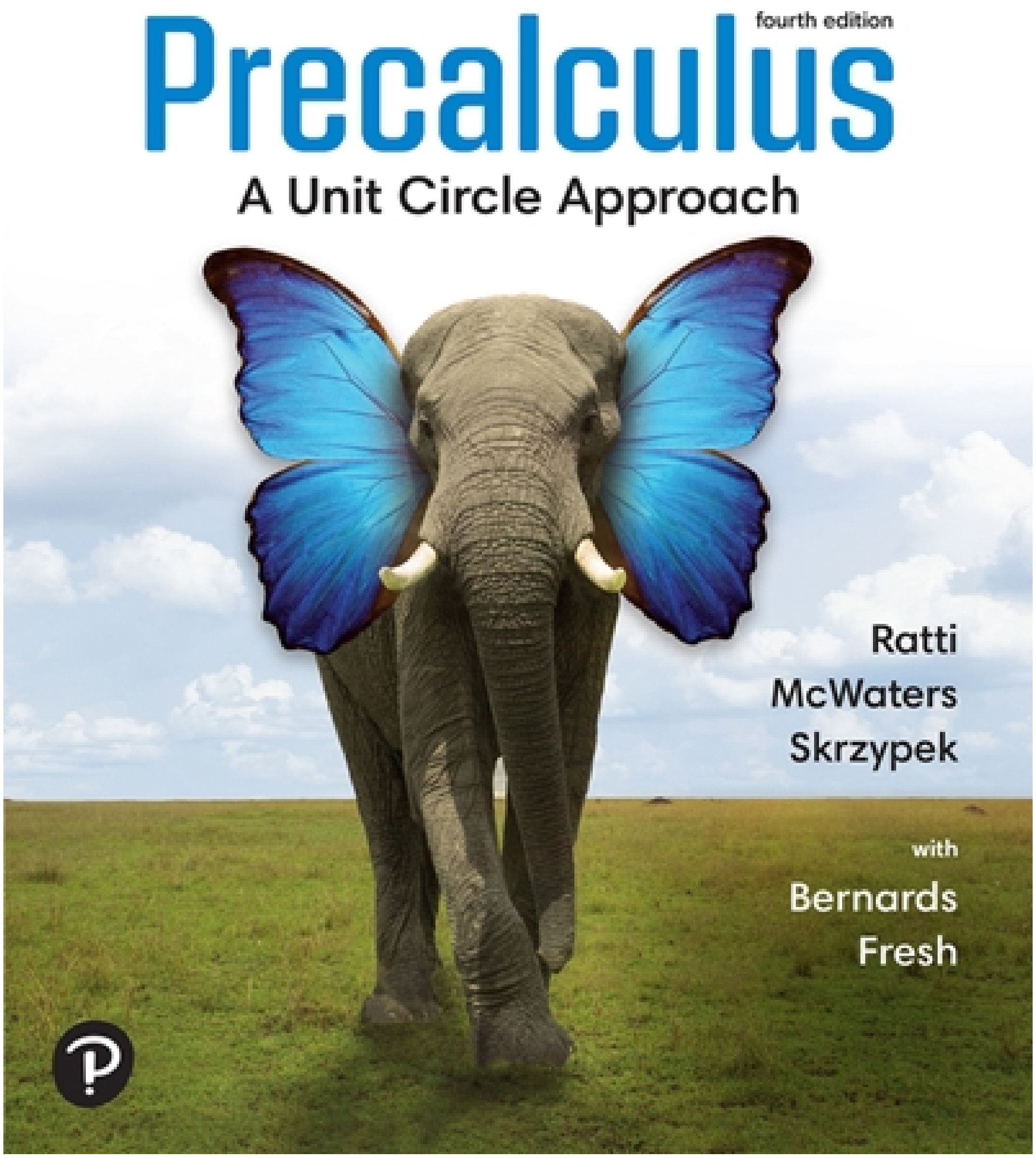

Solutions

INSTRUCTOR'S SOLUTIONS MANUAL

BEVERLY FUSFIELD

PRECALCULUS
A UNIT CIRCLE APPROACH
FOURTH EDITION

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Chapter 1 Graphs and Functions

Getting Ready for the Next Section

GR1. $\dfrac{2+5}{2}=\dfrac{7}{2}=3.5$

GR2. $\dfrac{-3+7}{2}=\dfrac{4}{2}=2$

GR3. $\dfrac{-3-7}{2}=\dfrac{-10}{2}=-5$

GR4. $\dfrac{\left(3-\sqrt{2}\right)-\left(3+\sqrt{2}\right)}{2}=\dfrac{-2\sqrt{2}}{2}=-\sqrt{2}$

GR5. $\sqrt{(5-2)^2+(3-7)^2}=\sqrt{3^2+(-4)^2}$
$=\sqrt{9+16}=\sqrt{25}=5$

GR6. $\sqrt{(-8+3)^2+(-5-7)^2}=\sqrt{(-5)^2+(-12)^2}$
$=\sqrt{25+144}=\sqrt{169}$
$=13$

GR7. $\sqrt{(2-5)^2+(8-6)^2}=\sqrt{(-3)^2+2^2}=\sqrt{9+4}$
$=\sqrt{13}$

GR8. $\sqrt{\left(\sqrt{3}-\sqrt{12}\right)^2+\left(\sqrt{2}+\sqrt{8}\right)^2}$
$=\sqrt{\left(\sqrt{3}-2\sqrt{3}\right)^2+\left(\sqrt{2}+2\sqrt{2}\right)^2}$
$=\sqrt{\left(-\sqrt{3}\right)^2+\left(3\sqrt{2}\right)^2}=\sqrt{3+9\cdot 2}$
$=\sqrt{3+18}=\sqrt{21}$

GR9. $x^2+4x+\underline{4}=(x+2)^2$

GR10. $x^2-6x+\underline{9}=(x-3)^2$

GR11. $x^2-5x+\left(\dfrac{-5}{2}\right)^2=x^2-5x+\underline{\dfrac{25}{4}}=\left(x-\dfrac{5}{2}\right)^2$

GR12. $x^2+7x+\left(\dfrac{7}{2}\right)^2=x^2+7x+\underline{\dfrac{49}{4}}=\left(x+\dfrac{7}{2}\right)^2$

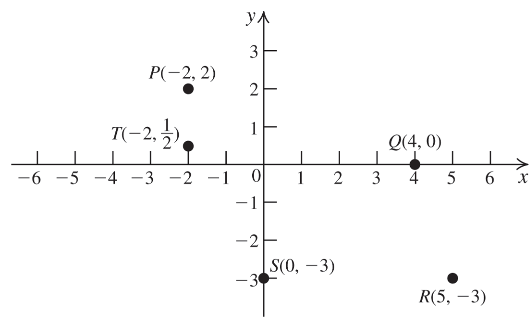
GR13. $x^2+\dfrac{3}{2}x+\left(\dfrac{\frac{3}{2}}{2}\right)^2=x^2+\dfrac{3}{2}x+\left(\dfrac{3}{4}\right)^2$
$=x^2+\dfrac{3}{2}x+\underline{\dfrac{9}{16}}=\left(x+\dfrac{3}{4}\right)^2$

GR14. $x^2-\dfrac{4}{5}x+\left(\dfrac{-\frac{4}{5}}{2}\right)^2=x^2-\dfrac{4}{5}x+\left(-\dfrac{2}{5}\right)^2$
$=x^2-\dfrac{4}{5}x+\underline{\dfrac{4}{25}}$
$=\left(x-\dfrac{2}{5}\right)^2$

1.1 The Coordinate Plane

Practice Problems

1.



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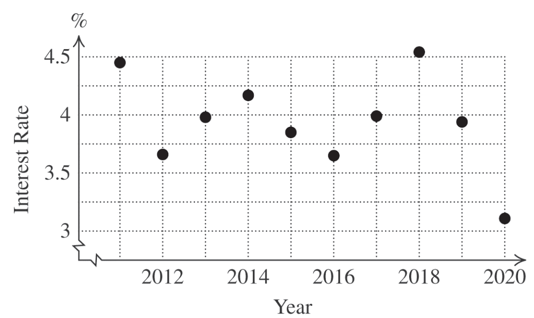
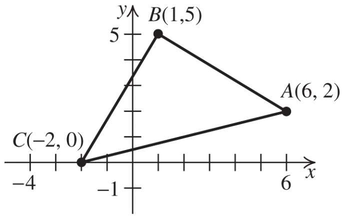


3. $(x_1,y_1)=(-5,2);\ (x_2,y_2)=(-4,1)$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(-4-(-5))^2+(1-2)^2}$
$=\sqrt{1^2+(-1)^2}=\sqrt{2}\approx 1.4$

4.



$(x_1, y_1) = (6, 2);\ (x_2, y_2) = (-2, 0)$
$(x_3, y_3) = (1, 5)$

$$\begin{aligned} AB &= \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2} \\ &= \sqrt{(1-6)^2 + (5-2)^2} \\ &= \sqrt{(-5)^2 + (3)^2} = \sqrt{34} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2} \\ &= \sqrt{(1-(-2))^2 + (5-0)^2} \\ &= \sqrt{(3)^2 + (5)^2} = \sqrt{34} \end{aligned}$$

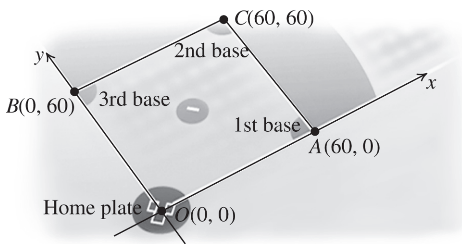
$$\begin{aligned} AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2-6)^2 + (0-2)^2} \\ &= \sqrt{(-8)^2 + (-2)^2} = \sqrt{68} \end{aligned}$$

Yes, the triangle is an isosceles triangle. Now check to see is the Pythagorean theorem holds for this triangle.

$$\begin{aligned} [d(A, B)]^2 + [d(B, C)]^2 &= (\sqrt{34})^2 + (\sqrt{34})^2 \\ &= 34 + 34 = 68 \\ &= [d(A, C)]^2 \end{aligned}$$

So, the triangle is an isosceles right triangle.

5.



We are asked to find the distance between the points $A(60, 0)$ and $B(0, 60)$.

$$\begin{aligned} d(A, B) &= \sqrt{(60-0)^2 + (0-60)^2} \\ &= \sqrt{(60)^2 + (-60)^2} = \sqrt{2(60)^2} \\ &= 60\sqrt{2} \approx 84.85 \text{ ft} \end{aligned}$$

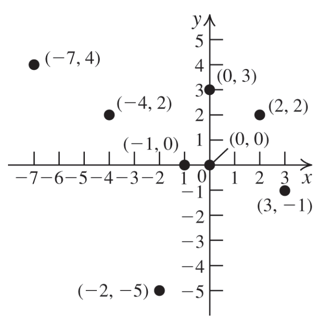
6. $M = \left(\dfrac{5+6}{2}, \dfrac{-2+(-1)}{2}\right) = \left(\dfrac{11}{2}, -\dfrac{3}{2}\right)$

Concepts and Vocabulary

1. A point with a negative first coordinate and a positive second coordinate lies in the second quadrant.
2. Any point on the x-axis has second coordinate 0.
3. The distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the formula $d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
4. The coordinates of the midpoint $M(x, y)$ of the line segment joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ are given by $(x, y) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.
5. True
6. False. The point $(7, -4)$ is 4 units to the right and 6 units below the point $(3, 2)$.
7. False. Every point in quadrant II has a negative x-coordinate.
8. True.

Building Skills

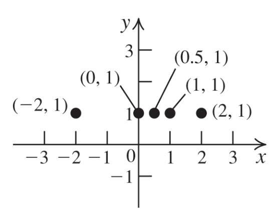
9.



$(2, 2)$: Q1; $(3, -1)$: Q4; $(-1, 0)$: x-axis
$(-2, -5)$: Q3; $(0, 0)$: origin; $(-7, 4)$: Q2
$(0, 3)$: y-axis; $(-4, 2)$: Q2

10. a. Answers will vary. Sample answer: $(-2, 0), (-1, 0), (0, 0), (1, 0), (2, 0)$
The y-coordinate is 0.

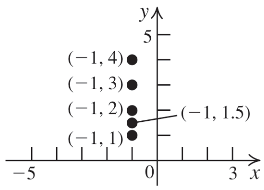
b.



The set of all points of the form $(x, 1)$ is a horizontal line that intersects the y-axis at 1.

11. a. If the x-coordinate of a point is 0, the point lies on the y-axis.

b.



The set of all points of the form $(-1, y)$ is a vertical line that intersects the x-axis at –1.

12. a. A vertical line that intersects the x-axis at –3.

b. A horizontal line that intersects the y-axis at 4.

13. a. $y > 0$ **b.** $y < 0$

c. $x < 0$ **d.** $x > 0$

14. a. Quadrant III **b.** Quadrant I

c. Quadrant IV **d.** Quadrant II

In Exercises 15–24, use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and the midpoint formula, $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

15. a. $d = \sqrt{(2-2)^2 + (5-1)^2} = \sqrt{4^2} = 4$

b. $M = \left(\frac{2+2}{2}, \frac{1+5}{2}\right) = (2, 3)$

16. a. $d = \sqrt{(-2-3)^2 + (5-5)^2} = \sqrt{(-5)^2} = 5$

b. $M = \left(\frac{3+(-2)}{2}, \frac{5+5}{2}\right) = (0.5, 5)$

17. a. $d = \sqrt{(2-(-1))^2 + (-3-(-5))^2}$
$= \sqrt{3^2 + 2^2} = \sqrt{13}$

b. $M = \left(\frac{-1+2}{2}, \frac{-5+(-3)}{2}\right) = (0.5, -4)$

18. a. $d = \sqrt{(-7-(-4))^2 + (-9-1)^2}$
$= \sqrt{(-3)^2 + (-10)^2} = \sqrt{109}$

b. $M = \left(\frac{-4+(-7)}{2}, \frac{1+(-9)}{2}\right) = (-5.5, -4)$

19. a. $d = \sqrt{(3-(-1))^2 + (-6.5-1.5)^2}$
$= \sqrt{4^2 + (-8)^2} = \sqrt{80} = 4\sqrt{5}$

b. $M = \left(\frac{-1+3}{2}, \frac{1.5+(-6.5)}{2}\right) = (1, -2.5)$

20. a. $d = \sqrt{(1-0.5)^2 + (-1-0.5)^2}$
$= \sqrt{(0.5)^2 + (-1.5)^2} = \sqrt{2.5} = \sqrt{\frac{5}{2}} = \frac{\sqrt{10}}{2}$

b. $M = \left(\frac{0.5+1}{2}, \frac{0.5+(-1)}{2}\right) = (0.75, -0.25)$

21. a. $d = \sqrt{\left(\sqrt{2} - \sqrt{2}\right)^2 + (5-4)^2} = \sqrt{1^2} = 1$

b. $M = \left(\frac{\sqrt{2}+\sqrt{2}}{2}, \frac{4+5}{2}\right) = \left(\sqrt{2}, 4.5\right)$

22. a. $d = \sqrt{((v+w)-(v-w))^2 + (t-t)^2}$
$= \sqrt{(2w)^2} = 2|w|$

b. $M = \left(\frac{(v-w)+(v+w)}{2}, \frac{t+t}{2}\right) = (v, t)$

23. a. $d = \sqrt{(k-t)^2 + (t-k)^2}$
$= \sqrt{(t-k)^2 + (t-k)^2}$
$= \sqrt{2(t-k)^2} = \sqrt{2}\,|t-k|$

b. $M = \left(\frac{t+k}{2}, \frac{k+t}{2}\right)$

24. a. $d = \sqrt{(-n-m)^2 + (-m-n)^2}$
$= \sqrt{\left(n^2 + 2mn + m^2\right) + \left(m^2 + 2mn + n^2\right)}$
$= \sqrt{2m^2 + 4mn + 2n^2}$
$= \sqrt{2\left(m^2 + 2mn + n^2\right)}$
$= \sqrt{2(m+n)^2} = \sqrt{2}\,|m+n|$

b. $M = \left(\frac{m+(-n)}{2}, \frac{n+(-m)}{2}\right)$
$= \left(\frac{m-n}{2}, \frac{n-m}{2}\right)$

25. $P=(-1,-2), Q=(0,0), R=(1,2)$

$d(P,Q)=\sqrt{(0-(-1))^2+(0-(-2))^2}=\sqrt{5}$

$d(Q,R)=\sqrt{(1-0)^2+(2-0)^2}=\sqrt{5}$

$d(P,R)=\sqrt{(1-(-1))^2+(2-(-2))^2}$
$=\sqrt{2^2+4^2}=\sqrt{20}=2\sqrt{5}$

Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

26. $P=(-3,-4), Q=(0,0), R=(3,4)$

$d(P,Q)=\sqrt{(0-(-3))^2+(0-(-4))^2}$
$=\sqrt{25}=5$

$d(Q,R)=\sqrt{(3-0)^2+(4-0)^2}=\sqrt{25}=5$

$d(P,R)=\sqrt{(3-(-3))^2+(4-(-4))^2}$
$=\sqrt{6^2+8^2}=\sqrt{100}=10$

Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

27. $P=(4,-2), Q=(1,3), R=(-2,8)$

$d(P,Q)=\sqrt{(1-4)^2+(3-(-2))^2}=\sqrt{34}$

$d(Q,R)=\sqrt{(-2-1)^2+(8-3)^2}=\sqrt{34}$

$d(P,R)=\sqrt{(-2-4)^2+(8-(-2))^2}$
$=\sqrt{(-6)^2+10^2}=\sqrt{136}=2\sqrt{34}$

Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

28. It is not possible to arrange the points in such a way so that $d(P, Q) + d(Q, R) = d(P, R)$, so the points are not collinear.

29. $P=(-1,4), Q=(3,0), R=(11,-8)$

$d(P,Q)=\sqrt{(3-(-1))^2+(0-4)^2}=4\sqrt{2}$

$d(Q,R)=\sqrt{(11-3)^2+((-8)-0)^2}=8\sqrt{2}$

$d(P,R)=\sqrt{(11-(-1))^2+(-8-4)^2}$
$=\sqrt{(12)^2+(-12)^2}=\sqrt{288}=12\sqrt{2}$

Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

30. It is not possible to arrange the points in such a way so that $d(P, Q) + d(Q, R) = d(P, R)$, so the points are not collinear.

31. It is not possible to arrange the points in such a way so that $d(P, Q) + d(Q, R) = d(P, R)$, so the points are not collinear.

32. $P=(1,7), Q=(-3,7.5), R=(-7,8)$

$d(P,Q)=\sqrt{(-3-1)^2+(7.5-7)^2}=\sqrt{16.25}$

$d(Q,R)=\sqrt{(-7-(-3))^2+(8-7.5)^2}$
$=\sqrt{16.25}$

$d(P,R)=\sqrt{(-7-1)^2+(8-7)^2}$
$=\sqrt{(-8)^2+1^2}=\sqrt{65}=2\sqrt{16.25}$

Because $d(P, Q) + d(Q, R) = d(P, R)$, the points are collinear.

33. First, find the midpoint M of PQ.

$M=\left(\frac{-4+0}{2},\frac{0+8}{2}\right)=(-2,4)$

Now find the midpoint R of PM.

$R=\left(\frac{-4+(-2)}{2},\frac{0+4}{2}\right)=(-3,2)$

Finally, find the midpoint S of MQ.

$S=\left(\frac{-2+0}{2},\frac{4+8}{2}\right)=(-1,6)$

Thus, the three points are $(-3, 2)$, $(-2, 4)$, and $(-1, 6)$.

34. First, find the midpoint M of PQ.

$M=\left(\frac{-8+16}{2},\frac{4+(-12)}{2}\right)=(4,-4)$

Now find the midpoint R of PM.

$R=\left(\frac{-8+4}{2},\frac{4+(-4)}{2}\right)=(-2,0)$

Finally, find the midpoint S of MQ.

$S=\left(\frac{4+16}{2},\frac{-4+(-12)}{2}\right)=(10,-8)$

Thus, the three points are $(-2, 0)$, $(4, -4)$, and $(10, -8)$.

35. $d(P,Q)=\sqrt{(-1-(-5))^2+(4-5)^2}=\sqrt{17}$

$d(Q,R)=\sqrt{(-4-(-1))^2+(1-4)^2}=3\sqrt{2}$

$d(P,R)=\sqrt{(-4-(-5))^2+(1-5)^2}=\sqrt{17}$

The triangle is isosceles.

36. $d(P,Q)=\sqrt{(6-3)^2+(6-2)^2}=5$

$d(Q,R)=\sqrt{(-1-6)^2+(5-6)^2}=5\sqrt{2}$

$d(P,R)=\sqrt{(-1-3)^2+(5-2)^2}=5$

The triangle is an isosceles triangle.

37. $d(P,Q)=\sqrt{(0-(-4))^2+(7-8)^2}=\sqrt{17}$

$d(Q,R)=\sqrt{(-3-0)^2+(5-7)^2}=\sqrt{13}$

$d(P,R)=\sqrt{(-3-(-4))^2+(5-8)^2}=\sqrt{10}$

The triangle is scalene.

38. $d(P,Q)=\sqrt{(-1-6)^2+(-1-6)^2}=7\sqrt{2}$

$d(Q,R)=\sqrt{(-5-(-1))^2+(3-(-1))^2}=4\sqrt{2}$

$d(P,R)=\sqrt{(-5-6)^2+(3-6)^2}=\sqrt{130}$

The triangle is scalene.

39. $d(P,Q)=\sqrt{(9-0)^2+(-9-(-1))^2}=\sqrt{145}$

$d(Q,R)=\sqrt{(5-9)^2+(1-(-9))^2}=2\sqrt{29}$

$d(P,R)=\sqrt{(5-0)^2+(1-(-1))^2}=\sqrt{29}$

The triangle is scalene.

40. $d(P,Q)=\sqrt{(4-(-4))^2+(5-4)^2}=\sqrt{65}$

$d(Q,R)=\sqrt{(0-4)^2+(-2-5)^2}=\sqrt{65}$

$d(P,R)=\sqrt{(0-(-4))^2+(-2-4)^2}=2\sqrt{13}$

The triangle is isosceles.

41. $d(P,Q)=\sqrt{(-1-1)^2+(1-(-1))^2}=2\sqrt{2}$

$d(Q,R)=\sqrt{\left(-\sqrt{3}-(-1)\right)^2+\left(-\sqrt{3}-1\right)^2}=\sqrt{\left(3-2\sqrt{3}+1\right)+\left(3+2\sqrt{3}+1\right)}=\sqrt{8}=2\sqrt{2}$

$d(P,R)=\sqrt{\left(-\sqrt{3}-1\right)^2+\left(-\sqrt{3}-(-1)\right)^2}=\sqrt{\left(3+2\sqrt{3}+1\right)+\left(3-2\sqrt{3}+1\right)}=\sqrt{8}=2\sqrt{2}$

The triangle is equilateral.

42. $d(P,Q)=\sqrt{(-1.5-(-0.5))^2+(1-(-1))^2}=\sqrt{5}$

$d(Q,R)=\sqrt{\left(\left(\sqrt{3}-1\right)-(-1.5)\right)^2+\left(\frac{\sqrt{3}}{2}-1\right)^2}=\sqrt{\left(\left(\sqrt{3}-1\right)^2+3\left(\sqrt{3}-1\right)+2.25\right)+\left(\frac{3}{4}-\sqrt{3}+1\right)}$

$=\sqrt{\left(3-2\sqrt{3}+1+3\sqrt{3}-3+2.25\right)+\left(1.75-\sqrt{3}\right)}=\sqrt{5}$

$d(P,R)=\sqrt{\left(\left(\sqrt{3}-1\right)-(-0.5)\right)^2+\left(\frac{\sqrt{3}}{2}-(-1)\right)^2}=\sqrt{\left(\left(\sqrt{3}-1\right)^2+\left(\sqrt{3}-1\right)+0.25\right)+\left(\frac{3}{4}+\sqrt{3}+1\right)}$

$=\sqrt{\left(3-2\sqrt{3}+1+\sqrt{3}-1+0.25\right)+\left(1.75+\sqrt{3}\right)}=\sqrt{5}$

The triangle is equilateral.

43. First find the lengths of the sides:

$d(P,Q)=\sqrt{(-1-7)^2+(3-(-12))^2}=17$

$d(Q,R)=\sqrt{(14-(-1))^2+(11-3)^2}=17$

$d(R,S)=\sqrt{(22-14)^2+(-4-11)^2}=17$

$d(S,P)=\sqrt{(22-7)^2+(-4-(-12))^2}=17$

All the sides are equal, so the quadrilateral is either a square or a rhombus. Now find the length of the diagonals:

$d(P,R)=\sqrt{(14-7)^2+(11-(-12))^2}=17\sqrt{2}$

$d(Q,S)=\sqrt{(22-(-1))^2+(-4-3)^2}=17\sqrt{2}$

The diagonals are equal, so the quadrilateral is a square.

44. First find the lengths of the sides:

$d(P,Q)=\sqrt{(9-8)^2+(-11-(-10))^2}=\sqrt{2}$

$d(Q,R)=\sqrt{(8-9)^2+(-12-(-11))^2}=\sqrt{2}$

$d(R,S)=\sqrt{(7-8)^2+(-11-(-12))^2}=\sqrt{2}$

$d(S,P)=\sqrt{(8-7)^2+(-10-(-11))^2}=\sqrt{2}$

All the sides are equal, so the quadrilateral is either a square or a rhombus. Now find the length of the diagonals.

$d(P,R)=\sqrt{(8-8)^2+(-12-(-10))^2}=2$

$d(Q,S)=\sqrt{(7-9)^2+(-11-(-11))^2}=2$

The diagonals are equal, so the quadrilateral is a square.

45. $5=\sqrt{(x-2)^2+(2-(-1))^2}$
$=\sqrt{x^2-4x+4+9}\Rightarrow$
$5=\sqrt{x^2-4x+13}\Rightarrow 25=x^2-4x+13\Rightarrow$
$0=x^2-4x-12\Rightarrow 0=(x-6)(x+2)\Rightarrow$
$x=-2$ or $x=6$

46. $13=\sqrt{(2-(-10))^2+(y-(-3))^2}$
$=\sqrt{144+y^2+6y+9}$
$=\sqrt{y^2+6y+153}\Rightarrow$
$169=y^2+6y+153$
$0=y^2+6y-16\Rightarrow 0=(y+8)(y-2)\Rightarrow$
$y=-8$ or $y=2$

47. $P=(-5,2)$, $Q=(2,3)$, $R=(x,0)$ (R is on the x-axis, so the y-coordinate is 0).

$$d(P,R)=\sqrt{(x-(-5))^2+(0-2)^2}$$
$$d(Q,R)=\sqrt{(x-2)^2+(0-3)^2}$$
$$\sqrt{(x-(-5))^2+(0-2)^2}=\sqrt{(x-2)^2+(0-3)^2}$$
$$(x+5)^2+(0-2)^2=(x-2)^2+(0-3)^2$$
$$x^2+10x+25+4=x^2-4x+4+9$$
$$10x+29=-4x+13$$
$$14x=-16$$
$$x=-\frac{8}{7}$$

The coordinates of R are $\left(-\frac{8}{7},0\right)$.

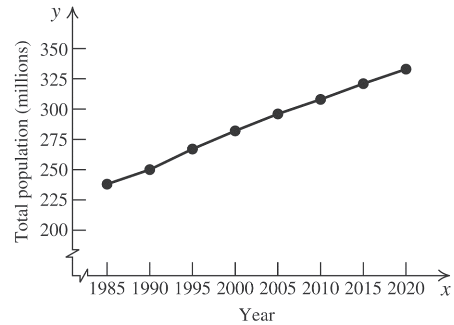
48. $P=(7,-4)$, $Q=(8,3)$, $R=(0,y)$ (R is on the y-axis, so the x-coordinate is 0).

$$d(P,R)=\sqrt{(0-7)^2+(y-(-4))^2}$$
$$d(Q,R)=\sqrt{(0-8)^2+(y-3)^2}$$
$$\sqrt{(0-7)^2+(y-(-4))^2}=\sqrt{(0-8)^2+(y-3)^2}$$
$$49+(y-(-4))^2=64+(y-3)^2$$
$$49+y^2+8y+16=64+y^2-6y+9$$
$$8y+65=-6y+73$$
$$14y=8$$
$$y=\frac{4}{7}$$

The coordinates of R are $\left(0,\frac{4}{7}\right)$.

Applying the Concepts

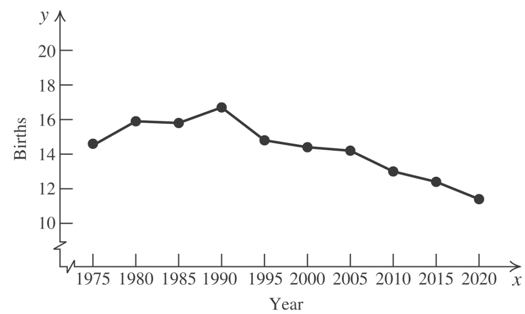
49.



50. $M=\left(\frac{2010+2020}{2},\frac{308+333}{2}\right)$
$=(2015,\ 320.5)$

The population is 2015 was about 320.5 million, which is very close to the table value for 2015.

51.



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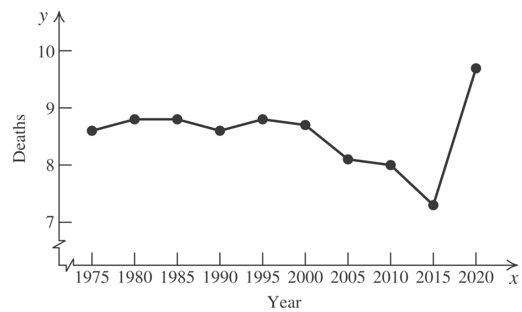

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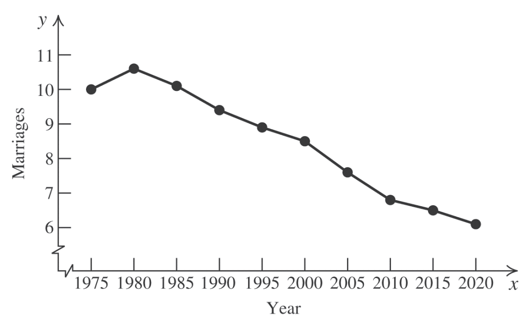
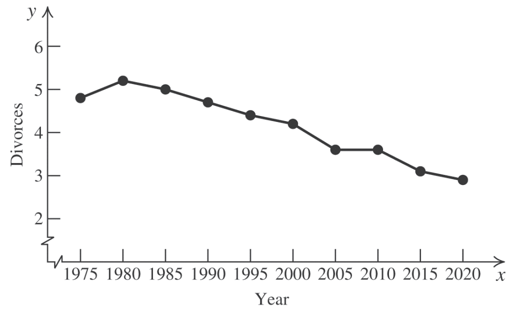


54.



55. 2018 is the midpoint of the initial range, so

$$M = \left(\frac{2016+2020}{2}, \frac{322+359}{2}\right) = (2018,\ 340.5)$$

Americans spent about \$341 billion on prescription drugs in 2018.

56. 2018 is the midpoint of the initial range, so

$$M = \left(\frac{2016+2020}{2}, \frac{3696+5053}{2}\right). = (2018,\ 4374.5)$$

There were about 4375 million Internet users in 2018.

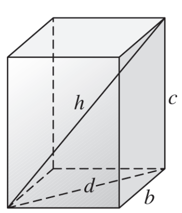
57. Percentage of Android sales in January 2014: 38.1%

58. Percentage of iPhone sales in January 2017: 53.9%

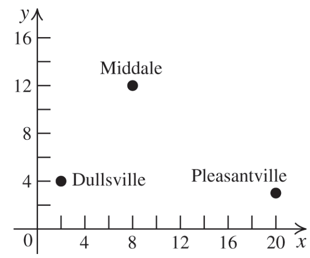
59. Android sales were at a maximum in January 2018.

60. iPhone sales were at a maximum in January 2021.

61. Denote the diagonal connecting the endpoints of the edges *a* and *b* by *d*. Then *a*, *b*, and *d* form a right triangle. By the Pythagorean theorem, $a^2+b^2=d^2$. The edge *c* and the diagonals *d* and *h* also form a right triangle, so $c^2+d^2=h^2$. Substituting d^2 from the first equation, we obtain $a^2+b^2+c^2=h^2$.



62. a.



b. $d(D,M)=\sqrt{(800-200)^2+(1200-400)^2} = 1000$

$d(M,P)=\sqrt{(2000-800)^2+(300-1200)^2} = 1500$

The distance traveled by the pilot = 1000 + 1500 = 2500 miles.

c. $d(D,P)=\sqrt{(2000-200)^2+(300-400)^2} = \sqrt{3,250,000} = \sqrt{325\cdot 10000} = 100\sqrt{325} = 100\cdot 5\sqrt{13} = 500\sqrt{13} \approx 1802.78$ miles

63. First, find the initial length of the rope using the Pythagorean theorem:

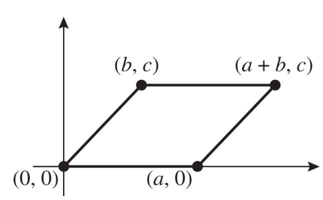
$c=\sqrt{24^2+10^2}=26$.

After *t* seconds, the length of the rope is $26-3t$. Now find the distance from the boat to the dock, *x*, using the Pythagorean theorem again and solving for *x*:

$$(26-3t)^2 = x^2+10^2$$
$$676-156t+9t^2 = x^2+100$$
$$576-156t+9t^2 = x^2$$
$$\sqrt{576-156t+9t^2} = x$$

Beyond the Basics

64. The midpoint of the diagonal connecting (0, 0) and (*a* + *b*, *c*) is $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. The midpoint of the diagonal connecting (*a*, 0) and (*b*, *c*) is also $\left(\frac{a+b}{2}, \frac{c}{2}\right)$. Because the midpoints of the two diagonals are the same, the diagonals bisect each other.



65. a. If AB is one of the diagonals, then DC is the other diagonal, and both diagonals have the same midpoint. The midpoint of AB is $\left(\frac{2+5}{2},\frac{3+4}{2}\right)=(3.5,3.5)$. The midpoint of

$DC = (3.5,3.5)=\left(\frac{x+3}{2},\frac{y+8}{2}\right)$.

So we have $3.5=\frac{x+3}{2}\Rightarrow x=4$ and

$3.5=\frac{y+8}{2}\Rightarrow y=-1$.

The coordinates of D are (4, –1).

b. If AC is one of the diagonals, then DB is the other diagonal, and both diagonals have the same midpoint. The midpoint of AC is $\left(\frac{2+3}{2},\frac{3+8}{2}\right)=(2.5,5.5)$. The midpoint of

$DB = (2.5,5.5)=\left(\frac{x+5}{2},\frac{y+4}{2}\right)$.

So we have $2.5=\frac{x+5}{2}\Rightarrow x=0$ and

$5.5=\frac{y+4}{2}\Rightarrow y=7$.

The coordinates of D are (0, 7).

c. If BC is one of the diagonals, then DA is the other diagonal, and both diagonals have the same midpoint. The midpoint of BC is $\left(\frac{5+3}{2},\frac{4+8}{2}\right)=(4,6)$. The midpoint of DA

is $(4,6)=\left(\frac{x+2}{2},\frac{y+3}{2}\right)$. So we have

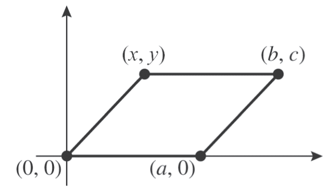
$4=\frac{x+2}{2}\Rightarrow x=6$ and $6=\frac{y+3}{2}\Rightarrow y=9$.

The coordinates of D are (6, 9).

66. The midpoint of the diagonal connecting (0, 0) and (b, c) is $\left(\frac{b}{2},\frac{c}{2}\right)$. The midpoint of the diagonal connecting $(a, 0)$ and (x, y) is $\left(\frac{a+x}{2},\frac{y}{2}\right)$. Because the diagonals bisect each other, the midpoints coincide. So

$\frac{b}{2}=\frac{a+x}{2}\Rightarrow x=b-a \quad x=a+b$, and

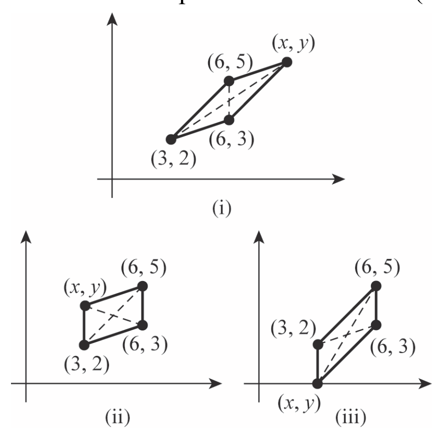
$\frac{y}{2}=\frac{c}{2}\Rightarrow y=c$. Therefore, the quadrilateral is a parallelogram.



67. a. The midpoint of the diagonal connecting (1, 2) and (5, 8) is $\left(\frac{1+5}{2},\frac{2+8}{2}\right)=(3,5)$.

The midpoint of the diagonal connecting (–2, 6) and (8, 4) is $\left(\frac{-2+8}{2},\frac{6+4}{2}\right)=(3,5)$. Because the midpoints are the same, the figure is a parallelogram.

b. There are three possible locations for (x, y).



Not drawn to scale

For part (i), the midpoint of the diagonal connecting (3, 2) and (x, y) is $\left(\frac{3+x}{2},\frac{2+y}{2}\right)$. The midpoint of the diagonal connecting (6, 3) and (6, 5) is (6, 4). So $\frac{3+x}{2}=6\Rightarrow x=9$ and

$\frac{2+y}{2}=4\Rightarrow y=6$.

For part (ii), the midpoint of the diagonal connecting (3, 2) and (6, 5) is $\left(\frac{9}{2},\frac{7}{2}\right)$, while the midpoint of the diagonal connecting (6, 3) and (x, y) is $\left(\frac{6+x}{2},\frac{3+y}{2}\right)$. $\frac{9}{2}=\frac{6+x}{2}\Rightarrow x=3$ and

$\frac{7}{2}=\frac{3+y}{2}\Rightarrow y=4$.

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For part (iii), the midpoint of the diagonal connecting (3, 2) and (6, 3) is $\left(\frac{9}{2}, \frac{5}{2}\right)$, while the midpoint of the diagonal connecting (6, 5) and (x, y) is $\left(\frac{6+x}{2}, \frac{5+y}{2}\right)$. $\frac{6}{2} = \frac{3+x}{2} \Rightarrow x = 3$ and $\frac{5}{2} = \frac{5+y}{2} \Rightarrow y = 0$. Thus, the possible values for (x, y) are (9, 6), (3, 4), and (3, 0).

68. Let $P(0, 0)$, $Q(a, 0)$, $R(a + b, c)$, and $S(b, c)$ be the vertices of the parallelogram.

$PQ = RS = \sqrt{(a-0)^2 + (0-0)^2} = |a|$.

$QR = PS = \sqrt{((a+b)-a)^2 + (c-0)^2}$
$= \sqrt{b^2 + c^2}$.

The sum of the squares of the lengths of the sides $= 2(a^2 + b^2 + c^2)$.

$d(P, R) = \sqrt{(a+b)^2 + c^2}$.

$d(Q, S) = \sqrt{(a-b)^2 + (0-c)^2}$.

The sum of the squares of the lengths of the diagonals is

$\left((a+b)^2 + c^2\right) + \left((a-b)^2 + c^2\right) =$
$a^2 + 2ab + b^2 + c^2 + a^2 - 2ab + b^2 + c^2 =$
$2a^2 + 2b^2 + 2c^2 = 2(a^2 + b^2 + c^2)$.

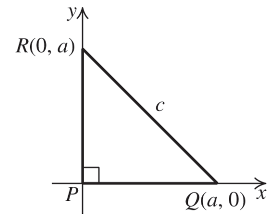
69. Let $P(0, 0)$, $Q(a, 0)$, and $R(0, b)$ be the vertices of the right triangle. The midpoint M of the hypotenuse is $\left(\frac{a}{2}, \frac{b}{2}\right)$.

$$d(Q, M) = \sqrt{\left(a - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2} = \frac{\sqrt{a^2+b^2}}{2}$$

$$d(R, M) = \sqrt{\left(0 - \frac{a}{2}\right)^2 + \left(b - \frac{b}{2}\right)^2} = \sqrt{\left(-\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2} = \frac{\sqrt{a^2+b^2}}{2}$$

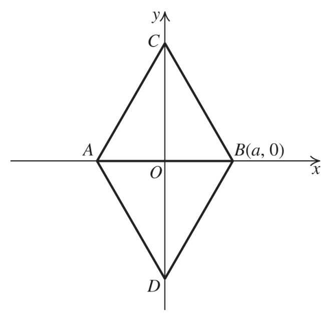
$$d(P, M) = \sqrt{\left(0 - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \sqrt{\left(-\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2} = \frac{\sqrt{a^2+b^2}}{2}$$

70. Let $P(0, 0)$, $Q(a, 0)$, and $R(0, a)$ be the vertices of the triangle.



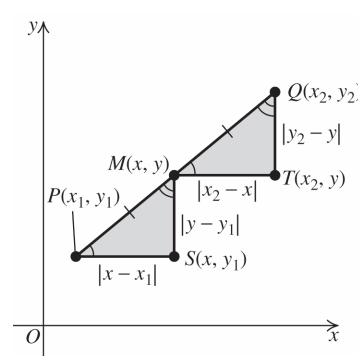
Using the Pythagorean theorem, we have
$c^2 = a^2 + a^2 \Rightarrow c^2 = 2a^2 \Rightarrow c = \sqrt{2}a \Rightarrow$
$a = \frac{1}{\sqrt{2}}c = \frac{\sqrt{2}}{2}c$

71. Since ABC is an equilateral triangle and O is the midpoint of AB, then the coordinates of A are $(-a, 0)$.



$AB = AC = AB = 2a$. Using triangle BOC and the Pythagorean theorem, we have
$BC^2 = OB^2 + OC^2 \Rightarrow (2a)^2 = a^2 + OC^2 \Rightarrow$
$4a^2 = a^2 + OC^2 \Rightarrow 3a^2 = OC^2 \Rightarrow OC = \sqrt{3}a$
Thus, the coordinates of C are $\left(0, \sqrt{3}a\right)$ and the coordinates of D are $\left(0, -\sqrt{3}a\right)$.

72.



To show that M is the midpoint of the line segment PQ, we need to show that the distance between M and Q is the same as the distance between M and P and that this distance is half the distance from P to Q.

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$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{aligned} MP &= \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2} \\ &= \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2} \\ &= \sqrt{\frac{(x_2 - x_1)^2}{4} + \frac{(y_2 - y_1)^2}{4}} \\ &= \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{aligned}$$

Thus, we have $MP = \frac{1}{2}PQ$.

Similarly, we can show that $MQ = \frac{1}{2}PQ$.

Thus, M is the midpoint of PQ, and

$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Critical Thinking/Discussion/Writing

73. a. y-axis **b.** x-axis

74. a. The union of the x-axis and y-axis

b. The plane without the x-axis and y-axis

75. a. Quadrants I and III

b. Quadrants II and IV

76. a. The origin, $\{(0, 0)\}$

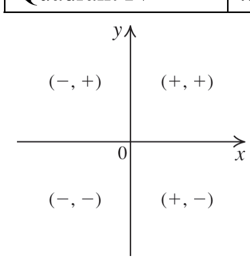
b. The plane without the origin

77. a. Right half-plane

b. Upper half-plane

78. Let (x, y) be the point.

The point lies in	if
Quadrant I	$x > 0$ and $y > 0$
Quadrant II	$x < 0$ and $y > 0$
Quadrant III	$x < 0$ and $y < 0$
Quadrant IV	$x > 0$ and $y < 0$



Getting Ready for the Next Section

GR1. a. $x^2 + y^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

b. $x^2 + y^2 = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} + \frac{2}{4} = 1$

GR2. a. $(x-1)^2 + (y+2)^2 = [(-1) - 1]^2 + (1+2)^2 = (-2)^2 + 3^2 = 4 + 9 = 13$

b. $(x-1)^2 + (y+2)^2 = (4-1)^2 + (2+2)^2 = 3^2 + 4^2 = 9 + 16 = 25$

GR3. a. $\frac{x}{|x|} + \frac{|y|}{y} = \frac{2}{|2|} + \frac{|-3|}{-3} = \frac{2}{2} + \frac{3}{-3} = 1 - 1 = 0$

b. $\frac{x}{|x|} + \frac{|y|}{y} = \frac{-4}{|-4|} + \frac{|3|}{3} = \frac{-4}{4} + \frac{3}{3} = -1 + 1 = 0$

GR4. a. $\frac{|x|}{x} + \frac{|y|}{y} = \frac{|-1|}{-1} + \frac{|-2|}{-2} = \frac{1}{-1} + \frac{2}{-2} = -1 + (-1) = -2$

b. $\frac{|x|}{x} + \frac{|y|}{y} = \frac{|3|}{3} + \frac{|2|}{2} = \frac{3}{3} + \frac{2}{2} = 1 + 1 = 2$

GR5. $x^2 - 6x + \left(\frac{-6}{2}\right)^2 = x^2 - 6x + 3^2 = x^2 - 6x + \underline{9}$

GR6. $x^2 - 8x + \left(\frac{-8}{2}\right)^2 = x^2 - 8x + (-4)^2 = x^2 - 8x + \underline{16}$

GR7. $y^2 + 3y = y^2 + 3y + \left(\frac{3}{2}\right)^2 = y^2 + 3y + \underline{\frac{9}{4}}$

GR8. $y^2 + 5y + \left(\frac{5}{2}\right)^2 = y^2 + 5y + \underline{\frac{25}{4}}$

GR9. $x^2 - ax + \left(\frac{-a}{2}\right)^2 = x^2 - ax + \underline{\frac{a^2}{4}}$

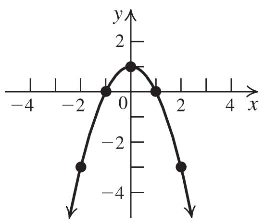
GR10. $x^2 + xy + \left(\frac{y}{2}\right)^2 = x^2 + xy + \underline{\frac{y^2}{4}}$

1.2 Graphs of Equations

Practice Problems

1. $y = -x^2 + 1$

x	$y = -x^2 + 1$	(x, y)
-2	$y = -(-2)^2 + 1$	$(-2, -3)$
-1	$y = -(-1)^2 + 1$	$(-1, 0)$
0	$y = -(0)^2 + 1$	$(0, 1)$
1	$y = -(1)^2 + 1$	$(1, 0)$
2	$y = -(2)^2 + 1$	$(2, -3)$



2. To find the x-intercept, let $y = 0$, and solve the equation for x: $0 = 2x^2 + 3x - 2 \Rightarrow$

$0 = (2x-1)(x+2) \Rightarrow x = \frac{1}{2}$ or $x = -2$.

To find the y-intercept, let $x = 0$, and solve the equation for y:

$y = 2(0)^2 + 3(0) - 2 \Rightarrow y = -2.$

The x-intercepts are $\frac{1}{2}$ and -2; the y-intercept is -2.

3. To test for symmetry about the y-axis, replace x with $-x$ to determine if $(-x, y)$ satisfies the equation.

$(-x)^2 - y^2 = 1 \Rightarrow x^2 - y^2 = 1,$ which is the same as the original equation. So the graph is symmetric about the y-axis.

4. x-axis: $x^2 = (-y)^3 \Rightarrow x^2 = -y^3,$ which is not the same as the original equation, so the equation is not symmetric about the x-axis.

y-axis: $(-x)^2 = y^3 \Rightarrow x^2 = y^3,$ which is the same as the original equation, so the equation is symmetric about the y-axis.

origin: $(-x)^2 = (-y)^3 \Rightarrow x^2 = -y^3,$ which is not the same as the original equation, so the equation is not symmetric about the origin.

5. $y = -t^4 + 77t^2 + 324$

a. First, find the intercepts. If $t = 0$, then $y = 324$, so the y-intercept is $(0, 324)$. If $y = 0$, then we have

$$0 = -t^4 + 77t^2 + 324$$
$$t^4 - 77t^2 - 324 = 0$$
$$(t^2 - 81)(t^2 + 4) = 0$$
$$(t+9)(t-9)(t^2+4) = 0 \Rightarrow t = -9, 9, \pm 2i$$

So, the t-intercepts are $(-9, 0)$ and $(9, 0)$. Next, check for symmetry.

t-axis: $-y = -t^4 + 77t^2 + 324$ is not the same as the original equation, so the equation is not symmetric with respect to the t-axis.

y-axis: $y = -(-t)^4 + 77(-t)^2 + 324 \Rightarrow$

$y = -t^4 + 77t^2 + 324,$ which is the same as the original equation. So the graph is symmetric with respect to the y-axis.

origin: $-y = -(-t)^4 + 77(-t)^2 + 324 \Rightarrow$

$-y = -t^4 + 77t^2 + 324,$ which is not the same as the original equation. So the graph is not symmetric with respect to the origin. Now, make a table of values. The graph is symmetric with respect to the y-axis, so, if (t, y) is on the graph, then so is $(-t, y)$. However, the graph pertaining to the physical aspects of the problem consists only of those values for $t \geq 0$.

t	$y = -t^4 + 77t^2 + 324$	(t, y)
0	324	(0, 324)
1	400	(1, 400)
2	616	(2, 616)
3	936	(3, 936)
4	1300	(4, 1300)
5	1624	(5, 1624)
6	1800	(6, 1800)
7	1696	(7, 1696)
8	1156	(8, 1156)
9	0	(9, 0)

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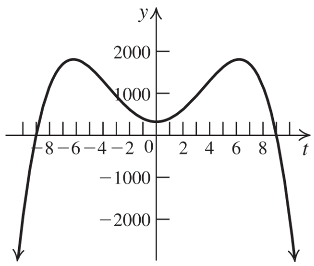
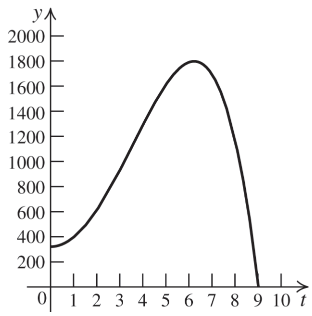


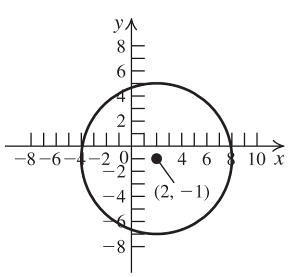
b.



c. The population becomes extinct after 9 years.

6. The standard form of the equation of a circle is $(x-h)^2+(y-k)^2=r^2$.
$(h, k) = (3, -6)$ and $r = 10$
The equation of the circle is
$(x-3)^2+(y+6)^2=100$.

7. $(x-2)^2+(y+1)^2=36 \Rightarrow (h,k)=(2,-1),\ r=6$
This is the equation of a circle with center $(2, -1)$ and radius 6.



8. $x^2+y^2+4x-6y-12=0 \Rightarrow$
$x^2+4x+y^2-6y=12$
Now complete the square:
$\left(x^2+4x+4\right)+\left(y^2-6y+9\right)=12+4+9 \Rightarrow$
$(x+2)^2+(y-3)^2=25$
This is a circle with center $(-2, 3)$ and radius 5.

Concepts and Vocabulary

1. The graph of an equation in two variables, such as x and y, is the set of all ordered pairs (a, b) that satisfy the equation.

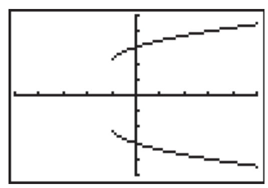
2. If $(-2, 4)$ is a point on a graph that is symmetric with respect to the y-axis, then the point $(2, 4)$ is also on the graph.

3. If $(0, -5)$ is a point of a graph, then -5 is a y- intercept of the graph.

4. An equation in standard form of a circle with center $(1, 0)$ and radius 2 is $(x-1)^2+y^2=4$.

5. False. The equation of a circle has both an x^2-term and a y^2-term. The given equation does not have a y^2-term.

6. False. The graph below is an example of a graph that is symmetric about the x-axis, but does not have an x-intercept.



7. False. The center of the circle with equation $(x+3)^2+(y+4)^2=9$ is $(-3, -4)$.

8. True

Building Skills

In exercises 9−14, to determine if a point lies on the graph of the equation, substitute the point's coordinates into the equation to see if the resulting statement is true.

9. on the graph: $(-3, -4)$, $(1, 0)$, $(4, 3)$; not on the graph: $(2, 3)$

10. on the graph: $(-1, 1)$, $(1, 4)$, $\left(-\frac{5}{3}, 0\right)$; not on the graph: $(0, 2)$

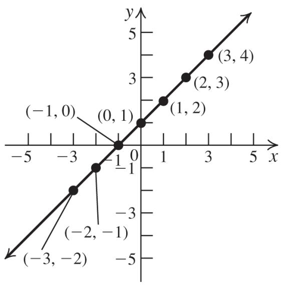
11. on the graph: $(3, 2)$, $(0, 1)$, $(8, 3)$; not on the graph: $(8, -3)$

12. on the graph: $(1, 1)$, $\left(2, \frac{1}{2}\right)$; not on the graph: $(0, 0)$, $\left(-3, \frac{1}{3}\right)$

13. on the graph: (1, 0), $\left(2, \sqrt{3}\right), \left(2, -\sqrt{3}\right)$; not on the graph: (0, –1)

14. Each point is on the graph.

15.



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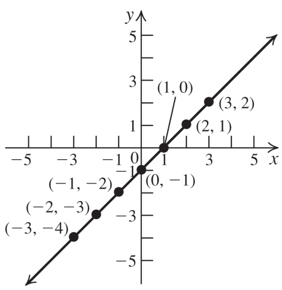

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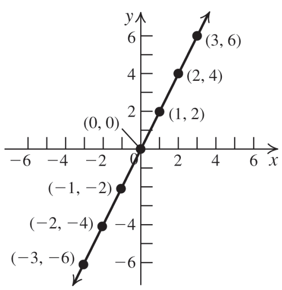

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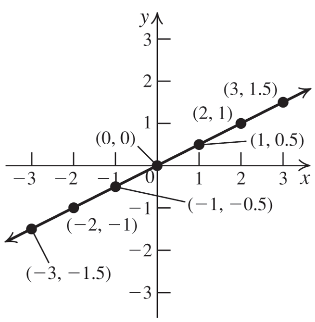
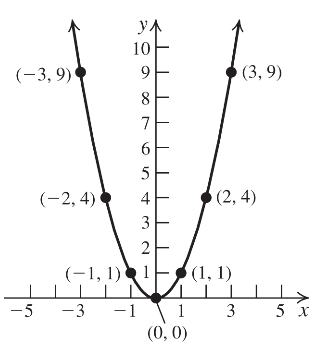


19.



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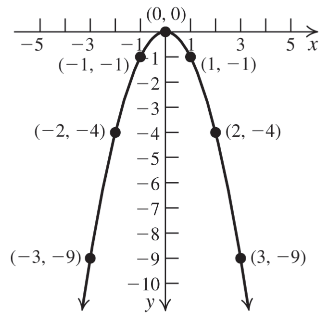

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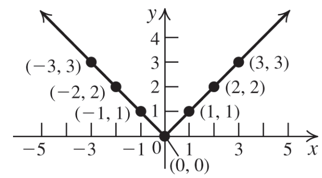

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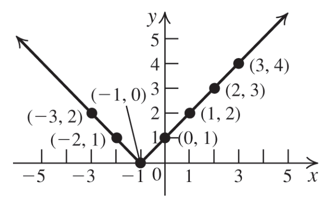

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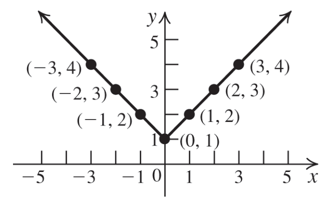

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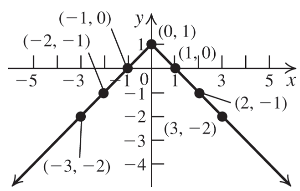

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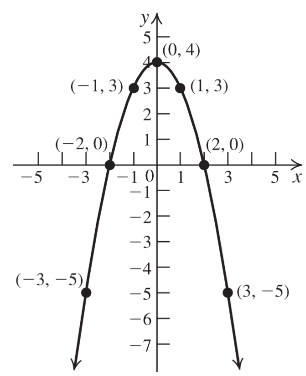

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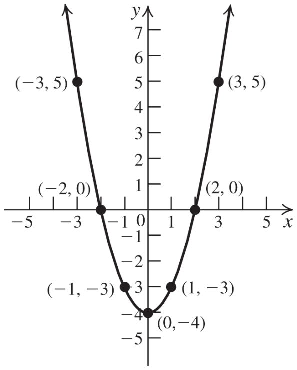

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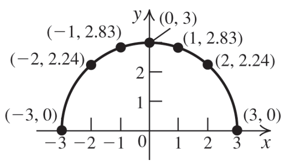

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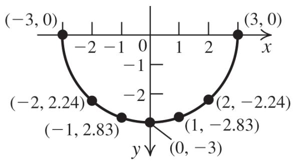

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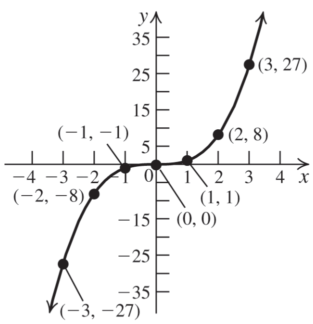

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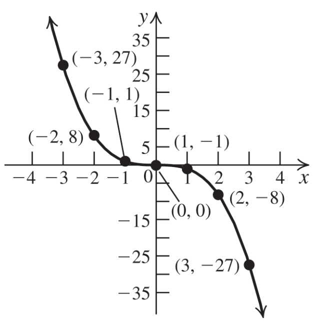

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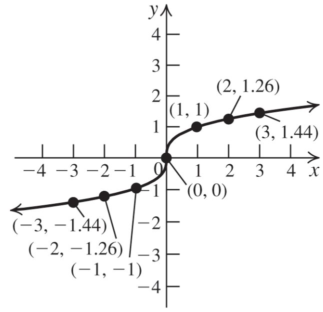

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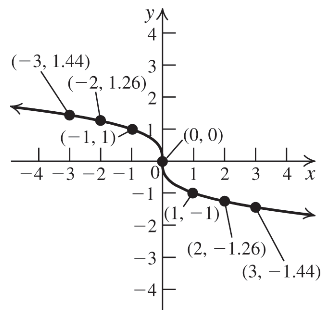

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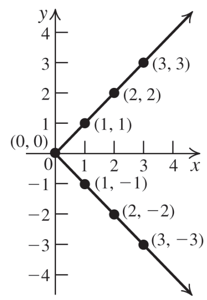

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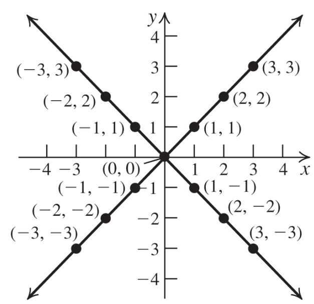

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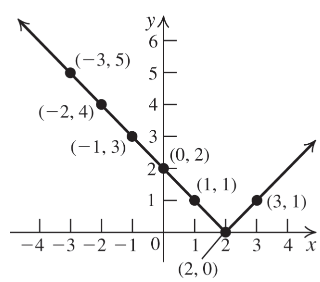

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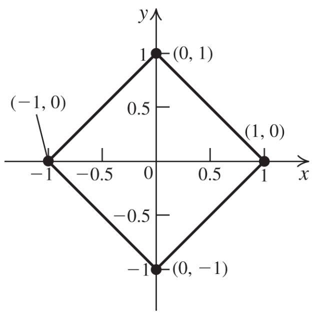


For exercises 37–46, read the answers directly from the given graphs.

37. x-intercepts: –1, 1
y-intercepts: none
symmetries: y-axis

38. x-intercepts: none
y-intercepts: –1, 1
symmetries: x-axis

39. x-intercepts: $-\pi$, 0, π
y-intercepts: 0
symmetries: origin

40. x-intercepts: $-\dfrac{\pi}{2}, \dfrac{\pi}{2}$
y-intercepts: 2
symmetries: y-axis

41. x-intercepts: –3, 3
y-intercepts: –2, 2
symmetries: x-axis, y-axis, origin

42. x-intercepts: –2, 2
y-intercepts: –3, 3
symmetries: x-axis, y-axis, origin

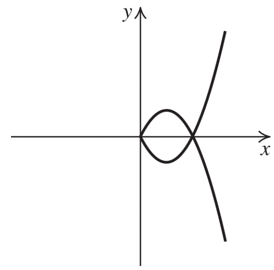
43. x-intercepts: –2, 0, 2
y-intercepts: 0
symmetries: origin

44. x-intercepts: –2, 0, 2
y-intercepts: 0
symmetries: origin

45. x-intercepts: –2, 0, 2
y-intercepts: 0, 3
symmetries: y-axis

46. x-intercepts: 0, 3
y-intercepts: –2, 0, 2
symmetries: x-axis

47.



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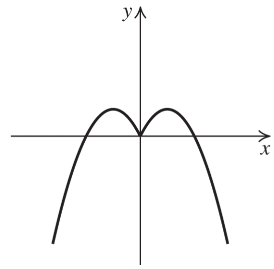

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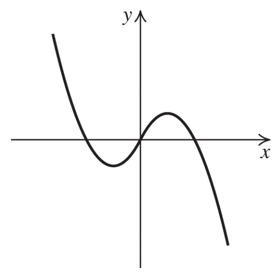

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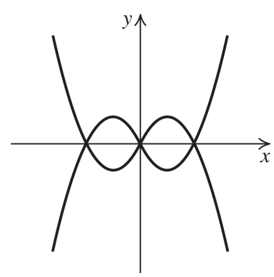


51. To find the x-intercept, let $y = 0$, and solve the equation for x: $3x + 4(0) = 12 \Rightarrow x = 4$.
To find the y-intercept, let $x = 0$, and solve the equation for y: $3(0) + 4y = 12 \Rightarrow y = 3$.
The x-intercept is 4; the y-intercept is 3.

52. To find the x-intercept, let $y = 0$, and solve the equation for x: $2x + 3(0) = 5 \Rightarrow x = \dfrac{5}{2}$.
To find the y-intercept, let $x = 0$, and solve the equation for y: $2(0) + 3y = 5 \Rightarrow y = \dfrac{5}{3}$.
The x-intercept is $5/2$; the y-intercept is $5/3$.

53. To find the *x*-intercept, let $y = 0$, and solve the equation for *x*: $\frac{x}{5}+\frac{0}{3}=1 \Rightarrow x=5.$ To find the *y*-intercept, let $x = 0$, and solve the equation for *y*: $\frac{0}{5}+\frac{y}{3}=1 \Rightarrow y=3.$ The *x*-intercept is 5; the *y*-intercept is 3.

54. To find the *x*-intercept, let $y = 0$, and solve the equation for *x*: $\frac{x}{2}-\frac{0}{3}=1 \Rightarrow x=2.$ To find the *y*-intercept, let $x = 0$, and solve the equation for *y*: $\frac{0}{2}-\frac{y}{3}=1 \Rightarrow y=-3.$ The *x*-intercept is 2; the *y*-intercept is -3.

55. To find the *x*-intercept, let $y = 0$, and solve the equation for *x*: $0=\frac{x+2}{x-1} \Rightarrow x=-2.$ To find the *y*-intercept, let $x = 0$, and solve the equation for *y*: $y=\frac{0+2}{0-1}=-2.$
The *x*-intercept is –2; the *y*-intercept is –2.

56. To find the *x*-intercept, let $y = 0$, and solve the equation for *x*: $x=\frac{0-2}{0+1} \Rightarrow x=-2.$ To find the *y*-intercept, let $x = 0$, and solve the equation for *y*: $0=\frac{y-2}{y+1} \Rightarrow y=2.$
The *x*-intercept is –2; the *y*-intercept is 2.

57. To find the *x*-intercept, let $y = 0$, and solve the equation for *x*: $0=x^2-6x+8 \Rightarrow x=4$ or $x=2.$ To find the *y*-intercept, let $x = 0$, and solve the equation for *y*: $y=0^2-6(0)+8 \Rightarrow y=8.$ The *x*-intercepts are 2 and 4; the *y*-intercept is 8.

58. To find the *x*-intercept, let $y = 0$, and solve the equation for *x*: $x=0^2-5(0)+6 \Rightarrow x=6.$
To find the *y*-intercept, let $x = 0$, and solve the equation for *y*: $0=y^2-5y+6 \Rightarrow y=2$ or $y=3.$ The *x*-intercept is 6; the *y*-intercepts are 2 and 3.

59. To find the *x*-intercept, let $y = 0$, and solve the equation for *x*: $x^2+0^2=4 \Rightarrow x=\pm 2.$
To find the *y*-intercept, let $x = 0$, and solve the equation for *y*: $0^2+y^2=4 \Rightarrow y=\pm 2.$
The *x*-intercepts are –2 and 2; the *y*-intercepts are 2 and 2.

60. To find the *x*-intercept, let $y = 0$, and solve the equation for *x*:
$(x-1)^2+0^2=9 \Rightarrow x-1=\pm 3 \Rightarrow$
$x=-2$ or $x=4$
To find the *y*-intercept, let $x = 0$, and solve the equation for *y*:
$(0-1)^2+y^2=9 \Rightarrow 1+y^2=9 \Rightarrow y^2=8 \Rightarrow$
$y=\pm\sqrt{8}=\pm 2\sqrt{2}$
The *x*-intercepts are –2 and 4; the *y*-intercepts are $\pm 2\sqrt{2}.$

61. To find the *x*-intercept, let $y = 0$, and solve the equation for *x*: $0=\sqrt{9-x^2} \Rightarrow x=\pm 3.$
To find the *y*-intercept, let $x = 0$, and solve the equation for *y*: $y=\sqrt{9-0^2} \Rightarrow y=3.$
The *x*-intercepts are –3 and 3; the *y*-intercept is 3.

62. To find the *x*-intercept, let $y = 0$, and solve the equation for *x*: $0=\sqrt{x^2-1} \Rightarrow x=\pm 1.$ To find the *y*-intercept, let $x = 0$, and solve the equation for *y*: $y=\sqrt{0^2-1} \Rightarrow$ no solution.
The *x*-intercepts are –1 and 1; there is no *y*-intercept.

63. To find the *x*-intercept, let $y = 0$, and solve the equation for *x*: $x(0)=1 \Rightarrow$ no solution. To find the *y*-intercept, let $x = 0$, and solve the equation for *y*: $(0)y=1 \Rightarrow$ no solution. There is no *x*-intercept; there is no *y*-intercept.

64. To find the *x*-intercept, let $y = 0$, and solve the equation for *x*: $0=x^2+1 \Rightarrow x^2=-1 \Rightarrow$ there is no real solution. To find the *y*-intercept, let $x = 0$, and solve the equation for *y*:
$y=0^2+1 \Rightarrow y=1.$ There is no *x*-intercept; the *y*-intercept is 1.

In exercises 65–74, to test for symmetry with respect to the *x*-axis, replace *y* with –*y* to determine if (*x*, –*y*) satisfies the equation. To test for symmetry with respect to the *y*-axis, replace *x* with –*x* to determine if (–*x*, *y*) satisfies the equation. To test for symmetry with respect to the origin, replace *x* with –*x* and *y* with –*y* to determine if (–*x*, –*y*) satisfies the equation.

65. $-y=x^2+1$ is not the same as the original equation, so the equation is not symmetric with respect to the *x*-axis.
$y=(-x)^2+1 \Rightarrow \; y=x^2+1$, so the equation is symmetric with respect to the *y*-axis.
(*continued on next page*)

(*continued*)

$-y = (-x)^2 + 1 \Rightarrow -y = x^2 + 1$, is not the same as the original equation, so the equation is not symmetric with respect to the origin.

66. $x = (-y)^2 + 1 \Rightarrow x = y^2 + 1$, so the equation is symmetric with respect to the *x*-axis.
$-x = y^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the *y*-axis.
$-x = (-y)^2 + 1 \Rightarrow -x = y^2 + 1$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

67. $-y = x^3 + x$ is not the same as the original equation, so the equation is not symmetric with respect to the *x*-axis.
$y = (-x)^3 - x \Rightarrow y = -x^3 - x \Rightarrow$
$y = -(x^3 + x)$ is not the same as the original equation, so the equation is not symmetric with respect to the *y*-axis.
$-y = (-x)^3 - x \Rightarrow -y = -x^3 - x \Rightarrow$
$-y = -(x^3 + x) \Rightarrow y = x^3 + x,$ so the equation is symmetric with respect to the origin.

68. $-y = 2x^3 - x$ is not the same as the original equation, so the equation is not symmetric with respect to the *x*-axis.
$y = 2(-x)^3 - (-x) \Rightarrow y = -2x^3 + x \Rightarrow$
$y = -2(x^3 - x)$ is not the same as the original equation, so the equation is not symmetric with respect to the *y*-axis.
$-y = 2(-x)^3 - (-x) \Rightarrow -y = -2x^3 + x \Rightarrow$
$-y = -2(x^3 - x) \Rightarrow y = 2x^3 - x$, so the equation is symmetric with respect to the origin.

69. $-y = 5x^4 + 2x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the *x*-axis.
$y = 5(-x)^4 + 2(-x)^2 \Rightarrow y = 5x^4 + 2x^2$, so the equation is symmetric with respect to the *y*-axis.
$-y = 5(-x)^4 + 2(-x) \Rightarrow -y = 5x^4 + 2x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

70. $-y = -3x^6 + 2x^4 + x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the *x*-axis.
$y = -3(-x)^6 + 2(-x)^4 + (-x)^2 \Rightarrow$
$y = -3x^6 + 2x^4 + x^2$, so the equation is symmetric with respect to the *y*-axis.
$-y = -3(-x)^6 + 2(-x)^4 + (-x)^2 \Rightarrow$
$-y = -3x^6 + 2x^4 + x^2$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

71. $-y = -3x^5 + 2x^3$ is not the same as the original equation, so the equation is not symmetric with respect to the *x*-axis.
$y = -3(-x)^5 + 2(-x)^3 \Rightarrow y = 3x^5 - 2x^3$ is not the same as the original equation, so the equation is not symmetric with respect to the *y*-axis.
$-y = -3(-x)^5 + 2(-x)^3 \Rightarrow -y = 3x^5 - 2x^3 \Rightarrow$
$-y = -(-3x^5 + 2x^3) \Rightarrow y = -3x^5 + 2x^3,$ so the equation is symmetric with respect to the origin.

72. $-y = 2x^2 - |x|$ is not the same as the original equation, so the equation is not symmetric with respect to the *x*-axis.
$y = 2(-x)^2 - |-x| \Rightarrow y = 2x^2 - |x|$, so the equation is symmetric with respect to the *y*-axis.
$-y = 2(-x)^2 - |-x| \Rightarrow -y = 2x^2 - |x|$ is not the same as the original equation, so the equation is not symmetric with respect to the origin.

73. $x^2(-y)^2 + 2x(-y) = 1 \Rightarrow x^2y^2 - 2xy = 1$ is not the same as the original equation, so the equation is not symmetric with respect to the *x*-axis.
$(-x)^2y^2 + 2(-x)y = 1 \Rightarrow x^2y^2 - 2xy = 1$ is not the same as the original equation, so the equation is not symmetric with respect to the *y*-axis.
$(-x)^2(-y)^2 + 2(-x)(-y) = 1 \Rightarrow$
$x^2y^2 + 2xy = 1$, so the equation is symmetric with respect to the origin.

74. $x^2 + (-y)^2 = 16 \Rightarrow x^2 + y^2 = 16$, so the equation is symmetric with respect to the x-axis.
$(-x)^2 + y^2 = 16 \Rightarrow x^2 + y^2 = 16$, so the equation is symmetric with respect to the y-axis.
$(-x)^2 + (-y)^2 = 16 \Rightarrow x^2 + y^2 = 16$, so the equation is symmetric with respect to the origin.

For exercises 75–78, use the standard form of the equation of a circle, $(x-h)^2 + (y-k)^2 = r^2$.

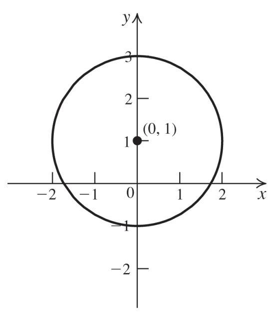
75. Center (2, 3); radius = 6

76. Center (–1, 3); radius = 4

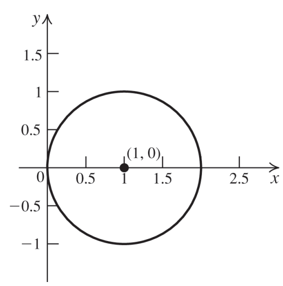
77. Center (–2, –3); radius = $\sqrt{11}$

78. Center $\left(\frac{1}{2}, -\frac{3}{2}\right)$; radius = $\frac{\sqrt{3}}{2}$

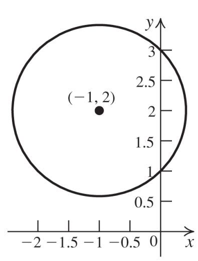
79. $x^2 + (y-1)^2 = 4$



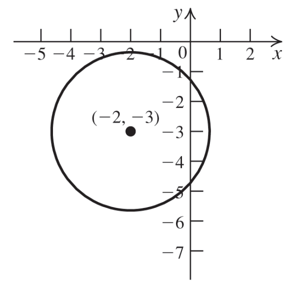
80. $(x-1)^2 + y^2 = 1$



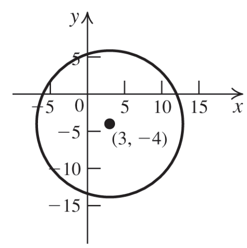
81. $(x+1)^2 + (y-2)^2 = 2$



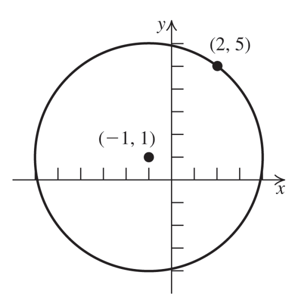
82. $(x+2)^2 + (y+3)^2 = 7$



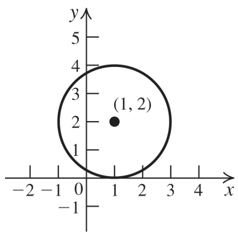
83. The standard form of the equation of a circle with center $(3, -4)$ is $(x-3)^2 + (y+4)^2 = r^2$. Substitute $(-1, 5)$ for (x, y) and solve for r^2.
$(-1-3)^2 + (5+4)^2 = r^2 \Rightarrow$
$(-4)^2 + 9^2 = r^2 \Rightarrow 97 = r^2$
The equation of the circle is
$(x-3)^2 + (y+4)^2 = 97$.



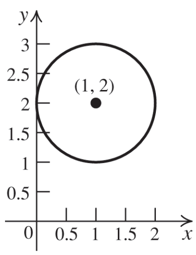
84. The standard form of the equation of a circle with center $(-1, 1)$ is $(x+1)^2 + (y-1)^2 = r^2$. Substitute $(2, 5)$ for (x, y) and solve for r^2.
$(2+1)^2 + (5-1)^2 = r^2 \Rightarrow$
$(3)^2 + 4^2 = r^2 \Rightarrow 25 = r^2$
The equation of the circle is
$(x+1)^2 + (y-1)^2 = 25$.



85. The circle touches the x-axis, so the radius is 2. The equation of the circle is $(x-1)^2+(y-2)^2=4$.



86. The circle touches the y-axis, so the radius is 1. The equation of the circle is $(x-1)^2+(y-2)^2=1$

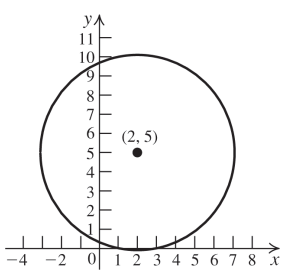


87. Find the diameter by using the distance formula:

$d=\sqrt{(-3-7)^2+(6-4)^2}=\sqrt{104}=2\sqrt{26}$.

So the radius is $\sqrt{26}$. Use the midpoint formula to find the center:

$M=\left(\frac{7+(-3)}{2},\frac{4+6}{2}\right)=(2,5)$. The equation of the circle is $(x-2)^2+(y-5)^2=26$.

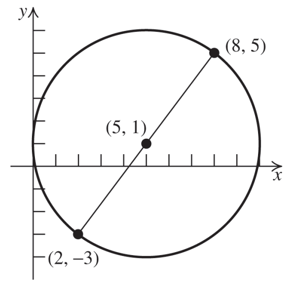


88. Find the center by finding the midpoint of the diameter: $C=\left(\frac{2+8}{2},\frac{-3+5}{2}\right)=(5,1)$

Find the length of the radius by finding the length of the diameter and dividing that by 2.

$d=\sqrt{(2-8)^2+(-3-5)^2}=\sqrt{100}=10$

Thus, the length of the radius is 5, and the equation of the circle is $(x-5)^2+(y-1)^2=25$.



89. a. $x^2+y^2-2x-2y-4=0\Rightarrow$
$x^2-2x+y^2-2y=4$
Now complete the square:
$x^2-2x+1+y^2-2y+1=4+1+1\Rightarrow$
$(x-1)^2+(y-1)^2=6$. This is a circle with center (1, 1) and radius $\sqrt{6}$.

b. To find the x-intercepts, let $y=0$ and solve for x:
$(x-1)^2+(0-1)^2=6\Rightarrow(x-1)^2+1=6\Rightarrow$
$(x-1)^2=5\Rightarrow x-1=\pm\sqrt{5}\Rightarrow x=1\pm\sqrt{5}$
Thus, the x-intercepts are $(1+\sqrt{5},\,0)$ and $(1-\sqrt{5},\,0)$.

To find the y-intercepts, let $x=0$ and solve for y:
$(0-1)^2+(y-1)^2=6\Rightarrow 1+(y-1)^2=6\Rightarrow$
$(y-1)^2=5\Rightarrow y-1=\pm\sqrt{5}\Rightarrow y=1\pm\sqrt{5}$
Thus, the y-intercepts are $(0,\,1+\sqrt{5})$ and $(0,\,1-\sqrt{5})$.

90. a. $x^2+y^2-4x-2y-15=0\Rightarrow$
$x^2-4x+y^2-2y=15$
Now complete the square:
$x^2-4x+4+y^2-2y+1=15+4+1\Rightarrow$
$(x-2)^2+(y-1)^2=20$. This is a circle with center (2, 1) and radius $2\sqrt{5}$.

b. To find the x-intercepts, let $y=0$ and solve for x: $(x-2)^2+(0-1)^2=20\Rightarrow$
$(x-2)^2+1=20\Rightarrow(x-2)^2=19\Rightarrow$
$x-2=\pm\sqrt{19}\Rightarrow x=2\pm\sqrt{19}$
Thus, the x-intercepts are $(2+\sqrt{19},\,0)$ and $(2-\sqrt{19},\,0)$.

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(*continued*)

To find the y-intercepts, let $x = 0$ and solve for y: $(0-2)^2 + (y-1)^2 = 20 \Rightarrow$
$4 + (y-1)^2 = 20 \Rightarrow (y-1)^2 = 16 \Rightarrow$
$y - 1 = \pm 4 \Rightarrow y = -3, 5$
Thus, the y-intercepts are (0, −3) and (0, 5).

91. a. $2x^2 + 2y^2 + 4y = 0 \Rightarrow$
$2(x^2 + y^2 + 2y) = 0 \Rightarrow x^2 + y^2 + 2y = 0.$
Now complete the square:
$x^2 + y^2 + 2y + 1 = 0 + 1 \Rightarrow x^2 + (y+1)^2 = 1.$
This is a circle with center (0, –1) and radius 1.

b. To find the x-intercepts, let $y = 0$ and solve for x: $x^2 + (0+1)^2 = 1 \Rightarrow x^2 = 0 \Rightarrow x = 0$
Thus, the x-intercept is (0, 0).
To find the y-intercepts, let $x = 0$ and solve for y:
$0^2 + (y+1)^2 = 1 \Rightarrow y + 1 = \pm 1 \Rightarrow y = 0, -2$
Thus, the y-intercepts are (0, 0) and (0, −2).

92. a. $3x^2 + 3y^2 + 6x = 0 \Rightarrow$
$3(x^2 + y^2 + 2x) = 0 \Rightarrow x^2 + 2x + y^2 = 0.$
Now complete the square:
$x^2 + 2x + 1 + y^2 = 0 + 1 \Rightarrow (x+1)^2 + y^2 = 1.$
This is a circle with center (–1, 0) and radius 1.

b. To find the x-intercepts, let $y = 0$ and solve for x:
$(x+1)^2 + 0^2 = 1 \Rightarrow x + 1 = \pm 1 \Rightarrow x = 0, -2$
Thus, the x-intercepts are (0, 0) and (−2, 0).
To find the y-intercepts, let $x = 0$ and solve for y: $(0+1)^2 + y^2 = 1 \Rightarrow y^2 = 0 \Rightarrow y = 0$
Thus, the y-intercept is (0, 0).

93. a. $x^2 + y^2 - x = 0 \Rightarrow x^2 - x + y^2 = 0.$
Now complete the square:
$x^2 - x + \frac{1}{4} + y^2 = 0 + \frac{1}{4} \Rightarrow$
$\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}.$ This is a circle with center $\left(\frac{1}{2}, 0\right)$ and radius $\frac{1}{2}$.

b. To find the x-intercepts, let $y = 0$ and solve for x: $\left(x - \frac{1}{2}\right)^2 + 0^2 = \frac{1}{4} \Rightarrow x - \frac{1}{2} = \pm\frac{1}{2} \Rightarrow$
$x = 0, 1.$ Thus, the x-intercepts are (0, 0) and (1, 0).
To find the y-intercepts, let $x = 0$ and solve for y: $\left(0 - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4} \Rightarrow y^2 + \frac{1}{4} = \frac{1}{4} \Rightarrow$
$y^2 = 0 \Rightarrow y = 0.$ The y-intercept is (0, 0).

94. a. $x^2 + y^2 + 1 = 0 \Rightarrow x^2 + y^2 = -1.$ The radius cannot be negative, so there is no graph.

b. There are no intercepts.

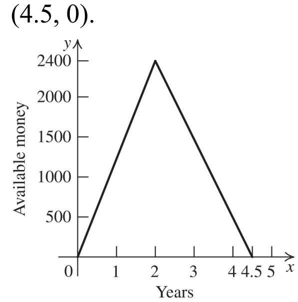
Applying the Concepts

95. The distance from $P(x, y)$ to the x-axis is $|x|$ while the distance from P to the y-axis is $|y|$. So the equation of the graph is $|x| = |y|$.

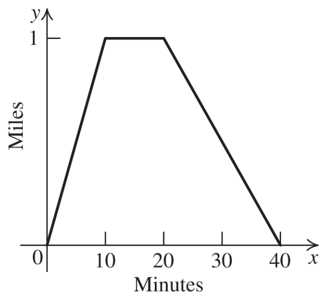
96. The distance from $P(x, y)$ to (1, 2) is $\sqrt{(x-1)^2 + (y-2)^2}$ while the distance from P to (3, –4) is $\sqrt{(x-3)^2 + (y+4)^2}$.

So the equation of the graph is
$$\sqrt{(x-1)^2 + (y-2)^2} = \sqrt{(x-3)^2 + (y+4)^2} \Rightarrow$$
$$(x-1)^2 + (y-2)^2 = (x-3)^2 + (y+4)^2 \Rightarrow$$
$$x^2 - 2x + 1 + y^2 - 4y + 4 = x^2 - 6x + 9 + y^2 + 8y + 16 \Rightarrow$$
$$-2x - 4y + 5 = -6x + 8y + 25 \Rightarrow$$
$$4x - 20 = 12y \Rightarrow y = \frac{1}{3}x - \frac{5}{3}.$$

97. If you save \$100 each month, it will take 24 months (or two years) to save \$2400. So, the graph starts at (0, 0) and increases to (2, 2400). It will take another 30 months (or 2.5) years to withdraw \$80 per month until the \$2400 is gone. Thus, the graph passes through (4.5, 0).



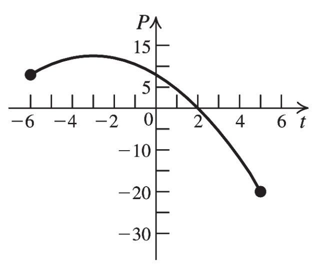
98. If you jog at 6 mph for 10 minutes, then you have traveled $6\left(\frac{1}{6}\right)=1$ mile. So the graph starts at (0, 0) and increases to (10, 1). Resting for 10 minutes takes the graph to (20, 1). It will take 20 minutes to walk one mile at 3 mph back to the starting point. Thus, the graph passes through (40, 0).



99. a. July 2022 is represented by $t = 0$, so March 2022 is represented by $t = -4$. The monthly profit for March is determined by $P=-0.5(-4)^2-3(-4)+8=\$12$ million.

b. July 2018 is represented by $t = 0$, so October 2018 is represented by $t = 3$. So the monthly profit for October is determined by $P=-0.5(3)^2-3(3)+8=-\$5.5$ million. This is a loss.

c.



Because $t = 0$ represents July 2022, $t = -6$ represents January 2022, and $t = 5$ represents December 2022.

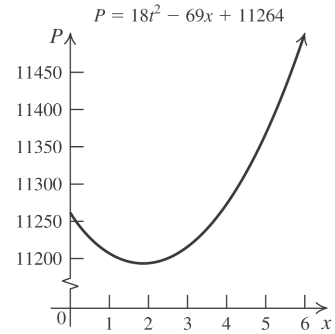
d. To find the t-intercept, set $P = 0$ and solve for t: $0=-0.5t^2-3t+8 \Rightarrow$

$$t=\frac{3\pm\sqrt{(-3)^2-4(-0.5)(8)}}{2(-0.5)}=\frac{3\pm\sqrt{25}}{-1}$$
$$=2 \text{ or } -8$$

The t-intercepts represent the months with no profit and no loss. In this case, $t = -8$ makes no sense in terms of the problem, so we disregard this solution. $t = 2$ represents Sept 2022.

e. To find the P-intercept, set $t = 0$ and solve for P: $P=-0.5(0)^2-3(0)+8 \Rightarrow P=8$. The P-intercept represents the profit in July 2022.

100. a.

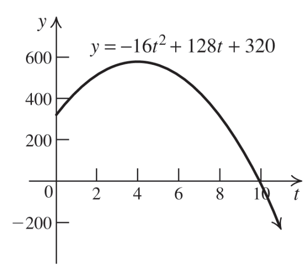


b. To find the P-intercept, set $t = 0$ and solve for P: $P=-18(0)^2-69(0)+11,264 \Rightarrow$ $P=11,264$. The P-intercept represents the number of female college students (in thousands) in 2015.

101. a.

T sec	Height $=-16t^2+128t+320$
0	320 feet
1	432 feet
2	512 feet
3	560 feet
4	576 feet
5	560 feet
6	512 feet

b.



c. $0 \le t \le 10$

d. To find the t-intercept, set $y = 0$ and solve for t:

$0 = -16t^2 + 128t + 320 \Rightarrow$

$0 = -16(t^2 - 8t - 20) \Rightarrow$

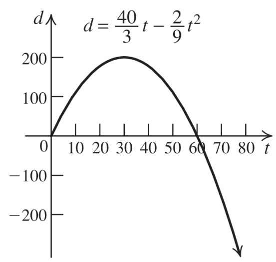
$0 = (t - 10)(t + 2) \Rightarrow \ t = 10$ or $t = -2$.

The graph does not apply if $t < 0$, so the t-intercept is 10. This represents the time when the object hits the ground. To find the y-intercept, set $t = 0$ and solve for y:

$y = -16(0)^2 + 128(0) + 320 \Rightarrow y = 320.$

This represents the height of the building.

102. a.



b. $0 \le t \le 60$

c. The total time of the experiment is 60 minutes or 1 hour.

Beyond the Basics

103. $x^2 + y^2 - 4x + 2y - 20 = 0 \Rightarrow$

$x^2 - 4x + y^2 + 2y = 20 \Rightarrow$

$x^2 - 4x + 4 + y^2 + 2y + 1 = 20 + 4 + 1 \Rightarrow$

$(x - 2)^2 + (y + 1)^2 = 25$

This is the graph of a circle with center (2, −1) and radius 5. The area of this circle is 25π.

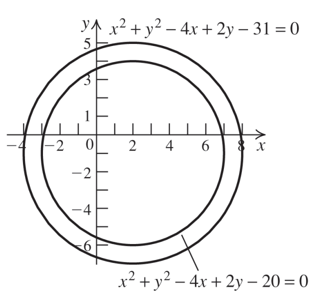
$x^2 + y^2 - 4x + 2y - 31 = 0 \Rightarrow$

$x^2 - 4x + y^2 + 2y = 31 \Rightarrow$

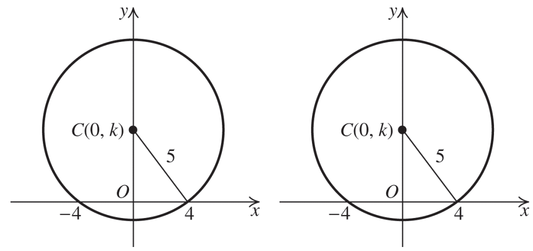
$x^2 - 4x + 4 + y^2 + 2y + 1 = 31 + 4 + 1 \Rightarrow$

$(x - 2)^2 + (y + 1)^2 = 36$

This is the graph of a circle with center (2, −1) and radius 6. The area of this circle is 36π. Both circles have the same center, so the area of the region bounded by the two circles equals $36\pi - 25\pi = 11\pi$.



104. Using the hint, we know that the center of the circle will have coordinates (0, k).



Use the Pythagorean theorem to find k.

$k^2 + 4^2 = 5^2 \Rightarrow k^2 + 16 = 25 \Rightarrow k^2 = 9 \Rightarrow$

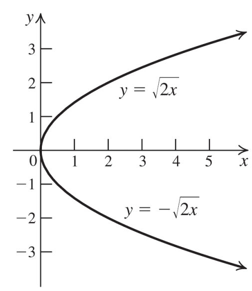
$k = \pm 3$

The equations of the circles are

$x^2 + (y \pm 3)^2 = 5^2.$

Critical Thinking/Discussion/Writing

105. The graph of $y^2 = 2x$ is the union of the graphs of $y = \sqrt{2x}$ and $y = -\sqrt{2x}$.



106. Let (x, y) be a point on the graph. The graph is symmetric with regard to the x-axis, so the point (x, −y) is also on the graph. Because the graph is symmetric with regard to the y-axis, the point (−x, y) is also on the graph. Therefore the point (−x, −y) is on the graph, and the graph is symmetric with respect to the origin. The converse, however, is not true. If a graph is symmetric with respect to the origin, it is not necessarily symmetric with respect to the x-axis and y-axis. For example, the graph of $y = x^3$ is symmetric with respect to the origin but is not symmetric with respect to the x- and y-axes.

107. a. First find the radius of the circle:

$d(A, B) = \sqrt{(6 - 0)^2 + (8 - 1)^2} = \sqrt{85} \Rightarrow$

$r = \dfrac{\sqrt{85}}{2}.$

The center of the circle is

$\left(\dfrac{6+0}{2}, \dfrac{1+8}{2}\right) = \left(3, \dfrac{9}{2}\right).$

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The equation of the circle is

$$(x-3)^2+\left(y-\frac{9}{2}\right)^2=\frac{85}{4}.$$

To find the x-intercepts, set $y = 0$, and solve for x:

$$(x-3)^2+\left(0-\frac{9}{2}\right)^2=\frac{85}{4}\Rightarrow$$

$$(x-3)^2+\frac{81}{4}=\frac{85}{4}\Rightarrow x^2-6x+9=1\Rightarrow$$

$$x^2-6x+8=0$$

The x-intercepts are the roots of this equation.

b. First find the radius of the circle:

$$d(A,B)=\sqrt{(a-0)^2+(b-1)^2}$$
$$=\sqrt{a^2+(b-1)^2}\Rightarrow$$

$$r=\frac{\sqrt{a^2+(b-1)^2}}{2}.$$

The center of the circle is

$$\left(\frac{a+0}{2},\frac{b+1}{2}\right)=\left(\frac{a}{2},\frac{b+1}{2}\right)$$

So the equation of the circle is

$$\left(x-\frac{a}{2}\right)^2+\left(y-\frac{b+1}{2}\right)^2=\frac{a^2+(b-1)^2}{4}.$$

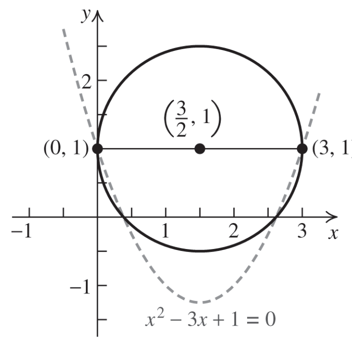
To find the x-intercepts, set $y = 0$ and solve for x:

$$\left(x-\frac{a}{2}\right)^2+\left(0-\frac{b+1}{2}\right)^2=\frac{a^2+(b-1)^2}{4}$$
$$x^2-ax+\frac{a^2}{4}+\frac{(b+1)^2}{4}=\frac{a^2+(b-1)^2}{4}$$
$$4x^2-4ax+a^2+b^2+2b+1=a^2+b^2-2b+1$$
$$4x^2-4ax+4b=0$$
$$x^2-ax+b=0$$

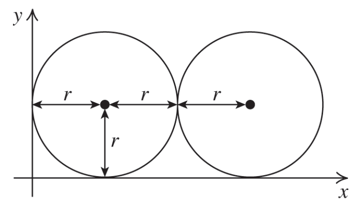
The x-intercepts are the roots of this equation.

c. $a = 3$ and $b = 1$. Approximate the roots of the equation by drawing a circle whose diameter has endpoints $A(0, 1)$ and $B(3, 1)$.

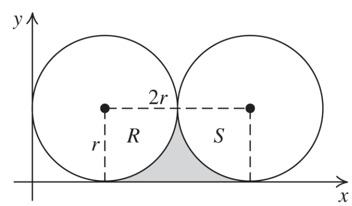
The center of the circle is $\left(\frac{3}{2},1\right)$ and the radius is $\frac{3}{2}$. The roots are approximately $(0.4, 0)$ and $(2.6, 0)$.



108. a. The coordinates of the center of each circle are (r, r) and $(3r, r)$.



b. To find the area of the shaded region, first find the area of the rectangle shown in the figure below, and then subtract the sum of the areas of the two sectors, A and B.



$$A_{\text{rect}}=r(2r)=2r^2$$

$$A_{\text{sector }R}=A_{\text{sector }S}=\frac{1}{4}\pi r^2$$

$$A_{\text{shaded region}}=2r^2-\left(\frac{1}{4}\pi r^2+\frac{1}{4}\pi r^2\right)$$
$$=2r^2-\frac{1}{2}\pi r^2=\left(2-\frac{\pi}{2}\right)r^2$$

Getting Ready for the Next Section

GR1. $\frac{5-3}{6-2}=\frac{2}{4}=\frac{1}{2}$

GR2. $\frac{1-2}{-2-2}=\frac{-1}{-4}=\frac{1}{4}$

GR3. $\frac{2-(-3)}{3-13}=\frac{5}{-10}=-\frac{1}{2}$

GR4. $\frac{3-1}{-2-(-6)}=\frac{2}{4}=\frac{1}{2}$

GR5. $\dfrac{\frac{1}{2}-\frac{1}{4}}{\frac{3}{8}-\left(-\frac{1}{4}\right)}=\dfrac{\frac{1}{4}}{\frac{5}{8}}=\dfrac{1}{4}\cdot\dfrac{8}{5}=\dfrac{2}{5}$

GR6. $\dfrac{\frac{3}{4}-1}{\frac{1}{2}-\frac{1}{6}}=\dfrac{-\frac{1}{4}}{\frac{1}{3}}=\left(-\frac{1}{4}\right)(3)=-\dfrac{3}{4}$

GR7. $2x+3y=6\Rightarrow 3y=-2x+6\Rightarrow y=-\frac{2}{3}x+2$

GR8. $\frac{x}{2}-\frac{y}{5}=3\Rightarrow -\frac{y}{5}=-\frac{x}{2}+3\Rightarrow y=\frac{5}{2}x-15$

GR9. $y-2-\frac{2}{3}(x+1)=0\Rightarrow y-2=\frac{2}{3}(x+1)\Rightarrow$
$y=\frac{2}{3}x+\frac{2}{3}+2=\frac{2}{3}x+\frac{8}{3}$

GR10. $0.1x+0.2y=0\Rightarrow 0.2y=-0.1x\Rightarrow y=-0.5x$

1.3 Lines

Practice Problems

1. $m=\dfrac{-3-5}{6-(-7)}=-\dfrac{8}{13}$

A slope of $-\frac{8}{13}$ means that the value of y decreases 8 units for every 13 units increase in x.

2. $P(-2,-3),\ m=-\frac{2}{3}$

$$y-(-3)=-\frac{2}{3}[x-(-2)]$$
$$y+3=-\frac{2}{3}(x+2)$$
$$y+3=-\frac{2}{3}x-\frac{4}{3}$$
$$y=-\frac{2}{3}x-\frac{13}{3}$$

3. $m=\dfrac{-4-6}{-3-(-1)}=\dfrac{-10}{-2}=5$

Use either point to determine the equation of the line. Using (−3, −4), we have

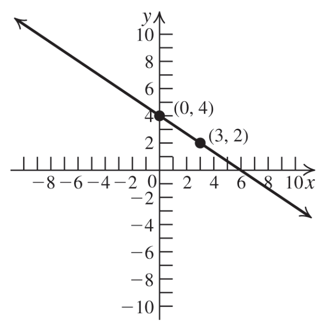
$y-(-4)=5[x-(-3)]$
$y+4=5(x+3)$ (point-slope form)
$y+4=5x+15\Rightarrow y=5x+11$

Using (−1, 6), we have

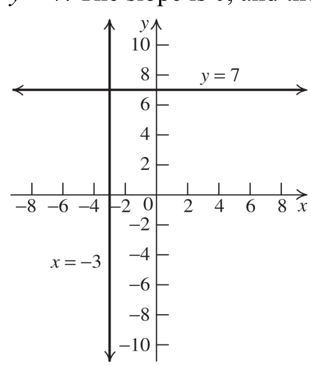
$y-6=5[x-(-1)]$
$y-6=5(x+1)$ (point-slope form)
$y-6=5x+5\Rightarrow y=5x+11$

4. $y-y_1=m(x-x_1)$
$y-(-3)=2(x-0)$ (point-slope form)
$y-(-3)=2(x-0)$
$y+3=2x\Rightarrow y=2x-3$

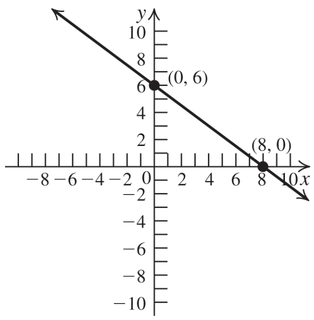
5. The slope is $-\frac{2}{3}$ and the y-intercept is 4. The line goes through (0, 4), so locate a second point by moving two units down and three units right. Thus, the line goes through (3, 2).



6. $x=-3$. The slope is undefined, and there is no y-intercept. The x-intercept is −3.
$y=7$. The slope is 0, and the y-intercept is 7.



7. First, solve for y to write the equation in slope-intercept form:
$3x + 4y = 24 \Rightarrow 4y = -3x + 24 \Rightarrow$
$y = -\frac{3}{4}x + 6$. The slope is $-\frac{3}{4}$, and the y-intercept is 6. Find the x-intercept by setting $y = 0$ and solving the equation for x:
$0 = -\frac{3}{4}x + 6 \Rightarrow 6 = \frac{3}{4}x \Rightarrow 8 = x$. Thus, the graph passes through the points (0, 6) and (8, 0).



8. Use the equation $H = 2.5x + 54$.
$H_1 = 2.5(43) + 54 = 161.5$
$H_2 = 2.5(44) + 54 = 164$
The person is between 161.5 cm and 164 cm tall, or 1.615 m and 1.64 m.

9. Parallel lines have the same slope, so the slope of the line is $m = \frac{3-7}{2-5} = \frac{-4}{-3} = \frac{4}{3}$. Using the point-slope form, we have
$y - 5 = \frac{4}{3}[x - (-2)] \Rightarrow 3y - 15 = 4(x+2) \Rightarrow$
$3y - 15 = 4x + 8 \Rightarrow 4x - 3y + 23 = 0$

10. The slopes of perpendicular lines are negative reciprocals. Write the equation $4x + 5y + 1 = 0$ in slope-intercept form to find its slope: $4x + 5y + 1 = 0 \Rightarrow$
$5y = -4x - 1 \Rightarrow y = -\frac{4}{5}x - \frac{1}{5}$. The slope of a line perpendicular to this lines is $\frac{5}{4}$. Using the point-slope form, we have
$y - (-4) = \frac{5}{4}(x-3) \Rightarrow 4(y+4) = 5(x-3) \Rightarrow$
$4y + 16 = 5x - 15 \Rightarrow 5x - 4y - 31 = 0$

11. Because 2026 is 12 years after 2014, set $x = 12$. Then $y = 631.1(12) + 3376.25 =$ 10949.45
There will be 10,949.45 million Facebook users worldwide in 2026.

Concepts and Vocabulary

1. The slope of a horizontal line is 0; the slope of a vertical line is undefined.
2. The slope of the line passing through the points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.
3. Every line parallel to the line $y = 3x - 2$ has slope, m, equal to 3.
4. Every line perpendicular to the line $y = 3x - 2$ has slope, m, equal to $-\frac{1}{3}$.
5. False. The slope of the line $y = -\frac{1}{4}x + 5$ is equal to $-\frac{1}{4}$.
6. False. The y-intercept of the line $y = 2x - 3$ is equal to –3.
7. True
8. True

Building Skills

9. $m = \frac{7-3}{4-1} = \frac{4}{3}$; the graph is rising.

10. $m = \frac{0-4}{2-0} = \frac{-4}{2} = -2$; the graph is falling.

11. $m = \frac{-2-(-2)}{-2-6} = \frac{0}{-8} = 0$; the graph is horizontal.

12. $m = \frac{7-(-4)}{-3-(-3)} = \frac{11}{0} \Rightarrow$ slope is undefined; the graph is vertical.

13. $m = \frac{-3.5-2}{3-0.5} = \frac{-5.5}{2.5} = -2.2$; the graph is falling.

14. $m = \frac{-3-(-2)}{2-3} = \frac{-1}{-1} = 1$; the graph is rising.

15. $m = \dfrac{5-1}{\left(1+\sqrt{2}\right)-\sqrt{2}} = \dfrac{4}{1} = 4$; the graph is rising.

16. $m = \dfrac{3\sqrt{3}-0}{\left(1+\sqrt{3}\right)-\left(1-\sqrt{3}\right)} = \dfrac{3\sqrt{3}}{2\sqrt{3}} = \dfrac{3}{2}$; the graph is rising.

17. ℓ_3

18. ℓ_2

19. ℓ_4

20. ℓ_1

21. ℓ_1 passes through the points (2, 3) and (−5, −4). $m_{\ell_1} = \dfrac{-4-3}{-5-2} = \dfrac{-7}{-7} = 1.$

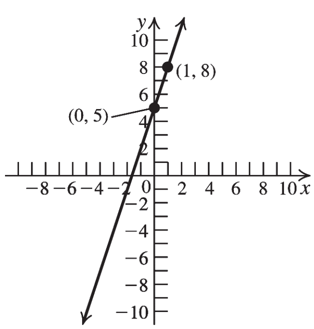
22. ℓ_2 is a horizontal line, so it has slope 0.

23. ℓ_3 passes through the points (2, 3) and (0, −1). $m_{\ell_3} = \dfrac{-1-3}{0-2} = 2.$

24. ℓ_4 passes through the points (−3, 3) and (0, −1). $m_{\ell_4} = \dfrac{-1-3}{0-(-3)} = -\dfrac{4}{3}.$

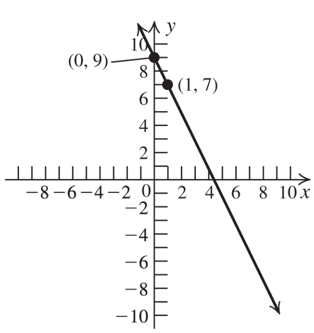
25. $(0, 5)$; $m = 3$

$y = 3x + 5$

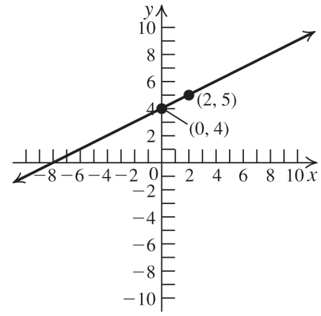


26. $(0, 9)$; $m = -2$

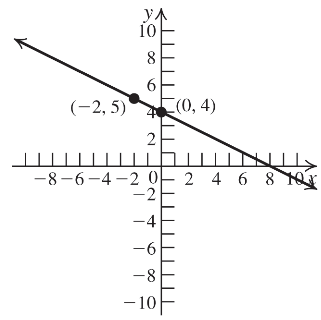
$y = -2x + 9$



27. $y = \dfrac{1}{2}x + 4$

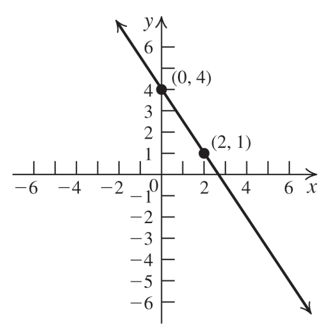


28. $y = -\dfrac{1}{2}x + 4$

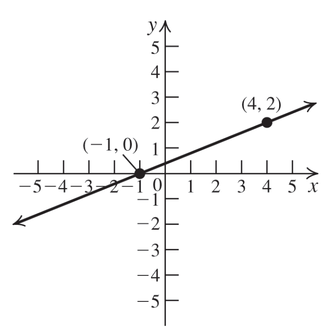


29. $y - 1 = -\dfrac{3}{2}(x-2) \Rightarrow y - 1 = -\dfrac{3}{2}x + 3 \Rightarrow$

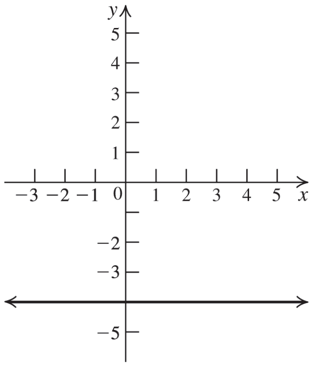
$y = -\dfrac{3}{2}x + 4$



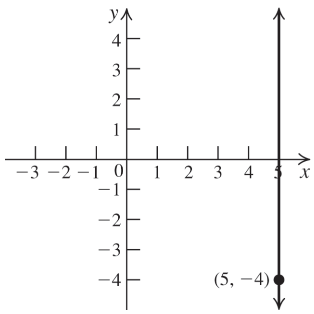
30. $y = \dfrac{2}{5}(x+1) \Rightarrow y = \dfrac{2}{5}x + \dfrac{2}{5}$



31. $y+4=0(x-5) \Rightarrow y+4=0 \Rightarrow y=-4$



32. Because the slope is undefined, the graph is vertical. The equation is $x=5$.



33. $m=\dfrac{0-1}{1-0}=-1$. The y-intercept is (0, 1), so the equation is $y=-x+1$.

34. $m=\dfrac{3-1}{1-0}=2$. The y-intercept is (0, 1), so the equation is $y=2x+1$.

35. $m=\dfrac{3-3}{3-(-1)}=0$ Because the slope = 0, the line is horizontal. Its equation is $y=3$.

36. $m=\dfrac{7-1}{2-(-5)}=\dfrac{6}{7}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.

$$y-1=\frac{6}{7}(x+5) \Rightarrow y-1=\frac{6}{7}x+\frac{30}{7} \Rightarrow$$
$$y=\frac{6}{7}x+\frac{37}{7}$$

37. $m=\dfrac{1-(-1)}{1-(-2)}=\dfrac{2}{3}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.

$$y+1=\frac{2}{3}(x+2) \Rightarrow y+1=\frac{2}{3}x+\frac{4}{3} \Rightarrow$$
$$y=\frac{2}{3}x+\frac{1}{3}$$

38. $m=\dfrac{-9-(-3)}{6-(-1)}=-\dfrac{6}{7}$. Now write the equation in point-slope form, and then solve for y to write the equation in slope-intercept form.

$$y+3=-\frac{6}{7}(x+1) \Rightarrow y+3=-\frac{6}{7}x-\frac{6}{7} \Rightarrow$$
$$y=-\frac{6}{7}x-\frac{27}{7}$$

39. $m=\dfrac{2-\frac{1}{4}}{0-\frac{1}{2}}=\dfrac{\frac{7}{4}}{-\frac{1}{2}}=-\dfrac{7}{2}$.

The y-intercept is 2, so the equation is $y=-\dfrac{7}{2}x+2$.

40. $m=\dfrac{3-(-7)}{4-4}=\dfrac{10}{0} \Rightarrow$ the slope is undefined. So the graph is a vertical line. The equation is $x=4$.

41. $x=5$

42. $y=1.5$

43. $y=0$

44. $x=0$

45. $y=14$

46. $y=2x+5$

47. $y=-\dfrac{2}{3}x-4$

48. $y=-6x-3$

49. $m=\dfrac{4-0}{0-(-3)}=\dfrac{4}{3}$; $y=\dfrac{4}{3}x+4$

50. $m=\dfrac{-2-0}{0-(-5)}=-\dfrac{2}{5}$; $y=-\dfrac{2}{5}x-2$

51. $y=7$

52. $x=4$

53. $y=-5$

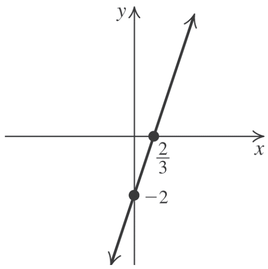
54. $x=-3$

55. $y = 3x - 2$

The slope is 3 and the y-intercept is (0, –2).

$0 = 3x - 2 \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$

The x-intercept is $\left(\frac{2}{3}, 0\right)$.

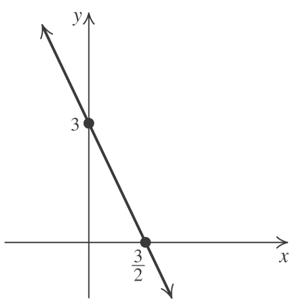


56. $y = -2x + 3$

The slope is –2 and the y-intercept is (0, 3).

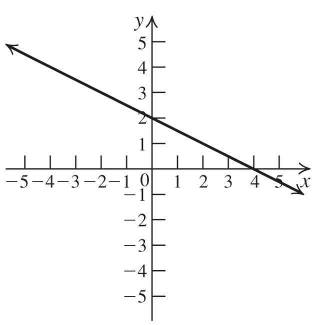
$0 = -2x + 3 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$

The x-intercept is $\left(\frac{3}{2}, 0\right)$.



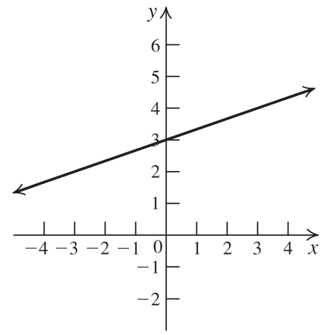
57. $x + 2y - 4 = 0 \Rightarrow 2y = -x + 4 \Rightarrow y = -\frac{1}{2}x + 2.$

The slope is $-1/2$, and the y-intercept is (0, 2). To find the x-intercept, set $y = 0$ and solve for x: $x + 2(0) - 4 = 0 \Rightarrow x = 4$.



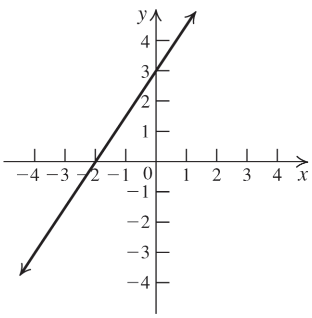
58. $x = 3y - 9 \Rightarrow x + 9 = 3y \Rightarrow y = \frac{1}{3}x + 3$

The slope is $1/3$, and the y-intercept is (0, 3). To find the x-intercept, set $y = 0$ and solve for x: $x = 3(0) - 9 \Rightarrow x = -9$.



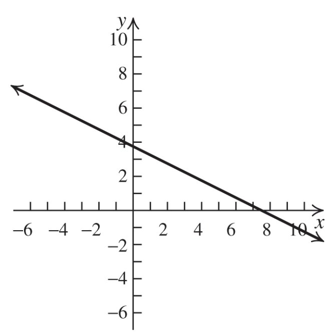
59. $3x - 2y + 6 = 0 \Rightarrow 3x + 6 = 2y \Rightarrow \frac{3}{2}x + 3 = y$.

The slope is $3/2$, and the y-intercept is (0, 3). To find the x-intercept, set $y = 0$ and solve for x: $3x - 2(0) + 6 = 0 \Rightarrow 3x = -6 \Rightarrow x = -2$.

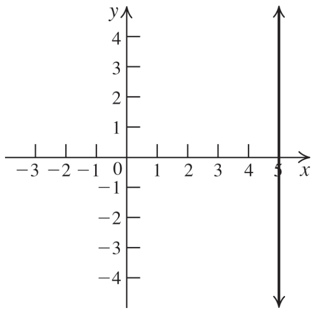


60. $2x = -4y + 15 \Rightarrow 2x - 15 = -4y \Rightarrow$

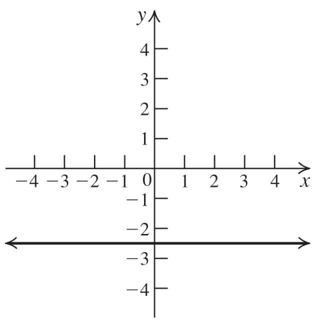
$-\frac{1}{2}x + \frac{15}{4} = y$. The slope is $-1/2$, and the y-intercept is $15/4$. To find the x-intercept, set $y = 0$ and solve for x: $2x = -4(0) + 15 \Rightarrow$ $x = 15/2$.



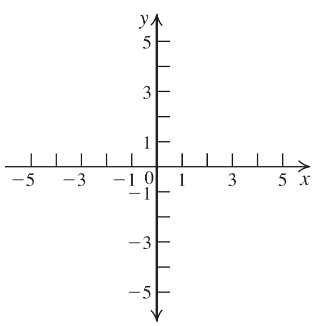
61. $x - 5 = 0 \Rightarrow x = 5$. The slope is undefined, and there is no *y*-intercept. The *x*-intercept is 5.



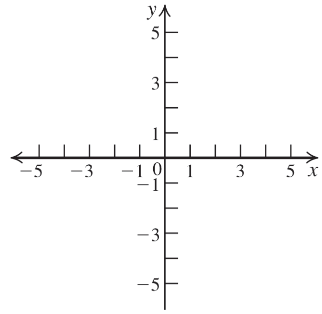
62. $2y + 5 = 0 \Rightarrow y = -\frac{5}{2}$. The slope is 0, and the *y*-intercept is $-5/2$. This is a horizontal line, so there is no *x*-intercept.



63. $x = 0$. The slope is undefined, and the *y*-intercepts are the *y*-axis. This is a vertical line whose *x*-intercept is 0.



64. $y = 0$. The slope is 0, and the *x*-intercepts are the *x*-axis. This is a horizontal line whose *y*-intercept is 0.



For exercises 65–68, the two-intercept form of the equation of a line is $\frac{x}{a} + \frac{y}{b} = 1$, where a is the *x*-intercept and b is the *y*-intercept.

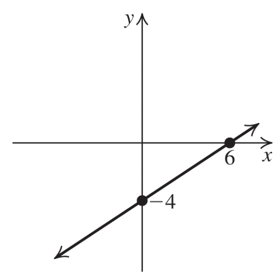
65. $\frac{x}{4} + \frac{y}{3} = 1$

66. $-\frac{x}{3} + \frac{y}{2} = 1$

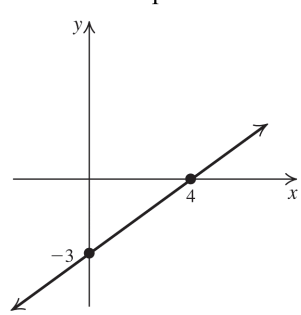
67. $2x + 3y = 6 \Rightarrow \frac{2x}{6} + \frac{3y}{6} = \frac{6}{6} \Rightarrow \frac{x}{3} + \frac{y}{2} = 1$;
x-intercept = 3; *y*-intercept = 2

68. $3x - 4y + 12 = 0 \Rightarrow 3x - 4y = -12 \Rightarrow$
$\frac{3x}{-12} - \frac{4y}{-12} = \frac{-12}{-12} \Rightarrow -\frac{x}{4} + \frac{y}{3} = 1$;
x-intercept = –4; *y*-intercept = 3

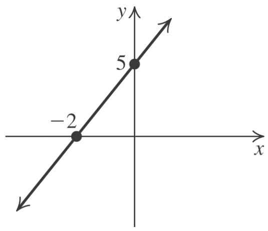
69. $2x - 3y = 12 \Rightarrow \frac{2x}{12} - \frac{3y}{12} = 1 \Rightarrow \frac{x}{6} - \frac{y}{4} = 1$
The *x*-intercept is 6 and the *y*-intercept is –4.



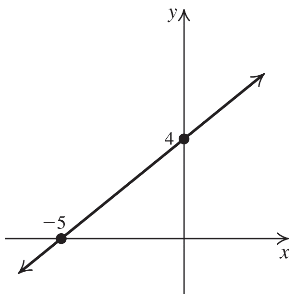
70. $3x - 4y = 12 \Rightarrow \frac{3x}{12} - \frac{4y}{12} = 1 \Rightarrow \frac{x}{4} - \frac{y}{3} = 1$
The *x*-intercept is 4 and the *y*-intercept is –3.



71. $-5x+2y=10 \Rightarrow -\frac{5x}{10}+\frac{2y}{10}=1 \Rightarrow -\frac{x}{2}+\frac{y}{5}=1$
The x-intercept is –2 and the y-intercept is 5.



72. $-4x+5y=20 \Rightarrow -\frac{4x}{20}+\frac{5y}{20}=1 \Rightarrow -\frac{x}{5}+\frac{y}{4}=1$
The x-intercept is –5 and the y-intercept is 4.



73. $m=\frac{9-4}{7-2}=\frac{5}{5}=1.$ The equation of the line through (2, 4) and (7, 9) is $y-4=1(x-2) \Rightarrow$ $y=x+2$. Check to see if (–1, 1) satisfies the equation by substituting $x=-1$ and $y=1$: $1=-1+2 \Rightarrow 1=1.$ So (–1, 1) lies on the line.

74. $m=\frac{-3-2}{2-7}=\frac{-5}{-5}=1.$ The equation of the line through (7, 2) and (2, –3) is $y-2=1(x-7) \Rightarrow y=x-5.$ Check to see if (5, 1) satisfies the equation by substituting $x=5$ and $y=1$:
$1=5-5 \Rightarrow 1 \neq 0.$
So (5, 1) does not lie on the line, and the points are not collinear.

75. The given line passes through the points (0, 3) and (4, 0), so its slope is $-\frac{3}{4}.$ Any line parallel to this line will have the same slope. The line that passes through the origin and is parallel to the given line has equation $y=-\frac{3}{4}x.$

76. From exercise 75, the slope of the given line is $-\frac{3}{4}.$ Any line perpendicular to this line will have slope $\frac{4}{3}.$ The line that passes through the origin and is perpendicular to the given line has equation $y=\frac{4}{3}x.$

77. The red line passes through the points (–2, 0) and (0, 3), so its slope is $\frac{3}{2}.$ The blue line passes through (4, 2) and has the same slope, so its equation is
$y-2=\frac{3}{2}(x-4) \Rightarrow 2y-4=3x-12 \Rightarrow$
$2y=3x-8 \Rightarrow y=\frac{3}{2}x-4$

78. The red line passes through the points (–2, 0) and (0, 3), so its slope is $\frac{3}{2}.$ The green line passes through (4, 2) and has slope $-\frac{2}{3},$ so its equation is
$y-2=-\frac{2}{3}(x-4) \Rightarrow 3y-6=-2x+8 \Rightarrow$
$3y=-2x+14 \Rightarrow y=-\frac{2}{3}x+\frac{14}{3}$

79. The slope of $y=3x-1$ is 3. The slope of $y=3x+2$ is also 3. The lines are parallel.

80. The slope of $y=2x+2$ is 2. The slope of $y=-2x+2$ is –2. The lines are neither parallel nor perpendicular.

81. The slope of $y=2x-4$ is 2. The slope of $y=-\frac{1}{2}x+4$ is $-\frac{1}{2}.$ The lines are perpendicular.

82. The slope of $y=3x+1$ is 3. The slope of $y=\frac{1}{3}x-1$ is $\frac{1}{3}.$ The lines are neither parallel nor perpendicular.

83. The slope of $3x+8y=7$ is $-3/8$, while the slope of $5x-7y=0$ is $5/7$. The lines are neither parallel nor perpendicular.

84. The slope of $10x+2y=3$ is –5. The slope of $5x+y=-1$ is also –5, so the lines are parallel.

85. The slope of $x=4y+8$ is $1/4$. The slope of $y=-4x+1$ is -4, so the lines are perpendicular.

86. The slope of $y=3x+1$ is 3. The slope of $6y+2x=0$ is $-1/3$. The lines are perpendicular.

87. Both lines are vertical lines. The lines are parallel.

88. The slope of $2x+3y=7$ is $-2/3$, while $y=2$ is a horizontal line. The lines are neither parallel nor perpendicular.

89. The equation of the line through (2, –3) with slope 3 is
$y+3=3(x-2)\Rightarrow y+3=3x-6\Rightarrow$
$y=3x-9.$

90. The equation of the line through (–1, 3) with slope –2 is
$y-3=-2(x-(-1))\Rightarrow y-3=-2(x+1)\Rightarrow$
$y-3=-2x-2\Rightarrow y=-2x+1.$

91. A line perpendicular to a line with slope $-\frac{1}{2}$ has slope 2. The equation of the line through (–1, 2) with slope 2 is $y-2=2(x-(-1))\Rightarrow$
$y-2=2(x+1)\Rightarrow y-2=2x+2\Rightarrow$
$y=2x+4.$

92. A line perpendicular to a line with slope $\frac{1}{3}$ has slope –3. The equation of the line through (2, –1) with slope –3 is
$y-(-1)=-3(x-2)\Rightarrow$
$y+1=-3(x-2)\Rightarrow y+1=-3x+6\Rightarrow$
$y=-3x+5.$

93. The slope of the line joining (1, –2) and (–3, 2) is $\frac{2-(-2)}{-3-1}=-1.$ The equation of the line through (–2, –5) with slope –1 is
$y-(-5)=-(x-(-2))\Rightarrow y+5=-(x+2)\Rightarrow$
$y+5=-x-2\Rightarrow y=-x-7.$

94. The slope of the line joining (–2, 1) and (3, 5) is $\frac{5-1}{3-(-2)}=\frac{4}{5}.$
The equation of the line through (1, 2) with slope $\frac{4}{5}$ is
$y-2=\frac{4}{5}(x-1)\Rightarrow 5(y-2)=4(x-1)\Rightarrow$
$5y-10=4x-4\Rightarrow 5y=4x+6\Rightarrow$
$y=\frac{4}{5}x+\frac{6}{5}.$

95. The slope of the line joining (–3, 2) and (–4, –1) is
$\frac{-1-2}{-4-(-3)}=3.$
A line perpendicular to this line has slope $-\frac{1}{3}$.
The equation of the line through (1, –2) with slope $-\frac{1}{3}$ is
$y-(-2)=-\frac{1}{3}(x-1)\Rightarrow 3(y+2)=-(x-1)\Rightarrow$
$3y+6=-x+1\Rightarrow 3y=-x-5\Rightarrow$
$y=-\frac{1}{3}x-\frac{5}{3}.$

96. The slope of the line joining (2, 1) and (4, –1) is
$\frac{-1-1}{4-2}=-1.$
A line perpendicular to this line has slope 1
The equation of the line through (–1, 2) with slope 1 is
$y-2=x-(-1)\Rightarrow y-2=x+1\Rightarrow y=x+3.$

97. The slope of the line $y=6x+5$ is 6. The lines are parallel, so the slope of the new line is also 6. The equation of the line with slope 6 and y-intercept 4 is $y=6x+4$.

98. The slope of the line $y=-\frac{1}{2}x+5$ is $-\frac{1}{2}$. The lines are parallel, so the slope of the new line is also $-\frac{1}{2}$. The equation of the line with slope $-\frac{1}{2}$ and y-intercept 2 is $y=-\frac{1}{2}x+2$.

99. The slope of the line $y=6x+5$ is 6. The lines are perpendicular, so the slope of the new line is $-\frac{1}{6}$. The equation of the line with slope $-\frac{1}{6}$ and y-intercept 4 is $y=-\frac{1}{6}x+4$.

100. The slope of the line $y=-\frac{1}{2}x+5$ is $-\frac{1}{2}$. The lines are perpendicular, so the slope of the new line is 2. The equation of the line with slope 2 and y-intercept –4 is $y=2x-4$.

101. The slope of $x+y=1$ is -1. The lines are parallel, so they have the same slope. The equation of the line through (1, 1) with slope -1 is $y-1=-(x-1)\Rightarrow y-1=-x+1\Rightarrow$
$y=-x+2$.

102. The slope of $-2x+3y=7$ is $2/3$. The lines are parallel, so they have the same slope. The equation of the line through (1, 0) with slope $2/3$ is $y-0=\frac{2}{3}(x-1) \Rightarrow y=\frac{2}{3}x-\frac{2}{3}$.

103. The slope of $3x-9y=18$ is $1/3$. The lines are perpendicular, so the slope of the new line is -3. The equation of the line through (–2, 4) with slope -3 is $y-4=-3(x-(-2)) \Rightarrow$ $y-4=-3x-6 \Rightarrow y=-3x-2$.

104. The slope of $-2x+y=14$ is 2. The lines are perpendicular, so the slope of the new line is $-1/2$. The equation of the line through (0, 2) with slope $-1/2$ is $y=-\frac{1}{2}x+2$.

Applying the Concepts

In exercises 105 and 106, recall that 1 ft ≈ 30.48 cm.

105. $H=2.42x+81.83$
Substitute 43 for x:
$H=2.42(43)+81.83=185.89$ cm

Now convert to feet: $\frac{185.89}{30.48} \approx 6.1$ ft
The male is approximately 6.1 ft tall.

106. $H=2.9x+61.53$
Substitute 36 for x:
$H=2.9(36)+61.53=165.93$ cm

Now convert to feet: $\frac{165.93}{30.48} \approx 5.4$ ft
The female is approximately 5.4 ft tall.

107. a. The y-intercept represents the initial expenses.

b. The x-intercept represents the point at which the teacher breaks even, i.e., the expenses equal the income.

c. The teacher's profit if there are 16 students in the class is \$640.

d. The slope of the line is
$$\frac{640-(-750)}{16-0}=\frac{1390}{16}=\frac{695}{8}$$
The equation of the line is
$$P=\frac{695}{8}n-750.$$

108. a. The y-intercept represents the initial prepaid amount.

b. The x-intercept represents the total number of minutes the cellphone can be used.

c. The slope of the line is
$\frac{0-15}{75-0}=-\frac{15}{75}=-\frac{1}{5}$.

The equation of the line is $P=-\frac{1}{5}t+15$.

d. The cost per minute is $\$\frac{1}{5}=20$¢.

109. slope $=\frac{\text{rise}}{\text{run}} \Rightarrow \frac{4}{40}=\frac{1}{10}$

110. 4 miles = 21,120 feet.
$|\text{slope}|=\frac{\text{rise}}{\text{run}} \Rightarrow \frac{2000}{21{,}120}=\frac{25}{264}$

111. 8 in. in two weeks $\Rightarrow$ the plant grows 4 in. per week. John wants to trim the hedge when it grows 6 in., so he should trim it every $\frac{6}{4}=1.5$ weeks ≈ 10 days.

112. $\frac{2\text{ min.}}{5\text{ in.}}=\frac{x\text{ min.}}{31\text{ in.}} \Rightarrow x=\frac{2\cdot 31}{5}=12.4$ min.
The water will overflow in about 12 min.

113. a. x = the number of weeks; y = the amount of money in the account after x weeks;
$y=7x+130$

b. The slope is the amount of money deposited each week; the y-intercept is the initial deposit.

114. a. x = the number of sessions of golf; y = the yearly payment to the club; $y=35x+1000$

b. The slope is the cost per golf session; the y-intercept is the yearly membership fee.

115. a. x = the number of months owed to pay off the refrigerator; y = the amount owed after x months
$y=-15x+600$

b. The slope is the amount that the balance due changes per month; the y-intercept is the initial amount owed.

116. a. x = the number of rupees; y = the number of dollars equal to x rupees.
$y=\frac{1}{72.5}x \approx 0.0138x$

b. The slope is the number of dollars per rupee. The y-intercept is the number of dollars for 0 rupees.

117. a. x = the number of years after 2010; y = the life expectancy of a female born in the U.S. in the year 2010 + x; $y = 0.17x + 80.8$

b. The slope is the rate of increase in life expectancy; the y-intercept is the current life expectancy of a female born in the U.S. in 2010.

118. a. $v = -1400(2) + 14,000 = \$11,200$

b. $v = -1400(6) + 14,000 = \$5600$

To find when the tractor will have no value, set $v = 0$ and solve the equation for t:
$0 = -1400t + 14,000 \Rightarrow t = 10$ years

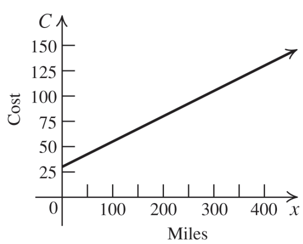
119. There are 30 days in June. For the first 13 days, you used data at a rate of $\frac{435}{13} \approx 33.5$ MB/day. At the same rate, you will use $33.5(17) = 569.5$ MB for the rest of the month.
435 + 569.5 = 1004.5
So, you don't need to buy extra data. You will have about 20 MB left.

120. For the first three hours, you traveled at $\frac{195}{3} = 65$ mph.
$d = rt \Rightarrow 520 - 195 = 65t \Rightarrow 325 = 65t \Rightarrow$
$t = 5$
You will arrive at your destination five hours after 12 pm or 5 pm.

121. $y = 5x + 40,000$

122. a. $C = 0.25x + 30$

b.



c. $y = 0.25(60) + 30 = \$45$

d. $47.75 = 0.25x + 30 \Rightarrow x = 71$ miles

123. a. The two points are (100, 212) and (0, 32). So the slope is $\frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$.
The equation is
$F - 32 = \frac{9}{5}(C - 0) \Rightarrow F = \frac{9}{5}C + 32$

b. One degree Celsius change in the temperature equals $9/5$ degrees change in degrees Fahrenheit.

c.

C	$F = \frac{9}{5}C + 32$
40ºC	104°F
25º C	77°F
–5ºC	23°F
–10ºC	14°F

d. $100°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 37.78°\text{C} \approx 38°\text{C}$

$90°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 32.22°\text{C} \approx 32°\text{C}$

$75°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = 23.89°\text{C} \approx 24°\text{C}$

$-10°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = -23.33°\text{C} \approx -24°\text{C}$

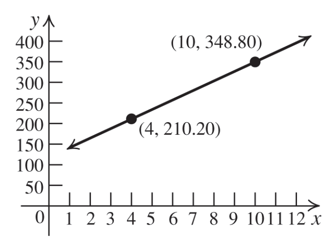
$-20°\text{F} = \frac{9}{5}C + 32 \Rightarrow C = -28.89°\text{C} \approx -29°\text{C}$

e. $97.6°\text{F} = \frac{9}{5}C + 32 \Rightarrow C \approx 36.44°\text{C}$;

$99.6°\text{F} = \frac{9}{5}C + 32 \Rightarrow C \approx 37.56°\text{C}$

f. Let x = °F = °C. Then $x = \frac{9}{5}x + 32 \Rightarrow$
$-\frac{4}{5}x = 32 \Rightarrow x = -40$. At –40°, °F = °C.

124. a. The two points are (4, 210.20) and (10, 348.80). So the slope is $\frac{348.80 - 210.20}{10 - 4} = \frac{138.6}{6} = 23.1$. The equation is $y - 348.8 = 23.1(x - 10) \Rightarrow$
$y = 23.1x + 117.8$

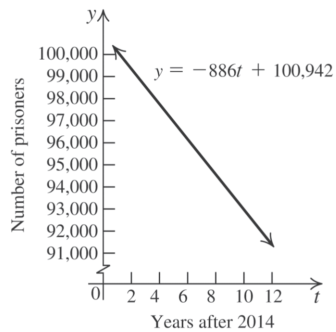


b. The slope represents the cost of producing customized skateboard deck. The y-intercept represents the fixed cost.

c. $y = 23.1(12) + 117.8 \Rightarrow y = \$395$

125. a. The year 2014 is represented by $t = 0$, and the year 2019 is represented by $t = 5$. The points are (0, 100942) and (5, 96512). So the slope is $\frac{96512 - 100942}{5} = -886$ The equation is $y - 100942 = -886(t - 0) \Rightarrow$ $y = -886t + 100,942$.

b.



c. The year 2017 is represented by $t = 3$.
$y = -886(3) + 100942 = 98284$
There were 98,284 prisoners in 2017.

d. The year 2025 is represented by $t = 11$.
$y = -886(11) + 100942 = 91196$
There will be 91,196 prisoners in 2025.

126. a. The two points are (5, 5.73) and (8, 6.27). The slope is $\frac{6.27 - 5.73}{8 - 5} = \frac{0.54}{3} = 0.18$. The equation is $V - 5.73 = 0.18(x - 5) \Rightarrow$ $V = 0.18x + 4.83$.

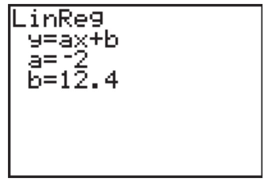
b. The slope represents the monthly change in the number of viewers. The V-intercept represents the number of viewers when the show first started.

c. $V = 0.18(11) + 4.83 \Rightarrow V = 6.81$ million

127. Let p represent the number of people not covered by health insurance. Let t represent the number of years after 2015, with $t = 0$ representing 2015. The two data points are (0, 29) and (5, 31). So the slope is $\frac{31 - 29}{5} = 0.4$. The equation for the model is $p - 29 = 0.4(t - 0) \Rightarrow$ $p = 0.4t + 29$.
The year 2025 is represented by $t = 10$.
$p = 0.4(10) + 29 = 33$
In 2025, 33 million people in the U.S. will not be covered by health insurance.

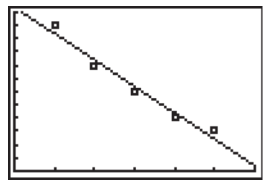
128. The year 2010 is represented by $t = 0$, so the year 2021 is represented by $t = 11$. The two data points are (0, 86.4) and (11, 97.1). So the slope is $\frac{97.1 - 86.4}{11 - 0} = \frac{10.7}{11} \approx 0.97$.
The equation is $y - 86.4 = 0.97(t - 0) \Rightarrow$
$y = 0.97t + 86.4$

129. a.



$y = -2x + 12.4$

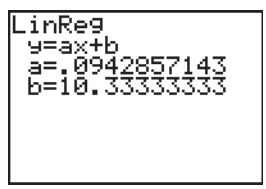
b.



[0, 6, 1] by [0, 12, 1]

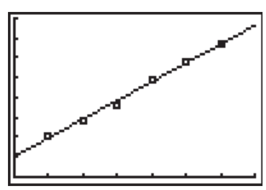
c. The price in the table is given as the number of nickels. 35¢ = 7 nickels, so let $x = 7$. $y = -2(7) + 12.4 = -1.6$
Thus, no newspapers will be sold if the price per copy is 35¢. Note that this is also clear from the graph, which appears to cross the x-axis at approximately $x = 6$.

130. a.



$y \approx 0.09x + 10.3$

b.



[0, 700, 100] by [0, 80, 10]

c. The advertising expenses in the table are given as thousands of dollars, so let $x = 700$. $y \approx 0.09(700) + 10.3 = 73.3$
Sales are given in thousands, so approximately $73.3 \times 1000 = 73,300$ computers will be sold.

Beyond the Basics

131. $3 = \dfrac{c-3}{1-(-2)} \Rightarrow 9 = c - 3 \Rightarrow c = 12$

132. The y-intercept is –4, so its coordinates are (0, –4). Substitute $x = 0$, $y = -4$ into the equation and solve for c.

$3x - cy - 2 = 0 \Rightarrow 3(0) - c(-4) - 2 = 0 \Rightarrow$
$4c - 2 = 0 \Rightarrow 4c = 2 \Rightarrow c = \dfrac{1}{2}$

133. a. Let $A = (0, 1)$, $B = (1, 3)$, $C = (-1, -1)$.

$m_{AB} = \dfrac{3-1}{1-0} = 2; m_{BC} = \dfrac{-1-3}{-1-1} = \dfrac{-4}{-2} = 2$

$m_{AC} = \dfrac{-1-1}{-1-0} = 2$

The slopes of the three segments are the same, so the points are collinear.

b. $d(A,B) = \sqrt{(1-0)^2 + (3-1)^2} = \sqrt{5}$

$d(B,C) = \sqrt{(-1-1)^2 + (-1-3)^2} = 2\sqrt{5}$

$d(A,C) = \sqrt{(-1-0)^2 + (-1-1)^2} = \sqrt{5}$

Because $d(B, C) = d(A, B) + d(A, C)$, the three points are collinear.

134. a. Let $A = (1, 0.5)$, $B = (2, 0)$, $C = (0.5, 0.75)$.

$m_{AB} = \dfrac{0-0.5}{2-1} = -0.5; m_{BC} = \dfrac{0.75-0}{0.5-2} = -0.5$

$m_{AC} = \dfrac{0.75-0.5}{0.5-1} = -0.5$

The slopes of the three segments are the same, so the points are collinear.

b. $d(A,B) = \sqrt{(1-2)^2 + \left(\dfrac{1}{2} - 0\right)^2} = \sqrt{\dfrac{5}{4}} = \dfrac{\sqrt{5}}{2}$

$d(B,C) = \sqrt{\left(\dfrac{1}{2} - 2\right)^2 + \left(\dfrac{3}{4} - 0\right)^2}$
$= \sqrt{\dfrac{45}{16}} = \dfrac{3\sqrt{5}}{4}$

$d(A,C) = \sqrt{\left(\dfrac{1}{2} - 1\right)^2 + \left(\dfrac{3}{4} - \dfrac{1}{2}\right)^2}$
$= \sqrt{\dfrac{5}{16}} = \dfrac{\sqrt{5}}{4}$

Because $d(B, C) = d(A, B) + d(A, C)$, the three points are collinear.

135. a. $m_{AB} = \dfrac{4-1}{-1-1} = -\dfrac{3}{2}; m_{BC} = \dfrac{8-4}{5-(-1)} = \dfrac{2}{3}$.

The product of the slopes = –1, so $AB \perp BC$.

b. $d(A,B) = \sqrt{(-1-1)^2 + (4-1)^2} = \sqrt{13}$

$d(B,C) = \sqrt{(5-(-1))^2 + (8-4)^2} = \sqrt{52}$

$d(A,C) = \sqrt{(5-1)^2 + (8-1)^2} = \sqrt{65}$

$(d(A,B))^2 + (d(B,C))^2 = (d(A,C))^2$, so the triangle is a right triangle.

136. $m_{\overline{AB}} = \dfrac{2-(-1)}{1-(-4)} = \dfrac{3}{5}; m_{\overline{BC}} = \dfrac{1-2}{3-1} = -\dfrac{1}{2}$

$m_{\overline{CD}} = \dfrac{-2-1}{-2-3} = \dfrac{3}{5}; m_{\overline{AD}} = \dfrac{-2-(-1)}{-2-(-4)} = -\dfrac{1}{2}$

So, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$, and $ABCD$ is a parallelogram.

For exercises 137 and 138, refer to the figures accompanying the exercises in your text.

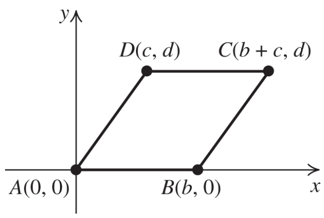
137. $\overline{AD}$ and $\overline{BC}$ are parallel because they lie on parallel lines l_1 and l_2. $\overline{AB}$ and $\overline{CD}$ are parallel because they are parallel to the x-axis. Therefore, $ABCD$ is a parallelogram.
$\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$ because opposite sides of a parallelogram are congruent.
$\triangle ABD \cong \triangle CDB$ by SSS. Then
$m_1 = \dfrac{\text{rise}}{\text{run}} = \dfrac{BD}{CD}$ and $m_2 = \dfrac{\text{rise}}{\text{run}} = \dfrac{BD}{AB}$. Since
$AB = CD$, $m_1 = \dfrac{BD}{CD} = \dfrac{BD}{AB} = m_2$.

138. $\triangle OKA \sim \triangle BLO$ because $OL = AK = d$ and $BL = OK = c$. Then, $m_1 = \dfrac{\text{rise}}{\text{run}} = \dfrac{d}{c}$ and

$m_2 = \dfrac{\text{rise}}{\text{run}} = \dfrac{c}{-d} = -\dfrac{c}{d}$.

$m_1 \cdot m_2 = \dfrac{d}{c}\left(-\dfrac{c}{d}\right) = -1$.

139. Let the quadrilateral $ABCD$ be such that $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$. Locate the points as shown in the figure.



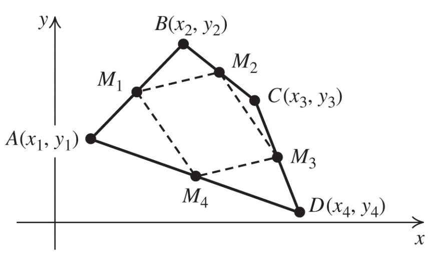
Because $\overline{AB} \parallel \overline{CD}$, the y-coordinates of C and D are equal. Because $\overline{AB} \cong \overline{CD}$, the x-coordinates of the points are as shown in the figure. The slope of AD is d/c. The slope of BC is $\dfrac{d-0}{b+c-b} = \dfrac{d}{c}$. So $\overline{AD} \parallel \overline{BC}$.

$d(A, D) = \sqrt{d^2 + c^2}$.

$d(B, C) = \sqrt{d^2 + ((b+c) - b)^2} = \sqrt{d^2 + c^2}$.

So $\overline{AD} \cong \overline{BC}$.

140. Let $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ and $D(x_4, y_4)$ be the vertices of the quadrilateral.



Then the midpoint M_1 of $\overline{AB}$ is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$; the midpoint M_2 of $\overline{BC}$ is $\left(\dfrac{x_2 + x_3}{2}, \dfrac{y_2 + y_3}{2}\right)$; the midpoint M_3 of $\overline{CD}$ is $\left(\dfrac{x_3 + x_4}{2}, \dfrac{y_3 + y_4}{2}\right)$; and the midpoint M_4 of $\overline{AD}$ is $\left(\dfrac{x_1 + x_4}{2}, \dfrac{y_1 + y_4}{2}\right)$.

The slope of $\overline{M_1M_2}$ is

$$\frac{\frac{y_1 + y_2}{2} - \frac{y_2 + y_3}{2}}{\frac{x_1 + x_2}{2} - \frac{x_2 + x_3}{2}} = \frac{y_1 - y_3}{x_1 - x_3}.$$

The slope of $\overline{M_2M_3}$ is

$$\frac{\frac{y_2 + y_3}{2} - \frac{y_3 + y_4}{2}}{\frac{x_2 + x_3}{2} - \frac{x_3 + x_4}{2}} = \frac{y_2 - y_4}{x_2 - x_4}.$$

The slope of $\overline{M_3M_4}$ is

$$\frac{\frac{y_3 + y_4}{2} - \frac{y_1 + y_4}{2}}{\frac{x_3 + x_4}{2} - \frac{x_1 + x_4}{2}} = \frac{y_3 - y_1}{x_3 - x_1} = \frac{y_1 - y_3}{x_1 - x_3}.$$

The slope of $\overline{M_1M_4}$ is

$$\frac{\frac{y_1 + y_2}{2} - \frac{y_1 + y_4}{2}}{\frac{x_1 + x_2}{2} - \frac{x_1 + x_4}{2}} = \frac{y_2 - y_4}{x_2 - x_4}.$$

So $\overline{M_1M_2} \parallel \overline{M_3M_4}$ and $\overline{M_2M_3} \parallel \overline{M_1M_4}$, and $M_1M_2M_3M_4$ is a parallelogram.

141. Let (x, y) be the coordinates of point B. Then

$d(A, B) = 12.5 = \sqrt{(x-2)^2 + (y-2)^2} \Rightarrow$

$(x-2)^2 + (y-2)^2 = 156.25$ and

$m_{\overline{AB}} = \dfrac{4}{3} = \dfrac{y-2}{x-2} \Rightarrow 4(x-2) = 3(y-2) \Rightarrow$

$y = \dfrac{4}{3}x - \dfrac{2}{3}$. Substitute this into the first equation and solve for x:

$$(x-2)^2 + \left(\left(\frac{4}{3}x - \frac{2}{3}\right) - 2\right)^2 = 156.25$$

$$(x-2)^2 + \left(\frac{4}{3}x - \frac{8}{3}\right)^2 = 156.25$$

$$x^2 - 4x + 4 + \frac{16}{9}x^2 - \frac{64}{9}x + \frac{64}{9} = 156.25$$

$$9x^2 - 36x + 36 + 16x^2 - 64x + 64 = 1406.25$$

$$25x^2 - 100x - 1306.25 = 0$$

Solve this equation using the quadratic formula:

$$x = \frac{100 \pm \sqrt{100^2 - 4(25)(-1306.25)}}{2(25)}$$
$$= \frac{100 \pm \sqrt{10,000 + 130,625}}{50}$$
$$= \frac{100 \pm \sqrt{140,625}}{50} = \frac{100 \pm 375}{50}$$
$$= 9.5 \text{ or } -5.5$$

(*continued on next page*

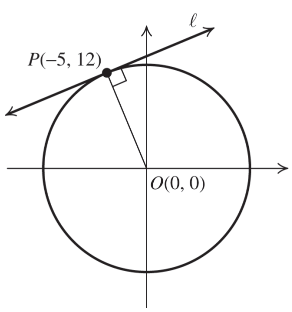
(*continued*)

Now find y by substituting the x-values into the slope formula: $\frac{4}{3} = \frac{y-2}{9.5-2} \Rightarrow y = 12$ or $\frac{4}{3} = \frac{y-2}{-5.5-2} \Rightarrow y = -8$. So the coordinates of B are (9.5, 12) or (–5.5, –8).

142. Let (x, y) be a point on the circle with (x_1, y_1) and (x_2, y_2) as the endpoints of a diameter. Then the line that passes through (x, y) and (x_1, y_1) is perpendicular to the line that passes through (x, y) and (x_2, y_2), and their slopes are negative reciprocals. So

$$\frac{y-y_1}{x-x_1} = -\frac{x-x_2}{y-y_2} \Rightarrow$$
$$(y-y_1)(y-y_2) = -(x-x_1)(x-x_2) \Rightarrow$$
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2) = 0.$$

143.

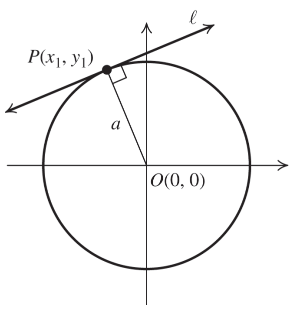


$$m_{\overline{OP}} = \frac{12-0}{-5-0} = -\frac{12}{5}$$

Since the tangent line ℓ is perpendicular to $\overline{OP}$, the slope of ℓ is the negative reciprocal of $-\frac{12}{5}$ or $\frac{5}{12}$. Using the point-slope form, we have

$$y-12 = \frac{5}{12}\left[x-(-5)\right] \Rightarrow y-12 = \frac{5}{12}(x+5) \Rightarrow$$
$$y-12 = \frac{5}{12}x + \frac{25}{12} \Rightarrow y = \frac{5}{12}x + \frac{169}{12}$$

144.



$$m_{\overline{OP}} = \frac{y_1-0}{x_1-0} = \frac{y_1}{x_1}$$

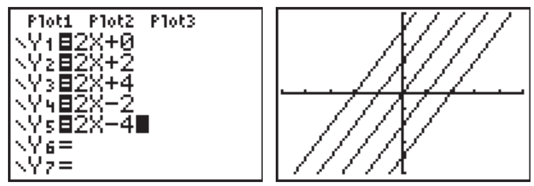
Since the tangent line ℓ is perpendicular to $\overline{OP}$, the slope of ℓ is the negative reciprocal of $\frac{y_1}{x_1}$ or $-\frac{x_1}{y_1}$.

Using the point-slope form, we have

$$y-y_1 = -\frac{x_1}{y_1}(x-x_1) \Rightarrow$$
$$yy_1 - y_1^2 = -xx_1 + x_1^2 \Rightarrow xx_1 + yy_1 = x_1^2 + y_1^2$$

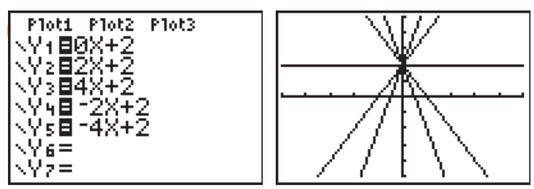
Since the equation of the circle is $x^2 + y^2 = a^2$, we substitute a^2 for $x_1^2 + y_1^2$ to obtain $xx_1 + yy_1 = a^2$.

145.



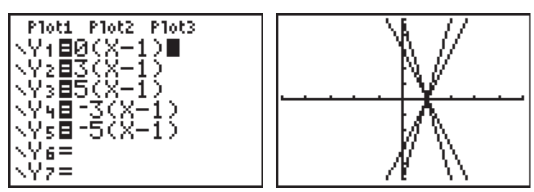
The family of lines has slope 2. The lines have different y-intercepts.

146.



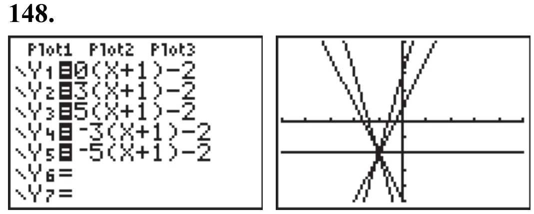
The family of lines has y-intercept 2. The lines have different slopes.

147.



The lines pass through (1, 0). The lines have different slopes.

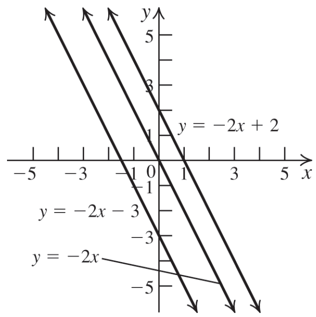
148.



The lines pass through (−1, −2). The lines have different slopes.

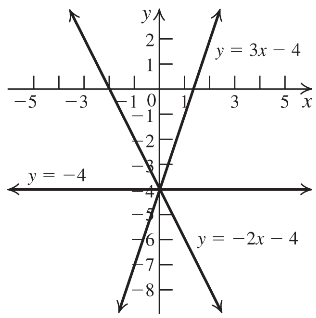
Critical Thinking/Discussion/Writing

149. a.



This is a family of lines parallel to the line $y = -2x$. They all have slope –2.

b.



This is a family of lines that passes through the point (0, –4). Their y-intercept is –4.

150. $\left.\begin{array}{l} y = m_1x + b_1 \\ y = m_2x + b_2 \end{array}\right\} \Rightarrow m_1x + b_1 = m_2x + b_2 \Rightarrow$
$m_1x - m_2x = b_2 - b_1 \Rightarrow x(m_1 - m_2) = b_2 - b_1 \Rightarrow$
$x = \dfrac{b_2 - b_1}{m_1 - m_2}$

a. If $m_1 > m_2 > 0$ and $b_1 > b_2$, then
$x = \dfrac{b_2 - b_1}{m_1 - m_2} = -\dfrac{b_1 - b_2}{m_1 - m_2} < 0.$

b. If $m_1 > m_2 > 0$ and $b_1 < b_2$, then
$x = \dfrac{b_2 - b_1}{m_1 - m_2} > 0.$

c. If $m_1 < m_2 < 0$ and $b_1 > b_2$, then
$x = \dfrac{b_2 - b_1}{m_1 - m_2} = \dfrac{b_1 - b_2}{m_2 - m_1} > 0.$

d. If $m_1 < m_2 < 0$ and $b_1 < b_2$, then
$x = \dfrac{b_2 - b_1}{m_1 - m_2} = -\dfrac{b_2 - b_1}{m_2 - m_1} < 0.$

Active Learning

151. a.–c. Refer to the app using the QR code in your text.

Getting Ready for the Next Section

GR1. $x^2 - 4 = 0 \Rightarrow (x+2)(x-2) = 0 \Rightarrow$
$x + 2 = 0 \Rightarrow x = -2$ or
$x - 2 = 0 \Rightarrow x = 2$
Solution: {–2, 2}

GR2. $1 - x^2 = 0 \Rightarrow (1+x)(1-x) = 0 \Rightarrow$
$1 + x = 0 \Rightarrow x = -1$ or
$1 - x = 0 \Rightarrow x = 1$
Solution: {–1, 1}

GR3. $x^2 - x - 2 = 0$
$(x+1)(x-2) = 0$
$x + 1 = 0 \Rightarrow x = -1$ or $x - 2 = 0 \Rightarrow x = 2$
Solution: {–1, 2}

GR4. $x^2 + 2x - 3 = 0$
$(x+3)(x-1) = 0$
$x + 3 = 0 \Rightarrow x = -3$
$x - 1 = 0 \Rightarrow x = 1$
Solution: {–3, 1}

GR5. $(3(a+h)+1) - (3a+1) = (3a+3h+1) - (3a+1)$
$= 3h$

GR6. $\left(2(a+h)^2+1\right) - \left(2a^2+1\right)$
$= \left(2\left(a^2+2ah+h^2\right)+1\right) - \left(2a^2+1\right)$
$= \left(2a^2+4ah+2h^2+1\right) - \left(2a^2+1\right)$
$= 4ah + 2h^2$

GR7. $\dfrac{-(a+h)^2 + a^2}{h} = \dfrac{-\left(a^2+2ah+h^2\right)+a^2}{h}$
$= \dfrac{-2ah - h^2}{h} = \dfrac{h(-2a-h)}{h}$
$= -2a - h$

GR8. $\dfrac{1}{h}\left(\dfrac{1}{a+h} - \dfrac{1}{a}\right) = \dfrac{1}{h}\left(\dfrac{a-(a+h)}{a(a+h)}\right) = \dfrac{-h}{h(a(a+h))}$
$= -\dfrac{1}{a(a+h)}$

GR9. $(x-1)(x-3) < 0$

Solve the associated equation:
$(x-1)(x-3) = 0 \Rightarrow x = 1$ or $x = 3$.
So, the intervals are
$(-\infty, 1), (1, 3)$, and $(1, \infty)$.

Interval	Test point	Value of $(x-1)(x-3)$	Result
$(-\infty, 1)$	0	3	+
$(1, 3)$	2	–2	–
$(3, \infty)$	4	3	+

The solution set is $(1, 3)$.

GR10. $x^2 - 2x - 3 \geq 0$

$x^2 - 2x - 3 \geq 0 \Rightarrow (x-3)(x+1) \geq 0$
Now solve the associated equation:
$(x-3)(x+1) = 0 \Rightarrow x = 3$ or $x = -1$.
So, the intervals are
$(-\infty, -1], [-1, 3]$, and $[3, \infty)$.

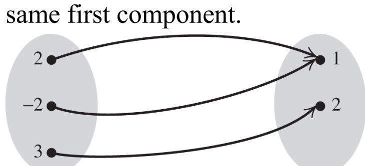
Interval	Test point	Value of $x^2 - 2x - 3$	Result
$(-\infty, -1]$	–2	5	+
$[-1, 3]$	0	–3	–
$[3, \infty)$	5	12	+

The solution set is $(-\infty, -1] \cup [3, \infty)$.

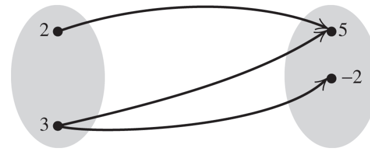
1.4 Functions

Practice Problems

1. a. The domain of R is $\{2, -2, 3\}$ and its range is $\{1, 2\}$. The relation R is a function because no two ordered pairs in R have the same first component.



b. The domain of S is $\{2, 3\}$ and its range is $\{5, -2\}$. The relation S is not a function because the ordered paired (3, −2) and (3, 5) have the same first component.



2. Solve each equation for y.

a. $2x^2 - y^2 = 1 \Rightarrow 2x^2 - 1 = y^2 \Rightarrow$
$\pm\sqrt{2x^2 - 1} = y$; not a function

b. $x - 2y = 5 \Rightarrow x - 5 = 2y \Rightarrow \frac{1}{2}(x-5) = y$;
a function

3. $g(x) = -2x^2 + 5x$

a. $g(0) = -2(0)^2 + 5(0) = 0$

b. $g(-1) = -2(-1)^2 + 5(-1) = -7$

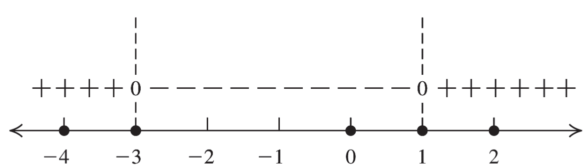
c. $g(x+h) = -2(x+h)^2 + 5(x+h)$
$= -2(x^2 + 2xh + h^2) + 5x + 5h$
$= -2x^2 - 4hx + 5x - 2h^2 + 5h$

4. $A_{TLMS} = (\text{length})(\text{height}) = (|3-1|)(22)$
$= (2)(22) = 44$ sq. units

5. a. The function f puts no restrictions on the variable x. You can multiply any real number by 3 and then subtract 5. So, the domain is the set of all real numbers, in interval notation $(-\infty, \infty)$.

b. $f(x) = \frac{1}{\sqrt{1-x}}$ is not defined when
$1 - x = 0 \Rightarrow x = 1$ or
when $1 - x < 0 \Rightarrow 1 < x$. . Thus, the domain of f is $(-\infty, 1)$.

c. $g(x) = \sqrt{x^2 + 2x - 3}$ is not defined when
$x^2 + 2x - 3 < 0$.
Use the test point method to see that
$x^2 + 2x - 3 < 0$ on the interval (–3, 1).
Thus, the domain of g is
$(-\infty, -3] \cup [1, \infty)$.



$x^2 + 2x - 3 = (x+3)(x-1) \geq 0$ or $(-\infty, -3] \cup [1, \infty)$

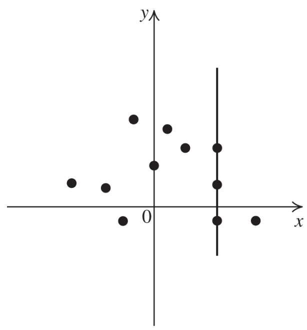
6. $f(x) = x^2$, domain $X = [-3, 3]$

a. $f(x) = 10 \Rightarrow x^2 = 10 \Rightarrow x = \pm\sqrt{10} \approx \pm 3.16$
Since $\sqrt{10} > 3$ and $-\sqrt{10} < -3$, neither solution is in the interval $X = [-3, 3]$. Therefore, 10 is not in the range of *f*.

b. $f(x) = 4 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$
Since $-3 < -2 < 2 < 3$, 4 is in the range of *f*.

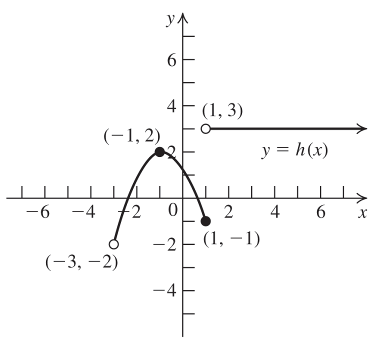
c. The range of *f* is the interval [0, 9] because for each number *y* in this interval, the number $x = \sqrt{y}$ is in the interval [−3, 3].

7.



The graph is not a function because a vertical line can be drawn through three points, as shown.

8.



Domain: $(-3, \infty)$; range: $(-2, 2] \cup \{3\}$

9. $y = f(x) = x^2 + 4x - 5$

a. Check whether the ordered pair (2, 7) satisfies the equation:

$7 \overset{?}{=} 2^2 + 4(2) - 5$
$7 = 7$ ✓

The point (2, 7) is on the graph.

b. Let $y = -8$, then solve for *x*:
$-8 = x^2 + 4x - 5 \Rightarrow 0 = x^2 + 4x + 3 \Rightarrow$
$0 = (x+3)(x+1) \Rightarrow x = -3$ or $x = -1$
The points (−3, −8) and (−1, −8) lie on the graph.

c. Let $x = 0$, then solve for *y*:
$y = 0^2 + 4(0) - 5 = -5$
The *y*-intercept is −5.

d. Let $y = 0$, then solve for *x*:
$0 = x^2 + 4x - 5 \Rightarrow 0 = (x+5)(x-1) \Rightarrow$
$x = -5$ or $x = 1$
The *x*-intercepts are −5 and 1.

10. The range of *C*(*t*) is [6, 12).
$C(11) = \frac{1}{2}C(10) + 6 = \frac{1}{2}(11.989) + 6 \approx 11.995$.

11. From Example 11, we have
$AP = \sqrt{500^2 + x^2}$ and $\overline{PD} = 1200 - x$ feet. If c = the cost on land, the total cost *C* is given by $C = 1.3c(PD) + c(AP)$

$$= 1.3c\sqrt{500^2 + x^2} + c(1200 - x)$$
$$= c\left[1.3\sqrt{500^2 + x^2} + 1200 - x\right]$$

12. a. $C(x) = 1200x + 100{,}000$

b. $R(x) = 2500x$

c. $P(x) = R(x) - C(x)$
$= 2500x - (1200x + 100{,}000)$
$= 1300x - 100{,}000$

d. The break-even point occurs when $C(x) = R(x)$.
$1200x + 100{,}000 = 2500x$
$100{,}000 = 1300x \Rightarrow x \approx 77$
Metro needs 77 shows to break even.

Concepts and Vocabulary

1. In the functional notation $y = f(x)$, *x* is the <u>independent</u> variable.

2. If $f(-2) = 7$, then −2 is in the <u>domain</u> of the function *f*, and 7 is in the <u>range</u> of *f*.

3. If the point (9, −14) is on the graph of a function *f*, then <u>$f(9) = -14$</u>.

4. If (3, 7) and (3, 0) ar both points on a graph, then the graph cannot be the graph of a <u>function</u>.

5. False.

6. False. For example, if $f(x) = \frac{1}{x}$, then $a = 1$ and $b = -1$ are both in the domain of f. However, $a + b = 0$ is not in the domain of f.

7. True. $-x = 7$ and the square root function is defined for all positive numbers.

8. False. The domain of f is all real x for $x > -2$. Values of $x \le -2$ make the fraction undefined.

Building Skills

9. Domain: $\{a, b, c\}$; range: $\{d, e\}$; function

10. Domain: $\{a, b, c\}$; range: $\{d, e, f\}$; function

11. Domain: $\{a, b, c\}$; range: $\{1, 2\}$; function

12. Domain: $\{1, 2, 3\}$; range: $\{a, b, c, d\}$; not a function

13. Domain: $\{0, 3, 8\}$; range: $\{-3, -2, -1, 1, 2\}$; not a function

14. Domain: $\{-3, -1, 0, 1, 2, 3\}$; range: $\{-8, -3, 0, 1\}$; function

15. $x + y = 2 \Rightarrow y = -x + 2$; a function

16. $x = y - 1 \Rightarrow y = x + 1$; a function

17. $y = \frac{1}{x}$; a function

18. $xy = -1 \Rightarrow y = -\frac{1}{x}$; a function

19. $y^2 = x^2 \Rightarrow y = \pm\sqrt{x^2} \Rightarrow y = \pm x$; not a function

20. $x = |y| \Rightarrow y = x$ or $y = -x$; not a function

21. $y = \frac{1}{\sqrt{2x-5}}$; a function

22. $y = \frac{1}{\sqrt{x^2-1}}$; a function

23. $2 - y = 3x \Rightarrow y = 2 - 3x$; a function

24. $3x - 5y = 15 \Rightarrow y = \frac{3}{5}x - 3$; a function

25. $x + y^2 = 8 \Rightarrow y = \pm\sqrt{8-x}$; not a function

26. $x = y^2 \Rightarrow y = \sqrt{x}$ or $y = -\sqrt{x}$; not a function

27. $x^2 + y^3 = 5 \Rightarrow y = \sqrt[3]{5 - x^2}$; a function

28. $x + y^3 = 8 \Rightarrow y = \sqrt[3]{8 - x}$; a function

In exercises 29–32, $f(x) = x^2 - 3x + 1$, $g(x) = \frac{2}{\sqrt{x}}$, and $h(x) = \sqrt{2-x}$.

29. $f(0) = 0^2 - 3(0) + 1 = 1$
$g(0) = \frac{2}{\sqrt{0}} \Rightarrow g(0)$ is undefined
$h(0) = \sqrt{2-0} = \sqrt{2}$
$f(a) = a^2 - 3a + 1$
$f(-x) = (-x)^2 - 3(-x) + 1 = x^2 + 3x + 1$

30. $f(1) = 1^2 - 3(1) + 1 = -1$; $g(1) = \frac{2}{\sqrt{1}} = 2$;
$h(1) = \sqrt{2-1} = 1$; $g(a) = \frac{2}{\sqrt{a}} = \frac{2\sqrt{a}}{a}$;
$g(x^2) = \frac{2}{\sqrt{x^2}} = \frac{2}{|x|}$

31. $f(-1) = (-1)^2 - 3(-1) + 1 = 5$;
$g(-1) = \frac{2}{\sqrt{-1}} \Rightarrow g(-1)$ is undefined;
$h(-1) = \sqrt{2-(-1)} = \sqrt{3}$; $h(c) = \sqrt{2-c}$;
$h(-x) = \sqrt{2-(-x)} = \sqrt{2+x}$

32. $f(4) = 4^2 - 3(4) + 1 = 5$; $g(4) = \frac{2}{\sqrt{4}} = 1$;
$h(4) = \sqrt{2-4} = \sqrt{-2} \Rightarrow h(4)$ is undefined;
$g(2+k) = \frac{2}{\sqrt{2+k}}$;
$f(a+k) = (a+k)^2 - 3(a+k) + 1$
$= a^2 + 2ak + k^2 - 3a - 3k + 1$

33. a. $f(0) = \frac{2(0)}{\sqrt{4-0^2}} = 0$

b. $f(1) = \frac{2(1)}{\sqrt{4-1^2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

c. $f(2) = \frac{2(2)}{\sqrt{4-2^2}} = \frac{4}{0} \Rightarrow f(2)$ is undefined

d. $f(-2)=\dfrac{2(-2)}{\sqrt{4-(-2)^2}}=\dfrac{-4}{0}\Rightarrow f(-2)$ is undefined

e. $f(-x)=\dfrac{2(-x)}{\sqrt{4-(-x)^2}}=\dfrac{-2x}{\sqrt{4-x^2}}$

34. a. $g(0)=2(0)+\sqrt{0^2-4}\Rightarrow g(0)$ is undefined

b. $g(1)=2(1)+\sqrt{1^2-4}\Rightarrow g(1)$ is undefined

c. $g(2)=2(2)+\sqrt{2^2-4}=4$

d. $g(-3)=2(-3)+\sqrt{(-3)^2-4}=-6+\sqrt{5}$

e. $g(-x)=2(-x)+\sqrt{(-x)^2-4}$
$=-2x+\sqrt{x^2-4}$

35. The width of each rectangle is 1. The height of the left rectangle is $f(1)=1^2+2=3$. The height of the right rectangle is $f(2)=2^2+2=6$.
$A=(1)(f(1))+(1)(f(2))$
$=1(3)+(1)(6)=9$ sq. units

36. The width of each rectangle is 1. The height of the left rectangle is $f(0)=0^2+2=2$. The height of the right rectangle is $f(1)=1^2+2=3$.
$A=(1)(f(0))+(1)(f(1))$
$=1(2)+(1)(3)=5$ sq. units

37. $(-\infty,\infty)$

38. $(-\infty,\infty)$

39. The denominator is not defined for $x=9$. The domain is $(-\infty,9)\cup(9,\infty)$

40. The denominator is not defined for $x=-9$. The domain is $(-\infty,-9)\cup(-9,\infty)$

41. The denominator is not defined for $x=-1$ or $x=1$. The domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

42. The denominator is not defined for $x=-2$ or $x=2$. The domain is $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$.

43. The numerator is not defined for $x<3$, and the denominator is not defined for $x=-2$. The domain is $[3,\infty)$

44. The denominator is not defined for $x\ge 4$. The domain is $(-\infty,4)$

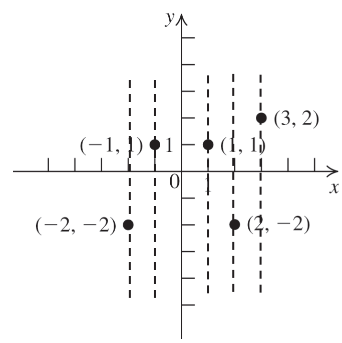
45. The denominator equals 0 if $x=-1$ or $x=-2$. The domain is $(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)$.

46. The denominator equals 0 if $x=-2$ or $x=-3$. The domain is $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$.

47. The denominator is not defined for $x=0$. The domain is $(-\infty,0)\cup(0,\infty)$

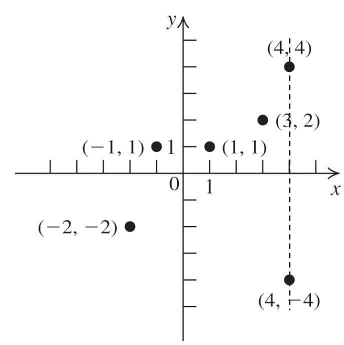
48. The denominator is defined for all values of x. The domain is $(-\infty,\infty)$.

49.



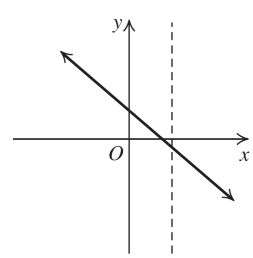
a function

50.

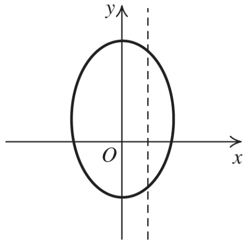


not a function

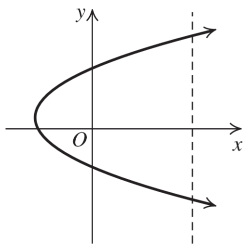
51.



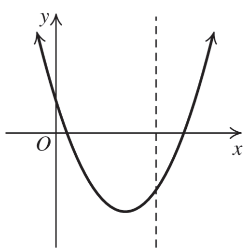
a function

52. 

not a function

53. 

not a function

54. 

a function

55. $f(-4) = -2; f(-1) = 1; f(3) = 5; f(5) = 7$

56. $g(-2) = 5; g(1) = -4; g(3) = 0; g(4) = 5$

57. $h(-2) = -5; h(-1) = 4; h(0) = 3; h(1) = 4$

58. $f(-1) = 4; f(0) = 0; f(1) = -4$

59. $h(x) = 7$, so solve the equation
$7 = x^2 - x + 1$.
$x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0 \Rightarrow x = -2$
or $x = 3$.

60. $H(x) = 7$, so solve the equation
$7 = x^2 + x + 8$.
$x^2 + x + 1 = 0 \Rightarrow x = \dfrac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)} \Rightarrow$
$x = \dfrac{-1 \pm \sqrt{-3}}{2} \Rightarrow$ there is no real solution.

61. a. $1 = -2(1+1)^2 + 7 \Rightarrow 1 = -1$, which is false. Therefore, (1, 1) does not lie on the graph of *f*.

b. $1 = -2(x+1)^2 + 7 \Rightarrow 2(x+1)^2 = 6 \Rightarrow$
$(x+1)^2 = 3 \Rightarrow x + 1 = \pm\sqrt{3} \Rightarrow x = -1 \pm \sqrt{3}$
The points $\left(-1-\sqrt{3}, 1\right)$ and $\left(-1+\sqrt{3}, 1\right)$ lie on the graph of *f*.

c. $y = -2(0+1)^2 + 7 \Rightarrow y = 5$
The *y*-intercept is (0, 5).

d. $0 = -2(x+1)^2 + 7 \Rightarrow -7 = -2(x+1)^2 \Rightarrow$
$\dfrac{7}{2} = (x+1)^2 \Rightarrow \pm\sqrt{\dfrac{7}{2}} = \pm\dfrac{\sqrt{14}}{2} = x + 1 \Rightarrow$
$x = -1 \pm \dfrac{\sqrt{14}}{2}$
The *x*-intercepts are $\left(-1 - \dfrac{\sqrt{14}}{2}, 0\right)$ and $\left(-1 + \dfrac{\sqrt{14}}{2}, 0\right)$.

62. a. $10 = -3(-2)^2 - 12(-2) \Rightarrow 10 = 12$, which is false. Therefore, (−2, 10) does not lie on the graph of *f*.

b. $f(x) = 12$, so solve the equation
$-3x^2 - 12x = 12$.
$-3x^2 - 12x = 12 \Rightarrow x^2 + 4x = -4 \Rightarrow$
$x^2 + 4x + 4 = 0 \Rightarrow (x+2)^2 = 0 \Rightarrow$
$x + 2 = 0 \Rightarrow x = -2$

c. $y = -3(0)^2 - 12(0) \Rightarrow y = 0$
The *y*-intercept is (0, 0).

d. $0 = -3x^2 - 12x \Rightarrow 0 = -3x(x+4) \Rightarrow$
$x = 0$ or $x = -4$
The *x*-intercepts are (0, 0) and (−4, 0).

63. Domain: [−3, 2]; range: [−3, 3]

64. Domain: [−1, 3]; range: [−2, 4]

65. Domain: $[-4, \infty)$; range: [−2, 3]

66. Domain: $(-\infty, 4]$; range: [−1, 3]

67. Domain: $[-3, \infty)$; range: $[-1, 4] \cup \{-3\}$

68. Domain: $(-\infty, -1) \cup [1, 4)$
Range: (−2, 4]

69. Domain: $(-\infty, -4] \cup [-2, 2] \cup [4, \infty)$
Range: $[-2, 2] \cup \{3\}$

70. Domain: $(-\infty, -2) \cup [-1, \infty)$
Range: $(-\infty, \infty)$

71. $[-9, \infty)$

72. $[-1, 7]$

73. $-3, 4, 7, 9$

74. 6

75. $f(-7) = 4,\ f(1) = 5,\ f(5) = 2$

76. $f(-4) = 4,\ f(-1) = 7,\ f(3) = 3$

77. $\{-3.75, \sim -2.14, 3\} \cup [12, \infty)$

78. $\varnothing$

79. $[-9, \infty)$

80. $\{-4\} \cup [-2, 6]$

81. $g(-4) = -1,\ g(1) = 3,\ g(3) = 4$

82. $|g(-5) - g(5)| = |-2 - 6| = 8$

83. $[-9, -5)$

84. $[5, \infty)$

Applying the Concepts

85. A function because there is only one high temperature per day.

86. A function because there is only one cost of a first-class stamp on January 1 each year.

87. Not a function because there are several states that begin with N (i.e., New York, New Jersey, New Mexico, Nevada, North Carolina, North Dakota); there are also several states that begin with T and S.

88. Not a function because people with different first names may have the same birthday.

89. $A(x) = x^2; A(4) = 16$
$A(4)$ represents the area of a tile with side 4.

90. $V(x) = x^3; V(3) = 27 \text{ in.}^3$
$V(3)$ represents the volume of a cube with edge 3.

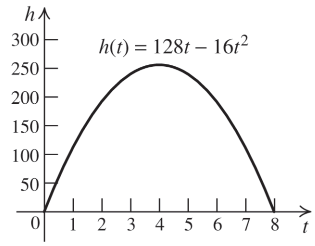
91. It is a function. $S(x) = 6x^2; S(3) = 54$

92. $f(x) = \dfrac{x}{39.37}; f(59) \approx 1.5$ meters

93. a. The domain is [0, 8].

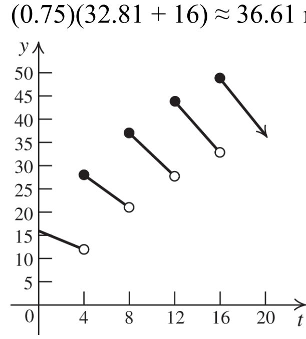
b. $h(2) = 128(2) - 16(2^2) = 192$
$h(4) = 128(4) - 16(4^2) = 256$
$h(6) = 128(6) - 16(6^2) = 192$

c. $0 = 128t - 16t^2 \Rightarrow 0 = 16t(8 - t) \Rightarrow$
$t = 0$ or $t = 8$. It will take 8 seconds for the stone to hit the ground.

d.


94. After the first shot, there is 100% of 0.5 mg of the drug in Andrew's bloodstream.
After 12 hours (filter #1), there is
$0.5 \text{ mg} - 0.5(0.5 \text{ mg}) = 0.25 \text{ mg}$ in his bloodstream.
After the next 12 hours (filter #2), there is
$0.25 - 0.5(0.25) = 0.125 \text{ mg}$
He then gets shot #2, so there is
$0.125 \text{ mg} + 0.5 \text{ mg} = 0.625 \text{ mg}$ in his bloodstream.
After the next 12 hours (filter #3), there is
$0.625 - 0.5(0.625) = 0.3125 \text{ mg}$
After the next 12 hours (filter #4), there is
$0.3125 - 0.5(0.3125) = 0.15625 \text{ mg}$ in his bloodstream.
He then gets shot #3, so there is
$0.5 + 0.15625 = 0.65625 \text{ mg}$ of the drug in Andrew's bloodstream.

95. After 4 hours, there are (0.75)(16) = 12 ml of the drug.
After 8 hours, there are (0.75)(12 + 16) = 21 ml. After 12 hours, there are (0.75)(21 + 16) = 27.75 ml.
After 16 hours, there are (0.75)(27.75 + 16) ≈ 32.81 ml.
After 20 hours, there are (0.75)(32.81 + 16) ≈ 36.61 ml.

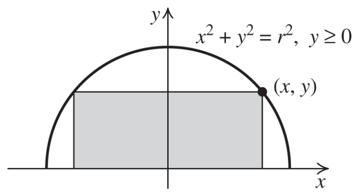


96. $x + y = 28 \Rightarrow y = 28 - x$
$P = x(28 - x) = 28x - x^2$

97. $P = 60 = 2(x+y) \Rightarrow 30 = x + y \Rightarrow y = 30 - x$
$A = x(30 - x) = 30x - x^2$

98. Note that the length of the base = the width of the base = x.
$V = lwh = x^2h = 64 \Rightarrow h = \frac{64}{x^2}$
$S = 2lw + 2lh + 2wh$
$= 2x^2 + 2x\left(\frac{64}{x^2}\right) + 2x\left(\frac{64}{x^2}\right)$
$= 2x^2 + \frac{128}{x} + \frac{128}{x} = 2x^2 + \frac{256}{x}$

99.



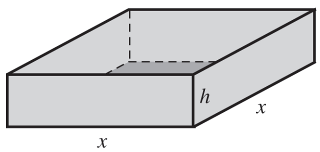
a. $x^2 + y^2 = r^2 \Rightarrow y^2 = r^2 - x^2 \Rightarrow$
$y = \sqrt{r^2 - x^2}$
The length of the rectangle is $2x$ and its height is $y = \sqrt{r^2 - x^2}$.
$P = 2l + 2w = 2(2x) + 2\sqrt{r^2 - x^2}$
$= 4x + 2\sqrt{r^2 - x^2}$

b. $A = lw = 2x\sqrt{r^2 - x^2}$

100. The piece with length x is formed into a circle, so $C = x = 2\pi r \Rightarrow r = \frac{x}{2\pi}$. Thus, the area of the circle is $A = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}$.
The piece with length $20 - x$ is formed into a square, so $P = 20 - x = 4s \Rightarrow s = \frac{1}{4}(20 - x)$.
Thus, the area of the square is
$s^2 = \left[\frac{1}{4}(20 - x)\right]^2 = \frac{1}{16}(20 - x)^2$.
The sum of the areas is $A = \frac{x^2}{4\pi} + \frac{1}{16}(20 - x)^2$

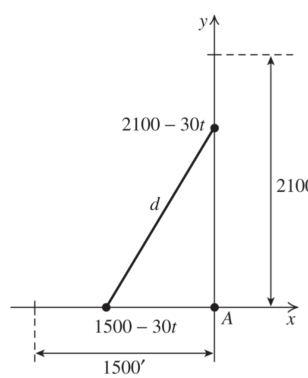
101. The volume of the tank is $V = 64 = \pi r^2 h$, so $h = \frac{64}{\pi r^2}$. The top is open, so the surface area is given by
$\pi r^2 + 2\pi rh = \pi r^2 + 2\pi r\left(\frac{64}{\pi r^2}\right)$
$= \pi r^2 + \frac{128}{r}$.

102. The volume of the pool is
$V = 288 = x^2 h \Rightarrow h = \frac{288}{x^2}$.



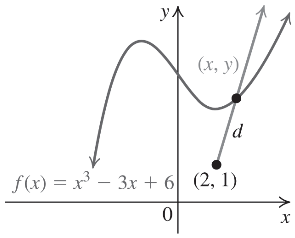
The total area to be tiled is
$4xh = 4x\left(\frac{288}{x^2}\right) = \frac{1152}{x}$
The cost of the tile is $6\left(\frac{1152}{x}\right) = \frac{6912}{x}$.
The area of the bottom of the pool is x^2, so the cost of the cement is $2x^2$. Therefore, the total cost is $C = 2x^2 + \frac{6912}{x}$.

103.



Using the Pythagorean theorem, we have
$d^2 = (1500 - 30t)^2 + (2100 - 30t)^2 \Rightarrow$
$d = \left[(1500 - 30t)^2 + (2100 - 30t)^2\right]^{1/2}$

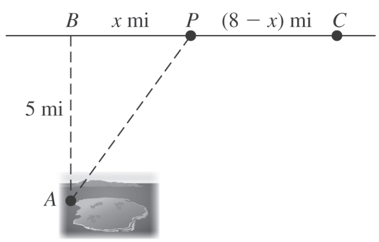
104.



Using the distance formula we have

$$d = \sqrt{(x-2)^2 + (y-1)^2}$$
$$= \sqrt{(x-2)^2 + \left[\left(x^3 - 3x + 6\right) - 1\right]^2}$$
$$= \sqrt{(x-2)^2 + \left(x^3 - 3x + 5\right)^2}$$
$$= \left[(x-2)^2 + \left(x^3 - 3x + 5\right)^2\right]^{1/2}$$

105.



The distance from A to P is $\sqrt{x^2 + 5^2} = \sqrt{x^2 + 25}$ mi. At 4 mi/hr, it will take Julio $\dfrac{\sqrt{x^2 + 25}}{4}$ hr to row that distance.

The distance from P to C is $(8 - x)$ mi, so it will take Julio $\dfrac{8 - x}{5}$ hr to walk that distance.

The total time it will take him to travel is

$$T = \frac{\sqrt{x^2 + 25}}{4} + \frac{8 - x}{5}.$$

106. a. $p(5) = 1275 - 25(5) = 1150$

If 5000 TVs can be sold, the price per TV is \$1150.

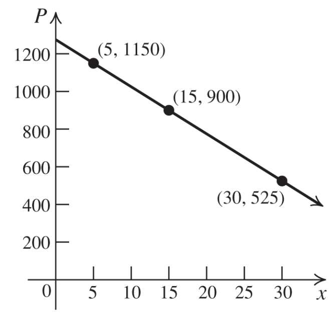
$p(15) = 1275 - 25(15) = 900$

If 15,000 TVs can be sold, the price per TV is \$900.

$p(30) = 1275 - 25(30) = 525$

If 30,000 TVs can be sold, the price per TV is \$525.

b.



c. $650 = 1275 - 25x \Rightarrow -625 = -25x \Rightarrow x = 25$

25,000 TVs can be sold at \$650 per TV.

107. a. $R(x) = (1275 - 25x)x = 1275x - 25x^2$

domain [1, 30]

b. $R(1) = 1275(1) - 25(1^2) = 1250$

$R(5) = 1275(5) - 25(5^2) = 5750$

$R(10) = 1275(10) - 25(10^2) = 10{,}250$

$R(15) = 1275(15) - 25(15^2) = 13{,}500$

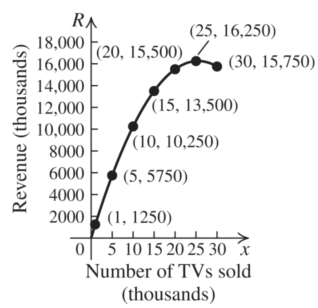
$R(20) = 1275(20) - 25(20^2) = 15{,}500$

$R(25) = 1275(25) - 25(25^2) = 16{,}250$

$R(30) = 1275(30) - 25(30^2) = 15{,}750$

This is the amount of revenue (in thousands of dollars) for the given number of TVs sold (in thousands).

c.



d. $4700 = 1275x - 25x^2 \Rightarrow$

$x^2 - 51x + 188 = 0 \Rightarrow$

$$\frac{51 \pm \sqrt{51^2 - 4(1)(188)}}{2(1)} = x \Rightarrow$$

$x = 4$ or $x = 47$

47 is not in the domain, so 4000 TVs must be sold in order to generate revenue of 4.7 million dollars.

108. a. $C(x) = 5.5x + 75{,}000$

b. $R(x) = 0.6(15)x = 9x$

c. $P(x) = R(x) - C(x) = 9x - (5.5x + 75{,}000)$
$= 3.5x - 75{,}000$

d. The break-even point is when the profit is zero: $3.5x - 75,000 = 0 \Rightarrow x = 21,429$

e. $P(46,000) = 3.5(46,000) - 75,000 = \$86,000$

The company's profit is \$86,000 when 46,000 copies are sold.

109. a. $C(x) = 0.5x + 500,000; R(x) = 5x$

The break-even point is when the profit is zero (when the revenue equals the cost):
$5x = 0.5x + 500,000 \Rightarrow 4.5x = 500,000 \Rightarrow$
$x = 111,111.11$
Because a fraction of a CD cannot be sold, 111,112 CD's must be sold.

b.
$$P(x) = R(x) - C(x)$$
$$750,000 = 5x - (0.5x + 500,000)$$
$$1,250,000 = 4.5x \Rightarrow x = 277,778$$
The company must sell 277,778 CDs in order to make a profit of \$750,000.

Beyond the Basics

110. $x = \dfrac{2}{y-4} \Rightarrow xy - 4x = 2 \Rightarrow xy = 2 + 4x \Rightarrow$
$y = \dfrac{4x+2}{x} \Rightarrow f(x) = \dfrac{4x+2}{x}$;
Domain: $(-\infty, 0) \cup (0, \infty)$. $f(4) = \dfrac{9}{2}$.

111. $xy - 3 = 2y \Rightarrow 2y - xy = -3 \Rightarrow$
$y(2-x) = -3 \Rightarrow y = -\dfrac{3}{2-x} \Rightarrow f(x) = \dfrac{3}{x-2}$
Domain: $(-\infty, 2) \cup (2, \infty)$. $f(4) = \dfrac{3}{2}$

112. $\left(x^2+1\right)y + x = 2 \Rightarrow y = \dfrac{2-x}{x^2+1} \Rightarrow$
$f(x) = \dfrac{2-x}{x^2+1}$; Domain: $(-\infty, \infty)$; $f(4) = -\dfrac{2}{17}$

113. $yx^2 - \sqrt{x} = -2y \Rightarrow yx^2 + 2y = \sqrt{x} \Rightarrow$
$y\left(x^2+2\right) = \sqrt{x} \Rightarrow y = \dfrac{\sqrt{x}}{x^2+2} \Rightarrow f(x) = \dfrac{\sqrt{x}}{x^2+2}$
Domain: $[0, \infty)$; $f(4) = \dfrac{1}{9}$

114. $f(x) \neq g(x)$ because they have different domains.

115. $f(x) \neq g(x)$ because they have different domains.

116. $f(x) \neq g(x)$ because they have different domains. $g(x)$ is not defined for $x = -1$, while $f(x)$ is defined for all real numbers.

117. $f(x) \neq g(x)$ because they have different domains. $g(x)$ is not defined for $x = 3$, while $f(x)$ is not defined for $x = 3$ or $x = -2$.

118. $f(x) = g(x)$ because $f(3) = 10 = g(3)$ and $f(5) = 26 = g(5)$.

119. $f(x) \neq g(x)$ because $f(2) = 16$ while $g(2) = 13$.

120. $f(2) = 15 = a(2^2) + 2a - 3 \Rightarrow 15 = 6a - 3 \Rightarrow a = 3$.

121. $g(6) = 28 = 6^2 + 6b + b^2 \Rightarrow b^2 + 6b + 8 = 0 \Rightarrow (b+2)(b+4) = 0 \Rightarrow b = -2$ or $b = -4$.

122. $h(6) = 0 = \dfrac{3(6)+2a}{2(6)-b} \Rightarrow 0 = 18 + 2a \Rightarrow a = -9$

$h(3)$ is undefined $\Rightarrow \dfrac{3(3)+2(-9)}{2(3)-b}$ has a zero in the denominator. So $6 - b = 0 \Rightarrow b = 6$.

123. $f(x) = 2x - 3 \Rightarrow f\left(x^2\right) = 2x^2 - 3$
$\left(f(x)\right)^2 = (2x-3)^2 = 4x^2 - 12x + 9$

124. $g(x) = x^2 - \dfrac{1}{x^2} \Rightarrow g\left(\dfrac{1}{x}\right) = \dfrac{1}{x^2} - \dfrac{1}{\frac{1}{x^2}} = \dfrac{1}{x^2} - x^2$

$g(x) + g\left(\dfrac{1}{x}\right) = \left(x^2 - \dfrac{1}{x^2}\right) + \left(\dfrac{1}{x^2} - x^2\right) = 0$

125. $f(x) = \dfrac{x-1}{x+1} \Rightarrow f\left(\dfrac{x-1}{x+1}\right) = \dfrac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1}$
$$= \dfrac{\frac{(x-1)-(x+1)}{x+1}}{\frac{(x-1)+(x+1)}{x+1}} = -\dfrac{2}{2x} = -\dfrac{1}{x}$$

126. $f(x) = \dfrac{x+3}{4x-5} \Rightarrow$

$$f(t) = \frac{\frac{3+5x}{4x-1}+3}{4\left(\frac{3+5x}{4x-1}\right)-5} = \frac{\frac{(3+5x)+3(4x-1)}{4x-1}}{\frac{(12+20x)-(5(4x-1))}{4x-1}}$$
$$= \frac{(3+5x)+(12x-3)}{(12+20x)-(20x-5)} = \frac{17x}{17} = x$$

Critical Thinking/Discussion/Writing

127. Answers may vary. Sample answers are given

a. $y = \sqrt{x-2}$ **b.** $y = \dfrac{1}{\sqrt{x-2}}$

c. $y = \sqrt{2-x}$ **d.** $y = \dfrac{1}{\sqrt{2-x}}$

128. a. $ax^2 + bx + c = 0$

b. $y = c$

c. The equation will have no x-intercepts if $b^2 - 4ac < 0$.

d. It is not possible for the equation to have no y-intercepts because $y = f(x)$.

129. a. $f(x) = |x|$ **b.** $f(x) = 0$

c. $f(x) = x$

d. $f(x) = \sqrt{-x^2}$ (Note: the point is the origin.)

e. $f(x) = 1$

f. A vertical line is not a function.

130. a. {(a, 1), (b, 1)} {(a, 2), (b, 1)}
{(a, 1), (b, 2)} {(a, 2), (b, 2)}
{(a, 1), (b, 3)} {(a, 2), (b, 3)}

{(a, 3), (b, 1)}
{(a, 3), (b, 2)}
{(a, 3), (b, 3)}

There are nine functions from X to Y.

b. {(1, a)}, {(2, a)}, {(3, a)}
{(1, a)}, {(2, a)}, {(3, b)}
{(1, a)}, {(2, b)}, {(3, a)}
{(1, a)}, {(2, a)}, {(3, b)}
{(1, b)}, {(2, a)}, {(3, a)}
{(1, b)}, {(2, a)}, {(3, b)}
{(1, b)}, {(2, b)}, {(3, a)}
{(1, b)}, {(2, b)}, {(3, b)}

There are eight functions from Y to X.

131. If a set X has m elements and a set of Y has n elements, there are n^m functions that can be defined from X to Y. This is true since a function assigns each element of X to an element of Y. There are m possibilities for each element of X, so there are $\underbrace{n \cdot n \cdot n \cdot \cdots \cdot n}_{m} = n^m$ possible functions.

Active Learning

132. a.–c. Refer to the app using the QR code in your text.

Getting Ready for the Next Section

GR1. $2x - 4 < 12 \Rightarrow 2x < 16 \Rightarrow x < 8$
The solution set is $(-\infty, 8)$.

GR2. $5x + 9 \le 7(x+1) \Rightarrow 5x + 9 \le 7x + 7 \Rightarrow$
$2 \le 2x \Rightarrow 1 \le x$ or $x \ge 1$
The solution set is $[1, \infty)$.

GR3. $x^2 > 0$
Solve the associated equation:
$x^2 = 0 \Rightarrow x = 0.$
So, the intervals are $(-\infty, 0)$ and $(0, \infty)$.

Interval	Test point	Value of x^2	Result
$(-\infty, 0)$	–1	1	+
$(0, \infty)$	1	1	+

The solution set is $(-\infty, 0) \cup (0, \infty)$

GR4. $(3-x)(x+5) \ge 0$
Solve the associated equation:
$(3-x)(x+5) = 0 \Rightarrow x = 3$ or $x = -5$.
So, the intervals are
$(-\infty, -5]$, $[-5, 3]$, and $[3, \infty)$.

Interval	Test point	Value of $(3-x)(x+5)$	Result
$(-\infty, -5]$	–10	–65	–
$[-5, 3]$	0	15	+
$[3, \infty)$	5	–20	–

The solution set is $[-5, 3]$.

For exercises GR5–GR10, $f(x) = 3 - 2x^2$, $g(x) = \sqrt{x+3}$, and $h(x) = \dfrac{2}{x^2+1}$.

GR5. $f(0) = 3 - 2(0)^2 = 3$
$g(0) = \sqrt{0+3} = \sqrt{3}$
$h(0) = \dfrac{2}{0^2+1} = 2$

GR6. $f(1) = 3 - 2(1)^2 = 1$
$g(1) = \sqrt{1+3} = \sqrt{4} = 2$
$h(1) = \dfrac{2}{1^2+1} = 1$

GR7. $f(-2) = 3 - 2(-2)^2 = 3 - 8 = -5$
$g(-2) = \sqrt{-2+3} = \sqrt{1} = 1$
$h(-2) = \dfrac{2}{(-2)^2+1} = \dfrac{2}{5}$

GR8. $f(x+1) = 3 - 2(x+1)^2 = 3 - 2\left(x^2+2x+1\right)$
$= 3 - 2x^2 - 4x - 2 = -2x^2 - 4x + 1$
$g(x+1) = \sqrt{(x+1)+3} = \sqrt{x+4}$
$h(x+1) = \dfrac{2}{(x+1)^2+1} = \dfrac{2}{\left(x^2+2x+1\right)+1}$
$= \dfrac{2}{x^2+2x+2}$

GR9. $f(-x) = 3 - 2(-x)^2 = 3 - 2x^2$
$g(-x) = \sqrt{-x+3}$
$h(-x) = \dfrac{2}{(-x)^2+1} = \dfrac{2}{x^2+1}$

GR10. $-f(x) = -\left(3 - 2x^2\right) = 2x^2 - 3$
$-g(x) = -\sqrt{x+3}$
$-h(x) = -\dfrac{2}{x^2+1}$

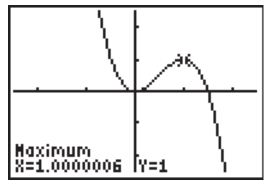
1.5 Properties of Functions

Practice Problems

1. The function is decreasing on $(0, 3)$, $(12, 13)$, and $(15, 24)$; increasing on $(3, 12)$ and $(13, 15)$

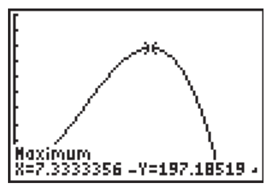
2. Relative maxima of 3640 at $x = 12$ and 4070 at $x = 15$; relative minima of 40 at $x = 3$ and 3490 at $x = 13$.

3.



Relative minimum of 0 at $x = 0$
Relative maximum of 1 at $x = 1$

4. $v = (11 - r)r^2$



[0, 13, 1] by [0, 250, 25]

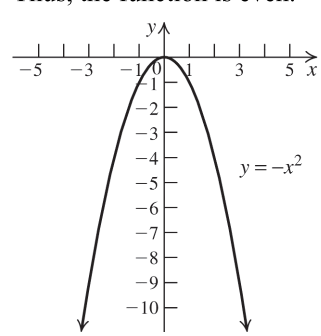
Mrs. Osborn's windpipe should be contracted to a radius of 7.33 mm for maximizing the airflow velocity.

5. $f(x) = -x^2$

Replace x with $-x$:

$f(-x) = -(-x)^2 = -x^2 = f(x)$

Thus, the function is even.

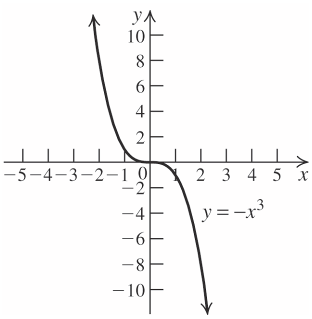


6. $f(x)=-x^3$

Replace x with $-x$:

$f(-x)=-(-x)^3=x^3=-f(x)$

Thus, the function is odd.



7. a. $g(-x)=3(-x)^4-5(-x)^2$
$=3x^4-5x^2=f(x)\Rightarrow$
$g(x)$ is even.

b. $f(-x)=4(-x)^5+2(-x)^3=-4x^5-2x^3$
$=-\left(4x^5+2x^3\right)=-f(x)\Rightarrow$
$f(x)$ is odd.

c. $h(-x)=2(-x)+1=-2x+1$
$\neq h(x)$
$\neq h(-x)\Rightarrow h$ is neither even nor odd.

8. $f(x)=1-x^2;\ a=2, b=4$

$f(2)=1-2^2=-3;\ f(4)=1-4^2=-15$

$\dfrac{f(b)-f(a)}{b-a}=\dfrac{-15-(-3)}{4-2}=\dfrac{-12}{2}=-6$

The average rate of change is −6.

9. $f(t)=1-t;\ a=2,\ b=x,\ x\neq 2$

$f(a)=f(2)=1-2=-1$

$f(b)=f(x)=1-x$

$\dfrac{f(b)-f(a)}{b-a}=\dfrac{(1-x)-(-1)}{x-2}=\dfrac{2-x}{x-2}$
$=\dfrac{-1(x-2)}{x-2}=-1$

The average rate of change is −1.

10. $f(x)=100x^2-800x+2000$

$f(0)=100(0)^2-800(0)+2000=2000$

$f(3)=100(3)^2-800(3)+2000=500$

$\dfrac{f(3)-f(0)}{3-0}=\dfrac{500-2000}{3}=\dfrac{-1500}{3}=-500$

The average rate of change is –500, so the number of bacteria per cubic centimeter decreases by 500 each day after adding the chlorine.

11. $f(x)=-x^2+x-3$

$f(x+h)=-(x+h)^2+(x+h)-3$
$=-x^2-2xh-h^2+x+h-3$

$\dfrac{f(x+h)-f(x)}{h}$
$=\dfrac{\left(-x^2-2xh-h^2+x+h-3\right)-\left(-x^2+x-3\right)}{h}$
$=\dfrac{-2xh-h^2+h}{h}=-2x-h+1$

Concepts and Vocabulary

1. A function f is decreasing if $x_1<x_2$ implies that $\underline{f(x_1)>f(x_2)}$.

2. $f(a)$ is a relative maximum of f if there is an interval $(x_1,\ x_2)$ containing a such that $\underline{f(a)\geq f(x)}$ for every x in the interval $(x_1,\ x_2)$.

3. A function f is even if $\underline{f(-x)=f(x)\text{ for all } x \text{ in the domain of } f}$.

4. The average rate of change of f as x changes from $x=a$ to $x=b$ is $\underline{\dfrac{f(b)-f(a)}{b-a}}$, $a\neq b$.

5. True

6. False. A relative maximum or minimum could occur at an endpoint of the domain of the function.

7. True

8. False. The graph of an odd function is symmetric with respect to the origin.

Building Skills

9. Increasing on $(-\infty,\ \infty)$

10. Decreasing on $(-\infty,\ \infty)$

11. Increasing on $(-\infty,\ 2)$, decreasing on $(2,\ \infty)$

12. Decreasing on $(-\infty,\ 3)$, increasing on $(3,\ \infty)$

13. Increasing on $(-\infty, -2)$, constant on $(-2, 2)$, increasing on $(2, \infty)$

14. Decreasing on $(-\infty, -1)$, constant on $(-1, 4)$, decreasing on $(4, \infty)$

15. Increasing on $(-\infty, -3)$ and $\left(-\frac{1}{2}, 2\right)$, decreasing on $\left(-3, -\frac{1}{2}\right)$ and $(2, \infty)$

16. Increasing on $(-3, -1)$, $(0, 1)$, and $(2, \infty)$. Decreasing on $(-\infty, -3)$, $(-1, 0)$, and $(1, 2)$.

17. No relative extrema

18. No relative extrema

19. (2, 10) is a relative maximum point and a turning point.

20. (3, 2) is a relative minimum point and a turning point.

21. Any point on (x, 2) is a relative maximum and a relative minimum point on the interval (–2, 2). Relative maximum at (−2, 2); relative minimum at (2, 2). None of these points are turning points.

22. Any point on (x, 3) is a relative maximum and a relative minimum point on the interval (–1, 4). Relative maximum at (4, 3); relative minimum at (−1, 3). None of these points are turning points.

23. (–3, 4) and (2, 5) are relative maxima points and turning points. $\left(-\frac{1}{2}, -2\right)$ is a relative minimum and a turning point.

24. (–3, –2), (0, 0), and (2, –3) are relative minimum points and turning points. (–1, 1) and (1, 2) are relative maximum points and turning points.

For exercises 25−34, recall that the graph of an even function is symmetric about the y-axis, and the graph of an odd function is symmetric about the origin.

25. The graph is symmetric with respect to the origin. The function is odd.

26. The graph is symmetric with respect to the origin. The function is odd.

27. The graph has no symmetries, so the function is neither odd nor even.

28. The graph has no symmetries, so the function is neither odd nor even.

29. The graph is symmetric with respect to the origin. The function is odd.

30. The graph is symmetric with respect to the origin. The function is odd.

31. The graph is symmetric with respect to the y-axis. The function is even.

32. The graph is symmetric with respect to the y-axis. The function is even.

33. The graph is symmetric with respect to the origin. The function is odd.

34. The graph is symmetric with respect to the origin. The function is odd.

For exercises 35−48, $f(-x) = f(x) \Rightarrow f(x)$ is even and $f(-x) = -f(x) \Rightarrow f(x)$ is odd.

35. $f(-x) = 2(-x)^4 + 4 = 2x^4 + 4 = f(x) \Rightarrow$ $f(x)$ is even.

36. $g(-x) = 3(-x)^4 - 5 = 3x^4 - 5 = g(x) \Rightarrow$ $g(x)$ is even.

37. $f(-x) = 5(-x)^3 - 3(-x) = -5x^3 + 3x = -(5x^3 - 3x) = -f(x) \Rightarrow$ $f(x)$ is odd.

38. $g(-x) = 2(-x)^3 + 4(-x) = -2x^3 - 4x = -g(x) \Rightarrow$ $g(x)$ is odd.

39. $f(-x) = 2(-x) + 4 = -2x + 4 \neq -f(x) \neq f(x) \Rightarrow$ $f(x)$ is neither even nor odd.

40. $g(-x) = 3(-x) + 7 = -3x + 7 \neq -g(x) \neq g(x) \Rightarrow$ $g(x)$ is neither even nor odd.

41. $f(-x) = \dfrac{1}{(-x)^2 + 4} = \dfrac{1}{x^2 + 4} = f(x) \Rightarrow$ $f(x)$ is even.

42. $g(-x) = \dfrac{(-x)^2 + 2}{(-x)^4 + 1} = \dfrac{x^2 + 2}{x^4 + 1} = g(x) \Rightarrow$ $g(x)$ is even.

43. $f(-x) = \dfrac{(-x)^3}{(-x)^2 + 1} = -\dfrac{x^3}{x^2 + 1} = -f(x) \Rightarrow$ $f(x)$ is odd.

44. $g(-x)=\dfrac{(-x)^4+3}{2(-x)^3-3(-x)}=\dfrac{x^4+3}{-2x^3+3x}$
$=-\dfrac{x^4+3}{2x^3-3x}=-f(x)\Rightarrow f(x)$ is odd.

45. $f(-x)=\dfrac{-x}{(-x)^5-3(-x)^3}=\dfrac{-x}{-x^5+3x^3}$
$=\dfrac{(-1)(-x)}{(-1)\left(-x^5+3x^3\right)}=\dfrac{x}{x^5-3x^3}=f(x)$

Thus, $f(x)$ is even.

46. $g(-x)=\dfrac{(-x)^3+2(-x)}{2(-x)^5-3(-x)}=\dfrac{-x^3-2x}{-2x^5+3x}$
$=\dfrac{(-1)\left(-x^3-2x\right)}{(-1)\left(-2x^5+3x\right)}=\dfrac{x^3+2x}{2x^5-3x}=f(x)$

Thus, $f(x)$ is even.

47. $f(-x)=\dfrac{(-x)^2-2(-x)}{5(-x)^4+7}=\dfrac{x^2+2x}{5x^4+7}$
$\neq -f(x)\neq f(x)$

Thus, $f(x)$ is neither even nor odd.

48. $g(-x)=\dfrac{3(-x)^2+7}{(-x)-3}=\dfrac{3x^2+7}{-x-3}\neq -g(x)\neq g(x)$

Thus, $g(x)$ is neither even nor odd.

49. a. domain: $(-\infty,\infty)$; range: $(-\infty, 3]$

b. x-intercepts: (−3, 0), (3, 0)
y-intercept: (0, 3)

c. increasing on $(-\infty, 0)$, decreasing on $(0, \infty)$

d. relative maximum at (0, 3)

e. even

50. a. domain: $(-\infty,\infty)$; range: $(-\infty,\infty)$

b. x-intercepts: (−4, 0), (0, 0), (4, 0)
y-intercept: (0, 0)

c. decreasing on $(-\infty, -2)$ and $(2, \infty)$, increasing on $(-2, 2)$

d. relative maximum at (2,3); relative minimum at (−2, −3)

e. odd

51. a. domain: (−3, 4); range: [−2, 2]

b. x-intercept: (1, 0); y-intercept: (0, −1)

c. constant on (−3, −1) and (3, 4)
increasing on (−1, 3)

d. Because the function is constant on (−3, −1), any point (x, −2) is both a relative maximum and a relative minimum on that interval. Since the function is constant on (3, 4), any point (x, 2) is both a relative maximum and a relative minimum on that interval.

e. neither even not odd

52. a. domain: (−3, 3); range: {−2, 0, 2}

b. x-intercept: (0, 0); y-intercept: (0, 0)

c. constant on (−3, 0) and (0, 3)

d. Because the function is constant on (−3, 0), any point (x, 2) is both a relative maximum and a relative minimum on that interval. Since the function is constant on (0, 3), any point (x, −2) is both a relative maximum and a relative minimum on that interval.

e. odd

53. a. domain: (−2, 4); range: [−2, 3]

b. x-intercept: (0, 0); y-intercept: (0, 0)

c. decreasing on (−2, −1) and (3, 4)
increasing on (−1, 3)

d. relative maximum: (3, 3)
relative minimum: (−1, −2)

e. neither even nor odd

54. a. domain: $(-\infty, \infty)$
range: $(-\infty, \infty)$

b. x-intercepts: (2, 0), (3, 0)
y-intercept: (0, 3)

c. decreasing on $(-\infty, 0)\cup(0, 3)\cup(3, \infty)$

d. no relative minimum
relative maximum: (0, 3)

e. neither even nor odd

55. a. domain: $(-\infty,\infty)$; range: $(0, \infty)$

b. no x-intercept; y-intercept: (0, 1)

c. increasing on $(-\infty, \infty)$

d. no relative minimum or relative maximum

e. neither even nor odd

56. a. domain: $(-\infty, 0)\cup(0, \infty)$

range: $(-\infty, \infty)$

b. x-intercepts: $(-1.5, 0)$, $(1.5, 0)$
no y-intercept

c. decreasing on $(-\infty, 0)$
increasing on $(0, \infty)$

d. no relative minimum or relative maximum

e. even

57. $f(x) = -2x + 7;\ a = -1,\ b = 3$
$f(3) = -2(3) + 7 = 1;\ f(-1) = -2(-1) + 7 = 9$

$$\text{average rate of change} = \frac{f(3) - f(-1)}{3 - (-1)} = \frac{1 - 9}{4} = -2$$

58. $f(x) = 4x - 9;\ a = -2,\ b = 2$
$f(2) = 4(2) - 9 = -1;\ f(-2) = 4(-2) - 9 = -17$

$$\text{average rate of change} = \frac{f(2) - f(-2)}{2 - (-2)} = \frac{-1 - (-17)}{4} = 4$$

59. $f(x) = 3x + c;\ a = 1,\ b = 5$
$f(5) = 3\cdot 5 + c = 15 + c;\ f(1) = 3\cdot 1 + c = 3 + c$

$$\text{average rate of change} = \frac{f(5) - f(1)}{5 - 1} = \frac{15 + c - (3 + c)}{4} = \frac{12}{4} = 3$$

60. $f(x) = mx + c;\ a = -1,\ b = 7$
$f(7) = 7m + c;\ f(-1) = -m + c$

$$\text{average rate of change} = \frac{f(7) - f(-1)}{7 - (-1)} = \frac{7m + c - (-m + c)}{8} = \frac{8m}{8} = m$$

61. $h(x) = x^2 - 1;\ a = -2,\ b = 0$
$h(0) = 0^2 - 1 = -1;\ h(-2) = (-2)^2 - 1 = 3$

$$\text{average rate of change} = \frac{h(0) - h(-2)}{0 - (-2)} = \frac{-1 - 3}{2} = -2$$

62. $h(x) = 2 - x^2;\ a = 3,\ b = 4$
$h(4) = 2 - 4^2 = -14;\ h(3) = 2 - 3^2 = -7$

$$\text{average rate of change} = \frac{h(4) - h(3)}{4 - 3} = \frac{-14 - (-7)}{1} = -7$$

63. $f(x) = (3 - x)^2;\ a = 1,\ b = 3$
$f(4) = (3 - 3)^2 = 0;\ f(1) = (3 - 1)^2 = 4$

$$\text{average rate of change} = \frac{f(3) - f(1)}{3 - 1} = \frac{0 - 4}{2} = -2$$

64. $f(x) = (x - 2)^2;\ a = -1,\ b = 5$
$f(5) = (5 - 2)^2 = 9;\ f(-1) = (-1 - 2)^2 = 9$

$$\text{average rate of change} = \frac{f(5) - f(-1)}{5 - (-1)} = \frac{9 - 9}{6} = 0$$

65. $g(x) = x^3;\ a = -1,\ b = 3$
$g(3) = 3^3 = 27;\ g(-1) = (-1)^3 = -1$

$$\text{average rate of change} = \frac{g(3) - g(-1)}{3 - (-1)} = \frac{27 - (-1)}{4} = 7$$

66. $g(x) = -x^3;\ a = -1,\ b = 3$
$g(3) = -3^3 = -27;\ g(-1) = -(-1)^3 = 1$

$$\text{average rate of change} = \frac{g(3) - g(-1)}{3 - (-1)} = \frac{-27 - 1}{4} = -7$$

67. $h(x) = \frac{1}{x};\ a = 2,\ b = 6$

$h(2) = \frac{1}{2};\ h(6) = \frac{1}{6}$

$$\text{average rate of change} = \frac{h(6) - h(2)}{6 - 2} = \frac{\frac{1}{6} - \frac{1}{2}}{4} = -\frac{1}{12}$$

68. $h(x)=\dfrac{4}{x+3};\ a=-2,\ b=4$

$h(4)=\dfrac{4}{4+3}=\dfrac{4}{7};\ h(-2)=\dfrac{4}{-2+3}=4$

$$\text{average rate of change}=\frac{h(4)-h(-2)}{4-(-2)}=\frac{\frac{4}{7}-4}{6}=-\frac{4}{7}$$

69. $f(x+h)=x+h$

$f(x+h)-f(x)=x+h-x=h$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{h}{h}=1$

70. $f(x+h)=3(x+h)+2=3x+3h+2$

$f(x+h)-f(x)=3x+3h+2-(3x+2)=3h$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{3h}{h}=3$

71. $f(x+h)=-2(x+h)+3=-2x-2h+3$

$f(x+h)-f(x)=-2x-2h+3-(-2x+3)=-2h$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{-2h}{h}=-2$

72. $f(x+h)=-5(x+h)-6=-5x-5h-6$

$f(x+h)-f(x)=-5x-5h-6-(-5x-6)=-5h$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{-5h}{h}=-5$

73. $f(x+h)=m(x+h)+b=mx+mh+b$

$f(x+h)-f(x)=mx+mh+b-(mx+b)=mh$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{mh}{h}=m$

74. $f(x+h)=-2a(x+h)+c=-2ax-2ah+c$

$f(x+h)-f(x)=-2ax-2ah+c-(-2ax+c)=-2ah$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{-2ah}{h}=-2a$

75. $f(x+h)=(x+h)^2=x^2+2xh+h^2$

$f(x+h)-f(x)=x^2+2xh+h^2-x^2=2xh+h^2$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{2xh+h^2}{h}=2x+h$

76. $f(x+h)=(x+h)^2-(x+h)=x^2+2xh+h^2-x-h$

$f(x+h)-f(x)=x^2+2xh+h^2-x-h-(x^2-x)=2xh+h^2-h$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{2xh+h^2-h}{h}=2x+h-1$

77. $f(x+h)=2(x+h)^2+3(x+h)=2x^2+4xh+2h^2+3x+3h$

$f(x+h)-f(x)=2x^2+4xh+2h^2+3x+3h-(2x^2+3x)=4xh+2h^2+3h$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{4xh+2h^2+3h}{h}=4x+2h+3$

78. $f(x+h)=3(x+h)^2-2(x+h)+5=3x^2+6xh+3h^2-2x-2h+5$

$f(x+h)-f(x)=3x^2+6xh+3h^2-2x-2h+5-(3x^2-2x+5)=6xh+3h^2-2h$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{6xh+3h^2-2h}{h}=6x+3h-2$

79. $f(x+h)=4$

$f(x+h)-f(x)=4-4=0$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{0}{h}=0$

80. $f(x+h)=-3$

$f(x+h)-f(x)=-3-(-3)=0$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{0}{h}=0$

81. $f(x+h)=\dfrac{1}{x+h}$

$$f(x+h)-f(x)=\frac{1}{x+h}-\frac{1}{x}=\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}=-\frac{h}{x(x+h)}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{-\frac{h}{x(x+h)}}{h}=-\frac{1}{x(x+h)}$$

82. $f(x+h)=-\dfrac{1}{x+h}$

$$f(x+h)-f(x)=-\frac{1}{x+h}-\left(-\frac{1}{x}\right)=-\frac{x}{x(x+h)}+\frac{x+h}{x(x+h)}=\frac{h}{x(x+h)}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{h}{x(x+h)}}{h}=\frac{1}{x(x+h)}$$

83. $f(x)=2x,\ a=3$

$$\frac{f(x)-f(a)}{x-a}=\frac{2x-2(3)}{x-3}=\frac{2(x-3)}{x-3}=2$$

84. $f(x)=3x+2,\ a=2$

$$\frac{f(x)-f(a)}{x-a}=\frac{(3x+2)-(3(2)+2)}{x-2}=\frac{3x-6}{x-2}=\frac{3(x-2)}{x-2}=3$$

85. $f(x)=-x^2,\ a=1$

$$\frac{f(x)-f(a)}{x-a}=\frac{-x^2-\left[-\left(1^2\right)\right]}{x-1}=\frac{-x^2+1}{x-1}=\frac{-\left(x^2-1\right)}{x-1}=\frac{-(x-1)(x+1)}{x-1}=-(x+1)=-x-1$$

86. $f(x)=2x^2,\ a=-1$

$$\frac{f(x)-f(a)}{x-a}=\frac{2x^2-2(-1)^2}{x-(-1)}=\frac{2x^2-2}{x+1}=\frac{2(x-1)(x+1)}{x+1}=2(x-1)=2x-2$$

87. $f(x)=3x^2+x,\ a=2$

$$\frac{f(x)-f(a)}{x-a}=\frac{\left(3x^2+x\right)-\left(3(2)^2+2\right)}{x-2}=\frac{3x^2+x-14}{x-2}=\frac{(3x+7)(x-2)}{x-2}=3x+7$$

88. $f(x)=-2x^2+x,\ a=3$

$$\frac{f(x)-f(a)}{x-a}=\frac{\left(-2x^2+x\right)-\left(-2(3)^2+3\right)}{x-3}=\frac{-2x^2+x+15}{x-3}=\frac{(-2x-5)(x-3)}{x-3}=-2x-5$$

89. $f(x)=\dfrac{4}{x},\ a=1$

$$\frac{f(x)-f(a)}{x-a}=\frac{\frac{4}{x}-\frac{4}{1}}{x-1}=\frac{\frac{4}{x}-4}{x-1}=\frac{\frac{4-4x}{x}}{x-1}=\frac{4-4x}{x(x-1)}=\frac{-4(x-1)}{x(x-1)}=-\frac{4}{x}$$

90. $f(x)=-\dfrac{4}{x},\ a=1$

$$\frac{f(x)-f(a)}{x-a}=\frac{-\frac{4}{x}-\left(-\frac{4}{1}\right)}{x-1}=\frac{-\frac{4}{x}+4}{x-1}=\frac{\frac{-4+4x}{x}}{x-1}=\frac{4x-4}{x(x-1)}=\frac{4(x-1)}{x(x-1)}=\frac{4}{x}$$

Applying the Concepts

91. a. Increasing: (Feb 5, Feb 16), (Feb 23, Mar 13), (Mar 20, Mar 31)
Decreasing: (Feb 1, Feb 5), (Feb 16, Feb 23), (Mar 13, Mar 20)

b. Relative maximum of 0.51 on Feb 16, relative maximum of 0.76 on Mar 13, relative minimum of 0.38 on Feb 5, relative minimum of 0.42 on Feb 23, relative minimum of 0.67 on Mar 20

92. a. Increasing: (Jan., June), (July, Sept.)
Decreasing: (June, July), (Sept., Dec.)

b. Relative maxima: 185 in June, 185 in Sept.
Relative minima: 132 in July

93. The area of a cross section of the hose is

$$A=\pi r^2\approx 3.14(1)^2=3.14\text{ sq in.}=3.14\text{ sq in.}\times\frac{1\text{ sq ft}}{144\text{ sq in.}}\approx 0.02181\text{ sq ft}$$

$$Q=5\text{ gal/min}=5\times(0.134)\text{ ft}^3/\text{min}=0.67\text{ ft}^3/\text{min}$$

$Q=Av\Rightarrow$

$$v=\frac{Q}{A}=\frac{0.67\text{ ft}^3/\text{min}}{0.02181\text{ ft}^2}\approx 30.72\text{ ft/min}$$

Answers may vary slightly due to rounding.

94. The area of a cross section of the nozzle is

$A = \pi r^2 \approx 3.14(0.5)^2$

$= 0.785 \text{ sq in.} = 0.785 \text{ sq in.} \times \frac{1 \text{ sq ft}}{144 \text{ sq in.}}$

$\approx 0.00545 \text{ sq ft}$

$Q = 5 \text{ gal/min} = 5 \times (0.134) \text{ ft}^3 / \text{min}$

$= 0.67 \text{ ft}^3 / \text{min}$

$Q = Av \Rightarrow$

$v = \frac{Q}{A} = \frac{0.67 \text{ ft}^3 / \text{min}}{0.00545 \text{ ft}^2} \approx 122.9 \text{ ft/min}$

Answers may vary slightly due to rounding.

95. domain: $[0, \infty)$

The particle's motion is tracked indefinitely from time $t = 0$.

96. range: $[-7, 5]$

The particle takes on all velocities between –7 an 5. Note that a negative velocity indicates that the particle is moving backward.

97. The graph is above the t-axis on the intervals (0, 9) and (21, 24). This means that the particle was moving forward between 0 and 9 seconds and between 21 and 24 seconds.

98. The graph is below the t-axis on the interval (11, 19). This means that the particle is moving backward between 11 and 19 seconds.

99. The function is increasing on (0, 3), (5, 6), (16, 19), and (21, 23). However, the speed $|v|$ of the particle is increasing on (0, 3), (5, 6), (11, 15), and (21, 23). Note that the particle is moving forward on (0, 3), (5, 6), and (21, 23), and moving backward on (11, 15).

100. The function is decreasing on (6, 9), (11, 15), and (23, 24). However, the speed $|v|$ of the particle is decreasing on (6, 9), (16, 19), and (23, 24). Note that the particle is moving forward on (6, 9) and (23, 24), and moving backward on (16, 19).

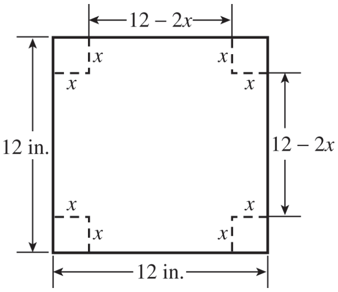
101. The maximum speed is between times $t = 15$ and $t = 16$.

102. The minimum speed is 0 on the intervals (9, 11), (19,21), and (24, ∞).

103. The particle is moving forward with increasing velocity.

104. The particle is moving backward with decreasing speed.

105.



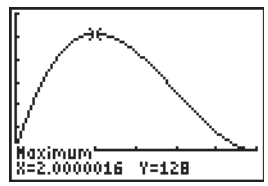
a. $V = lwh = (12 - 2x)(12 - 2x)x$

$= (144 - 48x + 4x^2)x$

$= 4x^3 - 48x^2 + 144x$

b. The length of the squares in the corners must be greater than 0 and less than 6, so the domain of V is (0, 6).

c.



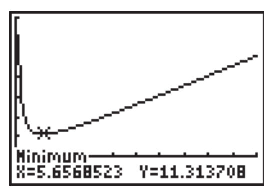
[0, 6, 1] by [−25, 150, 25]

range: [0, 128]

d. V is at its maximum when $x = 2$.

106. a. Let x = one of the numbers. Then $\frac{32}{x}$ = the other number. The sum of the numbers is $S = x + \frac{32}{x}$.

b.



[0, 50, 10] by [−10, 70, 10]

The minimum value of approximately 11.31 occurs at $x \approx 5.66$.

107. a. $C(x) = 210x + 10{,}500$

b. $C(50) = 210(50) + 10{,}500 = \$21{,}000$

It costs $21,000 to produce 50 notebooks per day.

c. average cost $= \frac{\$21{,}000}{50} = \$420$

d. $\frac{210x+10,500}{x}=315$
$210x+10,500=315x$
$10,500=105x \Rightarrow x=100$
The average cost per notebook will be $315 when 100 notebooks are produced.

108. $f(x)=-2x^2+3x+4$
$f(1)=-2(1)^2+3(1)+4=5$
$f(3)=-2(3)^2+3(3)+4=-5$
The secant passes through the points (1, 5) and (3, −5).
$m=\frac{-5-5}{3-1}=\frac{-10}{2}=-5$
The equation of the secant is
$y-5=-5(x-1) \Rightarrow y-5=-5x+5 \Rightarrow$
$y=-5x+10$

109. average rate of increase $=\frac{f(2019)-f(2015)}{2019-2015}$
$=\frac{992,000-935,133}{4}$
$=\frac{56,867}{4}\approx 14,217$
The average rate of increase was about 14,217 students per year.

110. $f(t)=\frac{60}{t}-5$
$f(5)=\frac{60}{5}-5=7;\quad f(1)=\frac{60}{1}-5=55$
average rate of decrease $=\frac{f(5)-f(1)}{5-1}$
$=\frac{7-55}{4}=-12$
The average rate of decrease is 12 gallons per minute.

111. a. $f(0)=0^2+3(0)+4=4$
The particle is 4 ft to the right from the origin.

b. $f(4)=4^2+3(4)+4=32$
The particle started 4 ft from the origin, so it traveled 32 − 4 = 28 ft in four seconds.

c. $f(3)=3^2+3(3)+4=22$
The particle started 4 ft from the origin, so it traveled 22 − 4 = 18 ft in three seconds.
The average velocity is $18/3=6$ ft/sec

d. $f(2)=2^2+3(2)+4=14$
$f(5)=5^2+3(5)+4=44$
The particle traveled 44 − 14 = 30 ft between the second and fifth seconds. The average velocity is $30/(5-2)=10$ ft/sec

112. a. $P(0)=0.02(0)^2+0.2(0)+50=50$
$P(5)=0.02(5)^2+0.2(5)+50=51.5$
The population of Sardonia was 50 million in 2015 and 51.5 million in 2020.

b. The average rate of growth from 2015 to 2020 was $\frac{51.5-50}{5}=0.3$ million per year.

Beyond the Basics

113. $f(x)=\frac{x-1}{x+1}$
$f(2x)=\frac{2x-1}{2x+1}$
$$\frac{3f(x)+1}{f(x)+3}=\frac{3\left(\frac{x-1}{x+1}\right)+1}{\frac{x-1}{x+1}+3}=\frac{\frac{3x-3}{x+1}+1}{\frac{x-1+3(x+1)}{x+1}}$$
$$=\frac{\frac{3x-3+x+1}{x+1}}{\frac{x-1+3(x+1)}{x+1}}=\frac{4x-2}{4x+2}$$
$$=\frac{2(2x-1)}{2(2x+1)}=\frac{2x-1}{2x+1}=f(2x)$$

114. $f(x)=0$ is both even and odd.

115. In order to find the relative maximum, first observe that the relative maximum of $-(x+1)^2\le 0$. Then $-(x+1)^2\le 0 \Rightarrow$
$(x+1)^2\ge 0 \Rightarrow x\ge -1$.
Thus, the x-coordinate of the relative maximum is −1. $f(-1)=-(-1+1)^2+5=5$
The relative maximum is (−1, 5).
There is no relative minimum.

116. Answers will vary.
$$f(x)=\begin{cases}x+10 & \text{if } x<-5\\ 5 & \text{if } -5\le x\le 5\\ -x & \text{if } x>5\end{cases}$$
The point (0, 5) is a relative maximum, but not a turning point.

117. $f(x)=\sqrt{x}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

118. $f(x)=\sqrt{x-1}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x+h-1}-\sqrt{x-1}}{h}\cdot\frac{\sqrt{x+h-1}+\sqrt{x-1}}{\sqrt{x+h-1}+\sqrt{x-1}}=\frac{x+h-1-(x-1)}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}=\frac{h}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}=\frac{1}{\sqrt{x+h-1}+\sqrt{x-1}}$$

119. $f(x)=-\frac{1}{\sqrt{x}}$

$$\frac{f(x+h)-f(x)}{h}=\frac{-\frac{1}{\sqrt{x+h}}+\frac{1}{\sqrt{x}}}{h}=\frac{\frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x}\sqrt{x+h}}}{h}=\frac{\frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x(x+h)}}}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h\sqrt{x(x+h)}}$$
$$=\frac{\sqrt{x+h}-\sqrt{x}}{h\sqrt{x(x+h)}}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=\frac{(x+h)-x}{h\sqrt{x(x+h)}\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{1}{\sqrt{x(x+h)}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

120. $f(x)=\frac{1}{x^2}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\frac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}=\frac{x^2-(x+h)^2}{hx^2(x+h)^2}=\frac{x^2-\left(x^2+2xh+h^2\right)}{hx^2(x+h)^2}=\frac{-2xh-h^2}{hx^2(x+h)^2}=-\frac{2x+h}{x^2(x+h)^2}$$

Critical Thinking/Discussion/Writing

121. f has a relative maximum at $x=a$ if there is an interval $[a, x_1)$ with $a<x_1<b$ such that $f(a)\ge f(x)$, or $f(x)\le f(a)$, for every x in the interval $(x_1, b]$.

122. f has a relative minimum at $x=b$ if there is x_1 in $[a, b]$ such that $f(x)\ge f(b)$ for every x in the interval $(x_1, b]$.

123. Answers will vary. Sample answers are given.

a. $f(x)=x$ on the interval $[-1, 1]$

b. $f(x)=\begin{cases}x & \text{if } 0\le x<1\\ 0 & \text{if } x=1\end{cases}$

c. $f(x)=\begin{cases}x & \text{if } 0<x\le 1\\ 1 & \text{if } x=0\end{cases}$

d. $f(x)=\begin{cases}0 & \text{if } x=0 \text{ or } x=1\\ 1 & \text{if } 0<x<1 \text{ and } x \text{ is rational}\\ -1 & \text{if } 0<x<1 \text{ and } x \text{ is irrational}\end{cases}$

Active Learning

124. a.–b. Refer to the app using the QR code in your text.

Getting Ready for the Next Section

GR1. $m = \dfrac{-2-3}{4-(-1)} = \dfrac{-5}{5} = -1$

$y-3 = -(x-(-1)) \Rightarrow y-3 = -(x+1) \Rightarrow$
$y-3 = -x-1 \Rightarrow y = -x+2$

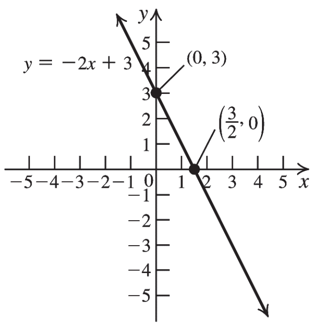
GR2. $m = \dfrac{-1-2}{7-6} = \dfrac{-3}{1} = -3$

$y-2 = -3(x-6) \Rightarrow y-2 = -3x+18 \Rightarrow$
$y = -3x+20$

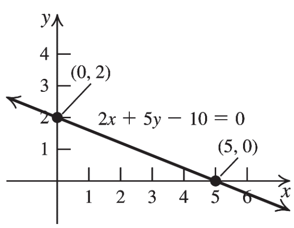
GR3. $m = \dfrac{-3-(-5)}{6-3} = \dfrac{2}{3}$

$y-(-5) = \dfrac{2}{3}(x-3) \Rightarrow y+5 = \dfrac{2}{3}x-2 \Rightarrow$
$y = \dfrac{2}{3}x-7$

GR4.



GR5.



GR6. If $x=-3$, then $|x+1| = \underline{2}$.

GR7. $|2x-8| = 0 \Rightarrow 2x-8 = 0 \Rightarrow 2x = 8 \Rightarrow x = 4$

GR8. $f(x) = x^{3/2}$

(i) $f(2) = 2^{3/2} = \left(\sqrt{2}\right)^3 = \sqrt{8} = 2\sqrt{2}$

(ii) $f(4) = 4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

(iii) $f(-4) = (-4)^{3/2} = \left(\sqrt{-4}\right)^3 = (2i)^3 = -8i$

GR9. $f(x) = x^{2/3}$

(i) $f(2) = 2^{2/3} = 4^{1/3} = \sqrt[3]{4}$

(ii) $f(8) = 8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

(iii) $f(-8) = (-8)^{2/3} = \left(\sqrt[3]{-8}\right)^2 = (-2)^2 = 4$

1.6 A Library of Functions

Practice Problems

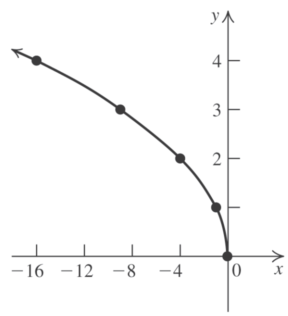
1. Because $g(-2) = 2$ and $g(1) = 8$, the line passes through the points $(-2, 2)$ and $(1, 8)$.

$m = \dfrac{8-2}{1-(-2)} = \dfrac{6}{3} = 2$

Using the point-slope form:
$y-8 = 2(x-1) \Rightarrow y-8 = 2x-2 \Rightarrow$
$y = 2x+6 \Rightarrow g(x) = 2x+6$

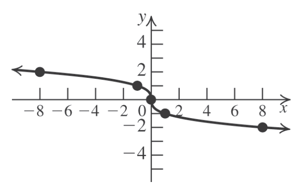
2. Using the formula
Shark length = (0.96)(tooth height) − 0.22,
gives:
Shark length = (0.96)(16.4) − 0.22 = 15.524 m

3.



Domain: $(-\infty, 0]$; range: $[0, \infty)$

4.



Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

5. $f(x) = \begin{cases} x^2 & \text{if } x \le -1 \\ 2x & \text{if } x > -1 \end{cases}$

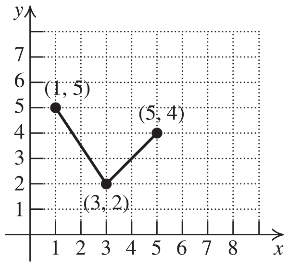
$f(-2) = (-2)^2 = 4;\quad f(3) = 2(3) = 6$

6. a. $f(x) = \begin{cases} 50 + 4(x-55), & 56 \le x < 75 \\ 200 + 5(x-75), & x \ge 75 \end{cases}$

b. The fine for driving 60 mph is
$50 + 4(60-55) = \$70$.

c. The fine for driving 90 mph is $200 + 5(90 - 75) = \$275$.

7. The graph of *f* is made up of two parts: a line segment passing through (1, 5) and (3, 2) on the interval [1, 3], and a line segment passing through (3, 2) and (5, 4) on the interval [3, 5].



For the first line segment:

$$m = \frac{2-5}{3-1} = -\frac{3}{2}$$

$$y - 5 = -\frac{3}{2}(x-1) \Rightarrow 2y - 10 = -3(x-1) \Rightarrow$$

$$2y - 10 = -3x + 3 \Rightarrow 2y = -3x + 13 \Rightarrow$$

$$y = -\frac{3}{2}x + \frac{13}{2}$$

For the second line segment:

$$m = \frac{4-2}{5-3} = 1$$

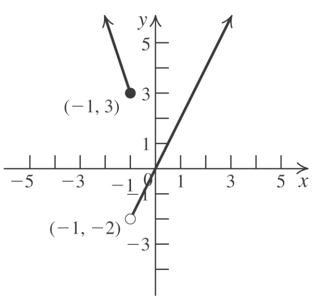
$$y - 4 = x - 5 \Rightarrow y = x - 1$$

The piecewise function is

$$g(x) = \begin{cases} -\frac{3}{2}x + \frac{13}{2} & \text{if } 1 \le x \le 3 \\ x - 1 & \text{if } 3 < x \le 5 \end{cases}$$

8. $f(x) = \begin{cases} -3x & \text{if } x \le -1 \\ 2x & \text{if } x > -1 \end{cases}$

Graph $f(x) = -3x$ on the interval $(-\infty, -1]$, and graph $f(x) = 2x$ on the interval $(-1, \infty)$.

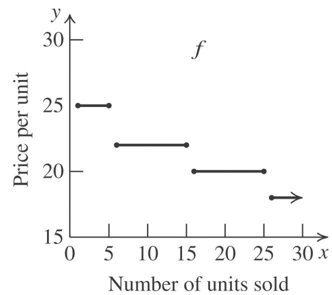


9. $f(x) = [\![x]\!]$

$f(-3.4) = -4;\ f(4.7) = 4$

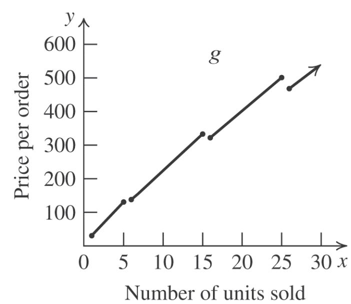
10. a. Each line of the piecewise function gives the purchase price per backpack for *x* backpacks.

$$f(x) = \begin{cases} 25 & \text{if } 1 \le x \le 5 \\ 22 & \text{if } 6 \le x \le 15 \\ 20 & \text{if } 16 \le x \le 25 \\ 18 & \text{if } 26 \le x \end{cases}$$



b. Each line of the piecewise function gives the total purchase price for *x* backpacks.

$$g(x) = \begin{cases} 25x & \text{if } 1 \le x \le 5 \\ 22x & \text{if } 6 \le x \le 15 \\ 20x & \text{if } 16 \le x \le 25 \\ 18x & \text{if } 26 \le x \end{cases}$$



Concepts and Vocabulary

1. The graph of the linear function $f(x) = b$ is a <u>horizontal</u> line.

2. The absolute value function can be expressed as a piecewise function by writing $f(x) = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}$.

3. The graph of the function

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x \le 1 \\ ax & \text{if } x > 1 \end{cases}$$

will have a break at $x = 1$ unless $a = \underline{3}$.

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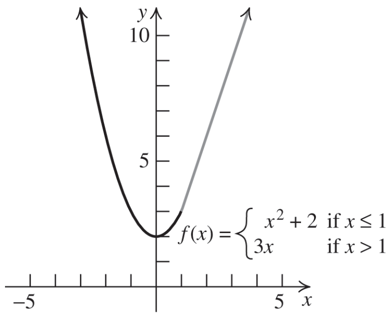


4. The line that is the graph of $f(x) = -2x + 3$ has slope <u>–2</u>.

5. True. The equation of the graph of a vertical line has the format $x = a$.

6. False. The absolute value function, $f(x) = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}$ is an example of a piecewise function with domain $(-\infty, \infty)$.

7. True

8. False. The function is constant on [0, 1), [1, 2), and [2, 3).

Building Skills

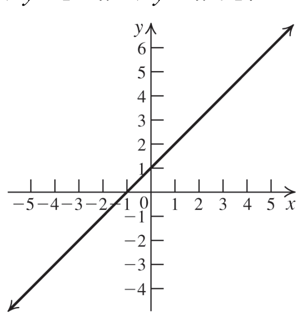
In exercises 9−18, first find the slope of the line using the two points given. Then substitute the coordinates of one of the points into the point-slope form of the equation to determine the equation.

9. The two points are (0, 1) and (–1, 0).

$m = \dfrac{0-1}{-1-0} = 1.$

$y - 1 = 1(x - 0) \Rightarrow y - 1 = x \Rightarrow y = x + 1.$

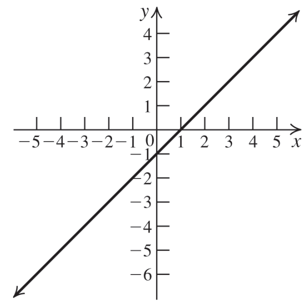
$f(x) = x + 1$



10. The two points are (1, 0) and (2, 1).

$m = \dfrac{1-0}{2-1} = 1. \quad y - 0 = 1(x - 1) \Rightarrow y = x - 1.$

$f(x) = x - 1$



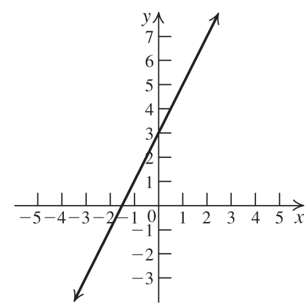
11. The two points are (–1, 1) and (2, 7).

$m = \dfrac{7-1}{2-(-1)} = 2.$

$y - 1 = 2(x - (-1)) \Rightarrow y - 1 = 2(x + 1) \Rightarrow$

$y - 1 = 2x + 2 \Rightarrow y = 2x + 3$

$f(x) = 2x + 3$



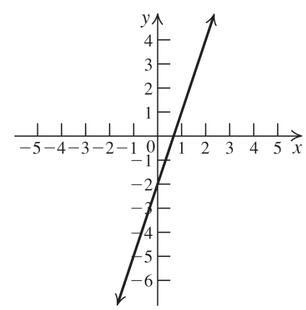
12. The two points are (–1, –5) and (2, 4).

$m = \dfrac{4-(-5)}{2-(-1)} = 3.$

$y - 4 = 3(x - 2) \Rightarrow y - 4 = 3x - 6 \Rightarrow$

$y = 3x - 2$

$f(x) = 3x - 2$



13. The two points are (1, 1) and (2, –2).

$m = \dfrac{-2-1}{2-1} = -3.$

$y - 1 = -3(x - 1) \Rightarrow y - 1 = -3x + 3 \Rightarrow$

$y = -3x + 4$

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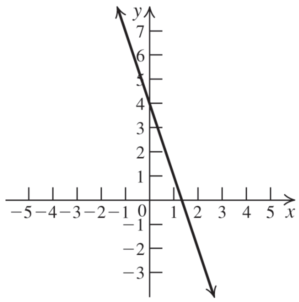
$f(x) = -3x + 4$

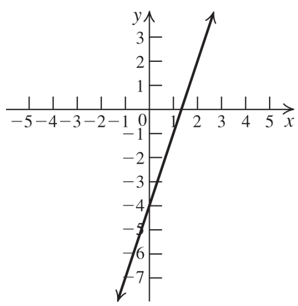


14. The two points are (1, –1) and (3, 5).

$m = \dfrac{5-(-1)}{3-1} = 3.$

$y - 5 = 3(x-3) \Rightarrow y - 5 = 3x - 9 \Rightarrow$
$y = 3x - 4$

$f(x) = 3x - 4$



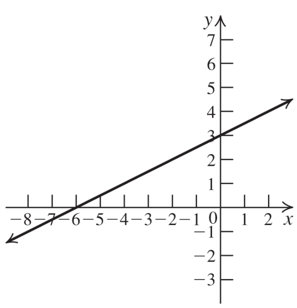
15. The two points are (–2, 2) and (2, 4).

$m = \dfrac{4-2}{2-(-2)} = \dfrac{1}{2}.$

$y - 4 = \dfrac{1}{2}(x-2) \Rightarrow y - 4 = \dfrac{1}{2}x - 1 \Rightarrow$

$y = \dfrac{1}{2}x + 3$

$f(x) = \dfrac{1}{2}x + 3$



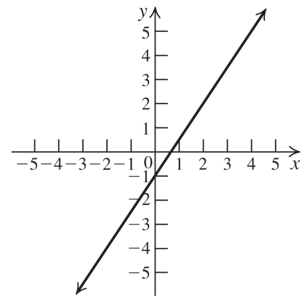
16. The two points are (2, 2) and (4, 5).

$m = \dfrac{5-2}{4-2} = \dfrac{3}{2}.$

$y - 2 = \dfrac{3}{2}(x-2) \Rightarrow y - 2 = \dfrac{3}{2}x - 3 \Rightarrow$

$y = \dfrac{3}{2}x - 1$

$f(x) = \dfrac{3}{2}x - 1$



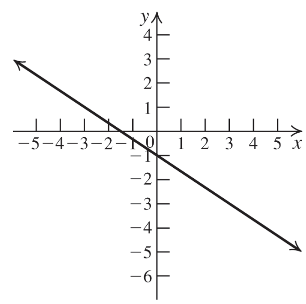
17. The two points are (0, –1) and (3, –3).

$m = \dfrac{-3-(-1)}{3-0} = -\dfrac{2}{3}.$

$y - (-1) = -\dfrac{2}{3}(x-0) \Rightarrow y + 1 = -\dfrac{2}{3}x \Rightarrow$

$y = -\dfrac{2}{3}x - 1$

$f(x) = -\dfrac{2}{3}x - 1$



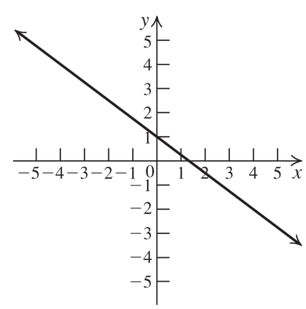
18. The two points are (1, 1/4) and (4, –2).

$m = \dfrac{-2-1/4}{4-1} = \dfrac{-9/4}{3} = -\dfrac{3}{4}.$

$y - (-2) = -\dfrac{3}{4}(x-4) \Rightarrow y + 2 = -\dfrac{3}{4}x + 3 \Rightarrow$

$y = -\dfrac{3}{4}x + 1$

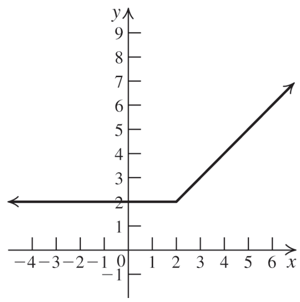
$f(x) = -\dfrac{3}{4}x + 1.$



19. $f(x) = \begin{cases} x & \text{if } x \ge 2 \\ 2 & \text{if } x < 2 \end{cases}$

a. $f(1) = 2;\ f(2) = 2;\ f(3) = 3$

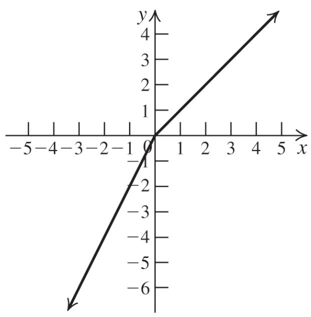
b.



20. $g(x)=\begin{cases}2x & \text{if } x<0\\ x & \text{if } x\ge 0\end{cases}$

a. $g(-1)=-2;\ g(0)=0;\ g(1)=1$

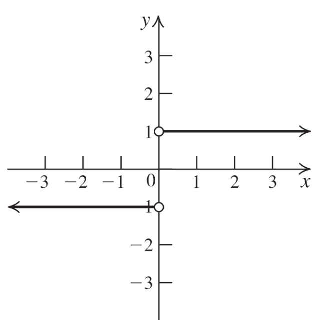
b.



21. $g(x)=\begin{cases}1 & \text{if } x>0\\ -1 & \text{if } x<0\end{cases}$

a. $f(-15)=-1;\ f(12)=1$

b.



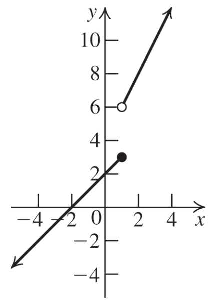
c. domain: $(-\infty,\ 0)\cup(0,\ \infty)$

range: $\{-1,\ 1\}$

22. $g(x)=\begin{cases}2x+4 & \text{if } x>1\\ x+2 & \text{if } x\le 1\end{cases}$

a. $g(-3)=-1;\ g(1)=3;\ g(3)=10$

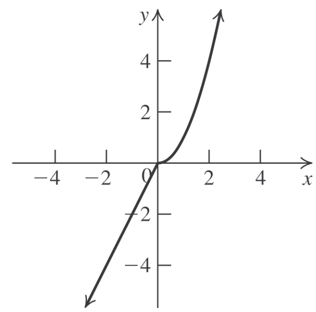
b.



c. domain: $(-\infty,\ \infty)$

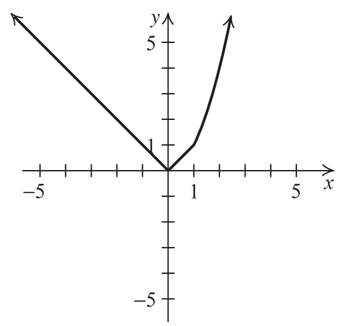
range: $(-\infty,\ 3]\cup(6,\ \infty)$

23. $f(x)=\begin{cases}2x & \text{if } x<0\\ x^2 & \text{if } x\ge 0\end{cases}$



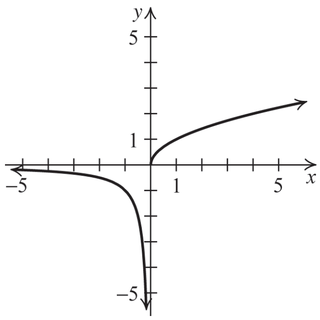
Domain: $(-\infty,\ \infty)$; range: $(-\infty,\ \infty)$

24. $f(x)=\begin{cases}|x| & \text{if } x<1\\ x^2 & \text{if } x\ge 1\end{cases}$



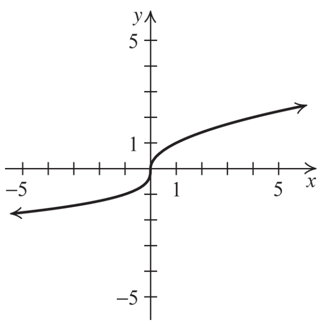
Domain: $(-\infty,\ \infty)$; range: $[0,\ \infty)$

25. $g(x)=\begin{cases}\dfrac{1}{x} & \text{if } x<0\\ \sqrt{x} & \text{if } x\ge 0\end{cases}$



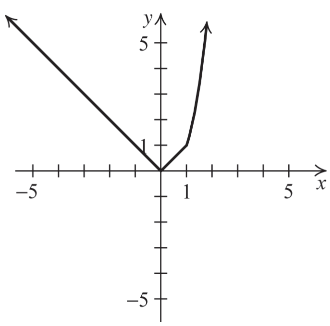
Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

26. $h(x)=\begin{cases}\sqrt[3]{x} & \text{if } x<1\\ \sqrt{x} & \text{if } x\ge 1\end{cases}$



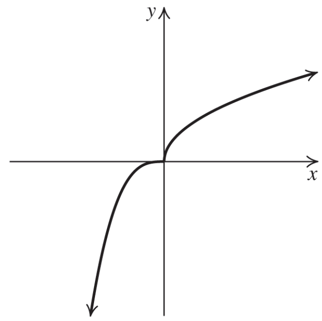
Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

27. $g(x)=\begin{cases}|x| & \text{if } x<1\\ x^3 & \text{if } x\ge 1\end{cases}$



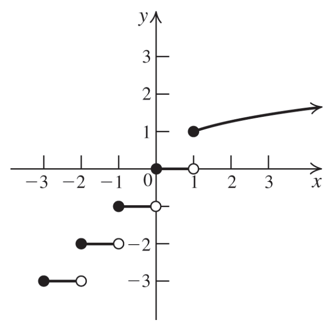
Domain: $(-\infty, \infty)$; range: $[0, \infty)$

28. $g(x)=\begin{cases}x^3 & \text{if } x<0\\ \sqrt{x} & \text{if } x\ge 0\end{cases}$



Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

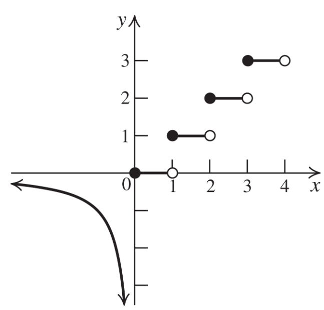
29. $f(x)=\begin{cases}[\![x]\!] & \text{if } x<1\\ \sqrt[3]{x} & \text{if } x\ge 1\end{cases}$



Domain: $(-\infty, \infty)$;

range: $\{\ldots, -3, -2, -1, 0\}\cup[1, \infty)$

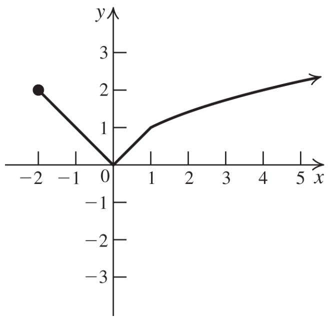
30. $f(x)=\begin{cases}\dfrac{1}{x} & \text{if } x<0\\ [\![x]\!] & \text{if } x\ge 0\end{cases}$



Domain: $(-\infty, \infty)$; range:

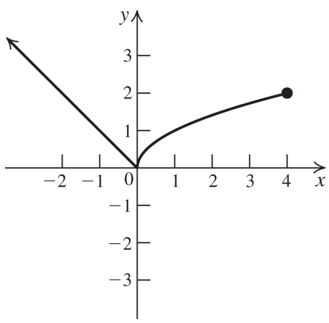
$(-\infty, 0]\cup\{1, 2, 3, \ldots\}$

31. $f(x)=\begin{cases}|x| & \text{if } -2\le x<1\\ \sqrt{x} & \text{if } x\ge 1\end{cases}$



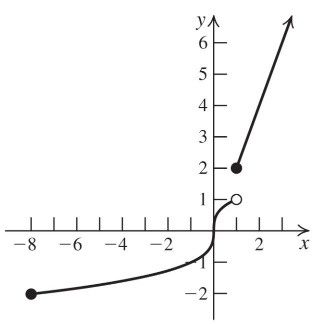
Domain: $[-2,\infty)$; range: $[0,\infty)$

32. $h(x)=\begin{cases}|x| & \text{if } x<0\\ \sqrt{x} & \text{if } 0\le x\le 4\end{cases}$



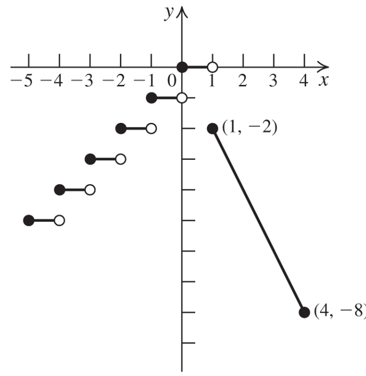
Domain: $(-\infty, 4]$; range: $[0,\infty)$

33. $g(x)=\begin{cases}\sqrt[3]{x} & \text{if } -8\le x<1\\ 2x & \text{if } x\ge 1\end{cases}$



Domain: $[-8,\infty)$; range: $[-2, 1)\cup[2,\infty)$

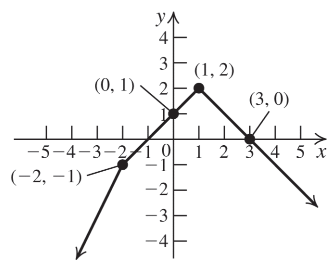
34. $f(x)=\begin{cases}[\![x]\!] & \text{if } x<1\\ -2x & \text{if } 1\le x\le 4\end{cases}$



Domain: $(-\infty, 4]$;

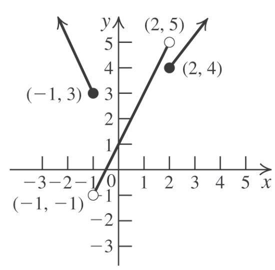
range: $\{\ldots,-3,-2,-1,0\}\cup[-8,-2]$

35. $f(x)=\begin{cases}2x+3 & \text{if } x<-2\\ x+1 & -2\le x<1\\ -x+3 & \text{if } x\ge 1\end{cases}$



Domain: $(-\infty,\infty)$; range: $(-\infty, 2]$

36. $f(x)=\begin{cases}-2x+1 & \text{if } x\le -1\\ 2x+1 & -1<x<2\\ x+2 & \text{if } x\ge 2\end{cases}$



Domain: $(-\infty,\infty)$; range: $(-1,\infty)$

37. The graph of f is made up of two parts. For $x<2$, the graph is made up of the half-line passing through the points $(-1, 0)$ and $(2, 3)$.

$$m=\frac{3-0}{2-(-1)}=\frac{3}{3}=1$$

$$y-0=x-(-1)\Rightarrow y=x+1$$

(*continued on next page*)

(*continued*)

For $x \geq 2$, the graph is a half-line passing through the points (2, 3) and (3, 0).

$m = \frac{0-3}{3-2} = -3$

$y - 0 = -3(x-3) \Rightarrow y = -3x + 9$

Combining the two parts, we have

$f(x) = \begin{cases} x+1 & \text{if } x < -2 \\ -3x+9 & \text{if } x \geq 2 \end{cases}$

38. The graph of *f* is made up of two parts. For $x < 2$, the graph is made up of the half-line passing through the points (2, −1) and (0, 3).

$m = \frac{3-(-1)}{0-2} = \frac{4}{-2} = -2$

$y = -2x + 3$

For $x \geq 2$, the graph is a half-line passing through the points (2, −1) and (4, 0).

$m = \frac{-1-0}{2-4} = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x-4) \Rightarrow y = \frac{1}{2}x - 2$

Combining the two parts, we have

$f(x) = \begin{cases} -2x+3 & \text{if } x < 2 \\ \frac{1}{2}x - 2 & \text{if } x \geq 2 \end{cases}$

39. The graph of *f* is made up of three parts. For $x < -2$, the graph is the half-line passing through the points (−2, 2) and (−3, 0).

$m = \frac{0-2}{-3-(-2)} = \frac{-2}{-1} = 2$

$y - 0 = 2(x-(-3)) \Rightarrow y = 2(x+3) \Rightarrow$

$y = 2x + 6$

For $-2 \leq x < 2$, the graph is a horizontal line segment passing through the points (−2, 4) and (2, 4), so the equation is $y = 4$.

For $x \geq 2$, the graph is the half-line passing through the points (2, 1) and (3, 0).

$m = \frac{0-1}{3-2} = -1$

$y - 0 = -(x-3) \Rightarrow y = -x + 3$

Combining the three parts, we have

$f(x) = \begin{cases} 2x+6 & \text{if } x < -2 \\ 4 & \text{if } -2 \leq x < 2 \\ -x+3 & \text{if } x \geq 2 \end{cases}$

40. The graph of *f* is made up of four parts. For $x \leq -2$, the graph is the half-line passing through the points (−2, 0) and (−4, 3).

$m = \frac{3-0}{-4-(-2)} = -\frac{3}{2}$

$y - 0 = -\frac{3}{2}(x-(-2)) \Rightarrow y = -\frac{3}{2}(x+2) \Rightarrow$

$y = -\frac{3}{2}x - 3$

For $-2 < x \leq 0$, the graph is a line segment passing through the points (−2, 0) and (0, 3).

$m = \frac{3-0}{0-(-2)} = \frac{3}{2}$

The *y*-intercept is 3, so $y = \frac{3}{2}x + 3$.

For $0 < x \leq 2$, the graph is a line segment passing through the points (0, 3) and (2, 0).

$m = \frac{0-3}{2-0} = -\frac{3}{2}$

The *y*-intercept is 3, so $y = -\frac{3}{2}x + 3$

For $x \geq 2$, the graph is the half-line passing through the points (2, 0) and (4, 3).

$m = \frac{3-0}{4-2} = \frac{3}{2}$

$y - 0 = \frac{3}{2}(x-2) \Rightarrow y = \frac{3}{2}x - 3$

Combining the four parts, we have

$f(x) = \begin{cases} y = -\frac{3}{2}x - 3 & \text{if } x \leq -2 \\ y = \frac{3}{2}x + 3 & \text{if } -2 < x \leq 0 \\ y = -\frac{3}{2}x + 3 & \text{if } 0 < x \leq 2 \\ y = \frac{3}{2}x - 3 & \text{if } x > 2 \end{cases}$

Applying the Concepts

41. First, convert 7.5 inches to centimeters.

$7.5 \text{ in.} = 2.54 \text{ cm/in.} \times 7.5 \text{ in.} = 19.05 \text{ cm}$

Now apply the equation.

$H = 0.920x + 145$
$= 0.920(19.05) + 145 = 162.526 \text{ cm}$

Convert 162.526 cm to ft.

$162.526 \text{ cm} = 162.526 \text{ cm} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in.}}$
$\approx 5.33 \text{ ft} \approx 5'4''$

Answers may vary slightly due to rounding.

42. First, convert 9 inches to centimeters.
$9\text{ in.} = 2.54\text{ cm/in.} \times 9\text{ in.} = 22.86\text{ cm}$
Now apply the equation.
$$H = 0.85x + 155 = 0.85(22.86) + 155 = 174.431\text{ cm}$$
Convert 174.431 cm to ft.
$$174.431\text{ cm} = 174.431\text{ cm} \times \frac{1\text{ in.}}{2.54\text{ cm}} \times \frac{1\text{ ft}}{12\text{ in.}} \approx 5.72\text{ ft} \approx 5'8.67'' \approx 5'9''$$
Answers may vary slightly due to rounding.

43. a. $f(x) = \frac{x}{33.81}$
Domain: $[0, \infty)$; range: $[0, \infty)$.

b. $f(3) = \frac{3}{33.81} \approx 0.0887$
This means that 3 oz $\approx$ 0.0887 liter.

c. $f(12) = \frac{12}{33.81} \approx 0.3549$ liter.

44. a. $B(0) = -1.8(0) + 212 = 212.$
The y-intercept is 212. This means that water boils at 212°F at sea level.
$0 = -1.8h + 212 \Rightarrow h \approx 117.8$
The h-intercept is approximately 117.80. This means that water boils at 0°F at approximately 117,800 feet above sea level.

b. Domain: closed interval from 0 to the end of the atmosphere, in thousands of feet.

c. $98.6 = -1.8h + 212 \Rightarrow h = 63.$
Water boils at 98.6°F at 63,000 feet. It is dangerous because 98.6°F is the temperature of human blood.

45. a. $P(0) = \frac{1}{33}(0) + 1 = 1.$
The y-intercept is 1. This means that the pressure at sea level ($d = 0$) is 1 atm.
$0 = \frac{1}{33}d + 1 \Rightarrow d = -33.$
Because d can't be negative, there is no d-intercept.

b. $P(0) = 1\text{ atm}; P(10) = \frac{1}{33}(10) + 1 \approx 1.3\text{ atm};$
$P(33) = \frac{1}{33}(33) + 1 = 2\text{ atm};$
$P(100) = \frac{1}{33}(100) + 1 \approx 4.03\text{ atm}.$

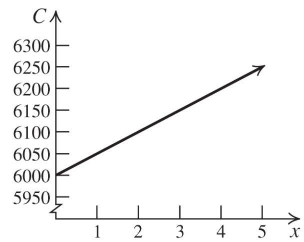
c. $5 = \frac{1}{33}d + 1 \Rightarrow d = 132$ feet
The pressure is 5 atm at 132 feet.

46. a. $V(90) = 1055 + 1.1(90) = 1154$ ft/sec
The speed of sound at 90ºF is 1154 feet per second.

b. $1100 = 1055 + 1.1T \Rightarrow T \approx 40.91°\text{F}$
The speed of sound is 1100 feet per second at approximately 40.91ºF.

47. a. $C(x) = 50x + 6000$

b. The y-intercept is the fixed overhead cost.



c. $11{,}500 = 50x + 6000 \Rightarrow 110$
110 printers were manufactures on a day when the total cost was $11,500.

48. a. The rate of change (slope) is 100. Using point-slope form, (10, 750):
$y - 750 = 100(p - 10) \Rightarrow$
$y - 750 = 100p - 1000 \Rightarrow y = 100p - 250$
The equation is $f(p) = 100p - 250.$

b. $f(15) = 100(15) - 250 = 1250$
When the price is $15 per unit, there are 1250 units.

c. $1750 = 100p - 250 \Rightarrow p = \$20.$
1750 units can be supplied at $20 per unit.

49. a. $R = 900 - 30x$

b. $R(6) = 900 - 30(6) = 720$
If you move in 6 days after the first of the month, the rent is $720.

c. $600 = 900 - 30x \Rightarrow x = 10$
You moved in ten days after first of the month.

50. a. Let $t = 0$ represent July 2020. Then September 2020 is represented by $t = 2$. The rate of change (slope) is $\frac{6.07-2.69}{2-0} = 1.69$. Using the point-slope form, $y - 2.69 = 1.69(t-0) \Rightarrow$
$y = 1.69t + 2.69 \Rightarrow f(t) = 1.69t + 2.69$

b. November 2020 is represented by $t = 4$.
$f(4) = 1.69(4) + 2.69 = 9.45$
If the trend continued, there were 9.45 million confirmed case in November 2020.

c. $13 = 1.69t + 2.69 \Rightarrow 10.31 = 1.69t \Rightarrow t \approx 6.1$
If the trend continued, the number of confirmed cases was approximately 13 million six months after July 2020, or in January 2021.

51. The rate of change (slope) is $\frac{100-40}{20-80} = -1$.
Using the point-slope form,
$y - 100 = -(x-20) \Rightarrow y - 100 = -x + 20 \Rightarrow$
$y = -x + 120$
Now solve $50 = -x + 120 \Rightarrow x = 70$.
Age 70 corresponds to 50% capacity.

52. a. $y = \frac{2}{25}(5)(60) = 24$
The dosage for a five-year-old child is 24 mg.

b. $60 = \frac{2}{25}(60)a \Rightarrow a = 12.5$
A child would have to be 12.5 years old to be prescribed an adult dosage.

53. a. The rate of change (slope) is
$\frac{50-30}{420-150} = \frac{2}{27}$.
The equation of the line is
$y - 30 = \frac{2}{27}(x-150) \Rightarrow$
$y = \frac{2}{27}(x-150) + 30.$

b. $y = \frac{2}{27}(350-150) + 30 \Rightarrow y = \frac{1210}{27} \approx 44.8$
There can't be a fractional number of deaths, so round up. There will be about 45 deaths when $x = 350$ milligrams per cubic meter.

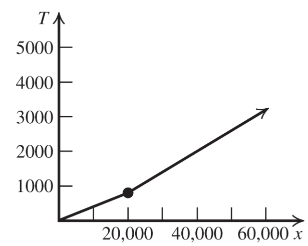
c. $70 = \frac{2}{27}(x-150) + 30 \Rightarrow x = 690$
If the number of deaths per month is 70, the concentration of sulfur dioxide in the air is 690 mg/m^3.

54. a. The rate of change is $\frac{1}{3}$. The data point given is $\left(0, 3\frac{11}{12}\right)$ or $\left(0, \frac{47}{12}\right)$.
The equation is
$y - \frac{47}{12} = \frac{1}{3}(S-0) \Rightarrow y = L(S) = \frac{1}{3}S + \frac{47}{12}$.

b. $L(4) = \frac{1}{3}(4) + \frac{47}{12} = 5.25$
A child's size 4 shoe has insole length 5.25 inches.

c. $5.7 = \frac{57}{10} = \frac{1}{3}x + \frac{47}{12} \Rightarrow \frac{107}{60} = \frac{1}{3}x \Rightarrow$
$x = \frac{107}{20} = 5.35$
A child whose insole length is 5.7 inches wears a size 5.5 shoe.

55. a.



b. **(i)** $T(12,000) = 0.04(12,000) = \$480$
(ii)
$T(20,000) = 800 + 0.06(20,000 - 20,000) = \$800$
(iii)
$T(50,000) = 800 + 0.06(50,000 - 20,000) = \$2600$

c. **(i)** $600 = 0.04x \Rightarrow x = \$15,000$
(ii) $1200 = 0.04x \Rightarrow x = \$30,000$, which is outside of the domain. Try
$1200 = 800 + 0.06(x - 20,000)$
$400 = 0.06(x - 20,000)$
$6666.67 = x - 20,000 \Rightarrow x \approx \$26,667$
(iii) $2300 = 800 + 0.06(x - 20,000)$
$1500 = 0.06(x - 20,000)$
$25,000 = 20,000 \Rightarrow$
$x = \$45,000$

56. a. To find the each part of the piece-wise function, apply the process shown in Example 7 using the data points (0, 5), (1, 4.5), (2, 3.6), (3, 2.3), (4, 1.2), (5, 0.5), and (6, 0).

Between (0, 5) and (1, 4.5):

$m = \dfrac{4.5-5}{1-0} = -0.5;$

$y - 5 = -0.5(x-0) \Rightarrow y = -0.5x + 5$

Between (1, 4.5) and (2, 3.6):

$m = \dfrac{3.6-4.5}{2-1} = -0.9;$

$y - 4.5 = -0.9(x-1) \Rightarrow y = -0.9x + 5.4$

Between (2, 3.6) and (3, 2.3):

$m = \dfrac{2.3-3.6}{3-2} = -1.3;$

$y - 3.6 = -1.3(x-2) \Rightarrow y = -1.3x + 6.2$

Between (3, 2.3) and (4, 1.2):

$m = \dfrac{1.2-2.3}{4-3} = -1.1;$

$y - 2.3 = -1.1(x-3) \Rightarrow y = -1.1x + 5.6$

Between (4, 1.2) and (5, 0.5):

$m = \dfrac{0.5-1.2}{5-4} = -0.7$

$y - 1.2 = -0.7(x-4) \Rightarrow y = -0.7x + 4$

Between (5, 0.5) and (6, 0):

$m = \dfrac{0-0.5}{6-5} = -0.5$

$y - 0 = -0.5(x-6) \Rightarrow y = -0.5x + 3$

$$f(x) = \begin{cases} -0.5x+5 & \text{if } 0 < x \le 1 \\ -0.9x+5.4 & \text{if } 1 < x \le 2 \\ -1.3x+6.2 & \text{if } 2 < x \le 3, \\ -1.1x+5.6 & \text{if } 3 < x \le 4 \\ -0.7x+4 & \text{if } 4 < x \le 5 \\ -0.5x+3 & \text{if } 5 < x \le 6 \end{cases}$$

b. $f(2.5) = -1.3(2.5) + 6.2 = 2.95$

After two and a half months, there are 2.95 gallons remaining in the lake.

Beyond the Basics

57. $2(3) - 1 = a - 3(3) \Rightarrow 5 = a - 9 \Rightarrow a = 14$

58. $1 - 3 = 3a + 3 \Rightarrow -2 = 3a + 3 \Rightarrow -5 = 3a \Rightarrow a = -\dfrac{5}{3}$

59. a. Domain: $(-\infty, \infty)$; range: $[0, 1)$

b. The function is increasing on $(n, n+1)$ for every integer n.

c. $f(-x) = -x - [\![-x]\!] \ne -f(x) \ne f(x),$ so the function is neither even nor odd.

60. a. Domain: $(-\infty, 0) \cup [1, \infty)$

range: $\left\{\dfrac{1}{n} : n \ne 0, n \text{ an integer}\right\}$

b. The function is constant on $(n, n+1)$ for every nonzero integer n.

c. $f(-x) = \dfrac{1}{[\![-x]\!]} \ne -f(x) \ne f(x),$ so the function is neither even nor odd.

61. a. (i) $WCI(2) = 40$

(ii)

$$\begin{aligned} WCI(16) &= 91.4 + (91.4 - 40) \\ &\quad \cdot \left(0.0203(16) - 0.304\sqrt{16} - 0.474\right) \\ &\approx 21 \end{aligned}$$

(iii) $WCI(50) = 1.6(40) - 55 = 9$

b. (i)

$$\begin{aligned} -58 &= 91.4 + (91.4 - T) \\ &\quad \cdot (0.0203(36) - 0.304\sqrt{36} - 0.474 \\ -58 &= 91.4 + (91.4 - T)(-1.5672) \\ -58 &= 91.4 - 143.24 + 1.5672T \\ -58 &= -51.84 + 1.5672T \Rightarrow T \approx -4°\text{F} \end{aligned}$$

(ii) $-10 = 1.6T - 55 \Rightarrow T \approx 28°\text{F}$

62. a.

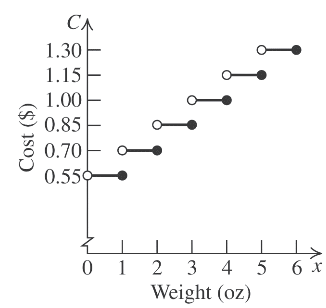
$$\begin{aligned} C(x) &= 15(f(x) - 1) + 55 \\ &= 15(-[\![-x]\!] - 1) + 55 \end{aligned}$$

b.

$$\begin{aligned} C(2.3) &= 15(-[\![-2.3]\!] - 1) + 55 \\ &= 15[-(-3) - 1] + 55 \\ &= 15(2) + 55 = 85 \end{aligned}$$

It will cost 85¢ to mail a first-class letter weighing 2.3 oz.

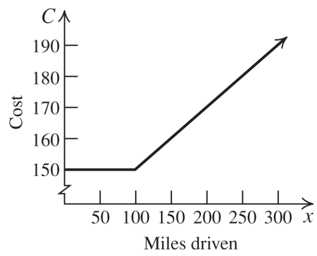
c.



63. $C(x) = 2[\![x]\!] + 4$

64. a. $C(x) = \begin{cases} 150 & \text{if } x \le 100 \\ 0.2[\![x - 100]\!] + 150 & \text{if } x > 100 \end{cases}$

b.



c. $190 = 0.2[\![x-99]\!]+150$

$40 = 0.2[\![x-99]\!] \Rightarrow 200 = [\![x-99]\!]$

Because 200 is the greatest integer less than $x-99$, $299 < x < 300$.

Between 299 and 300 miles were driven.

Critical Thinking/Discussion/Writing

65. D **66.** C

67. a. If f is even, then f is symmetric about the y-axis. So f is increasing on $(-\infty, -1)$ and decreasing on $(-1, 0)$.

b. If f is odd, then f is symmetric about the origin. So f is decreasing on $(-\infty, -1)$ and increasing on $(-1, 0)$.

68. a. If f is even, then f has a relative maximum at $x = -1$ and a relative minimum at $x = -3$.

b. If f is odd, then f has a relative minimum at $x = -1$ and a relative maximum at $x = -3$.

69. $f(x) = \dfrac{x+|x|}{2}$

70. $f(x) = \dfrac{x-|x|}{2}$

71. $f(x) = [\![x+0.5]\!]$

72. $f(x) = [\![x]\!] + [\![-x]\!] + 1$

Getting Ready for the Next Section

For exercises GR1–GR4, refer to section 1.3 in your text for help on completing the square.

GR1. $x^2 + 8x + \left(\dfrac{8}{2}\right)^2 = x^2 + 8x + \underline{16} = (x+4)^2$

GR2. $x^2 - 6x + \left(\dfrac{6}{2}\right)^2 = x^2 - 6x + \underline{9} = (x-3)^2$

GR3. $x^2 - \dfrac{2}{3}x + \left(\dfrac{1}{2}\cdot\dfrac{2}{3}\right)^2 = x^2 - \dfrac{2}{3}x + \underline{\dfrac{1}{9}} = \left(x - \dfrac{1}{3}\right)^2$

GR4. $x^2 + \dfrac{4}{5}x + \left(\dfrac{1}{2}\cdot\dfrac{4}{5}\right)^2 = x^2 + \dfrac{4}{5}x + \underline{\dfrac{4}{25}} = \left(x + \dfrac{2}{5}\right)^2$

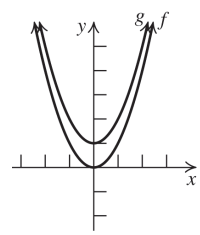
GR5. If we add 3 to each y-coordinate of the graph of f, we will obtain the graph of $y = \underline{f(x)+3}.$

GR6. If we subtract 2 from each x-coordinate of the graph of f, we will obtain the graph of $y = \underline{f(x+2)}.$

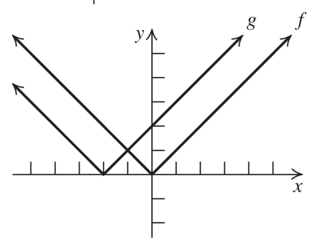
GR7. If we replace each x-coordinate with its opposite in the graph of f, we will obtain the graph of $y = \underline{f(-x)}.$

GR8. If we replace each y-coordinate with its opposite in the graph of f, we will obtain the graph of $y = \underline{-f(x)}.$

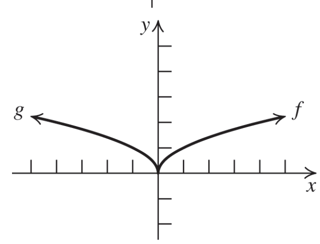
GR9.



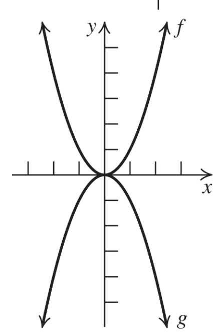
GR10.



GR11.



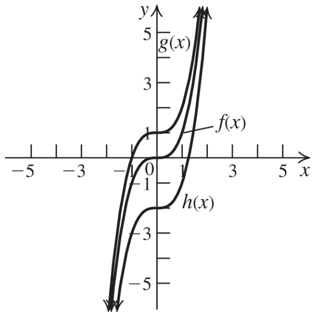
GR12.



1.7 Transformations of Functions

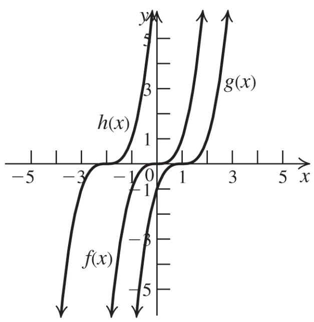
Practice Problems

1.



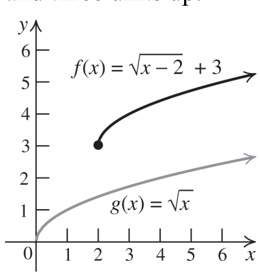
The graph of g is the graph of f shifted one unit up. The graph of h is the graph of f shifted two units down.

2.



The graph of g is the graph of f shifted one unit to the right. The graph of h is the graph of f shifted two units to the left.

3. The graph of $f(x)=\sqrt{x-2}+3$ is the graph of $g(x)=\sqrt{x}$ shifted two units to the right and three units up.

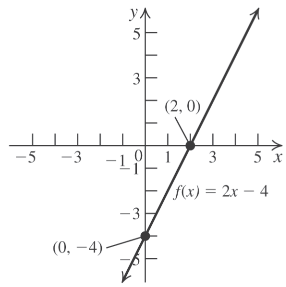


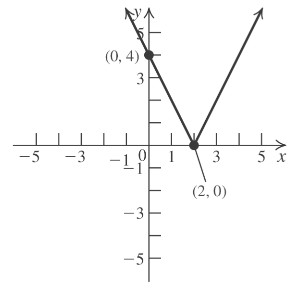
4. The graph of $y=-(x-1)^2+2$ can be obtained from the graph of $y=x^2$ by first shifting the graph of $y=x^2$ one unit to the right. Reflect the resulting graph about the x-axis, and then shift the graph two units up

5. The graph of $y=2x-4$ is obtained from the graph of $y=2x$ by shifting it down by four units. We know that

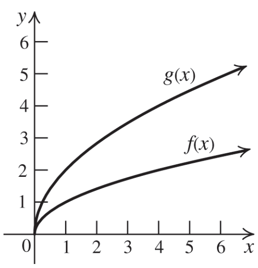
$$|y|=\begin{cases} y & \text{if } y\ge 0\\ -y & \text{if } y<0.\end{cases}$$

This means that the portion of the graph on or above the x-axis $(y\ge 0)$ is unchanged while the portion of the graph below the x-axis $(y<0)$ is reflected above the x-axis. The graph of $y=|f(x)|=|2x-4|$ is given on the right.



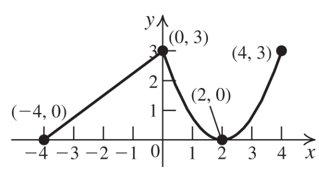


6.

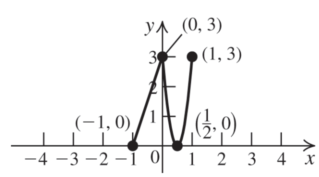


The graph of g is the graph of f stretched vertically by multiplying each of its y-coordinates by 2.

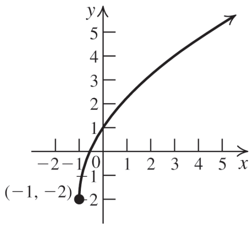
7. a. $f\left(\frac{1}{2}x\right)$



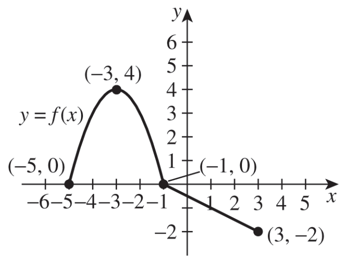
b. $f(2x)$



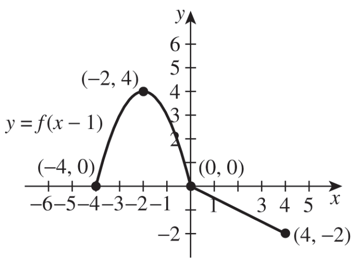
8. Start with the graph of $y=\sqrt{x}$. Shift the graph one unit to the left, then stretch the graph vertically by a factor of three. Shift the resulting graph down two units.



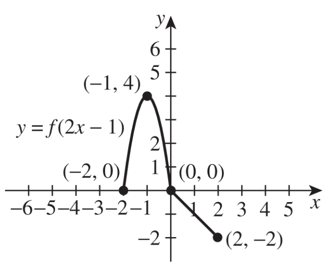
9.



Shift the graph one unit right to graph $y=f(x-1)$.

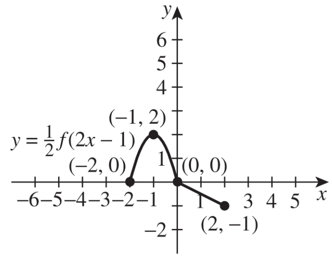


Compress horizontally by a factor of 2. Multiply each x-coordinate by $\frac{1}{2}$ to graph $y=f(2x-1)$.

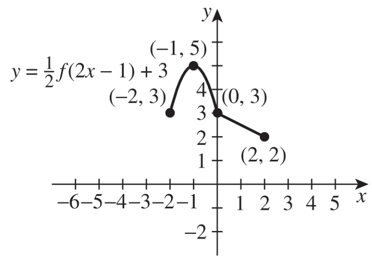


Compress vertically by a factor of $\frac{1}{2}$.

Multiply each y-coordinate by $\frac{1}{2}$ to graph $y=\frac{1}{2}f(2x-1)$.

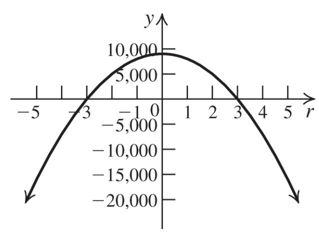


Shift the graph up three units to graph $y=\frac{1}{2}f(2x-1)+3$.



$$y=f(x)\to y=f(x-1)\to y=f(2x-1)\to$$
$$y=\frac{1}{2}f(2x-1)\Rightarrow y=\frac{1}{2}f(2x-1)+3$$

10. The equation is $v(r)=10^3\left(3^2-r^2\right)=9000-1000r^2$. The graph is obtained by stretching the graph of $y=r^2$ vertically by a factor of 1000, reflecting the resulting graph in the x-axis, and then shifting the graph 9000 units up.



Concepts and Vocabulary

1. The graph of $y=f(x)-3$ is found by vertically shifting the graph of $y=f(x)$ three units <u>down</u>.

2. The graph of $y=f(x+5)$ is found by horizontally shifting the graph of $y=f(x)$ five units to the <u>left</u>.

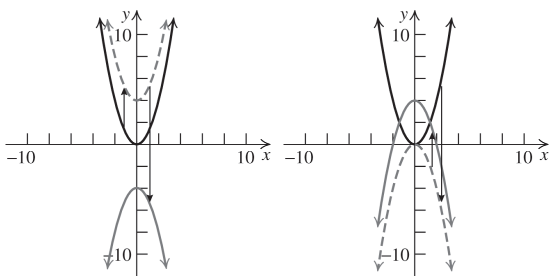
3. The graph of $y=f(bx)$ is a horizontal compression of the graph of $y=f(x)$ is b <u>is greater than 1</u>.

4. The graph of $y = f(-x)$ is found by reflecting the graph of $y = f(x)$ about the <u>y-axis</u>.

5. False. The graphs are the same if the function is an even function.

6. True

7. False. The graph on the left shows $y = x^2$ first shifted up two units and then reflected about the x-axis, while the graph on the right shows $y = x^2$ reflected about the x-axis and then shifted up two units.



8. True

Building Skills

9. a. The graph of g is the graph of f shifted two units up.

b. The graph of h is the graph of f shifted one unit down.

10. a. The graph of g is the graph of f shifted one unit up.

b. The graph of h is the graph of f shifted two units down.

11. a. The graph of g is the graph of f shifted one unit to the left.

b. The graph of h is the graph of f shifted two units to the right.

12. a. The graph of g is the graph of f shifted two units to the left.

b. The graph of h is the graph of f shifted three units to the right.

13. a. The graph of g is the graph of f shifted one unit left, then two units down.

b. The graph of h is the graph of f shifted one unit right, then three units up.

14. a. The graph of g is the graph of f reflected about the x-axis.

b. The graph of h is the graph of f reflected about the y-axis.

15. a. The graph of g is the graph of f reflected about the x-axis.

b. The graph of h is the graph of f reflected about the y-axis.

16. a. The graph of g is the graph of f stretched vertically by a factor of 2.

b. The graph of h is the graph of f compressed horizontally by a factor of 2.

17. a. The graph of g is the graph of f vertically stretched by a factor of 2.

b. The graph of h is the graph of f horizontally compressed by a factor of 2.

18. a. The graph of g is the graph of f shifted two units to the right, then one unit up.

b. The graph of h is the graph of f shifted one unit to the left, reflected about the x-axis, and then shifted two units up.

19. a. The graph of g is the graph of f reflected about the x-axis and then shifted one unit up.

b. The graph of h is the graph of f reflected about the y-axis and then shifted one unit up.

20. a. The graph of g is the graph of f shifted one unit to the right and then shifted two units up.

b. The graph of h is the graph of f stretched vertically by a factor of three and then shifted one unit down.

21. a. The graph of g is the graph of f shifted one unit up.

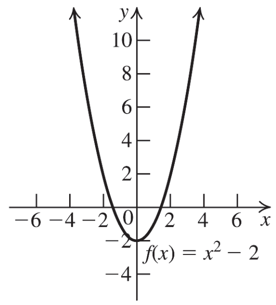
b. The graph of h is the graph of f shifted one unit to the left.

22. a. The graph of g is the graph of f shifted one unit left, vertically stretched by a factor of 2, reflected about the y-axis, and then shifted 4 units up.

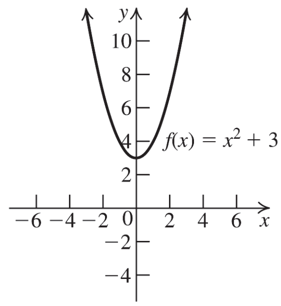
b. The graph of h is the graph of f shifted one unit to the right, reflected about the x-axis, and then shifted three units up.

23. e	**24.** c	**25.** g	**26.** h
27. i	**28.** a	**29.** b	**30.** k
31. l	**32.** f	**33.** d	**34.** j

35.



36.



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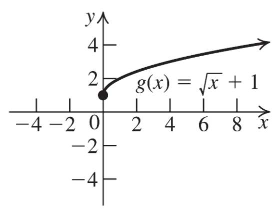
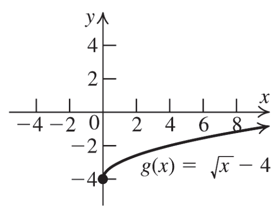
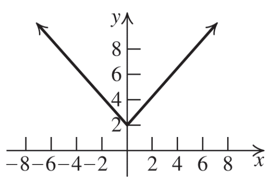


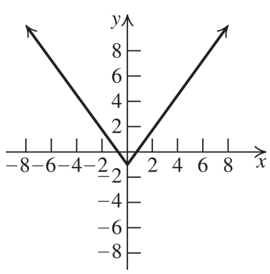
38.



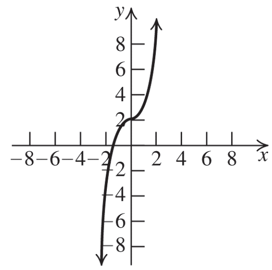
39. $f(x) = |x| + 2$



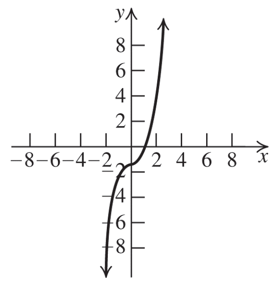
40. $f(x) = |x| - 1$



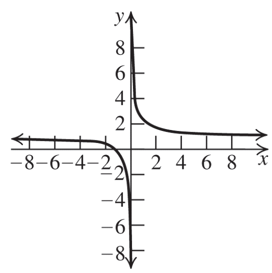
41. $f(x) = x^3 + 2$



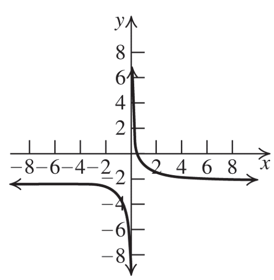
42. $f(x) = x^3 - 1$



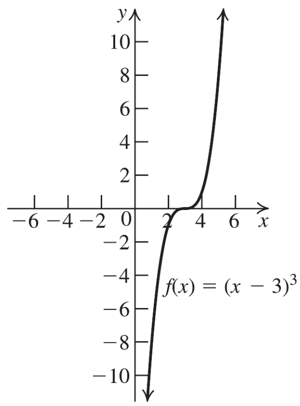
43. $f(x) = \dfrac{1}{x} + 1$



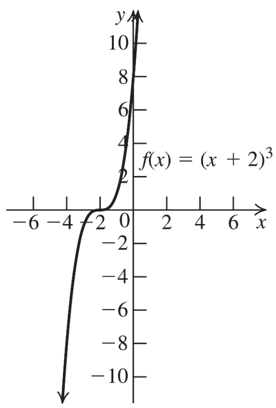
44. $f(x) = \dfrac{1}{x} - 2$



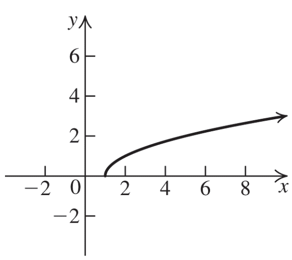
45.



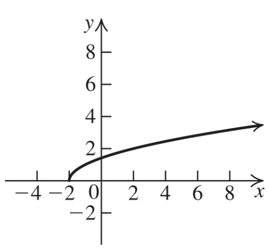
46.



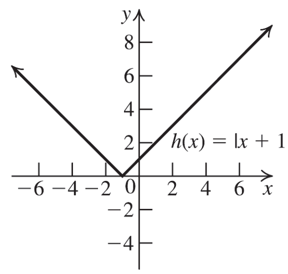
47. $f(x)=\sqrt{x-1}$



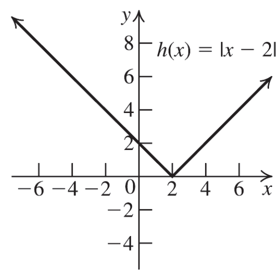
48. $f(x)=\sqrt{x+2}$



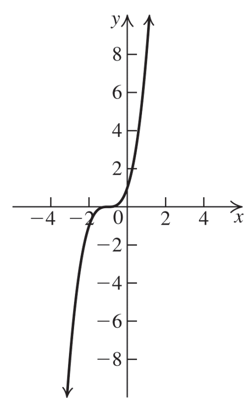
49.



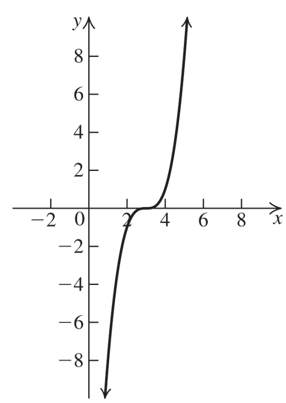
50.



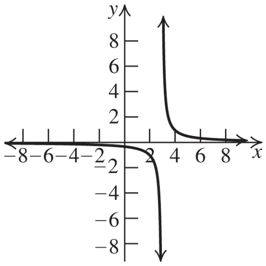
51. $f(x)=(x+1)^3$



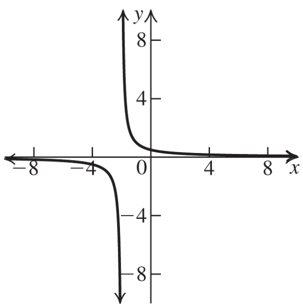
52. $f(x)=(x-3)^3$



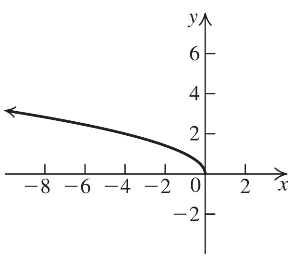
53. $f(x)=\dfrac{1}{x-3}$



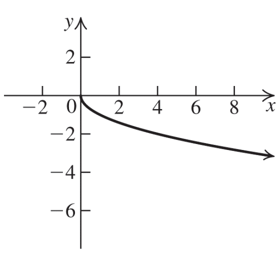
54. $f(x)=\dfrac{1}{x+2}$



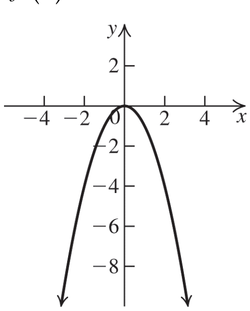
55. $f(x)=\sqrt{-x}$



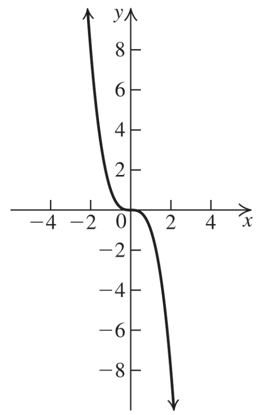
56. $f(x)=-\sqrt{x}$



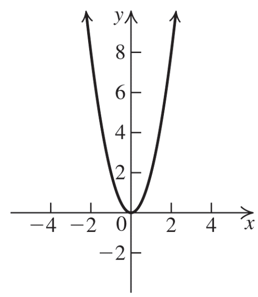
57. $f(x)=-x^2$



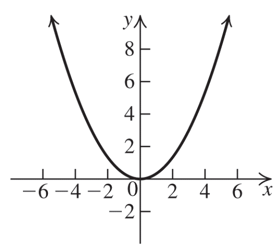
58. $f(x)=-x^3$



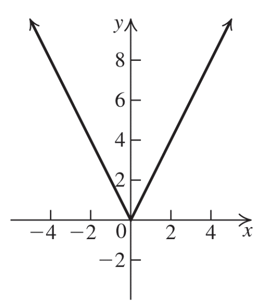
59. $f(x)=2x^2$



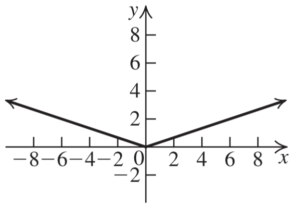
60. $f(x)=\dfrac{1}{3}x^2$



61. $f(x)=2|x|$

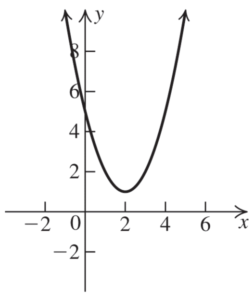


62. $f(x) = \frac{1}{3}|x|$



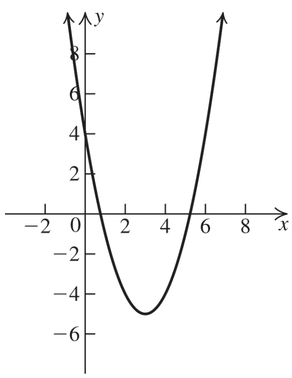
63. $f(x) = (x-2)^2 + 1$

Start with the graph of $f(x) = x^2$, then shift it two units right and one unit up.



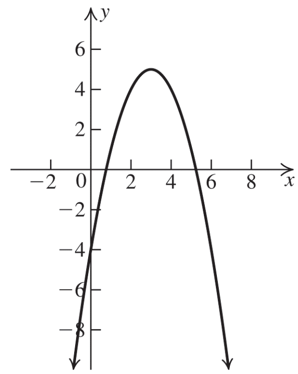
64. $f(x) = (x-3)^2 - 5$

Start with the graph of $f(x) = x^2$, then shift it three units right and five units down.



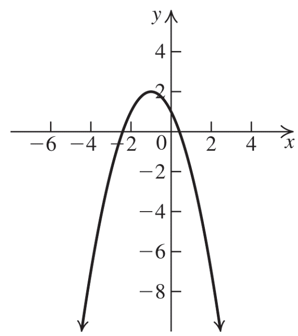
65. $f(x) = 5 - (x-3)^2$

Start with the graph of $f(x) = x^2$, then shift it three units right. Reflect the graph across the x-axis. Then, shift it five units up.



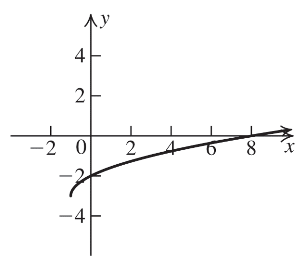
66. $f(x) = 2 - (x+1)^2$

Start with the graph of $f(x) = x^2$, then shift it one unit left. Reflect the graph across the x-axis. Shift it two units up.



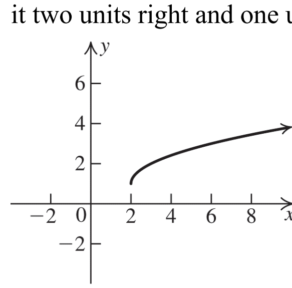
67. $f(x) = \sqrt{x+1} - 3$

Start with the graph of $f(x) = \sqrt{x}$, then shift it one unit left and three units down.



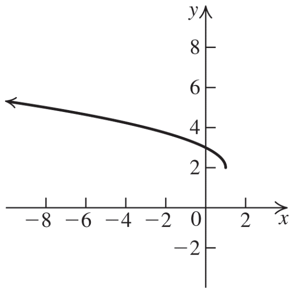
68. $f(x) = \sqrt{x-2} + 1$

Start with the graph of $f(x) = \sqrt{x}$, then shift it two units right and one unit up



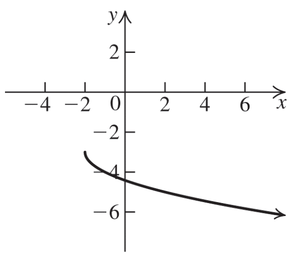
69. $f(x)=\sqrt{1-x}+2$

Start with the graph of $f(x)=\sqrt{x}$, then shift it one unit left. Reflect the graph across the y-axis, and then shift it two units up.



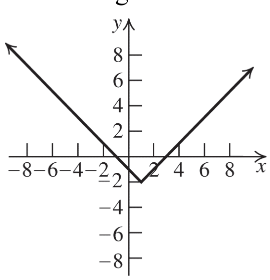
70. $f(x)=-\sqrt{x+2}-3$

Start with the graph of $f(x)=\sqrt{x}$, then shift it two units left. Reflect the graph across the x-axis, and then shift it three units down.



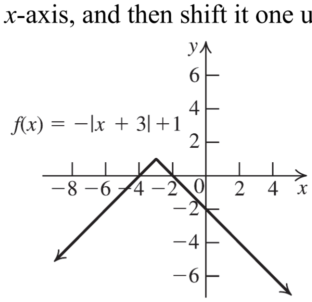
71. $f(x)=|x-1|-2$

Start with the graph of $f(x)=|x|$, then shift it one unit right and two units down.



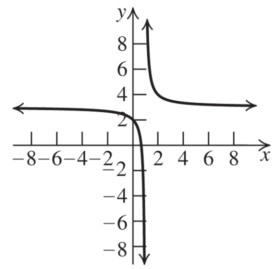
72. $f(x)=-|x+3|+1$

Start with the graph of $f(x)=|x|$, then shift it three units left. Reflect the graph across the x-axis, and then shift it one unit up.



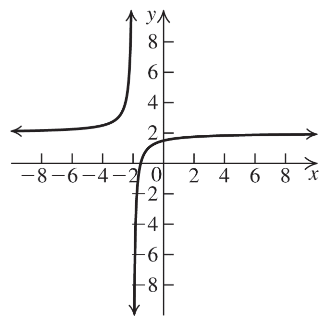
73. $f(x)=\dfrac{1}{x-1}+3$

Start with the graph of $f(x)=\dfrac{1}{x}$, then shift it one unit right and three units up.



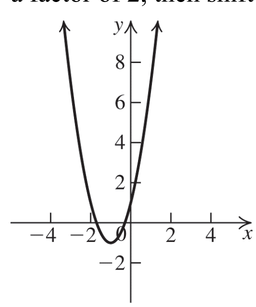
74. $f(x)=2-\dfrac{1}{x+2}$

Start with the graph of $f(x)=\dfrac{1}{x}$, then shift it two units left. Reflect the graph across the x-axis and then shift up two units up.



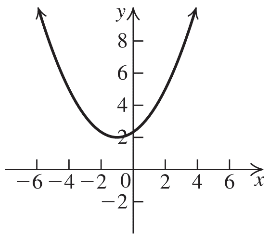
75. $f(x)=2(x+1)^2-1$

Start with the graph of $f(x)=x^2$, then shift it one unit left. Stretch the graph vertically by a factor of 2, then shift it one unit down.



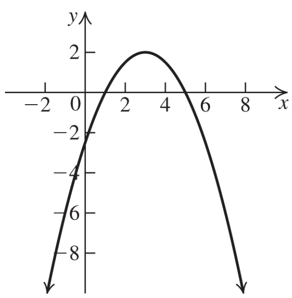
76. $f(x)=\frac{1}{3}(x+1)^2+2$

Start with the graph of $f(x)=x^2$, then shift it one unit left. Compress the graph vertically by a factor of 1/3, then shift it two units up.



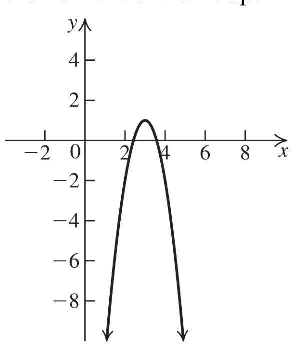
77. $f(x)=2-\frac{1}{2}(x-3)^2$

Start with the graph of $f(x)=x^2$, then shift it three units right. Compress the graph vertically by a factor of 1/2, reflect it across the *x*-axis, then shift it two units up.



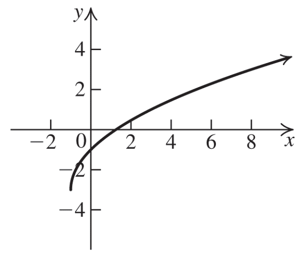
78. $f(x)=1-3(x-3)^2$

Start with the graph of $f(x)=x^2$, then shift it three units right. Stretch the graph vertically by a factor of 3, reflect it across the *x*-axis, then shift it one unit up.



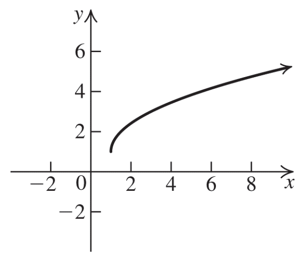
79. $f(x)=2\sqrt{x+1}-3$

Start with the graph of $f(x)=\sqrt{x}$, then shift it one unit left. Stretch the graph vertically by a factor of 2, and then shift it three units down.



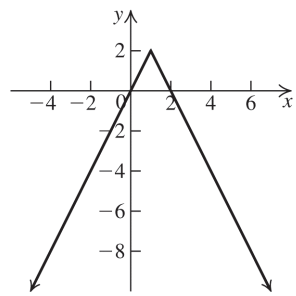
80. $f(x)=\sqrt{2x-2}+1$

Start with the graph of $f(x)=\sqrt{x}$, then shift it two units right. Compress the graph horizontally by a factor of 1/2, and then shift it one unit up.



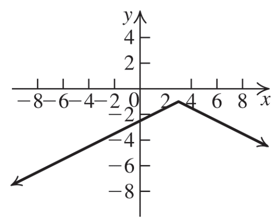
81. $f(x)=-2|x-1|+2$

Start with the graph of $f(x)=|x|$, then shift it one unit right. Stretch the graph vertically by a factor of 2, then reflect it across the *x*-axis. Shift the graph up two units.



82. $f(x)=-\frac{1}{2}|3-x|-1$

Start with the graph of $f(x)=|x|$, then shift it three units left. Compress the graph vertically by a factor of 1/2, then reflect it across the *y*-axis. Reflect the graph across the *x*-axis, and then shift the graph down one unit.



83. $y = x^3 + 2$

84. $y = \sqrt{x+3}$

85. $y = -|x|$

86. $y = \sqrt{-x}$

87. $y = (x-3)^2 + 2$

88. $y = -(x+2)^2$

89. $y = -\sqrt{x+3} - 2$

90. $y = -\frac{1}{2}\left(\sqrt{x} - 2\right)$

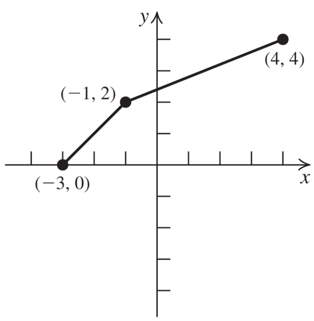
91. $y = 3(-x+4)^3 + 2$

92. $y = -(-x+1)^3 + 1$

93. $y = -|2x-4| - 3$

94. $y = \left|-\frac{1}{2}x - 2\right| - 3$

95.



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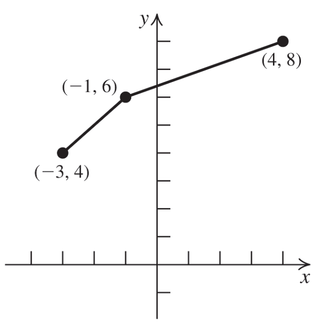

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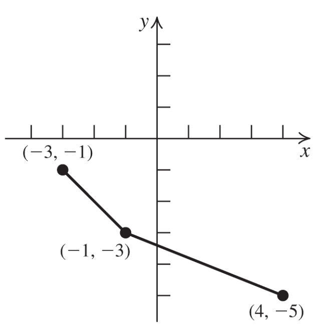

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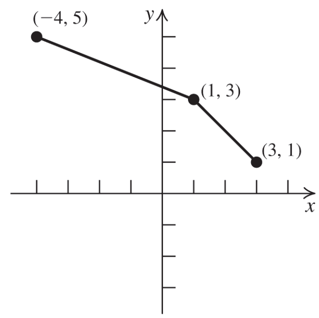

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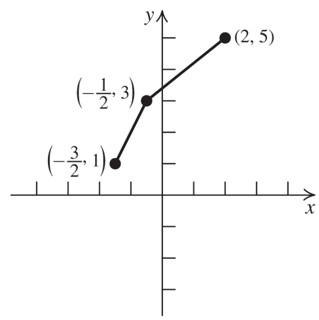

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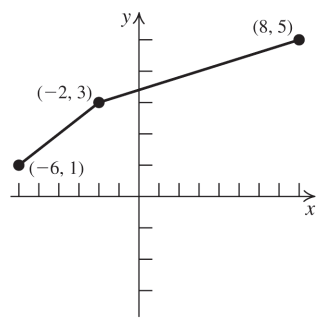

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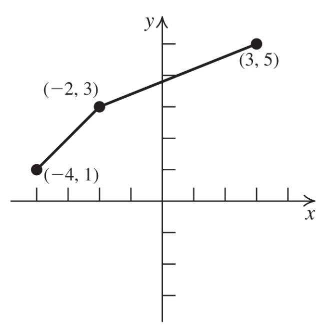

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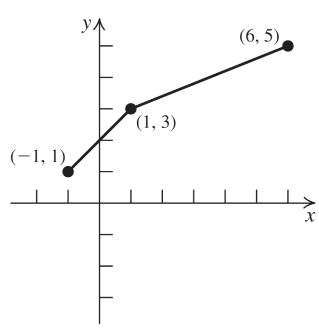

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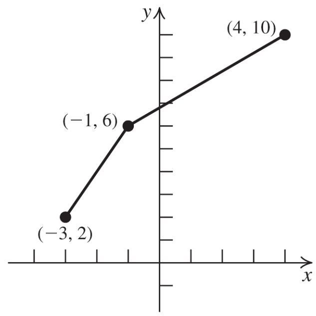

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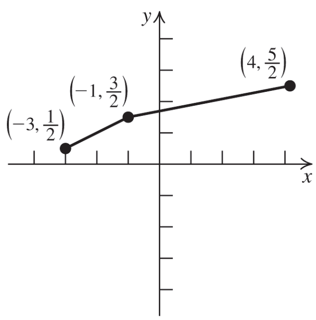

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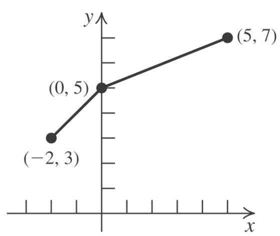

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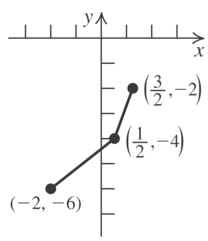

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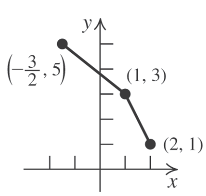

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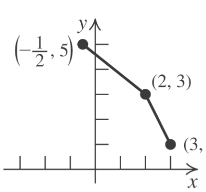
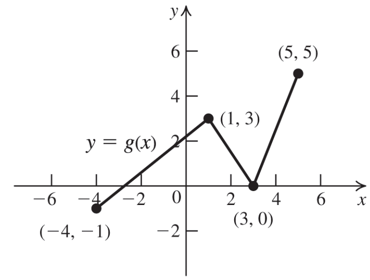
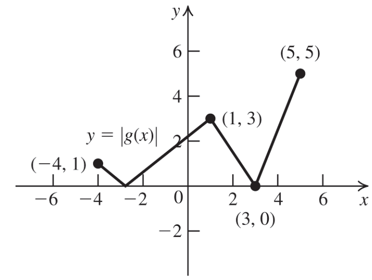


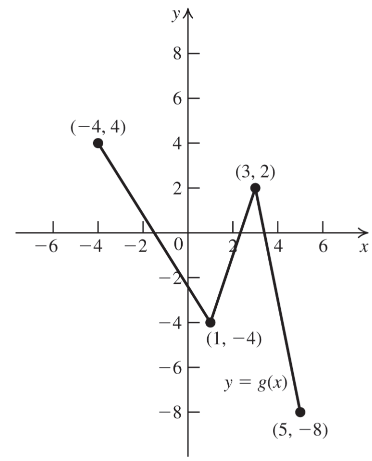
109. a.



b.



110. a.



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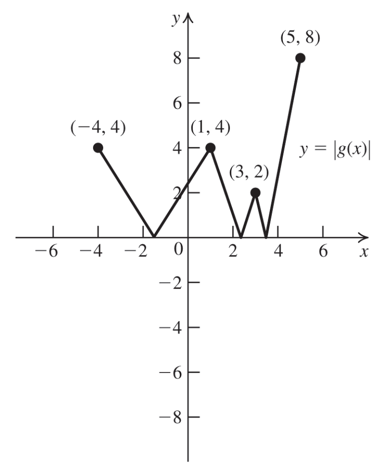
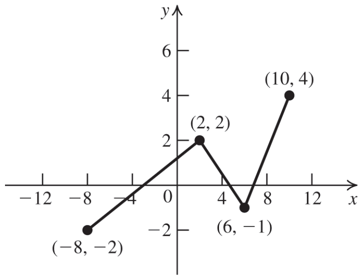
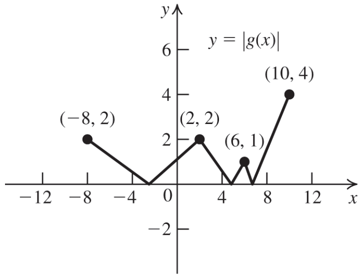


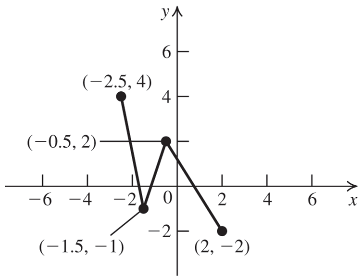
111. a.



b.



112. a.



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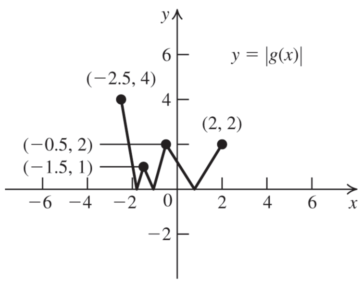
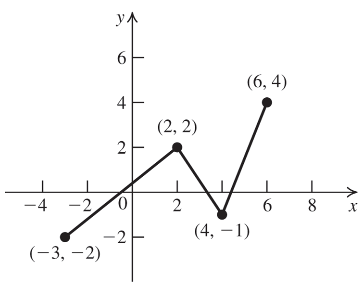


113. a.



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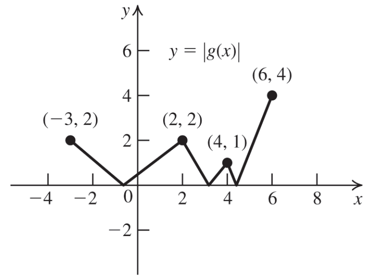
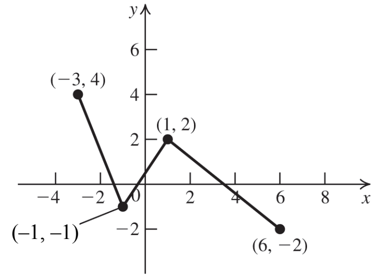


114. a.



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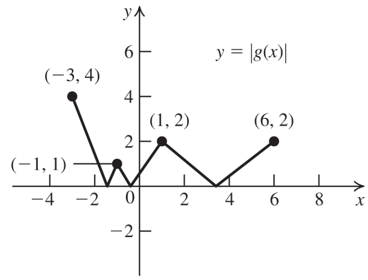
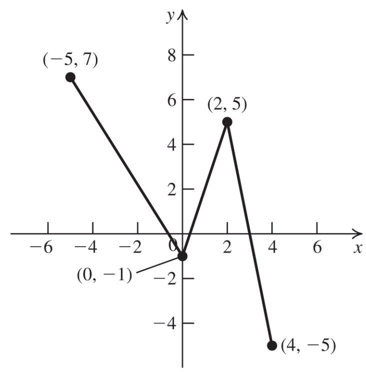


115. a.



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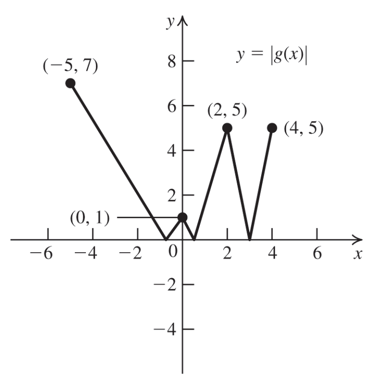
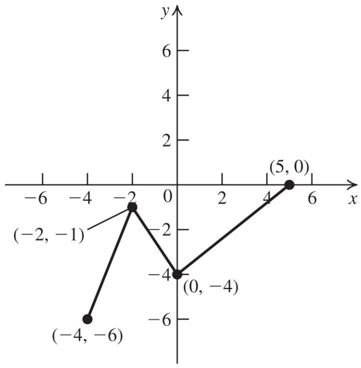


116. a.



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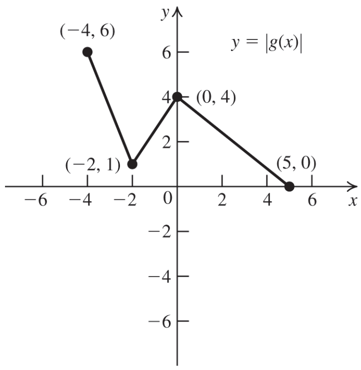
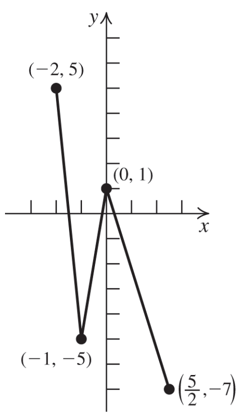


117. a.



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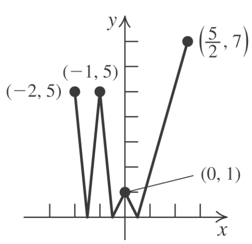
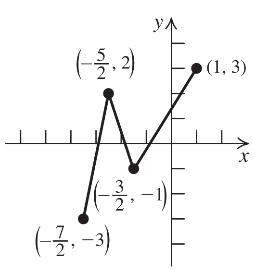


118. a.



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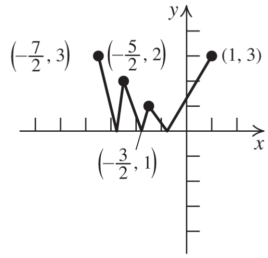


Applying the Concepts

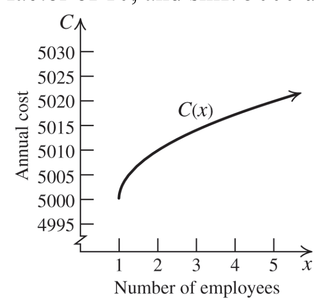
119. $g(x) = f(x) + 800$

120. $h(x) = 1.05f(x)$

121. $p(x) = 1.02\left(f(x) + 500\right)$

122. $g(x) = \begin{cases} 1.1f(x) & \text{if } f(x) < 30,000 \\ 1.02f(x) & \text{if } f(x) \geq 30,000 \end{cases}$

123. a. Shift one unit right, stretch vertically by a factor of 10, and shift 5000 units up.



b. $C(400) = 5000 + 10\sqrt{400 - 1} = \$5199.75$

124. a. For the center of the artery, $R = 3$ mm and $r = 0$.

$v = 1000\left(3^2 - 0^2\right) = 9000$ mm/minute

b. For the inner linings of the artery, $R = 3$ mm and $r = 3$ mm

$v = 1000\left(3^2 - 3^2\right) = 0$ mm/minute

c. Midway between the center and the inner linings, $R = 3$ mm and $r = 1.5$ mm

$v = 1000\left(3^2 - 1.5^2\right) = 6750$ mm/minute

125. a. For the center of the artery, $R = 2$ mm and $C = 750$.

$v = 750\left(2^2 - 0^2\right) = 3000$ mm/minute

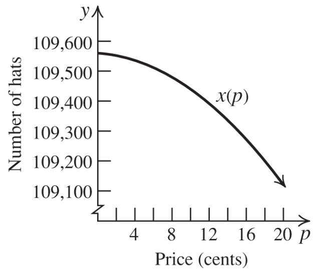
b. For the inner linings of the artery, $R = 2$ mm and $r = 2$ mm

$v = 750\left(2^2 - 2^2\right) = 0$ mm/minute

c. Midway between the center and the inner linings, $R = 2$ mm and $r = 1$ mm

$v = 750\left(2^2 - 1^2\right) = 2250$ mm/minute

126. a. Shift one unit left, reflect across the x-axis, and shift up 109,561 units.



b. $69{,}160 = 109{,}561 - (p+1)^2$

$40{,}401 = (p+1)^2$

$201 = p + 1 \Rightarrow p = 200¢ = \$2.00$

c. $0 = 109{,}561 - (p+1)^2$

$109{,}561 = (p+1)^2$

$331 = p + 1 \Rightarrow p = 330¢ = \$3.30$

127. Write $R(p)$ in the form $-3(p-h)^2 + k$:

$R(p) = -3p^2 + 600p = -3(p^2 - 200p)$

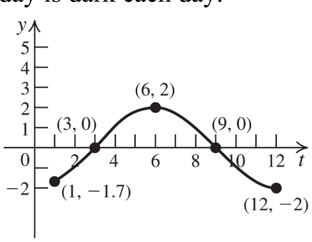
Complete the square

$= -3(p^2 - 200p + 10{,}000) + 30{,}000$

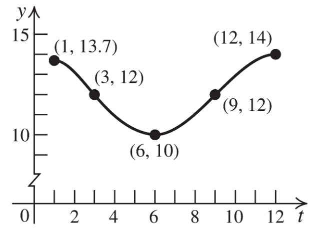
$= -3(p - 100)^2 + 30{,}000$

To graph $R(p)$, shift the graph of $y = p^2$ 100 units to the right, stretch by a factor of 3, reflect about the x-axis, and shift by 30,000 units up.

128. The first coordinate gives the month; the second coordinate gives the hours of daylight. From March to September, there is daylight more than half of the day each day. From September to March, more than half of the day is dark each day.



129. The graph shows the number of hours of darkness.

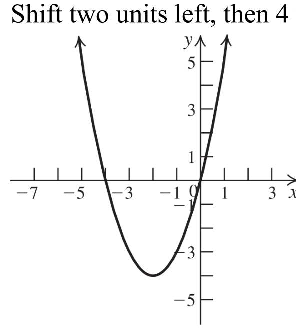


Beyond the Basics

130. The graph is shifted one unit right, then reflected about the x-axis, and finally reflected about the y-axis. The equation is $g(x) = -f(1-x)$.

131. The graph is shifted two units right and then reflected about the y-axis.
The equation is $g(x) = f(-2-x)$.

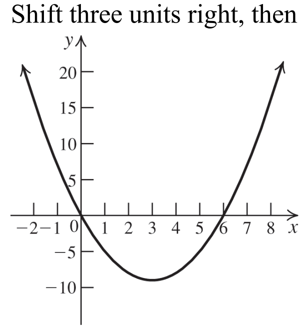
132. Shift two units left, then 4 units down.



133. $f(x) = x^2 - 6x = \left(x^2 - 6x + 9\right) - 9$

$= (x-3)^2 - 9$

Shift three units right, then 9 units down.



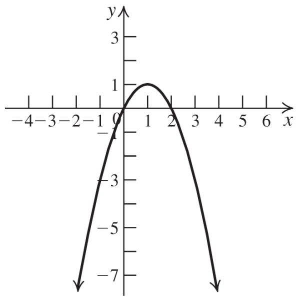
134. $f(x) = -x^2 + 2x = -\left(x^2 - 2x + 1\right) + 1$

$= -(x-1)^2 + 1$

Shift one unit right, reflect about the x-axis, then shift one unit up.

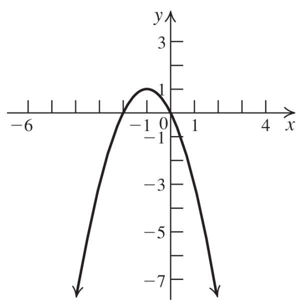
(*continued on next page*)

(*continued*)



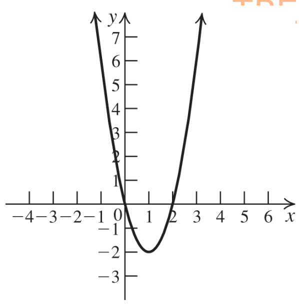
135. $f(x) = -x^2 - 2x = -\left(x^2 + 2x + 1\right) + 1$
$= -(x+1)^2 + 1$

Shift one unit left, reflect about the x-axis, then shift one unit up.



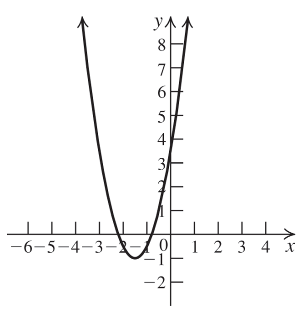
136. $f(x) = 2x^2 - 4x = 2\left(x^2 - 2x + 1\right) - 2$
$= 2(x-1)^2 - 2$

Shift one unit right, stretch vertically by a factor of 2, then shift two units down.



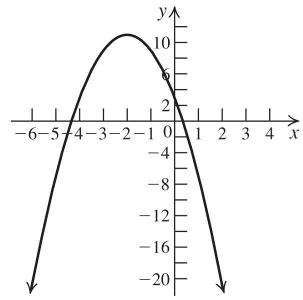
137. $f(x) = 2x^2 + 6x + 3.5$
$= 2\left(x^2 + 3x\right) + 3.5$
$= 2\left(x^2 + 3x + 2.25\right) + 3.5 - 2(2.25)$
$= 2(x+1.5)^2 - 1$

Shift 1.5 units left, stretch vertically by a factor of 2, then shift one unit down.



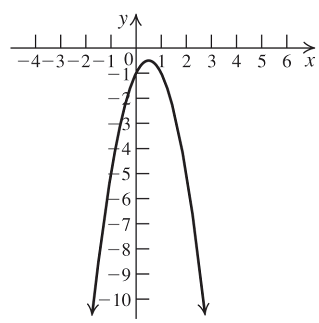
138. $f(x) = -2x^2 - 8x + 3 = -2\left(x^2 + 4x\right) + 3$
$= -2\left(x^2 + 4x + 4\right) + 3 + 2(4)$
$= -2(x+2)^2 + 11$

Shift two units left, stretch vertically by a factor of 2, reflect across the x-axis, then shift eleven units up.



139. $f(x) = -2x^2 + 2x - 1 = -2\left(x^2 - x\right) - 1$
$= -2\left(x^2 - x + 0.25\right) - 1 + 2(0.25)$
$= -2\left(x^2 - x + 0.25\right) - 0.5$
$= -2(x-0.5)^2 - 0.5$

Shift 0.5 unit right, stretch vertically by a factor of 2, reflect across the x-axis, then shift 0.5 unit down.



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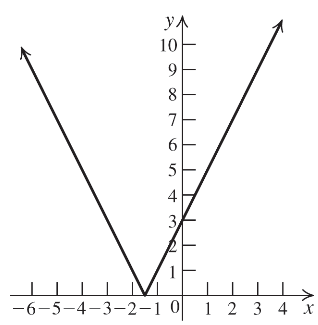
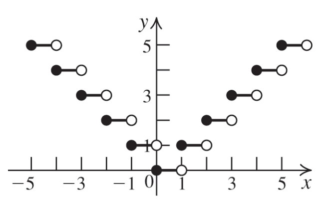


141.



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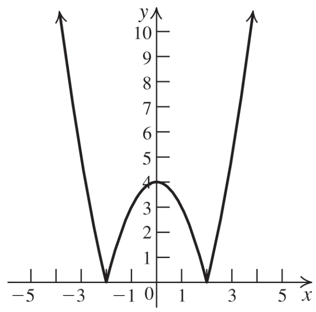

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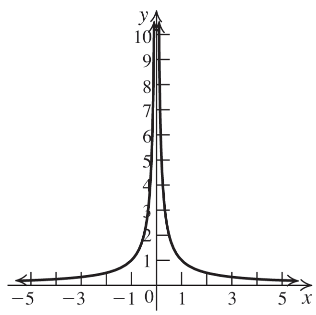
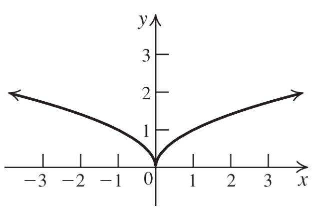
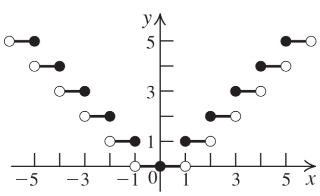


144.



145.



Critical Thinking/Discussion/Writing

146. a. $y = f(x+2)$ is the graph of $y = f(x)$ shifted two units left. So the x-intercepts are –1 – 2 = –3, 0 – 2 = –2, and 2 – 2 = 0.

b. $y = f(x-2)$ is the graph of $y = f(x)$ shifted two units right. So the x-intercepts are –1 + 2 = 1, 0 + 2 = 2, and 2 + 2 = 4.

c. $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis. The x-intercepts are the same, –1, 0, 2.

d. $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis. The x-intercepts are the opposites, 1, 0, –2.

e. $y = f(2x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of 1/2. The x-intercepts are $-\frac{1}{2}$, 0, 1.

f. $y = f\left(\frac{1}{2}x\right)$ is the graph of $y = f(x)$ stretched horizontally by a factor of 2. The x-intercepts are –2, 0, 4.

147. a. $y = f(x)+2$ is the graph of $y = f(x)$ shifted two units up. The y-intercept is 2 + 2 = 4.

b. $y = f(x)-2$ is the graph of $y = f(x)$ shifted two units down. The y-intercept is 2 – 2 = 0.

c. $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis. The y-intercept is the opposite, –2.

d. $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis. The y-intercept is the same, 2.

e. $y = 2f(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 2. The y-intercept is 4.

f. $y = \frac{1}{2}f(x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of 1/2. The y-intercept is 1.

148. a. $y = f(x+2)$ is the graph of $y = f(x)$ shifted two units left. The domain is $[-1-2, 3-2] = [-3, 1]$. The range is the same, $[-2, 1]$.

b. $y = f(x)-2$ is the graph of $y = f(x)$ shifted two units down. The domain is the same, $[-1, 3]$. The range is $[-2-2, 1-2] = [-4, -1]$.

c. $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis. The domain is the same, $[-1, 3]$. The range is the opposite, $[-1, 2]$.

d. $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis. The domain is the opposite, $[-3, 1]$. The range is the same, $[-2, 1]$.

e. $y = 2f(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 2. The domain is the same, $[-1, 3]$. The range is $[2(-2), 2(1)] = [-4, 2]$.

f. $y = \frac{1}{2}f(x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of 1/2. The domain is the same, $[-1, 3]$. The range is $\left[\frac{1}{2}(-2), \frac{1}{2}(1)\right] = \left[-1, \frac{1}{2}\right]$.

149. a. $y = f(x+2)$ is the graph of $y = f(x)$ shifted two units left. So the relative maximum is at $x = 1 - 2 = -1$, and the relative minimum is at $x = 2 - 2 = 0$.

b. $y = f(x) - 2$ is the graph of $y = f(x)$ shifted two units down. The locations of the relative maximum and minimum do not change. Relative maximum at $x = 1$, relative minimum at $x = 2$.

c. $y = -f(x)$ is the graph of $y = f(x)$ reflected across the x-axis. The relative maximum and relative minimum switch. The relative maximum occurs at $x = 2$, and the relative minimum occurs at $x = 1$.

d. $y = f(-x)$ is the graph of $y = f(x)$ reflected across the y-axis. The relative maximum and relative minimum occur at their opposites. The relative maximum occurs at $x = -1$, and the relative minimum occurs at $x = -2$.

e. $y = 2f(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 2. The locations of the relative maximum and minimum do not change. Relative maximum at $x = 1$, relative minimum at $x = 2$.

f. $y = \frac{1}{2}f(x)$ is the graph of $y = f(x)$ compressed horizontally by a factor of 1/2. The locations of the relative maximum and minimum do not change. Relative maximum at $x = 1$, relative minimum at $x = 2$.

Active Learning

150. a.–b. Refer to the app using the QR code in your text.

Getting Ready for the Next Section

GR1. $(5x^2 + 5x + 7) + (x^2 + 9x - 4) = 6x^2 + 14x + 3$

GR2. $(x^2 + 2x) + (6x^3 - 2x + 5) = 6x^3 + x^2 + 5$

GR3. $(5x^2 + 6x - 2) - (3x^2 - 9x + 1) = 2x^2 + 15x - 3$

GR4. $(x^3 + 2) - (2x^3 + 5x - 3) = -x^3 - 5x + 5$

GR5. $(x-2)(x^2 + 2x + 4)$
$= x^3 + 2x^2 + 4x - 2x^2 - 4x - 8$
$= x^3 - 8$

GR6. $(x^2 + x + 1)(x^2 - x + 1)$
$= x^4 - x^3 + x^2 + x^3 - x^2 + x + x^2 - x + 1$
$= x^4 + x^2 + 1$

GR7. $f(x) = \dfrac{2x - 3}{x^2 - 5x + 6}$
The function is not defined when the denominator is zero.
$x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x = 2, 3$
The domain is $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$.

GR8. $f(x) = \dfrac{x - 2}{x^2 - 4}$
The function is not defined when the denominator is zero.
$x^2 - 4 = 0 \Rightarrow (x+2)(x-2) = 0 \Rightarrow x = -2, 2$
The domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

GR9. $f(x) = \sqrt{2x - 3}$
The function is defined only if $2x - 3 \geq 0$.
$2x - 3 \geq 0 \Rightarrow 2x \geq 3 \Rightarrow x \geq \dfrac{3}{2}$
The domain is $\left[\dfrac{3}{2}, \infty\right)$.

GR10. $f(x)=\dfrac{1}{\sqrt{5-2x}}$

The function is defined only if $5-2x>0$.

$5-2x>0 \Rightarrow -2x>-5 \Rightarrow x<\dfrac{5}{2}$

The domain is $\left(-\infty, \dfrac{5}{2}\right)$.

GR11. $\dfrac{x-1}{x-10}<0$

First, solve $x-1=0 \Rightarrow x=1$ and $x-10=0 \Rightarrow x=10$.
So the intervals are $(-\infty, 1), (1, 10)$, and $(10, \infty)$.

Interval	Test point	Value of $\dfrac{x-1}{x-10}$	Result
$(-\infty, 1)$	0	$\frac{1}{10}$	+
$(1, 10)$	5	$-\frac{4}{5}$	–
$(10, \infty)$	15	$\frac{14}{5}$	+

Note that the fraction is undefined if $x = 10$.
The solution set is $(1, 10)$.

GR12. $\dfrac{-3}{(1-x)^2}>0$

Set the denominator equal to zero and solve for x.

$(1-x)^2=0 \Rightarrow x=1$

The intervals are $(-\infty, 1)$ and $(1, \infty)$.

Interval	Test point	Value of $\dfrac{-3}{(1-x)^2}$	Result
$(-\infty, 1)$	0	-3	–
$(1, \infty)$	2	-3	–

There is no solution. The solution set is $\varnothing$.

GR13. $\dfrac{-2x+8}{x^2+1}\le 0$

Set the numerator and denominator equal to zero and solve for x.
$-2x+8=0 \Rightarrow -2x=-8 \Rightarrow x=4$

$x^2+1=0$ has no real solution.

The intervals are $(-\infty, 4]$ and $[4, \infty)$.

Interval	Test point	Value of $\dfrac{-2x+8}{x^2+1}$	Result
$(-\infty, 4]$	0	8	+
$[4, \infty)$	5	$-\frac{1}{13}$	–

The solution set is $[4, \infty)$.

GR14. $\dfrac{(x-3)(x-1)}{(x-5)(x+1)}\ge 0$

Set the numerator and denominator equal to zero and solve for x.

$(x-3)(x-1)=0 \Rightarrow x=3, 1$

$(x-5)(x+1)=0 \Rightarrow x=5, -1$

The intervals are $(-\infty, -1)$, $(-1, 1]$, $[1, 3]$, $[3, 5)$, and $(5, \infty)$. Note that the original fraction is undefined for $x=-1$ and $x=5$, so those points are not included.

Interval	Test point	Value of $\dfrac{(x-3)(x-1)}{(x-5)(x+1)}$	Result
$(-\infty, -1)$	-2	$\frac{15}{7}$	+
$(-1, 1]$	0	$-\frac{3}{5}$	–
$[1, 3]$	2	$\frac{1}{9}$	+
$[3, 5)$	4	$-\frac{3}{5}$	–
$(5, \infty)$	6	$\frac{15}{7}$	+

The solution set is $(-\infty, -1)\cup[1, 3]\cup(5, \infty)$.

1.8 Combining Functions; Composite Functions

Practice Problems

1. $f(x)=3x-1, \quad g(x)=x^2+2$

$$\begin{aligned}(f+g)(x)&=f(x)+g(x)\\&=(3x-1)+(x^2+2)\\&=x^2+3x+1\end{aligned}$$

$$\begin{aligned}(f-g)(x)&=f(x)-g(x)\\&=(3x-1)-(x^2+2)\\&=-x^2+3x-3\end{aligned}$$

(*continued on next page*)

(*continued*)

$(fg)(x) = f(x)\cdot g(x)$
$= (3x-1)(x^2+2)$
$= 3x^3 - x^2 + 6x - 2$

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x-1}{x^2+2}$

2. $f(x) = \sqrt{x-1},\ g(x) = \sqrt{3-x}$

The domain of f is $[1, \infty)$ and the domain of g is $(-\infty, 3]$. The intersection of D_f and D_g, $D_f \cap D_g = [1, 3]$.

The domain of fg is [1, 3].

For the domain of $\dfrac{f}{g}$, we must exclude from $D_f \cap D_g$ those values where $g(x) = \sqrt{3-x} = 0$. Thus, the domain of $\dfrac{f}{g}$ is [1, 3).

For the domain of $\dfrac{g}{f}$, we must exclude from $D_f \cap D_g$ those values where $f(x) = \sqrt{x-1} = 0$. Thus, the domain of $\dfrac{g}{f}$ is (1, 3].

3. $f(x) = -5x,\ g(x) = x^2 + 1$

a. $(f \circ g)(0) = f(g(0)) = -5(g(0))$
$= -5(0^2+1) = -5$

b. $(g \circ f)(0) = g(f(0)) = [f(0)]^2 + 1$
$= (-5\cdot 0)^2 + 1 = 1$

4. $f(x) = 2 - x,\ g(x) = 2x^2 + 1$

a. $(g \circ f)(x) = g(f(x)) = 2(f(x))^2 + 1$
$= 2(2-x)^2 + 1$
$= 2(4 - 4x + x^2) + 1$
$= 8 - 8x + 2x^2 + 1$
$= 2x^2 - 8x + 9$

b. $(f \circ g)(x) = f(g(x)) = 2 - g(x)$
$= 2 - (2x^2+1) = 1 - 2x^2$

c. $(g \circ g)(x) = g(g(x)) = 2(g(x))^2 + 1$
$= 2(2x^2+1)^2 + 1$
$= 2(4x^4 + 4x^2 + 1) + 1$
$= 8x^4 + 8x^2 + 3$

5. $f(x) = \sqrt{x+1},\ g(x) = \dfrac{2}{x-3}$

Let $A = \{x \mid g(x) \text{ is defined}\}$.

$g(x)$ is not defined if $x = 3$, so $A = (-\infty, 3) \cup (3, \infty)$.

Let $B = \{x \mid f(g(x)) \text{ is defined}\}$.

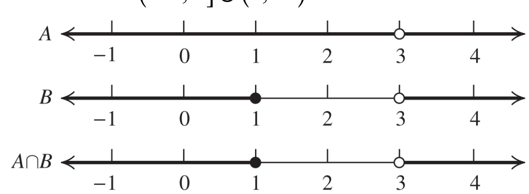
$f(g(x)) = \sqrt{g(x)+1} = \sqrt{\dfrac{2}{x-3}+1}$
$= \sqrt{\dfrac{2+x-3}{x-3}} = \sqrt{\dfrac{x-1}{x-3}}$

$f(g(x))$ is not defined if $x = 3$ or if $\dfrac{x-1}{x-3} < 0$.

$x - 1 = 0 \Rightarrow x = 1$

Interval	Test point	Value of $\dfrac{x-1}{x-3}$	Result
$(-\infty, 1]$	0	$\frac{1}{3}$	+
$[1, 3)$	2	−1	−
$(3, \infty)$	4	3	+

$f(g(x))$ is not defined for $[1, 3)$, so $B = (-\infty, 1] \cup (3, \infty)$.



The domain of $f \circ g$ is $A \cap B = (-\infty, 1] \cup (3, \infty)$.

6. $f(x) = \sqrt{x};\ \ g(x) = 3 - x$

$f \circ g = \sqrt{g(x)} = \sqrt{3-x}$

$f \circ g$ is not defined if $3 - x < 0 \Rightarrow 3 < x$ or $x > 3$. $f \circ g$ is defined for $x < 3$. Thus, the domain of $f \circ g$ is $(-\infty, 3]$.

7. $f(x)=\sqrt{x-1},\quad g(x)=\sqrt{4-x^2}$

a. $(f\circ g)(x)=f(g(x))=\sqrt{g(x)-1}$
$=\sqrt{\sqrt{4-x^2}-1}$

The function $g(x)=\sqrt{4-x^2}$ is defined for $4-x^2\geq 0\Rightarrow x^2\leq 4\Rightarrow -2\leq x\leq 2.$ So, $A=[-2,\ 2].$

The function $f(g(x))$ is defined for
$\sqrt{4-x^2}-1\geq 0\Rightarrow\sqrt{4-x^2}\geq 1\Rightarrow$
$4-x^2\geq 1\Rightarrow -x^2\geq -3\Rightarrow x^2\leq 3\Rightarrow$
$-\sqrt{3}\leq x\leq\sqrt{3}$

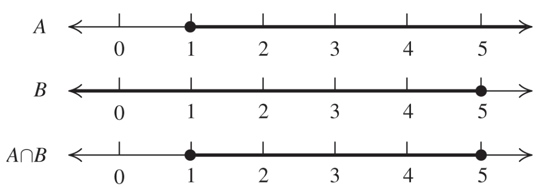
So, $B=\left[-\sqrt{3},\ \sqrt{3}\right].$

The domain of $f\circ g$ is
$A\cap B=\left[-\sqrt{3},\ \sqrt{3}\right].$

b. $(g\circ f)(x)=g(f(x))=\sqrt{4-(f(x))^2}$
$=\sqrt{4-(x-1)}=\sqrt{5-x}$

The function $f(x)=\sqrt{x-1}$ is defined for $x-1\geq 0\Rightarrow x\geq 1.$ So, $A=[1,\ \infty).$

The function $g(f(x))$ is defined for $5-x\geq 0\Rightarrow 5\geq x,$ or $x\leq 5.$ So, $B=(-\infty,\ 5].$



The domain of $g\circ f$ is $A\cap B=[1,\ 5].$

8. $H(x)=\dfrac{1}{\sqrt{2x^2+1}},\quad g(x)=\sqrt{2x^2+1}$

To get $f(x)$ from the defining equation for *H*, replace the letter *H* with *f* and then replace the expression chosen for $g(x)$ with *x*. Thus, $H(x)$ becomes $f(x)=\dfrac{1}{x}.$

$H(x)=(f\circ g)(x)=f(g(x))$
$=\dfrac{1}{g(x)}=\dfrac{1}{\sqrt{2x^2+1}}$

9. $f(x)=2-x^2;\ g(x)=3x-1$

First compute the component related to the average rate of change of the inner function.
$g(-1)=3(-1)-1=-4$ and
$g(2)=3(2)-1=5.$

ARC of g from -1 to $2=\dfrac{g(2)-g(-1)}{2-(-1)}$
$=\dfrac{5-(-4)}{3}=\dfrac{9}{3}=3$

To compute the component related to the average rate of change of the outer function, we need to use the range for *f* as from $g(-1)=-4$ to $g(2)=5.$ We have
$f(-4)=2-(-4)^2=-14$ and
$f(5)=2-5^2=-23.$

ARC of f from $g(-1)=-4$ to $g(2)=5$
$=\dfrac{f(5)-f(-4)}{5-(-4)}=\dfrac{-23-(-14)}{9}=\dfrac{-9}{9}=-1$

Finally, we can compute the average rate of change of the composite function as
ARC of $f\circ g$ from -1 to 2
$=$ (ARC of f from $g(-1)=-4$ to $g(2)=5$)
$\cdot$ (ARC of g from -1 to 2)
$=-1\cdot 3=-3$

10. a. $\text{CBR}=0.32(11.55+144.84g)$

Because *g* is constant over the whole time period, the woman will burn the calories at the same rate given by the equation.
$\text{CBR}=0.32(11.55+144.84\cdot 0.03)\approx 5.09$

The exercise lasts 1 hour, so she will burn $60\cdot 5.09\approx 305$ calories.

b. Express *g* as a piecewise function:

$$g(t)=\begin{cases}0.02 & \text{if } 0\leq t\leq 10\\ 0.05 & \text{if } 10<t\leq 40\\ 0.04 & \text{if } 40<t\leq 60\end{cases}$$

On each of the intervals, the woman will burn calories at a different rate. That rate can be computed based on the main formula as follows.

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(*continued*)

$$\text{CBR}(t)=\begin{cases}0.32(11.55+144.84\cdot 0.02)\approx 4.62 \\ \quad \text{if } 0\le t\le 10 \\ 0.32(11.55+144.84\cdot 0.05)\approx 6.01 \\ \quad \text{if } 10< t\le 40 \\ 0.32(11.55+144.84\cdot 0.04)\approx 5.55 \\ \quad \text{if } 40< t\le 60\end{cases}$$

Taking into account the length of the intervals, estimate the total number of burned calories:

$4.62(10)+6.01(30)+5.55(20)\approx 338$ cal.

11. a. $A=f(g(t))=f(g(3))=\pi(g(3))^3$
$=\pi(3t)^2=9\pi t^2$

b. $A=9\pi t^2=9\pi(6)^2=324\pi$

The area covered by the oil slick is $324\pi\approx 1018$ square miles.

12. a. $r(x)=x-4500$

b. $d(x)=x-0.06x=0.94x$

c. i. $(r\circ d)(x)=r(dx)=d(x)-4500$
$=0.94x-4500$

This represents the price of a car when the dealer's discount is applied first and then the manufacturer's rebate is applied.

ii. $(d\circ r)(x)=d(r(x))=d(x-4500)$
$=0.94(x-4500)$
$=0.94x-4230$

This represents the price of a car when the manufacturer's rebate is applied first and then the dealer's discount is applied.

d. $(d\circ r)(x)-(r\circ d)(x)$
$=(0.94x-4230)-(0.94x-4500)$
$=270$

This equation shows that any car, regardless of its price, will cost \$270 more if you apply the \$4500 rebate first and then the 6% discount.

Concepts and Vocabulary

1. $(f\cdot g)(x)=\underline{f(x)\cdot g(x)}$.

2. The domain of the function $f+g$ consists of those values of x that are <u>common</u> to the domains of f and g.

3. The composition of the function f with the function g is written as $f\circ g$ and is defined by $f\circ g(x)=\underline{f(g(x))}$.

4. The domain of the composite function $f\circ g$ consists of those values of x in the domain of g for which $g(x)$ <u>is in the domain of f</u>.

5. False. For example, if $f(x)=2x$ and $g(x)=x^2$, then $(f\circ g)(x)=f(g(x))=f(x^2)=2x^2$, while $(g\circ f)(x)=g(f(x))=g(2x)=4x^2$.

6. True

7. False. The domain of $f\cdot g$ may include $g(x)=0$, but the domain of $\frac{f}{g}$ cannot include $g(x)=0$.

8. True

Building Skills

9. $(f+g)(-2)=f(-2)+g(-2)=1+2=3$

10. $(f+g)(2)=f(2)+g(2)=-2+(-1)=-3$

11. $(f-g)(4)=f(4)-g(4)=-2-1=-3$

12. $(f-g)(-1)=f(-1)-g(-1)=1-(-4)=5$

13. $(f\cdot g)(-1)=f(-1)\cdot g(-1)=1\cdot(-4)=-4$

14. $(f\cdot g)(2)=f(2)\cdot g(2)=-2\cdot(-1)=2$

15. $\left(\frac{f}{g}\right)(-2)=\frac{f(-2)}{g(-2)}=\frac{1}{2}$

16. $\left(\frac{f}{g}\right)(2)=\frac{f(2)}{g(2)}=\frac{-2}{-1}=2$

17. $(f\circ g)(1)=f(g(1))=f(-2)=1$

18. $(g\circ f)(1)=g(f(1))=g(-2)=2$

19. $(f\circ g)(-3)=f(g(-3))=f(0)=0$

20. $(g\circ f)(-3)=g(f(-3))=g(1)=-2$

21. a. $(f+g)(-1)=f(-1)+g(-1)$
$=2(-1)+-(-1)=-2+1=-1$

b. $(f-g)(0) = f(0) - g(0) = 2(0) - (-0) = 0$

c. $(f \cdot g)(2) = f(2) \cdot g(2) = 2(2) \cdot (-2) = -8$

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{2(1)}{-1} = -2$

22. a. $(f+g)(-1) = f(-1) + g(-1)$
$= \left(1-(-1)^2\right) + (-1+1) = 0$

b. $(f-g)(0) = f(0) - g(0)$
$= (1-0^2) - (0+1) = 0$

c. $(f \cdot g)(2) = f(2) \cdot g(2)$
$= (1-2^2) \cdot (2+1) = -9$

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{1-1^2}{1+1} = 0$

23. a. $(f+g)(-1) = f(-1) + g(-1)$
$= \frac{1}{\sqrt{-1+2}} + (2(-1)+1) = 0$

b. $(f-g)(0) = f(0) - g(0)$
$= \frac{1}{\sqrt{0+2}} - (2(0)+1) = \frac{\sqrt{2}}{2} - 1$

c. $(f \cdot g)(2) = f(2) \cdot g(2)$
$= \frac{1}{\sqrt{2+2}} \cdot (2(2)+1) = \frac{5}{2}$

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{\frac{1}{\sqrt{1+2}}}{2(1)+1} = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{9}$

24. a. $(f+g)(-1) = f(-1) + g(-1)$
$= \frac{-1}{(-1)^2 - 6(-1) + 8} + (3-(-1))$
$= -\frac{1}{15} + 4 = \frac{59}{15}$

b. $(f-g)(0) = f(0) - g(0)$
$= \frac{0}{0^2 - 6(0) + 8} - (3-0) = -3$

c. $(f \cdot g)(2) = f(2) \cdot g(2)$
$= \frac{2}{2^2 - 6(2) + 8} \cdot (3-2) = \frac{2}{0} \cdot 1 \Rightarrow$
the product does not exist.

d. $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{\frac{1}{1^2 - 6(1) + 8}}{3-1} = \frac{\frac{1}{3}}{2} = \frac{1}{6}$

25. a. $f + g = x^2 + x - 3$; domain: $(-\infty, \infty)$

b. $f - g = x - 3 - x^2 = -x^2 + x - 3$;
domain: $(-\infty, \infty)$

c. $f \cdot g = (x-3)x^2 = x^3 - 3x^2$;
domain: $(-\infty, \infty)$

d. $\frac{f}{g} = \frac{x-3}{x^2}$; domain: $(-\infty, 0) \cup (0, \infty)$

e. $\frac{g}{f} = \frac{x^2}{x-3}$; domain: $(-\infty, 3) \cup (3, \infty)$

26. a. $f + g = x^2 + 2x - 1$; domain: $(-\infty, \infty)$

b. $f - g = 2x - 1 - x^2 = -x^2 + 2x - 1$
domain: $(-\infty, \infty)$

c. $f \cdot g = (2x-1)x^2 = 2x^3 - x^2$;
domain: $(-\infty, \infty)$

d. $\frac{f}{g} = \frac{2x-1}{x^2}$; domain: $(-\infty, 0) \cup (0, \infty)$

e. $\frac{g}{f} = \frac{x^2}{2x-1}$; domain: $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

27. a. $f + g = \left(x^3 - 1\right) + \left(2x^2 + 5\right) = x^3 + 2x^2 + 4$
domain: $(-\infty, \infty)$

b. $f - g = \left(x^3 - 1\right) - \left(2x^2 + 5\right) = x^3 - 2x^2 - 6$
domain: $(-\infty, \infty)$

c. $f \cdot g = \left(x^3 - 1\right)\left(2x^2 + 5\right)$
$= 2x^5 + 5x^3 - 2x^2 - 5$
domain: $(-\infty, \infty)$

d. $\frac{f}{g} = \frac{x^3 - 1}{2x^2 + 5}$; domain: $(-\infty, \infty)$

e. $\frac{g}{f} = \frac{2x^2 + 5}{x^3 - 1}$; domain: $(-\infty, 1) \cup (1, \infty)$

28. a. $f + g = \left(x^2 - 4\right) + \left(x^2 - 6x + 8\right)$
$= 2x^2 - 6x + 4$;
domain: $(-\infty, \infty)$

b. $f-g=\left(x^2-4\right)-\left(x^2-6x+8\right)=6x-12$
domain: $(-\infty,\infty)$

c. $f\cdot g=\left(x^2-4\right)\left(x^2-6x+8\right)$
$=x^4-6x^3+4x^2+24x-32$
domain: $(-\infty,\infty)$

d. $\dfrac{f}{g}=\dfrac{x^2-4}{x^2-6x+8}=\dfrac{(x+2)(x-2)}{(x-2)(x-4)}=\dfrac{x+2}{x-4}$
The denominator of the original fraction equals 0 if $x=2$ or $x=4$, so the domain is $(-\infty,2)\cup(2,4)\cup(4,\infty)$.

e. $\dfrac{g}{f}=\dfrac{x^2-6x+8}{x^2-4}=\dfrac{(x-2)(x-4)}{(x-2)(x+2)}=\dfrac{x-4}{x+2}$
The denominator of the original fraction equals 0 if $x=-2$ or $x=2$, so the domain is $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$.

29. a. $f+g=2x+\sqrt{x}-1$; domain: $[0,\infty)$

b. $f-g=2x-\sqrt{x}-1$; domain: $[0,\infty)$

c. $f\cdot g=(2x-1)\sqrt{x}=2x\sqrt{x}-\sqrt{x}$
domain: $[0,\infty)$

d. $\dfrac{f}{g}=\dfrac{2x-1}{\sqrt{x}}$; domain: $(0,\infty)$

e. $\dfrac{g}{f}=\dfrac{\sqrt{x}}{2x-1}$; the numerator is defined only for $x\ge 0$, while the denominator = 0 when $x=\dfrac{1}{2}$, so the domain is $\left[0,\dfrac{1}{2}\right)\cup\left(\dfrac{1}{2},\infty\right)$.

30. a. $f+g=x-1+\sqrt{x}$; domain: $[0,\infty)$

b. $f-g=x-1-\sqrt{x}$; domain: $[0,\infty)$

c. $f\cdot g=(x-1)\sqrt{x}$; domain: $[0,\infty)$

d. $\dfrac{f}{g}=\dfrac{x-1}{\sqrt{x}}$; domain: $(0,\infty)$

e. $\dfrac{g}{f}=\dfrac{\sqrt{x}}{x-1}$; the numerator is defined only for $x\ge 0$, while the denominator = 0 when $x=1$, so the domain is $[0,1)\cup(1,\infty)$.

31. a. $f+g=x-6+\sqrt{x-3}$; domain: $[3,\infty)$

b. $f-g=x-6-\sqrt{x-3}$; domain: $[3,\infty)$

c. $f\cdot g=(x-6)\sqrt{x-3}$; domain: $[3,\infty)$

d. $\dfrac{f}{g}=\dfrac{x-6}{\sqrt{x-3}}$; domain: $(3,\infty)$

e. $\dfrac{g}{f}=\dfrac{\sqrt{x-3}}{x-6}$; the numerator is defined only for $x\ge 3$, while the denominator = 0 when $x=6$, so the domain is $[3,6)\cup(6,\infty)$.

32. a. $f+g=x+2+\sqrt{1-x}$; domain: $(-\infty,1]$

b. $f-g=x+2-\sqrt{1-x}$; domain: $(-\infty,1]$

c. $f\cdot g=(x+2)\sqrt{1-x}$; domain: $(-\infty,1]$

d. $\dfrac{f}{g}=\dfrac{x+2}{\sqrt{1-x}}$; domain: $(-\infty,1)$

e. $\dfrac{g}{f}=\dfrac{\sqrt{1-x}}{x+2}$; the numerator is defined only for $x\le 1$, while the denominator = 0 when $x=-2$, so the domain is $(-\infty.-2)\cup(-2,1]$.

33. a. $f+g=1-\dfrac{2}{x+1}+\dfrac{1}{x}$
$=\dfrac{x(x+1)-2x+(x+1)}{x(x+1)}$
$=\dfrac{x^2+x-2x+x+1}{x(x+1)}=\dfrac{x^2+1}{x(x+1)}$
domain: $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$

b. $f-g=1-\dfrac{2}{x+1}-\dfrac{1}{x}$
$=\dfrac{x(x+1)-2x-(x+1)}{x(x+1)}$
$=\dfrac{x^2+x-2x-x-1}{x(x+1)}=\dfrac{x^2-2x-1}{x(x+1)}$
domain: $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$

c. $f\cdot g=\left(1-\dfrac{2}{x+1}\right)\dfrac{1}{x}=\left(\dfrac{x+1-2}{x+1}\right)\dfrac{1}{x}$
$=\left(\dfrac{x-1}{x+1}\right)\dfrac{1}{x}=\dfrac{x-1}{x(x+1)}$
domain: $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$

d. $\dfrac{f}{g}=\dfrac{1-\frac{2}{x+1}}{\frac{1}{x}}=\left(1-\dfrac{2}{x+1}\right)(x)$
$=\left(\dfrac{x+1-2}{x+1}\right)x=\dfrac{x(x-1)}{x+1}$
domain: $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$

e. $\dfrac{g}{f}=\dfrac{\frac{1}{x}}{1-\frac{2}{x+1}}=\dfrac{\frac{1}{x}(x+1)}{\left(1-\frac{2}{x+1}\right)(x+1)}=\dfrac{\frac{x+1}{x}}{x+1-2}$
$=\dfrac{\frac{x+1}{x}}{x-1}=\dfrac{x+1}{x(x-1)}$
The denominator equals zero when $x=0$ or $x=1$, so the domain is
$(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$.

34. a. $f+g=\left(1-\dfrac{1}{x}\right)+\dfrac{1}{x}=1$
Neither *f* nor *g* is defined for $x=0$, so the domain is $(-\infty,0)\cup(0,\infty)$.

b. $f-g=\left(1-\dfrac{1}{x}\right)-\dfrac{1}{x}=1-\dfrac{2}{x}$;
domain: $(-\infty,0)\cup(0,\infty)$.

c. $f\cdot g=\left(1-\dfrac{1}{x}\right)\left(\dfrac{1}{x}\right)=\dfrac{1}{x}-\dfrac{1}{x^2}=\dfrac{x-1}{x^2}$;
domain: $(-\infty,0)\cup(0,\infty)$.

d. $\dfrac{f}{g}=\dfrac{1-\frac{1}{x}}{\frac{1}{x}}=\dfrac{\frac{x-1}{x}}{\frac{1}{x}}=x-1$
Neither *f* nor *g* is defined for $x=0$, so the domain is $(-\infty,0)\cup(0,\infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{1}{x}}{1-\frac{1}{x}}=\dfrac{\frac{1}{x}}{\frac{x-1}{x}}=\dfrac{1}{x-1}$
Neither *f* nor *g* is defined for $x=0$, and *g*/*f* is not defined for $x=1$, so the domain is $(-\infty,0)\cup(0,1)\cup(1,\infty)$.

35. a. $f+g=\dfrac{2}{x+1}+\dfrac{x}{x+1}=\dfrac{2+x}{x+1}$
Neither *f* nor *g* is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

b. $f-g=\dfrac{2}{x+1}-\dfrac{x}{x+1}=\dfrac{2-x}{x+1}$.
domain: $(-\infty,-1)\cup(-1,\infty)$.

c. $f\cdot g=\left(\dfrac{2}{x+1}\right)\left(\dfrac{x}{x+1}\right)=\dfrac{2x}{(x+1)^2}$.
domain: $(-\infty,-1)\cup(-1,\infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{2}{x+1}}{\frac{x}{x+1}}=\dfrac{2}{x}$. Neither *f* nor *g* is defined for $x=-1$, and f/g is not defined for $x=0$, so the domain is $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$.

e. $\dfrac{f}{g}=\dfrac{\frac{x}{x+1}}{\frac{2}{x+1}}=\dfrac{x}{2}$. Neither *f* nor *g* is defined for $x=-1$, so the domain is
$(-\infty,-1)\cup(-1,\infty)$.

36. $f(x)=\dfrac{5x-1}{x+1};\ g(x)=\dfrac{4x+10}{x+1}$

a. $f+g=\dfrac{5x-1}{x+1}+\dfrac{4x+10}{x+1}=\dfrac{9x+9}{x+1}$
$=\dfrac{9(x+1)}{x+1}=9$
Neither *f* nor *g* is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

b. $f-g=\dfrac{5x-1}{x+1}-\dfrac{4x+10}{x+1}=\dfrac{x-11}{x+1}$
Neither *f* nor *g* is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

c. $f\cdot g=\dfrac{5x-1}{x+1}\cdot\dfrac{4x+10}{x+1}=\dfrac{20x^2+46x-10}{x^2+2x+1}$
Neither *f* nor *g* is defined for $x=-1$, so the domain is $(-\infty,-1)\cup(-1,\infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{5x-1}{x+1}}{\frac{4x+10}{x+1}}=\dfrac{5x-1}{4x+10}$
Neither *f* nor *g* is defined for $x=-1$ and f/g is not defined for $x=-5/2$, so the domain is
$\left(-\infty,-\dfrac{5}{2}\right)\cup\left(-\dfrac{5}{2},-1\right)\cup(-1,\infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{4x+10}{x+1}}{\frac{5x-1}{x+1}}=\dfrac{4x+10}{5x-1}$

Neither *f* nor *g* is defined for $x=-1$ and g/f is not defined for $x=1/5$, so the domain is $(-\infty,-1)\cup\left(-1,\frac{1}{5}\right)\cup\left(\frac{1}{5},\infty\right)$.

37. $f(x)=\dfrac{x^2}{x+1};\ g(x)=\dfrac{2x}{x^2-1}$

a. $f+g=\dfrac{x^2}{x+1}+\dfrac{2x}{x^2-1}=\dfrac{x^2(x-1)}{x^2-1}+\dfrac{2x}{x^2-1}$
$=\dfrac{x^3-x^2+2x}{x^2-1}$

f is not defined for $x=-1$, *g* is not defined for $x=\pm1$, and $f+g$ is not defined for either -1 or 1, so the domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

b. $f-g=\dfrac{x^2}{x+1}-\dfrac{2x}{x^2-1}=\dfrac{x^2(x-1)}{x^2-1}-\dfrac{2x}{x^2-1}$
$=\dfrac{x^3-x^2-2x}{x^2-1}=\dfrac{x(x^2-x-2)}{x^2-1}$
$=\dfrac{x(x-2)(x+1)}{(x-1)(x+1)}=\dfrac{x^2-2x}{x-1}$

f is not defined for $x=-1$, *g* is not defined for $x=\pm1$, and $f-g$ is not defined for 1, so the domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

c. $f\cdot g=\dfrac{x^2}{x+1}\cdot\dfrac{2x}{x^2-1}=\dfrac{2x^3}{x^3+x^2-x-1}$

f is not defined for $x=-1$, *g* is not defined for $x=\pm1$, and *fg* is not defined for either -1 or 1, so the domain is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

d. $\dfrac{f}{g}=\dfrac{\frac{x^2}{x+1}}{\frac{2x}{x^2-1}}=\dfrac{x^2}{x+1}\cdot\dfrac{x^2-1}{2x}$
$=\dfrac{x(x-1)}{2}=\dfrac{x^2-x}{2}$

f is not defined for $x=-1$, *g* is not defined for $x=\pm1$, and f/g is not defined for either -1, 0, or 1, so the domain is $(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$.

e. $\dfrac{g}{f}=\dfrac{\frac{2x}{x^2-1}}{\frac{x^2}{x+1}}=\dfrac{2x}{x^2-1}\cdot\dfrac{x+1}{x^2}$
$=\dfrac{2}{x(x-1)}=\dfrac{2}{x^2-x}$

Neither *f* nor *g* is defined for $x=-1$ and g/f is not defined for $x=0$ or $x=1$, so the domain is $(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$.

38. $f(x)=\dfrac{x-3}{x^2-25};\ g(x)=\dfrac{x-3}{x^2+9x+20}$

a. $f+g=\dfrac{x-3}{x^2-25}+\dfrac{x-3}{x^2+9x+20}$
$=\dfrac{x-3}{(x-5)(x+5)}+\dfrac{x-3}{(x+4)(x+5)}$
$=\dfrac{(x-3)(x+4)+(x-3)(x-5)}{(x-5)(x+5)(x+4)}$
$=\dfrac{x^2+x-12+x^2-8x+15}{x^3+4x^2-25x-100}$
$=\dfrac{2x^2-7x+3}{x^3+4x^2-25x-100}$

f is not defined for $x=-5$ and $x=5$, *g* is not defined for $x=-4$ and $x=-5$, and $f+g$ is not defined for -5, 5 or -4, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,5)\cup(5,\infty)$.

b. $f-g=\dfrac{x-3}{x^2-25}-\dfrac{x-3}{x^2+9x+20}$
$=\dfrac{x-3}{(x-5)(x+5)}-\dfrac{x-3}{(x+4)(x+5)}$
$=\dfrac{(x-3)(x+4)-(x-3)(x-5)}{(x-5)(x+5)(x+4)}$
$=\dfrac{x^2+x-12-(x^2-8x+15)}{x^3+4x^2-25x-100}$
$=\dfrac{9x-27}{x^3+4x^2-25x-100}$

f is not defined for $x=-5$ and $x=5$, *g* is not defined for $x=-4$ and $x=-5$, and $f+g$ is not defined for -5, 5, or -4, so the domain is $(-\infty,-5)\cup(-5,-4)\cup(-4,5)\cup(5,\infty)$.

c. $f \cdot g = \dfrac{x-3}{x^2-25} \cdot \dfrac{x-3}{x^2+9x+20}$
$= \dfrac{(x-3)^2}{(x^2-25)(x^2+9x+20)}$
$= \dfrac{x^2-6x+9}{x^4+9x^3-5x^2-225x-500}$

f is not defined for $x = -5$ and $x = 5$, *g* is not defined for $x = -4$ and $x = -5$, and *fg* is not defined for -5, 5, or -4, so the domain is $(-\infty, -5) \cup (-5, -4) \cup (-4, 5) \cup (5, \infty)$.

d. $\dfrac{f}{g} = \dfrac{\frac{x-3}{x^2-25}}{\frac{x-3}{x^2+9x+20}}$
$= \dfrac{x-3}{(x-5)(x+5)} \cdot \dfrac{(x+5)(x+4)}{x-3} = \dfrac{x+4}{x-5}$

f is not defined for $x = -5$ and $x = 5$, *g* is not defined for $x = -4$ and $x = -5$, and f/g is not defined for $x = 3$ and $x = 5$, so the domain is $(-\infty, -5) \cup (-5, -4) \cup (-4, 3) \cup (3, 5) \cup (5, \infty)$.

e. $\dfrac{g}{f} = \dfrac{\frac{x-3}{x^2+9x+20}}{\frac{x-3}{x^2-25}}$
$= \dfrac{x-3}{(x+5)(x+4)} \cdot \dfrac{(x-5)(x+5)}{x-3} = \dfrac{x-5}{x+4}$

f is not defined for $x = -5$ and $x = 5$, *g* is not defined for $x = -4$ and $x = -5$, and g/f is not defined for $x = -4$ and $x = 3$, so the domain is $(-\infty, -5) \cup (-5, -4) \cup (-4, 3) \cup (3, 5) \cup (5, \infty)$.

39. $f(x) = \sqrt{x-1}$; $g(x) = \sqrt{5-x}$

a. $f \cdot g = \sqrt{x-1} \cdot \sqrt{5-x}$
f is not defined for $x < 1$, *g* is not defined for $x > 5$. The domain is [1, 5].

b. $\dfrac{f}{g} = \dfrac{\sqrt{x-1}}{\sqrt{5-x}}$
f is not defined for $x < 1$, *g* is not defined for $x > 5$. The denominator is zero when $x = 5$. The domain is [1, 5).

40. $f(x) = \sqrt{x-2}$; $g(x) = \sqrt{x+2}$

a. $f \cdot g = \sqrt{x-2} \cdot \sqrt{x+2} = \sqrt{x^2-4}$
f is not defined for $x < 2$, *g* is not defined for $x < -2$. The domain is $[2, \infty)$.

b. $\dfrac{f}{g} = \dfrac{\sqrt{x-2}}{\sqrt{x+2}}$
f is not defined for $x < 2$, *g* is not defined for $x < -2$. The denominator is zero when $x = -2$. The domain is $[2, \infty)$.

41. $f(x) = \sqrt{x+2}$; $g(x) = \sqrt{9-x^2}$

a. $f \cdot g = \sqrt{x+2} \cdot \sqrt{9-x^2}$
f is not defined for $x < -2$, *g* is defined for [–3, 3]. The domain is $[-2, 3]$.

b. $\dfrac{f}{g} = \dfrac{\sqrt{x+2}}{\sqrt{9-x^2}}$
f is not defined for $x < -2$, *g* is defined for [–3, 3]. The denominator is zero when $x = -3$ or $x = 3$. The domain is $[-2, 3)$.

42. $f(x) = \sqrt{x^2-4}$; $g(x) = \sqrt{25-x^2}$

a. $f \cdot g = \sqrt{x^2-4} \cdot \sqrt{25-x^2}$
f is defined for $x \le -2$ or $x \ge 2$, *g* is defined for [–5, 5]. The domain is $[-5, -2] \cup [2, 5]$.

b. $\dfrac{f}{g} = \dfrac{\sqrt{x^2-4}}{\sqrt{25-x^2}}$
f is defined for $x \le -2$ or $x \ge 2$, *g* is defined for [–5, 5]. The denominator is zero when $x = -5$ or $x = 5$. The domain is $(-5, -2] \cup [2, 5)$.

43. $(g \circ f)(x) = g(f(x)) = 2f(x) + 3$
$= 2(x^2 - 1) + 3 = 2x^2 + 1$
$(g \circ f)(2) = g(f(2)) = 2f(2) + 3$
$= 2(2^2 - 1) + 3 = 9$
$(g \circ f)(-3) = g(f(-3)) = 2f(-3) + 3$
$= 2((-3)^2 - 1) + 3 = 19$

44. $(g\circ f)(x)=g(f(x))=3(f(x))^2-1$
$=3|x+1|^2-1=3|x^2+2x+1|-1$
$(g\circ f)(2)=3(f(2))^2-1$
$=3|2+1|^2-1=26$
$(g\circ f)(-3)=3(f(-3))^2-1$
$=3|(-3)+1|^2-1=11$

45. $(f\circ g)(2)=f(g(2))=2[g(2)]+1$
$=2(2(2)^2-3)+1=11$

46. $(g\circ f)(2)=g(f(2))=2[f(2)]^2-3$
$=2(2(2)+1)^2-3=47$

47. $(f\circ g)(-3)=f(g(-3))=2(g(-3))+1$
$=2(2(-3)^2-3)+1=31$

48. $(g\circ f)(-5)=g(f(-5))=2[f(-5)]^2-3$
$=2(2(-5)+1)^2-3=159$

49. $(f\circ g)(0)=f(g(0))=2(g(0))+1$
$=2(2(0^2)-3)+1=-5$

50. $(g\circ f)\left(\frac{1}{2}\right)=g\left(f\left(\frac{1}{2}\right)\right)=2\left[f\left(\frac{1}{2}\right)\right]^2-3$
$=2\left(2\left(\frac{1}{2}\right)+1\right)^2-3=5$

51. $(f\circ g)(-c)=f(g(-c))=2(g(-c))+1$
$=2(2(-c)^2-3)+1=4c^2-5$

52. $(f\circ g)(c)=f(g(c))=2(g(c))+1$
$=2(2c^2-3)+1=4c^2-5$

53. $(g\circ f)(a)=g(f(a))=2[f(a)]^2-3$
$=2(2a+1)^2-3$
$=2(4a^2+4a+1)-3$
$=8a^2+8a-1$

54. $(g\circ f)(-a)=g(f(-a))=2[f(-a)]^2-3$
$=2(2(-a)+1)^2-3$
$=2(4a^2-4a+1)-3$
$=8a^2-8a-1$

55. $(f\circ f)(1)=f(f(1))=2(f(1))+1$
$=2(2(1)+1)+1=7$

56. $(g\circ g)(-1)=g(g(-1))=2(g(-1))^2-3$
$=2(2(-1)^2-3)^2-3=-1$

57. $f(x)=\frac{1}{x};\ g(x)=10-5x$
$(f\circ g)(x)=\frac{1}{10-5x}$
The domain of $f\circ g$ is the set of all real numbers such that $10-5x\neq 0$, or $x\neq 2$. The domain of $f\circ g$ is $(-\infty, 2)\cup(2, \infty)$.

58. $f(x)=\frac{1}{x};\ g(x)=\sqrt{x}$
$(f\circ g)(x)=\frac{1}{\sqrt{x}}$
The domain of *f* is the set of all real numbers such that $x\neq 0$. The domain of $g(x)$ is $[0, \infty)$. Therefore, the domain of $f\circ g$ is $(0, \infty)$.

59. $f(x)=\sqrt{x};\ g(x)=2x-8$
$(f\circ g)(x)=\sqrt{2x-8}$
The domain of $f\circ g$ is the set of all real numbers such that $2x-8\geq 0$, or $x\geq 4$. The domain of $f\circ g$ is $[4, \infty)$.

60. $f(x)=\sqrt{x};\ g(x)=-x$
$(f\circ g)(x)=\sqrt{-x}$
The domain of $f\circ g$ is the set of all real numbers such that $-x\geq 0$, or $x\leq 0$. The domain of $f\circ g$ is $(-\infty, 0]$.

61. $(f\circ g)(x)=\frac{2}{g(x)+1}=\frac{2}{\frac{1}{x}+1}=\frac{2}{\frac{x+1}{x}}=\frac{2x}{x+1}$
The domain of *g* is $(-\infty, 0)\cup(0, \infty)$. Because –1 is not in the domain of *f*, we must exclude those values of *x* that make $g(x)=-1$.
$\frac{1}{x}=-1\Rightarrow x=-1$
Thus, the domain of $f\circ g$ is $(-\infty, -1)\cup(-1, 0)\cup(0, \infty)$.

62. $(f \circ g)(x) = \dfrac{1}{g(x)-1} = \dfrac{1}{\frac{2}{x+3}-1}$
$= \dfrac{1}{\frac{2-(x+3)}{x+3}} = \dfrac{x+3}{-x-1} = -\dfrac{x+3}{x+1}$

The domain of g is $(-\infty, -3) \cup (-3, \infty)$. Because 1 is not in the domain of f, we must exclude those values of x that make $g(x) = 1$.

$\dfrac{2}{x+3} = 1 \Rightarrow 2 = x+3 \Rightarrow x = -1$

Thus, the domain of $f \circ g$ is $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$.

63. $(f \circ g)(x) = \sqrt{g(x)-3}$
$= \sqrt{(2-3x)-3} = \sqrt{-1-3x}$

The domain of g is $(-\infty, \infty)$. Because f is not defined for $(-\infty, 3)$, we must exclude those values of x that make $g(x) < 3$.

$2 - 3x < 3 \Rightarrow -3x < 1 \Rightarrow x > -\dfrac{1}{3}$

Thus, the domain of $f \circ g$ is $\left(-\infty, -\dfrac{1}{3}\right]$.

64. $(f \circ g)(x) = \dfrac{g(x)}{g(x)-1} = \dfrac{2+5x}{(2+5x)-1} = \dfrac{2+5x}{1+5x}$

The domain of g is $(-\infty, \infty)$. Since f is not defined for $x = 1$ we must exclude those values of x that make $g(x) = 1$.

$2 + 5x = 1 \Rightarrow 5x = -1 \Rightarrow x = -\dfrac{1}{5}$

Thus, the domain of $f \circ g$ is $\left(-\infty, -\dfrac{1}{5}\right) \cup \left(-\dfrac{1}{5}, \infty\right)$.

65. $(f \circ g)(x) = |g(x)| = |x^2 - 1|$
domain: $(-\infty, \infty)$

66. $(f \circ g)(x) = 3g(x) - 2 = 3|x-1| - 2$
domain: $(-\infty, \infty)$

67. a. $(f \circ g)(x) = 2(g(x)) - 3$
$= 2(x+4) - 3 = 2x + 5$
domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = f(x) + 4$
$= (2x-3) + 4 = 2x + 1$
domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 2f(x) - 3$
$= 2(2x-3) - 3 = 4x - 9$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = g(x) + 4$
$= (x+4) + 4 = x + 8$
domain: $(-\infty, \infty)$

68. a. $(f \circ g)(x) = g(x) - 3$
$= (3x-5) - 3 = 3x - 8$
domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 3f(x) - 5$
$= 3(x-3) - 5 = 3x - 14$
domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = f(x) - 3$
$= (x-3) - 3 = x - 6$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 3g(x) - 5$
$= 3(3x-5) - 5 = 9x - 20$
domain: $(-\infty, \infty)$

69. a. $(f \circ g)(x) = 1 - 2g(x)$
$= 1 - 2(1+x^2) = -2x^2 - 1$
domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 1 + (f(x))^2$
$= 1 + (1-2x)^2 = 4x^2 - 4x + 2$
domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = 1 - 2f(x)$
$= 1 - 2(1-2x) = 4x - 1$
domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = 1 + (g(x))^2$
$= 1 + (1+x^2)^2 = x^4 + 2x^2 + 2$
domain: $(-\infty, \infty)$

70. a. $(f \circ g)(x) = 2g(x) - 3$
$= 2(2x^2) - 3 = 4x^2 - 3$
domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = 2(f(x))^2 = 2(2x-3)^2$
$= 8x^2 - 24x + 18$
domain: $(-\infty, \infty)$

c. $(f\circ f)(x)=2f(x)-3$
$=2(2x-3)-3=4x-9$
domain: $(-\infty,\infty)$

d. $(g\circ g)(x)=2(g(x))^2=2(2x^2)^2=8x^4$
domain: $(-\infty,\infty)$

71. a. $(f\circ g)(x)=2(g(x))^2+3g(x)$
$=2(2x-1)^2+3(2x-1)$
$=2(4x^2-4x+1)+6x-3$
$=8x^2-2x-1$
domain: $(-\infty,\infty)$

b. $(g\circ f)(x)=2f(x)-1$
$=2(2x^2+3x)-1=4x^2+6x-1$
domain: $(-\infty,\infty)$

c. $(f\circ f)(x)=2(f(x))^2+3f(x)$
$=2\left(2x^2+3x\right)^2+3\left(2x^2+3x\right)$
$=2\left(4x^4+12x^3+9x^2\right)+6x^2+9x$
$=8x^4+24x^3+24x^2+9x$
domain: $(-\infty,\infty)$

d. $(g\circ g)(x)=2(g(x))-1$
$=2(2x-1)-1=4x-3$
domain: $(-\infty,\infty)$

72. a. $(f\circ g)(x)=[g(x)]^2+3g(x)$
$=(2x)^2+3(2x)=4x^2+6x$
domain: $(-\infty,\infty)$

b. $(g\circ f)(x)=2f(x)$
$=2(x^2+3x)=2x^2+6x$
domain: $(-\infty,\infty)$

c. $(f\circ f)(x)=[f(x)]^2+3f(x)$
$=\left(x^2+3x\right)^2+3\left(x^2+3x\right)$
$=x^4+6x^3+9x^2+3x^2+9x$
$=x^4+6x^3+12x^2+9x$
domain: $(-\infty,\infty)$

d. $(g\circ g)(x)=2g(x)=2(2x)=4x$
domain: $(-\infty,\infty)$

73. a. $(f\circ g)(x)=[g(x)]^2=\left(\sqrt{x}\right)^2=x$
domain: $[0,\infty)$

b. $(g\circ f)(x)=\sqrt{f(x)}=\sqrt{x^2}=|x|$
domain: $(-\infty,\infty)$

c. $(f\circ f)(x)=[f(x)]^2=\left(x^2\right)^2=x^4$
domain: $(-\infty,\infty)$

d. $(g\circ g)(x)=\sqrt{g(x)}=\sqrt{\sqrt{x}}=\sqrt[4]{x}$
domain: $[0,\infty)$

74. a. $(f\circ g)(x)=[g(x)]^2+2g(x)$
$=\left(\sqrt{x+2}\right)^2+2\sqrt{x+2}$
$=x+2+2\sqrt{x+2}$
domain: $[-2,\infty)$

b. $(g\circ f)(x)=\sqrt{f(x)+2}=\sqrt{x^2+2x+2}$
domain: $(-\infty,\infty)$

c. $(f\circ f)(x)=[f(x)]^2+2f(x)$
$=(x^2+2x)^2+2(x^2+2x)$
$=x^4+4x^3+4x^2+2x^2+4x$
$=x^4+4x^3+6x^2+4x$
domain: $(-\infty,\infty)$

d. $(g\circ g)(x)=\sqrt{g(x)+2}=\sqrt{\sqrt{x+2}+2}$
domain: $[-2,\infty)$

75. a. $(f\circ g)(x)=\dfrac{1}{2g(x)-1}=\dfrac{1}{2\left(\frac{1}{x^2}\right)-1}$
$=\dfrac{1}{\frac{2-x^2}{x^2}}=\dfrac{x^2}{2-x^2}=-\dfrac{x^2}{x^2-2}$

The domain of g is $(-\infty,0)\cup(0,\infty)$. Since $\frac{1}{2}$ is not in the domain of f, we must find those values of x that make $g(x)=\frac{1}{2}$.

$\dfrac{1}{x^2}=\dfrac{1}{2}\Rightarrow x^2=2\Rightarrow x=\pm\sqrt{2}$

Thus, the domain of $f\circ g$ is $\left(-\infty,-\sqrt{2}\right)\cup\left(-\sqrt{2},0\right)\cup\left(0,\sqrt{2}\right)\cup\left(\sqrt{2},\infty\right)$.

b. $(g \circ f) = \dfrac{1}{[f(x)]^2} = \dfrac{1}{\left(\dfrac{1}{2x-1}\right)^2} = (2x-1)^2$

The domain of f is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$. Since 0 is not in the domain of g, we must find those values of x that make $f(x) = 0$. However, there are no such values, so the domain of $g \circ f$ is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

c. $(f \circ f)(x) = \dfrac{1}{2f(x)-1} = \dfrac{1}{2\left(\dfrac{1}{2x-1}\right)-1}$

$= \dfrac{1}{\dfrac{2-2x+1}{2x-1}} = \dfrac{2x-1}{3-2x} = -\dfrac{2x-1}{2x-3}$

The domain of f is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

$-\dfrac{2x-1}{2x-3}$ is defined for $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$, so the domain of $f \circ f$ is $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$.

d. $(g \circ g)(x) = \dfrac{1}{[g(x)]^2} = \dfrac{1}{\left(\dfrac{1}{x^2}\right)^2} = x^4$

The domain of g is $(-\infty, 0) \cup (0, \infty)$, while $g \circ g = x^4$ is defined for all real numbers. Thus, the domain of $g \circ g$ is $(-\infty, 0) \cup (0, \infty)$.

76. a. $(f \circ g)(x) = g(x) - 1 = \dfrac{x}{x+1} - 1$

$= \dfrac{x-(x+1)}{x+1} = -\dfrac{1}{x+1}$

The domain of g is $(-\infty, -1) \cup (-1, \infty)$. Since f is defined for all real numbers, there are no values that must be excluded. Thus, the domain of $f \circ g$ is $(-\infty, -1) \cup (-1, \infty)$.

b. $(g \circ f)(x) = \dfrac{f(x)}{f(x)+1} = \dfrac{x-1}{(x-1)+1} = \dfrac{x-1}{x}$

The domain of f is all real numbers. Since g is not defined for $x = -1$, we must exclude those values of x that make $f(x) = -1$.

$x - 1 = -1 \Rightarrow x = 0$

Thus, the domain of $g \circ f$ is $(-\infty, 0) \cup (0, \infty)$.

c. $(f \circ f)(x) = f(x) - 1 = (x-1) - 1 = x - 2$

domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \dfrac{g(x)}{g(x)+1} = \dfrac{\dfrac{x}{x+1}}{\dfrac{x}{x+1}+1}$

$= \dfrac{\dfrac{x}{x+1}}{\dfrac{x+x+1}{x+1}} = \dfrac{x}{2x+1}$

The domain of g is $(-\infty, -1) \cup (-1, \infty)$, while $\dfrac{x}{2x+1}$ is defined for $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

The domain of $g \circ g$ is $(-\infty, -1) \cup \left(-1. -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

77. a. $(f \circ g)(x) = \sqrt{g(x) - 1} = \sqrt{\sqrt{4-x} - 1}$

domain: $(-\infty, 3]$

b. $(g \circ f)(x) = \sqrt{4 - f(x)} = \sqrt{4 - \sqrt{x-1}}$

domain: $[1, 17]$

c. $(f \circ f)(x) = \sqrt{f(x) - 1} = \sqrt{\sqrt{x-1} - 1}$

domain: $[2, \infty)$

d. $(g \circ g)(x) = \sqrt{4 - g(x)} = \sqrt{4 - \sqrt{4-x}}$

domain: $[-12, 4]$

78. a. $(f \circ g)(x) = [g(x)]^2 - 4$

$= \left(\sqrt{4-x^2}\right)^2 - 4 = -x^2$

domain: $[-2, 2]$

b. $(g \circ f)(x) = \sqrt{4 - [f(x)]^2}$

$= \sqrt{4 - (x^2 - 4)^2}$

domain: $[-\sqrt{6}, -\sqrt{2}] \cup [\sqrt{2}, \sqrt{6}]$

c. $(f \circ f)(x) = [f(x)]^2 - 4 = (x^2 - 4)^2 - 4$

$= (x^4 - 8x^2 + 16) - 4$

$= x^4 - 8x^2 + 12$

domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = \sqrt{4 - [g(x)]^2}$

$= \sqrt{4 - \left(\sqrt{4 - x^2}\right)^2}$

$= \sqrt{4 - (4 - x^2)}$

$= \sqrt{4 - 4 + x^2} = \sqrt{x^2} = |x|$

domain: $[-2, 2]$

79. a. $(f \circ g)(x) = \dfrac{1 - g(x)}{g(x) + 2} = \dfrac{1 - \frac{x+3}{x-4}}{\frac{x+3}{x-4} + 2}$

$= \dfrac{\left(1 - \frac{x+3}{x-4}\right)(x-4)}{\left(\frac{x+3}{x-4} + 2\right)(x-4)}$

$= \dfrac{(x-4) - (x+3)}{(x+3) + 2(x-4)} = -\dfrac{7}{3x-5}$

The domain of g is $(-\infty, 4) \cup (4, \infty)$. The denominator of $f \circ g$ is 0 when $x = \frac{5}{3}$, so the domain of $f \circ g$ is $\left(-\infty, \frac{5}{3}\right) \cup \left(\frac{5}{3}, 4\right) \cup (4, \infty)$.

b. $(g \circ f)(x) = \dfrac{f(x) + 3}{f(x) - 4} = \dfrac{\frac{1-x}{x+2} + 3}{\frac{1-x}{x+2} - 4}$

$= \dfrac{\left(\frac{1-x}{x+2} + 3\right)(x+2)}{\left(\frac{1-x}{x+2} - 4\right)(x+2)}$

$= \dfrac{(1-x) + 3(x+2)}{(1-x) - 4(x+2)} = \dfrac{2x+7}{-5x-7}$

$= -\dfrac{2x+7}{5x+7}$

The domain of f is $(-\infty, -2) \cup (-2, \infty)$. The denominator of $g \circ f$ is 0 when $x = -\frac{7}{5}$, so, the domain of $g \circ f$ is $(-\infty, -2) \cup \left(-2, -\frac{7}{5}\right) \cup \left(-\frac{7}{5}, \infty\right)$.

c. $(f \circ f)(x) = \dfrac{1 - f(x)}{f(x) + 2} = \dfrac{1 - \frac{1-x}{x+2}}{\frac{1-x}{x+2} + 2}$

$= \dfrac{\left(1 - \frac{1-x}{x+2}\right)(x+2)}{\left(\frac{1-x}{x+2} + 2\right)(x+2)}$

$= \dfrac{(x+2) - (1-x)}{(1-x) + 2(x+2)} = \dfrac{2x+1}{x+5}$

The domain of f is $(-\infty, -2) \cup (-2, \infty)$. The denominator of $f \circ f$ is 0 when $x = -5$, so, the domain of $f \circ f$ is $(-\infty, -5) \cup (-5, -2) \cup (-2, \infty)$.

d. $(g \circ g)(x) = \dfrac{g(x) + 3}{g(x) - 4} = \dfrac{\frac{x+3}{x-4} + 3}{\frac{x+3}{x-4} - 4}$

$= \dfrac{\left(\frac{x+3}{x-4} + 3\right)(x-4)}{\left(\frac{x+3}{x-4} - 4\right)(x-4)}$

$= \dfrac{(x+3) + 3(x-4)}{(x+3) - 4(x-4)}$

$= \dfrac{4x-9}{-3x+19} = -\dfrac{4x-9}{3x-19}$

The domain of g is $(-\infty, 4) \cup (4, \infty)$. The denominator of $g \circ g$ is 0 when $x = \frac{19}{3}$, so the domain of $g \circ g$ is $(-\infty, 4) \cup \left(4, \frac{19}{3}\right) \cup \left(\frac{19}{3}, \infty\right)$.

80. a. $(f \circ g)(x) = \dfrac{g(2) + 2}{g(2) - 3} = \dfrac{\frac{x+1}{x-1} + 2}{\frac{x+1}{x-1} - 3}$

$= \dfrac{\left(\frac{x+1}{x-1} + 2\right)(x-1)}{\left(\frac{x+1}{x-1} - 3\right)(x-1)}$

$= \dfrac{(x+1) + 2(x-1)}{(x+1) - 3(x-1)} = \dfrac{3x-1}{-2x+4}$

$= -\dfrac{3x-1}{2x-4}$

The domain of g is $(-\infty, 1) \cup (1, \infty)$. The denominator of $f \circ g$ is 0 when $x = 2$, so the domain of $f \circ g$ is $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$.

b. $(g \circ f)(x) = \dfrac{f(x)+1}{f(x)-1} = \dfrac{\frac{x+2}{x-3}+1}{\frac{x+2}{x-3}-1}$
$= \dfrac{\left(\frac{x+2}{x-3}+1\right)(x-3)}{\left(\frac{x+2}{x-3}-1\right)(x-3)}$
$= \dfrac{(x+2)+(x-3)}{(x+2)-(x-3)} = \dfrac{2x-1}{5}$

The domain of f is $(-\infty, 3) \cup (3, \infty)$. The denominator of $g \circ f$ is never 0, so, the domain of $g \circ f$ is $(-\infty, 3) \cup (3, \infty)$.

c. $(f \circ f)(x) = \dfrac{f(x)+2}{f(x)-3} = \dfrac{\frac{x+2}{x-3}+2}{\frac{x+2}{x-3}-3}$
$= \dfrac{\left(\frac{x+2}{x-3}+2\right)(x-3)}{\left(\frac{x+2}{x-3}-3\right)(x-3)}$
$= \dfrac{(x+2)+2(x-3)}{(x+2)-3(x-3)}$
$= \dfrac{3x-4}{-2x+11} = -\dfrac{3x-4}{2x-11}$

The domain of f is $(-\infty, 3) \cup (3, \infty)$. The denominator of $f \circ f$ is 0 when $x = \frac{11}{2}$, so, the domain of $f \circ f$ is $(-\infty, 3) \cup \left(3, \frac{11}{2}\right) \cup \left(\frac{11}{2}, \infty\right)$.

d. $(g \circ g)(x) = \dfrac{g(x)+1}{g(x)-1} = \dfrac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$
$= \dfrac{\left(\frac{x+1}{x-1}+1\right)(x-1)}{\left(\frac{x+1}{x-1}-1\right)(x-1)}$
$= \dfrac{(x+1)+(x-1)}{(x+1)-(x-1)} = \dfrac{2x}{2} = x$

The domain of g is $(-\infty, 1) \cup (1, \infty)$. The denominator of $g \circ g$ is never 0 so the domain of $g \circ g$ is $(-\infty, 1) \cup (1, \infty)$.

81. a. $(f \circ g)(x) = 1 + \dfrac{1}{\frac{1+x}{1-x}} = 1 + \dfrac{1-x}{1+x}$
$= \dfrac{1+x+1-x}{1+x} = \dfrac{2}{1+x}$.

The domain of g is $(-\infty, 1) \cup (1, \infty)$. Because 0 is not in the domain of f, we must find those values of x that make $g(x) = 0$.

$\dfrac{1+x}{1-x} = 0 \Rightarrow 1+x = 0 \Rightarrow x = -1$

Thus, the domain of $f \circ g$ is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

b. $(g \circ f)(x) = \dfrac{1+\left(1+\frac{1}{x}\right)}{1-\left(1+\frac{1}{x}\right)} = \dfrac{2+\frac{1}{x}}{-\frac{1}{x}} \cdot \dfrac{x}{x}$
$= \dfrac{2x+1}{-1} = -2x-1$

The domain of f is $(-\infty, 0) \cup (0, \infty)$. Since 1 is not in the domain of g, we must find those values of x that make $f(x) = 1$.

$1 + \dfrac{1}{x} = 1 \Rightarrow \dfrac{1}{x} = 0$

There are no values of x that make this true, so there are no additional values to be excluded from the domain of $g \circ f$. Thus, the domain of $g \circ f$ is $(-\infty, 0) \cup (0, \infty)$.

c. $(f \circ f)(x) = 1 + \dfrac{1}{1+\frac{1}{x}} = 1 + \dfrac{1}{\frac{x+1}{x}} = 1 + \dfrac{x}{x+1}$
$= \dfrac{2x+1}{x+1}$

The domain of f is $(-\infty, 0) \cup (0, \infty)$.

$\dfrac{2x+1}{x+1}$ is undefined for $(-\infty, -1) \cup (-1, \infty)$, so the domain of $f \circ f$ is $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$.

d. $(g \circ g)(x) = \dfrac{1+\frac{1+x}{1-x}}{1-\frac{1+x}{1-x}} = \dfrac{\frac{2}{1-x}}{\frac{-2x}{1-x}} = -\dfrac{1}{x}$.

The domain of g is $(-\infty, 1) \cup (1, \infty)$, while $-\dfrac{1}{x}$ is defined for $(-\infty, 0) \cup (0, \infty)$. The domain of $g \circ g$ is $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$.

82. a. $(f \circ g)(x) = \sqrt[3]{(x^3+1)+1} = \sqrt[3]{x^3+2}$; domain: $(-\infty, \infty)$

b. $(g \circ f)(x) = \left(\sqrt[3]{x+1}\right)^3 + 1 = x+2$; domain: $(-\infty, \infty)$

c. $(f \circ f)(x) = \sqrt[3]{\sqrt[3]{x+1}+1}$; domain: $(-\infty, \infty)$

d. $(g \circ g)(x) = (x^3+1)^3 + 1$; domain: $(-\infty, \infty)$

83. The domain of f is $(-\infty, 3)\cup(3, \infty)$. Because the domain of g is $[0, 6]$, the domain of $f \circ g$ cannot extend beyond those values. The value of g is 3 for $(1, 3]$, so those values are not in the domain of $f \circ g$. Therefore, the domain of $f \circ g$ is $[0, 1]\cup(3, 6]$.

84. The domain of f is $(-\infty, 2)\cup(2, \infty)$. Because the domain of g is $[0, 6]$, the domain of $f \circ g$ cannot extend beyond those values. The value of g is 2 for $[0, 1]$, so those values are not in the domain of $f \circ g$. Therefore, the domain of $f \circ g$ is $(1, 6]$.

85. The domain of f is $(-\infty, 3]$. Because the domain of g is $[0, 6]$, the domain of $f \circ g$ cannot extend beyond those values. $g(x) > 3$ for $x > 4$, so the domain of $f \circ g$ is $[0, 4]$.

86. The domain of f is $[2, \infty)$. Because the domain of g is $[0, 6]$, the domain of $f \circ g$ cannot extend beyond those values. $g(x) < 2$ for $3 < x \le 4$, so the domain of $f \circ g$ is $[0, 3]\cup(4, 6]$.

87. $$(f \circ g) = \begin{cases} 2^2 - 2 = 2 & \text{if } 0 \le x \le 1 \\ 4^2 - 2 = 14 & \text{if } 1 < x \le 3 \\ 3^2 - 2 = 7 & \text{if } 3 < x \le 6 \end{cases}$$

88. $$(f \circ g) = \begin{cases} 1 - 2^2 = -3 & \text{if } 0 \le x \le 1 \\ 1 - 4^2 = -15 & \text{if } 1 < x \le 3 \\ 1 - 3^2 = -8 & \text{if } 3 < x \le 6 \end{cases}$$

89. $$(f \circ g) = \begin{cases} \dfrac{1}{2+2} = \dfrac{1}{4} & \text{if } 0 \le x \le 1 \\ \dfrac{1}{2+4} = \dfrac{1}{6} & \text{if } 1 < x \le 3 \\ \dfrac{1}{2+3} = \dfrac{1}{5} & \text{if } 3 < x \le 6 \end{cases}$$

90. $$(f \circ g) = \begin{cases} \dfrac{1}{1-2} = -1 & \text{if } 0 \le x \le 1 \\ \dfrac{1}{1-4} = -\dfrac{1}{3} & \text{if } 1 < x \le 3 \\ \dfrac{1}{1-3} = -\dfrac{1}{2} & \text{if } 3 < x \le 6 \end{cases}$$

In exercises 91−100, sample answers are given. Other answers are possible.

91. $H(x) = \sqrt{x+2} \Rightarrow f(x) = \sqrt{x}, g(x) = x + 2$

92. $H(x) = |3x+2| \Rightarrow f(x) = |x|, g(x) = 3x + 2$

93. $H(x) = \left(x^2 - 3\right)^{10} \Rightarrow f(x) = x^{10}, g(x) = x^2 - 3$

94. $H(x) = \sqrt{3x^2} + 5 \Rightarrow f(x) = \sqrt{x} + 5, g(x) = 3x^2$

95. $H(x) = \dfrac{1}{3x-5} \Rightarrow f(x) = \dfrac{1}{x}, g(x) = 3x - 5$

96. $H(x) = \dfrac{5}{2x+3} \Rightarrow f(x) = \dfrac{5}{x}, g(x) = 2x + 3$

97. $H(x) = \sqrt[3]{x^2 - 7} \Rightarrow f(x) = \sqrt[3]{x}, g(x) = x^2 - 7$

98. $H(x) = \sqrt[4]{x^2 + x + 1} \Rightarrow f(x) = \sqrt[4]{x},$
$g(x) = x^2 + x + 1$

99. $H(x) = \dfrac{1}{\left|x^3 - 1\right|} \Rightarrow f(x) = \dfrac{1}{|x|}, g(x) = x^3 - 1$

100. $H(x) = \sqrt[3]{1 + \sqrt{x}} \Rightarrow f(x) = \sqrt[3]{x}, g(x) = 1 + \sqrt{x}$

101. $f(x) = x^2 + 2;\ \ g(x) = 1 - 2x$

First compute the component related to the average rate of change of the inner function.

$g(1) = 1 - 2(1) = -1$ and

$g(2) = 1 - 2(2) = -3.$

$$\text{ARC of } g \text{ from 1 to } 2 = \frac{g(2) - g(1)}{2 - 1} = \frac{-3 - (-1)}{1} = -2$$

To compute the component related to the average rate of change of the outer function, we need to use the range for f as from $g(1) = -1$ to $g(2) = -3$. We have

$f(-1) = (-1)^2 + 2 = 3$ and

$f(-3) = (-3)^2 + 2 = 11.$

ARC of f from $g(1) = -1$ to $g(2) = -3$

$$= \frac{f(-3) - f(-1)}{-3 - (-1)} = \frac{11 - 3}{-2} = -4$$

(*continued on next page*)

(*continued*)

Finally, we can compute the average rate of change of the composite function as

ARC of $f \circ g$ from 1 to 2
$= (\text{ARC of } f \text{ from } g(1) = -1 \text{ to } g(2) = -3)$
$\quad \cdot (\text{ARC of } g \text{ from 1 to 2})$
$= -4 \cdot (-2) = 8$

102. $f(x) = 1 - x^2;\; g(x) = 1 + 3x$

First compute the average rate of change of g from $x = -1$ to $x = 1$.
First compute the component related to the average rate of change of the inner function.

$g(-1) = 1 + 3(-1) = -2$ and
$g(1) = 1 + 3(1) = 4.$

$$\text{ARC of } g \text{ from } -1 \text{ to } 1 = \frac{g(1) - g(-1)}{1 - (-1)} = \frac{4 - (-2)}{2} = 3$$

To compute the component related to the average rate of change of the outer function, we need to use the range for f as from $g(-1) = -2$ to $g(1) = 4.$ We have

$f(-2) = 1 - (-2)^2 = -3$ and
$f(4) = 1 - 4^2 = -15.$

ARC of f from $g(-1) = -2$ to $g(1) = 4$
$$= \frac{f(4) - f(-2)}{4 - (-2)} = \frac{-15 - (-3)}{6} = -2$$

Finally, we can compute the average rate of change of the composite function as

ARC of $f \circ g$ from -1 to 1
$= (\text{ARC of } f \text{ from } g(-1) = -2 \text{ to } g(1) = 4)$
$\quad \cdot (\text{ARC of } g \text{ from } -1 \text{ to } 1)$
$= -2 \cdot 3 = -6$

103. $f(x) = x^3 + 2;\; g(x) = 1 - x^2$

First compute the component related to the average rate of change of the inner function.

$g(1) = 1 - 1^2 = 0$ and $g(2) = 1 - 2^2 = -3.$

$$\text{ARC of } g \text{ from 1 to } 2 = \frac{g(2) - g(1)}{2 - 1} = \frac{-3 - 0}{1} = -3$$

To compute the component related to the average rate of change of the outer function, we need to use the range for f as from $g(1) = 0$ to $g(2) = -3.$ We have

$f(0) = 0^3 + 2 = 2$ and
$f(-3) = (-3)^3 + 2 = -25.$

ARC of f from $g(1) = -1$ to $g(2) = -3$
$$= \frac{f(0) - f(-3)}{0 - (-3)} = \frac{2 - (-25)}{3} = 9$$

Finally, we can compute the average rate of change of the composite function as

ARC of $f \circ g$ from 1 to 2
$= (\text{ARC of } f \text{ from } g(1) = 0 \text{ to } g(2) = -3)$
$\quad \cdot (\text{ARC of } g \text{ from 1 to 2})$
$= 9 \cdot (-3) = -27$

104. $f(x) = 1 - x^3;\; g(x) = x^2 + 1$

First compute the component related to the average rate of change of the inner function.

$g(-1) = (-1)^2 + 1 = 2$ and $g(0) = 0^2 + 1 = 1.$

ARC of g from -1 to 0
$$= \frac{g(0) - g(-1)}{0 - (-1)} = \frac{1 - 2}{1} = -1$$

To compute the component related to the average rate of change of the outer function, we need to use the range for f as from $g(-1) = 2$ to $g(0) = 1.$ We have $f(2) = 1 - 2^3 = -7$ and $f(1) = 1 - 1^3 = 0.$

ARC of f from $g(-1) = 2$ to $g(0) = 1$
$$= \frac{f(2) - f(1)}{2 - 1} = \frac{-7 - 0}{1} = -7$$

Finally, we can compute the average rate of change of the composite function as

ARC of $f \circ g$ from -1 to 0
$= (\text{ARC of } f \text{ from } g(-1) = 2 \text{ to } g(0) = 1)$
$\quad \cdot (\text{ARC of } g \text{ from } -1 \text{ to } 0)$
$= -7(-1) = 7$

105. $f(x)=\frac{1}{4+x};\ g(x)=x^2-1$

First compute the component related to the average rate of change of the inner function.

$g(1)=1^2-1=0$ and $g(2)=2^2-1=3.$

$$\text{ARC of } g \text{ from 1 to 2} = \frac{g(2)-g(1)}{2-1}=\frac{3-0}{1} = 3$$

To compute the component related to the average rate of change of the outer function, we need to use the range for *f* as from $g(1)=0$ to $g(2)=3$. We have

$f(0)=\frac{1}{4+0}=\frac{1}{4}$ and $f(3)=\frac{1}{4+3}=\frac{1}{7}.$

ARC of f from $g(1)=0$ to $g(2)=3$

$$=\frac{f(0)-f(3)}{0-3}=\frac{\frac{1}{4}-\frac{1}{7}}{-3}=\frac{\frac{3}{28}}{-3}=-\frac{1}{28}$$

Finally, we can compute the average rate of change of the composite function as

ARC of $f\circ g$ from 1 to 2

$=(\text{ARC of } f \text{ from } g(1)=0 \text{ to } g(2)=3)$

$\cdot(\text{ARC of } g \text{ from 1 to 2})$

$=-\frac{1}{28}\cdot 3=-\frac{3}{28}$

106. $f(x)=\frac{1}{2+x};\ g(x)=x^2+1$

First compute the component related to the average rate of change of the inner function.

$g(0)=0^2+1=1$ and $g(2)=2^2+1=5.$

$$\text{ARC of } g \text{ from 0 to 2} = \frac{g(2)-g(0)}{2-0}=\frac{5-1}{2} = 2$$

To compute the component related to the average rate of change of the outer function, we need to use the range for *f* as from $g(0)=1$ to $g(2)=5$. We have

$f(1)=\frac{1}{2+1}=\frac{1}{3}$ and $f(5)=\frac{1}{2+5}=\frac{1}{7}.$

ARC of f from $g(0)=1$ to $g(2)=5$

$$=\frac{f(1)-f(5)}{1-5}=\frac{\frac{1}{3}-\frac{1}{7}}{-4}=\frac{\frac{4}{21}}{-4}=-\frac{1}{21}$$

Finally, we can compute the average rate of change of the composite function as

ARC of $f\circ g$ from 0 to 2

$=(\text{ARC of } f \text{ from } g(0)=1 \text{ to } g(2)=5)$

$\cdot(\text{ARC of } g \text{ from 0 to 5})$

$=-\frac{1}{21}\cdot 2=-\frac{2}{21}$

Applying the Concepts

107. a. $f(x)$ is the cost function.

b. $g(x)$ is the revenue function.

c. $h(x)$ is the selling price of x shirts including sales tax.

d. $P(x)$ is the profit function.

108. a.
$$\begin{aligned}C(p)&=4x+12{,}000\\&=4(5000-5p)+12{,}000\\&=20{,}000-20p+12{,}000\\&=32{,}000-20p\end{aligned}$$

b. $R(p)=px=p(5000-5p)=5000p-5p^2$

c.
$$\begin{aligned}P(p)&=R(p)-C(p)\\&=5000p-5p^2-(32{,}000-20p)\\&=-5p^2+5020p-32{,}000\end{aligned}$$

109. a. $P(x)=R(x)-C(x)=25x-(350+5x)$
$=20x-350$

b. $P(20)=20(20)-350=50.$
This represents the profit when 20 radios are sold.

c. $P(x)=20x-350; 500=20x-350\Rightarrow x=43$

d. $C=350+5x\Rightarrow x=\frac{C-350}{5}=x(C).$

$(R\circ x)(C)=25\left(\frac{C-350}{5}\right)=5C-1750.$

This function represents the revenue in terms of the cost *C*.

110. a. $g(x)=0.04x$

b. $h(x)$ is the after tax selling price of merchandise worth x dollars.

c. $f(x)=0.02h(x)+3$

d. $T(x)$ represents the total price of merchandise worth x dollars, including the shipping and handling fee.

111. a. $f(x) = 0.7x$

b. $g(x) = x - 5$

c. $(g \circ f)(x) = 0.7x - 5$

d. $(f \circ g)(x) = 0.7(x - 5)$

e. $$(f \circ g) - (g \circ f) = 0.7(x-5) - (0.7x - 5) = 0.7x - 3.5 - 0.7x + 5 = \$1.50$$

112. a. $f(x) = 0.8x$

b. $g(x) = 0.9x$

c. $(g \circ f)(x) = 0.9f(x) = 0.9(0.8x) = 0.72x$

d. $(f \circ g)(x) = 0.8g(x) = 0.8(0.9x) = 0.72x$

e. They are the same.

113. a. $f(x) = 1.1x; g(x) = x + 8$

b. $(f \circ g)(x) = 1.1g(x) = 1.1(x + 8) = 1.1x + 8.8$

This represents a final test score, with the original score represented by x, computed by first adding 8 points to the original score and then increasing the total by 10%.

c. $(g \circ f)(x) = f(x) + 8 = 1.1x + 8$

This represents a final test score, with the original score represented by x, computed by first increasing the original score by 10% and then adding 8 points.

d. $(f \circ g)(70) = 1.1(70 + 8) = 85.8;$
$(g \circ f)(70) = 1.1(70) + 8 = 85.0;$

e. $(f \circ g)(x) \neq (g \circ f)(x)$

f. **(i)** $(f \circ g)(x) = 1.1x + 8.8 \geq 90 \Rightarrow x \geq 73.82$

(ii) $(g \circ f)(x) = 1.1x + 8 \geq 90 \Rightarrow x \geq 74.55$

114. a. $f(x)$ is a function that models 3% of an amount x.

b. $g(x)$ represents the amount of money that qualifies for a 3% bonus.

c. Her bonus is represented by $(f \circ g)(x)$.

d. $200 + 0.03(17,500 - 8000) = \$485$

e. $521 = 200 + 0.03(x - 8000) \Rightarrow x = \$18,700$

115. a. $f(x) = \pi x^2$

b. $g(x) = \pi(x + 30)^2$

c. $g(x) - f(x)$ represents the area between the fountain and the fence.

d. The circumference of the fence is $2\pi(x + 30)$.

$10.5(2\pi(x+30)) = 4200 \Rightarrow$
$\pi(x + 30) = 200 \Rightarrow$
$\pi x + 30\pi = 200 \Rightarrow \pi x = 200 - 30\pi.$

$$g(x) - f(x) = \pi(x+30)^2 - \pi x^2 = \pi(x^2 + 60x + 900) - \pi x^2 = 60\pi x + 900\pi.$$ Now substitute $200 - 30\pi$ for πx to compute the estimate:

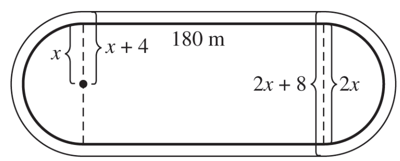
$$1.75[60(200 - 30\pi) + 900\pi] = 1.75(12,000 - 900\pi) \approx \$16,052.$$

116. a. $$f(x) = 180(2x + 8) + \pi(x+4)^2 = 1440 + 360x + \pi(x+4)^2$$

b. $g(x) = 2x(180) + \pi x^2 = 360x + \pi x^2$

c. $f(x) - g(x)$ represents the area of the track.

d.



(i) First find the radius of the inner track:

$900 = 2\pi x + 360 \Rightarrow \dfrac{270}{\pi} = x$. Use this value to compute $f(x) - g(x)$.

$$f\left(\frac{270}{\pi}\right) - g\left(\frac{270}{\pi}\right)$$
$$= \left(1440 + 360\left(\frac{270}{\pi}\right) + \pi\left(\frac{270}{\pi} + 4\right)^2\right) - \left(360\left(\frac{270}{\pi}\right) + \pi\left(\frac{270}{\pi}\right)^2\right)$$
$$= 1440 + 360\left(\frac{270}{\pi}\right) + \frac{270^2}{\pi} + 2160 + 16\pi - 360\left(\frac{270}{\pi}\right) - \frac{270^2}{\pi}$$
$$= 3600 + 16\pi \approx 3650.27 \text{ square meters}$$

(ii) The outer perimeter

$$= 360 + 2\pi\left(\frac{270}{\pi} + 4\right) \approx 925.13 \text{ meters}$$

117. a. $(f \circ g)(t) = \pi(2t+1)^2$

b. $A(t) = f(2t+1) = \pi(2t+1)^2$

c. They are the same.

118. a. $(f \circ g)(t) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3$

b. $V(t) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3$

c. They are the same.

Beyond the Basics

119. a. When you are looking for the domain of the sum of two functions that are given as sets, you are looking for the intersection of their domains. Because the x-values that f and g have in common are −2, 1, and 3, the domain of $f + g$ is {−2, 1, 3}. Now add the y-values.
$(f+g)(-2) = 3+0 = 3$
$(f+g)(1) = 2+(-2) = 0$
$(f+g)(3) = 0+2 = 2$
Thus, $f + g = \{(-2, 3), (1, 0), (3, 2)\}$.

b. When you are looking for the domain of the product of two functions that are given as sets, you are looking for the intersection of their domains. Because the x-values that f and g have in common are −2, 1, and 3, the domain of $f + g$ is {−2, 1, 3}.Now multiply the y-values.
$(fg)(-2) = 3 \cdot 0 = 0$
$(fg)(1) = 2 \cdot (-2) = -4$
$(fg)(3) = 0 \cdot 2 = 0$
Thus, $fg = \{(-2, 0), (1, -4), (3, 0)\}$.

c. When you are looking for the domain of the quotient of two functions that are given as sets, you are looking for the intersection of their domains and values of x that do not cause the denominator to equal zero. The x-values that f and g have in common are −2, 1, and 3; however, $g(-2) = 0$, so the domain is {1, 3}. Now divide the y-values.
$\left(\frac{f}{g}\right)(1) = \frac{2}{-2} = -1$
$\left(\frac{f}{g}\right)(3) = \frac{0}{2} = 0$
Thus, $\frac{f}{g} = \{(1, -1), (3, 0)\}$.

d. When you are looking for the domain of the composition of two functions that are given as sets, you are looking for values that come from the domain of the inside function and when you plug those values of x into the inside function, the output is in the domain of the outside function.
$f(g(-2)) = f(0)$, which is undefined
$f(g(0)) = f(2) = 1$,
$f(g(1)) = f(-2) = 3$,
$f(g(3)) = f(2) = 1$
Thus, $f \circ g = \{(0, 1), (1, 3), (3, 1)\}$.

120. When you are looking for the domain of the sum of two functions, you are looking for the intersection of their domains. The domain of f is [−2, 3], while the domain of g is [−3, 3]. The intersection of the two domains is [−2, 3], so the domain of $f + g$ is [−2, 3].
For the interval [−2, 1],
$f + g = 2x + (x + 1) = 3x + 1$.
For the interval (1, 2)
$f + g = (x + 1) + (x + 1) = 2x + 2$.
For the interval [2, 3],
$f + g = (x + 1) + (2x - 1) = 3x$.
Thus,
$$(f+g)(x) = \begin{cases} 3x+1 & \text{if } -2 \le x \le 1 \\ 2x+2 & \text{if } 1 < x < 2 \\ 3x & \text{if } 2 \le x \le 3. \end{cases}$$

121. a. $f(-x) = h(-x) + h(-(-x)) = h(-x) + h(x) = f(x) \Rightarrow f(x)$ is an even function.

b. $g(-x) = h(-x) - h(-(-x)) = h(-x) - h(x) = -g(x) \Rightarrow g(x)$ is an odd function.

c. $\begin{cases} f(x) = h(x) + h(-x) \\ g(x) = h(x) - h(-x) \end{cases} \Rightarrow$
$f(x) + g(x) = 2h(x) \Rightarrow$
$h(x) = \frac{f(x)+g(x)}{2} = \frac{f(x)}{2} + \frac{g(x)}{2} \Rightarrow$
$h(x)$ is the sum of an even function and an odd function.

122. a. $h(x) = x^2 - 2x + 3 \Rightarrow f(x) + g(x)$, where $f(x) = x^2 + 3$ (even), $g(x) = -2x$ (odd)

b. $h(x) = [\![x]\!] + x \Rightarrow f(x) = \frac{[\![x]\!] + [\![-x]\!]}{2}$ (even),
$g(x) = x + \frac{[\![x]\!] - [\![-x]\!]}{2}$ (odd)

123. $f(x)=\sqrt{\dfrac{1-|x|}{2-|x|}}$

$f(x)$ is defined if $\dfrac{1-|x|}{2-|x|}\ge 0$ and $2-|x|\ne 0$.

$2-|x|=0\Rightarrow 2=|x|\Rightarrow x=\pm 2$

Thus, the values −2 and 2 are not in the domain of *f*.

$\dfrac{1-|x|}{2-|x|}\ge 0$ if $1-|x|\ge 0$ and $2-|x|>0$, or if $1-|x|\le 0$ and $2-|x|<0$.

Case 1: $1-|x|\ge 0$ and $2-|x|>0$.

$1-|x|\ge 0\Rightarrow 1\ge |x|\Rightarrow -1\le x\le 1$

$2-|x|>0\Rightarrow 2>|x|\Rightarrow -2<x<2$

Thus, $1-|x|\ge 0$ and $2-|x|>0\Rightarrow -1\le x\le 1$.

Case 2: $1-|x|\le 0$ and $2-|x|<0$.

$1-|x|\le 0\Rightarrow 1\le |x|\Rightarrow (-\infty,-1]\cup[1,\infty)$

$2-|x|<0\Rightarrow 2\le |x|\Rightarrow (-\infty,-2)\cup(2,\infty)$

Thus, $1-|x|\le 0$ and $2-|x|<0\Rightarrow$ $(-\infty,-2)\cup(2,\infty)$.

The domain of *f* is $(-\infty,-2)\cup[-1,1]\cup(2,\infty)$.

124. $f(x)=\begin{cases}-1 & \text{if } -2\le x\le 0\\ x-1 & \text{if } 0<x\le 2\end{cases}$

$f(|x|)=|x|-1,\ -2\le x\le 2$

$|f(x)|=\begin{cases}1 & \text{if } -2\le x\le 0\\ |x-1|=1-x & \text{if } 0<x<1\\ x-1 & \text{if } 1\le x\le 2\end{cases}$

$g(x)=f(|x|)+|f(x)|$

If $-2\le x\le 0$, then $g(x)=|x|-1+1=|x|=-x$.

If $0<x<1$, then $g(x)=(1-x)+(x-1)=0$.

If $1\le x\le 2$, then $g(x)=(x-1)+(x-1)=2(x-1)$.

Writing *g* as a piecewise function, we have

$g(x)=\begin{cases}|x|=-x & \text{if } -2\le x\le 0\\ 0 & \text{if } 0<x<1\\ 2(x-1) & \text{if } 1\le x\le 2\end{cases}$

Critical Thinking/Discussion/Writing

125. a. The domain of $f(x)$ is $(-\infty,0)\cup[1,\infty)$.

b. The domain of $g(x)$ is [0, 2].

c. The domain of $f(x)+g(x)$ is [1, 2].

d. The domain of $\dfrac{f(x)}{g(x)}$ is [1, 2).

126. a. The domain of *f* is $(-\infty,0)$. The domain of $f\circ f$ is $\varnothing$ because $f\circ f=\dfrac{1}{\sqrt{-\dfrac{1}{\sqrt{-x}}}}$ and the denominator is the square root of a negative number.

b. The domain of *f* is $(-\infty,1)$. The domain of $f\circ f$ is $(-\infty,0)$ because $f\circ f=\dfrac{1}{\sqrt{1-\dfrac{1}{\sqrt{1-x}}}}$ and the denominator must be greater than 0. If $x=0$, then the denominator = 0.

127. a. The sum of two even functions is an even function.
$f(x)=f(-x)$ and $g(x)=g(-x)\Rightarrow$
$(f+g)(x)=f(x)+g(x)=f(-x)+g(-x)$
$=(f+g)(-x)$.

b. The sum of two odd functions is an odd function.
$f(-x)=-f(x)$ and $g(-x)=-g(x)\Rightarrow$
$(f+g)(-x)=f(-x)+g(-x)=-f(x)-g(x)$
$=-(f+g)(x)$.

c. The sum of an even function and an odd function is neither even nor odd.
$f(x)$ even $\Rightarrow f(x)=f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x)=-g(x)\Rightarrow f(-x)+g(-x)=$
$f(x)+(-g(x))$, which is neither even nor odd.

d. The product of two even functions is an even function.
$f(x)=f(-x)$ and $g(x)=g(-x)\Rightarrow$
$(f\cdot g)(x)=f(x)\cdot g(x)=f(-x)\cdot(g(-x))$
$=(f\cdot g)(-x)$.

e. The product of two odd functions is an even function.
$f(-x) = -f(x)$ and $g(-x) = -g(x) \Rightarrow$
$(f \cdot g)(-x) = f(-x) \cdot g(-x) = -f(x) \cdot (-g(x))$
$= (f \cdot g)(x).$

f. The product of an even function and an odd function is an odd function.
$f(x)$ even $\Rightarrow f(x) = f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x) = -g(x) \Rightarrow$
$f(-x) \cdot g(-x) = f(x) \cdot (-g(x)) = -(f \cdot g)(x)$

128. a. $f(-x) = -f(x)$ and $g(-x) = -g(x) \Rightarrow$
$(f \circ g)(-x) = f(g(-x)) = f(-g(x)) =$
$-f(g(x)) \Rightarrow (f \circ g)(x)$ is odd.

b. $f(x) = f(-x)$ and $g(x) = g(-x) \Rightarrow$
$(f \circ g)(-x) = f(g(-x)) = f(g(x)) \Rightarrow$
$(f \circ g)(x)$ is even.

c. $f(x)$ odd $\Rightarrow f(-x) = -f(x)$ and
$g(x)$ even $\Rightarrow g(x) = g(-x) \Rightarrow (f \circ g)(-x)$
$f(g(x)) = f(g(-x)) \Rightarrow (f \circ g)(x)$ is even.

d. $f(x)$ even $\Rightarrow f(x) = f(-x)$ and $g(x)$ odd $\Rightarrow$
$g(-x) = -g(x) \Rightarrow (f \circ g)(-x) = f(-g(x))$
$= f(g(x)) = (f \circ g)(-x) \Rightarrow (f \circ g)(x)$ is even.

Getting Ready for the Next Section

GR1. a. Yes, R defines a function.

b. $S = \{(2, -3), (1, -1), (3, 1), (1, 2)\}$
No, S does not define a function because the value 1 maps to two different second values, −1 and 2.

GR2. The slope of $PP' = \frac{2-5}{5-2} = -1,$ while the slope of $y = x$ is 1. Because the slopes are negative reciprocals, the lines are perpendicular. The midpoint of PP' is $\left(\frac{2+5}{2}, \frac{5+2}{2}\right) = \left(\frac{7}{2}, \frac{7}{2}\right),$ which lies on the line $y = x$. Thus, $y = x$ is the perpendicular bisector of PP'.

GR3. $x = 2y + 3 \Rightarrow x - 3 = 2y \Rightarrow \frac{x-3}{2} = y$

GR4. $x = y^2 + 1,\ y \ge 0 \Rightarrow x - 1 = y^2 \Rightarrow \sqrt{x-1} = y$

GR5. $x^2 + y^2 = 4,\ x \le 0 \Rightarrow x^2 = 4 - y^2 \Rightarrow$
$x = -\sqrt{4 - y^2}$

GR6. $2x - \frac{1}{y} = 3 \Rightarrow -\frac{1}{y} = 3 - 2x \Rightarrow \frac{1}{y} = -3 + 2x \Rightarrow$
$y = \frac{1}{2x-3}$

1.9 Inverse Functions

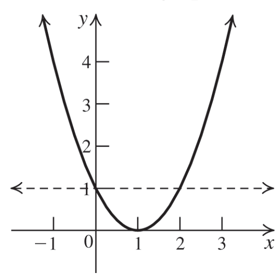
Practice Problems

1. Algebraic approach**:** Let x_1 and x_2 be two numbers in the domain of the function. Suppose $f(x_1) = f(x_2)$. Then
$(x_1 - 1)^2 = (x_2 - 1)^2 \Rightarrow x_1 - 1 = \pm(x_2 - 1) \Rightarrow$

$x_1 - 1 = +(x_2 - 1)$	$x_1 - 1 = -(x_2 - 1)$
$x_1 - 1 = x_2 - 1$	$x_1 - 1 = -x_2 + 1$
$x_1 = x_2$	$x_1 = x_2 + 2 \Rightarrow$
	$x_2 = x_1 - 2$

This means that for two distinct numbers, x_1 and $x_2 = x_1 - 2,$ $f(x_1) = f(x_2)$. Therefore, f is not a one-to-one function.

Geometric approach: $f(x) = (x-1)^2$ is not one-to-one because the horizontal line $y = 1$ intersects the graph at two different points.

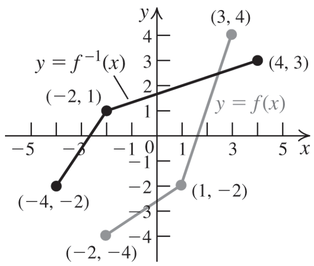


2. a. $f^{-1}(12) = -3$

b. $f(9) = 4$

3. $f(x) = 3x - 1,\ g(x) = \frac{x+1}{3}$
$(f \circ g)(x) = 3g(x) - 1 = 3\left(\frac{x+1}{3}\right) - 1 = x$
$(g \circ f)(x) = \frac{f(x)+1}{3} = \frac{(3x-1)+1}{3} = x$
Because $f(g(x)) = g(f(x)) = x,$ the two functions are inverses.

4. The graph of f^{-1} is the reflection of the graph of *f* about the line $y = x$.



5. $f(x) = -2x + 3$ is a one-to-one function, so the function has an inverse. Interchange the variables and solve for *y*:

$f(x) = y = -2x + 3 \Rightarrow x = -2y + 3 \Rightarrow$
$\frac{x-3}{-2} = y \Rightarrow y = f^{-1}(x) = \frac{3-x}{2}$.

6. Interchange the variables and solve for *y*:

$$f(x) = y = \frac{x}{x+3}, x \neq -3$$
$$x = \frac{y}{y+3} \Rightarrow xy + 3x = y \Rightarrow 3x = y - xy \Rightarrow$$
$$3x = y(1-x) \Rightarrow \frac{3x}{1-x} = y \Rightarrow$$
$$f^{-1}(x) = \frac{3x}{1-x},\ x \neq 1$$

7. $f(x) = \frac{x}{x+3}$

The function is not defined if the denominator is zero, so the domain is $(-\infty, -3) \cup (-3, \infty)$.

From Practice Problem 6,

$f^{-1}(x) = \frac{3x}{1-x},\ x \neq 1$. The range of the function is the same as the domain of the inverse, thus the range is $(-\infty, 1) \cup (1, \infty)$.

8. *G* is one-to-one because the domain is restricted, so an inverse exists.

$G(x) = y = x^2 - 1, x \leq 0$. Interchange the variables and solve for y:

$x = y^2 - 1,\ y \leq 0 \Rightarrow y = G^{-1}(x) = -\sqrt{x+1}$.

9. $f(x) = 2 - (3x-1)^3$

$f(0) = 3; \quad f(1) = -6$

First we compute the corresponding average rate of *f*.

$$\text{ARC of } f \text{ from 0 to 1} = \frac{f(1) - f(0)}{1-0} = \frac{-6-3}{1} = -9$$

Now compute the average rate of change of f^{-1} as

$$\text{ARC of } f^{-1} \text{ from } f(0) = -6 \text{ to } f(1) = 3 = \frac{1}{\text{ARC of } f \text{ from 0 to 1}} = -\frac{1}{9}$$

10. From the text, we have $d = \frac{11p}{5} - 33$.

$$d = \frac{11 \cdot 1650}{5} - 33 = 3597$$

The bell was 3597 feet below the surface when the gauge failed.

Concepts and Vocabulary

1. If no horizontal line intersects the graph of a function *f* in more than one point, the *f* is a one-to-one function.
2. A function *f* is one-to-one if different *x*-values correspond to different *y*-values.
3. If $f(x) = 3x$, then $f^{-1}(x) = \frac{1}{3}x$.

4. The graphs of a function *f* and its inverse f^{-1} are symmetric about the line $y = x$.
5. True
6. True. For example, the inverse of $f(x) = x$ is $f^{-1}(x) = x$.
7. False. $f^{-1}(x)$ means the inverse of *f*.
8. True

Building Skills

9. One-to-one	**10.** Not one-to-one
11. Not one-to-one	**12.** One-to-one
13. Not one-to-one	**14.** Not one-to-one
15. One-to-one	**16.** Not one-to-one

17. $f(2) = 7 \Rightarrow f^{-1}(7) = 2$

18. $f^{-1}(4) = -7 \Rightarrow f(-7) = 4$

19. $f(-1) = 2 \Rightarrow f^{-1}(2) = -1$

20. $f^{-1}(-3)=5 \Rightarrow f(5)=-3$

21. $f(a)=b \Rightarrow f^{-1}(b)=a$

22. $f^{-1}(c)=d \Rightarrow f(d)=c$

23. $\left(f^{-1}\circ f\right)(337)=f^{-1}\left(f(337)\right)=337$

24. $\left(f\circ f^{-1}\right)(25\pi)=f\left(f^{-1}(25\pi)\right)=25\pi$

25. a. $f(3)=2(3)-3=3$

b. Using the result from part (a), $f^{-1}(3)=3$.

c. $\left(f\circ f^{-1}\right)(19)=f\left(f^{-1}(19)\right)=19$

d. $\left(f\circ f^{-1}\right)(5)=f\left(f^{-1}(5)\right)=5$

26. a. $f(2)=2^3=8$

b. Using the result from part (a), $f^{-1}(8)=2$.

c. $\left(f\circ f^{-1}\right)(15)=f\left(f^{-1}(15)\right)=15$

d. $\left(f\circ f^{-1}\right)(27)=f\left(f^{-1}(27)\right)=27$

27. a. $f(1)=1^3+1=2$

b. Using the result from part (a), $f^{-1}(2)=1$.

c. $\left(f\circ f^{-1}\right)(269)=f\left(f^{-1}(269)\right)=269$

28. a. $g(1)=\sqrt[3]{2\left(1^3\right)-1}=\sqrt[3]{1}=1$

b. Using the result from part (a), $g^{-1}(1)=1$.

c. $\left(g^{-1}\circ g\right)(135)=g^{-1}\left(g(135)\right)=135$

29.
$$f\left(g(x)\right)=3g(x)+1=3\left(\frac{x-1}{3}\right)+1$$
$$=x-1+1=x$$
$$g\left(f(x)\right)=\frac{f(x)-1}{3}=\frac{(3x+1)-1}{3}=\frac{3x}{3}=x$$

30.
$$f\left(g(x)\right)=2-3g(x)=2-3\left(\frac{2-x}{3}\right)$$
$$=2-2+x=x$$
$$g\left(f(x)\right)=\frac{2-f(x)}{3}=\frac{2-(2-3x)}{3}=\frac{3x}{3}=x$$

31.
$$f\left(g(x)\right)=\left[g(x)\right]^3=\left(\sqrt[3]{x}\right)^3=x$$
$$g\left(f(x)\right)=\sqrt[3]{f(x)}=\sqrt[3]{x^3}=x$$

32.
$$f\left(g(x)\right)=\frac{1}{g(x)}=\frac{1}{\frac{1}{x}}=x$$
$$g\left(f(x)\right)=\frac{1}{f(x)}=\frac{1}{\frac{1}{x}}=x$$

33.
$$f\left(g(x)\right)=\frac{g(x)-1}{g(x)+2}=\frac{\frac{1+2x}{1-x}-1}{\frac{1+2x}{1-x}+2}$$
$$=\frac{\frac{1+2x-(1-x)}{1-x}}{\frac{1+2x+2(1-x)}{1-x}}=\frac{3x}{3}=x$$
$$g\left(f(x)\right)=\frac{1+2f(x)}{2-f(x)}=\frac{1+2\left(\frac{x-1}{x+2}\right)}{1-\frac{x-1}{x+2}}$$
$$=\frac{1+\frac{2x-2}{x+2}}{1-\frac{x-1}{x+2}}=\frac{\frac{x+2+2x-2}{x+2}}{\frac{x+2-(x-1)}{x+2}}=\frac{3x}{3}$$
$$=x$$

34.
$$f\left(g(x)\right)=\frac{3g(x)+2}{g(x)-1}=\frac{3\left(\frac{x+2}{x-3}\right)+2}{\frac{x+2}{x-3}-1}$$
$$=\frac{\frac{3x+6}{x-3}+\frac{2(x-3)}{x-3}}{\frac{x+2}{x-3}-\frac{1(x-3)}{x-3}}=\frac{\frac{3x+6+2x-6}{x-3}}{\frac{x+2-x+3}{x-3}}$$
$$=\frac{5x}{5}=x$$
$$g\left(f(x)\right)=\frac{f(x)+2}{f(x)-3}=\frac{\frac{3x+2}{x-1}+2}{\frac{3x+2}{x-1}-3}$$
$$=\frac{\frac{3x+2}{x-1}+\frac{2(x-1)}{x-1}}{\frac{3x+2}{x-1}-\frac{3(x-1)}{x-1}}=\frac{\frac{3x+2+2x-2}{x-1}}{\frac{3x+2-3x+3}{x-1}}$$
$$=\frac{5x}{5}=x$$

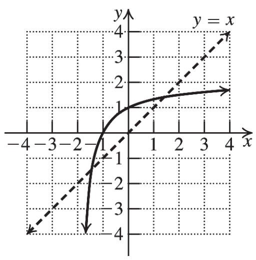
35. $f\left(g(x)\right)=2\left(\sqrt[5]{\dfrac{x-1}{2}}\right)^5+1=2\left(\dfrac{x-1}{2}\right)+1$
$=x-1+1=x$

$g\left(f(x)\right)=\sqrt[5]{\dfrac{\left(2x^5+1\right)-1}{2}}=\sqrt[5]{\dfrac{2x^5}{2}}=\sqrt[5]{x^5}=x$

36. $f\left(g(x)\right)=\left(1-3\left(\dfrac{1-\sqrt[3]{x}}{3}\right)\right)^3$
$=\left(1-1+\sqrt[3]{x}\right)^3=x$

$g\left(f(x)\right)=\dfrac{1-\sqrt[3]{(1-3x)^3}}{3}=\dfrac{1-(1-3x)}{3}$
$=\dfrac{3x}{3}=x$

37.



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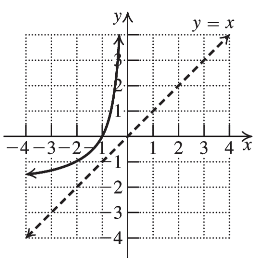

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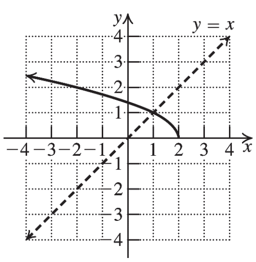

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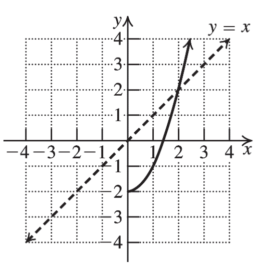

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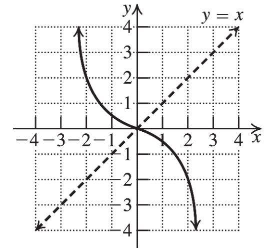

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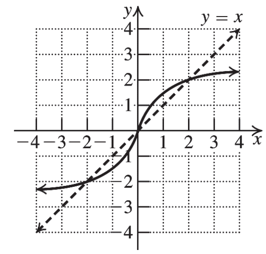

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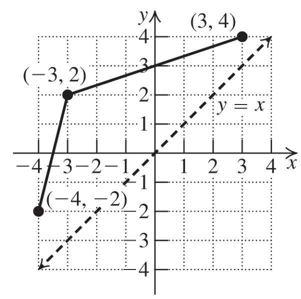

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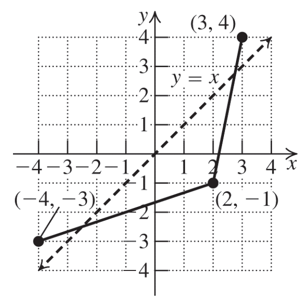

45. $f(x)=3x-1$

Domain: $(-\infty,\infty)$; range: $(-\infty,\infty)$

Replace $f(x)$ with y, interchange x and y, and then solve for y.

$y=3x-1\Rightarrow x=3y-1\Rightarrow x+1=3y\Rightarrow$

$y=f^{-1}(x)=\dfrac{x+1}{3}$

46. $f(x)=2x+3$

Domain: $(-\infty,\infty)$; range: $(-\infty,\infty)$

Replace $f(x)$ with y, interchange x and y, and then solve for y.

$y=2x+3\Rightarrow x=2y+3\Rightarrow x-3=2y\Rightarrow$

$y=f^{-1}(x)=\dfrac{x-3}{2}$

47. $f(x)=\sqrt[3]{\frac{x+1}{3}}+2$

Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

Replace $f(x)$ with *y*, interchange *x* and *y*, and then solve for *y*.

$y=\sqrt[3]{\frac{x+1}{3}}+2 \Rightarrow x=\sqrt[3]{\frac{y+1}{3}}+2 \Rightarrow$

$x-2=\sqrt[3]{\frac{y+1}{3}} \Rightarrow (x-2)^3=\frac{y+1}{3} \Rightarrow$

$3(x-2)^3=y+1 \Rightarrow$

$y=f^{-1}(x)=3(x-2)^3-1$

48. $f(x)=\sqrt[3]{\frac{x-2}{3}}-1$

Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

Replace $f(x)$ with *y*, interchange *x* and *y*, and then solve for *y*.

$y=\sqrt[3]{\frac{x-2}{3}}-1 \Rightarrow x=\sqrt[3]{\frac{y-2}{3}}-1 \Rightarrow$

$x+1=\sqrt[3]{\frac{y-2}{3}} \Rightarrow (x+1)^3=\frac{y-2}{3} \Rightarrow$

$3(x+1)^3=y-2 \Rightarrow$

$y=f^{-1}(x)=3(x+1)^3+2$

49. $f(x)=(3x-1)^3+2$

Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

Replace $f(x)$ with *y*, interchange *x* and *y*, and then solve for *y*.

$y=(3x-1)^3+2 \Rightarrow x=(3y-1)^3+2 \Rightarrow$

$x-2=(3y-1)^3 \Rightarrow \sqrt[3]{x-2}=3y-1 \Rightarrow$

$\sqrt[3]{x-2}+1=3y \Rightarrow$

$y=f^{-1}(x)=\frac{\sqrt[3]{x-2}+1}{3}$

50. $f(x)=(2x+1)^3-3$

Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$

Replace $f(x)$ with *y*, interchange *x* and *y*, and then solve for *y*.

$y=(2x+1)^3-3 \Rightarrow x=(2y+1)^3-3 \Rightarrow$

$x+3=(2y+1)^3 \Rightarrow \sqrt[3]{x+3}=2y+1 \Rightarrow$

$\sqrt[3]{x+3}-1=2y \Rightarrow$

$y=f^{-1}(x)=\frac{\sqrt[3]{x+3}-1}{2}$

51. $f(x)=\frac{2}{1+x}$

Domain: $(-\infty, -1)\cup(-1, \infty)$

Range: $(-\infty, 0)\cup(0, \infty)$

Replace $f(x)$ with *y*, interchange *x* and *y*, and then solve for *y*.

$y=\frac{2}{1+x} \Rightarrow x=\frac{2}{1+y} \Rightarrow x(1+y)=2 \Rightarrow$

$1+y=\frac{2}{x} \Rightarrow y=f^{-1}(x)=\frac{2}{x}-1$

52. $f(x)=1-\frac{1}{x+1}$

Domain: $(-\infty, -1)\cup(-1, \infty)$

Range: $(-\infty, 1)\cup(1, \infty)$

Replace $f(x)$ with *y*, interchange *x* and *y*, and then solve for *y*.

$y=1-\frac{1}{x+1} \Rightarrow x=1-\frac{1}{y+1} \Rightarrow$

$x-1=-\frac{1}{y+1} \Rightarrow (x-1)(y+1)=-1 \Rightarrow$

$y+1=-\frac{1}{x-1} \Rightarrow y=f^{-1}(x)=-\frac{1}{x-1}-1$

53. $f(x)=\frac{x+1}{x-2}$

Domain: $(-\infty, 2)\cup(2, \infty)$

Range: $(-\infty, 1)\cup(1, \infty)$

Replace $f(x)$ with *y*, interchange *x* and *y*, and then solve for *y*.

$y=\frac{x+1}{x-2} \Rightarrow x=\frac{y+1}{y-2} \Rightarrow x(y-2)=y+1 \Rightarrow$

$xy-2x=y+1 \Rightarrow xy-y=2x+1 \Rightarrow$

$y(x-1)=2x+1 \Rightarrow y=f^{-1}(x)=\frac{2x+1}{x-1}$

54. $f(x)=\frac{1-2x}{1+x}$

Domain: $(-\infty, -1)\cup(-1, \infty)$

Range: $(-\infty, -2)\cup(-2, \infty)$

Replace $f(x)$ with *y*, interchange *x* and *y*, and then solve for *y*.

$y=\frac{1-2x}{1+x} \Rightarrow x=\frac{1-2y}{1+y} \Rightarrow$

$x(1+y)=1-2y \Rightarrow x+xy=1-2y \Rightarrow$

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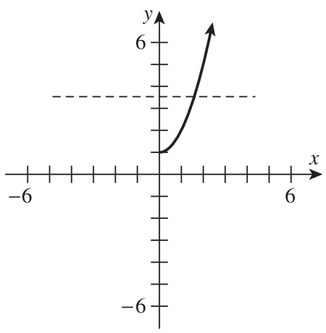
$$xy + 2y = 1 - x \Rightarrow y(x+2) = 1 - x \Rightarrow$$
$$y = f^{-1}(x) = \frac{1-x}{x+2} = -\frac{x-1}{x+2}$$

In exercises 55–58, note that the domain of *f* is the range of f^{-1}, and the range of *f* is the domain of f^{-1}.

55. $f(x) = x^2 + 1, \ x \geq 0$



The horizontal line test confirms that *f* is one-to-one.
Domain of *f*: $[0, \infty)$

Range of *f*: $[1, \infty)$

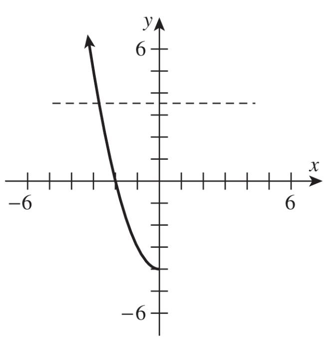
Find $f^{-1}(x)$ by interchanging the variables and solving for *y*.

$$y = x^2 + 1 \Rightarrow x = y^2 + 1 \Rightarrow x - 1 = y^2 \Rightarrow$$
$$y = f^{-1}(x) = \sqrt{x-1}$$

Domain of f^{-1}: $[1, \infty)$

Range of f^{-1}: $[0, \infty)$

56. $f(x) = x^2 - 4, \ x \leq 0$



The horizontal line test confirms that *f* is one-to-one.
Domain of *f*: $(-\infty, 0]$

Range of *f*: $[-4, \infty)$

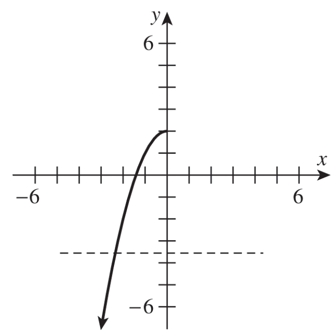
Find $f^{-1}(x)$ by interchanging the variables and solving for *y*.

$$y = x^2 - 4 \Rightarrow x = y^2 - 4 \Rightarrow x + 4 = y^2 \Rightarrow$$
$$y = f^{-1}(x) = -\sqrt{x+4}$$

Domain of f^{-1}: $[-4, \infty)$

Range of f^{-1}: $(-\infty, 0]$

57. $f(x) = -x^2 + 2, \ x \leq 0$



The horizontal line test confirms that *f* is one-to-one.
Domain of *f*: $(-\infty, 0]$; range of *f*: $(-\infty, 2]$

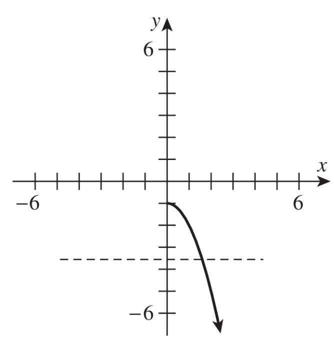
Find $f^{-1}(x)$ by interchanging the variables and solving for *y*.

$$y = -x^2 + 2 \Rightarrow x = -y^2 + 2 \Rightarrow x - 2 = -y^2 \Rightarrow$$
$$y^2 = 2 - x \Rightarrow y = f^{-1}(x) = -\sqrt{2-x}$$

Domain of f^{-1}: $(-\infty, 2]$

Range of f^{-1}: $(-\infty, 0]$

58. $f(x) = -x^2 - 1, \ x \geq 0$



The horizontal line test confirms that *f* is one-to-one.

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Domain of *f*: $[0, \infty)$

Range of *f*: $(-\infty, -1]$

Find $f^{-1}(x)$ by interchanging the variables and solving for *y*.

$y = -x^2 - 1 \Rightarrow x = -y^2 - 1 \Rightarrow x + 1 = -y^2 \Rightarrow$
$y^2 = -x - 1 \Rightarrow y = f^{-1}(x) = \sqrt{-x-1}$

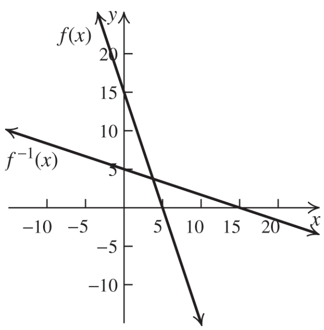
Domain of $f^{-1}: (-\infty, -1]$

Range of $f^{-1}: [0, \infty)$

59. a. One-to-one

b. $f(x) = y = 15 - 3x$. Interchange the variables and solve for $y: x = 15 - 3y \Rightarrow$
$y = f^{-1}(x) = \dfrac{15 - x}{3} = 5 - \dfrac{1}{3}x.$

c.

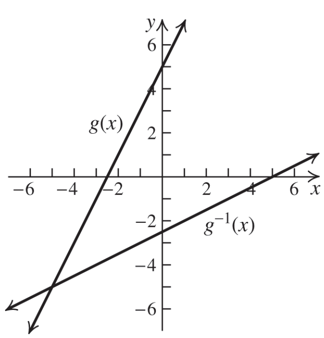


d. Domain of *f*: $(-\infty, \infty)$; *x*-intercept of *f*: 5;
y-intercept of *f*: 15
domain of f^{-1}: $(-\infty, \infty)$; *x*-intercept of
f^{-1}: 15; *y*-intercept of f^{-1}: 5

60. a. One-to-one

b. $g(x) = y = 2x + 5$. Interchange the variables and solve for $y: x = 2y + 5 \Rightarrow$
$y = g^{-1}(x) = \dfrac{x-5}{2} = \dfrac{1}{2}x - \dfrac{5}{2}.$

c.



d. Domain of *g*: $(-\infty, \infty)$

x-intercept of *g*: $-\dfrac{5}{2}$

y-intercept of *g*: 5

domain of g^{-1}: $(-\infty, \infty)$; *x*-intercept of g^{-1}: 5; *y*-intercept of g^{-1}: $-\dfrac{5}{2}$

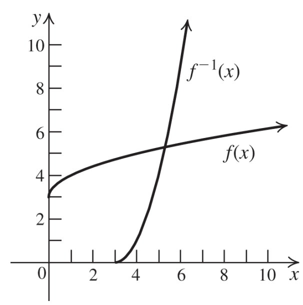
61. a. Not one-to-one because
$f(1) = \sqrt{4 - 1^2} = \sqrt{3}$
$f(-1) = \sqrt{4 - (-1)^2} = \sqrt{4 - 1^2} = \sqrt{3}$

62. a. Not one-to-one because
$f(1) = -\sqrt{9 - 1^2} = -\sqrt{8}$
$f(-1) = -\sqrt{9 - (-1)^2} = -\sqrt{9 - 1^2} = -\sqrt{8}$

63. a. One-to-one

b. $f(x) = y = \sqrt{x} + 3$ Interchange the variables and solve for $y: x = \sqrt{y} + 3 \Rightarrow$
$x - 3 = \sqrt{y} \Rightarrow y = f^{-1}(x) = (x-3)^2.$

c.

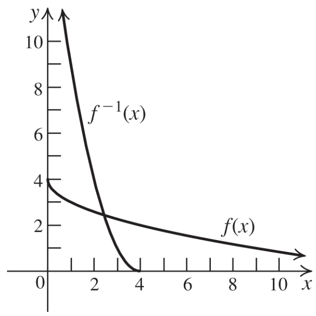


d. Domain of *f*: $[0, \infty)$; *x*-intercept of *f*: none;
y-intercept of *f*: 3
domain of f^{-1}: $[3, \infty)$; *x*-intercept of
f^{-1}: 3; *y*-intercept of f^{-1}: none

64. a. One-to-one

b. $f(x) = y = 4 - \sqrt{x}$. Interchange the variables and solve for $y: x = 4 - \sqrt{y} \Rightarrow$
$-4 + x = -\sqrt{y} \Rightarrow y = f^{-1}(x) = (x-4)^2$

c.

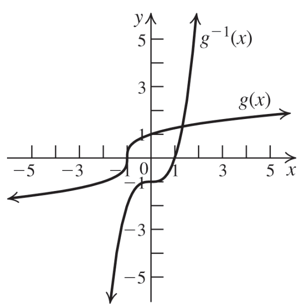


d. Domain of f: $[0, \infty)$; x-intercept of f: 16; y-intercept of f: 4
domain of f^{-1}: $(-\infty, 4]$; x-intercept of f^{-1}: 4; y-intercept of f^{-1}: 16

65. a. One-to-one

b. $g(x) = y = \sqrt[3]{x+1}$. Interchange the variables and solve for $y: x = \sqrt[3]{y+1} \Rightarrow$
$x^3 = y + 1 \Rightarrow y = g^{-1}(x) = x^3 - 1$

c.

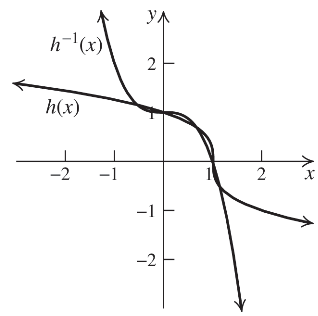


d. Domain of g: $(-\infty, \infty)$
x-intercept of g: −1
y-intercept of g: 1
domain of g^{-1}: $(-\infty, \infty)$; x-intercept of g^{-1}: 1; y-intercept of g^{-1}: −1

66. a. One-to-one

b. $h(x) = y = \sqrt[3]{1-x}$. Interchange the variables and solve for $y: x = \sqrt[3]{1-y} \Rightarrow$
$x^3 = 1 - y \Rightarrow y = g^{-1}(x) = 1 - x^3$.

c.

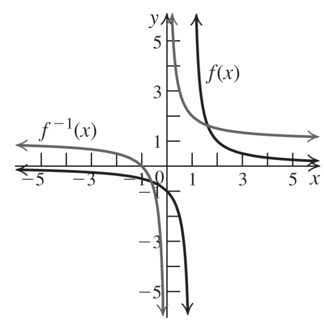


d. Domain of h: $(-\infty, \infty)$; x-intercept of h: 1; y-intercept of h: 1
domain of h^{-1}: $(-\infty, \infty)$; x-intercept of h^{-1}: 1; y-intercept of h^{-1}: 1

67. a. One-to-one

b. $f(x) = y = \dfrac{1}{x-1}$. Interchange the variables and solve for $y: x = \dfrac{1}{y-1} \Rightarrow x(y-1) = 1 \Rightarrow$
$\dfrac{1}{x} = y - 1 \Rightarrow y = f^{-1}(x) = \dfrac{1}{x} + 1 = \dfrac{1+x}{x}$.

c.

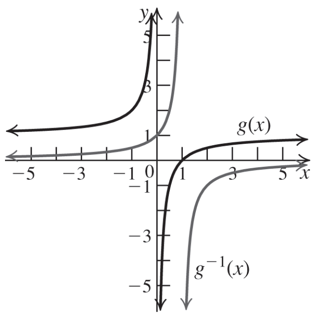


d. Domain of f: $(-\infty, 1) \cup (1, \infty)$
x-intercept of f: none; y-intercept of f: −1
domain of f^{-1}: $(-\infty, 0) \cup (0, \infty)$
x-intercept of f^{-1}: −1
y-intercept of f^{-1}: none

68. a. One-to-one

b. $g(x) = y = 1 - \dfrac{1}{x}$. Interchange the variables and solve for $y: x = 1 - \dfrac{1}{y} \Rightarrow x = \dfrac{y-1}{y} \Rightarrow$
$xy = y - 1 \Rightarrow xy - y = -1 \Rightarrow y(x-1) = -1 \Rightarrow$
$y = g^{-1}(x) = -\dfrac{1}{x-1} = \dfrac{1}{1-x}$.

c.

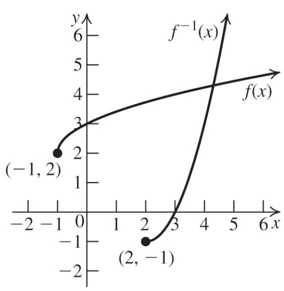


d. Domain of g: $(-\infty, 0) \cup (0, \infty)$
x-intercept of g: 1; y-intercept of g: none
domain of g^{-1}: $(-\infty, 1) \cup (1, \infty)$
x-intercept of g^{-1}: none
y-intercept of g^{-1}: 1

69. a. One-to-one

b. $f(x) = y = 2 + \sqrt{x+1}$. Interchange the variables and solve for y: $x = 2 + \sqrt{y+1} \Rightarrow$
$x - 2 = \sqrt{y+1} \Rightarrow (x-2)^2 = y + 1 \Rightarrow$
$y = f^{-1}(x) = (x-2)^2 - 1 = x^2 - 4x + 3$

c.

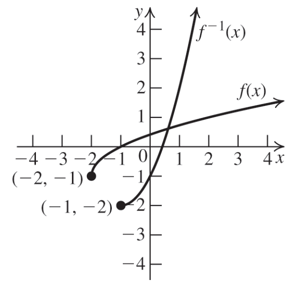


d. Domain of f: $[-1, \infty)$; x-intercept of f: none; y-intercept of f: 3
Domain of f^{-1}: $[2, \infty)$;
x-intercept of f^{-1}: 3
y-intercept of f^{-1}: none

70. a. One-to-one

b. $f(x) = y = -1 + \sqrt{x+2}$. Interchange the variables and solve for y:
$x = -1 + \sqrt{y+2} \Rightarrow x + 1 = \sqrt{y+2} \Rightarrow$
$(x+1)^2 = y + 2 \Rightarrow$
$y = f^{-1}(x) = (x+1)^2 - 2 = x^2 + 2x - 1$

c.



d. Domain of f: $[-2, \infty)$; x-intercept of f: -1;
y-intercept of f: $-1 + \sqrt{2}$
Domain of f^{-1}: $[-1, \infty)$
x-intercept of f^{-1}: $-1 + \sqrt{2}$
y-intercept of f^{-1}: -1

71. Domain: $(-\infty, -2) \cup (-2, \infty)$
Range: $(-\infty, 1) \cup (1, \infty)$

72. Domain: $(-\infty, 1) \cup (1, \infty)$
Range: $(-\infty, 3) \cup (3, \infty)$

In exercises 73–76, use the fact that the range of f is the same as the domain of f^{-1}.

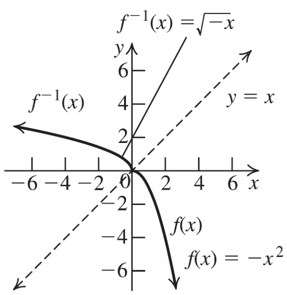
73. $f(x) = y = \dfrac{x+1}{x-2}$. Interchange the variables and solve for y: $x = \dfrac{y+1}{y-2} \Rightarrow xy - 2x = y + 1 \Rightarrow$
$xy - y = 2x + 1 \Rightarrow y(x-1) = 2x + 1 \Rightarrow$
$y = f^{-1}(x) = \dfrac{2x+1}{x-1}$.
Domain of f: $(-\infty, 2) \cup (2, \infty)$
Range of f: $(-\infty, 1) \cup (1, \infty)$.

74. $g(x) = y = \dfrac{x+2}{x+1}$. Interchange the variables and solve for y: $x = \dfrac{y+2}{y+1} \Rightarrow xy + x = y + 2 \Rightarrow$
$xy - y = -x + 2 \Rightarrow y(x-1) = -x + 2 \Rightarrow$
$y = g^{-1}(x) = \dfrac{-x+2}{x-1} = \dfrac{x-2}{1-x}$.
Domain of g: $(-\infty, -1) \cup (-1, \infty)$
Range of g: $(-\infty, 1) \cup (1, \infty)$.

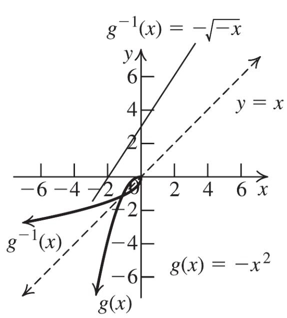
75. $f(x) = y = \dfrac{1-2x}{1+x}$. Interchange the variables and solve for $y: x = \dfrac{1-2y}{1+y} \Rightarrow$
$x + xy = 1 - 2y \Rightarrow xy + 2y = 1 - x \Rightarrow$
$y(x+2) = 1 - x \Rightarrow y = f^{-1}(x) = \dfrac{1-x}{x+2}$.
Domain of *f*: $(-\infty, -1) \cup (-1, \infty)$
Range of *f*: $(-\infty, -2) \cup (-2, \infty)$.

76. $h(x) = y = \dfrac{x-1}{x-3}$. Interchange the variables and solve for $y: x = \dfrac{y-1}{y-3} \Rightarrow xy - 3x = y - 1 \Rightarrow$
$xy - y = 3x - 1 \Rightarrow y(x-1) = 3x - 1 \Rightarrow$
$y = h^{-1}(x) = \dfrac{3x-1}{x-1}$.
Domain of *h*: $(-\infty, 3) \cup (3, \infty)$
Range of *h*: $(-\infty, 1) \cup (1, \infty)$.

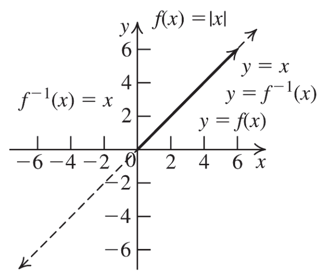
77. *f* is one-to-one because the domain is restricted, so an inverse exists.
$f(x) = y = -x^2, x \ge 0$. Interchange the variables and solve for y:
$x = -y^2 \Rightarrow y = \sqrt{-x},\ x \le 0$.



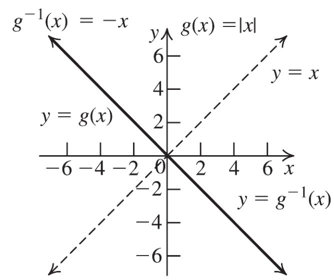
78. *g* is one-to-one because the domain is restricted, so an inverse exists.
$g(x) = y = -x^2,\ x \le 0$. Interchange the variables and solve for y:
$x = -y^2 \Rightarrow y = -\sqrt{-x},\ x \le 0$.



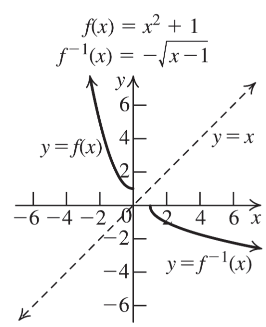
79. *f* is one-to-one because the domain is restricted, so an inverse exists.
$f(x) = y = |x| = x,\ x \ge 0$. Interchange the variables and solve for $y: y = x,\ x \ge 0$.



80. *g* is one-to-one because the domain is restricted, so an inverse exists.
$g(x) = y = |x| = -x,\ x \le 0$. Interchange the variables and solve for $y: y = -x,\ x \ge 0$.



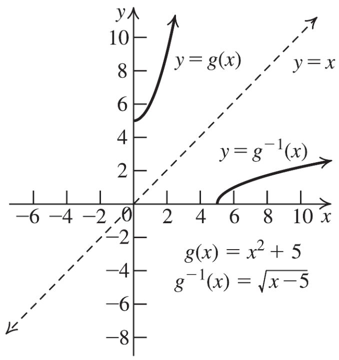
81. *f* is one-to-one because the domain is restricted, so an inverse exists.
$f(x) = y = x^2 + 1,\ x \le 0$. Interchange the variables and solve for y:
$x = y^2 + 1 \Rightarrow y = -\sqrt{x-1},\ x \ge 1$.



82. g is one-to-one because the domain is restricted, so an inverse exists.

$g(x) = y = x^2 + 5,\ x \geq 0$. Interchange the variables and solve for y:

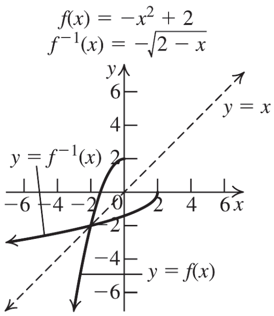
$x = y^2 + 5 \Rightarrow y = \sqrt{x-5},\ x \geq 5.$



83. f is one-to-one because the domain is restricted, so an inverse exists.

$f(x) = y = -x^2 + 2,\ x \leq 0$. Interchange the variables and solve for y:

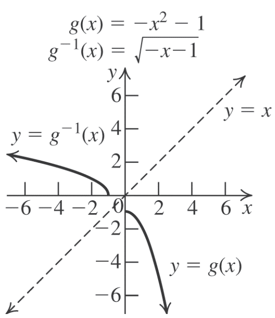
$x = -y^2 + 2 \Rightarrow y = -\sqrt{2-x},\ x \leq 2.$



84. g is one-to-one since the domain is restricted, so an inverse exists.

$g(x) = y = -x^2 - 1,\ x \geq 0$. Interchange the variables and solve for y:

$x = -y^2 - 1 \Rightarrow y = \sqrt{-x-1},\ x \leq -1.$



85. $f(x) = (3x-1)^3 + 2$

$f(0) = 1; \quad f(1) = 10$

First we compute the corresponding average rate of f.

$$\text{ARC of } f \text{ from 0 to 1} = \frac{f(1) - f(0)}{1-0} = \frac{10-1}{1} = 9$$

Now compute the average rate of change of f^{-1} as

$$\text{ARC of } f^{-1} \text{ from } f(0) = 1 \text{ to } f(1) = 10 = \frac{1}{\text{ARC of } f \text{ from 0 to 1}} = \frac{1}{9}$$

86. $f(x) = 1 - (2x-1)^3$

$f(0) = 2; \quad f(1) = 0$

First we compute the corresponding average rate of f.

$$\text{ARC of } f \text{ from 0 to 1} = \frac{f(1) - f(0)}{1-0} = \frac{0-2}{1} = -2$$

Now compute the average rate of change of f^{-1} as

$$\text{ARC of } f^{-1} \text{ from } f(0) = 2 \text{ to } f(1) = 0 = \frac{1}{\text{ARC of } f \text{ from 0 to 1}} = -\frac{1}{2}$$

87. $f(x) = \dfrac{3}{x-1}$

$f(4) = 1; \quad f(9) = \dfrac{3}{8}$

First we compute the corresponding average rate of f.

$$\text{ARC of } f \text{ from 4 to 9} = \frac{f(9) - f(4)}{9-4} = \frac{\frac{3}{8} - 1}{5} = -\frac{1}{8}$$

Now compute the average rate of change of f^{-1} as

$$\text{ARC of } f^{-1} \text{ from } f(4) = 1 \text{ to } f(9) = \frac{3}{8} = \frac{1}{\text{ARC of } f \text{ from 4 to 9}} = -8$$

88. $f(x) = 1 - \frac{2}{x}$

$f(1) = -1; \quad f(3) = \frac{1}{3}$

First we compute the corresponding average rate of *f*.

$$\text{ARC of } f \text{ from 1 to 3} = \frac{f(3) - f(1)}{3 - 1} = \frac{\frac{1}{3} - (-1)}{2} = \frac{2}{3}$$

Now compute the average rate of change of f^{-1} as

$$\text{ARC of } f^{-1} \text{ from } f(1) = -1 \text{ to } f(3) = \frac{1}{3} = \frac{1}{\text{ARC of } f \text{ from 1 to 3}} = \frac{3}{2}$$

89. $f(x) = \frac{2x+3}{x-2}$

$f(3) = 9; \quad f(5) = \frac{13}{3}$

First we compute the corresponding average rate of *f*.

$$\text{ARC of } f \text{ from 3 to 5} = \frac{f(3) - f(1)}{3 - 1} = \frac{\frac{13}{3} - 9}{2} = -\frac{7}{3}$$

Now compute the average rate of change of f^{-1} as

$$\text{ARC of } f^{-1} \text{ from } f(3) = 9 \text{ to } f(5) = \frac{13}{3} = \frac{1}{\text{ARC of } f \text{ from 3 to 5}} = -\frac{3}{7}$$

90. $f(x) = \frac{3x+1}{x+1}$

$f(0) = 1; \quad f(2) = \frac{7}{3}$

First we compute the corresponding average rate of *f*.

$$\text{ARC of } f \text{ from 0 to 2} = \frac{f(2) - f(0)}{2 - 0} = \frac{\frac{7}{3} - 1}{2} = \frac{2}{3}$$

Now compute the average rate of change of f^{-1} as

$$\text{ARC of } f^{-1} \text{ from } f(0) = 1 \text{ to } f(2) = \frac{7}{3} = \frac{1}{\text{ARC of } f \text{ from 0 to 2}} = \frac{3}{2}$$

Applying the Concepts

91. a. $K(C) = C + 273 \Rightarrow C(K) = K - 273 = K^{-1}(C).$ This represents the Celsius temperature corresponding to a given Kelvin temperature.

b. $C(300) = 300 - 273 = 27°\text{C}$

c. $K(22) = 22 + 273 = 295°\text{K}$

92. a. The two points are (212, 373) and (32, 273). The rate of change is $\frac{373 - 273}{212 - 32} = \frac{100}{180} = \frac{5}{9}$.

$$273 = \frac{5}{9}(32) + b \Rightarrow b = \frac{2297}{9} \Rightarrow$$

$$K(F) = \frac{5}{9}F + \frac{2297}{9}.$$

b. $K = \frac{5}{9}F + \frac{2297}{9} \Rightarrow K - \frac{2297}{9} = \frac{5}{9}F \Rightarrow$

$9K - 2297 = 5F \Rightarrow F(K) = \frac{9}{5}K - \frac{2297}{5}$

This represents the Fahrenheit temperature corresponding to a given Kelvin temperature.

c. $K(98.6) = \frac{5}{9}(98.6) + \frac{2297}{9} = 310°\text{K}$

93. a.

$$F(K(C)) = \frac{9}{5}K(C) - \frac{2297}{5} = \frac{9}{5}(C + 273) - \frac{2297}{5} = \frac{9}{5}C + \frac{9(273)}{5} - \frac{2297}{5} = \frac{9}{5}C + \frac{160}{5} = \frac{9}{5}C + 32$$

b.

$$C(K(F)) = K(F) - 273 = \frac{5}{9}F + \frac{2297}{9} - 273 = \frac{5}{9}F + \frac{2297 - 2457}{9} = \frac{5}{9}F - \frac{160}{9} = \frac{5}{9}(F - 32)$$

94. $F(C(x)) = \frac{9}{5}(C(x)) + 32 = \frac{9}{5}\left(\frac{5}{9}x - \frac{160}{9}\right) + 32$
$= x - 32 + 32 = x$
$C(F(x)) = \frac{5}{9}(F(x)) - \frac{160}{9} = x + \frac{160}{9} - \frac{160}{9}$
$= x$
Therefore, F and C are inverses of each other.

95. a. $E(x) = 0.75x,$ where x represents the number of dollars
$D(x) = 1.25x,$ where x represents the number of euros.

b. $E(D(x)) = 0.75(1.25x) = 0.9375x \neq x.$
Therefore, the two functions are not inverses.

c. She loses money both ways.

96. a. $w = 4 + 0.05x \Rightarrow w - 4 = 0.05x \Rightarrow$
$x = 20w - 80.$
This represents the food sales in terms of his hourly wage.

b. $x = 20(12) - 80 = \$160$

97. a. $7 = 4 + 0.05x \Rightarrow x = \$60$. This means that if food sales $\leq \$60$, he will receive the minimum hourly wage. If food sales $> \$60$, his wages will be based on food sales.
$$w = \begin{cases} 4 + 0.05x & \text{if } x > 60 \\ 7 & \text{if } x \leq 60 \end{cases}$$

b. The function does not have an inverse because it is constant on (0, 60), and thus, it is not one-to-one.

c. If the domain is restricted to $[60, \infty),$ the function has an inverse.

98. a. $T = 1.11\sqrt{l} \Rightarrow l = \left(\frac{T}{1.11}\right)^2$
This shows the length as the function of the period.

b. $l = \left(\frac{2}{1.11}\right)^2 \approx 3.2$ ft

c. $T = 1.11\sqrt{70} \approx 9.3$ sec

99. a. $\text{CO} = \text{PP} \cdot \text{HR} \cdot 0.002 \Rightarrow \text{HR} = \frac{\text{CO}}{\text{PP} \cdot 0.002}$

b. $\text{PP} = \text{SBP} - \text{DBP} = 115.5 - 70.5 = 45$
$\text{HR} = \frac{\text{CO}}{\text{PP} \cdot 0.002} = \frac{6.48}{45 \cdot 0.002}$
$= 72$ beats per minute

100. a. $\text{CO} = \text{PP} \cdot \text{HR} \cdot 0.002 \Rightarrow \text{PP} = \frac{\text{CO}}{\text{HR} \cdot 0.002}$

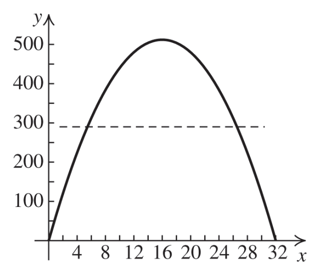
b. $\text{HR} = \frac{\text{CO}}{\text{PP} \cdot 0.002} \Rightarrow 80 = \frac{6.08}{\text{PP} \cdot 0.002} \Rightarrow$
$\text{PP} = \frac{6.08}{80 \cdot 0.002} = 38$ mmHg

101. a. $V = 8\sqrt{x} \Rightarrow \frac{V}{8} = \sqrt{x} \Rightarrow \frac{1}{64}V^2 = x = V^{-1}(x)$
This represents the height of the water in terms of the velocity.

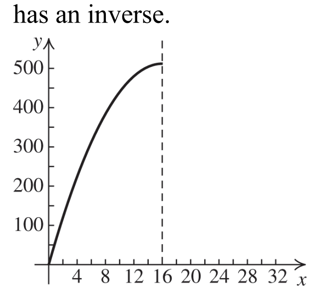
b. **(i)** $x = \frac{1}{64}\left(30^2\right) = 14.0625$ ft

(ii) $x = \frac{1}{64}\left(20^2\right) = 6.25$ ft

102. a. $y = 64x - 2x^2$ has no inverse because it is not one-to-one across its domain, [0, 32]. (It fails the horizontal line test.)



However, if the domain is restricted to [0, 16], the function is one-to-one, and it has an inverse.



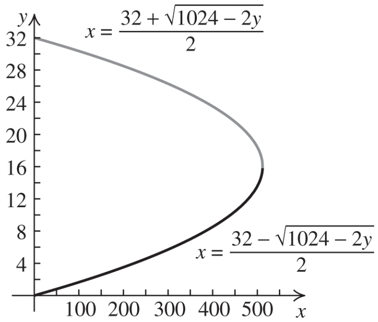
$y = 64x - 2x^2 \Rightarrow 2x^2 - 64x + y = 0 \Rightarrow$
$x = \frac{64 \pm \sqrt{64^2 - 8y}}{4} \Rightarrow$
$x = \frac{64 \pm \sqrt{4096 - 8y}}{4} = \frac{64 \pm 2\sqrt{1024 - 2y}}{4}$
$= \frac{32 \pm \sqrt{1024 - 2y}}{2}$
$1024 - 2y \geq 0 \Rightarrow 0 \leq y \leq 512.$

(*continued on next page*)

(*continued*)

(Because y is a number of feet, it cannot be negative.) This is the range of the original function. The domain of the original function is [0, 16], which is the range of the inverse.

The range of $x = \frac{32+\sqrt{1024-2y}}{2}$ is [16, 32], so this is not the inverse.
The range of
$x = \frac{32-\sqrt{1024-2y}}{2},\ 0 \le y \le 512,$ is [0, 16], so this is the inverse.



Note that the bottom half of the graph is the inverse.

b. **(i)** $x = \frac{32-\sqrt{1024-2(32)}}{2} \approx 0.51$ ft

(ii) $x = \frac{32-\sqrt{1024-2(256)}}{2} \approx 4.69$ ft

(iii) $x = \frac{32-\sqrt{1024-2(512)}}{2} \approx 16$ ft

103. a. The function represents the amount she still owes after x months.

b. $y = 36{,}000 - 600x$. Interchange the variables and solve for y: $x = 36{,}000 - 600y \Rightarrow$

$600y = 36{,}000 - x \Rightarrow y = 60 - \frac{x}{600} \Rightarrow$

$f^{-1}(x) = 60 - \frac{1}{600}x.$

This represents the number of months that have passed from the first payment until the balance due is \$$x$.

c. $y = 60 - \frac{1}{600}(22{,}000) \approx 23.33 \approx 24$ months

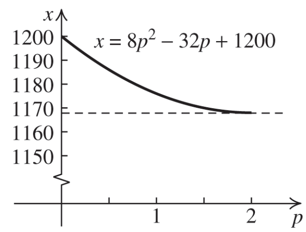
There are approximately 24 months remaining.

104. a. To find the inverse, solve $x = 8p^2 - 32p + 1200$ for p:

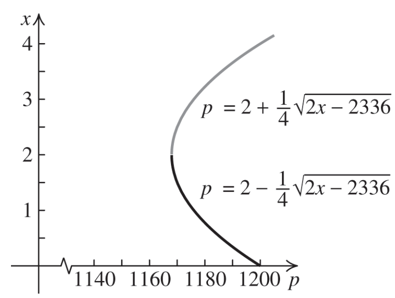
$8p^2 - 32p + 1200 - x = 0 \Rightarrow$

$$p = \frac{32 \pm \sqrt{(-32)^2 - 4(8)(1200-x)}}{2(8)}$$
$$= \frac{32 \pm \sqrt{1024 - 38{,}400 + 32x}}{16}$$
$$= \frac{32 \pm \sqrt{32x - 37376}}{16} = \frac{32 \pm 4\sqrt{2x-2336}}{16}$$
$$= 2 \pm \frac{1}{4}\sqrt{2x-2336}$$

Because the domain of the original function is (0, 2], its range is [1168, 1200).



So the domain of the inverse is [1168, 1200), and its range is (0, 2]. The range of $p = 2 + \frac{1}{4}\sqrt{2x-2336}$ is (2, 4], so it is not the inverse. The range of $p = 2 - \frac{1}{4}\sqrt{2x-2336}, 1168 \le x < 1200,$ is (0, 2], so it is the inverse. This gives the price of computer chips in terms of the demand x.



Note that the bottom half of the graph is the inverse.

b. $p = 2 - \frac{1}{4}\sqrt{2(1180.5) - 2336} = \$0.75$

105. a. $F(r) = GMm\frac{1}{r^2}$ is a one-to-one function because it is decreasing on $(0, \infty)$.

b. $F = GMm\frac{1}{r^2} \Rightarrow \frac{F}{GMm} = \frac{1}{r^2} \Rightarrow$
$r^2 = \frac{GMm}{F} \Rightarrow r = \sqrt{\frac{GMm}{F}}$

c. $y = GMm\frac{1}{x^2} \Rightarrow r = GMm\frac{1}{y^2} \Rightarrow$
$\frac{x}{GMm} = \frac{1}{y^2} \Rightarrow y^2 = \frac{GMm}{x} \Rightarrow$
$y = f^{-1}(x) = \sqrt{\frac{GMm}{x}}$

106. a. $V(r) = \frac{4}{3}\pi r^3$ is a one-to-one function because it is increasing on $(0, \infty)$.

b. $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{3V}{4\pi} = r^3 \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$

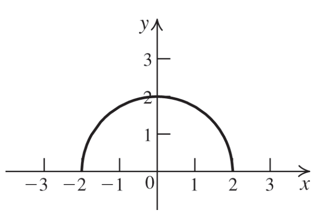
c. $V(x) = y = \frac{4}{3}\pi x^3 \Rightarrow x = \frac{4}{3}\pi y^3 \Rightarrow$
$y^3 = \frac{3x}{4\pi} \Rightarrow y = V^{-1}(x) = \sqrt[3]{\frac{3x}{4\pi}}$

Beyond the Basics

107. $f(g(3)) = f(1) = 3, f(g(5)) = f(3) = 5,$ and $f(g(2)) = f(4) = 2 \Rightarrow f(g(x)) = x$ for each x.
$g(f(1)) = g(3) = 1, g(f(3)) = g(5) = 3,$ and $g(f(4)) = g(2) = 4 \Rightarrow g(f(x)) = x$ for each x.
So, f and g are inverses.

108. $f(g(-2)) = f(1) = -2, f(g(0)) = f(2) = 0,$
$f(g(-3)) = f(3) = -3,$ and
$f(g(1)) = f(4) = 1 \Rightarrow f(g(x)) = x$
for each x.
$g(f(1)) = g(-2) = 1, g(f(2)) = g(0) = 2,$
$g(f(3)) = g(-3) = 3,$ and $g(f(4)) = g(1) = 4$
$\Rightarrow g(f(x)) = x$ for each x.
So f and g are inverses.

109. a. $f(x) = \sqrt{4 - x^2}$



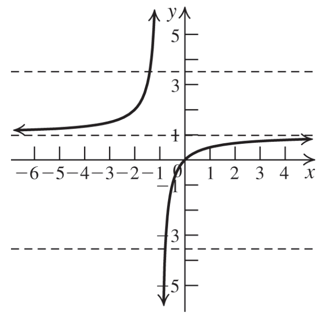
b. f is not one-to-one

c. Domain: [–2, 2]; range: [0, 2]

110. a. Domain: $(-\infty, 2) \cup [3, \infty)$. Note that the domain is not $(-\infty, 2) \cup (2, \infty)$ because $[\![x]\!] = 2$ for $2 \le x < 3$.

b. The function is not one-to-one. The function is constant on each interval $[n, n + 1)$, n an integer.

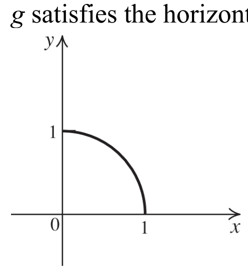
111. a. f satisfies the horizontal line test.



b. $y = 1 - \frac{1}{x+1}$. Interchange the variables and solve for $y: x = 1 - \frac{1}{y+1} \Rightarrow$
$\frac{1}{y+1} = 1 - x \Rightarrow 1 = y + 1 - xy - x \Rightarrow$
$xy - y = -x \Rightarrow y(x-1) = -x \Rightarrow$
$y = f^{-1}(x) = -\frac{x}{x-1} = \frac{x}{1-x}$

c. Domain of f: $(-\infty, -1) \cup (-1, \infty)$; range of f: $(-\infty, 1) \cup (1, \infty)$.

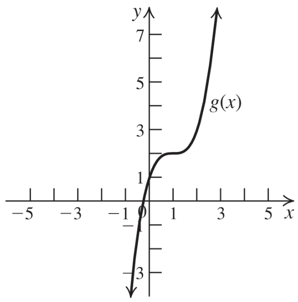
112. a. g satisfies the horizontal line test.



b. $y = \sqrt{1 - x^2}$. Interchange the variables and solve for $y: x = \sqrt{1 - y^2} \Rightarrow$
$x^2 = 1 - y^2 \Rightarrow y^2 = 1 - x^2 \Rightarrow$
$y = g^{-1}(x) = \sqrt{1 - x^2}$

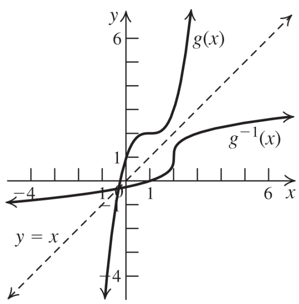
c. Domain of f = range of f: [0, 1]

113. a. The graph of g is the graph of f shifted one unit to the right and two units up.



b. $g(x) = y = (x-1)^3 + 2$. Interchange the variables and solve for y: $x = (y-1)^3 + 2 \Rightarrow$
$y = g^{-1}(x) = \sqrt[3]{x-2} + 1$.

c.



114. Each of the functions $1,\ 2x^3,\ 3x^5,$ and $4x^7$ is increasing. Therefore, f is increasing as a sum of increasing functions. Because f is increasing, it is a one-to-one function. We know that f^{-1} exists because f is a one-to-one function and an inverse exists. However, to find f^{-1} algebraically, we would have to solve the equation $x = 1 + 2y^3 + 3y^5 + 4y^7$ for y, which is not possible.

115. a. $M = \left(\frac{3+7}{2}, \frac{7+3}{2}\right) = (5, 5)$.

The coordinates of M satisfy the equation $y = x$, so M lies on the line.

b. The slope of $y = x$ is 1, while the slope of $\overline{PQ}$ is $\frac{3-7}{7-3} = -1$. So, $y = x$ is perpendicular to $\overline{PQ}$.

116. $M = \left(\frac{a+b}{2}, \frac{b+a}{2}\right)$.

Because the coordinates of M satisfy the equation $y = x$, M lies on the line. The slope of the line segment between the two points is $\frac{b-a}{a-b} = -1$, while the slope of $y = x$ is 1. So the two lines are perpendicular, and the points (a, b) and (b, a) are symmetric about the line $y = x$.

117. a. (i) $f(x) = y = 2x - 1$. Interchange the variables and solve for y: $x = 2y - 1 \Rightarrow$
$y = f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$

(ii) $g(x) = y = 3x + 4$. Interchange the variables and solve for y: $x = 3y + 4 \Rightarrow$
$y = g^{-1}(x) = \frac{1}{3}x - \frac{4}{3}$

(iii) $(f \circ g)(x) = 2(3x+4) - 1 = 6x + 7$

(iv) $(g \circ f)(x) = 3(2x-1) + 4 = 6x + 1$

(v) $(f \circ g)(x) = y = 6x + 7$. Interchange the variables and solve for y:
$x = 6y + 7 \Rightarrow (f \circ g)^{-1}(x) = \frac{1}{6}x - \frac{7}{6}$

(vi) $(g \circ f)(x) = y = 6x + 1$. Interchange the variables and solve for y:
$x = 6y + 1 \Rightarrow (f \circ g)^{-1}(x) = \frac{1}{6}x - \frac{1}{6}$

(vii) $\left(f^{-1} \circ g^{-1}\right)(x) = \frac{1}{2}\left(\frac{1}{3}x - \frac{4}{3}\right) + \frac{1}{2}$
$= \frac{1}{6}x - \frac{2}{3} + \frac{1}{2} = \frac{1}{6}x - \frac{1}{6}$

(viii) $\left(g^{-1} \circ f^{-1}\right)(x) = \frac{1}{3}\left(\frac{1}{2}x + \frac{1}{2}\right) - \frac{4}{3}$
$= \frac{1}{6}x + \frac{1}{6} - \frac{4}{3} = \frac{1}{6}x - \frac{7}{6}$

b. (i) $(f \circ g)^{-1}(x) = \frac{1}{6}x - \frac{7}{6}$
$= \left(g^{-1} \circ f^{-1}\right)(x)$

(ii) $(g \circ f)^{-1}(x) = \frac{1}{6}x - \frac{1}{6}$
$= \left(f^{-1} \circ g^{-1}\right)(x)$

118. a. (i) $f(x) = y = 2x + 3$. Interchange the variables and solve for y: $x = 2y + 3 \Rightarrow$
$y = f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$

(ii) $g(x) = y = x^3 - 1$. Interchange the variables and solve for y: $x = y^3 - 1 \Rightarrow$
$y = g^{-1}(x) = \sqrt[3]{x+1}$

(iii) $(f \circ g)(x) = 2(x^3 - 1) + 3 = 2x^3 + 1$

(iv) $(g \circ f)(x) = (2x+3)^3 - 1$
$= 8x^3 + 36x^2 + 54x + 26$

(v) $(f \circ g)(x) = y = 2x^3 + 1$. Interchange the variables and solve for y:
$x = 2y^3 + 1 \Rightarrow (f \circ g)^{-1}(x) = \sqrt[3]{\frac{x-1}{2}}$

(vi) $(g \circ f)(x) = y$
$= 8x^3 + 36x^2 + 54x + 26$
Interchange the variables and solve for y:
$x = 8y^3 + 36y^2 + 54y + 26 \Rightarrow$
$x + 1 = 8y^3 + 36y^2 + 54y + 27 \Rightarrow$
$x + 1 = (2y+3)^3 \Rightarrow \sqrt[3]{x+1} = 2y + 3 \Rightarrow$
$y = (g \circ f)^{-1}(x) = \frac{1}{2}\sqrt[3]{x+1} - \frac{3}{2}$

(vii) $(f^{-1} \circ g^{-1})(x) = \frac{1}{2}(\sqrt[3]{x+1}) - \frac{3}{2}$

(viii) $(g^{-1} \circ f^{-1})(x) = \sqrt[3]{\frac{1}{2}x - \frac{3}{2} + 1}$
$= \sqrt[3]{\frac{1}{2}x - \frac{1}{2}} = \sqrt[3]{\frac{x-1}{2}}$

b. (i) $(f \circ g)^{-1}(x) = \sqrt[3]{\frac{1}{2}x - \frac{1}{2}} = \sqrt[3]{\frac{x-1}{2}}$
$= (g^{-1} \circ f^{-1})(x)$

(ii) $(g \circ f)^{-1}(x) = \frac{1}{2}(\sqrt[3]{x+1}) - \frac{3}{2}$
$= (f^{-1} \circ g^{-1})(x)$

Critical Thinking/Discussion/Writing

119. No. For example, $f(x) = x^3 - x$ is odd, but it does not have an inverse, because $f(0) = f(1)$, so it is not one-to-one.

120. Yes. The function $f = \{(0,1)\}$ is even, and it has an inverse: $f^{-1} = \{(1,0)\}$.

121. Yes, because increasing and decreasing functions are one-to-one.

122. a. $R = \{(-1,1), (0,0), (1,1)\}$

b. $R = \{(-1,1), (0,0), (1,2)\}$

Active Learning

123. a.–c. Refer to the app using the QR code in your text.

Chapter 1 Key Ideas at a Glance

K1. $m = \frac{-4-0}{0-3} = \frac{-4}{-3} = \frac{4}{3}$
Note that 3 is the x-intercept of the line and –4 is the y-intercept of the line.
The slope-intercept form of the equation of the line through (3, 0) and (0, –4) is
$y = mx + b = \frac{4}{3}x - 4.$
Alternatively, we can use the point-slope format and solve for y.
$y - 0 = \frac{4}{3}(x-3) \Rightarrow y = \frac{4}{3}x - 4$
The two-intercept form of the equation of the line through (3, 0) and (0, –4) is
$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{3} + \frac{y}{-4} = 1 \Rightarrow \frac{x}{3} - \frac{y}{4} = 1.$

K2. $m = \frac{5-4}{0-2} = \frac{1}{-2} = -\frac{1}{2}$
Note that 5 is the y-intercept of the line. We are not given the x-intercept, so we will have to compute it.
The slope-intercept form of the equation of the line through (2, 4) and (0, 5) is
$y = mx + b = -\frac{1}{2}x + 5.$
Alternatively, we can use the point-slope format and solve for y.
$y - 4 = -\frac{1}{2}(x-2) \Rightarrow y - 4 = -\frac{1}{2}x + 1 \Rightarrow$
$y = -\frac{1}{2}x + 5$
Letting $y = 0$ gives
$0 = -\frac{1}{2}x + 5 \Rightarrow -5 = -\frac{1}{2}x \Rightarrow 10 = x.$
Thus, the x-intercept is 10.

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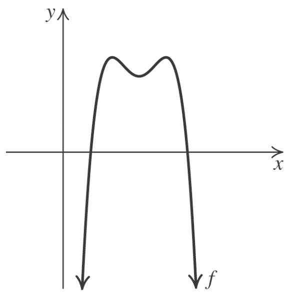
(*continued*)

The two-intercept form of the equation of the line through (2, 4) and (0, 5) is

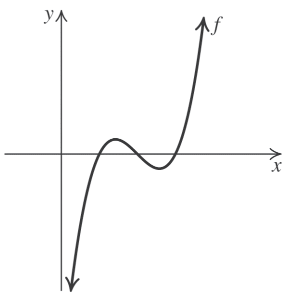
$$\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{10}+\frac{y}{5}=1.$$

For exercises K3 and K4, answers will vary. Sample answers are given.

K3.

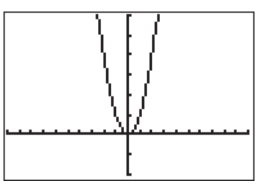


K4.

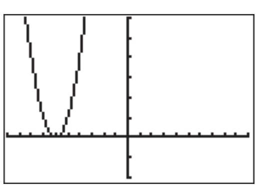


K5. The grapher screens shown below are graphed on the window [–10, 10] by [–2, 6].

Start with $f(x)=x^2$.

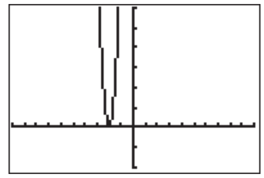


Shift the graph 6 units left to graph $f(x)=(x+6)^2$.

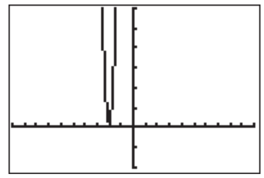


Then compress horizontally by a factor of 3 by multiplying each *x*-coordinate by $\frac{1}{3}$.

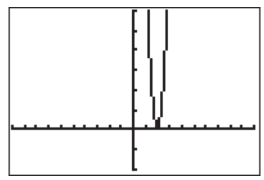
$f(x)=(3x+6)^2$



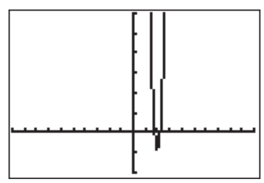
Next, stretch vertically by a factor of 2 to graph $f(x)=2(3x+6)^2$.

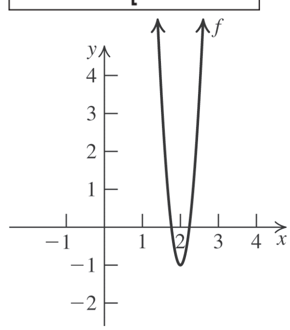


Now reflect the graph about the *y*-axis to graph $f(x)=2(-3x+6)^2$.



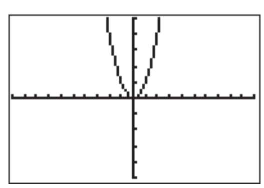
Finally, shift the graph down one unit to obtain the graph of $f(x)=2(-3x+6)^2-1$.





K6. The grapher screens shown below are graphed on the window [–10, 10] by [–5, 5].

Start with $f(x)=x^2$.



Shift the graph 4 units left to graph $f(x)=(x+4)^2$.

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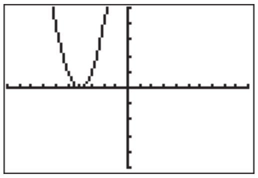
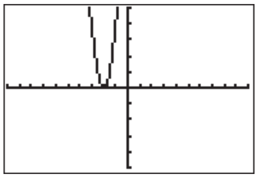
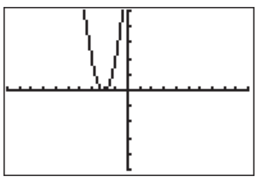


Then compress horizontally by a factor of 2 by multiplying each x-coordinate by $\frac{1}{2}$.

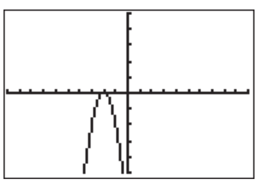
$f(x) = (2x+4)^2$



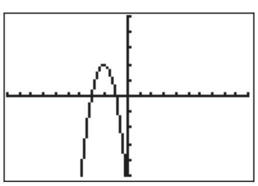
Next, compress vertically by a factor of $\frac{1}{2}$ to graph $f(x) = \frac{1}{2}(2x+4)^2$.

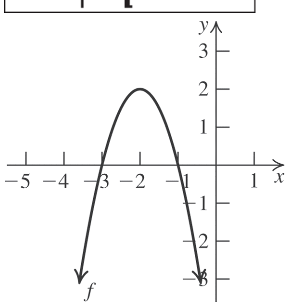


Now reflect the graph about the x-axis to graph $f(x) = -\frac{1}{2}(2x+4)^2$.



Finally, shift the graph up two units to obtain the graph of $f(x) = -\frac{1}{2}(2x+4)^2 + 2$.





Chapter 1 Review Exercises

Building Skills

1. False. The midpoint is $\left(\frac{-3+3}{2}, \frac{1+11}{2}\right) = (0, 6)$.

2. False. The equation is a circle with center $(-2, -3)$ and radius $\sqrt{5}$.

3. True

4. False. A graph that is symmetric with respect to the origin is the graph of an odd function. A graph that is symmetric with respect to the y-axis is the graph of an even function.

5. False.
The slope is 4/3 and the y-intercept is 3.

6. False. The slope of a line that is perpendicular to a line with slope 2 is –1/2.

7. True

8. False. There is no graph because the radius cannot be negative.

9. a. $d(P,Q) = \sqrt{(-1-3)^2 + (3-5)^2} = 2\sqrt{5}$

b. $M = \left(\frac{3+(-1)}{2}, \frac{5+3}{2}\right) = (1, 4)$

c. $m = \frac{3-5}{-1-3} = \frac{1}{2}$

10. a. $d(P,Q) = \sqrt{(3-(-3))^2 + (-1-5)^2} = 6\sqrt{2}$

b. $M = \left(\frac{-3+3}{2}, \frac{5+(-1)}{2}\right) = (0, 2)$

c. $m = \frac{-1-5}{3-(-3)} = -1$

11. a. $d(P,Q) = \sqrt{(9-4)^2 + (-8-(-3))^2} = 5\sqrt{2}$

b. $M = \left(\frac{4+9}{2}, \frac{-3+(-8)}{2}\right) = \left(\frac{13}{2}, -\frac{11}{2}\right)$

c. $m = \frac{-8-(-3)}{9-4} = -1$

12. a. $d(P,Q) = \sqrt{(-7-2)^2 + (-8-3)^2} = \sqrt{202}$

b. $M = \left(\frac{2+(-7)}{2}, \frac{3+(-8)}{2}\right) = \left(-\frac{5}{2}, -\frac{5}{2}\right)$

c. $m=\frac{-8-3}{-7-2}=\frac{11}{9}$

13. a. $D(P,Q)=\sqrt{(5-2)^2+(-2-(-7))^2}=\sqrt{34}$

b. $M=\left(\frac{2+5}{2},\frac{-7+(-2)}{2}\right)=\left(\frac{7}{2},-\frac{9}{2}\right)$

c. $m=\frac{-2-(-7)}{5-2}=\frac{5}{3}$

14. a. $d(P,Q)=\sqrt{(10-(-5))^2+(-3-4)^2}=\sqrt{274}$

b. $M=\left(\frac{-5+10}{2},\frac{4+(-3)}{2}\right)=\left(\frac{5}{2},\frac{1}{2}\right)$

c. $m=\frac{-3-4}{10-(-5)}=-\frac{7}{15}$

15. $d(A,B)=\sqrt{(-2-0)^2+(-3-5)^2}=\sqrt{68}$
$d(A,C)=\sqrt{(3-0)^2+(0-5)^2}=\sqrt{34}$
$d(B,C)=\sqrt{(3-(-2))^2+(0-(-3))^2}=\sqrt{34}$
Using the Pythagorean theorem, we have
$AC^2+BC^2=\left(\sqrt{34}\right)^2+\left(\sqrt{34}\right)^2$
$=68=\left(\sqrt{68}\right)^2=AB^2$

Alternatively, we can show that $\overline{AC}$ and $\overline{CB}$ are perpendicular using their slopes.
$m_{\overline{AC}}=\frac{0-5}{3-0}=-\frac{5}{3}; m_{\overline{CB}}=\frac{0-(-3)}{3-(-2)}=\frac{3}{5}$
$m_{\overline{AC}}\cdot m_{\overline{CB}}=-1\Rightarrow \overline{AC}\perp\overline{CB}$, so $\triangle ABC$ is a right triangle.

16. $d(A,B)=\sqrt{(4-1)^2+(8-2)^2}=3\sqrt{5}$
$d(C,D)=\sqrt{(10-7)^2+(5-(-1))^2}=3\sqrt{5}$
$d(A,C)=\sqrt{(7-1)^2+(-1-2)^2}=3\sqrt{5}$
$d(B,D)=\sqrt{(10-4)^2+(5-8)^2}=3\sqrt{5}$
The four sides have equal lengths, so the quadrilateral is a rhombus.

17. $A=(-6,3), B=(4,5)$
$d(A,O)=\sqrt{(-6-0)^2+(3-0)^2}=\sqrt{45}$
$d(B,O)=\sqrt{(4-0)^2+(5-0)^2}=\sqrt{41}$
(4, 5) is closer to the origin.

18. $A=(-6,4), B=(5,10), C=(2,3)$
$d(A,C)=\sqrt{(2-(-6))^2+(3-4)^2}=\sqrt{65}$
$d(B,C)=\sqrt{(2-5)^2+(3-10)^2}=\sqrt{58}$
(5, 10) is closer to (2, 3).

19. $A=(-5,3), B=(4,7), C=(x,0)$
$d(A,C)=\sqrt{(x-(-5))^2+(0-3)^2}$
$=\sqrt{(x+5)^2+9}$
$d(B,C)=\sqrt{(x-4)^2+(0-7)^2}$
$=\sqrt{(x-4)^2+49}$
$d(A,C)=d(B,C)\Rightarrow$
$\sqrt{(x+5)^2+9}=\sqrt{(x-4)^2+49}$
$(x+5)^2+9=(x-4)^2+49$
$x^2+10x+34=x^2-8x+65$
$x=\frac{31}{18}\Rightarrow$ The point is $\left(\frac{31}{18},0\right)$.

20. $A=(-3,-2), B(2,-1), C(0,y)$
$d(A,C)=\sqrt{(0-(-3))^2+(y-(-2))^2}$
$=\sqrt{(y+2)^2+9}$
$d(B,C)=\sqrt{(0-(2))^2+(y-(-1))^2}$
$=\sqrt{(y+1)^2+4}$
$d(A,C)=d(B,C)\Rightarrow$
$\sqrt{(y+2)^2+9}=\sqrt{(y+1)^2+4}$
$(y+2)^2+9=(y+1)^2+4$
$y^2+4y+13=y^2+2y+5$
$y=-4\Rightarrow$ The point is $(0,-4)$.

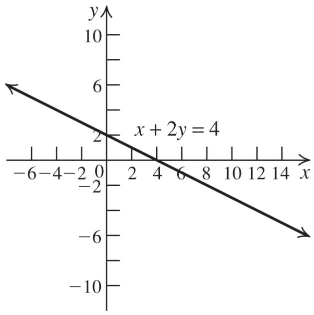
21. Not symmetric with respect to the *x*-axis; symmetric with respect to the *y*-axis; not symmetric with respect to the origin.

22. Not symmetric with respect to the *x*-axis; not symmetric with respect to the *y*-axis; symmetric with respect to the origin.

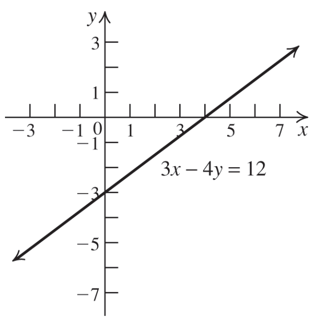
23. Symmetric with respect to the *x*-axis; not symmetric with respect to the *y*-axis; not symmetric with respect to the origin.

24. Symmetric with respect to the *x*-axis; symmetric with respect to the *y*-axis; symmetric with respect to the origin.

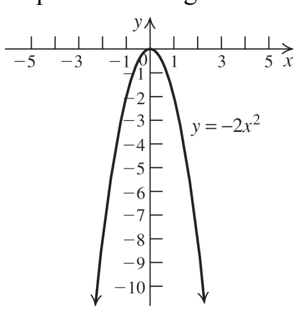
25. x-intercept: 4; y-intercept: 2; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



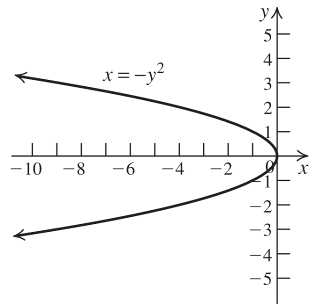
26. x-intercept: 4; y-intercept: –3; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



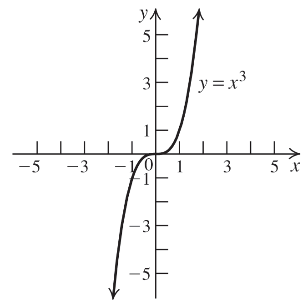
27. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.



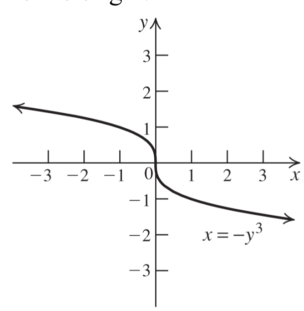
28. x-intercept: 0; y-intercept: 0; symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.



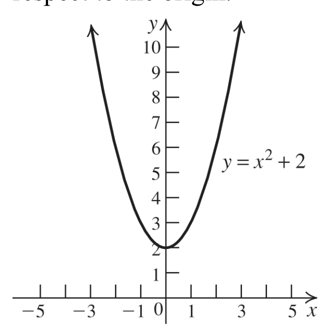
29. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.



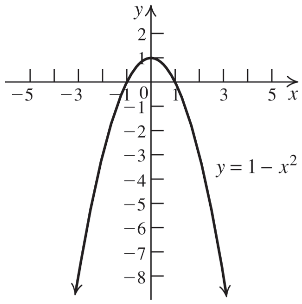
30. x-intercept: 0; y-intercept: 0; not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.



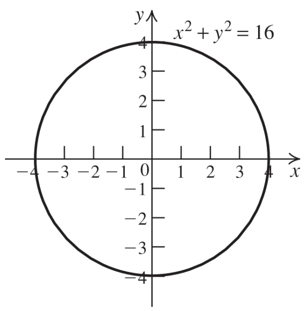
31. No x-intercept; y-intercept: 2; not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.



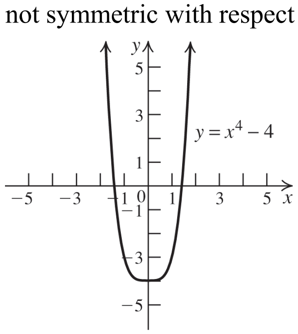
32. x-intercepts: –1, 1; y-intercept: 1;
not symmetric with respect to the x-axis;
symmetric with respect to the y-axis;
not symmetric with respect to the origin.



33. x-intercepts: –4, 4; y-intercepts: –4, 4;
symmetric with respect to the x-axis;
symmetric with respect to the y-axis;
symmetric with respect to the origin.



34. x-intercepts: $-\sqrt{2}$, $\sqrt{2}$; y-intercept: –4
not symmetric with respect to the x-axis
symmetric with respect to the y-axis
not symmetric with respect to the origin.



35. $(x-2)^2 + (y+3)^2 = 25$

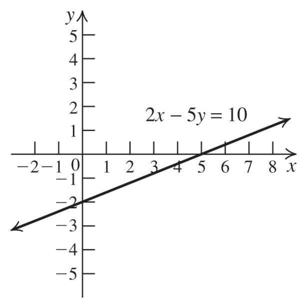
36. The center of the circle is the midpoint of the diameter. $M = \left(\frac{5+(-5)}{2}, \frac{2+4}{2}\right) = (0, 3).$

The length of the radius is the distance from the center to one of the endpoints of the diameter $= \sqrt{(5-0)^2 + (2-3)^2} = \sqrt{26}$. The equation of the circle is $x^2 + (y-3)^2 = 26$.

37. The radius is 2, so the equation of the circle is $(x+2)^2 + (y+5)^2 = 4$.

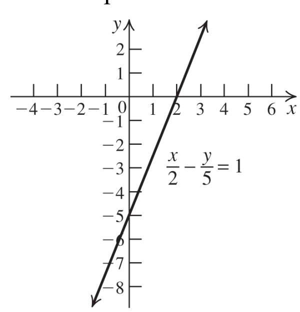
38. $2x - 5y = 10 \Rightarrow y = \frac{2}{5}x - 2$

Line with slope 2/5; y-intercept –2
x-intercept 5

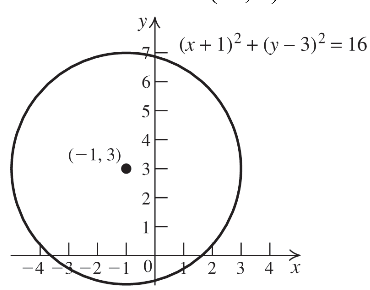


39. $\frac{x}{2} - \frac{y}{5} = 1 \Rightarrow 5x - 2y = 10 \Rightarrow y = \frac{5}{2}x - 5$

Line with slope 5/2; y-intercept –5
x-intercept: 2



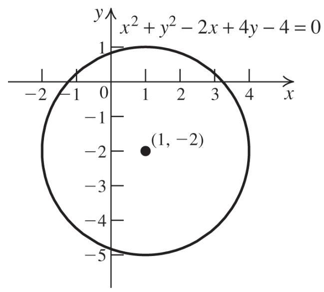
40. Circle with center (–1, 3) and radius 4.



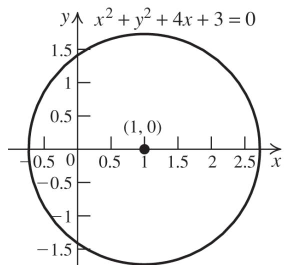
41. $x^2 + y^2 - 2x + 4y - 4 = 0 \Rightarrow$
$x^2 - 2x + 1 + y^2 + 4y + 4 = 4 + 1 + 4 \Rightarrow$
$(x-1)^2 + (y+2)^2 = 9.$
Circle with center (1, –2) and radius 3.

(*continued on next page*)

(*continued*)



42. $3x^2 + 3y^2 - 6x - 6 = 0 \Rightarrow x^2 - 2x + y^2 = 2 \Rightarrow$
$x^2 - 2x + 1 + y^2 = 2 + 1 \Rightarrow (x-1)^2 + y^2 = 3.$
Circle with center (1, 0) and radius $\sqrt{3}$.



43. $y - 2 = -2(x-1) \Rightarrow y = -2x + 4$

44. $\frac{x}{2} + \frac{y}{5} = 1 \Rightarrow 5x + 2y = 10 \Rightarrow 2y = -5x + 10 \Rightarrow$
$y = -\frac{5}{2}x + 5$

45. $m = \frac{7-3}{-1-1} = -2$
$y - 3 = -2(x-1) \Rightarrow y - 3 = -2x + 2 \Rightarrow$
$y = -2x + 5$

46. $x = 1$

47. a. $y = 3x - 2 \Rightarrow m = 3; y = 3x + 2 \Rightarrow m = 3$
The slopes are equal, so the lines are parallel.

b. $3x - 5y + 7 \Rightarrow m = 3/5;$
$5x - 3y + 2 = 0 \Rightarrow m = 5/3$
The slopes are neither equal nor negative reciprocals, so the lines are neither parallel nor perpendicular.

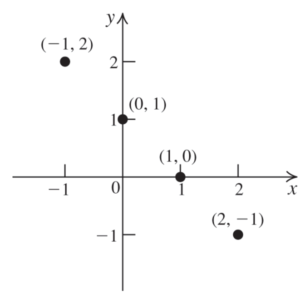
c. $ax + by + c = 0 \Rightarrow m = -a/b;$
$bx - ay + d = 0 \Rightarrow m = b/a$
The slopes are negative reciprocals, so the lines are perpendicular.

d. $y + 2 = \frac{1}{3}(x-3) \Rightarrow m = \frac{1}{3};$
$y - 5 = 3(x-3) \Rightarrow m = 3$
The slopes are neither equal nor negative reciprocals, so the lines are neither parallel nor perpendicular.

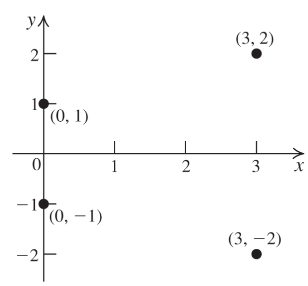
48. a. The equation with x-intercept 4 passes through the points (0, 2) and (4, 0), so its slope is $\frac{0-2}{4-0} = -\frac{1}{2}$. Thus, the slope of the line we are seeking is also $-\frac{1}{2}$. The line passes through (0, 1), which is the y-intercept, so its equation is $y = -\frac{1}{2}x + 1$.

b. The slope of the line we are seeking is 2 and the line passes through the origin, so its equation is $y - 0 = 2(x - 0)$, or $y = 2x$.

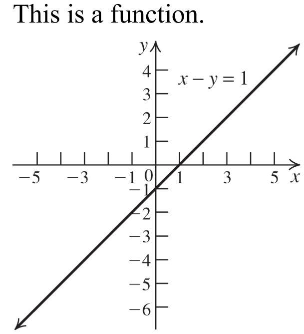
49. Domain: {–1, 0, 1, 2}; range: {–1, 0, 1, 2}.
This is a function.



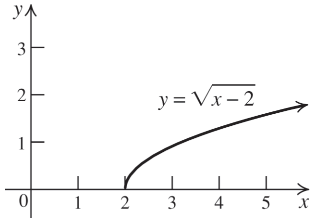
50. Domain: {0, 3}; range: {–2, –1, 1, 2}.
This is not a function.



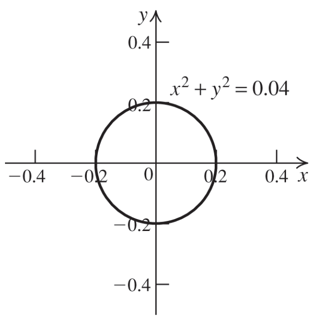
51. Domain: $(-\infty, \infty)$; range: $(-\infty, \infty)$.
This is a function.



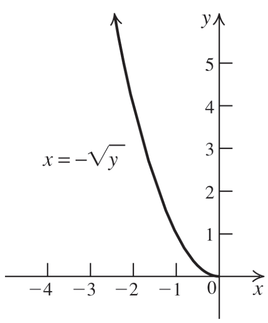
52. Domain: $[2, \infty)$; range: $[0, \infty)$.
This is a function.



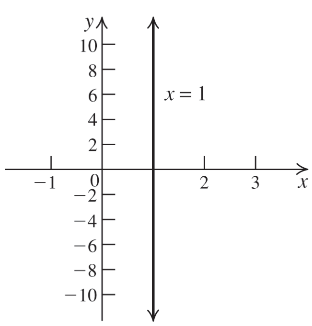
53. Domain: $[-0.2, 0.2]$; range: $[-0.2, 0.2]$.
This is not a function.



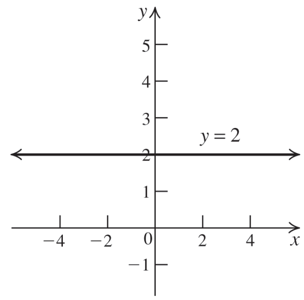
54. Domain: $(-\infty, 0]$; range: $[0, \infty)$.
This is a function.



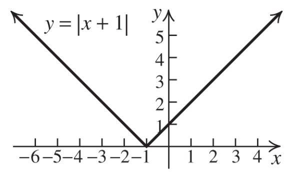
55. Domain: $\{1\}$; range: $(-\infty, \infty)$.
This is not a function.



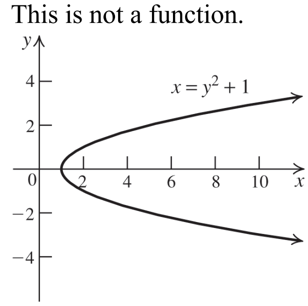
56. Domain: $(-\infty, \infty)$; range: $\{2\}$.
This is a function.



57. Domain: $(-\infty, \infty)$; range: $[0, \infty)$.
This is a function.



58. Domain: $[1, \infty)$; range: $(-\infty, \infty)$.
This is not a function.



59. $f(-2) = 3(-2) + 1 = -5$

60. $g(-2) = (-2)^2 - 2 = 2$

61. $f(x) = 4 \Rightarrow 3x + 1 = 4 \Rightarrow x = 1$

62. $g(x) = 2 \Rightarrow x^2 - 2 = 2 \Rightarrow x = \pm 2$

63. $$(f+g)(1) = f(1) + g(1) = (3(1)+1) + \left(1^2 - 2\right) = 3$$

64. $$(f-g)(-1) = f(-1) - g(-1) = (3(-1)+1) - \left((-1)^2 - 2\right) = -1$$

65. $$(f \cdot g)(-2) = f(-2) \cdot g(-2) = (3(-2)+1) \cdot \left((-2)^2 - 2\right) = -10$$

66. $$(g \cdot f)(0) = g(0) \cdot f(0) = \left(0^2 - 2\right) \cdot (3(0)+1) = -2$$

67. $(f \circ g)(3) = 3g(3) + 1 = 3\left(3^2 - 2\right) + 1 = 22$

68. $(g \circ f)(-2) = \left[f(-2)\right]^2 - 2$
$= (3(-2)+1)^2 - 2 = 23$

69. $(f \circ g)(x) = 3g(x) + 1 = 3\left(x^2 - 2\right) + 1$
$= 3x^2 - 5$

70. $(g \circ f)(x) = \left[f(x)\right]^2 - 2$
$= (3x+1)^2 - 2 = 9x^2 + 6x - 1$

71. $(f \circ f)(x) = 3f(x) + 1$
$= 3(3x+1) + 1 = 9x + 4$

72. $(g \circ g)(x) = \left[g(2)\right]^2 - 2$
$= \left(x^2 - 2\right)^2 - 2 = x^4 - 4x^2 + 2$

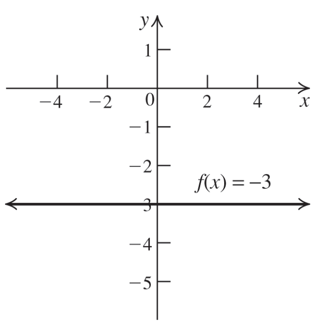
73. $f(a+h) = 3(a+h) + 1 = 3a + 3h + 1$

74. $g(a-h) = (a-h)^2 - 2 = a^2 - 2ah + h^2 - 2$

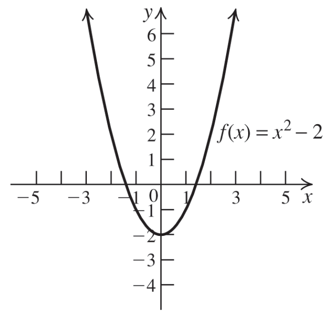
75. $$\frac{f(x+h) - f(x)}{h} = \frac{(3(x+h)+1) - (3x+1)}{h} = \frac{3x + 3h + 1 - 3x - 1}{h} = \frac{3h}{h} = 3$$

76. $$\frac{g(x+h) - g(x)}{h} = \frac{\left((x+h)^2 - 2\right) - (x^2 - 2)}{h} = \frac{x^2 + 2xh + h^2 - 2 - x^2 + 2}{h} = \frac{h^2 + 2xh}{h} = h + 2x = 2x + h$$

77. Domain: $(-\infty, \infty)$; range: $\{-3\}$.
Constant on $(-\infty, \infty)$.

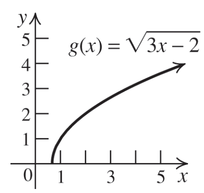


78. Domain: $(-\infty, \infty)$; range: $[-2, \infty)$.
Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.

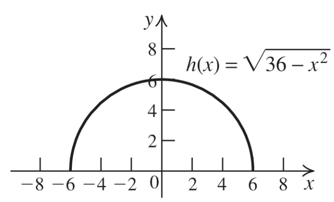


79. Domain: $\left[\frac{2}{3}, \infty\right)$; range: $[0, \infty)$

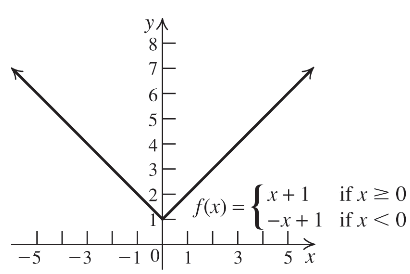
Increasing on $\left(\frac{2}{3}, \infty\right)$.



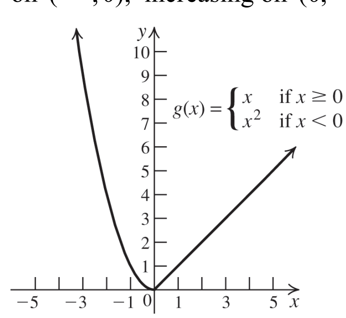
80. Domain: $[-6, 6]$; range: $[0, 6]$. Increasing on $(-6, 0)$; decreasing on $(0, 6)$.



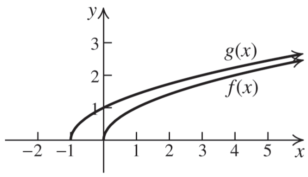
81. Domain: $(-\infty, \infty)$; range: $[1, \infty)$. Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.



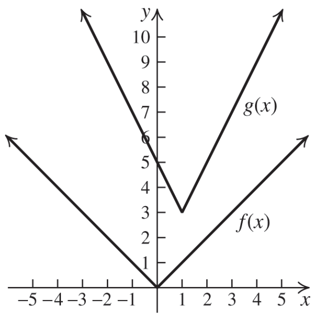
82. Domain: $(-\infty, \infty)$; range: $[0, \infty)$. Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$.



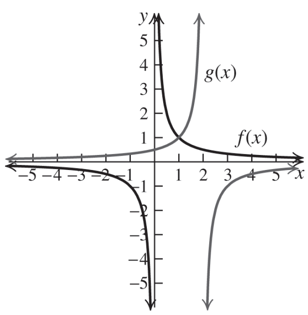
83. The graph of g is the graph of f shifted one unit left.



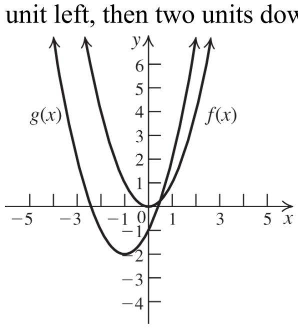
84. The graph of g is the graph of f shifted one unit right, stretched vertically by a factor of 2, then shifted three units up.



85. The graph of g is the graph of f shifted two units right, and then reflected across the x-axis.



86. The graph of g is the graph of f shifted one unit left, then two units down.



87. $f(-x)=(-x)^2-(-x)^4=x^2-x^4=f(x)\Rightarrow$ $f(x)$ is even. Not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

88. $f(-x)=(-x)^3+(-x)=-x^3-x=-f(x)\Rightarrow$ $f(x)$ is odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.

89. $f(-x)=|-x|+3=|x|+3=f(x)\Rightarrow$ $f(x)$ is even. Not symmetric with respect to the x-axis; symmetric with respect to the y-axis; not symmetric with respect to the origin.

90. $f(-x)=-3x+5\neq f(x)$ or $-f(x)\Rightarrow f(x)$ is neither even nor odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.

91. $f(-x)=\sqrt{-x}\neq f(x)$ or $-f(x)\Rightarrow f(x)$ is neither even nor odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; not symmetric with respect to the origin.

92. $f(-x)=-\dfrac{2}{x}=-f(x)\Rightarrow f(x)$ is odd. Not symmetric with respect to the x-axis; not symmetric with respect to the y-axis; symmetric with respect to the origin.

For exercises 93–96, answers may vary. Sample answers are given.

93. $f(x)=\sqrt{x^2-4}\Rightarrow f(x)=(g\circ h)(x)$ where $g(x)=\sqrt{x}$ and $h(x)=x^2-4$.

94. $g(x)=\left(x^2-x+2\right)^{50}\Rightarrow g(x)=(f\circ h)(x)$ where $f(x)=x^{50}$ and $h(x)=x^2-x+2$.

95. $h(x)=\sqrt{\dfrac{x-3}{2x+5}}\Rightarrow h(x)=(f\circ g)(x)$ where $f(x)=\sqrt{x}$ and $g(x)=\dfrac{x-3}{2x+5}$.

96. $H(x)=(2x-1)^3+5\Rightarrow H(x)=(f\circ g)(x)$ where $f(x)=x^3+5$ and $g(x)=2x-1$.

97. $f(x) = x^2 - 2; \quad g(x) = 2 - 3x$

First compute the average rate of change of g from $x = -1$ to $x = 1$, the component related to the average rate of change of the inner function.

$g(-1) = 2 - 3(-1) = 5$ and

$g(1) = 2 - 3(1) = -1.$

$$\text{ARC of } g \text{ from } -1 \text{ to } 1 = \frac{g(1) - g(-1)}{1 - (-1)} = \frac{5 - (-1)}{2} = 3$$

To compute the component related to the average rate of change of the outer function, we need to use the range for f as from $g(1) = -1$ to $g(-1) = 5.$ We have

$f(-1) = (-1)^2 - 2 = -1$ and

$f(5) = 5^2 - 2 = 23.$

ARC of f from $g(1) = -1$ to $g(-1) = 5$

$$= \frac{f(5) - f(-1)}{5 - (-1)} = \frac{23 - (-1)}{6} = 4$$

Finally, we can compute the average rate of change of the composite function as

ARC of $f \circ g$ from -1 to 1

$= (\text{ARC of } f \text{ from } g(1) = -1 \text{ to } g(-1) = 5)$

$\quad \cdot (\text{ARC of } g \text{ from } -1 \text{ to } 1)$

$= 4 \cdot 3 = 12$

98. $f(x) = x^3 - 1; \; g(x) = 2x + 1$

First compute the average rate of change of g from $x = -2$ to $x = 0$, the component related to the average rate of change of the inner function.

$g(-2) = 2(-2) + 1 = -3$ and

$g(0) = 2(0) + 1 = 1.$

ARC of g from -2 to 0

$$= \frac{g(0) - g(-2)}{0 - (-2)} = \frac{1 - (-3)}{2} = 2$$

To compute the component related to the average rate of change of the outer function, we need to use the range for f as from $g(-2) = -3$ to $g(0) = 1.$ We have $f(-3) = (-3)^3 - 1 = -28$ and $f(1) = 1^3 - 1 = 0.$

ARC of f from $g(-2) = -3$ to $g(0) = 1$

$$= \frac{f(-3) - f(1)}{-3 - 1} = \frac{-28 - 0}{-4} = 7$$

Finally, we can compute the average rate of change of the composite function as

ARC of $f \circ g$ from -2 to 0

$= (\text{ARC of } f \text{ from } g(-2) = -3 \text{ to } g(0) = 1)$

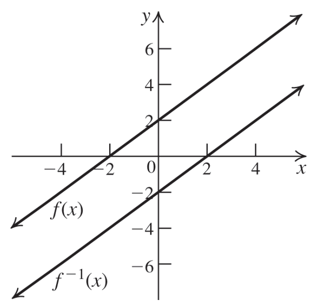
$\quad \cdot (\text{ARC of } g \text{ from } -2 \text{ to } 0)$

$= 7(2) = 14$

99. $f(x)$ is one-to-one. $f(x) = y = x + 2.$

Interchange the variables and solve for y:

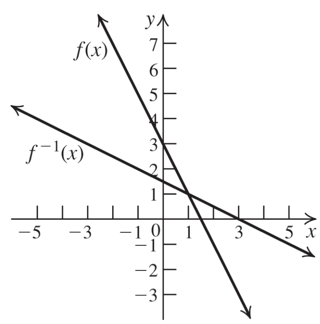
$x = y + 2 \Rightarrow y = x - 2 = f^{-1}(x).$



100. $f(x)$ is one-to-one. $f(x) = y = -2x + 3.$

Interchange the variables and solve for y:

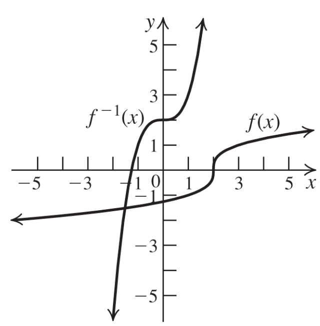
$x = -2y + 3 \Rightarrow y = -\frac{1}{2}x + \frac{3}{2} = f^{-1}(x).$



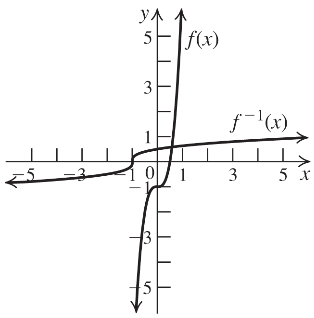
101. $f(x)$ is one-to-one. $f(x) = y = \sqrt[3]{x - 2}.$

Interchange the variables and solve for y:

$x = \sqrt[3]{y - 2} \Rightarrow y = x^3 + 2 = f^{-1}(x).$



102. $f(x)$ is one-to-one. $f(x) = y = 8x^3 - 1$.
Interchange the variables and solve for y:
$x = 8y^3 - 1 \Rightarrow y = \sqrt[3]{\dfrac{x+1}{8}} \Rightarrow$
$y = \dfrac{1}{2}\sqrt[3]{x+1} = f^{-1}(x).$



103. $f(x) = y = \dfrac{x-1}{x+2}, x \neq -2.$
Interchange the variables and solve for y.
$x = \dfrac{y-1}{y+2} \Rightarrow xy + 2x = y - 1 \Rightarrow$
$xy - y = -2x - 1 \Rightarrow y(x-1) = -2x - 1 \Rightarrow$
$y = \dfrac{-2x-1}{x-1} \Rightarrow y = f^{-1}(x) = \dfrac{2x+1}{1-x}$
Domain of f: $(-\infty, -2) \cup (-2, \infty)$
Range of f: $(-\infty, 1) \cup (1, \infty)$

104. $f(x) = y = \dfrac{2x+3}{x-1}, x \neq 1.$
Interchange the variables and solve for y.
$x = \dfrac{2y+3}{y-1} \Rightarrow xy - x = 2y + 3 \Rightarrow$
$xy - 2y = x + 3 \Rightarrow y(x-2) = x + 3 \Rightarrow$
$y = f^{-1}(x) = \dfrac{x+3}{x-2}$
Domain of f: $(-\infty, 1) \cup (1, \infty)$
Range of f: $(-\infty, 2) \cup (2, \infty)$

105. $f(x) = (x+1)^3 + 2$
$f(-1) = 2; \qquad f(1) = 10$
First we compute the corresponding average rate of f.

$$\text{ARC of } f \text{ from 0 to 1} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{10-2}{2} = 4$$

Now compute the average rate of change of f^{-1} as

$$\text{ARC of } f^{-1} \text{ from } f(-1) = 2 \text{ to } f(1) = 10 = \frac{1}{\text{ARC of } f \text{ from } -1 \text{ to } 1} = \frac{1}{4}$$

106. $f(x) = \sqrt[3]{x} + 1$
$f(1) = 2; \qquad f(8) = 3$
First we compute the corresponding average rate of f.

$$\text{ARC of } f \text{ from 1 to 8} = \frac{f(8) - f(1)}{8 - 1} = \frac{3-2}{7} = \frac{1}{7}$$

Now compute the average rate of change of f^{-1} as

$$\text{ARC of } f^{-1} \text{ from } f(1) = 2 \text{ to } f(8) = 3 = \frac{1}{\text{ARC of } f \text{ from 1 to 8}} = 7$$

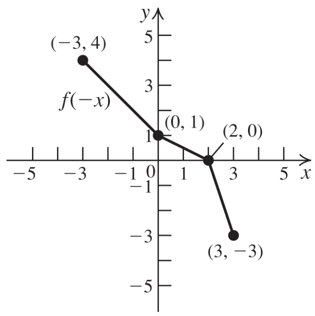
107. a. $A = (-3, -3), B = (-2, 0), C = (0, 1), D = (3, 4).$
Find the equation of each segment:
$m_{\overline{AB}} = \dfrac{0 - (-3)}{-2 - (-3)} = 3.0 = 3(-2) + b \Rightarrow b = 6.$
The equation of $\overline{AB}$ is $y = 3x + 6$.
$m_{\overline{BC}} = \dfrac{1-0}{0-(-2)} = \dfrac{1}{2}; b = 1.$
The equation of $\overline{BC}$ is $y = \dfrac{1}{2}x + 1$.
$m_{\overline{CD}} = \dfrac{4-1}{3-0} = 1; b = 1.$
The equation of $\overline{CD}$ is $y = x + 1$.
So,

$$f(x) = \begin{cases} 3x + 6 & \text{if } -3 \le x \le -2 \\ \dfrac{1}{2}x + 1 & \text{if } -2 < x < 0 \\ x + 1 & \text{if } 0 \le x \le 3 \end{cases}$$

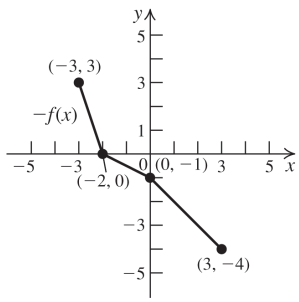
b. Domain: [–3, 3]; range: [–3, 4]

c. x-intercept: –2; y-intercept: 1

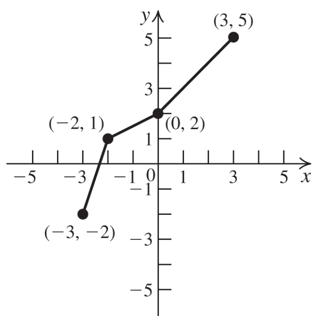
d.



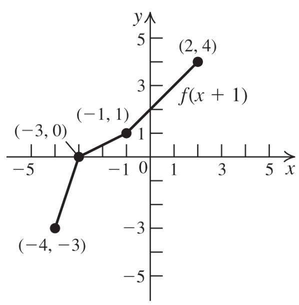
e.



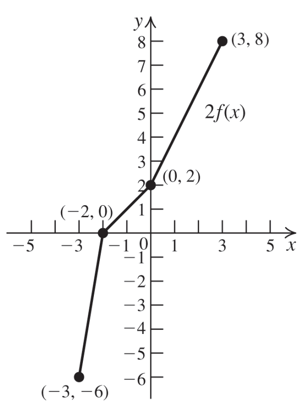
f.



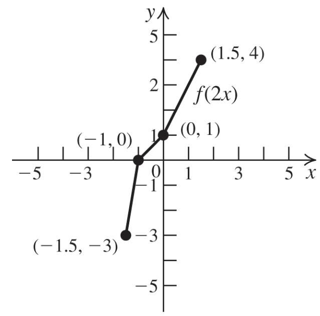
g.



h.



i.



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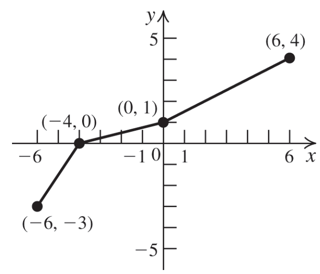
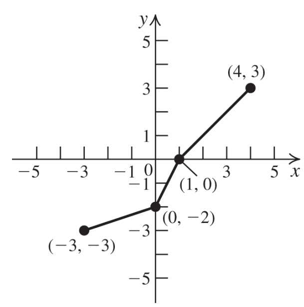


k. f is one-to-one because it satisfies the horizontal line test.

l.



Applying the Concepts

108. a. rate of change (slope) $= \dfrac{25.95 - 19.2}{25 - 10} = 0.45.$

Using the data point (10, 19.2), we have

$$P - 19.2 = 0.45(d - 10)$$
$$P - 19.2 = 0.45d - 4.5$$
$$P = 0.45d + 14.7$$

The equation is $P = 0.45d + 14.7$.

b. The slope represents the amount of increase in pressure (in pounds per square inch) as the diver descends one foot deeper. The y-intercept represents the pressure at the surface of the sea.

c. $P = 0.45(160) + 14.7 = 86.7 \text{ lb/in.}^2$

d. $104.7 = 0.45d + 14.7 \Rightarrow 200$ feet

109. a. Let the year 2010 be represented by $Y = 0$. Then the year 2018 is represented by $Y = 8$.

$$\text{rate of change (slope)} = \frac{94-85}{8-0} = \frac{9}{8} = 1.125$$

Using the data point (0, 85), we have

$$w - 85 = 1.125(Y - 0) \Rightarrow w = 1.125Y + 85.$$

b. The slope represents the cost to dispose of one pound of waste. The x-intercept represents the amount of waste that can be disposed with no cost. The y-intercept represents the fixed cost.

c. The year 2023 is represented by $Y = 13$.

$$w = 1.125(13) + 85 = 99.625$$

Approximately 100 million tons will be recycled in 2023.

110. a. At 60 mph = 1 mile per minute, so if the speedometer is correct, the number of minutes elapsed is equal to the number of miles driven.

b. The odometer is based on the speedometer, so if the speedometer is incorrect, so is the odometer.

111. a. $f(2) = 100 + 55(2) - 3(2)^2 = \$198.$
She started with \$100, so she won \$98.

b. She was winning at a rate of \$49/hour.

c. $0 = 100 + 55t - 3t^2 \Rightarrow (-t + 20)(3t + 5) \Rightarrow$
$t = 20,\ t = -5/3$. Since t represents the amount of time, we reject $t = -5/3$.
Chloe will lose all her money after playing for 20 hours.

d. \$100/20 = \$5/hour.

112. If $100 < x \le 500$, then the sales price per case is \$4 – 0.2(4) = \$3.20.

$$f(x) = \begin{cases} 4x & \text{if } 0 \le x \le 100 \\ 3.2x & \text{if } 100 < x \le 500 \\ 3x & \text{if } x > 500 \end{cases}$$

113. a.
$$(L \circ x)(t) = 0.5\sqrt{[x(t)]^2 + 4} = 0.5\sqrt{\left(1 + 0.002t^2\right)^2 + 4} = 0.5\sqrt{0.000004t^4 + 0.004t^2 + 5}$$

b.
$$(L \circ x)(5) = 0.5\sqrt{[x(5)]^2 + 4} = 0.5\sqrt{\left(1 + 0.002(5^2)\right)^2 + 4} = 0.5\sqrt{(1.05)^2 + 4} = 0.5\sqrt{5.1025} \approx 1.13$$

114 a. Revenue = number of units × price per unit:

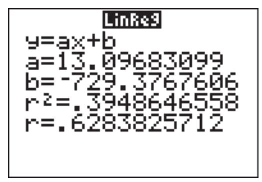
$$x \bullet p = (5000 + 50t + 10t^2)(10 + 0.5t) = 5t^3 + 125t^2 + 3000t + 50{,}000$$

b. $p = 10 + 0.5t \Rightarrow t = 2p - 20.$

$$x(t) = x(2p - 20) = 5000 + 50(2p - 20) + 10(2p - 20)^2 = 40p^2 - 700p + 8000,$$ which is the number of toys made at price p. The revenue is $p\left(40p^2 - 700p + 8000\right) =$

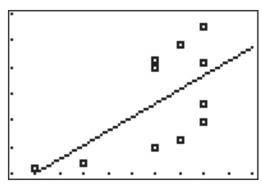
$$40p^3 - 700p^2 + 8000p.$$

115. a.



$y \approx 13.1x - 729.4$

b.



[70, 80, 1] by [200, 350, 25]

c. $y \approx 13.1(75) - 729.4 = 253.1$

A player whose height is 75 inches weighs about 253.1 pounds. (Answer may vary due to rounding.)

Chapter 1 Practice Test A

1. The endpoints of the diameter are (−2, 3) and (−4, 5), so the center of the circle is

$$C = \left(\frac{-2 + (-4)}{2}, \frac{3+5}{2}\right) = (-3, 4).$$

The length of the diameter is

$$\sqrt{(-4 - (-2))^2 + (5-3)^2} = \sqrt{8} = 2\sqrt{2}.$$

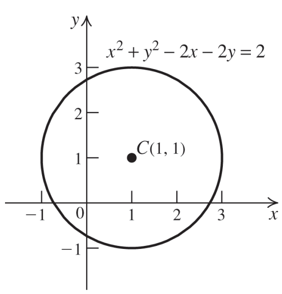
Therefore, the length of the radius is $\sqrt{2}$.
The equation of the circle is

$$(x+3)^2 + (y-4)^2 = 2.$$

2. To test if the graph is symmetric with respect to the y-axis, replace x with $-x$:
$3(-x)+2(-x)y^2 = 1 \Rightarrow -3x-2xy^2 = 1,$ which is not the same as the original equation, so the graph is not symmetric with respect to the y-axis. To test if the graph is symmetric with respect to the x-axis, replace y with $-y$:
$3x+2x(-y)^2 = 1 \Rightarrow 3x+2xy^2 = 1,$ which is the same as the original equation, so the graph is symmetric with respect to the x-axis. To test if the graph is symmetric with respect to the origin, replace x with $-x$ and y with $-y$:
$3(-x)+2(-x)(-y)^2 = 1 \Rightarrow -3x-2xy^2 = 1,$ which is not the same as the original equation, so the graph is not symmetric with respect to the origin.

3. $0 = x^2(x-3)(x+1) \Rightarrow x = 0$ or $x = 3$ or $x = -1$
$y = 0^2(0-3)(0+1) \Rightarrow y = 0.$ The x-intercepts are 0, 3, and –1; the y-intercept is 0.

4.
$$x^2-2x+y^2-2y = 2$$
$$\left(x^2-2x+1\right)+\left(y^2-2y+1\right) = 2+1+1$$
$$(x-1)^2+(y-1)^2 = 4$$
This is a circle with center (1, 1) and radius 2.



5. $y-7 = -1(x-2) \Rightarrow y-7 = -x+2 \Rightarrow$
$y = -x+9$
The equation is $y = -x+9.$

6. $8x-2y = 7 \Rightarrow y = 4x-\frac{7}{2} \Rightarrow$ the slope of the line is 4.
$y-(-1) = 4(x-2) \Rightarrow y+1 = 4x-8 \Rightarrow$
$y = 4x-9$
So the equation of the line parallel to the line $8x-2y = 7$ and passing through $(2, -1)$ is $y = 4x-9.$

7.
$$\begin{aligned}(fg)(2) &= f(2)\cdot g(2)\\ &= (-2(2)+1)\left(2^2+3(2)+2\right)\\ &= (-3)(12) = -36\end{aligned}$$

8.
$$\begin{aligned}g(f(2)) &= 1-2(f(2))^2 = 1-2(2(2)-3)^2\\ &= 1-2(1)^2 = -1\end{aligned}$$

9.
$$\begin{aligned}(f\circ f)(x) &= [f(x)]^2-2f(x)\\ &= (x^2-2x)^2-2(x^2-2x)\\ &= x^4-4x^3+4x^2-2x^2+4x\\ &= x^4-4x^3+2x^2+4x\end{aligned}$$

10. a. $f(-1) = (-1)^3-2(-1) = -1+2 = 1$

b. $f(0) = 0^3-2(0) = 0$

c. $f(1) = 1-2(1)^2 = -1$

11. $1-x > 0 \Rightarrow x < 1; x$ must also be greater than or equal to 0, so the domain is [0, 1).

12. $\dfrac{f(4)-f(1)}{4-1} = \dfrac{(2(4)+7)-(2(1)+7)}{3} = 2$

13. $f(-x) = 2(-x)^4-\dfrac{3}{(-x)^2} = 2x^4-\dfrac{3}{x^2} = f(x) \Rightarrow$
$f(x)$ is even.

14. Increasing on $(-\infty, 0)$ and $(2, \infty)$; decreasing on $(0, 2)$.

15. Shift the graph of $y = \sqrt{x}$ three units to the right, then stretch the graph of $y = \sqrt{x-3}$ vertically by a factor of 2, and then shift the resulting graph, $y = 2\sqrt{x-3},$ four units up.

16. $25 = 25-(2t-5)^2 \Rightarrow 0 = -(2t-5)^2 \Rightarrow$
$0 = 2t-5 \Rightarrow t = \frac{5}{2} = 2.5$ seconds

17. $f(2) = 7 \Rightarrow f^{-1}(7) = 2$

18. $f(x) = y = \dfrac{2x}{x-1}.$ Interchange the variables and solve for y: $x = \dfrac{2y}{y-1} \Rightarrow$
$xy-x = 2y \Rightarrow xy-2y = x \Rightarrow$
$y(x-2) = x \Rightarrow y = f^{-1}(x) = \dfrac{x}{x-2}$

19. $A(x) = 100x+1000$

20. a. $C(230) = 0.25(230) + 30 = \$87.50$

b. $57.50 = 0.25m + 30 \Rightarrow m = 110$ miles

Chapter 1 Practice Test B

1. To test if the graph is symmetric with respect to the y-axis, replace x with $-x$: $|-x| + 2|y| = 2 \Rightarrow |x| + 2|y| = 2,$ which is the same as the original equation, so the graph is symmetric with respect to the y-axis. To test if the graph is symmetric with respect to the x-axis, replace y with $-y$: $|x| + 2|-y| = 2 \Rightarrow |x| + 2|y| = 2,$ which is the same as the original equation, so the graph is symmetric with respect to the x-axis. To test if the graph is symmetric with respect to the origin, replace x with $-x,$ and y with $-y$: $|-x| + 2|-y| = 2 \Rightarrow |x| + 2|y| = 2,$ which is the same as the original equation, so the graph is symmetric with respect to the origin. The answer is D.

2. $0 = x^2 - 9 \Rightarrow x = \pm 3; y = 0^2 - 9 \Rightarrow y = -9.$ The x-intercepts are ± 3; the y-intercept is -9. The answer is B.

3. D **4.** D **5.** C

6. Suppose the coordinates of the second point are (a, b). Then $-\frac{1}{2} = \frac{b-2}{a-3}$. Substitute each of the points given into this equation to see which makes it true.
$-\frac{1}{2} = \frac{1-2}{5-3}$
The answer is B.

7. Find the slope of the original line:
$6x - 3y = 5 \Rightarrow y = 2x - \frac{5}{3}$. The slope is 2.
The equation of the line with slope 2, passing through $(-1, 2)$ is $y - 2 = 2(x + 1)$.
The answer is D.

8. $(f \circ g)(x) = 3\left(2 - x^2\right) - 5 = 1 - 3x^2.$
The answer is B.

9. $(f \circ f)(x) = 2\left(2x^2 - x\right)^2 - \left(2x^2 - x\right)$
$= 8x^4 - 8x^3 + x$
The answer is A.

10. $g(a-1) = \frac{1-(a-1)}{1+(a-1)} = \frac{2-a}{a}.$
The answer is C.

11. $1 - x \geq 0 \Rightarrow x \leq 1;$ x must also be greater than or equal to 0, so the domain is [0, 1].
The answer is A.

12. $f(x) = \sqrt{x^2 + 6x - 7} = \sqrt{(x+7)(x-1)} \Rightarrow$
$(x+7)(x-1) \geq 0$
So, the domain of the function $(-\infty, -7] \cup [1, \infty)$.
The answer is B.

13. A **14.** A **15.** B
16. D **17.** C

18. $f(x) = y = \frac{1-3x}{5+2x}$. Interchange the variables and solve for y: $x = \frac{1-3y}{5+2y} \Rightarrow$
$5x + 2xy = 1 - 3y \Rightarrow 2xy + 3y = 1 - 5x \Rightarrow$
$y(2x+3) = 1 - 5x \Rightarrow y = f^{-1}(x) = \frac{1-5x}{2x+3}$
The answer is C.

19. $w = 5x - 190; w = 5(70) - 190 = 160.$
The answer is B.

20. $50 = 0.2m + 25 \Rightarrow m = 125.$ The answer is A.

Chapter 2 Polynomial and Rational Functions

Getting Ready for the Next Section

GR1. $x^2 - x - 12 = (x+3)(x-4)$

GR2. $x^2 - 5x + 6 = (x-2)(x-3)$

GR3. $x^2 + 2x - 8 = (x+4)(x-2)$

GR4. $x^2 + 7x + 10 = (x+2)(x+5)$

GR5. $x^2 - 6x$

One-half of -6 is -3. $(-3)^2 = 9$, so add 9 to obtain $x^2 - 6x + 9 = (x-3)^2$.

GR6. $x^2 + 8x$

One-half of 8 is 4. $4^2 = 16$, so add 16 to obtain $x^2 + 8x + 16 = (x+4)^2$.

GR7. $x^2 - 5x$

One-half of -5 is $-\frac{5}{2}$. $\left(-\frac{5}{2}\right)^2 = \frac{25}{4}$, so add 9 to obtain $x^2 - 5x + \frac{25}{4} = \left(x - \frac{5}{2}\right)^2$.

GR8. $x^2 + \dfrac{3}{2}x$

One-half of $\frac{3}{2}$ is $\frac{3}{4}$. $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$, so add $\frac{9}{16}$ to obtain $x^2 + \frac{3}{2}x + \frac{9}{16} = \left(x + \frac{3}{4}\right)^2$.

GR9. $x^2 - 7x + 12 = 0$

$(x-3)(x-4) = 0$

$x - 3 = 0 \quad | \quad x - 4 = 0$

$x = 3 \quad | \quad x = 4$

Solution: {3, 4}

GR10. $x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x + 2 = 0 \quad | \quad x - 3 = 0$

$x = -2 \quad | \quad x = 3$

Solution: {–2, 3}

GR11. $3x^2 + 7x + 2 = 0$

$(3x+1)(x+2) = 0$

$3x + 1 = 0 \quad | \quad x + 2 = 0$

$x = -\dfrac{1}{3} \quad | \quad x = -2$

Solution: $\left\{-2, -\frac{1}{3}\right\}$

GR12. $x^2 - 4x + 1 = 0$

Use the quadratic formula.

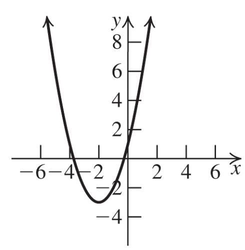
$a = 1, b = -4, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

Solution: $\left\{2 - \sqrt{3}, 2 + \sqrt{3}\right\}$

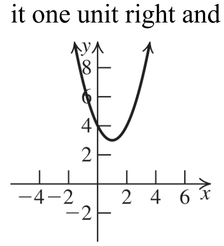
GR13. $y = (x+2)^2 - 3$

Start with the graph of $f(x) = x^2$, then shift it two units left and three units down.



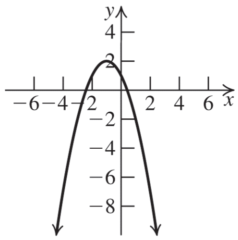
GR14. $y = (x-1)^2 + 3$

Start with the graph of $f(x) = x^2$, then shift it one unit right and three units up.



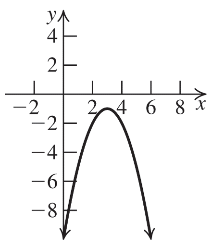
GR15. $y = -(x+1)^2 + 2$

Start with the graph of $f(x) = x^2$, then shift it one unit left. Reflect the graph across the x-axis and then shift it two units up



GR16. $y = -(x-3)^2 - 1$

Start with the graph of $f(x) = x^2$, then shift it three units right. Reflect the graph across the x-axis and then shift it one unit down.



2.1 Quadratic Functions

Practice Problems

1. Substitute 1 for h, −5 for k, 3 for x, and 7 for y in the standard form for a quadratic equation to solve for a:

   $7 = a(3-1)^2 - 5 \Rightarrow 7 = 4a - 5 \Rightarrow \; a = 3$. The equation is $y = 3(x-1)^2 - 5$.

   Because $a = 3 > 0$, f has a minimum value of −5 at $x = 1$.

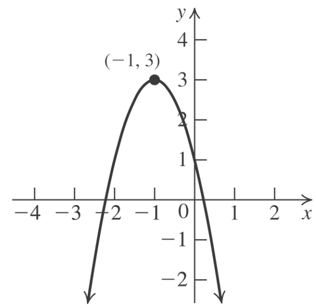
2. The graph of $f(x) = -2(x+1)^2 + 3$ is a parabola with $a = -2$, $h = -1$ and $k = 3$. Thus, the vertex is (−1, 3), and the maximum value of the function is 3. The parabola opens down because $a < 0$. Now, find the x-intercepts:

   $0 = -2(x+1)^2 + 3 \Rightarrow 2(x+1)^2 = 3 \Rightarrow$

   $(x+1)^2 = \frac{3}{2} \Rightarrow x+1 = \pm\sqrt{\frac{3}{2}} \Rightarrow x = \pm\sqrt{\frac{3}{2}} - 1 \Rightarrow$

   $x \approx 0.22$ or $x \approx -2.22$. Next, find the y-intercept: $f(0) = -2(0+1)^2 + 3 = 1$

   Plot the vertex, the x-intercepts, and the y-intercept, and join them with a parabola.



3. The graph of $f(x) = 3x^2 - 3x - 6$ is a parabola with $a = 3$, $b = -3$ and $c = -6$. The parabola opens up because $a > 0$. Now, find the vertex:

   $h = -\frac{b}{2a} = -\frac{-3}{2(3)} = \frac{1}{2}$

   $k = f(h) = f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) - 6 = -\frac{27}{4}$

   Thus, the vertex (h, k) is $\left(\frac{1}{2}, -\frac{27}{4}\right)$, and the minimum value of the function is $-\frac{27}{4}$.

   Next, find the x-intercepts:

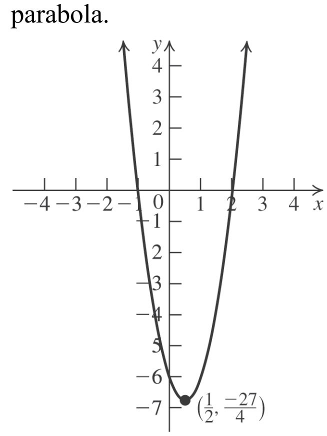
   $3x^2 - 3x - 6 = 0 \Rightarrow 3(x^2 - x - 2) = 0 \Rightarrow$

   $(x-2)(x+1) = 0 \Rightarrow x = 2$ or $x = -1$

   Now, find the y-intercept:

   $f(0) = 3(0)^2 - 3(0) - 6 = -6$.

   Thus, the intercepts are (−1, 0), (2, 0) and (0, −6). Use the fact that the parabola is symmetric with respect to its axis, $x = \frac{1}{2}$, to locate additional points. Plot the vertex, the x-intercepts, the y-intercept, and any additional points, and join them with a parabola.



4. The graph of $f(x) = 3x^2 - 6x - 1$ is a parabola with $a = 3$, $b = -6$ and $c = -1$. The parabola opens up because $a > 0$. Complete the square to write the equation in standard form:

$$f(x) = 3x^2 - 6x - 1 = 3(x^2 - 2x) - 1$$
$$= 3(x^2 - 2x + 1) - 1 - 3 = 3(x-1)^2 - 4$$

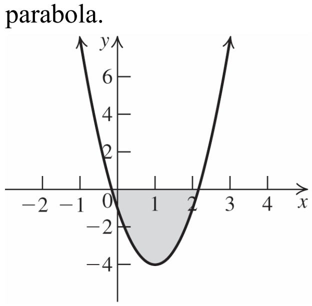
Thus, the vertex is $(1, -4)$. The domain of f is $(-\infty, \infty)$ and the range is $[-4, \infty)$.

Next, find the x-intercepts:

$$0 = 3(x-1)^2 - 4 \Rightarrow \frac{4}{3} = (x-1)^2 \Rightarrow$$
$$\pm\frac{2\sqrt{3}}{3} = x - 1 \Rightarrow 1 \pm \frac{2\sqrt{3}}{3} = x \Rightarrow x \approx 2.15 \text{ or}$$

$x \approx -0.15$. Now, find the y-intercept: $f(0) = 3(0)^2 - 6(0) - 1 = -1$.

Use the fact that the parabola is symmetric with respect to its axis, $x = 1$, to locate additional points. Plot the vertex, the x-intercepts, the y-intercept, and any additional points, and join them with a parabola.



The graph of f is below the x-axis between the x-intercepts, so the solution set for $f(x) = 3x^2 - 6x - 1 \le 0$ is

$$\left[1 - \frac{2\sqrt{3}}{3}, 1 + \frac{2\sqrt{3}}{3}\right] \text{ or } \left[\frac{3 - 2\sqrt{3}}{3}, \frac{3 + 2\sqrt{3}}{3}\right].$$

5. $h(t) = -\frac{g_E}{2}t^2 + v_0 t + h_0$

$g_E = 32 \text{ ft/s}^2$, $h_0 = 100$ ft, max height $= 244$ ft

a. Using the given values, we have

$$h(t) = -\frac{32}{2}t^2 + v_0 t + 100$$
$$= -16t^2 + v_0 t + 100$$

Using the formula for the vertex of a parabola gives $t = -\frac{v_0}{2(-16)} = \frac{v_0}{32}$. This is the time at which the maximum height $h(t) = 244$ ft is attained. Thus,

$$h(t) = 244 = h\left(\frac{v_0}{32}\right)$$
$$= -16\left(\frac{v_0}{32}\right)^2 + v_0\left(\frac{v_0}{32}\right) + 100$$
$$= -\frac{v_0^2}{64} + \frac{v_0^2}{32} + 100 = \frac{v_0^2}{64} + 100$$

Solving for v_0 yields

$$244 = \frac{v_0^2}{64} + 100 \Rightarrow 144 = \frac{v_0^2}{64} \Rightarrow$$
$$v_0^2 = 144 \cdot 64 \Rightarrow v_0 = \sqrt{144 \cdot 64} = 12 \cdot 8 = 96$$

Thus, $h(t) = -16t^2 + 96t + 100$ feet.

b. Using the formula for the vertex of a parabola gives $t = -\frac{96}{2(-16)} = \frac{96}{32} = 3$.

The ball reached its highest point 3 seconds after it was released.

6. Let x = the length of the playground and y = the width of the playground.

Then $2(x + y) = 1000 \Rightarrow x + y = 500 \Rightarrow$ $y = 500 - x$.

The area of the playground is $A(x) = xy = x(500 - x) = 500x - x^2$.

The vertex for the parabola is (h, k) where

$$h = -\frac{500}{2(-1)} = 250 \text{ and}$$
$$k = 500(250) - 250^2 = 62{,}500.$$

Thus, the maximum area that can be enclosed is 62,500 ft^2. The playground is a square with side length 250 ft.

Concepts and Vocabulary

1. The graph of $f(x) = ax^2 + bx + c$, $a \ne 0$, is a <u>parabola</u>.

2. The vertex of the graph of $f(x) = a(x-h)^2 + k$, $a \ne 0$, is <u>(h, k)</u>.

3. The graph of $f(x) = a(x-h)^2 + k$, $a \ne 0$, is symmetric with respect to the vertical line <u>$x = h$</u>.

4. The x-coordinate of the vertex of $f(x) = ax^2 + bx + c,\ a \neq 0,$ is given by $x = -\frac{b}{2a}$.

5. True.

6. False. The graph of $f(x) = 2x^2$ is the graph of $g(x) = x^2$ compressed horizontally by a factor of 2.

7. False. If $a > 0$, then the parabola opens up and has a minimum.

8. True

Building Skills

9. f **10.** d **11.** a **12.** e

13. h **14.** b **15.** g **16.** c

17. Substitute −8 for y and 2 for x to solve for a: $-8 = a(2)^2 \Rightarrow -2 = a.$

The equation is $y = -2x^2.$

18. Substitute 3 for y and –3 for x to solve for a: $3 = a(-3)^2 \Rightarrow \frac{1}{3} = a.$

The equation is $y = \frac{1}{3}x^2.$

19. Substitute 20 for y and 2 for x to solve for a: $20 = a(2)^2 \Rightarrow 5 = a.$

The equation is $y = 5x^2.$

20. Substitute –6 for y and –3 for x to solve for a: $-6 = a(-3)^2 \Rightarrow -\frac{2}{3} = a.$

The equation is $y = -\frac{2}{3}x^2.$

21. Substitute 0 for h, 0 for k, 8 for y, and –2 for x in the standard form for a quadratic equation to solve for a: $8 = a(-2-0)^2 + 0 \Rightarrow 8 = 4a \Rightarrow 2 = a.$ The equation is $y = 2x^2.$

22. Substitute 2 for h, 0 for k, 3 for y, and 1 for x in the standard form for a quadratic equation to solve for a: $3 = a(1-2)^2 + 0 \Rightarrow 3 = a.$

The equation is $y = 3(x-2)^2.$

23. Substitute –3 for h, 0 for k, –4 for y, and –5 for x in the standard form for a quadratic equation to solve for a:

$-4 = a(-5-(-3))^2 + 0 \Rightarrow -1 = a.$

The equation is $y = -(x+3)^2.$

24. Substitute 0 for h, 1 for k, 0 for y, and –1 for x in the standard form for a quadratic equation to solve for a: $0 = a(-1-0)^2 + 1 \Rightarrow -1 = a.$

The equation is $y = -x^2 + 1.$

25. Substitute 2 for h, 5 for k, 7 for y, and 3 for x in the standard form for a quadratic equation to solve for a: $7 = a(3-2)^2 + 5 \Rightarrow 2 = a.$

The equation is $y = 2(x-2)^2 + 5.$

26. Substitute –3 for h, 4 for k, 0 for y, and 0 for x in the standard form for a quadratic equation to solve for a:

$0 = a(0-(-3))^2 + 4 \Rightarrow -\frac{4}{9} = a.$ The equation is $y = -\frac{4}{9}(x+3)^2 + 4.$

27. Substitute 2 for h, –3 for k, 8 for y, and –5 for x in the standard form for a quadratic equation to solve for a: $8 = a(-5-2)^2 - 3 \Rightarrow \frac{11}{49} = a.$

The equation is $y = \frac{11}{49}(x-2)^2 - 3.$

28. Substitute –3 for h, –2 for k, –8 for y, and 0 for x in the standard form for a quadratic equation to solve for a:

$-8 = a(0-(-3))^2 - 2 \Rightarrow -\frac{2}{3} = a.$

The equation is $y = -\frac{2}{3}(x+3)^2 - 2.$

29. Substitute $\frac{1}{2}$ for h, $\frac{1}{2}$ for k, $-\frac{1}{4}$ for y, and $\frac{3}{4}$ for x in the standard form for a quadratic equation to solve for a:

$-\frac{1}{4} = a\left(\frac{3}{4} - \frac{1}{2}\right)^2 + \frac{1}{2} \Rightarrow -12 = a.$

The equation is $y = -12\left(x - \frac{1}{2}\right)^2 + \frac{1}{2}.$

30. Substitute $-\frac{3}{2}$ for h, $-\frac{5}{2}$ for k, $\frac{55}{8}$ for y, and 1 for x in the standard form for a quadratic equation to solve for a:

$$\frac{55}{8} = a\left(1-\left(-\frac{3}{2}\right)\right)^2 - \frac{5}{2} \Rightarrow \frac{3}{2} = a.$$

The equation is $y = \frac{3}{2}\left(x+\frac{3}{2}\right)^2 - \frac{5}{2}$.

31. The vertex is (–2, 0), and the graph passes through (0, 3). Substitute −2 for h, 0 for k, 3 for y, and 0 for x in the standard form for a quadratic equation to solve for a:

$$3 = a(0-(-2))^2 + 0 \Rightarrow \frac{3}{4} = a.$$

The equation is $y = \frac{3}{4}(x+2)^2$.

32. The vertex is (3, 0), and the graph passes through (0, 2). Substitute 3 for h, 0 for k, 2 for y, and 0 for x in the standard form for a quadratic equation to solve for a:

$$2 = a(0-3)^2 + 0 \Rightarrow \frac{2}{9} = a.$$

The equation is $y = \frac{2}{9}(x-3)^2$.

33. The vertex is (3, –1), and the graph passes through (5, 2). Substitute 3 for h, –1 for k, 2 for y, and 5 for x in the standard form for a quadratic equation to solve for a:

$$2 = a(5-3)^2 - 1 \Rightarrow \frac{3}{4} = a.$$

The equation is $y = \frac{3}{4}(x-3)^2 - 1$.

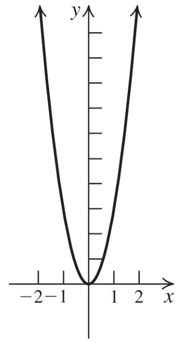
34. The vertex is (3, –1), and the graph passes through (0, 3). Substitute 3 for h, –1 for k, 3 for y, and 0 for x in the standard form for a quadratic equation to solve for a:

$$3 = a(0-3)^2 - 1 \Rightarrow 9a = 4 \Rightarrow a = \frac{4}{9}.$$

The equation is $y = \frac{4}{9}(x-3)^2 - 1$.

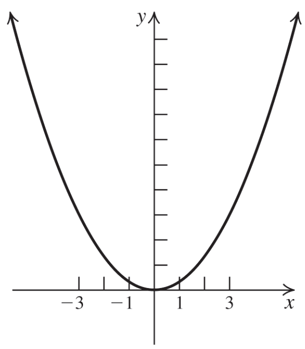
35. $f(x) = 3x^2$

Stretch the graph of $y = x^2$ vertically by a factor of 3.



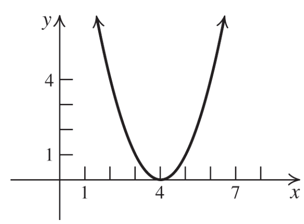
36. $f(x) = \frac{1}{3}x^2$

Compress the graph of $y = x^2$ vertically by a factor of 1/3.



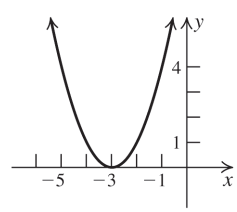
37. $g(x) = (x-4)^2$

Shift the graph of $y = x^2$ right four units.



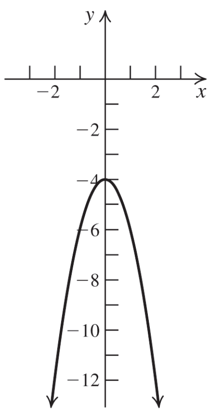
38. $g(x) = (x+3)^2$

Shift the graph of $y = x^2$ left three units.



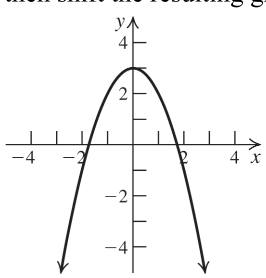
39. $f(x) = -2x^2 - 4$

Stretch the graph of $y = x^2$ vertically by a factor of 2, reflect the resulting graph about the x-axis, then shift the resulting graph down 4 units.



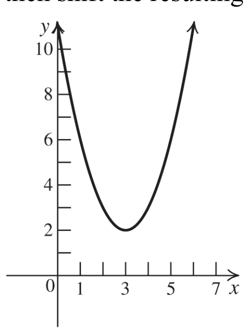
40. $f(x) = -x^2 + 3$

Reflect the graph of $y = x^2$ about the x-axis, then shift the resulting graph up 3 units.



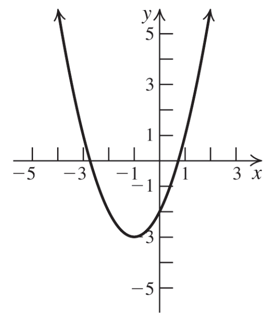
41. $g(x) = (x-3)^2 + 2$

Shift the graph of $y = x^2$ right three units, then shift the resulting graph up two units.



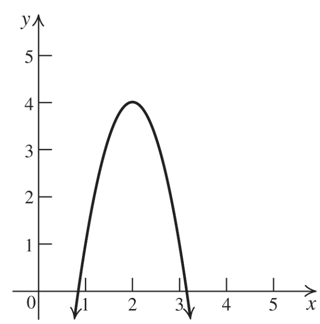
42. $g(x) = (x+1)^2 - 3$

Shift the graph of $y = x^2$ left one unit, then shift the resulting graph down three units.



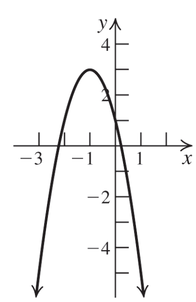
43. $f(x) = -3(x-2)^2 + 4$

Shift the graph of $y = x^2$ right two units, stretch the resulting graph vertically by a factor of 3, reflect the resulting graph about the x-axis, and then shift the resulting graph up four units.



44. $f(x) = -2(x+1)^2 + 3$

Shift the graph of $y = x^2$ left one unit, stretch the resulting graph vertically by a factor of 2, reflect the resulting graph about the x-axis, and then shift the resulting graph up three units.

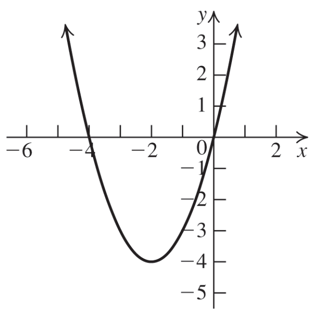


45. Complete the square to write the equation in standard form: $y = x^2 + 4x \Rightarrow$

$y = \left(x^2 + 4x + 4\right) - 4 \Rightarrow y = (x+2)^2 - 4.$ This is the graph of $y = x^2$ shifted two units left and four units down. The vertex is $(-2, -4)$. The axis of symmetry is $x = -2$. To find the x-intercepts, let $y = 0$ and solve

$0 = (x+2)^2 - 4 \Rightarrow (x+2)^2 = 4 \Rightarrow$
$x + 2 = \pm 2 \Rightarrow x = -4$ or $x = 0$

To find the y-intercept, let $x = 0$ and solve $y = (0+2)^2 - 4 \Rightarrow y = 0.$

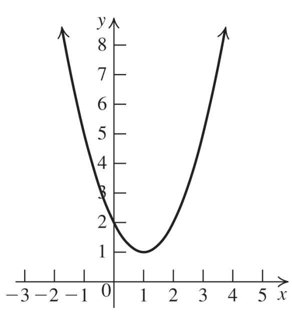


46. Complete the square to write the equation in standard form: $y = x^2 - 2x + 2 \Rightarrow$

$y = (x^2 - 2x + 1) - 1 + 2 \Rightarrow y = (x-1)^2 + 1.$

This is the graph of $y = x^2$ shifted one unit right and one unit up. The vertex is $(1, 1)$. The axis of symmetry is $x = 1$. To find the x-intercepts, let $y = 0$ and solve

$0 = (x-1)^2 + 1 \Rightarrow (x-1)^2 = -1 \Rightarrow$ there is no x-intercept. To find the y-intercept, let $x = 0$ and solve $y = (0-1)^2 + 1 \Rightarrow y = 2.$

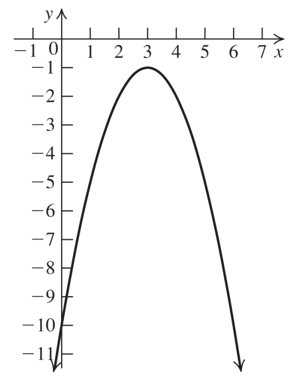


47. Complete the square to write the equation in standard form:

$y = 6x - 10 - x^2 \Rightarrow y = -(x^2 - 6x) - 10 \Rightarrow$
$y = -(x^2 - 6x + 9 - 9) - 10 \Rightarrow$
$y = -(x^2 - 6x + 9) + 9 - 10 \Rightarrow y = -(x-3)^2 - 1$

This is the graph of $y = x^2$ shifted three units right, reflected about the x-axis, and then shifted one unit down. The vertex is $(3, -1)$. The axis of symmetry is $x = 3$. To find the x-intercepts, let $y = 0$ and solve

$0 = -(x-3)^2 - 1 \Rightarrow -1 = (x-3)^2 \Rightarrow$ there is no x-intercept. To find the y-intercept, let $x = 0$ and solve $y = -(0-3)^2 - 1 \Rightarrow y = -10.$



48. Complete the square to write the equation in standard form:

$y = 8 + 3x - x^2 \Rightarrow y = -(x^2 - 3x) + 8 \Rightarrow$

$y = -\left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right) + 8 \Rightarrow$

$y = -\left(x^2 - 3x + \frac{9}{4}\right) + \frac{9}{4} + 8$

$y = -\left(x - \frac{3}{2}\right)^2 + \frac{41}{4}$

This is the graph of $y = x^2$ shifted $3/2$ units right, reflected about the x-axis, and then shifted $\frac{41}{4}$ units up. The vertex is $\left(\frac{3}{2}, \frac{41}{4}\right)$.

The axis of symmetry is $x = \frac{3}{2}$. To find the x-intercepts, let $y = 0$ and solve

$0 = -\left(x - \frac{3}{2}\right)^2 + \frac{41}{4} \Rightarrow \left(x - \frac{3}{2}\right)^2 = \frac{41}{4} \Rightarrow$

$x - \frac{3}{2} = \pm\frac{\sqrt{41}}{2} \Rightarrow x = \frac{3}{2} \pm \frac{1}{2}\sqrt{41}.$

To find the y-intercept, let $x = 0$ and solve

$y = -\left(0 - \frac{3}{2}\right)^2 + \frac{41}{4} \Rightarrow y = 8.$

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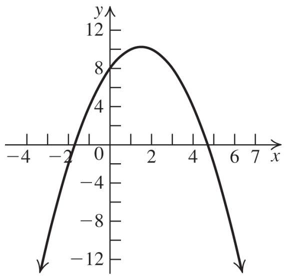
(*continued*)



49. Complete the square to write the equation in standard form:

$y = 2x^2 - 8x + 9 \Rightarrow y = 2(x^2 - 4x) + 9 \Rightarrow$

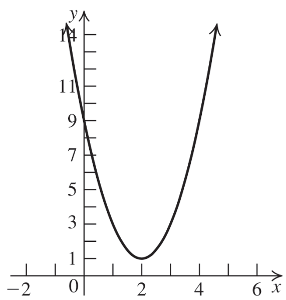
$y = 2(x^2 - 4x + 4 - 4) + 9$

$y = 2(x^2 - 4x + 4) - 2(4) + 9$

$y = 2(x - 2)^2 + 1$

This is the graph of $y = x^2$ shifted 2 units right, stretched vertically by a factor of 2, and then shifted one unit up. The vertex is (2, 1). The axis of symmetry is $x = 2$. To find the x-intercepts, let $y = 0$ and solve

$0 = 2(x-2)^2 + 1 \Rightarrow -\frac{1}{2} = (x-2)^2 \Rightarrow$ there is no x-intercept. To find the y-intercept, let $x = 0$ and solve $y = 2(0-2)^2 + 1 \Rightarrow y = 9$.



50. Complete the square to write the equation in standard form:

$y = 3x^2 + 12x - 7 \Rightarrow y = 3(x^2 + 4x) - 7 \Rightarrow$

$y = 3(x^2 + 4x + 4 - 4) - 7$

$y = 3(x^2 + 4x + 4) - 3(4) - 7$

$y = 3(x+2)^2 - 19$

This is the graph of $y = x^2$ shifted 2 units left, stretched vertically by a factor of 3, and then shifted 19 units down. The vertex is (–2, –19). The axis of symmetry is $x = -2$. To find the x-intercepts, let $y = 0$ and solve

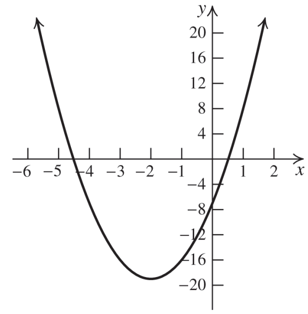
$0 = 3(x+2)^2 - 19$.

$0 = 3(x+2)^2 - 19 \Rightarrow \frac{19}{3} = (x+2)^2 \Rightarrow$

$\pm\sqrt{\frac{19}{3}} = x + 2 \Rightarrow \pm\frac{\sqrt{57}}{3} = x + 2 \Rightarrow$

$-2 \pm \frac{1}{3}\sqrt{57} = x.$

To find the y-intercept, let $x = 0$ and solve $y = 3(0+2)^2 - 19 \Rightarrow y = -7$.



51. Complete the square to write the equation in standard form:

$y = -3x^2 + 18x - 11 \Rightarrow y = -3(x^2 - 6x) - 11 \Rightarrow$

$y = -3(x^2 - 6x + 9 - 9) - 11$

$y = -3(x^2 - 6x + 9) + 3(9) - 11$

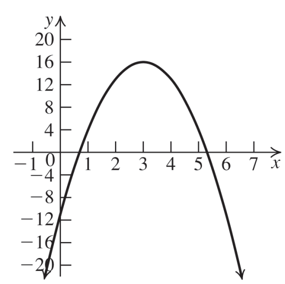
$y = -3(x-3)^2 + 16.$

This is the graph of $y = x^2$ shifted three units right, stretched vertically by a factor of three, reflected about the x-axis, and then shifted 16 units up. The vertex is (3, 16). The axis of symmetry is
$x = 3$. To find the x-intercepts, let $y = 0$ and solve $y = 0 = -3(x-3)^2 + 16 \Rightarrow$

$\frac{16}{3} = (x-3)^2 \Rightarrow \pm\frac{4\sqrt{3}}{3} = x - 3 \Rightarrow$

$3 \pm \frac{4}{3}\sqrt{3} = x.$

To find the y-intercept, let $x = 0$ and solve $y = -3(0-3)^2 + 16 \Rightarrow y = -11$.



52. Complete the square to write the equation in standard form:

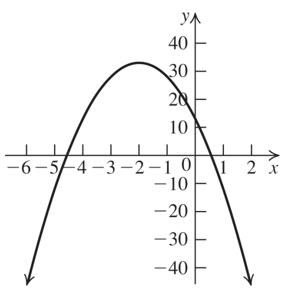
$y=-5x^2-20x+13 \Rightarrow y=-5(x^2+4x)+13 \Rightarrow$

$y=-5(x^2+4x+4-4)+13$

$y=-5(x^2+4x+4)+5(4)+13$

$y=-5(x+2)^2+33$

This is the graph of $y=x^2$ shifted two units left, stretched vertically by a factor of five, reflected about the x-axis, and then shifted 33 units up. The vertex is (–2, 33). The axis is $x=-2$. To find the x-intercepts, let $y = 0$ and solve $0=-5(x+2)^2+33 \Rightarrow \frac{33}{5}=(x+2)^2 \Rightarrow$ $\pm\sqrt{\frac{33}{5}}=x+2 \Rightarrow -2\pm\frac{1}{5}\sqrt{165}=x.$ To find the y-intercept, let $x = 0$ and solve $y=-5(0+2)^2+33 \Rightarrow y=13.$



53. $y=x^2-8x+15 \Rightarrow a=1,\ b=-8,\ c=15$

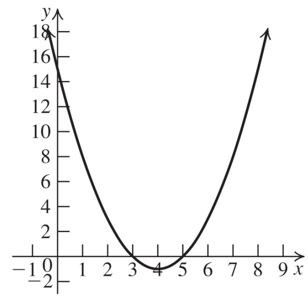
a. $a = 1 > 0$, so the graph opens up.

b. The vertex is $\left(-\frac{-8}{2(1)}, f\left(-\frac{-8}{2(1)}\right)\right)=(4,-1).$

c. The axis of symmetry is $x = 4$.

d. To find the x-intercepts, let $y = 0$ and solve $0=x^2-8x+15 \Rightarrow 0=(x-3)(x-5) \Rightarrow$ $x=3$ or $x=5.$ To find the y-intercept, let $x=0$ and solve $y=0^2-8(0)+15 \Rightarrow$ $y=15.$

e.



54. $y=x^2+8x+13 \Rightarrow a=1,\ b=8,\ c=13$

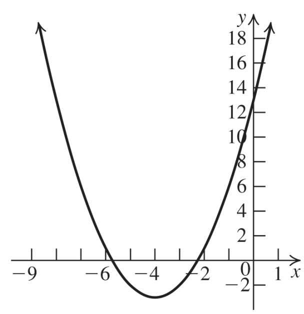
a. $a = 1 > 0$, so the graph opens up.

b. The vertex is $\left(-\frac{8}{2(1)}, f\left(-\frac{8}{2(1)}\right)\right)=(-4,-3).$

c. The axis of symmetry is $x = –4$.

d. To find the x-intercepts, let $y = 0$ and solve $0=x^2+8x+13 \Rightarrow x=\frac{-8\pm\sqrt{8^2-4(1)(13)}}{2(1)} \Rightarrow$ $x=\frac{-8\pm\sqrt{12}}{2}=-4\pm\sqrt{3}.$ To find the y-intercept, let $x = 0$ and solve $y=0^2+8(0)+13=13.$

e.



55. $y=x^2-x-6 \Rightarrow a=1,\ b=-1,\ c=-6$

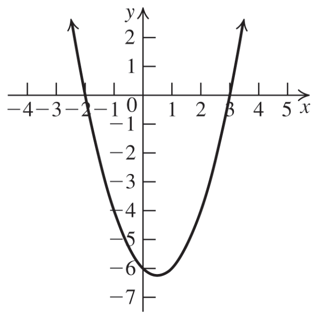
a. $a = 1 > 0$, so the graph opens up.

b. The vertex is $\left(-\frac{-1}{2(1)}, f\left(-\frac{-1}{2(1)}\right)\right)=\left(\frac{1}{2},-\frac{25}{4}\right).$

c. The axis of symmetry is $x=\frac{1}{2}.$

d. To find the x-intercepts, let $y = 0$ and solve $0=x^2-x-6 \Rightarrow 0=(x-3)(x+2) \Rightarrow x=3$ or $x=-2.$ To find the y-intercept, let $x = 0$ and solve $y=0^2-(0)-6 \Rightarrow y=-6.$

e.



56. $y = x^2 + x - 2 \Rightarrow a = 1,\ b = 1,\ c = -2$

a. $a = 1 > 0$, so the graph opens up.

b. The vertex is

$$\left(-\frac{1}{2(1)}, f\left(-\frac{1}{2(1)}\right)\right) = \left(-\frac{1}{2}, -\frac{9}{4}\right).$$

c. The axis of symmetry is $x = -\frac{1}{2}$.

d. To find the x-intercepts, let $y = 0$ and solve

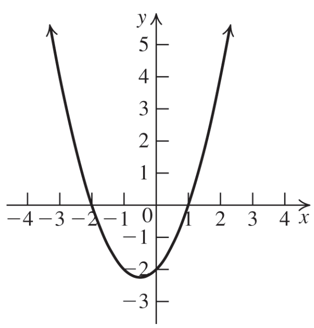
$0 = x^2 + x - 2 \Rightarrow 0 = (x-1)(x+2) \Rightarrow$

$x = 1$ or $x = -2$.

To find the y-intercept, let $x = 0$ and solve

$y = 0^2 + (0) - 2 \Rightarrow y = -2.$

e.



57. $y = x^2 - 2x + 4 \Rightarrow a = 1,\ b = -2,\ c = 4$

a. $a = 1 > 0$, so the graph opens up.

b. The vertex is $\left(-\frac{-2}{2(1)}, f\left(-\frac{-2}{2(1)}\right)\right) = (1, 3).$

c. The axis of symmetry is $x = 1$.

d. To find the x-intercepts, let $y = 0$ and solve

$0 = x^2 - 2x + 4 \Rightarrow$

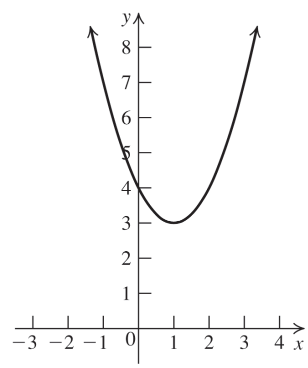
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} \Rightarrow$$

$$x = \frac{2 \pm \sqrt{-12}}{2} \Rightarrow \text{ there are no } x\text{-intercepts.}$$

To find the y-intercept, let $x = 0$ and solve

$y = 0^2 - 2(0) + 4 \Rightarrow y = 4.$

e.



58. $y = x^2 - 4x + 5 \Rightarrow a = 1,\ b = -4,\ c = 5$

a. $a = 1 > 0$, so the graph opens up.

b. The vertex is $\left(-\frac{-4}{2(1)}, f\left(-\frac{-4}{2(1)}\right)\right) = (2, 1).$

c. The axis of symmetry is $x = 2$.

d. To find the x-intercepts, let $y = 0$ and solve

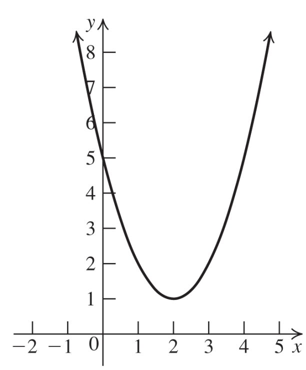
$0 = x^2 - 4x + 5 \Rightarrow$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} \Rightarrow$$

$x = \frac{4 \pm \sqrt{-4}}{2} \Rightarrow$ there are no x-intercepts. To find the y-intercept, let $x = 0$ and solve

$y = 0^2 - 4(0) + 5 \Rightarrow y = 5.$

e.



59. $y = 6 - 2x - x^2 \Rightarrow a = -1,\ b = -2,\ c = 6$

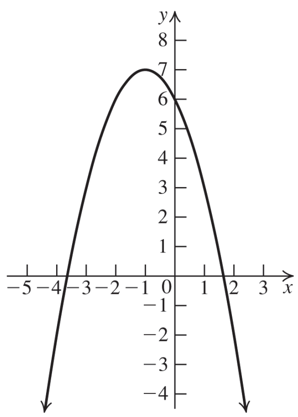
a. $a = -1 < 0$, so the graph opens down.

b. The vertex is $\left(-\frac{-2}{2(-1)}, f\left(-\frac{-2}{2(-1)}\right)\right) = (-1, 7)$.

c. The axis of symmetry is $x = -1$.

d. To find the x-intercepts, let $y = 0$ and solve $0 = 6 - 2x - x^2 \Rightarrow$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-1)(6)}}{2(-1)} \Rightarrow x = \frac{2 \pm \sqrt{28}}{-2} \Rightarrow$ $x = -1 \pm \sqrt{7}$.
To find the y-intercept, let $x = 0$ and solve $y = 6 - 2(0) - 0^2 \Rightarrow y = 6$.

e.



60. $y = 2 + 5x - 3x^2 \Rightarrow a = -3,\ b = 5,\ c = 2$

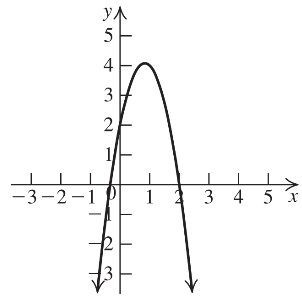
a. $a = -3 < 0$, so the graph opens down.

b. The vertex is $\left(-\frac{5}{2(-3)}, f\left(-\frac{5}{2(-3)}\right)\right) = \left(\frac{5}{6}, \frac{49}{12}\right)$.

c. The axis of symmetry is $x = \frac{5}{6}$.

d. To find the x-intercepts, let $y = 0$ and solve $0 = 2 + 5x - 3x^2 \Rightarrow 0 = -(3x^2 - 5x - 2) \Rightarrow$ $0 = -(3x + 1)(x - 2) \Rightarrow x = -\frac{1}{3}$ or $x = 2$. To find the y-intercept, let $x = 0$ and solve $y = 2 + 5(0) - 3(0)^2 \Rightarrow y = 2$.

e.



61. a. $a = 1 > 0$, so the graph opens up and has a minimum value. Find the minimum value by finding the vertex: $\left(-\frac{-4}{2(1)}, f\left(-\frac{-4}{2(1)}\right)\right) = (2, -1)$
The minimum value is –1.

b. The range of f is $[-1, \infty)$.

62. a. $a = -1 < 0$, so the graph opens down and has a maximum value. Find the maximum value by finding the vertex: $\left(-\frac{6}{2(-1)}, f\left(-\frac{6}{2(-1)}\right)\right) = (3, 1)$
The maximum value is 1.

b. The range of f is $(-\infty, 1]$.

63. a. $a = -1 < 0$, so the graph opens down and has a maximum value. Find the maximum value by finding the vertex: $\left(-\frac{4}{2(-1)}, f\left(-\frac{4}{2(-1)}\right)\right) = (2, 0)$
The maximum value is 0.

b. The range of f is $(-\infty, 0]$.

64. a. $a = 1 > 0$, so the graph opens up and has a minimum value. Find the minimum value by finding the vertex: $\left(-\frac{-6}{2(1)}, f\left(-\frac{-6}{2(1)}\right)\right) = (3, 0)$
The minimum value is 0.

b. The range of f is $[0, \infty)$.

65. a. $a = 2 > 0$, so the graph opens up and has a minimum value. Find the minimum value by finding the vertex: $\left(-\frac{-8}{2(2)}, f\left(-\frac{-8}{2(2)}\right)\right) = (2, -5)$
The minimum value is –5.

b. The range of f is $[-5, \infty)$.

66. a. $a = 3 > 0$, so the graph opens up and has a minimum value. Find the minimum value by finding the vertex:
$$\left(-\frac{12}{2(3)}, f\left(-\frac{12}{2(3)}\right)\right) = (-2, -17)$$
The minimum value is –17.

b. The range of f is $[-17, \infty)$.

67. a. $a = -4 < 0$, so the graph opens down and has a maximum value. Find the maximum value by finding the vertex:
$$\left(-\frac{12}{2(-4)}, f\left(-\frac{12}{2(-4)}\right)\right) = \left(\frac{3}{2}, 16\right)$$
The maximum value is 16.

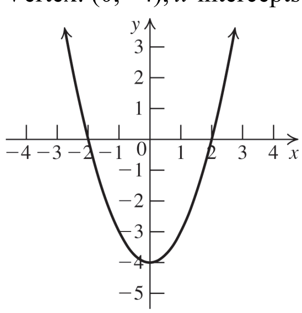
b. The range of f is $(-\infty, 16]$.

68. a. $a = -2 < 0$, so the graph opens down and has a maximum value. Find the maximum value by finding the vertex:
$$\left(-\frac{8}{2(-2)}, f\left(-\frac{8}{2(-2)}\right)\right) = (2, 3)$$
The maximum value is 3.

b. The range of f is $(-\infty, 3]$.

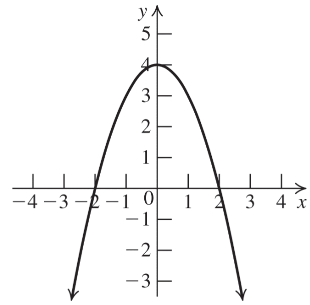
For exercises 69–82, refer to Example 4 for the process. The x-intercepts of the function are the boundaries of the intervals.

69. $x^2 - 4 \le 0$
Vertex: (0, –4); x-intercepts: –2, 2



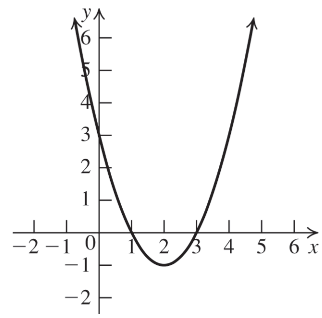
Solution: [–2, 2]

70. $4 - x^2 \ge 0$
Vertex: (0, 4); x-intercepts: –2, 2



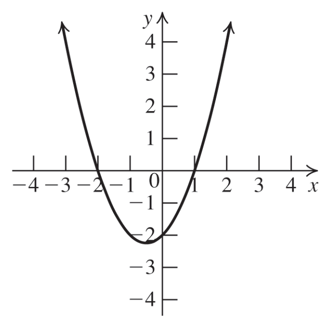
Solution: [–2, 2]

71. $x^2 - 4x + 3 < 0$
Vertex: (2, –1); x-intercepts: 1, 3



Solution: $(1, 3)$

72. $x^2 + x - 2 > 0$
Vertex: $\left(-\frac{1}{2}, -\frac{9}{4}\right)$, x-intercepts: –2, 1



Solution: $(-\infty, -2) \cup (1, \infty)$

73. $x^2 - 3x - 7 \ge 0$
Vertex: $\left(\frac{3}{2}, -\frac{37}{4}\right)$
To find the x-intercepts, use the quadratic equation to solve $x^2 - 3x - 7 = 0$.
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-7)}}{2(1)} = \frac{3 \pm \sqrt{37}}{2}$$

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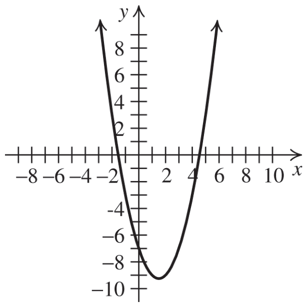


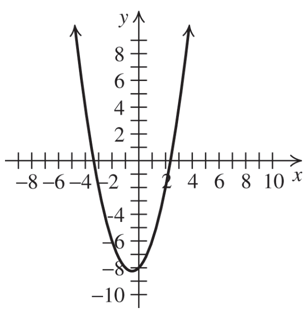
The solution of the inequality $x^2 - 3x - 7 \geq 0$ is $\left(-\infty, \frac{3}{2} - \frac{\sqrt{37}}{2}\right] \cup \left[\frac{3}{2} + \frac{\sqrt{37}}{2}, \infty\right)$.

74. $x^2 + x - 8 \leq 0$

Vertex: $\left(-\frac{1}{2}, -\frac{33}{4}\right)$

To find the x-intercepts, use the quadratic equation to solve $x^2 + x - 8 = 0$.

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-8)}}{2(1)} = \frac{-1 \pm \sqrt{33}}{2}$$



The solution of the inequality $x^2 + x - 8 \leq 0$ is $\left[-\frac{1}{2} - \frac{\sqrt{33}}{2}, -\frac{1}{2} + \frac{\sqrt{33}}{2}\right]$.

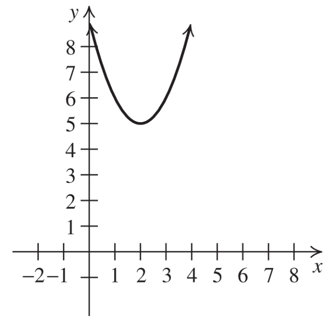
75. $x^2 - 4x + 9 \geq 0$

Vertex: (2, 5)

To find the x-intercepts, use the quadratic equation to solve $x^2 - 4x + 9 = 0$.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(9)}}{2(1)} = \frac{4 \pm \sqrt{-20}}{2}$$

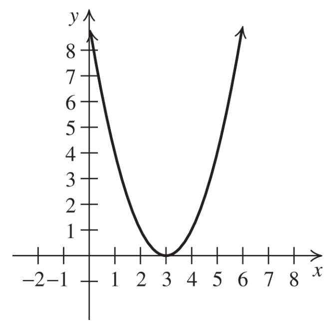
There are no x-intercepts.



Solution: $(-\infty, \infty)$

76. $x^2 - 6x + 9 \leq 0$

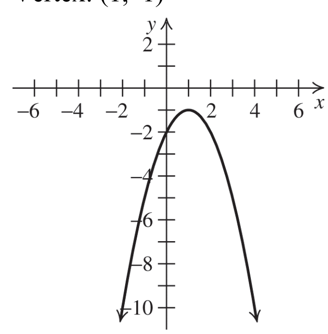
Vertex: (3, 0); x-intercept: (3, 0)



Solution: $\{3\}$

77. $-x^2 + 2x - 2 > 0$

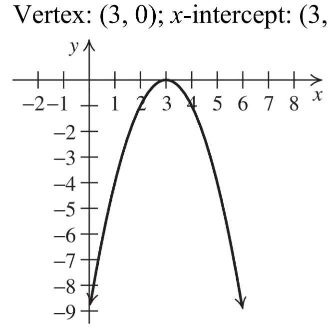
Vertex: (1,–1)



Solution: $\varnothing$

78. $-x^2 + 6x - 9 < 0$

Vertex: (3, 0); x-intercept: (3, 0)

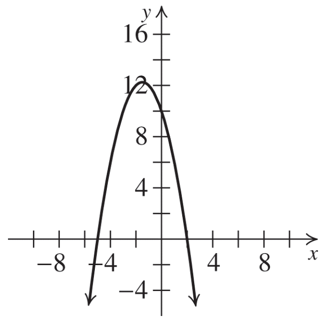


Solution: $(-\infty, 3) \cup (3, \infty)$

79. $-x^2 - 3x + 10 \geq 0$

Vertex: $\left(-\frac{3}{2}, \frac{49}{4}\right)$

x-intercepts: –5, 2

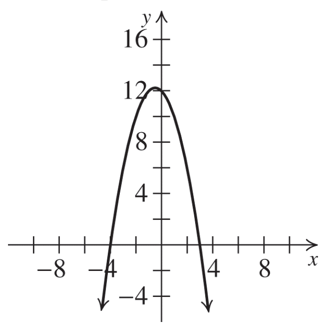


Solution: [–5, 2]

80. $-x^2 - x + 12 \leq 0$

Vertex: $\left(-\frac{1}{2}, \frac{49}{4}\right)$

x-intercepts: –4, 3

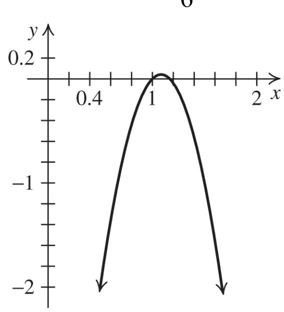


Solution: $(-\infty, -4] \cup [3, \infty)$

81. $-6x^2 + 13x - 7 < 0$

Vertex: $\left(\frac{13}{12}, \frac{1}{24}\right)$

x-intercepts: 1, $\frac{7}{6}$

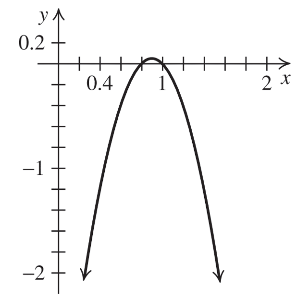


Solution: $(-\infty, 1) \cup \left(\frac{7}{6}, \infty\right)$

82. $-5x^2 + 9x - 4 > 0$

Vertex: $\left(\frac{9}{10}, \frac{1}{20}\right)$

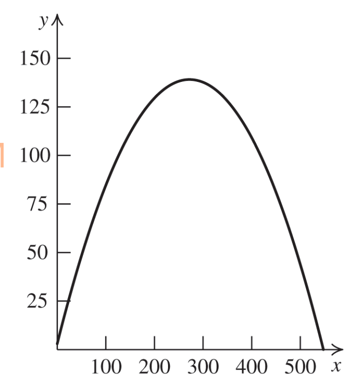
x-intercepts: $\frac{4}{5}$, 1



Solution: $\left(\frac{4}{5}, 1\right)$

Applying the Concepts

83. $h(x) = -\frac{32}{132^2}x^2 + x + 3$



a. Using the graph, we see that the ball traveled approximately 550 ft horizontally.

b. Using the graph, we see that the ball went approximately 140 high.

c. $0 = -\frac{32}{132^2}x^2 + x + 3$

$$x = \frac{-1 \pm \sqrt{1^2 - 4\left(-\frac{32}{132^2}\right)(3)}}{2\left(-\frac{32}{132^2}\right)}$$

≈ -3 or 547

Thus, the ball traveled approximately 547 ft horizontally.

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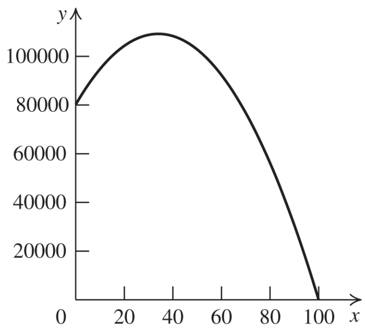
The ball reached its maximum height at the vertex of the function, $\left(-\frac{b}{2a}, h\left(-\frac{b}{2a}\right)\right)$.

$$-\frac{b}{2a} = -\frac{1}{2\left(-32/\left(132^2\right)\right)} = 272.25$$

$$h(272.25) = -\frac{32}{132^2}(272.25)^2 + 272.25 + 3 \approx 139$$

The ball reached approximately 139 ft

84. $R(x) = -25x^2 + 1700x + 80{,}000$



a. Using the graph, we see that the maximum revenue from the apartments is approximately \$110,000

b. Using the graph, we see that the maximum revenue is generated by about 35 \$25-increases.

c. The maximum revenue is at the vertex of the function, $\left(-\frac{b}{2a}, R\left(-\frac{b}{2a}\right)\right)$.

$$-\frac{b}{2a} = -\frac{1700}{2(-25)} = 34$$

$$R(34) = -25(34)^2 + 1700(34) + 80{,}000 = 108{,}900$$

The maximum revenue of \$108,900 is reached at 34 \$25 increases.

85. The vertex of the function is $\left(-\frac{114}{2(-3)}, f\left(-\frac{114}{2(-3)}\right)\right) = (19, 1098)$.

The revenue is at its maximum when $x = 19$.

86. $p = 200 - 4x \Rightarrow R(x) = 200x - 4x^2$.

The vertex of the revenue function is $\left(-\frac{200}{2(-4)}, f\left(-\frac{200}{2(-4)}\right)\right) = (25, 2500)$.

The revenue is at its maximum when $x = 25$.

87. $\left(-\frac{-50}{2(1)}, f\left(-\frac{-50}{2(1)}\right)\right) = (25, -425)$.

The total cost is minimum when $x = 25$.

88. $p = 100 - x \Rightarrow R(x) = 100x - x^2$.

Profit = revenue – cost

$\Rightarrow P(x) = \left(100x - x^2\right) - (50 + 2x) \Rightarrow$

$P(x) = -x^2 + 98x - 50$.

Now find the vertex of the profit function:

$\left(-\frac{98}{2(-1)}, f\left(-\frac{98}{2(-1)}\right)\right) = (49, 2351)$.

So the maximum profit occurs when $x = 49$. The maximum profit is \$2351.

89. Let x = the length of the rectangle. Then $\frac{80 - 2x}{2} = 40 - x =$ the width of the rectangle.

The area of the rectangle = $x(40 - x)$

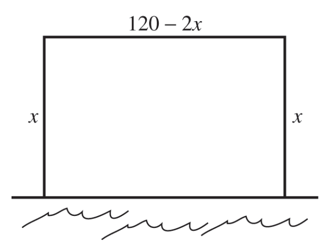
$= 40x - x^2$.

Find the vertex to find the maximum value:

$\left(-\frac{40}{2(-1)}, f\left(-\frac{40}{2(-1)}\right)\right) = (20, 400)$.

The rectangle with the maximum area is a square with sides of length 20 units. Its area is 400 square units.

90. The fence encloses three sides of the region. Let x = the width of the region. Then $120 - 2x$ = the length of the region. The area of the region = $x(120 - 2x) = 120x - 2x^2$.

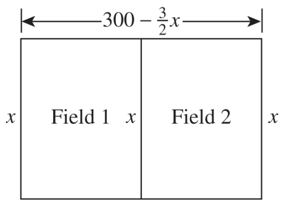


Find the vertex to find the maximum value:

$\left(-\frac{120}{2(-2)}, f\left(-\frac{120}{2(-2)}\right)\right) = (30, 1800)$.

The maximum area that can be enclosed is 1800 square meters.

91. Let x = the width of the fields. Then, $\frac{600-3x}{2} = 300 - \frac{3}{2}x =$ the length of the two fields together. (Note that there is fencing between the two fields, so there are three "widths.")



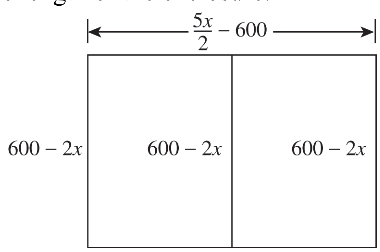
The total area $= x\left(300 - \frac{3}{2}x\right) = 300x - \frac{3}{2}x^2.$

Find the vertex to find the dimensions and maximum value:

$$\left(-\frac{300}{2(-3/2)}, f\left(-\frac{300}{2(-3/2)}\right)\right) = (100, 15,000).$$

So the width of each field is 100 meters. The length of the two fields together is 300 – 1.5(100) = 150 meters, so the length of each field is 150/2 = 75 meters. The area of each field is 100(75) = 7500 square meters.

92. Let x = the amount of high fencing. Then $8x$ = the cost of the high fencing. The cost of the low fencing = 2400 – $8x$, and $\frac{2400-8x}{4} = 600 - 2x$ = the amount of low fencing. This is also the width of the enclosure. So $\frac{x-2(600-2x)}{2} = \frac{5x-1200}{2} = \frac{5}{2}x - 600 =$ the length of the enclosure.



The area of the entire enclosure is

$$(600-2x)\left(\frac{5}{2}x - 600\right)$$
$$= -5x^2 + 2700x - 360,000.$$

Use the vertex to find the dimensions and maximum area:

$$\left(-\frac{2700}{2(-5)}, f\left(-\frac{2700}{2(-5)}\right)\right) = (270, 4500).$$

So the area of the entire enclosure is 4500 square feet. There are 270 feet of high fencing, so the dimensions of the enclosure are $\frac{5(270)}{2} - 600 = 75$ feet by $600 - 2(270) = 60$ feet. The question asks for the dimensions and maximum area of each half of the enclosure, so each half has maximum area 2250 sq ft with dimensions 37.5 ft by 60 ft.

93. The yield per tree is modeled by the equation of a line passing through (26, 500) where the x-coordinate represents the number of trees planted, and the y-coordinate represents the number of apples per tree. The rate of change is –10; that is, for each tree planted the yield decreases by 10. So, $y - 500 = -10(x - 26) \Rightarrow y = -10x + 760.$ Since there are x trees, the total yield = $x(-10x + 760) = -10x^2 + 760x.$ Use the vertex to find the number of trees that will maximize the yield:

$$\left(-\frac{760}{2(-10)}, f\left(-\frac{760}{2(-10)}\right)\right) = (38, 14, 440).$$

So the maximum yield occurs when 38 trees are planted per acre.

94. Let x = the number of days. The original price is \$1.50 per pound and the price decreases \$0.02 per pound each day, so the price per pound is (1.5 – $0.02x$). The weight of the steer after x days is 300 + $8x$. So the selling price = number of pounds times price per pound $= (300 + 8x)(1.5 - 0.02x).$ The original cost of the steer is 1.5(300) = \$450, and the daily cost of the steer is x, so the total cost of the steer after x days is x + 450. The profit is selling price – cost

$$= (300 + 8x)(1.5 - 0.02x) - (x + 450)$$
$$= (-0.16x^2 + 6x + 450) - (x + 450)$$
$$= -0.16x^2 + 5x.$$

The maximum profit occurs at the x-coordinate of the vertex:

$$\left(-\frac{5}{2(-0.16)}\right) = 15.625.$$

The maximum profit occurs after 16 days.

95. If 20 students or less go on the trip, the cost is $72 per students. If more than 20 students go on the trip, the cost is reduced by $2 per the number of students over 20. So the cost per student is a piecewise function based on the number of students, n, going on the trip:

$$f(n) = \begin{cases} 72 & \text{if } n \le 20 \\ 72 - 2(n-20) = 112 - 2n & \text{if } n > 20 \end{cases}$$

The total revenue is

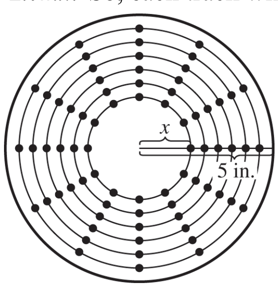
$$nf(n) = \begin{cases} 72n & \text{if } n \le 20 \\ n(112 - 2n) = 112n - 2n^2 & \text{if } n > 20 \end{cases}$$

The maximum revenue is either 1440 (the revenue if 20 students go on the trip) or the maximum of $112n - 2n^2$. Find this by using the vertex:

$$\left(-\frac{112}{(2)(-2)}, f\left(-\frac{112}{(2)(-2)}\right)\right) = (28, 1568).$$

The maximum revenue is $1568 when 28 students go on the trip.

96. Assume that m bytes per inch can be put on any track. Let x = the radius of the innermost track. Then the maximum number of bytes that can be put on the innermost track is $2\pi mx$. So, each track will have $2\pi mx$ bytes.



The total number of bytes on the disk is the number of bytes on each track times the number of tracks per inch times the radius (in inches): $2\pi mx(p(5-x)) = 2\pi mp(5x - x^2)$

$= -2\pi mpx^2 + 10\pi mpx$.

The maximum occurs at the x-coordinate of the vertex: $-\frac{b}{2a} = -\frac{10\pi mp}{2(-2\pi mp)} = \frac{10}{4} = 2.5$ inches from the center.

97. $h(t) = -\frac{g_M}{2}t^2 + v_0 t + h_0$

$g_M = 1.6\,\text{m/s}^2$, $h_0 = 5$ m, max height = 25 m

a. Using the given values, we have

$$h(t) = -\frac{1.6}{2}t^2 + v_0 t + 5 = -0.8t^2 + v_0 t + 5$$

Using the formula for the vertex of a parabola gives $t = -\frac{v_0}{2(-0.8)} = \frac{v_0}{1.6}$. This is the time at which the height $h(t) = 25$ m is attained. Thus,

$$\begin{aligned} h(t) = 25 = h\left(\frac{v_0}{1.6}\right) &= -0.8\left(\frac{v_0}{1.6}\right)^2 + v_0\left(\frac{v_0}{1.6}\right) + 5 \\ &= -\frac{v_0^2}{3.2} + \frac{v_0^2}{1.6} + 5 = \frac{v_0^2}{3.2} + 5 \end{aligned}$$

Solving for v_0 yields

$$25 = \frac{v_0^2}{3.2} + 5 \Rightarrow 20 = \frac{v_0^2}{3.2} \Rightarrow$$
$$v_0^2 = 64 \Rightarrow v_0 = 8$$

Thus, $h(t) = -0.8t^2 + 8t + 5$.

b. Using the formula for the vertex of a parabola gives $t = -\frac{8}{2(-0.8)} = \frac{8}{1.6} = 5$.

The ball reached its highest point 5 seconds after it was released.

98. $h(t) = -\frac{g_M}{2}t^2 + v_0 t + h_0$

$g_M = 1.6\,\text{m/s}^2$, $h_0 = 7$ m, max height = 52 m

a. Using the given values, we have

$$h(t) = -\frac{1.6}{2}t^2 + v_0 t + 7 = -0.8t^2 + v_0 t + 7$$

Using the formula for the vertex of a parabola gives $t = -\frac{v_0}{2(-0.8)} = \frac{v_0}{1.6}$. This is the time at which the height $h(t) = 52$ m is attained. Thus,

$$\begin{aligned} h(t) = 52 = h\left(\frac{v_0}{1.6}\right) &= -0.8\left(\frac{v_0}{1.6}\right)^2 + v_0\left(\frac{v_0}{1.6}\right) + 7 \\ &= -\frac{v_0^2}{3.2} + \frac{v_0^2}{1.6} + 7 = \frac{v_0^2}{3.2} + 7 \end{aligned}$$

Solving for v_0 yields

$$52 = \frac{v_0^2}{3.2} + 7 \Rightarrow 45 = \frac{v_0^2}{3.2} \Rightarrow$$
$$v_0^2 = 144 \Rightarrow v_0 = 12$$

Thus, $h(t) = -0.8t^2 + 12t + 7$.

b. Using the formula for the vertex of a parabola gives $t = -\frac{12}{2(-0.8)} = \frac{12}{1.6} = 7.5.$ The ball reached its highest point 7.5 seconds after it was released.

99. a. The maximum height occurs at the vertex: $\left(-\frac{64}{2(-16)}, f\left(-\frac{64}{2(-16)}\right)\right) = (2, 64),$ so the maximum height is 64 feet.

b. When the projectile hits the ground, $h = 0$, so solve

$0 = -16t^2 + 64t \Rightarrow -16t(t-4) = 0 \Rightarrow$
$t = 0$ or $t = 4$.

The projectile hits the ground at 4 seconds.

100. a. The maximum height occurs at the vertex: $\left(-\frac{64}{2(-16)}, f\left(-\frac{64}{2(-16)}\right)\right) = (2, 464).$

The maximum height is 464 feet.

b. When the projectile hits the ground, $h = 0$, so solve $0 = -16t^2 + 64t + 400 \Rightarrow$

$-16(t^2 - 4t - 25) = 0 \Rightarrow t^2 - 4t - 25 = 0 \Rightarrow$

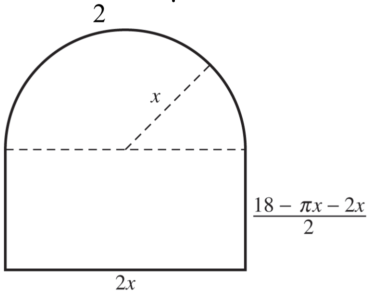
$t = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-25)}}{2(1)} \Rightarrow$

$t = \frac{4 \pm \sqrt{116}}{2} \Rightarrow t = 2 \pm \sqrt{29} \Rightarrow$

$t \approx -3.39$ or $t \approx 7.39$.

Reject the negative solution (time cannot be negative). The projectile hits the ground at about 7.39 seconds.

101. Let x = the radius of the semicircle. Then the length of the rectangle is $2x$. The circumference of the semicircle is πx, so the perimeter of the rectangular portion of the window is $18 - \pi x$. The width of the rectangle $= \frac{18 - \pi x - 2x}{2}$.



The area of the semicircle is $\pi x^2/2$, and the area of the rectangle is $2x\left(\frac{18 - \pi x - 2x}{2}\right) = 18x - \pi x^2 - 2x^2$. So the total area is

$18x - \pi x^2 - 2x^2 + \frac{\pi x^2}{2} = 18x - 2x^2 - \frac{\pi x^2}{2} \Rightarrow$

$18x - \left(2 + \frac{\pi}{2}\right)x^2.$

The maximum area occurs at the x-coordinate of the vertex:

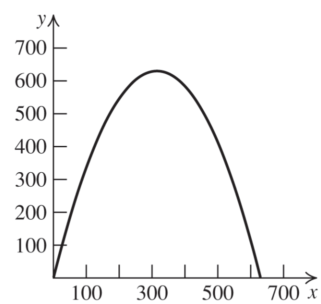
$-\frac{18}{2\left(-2 - \frac{\pi}{2}\right)} = -\frac{18}{-4-\pi} = \frac{18}{4+\pi}.$

This is the radius of the semicircle. The length of the rectangle is $2\left(\frac{18}{4+\pi}\right) = \frac{36}{4+\pi}$ ft.

The width of the rectangle is

$\frac{18 - \pi\left(\frac{18}{4+\pi}\right) - 2\left(\frac{18}{4+\pi}\right)}{2} = \frac{18}{4+\pi}$ ft.

102. a.



b. The width of the arch is the difference between the two x-intercepts:

$0 = -0.00635x^2 + 4x \Rightarrow$
$0 = 4x(-0.0015875x + 1) \Rightarrow x = 0$ or
$x \approx 629.92.$

The arch is approximately 629.92 feet wide.

c. The maximum height occurs at the vertex of the arch. We know that the arch is approximately 629.92 feet wide, the vertex occurs at $x \approx 629.92/2 = 314.96$.

$f(314.96) = -0.00635(314.96)^2 + 4(314.96)$
≈ 629.92 feet.

Note that the arch is as tall as it is wide!

103. a. $V = \left(-\frac{1.18}{2(-0.01)}, f\left(-\frac{1.18}{2(-0.01)}\right)\right)$
$= (59, 36.81)$

b. $0 = -0.01x^2 + 1.18x + 2 \Rightarrow$

$$x = \frac{-1.18 \pm \sqrt{1.18^2 - 4(-0.01)(2)}}{2(-0.01)} = \frac{-1.18 \pm \sqrt{1.4724}}{-0.02} \approx \frac{-1.18 \pm 1.21}{-0.02} \approx -1.5 \text{ or } 119.5.$$

Reject the negative answer, and round the positive answer to the nearest whole number. The ball hits the ground approximately 120 feet from the punter.

c. The maximum height occurs at the vertex. (See part (a).) The maximum height is approximately 37 ft.

d. The player is at $x = 6$ feet.

$f(6) = -0.01(6)^2 + 1.18(6) + 2 = 8.72.$

The player must reach approximately 9 feet to block the ball.

e. $7 = -0.01x^2 + 1.18x + 2$

$0 = -0.01x^2 + 1.18x - 5$

$$x = \frac{-1.18 \pm \sqrt{1.18^2 - 4(-0.01)(-5)}}{2(-0.01)} = \frac{-1.18 \pm \sqrt{1.1924}}{-0.02} \Rightarrow$$

$x \approx 4.4$ feet or $x \approx 113.6$ feet

104. The maximum height occurs at the vertex:

$$\left(-\frac{30}{2(-16)}, f\left(-\frac{30}{2(-16)}\right)\right) = \left(\frac{15}{16}, \frac{225}{16}\right).$$

The maximum height is approximately 14 feet, so it will never reach a height of 16 feet.

105. $f(x) = 15.15 + 1.216x - 0.152x^2$

a. We must complete the square to write the function in standard form.

$$\begin{aligned} &-0.152x^2 + 1.216x + 15.15 \\ &= -0.152\left(x^2 - \frac{1.216}{0.152}x\right) + 15.15 \\ &= -0.152\left(x^2 - \frac{1.216}{0.152}x + \frac{1.216^2}{(2\cdot 0.152)^2}\right) \\ &\qquad + 15.15 + (0.152)\left(\frac{1.216^2}{(2\cdot 0.152)^2}\right) \\ &\approx -0.152\left(x - \frac{1.216}{2\cdot 0.152}\right)^2 + 17.582 \\ &\approx -0.152(x-4)^2 + 17.582 \end{aligned}$$

b. The vertex of the function is (4, 17.582). Sales were at a maximum at $x = 4$, which is year 4, 2016. This fits the original data.

c. The year 2015 is represented by $x = 3$.

$$\begin{aligned} f(3) &= 15.15 + 1.216(3) - 0.152(3)^2 \\ &= 17.43 \end{aligned}$$

In 2015, there were about 17.43 million trucks and cars sold.

106. $f(x) = 65.3 - 0.8x + 0.11x^2$

a. We must complete the square to write the function in standard form.

$$\begin{aligned} &0.11x^2 - 0.8x + 65.3 \\ &= 0.11\left(x^2 - \frac{0.8}{0.11}x\right) + 65.3 \\ &= 0.11\left(x^2 - \frac{0.8}{0.11}x + \frac{0.8^2}{(2\cdot 0.11)^2}\right) \\ &\qquad + 65.3 - 0.11\left(\frac{0.8^2}{(2\cdot 0.11)^2}\right) \\ &= 0.11\left(x - \frac{0.8}{2\cdot 0.11}\right)^2 + 63.8 \\ &= 0.11(x-3.64)^2 + 63.8 \end{aligned}$$

b. The vertex of the function is $(3.64, -63.8)$. The home-ownership rate was at a minimum at $x \approx 3.64$, or during year 4, 2016. This fits the original data.

c. The year 2011 is represented by $x = -1$.

$$\begin{aligned} f(-1) &= 65.3 - 0.8(-1) + 0.11(-1)^2 \\ &\approx 66.2 \end{aligned}$$

In 2011, about 66.2% of homes were owned by their occupants.

Beyond the Basics

107. $y = 3(x+2)^2 + 3 \Rightarrow y = 3(x^2 + 4x + 4) + 3 \Rightarrow$
$y = 3x^2 + 12x + 15$

108. Substitute the coordinates (1, 5) for x and y and (–3, –2) for h and k into the standard form $y = a(x-h)^2 + k$ to solve for a:

$$5 = a(1+3)^2 - 2 \Rightarrow 7 = 16a \Rightarrow \frac{7}{16} = a$$

The equation is $y = f(x) = \frac{7}{16}(x+3)^2 - 2 \Rightarrow$

$$y = \frac{7}{16}x^2 + \frac{21}{8}x + \frac{31}{16}.$$

109. The y-intercept is 4, so the graph passes through (0, 4). Substitute the coordinates (0, 4) for x and y and (1, –2) for h and k into the standard form $y = a(x-h)^2 + k$ to solve for a: $4 = a(0-1)^2 - 2 \Rightarrow 6 = a.$
The x-coordinate of the vertex is
$1 = -\frac{b}{2a} = -\frac{b}{2(6)} \Rightarrow b = -12.$
The y-intercept $= c$, so the equation is
$f(x) = 6x^2 - 12x + 4.$

110. The x-coordinate of the vertex of the graph is halfway between the x-intercepts:
$\frac{2+6}{2} = 4 = h.$ Substitute the coordinates of one of the x-intercepts and the x-coordinate of the vertex into the standard form $y = a(x-h)^2 + k$ to find an expression relating a and k: $0 = a(2-4)^2 + k \Rightarrow -4a = k.$
Now substitute the coordinates of the y-intercept, the x-coordinate of the vertex, and the expression for k into the standard form $y = a(x-h)^2 + k$ to solve for a:
$24 = a(0-4)^2 - 4a \Rightarrow$
$24 = 16a - 4a \Rightarrow 2 = a.$ Use this value to find k: $k = -4(2) = -8.$
The equation is
$y = 2(x-4)^2 - 8 \Rightarrow y = 2x^2 - 16x + 24.$

111. The x-coordinate of the vertex is 3. Substitute the coordinates of the x-intercept and the x-coordinate of the vertex into the standard form $y = a(x-h)^2 + k$ to find an expression relating a and k:
$0 = a(7-3)^2 + k \Rightarrow -16a = k.$ Now substitute the coordinates of the y-intercept, the x-coordinate of the vertex, and the expression for k into the standard form $y = a(x-h)^2 + k$ to solve for a: $14 = a(0-3)^2 - 16a \Rightarrow -2 = a.$
Use this value to find k: $k = -16(-2) = 32.$
The equation is
$y = -2(x-3)^2 + 32 \Rightarrow y = -2x^2 + 12x + 14.$

112. First, write $y = x^2 + 2x + 2$ in standard form by completing the square:
$y = \left(x^2 + 2x + 1\right) + 1 \Rightarrow y = (x+1)^2 + 1.$
Move the curve three units to the right by subtracting 3 from $x + 1$; move the curve two units down by subtracting 2 from 1:
$y = (x+1-3)^2 + (1-2) = (x-2)^2 - 1 \Rightarrow$
$y = x^2 - 4x + 3.$

113. The x-coordinate of the vertex is $\frac{-2+6}{2} = 2.$
Let $a = 1$ and then substitute the coordinates of one of the x-intercepts and the x-coordinate of the vertex into the standard form $y = a(x-h)^2 + k$ to find a value for k:
$0 = 1(-2-2)^2 + k \Rightarrow -16 = k.$
So one equation is
$y = 1(x-2)^2 - 16 = x^2 - 4x - 12.$ The graph of this equation opens upward. To find the equation of the graph that opens downward, multiply the equation by –1:
$y = -x^2 + 4x + 12.$

114. The x-coordinate of the vertex is $\frac{-3+5}{2} = 1.$
Let $a = 1$ and then substitute the coordinates of one of the x-intercepts and the x-coordinate of the vertex into the standard form $y = a(x-h)^2 + k$ to find a value for k:
$0 = 1(-3-1)^2 + k \Rightarrow -16 = k.$ So one equation is $y = (x-1)^2 - 16 = x^2 - 2x - 15.$
The graph of this equation opens upward. To find the equation of the graph that opens downward, multiply the equation by –1:
$y = -x^2 + 2x + 15.$

115. The x-coordinate of the vertex is $\frac{-7-1}{2} = -4.$
Let $a = 1$ and then substitute the coordinates of one of the x-intercepts and the x-coordinate of the vertex into the standard form $y = a(x-h)^2 + k$ to find a value for k:
$0 = 1(-7+4)^2 + k \Rightarrow -9 = k.$ So one equation is $y = (x+4)^2 - 9 = x^2 + 8x + 7.$ The graph of this equation opens upward. To find the equation of the graph that opens downward, multiply the equation by –1:
$y = -x^2 - 8x - 7.$

116. The x-coordinate of the vertex is $\frac{2+10}{2} = 6$. Let $a = 1$ and then substitute the coordinates of one of the x-intercepts and the x-coordinate of the vertex into the standard form $y = a(x-h)^2 + k$ to find a value for k: $0 = 1(2-6)^2 + k \Rightarrow -16 = k$. So one equation is $y = (x-6)^2 - 16 = x^2 - 12x + 20$. The graph of this equation opens upward. To find the equation of the graph that opens downward, multiply the equation by –1: $y = -x^2 + 12x - 20$.

117. First, complete the squares on x and y.

$$\begin{aligned}2x^2 + y^2 - 4x + 6y + 15 = 2x^2 - 4x + y^2 + 6y + 15 &= 2\left(x^2 - 2x\right) + \left(y^2 + 6y\right) + 15\\ &= 2\left(x^2 - 2x + 1\right) + \left(y^2 + 6y + 9\right) + 15 - 2(1) - 9\\ &= 2(x-1)^2 + (y+3)^2 + 4\end{aligned}$$

Now, consider the two functions $f = 2(x-1)^2$ and $g = (y+3)^2 + 4$.
The minimum of f is 0 and the minimum of g is 4, so the minimum of $f + g = 4$.

118. First, complete the squares on x and y.

$$\begin{aligned}-3x^2 - y^2 - 12x + 2y - 11 = -3x^2 - 12x - y^2 + 2y - 11 &= -3\left(x^2 + 4x\right) - \left(y^2 - 2y\right) - 11\\ &= -3\left(x^2 + 4x + 4\right) - \left(y^2 - 2y + 1\right) - 11 + 12 + 1\\ &= -3(x+2)^2 - (y-1)^2 + 2\end{aligned}$$

Now consider the two functions $f = -3(x+2)^2$ and $g = -(x-1)^2 + 2$. The maximum of f is 0 and the maximum of g is 2, so the maximum of $f + g = 2$.

Critical Thinking/Discussion/Writing

119. $f(h+p) = a(h+p)^2 + b(h+p) + c;\quad f(h-p) = a(h-p)^2 + b(h-p) + c$

$$\begin{aligned}f(h+p) &= f(h-p)\\ a(h+p)^2 + b(h+p) + c &= a(h-p)^2 + b(h-p) + c\\ ah^2 + 2ahp + ap^2 + bh + bp + c &= ah^2 - 2ahp + ap^2 + bh - bp + c\\ 4ahp &= -2bp \Rightarrow 2ah = -b.\ \text{(We can divide by } p \text{ because } p \neq 0.)\end{aligned}$$

Because $a \neq 0$, $2ah = -b \Rightarrow h = -\frac{b}{2a}$.

120. Write $y = 2x^2 - 8x + 9$ in standard form to find the axis of symmetry:
$y = 2x^2 - 8x + 9 \Rightarrow y = 2(x^2 - 4x + 4) - 2(4) + 9 \Rightarrow y = 2(x-2)^2 + 1$.
The axis of symmetry is $x = 2$.
Using the results of exercise 119, we know that $f(2+p) = f(-1) = f(2-p)$.
$2 + p = -1 \Rightarrow p = -3$, so $2 - p = 2 - (-3) = 5$.
The point symmetric to the point (–1, 19) across the axis of symmetry is (5, 19).

121. a.

$$\begin{aligned}(f \circ g)(x) = f(mx+b) &= a\left[(mx+b) - h\right]^2 + k\\ &= a\left[(mx+b)^2 - 2h(mx+b) + h^2\right] + k\\ &= a\left[m^2x^2 + 2bmx + b^2 - 2hmx - 2hb + h^2\right] + k\\ &= am^2x^2 + 2abmx + ab^2 - 2ahmx - 2ahb + ah^2 + k\\ &= am^2x^2 + (2abm - 2ahm)x + \left(ab^2 - 2ahb + ah^2 + k\right)\end{aligned}$$

(*continued on next page*)

(*continued*)

This is the equation of a parabola. The x-coordinate of the vertex is
$-\frac{2abm-2ahm}{2am^2}=-\frac{2am(b-h)}{2am^2}=-\frac{b-h}{m}$ or $\frac{h-b}{m}$.
The y-coordinate of the vertex is

$$\begin{aligned}am^2\left(\frac{h-b}{m}\right)^2&+(2abm-2ahm)\left(\frac{h-b}{m}\right)+\left(ab^2-2ahb+ah^2+k\right)\\&=a(h-b)^2+2a(b-h)(h-b)+\left(ab^2-2ahb+ah^2+k\right)\\&=ah^2-2ahb+ab^2-2ab^2+4abh-2ah^2+ab^2-2ahb+ah^2+k=k\end{aligned}$$

The vertex is $\left(\frac{h-b}{m},\ k\right)$.

b. $$\begin{aligned}(g\circ f)(x)&=g\left[a(x-h)^2+k\right]=m\left[a(x-h)^2+k\right]+b\\&=m\left(ax^2-2ahx+ah^2+k\right)+b=max^2-2mahx+mah^2+mk+b\end{aligned}$$

This is the equation of a parabola. The x-coordinate of the vertex is $-\frac{-2mah}{2ma}=h$.

The y-coordinate of the vertex is $mah^2-2mah^2+mah^2+mk+b=mk+b$
The vertex is $(h,\ mk+b)$.

122. If the discriminant equals zero, there is exactly one real solution. Thus, the vertex of $y=f(x)$ lies on the x-axis at $x=-\frac{b}{2a}$. If the discriminant >0, there are two unequal real solutions. This means that the graph of $y=f(x)$ crosses the x-axis in two places. If $a>0$, then the vertex lies below the x-axis and the parabola crosses the x-axis; if $a<0$, then the vertex lies above the x-axis and the parabola crosses the x-axis. If the discriminant <0, there are two nonreal complex solutions. If $a>0$, then the vertex lies above the x-axis and the parabola does not cross the x-axis (it opens upward); if $a<0$, then the vertex lies below the x-axis and the parabola does not cross the x-axis (it opens downward).

123. Start by writing the function in standard form.

$$\begin{aligned}f(x)&=2x^2-2ax+a^2\\&=2\left(x^2-ax\right)+a^2\\&=2\left(x^2-ax+\frac{a^2}{4}\right)+a^2-\frac{a^2}{2}\\&=2\left(x-\frac{a}{2}\right)^2+\frac{a^2}{2}\end{aligned}$$

The minimum value of this function is $\frac{a^2}{2}$.

Now, rewrite the function as

$$\begin{aligned}f(x)&=a^2-2ax+2x^2\\&=\left(a^2-2ax\right)+2x^2\\&=\left(a^2-2ax+x^2\right)+2x^2-x^2\\&=(a-x)^2+x^2\end{aligned}$$

This is the same function, so
$(a-x)^2+x^2\ge\frac{a^2}{2}$, or $x^2+(a-x)^2\ge\frac{a^2}{2}$.

124. Start by writing the function in standard form.

$$\begin{aligned}f(x)&=-x^2+ax=-\left(x^2-ax\right)\\&=-\left(x^2-ax+\frac{a^2}{4}\right)+\frac{a^2}{4}\\&=-\left(x-\frac{a}{2}\right)^2+\frac{a^2}{4}\end{aligned}$$

The maximum value of this function is $\frac{a^2}{4}$.

Now, rewrite the function as
$f(x)=ax-x^2=x(a-x)$

This is the same function, so $x(a-x)\le\frac{a^2}{4}$.

Active Learning

125. a.–b. Refer to the app using the QR code in your text.

Getting Ready for the Next Section

GR1. $\left(-2x^2\right)\left(-3x^4\right)=(-2)(-3)\left(x^2\right)\left(x^4\right)$
$=6x^{2+4}=6x^6$

GR2. $\left(2x^2\right)(3x)\left(-x^3\right)=(2)(3)(-1)\left(x^2\right)(x)\left(x^3\right)$
$=-6x^{2+1+3}=-6x^6$

GR3. $x^2\left(3-\frac{3}{4}\right)=x^2\left(\frac{12}{4}-\frac{3}{4}\right)=\frac{9}{4}x^2$

GR4. $x^4\left(1+\frac{3}{x^2}-\frac{5}{x^3}\right)=x^4(1)+x^4\left(\frac{3}{x^2}\right)+x^4\left(-\frac{5}{x^3}\right)$
$=x^4+3x^2-5x$

GR5. $4x^2-9=(2x+3)(2x-3)$

GR6. $x^2+6x+9=(x+3)^2$

GR7. $15x^2+11x-12=(3x+4)(5x-3)$

GR8. $14x^2-3x-2=(2x-1)(7x+2)$

GR9. $x^2(x-1)-4(x-1)=\left(x^2-4\right)(x-1)$
$=(x+2)(x-2)(x-1)$

GR10. $9x^2(2x+7)-25(2x+7)$
$=\left(9x^2-25\right)(2x+7)$
$=(3x+5)(3x-5)(2x+7)$

GR11. $x^3+4x^2+3x+12=\left(x^3+4x^2\right)+(3x+12)$
$=x^2(x+4)+3(x+4)$
$=\left(x^2+3\right)(x+4)$

GR12. $2x^3-3x^2+2x-3=\left(2x^3-3x^2\right)+(2x-3)$
$=x^2(2x-3)+(2x-3)$
$=\left(x^2+1\right)(2x-3)$

GR13. $3x-5=3-2x$
$3x-5+2x+5=3-2x+2x+5$
$5x=8\Rightarrow x=\frac{8}{5}$

Solution set: $\left\{\frac{8}{5}\right\}$

GR14. $5x-1=7x+6$
$5x-1-7x+1=7x+6-7x+1$
$-2x=7\Rightarrow x=-\frac{7}{2}$

Solution set: $\left\{-\frac{7}{2}\right\}$

GR15. $6x^2-x-2=0\Rightarrow(3x-2)(2x+1)=0\Rightarrow$
$x=\frac{2}{3},\ x=-\frac{1}{2}$

Solution set: $\left\{-\frac{1}{2},\frac{2}{3}\right\}$

GR16. $12x^2+5x-3=0\Rightarrow(4x+3)(3x-1)=0\Rightarrow$
$x=-\frac{3}{4},\ x=\frac{1}{3}$

Solution set: $\left\{-\frac{3}{4},\frac{1}{3}\right\}$

2.2 Polynomial Functions

Practice Problems

1. a. $f(x)=\frac{x^2+1}{x-1}$ is not a polynomial function because its domain is not $(-\infty,\infty)$.

b. $g(x)=2x^7+5x^2-17$ is a polynomial function. Its degree is 7, the leading term is $2x^7$, and the leading coefficient is 2.

2. $P(x)=4x^3+2x^2+5x-17$
$=x^3\left[4+\frac{2}{x}+\frac{5}{x^2}-\frac{17}{x^3}\right]$

When $|x|$ is large, the terms $\frac{2}{x}$, $\frac{5}{x^2}$, and $-\frac{17}{x^3}$ are close to 0. Therefore,
$P(x)=x^3(4+0+0-0)\approx 4x^3$.

3. Use the leading-term test to determine the end behavior of $y=f(x)=-2x^4+5x^2+3$. Here $n=4$ and $a_n=-2<0$. Thus, Case 2 applies. The end behavior is described as $y\to-\infty$ as $x\to-\infty$ and $y\to-\infty$ as $x\to\infty$.

4. First group the terms, then factor and solve $f(x)=0$:

$$\begin{aligned} f(x) &= 2x^3 - 3x^2 + 4x - 6 \\ &= 2x^3 + 4x - 3x^2 - 6 \\ &= 2x\left(x^2+2\right) - 3\left(x^2+2\right) \\ &= (2x-3)\left(x^2+2\right) \end{aligned}$$

$0=(2x-3)\left(x^2+2\right)$

$0=2x-3 \Rightarrow x=\frac{3}{2}$ or

$0=x^2+2$ (no real solution)

The only real zero is $\frac{3}{2}$.

5. $f(x)=2x^3-3x-6$

$f(1)=-7$ and $f(2)=4$. Because $f(1)$ and $f(2)$ have opposite signs, by the Intermediate Value Theorem, *f* has a real zero between 1 and 2.

6. $f(x)=(x+1)^2(x-3)(x+5)=0 \Rightarrow$

$(x+1)^2=0$ or $x-3=0$ or $x+5=0 \Rightarrow$

$x=-1$ or $x=3$ or $x=-5$

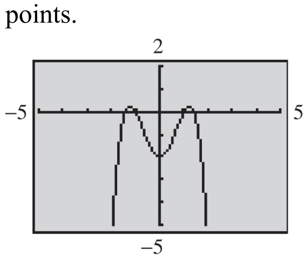
Thus, $f(x)$ has three distinct zeros.

7. $f(x)=(x-1)^2(x+3)(x+5)=0 \Rightarrow$

$(x-1)^2=0$ or $x+3=0$ or $x+5=0 \Rightarrow$

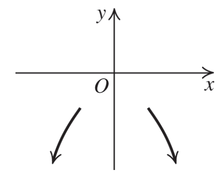
$x=1$ (multiplicity 2) or $x=-3$ (multiplicity 1) or $x=-5$ (multiplicity 1)

8. $f(x)=-x^4+3x^2-2$ has at most three turning points. Using a graphing calculator, we see that there are indeed, three turning points.



9. $f(x)=-x^4+5x^2-4$

Since the degree, 4, is even and the leading coefficient is −1, the end behavior is as shown:



$y \to -\infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to \infty$.

Now find the zeros of the function:

$0=-x^4+5x^2-4 \Rightarrow 0=-\left(x^4-5x^2+4\right)$

$=-\left(x^2-4\right)\left(x^2-1\right) \Rightarrow$

$0=x^2-4$ or $0=x^2-1 \Rightarrow$

$x=\pm 2$ or $x=\pm 1$

There are four zeros, each of multiplicity 1, so the graph crosses the *x*-axis at each zero.

Next, find the *y*-intercept:

$f(0)=-(0)^4+5(0)^2-4=-4$

Check for symmetries.

$$\begin{aligned} f(-x) &= -(-x)^4+5(-x)^2-4 \\ &= -x^4+5x^2-4 \end{aligned}$$

So, $f(-x)=f(x)$, but $f(-x)\neq -f(x)$.

Therefore, the graph is symmetric with respect to the *y*-axis.

Now find the intervals on which the graph lies above or below the *x*-axis. The four zeros divide the *x*-axis into five intervals, $(-\infty,-2)$, $(-2,-1)$, $(-1,1)$, $(1,2)$, and $(2,\infty)$.

Determine the sign of a test value in each interval and draw a sign graph as shown in Example 9, Step 5. We summarize the sign graph in the chart below.

Interval	Test point	Value of $f(x)$	Above/below *x*-axis
$(-\infty,-2)$	−3	−40	below
$(-2,-1)$	−1.5	2.1875	above
$(-1, 1)$	0	−4	below
$(1,2)$	1.5	2.1875	above
$(2,\infty)$	3	−40	below

Then plot the zeros, *y*-intercepts, and test points, and then join the points with a smooth curve.

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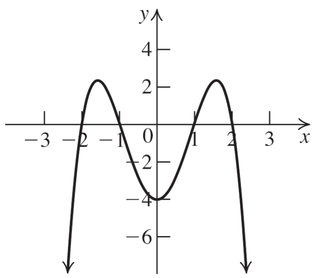
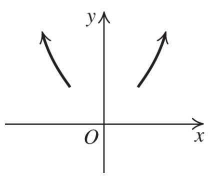


10. $f(x) = x(x+2)^3(x-1)^2$

We can determine the end behavior of the polynomial by finding its leading term as a product of the leading terms of each factor.

$f(x) = x(x+2)^3(x-1)^2 \approx x(x^3)(x^2) = x^6$

Therefore, the end behavior of $f(x)$ is Case 1:



Now find the zeros of the function.

$x(x+2)^3(x-1)^2 = 0 \Rightarrow$

$x = 0$ | $x + 2 = 0$, $x = -2$ | $x - 1 = 0$, $x = 1$

The function has three distinct zeros, –2, 1, and 0. The zero $x = –2$ has multiplicity 3, so the graph crosses the x-axis at –2, and near –2 the function looks like

$f(x) = -2(x+2)^3(-2-1)^2 = -18(x+2)^3$.

The zero $x = 0$ has multiplicity 1, so the graph crosses the x-axis at 0, and near 0 the function looks like $f(x) = x(0+2)^3(0-1)^2 = 8x$.

The zero $x = 1$ has multiplicity 2, so the graph touches the x-axis at 1, and near 1 the function looks like

$f(x) = 1(1+2)^3(x-1)^2 = 27(x-1)^2$.

Check for symmetries.

$f(-x) = -x(-x+2)^3(-x-1)^2 \neq f(x)$

$-f(x) = -\left(x(x+2)^3(x-1)^2\right)$

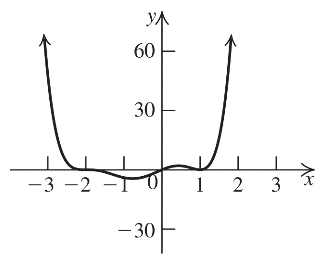
$= -x(x+2)^3(x-1)^2 \neq f(-x)$

There are no symmetries.

Now find the intervals on which the graph lies above or below the x-axis. The three zeros divide the x-axis into four intervals, $(-\infty, -2)$, $(-2, 0)$, $(0, 1)$, and $(1, \infty)$. Determine the sign of a test value in each interval and draw a sign graph as shown in Example 10, Step 5. We summarize the sign graph in the chart below.

Interval	Test point	Value of $f(x)$	Above/below x-axis
$(-\infty, -2)$	−3	48	above
$(-2, 0)$	−1	–4	below
$(0, 1)$	0.5	1.95	above
$(1, \infty)$	2	128	above

We can sketch the graph of the polynomial function *f* by connecting the pieces of information we gathered above (End behavior and Zero behavior) with a smooth curve. Finally, we can check our graph using the sign graph information. Plot the zeros, y-intercepts, and test points, and then join the points with a smooth curve.



11. 70 cm = 7 dm

$V = \frac{\pi}{3\sqrt{3}}x^3 = \frac{\pi}{3\sqrt{3}} \cdot 7^3 \approx 207.378 \text{ dm}^3$

≈ 207.378 L

Concepts and Vocabulary

1. Consider the polynomial $2x^5 - 3x^4 + x - 6$. The degree of this polynomial is 5, its leading term is $2x^5$, its leading coefficient is 2, and its constant term is −6.

2. A number *c* for which $f(c) = 0$ is called a zero of the polynomial function *f*.

3. If *c* is a zero of even multiplicity for a polynomial function *f*, then the graph of *f* touches the x-axis at *c*.

4. If c is a zero of odd multiplicity for a polynomial function f, then the graph of f crosses the x-axis at c.

5. False. The graph of a polynomial function is a smooth curve. That means it has no corners or cusps.

6. True

7. False. The polynomial function of degree n has, at most, n zeros.

8. True

Building Skills

9. Polynomial function; degree: 5; leading term: $2x^5$; leading coefficient: 2

10. Polynomial function; degree: 4; leading term: $-7x^4$; leading coefficient: –7

11. Polynomial function; degree: 3; leading term: $\frac{2}{3}x^3$; leading coefficient: $\frac{2}{3}$

12. Polynomial function; degree: 3; leading term: $\sqrt{2}x^3$; leading coefficient: $\sqrt{2}$

13. Polynomial function; degree: 4; leading term: πx^4; leading coefficient: π

14. Polynomial function; degree: 0; leading term: 5; leading coefficient: 5

15. Not a polynomial function because the graph has sharp corners. It is not a smooth because of the presence of $|x|$.

16. Not a polynomial function because the domain is not $(-\infty, \infty)$.

17. Not a polynomial function because the domain is not $(-\infty, \infty)$.

18. Not a polynomial function because the graph is not continuous.

19. Not a polynomial function because of the presence of $\sqrt{x}$.

20. Not a polynomial function because there is a noninteger exponent.

21. Not a polynomial function because the domain is not $(-\infty, \infty)$.

22. Not a polynomial function because there is a negative exponent.

23. Not a polynomial function because the graph is not continuous.

24. Not a polynomial function because the domain is not $(-\infty, \infty)$.

25. Not a polynomial function because the graph is not continuous.

26. Not a polynomial function because the graph has a sharp corner. It is not a smooth curve.

27. Not a polynomial function because it is not the graph of a function.

28. Not a polynomial function because the graph has sharp corners. It is not a smooth curve.

29. c **30.** f **31.** a

32. e **33.** d **34.** b

35. $f(x) = x - x^3$

The leading term is $-x^3$, so $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$.

36. $f(x) = 2x^3 - 2x^2 + 1$

The leading term is $2x^3$, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

37. $f(x) = 4x^4 + 2x^3 + 1$

The leading term is $4x^4$, so $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

38. $f(x) = -x^4 + 3x^3 + x$

The leading term is $-x^4$, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$.

39. $f(x) = (x+2)^2(2x-1)$

The leading term is $x^2(2x) = 2x^3$, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

40. $f(x) = (x-2)^3(2x+1)$

The leading term is $x^3(2x) = 2x^4$, so $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$.

41. $f(x) = (x+2)^2(4-x)$

The leading term is $x^2(-x) = -x^3$, so $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$.

42. $f(x) = (x+3)^3(2-x)$

The leading term is $x^3(-x) = -x^4$, so $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$.

43. $f(x) = 3(x-1)(x+2)(x-3)$

Zeros: –2, 1, 3
$x = -2$: multiplicity: 1, crosses the x-axis
$x = 1$: multiplicity: 1, crosses the x-axis
$x = 3$: multiplicity: 1, crosses the x-axis

44. $f(x) = -5(x+1)(x+2)(x-3)$

Zeros: –2, –1, 3
$x = -2$: multiplicity: 1, crosses the x-axis
$x = -1$: multiplicity: 1, crosses the x-axis
$x = 3$: multiplicity: 1, crosses the x-axis

45. $f(x) = (x+2)^2(2x-1)$

Zeros: –2, $\frac{1}{2}$

$x = -2$: multiplicity 2, touches but does not cross the x-axis
$x = \frac{1}{2}$: multiplicity 1, crosses the x-axis

46. $f(x) = (x-2)^3(2x+1)$

Zeros: $-\frac{1}{2}$, 2

$x = -\frac{1}{2}$: multiplicity 1, crosses the x-axis
$x = 2$: multiplicity 3, crosses the x-axis

47. $f(x) = x^2(x^2-9)(3x+2)^3$

Zeros: $-3, -\frac{2}{3}, 0, 3$

$x = -3$: multiplicity 1, crosses the x-axis
$x = -\frac{2}{3}$: multiplicity 3, crosses the x-axis
$x = 0$: multiplicity 2, touches but does not cross the x-axis
$x = 3$: multiplicity 1, crosses the x-axis

48. $f(x) = -x^3(x^2-4)(3x-2)^2$

Zeros: $-2, 0, \frac{2}{3}, 2$

$x = -2$: multiplicity 1, crosses the x-axis
$x = 0$: multiplicity 3, crosses the x-axis
$x = \frac{2}{3}$: multiplicity 2, touches but does not cross the x-axis
$x = 2$: multiplicity 1, crosses the x-axis

49. $f(x) = (x^2+1)(3x-2)^2$

Zero: $\frac{2}{3}$, multiplicity 2, touches but does not cross the x-axis

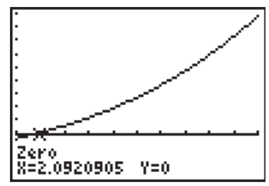
50. $f(x) = (x^2+1)(x+1)(x-2)$

Zeros: –1, 2
$x = -1$: multiplicity 1, crosses the x-axis
$x = 2$; multiplicity 1, crosses the x-axis

51. $f(2) = 2^4 - 2^3 - 10 = -2;$

$f(3) = 3^4 - 3^3 - 10 = 44$.

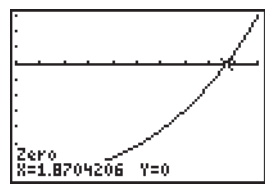
Because the sign changes, there is a real zero between 2 and 3. The zero is approximately 2.09.



52. $f(1) = 1^4 - 1^2 - 2(1) - 5 = -7;$

$f(2) = 2^4 - 2^2 - 2(2) - 5 = 3.$

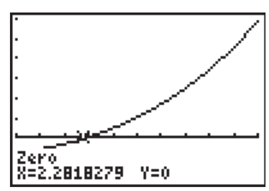
Because the sign changes, there is a real zero between 1 and 2. The zero is approximately 1.87.



53. $f(2) = 2^5 - 9(2)^2 - 15 = -19;$

$f(3) = 3^5 - 9(3)^2 - 15 = 147.$

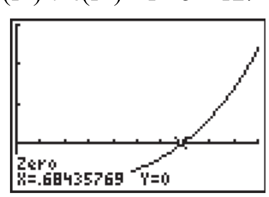
Because the sign changes, there is a real zero between 2 and 3. The zero is approximately 2.28.



54. $f(0) = 0^5 + 5(0^4) + 8(0^3) + 4(0^2) - 0 - 5 = -5;$

$f(1) = 1^5 + 5(1^4) + 8(1^3) + 4(1^2) - 1 - 5 = 12.$

Because the sign changes, there is a real zero between 0 and 1. The zero is approximately 0.68.



55. The least possible degree is 3. The zeros are –2, 1, and 3, each with multiplicity 1. The end behavior is $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$, so the leading coefficient is 1. The polynomial with smallest possible degree is $(x+2)(x-1)(x-3)$.

56. The least possible degree is 3.The zeros are –1, 1, and 3, each with multiplicity 1. The end behavior is $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$, so the leading coefficient is –1. The polynomial with smallest possible degree is $-(x+1)(x-1)(x-3)$.

57. The least possible degree is 3.The zeros are –3 (multiplicity 1) and 2 (multiplicity 2). The end behavior is $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$, so the leading coefficient is 1. The polynomial with smallest possible degree is $(x+3)(x-2)^2$.

58. The least possible degree is 3.The zeros are –1 (multiplicity 2) and 2 (multiplicity 1). The end behavior is $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$, so the leading coefficient is –1. The polynomial with smallest possible degree is $-(x+1)^2(x-2)$.

59. The least possible degree is 4. The zeros are –1 (multiplicity 2) and 2 (multiplicity 2). The end behavior is $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$, so the leading coefficient is 1. The polynomial with smallest possible degree is $(x+1)^2(x-2)^2$.

60. The least possible degree is 4. The zeros are –2 (multiplicity 2) and 2 (multiplicity 2). The end behavior is $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$, so the leading coefficient is –1. The polynomial with smallest possible degree is $-(x+2)^2(x-2)^2$.

61. The zeros are –2 (multiplicity 2), 2 (multiplicity 1), and 3 (multiplicity 1), so the least possible degree is 4. The end behavior is $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$, so the leading coefficient is 1. The polynomial with smallest possible degree is $(x+2)^2(x-2)(x-3)$.

62. The zeros are –3 (multiplicity 1), 0 (multiplicity 2), and 2 (multiplicity 1), so the least possible degree is 4. The end behavior is $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to \infty$, so the leading coefficient is –1. The polynomial with smallest possible degree is $-x^2(x+3)(x-2)$.

For exercises 63–74, use the procedures shown in Examples 9 and 10 in the text to graph the function.
Step 1: Determine the end behavior.
Step 2: Find the real zeros of the polynomial function.
Step 3: Find the *y*-intercept by computing $f(0)$.
Step 4: Use symmetry, if possible, by checking whether the function is odd, even, or neither.
Step 5: Determine the sign of $f(x)$ by using arbitrarily chosen "test numbers" in the intervals defined by the *x*-intercepts. Find on which of these intervals the graph lies above or below the *x*-axis. Draw a sign graph as shown in the Examples.
Step 6: Draw the graph using points from Steps 1–5.

63. $f(x) = 2(x+1)(x-2)(x+4)$

End behavior: like $y = x^3$

Zeros: $(-1,\ 2,\ -4)$

y-intercept: $f(0) = 2(1)(-2)(4) = -16$

symmetry:

$$\begin{aligned} f(x) &= 2(x+1)(x-2)(x+4) \\ &= 2\left(x^3 + 3x^2 - 6x - 8\right) \\ f(-x) &= 2(-x+1)(-x-2)(-x+4) \\ &= 2\left(-x^3 + 3x^2 + 6x - 8\right) \end{aligned}$$

$f(-x) \neq f(x)$, so $f(x)$ is not even.

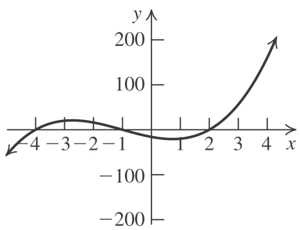
$f(-x) \neq -f(x)$, so $f(x)$ is not odd. There are no symmetries.

The three zeros $(-1,\ 2,\ -4)$ divide the *x*-axis into four intervals: $(-\infty,\ -4)$, $(-4,\ -1)$, $(-1,\ 2)$, and $(2,\ \infty)$. We choose –5,–3, 0, and 3 as test numbers.

(*continued on next page*)

(*continued*)

$f(-5) = -56$, below x-axis
$f(-3) = 20$, above x-axis
$f(0) = -16$, below x-axis
$f(3) = 56$, above x-axis



64. $f(x) = -(x-1)(x+3)(x-4)$

End behavior: like $y = -x^3$

Zeros: $(1, -3, 4)$

y-intercept: $f(0) = -(-1)(3)(-4) = -12$

symmetry:

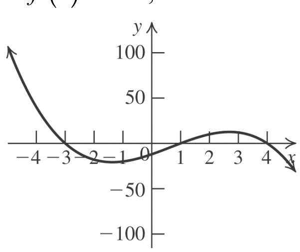
$$f(x) = -(x-1)(x+3)(x-4)$$
$$= -x^3 + 2x^2 + 11x - 12$$
$$f(-x) = -(-x-1)(-x+3)(-x-4)$$
$$x^3 + 2x^2 - 11x - 12$$

$f(-x) \neq f(x)$, so $f(x)$ is not even.

$f(-x) \neq -f(x)$, so $f(x)$ is not odd. There are no symmetries.

The three zeros $(1, -3, 4)$ divide the x-axis into four intervals: $(-\infty, -3)$, $(-3, 1)$, $(1, 4)$, and $(4, \infty)$. We choose -5, 0, 3, and 5 as test numbers.

$f(-5) = 108$, above x-axis
$f(0) = -12$, below x-axis
$f(3) = 12$, above x-axis
$f(5) = -32$, below x-axis



65. $f(x) = (x-1)^2(x+3)(x-4)$

End behavior: like $y = x^2$

Zeros: $(1, -3, 4)$; 1 has multiplicity 2

y-intercept: $f(0) = -(-1)(3)(-4) = -12$

symmetry:

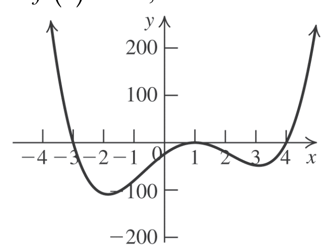
$$f(x) = (x-1)^2(x+3)(x-4)$$
$$= x^4 - 3x^3 - 9x^2 + 23x - 12$$
$$f(-x) = (-x-1)^2(-x+3)(-x-4)$$
$$= x^4 + 3x^3 - 9x^2 - 23x - 12$$

$f(-x) \neq f(x)$, so $f(x)$ is not even.

$f(-x) \neq -f(x)$, so $f(x)$ is not odd. There are no symmetries.

The three zeros $(1, -3, 4)$ divide the x-axis into four intervals: $(-\infty, -3)$, $(-3, 1)$, $(1, 4)$, and $(4, \infty)$. We choose -4, 0, 3, and 5 as test numbers.

$f(-4) = 200$, above x-axis
$f(0) = -12$, below x-axis
$f(3) = -24$, below x-axis
$f(5) = 128$, above x-axis



66. $f(x) = -x^2(x+1)(x-2)$

End behavior: like $y = -x^2$

Zeros: $(0, -1, 2)$; 0 has multiplicity 2

y-intercept: $f(0) = 0$

symmetry:

$$f(x) = -x^2(x+1)(x-2)$$
$$= -x^4 + x^3 + 2x^2$$
$$f(-x) = -(-x)^2(-x+1)(-x-2)$$
$$= -x^4 - x^3 + 2x^2$$

$f(-x) \neq f(x)$, so $f(x)$ is not even.

$f(-x) \neq -f(x)$, so $f(x)$ is not odd. There are no symmetries.

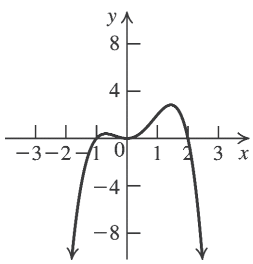
The three zeros $(0, -1, 2)$ divide the x-axis into four intervals: $(-\infty, -1)$, $(-1, 0)$, $(0, 2)$, and $(2, \infty)$. We choose -2, $-\frac{1}{2}$, 1, and 3 as test numbers.

$f(-2) = -16$, below x-axis

$f\left(-\frac{1}{2}\right) = \frac{5}{16}$, above x-axis

$f(1) = 2$, above x-axis

$f(3) = -36$, below x-axis



67. $f(x) = -x^2(x-3)^2$

End behavior: like $y = -x^2$

Zeros: $(0, 3)$; both zeros have multiplicity 2

y-intercept: $f(0) = 0$

symmetry:

$$f(x) = -x^2(x-3)^2 = -x^4 + 6x^3 - 9x^2$$

$$f(-x) = -(-x)^2(-x-3)^2 = -x^4 - 6x^3 - 9x^2$$

$f(-x) \neq f(x)$, so $f(x)$ is not even.

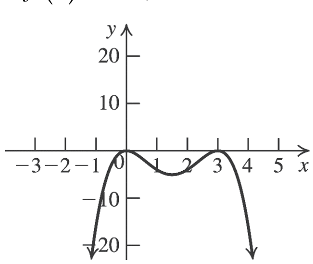
$f(-x) \neq -f(x)$, so $f(x)$ is not odd. There are no symmetries.

The two zeros $(0, 3)$ divide the x-axis into three intervals: $(-\infty, 0)$, $(0, 3)$, and $(3, \infty)$. We choose -1, 1, and 4 as test numbers.

$f(-1) = -16$, below x-axis

$f(1) = -4$, below x-axis

$f(4) = -16$, below x-axis



68. $f(x) = (x-2)^2(x+3)^2$

End behavior: like $y = x^2$

Zeros: $(2, -3)$; both zeros have multiplicity 2

y-intercept: $f(0) = 36$

symmetry:

$$f(x) = (x-2)^2(x+3)^2 = x^4 + 2x^3 - 11x^2 - 12x + 36$$

$$f(-x) = (-x-2)^2(-x+3)^2 = x^4 - 2x^3 - 11x^2 + 12x + 36$$

$f(-x) \neq f(x)$, so $f(x)$ is not even.

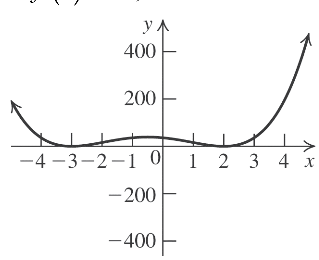
$f(-x) \neq -f(x)$, so $f(x)$ is not odd. There are no symmetries.

The two zeros $(2, -3)$ divide the x-axis into three intervals: $(-\infty, -3)$, $(-3, 2)$, and $(2, \infty)$. We choose -4, 0, and 3 as test numbers.

$f(-4) = 36$, above x-axis

$f(0) = 36$, above x-axis

$f(3) = 36$, above x-axis



69. $f(x) = (x-1)^2(x+2)^3(x-3)$

End behavior: like $y = x^2$

Zeros: $(1, -2, 3)$; 1 has multiplicity 2

y-intercept: $f(0) = -24$

symmetry:

$$f(x) = (x-1)^2(x+2)^3(x-3) = x^6 + x^5 - 11x^4 - 13x^3 + 26x^2 + 20x - 24$$

$$f(-x) = (-x-1)^2(-x+2)^3(-x-3) = x^6 - x^5 - 11x^4 + 13x^3 + 26x^2 - 20x - 24$$

$f(-x) \neq f(x)$, so $f(x)$ is not even.

$f(-x) \neq -f(x)$, so $f(x)$ is not odd. There are no symmetries.

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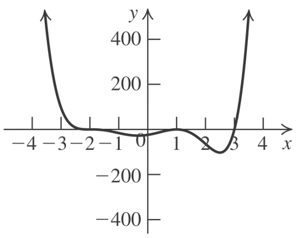
The three zeros $(1, -2, 3)$ divide the x-axis into four intervals: $(-\infty, -2), (-2, 1), (1, 3)$, and $(3, \infty)$. We choose -3, 0, 2, and 4 as test numbers.

$f(-3) = 96$, above x-axis
$f(0) = -24$, below x-axis
$f(2) = -64$, below x-axis
$f(4) = 1944$, above x-axis



70. $f(x) = (x-1)^3 (x+1)^3 (x-3)$

End behavior: like $y = x^2$

Zeros: $(1, -1, 3)$; -1 has multiplicity 2

y-intercept: $f(0) = 3$

symmetry:

$$f(x) = (x-1)^3 (x+1)^2 (x-3) = x^6 - 4x^5 + x^4 + 8x^3 - 5x^2 - 4x + 3$$

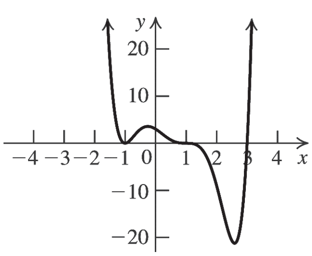
$$f(-x) = (-x-1)^3 (-x+1)^2 (-x-3) = x^6 + 4x^5 + x^4 - 8x^3 - 5x^2 + 4x + 3$$

$f(-x) \neq f(x)$, so $f(x)$ is not even.

$f(-x) \neq -f(x)$, so $f(x)$ is not odd. There are no symmetries.

The three zeros $(1, -1, 3)$ divide the x-axis into four intervals: $(-\infty, -1), (-1, 1), (1, 3)$, and $(3, \infty)$. We choose -2, 0, 2, and 4 as test numbers.

$f(-2) = 135$, above x-axis
$f(0) = 3$, above x-axis
$f(2) = -9$, below x-axis
$f(4) = 675$, above x-axis



71. $f(x) = -x^2 (x^2 - 1)(x+1)$

End behavior: like $y = -x^3$

Zeros: $(0, 1, -1)$; 0 and -1 have multiplicity 2

y-intercept: $f(0) = 0$

symmetry:

$$f(x) = -x^2 (x^2 - 1)(x+1) = -x^5 - x^4 + x^3 + x^2$$

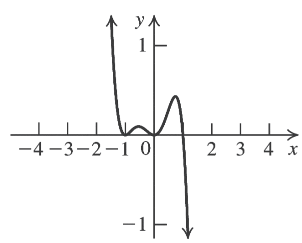
$$f(-x) = -(-x)^2 ((-x)^2 - 1)(-x+1) = x^5 - x^4 - x^3 + x^2$$

$f(-x) \neq f(x)$, so $f(x)$ is not even.

$f(-x) \neq -f(x)$, so $f(x)$ is not odd. There are no symmetries.

The three zeros $(0, 1, -1)$ divide the x-axis into four intervals: $(-\infty, -1), (-1, 0), (0, 1)$, and $(1, \infty)$. We choose -2, $-\frac{1}{2}$, $\frac{1}{2}$, and 2 as test numbers.

$f(-2) = 12$, above x-axis
$f\left(-\frac{1}{2}\right) = \frac{3}{32}$, above x-axis
$f\left(\frac{1}{2}\right) = \frac{9}{32}$, above x-axis
$f(2) = -36$, below x-axis



72. $f(x) = x^2 (x^2 - 4)(x+2)$

End behavior: like $y = x^3$

Zeros: $(0, 2, -2)$; 0 and -2 have multiplicity 2

y-intercept: $f(0) = 0$

symmetry:

$$f(x) = x^2 (x^2 - 4)(x+2) = x^5 + 2x^4 - 4x^3 - 8x^2$$

$$f(-x) = (-x)^2 ((-x)^2 - 4)(-x+2) = -x^5 + 2x^4 + 4x^3 - 8x^2$$

$f(-x) \neq f(x)$, so $f(x)$ is not even.

$f(-x) \neq -f(x)$, so $f(x)$ is not odd. There are no symmetries.

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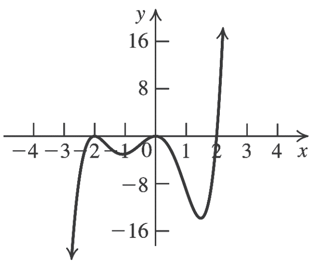
The three zeros $(0, 2, -2)$ divide the x-axis into four intervals: $(-\infty, -2), (-2, 0), (0, 2),$ and $(2, \infty)$. We choose −3, −1, 1 and 3 as test numbers.

$f(-3) = -45,$ below x-axis
$f(-1) = -3,$ below x-axis
$f(1) = -9,$ below x-axis
$f(3) = 225,$ above x-axis



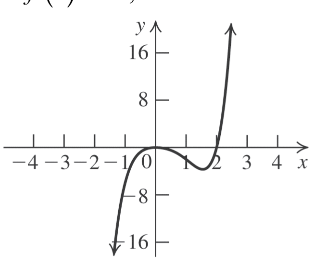
73. $f(x) = x^2(x^2+1)(x-2)$

End behavior: like $y = x^3$
Zeros: (0, 2); 0 has multiplicity 2
y-intercept: $f(0) = 0$
symmetry:

$$f(x) = x^2(x^2+1)(x-2) = x^5 - 2x^4 + x^3 - 2x^2$$
$$f(-x) = (-x)^2((-x)^2+1)(-x-2) = -x^5 - 2x^4 - x^3 - 2x^2$$

$f(-x) \neq f(x)$, so $f(x)$ is not even.
$f(-x) \neq -f(x)$, so $f(x)$ is not odd. There are no symmetries.
The two zeros (0, 2) divide the x-axis into three intervals: $(-\infty, 0), (0, 2),$ and $(2, \infty)$.
We choose −1, 1 and 3 as test numbers.

$f(-1) = -6,$ below x-axis
$f(1) = -2,$ below x-axis
$f(3) = 90,$ above x-axis



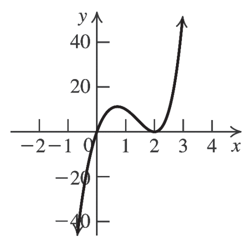
74. $f(x) = x(x^2+9)(x-2)^2$

End behavior: like $y = x^3$
Zeros: (0, 2); 2 has multiplicity 2
y-intercept: $f(0) = 0$
symmetry:

$$f(x) = x(x^2+9)(x-2)^2 = x^5 - 4x^4 + 13x^3 - 36x^2 + 36x$$
$$f(-x) = -x((-x)^2+9)(-x-2)^2 = -x^5 - 4x^4 - 13x^3 - 36x^2 - 36x$$

$f(-x) \neq f(x)$, so $f(x)$ is not even.
$f(-x) \neq -f(x)$, so $f(x)$ is not odd. There are no symmetries.
The two zeros (0, 2) divide the x-axis into three intervals: $(-\infty, 0), (0, 2),$ and $(2, \infty)$.
We choose −1, 1 and 3 as test numbers.

$f(-1) = -90,$ below x-axis
$f(1) = 10,$ above x-axis
$f(3) = 54,$ above x-axis



Applying the Concepts

75. a. $P = \dfrac{0.04825}{746}(40)^3 \approx 4.14$ hp

b. $P = \dfrac{0.04825}{746}(65)^3 \approx 17.76$ hp

c. $P = \dfrac{0.04825}{746}(80)^3 \approx 33.12$ hp

d. $\dfrac{P(2v)}{P(v)} = \dfrac{\frac{0.04825}{746}(2v)^3}{\frac{0.04825}{746}v^3} = \dfrac{8v^3}{v^3} = 8$

76. a. $E = 12800(1.75)^4 = 120{,}050$ J

b. $E = 12800(0.5833)^4 \approx 1481.76\text{ J}$

$E = 12800\left(\frac{L}{3}\right)^4 = 12800\left(\frac{1.75}{3}\right)^4$

$= 12800(1.75)^4\left(\frac{1}{81}\right) \approx 1482\text{ J}$

It reduced by a factor of $\frac{1}{81}$.

c. $E = 12800(0.07)^4 \approx 0.31\text{ J}$

$E = 12800\left(\frac{L}{25}\right)^4 = 12800\left(\frac{1.75}{25}\right)^4$

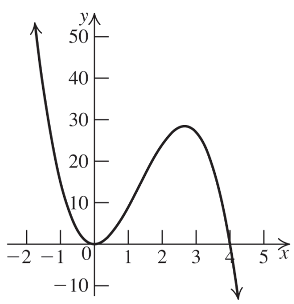
$= 12800(1.75)^4\left(\frac{1}{390,625}\right) \approx 0.31\text{ J}$

It reduced by a factor of $\frac{1}{390,625}$.

d. $\frac{E\left(\frac{L}{2}\right)}{E(L)} = \frac{12800\left(\frac{L}{2}\right)^4}{12800L^4} = \frac{\frac{L^4}{16}}{L^4} = \frac{1}{16}$

77. a. $3x^2(4-x) = 0 \Rightarrow x = 0$ or $x = 4$.
$x = 0$, multiplicity 2; $x = 4$, multiplicity 1.

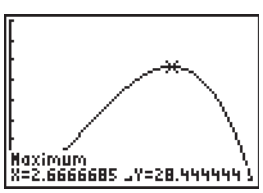
b.



c. There are 2 turning points.

d. Domain: [0, 4].
The portion between the x-intercepts is the graph of $R(x)$.

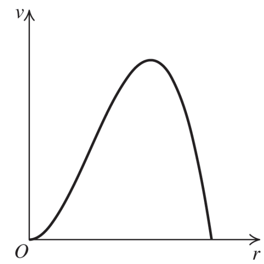
78.



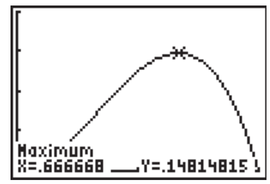
$R(x)$ is a maximum at $x \approx 2.67$.

79. a. Domain [0, 1]

b.



80.

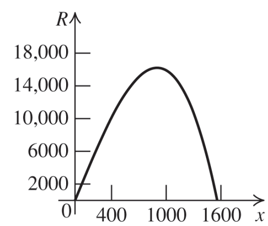


v is a maximum when $r \approx 0.67$.

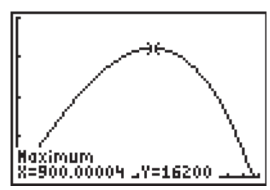
81. a. $R(x) = x\left(27 - \left(\frac{x}{300}\right)^2\right) = 27x - \frac{x^3}{90,000}$

b. The domain of $R(x)$ is the same as the domain of p. $p \geq 0$ when $p \leq 900\sqrt{3}$.
The domain is $\left[0, 900\sqrt{3}\right]$.

c.



82. a.



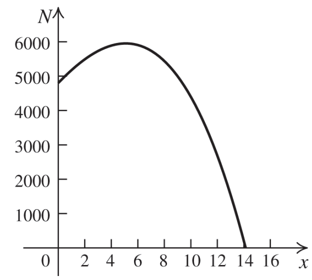
About 900 pairs of slacks were sold.

b. $16,200

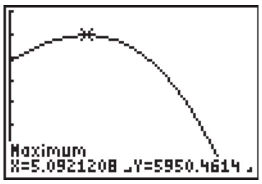
83. a. $N(x) = (x+12)(400 - 2x^2)$

b. The low end of the domain is 0. (There cannot be fewer than 0 workers.) The upper end of the domain is the value where productivity is 0, so solve $N(x) = 0$ to find the upper end of the domain.
$(x+12)(400-2x^2) = 0 \Rightarrow x = -12$ (reject this) or $400 - 2x^2 = 0 \Rightarrow 200 = x^2 \Rightarrow$
$x = \pm 10\sqrt{2}$ (reject the negative solution).
The domain is $[0, 10\sqrt{2}]$.

c.



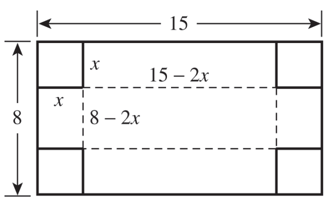
84. a.



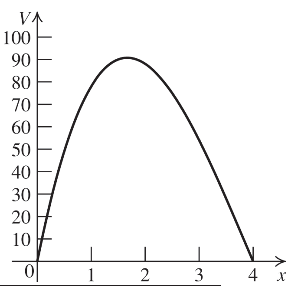
The maximum number of oranges that can be picked per hour is about 5950.

b. The number of employees = 12 + 5 = 17.

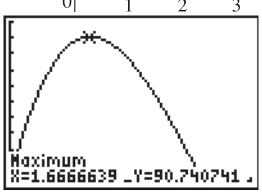
85. a. $V(x) = x(8-2x)(15-2x)$



b.



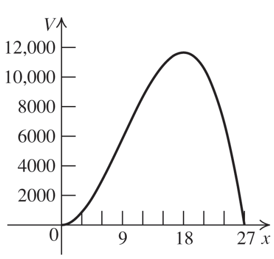
86.



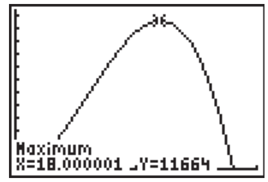
The largest possible value of the volume of the box is 90.74 cubic units.

87. a. $V(x) = lwh = x^2(108-4x)$

b.



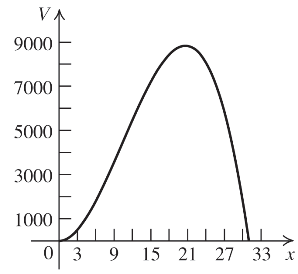
88.



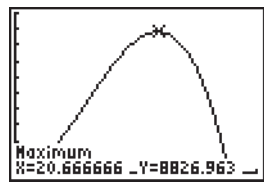
The largest possible volume of the box is 11,664 cubic inches.

89. a. $V(x) = x^2(62-2x)$

b.



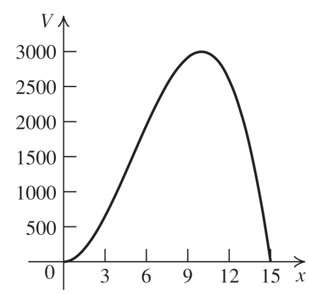
90.



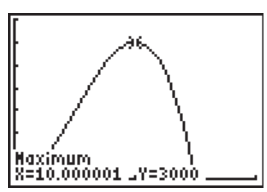
The volume is greatest when $x \approx 20.67$, so the dimensions of the suitcase with the largest volume are approximately 20.67 in. × 20.67 in. × 20.67 in.

91. a. $V(x) = 2x^2(45-3x)$

b.



92.



The volume is greatest when $x = 10$, so the width of the bag is 10 inches, the length is 2(10) = 20 inches, and the height is 45 – 20 – 10 = 15 inches.