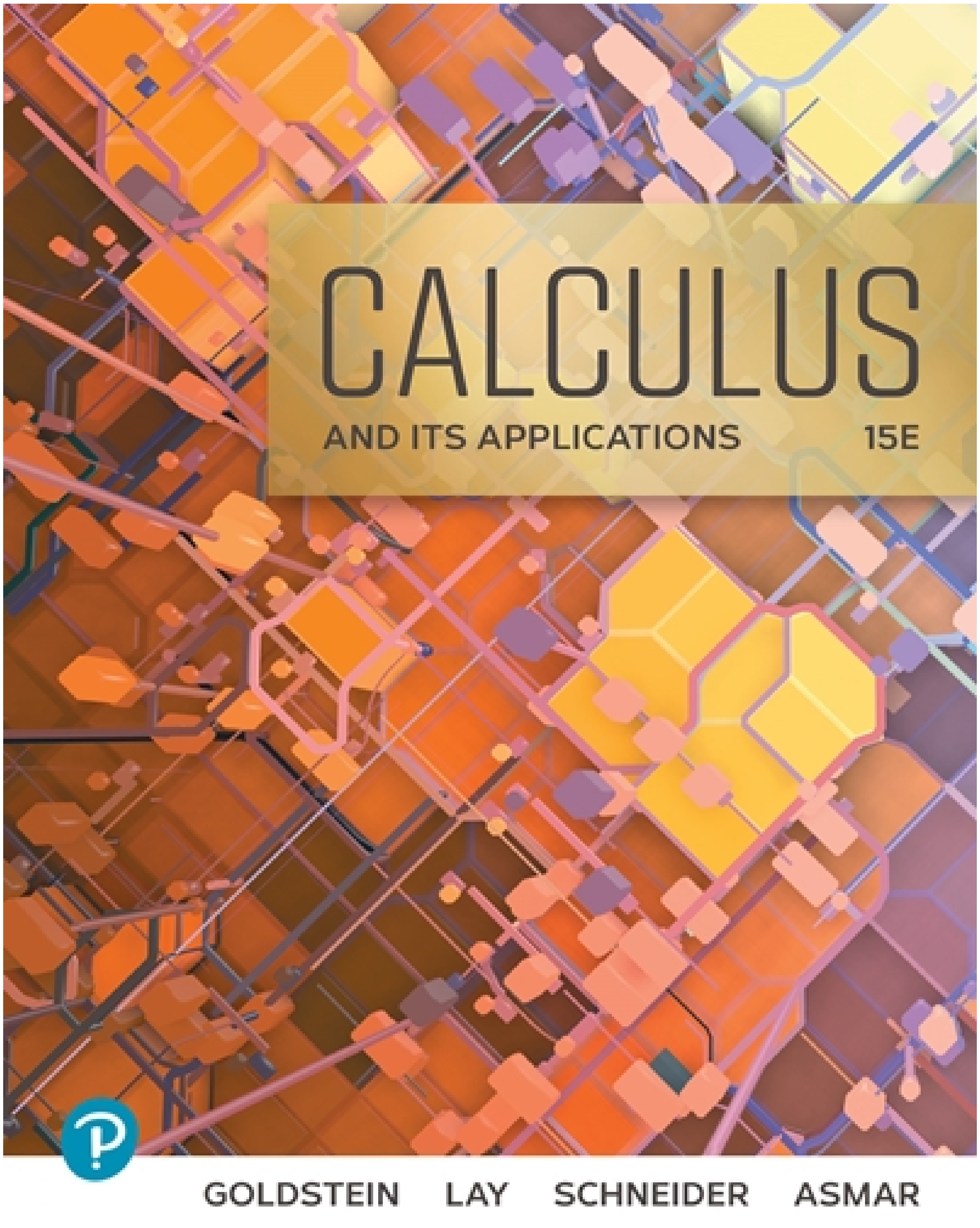
Solutions for Calculus and Its Applications 15th Edition by Goldstein



Solutions

Instructor's Solutions Manual

Calculus and Its Applications
Fifteenth Edition

Calculus and Its Applications, Brief Version
Fifteenth Edition

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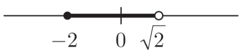
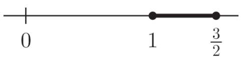
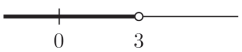
3. 

4. 

5. 

6. 

7. $[2, 3)$

8. $\left(-1, \frac{3}{2}\right)$

9. $[-1, 0)$

10. $[-1, 8)$

11. $(-\infty, 3)$

12. $\left[\sqrt{2}, \infty\right)$

13. $f(x) = x^2 - 3x$

$f(0) = 0^2 - 3(0) = 0$

$f(5) = 5^2 - 3(5) = 25 - 15 = 10$

$f(3) = 3^2 - 3(3) = 9 - 9 = 0$

$f(-7) = (-7)^2 - 3(-7) = 49 + 21 = 70$

14. $f(x) = x^3 + x^2 - x - 1$

$f(1) = 1^3 + 1^2 - 1 - 1 = 0$

$f(-1) = (-1)^3 + (-1)^2 - (-1) - 1 = 0$

$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 1 = -\frac{9}{8}$

$f(a) = a^3 + a^2 - a - 1$

15. $f(x) = x^2 - 2x$

$f(a+1) = (a+1)^2 - 2(a+1)$
$= (a^2 + 2a + 1) - 2a - 2 = a^2 - 1$

$f(a+2) = (a+2)^2 - 2(a+2)$
$= (a^2 + 4a + 4) - 2a - 4 = a^2 + 2a$

16. $h(s) = \frac{s}{(1+s)}$

$h\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\left(1+\frac{1}{2}\right)} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$

$h\left(-\frac{3}{2}\right) = \frac{-\frac{3}{2}}{1+\left(-\frac{3}{2}\right)} = \frac{-\frac{3}{2}}{-\frac{1}{2}} = 3$

$h(a+1) = \frac{a+1}{1+(a+1)} = \frac{a+1}{a+2}$

17. $f(x) = 3x + 2,\ h \neq 0$

$f(3+h) = 3(3+h) + 2 = 9 + 3h + 2 = 3h + 11$

$f(3) = 3(3) + 2 = 11$

$\frac{f(3+h) - f(3)}{h} = \frac{(3h+11) - 11}{h} = \frac{3h}{h} = 3$

18. $f(x) = x^2,\ h \neq 0$

$f(1+h) = (1+h)^2 = 1 + 2h + h^2$

$f(1) = 1^2 = 1$

$\frac{f(1+h) - f(1)}{h} = \frac{\left(1 + 2h + h^2\right) - 1}{h}$

$= \frac{2h + h^2}{h} = 2 + h$

19. a. $k(x) = x + 273$

$5933 = x + 273 \Rightarrow x = 5660$

The boiling point of tungsten is 5660°C.

b. $f(x) = \frac{9}{5}x + 32$

$f(x) = \frac{9}{5}(5660) + 32 = 10220$

The boiling point of tungsten is 10220°F.

20. a. $f(0)$ represents the number of laptops sold in 2015.

b. $f(5) = 150 + 2(5) + 5^2$
$= 150 + 10 + 25 = 185$

In 2020, the company will sell 185 laptops.

21. $f(x) = \frac{8x}{(x-1)(x-2)}$

all real numbers such that $x \neq 1, 2$ or $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

22. $f(t)=\dfrac{1}{\sqrt{t}}$

all real numbers such that $t > 0$ or $(0,\infty)$

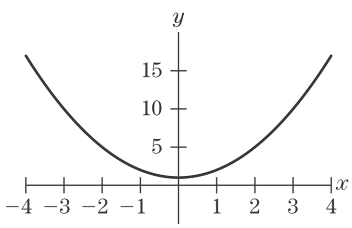
23. $g(x)=\dfrac{1}{\sqrt{3-x}}$

all real numbers such that $x < 3$ or $(-\infty,-3)$

24. $g(x)=\dfrac{4}{x(x+2)}$

all real numbers such that $x \neq 0, -2$ or $(-\infty,-2)\cup(-2,0)\cup(0,\infty)$

25.



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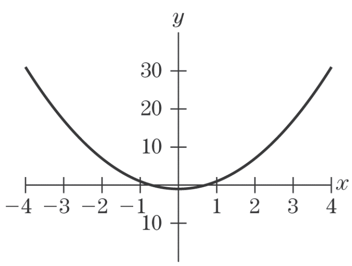

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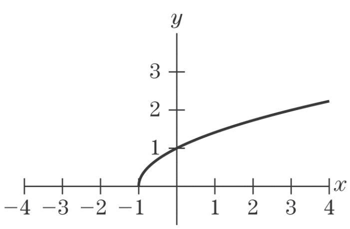

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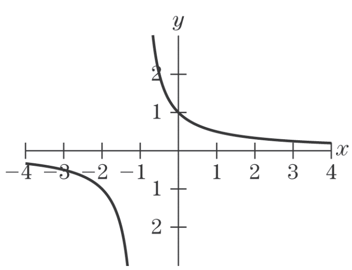


29. function **30.** not a function

31. not a function **32.** not a function

33. not a function **34.** function

35. $f(0)=1;\quad f(7)=-1$

36. $f(2)=3;\quad f(-1)=0$

37. positive **38.** negative

39. $[-1, 3]$ **40.** $-1, 5, 9$

41. $(-\infty, -1]\cup[5, 9]$ **42.** $[-1,5]\cup[9,\infty]$

43. $f(1)\approx .03;\quad f(5)\approx .037$

44. $f(6)\approx .03$

45. $[0, .05]$ **46.** $t\approx 3$

47. $f(x)=\left(x-\dfrac{1}{2}\right)(x+2)$

$f(3)=\left(3-\dfrac{1}{2}\right)(3+2)=\dfrac{25}{2}$

No, (3, 12) is not on the graph.

48. $f(x) = x(5 + x)(4 - x)$

$f(-2) = -2(5 + (-2))(4 - (-2)) = -36$

No, (–2, 12) is not on the graph.

49. $g(x)=\dfrac{3x-1}{x^2+1}$

$g(1)=\dfrac{3(1)-1}{(1)^2+1}=\dfrac{2}{2}=1$

Yes, $(1, 1)$ is on the graph.

50. $g(x)=\dfrac{x^2+4}{x+2}$

$g(4)=\dfrac{(4)^2+4}{4+2}=\dfrac{20}{6}=\dfrac{10}{3}$

No, $\left(4, \dfrac{1}{4}\right)$ is not on the graph.

51. $f(x)=x^3$

$f(a+1)=(a+1)^3$

52. $f(x)=\left(\dfrac{5}{x}\right)-x$

$f(2+h)=\dfrac{5}{(2+h)}-(2+h)$

$=\dfrac{5-(2+h)^2}{(2+h)}=\dfrac{1-4h-h^2}{2+h}$

53. $f(x)=\begin{cases}\sqrt{x} & \text{for } 0\le x<2\\ 1+x & \text{for } 2\le x\le 5\end{cases}$

$f(1)=\sqrt{1}=1$

$f(2)=1+2=3$

$f(3)=1+3=4$

54. $f(x)=\begin{cases}\dfrac{1}{x} & \text{for } 1\le x\le 2\\ x^2 & \text{for } 2<x\end{cases}$

$f(1)=\dfrac{1}{1}=1$

$f(2)=\dfrac{1}{2}$

$f(3)=3^2=9$

55. $f(x)=\begin{cases}\pi x^2 & \text{for } x<2\\ 1+x & \text{for } 2\le x\le 2.5\\ 4x & \text{for } 2.5<x\end{cases}$

$f(1)=\pi(1)^2=\pi$
$f(2)=1+2=3$
$f(3)=4(3)=12$

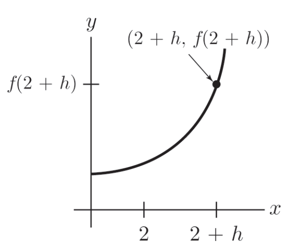
56. $f(x)=\begin{cases}\dfrac{3}{4-x} & \text{for } x<2\\ 2x & \text{for } 2\le x<3\\ \sqrt{x^2-5} & \text{for } 3\le x\end{cases}$

$f(1)=\dfrac{3}{4-1}=1$
$f(2)=2(2)=4$
$f(3)=\sqrt{3^2-5}=\sqrt{4}=2$

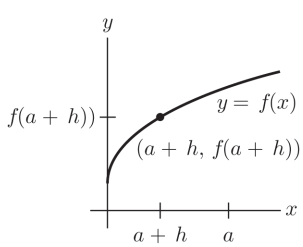
57. **a.** $f(x)=\begin{cases}0.06x & \text{for } 50\le x\le 3000\\ 0.02x+15 & \text{for } 3000<x\end{cases}$

b. $f(3000)=0.06(3000)=180$
$f(4500)=0.02(4500)+15=105$

58.



59.



60. $P(x)=\dfrac{110x-25}{10x+n}$

a. $P(30)=\dfrac{110(30)-25}{10(30)+5}=\dfrac{3275}{305}\approx 10.738$

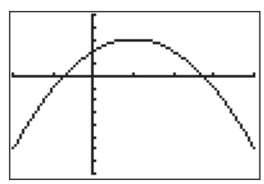
Each partner's profit was be approximately, $10.738 thousand or $10,738.

b. $5=\dfrac{110(30)-25}{10(30)+n}\Rightarrow n+300=\dfrac{3275}{5}\Rightarrow$
$n=\dfrac{3275}{5}-300=355$

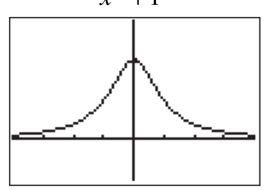
61. Entering **Y_1 = 1/X + 1** will graph the function $f(x)=\dfrac{1}{x}+1$. In order to graph the function $f(x)=\dfrac{1}{x+1}$, you need to include parentheses in the denominator: **Y_1 = 1/(X + 1)**.

62. Entering **Y_1 = X ^ 3 / 4** will graph the function $f(x)=\dfrac{x^3}{4}$. In order to graph the function $y=x^{3/4}$, you need to include parentheses in the exponent: **Y_1 = X ^ (3/4)**.

63. $f(x)=-x^2+2x+2$



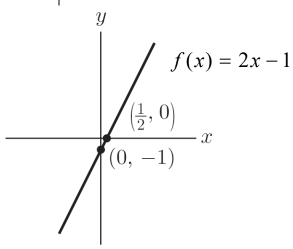
64. $f(x)=\dfrac{1}{x^2+1}$



0.2 Some Important Functions

1. $y = 2x - 1$

x	y
1	1
0	−1
−1	−3



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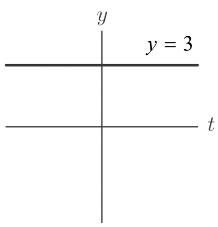
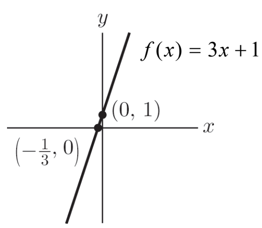


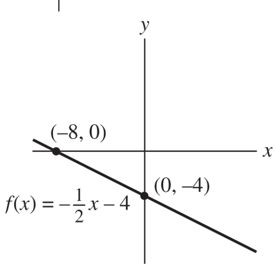
3. $y = 3x + 1$

x	y
1	4
0	1
−1	−2



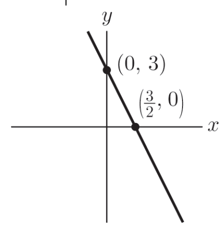
4. $y = -\frac{1}{2}x - 4$

x	y
2	−5
0	−4
−2	−3

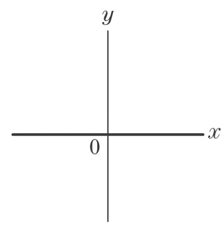


5. $y = -2x + 3$

x	y
−1	5
0	3
1	1

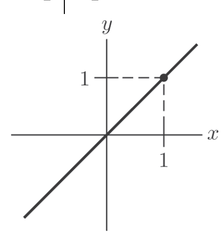


6. $y = 0$



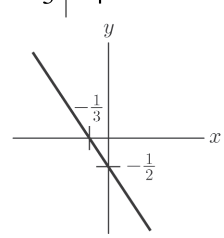
7. $x - y = 0$

x	y
1	1
0	0
−1	−1



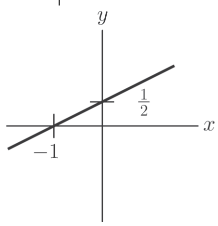
8. $3x + 2y = -1$

x	y
3	−5
1	−2
−3	4

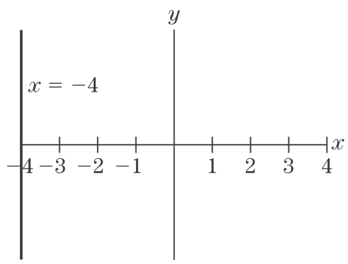


9. $x = 2y - 1$

x	y
3	2
1	1
–3	–1



10.



11. $f(x) = 9x + 3$
$f(0) = 9(9) + 3 = 3$
The y-intercept is (0, 3).
$9x + 3 = 0 \Rightarrow 9x = -3 \Rightarrow x = -\frac{1}{3}$
The x-intercept is $\left(-\frac{1}{3}, 0\right)$.

12. $f(x) = -\frac{1}{2}x - 1$
$f(0) = -\frac{1}{2}(0) - 1 = -1$
The y-intercept is (0, –1).
$-\frac{1}{2}x - 1 = 0 \Rightarrow -\frac{1}{2}x = 1 \Rightarrow x = -2$
The x-intercept is (–2, 0).

13. $f(x) = 5$
The y-intercept is (0, 5).
There is no x-intercept.

14. $f(x) = 14$
The y-intercept is (0, 14).
There is no x-intercept.

15. $x - 5y = 0$
$0 - 5y = 0 \Rightarrow y = 0$
The x- and y-intercept is (0, 0).

16. $2 + 3x = 2y$
$2 + 3(0) = 2y \Rightarrow y = 1$
The y-intercept is (0, 1).
$2 + 3x = 2(0) \Rightarrow 3x = -2 \Rightarrow x = -\frac{2}{3}$
The x-intercept is $\left(-\frac{2}{3}, 0\right)$.

17. **a.** Cost is \$(24 + 200(.45)) = \$114.

b. $f(x) = .45x + 24$

18. Let x be the volume of gas (in thousands of cubic feet) extracted.
$f(x) = 5000 + .10x$

19. Let x be the number of days of hospital confinement.
$f(x) = 700x + 1900$

20. **a.**
$$\begin{aligned}\text{cost per empl} &= 12(1000) + 12(25)(240)\\ &= 12{,}000 + 72{,}000\\ &= 84{,}000\end{aligned}$$

$$\begin{aligned}\text{total cost} &= 84{,}000(10)\\ &= 840{,}000\end{aligned}$$

b. $c(x) = 84{,}000x$

c.
$$\begin{aligned}c(25) &= 840{,}000(25)\\ &= 21{,}000{,}000\end{aligned}$$

21. $f(x) = \frac{50x}{105 - x},\ 0 \le x \le 100$
From example 6, we know that $f(70) = 100$. The cost to remove 75% of the pollutant is
$f(75) = \frac{50 \cdot 75}{105 - 75} = 125.$
The cost of removing an extra 5% is \$125 − \$100 = \$25 million. To remove the final 5% the cost is
$f(100) - f(95) = 1000 - 475 = \$525$ million.
This costs 21 times as much as the cost to remove the next 5% after the first 70% is removed.

22. **a.** $f(85) = \frac{20(85)}{102 - 85} = \$100\text{ million}$

b. $f(100) - f(95) = 1000 - 271.43 \approx \$728.57$ million

23. $f(x) = \left(\frac{K}{V}\right)x + \frac{1}{V}$

a. $f(x) = .2x + 50$

We have $\frac{K}{V} = .2$ and $\frac{1}{V} = 50$. If $\frac{1}{V} = 50$, then $V = \frac{1}{50}$. Now, $\frac{K}{V} = .2$ implies $\frac{K}{\frac{1}{50}} = .2$, so $K = \frac{1}{5} \cdot \frac{1}{50} = \frac{1}{250}$.

b. $y = \left(\frac{K}{V}\right)x + \frac{1}{V}$, $\left(\frac{K}{V}\right) \cdot 0 + \frac{1}{V} = \frac{1}{V}$, so the y-intercept is $\left(0, \frac{1}{V}\right)$.

Solving $\left(\frac{K}{V}\right)x + \frac{1}{V} = 0$, we get $\frac{K}{V}x = -\frac{1}{V} \Rightarrow x = -\frac{1}{K}$, so the x-intercept is $\left(-\frac{1}{K}, 0\right)$.

24. From 17(b), $\left(-\frac{1}{K}, 0\right)$ is the x-intercept. From the experimental data, (–500, 0) is also the x-intercept. Thus $-\frac{1}{K} = -500 \Rightarrow K = \frac{1}{500}$.

Again from 17(b), $\left(0, \frac{1}{V}\right)$ is the y-intercept. From the experimental data, (0, 60) is also the y-intercept. Thus $\frac{1}{V} = 60 \Rightarrow V = \frac{1}{60}$.

25. $y = 3x^2 - 4x$

$a = 3, b = -4, c = 0$

26. $y = \frac{x^2 - 6x + 2}{3} = \frac{1}{3}x^2 - 2x + \frac{2}{3}$

$a = \frac{1}{3},\ b = -2,\ c = \frac{2}{3}$

27. $y = 3x - 2x^2 + 1$

$a = -2, b = 3, c = 1$

28. $y = 3 - 2x + 4x^2$

$a = 4, b = -2, c = 3$

29. $y = 1 - x^2$

$a = -1, b = 0, c = 1$

30. $y = \frac{1}{2}x^2 + \sqrt{3}\,x - \pi$

$a = \frac{1}{2},\ b = \sqrt{3},\ c = -\pi$

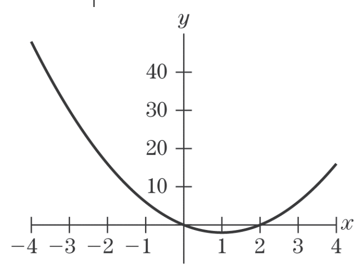
31. $f(x) = 2x^2 - 4x$

$a = 2, b = -4, c = 0$

vertex:

$$\left(\frac{-(-4)}{2(2)}, f\left(\frac{-(-4)}{2(2)}\right)\right) = (1, f(1)) = (1, -2)$$

x	y
0	0
2	0



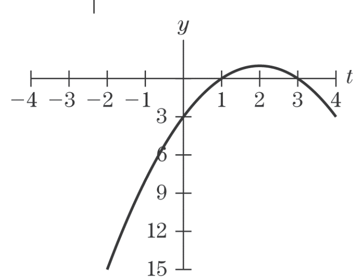
32. $g(t) = -t^2 + 4t - 3$

$a = -1, b = 4, c = -3$

vertex:

$$\left(\frac{-4}{2(-1)}, g\left(\frac{-4}{2(-1)}\right)\right) = (2, g(2)) = (2, 1)$$

x	y
0	–3
1	0
3	0



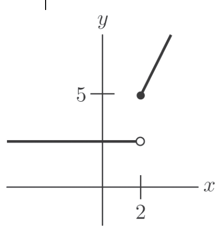
33. $f(x) = \begin{cases} 3 & \text{for } x < 2 \\ 2x + 1 & \text{for } x \ge 2 \end{cases}$

$x < 2$

x	$f(x) = 3$
1	3
0	3

$x \geq 2$

x	$f(x) = 2x + 1$
2	5
3	7



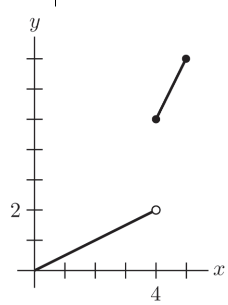
34. $f(x) = \begin{cases} \frac{1}{2}x & \text{for } 0 \leq x < 4 \\ 2x - 3 & \text{for } 4 \leq x \leq 5 \end{cases}$

$0 \leq x < 4$

x	$f(x) = \frac{1}{2}x$
0	0
2	1
3	$\frac{3}{2}$

$4 \leq x \leq 5$

x	$f(x) = 2x - 3$
4	5
5	7



35. $f(x) = \begin{cases} 4 - x & \text{for } 0 \leq x < 2 \\ 2x - 2 & \text{for } 2 \leq x < 3 \\ x + 1 & \text{for } x \geq 3 \end{cases}$

$0 \leq x < 2$

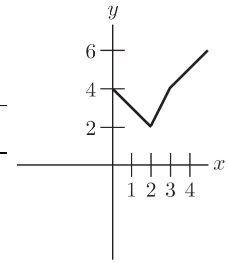
x	$f(x) = 4 - x$
0	4
1	3

$2 \leq x < 3$

x	$f(x) = 2x - 2$
2	2
$\frac{5}{2}$	3

$x \geq 3$

x	$f(x) = x + 1$
3	4
4	5



36. $f(x) = \begin{cases} 4x & \text{for } 0 \leq x < 1 \\ 8 - 4x & \text{for } 1 \leq x < 2 \\ 2x - 4 & \text{for } x \geq 2 \end{cases}$

$0 \leq x < 1$

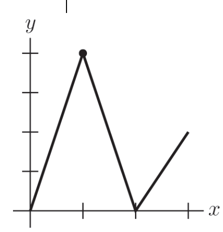
x	$f(x) = 4x$
0	0
$\frac{1}{2}$	2

$1 \leq x < 2$

x	$f(x) = 8 - 4x$
1	4
$\frac{3}{2}$	2

$x \geq 2$

x	$f(x) = 2x - 4$
2	0
3	2



37. $f(x) = x^{100}$, $x = -1$

$f(-1) = (-1)^{100} = 1$

38. $f(x) = x^5$, $x = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

39. $f(x) = |x|$, $x = 10^{-2}$

$f(10^{-2}) = |10^{-2}| = 10^{-2}$

40. $f(x) = |x|$, $x = \pi$

$f(\pi) = |\pi| = \pi$

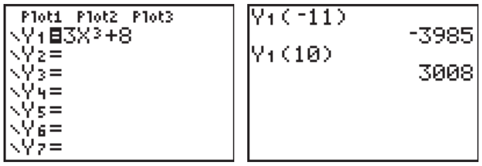
41. $f(x) = |x|$, $x = -2.5$

$f(-2.5) = |-2.5| = 2.5$

42. $f(x) = |x|,\ x = -\dfrac{2}{3}$

$f\left(-\dfrac{2}{3}\right) = \left|-\dfrac{2}{3}\right| = \dfrac{2}{3}$

43.



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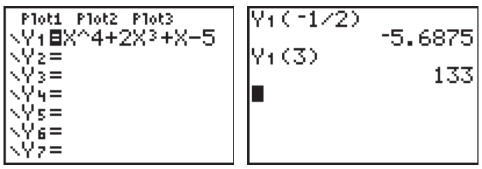

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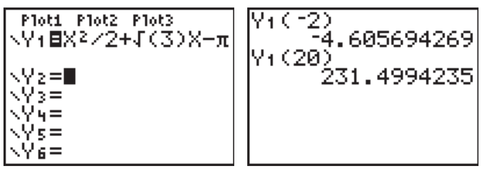

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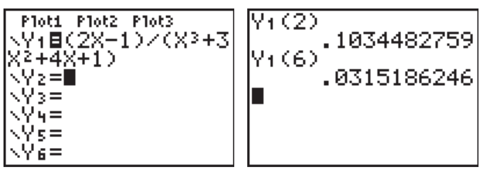


0.3 The Algebra of Functions

1. $f(x) + g(x) = (x^2 + 1) + 9x = x^2 + 9x + 1$

2. $f(x) - h(x) = (x^2 + 1) - (5 - 2x^2) = 3x^2 - 4$

3. $f(x)g(x) = (x^2 + 1)(9x) = 9x^3 + 9x$

4. $g(x)h(x) = (9x)(5 - 2x^2) = 45x - 18x^3$

5. $\dfrac{f(t)}{g(t)} = \dfrac{t^2 + 1}{9t} = \dfrac{t^2}{9t} + \dfrac{1}{9t} = \dfrac{t}{9} + \dfrac{1}{9t} = \dfrac{t^2 + 1}{9t}$

6. $\dfrac{g(t)}{h(t)} = \dfrac{9t}{5 - 2t^2}$

7. $\dfrac{2}{x - 3} + \dfrac{1}{x + 2} = \dfrac{2(x + 2) + (x - 3)}{(x - 3)(x + 2)}$

$= \dfrac{3x + 1}{x^2 - x - 6}$

8. $\dfrac{3}{x - 6} + \dfrac{-2}{x - 2} = \dfrac{3(x - 2) + (-2)(x - 6)}{(x - 6)(x - 2)}$

$= \dfrac{x + 6}{x^2 - 8x + 12}$

9. $\dfrac{x}{x - 8} + \dfrac{-x}{x - 4} = \dfrac{x(x - 4) + (-x)(x - 8)}{(x - 8)(x - 4)}$

$= \dfrac{4x}{x^2 - 12x + 32}$

10. $\dfrac{-x}{x + 3} + \dfrac{x}{x + 5} = \dfrac{(-x)(x + 5) + x(x + 3)}{(x + 3)(x + 5)}$

$= \dfrac{-2x}{x^2 + 8x + 15}$

11. $\dfrac{x + 5}{x - 10} + \dfrac{x}{x + 10} = \dfrac{(x + 5)(x + 10) + x(x - 10)}{(x - 10)(x + 10)}$

$= \dfrac{2x^2 + 5x + 50}{x^2 - 100}$

12. $\dfrac{x + 6}{x - 6} + \dfrac{x - 6}{x + 6} = \dfrac{(x + 6)(x + 6) + (x - 6)(x - 6)}{(x - 6)(x + 6)}$

$= \dfrac{2x^2 + 72}{x^2 - 36}$

13. $\dfrac{x}{x - 2} - \dfrac{5 - x}{5 + x} = \dfrac{x(5 + x) - (5 - x)(x - 2)}{(x - 2)(5 + x)}$

$= \dfrac{2x^2 - 2x + 10}{x^2 + 3x - 10}$

14. $\dfrac{t}{t - 2} - \dfrac{t + 1}{3t - 1} = \dfrac{t(3t - 1) - (t - 2)(t + 1)}{(t - 2)(3t - 1)}$

$= \dfrac{2t^2 + 2}{3t^2 - 7t + 2}$

15. $\dfrac{x}{x - 2} \cdot \dfrac{5 - x}{5 + x} = \dfrac{-x^2 + 5x}{x^2 + 3x - 10}$

16. $\dfrac{5 - x}{5 + x} \cdot \dfrac{x + 1}{3x - 1} = \dfrac{-x^2 + 4x + 5}{3x^2 + 14x - 5}$

17. $\dfrac{\frac{x}{x - 2}}{\frac{5 - x}{5 + x}} = \dfrac{x}{x - 2} \cdot \dfrac{5 + x}{5 - x} = \dfrac{x^2 + 5x}{-x^2 + 7x - 10}$

18. $\dfrac{\frac{s + 1}{3s - 1}}{\frac{s}{s - 2}} = \dfrac{s + 1}{3s - 1} \cdot \dfrac{s - 2}{s} = \dfrac{s^2 - s - 2}{3s^2 - s}$

19. $\dfrac{x+1}{(x+1)-2}\cdot\dfrac{5-(x+1)}{5+(x+1)}=\dfrac{x+1}{x-1}\cdot\dfrac{-x+4}{6+x}$
$=\dfrac{-x^2+3x+4}{x^2+5x-6}$

20. $\dfrac{x+2}{(x+2)-2}+\dfrac{5-(x+2)}{5+(x+2)}$
$=\dfrac{x+2}{x}+\dfrac{3-x}{x+7}$
$=\dfrac{(x+2)(x+7)+(3-x)(x)}{x(x+7)}=\dfrac{12x+14}{x^2+7x}$

21. $\dfrac{\dfrac{5-(x+5)}{5+(x+5)}}{\dfrac{x+5}{(x+5)-2}}=\dfrac{5-(x+5)}{5+(x+5)}\cdot\dfrac{(x+5)-2}{x+5}$
$=\dfrac{-x}{10+x}\cdot\dfrac{x+3}{x+5}$
$=\dfrac{-x^2-3x}{x^2+15x+50}$

22. $\dfrac{\frac{1}{t}}{\frac{1}{t}-2}=\dfrac{1}{t}\cdot\dfrac{t}{1-2t}=\dfrac{1}{1-2t},\ t\neq 0$

23. $\dfrac{5-\frac{1}{u}}{5+\frac{1}{u}}=\dfrac{5u-1}{u}\cdot\dfrac{u}{5u+1}=\dfrac{5u-1}{5u+1},\ u\neq 0$

24. $\dfrac{\frac{1}{x^2}+1}{3\left(\frac{1}{x^2}\right)-1}=\dfrac{1+x^2}{x^2}\cdot\dfrac{x^2}{3-x^2}=\dfrac{1+x^2}{3-x^2},\ x\neq 0$

25. $f\left(\dfrac{x}{1-x}\right)=\left(\dfrac{x}{1-x}\right)^6$

26. $h\left(t^6\right)=\left(t^6\right)^3-5\left(t^6\right)^2+1=t^{18}-5t^{12}+1$

27. $h\left(\dfrac{x}{1-x}\right)=\left(\dfrac{x}{1-x}\right)^3-5\left(\dfrac{x}{1-x}\right)^2+1$

28. $g\left(x^6\right)=\dfrac{x^6}{1-x^6}$

29. $g(t^3-5t^2+1)=\dfrac{t^3-5t^2+1}{1-(t^3-5t^2+1)}$
$=\dfrac{t^3-5t^2+1}{-t^3+5t^2}$

30. $f(x^3-5x^2+1)=(x^3-5x^2+1)^6$

31. $(x+h)^2-x^2=x^2+2xh+h^2-x^2$
$=2xh+h^2$

32. $\dfrac{1}{x+h}-\dfrac{1}{x}=\dfrac{x-x-h}{x(x+h)}=\dfrac{-h}{x(x+h)}$

33. $\dfrac{\left[4(t+h)-(t+h)^2\right]-\left(4t-t^2\right)}{h}$
$=\dfrac{4t+4h-(t^2+2th+h^2)-4t+t^2}{h}$
$=\dfrac{4h-2th-h^2}{h}=\dfrac{h(4-2t-h)}{h}$
$=4-2t-h$

34. $\dfrac{\left[(t+h)^3+5\right]-\left(t^3+5\right)}{h}$
$=\dfrac{t^3+3t^2h+3th^2+h^3+5-t^3-5}{h}$
$=\dfrac{3t^2h+3th^2+h^3}{h}=\dfrac{h(3t^2+3th+h^2)}{h}$
$=3t^2+3th+h^2$

35. a. $C\left(A(t)\right)=3000+80\left(20t-\dfrac{1}{2}t^2\right)$
$=3000+1600t-40t^2$

b. $C(2)=3000+1600(2)-40(2)^2$
$=3000+3200-160=\$6040$

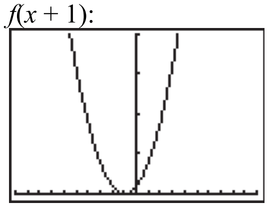
36. a. $C(f(t))$
$=.1(10t-5)^2+25(10t-5)+200$
$=.1(100t^2-100t+25)+250t-125+200$
$=10t^2+240t+77.5$

b. $C(4)=10(4)^2+240(4)+77.5=\$1197.50$

37. $h(x)=f(8x+1)=\left(\dfrac{1}{8}\right)(8x+1)=x+\dfrac{1}{8}$

$h(x)$ converts from British to U.S. sizes.

38. $f(x+1)$:

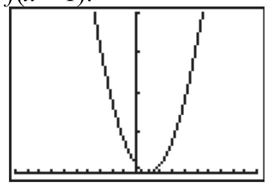


[−10, 10] by [0, 20]

(*continued on next page*)

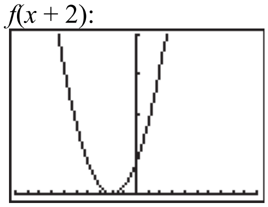
(*continued*)

$f(x-1)$:



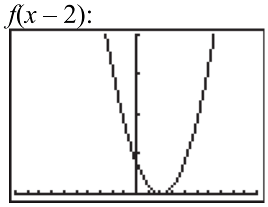
[−10, 10] by [0, 20]

$f(x+2)$:



[−10, 10] by [0, 20]

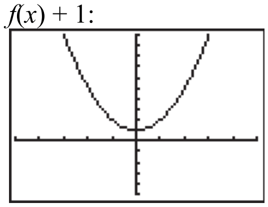
$f(x-2)$:



[−10, 10] by [0, 20]

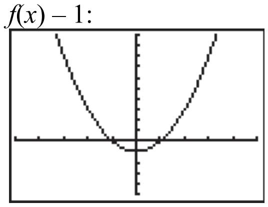
The graph of $f(x+a)$ is the graph of $f(x)$ shifted to the left (if $a > 0$) or to the right (if $a < 0$) by $|a|$ units.

39. $f(x)+1$:



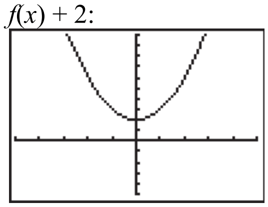
[−5, 5] by [−5, 15]

$f(x)-1$:



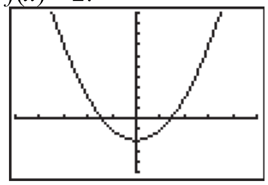
[−5, 5] by [−5, 15]

$f(x)+2$:



[−5, 5] by [−5, 15]

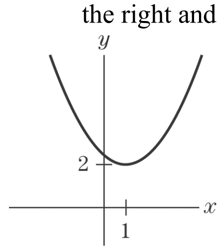
$f(x)-2$:

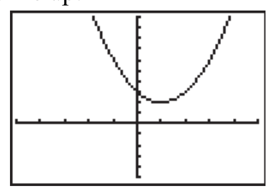


[−5, 5] by [−5, 15]

The graph of $f(x)+c$ is the graph of $f(x)$ shifted up (if $c > 0$) or down (if $c < 0$) by $|c|$ units.

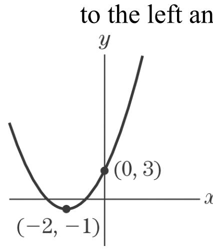
40. This is the graph of $f(x) = x^2$ shifted 1 unit to the right and 2 units up.

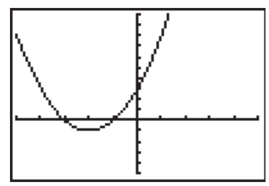




[−5, 5] by [−5, 15]

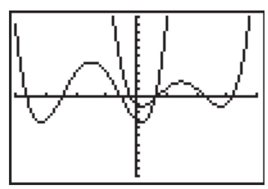
41. This is the graph of $f(x) = x^2$ shifted 2 units to the left and 1 unit down.





[−5, 5] by [−5, 15]

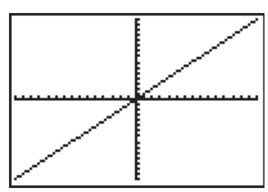
42.



[−4, 4] by [−10, 10]

They are not the same function.

43.



[−15, 15] by [−10, 10]

$$f(f(x)) = f\left(\frac{x}{x-1}\right) = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1}$$

$$= \frac{x}{x-(x-1)} = x,\ x \neq 1$$

0.4 Zeros of Functions—The Quadratic Formula and Factoring

1. $f(x)=2x^2-7x+6$

$2x^2-7x+6=0$

$a=2, b=-7, c=6$

$\sqrt{b^2-4ac}=\sqrt{49-4(2)(6)}=\sqrt{1}=1$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{7\pm1}{4}=2,\ \dfrac{3}{2}$

2. $f(x)=3x^2+2x-1$

$3x^2+2x-1=0$

$a=3, b=2, c=-1$

$\sqrt{b^2-4ac}=\sqrt{4^2-4(3)(-1)}=\sqrt{16}=4$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-2\pm4}{6}=\dfrac{1}{3},\ -1$

3. $f(t)=4t^2-12t+9$

$4t^2-12t+9=0$

$t=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{12\pm\sqrt{(-12)^2-4(4)(9)}}{2(4)}$

$=\dfrac{12\pm\sqrt{0}}{8}=\dfrac{3}{2}$

4. $f(x)=\dfrac{1}{4}x^2+x+1$

$\dfrac{1}{4}x^2+x+1=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-1\pm\sqrt{1^2-4\left(\frac{1}{4}\right)(1)}}{2\left(\frac{1}{4}\right)}$

$=\dfrac{-1\pm\sqrt{0}}{\frac{1}{2}}=-2$

5. $f(x)=-2x^2+3x-4$

$-2x^2+3x-4=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-3\pm\sqrt{3^2-4(-2)(-4)}}{2(-2)}$

$=\dfrac{-3\pm\sqrt{-23}}{-4}$

$\sqrt{-23}$ is undefined, so $f(x)$ has no real zeros.

6. $f(a)=11a^2-7a+1$

$11a^2-7a+1=0$

$a=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{7\pm\sqrt{(-7)^2-4(11)(1)}}{2(11)}$

$=\dfrac{7\pm\sqrt{5}}{22}=\dfrac{7+\sqrt{5}}{22},\ \dfrac{7-\sqrt{5}}{22}$

7. $5x^2-4x-1=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{4\pm\sqrt{(-4)^2-4(5)(-1)}}{2(5)}$

$=\dfrac{4\pm\sqrt{36}}{10}=\dfrac{4\pm6}{10}=1,\ -\dfrac{1}{5}$

8. $x^2-4x+5=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{4\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}$

$=\dfrac{4\pm\sqrt{-4}}{2}$

$\sqrt{-4}$ is undefined, so there is no real solution.

9. $15x^2-135x+300=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{135\pm\sqrt{(-135)^2-4(15)(300)}}{2(15)}$

$=\dfrac{135\pm\sqrt{225}}{30}=\dfrac{135\pm15}{30}=5,\ 4$

10. $z^2-\sqrt{2}\,z-\dfrac{5}{4}=0$

$z=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{\sqrt{2}\pm\sqrt{\left(-\sqrt{2}\right)^2-4(1)\left(-\frac{5}{4}\right)}}{2(1)}=\dfrac{\sqrt{2}\pm\sqrt{7}}{2}$

$=\dfrac{\sqrt{2}+\sqrt{7}}{2},\ \dfrac{\sqrt{2}-\sqrt{7}}{2}$

11. $\dfrac{3}{2}x^2-6x+5=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{6\pm\sqrt{(-6)^2-4\left(\frac{3}{2}\right)(5)}}{2\left(\frac{3}{2}\right)}$

$=\dfrac{6\pm\sqrt{6}}{3}=2+\dfrac{\sqrt{6}}{3},\ 2-\dfrac{\sqrt{6}}{3}$

12. $9x^2 - 12x + 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12 \pm \sqrt{(-12)^2 - 4(9)(4)}}{2(9)}$$
$$= \frac{12 \pm \sqrt{0}}{18} = \frac{2}{3}$$

13. $x^2 + 8x + 15 = (x+5)(x+3)$

14. $x^2 - 10x + 16 = (x-2)(x-8)$

15. $x^2 - 16 = (x-4)(x+4)$

16. $x^2 - 1 = (x+1)(x-1)$

17. $3x^2 + 12x + 12 = 3\left(x^2 + 4x + 4\right)$
$= 3(x+2)(x+2) = 3(x+2)^2$

18. $2x^2 - 12x + 18 = 2\left(x^2 - 6x + 9\right)$
$= 2(x-3)(x-3) = 2(x-3)^2$

19. $30 - 4x - 2x^2 = -2\left(-15 + 2x + x^2\right)$
$= -2(x-3)(x+5)$

20. $15 + 12x - 3x^2 = -3(-5 - 4x + x^2)$
$= -3(x-5)(x+1)$

21. $3x - x^2 = x(3-x)$

22. $4x^2 - 1 = (2x+1)(2x-1)$

23. $6x - 2x^3 = -2x(x^2 - 3)$
$= -2x\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)$

24. $16x + 6x^2 - x^3 = x\left(16 + 6x - x^2\right)$
$= x(8-x)(x+2)$
$= -x(x-8)(x+2)$

25. $x^3 - 1 = (x-1)\left(x^2 + x + 1\right)$

26. $x^3 + 125 = (x+5)\left(x^2 - 5x + 25\right)$

27. $8x^3 + 27 = (2x+3)\left(4x^2 - 6x + 9\right)$

28. $x^3 - \frac{1}{8} = \left(x - \frac{1}{2}\right)\left(x^2 + \frac{x}{2} + \frac{1}{4}\right)$

29. $x^2 - 14x + 49 = (x-7)^2$

30. $x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$

31. $2x^2 - 5x - 6 = 3x + 4$
$2x^2 - 8x - 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(-10)}}{2(2)}$$
$$= \frac{8 \pm \sqrt{144}}{4} = \frac{8 \pm 12}{4} = 5, -1$$

$y = 3x + 4 = 15 + 4 = 19$
$y = -3 + 4 = 1$
Points of intersection: (5, 19), (–1, 1)

32. $x^2 - 10x + 9 = x - 9$
$x^2 - 11x + 18 = 0$
$(x-9)(x-2) = 0$
$x = 9, 2$
$y = x - 9 = 9 - 9 = 0$
$y = 2 - 9 = -7$
Points of intersection: (9, 0), (2, –7)

33. $y = x^2 - 4x + 4$
$y = 12 + 2x - x^2$
$x^2 - 4x + 4 = 12 + 2x - x^2$
$2x^2 - 6x - 8 = 0$
$2(x^2 - 3x - 4) = 0$
$2(x-4)(x+1) = 0$
$x = 4, -1$
$y = x^2 - 4x + 4 = 4^2 - 4(4) + 4 = 4$
$y = (-1)^2 - 4(-1) + 4 = 9$
Points of intersection: (4, 4), (–1, 9)

34. $y = 3x^2 + 9$
$y = 2x^2 - 5x + 3$
$3x^2 + 9 = 2x^2 - 5x + 3$
$x^2 + 5x + 6 = 0$
$(x+3)(x+2) = 0$
$x = -3, -2$
$y = 3x^2 + 9 = 3(-3)^2 + 9 = 36$
$y = 3(-2)^2 + 9 = 21$
Points of intersection: (–3, 36), (–2, 21)

35. $y = x^3 - 3x^2 + x$

$y = x^2 - 3x$

$x^3 - 3x^2 + x = x^2 - 3x$

$x^3 - 4x^2 + 4x = 0$

$x(x^2 - 4x + 4) = 0$

$x(x-2)(x-2) = 0 \Rightarrow x = 0,\ 2$

$y = x^2 - 3x = 0^2 - 3(0) = 0$

$y = 2^2 - 3(2) = 4 - 6 = -2$

Points of intersection: (0, 0), (2, –2)

36. $y = \frac{1}{2}x^3 - 2x^2$

$y = 2x$

$\frac{1}{2}x^3 - 2x^2 = 2x$

$\frac{1}{2}x^3 - 2x^2 - 2x = 0$

$x\left(\frac{1}{2}x^2 - 2x - 2\right) = 0$

$x = 0$ or $\frac{1}{2}x^2 - 2x - 2 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - 4\left(\frac{1}{2}\right)(-2)}}{2\left(\frac{1}{2}\right)}$

$= \frac{2 \pm \sqrt{8}}{1} = 2 + 2\sqrt{2},\ 2 - 2\sqrt{2}$

$y = 2x = 2(0) = 0$

$y = 2\left(2 + 2\sqrt{2}\right) = 4 + 4\sqrt{2}$

$y = 2\left(2 - 2\sqrt{2}\right) = 4 - 4\sqrt{2}$

Points of intersection: (0, 0),
$\left(2 + 2\sqrt{2},\ 4 + 4\sqrt{2}\right),\ \left(2 - 2\sqrt{2},\ 4 - 4\sqrt{2}\right)$

37. $y = \frac{1}{2}x^3 + x^2 + 5$

$y = 3x^2 - \frac{1}{2}x + 5$

$\frac{1}{2}x^3 + x^2 + 5 = 3x^2 - \frac{1}{2}x + 5$

$\frac{1}{2}x^3 - 2x^2 + \frac{1}{2}x = 0$

$x\left(\frac{1}{2}x^2 - 2x + \frac{1}{2}\right) = 0$

$x = 0$ or $\frac{1}{2}x^2 - 2x + \frac{1}{2} = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - 4\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}}{2\left(\frac{1}{2}\right)}$

$= 2 \pm \sqrt{3}$

$y = 3x^2 - \frac{1}{2}x + 5 = 3(0)^2 - \frac{1}{2}(0) + 5 = 5$

$y = 3\left(2 + \sqrt{3}\right)^2 - \frac{1}{2}\left(2 + \sqrt{3}\right) + 5 = 25 + \frac{23\sqrt{3}}{2}$

$y = 3\left(2 - \sqrt{3}\right)^2 - \frac{1}{2}\left(2 - \sqrt{3}\right) + 5 = 25 - \frac{23\sqrt{3}}{2}$

Points of intersection: (0, 5),
$\left(2 - \sqrt{3},\ 25 - \frac{23\sqrt{3}}{2}\right),\ \left(2 + \sqrt{3},\ 25 + \frac{23\sqrt{3}}{2}\right)$

38. $y = 30x^3 - 3x^2$

$y = 16x^3 + 25x^2$

$30x^3 - 3x^2 = 16x^3 + 25x^2$

$14x^3 - 28x^2 = 0$

$14x^2(x - 2) = 0$

$x = 0$ or $x = 2$

$y = 30(0)^3 - 3(0)^2 = 0$

$y = 30(2)^3 - 3(2)^2 = 30(8) - 3(4) = 228$

Points of intersection: (0, 0), (2, 228)

39. $\frac{21}{x} - x = 4$

$21 - x^2 = 4x$

$x^2 + 4x - 21 = 0$

$(x+7)(x-3) = 0 \Rightarrow x = -7,\ 3$

40. $x + \frac{2}{x-6} = 3$

$x^2 - 6x + 2 = 3x - 18$

$x^2 - 9x + 20 = 0$

$(x-4)(x-5) = 0 \Rightarrow x = 4,\ 5$

41. $x + \frac{14}{x+4} = 5$

$x^2 + 4x + 14 = 5x + 20$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0 \Rightarrow x = 3,\ -2$

42. $1 = \frac{5}{x} + \frac{6}{x^2}$

$1 = \frac{5x + 6}{x^2}$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0 \Rightarrow x = 6,\ -1$

43. $\dfrac{x^2+14x+49}{x^2+1}=0$

$x^2+14x+49=0$

$(x+7)^2=0 \Rightarrow x=-7$

44. $\dfrac{x^2-8x+16}{1+\sqrt{x}}=0$

$x^2-8x+16=0$

$(x-4)^2=0 \Rightarrow x=4$

45. $C(x) = 275 + 12x$

$R(x)=32x-.21x^2$

$C(x)=R(x)$

$275+12x=32x-.21x^2$

$.21x^2-20x+275=0$

Thus

$$x=\frac{20\pm\sqrt{(-20)^2-4(.21)275}}{.42}$$

$=16,667$ or $78,571$ subscribers

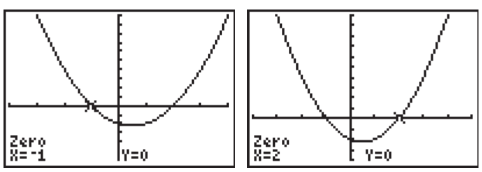
46. $x+\left(\dfrac{1}{20}\right)x^2=175$

$x^2+20x-3500=0$

$(x-50)(x+70)=0$

$x=50$ mph

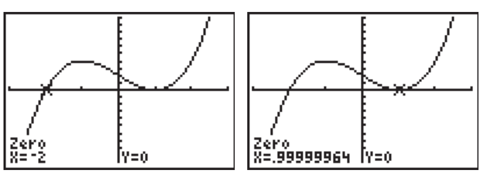
47.



$[-4, 5]$ by $[-4, 10]$

The zeros are –1 and 2.

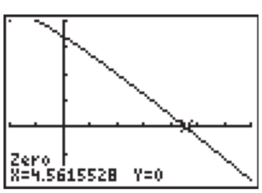
48.



$[-4, 5]$ by $[-4, 10]$

The zeros are –2 and 1.

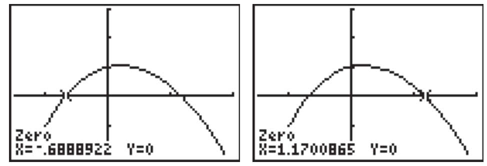
49.



$[-2, 7]$ by $[-2, 4]$

The zero is approximately 4.56.

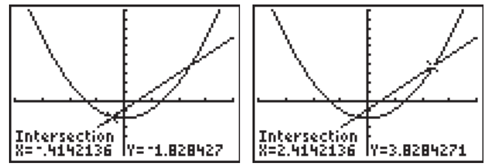
50.



$[-1.5, 2]$ by $[-2, 3]$

The zeros are approximately −.689 and 1.170.

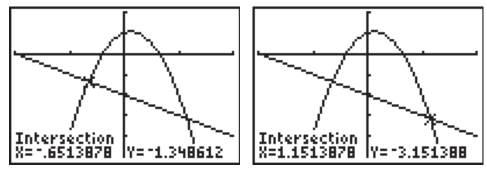
51.



$[-4, 4]$ by $[-6, 10]$

Approximate points of intersection:
(–0.41, –1.83) and (2.41, 3.83)

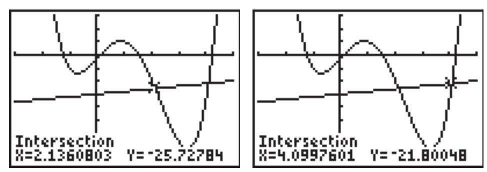
52.



$[-2, 2]$ by $[-5, 2]$

Approximate points of intersection:
(–.65, –1.35) and (1.15, –3.15)

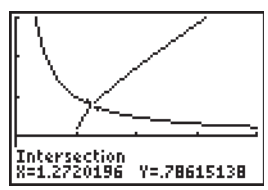
53.



$[-3, 5]$ by $[-80, 30]$

Approximate points of intersection:
(2.14, –25.73) and (4.10, –21.80)

54.

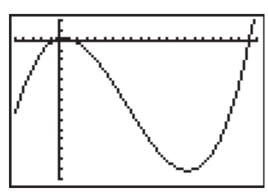


$[0, 4]$ by $[-1, 3]$

Approximate point of intersection: (1.27, .79)

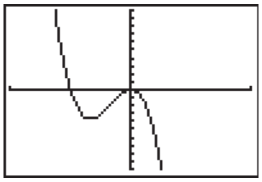
Answers may vary for exercises 55−58.

55.



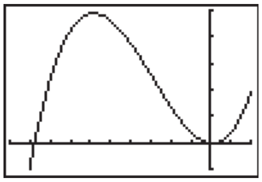
$[-5, 22]$ by $[-1400, 100]$

56.



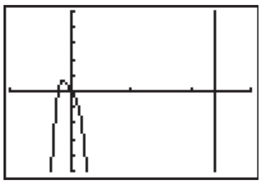
[−1, 1] by [−10, 10]

57.



[−20, 4] by [−500, 2500]

58.



[−5, 15] by [−100, 100]

0.5 Exponents and Power Functions

1. $3^3 = 27$

2. $(-2)^3 = -8$

3. $1^{100} = 1$

4. $0^{25} = 0$

5. $(.1)^4 = (.1)(.1)(.1)(.1) = .0001$

6. $(100)^4 = (100)(100)(100)(100) = 100,000,000$

7. $-4^2 = -16$

8. $(.01)^3 = .000001$

9. $(16)^{1/2} = \sqrt{16} = 4$

10. $(27)^{1/3} = \sqrt[3]{27} = 3$

11. $(.000001)^{1/3} = \sqrt[3]{.000001} = .01$

12. $\left(\frac{1}{125}\right)^{1/3} = \sqrt[3]{\frac{1}{125}} = \frac{1}{5}$

13. $6^{-1} = \frac{1}{6}$

14. $\left(\frac{1}{2}\right)^{-1} = \frac{1}{\frac{1}{2}} = 2$

15. $(.01)^{-1} = \frac{1}{.01} = 100$

16. $(-5)^{-1} = -\frac{1}{5}$

17. $8^{4/3} = \left(\sqrt[3]{8}\right)^4 = 16$

18. $16^{3/4} = \left(\sqrt[4]{16}\right)^3 = 8$

19. $(25)^{3/2} = \left(\sqrt{25}\right)^3 = 125$

20. $(27)^{2/3} = \left(\sqrt[3]{27}\right)^2 = 9$

21. $(1.8)^0 = 1$

22. $9^{1.5} = 9^{3/2} = \left(\sqrt{9}\right)^3 = 27$

23. $16^{0.5} = 16^{1/2} = 4$

24. $81^{0.75} = 81^{3/4} = 27$

25. $4^{-1/2} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

26. $\left(\frac{1}{8}\right)^{-2/3} = 8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 4$

27. $(.01)^{-1.5} = \frac{1}{(.01)^{3/2}} = \frac{1}{.001} = 1000$

28. $1^{-1.2} = \frac{1}{1^{1.2}} = 1$

29. $5^{1/3} \cdot 200^{1/3} = 1000^{1/3} = 10$

30. $(3^{1/3} \cdot 3^{1/6})^6 = (3^{1/2})^6 = 27$

31. $6^{1/3} \cdot 6^{2/3} = 6^1 = 6$

32. $(9^{4/5})^{5/8} = 9^{1/2} = 3$

33. $\frac{10^4}{5^4} = 2^4 = 16$

34. $\frac{3^{5/2}}{3^{1/2}} = 3^{(5/2)-(1/2)} = 3^{4/2} = 9$

35. $(2^{1/3} \cdot 3^{2/3})^3 = \left(\sqrt[3]{2}\sqrt[3]{9}\right)^3 = \left(\sqrt[3]{18}\right)^3 = 18$

36. $20^{0.5} \cdot 5^{0.5} = (100)^{1/2} = 10$

37. $\left(\frac{8}{27}\right)^{2/3} = \frac{8^{2/3}}{27^{2/3}} = \frac{4}{9}$

38. $(125 \cdot 27)^{1/3} = 125^{1/3} \cdot 27^{1/3} = 15$

39. $\frac{7^{4/3}}{7^{1/3}} = 7^{(4/3)-(1/3)} = 7^{3/3} = 7$

40. $(6^{1/2})^0 = 6^{(1/2)(0)} = 6^0 = 1$

41. $(xy)^6 = x^6 y^6$

42. $(x^{1/3})^6 = x^{(1/3)(6)} = x^2$

43. $\frac{x^4 \cdot y^5}{xy^2} = x^4 \cdot y^5 \cdot x^{-1} \cdot y^{-2} = x^3 y^3$

44. $\frac{1}{x^{-3}} = x^3$

45. $x^{-1/2} = \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$

46. $(x^3 \cdot y^6)^{1/3} = x^{3(1/3)} \cdot y^{6(1/3)} = xy^2$

47. $\left(\frac{x^4}{y^2}\right)^3 = \frac{x^{4(3)}}{y^{2(3)}} = \frac{x^{12}}{y^6}$

48. $\left(\frac{x}{y}\right)^{-2} = \frac{1}{x^2} \cdot y^2 = \frac{y^2}{x^2}$

49. $(x^3 y^5)^4 = x^{3(4)} \cdot y^{5(4)} = x^{12} y^{20}$

50. $\sqrt{1+x}(1+x)^{3/2} = (1+x)^{1/2}(1+x)^{3/2}$
$= (1+x)^{(1/2)+(3/2)} = (1+x)^2$
$= x^2 + 2x + 1$

51. $x^5 \cdot \left(\frac{y^2}{x}\right)^3 = \frac{x^5 \cdot y^{2(3)}}{x^3} = x^5 \cdot y^6 \cdot x^{-3} = x^2 y^6$

52. $x^{-3} \cdot x^7 = x^{7-3} = x^4$

53. $(2x)^4 = 2^4 \cdot x^4 = 16x^4$

54. $\frac{-3x}{15x^4} = -\frac{3}{15} \cdot \frac{x}{x^4} = -\frac{1}{5x^3}$

55. $\frac{-x^3 y}{-xy} = \frac{x^3}{x} \cdot \frac{y}{y} = x^2$

56. $\frac{x^3}{y^{-2}} = x^3 y^2$

57. $\frac{x^{-4}}{x^3} = \frac{1}{x^4} \cdot \frac{1}{x^3} = (-3)^3 \cdot x^3 = \frac{1}{x^7}$

58. $(-3x)^3 = -27x^3$

59. $\sqrt[3]{x} \cdot \sqrt[3]{x^2} = x^{1/3} \cdot x^{2/3} = x$

60. $(9x)^{-1/2} = \frac{1}{\sqrt{9x}} = \frac{1}{3\sqrt{x}}$

61. $\left(\frac{3x^2}{2y}\right)^3 = \frac{3^3 \cdot x^6}{2^3 \cdot y^3} = \frac{27x^6}{8y^3}$

62. $\frac{x^2}{x^5 y} = \frac{x^2}{x^5} \cdot \frac{1}{y} = \frac{1}{x^3 y}$

63. $\frac{2x}{\sqrt{x}} = 2x \cdot x^{-1/2} = 2\sqrt{x}$

64. $\frac{1}{yx^{-5}} = \frac{x^5}{y}$

65. $(16x^8)^{-3/4} = 16^{-3/4} \cdot x^{-6} = \frac{1}{8x^6}$

66. $(-8y^9)^{2/3} = (-8)^{2/3} y^{9(2/3)} = 4y^6$

67. $\sqrt{x}\left(\frac{1}{4x}\right)^{5/2} = \frac{x^{1/2}}{4^{5/2} x^{5/2}} = \frac{x^{1/2} \cdot x^{-5/2}}{32}$
$= \frac{1}{32x^2}$

68. $\frac{(25xy)^{3/2}}{x^2 y} = \frac{(25)^{3/2} x^{3/2} y^{3/2}}{x^2 y} = \frac{125\sqrt{y}}{\sqrt{x}}$

69. $\frac{(-27x^5)^{2/3}}{\sqrt[3]{x}} = \frac{(-27)^{2/3} x^{5(2/3)}}{x^{1/3}} = 9x^3$

70. $(-32y^{-5})^{3/5} = (-32)^{3/5} y^{-5(3/5)} = -\frac{8}{y^3}$

For exercises 71−82, $f(x) = \sqrt[3]{x}$ and $g(x) = \frac{1}{x^2}$.

71. $f(x)g(x) = \sqrt[3]{x} \cdot \frac{1}{x^2} = x^{1/3} \cdot x^{-2} = x^{-5/3} = \frac{1}{x^{5/3}}$

72. $\frac{f(x)}{g(x)} = \frac{\sqrt[3]{x}}{\frac{1}{x^2}} = x^{1/3} \cdot x^2 = x^{7/3}$

73. $\frac{g(x)}{f(x)} = \frac{\frac{1}{x^2}}{\sqrt[3]{x}} = x^{-2} \cdot x^{-1/3} = x^{-7/3} = \frac{1}{x^{7/3}}$

74. $[f(x)]^3 g(x) = (\sqrt[3]{x})^3 \cdot \frac{1}{x^2} = x \cdot x^{-2} = x^{-1} = \frac{1}{x}$

75. $\left[f(x)g(x)\right]^3 = \left(\sqrt[3]{x}\cdot\frac{1}{x^2}\right)^3 = \left(x^{1/3}\cdot x^{-2}\right)^3$
$= \left(x^{-5/3}\right)^3 = x^{-5} = \frac{1}{x^5}$

76. $\sqrt{\frac{f(x)}{g(x)}} = \left(\frac{\sqrt[3]{x}}{\frac{1}{x^2}}\right)^{1/2} = \left(x^{1/3}\cdot x^2\right)^{1/2} = \left(x^{7/3}\right)^{1/2}$
$= x^{7/6}$

77. $\sqrt{f(x)g(x)} = \left(\sqrt[3]{x}\cdot\frac{1}{x^2}\right)^{1/2} = \left(x^{1/3}\cdot x^{-2}\right)^{1/2}$
$= \left(x^{-5/3}\right)^{1/2} = x^{-5/6} = \frac{1}{x^{5/6}}$

78. $\sqrt[3]{f(x)g(x)} = \left(\sqrt[3]{x}\cdot\frac{1}{x^2}\right)^{1/3} = \left(x^{1/3}\cdot x^{-2}\right)^{1/3}$
$= \left(x^{-5/3}\right)^{1/3} = x^{-5/9} = \frac{1}{x^{5/9}}$

79. $f(g(x)) = f\left(\frac{1}{x^2}\right) = f\left(x^{-2}\right) = \sqrt[3]{x^{-2}}$
$= \left(x^{-2}\right)^{1/3} = x^{-2/3} = \frac{1}{x^{2/3}}$

80. $g(f(x)) = g\left(\sqrt[3]{x}\right) = g\left(x^{1/3}\right) = \left(\frac{1}{x^{1/3}}\right)^2 = \frac{1}{x^{2/3}}$

81. $f(g(x)) = f\left(\sqrt[3]{x}\right) = f\left(x^{1/3}\right) = \sqrt[3]{x^{1/3}}$
$= \left(x^{1/3}\right)^{1/3} = x^{1/9}$

82. $g(g(x)) = g\left(\frac{1}{x^2}\right) = g\left(x^{-2}\right) = \left(\frac{1}{x^{-2}}\right)^2$
$= \frac{1}{x^{-4}} = x^4$

83. $\sqrt{x} - \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}}(x-1)$

84. $2x^{2/3} - x^{-1/3} = x^{-1/3}(2x-1)$

85. $x^{-1/4} + 6x^{1/4} = x^{-1/4}\left(1+6\sqrt{x}\right)$

86. $\sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}} = \sqrt{xy}\left(\frac{1}{y} - \frac{1}{x}\right)$

87. $\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$
$a^{1/2}\cdot b^{1/2} = (ab)^{1/2}$ (Law 5)

88. $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
$\frac{a^{1/2}}{b^{1/2}} = \left(\frac{a}{b}\right)^{1/2}$ (Law 6)

89. $f(x) = x^2 \Rightarrow f(4) = (4)^2 = 16$

90. $f(x) = x^3 \Rightarrow f(4) = (4)^3 = 64$

91. $f(x) = x^{-1} \Rightarrow f(4) = (4)^{-1} = \frac{1}{4}$

92. $f(x) = x^{1/2} \Rightarrow f(4) = (4)^{1/2} = 2$

93. $f(x) = x^{3/2} \Rightarrow f(4) = (4)^{3/2} = 8$

94. $f(x) = x^{-1/2} \Rightarrow f(4) = (4)^{-1/2} = \frac{1}{2}$

95. $f(x) = x^{-5/2} \Rightarrow f(4) = (4)^{-5/2} = \frac{1}{32}$

96. $f(x) = x^0 \Rightarrow f(4) = 4^0 = 1$

In exercises 97−104, use the compound interest formula $A = P\left(1+\frac{r}{m}\right)^{mt}$, where P is the principal, r is the annual interest rate, m is the number of interest periods per year, and t is the number of years.

97. $A = 500\left(1+\frac{.06}{1}\right)^{1(6)} \approx \$709.26$

98. $A = 700\left(1+\frac{.08}{1}\right)^{1(8)} \approx \$1295.65$

99. $A = 50{,}000\left(1+\frac{.095}{4}\right)^{4(10)} \approx \$127{,}857.61$

100. $A = 20{,}000\left(1+\frac{.12}{4}\right)^{4(3)} \approx \$28{,}515.22$

101. $A = 100\left(1+\frac{.05}{12}\right)^{12(10)} \approx \$164.70$

102. $A = 500\left(1+\frac{.045}{12}\right)^{12(1)} \approx \$522.97$

103. $A = 1500\left(1+\frac{.06}{365}\right)^{365(1)} \approx \$1592.75$

104. $A = 1500\left(1+\frac{.06}{365}\right)^{365(3)} \approx \$1795.80$

105. $A = 1000\left(1+\frac{.068}{1}\right)^{1(18)} \approx \$3268.00$

106. At the end of the first year, there will be $A_1 = A_0(1+.08) = 4000(1.08) = \$4320$ in the account. At the end of the second year, there will be

$$A_2 = A_1(1+.08) = (4320+4000)(1.08) = \$8985.60$$

in the account. At the end of the third year, there will be

$$A_3 = A_2(1+0.8) = (8985.60+4000)(1.08) = 14,024.448$$

in the account. (Note that we hold the decimals since this is a partial answer. We will round at the end of the calculations.) At the end of the fourth year, there will be

$$A_4 = A_3(1+.08) = (14,024.448+4000)(1.08) \approx 19,466.40384$$

in the account. No additional deposits are made, so use the compound interest formula to compute the amount in the account after another four years:

$$A = 19,466.40384\left(1+\frac{.08}{1}\right)^{1(4)} \approx \$26,483.83.$$

107. $A = 500 + 500r + \frac{375}{2}r^2 + \frac{125}{4}r^3 + \frac{125}{64}r^4 = \frac{500}{256}\left(256 + 256r + 96r^2 + 16r^3 + r^4\right)$

108. $A = 1000 + 2000r + 1500r^2 + 500r^3 + \frac{125}{2}r^4 = \frac{125}{2}\left(16 + 32r + 24r^2 + 8r^3 + r^4\right)$

109. If the speed is $2x$, then $\frac{1}{20}(2x)^2 = \frac{1}{20}\left(4x^2\right) = 4\left(\frac{1}{20}x^2\right).$

110. 5E–5 $= 5\cdot 10^{-5} = .00005$

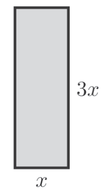
111. 8.103E–4 $= 8.103\cdot 10^{-4} = .0008103$

112. 1.35E13 $= 1.35\cdot 10^{13} = 13,500,000,000,000$

113. 8.23E–6 $= 8.23\cdot 10^{-6} = .00000823$

0.6 Functions and Graphs in Applications

1.



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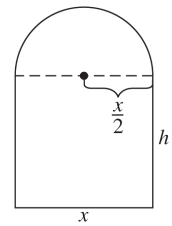
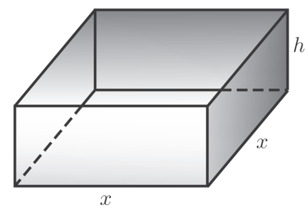


3.



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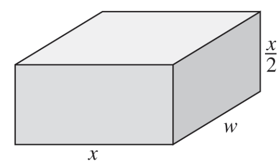
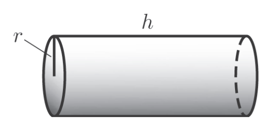


5.



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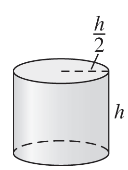


7. $P = 2(x+3x) = 8x$

$3x^2 = 25$

8. $A = 3x^2$

$8x = 30$

9. $A = \pi r^2$

$2\pi r = 15$

10. $P = 2r + 2h + \pi r$

The area of the window is represented by $A = 2rh + \frac{1}{2}\pi r^2.$

$$2rh + \left(\frac{1}{2}\right)\pi r^2 = 2.5$$

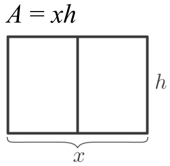
11. $V = x^2h$
The surface area of the box is represented by
$S = x^2 + 4xh.$
$x^2 + 4xh = 65$

12. $\text{SA} = 2xw + 2x\left(\frac{x}{2}\right) + 2w\left(\frac{x}{2}\right) = 3xw + x^2$
The volume is represented by
$xw\left(\frac{x}{2}\right) = \frac{1}{2}x^2w.$
$\left(\frac{1}{2}\right)wx^2 = 10$

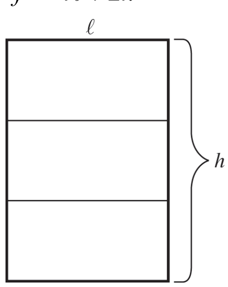
13. $\pi r^2h = 100$
$\text{Cost} = 5\pi r^2 + 6\pi r^2 + 7(2\pi rh)$
$= 11\pi r^2 + 14\pi rh$

14. $2\pi\left(\frac{h}{2}\right)^2 + 2\pi\left(\frac{h}{2}\right)h = \frac{\pi h^2}{2} + \pi h^2$
$= \frac{3\pi h^2}{2} = 30\pi$
$V = \pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{4}$

15. $2x + 3h = 5000$
$A = xh$



16. $\ell h = 2500$
$f = 4\ell + 2h$



17. $C = 10(2\ell + 2h) + 8(2\ell) = 36\ell + 20h$

18. $5x^2 + 4(4xh) = 5x^2 + 16xh = 150$

19. $8x = 40 \Rightarrow x = 5$
$A = 3x^2 = 3(25) = 75\ \text{cm}^2$

20. $V = 2\pi r^3 = 54\pi \Rightarrow r^3 = 27 \Rightarrow r = 3$
From exercise 14, we know that the surface area is equal to $6\pi r^2$. Thus, in this example $S = 6\pi(3^2) = 54\pi\ \text{in.}^2$

21. **a.** $73 + 4x = 225 \Rightarrow x = 38$
When 38 T-shirts are sold, the cost will be \$225.

b. $C(50) - C(40)$
$= (73 + 4(50)) - (73 + 4(40))$
$= 273 - 233 = \$40$
The cost will rise \$40.

22. **a.** $P(x) = 4x - C(x)$
$P(100) = 400 - (10 + 75) = \$315$

b. $P(101) = 404 - (10.1 + 75) = \$318.9$
Increase is \$3.90.

23. **a.** $.4x - 80 = 0 \Rightarrow x = \frac{80}{.4} = 200$
Sales will break-even when 200 scoops are sold.

b. $30 = .4x - 80 \Rightarrow x = 275$
Sales of 275 scoops will generate a daily profit of \$30.

c. $40 = .4x - 80 \Rightarrow x = 300$
To raise the daily profit to \$40, $300 - 275 = 25$ more scoops will have to be sold.

24. **a.** $160 = 12x - 200 \Rightarrow x = 30$
30 thousand subscribers are needed for a monthly profit of \$160 thousand

b. $166 = 12x - 200 \Rightarrow x = 30.5$ thousand
There will need to be 30,500 – 30,000 = 500 new subscribers.

25. **a.** $P(x) = R(x) - C(x) = 21x - 9x - 800$
$= 12x - 800$

b. $P(120) = 1440 - 800 = \$640$

c. $1000 = 12x - 800 \Rightarrow x = 150$
$R(150) = 21(150) = \$3150$

26. **a.** $P(x) = R(x) - C(x)$
$= 1200x - (550x + 6500)$
$= 650x - 6500$
$P(12) = 650(12) - 6500 = \$1300$
The company will earn \$1300.

b. $C(x) = 14,750 = 550x + 6500 \Rightarrow x = 15$
$P(15) = 650(15) - 6500 = \$3250$

27. $f(6) = 270$ cents

28. From the graph, $f(r) = 330$ for $r = 1$ and $r = 6.87$.

29. A 100-inch^3 cylinder with radius 3 inches costs $1.62 to construct.

30. The least expensive cylinder has radius 3 inches and costs $1.62 to construct.
The cost drops until the radius is 3 in. and then increases.

31. $f(3) = \$1.62$; $f(6) = \$2.70$, so the additional cost = 2.70 – 1.62 = $1.08

32. $f(1) = \$3.30$; $f(3) = \$1.62$, so the amount saved is 3.30 – 1.62 = $1.68

33. From the graph, we see that revenue = $1800 and cost = $1200.

34. The revenue is $1400 when production is 20 units.

35. The cost is $1400 when production is 40 units.

36. 1800 – 1200 = $600

37. $C(1000) = \$4000$

38. Find the x-coordinate of the point on the graph whose y-coordinate is 3500.

39. Find the y-coordinate of the point on the graph whose x-coordinate is 400.

40. $C(600) - C(500) = 3136 - 2875 = \$261$

41. The greatest profit, $52,500, occurs when 2500 units of goods are produced.

42. $P(1500) = \$42,500$

43. Find the x-coordinate of the point on the graph whose y-coordinate is 30,000.

44. Find the y-coordinate of the point on the graph whose x-coordinate is 2000.

45. Find $h(3)$. Find the y-coordinate of the point on the graph whose t-coordinate is 3.

46. Find t such that $h(t)$ is as large as possible. Find the t-coordinate of the highest point of the graph.

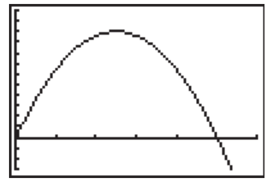
47. Find the maximum value of $h(t)$. Find the y-coordinate of the highest point of the graph.

48. Solve $h(t) = 0$. Find the t-intercept of the graph.

49. Solve $h(t) = 100$. Find the t-coordinates of the points whose y-coordinate is 100.

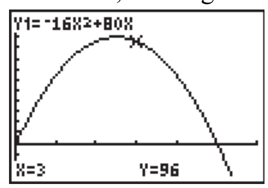
50. Find $h(0)$. Find the y-intercept of the graph.

51. **a.**

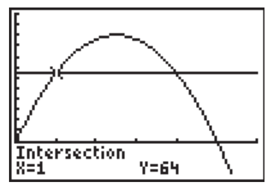


[0, 6] by [−30, 120]

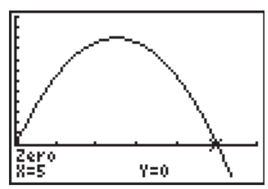
b. Using the Trace command or the Value command, the height is 96 feet.



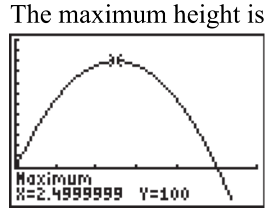
c. Graphing $Y_2 = 64$ and using the Intersect command, the height is 64 feet when $x = 1$ and $x = 4$ seconds.



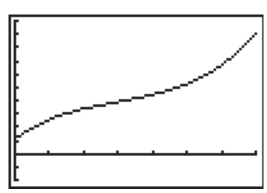
d. Using the Trace command or the Zero command, the ball hits the ground when $x = 5$ seconds.



e. Using the Trace command or the Maximum command, the maximum height is reached when $x = 2.5$ seconds. The maximum height is 100 feet.

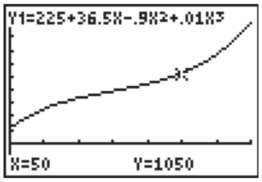


52. **a.**

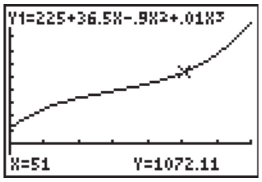


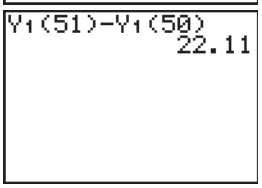
[0, 70] by [−400, 2000]

b. Using the Trace command or the Value command, the cost is \$1050.



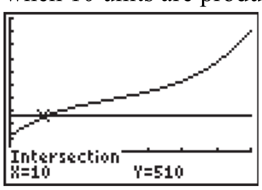
c.



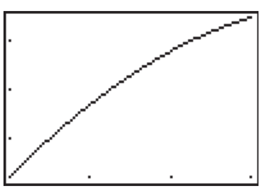


The additional cost is \$22.11.

d. Graphing $Y_2 = 510$ and using the Intersect command, the daily cost is \$510 when 10 units are produced.

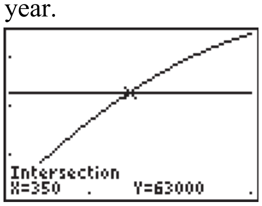


53. a.

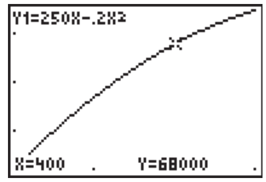


[200, 500] by [42000, 75000]

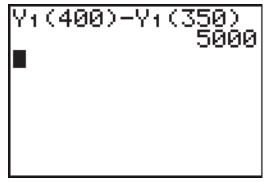
b. Graphing $Y_2 = 63{,}000$ and using the Intersect command, the revenue is \$63,000 when sales are 350 bicycles per year.



c. Using the Trace command or the Value command, the revenue is \$68,000 when 400 bicycles are sold per year.

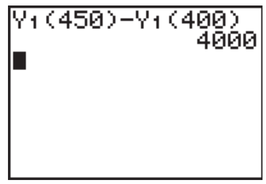


d.



$R(400) - R(350) = 5000$
Revenue would decrease by \$5000.

e.



$R(450) - R(400) = 4000$
No, the store should not spend \$5000 on advertising, since the revenues would only increase by \$4000.

Chapter 0 Fundamental Concept Check Exercises

1. Real numbers can be thought of as points on a number line, where each number corresponds to one point on the line, and each point determines one real number. Every real number has a decimal representation. A rational number is a real number with a finite or infinite repeating decimal, such as $-\frac{5}{2} = -2.5,\ 1,\ \frac{13}{3} = 4.33\overline{3}$. An irrational number is a real number with an infinite, non-repeating decimal representation, such as $-\sqrt{2} = -1.414213\ldots$ or $\pi = 3.14159\ldots$.

2. $x < y$ means *x* is less than *y*; $x \le y$ means *x* is less than or equal to *y*; $x > y$ means *x* is greater than *y*; $x \ge y$ means *x* is greater than or equal to *y*.

3. An open interval (a, b) does not contain its endpoints *a* and *b* but a closed interval $[a, b]$ does not contain *a* and *b*.

4. A function of a variable *x* is a rule *f* that assigns a unique number $f(x)$ to each value of *x*.

5. The value of a function at *x* is the unique number $f(x)$.

6. The domain of a function is the set of values that the independent variable x is allowed to assume. The range of a function is the set of values that the function assumes.

7. The graph of a function $f(x)$ is the curve that consists of the set of all points $(x, f(x))$ in the xy-plane. A curve is the graph of a function if and only if each vertical line cuts or touches the curve at no more than one point.

8. A linear function has the form $f(x) = mx + b$. When $m = 0$, the function is a constant function. $f(x) = 3x - .5$ is a linear function. $f = -2$ is a constant function.

9. An x-intercept is a point at which the graph of a function intersects the x-axis. A y-intercept is a point at which the graph intersects the y-axis. To find the x-intercept, set $f(x) = 0$ and solve for x, if possible. The y-intercept is the point $(0, f(0))$.

10. A quadratic function has the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. The graph is a parabola.

11. a. Quadratic function: $f(x) = ax^2 + bx + c$, where $a \neq 0$; $f(x) = -2x^2 + 4x + 9$

b. Polynomial function: $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$, where n is a nonnegative integer and $a_0, a_1, \ldots, a_n$ are real numbers, $a_n \neq 0$, and n is a nonnegative integer; $f(x) = x^5 + 3x^3 - 7x + 3$

c. Rational function: $h(x) = \dfrac{f(x)}{g(x)}$, where f and g are polynomials; $h(x) = \dfrac{2x-3}{x^2+1}$

d. Power function: $f(x) = x^r$, where r is a real number; $f(x) = \sqrt{x} = x^{1/2}$

12. $f(x) = |x|$ is defined as

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}.$$

13. Sum: $f(x) + g(x)$

Difference: $f(x) - g(x)$

Product: $f(x)g(x)$

Quotient: $\dfrac{f(x)}{g(x)}$

Composition: $f(g(x))$

If $f(x) = 3x^2$ and $g(x) = 3x + 1$, then

$$f(x) + g(x) = 3x^2 + 3x + 1$$
$$f(x) - g(x) = 3x^2 - (3x+1) = 3x^2 - 3x - 1$$
$$f(x)g(x) = 3x^2(3x+1) = 9x^3 + 3x^2$$
$$\frac{f(x)}{g(x)} = \frac{3x^2}{3x+1}$$
$$f(g(x)) = 3(3x+1)^2 = 3(9x^2 + 6x + 1)$$
$$= 27x^2 + 18x + 3$$

14. $x = a$ is a zero of $f(x)$ if $f(a) = 0$.

15. Two methods for finding the zeros of a quadratic function are using factoring or using the quadratic equation.

16. $b^r b^s = b^{r+s}$ $\quad b^{-r} = \dfrac{1}{b^r}$

$\dfrac{b^r}{b^s} = b^{r-s}$ $\quad (b^r)^s = b^{rs}$

$(ab)^r = a^r b^r$ $\quad \left(\dfrac{a}{b}\right)^r = \dfrac{a^r}{b^r}$

17. In the formula $A = P(1+i)^n$, A represents the compound amount, P represents the principal amount, i represents the interest rate, and n represents the number of interest periods.

18. To solve $f(x) = b$ geometrically from the graph of $y = f(x)$, draw the horizontal line $y = b$. The line intersects the graph at a point (a, b) if and only if $f(a) = b$. Thus, $x = a$ is a solution of $f(x) = b$.

19. To find $f(a)$ geometrically from the graph of $y = f(x)$, draw the vertical line $x = a$. This line intersects the graph at the point $(a, f(a))$.

Chapter 0 Review Exercises

1. $f(x) = x^3 + \frac{1}{x}$

$f(1) = 1^3 + \frac{1}{1} = 2$

$f(3) = 3^3 + \frac{1}{3} = \frac{82}{3} = 27\frac{1}{3}$

$f(-1) = (-1)^3 + \frac{1}{(-1)} = -2$

$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - 2 = -\frac{17}{8} = -2\frac{1}{8}$

$f\left(\sqrt{2}\right) = \left(\sqrt{2}\right)^3 + \frac{1}{\sqrt{2}} = 2\sqrt{2} + \frac{1}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

2. $f(x) = 2x + 3x^2$

$f(0) = 2(0) + 3(0)^2 = 0$

$f\left(-\frac{1}{4}\right) = 2\left(-\frac{1}{4}\right) + 3\left(-\frac{1}{4}\right)^2 = -\frac{5}{16}$

$f\left(\frac{1}{\sqrt{2}}\right) = 2\left(\frac{1}{\sqrt{2}}\right) + 3\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{3+2\sqrt{2}}{2}$

3. $f(x) = x^2 - 2$

$f(a-2) = (a-2)^2 - 2 = a^2 - 4a + 2$

4. $f(x) = \frac{1}{x+1} - x^2$

$f(a+1) = \frac{1}{(a+1)+1} - (a+1)^2$

$= \frac{1}{a+2} - (a+1)^2$

5. $f(x) = \frac{1}{x(x+3)} \Rightarrow x \neq 0, -3$

6. $f(x) = \sqrt{x-1} \Rightarrow x \geq 1$

7. $f(x) = \sqrt{x^2+1}$, all values of x

8. $f(x) = \frac{1}{\sqrt{3x}},\ x > 0$

9. $h(x) = \frac{x^2-1}{x^2+1}$

$h\left(\frac{1}{2}\right) = \frac{\left(\frac{1}{2}\right)^2 - 1}{\left(\frac{1}{2}\right)^2 + 1} = -\frac{3}{5}$

Yes, the point $\left(\frac{1}{2}, -\frac{3}{5}\right)$ is on the graph.

10. $k(x) = x^2 + \frac{2}{x}$

$k(1) = 1^2 + \frac{2}{1} = 3$

No, the point (1, –2) is not on the graph.

11. $5x^3 + 15x^2 - 20x = 5x(x^2 + 3x - 4)$

$= 5x(x-1)(x+4)$

12. $3x^2 - 3x - 60 = 3(x^2 - x - 20)$

$= 3(x-5)(x+4)$

13. $18 + 3x - x^2 = (-x-3)(x-6)$

$= (-1)(x-6)(x+3)$

14. $x^5 - x^4 - 2x^3 = x^3(x^2 - x - 2)$

$= x^3(x-2)(x+1)$

15. $y = 5x^2 - 3x - 2 \Rightarrow 5x^2 - 3x - 2 = 0.$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{3 \pm \sqrt{(-3)^2 - 4(5)(-2)}}{2(5)}$

$= \frac{3 \pm 7}{10} \Rightarrow x = 1 \text{ or } x = -\frac{2}{5}$

16. $y = -2x^2 - x + 2 \Rightarrow -2x^2 - x + 2 = 0.$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{1 \pm \sqrt{(-1)^2 - 4(-2)(2)}}{2(-2)}$

$= \frac{1 \pm \sqrt{17}}{-4} \Rightarrow x = \frac{-1+\sqrt{17}}{4} \text{ or } x = \frac{-1-\sqrt{17}}{4}$

17. Substitute $2x - 1$ for y in the quadratic equation, then find the zeros:

$5x^2 - 3x - 2 = 2x - 1 \Rightarrow 5x^2 - 5x - 1 = 0.$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{5 \pm \sqrt{(-5)^2 - 4(5)(-1)}}{2(5)}$

$= \frac{5 \pm 3\sqrt{5}}{10}$

Now find the y-values for each x value:

$y = 2x - 1 = 2\left(\frac{5+3\sqrt{5}}{10}\right) - 1 = \frac{3\sqrt{5}}{5}$

$y = 2x - 1 = 2\left(\frac{5-3\sqrt{5}}{10}\right) - 1 = \frac{-3\sqrt{5}}{5}$

Points of intersection:

$\left(\frac{5+3\sqrt{5}}{10}, \frac{3\sqrt{5}}{5}\right), \left(\frac{5-3\sqrt{5}}{10}, -\frac{3\sqrt{5}}{5}\right)$

18. Substitute $x - 5$ for y in the quadratic equation, then find the zeros:
$-x^2 + x + 1 = x - 5 \Rightarrow x^2 - 6 = 0 \Rightarrow x = \pm\sqrt{6}$
Now find the y-values for each x value:
$y = x - 5 = \sqrt{6} - 5$
$y = -\sqrt{6} - 5$
Points of intersection:
$\left(\sqrt{6}, \sqrt{6} - 5\right), \left(-\sqrt{6}, -\sqrt{6} - 5\right)$

19. $f(x) + g(x) = \left(x^2 - 2x\right) + (3x - 1) = x^2 + x - 1$

20. $f(x) - g(x) = \left(x^2 - 2x\right) - (3x - 1)$
$= x^2 - 5x + 1$

21. $f(x)h(x) = \left(x^2 - 2x\right)\left(\sqrt{x}\right)$
$= x^2 \cdot x^{1/2} - 2x \cdot x^{1/2}$
$= x^{5/2} - 2x^{3/2}$

22. $f(x)g(x) = (x^2 - 2x)(3x - 1)$
$= 3x^3 - x^2 - 6x^2 + 2x$
$= 3x^3 - 7x^2 + 2x$

23. $\dfrac{f(x)}{h(x)} = \dfrac{x^2 - 2x}{\sqrt{x}} = x^{3/2} - 2x^{1/2}$

24. $g(x)h(x) = (3x - 1)\sqrt{x} = 3x \cdot x^{1/2} - x^{1/2}$
$= 3x^{3/2} - x^{1/2}$

25. $f(x) - g(x) = \dfrac{x}{x^2 - 1} - \dfrac{1 - x}{1 + x}$
$= \dfrac{x - (x - 1)(1 - x)}{x^2 - 1}$
$= \dfrac{x^2 - x + 1}{x^2 - 1} = \dfrac{x^2 - x + 1}{(x - 1)(x + 1)}$

26. $f(x) - g(x + 1) = \dfrac{x}{x^2 - 1} - \dfrac{1 - (x + 1)}{1 + (x + 1)}$
$= \dfrac{x(x + 2) - (-x)\left(x^2 - 1\right)}{\left(x^2 - 1\right)(x + 2)}$
$= \dfrac{x^3 + x^2 + x}{\left(x^2 - 1\right)(x + 2)}$

27. $g(x) - h(x) = \dfrac{1 - x}{1 + x} - \dfrac{2}{3x + 1}$
$= \dfrac{(1 - x)(3x + 1) - 2(1 + x)}{(1 + x)(3x + 1)}$
$= -\dfrac{3x^2 + 1}{(1 + x)(3x + 1)}$
$= -\dfrac{3x^2 + 1}{3x^2 + 4x + 1}$

28. $f(x) + h(x) = \dfrac{x}{x^2 - 1} + \dfrac{2}{3x + 1}$
$= \dfrac{x(3x + 1) + 2\left(x^2 - 1\right)}{\left(x^2 - 1\right)(3x + 1)}$
$= \dfrac{5x^2 + x - 2}{\left(x^2 - 1\right)(3x + 1)}$

29. $g(x) - h(x - 3) = \dfrac{1 - x}{1 + x} - \dfrac{2}{3(x - 3) + 1}$
$= \dfrac{(1 - x)(3x - 8) - 2(1 + x)}{(1 + x)(3x - 8)}$
$= \dfrac{-3x^2 + 9x - 10}{(1 + x)(3x - 8)}$
$= \dfrac{-3x^2 + 9x - 10}{3x^2 - 5x - 8}$

30. $f(x) + g(x) = \dfrac{x}{x^2 - 1} + \dfrac{1 - x}{1 + x} = \dfrac{x + (1 - x)(x - 1)}{x^2 - 1}$
$= \dfrac{-x^2 + 3x - 1}{x^2 - 1}$

For exercises 31–36, $f(x) = x^2 - 2x + 4$, $g(x) = \dfrac{1}{x^2}$ and $h(x) = \dfrac{1}{\sqrt{x} - 1}$.

31. $f(g(x)) = f\left(\dfrac{1}{x^2}\right) = \left(\dfrac{1}{x^2}\right)^2 - 2\left(\dfrac{1}{x^2}\right) + 4$
$= \dfrac{1}{x^4} - \dfrac{2}{x^2} + 4$

32. $g(f(x)) = g(x^2 - 2x + 4) = \dfrac{1}{\left(x^2 - 2x + 4\right)^2}$

33. $g(h(x)) = g\left(\dfrac{1}{\sqrt{x} - 1}\right) = \dfrac{1}{\left(\frac{1}{\sqrt{x}-1}\right)^2} = \dfrac{1}{\frac{1}{x - 2\sqrt{x} + 1}}$
$= x - 2\sqrt{x} + 1 = \left(\sqrt{x} - 1\right)^2$

34. $h(g(x)) = h\left(\frac{1}{x^2}\right) = \frac{1}{\sqrt{\frac{1}{x^2}} - 1} = \frac{1}{\frac{1}{|x|} - 1} = \frac{|x|}{1-|x|}$

35. $f(h(x)) = f\left(\frac{1}{\sqrt{x}-1}\right)$

$= \left(\frac{1}{\sqrt{x}-1}\right)^2 - 2\left(\frac{1}{\sqrt{x}-1}\right) + 4$

$= \frac{1}{\left(\sqrt{x}-1\right)^2} - \frac{2}{\sqrt{x}-1} + 4$

36. $h(f(x)) = h\left(x^2 - 2x + 4\right)$

$= \frac{1}{\sqrt{x^2 - 2x + 4} - 1}$

$= \left(\sqrt{x^2 - 2x + 4} - 1\right)^{-1}$

37. $(81)^{3/4} = \left(\sqrt[4]{81}\right)^3 = 27$

$8^{5/3} = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$

$(0.25)^{-1} = \left(\frac{1}{4}\right)^{-1} = 4$

38. $(100)^{3/2} = \left(\sqrt{100}\right)^3 = 1000$

$(.001)^{1/3} = \left(\sqrt[3]{.001}\right) = .1$

39. $C(x)$ = carbon monoxide level corresponding to population x

$P(t)$ = population of the city in t years

$C(x) = 1 + .4x$

$P(t) = 750 + 25t + .1t^2$

$C(P(t)) = 1 + .4\left(750 + 25t + .1t^2\right)$

$= 1 + 300 + 10t + .04t^2$

$= .04t^2 + 10t + 301$

40. $R(x) = 5x - x^2$

$f(d) = 6\left(1 - \frac{200}{d+200}\right)$

$R(f(d)) = 5 \cdot 6\left(1 - \frac{200}{d+200}\right) - \left[6\left(1 - \frac{200}{d+200}\right)\right]^2$

$= 30\left(1 - \frac{200}{d+200}\right) - 36\left(1 - \frac{200}{d+200}\right)^2$

41. $\left(\sqrt{x+1}\right)^4 = (x+1)^{4/2} = (x+1)^2 = x^2 + 2x + 1$

42. $\frac{xy^3}{x^{-5}y^6} = x \cdot x^5 \cdot y^3 \cdot y^{-6} = \frac{x^6}{y^3}$

43. $\frac{x^{3/2}}{\sqrt{x}} = x^{3/2} \cdot x^{-1/2} = x$

44. $\sqrt[3]{x}(8x^{2/3}) = x^{1/3} \cdot 8x^{2/3} = 8x$

45. a. $P = 15000,\ r = .04,\ m = 12$

$A(t) = 15000\left(1 + \frac{.04}{12}\right)^{12t}$

$= 15000(1.00333)^{12t}$

b. $A(2) = 15000\left(1 + \frac{.04}{12}\right)^{12 \cdot 2} \approx 16247.14$

$A(5) = 15000\left(1 + \frac{.04}{12}\right)^{12 \cdot 5} \approx 18314.94$

At the end of 2 years, the account balance is about \$16,247. At the end of 5 years, the account balance is about \$18,315.

46. a. $P = 7000,\ r = .09,\ m = 2$

$A(t) = 7000\left(1 + \frac{.09}{2}\right)^{2t} = 7000(1.045)^{2t}$

b. $A(10) = 7000(1.045)^{2 \cdot 10} = 16882.00$

$A(20) = 7000(1.045)^{2 \cdot 20} = 40714.55$

At the end of 10 years, the account balance is about \$16,882. At the end of 20 years, the account balance is about \$40,715.

47. a. $P = 15000,\ m = 1,\ t = 10$

$A(r) = 15000(1 + r)^{10}$

b. $A(.04) = 15000(1 + .04)^{10} = 22203.66$

$A(.06) = 15000(1 + .06)^{10} = 26862.72$

48. a. $P = 7000,\ m = 1,\ t = 20$

$A(r) = 7000(1 + r)^{20}$

b. $A(.07) = 7000(1 + .07)^{20} = 27087.79$

$A(.12) = 7000(1 + .12)^{20} = 67524.05$

Chapter 1 The Derivative

1.1 The Slope of a Straight Line

1. $y = 3 - 7x$; y-intercept: (0, 3), slope: −7

2. $y = \frac{3x+1}{5} = \frac{3}{5}x + \frac{1}{5}$; y-intercept: $\left(0, \frac{1}{5}\right)$, slope: $\frac{3}{5}$

3. $x = 2y - 3 \Rightarrow y = \frac{x+3}{2} \Rightarrow y = \frac{1}{2}x + \frac{3}{2}$; y-intercept: $\left(0, \frac{3}{2}\right)$, slope: $\frac{1}{2}$

4. $y = 6 \Rightarrow y = 0x + 6$; y-intercept: (0, 6), slope: 0

5. $y = \frac{x}{7} - 5 \Rightarrow y = \frac{1}{7}x - 5$; y-intercept: (0, −5), slope: $\frac{1}{7}$

6. $4x + 9y = -1 \Rightarrow y = \frac{-4x-1}{9} \Rightarrow y = -\frac{4}{9}x - \frac{1}{9}$; y-intercept: $\left(0, -\frac{1}{9}\right)$, slope $= -\frac{4}{9}$

7. slope = –1, (7, 1) on line.
Let $(x, y) = (7, 1)$, $m = -1$.
$y - y_1 = m(x - x_1) \Rightarrow y - 1 = -(x - 7) \Rightarrow$
$y = -x + 8$

8. slope = 2; (1, −2) on line.
Let $(x, y) = (1, -2)$, $m = 2$.
$y - y_1 = m(x - x_1) \Rightarrow y + 2 = 2(x - 1) \Rightarrow$
$y = 2x - 4$

9. slope $= \frac{1}{2}$; (2, 1) on line.
Let $(x_1, y_1) = (2, 1)$; $m = \frac{1}{2}$.
$y - y_1 = m(x - x_1) \Rightarrow y - 1 = \frac{1}{2}(x - 2) \Rightarrow$
$y = \frac{1}{2}x$

10. slope $= \frac{7}{3}$; $\left(\frac{1}{4}, -\frac{2}{5}\right)$ on line.
Let $(x_1, y_1) = \left(\frac{1}{4}, -\frac{2}{5}\right)$; $m = \frac{7}{3}$.
$y - y_1 = m(x - x_1) \Rightarrow y + \frac{2}{5} = \frac{7}{3}\left(x - \frac{1}{4}\right) \Rightarrow$
$y = \frac{7}{3}x - \frac{59}{60}$

11. $\left(\frac{5}{7}, 5\right)$ and $\left(-\frac{5}{7}, -4\right)$ on line.
slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-5}{-\frac{5}{7} - \frac{5}{7}} = \frac{-9}{-\frac{10}{7}} = \frac{63}{10}$
Let $(x_1, y_1) = \left(\frac{5}{7}, 5\right)$, $m = \frac{63}{10}$.
$y - y_1 = m(x - x_1) \Rightarrow y - 5 = \frac{63}{10}\left(x - \frac{5}{7}\right)$

12. $\left(\frac{1}{2}, 1\right)$ and (1, 4) on line.
slope$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{1 - \frac{1}{2}} = \frac{3}{\frac{1}{2}} = 6$
Let $(x_1, y_1) = (1, 4)$, $m = 6$.
$y - y_1 = m(x - x_1) \Rightarrow y - 4 = 6(x - 1) \Rightarrow$
$y = 6x - 2$

13. (0, 0) and (1, 0) on line.
slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-0}{1-0} = 0$
$y - 0 = 0(x - 0) \Rightarrow y = 0$

14. $\left(-\frac{1}{2}, -\frac{1}{7}\right)$ and $\left(\frac{2}{3}, 1\right)$ on line.
slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - \left(-\frac{1}{7}\right)}{\frac{2}{3} - \left(-\frac{1}{2}\right)} = \frac{\frac{8}{7}}{\frac{7}{6}} = \frac{48}{49} = m$
Let $(x_1, y_1) = \left(\frac{2}{3}, 1\right)$.
$y - y_1 = m(x - x_1) \Rightarrow y - 1 = \frac{48}{49}\left(x - \frac{2}{3}\right) \Rightarrow$
$y = \frac{48}{49}x + \frac{17}{49}$

15. Horizontal through (2, 9).
Let $(x_1, y_1) = (2, 9)$, $m = 0$ (horizontal line).
$y - y_1 = m(x - x_1) \Rightarrow y - 9 = 0(x - 2) \Rightarrow$
$y = 9$

16. x-intercept is 1; y-intercept is –3.
The intercepts (1, 0) and (0, –3) are on the line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-0}{0-1} = 3 = m$$

y-intercept $(0, b) = (0, -3)$

$y = mx + b \Rightarrow y = 3x - 3$

17. x-intercept is $-\pi$; y-intercept is 1.
The intercepts $(-\pi, 0)$ and (0, 1) are on the line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{0-(-\pi)} = \frac{1}{\pi}$$

y-intercept $(0, b) = (0, 1)$

$$y = mx + b \Rightarrow y = \frac{x}{\pi} + 1$$

18. Slope = 2; x-intercept is –3.
The x-intercept (–3, 0) is on the line.
Let $(x_1, y_1) = (-3, 0)$, $m = 2$.

$y - y_1 = m(x - x_1) \Rightarrow y - 0 = 2(x + 3) \Rightarrow$
$y = 2x + 6$

19. Slope = –2; x-intercept is –2.
The x-intercept (–2, 0) is on the line.
Let $(x_1, y_1) = (-2, 0)$, $m = -2$.

$y - y_1 = m(x - x_1) \Rightarrow y - 0 = -2(x + 2) \Rightarrow$
$y = -2x - 4$

20. Horizontal through $(\sqrt{7}, 2)$.

Let $(x_1, y_1) = (\sqrt{7}, 2)$, $m = 0$ (horizontal line).

$y - y_1 = m(x - x_1) \Rightarrow y - 2 = 0(x - \sqrt{7}) \Rightarrow$
$y = 2$

21. Parallel to $y = x$; (2, 0) on line.
Let $(x_1, y_1) = (2, 0)$; slope = m = 1.

$y - y_1 = m(x - x_1) \Rightarrow y - 0 = 1(x - 2) \Rightarrow$
$y = x - 2$

22. Parallel to $x + 2y = 0$; (1, 2) on line.

$$x + 2y = 0 \Rightarrow y = -\frac{1}{2}x;\ m = -\frac{1}{2}$$

Let $(x_1, y_1) = (1, 2)$.

$$y - y_1 = m(x - x_1) \Rightarrow y - 2 = -\frac{1}{2}(x - 1) \Rightarrow$$

$$y = -\frac{1}{2}x + \frac{5}{2}$$

23. Parallel to $y = 3x + 7$; x-intercept is 2.

slope = m = 3. Let $(x_1, y_1) = (2, 0)$.

$y - y_1 = m(x - x_1) \Rightarrow y - 0 = 3(x - 2) \Rightarrow$
$y = 3x - 6$

24. Parallel to $y - x = 13$; y-intercept is 0.

$y = x + 13$, slope = m = 1, b = 0.

$y = mx + b \Rightarrow y = x$

25. Perpendicular to $y + x = 0$; (2, 0) on line.

$y + x = 0 \Rightarrow y = -x \Rightarrow \text{slope} = m_1 = -1$
$m_1 \cdot m_2 = -1 \Rightarrow -1 \cdot m_2 = -1 \Rightarrow m_2 = 1$
Let $(x_2, y_2) = (2, 0)$.

$y - y_2 = m_2(x - x_2) \Rightarrow y - 0 = x - 2 \Rightarrow$
$y = x - 2$

26. Perpendicular to $y = -5x + 1$; (1, 5) on line.

slope = $m_1 = -5$

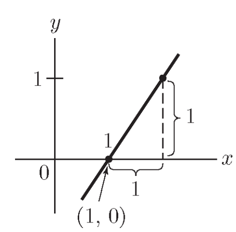
$$m_1 \cdot m_2 = -1 \Rightarrow -5m_2 = -1 \Rightarrow m_2 = \frac{1}{5}$$

Let $(x_2, y_2) = (1, 5)$.

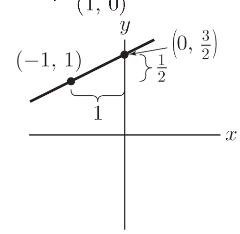
$$y - y_2 = m_2(x - x_2) \Rightarrow y - 5 = \frac{1}{5}(x - 1) \Rightarrow$$

$$y = \frac{1}{5}x + \frac{24}{5}$$

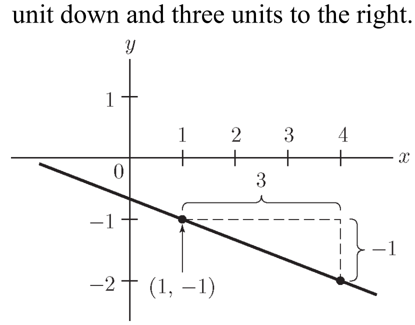
27. Start at (1, 0), then move one unit right and one unit up to (2, 1).



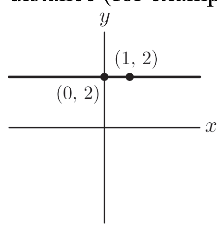
28. Start at (−1, 1), then move one unit up and two units to the right.



29. Start at (1, −1), then move one unit up and three units to the left. Alternatively, move one unit down and three units to the right.



30. Start at (0, 2), then move zero units up and any distance (for example, one unit) right.



31. (a)–(C) x- and y-intercepts are 1.
(b)–(B) x-intercept is 1, y-intercept is –1.
(c)–(D) x- and y-intercepts are –1.
(d)–(A) x-intercept is –1, y-intercept is 1.

32. $x+2y=0 \Rightarrow y=-\frac{1}{2}x \Rightarrow m=-\frac{1}{2}$

The slope of the line through (−1, 2) and (3, b) is also $-\frac{1}{2}$..

$m=-\frac{1}{2}=\frac{b-2}{3-(-1)} \Rightarrow -4=2b-4 \Rightarrow b=0$

33. $m=\frac{1}{3},\ h=3$

If you move 3 units in the x-direction, then you must move 1 unit in the y-direction to return to the line.

34. $m=2,\ h=\frac{1}{2}$

If you move $\frac{1}{2}$ unit in the x-direction, then you must move $\frac{1}{2}\cdot 2=1$ unit in the y-direction.

35. $m=-3,\ h=.25$
If you move .25 unit in the x-direction, then you must move $-3\cdot .25=-.75$ unit in the y-direction.

36. $m=\frac{2}{3},\ h=\frac{1}{2}$

If you move $\frac{1}{2}$ unit in the x-direction, then you must move $\frac{1}{2}\cdot\frac{2}{3}=\frac{1}{3}$ unit in the y-direction.

37. Slope = 2, (1, 3) on line.
$x_1=1,\ y_1=3$
If $x=2$, then $y-3=2(2-1) \Rightarrow y=5$.
If $x=3$, then $y-3=2(3-1) \Rightarrow y=7$.
If $x=0$, then $y-3=2(0-1) \Rightarrow y=1$.
The points are (2, 5), (3, 7), and (0, 1).

38. Slope = –3, (2, 2) on line.
$x_1=2,\ y_1=2$
If $x=3$, then $y-2=-3(3-2) \Rightarrow y=-1$.
If $x=4$, then $y-2=-3(4-2) \Rightarrow y=-4$.
If $x=1$, then $y-2=-3(1-2) \Rightarrow y=5$.
The points are (3, −1), (4, −4), and (1, 5).

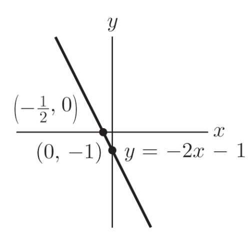
39. $f(1)=0 \Rightarrow (1,0)$ lies on the line.
$f(2)=1 \Rightarrow (2,1)$ lies on the line. Thus, the slope of the line is $\frac{1-0}{2-1}=1$. If $x=3$ and $y=f(3)$, then $1=\frac{y-1}{3-2} \Rightarrow 1=y-1 \Rightarrow y=2$.
Thus $f(3)=2$.

40. First find the slope of $2x+3y=0$.
$2x+3y=0 \Rightarrow 3y=-2x \Rightarrow y=-\frac{2}{3}x \Rightarrow$
$m_1=-\frac{2}{3}$
Now find the slope of the line through (3, 4) and (–1, 2).
$m_2=\frac{y_2-y_1}{x_2-x_1}=\frac{2-4}{-1-3}=\frac{-2}{-4}=\frac{1}{2}$
Since the slopes are not equal, the lines are not parallel.

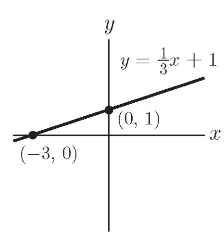
41. l_1

42. l_2

43. Slope = m = –2
y-intercept: (0, –1)
$y=mx+b$
$y=-2x-1$



44. Slope = $m=\frac{1}{3}$
y-intercept: (0, 1)
$y=mx+b$
$y=\frac{1}{3}x+1$



45. a is the x-coordinate of the point of intersection of $y=-x+4$ and $y=2$. Use substitution to find the x-coordinate.
$2=-x+4 \Rightarrow x=2$
So $a=2$. $f(a)$ is the y-coordinate of the intersection point. So $f(a)=2$.

46. a is the x-coordinate of the point of intersection of $y = x$ and $y = \frac{1}{2}x + 1$. Use substitution to find the x-coordinate.
$x = \frac{1}{2}x + 1 \Rightarrow \frac{1}{2}x = 1 \Rightarrow x = 2$
So $a = 2$. $f(a)$ is the y-coordinate of the intersection point. Substituting $x = 2$ into $y = x$ gives $y = 2$. So $f(a) = 2$.

47. $C(x) = 12x + 1100$

a. $C(10) = 12(10) + 1100 = \$1220$

b. The marginal cost is the slope of line. Marginal cost = m = \$12/unit

c. It would cost an additional \$12 to raise the daily production level from 10 units to 11 units.

48. $C(x+1) - C(x)$
$= (12(x+1) + 1100) - (12x + 1100)$
$= 12x + 12 + 1100 - 12x - 1100$
$= 12$
\$12 is the marginal cost. It is the additional cost incurred when the production level of this commodity is increased one unit, from x to $x + 1$, per day.

49. Let x be the number of months since January 1, 2020. Then (0, 3.19) is one point on the line. The slope is –.04 since the price fell \$.04 per month. Therefore, $P(x) = -.04x + 3.19$ gives the price of gasoline x months after January 1, 2020. On April 1, 2020, 3 months later, the cost of one gallon of gasoline is:
$P(3) = -.04(3) + 3.19 = \$3.07 / \text{gallon}$. So, 15 gallons cost $15 \cdot 3.07 = \$46.05$.
On September 1, 2020, 8 months after January 1, the cost of one gallon of gasoline is:
$P(8) = -.04(8) + 3.19 = \$2.87 / \text{gallon}$. So, 15 gallons cost $15 \cdot 2.87 = \$43.05$.

50. Let y be the value of monthly exports in millions of dollars. Let x be the number of months since Sept 1, 2003. Since the rate of change of y is constant, we conclude that y is a linear function of x whose slope is equal to its rate of change, $m = 42.5$. On September 1 (when $x = 0$), the value of monthly exports was 0 dollars, since the ban had just ended. So the point (0, 0) is on the graph of y. Using the point-slope form, we have
$y - 0 = 42.5(x - 0) \Rightarrow y = 42.5x$.
The end of December 2003 corresponds to $x = 4$, at which time the exports had reached the value $y = 42.5 \cdot 4 = 170$ million dollars

51. Let x = the cost of order. Then $C(x) = .03x + 5$.

52. a. The points (7.25, .2) and (8, .18) are on the line. The slope of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{.18 - .2}{8 - 7.25} = -\frac{2}{75}$$

Let $(x_1, y_1) = (8, .18)$. Then, the equation of the line is

$$y - y_1 = m(x - x_1)$$
$$y - .18 = -\frac{2}{75}(x - 8)$$
$$y = -\frac{2}{75}x + \frac{59}{150}$$

Thus, $Q(x) = -\frac{2}{75}x + \frac{59}{150}$.

b. Let $Q(x) = .1$ (10 employees per 100) and solve for x.

$$.1 = -\frac{2}{75}x + \frac{59}{150}$$
$$-\frac{22}{75} = -\frac{2}{75}x \Rightarrow x = 11$$

The hourly wage should be \$11 in order for the quit ratio to drop to 10 employees per 100.

53. The points (3.10, 1500) and (3.25, 1250) are on the line. The slope of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1500 - 1250}{3.10 - 3.25} = -\frac{5000}{3}.$$

Let $(x_1, y_1) = (3.10, 1500)$. The equation of the line is

$$y - y_1 = m(x - x_1)$$
$$y - 1500 = -\frac{5000}{3}(x - 3.10)$$
$$y = -\frac{5000}{3}x + \frac{20,000}{3}$$
$$G(x) = -\frac{5000}{3}x + \frac{20,000}{3}$$

Now find $G(3.34)$:

$$G(3.34) = -\frac{5000}{3}(3.34) + \frac{20,000}{3} = 1100$$

gallons.

54. Solve for x:

$$G(x) = 2200 = -\frac{5000}{3}x + \frac{20,000}{3}$$

$$x = \left(-\frac{13,400}{3}\right)\left(-\frac{3}{5000}\right) = 2.68$$

The owner should set the price at \$2.68 in order to sell 2200 gallons per day.

55. a. $C(x) = mx + b$

$b = \$1500$ (fixed costs)

Total cost of producing 100 rods is \$2200.

$C(100) = m(100) + 1500 = \$2200 \Rightarrow m = 7$

Thus, $C(x) = 7x + 1500.$

b. Marginal cost at $x = 100$ is $m = \$7$/rod

c. Since the marginal cost = \$7, the cost of raising the daily production level form 100 to 101 rods is \$7. Alternatively, $C(101) - C(100) = 2207 - 2200 = \$7.$

56. Each unit sold increases the pay by 5 dollars. Thus, the slope is her pay per unit sold. The weekly pay is 60 dollars if no units are sold. Thus, the y-intercept is her base pay.

57. If the monopolist wants to sell one more unit of goods, then the price per unit must be lowered by 2 cents. No one will pay 7 dollars or more for a unit of goods.

58. x = degrees Fahrenheit, y = degrees Celsius, so the points (32, 0) and (212, 100) lie on the line.

$$m = \frac{100-0}{212-32} = \frac{100}{180} = \frac{5}{9}$$

Now find b:

$$y = mx + b \Rightarrow 32 = \frac{5}{9}(0) + b \Rightarrow b = -\frac{160}{9}.$$

Thus, $y = \frac{9}{5}x + 32.$ $y = \frac{5}{9}(98.6) - \frac{160}{9} = 37$

98.6°F corresponds to 37°C.

59. The point (0, 1.5) is on the line and the slope is 6 (ml/min). Let y be the amount of drug in the body x minutes from the start of the infusion. Then $y - 1.5 = 6(x - 0) \Rightarrow y = 6x + 1.5.$

60. Eliminating 2 ml/hour means that the rate is $-\frac{1}{30}$ ml/min. (The rate given in exercise 59 is in ml/min.) $y = 6x + 1.5 - \frac{1}{30}x = \frac{179}{30}x + 1.5$

61. The diver starts at a depth of 212 ft, which is represented as −212. Thus, the function is $y(t) = 2t - 212.$

62. First we must determine how long it will take the diver to reach 150 feet depth.

$-150 = 2t - 212 \Rightarrow 62 = 2t \Rightarrow t = 31$ sec

The diver must then rest for 5 minutes or $5 \cdot 60 = 300$ sec, which is 331 sec after she started ascending. The remaining depth can be determined by $y = 2(t - 331) - 150 = 2t - 812.$

Thus, the function giving depth as a function of time is

$$y(t) = \begin{cases} 2t - 212 & 0 \le t \le 31 \\ -150 & 31 \le t \le 331 \\ 2t - 812 & t \ge 331 \end{cases}$$

The first 62 ft will take the diver 31 sec to ascend. To determine how long it will take the diver to ascend final 150 ft, solve $150 = 2t \Rightarrow t = 75$ sec. Therefore, it will take the diver 31 + 300 + 75 = 406 sec to reach the surface.

63. a. $C(x) = 7x + 230$

b. $R(x) = 12x$

64. $C(x) = R(x) \Rightarrow$

$7x + 230 = 12x \Rightarrow 230 = 5x \Rightarrow x = 46$

The business will break even when 46 t-shirts are sold.

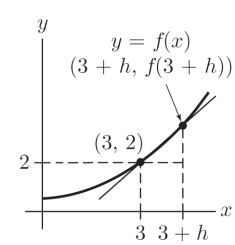
65. Using $\frac{f(x_2) - f(x_1)}{x_2 - x_1} = m$ and the hint,

$$\frac{f(x) - f(x_1)}{x - x_1} = m \Rightarrow f(x) - f(x_1) = m(x - x_1)$$

$$f(x) = m(x - x_1) + f(x_1)$$
$$= mx + (-mx_1 + f(x_1))$$

Let $b = -mx_1 + f(x_1)$. Then $f(x) = mx + b$.

66. a–c.



d. $\frac{f(3+h) - f(3)}{3 + h - 3} = \frac{f(3+h) - 2}{h}$

67. a. The points (0, 54) and (36, 66) lie on the line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{66 - 54}{36 - 0} = \frac{1}{3}$$

$$y - 54 = \frac{1}{3}(x - 0) \Rightarrow y = \frac{1}{3}x + 54$$

b. Every year since 2014, $\frac{1}{3}\% = .33\%$ more of the world population becomes urban.

c. The year 2020 is represented by $x = 6$.

$$f(6) = \frac{1}{3}(6) + 54 = 56$$

Thus, in 2020, 56% of the world's population will be urban.

d. $72 = \frac{1}{3}x + 54 \Rightarrow 18 = \frac{1}{3}x \Rightarrow x = 54$

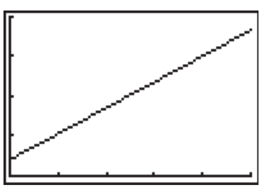
72% of the world's population will be urban 54 years after 2014, or in 2068.

68. a. (20,000, 729) and (50,000, 1380) on line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1380 - 729}{50,000 - 20,000} = \frac{651}{30,000} = .0217$$

$$y - 1380 = .0217(x - 50,000) \Rightarrow$$
$$y = .0217x + 295$$

b.



[0, 75000] by [0, 2000]

c. For every increase of $1 in reported income, the average itemized deductions increase by $.0217. (Alternatively, an increase of $100 in reported income corresponds to an average increase of $2.17 in itemized deductions.)

d. Using the **TRACE** or **VALUE** feature on a graphing calculator, the point (75,000, 1992.5) is on the line. Thus, the average amount of itemized deductions on a return reporting income of $75,000 is $1922.50.

e. Graphing the line $y = 5000$ and using the **INTERSECT** command, the point ($60,138.25, 1600) is on both lines. An average itemized deduction of $1600 corresponds to a reported income of $60,138.25.

f. An increase of $15,000 in income level will correspond to an increase of $15,000(.0217) = $325.50 in itemized deductions.

1.2 The Slope of a Curve at a Point

1. $-\frac{4}{3}$

2. 0

3. 1

4. 1

5. 1

6. $\frac{1}{2}$

7. –2

8. $-\frac{1}{3}$

9. Small positive slope; large positive slope

10. Zero slope; large negative slope

11. Zero slope; small negative slope

12. Let $m_P =$ slope at point *P*. Then, $m_A = 1, m_B = 8,$

$m_C = 0,\ m_D = -6,\ m_E = 0,\ m_F = -\frac{1}{2}.$

For 13–24, note that the slope of the line tangent to the graph of $y = x^2$ at the point (*x*, *y*) is 2*x*.

13. The slope at (–.4, .16) is 2(–.4) = –.8. Let $(x_1, y_1) = (-.4, .16),\ m = -.8.$

$$y - .16 = -.8(x - (-.4))$$
$$y - .16 = -.8(x + .4)$$
$$y = -.8x - .16$$

14. The slope at (–2, 4) is 2*x* = 2(–2) = –4. Let $(x_1, y_1) = (-2, 4),\ m = -4$.

$$y - 4 = -4(x - (-2)) \Rightarrow y - 4 = -4(x + 2) \Rightarrow$$
$$y = -4x - 4$$

15. The slope at $\left(\frac{1}{3}, \frac{1}{9}\right)$ is $m = 2x = 2\left(\frac{1}{3}\right) = \frac{2}{3}$.

Let $(x_1, y_1) = \left(\frac{1}{3}, \frac{1}{9}\right)$.

$y - \frac{1}{9} = \frac{2}{3}\left(x - \frac{1}{3}\right) \Rightarrow y - \frac{1}{9} = \frac{2}{3}x - \frac{2}{9} \Rightarrow$

$y = \frac{2}{3}x - \frac{1}{9}$

16. The slope at (–1.5, 2.25) is $2x = 2(-1.5) = -3$.
Let $(x_1, y_1) = (-1.5, 2.25)$, $m = -3$.

$y - 2.25 = -3(x - (-1.5)) \Rightarrow$

$y - 2.25 = -3(x + 1.5) \Rightarrow y = -3x - 2.25$

17. When $x = -\frac{1}{4}$, slope $= 2\left(-\frac{1}{4}\right) = -\frac{1}{2}$.

18. When $x = -.2$, slope $= 2(-.2) = -.4$.

19. When $x = 2.5$, slope $= 2(2.5) = 5$ and $y = (2.5)^2 = 6.25$. Let $(x_1, y_1) = (2.5, 6.25)$, $m = 5$.

$y - 6.25 = 5(x - 2.5) \Rightarrow y - 6.25 = 5x - 12.5 \Rightarrow$

$y = 5x - 6.25$

20. When $x = 2.1$, slope $= 2(2.1) = 4.2$ and $y = (2.1)^2 = 4.41$. Let $(x_1, y_1) = (2.1, 4.41)$, $m = 4.2$.

$y - 4.41 = 4.2(x - 2.1) \Rightarrow$

$y - 4.41 = 4.2x - 8.82 \Rightarrow y = 4.2x - 4.41$

21. The slope of the tangent is $2x$, so solve $2x = \frac{7}{2} \Rightarrow x = \frac{7}{4}$. The point is

$\left(\frac{7}{4}, \left(\frac{7}{4}\right)^2\right) = \left(\frac{7}{4}, \frac{49}{16}\right)$.

22. The slope of the tangent is $2x$, so solve $2x = -6 \Rightarrow x = -3$. The point is

$\left(-3, (-3)^2\right) = (-3, 9)$.

23. The slope of the line $2x + 3y = 4$ is $-\frac{2}{3}$, so the slope of the tangent line is also $-\frac{2}{3}$. Now solve $2x = -\frac{2}{3} \Rightarrow x = -\frac{1}{3}$. The point is

$\left(-\frac{1}{3}, \left(-\frac{1}{3}\right)^2\right) = \left(-\frac{1}{3}, \frac{1}{9}\right)$.

24. The slope of the line $3x - 2y = 2$ is $\frac{3}{2}$, so the slope of the tangent line is also $\frac{3}{2}$. Now solve $2x = \frac{3}{2} \Rightarrow x = \frac{3}{4}$. The point is

$\left(\frac{3}{4}, \left(\frac{3}{4}\right)^2\right) = \left(\frac{3}{4}, \frac{9}{16}\right)$.

25. March 1, 2020: about \$52.00
January 1, 2021: about \$27.00
The price decreased about \$25.00
The price was rising on both days.

26. Yes, the slope of the graph on these days are almost equal.

27. The price of a barrel of oil was about \$27.25. It was rising at a rate of about $\frac{27.50 - 27.25}{5} \approx \$.05$ per day.

28. The price of a barrel of oil was about \$27 on January 12, 2015. The slope at that point is about 0, so the price was holding steady.

For 29–31, note that the slope of the line tangent to the graph of $y = x^3$ at the point (x, y) is $3x^2$.

29. Slope $= 3x^2$

When $x = 2$, slope $= 3(2)^2 = 12$.

30. Slope $= 3x^2$

When $x = \frac{3}{2}$, slope $= 3\left(\frac{3}{2}\right)^2 = \frac{27}{4}$.

31. Slope $= 3x^2$

When $x = -\frac{1}{2}$, slope $= 3\left(-\frac{1}{2}\right)^2 = \frac{3}{4}$.

32. When $x = -1$, slope $= 3(-1)^2 = 3$.

$y = (-1)^3 = -1$.

Let $(x_1, y_1) = (-1, -1)$.

$y - (-1) = 3(x - (-1)) \Rightarrow y + 1 = 3(x + 1) \Rightarrow$

$y = 3x + 2$

33. The slope of the line tangent to $y = x^2$ at $x = a$ is $2a$. The slope of $y = 2x - 1$ is 2. Equating these gives $2a = 2 \Rightarrow a = 1$.

So, $f(a) = (1)^2 = 1$, $f'(1) = 2(1) = 2$

34. The slope of the line tangent to $y = x^2$ at $x = a$ is $2a$. The slope of $y = -x - \frac{1}{4}$ is -1.

Equating these gives $2a = -1 \Rightarrow a = -\frac{1}{2}$.

So, $f(a) = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$, and

$m = f'\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) = -1$

35. The slope of the curve $y = x^3$ at any point is $3x^2$. Solve $3x^2 = \frac{3}{2} \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$.

$x = \frac{1}{\sqrt{2}} \Rightarrow y = \left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{2\sqrt{2}}$.

$x = -\frac{1}{\sqrt{2}} \Rightarrow y = \left(-\frac{1}{\sqrt{2}}\right)^3 = -\frac{1}{2\sqrt{2}}$. The points are $\left(\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{2\sqrt{2}}\right)$.

36. The slope of $y = 2x$ is 2. Solve

$3x^2 = 2 \Rightarrow x^2 = \frac{2}{3} \Rightarrow x = \pm\sqrt{\frac{2}{3}}$.

$x = \sqrt{\frac{2}{3}} \Rightarrow y = \left(\sqrt{\frac{2}{3}}\right)^3 = \frac{2}{3}\sqrt{\frac{2}{3}}$.

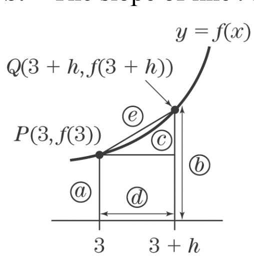
$x = -\sqrt{\frac{2}{3}} \Rightarrow y = \left(-\sqrt{\frac{2}{3}}\right)^3 = -\frac{2}{3}\sqrt{\frac{2}{3}}$. The points are $\left(\sqrt{\frac{2}{3}}, \frac{2}{3}\sqrt{\frac{2}{3}}\right)$ and $\left(-\sqrt{\frac{2}{3}}, -\frac{2}{3}\sqrt{\frac{2}{3}}\right)$.

37. **a.** $m = \frac{13-4}{5-2} = 3$

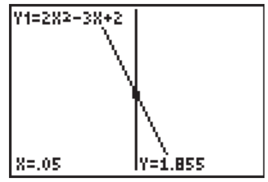
length of d is $13 - 4 = 9$

b. The slope of line l increases.

38.



39.



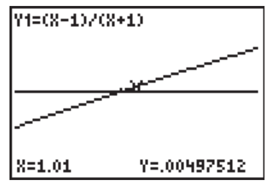
[−.078125, .078125] by [1.927923, 2.084173]

When $x = 0$, $y = 2$. Find a second point on the line using **VALUE**: $x = .05$, $y = 1.855$

$m = \frac{1.855-2}{.05-0} = -2.9$

The actual value of m is -3.

40.



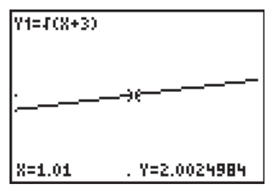
[.93251, 1.08876] by [−.078125, .078125]

When $x = 1$, $y = 0$. Find a second point on the line using **VALUE**: $x = .95$, $y = -.025641$

$m = \frac{.00497512-0}{1.01-1} \approx .5$

The actual value of m is $\frac{1}{2}$.

41.



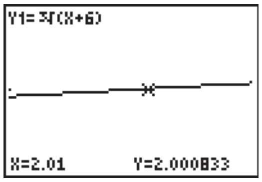
[.6463, 1.3963] by [1.5, 2.5]

When $x = 1$, $y = 2$

Find a second point on the line using **VALUE**: $x = 1.01$, $y = 2.0024984$

$m = \frac{2.0024984-2}{1.01-1} \approx .25$

The actual value of m is $\frac{1}{2\sqrt{3+1}} = .25$.

42. 

[1.2899, 2.5399] by [1.3679, 2.6179]

When $x = 2$, y=2.
Find a second point on the line using **value**:
$x = 2.01$, $y \approx 2.000833$

$$m = \frac{2.000833 - 2}{2.01 - 2} = .0833$$

The actual value of m is $\frac{1}{12}$.

1.3 The Derivative and Limits

For exercises 1– 16, refer to equations (1) and (2) section 1.3 in the text along with the Power Rule $f'(x) = rx^{r-1}$ for $f(x) = x^r$.

1. $f(x) = 3x + 7,\ f'(x) = 3$

2. $f(x) = -2x,\ f'(x) = -2$

3. $f(x) = \frac{3x}{4} - 2,\ f'(x) = \frac{3}{4}$

4. $f(x) = \frac{2x-6}{7} = \frac{2x}{7} - \frac{6}{7},\ f'(x) = \frac{2}{7}$

5. $f(x) = x^7,\ f'(x) = 7x^6$

6. $f(x) = x^{-2},\ f'(x) = -2x^{-3} = -\frac{2}{x^3}$

7. $f(x) = x^{2/3},\ f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

8. $f(x) = x^{-1/2},\ f'(x) = -\frac{1}{2}x^{-3/2} = -\frac{1}{2\sqrt{x^3}}$

9. $f(x) = \frac{1}{\sqrt{x^5}} = \frac{1}{x^{5/2}} = x^{-5/2}$,
$f'(x) = -\frac{5}{2}x^{-7/2} = -\frac{5}{2x^{7/2}}$

10. $f(x) = \frac{1}{x^3} = x^{-3},\ f'(x) = -3x^{-4} = -\frac{3}{x^4}$

11. $f(x) = \sqrt[3]{x} = x^{1/3},\ f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$

12. $f(x) = \frac{1}{\sqrt[5]{x}} = x^{-1/5}$,
$f'(x) = -\frac{1}{5}x^{-6/5} = -\frac{1}{5x^{6/5}}$

13. $f(x) = \frac{1}{x^{-2}} = x^2,\ f'(x) = 2x$

14. $f(x) = \sqrt[7]{x^2} = x^{2/7},\ f'(x) = \frac{2}{7}x^{-5/7} = \frac{2}{7x^{5/7}}$

15. $f(x) = 4^2 = 16,\ f'(x) = 0$

16. $f(x) = \pi,\ f'(x) = 0$

In exercises 17–24, first find the derivative of the function, then evaluate the derivative for the given value of x.

17. $f(x) = x^3$ at $x = \frac{1}{2}$
$f'(x) = 3x^2$
$f'\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 = \frac{3}{4}$

18. $f(x) = x^5$ at $x = \frac{3}{2}$
$f'(x) = 5x^4$
$f'\left(\frac{3}{2}\right) = 5\left(\frac{3}{2}\right)^4 = \frac{405}{16} = 25.3125$

19. $f(x) = \frac{1}{x}$ at $x = \frac{2}{3}$
$f(x) = x^{-1};\ f'(x) = -x^{-2} = -\frac{1}{x^2}$
$f'\left(\frac{2}{3}\right) = -\frac{1}{(2/3)^2} = -\frac{1}{4/9} = -\frac{9}{4}$

20. $f(x) = \frac{1}{3}$ at $x = 2$
$f'(x) = 0 \Rightarrow f'(2) = 0$

21. $f(x) = x + 11$ at $x = 0$
$f'(x) = 1 \Rightarrow f'(0) = 1$

22. $f(x) = x^{1/3}$ at $x = 8$
$f'(x) = \frac{1}{3}x^{-2/3}$
$f'(4) = \frac{1}{3}(8)^{-2/3} = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

23. $f(x)=\sqrt{x}$ at $x=\frac{1}{16}$

$f(x)=x^{1/2}$

$f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

$f'(16)=\frac{1}{2\sqrt{\frac{1}{16}}}=\frac{1}{2\cdot\frac{1}{4}}=2$

24. $f(x)=\frac{1}{\sqrt[5]{x^2}}$ at $x=32$

$f(x)=x^{-2/5}$

$f'(x)=-\frac{2}{5}x^{-7/5}$

$f'(32)=-\frac{2}{5}(32)^{-7/5}=-\frac{2}{5}\cdot\frac{1}{128}=-\frac{1}{320}$

In exercises 25 and 26, remember that the slope of a curve at a given point is the value of the derivative evaluated at that point.

25. $y=x^4$

slope $=y'=4x^3$

at $x=2$, $y'=4(2)^3=32$

26. $y=x^5$

slope $=y'=5x^4$

at $x=\frac{1}{3}$, $y'=5\left(\frac{1}{3}\right)^4=\frac{5}{81}$

27. $f(x)=x^3$; $f(-5)=(-5)^3=-125$

$f'(x)=3x^2$

$f'(-5)=3(-5)^2=75$

28. $f(x)=2x+6$

$f(0)=2(0)+6=6$

$f'(x)=2\Rightarrow f'(0)=2$

29. $f(x)=x^{1/3}$; $f(8)=8^{1/3}=2$

$f'(x)=\frac{1}{3}x^{-2/3}$

$f'(8)=\frac{1}{3}\cdot 8^{-2/3}=\frac{1}{3}\cdot\frac{1}{4}=\frac{1}{12}$

30. $f(x)=\frac{1}{x^2}=x^{-2}$; $f(1)=\frac{1}{1^2}=1$

$f'(x)=-2x^{-3}=-\frac{2}{x^3}$

$f'(1)=-\frac{2}{1^3}=-2$

31. $f(x)=\frac{1}{x^5}=x^{-5}$

$f(-2)=\frac{1}{(-2)^5}=-\frac{1}{32}$

$f'(x)=-5x^{-6}=-\frac{5}{x^6}$

$f'(-2)=-\frac{5}{(-2)^6}=-\frac{5}{64}$

32. $f(x)=x^{3/2}$

$f(16)=(16)^{3/2}=64$

$f'(x)=\frac{3}{2}x^{1/2}=\frac{3}{2}\sqrt{x}$

$f'(16)=\frac{3}{2}\sqrt{16}=\frac{12}{2}=6$

For exercises 33−40, refer to Example 4 on page 77 in the text.

33. $f(x)=x^3\Rightarrow f'(x)=3x^2$

When $x=-2$, $f(x)=(-2)^3=-8$.
The slope of the tangent at $x=-2$ is
$f'(-2)=3(-2)^2=12$. Thus, the equation of the tangent at $(-2,-8)$ in point-slope form is
$y+8=12(x+2)$.

34. $f(x)=x^2\Rightarrow f'(x)=2x$

When $x=-\frac{1}{2}$, $f(x)=\left(-\frac{1}{2}\right)^2=\frac{1}{4}$. The slope of the tangent at $x=-\frac{1}{2}$ is
$f'\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)=-1$. Thus, the equation of the tangent at $\left(-\frac{1}{2},\frac{1}{4}\right)$ in point-slope form is $y-\frac{1}{4}=-\left(x+\frac{1}{2}\right)$.

35. $f(x) = 3x + 1 \Rightarrow f'(x) = 3$
When $x = 4$, $f(x) = 3 \cdot 4 + 1 = 13$. The slope of the tangent at $x = 4$ is $f'(4) = 3$. Thus, the equation of the tangent at $(4, 13)$ in point-slope form is $y - 13 = 3(x - 4)$ or $y = 3x + 1$ in slope-intercept form.

36. $f(x) = 5 \Rightarrow f'(x) = 0$
When $x = -2$, $f(x) = 5$. The slope of the tangent at $x = -2$ is $f'(-2) = 0$. Thus, the equation of the tangent at $(4, 13)$ in point-slope form is $y - 5 = 0(x + 5)$ or $y = 5$.

37. $f(x) = \sqrt{x} = x^{1/2} \Rightarrow f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$
When $x = \frac{1}{9}$, $f(x) = \sqrt{\frac{1}{9}} = \frac{1}{3}$. The slope of the tangent at $x = \frac{1}{9}$ is
$f'\left(\frac{1}{9}\right) = \frac{1}{2}\left(\frac{1}{9}\right)^{-1/2} = \frac{1}{2}(9)^{1/2} = \frac{3}{2}$. Thus, the equation of the tangent at $\left(\frac{1}{9}, \frac{1}{3}\right)$ in point-slope form is $y - \frac{1}{3} = \frac{3}{2}\left(x - \frac{1}{9}\right)$.

38. $f(x) = \frac{1}{x} = x^{-1} \Rightarrow f'(x) = -x^{-2}$
When $x = .01$, $f(x) = \frac{1}{.01} = 100$. The slope of the tangent at $x = .01$ is
$f'(.01) = -(.01)^{-2} = -10{,}000$. Thus, the equation of the tangent at $(.01, 100)$ in point-slope form is $y - 100 = -10{,}000(x - .01)$.

39. $f(x) = \frac{1}{\sqrt{x}} = x^{-1/2} \Rightarrow f'(x) = -\frac{1}{2}x^{-3/2}$
When $x = 1$, $f(x) = \frac{1}{\sqrt{1}} = 1$. The slope of the tangent at $x = 1$ is $f'(1) = -\frac{1}{2}(1)^{-3/2} = -\frac{1}{2}$.
Thus, the equation of the tangent at $(1, 1)$ in point-slope form is $y - 1 = -\frac{1}{2}(x - 1)$.

40. $f(x) = \frac{1}{x^3} = x^{-3} \Rightarrow f'(x) = -3x^{-4}$
When $x = 3$, $f(x) = \frac{1}{3^3} = \frac{1}{27}$. The slope of the tangent at $x = 3$ is
$f'(3) = -3(3)^{-4} = -\frac{3}{81} = -\frac{1}{27}$.
Thus, the equation of the tangent at $\left(3, \frac{1}{27}\right)$ in point-slope form is $y - \frac{1}{27} = -\frac{1}{27}(x - 3)$.

41. Equation 6 on page 76 states that
$y - f(a) = f'(a)(x - a)$.
$y = f(x) = x^4 \Rightarrow y' = f'(x) = 4x^3$
For $a = 1$, $f(a) = f(1) = 1$ and
$f'(a) = f'(1) = 4$. Thus, the equation of the tangent at $(1, 1)$ in point-slope form is
$y - 1 = 4(x - 1)$.

42. The tangent is perpendicular to $y = 4x + 1$, so the slope of the tangent is $m = -\frac{1}{4}$ because the slopes of perpendicular lines is -1.
$f(x) = \frac{1}{x} = x^{-1} \Rightarrow f'(x) = -x^{-2}$.
The slope of the tangent at $x = a$ is
$f'(a) = -a^{-2}$. $-a^{-2} = -\frac{1}{4} \Rightarrow a = \pm 2$.
Therefore, $P = \left(2, \frac{1}{2}\right)$ or $P = \left(-2, -\frac{1}{2}\right)$.

43. The slope of the tangent is $m = 2$.
$f(x) = \sqrt{x} = x^{1/2} \Rightarrow f'(x) = \frac{1}{2}x^{-1/2}$. The slope of the tangent at $x = a$ is
$f'(a) = \frac{1}{2}a^{-1/2}$. Therefore $P = \left(\frac{1}{16}, \frac{1}{4}\right)$.
$\frac{1}{4} = 2\left(\frac{1}{16}\right) + b \Rightarrow b = \frac{1}{8}$.

44. The slope of the tangent is $m = a$.
$f(x) = x^3 \Rightarrow f'(x) = 3x^2$. The slope of the tangent at $(-3, -27)$ is $f'(-3) = 3(-3)^2 = 27$.
Therefore $a = 27$.
$-27 = 27(-3) + b \Rightarrow b = 54$.