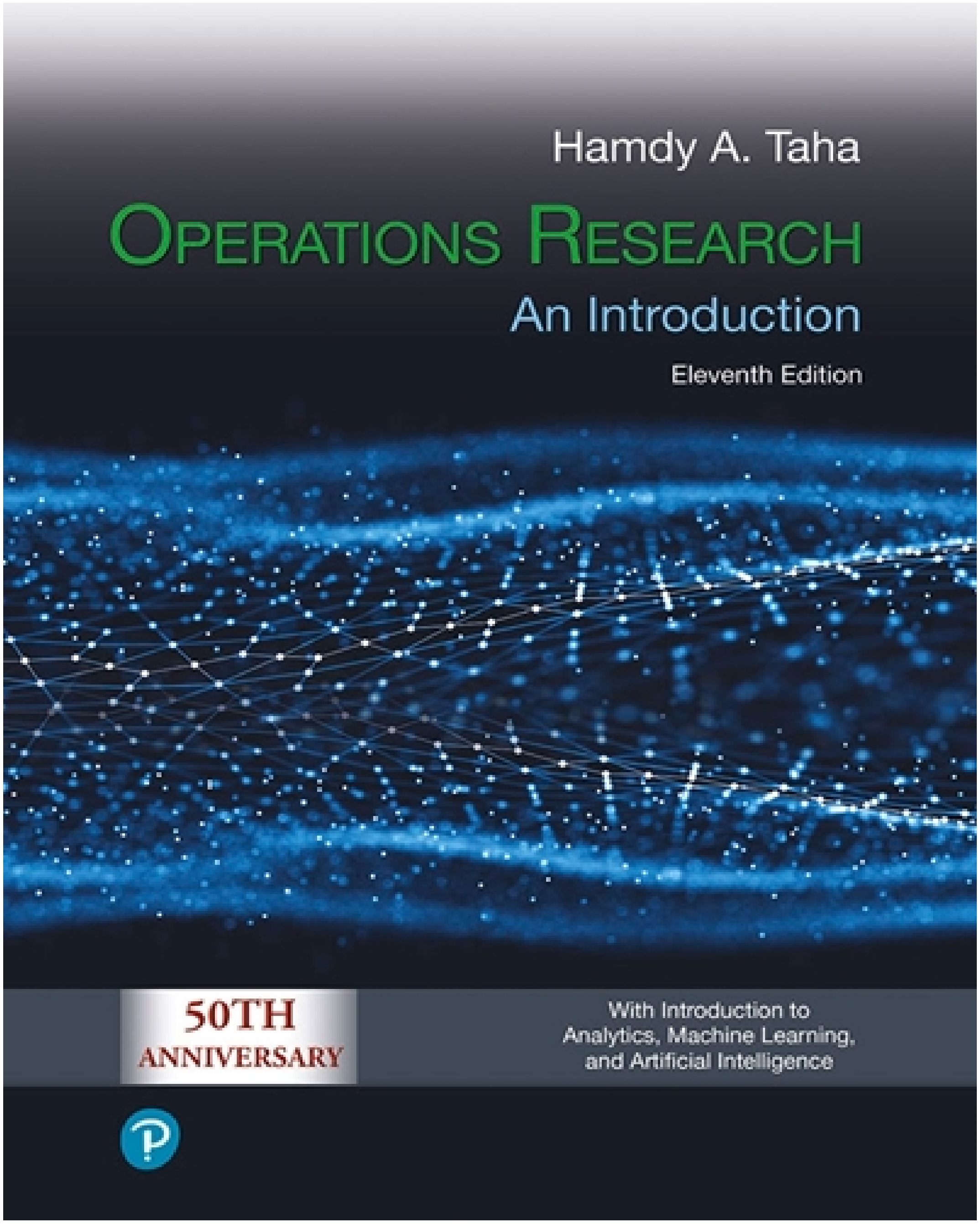
Solutions for Operations Research 11th Edition by Taha



Solutions

CHAPTER 2

Modeling with linear programming

Chapter 2

1

(a) $x_2 - x_1 \geq 1$ **or** $-x_1 + x_2 \geq 1$
(b) $x_1 + 2x_2 \geq 3$ **and** $x_1 + 2x_2 \leq 6$

$$\left.\begin{array}{lll} 6\times2+4\times2=20 & < & 24 \\ 1\times1+\ 2\times2\ =6 & = & 6 \\ -1\times1+\ 1\times2\ =0 & < & 1 \\ 1\times2\ =2 & < & 1 \end{array}\right\} \textit{feasible}$$

(c) $x_2 \geq x_1$ **or** $x_1 - x_2 \leq 0$
(d) $x_1 + x_2 \geq 3$
(e) $\dfrac{x_2}{x_1+x_2} \leq .5$ **or** $.5\,x_1 - x_2 \geq 0$

2

(a) $(x_1, x_2) = (1, 4)$
$(x_1, x_2) \geq 0$
$6\times1+4\times4=22 < 24$
$1\times1+2\times4 = 9 > 6$ $\Rightarrow$ infeasible
(b) $(x_1, x_2) = (2, 2)$
$(x_1, x_2) \geq 0$
$6\times2+4\times2=20 < 24$
$1\times2+2\times2 = 6 = 6$
$-1\times2+1\times2 = 0 < 1$
$1\times2 = 2 = 2$
Feasible, $z=5\times2+4\times2=\$18$
(c) $(x_1, x_2) = (3, 1.5)$
$(x_1, x_2) \geq 0$
$6\times3+4\times1.5=24 < 24$
$1\times3+2\times1.5 = 6 = 6$
$-1\times3+1\times1.5 = -1.5 < 1$
$1\times1.5 = 1.5 < 2$
Feasible, $z=5\times3+4\times1.5=$ **\$21**
(c) $(x_1, x_2) = (2, 1)$
$(x_1, x_2) \geq 0$
$6\times2+4\times1=16 < 24$
$1\times2+2\times1 = 4 < 6$
$-1\times2+1\times1 = -1 < 1$
$1\times1 = 1 < 2$
Feasible, $z=5\times2+4\times1=\$14$
(c) $(x_1, x_2) = (2, -1)$
$x_2 < 0$ $\Rightarrow$ infeasible

3

$(x_1, x_2) = (2, 2)$: Let s_1 and s_2 be the unused daily amounts of M1 and M2.
For M1: $s_1 = 24 - (6x_1 + 4x_2) = 24 - 2\times10 = 4$ tons/day
For M2: $s_2 = 6 - (x_1 + 2x_2) = 6 - 2\times3 = 0$ ton/day

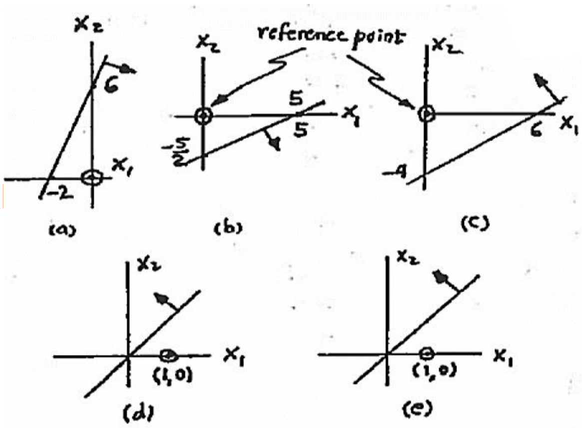
4

Quantity discount results in a nonlinear function:

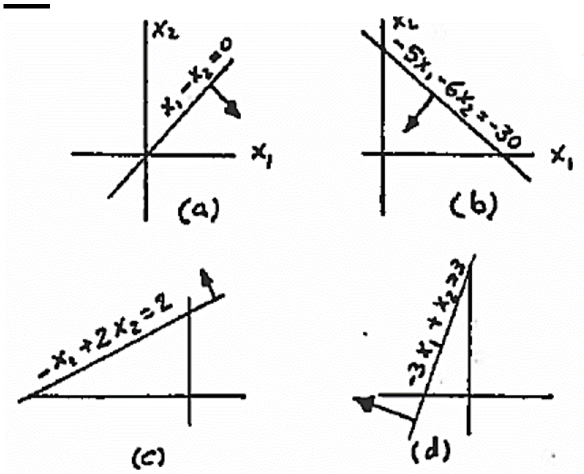
$$z = \begin{cases} 5x_1 + 4x_2, 0 \leq x_1 \leq 2 \\ 4.5x_1 + 4x_2, x_1 \geq 0 \end{cases}$$

The situation cannot be an LP. Mixed integer programming (Chapter 9) can handle this nonlinearity.

5



6

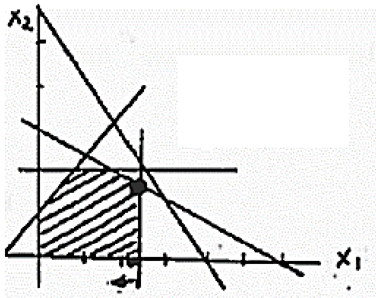


continue…

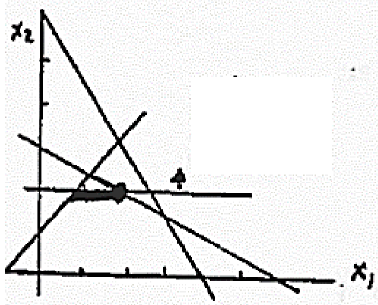
Chapter 2

7

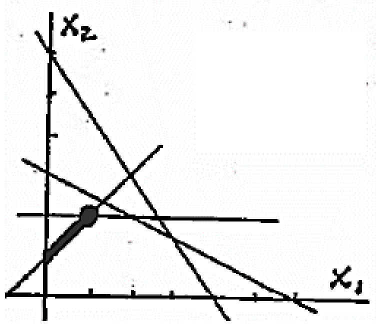
(a)
$x_1 \le 2.5$
Optimum:
$x_1 = 2.5$
$x_2 = 1.75$
$z = \$13$



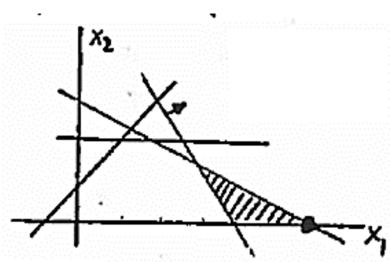
(b)
$x_2 \ge 2$
Optimum:
$x_1 = 2, x_2 = 2$
$z = \$18$



(c)
$-x_1 + x_2 = 1$
Optimum:
$x_1 = 1, x_2 = 2$
$z = \$13$



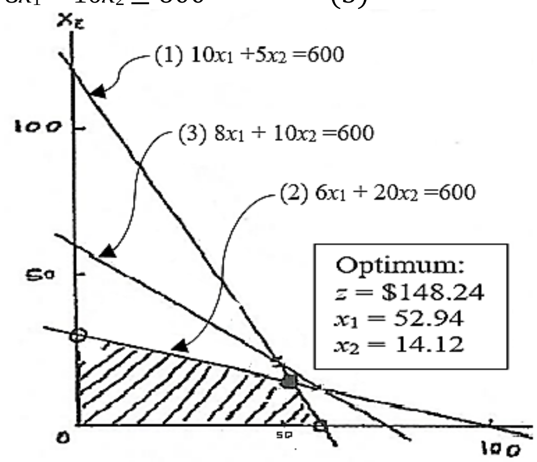
(d)
$6x_1 + 4x_2 \ge 24$
Optimum:
$x_1 = 6, x_2 = 0$
$z = \$30$



(e)
infeasible space

8

x_1=daily units of product 1
x_2=daily units of product 2
Max z = $2x_1 + 3x_2$ s.t.
$10x_1 + 5x_2 \le 600$ (1)
$6x_1 + 20x_2 \le 600$ (2)
$8x_1 + 10x_2 \le 600$ (3)



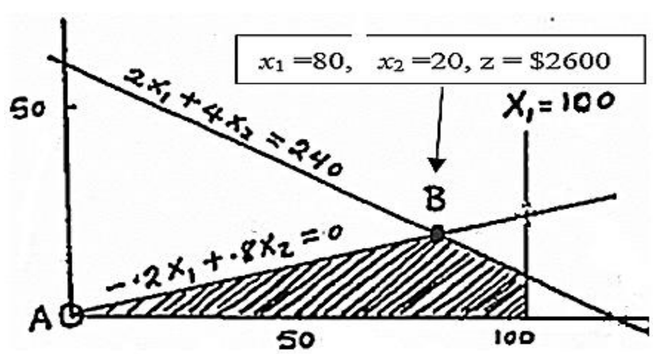
9

x_1= nbr. of units of A
x_2= nbr. of units of B
Max z = $20x_1 + 50x_2$ s.t. $x_1, x_2 \ge 0$

$$\frac{x_1}{x_1 + x_2} \ge .8 \quad \text{or} \quad -.2x_1 + .8x_2 \le 0$$

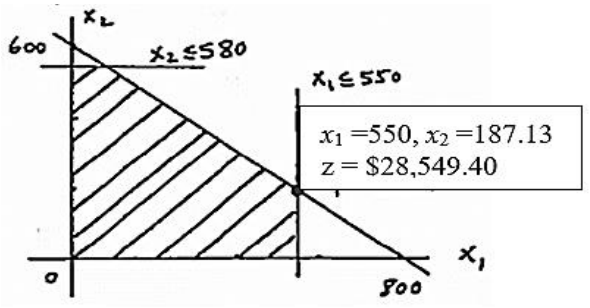
$$x_1 \le 100$$

$$2x_1 + 4x_2 \le 240$$



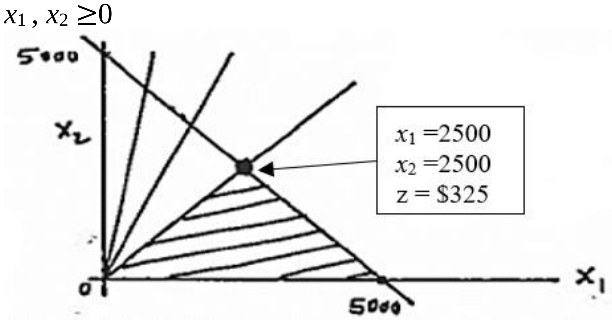
10

x_1=nbr. of sheets/day
x_2= nbr. of bars/day
Max z = $40x_1 + 35x_2$ s.t.
$\frac{x_1}{800} + \frac{x_2}{600} \leq 1$
$0 \leq x_1 \leq 550, 0 \leq x_2 \leq 580$



11

x_1=$ invested in A
x_2= $ invested in B
Max z = $.05x_1 + .08x_2$ s.t.
$x_1 \geq .25(x_1 + x_2)$
$x_2 \leq .5(x_1 + x_2)$
$x_1 \geq .5x_2$
$x_1 + x_2 \leq 5000$
$x_1, x_2 \geq 0$



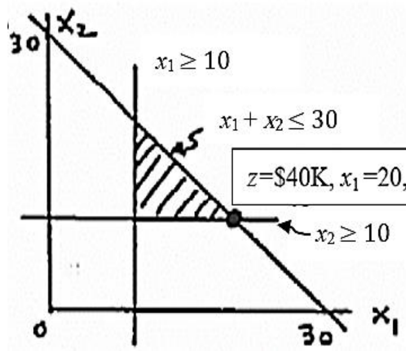
12

x_1= nbr of practical courses
x_2= nbr of humanistic courses
Max z = $1500x_1 + .1000x_2$ s.t.
$x_1 \geq .25(x_1 + x_2)$
$x_1 + x_2 \leq 30$
$x_1 \geq 10$,

continued…

12 cont'd

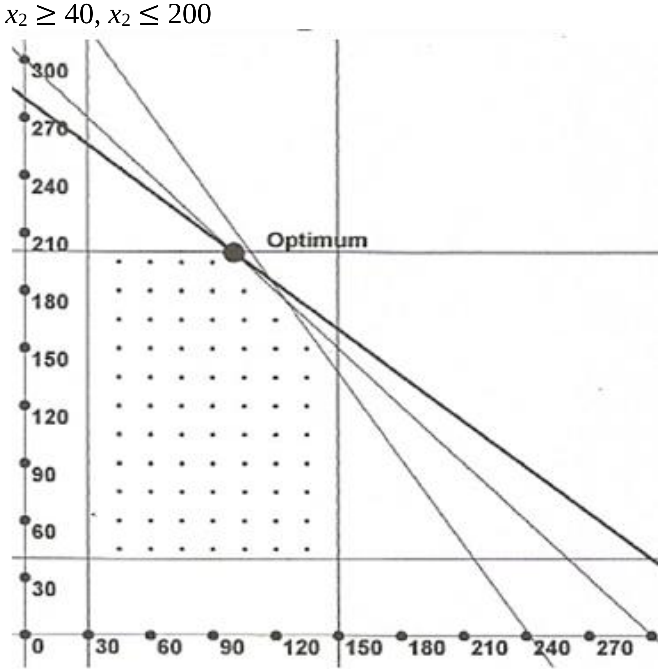
(a)



(b) Change $x_1 + x_2 \leq 30$ to $x_1 + x_2 \leq 31$
Optimum: z=$41,500, Δz=41500-40,000=$1500
Conclusion: any additional course will be 'practical'

13

x_1=units of solution A
x_2= units of solution B
Max z = $8x_1 + 10x_2$ s.t.
$.5x_1 + .5x_2 \leq 159$
$.6x_1 + .4x_2 \leq 145$
$x_1 \geq 30, x_1 \leq 150$
$x_2 \geq 40, x_2 \leq 200$



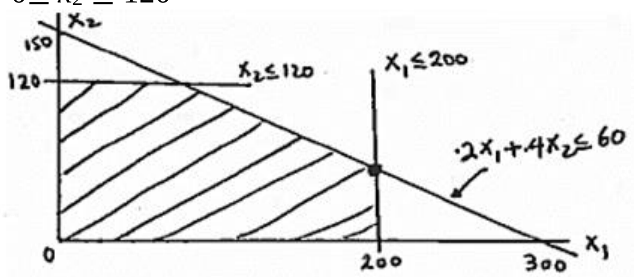
Optimum: z = 2800, x_1 = 100, x_2 = 200

continued…

Chapter 2

14

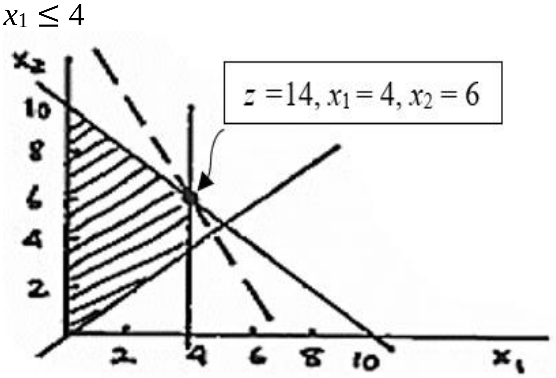
x_1= nbr of grano boxes
x_2= nbr of wheatie boxes
Max z = $x_1 + 1.35x_2$ s.t.
$.2x_1 + .4x_2 \leq 60$
$.6x_1 + .4x_2 \leq 145$
$0 \leq x_1 \leq 200$
$0 \leq x_2 \leq 120$



Optimum: $x_1 = 200$, $x_2 = 50$, z=\$267.50
Area allocation: 67% grano, 33% wheatie

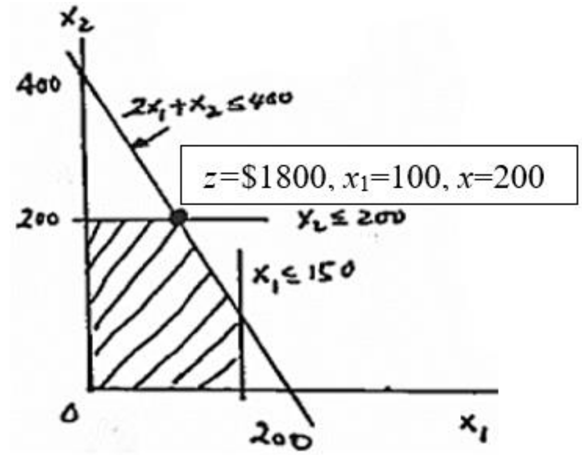
15

x_1= play hours/day
x_2 = wok hours/day
Max z = $2x_1 + x_2$ s.t.
$x_1 + x_2 \leq 10$
$x_1 - x_2 \leq 0$
$x_1 \leq 4$



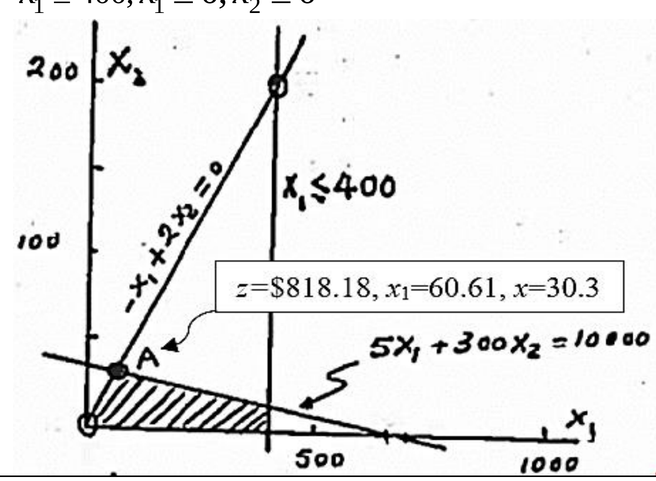
16

x_1=daily nbr. of type 1 hats
x_2= daily nbr. of type 2 hats
Max z = $8x_1 + 5x_2$ s.t.
$2x_1 + x_2 \leq 400$
$0 \leq x_1 \leq 150$
$0 \leq x_2 \leq 200$



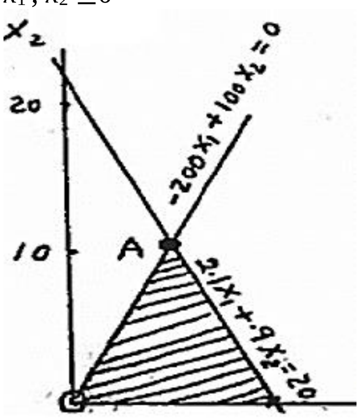
17

x_1=radio minutes
x_2= TV minutes
Max z = $x_1 + 25x_2$ s.t.
$15x_1 + 300x_2 \leq 10{,}000$
$\frac{x_1}{x_2} \geq 2$ or $-x_1 + 2x_2 \leq 0$
$x_1 \leq 400, x_1 \geq 0, x_2 \geq 0$



18

x_1=consumed tons of C1/hr
x_2= consumed tons of C2/hr
Max z = $12000x_1+9000x_2$ s.t.
$1800x_1+2100x_2 \le 2000(x_1 + x_2) \Rightarrow -200x_1+100x_2 \le 20$
$2.1\, x_1+.9x_2 \le 20$
$x_1 , x_2 \ge 0$



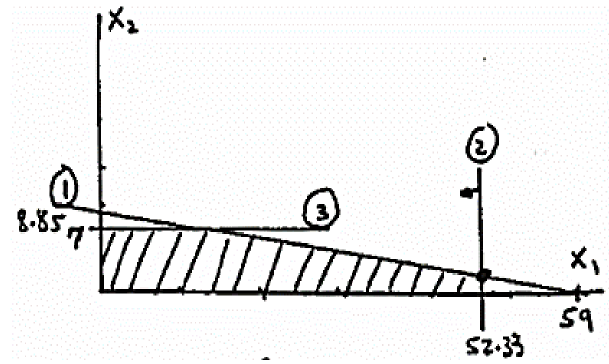
(a) optimum:
z=153,846 lb, x_1=5.128 ton/hr, x_2=10.256 ton/hr
optimal ration= 5.128/10.256=.5

(b) $2.1\, x_1+.9x_2 \le (20+1)$
z=161,538lb, Δz =161,538 − 153,846 =7,692 lb

19

x_1=nbr of radio commercials beyond the first
x_2= nbr of TV commercials beyond the first
Max z = $2000x_1+3000x_2+5000+2000$ s.t.
$300(x_1+1)+2000(x_2 +1) \le 20{,}000$
$300(x_1+1) \le .8\times 20{,}000$
$2000(x_2 +1) \le .8\times 20{,}000$
or,
Max z = $2000x_1+3000x_2+7000$ s.t.
$300x_1+200x_2 \le 17{,}700$
$300\, x_1 \le 15{,}700$
$2000x_2 \le 14{,}000$
$x_1 , x_2 \ge 0$

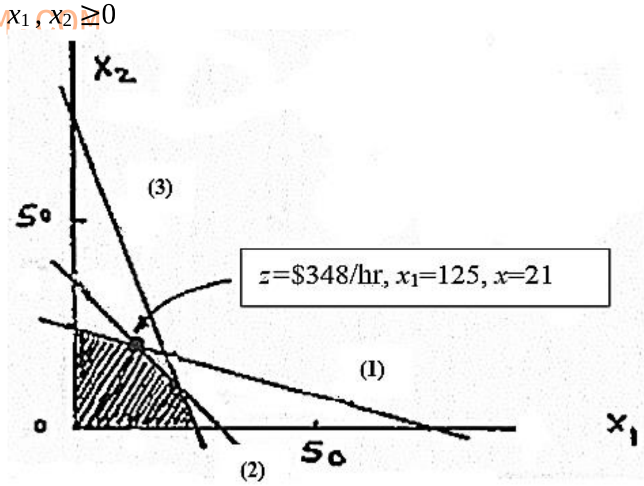
19 cont'd



Optimum: Radio=52.33+1=53.33
TV= 1+1=2, z=114,666.67

20

x_1=nbr of T-shirts/hr
x_2= nbr of jackets/hr
Max z = $8x_1+12x_2$ s.t.
$20x_1+60x_2 \le 25\times 60=1500$ (1)
$70x_1+60x_2 \le 35\times 60=2100$ (2)
$12x_1+4x_2 \le 5\times 60=300$ (3)
$x_1 , x_2 \ge 0$

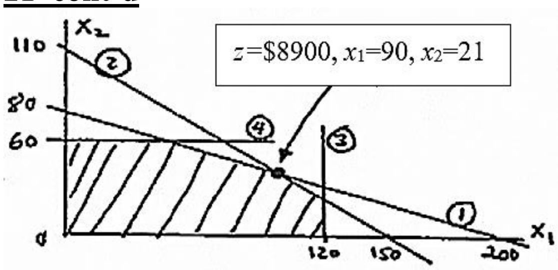


21

x_1=nbr of desks/day
x_2= nbr of chair/day
Max z = $50x_1+100x_2$ s.t.
$x_1/200+x_2/80 \le 1$ (1)
$x_1/150+x_2/110 \le 1$ (2)
$x_1 \le 120$, $x_2 \le 60$ (3&4)
$x_1 , x_2 \ge 0$

continued…

21 cont'd

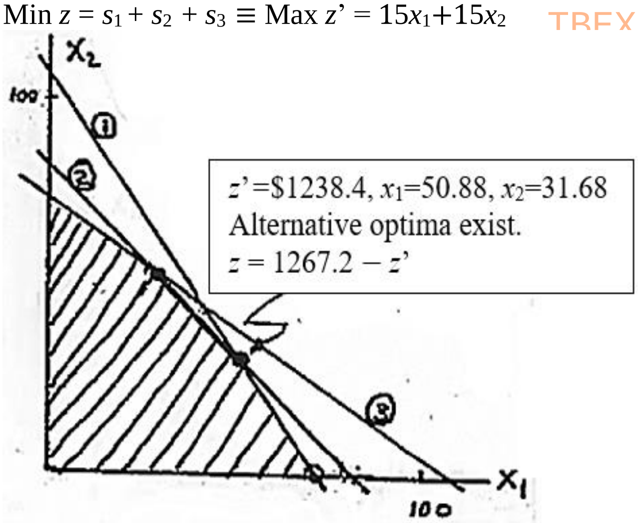


22

x_1=nbr of HiFi1 units
x_2= nbr of HiFi2 units
constraints:
Let s_i be a slack variable, i=1,2,3

$6x_1+4x_2 + s_1 = 480\times.9=432$ (1)
$5x_1+5x_2 + s_2 = 35\times60=2100$ (2)
$4x_1+6x_2 + s_3 = 5\times60=300$ (3)
$x_1, x_2 \geq 0$

Objective function:
Let $z = s_1 + s_2 + s_3$ (= $1267.2 - 15x_1 - 15x_2$, per constrains 1&2&3), hence
Min $z = s_1 + s_2 + s_3 \equiv$ Max $z' = 15x_1+15x_2$



23

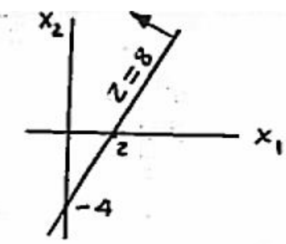
Corner point	(x_1, x_2)	z
A	(0, 0)	0
B	(4, 0)	20
C	**(3, 1.5)**	**21** (optimum)
D	(2, 2)	18
E	(1, 2)	13
F	(0, 1)	4

24 – 27

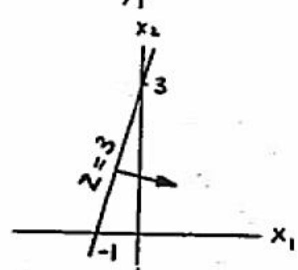
TORA experiments

28

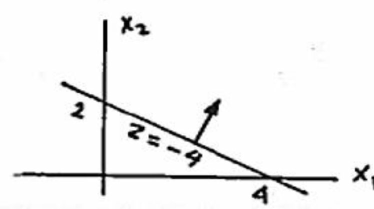
(a)



(b)

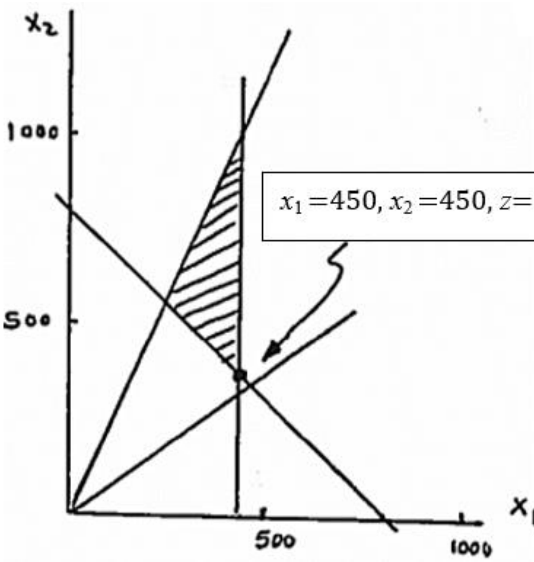


(c)



29

Additional constraint: $x_1 \leq 450$



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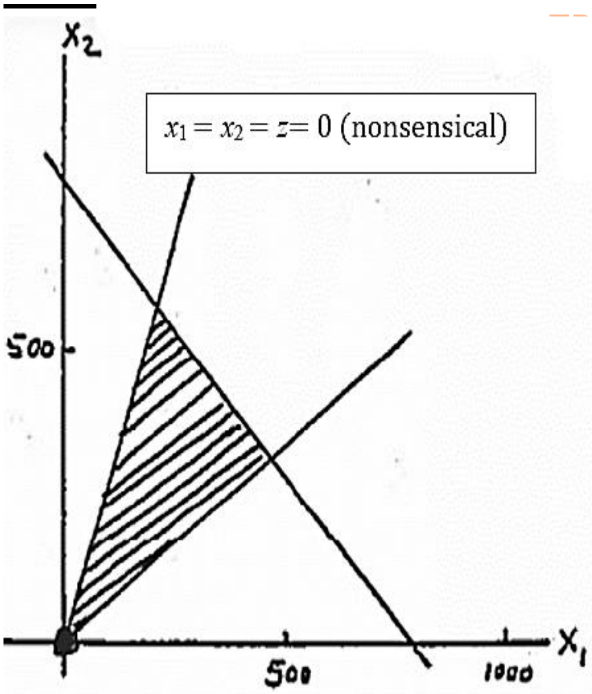
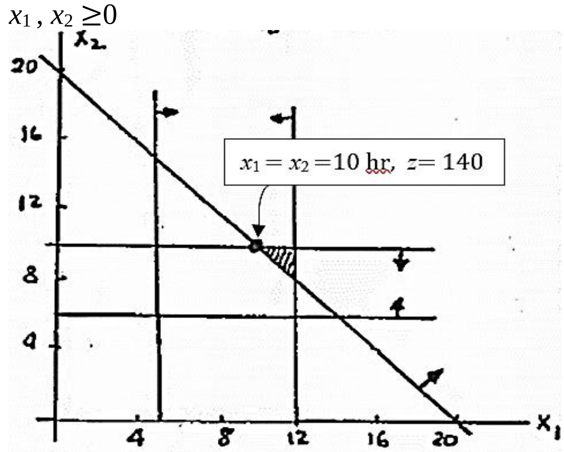


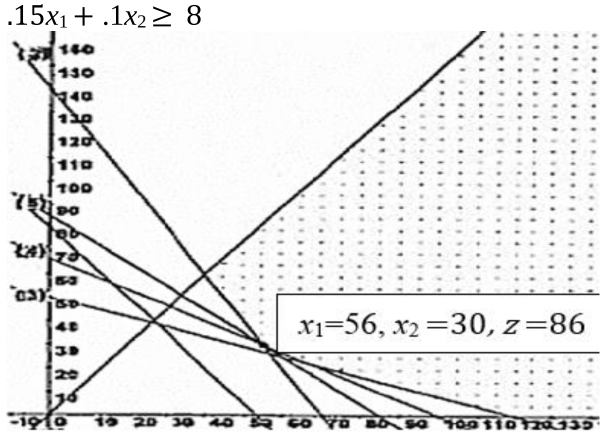
31

x_1=nbr of hours in store1
x_2= nbr of hours in store2
Min z = $8x_1+6x_2$ s.t.
$x_1+x_2 \geq 20$
$5 \leq x_1 \leq 12$
$6 \leq x_2 \leq 10$
$x_1, x_2 \geq 0$



32

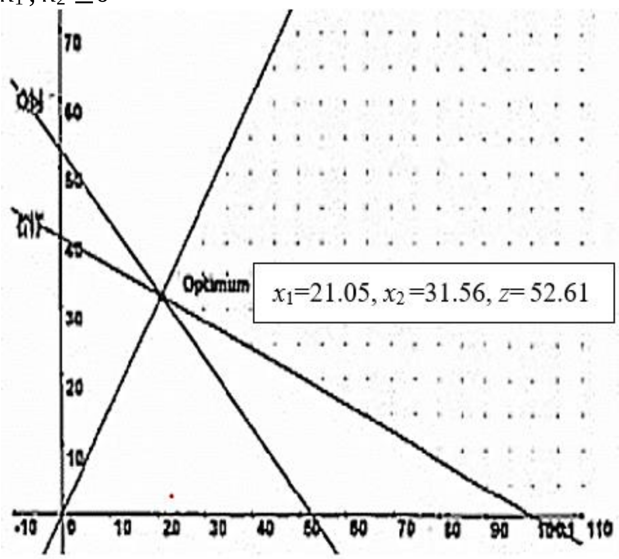
x_1 = amount in 1K bbl/day from Iran
x_2 = amount in 1K bbl/day from Dubai
Refinery capacity = (x_1+x_2)
Min z = x_1+x_2 s.t.
$x_1 \geq .4(x_1+x_2)$ or $.6x_1 - .4x_2 \geq 0$
$.2x_1 + .1x_2 \geq 14$
$.25x_1 + .6x_2 \geq 30$
$.1x_1 + .15x_2 \geq 10$
$.15x_1 + .1x_2 \geq 8$



33

x_1 = amount in \$1K invested in blue chips
x_2 = amount in \$1K invested in high-tech
Refinery capacity = (x_1+x_2)
Min $z = x_1+x_2$ s.t.
$.1x_1 + .25x_2 \geq 10$
$.6x_1 - .4x_2 \geq 0$
$x_1, x_2 \geq 0$

$x_1=.33,\ x_2=.67,\ z=86.67$



34

x_1= ratio of scrap A in alloy
x_2 = ratio of scrap B in alloy
Min z = $100x_1 + 80x_2$ s.t.
$.06x_1 + .03x_2 .03$
$.06x_1 + .03x_2 \leq .06$
$.03x_1 + .06x_2 \geq .03$
$.03x_1 + .06x_2 \leq .05$
$.04x_1 + .03x_2 \geq \geq .03$
$.04x_1 + .03x_2 \leq .07$
$x_1 + x_2 = 1$
$x_1, x_2 \geq 0$

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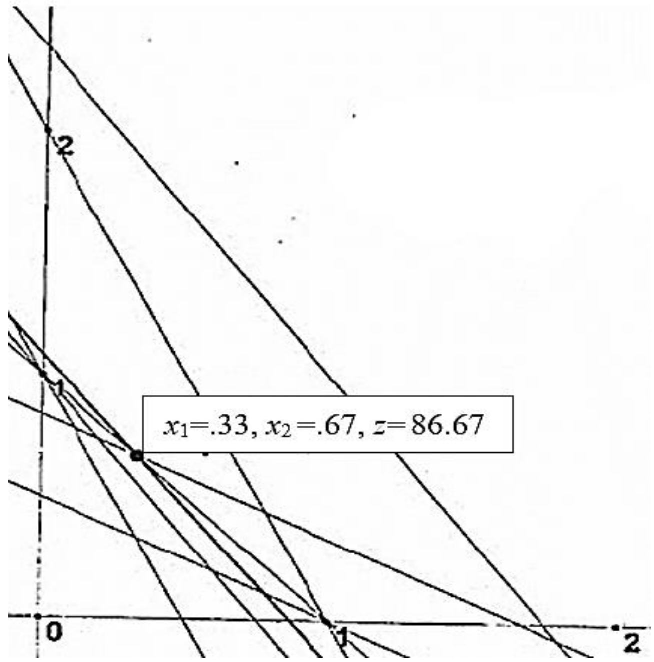


35-39 AMPL-Solver-TORA exercises

40

(a) x_j = executed portion of project j
Max $z= 32.4x_1 +35.8x_2 + 17.75x_3 +14.8x_4 +18.2x_5 +12.35x_6$
s.t.
$10.5x_1+8.3x_2+10.2x_3+7{,}2x_4+12.3x_5+9.2x_6 \leq 60$
$14.4x_1+12.6x_2+14.2x_3+10.5x_4+10.1x_5+7.8x_6 \leq 70$
$2.2x_1+9.5x_2+5.6x_3+7.5x_4+8.3x_5+6.9x_6 \leq 35$
$2.4x_1+3.1x_2+4.2x_3+5x_4+6.3x_5+5.1x_6 \leq 20$
$0 \leq x_i \leq 1,\ j = 1, 2, ..., 6$
Solution: $x_1 = x_2 = x_3 = x_4 =1$, $x_5 =.84$, $x_6 = 0$, z=116.06
(b) Add the constraint $x_2 \leq x_6$
Solution: $x_1 = x_2 = x_3 = x_4 = x_6$=1, x_5 =7 70+.03, z=113.68
(c) Let s_i =unused funds at the end of year i and change the RHS of constraints 2, 3, and 4 to $70+s_2$, $35s_3$, and $20+s_4$.
Solution: $x_1 = x_2 = x_3 = x_4 = x_5 =1$, x_6=.71, z=127.72

continued…

40 cont'd

Solution interpretation:

i	s_i	s_i- s_{i-1}	Decision
1	4.96	--	--
2	7.62	+2.66	Don't borrow from year 1
3	4.62	-3.00	Borrow 3 from year 2
4	0	-24.6	Borrow 4.62 from year 2

Availing excess money in current year for later years resulted in completing the first five projects + 71% of project 6. Total revenue increased from \$116,060 to \$127,720.

(d) Declare slack s_i unrestricted and re-solve.
Solution: s_1 = 2.3, s_2=.4, s_3=-5, s_4 = -6.1, z=131.3 ⇒ additional funds are needed in years 3 and 4.
Increase in return = 131.3-116.06 = 15.24. Ignoring time value of money, the amount borrowed [= 5+6.1- (2.3+.4) = 8.4] yields a rate of return = (15.28-8.4)/8.8 ≈ 81%

41

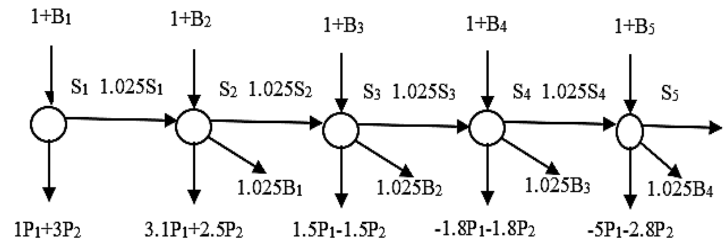
x_i = \$-amt invested in project i (=1,2,3,4)
y_j = \$-amt invested in in bank in year j (=1,2,3,4,5)
Max $z = y_5$
s.t.
$x_1 + x_2 + x_4 + y_1 \leq 10{,}000$
$.5x_1 + .6x_2 - x_3 + .4x_4 + 1.065\, y_1 - y_2 = 0$
$.3x_1 + .2x_2 + .8x_3 + .6x_4 + 1.065\, y_2 - y_3 = 0$
$1.8x_1 + 1.5x_2 + 1.9x_3 + 1.86x_4 + 1.065\, y_3 - y_4 = 0$
$1.2x_1 + 1.3x_2 + .8x_3 + .95x_4 + 1.065\, y_4 - y_5 = 0$
all vars ≥ 0
Solution: $x_1 = 0$, x_2 =\$10,000, x_3 =\$600, $x_4 = 0$
$y_1 = y_2 = 0$, y_3 =\$6800, y_4 =\$33,642
$y_5 = z$ = \$53,628.73 at the start of year 5

42

P_i = undertaken fraction of project i (=1,2)
B_j = million \$ borrowed in quarter j (=1,2,3,4)
S_j = surplus million \$ at the start of quarter j (=1,2,3,4)

continued…

42 cont'd



(a)
Max $z = S_5$
s.t.
$P_1 + 3P_2 - S_5 - B_1 = 1$
$3.1P_1 + 2.5P_2 - 1.02S_1 + S_2 + 1.025B_1 - B_2 = 1$
$1.5P_1 - 1.5P_2 - 1.02S_2 + S_3 + 1.025B_2 - B_3 = 1$
$-1.8P_1 - 1.8P_2 - 1.02S_3 + S_4 + 1.025B_3 - B_4 = 1$
$-5P_1 - 2.8P_2 - 1.02\, S_4 + S_5 + 1.025\, B_4 = 1$
$0 \leq P_i \leq 1$, $i = 1, 2$
$0 \leq B_j \leq 1$, $j = 1, 2,3,4$
Solution: z = 5.8366, P_1 =.7113, P_2 =0
B_1 =0, B_2 =.9104, B_3 =1, B_4=0

(b) B_1 =0, B_2 =.9104, B_3 =1, B_4=0
S_1 =.2887, S_2 =0, S_3 =0, S_4=1.2553
The solution shows that $B_iS_i = 0$, which means that it is not optimal to borrow and end up with surplus in any quarter. The result also makes sense because the cost of borrowing (=2.5%) is higher that the return on surplus funds (=2%).

43

Assume that the investment program ends at the start of year 11, so that the 6-year bond can be exercised in years 1 through 5 only. Similarly, the 9-year bond can be used in years 1 and 2 only. From years 6 on, the only investments available are insured saving at 7.5%.
I_i = insured savings in year i (=1,2, …, 10)
G_i = 6-year bond i (=1, 2, …, 5)
M_i = 9-year bond i (=1,2)
Objective: Maximize accumulation at end of year 10
Max $z = 1.075I_{10} + 1.079G_5 + 1.085M_2$

continued…

43 cont'd

Cash flow constraints:

$I_1 + .98G_1 + 1.02M_1 = 2$

$I_2 + .98G_2 + 1.02M_2 = 2 + 1.075\, I_1 + .079G_1 + .085M_1$

$I_3 + .98G_3 = 2.5 + 1.075\, I_2 + .079\,(G_1 + G_2) + .085(M_1 + M_2)$

$I_4 + .98G_4 = 2.5 + 1.075\, I_3 + .079\,(G_1 + G_2 + G_3) + .085(M_1 + M_2)$

$I_5 + .98G_5 = 3 + 1.075\, I_4 + .079\,(G_1 + G_2 + G_3 + G_4) + .085(M_1 + M_2)$

$I_6 = 3.5 + 1.075\, I_5 + .079\,(G_1 + G_2 + G_3 + G_4 + G_5) + .085(M_1 + M_2)$

$I_7 = 3.5 + 1.075\, I_6 + 1.079G_1 + .079\,(G_2 + G_3 + G_4 + G_5) + .085(M_1 + M_2)$

$I_8 = 4 + 1.075\, I_7 + 1.079G_2 + .079\,(G_3 + G_4 + G_5) + .085(M_1 + M_2)$

$I_9 = 4 + 1.075\, I_8 + 1.079G_3 + .079(+G_4 + G_5) + .085(M_1 + M_2)$

$I_{10} = 5 + 1.075\, I_9 + 1.079G_4 + .079G_5 + 1.085M_1 + .085M_2$

all vars nonnegative

Solution: $z = 46.85$

I_1 to $I_5 = 0$, $I_6 = 4.63$, $I_7 = 9.61$, $I_8 = 15.47$, $I_9 = 24.67$, $I_{10} = 37.52$

$G_1 = G_2 = 0$, $G_3 = 2.91$, $G_4 = 3.14$, $G_5 = 3.90$, $M_1 = 1.96$, $M_2 = 2.12$

Year	Recommendation
1	invest all in 9-yr bond
2	ditto
3	invest all in 6-yr bond
4	ditto
5	ditto
6	invest all in savings
7	ditto
8	ditto
9	ditto
10	ditto

44

x_{iA} = \$1K-amt invested in plan A, year i (=1,2,3)

x_{iB} = \$1K-amt invested in plan B, year i (=1,2)

Max $z = 1.7x_{3A} + 3x_{2B}$ s.t.

$x_{1A} + 3x_{1B} \le 100$

$-1.7x_{1A} + 3_{2A} + x_{2B} = 0$

$-3x_{1B} - 1.7x_{2A} + x_{3A} = 0$

all vars nonnegative

Solution: all values in \$1K

$z = 46.85$, $x_{1A} = 100$, $x_{2B} = 170$, all other vars = 0

Alternative solution: $x_{1B} = 100$, $x_{3A} = 300$, all other vars = 0

45

x_i = \$-amt allocated to choice i (=1,2,3,4)

y = minimum return

$$Max\ z = \min\begin{cases} -3x_1 + 4x_2 - 7x_3 + 15x_4 \\ 5x_1 - 3x_2 + 9x_3 + 4x_4 \\ 3x_1 - 9x_2 + 10x_3 - 8x_4 \end{cases} \quad \text{s.t.}$$

$$x_1 + x_2 + x_3 + x_4 \le 500$$

$$x_1, x_2, x_3, x_4 \ge 0$$

The problem can be converted to an LP as

Max $z = y$

$$y = \min\begin{cases} -3x_1 + 4x_2 - 7x_3 + 15x_4 \\ 5x_1 - 3x_2 + 9x_3 + 4x_4 \\ 3x_1 - 9x_2 + 10x_3 - 8x_4 \end{cases} \Rightarrow$$

$$-3x_1 + 4x_2 - 7x_3 + 15x_4 \ge y$$

$$5x_1 - 3x_2 + 9x_3 + 4x_4 \ge y$$

$$3x_1 - 9x_2 + 10x_3 - 8x_4 \ge y$$

$$x_1 + x_2 + x_3 + x_4 \le 500$$

$$x_1, x_2, x_3, x_4 \ge 0$$

Solution: z=\$1175, $x_1 = x_2 = 0$, x_3 = \$287.5, x_4 =\$212.5

46

$$i=\begin{cases}1, \text{ regular savings}\\ 2, \text{ 3-month CD}\\ 3, \text{ 6-month CD}\end{cases}$$

x_{it}=\$-deposit in plan i at the start of month t

$$i=\begin{cases}1,2, ...,12 \text{ if } i=1\\ 1,2, ...,10 \text{ if } i=2\\ 1,2, ..., 7 \text{ if } i=3\end{cases}$$

d_t = \$demand for period t

y_1 = initial amt on hand to ensure feasibility

r_i = interest rate for plan i (= 1, 2, 3)

$$J_i=\begin{cases}12, \text{ if } i=1\\ 10, \text{ if } i=2\\ 7, \text{ if } i=3\end{cases} \qquad P_i=\begin{cases}1, \text{ if } i=1\\ 3, \text{ if } i=2\\ 6, \text{ if } i=3\end{cases}$$

$$\text{Max } z=\sum_{\substack{t=1\\ t>P_i}}^{12}\sum_{i=1}^{3} r_i x_{i,t-p_i} - y_1 \text{ s.t.}$$

$$y_1 - x_{11} - x_{21} - x_{31} \geq d_1$$

$$1000+\sum_{i=1|t>P_i}^{3}(1+r_i)x_{i,t-P_i} - \sum_{i=1|t\leq J_i}^{3} x_{it} \geq d_t, t=2,...12$$

all variables nonnegative

Solution: (see file *amplProb2-46.txt*)

y_1 = \$1200, z = =\$1136.29

interest amt =1200-1136.29=\$63.71

Deposits:

t	x1t	x2t	x3t
1	0	0	0
2	0	200	0
3	286.48	313.53	0
4	0	587.43	0
5	314.37	289.30	0
6	0	734.69	0
7	0	98.20	0
8	0	294.60	
9	0	848.60	
10	0	0	
11	0		
12	0		

47

x_{w1} = wrenches/wk (regular time)

x_{w2} = wrenches/wk (overtime)

x_{w3} = wrenches/wk (subcontracting)

x_{c1} = chisels/wk (regular time)

x_{c2} = chisels /wk (overtime)

x_{c3} = chisels /wk (subcontracting)

Min z =$2x_{w1}+2.8x_{w2}+3x_{w3}+2.1x_{c1}+3.2x_{c2}+4.2\,x_{c3}$

s.t.

$x_{w1}\leq 550$, $x_{w2}\leq 250$, $x_{c1}\leq 620$, $x_{c2}\leq 280$

$$x_{w1}+x_{w2}+x_{w3}\geq 1500$$

$$x_{c1}+x_{c2}+x_{c3}\geq 1200$$

$$\frac{x_{c1}+x_{c2}+x_{c3}}{x_{w1}+x_{w2}+x_{w3}}\geq 2$$

$$\Rightarrow 2(x_{w1}+x_{w2}+x_{w3})-(x_{c1}+x_{c2}+x_{c3})\leq 0$$

all vars nonnegative

(a) Solution: z = \$14,918

$$x_{w1}=550, x_{w2}=250, x_{w3}=700$$

$$x_{c1}=620, x_{c2}=280, x_{c3}=2100$$

(b) Increasing marginal unit costs ensures using the less expensive capacity before the more expensive ones. Else, if the cost functions are not monotonically increasing, additional constraints are needed to ensure the capacity restriction is satisfied.

48

x_j=units produced of product j (=1,2,3,4)

Unit profit:

Product 1: 75−2×10 −3×5−7×4=\$12

Product 2: 70−3×10 −2×5−3×4=\$18

Product 3: 55−4×10 −1×5−2×4=\$2

Product 1: 45−2×10 −2×5−1×4=\$11

Max $z=12x_1+18x_2+2x_3+11x_4$ s.t.

$2x_1+3x_2+4x_3+2x_4 \leq 500$

$3x_1+2x_2+1x_3+2x_4 \leq 380$

$7x_1+3x_2+2x_3+1x_4 \leq 4500$

Solution: z=\$2950

x_1=0, x_2=133.33, x_3=0, x_4=50

49

x_j=units produced of model j (=1,2,3,4)
Max $z= 30x_1 + 20x_2 + 50x_3$ s.t.
(1) $2x_1 +3x_2 +5x_3 \le 4000$
(2) $4x_1 +2x_2 +7x_3 \le 6000$
(3) $x_1 +.5\, x_2 +(1/3)\, x_3 \le 1500$
(4) $x_1 /3= x_2 /2 \rightarrow 2x_1 -3x_2 = 0$
(5) $x_2 /3= x_3 /5 \rightarrow 5x_2 -2x_3 = 0$
$x_1 \ge 200\; x_2 \ge 200,\; x_3 \ge 150$
all vars nonnegative
Solution: z=\$4108.08
x_1 =324.32, x_2 =216.22, x_3 =540.54

50

For i= (1,2,3) and j=(1,2)
x_{ij}= cartons at start of month i from supplier j
I_i = end inventory in period i
c_{ij}=price per unit of x_{ij}
h=holding cost per unit per month
C=supplier capacity per month
d_i =demand for month i

$$\text{Min } z = \sum_{i=1}^{3}\sum_{j=1}^{2} c_{ij}x_{ij} + h\left[\sum_{i=1}^{3}\left(\sum_{j=1}^{2}\frac{(I_{i-1}+x_{ij})+I_i}{2}\right)\right] \text{ s.t.}$$

$$\sum_{j=1}^{2} I_{i-1} + x_{ij} + I_i = d_i, i = 1,2,3$$

$x_{ij} \le C$, for all i and j
Solution:

i	x_{i1}	x_{i2}	I_i
1	400	100	0
2	400	400	200
3	200	0	0

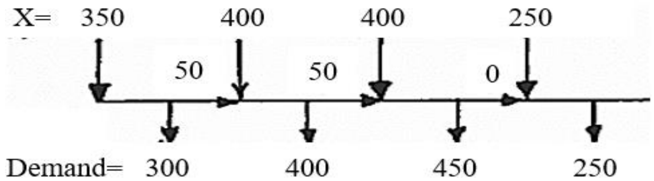
51

x_i = production in quarter i
I_i = end inventory in quarter i
Min $z= 20x_1 +22x_2 +24x_3 +26x_4 +3.5(I_1+ I_2+ I_3)$ s.t.
x_1=300+I_1, $I_1 + x_2$=400+ I_2, $I_2 + x_3$=450+I_3,
$I_3+ x_4$=250
$x_i \le 400$, i=1,2,3, $I_i \le 100$, i=1,2, I_0= I_4=0

continued…

51 cont'd

Solution: z=\$32,250



52

For i= (1,2) and j=(1,2,3)
x_{ij}=quantity of product i in month j
I_{ij}=end inventory of product i in month j

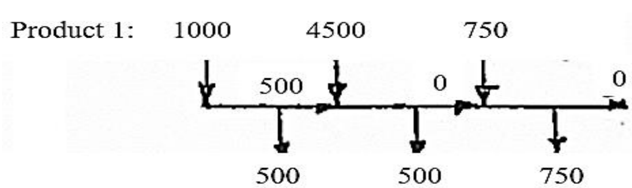
$$\text{Min } z = 30\sum_{j=1}^{3} x_{1j} + 28\sum_{j=1}^{3} x_{2j} + .9\sum_{j=1}^{3} I_{1j} + .75\sum_{j=1}^{3} I_{2j} \quad \text{s.t.}$$

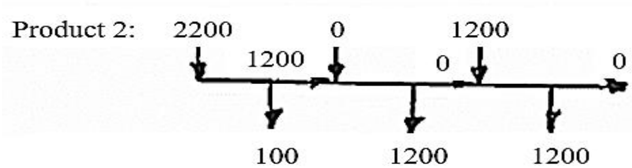
$$\frac{x_{1j}}{1.25} + x_{2j} \le \begin{cases} 3000, & j=1 \\ 3500, & j=2 \\ 3000, & j=3 \end{cases}$$

$$\left.\begin{array}{l} I_{1,j-1} + x_{1j} - I_{1j} \le \begin{cases} 500, & j=1 \\ 5000, & j=2 \\ 750, & j=3 \end{cases} \\ I_{2,j-1} + x_{2j} - I_{2j} \le \begin{cases} 1000, & j=1 \\ 1200, & j=2 \\ 1200, & j=3 \end{cases} \end{array}\right\} I_{i0} = 0, i = 1,2$$

all vars nonnegative

Solution: Cost z = \$39720





53

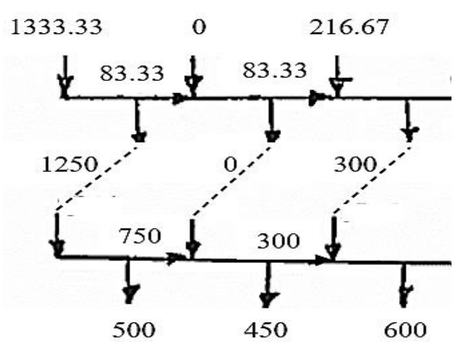
For i= (1,2) and j=(1,2,3)
x_{ij}=quantity by operation i in month j
I_{ij}=entering inventory of operation i in month j

$$\text{Min } z = .2\sum_{j=1}^{2} I_{1j} + .4\sum_{j=1}^{2} I_{2j}$$
$$+10x_{11} + 12x_{12} + 11x_{13} + 15x_{21} + 18x_{22} + 16x_{23}$$

s.t.
$.6x_{11} \le 800$, $.6x_{12} \le 700$, $.6x_{13} \le 550$
$.8\,x_{21} \le 1000$, $.8\,x_{22} \le 850$, $.8\,x_{23} \le 700$

$$\left.\begin{matrix} x_{1j} + I_{1,j-1} = x_{2j} + I_{1j} \\ x_{2j} + I_{2,j-1} = x_{2j} + I_{2j} + d_j \end{matrix}\right\}, I_{i0} = 0 \text{ for } i=1, 2$$

Solution: Cost z = $39,720



54

x_j=units of product j, j = 1, 2
y_i^- = unused hours of machine i
y_i^+ = overtime hours of machine i $\quad i$=1, 2

$$\text{Max } z = 110x_1 + 118x_2 - 100(y_1^+ + y_2^+) \text{ s.t.}$$
$$\frac{x_1}{5} + \frac{x_2}{5} + y_1^- - y_1^+ = 8$$
$$\frac{x_1}{8} + \frac{x_2}{4} + y_2^- - y_2^+ = 8$$

all vars. nonnegative
Solution: Revenue z = $6,232
$x_1 = 56, x_2 = 4, y_1^+ = 4$ hrs, $y_2^+ = 0, y_1^- = 0, y_2^- = 0$

55

x_i = nbr of 8-hr shift buses in period i
y_i = nbr of 12-hr-shift buses in period i
p = regular pay rate/hr
8-hour pay = $8p
12-hour pay = 8p +4×(1.5p) = $14p

$$\text{Min } z = p\left(8\sum_{i=1}^{6} x_i + 14\sum_{i=1}^{6} y_i\right) \text{ s.t.}$$

x						y							
1	2	3	4	5	6	1	2	3	4	5	6		
1						1				1	1	≥	4
1	1					1	1				1	≥	8
	1	1				1	1	1				≥	10
		1	1				1	1	1			≥	7
			1	1				1	1	1		≥	12
				1	1				1	1	1	≥	4

Solution: z = 196p$
$x_1 = 4$, $x_2 = 4$, $x_3 = 0$, $x_4 = 2$, $x_5 = 4$, $x_6 = 0$, sum =14 8-hr
$y_1 = 0$, $y_2 = 0$, $y_3 = 6$, $y_4 = 0$, $y_5 = 0$, $y_6 = 0$, sum =6 12-hr
Total nbr of buses= 14 8-hr and 6 12-hr = 20

8-hr buses only solution (see Example 2.4-5):
nbr of buses = 26
cost: 8p×26 =208p$
Conclusion: A mix of 8-hr and 12-hr buses is cheaper.

Chapter 2

56

x_i = nbr of volunteers starting in hour *i*

$$\text{Min } z = \sum_{i=1}^{14} x_i \text{ s.t.}$$

	x														
	1	2	**3**	**4**	5	**6**	**7**	8	9	**10**	11	**12**	13		
8:00	1													≥	4
9:00	1	1												≥	4
10:00	1	1												≥	6
11:00		1	1											≥	6
12:00			1	1										≥	8
1:00			1	1	1									≥	8
2:00				1	1	1								≥	6
3:00					1	1	1							≥	6
4:00						1	1	1						≥	4
5:00							1	1	1	1				≥	4
6:00								1	1	1	1			≥	6
7:00									1	1	1	1		≥	6
8:00											1	1	1	≥	8
9:00												1	1	≥	8

Solution: 32 volunteers
x_1 =4, x_3 =2, x_4 = 6, x_6 =2,
x_7 = 4, x_{10} = 6, x_{12} = 8
All other variables are zero

57

Add the constraints x_5=0 and x_{11}=0 in Problem 2-56. The solution in 2-56 happened to satisfy these constraints.

58

x_i = nbr of casuals starting on day *i* (*i*=1:Monday…)
Min $z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$ s.t.

	x								
	1	2	3	4	5	6	7		
M	1			1	1	1	1	≥	20
T	1	1			1	1	1	≥	14
W	1	1	1			1	1	≥	10
Th	1	1	1	1			1	≥	15
F	1	1	1	1	1			≥	18
Sat		1	1	1	1	1		≥	10
Sun			1	1	1	1	1	≥	12

Solution: z = 20 worker
$x_1 = 8$, $x_4 = 6$, $x_5 = 4$, $x_6 = 1$, $x_7 = 1$, all others = 0

59

x_i = nbr of casuals starting on day *i*
(*i*=1:Monday…)

Min $z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$
s.t.

	x										
	1	2	3	4	**5**	6	7	8	9		
8:01	1									≥	2
9:01	1	1								≥	2
10:01	1	1	1							≥	3
11:01		1	1	1						≥	4
12:01			1	1						≥	4
1:01				1		1				≥	3
2:01						1	1			≥	3
3:01						1	1	1		≥	3
4:01							1	1	1	≥	3

Solution: z = 9 worker
$x_1 = 2$, $x_3 = 1$, $x_4 = 3$, $x_7 = 3$, all others = 0

60

Let x_i = Nbr. of workers starting shift on day i, lasting for 7 days

y_{ij} = Nbr. of worker starting shift on day i and starting their 2 days of on day j, $i \neq j$

Thus, of the x_i workers who start on Monday, y12 will take T and W off, y13 will take W and Th off, as the following table shows:

	x1	x2	x3	4	x5	x6	x7
1	start x1 on **M**	y12	y12+y13	y13+y14	y14+y15	y15+y16	y16+y17
2	y27	start x2 on **T**	y23	y23+y24	y24+y25	y25+y26	y26+y27
3	y31+y37	y31	start x3 on **W**	y34	y34+y35	y35+y36	y36+y37
4	y41+y47	y41+y42	y42	start x4 on **Th**	y45	y45+y46	y46+y47
5	y51+y57	y51+y52	y53+y54		start x5 on **F**	y56	y56+y57
6	y61+y67	y61+y62	y62+y63	Y63+y64	y64	x6 start on **S**	y67
7	y71	y71+y72	y72+y73	y73+y74	y74+y75	y75	x7 start on **Su**
	12	18	20	28	32	40	40

Let

b_i = min number of workers needed for day i

$s = \sum_{i=1}^{7} x_i$

LP model: Minimize $z = s$, subject to

$$s - \sum_{i=1}^{7} x_i = 0$$

$$s - \sum_{i \neq j} \{\text{all } y_{ij} \text{ in column (day) } i\} \geq b_i, i = 1, 2, \ldots, 7$$

$$x_i - \sum_{\substack{j=1 \\ j \neq i}}^{7} y_{ij} = 0, i = 1, 2, \ldots, 7$$

continued…

continued…

60 cont'd

Solution: 42 workers

Starting on Day	Nbr	Non	Tue	Wed	Th	Fri	Sat	Sun
Mon	16		16	16				
Tu	8				8	8		
Wed	8	8	8					
Thu	0							
Fri	6			6	6			
Sat	2	2						2
Sun	2					2	2	
Nbr off		10	24	22	14	10	2	2
Nbr at work		32	18	20	28	32	40	40
Surplus		22	0	0	0	0	0	0

61

x_e = nbr of efficiency apartments
x_d = nbr of duplxes
x_s = nbr of single-family homes
x_r = retail space in ft^2

Max $z = 600x_e + 750x_d + 1200x_s + 100x_r$
s.t.
$x_e \le 500,\ x_d \le 300,\ x_s \le 250$
$x_r \ge 10x_e + 15x_d + 18x_s$
$x_r \le 10{,}000$
$$x_d \ge \frac{x_e + x_s}{2}$$
all var nonnegative
Solution: z= $1,595,714.29
$x_e = 207.14 \approx 207,\ x_d = 228.57 \approx 229,\ x_s = 250,$
$x_r = 10{,}000$
LP does not generate integer solution, hence is the rounding. See Chapter 9 for ILP.

62

x_i = acquired potion of property i
Each site is represented by a separate LP. The site that yields the smallest objective value is selected.
Site 1 LP:
Min $z = 25 + x_1 + 2.1x_2 + 2.35x_3 + 1.85x_4 + 2.95x_5$
s.t.
$x_4 \ge .75,\ x_i \le 1,\ i = 1, 2, \ldots, 5$
$20x_1 + 50x_2 + 50x_3 + 30x_4 + 60x_5 \ge 200$
Solution: z= $34.6625M
$x_1 = .875,\ x_2 = 1,\ x_3 = 1,\ x_4 = .75,\ x_5 = 1$
Site 2 LP:
Min $z = 27 + x_1 + 2.8x_2 + 1.9x_3 + 2.85x_4 + 2.5x_5$
s.t.
$x_3 \ge .5,\ x_i \le 1,\ i = 1, 2, 3, 4$
$80x_1 + 60x_2 + 50x_3 + 70x_4 \ge 200$
Solution: z= $34.35M
$x_1 = 1,\ x_2 = 1,\ x_3 = .5,\ x_4 = .5$
Select site 2

63

x_{ij}=portion of project i completed in year j

Years of income for project i = $\sum_{j=1}^{5}(5-j)x_{ij}$

Max $z = .05(4x_{11} + 3x_{12} + 2x_{13}) +$
$.07(3x_{22} + 2x_{23} + x_{24}) +$
$.15(4x_{31} + 3x_{32} + 2x_{33} + x_{34}) +$
$.02(2x_{43} + x_{44})$
s.t.
Completion:
$x_{11} + x_{12} + x_{13} = 1$ (project 1)
$x_{43} + x_{44} = 1$ (project 4)
$.25 \le x_{22} + x_{23} + x_{24} + x_{25} \le 1$ (project 2)
$.25 \le x_{31} + x_{32} + x_{33} + x_{34} + x_{35} \le 1$ (project 3)
Budget:
$5x_{11} + 15x_{31} \le 3$ (year 1)
$5x_{12} + 8x_{22} + 15x_{32} \le 6$ (year 2)
$5x_{13} + 8x_{23} + 15x_{33} + 1.2x_{43} \le 7$ (year 3)
$8x_{24} + 15x_{34} + 1.2x_{44} \le 7$ (year 4)
$8x_{25} + 15x_{35} \le 7$ (year 5)
all vars nonnegative
Solution: z=$523,750
$x_{11} = .6,\ x_{12} = .4$ (Project 1 completed in yr 2)
$x_{24} = .225,\ x_{25} = .025$ (Project 2 25% done in yr 5)
$x_{32} = .267,\ x_{33} = .387,\ x_{34} = .346$ (Project 3 completed in yr 5)
$x_{43} = 1$ (Project 4 completed in yr 3)

64

x_L = nbr of low income units
x_m = nbr of middle income units
x_u = nbr of upper income units
x_p = nbr of public housing units
x_s = nbr of school rooms
x_r = nbr of retail units
x_c = nbr of condemned homes
Max $z = 7x_L + 12x_m + 20x_u + 5x_p - 10x_s + 15x_r - 7x_c$ s.t.
$100 \le x_L \le 200,\ 125 \le x_m \le 190,$
$75 \le x_u \le 260,\ 0 \le x_p \le 600,\ 0 \le x_c \le 2/.045$
$.05x_L + .07x_m + .03x_u + .025x_p + .045x_s + .1x_r \le .85(50 + .25x_c)$
$x_r \ge .023x_L + .034x_m + .046x_u + .023x_p + .034x_s$
$25x_s \ge 1.3x_L + 1.2x_m + .5x_u + 1.4x_p$
continued…

64 cont'd

Solution: z= $8290.30

x_L =100, x_m =125, x_u =227.0, x_p=300, x_s= 32.54, x_r =25, x_c = 0.

65

x_1 =nbr of single-family homes
x_2 = nbr of double-family homes
x_3 = nbr of triple-family homes
x_4 = nbr of recreations areas

Max z= 10,000 x_1 +12,000 x_2 +15,000 x_3 s.t.

$2x_1 + 3x_2 + 4x_3 + x_4 \leq .85\times800$

$\frac{x_1}{x_1+x_2+x_3} \geq .5$ or $.5x_1 - .5x_2 - .5x_3 \geq 0$

$x_4 \geq \frac{x_1+2x_2+3x_3}{200}$ or $200x_4 - x_1 - 2x_2 - 3x_3 \geq 0$

$1000x_1 + 1200x_2 + 1400x_3 + 450x_4 \geq 100,000$

$400x_1 + 600x_2 + 840x_3 + 450x_4 \leq 200,000$

all vars nonnegative

Solution: z=$3,391,521.20

x_1 =339.15, x_2 =0, x_3 =0, x_4 =1.69 areas

66

New land use constraint: $x_1 + x_2 + x_3 + x_4$

$2x_1 + 3x_2 + 4x_3 + x_4 \leq .85\times(800+100)$

Solution: z=$3,815,461.35

x_1 =381.54, x_2 =0, x_3 =0, x_4 =1.91 areas

Δz= 3,815,461.35 − 3,391,521.20 = $423,940

Δz < $450,000. The purchasing cost of 100 new acres. Purchase of additional acres is not recommended.

67

x_s = tons of strawberry/day
x_g = tons of grapes/day
x_a = tons of apples/day
x_A = cans of drink A/day
x_B = cans of drink B/day
x_C = cans of drink C/day
x_{sA} = lb strawberry in drink A/day
x_{sB} = lb strawberry in drink B/day
x_{gA} = lb grapes in drink A/day
x_{gB} = lb grapes in drink B/day
x_{gC} = lb grapes in drink C/day
x_{aB} = lb apples in drink B/day
x_{aC} = lb apples in drink C/day

Max z= 1.15 x_A+1.25x_B+1.2x_C−200 x_s−100 x_g−90 x_a

s.t.

$x_a \leq 200$, $x_g \leq 100$, $x_a \leq 150$
$x_{sA}+x_{sB}$ = 1500 x_s
$x_{gA}+x_{gB}+x_{gC}$ = 1200x_g
$x_{aB}+x_{aC}$ =1000 x_a

$x_A = x_{sA} + x_{gA}$
$x_B = x_{sB}+x_{gB}+x_{aB}$
$x_C = x_{gC}+x_{aC}$
$x_{sA} = x_{gA}$
$x_{sB} = x_{gB}$
$x_{sB} = .5x_{aB}$
$3 x_{gC} = 2 x_{aC}$

Solution: $x_A = x_{sA} + x_{gA}$
$x_B = x_{sB}+x_{gB}+x_{aB}$
x_A = 90,000 cans, x_B =300,000 cans, x_C = 0

		j		
	i	A	B	C
x_{ij}	s	45,000	75,000	0
	g	45,000	75,000	0
	a	0	150,000	0

68

x_s = lb of screws/package
x_b =lb of bolts/package
x_n = lb of nuts/package
x_w = lb of washers/package
Min z =1.1 x_s +1.5 x_b +(70/80)x_n +(20/30) x_w s.t.
$Y = x_s + x_b + x_n + x_w$
$x_s \geq .1Y,\ x_b \geq .25Y, x_b \leq 50\ x_w,\ x_b \leq 10x_n$
$x_n \leq .15Y,\ Y \geq 1$
all vars nonnegative
Solution: z = \$1.12
$Y =1,\ x_s =.5,\ x_b = .25,\ x_n = .15,\ x_w =.1$

69

x_{oA}, x_{oB}, x_{oC} = lb of oats in cereal $A,\ B,\ C$
x_{rA}, x_{rC} = lb of rasins in cereal $A,\ C$
x_{cB}, x_{cC} = lb of coconuts in cereal $B,\ C$
x_{aA}, x_{aB}, x_{aC} = lb of almond in cereal $A,\ B,\ C$

$$\text{Max } z = \tfrac{1}{5}(2W_A + 2.5W_B + 3W_C) - \tfrac{1}{2000}(100Y_o + 120Y_r + 110Y_c + 200Y_a) \text{ s.t.}$$

$Y_o = x_{oA} + x_{oB} + x_{oC}$
$Y_r = x_{rA} + x_{rC}$
$Y_c = x_{cB} + x_{cC}$
$Y_a = x_{aA} + x_{aB} + x_{aC}$
$W_A = x_{oA} + x_{rA} + x_{aA}$
$W_B = x_{oB} + x_{cB} + x_{aB}$
$W_C = x_{oC} + x_{rC} + x_{cC} + x_{aC}$
$W_A \geq 500 \times 5 (= 2500)$
$W_B \geq 600 \times 5 (= 3000)$
$W_C \geq 800 \times 5 (= 4000)$
$Y_o \leq 5 \times 2000 (= 10{,}000)$
$Y_r \leq 2 \times 2000 (= 4{,}000)$
$Y_c \leq 1 \times 2000 (= 2{,}000)$
$Y_a \leq 1 \times 2000 (= 2{,}000)$

continued…

69 cont'd

$x_{oA} = \frac{50}{5} x_{rA},\ x_{oA} = \frac{50}{2} x_{aA}$
$x_{oB} = \frac{60}{5} x_{cB},\ x_{oB} = \frac{60}{3} x_{aB}$
$x_{oC} = \frac{60}{3} x_{rC},\ x_{oC} = \frac{60}{4} x_{cC},\ x_{oC} = \frac{60}{2} x_{aC}$
all vars nonnegative
Solution: z= \$5384.84/day
W_A=2500 lb *or* 500 boxes/day
W_B=3000 lb *or* 600 boxes/day
W_C=5793.45 lb *or* ≈1158 boxes/day
Y_o=10,000 lb *or* 5 tons/day
Y_r= 471.19 lb *or* .214 ton/day
Y_c= 428.16 lb *or* .236 ton/day
Y_a= 394.11 lb *or* .197 ton/day

70

For $i = 1,\ 2,$
x_{Ai} = bbl of gasoline A in fuel i
x_{Bi} = bbl of gasoline B in fuel i
x_{Ci} = bbl of gasoline C in fuel i
x_{Di} = bbl of gasoline D in fuel i

$$\text{Max } z = 200F_1 + 250F_2 - (120Y_A + 90Y_B + 100Y_C + 150Y_D) \text{ s.t.}$$

$Y_A = x_{A1} + x_{A2}, Y_B = x_{B1} + x_{B2}$
$Y_C = x_{C1} + x_{C2}, Y_D = x_{D1} + x_{D2}$
$F_1 = x_{A1} + x_{B1} + x_{C1} + x_{D1}$
$F_2 = x_{A2} + x_{B2} + x_{C2} + x_{D2}$
$x_{A1} = x_{B1}, x_{A1} = .5x_{C1}, x_{A1} = .25x_{D1}$
$x_{A2} = x_{B2}, x_{A2} = 2x_{C2}, x_{A2} = (2/3)x_{D1}$
$Y_A \leq 1000, Y_B \leq 1200, Y_C \leq 900, Y_D \leq 1500$
$F_1 \geq 200, F_2 \geq 400$
Solution: z= 495,416.67
Y_A = 958.33 bbl/day, Y_B = 958.33 bbl/day
Y_C = 516.67 bbl/day, Y_D = 1500 bbl/day .
F_1 = 200 bbl/day, F_2 = 3733.33 bbl/day

71

$A =$ bbl crude A/day

$B =$ bbl crude B/day

$R =$ bbl regular/day

$P =$ bbl premium/day

$J =$ bbl jet/day

$+/-$ superscript $\equiv$ (R, P, J)surplus/shortage

$$\text{Max } z = 50(R - R^+) + 70(P - P^+) + 120(J - J^+)$$
$$- (10R^- + 15P^- + 20J^-) - (2R^+ + 3P^+ + 4J^+)$$
$$- (30A + 40B) \quad \text{s.t.}$$

$A \le 2500, B \le 3000$

$R = .2A + .25B, R + R^- - R^+ = 500$

$P = .1A + .3B, P + P^- - P^+ = 700$

$J = .25A + .1B, J + J^- - J^+ = 400$

Solution: $z = \$21,852.94$

$A = 1176.47$ bbl/day, $B = 1058.82$ bbl/day

$R = 500$ bbl/day, $P = 435.29$ bbl/day

$J = 400$ bbl/day

72

$NR =$ nafta bbl/day in regular

$NP =$ nafta bbl/day in premium

$NJ =$ nafta bbl/day in jet

$LR =$ light oil bbl/day in regular

$LP =$ light oil bbl/day in premium

$LJ =$ light oil bbl/day in jet

$+/-$ superscript $\equiv$ (R, P, J)surplus/shortage

72 cont'd

$$\text{Max } z = 50(R - R^+) + 70(P - P^+) + 120(J - J^+)$$
$$- (10R^- + 15P^- + 20J^-) - (2R^+ + 3P^+ + 4J^+)$$
$$- (30A + 40B) \quad \text{s.t.}$$

$A \le 2500, B \le 3000$

$R + R^- - R^+ = 500$

$P + P^- - P^+ = 700$

$J + J^- - J^+ = 400$

$.35A + .45B = NR + NP + NJ$

$.6A + .5B = LR + LP + LJ$

$R = NR + LR, P = NP + LP, J = NJ + LJ$

Solution: $z = \$71,473.68$

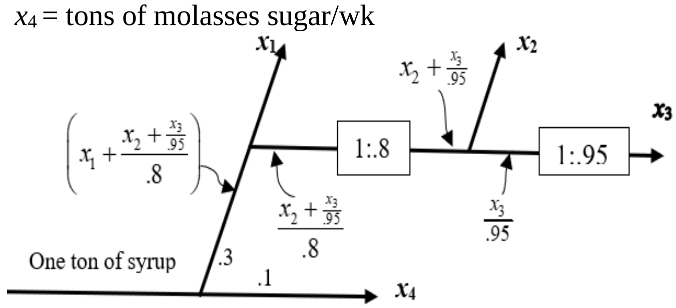
$A = 1684.21$ bbl/day, $B = 0$

$R = 500$ bbl/day, $P = 700$ bbl/day

$J = 400$ bbl/day

73

$x_1 =$ tons of brown sugar/wk
$x_2 =$ tons of white sugar/wk
$x_3 =$ tons of powdered sugar/wk
$x_4 =$ tons of molasses sugar/wk



$$\text{Max } z = 150x_1 + 200x_2 + 230x_3 + 35x_4 \quad \text{s.t.}$$

$x_4 \le 4000 \times .1 (= 400)$

$x_1 + \left(\frac{x_2 + \frac{x_3}{0.95}}{.8} \right) \le .3 \times 4000$ *or* $.76x_1 + .95x_2 + x_3 \le 912$

$x_1 \ge 25, x_2 \ge 25, x_3 \ge 25, x_4 \ge 25$

Solution: $z = \$222,667.50$

$x_1 = 25, x_2 = 25, x_3 = 869.25, x_4 = 400 / wk$

74

$A =$ bbl/hr of stock A

$B =$ bbl/hr of sock B

for $i = (1, 2$:

$Y_{Ai} =$ bbl/hr of A in gasoline i

$Y_{Bi} =$ bbl/hr of B in gasoline i

Max $z = 7(Y_{A1} + Y_{B1}) + 10(Y_{A2} + Y_{B2})$ s.t.

$A = Y_{A1} + Y_{A2}, A \le 450$

$B = Y_{B1} + Y_{B2}, B \le 700$

$98Y_{A1} + 89Y_{B1} \ge 91(Y_{A1} + Y_{B1})$

$98Y_{A2} + 89Y_{B2} \ge 93(Y_{A2} + Y_{B2})$

$10 \ge 12(Y_{A1} + Y_{B1})$

$10Y_{A2} + 8Y_{B2} \ge 12(Y_{A2} + Y_{B2})$

all vars no$Y_{A1} + 8Y_{B1}$nnegative

Solution: $z = \$10,675$

$A = 450$ bl/hr, $B = 700$ bl/hr

gas 1 produced $= Y_{A1} + Y_{B1}$

$= 61.11 + 213.89 = 275$ bbl/hr

gas 2 produced $= Y_{A2} + Y_{B2}$

$= 385.89 + 486.11 = 875$ bbl/hr

75

$S =$ steel scrap tons/day

$A =$ aluminum scrap tons/day

$C =$ cast iron scrap tons/day

$Ab =$ aluminum briquette tons/day

$Sb =$ silicon briquette tons/day

$a =$ tons of aluminum/day

$g =$ tons of graphite/day

$s =$ tons of silicon/day

continued…

75 cont'd

$aI =$ tons of alum. ingots-I/day

$aII =$ tons of alum. ingots-II/day

$gI =$ tons of graph. ingots-I/day

$gII =$ tons of graph. ingots-II/day

$sI =$ tons of silicon ingots-I/day

$sII =$ tons of silicon ingots-II/day

$\mathrm{I}_1 =$ tons of ingot-I/day

$\mathrm{I}_2 =$ tons of ingot-II/day

Min $z = 100S + 150A + 75C + 900Ab + 380Sb$ s.t.

$a = .1S + .95A + Ab$

$g = .05S + .01A + .15C$

$s = .04S + .02A + .08C + Sb$

$I_1 = aI + gI + sI$

$_2 = aII + gII + sII$

$aI + aII \le a, sI + sII \le s, gI + gII \le g$

$.081I_1 \le aI \le .108I_1$

$.015I_1 \le gI \le .03I_1$

$.025I_1 \le sI \le \infty$

$.062I_2 \le aII \le .089I_2$

$.015I_2 \le gII \le \infty$

$.025I_2 \le sII \le .041I_2$

$I_1 \ge 130, I_2 \ge 250$

Solution: $z = \$117,435.65$

$S = 0, A = 38.2, C = 1489.41$

$Ab = Sb = 0$

$I_1 = 130, I_2 = 250$

$a = 36.29, g = 223.79, s = 119.92$

76

For i = 1, 2, 3; j = A, B; k = I, II, III, IV:

x_{ij} = tons of ore i allocated to alloy j

W_j = tons of alloy j produced

M_k = tons of metal k extracted from all three ores

continued…

76 cont'd

Min $z = 200W_A + 300W_B - 30(x_{1A} + x_{1B})$

$- 40(x_{2A} + x_{2B}) - 50(x_{3A} + x_{3B})$ s.t.

Alloy A specs:

$\left.\begin{array}{l} M_I = .2x_{1A} + .1x_{2A} + .05x_{3A} \\ M_I \le .8W_A \end{array}\right\} \equiv .2x_{1A} + .1x_{2A} + .05x_{3A} \le .8W_A$

M_{II}: $.1x_{1A} + .2x_{2A} + .05x_{3A} \le .3W_A$

M_{IV}: $.3x_{1A} + .3x_{2A} + .2x_{3A} \le .5W_A$

Alloy B specs:

M_{II}: $.1x_{1B} + .2x_{2B} + .05x_{3B} \ge .4W_B$

M_{II}: $.1x_{1B} + .2x_{2B} + .05x_{3B} \le .6W_B$

M_{III}: $.3x_{1B} + .3x_{2B} + .7x_{3B} \ge .3W_B$

M_{IV}: $.3x_{1B} + .3x_{2B} + .2x_{3B} \le .7W_B$

Ore constraints:

$x_{1A} + x_{1B} \le 1000$

$x_{2A} + x_{2B} \le 2000$

$x_{3A} + x_{3B} \le 3000$

Solution: $z = 400,000$

$W_A = 1800, W_B = 1000$

$x_{1A} = 1000$, $x_{1B} = 0$, $x_{2A} = 0$

$x_{2B} = 2000$, $x_{3A} = 3000$, $x_{3B} = 0$

77

x_i = Shelf space (in^2) allocated to cereal i

Max $z = 1.1x_1 + 1.3x_2 + 1.08x_3 + 1.25x_4 + 1.25x_5$ s.t.

$16x_1 + 24x_2 + 14x_3 + 22x_4 + 20x_5 \le 5000$

$x_1 \le 100$, $x_2 \le 85$, $x_3 \le 140$, $x_4 \le 80$, $x_5 \le 90$

all vars nonnegative

Solution: z= \$314/day

$x_1 = 100$, $x_2 = 0$, $x_3 = 140$, $x_4 = 0$, $x_5 = 44$

78

x_i = nbr of ads in issue i (=1,2,3,4)

Min $z = S_1^- + S_2^- + S_3^- + S_4^-$ s.t.

$(-30 + 60 + 30)x_1 + S_1^- - S_1^+ = .51(400)$

$(80 + 30 - 45)x_2 + S_2^- - S_2^+ = .51(400)$

$(40 + 10)x_3 + S_3^- - S_3^+ = .51(400)$

$(90 - 25)x_4 + S_4^- - S_4^+ = .51(400)$

$1500(x_1 + x_2 + x_3 + x_4) \le 100,000$

all vars nonnegative

Solution: $x_1 = 3.4, x_2 = 3.14, x_3 = 4.08, x_4 = 3.14$

79

For i =1,2 and j = 1,2,3

x_{ij}= units of part j produced by department i

Max z= min$\{x_{11} + x_{21}, x_{12} + x_{22}, x_{13} + x_{23}\}$

or

Max $z = y$ s.t.

$y \le x_{11} + x_{21}$

$y \le x_{12} + x_{22}$

$y \le x_{13} + x_{23}$

$\frac{x_{11}}{8} + \frac{x_{12}}{5} + \frac{x_{13}}{10} \le 100$

$\frac{x_{21}}{6} + \frac{x_{22}}{12} + \frac{x_{23}}{4} \le 80$

all vars nonnegative

Solution:

nbr of assembly units=y=556.2=556

$x_{11} = 354.78, x_{12} = 0, x_{13} = 556.52$

$x_{21} = 201.79, x_{22} = 556.52, x_{23} = 556.52 = 0$

80

x_i = tons of coal i (=1,2,3)

Min $z = 30x_1 + 35x_2 + 33x_3$ s.t.

$2500x_1 + 1500x_2 + 1600x_3 \le 2000(x_1 + x_2 + x_3)$

$x_1 \le 30$, $x_2 \le 30$, $x_3 \le 30$

$x_1 + x_2 + x_3 \ge 50$

continued…

80 cont'd

Solution: z= 1361.11

x_1=22.22, x_2=0, x_3=27.78

81

t_i = green time in sec for highway i (=1,2,3)

$$\max\ z = 3\left(\tfrac{500}{3600}\right)t_1 + 4\left(\tfrac{600}{3600}\right)t_2 + 5\left(\tfrac{400}{3600}\right)t_3$$

$$\left(\tfrac{500}{3600}\right)t_1 + \left(\tfrac{600}{3600}\right)t_2 + \left(\tfrac{400}{3600}\right)t_3 \le \left(\tfrac{510}{3600}\right)(2.2\times 60 - 3\times 10)$$

$$t_1 + t_2 + t_3 + 3\times 10 \le 2.2\times 20$$

Solution: z =$58.04/hr

$t_1 = 25, t_2 = 43.6, t_3 = 33.4$

82

y_i = observation i (=1, 2, …, 10)

fitted straight line: $\hat{y}_i = ai + b$

Let $d_i = |y_i - (ai+b)|, i = 1, 2, ..., 10$

Min $z = d_1 + d_2 + ... + d_{10}$

$\sum_{i=1}^{10} |y_i - (ai+b)|$ s.t.

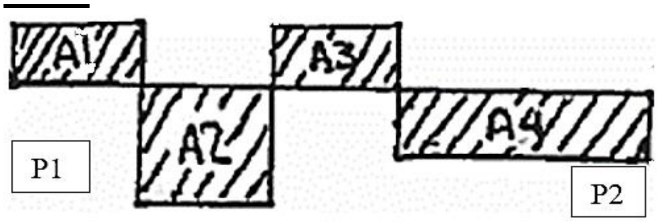
$y_i - (ai+b) \le d_i$

$y_i - (ai+b) \ge -d_i$

all $d_i \ge 0, a$ and b unrestricted

Solution:$\hat{y}_i = 2.85714i + 6.42857$

83



A1=(2mile×1760yard/mile)×10yard×50yard
=+1760K cubic yard

A2=−3520K, A3=+1760K, A4=−3520K

continued…

83 cont'd

center-to-center distances in miles=

	(5)A2	(6)A4
(1)A1	**2**	**7**
(2)A2	**2**	**3**
(3)P1	**3**	**8**
(4)P2	**7**	**2**

$ transportation/cubic yard =

	(5)A2	(6)A4
(1)A1	.2+**2**×.15=.50	.2+**7**×.15=1.25
(2)A2	.2+**2**×.15=.50	.2+**3**×.15=.65
(3)P1	(1.5+.2)+**3**×.15=2.15	(1.5+.2)+**8**×.15=2.90
(4)P2	(1.9+.2)+**7**×.15=3.15	(1.9+.2)+**2**×.15=2.40

Solution below in **bf:** see file *solverProb2-83.xslx*

x15	x16	x25	x26	x35	x36	x45	x46	z
1760	**0**	**0**	**1760**	**1760**	**0**	**0**	**1760**	**10032**
0.5	1.25	0.5	0.65	2.15	2.9	3.15	2.4	10032
1	1							<= 1760
		1	1					<=1760
				1	1			<= 20000
						1	1	<= 15000
1		1		1		1		>= 3520
	1		1		1		1	>= 3520

x_{15}=A1→A2, x_{26}=A3→A4

x_{55}=P1→A2, x_{46}=P2→A4

84

x_{ij}= blue regulars on front i defending line j

y_{ij}= blue reserves on front i defending line j

t_{ij}= delay days on front i defending line j

Max $z = \min\{\ t_{11} + t_{12} + t_{13},\ t_{21} + t_{22} + t_{23}\}$ s.t.

$x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} \le 200$

$y_{12} + y_{13} + y_{22} + y_{23} \le 200$

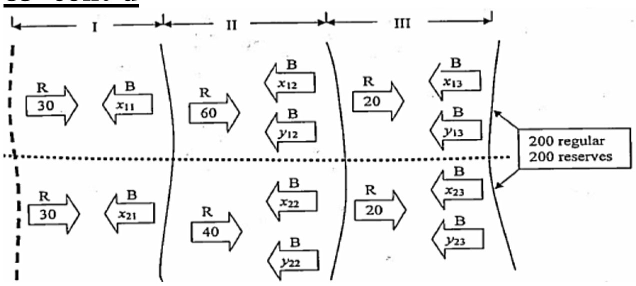
$t_{11} = .5 + 8.8\frac{x_{11}}{30},\quad t_{12} = .75 + 7.9\frac{x_{12}+y_{12}}{60}$

$t_{13} = .55 + 10.2\frac{x_{13}+y_{13}}{20},\quad t_{21} = 1.1 + 10.5\frac{x_{21}}{30}$

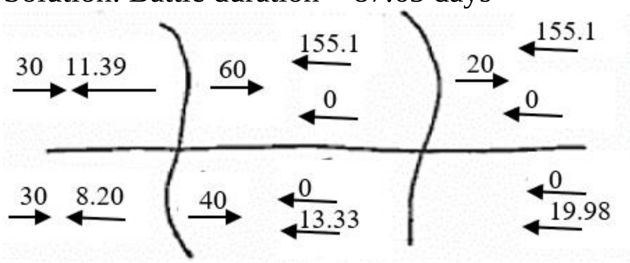
$t_{22} = 1.3 + 8.1\frac{x_{22}+y_{22}}{40},\quad t_{23} = 1.5 + 9.2\frac{x_{23}+y_{23}}{20}$

continued…

83 cont'd



Solution: Battle duration = 87.65 days



85

x_i = efficiency of plant i (=1,2,3,4)

Min $z = .2\times500x_1 + .25\times3000x_2 + .15\times6000x_3 + .18\times1000x_4$ s.t.

$.94\times500(1-x_1) + 3000(1-x_2) \le .0009\times220000$

$.94^2\times500(1-x_1) + .94\times3000(1-x_2) + 6000(1-x_3) \le .0008\times200000$

$.94^3\times500(1-x_1) + .94^2\times3000(1-x_2) + .94\times6000(1-x_3) + 1000(1-x_4) \le .0008\times210000$

$0 \le x_1 \le .99$, $0 \le x_2 \le .99$, $0 \le x_3 \le .99$, , $0 \le x_4 \le .99x_4$

Solution: cost/hr = \$1891.41

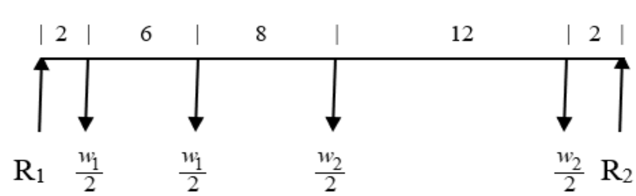
efficiency: $x_1 = .99$, $x_2 = .9661$, $x_3 = .99$ $x_4 = .9824$

86

w_i = capacity of yoke i in kips

R_1 = reaction in kips at left end

R_2 = reaction in kips at right end



continued…

86 cont'd

Max $z = w_1 + w_2$ s.t.

$$R_1 + R_2 = w_1 + w_2$$

$$2\left(\tfrac{w_1}{2}\right) + 8\left(\tfrac{w_1}{2}\right) + 16\left(\tfrac{w_2}{2}\right) + 28\left(\tfrac{w_2}{2}\right) = 30R_2$$

$$R_1 \le 25, R_2 \le 25, \tfrac{w_1}{2} \le 20, \tfrac{w_2}{2} \le 20$$

Solution: $w_1 = 20.59$ kips, w_2 29.41 kips

87

For i=1,2,3, j=1,2,3,4:

x_{ij}=nbr of aircraft type i allocated to route j

S_j = nbr of passengers denied service on route j

Min $z = 1000(3x_{11}) + 1100(2x_{12}) + 1200(2x_{13}) + 1500(x_{14}) + 800(4x_{21}) + 900(3x_{22}) + 1000(3x_{23}) + 1000(2x_{24}) + 600(5x_{31}) + 800(5x_{32}) + 800(4x_{33}) + 900(2x_{34}) + 40S_1 + 50S_2 + 45S_2 + 70S_4$ s.t.

$$\sum_{j=1}^{4} x_{1j} \le 5, \sum_{j=1}^{4} x_{2j} \le 5, \sum_{j=1}^{4} x_{3j} \le 5,$$

$50((3x_{11}) + 30(4x_{21}) + 20(5x_{31}) + S_1 \le 1000$

$50(2x_{12}) + 30(3x_{22}) + 20(5x_{32}) + S_2 \le 2000$

$50(2x_{13}) + 30(3x_{23}) + 20(4x_{33}) + S_3 \le 900$

$50(x_{14}) + 30(3x_{24}) + 20(2x_{34}) + S_4 \le 1200$

all vars nonnegative

Solution: cost = \$221,900

Type	Route	Nbr planes
1	1	5
2	4	8
3	1	2.5≈3
3	2	7.5≈8