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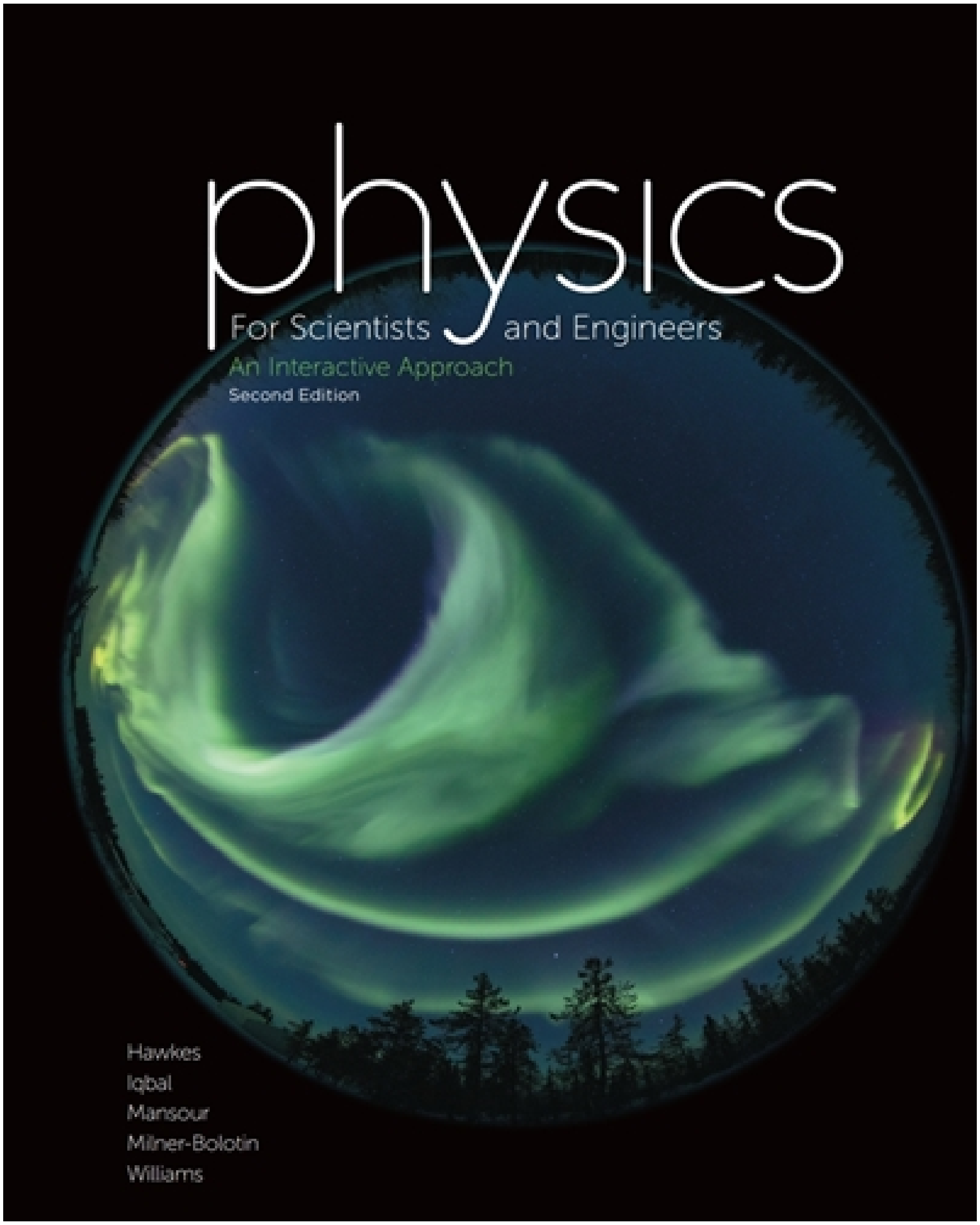

Solutions

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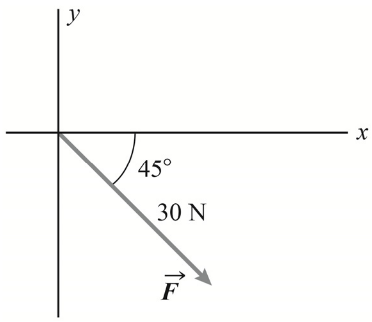
Chapter 2—SCALARS AND VECTORS

1. (a) $F = 30$ N, $\theta = 315°$

   (b) $F_x = 21$ N, $F_y = -21$ N

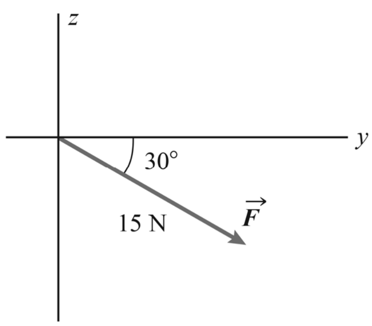
   (c) $\vec{F} = (21\text{ N})\hat{i} - (21\text{ N})\hat{j}$



2. (a) $F = 15$ N, $\theta = 330°$

   (b) $F_x = 13$ N, $F_y = -7.5$ N

   (c) $\vec{F} = (13\text{ N})\hat{j} - (7.5\text{ N})\hat{k}$



3. A parametric equation for the line is

$$\begin{cases} x = t \\ y = -3t + 4 \end{cases}$$

So a vector parallel to the given force vector will have components (1, −3), with magnitude

$$\sqrt{1^2 + (-3)^2} = 3.162$$

A unit vector parallel to the force is therefore given by

$$\hat{u}_F = \frac{1}{3.162}\hat{i} - \frac{3}{3.162}\hat{j} = 0.316\hat{i} - 0.949\hat{j}$$

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There are two possible force vectors, because the force can point in two possible directions along the line, either in the direction of $\hat{\boldsymbol{n}}$ or in the opposite direction. That is,

$$\vec{\boldsymbol{F}} = (\pm 8.00 \text{ N})\hat{\boldsymbol{u}}_{\text{F}}$$

$$\vec{\boldsymbol{F}}_{1,2} = \pm 8.00[(0.316 \text{ N})\hat{\boldsymbol{i}} - (0.949 \text{ N})]\hat{\boldsymbol{j}} = \pm[(2.53 \text{ N})\hat{\boldsymbol{i}} - (7.59 \text{ N})]\hat{\boldsymbol{j}}$$

4. $r = \sqrt{(3.00 \text{ m})^2 + (-4.00 \text{ m})^2 + 0} = 5.00 \text{ m}$

$$\theta = \tan^{-1}\left(-\frac{4.00 \text{ m}}{3.00 \text{ m}}\right) + 360° = -53.13° + 360° = 307°$$

counter-clockwise relative to the positive x-axis

5. To find the magnitude of the displacement vector:

$$\vec{\boldsymbol{D}} = [(3-[-2]) \text{ m}]\hat{\boldsymbol{i}} + [(0-2) \text{ m}]\hat{\boldsymbol{j}} + [(-2-0) \text{ m}]\hat{\boldsymbol{k}} = (5 \text{ m})\hat{\boldsymbol{i}} - (2 \text{ m})\hat{\boldsymbol{j}} - (2 \text{ m})\hat{\boldsymbol{k}}$$

$$D = \sqrt{5^2 + (-2)^2 + (-2)^2} \text{ m} = 5.74 \text{ m}$$

To find the direction angles of the displacement vector, we use Equation 2-12:

$$\cos\alpha = \frac{5}{5.74} \Rightarrow \alpha = \cos^{-1}\left(\frac{5}{5.74}\right) = 29.4°$$

$$\cos\beta = \frac{-2}{5.74} \Rightarrow \beta = \cos^{-1}\left(\frac{-2}{5.74}\right) = 110.°$$

$$\cos\gamma = \frac{-2}{5.74} \Rightarrow \gamma = \cos^{-1}\left(\frac{-2}{5.74}\right) = 110.°$$

6. (a) $r_{\text{AB}} = 3.00 \text{ m};\ r_{\text{BC}} = 7.00 \text{ m};\ r_{\text{CD}} = 6.00 \text{ m};\ r_{\text{DE}} = \sqrt{98} \text{ m} = 9.90 \text{ m};$

$r_{\text{EA}} = \sqrt{178} \text{ m} = 13.3 \text{ m}$

Thus,

$$r_{\text{EA}} > r_{\text{DE}} > r_{\text{BC}} > r_{\text{CD}} > r_{\text{AB}}$$

(b) No

(c) No

(d) There are no two displacements with the same magnitude, as shown in part (a).

7. (a) $\vec{\boldsymbol{D}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} + \vec{\boldsymbol{C}} = (1,-3,4) + (5,-2,1) + (-2,3,6) = (4,-2,11)$

(b) $\vec{\boldsymbol{E}} = -\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} - \vec{\boldsymbol{C}} = -\vec{\boldsymbol{D}} = (-4,2,-11)$

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(c) $\vec{F} = 2\vec{A} - 4\vec{B} + 3\vec{C} = (2,-6,8) - (20,-8,4) + (-6,9,18) = (-24,11,22)$

(d) $\vec{G} = 2(\vec{A} - 2\vec{B}) + \vec{C} = \vec{F} - 2\vec{C} = (-24,11,22) - (-4,6,12) = (-20,5,10)$

8. (a) The horizontal components of the given forces balance out. The vertical components of the forces are both directed upward, and each component equals 5 N. Therefore, to balance the vertical components, the third force must be 10 N and it must be at an angle of 120° to each force.

9. (c) Vector magnitudes are invariant with respect to translations or rotations of coordinate systems.

None of the other statements are correct. We will give one counterexample for each incorrect statement.

(a) Let us consider vector $\vec{A} = (1,0,0)$. Its magnitude is 1, and one of its components is also 1, which contradicts the given statement.

(b) Adding vector components is meaningless. To find a vector's magnitude we use Equation 2-3.

(c) Correct statement

(d) As we proved in Example 2-6, this statement is incorrect.

(e) There are an infinite number of vectors that have the same magnitude. They might be parallel or antiparallel to each other, they can be perpendicular, or they can be directed at arbitrary angles.

10. Two vectors are orthogonal if $\vec{A} \bullet \vec{B} = 0$. The pairs of vectors in parts (b) and (c) are orthogonal.

(a) $(2,3,-6) \bullet (-2,3,-6) = 2(-2) + 3(3) + (-6)(-6) = -4 + 9 + 36 = 41$

(b) $(2,3,-1) \bullet (-2,-1,-7) = 2(-2) + 3(-1) + (-1)(-7) = -4 - 3 + 7 = 0$

(c) $(-2,-3,-1) \bullet (-2,-1,7) = (-2)(-2) + (-3)(-1) + (-1)7 = 4 + 3 - 7 = 0$

(d) $(1,0,-6) \bullet (-2,3,0) = 1(-2) + 0 \cdot 3 + (-6)0 = -2 + 0 + 0 = -2$

(e) $(2,3,0) \bullet (0,-3,6) = 2(0) + 3(-3) + 0(6) = 0 - 9 + 0 = -9$

11. Student 2: Consider a vector in each plane directed such that they both form a 45° angle with the line of intersection of the two orthogonal planes. These two vectors are not perpendicular to each other. You can also consider two vectors that are both parallel to the line of intersection of the two orthogonal planes. These two vectors are parallel (or antiparallel) to each other and are not perpendicular to each other.

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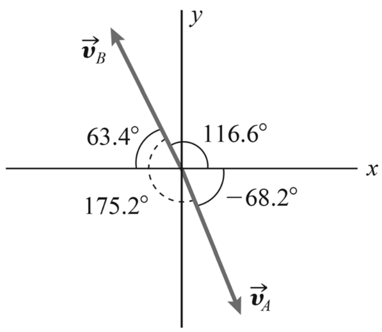
12.



(a) Object A is moving faster since the magnitude of its velocity is higher.

$$v_A = \sqrt{2^2 + (-5)^2}\ \text{m/s} = \sqrt{29}\ \text{m/s};\ v_B = \sqrt{(-2)^2 + 4^2}\ \text{m/s} = \sqrt{20}\ \text{m/s}$$

(b) Solution 1 (draw the vectors to identify the angles):

$$\theta_A = \tan^{-1}\left(\frac{-5.000}{2.000}\right) = -68.20°,\ \theta_B = \tan^{-1}\left(\frac{4.000}{-2.000}\right) + 180.0° = -63.43° + 180.0° = 116.6°$$

$$\Rightarrow \theta_{AB} = \Delta\theta = |\theta_B - \theta_A| = 116.6° - (-68.20°) = 184.8°$$

Solution 2:

$$\vec{v}_A \bullet \vec{v}_B = v_A v_B \cos(\theta_{AB})$$

Notice that the units on both sides are $(\text{m/s})^2$, so we can cancel them out.

$$2.000(-2.000) + (-5.000)4.000 = \sqrt{29}\sqrt{20}\cos(\theta_{AB})$$

$$\cos(\theta_{AB}) = \frac{-24.00}{\sqrt{29}\sqrt{20}} = -0.9965$$

$$\theta_{AB} = \cos^{-1}(-0.9965) = 175.2°$$

Notice that the angle between two vectors is often given as an angle smaller than 180°; out of the two possible ways to give the answer to the problem (175.2° and 184.8°), we choose 175.2°.

13. (a) $\vec{A}_x = (2,0,0),\ \vec{A}_y = (0,2,0),\ \vec{A}_z = (0,0,-2),\ \vec{B}_x = (-1,0,0),\ \vec{B}_y = (0,3,0),\ \vec{B}_z = (0,0,-2)$

Note: The projection of a vector onto a coordinate axis, e.g., the x-axis, is a vector directed in the x-direction (thus it has zero y- and z-components) and has the same x-component as the original vector.

(b) $\vec{A} \bullet \vec{B} = 2(-1) + 2(3) + (-2)(-2) = 8$

(c) Using the determinant form of a vector product (Equation 2-34), we get

$$\vec{A} \times \vec{B} = (-4 - (-6))\hat{i} + (2 - (-4))\hat{j} + (6 - (-2))\hat{k} = 2\hat{i} + 6\hat{j} + 8\hat{k}$$

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(d) The scalar product represents the level of collinearity between the two vectors—how much of one vector is directed in the direction of another vector (Equations 2-24 and 2-25). Based on the definition of the scalar product (Equation 2-17), if the vectors are perpendicular, the scalar product is zero. If the vectors are parallel, the scalar product is at its maximum value, as the vectors are collinear. If the vectors are antiparallel, the scalar product is at its minimum value. The scalar product of a given vector multiplied by itself gives the square of its magnitude. Thus,

$$\vec{A}\bullet\vec{A} = A \cdot A \cdot \cos(0°) = A^2 \Rightarrow A = \sqrt{\vec{A}\bullet\vec{A}}$$

The vector product represents a vector perpendicular to both given vectors (right-hand rule; Figure 2-18 in the textbook) and whose magnitude is equal to the area of a parallelogram whose sides are formed by the original vectors.

14. $$\cos\alpha = \frac{2.00}{\sqrt{2.00^2 + 5.00^2 + 3.00^2}} = 0.324 \Rightarrow \alpha = 71.1°$$

$$\cos\beta = \frac{-5.00}{\sqrt{2.00^2 + 5.00^2 + 3.00^2}} = -0.811 \Rightarrow \beta = 144°$$

$$\cos\gamma = \frac{-3.00}{\sqrt{2.00^2 + 5.00^2 + 3.00^2}} = -0.487 \Rightarrow \gamma = 119°$$

To verify that Equation 2-11 is satisfied, note that the magnitude of the position vector is $r = \sqrt{2.00^2 + 5.00^2 + 3.00^2} = \sqrt{38.0}$. Therefore, we can express Equation 2-11 as

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \left(\frac{2.00}{\sqrt{38.0}}\right)^2 + \left(\frac{-5.00}{\sqrt{38.0}}\right)^2 + \left(\frac{-3.00}{\sqrt{38.0}}\right)^2$$

$$= \frac{2.00^2}{38.0} + \frac{5.00^2}{38.0} + \frac{3.00^2}{38.0} = \frac{38.0}{38.0} = 1.00$$

15. Disagree; if all of the components of a vector equal 1, then the vector is not a unit vector. For example, if we have $\vec{A}(1,1,1)$, then $A = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \neq 1$, so it is not a unit vector.

16. $$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (2.00\text{ N})\hat{i} + (3.00\text{ N})\hat{j} - (6.00\text{ N})\hat{k}$$

$$\left|\vec{F}_1 + \vec{F}_2 + \vec{F}_3\right| = \sqrt{2.00^2 + 3.00^2 + 6.00^2}\text{ N} = 7.00\text{ N}$$

17. $$\vec{F}_1 = (1.00\text{ N})\hat{i} - (3.00\text{ N})\hat{j} + (1.00\text{ N})\hat{k}$$

$$\vec{F}_2 = (-2.00\text{ N})\hat{i} - (4.00\text{ N})\hat{j} + (3.00\text{ N})\hat{k}$$

$$\vec{F}_3 = (1.00\text{ N})\hat{i} + (2.00\text{ N})\hat{j} + (0.00\text{ N})\hat{k}\text{ N}$$

$$F_1 = \sqrt{11.00}\text{ N},\ F_2 = \sqrt{29.00}\text{ N},\ F_3 = \sqrt{5.00}\text{ N}$$

$$\vec{F}_R = (-5.00\text{ N})\hat{j} + (4.00\text{ N})\hat{k},\ F_R = \sqrt{41.00}\text{ N}$$

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18. Let $\vec{A}$ be parallel to the x-axis. Let us consider $\vec{A}$ to be the base of this parallelogram. Then the height of the parallelogram can be expressed as the projection of $\vec{B}$ onto the y-axis as $h = B \sin(\theta)$. The area of the parallelogram can then be expressed as Area = base · height $= AB \sin(\theta) = \left|\vec{A} \times \vec{B}\right|$.

19. Disagree. The magnitude of the resultant is not equal to the sum of the radius vectors of individual vectors. Similarly, the resultant angle is not equal to the sum of the individual vector angles. You may want to use the PhET computer simulation "Vector Addition" to check this.

20. The polar coordinate system is advantageous when the situation to be represented has circular symmetry, such as movement in a circle. However, representing a simple straight line in polar coordinates can become fairly complicated. A straight line in Cartesian coordinates can be written as $y = mx + c$, where m represents the slope and c represents the y intercept. But a straight line in polar coordinates is expressed as $r = \dfrac{a}{b\cos\theta + c\sin\theta}$, which can be difficult to handle in calculations.

21. Mathematically, the head-to-tail rule is equivalent to addition of vectors in Cartesian form. The associative property of vector addition, that is, $\vec{V}_1 + \vec{V}_2 + \vec{V}_3 = (\vec{V}_1 + \vec{V}_2) + \vec{V}_3 = \vec{V}_1 + (\vec{V}_2 + \vec{V}_3)$, proves the head-to-tail rule for more than two vectors. You may want to use the PhET computer simulation "Vector Addition" to check this.

22. If the vectors are opposite, then the angles between corresponding direction angles are 180°: $|\alpha_1 - \alpha_2| = 180°,\ |\beta_1 - \beta_2| = 180°,\ |\gamma_1 - \gamma_2| = 180°$

23. The scalar product of two vectors is defined as the product of the magnitudes of the two vectors and the cosine of the angle between them. Because the definition involves the product of three real numbers, and the product of real numbers is commutative, the scalar product of two vectors is also commutative.

    The vector product of two vectors is not commutative because the direction of the cross product is defined using the right-hand rule, and the direction is opposite if the two vectors are multiplied in opposite order.

    The vector product of two vectors is actually anti-commutative, which means when you change the order of the factors, the signs changes as well: $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$.

24. If two vectors are parallel, their vector product is zero because the angle between them is either 0° or 180°. In this case, the sine of the angle that appears in the formula for the magnitude of the vector product is zero, and so the vector product of the two vectors is zero. You can visualize this using Figure 2-18: the area of a parallelogram formed by two parallel vectors is zero.

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25. $F_x = 5\cos(30°) = 4.33\text{ N}$

$F_y = 5\cos(-40°) = 3.83\text{ N}$

$F_z = 5\cos(-60°) = 2.50\text{ N}$

$\Rightarrow \vec{F} = (4.33\text{ N})\hat{i} + (3.83\text{ N})\hat{j} + (2.50\text{ N})\hat{k}$

26. (a) $\vec{A} = (3.00, 5.00, -2.00)$, $\vec{B} = (1.00, 2.00, 4.00)$, $\vec{C} = (4.00, -2.00, 1.00)$; therefore,

$$\vec{A} + 2\vec{B} - 3\vec{C} = (3.00, 5.00, -2.00) + 2(1.00, 2.00, 4.00) - 3(4.00, -2.00, 1.00)$$
$$= (-7.00, 15.0, 3.00)$$

Notice that we add up corresponding coordinates.

(b) In Cartesian notation, vector $\vec{D}(-7.000, 15.00, 3.000)$ can be expressed as $\vec{D} = -7.000\hat{i} + 15.00\hat{j} + 3.000\hat{k}$.

(c) In polar notation, we need to know the magnitude of the vector and the angles it makes with the coordinate axes: the coordinate direction angles. The magnitude of vector $\vec{D}$ is $D = \sqrt{7.00^2 + 15.0^2 + 3.00^2} = \sqrt{283} = 16.8$. Its coordinate direction angles are

$$\alpha_{\vec{D}} = \cos^{-1}\left(\frac{-7.00}{\sqrt{283}}\right) = 115°;\ \beta_{\vec{D}} = \cos^{-1}\left(\frac{15.0}{\sqrt{283}}\right) = 26.9°;\ \gamma_{\vec{D}} = \cos^{-1}\left(\frac{3.00}{\sqrt{283}}\right) = 79.7°$$

(d) Antiparallel means directed in the opposite direction. In Cartesian notation, vector $-\vec{D}(7.00, -15.0, -3.00)$ can be expressed as $-\vec{D} = 7.00\hat{i} - 15.0\hat{j} - 3.00\hat{k}$.

(e) Describe the vector in part (d) using polar notation. It has the same magnitude as the vector in part (c), with direction angles

$$\alpha_{-\vec{D}} = \cos^{-1}\left(\frac{7.00}{\sqrt{283}}\right) = 65.4°$$

$$\beta_{-\vec{D}} = \cos^{-1}\left(\frac{-15.0}{\sqrt{283}}\right) = 153°$$

$$\gamma_{-\vec{D}} = \cos^{-1}\left(\frac{-3.00}{\sqrt{283}}\right) = 100.°$$

Notice that the corresponding angles of two antiparallel angles are related:

$$\alpha_{\vec{D}} + \alpha_{-\vec{D}} = 180°;\ \beta_{\vec{D}} + \beta_{-\vec{D}} = 180°;\ \gamma_{\vec{D}} + \gamma_{-\vec{D}} = 180°$$

27. Given: $\vec{A}(5,10)$; $\vec{B}(-5,10)$; $\vec{C}(10,-5)$

(a) $$\vec{A} + \vec{B} + \vec{C} = (5,10) + (-5,10) + (10,-5)$$
$$= (10,15)$$

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(b) $\vec{A}+2\vec{B}+\vec{C}=(5,10)+2(-5,10)+(10,-5)$
$=(5,25)$

(c) $2\vec{A}-\vec{B}+3\vec{C}=2(5,10)-(-5,10)+3(10,-5)$
$=(45,-5)$

(d) $-3\vec{A}+\vec{B}-4\vec{C}=-3(5,10)+(-5,10)-4(10,-5)$
$=(-60,0)$

28. (a) First we express the components of the given vectors in Cartesian (algebraic) notation:
$\vec{F}_A=(2,0,-1)$ N, $\vec{F}_B=(0,3,0)$ N, $\vec{F}_C=(0,2,2)$ N, $\vec{F}_D=(0,-3,3)$ N

Notice that since each force is located in a coordinate plane, at least one of its components must be zero.

Then we are ready to find the sum of the four forces:

$$\begin{aligned}\vec{F}_R&=\vec{F}_A+\vec{F}_B+\vec{F}_C+\vec{F}_D\\&=(2,0,-1)\text{ N}+(0,3,0)\text{ N}+(0,2,2)\text{ N}+(0,-3,3)\text{ N}\\&=(2,2,4)\text{ N}\end{aligned}$$

(b) $\vec{F}_R=\left(2\hat{i}+2\hat{j}+4\hat{k}\right)$ N

(c) To express the net force in polar notation, we need to find its magnitude and the coordinate direction angles:

$$\vec{F}_R=(2,2,4)\text{ N}\Rightarrow F_R=\sqrt{2^2+2^2+4^2}\text{ N}=\sqrt{24}\text{ N}=2\sqrt{6}\text{ N}$$

The coordinate direction angles can be found as follows:

$$\alpha=\cos^{-1}\left(\frac{2}{\sqrt{24}}\right)\approx 66°;\ \beta=\cos^{-1}\left(\frac{2}{\sqrt{24}}\right)\approx 66°;\ \gamma=\cos^{-1}\left(\frac{4}{\sqrt{24}}\right)\approx 35°$$

Note that, based on the givens, the values of the coordinate degree angles are only known to one significant figure, so we use the approximately equal sign.

(d) To find the unit vector in the direction of the net force we use the results from (c):

$$\vec{F}_R=\left(2\hat{i}+2\hat{j}+4\hat{k}\right)\text{ N and }F_R=\sqrt{24}\text{ N }=2\sqrt{6}\text{ N }\Rightarrow$$

$$\hat{u}_{F_R}=\frac{2}{2\sqrt{6}}\hat{i}+\frac{2}{2\sqrt{6}}\hat{j}+\frac{4}{2\sqrt{6}}\hat{k}=\frac{1}{\sqrt{6}}\hat{i}+\frac{1}{\sqrt{6}}\hat{j}+\frac{2}{\sqrt{6}}\hat{k}$$

It is possible, but highly impractical to add these vectors by hand, as it is difficult to accurately draw vectors head-to-tail in three dimensions.

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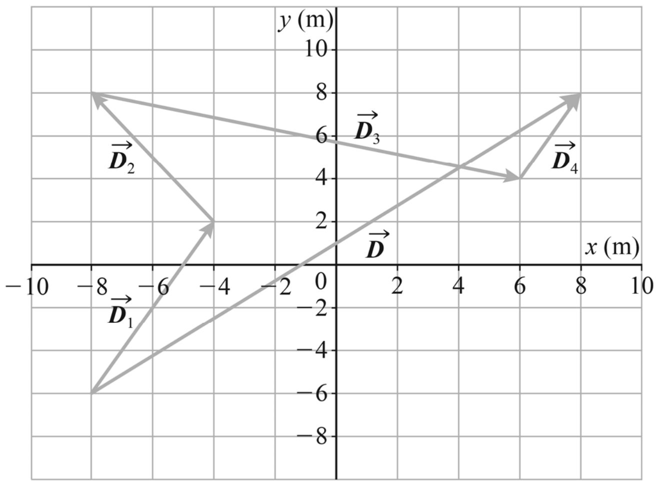
29. (a) The final displacement is a vector that coincides with the tail of the initial vector and the head of the final vector. Therefore, the displacement vector can be found by looking at the horizontal and vertical coordinates of the vector connecting the initial and final points of the trajectory: $\vec{D} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3 + \vec{D}_4 = 16\hat{i} + 14\hat{j}$



(b) The algebraic approach for finding the displacement is

$$\vec{D} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3 + \vec{D}_4 = [(4.0 \text{ m})\hat{i} + (8.0 \text{ m})\hat{j}] + [(-4.0 \text{ m})\hat{i} + (6.0 \text{ m})\hat{j}]$$
$$+[(14 \text{ m})\hat{i} - (4.0 \text{ m})\hat{j}] + [(2.0 \text{ m})\hat{i} + (4.0 \text{ m})\hat{j}]$$
$$= (16 \text{ m})\hat{i} + (14 \text{ m})\hat{j}$$

Both approaches give a resultant displacement vector with components (16 m, 14 m). Thus, the magnitude of the resultant displacement vector is

$$\sqrt{16^2 + 14^2} \text{ m} \approx 21 \text{ m}$$

and the angle that the displacement vector makes with the positive *x*-axis is

$$\theta = \tan^{-1}\left(\frac{14}{16}\right) = 41°$$

30. $$\begin{cases} x = -[10.0\sin(20.0°) + 7.00 + 20.0\sin(30.0°)] = -20.4 \text{ m} \\ y = 10.0\cos(20.0°) - 20.0\cos(30.0°) = -7.92 \text{ m} \end{cases}$$

$$\Rightarrow D = \sqrt{(20.4 \text{ m})^2 + (7.92 \text{ m})^2} = 21.9 \text{ m}$$

You can use the PhET "Vector Addition" to visualize the problem. However, note that this simulation only allows us to construct vectors that have whole-number components. So the simulation should only be used to conceptualize the problem, not to find the exact values.

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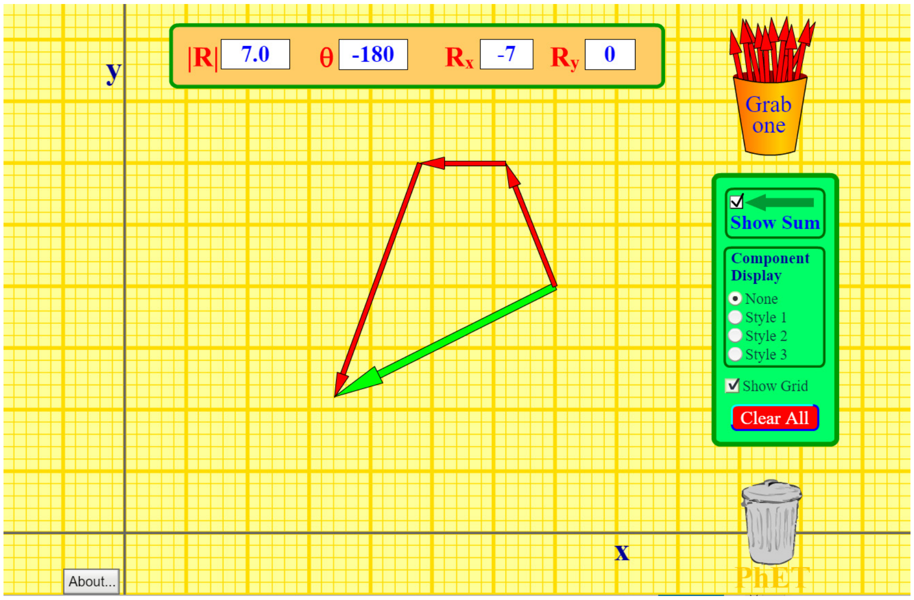
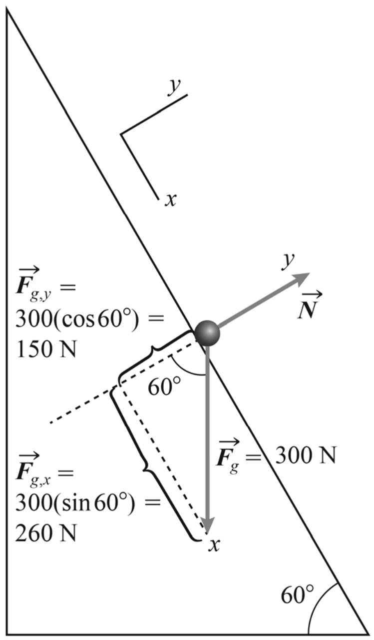


Figure for Problem 30: A screenshot of PhET "Vector Addition" representing the problem.

This screen grab was taken from PhET Interactive Simulations, University of Colorado, http://phet.colorado.edu

31. (a)



(b) $F_{g,x} = mg \sin(60.0°) = (300.\text{ N}) \sin(60.0°) = 260.\text{ N}$

$F_{g,y} = mg \cos(60.0°) = (300.\text{ N}) \cos(60.0°) = 150.\text{ N}$

(c) $N_x = 0$, $N_y = 150.\text{ N}$

(d) $F_{\text{net},x} = F_{g,x} - N_x = 260.\text{ N}$, $F_{\text{net},y} = F_{g,y} - N_y = 0$

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The net force in the y-direction is zero, but there is a non-zero net force in the x-direction. Thus, according to Newton's second law, the child accelerates in the x-direction (down the slope).

32. (a) $A_x = 20.0\cos(360° - 15°) = 19.3$; $A_y = 20.0\sin(360° - 15°) = -5.18$

$B_x = 15.0\cos(35°) = 12.3$; $B_y = 15.0\sin(35°) = 8.60$

$C_x = 25.0\cos(125°) = -14.3$; $C_y = 25.0\sin(125°) = 20.5$

(b) $\vec{\boldsymbol{A}} = (19.3\hat{\boldsymbol{i}} - 5.18\hat{\boldsymbol{j}})$; $\vec{\boldsymbol{B}} = (12.3\hat{\boldsymbol{i}} + 8.60\hat{\boldsymbol{j}})$; $\vec{\boldsymbol{C}} = (-14.3\hat{\boldsymbol{i}} + 20.5\hat{\boldsymbol{j}})$

$\vec{\boldsymbol{U}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} + \vec{\boldsymbol{C}} = (17.3\hat{\boldsymbol{i}} + 23.9\hat{\boldsymbol{j}})$

$U = \sqrt{17.3^2 + 23.9^2} = 29.5$

$\theta = \tan^{-1}\left(\dfrac{23.9}{17.3}\right) = 54.1°$

$\Rightarrow \vec{\boldsymbol{U}} = (29.5,\ 54.1°)$

(c) $\vec{\boldsymbol{A}} = (19.3\hat{\boldsymbol{i}} - 5.18\hat{\boldsymbol{j}})$; $\vec{\boldsymbol{B}} = (12.3\hat{\boldsymbol{i}} + 8.60\hat{\boldsymbol{j}})$; $\vec{\boldsymbol{C}} = (-14.3\hat{\boldsymbol{i}} + 20.5\hat{\boldsymbol{j}})$

$\vec{\boldsymbol{D}} = 2\vec{\boldsymbol{A}} - 3\vec{\boldsymbol{B}} + \vec{\boldsymbol{C}} = (-12.6\hat{\boldsymbol{i}} - 15.7\hat{\boldsymbol{j}})$

$D = \sqrt{12.6^2 + 15.7^2} = 20.1$

$\theta = \tan^{-1}\left(\dfrac{15.7}{12.6}\right) = 51.3°$

33. (a) $F_1 = \sqrt{3.00^2 + 5.00^2 + 6.00^2} = 8.37\text{ N}$

$\alpha_{\vec{F}_1} = \cos^{-1}\dfrac{3.00}{8.37} = 69.0°,\ \beta_{\vec{F}_1} = \cos^{-1}\dfrac{-5.00}{8.37} = 127°,\ \gamma_{\vec{F}_1} = \cos^{-1}\dfrac{6.00}{8.37} = 44.2°$

(b) $F_2 = \sqrt{3.00^2 + 5.00^2 + 6.00^2} = 8.37\text{ N}$

$\alpha_{\vec{F}_2} = \cos^{-1}\dfrac{-3.00}{8.37} = 111°,\ \beta_{\vec{F}_2} = \cos^{-1}\dfrac{5.00}{8.37} = 53.3°,\ \gamma_{\vec{F}_2} = \cos^{-1}\dfrac{-6.00}{8.37} = 136°$

(c) The two vectors have the same magnitude but opposite directions. Both magnitudes equal 8.37 N. The fact that the vectors have opposite directions can be seen from verifying that the sum of the corresponding direction angles is 180°:

α: 69.0° + 111° = 180°

β: 127° + 53.3° = 180°

γ: 44.2° + 136° = 180°

The discrepancies we observe are caused by the rounding errors in the calculations.

(d) We get 1 in both cases, which is the expected result—equation 2-13.

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34. (a) $F = \sqrt{3.00^2 + 5.00^2 + 6.00^2} = 8.37\text{ N}$

$$\Rightarrow \hat{\boldsymbol{u}}_{\vec{F}} = \frac{3.00}{8.37}\hat{\boldsymbol{i}} - \frac{5.00}{8.37}\hat{\boldsymbol{j}} + \frac{6}{8.37}\hat{\boldsymbol{k}} = 0.359\hat{\boldsymbol{i}} - 0.598\hat{\boldsymbol{j}} + 0.717\hat{\boldsymbol{k}}$$

(b) Any unit vector $\hat{\boldsymbol{u}}$ that satisfies the condition $\hat{\boldsymbol{u}} \bullet (3.00\hat{\boldsymbol{i}} - 5.00\hat{\boldsymbol{j}} + 6.00\hat{\boldsymbol{k}}) = 0$ is perpendicular to the given vector. To find it, we use Equation 2-23 for dot product: $3.00u_x - 5.00u_y + 6.00u_k = 0$. At the same time, since $\hat{\boldsymbol{u}}$ is a unit vector, we can write $u_x^2 + u_y^2 + u_k^2 = 1$. Thus, we have a system of two equations with three unknowns:

$$\begin{cases} 3.00u_x - 5.00u_y + 6.00u_k = 0 \\ u_x^2 + u_y^2 + u_k^2 = 1 \end{cases}$$

This system of equations has an infinite number of solutions.

Therefore, this question thus has an infinite number of possible answers, such as $\dfrac{3.00\hat{\boldsymbol{i}} + 3.00\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}}{\sqrt{19}}$.

(c) There is also an infinite number of unit vectors parallel to the plane. Any unit vector $\hat{\boldsymbol{u}} = a\hat{\boldsymbol{i}} + b\hat{\boldsymbol{j}} + c\hat{\boldsymbol{k}}$ whose coefficients satisfy the relation $3a + 2b - 4c = 0$ is a unit vector parallel to the given plane; an example is $\frac{1}{3}(2\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}} + 2\hat{\boldsymbol{k}})$.

(d) The vector (3, 2, −4) is perpendicular to the plane. You can check this by calculating its scalar product with one of the vectors found in part (c). Thus, the following two unit vectors are perpendicular to the given plane:

$$\pm\frac{1}{\sqrt{29}}(3\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} - 4\hat{\boldsymbol{k}})$$

35. (a) $\vec{\boldsymbol{v}}_{\text{avg}} = \dfrac{\vec{\boldsymbol{r}}_{\text{AB}}}{\Delta t} = \dfrac{(-3.00\hat{\boldsymbol{i}} + 2.00\hat{\boldsymbol{j}}) - (3.00\hat{\boldsymbol{i}} + 2.00\hat{\boldsymbol{j}})}{5.00}\text{ m/s}$

$$= -\frac{6.00\hat{\boldsymbol{i}}}{5.00}\text{ m/s} = -1.20\hat{\boldsymbol{i}}\text{ m/s}$$

(b) To find the average speed, we need to know the total distance covered by the object in the given time interval. Since the object moved along a straight line, this distance is represented by the magnitude of the object's displacement:

$$v_{\text{avg}} = \frac{|\vec{\boldsymbol{r}}_{\text{AB}}|}{\Delta t} = \frac{6.00}{5.00}\text{ m/s} = 1.20\text{ m/s}$$

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(c) Since the object moved along a straight line without turning, the magnitude of its average velocity equals its average speed. However, the average velocity is a vector, while average speed is a scalar. Note that, if the object moved along a curved path, the magnitude of its average velocity would be different from its average speed.

36. For this we compare the *x*-, *y*-, and *z*-coordinates.

(a) $F_x\hat{\boldsymbol{i}} + 3\hat{\boldsymbol{j}} + \sqrt{2}\hat{\boldsymbol{i}} - F_y\hat{\boldsymbol{j}} + F_z\hat{\boldsymbol{k}} - 5\hat{\boldsymbol{k}} = 0$

$$(F_x + \sqrt{2})\hat{\boldsymbol{i}} + (3 - F_y)\hat{\boldsymbol{j}} + (F_z - 5)\hat{\boldsymbol{k}} = 0$$

$$F_x + \sqrt{2} = 0 \Rightarrow F_x = -\sqrt{2}$$

$$3 - F_y = 0 \Rightarrow F_y = 3$$

$$F_z - 5 = 0 \Rightarrow F_z = 5$$

(b) The same reasoning applies to solving the second equation:

$$3\hat{\boldsymbol{i}} - 5\hat{\boldsymbol{j}} + F_x\hat{\boldsymbol{i}} - 2F_y\hat{\boldsymbol{j}} + F_z\hat{\boldsymbol{k}} - 3\hat{\boldsymbol{k}} = 0$$

$$(3 + F_x)\hat{\boldsymbol{i}} - (5 + 2F_y)\hat{\boldsymbol{j}} + (F_z - 3)\hat{\boldsymbol{k}} = 0$$

$$F_x = -3,\ F_y = -\frac{5}{2},\ F_z = 3$$

37. $\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = (3)(2) + (4)(-2) + (-5)(4) = -22$

The dot product represents the degree of collinearity of two vectors. If we calculate the magnitudes of the vectors, we find $A = 7.07$, $B = 4.89$. Thus, if the vectors were parallel to each other, the scalar product would have been equal to 34.64, However, since its magnitude is only 22, the vectors must be directed at an angle to each other.

$$\cos(\theta) = \frac{-22}{34.64} = -0.635 \Rightarrow \theta = \cos^{-1}(-0.635) = 129.4°$$

38. $3 \times 4 - 5 \times 4 + F_z^2 = 0$

$$12 - 20 + F_z^2 = 0$$

$$F_z = \pm\sqrt{8} = \pm 2\sqrt{2}$$

39. $\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = (10)(5)\cos(20°) = 47$

The dot product represents the degree of collinearity of two vectors. If the vectors were collinear (parallel and directed in the same direction), their dot product would have been equal to 50. However, since the dot product equals 47, we can see that the vectors are "almost parallel"—they are directed at a 20° angle to each other.

40. (a) Taking a careful look at Figure 2-24, you can see the following relationships: $\vec{\boldsymbol{A}} \perp \vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{B}} \perp \vec{\boldsymbol{C}}$, so $\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} \bullet \vec{\boldsymbol{A}} = 0$ and $\vec{\boldsymbol{B}} \bullet \vec{\boldsymbol{C}} = \vec{\boldsymbol{C}} \bullet \vec{\boldsymbol{B}} = 0$. At the same time, the angle

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between vectors is more than 0° and less than 90°. Therefore, considering that the dot product is commutative, we deduce that $\vec{C}\bullet\vec{A} = \vec{A}\bullet\vec{C} > \vec{B}\bullet\vec{C} = \vec{C}\bullet\vec{B} = \vec{A}\bullet\vec{B} = \vec{B}\bullet\vec{A} = 0$.

(b) $\vec{A}\bullet\vec{B} = (-2)(0) + (0)(4) + (3)(0) = 0$

Similarly,

$\vec{B}\bullet\vec{C} = 0,\ \vec{A}\bullet\vec{C} = 10,\ \vec{B}\bullet\vec{A} = 0,\ \vec{C}\bullet\vec{B} = 0,\ \vec{C}\bullet\vec{A} = 10$

We can see that the answers to part (b) support our ranking in part (a).

41. Since this unit vector is located in the *xy*-plane, it must have the form

$\hat{\boldsymbol{u}} = x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}} + 0\hat{\boldsymbol{k}} = x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}}$

Since it is a unit vector, it has magnitude one, so it must satisfy the relation

$$x^2 + y^2 = 1 \tag{1}$$

Since it is perpendicular to the given vector, $\vec{A} = 3\hat{\boldsymbol{i}} - 2\hat{\boldsymbol{j}} + 5\hat{\boldsymbol{k}}$, the scalar product of the unit vector and vector $\vec{A}$ must be zero. Therefore,

$$3x - 2y = 0 \tag{2}$$

Solving (1) and (2) simultaneously gives

$$x^2 + \frac{9}{4}x^2 = 1 \Rightarrow 13x^2 = 4 \Rightarrow x = \pm\frac{2\sqrt{13}}{13}$$

By plugging the values of x in Equation (2), we find the corresponding values of y:

$y = \pm\frac{3\sqrt{13}}{13}$. Thus, these values are $x = \pm\frac{2\sqrt{13}}{13}$ and $y = \pm\frac{3\sqrt{13}}{13}$.

So the required unit vectors are

$$\hat{\boldsymbol{u}} = \frac{2\sqrt{13}}{13}\hat{\boldsymbol{i}} + \frac{3\sqrt{13}}{13}\hat{\boldsymbol{j}} \text{ and } -\frac{2\sqrt{13}}{13}\hat{\boldsymbol{i}} - \frac{3\sqrt{13}}{13}\hat{\boldsymbol{j}}$$

42. The line in parametric form can be expressed as

$$\begin{cases} x = t \\ y = -2t + 5 \end{cases}$$

So a vector parallel to the line is (1, −2) with magnitude

$$\sqrt{1^2 + (-2)^2} = \sqrt{5}$$

A unit vector parallel to the line is therefore given by

$$\hat{\boldsymbol{u}} = \frac{1}{\sqrt{5}}\hat{\boldsymbol{i}} - \frac{2}{\sqrt{5}}\hat{\boldsymbol{j}}$$

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The projection of the given vector, $\vec{F}(3.00, 4.00)$, on the line can be expressed as the dot product of the force vector and a unit vector parallel to the line. Thus, it is

$$(3.00)\left(\frac{1}{\sqrt{5}}\right)+(4.00)\left(-\frac{2}{\sqrt{5}}\right)=-\sqrt{5.00}$$

43. We can use either a dot product calculation or a cross product calculation to find the angle between the two given vectors.

We can use Equations 2-17 and 2-24 to calculate the dot product:

$$A=\sqrt{(-2.00)^2+3.00^2+(-5.00)^2}=\sqrt{38.0}$$
$$B=\sqrt{(-2.00)^2+2.00^2+(-5.00)^2}=\sqrt{33.0}$$
$$AB\cos\theta=\vec{A}\bullet\vec{B}\Rightarrow AB\cos\theta=A_xB_x+A_yB_y+A_zB_z$$
$$\sqrt{38\cdot 33}\cos\theta=4.00+6.00+25.0$$
$$\cos\theta=\frac{35.0}{\sqrt{38.0\cdot 33.0}}$$
$$\theta=8.75°$$

Cross product calculations can be done using the matrix form of the cross product—Equation 2-34. Note that we are not following the significant digit rules in the intermediate calculations to simplify the appearance of the equations. However, we keep three significant digits in the final answer:

$$\vec{C}=\vec{A}\times\vec{B}=\begin{pmatrix}\hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}}\\ -2.00 & 3.00 & -5.00\\ -2.00 & 2.00 & -5.00\end{pmatrix}$$
$$=\hat{\boldsymbol{i}}(3(-5)-(-5)(2))-\hat{\boldsymbol{j}}((-2)(-5)-(-2)(-5))+\hat{\boldsymbol{k}}(2(-2)-3(-2))$$
$$=-5.00\hat{\boldsymbol{i}}-0\hat{\boldsymbol{j}}+2.00\hat{\boldsymbol{k}}$$
$$C=\sqrt{5.00^2+2.00^2}=\sqrt{29.0}$$
$$\left|\vec{A}\times\vec{B}\right|=AB\sin\theta\Rightarrow\sin\theta=\frac{\left|\vec{A}\times\vec{B}\right|}{AB}\Rightarrow\theta=\sin^{-1}\left(\frac{\left|\vec{A}\times\vec{B}\right|}{AB}\right)=\sin^{-1}\left(\frac{\sqrt{29.0}}{\sqrt{38.0\cdot 33.0}}\right)=8.75°$$

Both methods produce the same result, yet it is clear that it is easier to use the dot product method to find the angle.

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44. Since we are dealing with a mathematical problem, we can assume that the values are exact. The first and the most elegant method is using the determinant form of the vector product (Equation 2-34):

$$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & -5 \\ 2 & -2 & 4 \end{vmatrix}$$

$$= \hat{i}(4(4)-(-5)(-2)) - \hat{j}((3(4)-(-5)(2)) + \hat{k}(3(-2)-4(2))$$

$$= 6\hat{i} - 22\hat{j} - 14\hat{k}$$

Comment: $C = \sqrt{6^2 + 22^2 + 14^2} = \sqrt{716} = 26.76$

We have left four digits in the answer for the purpose of the proof.

The coordinate direction angles can be found using Equation 2-12:

$$\alpha = \cos^{-1}\left(\frac{6}{\sqrt{716}}\right) = 77.04°;\ \beta = \cos^{-1}\left(\frac{-22}{\sqrt{716}}\right) = 145.3°;\ \gamma = \cos^{-1}\left(\frac{-14}{\sqrt{716}}\right) = 121.5°$$

Another way of calculating the cross product is to use its basic definition—Equation 2-26:

$$C = \left|\vec{A}\times\vec{B}\right| = AB\sin(\theta)$$

$$A = \sqrt{3^2 + 4^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$$

$$B = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{24} = 2\sqrt{6}$$

$$C = 5\sqrt{2}(2\sqrt{6})\sin(\theta) = 20\sqrt{3}\sin(\theta)$$

We can find the angle between the two given vectors using the dot product (see problem 43):

$$AB\cos(\theta) = \vec{A}\bullet\vec{B}$$

$$= \sqrt{50(24)}\cos(\theta) = 6 - 8 - 20 = -22$$

$$\cos(\theta) = \frac{-22}{\sqrt{1200}}$$

$$\theta = \cos^{-1}\left(\frac{-22}{\sqrt{1200}}\right) = 129.4°$$

$$C = 5\sqrt{2}(2\sqrt{6})\sin(\theta) = 20\sqrt{3}\sin(129.4°) = 26.77$$

We can see that the magnitudes of vector $\vec{C}$ calculated by both methods are practically the same. The discrepancy is caused by rounding error. To find the direction of the cross

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product vector $\vec{C}$ using this method, we have to find a unit ($u = 1$) vector that is at the same time perpendicular to each of the given vectors:

$$\begin{cases} \hat{u}\bullet(3\hat{i} + 4\hat{j} - 5\hat{k}) = 0 \\ \hat{u}\bullet(2\hat{i} - 2\hat{j} + 4\hat{k})\hat{k} = 0 \end{cases}$$

While this can be done using the dot product, it is cumbersome. In addition, at the end we will get two solutions, and we will need to choose the one that obeys the right-hand rule. From this we can see that it is much easier to use the determinant method for finding the cross product of two given vectors if we want to know both the magnitude and the direction of the result.

45. $C = \left|\vec{A} \times \vec{B}\right| = 10.0(5.00)\sin(20.0°) = 17.1$

However, we do not have enough information to determine the exact direction of the cross product. We know it has to be directed along the *y*-axis (perpendicular to the *xz*-plane), but we do not know if it is directed in the positive or negative direction without knowing the directions of the original vectors. So the answer can be either $\vec{C}_1(0, 17.1, 0)$ or $\vec{C}_2(0, -17.1, 0)$.

46. (a) The volume of a prism (V) can be found as

$$V = \vec{A}\bullet(\vec{B} \times \vec{C})$$

$$\vec{B} \times \vec{C} = (20 - 6)\hat{i} - (-10 - 2)\hat{j} + (6 + 4)\hat{k} = 14\hat{i} + 12\hat{j} + 10\hat{k}$$

$$V = (4\hat{i} + 3\hat{j} + \hat{k})\bullet(14\hat{i} + 12\hat{j} + 10\hat{k}) = 102 \text{ cubic units}$$

(b) The surface area of a prism (SA) consists of the areas of all six of its faces. The area of each of the faces can be found as the absolute value of the vector product of the vectors describing its edges:

$$\vec{A} \times \vec{B} = -10\hat{i} + 6\hat{j} + 22\hat{k} \Rightarrow \left|\vec{A} \times \vec{B}\right| = \sqrt{620} = 24.9$$

$$\vec{B} \times \vec{C} = 14\hat{i} + 12\hat{j} + 10\hat{k} \Rightarrow \left|\vec{B} \times \vec{C}\right| = \sqrt{440} = 21.0$$

$$\vec{A} \times \vec{C} = 18\hat{i} - 21\hat{j} - 9\hat{k} \Rightarrow \left|\vec{A} \times \vec{C}\right| = \sqrt{620} = 29.1$$

$$SA = 2\left|\vec{A} \times \vec{B}\right| + 2\left|\vec{B} \times \vec{C}\right| + 2\left|\vec{A} \times \vec{C}\right| = 2(24.9) + 2(21.0) + 2(29.1) = 150$$

47. (a) First of all, we should describe each vector in Figure 2-25 using Cartesian vector notation:

$$\vec{A} = -2\hat{i} + 0\hat{j} + 3\hat{k}$$

$$\vec{B} = 0\hat{i} + 4\hat{j} + 0\hat{k}$$

$$\vec{C} = -5\hat{i} + 0\hat{j} + 0\hat{k}$$

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To find the vector products, the easiest way is to use Cartesian vector notation and Equation 2-33 or Equation 2-34:

$$\vec{A}\times\vec{B} = -12\hat{i} - 8\hat{k},\ \vec{B}\times\vec{C} = 20\hat{k},\ \vec{A}\times\vec{C} = -15\hat{j}$$

$$\vec{B}\times\vec{A} = 12\hat{i} + 8\hat{k},\ \vec{C}\times\vec{B} = -20\hat{k},\ \vec{C}\times\vec{A} = 15\hat{j}$$

(b) We can also perform the calculations using the original definitions. We will need to find the magnitude of each vector and the angles between them. An example of how one might do that can be found in problem 43.

(c) Both methods will produce the same results.

48. (a) The cross product gives $-14\hat{i} + \frac{F_x}{2}\hat{j}$.

Hence, the given equation becomes

$$-14\hat{i} + \frac{F_x}{2}\hat{j} + F_y\hat{i} - \sqrt{2}\hat{j} = 0$$

$$\Rightarrow F_x = 2\sqrt{2},\ F_y = 14$$

We can also solve this problem by using the distributive property of the vector product and Equation 2-30.

(b) The cross product gives $20\hat{i} - 4F_x\hat{k}$.

Hence, the given equation becomes

$$20\hat{i} - 4F_x\hat{k} + F_z\hat{k} + \sqrt{3}F_x\hat{i} = 0$$

$$(20+\sqrt{3})\hat{i} + (F_z - 4F_x)\hat{k} = 0$$

$$\Rightarrow F_x = -\frac{20}{\sqrt{3}},\ F_z = -\frac{80}{\sqrt{3}}$$

49. (a)

$$\vec{A}\times\vec{B} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 5 \\ -4 & 2 & -4 \end{pmatrix}$$

$$= \hat{i}((-1)(-4) - 5(2)) - \hat{j}(3(-4) - 5(-4)) + \hat{k}(3(2) - (-1)(-4))$$

$$= -6\hat{i} - 8\hat{j} + 2\hat{k}$$

(b)

$$\vec{A}\times\vec{B} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -5 \\ -1 & 0 & -4 \end{pmatrix}$$

$$= -12\hat{i} + 5\hat{j} + 3\hat{k}$$

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(c) $\vec{A}\times\vec{B}=\begin{pmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 5\\ -4 & 2 & -10\end{pmatrix}=0$

This calculation confirms that the cross product of two parallel vectors is zero. We could have predicted this without doing any calculations.

50. Assume the cube's edges have unit lengths, and assume that one corner is at the origin of the coordinate system. The diagonal of the cube can be expressed as (1, 1, 1). The edges of the cube that share a vertex with that diagonal can be expressed as (1, 0, 0), (0, 1, 0), and (0, 0, 1).

The magnitude of each edge is one and of the diagonal vector is $\sqrt{1^2+1^2+1^2}=\sqrt{3}$.

The angle between the diagonal and each edge can be found using the scalar product of the diagonal and each edge. These products for all the edges that share a vertex with the diagonal are equal to one. Therefore,

$$\Rightarrow \theta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)=54.7°$$

51. (a) The translation matrix of a coordinate system in the *xy*-plane from the origin O to point O′(4, 5) can be expressed using the following translation matrix:

$$T=\begin{pmatrix}1 & 0 & 4\\ 0 & 1 & 5\\ 0 & 0 & 1\end{pmatrix}$$

Rotation of a coordinate system around the *z*-axis counter-clockwise by 60° can be expressed using the following rotation matrix:

$$R=\begin{pmatrix}\cos(60°) & -\sin(60°) & 0\\ \sin(60°) & \cos(60°) & 0\\ 0 & 0 & 1\end{pmatrix}$$

When two linear transformations are represented by matrices, as we have here, the matrix product represents the composition of two transformations. In other words, the transformation matrix for translation and rotation can be expressed by the product of these two matrices. Therefore,

$$\begin{pmatrix}\cos(60°) & -\sin(60°) & 4\\ \sin(60°) & \cos(60°) & 5\\ 0 & 0 & 1\end{pmatrix}$$

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The position of any point in the new coordinate system is then given by

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(60°) & -\sin(60°) & 4 \\ \sin(60°) & \cos(60°) & 5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

(b) From the above matrix, we can see how the components of a vector will transform:

$$x' = x\cos(60°) - y\sin(60°) + 4 = \frac{1}{2}x - \frac{\sqrt{3}}{2}y + 4$$

$$y' = x\sin(60°) + y\cos(60°) + 5 = \frac{\sqrt{3}}{2}x + \frac{1}{2}y + 5$$

52. $$\vec{\boldsymbol{B}} \times \vec{\boldsymbol{C}} = \begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = \hat{\boldsymbol{i}}(b_y c_z - b_z c_y) - \hat{\boldsymbol{j}}(b_x c_z - b_z c_x) + \hat{\boldsymbol{k}}(b_x c_y - b_y c_x)$$

$$\begin{aligned} \vec{\boldsymbol{A}} \bullet (\vec{\boldsymbol{B}} \times \vec{\boldsymbol{C}}) &= (a_x\hat{\boldsymbol{i}} + a_y\hat{\boldsymbol{j}} + a_z\hat{\boldsymbol{k}}) \bullet [\hat{\boldsymbol{i}}(b_y c_z - b_z c_y) - \hat{\boldsymbol{j}}(b_x c_z - b_z c_x) + \hat{\boldsymbol{k}}(b_x c_y - b_y c_x)] \\ &= a_x(b_y c_z - b_z c_y) - a_y(b_x c_z - b_z c_x) + a_z(b_x c_y - b_y c_x) \\ &= \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \end{aligned}$$

53. This fact can be proven by definition of the vector product—see Equation 2-26 and Figure 2-18. This fact can also be proven using the algebraic definition of cross product and the fact that the scalar product of two orthogonal vectors is zero. The proof below is for vector $\vec{\boldsymbol{A}}$; the same can be done for vector $\vec{\boldsymbol{B}}$:

$$\begin{aligned} \vec{\boldsymbol{A}} \bullet (\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}) &= (A_x\hat{\boldsymbol{i}} + A_y\hat{\boldsymbol{j}} + A_z\hat{\boldsymbol{k}}) \bullet [(A_y B_z - A_z B_y)\hat{\boldsymbol{i}} - (A_x B_z - A_z B_x)\hat{\boldsymbol{j}} + (A_x B_y - A_y B_x)\hat{\boldsymbol{k}}] \\ &= A_x A_y B_z - A_x A_z B_y - A_y A_x B_z + A_y A_z B_x + A_z A_x B_y - A_z A_y B_x \\ &= 0 \end{aligned}$$

Therefore,

$$\vec{\boldsymbol{A}} \perp \vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}$$

54. For these proofs, we will use $\vec{\boldsymbol{U}} \times \vec{\boldsymbol{V}} = -\vec{\boldsymbol{V}} \times \vec{\boldsymbol{U}}$ repeatedly.

(a) $(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}) \times \vec{\boldsymbol{C}} = -(\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}}) \times \vec{\boldsymbol{C}}$

(b) $(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}) \times \vec{\boldsymbol{C}} = -\vec{\boldsymbol{C}} \times (\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}) = -\vec{\boldsymbol{C}} \times [-(\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}})] = \vec{\boldsymbol{C}} \times (\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}})$

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(c) To prove that $(\vec{A}\times\vec{B})\times\vec{C}\neq\vec{A}\times(\vec{B}\times\vec{C})$, find a counterexample. Let's take the three vectors $\vec{A}$ = (1, 1, 1), $\vec{B}$ = (1, 0, 1), and $\vec{C}$ = (1, 0, 0), for example:

$$\begin{aligned}(\vec{A}\times\vec{B})\times\vec{C} &= [(1,1,1)\times(1,0,1)]\times(1,0,0)\\ &= (1,0,-1)\times(1,0,0)\\ &= (0,-1,0)\end{aligned}$$

$$\begin{aligned}\vec{A}\times(\vec{B}\times\vec{C}) &= (1,1,1)\times[(1,0,1)\times(1,0,0)]\\ &= (1,1,1)\times(0,1,0)\\ &= (-1,0,1)\end{aligned}$$

Thus, $(\vec{A}\times\vec{B})\times\vec{C}\neq\vec{A}\times(\vec{B}\times\vec{C})$.

(d) We can use the same counterexample as in part (c):

$$(\vec{A}\times\vec{B})\times\vec{C} = (0,-1,0)$$

$$\begin{aligned}\vec{A}\times(\vec{C}\times\vec{B}) &= \vec{A}\times[-(\vec{B}\times\vec{C})]\\ &= -\vec{A}\times(\vec{B}\times\vec{C})\\ &= (0,1,0)\\ &\neq(\vec{A}\times\vec{B})\times\vec{C}\end{aligned}$$

55. For simplicity, we will assume that the z-components of both vectors are zero, that is, $A_z = B_z = 0$. There is no loss of generality, because the coordinate system can always be rotated so that the two vectors can be situated in the xy-plane. Using the determinant formula, the magnitude of the cross product is then given by

$$\left|\vec{A}\times\vec{B}\right| = \sqrt{A_x^2B_y^2 + A_y^2B_x^2 - 2A_xA_yB_xB_y}$$

The scalar product between the two vectors is given by Equations 2-17 and 2-24:

$$\vec{A}\bullet\vec{B} = AB\cos\theta$$

$$A_xB_x + A_yB_y = AB\cos\theta$$

Squaring both sides of the second equation gives

$$(A_xB_x + A_yB_y)^2 = A^2B^2\cos^2(\theta)$$

$$(A_xB_x + A_yB_y)^2 = A^2B^2[1-\sin^2(\theta)]$$

$$A^2B^2\sin^2(\theta) = A^2B^2 - (A_xB_x + A_yB_y)^2$$

With $A^2 = A_x^2 + A_y^2$ and $B^2 = B_x^2 + B_y^2$, the above expression becomes

$$\begin{aligned}A^2B^2\sin^2(\theta) &= (A_x^2 + A_y^2)(B_x^2 + B_y^2) - (A_xB_x + A_yB_y)^2\\ &= A_x^2B_x^2 + A_x^2B_y^2 + A_y^2B_x^2 + A_y^2B_y^2 - (A_x^2B_x^2 + A_y^2B_y^2 + 2A_xB_xA_yB_y)\\ &= A_x^2B_y^2 + A_y^2B_x^2 - (2A_xB_xA_yB_y)\end{aligned}$$

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But we saw earlier that the right-hand side is equal to $\left|\vec{A}\times\vec{B}\right|^2$. Thus,

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

The same is true in general, even for vectors not in the *xy*-plane.

56. The determinant form of the vector product is

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

The Cartesian form using Equation 2-30 gives

$$\begin{aligned}\vec{A}\times\vec{B} &= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\times(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\ &= A_xB_x(\hat{i}\times\hat{i}) + A_xB_y(\hat{i}\times\hat{j}) + A_xB_z(\hat{i}\times\hat{k}) \\ &\quad + A_yB_x(\hat{j}\times\hat{i}) + A_yB_y(\hat{j}\times\hat{j}) + A_yB_z(\hat{j}\times\hat{k}) + A_zB_x(\hat{k}\times\hat{i}) + A_zB_y(\hat{k}\times\hat{j}) + A_zB_z(\hat{k}\times\hat{k}) \\ &= A_xB_x(0) + A_xB_y(\hat{k}) + A_xB_z(-\hat{j}) \\ &\quad + A_yB_x(-\hat{k}) + A_yB_y(0) + A_yB_z(\hat{i}) + A_zB_x(\hat{j}) + A_zB_y(-\hat{i}) + A_zB_z(0) \\ &= (A_yB_z - A_zB_y)\hat{i} - (A_xB_z - A_zB_x)\hat{j} + (A_xB_y - A_yB_x)\hat{k}\end{aligned}$$

This is equal to the determinant form given in Equation 2-32.

57. (a) $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -5 \\ -6 & -4 & 10 \end{vmatrix} = \hat{i}(20-20) - \hat{j}(-30+30) + \hat{k}(-12+12) = 0$

(b) The two vectors are antiparallel—they are directed in opposite directions—and their lengths are related as follows: $2\vec{A} = -\vec{B}$. The cross product of two parallel or antiparallel vectors is always zero because sin 0° = sin 180° = 0. Notice that it doesn't matter that they have different magnitudes; what matters in this case is their direction.

58. Line *a* in parametric form is

$$\begin{cases} x = t \\ y = m_1t + b_1 \end{cases}$$

Hence, the vector representing line *a* can be expressed as $\vec{a} = \hat{i} + m_1\hat{j}$.

Similarly, line *b* in parametric form is

$$\begin{cases} x = t \\ y = m_2t + b_2 \end{cases}$$

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Hence, the vector representing line b can be expressed as $\vec{b} = \hat{i} + m_2\hat{j}$.

The lines are perpendicular if and only if the scalar product of the two direction vectors is zero:

$$\begin{aligned}\vec{a}\bullet\vec{b} &= 0\\(\hat{i} + m_1\hat{j})\bullet(\hat{i} + m_2\hat{j}) &= 0\\1 + m_1m_2 &= 0\\m_1m_2 &= -1\end{aligned}$$

59. (a) Let us prove that both points are located along the given line. To do that, we will find the value of parameter t for each point:

$$\vec{r} = \vec{a} + t\vec{b} = (2.00\hat{i} - 3.00\hat{j} - 4.00\hat{k}) + (3.00\hat{i} - 2.00\hat{j} + 4.00\hat{k})t$$

$$5.00\hat{i} - 5.00\hat{j} + 0\hat{k} = (2.00 + 3.00t)\hat{i} + (-3.00 - 2.00t)\hat{j} + (-4.00 + 4.00t)\hat{k}$$

Thus, $2.00 + 3.00t = 5.00$, $-3.00 - 2.00t = -5.00$, and $-4.00 + 4.00t = 0$. All three of these equations give $t = 1.00$, so point A is on the line when $t = 1.00$.

In the case of point B(8.00 m, −7.00 m, 4.00 m), $2.00 + 3.00t = 8.00$, $-3.00 - 2.00t = -7.00$, and $-4.00 + 4.00t = 4.00$. All three equations give $t = 2.00$, so point B is on the line when $t = 2.00$.

(b) Considering the definition of work, $W_F = \vec{F}\bullet\Delta\vec{r}$ by a constant force $\vec{F}$, which is given, we have to calculate the displacement of the object first. The displacement can be found as

$$\begin{aligned}\Delta\vec{r}_{A\to B} &= \text{B}(8.00\text{ m}, -7.00\text{ m}, 4.00\text{ m}) - \text{A}(5.00\text{ m}, -5.00\text{ m}, 0\text{ m})\\&= (3.00\text{ m}, -2.00\text{ m}, 4.00\text{ m}) = (3.00\text{ m})\hat{i} - (2.00\text{ m})\hat{j} + (4.00\text{ m})\hat{k}\end{aligned}$$

Then the work can be calculated as follows:

$$\begin{aligned}W_F &= \vec{F}\bullet\Delta\vec{r} = [(3.00\text{ N})\hat{i} + (4.00\text{ N})\hat{j} - (5.00\text{ N})\hat{k}]\bullet[(3.00\text{ m})\hat{i} - (2.00\text{ m})\hat{j} + (4.00\text{ m})\hat{k}]\\&= (9.00 - 8.00 - 20.00)\text{ N}\cdot\text{m} = -19.0\text{ N}\cdot\text{m} = -19.0\text{ J}\end{aligned}$$

Notice that the units of work are called joules: 1 N·m ≡ 1 J.

60. Let $\vec{r}$ represent a vector along the horizontal side of the triangle, with length equal to the radius of the circle. Let $\vec{R}$ represent a vector from the centre of the circle to the vertex of a triangle that is not located on the diameter of the circle. The two vectors $\vec{A}$ and $\vec{B}$ shown in the figure represent the sides of the triangle. They are given by

$$\begin{aligned}\vec{A} &= \vec{R} + \vec{r}\\\vec{B} &= \vec{R} - \vec{r}\end{aligned}$$

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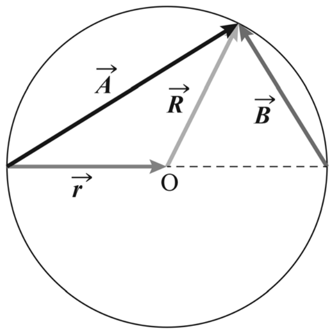
Note that

$$\begin{aligned}\vec{A}\bullet\vec{B} &= (\vec{R}+\vec{r})\bullet(\vec{R}-\vec{r})\\ &= \vec{R}\bullet\vec{R}+\vec{r}\bullet\vec{R}-\vec{R}\bullet\vec{r}-\vec{r}\bullet\vec{r} = R^2-r^2\\ &= R^2-r^2\\ &= 0\end{aligned}$$



Since the magnitudes of vectors $\vec{r}$ and $\vec{R}$ are both equal to the radius of the circle, $\vec{A}\bullet\vec{B}=0$ Thus, vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other, and so the triangle containing these vectors as its sides must be a right triangle.

We can use another method to prove that this triangle must be a right triangle.Let the centre of the circle be at the origin of an xy coordinate system, and let its radius be r. Let the diameter be the base of the triangle. The ends of the diameter are at $(-r, 0)$ and $(r, 0)$, and these are also the two vertices of the triangle. Let the other vertex of the triangle be at (x, y). Then the vectors representing the two sides that are not the diameter of the circle are $(x + r, y)$ and $(x - r, y)$. Find their dot product:

$$(x+r, y)\bullet(x-r, y) = x^2 - r^2 + y^2 = x^2 + y^2 - r^2$$

Since (x, y) is on the circle, $x^2 + y^2 = r^2$. Thus,

$$(x+r, y)\bullet(x-r, y) = 0$$

Since the dot product of the two vectors is zero, the angle between them is 90°. Thus, it is a right triangle.

61. We know that $|\vec{A}\times\vec{B}| = AB\sin\theta$. Thus,

$$\begin{aligned}|\vec{A}\times\vec{B}|^2 &= A^2B^2\sin^2(\theta)\\ |\vec{A}\times\vec{B}|^2 &= A^2B^2[1-\cos^2(\theta)]\\ |\vec{A}\times\vec{B}|^2 &= A^2B^2 - A^2B^2\cos^2(\theta)\\ A^2B^2 - |\vec{A}\times\vec{B}|^2 &= A^2B^2\cos^2\theta\end{aligned}$$

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But $A^2B^2\cos^2\theta = (\vec{A}\bullet\vec{B})^2$. So

$$A^2B^2 - \left|\vec{A}\times\vec{B}\right|^2 = (\vec{A}\bullet\vec{B})^2$$

$$\left|\vec{A}\times\vec{B}\right|^2 = A^2B^2 - (\vec{A}\bullet\vec{B})^2$$

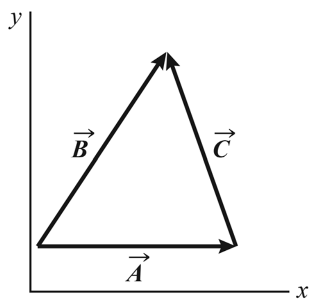
62. Any triangle can be oriented with respect to a coordinate system, as shown in the figure. The sides of the triangles can be represented as vectors. The law of vector addition then gives

$$\vec{C} = \vec{B} - \vec{A}$$

$$\vec{C}\bullet\vec{C} = (\vec{B}-\vec{A})\bullet(\vec{B}-\vec{A})$$

But $\vec{C}\bullet\vec{C} = C^2$ (and similarly for $\vec{A}$ and $\vec{B}$). Thus,

$$\begin{aligned} C^2 &= B^2 + A^2 - \vec{A}\bullet\vec{B} - \vec{B}\bullet\vec{A} \\ &= A^2 + B^2 - 2\vec{A}\bullet\vec{B} \\ &= A^2 + B^2 - 2AB\cos(\theta) \end{aligned}$$



63. We know that the differentiation of the product of two variables is $d(uv) = udv + vdu$.

The dot product of two vectors can be differentiated in the same way, as the following proof shows:

$$\begin{aligned} d(\vec{A}\bullet\vec{B}) &= d\left(A_xB_x + A_yB_y + A_zB_z\right) \\ &= A_xdB_x + B_xdA_x + A_ydB_y + B_ydA_y + A_zdB_z + B_zdA_z \\ &= (A_xdB_x + A_ydB_y + A_zdB_z) + (B_xdA_x + B_ydA_y + B_zdA_z) \\ &= (A_x, A_y, A_z)\bullet(dB_x, dB_y, dB_z) + (dA_x, dA_y, dA_z)\bullet(B_x, B_y, B_z) \\ &= \vec{A}\bullet(d\vec{B}) + (d\vec{A})\bullet\vec{B} \end{aligned}$$

Thus,

$$d(\vec{v}\bullet\vec{v}) = \vec{v}\bullet(d\vec{v}) + (d\vec{v})\bullet\vec{v}$$

Since $\vec{a}\bullet\vec{b} = \vec{b}\bullet\vec{a}$,

$$d(\vec{v}\bullet\vec{v}) = 2\vec{v}\bullet(d\vec{v})$$

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Thus,

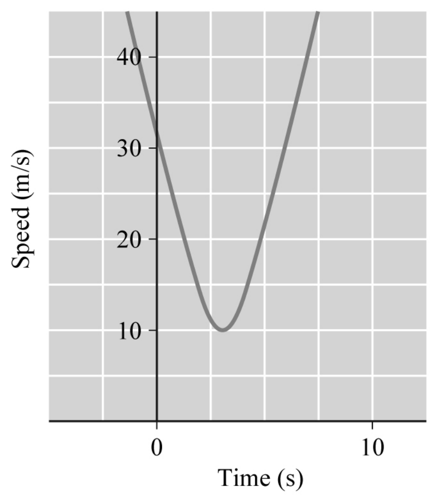
$$\frac{d}{dt}(\vec{v} \bullet \vec{v}) = 2\vec{v} \bullet \frac{d\vec{v}}{dt} = 2\vec{v} \bullet \vec{a}$$

64. (a) We are given that $\vec{v} = v_{0x}\hat{i} + (v_{0y} - gt)\hat{j}$. Thus,

$$v_x = v_{0x} \text{ and } v_y = v_{0y} - gt$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_x^2 + (v_{0y} - gt)^2}$$

(b) Assuming that v_{0x} is positive, a graph of the magnitude of the velocity will have a parabola shape similar to the example shown below. The curve shown has v_{0x} = 10 m/s and v_{0y} = 30 m/s.



(c) Since the x-component of velocity is constant during the motion, the speed is smallest when the y-component of velocity vanishes, that is,

$$v_{0y} - gt = 0$$

$$t = \frac{v_{0y}}{g}$$

Under the conditions in part (b), this would have happened when time t = 3.06 s. Notice that the speed at this point is 10 m/s (it equals the value of the horizontal component of the velocity).

(d) This model represents the motion of a projectile, assuming that there is no air resistance.

65. (a) $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (11.0 \text{ N})\hat{j} - (14.0 \text{ N})\hat{k}$

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Thus,

$$\vec{a} = \frac{(11.0\text{ N})\hat{\boldsymbol{j}} - (14.0\text{ N})\hat{\boldsymbol{k}}}{m} = \frac{(11.0\text{ N})\hat{\boldsymbol{j}} - (14.0\text{ N})\hat{\boldsymbol{k}}}{0.100\text{ kg}}$$

$$= (110\text{ N/kg})\hat{\boldsymbol{j}} - (140\text{ N/kg})\hat{\boldsymbol{k}}$$

$$a = \sqrt{110^2 + 140^2}\text{ N/kg} = 178\text{ N/kg}$$

(b) $\alpha = \cos^{-1}\left(\frac{0}{178}\right) = 90.0°$

$\beta = \cos^{-1}\left(\frac{110}{178}\right) = 51.8°$

$\gamma = \cos^{-1}\left(\frac{-140}{178}\right) = 142°$

(c) As in part (a),

$\vec{a} = (110\text{ N/kg})\hat{\boldsymbol{j}} - (140\text{ N/kg})\hat{\boldsymbol{k}}$

(d) $\vec{a}_{xy} = \vec{a} - (-140\text{ N/kg})\hat{\boldsymbol{k}} = (110\text{ N/kg})\hat{\boldsymbol{j}}$

$\vec{a}_{xz} = \vec{a} - (110\text{ N/kg})\hat{\boldsymbol{j}} = (-140\text{ N/kg})\hat{\boldsymbol{k}}$

$\vec{a}_{yz} = \vec{a} - (0\text{ N/kg})\hat{\boldsymbol{i}} = (110\text{ N/kg})\hat{\boldsymbol{j}} - (140\text{ N/kg})\hat{\boldsymbol{k}}$

66. (a) (i) $\vec{A}\times\vec{B} = \hat{\boldsymbol{i}}(-6+20) - \hat{\boldsymbol{j}}(-4-5) + \hat{\boldsymbol{k}}(8+3) = 14\hat{\boldsymbol{i}} + 9\hat{\boldsymbol{j}} + 11\hat{\boldsymbol{k}}$

(ii) $\vec{A}\times\vec{B} + \vec{C} = (14\hat{\boldsymbol{i}} + 9\hat{\boldsymbol{j}} + 11\hat{\boldsymbol{k}}) + (\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}) = 15\hat{\boldsymbol{i}} + 11\hat{\boldsymbol{j}} + 12\hat{\boldsymbol{k}}$

(iii)
$$\begin{aligned} 2\vec{A}\times\vec{B} + \vec{C} &= 2(2\hat{\boldsymbol{i}} + 3\hat{\boldsymbol{j}} - 5\hat{\boldsymbol{k}})\times(-\hat{\boldsymbol{i}} + 4\hat{\boldsymbol{j}} - 2\hat{\boldsymbol{k}}) + (\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}) \\ &= (-12+40)\hat{\boldsymbol{i}} - (-8-10)\hat{\boldsymbol{j}} + (16+6)\hat{\boldsymbol{k}} + (\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}) \\ &= 29\hat{\boldsymbol{i}} + 20\hat{\boldsymbol{j}} + 23\hat{\boldsymbol{k}} \end{aligned}$$

Another way to calculate this is to use the information in part (a):

$$\begin{aligned} 2\vec{A}\times\vec{B} + \vec{C} &= 2(\vec{A}\times\vec{B}) + \vec{C} \\ &= 2(14\hat{\boldsymbol{i}} + 9\hat{\boldsymbol{j}} + 11\hat{\boldsymbol{k}}) + (\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}) \\ &= 29\hat{\boldsymbol{i}} + 20\hat{\boldsymbol{j}} + 23\hat{\boldsymbol{k}} \end{aligned}$$

(iv) $\vec{G} \equiv (\vec{A}\times\vec{B})\bullet\vec{C} = (14\hat{\boldsymbol{i}} + 9\hat{\boldsymbol{j}} + 11\hat{\boldsymbol{k}})\bullet(\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}) = 14 + 18 + 11 = 43$

(v) $\vec{H} \equiv (\vec{A}\times\vec{B})\times\vec{C} = (14\hat{\boldsymbol{i}} + 9\hat{\boldsymbol{j}} + 11\hat{\boldsymbol{k}})\times(\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}) = -13\hat{\boldsymbol{i}} - 3\hat{\boldsymbol{j}} + 19\hat{\boldsymbol{k}}$

(vi) This expression is meaningless. The result of a dot product is a scalar, and the cross product of a scalar and a vector is undefined.

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(b) Parts (i), (ii), (iii), and (v) are all vectors, while part (iv) is a dot product, which must be a scalar. See part (iv) for an explanation of an impossible calculation.

67. We will use two different approaches to prove this trigonometric identity. The first approach does not require the knowledge of vectors, while the second approach uses vector properties and vector multiplication. You will see that the second approach is simpler.

The length of $\vec{B}$ in the figure below can be obtained from the coordinates of its two end points, which are (cos α, sin α) and (cos β, sin β):

$$B^2 = (\cos\beta - \cos\alpha)^2 + (\sin\beta - \sin\alpha)^2$$
$$= \cos^2\beta + \cos^2\alpha - 2\cos\beta\cos\alpha + \sin^2\beta + \sin^2\alpha - 2\sin\beta\sin\alpha$$
$$= \cos^2\beta + \sin^2\beta + \cos^2\alpha + \sin^2\alpha - 2\cos\beta\cos\alpha - 2\sin\beta\sin\alpha$$
$$B^2 = 2 - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta \qquad (1)$$

The cosine law gives

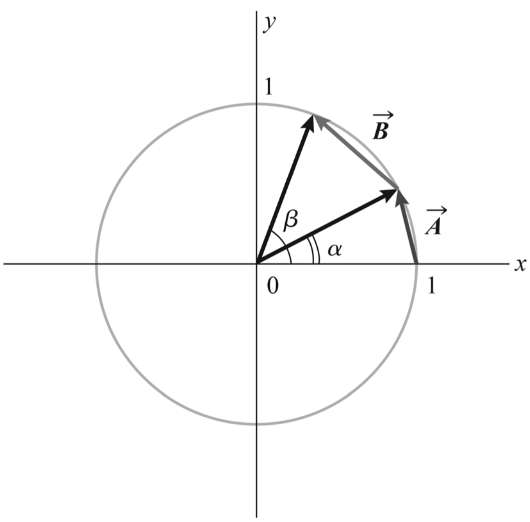
$$B^2 = 1 + 1 - 2\cos(\beta - \alpha)$$

Substituting the value of B^2 from (1), we get

$$1 + 1 - 2\cos(\beta - \alpha) = 2 - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta$$

Simplifying, we get

$$\cos(\beta - \alpha) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$



We can also use a different approach. Let the radius vector at angle α be $\vec{r}_1$, and let the radius vector at angle β be $\vec{r}_2$. The coordinates of these two vectors are (cos (α), sin (α)) and (cos (β), sin (β)). Thus,

$$\vec{r}_1 \cdot \vec{r}_2 = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

But the dot product can also be defined using the cosine of the angle between the two vectors:

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$\vec{r}_1 \bullet \vec{r}_2 = r_1 r_2 \cos(\beta - \alpha) = (1)(1)\cos(\beta - \alpha) = \cos(\beta - \alpha)$

Thus,

$\cos(\beta - \alpha) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

68. Let us denote the vectors we are looking for as $\vec{D}(x, y, z)$.

First, we find the magnitude of vector $\vec{C}$:

$C = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

Thus,

$x^2 + y^2 + z^2 = 14$ (1)

Since the vectors we are looking for are perpendicular to both $\vec{A}$ and $\vec{B}$, their scalar products with vectors $\vec{A}$ and $\vec{B}$ must be equal to zero, so their coordinates, x, y, and z, must satisfy the following equations:

$2x - 3y - 6z = 0$ (2)

$x - 4y + 2z = 0$ (3)

Eliminate x from (2) and (3) to get z in terms of y:

$$\begin{aligned} 2x - 3y - 6z &= 0 \\ 2x - 8y + 4z &= 0 \\ 5y - 10z &= 0 \\ z &= \frac{y}{2} \end{aligned}$$

Eliminate z from (2) and (3) to get x in terms of y:

$$\begin{aligned} 2x - 3y - 6z &= 0 \\ 3x - 12y + 6z &= 0 \\ 5x - 15y &= 0 \\ x &= 3y \end{aligned}$$

Substitute these into (1):

$$\begin{aligned} x^2 + y^2 + z^2 &= 14 \\ (3y)^2 + y^2 + \left(\frac{y}{2}\right)^2 &= 14 \\ 9y^2 + y^2 + \frac{y^2}{4} &= 14 \\ 41y^2 &= 56 \\ y &= \pm 2\sqrt{\frac{14}{41}} \end{aligned}$$

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Thus, $x = \pm 6\sqrt{\frac{14}{41}}$ and $z = \pm\sqrt{\frac{14}{41}}$.

So,

$$\vec{\boldsymbol{D}} = 6\sqrt{\frac{14}{41}}\hat{\boldsymbol{i}} + 2\sqrt{\frac{14}{41}}\hat{\boldsymbol{j}} + \sqrt{\frac{14}{41}}\hat{\boldsymbol{k}} \quad \text{or} \quad \vec{\boldsymbol{D}} = -6\sqrt{\frac{14}{41}}\hat{\boldsymbol{i}} - 2\sqrt{\frac{14}{41}}\hat{\boldsymbol{j}} - \sqrt{\frac{14}{41}}\hat{\boldsymbol{k}}$$

These solutions make sense as the two vectors are opposite vectors.

69. Solution using the Pythagorean theorem:

Let us look at the triangle formed by one Cl-C-Cl combination. We can cut this triangle in half by dropping a perpendicular to the base from the vertex. Let us consider one of these triangles and find the angle α it subtends at the vertex. Then we will find the required angle as $\theta = 2\alpha$.

If we assume each side to be of unit length, then the base of this triangle will be 0.5 units and its height will be 0.707 units (using the Pythagorean theorem):

$$\alpha = \tan^{-1}\left(\frac{0.707}{0.5}\right) = 54.7°$$
$$\theta = 2\alpha = 109.5°$$

Solution using vectors:

Let us imagine that the cube in Figure 2-26 has an edge length equal to one. Let us imagine a coordinate system with the origin at the left bottom vertex of the cube. Then let us choose two vectors that coincide with the side of the triangle Cl-C-Cl (bottom left and right Cl atoms in the figure):

$$\mathrm{C}\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right); \mathrm{Cl}_{\text{bottom-left}} = (0,0,0); \mathrm{Cl}_{\text{bottom-right}} = (1,1,0)$$

From this, we have

$$\overrightarrow{\boldsymbol{CCl}}_{\text{left}} = \left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right) \text{ and } \overrightarrow{\boldsymbol{CCl}}_{\text{right}} = \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$$

So,

$$\mathrm{CCl}_{\text{left}} = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}}$$

$$\mathrm{CCl}_{\text{right}} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}} = \mathrm{CCl}_{\text{left}}$$

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From the definitions of the dot product,

$$\overrightarrow{\boldsymbol{CCl}}_{\text{left}} \bullet \overrightarrow{\boldsymbol{CCl}}_{\text{right}} = -\frac{1}{4} - \frac{1}{4} + \frac{1}{4} = -\frac{1}{4}$$

and

$$\begin{aligned}\overrightarrow{\boldsymbol{CCl}}_{\text{left}} \bullet \overrightarrow{\boldsymbol{CCl}}_{\text{right}} &= \text{CCl}_{\text{left}}\text{CCl}_{\text{right}} \cos\theta \\ &= \text{CCl}_{\text{left}}\text{CCl}_{\text{left}} \cos\theta\end{aligned}$$

Thus,

$$\text{CCl}_{\text{left}}\text{CCl}_{\text{left}} \cos\theta = -\frac{1}{4}$$

$$\left(\sqrt{\frac{3}{4}}\right)\left(\sqrt{\frac{3}{4}}\right)\cos\theta = -\frac{1}{4}$$

$$\cos\theta = \frac{-\frac{1}{4}}{\frac{3}{4}} = -\frac{1}{3}$$

$$\theta = \cos^{-1}\left(-\frac{1}{3}\right) = 109.5°$$

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