Solutions for Elements of Electromagnetics 7th Edition by Sadiku

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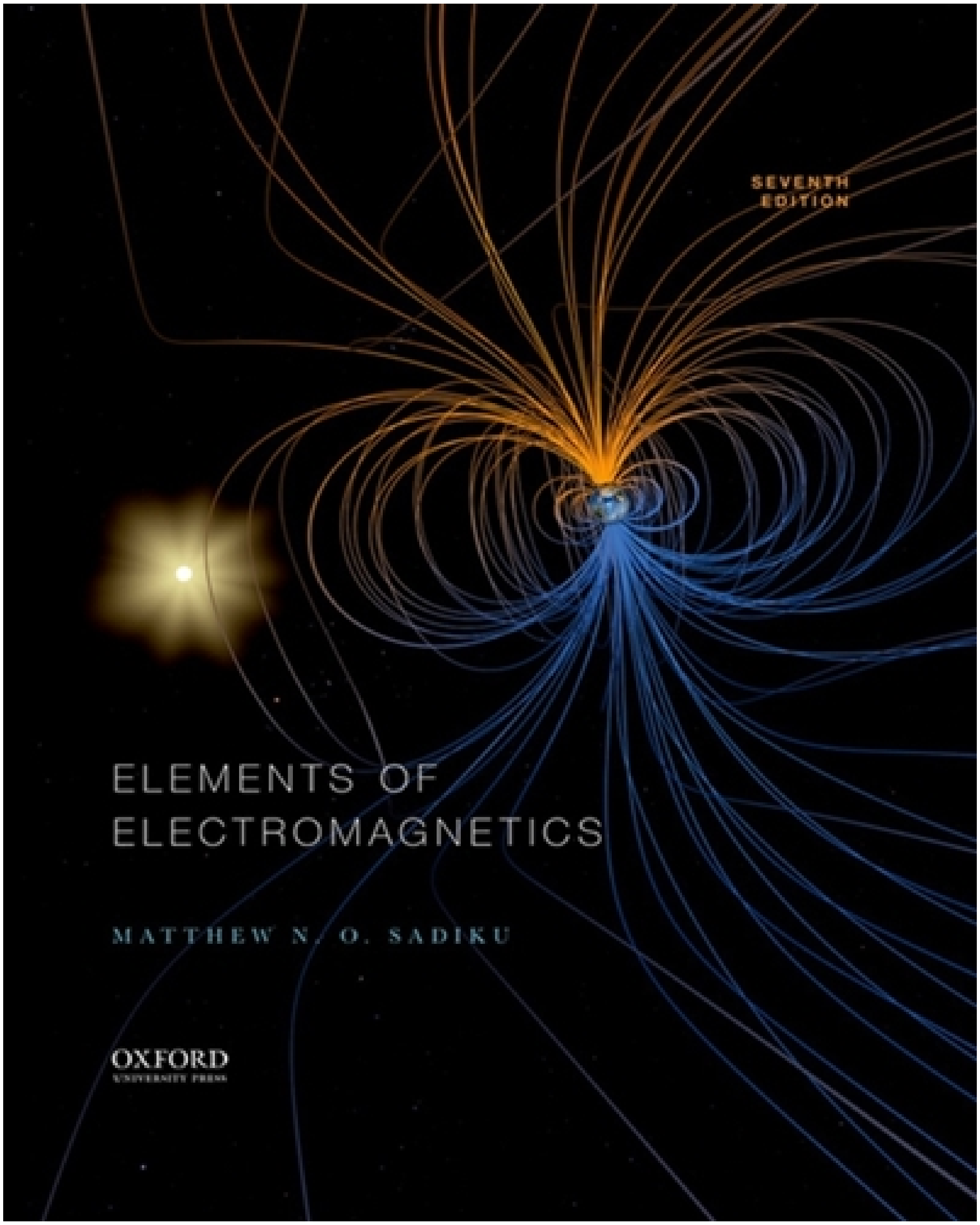


Solutions

CHAPTER 10

P. E. 10.1 (a)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\times10^8} = \underline{\underline{31.42\ \text{ns}}},$$

$$\lambda = uT = 3\times10^8 \times 31.42\times10^{-9} = \underline{\underline{9.425\ \text{m}}}$$

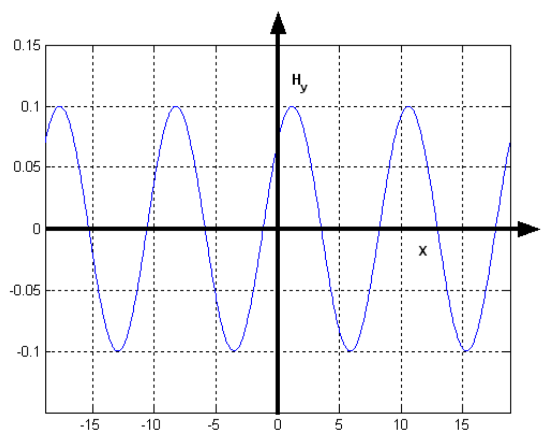
$$k = \beta = 2\pi/\lambda = \underline{\underline{0.6667\ \text{rad/m}}}$$

(b) $t_1 = T/8 = \underline{\underline{3.927\ \text{ns}}}$

(c)

$$\boldsymbol{H}(t=t_1) = 0.1\cos(2\times10^8 \frac{\pi}{8\times10^8} - 2x/3)\boldsymbol{a}_y = 0.1\cos(2x/3 - \pi/4)\boldsymbol{a}_y$$

as sketched below.



P. E. 10.2 Let $x_o = \sqrt{1+(\sigma/\omega\varepsilon)^2}$, then

$$\alpha = \omega\sqrt{\frac{\mu_o\varepsilon_o}{2}\mu_r\varepsilon_r(x_o - 1)} = \frac{\omega}{c}\sqrt{\frac{16}{2}}\sqrt{x_o - 1}$$

or $\sqrt{x_o - 1} = \frac{\alpha c}{\omega\sqrt{8}} = \frac{1/3\times3\times10^8}{10^8\sqrt{8}} = \frac{1}{\sqrt{8}} \longrightarrow x_o = 9/8$

$$x_o^2 = \frac{81}{64} = 1+(\sigma/\omega\varepsilon)^2 \longrightarrow \frac{\sigma}{\omega\varepsilon} = 0.5154$$

$\tan 2\theta_\eta = 0.5154 \longrightarrow \theta_\eta = 13.63^o$

$$\frac{\beta}{\alpha} = \sqrt{\frac{x_o + 1}{x_o - 1}} = \sqrt{17}$$

(a) $\beta = \alpha\sqrt{17} = \frac{\sqrt{17}}{3} = \underline{\underline{1.374 \text{ rad/m}}}$

(b) $\frac{\sigma}{\omega\varepsilon} = \underline{\underline{0.5154}}$

(c) $|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{x_o}} = \frac{120\pi\sqrt{2/8}}{\sqrt{9/8}} = 177.72$

$\eta = \underline{\underline{177.72\angle 13.63^o\,\Omega}}$

(d) $u = \frac{\omega}{\beta} = \frac{10^8}{1.374} = \underline{\underline{7.278\times 10^7}}$ m/s

(e) $\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E \longrightarrow \mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_H \longrightarrow \mathbf{a}_H = \mathbf{a}_y$

$$\mathbf{H} = \frac{0.5}{177.5} e^{-z/3}\sin(10^8 t - \beta z - 13.63^o)\mathbf{a}_y = \underline{\underline{2.817 e^{-z/3}\sin(10^8 t - \beta z - 13.63^o)\mathbf{a}_y \text{ mA/m}}}$$

P. E. 10.3 (a) Along $\underline{\underline{\text{-z direction}}}$

(b) $\lambda = \frac{2\pi}{\beta} = 2\pi/2 = \underline{\underline{3.142 \text{ m}}}$

$f = \frac{\omega}{2\pi} = \frac{10^8}{2\pi} = \underline{\underline{15.92 \text{ MHz}}}$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o}\sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\sqrt{(1)\varepsilon_r}$$

or $\sqrt{\varepsilon_r} = \beta c/\omega = \frac{3\times 10^8 \times 2}{10^8} = 6 \longrightarrow \underline{\underline{\varepsilon_r = 36}}$

(c) $\theta_\eta = 0, |\eta| = \sqrt{\mu/\varepsilon} = \sqrt{\mu_o/\varepsilon_o}\sqrt{1/\varepsilon_r} = \frac{120\pi}{6} = 20\pi$

$\mathbf{a}_k = \mathbf{a}_E \times \mathbf{a}_H \longrightarrow -\mathbf{a}_z = \mathbf{a}_y \times \mathbf{a}_H \longrightarrow \mathbf{a}_H = \mathbf{a}_x$

$$\boldsymbol{H} = \frac{50}{20\pi}\sin(\omega t + \beta z)\boldsymbol{a}_x = \underline{\underline{795.8\sin(10^8 t + 2z)\boldsymbol{a}_x}} \quad \text{mA/m}$$

P. E. 10.4 (a)

$$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-2}}{10^9 \pi \times 4 \times \frac{10^{-9}}{36\pi}} = 0.09$$

$$\alpha \cong \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = \frac{\omega}{2c}\sqrt{\mu_r\varepsilon_r}\,\frac{\sigma}{\omega\varepsilon} = \frac{10^9\pi}{2\times 3\times 10^8}(2)(0.09) = 0.9425 \text{ Np/m}$$

$$\beta \cong \omega\sqrt{\frac{\mu\varepsilon}{2}\left[1+\frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2 + 1\right]} = \frac{10^9\pi}{3\times 10^8}\sqrt{2[2+0.5(0.09)^2]} = 20.965 \text{ rad/m}$$

$$\boldsymbol{E} = 30e^{-0.9425y}\cos(10^9\pi t - 20.96y + \pi/4)\boldsymbol{a}_z$$

At t = 2ns, y = 1m,

$$\boldsymbol{E} = 30e^{-0.9425}\cos(2\pi - 20.96 + \pi/4)\boldsymbol{a}_z = \underline{\underline{2.844\boldsymbol{a}_z}} \quad \text{V/m}$$

(b) $\beta y = 10^o = \frac{10\pi}{180}$ rad

or

$$y = \frac{\pi}{18}\frac{1}{\beta} = \frac{\pi}{18\times 20.965} = \underline{\underline{8.325 \text{ mm}}}$$

(c) $30(0.6) = 30\ e^{-\alpha y}$

$$y = \frac{1}{\alpha}\ln(1/0.6) = \frac{1}{0.9425}\ln\frac{1}{0.6} = \underline{\underline{542 \text{ mm}}}$$

(d)

$$|\eta| \cong \frac{\sqrt{\mu/\varepsilon}}{[1+\frac{1}{4}(0.09)^2]} = \frac{60\pi}{1.002} = 188.11\ \Omega$$

$2\theta_\eta = \tan^{-1} 0.09 \longrightarrow \theta_\eta = 2.571^o$

$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_y \times \boldsymbol{a}_z = \boldsymbol{a}_x$

$$\boldsymbol{H} = \frac{30}{188.11} e^{-0.9425y} \cos(10^9 \pi t - 20.96y + \pi/4 - 2.571^o)\boldsymbol{a}_x$$

At y = 2m, t = 2ns,

$$\boldsymbol{H} = (0.1595)(0.1518)\ \cos(-34.8963 rad)\boldsymbol{a}_x = \underline{\underline{-22.83\boldsymbol{a}_x}}\ \text{mA/m}$$

P. E. 10.5

$$I_s = \int_0^w\int_0^\infty J_{xs} dydz = J_{xs}(0)\int_0^w dy \int_0^\infty e^{-z(1+j)/\delta} dz = \frac{J_{xs}(0)w\delta}{1+j}$$

$$\underline{\underline{|I_s| = \frac{J_{xs}(0)w\delta}{\sqrt{2}}}}$$

P. E. 10.6 (a)

$$\frac{R_{ac}}{R_{dc}} = \frac{a}{2\delta} = \frac{a}{2}\sqrt{\pi f \mu\sigma} = \frac{1.3\times10^{-3}}{2}\sqrt{\pi\times10^7\times4\pi\times10^{-7}\times3.5\times10^7} = \underline{\underline{24.16}}$$

(b)

$$\frac{R_{ac}}{R_{dc}} = \frac{1.3\times10^{-3}}{2}\sqrt{\pi\times2\times10^9\times4\pi\times10^{-7}\times3.5\times10^7} = \underline{\underline{341.7}}$$

P. E. 10.7

$$E = \text{Re}[\boldsymbol{E}_s e^{j\omega t}] = \text{Re}\left[E_o e^{j\omega t} e^{-j\beta z}\boldsymbol{a}_x + E_o e^{-j\pi/2} e^{j\omega t} e^{-j\beta z}\boldsymbol{a}_y\right]$$

$$= E_o \cos(\omega t - \beta z)\boldsymbol{a}_x + E_o \cos(\omega t - \beta z - \pi/2)\boldsymbol{a}_y$$

$$= E_o \cos(\omega t - \beta z)\boldsymbol{a}_x + E_o \sin(\omega t - \beta z)\boldsymbol{a}_y$$

At z = 0, $E_x = E_o \cos\omega t, E_y = E_o \sin\omega t$

$$\cos^2\omega t + \sin^2\omega t = 1 \longrightarrow \left(\frac{E_x}{E_o}\right)^2 + \left(\frac{E_y}{E_o}\right)^2 = 1$$

which describes a circle. Hence the polarization is circular.

P. E. 10.8

$$\mathcal{P}_{ave} = \frac{1}{2}\eta H_o{}^2 \mathbf{a}_x$$

(a) Let $f(x,z) = x + y - 1 = 0$

$$\mathbf{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{\mathbf{a}_x + \mathbf{a}_y}{\sqrt{2}}, \qquad d\mathbf{S} = \mathrm{dS}\mathbf{a}_n$$

$$P_t = \int \mathbf{P}.\mathrm{d}\mathbf{S} = \mathbf{P}.S\mathbf{a}_n = \frac{1}{2}\eta H_o{}^2 \mathbf{a}_x . \frac{\mathbf{a}_x + \mathbf{a}_y}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}(120\pi)(0.2)^2(0.1)^2 = \underline{\underline{53.31 \text{ mW}}}$$

(b) $d\mathbf{S} = \mathrm{dydz}\mathbf{a}_x$, $P_t = \int \mathcal{P}.\mathrm{d}\mathbf{S} = \frac{1}{2}\eta H_o{}^2 S$

$$P_t = \frac{1}{2}(120\pi)(0.2)^2 \pi (0.05)^2 = \underline{\underline{59.22 \text{ mW}}}$$

P. E. 10.9 $\eta_1 = \eta_o = 120\pi, \eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_o}{2}$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 2/3, \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1/3$$

$$E_{ro} = \Gamma E_{io} = -\frac{10}{3}$$

$$\underline{\underline{\mathbf{E}_{rs} = -\frac{10}{3} e^{j\beta_1 z} \mathbf{a}_x \text{ V/m}}}$$

where $\beta_1 = \omega / c = 100\pi / 3$.

$$E_{to} = \tau E_{io} = \frac{20}{3}$$

$$\underline{\underline{\mathbf{E}_{ts} = \frac{20}{3} e^{-j\beta_2 z} \mathbf{a}_x \text{ V/m}}}$$

where $\beta_2 = \omega \sqrt{\varepsilon_r} / c = 2\beta_1 = 200\pi / 3$.

P. E. 10.10

$$\alpha_1 = 0, \quad \beta_1 = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = \frac{2\omega}{c} = 5 \longrightarrow \omega = 5c/2 = 7.5\times10^8$$

$$\frac{\sigma_2}{\omega\varepsilon_2} = \frac{0.1}{7.5\times10^8 \times 4 \times \frac{10^{-9}}{36\pi}} = 1.2\pi$$

$$\alpha_2 = \frac{\omega}{c}\sqrt{\frac{4}{2}\left[\sqrt{1+1.44\pi^2} - 1\right]} = 6.021$$

$$\beta_2 = \frac{\omega}{c}\sqrt{\frac{4}{2}\left[\sqrt{1+1.44\pi^2} + 1\right]} = 7.826$$

$$|\eta_2| = \frac{60\pi}{\sqrt[4]{1+1.44\pi^2}} = 95.445, \eta_1 = 120\pi\sqrt{\varepsilon_{r1}} = 754$$

$$\tan 2\theta_{\eta_2} = 1.2\pi \longrightarrow \theta_{\eta_2} = 37.57^o$$

$$\eta_2 = 95.445\angle 37.57^o$$

(a)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{95.445\angle 37.57^o - 754}{95.445\angle 37.57^o + 754} = \underline{\underline{0.8186\angle 171.08^o}}$$

$$\tau = 1 + \Gamma = \underline{\underline{0.2295\angle 33.56^o}}$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.8186}{1-0.8186} = \underline{\underline{10.025}}$$

(b) $\boldsymbol{E}_i = 50\sin(\omega t - 5x)\boldsymbol{a}_y = \text{Im}(\boldsymbol{E}_{is}e^{j\omega t})$, where $E_{is} = 50e^{-j5x}a_y$.

$$E_{ro} = \Gamma E_{io} = 0.8186e^{j171.08^o}(50) = 40.93e^{j171.08^o}$$

$$\boldsymbol{E}_{rs} = 40.93e^{j5x + j171.08^o}\boldsymbol{a}_y$$

$$\boldsymbol{E}_r = \text{Im}(\boldsymbol{E}_{rs}e^{j\omega t}) = \underline{\underline{40.93\sin(\omega t + 5x + 171.1^o)\boldsymbol{a}_y}} \text{ V/m}$$

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = -\boldsymbol{a}_x \times \boldsymbol{a}_y = -\boldsymbol{a}_z$$

$$\boldsymbol{H}_r = -\frac{40.93}{754}\sin(\omega t + 5x + 171.1^o)\boldsymbol{a}_z = \underline{\underline{-0.0543\sin(\omega t + 5x + 171.1^o)\boldsymbol{a}_z}} \quad \text{A/m}$$

(c)

$$E_{to} = \tau E_{io} = 0.229e^{j33.56^o}(50) = 11.475e^{j33.56^o}$$

$$\boldsymbol{E}_{ts} = 11.475e^{-j\beta_2 x + j33.56^o}e^{-\alpha_2 x}\boldsymbol{a}_y$$

$$\boldsymbol{E}_t = \text{Im}(\boldsymbol{E}_{ts}e^{j\omega t}) = \underline{\underline{11.475e^{-6.021x}\sin(\omega t - 7.826x + 33.56^o)\boldsymbol{a}_y}} \quad \text{V/m}$$

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_x \times \boldsymbol{a}_y = \boldsymbol{a}_z$$

$$\boldsymbol{H}_t = \frac{11.495}{95.445}e^{-6.021x}\sin(\omega t - 7.826x + 33.56^o - 37.57^o)\boldsymbol{a}_z$$

$$= \underline{\underline{0.1202e^{-6.021x}\sin(\omega t - 7.826x - 4.01^o)\boldsymbol{a}_z}} \quad \text{A/m}$$

(d)

$$\mathcal{P}_{1\text{ave}} = \frac{E_{io}^{\ 2}}{2\eta_1}\boldsymbol{a}_x + \frac{E_{ro}^{\ 2}}{2\eta_1}(-\boldsymbol{a}_x) = \frac{1}{2(240\pi)}[50^2\boldsymbol{a}_x - 40.93^2\boldsymbol{a}_x] = \underline{\underline{0.5469\boldsymbol{a}_x}} \quad \text{W/m}^2$$

$$\boldsymbol{P}_{2\text{ave}} = \frac{E_{to}^{\ 2}}{2|\eta_2|}e^{-2\alpha_2 x}\cos\theta_{\eta_2}\boldsymbol{a}_x = \frac{(11.475)^2}{2(95.445)}\cos 37.57^o e^{-2(6.021)x}\boldsymbol{a}_x = \underline{\underline{0.5469e^{-12.04x}\boldsymbol{a}_x}} \quad \text{W/m}^2$$

P. E. 10.11 (a)

$$\boldsymbol{k} = -2\boldsymbol{a}_y + 4\boldsymbol{a}_z \longrightarrow k = \sqrt{2^2 + 4^2} = \sqrt{20}$$

$$\omega = kc = 3\times 10^8\sqrt{20} = \underline{\underline{1.342\times 10^9 \text{ rad/s}}},$$

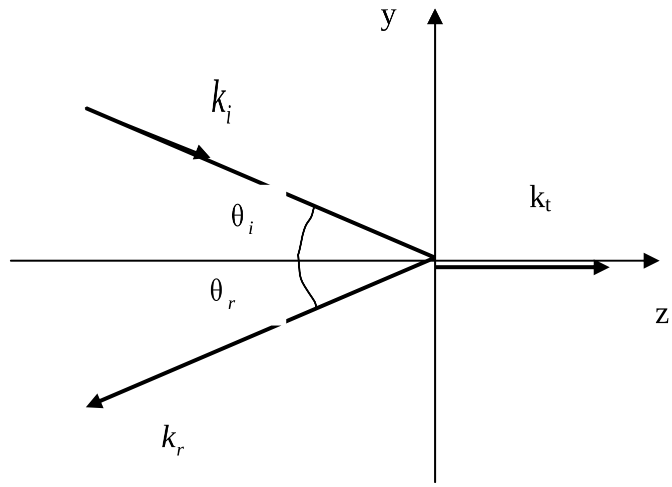
$$\lambda = 2\pi / k = \underline{\underline{1.405\text{m}}}$$

(b) $$\boldsymbol{H} = \frac{\boldsymbol{a}_k \times \boldsymbol{E}}{\eta_o} = \frac{(-2\boldsymbol{a}_y + 4\boldsymbol{a}_z)}{\sqrt{20}(120\pi)} \times (10\boldsymbol{a}_y + 5\boldsymbol{a}_z)\cos(\omega t - \boldsymbol{k}.\boldsymbol{r})$$

$$= \underline{\underline{-29.66\cos(1.342x10^9 t + 2y - 4z)\boldsymbol{a}_x}} \quad \text{mA/m}$$

(c) $\mathscr{P}_{ave} = \frac{|E_o|^2}{2\eta_o}\boldsymbol{a}_k = \frac{125}{2(120\pi)}\frac{(-2\boldsymbol{a}_y + 4\boldsymbol{a}_z)}{\sqrt{20}} = \underline{\underline{-74.15\boldsymbol{a}_y + 148.9\boldsymbol{a}_z}}$ mW/m^2

P. E. 10.12 (a)



$$\tan\theta_i = \frac{k_{iy}}{k_{iz}} = \frac{2}{4} \longrightarrow \underline{\underline{\theta_i = 26.56 = \theta_r}}$$

$$\sin\theta_t = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\sin\theta_i = \frac{1}{2}\sin 26.56^o \longrightarrow \underline{\underline{\theta_t = 12.92^o}}$$

(b) $\eta_1 = \eta_o, \eta_2 = \eta_o/2$ **E** is parallel to the plane of incidence. Since $\mu_1 = \mu_2 = \mu_o$,

we may use the result of Prob. 10.42, i.e.

$$\Gamma_{\backslash\backslash} = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} = \frac{\tan(-13.64^o)}{\tan(39.48^o)} = \underline{\underline{-0.2946}}$$

$$\tau_{\backslash\backslash} = \frac{2\cos 26.56^o \sin 12.92^o}{\sin 39.48^o \cos(-13.64^o)} = \underline{\underline{0.6474}}$$

(c) $\boldsymbol{k}_r = -\beta_1 \sin\theta_r \boldsymbol{a}_y - \beta_1\cos\theta_r\boldsymbol{a}_z$. Once k_r is known, E_r is chosen such that

$\boldsymbol{k}_r . \boldsymbol{E}_r = 0$ or $\nabla . \boldsymbol{E}_r = 0$. Let

$\boldsymbol{E}_r = \pm E_{or}(-\cos\theta_r\boldsymbol{a}_y + \sin\theta_r\boldsymbol{a}_z)\cos(\omega t + \beta_1\sin\theta_r y + \beta_1\cos\theta_r z)$

Only the positive sign will satisfy the boundary conditions. It is evident that

$$\boldsymbol{E}_i = E_{oi}(\cos\theta_i \boldsymbol{a}_y + \sin\theta_i \boldsymbol{a}_z)\cos(\omega t + 2y - 4z)$$

Since $\theta_r = \theta_i$,

$$E_{or}\cos\theta_r = \Gamma_{//}E_{oi}\cos\theta_i = 10\Gamma_{//} = -2.946$$

$$E_{or}\sin\theta_r = \Gamma_{//}E_{oi}\sin\theta_i = 5\Gamma_{//} = -1.473$$

$$\beta_1\sin\theta_r = 2, \beta_1\cos\theta_r = 4$$

i.e.

$$E_r = -(2.946a_y - 1.473a_z)\cos(\omega t + 2y + 4z)$$

$$\boldsymbol{E}_1 = \boldsymbol{E}_i + \boldsymbol{E}_r = \underline{\underline{(10\boldsymbol{a}_y + 5\boldsymbol{a}_z)\cos(\omega t + 2y - 4z) + (-2.946\boldsymbol{a}_y + 1.473\boldsymbol{a}_z)\cos(\omega t + 2y + 4z)}}$$

V/m

(d) $\boldsymbol{k}_t = -\beta_2\sin\theta_t \boldsymbol{a}_y + \beta_2\cos\theta_t \boldsymbol{a}_z$. Since $k_r \bullet E_r = 0$, let

$$\boldsymbol{E}_t = E_{ot}(\cos\theta_t \boldsymbol{a}_y + \sin\theta_t \boldsymbol{a}_z)\cos(\omega t + \beta_2 y\sin\theta_t - \beta_2 z\cos\theta_t)$$

$$\beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = \beta_1\sqrt{\varepsilon_{r2}} = 2\sqrt{20}$$

$$\sin\theta_t = \frac{1}{2}\sin\theta_i = \frac{1}{2\sqrt{5}}, \qquad \cos\theta_t = \frac{\sqrt{9}}{\sqrt{20}}$$

$$\beta_2\cos\theta_t = 2\sqrt{20}\sqrt{\frac{19}{20}} = 8.718$$

$$E_{ot}\cos\theta_t = \tau_{//}E_{oi}\cos\theta_t = 0.6474\sqrt{125}\sqrt{\frac{19}{20}} = 7.055$$

$$E_{ot}\sin\theta_t = \tau_{//}E_{oi}\sin\theta_t = 0.6474\sqrt{125}\sqrt{\frac{1}{20}} = 1.6185$$

Hence

$$\boldsymbol{E}_2 = \boldsymbol{E}_t = \underline{\underline{(7.055\boldsymbol{a}_y + 1.6185\boldsymbol{a}_z)\cos(\omega t + 2y - 8.718z)}} \quad \text{V/m}$$

(d) $\tan\theta_{B//} = \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = 2 \longrightarrow \underline{\underline{\theta_{B//} = 63.43^o}}$

P.E. 10.13

$$S_i = \frac{1+0.4}{1-0.4} = \frac{1.4}{0.6} = \underline{\underline{2.333}}$$

$$S_o = \frac{1+0.2}{1-0.2} = \frac{1.2}{0.8} = \underline{\underline{1.5}}$$

Prob. 10.1 (a) Wave propagates $\underline{\underline{\text{along } +\mathbf{a}_x}}$.

(b)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi \times 10^6} = \underline{\underline{1\mu s}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6} = \underline{\underline{1.047\text{m}}}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi \times 10^6}{6} = \underline{\underline{1.047 \times 10^6\,\text{m/s}}}$$

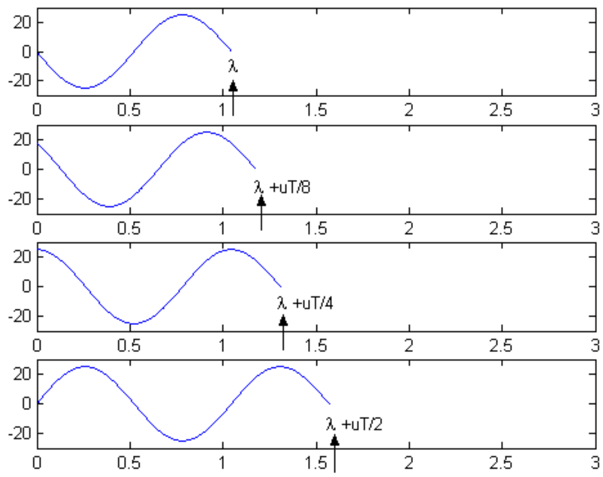
(c) At t=0, $E_z = 25\sin(-6x) = -25\sin 6x$

At t=T/8, $E_z = 25\sin(\frac{2\pi}{T}\frac{T}{8} - 6x) = 25\sin(\frac{\pi}{4} - 6x)$

At t=T/4, $E_z = 25\sin(\frac{2\pi}{T}\frac{T}{4} - 6x) = 25\sin(-6x + 90^o) = 25\cos 6x$

At t=T/2, $E_z = 25\sin(\frac{2\pi}{T}\frac{T}{2} - 6x) = 25\sin(-6x + \pi) = 25\sin 6x$

These are sketched below.



Prob. 10.2

(a) $\lambda = \frac{c}{f} = \frac{3\times10^8}{60} = \underline{\underline{5\times10^6\ \text{m}}}$

(b) $\lambda = \frac{3\times10^8}{2\times10^6} = \underline{\underline{150\ \text{m}}}$

(c) $\lambda = \frac{3\times10^8}{120\times10^6} = \underline{\underline{2.5\ \text{m}}}$

(d) $\lambda = \frac{3\times10^8}{2.4\times10^9} = \underline{\underline{0.125\ \text{m}}}$

Prob. 10.3

(a) $\omega = \underline{\underline{10^8\ \text{rad/s}}}$

(b) $\beta = \frac{\omega}{c} = \frac{10^8}{3\times10^8} = \underline{\underline{0.333\ \text{rad/m}}}$

(c) $\lambda = \frac{2\pi}{\beta} = 6\pi = \underline{\underline{18.85\ \text{m}}}$

(d) Along **-a**$_y$

At y=1, t=10ms,

(e) $H = 0.5\cos(10^8 t\times10\times10^{-9} + \frac{1}{3}\times3) = 0.5\cos(1+1)$

$= \underline{\underline{-0.1665\ \text{A/m}}}$

Prob. 10.4
(a)

$$\frac{\partial E}{\partial x} = -\sin(x+\omega t) - \sin(x-\omega t)$$

$$\frac{\partial^2 E}{\partial x^2} = -\cos(x+\omega t) - \cos(x-\omega t) = -E$$

$$\frac{\partial E}{\partial t} = -\omega \sin(x+\omega t) - \omega \sin(x-\omega t)$$

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 \cos(x+\omega t) - \omega^2 \cos(x-\omega t) = -\omega^2 E$$

$$\frac{\partial^2 E}{\partial t^2} - u^2 \frac{\partial^2 E}{\partial x^2} = -\omega^2 E + u^2 E = 0$$

if $\omega^2 = u^2$ and hence, eq. (10.1) is satisfied.

(b) $\underline{\underline{u = \omega}}$

Prob. 10.5 If

$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon) = -\omega^2\mu\varepsilon + j\omega\mu\sigma$ and $\gamma = \alpha + j\beta$, then

$$|\gamma^2| = \sqrt{(\alpha^2 - \beta^2) + 4\alpha^2\beta^2} = \sqrt{(\alpha^2 + \beta^2)^2} = \alpha^2 + \beta^2$$

i.e.

$$\alpha^2 + \beta^2 = \omega\mu\sqrt{(\sigma^2 + \omega^2\varepsilon^2)} \qquad (1)$$

$$\text{Re}(\gamma^2) = \alpha^2 - \beta^2 = -\omega^2\mu\varepsilon$$

$$\beta^2 - \alpha^2 = \omega^2\mu\varepsilon \qquad (2)$$

Subtracting and adding (1) and (2) lead respectively to

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

(b) From eq. (10.25), $\boldsymbol{E}_s(z) = E_o e^{-\gamma z}\boldsymbol{a}_x$.

$\nabla \times \boldsymbol{E} = -j\omega\mu \boldsymbol{H}_s \longrightarrow \boldsymbol{H}_s = \frac{j}{\omega\mu}\nabla \times \boldsymbol{E}_s = \frac{j}{\omega\mu}(-\gamma E_o e^{-\gamma z}\boldsymbol{a}_y)$

But $\boldsymbol{H}_s(z) = H_o e^{-\gamma z}\boldsymbol{a}_y$, hence $H_o = \frac{E_o}{\eta} = -\frac{j\gamma}{\omega\mu}E_o$

$$\eta = \frac{j\omega\mu}{\gamma}$$

(c) From (b),

$$\eta = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}}, \tan 2\theta_\eta = \left(\frac{\omega\varepsilon}{\sigma}\right)^{-1} = \frac{\sigma}{\omega\varepsilon}$$

Prob. 10.6 (a)

From eq. (10.18),

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \sqrt{j\omega\mu\, j\omega\varepsilon(1 - \frac{j\sigma}{\omega\varepsilon})} = j\omega\sqrt{\mu\varepsilon}\left(1 - \frac{j\sigma}{\omega\varepsilon}\right)^{1/2}$$

Assuming that $\frac{\sigma}{\omega\varepsilon}$ □ 1, we include up to the second power in $\frac{\sigma}{\omega\varepsilon}$ and neglect higher-order terms.

$$\gamma = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}\left(1 - \frac{\sigma^2}{\omega^2\varepsilon}\right) + j\omega\sqrt{\mu\varepsilon}\left(1 + \frac{\sigma^2}{8\omega^2\varepsilon^2}\right) = \alpha + j\beta$$

Thus,

$$\beta = \omega\sqrt{\mu\varepsilon}\left(1 + \frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

Prob. 10.7

(a)

$$\frac{\sigma}{\omega\varepsilon} = \frac{8\times 10^{-2}}{2\pi \times 50 \times 10^6 \times 3.6 \times \frac{10^{-9}}{36\pi}} = 8$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]} = \frac{2\pi\times 50\times 10^6}{3\times 10^8}\sqrt{\frac{2.1\times 3.6}{2}[\sqrt{65}-1]} = 5.41$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]} = 6.129$$

$$\gamma = \alpha + j\beta = \underline{\underline{5.41 + j6.129}} \text{ /m}$$

(b) $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{6.129} = \underline{\underline{1.025}}\text{ m}$

(c) $u = \dfrac{\omega}{\beta} = \dfrac{2\pi\times 50\times 10^6}{6.129} = \underline{\underline{5.125\times 10^7}}\text{ m/s}$

(d) $$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}} = \frac{120\pi\sqrt{\frac{2.1}{3.6}}}{\sqrt[4]{65}} = 101.4$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = 8 \longrightarrow \theta_\eta = 41.44^o$$

$$\eta = \underline{\underline{101.41\angle 41.44^o\ \Omega}}$$

(e) $$\mathbf{H}_s = \mathbf{a}_k \times \frac{\mathbf{E}_s}{\eta} = \mathbf{a}_x \times \frac{6}{\eta}e^{-\gamma z}\mathbf{a}_z = -\frac{6}{\eta}e^{-\gamma z}\mathbf{a}_y = \underline{\underline{-59.16e^{-j41.44^o}e^{-\gamma z}\mathbf{a}_y}} \text{ mA/m}$$

Prob. 10.8

(a) $\tan\theta = \dfrac{\sigma}{\omega\varepsilon} = \dfrac{10^{-2}}{2\pi\times 12\times 10^6\times 10\times\dfrac{10^{-9}}{36\pi}} = \underline{\underline{1.5}}$

(b) $\tan\theta = \dfrac{10^{-4}}{2\pi\times 12\times 10^6\times 4\times\dfrac{10^{-9}}{36\pi}} = \underline{\underline{3.75\times 10^{-2}}}$

(c) $\tan\theta = \dfrac{4}{2\pi \times 12\times 10^{6} \times 81 \times \dfrac{10^{-9}}{36\pi}} = \underline{\underline{74.07}}$

Prob. 10.9

(a)

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^{2}} - 1\right]} = \frac{2\pi\times 15\times 10^{9}}{3\times 10^{8}}\sqrt{\frac{1\times 9.6}{2}\left[\sqrt{1+9\times 10^{-8}} - 1\right]}$$

$$= 100\pi\sqrt{4.8\left(\frac{1}{2}\times 9\times 10^{-8}\right)} = 0.146$$

$$\delta = \frac{1}{\alpha} = \underline{\underline{6.85\text{ m}}}$$

(b) $A = \alpha\ell = 0.146\times 5\times 10^{-3} = \underline{\underline{0.73\times 10^{-3}\text{ Np}}}$

Prob. 10.10

If $\sigma = \varepsilon\omega$, the loss tangent is $\dfrac{\sigma}{\varepsilon\omega} = 1$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^{2}} - 1\right]} = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+1^{2}} - 1\right]} = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{2} - 1\right]}$$

But $\lambda_o = \dfrac{2\pi}{\beta} = \dfrac{2\pi c}{\omega} \quad \rightarrow \quad \omega = \dfrac{2\pi c}{\lambda_o}$

$$\alpha = \frac{2\pi c}{\lambda_o}\sqrt{\frac{\mu\varepsilon}{2}}\sqrt{\sqrt{2}-1} = \frac{2\pi\times 3\times 10^{8}}{12\times 10^{-2}}\sqrt{\frac{1}{2}\times 4\pi\times 10^{-7}\times 4\times\frac{10^{-9}}{36\pi}}(0.6436) = \underline{\underline{47.66\ \text{Np/m}}}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^{2}} + 1\right]} = \frac{2\pi c}{\lambda_o}\sqrt{\frac{\mu\varepsilon}{2}}\sqrt{\sqrt{2}+1}$$

$$= \frac{2\pi\times 3\times 10^{8}}{12\times 10^{-2}}\sqrt{\frac{1}{2}\times 4\pi\times 10^{-7}\times 4\times\frac{10^{-9}}{36\pi}}(1.5538)$$

$$= \underline{\underline{115.06\text{ rad/m}}}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi c}{\lambda_o\beta} = \frac{2\pi\times 3\times 10^{8}}{12\times 10^{-2}\times 115.06} = \underline{\underline{1.3652\times 10^{8}\ \text{m/s}}}$$

Prob. 10.11

For silver, the loss tangent is

$$\frac{\sigma}{\omega\varepsilon} = \frac{6.1\times10^{7}}{2\pi\times10^{8}\times\dfrac{10^{-9}}{36\pi}} = 6.1\times18\times10^{8} \Box\ 1$$

Hence, silver is a good conductor

For rubber,

$$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-15}}{2\pi\times10^{8}\times3.1\times\dfrac{10^{-9}}{36\pi}} = \frac{18}{3.1}\times10^{-14} \Box\ 1$$

Hence, rubber is a poor conductor or a good insulator.

Prob. 10.12

$$\frac{\sigma}{\omega\varepsilon} = \frac{4}{2\pi\times10^{5}\times80\times10^{-9}/36\pi} = 9,000 >> 1$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi\times10^{5}}{2}\times4\pi\times10^{-7}\times4} = 0.4\pi$$

(a) $u = \omega/\beta = \dfrac{2\pi\times10^{5}}{0.4\pi} = \underline{\underline{5\times10^{5}}}$ m/s

(b) $\lambda = 2\pi/\beta = \dfrac{2\pi}{0.4\pi} = \underline{\underline{5}}$ m

(c) $\delta = 1/\alpha = \dfrac{1}{0.4\pi} = \underline{\underline{0.796}}$ m

(d) $\eta = |\eta|\angle\theta_{\eta}, \theta_{\eta} = 45^{o}$

$$|\eta| = \frac{\sqrt{\dfrac{\mu}{\varepsilon}}}{\sqrt[4]{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^{2}}} \cong \sqrt{\frac{\mu}{\varepsilon}\frac{\omega\varepsilon}{\sigma}} = \sqrt{\frac{4\pi\times10^{-7}\times2\pi\times10^{5}}{4}} = 0.4443$$

$$\eta = \underline{\underline{0.4443\angle45^{o}\quad\Omega}}$$

Prob. 10.13 (a)

$$T = 1/f = 2\pi/\omega = \frac{2\pi}{\pi x 10^8} = \underline{\underline{20 \text{ ns}}}$$

(b) Let $x = \sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}$

$$\frac{\alpha}{\beta} = \left(\frac{x-1}{x+1}\right)^{1/2}$$

But $\alpha = \frac{\omega}{c}\sqrt{\frac{\mu_r \varepsilon_r}{2}}\sqrt{x-1}$

$$\sqrt{x-1} = \frac{\alpha c}{\omega\sqrt{\frac{\mu_r \varepsilon_r}{2}}} = \frac{0.1 \times 3 \times 10^8}{\pi \times 10^8 \sqrt{2}} = 0.06752 \longrightarrow x = 1.0046$$

$$\beta = \left(\frac{x+1}{x-1}\right)^{1/2} \alpha = \left(\frac{2.0046}{0.0046}\right)^{1/2} 0.1 = 2.088$$

$$\lambda = 2\pi/\beta = \frac{2\pi}{2.088} = \underline{\underline{3}} \text{ m}$$

(c) $|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{x}} = \frac{377}{2\sqrt{1.0046}} = 188.1$

$$x = \sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} = 1.0046$$

$$\frac{\sigma}{\omega\varepsilon} = 0.096 = \tan 2\theta_\eta \longrightarrow \theta_\eta = 2.74^o$$

$$\eta = 188.1\angle 2.74^o \quad \Omega$$

$$E_o = \eta H_o = 12 \times 188.1 = 2257.2$$

$$\boldsymbol{a}_E \times \boldsymbol{a}_H = \boldsymbol{a}_k \longrightarrow \boldsymbol{a}_E \times \boldsymbol{a}_x = \boldsymbol{a}_y \longrightarrow \boldsymbol{a}_E = \boldsymbol{a}_z$$

$$\boldsymbol{E} = \underline{\underline{2.257 e^{-0.1y} \sin(\pi \times 10^8 t - 2.088y + 2.74^o)\boldsymbol{a}_z}} \text{ kV/m}$$

(d) The phase difference is $\underline{\underline{2.74^o}}$.

Prob. 10.14

This is a lossy medium in which $\mu=\mu_o$.

Let $x = \left(\dfrac{\sigma}{\omega\varepsilon}\right)^2$

$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j2\pi\times10^9\times4\pi}{100+j200} = 35.31\angle 26.57^o$$

$$E_o = 0.05\times35.31 = 1.765$$

$$\boldsymbol{a}_E = \boldsymbol{a}_H \times \boldsymbol{a}_k = -\boldsymbol{a}_z$$

Thus, we obtain

$$\underline{\underline{\boldsymbol{E} = \text{-}1.765\cos(2\pi\times10^9 t - 200x + 26.57^o)\boldsymbol{a}_z \quad \text{V/m}}}$$

$$\sqrt{\varepsilon_r(1/3)} = \frac{\alpha c}{\omega} = \frac{100\times3\times10^8}{2\pi\times10^9} = \frac{15}{\pi}$$

$$\varepsilon_r\frac{1}{3} = 4.776 \quad\longrightarrow\quad \varepsilon_r = 14.32$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = \frac{4}{3} \quad\longrightarrow\quad \theta_\eta = 26.57^o$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt[4]{1+x}} = \frac{\dfrac{377}{\sqrt{14.32}}}{\sqrt{5/3}} = 77.175$$

$$E_o = |\eta| H_o = 77.175\times50\times10^{-3} = 3.858$$

$$\boldsymbol{a}_E = -(\boldsymbol{a}_k\times\boldsymbol{a}_H) = -(\boldsymbol{a}_x\times\boldsymbol{a}_y) = -\boldsymbol{a}_z$$

$$\boldsymbol{E} = \underline{\underline{-3.858e^{-100x}\cos(2\pi\times10^9 t - 200x + 26.57^o)\boldsymbol{a}_z \quad \text{V/m}}}$$

Prob. 10.15

$$\frac{\sigma}{\omega\varepsilon} = \frac{1}{2\pi\times10^9\times4\times\dfrac{10^{-9}}{36\pi}} = 4.5$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

$$= 2\pi\times10^9\sqrt{\frac{4\pi}{2}\times10^{-7}\times4\times9\times\frac{10^{-9}}{36\pi}\left[\sqrt{1+4.5^2}-1\right]}$$

$$= 20\pi\sqrt{2[\sqrt{21.25}-1]} = \underline{\underline{168.8 \text{ Np/m}}}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]} = 20\pi\sqrt{2[\sqrt{21.25}+1]} = \underline{\underline{210.5 \text{ rad/m}}}$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = 4.5 \longrightarrow \theta_\eta = 38.73^o$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt[4]{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2}} = \frac{120\pi\sqrt{9/4}}{\sqrt[4]{1+4.5^2}} = 263.38$$

$$\eta = \underline{\underline{263.38\angle38.73^o \ \ \Omega}}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi\times10^9}{210.5} = \underline{\underline{2.985\times10^7 \text{ m/s}}}$$

Prob. 10.16

(a) $\beta = 6.5 = \omega\sqrt{\mu_o\varepsilon_o} = \dfrac{\omega}{c}$

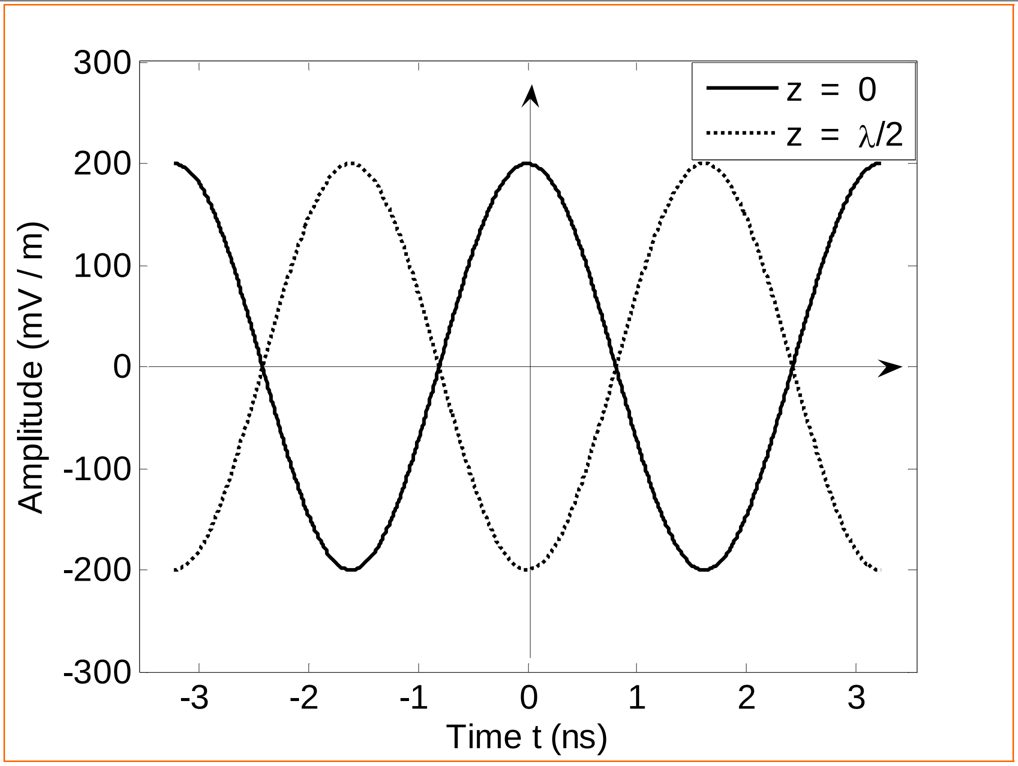
$$\omega = \beta c = 6.5\times3\times10^8 = \underline{\underline{1.95\times10^9 \text{ rad/s}}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6.5} = \underline{\underline{0.9666 \text{ m}}}$$

(b) For z=0, $E_z = 0.2\cos\omega t$

For z=λ/2, $E_z = 0.2\cos(\omega t - \dfrac{2\pi}{\lambda}\dfrac{\lambda}{2}) = -0.2\cos\omega t$

The two waves are sketched below.



(c) $\boldsymbol{H} = H_o \cos(\omega t - 6.5z)\boldsymbol{a}_H$

$$H_o = \frac{E_o}{\eta_o} = \frac{0.2}{377} = 5.305 \times 10^{-4}$$

$$\boldsymbol{a}_E \times \boldsymbol{a}_H = \boldsymbol{a}_k \quad \longrightarrow \quad \boldsymbol{a}_x \times \boldsymbol{a}_H = \boldsymbol{a}_z \quad \longrightarrow \quad \boldsymbol{a}_H = \boldsymbol{a}_y$$

$$\boldsymbol{H} = \underline{\underline{0.5305 \cos(\omega t - 6.5z)\boldsymbol{a}_y \text{ mA/m}}}$$

Prob. 10.17

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}}$$

$$\frac{\lambda_o}{\lambda} = \frac{\sqrt{\mu_o \varepsilon_o \varepsilon_r}}{\sqrt{\mu_o \varepsilon_o}} = \sqrt{\varepsilon_r}$$

$$\sqrt{\varepsilon_r} = \frac{\lambda_o}{\lambda} = \frac{6.4}{2.8} = 2.286 \quad \rightarrow \quad \underline{\underline{\varepsilon_r = 5.224}}$$

Prob. 10.18 (a) Along <u>-x direction.</u>

(b) $\beta = 6, \quad \omega = 2\times 10^8$,

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r}$$

$$\sqrt{\varepsilon_r} = \beta c/\omega = \frac{6\times 3\times 10^8}{2\times 10^8} = 9 \longrightarrow \varepsilon_r = 81$$

$$\varepsilon = \varepsilon_o\varepsilon_r = \frac{10^{-9}}{36\pi}\times 81 = \underline{\underline{7.162\times 10^{-10}}} \text{ F/m}$$

(c) $\eta = \sqrt{\mu/\varepsilon} = \sqrt{\mu_o/\varepsilon_o}\sqrt{\mu_r/\varepsilon_r} = \dfrac{120\pi}{9} = 41.89\ \Omega$

$$E_o = H_o\eta = 25\times 10^{-3}\times 41.88 = 1.047$$

$$\mathbf{a}_E\times\mathbf{a}_H = \mathbf{a}_k \longrightarrow \mathbf{a}_E\times\mathbf{a}_y = -\mathbf{a}_x \longrightarrow \mathbf{a}_E = \mathbf{a}_z$$

$$\mathbf{E} = \underline{\underline{1.047\sin(2\times 10^8 t + 6x)\mathbf{a}_z}} \text{ V/m}$$

Prob. 10.19 (a) $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{10^{-6}}{2\pi\times 10^7\times 5\times\dfrac{10^{-9}}{36\pi}} = 3.6\times 10^{-4} << 1$

Thus, the material is <u>lossless</u> at this frequency.

(b) $\beta = \omega\sqrt{\mu\varepsilon} = \dfrac{2\pi\times 10^7}{3\times 10^8}\sqrt{5\times 750} = \underline{\underline{12.83}}$ rad/m

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{12.83} = \underline{\underline{0.49}} \text{ m}$$

(c) Phase difference = βl = <u>25.66 rad</u>

(d) $\eta = \sqrt{\mu/\varepsilon} = 120\pi\sqrt{\dfrac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\dfrac{750}{5}} = \underline{\underline{4.62\text{ k}\Omega}}$

Prob. 10.20

(a)

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{10^8\pi}{3\times10^8} = \frac{\pi}{3} = \underline{\underline{1.0472 \text{ rad/m}}}$$

(b)

$$E = 0 \quad \longrightarrow \quad \sin(10^8\pi t_o - \beta x_o) = 0 = \sin(n\pi), n = 1, 2, 3, ...$$

$$10^8\pi t_o - \beta x_o = \pi$$

$$10^8\pi\times5\times10^{-3} - \frac{\pi}{3}x_o = \pi \quad \longrightarrow \quad x_o \approx \underline{\underline{5\times10^5 \text{ m}}}$$

(c)

$$\boldsymbol{H} = H_o\sin(10^8\pi t - \beta x)\boldsymbol{a}_H$$

$$H_o = \frac{E_o}{\eta} = \frac{50\times10^{-3}}{120\pi} = 132.63\ \mu\text{A/m}$$

$$\boldsymbol{a}_H = \boldsymbol{a}_k\times\boldsymbol{a}_E = \boldsymbol{a}_x\times\boldsymbol{a}_z = -\boldsymbol{a}_y$$

$$\underline{\underline{\boldsymbol{H} = -132.63\sin(10^8\pi t - 1.0472x)\boldsymbol{a}_y\ \mu\text{A/m}}}$$

Prob. 10.21

This is a lossless material.

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = 377\sqrt{\frac{\mu_r}{\varepsilon_r}} = 105 \qquad (1)$$

$$u = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = 7.6\times10^7 \qquad (2)$$

From (1),

$$\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{105}{377} = 0.2785 \qquad (1)a$$

From (2),

$$\frac{1}{\sqrt{\mu_r\varepsilon_r}} = \frac{7.6\times10^7}{3\times10^8} = 0.2533 \qquad (2)a$$

Multiplying (1)a by (2)a,

$$\frac{1}{\varepsilon_r} = 0.2785\times0.2533 = 0.07054 \quad \longrightarrow \quad \varepsilon_r = \underline{\underline{14.175}}$$

Dividing (1)a by (2)a,

$$\mu_r = \frac{0.2785}{0.2533} = \underline{\underline{1.0995}}$$

Prob. 10.22

(a) $$\nabla \times \boldsymbol{E} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(z,t) & E_y(z,t) & 0 \end{vmatrix} = -\frac{\partial E_y}{\partial z}\boldsymbol{a}_x + \frac{\partial E_x}{\partial z}\boldsymbol{a}_y$$

$$= -6\beta\cos(\omega t - \beta z)\boldsymbol{a}_x + 8\beta\sin(\omega t - \beta z)\boldsymbol{a}_y$$

But $\nabla \times \boldsymbol{E} = -\mu\frac{\partial \boldsymbol{H}}{\partial t} \longrightarrow \boldsymbol{H} = -\frac{1}{\mu}\int \nabla \times \boldsymbol{E}dt$

$$\boldsymbol{H} = \underline{\underline{\frac{6\beta}{\mu\omega}\sin(\omega t - \beta z)\boldsymbol{a}_x + \frac{8\beta}{\mu\omega}\cos(\omega t - \beta z)\boldsymbol{a}_y}}$$

(b) $\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi f}{c}\sqrt{4.5} = \frac{2\pi \times 40\times 10^6}{3\times 10^8}\sqrt{4.5} = \underline{\underline{1.777 \text{ rad/m}}}$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{1.777} = \underline{\underline{3.536 \text{ m}}}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{4.5}} = \underline{\underline{177.72\ \Omega}}$$

$$u = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{4.5}} = \frac{3\times 10^8}{\sqrt{4.5}} = \underline{\underline{1.4142\times 10^8 \text{ m/s}}}$$

Prob. 10.23

(a) $\boldsymbol{E} = \text{Re}[\boldsymbol{E}_s e^{j\omega t}] = (5\boldsymbol{a}_x + 12\boldsymbol{a}_y)e^{-0.2z}\cos(\omega t - 3.4z)$

At $z = 4$m, $t = T/8$, $\omega t = \frac{2\pi}{T}\frac{T}{8} = \frac{\pi}{4}$

$$\boldsymbol{E} = (5\boldsymbol{a}_x + 12\boldsymbol{a}_y)e^{-0.8}\cos(\pi/4 - 13.6)$$

$$|E| = 13e^{-0.8}|\cos(\pi/4 - 13.6)| = \underline{\underline{5.662}} \text{ V/m}$$

(b) loss $= \alpha\Delta z = 0.2(3) = 0.6$ Np. Since 1 Np = 8.686 dB,

loss = 0.6 x 8.686 = $\underline{\underline{5.212 \text{ dB}}}$

(c) Let $x = \sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2}$

$$\frac{\alpha}{\beta} = \left(\frac{x-1}{x+1}\right)^{1/2} = 0.2/3.4 = \frac{1}{17}$$

$$\frac{x-1}{x+1} = 1/289 \longrightarrow x = 1.00694$$

$$\alpha = \omega\sqrt{\mu\varepsilon/2}\sqrt{x-1} = \frac{\omega}{c}\sqrt{\varepsilon_r/2}\sqrt{x-1}$$

$$\sqrt{\frac{\varepsilon_r}{2}} = \frac{\alpha c}{\omega\sqrt{x-1}} = \frac{0.2\times3\times10^8}{10^8\sqrt{0.00694}} = 7.2 \longrightarrow \varepsilon_r = 103.68$$

$$|\eta| = \frac{\sqrt{\frac{\mu_o}{\varepsilon_o}}\cdot\frac{1}{\sqrt{\varepsilon_r}}}{\sqrt{x}} = \frac{120\pi}{\sqrt{103.68\times1.00694}} = 36.896$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = \sqrt{x^2-1} = 0.118 \longrightarrow \theta_\eta = 3.365^o$$

$$\eta = \underline{\underline{36.896\angle 3.365^o\ \Omega}}$$

Prob. 10.24

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \longrightarrow \sigma = \frac{2\alpha^2}{\omega\mu} = \frac{2\times12^2}{2\pi\times10^6\times4\pi\times10^{-7}} = \underline{\underline{36.48}}$$

$$\eta = |\eta|\angle\theta_\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$$

$$|\eta| = \sqrt{\frac{\omega\mu}{\sigma}} = \sqrt{\frac{2\pi\times10^6\times4\pi\times10^{-7}}{36.48}} = 0.4652$$

$$E_o = |\eta| H_o = 0.4652\times20\times10^{-3} = 9.305\times10^{-3}$$

$$\mathbf{a}_E = \mathbf{a}_H\times\mathbf{a}_k = \mathbf{a}_y\times(-\mathbf{a}_z) = -\mathbf{a}_x$$

$$\mathbf{E} = E_o e^{-\alpha z}\sin(\omega t+\beta z)\mathbf{a}_E$$

$$\underline{\underline{= -9.305 e^{-12z}\sin(2\pi\times10^6 t + 12z + 45^o)\mathbf{a}_x\ \text{mV/m}}}$$

Prob. 10.25 For a good conductor, $\frac{\sigma}{\omega\varepsilon} >> 1$, say $\frac{\sigma}{\omega\varepsilon} > 100$

(a) $\frac{\sigma}{\omega\varepsilon} = \frac{10^{-2}}{2\pi\times 8\times 10^{6}\times 15\times \frac{10^{-9}}{36\pi}} = 1.5 \longrightarrow$ lossy

No, not conducting.

(b) $\frac{\sigma}{\omega\varepsilon} = \frac{0.025}{2\pi\times 8\times 10^{6}\times 16\times \frac{10^{-9}}{36\pi}} = 3.515 \longrightarrow$ lossy

No, not conducting.

(c) $\frac{\sigma}{\omega\varepsilon} = \frac{25}{2\pi\times 8\times 10^{6}\times 81\times \frac{10^{-9}}{36\pi}} = 694.4 \longrightarrow$ conducting

Yes, conducting.

Prob. 10.26

$$\alpha = \beta = \frac{1}{\delta}$$

But $u = \frac{\omega}{\beta} = \delta\omega = 0.02\times 10^{-3}\times 2\pi\times 100\times 10^{6} = 4\pi\times 10^{3} = \underline{\underline{1.256\times 10^{4}\text{ m/s}}}$

Prob. 10.27 (a)

$$R_{dc} = \frac{l}{\sigma S} = \frac{l}{\sigma\pi a^2} = \frac{600}{5.8\times 10^{7}\times\pi\times(1.2)^2\times 10^{-6}} = \underline{\underline{2.287\ \Omega}}$$

(b) $R_{ac} = \frac{l}{\sigma 2\pi a\delta}$. At 100 MHz, $\delta = 6.6\times 10^{-3}$ mm $=6.6\times 10^{-6}$ m mm for copper (see Table 10.2).

$$R_{ac} = \frac{600}{5.8\times 10^{7}\times 2\pi\times(1.2\times 10^{-3})\times 6.6\times 10^{-6}} = \underline{\underline{207.61\ \Omega}}$$

(c) $\frac{R_{ac}}{R_{dc}} = \frac{a}{2\delta} = 1 \longrightarrow \delta = a/2 = \frac{66.1\times 10^{-3}}{\sqrt{f}}$

$$\sqrt{f} = \frac{66.1\times 2\times 10^{-3}}{a} = \frac{66.1\times 2}{1.2} \longrightarrow f = \underline{\underline{12.137}} \text{ kHz}$$

Prob. 10.28

(a) Copper is a good conductor.

$$\alpha = \sqrt{\pi f \mu\sigma} = \sqrt{\pi\times 10^{10}\times 4\pi\times 10^{-7}\times 5.8\times 10^{7}} = 2\pi\times 10^{5}\sqrt{5.8}$$

$$\underline{\underline{\alpha = 1.513\times 10^{6} \text{ Np/m}}}$$

$$(b) \qquad \delta = \frac{1}{\alpha} = \frac{1}{1.513\times 10^{6}} = \underline{\underline{6.609\times 10^{-7} \text{ m}}}$$

$$(c) \qquad \eta = \frac{1+j}{\sigma\delta} = \frac{1+j}{5.8\times 10^{7}\times 6.609\times 10^{-7}} = \underline{\underline{26.09(1+j)\times 10^{-3}\ \Omega}}$$

Prob. 10.29

$$\delta = \frac{1}{\sqrt{\pi f \mu\sigma}} \longrightarrow f = \frac{1}{\delta^{2}\pi\mu\sigma}$$

$$f = \frac{1}{4\times 10^{-6}\times\pi\times 4\pi\times 10^{-7}\times 6.1\times 10^{7}} = \underline{\underline{1.038 \text{ kHz}}}$$

Prob. 10.30

$$\delta = \frac{1}{\sqrt{\pi f \mu\sigma}} = \frac{1}{\sqrt{\pi\times 5.8\times 10^{7}(60)4\pi\times 10^{-7}}} = \frac{1}{2\pi\sqrt{5.8(60)}} = \underline{\underline{8.531 \text{ mm}}}$$

Prob. 10.31

This is a good conductor.

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$$

$$24.6 = \sqrt{\frac{2\pi \times 12 \times 10^6 \times 4\pi \times 10^{-7}}{\sigma}} = \sqrt{\frac{16\pi^2(0.6)}{\sigma}}$$

$$\sigma = \frac{16\pi^2(0.6)}{24.6^2} = \underline{\underline{0.1566 \text{ S/m}}}$$

$$\beta = \alpha = \frac{1}{\delta} = \frac{1}{0.12} = \underline{\underline{8.333 \text{ rad/m}}}$$

$$\lambda = \frac{2\pi}{\beta} = 2\pi(0.12) = \underline{\underline{0.754 \text{ m}}}$$

$$u = \frac{\omega}{\beta} = \omega\delta = 2\pi \times 12 \times 10^6 \times 0.12 = \underline{\underline{9.05 \times 10^6 \text{ m/s}}}$$

Prob. 10.32

$$\frac{\sigma}{\omega\varepsilon} = \frac{0.12}{2\pi \times 2.42 \times 10^9 \times 5.5 \times \dfrac{10^{-9}}{36\pi}} = 0.1623$$

This is a lossy material.

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$= 2\pi \times 2.42 \times 10^9 \sqrt{\frac{4\pi}{2} \times 10^{-7} \times 5.5 \times \frac{10^{-9}}{36\pi}\left[\sqrt{1+0.1623^2} - 1\right]}$$

$$= 15.21(10^9)(10^{-8})\sqrt{0.01308}$$

$$= 17.39$$

$$\delta = \frac{1}{\alpha} = \underline{\underline{57.5 \text{ mm}}}$$

Prob. 10.33

$$t = 5\delta = \frac{5}{\sqrt{\pi f \mu\sigma}} = \frac{5}{\sqrt{\pi \times 12 \times 10^9 \times 4\pi \times 10^{-7} \times 6.1 \times 10^7}} = \underline{\underline{2.94 \times 10^{-6} \text{ m}}}$$

Prob. 10.34

$$\frac{\sigma}{\omega\varepsilon} = \frac{4}{2\pi \times 2\times 10^9 \times 24 \times \dfrac{10^{-9}}{36\pi}} = 1.5$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} = 2\pi\times 2\times 10^9\sqrt{\frac{4\pi\times 10^{-7}\times 24\times \dfrac{10^{-9}}{36\pi}}{2}\left[\sqrt{1+(1.5)^2} - 1\right]}$$

$$= 130.01 \text{ Np/m}$$

$$10^{-5}E_o = E_o e^{-\alpha d}$$

Taking the log of both sides gives

$$\text{-5}\ln 10 = -\alpha d \qquad \longrightarrow \qquad d = \frac{5\ln 10}{\alpha} = \frac{5\ln 10}{130.01} = \underline{\underline{0.0886 \text{ m}}}$$

Prob. 10.35

(a) Linearly polarized along $\mathbf{a}_z$

(b) $\omega = 2\pi f = 2\pi\times 10^7 \qquad \longrightarrow \qquad f = 10^7 = \underline{\underline{10 \text{ MHz}}}$

(c)
$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o}\sqrt{\varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r}$$

$$\sqrt{\varepsilon_r} = \frac{\beta c}{\omega} = \frac{3\times 3\times 10^8}{2\pi\times 10^7} = 14.32 \qquad \longrightarrow \qquad \varepsilon_r = \underline{\underline{205.18}}$$

Let $\boldsymbol{H} = H_o \sin(\omega t - 3y)\boldsymbol{a}_H$

$$H_o = \frac{E_o}{\eta}, \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_o}{\sqrt{\varepsilon_r}} = \frac{120\pi}{14.32} = 26.33$$

(d) $H_o = \dfrac{12}{26.33} = 0.456$

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_y \times \boldsymbol{a}_z = \boldsymbol{a}_x$$

$$\boldsymbol{H} = \underline{\underline{0.456\sin(2\pi\times 10^7 t - 3y)\boldsymbol{a}_x \text{ A/m}}}$$

Prob. 10.36

$$\boldsymbol{E} = (2\boldsymbol{a}_y - 5\boldsymbol{a}_z)\sin(\omega t - \beta x)$$

The ratio E_y / E_z remains the same as t changes. Hence the wave is linearly polarized

Prob. 10.37

(a)

$$E_x = E_o\cos(\omega t + \beta y), \qquad E_y = E_o\sin(\omega t + \beta y)$$

$$E_x(0,t) = E_o\cos\omega t \qquad \longrightarrow \qquad \cos\omega t = \frac{E_x(0,t)}{E_o}$$

$$E_y(0,t) = E_o\sin\omega t \qquad \longrightarrow \qquad \sin\omega t = \frac{E_y(0,t)}{E_o}$$

$$\cos^2\omega t + \sin^2\omega t = 1 \qquad \longrightarrow \qquad \left(\frac{E_x}{E_o}\right)^2 + \left(\frac{E_y}{E_o}\right)^2 = 1$$

Hence, we have circular polarization.

(b)

$$E_x = E_o\cos(\omega t - \beta y), \qquad E_y = -3E_o\sin(\omega t - \beta y)$$

In the y=0 plane,

$$E_x(0,t) = E_o\cos\omega t \qquad \longrightarrow \qquad \cos\omega t = \frac{E_x(0,t)}{E_o}$$

$$E_y(0,t) = E_o\sin\omega t \qquad \longrightarrow \qquad \sin\omega t = \frac{-E_y(0,t)}{3E_o}$$

$$\cos^2\omega t + \sin^2\omega t = 1 \qquad \longrightarrow \qquad \left(\frac{E_x}{E_o}\right)^2 + \frac{1}{9}\left(\frac{E_y}{E_o}\right)^2 = 1$$

Hence, we have elliptical polarization.

Prob. 10.38

(a) We can write

$$\boldsymbol{E} = \mathrm{Re}(E_s e^{j\omega t}) = (40\boldsymbol{a}_x + 60\boldsymbol{a}_y)\cos(\omega t - 10z)$$

Since E_x / E_y does not change with time, the wave is linearly polarized.

(b) This is elliptically polarized.

Prob. 10.39

(a) The wave is elliptically polarized.

(b)

Let $\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2$,

where $\boldsymbol{E}_1 = 40\cos(\omega t - \beta z)\boldsymbol{a}_x$, $\quad \boldsymbol{E}_2 = 60\sin(\omega t - \beta z)\boldsymbol{a}_y$

$\boldsymbol{H}_1 = H_{o1}\cos(\omega t - \beta z)\boldsymbol{a}_{H1}$

$$H_{o1} = \frac{40}{\eta_o} = \frac{40}{120\pi} = 0.106$$

$\boldsymbol{a}_{H1} = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_z \times \boldsymbol{a}_x = \boldsymbol{a}_y$

$\boldsymbol{H}_1 = 0.106\cos(\omega t - \beta z)\boldsymbol{a}_y$

$\boldsymbol{H}_2 = H_{o2}\sin(\omega t - \beta z)\boldsymbol{a}_{H2}$

$$H_{o1} = \frac{60}{\eta_o} = \frac{60}{120\pi} = 0.1592$$

$\boldsymbol{a}_{H2} = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_z \times \boldsymbol{a}_y = -\boldsymbol{a}_x$

$\boldsymbol{H}_2 = -0.1592\sin(\omega t - \beta z)\boldsymbol{a}_x$

$\boldsymbol{H} = \boldsymbol{H}_1 + \boldsymbol{H}_2 = \underline{\underline{-159.2\sin(\omega t - \beta z)\boldsymbol{a}_x + 106\cos(\omega t - \beta z)\boldsymbol{a}_y \text{ mA/m}}}$

Prob. 10.40

We can write $\mathbf{E}_s$ as

$\boldsymbol{E}_s = \boldsymbol{E}_1(z) + \boldsymbol{E}_2(z)$

where

$$\boldsymbol{E}_1(z) = \frac{1}{2}E_o(\boldsymbol{a}_x - j\boldsymbol{a}_y)e^{-j\beta z}$$

$$\boldsymbol{E}_2(z) = \frac{1}{2}E_o(\boldsymbol{a}_x + j\boldsymbol{a}_y)e^{-j\beta z}$$

We recognize that $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are circularly polarized waves. The problem is therefore proved.

Prob. 10.41

(a)

When $\phi = 0$,

$\boldsymbol{E}(y,t) = (E_{o1}\boldsymbol{a}_x + E_{o2}\boldsymbol{a}_z)\cos(\omega t - \beta y)$

The two components are in phase and the wave is linearly polarized.

(b)

When $\phi = \pi/2$,

$E_z = E_{o2}\cos(\omega t - \beta y + \pi/2) = -E_{o2}\sin(\omega t - \beta y)$

We can combine E_x and E_z to show that the wave is elliptically polarized.

(c)
When $\phi = \pi$,

$$\boldsymbol{E}(y,t) = E_{o1}\cos(\omega t - \beta y)\boldsymbol{a}_x + E_{o2}\cos(\omega t - \beta y + \pi)\boldsymbol{a}_z$$
$$= (E_{o1}\,\boldsymbol{a}_x - E_{o2}\boldsymbol{a}_y)\cos(\omega t - \beta y)$$

Thus, the wave is linearly polarized.

Prob. 10.42

Let $\boldsymbol{E}_s = \boldsymbol{E}_r + j\boldsymbol{E}_i$ and $\boldsymbol{H}_s = \boldsymbol{H}_r + j\boldsymbol{H}_i$

$$\boldsymbol{E} = \text{Re}(\boldsymbol{E}_s e^{j\omega t}) = \boldsymbol{E}_r \cos\omega t - \boldsymbol{E}_i \sin\omega t$$

Similarly,

$$\boldsymbol{H} = \boldsymbol{H}_r \cos\omega t - \boldsymbol{H}_i \sin\omega t$$

$$\mathscr{P} = \boldsymbol{E}\times\boldsymbol{H} = \boldsymbol{E}_r\times\boldsymbol{H}_r\cos^2\omega t + \boldsymbol{E}_i\times\boldsymbol{H}_i\sin^2\omega t - \frac{1}{2}(\boldsymbol{E}_r\times\boldsymbol{H}_i + \boldsymbol{E}_i\times\boldsymbol{H}_r)\sin\ 2\omega t$$

$$\mathscr{P}_{\text{ave}} = \frac{1}{T}\int_0^T \mathscr{P}dt = \frac{1}{T}\int_0^T \cos^2\omega dt(\boldsymbol{E}_r\times\boldsymbol{H}_r) + \frac{1}{T}\int_0^T \sin^2\omega dt(\boldsymbol{E}_i\times\boldsymbol{H}_i) - \frac{1}{2T}\int_0^T \sin 2\omega dt(\boldsymbol{E}_i\times\boldsymbol{H}_i + \boldsymbol{E}_i\times\boldsymbol{H}_r)$$

$$= \frac{1}{2}(\boldsymbol{E}_r\times\boldsymbol{H}_r + \boldsymbol{E}_i\times\boldsymbol{H}_i) = \frac{1}{2}\text{Re}[(\boldsymbol{E}_r + j\boldsymbol{E}_i)\times(\boldsymbol{H}_r - j\boldsymbol{H}_i)]$$

$$\mathscr{P}_{\text{ave}} = \frac{1}{2}\ \text{Re}(\boldsymbol{E}_s\times\boldsymbol{H}_s^*)$$

as required.

Prob. 10.43

(a)

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_o\varepsilon_o}\sqrt{\varepsilon_r} = \frac{\omega}{c}\sqrt{\varepsilon_r}$$

$$\sqrt{\varepsilon_r} = \frac{\beta c}{\omega} = \frac{8\times 3\times 10^8}{10^9} = 2.4$$

$$\varepsilon_r = \underline{\underline{5.76}}$$

(b) $\eta = \sqrt{\dfrac{\mu}{\varepsilon}} = \sqrt{\dfrac{\mu_o}{\varepsilon_o}}\dfrac{1}{\sqrt{\varepsilon_r}} = \dfrac{377}{2.4} = \underline{\underline{157.1\ \Omega}}$

(c) $u = \dfrac{\omega}{\beta} = \dfrac{10^9}{8} = \underline{\underline{1.25\times 10^8\ \text{m/s}}}$

(d)

Let $\boldsymbol{H} = H_o \cos(10^9 t + 8x)\boldsymbol{a}_H$

$$H_o = \frac{E_o}{\eta} = \frac{150}{157.1} = 0.955$$

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = -\boldsymbol{a}_x \times \boldsymbol{a}_z = \boldsymbol{a}_y$$

$$\boldsymbol{H} = \underline{\underline{0.955\cos(10^9 t + 8x)\boldsymbol{a}_y \text{ A/m}}}$$

(e)

$$\mathscr{P} = \boldsymbol{E} \times \boldsymbol{H} = -150(0.955)\cos^2(10^9 t + 8x)\boldsymbol{a}_x$$

$$= \underline{\underline{-143.25\cos^2(10^9 t + 8x)\boldsymbol{a}_x \text{ W/m}^2}}$$

Prob. 10.44

$$\mathscr{P} = \boldsymbol{E} \times \boldsymbol{H} = \frac{E_o^2}{\eta}\cos^2(\omega t - 10z)\boldsymbol{a}_z$$

$$\mathscr{P}_{ave} = \frac{1}{T}\int_0^T \mathscr{P}\, dt = \frac{E_o^2}{2\eta}\boldsymbol{a}_z$$

$$P = \int_S \mathscr{P}_{ave} \bullet d\boldsymbol{S} = \frac{E_o^2 S}{2\eta} = \frac{(40)^2 \times \pi(1.5)^2}{2 \times 120\pi} = \frac{(60)^2}{240} = \underline{\underline{15 \text{ W}}}$$

Prob. 10.45

(a)

Let $\boldsymbol{H}_s = \dfrac{H_o}{r}\sin\theta e^{-j3r}\boldsymbol{a}_H$

$$H_o = \frac{E_o}{\eta_o} = \frac{10}{120\pi} = \frac{1}{12\pi}$$

$$\boldsymbol{a}_H = \boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_r \times \boldsymbol{a}_\theta = \boldsymbol{a}_\phi$$

$$\boldsymbol{H}_s = \underline{\underline{\frac{1}{12\pi r}\sin\theta e^{-j3r}\boldsymbol{a}_\phi \text{ A/m}}}$$

(b)

$$\mathscr{P}_{ave} = \frac{1}{2}\text{Re}(\boldsymbol{E}_s \times \boldsymbol{H}_s) = \frac{10}{2 \times 12\pi r^2}\sin^2\theta \boldsymbol{a}_r$$

$$P_{ave} = \int_S \mathscr{P}_{ave} \cdot d\boldsymbol{S}, \qquad d\boldsymbol{S} = r^2 \sin\theta d\theta d\phi \boldsymbol{a}_r$$

$$P_{ave} = \frac{10}{24\pi}\int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi/6} r^2 \sin^3\theta d\theta \Big|_{r=2} = \frac{5}{8} - \frac{5\sqrt{3}}{32} = 0.007145$$

$$= \underline{\underline{7.145 \text{ mW}}}$$

Prob. 10.46

(a) $P_{ave} = \frac{1}{2}\text{Re}(E_s H_s^*) = \frac{1}{2}\text{Re}(\frac{|E_s|}{|\eta|}) = \frac{8^2}{2|\eta|}e^{-0.2z}$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

Let $\quad x = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}$

$$\frac{\alpha}{\beta} = \frac{\sqrt{x-1}}{\sqrt{x+1}} = 0.1/0.3 = 1/3$$

$$\frac{x-1}{x+1} = \frac{1}{9} \qquad \longrightarrow \qquad x = 5/4$$

$$\frac{5}{4} = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} \qquad \longrightarrow \qquad \frac{\sigma}{\omega\varepsilon} = 3/4$$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}} = \frac{120\pi/\sqrt{81}}{\sqrt{\frac{5}{4}}} = 37.4657$$

$$P_{ave} = \frac{64}{2(37.4657)}e^{-0.2z} = \underline{\underline{0.8541e^{-0.2z}\ \text{W/m}^2}}$$

(b) $20dB = 10\log\frac{P_1}{P_2} \qquad \longrightarrow \qquad \frac{P_1}{P_2} = 100$

$$\frac{P_2}{P_1} = e^{-0.2z} = \frac{1}{100} \qquad \longrightarrow \qquad e^{0.2z} = 100$$

$$z = 5\log 100 = \underline{\underline{23\ \text{m}}}$$

Prob. 10.47

(a) $u = \omega / \beta \longrightarrow \omega = u\beta = \frac{\beta c}{\sqrt{4.5}} = \frac{2\times 3\times 10^8}{\sqrt{4.5}} = \underline{\underline{2.828\times 10^8}}$ rad/s

$$\eta = \frac{120\pi}{\sqrt{4.5}} = 177.7\Omega$$

$$\boldsymbol{H} = \boldsymbol{a}_k \times \frac{\boldsymbol{E}}{\eta} = \frac{\boldsymbol{a}_z}{\eta} \times \frac{40}{\rho}\sin(\omega t - 2z)\boldsymbol{a}_\rho = \underline{\underline{\frac{0.225}{\rho}\sin(\omega t - 2z)\boldsymbol{a}_\phi}} \text{ A/m}$$

(b) $\boldsymbol{P} = \boldsymbol{E}\times\boldsymbol{H} = \underline{\underline{\frac{9}{\rho^2}\sin^2(\omega t - 2z)\boldsymbol{a}_z}}$ W/m^2

(c) $\mathcal{P}_{\text{ave}} = \frac{4.5}{\rho^2}\boldsymbol{a}_z$, $d\mathbf{S} = \rho d\phi d\rho \boldsymbol{a}_z$

$$P_{\text{ave}} = \int \mathbf{P}_{\text{ave}} \bullet dS = 4.5 \int_{2\text{mm}}^{3\text{mm}} \frac{d\rho}{\rho} \int_0^{2\pi} d\phi = 4.5\ln(3/2)(2\pi) = \underline{\underline{11.46 \text{ W}}}$$

Prob. 10.48

$$\mathcal{P} = \boldsymbol{E}\times\boldsymbol{H} = \frac{E_o^2}{\eta r^2}\sin^2\theta \sin^2\omega(t - r/c)\boldsymbol{a}_r$$

$$\boldsymbol{P}_{ave} = \frac{1}{T}\int_0^T \mathcal{P}\, dt = \underline{\underline{\frac{E_o^2}{2\eta r^2}\sin^2\theta \boldsymbol{a}_r}}$$

Prob. 10.49

$\beta = \frac{\omega}{c} \longrightarrow \omega = \beta c = 40(3\times 10^8) = \underline{\underline{12\times 10^9 \text{ rad/s}}}$

$$\boldsymbol{E} = \frac{1}{\varepsilon}\int \nabla\times\boldsymbol{H} dt$$

$$\nabla\times\boldsymbol{H} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 10\sin(\omega t - 40x) & -20\sin(\omega t - 40x) \end{vmatrix}$$

$$= -800\cos(\omega t - 40x)\boldsymbol{a}_y - 400\cos(\omega t - 40x)\boldsymbol{a}_z$$

$$\boldsymbol{E} = \frac{1}{\varepsilon}\int \nabla\times \boldsymbol{H}dt = -\frac{800}{\omega\varepsilon}\sin(\omega t-40x)\boldsymbol{a}_y - \frac{400}{\omega\varepsilon}\sin(\omega t-40x)\boldsymbol{a}_z$$

$$= -\frac{800}{12\times10^9\times\frac{10^{-9}}{36\pi}}\sin(\omega t-40x)\boldsymbol{a}_y - \frac{400}{12\times10^9\times\frac{10^{-9}}{36\pi}}\sin(\omega t-40x)\boldsymbol{a}_z$$

$$= -7.539\sin(\omega t-40x)\boldsymbol{a}_y - 3.77\sin(\omega t-40x)\boldsymbol{a}_z \text{ kV/m}$$

$$\boldsymbol{P} = \boldsymbol{E}\times\boldsymbol{H} = \begin{vmatrix} 0 & E_y & E_z \\ 0 & H_y & H_z \end{vmatrix} = (E_yH_z - E_zH_y)\boldsymbol{a}_x$$

$$= \left[20(7.537)\sin^2(\omega t-40x) + 37.7\sin^2(\omega t-40x)\right]\boldsymbol{a}_x 10^3$$

$$\boldsymbol{P}_{\text{ave}} = \frac{1}{2}\left[20(7.537)+37.7\right]\boldsymbol{a}_x 10^3 = \underline{\underline{94.23\,\boldsymbol{a}_x \text{ kW/m}^2}}$$

Prob. 10.50

$$P = \frac{E_o^2}{2\eta_o} \longrightarrow E_o^2 = 2\eta_o P = 2(120\pi)10\times10^{-3} = 7.539$$

$$E_o = \underline{\underline{2.746 \text{ V/m}}}$$

Prob. 10.51

Let $T = \omega t - \beta z.$

$$-\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times\boldsymbol{E} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos T & \sin T & 0 \end{vmatrix}$$

$$-\mu\frac{\partial \boldsymbol{H}}{\partial t} = \beta\cos T\boldsymbol{a}_x + \beta\sin T\boldsymbol{a}_y$$

$$\boldsymbol{H} = -\frac{\beta}{\mu}\int\left[\cos T\boldsymbol{a}_x + \sin T\boldsymbol{a}_y\right]dt = -\frac{\beta}{\mu\omega}\sin T\boldsymbol{a}_x + \frac{\beta}{\mu\omega}\cos T\boldsymbol{a}_y$$

$$\mathscr{P} = \boldsymbol{E}\times\boldsymbol{H} = \begin{vmatrix} cosT & sinT & 0 \\ -\frac{\beta}{\mu\omega}\sin T & \frac{\beta}{\mu\omega}\cos T & 0 \end{vmatrix} = \frac{\beta}{\mu\omega}(\cos^2 T + \sin^2 T)\boldsymbol{a}_z$$

$$= \frac{\beta}{\mu\omega}\boldsymbol{a}_z = \sqrt{\frac{\varepsilon}{\mu}}\boldsymbol{a}_z$$

which is constant everywhere.

Prob. 10.52

$$\mathcal{P} = \frac{E_o^2}{2\eta_o}$$

$$P = \mathcal{P}S = \frac{E_o^2 S}{2\eta_o} = \frac{(2.4\times10^3)^2\times450\times10^{-4}}{2\times377} = \underline{\underline{343.8\text{ W}}}$$

Prob. 10.53

$$\mathscr{P} = \mathbf{E}\times\mathbf{H} = \frac{V_o I_o}{2\pi\rho^2\ln(b/a)}\sin^2(\omega t - \beta z)\mathbf{a}_z$$

(a) $$\mathscr{P}_{ave} = \frac{1}{T}\int_0^T \mathscr{P}\,dt = \frac{V_o I_o}{2\pi\rho^2\ln(b/a)}\frac{1}{T}\int_0^T \sin^2(\omega t-\beta z)dt\,\mathbf{a}_z = \frac{V_o I_o}{2\pi\rho^2\ln(b/a)}\frac{1}{2}\mathbf{a}_z$$

$$= \underline{\underline{\frac{V_o I_o}{4\pi\rho^2\ln(b/a)}\mathbf{a}_z}}$$

(b)

$$P_{ave} = \int_S \mathscr{P}_{ave}\cdot d\mathbf{S}, \qquad d\mathbf{S} = \rho\, d\rho\, d\phi\, \mathbf{a}_z$$

$$= \frac{V_o I_o}{4\pi\ln(b/a)}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b}\frac{1}{\rho^2}\rho\, d\rho\, d\phi = \frac{V_o I_o}{4\pi\ln(b/a)}(2\pi)\ln(b/a)$$

$$= \underline{\underline{\frac{1}{2}V_o I_o}}$$

Prob. 10.54

(a) $$P_{i,ave} = \frac{E_{io}^2}{2\eta_1}, \qquad P_{r,ave} = \frac{E_{ro}^2}{2\eta_1}, \qquad P_{t,ave} = \frac{E_{to}^2}{2\eta_2}$$

$$R = \frac{P_{r,ave}}{P_{i,ave}} = \frac{E_{ro}^2}{E_{io}^2} = \Gamma^2 = \underline{\underline{\left(\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right)^2}}$$

$$R = \left(\frac{\sqrt{\frac{\mu_o}{\varepsilon_2}} - \sqrt{\frac{\mu_o}{\varepsilon_1}}}{\sqrt{\frac{\mu_o}{\varepsilon_2}} + \sqrt{\frac{\mu_o}{\varepsilon_1}}}\right)^2 = \left(\frac{\sqrt{\mu_o\varepsilon_1} - \sqrt{\mu_o\varepsilon_2}}{\sqrt{\mu_o\varepsilon_1} + \sqrt{\mu_o\varepsilon_2}}\right)^2$$

Since $n_1 = c\sqrt{\mu_1\varepsilon_1} = c\sqrt{\mu_o\varepsilon_1}$, $\qquad n_2 = c\sqrt{\mu_o\varepsilon_2}$,

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T = \frac{P_{t,ave}}{P_{i,ave}} = \frac{\eta_1}{\eta_2}\frac{E_{to}^{\ 2}}{E_{io}^{\ 2}} = \frac{\eta_1}{\eta_2}\tau^2 = \frac{\eta_1}{\eta_2}(1+\Gamma)^2 = \frac{4n_1n_2}{(n_1+n_2)^2}$$

(b) If $P_{r,ave} = P_{t,ave} \longrightarrow RP_{i,ave} = TP_{i.ave} \longrightarrow R = T$

i.e. $(n_1 - n_2)^2 = 4n_1n_2 \qquad \longrightarrow \qquad n_1^{\ 2} - 6n_1n_2 + n_2^{\ 2} = 0$

or $\left(\frac{n_1}{n_2}\right)^2 - 6\left(\frac{n_1}{n_2}\right) + 1 = 0,$ so

$$\frac{n_1}{n_2} = 3 \pm \sqrt{8} = \underline{\underline{5.828 \quad \text{or} \quad 0.1716}}$$

(Note that these values are mutual reciprocals, reflecting the inherent symmetry of the problem.)

Prob. 10.55

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{2\mu_o}{8\varepsilon_o}} = \frac{\eta_o}{2}, \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{\mu_o}{16\varepsilon_o}} = \frac{\eta_o}{4}$$

$$\Gamma = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \frac{\eta_o/4 - \eta_o/2}{\eta_o/4 + \eta_o/2} = -\frac{1}{3}, \qquad \tau = \Gamma + 1 = \frac{2}{3}$$

$$\mathbf{E}_r = -\frac{1}{3}(60)\sin(\omega t + 10z)\mathbf{a}_x - \frac{1}{3}(30)\sin(\omega t + 10z + \pi/6)\mathbf{a}_y$$

$$\underline{\underline{= -20\sin(\omega t + 10z)\mathbf{a}_x - 10\sin(\omega t + 10z + \pi/6)\mathbf{a}_y \ \text{V/m}}}$$

$$\mathbf{E}_t = \frac{2}{3}(60)\sin(\omega t - 10z)\mathbf{a}_x + \frac{2}{3}(30)\sin(\omega t - 10z + \pi/6)\mathbf{a}_y$$

$$\underline{\underline{= 40\sin(\omega t - 10z)\mathbf{a}_x + 20\sin(\omega t - 10z + \pi/6)\mathbf{a}_y \ \text{V/m}}}$$

Prob. 10.56

$$\eta_1 = \eta_o = 120\pi, \qquad \eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_o}{9\varepsilon_o}} = \frac{\eta_o}{3} = 40\pi$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_o/3 - \eta_o}{\eta_o/3 + \eta_o} = -\frac{1}{2}, \qquad \tau = 1 + \Gamma = \frac{1}{2}$$

$$E_r = \Gamma E_o = -E_o/2, \qquad E_t = \tau E_o = E_o/2$$

$$P_{iave} = \frac{|E_o|^2}{2\eta_1} = \frac{E_o^2}{2\eta_o}$$

$$P_{tave} = \frac{|E_t|^2}{2\eta_2} = \frac{\frac{1}{4}E_o^2}{2(\eta_o/3)} = \frac{E_o^2}{2\eta_o}\frac{3}{4}$$

$$\frac{P_{tave}}{P_{iave}} = \frac{3}{4} = \underline{\underline{0.75}}$$

Prob. 10.57

(a) $\eta_1 = \eta_o$

$$\boldsymbol{E}_i = E_{io}\sin(\omega t - 5x)\boldsymbol{a}_E$$

$$E_{io} = H_{io}\eta_o = 120\pi \times 4 = 480\pi$$

$$\boldsymbol{a}_E \times \boldsymbol{a}_H = \boldsymbol{a}_k \longrightarrow \boldsymbol{a}_E \times \boldsymbol{a}_y = \boldsymbol{a}_x \longrightarrow \boldsymbol{a}_E = -\boldsymbol{a}_z$$

$$\boldsymbol{E}_i = -480\pi\sin(\omega t - 5x)\boldsymbol{a}_z$$

$$\eta_2 = \sqrt{\frac{\mu_o}{4\varepsilon_o}} = \frac{120\pi}{\sqrt{4}} = 60\pi$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = -1/3, \qquad \tau = 1 + \Gamma = 2/3$$

$$E_{ro} = \Gamma E_{io} = (-1/3)(-480\pi) = 160\pi$$

$$\boldsymbol{E}_r = 160\pi\sin(\omega t + 5x)\boldsymbol{a}_z$$

$$\boldsymbol{E}_1 = \boldsymbol{E}_i + \boldsymbol{E}_r = \underline{\underline{-1.508\sin(\omega t - 5x)\boldsymbol{a}_z + 0.503\sin(\omega t + 5x)\boldsymbol{a}_z}} \text{ kV/m}$$

(b) $E_{to} = \tau E_{io} = (2/3)(480\pi) = 320\pi$

$$\mathscr{P} = \frac{E_{to}^{\ 2}}{2\eta_2}\boldsymbol{a}_x = \frac{(320\pi)^2}{2(60\pi)}\boldsymbol{a}_x = \underline{\underline{2.68\boldsymbol{a}_x \text{ kW/m}^2}}$$

(c) $s = \dfrac{1+|\Gamma|}{1-|\Gamma|} = \dfrac{1+1/3}{1-1/3} = \underline{\underline{2}}$

Prob. 10.58 $\eta_1 = \sqrt{\dfrac{\mu_1}{\varepsilon_1}} = \eta_o/2, \qquad \eta_2 = \eta_o$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 1/3, \qquad \tau = 1 + \Gamma = 4/3$$

$$E_{or} = \Gamma E_{io} = (1/3)(5) = 5/3, \qquad E_{ot} = \tau E_{io} = 20/3$$

$$\beta = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = \frac{10^8}{3\times 10^8}\sqrt{4} = 2/3$$

(a) $\boldsymbol{E}_r = \dfrac{5}{3}\cos(10^8 t - 2y/3)\boldsymbol{a}_z$

$$\boldsymbol{E}_1 = \boldsymbol{E}_i + \boldsymbol{E}_r = \underline{\underline{5\cos(10^8 t + \frac{2}{3}y)\boldsymbol{a}_z + \frac{5}{3}\cos(10^8 t - \frac{2}{3}y)\boldsymbol{a}_z}} \text{ V/m}$$

(b) $\mathscr{P}_{\text{ave1}} = \dfrac{E_{io}^{\ 2}}{2\eta_1}(-\boldsymbol{a}_y) + \dfrac{E_{ro}^{\ 2}}{2\eta_1}(+\boldsymbol{a}_y) = \dfrac{25}{2(60\pi)}(1 - \dfrac{1}{9})(-\boldsymbol{a}_y) = \underline{\underline{-0.0589\boldsymbol{a}_y \text{ W/m}^2}}$

(c) $\mathscr{P}_{\text{ave2}} = \dfrac{E_{to}^{\ 2}}{2\eta_2}(-\boldsymbol{a}_y) = \dfrac{400}{9(2)(120\pi)}(-\boldsymbol{a}_y) = \underline{\underline{-0.0589\boldsymbol{a}_y \text{ W/m}^2}}$

Prob. 10.59

$$\beta_1 = \omega\sqrt{\mu_1\varepsilon_1} = \omega\sqrt{\mu_o\varepsilon_o}\sqrt{\mu_{r1}\varepsilon_{r1}} = \frac{\omega}{c}\sqrt{16} = \frac{90\times10^9(4)}{3\times10^8} = 300(4) = 1200$$

$$\beta_2 = \omega\sqrt{\mu_o\varepsilon_o} = \frac{90\times10^9}{3\times10^8} = 300$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{\mu_o}{\varepsilon_o}}\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}} = \frac{\eta_o}{4} = 30\pi, \qquad \eta_2 = \eta_o = 120\pi$$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\eta_o-\eta_o/4}{\eta_o+\eta_o/4} = \frac{3}{5}, \qquad \tau = 1+\Gamma = \frac{8}{5}$$

$$\mathscr{P} = \boldsymbol{E}\times\boldsymbol{H}$$

$$\mathscr{P}_1 = \mathscr{P}_i + \mathscr{P}_r = \frac{60^2}{\eta_1}\cos^2(\omega t-\beta_1 z)\boldsymbol{a}_x + \frac{(60\times\frac{3}{5})^2}{\eta_1}\cos^2(\omega t+\beta_1 z)\boldsymbol{a}_x$$

$$\frac{60^2}{\eta_1} = \frac{60^2}{30\pi} = 38.197, \qquad \frac{(60\times\frac{3}{5})^2}{\eta_1} = 13.75$$

$$\underline{\underline{\mathscr{P}_1 = 38.197\cos^2(\omega t-\beta_1 z)\boldsymbol{a}_x + 13.75\cos^2(\omega t+\beta_1 z)\boldsymbol{a}_x \text{ W/m}^2,}} \quad \text{where } \beta_1 = 1200$$

$$\mathscr{P}_2 = \mathscr{P}_t = \frac{(60\times\frac{8}{5})^2}{\eta_o}\cos^2(\omega t-\beta_1 z)\boldsymbol{a}_x$$

$$\frac{(60\times\frac{8}{5})^2}{\eta_o} = \frac{96^2}{120\pi} = 24.46$$

$$\underline{\underline{\mathscr{P}_2 = 24.46\cos^2(\omega t-\beta_2 z)\boldsymbol{a}_x \text{ W/m}^2,}} \quad \text{where } \beta_2 = 300$$

Prob. 10.60

(a) In air, $\beta_1 = 1, \lambda_1 = 2\pi/\beta_1 = 2\pi = \underline{\underline{6.283 \text{ m}}}$

$$\omega = \beta_1 c = \underline{\underline{3\times10^8 \text{ rad/s}}}$$

In the dielectric medium, ω is the same .

$$\omega = \underline{\underline{3\times10^8 \text{ rad/s}}}$$

$$\beta_2 = \frac{\omega}{c}\sqrt{\varepsilon_{r2}} = \beta_1\sqrt{\varepsilon_{r2}} = \sqrt{3}$$

$$\lambda_2 = \frac{2\pi}{\beta_2} = \frac{2\pi}{\sqrt{3}} = \underline{\underline{3.6276 \text{ m}}}$$

(b) $H_o = \frac{E_o}{\eta_o} = \frac{10}{120\pi} = 0.0265$

$$\mathbf{a}_H = \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_z \times \mathbf{a}_y = -\mathbf{a}_x$$

$$\mathbf{H}_i = \underline{\underline{-26.5\cos(\omega t - z)\mathbf{a}_x \text{ mA/m}}}$$

(c) $\eta_1 = \eta_o, \quad \eta_2 = \eta_o / \sqrt{3}$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{(1/\sqrt{3}) - 1}{(1/\sqrt{3}) + 1} = \underline{\underline{-0.268}}, \qquad \tau = 1 + \Gamma = \underline{\underline{0.732}}$$

(d) $E_{to} = \tau E_{io} = 7.32, \quad E_{ro} = \Gamma E_{io} = -2.68$

$$\mathbf{E}_1 = \mathbf{E}_i + \mathbf{E}_r = \underline{\underline{10\cos(\omega t - z)\mathbf{a}_y - 2.68\cos(\omega t + z)\mathbf{a}_y \text{ V/m}}}$$

$$\mathbf{E}_2 = \mathbf{E}_t = \underline{\underline{7.32\cos(\omega t - z)\mathbf{a}_y \text{ V/m}}}$$

$$\mathcal{P}_{ave1} = \frac{1}{2\eta_1}(\mathbf{a}_z)[E_{io}^2 - E_{ro}^2] = \frac{1}{2(120\pi)}(\mathbf{a}_z)(10^2 - 2.68^2) = \underline{\underline{0.1231\mathbf{a}_z \text{ W/m}^2}}$$

$$\mathcal{P}_{ave2} = \frac{E_{to}^2}{2\eta_2}(\mathbf{a}_z) = \frac{\sqrt{3}}{2\times 120\pi}(7.32)^2(\mathbf{a}_z) = \underline{\underline{0.1231\mathbf{a}_z \text{ W/m}^2}}$$

Prob. 10.61

$\eta_1 = \eta_o = 120\pi$

For seawater (lossy medium),

$$\eta_2 = \sqrt{\frac{j\omega\mu_o}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j2\pi \times 10^8 \times 4}{4 + j2\pi \times 10^8 \times 81 \times \frac{10^{-9}}{36\pi}}} = 10.44 + j9.333$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 0.9461\angle 177.16$$

$$|\Gamma|^2 = 0.8952, \qquad 1 - |\Gamma| = 0.1084$$

$$\frac{P_r}{P_i} = \underline{\underline{89.51\%}}, \qquad \frac{P_t}{P_i} = \underline{\underline{10.84\%}},$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{7.924\angle 43.975 - 377}{7.924\angle 43.975 + 377} = 0.9702\angle 178.2^o$$

The fraction of the incident power reflected is

$$\frac{P_r}{P_i} = |\Gamma|^2 = 0.9702^2 = \underline{\underline{0.9413}}$$

The transmitted fraction is

$$\frac{P_t}{P_i} = 1 - |\Gamma|^2 = 1 - 0.9702^2 = \underline{\underline{0.0587}}$$

Prob. 10.62

(a)

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \frac{120\pi}{\sqrt{4}} = 188.5, \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{120\pi}{\sqrt{3.2}} = 210.75$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{210.75 - 188.5}{210.75 + 188.5} = 0.0557, \qquad \tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times 210.75}{210.75 + 188.5} = 1.0557$$

$$E_{ro} = \Gamma E_{io} = (0.0557)(12) = 0.6684 \qquad E_{to} = \tau E_{io} = 1.0557(12) = 12.668$$

$$\beta_1 = \omega\sqrt{\mu_1\varepsilon_1} = \frac{\omega}{c}\sqrt{4} \longrightarrow \omega = \frac{\beta_1 c}{2} = \frac{40\pi(3\times 10^8)}{2} = \underline{\underline{6\pi \times 10^9 \text{ rad/s}}}$$

(b)

$$\beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = \frac{\omega}{c}\sqrt{3.2} = \frac{6\pi\times 10^9\sqrt{3.2}}{3\times 10^8} = 112.4$$

$$\boldsymbol{E}_r = E_{ro}\cos(\omega t + 40\pi x)\boldsymbol{a}_z = \underline{\underline{0.6684\cos(6\pi\times 10^9 t + 40\pi x)\boldsymbol{a}_z \text{ V/m}}}$$

$$\boldsymbol{E}_t = E_{to}\cos(\omega t - \beta_2 x)\boldsymbol{a}_z = \underline{\underline{12.668\cos(6\pi\times 10^9 t - 112.4x)\boldsymbol{a}_z \text{ V/m}}}$$

Prob. 10.63 (a) $\omega = \beta c = 3\times 3\times 10^8 = \underline{\underline{9\times 10^8 \text{ rad/s}}}$

(b) $\lambda = 2\pi/\beta = 2\pi/3 = \underline{\underline{2.094 \text{ m}}}$

(c) $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{4}{9\times 10^8 \times 80 \times 10^{-9}/36\pi} = 2\pi = \underline{\underline{6.288}}$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon} = 6.288 \longrightarrow \theta_\eta = 40.47^o$$

$$|\eta_2| = \frac{\sqrt{\mu_2/\varepsilon_2}}{\sqrt[4]{1 + \left(\frac{\sigma_2}{\omega\varepsilon_2}\right)^2}} = \frac{377/\sqrt{80}}{\sqrt[4]{1 + 4\pi^2}} = 16.71$$

$\eta_2 = \underline{\underline{16.71\angle 40.47^o\ \Omega}}$

(d) $\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \dfrac{16.71\angle 40.47^o - 377}{16.71\angle 40.47^o + 377} = 0.935\angle 179.7^o$

$E_{or} = \Gamma E_{oi} = 9.35\angle 179.7^o$

$$\boldsymbol{E}_r = \underline{\underline{9.35\sin(\omega t - 3z + 179.7)\boldsymbol{a}_x\ \text{V/m}}}$$

$$\alpha_2 = \frac{\omega}{c}\sqrt{\frac{\mu_{r2}\varepsilon_{r2}}{2}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\varepsilon_2}\right)^2} - 1\right]} = \frac{9\times10^8}{3\times10^8}\sqrt{\frac{80}{2}\left[\sqrt{1+4\pi^2} - 1\right]} = 43.94\ \text{Np/m}$$

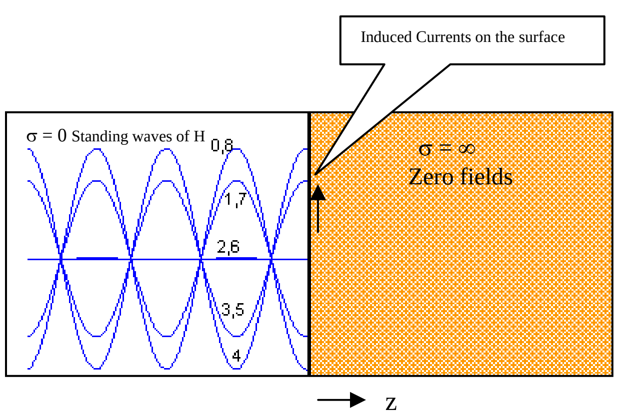
$$\beta_2 = \frac{9\times10^8}{3\times10^8}\sqrt{\frac{80}{2}\left[\sqrt{1+4\pi^2} + 1\right]} = 51.48\ \text{rad/m}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2\times16.71\angle 40.47^o}{16.71\angle 40.47^o + 377} = 0.0857\angle 38.89^o$$

$E_{ot} = \tau E_o = 0.857\angle 38.89^o$

$$\underline{\underline{\boldsymbol{E}_t = 0.857 e^{43.94z}\sin(9\times10^8 t + 51.48z + 38.89^o)\boldsymbol{a}_x\ \ \text{V/m}}}$$

Prob. 10.64



Curve 0 is at t = 0; curve 1 is at t = T/8; curve 2 is at t = T/4; curve 3 is at t = 3T/8, etc.

Prob. 10.65 Since $\mu_o = \mu_1 = \mu_2$,

$$\sin\theta_{t1} = \sin\theta_i \sqrt{\frac{\varepsilon_o}{\varepsilon_1}} = \frac{\sin 45^o}{\sqrt{4.5}} = 0.3333 \longrightarrow \underline{\underline{\theta_{t1} = 19.47^o}}$$

$$\sin\theta_{t2} = \sin\theta_{t1}\sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = \frac{1}{3}\sqrt{\frac{4.5}{2.25}} = 0.4714 \longrightarrow \underline{\underline{\theta_{t2} = 28.13^o}}$$

Prob. 10.66

$$\boldsymbol{E}_s = \frac{20(e^{jk_x x} - e^{-jk_x x})}{j2}\frac{(e^{jk_y y} + e^{-jk_y y})}{2}\boldsymbol{a}_z$$

$$= -j5\left[e^{j(k_x x + k_y y)} + e^{j(k_x x - k_y y)} - e^{-j(k_x x - k_y y)} - e^{-j(k_x x + k_y y)}\right]\boldsymbol{a}_z$$

which consists of four plane waves.

$$\nabla\times\boldsymbol{E}_s = -j\omega\mu_o\boldsymbol{H}_s \longrightarrow \boldsymbol{H}_s = \frac{j}{\omega\mu_o}\nabla\times\boldsymbol{E}_s = \frac{j}{\omega\mu_o}\left(\frac{\partial E_z}{\partial y}\boldsymbol{a}_x - \frac{\partial E_z}{\partial x}\boldsymbol{a}_y\right)$$

$$\underline{\underline{\boldsymbol{H}_s = -\frac{j20}{\omega\mu_o}\left[k_y \sin(k_x x)\sin(k_y y)\boldsymbol{a}_x + k_x\cos(k_x x)\cos(k_y y)\boldsymbol{a}_y\right]}}$$

Prob. 10.67

$\eta_1 = \eta_o = 377\Omega$

For η_2,

$$\frac{\sigma_2}{\omega\varepsilon_2} = \frac{4}{2\pi\times 1.2\times 10^9 \times 50\times\frac{10^{-9}}{36\pi}} = 1.2$$

$$\tan 2\theta_{\eta_2} = \frac{\sigma_2}{\omega\varepsilon_2} = 1.2 \longrightarrow \theta_{\eta_2} = 25.1^o$$

$$|\eta_2| = \frac{\sqrt{\mu/\varepsilon}}{\sqrt[4]{1+\left(\frac{\sigma_2}{\omega\varepsilon_2}\right)^2}} = \frac{120\pi\sqrt{1/50}}{\sqrt[4]{1+1.2^2}} = 42.658$$

$\eta_2 = 42.658\angle 25.1^o$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{42.658\angle 25.1^o - 377}{42.658\angle 25.1^o + 377} = \underline{\underline{0.8146\angle 174.4^o}}$$

Prob. 10.68

(a)

$$P_t = (1-|\Gamma|^2)P_i$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} \longrightarrow |\Gamma| = \frac{s-1}{s+1}$$

$$\frac{P_t}{P_i} = 1 - \left(\frac{s-1}{s+1}\right)^2 = \underline{\underline{\frac{4s}{(s+1)^2}}}$$

(b) $P_i = P_r + P_t \longrightarrow \frac{P_r}{P_i} = 1 - \frac{P_t}{P_i} = \underline{\underline{\left(\frac{s-1}{s+1}\right)^2}}$

Prob. 10.69

If **A** is a uniform vector and $\Phi(r)$ is a scalar,

$$\nabla \times (\Phi \boldsymbol{A}) = \nabla\Phi \times \boldsymbol{A} + \Phi(\nabla \times \boldsymbol{A}) = \nabla\Phi \times \boldsymbol{A}$$

since $\nabla \times \boldsymbol{A} = \boldsymbol{0}$.

$$\nabla \times \boldsymbol{E} = \left(\frac{\partial}{\partial x}\boldsymbol{a}_x + \frac{\partial}{\partial y}\boldsymbol{a}_y + \frac{\partial}{\partial z}\mathrm{a}_z\right) \times \boldsymbol{E}_o e^{j(k_x x + k_y y + k_z z - \omega t)} = j(k_x \boldsymbol{a}_x + k_y \boldsymbol{a}_y + k_z \boldsymbol{a}_z) e^{j(\mathbf{k}\bullet\mathbf{r} - \omega t)} \times \boldsymbol{E}_o$$

$$= j\boldsymbol{k} \times \boldsymbol{E}_o e^{j(\mathbf{k}\bullet\mathbf{r} - \omega t)} = j\boldsymbol{k} \times \boldsymbol{E}$$

Also, $-\frac{\partial \boldsymbol{B}}{\partial t} = j\omega\mu\boldsymbol{H}$. Hence $\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$ becomes $\boldsymbol{k} \times \boldsymbol{E} = \omega\mu\boldsymbol{H}$

From this, $\underline{\underline{\boldsymbol{a}_k \times \boldsymbol{a}_E = \boldsymbol{a}_H}}$

Prob. 10.70

$$k = |\mathrm{k}| = \sqrt{124^2 + 124^2 + 263^2} = 316.1$$

$$\lambda = \frac{2\pi}{k} = \underline{\underline{19.88 \text{ mm}}}$$

$$k = \omega\sqrt{\mu\varepsilon} = \frac{2\pi f}{c} \longrightarrow f = \frac{kc}{2\pi} = \frac{316.1 \times 3 \times 10^8}{2\pi} = \underline{\underline{15.093 \text{ GHz}}}$$

$$\boldsymbol{k} \bullet \boldsymbol{a}_x = k\cos\theta_x \longrightarrow \cos\theta_x = \frac{124}{316.1} \longrightarrow \theta_x = 66.9^o = \theta_y$$

$$\theta_z = \cos^{-1}\frac{263}{361.1} = 33.69^o$$

Thus,

$$\underline{\underline{\theta_x = \theta_y = 66.9^{\mathrm{o}},\ \theta_z = 33.69^o}}$$

Prob. 10.71

$\boldsymbol{k} = -3.4\boldsymbol{a}_x + 4.2\boldsymbol{a}_y$

$\boldsymbol{k} \cdot \boldsymbol{E} = 0 \longrightarrow 0 = -3.4E_o + 4.2$

$E_o = \dfrac{4.2}{3.4} = \underline{\underline{1.235}}$

$k = |\boldsymbol{k}| = \beta = \sqrt{(-3.4)^2 + (4.2)^2} = 5.403$

$\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{5.403} = 1.162$

$f = \dfrac{c}{\lambda} = \dfrac{3\times 10^8}{1.162} = \underline{\underline{258 \text{ MHz}}}$

$$\boldsymbol{H}_s = \frac{1}{\mu\omega}\boldsymbol{k}\times\boldsymbol{E}_s = \frac{1}{\mu k c}\boldsymbol{k}\times\boldsymbol{E}_s$$

$$= \frac{1}{4\pi\times 10^{-7}\times 5.403\times 3\times 10^8}\begin{vmatrix} -3.4 & 4.2 & 0 \\ E_o & 1 & 3+j4 \end{vmatrix} A_o$$

where $A_o = e^{-j3.4x+4.2y}$

$$\boldsymbol{H}_s = 4.91A_o\times 10^{-4}\left[4.2(3+j4)\boldsymbol{a}_x + 3.4(3+j4)\boldsymbol{a}_y + (-3.4-4.2E_o)\boldsymbol{a}_z\right]$$

$$= 0.491\left[(12.6+j16.8)\boldsymbol{a}_x + (10.2+j13.6)\boldsymbol{a}_y - 8.59\boldsymbol{a}_z\right]e^{-j3.4x+4.2y} \text{ mA/m}$$

Prob.10.72

$$\nabla\bullet\boldsymbol{E} = \left(\frac{\partial}{\partial x}\boldsymbol{a}_x + \frac{\partial}{\partial y}\boldsymbol{a}_y + \frac{\partial}{\partial z}\boldsymbol{a}_z\right)\bullet\boldsymbol{E}_o e^{j(k_x x + k_y y + k_z z - \omega t)} = j(k_x\boldsymbol{a}_x + k_y\boldsymbol{a}_y + k_z\boldsymbol{a}_z)e^{j(\mathbf{k}\bullet\mathbf{r}-\omega t)}\bullet\boldsymbol{E}_o$$

$$= j\boldsymbol{k}\bullet\boldsymbol{E}_o e^{j(\mathbf{k}\bullet\mathbf{r}-\omega t)} = j\boldsymbol{k}\bullet\boldsymbol{E} = 0 \longrightarrow \boldsymbol{k}\bullet\boldsymbol{E} = 0$$

Similarly,

$$\nabla\bullet\boldsymbol{H} = j\boldsymbol{k}\bullet\boldsymbol{H} = 0 \longrightarrow \boldsymbol{k}\bullet\boldsymbol{H} = 0$$

It has been shown in the previous problem that

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t} \longrightarrow \boldsymbol{k}\times\boldsymbol{E} = \omega\mu\boldsymbol{H}$$

Similarly,

$$\nabla\times\boldsymbol{H} = \frac{\partial\boldsymbol{D}}{\partial t} \longrightarrow kxH = -\varepsilon\omega E$$

From $\mathbf{k}\times\mathbf{E}=\omega\mu\mathbf{H}$, $\quad \mathbf{a}_k\times\mathbf{a}_E=\mathbf{a}_H$ and

From $\mathbf{k}\times\mathbf{H}=-\varepsilon\omega\mathbf{E}$, $\quad \mathbf{a}_k\times\mathbf{a}_H=-\mathbf{a}_E$

Prob. 10.73

If $\mu_o=\mu_1=\mu_2,\quad \eta_1=\dfrac{\eta_o}{\sqrt{\varepsilon_{r1}}},\eta_2=\dfrac{\eta_o}{\sqrt{\varepsilon_{r2}}}$

$$\Gamma_{\backslash\backslash}=\frac{\dfrac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_t-\dfrac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_i}{\dfrac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_t+\dfrac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_i}$$

$$\sqrt{\varepsilon_{r1}}\sin\theta_i=\sqrt{\varepsilon_{r2}}\sin\theta_t \quad\longrightarrow\quad \frac{\sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}}}=\frac{\sin\theta_i}{\sin\theta_t}$$

$$\Gamma_{\backslash\backslash}=\frac{\cos\theta_t-\dfrac{\sin\theta_i}{\sin\theta_t}\cos\theta_i}{\cos\theta_t+\dfrac{\sin\theta_i}{\sin\theta_t}\cos\theta_i}=\frac{\sin\theta_t\cos\theta_t-\sin\theta_i\cos\theta_i}{\sin\theta_t\cos\theta_t+\sin\theta_i\cos\theta_i}$$

$$=\frac{\sin 2\theta_t-\sin 2\theta_i}{\sin 2\theta_t+\sin 2\theta_i}=\frac{\sin(\theta_t-\theta_i)\cos(\theta_t+\theta_i)}{\cos(\theta_t-\theta_i)\sin(\theta_t+\theta_i)}=\frac{\tan(\theta_t-\theta_i)}{\tan(\theta_t+\theta_i)}$$

Similarly,

$$\tau_{\backslash\backslash}=\frac{\dfrac{2}{\sqrt{\varepsilon_{r2}}}\cos\theta_i}{\dfrac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_t+\dfrac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_i}=\frac{2\cos\theta_i}{\cos\theta_t+\dfrac{\sin\theta_i}{\sin\theta_t}\cos\theta_i}$$

$$=\frac{2\cos\theta_i\sin\theta_t}{\sin\theta_t\cos\theta_t(\sin^2\theta_i+\cos^2\theta_i)+\sin\theta_i\cos\theta_i(\sin^2\theta_t+\cos^2\theta_t)}$$

$$=\frac{2\cos\theta_i\sin\theta_t}{(\sin\theta_i\cos\theta_t+\sin\ \theta_t\cos\ \theta_i)(\cos\theta_i\cos\theta_t+\sin\ \theta_i\sin\ \theta_t)}$$

$$=\frac{2\cos\theta_i\sin\theta_t}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}$$

$$\Gamma_\perp = \frac{\frac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_i - \frac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_t}{\frac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_i + \frac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_t} = \frac{\cos\theta_i - \frac{\sin\theta_i}{\sin\theta_t}\cos\theta_t}{\cos\theta_i + \frac{\sin\theta_i}{\sin\theta_t}\cos\theta_t} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}$$

$$\tau_\perp = \frac{\frac{2}{\sqrt{\varepsilon_{r2}}}\cos\theta_i}{\frac{1}{\sqrt{\varepsilon_{r2}}}\cos\theta_i + \frac{1}{\sqrt{\varepsilon_{r1}}}\cos\theta_t} = \frac{2\cos\theta_i}{\cos\theta_i + \frac{\sin\theta_i}{\sin\theta_t}\cos\theta_t} = \frac{2\cos\theta_i \sin\theta_i}{\sin(\theta_t + \theta_i)}$$

Prob. 10.74

(a) $n_1 = 1, \qquad n_2 = c\sqrt{\mu_2\varepsilon_2} = c\sqrt{6.4\varepsilon_o \times \mu_o} = \sqrt{6.4} = 2.5298$

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i = \frac{1}{2.5298}\sin 12^o = 0.082185 \quad \longrightarrow \quad \theta_t = 4.714^o$$

$$\eta_1 = 120\pi, \qquad \eta_2 = 120\pi\sqrt{\frac{1}{6.4}} = 47.43\pi$$

$$\frac{E_{ro}}{E_{io}} = \Gamma = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{47.43\pi\cos 4.714^o - 120\pi\cos 12^o}{47.43\pi\cos 4.714^o + 120\pi\cos 12^o}$$

$$= \frac{47.27 - 117.38}{47.27 + 117.38} = \underline{\underline{-0.4258}}$$

$$\frac{E_{to}}{E_{io}} = \tau = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{2x47.43\cos 12^o}{47.27 + 117.33} = \frac{92.787}{164.65} = \underline{\underline{0.5635}}$$

Prob. 10.75

(a) $\mathbf{k}_i = 4\mathbf{a}_y + \mathbf{3}a_z$

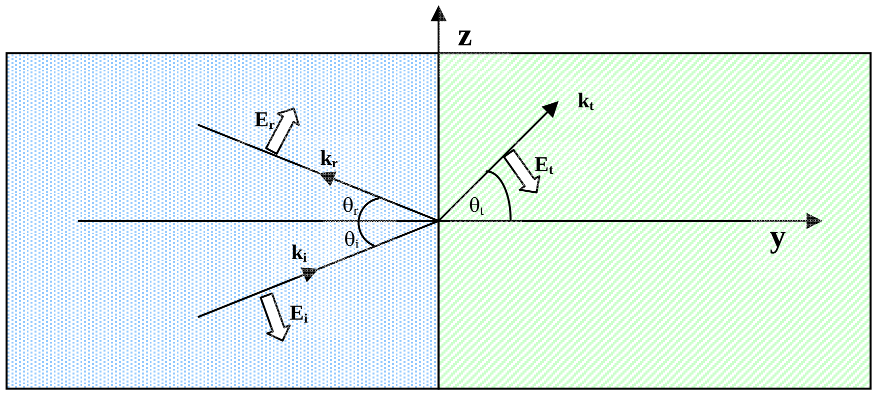
$$\mathbf{k}_i \bullet \mathbf{a}_n = k_i\cos\theta_i \quad \longrightarrow \quad \cos\theta_i = 4/5 \quad \longrightarrow \quad \underline{\underline{\theta_i = 36.87^o}}$$

(b)

$$\mathbf{P}_{\text{ave}} = \frac{1}{2}\,\text{Re}(\,\mathbf{E}_s \times \mathbf{H}_s^{\,*}\,) = \frac{E_o^{\,2}}{2\eta}\mathbf{a}_k = \frac{(\sqrt{8^2+6^2})^2}{2\times 120\pi}\,\frac{(4\mathbf{a}_y + 3\mathbf{a}_z)}{5} = \underline{\underline{106.1\mathbf{a}_y + 79.58\mathbf{a}_z \text{ mW/m}^2}}$$

(c) $\theta_r = \theta_i = 36.87^o$. Let

$$\mathbf{E}_r = (E_{ry}\mathbf{a}_x + E_{rz}\mathbf{a}_z)\sin(\omega t - \mathbf{k}_r \bullet \mathbf{r})$$



From the figure, $\boldsymbol{k}_r = k_{rz}\boldsymbol{a}_z - k_{ry}\boldsymbol{a}_y$. But $k_r = k_i = 5$

$$k_{rz} = k_r \sin\theta_r = 5(3/5) = 3, \qquad k_{ry} = k_r \cos\theta_r = 5(4/5) = 4,$$

Hence, $\boldsymbol{k}_r = -4\boldsymbol{a}_y + 3\boldsymbol{a}_z$

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i = \frac{c\sqrt{\mu_1\varepsilon_1}}{c\sqrt{\mu_2\varepsilon_2}}\sin\theta_i = \frac{3/5}{\sqrt{4}} = 0.3$$

$$\theta_t = 17.46, \quad \cos\theta_t = 0.9539, \qquad \eta_1 = \eta_o = 120\pi, \eta_2 = \eta_o/2 = 60\pi$$

$$\Gamma_{//} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{\frac{\eta_o}{2}(0.9539) - \eta_o(0.8)}{\frac{\eta_o}{2}(0.9539) + \eta_o(0.8)} = -0.253$$

$$E_{ro} = \Gamma_{//}E_{io} = -0.253(10) = -2.53$$

But $(E_{ry}\boldsymbol{a}_y + E_{rz}\boldsymbol{a}_z) = E_{ro}(\sin\theta_r\boldsymbol{a}_y + \cos\theta_r\boldsymbol{a}_z) = -2.53(\frac{3}{5}\boldsymbol{a}_y + \frac{4}{5}\boldsymbol{a}_z)$

$$\underline{\underline{\boldsymbol{E}_r = -(1.518\boldsymbol{a}_y + 2.024\boldsymbol{a}_z)\sin(\omega t + 4y - 3z)}} \text{ V/m}$$

Similarly, let

$$\boldsymbol{E}_t = (E_{ty}\boldsymbol{a}_y + E_{tz}\boldsymbol{a}_z)\sin(\omega t - \boldsymbol{k}_t \bullet \boldsymbol{r})$$

$$k_t = \beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = \omega\sqrt{4\mu_o\varepsilon_o}$$

But $k_i = \beta_1 = \omega\sqrt{\mu_o\varepsilon_o}$

$$\frac{k_t}{k_i} = 2 \longrightarrow k_t = 2k_i = 10$$

$$k_{ty} = k_t\cos\theta_t = 9.539, \qquad k_{tz} = k_t\sin\theta_t = 3,$$

$$k_t = 9.539a_y + 3a_z$$

Note that $k_{iz} = k_{rz} = k_{tz} = 3$

$$\tau_{\backslash\backslash} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{\eta_o(0.8)}{\frac{\eta_o}{2}(0.9539) + \eta_o(0.8)} = 0.6265$$

$$E_{to} = \tau_{\backslash\backslash}E_{io} = 6.265$$

But

$$(E_{ty}\boldsymbol{a}_y + E_{tz}\boldsymbol{a}_z) = E_{to}(\sin\theta_t\boldsymbol{a}_y - \cos\theta_t\boldsymbol{a}_z) = 6.256(0.3\boldsymbol{a}_y - 0.9539\boldsymbol{a}_z)$$

Hence,

$$\underline{\underline{\boldsymbol{E}_t = (1.879\boldsymbol{a}_y - 5.968\boldsymbol{a}_z)\sin(\omega t - 9.539y - 3z)}} \text{ V/m}$$

Prob. 10.76

(a)

$$\tan\theta_i = \frac{k_{ix}}{k_{iz}} = \frac{1}{\sqrt{8}} \longrightarrow \underline{\underline{\theta_i = \theta_r = 19.47^o}}$$

$$\sin\theta_t = \sin\theta_i\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} = \frac{1}{3}(3) = 1 \longrightarrow \underline{\underline{\theta_t = 90^o}}$$

(b) $\beta_1 = \frac{\omega}{c}\sqrt{\varepsilon_{r1}} = \frac{10^9}{3\times10^8}\times 3 = 10 = k\sqrt{1+8} = 3k \longrightarrow \underline{\underline{k = 3.333}}$

(c) $\lambda = 2\pi/\beta, \qquad \lambda_1 = 2\pi/\beta_1 = 2\pi/10 = \underline{\underline{0.6283}}$ m

$\beta_2 = \omega/c = 10/3, \qquad \lambda_2 = 2\pi/\beta_2 = 2\pi\times 3/10 = \underline{\underline{1.885}}$ m

(d) $$\mathbf{E}_i = \eta_1 \mathbf{H}_x \times \mathbf{a}_k = 40\pi(0.2)\cos(\omega t - \mathbf{k}\bullet\mathbf{r})\mathbf{a}_y \times \frac{(\mathbf{a}_x + \sqrt{8}\mathbf{a}_z)}{3}$$
$$= \underline{\underline{(23.6954\mathbf{a}_x - 8.3776\mathbf{a}_z)\cos(10^9 t - kx - k\sqrt{8}z) \text{ V/m}}}$$

(e) $$\tau_{//} = \frac{2\cos\theta_i \sin\theta_t}{\sin(\theta_i + \theta_t)\cos(\theta_t - \theta_i)} = \frac{2\cos 19.47^o \sin 90^o}{\sin 19.47^o \cos 19.47^o} = 6$$

$$\Gamma_{//} = -\frac{\cot 19.47^o}{\cot 19.47^o} = -1$$

Let $\mathbf{E}_t = -E_{io}(\cos\theta_t \mathbf{a}_x - \sin\theta_t \mathbf{a}_z)\cos(10^9 t - \beta_2 x \sin\theta_t - \beta_2 z\cos\theta_t)$

where

$$\mathbf{E}_t = -E_{io}(\cos\theta_i \mathbf{a}_x - \sin\theta_i \mathbf{a}_z)\cos(10^9 t - \beta_1 x \sin\theta_i - \beta_1 z\cos\theta_i)$$

$$\sin\theta_t = 1, \quad \cos\theta_t = 0, \quad \beta_2 \sin\theta_t = 10/3$$

$$E_{to}\sin\theta_t = \tau_{\backslash\backslash} E_{io} = 6(24\pi)(3)(1) = 1357.2$$

Hence,

$$\underline{\underline{\mathbf{E}_t = 1357\cos(10^9 t - 3.333x)\mathbf{a}_z}} \text{ V/m}$$

Since $\Gamma = -1, \quad \theta_r = \theta_i$

$$\underline{\underline{\mathbf{E}_r = (213.3\mathbf{a}_x + 75.4\mathbf{a}_z)\cos(10^9 t - kx + k\sqrt{8}z)}} \text{ V/m}$$

(f) $$\tan\theta_{B//} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{\frac{\varepsilon_o}{9\varepsilon_o}} = 1/3 \longrightarrow \underline{\underline{\theta_{B//} = 18.43^o}}$$

Prob. 10.77

(*a*) $$\underline{\underline{\mathbf{E}_{i\perp} = 5\cos(\omega t - 0.5\pi x - 0.866\pi z)\mathbf{a}_y}}$$

$$\underline{\underline{\mathbf{E}_{i//} = (4\mathbf{a}_x - 3\mathbf{a}_z)\cos(\omega t - 0.5\pi x - 0.866\pi z)}}$$

(*b*) Comparing $\mathbf{E}_{i//}$ with eq. (10.115a),

$$4\mathbf{a}_x - 3\mathbf{a}_z = (\cos\theta_i \mathbf{a}_x - \sin\theta_i \mathbf{a}_z)E_{io}$$

$$\tan\theta_i = \frac{\sin\theta_i}{\cos\theta_i} = \frac{3}{4} \quad \rightarrow \quad \underline{\underline{\theta_i = 36.87^o}}$$

Prob. 10.78

$$\tan\theta_B = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = \sqrt{\frac{\varepsilon_o\varepsilon_r}{\varepsilon_o}} = \sqrt{\varepsilon_r}$$

$$\varepsilon_r = \tan^2\theta_B = \tan^2 68 = \underline{\underline{6.126}}$$

Prob. 10.79

(a) $n = \frac{c}{u} = \sqrt{\mu_r\varepsilon_r} = \sqrt{2.1\times 1} = \underline{\underline{1.45}}$

(b) $n = \sqrt{\mu_r\varepsilon_r} = \sqrt{1\times 81} = \underline{\underline{9}}$

(c) $n = \sqrt{\varepsilon_r} = \sqrt{2.7} = \underline{\underline{1.643}}$

Prob.10.80

Microwave is used:

(1) For surveying land with a piece of equipment called the *tellurometer*. This radar system can precisely measure the distance between two points.

(2) For guidance. The guidance of missiles, the launching and homing guidance of space vehicles, and the control of ships are performed with the aid of microwaves.

(3) In semiconductor devices. A large number of new microwave semiconductor devices have been developed for the purpose of microwave oscillator, amplification, mixing/detection, frequency multiplication, and switching. Without such achievement, the majority of today's microwave systems could not exist.

Prob.10.81

(a) In terms of the S-parameters, the T-parameters are given by

$T_{11} = 1/S_{21}$, $T_{12} = -S_{22}/S_{21}$, $T_{21} = S_{11}/S_{21}$, $T_{22} = S_{12} - S_{11} S_{22}/S_{21}$

(b) $T_{11} = 1/0.4 = 2.5$, $T_{12} = -0.2/0.4$,

$T_{21} = 0.2/0.4$, $T_{22} = 0.4 - 0.2 \times 0.2/0.4 = 0.3$

Hence,

$$T = \underline{\underline{\begin{bmatrix} 2.5 & -0.5 \\ 0.5 & 0.3 \end{bmatrix}}}$$

Prob. 10.82

Since $Z_L = Z_o$, $\Gamma_L = 0$.

$\Gamma_i = S_{11} = \underline{0.33 - j0.15}$

$\Gamma_g = (Z_g - Z_o)/(Z_g + Z_o) = (2 - 1)/(2 + 1) = 1/3$

$\Gamma o = S_{22} + S_{12}S_{21}\Gamma_g /(1 - S_{11} \Gamma_g)$

$= 0.44 - j0.62 + 0.56x0.56\ x(1/3)/[1 - (0.11 - j0.05)]$

$= \underline{0.5571 - j0.6266}$

Prob. 10.83 The microwave wavelengths are of the same magnitude as the circuit components. The wavelength in air at a microwave frequency of 300 GHz, for example, is 1 mm. The physical dimension of the lumped element must be in this range to avoid interference. Also, the leads connecting the lumped element probably have much more inductance and capacitance than is needed.

Prob. 10.84

$$\lambda = c/f = \frac{3\times10^8}{8.4\times10^9} = \underline{35.71\text{ mm}}$$

CHAPTER 12

P. E. 12.1 (a) For TE_{10}, $f_c = 3$ GHz,

$$\sqrt{1-(f_c/f)^2} = \sqrt{1-(3/15)^2} = \sqrt{0.96}\,, \quad \beta_o = \omega/u_o = 4\pi f/c$$

$$\beta = \frac{4\pi f}{c}\sqrt{0.96} = \frac{4\pi\times 15\times 10^9}{3\times 10^8}\sqrt{0.96} = \underline{\underline{615.6}} \text{ rad/m}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi\times 15\times 10^9}{615.6} = \underline{\underline{1.531\times 10^8}} \text{ m/s}$$

$$\eta' = \sqrt{\frac{\mu}{\varepsilon}} = 60\pi, \quad \eta_{TE} = \frac{60\pi}{\sqrt{0.96}} = \underline{\underline{192.4\Omega}}$$

(b)For TM_{11}, $f_c = 3\sqrt{7.25}$ GHz, $\sqrt{1-(f_c/f)^2} = 0.8426$

$$\beta = \frac{4\pi f}{c}(0.8426) = \frac{4\pi\times 15\times 10^9 (0.8426)}{3\times 10^8} = \underline{\underline{529.4}} \text{ rad/m}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi\times 15\times 10^9}{529.4} = \underline{\underline{1.78\times 10^8}} \text{ m/s}$$

$$\eta_{TM} = 60\pi(0.8426) = \underline{\underline{158.8\Omega}}$$

P. E. 12.2 (a) Since $E_z \neq 0$, this is a TM mode

$$E_{zs} = E_o \sin(m\pi x/a)\sin(n\pi y/b)e^{-j\beta z}$$

$E_o = 20$, $\dfrac{m\pi}{a} = 40\pi \longrightarrow$ m=2, $\dfrac{n\pi}{b} = 50\pi \longrightarrow$ n=1

i.e. $\underline{TM_{21}\text{ mode}}$.

(b) $f_c = \dfrac{u'}{2}\sqrt{(m/a)^2+(n/b)^2} = \dfrac{3\times 10^8}{2}\sqrt{40^2+50^2} = 1.5\sqrt{41}$ GHz

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1-(f_c/f)^2} = \frac{2\pi f}{c}\sqrt{f^2-f_c^2} = \frac{2\pi\times 10^9}{3\times 10^8}\sqrt{225-92.25} = \underline{\underline{241.3 \text{ rad/m.}}}$$

(c)

$$E_{xs} = \frac{-j\beta}{h^2}(40\pi)20\cos 40\pi x \sin 50\pi y e^{-j\beta z}$$

$$E_{ys} = \frac{-j\beta}{h^2}(50\pi)20\sin 40\pi x \cos 50\pi y e^{-j\beta z}$$

$$\frac{E_y}{E_x} = \underline{\underline{1.25\tan 40\pi x \cot 50\pi y}}$$

P. E. 12.3 If TE_{13} mode is assumed, f_c and β remain the same.

$\underline{\underline{f_c = 28.57 \text{ GHz}, \ \beta = 1718.81 \text{ rad/m}, \ \gamma = j\beta}}$

$$\eta_{TE13} = \frac{377/2}{\sqrt{1-(28.57/50)^2}} = \underline{\underline{229.69\ \Omega}}$$

For m=1, n=3, the field components are:

$E_z = 0$

$$H_z = H_o \cos(\pi x/a)\cos(3\pi y/b)\cos(\omega t - \beta z)$$

$$E_x = -\frac{\omega\mu}{h^2}\left(\frac{3\pi}{b}\right)H_o \cos(\pi x/a)\sin(3\pi y/b)\sin(\omega t - \beta z)$$

$$E_y = \frac{\omega\mu}{h^2}\left(\frac{\pi}{a}\right)H_o \sin(\pi x/a)\cos(3\pi y/b)\sin(\omega t - \beta z)$$

$$H_x = -\frac{\beta}{h^2}\left(\frac{\pi}{a}\right)H_o \sin(\pi x/a)\cos(3\pi y/b)\sin(\omega t - \beta z)$$

$$H_y = -\frac{\beta}{h^2}\left(\frac{3\pi}{a}\right)H_o \cos(\pi x/a)\sin(3\pi y/b)\sin(\omega t - \beta z)$$

Given that $H_{ox} = 2 = -\frac{\beta}{h^2}(\pi/a)H_o$,

$$H_{oy} = -\frac{\beta}{h^2}(3\pi/b)H_o = 6a/b = 6(1.5)/8 = 11.25$$

$$H_{oz} = H_o = -\frac{2h^2a}{\beta\pi} = \frac{-2\times 14.51\pi^2 \times 10^4 \times 1.5\times 10^{-2}}{1718.81\pi} = -7.96$$

$$E_{oy} = \frac{\omega\mu}{h^2}\left(\frac{\pi}{a}\right)H_o = -\frac{2\omega\mu}{\beta} = 2\eta_{TE} = -459.4$$

$$E_{ox} = -E_{oy}\frac{3a}{b} = 459.4(4.5/0.8) = 2584.1$$

$$E_x = 2584.1\cos(\pi x/a)\sin(3\pi y/b)\sin(\omega t - \beta z) \text{ V/m},$$

$$E_y = -459.4\sin(\pi x/a)\cos(3\pi y/b)\sin(\omega t - \beta z) \text{ V/m},$$

$E_z = 0$,

$$H_y = 11.25\cos(\pi x/a)\sin(3\pi y/b)\sin(\omega t - \beta z) \quad \text{A/m},$$

$$H_z = -7.96\cos(\pi x/a)\cos(3\pi y/b)\cos(\omega t - \beta z) \quad \text{A/m}$$

P. E. 12.4

$$f_{c11} = \frac{u'}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = \frac{3\times10^8\times10^2}{2}\sqrt{1/8.636^2+1/4.318^2} = 3.883 \quad \text{GHz}$$

$$u_p = \frac{3\times10^8}{\sqrt{1-(3.883/4)^2}} = \underline{\underline{12.5\times10^8}} \quad \text{m/s},$$

$$u_g = \frac{9\times10^{16}}{12.5\times10^8} = \underline{\underline{7.2\times10^7}} \quad \text{m/s}$$

P. E. 12.5 The dominant mode becomes TE_{01} mode

$$f_{c01} = \frac{c}{2b} = 3.75 \quad \text{GHz}, \quad \eta_{TE} = 406.7\Omega$$

From Example 12.2,

$$E_x = -E_o\sin(3\pi y/b)\sin(\omega t - \beta z), \quad \text{where } E_o = \frac{\omega\mu b}{\pi}H_o.$$

$$\mathcal{P}_{ave} = \int_{x=0}^{a}\int_{y=0}^{b}\frac{|E_{xs}|^2}{2\eta}dxdy = \frac{E_o^{\,2}\,ab}{4\eta}$$

Hence $E_o = 63.77$ V/m as in Example 12.5.

$$H_o = \frac{\pi E_o}{\omega\mu b} = \frac{\pi\times63.77}{2\pi\times10^{10}\times4\pi\times10^{-7}\times4\times10^{-2}} = \underline{\underline{63.34}} \quad \text{mA/m}$$

P. E. 12.6 (a) For m=1, n=0, $f_c = u'/(2a)$

$$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-15}}{2\pi\times9\times10^9\times2.6\times10^{-9}/(36\pi)} = \frac{10^{-15}}{1.3} << 1$$

Hence,

$u' \cong \dfrac{1}{\sqrt{\mu\varepsilon}} = c/\sqrt{2.6}, \quad f_c = \dfrac{3\times10^8}{2\times2.4\times10^{-2}\sqrt{2.6}} = 2.2149 \text{ GHz}$

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1-(f_c/f)^2}} = \frac{10^{-15}\times377/\sqrt{2.6}}{2\sqrt{1-(2.2149/9)^2}} = 1.205\times10^{-13} \text{ Np/m}$$

For n = 0, m=1,

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1-(f_c/f)^2}}[\frac{1}{2}+\frac{b}{a}(f_c/f)^2]$$

$$= \frac{2\sqrt{2.6}\sqrt{\pi\times9\times10^9\times1.1\times10^7\times4\pi\times10^{-7}}}{377\times1.5\times10^{-2}\times1.1\times10^7\sqrt{1-(2.2149/9)^2}}[0.5+(2.4/1.5)(2.2148/9)^2] = \underline{\underline{2\times10^{-2}}} \text{ Np/m}$$

(b)Since $\alpha_c >> \alpha_d, \alpha = \alpha_c + \alpha_d \cong \alpha_c = 2x10^{-2}$

loss = $\alpha l = 2\times10^{-2}\times0.4 = 0.8\times10^{-2}$ Np = $\underline{0.06945 \text{ dB}}$

P. E. 12.7 For TE_{11} , m = 1 = n,

$$H_{zs} = H_o\cos(\pi x/a)\cos(\pi y/b)e^{-\gamma z}$$

$$E_{xs} = \frac{j\omega}{h^2}(\pi/b)H_o\cos(\pi x/a)\sin(\pi y/b)e^{-\gamma z}$$

$$E_{ys} = -\frac{j\omega\mu}{h^2}(\pi/a)H_o\sin(\pi x/a)\cos(\pi y/b)e^{-\gamma z}$$

$$H_{xs} = \frac{j\beta}{h^2}(\pi/a)H_o\sin(\pi x/a)\cos(\pi y/b)e^{-\gamma z}$$

$$H_{ys} = \frac{j\beta}{h^2}(\pi/b)H_o\cos(\pi x/a)\sin(\pi y/b)e^{-\gamma z}$$

$$E_{zs} = 0$$

For the electric field lines,

$$\frac{dy}{dx} = \frac{E_y}{E_x} = (a/b)\tan(\pi x/a)\cot(\pi y/b)$$

For the magnetic field lines

$$\frac{dy}{dx} = \frac{H_y}{H_x} = -(a/b)\cot(\pi x/a)\tan(\pi y/b)$$

Notice that $(\frac{E_y}{E_x})(\frac{H_y}{H_x}) = -1$

showing that the electric and magnetic field lines are mutually orthogonal. The field lines are as shown in Fig. 12.14.

P. E. 12.8

$$u' = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\varepsilon_r}}$$

$$f_{TE101} = \frac{1.5\times10^{10}}{\sqrt{3}}\sqrt{1/25+0+1/100} = \underline{\underline{1.936}} \text{ GHz}$$

$$Q_{TE101} = \frac{1}{61\delta}, \text{ where}$$

$$\delta = \frac{1}{\sqrt{\pi f_{101}\mu\sigma_c}} = \frac{1}{\sqrt{\pi\times1.936\times10^9\times4\pi\times10^{-7}\times5.8\times10^7}} = 1.5\times10^{-6}$$

$$Q_{TE101} = \frac{10^6}{61\times1.5} = \underline{\underline{10{,}929}}$$

P. E. 12.9

(a) By Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Thus

$\theta_2 = 90^o \longrightarrow \sin\theta_2 = 1$

$\sin\theta_1 = n_2/n_1$, $\theta_1 = \sin^{-1} n_2/n_1 = \sin^{-1} 1.465/1.48 = \underline{\underline{81.83^o}}$

(b) NA $= \sqrt{n_1^2 - n_2^2} = \sqrt{1.48^2 - 1.465^2} = \underline{\underline{0.21}}$

P. E. 12.10

$\alpha l = 10 \log P(0)/P(l) = 0.2 \text{ X } 10 = 2$

$P(0)/P(l) = 10^{0.2}$, i.e. $P(l) = P(0)\, 10^{-0.2} = 0.631\, P(0)$

i.e. <u>63.1 %</u>

Prob. 12.1

(a) For TE_{10} mode, $f_c = \frac{u'}{2a} = \frac{3\times10^8}{2\times6\times10^{-2}} = \underline{\underline{2.5\text{ GHz}}}$

(b) $f = 3f_c = 7.5$ GHz

$$f_{cmn} = \frac{u^1}{2}\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}} = \frac{3\times10^8\times10^2}{2}\sqrt{\left(\frac{m}{6}\right)^2+\left(\frac{n}{4}\right)^2}$$

$$= 15\sqrt{\left(\frac{m}{6}\right)^2+\left(\frac{n}{4}\right)^2}\text{ GHz}$$

$f_{c20} = 15\times\frac{2}{6} = 5$ GHz

$f_{c01} = 3.75$ GHz, $f_{c02} = 7.5$ GHz

$f_{c10} = 2.5$ GHz, $f_{c20} = 5.0$ GHz

$f_{c21} = 6.25$ GHz, $f_{c30} = 7.5$ GHz

$$f_{12} = 15\sqrt{\left(\frac{1}{6}\right)^2+\left(\frac{2}{4}\right)^2} = 7.91\text{ GHz}$$

$$f_{11} = 15\sqrt{\left(\frac{1}{6}\right)^2+\left(\frac{1}{4}\right)^2} = 4.507\text{ GHz}$$

The following modes are transmitted

$TE_{01}, TE_{02}, TE_{10}, TE_{11}, TE_{20}, TE_{21}, TE_{30}$

TM_{11}, TM_{21}

i.e. 7 TE modes and 2 TM modes

Prob.12.2

$$f_{c10} = \frac{u'}{2a},\ f_{c11} = \frac{u'}{2}\sqrt{\frac{1}{a^2}+\frac{1}{a^2}} = \frac{u'}{2a}\sqrt{2}$$

Since the guide can only propagate TE_{10} mode,

$$f_{c10} < f < f_{c11} \quad\rightarrow\quad \frac{u'}{2a} < f < \frac{u'}{2a}\sqrt{2} \quad\rightarrow\quad u' < 2af < u'\sqrt{2}$$

$$\frac{u'}{f} < 2a < \frac{u'}{f}\sqrt{2} \quad\rightarrow\quad \lambda < 2a < \lambda\sqrt{2}$$

$$\frac{\lambda}{2} < a < \frac{\lambda}{\sqrt{2}}$$

Prob. 12.3

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2\sqrt{2.25}\times10^{-2}}\left[\left(\frac{m}{2.28}\right)^2 + \left(\frac{n}{1.01}\right)^2\right]^{1/2}$$

$$= \frac{15}{\sqrt{2.25}}\left[\left(\frac{m}{2.28}\right)^2 + \left(\frac{n}{1.01}\right)^2\right]^{1/2} \text{ GHz}$$

Using this formula, we obtain the cutoff frequencies for the given modes as shown below.

Mode	f_c (GHz)
TE_{01}	9.901
TE_{10}	4.386
TE_{11}	10.829
TE_{02}	19.802
TE_{22}	21.658
TM_{11}	10.829
TM_{12}	20.282
TM_{21}	13.228

Prob. 12.4

(a)

For TE_{10} mode,

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{1}{a}\right)^2} = \frac{3\times10^8}{2\times2.4\times10^{-2}} = \underline{\underline{6.25 \text{ GHz}}}$$

For TE_{01} mode,

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{1}{b}\right)^2} = \frac{3\times10^8}{2\times1.2\times10^{-2}} = \underline{\underline{12.5 \text{ GHz}}}$$

For TE_{20} mode,

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{2}{a}\right)^2} = 2\times6.25 = \underline{\underline{12.5 \text{ GHz}}}$$

For TE_{02} mode,

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{2}{b}\right)^2} = 2\times12.5 = \underline{\underline{25 \text{ GHz}}}$$

(b) Since f = 12 GHz, $\underline{\underline{\text{only } TE_{10} \text{ mode}}}$ will propagate.

Prob. 12.5 a/b = 3 ⟶ a = 3b

$$f_{c10} = \frac{u'}{2a} \longrightarrow a = \frac{u'}{2f_{c10}} = \frac{3\times10^8}{2\times18\times10^9} \text{ m} = 0.833\text{cm}$$

A design could be a = 9mm, b = 3mm.

Prob. 12.6 For the dominant mode,

$$f_c = \frac{c}{2a} = \frac{3\times10^8}{2\times8} = 18.75 \text{ MHz}$$

(a) It will not pass the AM signal, (b) it will pass the FM signal.

Prob. 12.7 (a) For TE_{10} mode, $f_c = \dfrac{u'}{2a}$

Or $a = \dfrac{u'}{2f_c} = \dfrac{3\times10^8}{2\times5\times10^9} = 3 \text{ cm}$

For TE_{01} mode, $f_c = \dfrac{u'}{2b}$

Or $b = \dfrac{u'}{2f_c} = \dfrac{3\times10^8}{2\times12\times10^9} = 1.25 \text{ cm}$

(b) Since a > b, 1/a < 1/b, the next higher modes are calculated as shown below.

Mode	f_c (GHz)
TE_{10}	5
*TE_{20}	10
TE_{30}	15
TE_{40}	20
*TE_{01}	12
TE_{02}	24
*TE_{11}	13
TE_{21}	15.62

The next three higher modes are starred ones, i.e. TE_{20}, TE_{01}, TE_{11}

(c) $u' = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{c}{\sqrt{2.25}} = 2\times10^8$ m/s

For TE_{11} modes,

$$f_c = \frac{3\times10^8}{2\times10^{-2}\sqrt{2.25}}\sqrt{\frac{1}{3^2}+\frac{1}{1.25^2}} = \underline{\underline{8.67}} \text{ GHz}$$

Prob. 12.8

$$\text{Let } F_{12} = \sqrt{1-\left(\frac{f_{c12}}{f}\right)^2} = \sqrt{1-\left(\frac{25}{40}\right)^2} = 0.7806$$

$$\lambda' = \frac{c}{f} = \frac{3\times10^8}{40\times10^9} = 0.0075 \text{ m} = 7.5\times10^{-3} \text{ m}$$

$$\lambda_{12} = \frac{\lambda'}{F_{12}} = \frac{7.5\times10^{-3} \text{ m}}{0.7806} = \underline{\underline{9.608\times10^{-3} \text{ m}}}$$

$$u_{12} = \frac{u'}{F_{12}} = \frac{3\times10^8}{0.7806} = \underline{\underline{3.843\times10^8 \text{ m/s}}}$$

$$\beta_{12} = \frac{2\pi}{\lambda_{12}} = \frac{2\pi}{9.608\times10^{-3}} = \underline{\underline{653.95 \text{ rad/m}}}$$

$$\eta_{TE12} = \frac{\eta'}{F_{12}} = \frac{120\pi}{0.7806} = \underline{\underline{482.95\Omega}}$$

Prob. 12.9

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

For TE_{10} mode, m=1, n=0,

$$f_c = \frac{u'}{2a} = \frac{c}{2a} = \frac{3\times10^8}{2\times5\times10^{-2}} = 3GHz$$

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 2\pi\times12.5\times10^9\sqrt{4\pi\times10^{-7}\times\frac{10^{-9}}{36\pi}}\sqrt{1-\left(\frac{3}{12.5}\right)^2} = \frac{785.4}{3}(0.9708)$$

$$\underline{\underline{\beta \quad = 254.15 \text{ rad/m}}}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi\times12.5\times10^9}{254.15} = \underline{\underline{3.09\times10^8 \text{ m/s}}}$$

$$\eta_{TE} = \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{120\pi}{0.9708} = \underline{\underline{388.3\ \Omega}}$$

Prob. 12.10

$$f_c = \frac{u'}{2a} = \frac{c}{2a} = \frac{3\times10^8}{2\times4\times10^{-2}} = \underline{\underline{3.75 \text{ GHz}}}$$

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi\times24\times10^9}{3\times10^8}\sqrt{1-\left(\frac{3.75}{24}\right)^2} = \frac{480\pi}{3}(0.9877)$$

$$\underline{\underline{\beta \quad = 496.48 \text{ rad/m}}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{496.48} = \underline{\underline{0.0127 \text{ m}}}$$

Prob. 12.11

$$u = \frac{\omega}{\beta} = \frac{u'}{\sqrt{1-(f_c/f)^2}} = \frac{3\times10^8}{\sqrt{1-(6.5/7.2)^2}} = 6.975\times10^8 \text{ m/s}$$

$$u_g = \frac{9\times10^{16}}{u} = 1.2903\times10^8 \text{ m/s}$$

$$t = \frac{2l}{u_g} = \frac{300}{1.2903\times10^8} = \underline{\underline{2.325}}\ \mu\text{s}$$

Prob. 12.12

$$f_c = \frac{u'}{2a} = \frac{c}{2a} = \frac{3\times10^8}{2\times2.4\times10^{-2}} = \underline{\underline{6.25 \text{ GHz}}}$$

$$f = 1.25 f_c = \underline{\underline{7.813 \text{ GHz}}}$$

$$\eta = \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{1}{1.25}\right)^2}} = \frac{377}{0.6} = \underline{\underline{628.32\ \Omega}}$$

Prob. 12.13

$$(a)\ f_{c10} = \frac{u'}{2a} = \frac{c}{2a\sqrt{\varepsilon_r}}$$

$$= \frac{3\times10^8}{2\times1.067\times10^{-2}\sqrt{6.8}}$$

$$= \frac{30}{2\times1.067\sqrt{6.8}} \text{ GHz}$$

$$\underline{\underline{= 5.391 \text{ GHz}}}$$

(b)

$$F = \sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= \sqrt{1-\left(\frac{5.391}{6}\right)^2}$$

$$= 0.439$$

$$u_\rho = \frac{u'}{F} = \frac{c}{F\sqrt{\varepsilon_r}} = \frac{3\times10^8}{0.439\times\sqrt{6.8}} = \underline{\underline{2.62\times10^8 \text{ m/s}}}$$

(c)

$$\lambda = \frac{\lambda'}{F} = \frac{u'/f}{F} = \frac{c}{fF\sqrt{\varepsilon_r}} = \frac{3\times10^8}{0.439\times6\times10^9\times\sqrt{6.8}}$$

$$= \frac{10^{-1}}{2\times0.439\sqrt{6.8}} = 0.04368 \text{ m} = \underline{\underline{4.368 \text{ cm}}}$$

Prob.12.14
In evanescent mode,

$$k^2 = \omega^2\mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\beta = 0, \qquad \gamma = \alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2} = \sqrt{4\pi^2\mu\varepsilon f_c^2 - \omega^2\mu\varepsilon}$$

$$\alpha = \sqrt{\mu\varepsilon}\sqrt{4\pi^2 f_c^2 - 4\pi^2 f^2} = \underline{\underline{2\pi\sqrt{\mu\varepsilon}\, f_c\sqrt{1-\left(\frac{f}{f_c}\right)^2}}}$$

Prob. 12.15
$E_z \neq 0$. This must be TM_{23} mode (m=2, n=3). Since a= 2b,

$$f_c = \frac{c}{4b}\sqrt{m^2+4n^2} = \frac{3\times10^8}{4\times3\times10^{-2}}\sqrt{4+36} = 15.81 \text{ GHz}, \quad f = \frac{\omega}{2\pi} = \frac{10^{12}}{2\pi} = 159.2 \text{ GHz}$$

$$\eta_{TM} = 377\sqrt{1-(15.81/159.2)^2} = \underline{\underline{375.1\ \Omega}}$$

$$\mathcal{P}_{ave} = \frac{|E_{xs}|^2 + |E_{ys}|^2}{2\eta_{TM}}\mathbf{a}_z$$

$$= \frac{\beta^2 E_o^{\ 2}}{2h^4 \eta_{TM}} \left[(2\pi/a)^2 \cos^2(2\pi x/a) \sin^2(3\pi y/b) + (3\pi/b)^2 \sin^2(2\pi x/a) \cos^2(3\pi y/b) \right] \mathbf{a}_z$$

$$P_{ave} = \int \mathcal{P}_{ave} . d\mathbf{S} = \int_{x=0}^{a} \int_{y=0}^{b} \mathcal{P}_{ave} \, dxdy\mathbf{a}_z$$

$$= \frac{\beta^2 E_o^{\ 2}}{2h^4 \eta_{TM}} \frac{ab}{4} \left[\frac{4\pi^2}{a^2} + \frac{9\pi^2}{b^2} \right] = \frac{\beta^2 E_o^{\ 2} ab}{8h^2 \eta_{TM}}$$

But

$$\beta = \frac{\omega}{c} \sqrt{1-(f_c/f)^2} = \frac{10^{12}}{3\times 10^8} \sqrt{1-(15.81/159.2)^2} = 3.317\times 10^3$$

$$h^2 = \frac{4\pi^2}{a^2} + \frac{9\pi^2}{b^2} = \frac{10\pi^2}{b^2} = 1.097\times 10^5$$

$$P_{ave} = \frac{(3.317)^2 \times 10^6 \times 5^2 \times 18\times 10^{-4}}{8\times (1.098\times 10^5)\times 375.1} = \underline{\underline{1.5 \text{ mW}}}$$

Prob. 12.16 (a) Since m=2 and n=1, we have $\underline{\underline{TE_{21} \text{ mode}}}$

(b) $\beta = \beta' \sqrt{1-(f_c/f)^2} = \omega \sqrt{\mu_o \varepsilon_o} \sqrt{1-(\omega_c/\omega)^2}$

$\beta c = \sqrt{\omega^2 - \omega^2_c}$ ⟶ $\omega_c^{\ 2} = \sqrt{\omega^2 - \beta^2 c^2}$

$$f_c = \frac{\omega_c}{2\pi} = \sqrt{f^2 - \frac{\beta^2 c^2}{4\pi^2}} = \sqrt{36\times 10^{18} - \frac{144\times 9\times 10^{16}}{4\pi^2}} = \underline{\underline{5.973}} \text{ GHz}$$

(c) $\eta_{TE} = \dfrac{\eta}{\sqrt{1-(f_c/f)^2}} = \dfrac{377}{\sqrt{1-(5.973/6)^2}} = \underline{\underline{3978\Omega}}$

(d)For TE mode,

$$E_y = \frac{\omega\mu}{h^2}(m\pi/a)H_o \sin(m\pi x/a)\cos(n\pi y/b)\sin(\omega t - \beta z)$$

$$H_x = \frac{-\beta}{h^2}(m\pi/a)H_o \sin(m\pi x/a)\cos(n\pi y/b)\sin(\omega t - \beta z)$$

$\beta = 12$, m = 2, n =1

$$E_{oy} = \frac{\omega\mu}{h^2}(m\pi / a)H_o, \quad H_{ox} = \frac{\beta}{h^2}(m\pi / a)H_o$$

$$\eta_{TE} = \frac{E_{oy}}{H_{ox}} = \frac{\omega\mu}{\beta} = \frac{2\pi\times 6\times 10^9 \times 4\pi\times 10^{-7}}{12} = 4\pi^2 \times 100$$

$$H_{ox} = \frac{E_{oy}}{\eta_{TE}} = \frac{5}{4\pi^2\times 100} = 1.267 \text{ mA/m}$$

$$\underline{\underline{H_x = -1.267\sin(m\pi x / a)\cos(n\pi y / b)\sin(\omega t - \beta z)}} \text{ mA/m}$$

Prob. 12.17 (a) Since m=2, n=3, the mode is $\underline{\underline{TE_{23}}}$.

(b) $\beta = \beta'\sqrt{1-(f_c / f)^2} = \dfrac{2\pi f}{c}\sqrt{1-(f_c / f)^2}$

But

$$f_c = \frac{u'}{2}\sqrt{(m/a)^2 + (n/b)^2} = \frac{3\times 10^8}{2\times 10^{-2}}\sqrt{(2/2.86)^2 + (3/1.016)^2} = 46.19 \text{ GHz}, \quad f = 50 \text{ GHz}$$

$$\beta = \frac{2\pi\times 50\times 10^9}{3\times 10^8}\sqrt{1-(46.19/50)^2} = 400.68 \text{ rad/m}$$

$$\gamma = j\beta = \underline{\underline{j400.7}} \text{ /m}$$

(c) $\eta = \dfrac{\eta'}{\sqrt{1-(f_c / f)^2}} = \dfrac{377}{\sqrt{1-(46.19/50)^2}} = \underline{\underline{985.3\Omega}}$

Prob. 12.18 In free space,

$$\eta_1 = \frac{\eta_o}{\sqrt{1-(f_c / f)^2}}, \qquad f_c = \frac{c}{2a} = \frac{3\times 10^8}{2\times 5\times 10^{-2}} = 3 \text{ GHz}$$

$$\eta_1 = \frac{377}{\sqrt{1-(3/8)^2}} = 406.7\Omega$$

$$\eta_2 = \frac{\eta'_1}{\sqrt{1-(f_c / f)^2}}, \eta' = \frac{120\pi}{\sqrt{2.25}} = 80\pi, f_c = \frac{u'}{2a}, u' = \frac{c}{\sqrt{\varepsilon_r}}$$

$$f_c = \frac{3\times 10^8}{2\times 5\times 10^{-2}\sqrt{2.25}} = 2 \text{ GHz}, \quad \eta_2 = \frac{80\pi}{\sqrt{1-(2/8)^2}} = 259.57\Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.2208$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \underline{\underline{1.5667}}$$

Prob. 12.19 Substituting $E_z = R\Phi Z$ into the wave equation,

$$\frac{\Phi Z}{\rho}\frac{d}{d\rho}(\rho R') + \frac{RZ}{\rho^2}\Phi'' + R\Phi Z'' + k^2 R\Phi Z = 0$$

Dividing by $R\Phi Z$,

$$\frac{1}{R\rho}\frac{d}{d\rho}(\rho R') + \frac{\Phi''}{\Phi\rho^2} + k^2 = -\frac{Z''}{Z} = -k_z^2$$

i.e. $\underline{\underline{Z'' - k_z^2 Z = 0}}$

$$\frac{1}{R\rho}\frac{d}{d\rho}(\rho R') + \frac{\Phi''}{\Phi\rho^2} + (k^2 + k_z^2) = 0$$

$$\frac{\rho}{R}\frac{d}{d\rho}(\rho R') + (k^2 + k_z^2)\rho^2 = -\frac{\Phi''}{\Phi} = k_\phi^2$$

or

$$\underline{\underline{\Phi'' + k_\phi^2 \Phi = 0}}$$

$\rho\frac{d}{d\rho}(\rho R') + (k_\rho^2\rho^2 - k_\phi^2)R = 0$, where $k_\rho^2 = k^2 + k_z^2$. Hence

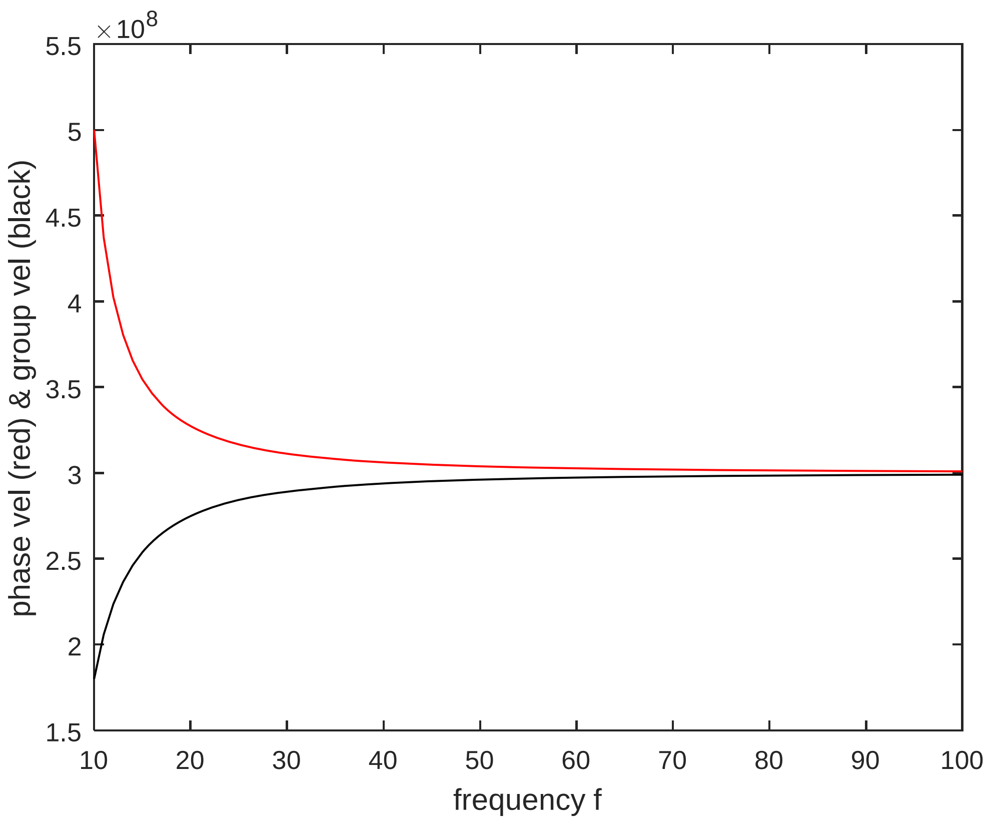
$$\underline{\underline{\rho^2 R'' + \rho R' + (k_\rho^2\rho^2 - k_\phi^2)R = 0}}$$

Prob. 12.20

The MATLAB code and the plot of the phase and group velocities are presented below.

```
%    Plot U_p and U_g versus frequency f  (10<f<100) in GHz

c=3*10^8;
for k=1:91
    f(k)=9+k
    fac = sqrt( 1 - (8/f(k))^2 );
    up(k) = c/fac;
    ug(k) = c*fac;
end
plot(f,up, 'r', f, ug, 'k')
xlabel('frequency f')
ylabel('phase vel (red) & group vel (black)')
```



Prob. 12.21

(a)

For TE_{10} mode,

$$f_c = \frac{u'}{2a} = \frac{3\times10^8}{2\times7.2\times10^{-2}} = 2.083 \text{ GHz}$$

Let $F = \sqrt{1-\left(\frac{f_c}{f}\right)^2} = \sqrt{1-\left(\frac{2.083}{6.2}\right)^2} = 0.942$

$$\beta = \omega\sqrt{\mu\varepsilon}F = \frac{\omega F}{c} = \frac{2\pi\times6.2\times10^9\times0.942}{3\times10^8} = \underline{\underline{122.32 \text{ rad/m}}}$$

$$u_p = \frac{\omega}{\beta} = \frac{c}{F} = \frac{3\times10^8}{0.942} = \underline{\underline{3.185\times10^8 \text{ m/s}}}$$

$$u_g = u'F = 3\times10^8(0.942) = \underline{\underline{2.826\times10^8 \text{ m/s}}}$$

$$\eta_{TE} = \frac{\eta'}{F} = \frac{377}{0.942} = \underline{\underline{400.21\ \Omega}}$$

(b)

$$u' = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times10^8}{\sqrt{2.25}} = 2\times10^8$$

$$f_c = \frac{u'}{2a} = \frac{2\times10^8}{2\times7.2\times10^{-2}} = 1.389 \text{ GHz}$$

Let $F = \sqrt{1-\left(\frac{f_c}{f}\right)^2} = \sqrt{1-\left(\frac{1.389}{6.2}\right)^2} = 0.9746$

$$\beta = \omega\sqrt{\mu\varepsilon}F = \frac{\omega F\sqrt{\varepsilon_r}}{c} = \frac{2\pi\times6.2\times10^9\times0.9746\times1.5}{3\times10^8} = \underline{\underline{189.83 \text{ rad/m}}}$$

$$u_p = \frac{\omega}{\beta} = \frac{2\pi\times6.2\times10^9}{189.83} = \underline{\underline{2.052\times10^8 \text{ m/s}}}$$

$$u_g = u'F = 2\times10^8(0.9746) = \underline{\underline{1.949\times10^8 \text{ m/s}}}$$

$$\eta_{TE} = \frac{\eta'}{F} = \frac{377}{1.5\times0.9746} = \underline{\underline{257.88\ \Omega}}$$

Prob. 12.22

$$f_{c10} = \frac{u'}{2a} = \frac{\frac{1}{\sqrt{\mu\varepsilon}}}{2a} = \frac{c}{2a\sqrt{\varepsilon_r}} = \frac{3\times10^8}{2\times7.214\times10^{-2}\sqrt{2.5}} = 1.315 \text{ GHz}$$

$$\text{Let} \quad F = \sqrt{1-\left(\frac{f_c}{f}\right)^2} = \sqrt{1-\left(\frac{1.315}{4}\right)^2} = 0.9444$$

$$\beta = \omega\sqrt{\mu\varepsilon}F = \frac{\omega\sqrt{\varepsilon_r}F}{c}$$

$$u_p = \frac{\omega}{\beta} = \frac{c}{F\sqrt{\varepsilon_r}} = \frac{3\times10^8}{0.9444\times\sqrt{2.5}} = \underline{\underline{2.009\times10^8 \text{ m/s}}}$$

$$u_g = u'F = \frac{cF}{\sqrt{\varepsilon_r}} = \frac{3\times10^8\times0.9444}{\sqrt{2.5}} = \underline{\underline{1.792\times10^8 \text{ m/s}}}$$

Prob. 12.23

$$u_p = \frac{\omega}{\beta} = \frac{\omega\lambda}{2\pi} \quad \rightarrow \quad \omega = \beta u_p = \beta c\frac{\lambda_o^2}{\lambda^2} = \frac{c\lambda_o^2\beta^3}{4\pi^2}, \qquad (\lambda = 2\pi/\beta)$$

$$u_g = \frac{d\omega}{d\beta} = 3\left(\frac{c\lambda_o^2}{4\pi^2}\right)\beta^2 = 3c\left(\frac{\lambda_o}{\lambda}\right)^2 = \underline{\underline{3u_p}}$$

Prob. 12.24

$$u_g = \frac{1}{4}c = u'\sqrt{1-\left(\frac{f_c}{f_1}\right)^2} \qquad (1)$$

$$u_g = \frac{1}{3}c = u'\sqrt{1-\left(\frac{f_c}{f_2}\right)^2} \qquad (2)$$

Dividing (1) by (2),

$$\frac{1/4}{1/3} = \frac{\sqrt{1-\left(\frac{f_c}{f_1}\right)^2}}{\sqrt{1-\left(\frac{f_c}{f_2}\right)^2}} \quad \rightarrow \quad \left(\frac{3}{4}\right)^2 = 0.5625 = \frac{1-\left(\frac{f_c}{f_1}\right)^2}{1-\left(\frac{f_c}{f_2}\right)^2}$$

$$1-\left(\frac{f_c}{f_1}\right)^2 = 0.5625\left[1-\left(\frac{f_c}{f_2}\right)^2\right]$$

Assuming f_c is in GHz,

$$1-\frac{f_c^2}{144} = 0.5625 - \frac{0.5625 f_c^2}{225} \quad \rightarrow \quad f_c^2 = 98.44 \quad \rightarrow \quad \underline{\underline{f_c = 9.9216 \text{ GHz}}}$$

From (1),

$$u' = \frac{0.25c}{\sqrt{1-\left(\frac{f_c}{f_1}\right)^2}} = \frac{0.25c}{\sqrt{1-\left(\frac{9.9216}{12}\right)^2}} = 0.4444c$$

But $\quad u' = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\varepsilon_r}}$

$$\frac{c}{\sqrt{\varepsilon_r}} = 0.4444c \quad \rightarrow \quad \varepsilon_r = \left(\frac{1}{0.4444}\right)^2 = \underline{\underline{5.0625}}$$

Prob. 12.25

$$f_c = \frac{u'}{2a}$$

$$u_g = u'\sqrt{1-\left(\frac{f_c}{f}\right)^2} \quad \longrightarrow \quad \left(\frac{f_c}{f}\right)^2 = 1-\left(\frac{u_g}{u'}\right)^2 = 1-\left(\frac{1.8\times10^8}{\frac{3\times10^8}{\sqrt{2.2}}}\right)^2 = 0.208$$

$$f_c = \sqrt{0.208}\, f = 2.0523 \text{ GHz}$$

$$a = \frac{u'}{2f_c} = \frac{3\times10^8}{2\sqrt{2.2}\times2.053\times10^9} = \underline{\underline{4.927 \text{ cm}}}$$

Prob. 12.26

Let $F = \sqrt{1-(f_c/f)^2} = \sqrt{1-(16/24)^2} = 0.7453$

$$u' = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{3\times10^8}{\sqrt{2.25}} = 2\times10^8, \qquad u_p = \frac{u'}{F}, \qquad u_g = u'F = 2\times10^8\times0.7453 = \underline{\underline{1.491\times10^8}}$$

m/s

$$\eta_{TE} = \eta'/F = \frac{377}{1.5x0.7453} = \underline{\underline{337.2\Omega}}$$

Prob. 12.27

For the TE_{10} mode,

$$H_{zs} = H_o\cos\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

$$H_{xs} = \frac{j\beta a}{\pi}H_o\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

$$E_{ys} = -\frac{j\omega\mu a}{\pi}H_o\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}$$

$$E_{xs} = 0 = E_{zs} = H_{ys}$$

$$\boldsymbol{E}_s\times\boldsymbol{H}_s^* = \begin{vmatrix} 0 & E_{ys} & 0 \\ H_{xs}^* & 0 & H_{zs}^* \end{vmatrix} = E_{ys}H_{zs}^*\boldsymbol{a}_x - E_{ys}H_{xs}^*\boldsymbol{a}_z$$

$$= -\frac{j\omega\mu a}{\pi}H_o^2\cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi x}{a}\right)\boldsymbol{a}_x + \frac{\omega\mu\beta a^2}{\pi^2}H_o^2\sin^2\left(\frac{\pi x}{a}\right)\boldsymbol{a}_z$$

$$\mathscr{P}_{ave} = \frac{1}{2}\mathrm{Re}\left[\boldsymbol{E}_s\times\boldsymbol{H}_s^*\right] = \underline{\underline{\frac{\omega\mu\beta a^2}{2\pi^2}H_o^2\sin^2\left(\frac{\pi x}{a}\right)\boldsymbol{a}_z}}$$

Prob. 12.28

$$\boldsymbol{P}_{\text{ave}} = \frac{|E_{xs}|^2+|E_{ys}|^2}{2\eta}\boldsymbol{a}_z = \underline{\underline{\frac{\omega^2\mu^2\pi^2}{2\eta b^2h^4}H_o{}^2\sin^2\pi y/b\boldsymbol{a}_z}}$$

where $\eta = \eta_{TE10}$.

$$\mathrm{P}_{\text{ave}} = \int \boldsymbol{P}_{\text{ave}}.d\boldsymbol{S} = \frac{\omega^2\mu^2\pi^2}{2\eta b^2h^4}H_o{}^2\int_{x=0}^{a}\int_{y=0}^{b}\sin^2\pi y/b\,dxdy$$

$$P_{ave} = \frac{\omega^2 \mu^2 \pi^2}{2\eta b^2 h^4} H_o^2 ab/2$$

But $h^2 = (m\pi/a)^2 + (n\pi/b)^2 = \dfrac{\pi^2}{b^2}$,

$$P_{ave} = \frac{\omega^2 \mu^2 a b^3 H_o^2}{4\pi^2 \eta}$$

Prob. 12.29

$$R_s = \sqrt{\frac{\pi \mu f}{\sigma_c}} = \sqrt{\frac{\pi \times 12\times 10^9 \times 4\pi \times 10^{-7}}{5.8\times 10^7}} = 2.858\times 10^{-2}$$

$$f_{c10} = \frac{u'}{2a} = \frac{3\times 10^8}{2\sqrt{2.6}\times 2\times 10^{-2}} = 4.651 \text{ GHz}$$

$$f_{c11} = \frac{u'}{2}\left[\frac{1}{a^2} + \frac{1}{b^2}\right]^{1/2} = 10.4 \text{ GHz}$$

$$\eta' = \sqrt{\frac{\mu}{\varepsilon}} = \frac{377}{\sqrt{2.6}} = 233.81\Omega$$

(a) For TE_{10} mode, eq.(12.57) gives

$$\alpha_d + j\beta_d = \sqrt{-\omega^2 \mu\varepsilon + k_x^2 + k_y^2 + j\omega\mu\sigma_d}$$

$$= \sqrt{-\omega^2/u^2 + \frac{\pi^2}{a^2} + j\omega\mu\sigma_d}$$

$$= \sqrt{-\left(\frac{2\pi\times 12\times 10^9}{3\times 10^8}\right)^2 (2.6) + \frac{\pi^2}{(2\times 10^{-2})^2} + j2\pi\times 12\times 10^9 \times 4\pi \times 10^{-7} \times 10^{-4}}$$

$= 0.012682 + j373.57$

$\alpha_d = 0.012682$ Np/m

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1-(f_c/f)^2}}\left[\frac{1}{2} + \frac{b}{a}\left(\frac{f_c}{f}\right)^2\right]$$

$$= \frac{2x2.858x10^{-2}}{10^{-2}(233.81)\sqrt{1-(4.651\ /12)^2}}\left[\frac{1}{2}+\frac{1}{2}(\frac{4.651}{12})^2\right] = \underline{\underline{0.0153}}\ \text{Np/m}$$

(b) For TE_{11} mode,

$$\alpha_d + j\beta_d = \sqrt{-\omega^2/u^2 + 1/a^2 + 1/b^2 + j\omega\mu\sigma_d}$$

$$= \sqrt{-139556.21 + \frac{\pi^2}{(10^{-2})^2} + j9.4748} = 0.02344 + j202.14$$

$\underline{\underline{\alpha_d = 0.02344}}$ Np/m

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1-(f_c/f)^2}}\left[\frac{(b/a)^3+1}{(b/a)^2+1}\right] = \frac{2\times 2.858\times 10^{-2}}{10^{-2}(233.81)\sqrt{1-(10.4/12)^2}}\left[\frac{(1/8)+1}{(1/4)\ +1}\right]$$

$\underline{\underline{\alpha_c = 0.0441}}$ Np/m

Prob. 12.30 $\varepsilon_c = \varepsilon' - j\varepsilon'' = \varepsilon - j\dfrac{\sigma}{\omega}$

Comparing this with

$$\varepsilon_c = 16\varepsilon_o(1-j10^{-4}) = 16\varepsilon_o - j16\varepsilon_o\times 10^{-4}$$

$$\varepsilon = 16\varepsilon_o, \qquad \frac{\sigma}{\omega} = 16\varepsilon_o x10^{-4}$$

For TM_{21} mode,

$$f_c = \frac{u'}{2}\left[\frac{m^2}{a^2}+\frac{n^2}{b^2}\right]^{1/2} = 2.0963\ \text{GHz}, \quad f = 1.1 f_c = 2.3059\ \text{GHz}$$

$$\sigma = 16\varepsilon_o\omega\times 10^{-4} = 16\times 2\pi\times 2.3059\times 10^9\times\frac{10^{-9}}{36\pi}\times 10^{-4} = 2.0525\times 10^{-4}$$

$$\eta' = \sqrt{\frac{\mu}{\varepsilon}} = 30\pi\ \Omega$$

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1-(f_c/f)^2}} = \frac{4.1\times 10^{-4}\times 30\pi}{2\sqrt{1-1/1.12}} = \underline{\underline{0.0231}}\ \text{Np/m}$$

$$E_o e^{-\alpha_d z} = 0.8E_o \longrightarrow z = \frac{1}{\alpha_d}\ln(1/0.8) = \underline{\underline{9.66\ \text{m}}}$$

Prob. 12.31

For TM_{21} mode,

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1-(f_c/f)^2}}$$

$$R_s = \frac{1}{\sigma_c\delta} = \sqrt{\frac{\pi f\mu}{\sigma_c}} = \sqrt{\frac{\pi\times 2.3059\times 10^9\times 4\pi\times 10^{-7}}{1.5\times 10^7}} = 0.0246\ \Omega$$

$$\alpha_c = \frac{2\times 0.0246}{4\pi\times 10^{-2}\times 30\pi\times 0.4166} = 0.0314\ \text{Np/m}$$

$$E_o e^{-(\alpha_c+\alpha_d)z} = 0.7E_o \longrightarrow z = \frac{1}{\alpha_c+\alpha_d}\ln(1/0.7) = \underline{\underline{6.5445\ \text{m}}}$$

Prob. 12.32

$$f_{c10} = \frac{u'}{2a} = \frac{3\times 10^8}{2\times 6\times 10^{-2}} = 2.5\ \text{GHz}$$

$$\eta_{TE} = \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{2.5}{4}\right)^2}} = 483\ \Omega$$

From Example 12.5,

$$P_{ave} = \frac{E_o^2 ab}{2\eta} = \frac{(2.2)^2\times 10^6\times 6\times 3\times 10^{-4}}{2\times 483} = \underline{\underline{9.0196\ \text{mW}}}$$

Prob. 12.33

For TE_{10} mode,

$$f_c = \frac{u'}{2a} = \frac{3\times 10^8}{2\sqrt{2.11}\times 4.8\times 10^{-2}} = 2.151\ \text{GHz}$$

(a) loss tangent $= \dfrac{\sigma}{\omega\varepsilon} = d$

$$\sigma = d\omega\varepsilon = 3\times 10^{-4}\times 2\pi\times 4\times 10^9\times 2.11\times \frac{10^{-9}}{36\pi} = 1.4086\times 10^{-4}$$

$$\eta' = \frac{120\pi}{\sqrt{2.11}} = 259.53\ \Omega$$

$$\alpha_d = \frac{\sigma\eta'}{2\sqrt{1-(f_c/f)^2}} = \frac{1.4067\times10^{-4}\times259.53}{2\sqrt{1-(2.151/4)^2}} = \underline{\underline{2.165\times10^{-2}\text{ Np/m}}}$$

(b) $$R_s = \sqrt{\frac{\mu f \pi}{\sigma_c}} = \sqrt{\frac{\pi\times4\times10^9\times4\pi\times10^{-7}}{4.1\times10^7}} = 1.9625\times10^{-2}\ \Omega$$

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1-(f_c/f)^2}}\left[\frac{1}{2}+\frac{b}{a}(f_c/f)^2\right] = \frac{3.925\times10^{-2}(0.5+0.5\times0.2892)}{2.4\times10^{-2}\times259.53\times0.8431}$$

$$= \underline{\underline{4.818\times10^{-3}\text{ Np/m}}}$$

Prob.12.34

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[\frac{1}{2}+\frac{b}{a}\left(\frac{f_c}{f}\right)^2\right] = \frac{2\sqrt{\frac{\pi f\mu}{\sigma_c}}}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[\frac{1}{2}+\frac{1}{2}\left(\frac{f_c}{f}\right)^2\right]$$

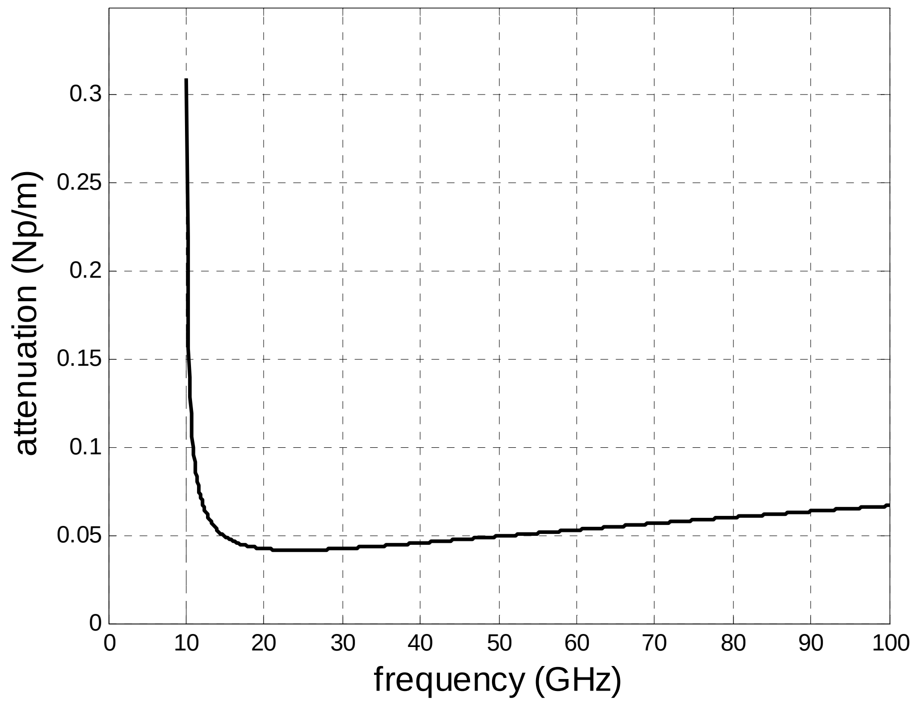
$$= \frac{2\sqrt{4\pi\times10^{-7}\times\pi}\sqrt{f}\times\frac{1}{2}}{0.5\times10^{-2}\times(120\pi/\sqrt{2.25})\sqrt{5.8\times10^7}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[1+\left(\frac{f_c}{f}\right)^2\right]$$

$$= \frac{10^{-5}\sqrt{f}}{30\sqrt{(5.8/2.25)}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[1+\left(\frac{f_c}{f}\right)^2\right]$$

The MATLAB code is shown below

```
k=10^(-5)/(30*sqrt(5.8/2.25));
fc=10^10;
for  n=1:1000
   f(n)=fc*(n/100+1);
   fn=f(n);
   num=sqrt(fn)*(1 +(fc/fn)^2 );
   den=sqrt(1- (fc/fn)^2 );
   alpha(n) =k*num/den;
end
plot(f/10^9,alpha)
xlabel('frequency (GHz)')
ylabel('attenuation')
grid
```

The plot of attenuation versus frequency is shown below.



Prob. 12.35

The cutoff frequency of the dominant mode is

$$f_{c10} = \frac{u}{2a} = \frac{3\times10^8}{4.576\times10^{-2}} = 6.56 \text{ GHz}$$

The surface resistance is

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma_c}} = \sqrt{\frac{\pi\times8.4\times10^9\times4\pi\times10^{-7}}{5.8\times10^7}} = 23.91\times10^{-3}\ \Omega$$

For TE_{10} mode,

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[0.5+\frac{b}{a}\left(\frac{f_c}{f}\right)^2\right]$$

$$\frac{f_c}{f} = \frac{6.56}{8.4} = 0.781, \qquad \eta' = \eta_o = 377$$

$$\alpha_c = \frac{2\times 23.91\times 10^{-3}}{1.016\times 10^{-2}\times 377\sqrt{1-0.781^2}}\left[0.5+\frac{1.016}{2.286}(0.781)^2\right]$$

$$= \frac{47.82\times 10^{-3}(0.5+0.2711)}{3.83\times 0.6245} = 15.42\times 10^{-3} \text{ Np/m}$$

$$= 15.42\times 10^{-3}\times 8.686 \text{ dB/m} = \underline{\underline{0.1339 \text{ dB/m}}}$$

Prob. 12.36

(a) $f_{c10} = \dfrac{u'}{2a} = \dfrac{3\times 10^8}{2\times 3.8\times 10^{-2}} = 3.947$ GHz

$$u_g = u'\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 3\times 10^8\sqrt{1-(0.3947)^2} = \underline{\underline{2.756\times 10^8 \text{ m/s}}}$$

(b) $\alpha = \alpha_d + \alpha_c$

$\alpha_d = 0$ since the guide is air-filled.

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma_c}} = \sqrt{\frac{\pi\times 10^{10}\times 4\pi\times 10^{-7}}{5.8\times 10^7}} = 2.609\times 10^{-2}\,\Omega$$

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[0.5+\frac{b}{a}\left(\frac{f_c}{f}\right)^2\right]$$

$$= \frac{2\times 2.609\times 10^{-2}}{1.6\times 10^{-2}(377)\sqrt{1-(0.3947)^2}}\left[0.5+\frac{1.6}{3.8}(0.3947)^2\right] = \frac{5.218\times 0.5656}{554.23}$$

$$\underline{\underline{= 5.325\times 10^{-3} \text{ Np/m}}}$$

$$\alpha_c(dB) = 8.686\times 5.325\times 10^{-3} = 0.04626 \text{ dB/m}$$

Prob.12.37

$$f_{c10} = \frac{u'}{2a} = \frac{c}{2a\sqrt{\varepsilon_r\mu_r}} = \frac{3\times 10^8}{2\times 2.5\times 10^{-2}\sqrt{2.26}} = 3.991 \text{ GHz}$$

$$\beta' = \frac{\omega}{u'} = \frac{2\pi f\sqrt{\varepsilon_r}}{c}$$

$$F = \sqrt{1-\left(\frac{f_c}{f}\right)^2} = \sqrt{1-\left(\frac{3.991}{7.5}\right)^2} = 0.8467$$

$$\beta = \beta' F = \frac{2\pi\times 7.5\times 10^9\sqrt{2.26}}{3\times 10^8}0.8467 = \underline{\underline{199.94 \text{ rad/m}}}$$

$$\alpha_d = \frac{\sigma\eta'}{2F} = \frac{\sigma\eta_o}{2F\sqrt{\varepsilon_r}} = \frac{10^{-4}(377)}{2\times 0.8467\sqrt{2.26}} = \underline{\underline{1.481\times 10^{-2}\ \text{Np/m}}}$$

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}\left[0.5+\frac{b}{a}\left(\frac{f_c}{f}\right)^2\right]$$

$$R_s = \sqrt{\frac{\pi f\mu}{\sigma_c}} = \sqrt{\frac{\pi\times 7.5\times 10^9\times 4\pi\times 10^{-7}}{1.1\times 10^7}} = 0.0519\ \Omega$$

$$\alpha_c = \frac{2\times 0.0519\left[0.5+\frac{1.5}{2.5}\left(\frac{3.991}{7.5}\right)^2\right]}{1.5\times 10^{-2}\times\frac{377}{\sqrt{2.66}}\times 0.8467} = \frac{0.1038\times 0.6698}{3.1848}$$

$$\underline{\underline{= 0.02183\ \text{Np/m}}}$$

$$u_p = \frac{u'}{F} = \frac{c}{F\sqrt{\varepsilon_r}} = \frac{3\times 10^8}{0.8467\sqrt{2.26}} = \underline{\underline{2.357\times 10^8\ \text{m/s}}}$$

$$u_g = u'F = \frac{3\times 10^8\times 0.8467}{\sqrt{2.26}} = \underline{\underline{1.689\times 10^8\ \text{m/s}}}$$

$$\lambda_c = \frac{u'}{f_c} = \frac{c}{f_c\sqrt{\varepsilon_r}} = \frac{3\times 10^8}{3.991\times 10^9\sqrt{2.26}} = 0.05\ \text{m} = \underline{\underline{5\ \text{cm}}}(=2a,\ \text{as expected})$$

Prob. 12.38 (a) For TE_{10} mode,

$$f_c = \frac{u'}{2a}, \qquad u' = \frac{c}{\sqrt{2.11}}$$

$$f_c = \frac{3\times 10^8}{\sqrt{2.11}(2\times 2.25\times 10^{-2})} = \underline{\underline{4.589\ \text{GHz}}}$$

(b)
$$\alpha_{cTE10} = \frac{2R_s}{b\eta'\sqrt{1-(f_c/f)^2}}\left[\frac{1}{2}+\frac{b}{a}(f_c/f)^2\right]$$

$$R_s = \sqrt{\frac{\pi f\mu}{\sigma_c}} = \sqrt{\frac{\pi\times 5\times 10^9\times 4\pi\times 10^{-7}}{1.37\times 10^7}} = 3.796\times 10^{-2}\ \Omega$$

$$\eta' = \frac{377}{\sqrt{2.11}} = 259.54\ \Omega$$

$$\alpha_c = \frac{2\times 3.796\times 10^{-2}[0.5+\frac{1.5}{2.25}(4.589/5)^2]}{1.5\times 10^{-4}(259.54)\sqrt{1-(4.589/5)^2}} = \underline{\underline{0.05217}} \text{ Np/m}$$

Prob. 12.39 For TE_{10} mode,

$$\alpha_c = \frac{2R_s}{b\eta'\sqrt{1-(f_c/f)^2}}\left[\frac{1}{2}+\frac{b}{a}\left(\frac{f_c}{f}\right)^2\right]$$

But a = b, $R_s = \frac{1}{\sigma_c\delta} = \sqrt{\frac{\pi f\mu}{\sigma_c}}$

$$\alpha_c = \frac{2\sqrt{\frac{\pi f\mu}{\sigma_c}}}{a\eta'\sqrt{1-(f_c/f)^2}}\left[\frac{1}{2}+\left(\frac{f_c}{f}\right)^2\right] = \frac{k\sqrt{f}\left[\frac{1}{2}+\left(\frac{f_c}{f}\right)^2\right]}{\sqrt{1-(f_c/f)^2}}$$

where k is a constant.

$$\frac{d\alpha_c}{df} = \frac{k[1-(\frac{f_c}{f})^2]^{1/2}[\frac{1}{4}f^{-1/2}-\frac{3}{2}f_c^2f^{-5/2}]-\frac{k}{2}[\frac{1}{2}f^{1/2}+f_c^2f^{-3/2}](2f_c^2f^{-3})[1-(\frac{f_c}{f})^2]^{-1/2}}{1-(f_c/f)^2}$$

For minimum value, $\frac{d\alpha_c}{df} = 0$. This leads to $\underline{\underline{f = 2.962 f_c}}$.

Prob. 12.40 For the TE mode to z,

$$E_{zs} = 0, H_{zs} = H_o\cos(m\pi x/a)\cos(n\pi y/b)\sin(p\pi z/c)$$

$$E_{ys} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x} = \underline{\underline{-\frac{j\omega\mu}{h^2}(m\pi/a)H_o\sin(m\pi x/a)\cos(n\pi y/b)\sin(p\pi z/c)}}$$

as required.

$$E_{xs} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y} = \frac{j\omega\mu}{h^2}(n\pi/b)H_o\cos(m\pi x/a)\sin(n\pi y/b)\sin(p\pi z/c)$$

From Maxwell's equation,

$$-j\omega\mu \boldsymbol{H}_s = \nabla \times \boldsymbol{E}_s = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{xs} & E_{ys} & 0 \end{vmatrix}$$

$$H_{xs} = \frac{1}{j\omega\mu}\frac{\partial E_{ys}}{\partial z} = -\frac{1}{h^2}(m\pi/a)(p\pi/c)H_o \sin(m\pi x/a)\cos(n\pi y/b)\cos(p\pi z/c)$$

Prob. 12.41 Maxwell's equation can be written as

$$H_{xs} = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_{zs}}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial x}$$

For a rectangular cavity,

$$h^2 = k_x^{\,2} + k_y^{\,2} = (m\pi/a)^2 + (n\pi/b)^2$$

For TM mode, $H_{zs} = 0$ and

$$E_{zs} = E_o \sin(m\pi x/a)\sin(n\pi y/b)\cos(p\pi z/c)$$

Thus

$$H_{xs} = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_{zs}}{\partial y} = \frac{j\omega\varepsilon}{h^2}(n\pi/b)E_o \sin(m\pi x/a)\cos(n\pi y/b)\cos(p\pi z/c)$$

as required.

$$H_{xs} = -\frac{j\omega\varepsilon}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial y}$$

$$= -\frac{j\omega\varepsilon}{h^2}(m\pi/a)E_o \cos(m\pi x/a)\sin(n\pi y/b)\cos(p\pi z/c)$$

From Maxwell's equation,

$$j\omega\varepsilon \boldsymbol{E}_s = \nabla \times \boldsymbol{H}_s = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_{xs} & H_{ys} & 0 \end{vmatrix}$$

$$E_{ys} = \frac{1}{j\omega\varepsilon}\frac{\partial H_{xs}}{\partial z} = \frac{1}{h^2}(n\pi/b)(p\pi/c)E_o \sin(m\pi x/a)\cos(n\pi y/b)\cos(p\pi z/c)$$

Prob. 12.42

$$f_r = \frac{u'}{2}\sqrt{(m/a)^2 + (n/b)^2 + (p/c)^2}$$

where for TM mode to z, m = 1, 2, 3,…, n=1, 2, 3, …., p = 0, 1, 2, ….

and for TE mode to z, m = 0,1, 2, 3,…, n=0,1, 2, 3, …., p = 1, 2, 3, … , $(m+n) \neq 0$.

(a) If a < b < c, 1/a > 1/b > 1/c,

The lowest TM mode is TM_{110} with $f_r = \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

The lowest TE mode is TE_{011} with $f_r = \frac{u'}{2}\sqrt{\frac{1}{b^2} + \frac{1}{c^2}} < \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

Hence the dominant mode is TE_{011}.

(b) If a > b > c, 1/a < 1/b < 1/c,

The lowest TM mode is TM_{110} with $f_r = \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

The lowest TE mode is TE_{101} with $f_r = \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{c^2}} > \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

Hence the dominant mode is TM_{110}.

(c) If a = c > b, 1/a = 1/c < 1/b,

The lowest TM mode is TM_{110} with $f_r = \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

The lowest TE mode is TE_{101} with $f_r = \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{c^2}} < \frac{u'}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

Hence the dominant mode is TE_{101}.

Prob. 12.43

(*a*) $u' = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times 10^8}{\sqrt{4.6}}$

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

For the dominant mode, m = 1, n=0, p=1

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2} = \frac{3\times 10^8}{2\sqrt{4.6}}\sqrt{\frac{1}{9\times 10^{-4}} + \frac{1}{36\times 10^{-4}}} = \frac{3\times 10^{10}}{2(2.1447)}(0.37267) = \underline{\underline{2.606 \text{ GHz}}}$$

(*b*) $Q = \frac{(a^2 + c^2)abc}{\delta\left[2b(a^3 + c^3) + ac(a^2 + c^2)\right]}$

$$\delta = \frac{1}{\sqrt{\pi f_{r101}\sigma\mu_o}} = \frac{1}{\sqrt{\pi \times 2.606\times 10^9 \times 1.57\times 10^7 \times 4\pi\times 10^{-7}}} = 2.49\times 10^{-6} \text{ m}$$

$$Q = \frac{(9+36)(72)\times 10^{-2}}{\delta\left[8(27+216) + 18(9+36)\right]} = \frac{32.42}{2.49\times 10^{-6}(2754)} = \underline{\underline{4727.7}}$$

Prob. 12.44

(*a*)

$$f_r = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$f_{rTE101} = 1.5\times 10^{10}\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2} = 1.5\times 10^{10}\sqrt{0.1736} = \underline{\underline{6.25 \text{ GHz}}}$$

$$f_{rTE011} = 1.5\times 10^{10}\sqrt{\frac{1}{6.25} + \frac{1}{16}} = 1.5\times 10^{10}\sqrt{0.2225} = \underline{\underline{7.075 \text{ GHz}}}$$

$$f_{rTE110} = 1.5\times 10^{10}\sqrt{\frac{1}{9} + \frac{1}{6.25}} = 1.5\times 10^{10}\sqrt{0.2711} = \underline{\underline{7.81 \text{ GHz}}}$$

Prob. 12.45

$$u' = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{2.5}} = \frac{3\times10^8}{\sqrt{2.5}} = 1.897\times10^8$$

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2} = \frac{1.897\times10^8\times10^2}{2}\sqrt{\left(\frac{m}{1}\right)^2 + \left(\frac{n}{2}\right)^2 + \left(\frac{p}{3}\right)^2}$$

$$= 9.485\sqrt{m^2 + 0.25n^2 + 0.111p^2} \quad \text{GHz}$$

$$f_{r101} = 9.485\sqrt{1+0+0.111} = 10 \text{ GHz}$$

$$f_{r011} = 9.485\sqrt{0+0.25+0.111} = 5.701 \text{ GHz}$$

$$f_{r012} = 9.485\sqrt{0+0.25+0.444} = 7.906 \text{ GHz}$$

$$f_{r013} = 9.485\sqrt{0+0.25+0.999} = 10.61 \text{ GHz}$$

$$f_{r021} = 9.485\sqrt{0+1+0.111} = 10 \text{ GHz}$$

Thus, the first five resonant frequencies are:

5.701 GHz(TE_{011})

7.906 GHz (TE_{012})

10 GHz (TE_{101} and TE_{021})

10.61 GHz (TE_{013} or TM_{110})

11.07 GHz (TE_{111} or TM_{111})

Prob. 12.46

$$Q = \frac{(a^2+c^2)abc}{\delta\left[2b(a^3+c^3)+ac(a^2+c^2)\right]}$$

When a = b = c,

$$Q = \frac{2a^2a^3}{\delta\left[2a\times2a^3+a^2\times2a^2\right]} = \frac{2a^5}{6\delta a^4} = \frac{a}{3\delta}$$

Prob. 12.47

(a) Since a > b < c, the dominant mode is TE_{101}

$$f_{r101} = \frac{u'}{2}\sqrt{\frac{1}{a^2}+0+\frac{1}{c^2}} = \frac{3\times10^8\times10^2}{2}\sqrt{\frac{1}{2^2}+\frac{1}{1^2}} = \underline{16.77 \text{ GHz}}$$

(b) $$Q_{TE101} = \frac{(a^2 + c^2)abc}{\delta\left[2b(a^3 + c^3) + ac(a^2 + c^2)\right]}$$

$$= \frac{(400+100)20\times 8\times 10\times 10^{-3}}{\delta[16(8000+1000)+200(400+100)]} = \frac{3.279\times 10^{-3}}{\delta}$$

But $$\delta = \frac{1}{\sqrt{\pi f_{r101}\mu_o\sigma}} = \frac{1}{\sqrt{\pi 16.77\times 10^9 \times 4\pi\times 10^{-7}\times 6.1\times 10^7}} = \frac{10^{-4}}{200.961} \text{ m}$$

$$Q_{TE101} = 3.279\times 10^{-3}\frac{200.961}{10^{-4}} = \underline{\underline{6589.51}}$$

Prob. 12.48

$$f_r = \frac{c}{2a}\sqrt{m^2 + n^2 + p^2}$$

The lowest possible modes are TE_{101}, TE_{011}, and TM_{110}. Hence

$$f_r = \frac{c}{2a}\sqrt{2} \longrightarrow a = \frac{c}{f_r\sqrt{2}} = \frac{3\times 10^8}{\sqrt{2}\times 3\times 10^9} = 7.071 \text{ cm}$$

$\underline{\underline{a = b = c = 7.071 \text{ cm}}}$

Prob. 12.49

(a) a = b = c

$$f_r = \frac{u'}{2a}\sqrt{m^2 + n^2 + p^2}$$

For the dominant mode TE_{101},

$$f_r = \frac{u'}{2a}\sqrt{1+1} = \frac{c}{2a}\sqrt{2}$$

$$a = \frac{c\sqrt{2}}{2f_r} = \frac{3\times 10^8\sqrt{2}}{2\times 5.6\times 10^9} = 0.03788 \text{ m}$$

$\underline{\underline{a = b = c = 3.788 \text{ cm}}}$

(b)

For $\varepsilon_r = 2.05$, $u' = \frac{c}{\sqrt{\varepsilon_r}}$

$$a = \frac{c\sqrt{2}}{2f_r\sqrt{\varepsilon_r}} = \frac{0.03788}{\sqrt{2.05}} = 0.02646$$

$\underline{\underline{a = b = c = 2.646 \text{ cm}}}$

Prob. 12.50

(a)
This is a TM mode to z. From Maxwell's equations,

$$\nabla \times \boldsymbol{E}_s = -j\omega\mu \boldsymbol{H}_s$$

$$\boldsymbol{H}_s = -\frac{1}{j\omega\mu}\nabla \times \boldsymbol{E}_s = \frac{j}{\omega\mu}\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_{zs}(x,y) \end{vmatrix} = \frac{j}{\omega\mu}\left(\frac{\partial E_{zs}}{\partial y}\boldsymbol{a}_x - \frac{\partial E_{zs}}{\partial x}\boldsymbol{a}_y\right)$$

But

$$E_{zs} = 200\sin 30\pi x \sin 30\pi y, \quad \frac{1}{\omega\mu} = \frac{1}{6\times 10^9 \times 4\pi \times 10^{-7}} = \frac{10^{-2}}{24\pi}$$

$$\boldsymbol{H}_s = \frac{j10^{-2}}{24\pi}\times 200 \times 30\pi\left\{\sin 30\pi x \cos 30\pi y \boldsymbol{a}_x - \cos 30\pi x \sin 30\pi y \boldsymbol{a}_y\right\}$$

$$\mathbf{H} = \mathrm{Re}\,(\mathbf{H}_s\, e^{j\omega t})$$

$$\underline{\underline{\boldsymbol{H} = 2.5\left\{-\sin 30\pi x \cos 30\pi y \boldsymbol{a}_x + \cos 30\pi x \sin 30\pi y \boldsymbol{a}_y\right\}\sin 6\times 10^9 \pi t}} \quad \text{A/m}$$

(*b*)

$$\boldsymbol{E} = E_z \boldsymbol{a}_z, \qquad \boldsymbol{H} = H_x \boldsymbol{a}_x + H_y \boldsymbol{a}_y$$

$$\boldsymbol{E} \cdot \boldsymbol{H} = 0$$

Prob. 12.51

(a) $a = b = c \longrightarrow f_{r101} = \frac{3\times 10^8}{a\sqrt{2}} = 12\times 10^9$

$$a = \frac{3\times 10^8}{\sqrt{2}\times 12\times 10^9} = \underline{\underline{1.77 \text{ cm}}}$$

(b) $Q_{TE101} = \frac{a}{3\delta} = \frac{a\sqrt{\pi f_{r101}\mu\sigma}}{3}$

$$= \frac{1.77\times 10^{-2}\sqrt{\pi \times 12\times 10^9 \times 4\pi \times 10^{-7} \times 5.8\times 10^7}}{3} = \underline{\underline{9767.61}}$$

Prob. 12.52

$$f_r = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$f_{r101} = \frac{3\times10^8}{2}\sqrt{\frac{1}{(10.2)^2} + \frac{1}{(3.6)^2}} = 44.186 \text{ MHz}$$

$$f_{r011} = 150\sqrt{\frac{1}{(8.7)^2} + \frac{1}{(3.6)^2}} \text{ MHz} = 45.093 \text{ MHz}$$

$$f_{r111} = 150\sqrt{\frac{1}{(10.2)^2} + \frac{1}{(8.7)^2} + \frac{1}{(3.6)^2}} \text{ MHz} = 47.43 \text{ MHz}$$

$$f_{r110} = 150\sqrt{\frac{1}{(10.2)^2} + \frac{1}{(8.7)^2}} \text{ MHz} = 22.66 \text{ MHz}$$

$$f_{r102} = 150\sqrt{\frac{1}{(10.2)^2} + \frac{4}{(3.6)^2}} \text{ MHz} = 84.62 \text{ MHz}$$

$$f_{r201} = 150\sqrt{\frac{4}{(10.2)^2} + \frac{1}{(3.6)^2}} \text{ MHz} = 51 \text{ MHz}$$

Thus, the resonant frequences below 50 MHz are

$\underline{\underline{f_{r110}, f_{r101}, f_{r011}, \text{ and } f_{r111}}}$

Prob. 12.53

$$n = c/u_m = \frac{3\times10^8}{2.1\times10^8} = \underline{\underline{1.4286}}$$

Prob. 12.54

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.51^2 - 1.45^2} = \sqrt{0.1776} = 0.421$$

Prob. 12.55

(a) $NA = \sqrt{n_1{}^2 - n_2{}^2} = \sqrt{1.62^2 - 1.604^2} = \underline{\underline{0.2271}}$

(b) $NA = \sin\theta_a = 0.2271$ or $\theta_a = \sin^{-1} 0.2271 = \underline{\underline{13.13^{\circ}}}$

(c) $V = \dfrac{\pi d}{\lambda} NA = \dfrac{\pi\times50\times10^{-6}\times0.2271}{1300\times10^{-9}} = 27.441$

$N = V^2/2$ $\underline{\underline{6 \text{ modes}}}$

Prob. 12.56

$$V = \frac{\pi d}{\lambda}\sqrt{n_1^2 - n_2^3} = \frac{\pi \times 2 \times 5 \times 10^{-6}}{1300 \times 10^{-9}}\sqrt{1.48^2 - 1.46^2} = 5.86$$

$$N = \frac{V^2}{2} = 17.17 \text{ or } \underline{\underline{17 \text{ modes}}}$$

Prob. 12.57

(a) NA = sin θ_a = $\sqrt{n_1{}^2 - n_2{}^2}$ = $\sqrt{1.53^2 - 1.45^2}$ = 0.4883

θ_a = $\sin^{-1} 0.4883$ = $\underline{\underline{29.23^o}}$

(b) P(l)/P(0) = $10^{-\alpha l/10}$ = $10^{-0.4X5/10}$ = 0.631

i.e. $\underline{\underline{63.1\ \%}}$

Prob. 12.58

$$P(\ell) = P(0)10^{-\alpha\ell/10} = 10 \times 10^{-0.5 \times 0.85/10} = \underline{\underline{9.0678 \text{ mW}}}$$

Prob. 12.59

As shown in Eq. (10.35), $\log_{10}$ P_1/P_2 = 0.434 ln P_1/P_2 ,

1 Np = 20 $\log_{10}$ e = 8.686 dB or 1 Np/km = 8.686 dB/km,

or 1Np/m = 8686 dB/km. Thus,

$$\alpha_{12} = \underline{\underline{8686\alpha_{10}}}$$

Prob. 12.60

$$\alpha\ell = 10\log_{10}\frac{P_{in}}{P_{out}} = 10\log_{10}\frac{1.2 \times 10^{-3}}{1 \times 10^{-6}} = 30.792$$

$$\alpha = 0.4 \text{ dB/km} = \frac{0.4}{8.686} \text{ Np/km}$$

$$\ell = \frac{30.792}{\alpha} = \frac{30.392 \text{ dB}}{0.4 \text{ dB/km}} = \underline{\underline{76.98 \text{ km}}}$$

Prob. 12.61

P(0) = P(l) $10^{\alpha\ \mathrm{l}/10}$ = 0.2 x $10^{0.4 \times 30/10}$ mW = $\underline{\underline{3.1698 \text{ mW}}}$

Prob. 12.62 See text.

CHAPTER 2

P. E. 2.1

(a) At P(1,3,5), x = 1, y = 3, z = 5,

$$\rho = \sqrt{x^2 + y^2} = \sqrt{10}, \quad z = 5, \quad \phi = \tan^{-1} y/x = \tan^{-1} 3 = 71.6^o$$

$$P(\rho, \phi, z) = P(\sqrt{10}, \tan^{-1} 3, 5) = \underline{\underline{P(3.162, 71.6^o, 5)}}$$

Spherical system:

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{35} = 5.916$$

$$\theta = \tan^{-1} \sqrt{x^2 + y^2}/z = \tan^{-1} \sqrt{10}/5 = \tan^{-1} 0.6325 = 32.31°$$

$$P(r, \theta, \varphi) = \underline{\underline{P(5.916, 32.31°, 71.57°)}}$$

At T(0,-4,3), x = 0 y = -4, z = 3;

$$\rho = \sqrt{x^2 + y^2} = 4, z = 3, \varphi = \tan^{-1} y/x = \tan^{-1} -4/0 = 270°$$

$$\underline{\underline{T(\rho, \varphi, z) = T(4, 270°, 3).}}$$

Spherical system:

$$r = \sqrt{x^2 + y^2 + z^2} = 5, \theta = \tan^{-1} \rho/z = \tan^{-1} 4/3 = 53.13°.$$

$$\underline{\underline{T(r, \theta, \varphi) = T(5, 53.13°, 270°).}}$$

At S(-3-4-10), x = -3, y = -4, z = -10;

$$\rho = \sqrt{x^2 + y^2} = 5, \phi = \tan^{-1}\left(\frac{-4}{-3}\right) = 233.1°$$

$$\underline{\underline{S(\rho, \phi, z) = S(5, 233.1, -10).}}$$

Spherical system:

$$r = \sqrt{x^2 + y^2 + z^2} = 5\sqrt{5} = 11.18.$$

$$\theta = \tan^{-1} \rho/z = \tan^{-1} \frac{5}{-10} = 153.43°;$$

$$\underline{\underline{S(r, \theta, \phi) = S(11.18, 153.43°, 233.1°).}}$$

(b) In Cylindrical system, $\rho = \sqrt{x^2 + y^2}$; $yz = z\rho \sin\phi$,

$$Q_x = \frac{\rho}{\sqrt{\rho^2 + z^2}}; \quad Q_y = 0; \quad Q_z = -\frac{z\rho \sin\phi}{\sqrt{\rho^2 + z^2}};$$

$$\begin{bmatrix} Q_\rho \\ Q_\phi \\ Q_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_x \\ 0 \\ Q_z \end{bmatrix};$$

$$Q_\rho = Q_x \cos\phi = \frac{\rho\cos\phi}{\sqrt{\rho^2 + z^2}}, \qquad Q_\phi = -Q_x \sin\phi = \frac{-\rho\sin\phi}{\sqrt{\rho^2 + z^2}}$$

Hence,

$$\mathbf{Q} = \frac{\rho}{\sqrt{\rho^2 + z^2}}(\cos\phi\,\boldsymbol{a}_\rho - \sin\phi\,\boldsymbol{a}_\phi - z\sin\phi\,\boldsymbol{a}_z).$$

In Spherical coordinates:

$$Q_x = \frac{r\sin\theta}{r} = \sin\theta;$$

$$Q_z = -r\sin\phi\sin\theta\, r\cos\theta\,\frac{1}{r} = -r\sin\theta\cos\theta\sin\phi.$$

$$\begin{bmatrix} Q_r \\ Q_\theta \\ Q_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} Q_x \\ 0 \\ Q_z \end{bmatrix};$$

$$Q_r = Q_x \sin\theta\cos\phi + Q_z\cos\theta = \sin^2\theta\cos\phi - r\sin\theta\cos^2\theta\sin\phi.$$

$$Q_\theta = Q_x\cos\theta\cos\phi - Q_z\sin\theta = \sin\theta\cos\theta\cos\phi + r\sin^2\theta\cos\theta\sin\phi.$$

$$Q_\phi = -Q_x\sin\phi = -\sin\theta\sin\phi.$$

$$\therefore\ \mathbf{Q} = \sin\theta\left(\sin\theta\cos\phi - r\cos^2\theta\sin\phi\right)\boldsymbol{a}_r + \sin\theta\cos\theta(\cos\phi + r\sin\theta\sin\phi)\boldsymbol{a}_\theta - \sin\theta\sin\phi\,\boldsymbol{a}_\phi\ .$$

At T :

$$\mathbf{Q}(x,y,z) = \frac{4}{5}\boldsymbol{a}_x + \frac{12}{5}\boldsymbol{a}_z = 0.8\boldsymbol{a}_x + 2.4\boldsymbol{a}_z;$$

$$\mathbf{Q}(\rho,\phi,z) = \frac{4}{5}(\cos 270°\,\boldsymbol{a}_\rho - \sin 270°\,\boldsymbol{a}_\phi - 3\sin 270°\,\boldsymbol{a}_z$$

$$= 0.8\boldsymbol{a}_\phi + 2.4\boldsymbol{a}_z;$$

$$\mathbf{Q}(r,\theta,\phi) = \frac{4}{5}(0 - \frac{45}{25}(-1))\boldsymbol{a}_r + \frac{4}{5}(\frac{3}{5})(0 + \frac{20}{5}(-1))\boldsymbol{a}_\theta - \frac{4}{5}(-1)\boldsymbol{a}_\phi$$

$$= \frac{36}{25}\boldsymbol{a}_r - \frac{48}{25}\boldsymbol{a}_\theta + \frac{4}{5}\boldsymbol{a}_\phi = 1.44\boldsymbol{a}_r - 1.92\boldsymbol{a}_\theta + 0.8\boldsymbol{a}_\phi;$$

Note, that the magnitude of vector Q = 2.53 in all 3 cases above.

P.E. 2.2 (a)

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho z\sin\phi \\ 3\rho\cos\phi \\ \rho\cos\phi\sin\phi \end{bmatrix}$$

$$\boldsymbol{A} = (\rho z\cos\phi\sin\phi - 3\rho\cos\phi\sin\phi)\boldsymbol{a}_x + (\rho z\sin^2\phi + 3\rho\cos^2\phi)\boldsymbol{a}_y + \rho\cos\phi\sin\phi\,\boldsymbol{a}_z.$$

But $\rho = \sqrt{x^2+y^2}$, $\tan\phi = \dfrac{y}{x}$, $\cos\phi = \dfrac{x}{\sqrt{x^2+y^2}}$, $\sin\phi = \dfrac{y}{\sqrt{x^2+y^2}}$;

Substituting all this yields:

$$\boldsymbol{A} = \frac{1}{\sqrt{x^2+y^2}}[(xyz - 3xy)\boldsymbol{a}_x + (zy^2 + 3x^2)\boldsymbol{a}_y + xy\,\boldsymbol{a}_z].$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} r^2 \\ 0 \\ \sin\theta \end{bmatrix}$$

Since $r = \sqrt{x^2+y^2+z^2}$, $\tan\theta = \dfrac{\sqrt{x^2+y^2}}{z}$, $\tan\phi = \dfrac{y}{z}$;

and $\sin\theta = \dfrac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$, $\cos\theta = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$;

and $\sin\phi = \dfrac{y}{\sqrt{x^2+y^2}}$, $\cos\phi = \dfrac{x}{\sqrt{x^2+y^2}}$;

$$B_x = r^2\sin\theta\cos\phi - \sin\theta\sin\phi = rx - \frac{y}{r} = \frac{1}{r}(r^2x - y).$$

$$B_y = r^2\sin\theta\sin\phi + \sin\theta\cos\phi = ry + \frac{x}{r} = \frac{1}{r}(r^2y + x).$$

$$B_z = r^2\cos\theta = rz = \frac{1}{r}(r^2z).$$

Hence,

$$\boldsymbol{B} = \frac{1}{\sqrt{x^2+y^2+z^2}}[\{x(x^2+y^2+z^2) - y\}\boldsymbol{a}_x + \{y(x^2+y^2+z^2) + x\}\boldsymbol{a}_y + z(x^2+y^2+z^2)\boldsymbol{a}_z].$$

P.E.2.3 (a) At:

$(1, \pi/3, 0), \quad \boldsymbol{H} = (0, 0.06767, 1)$

$$\boldsymbol{a}_x = \cos\phi\,\boldsymbol{a}_\rho - \sin\phi\,\boldsymbol{a}_\phi = \frac{1}{2}(\boldsymbol{a}_\rho - \sqrt{3}\,\boldsymbol{a}_\phi)$$

$\boldsymbol{H} \bullet \boldsymbol{a}_x = \underline{\underline{-0.0586.}}$

(b) At:

$(1, \pi/3, 0), \quad \boldsymbol{a}_\theta = \cos\theta\,\boldsymbol{a}_\rho - \sin\theta\,\boldsymbol{a}_z = -\boldsymbol{a}_z\,.$

$$H \times \boldsymbol{a}_z = \begin{vmatrix} \boldsymbol{a}_\rho & a_\phi & a_z \\ 0 & 0.06767 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \underline{\underline{-0.06767\,a_{\rho.}}}$$

(c) $(\boldsymbol{H} \bullet \boldsymbol{a}_\rho)\boldsymbol{a}_\rho = \underline{\underline{0\boldsymbol{a}_\rho}}.$

(d)

$$\boldsymbol{H} \times \boldsymbol{a}_z = \begin{vmatrix} \boldsymbol{a}_\rho & \boldsymbol{a}_\phi & \boldsymbol{a}_z \\ 0 & 0.06767 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0.06767\,\boldsymbol{a}_\rho.$$

$|\boldsymbol{H} \times \boldsymbol{a}_z| = \underline{\underline{0.06767}}$

P.E. 2.4

(a)

$A \square \boldsymbol{B} = (3, 2, -6) \bullet (4, 0, 3) = \underline{\underline{-6.}}$

(b)

$$|\boldsymbol{A} \times B| = \begin{vmatrix} 3 & 2 & -6 \\ 4 & 0 & 3 \end{vmatrix} = |6\boldsymbol{a}_r - 33\boldsymbol{a}_\theta - 8\boldsymbol{a}_\phi|.$$

Thus the magnitude of $A \times B = \underline{\underline{34.48.}}$

(c)

$At \;(1, \;\pi/3, \;5\pi/4), \quad \theta = \pi/3,$

$$\boldsymbol{a}_z = \cos\theta\,\boldsymbol{a}_r - \sin\theta\,\boldsymbol{a}_\theta = \frac{1}{2}\boldsymbol{a}_r - \frac{\sqrt{3}}{2}\boldsymbol{a}_\theta.$$

$$(A \square \boldsymbol{a}_z)\boldsymbol{a}_z = \left(\frac{3}{2} - \sqrt{3}\right)\left(\frac{1}{2}\boldsymbol{a}_r - \frac{\sqrt{3}}{2}\boldsymbol{a}_\theta\right) = \underline{\underline{-0.116\boldsymbol{a}_r + 0.201\boldsymbol{a}_\theta}}$$

Prob. 2.1
(a)

$$\rho = \sqrt{x^2+y^2} = \sqrt{4+25} = 5.3852, \qquad \phi = \tan^{-1}\frac{y}{x} = \tan^{-1} 2.5 = 68.2^o$$

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{4+25+1} = 5.477, \qquad \theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} = \tan^{-1}\frac{5.3852}{1} = 79.48^o$$

$$P(\rho,\phi,z) = \underline{\underline{P(5.3852, 68.2^o, 1)}}, \qquad P(r,\theta,\phi) = \underline{\underline{P(5.477, 79.48^o, 68.2^o)}}$$

(b)

$$\rho = \sqrt{x^2+y^2} = \sqrt{9+16} = 5, \qquad \phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{4}{-3} = 360^o - 53.123^o = 306.88^o$$

$$r = \sqrt{x^2+y^2+z^2} = 5, \qquad \theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} = \tan^{-1}\infty = 90^o$$

$$\mathrm{Q}(\rho,\phi,z) = \underline{\underline{\mathrm{Q}(5, 306.88^o, 0)}}, \qquad P(r,\theta,\phi) = \underline{\underline{P(5, 90^o, 306.88^o)}}$$

(c)

$$\rho = \sqrt{x^2+y^2} = \sqrt{36+4} = 6.325, \qquad \phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{2}{6} = 18.43^o$$

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{36+4+16} = 7.483,$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} = \tan^{-1}\frac{6.325}{-4} = 180^o - 57.69^o = 122.31^o$$

$$\mathrm{R}(\rho,\phi,z) = \underline{\underline{\mathrm{R}(6.325, 18.43^o, -4)}}, \qquad \mathrm{R}(r,\theta,\phi) = \underline{\underline{\mathrm{R}(7.483, 122.31^o, 18.43^o)}}$$

Prob. 2.2

(a)

$$x = \rho\cos\phi = 2\cos 30^\circ = 1.732;$$

$$y = \rho\sin\phi = 2\sin 30^\circ = 1;$$

$$z = 5;$$

$$P_1(x,y,z) = \underline{\underline{P_1(1.732, 1, 5)}}.$$

(b)

$$x = 1\cos 90^\circ = 0; \qquad y = 1\sin 90^\circ = 1; \qquad z = -3.$$

$$P_2(x,y,z) = \underline{\underline{P_2(0, 1, -3)}}.$$

(c)

$$x = r\sin\theta\cos\phi = 10\sin(\pi/4)\cos(\pi/3) = 3.535;$$
$$y = r\sin\theta\sin\phi = 10\sin(\pi/4)\sin(\pi/3) = 6.124;$$
$$z = r\cos\theta = 10\cos(\pi/4) = 7.0711$$
$$P_3(x,y,z) = \underline{\underline{P_3(3.535,\ 6.124, 7.0711).}}$$

(d)

$$x = 4\sin 30^\circ \cos 60^\circ = 1$$
$$y = 4\sin 30^\circ \sin 60^\circ = 1.7321$$
$$z = r\cos\theta = 4\cos 30^\circ = 3.464$$
$$P_4(x,y,z) = \underline{\underline{P_4(1, 1.7321, 3.464).}}$$

Prob. 2.3

$$\rho = \sqrt{x^2 + y^2} = \sqrt{4+36} = 6.324$$

(a) $\phi = \tan^{-1}\dfrac{y}{x} = \tan^{-1}\dfrac{6}{2} = 71.56^o$

P is $\underline{\underline{(6.324, 71.56^o, -4)}}$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{4+36+16} = 7.485$$

(b) $\theta = \tan^{-1}\dfrac{\sqrt{x^2+y^2}}{z} = \tan^{-1}\dfrac{6.324}{-4} = 90^o + \tan^{-1}\dfrac{4}{6.324} = 122.3^o$

P is $\underline{\underline{(7.483, 122.3^o, 71.56^o)}}$

Prob. 2.4

(a)

$$x = \rho\cos\phi = 5\cos 120^o = -2.5$$
$$y = \rho\sin\phi = 5\sin 120^o = 4.33$$
$$z = 1$$

Hence Q $= \underline{\underline{(-2.5, 4.33, 1)}}$

(b)

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{\rho^2 + z^2} = \sqrt{25+1} = 5.099$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} = \tan^{-1}\frac{\rho}{z} = \tan^{-1}\frac{5}{1} = 78.69^o$$

$$\phi = 120^o$$

Hence Q $= \underline{\underline{(5.099, 78.69^o, 120^o)}}$

Prob. 2.5

$$T(r,\theta,\phi) \longrightarrow r = 10, \theta = 60^o, \phi = 30^o$$

$$x = r\sin\theta\cos\phi = 10\sin 60^o \cos 30^o = 7.5$$

$$y = r\sin\theta\sin\phi = 10\sin 60^o \sin 30^o = 4.33$$

$$z = r\cos\theta = 10\cos 60^o = 5$$

$$T(x,y,z) = \underline{\underline{(7.5, 4.33, 5)}}$$

$$\rho = r\sin\theta = 10\sin 60^o = 8.66$$

$$T(\rho,\phi,z) = \underline{\underline{(8.66, 30^o, 5)}}$$

Prob. 2.6

(a)

$$x = \rho\cos\phi, \qquad y = \rho\sin\phi,$$

$$V = \underline{\underline{\rho z\cos\phi - \rho^2\sin\phi\cos\phi + \rho z\sin\phi}}$$

(b)

$$U = x^2 + y^2 + z^2 + y^2 + 2z^2$$

$$= r^2 + r^2\sin^2\theta\sin^2\phi + 2r^2\cos^2\theta$$

$$= \underline{\underline{r^2[1 + \sin^2\theta\sin^2\phi + 2\cos^2\theta]}}$$

Prob. 2.7
(a)

$$\begin{bmatrix} F_\rho \\ F_\phi \\ F_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{x}{\sqrt{\rho^2+z^2}} \\ \frac{y}{\sqrt{\rho^2+z^2}} \\ \frac{4}{\sqrt{\rho^2+z^2}} \end{bmatrix}$$

$$F_\rho = \frac{1}{\sqrt{\rho^2+z^2}}[\rho\cos^2\phi + \rho\sin^2\phi] = \frac{\rho}{\sqrt{\rho^2+z^2}};$$

$$F_\phi = \frac{1}{\sqrt{\rho^2+z^2}}[-\rho\cos\phi\sin\phi + \rho\cos\phi\sin\phi] = 0;$$

$$F_z = \frac{4}{\sqrt{\rho^2+z^2}};$$

$$\bar{F} = \frac{1}{\sqrt{\rho^2+z^2}}(\rho\mathbf{a}_\rho + 4\mathbf{a}_z)$$

In Spherical:

$$\begin{bmatrix} F_r \\ F_\theta \\ F_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} \frac{x}{r} \\ \frac{y}{r} \\ \frac{4}{r} \end{bmatrix}$$

$$F_r = \frac{r}{r}\sin^2\theta\cos^2\phi + \frac{r}{r}\sin^2\theta\sin^2\phi + \frac{4}{r}\cos\theta = \sin^2\theta + \frac{4}{r}\cos\theta;$$

$$F_\theta = \sin\theta\cos\theta\cos^2\phi + \sin\theta\cos\theta\sin^2\phi - \frac{4}{r}\sin\theta = \sin\theta\cos\theta - \frac{4}{r}\sin\theta;$$

$$F_\phi = -\sin\theta\cos\phi\sin\phi + \sin\theta\sin\phi\cos\phi = 0;$$

$$\therefore \bar{F} = (\sin^2\theta + \frac{4}{r}\cos\theta)\mathbf{a}_r + \sin\theta(\cos\theta - \frac{4}{r})\mathbf{a}_\theta$$

(b)

$$\begin{bmatrix} G_\rho \\ G_\phi \\ G_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dfrac{x\rho^2}{\sqrt{\rho^2+z^2}} \\ \dfrac{y\rho^2}{\sqrt{\rho^2+z^2}} \\ \dfrac{z\rho^2}{\sqrt{\rho^2+z^2}} \end{bmatrix}$$

$$G_\rho = \frac{\rho^2}{\sqrt{\rho^2+z^2}}[\rho\cos^2\phi + \rho\sin^2\phi] = \frac{\rho^3}{\sqrt{\rho^2+z^2}};$$

$$G_\phi = 0;$$

$$G_z = \frac{z\rho^2}{\sqrt{\rho^2+z^2}};$$

$$\underline{\underline{\boldsymbol{G} = \frac{\rho^2}{\sqrt{\rho^2+z^2}}(\rho\,\boldsymbol{a}_\rho + z\,\boldsymbol{a}_z)}}$$

Spherical :

$$\boldsymbol{G} = \frac{\rho^2}{r}(x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z) = \frac{r^2\sin^2\theta}{r} r\boldsymbol{a}_r = \underline{\underline{r^2\sin^2\theta\boldsymbol{a}_r}}$$

Prob. 2.8

$$\boldsymbol{B} = \rho\boldsymbol{a}_x + \frac{y}{\rho}\boldsymbol{a}_y + z\boldsymbol{a}_z$$

$$\begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho \\ y/\rho \\ z \end{bmatrix}$$

$$B_\rho = \rho\cos\phi + \frac{y}{\rho}\sin\phi$$

$$B_\phi = -\rho\sin\phi + \frac{y}{\rho}\cos\phi$$

$$B_z = z$$

But $y = \rho\sin\phi$

$$B_\rho = \rho\cos\phi + \sin^2\phi, B_\phi = -\rho\sin\phi + \sin\phi\cos\phi$$

Hence,

$$\underline{\underline{\boldsymbol{B} = (\rho\cos\phi + \sin^2\phi)\boldsymbol{a}_\rho + \sin\phi(\cos\phi - \rho)\boldsymbol{a}_\phi + z\boldsymbol{a}_z}}$$

Prob. 2.9

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

At P, $\rho = 2, \quad \phi = \pi/2, \quad z = -1$

$$A_x = 2\cos\phi - 3\sin\phi = 2\cos 90^o - 3\sin 90^o = -3$$

$$A_y = 2\sin\phi + 3\cos\phi = 2\sin 90^o + 3\cos 90^o = 2$$

$$A_z = 4$$

Hence, $\quad \underline{\underline{\boldsymbol{A} = -3\boldsymbol{a}_x + 2\boldsymbol{a}_y + 4\boldsymbol{a}_z}}$

Prob. 2.10

(a)

$$\begin{bmatrix} A_x \\ A_y \\ A_y \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho\sin\phi \\ \rho\cos\phi \\ -2z \end{bmatrix}$$

$$A_x = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi = 0$$

$$A_y = \rho\sin^2\phi + \rho\cos^2\phi = \rho = \sqrt{x^2 + y^2}$$

$$A_z = -2z$$

Hence,

$$\underline{\underline{\boldsymbol{A} = \sqrt{x^2 + y^2}\,\boldsymbol{a}_y - 2z\boldsymbol{a}_z}}$$

(b)

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 4r\cos\phi \\ r \\ 0 \end{bmatrix}$$

$$B_x = 4r\sin\theta\cos^2\phi + r\cos\theta\cos\phi$$

$$B_y = 4r\sin\theta\sin\phi\cos\phi + r\cos\theta\sin\phi$$

$$B_z = 4r\cos\theta\cos\phi - r\sin\theta$$

But $\quad r = \sqrt{x^2 + y^2 + z^2}, \quad \sin\theta = \dfrac{\sqrt{x^2 + y^2}}{r}, \quad \cos\theta = \dfrac{z}{r}$

$$\sin\phi = \frac{y}{\sqrt{x^2 + y^2}}, \quad \cos\phi = \frac{x}{\sqrt{x^2 + y^2}}$$

$$B_x = 4\sqrt{x^2+y^2}\,\frac{x^2}{x^2+y^2} + \frac{zx}{\sqrt{x^2+y^2}}$$

$$B_y = 4\sqrt{x^2+y^2}\,\frac{xy}{x^2+y^2} + \frac{zy}{\sqrt{x^2+y^2}}$$

$$B_z = 4z\frac{x}{\sqrt{x^2+y^2}} - \sqrt{x^2+y^2}$$

$$\boldsymbol{B} = \frac{1}{\sqrt{x^2+y^2}}\left[x(4x+z)\mathbf{a}_x + y(4x+z)\mathbf{a}_y + (4xz - x^2 - y^2)\mathbf{a}_z\right]$$

Prob. 2.11

Method 1:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 4/r^2 \\ 0 \\ 0 \end{bmatrix}$$

$$F_x = \frac{4}{r^2}\sin\theta\cos\phi, \qquad F_y = \frac{4}{r^2}\sin\theta\sin\phi, \qquad F_z = \frac{4}{r^2}\cos\theta$$

$$r^2 = x^2+y^2+z^2, \qquad \sin\theta = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}, \qquad \cos\theta = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\sin\phi = \frac{y}{\sqrt{x^2+y^2}}, \qquad \cos\phi = \frac{x}{\sqrt{x^2+y^2}}$$

$$F_x = \frac{4}{x^2+y^2+z^2}\,\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\,\frac{x}{\sqrt{x^2+y^2}} = \frac{4x}{(x^2+y^2+z^2)^{3/2}}$$

$$F_y = \frac{4}{x^2+y^2+z^2}\,\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\,\frac{y}{\sqrt{x^2+y^2}} = \frac{4y}{(x^2+y^2+z^2)^{3/2}}$$

$$F_x = \frac{4}{x^2+y^2+z^2}\,\frac{z}{(x^2+y^2+z^2)} = \frac{4z}{(x^2+y^2+z^2)^{3/2}}$$

Thus,

$$\boldsymbol{F} = \frac{4}{(x^2+y^2+z^2)^{3/2}}\left[x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z\right]$$

Method 2:

$$\boldsymbol{F} = \frac{4\boldsymbol{a}_r}{r^2} \cdot \frac{\boldsymbol{r}}{r} = \frac{4r\boldsymbol{a}_r}{r^3}$$

$$\boldsymbol{F} = \frac{4}{(x^2+y^2+z^2)^{3/2}}\left[x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z\right]$$

Prob. 2.12

(a)
$$r = 2, \qquad \theta = \pi/2, \qquad \phi = 3\pi/2$$
$$\boldsymbol{B} = 2\sin(\pi/2)\boldsymbol{a}_r - 4\cos(3\pi/2)\boldsymbol{a}_\phi = 2\boldsymbol{a}_r$$

(b)

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \cos\theta\cos\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} r\sin\theta \\ 0 \\ -r^2\cos\phi \end{bmatrix}$$

$$B_x = r\sin^2\theta\cos\phi - r^2\sin\phi\cos\phi, \qquad B_y = r\sin\theta\cos\theta\cos\phi - r^2\cos^2\phi$$

$$B_z = r\sin\theta\cos\theta$$

But $\quad r = \sqrt{x^2+y^2+z^2}, \cos\theta = \dfrac{z}{r}, \sin\theta = \dfrac{\rho}{r} = \dfrac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$

$$\cos\phi = \frac{x}{\rho} = \frac{x}{\sqrt{x^2+y^2}}, \qquad \sin\phi = \frac{y}{\rho} = \frac{y}{\sqrt{x^2+y^2}}$$

$$B_x = \sqrt{x^2+y^2+z^2}\,\frac{x^2+y^2}{x^2+y^2+z^2}\,\frac{x}{\sqrt{x^2+y^2}} - (x^2+y^2+z^2)\frac{xy}{x^2+y^2}$$

$$= \frac{x\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}} - \frac{xy(x^2+y^2+z^2)}{x^2+y^2}$$

$$B_y = \sqrt{x^2+y^2+z^2}\,\frac{z\sqrt{x^2+y^2}}{x^2+y^2+z^2}\,\frac{x}{\sqrt{x^2+y^2}} - (x^2+y^2+z^2)\frac{x^2}{x^2+y^2}$$

$$= \frac{xz}{\sqrt{x^2+y^2+z^2}} - \frac{x^2(x^2+y^2+z^2)}{x^2+y^2}$$

$$B_z = \sqrt{x^2+y^2+z^2}\,\frac{z\sqrt{x^2+y^2}}{x^2+y^2+z^2} = \frac{z\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$$

$$\boldsymbol{B} = B_x\boldsymbol{a}_x + B_y\boldsymbol{a}_y + B_z\boldsymbol{a}_z$$

Prob. 2.13

(a) $x = \rho\cos\phi$

$$\underline{\underline{\boldsymbol{B} = \rho\cos\phi\boldsymbol{a}_z}}$$

$x = r\sin\theta\cos\phi$

(b) $\boldsymbol{B} = r\sin\theta\cos\phi\boldsymbol{a}_z, \qquad B_x = 0 = B_y, B_z = r\sin\theta\cos\phi$

$$\begin{bmatrix} B_r \\ B_\theta \\ B_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ r\sin\theta\cos\phi \end{bmatrix}$$

$$B_r = r\sin\theta\cos\theta\cos\phi = 0.5r\sin(2\theta)\cos\phi$$

$$B_\theta = -r\sin^2\theta\cos\phi, \qquad B_\phi = 0$$

$$\underline{\underline{\boldsymbol{B} = 0.5r\sin(2\theta)\cos\phi\boldsymbol{a}_r - r\sin^2\theta\cos\phi\boldsymbol{a}_\theta}}$$

Prob. 2.14

(a)

$$\boldsymbol{a}_x \times \boldsymbol{a}_\rho = (\cos\phi\,\boldsymbol{a}_\rho - \sin\phi\boldsymbol{a}_\phi)\times\boldsymbol{a}_\rho = \cos\phi$$

$$\boldsymbol{a}_x \times \boldsymbol{a}_\phi = (\cos\phi\boldsymbol{a}_\rho - \sin\phi\,\boldsymbol{a}_\phi)\times\boldsymbol{a}_\phi = -\sin\phi$$

$$\boldsymbol{a}_y \times \boldsymbol{a}_\rho = (\sin\phi\,\boldsymbol{a}_\rho + \cos\phi\boldsymbol{a}_\phi)\times\boldsymbol{a}_\rho = \sin\phi$$

$$\bar{a}_y \times \bar{a}_\phi = (\sin\phi\,\boldsymbol{a}_\rho + \sin\phi\boldsymbol{a}_\phi)\times\boldsymbol{a}_\phi = \cos\phi$$

(b) and (c)

In spherical system:

$$\boldsymbol{a}_x = \sin\theta\cos\phi\boldsymbol{a}_r + \cos\theta\cos\phi\,\boldsymbol{a}_\theta - \sin\phi\boldsymbol{a}_\phi.$$

$$\boldsymbol{a}_y = \sin\theta\sin\phi\,\boldsymbol{a}_r + \cos\theta\sin\phi\boldsymbol{a}_\theta - \cos\phi\boldsymbol{a}_\phi.$$

$$\boldsymbol{a}_z = \cos\theta\,\boldsymbol{a}_x - \sin\theta\boldsymbol{a}_\theta\,.$$

Hence,

$$\boldsymbol{a}_x \times \boldsymbol{a}_r = \sin\theta\cos\phi;$$

$$\boldsymbol{a}_x \times \boldsymbol{a}_\theta = \cos\theta\cos\phi;$$

$$\boldsymbol{a}_y \times \boldsymbol{a}_r = \sin\theta\sin\phi;$$

$$\boldsymbol{a}_y \times \boldsymbol{a}_\theta = \cos\theta\sin\phi;$$

$$\bar{a}_z \times \bar{a}_r = \cos\theta;$$

$$\bar{a}_z \times \bar{a}_\theta = -\sin\theta;$$

Prob. 2.15

(a)

$$\boldsymbol{a}_\rho = \cos\phi\boldsymbol{a}_x + \sin\phi\boldsymbol{a}_y, \qquad \boldsymbol{a}_\phi = -\sin\phi\boldsymbol{a}_x + \cos\phi\boldsymbol{a}_y$$

$$\boldsymbol{a}_\rho \times \boldsymbol{a}_\phi = \begin{vmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \end{vmatrix} = (\cos^2\phi + \sin^2\phi)\boldsymbol{a}_z = \boldsymbol{a}_z$$

$$\boldsymbol{a}_z \times \boldsymbol{a}_\rho = \begin{vmatrix} 0 & 0 & 1 \\ \cos\phi & \sin\phi & 0 \end{vmatrix} = -\sin\phi\boldsymbol{a}_x + \cos\phi\boldsymbol{a}_y = \boldsymbol{a}_\phi$$

$$\boldsymbol{a}_\phi \times \boldsymbol{a}_z = \begin{vmatrix} -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos\phi\boldsymbol{a}_x + \sin\phi\boldsymbol{a}_y = \boldsymbol{a}_\rho$$

(b)

$$\boldsymbol{a}_r = \sin\theta\cos\phi\boldsymbol{a}_x + \sin\theta\sin\phi\boldsymbol{a}_y + \cos\theta\boldsymbol{a}_z$$

$$\boldsymbol{a}_\theta = \cos\theta\cos\phi\boldsymbol{a}_x + \cos\theta\sin\phi\boldsymbol{a}_y - \sin\theta\boldsymbol{a}_z$$

$$\boldsymbol{a}_\phi = -\sin\phi\boldsymbol{a}_x + \cos\phi\boldsymbol{a}_y$$

$$\boldsymbol{a}_r \times \boldsymbol{a}_\theta = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \end{vmatrix}$$

$$= (-\sin^2\theta\sin\phi - \cos^2\theta\sin\phi)\boldsymbol{a}_x + (\cos^2\theta\cos\phi + \sin^2\theta\cos\phi)\boldsymbol{a}_y$$

$$+(\sin\theta\cos\theta\sin\phi\cos\phi - \sin\theta\cos\theta\sin\phi\cos\phi)\boldsymbol{a}_z$$

$$= -\sin\phi\boldsymbol{a}_x + \cos\phi\boldsymbol{a}_y = \boldsymbol{a}_\phi$$

$$\boldsymbol{a}_\phi \times \boldsymbol{a}_r = \begin{vmatrix} -\sin\phi & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{vmatrix}$$

$$= \cos\theta\cos\phi\ \boldsymbol{a}_x + \cos\theta\sin\phi\boldsymbol{a}_y + (-\sin\theta\sin^2\phi - \sin\theta\cos^2\phi)\boldsymbol{a}_z$$

$$= \cos\theta\cos\phi\boldsymbol{a}_x + \cos\theta\sin\phi\boldsymbol{a}_y - \sin\theta\boldsymbol{a}_z = \boldsymbol{a}_\theta$$

$$\boldsymbol{a}_\theta \times \boldsymbol{a}_\phi = \begin{vmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{vmatrix}$$

$$= \sin\theta\cos\phi\ \boldsymbol{a}_x + \sin\theta\sin\phi\boldsymbol{a}_y + (\cos\theta\cos^2\phi + \cos\theta\sin^2\phi)\boldsymbol{a}_z$$

$$= \sin\theta\cos\phi\boldsymbol{a}_x + \sin\theta\sin\phi\boldsymbol{a}_y + \cos\theta\boldsymbol{a}_z = \boldsymbol{a}_r$$

Prob. 2.16

(a)

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{\rho^2 + z^2}.$$

$$\theta = \tan^{-1}\frac{\rho}{z}; \qquad \phi = \phi.$$

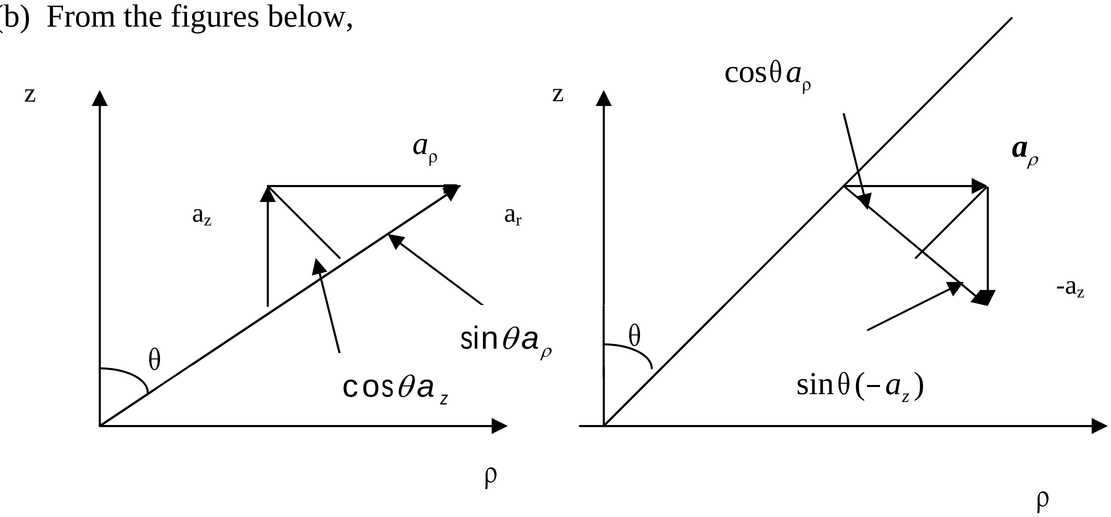
or

$$\rho = \sqrt{x^2 + y^2} = \sqrt{r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi}.$$

$$= r\sin\theta;$$

$$z = r\cos\theta; \qquad \phi = \phi.$$

(b) From the figures below,



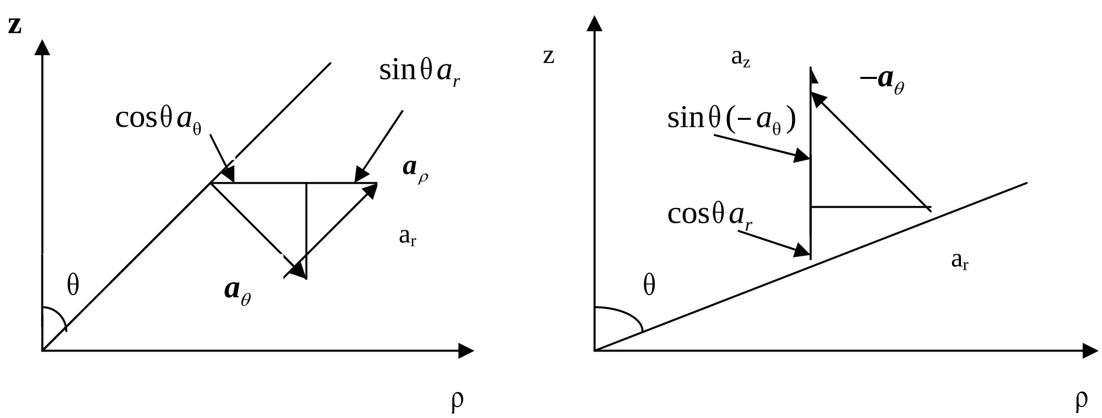
$\mathbf{a}_r = \sin\theta\, \mathbf{a}_\rho + \cos\theta\, \mathbf{a}_z; \quad \mathbf{a}_\theta = \cos\theta\, \mathbf{a}_\rho - \sin\theta\, \mathbf{a}_z; \quad \mathbf{a}_\phi = \mathbf{a}_\phi.$

Hence,

$$\begin{bmatrix} \mathbf{a}_r \\ \mathbf{a}_\theta \\ \mathbf{a}_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{a}_\rho \\ \mathbf{a}_\phi \\ \mathbf{a}_z \end{bmatrix}$$

From the figures below,

$\mathbf{a}_\rho = \cos\theta\, \mathbf{a}_\theta + \sin\theta\, \mathbf{a}_r; \mathbf{a}_z = \cos\theta\, \mathbf{a}_r - \sin\theta\, \mathbf{a}_\theta; \mathbf{a}_\phi = \mathbf{a}_\phi.$



$$\begin{bmatrix} \mathbf{a}_\rho \\ \mathbf{a}_\phi \\ \mathbf{a}_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \mathbf{a}_r \\ \mathbf{a}_\theta \\ \mathbf{a}_z \end{bmatrix}$$

Prob. 2.17

At $P(2,0,-1)$, $\quad \phi = 0, \quad \theta = \cos^{-1}\left(\dfrac{z}{\sqrt{x^2+y^2+z^2}}\right) = \cos^{-1}\left(\dfrac{-1}{\sqrt{5}}\right) = 116.56^o$

(a) $\mathbf{a}_\rho \bullet \mathbf{a}_x = \cos\phi = \underline{\underline{1}}$

(b) $\mathbf{a}_\phi \bullet \mathbf{a}_y = \cos\phi = \underline{\underline{1}}$

(c) $\mathbf{a}_r \bullet \mathbf{a}_z = \cos\theta = \underline{\underline{-0.4472}}$

Prob. 2.18

If **A** and **B** are perpendicular to each other, **A**□**B** = 0

$$\boldsymbol{A}\square\boldsymbol{B} = \rho^2 \sin^2\phi + \rho^2 \cos^2\phi - \rho^2$$
$$= \rho^2(\sin^2\phi + \cos^2\phi) - \rho^2$$
$$= \rho^2 - \rho^2$$
$$= 0$$

As expected.

Prob. 2.19

$$(a)\ \boldsymbol{A}+\boldsymbol{B} = \underline{\underline{8\boldsymbol{a}_\rho + 2\boldsymbol{a}_\phi - 7\boldsymbol{a}_z}}$$

$$(b)\ \boldsymbol{A}\square\boldsymbol{B} = 15 + 0 - 8 = \underline{\underline{7}}$$

$$(c)\ \boldsymbol{A}\times\boldsymbol{B} = \begin{vmatrix} 3 & 2 & 1 \\ 5 & 0 & -8 \end{vmatrix}$$
$$= -16\mathbf{a}_\rho + (5+24)\boldsymbol{a}_\phi - 10\boldsymbol{a}_z$$
$$= \underline{\underline{-16\mathbf{a}_\rho + 29\boldsymbol{a}_\phi - 10\boldsymbol{a}_z}}$$

$$(d)\ \cos\theta_{AB} = \frac{\boldsymbol{A}\cdot\boldsymbol{B}}{AB} = \frac{7}{\sqrt{9+4+1}\sqrt{25+64}} = \frac{7}{\sqrt{14}\sqrt{89}}$$
$$= 0.19831$$
$$\underline{\underline{\theta_{\mathrm{AB}} = 78.56^o}}$$

Prob. 2.20

$$\begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} G_\rho \\ G_\phi \\ G_z \end{bmatrix}$$

$$G_x = G_\rho \cos\phi - G_\phi \sin\phi = 3\rho\cos\phi - \rho\cos\phi\sin\phi$$
$$= 3x - x\sin\phi = 3(3) - (3)\sin(306.87^o) = 11.4$$
$$\boldsymbol{G}_x = G_x\boldsymbol{a}_x = \underline{\underline{11.4\boldsymbol{a}_x}}$$

Prob. 2.21

$$\begin{bmatrix} G_\rho \\ G_\phi \\ G_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} yz \\ xz \\ xy \end{bmatrix}$$

$$G_\rho = yz\cos\phi + xz\sin\phi$$

$$x = \rho\cos\phi,\, y = \rho\sin\phi, \qquad yz = \rho z\sin\phi,\, xz = \rho z\cos\phi$$

$$G_\rho = \rho z\sin\phi\cos\phi + \rho z\cos\phi\sin\phi = 2\rho z\sin\phi\cos\phi = \rho z\sin 2\phi$$

$$G_\phi = -yz\sin\phi + xz\cos\phi = \rho z(\cos^2\phi - \sin^2\phi) = \rho z\cos 2\phi$$

$$G_z = xy = \rho^2\cos\phi\sin\phi = 0.5\rho^2\sin 2\phi$$

$$\underline{\underline{\boldsymbol{G} = \rho z\sin 2\phi \boldsymbol{a}_y + \rho z\cos 2\phi \boldsymbol{a}_\phi + 0.5\rho^2\sin 2\phi \boldsymbol{a}_z}}$$

Prob. 2.22

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{x}{\sqrt{x^2+y^2}} & -\dfrac{y}{\sqrt{x^2+y^2}} & 0 \\ \dfrac{y}{\sqrt{x^2+y^2}} & \dfrac{x}{\sqrt{x^2+y^2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{x}{\sqrt{x^2+y^2+z^2}} & \dfrac{xz}{\sqrt{x^2+y^2}\sqrt{x^2+y^2+z^2}} & \dfrac{-y}{\sqrt{x^2+y^2}} \\ \dfrac{y}{\sqrt{x^2+y^2+z^2}} & \dfrac{yz}{\sqrt{x^2+y^2}\sqrt{x^2+y^2+z^2}} & \dfrac{x}{\sqrt{x^2+y^2}} \\ \dfrac{z}{\sqrt{x^2+y^2+z^2}} & -\dfrac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}} & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\phi \\ A_\phi \end{bmatrix}$$

Prob. 2.23 (a) Using the results in Prob.2.14,

$$A_\rho = \rho z \sin\phi = r^2 \sin\theta \cos\theta \sin\phi$$
$$A_\phi = 3\rho\cos\phi = 3r\sin\theta\cos\phi$$
$$A_z = \rho\cos\phi\sin\phi = r\sin\theta\cos\phi\sin\phi$$

Hence,

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r^2\sin\theta\cos\theta\sin\phi \\ 3r\sin\theta\cos\phi \\ r\sin\theta\cos\phi\sin\phi \end{bmatrix}$$

$$\mathbf{A}(r,\theta,\phi) = \underline{\underline{r\sin\theta\left[\sin\phi\cos\theta\left(r\sin\theta + \cos\phi\right)\mathbf{a}_r + \sin\phi\left(r\cos^2\theta - \sin\theta\cos\phi\right)\mathbf{a}_\theta + 3\cos\phi\mathbf{a}_\phi\right]}}$$

At $(10, \pi/2, 3\pi/4)$, $r = 10, \theta = \pi/2, \phi = 3\pi/4$

$$\mathbf{A} = 10(0\mathbf{a}_r + 0.5\mathbf{a}_\theta - \frac{3}{\sqrt{2}}\mathbf{a}_\phi) = \underline{\underline{5\mathbf{a}_\theta - 21.21\mathbf{a}_\phi}}$$

(b) $B_r = r^2 = (\rho^2 + z^2)$, $B_\theta = 0$, $B_\phi = \sin\theta = \dfrac{\rho}{\sqrt{\rho^2 + z^2}}$

$$\begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} B_r \\ B_\theta \\ B_\phi \end{bmatrix}$$

$$\mathbf{B}(\rho,\phi,z) = \underline{\underline{\sqrt{\rho^2 + z^2}\left(\rho\mathbf{a}_\rho + \frac{\rho}{\rho^2 + z^2}\mathbf{a}_\phi + z\mathbf{a}_z\right)}}$$

At $(2, \pi/6, 1)$, $\rho = 2, \phi = \pi/6, z = 1$

$$\mathbf{B} = \sqrt{5}(2\mathbf{a}_\rho + 0.4\mathbf{a}_\phi + \mathbf{a}_z) = \underline{\underline{4.472\mathbf{a}_\rho + 0.8944\mathbf{a}_\phi + 2.236\mathbf{a}_z}}$$

Prob. 2.24

(a) $d = \sqrt{(6-2)^2 + (-1-1)^2 + (2-5)^2} = \sqrt{29} = \underline{\underline{5.385}}$

(b)
$$d^2 = 3^2 + 5^2 - 2(3)(5)\cos\pi + (-1-5)^2 = 100$$
$$d = \sqrt{100} = \underline{\underline{10}}$$

(c)

$$d^2 = 10^2 + 5^2 - 2(10)(5)\cos\frac{\pi}{4}\cos\frac{\pi}{6} - 2(10)(5)\sin\frac{\pi}{4}\sin\frac{\pi}{6}\cos(7\frac{\pi}{4} - \frac{3\pi}{4})$$

$$= 125 - 100(0.7071)(0.866) - 100(0.7071)(0.5)(-0.2334)$$

$$= 125 - 61.23 + 35.33 = 99.118$$

$$d = \sqrt{99.118} = \underline{\underline{9.956.}}$$

Prob. 2.25

Using eq. (2.33),

$$d^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\theta_1\cos\theta_2 - 2r_1r_2\sin\theta_1\sin\theta_2\cos(\phi_2 - \phi_1)$$

$$= 16 + 36 - 2(4)(6)\cos 30^o\cos 90^o - 2(4)(6)\sin 30^o\sin 90^o\cos(180^o)$$

$$= 16 + 36 - 0 - 48(0.5)(1)(-1) = 52 + 24 = 76$$

$$d = 8.718$$

Prob. 2.26

$$\begin{bmatrix} \boldsymbol{a}_\rho \\ \boldsymbol{a}_\phi \\ \boldsymbol{a}_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_x \\ \boldsymbol{a}_y \\ \boldsymbol{a}_z \end{bmatrix}$$

$$\boldsymbol{a}_\rho = \cos\phi\boldsymbol{a}_x + \sin\phi\boldsymbol{a}_y, \qquad \boldsymbol{a}_\phi = -\sin\phi\boldsymbol{a}_x + \cos\phi\boldsymbol{a}_y$$

At (0, 4, -1), $\phi = 90^o$

$$\boldsymbol{a}_\rho = \sin 90^o\boldsymbol{a}_y = \underline{\underline{\boldsymbol{a}_y}}$$

$$\boldsymbol{a}_\phi = -\sin 90^o\boldsymbol{a}_x = \underline{\underline{-\boldsymbol{a}_x}}$$

Prob. 2.27

At $(1, 60^o, -1)$, $\rho = 1, \phi = 60^o, z = -1,$

(a) $\boldsymbol{A} = (-2 - \sin 60^o)\boldsymbol{a}_\rho + (4 + 2\cos 60^o)\boldsymbol{a}_\phi - 3(1)(-1)\boldsymbol{a}_z$

$$= -2.866\boldsymbol{a}_\rho + 5\boldsymbol{a}_\phi + 3\boldsymbol{a}_z$$

$$\boldsymbol{B} = 1\cos 60^o\boldsymbol{a}_\rho + \sin 60^o\boldsymbol{a}_\phi + \boldsymbol{a}_z = 0.5\boldsymbol{a}_\phi + 0.866\boldsymbol{a}_\phi + \boldsymbol{a}_z$$

$$\boldsymbol{A}\cdot\boldsymbol{B} = -1.433 + 4.33 + 3 = 5.897$$

$$AB = \sqrt{2.866^2 + 26 + 9}\sqrt{0.25 + 1 + 0.866^2} = 9.1885$$

$$\cos\theta_{AB} = \frac{\boldsymbol{A}\cdot\boldsymbol{B}}{AB} = \frac{5.897}{9.1885} = 0.6419 \longrightarrow \theta_{AB} = \underline{\underline{50.07^o}}$$

Let $\mathbf{D} = \mathbf{A} \times \mathbf{B}$. At $(1,90^o,0)$, $\rho = 1, \phi = 90^o, z = 0$

(b) $\boldsymbol{A} = -\sin 90^o \boldsymbol{a}_\rho + 4\boldsymbol{a}_\phi = -\boldsymbol{a}_\rho + 4\boldsymbol{a}_\phi$

$\boldsymbol{B} = 1\cos 90^o \boldsymbol{a}_\rho + \sin 90^o \boldsymbol{a}_\phi + \boldsymbol{a}_z = \boldsymbol{a}_\phi + \boldsymbol{a}_z$

$$\boldsymbol{D} = \boldsymbol{A} \times \boldsymbol{B} = \begin{vmatrix} \boldsymbol{a}_\rho & \boldsymbol{a}_\phi & \boldsymbol{a}_z \\ -1 & 4 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 4\boldsymbol{a}_\rho + \boldsymbol{a}_\phi - \boldsymbol{a}_z$$

$$\boldsymbol{a}_D = \frac{\boldsymbol{D}}{D} = \frac{(4,1,-1)}{\sqrt{16+1+1}} = \underline{\underline{0.9428\boldsymbol{a}_\rho + 0.2357\boldsymbol{a}_\phi - 0.2357\boldsymbol{a}_z}}$$

Prob.2.28

$At\ P(0,2,-5), \quad \phi = 90°;$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 1 \\ -3 \end{bmatrix}$$

$\boldsymbol{B} = -\boldsymbol{a}_x - 5\boldsymbol{a}_y - 3\boldsymbol{a}_z$

(a) $\boldsymbol{A} + \boldsymbol{B} = (2,4,10) + (-1,-5,-3)$

$= \underline{\underline{\boldsymbol{a}_x - \boldsymbol{a}_y + 7\boldsymbol{a}_z}}.$

(b) $\cos\theta_{AB} = \frac{\boldsymbol{A} \bullet \boldsymbol{B}}{AB} = \frac{-52}{\sqrt{4200}}$

$\theta_{AB} = \cos^{-1}\left(\frac{-52}{\sqrt{4200}}\right) = \underline{\underline{143.36°}}.$

(c) $A_B = \boldsymbol{A} \bullet \boldsymbol{a}_B = \frac{\boldsymbol{A} \bullet \boldsymbol{B}}{B} = -\frac{52}{\sqrt{35}} = \underline{\underline{-8.789}}.$

Prob. 2.29

$$\mathbf{B}\bullet\mathbf{a}_x = B_x$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix}$$

$$B_x = B_\rho\cos\phi - B_\phi\sin\phi = \rho^2\sin\phi\cos\phi - (z-1)\cos\phi\sin\phi$$

$$= 16(0.5) - (-2)(0.5) = 8 + 1 = \underline{\underline{9}}$$

Prob. 2.30

$$\bar{G} = \cos^2\phi\,\mathbf{a}_x + \frac{2r\cos\theta\sin\phi}{r\sin\theta}\bar{a}_y + (1-\cos^2\phi)\bar{a}_z$$

$$= \cos^2\phi\,\bar{a}_x + 2\cot\theta\sin\phi\,\bar{a}_y + \sin^2\phi\,\bar{a}_z$$

$$\begin{pmatrix} G_r \\ G_\theta \\ G_\phi \end{pmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} \cos^2\phi \\ 2\cot\theta\sin\phi \\ \sin^2\phi \end{bmatrix}$$

$$G_r = \sin\theta\cos^3\phi + 2\cos\theta\sin^2\phi + \cos\theta\sin^2\phi$$

$$= \sin\theta\cos^3\phi + 3\cos\theta\sin^2\phi$$

$$G_\theta = \cos\theta\cos^3\phi + 2\cot\theta\cos\theta\sin^2\phi - \sin\theta\sin^2\phi$$

$$G_\phi = -\sin\phi\cos^2\phi + 2\cot\theta\sin\phi\cos\phi$$

$$\bar{G} = [\sin\theta\cos^3\phi + 3\cos\theta\sin^2\phi]\mathbf{a}_r$$

$$+[\cos\theta\cos^3\phi + 2\cot\theta\cos\theta\sin^2\phi - \sin\theta\sin^2\phi]\mathbf{a}_\theta$$

$$\underline{\underline{+\sin\phi\cos\phi(2\cot\theta - \cos\phi)\mathbf{a}_\phi}}$$

Prob. 2.31

(a) An infinite line parallel to the z-axis.

(b) Point (2,-1,10).

(c) A circle of radius $r\sin\theta = 5$, i.e. the intersection of a cone and a sphere.

(d) An infinite line parallel to the z-axis.

(e) A semi-infinite line parallel to the x-y plane.

(f) A semi-circle of radius 5 in the y-z plane.

Prob. 2.32

(a) $\boldsymbol{J}_z = (\boldsymbol{J} \bullet \boldsymbol{a}_z)\boldsymbol{a}_z$.

$At\,(2,\ \ \pi/2,\ \ 3\pi/2),\ \ \boldsymbol{a}_z = \cos\theta\boldsymbol{a}_r - \sin\theta\boldsymbol{a}_\theta = -\boldsymbol{a}_\theta.$

$\boldsymbol{J}_z = -\cos 2\theta \sin\phi \boldsymbol{a}_\theta = -\cos\pi \sin(3\pi/2)\boldsymbol{a}_\theta = -\boldsymbol{a}_\theta.$

$(b)\boldsymbol{J}_\phi = \tan\frac{\theta}{2}\ln r\,\boldsymbol{a}_\phi = \tan\frac{\pi}{4}\ln 2\,\boldsymbol{a}_\phi = \ln 2\boldsymbol{a}_\phi = 0.6931\boldsymbol{a}_\phi\,.$

$(c)\boldsymbol{J}_t = \boldsymbol{J} - \boldsymbol{J}_n = \boldsymbol{J} - \boldsymbol{J}_r = -\boldsymbol{a}_\theta + \ln 2\,\boldsymbol{a}_\phi = \underline{\underline{-\boldsymbol{a}_\theta + 0.6931\boldsymbol{a}_\phi}}$

$(d)\quad \boldsymbol{J}_P = (\boldsymbol{J} \bullet \boldsymbol{a}_x)\boldsymbol{a}_x$

$\boldsymbol{a}_x = \sin\theta\cos\phi\boldsymbol{a}_r + \cos\theta\cos\phi\boldsymbol{a}_\theta - \sin\phi\boldsymbol{a}_\phi = \boldsymbol{a}_\phi.$

At $(2,\ \ \pi/2,\ \ 3\pi/2)$,

$\boldsymbol{J}_P = \underline{\underline{\ln 2\boldsymbol{a}_\phi.}}$

Prob. 2.33

$\boldsymbol{H} \cdot \boldsymbol{a}_x = H_x$

$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho^2\cos\phi \\ -\rho\sin\phi \\ 0 \end{bmatrix}$$

$H_x = \rho^2\cos^2\phi + \rho\sin^2\phi$

At P, $\rho = 2, \phi = 60^o, z = -1$

$H_x = 4(1/4) + 2(3/4) = 1 + 1.5 = \underline{\underline{2.5}}$

Prob. 2.34

(a) $5 = \boldsymbol{r}\cdot\boldsymbol{a}_x + \boldsymbol{r}\cdot\boldsymbol{a}_y = x + y$ $\underline{\underline{\text{a plane}}}$

(b) $10 = |\boldsymbol{r} x \boldsymbol{a}_z| = \begin{vmatrix} x & y & z \\ 0 & 0 & 1 \end{vmatrix} = |y\boldsymbol{a}_x - x\boldsymbol{a}_y| = \sqrt{x^2 + y^2} = \rho$

$\underline{\underline{\text{a cylinder of infinite length}}}$