Test Bank for Introduction to Formal Logic 1st Edition by Marcus

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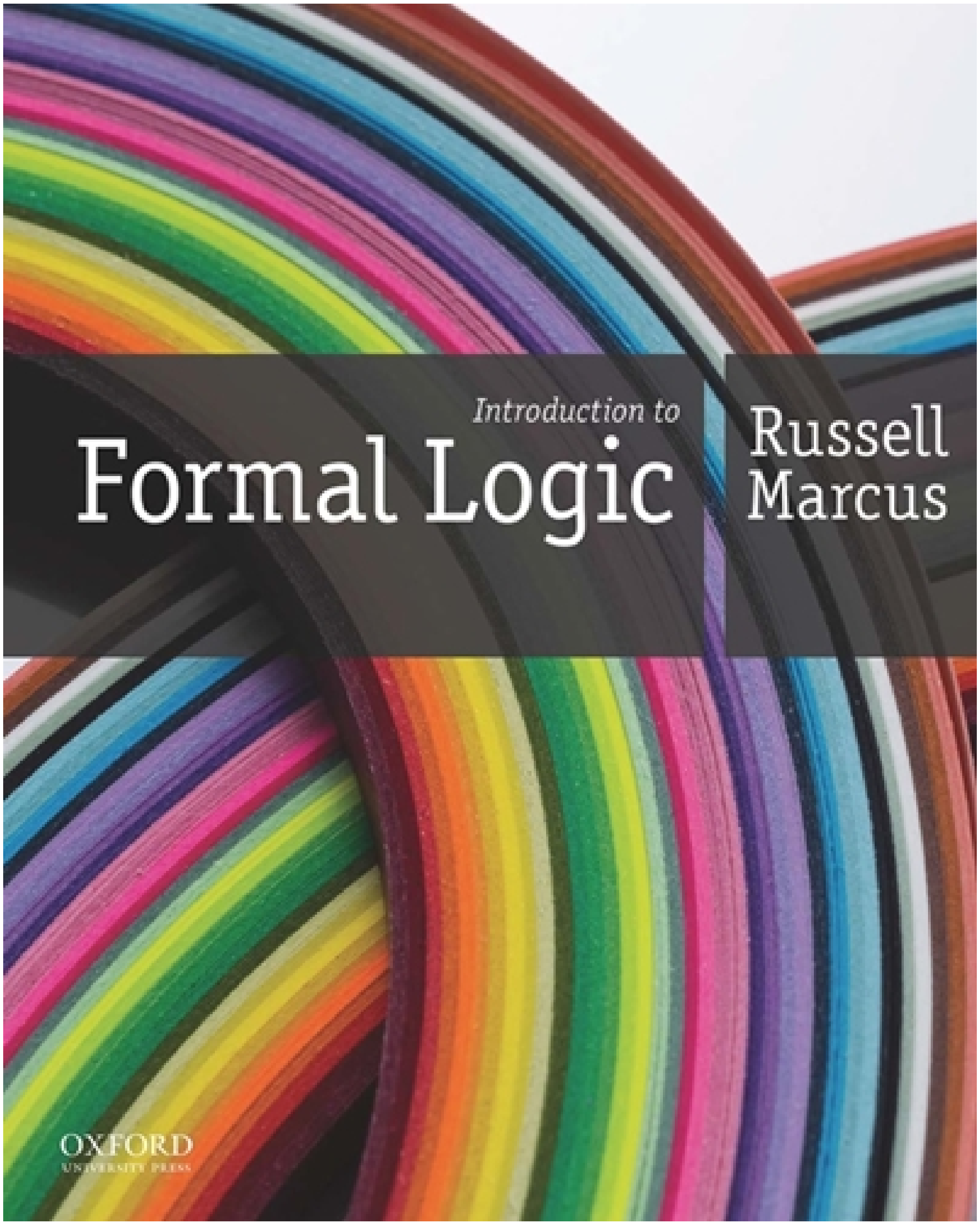

Test Bank

Introduction to Formal Logic with Philosophical Applications
Instructor's Manual
Chapter 2

Contents

Chapter 2 - Propositional Logic: Syntax and Semantics

Chapter 2 Summary:

- Propositional logic is the logic of propositions and their inferential relations.
- A proposition is a statement, often expressed by a declarative sentence, which has a truth value.
- **PL** is the formal object language of propositional logic.
- The syntax of **PL** specifies its vocabulary and rules for making formulas.
- The vocabulary of **PL** includes uppercase letters, five operators— tilde (~), dot (•), vel (∨), horseshoe (⊃), triple-bar (≡)—and punctuation marks (), [], {}.
- Logical operators are tools for combining propositions or terms.
- Unary operators apply only to a single proposition. They never relate or connect two propositions. Negation, ~, is the only unary operator in **PL**.
- Binary operators relate or connect two propositions. All operators of **PL**, except negation, are binary operators.
- Negation, ~, is the logical operator used to translate 'not', 'it is not the case that', 'it is false that', and related terms. It is the only unary operator.
- Conjunction, •, is the logical operator used to translate 'and', 'but', and related terms. It is a binary operator. The formulas joined by a conjunction are called conjuncts.
- Disjunction, ∨, is the logical operator used to translate 'or', 'unless', and related terms. It is a binary operator. The formulas joined by a disjunction are called disjuncts.
- Material implication, ⊃, is the logical operator used to translate conditionals, 'if . . . then . . . statements', and related terms. It is a binary operator. The formula preceding the ⊃ is called the antecedent; the formula following the ⊃ is called the consequent. The order of the antecedent and consequent is significant; S ⊃ P is not logically equivalent to P ⊃ S.
- The biconditional, ≡, is the logical operator used for 'if-and-only-if', and related terms. It is a binary operator. The biconditional is a conjunction of a conditional with its converse; 'A ≡ B' is short for '(A ⊃ B) • (B ⊃ A)'.
- Formation rules specify how to combine the vocabulary of a language into well-formed formulas (wffs).
- A wff, or well-formed formula, is any logical symbol or string of symbols that are constructed properly. A wff is analogous to a grammatically correct sentence.
- **PL** has four formation rules. PL1: A single capital English letter is a wff. PL2: If α is a wff, so is ~α. PL3: If α and β are wffs, then so are: (α • β), (α ∨ β), (α ⊃ β), and (α ≡ β). PL4: These are the only ways to make wffs.
- An atomic formula of **PL** is any wff formed by a single use of PL1: a single capital English letter.
- A complex formula of **PL** is a wff formed in any way besides a single use of PL1.
- The main operator is the last operator added to a wff according to the formation rules.
- The semantics of **PL** specifies the rules for interpreting the symbols and formulas of the language.
- Bivalent logic is a two-valued logic. Every statement is interpreted as either true or false, and not both. The logic of **PL** is interpreted as bivalent.
- Compositionality is a semantic principle stating that the meaning of a complex sentence is determined by the meanings of its component parts. The language of **PL** is compositional.

- The truth value of a complex proposition is the truth value of its main operator.
- A truth table shows the truth value for a complex proposition given any truth values of its component propositions.

- The basic truth table is a way of representing the semantic rules governing each operator by showing the truth value of the operation, given any possible distribution of truth values of the component propositions.
- Negation, ~, is interpreted as true when the formula to which it applies is false; it is interpreted as false when the formula to which it applies is true.
- Conjunction, •, is interpreted as true only when both conjuncts are true; otherwise it is false.
- Disjunction, ∨, is interpreted as false only when both disjuncts are false; otherwise it is true.
- Material implication, ⊃, is interpreted as false only when the antecedent is true and the consequent is false; otherwise it is true.
- The biconditional, ≡, is interpreted as true when the component statements share the same truth value, (when they are both true or both false); otherwise it is false.
- A sufficient condition is something adequate or enough (though not necessarily required) for something else to obtain. In a material implication, the truth of the antecedent is the sufficient condition of the truth of the consequent.
- A necessary condition is something required (though not necessarily adequate or enough) for something else to obtain. In a material implication, the truth of the consequent is the necessary condition of the truth of the antecedent.
- A tautology is a proposition that is true in every row of its truth table. They are the logical truths of **PL**.
- Logical truths are propositions that are true on any interpretation.
- A contingency is a proposition that is true in some rows of its truth table and false in others.
- A contradiction is a proposition that is false in every row of its truth table.
- Two or more propositions are logically equivalent when they have the same truth values in every row of their truth tables.
- Two propositions are contradictory when they have opposite truth values in every row of their truth tables.
- Two or more propositions are consistent when they are true in at least one common row of their truth tables.
- Two propositions are inconsistent when there is no row of their truth tables in which both statements are true.
- A valuation is an assignment of truth values to simple component propositions.
- An invalid argument is one in which it is possible for true premises to yield a false conclusion.
- A valid argument has no row of its truth table in which the premises are true and the conclusion is false. In a valid argument, if the premises are true then the conclusion must be true.
- A counterexample to an argument is a valuation that makes the premises true and the conclusion false.
- If an argument has a counterexample, it is invalid. If an argument is invalid, it has at least one counterexample.
- A consistent valuation is an assignment of truth values to atomic propositions that makes a set of propositions all true. If it is not possible to make each statement true, then the set is inconsistent.

Chapter 2 Key Terms

antecedent
atomic formula
basic truth table
biconditional
binary operator
bivalent logic
complex formula
compositionality
conditional
conjunction
consequent
consistent
consistent valuation
contingency
contradiction
contradictory
counterexample
disjunction
formation rules
inconsistent pair
logical truths
logically equivalent
main operator
material implication
negation
neither
not both
operators
semantics
syntax
tautology
truth table
truth value
unary operator
unless
valid argument
valuation
wff

Chapter 2 Instructor Test Bank

(Please note this test bank is unique from the self quizzes we provide to students on the ARC and unique from the material in our Dashboard software)

Chapter 1 Multiple Choice

Section 2.1

Instructions: For exercises 1–5, use the following key to determine which of the translations of the given English argument to **PL** is best.

B: Brouwer is an intuitionist.
F: Frege is a logicist.
G: Gödel is a platonist.
H: Hilbert is a formalist.

1. It is not the case that either Frege is a logicist or Brouwer is an intuitionist. Gödel being a platonist is necessary and sufficient for Brouwer being an intuitionist. Hilbert is a formalist. So, Gödel is not a platonist; however, Hilbert is a formalist.

A. $\sim F \lor B$
$G \equiv B$
H $\quad$ / $\sim G \bullet H$

B. $\sim F \lor B$
$G \equiv B$
H $\quad$ / $\sim G \equiv H$

C. $\sim (F \lor B)$
$G \equiv B$
H $\quad$ / $\sim G \bullet H$

D. $\sim(F \vee B)$
$G \equiv B$
H / $\sim G \supset H$

E. $\sim(F \vee B)$
$G \equiv B$
H / $H \supset \sim G$

Answer: C

2. Hilbert is a formalist if, and only if, Gödel is a platonist. Hilbert is not a formalist and Brouwer is an intuitionist. Hilbert is a formalist if Frege is a logicist. Therefore, Frege is not a logicist and Gödel is not a platonist.

A. $H \equiv G$
$\sim H \bullet B$
$H \supset F$ / $\sim F \bullet \sim G$

B. $H \equiv G$
$\sim H \bullet B$
$F \supset H$ / $\sim F \bullet \sim G$

C. $G \supset H$
$\sim H \bullet B$
$F \supset H$ / $\sim F \bullet \sim G$

D. $H \supset G$
$\sim H \bullet B$
$F \supset H$ / $\sim F \bullet \sim G$

E. $H \supset G$
$\sim H \bullet B$
$H \supset F$ / $\sim F \bullet \sim G$

Answer: B

3. If Gödel is a platonist, then Frege is a logicist. If Frege is a logicist, Brouwer being an intuitionist is a sufficient condition for Hilbert being a formalist. Gödel is a platonist. Gödel is a platonist if Hilbert is a formalist. Therefore, Gödel is a platonist if Brouwer is an intuitionist.

A. $G \supset F$
$F \supset (B \equiv H)$
G
$H \supset G$ / $B \supset G$

B. $G \supset F$
$F \supset (B \equiv H)$
G
$G \supset H$ / $G \supset B$

C. $G \supset F$
$F \supset (H \supset B)$

G
H ⊃ G   / B ⊃ G

D. G ⊃ F
F ⊃ (B ⊃ H)
G
H ⊃ G   / B ⊃ G

E. G ⊃ F
F ⊃ (B ⊃ H)
G
G ⊃ H   / G ⊃ B

Answer: D

4. If Frege is a logicist, then Brouwer is an intuitionist. If Brouwer is an intuitionist, then Gödel is a platonist only if Hilbert is a formalist. Gödel is a platonist. Frege is a logicist. So, Hilbert is a formalist.

A. F ⊃ B
B ⊃ (H ⊃ G)
G
F   / H

B. F ⊃ B
B ⊃ (G ⊃ H)
G
F   / H

C. F ⊃ B
B ⊃ (G ≡ H)
G
F   / H

D. F ⊃ B
(B ⊃ G) ⊃ H
G
F   / H

E. F ⊃ B
(B ⊃ H) ⊃ G
G
F   / H

Answer: B

5. If Frege is a logicist and Brouwer is an intuitionist, then Hilbert is a formalist and Gödel is a platonist. Hilbert is not a formalist. Brouwer is an intuitionist. Either Frege is a logicist or both Gödel is not a platonist and Brouwer is an intuitionist. Therefore, Gödel is not a platonist.

A. F • [B ⊃ (H • G)]

~H
B
F ∨ (~G • B)   /~G

B. (F • B) ⊃ (H ⊃ G)
~H
B
(F ∨ ~G) • B   /~G

C. (F • B) ⊃ (H • G)
~H
B
F ∨ (G • B)   / ~G

D. (F • B) ⊃ (H • G)
~H
B
(F ∨ ~G) • B   / ~G

E. (F • B) ⊃ (H • G)
~H
B
F ∨ (~G • B)   / ~G

Answer: E

Instructions: For questions 6–12, use the following key to determine which English sentence best represents the given formula of **PL**.

A: Peirce studied logic.
B: James was a pluralist.
C: Dewey wrote about thirdness.
D: Dewey denigrated the quest for certainty.
E: Peirce emphasized education.

6. D ∨ ~E
A. If Dewey denigrated the quest for certainty, then Peirce did not emphasize education.
B. If Dewey denigrated the quest for certainty, then Peirce emphasized education.
C. Either Dewey denigrated the quest for certainty or Peirce emphasized education.
D. Either Dewey denigrated the quest for certainty or Peirce did not emphasize education.
E. Dewey denigrated the quest for certainty unless Peirce emphasized education.
Answer: D

7. ~(A • E)
A. It is not the case that Peirce either studied logic or emphasized education.
B. It is not the case that Peirce both studied logic and emphasized education.
C. Peirce neither studied logic nor emphasized education.
D. Peirce did not both not study logic and not emphasize education.
E. Peirce did not study logic and James was not a pluralist.
Answer: B

8. ~C ⊃ (A ≡ B)
A. If Dewey wrote about thirdness, then Peirce studied logic just in case James was a pluralist.
B. If Dewey did not write about thirdness, then Peirce studying logic is a necessary condition for James to be a pluralist.
C. Dewey did not write about thirdness provided that Peirce studied logic if, and only if, James was a pluralist.
D. Dewey not writing about thirdness entails Pierce studying logic and James being a pluralist.
E. If Dewey did not write about thirdness, then Peirce studied logic if, and only if, James was a pluralist.
Answer: E

9. [~D • (~E ⊃ ~C)]
A. Both Dewey did not denigrate the quest for certainty and Peirce did not emphasize education, given that Dewey did not write about thirdness.
B. Dewey not denigrating the quest for certainty and Peirce not emphasizing education are necessary conditions for Dewey not writing about thirdness.
C. Dewey did not denigrate the quest for certainty just in case if Peirce did not emphasize education then Dewey did not write about thirdness.
D. Dewey did not denigrate the quest for certainty if, and only if, Peirce not emphasizing education entails Dewey not writing about thirdness.
E. Dewey did not denigrate the quest for certainty and Dewey did not write about thirdness given that Peirce did not emphasize education.
Answer: E

10. (A ≡ B) ⊃ C
A. Peirce studying logic is necessary and sufficient for James being a pluralist, given that Dewey wrote about thirdness.
B. If Peirce studied logic just in case James was a pluralist, then Dewey did not write about thirdness.
C. If Peirce studied logic if, and only if, James was a pluralist, then Dewey wrote about thirdness.
D. If Peirce studying logic is necessary for James being a pluralist, then Dewey wrote about thirdness.
E. If James being a pluralist is necessary for Peirce studying logic, then Dewey wrote about thirdness.
Answer: C

11. ~(C ∨ D)
A. It is not the case that Dewey writing about thirdness entails his denigrating the quest for certainty.
B. Dewey did not both write about thirdness and denigrate the quest for certainty.
C. Dewey neither wrote about thirdness nor denigrated the quest for certainty.
D. Either Dewey did not write about thirdness or Dewey did not denigrate the quest for certainty.
E. Dewey not writing about thirdness entails his not denigrating the quest for certainty.
Answer: C

12. ~B ⊃ (~D • ~E)
A. If James was not a pluralist, then neither Dewey denigrated the quest for certainty nor Peirce emphasized education.
B. Dewey did not denigrate the quest for certainty and Peirce did not emphasize education given that James was not a pluralist.
C. James not being a pluralist entails neither Dewey denigrated the quest for certainty nor Peirce did not emphasize education.
D. James was not a pluralist provided that Dewey did not denigrate the quest for certainty and Peirce did not emphasize education.
E. If James was not a pluralist, then either Dewey did not denigrate the quest for certainty or Peirce did not emphasize education.

Answer: B

Section 2.2

Instructions: For each of the following questions, determine whether the given formula is a wff or not. If it is a wff, indicate its main operator.

1. ~A ∨ (B • D)
A. It's a wff. The main operator is the ~.
B. It's a wff. The main operator is the ∨.
C. It's a wff. The main operator is the •.
D. Not a wff
Answer: B

2. I • E ⊃ ~F
A. It's a wff. The main operator is the •.
B. It's a wff. The main operator is the ⊃.
C. It's a wff. The main operator is the ~.
D. Not a wff
Answer: D

3. {~R ⊃ [(Q • P) ⊃ S]}
A. It's a wff. The main operator is the ~.
B. It's a wff. The main operator is the first ⊃, reading left to right.
C. It's a wff. The main operator is the •.
D. It's a wff. The main operator is the second ⊃, reading left to right.
E. Not a wff
Answer: B

4. [(P ≡ Q) • ~Q] ⊃ (P ⊃ R)
A. It's a wff. The main operator is the ≡.
B. It's a wff. The main operator is the •.
C. It's a wff. The main operator is the ~.
D. It's a wff. The main operator is the first ⊃, reading left to right.
E. Not a wff
Answer: D

5. [A ∨ B • C] ⊃ (A ∨ C)
A. It's a wff. The main operator is the first ∨, reading left to right.
B. It's a wff. The main operator is the •.
C. It's a wff. The main operator is the ⊃.
D. It's a wff. The main operator is the second ∨, reading left to right.
E. Not a wff
Answer: E

6. P ⊃ (Q ⊃ R) ⊃ [(P • ~R) ⊃ ~Q]
A. It's a wff. The main operator is the first ∨, reading left to right.
B. It's a wff. The main operator is the ≡.
C. It's a wff. The main operator is the second ∨, reading left to right.
D. It's a wff. The main operator is the •.
E. Not a wff

Answer: E

7. [(D ⊃ ~E) • (F ⊃ E)] ⊃ [D ⊃ (~F ∨ G)]
A. It's a wff. The main operator is the first ⊃, reading left to right.
B. It's a wff. The main operator is the second ⊃, reading left to right.
C. It's a wff. The main operator is the third ⊃, reading left to right.
D. It's a wff. The main operator is the fourth ⊃, reading left to right.
E. Not a wff
Answer: C

8. ~{[(H ⊃ I) ⊃ ~(I ∨ ~J)] ⊃ (~H ⊃ J)}
A. It's a wff. The main operator is the first ~, reading left to right.
B. It's a wff. The main operator is the ≡.
C. It's a wff. The main operator is the second ∨, reading left to right.
D. It's a wff. The main operator is the •.
E. Not a wff
Answer: A

9. ~[(R • S) ⊃ U] ⊃ {{~U ⊃ [R ⊃ (S ⊃ T)]}
A. It's a wff. The main operator is the first ⊃, reading left to right.
B. It's a wff. The main operator is the first ~, reading left to right.
C. It's a wff. The main operator is the second ⊃, reading left to right.
D. It's a wff. The main operator is the third ⊃, reading left to right.
E. Not a wff
Answer: E

10. [(W ⊃ X) • (Y ∨ ~X)] ≡ [~(Z ∨ Y) ⊃ ~W]
A. It's a wff. The main operator is the •.
B. It's a wff. The main operator is the ≡.
C. It's a wff. The main operator is the first ~, reading left to right.
D. It's a wff. The main operator is the second ⊃, reading left to right.
E. Not a wff
Answer: B

Section 2.3

Assume A, B, C are true; X, Y, Z are false; and P and Q are unknown. Evaluate the truth value of each complex expression.

1. ~B ⊃ Y
A. True
B. False
C. Unknown
Answer: True

2. ~X ≡ A
A. True
B. False
C. Unknown
Answer: True

3. X ∨ [A • (B ⊃ Y)]

A. True
B. False
C. Unknown
Answer: False

4. X ∨ [A • (Y ⊃ B)]
A. True
B. False
C. Unknown
Answer: True

5. ~Y ⊃ [A ≡ (Y • B)]
A. True
B. False
C. Unknown
Answer: False

6. X ⊃ [(~X ∨ A) ⊃ X]
A. True
B. False
C. Unknown
Answer: True

7. (Z ∨ ~A) ≡ [(A ∨ ~Z) ⊃ (X ≡ ~X)]
A. True
B. False
C. Unknown
Answer: True

8. [(Y ⊃ ~Y) ⊃ (B ⊃ ~B)] ⊃ [(B ∨ Y) ≡ (~B ∨ ~Y)]
A. True
B. False
C. Unknown
Answer: True

9. ~{~[(~A ∨ ~X) • ~A] • ~X}
A. True
B. False
C. Unknown
Answer: False

10. {X ∨ [C • (Y ⊃ B)]} ⊃ {Z ⊃ [Z ⊃ (Z ⊃ Z)]}
A. True
B. False
C. Unknown
Answer: True

11. Q • (~A ≡ Q)
A. True
B. False
C. Unknown

Answer: False

12. Q • (~A • ~Q)
A. True
B. False
C. Unknown
Answer: False

13. ~Q ∨ (~X ∨ Q)
A. True
B. False
C. Unknown
Answer: True

14. (C ∨ X) ⊃ (Q ∨ A)
A. True
B. False
C. Unknown
Answer: True

15. [(P ⊃ X) ⊃ P] ⊃ P
A. True
B. False
C. Unknown
Answer: True

16. (Y ∨ P) ⊃ (B • P)
A. True
B. False
C. Unknown
Answer: True

17. (~P ⊃ P) ∨ (A ⊃ P)
A. True
B. False
C. Unknown
Answer: Unknown

18. (~P ⊃ P) ∨ (P ⊃ A)
A. True
B. False
C. Unknown
Answer: True

19. ~(Q ⊃ C) ∨ (Z • ~X)
A. True
B. False
C. Unknown
Answer: False

20. ~[(Z ⊃ B) • (P ⊃ C)] ∨ [(X • Y) ≡ A]

A. True
B. False
C. Unknown
Answer: False

21. (P ⊃ ~Q) ∨ ~P
A. True
B. False
C. Unknown
Answer: Unknown

22. (P ≡ Q) ∨ (P ≡ ~Q)
A. True
B. False
C. Unknown
Answer: True

23. [(P ∨ Q) ⊃ X] ≡ ~(P ∨ Q)

A. True
B. False
C. Unknown
Answer: True

24. [(P • Q) ∨ (~P • Q)] ∨ [(P • ~Q) ∨ (~P • ~Q)]
A. True
B. False
C. Unknown
Answer: True

25. ~[(A ⊃ P) ∨ (A ⊃ Q)] • (P • Q)
A. True
B. False
C. Unknown
Answer: False

Section 2.4

Note: the solutions to most of the multiple choice questions in these sections use what I call the standard assignment of truth values to atomic propositions. The standard assignment of truth values assigns the values given here to the variables in the wffs in the exercises, when read left to right. So, the first variable in the formula read left to right gets the α assignment; the second variable in the formula read left to right (if any) gets the β assignment; the third variable in the formula read left to right (if any) gets the γ assignment; and the fourth variable in the formula read left to right (if any) gets the δ assignment.

For exercises with only one propositional variable, the standard assignment is:

α
1
0

For exercises with two propositional variables, the standard assignment is:

α	β
1	1
1	0
0	1
0	0

For exercises with three propositional variables, the standard assignment is:

α	β	γ
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

For exercises with four propositional variables, the standard assignment is:

α	β	γ	δ
1	1	1	1
1	1	1	0
1	1	0	1
1	1	0	0
1	0	1	1
1	0	1	0
1	0	0	1
1	0	0	0
0	1	1	1
0	1	1	0

0	1	0	1
0	1	0	0
0	0	1	1
0	0	1	0
0	0	0	1
0	0	0	0

0	1	0	1

Instructions: For each of the given propositions, determine which of the given sequences properly describes the column under the main operator, given the standard assignment of truth values to atomic propositions.

1. (B ⊃ B) ∨ ~B
   A. 11
   B. 10
   C. 01
   D. 00
   Answer: A

2. (D ≡ ~D) ≡ (~D ≡ D)
   A. 11
   B. 10
   C. 01
   D. 00
   Answer: A

3. ~[F • (F ⊃ F)]
   A. 11
   B. 10
   C. 01
   D. 00
   Answer: C

4. U ≡ (Z ⊃ U)
   A. 1100
   B. 0011
   C. 0001
   D. 1110
   Answer: D

5. (I ⊃ J) ∨ (J ⊃ I)
   A. 1111
   B. 1100
   C. 1010
   D. 0000
   Answer: A

6. (K ⊃ L) ≡ (~K ⊃ L)
A. 1111
B. 1010
C. 0110
D. 1000
E. 0001
Answer: B

7. (M ∨ N) • (M ∨ ~N)
A. 1111
B. 1110
C. 1010
D. 1100
E. 1000
Answer: D

8. (V ≡ W) • (V ≡ ~W)
A. 1111
B. 1001
C. 0110
D. 1000
E. 0000
Answer: E

9. (~X • Y) ⊃ (X • Y)
A. 1111
B. 1101
C. 1011
D. 1000
E. 0010
Answer: B

10. A ≡ [(~B ∨ A) ⊃ ~A]
A. 1100
B. 1001
C. 0011
D. 0001
E. 0000
Answer: E

11. (~C ⊃ ~D) ⊃ [D ⊃ (C ⊃ D)]
A. 1111
B. 1101
C. 1011
D. 0011
E. 0101
Answer: A

12. [E ∨ ~(E • ~F)] ≡ [F ⊃ ~(E ∨ ~F)]
A. 1111
B. 1000

C. 0010
D. 0111
E. 0000
Answer: D

13. G ⊃ {(H ⊃ ~G) ⊃ [(G • ~H) ∨ (H • ~G)]}
A. 1111
B. 1100
C. 0111
D. 0100
E. 0000
Answer: A

14. [J ⊃ (K ∨ L)] ⊃ [~L ⊃ (K ∨ J)]
A. 1111 1111
B. 1111 1110
C. 1110 1111
D. 1111 1100
E. 0101 0101
Answer: B

15. [M ∨ (N • O)] ∨ [(M • N) ∨ (~M • N)]
A. 1111 1111
B. 1100 1100
C. 1111 1100
D. 1100 0000
E. 0000 1100
Answer: C

16. [P ⊃ (~R ⊃ Q)] ∨ [(Q ∨ R) ∨ (~P • R)]
A. 1111 1111
B. 1110 1111
C. 1110 1110
D. 0000 0011
E. 0000 1100
Answer: B

17. (S ⊃ ~T) ⊃ {[V ⊃ (~S • T)] ∨ [V ⊃ (~T • S)]}
A. 0011 1111
B. 0011 0000
C. 0101 1101
D. 1111 0101
E. 1111 1101
Answer: E

18. {[X ≡ ~(Y ∨ Z)] • [Y ≡ ~(X ∨ Z)]} ⊃ [(X • Z) ∨ ~(X • ~Z)]
A. 1111 1111
B. 1110 1111
C. 1010 1111
D. 0001 1110
E. 0001 0110
Answer: B

19. [(E ≡ ~F) ∨ (G ≡ ~H)] ∨ [(~E • ~F) • (~G • ~H)]
A. 1001 1111 1111 1001
B. 1001 1111 1111 1000
C. 0110 1111 1111 0110
D. 0110 1111 1111 0111
E. 0110 1111 1111 0000
Answer: D

20. [(A ⊃ B) ≡ (~C ⊃ D)] ⊃ {[A • (B ∨ D)] ≡ [D • (B ∨ ~D)]}
A. 1111 1010 0000 0000
B. 1110 0001 1110 1110
C. 1011 1111 0101 1111
D. 1100 0100 1100 0100
E. 1110 1111 0011 1010
Answer: C

Section 2.5

Instructions: For questions 1–10, construct a complete truth table for each of the following propositions. Then, using the truth table, classify each proposition as a tautology, a contingency, or a contradiction.

1. (I ⊃ ~I) ⊃ [I ⊃ (I ⊃ ~I)]
A. Tautology
B. Contingency
C. Contradiction
Answer: A

2. (G • ~G) ⊃ (G ∨ ~G)
A. Tautology
B. Contingency
C. Contradiction
Answer: A

3. (~A • B) ≡ (B ⊃ A)
A. Tautology
B. Contingency
C. Contradiction
Answer: C

4. (A • ~B) • (B ∨ ~A)
A. Tautology
B. Contingency
C. Contradiction
Answer: C

5. (A ≡ ~B) ⊃ ~(B • A)
A. Tautology
B. Contingency
C. Contradiction
Answer: A

6. (C ⊃ ~D) ∨ (~D ⊃ C)
A. Tautology
B. Contingency
C. Contradiction
Answer: A

7. (G ≡ H) ⊃ ~[(G • H) ∨ (~G • ~H)]
A. Tautology
B. Contingency
C. Contradiction
Answer: B

8. [N ⊃ (O ⊃ P)] ⊃ (N ⊃ O)
A. Tautology
B. Contingency
C. Contradiction
Answer: B

9. [(Q ⊃ R) ⊃ (R ⊃ S)] ≡ ~(~Q ∨ S)
A. Tautology
B. Contingency
C. Contradiction
Answer: B

10. [(P ∨ Q) • (R • S)] ⊃ [(P • R) ∨ (Q • S)]
A. Tautology
B. Contingency
C. Contradiction
Answer: A

Instructions: For questions 11–20, construct a complete truth table for each of the following pairs of propositions. Then, using the truth table, determine whether the statements are logically equivalent or contradictory. If neither, determine whether they are consistent or inconsistent.

11. A ⊃ ~A and ~A ⊃ A
A. Logically equivalent
B. Contradictory
C. Neither logically equivalent nor contradictory, but consistent
D. Inconsistent
Answer: B

12. D • ~E and ~(E ⊃ ~D)
A. Logically equivalent
B. Contradictory
C. Neither logically equivalent nor contradictory, but consistent
D. Inconsistent
Answer: D

13. G ≡ ~H and (H • ~G) ∨ (G • ~H)
A. Logically equivalent
B. Contradictory

C. Neither logically equivalent nor contradictory, but consistent
D. Inconsistent
Answer: A

14. J ⊃ (K ⊃ ~J) and J • K
A. Logically equivalent
B. Contradictory
C. Neither logically equivalent nor contradictory, but consistent
D. Inconsistent
Answer: B

15. (D ⊃ E) • (E ⊃ D) and (E ∨ ~D) • (D ∨ ~E)
A. Logically equivalent
B. Contradictory
C. Neither logically equivalent nor contradictory, but consistent
D. Inconsistent
Answer: A

16. R ⊃ (~S ⊃ R) and ~S ⊃ ~(R ∨ ~R)
A. Logically equivalent
B. Contradictory
C. Neither logically equivalent nor contradictory, but consistent
D. Inconsistent
Answer: C

17. ~I ⊃ (G ∨ ~H) and I • (~H • G)
A. Logically equivalent
B. Contradictory
C. Neither logically equivalent nor contradictory, but consistent
D. Inconsistent
Answer: C

18. F • (~G ∨ H) and F ≡ (~H • G)
A. Logically equivalent
B. Contradictory
C. Neither logically equivalent nor contradictory, but consistent
D. Inconsistent
Answer: D

19. L ∨ (M ⊃ N) and ~L • (M • ~N)
A. Logically equivalent
B. Contradictory
C. Neither logically equivalent nor contradictory, but consistent
D. Inconsistent
Answer: B

20. P ⊃ [Q ⊃ (~S ⊃ ~R)] and ~P ∨ ~[(Q • ~S) • R]
A. Logically equivalent
B. Contradictory
C. Neither logically equivalent nor contradictory, but consistent
D. Inconsistent

Answer: A

Section 2.6

Instructions: Construct a complete truth table for each of the following arguments. Then, using the truth table, determine whether the argument is valid or invalid. If the argument is invalid, choose an option which presents a counterexample. (There may be other counterexamples as well.)

1. A / A • ~A

A. Valid
B. Invalid. Counterexample when A is true
C. Invalid. Counterexample when A is false
D. Invalid. Counterexamples when A is either true or false.
Answer: B

2. B ∨ ~C
~~C / ~B

A. Valid
B. Invalid. Counterexample when B and C are true
C. Invalid. Counterexample when B is true and C is false
D. Invalid. Counterexample when C is true and B is false
E. Invalid. Counterexample when B and C are false
Answer: B

3. E ≡ F
F ⊃ ~E / E

A. Valid
B. Invalid. Counterexample when E and F are true
C. Invalid. Counterexample when E is true and F is false
D. Invalid. Counterexample when E is false and F is true
E. Invalid. Counterexample when E and F are false
Answer: E

4. I ⊃ J
~(J • I) / J ⊃ I

A. Valid
B. Invalid. Counterexample when I and J are true
C. Invalid. Counterexample when I is true and J is false
D. Invalid. Counterexample when I is false and J is true
E. Invalid. Counterexample when I and J are false
Answer: D

5. P ≡ ~Q
~(Q ∨ ~P)
Q ⊃ ~Q / P

A. Valid
B. Invalid. Counterexample when P and Q are true
C. Invalid. Counterexample when P is true and Q is false
D. Invalid. Counterexample when P is false and Q is true
E. Invalid. Counterexample when P and Q are false
Answer: A

6. E ∨ ~F
E ⊃ ~E
F / E
A. Valid
B. Invalid. Counterexample when E and F are true
C. Invalid. Counterexample when E is true and F is false
D. Invalid. Counterexample when E is false and F is true
E. Invalid. Counterexample when E and F are false
Answer: A

7. ~J ⊃ K
~K ∨ (J • K)
~K ⊃ K / ~J
A. Valid
B. Invalid. Counterexample when J and K are true
C. Invalid. Counterexample when J is true and K is false
D. Invalid. Counterexample when J is false and K is true
E. Invalid. Counterexample when J and K are false
Answer: B

8. G ⊃ H
~I ⊃ ~G / G ⊃ (H • I)
A. Valid
B. Invalid. Counterexample when G, H, and I are true
C. Invalid. Counterexample when H and G are true and I is false
D. Invalid. Counterexample when I and G are true and H is false
E. Invalid. Counterexample when H and I are true and G is false
Answer: A

9. M ⊃ (N • ~O)
N ⊃ (O ∨ M) / O ⊃ ~N
A. Valid
B. Invalid. Counterexample when M, N, and O is true
C. Invalid. Counterexample when M and N are true and O is false
D. Invalid. Counterexample when M is false and N and O are true
E. Invalid. Counterexample when M and N are false and O is true
Answer: D

10. P ⊃ ~Q
R ⊃ P / R ⊃ ~Q
A. Valid
B. Invalid. Counterexample when R, P, and Q are true
C. Invalid. Counterexample when R and P are true and Q is false
D. Invalid. Counterexample when Q is true and R and P are false
E. Invalid. Counterexample when R, P, and Q are false
Answer: A

11. S
~T • U / ~(T • U) • S
A. Valid
B. Invalid. Counterexample when T, U, and S are true
C. Invalid. Counterexample when T and S are true and U is false

D. Invalid. Counterexample when U and S are true and T is false
E. Invalid. Counterexample when S is true and T and U are false
Answer: A

12. X / X ∨ [Y ≡ (Z • ~X)]
A. Valid
B. Invalid. Counterexample when X, Y, and Z are true
C. Invalid. Counterexample when X and Y are true and Z is false
D. Invalid. Counterexample when Z is true and Z and Y are false
E. Invalid. Counterexample when X, Y, and Z are false
Answer: A

13. ~A ≡ B
(~A ∨ B) ⊃ C / C
A. Valid
B. Invalid. Counterexample when A and B are true and C is false
C. Invalid. Counterexample when A is true and B and C are false
D. Invalid. Counterexample when B is true and A and C are false
E. Invalid. Counterexample when A, B, and C are false
Answer: C

14. D ⊃ ~E
~E ⊃ F / ~F ⊃ D
A. Valid
B. Invalid. Counterexample when E and F are true and D is false
C. Invalid. Counterexample when E is true and D and F are false
D. Invalid. Counterexample when F is true and D and E are false
E. Invalid. Counterexample when D, E, and F are false
Answer: C

15. (G ∨ ~H) ∨ ~~I / G ∨ ~H
A. Valid
B. Invalid. Counterexample when G, H, and I are true
C. Invalid. Counterexample when H and I are true and G is false
D. Invalid. Counterexample when H is true and G and I are false
E. Invalid. Counterexample when I is true and G and are false
Answer: C

16. J ≡ (~K • L)
L ⊃ J / L ⊃ K
A. Valid
B. Invalid. Counterexample when J and L are true and K is false
C. Invalid. Counterexample when J is true and L and K are false
D. Invalid. Counterexample when L is true and J and K are false
E. Invalid. Counterexample when J, K, and L are false
Answer: B

17. M ⊃ N
O ⊃ M
~N
M ⊃ ~O / M

A. Valid
B. Invalid. Counterexample when N and O are true and M is false
C. Invalid. Counterexample when N is true and O and M are false
D. Invalid. Counterexample when O is true and N and M are false
E. Invalid. Counterexample when M, N, and O are false
Answer: E

18. P ∨ ~Q
P ⊃ R
~R / ~Q
A. Valid
B. Invalid. Counterexample when P, Q, and R are true
C. Invalid. Counterexample when P and Q are true and R is false
D. Invalid. Counterexample when Q and R are true and P is false
E. Invalid. Counterexample when Q is true and P and R are false
Answer: A

19. S ⊃ (T ∨ U)
~T
T ∨ S / U • S
A. Valid
B. Invalid. Counterexample when S and U are true and T is false
C. Invalid. Counterexample when S is true and T and U are false
D. Invalid. Counterexample when U is true and S and T are false
E. Invalid. Counterexample when S, T, and U are false
Answer: A

20. X ∨ Z
X ⊃ (Y ∨ Z)
~Z ⊃ Y / Y
A. Valid
B. Invalid. Counterexample when X, Y, and Z are true
C. Invalid. Counterexample when X is true and Y and Z are false
D. Invalid. Counterexample when Z is true and X and Y are false
E. Invalid. Counterexample when X, Y, and Z are false
Answer: D

Section 2.7

Instructions: For 1–10, use indirect truth tables to determine whether each of the following arguments is valid. If the argument is invalid, choose an option which presents a counterexample. (There may be other counterexamples as well.)

1. (U • ~V) ∨ W
W ≡ ~V / ~U
A. Valid
B. Invalid. Counterexample when U, V, and W are true
C. Invalid. Counterexample when U and V are true and W is false
D. Invalid. Counterexample when U and W are true and V is false
E. Invalid. Counterexample when U is true and V and W are false
Answer: D

2. (A ≡ B) • (A ∨ C)
C / B
A. Valid
B. Invalid. Counterexample when A and C are true and B is false
C. Invalid. Counterexample when A is true and B and C are false
D. Invalid. Counterexample when C is true and A and B are false
E. Invalid. Counterexample when A, B, and C are false
Answer: D

3. R ⊃ S
V ⊃ W
X ⊃ (S • ~W) / (R • V) ≡ X
A. Valid
B. Invalid. Counterexample when R, S, V, W, and X are true
C. Invalid. Counterexample when R, S, V, and X are true and W is false
D. Invalid. Counterexample when R, S, and X are true and V and W are false
E. Invalid. Counterexample when R and S are true and X, V, and W are false
Answer: D

4. D ⊃ (E ∨ F)
D ⊃ (G ∨ F)
~(F ∨ H) / D ⊃ (E • G)
A. Valid
B. Invalid. Counterexample when D, E and F are true and G and H are false
C. Invalid. Counterexample when D and E are true and F, G, and H are false
D. Invalid. Counterexample when D, G, and H are true and E and F are false
E. Invalid. Counterexample when D and G are true and E, F, and H are false
Answer: A

5. G ⊃ (J • ~K)
(I ⊃ H) • G
H ⊃ (K ∨ I)
J • ~I / I ∨ K
A. Valid
B. Invalid. Counterexample when G, H, I, J, and K are true
C. Invalid. Counterexample when G, H, J, and I are true and K is false
D. Invalid. Counterexample when G, H, and J are true and I and K are false
E. Invalid. Counterexample when G and J are true and H, I, and K are false
Answer: E

6. (P ∨ Q) ⊃ R
(S ∨ ~U) ⊃ (~R • ~W)
S ⊃ (P • T) / ~S
A. Valid
B. Invalid. Counterexample when P, Q, R, S, T, and U are true and W is false
C. Invalid. Counterexample when P, S, T and W are true and Q, R, and U are false
D. Invalid. Counterexample when S, T, and U are true and P, Q, R, and W are false
E. Invalid. Counterexample when S, T, and U are true and P, Q, R, and W are false
Answer: A

7. (W ∨ X) ⊃ ~(Y • ~Z)

~(Y • W) ⊃ ~Z
~(W ∨ X) ∨ Z / (Z ∨ W) ⊃ (X • Y)

A. Valid
B. Invalid. Counterexample when W is true and X, Y and Z are false
C. Invalid. Counterexample when W and Y are true and X and Z are false
D. Invalid. Counterexample when Y and Z are true and W and X are false
E. Invalid. Counterexample when W, Y, and Z are true and X is false
Answer: E

8. A ⊃ B
C ⊃ ~B
~A ⊃ D
E ⊃ ~F
~E ⊃ G
~F ⊃ C / D ∨ G

A. Valid
B. Invalid. Counterexample when A, B, C, E, and F are true and D and G are false
C. Invalid. Counterexample when A, B, and F are true and C, D, E, and G are false
D. Invalid. Counterexample when B and E are true and A, C, D, F, and G are false
E. Invalid. Counterexample when C is true and A, B, D, E, F, and G are false
Answer: A

9. (H ⊃ ~I) • ~J
~I ⊃ [(J ∨ K) • (J ∨ H)]
~H ∨ (K ⊃ L) / L ⊃ ~H

A. Valid
B. Invalid. Counterexample when H, I, K, and L are true and I is false
C. Invalid. Counterexample when H, I, and L are true and J and K are false
D. Invalid. Counterexample when H, K, and L are true and I and J are false
E. Invalid. Counterexample when H, J, and L are true and I and K are false
Answer: D

10. (M • N) ⊃ (O • P)
(M • O) ⊃ (Q ∨ ~P)
R ⊃ (Q ⊃ N) / (~M • ~N) ∨ (O • P)

A. Valid
B. Invalid. Counterexample when M, P, and Q are true and N, O, and R are false
C. Invalid. Counterexample when M, N P, Q, and R are true and O is false
D. Invalid. Counterexample when M, N, Q, and R are true and O and P are false
E. Invalid. Counterexample when M, N, O, Q, and R are true and P is false
Answer: B

For 11–20, use indirect truth tables to determine, for each given set of propositions, whether it is consistent. If the set is consistent, choose an option with a consistent valuation. (There may be other consistent valuations.)

11. X ≡ Y
Y ≡ ~Z
Z ≡ ~X

A. Inconsistent

B. Consistent. Consistent valuation when X, Y, and Z are true
C. Consistent. Consistent valuation when X and Y are true and Z is false
D. Consistent. Consistent valuation when X and Z are true and Y is false
E. Consistent. Consistent valuation when Y and Z are true and X is false
Answer: C

12. (G • H) ≡ I
G ≡ ~J
J • H
(I ∨ K) ⊃ G
A. Inconsistent
B. Consistent. Consistent valuation when H and J are true and G, I, and K are false
C. Consistent. Consistent valuation when H, I, and J are true and G and K are false
D. Consistent. Consistent valuation when H, J, and K are true and G and I are false
E. Consistent. Consistent valuation when H, I, J, and K are true and G is false
Answer: B

13. L ⊃ M
~N ⊃ (O ∨ L)
N ∨ P
~M ∨ ~P
A. Inconsistent
B. Consistent. Consistent valuation when L, M, and O are true and N and P are false
C. Consistent. Consistent valuation when M, N, and O are true and L and P are false
D. Consistent. Consistent valuation when N and P are true and L, M, and O are false
E. Consistent. Consistent valuation when L, O, and P are true and M and N are false
Answer: C

14. F • (A ⊃ D)
E ∨ ~B
~[C ⊃ (D ∨ F)]
A ∨ (B • D)
E ⊃ A
A. Inconsistent
B. Consistent. Consistent valuation when F and D are true and A, B, C, and E are false
C. Consistent. Consistent valuation when A, B, D, E, and F are true and C is false
D. Consistent. Consistent valuation when A, B, E, and F are true and C and D are false
E. Consistent. Consistent valuation when B and C are true and A, D, E, and F are false
Answer: A

15. (L • N) ∨ I
L ≡ ~K
K ⊃ (I ≡ ~M)
(J ∨ K) • ~N
A. Inconsistent
B. Consistent. Consistent valuation when I, J, and K are true and L, M, and N are false
C. Consistent. Consistent valuation when J, K, and M are true and I, L, and N are false
D. Consistent. Consistent valuation when J, L, M, and N are true and I and L are false
E. Consistent. Consistent valuation when I, J, L, and N are true and K and M are false
Answer: B

16. P ≡ (Q • ~R)
P ⊃ ~Q
(~P • Q) ⊃ S
A. Inconsistent
B. Consistent. Consistent valuation when P, Q, and S are true and R is false
C. Consistent. Consistent valuation when P and Q are true and S and R are false
D. Consistent. Consistent valuation when Q, R and S are true and P is false
E. Consistent. Consistent valuation when Q and R are false and P and S are true
Answer: D

17. A ∨ B
(C • D) ≡ (E • F)
F ⊃ ~A
D ⊃ B
A. Inconsistent
B. Consistent. Consistent valuation when A, C, and E are true and B, D, and F are false
C. Consistent. Consistent valuation when A, D, and E are true and B, C, and F are false
D. Consistent. Consistent valuation when B, C, and F are true and A, D, and E are false
E. Consistent. Consistent valuation when B, D, and E are true and A, C, and F are false
Answer: E

18. A ⊃ (B • C)
D ⊃ (E ∨ F)
E ⊃ ~B
F ⊃ ~C
A ∨ D
A. Inconsistent
B. Consistent. Consistent valuation when A, B, C, and D are true and E and F are false
C. Consistent. Consistent valuation when A, B, C, and F are true and D and E are false
D. Consistent. Consistent valuation when A, B, C, and E are true and D and F are false
E. Consistent. Consistent valuation when D, E, and F are true and A, B, and C are false
Answer: E

19. P ⊃ Q
R ⊃ Q
~S ⊃ (P ∨ R)
(Q • S) ⊃ T
~(U ⊃ T)
~W ≡ S
A. Inconsistent
B. Consistent. Consistent valuation when P, Q, S, U, and W are true and R and T are false
C. Consistent. Consistent valuation when Q, R, and U are true and P, S, T, and W are false
D. Consistent. Consistent valuation when P, Q, U, and W are true and R, S, and T are false
E. Consistent. Consistent valuation when S and U are true and P, Q, R, T, and W are false
Answer: D

20. D ≡ (A • B)
D ∨ (~E • F)
~E ⊃ A
F ⊃ B

A ∨ B
~A ∨ ~B

A. Inconsistent
B. Consistent. Consistent valuation when A and F are true and B, D, and E are false
C. Consistent. Consistent valuation when A, D, and E are true and B and F are false
D. Consistent. Consistent valuation when B, D, and E are true and A and F are false
E. Consistent. Consistent valuation when B and F are true and A, D, and E are false
Answer: A

Chapter 2 Traditional

Section 2.1

Instructions: For exercises 1–5, use the following key to translate each of the given arguments into symbols of **PL**.

B: Brouwer is an intuitionist.
F: Frege is a logicist.
G: Gödel is a platonist.
H: Hilbert is a formalist.

1. It is not the case that either Frege is a logicist or Brouwer is an intuitionist. Gödel being a platonist is necessary and sufficient for Brouwer being an intuitionist. Hilbert is a formalist. So, Gödel is not a platonist; however, Hilbert is a formalist.

Solution:
~(F ∨ B)
G ≡ B
H / ~G • H

2. Hilbert is a formalist if, and only if, Gödel is a platonist. Hilbert is not a formalist and Brouwer is an intuitionist. Hilbert is a formalist if Frege is a logicist. Therefore, Frege is not a logicist and Gödel is not a platonist.

Solution:
H ≡ G
~H • B
F ⊃ H / ~F • ~G

3. If Gödel is a platonist, then Frege is a logicist. If Frege is a logicist, Brouwer being an intuitionist is a sufficient condition for Hilbert being a formalist. Gödel is a platonist. Gödel is a platonist if Hilbert is a formalist. Therefore, Gödel is a platonist if Brouwer is an intuitionist.

Solution:
G ⊃ F
F ⊃ (B ⊃ H)
G
H ⊃ G / B ⊃ G

4. If Frege is a logicist, then Brouwer is an intuitionist. If Brouwer is an intuitionist, then Gödel is a platonist only if Hilbert is a formalist. Gödel is a platonist. Frege is a logicist. So, Hilbert is a formalist.

Solution:
F ⊃ B
B ⊃ (G ⊃ H)
G
F / H

5. If Frege is a logicist and Brouwer is an intuitionist, then Hilbert is a formalist and Gödel is a platonist. Hilbert is not a formalist. Brouwer is an intuitionist. Either Frege is a logicist or both Gödel is not a platonist and Brouwer is an intuitionist. Therefore, Gödel is not a platonist.

Solution: (F • B) ⊃ (H • G)
~H
E
F ∨ (~G • B) / ~G

Instructions: For exercises 6–7, use the following key to translate each of the arguments of **PL** into natural, English arguments.

A: Peirce studied logic.
B: James was a pluralist.
C: Dewey wrote about thirdness.
D: Dewey denigrated the quest for certainty.
E: Peirce emphasized education.

6. 1. ~C ⊃ (A ≡ B)
2. D ∨ ~E
3. ~D • (~E ⊃ ~C) / B ⊃ A

Solution: If Dewey didn't write about thirdness, then Peirce studied logic if, and only if, James was a pluralist. Either Dewey denigrated the quest for certainty or Peirce did not emphasize education. Dewey did not denigrate the quest for certainty, and Dewey did not write about thirdness given that Peirce did not emphasize education. So, James being a pluralist entails that Peirce studied logic.

7. 1. (A ≡ B) ⊃ C
2. ~(C ∨ D)
3. ~(A • E)
4. ~B / ~D • ~E

Solution: If Peirce studied logic just in case James was a pluralist, then Dewey wrote about thirdness. Dewey neither wrote about thirdness nor denigrated the quest for certainty. It's not the case that Peirce both studied logic and emphasized education. James was not a pluralist. So, Dewey did not denigrate the quest for certainty and Peirce didn't emphasize education.

Section 2.2

Instructions: For each of the following questions, determine whether the given formulas is a wff or not. If it is a wff, indicate its main operator.

1. ~A ∨ (B • D) — It's a wff. The main operator is the ∨.

2. I • E ⊃ ~F — Not a wff

3. {~R ⊃ [(Q • P) ⊃ S]} — It's a wff. The main operator is the first ⊃, reading left to right.

4. [(P ≡ Q) • ~Q] ⊃ (P ⊃ R) — It's a wff. The main operator is the first ⊃, reading left to right.

5. [A ∨ B • C] ⊃ (A ∨ C) — Not a wff

6. P ⊃ (Q ⊃ R) ⊃ [(P • ~R) ⊃ ~Q] — Not a wff

7. [(D ⊃ ~E) • (F ⊃ E)] ⊃ [D ⊃ (~F ∨ G)] — It's a wff. The main operator is the third ⊃, reading left to right.

8. ~{[(H ⊃ I) ⊃ ~(I ∨ ~J)] ⊃ (~H ⊃ J)} — It's a wff. The main operator is the first ~, reading left to right.

9. [(R • S) ⊃ U] ⊃ {{~U ⊃ [R ⊃ (S ⊃ T)]} — Not a wff

10. [(W ⊃ X) • (Y ∨ ~X)] ≡ [~(Z ∨ Y) ⊃ ~W] — It's a wff. The main operator is the ≡.

Section 2.3

Instructions: Assume A, B, C are true; X, Y, Z are false; and P and Q are unknown. Evaluate the truth value of each complex expression.

1. ~B ⊃ Y — True
2. ~X ≡ A — True
3. X ∨ [A • (B ⊃ Y)] — False
4. X ∨ [A • (Y ⊃ B)] — True
5. ~Y ⊃ [A ≡ (Y • B)] — False
6. X ⊃ [(~X ∨ A) ⊃ X] — True
7. (Z ∨ ~A) ≡ [(A ∨ ~Z) ⊃ (X ≡ ~X)] — True
8. [(Y ⊃ ~Y) ⊃ (B ⊃ ~B)] ⊃ [(B ∨ Y) ≡ (~B ∨ ~Y)] — True
9. ~{~[(~A ∨ ~X) • ~A] • ~X} — False
10. {X ∨ [C • (Y ⊃ B)]} ⊃ {Z ⊃ [Z ⊃ (Z ⊃ Z)]} — True
11. Q • (~A ≡ Q) — False
12. Q • (~A • ~Q) — False
13. ~Q ∨ (~X ∨ Q) — True
14. (C ∨ X) ⊃ (Q ∨ A) — True
15. [(P ⊃ X) ⊃ P] ⊃ P — True

16. (Y ∨ P) ⊃ (B • P)	True
17. (~P ⊃ P) ∨ (A ⊃ P)	Unknown
18. (~P ⊃ P) ∨ (P ⊃ A)	True
19. ~(Q ⊃ C) ∨ (Z • ~X)	False
20. ~[(Z ⊃ B) • (P ⊃ C)] ∨ [(X • Y) ≡ A]	False
21. (P ⊃ ~Q) ∨ ~P	Unknown
22. (P ≡ Q) ∨ (P ≡ ~Q)	True
23. [(P ∨ Q) ⊃ X] ≡ ~(P ∨ Q)	True
24. [(P • Q) ∨ (~P • Q)] ∨ [(P • ~Q) ∨ (~P • ~Q)]	True
25. ~[(A ⊃ P) ∨ (A ⊃ Q)] • (P • Q)	False

Section 2.4

Note: the solutions to most of the multiple choice questions in these sections use what I call the standard assignment of truth values to atomic propositions. The standard assignment of truth values assigns the values given here to the variables in the wffs in the exercises, when read left to right. So, the first variable in the formula read left to right gets the α assignment; the second variable in the formula read left to right (if any) gets the β assignment; the third variable in the formula read left to right (if any) gets the γ assignment; and the fourth variable in the formula read left to right (if any) gets the δ assignment.

For exercises with only one propositional variable, the standard assignment is:

α
1
0

For exercises with two propositional variables, the standard assignment is:

α	β
1	1
1	0
0	1
0	0

For exercises with three propositional variables, the standard assignment is:

α	β	γ
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

For exercises with four propositional variables, the standard assignment is:

α	β	γ	δ
1	1	1	1
1	1	1	0
1	1	0	1
1	1	0	0
1	0	1	1
1	0	1	0
1	0	0	1
1	0	0	0
0	1	1	1
0	1	1	0
0	1	0	1
0	1	0	0
0	0	1	1
0	0	1	0
0	0	0	1
0	0	0	0

Instructions: Complete truth tables for each of the following propositions.

1. (B ⊃ B) ∨ ~B

(B ⊃ B) ∨ ~B

B	**(B**	⊃	**B)**	∨	~	**B**
1	1	**1**	1	**1**	**0**	1
0	0	**1**	0	**1**	**1**	0

2. (D ≡ ~D) ≡ (~D ≡ D)

D	**(D**	**≡**	**~**	**D)**	**≡**	**(~**	**D**	**≡**	**D)**
1	1	**0**	**0**	1	**1**	**0**	1	**0**	1
0	0	**0**	**1**	0	**1**	**1**	0	**0**	0

3. ~[F • (F ⊃ F)]

~[F • (F ⊃ F)]

F		**~**	**[F**	**•**	**(F**	**⊃**	**F)]**
1		**0**	1	**1**	1	**1**	1
0		**1**	0	**0**	0	**1**	0

4. U ≡ (Z ⊃ U)

U	**Z**	**U**	**≡**	**(Z**	**⊃**	**U)**
1	**1**	1	**1**	1	**1**	1
1	**0**	1	**1**	0	**1**	1
0	**1**	0	**1**	1	**0**	0
0	**0**	0	**0**	0	**1**	0

5. (I ⊃ J) ∨ (J ⊃ I)

I	**J**	**(I**	**⊃**	**J)**	**∨**	**(J**	**⊃**	**I)**
1	**1**	1	**1**	1	**1**	1	**1**	1
1	**0**	1	**0**	0	**1**	0	**1**	1
0	**1**	0	**1**	1	**1**	1	**0**	0
0	**0**	0	**1**	0	**1**	0	**1**	0

6. (K ⊃ L) ≡ (~K ⊃L)

K	**L**	**(K**	**⊃**	**L)**	**≡**	**(~**	**K**	**⊃**	**L)**
1	**1**	1	**1**	1	**1**	**0**	1	**1**	1
1	**0**	1	**0**	0	**0**	**0**	1	**1**	0
0	**1**	0	**1**	1	**1**	**1**	0	**1**	1
0	**0**	0	**1**	0	**0**	**1**	0	**0**	0

7. (M ∨ N) • (M ∨ ~N)

M	N	(M	∨	N)	•	(M	∨	~	N)
1	1	1	1	1	1	1	1	0	1
1	0	1	1	0	1	1	1	1	0
0	1	0	1	1	0	0	0	0	1
0	0	0	0	0	0	0	1	1	0

8. (V ≡ W) • (V ≡ ~W)

V	W	(V	≡	W)	•	(V	≡	~	W)
1	1	1	1	1	0	1	0	0	1
1	0	1	0	0	0	1	1	1	0
0	1	0	0	1	0	0	1	0	1
0	0	0	1	0	0	0	0	1	0

9. (~X • Y) ≡ (X • Y)

X	Y	(~	X	•	Y)	≡	(X	•	Y)
1	1	0	1	0	1	0	1	1	1
1	0	0	1	0	0	1	1	0	0
0	1	1	0	1	1	0	0	0	1
0	0	1	0	0	0	1	0	0	0

10. A ≡ [(~B ∨ A) ⊃ ~A]

A	B	A	≡	[(~	B	∨	A)	⊃	~	A]
1	1	1	0	0	1	1	1	0	0	1
1	0	1	0	1	0	1	1	0	0	1
0	1	0	0	0	1	0	0	1	1	0
0	0	0	0	1	0	1	0	1	1	0

11. (~C ⊃ ~D) ⊃ [D ⊃ (C ⊃ D)]

C	D	(~	C	⊃	~	D)	⊃	[D	⊃	(C	⊃	D)]
1	1	0	1	1	0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1	0	1	1	0	0
0	1	1	0	0	0	1	1	1	1	0	1	1
0	0	1	0	1	1	0	1	0	1	0	1	0

12. [E ∨ ~(E • ~F)] ≡ [F ⊃ ~(E ∨ ~F)]

E	F	[E	∨	~	(E	•	~	F)]	≡	[F	⊃	~	(E	∨	~	F)]
1	1	1	1	1	1	0	0	1	0	1	0	0	1	1	0	1
1	0	1	1	0	1	1	1	0	1	0	1	0	1	1	1	0
0	1	0	1	1	0	0	0	1	1	1	1	1	0	0	0	1
0	0	0	1	1	0	0	1	0	1	0	1	0	0	1	1	0

13. G ⊃ {(H ⊃ ~G) ⊃ [(G • ~H) ∨ (H • ~G)]}

G	H	G	⊃	{(H	⊃	~	G)	⊃	[(G	•	~	H)	∨	(H	•	~	G)]}
1	1	1	1	1	0	0	1	1	1	0	0	1	0	1	0	0	1
1	0	1	1	0	1	0	1	1	1	1	1	0	1	0	0	0	1
0	1	0	1	1	1	1	0	1	0	0	0	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0	1	0	0	0	0	1	0

14. [J ⊃ (K ∨ L)] ⊃ [~L ⊃ (K ∨ J)]

J	K	L	[J	⊃	(K	∨	L)]	⊃	[~	L	⊃	(K	∨	J)]
1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
1	1	0	1	1	1	1	0	1	1	0	1	1	1	1
1	0	1	1	1	0	1	1	1	0	1	1	0	1	1
1	0	0	1	0	0	0	0	1	1	0	1	0	1	1
0	1	1	0	1	1	1	1	1	0	1	1	1	1	0
0	1	0	0	1	1	1	0	1	1	0	1	1	1	0
0	0	1	0	1	0	1	1	1	0	1	1	0	0	0
0	0	0	0	1	0	0	0	0	1	0	0	0	0	0

15. [M ∨ (N • O)] ∨ [(M • N) ∨ ~(M • N)]

M	N	O	[M	∨	(N	•	O)]	∨	[(M	•	N)	∨	~	(M	•	N)]
1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
1	1	0	1	1	1	0	0	1	1	1	1	1	0	1	1	1
1	0	1	1	1	0	0	1	1	1	0	0	1	1	1	0	0
1	0	0	1	1	0	0	0	1	1	0	0	1	1	1	0	0
0	1	1	0	1	1	1	1	1	0	0	1	1	1	0	0	1
0	1	0	0	0	1	0	0	1	0	0	1	1	1	0	0	1
0	0	1	0	0	0	0	1	1	0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0

16. [P ⊃ (~R ⊃ Q)] ⊃ [(Q ∨ R) ∨ (~P • R)]

P	R	Q	[P	⊃	(~	R	⊃	Q)]	⊃	[(Q	∨	R)	∨	(~	P	•	R)]
1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1	0	1
1	1	0	1	1	0	1	1	0	1	0	1	1	1	0	1	0	1
1	0	1	1	1	1	0	1	1	1	1	1	0	1	0	1	0	0
1	0	0	1	0	1	0	0	0	1	0	0	0	0	0	1	0	0
0	1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	1	1
0	1	0	0	1	0	1	1	0	1	0	1	1	1	1	0	1	1
0	0	1	0	1	1	0	1	1	1	1	1	0	1	1	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0

17. (S ⊃ ~T) ⊃ {[V ⊃ (~S ⊃ T)] ∨ [V ⊃ (~T ⊃ S)]}

S	T	V	(S	⊃	~	T)	⊃	{[V	⊃	(~	S	⊃	T)]	∨	[V	⊃	(~	T	⊃	S)]}
1	1	1	1	0	0	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1
1	1	0	1	0	0	1	1	0	1	0	1	1	1	1	0	1	0	1	1	1
1	0	1	1	1	1	0	1	1	1	0	1	1	0	1	1	1	1	0	1	1
1	0	0	1	1	1	0	1	0	1	0	1	1	0	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1	0	1	1	1	1	1	0	1	1	0
0	1	0	0	1	0	1	1	0	1	1	0	1	1	1	0	1	0	1	1	0
0	0	1	0	1	1	0	0	1	0	1	0	0	0	0	1	0	1	0	0	0
0	0	0	0	1	1	0	1	0	1	1	0	0	0	1	0	1	1	0	0	0

18. {[X ≡ ~(Y ∨ Z)] • [Y ≡ ~(X ∨ Z)]} ⊃ [(X • Z) ∨ ~(X • ~Z)]

X	Y	Z	{[X	≡	~	(Y	∨	Z)]	•	[Y	≡	~	(X	∨	Z)]}	⊃	[(X	•	Z)	∨	~	(X	•	~	Z)]
1	1	1	1	0	0	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1
1	1	0	1	0	0	1	1	0	0	1	0	0	1	1	0	1	1	0	0	0	0	1	1	1	0
1	0	1	1	0	0	0	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	0	0	1
1	0	0	1	1	1	0	0	0	1	0	1	0	1	1	0	0	1	0	0	0	0	1	1	1	0
0	1	1	0	1	0	1	1	1	0	1	0	0	0	1	1	1	0	0	1	1	1	0	0	0	1
0	1	0	0	1	0	1	1	0	1	1	1	1	0	0	0	1	0	0	0	1	1	0	0	1	0
0	0	1	0	1	0	0	1	1	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	1
0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	1	0

19. [(E ≡ ~F) ∨ (G ≡ ~H)] ∨ [(~E • ~F) • (~G • H)]

E	F	G	H	[(E	≡	~	F)	∨	(G	≡	~	H)]	∨	[(~	E	•	~	F)	•	(~	G	•	H)]
1	1	1	1	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	1
1	1	1	0	1	0	0	1	1	1	1	1	0	1	0	1	0	0	1	0	0	1	0	0
1	1	0	1	1	0	0	1	1	0	1	0	1	1	0	1	0	0	1	0	1	0	1	1
1	1	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0	0	1	0	1	0	0	0
1	0	1	1	1	1	1	0	1	1	0	0	1	1	0	1	0	1	0	0	0	1	0	1
1	0	1	0	1	1	1	0	1	1	1	1	0	1	0	1	0	1	0	0	0	1	0	0
1	0	0	1	1	1	1	0	1	0	1	0	1	1	0	1	0	1	0	0	1	0	1	1
1	0	0	0	1	1	1	0	1	0	0	1	0	1	0	1	0	1	0	0	1	0	0	0
0	1	1	1	0	1	0	1	1	1	0	0	1	1	1	0	0	0	1	0	0	1	0	1
0	1	1	0	0	1	0	1	1	1	1	1	0	1	1	0	0	0	1	0	0	1	0	0
0	1	0	1	0	1	0	1	1	0	1	0	1	1	1	0	0	0	1	0	1	0	1	1
0	1	0	0	0	1	0	1	1	0	0	1	0	1	1	0	0	0	1	0	1	0	0	0
0	0	1	1	0	0	1	0	0	1	0	0	1	0	1	0	1	1	0	0	0	1	0	1
0	0	1	0	0	0	1	0	1	1	1	1	0	1	1	0	1	1	0	0	0	1	0	0
0	0	0	1	0	0	1	0	1	0	1	0	1	1	1	0	1	1	0	1	1	0	1	1
0	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	1	0	0	1	0	0	0

Section 2.5

Instructions: For questions 1–10, construct a complete truth table for each of the following propositions. Then, using the truth table, classify each proposition as a tautology, a contingency, or a contradiction. Justify your answers by appeal to the meanings of those terms.

1. (I ⊃ ~I) ⊃ [I ⊃ (I ⊃ ~I)] Tautology

I	(I	⊃	~	I)	⊃	[1	⊃	(I	⊃	~	I)]
1	1	0	0	1	1	1	0	1	0	0	1
0	0	1	1	0	1	0	1	0	1	1	0

2. (G • ~G) ⊃ (G ∨ ~G) Tautology

G	(G	•	~	G)	⊃	(G	∨	~	G)
1	1	0	0	1	1	1	1	0	1
0	0	0	1	0	1	0	1	1	0

3. (~A • B) ≡ (B ⊃ A) Contradiction

A	B	(~	A	•	B)	≡	(B	⊃	A)

1	1	0	1	0	1	0	1	1	1
1	0	0	1	0	0	0	0	1	1
0	1	1	0	1	1	0	1	0	0
0	0	1	0	0	0	0	0	1	0

4. (A • ~B) • (B ∨ ~A) Contradiction

A	B	(A	•	~	B)	•	(B	∨	~	A)
1	1	1	0	0	1	0	1	1	0	1
1	0	1	1	1	0	0	0	0	0	1
0	1	0	0	0	1	0	1	1	1	0
0	0	0	0	1	0	0	0	1	1	0

5. (A ≡ ~B) ⊃ ~(B • A) Tautology

A	B	(A	≡	~	B)	⊃	~	(B	•	A)
1	1	1	0	0	1	1	0	1	1	1
1	0	1	1	1	0	1	1	0	0	1
0	1	0	1	0	1	1	1	1	0	0
0	0	0	0	1	0	1	1	0	0	0

6. (C ⊃ ~D) ∨ (~D ⊃ C) Tautology

C	D	(C	⊃	~	D)	∨	(~	D	⊃	C)
1	1	1	0	0	1	1	0	1	1	1
1	0	1	1	1	0	1	1	0	1	1
0	1	0	1	0	1	1	0	1	1	0
0	0	0	1	1	0	1	1	0	0	0

7. (G ≡ H) ⊃ ~ [(G • H) ∨ (~G • ~H)] Contingency

G	H	(G	≡	H)	⊃	~	[(G	•	H)	∨	(~	G	•	~	H)]
1	1	1	1	1	0	0	1	1	1	1	0	1	0	0	1
1	0	1	0	0	1	1	1	0	0	0	0	1	0	1	0
0	1	0	0	1	1	1	0	0	1	0	1	0	0	0	1
0	0	0	1	0	0	0	0	0	0	1	1	0	1	1	0

8. [N ⊃ (O ⊃ P)] ⊃ (N ⊃ O) Contingency

N	O	P	[N	⊃	(O	⊃	P)]	⊃	(N	⊃	O)
1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	0	1	0	0	1	1	1	1
1	0	1	1	1	0	1	1	0	1	0	0
1	0	0	1	1	0	1	0	0	1	0	0
0	1	1	0	1	1	1	1	1	0	1	1
0	1	0	0	1	1	0	0	1	0	1	1
0	0	1	0	1	0	1	1	1	0	1	0
0	0	0	0	1	0	1	0	1	0	1	0

9. [(Q ⊃ R) ⊃ (R ⊃ S)] ≡ ~(~Q ∨ S) Contingency

Q	R	S	[(Q	⊃	R)	⊃	(R	⊃	S)]	≡	~	(~	Q	∨	S)
1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1
1	1	0	1	1	1	0	1	0	0	0	1	0	1	0	0
1	0	1	1	0	0	1	0	1	1	0	0	0	1	1	1
1	0	0	1	0	0	1	0	1	0	1	1	0	1	0	0
0	1	1	0	1	1	1	1	1	1	0	0	1	0	1	1
0	1	0	0	1	1	0	1	0	0	1	0	1	0	1	0
0	0	1	0	1	0	1	0	1	1	0	0	1	0	1	1
0	0	0	0	1	0	1	0	1	0	0	0	1	0	1	0

10. [(P ∨ Q) • (R • S)] ⊃ [(P • R) ∨ (Q • S)] Tautology

P	Q	R	S	[(P	∨	Q)	•	(R	•	S)]	⊃	[(P	•	R)	∨	(Q	•	S)]
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	1	1	1	0	1	0	0	1	1	1	1	1	1	0	0
1	1	0	1	1	1	1	0	0	0	1	1	1	0	0	1	1	1	1
1	1	0	0	1	1	1	0	0	0	0	1	1	0	0	0	1	0	0
1	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	0	1
1	0	1	0	1	1	0	0	1	0	0	1	1	1	1	1	0	0	0
1	0	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0	1
1	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0
0	1	1	1	0	1	1	1	1	1	1	1	0	0	1	1	1	1	1
0	1	1	0	0	1	1	0	1	0	0	1	0	0	1	0	1	0	0
0	1	0	1	0	1	1	0	0	0	1	1	0	0	0	1	1	1	1
0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	1	0	0
0	0	1	1	0	0	0	0	1	1	1	1	0	0	1	0	0	0	1
0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0

0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0

Instructions: For questions 11–20, construct complete a truth table for each of the following pairs of propositions. Then, using the truth table, determine whether the statements are logically equivalent or contradictory. If neither, determine whether they are consistent or inconsistent. Justify your answers.

11. A ⊃ ~A and ~A⊃ A Contradictory

A	A	⊃	~	A	/	~	A	⊃	A
1	1	0	0	1		0	1	1	1
0	0	1	1	0		1	0	0	0

12. D • ~E and ~(E ⊃ ~D) Inconsistent

D	E	D	•	~	E	/	~	(E	⊃	~	D)
1	1	1	0	0	1		1	1	0	0	1
1	0	1	1	1	0		0	0	1	0	1
0	1	0	0	0	1		0	1	1	1	0
0	0	0	0	1	0		0	0	1	1	0

13. G ≡ ~H and (H • ~G) ∨ (G • ~H) Logically equivalent

G	H	G	≡	~	H	/	(H	•	~	G)	∨	(G	•	~	H)
1	1	1	0	0	1		1	0	0	1	0	1	0	0	1
1	0	1	1	1	0		0	0	0	1	1	1	1	1	0
0	1	0	1	0	1		1	1	1	0	1	0	0	0	1
0	0	0	0	1	0		0	0	1	0	0	0	0	1	0

14. J ⊃ (K ⊃ ~J) and J • K Contradictory

J	K	J	⊃	(K	⊃	~	J)	/	J	•	K
1	1	1	0	1	0	0	1		1	1	1
1	0	1	1	0	1	0	1		1	0	0
0	1	0	1	1	1	1	0		0	0	1
0	0	0	1	0	1	1	0		0	0	0

15. (D ⊃ E) • (E ⊃ D) and (E ∨ ~D) • (D ∨ ~E) Logically equivalent

D	E	(D	⊃	E)	•	(E	⊃	D)	/	(E	∨	~	D)	•	(D	∨	~	E)
1	1	1	1	1	1	1	1	1		1	1	0	1	1	1	1	0	1
1	0	1	0	0	0	0	1	1		0	0	0	1	0	1	1	1	0
0	1	0	1	1	0	1	0	0		1	1	1	0	0	0	0	0	1
0	0	0	1	0	1	0	1	0		0	1	1	0	1	0	1	1	0

16. R ⊃ (~S ⊃ R) and ~S ⊃ ~(R ∨ ~R) Consistent

R	S	R	⊃	(~	S	⊃	R)	/	~	S	⊃	~	(R	∨	~	R)
1	1	1	1	0	1	1	1		0	1	1	0	1	1	0	1
1	0	1	1	1	0	1	1		1	0	0	0	1	1	0	1
0	1	0	1	0	1	1	0		0	1	1	0	0	1	1	0
0	0	0	1	1	0	0	0		1	0	0	0	0	1	1	0

17. ~I ⊃ (G ∨ ~H) and I • (~H • G) Consistent

I	G	H	~	I	⊃	(G	∨	~	H)	/	I	•	(~	H	•	G)
1	1	1	0	1	1	1	1	0	1		1	0	0	1	0	1
1	1	0	0	1	1	1	1	1	0		1	1	1	0	1	1
1	0	1	0	1	1	0	0	0	1		1	0	0	1	0	0
1	0	0	0	1	1	0	1	1	0		1	0	1	0	0	0
0	1	1	1	0	1	1	1	0	1		0	0	0	1	0	1
0	1	0	1	0	1	1	1	1	0		0	0	1	0	1	1
0	0	1	1	0	0	0	0	0	1		0	0	0	1	0	0
0	0	0	1	0	1	0	1	1	0		0	0	1	0	0	0

18. F • (~G ∨ H) and F ≡ (~H • G) Inconsistent

F	G	H	F	•	(~	G	∨	H)	/	F	≡	(~	H	•	G)
1	1	1	1	1	0	1	1	1		1	0	0	1	0	1
1	1	0	1	0	0	1	0	0		1	1	1	0	1	1
1	0	1	1	1	1	0	1	1		1	0	0	1	0	0
1	0	0	1	1	1	0	1	0		1	0	1	0	0	0
0	1	1	0	0	0	1	1	1		0	1	0	1	0	1
0	1	0	0	0	0	1	0	0		0	0	1	0	1	1
0	0	1	0	0	1	0	1	1		0	1	0	1	0	0
0	0	0	0	0	1	0	1	0		0	1	1	0	0	0

19. L ∨ (M ⊃ N) and ~L • (M • ~N) Contradictory

L	M	N	L	∨	(M	⊃	N)	/	~	L	•	(M	•	~	N)
1	1	1	1	1	1	1	1		0	1	0	1	0	0	1
1	1	0	1	1	1	0	0		0	1	0	1	1	1	0
1	0	1	1	1	0	1	1		0	1	0	0	0	0	1
1	0	0	1	1	0	1	0		0	1	0	0	0	1	0
0	1	1	0	1	1	1	1		1	0	0	1	0	0	1
0	1	0	0	0	1	0	0		1	0	1	1	1	1	0
0	0	1	0	1	0	1	1		1	0	0	0	0	0	1
0	0	0	0	1	0	1	0		1	0	0	0	0	1	0

20. P ⊃ [Q ⊃ (~S ⊃ ~R)] and ~P ∨ ~[(Q • ~S) • R] Logically equivalent

P	Q	S	R	P	⊃	[Q	⊃	(~	S	⊃	~	R)]	/	~	P	∨	~	[(Q	•	~	S)	•	R]
1	1	1	1	1	1	1	1	0	1	1	0	1		0	1	1	1	1	0	0	1	0	1
1	1	1	0	1	1	1	1	0	1	1	1	0		0	1	1	1	1	0	0	1	0	0
1	1	0	1	1	0	1	0	1	0	0	0	1		0	1	0	0	1	1	1	0	1	1
1	1	0	0	1	1	1	1	1	0	1	1	0		0	1	1	1	1	1	1	0	0	0
1	0	1	1	1	1	0	1	0	1	1	0	1		0	1	1	1	0	0	0	1	0	1
1	0	1	0	1	1	0	1	0	1	1	1	0		0	1	1	1	0	0	0	1	0	0
1	0	0	1	1	1	0	1	1	0	0	0	1		0	1	1	1	0	0	1	0	0	1
1	0	0	0	1	1	0	1	1	0	1	1	0		0	1	1	1	0	0	1	0	0	0
0	1	1	1	0	1	1	1	0	1	1	0	1		1	0	1	1	1	0	0	1	0	1
0	1	1	0	0	1	1	1	0	1	1	1	0		1	0	1	1	1	0	0	1	0	0
0	1	0	1	0	1	1	0	1	0	0	0	1		1	0	1	0	1	1	1	0	1	1
0	1	0	0	0	1	1	1	1	0	1	1	0		1	0	1	1	1	1	1	0	0	0
0	0	1	1	0	1	0	1	0	1	1	0	1		1	0	1	1	0	0	0	1	0	1
0	0	1	0	0	1	0	1	0	1	1	1	0		1	0	1	1	0	0	0	1	0	0
0	0	0	1	0	1	0	1	1	0	0	0	1		1	0	1	1	0	0	1	0	0	1
0	0	0	0	0	1	0	1	1	0	1	1	0		1	0	1	1	0	0	1	0	0	0

Section 2.6

Instructions: Construct a complete truth table for each of the following arguments. Then, using the truth table, determine whether the argument is valid or invalid. If the argument is invalid, specify a counterexample.

1. A / A • ~A Invalid. Counterexample when A is true

A	A	//	A	•	~	A
1	**1**		1	**0**	**0**	1
0	**0**		0	**0**	**1**	0

2. B ∨ ~C

~~C / ~B Invalid. Counterexample when B and C are true

B	C	B	∨	~	C	/	~	~	C	//	~	B
1	**1**	1	**1**	**0**	1		**1**	**0**	1		**0**	1
1	**0**	1	**1**	**1**	0		**0**	**1**	0		**0**	1
0	**1**	0	**0**	**0**	1		**1**	**0**	1		**1**	0
0	**0**	0	**1**	**1**	0		**0**	**1**	0		**1**	0

3. E ≡ F

F ⊃ ~E / E Invalid. Counterexample when E and F are false

E	F	E	≡	F	/	F	⊃	~	E	//	E
1	**1**	1	**1**	1		1	**0**	**0**	1		**1**
1	**0**	1	**0**	0		0	**1**	**0**	1		**1**
0	**1**	0	**0**	1		1	**1**	**1**	0		**0**
0	**0**	0	**1**	0		0	**1**	**1**	0		**0**

4. I ⊃ J

~(J • I) / J ⊃ I Invalid. Counterexample when I is false and J is true

I	J	I	⊃	J	/	~	(J	•	I)	//	J	⊃	I
1	**1**	1	**1**	1		**0**	1	**1**	1		1	**1**	1
1	**0**	1	**0**	0		**1**	0	**0**	1		0	**1**	1
0	**1**	0	**1**	1		**1**	1	**0**	0		1	**0**	0
0	**0**	0	**1**	0		**1**	0	**0**	0		0	**1**	0

5. P ≡ ~Q

~(Q ∨~P)

Q ⊃~Q / P Valid

P	Q	P	≡	~	Q	/	~	(Q	∨	~	P)	/	Q	⊃	~	Q	//	P
1	**1**	1	**0**	**0**	1		**0**	1	**1**	**0**	1		1	**0**	**0**	1		**1**

1	0	1	1	1	0	1	0	0	0	1	0	1	1	0	1
0	1	0	1	0	1	0	1	1	1	0	1	0	0	1	0
0	0	0	0	1	0	0	0	1	1	0	0	1	1	0	0

6. E ∨ ~F

E ⊃ ~E

F / E Valid

E	F	E	∨	~	F	/	E	⊃	~	E	/	F	//	E
1	1	1	1	0	1		1	0	0	1		1		1
1	0	1	1	1	0		1	0	0	1		0		1
0	1	0	0	0	1		0	1	1	0		1		0
0	0	0	1	1	0		0	1	1	0		0		0

7. ~J ⊃ K

~K ∨ (J • K)

~K ⊃ K / ~J Invalid. Counterexample when J and K are true

J	K	~	J	⊃	K	/	~	K	∨	(J	•	K)	/	~	K	⊃	K	//	~	J
1	1	0	1	1	1		0	1	1	1	1	1		0	1	1	1		0	1
1	0	0	1	1	0		1	0	1	1	0	0		1	0	0	0		0	1
0	1	1	0	1	1		0	1	0	0	0	1		0	1	1	1		1	0
0	0	1	0	0	0		1	0	1	0	0	0		1	0	0	0		1	0

8. G ⊃ H

~I ⊃ ~G / G ⊃ (H • I) Valid

G	H	I	G	⊃	H	/	~	I	⊃	~	G	//	G	⊃	(H	•	I)
1	1	1	1	1	1		0	1	1	0	1		1	1	1	1	1
1	1	0	1	1	1		1	0	0	0	1		1	0	1	0	0
1	0	1	1	0	0		0	1	1	0	1		1	0	0	0	1
1	0	0	1	0	0		1	0	0	0	1		1	0	0	0	0
0	1	1	0	1	1		0	1	1	1	0		0	1	1	1	1
0	1	0	0	1	0		1	0	1	1	0		0	1	1	0	0
0	0	1	0	1	0		0	1	1	1	0		0	1	0	0	1
0	0	0	0	1	0		1	0	1	1	0		0	1	0	0	0

9. M ⊃ (N • ~O)

N ⊃ (O ∨ M)   / O ⊃~N   Invalid. Counterexample when M is false and N and O are true

M	N	O	M	⊃	(N	•	~	O)	/	N	⊃	(O	∨	M)	//	O	⊃	~	N
1	1	1	1	0	1	0	0	1		1	1	1	1	1		1	0	0	1
1	1	0	1	1	1	1	1	0		1	1	0	1	1		0	1	0	1
1	0	1	1	0	0	0	0	1		0	1	1	1	1		1	1	1	0
1	0	0	1	0	0	0	1	0		0	1	0	1	1		0	1	1	0
0	1	1	0	1	1	0	0	1		1	1	1	1	0		1	0	0	1
0	1	0	0	1	1	1	1	0		1	0	0	0	0		0	1	0	1
0	0	1	0	1	0	0	0	1		0	1	1	1	0		1	1	1	0
0	0	0	0	1	0	0	1	0		0	1	0	0	0		0	1	1	0

10. P ⊃~Q

R ⊃ P   / R ⊃~Q   Valid

P	Q	R	P	⊃	~	Q	/	R	⊃	P	//	R	⊃	~	Q
1	1	1	1	0	0	1		1	1	1		1	0	0	1
1	1	0	1	0	0	1		0	1	1		0	1	0	1
1	0	1	1	1	1	0		1	1	1		1	1	1	0
1	0	0	1	1	1	0		0	1	1		0	1	1	0
0	1	1	0	1	0	1		1	0	0		1	0	0	1
0	1	0	0	1	0	1		1	0	0		1	0	0	1
0	0	1	0	1	1	0		1	0	0		1	1	1	0
0	0	0	0	1	1	0		0	1	0		0	1	1	0

11. S

~T • U   / ~(T • U) • S   Valid

S	T	U	S	/	~	T	•	U	//	~	(T	•	U)	•	S
1	1	1	1		0	1	0	1		0	1	1	1	0	1
1	1	0	1		0	1	0	0		1	1	0	0	1	1
1	0	1	1		1	0	1	1		1	0	0	1	1	1
1	0	0	1		1	0	0	0		1	0	0	0	1	1
0	1	1	0		0	1	0	1		0	1	1	1	0	0
0	1	0	0		0	1	0	1		0	1	1	1	0	0
0	0	1	0		1	0	1	1		1	0	0	1	0	0
0	0	0	0		1	0	0	0		1	0	0	0	0	0

12. X / X ∨ [Y ≡ (Z • ~X)] Valid

X	Y	Z	X	//	X	∨	[Y	≡	(Z	•	~	X)]
1	1	1	1		1	1	1	0	1	0	0	1
1	1	0	1		1	1	1	0	0	0	0	1
1	0	1	1		1	1	0	1	1	0	0	1
1	0	0	1		1	1	0	1	0	0	0	1
0	1	1	0		0	1	1	1	1	1	1	0
0	1	0	0		0	0	1	0	0	0	1	0
0	0	1	0		0	0	0	0	1	1	1	0
0	0	0	0		0	1	0	1	0	0	1	0

13. ~A ≡ B

(~A ∨ B) ⊃ C / C Invalid. Counterexample when A is true and B and C are false

A	B	C	~	A	≡	B	/	(~	A	∨	B)	⊃	C	//	C
1	1	1	0	1	0	1		0	1	1	1	1	1		1
1	1	0	0	1	0	1		0	1	1	1	0	0		0
1	0	1	0	1	1	0		0	1	0	0	1	1		1
1	0	0	0	1	1	0		0	1	0	0	1	0		0
0	1	1	1	0	1	1		1	0	1	1	1	1		1
0	1	0	1	0	1	1		1	0	1	1	0	0		0
0	0	1	1	0	0	0		1	0	1	0	1	1		1
0	0	0	1	0	0	0		1	0	1	0	0	0		0

14. D ⊃~E

~E ⊃ F / ~F ⊃ D Invalid. Counterexample when E is true and D and F are false

D	E	F	D	⊃	~	E	/	~	E	⊃	F	//	~	F	⊃	D
1	1	1	1	0	0	1		0	1	1	1		0	1	1	1
1	1	0	1	0	0	1		0	1	1	0		1	0	1	1
1	0	1	1	1	1	0		1	0	1	1		0	1	1	1
1	0	0	1	1	1	0		1	0	0	0		1	0	1	1
0	1	1	0	1	0	1		0	1	1	1		0	1	1	0
0	1	0	0	1	0	1		0	1	1	0		1	0	0	0
0	0	1	0	1	1	0		1	0	1	1		0	1	1	0
0	0	0	0	1	1	0		1	0	0	0		1	0	0	0

15. (G ~H) ∨~~I / G ∨~H

Invalid. Counterexample when H and I are true and G is false

G	H	I	(G	∨	~	H)	∨	~	~	I	//	G	∨	~	H
1	**1**	**1**	1	**1**	**0**	1	**1**	**1**	**0**	1		1	**1**	**0**	1
1	**1**	**0**	1	**1**	**0**	1	**1**	**0**	**1**	0		1	**1**	**0**	1
1	**0**	**1**	1	**1**	**1**	0	**1**	**1**	**0**	1		1	**1**	**1**	0
1	**0**	**0**	1	**1**	**1**	0	**1**	**0**	**1**	0		1	**1**	**1**	0
0	**1**	**1**	0	**0**	**0**	1	**1**	**1**	**0**	1		0	**0**	**0**	1
0	**1**	**0**	0	**0**	**0**	1	**0**	**0**	**1**	0		0	**0**	**0**	1
0	**0**	**1**	0	**1**	**1**	0	**1**	**1**	**0**	1		0	**1**	**1**	0
0	**0**	**0**	0	**1**	**1**	0	**1**	**0**	**1**	0		0	**1**	**1**	0

16. J ≡ (~K • L)

L ⊃ J / L ⊃ K

Invalid. Counterexample when J and L are true and K is false

J	K	L	J	≡	(~	K	•	L	/	L	⊃	J	//	L	⊃	K
1	**1**	**1**	1	**0**	**0**	1	**0**	1		1	**1**	1		1	**1**	1
1	**1**	**0**	1	**0**	**0**	1	**0**	0		0	**1**	1		0	**1**	1
1	**0**	**1**	1	**1**	**1**	0	**1**	1		1	**1**	1		1	**0**	0
1	**0**	**0**	1	**0**	**1**	0	**0**	0		0	**1**	1		0	**1**	0
0	**1**	**1**	0	**1**	**0**	1	**0**	1		1	**0**	0		0	**1**	1
0	**1**	**0**	0	**1**	**0**	1	**0**	0		0	**1**	0		0	**1**	1
0	**0**	**1**	0	**0**	**1**	0	**1**	1		1	**0**	0		1	**0**	0
0	**0**	**0**	0	**1**	**1**	0	**0**	0		0	**1**	0		0	**1**	0

17. M ⊃ N

O ⊃ M

~N

M ⊃~O / M

Invalid. Counterexample when M, N, and O are false

M	N	O	M	⊃	N	/	O	⊃	M	/	~	N	/	M	⊃	~	O	//	M
1	**1**	**1**	**1**	**1**	1		1	**1**	1		**0**	1		1	**0**	**0**	1		**1**
1	**1**	**0**	1	**1**	1		0	**1**	1		**0**	1		1	**1**	**1**	0		**1**
1	**0**	**1**	1	**0**	0		1	**1**	1		**1**	0		1	**0**	**0**	1		**1**
1	**0**	**0**	1	**0**	0		0	**1**	1		**1**	0		1	**1**	**1**	0		**1**

CLICK HERE TO ACCESS THE COMPLETE Test Bank

0	**1**	**1**	0	**1**	1	1	**0**	0	**0**	1	0	**1**	**0**	1	**0**
0	**1**	**0**	0	**1**	1	0	**1**	0	**0**	1	0	**1**	**1**	0	**0**
0	**0**	**1**	0	**1**	0	1	**0**	0	**1**	0	0	**1**	**0**	1	**0**
0	**0**	**0**	0	**1**	0	0	**1**	0	**1**	0	0	**1**	**1**	0	**0**

18. P ∨~Q

P ⊃ R

~R / ~Q Valid

P	**Q**	**R**	**P**	**∨**	**~**	**Q**	**/**	**P**	**⊃**	**R**	**/**	**~**	**R**	**//**	**~**	**Q**
1	**1**	**1**	1	**1**	**0**	1		1	**1**	1		**0**	1		**0**	1
1	**1**	**0**	1	**1**	**0**	1		1	**0**	0		**1**	0		**0**	1
1	**0**	**1**	1	**1**	**1**	0		1	**1**	1		**0**	1		**1**	0
1	**0**	**0**	1	**1**	**1**	0		1	**0**	0		**1**	0		**1**	0
0	**1**	**1**	0	**0**	**0**	1		0	**1**	1		**0**	1		**0**	1
0	**1**	**0**	0	**0**	**0**	1		0	**1**	0		**1**	0		**0**	1
0	**0**	**1**	0	**1**	**1**	0		0	**1**	1		**0**	1		**1**	0
0	**0**	**0**	0	**1**	**1**	0		0	**1**	0		**1**	0		**1**	0

19. S ⊃ (T ∨ U)

~T

T ∨ S / U • S Valid

S	**T**	**U**	**S**	**⊃**	**(T**	**∨**	**U)**	**/**	**~**	**T**	**/**	**T**	**∨**	**S**	**//**	**U**	**•**	**S**
1	**1**	**1**	1	**1**	1	**1**	1		**0**	1		1	**1**	1		1	**1**	1
1	**1**	**0**	1	**1**	1	**1**	0		**0**	1		1	**1**	1		0	**0**	1
1	**0**	**1**	1	**1**	0	**1**	1		**1**	0		0	**1**	1		1	**1**	1
1	**0**	**0**	1	**0**	0	**0**	0		**1**	0		0	**1**	1		0	**0**	1
0	**1**	**1**	0	**1**	1	**1**	1		**0**	1		1	**1**	0		1	**0**	0
0	**1**	**0**	0	**1**	1	**1**	0		**0**	1		1	**1**	0		0	**0**	0
0	**0**	**1**	0	**1**	0	**1**	1		**1**	0		0	**0**	0		1	**0**	0
0	**0**	**0**	0	**1**	0	**0**	0		**1**	0		0	**0**	0		0	**0**	0

20. X ∨ Z

X ⊃ (Y ∨ Z)

~Z ⊃ Y / Y

Invalid. Counterexamples when Z is true and X and Y are false or when X and Z are true and Y is false.

X	Z	Y	X	∨	Z	/	X	⊃	(Y	∨	Z)	/	~	Z	⊃	Y	//	Y
1	**1**	**1**	1	**1**	1		1	**1**	1	**1**	1		**0**	1	**1**	1		**1**
1	**1**	**0**	1	**1**	1		1	**1**	0	**1**	1		**0**	1	**1**	0		**0**
1	**0**	**1**	1	**1**	0		1	**1**	1	**1**	0		**1**	0	**1**	1		**1**
1	**0**	**0**	1	**1**	0		1	**0**	0	**0**	0		**1**	0	**0**	0		**0**
0	**1**	**1**	0	**1**	1		0	**1**	1	**1**	1		**0**	1	**1**	1		**1**
0	**1**	**0**	0	**1**	1		0	**1**	0	**1**	1		**0**	1	**1**	0		**0**
0	**0**	**1**	0	**0**	0		0	**1**	1	**1**	0		**1**	0	**1**	1		**1**
0	**0**	**0**	0	**0**	0		0	**1**	0	**0**	0		**1**	0	**0**	0		**0**

Section 2.7

Instructions: For 1–10, use indirect truth tables to determine whether each of the following arguments is valid. If the argument is invalid, specify a counterexample.

1. (U • ~V) ∨ W
W ≡ ~V / ~U

Invalid. Counterexample when U and W are true and V is false

2. (A ≡ B) • (A ∨ C)
C / B

Invalid. Counterexample when C is true and A and B are false

3. R ⊃ S
V ⊃ W
X ⊃ (S • ~W) / (R • V) ≡ X

Invalid. Counterexamples when R, S, and X are true and V and W are false; or when R, S, V, and W are true and X is false; or when S and X are true and R, V, and W are false.

4. D ⊃ (E ∨ F)

D ⊃ (G ∨ F)
~(F ∨ H) / D ⊃ (E • G)

Valid

5. G ⊃ (J • ~K)
(I ⊃ H) • G
H ⊃ (K ∨ I)
J • ~I / I ∨ K

Invalid. Counterexample when G and J are true and H, I, and K are false

6. (P ∨ Q) ⊃ R
(S ∨~U) ⊃ (~R • ~W)
S ⊃ (P • T) / ~S Valid

7. (W ∨ X) ⊃~(Y • ~Z)
~(Y • W) ⊃~Z
~(W ∨ X) ∨ Z / (Z ∨ W) ⊃ (X • Y) Invalid. Counterexample when W, Y, and Z are true and X is false

8. A ⊃ B
C ⊃~B
~A ⊃ D
E ⊃~F
~E ⊃ G
~F ⊃ C / D ∨ G Valid

9. (H ⊃~I) • ~J
~I ⊃ [(J ∨ K) • (J ∨ H)]
~H ∨ (K ⊃ L) / L ⊃~H Invalid. Counterexample when H, K, and L are true and I and J are false

10. (M • N) ⊃ (O • P)
(M • O) ⊃ (Q ∨~P)
R ⊃ (Q ⊃ N) / (~M • ~N) ∨ (O • P) Invalid. There are 21 counterexamples listed in the table below.

M	N	O	P	Q	R
1	0	1	0	0	1
1	0	1	0	0	0
1	0	1	0	1	0
1	0	0	1	1	0
1	0	0	1	0	1
1	0	0	1	0	0
1	0	0	0	1	0
1	0	0	0	0	1
1	0	0	0	0	0
0	1	1	0	1	1
0	1	1	0	1	0
0	1	1	0	0	1
0	1	1	0	0	0
0	1	0	1	1	1

0	1	0	1	1	0
0	1	0	1	0	1
0	1	0	1	0	0
0	1	0	0	1	1
0	1	0	0	1	0
0	1	0	0	0	1
0	1	0	0	0	0

Instructions: For 11–20, use indirect truth tables to determine, for each given set of propositions, whether it is consistent. If the set is consistent, provide a consistent valuation.

11. X ≡ Y
Y ≡ ~Z
Z ≡ ~X

Consistent. Consistent valuations when X and Y are true and Z is false; or when X and Y are false and Z is true

12. (G • H) ≡ I
G ≡ ~J
J • H
(I ∨ K) ⊃ G

Consistent. Consistent valuation when H and J are true and G, I, and K are false

13. L ⊃ M
~N ⊃ (O ∨ L)
N ∨ P
~M ∨~P

Consistent. Consistent valuations when N and P are true, L and M are false, and O is either true or false; or when N is true, P is false, and it is not the case that L is true and M is false (six possibilities); or when O and P are true and L, M, and N are false.

14. F • (A ⊃ D)

E ∨~B
~[C ⊃ (D ∨ F)]
A ∨ (B • D)
E ⊃ A

Inconsistent

15. (L • N) ∨ I
L ≡ ~K
K ⊃ (I ≡ ~M)
(J ∨ K) • ~N

Consistent. Consistent valuations when I, J, and K are true and L, M, and N are false; or when I and K are true and J, L, M, and N are false; or when I, J, L, and M are true and K and N are false; or when I, J, and L are true and K, M, and N are false

16. P ≡ (Q • ~R)

P ⊃ ~Q
(~P • Q) ⊃ S

Consistent. Consistent valuations when Q, R, and S are true and P is false; or when R and S are true and P and Q are false; or when R is true and P, Q, and S are false; or when S is true and P, Q, and R are false; or when P, Q, R, and S are all false

17. A ∨ B
(C • D) ≡ (E • F)
F ⊃ ~A
D ⊃ B

Consistent. There are twenty consistent valuations listed in the table below.

A	B	C	D	E	F
1	1	1	0	1	0
1	1	1	0	0	0
1	1	0	1	1	0
1	1	0	1	0	0
1	1	0	0	1	0
1	1	0	0	0	0
1	0	1	0	1	0
1	0	1	0	0	0
1	0	0	0	1	0
1	0	0	0	0	0
0	1	1	1	1	1
0	1	1	0	1	0
0	1	1	0	0	1
0	1	1	0	0	0
0	1	0	1	1	0
0	1	0	1	0	1
0	1	0	1	0	0
0	1	0	0	1	0
0	1	0	0	0	1
0	1	0	0	0	0

18. A ⊃ (B • C)
D ⊃ (E ∨ F)

	E ⊃ ~B F ⊃ ~C A ∨ D	Consistent. Consistent valuations when A, B, C, and D are true and E and F are false; or when A, B, and C are true and D, E, and F are false; or when D, E, and F are true and A, B, and C are false; C, D, and E are true and A, B, and F are false; or when D and E are true and A, B, C, and F are false; or when B, D, and F are true and A, C, and E are false; or when D and F are true and A, B, C, and E are false
19.	P ⊃ Q R ⊃ Q ~S ⊃ (P ∨ R) (Q • S) ⊃ T ~(U ⊃ T) ~W ≡ S	Consistent. Consistent valuations when P, Q, U, and W are true and R, S and T are false; or when Q, R, U, and W are true and P, S, and T are false; or when S and U are true and P, Q, R, T, and W are false.
20.	D ≡ (A • B) D ∨ (~E • F) ~E ⊃ A F ⊃ B A ∨ B ~A ∨ ~B	Inconsistent