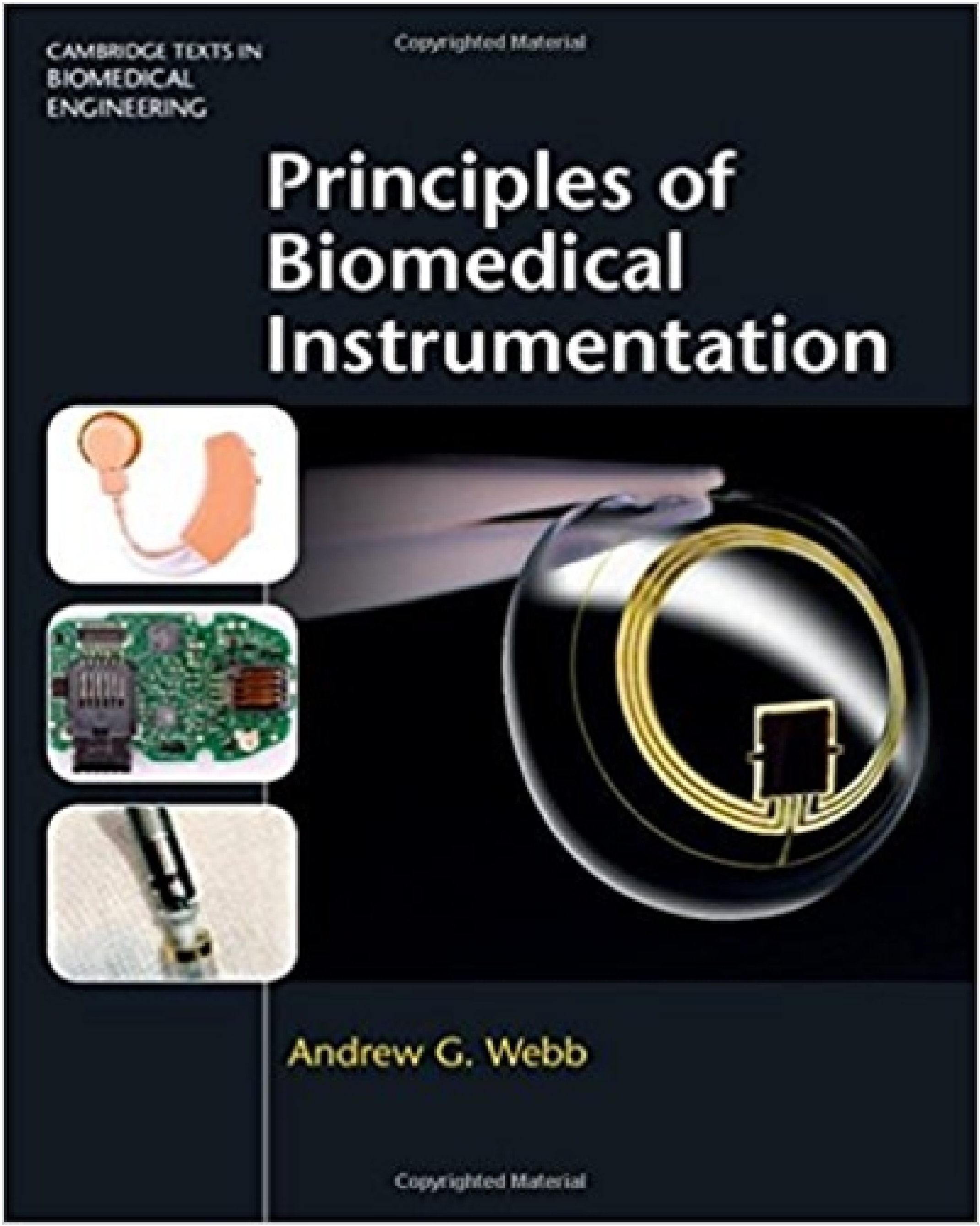
Solutions for Principles of Biomedical Instrumentation 1st Edition by Webb



Solutions

random guess, which is represented by the dotted line at 45^{o} to the axes. One rather trivial example might be that the wiring in a device is the wrong polarity (producing for example a negative voltage rather than a positive one), or there is a systematic DCoffset in the electronics which is not correctly accounted for before the values of the particular measurement are displayed.

Chapter 2

2.1 *From a biochemical point-of-view explain the mechanisms behind the absolute and relative refractory periods for an action potential. If there were no refractory periods, what could happen?*

When the membrane potential rises above the threshold value, the voltage-gated sodium channels open and sodium ions enter the cell. These sodium channels are either open or inactive, and for a certain time another stimulus cannot change this situation, i.e. they cannot be "opened further". This time is referred to as the absolute refractory period. During the depolarization stage more of the voltage-gated sodium channels reset to their closed state, and now another stimulus can open these channels, causing a second action potential. However, since only a few of the channels are closed, it takes a larger stimulus than normal in order to produce the action potential. As the depolarization process continues, an increasing number of sodium channels are reset to their closed state, and so the required stimulus becomes smaller. When all channels are reset, then the relative refractory period is over. If there were no refractory period then neurons would be able to fire continuously, and processes which require spatially-clustered or temporally-linked neuronal firing (such as occurs in the heart) could not happen.

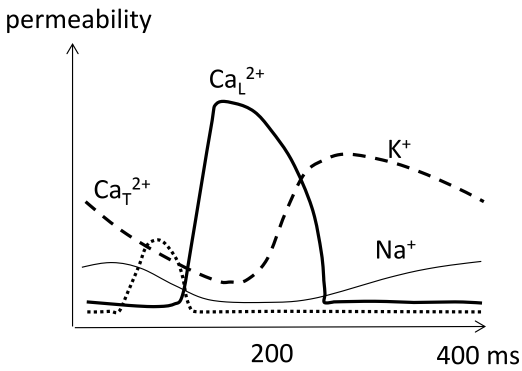
2.2 *Using the GHK equation, calculate the resting membrane potentials for cells which are impermeable to all ions except for: (i) sodium, (ii) potassium and (iii) calcium.*

Using the intracellular and extracellular concentrations of the ions gives:
Na^+ (intra) 15 mM (extra) 145 mM, V_{Na} = +60.60 mV
K^+ (intra) 150 mM (extra) 4 mM, V_K = −96.81 mV
Ca^{2+} (intra) 70 nM (extra) 2 mM, V_{Ca} = +137.04 mV

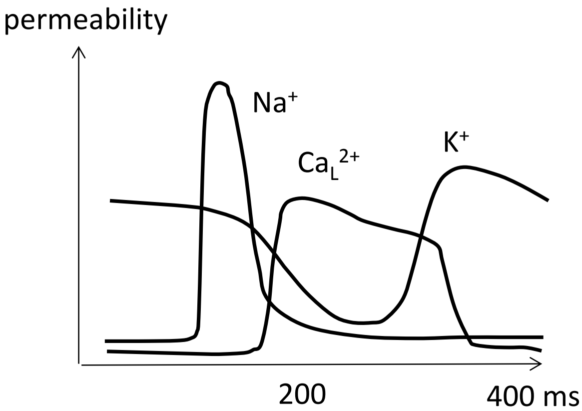
2.3 *What effect would increasing the membrane permeability of K^+ and Cl^- have on threshold potential? Would this make it harder or easier to generate an action potential?*

Increasing the membrane permeabilities would cause the threshold potentials to be higher, which would make it more difficult to generate an action potential.

2.4.1 *Plot the permeabilities over time of K^+, Na^+ and Ca^{2+} for cardiac pacemaker cells.*



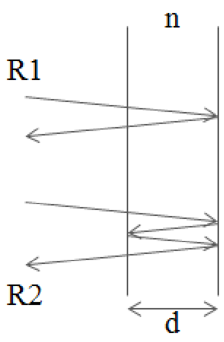
2.5 Plot the permeabilities over time of K^+, Na^+ and Ca^{2+} for cardiac ventricular cells.

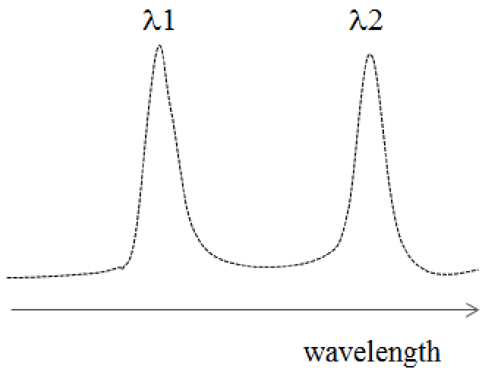


2.6 *Motion of an electrode can cause a disturbance of the Helmholz double layer surrounding an ECG electrode. Explain what effect this has on the signal recorded by the electrode.*

As the electrode moves with respect to the skin the charge distribution at the interface would be disturbed. If the double layer is disturbed, then the half-potential is reduced with respect to its equilibrium value, meaning that the DC offset of the ECG signal changes. The double layer quickly reforms, returning the offset to normal. Therefore, the disturbance causes a decrease and then an increase in the recorded ECG signal in terms of the DC offset signal.

2.7 *In a fiber-optic pressure sensor, changes in pressure can be detected by looking at changes in the spectral output as a function of displacement of a flexible diaphragm. In the figure below, waves can undergo multiple reflections, giving two different spectral peaks. Explain how this type of sensor works, and derive an equation to determine the sensitivity of the device. (n is the refractive index of the material, assume a change in dimensions of Δd).*





There are two diaphragms, one which is fixed in position and the other can move in response to changes in pressure. Assuming that the peak corresponding to λ_1 arises from a reflection from the back diaphragm, and the peak corresponding to λ_2 from a single back-to-front-to-back reflection, then the difference in pathlength between the two relections is simply given by 2d. The time difference in the two paths, given a refractive index of n, is given by 2d/nc. If the system assumes a constant frequency, then the difference in time is interpreted as two different velocities, in turn corresponding to two different wavelengths, which is how the data on the right are presented.

Since any change in displacement produces a linear change in time, which in turn corresponds to a linear change in velocity and wavelength, the overall system is linear.

2.8 *Think of three physical or physiological factors that could lead to an incorrect pulse oximeter reading. For each factor describe whether the reading would be too high or too low.*

Possibilities include: (i) nail varnish which reduces the absorption of light and produces a lower SpO_2 reading than it should, (ii) a bright light source in the vicinity of the probe which produces an artificially high value, (iii) low blood perfusion in the finger which produces a lower reading, or (iv) carbon monoxide poisoning which produces an overestimate of the SpO_2.

2.9 *A 10 cm long elastic resistive transducer with a 5 mm diameter has a resistance of 1 kΩ.*

(i) calculate the resistivity of the conductor,

(ii) calculate the resistance after it has been wrapped around the patient's chest which has a circumference of 1.2 m, assuming that the cross-section of the conductor does not change

(iii) calculate the change in voltage across the transducer assuming that breathing produces a 10% increase in chest circumference and a constant current of 0.5 mA is passed through the conductor.

(i) Resistivity = RA/l = 0.196 Ωm.

(ii) $R_{stretched}$ = 12 kΩ.

(iii) Using Ohms law, the change in voltage is 0.6 volts.

2.10 *In the Wheatstone bridge circuit shown in Figure 2.13 the change in output voltage was analyzed in terms of a change in the value of only one resistor, in this case R_4. The Wheatstone bridge can also be configured as:*

(i) Two-element half-bridge: $R_1=R_3=R$, $R_2=R-\Delta R$, $R_4=R+\Delta R$.

(ii) Four-element full-bridge: $R_1=R+\Delta R$, $R_4=R+\Delta R$, $R_2=R-\Delta R$, $R_3=R-\Delta R$.

For each of these three configurations (single element, two-element and four-element) calculate the output voltages as a function of ΔR.Comment on the linearity and sensitivity of the output as a function of ΔR.

For the two-element half-bridge, voltage division gives the result:

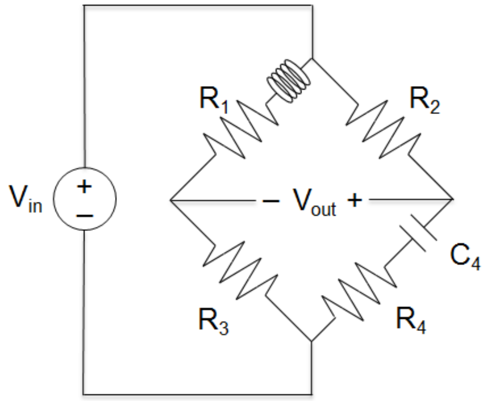
$$\Delta V_{out} = -\frac{V_{in}}{2}\frac{\Delta R}{R}$$

For the four-element half-bridge, the result is:

$$\Delta V_{out} = -V_{in}\frac{\Delta R}{R}$$

For the cases of two or four variable resistors the change in output is exactly linear with changes in input resistance (unlike the case with one resistor which has a small degree of non-linearity). The sensitivity increases proportionally with the number of variable resistors.

2.11 *The circuit below shows a Hays bridge which is used to measure an unknown inductance. Derive the value of L in terms of the other parameters outlined in the Figure. Can the circuit be used to measure changes in inductance? Derive appropriate equations.*



There are several ways to approach this. One simple one is to assume that the bridge is balanced so that V_{out}=0, in which case one can write:

$$\left(R_1 + jwL_1\right)\left(R_4 - \frac{j}{wC_4}\right) = R_2R_3$$

$$R_1R_4 + \frac{L_1}{C_4} + jwL_1R_4 - \frac{jR_1}{wC_4} = R_2R_3$$

Equating the real and imaginary components:

$$R_1R_4 + \frac{L_1}{C_4} = R_2R_3, L_1 = \frac{R_1}{w^2R_4C_4}$$

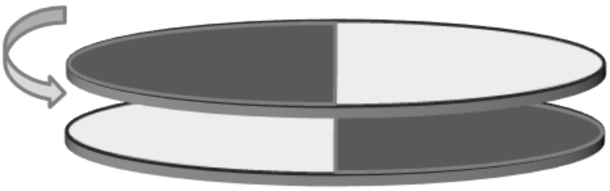
Solving gives:

$$L_1 = \frac{R_2R_3C_4}{1 + w^2C_4^2R_4^2}$$

If we assume that R_4 is very small, i.e. that the Q of the inductor is large, then the value of L is given simply by $R_2R_3C_4$.

If the inductance changes from L to ΔL, then it can be shown (tedious) that the output voltage varies non-linearly as it does for a change for a single variable resistor in the Wheatstone bridge circuit. Therefore the best way to measure the change is to have a variable capacitor C_4 and to change this value to keep the voltage zero: there is obviously a simple linear relationship between ΔL and ΔC_4.

2.12 *Calculate the sensitivity of the rotary capacitor shown below with respect to a change in rotation angle. The shaded areas represent copper with the unshaded area of the capacitor having zero conductivity. To what degree is the system linear? Is there a design which can make the system more linear?*

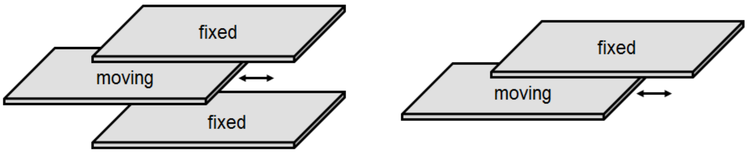


The capacitance is given by:

$$C = \frac{\varepsilon_r\varepsilon_0 A}{d} = \frac{\varepsilon_r\varepsilon_0 A_{\max}\frac{\theta}{\pi}}{d}$$

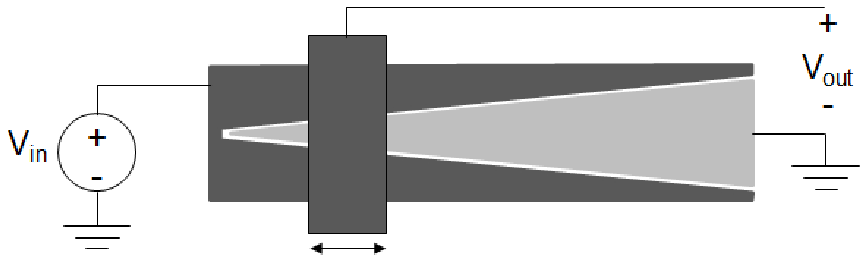
where A is the area of overlap, with A_{max} corresponding to full overlap. The rotation angle θ (measured in radians) is assumed to be zero in the setup in the figure above. This is, of course, a linear system for $0<\theta<\pi$, but non-linear over the range $0<\theta<2\pi$. There are many variations that can extend the range of linearity, for example by including a third disc which either has a different geometry of the conductor (e.g. a 90^o segment covered in copper) and is rotated in tandem with the top capacitor, with the middle one fixed.

2.13 *Compare the sensitivities of a three-plate capacitor on the left with a two-plate capacitor on the right.*

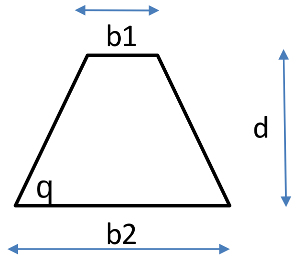


Assuming that the distance between the plates is the same, then the setup on the left essentially consists of two capacitors in parallel, giving double the capacitance value of the setup on the right. Therefore, the sensitivity of the left hand setup is twice as high as that on the right. Both systems are linear, with a change in displacement dx giving a change in capacitance $dC/dx=\varepsilon_r\varepsilon_0 w/d$, where w is the width of the metal plates, and d their separation.

2.14 The device below shows a "Chevron-shaped" capacitive sensor in which the capacitance changes as the rectangular bar slides from left-to-right and vice versa. Is this system linear? Discuss one advantage of this particular design.



If we consider the area of overlap as a function of distance along the chevron:



The overlap is given by d(b1+b2)/2. Also $\tan\theta=2d/(b2-b1)$

Using these simple equations, the overlap area is linearly dependent upon the position of the moving bar, and so the capacitance is also linear with respect to displacement. One of the advantages of this type of device is that it is immune to any slight tilting of the moving bar, since an increase in capacitance due to one part being closer to the base is compensated by a decrease in resistance from the part tilted away from the base.

2.15 *Design a MEMS device to produce a range of capacitances from 1pF to 5 pF.*

One possibility is to take the sliding capacitor in problem 2.13. Assume a very small device with maximum overlap of 2 x 2 mm^2. A material with a relatively high dielectric constant, e.g. ε_r=10, can be deposited on the top of one of the surfaces. For a capacitance of 5 pF, the thickness of the dielectric is approximately 70 μm, which is a reasonable deposition thickness. The movable plate must be able to move by a distance of approximately 1.6 mm in order to produce a capacitance range of 1 to 5 pF.

2.16 *In capacitance plethysmography a cuff electrode is placed around the limb with a small gap in-between, and the change in capacitance between the leg and the electrode measured. Assume that the limb is a uniform circular cylinder with diameter D, and the spacing s is uniform, calculate the sensitivity of the system in terms of the relationship between the change in capacitance to the change in volume.*

The capacitance is given by:

$$C = \frac{\varepsilon L \pi D}{s}$$

where ε is the permittivity of the spacer and L is the length of the electrode. The volume (V) of the leg located underneath the electrode is given by:

$$V = \frac{\pi L (D - 2s)^2}{4}$$

Assuming that D>>2s, then the sensitivity is given by:

$$\frac{dC}{dV} = \frac{dC}{ds} \Big/ \frac{dV}{ds} = \frac{\varepsilon}{s^2} \frac{D}{D - 2s} \approx \frac{\varepsilon}{s^2}$$

2.17 *Impedance plethysmography uses a four-band electrode placed around the leg. A constant current is supplied to the two outer electrodes, and the voltage between the two inner electrodes is measured. The ratio of the voltage to current gives the limb impedance. Assume that the electrodes cover a length L, with volume V_0 and tissue resistivity ρ_0. Additional blood with volume V_b and resistivity ρ_b enters the leg and is distributed uniformly in the leg. Derive an expression which relates the change in volume to the measured change in impedance.*

Assuming that the additional blood is distributed uniformly in the leg, it forms a parallel conductor. The original impedance Z_0 is given by:

$$Z_0 = \frac{\rho_0 L}{A_0} = \frac{\rho_0 L^2}{V_0}$$

The impedance of the additional blood is given by:

$$Z_b = \frac{\rho_b L}{A_b} = \frac{\rho_b L^2}{V_b}$$

The total impedance is given by the parallel combination of Z_0 and Z_b:

$$Z_{total} = \frac{Z_0 Z_b}{Z_0 + Z_b}$$

So the change in impedance ΔZ with the additional blood is given by:

$$\Delta Z = Z_{total} - Z_0 = \frac{Z_0 Z_b}{Z_0 + Z_b} - Z_0 = -\frac{Z_0^2}{Z_0 + Z_b}$$

If $\Delta Z << Z_0$ then:

$$V = -\frac{\rho_b L^2}{Z_0^2} \Delta Z$$

2.18 *Glucose measurements can be inaccurate due to a high hematocrit level in the blood. By considering separately (a) the electrochemical effect, and (b) the increased blood viscosity, at high hematocrit levels, explain whether these two effects produce an overestimate or underestimate of the true glucose concentration.*

Electrochemically, when the hematocrit is low the amount of glucose per unit blood is artificially high due to a decreased number of erythrocytes in the blood. This means that higher currents are produced, resulting in an overestimate of the glucose concentration. In terms of the blood viscosity, an increased hematocrit might impede diffusion of plasma into the reagent layer or decrease the volume of plasma which can diffuse. This would decrease the amount of glucose interacting with the sensor, therefore lowering the apparent glucose level

2.19 In a microphone, adding a hole in the diaphragm reduces the gain at low frequencies. Why does this occur?

A hole allows the pressure on either side of the diaphragm to equalize if the wavelength is long. This means that the gain is reduced at large wavelengths, i.e. at low frequencies.

Chapter 3