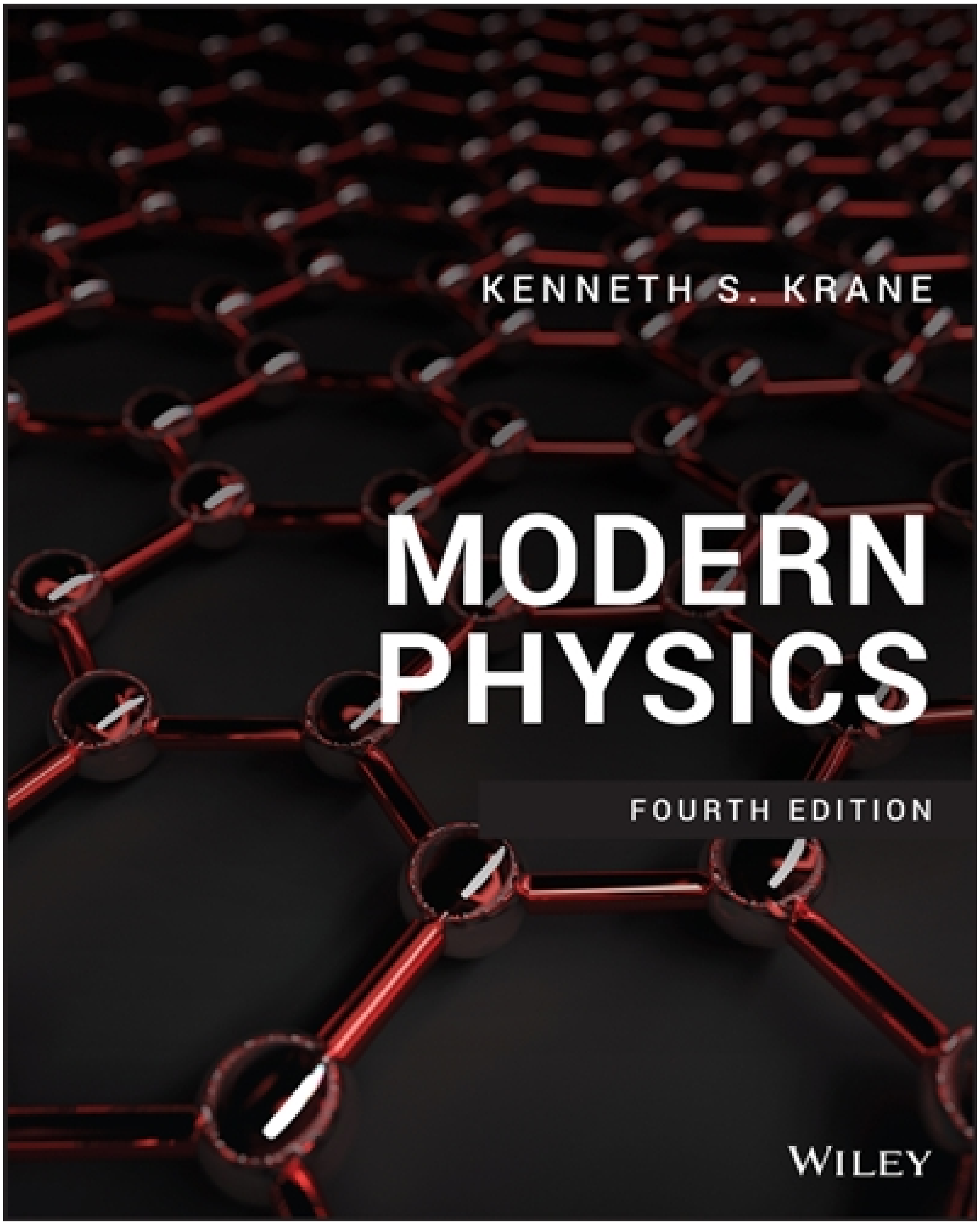
Solutions for Modern Physics 4th Edition by Krane






# Solutions

# Instructor's Solutions Manual

**to accompany**

# Modern Physics, 4th Edition

**Kenneth S. Krane**
**Department of Physics**
**Oregon State University**

# Chapter 1

1. (a) Conservation of momentum gives $p_{x,\text{initial}} = p_{x,\text{final}}$, or

$$m_{\text{H}} v_{\text{H,initial}} + m_{\text{He}} v_{\text{He,initial}} = m_{\text{H}} v_{\text{H,final}} + m_{\text{He}} v_{\text{He,final}}$$

Solving for $v_{\text{He,final}}$ with $v_{\text{He,initial}} = 0$, we obtain

$$v_{\text{He,final}} = \frac{m_{\text{H}}(v_{\text{H,initial}} - v_{\text{H,final}})}{m_{\text{He}}}$$
$$= \frac{(1.674\times10^{-27}\ \text{kg})[1.1250\times10^{7}\ \text{m/s} - (-6.724\times10^{6}\ \text{m/s})]}{6.646\times10^{-27}\ \text{kg}} = 4.527\times10^{6}\ \text{m/s}$$

(b) Kinetic energy is the only form of energy we need to consider in this elastic collision. Conservation of energy then gives $K_{\text{initial}} = K_{\text{final}}$, or

$$\tfrac{1}{2} m_{\text{H}} v_{\text{H,initial}}^2 + \tfrac{1}{2} m_{\text{He}} v_{\text{He,initial}}^2 = \tfrac{1}{2} m_{\text{H}} v_{\text{H,final}}^2 + \tfrac{1}{2} m_{\text{He}} v_{\text{He,final}}^2$$

Solving for $v_{\text{He,final}}$ with $v_{\text{He,initial}} = 0$, we obtain

$$v_{\text{He,final}} = \sqrt{\frac{m_{\text{H}}(v_{\text{H,initial}}^2 - v_{\text{H,final}}^2)}{m_{\text{He}}}}$$
$$= \sqrt{\frac{(1.674\times10^{-27}\ \text{kg})[(1.1250\times10^{7}\ \text{m/s})^2 - (-6.724\times10^{6}\ \text{m/s})^2]}{6.646\times10^{-27}\ \text{kg}}} = 4.527\times10^{6}\ \text{m/s}$$

2. (a) Let the helium initially move in the *x* direction. Then conservation of momentum gives:

$p_{x,\text{initial}} = p_{x,\text{final}}$: $\quad m_{\text{He}} v_{\text{He,initial}} = m_{\text{He}} v_{\text{He,final}} \cos\theta_{\text{He}} + m_{\text{O}} v_{\text{O,final}} \cos\theta_{\text{O}}$

$p_{y,\text{initial}} = p_{y,\text{final}}$: $\quad 0 = m_{\text{He}} v_{\text{He,final}} \sin\theta_{\text{He}} + m_{\text{O}} v_{\text{O,final}} \sin\theta_{\text{O}}$

From the second equation,

$$v_{\text{O,final}} = -\frac{m_{\text{He}} v_{\text{He,final}} \sin\theta_{\text{He}}}{m_{\text{O}} \sin\theta_{\text{O}}} = -\frac{(6.6465\times10^{-27}\ \text{kg})(6.636\times10^{6}\ \text{m/s})(\sin 84.7°)}{(2.6560\times10^{-26}\ \text{kg})[\sin(-40.4°)]} = 2.551\times10^{6}\ \text{m/s}$$

(b) From the first momentum equation,

$$v_{\text{He,initial}} = \frac{m_{\text{He}} v_{\text{He,final}} \cos\theta_{\text{He}} + m_{\text{O}} v_{\text{O,final}} \cos\theta_{\text{O}}}{m_{\text{He}}}$$

$$= \frac{(6.6465\times 10^{-27}\text{ kg})(6.636\times 10^{6}\text{ m/s})(\cos 84.7°) + (2.6560\times 10^{-26}\text{ kg})(2.551\times 10^{6}\text{ m/s})[\cos(-40.4°)]}{6.6465\times 10^{-27}\text{ kg}}$$

$$= 8.376\times 10^{6}\text{ m/s}$$

3. (a) Using conservation of momentum for this one-dimensional situation, we have $p_{x,\text{initial}} = p_{x,\text{final}}$, or

$$m_{\text{He}} v_{\text{He}} + m_{\text{N}} v_{\text{N}} = m_{\text{D}} v_{\text{D}} + m_{\text{O}} v_{\text{O}}$$

Solving for $v_{\text{O}}$ with $v_{\text{N}} = 0$, we obtain

$$v_{\text{O}} = \frac{m_{\text{He}} v_{\text{He}} - m_{\text{D}} v_{\text{D}}}{m_{\text{O}}} = \frac{(3.016\text{ u})(6.346\times 10^{6}\text{ m/s}) - (2.014\text{ u})(1.531\times 10^{7}\text{ m/s})}{15.003\text{ u}} = -7.79\times 10^{5}\text{ m/s}$$

(b) The kinetic energies are:

$$K_{\text{initial}} = \tfrac{1}{2} m_{\text{He}} v_{\text{He}}^2 + \tfrac{1}{2} m_{\text{N}} v_{\text{N}}^2 = \tfrac{1}{2}(3.016\text{ u})(1.6605\times 10^{-27}\text{ kg/u})(6.346\times 10^{6}\text{ m/s})^2 = 1.008\times 10^{-13}\text{ J}$$

$$K_{\text{final}} = \tfrac{1}{2} m_{\text{D}} v_{\text{D}}^2 + \tfrac{1}{2} m_{\text{O}} v_{\text{O}}^2 = \tfrac{1}{2}(2.014\text{ u})(1.6605\times 10^{-27}\text{ kg/u})(1.531\times 10^{7}\text{ m/s})^2 + \tfrac{1}{2}(15.003\text{ u})(1.6605\times 10^{-27}\text{ kg/u})(7.79\times 10^{5}\text{ m/s})^2 = 3.995\times 10^{-13}\text{ J}$$

As in Example 1.2, this is also a case in which nuclear energy turns into kinetic energy. The gain in kinetic energy is exactly equal to the loss in nuclear energy.

4. Let the two helium atoms move in opposite directions along the *x* axis with speeds $v_1$ and $v_2$. Conservation of momentum along the *x* direction ($p_{x,\text{initial}} = p_{x,\text{final}}$) gives

$$0 = m_1 v_1 - m_2 v_2 \qquad \text{or} \qquad v_1 = v_2$$

The energy released is in the form of the total kinetic energy of the two helium atoms:

$$K_1 + K_2 = 92.2\text{ keV}$$

Because $v_1 = v_2$, it follows that $K_1 = K_2 = 46.1\text{ keV}$, so

$$v = \sqrt{\frac{2K_1}{m_1}} = \sqrt{\frac{2(46.1\times 10^{3}\text{ eV})(1.602\times 10^{-19}\text{ J/eV})}{(4.00\text{ u})(1.6605\times 10^{-27}\text{ kg/u})}} = 1.49\times 10^{6}\text{ m/s}$$

$$v_2 = v_1 = 1.49\times 10^{6}\text{ m/s}$$

5. (a) The kinetic energy of the electrons is

$$K_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(9.11\times10^{-31}\text{ kg})(1.76\times10^6\text{ m/s}) = 14.11\times10^{-19}\text{ J}$$

In passing through a potential difference of $\Delta V = V_f - V_i = +4.15$ volts, the potential energy of the electrons changes by

$$\Delta U = q\Delta V = (-1.602\times10^{-19}\text{ C})(+4.15\text{ V}) = -6.65\times10^{-19}\text{ J}$$

Conservation of energy gives $K_i + U_i = K_f + U_f$, so

$$K_f = K_i + (U_i - U_f) = K_i - \Delta U = 14.11\times10^{-19}\text{ J} + 6.65\times10^{-19}\text{ J} = 20.76\times10^{-19}\text{ J}$$

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(20.76\times10^{-19}\text{ J})}{9.11\times10^{-31}\text{ kg}}} = 2.13\times10^6\text{ m/s}$$

(b) In this case $\Delta V = -4.15$ volts, so $\Delta U = +6.65\times10^{-19}$ J and thus

$$K_f = K_i - \Delta U = 14.11\times10^{-19}\text{ J} - 6.65\times10^{-19}\text{ J} = 7.46\times10^{-19}\text{ J}$$

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(7.46\times10^{-19}\text{ J})}{9.11\times10^{-31}\text{ kg}}} = 1.28\times10^6\text{ m/s}$$

6. (a) $\Delta x_A = v\Delta t_A = (0.624)(2.997\times10^8\text{ m/s})(124\times10^{-9}\text{ s}) = 23.2\text{ m}$
(b) $\Delta x_B = v\Delta t_B = (0.624)(2.997\times10^8\text{ m/s})(159\times10^{-9}\text{ s}) = 29.7\text{ m}$

7. With $T = 35°\text{C} = 308\text{ K}$ and $P = 1.22\text{ atm} = 1.23\times10^5\text{ Pa}$,

$$\frac{N}{V} = \frac{P}{kT} = \frac{1.23\times10^5\text{ Pa}}{(1.38\times10^{-23}\text{ J/K})(308\text{ K})} = 2.89\times10^{25}\text{ atoms/m}^3$$

so the volume available to each atom is $(2.89\times10^{25}/\text{m}^3)^{-1} = 3.46\times10^{-26}\text{ m}^3$. For a spherical atom, the volume would be

$$\tfrac{4}{3}\pi R^3 = \tfrac{4}{3}\pi(0.710\times10^{-10}\text{ m})^3 = 1.50\times10^{-30}\text{ m}^3$$

The fraction is then

$$\frac{1.50\times10^{-30}}{3.46\times10^{-26}} = 4.34\times10^{-5}$$

8. Differentiating $N(E)$ from Equation 1.23, we obtain

$$\frac{dN}{dE}=\frac{2N}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}\left[\tfrac{1}{2}E^{-1/2}e^{-E/kT}+E^{1/2}\left(-\frac{1}{kT}\right)e^{-E/kT}\right]$$

To find the maximum, we set this function equal to zero:

$$\frac{2N}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}E^{-1/2}e^{-E/kT}\left(\frac{1}{2}-\frac{E}{kT}\right)=0$$

Solving, we find the maximum occurs at $E=\tfrac{1}{2}kT$. Note that $E = 0$ and $E = \infty$ also satisfy the equation, but these solutions give minima rather than maxima.

9. For this case $kT=(280\text{ K})(8.617\times10^{-5}\text{ eV/K})=0.0241\text{ eV}$. We take $dE$ as the width of the interval (0.0012 eV) and $E$ as its midpoint (0.0306 eV). Then

$$dN=N(E)\,dE=\frac{2N}{\sqrt{\pi}}\frac{1}{(0.0241\text{ eV})^{3/2}}(0.0306\text{ eV})^{1/2}e^{-(0.0306\text{ eV})/(0.0241\text{ eV})}(0.0012\text{ eV})=1.8\times10^{-2}N$$

10. (a) From Eq. 1.33,

$$\Delta E_{\text{int}}=\tfrac{5}{2}nR\,\Delta T=\tfrac{5}{2}(2.37\text{ moles})(8.315\text{ J/mol}\cdot\text{K})(65.2\text{ K})=3.21\times10^{3}\text{ J}$$

(b) From Eq. 1.34,

$$\Delta E_{\text{int}}=\tfrac{7}{2}nR\,\Delta T=\tfrac{7}{2}(2.37\text{ moles})(8.315\text{ J/mol}\cdot\text{K})(65.2\text{ K})=4.50\times10^{3}\text{ J}$$

(c) For both cases, the change in the translational part of the kinetic energy is given by Eq. 1.31:

$$\Delta E_{\text{int}}=\tfrac{3}{2}nR\,\Delta T=\tfrac{5}{2}(2.37\text{ moles})(8.315\text{ J/mol}\cdot\text{K})(65.2\text{ K})=1.93\times10^{3}\text{ J}$$

11. After the collision, $m_1$ moves with speed $v_1'$ (in the $y$ direction) and $m_2$ with speed $v_2'$ (at an angle $\theta$ with the $x$ axis). Conservation of energy then gives $E_{\text{initial}} = E_{\text{final}}$:

$$\tfrac{1}{2}m_1v_1^2=\tfrac{1}{2}m_1v_1'^2+\tfrac{1}{2}m_2v_2'^2 \qquad \text{or} \qquad v^2=v_1'^2+3v_2'^2$$

Conservation of momentum gives:

$$p_{x,\text{initial}} = p_{x,\text{final}}: \quad m_1 v_1 = m_2 v_2' \cos\theta \qquad \text{or} \qquad v = 3v_2' \cos\theta$$

$$p_{y,\text{initial}} = p_{y,\text{final}}: \quad 0 = m_1 v_1' - m_2 v_2' \sin\theta \quad \text{or} \qquad v_1' = 3v_2' \sin\theta$$

We first solve for the speeds by eliminating $\theta$ from these equations. Squaring the two momentum equations and adding them, we obtain $v^2 + v_1'^2 = 9v_2'^2$, and combining this result with the energy equation allows us to solve for the speeds:

$$v_1' = v/\sqrt{2} \qquad \text{and} \qquad v_2' = v/\sqrt{6}$$

By substituting this value of $v_2'$ into the first momentum equation, we obtain

$$\cos\theta = \sqrt{2/3} \qquad \text{or} \qquad \theta = 35.3^\circ$$

12. The combined particle, with mass $m' = m_1 + m_2 = 3m$, moves with speed $v'$ at an angle $\theta$ with respect to the $x$ axis. Conservation of momentum then gives:

$$p_{x,\text{initial}} = p_{x,\text{final}}: \quad m_1 v_1 = m' v' \cos\theta \qquad \text{or} \qquad v = 3v' \cos\theta$$

$$p_{y,\text{initial}} = p_{y,\text{final}}: \quad m_2 v_2 = m' v' \sin\theta \qquad \text{or} \qquad \tfrac{4}{3} v = 3v' \sin\theta$$

We can first solve for $\theta$ by dividing these two equations to eliminate the unknown $v'$:

$$\tan\theta = \tfrac{4}{3} \qquad \text{or} \qquad \theta = 53.1^\circ$$

Now we can substitute this result into either of the momentum equations to find

$$v' = 5v/9$$

The kinetic energy lost is the difference between the initial and final kinetic energies:

$$K_{\text{initial}} - K_{\text{final}} = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 - \tfrac{1}{2} m' v'^2 = \tfrac{1}{2} m v^2 + \tfrac{1}{2}(2m)(\tfrac{2}{3} v)^2 - \tfrac{1}{2}(3m)(\tfrac{5}{9} v)^2 = \tfrac{26}{27}(\tfrac{1}{2} m v^2)$$

The total initial kinetic energy is $\tfrac{1}{2} m v^2 + \tfrac{1}{2}(2m)(\tfrac{2}{3} v)^2 = \tfrac{17}{9}(\tfrac{1}{2} m v^2)$. The loss in kinetic energy is then $\tfrac{26}{51} = 51\%$ of the initial kinetic energy.

13. (a) Let $v_1$ represent the helium atom that moves in the $+x$ direction, and let $v_2$ represent the other helium atom (which might move either in the positive or negative $x$ direction). Then conservation of momentum ($p_{x,\text{initial}} = p_{x,\text{final}}$) gives

$$m v = m_1 v_1 + m_2 v_2 \qquad \text{or} \qquad 2v = v_1 + v_2$$

where $v_2$ may be positive or negative. The initial velocity $v$ is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(40.0\times10^{3}\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}{(8.00\ \text{u})(1.6605\times10^{-27}\ \text{kg/u})}} = 9.822\times10^{5}\ \text{m/s}$$

The energy available to the two helium atoms after the decay is the initial kinetic energy of the beryllium atom plus the energy released in its decay:

$$K + 92.2\ \text{keV} = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2(2v - v_1)^2$$

where the last substitution is made from the momentum equation. Solving this quadratic equation for $v_1$, we obtain $v_1 = 2.47\times10^{6}\ \text{m/s}$ or $-0.508\times10^{6}\ \text{m/s}$. Because we identified $m_1$ as the helium moving in the positive $x$ direction, it is identified with the positive root and thus (because the two heliums are interchangeable in the equation) the second value represents the velocity of $m_2$:

$$v_1 = 2.47\times10^{6}\ \text{m/s},\ v_2 = -0.508\times10^{6}\ \text{m/s}$$

(b) Suppose we were to travel in the positive $x$ direction at a speed of $v = 9.822\times 10^5$ m/s, which is the original speed of the beryllium from part (a). If we travel at the same speed as the beryllium, it appears to be at rest, so its initial momentum is zero in this frame of reference. The two heliums then travel with equal speeds in opposite directions along the $x$ axis. Because they share the available energy equally, each helium has a kinetic energy of 46.1 keV and a speed of $\sqrt{2K/m} = 1.49\times10^{6}\ \text{m/s}$, as we found in Problem 4. Let's represent these velocities in this frame of reference as $v_1' = +1.49\times10^{6}\ \text{m/s}$ and $v_2' = -1.49\times10^{6}\ \text{m/s}$. Transforming back to the original frame, we find

$$v_1 = v_1' + v = 1.49\times10^{6}\ \text{m/s} + 9.822\times10^{5}\ \text{m/s} = 2.47\times10^{6}\ \text{m/s}$$

$$v_2 = v_2' + v = -1.49\times10^{6}\ \text{m/s} + 9.822\times10^{5}\ \text{m/s} = -0.508\times10^{6}\ \text{m/s}$$

14. (a) Let the second helium move in a direction at an angle $\theta$ with the $x$ axis. (We'll assume that the 30° angle for $m_1$ is measured above the $x$ axis, while the angle $\theta$ for $m_2$ is measured below the $x$ axis. Then conservation of momentum gives:

$$p_{x,\text{initial}} = p_{x,\text{final}}:\quad mv = m_1v_1\cos 30° + m_2v_2\cos\theta \qquad \text{or} \qquad 2v - \frac{\sqrt{3}}{2}v_1 = v_2\cos\theta$$

$$p_{y,\text{initial}} = p_{y,\text{final}}:\quad 0 = m_1v_1\sin 30° - m_2v_2\sin\theta \qquad \text{or} \qquad \frac{1}{2}v_1 = v_2\sin\theta$$

We can eliminate the angle $\theta$ by squaring and adding the two momentum equations:

$$4v^2 + v_1^2 - 2\sqrt{3}vv_1 = v_2^2$$

The kinetic energy given to the two heliums is equal to the original kinetic energy $\frac{1}{2}mv^2$ of the beryllium plus the energy released in the decay:

$$\tfrac{1}{2}mv^2 + 92.2\text{ keV} = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2(4v^2 + v_1^2 - 2\sqrt{3}vv_1)$$
$$\tfrac{1}{2}(m_1 + m_2)v_1^2 - \sqrt{3}m_2vv_1 + (2m_2v^2 - \tfrac{1}{2}mv^2 - 92.2\text{ keV}) = 0$$

Solving this quadratic equation gives

$$v_1 = 2.405\times10^6\text{ m/s}, \ -0.321\times10^6\text{ m/s}$$

Based on the directions assumed in writing the momentum equations, only the positive root is meaningful. We can substitute this value for $v_1$ into either the momentum or the energy equations to find $v_2$ and so our solution is:

$$v_1 = 2.41\times10^6\text{ m/s}, v_2 = 1.25\times10^6\text{ m/s}$$

The angle $\theta$ can be found by substituting these values into either of the momentum equations, for example

$$\theta = \sin^{-1}\frac{v_1}{2v_2} = \sin^{-1}\frac{2.41\times10^6\text{ m/s}}{2(1.25\times10^6\text{ m/s})} = 74.9°$$

(b) The original speed of the beryllium atom is $v = \sqrt{2K/m} = 1.203\times10^6\text{ m/s}$. If we were to view the experiment from a frame of reference moving at this velocity, the original beryllium atom would appear to be at rest. In this frame of reference, in which the initial momentum is zero, the two helium atoms are emitted in opposite directions with equal speeds. Each helium has a kinetic energy of 46.1 keV and a speed of $v_1' = v_2' = 1.49\times10^6\text{ m/s}$. Let $\phi$ represent the angle that each of the helium atoms makes with the $x$ axis in this frame of reference. Then the relationship between the $x$ components of the velocity of $m_1$ in this frame of reference and the original frame of reference is

$$v_1\cos 30° = v_1'\cos\phi + v$$

and similarly for the $y$ components

$$v_1\sin 30° = v_1'\sin\phi$$

We can divide these two equations to get

$$\cot 30° = \frac{v_1' \cos\phi + v}{v_1' \sin\phi}$$

which can be solved to give $\phi = 53.8°$. Using this value of $\phi$, we can then find $v_1 = 2.41\times10^6$ m/s. We can also write the velocity addition equations for $m_2$:

$$v_2 \cos\theta = -v_2' \cos\phi + v \qquad \text{and} \qquad v_2 \sin\theta = -v_2' \sin\phi$$

which describe respectively the $x$ and $y$ components. Solving as we did for $m_1$, we find $v_2 = 1.25\times10^6$ m/s and $\theta = 74.9°$.

15. (a) With $K = \frac{3}{2}kT$,

$$\Delta K = \tfrac{3}{2}k\Delta T = \tfrac{3}{2}(1.38\times10^{-23}\ \text{J/K})(80\ \text{K}) = 1.66\times10^{-21}\ \text{J} = 0.0104\ \text{eV}$$

(b) With $U = mgh$,

$$h = \frac{U}{mg} = \frac{1.66\times10^{-21}\ \text{J}}{(40.0\ \text{u})(1.66\times10^{-27}\ \text{kg/u})(9.80\ \text{m/s}^2)} = 2550\ \text{m}$$

16. We take $dE$ to be the width of this small interval: $dE = 0.04kT - 0.02kT = 0.02kT$, and we evaluate the distribution function at an energy equal to the midpoint of the interval ($E = 0.03kT$):

$$\frac{dN}{N} = \frac{N(E)\,dE}{N} = \frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}(0.03kT)^{1/2} e^{-(0.03kT)/kT}(0.02kT) = 3.79\times10^{-3}$$

17. If we represent the molecule as two atoms considered as point masses $m$ separated by a distance $2R$, the rotational inertia about one of the axes is $I_{x'} = mR^2 + mR^2 = 2mR^2$. On average, the rotational kinetic energy about any one axis is $\frac{1}{2}kT$, so $\frac{1}{2}I_{x'}\omega_{x'}^2 = \frac{1}{2}kT$ and

$$\omega_{x'} = \sqrt{\frac{kT}{I_{x'}}} = \sqrt{\frac{kT}{2mR^2}} = \sqrt{\frac{(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})}{2(15.995\ \text{u})(1.6605\times10^{-27}\ \text{kg/u})(0.0605\times10^{-9}\ \text{m})^2}} = 4.61\times10^{12}\ \text{rad/s}$$

# Chapter 2

1. Your air speed in still air is (750 km)/(3.14 h) = 238.8 km/h. With the nose of the plane pointed 22° west of north, you would be traveling at this speed in that direction if there were no wind. With the wind blowing, you are actually traveling due north at an effective speed of (750 km)/(4.32 h) = 173.6 km/h. The wind must therefore have a north-south component of (238.8 km/h)(cos 22°) – 173.6 km/h = 47.8 km/h (toward the south) and an east-west component of (238.8 km/h)(sin 22°) = 89.5 km/h (toward the east). The wind speed is thus

$$v=\sqrt{(47.8\text{ km/h})^2+(89.5\text{ km/h})^2}=101\text{ km/h}$$

in a direction that makes an angle of

$$\theta=\tan^{-1}\frac{89.5\text{ km/h}}{47.8\text{ km/h}}=62°\text{ east of south}$$

2. (a) $$\frac{95\text{ m}}{0.53\text{ m/s}}=179\text{ s}$$

(b) $$\frac{95\text{ m}}{1.24\text{ m/s}+0.53\text{ m/s}}=54\text{ s}$$

(c) $$\frac{95\text{ m}}{2.48\text{ m/s}-0.53\text{ m/s}}=49\text{ s}$$

3. $$\Delta t=t_{\text{up}}+t_{\text{down}}-2t_{\text{across}}=\frac{2L}{c}\left[\frac{1}{1-u^2/c^2}-\frac{1}{\sqrt{1-u^2/c^2}}\right]$$

Assuming $u \ll c$,

$$\frac{1}{1-u^2/c^2}\cong 1+\frac{u^2}{c^2}\qquad\text{and}\qquad\frac{1}{\sqrt{1-u^2/c^2}}\cong 1+\frac{1}{2}\frac{u^2}{c^2}$$

$$\Delta t\cong\frac{2L}{c}\left[1+\frac{u^2}{c^2}-\left(1+\frac{1}{2}\frac{u^2}{c^2}\right)\right]=\frac{Lu^2}{c^3}$$

$$u=\sqrt{\frac{c^3\Delta t}{L}}=\sqrt{\frac{(3\times10^8\text{ m/s})^3(2\times10^{-15}\text{ s})}{11\text{ m}}}=7\times10^4\text{ m/s}$$

4. (a) $u = 100 \text{ km/h} = 28 \text{ m/s} \ll c$

$$\sqrt{1-u^2/c^2} \cong 1-\frac{1}{2}\frac{u^2}{c^2} = 1-\frac{1}{2}\frac{(28 \text{ m/s})^2}{(3\times10^8 \text{ m/s})^2} = 1-4.3\times10^{-15}$$

$$L = L_0\sqrt{1-u^2/c^2} = L_0(1-4.3\times10^{-15})$$

$$L_0 - L = (4.3\times10^{-15})L_0 = (4.3\times10^{-15})(4\times10^6 \text{ m}) = 1.7\times10^{-8} \text{ m}$$

This is less than the wavelength of light.

(b) $$\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} \cong \frac{\Delta t_0}{1-4.3\times10^{-15}} \cong \Delta t_0(1+4.3\times10^{-15})$$

$$\Delta t - \Delta t_0 = (4.3\times10^{-15})\Delta t_0 = (4.3\times10^{-15})(40 \text{ h})(3600 \text{ s/h}) = 6.2\times10^{-10} \text{ s}$$

5. With $L = \frac{1}{2}L_0$, the length contraction formula gives $\frac{1}{2}L_0 = L_0\sqrt{1-u^2/c^2}$, so

$$u = \sqrt{3/4}\,c = 2.6\times10^8 \text{ m/s}$$

6. The astronaut must travel 600 light-years at a speed close to the speed of light and must age only 12 years. To an Earth-bound observer, the trip takes about $\Delta t$ = 600 years, but this is a dilated time interval; in the astronaut's frame of reference, the elapsed time is the proper time interval $\Delta t_0$ of 12 years. Thus, with $\Delta t = \Delta t_0/\sqrt{1-u^2/c^2}$,

$$600 \text{ years} = \frac{12 \text{ years}}{\sqrt{1-u^2/c^2}} \quad \text{or} \quad 1-\frac{u^2}{c^2} = \left(\frac{12}{600}\right)^2$$

$$u = \sqrt{1-(12/600)^2}\,c = 0.9998c$$

7. (a) $$\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} = \frac{120.0 \text{ ns}}{\sqrt{1-(0.950)^2}} = 384 \text{ ns}$$

(b) $$d = v\,\Delta t = 0.950(3.00\times10^8 \text{ m/s})(384\times10^{-9} \text{ s}) = 109 \text{ m}$$

(c) $$d_0 = v\,\Delta t_0 = 0.950(3.00\times10^8 \text{ m/s})(120.0\times10^{-9} \text{ s}) = 34.2 \text{ m}$$

8. In the laboratory reference frame, the lifetime is

$$\Delta t = \frac{d}{u} = \frac{1.15 \text{ mm}}{0.993(3.00\times 10^{8} \text{ m/s})} = 0.386\times 10^{-11} \text{ s}$$

$$\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = (0.386\times 10^{-11} \text{ s})\sqrt{1-(0.993)^2} = 4.56\times 10^{-13} \text{ s}$$

9. From Equation 2.15, $\Delta t_1 = L/(v-u)$, and from Equation 2.16, $\Delta t_2 = L/(c+u)$.

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{L}{v-u} + \frac{L}{c+u} = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$$

With $\Delta t_0 = L_0/v' + L_0/c$ and $L = L_0\sqrt{1-u^2/c^2}$, this becomes

$$\frac{1}{v-u} + \frac{1}{c+u} = \frac{1}{1-u^2/c^2}\left(\frac{1}{v'} + \frac{1}{c}\right)$$

Solving for *v*, we obtain

$$v = \frac{v'+u}{1+v'u/c^2}$$

10. Let ship *A* represent observer *O*, and let observer *O*′ be on Earth. Then $v' = 0.831c$ and $u = -0.743c$, and so

$$v = \frac{v'+u}{1+v'u/c^2} = \frac{0.831c + 0.743c}{1+(0.831)(0.743)} = 0.973c$$

If now ship *B* represents observer *O*, then $v' = -0.743c$ and $u = -0.831c$.

$$v = \frac{v'+u}{1+v'u/c^2} = \frac{-0.743c - 0.831c}{1+(-0.743)(-0.831)} = -0.973c$$

11. Let *O*′ be the observer on the space station, and let *O* be the observer on ship *B*. Then $v' = 0.811c$ and $u = -0.665c$.

$$v = \frac{v'+u}{1+v'u/c^2} = \frac{0.811c - 0.665c}{1+(0.811)(-0.665)} = 0.317c$$

12. (a) With $f' = f\sqrt{(1-u/c)/(1+u/c)}$ and $\lambda = c/f$, we obtain

$$\lambda' = \lambda\sqrt{\frac{1+u/c}{1-u/c}} \qquad \text{or} \qquad 366\text{ nm} = 122\text{ nm}\sqrt{\frac{1+u/c}{1-u/c}}$$

Solving, we get $u/c = 0.800$ or $u = 2.40 \times 10^8$ m/s.

(b) $\lambda' = \lambda\sqrt{\frac{1-u/c}{1+u/c}} = 122\text{ nm}\sqrt{\frac{1-0.800}{1+0.800}} = 40.7\text{ nm}$

13. With $f' = f\sqrt{(1-u/c)/(1+u/c)}$ and $\lambda = c/f$, we obtain

$$\frac{1-u/c}{1+u/c} = \left(\frac{f'}{f}\right)^2 = \left(\frac{\lambda}{\lambda'}\right)^2 = \left(\frac{650\text{ nm}}{550\text{ nm}}\right)^2 = 1.397$$

Solving, $u/c = 0.166$ or $u = 5.\ 0 \times 10^7$ m/s.

14. According to $O$, the rest length $L_0$ of the hypotenuse is shortened to

$$L = L_0\sqrt{1-u^2/c^2} = L_0\sqrt{1-(0.92)^2} = 0.392L_0$$

The height of the triangle, which is $0.5L_0$, is not affected by the length contraction. The base angles of the triangle are then

$$\theta = \tan^{-1}\left(\frac{0.5L_0}{\frac{1}{2}\times 0.392L_0}\right) = \tan^{-1}(2.55) = 68.6°$$

The apex angle is then 180° – 2(68.6°) = 42.8°.

15. In the direction of motion, the radius of the gold nucleus is reduced to

$$(7.0\text{ fm})\sqrt{1-(0.99995)^2} = 0.070\text{ fm}$$

At this speed the formerly spherical gold nucleus looks like a pancake whose thickness is only 1% that of the original sphere and thus whose density has been increased by a factor of 100. The collision of two such compressed, high-energy nuclei is thought to create for a fraction of a second the conditions of density and temperature that occurred in the very early universe.

16. $$dx' = \frac{dx - u\,dt}{\sqrt{1-u^2/c^2}} \quad \text{and} \quad dt' = \frac{dt - u\,dx/c^2}{\sqrt{1-u^2/c^2}}$$

$$v'_x = \frac{dx'}{dt'} = \frac{dx - u\,dt}{dt - u\,dx/c^2} = \frac{dx/dt - u}{1 - u(dx/dt)/c^2} = \frac{v_x - u}{1 - uv_x/c^2}$$

With $dz' = dz$, we obtain

$$v'_z = \frac{dz'}{dt'} = \frac{dz}{(dt - u\,dx/c^2)/\sqrt{1-u^2/c^2}} = \frac{(dz/dt)\sqrt{1-u^2/c^2}}{1-uv_x/c^2} = \frac{v_z\sqrt{1-u^2/c^2}}{1-uv_x/c^2}$$

17. For the light beam, observer $O$ measures $v_x = 0$, $v_y = c$. Observer $O'$ measures

$$v'_x = \frac{v_x - u}{1-uv_x/c^2} = 0 - u = -u \quad \text{and} \quad v'_y = \frac{v_y\sqrt{1-u^2/c^2}}{1-uv_x/c^2} = c\sqrt{1-u^2/c^2}$$

According to $O'$, the speed of the light beam is

$$v' = \sqrt{(v'_x)^2 + (v'_y)^2} = \sqrt{u^2 + c^2(1-u^2/c^2)} = c$$

18. $O$ measures times $t_1$ and $t_2$ for the beginning and end of the interval, while $O'$ measures $t'_1$ and $t'_2$. Using Equation 2.23*d*,

$$t'_1 = \frac{t_1 - ux/c^2}{\sqrt{1-u^2/c^2}} \quad \text{and} \quad t'_2 = \frac{t_2 - ux/c^2}{\sqrt{1-u^2/c^2}}$$

The same coordinate $x$ appears in both expressions, because the bulb is at rest according to $O$ (so $\Delta t$ is the proper time interval). Subtracting these two equations, we obtain

$$t'_2 - t'_1 = \frac{t_2 - t_1}{\sqrt{1-u^2/c^2}} \quad \text{or} \quad \Delta t' = \frac{\Delta t}{\sqrt{1-u^2/c^2}}$$

19. Suppose observer $O$ is moving with the $K$ meson; to this observer, the $K$ meson appears to be at rest, and so $O$ measures $v_1 = +0.815c$ and $v_2 = -0.815c$ for the two $\pi$ mesons. Observer $O'$ is moving relative to $O$ with a velocity $u = -0.453c$; in the reference frame of $O'$, observer $O$ and the $K$ meson are moving in the positive $x$ direction with a velocity of $0.453c$. We can use the Lorentz velocity transformation (Equation 2.28*a*) to find the velocities of the two $\pi$ mesons according to $O'$:

$$v_1' = \frac{v_1 - u}{1 - v_1 u / c^2} = \frac{+0.815c - (-0.453c)}{1 - (0.815)(-0.453)} = +0.926c$$

$$v_2' = \frac{v_2 - u}{1 - v_2 u / c^2} = \frac{-0.815c - (-0.453c)}{1 - (-0.815)(-0.453)} = -0.575c$$

20. Imagine the rod to be the hypotenuse of a right triangle having sides $L_x$ along the $x$ axis and $L_y$ in the $y$ direction. According to $O'$, the length $L_x$ is shortened by the length contraction, but the length $L_y$ is unaffected because it is perpendicular to the direction of motion. For $O$, $L_y = L_x \tan 34°$, while for $O'$, $L_y' = L_x' \tan 52°$ where $L_x' = L_x \sqrt{1 - u^2 / c^2}$. Because $L_y = L_y'$, we have

$$L_x \tan 34° = L_x' \tan 52° = L_x \sqrt{1 - u^2 / c^2} \tan 52°$$

or

$$u = c\sqrt{1 - (\tan 34°)^2 / (\tan 52°)^2} = 0.850c$$

21. (a) Changing the coordinates in Equation 2.23*d* to intervals, we have

$$\Delta t' = \frac{\Delta t - u\,\Delta x / c^2}{\sqrt{1 - u^2 / c^2}} = \frac{0.528\,\mu\text{s} - (0.685)(49.5\text{ m}) / (300\text{ m}/\mu\text{s})}{\sqrt{1 - (0.685)^2}} = +0.570\,\mu\text{s}$$

(b) Changing coordinates to intervals in Equation 2.23*a*,

$$\Delta x' = \frac{\Delta x - u\,\Delta t}{\sqrt{1 - u^2 / c^2}} = \frac{49.5\text{ m} - (0.685 \times 300\text{ m}/\mu\text{s})(0.528\,\mu\text{s})}{\sqrt{1 - (0.685)^2}} = -81.0\text{ m}$$

The negative sign of $\Delta x'$ indicates that $O'$ finds the two events in inverted locations compared with $O$; for example, if $O$ finds that event 1 occurs at a smaller $x$ coordinate than event 2, then $O'$ finds that event 1 occurs at a *larger* $x'$ coordinate than event 2. That is, $O$ sees event 1 to the left of event 2, while $O'$ sees event 1 to the right of event 2. Note than both observers find the time interval to be positive – event 2 occurs after event 1 to both observers.

22. From Equation 2.23*d* written in terms of intervals, for $O'$ to find $\Delta t' = 0$, it must be true that $\Delta t - (u / c^2)\Delta x = 0$. Thus

$$u = \frac{c^2 \Delta t}{\Delta x} = c \frac{(300 \text{ m}/\mu\text{s})(0.138\ \mu\text{s} - 0.124\ \mu\text{s})}{(23.6 \text{ m} - 10.4 \text{ m})} = +0.32c$$

23. (a) In your frame of reference on the plane, the distance between Los Angeles and Boston is contracted to $(3000 \text{ mi})\sqrt{1-[(600 \text{ mi/h})/(1000 \text{ mi/h})]^2} = 2400 \text{ mi}$, and you measure the time for the trip to be (2400 mi)/(600 mi/h) = 4 h. So your watch reads 2:00 pm.
(b) According to an observer on the ground, the trip takes (3000 mi)/(600 mi/h) = 5 h. So the airport clock reads 3:00 pm.
(c) The return trip again takes 4 h as measured in your frame of reference and 5 h from the frame of reference of the ground. When you depart Boston, the airport clock reads 10:00 am but your watch reads 9:00 am, so when you land in Los Angeles, your watch reads 1:00 pm and the airport clock reads 3:00 pm.

This analysis duplicates the result of the twin paradox: If your twin wearing an identical watch had remained at the Los Angeles airport, all observers would agree that you were 2 hours younger than your twin.

24. (a) On the outward journey at 0.60*c*, the rate at which signals are received is

$$f' = f\sqrt{\frac{1-u/c}{1+u/c}} = (1/\text{year})\sqrt{\frac{1-0.60}{1+0.60}} = 0.5/\text{year}$$

(b) During the return journey,

$$f' = f\sqrt{\frac{1+u/c}{1-u/c}} = (1/\text{year})\sqrt{\frac{1+0.60}{1-0.60}} = 2/\text{year}$$

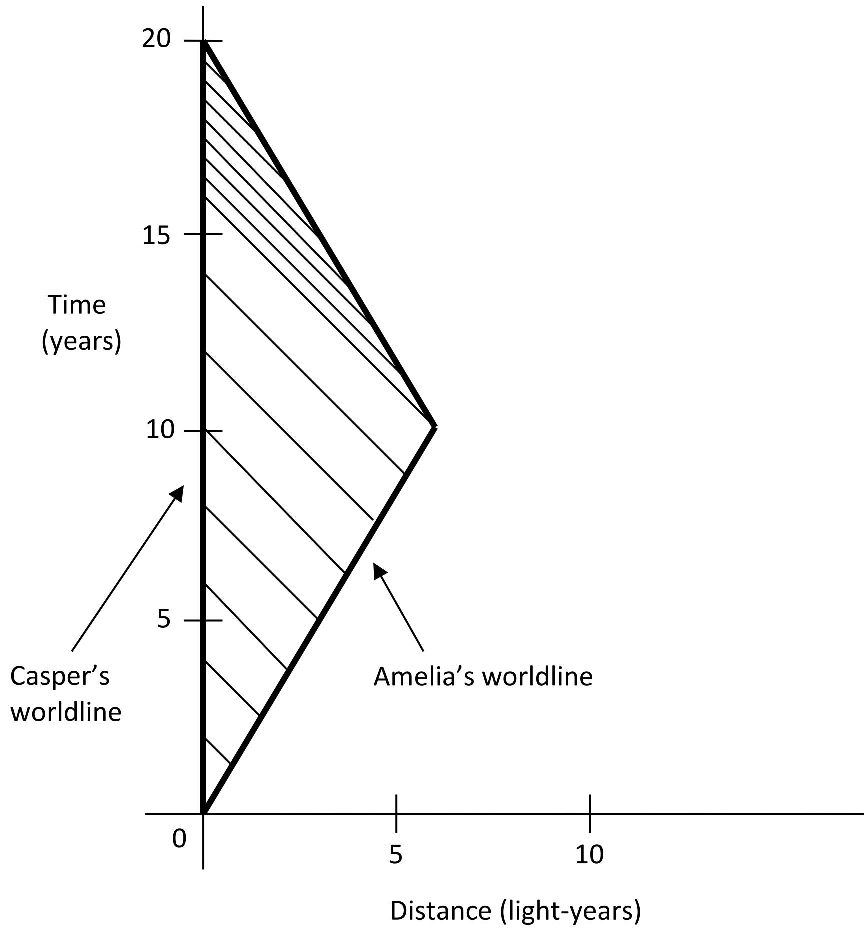
(c) According to Casper, Amelia's outward journey lasts 16 years (it is 16 years before he sees her arrive at the planet), during which he receives 8 signals (0.5/year × 16 years). Her total journey lasts 20 Earth years, so the return journey lasts 4 Earth years, during which he receives 8 signals (2/year × 4 years). Thus Casper receives 16 signals (8 during the outward trip and 8 during the return) and concludes that his sister has aged 16 years.

25. According to Amelia, the distance to the star is shortened to

$$L = L_0\sqrt{1-v^2/c^2} = (8.0 \text{ l-y})\sqrt{1-(0.80)^2} = 4.8 \text{ l-y}$$

and at a speed of 0.80*c* Amelia's travel time to the star is (4.8 l-y)/(0.80*c*) = 6.0 y. The total round-trip time in Amelia's frame of reference is 12 years, so she is 8 years younger than her brother when she returns.

26. (a)




(b) 16 years

(c) 4 years

27. (a) To an Earth-bound observer Alice's round trip takes 20 years each way (20 years × 0.6$c$ = 12 light-years) for a total time of 40 years. Bob's travel time is 15 years each way (15 years × 0.8$c$ = 12 light-years) for a total travel time of 30 years. With Bob's 10-year delay in departing, the two arrive on Earth simultaneously.
(b) To Alice, the distance to the star is contracted to

$$L = L_0\sqrt{1-v^2/c^2} = 12 \text{ light-years}\sqrt{1-(0.6)^2} = 9.6 \text{ light-years}$$

So in Alice's frame of reference the trip takes a time of (9.6 light years)/0.6$c$ = 16 years each way. To Bob, the distance to the star is

$$L = L_0\sqrt{1-v^2/c^2} = 12 \text{ light-years}\sqrt{1-(0.8)^2} = 7.2 \text{ light-years}$$

and in Bob's frame the travel time is (7.2 light-years)/0.8$c$ = 9 years each way. Relative to Alice's original departure time, Alice has aged 32 years while Bob has aged 10 + 18 = 28 years. So Bob is younger by 4 years.

28. (a) Suppose Agnes travels at speed $v$. Then in her reference frame the distance to the star is shortened to $L = L_0\sqrt{1 - v^2/c^2}$, so the time for her one-way trip is $L/v$ and thus

$$\frac{16\ \text{light-years}\sqrt{1-v^2/c^2}}{v} = 10\ \text{y} \qquad \text{or} \qquad \sqrt{\frac{c^2}{v^2} - 1} = \frac{10}{16}$$

Solving, we find $v = 0.848c$.
(b) According to Bert, Agnes traveled on a journey of 32 light-years at a speed of 0.848$c$ which corresponds to a time of (32 light-years)/0.848$c$ = 37.7 years.

29. (a)

$$K'_\text{i} = K'_{1\text{i}} + K'_{2\text{i}} = \frac{m_1c^2}{\sqrt{1 - v'^2_{1\text{i}}/c^2}} - m_1c^2 + \frac{m_2c^2}{\sqrt{1 - v'^2_{2\text{i}}/c^2}} - m_2c^2$$

$$= \frac{(2m)c^2}{\sqrt{1 - 0^2}} - (2m)c^2 + \frac{mc^2}{\sqrt{1 - (0.750)^2}} - mc^2 = 0.512mc^2$$

$$K'_\text{f} = K'_{1\text{f}} + K'_{2\text{f}} = \frac{m_1c^2}{\sqrt{1 - v'^2_{1\text{f}}/c^2}} - m_1c^2 + \frac{m_2c^2}{\sqrt{1 - v'^2_{2\text{f}}/c^2}} - m_2c^2$$

$$= \frac{(2m)c^2}{\sqrt{1 - (-0.585)^2}} - (2m)c^2 + \frac{mc^2}{\sqrt{1 - (0.294)^2}} - mc^2 = 0.512mc^2$$

(b)

$$K_\text{i} = K_{1\text{i}} + K_{2\text{i}} = \frac{m_1c^2}{\sqrt{1 - v^2_{1\text{i}}/c^2}} - m_1c^2 + \frac{m_2c^2}{\sqrt{1 - v^2_{2\text{i}}/c^2}} - m_2c^2$$

$$= \frac{(2m)c^2}{\sqrt{1 - (0.550)^2}} - (2m)c^2 + \frac{mc^2}{\sqrt{1 - (-0.340)^2}} - mc^2 = 0.458mc^2$$

$$K_\text{f} = K_{1\text{f}} + K_{2\text{f}} = \frac{m_1c^2}{\sqrt{1 - v^2_{1\text{f}}/c^2}} - m_1c^2 + \frac{m_2c^2}{\sqrt{1 - v^2_{2\text{f}}/c^2}} - m_2c^2$$

$$= \frac{(2m)c^2}{\sqrt{1 - (-0.051)^2}} - (2m)c^2 + \frac{mc^2}{\sqrt{1 - (0.727)^2}} - mc^2 = 0.458mc^2$$

30. $$p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \frac{1}{c}\frac{(mc^2)(v/c)}{\sqrt{1 - v^2/c^2}} = \frac{1}{c}\frac{(938.3\ \text{MeV})(0.835)}{\sqrt{1 - (0.835)^2}} = 1424\ \text{MeV}/c$$

$$K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = \frac{938.3\text{ MeV}}{\sqrt{1-(0.835)^2}} - 938.3\text{ MeV} = 767\text{ MeV}$$

$$E = K + mc^2 = 767\text{ MeV} + 938.3\text{ MeV} = 1705\text{ MeV}$$

31. $E = K + mc^2 = 0.923\text{ MeV} + 0.511\text{ MeV} = 1.434\text{ MeV}$

Solving Equation 2.36 for *v*, we obtain

$$v = c\sqrt{1-\left(\frac{mc^2}{E}\right)^2} = c\sqrt{1-\left(\frac{0.511\text{ MeV}}{1.434\text{ MeV}}\right)^2} = 0.934c$$

32. $$W = \int F\,dx = \int \frac{dp}{dt}\,dx = \int dp\,\frac{dx}{dt} = \int v\,dp$$

$$K = \int_0^v v\,dp = pv - \int_0^v p\,dv = \frac{mv^2}{\sqrt{1-v^2/c^2}} - \int_0^v \frac{mv}{\sqrt{1-v^2/c^2}}\,dv$$

$$= \frac{mv^2}{\sqrt{1-v^2/c^2}} + mc^2\sqrt{1-v^2/c^2} - mc^2 = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2$$

33. For what range of velocities is $K - \frac{1}{2}mv^2 \le 0.01K$? At the upper limit of this range, where $K - \frac{1}{2}mv^2 = 0.01K$, we have

$$0.99K = 0.99\left(\frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2\right) = \tfrac{1}{2}mv^2$$

With $x = v^2/c^2$, $0.99\left(\frac{1}{\sqrt{1-x}} - 1\right) = \frac{1}{2}x$ which gives $\frac{1}{1-x} = \left(1+\frac{0.5}{0.99}x\right)^2$

$$1 = (1-x)(1+1.0101x+0.2551x^2) \qquad \text{or} \qquad 0.2551x^2 + 0.7550x - 0.0101 = 0$$

Solving using the quadratic formula, we find $x$ = 0.0133 or -2.97. Only the positive solution is physically meaningful, so

$$v = \sqrt{0.0133}\,c = 0.115c$$

That is, for speeds smaller than 0.115*c*, the classical kinetic energy is accurate to within 1%. For a different approach to that same type of calculation, see Problem 36.

34. As in Problem 33, let us now find the *lower* limit on the momentum such that

$$\sqrt{(pc)^2+(mc^2)^2}-pc \le 0.01\sqrt{(pc)^2+(mc^2)^2}$$

From the lower limit, we obtain $0.99\sqrt{(pc)^2+(mc^2)^2} = pc$, which can be written as

$$(pc)^2 = \frac{m^2c^4}{1/(0.99)^2-1} \qquad \text{or} \qquad pc = 7.02mc^2$$

With $mvc/\sqrt{1-v^2/c^2} = 7.02mc^2$, we obtain

$$\frac{v^2}{c^2} = 49.25\left(1-\frac{v^2}{c^2}\right) \qquad \text{or} \qquad v/c = 0.990$$

Whenever $v/c \ge 0.990$, the expression $E = pc$ will be accurate to within 1%.

35. $$E^2 = \frac{(mc^2)^2}{1-v^2/c^2} = (mc^2)^2\left(\frac{1-v^2/c^2+v^2/c^2}{1-v^2/c^2}\right) = (mc^2)^2 + \frac{m^2c^2v^2}{1-v^2/c^2} = (mc^2)^2+(pc)^2$$

$$E = \sqrt{(mc^2)^2+(pc)^2}$$

36. With $(1+x)^n = 1+nx+n(n-1)x^2/2!$ we have

$$\frac{1}{\sqrt{1-v^2/c^2}} = 1+\frac{1}{2}\frac{v^2}{c^2}+\frac{(-1/2)(-3/2)}{2}\left(\frac{v^2}{c^2}\right)^2+\cdots$$

and so

$$K = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right) = mc^2\left(1+\frac{1}{2}\frac{v^2}{c^2}+\frac{3}{8}\frac{v^4}{c^4}+\cdots-1\right) = \tfrac{1}{2}mv^2\left(1+\frac{3}{4}\frac{v^2}{c^2}+\cdots\right)$$

The correction term is $3v^2/4c^2$, which has the value 0.1% when $3v^2/4c^2 = 0.001$, or

$$v = \sqrt{0.001(4/3)}\,c = 0.0365c$$

37. (a) With $E$ = 1351 MeV and $pc$ = 1256 MeV, Equation 2.39 gives

$$m = \frac{1}{c^2}\sqrt{E^2-(pc)^2} = \frac{1}{c^2}\sqrt{(1351\text{ MeV})^2-(1256\text{ MeV})^2} = 498\text{ MeV}/c^2$$

(b) $$E = \sqrt{(pc)^2 + (mc^2)^2} = \sqrt{(857\text{ MeV})^2 + (498\text{ MeV})^2} = 991\text{ MeV}$$

38.

$$K_\text{f} - K_\text{i} = (E_\text{f} - mc^2) - (E_\text{i} - mc^2) = E_\text{f} - E_\text{i}$$

$$= \frac{mc^2}{\sqrt{1 - v_\text{f}^2/c^2}} - \frac{mc^2}{\sqrt{1 - v_\text{i}^2/c^2}} = \frac{0.511\text{ MeV}}{\sqrt{1-(0.91)^2}} - \frac{0.511\text{ MeV}}{\sqrt{1-(0.85)^2}} = 0.262\text{ MeV}$$

39. $$\Delta E = mc\,\Delta T = (1\text{ g})(0.40\text{ J/g}\cdot\text{K})(100\text{ K}) = 40\text{ J}$$

$$\Delta m = \frac{\Delta E}{c^2} = \frac{40\text{ J}}{9\times 10^{16}\text{ m}^2/\text{s}^2} = 4.4\times 10^{-16}\text{ kg}$$

40. (a) At such low speed, the classical approximation is valid.

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}mc^2\left(\frac{v^2}{c^2}\right) = \tfrac{1}{2}(0.511\text{ MeV})(1.00\times 10^{-4})^2 = 2.56\times 10^{-3}\text{ eV}$$

(b) The relativistic expression gives

$$K = mc^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right) = 0.511\text{ MeV}\left(\frac{1}{\sqrt{1-(0.01)^2}} - 1\right) = 25.6\text{ eV}$$

For this speed, the classical expression $\frac{1}{2}mv^2$ also gives 25.6 eV, so the two calculations agree to at least three significant figures. (Actually they agree to four significant figures, but not to five.)

(c) The relativistic expression gives

$$K = mc^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right) = 0.511\text{ MeV}\left(\frac{1}{\sqrt{1-(0.3)^2}} - 1\right) = 24.7\text{ keV}$$

For this speed, the classical expression gives 23.0 keV, which is incorrect by about 7%.

(d) $$K = mc^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right) = 0.511\text{ MeV}\left(\frac{1}{\sqrt{1-(0.999)^2}} - 1\right) = 10.9\text{ MeV}$$

41. Because the electrons and the protons have charges of the same magnitude $e$, after acceleration through a potential difference of magnitude $\Delta V$ = 12.0 million volts (a positive difference for the electron, a negative difference for the proton), each loses potential energy of $\Delta U = -e\,\Delta V$ = -12.0 MeV and thus each acquires a kinetic energy of $K$ = +12.0 MeV. For the electron, $E = K + mc^2$ = 12.0 MeV + 0.511 MeV = 12.5 MeV. The momentum is then

$$p = \frac{1}{c}\sqrt{E^2 - (mc^2)^2} = \frac{1}{c}\sqrt{(12.5\text{ MeV})^2 - (0.511\text{ MeV})^2} = 12.5\text{ MeV}/c$$

The classical formula $K = p^2/2m$ gives

$$p = \sqrt{2mK} = \sqrt{2(0.511\text{ MeV}/c^2)(12.0\text{ MeV})} = 3.50\text{ MeV}/c$$

which is far from the correct result (a discrepancy we would expect for such highly relativistic electrons). For the protons, $E = K + mc^2$ = 12.0 MeV + 938.3 MeV= 950.3 MeV, and the momentum is

$$p = \frac{1}{c}\sqrt{E^2 - (mc^2)^2} = \frac{1}{c}\sqrt{(950.3\text{ MeV})^2 - (938.3\text{ MeV})^2} = 150.5\text{ MeV}/c$$

The classical formula gives

$$p = \sqrt{2mK} = \sqrt{2(938.3\text{ MeV}/c^2)(12.0\text{ MeV})} = 150.1\text{ MeV}/c$$

The difference between the classical and relativistic formulas appears only in the fourth significant figure.

42. The mass of a uranium atom is about (235 u)($1.66 \times 10^{-27}$ kg/u) = $3.90 \times 10^{-25}$ kg, so 1.50 kg contains 1.50 kg/$3.90 \times 10^{-25}$ kg = $3.84 \times 10^{24}$ atoms. The total energy released is

$$\Delta E = (210\text{ MeV/atom})(3.84\times10^{24}\text{ atoms}) = 8.06\times10^{26}\text{ MeV}$$

and the change in mass is

$$\Delta m = \frac{\Delta E}{c^2} = \frac{(8.06\times10^{26}\text{ MeV})(1.602\times10^{-13}\text{ J/MeV})}{(2.998\times10^{8}\text{ MeV})^2} = 1.44\times10^{-3}\text{ kg}$$

About one gram of matter vanishes for each kilogram that fissions!

43. (a) The change in mass is