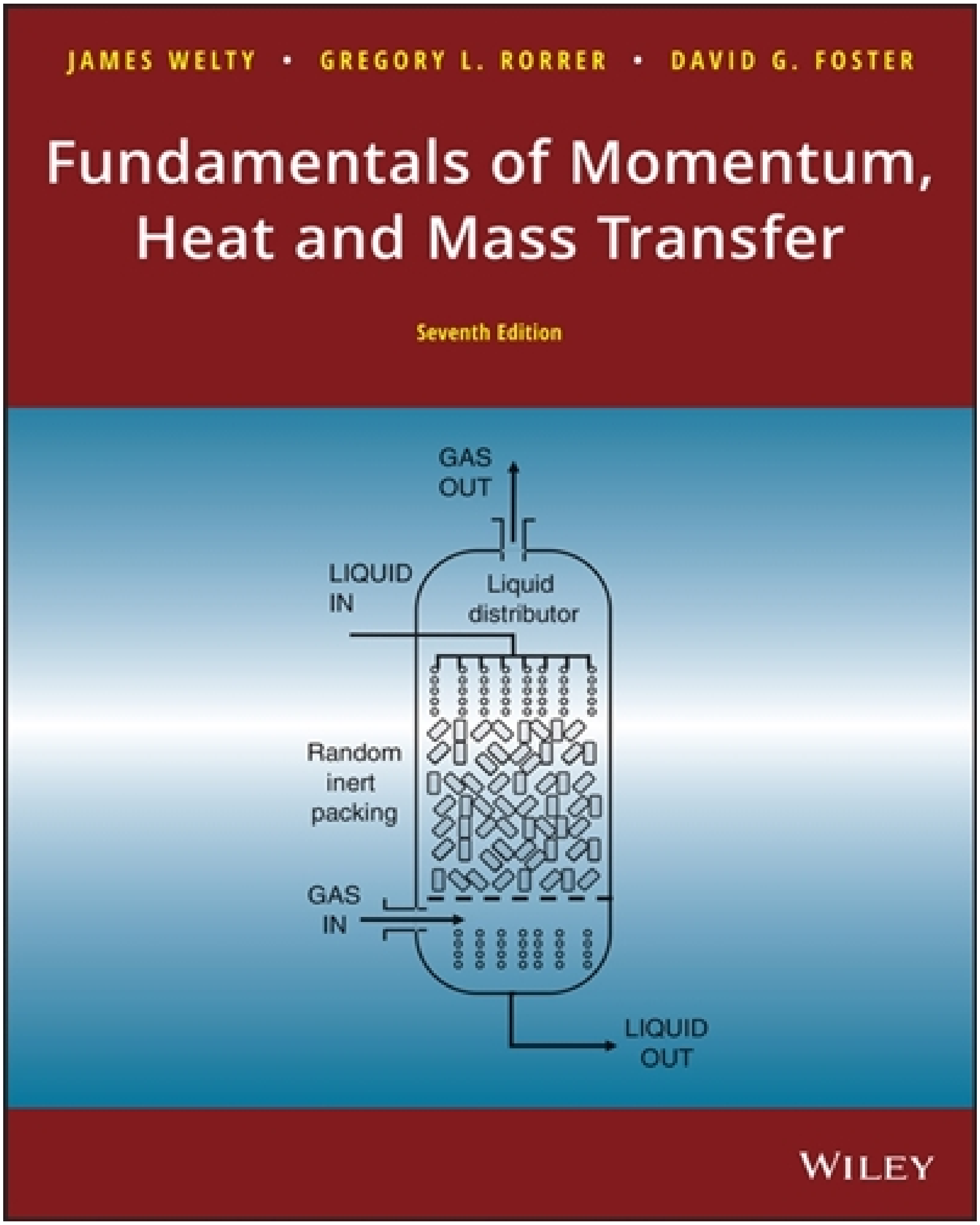
Solutions for Fundamentals of Momentum Heat and Mass Transfer 7th Edition by Welty



Solutions

Chapter 2 End of Chapter Problem Solutions

2.1

Assume Ideal Gas Behavior

$$\frac{dP}{dy} = -\rho g = -\frac{Pg}{RT}$$

For $T = a + by$

$\Rightarrow$ T= 530 - 24 y/n

$$\frac{dP}{P} = -\frac{g}{R} - \frac{dy}{530-24(\frac{y}{n})}$$

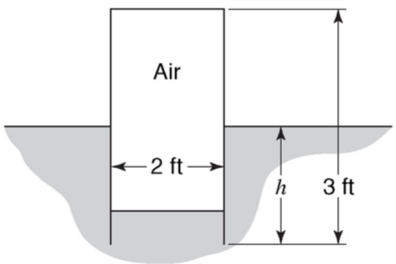
$$\int_{P_o}^{P} \frac{dP}{P} = \frac{gh}{24R} \int_0^1 \frac{-24\, d(\frac{y}{n})}{530-24(\frac{y}{n})}$$

$$\ln\frac{P}{P_o} = \frac{gh}{24R} \ln \frac{506}{530}$$

With $P = 10.6$ PSIA, $P_o = 30.1$ in Hg

h = 9192 ft.

2.2



(a) $\sum F=0$ on tank

$$P\frac{\pi d^2}{4} - P_{atm}\frac{\pi d^2}{4} - 250 = 0 \qquad (1)$$

At H_20 Level in Tank: $P=P_{atm}+\rho_w g(h-y)$ (2)

From (1) & (2): h-y=1.275 ft. (3)

For Isothermal Compression of Air
$P_{atm}V_{tank}=P(V_{air})$

$$P = \frac{3}{3-y}P_{atm} \qquad (4)$$

Combing (1) & (4): y=0.12 ft. and h=1.395 ft.

(b) For Top of Tank Flush with H_20 Level

$\sum F=0$
$P=P_{atm}+\frac{250+F}{\pi d^2/4}$

At H_20 Level in Tank: $P=P_{atm}+\rho_w g(3-y)$

Combining Equations:
F= 196(3-y)- 250

For Isothermal Compression of Air:
(As in Part (a))
3-y = 2.8 ft.

F = 196(2.8) - 250 = 293.6 LB_f

2.3

When New Force on Tank = 0

Wt. = Buoyant Force = 250 Lb_f

V_w Displaced = $250/\rho_w g$ = 4.01 ft^3

Assuming Isothermal Compression
$P_{atm}A(3ft.) = P(4.01\ ft^3) = (P_{atm}+\rho g y)(4.01)$
y=45.88 ft.

Top is at Level: $y-\frac{4.01}{\pi d^2/4}$

or at 44.6 ft. Below Surface

2.4

$\frac{dP}{dy} = \rho g = \rho_o e^{\Delta P/\beta}$

$\int_0^{\Delta P} \frac{-\Delta P/\beta}{e^{\Delta P/\beta}} = \int_0^y \frac{-\rho_o g \Delta y}{\beta}$

$e^{-\Delta P/\beta} = 1 - \frac{\rho_o g y}{\beta g}$

$\Delta P = -\beta \ln(1 - \frac{\rho_o y}{\beta}) = 300{,}000 \ln(1-0.0462) = 14190$ Psi

Density Ratio:

$\frac{\rho}{\rho_o} = e^{-\Delta P/\beta} = 1.0484$

so $P = 1.0484\rho_o$

2.5

Buoyant Force:

$F_B = \rho V = \frac{PV}{RT}$

For constant volume: F varies inversely with T

2.6

Sea H_20: S.G.=1.025

At Depth y=185m

$P_g = 1.025\rho_w gy = 1.025(1000)(9.81)(185) = 1.86x10^6$ Pa = 1.86 MPa

2.7

r = Measured from Earth's Surface
R = Radius of Earth

$\frac{dP}{dr} = \rho g = \rho g_o \frac{r}{R}$

$P - P_{atm} = \frac{\rho g_o r^2}{2R}$

At Center of Earth: r = R

$P_{Ctr} - P_{atm} = \frac{\rho g_o R}{2}$

Since $P_{Ctr} >> P_{atm}$

$P_{Ctr} \cong \frac{\rho g_o R}{2} = \frac{(5.67)(1000)(9.81)(6330x10^3)}{2} = 176x10^9$ Pa = 176 MPa

2.8

$$\frac{dP}{dy} = -\rho g$$

$$\int_{P_{atm}}^{P} dP = -\rho g \int_{0}^{-h} dy$$

$P - P_{atm} = \rho g(+h) = (1050)(9.81)(11034) = 113.7$ MPa $\cong$ 1122 Atmospheres

2.9

As in Previous Problem
$P-P_{atm} = \rho g h$

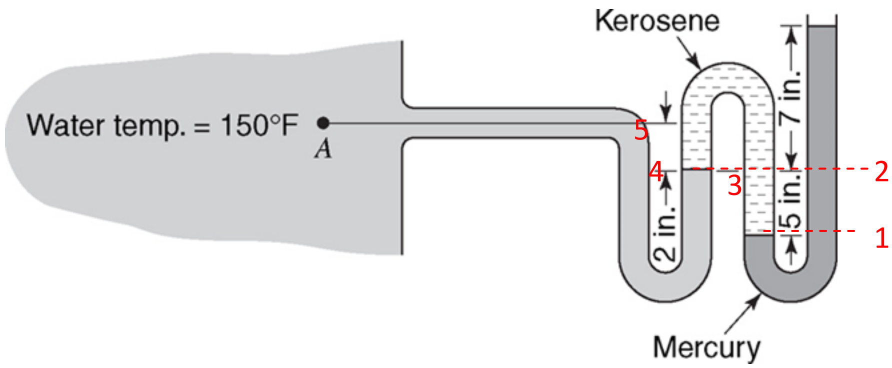
For $P-P_{atm} = 101.33$ kPa
$h = 101.33/\rho g$

for H_20: $h = \frac{101.33}{(1000)(9.81)} = 10.33m$

Sea H_20: $h = \frac{101.33}{(1.025)(1000)(9.81)} = 10.08m$

Hg: $h = \frac{101.33}{(13.6)(1000)(9.81)} = 0.80m$

2.10



$P_1 = P_{atm} + \rho_{Hg}\, g(12'')$ $\quad$ $P_1 = P_2$

$P_2 = P_3 + \rho_K\, g(5'')$ $\quad$ $P_3 = P_4$

$P_4 = P_A + \rho_w\, g(2'')$ $\quad$ $P_4 = P_5$

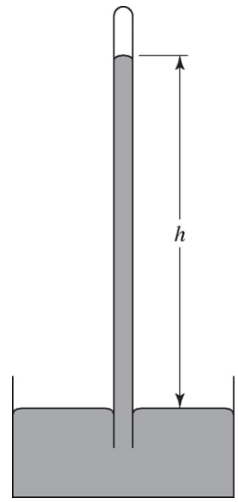
$P_{atm} + \rho_{Hg}\, g(12'') = P_A + \rho_w\, g(2'') + \rho_K\, g(5'')$

$P_A = P_{atm} + \rho_w g[(13.6)(12) - 2 - 0.75(5)] = P_{atm} + 5.81$

$P_A = 5.81$ PSIG

2.11

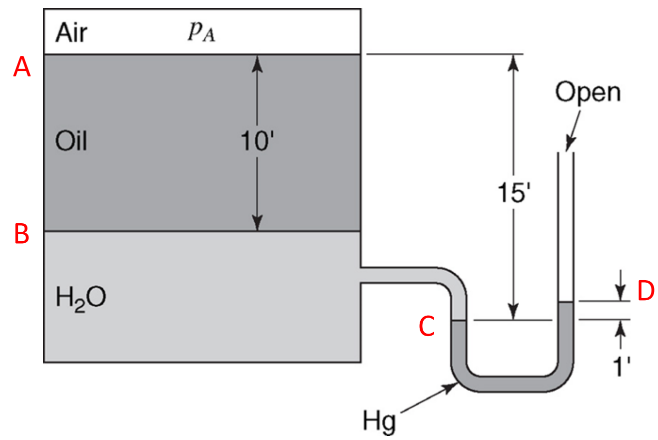
Force Balance on Liquid Column: A=Area of Tube



$-3A + 14.7A - \rho g h A = 0$

$h = \frac{11.7(144)}{62.4(12.2)} = 26.6$ in.

2.12



$P_A = P_B - \rho_o\, g(10 \text{ ft.})$

$P_C = P_B + \rho_w\, g(5 \text{ ft.})$

$P_D = P_C - \rho_{Hg}\, g(1 \text{ ft.})$

$P_A - P_D = \rho_{Hg}\, g(1) - \rho_w\, g(5) - \rho_o\, g(10)$

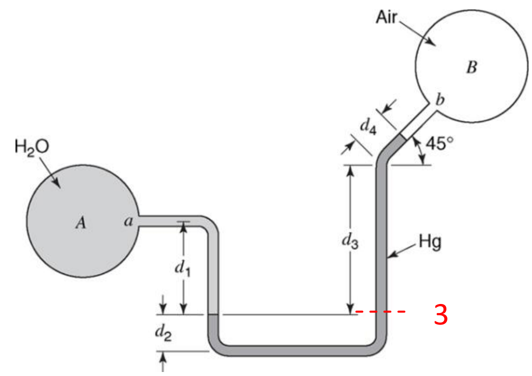
$P_A - P_{Atm} = \rho_w\, g(13.6 \text{ x}1 - 5 - 0.8 \text{ x } 10 \text{ x } 1) = 37.4 \text{ LB}_f/\text{ft}^2$

2.13

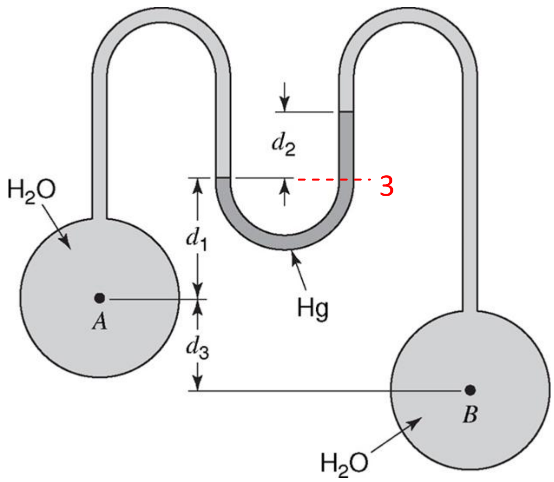
$P_3 = P_A - d_1 g\, \rho_w = P_B + (\rho_{Hg}\, g) x (d_3 + d_4 \sin 45)$

$$P_A - P_B = \frac{(62.4)(32.2)}{32.2}\left[\frac{(2.4+4\sin 45)}{12}\ 13.6 - 2\right]$$

$= 245\ LB_f/ft^2 = 1.70$ Psi



2.14



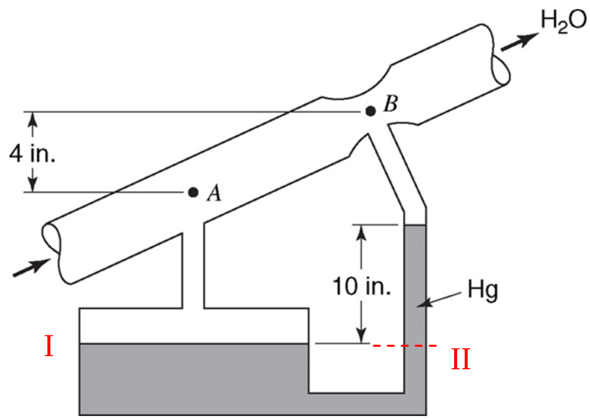
$P_3 = P_A - \rho_w g d_1$

$P_3 = P_B - \rho_w g(d_1+d_2+d_3) + \rho_{Hg}\, g d_2$

Equating:

$P_A - P_B = \rho_{Hg}\, g d_2 - \rho_w g(d_2+d_3) = \rho_w g[(13.6)(1/12)-7.3/12] = 32.8\ LB_f/ft^2 = 0.227$ Psi

2.15



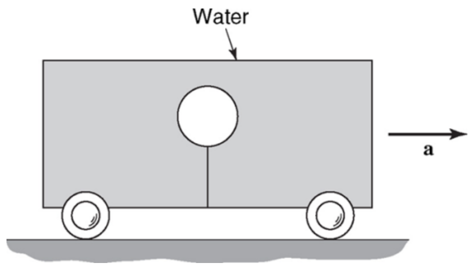
$P_I = P_A + \rho_w g(10")$

$P_{II} = P_B + \rho_w g(4") + \rho_{Hg} g(10")$

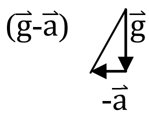
$P_I = P_{II}$

$P_A - P_B = \rho_w g[-6 + 13.6(10)] = 56.3$ psi

2.16

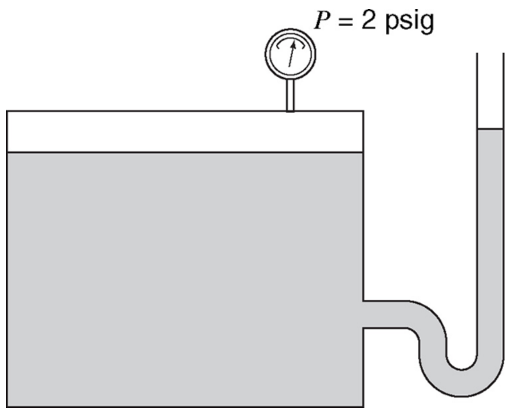


Pressure Gradient is in direction of $\vec{g}$-$\vec{a}$ & isobars are perpendicular to ($\vec{g}$-$\vec{a}$)



String will assume the ($\vec{g}$-$\vec{a}$) direction & Balloon will move <u>forward.</u>

2.17



At Rest: $P=\rho g y_o$

Accelerating: $P=\rho|(\vec{g}-\vec{a})|=\rho(g+a)y_a$

Equating: $y_a=\frac{g}{g-a}$ which $<y_o$

Level goes <u>down</u>.

2.18

$F = P_{6.6}A - P_{atm}A = \rho gh(\pi r^2)$

h=2m
r=0.3m

F=5546N

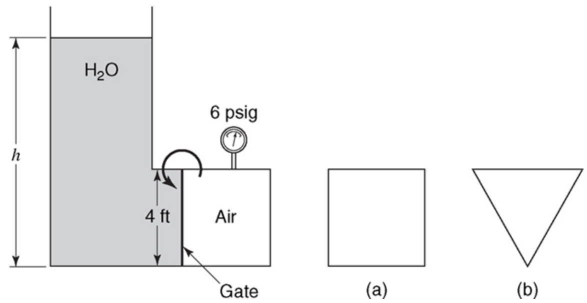
$y_{C.P.} = \bar{y} + I_{bb}/A\bar{y}$

For a circle: $I_{bb} = \pi r^4/4$

$$y_{C.P.} = 2m + \frac{\pi(0.3m)^4}{4\pi(0.3m)^2(2m)} = 2.011m$$

2.19

Height of H_20 column above differential element= h-4+y



For (a) - Rectangular gate- dA = 4dy

$dF_w=[\rho_w g(h-4+y)+P_{atm}]dA$

$dF_A=[P_{atm}+(6Psig)(144)]dA$

$\sum M_o= \int_A^0 y(dF_w - dF_A)=0$

$\int_0^4 y[\rho g(h-4+y)-864](4dy)=0$

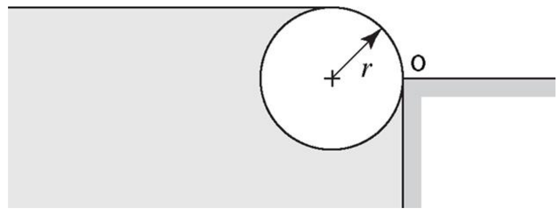
<u>h=15.18 ft.</u>

For (b): dA=(4-y)dy

$\int_0^4 y[\rho g(h-4+y)-864](4-y)dy=0$

<u>h=15.85 ft.</u>

2.20



Per unit depth: $\sum F_y = 0$

$F_y|_{up} = \rho_w g\, \pi r^2/2$ {buoyancy}

$F_y|_{down} = \rho g\, \pi r^2 + \rho_w g(r^2 - \pi r^2/4)$

Equating: $\frac{\rho_w g\, \pi r^2}{2} = \rho g\, \pi r^2 + \rho_w g r^2(1 - \pi/4)$

$\rho = \rho_w\left(\frac{\pi}{2} - 1 + \frac{\pi}{4}\right)/\pi = \rho_w\left(\frac{3}{4} - \frac{1}{\pi}\right) = 0.432\rho_w = 432 \text{ kg/m}^3$

2.21

a) To lift block from bottom

F = {wt. of concrete}+{wt. of H_20}

= $\rho_c gV+[\ \rho_w g(22.75')+P_{atm}]A$

= (150)g(3x3x0.5)+[62.4g(22.75)+14.7(144)]x(3x3)

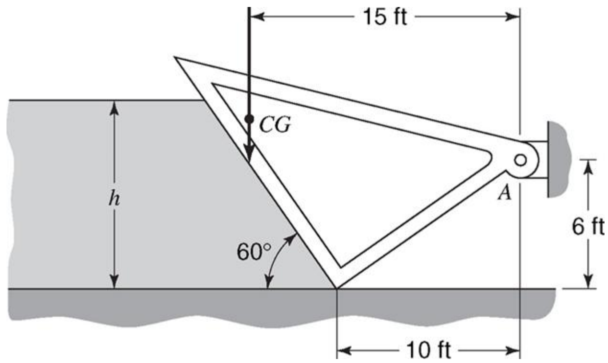
= 675+31828 = 32503 lb_f

b) To maintain block in free position

F = {wt. of concrete}-{Buoyant force of H_20)}

= 675- $\rho_w gV$ = 675-[62.4g(3x3x0.5)] = 675 - 281 = 394 lb_f

2.22



Distance z measured along gate surface from bottom

$\sum M_A = 500(15) - \int_0^{h/\sin 60} z\,\rho g(h - z\sin 60)dz = 0$

$\rho g \int_0^{h/\sin 60} (zh - z^2 \sin 60)\, dz = 7500$

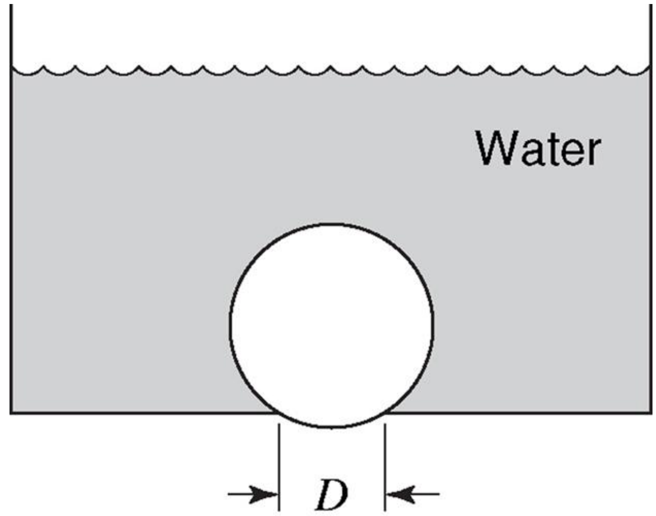
$\rho g \left[h\frac{z^2}{2} - \frac{z^3}{3}\sin 60 \right]_0^{h/\sin 60} = 7500$

$(62.4)g\left[\frac{h^3}{(\sin 60)^2}\left(\frac{1}{2} - \frac{1}{3}\right)\right] = 7500$

$h^3 = \frac{7500(6)(\sin 60)^2}{62.4g} = 541$

h=8.15 ft.

2.23



Using spherical coordinates for a ring
At y=constant

$dA = 2\pi r^2 \sin\theta d\theta$

$P = \rho g[h - r\cos\alpha + r\cos\theta]$

$dF_y = dF\cos\theta$

$F_y = \int \rho g(h - r\cos\alpha + r\cos\theta)(2\pi r^2 \sin\theta\cos\theta d\theta)$
$= 2\pi\ \rho g r^2 \int_x^{\pi} (h - r\cos\alpha + r\cos\theta)(\sin\theta\cos\theta d\theta)$

Let: $c = 2\pi\ \rho g r^2$

$= c[\int_\alpha^\pi (h - r\cos\alpha) \sin\theta\cos\theta d\theta + r\int_\alpha^\pi \sin\theta \cos^2\theta d\theta]$

$= c[(h - r\cos\alpha) \sin^2\theta\, |_\alpha^\pi + r(-\frac{1}{3}\cos^3\theta)|_\alpha^\pi]$

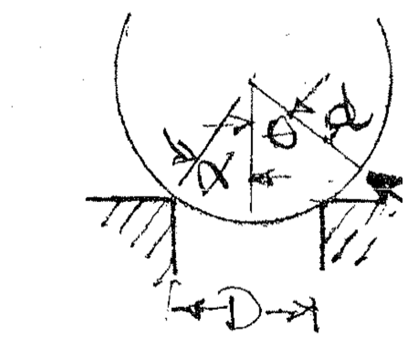
$= c[(h - r\cos\alpha)(1 - \sin^2\alpha) - \frac{r}{3}(0 - \cos^3\alpha)]$

Now for $F_y = 0$

$\sin\alpha = \frac{D}{d}\ \cos\alpha = [1 - \left(\frac{D}{d}\right)^2]^{1/2}$ & $r = d/2$

$0 = (h - \frac{d}{2}\cos\alpha)(\cos^2\alpha) + \frac{d}{6}(\cos^3\alpha) = h - \frac{d}{2}\cos\alpha + \frac{d}{6}\cos\alpha$

Giving $h = \frac{d}{3}\cos\alpha$

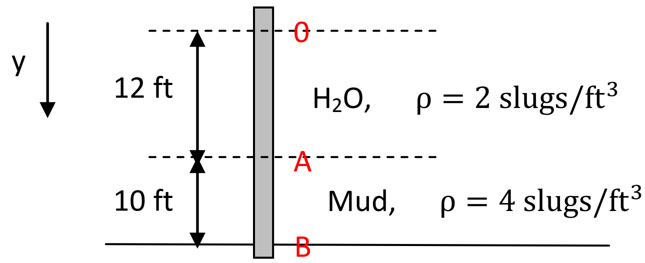


$\frac{h}{d} = \frac{\cos\alpha}{3} = \frac{1}{3}\left[1 - \left(\frac{D}{d}\right)^2\right]^{1/2}$

For d=0.6m

$h = \frac{0.6}{3} = \frac{1}{3}\left[1 - \left(\frac{5D}{3}\right)^2\right]^{1/2} = \frac{1}{5}\left[1 - \left(\frac{5D}{3}\right)^2\right]^{1/2}$

2.24



$P_A - P_{atm} = \rho_w g(12) = 24g$
$P_B - P_{atm} = 24g + 40g = 64g$

Between 0 & A: $P - P_{atm} = \rho_w g y$
Between A & B: $P = \rho_w g(12) + \rho_m g(y-12)$

Per unit depth:
$F = \int_{12}^{0} (P - P_{atm}) dA$

$= \int_0^{12} \rho_w g y dy + \int_{12}^{22} [\rho_w g(12) + \rho_m g(y - 12)] dy$

$= \rho_w g(192) + \rho_m g(50)$

$= 18790 \text{ lb}_f$

Force Location:
$Fxy = \int_0^{22} y(P - P_{atm}) dA$

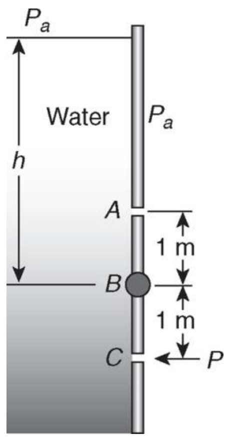
$= \int_0^{12} \rho_w g y^2 dy + \int_{12}^{22} \rho_w g 12 y dy + \int_{12}^{22} \rho_m g(y^2 - 12y) dy$

$= \rho_w g(576 + 2040) + \rho_m g(2973 - 2040)$

$= 288{,}400 \text{ ft. LB}_f$

$\bar{\bar{y}} = \frac{288{,}400}{18{,}790} = 15.35$ ft.

2.25



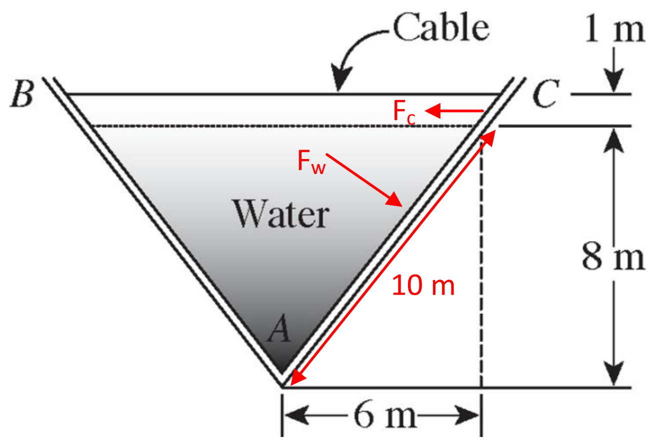
Force on gate= $\rho g \bar{y} A = (1000)(9.81)(12)\frac{\pi}{4}(2)^2 = 369.8$ kN

$y_{C.P.} = \frac{I_{\infty}}{\bar{y}A} = \frac{\frac{\pi}{4}(1)^4}{(12)\frac{\pi}{4}(2)^2} = 0.0208$m (Below axis B)

$\sum M_B = 0$

$P(1) = (369.8 \text{x} 10^3)(0.0208) = 7.70$ kN

2.26



$F_w = \rho_g g\bar{\bar{y}}A = (1000)(9.81)(4)(10)(1) = 392$ kN

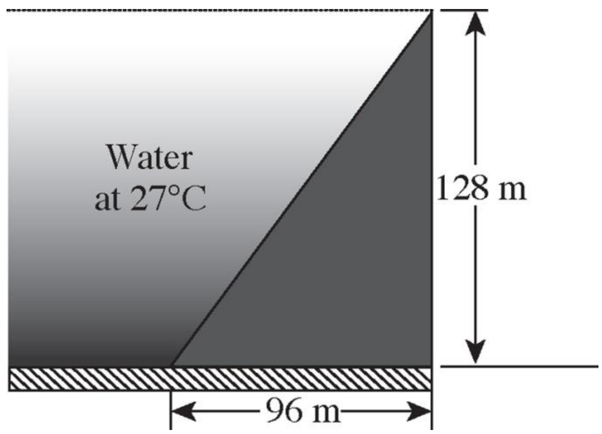
y_{cp} is 2/3 distance form water line to A

~ 6.66m down form H_20 line
~ 3.33m up from A

$\sum M_A = F_c(9) = 392(3.33)$

$F_c = 145.2$ kN

2.27



Width = 100m
H_20@27℃   ρ=997 kg/m^3

F = $\rho_g g\bar{y}A$ = (997)(9.81)(64)x(160)(100) = 10.016 x 10^9 N = 10.02x10^3 MN

For a free H_20 surface

y_{cp}= $\frac{2}{3}$(128m) = 85.3m {below H_20 surface} =106.7 m {measured along dam surface}

2.28 Spherical Float

Upward forces ~ $F + F_{Buoyant}$

Downward forces ~ W_T

$W = \rho g V = \rho g(\frac{4}{3}\pi R^3)$

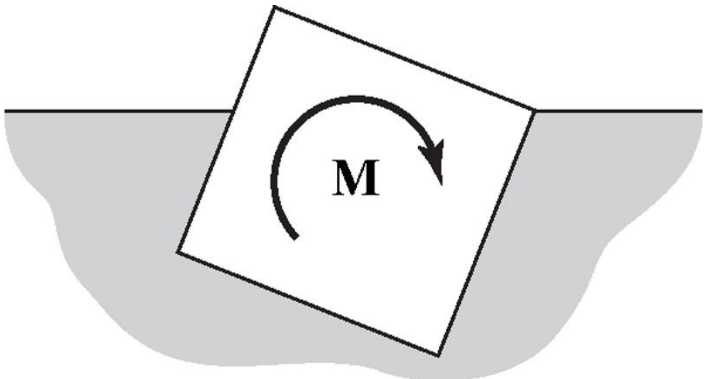
$F_b = \rho_w g V z = \rho_w g(\frac{4}{3}\pi R^3)z$

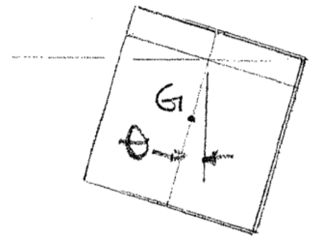
z= fraction submerged

$F = \rho g(\frac{4}{3}\pi R^3) - \rho_w g z(\frac{4}{3}\pi R^3)$

$z = \frac{\rho g\left(\frac{4}{3}\pi R^3\right) - F}{\rho_w g(\frac{4}{3}\pi R^3)}$

2.29





$\theta = \tan^{-1}\left(\frac{0.1}{0.5}\right)$

G is center of mass of solid

$\sum M_G = 2[1/2(L/2)(0.1L)(L)\ \rho g(\frac{2}{3}\frac{L}{2}\sin\theta) - (0.9L)(L)(L)\ \rho g(0.05L\sin\theta)] + M$

{Part of original submerged volume is now out of H_20}
(Part that was originally out is now submerged}

$M = \rho g\ L^4 \sin\theta[-\frac{1}{60} + 0.045] = \rho g L^4 \sin\theta(0.02833) = 0.00556\ \rho g L^4$

2.30

A large industrial waste collection tank contains butyl alcohol, benzene and water at 80°F that have separated into three distinct phases as shown in the figure. The diameter of the circular tank is 10 feet and it has total depth is 95 feet. The gauge at the top of the tank reads 2116 lb_f/ft^2. Please calculate (a) the pressure at the butyl alcohol/benzene interface, (b) the pressure at the benzene/water interface and (c) the pressure at the bottom of the tank.

Solution

(a) the pressure at the butyl alcohol/benzene interface

$$\Delta P = \rho g h$$

$$P_{top} - P_{Air-ButylAlcohol\ Interface} = -\rho_{air} g(17 feet)$$

$$P_{Air-ButylAlcohol\ Interface} - P_{ButylAlcohol-Benzene\ Interface} = -\rho_{ButylAlcohol} g(19 feet)$$

$$P_{ButylAlcohol-Benzene\ Interface} = P_{top} + \rho_{air} g(17 feet) + \rho_{ButylAlcohol} g(19 feet)$$

$$= 2116 \frac{lb_f}{ft^2} + \frac{\left(0.0735 \frac{lb_m}{ft^3}\right)(32.2\ ft/s^2)(17\ ft)}{32.174 \frac{lb_m ft}{lb_f s^2}}$$

$$+ \frac{\left(50.0 \frac{lb_m}{ft^3}\right)(32.2\ ft/s^2)(19\ ft)}{32.174 \frac{lb_m ft}{lb_f s^2}} = 3068.0 \frac{lb_f}{ft^2} = 21.3\ psi$$

(b) the pressure at the benzene/water interface

$$P_{ButylAlcohol-Benzene\ Interface} - P_{Benzene-Water\ Interface} = -\rho_{Benzene} g(34 feet)$$

$$P_{Benzene-Water\ Interface} = P_{ButylAlcohol-Benzene\ Interface} + \rho_{Benzene} g(34 feet)$$

$$= 3068.0 \frac{lb_f}{ft^2} + \frac{\left(54.6 \frac{lb_m}{ft^3}\right)(32.2\ ft/s^2)(34\ ft)}{32.174 \frac{lb_m ft}{lb_f s^2}} = 4925.9 \frac{lb_f}{ft^2} = 34.2\ psi$$

(c) the pressure at the bottom of the tank

$$P_{Benzene-Water\ Interface} - P_{Bottom\ of\ Tank} = -\rho_{Water} g(25 feet)$$

$$P_{Bottom\ of\ Tank} = P_{Benzene-Water\ Interface} + \rho_{Benzene} g(25 feet)$$

$$= 4925.9 \frac{lb_f}{ft^2} + \frac{\left(62.2 \frac{lb_m}{ft^3}\right)(32.2\ ft/s^2)(25\ ft)}{32.174 \frac{lb_m ft}{lb_f s^2}} = 6482.16 \frac{lb_f}{ft^2} = 45\ psi$$

2.31

The maximum blood pressure in the upper arm of a healthy person is about 120 mm Hg (this is a gauge pressure). If a vertical tube open to the atmosphere is connected to the vein in the arm of a person, determine how high the blood will rise in the tube. Take the density of the blood to be constant and equal to 1050 kg/m^3. (The fact that blood can rise in a tube explains why IV tubes must be placed high to force fluid into the vein of a patient.) Assume the system is at 80^{o}F.

Solution

$$\Delta P = \rho g h$$

For Blood: $P = \rho_B g h_B$

For Mercury: $P = \rho_M g h_M$

So,

$$\Delta P = \rho_B g h_B = \rho_M g h_M$$

Solve for the height of the blood,

$$\rho_B g h_B = \rho_M g h_M$$

Eliminate the gravity terms,

$$\rho_B h_B = \rho_M h_M$$

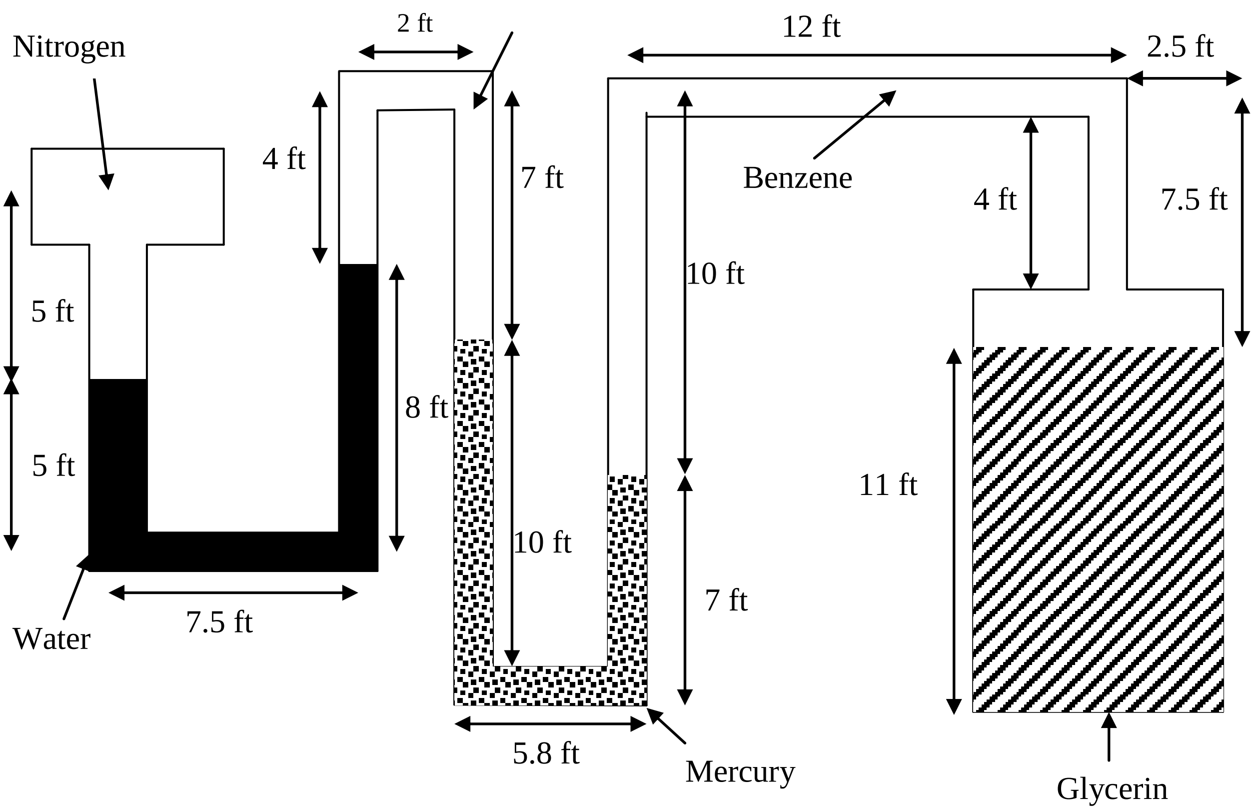
$$h_B = \frac{\rho_M}{\rho_B} h_M$$

We take 120 mm Hg as the height here, and 120 mm = 0.12 m,
and the density of mercury is 845 lb_m/ft^3.

$$h_B = \frac{(13535.6\ kg/m^3)}{(1050\ kg/m^3)}(0.12\ m) = 1.55\ m$$

2.32

If the nitrogen tank in the figure below is at a pressure of 4500 lb_f/ft^2 and the entire system is at 100°F, please calculate the pressure at the bottom of the tank of glycerin.



Solution

$$P_N - P_1 = -\rho_{Nitrogen} g(5\text{ ft}) \text{ so } P_1 = P_N + \rho_{Nitrogen} g(5\text{ ft})$$
$$P_2 - P_1 = -\rho_{H2O} g(8-5\text{ ft}) \text{ so } P_2 = P_1 - \rho_{H2O} g(8-5\text{ ft})$$
$$P_2 - P_3 = -\rho_{Freon} g(7-4\text{ ft}) \text{ so } P_3 = P_2 + \rho_{Freon} g(7-4\text{ ft})$$
$$P_3 - P_4 = -\rho_{Mercury} g(10-7\text{ ft}) \text{ so } P_4 = P_3 + \rho_{Mercury} g(10-7\text{ ft})$$
$$P_5 - P_4 = -\rho_{Benzene} g(10-7.5\text{ ft}) \text{ so } P_5 = P_4 - \rho_{Benzene} g(10-7.5\text{ ft})$$
$$P_5 - P_A = -\rho_A g(11\text{ ft}) \text{ so } P_A = P_6 + \rho_A g(11\text{ ft})$$
$$P_A = \rho_A g(11\text{ ft}) - \rho_{Benzene} g(10-7.5\text{ ft}) + \rho_{Mercury} g(10-7\text{ ft}) + +\rho_{Freon} g(7-4\text{ ft}) - \rho_{H2O} g(8-5\text{ ft}) + \rho_{Nitrogen} g(5\text{ ft}) + P_N$$

$$P_A = \frac{(78.2\ lb_m/ft^3)}{(32.174\ lb_m ft/lb_f s^2)}(32.2\ ft/s^2)(11\text{ ft}) - \frac{(53.6\ lb_m/ft^3)}{(32.174\ lb_m ft/lb_f s^2)}(32.2\ ft/s^2)(10-7.5\text{ ft}) + \frac{(843\ lb_m/ft^3)}{(32.174\ lb_m ft/lb_f s^2)}(32.2\ ft/s^2)(10-7\text{ ft}) + \frac{\left(78.7\frac{lb_m}{ft^3}\right)}{\left(32.174\frac{lb_m ft}{lb_f s^2}\right)}\left(32.2\frac{ft}{s^2}\right)(7-4\text{ ft}) - \frac{\left(62.1\frac{lb_m}{ft^3}\right)}{\left(32.174\frac{lb_m ft}{lb_f s^2}\right)}\left(32.2\frac{ft}{s^2}\right)(8-5\text{ ft}) + \frac{\left(0.0685\frac{lb_m}{ft^3}\right)}{\left(32.174\frac{lb_m ft}{lb_f s^2}\right)}\left(32.2\frac{ft}{s^2}\right)(5\text{ ft}) + 4500\frac{lb_f}{ft^2} = 7808.0\ lb_f/ft^2$$

2.33

The water in a lake has an average temperature of 60°F. If the barometric pressure of the atmosphere is 760 mm Hg (which is equal to 2.36 feet). Determine the gage pressure and the absolute pressure at a water depth of 46 feet.

Solution

From Appendix I, $\rho = 62.3\ lb_m/ft^3$ at 60°F.

$$P_g = \rho_w g h = \frac{\left(62.3 \frac{lb_m}{ft^3}\right)\left(32.2 \frac{ft}{s^2}\right)(46\ ft)}{32.174 \frac{lb_m ft}{lb_f s^2}} = 2868\ \frac{lb_f}{ft^2}$$

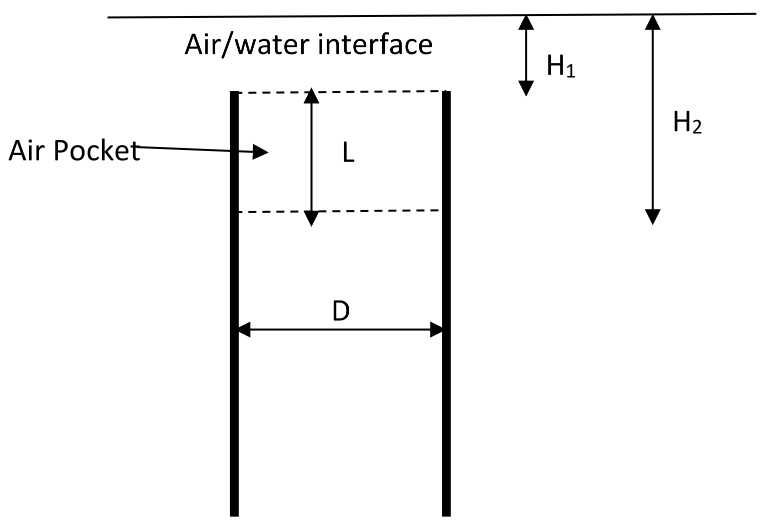
$$P_{atm} = \rho_{Hg} g h = \frac{\left(847\ \frac{lb_m}{ft^3}\right)\left(32.2 \frac{ft}{s^2}\right)\left(\frac{760\ mm}{1000\ mm/m}\right)\left(\frac{3.28\ ft}{m}\right)}{32.174 \frac{lb_m ft}{lb_f s^2}} = 2113\ \ \frac{lb_f}{ft^2}$$

$$P_{absolute} = P_{atm} + P_g = 644.24\ \frac{lb_f}{ft^2} + 2868\ \frac{lb_f}{ft^2} = 3512 \frac{lb_f}{ft^2}$$

2.34

Air initially fills a very long vertical capillary tube of inside diameter D. The tube is suddenly immersed in a large body of water, still in the vertical position. Water wets the tube surface and as soon as the ends of the tube are submerged, water enters the tube. When equilibrium is reached, what is the length, if any, of the air column that remains in the tube? For this problem we know that the tube diameter is 0.1 cm and the surface tension of water is 0.072 N/m and the density of water is 1000 kg/m^3. We can assume that the figure is not drawn to scale and that the capillary tube is drawn much bigger than reality, such that D<<H_1, and that the maximum bubble pressure occurs when the radius of curvature equals the tube radius so that R_1=R_2=D/2. Assume the system is at 273 K.

After immersion, the system looks like this:



Solution

Begin with the Young-Laplace Equation:

$$P_{H_2} - P_{H_2} = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Since $R_1 = R_2$ = D/2

$$P_{H_2} - P_{H_2} = \frac{2\sigma}{D/2} = \frac{4\sigma}{D}$$

Realizing that $\Delta P = \rho g h$

$$\rho g H_2 - \rho g H_1 = \frac{4\sigma}{D}$$

Realizing that

$$H_2 - H_1 = L$$

and

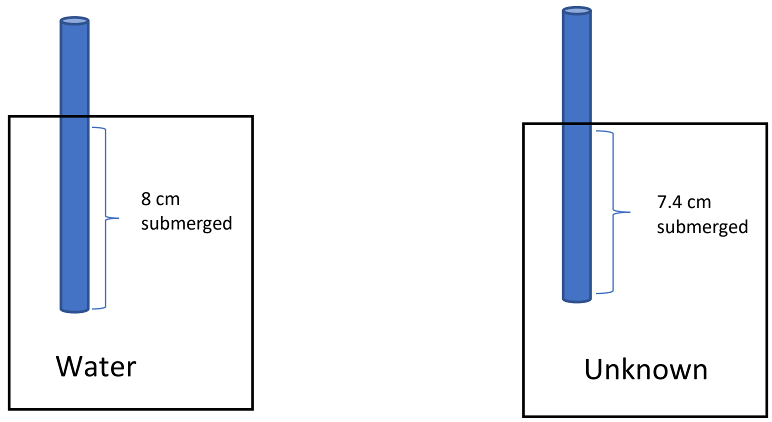
$$0.072\,\frac{N}{m} = 0.072\,\frac{kg}{s^2} = 72\,\frac{g}{s^2}$$

So,

$$H_2 - H_1 = L = \frac{4\sigma}{\rho g D} = \frac{4(72\;g/s^2)}{(1\;g/cm^3)(980\;cm/s^2)(0.1\;cm)} = 2.94\;cm$$

2.35

Hydrometers are inexpensive and easy to use to measure specific gravity and then density. You want to measure the density of an unknown liquid but all you have is a test tube that is 1 cm in diameter. You fill the test tube with water and drop it into a known sample of water with a temperature of 20°C, and measure the distance from the bottom of the test tube to the surface of the water to be 8 cm. Then you take this same test tube (after cleaning on the outside) and drop into an unknown liquid which is also at 20°C and measure the distance submerged to be 7.4 cm. Hydrometers floating in liquids are in static equilibrium. What is the density of the unknown liquid?



<u>Solution</u>

A hydrometer floating in water is in static equilibrium and the buoyant force F_B exerted by the liquid must always be equal to the weight W of the hydrometer, so that F_B = W.

$$\mathrm{F = PA = \rho g h A = \rho g W}$$
$$\mathrm{F_B = \rho g V_{submerged} = \rho g h A_{TT}}$$

Where h is the height of the submerged portion of the test tube and A_{TT} is the cross-sectional area of the test tube which is constant.

For the pure water: $W = \rho_w g h_w A_{TT}$

For unknown: $W = \rho_{unknown} g h_{unknown} A_{TT}$

Setting the equations equal since the weight of the test tube does not change,

$$\rho_w g h_w A_{TT} = \rho_{unknown} g h_{unknown} A_{TT}$$

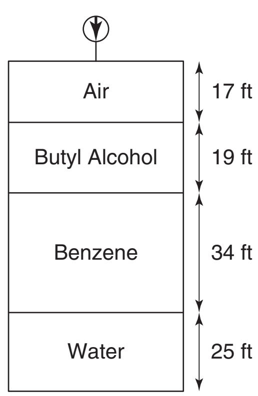
Solve for the density of the unknown liquid,

$$h_{unknown} = \frac{h_w}{h_{unknown}} \rho_w = \frac{8\ cm}{7.4\ cm}(998.2\ kg/m^3) = 1079\ kg/m^3$$

Instructor's Supplement Problems

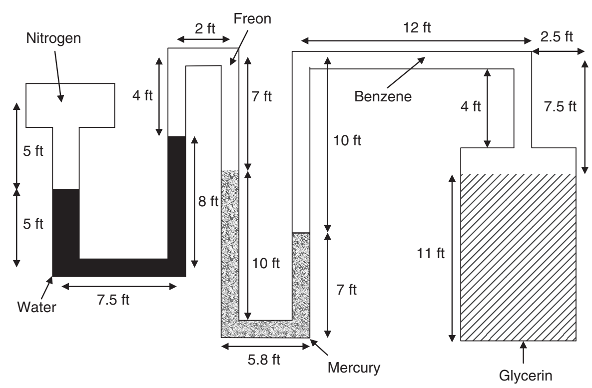
CHAPTER 2

2.1 A large industrial waste collection tank contains butyl alcohol, benzene, and water at 80F that have separated into three distinct phases as shown in the figure. The diameter of the circular tank is 10 ft and its total depth is 95 ft. The gauge at the top of the tank reads 2116 lb_f/ft^3. Calculate the pressure (a) at the butyl alcohol/benzene interface, (b) at the benzene/water interface, and (c) at the bottom of the tank.



2.2 The maximum blood pressure in the upper arm of a healthy person is about 120 mm Hg (this is a gage pressure). If a vertical tube open to the atmosphere is connected to the vein in the arm of a person, determine how high the blood will rise in the tube. Take the density of the blood to be both constant and equal to 1060 kg/m^3. (The fact that blood can rise in a tube explains why IV tubes must be placed high to force fluid into the vein of a patient.) Assume the system is at 80F.

2.3 If the nitrogen tank in the figure below is at a pressure of 4500 lb_f/ft^2 and the entire system is at 100F, calculate the pressure at the bottom of the tank of glycerin.



2.4 The water in a lake has an average temperature of 60 F. If the barometric pressure of the atmosphere is 760 mm Hg (which is equal to 2.36 feet), determine the gage pressure and the absolute pressure at a water depth of 46 ft.

2.5 Air initially fills a very long vertical capillary tube of inside diameter D. The tube is suddenly immersed in a large body of water, still in the vertical position. Water wets the tube surface, and as soon as the ends of the tube are submerged, water enters the tube. When equilibrium is reached, what is the length, if any, of the air column that remains in the tube? For this problem, we know that the tube diameter is 0.1 cm and the surface tension of water is 0.072 N/m and the density of water is 1000 kg/m^3. We can assume that the figure is not drawn to scale and that the capillary tube is drawn much bigger than reality, such that $D << H_1$, and that the maximum bubble pressure occurs when the radius of curvature equals the tube radius so that $R_1 = R_2 = D/2$. Assume the system is at 273K.

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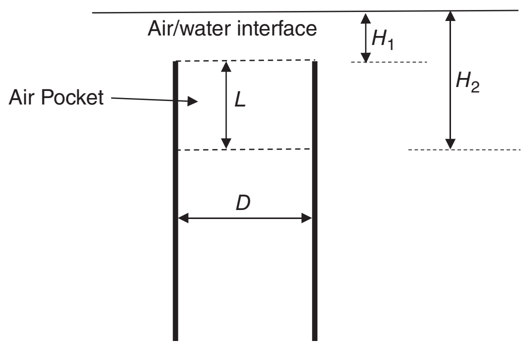
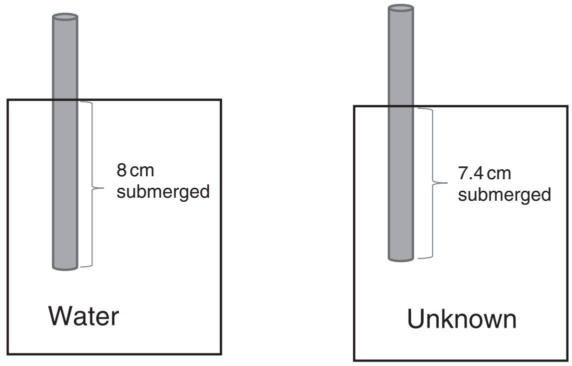


2.6 Hydrometers are inexpensive and easy to use to measure specific gravity and then density. You want to measure the density of an unknown liquid, but all you have is a test tube that is 1 cm in diameter. You fill the test tube with water and drop it into a known sample of water with a temperature of 20C, and measure the distance from the bottom of the test tube to the surface of the water to be 8 cm. Then, you take this same test tube (after cleaning on the outside) and drop into an unknown liquid, which is also at 20C and measure the distance submerged to be 7.4 cm. Hydrometers floating in liquids are in static equilibrium. What is the density of the unknown liquid?



Chapter 2
Show/Hide Problems

2.6
The practical depth limit for a suited diver is about 185 meters. What is the gage pressure of sea water at that depth? Assume that the specific gravity of sea water is constant at 1.025.

Solution

$$\frac{dP}{dy} = -\rho g$$

$$\int_{P_{atm}}^{P} dP = -\rho g \int_{h}^{0} dy$$

$$P - P_{atm} = -\rho g(0 - h) = \rho g h$$

$$P - P_{atm} = 1.025(1000\ kg/m^3)(9.8\ m/s^2)(185\ meters) = 1.86x10^6 \frac{kg}{ms^2} = 1.86x10^6\ Pa$$

2.9
Determine the depth change to cause a pressure increase of 1 atm for (a) water, (b) sea water (SG=1.0250) and (c) mercury (SG=13.6). (Assume the temperature is 30°C.)

Solution

$$P - P_{atm} = \rho g h$$

$\rho_{water} = 995.2\ kg/m^3$
$1\ atm = 101325 \frac{N}{m^2} = 101325\ \ kg/s^2m$

$$h = \frac{P - P_{atm}}{\rho g}$$

(a) water

$$h = \frac{P - P_{atm}}{\rho g} = \frac{101325\ \ kg/s^2m}{(995.2\ kg/m^3)(9.8\ m/s^2)} = 10.4\ m$$

(b) sea water (SG=1.0250)

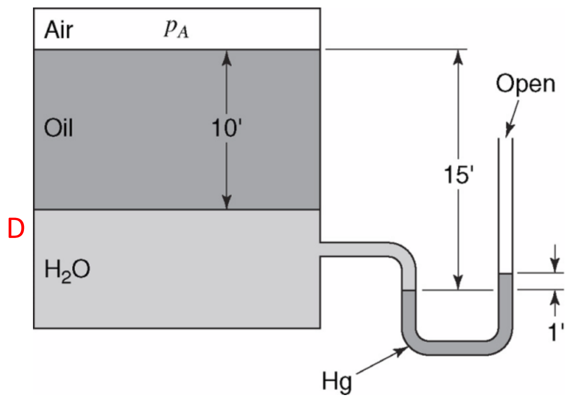
CLICK HERE TO ACCESS THE COMPLETE Solutions

$$h = \frac{P - P_{atm}}{\rho g} = \frac{101325\ kg/s^2 m}{(995.2\ kg/m^3)(1.0250)(9.8\ m/s^2)} = 10.13\ m$$

(c) mercury (SG=13.6)

$$h = \frac{P - P_{atm}}{\rho g} = \frac{101325\ kg/s^2 m}{(995.2\ kg/m^3)(13.6)(9.8\ m/s^2)} = 0.764\ m$$

2.12



What is the pressure p_A in the figure? The oil in the middle tank has a specific gravity of 0.8. Assume that the entire system is at 80°F.

Solution

$$P_E = P_B - \rho_{oil} g(10\ feet) \qquad (1)$$

$$P_C = P_B + \rho_{H2O} g(15 - 10\ feet) \qquad (2)$$

$$P_D = P_C - \rho_{Hg} g(1\ foot) \qquad (3)$$

Insert (2) into (3) then into (1):

$$P_D = P_B + \rho_{H2O} g(15 - 10\ feet) - \rho_{Hg} g(1\ foot)$$

$$P_E = P_D - \rho_{H2O} g(15 - 10\ feet) + \rho_{Hg} g(1\ foot) - \rho_{oil} g(10\ feet)$$

From Appendix I, at 80°F: ρ_{H2O}=62.2 lb_m/ft^3, ρ_{Hg}=845 lb_m/ft^3

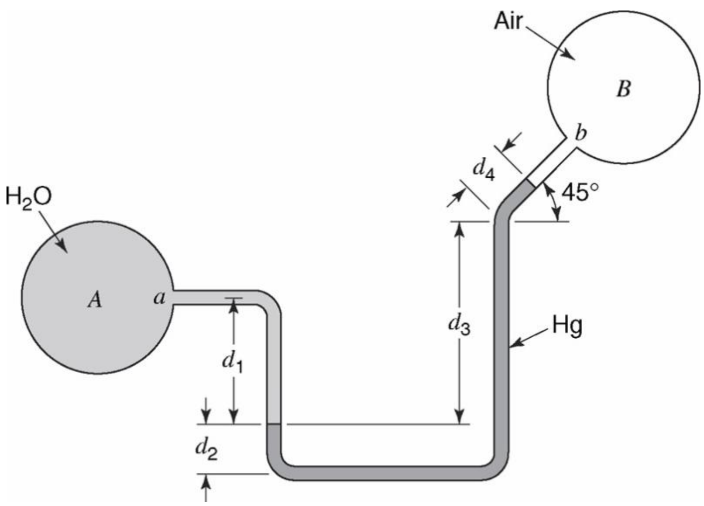
Thus,

$$P_A = P_E - P_D = -\frac{(62.2\ lb_m/\text{ft}^3)(32.2\ ft/s^2)(15 - 10\ ft)}{32.174\ lb_m ft/lb_f s^2} + \frac{(845\ lb_m/\text{ft}^3)(32.2\ ft/s^2)(1\ ft)}{32.174\ lb_m ft/lb_f s^2} - \frac{0.8(62.2\ lb_m/\text{ft}^3)(32.2\ ft/s^2)(10\ ft)}{32.174\ lb_m ft/lb_f s^2} = 36.45 \frac{lb_f}{ft^2} (gauge)$$

P_A is a gauge pressure because P_D is open to the atmosphere.

2.13

Assume that the entire system is at 80°F.



Referring to the figure at left, please find the pressure difference between tanks A and B if d_1=2 feet, d_2=6 inches, d_3=2.4 inches and d_4=4 inches.

Solution

$$P_A - P_3 = -\rho_{H_2O} g d_1 \qquad (1)$$

$$P_B - P_3 = -\rho_{Hg} g d_3 - \rho_{Hg} g d_4 sin\theta \qquad (2)$$

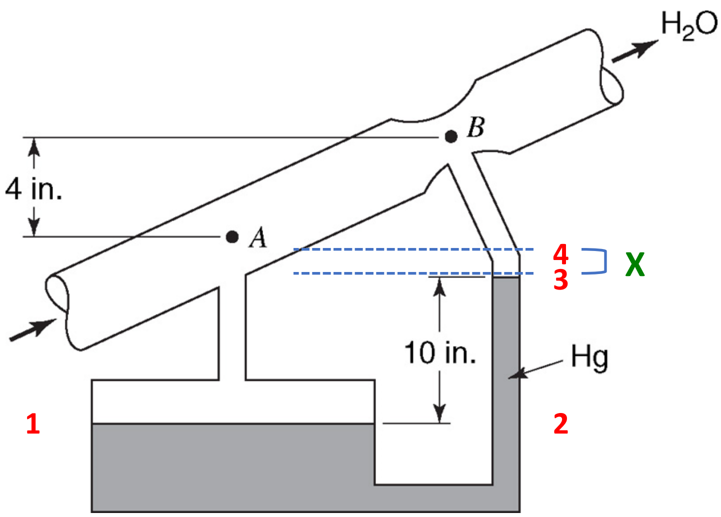
Subtract (1) -(2):

$$P_A - P_B = -\rho_{H_2O} g d_1 + \rho_{Hg} g d_3 + \rho_{Hg} g d_4 \sin(45)$$

From Appendix I, at 80°F: ρ_{H2O}=62.2 lb_m/ft^3, ρ_{Hg}=845 lb_m/ft^3

$$P_A - P_B = \frac{-(62.2\ lb_m/\text{ft}^3)(32.2\ ft/s^2)(2\ ft)}{32.174\ lb_m ft/lb_f s^2} + \frac{(845\ lb_m/\text{ft}^3)(32.2\ ft/s^2)(0.2\ ft)}{32.174\ lb_m ft/lb_f s^2}$$
$$+ \frac{(845\ lb_m/\text{ft}^3)(32.2\ ft/s^2)(0.333\ ft)\sin(45)}{32.174\ lb_m ft/lb_f s^2} = 244\frac{lb_f}{ft^2} = 1.70\frac{lb_f}{in^2}$$
$$= 1.7\ psi$$

2.15
Assume the system is at 80°F.
Points A and 3 are not necessarily at the same height.



A differential manometer is used to measure the pressure change caused by a flow constriction in a piping system as shown. Determine the pressure difference between points A and B in psi. Which section has the higher pressure?

<u>*Solution*</u>

$$P_1 - P_A = \rho_{H_2O} g(d_x + d_1) \rightarrow P_1 = P_A + \rho_{H_2O} g(d_1 + d_1) = P_A + \rho_{H_2O} g(10 + x\ in)$$

$$P_2 - P_B = \rho_{H_2O} g d_1 + \rho_{H_2O} g d_x + \rho_{Hg} g d_2 \rightarrow P_2 = P_B + \rho_{H_2O} g d_1 + \rho_{H_2O} g d_x + \rho_{Hg} g d_2$$
$$= P_B + \rho_{H_2O} g(4\ in) + \rho_{H_2O} g(x\ in) + \rho_{Hg} g(10\ in)$$

Equate the two equations by realizing that $P_1 = P_2$

$$P_A + \rho_{H_2O} g(10\ in) + \rho_{H_2O} g(x\ in) = P_B + \rho_{H_2O} g(4\ in) + \rho_{H_2O} g(x\ in) + \rho_{Hg} g(10\ in)$$

Rearrange and cancel $\rho_{H_2O} g(x\ in)$,

$$P_A - P_B = -\rho_{H_2O} g(10\ in) + \rho_{H_2O} g(4\ in) + \rho_{Hg} g(10\ in) = -\rho_{H_2O} g(6\ in) + \rho_{Hg} g(10\ in)$$

From Appendix I:

$$\rho_{H_2O}\ at\ 80°F = 62.2\ lb_m/ft^3$$
$$\rho_{Hg} at\ 80°F = 845\ lb_m/ft^3$$

In addition, we know that 10 inches = 0.83 ft and 6 inches = 0.5 ft.

$$P_A - P_B = \frac{-(62.2\ lb_m/ft^3)(32.2\ ft/s^2)(0.5\ ft) + (845\ lb_m/ft^3)(32.2\ ft/s^2)(0.83\ ft)}{32.174\ lb_m ft/lb_f s^2}$$

$$= 670.82\ lb_f/ft^2$$

We were asked to state the problem in terms of psi (pounds per square inch):

$$P_A - P_B = 670.82\ \frac{lb_f}{ft^2} x \frac{ft^2}{144\ in^2} = 4.66\ \frac{lb_f}{in^2} = 4.66\ psi$$

Since

$$P_A - P_B = 4.66\ psi$$

The pressure at A must be greater than the pressure at B.