Solutions for Statics and Mechanics of Materials 1st Edition by Goodno

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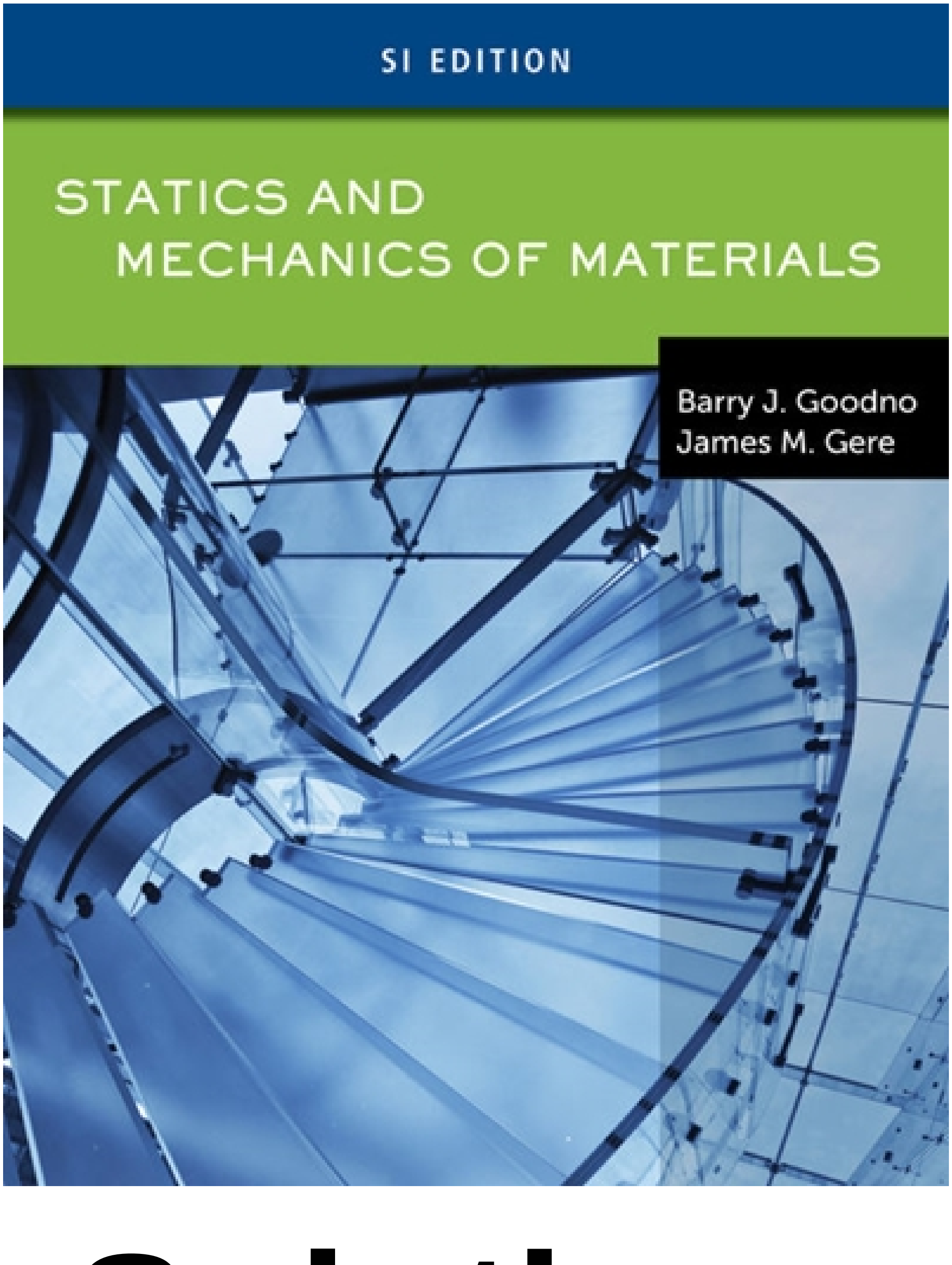


Solutions

Chapter 2 Solutions

Problem 2.2-1

Part (a)

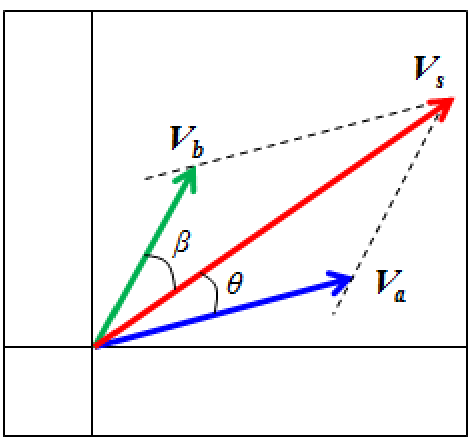
$$V_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \qquad V_2 = \begin{pmatrix} 2 \\ \alpha \end{pmatrix} \qquad V_1 \cdot V_2 = 0 \text{ solve}, \alpha \rightarrow -8$$

Part (b)

$$\gamma = \operatorname{atan}\left(\frac{1}{4}\right) = 14.036 \cdot \text{deg} \qquad \varphi = 30\text{deg} \qquad \beta = \varphi + \gamma = 44.036 \cdot \text{deg}$$

$$\tan(\beta) = \frac{\alpha}{2} \left| \begin{array}{l} \text{solve}, \alpha \\ \text{float}, 4 \end{array} \right. \rightarrow 1.934$$

Part (c) Change V_a to V_1 & V_b to V_2 in figure



Vectors V_a and V_b in Part (c)

$$V_1 = 4 \qquad V_2 = 8 \qquad V_s = 12$$

Law of Cosines

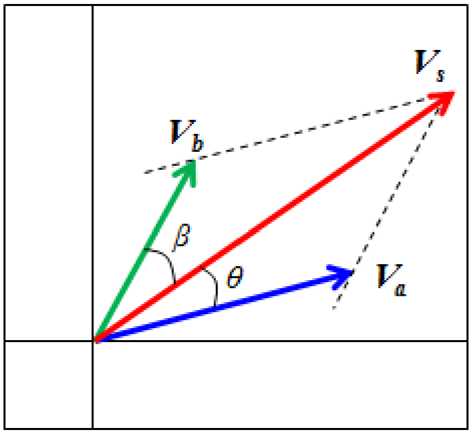
$$\theta = \operatorname{acos}\left(\frac{V_1^{\,2} + V_s^{\,2} - V_2^{\,2}}{2 \cdot V_1 \cdot V_s}\right) = 0 \cdot \text{deg}$$

Next, Law of Sines

$$\beta = \operatorname{asin}\left(\frac{V_1}{V_2} \cdot \sin(\theta)\right) = 0 \cdot \text{deg}$$

$$\phi = \theta + \beta = 0 \cdot \text{deg}$$

Problem 2.1-2



Vectors V_a and V_b in Part (c)

$V_a = 5 \qquad V_b = 7 \qquad V_s = 10$

Law of Cosines

$$\theta = \operatorname{acos}\left(\frac{V_a^{\,2} + V_s^{\,2} - V_b^{\,2}}{2 \cdot V_a \cdot V_s}\right) = 40.5 \cdot \text{deg}$$

Next, Law of Sines

$$\beta = \operatorname{asin}\left(\frac{V_a}{V_b} \cdot \sin(\theta)\right) = 27.7 \cdot \text{deg}$$

$$\phi = \theta + \beta = 68.2 \cdot \text{deg}$$

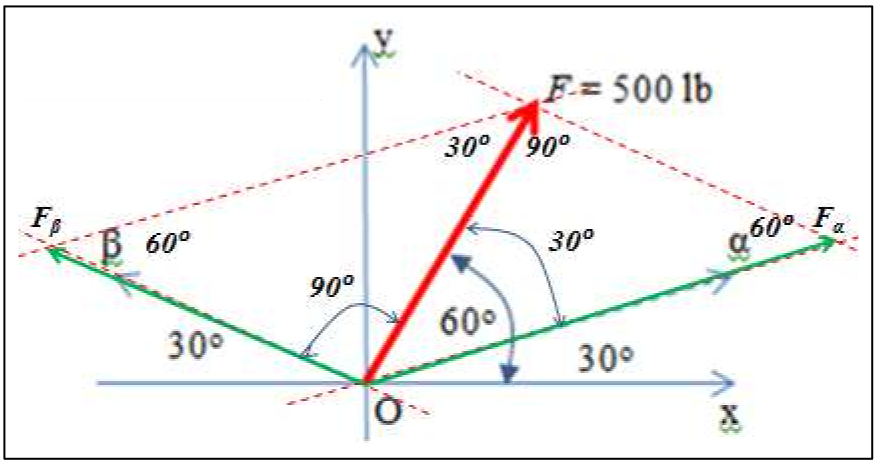
Problem 2.1-3

Part (a)

$$F = 500\text{lb}\cdot\begin{pmatrix}\cos(60\text{deg})\\ \sin(60\text{deg})\end{pmatrix} = \begin{pmatrix}250\\ 433\end{pmatrix}\cdot\text{lb}$$

Part (b) - components

parallelogram law



Law of Sines

$$\frac{F_\alpha}{\sin(90\text{deg})} = \frac{F}{\sin(60\text{deg})}$$

$$F_\alpha = 500\cdot\left(\frac{\sin(90\text{deg})}{\sin(60\text{deg})}\right)$$

$$F_\alpha = 577.35 \qquad 500\cdot\frac{2}{\sqrt{3}} = 577.35$$

$$\frac{F_\beta}{\sin(30\text{deg})} = \frac{F}{\sin(60\text{deg})}$$

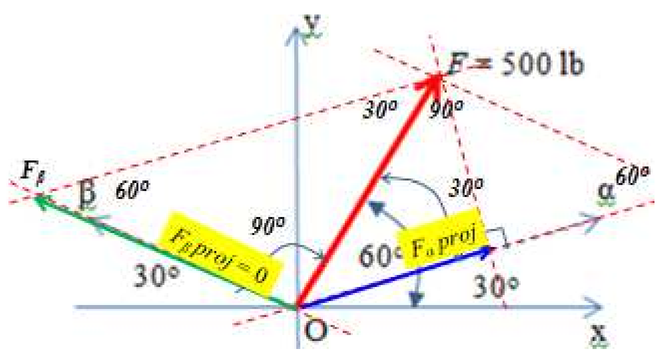
$$F_\beta = 500\cdot\left(\frac{\sin(30\text{deg})}{\sin(60\text{deg})}\right)$$

$$F_\beta = 288.675 \qquad \frac{500}{\sqrt{3}} = 288.675$$

Law of cosines check

$$\sqrt{F_\alpha^{\,2} + F_\beta^{\,2} - 2\cdot F_\alpha\cdot F_\beta\cdot\cos(60\text{deg})} = 500$$

Part (c) - projections



$$F_{\alpha proj} = 500\cdot\cos(30\text{deg}) = 433$$

$$500\cdot\frac{\sqrt{3}}{2} = 433.013$$

$$F_{\beta proj} = 500\cdot\cos(90\text{deg}) = 3.062\times10^{-14}$$

$$F_{\beta proj} = 0$$

Problem 2.1-4

Move the coordinate system from O to A then find component of F in x-y or n-t systems

$F = 2.5\text{kN} \qquad \alpha = 45\text{deg}$

Part (a)

$F_x = -F\cdot\cos(\alpha) = -1.768\cdot\text{kN}$

$F_y = F\cdot\sin(\alpha) = 1.768\cdot\text{kN}$

Part (b)

$\theta = 60\text{deg} - 45\text{deg} = 15\cdot\text{deg}$

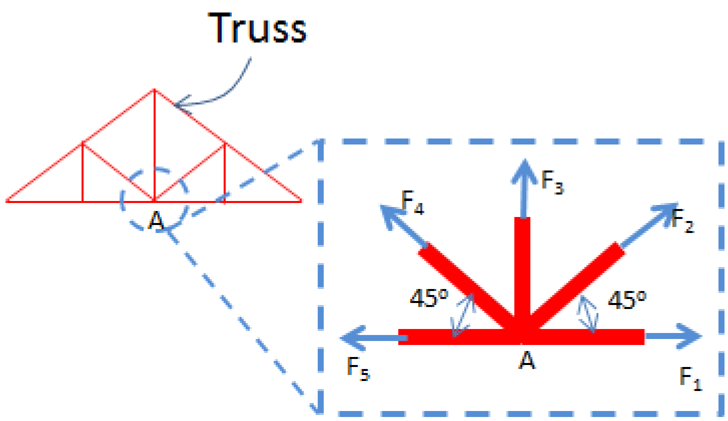
$F_n = -F\cdot\cos(\theta) = -2.415\cdot\text{kN}$

$F_t = -F\cdot\sin(\theta) = -0.647\cdot\text{kN}$

(a) $F_x = -1.768\cdot\text{kN} \qquad F_y = 1.768\cdot\text{kN}$

(b) $F_n = -2.415\cdot\text{kN} \qquad F_t = -0.647\cdot\text{kN}$

Problem 2.1-5



$F_1 = 100lb$ $F_5 = 150lb$ $F_3 = 20lb$ $\alpha = 45deg$

$\Sigma F_x = 0$

$F_1 + F_2 \cdot \cos(\alpha) - F_4 \cdot \cos(\alpha) - F_5 = 0$ ... (1) so $F_4 = \frac{1}{\cos(\alpha)} \cdot (F_1 + F_2 \cdot \cos(\alpha) - F_5)$

$\Sigma F_y = 0$

$F_2 \cdot \sin(\alpha) + F_4 \cdot \sin(\alpha) + F_3 = 0$ ... (2) so $F_2 = \frac{1}{\sin(\alpha)} \cdot (-F_3 - F_4 \cdot \sin(\alpha))$

Substitute expression for F_4 into Eq. (2)

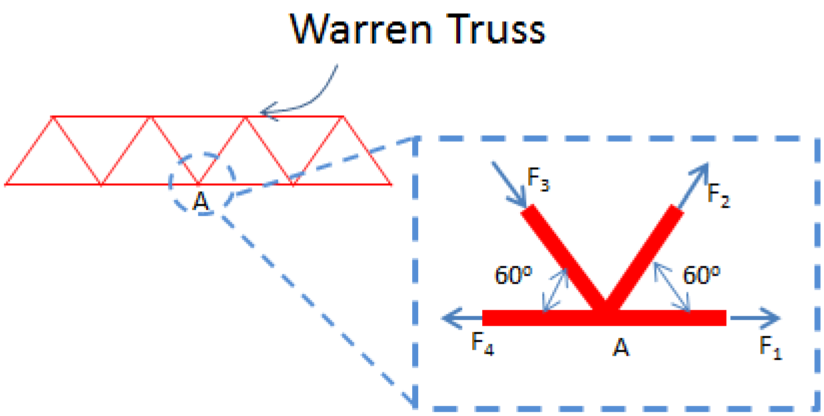
$F_2 = \frac{F_5 - F_1 - F_3}{(\cos(\alpha) + \sin(\alpha))} = 21.2 \cdot lb$ and $F_4 = \frac{-F_3 - F_2 \cdot \sin(\alpha)}{\sin(\alpha)} = -49.5 \cdot lb$

<u>CHECK</u> that resultant is zero

$\Sigma F_x = F_1 + F_2 \cdot \cos(\alpha) - F_4 \cdot \cos(\alpha) - F_5 = 0 \cdot lb$

$\Sigma F_y = F_2 \cdot \sin(\alpha) + F_3 + F_4 \cdot \sin(\alpha) = 0 \cdot lb$

Problem 2.1-6



Part (a): units = kN

$$F_1 = \begin{pmatrix} 100 \\ 0 \end{pmatrix} \quad F_2 = \begin{pmatrix} 60\cdot\cos(60deg) \\ 60\cdot\sin(60deg) \end{pmatrix} = \begin{pmatrix} 30 \\ 51.962 \end{pmatrix} \qquad R_{12} = F_1 + F_2 = \begin{pmatrix} 130 \\ 52 \end{pmatrix} \qquad |R_{12}| = 140$$

Law of sines: β_{12} = angle that R_{12} makes with x axis

$$\beta_{12} = \operatorname{asin}\left(\frac{|F_2|}{|R_{12}|}\cdot\sin(120deg)\right) = 21.79\cdot deg \qquad \beta_{12} = 21.8\cdot deg$$

Part (b)

$$F_3 = \begin{pmatrix} 60\cdot\cos(60deg) \\ -60\cdot\sin(60deg) \end{pmatrix} = \begin{pmatrix} 30 \\ -51.962 \end{pmatrix} \qquad F_4 = \begin{pmatrix} -100 \\ 0 \end{pmatrix} \quad R_{34} = F_3 + F_4 = \begin{pmatrix} -70 \\ -51.96 \end{pmatrix} \quad |R_{34}| = 87.18$$

γ_{34} = angle that R_{34} makes with (-x) axis

$$\gamma_{34} = \operatorname{asin}\left(\frac{|F_3|}{|R_{34}|}\cdot\sin(60deg)\right) = 36.587\cdot deg \quad \text{so} \quad \beta_{34} = 180deg + \gamma_{34} = 217\cdot deg$$

Part (c)

$$F_1 + F_2 + F_3 + F_4 = \begin{pmatrix} 60 \\ 0 \end{pmatrix} \qquad R_{12} + R_{34} = \begin{pmatrix} 60 \\ 0 \end{pmatrix} \qquad |F_1 + F_2 + F_3 + F_4| = 60$$

Problem 2.1-6 - cont'd

Solution #2

$F_1 = 100kN$ $F_2 = 60kN$ $\alpha = 60deg$ $F_4 = 100kN$

Part (a)

Use parallelogram rule & law of cosines

$$R_{12} = \sqrt{F_1^2 + F_2^2 + 2 \cdot F_1 \cdot F_2 \cdot \cos(\alpha)} = 140 \cdot kN$$

$$\beta = \operatorname{asin}\left(\frac{F_2}{R_{12}} \cdot \sin(120deg)\right) = 21.787 \cdot deg$$

Part (b)

$F_3 = 60kN$ $F_4 = 100kN$ $\alpha = 120deg$ $60deg - \beta = 38.213 \cdot deg$

$$R_{34} = \sqrt{F_3^2 + F_4^2 + 2 \cdot F_3 \cdot F_4 \cdot \cos(\alpha)} = 87.2 \cdot kN$$

$$\gamma = \operatorname{asin}\left(\frac{F_3}{R_{34}} \cdot \sin(60deg)\right) = 36.587 \cdot deg$$

Part (c): Sum of resultant forces at point A $\alpha = 60deg$

$$\Sigma F_x = F_1 - F_4 + F_2 \cdot \cos(\alpha) + F_3 \cdot \cos(\alpha) = 60 \cdot kN$$

$$\Sigma F_y = F_2 \cdot \sin(\alpha) - F_3 \cdot \sin(\alpha) = 0\,N$$

$$\Sigma F = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 60 \cdot kN$$

Problem 2.1-7

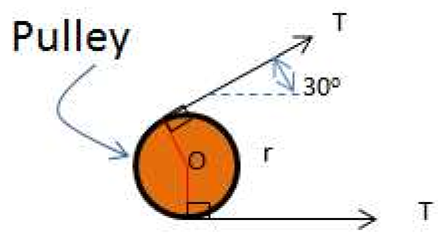
$T = 80lb \qquad r = 4in$

The tension force of the cable will induce no moment resultant at O.

The resultant of the cable forces is

$$R = \sqrt{T^2 + T^2 + 2 \cdot T \cdot T \cdot \cos(30deg)} = 154.5 \cdot lb$$

$$\alpha = atan\left(\frac{T \cdot \sin(30deg)}{T + T \cdot \cos(30deg)}\right) = 15 \cdot deg$$



or

$$\Sigma F_x = T \cdot \cos(30deg) + T = 149.282 \cdot lb$$

$$\Sigma F_y = T \cdot \sin(30deg) = 40 \cdot lb$$

$$R = \sqrt{\Sigma F_x^{\ 2} + \Sigma F_y^{\ 2}} = 154.548 \cdot lb$$

$$\alpha = atan\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = 15 \cdot deg$$

Problem 2.1-8

Part (a) magnitudes > $N_1 = 200N$ $N_2 = 347N$

vectors: $N_1 = N_1 \cdot \begin{pmatrix} \cos(30deg) \\ \sin(30deg) \\ 0 \end{pmatrix} = \begin{pmatrix} 173.2 \\ 100 \\ 0 \end{pmatrix} N$ $N_2 = N_2 \cdot \begin{pmatrix} -\sin(30deg) \\ \cos(30deg) \\ 0 \end{pmatrix} = \begin{pmatrix} -173.5 \\ 300.5 \\ 0 \end{pmatrix} N$

Resultant: $R = N_1 + N_2 = \begin{pmatrix} -0.3 \\ 400.5 \\ 0 \end{pmatrix} N$

angle with x axis:

$$180 \cdot deg + atan\left(\frac{R_2}{R_1}\right) = 90.042 \cdot deg$$

Part (b) $n = \begin{pmatrix} -\cos(60deg) \\ \sin(60deg) \\ 0 \end{pmatrix} = \begin{pmatrix} -0.5 \\ 0.866 \\ 0 \end{pmatrix}$ $t = \begin{pmatrix} \cos(30deg) \\ \sin(30deg) \\ 0 \end{pmatrix} = \begin{pmatrix} 0.866 \\ 0.5 \\ 0 \end{pmatrix}$

$R_n = R \cdot n = 347\,N$ $= N_2$ $R_t = R \cdot t = 200\,N$ $= N_1$

Problem 2.1-9

Cartesian vectors

$$T_1 = 800lb \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad T_2 = 600lb \cdot \begin{pmatrix} \cos(30deg) \\ \sin(30deg) \\ 0 \end{pmatrix} = \begin{pmatrix} 519.6 \\ 300 \\ 0 \end{pmatrix} \cdot lb \qquad T_3 = 1200lb \cdot \begin{pmatrix} \sin(30deg) \\ -\cos(30deg) \\ 0 \end{pmatrix} = \begin{pmatrix} 600 \\ -1039.2 \\ 0 \end{pmatrix} \cdot lb$$

Resultant

$$R = T_1 + T_2 + T_3 = \begin{pmatrix} 1119.6 \\ 60.8 \\ 0 \end{pmatrix} \cdot lb \qquad |R| = 1121 \cdot lb \qquad \operatorname{atan}\left(\frac{60.8}{1119.6}\right) = 3.11 \cdot deg$$

Angle with x axis

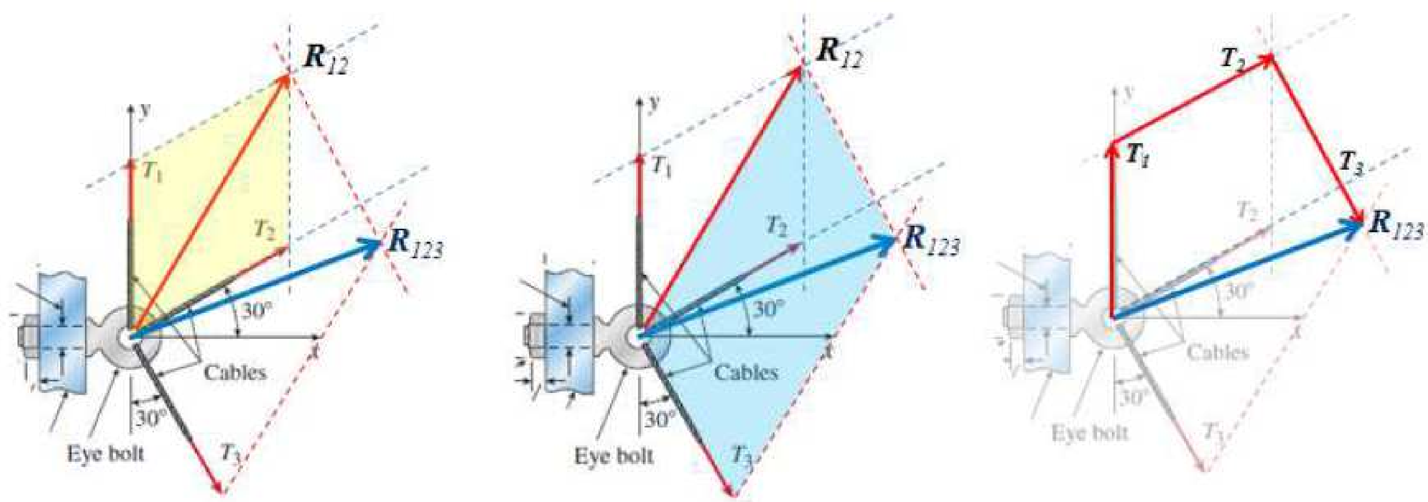
$$i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \theta_x = \operatorname{acos}\left(\frac{R}{|R|} \cdot i\right) = 3.11 \cdot deg$$

or direction cosines of R:

$$n_R = \frac{R}{|R|} = \begin{pmatrix} 1 \\ 0.05 \\ 0 \end{pmatrix}$$

$$\theta_x = \operatorname{acos}\left(n_{R_1}\right) = 3.11 \cdot deg \qquad \theta_y = \operatorname{acos}\left(n_{R_2}\right) = 86.893 \cdot deg$$

Polygon Law: *Successive application of parallelogram law*



Problem 2.1-10

Use Law of Cosines and Law of Sines to find lengths of each cable segment & angles between cables and line OB

$L_1 = \sqrt{2.5^2 + 1.5^2 - 2(2.5)\cdot(1.5)\cdot\cos(60deg)} = 2.179\ \text{m}$ $\quad L_2 = \sqrt{2.5^2 + 3^2 - 2(2.5)\cdot(3)\cdot\cos(60deg)} = 2.784\ \text{m}$

$\frac{L_1}{\sin(60deg)} = \frac{2.5}{\sin(\theta_1)}$ $\quad \theta_1 = \operatorname{asin}\left(\frac{2.5}{L_1}\cdot\sin(60deg)\right) = 83.413\cdot deg$ $\quad \theta_2 = \operatorname{asin}\left(\frac{2.5}{L_2}\cdot\sin(60deg)\right) = 51.052\cdot deg$

Subtract 30 deg. from each angle theta to find angles between each cable segment & x axis.

$\theta_{1x} = \theta_1 - 30deg = 53.413\cdot deg$ $\quad \theta_{2x} = \theta_2 - 30deg = 21.052\cdot deg$

Cartesian components for each cable

$T_1 = (440N)\cdot\begin{pmatrix} -\cos(\theta_{1x}) \\ \sin(\theta_{1x}) \end{pmatrix} = \begin{pmatrix} -262.257 \\ 353.3 \end{pmatrix} N$ $\quad T_2 = (560N)\cdot\begin{pmatrix} -\cos(\theta_{2x}) \\ \sin(\theta_{2x}) \end{pmatrix} = \begin{pmatrix} -522.624 \\ 201.158 \end{pmatrix} N$

Resultant

$R = T_1 + T_2 = \begin{pmatrix} -785 \\ 554 \end{pmatrix} N$

Direction cosines of R

$n_R = \frac{R}{|R|} = \begin{pmatrix} -0.817 \\ 0.577 \end{pmatrix}$ $\quad \theta_x = \operatorname{acos}\left(n_{R_1}\right) = 144.8\cdot deg$ $\quad 180deg - \theta_x = 35.238\cdot deg$

^ between θ_{1x} and θ_{2x}

Problem 2.1-11

$\theta = 80\text{deg}$ $\alpha = 38\text{deg}$ $T = 5000\text{lb}$ $a = 15\text{ft}$ $b = 25\text{ft}$

Geometry

$\beta = 180\text{deg} - (\theta + \alpha) = 62\cdot\text{deg}$

Resultant acts in y dir. so $T\cdot\sin(\beta_1) = T\cdot\sin(\beta_2)$ and $\beta_2 = \beta_1$ which gives $\beta_1 = \frac{\beta}{2} = 31\cdot\text{deg}$

Resultant of cable force T = 5000 ln. at B (in -y dir.)

$R = T\cdot(\cos(\beta_1) + \cos(\beta_2))$ $R = 8572\cdot\text{lb}$

Check that resultant component in x-dir. is zero $R_x = T\cdot(-\sin(\beta_1) + \sin(\beta_2)) = 0\cdot\text{lb}$

$-T\cdot\sin(\beta_1) = -2575\cdot\text{lb}$ $T\cdot\sin(\beta_2) = 2575\cdot\text{lb}$

Problem 2.1-12

$$F_1 = 250N \cdot \begin{pmatrix} -\cos(30deg) \\ \sin(30deg) \end{pmatrix} = \begin{pmatrix} -216.506 \\ 125 \end{pmatrix} N$$

$$F_3 = 250N \cdot \begin{pmatrix} \cos(30deg) \\ -\sin(30deg) \end{pmatrix} = \begin{pmatrix} 216.506 \\ -125 \end{pmatrix} N$$

$$F_2 = 180N \cdot \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$R = F_1 + F_2 + F_3 = \begin{pmatrix} 0 \\ -180 \end{pmatrix} N$$

Problem 2.1-13

$F_{EB} = 4500lb \qquad \alpha = 80deg \qquad \beta = 60deg$

Part (a) - Cartesian components

$$F_{EB} = F_{EB} \cdot \begin{pmatrix} -\cos(\alpha) \\ \sin(\alpha) \\ 0 \end{pmatrix} = \begin{pmatrix} -781 \\ 4432 \\ 0 \end{pmatrix} \cdot lb$$

Part (b) - projection on CA

$$n_{CA} = \begin{pmatrix} -\cos(\beta) \\ \sin(\beta) \\ 0 \end{pmatrix} = \begin{pmatrix} -0.5 \\ 0.866 \\ 0 \end{pmatrix} \qquad F_{CA} = F_{EB} \cdot n_{CA} = 4229 \cdot lb$$

Problem 2.1-14

$F = 8N \qquad T = 9N$

Fig. (a)

$$A = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} \qquad E = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$n_{AE} = \frac{E - A}{|E - A|} = \begin{pmatrix} -0.333 \\ 0.667 \\ -0.667 \end{pmatrix} \qquad R_a = F \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + T \cdot n_{AE} = \begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix} N$$

Fig. (b)

$$R_b = F \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + T \cdot (-n_{AE}) = \begin{pmatrix} -5 \\ -6 \\ 6 \end{pmatrix} N$$

Fig. (c)

$$E = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

$$n_{AE} = \frac{E - A}{|E - A|} = \begin{pmatrix} -0.728 \\ 0.485 \\ -0.485 \end{pmatrix} \qquad R_c = F \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + T \cdot n_{AE} = \begin{pmatrix} 1.45 \\ 4.37 \\ -4.37 \end{pmatrix} N$$

Answers

$$R_a = \begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix} N \qquad R_b = \begin{pmatrix} -5 \\ -6 \\ 6 \end{pmatrix} N \qquad R_c = \begin{pmatrix} 1.45 \\ 4.37 \\ -4.37 \end{pmatrix} N$$

Problem 2.1-15

$$A = \begin{pmatrix} 5 \\ 20 \\ 30 \end{pmatrix} \cdot ft \qquad n_{OA} = \frac{A}{|A|} = \begin{pmatrix} 0.137 \\ 0.549 \\ 0.824 \end{pmatrix} \qquad T = 2800lb \qquad T = T \cdot n_{OA} = \begin{pmatrix} 385 \\ 1538 \\ 2308 \end{pmatrix} \cdot lb \qquad |T| = 2800 \cdot lb$$

Problem 2.1-16

$$\alpha = \text{atan}\left(\frac{34}{27}\right) = 51.546 \cdot \text{deg} \qquad \beta = \text{atan}\left(\frac{17}{27}\right) = 32.196 \cdot \text{deg}$$

For resultant to be vertical, horizontal components of cable forces must sum to zero

$$T = \frac{6\text{kN} \cdot (\cos(\alpha)) + 4\text{kN} \cdot (\cos(\beta)) - 3.6\text{kN} \cdot (\cos(\beta))}{\cos(\alpha)} = 6.54 \cdot \text{kN}$$

Problem 2.1-17

$TCE = 375 lbf$ $a = 20 ft$ $b = 8 ft$ $c = 6 ft$

Position and unit vectors

$$r_{CE} = \begin{pmatrix} \frac{-a}{2} \\ c \\ -b \end{pmatrix} = \begin{pmatrix} -10 \\ 6 \\ -8 \end{pmatrix} \cdot ft \qquad n_{CE} = \frac{r_{CE}}{|r_{CE}|} = \begin{pmatrix} -0.707 \\ 0.424 \\ -0.566 \end{pmatrix} \qquad r_{CA} = \begin{pmatrix} 0 \\ 0 \\ 0-b \end{pmatrix} \qquad n_{CA} = \frac{r_{CA}}{|r_{CA}|} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$T_{CE} = TCE \cdot n_{CE} = \begin{pmatrix} -265 \\ 159 \\ -212 \end{pmatrix} \cdot lbf \qquad |T_{CE}| = 375 \cdot lbf$$

Angle between CE and line CA (= -z axis) $\theta = acos\left(n_{CE_3}\right) = 124.45 \cdot deg$ $acos\left[n_{CE} \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}\right] = 55.55 \cdot deg$

$124.45 + 55.55 = 180$

Projection onto line CA $TCE \cdot cos(55.55 deg) = 212 \cdot lbf$

^ in this case, projection = component (except for sign)

Problem 2.1-18

$a = 10m \qquad b = 4m \qquad c = 3m \qquad T_{CE} = 1700N \qquad T_{CF} = T_{CE}$

Symmetry about yz plane so R will lie in yz plane (x component is zero)

Joint coordinates

$$C = \begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \qquad E = \begin{pmatrix} \frac{-a}{2} \\ c \\ 0 \end{pmatrix} \qquad F = \begin{pmatrix} \frac{a}{2} \\ c \\ 0 \end{pmatrix}$$

unit vectors

$$n_{CE} = \frac{E - C}{|E - C|} = \begin{pmatrix} -0.707 \\ 0.424 \\ -0.566 \end{pmatrix} \qquad n_{CF} = \frac{F - C}{|F - C|} = \begin{pmatrix} 0.707 \\ 0.424 \\ -0.566 \end{pmatrix}$$

cable force vectors and resultant

$$T_{CE} = T_{CE} \cdot n_{CE} = \begin{pmatrix} -1202 \\ 721 \\ -962 \end{pmatrix} N \qquad T_{CF} = T_{CF} \cdot n_{CF} = \begin{pmatrix} 1202 \\ 721 \\ -962 \end{pmatrix} N \qquad R = T_{CE} + T_{CF} = \begin{pmatrix} 0 \\ 1442 \\ -1923 \end{pmatrix} N$$

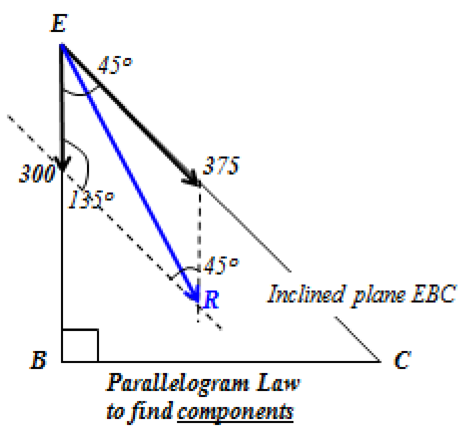
Problem 2.1-19

1. Note that EBC is a right angle
2. Length of EB is 10 ft = a/2 so angle CEB is 45 deg.
3. Use Parallelogram Law to find R (start with sketch)

Dimensions (ft)

$a = 20 \qquad b = 8 \qquad c = 6$

PART (a) - Parallelogram Law and Law of Cosines



$T_{EB} = 300 \qquad T_{EC} = 375 \quad \text{lb}$

$$R = \sqrt{T_{EB}^2 + T_{EC}^2 - 2 \cdot (T_{EC}) \cdot (T_{EB}) \cdot \cos(135\text{deg})} = 624.279$$

$R = 624 \text{ lb}$ < magnitude of resultant

Vectors

$$r_{EB} = \begin{pmatrix} 0 \\ -c \\ b \end{pmatrix} \quad r_{EC} = \begin{pmatrix} \frac{a}{2} \\ -c \\ b \end{pmatrix} \quad n_{EB} = \frac{r_{EB}}{|r_{EB}|} = \begin{pmatrix} 0 \\ -0.6 \\ 0.8 \end{pmatrix} \quad n_{EC} = \frac{r_{EC}}{|r_{EC}|} = \begin{pmatrix} 0.707 \\ -0.424 \\ 0.566 \end{pmatrix}$$

$$r_{FC} = \begin{pmatrix} \frac{-a}{2} \\ -c \\ b \end{pmatrix} \quad r_{FD} = \begin{pmatrix} 0 \\ -c \\ b \end{pmatrix} \quad n_{FC} = \frac{r_{FC}}{|r_{FC}|} = \begin{pmatrix} -0.707 \\ -0.424 \\ 0.566 \end{pmatrix} \quad n_{FD} = \frac{r_{FD}}{|r_{FD}|} = \begin{pmatrix} 0 \\ -0.6 \\ 0.8 \end{pmatrix}$$

$$T_{EB} = 300 \cdot n_{EB} \qquad T_{EC} = 375 \cdot n_{EC} \qquad R = T_{EB} + T_{EC} = \begin{pmatrix} 265 \\ -339 \\ 452 \end{pmatrix} \qquad |R| = 624 \text{ lb}$$

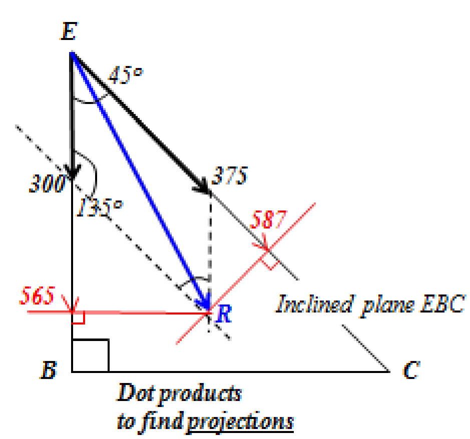
< agrees with above result

$|T_{EB}| = 300 \qquad |T_{EC}| = 375$

PART (b) - Projections of R onto lines EB and EC - use dot product

$R_{EB} = R \cdot n_{EB} = 565 \quad \text{lb}$ < not equal to component of R along EB (i.e., 300)

$R_{EC} = R \cdot n_{EC} = 587 \quad \text{lb}$ < not equal to component of R along EC (i.e., 375)



Problem 2.1-20

$F_s = 154N \quad h = 660mm \quad d = 150mm \quad H = 1041mm \quad c = 506mm$

Joint coordinates

$$A = \begin{pmatrix} h \\ h - H \\ d \end{pmatrix} = \begin{pmatrix} 660 \\ -381 \\ 150 \end{pmatrix} \cdot mm \qquad B = \begin{pmatrix} h \\ h \\ c \end{pmatrix} = \begin{pmatrix} 660 \\ 660 \\ 506 \end{pmatrix} \cdot mm$$

Unit vector

$$n_{AB} = \frac{B - A}{|B - A|} = \begin{pmatrix} 0 \\ 0.946 \\ 0.324 \end{pmatrix}$$

Part (a)

$$F_s = F_s \cdot n_{AB} = \begin{pmatrix} 0 \\ 145.7 \\ 49.8 \end{pmatrix} N \qquad |F_s| = 154\,N$$

Part (b)

$$n_{AD} = \frac{-A}{|A|} = \begin{pmatrix} -0.85 \\ 0.491 \\ -0.193 \end{pmatrix} \qquad F_{AD} = F_s \cdot n_{AD} = 61.9\,N \qquad \theta = acos\left(\frac{F_s \cdot n_{AD}}{|F_s|}\right) = 66.3 \cdot deg$$

Problem 2.1-21

$F = 350lb$ $\quad r = 2.5ft$ $\quad h = 7.5ft$ $\quad A = \begin{pmatrix} -r\cdot\sin(60deg) \\ r\cdot\cos(60deg) \\ h \end{pmatrix} = \begin{pmatrix} -2.165 \\ 1.25 \\ 7.5 \end{pmatrix}\cdot ft$

Part (a)

$$n_{OA} = \frac{A}{|A|} = \begin{pmatrix} -0.274 \\ 0.158 \\ 0.949 \end{pmatrix} \qquad F = F\cdot n_{OA} = \begin{pmatrix} -95.9 \\ 55.3 \\ 332 \end{pmatrix}\cdot lb \qquad |F| = 350\cdot lb$$

Part (b)

$$n_{BC} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad F_{BC} = F\cdot n_{BC} = 55.3\cdot lb \qquad \theta = acos\left(\frac{F\cdot n_{BC}}{|F|}\right) = 80.9\cdot deg$$

Problem 2.1-22

$T_B = 3700N$

Joint coordinates $B = \begin{pmatrix} -0.5 \\ -1 \\ 0 \end{pmatrix} \cdot m \qquad C = \begin{pmatrix} 0 \\ 0 \\ 2.5 \end{pmatrix} \cdot m$

Unit vector along cable B: $n_{BC} = \dfrac{C - B}{|C - B|} = \begin{pmatrix} 0.183 \\ 0.365 \\ 0.913 \end{pmatrix}$

Cable force vector: $T_B = T_B \cdot n_{BC} = \begin{pmatrix} 676 \\ 1351 \\ 3378 \end{pmatrix} \cdot N \qquad |T_B| = 3700 \cdot N$

Problem 2.1-23

$T_{AB} = 15\text{lb}$

Joint coordinates: $A = \begin{pmatrix} 8 \\ 20 - 20\cdot\cos(60\text{deg}) \\ -20\cdot\sin(60\text{deg}) \end{pmatrix}\cdot\text{in} = \begin{pmatrix} 8 \\ 10 \\ -17.321 \end{pmatrix}\cdot\text{in}$ $\quad B = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}\cdot\text{in}$ $\quad C = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}\cdot\text{in}$

<u>Part (a)</u>

$$n_{AB} = \frac{B - A}{|B - A|} = \begin{pmatrix} -0.1 \\ -0.498 \\ 0.862 \end{pmatrix} \qquad T_{AB} = T_{AB}\cdot n_{AB} = \begin{pmatrix} -1.493 \\ -7.463 \\ 12.926 \end{pmatrix}\cdot\text{lb} \qquad |T_{AB}| = 15\,\text{lbf}$$

<u>Part (b)</u>

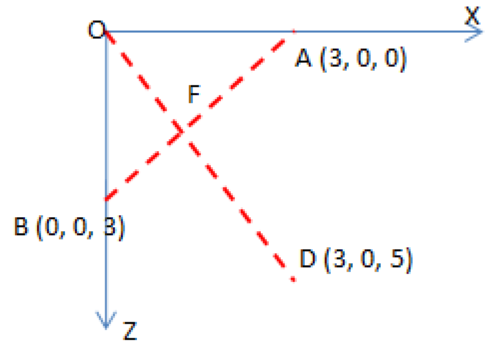
$$n_{CD} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad T_{CD} = T_{AB}\cdot n_{CD} = -1.493\cdot\text{lb}$$

<u>Part (c)</u>

$$n_{CB} = \frac{B - C}{|B - C|} = \begin{pmatrix} 0.287 \\ -0.958 \\ 0 \end{pmatrix} \qquad T_{CB} = T_{AB}\cdot n_{CB} = 6.72\cdot\text{lb}$$

Problem 2.1-24

Find the intersection line between OD and AB/ coordinate F in X-Z plane



line OD

$$z(x) = \frac{5}{3} \cdot x \quad \text{... (1)}$$

line AB

$$z(x) = 3 - x \quad \text{.... (2)}$$

equating equation (1) & (2)

Hence, $x = 3 \cdot \frac{3}{8} = 1.125$ $\quad z(x) = 1.875$

Part (a)

$$F = \begin{pmatrix} 1.125 \\ 0 \\ 1.875 \end{pmatrix} \quad C = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \quad E = \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \quad A = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$$

$$n_{CE} = \frac{E - C}{|E - C|} = \begin{pmatrix} 0.514 \\ 0 \\ 0.857 \end{pmatrix} \qquad n_{FE} = \frac{E - F}{|E - F|} = \begin{pmatrix} 0.397 \\ 0.636 \\ 0.662 \end{pmatrix}$$

$$P = (3\text{kN}) \cdot n_{CE} = \begin{pmatrix} 1.543 \\ 0 \\ 2.572 \end{pmatrix} \cdot \text{kN} \qquad Q = (4\text{kN}) \cdot n_{FE} = \begin{pmatrix} 1.589 \\ 2.542 \\ 2.648 \end{pmatrix} \cdot \text{kN}$$

Part (b)

$$R = P + Q = \begin{pmatrix} 3.13 \\ 2.54 \\ 5.22 \end{pmatrix} \cdot \text{kN} \qquad Y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \theta_Y = \text{acos}\left(\frac{R \cdot Y}{|R|}\right) = 67.3 \cdot \text{deg}$$

Part (c)

$$n_{CA} = \frac{A - C}{|A - C|} = \begin{pmatrix} 0.707 \\ -0.707 \\ 0 \end{pmatrix} \qquad P_{CApar} = P \cdot n_{CA} = 1.091 \cdot \text{kN} \qquad P_{CApar} \cdot n_{CA} = \begin{pmatrix} 0.772 \\ -0.772 \\ 0 \end{pmatrix} \cdot \text{kN}$$

$$P_{CAper} = P - P_{CApar} \cdot n_{CA} = \begin{pmatrix} 0.77 \\ 0.77 \\ 2.57 \end{pmatrix} \cdot \text{kN}$$

Problem 2.1-24 - cont'd

Part (d)

$$n_{BA} = \frac{A - B}{|A - B|} = \begin{pmatrix} 0.707 \\ 0 \\ -0.707 \end{pmatrix} \qquad Q_{BApar} = Q \cdot n_{BA} = -0.749 \cdot kN \qquad Q_{BApar} \cdot n_{BA} = \begin{pmatrix} -0.53 \\ 0 \\ 0.53 \end{pmatrix} \cdot kN$$

$$Q_{BAper} = Q - Q_{BApar} \cdot n_{BA} = \begin{pmatrix} 2.12 \\ 2.54 \\ 2.12 \end{pmatrix} \cdot kN$$

Part (e)

$$\theta = acos\left(\frac{P \cdot Q}{|P| \cdot |Q|}\right) = 39.5 \cdot deg$$

Problem 2.1-25

$T_2 = 5200\text{lb}$ $\quad B = \begin{pmatrix} 12 \\ 0 \\ 0 \end{pmatrix} \cdot \text{ft}$ $\quad D = \begin{pmatrix} 5 \\ 12 \\ 0 \end{pmatrix} \cdot \text{ft}$ $\quad Q = \begin{pmatrix} 5 \\ 5 \\ 7 \end{pmatrix} \cdot \text{ft}$

Unit vector along cable 2 $\quad n_{BQ} = \dfrac{Q - B}{|Q - B|} = \begin{pmatrix} -0.631 \\ 0.451 \\ 0.631 \end{pmatrix}$

Components of vector T_2 $\quad T_2 = T_2 \cdot n_{BQ} = \begin{pmatrix} -3282 \\ 2344 \\ 3282 \end{pmatrix} \cdot \text{lb}$ $\quad |T_2| = 5200 \cdot \text{lb}$

Projection on line BD

$n_{BD} = \dfrac{D - B}{|D - B|} = \begin{pmatrix} -0.504 \\ 0.864 \\ 0 \end{pmatrix}$ $\quad T_{BD} = T_2 \cdot n_{BD} = 3679 \cdot \text{lb}$

Problem 2.1-26

$$O = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad A = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} \qquad x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$r_{OA} = A - O = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} \qquad n_{OA} = \frac{r_{OA}}{|r_{OA}|} = \begin{pmatrix} 0.514 \\ 0.514 \\ 0.686 \end{pmatrix}$$

Part (a)

$$F = (25kN) \cdot n_{OA} = \begin{pmatrix} 12.86 \\ 12.86 \\ 17.15 \end{pmatrix} \cdot kN$$

Part (b)

$$n_{OA} = \begin{pmatrix} 0.514 \\ 0.514 \\ 0.686 \end{pmatrix}$$

Part (c)

$$x_p = \begin{pmatrix} \cos(30deg) \\ -\sin(30deg) \\ 0 \end{pmatrix} \qquad y_p = \begin{pmatrix} \sin(30deg) \\ \cos(30deg) \\ 0 \end{pmatrix} \qquad z_p = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$F_{xp} = F \cdot x_p = 4.71 \cdot kN \qquad F_{yp} = F \cdot y_p = 17.57 \cdot kN \qquad F_{zp} = F \cdot z_p = 17.15 \cdot kN$$

Problem 2.1-27

$$H = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \cdot in \qquad B = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} in \qquad C = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \cdot in \qquad P = 10kip$$

$$r_{HB} = B - H = \begin{pmatrix} 4 \\ 5 \\ -3 \end{pmatrix} \cdot in \qquad n_{HB} = \frac{r_{HB}}{|r_{HB}|} = \begin{pmatrix} 0.566 \\ 0.707 \\ -0.424 \end{pmatrix} \qquad x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

<u>Part (a)</u>

$$P = P \cdot n_{HB} = \begin{pmatrix} 5.657 \\ 7.071 \\ -4.243 \end{pmatrix} \cdot kip \qquad P = \begin{pmatrix} 5.66 \\ 7.07 \\ -4.24 \end{pmatrix} \cdot kip$$

<u>Part (b)</u>

$$n_{HC} = \frac{C - H}{|C - H|} = \begin{pmatrix} 0.8 \\ 0 \\ -0.6 \end{pmatrix} \qquad P_{HCpar} = P \cdot n_{HC} = 7.07 \cdot kip$$

$$P_{HCper} = P - P_{HCpar} \cdot n_{HC} = \begin{pmatrix} 0 \\ 7.07 \\ 0 \end{pmatrix} \cdot kip \qquad |P_{HCper}| = 7.07 \cdot kip$$

<u>Part (c)</u>

$$D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad G = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} \cdot in \qquad n_{DG} = \frac{G - D}{|G - D|} = \begin{pmatrix} 0.8 \\ 0 \\ 0.6 \end{pmatrix}$$

$$P_{DG} = P \cdot n_{DG} = 1.98 \cdot kip \qquad \theta = acos\left(\frac{P \cdot n_{DG}}{|P|}\right) = 78.6 \cdot deg$$

Problem 2.1-28

$$O = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad A = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 3 \\ 6 \end{pmatrix} \qquad C = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \qquad x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$n_{BC} = \frac{C - B}{|C - B|} = \begin{pmatrix} 0.442 \\ 0.147 \\ -0.885 \end{pmatrix}$$

Part (a)

$$P = \begin{pmatrix} 30 \\ 25 \\ -75 \end{pmatrix} \cdot kN$$

Part (b)

$$P \cdot n_{BC} = 83.3 \cdot kN$$

Part (c)

$$\theta = acos\left(\frac{P \cdot n_{BC}}{|P|}\right) = 9.88 \cdot deg$$

Problem 2.1-29

Solution

$$E = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \quad F = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix} \quad D = \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} \quad C = \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$$

$$n_{DB} = \frac{B - D}{|B - D|} = \begin{pmatrix} -0.728 \\ 0.485 \\ -0.485 \end{pmatrix} \quad n_{DC} = \frac{C - D}{|C - D|} = \begin{pmatrix} -0.728 \\ 0.485 \\ 0.485 \end{pmatrix} \quad n_{DE} = \frac{E - D}{|E - D|} = \begin{pmatrix} 0 \\ -0.707 \\ -0.707 \end{pmatrix}$$

$$n_{DF} = \frac{F - D}{|F - D|} = \begin{pmatrix} 0 \\ -0.707 \\ 0.707 \end{pmatrix}$$

$$T_{DB} = 10\text{kip} \quad T_{CD} = 10\text{kip} \quad T_{DE} = 15\text{kip} \quad T_{DF} = 15\text{kip}$$

Part (a)

$$T_{DB} = T_{DB} \cdot n_{DB} = \begin{pmatrix} -7.28 \\ 4.85 \\ -4.85 \end{pmatrix} \cdot \text{kip} \quad T_{CD} = T_{CD} \cdot n_{DC} = \begin{pmatrix} -7.28 \\ 4.85 \\ 4.85 \end{pmatrix} \cdot \text{kip}$$

$$T_{DE} = T_{DE} \cdot n_{DE} = \begin{pmatrix} 0 \\ -10.61 \\ -10.61 \end{pmatrix} \cdot \text{kip} \quad T_{DF} = T_{DF} \cdot n_{DF} = \begin{pmatrix} 0 \\ -10.61 \\ 10.61 \end{pmatrix} \cdot \text{kip}$$

Part (b)

$$R = T_{DB} + T_{CD} + T_{DE} + T_{DF} = \begin{pmatrix} -14.55 \\ -11.51 \\ 0 \end{pmatrix} \cdot \text{kip} \quad |R| = 18.6 \cdot \text{kip}$$

Problem 2.1-30

$$\text{mass} = 450\text{kg} \qquad P = 20\text{kN} \qquad D = \begin{pmatrix} 0 \\ 9 \\ 9\cdot\tan(55\text{deg}) \end{pmatrix}\cdot\text{m} = \begin{pmatrix} 0 \\ 9 \\ 12.853 \end{pmatrix}\cdot\text{m} \qquad E = \begin{pmatrix} 3 \\ 13 \\ 0 \end{pmatrix}\cdot\text{m}$$

Part (a)

$$W = \text{mass}\cdot g\cdot\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -4413 \end{pmatrix}\cdot\text{N} \qquad n_{DE} = \frac{E - D}{|E - D|} = \begin{pmatrix} 0.218 \\ 0.29 \\ -0.932 \end{pmatrix} \qquad P = P\cdot n_{DE} = \begin{pmatrix} 4350 \\ 5801 \\ -18639 \end{pmatrix}\cdot\text{N}$$

Part (b)

$$R = W + P = \begin{pmatrix} 4350 \\ 5801 \\ -23052 \end{pmatrix}\cdot\text{N} \qquad |R| = 24.2\cdot\text{kN}$$

Problem 2.2-1

$P = 10kip \qquad M = 25 \cdot kip \cdot in \qquad L = 6ft \qquad \theta = 45deg$

Resultant force

$$R_A = P \cdot \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + P \cdot \begin{pmatrix} \cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 7.07 \\ -17.07 \\ 0 \end{pmatrix} \cdot kip \qquad \left|R_A\right| = 18.48 \cdot kip \qquad \alpha = atan\left(\frac{R_{A_0}}{R_{A_1}}\right) = -22.5 \cdot deg$$

Resultant moment

$$M_A = \begin{pmatrix} \frac{3 \cdot L}{4} \\ 0 \\ 0 \end{pmatrix} \times \left[P \cdot \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}\right] + M \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} \frac{3 \cdot L}{2} \\ 0 \\ 0 \end{pmatrix} \times \left[P \cdot \begin{pmatrix} \cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix}\right] = \begin{pmatrix} 0 \\ 0 \\ -1329 \end{pmatrix} \cdot kip \cdot in$$

x intercept $\quad$ Given $\quad x = L$

$$\begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} \times R_A = M_A$$

$$x = Find(x) = 77.83 \cdot in \qquad d = \frac{\left|M_A\right|}{\left|R_A\right|} = 5.992\,ft \qquad \frac{d}{\cos(|\alpha|)} = 6.49\,ft \qquad x = \frac{\left|M_A\right|}{\left|R_y\right|} = 77.832 \cdot in$$

Answers

$$R_A = \begin{pmatrix} 7.07 \\ -17.07 \\ 0 \end{pmatrix} \cdot kip \qquad M_A = \begin{pmatrix} 0 \\ 0 \\ -1329 \end{pmatrix} \cdot kip \cdot in \qquad x = 77.8 \cdot in$$

Problem 2.2-2

$$\theta = \operatorname{atan}\left(\frac{3}{4}\right) = 36.87 \cdot deg \qquad P_B = 25kN \cdot \begin{pmatrix} \cos(45deg - \theta) \\ -\sin(45deg - \theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 24.749 \\ -3.536 \\ 0 \end{pmatrix} \cdot kN \qquad P_H = \begin{pmatrix} 0 \\ -50kN \\ 0 \end{pmatrix} \qquad P_F = \begin{pmatrix} 0 \\ -75kN \\ 0 \end{pmatrix}$$

$$P_D = 50kN \cdot \begin{pmatrix} \cos(60deg - \theta) \\ \sin(60deg - \theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 45.981 \\ 19.641 \\ 0 \end{pmatrix} \cdot kN$$

Resultants

$$R_A = P_B + P_D + P_H + P_F = \begin{pmatrix} 70.7 \\ -108.9 \\ 0 \end{pmatrix} \cdot kN \qquad \left|R_A\right| = 129.8 \cdot kN$$

$$M_A = \begin{pmatrix} 2m \\ \frac{3}{4} \cdot 2m \\ 0 \end{pmatrix} \times P_B + \begin{pmatrix} 2m \\ 0 \\ 0 \end{pmatrix} \times P_H + \begin{pmatrix} 6m \\ 0 \\ 0 \end{pmatrix} \times P_F + \begin{pmatrix} 6m \\ \frac{3}{4} \cdot 2m \\ 0 \end{pmatrix} \times P_D = \begin{pmatrix} 0 \\ 0 \\ -545 \end{pmatrix} \cdot kN \cdot m \qquad \left|M_A\right| = 545 \cdot kN \cdot m$$

y intercept

$$y = \frac{\left|M_A\right|}{R_x} = 7.71\ m$$

Problem 2.2-3

$L = 12\text{ft} \qquad \theta = 60\text{deg}$

$$R_A = \begin{pmatrix} 20 - 75\cdot\cos(\theta) \\ -45 - 75\cdot\sin(\theta) \\ 0 \end{pmatrix}\cdot\text{kip} = \begin{pmatrix} -17.5 \\ -110 \\ 0 \end{pmatrix}\cdot\text{kip} \qquad \left|R_A\right| = 111.3\cdot\text{kip}$$

$$M_A = \begin{pmatrix} 0 \\ 0 \\ -45\cdot\text{kip}\cdot\dfrac{L}{2} - 75\cdot\text{kip}\cdot\sin(\theta)\cdot\dfrac{3\cdot L}{4} - 100\cdot\text{kip}\cdot\text{ft} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -955 \end{pmatrix}\cdot\text{kip}\cdot\text{ft}$$

Problem 2.2-4

$\theta = 60\text{deg}$

$$R_O = \begin{pmatrix} 2-1 \\ 2-1 \\ 0 \end{pmatrix} \cdot F \qquad R_O = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} F$$

$$M_{Oz} = -2\cdot F\cdot(r\cdot\sin(60\text{deg})) - 2\cdot F\cdot(r\cdot\cos(60\text{deg})) - F\cdot r \qquad M_{Oz} = -\left(2+\sqrt{3}\right)\cdot F\cdot r$$

Problem 2.2-5

$F_{EB} = 2500lb \qquad \alpha = 80deg \qquad \beta = 45deg \qquad BC = 18ft$

$$F_{EB} = F_{EB} \cdot \begin{pmatrix} -\cos(\alpha) \\ \sin(\alpha) \\ 0 \end{pmatrix} = \begin{pmatrix} -434 \\ 2462 \\ 0 \end{pmatrix} \cdot lb \qquad r_{CB} = \begin{pmatrix} -BC \cdot \cos(\beta) \\ BC \cdot \sin(\beta) \\ 0 \end{pmatrix} = \begin{pmatrix} -12.728 \\ 12.728 \\ 0 \end{pmatrix} ft$$

Part (a)

$$M_C = r_{CB} \times F_{EB} = \begin{pmatrix} 0 \\ 0 \\ -25811 \end{pmatrix} \cdot lb \cdot ft$$

Part (b)

$$M_{Cz} = (2500 \cdot lb) \cdot [\cos(\alpha) \cdot (BC \cdot \sin(\beta)) - \sin(\alpha) \cdot (BC \cdot \cos(\beta))] = -25811 \cdot lb \cdot ft$$

Answers

$$M_C = \begin{pmatrix} 0 \\ 0 \\ -25811 \end{pmatrix} \cdot lb \cdot ft \qquad M_{Cz} = -25811 \cdot lb \cdot ft$$

Problem 2.2-6

$$F = 110\,\text{kN} \qquad A = \begin{pmatrix} -3 \\ -1 \\ 0 \end{pmatrix} \cdot \text{m} \qquad B = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \cdot \text{m} \qquad r_{OA} = A \qquad r_{OB} = B$$

$$F_A = F \cdot \begin{pmatrix} -\cos(30\text{deg}) \\ -\sin(30\text{deg}) \\ 0 \end{pmatrix} = \begin{pmatrix} -95.263 \\ -55 \\ 0 \end{pmatrix} \cdot \text{kN} \qquad F_B = F \cdot \begin{pmatrix} \cos(30\text{deg}) \\ \sin(30\text{deg}) \\ 0 \end{pmatrix} \qquad R_O = F_A + F_B = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{N}$$

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B = \begin{pmatrix} 0 \\ 0 \\ 139.5 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Problem 2.2-7

$F = 10\text{kip} \qquad A = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} \cdot \text{in} \qquad B = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \cdot \text{in} \qquad C = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \cdot \text{in}$

Part (a)

$n_{OA} = \dfrac{A}{|A|} = \begin{pmatrix} 0.514 \\ 0.514 \\ 0.686 \end{pmatrix} \qquad F_{OA} = F \cdot n_{OA} = \begin{pmatrix} 5.145 \\ 5.145 \\ 6.86 \end{pmatrix} \cdot \text{kip} \qquad r_{BO} = -B$

$M_B = r_{BO} \times F_{OA} = \begin{pmatrix} 0 \\ 34.3 \\ -25.7 \end{pmatrix} \cdot \text{kip} \cdot \text{in} \qquad |M_B| = 42.9 \cdot \text{kip} \cdot \text{in}$

Part (b)

$r_{CO} = -C \qquad M_C = r_{CO} \times F_{OA} = \begin{pmatrix} 15.43 \\ -15.43 \\ 0 \end{pmatrix} \cdot \text{kip} \cdot \text{in}$

Part (c)

$n_{CB} = \dfrac{B - C}{|C - B|} = \begin{pmatrix} 0.857 \\ 0 \\ -0.514 \end{pmatrix} \qquad M_{CB} = M_B \cdot n_{CB} = 13.24 \cdot \text{kip} \cdot \text{in} \quad = \quad M_C \cdot n_{CB} = 13.24 \cdot \text{kip} \cdot \text{in}$

Vectors $\quad M_{CBpar} = M_B \cdot n_{CB} \cdot n_{CB} = \begin{pmatrix} 11.35 \\ 0 \\ -6.81 \end{pmatrix} \cdot \text{kip} \cdot \text{in} \qquad M_{CBper} = M_B - M_{CBpar} = \begin{pmatrix} -11.35 \\ 34.3 \\ -18.92 \end{pmatrix} \cdot \text{kip} \cdot \text{in}$

$|M_{CBpar}| = 13.24 \cdot \text{kip} \cdot \text{in}$

If M_C used in Part (c)

$M_C - M_{CBpar} = \begin{pmatrix} 4.086 \\ -15.435 \\ 6.81 \end{pmatrix} \cdot \text{kip} \cdot \text{in}$

Problem 2.2-8

$$P = \begin{pmatrix} 30 \\ 25 \\ -75 \end{pmatrix} \cdot kN \qquad A = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} \cdot m \qquad B = \begin{pmatrix} 0 \\ 3 \\ 6 \end{pmatrix} \cdot m \qquad C = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \cdot m$$

Part (a)

$$r_{BA} = A - B = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} m \qquad M_B = r_{BA} \times P = \begin{pmatrix} 225 \\ 0 \\ 90 \end{pmatrix} \cdot kN \cdot m$$

Part (b)

$$M_{Bx} = (75kN) \cdot (3m) = 225 \cdot kN \cdot m \qquad M_{By} = 0 \qquad M_{Bz} = (30kN) \cdot (3m) = 90 \cdot kN \cdot m$$

Part (c)

$$n_{BC} = \frac{C - B}{|C - B|} = \begin{pmatrix} 0.442 \\ 0.147 \\ -0.885 \end{pmatrix} \qquad M_{BC} = M_B \cdot n_{BC} = 19.9 \cdot kN \cdot m$$

Vectors $$M_{BCpar} = M_B \cdot n_{BC} \cdot n_{BC} = \begin{pmatrix} 8.8 \\ 2.93 \\ -17.61 \end{pmatrix} \cdot kN \cdot m \qquad M_{BCper} = M_B - M_{BCpar} = \begin{pmatrix} 216.2 \\ -2.9 \\ 107.6 \end{pmatrix} \cdot kN \cdot m$$

$$|M_{BCpar}| = 19.9 \cdot kN \cdot m$$

Part (d)

$$r_{OA} = A \qquad M_O = r_{OA} \times P = \begin{pmatrix} -150 \\ 180 \\ 0 \end{pmatrix} \cdot kN \cdot m \qquad n_{OB} = \frac{B}{|B|} = \begin{pmatrix} 0 \\ 0.447 \\ 0.894 \end{pmatrix}$$

$$M_{OBpar} = M_O \cdot n_{OB} \cdot n_{OB} = \begin{pmatrix} 0 \\ 36 \\ 72 \end{pmatrix} \cdot kN \cdot m \qquad M_{OBper} = M_O - M_{OBpar} = \begin{pmatrix} -150 \\ 144 \\ -72 \end{pmatrix} \cdot kN \cdot m$$

Problem 2.2-9

Part (a)

$P = \begin{pmatrix} 6 \\ 12 \\ -18 \end{pmatrix} kip \quad A = \begin{pmatrix} 0 \\ 0 \\ 20 \end{pmatrix} ft \quad D = \begin{pmatrix} x_D \\ y_D \\ 0 \end{pmatrix} \quad r_{OA} = A \quad r_{OD} = D \quad M_O = r_{OA} \times P = \begin{pmatrix} -240 \\ 120 \\ 0 \end{pmatrix} \cdot kip \cdot ft$

Also $\quad r_{OD} \times P = M_O \quad r_{OD} \times P \rightarrow \begin{pmatrix} -18 \cdot kip \cdot y_D \\ 18 \cdot kip \cdot x_D \\ 12 \cdot kip \cdot x_D - 6 \cdot kip \cdot y_D \end{pmatrix}$ so $\quad x_D = \dfrac{120 \cdot kip \cdot ft}{18kip} = 6.67\ ft \quad y_D = \dfrac{-240kip \cdot ft}{-18kip} = 13.33\ ft$

(or use a geometry solution)

$12 \cdot x_D - 6 \cdot y_D = 0\ ft$

Part (b)

$P_A = P \quad P_O = -P \quad D = \begin{pmatrix} x_D \\ y_D \\ 0 \end{pmatrix} = \begin{pmatrix} 6.67 \\ 13.33 \\ 0 \end{pmatrix} ft \quad C = \begin{pmatrix} 15 \\ 10 \\ 0 \end{pmatrix} ft$

$|M_O| = 268 \cdot kip \cdot ft \quad M_A = -r_{OA} \times P_O \quad |M_A| = 268 \cdot kip \cdot ft$

$r_{DO} = -D \quad M_D = r_{DO} \times P_O \quad |M_D| = 268 \cdot kip \cdot ft$

$r_{CO} = -C \quad r_{CD} = D - C \quad M_C = r_{CD} \times P_A + r_{CO} \times P_O \quad |M_C| = 268 \cdot kip \cdot ft$

Part (c)

$d = \dfrac{|M_O|}{|P|} = 11.95\ ft$

Part (d)

Varignon's theorem

$M_{Ox} = -(12kip) \cdot (20ft) = -240 \cdot kip \cdot ft \quad M_{Oy} = (6 \cdot kip) \cdot (20ft) = 120 \cdot kip \cdot ft \quad M_O = \begin{pmatrix} -240 \\ 120 \\ 0 \end{pmatrix} \cdot kip \cdot ft$

$M_{Oz} = 0$ since P has line of action through z axis

Part (e)

$P = \begin{pmatrix} 6 \\ 12 \\ -18 \end{pmatrix} \cdot kip \quad Q = 0.8 \cdot P = \begin{pmatrix} 4.8 \\ 9.6 \\ -14.4 \end{pmatrix} \cdot kip \quad |P| = 22.45 \cdot kip \quad |Q| = 17.96 \cdot kip \quad \dfrac{|Q|}{|P|} = 0.8$

$B = \begin{pmatrix} 0 \\ 0 \\ z_Q \end{pmatrix} \quad r_{OB} = B \quad r_{OB} \times Q = M_O \quad |M_O| = 268.328 \cdot kip \cdot ft$

$\left| \begin{pmatrix} 0 \\ 0 \\ z_Q \end{pmatrix} \times \begin{pmatrix} 4.8 \\ 9.6 \\ -14.4 \end{pmatrix} \right| = 268.328 \quad \left| \begin{pmatrix} 0 \\ 0 \\ z_Q \end{pmatrix} \times \begin{pmatrix} 4.8 \\ 9.6 \\ -14.4 \end{pmatrix} \right| \rightarrow \sqrt{23.04 \cdot (|z_Q|)^2 + 92.16 \cdot (|z_Q|)^2} = 268.328$

$\sqrt{23.04 + 92.16} = 10.733 \quad \dfrac{268.328}{10.733} = 25\ ft$

Problem 2.2-10

$$P_A = \begin{pmatrix} 30 \\ 25 \\ -75 \end{pmatrix} kN \qquad P_O = -P_A = \begin{pmatrix} -30 \\ -25 \\ 75 \end{pmatrix} \cdot kN \qquad \left|P_A\right| = 84.558 \cdot kN \qquad F_C = \left|P_A\right| \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -84.558 \\ 0 \\ 0 \end{pmatrix} \cdot kN$$

$$A = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} m \qquad C = \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix} m \qquad r_{OA} = A \qquad r_{OC} = C$$

Resultants at O

$$R_O = P_A + P_O + F_C = \begin{pmatrix} -84.6 \\ 0 \\ 0 \end{pmatrix} \cdot kN \qquad M_O = r_{OA} \times P_A + r_{OC} \times F_C = \begin{pmatrix} -150 \\ 180 \\ 338 \end{pmatrix} \cdot kN \cdot m$$

Wrench resultant - components of M_O (kN-m) along & perpend. to R_O

$$n_R = \frac{R_O}{\left|R_O\right|} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \qquad M_{Rpar} = M_O \cdot n_R \cdot n_R = \begin{pmatrix} -150 \\ 0 \\ 0 \end{pmatrix} \cdot kN \cdot m \qquad M_{Rper} = M_O - M_{Rpar} = \begin{pmatrix} 0 \\ 180 \\ 338 \end{pmatrix} \cdot kN \cdot m$$

positive wrench resultant in (-x) direction

Coordinates (meters) of pt at which wrench pierces yz plane

$$\begin{pmatrix} 0 \\ y \\ z \end{pmatrix} \times R_O = M_{Oper} \text{ float}, 5 \ \rightarrow 0.0$$

$$y = \frac{338230.0 \cdot N \cdot m}{84558.0 \cdot N} = 4\,m$$

$$z = \frac{180000 \cdot N \cdot m}{-84558.0 \cdot N} = -2.13\,m$$

$$d = \frac{\left|M_{Rper}\right|}{\left|R_O\right|} = 4.53\,m$$ < perpend. dist. (m) from O to line of action of wrench

$$\sqrt{z^2 + y^2} = 4.53\,m$$ < same as d since wrench is perpendicular to yz plane

Answers

$$R_O = \begin{pmatrix} -84.6 \\ 0 \\ 0 \end{pmatrix} \cdot kN \qquad M_{Rpar} = \begin{pmatrix} -150 \\ 0 \\ 0 \end{pmatrix} \cdot kN \cdot m \qquad y = 4\,m \qquad z = -2.13\,m$$

Problem 2.2-11

$F = 350lb \qquad r = 2.5ft \qquad h = 7.5ft \qquad A = \begin{pmatrix} -r\cdot\sin(60deg) \\ r\cdot\cos(60deg) \\ h \end{pmatrix} = \begin{pmatrix} -2.165 \\ 1.25 \\ 7.5 \end{pmatrix} ft \qquad B = \begin{pmatrix} 0 \\ -r \\ h \end{pmatrix} \qquad C = \begin{pmatrix} 0 \\ r \\ h \end{pmatrix} \qquad D = \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix}$

$$n_{OA} = \frac{A}{|A|} = \begin{pmatrix} -0.274 \\ 0.158 \\ 0.949 \end{pmatrix}$$

Part (a)

$$F = F\cdot n_{OA} = \begin{pmatrix} -95.851 \\ 55.34 \\ 332.039 \end{pmatrix}\cdot lb \qquad F_x = F_1 \qquad F_y = F_2 \qquad F_z = F_3$$

$$r_{DO} = -D \qquad r_{DA} = A - D \qquad M_D = r_{DO} \times F = \begin{pmatrix} -830.098 \\ 0 \\ -239.629 \end{pmatrix}\cdot lb\cdot ft \quad = \quad r_{DA} \times F = \begin{pmatrix} -830.098 \\ 0 \\ -239.629 \end{pmatrix}\cdot lb\cdot ft$$

Part (b)

$$M_{Dx} = -F_z\cdot r = -830\cdot lb\cdot ft \qquad M_{Dy} = 0 \qquad M_{Dz} = F_x\cdot r = -240\cdot lb\cdot ft$$

Part (c)

$$n_{BD} = \frac{D - B}{|D - B|} = \begin{pmatrix} 0 \\ 0.555 \\ -0.832 \end{pmatrix} \qquad M_{BDpar} = M_D\cdot n_{BD}\cdot n_{BD} = \begin{pmatrix} 0 \\ 110.6 \\ -165.9 \end{pmatrix}\cdot lb\cdot ft \qquad M_{BDper} = M_D - M_{BDpar} = \begin{pmatrix} -830.1 \\ -110.6 \\ -73.7 \end{pmatrix}\cdot lb\cdot ft$$

$$\left|M_{BDpar}\right| = 199.4\cdot lb\cdot ft$$

Problem 2.2-12

$F = 520N \qquad A = \begin{pmatrix} 0 \\ 9 \\ 4 \end{pmatrix} m \qquad B = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} m \qquad C = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} m \qquad D = \begin{pmatrix} 5 \\ 9 \\ 0 \end{pmatrix} m \qquad n_{CA} = \frac{A - C}{|A - C|} = \begin{pmatrix} -0.453 \\ 0.815 \\ 0.362 \end{pmatrix}$

Part (a)

$F = F \cdot n_{CA} = \begin{pmatrix} -235.39 \\ 423.71 \\ 188.31 \end{pmatrix} N \qquad r_{BC} = C - B = \begin{pmatrix} 5 \\ 0 \\ -4 \end{pmatrix} m \qquad r_{BA} = A - B = \begin{pmatrix} 0 \\ 9 \\ 0 \end{pmatrix} m$

$M_B = r_{BC} \times F = \begin{pmatrix} 1695 \\ 0 \\ 2119 \end{pmatrix} \cdot N \cdot m$ check $r_{BA} \times F = \begin{pmatrix} 1695 \\ 0 \\ 2119 \end{pmatrix} \cdot N \cdot m$

Part (b)

$F_x = -235.39N \qquad F_y = 423.71N \qquad F_z = 188.31N$

$M_{Bx} = F_y \cdot (4m) = 1695 \cdot N \cdot m \qquad M_{By} = -F_z \cdot (5m) - F_x \cdot (4m) = 0 \cdot N \cdot m \qquad M_{Bz} = F_y \cdot (5m) = 2119 \cdot N \cdot m$

Part (c)

$n_{BD} = \frac{D - B}{|D - B|} = \begin{pmatrix} 0.453 \\ 0.815 \\ -0.362 \end{pmatrix} \qquad M_{BDpar} = M_B \cdot n_{BD} \cdot n_{BD} = \begin{pmatrix} -0 \\ -0 \\ 0 \end{pmatrix} \cdot N \cdot m \qquad M_{BDper} = M_B - M_{BDpar} = \begin{pmatrix} 1695 \\ 0 \\ 2119 \end{pmatrix} \cdot N \cdot m$

Part (d)

$r_{OC} = C \qquad M_O = r_{OC} \times F = \begin{pmatrix} 0 \\ -942 \\ 2119 \end{pmatrix} \cdot N \cdot m$

$n_{OD} = \frac{D}{|D|} = \begin{pmatrix} 0.486 \\ 0.874 \\ 0 \end{pmatrix} \qquad M_{ODpar} = M_O \cdot n_{OD} \cdot n_{OD} = \begin{pmatrix} -400 \\ -720 \\ 0 \end{pmatrix} \cdot N \cdot m \qquad M_{ODper} = M_O - M_{ODpar} = \begin{pmatrix} 400 \\ -222 \\ 2119 \end{pmatrix} \cdot N \cdot m$

Problem 2.2-13

$T_{AB} = 15lb$

Joint coordinates: $A = \begin{pmatrix} 8 \\ 20 - 20\cdot\cos(60deg) \\ -20\cdot\sin(60deg) \end{pmatrix}\cdot in = \begin{pmatrix} 8 \\ 10 \\ -17.321 \end{pmatrix}\cdot in$ $\quad B = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}\cdot in$ $\quad C = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}\cdot in$

Part (a) $\quad n_{AB} = \dfrac{B - A}{|B - A|} = \begin{pmatrix} -0.1 \\ -0.498 \\ 0.862 \end{pmatrix}$ $\quad T_{AB} = T_{AB}\cdot n_{AB} = \begin{pmatrix} -1.493 \\ -7.463 \\ 12.926 \end{pmatrix}\cdot lb$ $\quad r_{OB} = B \quad r_{OA} = A$

$M_O = r_{OB} \times T_{AB} = \begin{pmatrix} 0 \\ -77.6 \\ -44.8 \end{pmatrix}\cdot in\cdot lb$ or as a check $r_{OA} \times T_{AB} = \begin{pmatrix} 0 \\ -77.6 \\ -44.8 \end{pmatrix}\cdot in\cdot lb$

Part (b) $\quad T_{ABx} = -1.493lb \quad T_{ABy} = -7.463lb \quad T_{ABz} = 12.926lb$

$M_{Ox} = 0 \quad M_{Oy} = -T_{ABz}\cdot(8in) - T_{ABx}\cdot(20in\cdot\sin(60deg)) = -77.5\cdot in\cdot lb$

$M_{Oz} = -T_{ABx}\cdot(20in - 20in\cdot\cos(60deg)) + T_{ABy}\cdot(8in) = -44.8\cdot in\cdot lb$

Part (c)

$r_{CA} = A - C = \begin{pmatrix} 8 \\ -10 \\ -17.321 \end{pmatrix}\cdot in$ $\quad M_C = r_{CA} \times T_{AB} = \begin{pmatrix} -258.5 \\ -77.6 \\ -74.6 \end{pmatrix}\cdot in\cdot lb$ $\quad n_{CD} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$M_{CDpar} = M_C\cdot n_{CD}\cdot n_{CD} = \begin{pmatrix} -259 \\ 0 \\ 0 \end{pmatrix}\cdot in\cdot lb$ $\quad M_{CDper} = M_C - M_{CDpar} = \begin{pmatrix} 0 \\ -77.6 \\ -74.6 \end{pmatrix}\cdot in\cdot lb$

Answers

a), b) $M_O = \begin{pmatrix} 0 \\ -77.6 \\ -44.8 \end{pmatrix}\cdot in\cdot lb$

c) $M_{CDpar} = \begin{pmatrix} -259 \\ 0 \\ 0 \end{pmatrix}\cdot in\cdot lb$ $\quad M_{CDper} = \begin{pmatrix} 0 \\ -77.6 \\ -74.6 \end{pmatrix}\cdot in\cdot lb$

Problem 2.2-14

$F_{DO} = 2.5N$ $\quad T_{AE} = 4N$ $\quad M_y = 5N\cdot m$ $\quad A = \begin{pmatrix} 1.8 \\ 0 \\ 2.1 \end{pmatrix} m$ $\quad E = \begin{pmatrix} 2.6 \\ 0 \\ 0 \end{pmatrix} m$ $\quad r_{OA} = A$

$n_{AE} = \dfrac{E - A}{|E - A|} = \begin{pmatrix} 0.356 \\ 0 \\ -0.934 \end{pmatrix}$ $\quad T_{AE} = T_{AE} \cdot n_{AE} = \begin{pmatrix} 1.424 \\ 0 \\ -3.738 \end{pmatrix} N$ $\quad T_{AEx} = 1.424N$ $\quad T_{AEy} = 0$ $\quad T_{AEz} = -3.738N$

Part (a)

$$M_O = M_y \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + r_{OA} \times T_{AE} = \begin{pmatrix} 0 \\ 14.72 \\ 0 \end{pmatrix} \cdot N \cdot m$$

Part (b)

$M_{Ox} = 0$ $\quad M_{Oy} = M_y + T_{AEx} \cdot (2.1m) - T_{AEz} \cdot (1.8m) = 14.72 \cdot N \cdot m$ $\quad M_{Oz} = 0$

Problem 2.2-15

Part (a)

$$P = 40lb \qquad F_C = P\cdot\begin{pmatrix}0\\0\\-1\end{pmatrix} \qquad R_O = F_C = \begin{pmatrix}0\\0\\-40\end{pmatrix}\cdot lb \qquad C = \begin{pmatrix}5in\cdot\cos(40deg)\\1.25in + 2in\\5in\cdot\sin(40deg)\end{pmatrix} = \begin{pmatrix}3.83\\3.25\\3.214\end{pmatrix}\cdot in \qquad r_{OC} = C$$

$$M_O = r_{OC}\times F_C = \begin{pmatrix}-130\\153.2\\0\end{pmatrix}\cdot in\cdot lb$$

Part (b)

$$A = \begin{pmatrix}0\\1.25in\\0\end{pmatrix} \qquad B = \begin{pmatrix}5in\cdot\cos(40deg)\\1.25in\\5in\cdot\sin(40deg)\end{pmatrix} = \begin{pmatrix}3.83\\1.25\\3.214\end{pmatrix}\cdot in \qquad n_{AB} = \frac{B-A}{|B-A|} = \begin{pmatrix}0.766\\0\\0.643\end{pmatrix} \qquad r_{AC} = C - A$$

$$M_A = r_{AC}\times F_C = \begin{pmatrix}-80\\153.2\\0\end{pmatrix}\cdot in\cdot lb \qquad M_{AB} = M_A\cdot n_{AB}\cdot n_{AB} = \begin{pmatrix}-46.9\\0\\-39.4\end{pmatrix}\cdot in\cdot lb \qquad M_A\cdot n_{AB} = -61.3\cdot in\cdot lb$$

$$|M_{AB}| = 61.3\cdot in\cdot lb$$

Problem 2.2-16

$F_{DO} = 2.5N \qquad T_{AB} = 4N \qquad M_y = 5N\cdot m$

$A = \begin{pmatrix} 1.8 \\ 0 \\ 2.1 \end{pmatrix} m \qquad B = \begin{pmatrix} 2.6 \\ 1.4 \\ 0 \end{pmatrix} m \qquad C = \begin{pmatrix} 1.8 \\ 1.4 \\ 0 \end{pmatrix} m \qquad D = \begin{pmatrix} 0 \\ 0 \\ 2.1 \end{pmatrix} m \qquad r_{OA} = A$

$n_{AB} = \frac{B - A}{|B - A|} = \begin{pmatrix} 0.302 \\ 0.529 \\ -0.793 \end{pmatrix} \qquad T_{AB} = T_{AB} \cdot n_{AB} = \begin{pmatrix} 1.209 \\ 2.115 \\ -3.173 \end{pmatrix} N \qquad$ so $\qquad T_{ABx} = 1.209N \qquad T_{ABy} = 2.115N$

$T_{ABz} = -3.173N$

Part (a)

$$M_O = M_y \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + r_{OA} \times T_{AB} = \begin{pmatrix} -4.44 \\ 13.25 \\ 3.81 \end{pmatrix} \cdot N\cdot m$$

Part (b)

$M_{Ox} = -T_{ABy} \cdot (2.1m) = -4.44 N\cdot m \qquad M_{Oy} = M_y + T_{ABx} \cdot (2.1m) + \left(-T_{ABz}\right) \cdot (1.8m) = 13.25 N\cdot m$

$M_{Oz} = T_{ABy} \cdot (1.8m) = 3.81 \cdot N\cdot m$

Part (c)

$n_{OC} = \frac{C}{|C|} = \begin{pmatrix} 0.789 \\ 0.614 \\ 0 \end{pmatrix} \qquad M_{OCpar} = M_O \cdot n_{OC} \cdot n_{OC} = \begin{pmatrix} 3.65 \\ 2.84 \\ 0 \end{pmatrix} \cdot N\cdot m \qquad M_{OCper} = M_O - M_{OCpar} = \begin{pmatrix} -8.09 \\ 10.41 \\ 3.81 \end{pmatrix} \cdot N\cdot m$

Part (d) $\qquad M_O = M_O - M_y \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -4.442 \\ 8.249 \\ 3.807 \end{pmatrix} N\cdot m$

$$d = \frac{|M_O|}{|T_{AB}|} = 2.53m$$

Problem 2.2-17

$F = 15lb \qquad A = \begin{pmatrix} 1.8 \\ 0 \\ 2.1 \end{pmatrix} \cdot ft \qquad D = \begin{pmatrix} 0 \\ 0 \\ 2.1 \end{pmatrix} \cdot ft \qquad d = 1.8ft$

Varignon's theorem

$M_{Ay} = -F \cdot d = -27 \cdot lb \cdot ft \qquad M_{Dy} = M_{Ay} \qquad M_{Oy} = M_{Ay}$ < same moment about any point for force couple

Vector algebra

$F_A = F \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad F_D = -F_A$

$r_{AD} = D - A = \begin{pmatrix} -1.8 \\ 0 \\ 0 \end{pmatrix} ft \qquad M_A = r_{AD} \times F_D = \begin{pmatrix} 0 \\ -27 \\ 0 \end{pmatrix} \cdot lb \cdot ft$

$r_{DA} = -r_{AD} = \begin{pmatrix} 1.8 \\ 0 \\ 0 \end{pmatrix} ft \qquad M_D = r_{DA} \times F_A = \begin{pmatrix} 0 \\ -27 \\ 0 \end{pmatrix} \cdot lb \cdot ft$

$r_{OA} = A \qquad M_O = r_{OA} \times F_A = \begin{pmatrix} 0 \\ -27 \\ 0 \end{pmatrix} \cdot lb \cdot ft$

Problem 2.2-18

$$r_{CB} = \begin{pmatrix} -3 \\ -8 \\ 3 \end{pmatrix} \cdot m \qquad r_{CA} = \begin{pmatrix} -3 \\ 0 \\ 9 \end{pmatrix} \cdot m \qquad r_{BA} = \begin{pmatrix} 0 \\ 8 \\ 6 \end{pmatrix} \cdot m \qquad r_{DC} = \begin{pmatrix} -12 \\ 8 \\ -9 \end{pmatrix} \cdot m$$

$$F = 500N \qquad F_{BA} = F \cdot \frac{r_{BA}}{|r_{BA}|} = \begin{pmatrix} 0 \\ 400 \\ 300 \end{pmatrix} \cdot N$$

a) Find moment of F about point C

$$M_C = r_{CB} \times F_{BA} = \begin{pmatrix} -3600 \\ 900 \\ -1200 \end{pmatrix} \cdot N \cdot m \qquad r_{CA} \times F_{BA} = \begin{pmatrix} -3600 \\ 900 \\ -1200 \end{pmatrix} \cdot N \cdot m$$

< can use any pt along line of action of F

b) Find moment of F about line CD using a cross product

$$n_{CD} = \frac{r_{DC}}{|r_{DC}|} = \begin{pmatrix} -0.706 \\ 0.471 \\ -0.529 \end{pmatrix}$$

$$M_{CD} = M_C \cdot n_{CD} \cdot n_{CD} = \begin{pmatrix} -2541 \\ 1694 \\ -1906 \end{pmatrix} \cdot N \cdot m \qquad |M_{CD}| = 3600 \cdot N \cdot m$$

c) Find the shortest distance d between component F_{per} (perpendicular to line CD) and line CD

$$F_{parCD} = F_{BA} \cdot n_{CD} \cdot n_{CD} = \begin{pmatrix} -20.761 \\ 13.841 \\ -15.571 \end{pmatrix} \cdot N \qquad F_{perCD} = F_{BA} - F_{parCD} = \begin{pmatrix} 20.761 \\ 386.159 \\ 315.571 \end{pmatrix} \cdot N \qquad r_{DC} \times F_{parCD} = \begin{pmatrix} -0 \\ 0 \\ 0 \end{pmatrix} \cdot N \cdot m$$

^ component of F which is parallel to CD has no moment about CD

$$d = \frac{|M_{CD}|}{|F_{perCD}|} = 7.21 \cdot m \qquad |F_{perCD}| \cdot d = 3600 \cdot N \cdot m$$

< same as $|M_{CD}|$

Problem 2.2-19

$F = 20 \cdot lb \qquad M = 50 \cdot lb \cdot in \qquad r_{DB} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} \cdot in \qquad n_{DB} = \dfrac{r_{DB}}{|r_{DB}|} = \begin{pmatrix} 0.6 \\ -0.8 \\ 0 \end{pmatrix} \qquad F_{DB} = F \cdot n_{DB} = \begin{pmatrix} 12 \\ -16 \\ 0 \end{pmatrix} \cdot lb$

<u>Find unit vector normal to plane ABC</u>

$r_{BC} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \cdot in \qquad r_{BA} = \begin{pmatrix} -3 \\ 0 \\ 5 \end{pmatrix} \cdot in \qquad r_{AB} = -r_{BA}$

$n_{BCA} = \dfrac{r_{BC} \times r_{BA}}{|r_{BC} \times r_{BA}|} = \begin{pmatrix} 0.857 \\ 0 \\ 0.514 \end{pmatrix} \qquad M = M \cdot n_{BCA} = \begin{pmatrix} 42.875 \\ 0 \\ 25.725 \end{pmatrix} \cdot lb \cdot in$

<u>a) Find M_A due to both F and M</u>

$M_A = M + r_{AB} \times F_{DB} = \begin{pmatrix} -37.1 \\ -60 \\ -22.3 \end{pmatrix} \cdot lb \cdot in$

<u>b) Find moment about line AC</u>

$r_{AC} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} \cdot in \qquad n_{AC} = \dfrac{r_{AC}}{|r_{AC}|} = \begin{pmatrix} 0.424 \\ 0.566 \\ -0.707 \end{pmatrix} \qquad M_{AC} = M_A \cdot n_{AC} \cdot n_{AC} = \begin{pmatrix} -14.4 \\ -19.2 \\ 24 \end{pmatrix} \cdot lb \cdot in$

<u>c) What is the perpendicular distance d (inches) from force F to line AC?</u>

$d = \dfrac{|(M_A) \cdot n_{AC} \cdot n_{AC}|}{|F|} = 1.697 \cdot in$ < as expected distance is less than 1/2 of diagonal dist. at base (= 5/2 = 2.5)

Answers

a) $M_A = \begin{pmatrix} -37.1 \\ -60 \\ -22.3 \end{pmatrix} \cdot in \cdot lb$ b) $M_{AC} = \begin{pmatrix} -14.4 \\ -19.2 \\ 24 \end{pmatrix} \cdot in \cdot lb$ c) $d = 1.697 \cdot in$

Problem 2.2-20

Find unit vectors in directions of three couples   Let $b = 1$

$$n_1 = \frac{\begin{pmatrix} 0 \\ \frac{3 \cdot b}{2} \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \cdot b \\ 0 \\ 3 \cdot b \end{pmatrix}}{\left| \begin{pmatrix} 0 \\ \frac{3 \cdot b}{2} \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \cdot b \\ 0 \\ 3 \cdot b \end{pmatrix} \right|} \rightarrow \begin{pmatrix} \frac{3 \cdot \sqrt{13}}{13} \\ 0 \\ \frac{2 \cdot \sqrt{13}}{13} \end{pmatrix} = \begin{pmatrix} 0.832 \\ 0 \\ 0.555 \end{pmatrix}$$

$$n_2 = \frac{\begin{pmatrix} -2 \cdot b \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \cdot b \\ \frac{-3 \cdot b}{2} \\ 3 \cdot b \end{pmatrix}}{\left| \begin{pmatrix} -2 \cdot b \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} -2 \cdot b \\ \frac{-3 \cdot b}{2} \\ 3 \cdot b \end{pmatrix} \right|} \rightarrow \begin{pmatrix} 0 \\ \frac{2 \cdot \sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} \end{pmatrix} = \begin{pmatrix} 0 \\ 0.894 \\ 0.447 \end{pmatrix}$$

$$n_3 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

^ by inspection

$$M_1 = M_0 \cdot n_1 = \begin{pmatrix} 0.832 \\ 0 \\ 0.555 \end{pmatrix} M_0 \qquad M_2 = M_0 \cdot n_2 = \begin{pmatrix} 0 \\ 0.894 \\ 0.447 \end{pmatrix} M_0 \qquad M_3 = M_0 \cdot n_3 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} M_0$$

Single couple is sum of three vectors

$$M_{SE} = M_1 + M_2 + M_3 = \begin{pmatrix} 0.832 \\ 0.894 \\ 0.002 \end{pmatrix} M_0$$

Problem 2.2-21

(a) Moment of force $F_1 = 12$ lb about E

$$r_{EG} = \begin{pmatrix} 4 \\ 4 \\ -3 \end{pmatrix} \cdot ft \qquad F_1 = 12 \cdot lb \cdot \frac{\begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}\right|} = \begin{pmatrix} 0 \\ -9.6 \\ 7.2 \end{pmatrix} \cdot lb \qquad M_{EF1} = r_{EG} \times F_1 = \begin{pmatrix} -0 \\ -28.8 \\ -38.4 \end{pmatrix} \cdot ft \cdot lb$$

(b) Moment of force $F_2 = 18$ lb about E

$$F_2 = 18 \cdot lb \cdot \frac{\begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix}}{\left|\begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix}\right|} = \begin{pmatrix} -14.4 \\ 0 \\ 10.8 \end{pmatrix} \cdot lb \qquad M_{EF2} = r_{EG} \times F_2 = \begin{pmatrix} 43.2 \\ 0 \\ 57.6 \end{pmatrix} \cdot ft \cdot lb$$

(c) Moment of couple C = 14 lb-ft about E

$$r_{OG} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \cdot ft \qquad r_{OB} = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} \cdot ft$$

$$C = 14 \cdot ft \cdot lb \cdot \frac{r_{OG} \times r_{OB}}{\left|r_{OG} \times r_{OB}\right|} = \begin{pmatrix} 7.203 \\ -7.203 \\ 9.604 \end{pmatrix} \cdot ft \cdot lb \qquad M_{EC} = C = \begin{pmatrix} 7.2 \\ -7.2 \\ 9.6 \end{pmatrix} \cdot ft \cdot lb$$

(d) Moment of resultant of F_1, F_2 and C about line EG

$$n_{EG} = \frac{r_{EG}}{\left|r_{EG}\right|} = \begin{pmatrix} 0.625 \\ 0.625 \\ -0.469 \end{pmatrix} \qquad r_{EG} = \begin{pmatrix} 4 \\ 4 \\ -3 \end{pmatrix} ft$$

resultant moment about pt. E

$$M_{ER} = M_{EF1} + M_{EF2} + M_{EC} = \begin{pmatrix} 50.4 \\ -36 \\ 28.8 \end{pmatrix} \cdot ft \cdot lb \qquad M_{EG} = M_{ER} \cdot n_{EG} \cdot n_{EG} = \begin{pmatrix} -2.81 \\ -2.81 \\ 2.11 \end{pmatrix} \cdot ft \cdot lb$$

$$C \cdot n_{EG} \cdot n_{EG} = \begin{pmatrix} -2.81 \\ -2.81 \\ 2.11 \end{pmatrix} \cdot ft \cdot lb$$

(e) Moment of resultant of F_1, F_2 and C about line EB

$$n_{EB} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$M_{EB} = M_{ER} \cdot n_{EB} \cdot n_{EB} = \begin{pmatrix} 0 \\ -36 \\ 0 \end{pmatrix} \cdot ft \cdot lb$$ < y component of M_{ER} above

and $$\left(M_{EF1} + C\right) \cdot n_{EB} \cdot n_{EB} = \begin{pmatrix} 0 \\ -36 \\ 0 \end{pmatrix} \cdot ft \cdot lb$$ < F_2 passes through line EB so offers no contribution to M_{EB}

Problem 2.2-22

Part (a)

Resultant force (N)

$$R = (50 + 75 + 95 + 65 + 70)\cdot N\cdot\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\355\\0\end{pmatrix}N$$

Part (b)

Moment (N.m) about pt. B: note that r x F = -(F x r)

$$M_B = -\left[\begin{pmatrix}0\\50\\0\end{pmatrix}N\times\begin{pmatrix}0\\0\\\frac{3}{2}\end{pmatrix}m + \begin{pmatrix}0\\75\\0\end{pmatrix}N\times\begin{pmatrix}0\\\frac{6}{2}\\0\end{pmatrix}m + \begin{pmatrix}0\\95\\0\end{pmatrix}N\times\begin{pmatrix}\frac{5}{2}\\6\\0\end{pmatrix}m + \begin{pmatrix}0\\65\\0\end{pmatrix}N\times\begin{pmatrix}5\\6\\\frac{4}{2}\end{pmatrix}m + \begin{pmatrix}0\\70\\0\end{pmatrix}N\times\begin{pmatrix}5\\6-2\\4\end{pmatrix}m\right]$$

$$M_B = \begin{pmatrix}-485\\0\\913\end{pmatrix}\cdot N\cdot m$$

< all forces are parallel to y-axis so moment has x and z components only, as expected

Part (c)

$$|R| = 355\,N \qquad |M_B| = 1033\cdot N\cdot m \qquad d = \frac{|M_B|}{|R|} = 2.91\,m$$

$$\begin{pmatrix}x\\0\\z\end{pmatrix}\times R = M_B \;\Bigg|\begin{matrix}\text{solve}, x, z\\ \text{float}, 4\end{matrix} \rightarrow (2.57\cdot m \quad 1.366\cdot m) \qquad \sqrt{2.57^2 + 1.366^2} = 2.9105$$

^ move resultant R (acting in +y-dir.) a dist. d from B so that it pierces (x, z) plane at coordinates x = 2.57m, z = 1.366m

Problem 2.2-23

Part (a)

$R_{Ax} = -5lb$ $\quad R_{Ay} = -3lb + 2lb = -1 \cdot lb$ $\quad R_{Az} = 0$

$M_{Ax} = 3lb \cdot (4in) = 12 \cdot in \cdot lb$ $\quad M_{Ay} = -5lb \cdot (4in) = -20 \cdot in \cdot lb$ $\quad M_{Az} = (2-3)lb \cdot (1in) = -1 \cdot in \cdot lb$

Part (b)

$$r_{AB} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} in \qquad F_B = \begin{pmatrix} -5 \\ -3 \\ 0 \end{pmatrix} lb \qquad R_A = F_B + \begin{pmatrix} 0 \\ 2lb \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ 0 \end{pmatrix} \cdot lb \qquad M_A = r_{AB} \times F_B + \begin{bmatrix} 0 \\ 0 \\ 2 \cdot (1) \end{bmatrix} in \cdot lb = \begin{pmatrix} 12 \\ -20 \\ -1 \end{pmatrix} \cdot in \cdot lb$$

Part (c)

$$n_R = \frac{R_A}{|R_A|} = \begin{pmatrix} -0.981 \\ -0.196 \\ 0 \end{pmatrix} \qquad M_{Rpar} = M_A \cdot n_R \cdot n_R = \begin{pmatrix} 7.692 \\ 1.538 \\ 0 \end{pmatrix} \cdot in \cdot lb \qquad M_{Rper} = M_A - M_{Rpar} = \begin{pmatrix} 4.308 \\ -21.538 \\ -1 \end{pmatrix} \cdot in \cdot lb$$

$$\begin{pmatrix} 0 \\ y \\ z \end{pmatrix} \times R_A = M_{Rper} \qquad \begin{pmatrix} 0 \\ y \\ z \end{pmatrix} \times R_A \rightarrow \begin{pmatrix} 1.0 \cdot lbf \cdot z \\ -5.0 \cdot lbf \cdot z \\ 5.0 \cdot lbf \cdot y \end{pmatrix} \qquad y = \frac{-1}{5} in \qquad z = \frac{-21.538}{-5} in = 4.308 \cdot in$$

$$\begin{pmatrix} 0 \\ \frac{-1}{5} \\ 4.308 \end{pmatrix} in \times R_A = \begin{pmatrix} 4.308 \\ -21.54 \\ -1 \end{pmatrix} \cdot in \cdot lb = M_{Rper}$$

Answers

a), b) $\quad R_A = \begin{pmatrix} -5 \\ -1 \\ 0 \end{pmatrix} \cdot lb \qquad M_A = \begin{pmatrix} 12 \\ -20 \\ -1 \end{pmatrix} \cdot in \cdot lb$

c) $\quad y = -0.2 \cdot in \qquad z = 4.31 \cdot in$

Problem 2.2-24

Position and unit vectors

$$r_{AC} = \begin{pmatrix} 3 \\ 7 \\ 0 \end{pmatrix} m \qquad r_{AB} = \begin{pmatrix} 0 \\ 7 \\ -5 \end{pmatrix} m \qquad r_{CD} = \begin{pmatrix} 0 \\ -7 \\ -5 \end{pmatrix} m \qquad n_{AC} = \frac{r_{AC}}{|r_{AC}|} = \begin{pmatrix} 0.394 \\ 0.919 \\ 0 \end{pmatrix} \qquad n_{AB} = \frac{r_{AB}}{|r_{AB}|} = \begin{pmatrix} 0 \\ 0.814 \\ -0.581 \end{pmatrix}$$

$$n_{CD} = \frac{r_{CD}}{|r_{CD}|} = \begin{pmatrix} 0 \\ -0.814 \\ -0.581 \end{pmatrix}$$

Resultant force (N) and unit vector along R

$$R = 75N \cdot n_{AC} + 50N \cdot n_{AB} + 90N \cdot n_{CD} = \begin{pmatrix} 29.5 \\ 36.4 \\ -81.4 \end{pmatrix} N \qquad n_R = \frac{R}{|R|} = \begin{pmatrix} 0.315 \\ 0.387 \\ -0.867 \end{pmatrix}$$

Moment (N.m) about pt. O

$$M_O = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} m \times (75N \cdot n_{AC} + 50N \cdot n_{AB}) + \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} m \times (90N \cdot n_{CD}) = \begin{pmatrix} -548.1 \\ 304.7 \\ -219.7 \end{pmatrix} \cdot N \cdot m$$

Components of M_O along & perpend. to R

$$M_{Rpar} = M_O \cdot n_R \cdot n_R = \begin{pmatrix} 42.8 \\ 52.7 \\ -117.8 \end{pmatrix} \cdot N \cdot m \qquad M_{Rper} = M_O - M_{Rpar} = \begin{pmatrix} -590.9 \\ 252 \\ -101.9 \end{pmatrix} \cdot N \cdot m \qquad |M_{Rper}| = 650.4 \cdot N \cdot m$$

Coordinates (meters) of pt P where wrench pierces xy plane

$$\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \times R = M_{Rper} \text{ float}, 5 \rightarrow \begin{pmatrix} -81.373 \cdot N \cdot y \\ 81.373 \cdot N \cdot x \\ 36.387 \cdot N \cdot x + -29.544 \cdot N \cdot y \end{pmatrix} = \begin{pmatrix} -590.9 \cdot J \\ 251.96 \cdot J \\ -101.87 \cdot J \end{pmatrix}$$

$$x = \frac{251.96 N \cdot m}{81.373 N} = 3.1\, m \qquad y = \frac{590.9 N \cdot m}{81.373 N} = 7.26\, m$$

$$36.387 N \cdot x + -29.544 N \cdot y = -101.87 \cdot N \cdot m$$

$$d = \frac{|M_{Rper}|}{|R|} = 6.93\, m$$

< perpend. dist. (m) from O to line of action of wrench (note that R is not perpendicular to xy plane)

Problem 2.2-25

$$P = \begin{pmatrix} 6 \\ 12 \\ -18 \end{pmatrix} \text{kip} \qquad P_A = P \qquad P_O = -P \qquad R_O = P_A + P_O = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \cdot \text{kip} \qquad M_C = (200\text{kip}\cdot\text{ft})\cdot\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -200 \\ 0 \\ 0 \end{pmatrix} \cdot \text{kip}\cdot\text{ft}$$

$$M_O = \begin{pmatrix} 0 \\ 0 \\ 20 \end{pmatrix} \text{ft} \times P_A + M_C = \begin{pmatrix} -440 \\ 120 \\ 0 \end{pmatrix} \cdot \text{kip}\cdot\text{ft}$$

Problem 2.2-26

$$R_A = \begin{pmatrix} -140 \\ 80 \\ -120 \end{pmatrix} \text{N} \qquad M_A = \begin{bmatrix} -80\cdot(2) \\ 120\cdot(5) - 140\cdot(2) + 30 - 25 \\ 80\cdot(5) + 140\cdot(7) \end{bmatrix} = \begin{pmatrix} -160 \\ 325 \\ 1380 \end{pmatrix} \text{N}\cdot\text{m} \qquad n_R = \frac{R_A}{|R_A|} = \begin{pmatrix} -0.697 \\ 0.398 \\ -0.597 \end{pmatrix}$$

Wrench resultant (here, a negative wrench)

$$M_{Rpar} = M_A\cdot n_R\cdot n_R = \begin{pmatrix} 406 \\ -232 \\ 348 \end{pmatrix} \qquad M_{Rper} = M_A - M_{Rpar} = \begin{pmatrix} -566 \\ 557 \\ 1032 \end{pmatrix}$$

$$d = \frac{|M_{Rper}|}{|R_A|} = 6.479 \quad \text{m} \qquad \begin{pmatrix} 0 \\ y \\ z \end{pmatrix} \times R_A = M_{Rper} \text{ float}, 5 \;\rightarrow \begin{pmatrix} -120\cdot y - 80\cdot z \\ -140\cdot z \\ 140\cdot y \end{pmatrix} = \begin{pmatrix} -566.14 \\ 557.08 \\ 1031.9 \end{pmatrix}$$

$$y_P = \frac{1031.9}{140} = 7.37 \text{ m} \qquad z_P = \frac{557.08}{-140} = -3.98 \text{ m}$$

$$-120\cdot y_P - 80\cdot z_P = -566.154$$

Problem 2.2-27

$$R_O = (5)\cdot(175lb)\cdot\begin{pmatrix}0\\-1\\0\end{pmatrix} = \begin{pmatrix}0\\-875\\0\end{pmatrix}\cdot lb \qquad \beta = 120deg$$

Use Varignon's theorem

$$M_{Ox} = 175lb\cdot(4ft + 12ft + 20ft) - 175lb\cdot(10ft\cdot\sin(\beta - 90deg) + 22ft\cdot\sin(\beta - 90deg)) = 3500\cdot lb\cdot ft$$

$M_{Oy} = 0$ since all forces are parallel to y axis

$$M_{Oz} = -175lb\cdot(10ft\cdot\cos(\beta - 90deg) + 22ft\cdot\cos(\beta - 90deg)) = -4850\cdot lb\cdot ft$$

$$M_O = \begin{pmatrix}M_{Ox}\\M_{Oy}\\M_{Oz}\end{pmatrix} = \begin{pmatrix}3500\\0\\-4850\end{pmatrix}\cdot lb\cdot ft$$

Problem 2.2-28

Joint coordinates (meters)

$$A = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} \quad E = \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix} \quad F = \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} \quad G = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} \quad D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad n_{DB} = \frac{B}{|B|} = \begin{pmatrix} 0.625 \\ 0.781 \\ 0 \end{pmatrix}$$

Resultants

$$R_D = \begin{pmatrix} 25 \\ 30 \\ -10 \end{pmatrix} + 15 \cdot n_{DB} = \begin{pmatrix} 34.4 \\ 41.7 \\ -10 \end{pmatrix} \text{ kN} \qquad M_D = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} + A \times \begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix} + E \times \begin{pmatrix} 25 \\ 0 \\ 0 \end{pmatrix} + G \times \begin{pmatrix} 0 \\ 30 \\ 0 \end{pmatrix} = \begin{pmatrix} -140 \\ 75 \\ 0 \end{pmatrix} \text{ kN}\cdot\text{m}$$

Moments about and perpendicular to line DF

$$n_{DF} = \frac{F}{|F|} = \begin{pmatrix} 0.566 \\ 0.707 \\ 0.424 \end{pmatrix} \qquad M_{DFpar} = M_D \cdot n_{DF} \cdot n_{DF} = \begin{pmatrix} -14.8 \\ -18.5 \\ -11.1 \end{pmatrix} \text{ kN}\cdot\text{m} \qquad M_{DFper} = M_D - M_{DFpar} = \begin{pmatrix} -125.2 \\ 93.5 \\ 11.1 \end{pmatrix} \text{ kN}\cdot\text{m}$$

Wrench resultant (have negative wrench here)

$$n_R = \frac{R_D}{|R_D|} = \begin{pmatrix} 0.625 \\ 0.759 \\ -0.182 \end{pmatrix} \qquad M_{Rpar} = M_D \cdot n_R \cdot n_R = \begin{pmatrix} -19.15 \\ -23.24 \\ 5.57 \end{pmatrix} \text{ kN}\cdot\text{m} \qquad M_{Rper} = M_D - M_{Rpar} = \begin{pmatrix} -120.8 \\ 98.2 \\ -5.6 \end{pmatrix} \text{ kN}\cdot\text{m}$$

$$d = \frac{|M_{Rper}|}{|R_D|} = 2.84 \text{ m} \qquad \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \times R_D = M_{Rper} \text{ float}, 5 \rightarrow \begin{pmatrix} -10 \cdot y \\ 10 \cdot x \\ 41.713 \cdot x + -34.37 \cdot y \end{pmatrix} = \begin{pmatrix} -120.85 \\ 98.241 \\ -5.5717 \end{pmatrix}$$

$$x = 9.82 \text{ m} \qquad y = 12.08 \text{ m}$$

$$41.713 \cdot x + -34.37 \cdot y = -5.568$$

Answers

$$R_D = \begin{pmatrix} 34.4 \\ 41.7 \\ -10 \end{pmatrix} \text{ kN} \qquad M_D = \begin{pmatrix} -140 \\ 75 \\ 0 \end{pmatrix} \text{ kN}\cdot\text{m} \qquad M_{DFpar} = \begin{pmatrix} -14.8 \\ -18.5 \\ -11.1 \end{pmatrix} \text{ kN}\cdot\text{m}$$

$$M_{DFper} = \begin{pmatrix} -125.2 \\ 93.5 \\ 11.1 \end{pmatrix} \text{ kN}\cdot\text{m} \qquad x = 9.82 \text{ m} \qquad y = 12.08 \text{ m}$$

Problem 2.2-29

$$A = \begin{pmatrix} 1.25 \\ 0 \\ 0 \end{pmatrix} in \qquad B = \begin{pmatrix} 1.25 \\ 7.5 \\ 0 \end{pmatrix} in \qquad F = 25lb \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -25 \end{pmatrix} \cdot lb \qquad N = 15lb \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -15 \\ 0 \\ 0 \end{pmatrix} \cdot lb \qquad r_{OB} = B$$

<u>Part (a)</u>

$$R_O = F + N = \begin{pmatrix} -15 \\ 0 \\ -25 \end{pmatrix} \cdot lb \qquad M_O = r_{OB} \times F = \begin{pmatrix} -187.5 \\ 31.25 \\ 0 \end{pmatrix} \cdot in \cdot lb$$

<u>Part (b)</u>

$$n_R = \frac{R_O}{\left|R_O\right|} = \begin{pmatrix} -0.514 \\ 0 \\ -0.857 \end{pmatrix} \qquad M_{Rpar} = M_O \cdot n_R \cdot n_R = \begin{pmatrix} -49.6 \\ 0 \\ -82.7 \end{pmatrix} \cdot in \cdot lb \qquad M_{Rper} = M_O - M_{Rpar} = \begin{pmatrix} -137.868 \\ 31.25 \\ 82.721 \end{pmatrix} \cdot in \cdot lb$$

$$\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \times R_O = M_{Rper} \qquad \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \times R_O \text{ float}, 5 \rightarrow \begin{pmatrix} -25.0 \cdot lbf \cdot y \\ 25.0 \cdot lbf \cdot x \\ 15.0 \cdot lbf \cdot y \end{pmatrix} \qquad x = \frac{31.25}{25} = 1.25 \qquad y = \frac{-137.868}{-25} = 5.515$$

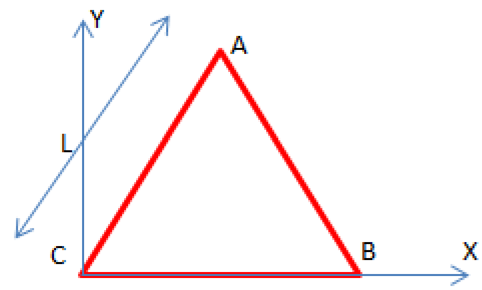
$$\frac{82.721}{15} = 5.515$$

$$d = \frac{\left|M_{Rper}\right|}{\left|R_O\right|} = 5.62 \cdot in$$

Problem 2.2-30

$F_1 = 150N$ $\quad F_2 = F_1$ $\quad F_3 = 75N$

ABC is an equilateral triangle



$R = F_1 + F_2 + F_3 = 375\,N$ $\quad R = R\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}$

Joint coordinates (assume that L = 1m) $\quad C = \begin{pmatrix}0\\0\\0\end{pmatrix}$ $\quad B = \begin{pmatrix}1\\0\\0\end{pmatrix}$ $\quad A = \begin{pmatrix}\cos(60deg)\\\sin(60deg)\\0\end{pmatrix} = \begin{pmatrix}0.5\\0.866\\0\end{pmatrix}$

$$M_C = B \times \begin{pmatrix}0\\0\\F_2\end{pmatrix} + A \times \begin{pmatrix}0\\0\\F_3\end{pmatrix} \qquad M_C \rightarrow \begin{pmatrix}64.951905283832895\cdot N\\-37.500000000000000825\cdot N - 150\cdot N\\0\end{pmatrix} = \begin{pmatrix}64.952\\-187.5\\0\end{pmatrix}N$$

$$\begin{pmatrix}x\\y\\0\end{pmatrix} \times R = M_C \left|\begin{matrix}solve, x, y\\float, 4\end{matrix}\right. \rightarrow (0.5 \quad 0.1732)$$

so resultant R passes through point with coordinates

$$x = \frac{L}{2} \qquad y = \frac{\sqrt{3}}{10}\cdot L$$

Problem 2.2-31

Joint coordinates

$$A = \begin{pmatrix} 0 \\ 0 \\ 12 \end{pmatrix} in \qquad B = \begin{pmatrix} 0 \\ 3.5 \\ 12 \end{pmatrix} in \qquad C = \begin{pmatrix} 3.5 \\ 3.5 \\ 12 \end{pmatrix} in \qquad r_{OA} = A \qquad r_{OC} = C$$

Part (a)

$$R_{Ox} = -10lb \qquad R_{Oy} = (7-5)lb = 2\cdot lb \qquad R_{Oz} = 0$$

$$M_{Ox} = -7lb\cdot(6in) + 5lb\cdot(12in) = 18\cdot in\cdot lb \qquad M_{Oy} = -10lb\cdot(12in) = -120\cdot in\cdot lb \qquad M_{Oz} = 14in\cdot lb - 5lb\cdot(3.5in) = -3.5\cdot in\cdot lb$$

Part (b)

$$R_O = \begin{pmatrix} -10 \\ 7-5 \\ 0 \end{pmatrix}\cdot lb = \begin{pmatrix} -10 \\ 2 \\ 0 \end{pmatrix}\cdot lb \qquad M_O = \left[\begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} in\right] \times \left[\begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} lb\right] + r_{OA} \times \left[\begin{pmatrix} -10 \\ 0 \\ 0 \end{pmatrix} lb\right] + r_{OC} \times \begin{pmatrix} 0 \\ -5 \\ 0 \end{pmatrix} lb + \begin{pmatrix} 0 \\ 0 \\ 14 \end{pmatrix} in\cdot lb = \begin{pmatrix} 18 \\ -120 \\ -3.5 \end{pmatrix}\cdot in\cdot lb$$

Answers

$$R_O = \begin{pmatrix} -10 \\ 2 \\ 0 \end{pmatrix}\cdot lb \qquad M_O = \begin{pmatrix} 18 \\ -120 \\ -3.5 \end{pmatrix}\cdot in\cdot lb$$

Problem 2.2-32

$$R_O = \begin{bmatrix} 0 \\ -2\cdot(300N) \\ 2\cdot(50N) - 2\cdot(50N) \end{bmatrix} = \begin{pmatrix} 0 \\ -600 \\ 0 \end{pmatrix} N$$

$$M_{Ox} = 4\cdot(50N)\cdot(1.5m) - 2\cdot(800N\cdot m) = -1300\cdot N\cdot m \qquad M_{Oy} = 0 \qquad M_{Oz} = 0 \qquad M_O = \begin{pmatrix} M_{Ox} \\ M_{Oy} \\ M_{Oz} \end{pmatrix} = \begin{pmatrix} -1300 \\ 0 \\ 0 \end{pmatrix} \cdot N\cdot m$$

Problem 2.2-33

Joint coordinates

$$C = \begin{pmatrix} 12 \\ 4 \\ 4 \end{pmatrix} ft \qquad D = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} ft$$

Unit vector

$$n_{CD} = \frac{D - C}{|D - C|} = \begin{pmatrix} -0.946 \\ -0.079 \\ -0.315 \end{pmatrix}$$

Force F

$$F_{CD} = (25kip) \cdot n_{CD} = \begin{pmatrix} -23.64 \\ -1.97 \\ -7.88 \end{pmatrix} \cdot kip$$

Part (a)

$$R_O = F_{CD} = \begin{pmatrix} -23.64 \\ -1.97 \\ -7.88 \end{pmatrix} \cdot kip \qquad M_x = (100 \cdot in \cdot kip) \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad r_{OD} = D$$

$$M_O = M_x + r_{OD} \times F_{CD} = \begin{pmatrix} -183.7 \\ 0 \\ 851.2 \end{pmatrix} \cdot in \cdot kip$$

Part (b) - select x and z components of M_O

$$M_{OApar} = M_O \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -183.7 \\ 0 \\ 0 \end{pmatrix} \cdot in \cdot kip \qquad M_{OAper} = M_O - M_{OApar} = \begin{pmatrix} 0 \\ 0 \\ 851 \end{pmatrix} \cdot in \cdot kip$$

Problem 2.2-34

Joint coordinates

$$F = \begin{pmatrix} 1.125 \\ 0 \\ 1.875 \end{pmatrix} m \quad C = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} m \quad E = \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} m \quad A = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} m \quad B = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} m \quad D = \begin{pmatrix} 3 \\ 0 \\ 5 \end{pmatrix} m$$

$$n_{CE} = \frac{E - C}{|E - C|} = \begin{pmatrix} 0.514 \\ 0 \\ 0.857 \end{pmatrix} \qquad n_{FE} = \frac{E - F}{|E - F|} = \begin{pmatrix} 0.397 \\ 0.636 \\ 0.662 \end{pmatrix}$$

$$P = (3kN) \cdot n_{CE} = \begin{pmatrix} 1.543 \\ 0 \\ 2.572 \end{pmatrix} \cdot kN \qquad Q = (4kN) \cdot n_{FE} = \begin{pmatrix} 1.589 \\ 2.542 \\ 2.648 \end{pmatrix} \cdot kN$$

Part (a)

$$R_O = P + Q = \begin{pmatrix} 3.13 \\ 2.54 \\ 5.22 \end{pmatrix} \cdot kN \qquad r_{OE} = E \qquad M_O = r_{OE} \times (P + Q) = \begin{pmatrix} 2.95 \\ 0 \\ -1.77 \end{pmatrix} \cdot kN \cdot m$$

Part (b)

$$r_{AE} = E - A = \begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix} m \qquad M_A = r_{AE} \times (P + Q) = \begin{pmatrix} 2.95 \\ 15.66 \\ -9.4 \end{pmatrix} \cdot kN \cdot m$$

$$n_{AD} = \frac{D - A}{|D - A|} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad M_{ADpar} = M_A \cdot n_{AD} \cdot n_{AD} = \begin{pmatrix} 0 \\ 0 \\ -9.4 \end{pmatrix} \cdot kN \cdot m \qquad M_{ADper} = M_A - M_{ADpar} = \begin{pmatrix} 2.95 \\ 15.66 \\ 0 \end{pmatrix} \cdot kN \cdot m$$

Answers

a) $R_O = \begin{pmatrix} 3.13 \\ 2.54 \\ 5.22 \end{pmatrix} \cdot kN \qquad M_O = \begin{pmatrix} 2.95 \\ 0 \\ -1.77 \end{pmatrix} \cdot kN \cdot m$

b) $M_{ADpar} = \begin{pmatrix} 0 \\ 0 \\ -9.4 \end{pmatrix} \cdot kN \cdot m \qquad M_{ADper} = \begin{pmatrix} 2.95 \\ 15.66 \\ 0 \end{pmatrix} \cdot kN \cdot m$

Problem 2.2-35

$H = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \cdot in$ $B = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} in$ $C = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \cdot in$ $G = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} \cdot in$ $A = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix} \cdot in$ $P = 10kip$

$n_{HB} = \frac{B - H}{|B - H|} = \begin{pmatrix} 0.566 \\ 0.707 \\ -0.424 \end{pmatrix}$ $n_{DG} = \frac{G}{|G|} = \begin{pmatrix} 0.8 \\ 0 \\ 0.6 \end{pmatrix}$ $r_{DH} = H$ $r_{DB} = B$

Part (a)

$P = P \cdot n_{HB} = \begin{pmatrix} 5.657 \\ 7.071 \\ -4.243 \end{pmatrix} \cdot kip$ $R_D = P = \begin{pmatrix} 5.66 \\ 7.07 \\ -4.24 \end{pmatrix} \cdot kip$ $M_D = r_{DH} \times P = \begin{pmatrix} -21.21 \\ 16.97 \\ 0 \end{pmatrix} \cdot kip \cdot in$ < no z component since P acts through z axis

and $r_{DB} \times P = \begin{pmatrix} -21.21 \\ 16.97 \\ 0 \end{pmatrix} \cdot kip \cdot in$

Part (b)

$M_{DGpar} = M_D \cdot n_{DG} \cdot n_{DG} = \begin{pmatrix} -13.58 \\ 0 \\ -10.18 \end{pmatrix} \cdot kip \cdot in$ $|M_{DGpar}| = 16.97 \cdot kip \cdot in$ $M_{DGper} = M_D - M_{DGpar} = \begin{pmatrix} -7.64 \\ 16.97 \\ 10.18 \end{pmatrix} \cdot kip \cdot in$

Part (c)

$r_{AB} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \cdot in$ $M_A = r_{AB} \times P = \begin{pmatrix} 0 \\ 16.971 \\ 28.284 \end{pmatrix} \cdot kip \cdot in$ $d = \frac{|M_A|}{|P|} = 3.3 \cdot in$

Answers

a) $R_D = \begin{pmatrix} 5.66 \\ 7.07 \\ -4.24 \end{pmatrix} \cdot kip$ $M_D = \begin{pmatrix} -21.21 \\ 16.97 \\ 0 \end{pmatrix} \cdot kip \cdot in$

b) $M_{DGpar} = \begin{pmatrix} -13.58 \\ 0 \\ -10.18 \end{pmatrix} \cdot kip \cdot in$ $M_{DGper} = \begin{pmatrix} -7.64 \\ 16.97 \\ 10.18 \end{pmatrix} \cdot kip \cdot in$ c) $d = 3.3 \cdot in$

Problem 2.2-36

$T_{CE} = 50kN$ $\quad a = 10m$ $\quad b = 4m$ $\quad c = 3m$

Position and unit vectors

$$r_{CE} = \begin{pmatrix} \frac{-a}{2} \\ c \\ -b \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \\ -4 \end{pmatrix} \cdot m \qquad n_{CE} = \frac{r_{CE}}{|r_{CE}|} = \begin{pmatrix} -0.707 \\ 0.424 \\ -0.566 \end{pmatrix} \qquad r_{CA} = \begin{pmatrix} 0 \\ 0 \\ 0 - b \end{pmatrix} \qquad n_{CA} = \frac{r_{CA}}{|r_{CA}|} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$T_{CE} = T_{CE} \cdot n_{CE} = \begin{pmatrix} -35.4 \\ 21.2 \\ -28.3 \end{pmatrix} \cdot kN \qquad |T_{CE}| = 50 \cdot kN$$

Part (a)

$$R_A = T_{CE} = \begin{pmatrix} -35.4 \\ 21.2 \\ -28.3 \end{pmatrix} \cdot kN \qquad r_{AC} = \begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \qquad M_A = r_{AC} \times T_{CE} = \begin{pmatrix} -84.9 \\ -141.4 \\ 0 \end{pmatrix} \cdot kN \cdot m$$

Part (b)

$$D = \begin{pmatrix} \frac{a}{2} \\ 0 \\ b \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix} m \qquad n_{AD} = \frac{D}{|D|} = \begin{pmatrix} 0.781 \\ 0 \\ 0.625 \end{pmatrix}$$

$$M_{ADpar} = M_A \cdot n_{AD} \cdot n_{AD} = \begin{pmatrix} -51.7 \\ 0 \\ -41.4 \end{pmatrix} \cdot kN \cdot m \qquad M_{ADper} = M_A - M_{ADpar} = \begin{pmatrix} -33.1 \\ -141.4 \\ 41.4 \end{pmatrix} \cdot kN \cdot m$$

$$|M_{ADpar}| = 66.3 \cdot kN \cdot m$$

Answers

a) $R_A = \begin{pmatrix} -35.4 \\ 21.2 \\ -28.3 \end{pmatrix} \cdot kN \qquad M_A = \begin{pmatrix} -84.9 \\ -141.4 \\ 0 \end{pmatrix} \cdot kN \cdot m$

b) $M_{ADpar} = \begin{pmatrix} -51.7 \\ 0 \\ -41.4 \end{pmatrix} \cdot kN \cdot m \qquad M_{ADper} = \begin{pmatrix} -33.1 \\ -141.4 \\ 41.4 \end{pmatrix} \cdot kN \cdot m$

Problem 2.2-37

Solution

$$E = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \quad F = \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix} \quad D = \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} \quad C = \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix} \quad A = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \quad G = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$n_{DB} = \frac{B - D}{|B - D|} = \begin{pmatrix} -0.728 \\ 0.485 \\ -0.485 \end{pmatrix} \quad n_{DC} = \frac{C - D}{|C - D|} = \begin{pmatrix} -0.728 \\ 0.485 \\ 0.485 \end{pmatrix} \quad n_{DE} = \frac{E - D}{|E - D|} = \begin{pmatrix} 0 \\ -0.707 \\ -0.707 \end{pmatrix}$$

$$n_{DF} = \frac{F - D}{|F - D|} = \begin{pmatrix} 0 \\ -0.707 \\ 0.707 \end{pmatrix} \quad n_{AG} = \frac{G - A}{|G - A|} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$T_{DB} = 10\text{kip} \quad T_{DC} = 10\text{kip} \quad T_{DE} = 15\text{kip} \quad T_{DF} = 15\text{kip}$$

Part (a)

$$T_{DB} = T_{DB} \cdot n_{DB} = \begin{pmatrix} -7.28 \\ 4.85 \\ -4.85 \end{pmatrix} \cdot \text{kip} \quad T_{DC} = T_{DC} \cdot n_{DC} = \begin{pmatrix} -7.28 \\ 4.85 \\ 4.85 \end{pmatrix} \cdot \text{kip}$$

$$T_{DE} = T_{DE} \cdot n_{DE} = \begin{pmatrix} 0 \\ -10.61 \\ -10.61 \end{pmatrix} \cdot \text{kip} \quad T_{DF} = T_{DF} \cdot n_{DF} = \begin{pmatrix} 0 \\ -10.61 \\ 10.61 \end{pmatrix} \cdot \text{kip}$$

$$R_A = T_{DB} + T_{DC} + T_{DE} + T_{DF} = \begin{pmatrix} -14.55 \\ -11.51 \\ 0 \end{pmatrix} \cdot \text{kip} \quad |R_A| = 18.6 \cdot \text{kip}$$

$$r_{AD} = D - A = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \text{ft} \quad M_A = r_{AD} \times R_A = \begin{pmatrix} 0 \\ 0 \\ -5.43 \end{pmatrix} \text{kip} \cdot \text{ft} \quad |M_A| = 5.43 \cdot \text{kip} \cdot \text{ft}$$

Part (b)

$M_A \cdot n_{AG} = 0 \cdot$

$T_{DB} = T_{DC}$ and $T_{DE} = T_{DF}$ so **symmetry** of loading (with respect to a vertical plane through ADG) means there is no moment component about AG axis

Problem 2.2-38

$F_s = 154N \quad h = 660mm \quad d = 150mm \quad H = 1041mm \quad c = 506mm \quad a = 760mm \quad h_o = 490mm \quad b = 254mm$

$mass = 20kg \quad g = 9.807\frac{m}{s^2}$

Joint coordinates

$$A = \begin{pmatrix} h \\ h-H \\ d \end{pmatrix} = \begin{pmatrix} 660 \\ -381 \\ 150 \end{pmatrix}\cdot mm \quad B = \begin{pmatrix} h \\ h \\ c \end{pmatrix} = \begin{pmatrix} 660 \\ 660 \\ 506 \end{pmatrix}\cdot mm \quad C = \begin{pmatrix} 0 \\ 0 \\ a+b+c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1520 \end{pmatrix}\cdot mm \quad D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} mm \quad O = \begin{pmatrix} h_o \\ \text{unknown} \\ b+c \end{pmatrix}$$

Unit vectors

$$n_{AB} = \frac{B-A}{|B-A|} = \begin{pmatrix} 0 \\ 0.946 \\ 0.324 \end{pmatrix} \quad n_{CD} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad n_{AC} = \frac{C-A}{|C-A|} = \begin{pmatrix} -0.421 \\ 0.243 \\ 0.874 \end{pmatrix}$$

Part (a)

$$W = mass\cdot g\cdot \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -196.13 \\ 0 \end{pmatrix} N \quad F_s = F_s \cdot n_{AB} = \begin{pmatrix} 0 \\ 145.71 \\ 49.83 \end{pmatrix} N \quad R_A = W + F_s = \begin{pmatrix} 0 \\ -50.4 \\ 49.8 \end{pmatrix} N \quad |R_A| = 70.9\,N$$

$$M_{Ax} = mass\cdot g\cdot (b+c-d) = 119.6\cdot N\cdot m \quad M_{Ay} = 0 \quad M_{Az} = -mass\cdot g\cdot (h-h_o) = -33.3\cdot N\cdot m$$

$$M_A = \begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = \begin{pmatrix} 119.6 \\ 0 \\ -33.3 \end{pmatrix}\cdot N\cdot m$$

Part (b)

$$r_{CA} = A - C = \begin{pmatrix} 660 \\ -381 \\ -1370 \end{pmatrix}\cdot mm \quad M_C = r_{CA} \times F_s + \begin{bmatrix} -mass\cdot g\cdot (a) \\ 0 \\ -mass\cdot g\cdot (h_o) \end{bmatrix} = \begin{pmatrix} 31.58 \\ -32.89 \\ 0.07 \end{pmatrix}\cdot N\cdot m \quad n_{CD} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$M_{CD} = M_C \cdot n_{CD} = -0.067\cdot N\cdot m$$

Problem 2.2-39

$P = 4500lb \qquad W = 975lb$

$$D = \begin{pmatrix} 0 \\ 27 \\ 27\cdot\tan(55deg) \end{pmatrix}\cdot ft = \begin{pmatrix} 0 \\ 27 \\ 38.56 \end{pmatrix} ft \qquad E = \begin{pmatrix} 9 \\ 39 \\ 0 \end{pmatrix} ft \qquad C = \begin{pmatrix} 0 \\ 15 \\ 15\cdot\tan(55deg) \end{pmatrix} ft = \begin{pmatrix} 0 \\ 15 \\ 21.422 \end{pmatrix} ft \qquad r_{OC} = C$$

$$n_{DE} = \frac{E - D}{|E - D|} = \begin{pmatrix} 0.218 \\ 0.29 \\ -0.932 \end{pmatrix} \qquad P = P\cdot n_{DE} = \begin{pmatrix} 979 \\ 1305 \\ -4194 \end{pmatrix}\cdot lb \qquad W = W\cdot\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -975 \end{pmatrix}\cdot lb \qquad r_{OE} = E$$

Resultants

$$R_O = P + W = \begin{pmatrix} 979 \\ 1305 \\ -5169 \end{pmatrix}\cdot lb \qquad M_O = r_{OC} \times W + r_{OE} \times P = \begin{pmatrix} -178185 \\ 37745 \\ -26429 \end{pmatrix}\cdot ft\cdot lb \qquad M_O = \begin{pmatrix} -178.2 \\ 37.7 \\ -26.4 \end{pmatrix}\cdot ft\cdot kip$$

$$|R_O| = 5420\cdot lb \qquad |M_O| = 184\cdot ft\cdot kip$$