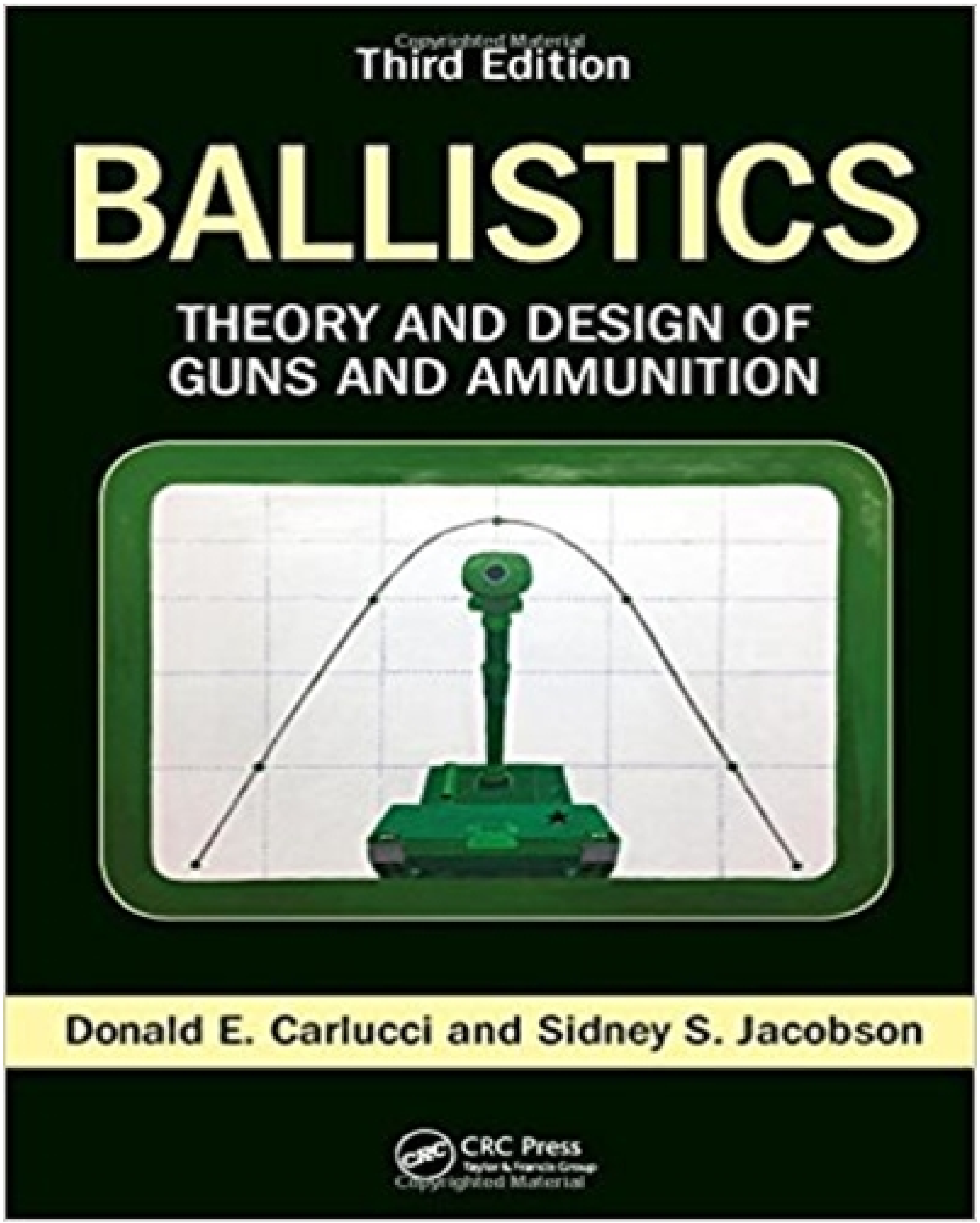
Solutions for Ballistics Theory and Design of Guns and Ammunition 3rd Edition by Carlucci

CLICK HERE TO ACCESS COMPLETE Solutions



Solutions

2.1 The Ideal Gas Law

Problem 1 - Assume we have a quantity of 10 grams of 11.1% nitrated nitrocellulose ($C_6H_8N_2O_9$) and it is heated to a temperature of 1000K and changes to gas somehow without changing chemical composition. If the process takes place in an expulsion cup with a volume of 10 in^3, assuming ideal gas behavior, what will the final pressure be in psi?

Answer $p = 292\left[\frac{\text{lbf}}{\text{in}^2}\right]$

Solution:

This problem is fairly straight-forward except for the units. We shall write our ideal gas law and let the units fall out directly. The easiest form to start with is equation (IG-4)

$$p\text{V} = m_g RT \qquad \text{(IG-4)}$$

Rearranging, we have

$$p = \frac{m_g RT}{\text{V}}$$

Here we go

$$p = \frac{(10)[\text{g}]\left(\frac{1}{1000}\right)\left[\frac{\text{kg}}{\text{g}}\right](8.314)\left[\frac{\text{kJ}}{\text{kgmol}\cdot\text{K}}\right]\left(\frac{1}{252}\right)\left[\frac{\text{kgmol}}{\text{kg}_{\text{C}_6\text{H}_8\text{N}_2\text{O}_9}}\right](737.6)\left[\frac{\text{ft}-\text{lbf}}{\text{kJ}}\right](12)\left[\frac{\text{in}}{\text{ft}}\right](1000)[\text{K}]}{(10)\left[\text{in}^3\right]}$$

$$p = 292\left[\frac{\text{lbf}}{\text{in}^2}\right]$$

You will notice that the units are all screwy – but that's half the battle when working these problems! Please note that this result is unlikely to happen. If the chemical composition were reacted we would have to balance the reaction equation and would have to use Dalton's law for the partial pressures of the gases as follows. First, assuming no air in the vessel we write the decomposition reaction.

$$\text{C}_6\text{H}_8\text{N}_2\text{O}_9 \rightarrow 4\text{H}_2\text{O} + 5\text{CO} + \text{N}_2 + \text{C(s)}$$

Then for each constituent (we ignore solid carbon) we have

$$p_i = \frac{N_i \Re T}{\text{V}}$$

So we can write

$$p_{H_2O} = \frac{(4)\left[\frac{\text{kgmol}_{H_2O}}{\text{kgmol}_{C_6H_8N_2O_9}}\right](8.314)\left[\frac{\text{kJ}}{\text{kgmol - K}}\right](1000)[\text{K}]\left(\frac{1}{252}\right)\left[\frac{\text{kgmol}_{C_6H_8N_2O_9}}{\text{kg}_{C_6H_8N_2O_9}}\right](10)\left[\text{g}_{C_6H_8N_2O_9}\right]\left(\frac{1}{1,000}\right)\left[\frac{\text{kg}_{C_6H_8N_2O_9}}{\text{g}_{C_6H_8N_2O_9}}\right]}{(10)\left[\text{in}^3\right]\left(\frac{1}{737.6}\right)\left[\frac{\text{kJ}}{\text{ft} - \text{lbf}}\right]\left(\frac{1}{12}\right)\left[\frac{\text{ft}}{\text{in}}\right]}$$

$$p_{H_2O} = 1,168\left[\frac{\text{lbf}}{\text{in}^2}\right]$$

$$p_{CO} = \frac{(5)\left[\frac{\text{kgmol}_{CO}}{\text{kgmol}_{C_6H_8N_2O_9}}\right](8.314)\left[\frac{\text{kJ}}{\text{kgmol - K}}\right](1000)[\text{K}]\left(\frac{1}{252}\right)\left[\frac{\text{kgmol}_{C_6H_8N_2O_9}}{\text{kg}_{C_6H_8N_2O_9}}\right](10)\left[\text{g}_{C_6H_8N_2O_9}\right]\left(\frac{1}{1,000}\right)\left[\frac{\text{kg}_{C_6H_8N_2O_9}}{\text{g}_{C_6H_8N_2O_9}}\right]}{(10)\left[\text{in}^3\right]\left(\frac{1}{737.6}\right)\left[\frac{\text{kJ}}{\text{ft} - \text{lbf}}\right]\left(\frac{1}{12}\right)\left[\frac{\text{ft}}{\text{in}}\right]}$$

$$p_{CO} = 1,460\left[\frac{\text{lbf}}{\text{in}^2}\right]$$

$$p_{N_2} = \frac{(1)\left[\frac{\text{kgmol}_{N_2}}{\text{kgmol}_{C_6H_8N_2O_9}}\right](8.314)\left[\frac{\text{kJ}}{\text{kgmol - K}}\right](1000)[\text{K}]\left(\frac{1}{252}\right)\left[\frac{\text{kgmol}_{C_6H_8N_2O_9}}{\text{kg}_{C_6H_8N_2O_9}}\right](10)\left[\text{g}_{C_6H_8N_2O_9}\right]\left(\frac{1}{1,000}\right)\left[\frac{\text{kg}_{C_6H_8N_2O_9}}{\text{g}_{C_6H_8N_2O_9}}\right]}{(10)\left[\text{in}^3\right]\left(\frac{1}{737.6}\right)\left[\frac{\text{kJ}}{\text{ft} - \text{lbf}}\right]\left(\frac{1}{12}\right)\left[\frac{\text{ft}}{\text{in}}\right]}$$

$$p_{N_2} = 292\left[\frac{\text{lbf}}{\text{in}^2}\right]$$

Then the total pressure is

$$p = p_{H_2O} + p_{CO} + p_{N_2}$$

$$p = 1,168\left[\frac{\text{lbf}}{\text{in}^2}\right] + 1,460\left[\frac{\text{lbf}}{\text{in}^2}\right] + 292\left[\frac{\text{lbf}}{\text{in}^2}\right] = 2,920\left[\frac{\text{lbf}}{\text{in}^2}\right]$$

2.2 Other Gas Laws

Problem 2 - Perform the same calculation as in problem 1 but use the Noble-Abel equation of state and assume the covolume to be 32.0 in^3/lbm

Answer: $p = 314.2\left[\frac{\text{lbf}}{\text{in}^2}\right]$

Solution:

This problem is again straight-forward except for those pesky units – but we've done this before. We start with equation (VW-2)

$$p(\mathrm{V} - cb) = m_g RT \qquad \text{(VW-2)}$$

Rearranging, we have

$$p = \frac{m_g RT}{\mathrm{V} - cb}$$

Here we go

$$p = \frac{(10)[\mathrm{g}]\left(\frac{1}{1000}\right)\left[\frac{\mathrm{kg}}{\mathrm{g}}\right](8.314)\left[\frac{\mathrm{kJ}}{\mathrm{kgmol}\cdot\mathrm{K}}\right]\left(\frac{1}{252}\right)\left[\frac{\mathrm{kgmol}}{\mathrm{kg}_{\mathrm{C_6H_8N_2O_9}}}\right](737.6)\left[\frac{\mathrm{ft-lbf}}{\mathrm{kJ}}\right](12)\left[\frac{\mathrm{in}}{\mathrm{ft}}\right](1000)[\mathrm{K}]}{(10)[\mathrm{in}^3] - \left\{(10)[\mathrm{g}]\left(\frac{1}{1000}\right)\left[\frac{\mathrm{kg}}{\mathrm{g}}\right](2.2)\left[\frac{\mathrm{lbm}}{\mathrm{kg}}\right](32.0)\left[\frac{\mathrm{in}^3}{\mathrm{lbm}}\right]\right\}}$$

$$p = 314.2\left[\frac{\mathrm{lbf}}{\mathrm{in}^2}\right]$$

So you can see that the real gas behavior is somewhat different than ideal gas behavior at this low pressure – it makes more of a difference at the greater pressures.

Again please note that this result is unlikely to happen. If the chemical composition were reacted we would have to balance the reaction equation and would again have to use Dalton's law for the partial pressures of the gases. Again, assuming no air in the vessel we write the decomposition reaction.

$$\mathrm{C_6H_8N_2O_9} \rightarrow 4\mathrm{H_2O} + 5\mathrm{CO} + \mathrm{N_2} + \mathrm{C(s)}$$

Then for each constituent (again ignoring solid carbon) we have

$$p_i = \frac{N_i \Re T}{(\mathrm{V} - cb)}$$

So we can write

$$p_{H_2O} = \frac{(4)\left[\frac{kgmol_{H_2O}}{kgmol_{C_6H_8N_2O_9}}\right](8.314)\left[\frac{kJ}{kgmol - K}\right](1000)[K]\left(\frac{1}{252}\right)\left[\frac{kgmol_{C_6H_8N_2O_9}}{kg_{C_6H_8N_2O_9}}\right](10)\left[g_{C_6H_8N_2O_9}\right]\left(\frac{1}{1{,}000}\right)\left[\frac{kg_{C_6H_8N_2O_9}}{g_{C_6H_8N_2O_9}}\right]}{\left((10)\left[in^3\right]-\left\{(10)[g]\left(\frac{1}{1000}\right)\left[\frac{kg}{g}\right](2.2)\left[\frac{lbm}{kg}\right](32.0)\left[\frac{in^3}{lbm}\right]\right\}\right)\left(\frac{1}{737.6}\right)\left[\frac{kJ}{ft-lbf}\right]\left(\frac{1}{12}\right)\left[\frac{ft}{in}\right]}$$

$$p_{H_2O} = 1{,}257\left[\frac{lbf}{in^2}\right]$$

$$p_{CO} = \frac{(5)\left[\frac{kgmol_{CO}}{kgmol_{C_6H_8N_2O_9}}\right](8.314)\left[\frac{kJ}{kgmol - K}\right](1000)[K]\left(\frac{1}{252}\right)\left[\frac{kgmol_{C_6H_8N_2O_9}}{kg_{C_6H_8N_2O_9}}\right](10)\left[g_{C_6H_8N_2O_9}\right]\left(\frac{1}{1{,}000}\right)\left[\frac{kg_{C_6H_8N_2O_9}}{g_{C_6H_8N_2O_9}}\right]}{\left((10)\left[in^3\right]-\left\{(10)[g]\left(\frac{1}{1000}\right)\left[\frac{kg}{g}\right](2.2)\left[\frac{lbm}{kg}\right](32.0)\left[\frac{in^3}{lbm}\right]\right\}\right)\left(\frac{1}{737.6}\right)\left[\frac{kJ}{ft-lbf}\right]\left(\frac{1}{12}\right)\left[\frac{ft}{in}\right]}$$

$$p_{CO} = 1{,}571\left[\frac{lbf}{in^2}\right]$$

$$p_{N_2} = \frac{(1)\left[\frac{kgmol_{N_2}}{kgmol_{C_6H_8N_2O_9}}\right](8.314)\left[\frac{kJ}{kgmol - K}\right](1000)[K]\left(\frac{1}{252}\right)\left[\frac{kgmol_{C_6H_8N_2O_9}}{kg_{C_6H_8N_2O_9}}\right](10)\left[g_{C_6H_8N_2O_9}\right]\left(\frac{1}{1{,}000}\right)\left[\frac{kg_{C_6H_8N_2O_9}}{g_{C_6H_8N_2O_9}}\right]}{\left((10)\left[in^3\right]-\left\{(10)[g]\left(\frac{1}{1000}\right)\left[\frac{kg}{g}\right](2.2)\left[\frac{lbm}{kg}\right](32.0)\left[\frac{in^3}{lbm}\right]\right\}\right)\left(\frac{1}{737.6}\right)\left[\frac{kJ}{ft-lbf}\right]\left(\frac{1}{12}\right)\left[\frac{ft}{in}\right]}$$

$$p_{N_2} = 314\left[\frac{lbf}{in^2}\right]$$

Then the total pressure is

$$p = p_{H_2O} + p_{CO} + p_{N_2}$$

$$p = 1{,}257\left[\frac{lbf}{in^2}\right] + 1{,}571\left[\frac{lbf}{in^2}\right] + 314\left[\frac{lbf}{in^2}\right] = 3{,}142\left[\frac{lbf}{in^2}\right]$$

Problem 3 – A hypothetical "air mortar" is to be made out of a tennis ball can using a tennis ball as the projectile. The can has a 2-1/2" inside diameter and is 8" long. If a tennis ball of the same diameter weighs 2 oz. and initially rests against the rear of the can, to what air pressure must one pressurize the can to in order to achieve a 30 ft/s launch of the tennis ball? Assume that the tennis ball can be held against this pressure until released, that it perfectly obturates and also assume an isentropic process and ideal gas behavior with $\gamma = 1.4$ for air.

Solution: First we need to get some parameters in order. We start with a chamber volume. Since the tennis ball rests against the bottom of the can, the chamber volume is the volume of a cylinder of 2-1/2" diameter by 1-1/4" length minus the volume of half a sphere of 2-1/2" diameter.

The empty chamber has a volume of

$$\mathrm{U} = \frac{\pi}{4}(2.5)^2\left[\mathrm{in}^2\right](1.25)[\mathrm{in}] = 6.136\left[\mathrm{in}^3\right]$$

$$\mathrm{V}_{\mathrm{sphere}} = \frac{\pi}{6}d^3 = \frac{\pi}{6}(2.5)^3\left[\mathrm{in}^3\right] = 8.181\left[\mathrm{in}^3\right]$$

$$\mathrm{V}_0 = \mathrm{U} - \frac{\mathrm{V}_{\mathrm{sphere}}}{2}$$

$$\mathrm{V}_0 = (6.136)\left[\mathrm{in}^3\right] - \frac{(8.181)}{2}\left[\mathrm{in}^3\right] = 2.045\left[\mathrm{in}^3\right]$$

There are several ways to solve this problem – the simplest one is to follow the method I described in the notes. We can determine our fake chamber length through

$$\mathrm{V}_0 = Al$$

$$l = \frac{\mathrm{V}_0}{A}$$

$$l = \frac{(2.045)\left[\mathrm{in}^3\right]}{\frac{\pi}{4}(2.5)^2\left[\mathrm{in}^2\right]} = 0.417[\mathrm{in}]$$

We shall assume muzzle exit of the projectile occurs as the equator of the tennis ball passes through the plane of the end of the can. Then the travel of the projectile is the length of the can less half the diameter of the tennis ball

$$L = (8)[\mathrm{in}] - \frac{(2.5)}{2}[\mathrm{in}] = 6.75[\mathrm{in}]$$

The formula for muzzle velocity given an isentropic expansion of air was provided in the notes as equation (IG-28)

$$V_m = \sqrt{\frac{2m_g RT_i l^{(\gamma-1)}}{m_p(1-\gamma)}\left[(l+L)^{(1-\gamma)} - l^{(1-\gamma)}\right]} \qquad \text{(IG-28)}$$

If we assume ideal gad behavior we recognize that the term $m_g RT_i$ is the initial pressure times the initial chamber volume thus we can write

$$p_i \mathrm{V}_0 = m_g RT_i$$

Making this substitution we have

$$V_m = \sqrt{\frac{2 p_i \mathrm{V}_0 l^{(\gamma-1)}}{m_p(1-\gamma)}\left[(l+L)^{(1-\gamma)} - l^{(1-\gamma)}\right]}$$

Rearranging and solving for the pressure allows us to write

$$p_i = \frac{V_m^2}{2\mathrm{V}_0 l^{(\gamma-1)}} \frac{m_p(1-\gamma)}{\left[(l+L)^{(1-\gamma)} - l^{(1-\gamma)}\right]}$$

We now have a direct substitution with the data provided and calculated.

$$p_i = \frac{(30)^2\left[\frac{\mathrm{ft}^2}{\mathrm{s}^2}\right](12)\left[\frac{\mathrm{in}}{\mathrm{ft}}\right]}{(2)(2.045)\left[\mathrm{in}^3\right](0.417)^{(1.4-1)}\left[\mathrm{in}^{0.4}\right]} \frac{(2)[\mathrm{oz}]\left(\frac{1}{16}\right)\left[\frac{\mathrm{lbm}}{\mathrm{oz}}\right](1-1.4)}{\left[(0.417+6.75)^{(1-1.4)} - (0.417)^{(1-1.4)}\right]\left[\mathrm{in}^{-0.4}\right](32.2)\left[\frac{\mathrm{lbm-ft}}{\mathrm{lbf-s}^2}\right]}$$

$$p_i = 6.033\left[\frac{\mathrm{lbf}}{\mathrm{in}^2}\right]$$

2.4 Thermodynamics

Problem 4 - The M898 SADARM projectile weighs 102.5 lb. The projectile was fired from a 56 caliber, 155mm weapon and a pressure-time trace was obtained. The area under the pressure-time curve was (after converting the time to distance) calculated to be 231,482 psi-m. Calculate the muzzle energy of the projectile in MegaJoules. Assume the bore area to be 29.83 in^2.

Answer $E = 30.7[\mathrm{MJ}]$

Solution:

As given in the problem statement

$W_p = 102.5 lb$

$L = 56 \cdot 0.155 m$

thus

$L = 8.68 \mathrm{m}$

and

$A = 29.83 in^2$

$g = 32.2 \frac{lb \cdot ft}{lbf \cdot s^2}$

If we were not given the average pressure we would now have to calculate it from the pressure-time trace, but since the individual who wrote the problem was such a nice guy and gave us the average pressure we shall use it directly to calculate the muzzle velocity.
We shall call the area under the curve A_{px}

$$A_{px} = 231482\, psi \cdot m$$

The average pressure is then the value of A_{px} divided by the length of travel

$$P := \frac{A_{px}}{L} \qquad P = 2.667 \times 10^{4}\, psi \qquad (1)$$

The muzzle velocity is then found from

$$V_m := \sqrt{\frac{2 \cdot g \cdot P \cdot A \cdot L}{W_p}} \qquad V_m = 1.15 \times 10^{3}\, \frac{\text{m}}{\text{s}} \qquad (2)$$

The muzzle energy is calculated from

$$E = \frac{W_p}{2 \cdot g} \cdot V_m^{\,2} \qquad E = 3.072 \times 10^{7}\, \text{J} \qquad (3)$$

To convert these units to Joules we used 1.356 Joules per foot pound force

Problem 5 - An 8” Mk. 14 Mod. 2 Navy cannon is used at NSWC Dahlgren, VA for “canister” firings. These firings are used to gun harden electronics which are carried in an 8” projectile. The projectile used weighs 260 lb. The measured muzzle velocity is around 2800 ft/s. Calculate the muzzle energy of the projectile in MegaJoules. Assume the bore area to be 51.53 in^2. The rifled length of the tube (distance of projectile travel) is 373.65 in.

Answer $E \approx 43[\text{MJ}]$

Solution:

As given in the problem statement

$$W_p = 260[\text{lbm}] \qquad L = 373.65[\text{in}]$$

and

$$g_c = 32.2\left[\frac{\text{lbm} - \text{ft}}{\text{lbf} - \text{s}^2}\right] \qquad A = 51.53[\text{in}^2]$$

The muzzle energy is calculated directly from

$$E = \frac{W_p}{2g_c} V_m^2$$

$$E = \frac{(260)[\text{lbm}]}{2(32.2)\left[\frac{\text{lbm}-\text{ft}}{\text{lbf}-\text{s}^2}\right]}(2800)^2\left[\frac{\text{ft}^2}{\text{s}^2}\right]$$

$$E = 31{,}652{,}000[\text{lbf}-\text{ft}](1.356)\left[\frac{\text{J}}{\text{ft}-\text{lbf}}\right] \approx 43[\text{MJ}]$$

2.5 Combustion

Problem 6 - Calculate the A-F ratio for the combustion of the following fuels. Calculate the ratio with both theoretical air and 10% excess air.

a.) Benzene – C_6H_6

b.) n-Butane – C_4H_{10}

c.) Ethyl Alcohol – C_2H_5OH

Answer a.) 13.24 and 14.56, b.) 15.42 and 12.5.96, c.) 8.98 and 9.88

Solution:

Benzene

$C_6H_6 + 7.50\ O_2 + (7.5)(3.76)N_2 \rightarrow 6CO_2 + 3H_2O + 28.20N_2$

1 Mole C_6H_6 = 78.11 lbm

Mass of Air = (7.5)(4.76)(28.97) = 1034.23 lbm

$$\text{A-F} = \frac{1034.23}{78.11} = 13.24$$

With 10% Excess Air:

$$\text{A-F} = \frac{(1034.23)(1.1)}{78.11} = 14.56$$

---

n – Butane

$C_4H_{10} + 6.50\ O_2 + (6.5)(3.76)N_2 \rightarrow 4CO_2 + 5H_2O + 24.44N_2$

1 Mole C_4H_{10} = 58.12 lbm

Mass of Air = (6.5)(4.76)(28.97) = 896.33 lbm

$$\text{A-F} = \frac{896.33}{58.12} = 15.42$$

With 10% Excess Air:

$$\text{A-F} = \frac{(896.33)(1.1)}{58.12} = 16.96$$

Ethyl Alchohol

$C_2H_5OH + 3O_2 + (3)(3.76)N_2 \rightarrow 2CO_2 + 3H_2O + 11.28N_2$

1 Mole C_2H_5OH = 46.07 lbm

Mass of Air = (3)(4.76)(28.97) = 413.69 lbm

$$\text{A-F} = \frac{413.69}{46.07} = 8.98$$

With 10% Excess Air:

$$\text{A-F} = \frac{(413.69)(1.1)}{46.07} = 9.88$$

Problem 7 – Let us examine a pressure vessel identical to the example problem in the text containing 0.001 kg of methane (CH_4) and 0.002 kg of air. The enthalpy of formation for the methane is -74,850 kJ/kgmol and its molecular weight is 16.04 kg/kgmol. The reaction will begin at 298 K and we shall remove enough heat from the vessel that the final temperature becomes 1,000 K.
a.) determine much heat is given off.
b.) compare the result in a.) above with the example problem in this chapter.

Answer a.) $Q = -0.386[\text{kJ}]$, b.) This situation removes 2.984 kJ more energy than the example.

Solution: We need to balance the chemical reaction on a molar basis so we shall determine how many moles of methane and air we have in the container. This is exactly the same procedure as in the chapter example problem. For the methane we have:

$$N_{CH_4} = \frac{(0.001)[kg_{CH_4}]}{(16.04)\left[\frac{kg}{kgmol}\right]} = 6.23\times10^{-5}[kgmol_{CH_4}]$$

For the air we have

$$N_{air} = \frac{(0.002)[kg_{air}]}{(28.97)\left[\frac{kg}{kgmol}\right]} = 6.90\times10^{-5}[kgmol_{air}]$$

Our balanced reaction is then

$$(6.23\times10^{-5})CH_4 + (6.90\times10^{-5})((0.21)O_2 + (0.79)N_2) \rightarrow$$
$$(2.9\times10^{-5})H_2O + (9.56\times10^{-5})H_2 + (6.23\times10^{-5})C(s) + (5.45\times10^{-5})N_2$$

We shall examine the reactants first. For the methane we have

$$N_{CH_4}\left(\bar{h}_f^0 + \Delta\bar{h}_{298\rightarrow T} - R_u T_{CH_4}\right) = (6.23\times10^{-5})[kgmol]\left(-74{,}850\left[\frac{kJ}{kgmol}\right] + 0 - (8.314)\left[\frac{kJ}{kgmol\cdot K}\right](298)[K]\right)$$
$$N_{CH_4}\left(\bar{h}_f^0 + \Delta\bar{h}_{298\rightarrow T} - R_u T_{CH_4}\right) = -4.82[kJ]$$

For the oxygen and nitrogen we have

$$N_{O_2}\left(\bar{h}_f^0 + \Delta\bar{h}_{298\rightarrow T} - R_u T_{O_2}\right) = (0.21)(6.90\times10^{-5})[kgmol]\left(0 + 0 - (8.314)\left[\frac{kJ}{kgmol\cdot K}\right](298)[K]\right)$$

$$N_{O_2}\left(\bar{h}_f^0 + \Delta\bar{h}_{298\rightarrow T} - R_u T_{O_2}\right) = -0.036[kJ]$$

$$N_{N_2}\left(\bar{h}_f^0 + \Delta\bar{h}_{298\rightarrow T} - R_u T_{N_2}\right) = (0.79)(6.90\times10^{-5})[kgmol]\left(0 + 0 - (8.314)\left[\frac{kJ}{kgmol\cdot K}\right](298)[K]\right)$$
$$N_{N_2}\left(\bar{h}_f^0 + \Delta\bar{h}_{298\rightarrow T} - R_u T_{N_2}\right) = -0.13[kJ]$$

The enthalpies of the reactants are the same as in the example problem therefore

$$\sum_i N_i\left(\bar{h}_{reac} - R_u T_{reac}\right) = -4.82[kJ] - 0.036[kJ] - 0.13[kJ] = -4.986[kJ]$$

For the products we have (using the tables in the appendix)

$$N_{H_2O}\left(\bar{h}_f^0 + \Delta\bar{h}_{298\rightarrow T} - R_u T_{H_2O}\right) = (2.9\times10^{-5})[kgmol]\left(-241{,}845 + 25{,}993 - (8.314)\left[\frac{kJ}{kgmol\cdot K}\right](1{,}000)[K]\right)$$

$$N_{H_2O}\left(\bar{h}_f^0 + \Delta\bar{h}_{298\to T} - R_u T_{H_2O}\right) = -6.50[\text{kJ}]$$

$$N_{H_2}\left(\bar{h}_f^0 + \Delta\bar{h}_{298\to T} - R_u T_{H_2O}\right) = \left(9.56\times10^{-5}\right)[\text{kgmol}]\left(0 + 20{,}664 - (8.314)\left[\frac{\text{kJ}}{\text{kgmol}\cdot\text{K}}\right](1{,}000)[\text{K}]\right)$$
$$N_{H_2}\left(\bar{h}_f^0 + \Delta\bar{h}_{298\to T} - R_u T_{H_2O}\right) = +1.18[\text{kJ}]$$

$$N_{N_2}\left(\bar{h}_f^0 + \Delta\bar{h}_{298\to T} - R_u T_{N_2}\right) = (0.79)\left(6.90\times10^{-5}\right)[\text{kgmol}]\left(0 + 21{,}468\left[\frac{\text{kJ}}{\text{kgmol}}\right] - (8.314)\left[\frac{\text{kJ}}{\text{kgmol}\cdot\text{K}}\right](1{,}000)[\text{K}]\right)$$
$$N_{N_2}\left(\bar{h}_f^0 + \Delta\bar{h}_{298\to T} - R_u T_{N_2}\right) = 0.72[\text{kJ}]$$

$$N_C\left(\bar{h}_f^0 + \Delta\bar{h}_{298\to T} - R_u T_C\right) = \left(6.23\times10^{-5}\right)[\text{kgmol}]\left(0 + 11{,}795\left[\frac{\text{kJ}}{\text{kgmol}}\right] - (8.314)\left[\frac{\text{kJ}}{\text{kgmol}\cdot\text{K}}\right](1{,}000)[\text{K}]\right)$$
$$N_C\left(\bar{h}_f^0 + \Delta\bar{h}_{298\to T} - R_u T_C\right) = +0.217[\text{kJ}]$$

The enthalpies of the products are then given by

$$\sum_i N_i\left(\bar{h}_{prod} - R_u T_{prod}\right) = -6.50[\text{kJ}] + 1.18[\text{kJ}] + 0.72[\text{kJ}] + 0.217[\text{kJ}] = -4.38[\text{kJ}]$$

The heat given off by the reaction is then calculated as

$$Q = (-4.38)[\text{kJ}] - (-4.986)[\text{kJ}] = +0.606[\text{kJ}]$$

The example in the text generated a heat output of 3.37 kJ so we have actually removed 2.764 kJ more energy by reducing the temperature of the products.

Problem 8 – A really interesting person takes the tennis ball mortar we built in problem 3 and modifies it – squirting in and igniting 0.003 oz. of acetylene gas ($C_2H_2(g)$). If we assume the combustion kinetics are fast enough such that the energy release occurs before the ball can move we want to determine the muzzle velocity of the tennis ball. Proceed along the following steps:

a.) Balance the stoichiometric reaction equation for acetylene.

b.) Balance the actual equation neglecting the volume the acetylene occupies in the chamber. Assume the air initially in the chamber is at 14.7 psia and 77 ºF.

c.) Determine the increase in internal energy of the gas as we have done in class

d.) Assuming the gas is calorically perfect ($\Delta U = m_g c_v \Delta T$) and that c_v = 0.33 BTU/lbm-ºR for the mixture, determine the increase in temperature of the gas.

NOTE: you will have to do c.) and d.) by iteration, first assuming a final reaction temperature, carrying out the calculation for Δu and seeing if the ΔT you get matches – if not iterate again – once you get an answer within say 10% that is good enough.

e.) Based on the result of d.) above, determine the initial pressure on the tennis ball assuming the specific gas constant of the products is R = 80 ft-lbf/lbm-ºR.

f.) Use the result of e.) and possibly your results from problem 2.) to determine the muzzle velocity of the tennis ball. Assume $\gamma = 1.4$

g.) Determine the temperature of the gases at shot exit.

For acetylene n = 26.038 lbm/lbmol, $\Delta h^0{}_f$ = +97,477 BTU/lbmol

Solution: We begin by balancing the stoichiometric equation.

$$C_2H_2(g) + 2.5O_2 + (3.76)(2.5)N_2 \rightarrow 2CO_2 + H_2O + 9.4N_2$$

As in the homework we will first need to determine the amount of air initially in the chamber. The volume of the chamber was determined in problem 2.) as

$$V_0 = (6.136)[in^3] - \frac{(8.181)}{2}[in^3] = 2.045[in^3]$$

The density of the air can be found from the ideal gas equation of state as

$$pv = RT \rightarrow \rho = \frac{p}{RT}$$

The air weighs 28.97 lbm/lbmol and the density of air is calculated from

$$\rho = \frac{(14.7)\left[\frac{\text{lbf}}{\text{in}^2}\right](28.97)\left[\frac{\text{lbm}}{\text{lbmol}}\right]}{(1545)\left[\frac{\text{ft}-\text{lbf}}{\text{lbmol}-\text{R}}\right](12)\left[\frac{\text{in}}{\text{ft}}\right](537)[\text{R}]} = 0.0000428\left[\frac{\text{lbm}}{\text{in}^3}\right]$$

So the amount of air we actually have is

$$m_{air} = \rho V_0 = (0.0000428)\left[\frac{\text{lbm}}{\text{in}^3}\right](2.045)[\text{in}^3] = 0.0000875[\text{lbm}]$$

The amount of fuel was given in ounces

$$m_{fuel} = (0.003)[\text{oz}](0.0625)\left[\frac{\text{lbm}}{\text{oz}}\right] = 0.0001875[\text{lbm}]$$

For the actual combustion we need to use our mass information from before and convert it to molar values.

For the fuel we have

$$N_{fuel} = \frac{m_{fuel}}{n_{fuel}} = (0.0001875)[\text{lbm}]\frac{1}{(26.038)\left[\frac{\text{lbm}}{\text{lbmol}}\right]} = 7.201\times10^{-6}[\text{lbmol}]$$

$$N_{air} = \frac{m_{air}}{n_{air}} = (0.0000875)[\text{lbm}]\frac{1}{(28.97)\left[\frac{\text{lbm}}{\text{lbmol}}\right]} = 3.020\times10^{-6}[\text{lbmol}]$$

for each lbmol of air we know that 1/4.76 lb-moles of it is oxygen so we have

$$N_{O_2} = \frac{1}{4.76}\left(3.020\times10^{-6}\right)[\text{lbmol}] = 6.344\times10^{-7}[\text{lbmol}]$$

$$N_{N_2} = \frac{3.76}{4.76}\left(3.020\times10^{-6}\right)[\text{lbmol}] = 2.385\times10^{-6}[\text{lbmol}]$$

Now we can write our actual equation as

$$\left(7.201\times10^{-6}\right)C_2H_2(g) + \left(6.344\times10^{-7}\right)O_2 + \left(2.385\times10^{-6}\right)N_2 \rightarrow$$
$$\left(1.269\times10^{-6}\right)H_2O + \left(1.440\times10^{-5}\right)C + \left(5.932\times10^{-6}\right)H_2 + \left(2.385\times10^{-6}\right)N_2$$

Now for the reaction calculation. Since the combustion occurs over a constant volume there is no work done on the projectile so our first law of thermodynamics is provided as

$$U_R = U_P + \Delta U$$

Since we are dealing with internal energies and not enthalpies we need to calculate R_uT for the initial and final state of the gases. Unfortunately we do not know the temperature of the products after the reaction takes place let's assume it is 2000 °R. For the reactants we have

$$R_uT = (1545)\left[\frac{\text{ft}-\text{lbf}}{\text{lbmol}-\text{R}}\right](537)[\text{R}](12)\left[\frac{\text{in}}{\text{ft}}\right] = 9{,}955{,}980\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

I used units of inches because everything else we have in the problem statement is in inches. Let's look at the internal energies for each of the reactants

Reactant	Enthalpy of formation (BTU/lbmol)	Enthalpy of formation (in-lbf/lbmol)
$C_2H_2(g)$	+97,477	+910,232,428
O_2	0	0
N_2	0	0

The conversion used here is as follows

$$(x)\left[\frac{\text{BTU}}{\text{lbmol}}\right](778.16)\left[\frac{\text{ft}-\text{lbf}}{\text{BTU}}\right](12)\left[\frac{\text{in}}{\text{ft}}\right]\rightarrow 9{,}337.9x\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

For the products we have

Product	Enthalpy of formation (BTU/lbmol)	Enthalpy of formation (in-lbf/lbmol)
H_2O (*g*)	-104,040	-971,515,116
N_2	0	0
C(*s*)	0	0
H_2	0	0

Since the reactants will be invariant here let's deal with them first.

We calculate U_R first

$$U_R = N_{C_2H_2}\left(\bar{h}_f^o + \Delta\bar{h} - R_uT\right) + N_{O_2}\left(\bar{h}_f^o + \Delta\bar{h} - R_uT\right) + N_{N_2}\left(\bar{h}_f^o + \Delta\bar{h} - R_uT\right)$$

Plugging in the numbers we have we get

$$U_R = \left(7.201\times10^{-6}\right)[\text{lbmol}](+910{,}232{,}428 + 0 - 9{,}955{,}980)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$
$$+\left(6.344\times10^{-7}\right)[\text{lbmol}](0 + 0 - 9{,}955{,}980)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$
$$+\left(2.385\times10^{-6}\right)[\text{lbmol}](0 + 0 - 9{,}955{,}980)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

$$U_R = +6{,}453[\text{in}-\text{lbf}]$$

At 2000 ºR we have the following values for R_uT and the products – we will neglect the effect of the solid carbon other than the effect with R_uT.

$$R_uT = (1545)\left[\frac{\text{ft}-\text{lbf}}{\text{lbmol}-\text{R}}\right](2000)[\text{R}](12)\left[\frac{\text{in}}{\text{ft}}\right] = 37{,}080{,}000\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

Product	Enthalpy at temperature (BTU/lbmol)	Enthalpy at temperature (in-lbf/lbmol)
H_2O (*g*)	13,183	123,102,854
N_2	10,804	100,887,752
C(*s*)	0	0
H_2	10,337	96,526,906

We calculate U_p in at 2000 ºR as

$$U_p = N_{H_2O}\left(\bar{h}_f^o + \Delta\bar{h} - R_u T\right) + N_{H_2}\left(\bar{h}_f^o + \Delta\bar{h} - R_u T\right) + N_{N_2}\left(\bar{h}_f^o + \Delta\bar{h} - R_u T\right) + N_C\left(\bar{h}_f^o + \Delta\bar{h} - R_u T\right)$$

$$U_p = \left(1.269\times10^{-6}\right)[\text{lbmol}]\left(-971{,}515{,}116 + 123{,}102{,}854 - 37{,}080{,}000\right)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

$$+\left(5.932\times10^{-6}\right)[\text{lbmol}]\left(0 + 96{,}526{,}906 - 37{,}080{,}000\right)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

$$+\left(2.385\times10^{-6}\right)[\text{lbmol}]\left(0 + 100{,}887{,}752 - 37{,}080{,}000\right)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

$$+\left(1.440\times10^{-5}\right)[\text{lbmol}]\left(0 + 0 - 37{,}080{,}000\right)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

$$U_p = -1{,}153[\text{in}-\text{lbf}]$$

Then

$$\Delta U = U_R - U_P$$

$$\Delta U = (6{,}453)[\text{in}-\text{lbf}] - (-1{,}153)[\text{in}-\text{lbf}] = 7{,}606[\text{in}-\text{lbf}]$$

Comparing this to the expression given in the problem statement

$$\Delta U = m_g c_v \Delta T$$

$$\Delta U = (0.0000875 + 0.0001875)[\text{lbm}](0.33)\left[\frac{\text{BTU}}{\text{lbm}\cdot\text{R}}\right](778.16)\left[\frac{\text{ft}-\text{lbf}}{\text{BTU}}\right](12)\left[\frac{\text{in}}{\text{ft}}\right](2000-537)[\text{R}]$$

$$\Delta U = 1{,}240[\text{in}-\text{lbf}]$$

Based on this let’s try a final temperature of 4000 ºR.

$$R_u T = (1545)\left[\frac{\text{ft}-\text{lbf}}{\text{lbmol}-\text{R}}\right](4{,}000)[\text{R}](12)\left[\frac{\text{in}}{\text{ft}}\right] = 74{,}160{,}000\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

Product	Enthalpy at temperature (BTU/lbmol)	Enthalpy at temperature (in-lbf/lbmol)
H_2O (*g*)	36,251	338,508,213
N_2	27,587	257,604,647
C(*s*)	0	0
H_2	26,071	243,448,391

We calculate U_p in at 4000 ºR as

$$U_p = N_{H_2O}\left(\bar{h}_f^o + \Delta\bar{h} - R_u T\right) + N_{H_2}\left(\bar{h}_f^o + \Delta\bar{h} - R_u T\right) + N_{N_2}\left(\bar{h}_f^o + \Delta\bar{h} - R_u T\right) + N_C\left(\bar{h}_f^o + \Delta\bar{h} - R_u T\right)$$

$$U_p = \left(1.269\times10^{-6}\right)[\text{lbmol}]\left(-971{,}515{,}116+338{,}508{,}213-74{,}160{,}000\right)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

$$+\left(5.932\times10^{-6}\right)[\text{lbmol}]\left(0+243{,}448{,}391-74{,}160{,}000\right)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

$$+\left(2.385\times10^{-6}\right)[\text{lbmol}]\left(0+257{,}604{,}647-74{,}160{,}000\right)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

$$+\left(1.440\times10^{-5}\right)[\text{lbmol}]\left(0+0-74{,}160{,}000\right)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

$$U_p = -523.5[\text{in}-\text{lbf}]$$

Then

$$\Delta U = U_R - U_P$$

$$\Delta U = (6{,}453)[\text{in}-\text{lbf}] - (-523.5)[\text{in}-\text{lbf}] = 6{,}976[\text{in}-\text{lbf}]$$

Comparing this to the expression given in the problem statement

$$\Delta U = m_g c_v \Delta T$$

$$\Delta U = (0.0000875+0.0001875)[\text{lbm}](0.33)\left[\frac{\text{BTU}}{\text{lbm}\cdot\text{R}}\right](778.16)\left[\frac{\text{ft}-\text{lbf}}{\text{BTU}}\right](12)\left[\frac{\text{in}}{\text{ft}}\right](4000-537)[\text{R}]$$

$$\Delta U = 2{,}934[\text{in}-\text{lbf}]$$

Now let's try a final temperature of 7000 ºR.

$$R_u T = (1545)\left[\frac{\text{ft}-\text{lbf}}{\text{lbmol}-\text{R}}\right](7{,}000)[\text{R}](12)\left[\frac{\text{in}}{\text{ft}}\right] = 129{,}780{,}000\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

Product	Enthalpy at temperature (BTU/lbmol)	Enthalpy at temperature (in-lbf/lbmol)
H_2O (*g*)	76,146	711,043,733
N_2	54,109	491,901,896
C(*s*)	0	0
H_2	52,678	505,264,431

We calculate U_p in at 7000 ºR as

$$U_p = N_{H_2O}\left(\bar{h}_f^o + \Delta\bar{h} - R_u T\right) + N_{H_2}\left(\bar{h}_f^o + \Delta\bar{h} - R_u T\right) + N_{N_2}\left(\bar{h}_f^o + \Delta\bar{h} - R_u T\right) + N_C\left(\bar{h}_f^o + \Delta\bar{h} - R_u T\right)$$

$$U_p = \left(1.269\times10^{-6}\right)[\text{lbmol}]\left(-971{,}515{,}116+711{,}043{,}733-129{,}780{,}000\right)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$
$$+\left(5.932\times10^{-6}\right)[\text{lbmol}]\left(0+505{,}264{,}431-129{,}780{,}000\right)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$
$$+\left(2.385\times10^{-6}\right)[\text{lbmol}]\left(0+491{,}901{,}896-129{,}780{,}000\right)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$
$$+\left(1.440\times10^{-5}\right)[\text{lbmol}]\left(0+0-129{,}780{,}000\right)\left[\frac{\text{in}-\text{lbf}}{\text{lbmol}}\right]$$

$$U_p = 679.6[\text{in}-\text{lbf}]$$

Then

$$\Delta U = U_R - U_P$$

$$\Delta U = (6{,}453)[\text{in}-\text{lbf}] - (679.6)[\text{in}-\text{lbf}] = 5{,}773[\text{in}-\text{lbf}]$$

Comparing this to the expression given in the problem statement

$$\Delta U = m_g c_v \Delta T$$

$$\Delta U = (0.0000875+0.0001875)[\text{lbm}](0.33)\left[\frac{\text{BTU}}{\text{lbm}\cdot\text{R}}\right](778.16)\left[\frac{\text{ft}-\text{lbf}}{\text{BTU}}\right](12)\left[\frac{\text{in}}{\text{ft}}\right](7000-537)[\text{R}]$$
$$\Delta U = 5{,}477[\text{in}-\text{lbf}]$$

This answer is close to 5% so we're good to go. The temperature is now 7,000 ºR – pretty hot! – that's why acetylene works so good as a cutting torch fuel. Now the initial pressure on the tennis ball comes through the ideal gas equation of state.

$$p_i = \rho R T$$

$$p_i = \frac{(0.0000875+0.0001875)[\text{lbm}]}{(2.045)[\text{in}^3]}(80)\left[\frac{\text{ft}-\text{lbf}}{\text{lbm}-\text{R}}\right](7{,}000)[\text{R}](12)\left[\frac{\text{in}}{\text{ft}}\right]$$

$$p_i = 903.5\left[\frac{\text{lbf}}{\text{in}^2}\right]$$

The muzzle velocity of the tennis ball is then

$$V_m = \sqrt{\frac{2 p_i V_0 l^{(\gamma-1)}}{m_p (1-\gamma)}\left[(l+L)^{(1-\gamma)} - l^{(1-\gamma)}\right]}$$

$$V_m = \sqrt{\frac{(2)(903.5)\left[\frac{\text{lbf}}{\text{in}^2}\right](2.045)\left[\text{in}^3\right](0.417)^{(1.4-1)}\left[\text{in}^{0.4}\right]}{(12)\left[\frac{\text{in}}{\text{ft}}\right](2)[\text{oz}]\left(\frac{1}{16}\right)\left[\frac{\text{lbm}}{\text{oz}}\right](1-1.4)}\left[(0.417+6.75)^{(1-1.4)} - (0.417)^{(1-1.4)}\right]\left[\text{in}^{-0.4}\right](32.2)\left[\frac{\text{lbm}-\text{ft}}{\text{lbf}-\text{s}^2}\right]}$$

$$V_m = 64.7\left[\frac{\text{ft}}{\text{s}}\right]$$

For the temperature of the gases when muzzle exit occurs we can use equation (IG-19)

$$T = T_i\left(\frac{V_c}{V}\right)^{(\gamma-1)} \qquad \text{(IG-19)}$$

Assuming muzzle exit occurs as we have stated earlier we can write

$$V_e = U_{can} - \frac{V_{sphere}}{2}$$

$$U_{can} = \frac{\pi}{4} d^2 L = \frac{\pi}{4}(2.5)^2\left[\text{in}^2\right](8)[\text{in}] = 39.27\left[\text{in}^3\right]$$

then we have

$$V_e = (39.27)\left[\text{in}^3\right] - \frac{(8.181)\left[\text{in}^3\right]}{2} = 35.179\left[\text{in}^3\right]$$

Then the temperature at muzzle exit is given by

$$T = (7{,}000)[\text{R}]\left(\frac{(2.045)\left[\text{in}^3\right]}{(35.179)\left[\text{in}^3\right]}\right)^{(1.4-1)}$$

$$T = 2{,}243\left[{}^\circ\text{R}\right] = 1{,}783\left[{}^\circ\text{F}\right]$$

Problem 9 – A potato is stuffed into the 3" diameter exhaust pipe of a car that is not running too well. 1 gram of incompletely combusted combustion products (assume gaseous Heptane) mixes with a stoichiometric amount of air behind the potato and ignites. If the potato is wedged 4" into the exhaust (i.e. it has 4" of travel) and weighs 0.25 lbm, and assuming the combustion takes place before the potato moves, determine the theoretical maximum "muzzle" velocity of the potato. Also calculate the muzzle velocity assuming an isentropic expansion. For gas expansion purposes you can assume the volume available initially behind the potato is equal to the volume between the point of obturation and the end of the exhaust and assume a "smeared" specific heat ratio of 1.3 for the product gases. Also assume the combustion begins at 500 K and completes at 1,500 K. Assume the total enthalpy at 500 K for *n*-Heptane (C_7H_{16}) is -120,000 kJ/kgmol.

Solution: We know that the combustion is occurring under stoichiometric conditions so we can write the balance reaction equation as follows:

$$C_7H_{16}(g)+11O_2+(3.76)(11)N_2 \rightarrow 7CO_2+8H_2O+41.36N_2$$

This is on a one mole of fuel basis. We need to write it for what we actually have which is 1 gram of fuel. First we determine the molecular weight of the Heptane to be 100.2 kg/kgmol. Therefore the number of moles of Heptane we have is

$$N_{C_7H_{16}}=\frac{(1)[g]\left(\frac{1}{1000}\right)\left[\frac{kg}{g}\right]}{(100.2)\left[\frac{kg}{kgmol}\right]}=1\times10^{-5}[kgmol]$$

So the balanced equation becomes

$$\left(1\times10^{-5}\right)C_7H_{16}(g)+\left(11\times10^{-5}\right)O_2+\left(41.36\times10^{-5}\right)N_2 \rightarrow \left(7\times10^{-5}\right)CO_2+\left(8\times10^{-5}\right)H_2O+\left(41.36\times10^{-5}\right)N_2$$

Now let's deal with the thermochemistry. We start with our second law of thermodynamics equation, simplified by the fact that there is no heat transfer and no shaft work. Then the energy of the fuel-air mixture equals the work done on the projectile plus the energy of the products of combustion plus the work done on the gas. (Note the difference here than in the text where we lumped the work done on the gas with the losses)

$$H_R=H_P+W_P$$

Let's look at the internal energies for each of the reactants

Reactant	Total Enthalpy (kJ/kgmol)
$C_7H_{16}(g)$	-120,000

	Enthalpy of formation (kJ/kgmol)	$\Delta h(500)$ (kJ/kgmol)
O_2	0	6,097
N_2	0	5,920

For the products we have

Product	Enthalpy of formation (kJ/kgmol)	$\Delta h(1{,}500)$ (kJ/kgmol)
H_2O (*g*)	-241,845	48,121

CO_2 (*g*)	-393,546	61,681
N_2	0	38,404

We will rearrange our second law equation as follows

$$W_p = H_R - H_p$$

We calculate H_R first

$$H_R = N_{C_7H_{16}}\left(\bar{h}_f^o + \Delta\bar{h}\right) + N_{O_2}\left(\bar{h}_f^o + \Delta\bar{h}\right) + N_{N_2}\left(\bar{h}_f^o + \Delta\bar{h}\right)$$

Plugging in the numbers we have we get

$$H_R = \left(1\times10^{-5}\right)[\text{kgmol}](-120{,}000)\left[\frac{\text{kJ}}{\text{kgmol}}\right] + \left(11\times10^{-5}\right)[\text{kgmol}](0+6{,}097)\left[\frac{\text{kJ}}{\text{kgmol}}\right]$$
$$+\left(41.36\times10^{-5}\right)[\text{kgmol}](0+5{,}920)\left[\frac{\text{kJ}}{\text{kgmol}}\right]$$

$$H_R = 1.919[\text{kJ}]$$

We calculate H_p in a similar manner

$$H_p = N_{H_2O}\left(\bar{h}_f^o + \Delta\bar{h}\right) + N_{CO_2}\left(\bar{h}_f^o + \Delta\bar{h}\right) + N_{N_2}\left(\bar{h}_f^o + \Delta\bar{h}\right)$$

$$H_p = \left(8\times10^{-5}\right)[\text{kgmol}](-241{,}845+48{,}121)\left[\frac{\text{kJ}}{\text{kgmol}}\right] + \left(7\times10^{-5}\right)[\text{kgmol}](-393{,}546+61{,}681)\left[\frac{\text{kJ}}{\text{kgmol}}\right]$$
$$+\left(41.36\times10^{-5}\right)[\text{kgmol}](0+38{,}404)\left[\frac{\text{kJ}}{\text{kgmol}}\right]$$

$$H_p = -22.8[\text{kJ}]$$

Then the work done on the projectile is

$$W_p = 1.919[\text{kJ}] - (-22.8)[\text{kJ}] = 24.8[\text{kJ}]$$

Now we need to recall that this work equals the muzzle energy of the projectile

$$W_p = \frac{1}{2}mV^2 = 24.8[\text{kJ}]$$

Therefore the theoretical maximum velocity for this reaction is

$$V=\sqrt{\frac{2W_p}{m}}=\sqrt{\frac{(2)(24.8)[\text{kJ}]}{(0.25)[\text{lbm}]\left(\frac{1}{2.2}\right)\left[\frac{\text{kg}}{\text{lbm}}\right]}}=\sqrt{\frac{(2)(24{,}800)\left[\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}\right]}{(0.25)[\text{lbm}]\left(\frac{1}{2.2}\right)\left[\frac{\text{kg}}{\text{lbm}}\right]}}=661\left[\frac{\text{m}}{\text{s}}\right]$$

This is over twice the speed of sound! To get a more realistic answer let's look at the ideal gas equation of state. If the projectile doesn't move, the volume behind it is

$$\text{V}_i=Al=\pi\frac{(3)^2}{4}\left[\text{in}^2\right](4)[\text{in}](0.0254)^3\left[\frac{\text{m}^3}{\text{in}^3}\right]=4.6\times10^{-4}\left[\text{m}^3\right]$$

The volume of the exhaust pipe when the potato is at the end is twice this value. The initial pressure is calculated from the ideal gas equation of state then using this volume

$$p\text{V}=NR_uT\rightarrow p=\frac{NR_uT}{\text{V}}$$

$$p=\frac{\left(56.36\times10^{-5}\right)[\text{kgmol}](8.314)\left[\frac{\text{kJ}}{\text{kgmol}\cdot\text{K}}\right](1{,}500)[\text{K}]}{\left(4.6\times10^{-4}\right)\left[\text{m}^3\right]}=15{,}280[\text{kPa}]$$

If we use the isentropic formulation for muzzle velocity we can start from equation (LG-180)

$$\frac{1}{2}\left(w_1+\frac{c}{3}\right)\left(V^2(x)-V^2(x_c)\right)=A\bar{p}_c\int_{x_c}^{x}\left(\frac{x+l}{x_c+l}\right)^{-\gamma}dx \qquad \text{(LG-180)}$$

In our case we can safely assume the gas is massless and the initial velocity is zero so we have

$$\frac{1}{2}mV^2(2l)=A\bar{p}_c\int_0^l\left(\frac{x+l}{l}\right)^{-\gamma}dx=\frac{A\bar{p}_c}{l^{-\gamma}}\int_0^l(x+l)^{-\gamma}dx$$

Integrating we have

$$\frac{1}{2}mV^2(2l)=\frac{A\bar{p}_c}{l^{-\gamma}}\int_0^l(x+l)^{-\gamma}dx=\frac{A\bar{p}_c}{l^{-\gamma}}\left[\frac{(x+l)^{1-\gamma}}{1-\gamma}\right]_0^l$$

Inserting the limits of integration we obtain

$$\frac{1}{2}mV^2 = \frac{A\bar{p}_c}{l^{-\gamma}}\left[\frac{(2l)^{1-\gamma} - l^{1-\gamma}}{1-\gamma}\right] = A\bar{p}_c\left[\frac{(2)^{1-\gamma}l - l}{1-\gamma}\right]$$

Rearranging we have

$$V = \sqrt{\frac{2A\bar{p}_c}{m}\left[\frac{2^{1-\gamma}l - l}{1-\gamma}\right]}$$

Inserting our numbers we have

$$V = \sqrt{\frac{(2)\frac{\pi(3)^2[\text{in}^2]}{4}(0.0254)^3\left[\frac{\text{m}^3}{\text{in}^3}\right](16{,}700{,}000)\left[\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{m}^2}\right]}{(0.25)[\text{lbm}]\left(\frac{1}{2.2}\right)\left[\frac{\text{kg}}{\text{lbm}}\right]}\left[\frac{(2)^{1-1.3}(4)-4}{1-1.3}\right][\text{in}]}$$

$$V = 601\left[\frac{\text{m}}{\text{s}}\right]$$

Still way high but we neglected friction and LOTS of losses.

Problem 10 – Assume we have a quantity of 29 pounds of 11.1% nitrated nitrocellulose ($C_6H_8N_2O_9$) and it is placed in an empty chamber of a gun at 77°F and 14.7 psia. The chamber is 1160 cubic inches in volume. The propellant density is 0.060 lbm/in^3. If the air in the chamber is NOT neglected and assuming the volume is fixed:

a.) Write the balanced equation for this combustion (assume the oxygen goes preferentially into CO_2 instead CO this time – you will find the difference later)
b.) Using the tables in the textbook, estimate the adiabatic flame temperature of the resultant gas (Hint: recall the definition of adiabatic flame temperature)

Solution:

First we need to convert the units for the amount of propellant and air into moles. For the propellant it is exactly as found in the notes 0.115 lbmol. For the air in the chamber we have to play with a few equations. First, the volume the propellant occupies in the chamber is

$$\text{V}_p = \frac{m}{\rho} \qquad (1)$$

$$\text{V}_p = \frac{(29)[\text{lbm}]}{(0.060)\left[\frac{\text{lbm}}{\text{in}^3}\right]} = 483[\text{in}^3] \qquad (2)$$

The volume the air occupies is then

$$\mathrm{V}_{air} = \mathrm{V}_c - \mathrm{V}_p \tag{3}$$

$$\mathrm{V}_{air} = 1160[\mathrm{in}^3] - 483[\mathrm{in}^3] = 677[\mathrm{in}^3] \tag{4}$$

The number of moles of air then can be found from the ideal gas equation of state as follows

$$N_{air} = \frac{p\mathrm{V}_{air}}{R_u T_{air}} \tag{5}$$

Noting that the air temperature must be in Rankine we have

$$N_{air} = \frac{(14.7)\left[\frac{\mathrm{lbf}}{\mathrm{in}^2}\right](677)[\mathrm{in}^3]}{(1{,}545)\left[\frac{\mathrm{ft}\cdot\mathrm{lbf}}{\mathrm{lbmol}\cdot\mathrm{R}}\right](12)\left[\frac{\mathrm{in}}{\mathrm{ft}}\right](537)[\mathrm{R}]} \tag{6}$$

$$N_{air} = 0.001[\mathrm{lbmol}] \tag{7}$$

Balancing the chemical equation we have

$$(0.115)\mathrm{C_6H_8N_2O_9} + (0.001)\left(\frac{1}{4.76}\right)\mathrm{O_2} + \left(\frac{3.76}{4.76}\right)(0.001)\mathrm{N_2} \rightarrow (4)(0.115)\mathrm{H_2O} + \{(3.5)(0.115) + (0.00021)\}\mathrm{CO_2} + \{(0.115) + (0.00079)\}\mathrm{N_2} + \{(3.5)(0.115) - (0.00021)\}\mathrm{C(s)} \tag{8}$$

or

$$(0.115)\mathrm{C_6H_8N_2O_9} + (0.00021)\mathrm{O_2} + (0.00079)\mathrm{N_2} \rightarrow (0.460)\mathrm{H_2O} + (0.403)\mathrm{CO_2} + (0.116)\mathrm{N_2} + (0.400)\mathrm{C(s)} \tag{9}$$

So this is our balanced reaction. To solve for the adiabatic flame temperature we need to write the first law of thermodynamics as in equation (CT-42)

$$Q_{C.V.} + H_R = H_P + W \tag{CT-42}$$

The problem statement stated that the volume was fixed so $W = 0$ and there is no flow work. Since we are looking for the adiabatic flame temperature we assume that $Q_{C.V.} = 0$ also. Then we see that

$$U_R = U_P \tag{10}$$

Looking at the notes we see that the enthalpy of formation for the propellant was given as 2,605.5 Btu/lbmol. The tables in the book are in kJ/kgmol so we have some unit conversions to do.

$$\bar{h}^0_{f\ C_6H_8N_2O_9} = (2{,}605.5)\left[\frac{\text{Btu}}{\text{lbmol}}\right](1.055)\left[\frac{\text{kJ}}{\text{Btu}}\right](2.2)\left[\frac{\text{lbmol}}{\text{kgmol}}\right] = 6{,}046\left[\frac{\text{kJ}}{\text{kgmol}}\right] \quad (11)$$

We can change this over to internal energy (because there was no flow work by using the definition of enthalpy, thus

$$U_R = (6{,}046)\left[\frac{\text{kJ}}{\text{kgmol}}\right] - (8.314)\left[\frac{\text{kJ}}{\text{kgmol}\cdot\text{K}}\right](298)[\text{K}] = 3{,}568\left[\frac{\text{kJ}}{\text{kgmol}}\right]$$

We can put equation (9) on a per mol of propellant basis by dividing by 0.115. We then have

$$C_6H_8N_2O_9 + (0.0018)O_2 + (0.00687)N_2 \rightarrow 4H_2O + (3.504)CO_2 + (1.007)N_2 + (2.496)C(s) \quad (12)$$

To get everything in gmol we would multiply by 52.27 but we will hold off on this for now. Since we know diatomic oxygen and nitrogen are entering at STP their enthalpies of formation are zero and the LHS boils down to equation (11). Now we have to create an equation for the RHS as a function of temperature. We have

$$U_p = N_{H_2O}\left(\bar{h}^o_f + \Delta\bar{h}\right) + N_{CO_2}\left(\bar{h}^o_f + \Delta\bar{h}\right) + N_{N_2}\left(\bar{h}^o_f + \Delta\bar{h}\right) + N_C\left(\bar{h}^o_f + \Delta\bar{h}\right) - \left(N_{H_2O} + N_{CO_2} + N_{N_2} + N_C\right)R_u T \quad (13)$$

Plugging in our values (in kJ/kgmol) we have

$$U_p = (4)\left(-241{,}845 + \Delta\bar{h}\right) + (2.513)\left(-393{,}546 + \Delta\bar{h}\right) + (1.007)\left(0 + \Delta\bar{h}\right) + (3.487)\left(0 + \Delta\bar{h}\right)$$
$$-(4 + 2.513 + 1.007 + 3.487)(8.314)T_0 \quad (14)$$

At this point we look in the tables in the book and apply temperatures to equation (14) until we achieve a balance. Because this problem ended up going beyond the upper limit of the table I simply extrapolated from the last entry to obtain T_0 = 5,848.7 K.

So the adiabatic flame temperature for this reaction is 5,848.7 K or 10,524 °R (REAL high). The reality is that the assumptions of the product species was incorrect.

2.6 Solid Propellant Combustion

Problem 11 - M1 propellant is measured in a closed bomb. Its adiabatic flame temperature is 3906° F. Its molar mass is 22.065 lbm/lbmol, what is the effective mean force constant in ft-lbf/lbm?

Answer $\lambda = 305{,}709\left[\frac{\text{ft - lbf}}{\text{lbm}}\right]$

Solution: This is just a straight use of the definition of the force constant from the notes.

$$\lambda = nR_u T_0 = \left(\frac{1}{22.065}\right)\left[\frac{\text{lbmol}}{\text{lbm}}\right](1545)\left[\frac{\text{ft - lbf}}{\text{lbmol - R}}\right](3906+460)[\text{R}]$$

$$\lambda = 305{,}709\left[\frac{\text{ft - lbf}}{\text{lbm}}\right]$$

Problem 12 - M15 propellant was tested in a strand burner to determine the linear burning rate. The average pressure evolved was 10,000 psi. If the burning exponent, α was known to be 0.693 and the pressure coefficient, β was known to be 0.00330 in/s/psi$^{0.693}$, determine the average linear burning rate, B in in/s.

Answer $B(p) = 1.952\left[\frac{\text{in}}{\text{s}}\right]$

Solution:

We can write the burning rate equation

$$B(p) = \beta p^{\alpha}$$

Then the average burning rate is

$$B(p) = (0.00330)\left[\frac{\frac{\text{in}}{\text{s}}}{\left(\frac{\text{lbf}}{\text{in}^2}\right)^{0.693}}\right](10{,}000)^{0.693}\left[\left(\frac{\text{lbf}}{\text{in}^2}\right)^{0.693}\right]$$

$$B(p) = 1.952\left[\frac{\text{in}}{\text{s}}\right]$$

Problem 13 - . Please derive the functional form of ϕ in terms of f for a flake propellant. Assume cylindrical geometry.

Hint Flake propellant consists of grains that have thicknesses muck smaller than any other characteristic dimension.

Answer $\phi(t) = 1 - f$

Solution: Given a short, (circular) flake of propellant, we can define the weight of the material as the product of the specific weight times the volume.

$$c = \rho_{grain} g \mathrm{V} \tag{1}$$

Here ρ_{grain} is the density of the propellant grain, g is the acceleration of gravity (the product $\rho_{grain}g$ is the specific weight), V is the volume of the grain and c is the weight of the propellant grain. If we define the cross-sectional area of the propellant grain as if it were a short, right circular cylinder as

$$A_{init} = \frac{\pi D^2}{4} \quad \text{and} \quad \mathrm{V}_{init} = A_{init}\tau = \frac{\pi D^2}{4}\tau \tag{2}$$

where, D is the diameter of the grain and τ is the thickness (length). Then the initial weight of the propellant grain is

$$c_{init} = \rho_{grain}\, g\, \frac{\pi D^2}{4}\tau \tag{3}$$

Since matter can neither be created nor destroyed, the weight of the gas generated by burning a propellant grain is equal to the initial weight of the grain minus the weight of solid grain left. This is described conveniently by the mass fraction *f*. In the case of a flake the surface area of the flats is much greater than that around the cylindrical section (this is in direct contrast to the long cylindrical grain).

Therefore the "web" of a flake is the thickness, τ, and the regression of the diameter (or the edges if the geometry is not circular for that matter) is insignificant with respect to the loss of volume due to the thickness change. This is the fraction of the thickness (and therefore mass if we neglect the burning of the circumference) remaining after a time *t* as depicted in Figure 1.

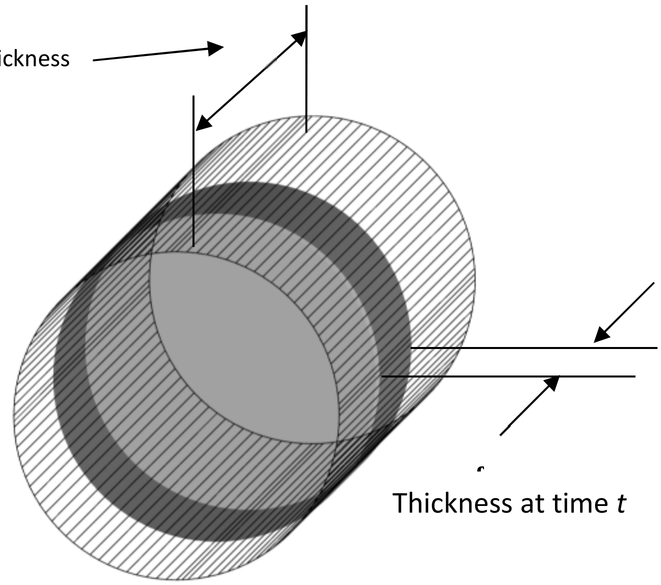


Figure 1. Propellant grain geometry

The weight of the unburnt propellant grain at time *t* is given by

$$c(t) = \rho_{grain} g \frac{\pi D^2}{4} f\tau \quad (4)$$

As stated earlier

$$c_{gas} = c_{init} - c(t) \quad (5)$$

Substitution of equations (3) and (4) into (5) yields

$$c_{gas} = \rho_{grain} g \frac{\pi D^2}{4}\{\tau - f\tau\} = \rho_{grain} g \frac{\pi D^2}{4}\tau(1 - f) \quad (6)$$

We now introduce the fraction of gas generated ϕ. Initially $\phi = 0$ and $\phi = 1$ at "all burnt". This function is defined as

$$\phi(t) = \frac{c_{gas}}{c_{init}} \quad (7)$$

Substitution of equations (3) and (6) into (7) yields the desired relation

$$\phi(t) = 1 - f \tag{8}$$

We need to note that this formulation will work for any geometry of flake propellant where the thickness is much smaller than any other dimension.

Problem 14 - An M60 projectile is to be fired from a 105mm M204 Howitzer. The propellant used in this semi-fixed piece of ammunition is 5.5 lbm of M1 propellant. M1 propellant consists of single perforated grains ($\theta = 0$) with a web thickness of 0.0165 inches. If the average pressure (over the launch of this projectile) developed in the weapon is 20,455 psi. Calculate the average burning rate coefficient in in^3/lbf-s if the burn rate is (we use a negative sign in the burn rate to make the form come out right later)

$$\frac{df}{dt} = -185.9\left[\mathrm{s}^{-1}\right]$$

Answer $$\beta = 1.50\times 10^{-4}\left[\frac{\mathrm{in}^3}{\mathrm{lbf\text{-}s}}\right]$$

Solution: We will use equation (2) from the notes to calculate β, thus we have

$$D\frac{df}{dt} = -\beta p \quad \text{or} \quad \beta = -\frac{D}{p_{avg}}\frac{df}{dt}$$

Inserting the information provided (and watching the units) above we can directly calculate β

$$\beta = -\frac{(0.0165)[\mathrm{in}]}{(20455)\left[\frac{\mathrm{lbf}}{\mathrm{in}^2}\right]}(-185.9)\left[\mathrm{s}^{-1}\right] = 1.50\times 10^{-4}\left[\frac{\mathrm{in}^3}{\mathrm{lbf\text{-}s}}\right]$$

Problem 15 β is actually a function of pressure and temperature (it is really given in tables at 25°F at this value). For simplification (and illustration) we will assume it is constant. Given this assumption, calculate the functional form of the web fraction, f from problem P 2.6.4 , above.

Answer $$f = 1 - \frac{\beta p_{avg}}{D}t$$

Solution: Since we have all the data on the burn rate from problem 1, we only need to integrate equation (2)

$$\frac{df}{dt} = -\frac{\beta p_{avg}}{D} \quad \text{or} \quad \int_1^f df = -\int_0^t \frac{\beta p_{avg}}{D}dt \quad \text{performing the integration we get}$$

$$f - 1 = -\frac{\beta p_{avg}}{D}t + \text{Const.}$$

We now need to evaluate the constant of integration by inserting the initial conditions of $f = 1$ at $t = 0$. Performing this task and substituting the constant back into the above equation yields the proper form

$$f = 1 - \frac{\beta p_{avg}}{D} t$$

Problem 16 - Given the data provided in problems P 2.6.4 and P 2.6.5, above, determine the proper form of the fraction of charge burnt.

Answer $\phi(t) = 185.9t$

Solution: This is determined by simply substituting the relation we obtained for f above and θ into equation (1) from the interior ballistics notes. Thus we have

$$\phi(t) = (1 - f)(1 + \theta f) = \left(1 - 1 + \frac{\beta p_{avg}}{D} t\right) = \frac{\beta p_{avg}}{D} t$$

Inserting numerical values into this relation gives us

$$\phi(t) = \frac{\left(1.5 \times 10^{-4}\right)\left[\frac{\text{in}^3}{\text{lbf - s}}\right](20455)\left[\frac{\text{lbf}}{\text{in}^2}\right]}{(0.0165)[\text{in}]} t = 185.9t$$

Note here that because θ is equal to zero, ϕ is equal to *df/dt*! (with the sign change)

Problem 17 – You are asked to characterize a commercial propellant. In order to do this you take one grain of the propellant and place it in a closed bomb of 0.5 in^3 volume, initially evacuated. You have a temperature and pressure sensor in the device. After 0.063 seconds you decide that the propellant has fully combusted. You read the data – pressure was measured to be 3.706 psi (this is not a big value but it was only one small grain of propellant) but it looks as though the temperature sensor is broken. The initial propellant grain weighed 0.003189 grains and it was 0.1 inches long by 0.01 inch diameter. Based on this data only –

a.) Estimate the propellant force, λ, in ft-lbf/lbm

b.) Estimate the linear burn rate coefficient, β in in/s/psi.

c.) List all assumptions and explain why you believe these estimates are too high or too low. (certain assumptions may make the estimates high while others make them low) – There are at least 4 buried in there

Solution: We can determine the propellant force for a cylindrical grain directly from equation (SP-23).

$$p_B(t)\mathrm{V} = m_g(t)RT_0 = \lambda m_g(t) \qquad \text{(SP-23)}$$

$$\lambda = \frac{p_B(t)\mathrm{V}}{m_g(t)}$$

Since all of the material is presumably gas at the end of the combustion we can simply insert values into this equation

$$\lambda = \frac{(3.706)\left[\frac{\text{lbf}}{\text{in}^2}\right](0.5)\left[\text{in}^3\right]\left(\frac{1}{12}\right)\left[\frac{\text{ft}}{\text{in}}\right]}{(0.003189)[\text{grains}]\left(\frac{1}{7{,}000}\right)\left[\frac{\text{lbm}}{\text{grains}}\right]}$$

$$\lambda = 339{,}000\left[\frac{\text{ft - lbf}}{\text{lbm}}\right]$$

A note here about why we used the gage pressure. This is because the bomb is usually evacuated to obtain a true measure – if there was air in there we would have accounted for that too. The burn rate coefficient can be approximated from equation (SP-38)

$$700 = \frac{2\beta\lambda c}{D\mathrm{V}}t_B \rightarrow t_B = 350\frac{D\mathrm{V}}{\beta\lambda c} \qquad \text{(SP-38)}$$

$$\beta \approx 350\frac{D\mathrm{V}}{t_B\lambda c}$$

Inserting our numbers we have

$$\beta \approx (350)\frac{(0.01)[\text{in}](0.5)\left[\text{in}^3\right](7{,}000)\left[\frac{\text{grains}}{\text{lbm}}\right]}{(0.063)[\text{s}](339{,}000)\left[\frac{\text{ft} - \text{lbf}}{\text{lbm}}\right](12)\left[\frac{\text{in}}{\text{ft}}\right](0.003189)[\text{grains}]}$$

$$\beta \approx 15\left[\frac{\frac{\text{in}}{\text{s}}}{\frac{\text{lbf}}{\text{in}^2}}\right]$$

The assumptions that went into these equations were as follows

1.) Ideal gas behavior – even though in the burn portion the Noble-Able equation of state was utilized – this will produce a temperature that is too high – thus the results will be slightly high

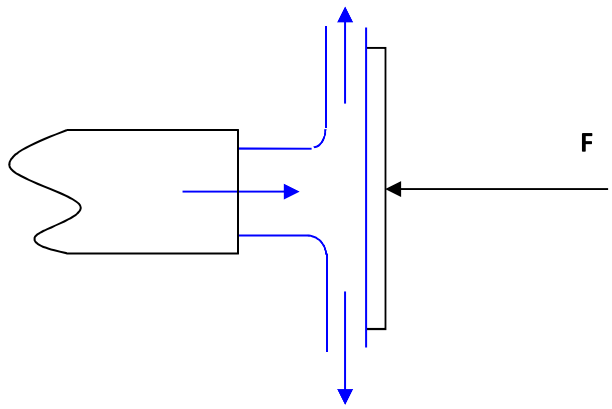
2.) Combustion products are generated at the adiabatic flame temperature of the propellant – again this is the most energy we can get out of the propellant – results will be slightly high

3.) No heat transfer occurred to the walls of the container – adiabatic behavior – There is always heat transfer so the actual pressure should be higher than we obtained – the answers will be slightly low

4.) The combustion of the propellant grain neglected end effects – so the propellant will have had more surface area burning in the time we assumed only the cylindrical portion was regressing – the burn rate will be too high thus the force constant will be too low.

2.7 Fluid Mechanics

Problem 18 - The principle behind a muzzle brake on a gun is to utilize some of the forward momentum of the propelling gases to reduce the recoil on the carriage. In the simple model below, the brake is assumed to be a flat plate with the jet of gases impinging upon it. If the jet diameter is 105mm and the velocity and density of the gas (assume air) are 750 m/s and 0.457 kg/m^3, find the force on the weapon in Newtons assuming the gases are directed 90° to the tube and the flow is steady.

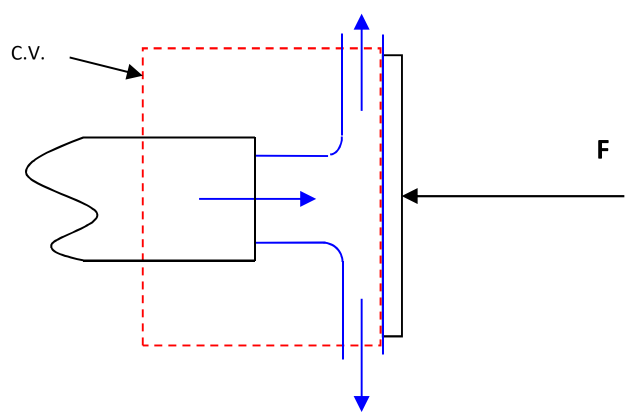


Answer -2,225.9 N

Solution: If we draw our control volume as shown below, we can write the following expression:

$$\mathbf{F} = \frac{d(m\mathbf{v})}{dt} = \frac{\partial}{\partial t}\int_{C.V} \rho\, \mathbf{v}\, d\mathrm{V} + \int_{C.S.} \mathbf{v}\, \rho \mathbf{v} \cdot d\mathbf{A}$$

= 0, Steady flow

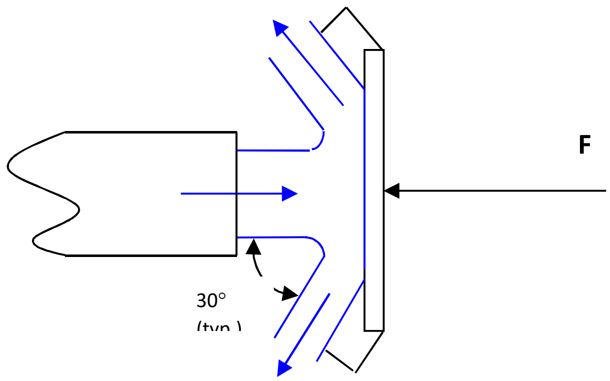


Let's look at the axial momentum to readily obtain the answer

$$F_x = \dot{m}(V_{in}\cos(0^\circ)) = -\rho V_{in} A (V_{in}\cos(0^\circ))$$

$$F_x = -(0.457)\left[\frac{\text{kg}}{\text{m}^3}\right](750)^2\left[\frac{\text{m}^2}{\text{s}^2}\right]\left(\frac{\pi(0.105)^2}{4}\right)[\text{m}^2] = -2225.9[\text{N}]$$

Problem 19 - Some engineer gets the idea that if deflecting the muzzle gases to the side is a good idea, then deflecting it rearward would be better (until of course an angry gun crew gets hold of him!). If the jet diameter is again 105mm and the velocity and density of the gas (again assume air) are 750 m/s and 0.457 kg/m^3, find the force on the weapon in Newtons assuming the gases are directed 150° to the tube and the flow is steady.

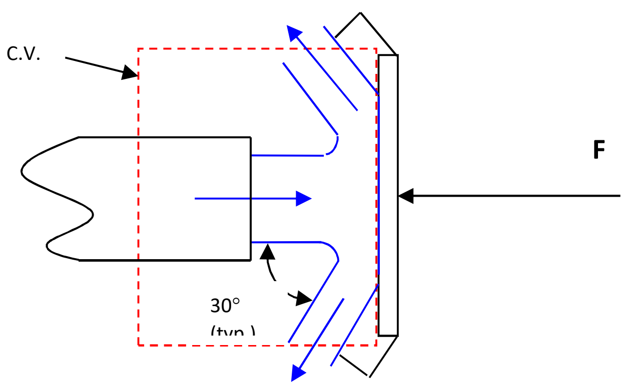


Answer -4,153.5 N

Solution: If we draw our control volume as shown below, we can write the following expression:

$$\mathbf{F} = \frac{d(m\mathbf{v})}{dt} = \frac{\partial}{\partial t}\int_{\text{C.V}} \rho\,\mathbf{v}\,d\text{V} + \int_{\text{C.S.}} \mathbf{v}\,\rho\mathbf{v}\cdot d\mathbf{A}$$

= 0, Steady flow



In the previous case, since the exiting fluid left at 90° to the control volume we were not concerned with the exiting area as cos(90°) = 0. Here we don't know what the exit area is but we do know from the continuity equation that the mass flow into the C.V. must equal the mass flow out of the C.V. and since we were not told otherwise, we assume that there is no loss in velocity, thus

$\dot{m}_{in} = \dot{m}_{out} = \rho_{in} V_{in} A_{in} = \rho_{out} V_{out} A_{out}$ and therefore

$A_{in} = A_{out}$ But since there are two outlets, each outlet is half of the area of the gun bore. Now we can invoke the momentum equation to obtain the axial force.

$$F_x = \rho V_{in} \left(V_x A_{in} \cos(180^\circ)\right) + (2)\rho V_{out} \left(V_{out} \frac{A_{out}}{2} \cos(150^\circ)\right)$$

It is sometimes helpful to write this equation as

$$F_x = \dot{m}_{in} \left(V_{in} \cos(0^\circ)\right) + (2)\dot{m}_{out} \left(\frac{V_{out}}{2} \cos(150^\circ)\right)$$

Now the answer is

$$F_x = -(0.457)\left[\frac{\text{kg}}{\text{m}^3}\right](750)^2\left[\frac{\text{m}^2}{\text{s}^2}\right]\left(\frac{\pi(0.105)^2}{4}\right)\left[\text{m}^2\right]$$

$$+(0.457)\left[\frac{\text{kg}}{\text{m}^3}\right](750)^2\left[\frac{\text{m}^2}{\text{s}^2}\right]\left(\frac{\pi(0.105)^2}{4}\right)\left[\text{m}^2\right](-0.866) = -4153.5[\text{N}]$$

Problem 20 - Consider a shock tube that is 6 feet long with a diaphragm at the center. Air is contained in both sections ($\gamma = 1.4$). The pressure in the high pressure region is 2,000 psi. The pressure in the low pressure region is 14.7 psi. The temperature in both sections is initially 68° F. When the diaphragm is burst
Determine

a) The velocity that the shock wave propagates into the low pressure region.

Answer 2,798 ft/s

b) The induced velocity behind the wave.

Answer 1,946 ft/s

c) The velocity of a wave reflected normally off the wall (relative to the laboratory).

Answer 1,232 ft/s

d) The temperature behind the incident wave.

Answer 657°F

e) Draw an *x-t* diagram of the event. Include the path of a particle located 2 feet from the diaphragm.

Solution: We need to determine the velocity of the shock wave but before we do we need the speed of sound in the still air and we must find p_2. The speed of sound comes directly from our hint.

$$a_1 = a_4 = \sqrt{\gamma RT}$$

$$a_1 = a_4 = \sqrt{(1.4)(1{,}716)\left[\frac{\text{ft}-\text{lbf}}{\text{slug}-\text{R}}\right](68+460)[\text{R}]} = 1{,}126\left[\frac{\text{ft}}{\text{s}}\right]$$

We need to iterate to determine the static pressure behind the wave. Since p_2 has to lie between p_1 and p_4 we shall use 20 psi as a starting point. The equation is:

$$\frac{p_4}{p_1} = \frac{p_2}{p_1}\left\{1-\frac{(\gamma_4-1)\left(\frac{a_1}{a_4}\right)\left(\frac{p_2}{p_1}-1\right)}{\sqrt{2\gamma_1\left[2\gamma_1+(\gamma_1+1)\left(\frac{p_2}{p_1}-1\right)\right]}}\right\}^{\frac{-2\gamma_4}{(\gamma_4-1)}}$$

It is best to set this up in a spread sheet. We have

$$\frac{p_4}{p_1} = \frac{(2{,}000)\left[\frac{\text{lbf}}{\text{in}^2}\right]}{(14.7)\left[\frac{\text{lbf}}{\text{in}^2}\right]} = 136.05$$

Without going into detail, noting that since air is on both sides of the shock wave ($\gamma_4 = \gamma_1$) we have.

Assumed p_2	Calculated p_4/p_1
20	1.87822
40	8.94021
60	25.5956
100	123.602
105	146.638
103	137.014
102	132.41
102.5	134.695
102.7	135.619
102.793	136.05

We can now calculate the speed of propagation of the wave into the low pressure region as

$$U = a_1\sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1}-1\right)+1} = (1{,}126)\left[\frac{\text{ft}}{\text{s}}\right]\sqrt{\frac{(1.4+1)}{(2)(1.4)}\left(\frac{102.793}{14.7}-1\right)+1} = 2{,}789\left[\frac{\text{ft}}{\text{s}}\right]$$

We can now calculate the induced velocity behind the wave

$$u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1}-1\right)\sqrt{\frac{\frac{2\gamma}{\gamma+1}}{\frac{\gamma-1}{\gamma+1}+\frac{p_2}{p_1}}} = \frac{(1{,}126)\left[\frac{\text{ft}}{\text{s}}\right]}{(1.4)}\left(\frac{102.793}{14.7}-1\right)\sqrt{\frac{\frac{2(1.4)}{1.4+1}}{\frac{1.4-1}{1.4+1}+\frac{102.793}{14.7}}} = 1{,}946\left[\frac{\text{ft}}{\text{s}}\right]$$

Now we need to see how this would reflect off of the rear wall. We will first find the incident wave Mach number

$$M_s = \frac{U}{a_1} = \frac{(2{,}789)\left[\frac{\text{ft}}{\text{s}}\right]}{(1{,}129)\left[\frac{\text{ft}}{\text{s}}\right]} = 2.47$$

We now need to to find the reflected wave Mach number

$$\frac{M_R}{M_R^2-1} = \frac{M_s}{M_s^2-1}\sqrt{1+\frac{2(\gamma-1)}{(\gamma+1)^2}\left(M_s^2-1\right)\left(\gamma+\frac{1}{M_s^2}\right)}$$

$$\frac{M_R}{M_R^2-1} = \frac{2.47}{(2.47)^2-1}\sqrt{1+\frac{2(1.4-1)}{(1.4+1)^2}\left((2.47)^2-1\right)\left(1.4+\frac{1}{(2.47)^2}\right)} = 0.703$$

We need to write this as a quadratic to solve it

$$M_R^2 - 1.422M_R - 1 = 0 \rightarrow M_R = 1.94$$

Now we need the speed of sound behind the incident shock to get the reflected wave speed. Thus we need to find T_2.

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left(\frac{\frac{\gamma+1}{\gamma-1}+\frac{p_2}{p_1}}{1+\frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1}}\right) = \left(\frac{102.793}{14.7}\right)\left(\frac{\frac{1.4+1}{1.4-1}+\frac{102.793}{14.7}}{1+\left(\frac{1.4+1}{1.4-1}\right)\left(\frac{102.793}{14.7}\right)}\right) = 2.115$$

Thus

$$T_2 = (2.115)(68+460) = 1{,}117^\circ\text{R} = 657^\circ\text{F}$$

This is pretty hot. We then get the speed of sound as

$$a_2 = \sqrt{(1.4)(1{,}716)\left[\frac{\text{ft}-\text{lbf}}{\text{slug}-\text{R}}\right](1{,}117)[\text{R}]} = 1{,}638\left[\frac{\text{ft}}{\text{s}}\right]$$

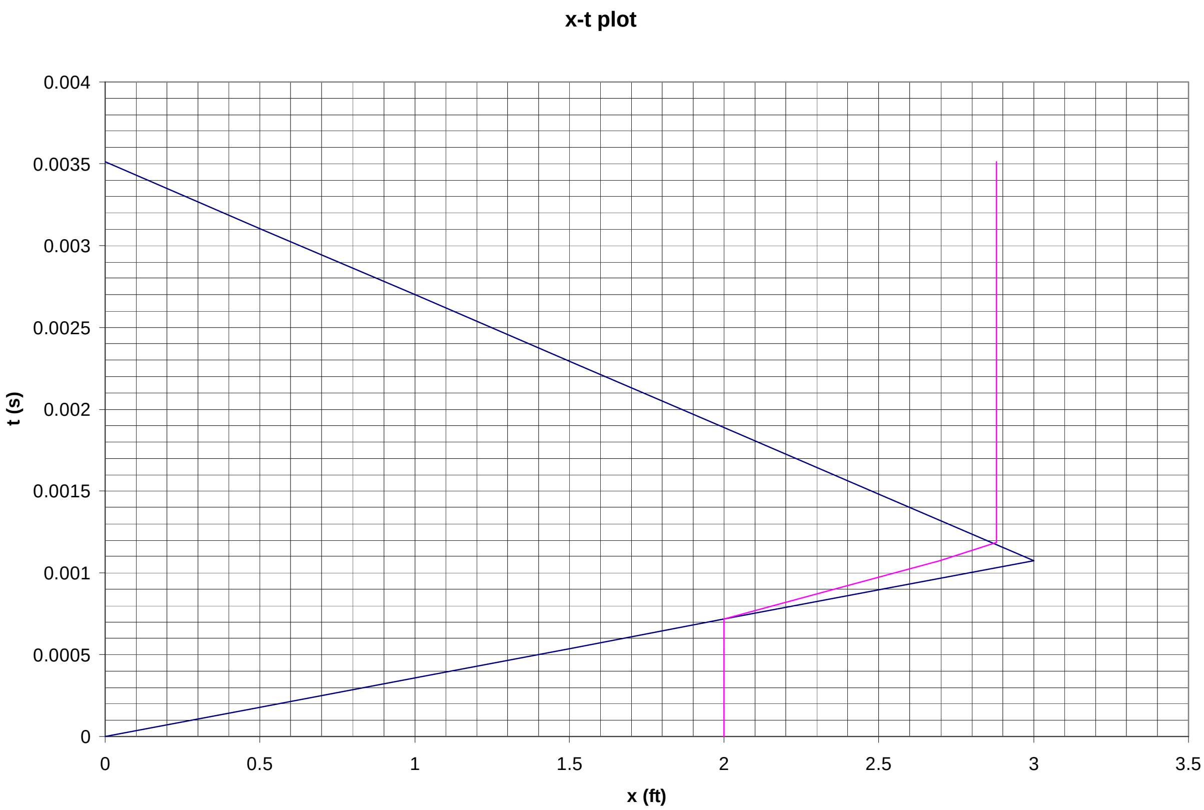
The velocity of the reflected wave would then be

$$U_R = M_R a_2 - u_p = (1.94)(1{,}638)\left[\frac{\text{ft}}{\text{s}}\right] - (1{,}946)\left[\frac{\text{ft}}{\text{s}}\right] = 1{,}232\left[\frac{\text{ft}}{\text{s}}\right]$$

Just as a check, remember that there was an upper limit on u_p thus we can see that

$$M_{u_p} \leq 1.89 = \frac{(1{,}232)\left[\frac{\text{ft}}{\text{s}}\right]}{(1{,}638)\left[\frac{\text{ft}}{\text{s}}\right]} = 0.75$$

The *x-t* plot is as follows



Problem 21 – An explosion generates a shock wave in still air. Assume we are far enough from the initial explosion that we can model the wave as a one-dimensional shock. Assume that the pressure generated by the explosion was 10,000 psi and the ambient atmospheric pressure, density and temperature are 14.7 psi, 0.06 lbm/ft^3 and 68° F, respectively.

Determine a.) The static pressure behind the wave (assume γ = 1.4 and since we are far away from the effects of the explosion assume $a_1/a_4 \approx$ 0.5)

b.) The velocity that the wave propagates in still air
c.) The induced velocity that a building would see after the wave passes
d.) The velocity of a wave reflected normally off a building

Answers a.) p_2 = 376.6 psi; b.) U = 5,294 ft/s; c.) u_p = 4,212 ft/s; d.) U_R = 1,921 ft/s

Solution: We need to iterate to determine the static pressure behind the wave. Since p_2 has to lie between p_1 and p_4 we shall use 200 psi as a starting point. The equation is:

$$\frac{p_4}{p_1} = \frac{p_2}{p_1}\left\{1 - \frac{(\gamma_4 - 1)\left(\frac{a_1}{a_4}\right)\left(\frac{p_2}{p_1} - 1\right)}{\sqrt{2\gamma_1\left[2\gamma_1 + (\gamma_1 + 1)\left(\frac{p_2}{p_1} - 1\right)\right]}}\right\}^{\frac{-2\gamma_4}{(\gamma_4 - 1)}}$$

It is best to set this up in a spread sheet. We have

$$\frac{p_4}{p_1} = \frac{(10{,}000)\left[\frac{\text{lbf}}{\text{in}^2}\right]}{(14.7)\left[\frac{\text{lbf}}{\text{in}^2}\right]} = 680.3$$

Without going into detail, noting that since air is on both sides of the shock wave ($\gamma_4 = \gamma_1$) we have.

Assumed p_2	Calculated p_4/p_1
200	114.175
400	831.675
300	337.124
350	537.794
375	671.005
380	700.791
377	682.786
376	676.874
376.5	679.824
376.6	680.416

We now determine the velocity of the shock wave but before we do we need the speed of sound in the still air.

$$a_1 = \sqrt{\gamma R T}$$

$$a_1 = \sqrt{(1.4)(1716)\left[\frac{\text{ft}-\text{lbf}}{\text{slug}-\text{R}}\right](68+460)[\text{R}]} = 1126\left[\frac{\text{ft}}{\text{s}}\right]$$

We can now calculate the speed of propagation of the wave through still air as

$$U = a_1\sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1}-1\right)+1} = (1126)\left[\frac{\text{ft}}{\text{s}}\right]\sqrt{\frac{(1.4+1)}{(2)(1.4)}\left(\frac{376.6}{14.7}-1\right)+1} = 5294\left[\frac{\text{ft}}{\text{s}}\right]$$

This is a pretty respectable speed. We can now calculate the induced velocity behind the wave.

$$u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1}-1\right)\sqrt{\frac{\frac{2\gamma}{\gamma+1}}{\frac{\gamma-1}{\gamma+1}+\frac{p_2}{p_1}}} = \frac{(1126)\left[\frac{\text{ft}}{\text{s}}\right]}{(1.4)}\left(\frac{376.6}{14.7}-1\right)\sqrt{\frac{\frac{2(1.4)}{1.4+1}}{\frac{1.4-1}{1.4+1}+\frac{376.6}{14.7}}} = 4212\left[\frac{\text{ft}}{\text{s}}\right]$$

This high of a velocity would certainly make itself felt. Now we need to see how this would reflect off of a building. We will first find the incident wave Mach number

$$M_s = \frac{W}{a_1} = \frac{(5294)\left[\frac{\text{ft}}{\text{s}}\right]}{(1129)\left[\frac{\text{ft}}{\text{s}}\right]} = 4.69$$

We now need to use equation (UW-16c) to find the reflected wave Mach number

$$\frac{M_R}{M_R^2-1} = \frac{M_s}{M_s^2-1}\sqrt{1+\frac{2(\gamma-1)}{(\gamma+1)^2}\left(M_s^2-1\right)\left(\gamma+\frac{1}{M_s^2}\right)}$$

$$\frac{M_R}{M_R^2-1} = \frac{4.69}{(4.69)^2-1}\sqrt{1+\frac{2(1.4-1)}{(1.4+1)^2}\left((4.69)^2-1\right)\left(1.4+\frac{1}{(4.69)^2}\right)} = 0.510$$

We need to write this as a quadratic to solve it

$$M_R^2 - 1.96M_R - 1 = 0 \rightarrow M_R = 2.38$$

Now we need the speed of sound behind the incident shock to get the reflected wave speed. Thus we need to find T_2.

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left(\frac{\frac{\gamma+1}{\gamma-1}+\frac{p_2}{p_1}}{1+\frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1}}\right) = \left(\frac{376.6}{14.7}\right)\left(\frac{\frac{1.4+1}{1.4-1}+\frac{376.6}{14.7}}{1+\left(\frac{1.4+1}{1.4-1}\right)\left(\frac{376.6}{14.7}\right)}\right) = 5.24$$

Thus

$$T_2 = (5.24)(68 + 460) = 2764^{\circ}\text{R} = 2304^{\circ}\text{F}$$

This is pretty hot. We then get the speed of sound as

$$a_2 = \sqrt{(1.4)(1716)\left[\frac{\text{ft} - \text{lbf}}{\text{slug} - \text{R}}\right](2304 + 460)[\text{R}]} = 2577\left[\frac{\text{ft}}{\text{s}}\right]$$

The velocity of the reflected wave would then be

$$U_R = M_R a_2 - u_p = (2.38)(2577)\left[\frac{\text{ft}}{\text{s}}\right] - (4212)\left[\frac{\text{ft}}{\text{s}}\right] = 1921\left[\frac{\text{ft}}{\text{s}}\right]$$

Just as a check, remember that there was an upper limit on u_p thus we can see that

$$M_{u_p} \le 1.89 = \frac{(4212)\left[\frac{\text{ft}}{\text{s}}\right]}{(2577)\left[\frac{\text{ft}}{\text{s}}\right]} = 1.63$$

In this example the numbers were fabricated to give you a feel for the problem. Thus these numbers should not be considered indicative of any particular explosive device.