Solutions for Business Mathematics In Canada 9th Edition by Jerome

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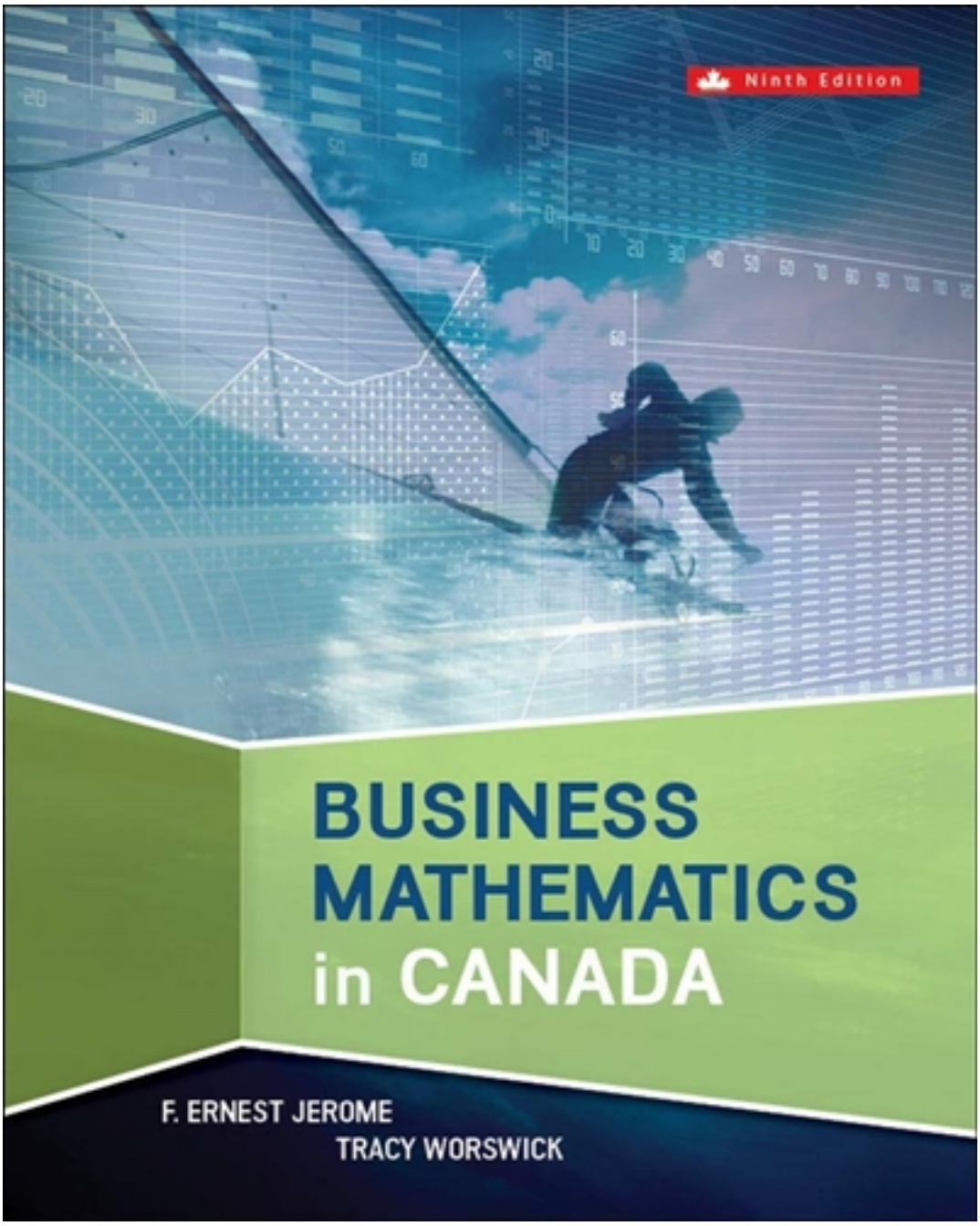

Solutions

2 Review and Applications of Algebra

Exercise 2.1

a. $(-p) + (-3p) + 4p = -p - 3p + 4p = \underline{0}$

b. $5s - 2t - 2s - 4t = 5s - 2s - 2t - 4t = \underline{3s - 6t}$

c. $4x^2y - 3x^2y + (-5x^2y) = 4x^2y - 3x^2y - 5x^2y = \underline{-4x^2y}$

d. $(5s - 2t) - (2s - 4t) = 5s - 2t - 2s + 4t = \underline{3s + 2t}$

e. $4x^2y + (-3x^2y) - (-5x^2y) = 4x^2y - 3x^2y + 5x^2y = \underline{6x^2y}$

f. $1 - (7e^2 - 5 + 3e - e^3) = 1 - 7e^2 + 5 - 3e + e^3 = \underline{e^3 - 7e^2 - 3e + 6}$

g. $(6x^2 - 3xy + 4y^2) - (8y^2 - 10xy - x^2) = 6x^2 - 3xy + 4y^2 - 8y^2 + 10xy + x^2$
$= \underline{7x^2 + 7xy - 4y^2}$

h. $(7m^3 - m - 6m^2 + 10) - (5m^3 - 9 + 3m - 2m^2)$
$= 7m^3 - m - 6m^2 + 10 - 5m^3 + 9 - 3m + 2m^2$
$= \underline{2m^3 - 4m^2 - 4m + 19}$

i. $2(7x - 3y) - 3(2x - 3y) = 14x - 6y - 6x + 9y = \underline{8x + 3y}$

j. $4(a^2 - 3a - 4) - 2(5a^2 - a - 6) = 4a^2 - 12a - 16 - 10a^2 + 2a + 12$
$= \underline{-6a^2 - 10a - 4}$

k. $15x - [4 - 5x - 6] = 15x - 4 + 5x + 6 = \underline{20x + 2}$

l. $6a - [3a - 2b - a] = 6a - 3a + 2b + a = \underline{4a + 2b}$

m. $15x - \left[4 - 2(5x - 6)\right] = 15x - 4 + 10x - 12 = \underline{25x - 16}$

n. $6a - \left[3a - 2(2b - a)\right] = 6a - 3a + 4b - 2a = \underline{a + 4b}$

o. $4a(3ab - 5a + 6b) = \underline{12a^2b - 20a^2 + 24ab}$

p. $9k(4 - 8k + 7k^2) = \underline{36k - 72k^2 + 63k^3}$

q. $-5xy(2x^2 - xy - 3y^2) = \underline{-10x^3y + 5x^2y^2 + 15xy^3}$

r. $-(p^2 - 4pq - 5p)\left(\frac{2q}{p}\right) = \underline{-2pq + 8q^2 + 10q}$

s. $(4r - 3t)(2t + 5r) = 8rt + 20r^2 - 6t^2 - 15rt = \underline{20r^2 - 7rt - 6t^2}$

t. $(3p^2 - 5p)(-4p + 2) = -12p^3 + 6p^2 + 20p^2 - 10p = \underline{-12p^3 + 26p^2 - 10p}$

u. $(4r - 3t) - (2t + 5r) = 4r - 3t - 2t - 5r = \underline{-r - 5t}$

v. $(3p^2 - 5p) + (-4p + 2) = 3p^2 - 5p - 4p + 2 = \underline{3p^2 - 9p + 2}$

w. $3(a - 2)(4a + 1) - 5(2a + 3)(a - 7) = 3(4a^2 + a - 8a - 2) - 5(2a^2 - 14a + 3a - 21)$
$= 12a^2 - 21a - 6 - 10a^2 + 55a + 105$
$= \underline{2a^2 + 34a + 99}$

x. $5(2x - y)(y + 3x) - 6x(x - 5y) = 5(2xy + 6x^2 - y^2 - 3xy) - 6x^2 + 30xy$
$= -5xy + 30x^2 - 5y^2 - 6x^2 + 30xy$

$= \underline{24x^2 + 25xy - 5y^2}$

y. $\dfrac{18x^2}{3x} = \underline{6x}$

z. $\dfrac{6a^2b}{-2ab^2} = \underline{\underline{-3\dfrac{a}{b}}}$

aa. $\dfrac{x^2y - xy^2}{xy} = \underline{x - y}$

bb. $\dfrac{-4x + 10x^2 - 6x^3}{-0.5x} = \underline{8 - 20x + 12x^2}$

cc. $\dfrac{12x^3 - 24x^2 + 36x}{48x} = \underline{\underline{\dfrac{x^2 - 2x + 3}{4}}}$

dd. $\dfrac{32a^2b - 8ab + 14ab^2}{2ab} = \underline{16a - 4 + 7b}$

ee. $\dfrac{4a^2b^3 - 6a^3b^2}{2ab^2} = \underline{2ab - 3a^2}$

ff. $\dfrac{120(1+i)^2 + 180(1+i)^3}{360(1+i)} = \underline{\underline{\dfrac{2(1+i) + 3(1+i)^2}{6}}}$

gg. $\dfrac{4a^2b^3 - 6ab^2 + 2ab^3}{2ab^3} = \underline{\underline{\dfrac{2ab - 3 + b}{b}}}$

hh. $\dfrac{12(1+i) + 18(1+i)^2}{36(1+i)^2} = \underline{\underline{\dfrac{2 + 3(1+i)}{6(1+i)}}}$

1. $3d^2 - 4d + \$15 = 3(\$2.50)^2 - 4(\$2.50) + \$15$
   $= \$18.75 - \$10 + \$15$
   $= \underline{\$23.75}$

2. $15g - 9h + \$3 = 15(\$14) - 9(\$15) + \$3 = \underline{\$78}$

3. $7x(4y - \$8) = 7(\$3.20)(4 \times \$1.50 - \$8) = \$22.4(\$6 - \$8) = \underline{\underline{-\$44.80}}$

4. $I \div Pr = \dfrac{\$13.75}{\$500 \times 0.11} = \underline{0.25}$

5. $\dfrac{I}{rt} = \dfrac{\$23.21}{0.095 \times \frac{283}{365}} = \dfrac{\$23.21}{0.073658} = \underline{\$315.11}$

6. $\dfrac{N}{1-d} = \dfrac{\$89.10}{1-0.10} = \underline{\$99.00}$

7. $L(1 - d_1)(1 - d_2)(1 - d_3) = \$490(1 - 0.125)(1 - 0.15)(1 - 0.05) = \underline{\$346.22}$

8. $P(1 + rt) = \$770\left(1 + 0.013 \times \dfrac{223}{365}\right) = \$770(1.0079425) = \underline{\$776.12}$

9. $\dfrac{S}{1+rt} = \dfrac{\$2500}{1 + 0.085 \times \frac{123}{365}} = \dfrac{\$2500}{1.028644} = \underline{\$2430.38}$

10. $\dfrac{S}{(1+i)^n} = \dfrac{\$850}{(1+0.0075)^6} = \dfrac{\$850}{1.045852} = \underline{\$812.73}$

11. $P(1+i)^n = \$1280(1+0.025)^3 = \underline{\$1378.42}$

12. $\dfrac{x}{2} - x^2 + \dfrac{4}{5} - 0.2x^2 - \dfrac{4}{5}x + \dfrac{1}{2} = 0.5x - x^2 + 0.8 - 0.2x^2 - 0.8x + 0.5$

$= \underline{-1.2x^2 - 0.3x + 1.3}$

13. $\dfrac{2x+9}{4} - 1.2(x-1) = 0.5x + 2.25 - 1.2x + 1.2 = \underline{-0.7x + 3.45}$

14. $\dfrac{2x}{1.045} - \dfrac{2.016x}{3} + \dfrac{x}{2} = 1.9139x - 0.6720x + 0.5x = \underline{1.7419x}$

15. $\dfrac{8x}{0.5} + \dfrac{5.5x}{11} + 0.5(4.6x - 17) = 16x + 0.5x + 2.3x - 8.5 = \underline{18.8x - 8.5}$

16. $y\left(1 - 0.125 \times \dfrac{213}{365}\right) + \dfrac{2y}{1 + 0.125 \times \frac{88}{365}} = 0.92705y + 1.94149y = \underline{2.8685y}$

17. $\dfrac{P}{1 + 0.095 \times \frac{5}{12}} + 2P\left(1 + 0.095 \times \dfrac{171}{365}\right) = 0.96192P + 2.08901P = \underline{3.0509P}$

18. $\dfrac{h}{(1+0.055)^2} - 3h(1+0.055)^3 = 0.89845h - 3.52272h = \underline{-2.6243h}$

19. $k(1+0.04)^2 + \dfrac{2k}{(1+0.04)^2} = 1.08160k + 1.84911k = \underline{2.9307k}$

20. $(1+i)^m - 1 = (1+0.0225)^4 - 1 = \underline{0.093083}$

21. $R\left[\dfrac{(1+i)^n - 1}{i}\right] = \$550\left(\dfrac{1.085^3 - 1}{0.085}\right) = \$550\left(\dfrac{0.2772891}{0.085}\right) = \underline{\$1794.22}$

22. $R\left[\dfrac{(1+i)^n - 1}{i}\right](1+i) = \$910\left(\dfrac{1.1038129^4 - 1}{0.1038129}\right)(1.1038129)$

$= \$910\left(\dfrac{0.4845057}{0.1038129}\right)(1.1038129)$

$= \underline{\$4687.97}$

23. $\dfrac{R}{i}\left[1 - \dfrac{1}{(1+i)^n}\right] = \dfrac{\$630}{0.115}\left(1 - \dfrac{1}{1.115^2}\right) = \underline{\$1071.77}$

24. $P(1 + rt_1) + \dfrac{S}{1 + rt_2} = \$470\left(1 + 0.075 \times \dfrac{104}{365}\right) + \dfrac{\$390}{1 + 0.075 \times \frac{73}{365}}$

$$= \$470(1.021370) + \frac{\$390}{1.01500}$$
$$= \$480.044 + \$384.236$$
$$= \underline{\$864.28}$$

Exercise 2.2

a. $a^2 \times a^3 = \underline{a^5}$

b. $(x^6)(x^{-4}) = \underline{x^2}$

c. $b^{10} \div b^6 = b^{10-6} = \underline{b^4}$

d. $h^7 \div h^{-4} = h^{7-(-4)} = \underline{h^{11}}$

e. $(1 + i)^4 \times (1 + i)^9 = \underline{(1 + i)^{13}}$

f. $(1 + i) \times (1 + i)^n = \underline{(1 + i)^{n+1}}$

g. $(x^4)^7 = x^{4x7} = \underline{x^{28}}$

h. $(y^3)^3 = \underline{y^9}$

i. $\left(t^6\right)^{\frac{1}{3}} = \underline{t^2}$

j. $(n^{0.5})^8 = \underline{n^4}$

k. $\dfrac{\left(x^5\right)\left(x^6\right)}{x^9} = x^{5+6-9} = \underline{x^2}$

l. $\dfrac{\left(x^5\right)^6}{x^9} = x^{5\times6-9} = \underline{x^{21}}$

m. $\left[2(1+i)\right]^2 = \underline{4(1+i)^2}$

n. $\left(\dfrac{1+i}{3i}\right)^3 = \underline{\dfrac{(1+i)^3}{27i^3}}$

o. $\dfrac{4r^5t^6}{\left(2r^2t\right)^3} = \dfrac{4r^5t^6}{8r^6t^3} = \dfrac{r^{5-6}t^{6-3}}{2} = \underline{\dfrac{t^3}{2r}}$

p. $\dfrac{\left(-r^3\right)(2r)^4}{\left(2r^{-2}\right)^2} = \dfrac{-r^3\left(16r^4\right)}{4r^{-4}} = -4r^{3+4-(-4)} = \underline{-4r^{11}}$

1. $8^{4/3} = \left(8^{1/3}\right)^4 = 2^4 = \underline{16.0000}$

2. $-27^{2/3} = -\left(27^{1/3}\right)^2 = \underline{-9.00000}$

3. $7^{3/2} = 7^{1.5} = \underline{18.5203}$

4. $5^{-\frac{3}{4}} = 5^{-0.75} = \underline{0.299070}$

5. $(0.001)^{-2} = \underline{1{,}000{,}000}$

6. $0.893^{-1/2} = 0.893^{-0.5} = \underline{1.05822}$

7. $(1.0085)^5(1.0085)^3 = 1.0085^8 = \underline{1.07006}$

8. $(1.005)^3(1.005)^{-6} = 1.005^{-3} = \underline{0.985149}$

9. $\sqrt[3]{103} = 103^{0.\overline{3}} = \underline{100990}$

10. $\sqrt[6]{105} = \underline{100816}$

11. $(4^4)(3^{-3})\left(-\frac{3}{4}\right)^3 = \frac{4^4}{3^3}\left(-\frac{3^3}{4^3}\right) = \underline{-4.00000}$

12. $\left[\left(-\frac{3}{4}\right)^2\right]^{-2} = \left(-\frac{3}{4}\right)^{-4} = \left(-\frac{4}{3}\right)^4 = \frac{256}{81} = \underline{3.16049}$

13. $\left(\frac{2}{3}\right)^3\left(-\frac{3}{2}\right)^2\left(-\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3\left(\frac{3}{2}\right)^2\left(-\frac{2}{3}\right)^3 = \frac{2}{3}\left(-\frac{2}{3}\right)^3 = -\frac{16}{81} = \underline{-0.197531}$

14. $\left(-\frac{2}{3}\right)^3, \left(\frac{3}{2}\right)^2 = \frac{\left(-\frac{2}{3}\right)^3}{\left(\frac{2}{3}\right)^2} = -\frac{2}{3} = \underline{-0.666667}$

15. $\frac{103^{16} - 1}{0.03} = \underline{20.1569}$

16. $\frac{\left(1008\overline{3}\right)^{30} - 1}{0.008\overline{3}} = \frac{0.2826960}{0.008333333} = \underline{33.9235}$

17. $\frac{1 - 10225^{-20}}{0.0225} = \frac{0.3591835}{0.0225} = \underline{15.9637}$

18. $\frac{1 - \left(100\overline{6}\right)^{-32}}{0.00\overline{6}} = \frac{0.1915410}{0.00\overline{6}} = \underline{28.7312}$

19. $(1 + 0.0275)^{1/3} = \underline{1.00908}$

20. $(1 + 0.055)^{1/6} - 1 = \underline{0.00896339}$

Exercise 2.3

a) $2a - 9 = a + 1$
$2a - a = 1 + 9$
$a = \underline{10}$

b) $5 + 3x = 20$
$3x = 20 - 5$
$3x = 15$
$x = \underline{5}$

c) $6(y - 4) = 0$
$6y - 24 = 0$
$6y = 24$
$y = \underline{4}$

d) $\frac{b}{3} - 1 = 5$
$\frac{b}{3} = 6$
$b = \underline{18}$

e) $-7x - 10 = 11$
$-7x = 21$
$x = \underline{-3}$

f) $4 = \frac{10-m}{3}$
$12 = 10 - m$
$m = 10 - 12$
$m = \underline{-2}$

g) $12 - 3x - 8 = -10x - 10$
$4 - 3x = -10x - 10$
$-3x + 10x = -10 - 4$
$7x = -14$
$x = \underline{-2}$

h) $2(4x - 5) = x - 3$
$8x - 10 = x - 3$
$8x - x = -3 + 10$
$7x = 7$
$x = \underline{1}$

i) $\frac{x+2}{x-1} = 4$
$x + 2 = 4(x - 1)$
$x + 2 = 4x - 4$
$x - 4x = -4 - 2$
$-3x = -6$
$x = \underline{2}$

j) $\frac{15p}{2} = 7p + 4$
$15p = 2(7p + 4)$
$15p = 14p + 8$
$15p - 14p = 8$
$p = \underline{8}$

1. $10a + 10 = 12 + 9a$
$10a - 9a = 12 - 10$
$a = \underline{2}$

2. $29 - 4y = 2y - 7$
$36 = 6y$
$y = \underline{6}$

3. $0.5 (x - 3) = 20$

$x - 3 = 40$
$x = \underline{43}$

4. $\frac{1}{3}(x-2) = 4$
$x - 2 = 12$
$x = \underline{14}$

5. $y = 192 + 0.04y$
$y - 0.04y = 192$
$y = \frac{192}{0.96} = \underline{200}$

6. $x - 0.025x = 341.25$
$0.975x = 341.25$
$x = \frac{341.25}{0.975} = \underline{350}$

7. $12x - 4(2x - 1) = 6(x + 1) - 3$
$12x - 8x + 4 = 6x + 6 - 3$
$-2x = -1$
$x = \underline{0.5}$

8. $3y - 4 = 3(y + 6) - 2(y + 3)$
$= 3y + 18 - 2y - 6$
$2y = 16$
$y = \underline{8}$

9. $8 - 0.5(x + 3) = 0.25(x - 1)$
$8 - 0.5x - 1.5 = 0.25x - 0.25$
$-0.75x = -6.75$
$x = \underline{9}$

10. $5(2 - c) = 10(2c - 4) - 6(3c + 1)$
$10 - 5c = 20c - 40 - 18c - 6$
$-7c = -56$
$c = \underline{8}$

11. $3.1t + 145 = 10 + 7.6t$
$-4.5t = -135$
$t = \underline{30}$

12. $1.25y - 20.5 = 0.5y - 11.5$
$0.75y = 9$
$y = \underline{12}$

13. $\frac{x+2}{5} = x + 0.8$
$x + 2 = 5(x + 0.8)$
$x + 2 = 5x + 4$
$x - 5x = 4 - 2$
$-4x = 2$
$x = \underline{-0.5}$

14. $\frac{3.5}{x-1} = 2.5$

3.5 = 2.5(x – 1)
3.5 = 2.5x – 2.5
-2.5x = -2.5 – 3.5
-2.5x = -6
x = 2.4

15. $\frac{10a}{2.2} + (2.2)^2 = 6 + a(2.2)^3$

$4.\overline{54}a + 4.84 = 6 + 10.648a$
$4.\overline{54}a$ - 10.648a = 6 – 4.84
-6.1025a = 1.16
a = -0.19

16. $21 - \frac{b}{1.45} = 5.5b - 9$
21 – 0.6897b = 5.5b – 9
-0.6897b-5.5b = -9 – 21
-6.1897b = -30
b = 4.85

17. $(1.065)^2x - \frac{x}{1.065} = \$35$
1.134225x – 0.93897x = $35
0.19526x = $35
x = $179.25

18. $12x - 4x(1.06)^4 = \$1800$
12x – 5.04991x = $1800
6.95001x = $1800
x = $258.99

19. $\frac{x}{1.1^2} + 2x(1.1)^3 = \$1000$
0.8264463x + 2.662x = $1000
3.488446x = $1000
x = $286.66

20. $\frac{3x}{1.025^6} + x(1.025)^8 = \$2641.35$
2.586891x + 1.218403x = $2641.35
x = $694.13

21. $\frac{2x}{1.03^7} + x + x\left(1.03^{10}\right) = \$1000 + \frac{\$2000}{1.03^4}$
1.626183x + x + 1.343916x = $1000 + $1776.974
3.970099x = $2776.974
x = $699.47

22. $x(1.05)^3 + \$1000 + \frac{x}{1.05^7} = \frac{\$5000}{1.05^2}$
1.157625x + 0.7106813x = $4535.147 – $1000
x = $1892.17

23. $x\left(1+0.095\times\frac{84}{365}\right)+\frac{2x}{1+0.095\times\frac{108}{365}}=\$1160.20$

$1.021863x + 1.945318x = \$1160.20$

$2.967181x = \$1160.20$

$x = \underline{\$391.01}$

24. $\frac{x}{1+0.115\times\frac{78}{365}}+3x\left(1+0.115\times\frac{121}{365}\right)=\$1000\left(1+0.115\times\frac{43}{365}\right)$

$0.9760141x + 3.114370x = \$1013.548$

$x = \underline{\$247.79}$

Exercise 2.4

1. $I = Prt$

$\$6.25 = P(0.05)0.25$

$\$6.25 = 0.0125P$

$P = \frac{\$6.25}{0.0125} = \underline{\$500.00}$

2. $PV = \frac{PMT}{i}$

$\$150{,}000 = \frac{\$900}{i}$

$\$150{,}000i = \$900$

$i = \frac{\$900}{\$150{,}000} = \underline{0.006}$

3. $S = P(1 + rt)$

$\$3626 = P(1 + 0.004\times9)$

$\$3626 = 1.036P$

$P = \frac{\$3626}{1.036} = \underline{\$3500.00}$

4. $N = L(1 – d)$

$\$891 = L(1 – 0.10)$

$\$891 = 0.90L$

$L = \frac{\$891}{0.90} = \underline{\$990.00}$

5. $N = L(1 – d)$

$\$410.85 = \$498(1 – d)$

$\frac{\$410.85}{\$498} = 1 – d$

$0.825 = 1 – d$

$d = 1 – 0.825 = \underline{0.175}$

6. $S = P(1 + rt)$

$\$5100 = \$5000(1 + 0.0025t)$

$\$5100 = \$5000 + \$12.5t$

$\$5100 – \$5000 = \$12.5t$

$t = \frac{\$100}{\$12.5} = \underline{8}$

7. $NI = (CM)X - FC$

$\$15{,}000 = CM(5000) - \$60{,}000$

$\$15{,}000 + \$60{,}000 = 5000CM$

$$CM = \frac{\$75{,}000}{5000} = \underline{\$15.00}$$

8. $NI = (CM)X - FC$

$-\$542.50 = (\$13.50)X - \$18{,}970$

$\$18{,}970 - \$542.50 = (\$13.50)X$

$$X = \frac{\$18{,}427.50}{\$13.50} = \underline{1365}$$

9. $N = L(1-d_1)(1-d_2)(1-d_3)$

$\$1468.80 = L(1 - 0.20)(1 - 0.15)(1 - 0.10)$

$\$1468.80 = L(0.80)(0.85)(0.90)$

$$L = \frac{\$1468.80}{0.6120} = \underline{\$2400.00}$$

10. $N = L(1-d_1)(1-d_2)(1-d_3)$

$\$70.29 = \$99.99(1 - 0.20)(1 - d_2)(1 - 0.05)$

$\$70.29 = \$75.9924(1 - d_2)$

$$\frac{\$70.29}{\$75.9924} = (1 - d_2)$$

$d_2 = 1 - 0.92496 = \underline{0.0750}$

11. $FV = PV(1+i_1)(1+i_2)(1+i_3)$

$\$1094.83 = \$1000(1 + i_1)(1 + 0.03)(1 + 0.035)$

$\$1094.83 = \$1066.05(1 + i_1)$

$$\frac{\$1094.83}{\$1066.05} = 1 + i_1$$

$i_1 = 1.02700 - 1 = \underline{0.0270}$

12. $$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\$1508.54 = PMT\left[\frac{(1+0.05)^4 - 1}{0.05}\right]$$

$$\$1508.54 = PMT\left(\frac{1.21550625 - 1}{0.05}\right)$$

$$PMT = \$1508.54 \times \frac{0.05}{0.21550625} = \underline{\$350.00}$$

13. $$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$\$6595.20 = PMT\left[\frac{1-(1+0.06)^{-20}}{0.06}\right]$$

$$\$6595.20 = PMT\left[\frac{1-0.31180473}{0.06}\right]$$

$$PMT = \$6595.20 \times \frac{0.06}{0.68819527} = \underline{\underline{\$575.00}}$$

14. $I = Prt$

$$\frac{I}{Pr} = \frac{Prt}{Pr}$$

$$t = \frac{I}{Pr}$$

15. $PV = \frac{PMT}{i}$

$$i(PV) = PMT$$

$$i = \frac{PMT}{PV}$$

16. $N = L(1-d)$

$$\frac{N}{L} = 1-d$$

$$d = 1-\frac{N}{L}$$

17. $NI = (CM)X - FC$

$$NI + FC = (CM)X$$

$$CM = \frac{NI + FC}{X}$$

18. $NI = (CM)X - FC$

$$NI + FC = (CM)X$$

$$X = \frac{NI + FC}{CM}$$

19. $S = P(1 + rt)$

$$S = P + Prt$$

$$S - P = Prt$$

$$r = (S - P)/Pt$$

20. $S = P(1 + rt)$

$$S = P + Prt$$

$$S - P = Prt$$

$$t = (S - P)/Pr$$

21. $N = L(1-d_1)(1-d_2)(1-d_3)$

$$\frac{N}{L(1-d_2)(1-d_3)} = (1-d_1)$$

$$d_1 = 1-\frac{N}{L(1-d_2)(1-d_3)}$$

22. $N = L(1-d_1)(1-d_2)(1-d_3)$

$$\frac{N}{L(1-d_1)(1-d_2)} = (1-d_3)$$

$$d_3 = 1-\frac{N}{L(1-d_1)(1-d_2)}$$

23. $FV = PV(1+i)^n$

$$\frac{FV}{(1+i)^n} = PV$$

$$PV = FV(1+i)^{-n}$$

24. $FV = PV(1+i)^n$

$\$9321.91 = \$2000(1 + i)^{20}$

$$\left(\frac{\$9321.91}{\$2000}\right)^{1/20} = 1 + i$$

$1.0800 = 1 + i$

$i = 1.08000 - 1 = \underline{0.08}$

$$PV = FV(1+i)^{-n}$$

$\$5167.20 = \$10{,}000(1 + i)^{-15}$

$$\frac{\$5167.20}{\$10{,}000} = \frac{1}{(1+i)^{15}}$$

$$(1+i)^{15} = \frac{\$10{,}000}{\$5167.20}$$

$1 + i = (1.935284)^{1/15} = 1.0450$

$i = \underline{0.045}$

25. $FV = PV(1+i)^{n}$

$$\left(\frac{FV}{PV}\right)^{1/n} = (1+i)$$

$$i = \left(\frac{FV}{PV}\right)^{1/n} - 1$$

Exercise 2.5

a. Step 2: Students on 4 buses = 177 less 9 in cars
Let number of students per bus be x.
Step 3: 4 buses x students per bus = students on buses less students in cars
Step 4: 4x = 177 – 9
Step 5: 4x = 168
x = 42
Each bus holds <u>42 students</u>.

b. Step 2: Cost of binder = \$9.50, Purchased 4 pencils
Let the cost of one pencil be p.
Step 3: Cost of order = cost of binder + cost of 4 pencils
Step 4: \$12.50 = \$9.50 + 4p
Step 5: 4p = \$12.50 - \$9.50
4p = \$3.00
p = \$0.75
Each pencil cost <u>\$0.75.</u>

c. Step 2: Weekly earnings = \$600 plus 2% commission on sales, Weekly salary = \$400
Let s represent weekly sales.
Step 3: Weekly earnings = weekly salary + 2% commission on sales
Step 4: \$600 = \$400 + 0.02s
Step 5: 0.02s = \$600 - \$400
0.02s = \$200
s = \$10,000
The salesman's weekly <u>sales were \$10,000</u>.

d. Step 2: Samuel's winnings = \$600 after sharing \$200 with each sibling.
Let s be the number of siblings.
Step 3: Total winnings = Samuel's winnings + siblings winnings

Step 4: $2000 = $600 + $200s

Step 5: $200s = $2000 - $600

$200s = $1400

s = 7

Samuel has 7 brothers and sisters.

e. Step 2: Money in pocket = $170 after ½ used to pay bill plus an extra $50.

Let c be the amount of his pay cheque.

Step 3: Money in pocket = ½ pay cheque + extra money earned

Step 4: $170 = 0.5c + $50

0.5c = $170 - $50

0.5c = $120

c = $240

Josh's weekly pay cheque is $240.

1. Step 2: Hits last month = 2655 after the $\frac{2}{7}$ increase.

Let the number of hits 1 year ago be n.

Step 3: Hits last month = Hits 1 year ago + $\frac{2}{7}$ (Hits 1 year ago)

Step 4: $2655 = n + \frac{2}{7}n$

Step 5: $2655 = \frac{9}{7}n$

Multiply both sides by $\frac{7}{9}$.

$n = 2655 \times \frac{7}{9} = 2065$

The Web site had 2065 hits in the same month 1 year ago.

2. Step 2: Retail price = $712; Markup = 60% of wholesale of cost.

Let the wholesale cost be C.

Step 3: Retail price = Cost + 0.60(Cost)

Step 4: $712 = C + 0.6C

Step 5: $712 = 1.6C

$C = \frac{\$712}{1.6} = \$445.00$. The wholesale cost is $445.00.

3. Step 2: Tag price = $39.55 (including 13% HST). Let the plant's pretax price be P.

Step 3: Tag price = Pre-tax price + HST

Step 4: $39.55 = P + 0.13P

Step 5: $39.55 = 1.13P

$P = \frac{\$39.55}{1.13} = \$35.00$

The amount of HST is $39.55 – $35.00 = $4.55

4. Step 2: Commission rate = 2.5% on the first $5000 and 1.5% on the remainder

Commission amount = $227. Let the transaction amount be x.

Step 3: Commission amount = 0.025($5000) + 0.015(Remainder)

Step 4: $227 = $125.00 + 0.015(x – $5000)

Step 5: $102 = 0.015x – $75.00

$102 + $75 = 0.015x

$$x = \frac{\$177}{0.015} = \underline{\$11{,}800.00}$$

The amount of the transaction was $11,800.00.

5. Step 2: Let the basic price be P. First 20 meals at P.
Next 20 meals at P – $2. Additional meals at P – $3.

Step 3: Total price for 73 meals = $1686

Step 4: 20P + 20(P – $2) + (73 – 40)(P – $3) = $1686

Step 5: 20P + 20P – $40 + 33P – $99 = $1686
73P = $1686 + $99 + $40

$$P = \frac{\$1825}{73} = \underline{\$25.00}$$

The basic price per meal is $25.00.

6. Step 2: Rental Plan 1: $295 per week + $0.15 × (Distance in excess of 1000 km)
Rental Plan 2: $389 per week
Let d represent the distance at which the costs of both plans are equal.

Step 3: Cost of Plan 1 = Cost of Plan 2

Step 4: $295 + $0.15($d$ – 1000) = $389

Step 5: $295 + 0.15d$ – $150 = $389
0.15d$ = $244
d = 1627 km

The unlimited driving plan will be cheaper if you drive more than 1626.7 km in the one-week interval.

7. Step 2: Tax rate = 38%; Overtime hourly rate = 1.5($23.50) = $35.25
Cost of canoe = $2750
Let h represent the hours of overtime Alicia must work.

Step 3: Gross overtime earnings – Income tax = Cost of the canoe

Step 4: 35.25h$ – 0.38(35.25h$) = $2750

Step 5: 21.855h$ = $2750
h = 125.83 hours

Alicia must work 125¾ hours of overtime to earn enough money to buy the canoe.

8. Step 2: Number of two-bedroom homes = 0.4(Number of three-bedroom homes)
Number of two-bedroom homes = 2(Number of four-bedroom homes)
Total number of homes = 96
Let h represent the number of two-bedroom homes

Step 3: # 2-bedroom homes + # 3-bedroom homes + # 4-bedroom homes = 96

Step 4: $$h + \frac{h}{0.4} + \frac{h}{2} = 96$$

Step 5: h + 2.5h + 0.5h = 96
4h = 96
h = 24

There should be 24 two-bedroom homes, 2.5(24) = 60 three-bedroom homes, and 0.5(24) = 12 four-bedroom homes.

9. Step 2: Cost of radio advertising = 0.5(Cost of newspaper advertising)
Cost of TV advertising = 0.6(Cost of radio advertising)
Total advertising budget = $160,000

Let r represent the amount allocated to radio advertising

Step 3: Radio advertising + TV advertising + Newspaper advertising = \$160,000

Step 4: $r + 0.6r + \frac{r}{0.5} = \$160{,}000$

Step 5: $3.6r = \$160{,}000$

$r = \$44{,}444.44$

The advertising budget allocations should be:

\$44,444 to radio advertising,

0.6(\$44,444.44) = \$26,667 to TV advertising, and

2(\$44,444.44) = \$88,889 to newspaper advertising.

10. Step 2: By-laws require: 5 parking spaces per 100 square meters,
4% of spaces for physically handicapped
In remaining 96%, # regular spaces = 1.4(# small car spaces)
Total area = 27,500 square meters

Let s represent the number of small car spaces.

Step 3: Total # spaces = # spaces for handicapped + # regular spaces + # small spaces

Step 4: $\frac{27{,}500}{100} \times 5 = 0.04 \times \frac{27{,}500}{100} \times 5 + s + 1.4s$

Step 5: $1375 = 55 + 2.4s$

$s = 550$

The shopping centre must have 55 parking spaces for the physically handicapped, 550 small-car spaces, and 770 regular parking spaces.

11. Step 2: Overall portfolio's rate return = 1.1%, equity fund's rate of return = –3.3%, bond fund's rate of return = 7.7%.
Let e represent the fraction of the portfolio initially invested in the equity fund.

Step 3: Overall rate of return = Weighted average rate of return
= (Equity fraction)(Equity return) + (Bond fraction)(Bond return)

Step 4: $1.1\% = e(-3.3\%) + (1 - e)(7.7\%)$

Step 5: $1.1\% = -3.3\%e + 7.7\% - 7.7\%e$

$-6.6\% = -11.0\%e$

$e = 0.600$

Therefore, 60.0% of Erin's original portfolio was invested in the equity fund.

12. Step 2: Pile A steel is 5.25% nickel; pile B steel is 2.84% nickel.
We want a 32.5-tonne mixture from A and B averaging 4.15% nickel.
Let A represent the tonnes of steel required from pile A.

Step 3: Wt. of nickel in 32.5 tonnes of mixture
= Wt. of nickel in steel from pile A + Wt. of nickel in steel from pile B
= (% nickel in pile A)(Amount from A) + (% nickel in pile B)(Amount from B)

Step 4: $0.0415(32.5) = 0.0525A + 0.0284(32.5 - A)$

Step 5: $1.34875 = 0.0525A + 0.9230 - 0.0284A$

$0.42575 = 0.0241A$

$A = 17.67$ tonnes

The recycling company should mix 17.67 tonnes from pile A with 14.83 tonnes from pile B.

13. Step 2: Total options = 100,000
# of options to an executive = 2000 + # of options to a scientist or engineer
# of options to a scientist or engineer = 1.5(# of options to a technician)
There are 3 executives, 8 scientists and engineers, and 14 technicians.

Let t represent the number of options to each technician.

Step 3: Total options = Total options to scientists and engineers
+ Total options to technicians + Total options to executives

Step 4: $100{,}000 = 8(1.5t) + 14t + 3(2000 + 1.5t)$

Step 5: $= 12t + 14t + 6000 + 4.5t$

$94{,}000 = 30.5t$

$t = 3082$ options

Each technician will receive 3082 options,
each scientist and engineer will receive $1.5(3082) =$ 4623 options,
and each executive will receive $2000 + 4623 =$ 6623 options.

14. Step 2: Plan X: 6.5 cents/minute (in business hours) and 4.5 cents/minute (at other times)
Plan Y: 5.3 cents/minute any time
Let b represent the fraction of business-hour usage at which costs are equal.

Step 3: Cost of Plan X = Cost of plan Y

Step 4: Pick any amount of usage in a month—say 1000 minutes.
$b(1000)\$0.065 + (1 - b)(1000)\$0.045 = 1000(\$0.053)$

Step 5: $\$65b + \$45 - \$45b = \$53$

$\$20b = \$8$

$b = 0.40$

If business-hour usage exceeds 40% of overall usage, plan Y will be cheaper.

15. Step 2: Raisins cost \$3.75 per kg; peanuts cost \$2.89 per kg.
Cost per kg of ingredients in 50 kg of "trail mix" is to be \$3.20.
Let p represent the weight of peanuts in the mixture.

Step 3: Cost of 50 kg of trail mix = Cost of p kg peanuts + Cost of $(50 - p)$ kg of raisins

Step 4: $50(\$3.20) = p(\$2.89) + (50 - p)(\$3.75)$

Step 5: $\$160.00 = \$2.89p + \$187.50 - \$3.75p$

$-\$27.50 = -\$0.86p$

$p = 31.98$ kg

32.0 kg of peanuts should be mixed with 18.0 kg of raisins.

16. Step 2: Total bill = \$3310. Total hours = 41.
Hourly rate = \$120 for CGA
= \$50 for technician.
Let x represent the CGA's hours.

Step 3: Total bill = (CGA hours x CGA rate) + (Technician hours x Technician rate)

Step 4: $\$3310 = x(\$120) + (41 - x)\$50$

Step 5: $\$3310 = \$120x + \$2050 - \$50x$

$1260 = 70x$

$x = 18$

The CGA worked 18 hours and the technician worked $41 - 18 =$ 23 hours.

17. Step 2: Total investment = \$32,760
Sue's investment = 1.2(Joan's investment)
Joan's investment = 1.2(Stella's investment)
Let L represent Stella's investment.

Step 3: Sue's investment + Joan's investment + Stella's investment = Total investment

Step 4: Joan's investment = 1.2L
Sue's investment = 1.2(1.2L) = 1.44L
1.44L + 1.2L + L = $32,760

Step 5: 3.64L = $32,760

$$L = \frac{\$32{,}760}{3.64} = \$9000$$

Stella will invest $9000, Joan will invest 1.2($9000) = $10,800, and Sue will invest 1.2($10,800) = $12,960

18. Step 2: Sven receives 30% less than George (or 70% of George's share).
Robert receives 25% more than George (or 1.25 times George's share).
Net income = $88,880
Let G represent George's share.

Step 3: George's share + Robert's share + Sven's share = Net income

Step 4: G + 1.25G + 0.7G = $88,880

Step 5: 2.95G = $88,880
G = $30,128.81
George's share is $30,128.81, Robert's share is 1.25($30,128.81) = $37,661.02, and Sven's share is 0.7($30,128.81) = $21,090.17.

19. Step 2: Time to make X is 20 minutes.
Time to make Y is 30 minutes.
Total time is 47 hours. Total units = 120. Let Y represent the number of units of Y.

Step 3: Total time = (Number of X) $\times$ (Time for X) + (Number of Y) $\times$ (Time for Y)

Step 4: 47 $\times$ 60 = (120 – Y)20 + Y(30)

Step 5: 2820 = 2400 – 20Y + 30Y
420 = 10Y
Y = 42
Forty-two units of product Y were manufactured.

20. Step 2: Price of blue ticket = $19.00. Price of red ticket = $25.50.
Total tickets = 4460. Total revenue = $93,450.
Let the number of tickets in the red section be R.

Step 3: Total revenue = (Number of red $\times$ Price of red) + (Number of blue $\times$ Price of blue)

Step 4: $93,450 = R($25.50) + (4460 – R)$19.00

Step 5: 93,450 = 25.5R + 84,740 – 19R
6.5R = 8710
R = 1340
1340 seats were sold in the red section and 4460 – 1340 = 3120 seats were sold in the blue section.

21. Step 2: Each of 4 children receive 0.5(Wife's share).

Each of 13 grandchildren receive $0.\overline{3}$ (Child's share).
Total distribution = $759,000. Let w represent the wife's share.

Step 3: Total amount = Wife's share + 4(Child's share) + 13(Grandchild's share)

Step 4: $759,000 = w + 4(0.5w) + 13$\left(0.\overline{3}\right)$(0.5w)

Step 5: $759,000 = w + 2w + 2.1\overline{6}w$

$= 5.1\overline{6}w$

$w = \$146{,}903.23$

Each child will receive $0.5(\$146{,}903.23) = \$73{,}451.62$

and each grandchild will receive $0.\overline{3}(\$73{,}451.62) = \$24{,}483.87$.

22. Step 2: Stage B workers = 1.6(Stage A workers)
Stage C workers = 0.75(Stage B workers)
Total workers = 114. Let A represent the number of Stage A workers.

Step 3: Total workers = A workers + B workers + C workers

Step 4: $114 = A + 1.6A + 0.75(1.6A)$

Step 5: $114 = 3.8A$

$A = 30$

30 workers should be allocated to Stage A, $1.6(30) = 48$ workers to Stage B, and $114 - 30 - 48 = 36$ workers to Stage C.

23. Step 2: Hillside charge = 2(Barnett charge) – \$1000
Westside charge = Hillside charge + \$2000
Total charges = \$27,600. Let B represent the Barnett charge.

Step 3: Total charges = Barnett charge + Hillside charge + Westside charge

Step 4: $\$27{,}600 = B + 2B - \$1000 + 2B - \$1000 + \$2000$

Step 5: $\$27{,}600 = 5B$

$B = \$5520$

Hence, the Westside charge is $2(\$5520) - \$1000 + \$2000 = \$12{,}040$

24. Step 2: There are 3 managers and 26 production workers. Total distribution = \$100,000.
Manager's share = 1.2 (Production worker's share).
Let p represent a production worker's share.

Step 3: 3(Manager's share) + 26(Production worker's share) = \$100,000

Step 4: $3(1.2p) + 26p = \$100{,}000$

Step 5: $29.6p = \$100{,}000$

$p = \$3378.38$

Each production worker will receive \$3378.38 and each manager will receive $1.2(\$3378.38) = \$4054.05$.

Concept Questions (Section 2.6)

1. If the portion is 4 times the size of the base then *rate* $= \frac{4}{1} \times 100 = 400\%$
2. If the portion is $\frac{1}{1000}$th of the size of the base then *rate* $= \frac{\frac{1}{1000}}{1} \times 100 = \frac{1}{10}$
3. If the percent rate is 1000% and the base is 1 then the portion is 10 times the base.
4. If the percent rate is 0.01% and the base is 1 then the portion is $\frac{1}{10{,}000}$th the size of the base.

Exercise 2.6

1. $Rate = \frac{Portion}{Base} \Rightarrow 0.0175 = \frac{Portion}{\$350}$

$Portion = 0.0175 \times \$350 = \underline{\$6.13}$

2. $Rate = \frac{Portion}{Base} \Rightarrow 0.06\overline{6} = \frac{Portion}{\$666.66}$

$Portion = 0.066667 \times \$666.66 = \underline{\$44.44}$

3. $Rate = \frac{Portion}{Base} = \frac{\$1.50}{\$11.50} = 0.130 = \underline{13.0\%}$

4. $Rate = \frac{Portion}{Base} = \frac{\$0.88}{\$44.00} = 0.0200 = \underline{2.00\%}$

5. $Rate = \frac{Portion}{Base} \Rightarrow 0.60 = \frac{\$45}{Base}$

$0.60(Base) = \$45$

$Base = \frac{\$45}{0.60} = \underline{\$75.00}$

6. $Rate = \frac{Portion}{Base} \Rightarrow 0.30 = \frac{\$69}{Base}$

$0.30(Base) = \$69$

$Base = \frac{\$69}{0.30} = \underline{\$230.00}$

7. $Rate = \frac{Portion}{Base} \Rightarrow 2.333 = \frac{Portion}{\$75}$

$Portion = 2.333 \times \$75 = \underline{\$174.98}$

8. $Rate = \frac{Portion}{Base} \Rightarrow 0.00075 = \frac{Portion}{\$1650}$

$Portion = 0.00075 \times \$1650 = \underline{\$1.24}$

9. $Rate = \frac{Portion}{Base} = \frac{\$134}{\$67} = 2.00 = \underline{200\%}$

10. $Rate = \frac{Portion}{Base} = \frac{\$1.34}{\$655} = 0.00205 = \underline{0.205\%}$

11. $Rate = \frac{Portion}{Base} \Rightarrow 1.50 = \frac{Portion}{\$60}$

$Portion = 1.50 \times \$60 = \underline{\$90.00}$

12. $Rate = \frac{Portion}{Base} \Rightarrow 0.00583 = \frac{Portion}{\$1500}$

$Portion = 0.00583 \times \$1500 = \underline{\$8.75}$

13. $Rate = \frac{Portion}{Base} \Rightarrow 0.075 = \frac{\$1.46}{Base}$

$0.075(Base) = \$1.46$

$$Base = \frac{\$1.46}{0.075} = \underline{\$19.47}$$

14. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.1275 = \frac{\$27.50}{Base}$

$$0.1275(Base) = \$27.50$$

$$Base = \frac{\$27.50}{0.1275} = \underline{\$215.69}$$

15. $Rate = \frac{Portion}{Base} = \frac{\$590}{\$950} = 0.621 = \underline{62.1\%}$

16. $Rate = \frac{Portion}{Base} = \frac{\$950}{\$590} = 1.61 = \underline{161\%}$

17. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.95 = \frac{\$100}{Base}$

$$Base = \frac{\$100}{0.95} = \underline{\$105.26}$$

18. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.08\overline{3} = \frac{\$10}{Base}$

$$Base = \frac{\$10}{0.08\overline{3}} = \underline{\$120.00}$$

19. $Rate = \frac{Portion}{Base} = \frac{30 \text{ metres}}{3000 \text{ metres}} = 0.0100 = \underline{1.00\%}$

20. $Rate = \frac{Portion}{Base} = \frac{500 \text{ grams}}{2800 \text{ grams}} = 0.179 = \underline{17.9\%}$

21. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.005 = \frac{Portion}{\$10.00}$

$$Portion = 0.005 \times \$10.00 = \underline{\$0.05}$$

22. $Rate = \frac{Portion}{Base} \Rightarrow \quad 0.0075 = \frac{Portion}{\$100}$

$$Portion = 0.0075 \times \$100 = \underline{\$0.75}$$

23. $Rate = \frac{Portion}{Base} \Rightarrow \quad 1.20 = \frac{\$180}{Base}$

$$Base = \frac{\$180}{1.20} = \underline{\$150.00}$$

24. $Rate = \frac{Portion}{Base} \Rightarrow \quad 1.13 = \frac{\$559.35}{Base}$

$$Base = \frac{\$559.35}{1.13} = \underline{\$495.00}$$

25. $Rate = \frac{Portion}{Base} \Rightarrow \quad 1.305 = \frac{Portion}{\$455}$

$$Portion = 1.305 \times \$455 = \underline{\$593.78}$$

26. $Rate = \frac{Portion}{Base} \Rightarrow 0.000505 = \frac{Portion}{\$50{,}000}$

$Portion = 0.000505 \times \$50{,}000 = \underline{\$25.25}$

27. $Rate = \frac{Portion}{Base} \Rightarrow 2.25 = \frac{\$281.25}{Base}$

$Base = \frac{\$281.25}{2.25} = \underline{\$125.00}$

28. $Rate = \frac{Portion}{Base} \Rightarrow 3.5 = \frac{\$1000}{Base}$

$Base = \frac{\$1000}{3.5} = \underline{\$285.71}$

29. $Rate = \frac{Portion}{Base} \Rightarrow 0.005 = \frac{\$10}{Base}$

$Base = \frac{\$10}{0.005} = \underline{\$2000.00}$

30. $Rate = \frac{Portion}{Base} \Rightarrow 0.0075 = \frac{\$1.25}{Base}$

$Base = \frac{\$1.25}{0.0075} = \underline{\$166.67}$

31. *a.* $Rate = \frac{Portion}{Base} = \frac{\$14{,}775}{\$8775} = 1.684 = \underline{168\%}$

The trip costs 168% of their gross monthly income.

b. Disposable income = 100% – 72% = 28% of gross income.
Annual disposable income = $12(0.28 \times \$8775) = \$29{,}484$
The cost of the trip is

$$\frac{Portion}{Base} \times 100\% = \frac{\$14{,}775}{\$29{,}484} \times 100\% = \underline{\underline{50.1\%}}$$

of Cecilia's and Nathan's annual disposable income.

32. Total sales per month = \$65,560 + \$36,740 = \$102,300

Gasoline sales are $\frac{\$65{,}560}{\$102{,}300} \times 100\% = \underline{64.1\%}$ of total sales.

33. Given: $Base = 540$ ml
$Rate = 100 - (28 + 15.5 + 6) = 50.5\%$ other ingredients

a) $Rate = \frac{Portion}{Base} \Rightarrow 0.505 = \frac{Portion}{540} = \underline{272.7 \text{ ml}}$

b) $Rate = 28\%$
$Base = \frac{5}{8} \text{ x } 540 \text{ ml} = 337.5 \text{ ml}$
$0.28 = \frac{Portion}{337.5} = \underline{94.5 \text{ ml}}$

34. Given: $Rate = 23.5\%$ and $Portion = \$2680$ million.

$$Rate = \frac{Portion}{Base} \Rightarrow \qquad 0.235 = \frac{\$2680}{\text{Total budget}}$$

$$\text{Total budget} = \frac{\$2680}{0.235} = \$11{,}404 \text{ million} = \underline{\$11.404 \text{ billion}}$$

The province's total budget is $11.404 billion.

35. The budgeted expenses are the *Base* while the actual expenses are the *Portion.*

$$Rate = \frac{Portion}{Base} \Rightarrow \qquad 1.27 = \frac{\$320{,}200}{\text{Budget}}$$

$$\text{Budget} = \frac{\$320{,}200}{1.27} = \$252{,}100 \quad \text{(to the nearest \$100)}$$

Brockton budgeted $\underline{\$252{,}100}$ for snow clearance.

36. The gross royalties ($99,736.41) are 5.7% of total revenue.

That is, $$0.057 = \frac{\$99{,}736.41}{\text{Total revenue}}$$

$$\text{Total revenue} = \frac{\$99{,}736.41}{0.057} = \$1{,}749{,}761.58$$

Number of downloads at $0.99 each is

$$\frac{\$1{,}749{,}761.58}{\$0.99} = \underline{1{,}767{,}436}$$

37. Total hours in a year = 52 weeks × 7 days/week × 24 hours/day = 8736 hours
Number of days worked = [(52 – 2) weeks × 5 days/week] – 7 holidays = 243 days
Total hours worked = 243 × 7.5 = 1822.5

Percentage of total hours that are worked = $\frac{1822.5}{8736} \times 100\% = \underline{20.9\%}$

38. Total Revenue for December = $9,820 + $4,025 + $1,830 = $15,675
Returns = 0.17 x $9,820 + 0.08 x $4,025 + 0.03 x $1,830 = $2046.30

$$Rate = \frac{Portion}{Base} = \frac{2{,}046.30}{15{,}675} = \underline{13.05\%}$$

39. Percentage of impurities = $100\% - 99\frac{44}{100}\% = \frac{56}{100}\% = 0.56\%$

Amount of impurities in a 150-g bar of soap = $Rate \times Base$ = 0.0056 × 150 g = 0.840 g = $\underline{840 \text{ mg}}$

40. Discount broker would charge $30 + 200($0.03) = $36.00
Full-service broker would charge 0.024(200 × $55.40) = $265.92
The discount broker charges only

$$\frac{\$36.00}{\$265.92} \times 100\% = \underline{13.5\%}$$

of the amount charged by the full-service broker.

41. Discount broker would charge $25 + 800($0.05) = $65.00
Full-service broker would charge 0.022(800 × $21.75) = $382.80
The discount broker charges $382.80 – $65.00 = $317.80 less
The percent saved is $\frac{\$317.80}{\$382.80} \times 100\% = \underline{83.0\%}$

42. *a.* Income tax = 0.16($15,000) + 0.26($33,000 – $15,000)
= $2400 + $4680

= $7080

This income tax is $\frac{\$7080}{\$33{,}000} \times 100\% = \underline{21.5\%}$ of taxable income.

b. Income tax = 0.16($15,000) + 0.26($20,000) + 0.35($66,000 – $35,000)
= $2400 + $5200 + $10,850
= $18,450

This income tax is $\frac{\$18{,}450}{\$66{,}000} \times 100\% = \underline{28.0\%}$ of taxable income.

c. Income tax = 0.16($15,000) + 0.26($20,000) + 0.35($40,000)
+ 0.45($99,000 – $75,000)
= $2400 + $5200 + $14,000 + $10,800
= $32,400

This income tax is $\frac{\$32{,}400}{\$99{,}000} \times 100\% = \underline{32.7\%}$ of taxable income.

43. Canada's population density = $\frac{35{,}750{,}000 \text{ people}}{9{,}093{,}500 \text{ square km}} = 3.931$ people per square km.

Japan's population density = $\frac{126{,}240{,}000 \text{ people}}{377{,}835 \text{ square km}} = 334.11$ people per square km.

Canada's population density is only $\frac{3.931}{334.11} \times 100\% = \underline{1.18\%}$ of Japan's population density.

44. The selling price is being compared to the original price. Hence, the original price is the *Base* and the selling price is the *Portion*.

Original price = $\frac{Portion}{Rate} = \frac{\$210{,}000}{2.50} = \underline{\$84{,}000.00}$

45. 13,020 seats represent 67.50% of capacity.
That is, 13,020 = 0.6750(Capacity)

Capacity = $\frac{13{,}020}{0.6750}$ = 19,289 seats

Seats not sold to season-ticket holders = 19,289 – 13,020 = 6269
Rounded to the nearest 100, 6300 seats were not sold to season-ticket holders.

46. Waking hours for Males = (24 – 7.5) x 365 x 78 years = 469,755 hours
Waking hours for Females = (24 – 7.5 +0.$\overline{3}$) x 365 x 82.7 years = 487,998.9167 hours

$Rate = \frac{Portion}{Base} = \frac{469{,}755}{487{,}998.9167} = \underline{96.26\%}$

47. Portion of commission retained = *Rate* × *Base* = 0.60 × 4.8% = 2.88%
Income of $150,480 (*Portion*) is 2.88% of sales (*Base*).
That is, $150,480 = 0.0288 × Sales
Stan's sales volume was $5,225,000.

48. The retained commission is $\frac{\$134.55}{\$11{,}500} \times 100\% = 1.17\%$ of the amount of the transaction.

This 1.17% (*Portion*) is 45% of the total commission rate (*Base*) charged to clients. Hence,

Rate of total commission = $\frac{Portion}{Rate} = \frac{1.17\%}{0.45} = \underline{2.60\%}$

49. *a.* The expected number of deaths (*Portion*) among 50,000 males (*Base*) is
$Rate \times Base = 0.0034 \times 50{,}000 = \underline{170}$

b. The number of 35-year-old males in the city of 1.45 million is
$0.0083 \times 1{,}450{,}000 = 12{,}035$.
The expected number of deaths in this group in a year is $0.0034 \times 12{,}035 = \underline{41}$.

Exercise 2.7

a) $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{110 - 100}{100} \times 100\% = \underline{10\%}$

b) $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{90 - 100}{100} \times 100\% = \underline{-10\%}$

c) $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{75 - 25}{25} \times 100\% = \underline{200\%}$

d) $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{0 - 50}{50} \times 100\% = \underline{-100\%}$

e) $V_f = V_i(1+c) = \$200[1+(-0.25)] = \underline{\$150}$

f) $V_f = V_i(1+c) = 80(1+0.50) = \underline{120\text{ g}}$

g) $V_f = V_i(1+c) = 300(1+2.0) = \underline{900\text{ cm}}$

h) $V_f = V_i(1+c) = \$400[1+(-0.50)] = \underline{\$200}$

i) $V_i = \dfrac{V_f}{1+c} = \dfrac{\$50}{1+1.0} = \dfrac{\$50}{2.0} = \underline{\$25}$

j) $V_i = \dfrac{V_f}{1+c} = \dfrac{\$300}{1+(-.50)} = \dfrac{\$300}{0.50} = \underline{\$600}$

1. $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{\$100 - \$95}{\$95} \times 100\% = \underline{5.26\%}$

2. $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{\$95 - \$100}{\$100} \times 100\% = \underline{-5.00\%}$

3. $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{135kg - 35kg}{35kg} \times 100\% = \underline{285.71\%}$

4. $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{35kg - 135kg}{135kg} \times 100\% = \underline{-74.07\%}$

5. $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{0.13 - 0.11}{0.11} \times 100\% = \underline{18.18\%}$

6. $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{0.085 - 0.095}{0.095} \times 100\% = \underline{-10.53\%}$

7. $V_f = V_i(1+c) = \$134.39[1 + (-0.12)] = \$134.39(0.88) = \underline{\$118.26}$

8. $V_f = V_i(1+c)$ = 112g(1 + 1.12) = 237.44 g

9. $V_f = V_i(1+c)$ = (26.3 cm)(1 + 3.00) = 105.2 cm

10. $V_f = V_i(1+c)$ = 0.043[1 + (–0.30)] = 0.0301

11. $V_i = \dfrac{V_f}{1+c} = \dfrac{\$75}{1+2.00} = \$25.00$

12. $V_i = \dfrac{V_f}{1+c} = \dfrac{\$75}{1+(-0.50)} = \$150.00$

13. Given: V_i = \$90, V_f = \$100

$$c = \frac{\$100 - \$90}{\$90} \times 100\% = 11.11\%$$

\$100 is 11.11% more than \$90.

14. Given: V_i = \$110, V_f = \$100

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$100 - \$110}{\$110} \times 100\% = -9.09\%$$

\$100 is 9.09% less than \$110.

15. Given: c = 25%, V_f = \$100

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+0.25} = \$80.00$$

\$80.00 increased by 25% equals \$100.00.

16. Given: c = 7%, V_f = \$52.43

$$V_i = \frac{V_f}{1+c} = \frac{\$52.43}{1+0.07} = \$49.00$$

\$49.00 increased by 7% equals \$52.43.

17. Given: V_f = \$75, c = 75%

$$V_i = \frac{V_f}{1+c} = \frac{\$75}{1+0.75} = \$42.86$$

\$75 is 75% more than \$42.86.

18. Given: V_i = \$56, c = 65%

$$V_f = V_i(1+c) = \$56(1.65) = \$92.40$$

\$56 after an increase of 65% is \$92.40.

19. Given: V_i = \$759.00, V_f = \$754.30

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$754.30 - \$759.00}{\$759.00} \times 100\% = -0.62\%$$

\$754.30 is 0.62% less than \$759.00.

20. Given: V_i = 77,400, V_f = 77,787

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{77{,}787 - 77{,}400}{77{,}400} \times 100\% = 0.50\%$$

77,787 is 0.50% more than 77,400.

21. Given: V_i = \$75, c = 75%

$$V_f = V_i(1 + c) = \$75(1 + 0.75) = \underline{\$131.25}$$

\$75.00 becomes \$131.25 after an increase of 75%.

22. Given: V_f = \$100, c = – 10%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+(-0.10)} = \underline{\underline{\$111.11}}$$

\$100.00 is 10% less than \$111.11.

23. Given: V_f = \$100, c = – 20%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+(-0.20)} = \underline{\underline{\$125.00}}$$

\$125 after a reduction of 20% equals \$100.

24. Given: V_f = \$50, c = – 25%

$$V_i = \frac{V_f}{1+c} = \frac{\$50}{1+(-0.25)} = \underline{\underline{\$66.67}}$$

\$66.67 after a reduction of 25% equals \$50.

25. Given: V_f = \$549, $c = -16.\overline{6}\%$

$$V_i = \frac{V_f}{1+c} = \frac{\$549}{1+\left(-0.1\overline{6}\right)} = \underline{\underline{\$658.80}}$$

\$658.80 after a reduction of $16.\overline{6}\%$ equals \$549.

26. Given: V_i = \$900, c = –90%

$$V_f = V_i(1 + c) = \$900[1 + (–0.9)] = \underline{\$90.00}$$

\$900 after a decrease of 90% is \$90.00.

27. Given: V_i = \$102, c = –2%

$$V_f = V_i(1 + c) = \$102(1 – 0.02) = \underline{\$99.96}$$

\$102 after a decrease of 2% is \$99.96.

28. Given: V_i = \$102, c = –100%

$$V_f = V_i(1 + c) = \$102[1 + (–1.00)] = \$102(0) = \underline{\$0.00}$$

Any positive amount after a decrease of 100% is zero.

29. Given: V_i = \$250, V_f = \$750

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$750 - \$250}{\$250} \times 100\% = \underline{\underline{200.00\%}}$$

\$750 is 200.00% more than \$250.

30. Given: V_i = \$750, V_f = \$250

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$250 - \$750}{\$750} \times 100\% = \underline{\underline{-66.67\%}}$$

\$250 is 66.67% less than \$750.

31. Given: c = 0.75%, V_i = \$10,000

$$V_f = V_i(1 + c) = \$10{,}000(1 + 0.0075) = \underline{\$10{,}075.00}$$

\$10,000 after an increase of $\frac{3}{4}$% is \$10,075.00.

32. Given: V_i = \$1045, c = – 0.5%

$V_f = V_i\,(1 + c)$ = $\$1045\left[1+\left(-0.005\right)\right]$ = \$1039.78

\$1045 after a decrease of 0.5% is \$1039.78.

33. Given: c = 150%, V_f = \$575

$$V_i = \frac{V_f}{1+c} = \frac{\$575}{1+1.5} = \$230.00$$

\$230.00 when increased by 150% equals \$575.

34. Given: c = 210%, V_f = \$465

$$V_i = \frac{V_f}{1+c} = \frac{\$465}{1+2.1} = \$150.00$$

\$150.00 after being increased by 210% equals \$465.

35. Given: V_i = \$150, c = 150%

$V_f = V_i\,(1 + c)$ = \$150(1 + 1.5) = \$375.00

\$150 after an increase of 150% is \$375.00.

36. Let the retail price be p. Then

p + 0.13 p = \$281.37

$$p = \frac{\$281.37}{1.13} = \$249.00$$

The coat's sticker price was \$249.00.

37. Let the TV's pre-tax price be p. Then

p + 0.05p + 0.07p = \$2797.76

$$p = \frac{\$2797.76}{1.12} = \$2498.00$$

Then, GST = 0.05p = 0.05(\$2498) = \$124.90

and PST = 0.07p = 0.07(\$2498) = \$174.86

38. Let the population figure for 2005 be p. Then

p + 0.0965p = 35,750,000

$$p = \frac{\$35{,}750{,}000}{1.0965} = 32{,}603{,}739$$

Rounded to the nearest 10,000, the population in 2005 was 32,600,000.

39. *a.* . Given: V_i = 32,400, V_f = 27,450

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{27{,}450 - 32{,}400}{32{,}400} \times 100\% = -15.28\%$$

The number of hammers sold declined by 15.28%.

b. Given: V_i = \$15.10, V_f = \$15.50

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$15.50 - \$15.10}{\$15.10} \times 100\% = 2.65\%$$

The average selling price increased by 2.65%.

c. Year 1 revenue = 32,400(\$15.10) = \$489,240

Year 2 revenue = 27,450(\$15.50) = \$425,475

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$425{,}475 - \$489{,}240}{\$489{,}240} \times 100\% = -13.03\%$$

The revenue decreased by 13.03%.

40. *a.* Given: V_i = \$0.55, V_f = \$1.55

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$1.55 - \$0.55}{\$0.55} \times 100\% = \underline{181.82\%}$$

The share price rose by 181.82% in the first year.

b. Given: V_i = \$1.55, V_f = \$0.75

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.75 - \$1.55}{\$1.55} \times 100\% = \underline{-51.61\%}$$

The share price declined by 51.61% in the second year.

c. Given: V_i = \$0.55, V_f = \$0.75

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.75 - \$0.55}{\$0.55} \times 100\% = \underline{36.36\%}$$

The share price rose by 36.36% over 2 years.

41. Pick an arbitrary price, say \$1.00, for a bar of the soap.

The former unit price was $V_i = \frac{\$1.00}{100\text{ g}} = \$0.01$ per gram.

The new unit price is $V_f = \frac{\$1.00}{90\text{ g}} = \$0.011111$ per gram.

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.011111 - \$0.01}{\$0.01} \times 100\% = \underline{11.11\%}$$

42. Initial unit price = $\frac{\$5.49}{1.65\,l}$ = \$3.327 per litre

Final unit price = $\frac{\$7.98}{2.2\,l}$ = \$3.627 per litre

The percent increase in the unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$3.627 - \$3.327}{\$3.327} \times 100\% = \underline{9.02\%}$$

43. Initial unit price = $\frac{\$7.98}{3.6\text{ kg}}$ = \$2.2167 per kg

Final unit price = $\frac{\$6.98}{3\text{ kg}}$ = \$2.3267 per kg

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$2.3267 - \$2.2167}{\$2.2167} \times 100\% = \underline{4.96\%}$$

44. Initial unit price = $\frac{1098\text{ cents}}{700\text{ g}}$ = 1.5686 cents per g

Final unit price = $\frac{998\text{ cents}}{600\text{ g}}$ = 1.6633 cents per g

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{1.6633 - 1.5686}{1.5686} \times 100\% = \underline{6.04\%}$$

45. Given: V_f = \$338,500, c = 8.7%

$$V_i = \frac{V_f}{1+c} = \frac{\$338{,}500}{1.087} = \underline{\$311{,}400}$$

The average price one year ago was \$311,400.

46. Given: V_f = \$348.60, c = –0.30

$$V_i = \frac{V_f}{1+c} = \frac{\$348.60}{1+(-0.30)} = \frac{\$348.60}{0.70} = \underline{\$498.00}$$

The regular price of the boots is \$498.00.

47. Canada's exports to US exceeded imports from the US by 23%.
That is, Exports = 1.23(Imports)
Therefore, Imports = $\frac{\text{Exports}}{1.23}$ = 0.8130(Exports)
That is, Canada's imports from US (= US exports to Canada) were
1 – 0.8130 = 0.1870 = <u>18.70%</u>
<u>less</u> than Canada's exports to US (= US imports from Canada.)

48. Given: For 2013, V_f (Apple) = 55.0 million, c = 35%

$$V_i = \frac{V_f}{1+c} = 40.7401 \text{ million} = 40{,}740{,}100 \text{ Galaxy phones}$$

Rounded to the nearest 10,000, Galaxy sold <u>40,740,000</u> phones in 2013.

49. Given: For 2015, V_f = 61.0 million, c = 40%

$$V_i = \frac{V_f}{1+c} = \frac{61.0 million}{1+0.40} = 43.5714 \text{ million} = 43{,}571{,}400$$

Rounded to the nearest 10,000, Apple sold <u>43,570,000</u> iPhones in the second quarter of 2014.

50. The fees to Fund A will be

$$\frac{(\text{Fees to Fund A}) - (\text{Fees to Fund B})}{(\text{Fees to Fund B})} \times 100\% = \frac{2.38\% - 1.65\%}{1.65\%} \times 100\% = \underline{44.24\%}$$

more than the fees to Fund B.

51. Percent change in the HST rate

$$= \frac{(\text{Final HST rate}) - (\text{Initial HST rate})}{(\text{Initial HST rate})} \times 100\% = \frac{14\% - 15\%}{15\%} \times 100\% = -6.67\%$$

The HST paid by consumers would have been reduced by <u>6.67%</u>.

52. Given: For June of 2015, V_f = 968,000,000 daily active users, c = 12.1%

Then, $V_i = \frac{V_f}{1+c} = \frac{968{,}000{,}000}{1+0.121} = 863{,}514{,}719$

That is, Facebook had 863,514,719 daily active users in June of 2014
Therefore, the absolute increase from June of 2015 to June of 2014 was
968,000,000 – 863,514,719 = <u>104,490,000</u> (rounded to the nearest 10,000)

53. Given: V_f = \$0.45, c = 76%

$$V_i = \frac{V_f}{1+c} = \frac{\$0.45}{1+(-0.76)} = \$1.88$$

Price decline = $V_i - V_f$ = \$1.88 – \$0.45 = <u>\$1.43</u>
The share price dropped by \$1.43.

54. Given: V_f = \$24,300, c = – 55%

$$V_i = \frac{V_f}{1+c} = \frac{\$24{,}300}{1+(-0.55)} = \$54{,}000$$

The amount of depreciation is \$54,000 – \$24,300 = <u>\$29,700</u>.

55. If General Paint's prices are marked down by 30%, then
General Paint's prices = 0.70(Cloverdale Paint's prices)
Hence, Cloverdale's prices = $\frac{\text{General Paint's prices}}{0.70}$ = 1.4286(General Paint's prices)
Therefore, you will pay <u>42.86% more</u> at Cloverdale Paint.

56. Given: January sales were 17.4% less than December sales
Hence, January sales = (1 – 0.174)(December sales) = 0.826(December sales)
Therefore, December sales = $\frac{\text{January sales}}{0.826}$ = 1.2107(January sales)
That is, December sales were <u>121.07%</u> of January sales.

57. Given: Operating expenses = 0.40(Revenue)
Then Revenue = $\frac{\text{Operating expenses}}{0.40}$ = 2.5(Operating expenses)
That is, Revenue is 250% of Operating expenses, or
Revenue exceeds Operating expenses by 250% – 100% = <u>150%</u>.

58. Given: Equity = (100% – 50%) of Debt = 50% of Debt = 0.50(Debt)
Therefore, $\frac{\text{Debt}}{\text{Equity}} = \frac{\text{Debt}}{0.5(\text{Debt})} = \frac{1}{0.5} = 2$
Since Debt is twice (or <u>200%</u> of) Equity, then debt financing is <u>100% more</u>
than equity financing.

59. Current unit price = $\frac{449 \text{ cents}}{500 \text{ ml}}$ = 0.8980 cents per ml
New unit price = 1.10(0.8980 cents per ml) = 0.9878 cents per ml
Price of a 425-ml container = (425 ml) × (0.9878 cents per ml) = 419.8 cents = <u>\$4.20</u>

60. Current unit price = $\frac{115 \text{ cents}}{100 \text{ g}}$ = 1.15 cents per g
New unit price = 1.075(1.15 cents per g) = 1.23625 cents per g
Price of an 80-g bar = (80 g) × (1.23625 cents per g) = 98.9 cents = <u>\$0.99</u>

61. For Year 1, V_f = \$6 and $V_f - V_i$ = – \$4
Therefore, $V_i = V_f$ + \$4 = \$6 + \$4 = \$10

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{-\$4}{\$10} \times 100\% = \underline{\underline{-40.00\%}}$$

For Year 2, V_i = \$6 and $V_f - V_i$ = \$4

Therefore, $c = \dfrac{V_f - V_i}{V_i} \times 100\% = \dfrac{\$4}{\$6} \times 100\% = \underline{\underline{66.67\%}}$

The percent change was -40.00% in Year 1 and 66.67% in Year 2.

62. If the Canadian dollar is worth 6.5% less than the US dollar,

Canadian dollar = (1 – 0.065)(US dollar) = 0.935(US dollar)

Hence, US dollar = $\frac{\text{Canadian dollar}}{0.935}$ = 1.0695(Canadian dollar)

Therefore, the US dollar is worth 6.95% more than the Canadian dollar.

63. Given: For the appreciation, V_i = Purchase price, c = 140%, V_f = List price

For the price reduction, V_i = List price, c = –10%, V_f = \$172,800

List price = $V_i = \frac{V_f}{1+c} = \frac{\$172,800}{1+(-0.1)} = \$192,000$

Original purchase price = $\frac{V_f}{1+c} = \frac{\$192,000}{1+1.4} = \$80,000$

The owner originally paid \$80,000 for the property.

64. Given: For the markup, V_i = Cost, c = 22%, V_f = List price

For the markdown, V_i = List price, c = –10%, V_f = \$17,568

List price = $\frac{V_f}{1+c} = \frac{\$17,568}{1+(-0.10)} = \$19,520$

Cost (to dealer) = $\frac{V_f}{1+c} = \frac{\$19,520}{1+0.22} = \$16,000$

The dealer paid \$16,000 for the car.

65. Suppose the initial ratio is $\frac{x}{y}$.

If the denominator is reduced by 20%, then

Final ratio = $\frac{x}{y-0.20y} = \frac{x}{0.8y} = 1.25\frac{x}{y}$

That is, the value of the ratio increases by 25%.

66. Next year there must be 15% fewer students per teacher.
With the same number of students,

$$\frac{\text{Students}}{\text{Teachers next year}} = 0.85\left(\frac{\text{Students}}{\text{Teachers now}}\right)$$

Therefore, Teachers next year = $\frac{\text{Teachers now}}{0.85}$ = 1.1765(Teachers now)

That is, if the number of students does not change, the number of teachers must be increased by 17.65%.

67. Use ppm as the abbreviation for “pages per minute”.
Given: Lightning printer prints 30% more ppm than the Reliable printer.
That is, the Lightning’s printing speed is 1.30 times faster than Reliable’s printing speed.
Therefore, the Reliable’s printing speed is

$\frac{1}{1.3}$ = 0.7692 = 76.92% of the Lightning’s printing speed

If Lightening can print 100 pages per minute then Reliable can print 76.92 pages per minute.

or Reliable is 100% – 76.92% = 23.08% slower than the Lighting’s speed.

The Lightning printer will require 23.08% less time than the Reliable for a long print job.

68. Given: Euro is worth 39% more than the Canadian dollar.
That is, Euro = 1.39(Canadian dollar)

Therefore, $\text{Canadian dollar} = \dfrac{\text{Euro}}{1.39} = 0.7194(\text{Euro}) = 71.94\%$ of a Euro.

That is, the Canadian dollar is worth 100% − 71.94% = 28.06% less than the Euro.

69. Let us use OT as an abbreviation for "overtime".
The number of OT hours permitted by this year's budget is

$$\text{OT hours (this year)} = \frac{\text{OT budget (this year)}}{\text{OT hourly rate (this year)}}$$

The number of overtime hours permitted by next year's budget is

$$\text{OT hours (next year)} = \frac{\text{OT budget (next year)}}{\text{OT hourly rate (next year)}} = \frac{1.03\left[\text{OT budget (this year)}\right]}{1.05\left[\text{OT hourly rate (this year)}\right]}$$

$$= 0.980952\,\frac{\text{OT budget (this year)}}{\text{OT hourly rate (this year)}}$$

= 98.0952% of this year's OT hours

The number of OT hours must be reduced by 100% − 98.0952% = 1.90%.

Concept Questions (Section 2.8)

1. Yes. If the expenses associated with an investment exceed the income from the investment, then the net income and the income yield will be negative. For example, if you hold a piece of raw land as an investment, you will have no income from the property but you must pay property taxes each year. The net income and income yield are then negative.
2. Yes. If the value of an investment more than doubles during the holding period, the capital gain is more than 100%.
3. Yes. Suppose, for example, you bought a $160,000 condominium as an investment property using $40,000 of your own money and $120,000 borrowed on a mortgage loan. Subsequently, the condo's market value fell to $100,000 because "leaky condo" problems were discovered in the building. At that point, you have lost more than 100% of your initial $40,000 investment because the condo's market value is less than the amount owed on the mortgage loan. You must still repay the balance on the loan after the proceeds of the sale are applied to the loan.
4. The overall percent change is larger because each successive change acts on previous increases as well as the original amount.
5. The magnitude of the overall percent change is smaller than the sum. To illustrate, consider two successive 10% decreases from a beginning value of $1000. The first 10% decrease causes a $100 decrease to $900. The second 10% decrease acts on $900 rather than on the initial $1000. The dollar amount of the second reduction is only $90. The overall reduction is $190, which is only 19% (not 20%) of the original $1000.

Exercise 2.8

1. Given: V_i = \$100, V_f = \$110, Income = \$10

   a. Income yield = $\frac{Income}{V_i} \times 100\% = \frac{\$10}{\$100} \times 100\% = \underline{\underline{10.00\%}}$

   b. Capital gain yield = $\frac{V_f - V_i}{V_i} \times 100\% = \frac{\$110 - \$100}{\$100} \times 100\% = \underline{10.00\%}$

   c. Rate of total return = Income yield + Capital gain yield = 10.00% + 10.00% = <u>20.00%</u>

2. Given: V_i = \$100, V_f = \$90, Income = \$10

   a. Income yield = $\frac{Income}{V_i} \times 100\% = \frac{\$10}{\$100} \times 100\% = \underline{\underline{10.00\%}}$

   b. Capital gain yield = $\frac{V_f - V_i}{V_i} \times 100\% = \frac{\$90 - \$100}{\$100} \times 100\% = \underline{-10.00\%}$

   c. Rate of total return = Income yield + Capital gain yield = 10.00% + (–10.00%) = <u>0.00%</u>

3. Given: V_i = \$8790, V_f = \$15,390, Income = \$280

   a. Income yield = $\frac{Income}{V_i} \times 100\% = \frac{\$280}{\$8790} \times 100\% = 3.185\% = \underline{3.19\%}$

   b. Capital gain yield = $\frac{V_f - V_i}{V_i} \times 100\% = \frac{\$15{,}390 - \$8790}{\$8790} \times 100\% = 75.085\% = \underline{75.09\%}$

   c. Rate of total return = Income yield + Capital gain yield = 3.185% + 75.085% = <u>78.27%</u>

4. Given: V_i = \$13,000, V_f = \$11,400, Income = \$260

a. Income yield = $\frac{Income}{V_i} \times 100\% = \frac{\$260}{\$13{,}000} \times 100\% = \underline{2.00\%}$

b. Capital gain yield = $\frac{V_f - V_i}{V_i} \times 100\% = \frac{\$11{,}400 - \$13{,}000}{\$13{,}000} \times 100\% = \underline{-12.31\%}$

c. Rate of total return = Income yield + Capital gain yld = 2.00% + (–12.31%) = <u>–10.31%</u>

5.

Security	Income yield	Capital gain yield	Rate of total return
	$\frac{Income}{V_i} \times 100\%$	$\frac{V_f - V_i}{V_i} \times 100\%$	Income yield + Capital gain yield
Potash Corp. shares (2014)	$\frac{\$0.35}{\$32.96} \times 100\%$ = <u>1.06%</u>	$\frac{\$35.32 - \$32.96}{\$32.96} \times 100\% =$ <u>7.16%</u>	1.06% + 7.16% = <u>8.22%</u>
Mawer New Canada Fund (2014)	$\frac{\$4.92}{\$69.92} \times 100\%$ = <u>7.04%</u>	$\frac{\$73.42 - \$69.92}{\$69.92} \times 100\% =$ <u>5.01%</u>	7.04% + 5.01% = <u>12.05%</u>

6.

Security	Income yield	Capital gain yield	Rate of total return
	$\frac{Income}{V_i} \times 100\%$	$\frac{V_f - V_i}{V_i} \times 100\%$	Income yield + Capital gain yield
Cameco shares (2013)	$\frac{\$0.10}{\$19.59} \times 100\%$ = <u>0.51%</u>	$\frac{\$22.04 - \$19.59}{\$19.59} \times 100\% =$ <u>12.51%</u>	0.51% + 12.51 = <u>13.02%</u>
Cameco shares (2014)	$\frac{\$0.10}{\$22.04} \times 100\%$ = <u>0.45%</u>	$\frac{\$19.05 - \$22.04}{\$22.04} \times 100\% =$ <u>-13.57%</u>	0.45% + (-13.57%) = <u>-13.12%</u>
Desjardins Div. Growth (2013)	$\frac{\$0.16}{\$14.37} \times 100\%$ = <u>1.11%</u>	$\frac{\$17.23 - \$14.37}{\$14.37} \times 100\% =$ <u>19.90%</u>	1.11% + 19.90% = <u>21.01%</u>
Desjardins Div. Growth (2014)	$\frac{\$0.46}{\$17.23} \times 100\%$ = <u>2.67%</u>	$\frac{\$18.36 - \$17.23}{\$17.23} \times 100\% =$ <u>6.56%</u>	2.67% + 6.56% = <u>9.23%</u>

7.

Security	Income yield	Capital gain yield	Rate of total return
	$\frac{Income}{V_i} \times 100\%$	$\frac{V_f - V_i}{V_i} \times 100\%$	Income yield + Capital gain yield
Blackberry shares (2013)	<u>0.0%</u>	$\frac{\$7.90 - \$11.80}{\$11.80} \times 100\% =$ <u>-33.05%</u>	= <u>–33.05%</u>
Blackberry shares (2014)	<u>0.0%</u>	$\frac{\$12.74 - \$7.90}{\$7.90} \times 100\% =$ <u>61.27%</u>%	= <u>61.27%</u>

Scotia Can. Bond Fund (2013)	$\frac{\$0.35}{\$11.71} \times 100\% = \underline{2.99\%}$	$\frac{\$11.16 - \$11.71}{\$11.71} \times 100\% = \underline{-4.70\%}$	2.99% + (-4.70%) = $\underline{-1.71\%}$
Scotia Can. Bond Fund (2014)	$\frac{\$0.36}{\$11.16} \times 100\% = \underline{3.23\%}$	$\frac{\$11.72 - \$11.16}{\$11.16} \times 100\% = \underline{5.02\%}$	3.23% + 5.02% = $\underline{8.25\%}$

8. Given: V_i = \$37, V_f = \$40, Income = \$0.60

Income yield = $\frac{Income}{V_i} \times 100\% = \frac{\$0.60}{\$37} \times 100\% = \underline{1.62\%}$

Capital gain yield = $\frac{V_f - V_i}{V_i} \times 100\% = \frac{\$40 - \$37}{\$37} \times 100\% = \underline{8.11\%}$

Rate of total return = Income yield + Capital gain yield = 1.62% + 8.11% = $\underline{9.73\%}$

9. Given: V_i = \$24.10, V_f = \$25.50, Income = \$0.83

Income yield = $\frac{Income}{V_i} \times 100\% = \frac{\$0.83}{\$24.10} \times 100\% = \underline{3.44\%}$

Capital gain yield = $\frac{V_f - V_i}{V_i} \times 100\% = \frac{\$25.50 - \$24.10}{\$24.10} \times 100\% = \underline{5.81\%}$

Rate of total return = Income yield + Capital gain yield = 3.44% + 5.81% = $\underline{9.25\%}$

10. Given: V_i = \$1053.25, V_f = \$1021.75, Income = 2(\$35) = \$70

Income yield = $\frac{Income}{V_i} \times 100\% = \frac{\$70}{\$1053.25} \times 100\% = \underline{6.65\%}$

Capital gain yield = $\frac{V_f - V_i}{V_i} \times 100\% = \frac{\$1021.75 - \$1053.25}{\$1053.25} \times 100\% = \underline{-2.99\%}$

Rate of total return = Income yield + Capital gain yield = 6.65% + (– 2.99%) = $\underline{3.66\%}$

11. Given: V_i = \$36.75, V_f = \$32.25, Income = \$0.50

Income yield = $\frac{Income}{V_i} \times 100\% = \frac{\$0.50}{\$36.75} \times 100\% = \underline{1.36\%}$

Capital gain yield = $\frac{V_f - V_i}{V_i} \times 100\% = \frac{\$32.25 - \$36.75}{\$36.75} \times 100\% = \underline{-12.24\%}$

Rate of total return = Income yield + Capital gain yield = 1.36% + (– 12.24%) = $\underline{-10.88\%}$

12. Given: V_i = \$56.49, V_f = \$65.75, Income = 4(\$0.30) = \$1.20

Capital gain = \$65.75 – \$56.49 = \$9.26 per share

Rate of total return = $\frac{Income + Capital\ gain}{V_i} \times 100\% = \frac{\$1.20 + \$9.26}{\$56.49} \times 100\% = \underline{18.52\%}$

13. Let V_i represent the share price at the beginning of 2011.

Let V_f represent the share price at the end of 2015.

a. $V_f = V_i\left(1+c_{2011}\right)\left(1+c_{2012}\right)\left(1+c_{2013}\right)\left(1+c_{2014}\right)\left(1+c_{2015}\right)$

$= V_i(1.0684)(1.1371)(1.135)(1.3604)(0.9887)$

$= V_i(1.85464)$

$= V_i(1 + 0.85464)$

Therefore, Microsoft shares rose 85.46% over the five years.

b. If V_f = \$45.57,

then $V_i = \dfrac{V_f}{1.85464} = \dfrac{\$45.57}{1.85464} = \$24.57$

14. No. A 20% increase does not offset a 20% decrease. Rather,

$$V_f = V_i(1 + c_1)(1 + c_2) = V_i(1 - 0.20)(1.20) = 0.96V_i$$

After the increases, the transfer payments will still be 4% below their level before the cuts started.

15. Given: $c_1 = c_2 = 0.25$

$V_f = V_i(1 + c_1)(1 + c_2) = V_i(1.25)^2 = 1.5625V_i$

The percent increase over the entire two years was 56.25%.

16. Given: $c_1 = c_2 = -0.25$

$V_f = V_i(1 - 0.25)^2 = 0.5625V_i$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{0.5625V_i - V_i}{V_i} \times 100\% = -43.75\%$$

17. Given: $c_1 = 0.50$, $V_f = V_i$

Substitute in $V_f = V_i(1 + c_1)(1 + c_2)$

$V_i = V_i(1.50)(1 + c_2)$

$1 = 1.50(1 + c_2)$

$c_2 = \frac{1}{1.5} - 1 = -0.3\overline{3} = -33.33\%$

A 33.33% decline in the second year will wipe out a 50% gain in the first year.

18. Given: $c_1 = 0.25$, $V_f = V_i$

Then $V_i = V_i(1.25)(1 + c_2)$

$c_2 = \frac{1}{1.25} - 1 = -0.2000$

A –20.00% return in the second year will nullify a 25% return in the first year.

19. Given: $c_1 = -0.50$, $V_f = V_i$

Substitute in $V_f = V_i(1 + c_1)(1 + c_2)$

$$V_i = V_i(1 - 0.5)(1 + c_2)$$

$$c_2 = \frac{1}{0.5} - 1 = 1.0000$$

A 100.00% gain in the second year is required to break even.

20. Given: $c_1 = -0.20$, $V_f = V_i$

Then $V_i = V_i(1 - 0.20)(1 + c_2)$

$$c_2 = \frac{1}{0.8} - 1 = 0.2500$$

A 25.00% return in the second year is required to break even.

21. Given: $c_1 = c_2 = 0.10$, $V_f = 1.30V_i$

Substitute in $V_f = V_i(1 + c_1)(1 + c_2)(1 + c_3)$

$$1.30V_i = V_i(1.10)(1.10)(1 + c_3)$$

$$1.30 = 1.21(1 + c_3)$$

$$c_3 = \frac{1.30}{1.21} - 1 = 0.0744$$

A gain of 7.44% in the third year will produce a cumulative gain of 30%.

22. Given: $c_1 = c_2 = -0.10$, $V_f = 0.70V_i$

Then $0.70V_i = V_i(1 - 0.10)^2(1 + c_3) = 0.81V_i(1 + c_3)$

$$c_3 = \frac{0.70}{0.81} - 1 = -0.1358$$

A return of –13.58% in the third year will produce a cumulative loss of 30%.

23. The value at the end of 2014 of an initial investment of V_i in the Mawer New Canada Fund was

$$V_f = V_i(1 + 0.2391)(1 + 0.0109)(1 + 0.1636)(1 + 0.4942)(1 + 0.1219) = 2.4433V_i$$

Therefore, the percent increase in the value of the fund was 144.33%.

Over the same period the S&P/TSX Total Return Index increased from V_i to

$$V_f = V_i(1 + 0.0882)(1 - 0.0149)(1 + 0.0817)(1 + 0.1148)(1 + 0.1206) = 1.44858V_i$$

Therefore, the percent increase in S&P/TSX Total Return Index was 44.86%.

The Mawer New Canada Fund outperformed the S&P/TSX Total Return Index by

144.33% – 44.86% = 99.47%

24. The value at the end of 2014 of an initial investment V_i in the BMO Dividend Fund was

$$V_f = V_i(1 + 0.0971)(1 - 0.0262)(1 + 0.0692)(1 + 0.1791)(1 + 0.1354) = 1.52924V_i$$

Therefore, the percent increase in the value of the fund was 52.92%

Over the same period the S&P/TSX Total Return Index increased from V_i to

$$V_f = V_i(1 + 0.0882)(1 - 0.0149)(1 + 0.0817)(1 + 0.1148)(1 + 0.1206) = 1.44858V_i$$

Therefore, the percent increase in S&P/TSX Total Return Index was 44.86%.

The BMO Dividend Fund outperformed the S&P/TSX Total Return Index by

52.92% – 44.86% = 8.06%

25. The value at the end of 2014 of an initial investment of V_i in the Fidelity Canadian Asset Allocation Fund was

$$V_f = V_i(1 + 0.1109)(1 - 0.0425)(1 + 0.0383)(1 + 0.1134)(1 + 0.1106) = 1.36567V_i$$

Therefore, the percent increase in the value of the fund was 36.57%.

Over the same period the S&P/TSX Total Return Index increased from V_i to

$$V_f = V_i(1 + 0.0882)(1 - 0.0149)(1 + 0.0817)(1 + 0.1148)(1 + 0.1206) = 1.44858V_i$$

Therefore, the percent increase in S&P/TSX Total Return Index was 44.86%.

The Fidelity Canadian Asset Allocation Fund fell short of the S&P/TSX Total Return Index by

44.86% – 36.57% = 8.29%

26. The value at the end of 2014 of an initial investment of V_i in the PH & N Bond Fund was

$$V_f = V_i(1 + 0.0685)(1 + 0.0833)(1 + 0.0343)(1 - 0.0126)(1 + 0.0866) = 1.28450V_i$$

Therefore, the percent increase in the value of the fund was 28.45%.

Over the same period the S&P/TSX Total Return Index increased from V_i to

$$V_f = V_i(1 + 0.0882)(1 - 0.0149)(1 + 0.0817)(1 + 0.1148)(1 + 0.1206) = 1.44858V_i$$

Therefore, the percent increase in S&P/TSX Total Return Index was 44.86%.

The PH & N Bond Fund fell short of the S&P/TSX Total Return Index by

44.86% – 28.45% = 16.41%

27. The income yield will be $\frac{Income}{V_i} \times 100\% = \frac{\$0.54}{\$36.90} \times 100\% = 1.46\%$

For a rate of total return = 10%,

Capital gain yield = Rate of total return – Income yield = 10% – 1.46% = 8.54%

The end-of-2015 Suncor Energy share price must be

$$V_i(1 + c) = \$36.90(1 + 0.0854) = \$40.05$$

28. The income yield will be $\frac{Income}{V_i} \times 100\% = \frac{\$0.48}{\$35.32} \times 100\% = 1.36\%$

For a rate of total return = 7%,

Capital gain yield = Rate of total return – Income yield = 7% – 1.36% = 5.64%

The end-of-2015 Potash Corp. share price must be

$$V_i(1 + c) = \$35.32(1 + 0.0564) = \$37.31$$

29. Given: V_i = \$5000 ÷ 400 = \$12.50 per unit, Rate of total return = 22%, V_f = \$13.75

$$\text{Income + Capital gain} = \frac{\text{Rate of total return}}{100\%} \times V_i = 0.22 \times \$12.50 = \$2.75$$

Since Capital gain = $V_f - V_i$ = \$13.75 – \$12.50 = \$1.25

Then Income = \$2.75 – \$1.25 = \$1.50

The fund distributed \$1.50 per unit during the year.

30. Given: $c_1 = 0.034$, $c_2 = -0.014$, $c_3 = 0.021$, $V_f = 9539$

a. $$V_i = \frac{V_f}{(1+c_1)(1+c_2)(1+c_3)} = \frac{9539}{(1.034)(1-0.014)(1.021)} = 9163.9$$

The S&P/TSX Index stood at 9163.9 at the beginning of the period.

b. After the first month,

$$V_f = V_i(1+c_1) = 9163.9(1.034) = 9475.5$$

The drop in the second month = $c_2(9475.5)$ = 132.7 points

31. Given: V_f = \$47,567, $c_1 = 0.154$, $c_2 = 0.243$, $c_3 = 0.321$, $c_4 = -0.033$

a. $$V_i = \frac{V_f}{(1+c_1)(1+c_2)(1+c_3)(1+c_4)}$$

$$V_i = \frac{\$47{,}567}{(1.154)(1.243)(1.321)(1-0.033)} = \frac{\$47{,}567}{1.8323407} = \$25{,}959.69$$

Victor's original investment was \$25,959.69.

b. The dollar amount of the increase in the third year was

c_3V_f(after 2 years) = $c_3V_i(1 + c_1)(1 + c_2)$ = 0.321(\$25,959.69)(1.154)(1.243) = \$11,953.13

The value of the fund increased by \$11,953.13 in Year 3.

32. Given: Rate of total return = 55%, Income = \$0.72, V_f = \$37.50

$$\text{Rate of total return} = = \frac{Income + Capital\ gain}{V_i} \times 100\% = \frac{Income + (V_f - V_i)}{V_i} \times 100\%$$

$$55\% = \frac{\$0.72 + (\$37.50 - V_i)}{V_i} \times 100\%$$

$$0.55V_i = \$38.22 - V_i$$

$$1.55V_i = \$38.22$$

$$V_i = \frac{\$38.22}{1.55} = \$24.66$$

The share price one year ago was \$24.66.

Review Problems

1. 4(3a + 2b) – 5a(2 – b) = 12a + 8b – 10a + 5ab
   = 2a + 5ab + 8b

2. 4(3a + 2b)(2b – a) –(2a – b)(a + 3b) = 4(6ab – 3a^2 + 4b^2 – 2ab) –(2a^2 + 6ab – ab – 3b^2)

$= 4(-3a^2 + 4ab + 4b^2) - (2a^2 + 5ab - 3b^2)$
$= -12a^2 + 16ab + 16b^2 - 2a^2 - 5ab + 3b^2$
$= \underline{-14a^2 + 11ab + 19b^2}$

3. *a.* $\frac{9y-7}{3} - 2.3(y-2) = 3y - 2.\overline{3} - 2.3y + 4.6 = \underline{0.7y + 2.2\overline{6}}$

b. $P\left(1 + 0.095 \times \frac{135}{365}\right) + \frac{2P}{1 + 0.095 \times \frac{75}{365}} = 1.035137P + 1.961706P = \underline{2.996843P}$

4. *a.* $6(4y-3)(2-3y) - 3(5-y)(1+4y) = 6(8y - 12y^2 - 6 + 9y) - 3(5 + 20y - y - 4y^2)$
$= \underline{-60y^2 + 45y - 51}$

b. $\frac{5b-4}{4} - \frac{25-b}{1.25} + \frac{7}{8}b = 1.25b - 1 - 20 + 0.8b + 0.875b = \underline{2.925b - 21}$

c. $\frac{x}{1 + 0.085 \times \frac{63}{365}} + 2x\left(1 + 0.085 \times \frac{151}{365}\right) = 0.985541x + 2.070329x = \underline{3.05587x}$

d. $\frac{96nm^2 - 72n^2m^2}{48n^2m} = \frac{4m - 3nm}{2n}$

5. $P(1+i)^n + \frac{S}{1+rt} = \$2500(1.1025)^2 + \frac{\$1500}{1 + 0.09 \times \frac{93}{365}} = \$3038.766 + \$1466.374 = \underline{\$4505.14}$

6. *a.* $L(1-d_1)(1-d_2)(1-d_3) = \$340(1-0.15)(1-0.08)(1-0.05) = \underline{\$252.59}$

b. $\frac{R}{i}\left[1 - \frac{1}{(1+i)^n}\right] = \frac{\$575}{0.085}\left[1 - \frac{1}{(1+0.085)^3}\right] = \$6764.706(1 - 0.7829081) = \underline{\$1468.56}$

7. *a.* $\frac{(-3x^2)^3(2x^{-2})}{6x^5} = \frac{(-27x^6)(2x^{-2})}{6x^5} = \underline{-\frac{9}{x}}$

b. $\frac{(-2a^3)^{-2}(4b^4)^{3/2}}{(-2b^3)(0.5a)^3} = \frac{\left(\frac{1}{4a^6}\right)(8b^6)}{(-2b^3)(0.125a^3)} = \underline{-\frac{8b^3}{a^9}}$

8. $\left(-\frac{2x^2}{3}\right)^{-2}\left(\frac{5^2}{6x^3}\right)\left(-\frac{15}{x^5}\right)^{-1} = \left(\frac{3}{2x^2}\right)^2\left(\frac{25}{6x^3}\right)\left(-\frac{x^5}{15}\right) = \underline{-\frac{5}{8x^2}}$

9. *a.* $1.0075^{24} = \underline{1.19641}$

b. $(1.05)^{1/6} - 1 = \underline{0.00816485}$

c. $\frac{(1+0.0075)^{36} - 1}{0.0075} = \underline{41.1527}$

d. $\frac{1 - (1+0.045)^{-12}}{0.045} = \underline{9.11858}$

10. *a.* $\frac{(1.00\overline{6})^{240} - 1}{0.00\overline{6}} = \frac{4.926802 - 1}{0.00\overline{6}} = \underline{589.020}$

b. $(1+0.025)^{1/3}-1=\underline{\underline{0.00826484}}$

a. $\dfrac{2x}{1+0.13\times\frac{92}{365}}+x\left(1+0.13\times\dfrac{59}{365}\right)=\$831$

$1.936545x + 1.021014x = \$831$

$2.957559x = \$831$

$x = \underline{\underline{\$280.97}}$

b. $3x(1.03^5) + \dfrac{x}{1.03^3} + x = \dfrac{\$2500}{1.03^2}$

$3.47782x + 0.91514x + x = \$2356.49$

$x = \underline{\underline{\$436.96}}$

11. *a.* $\dfrac{x}{1.08^3}+\dfrac{x}{2}(1.08)^4=\$850$

$0.793832x + 0.680245x = \$850$

$x = \underline{\underline{\$576.63}}$

Check: $\dfrac{\$576.63}{1.08^3}+\dfrac{\$576.63}{2}(1.08)^4=\$457.749+\$392.250=\$850.00$

b. $2x\left(1+0.085\times\dfrac{77}{365}\right)+\dfrac{x}{1+0.085\times\frac{132}{365}}=\$1565.70$

$2.03586x + 0.97018x = \$1565.70$

$x = \underline{\underline{\$520.85}}$

Check:

$2(\$520.85)\left(1+0.085\times\dfrac{77}{365}\right)+\dfrac{\$520.85}{1+0.085\times\frac{132}{365}}=\$1060.38+\$505.32=\$1565.70$

12. $N = L(1-d_1)(1-d_2)(1-d_3)$

$\$324.30 = \$498(1 - 0.20)(1 - d_2)(1 - 0.075)$

$\$324.30 = \$368.52(1 - d_2)$

$\dfrac{\$324.30}{\$368.52} = (1 - d_2)$

$d_2 = 1 - 0.8800 = \underline{\underline{0.120}} = \underline{\underline{12.0\%}}$

13. $V_f = V_i(1 + c_1)(1 + c_2)(1 + c_3)$

$\$586.64 = \$500(1 + 0.17)(1 + c_2)(1 + 0.09)$

$\$586.64 = \$637.65(1 + c_2)$

$1 + c_2 = \dfrac{\$586.64}{\$637.65}$

$c_2 = 0.9200 - 1 = \underline{\underline{-0.0800 = -8.00\%}}$

14. $FV = PV(1 + i_1)(1 + i_2)$

$\dfrac{FV}{PV(1+i_2)} = (1 + i_1)$

$i_1 = \dfrac{FV}{PV(1+i_2)} - 1$

15. $NI = (CM)X - FC$

$NI + FC = (CM)X$

$$CM = \frac{NI+FC}{X}$$

16. $Rate = \dfrac{Portion}{Base} = \dfrac{\$16.39}{\$6.39} \times 100\% = \underline{256.5\%}$

17. $Base = \dfrac{Portion}{Rate} = \dfrac{\$100}{0.80} = \underline{\$125.00}$

18. $Base = \dfrac{Portion}{Rate} = \dfrac{\$1.00}{0.0075} = \underline{\$133.33}$

19. Two hours = 2(60) = 120 minutes

$$Rate = \frac{Portion}{Base} = \frac{15 \text{ minutes}}{120 \text{ minutes}} \times 100\% = \underline{12.5\%}$$

20. $Base = \dfrac{Portion}{Rate} = \dfrac{\$150}{2.0} = \underline{\$75.00}$

21. *a.* Given: c = 17.5%, V_i = \$29.43
$V_f = V_i(1 + c) = \$29.43(1.175) = \underline{\$34.58}$
\$34.58 is 17.5% more than \$29.43.

b. Given: V_f = \$100, c = – 80%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1-0.80} = \underline{\$500.00}$$

80% off \$500 leaves \$100.

c. Given: V_f = \$100, c = – 15%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1-0.15} = \underline{\$117.65}$$

\$117.65 reduced by 15% equals \$100.

d. Given: V_i = \$47.50, c = 320%
$V_f = V_i(1 + c) = \$47.50(1 + 3.2) = \underline{\$199.50}$
\$47.50 after an increase of 320% is \$199.50.

e. Given: c = – 62%, V_f = \$213.56

$$V_i = \frac{V_f}{1+c} = \frac{\$213.56}{1-0.62} = \underline{\$562.00}$$

\$562 decreased by 62% equals \$213.56.

f. Given: c = 125%, V_f = \$787.50

$$V_i = \frac{V_f}{1+c} = \frac{\$787.50}{1+1.25} = \underline{\$350.00}$$

\$350 increased by 125% equals \$787.50.

g. Given: c = – 30%, V_i = \$300

$$V_f = V_i(1+c) = \$300(1-0.30) = \underline{\$210.00}$$

\$210 is 30% less than \$300.

22. Portion = 17 members, Rate = 11% of 42% = 0.11 x 42% = 4.62%

$$Base = \frac{Portion}{Rate} = \frac{17}{0.0462} = 367.965$$

Rounded to the nearest whole person, the building has 368 employees.

23. V_i = ice thickness in 1960's = 3.1 m

V_f = ice thickness in 1990's = 1.8 m

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{1.8 - 3.1}{3.1} \times 100\% = -41.94\%$$

24. a. V_i = 1970 price = \$1686

V_f = 2016 price = \$15,995

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$15,995 - \$1686}{\$1686} \times 100\% = 848.70\%$$

b. V_i = 1970 price + tax = \$1686(1 + 0.05) = \$1770.30

V_f = 2016 price + tax = \$15,995(1 + 13%) = \$18,074.35

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$18,074.35 - \$1770.30}{\$1770.30} \times 100\% = 920.98\%$$

25. Given:

	Year 1 value (V_i)	Year 2 *value* (V_f)
Gold produced:	34,300 oz.	23,750 oz.
Average price:	\$1160	\$1280

a. Percent change in gold production = $\frac{23,750 - 34,300}{34,300} \times 100\% = -30.76\%$

b. Percent change in price = $\frac{\$1280 - \$1160}{\$1160} \times 100\% = 10.34\%$

c. Year 1 revenue, V_i = 34,300(\$1160) = \$39.788 million
Year 2 revenue, V_f, = 23,750(\$1280) = \$30.400 million
Percent change in revenue = $\frac{\$30.400 - \$39.788}{\$39.788} \times 100\% = -23.60\%$

26. Given: For the first year, V_i = \$3.40, V_f = \$11.50.
For the second year, V_i = \$11.50, c = – 35%.

a. $c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$11.50 - \$3.40}{\$3.40} \times 100\% = 238.24\%$

The share price increased by 238.24% in the first year.

b. Current share price, $V_f = V_i$ (1 + c) = \$11.50(1 – 0.35) = \$7.48.

27. Given: For the first year, c = 150%
For the second year, c = – 40%, V_f = \$24
The price at the beginning of the second year was

$$V_i = \frac{V_f}{1+c} = \frac{\$24}{1-0.40} = \$40.00 = V_f$$ for the first year.

The price at the beginning of the first year was

$$V_i = \frac{V_f}{1+c} = \frac{\$40.00}{1+1.50} = \underline{\underline{\$16.00}}$$

Barry bought the stock for $16.00 per share.

28. Given: Last year's revenue = $2,347,000
Last year's expenses = $2,189,000

a. Given: Percent change in revenue = 10%; Percent change in expenses = 5%
Anticipated revenues, $V_f = V_i(1 + c)$ = $2,347,000(1.1) = $2,581,700
Anticipated expenses = $2,189,000(1.05) = $\underline{2,298,450}
Anticipated profit = $283,250
Last year's profit = $2,347,000 – $2,189,000 = $158,000

Percent increase in profit = $\frac{\$283,250 - \$158,000}{\$158,000} \times 100\% = \underline{\underline{79.27\%}}$

b. Given: c(revenue) = –10%; c(expenses) = – 5%
Anticipated revenues = $2,347,000(1 – 0.10) = $2,112,300
Anticipated expenses = $2,189,000(1 – 0.05) = $\underline{\$2,079,550}$
Anticipated profit $32,750

Percent change in profit = $\frac{\$32,750 - \$158,000}{\$158,000} \times 100\% = \underline{\underline{-79.27\%}}$

The operating profit will decline by 79.27%.

29. Given: For each unit, V_i = $20.35, V_f = $19.10, income = $0.40

a. Income yield = $\frac{\text{Income}}{V_i} \times 100\% = \frac{\$0.40}{\$20.35} \times 100\% = \underline{1.97\%}$

b. Capital gain yield = $\frac{V_f - V_i}{V_i} \times 100\% = \frac{\$19.10 - \$20.35}{\$20.35} \times 100\% = \underline{-6.14\%}$

c. Total return = Income + Capital gain = 1000($0.40) + 1000($19.10 – $20.35)
= $\underline{\underline{-\$850.00}}$

That is, an overall loss of $850.

d. Rate of total return = Income yield + Capital gain yield = 1.97% – 6.14% = $\underline{\underline{-4.17\%}}$

30. Given: c_1 = 0.23, c_2 = 0.10, c_3 = – 0.15, c_4 = 0.05, V_f = $30.50

a. Substitute in $V_f = V_i(1 + c_1).....(1 + c_4)$ and solve for V_i

$30.50 = V_i(1.23)(1.10)(0.85)(1.05)

$$V_i = \frac{\$30.50}{1.20755} = \underline{\underline{\$25.26}}$$

A share's initial price was $25.26.

b. The decline in the third year was 15% of the price at the end of the second year.

That is,

Price decline = 0.15×V_i (after 2 years) = 0.15($25.26)(1.23)(1.10) = $5.13

The share price declined by $5.13 in the third year.

31. The value, after 5 years, of an initial $100 investment was

$V_f = V_i(1 + c_1).....(1 + c_5)$

= $100(1 – 0.13)(1 + 0.18)(1 + 0.05)(1 + 0.24)(1 – 0.05)

= $126.98

The 5-year rate of return was the overall percent change in the five years

$$= \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$126.98 - \$100}{\$100} \times 100\% = 26.98\%$$

32. Suppose the initial investment in the AGF China Focus fund was $100.

Its value after 6 years was

$100(1 + c_1).....(1 + c_6) = $100(1.3053)(1 – 0.4167)(1.2775)(1.0127)(1 – 0.2071)(1.170)

= $91.38

The overall rate of return is $\frac{V_f - V_i}{V_i} \times 100\% = \frac{\$91.38 - \$100}{\$100} \times 100\% = -8.62\%$

33. Given: Ken's share = 0.80(Hugh's share) + $15,000; Total distribution = $98,430

Let H represent Hugh's share. Then

Hugh's share + Ken's share = Total distribution

H + 0.8H + $15,000 = $98,430

1.8H = $83,430

H = $46,350

Hugh should receive $46,350 and Ken should receive $98,430 – $46,350 = $52,080.

34. Given: Grace's share = 1.2(Kajsa's share); Mary Anne's share = $\frac{5}{8}$ (Grace's share)

Total allocated = $36,000

Let K represent Kajsa's share.

(Kajsa's share) + (Grace's share) + (Mary Anne's share) = $36,000

K + 1.2K + $\frac{5}{8}(1.2K)$ = $36,000

2.95 K = $36,000

K = $12,203.39

Kajsa's should receive $12,203.39. Grace should receive 1.2K = $14,644.07.

Mary Anne should receive $\frac{5}{8}$ ($14,644.07) = $9152.54.

35. 15% x (Income exceeding $68,000) = $6300

$$\text{Income exceeding \$68,000} = \frac{\$6300}{0.15} = \$42,000.00$$

Total net income = $68,000 + $42,000 = $110,000

36. Given: Income = $200; Income yield = 4%

Therefore, $\frac{\$200}{V_i} \times 100\% = 4\%$

and $V_i = \dfrac{\$200}{0.04} = \$5000$

Since the capital gain yield was 10%,

$V_f = V_i(1 + c) = \$5000(1 + 0.10) = \underline{\$5500.00}$

The investment's value after 1 year was \$5500.00.

37. Given: $V_i = \$8600$, $V_f = \$7900$; Capital loss = \$8600 – \$7900 = \$700

Total loss = $0.05(V_i) = 0.05(\$8600) = \$430.00$

Therefore, the \$700 capital loss was reduced to an overall loss of just \$430 by income from the investment.

Hence, Income = \$700 – \$430 = \underline{\$270}