Solutions for Introduction to Management Science 6th Edition by Hillier

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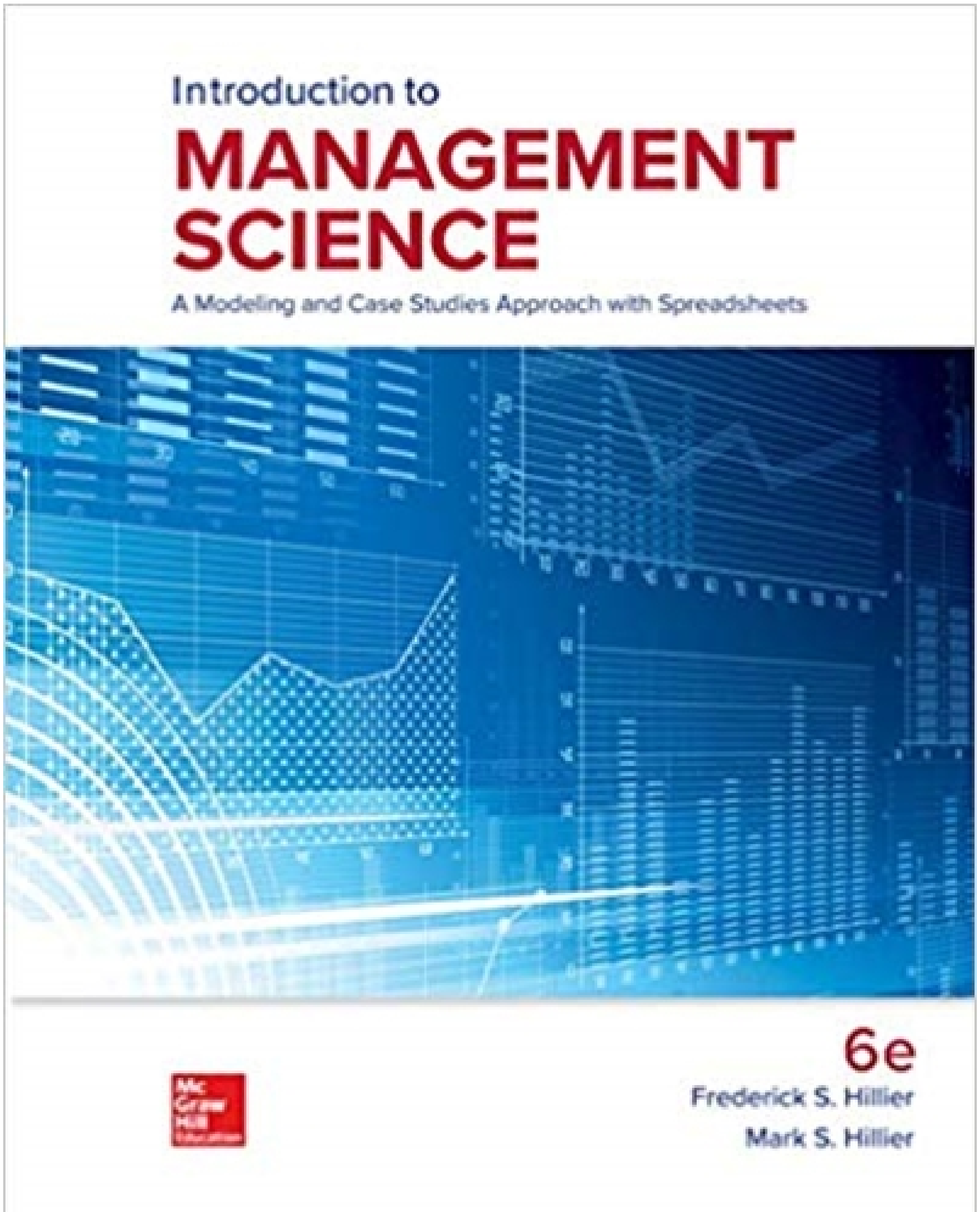


Solutions

CHAPTER 2
LINEAR PROGRAMMING: BASIC CONCEPTS

Review Questions

2.1-1 1) Should the company launch the two new products?

2) What should be the product mix for the two new products?

2.1-2 The group was asked to analyze product mix.

2.1-3 Which combination of production rates for the two new products would maximize the total profit from both of them.

2.1-4 1) available production capacity in each of the plants

2) how much of the production capacity in each plant would be needed by each product

3) profitability of each product

2.2-1 1) What are the decisions to be made?

2) What are the constraints on these decisions?

3) What is the overall measure of performance for these decisions?

2.2-2 When formulating a linear programming model on a spreadsheet, the cells showing the data for the problem are called the data cells. The changing cells are the cells that contain the decisions to be made. The output cells are the cells that provide output that depends on the changing cells. The objective cell is a special kind of output cell that shows the overall measure of performance of the decision to be made.

2.2-3 The Excel equation for each output cell can be expressed as a SUMPRODUCT function, where each term in the sum is the product of a data cell and a changing cell.

2.3-1 1) Gather the relevant data.

2) Identify the decisions to be made.

3) Identify the constraints on these decisions.

4) Identify the overall measure of performance for these decisions.

5) Convert the verbal description of the constraints and measure of performance into quantitative expressions in terms of the data and decisions

2.3-2 Algebraic symbols need to be introduced to represents the measure of performance and the decisions.

2.3-3 A decision variable is an algebraic variable that represents a decision regarding the level of a particular activity. The objective function is the part of a linear programming model that expresses what needs to be either maximized or minimized, depending on the objective for the problem. A nonnegativity constraint is a constraint that express the restriction that a particular decision variable must be greater than or equal to zero. All constraints that are not nonnegativity constraints are referred to as functional constraints.

2.3-4 A feasible solution is one that satisfies all the constraints of the problem. The best feasible solution is called the optimal solution.

2.4-1 Two.

2.4-2 The axes represent production rates for product 1 and product 2.

2.4-3 The line forming the boundary of what is permitted by a constraint is called a constraint boundary line. Its equation is called a constraint boundary equation.

2.4-4 The easiest way to determine which side of the line is permitted is to check whether the origin (0,0) satisfies the constraint. If it does, then the permissible region lies on the side of the constraint where the origin is. Otherwise it lies on the other side.

2.5-1 The Solver dialog box.

2.5-2 The Add Constraint dialog box.

2.5-3 The Simplex LP solving method and Make Variables Nonnegative option are selected.

2.6-1 The Objective button.

2.6-2 The Decisions button.

2.6-3 The Constraints button.

2.6-4 The Optimize button.

2.7-1 Cleaning products for home use.

2.7-2 Television and print media.

2.7-3 Determine how much to advertise in each medium to meet the market share goals at a minimum total cost.

2.7-4 The changing cells are in the column for the corresponding advertising medium.

2.7-5 The objective is to minimize total cost rather than maximize profit. The functional constraints contain ≥ rather than ≤.

2.8-1 No.

2.8-2 The graphical method helps a manager develop a good intuitive feeling for the linear programming is.

2.8-3 1) where linear programming is applicable

2) where it should not be applied

3) distinguish between competent and shoddy studies using linear programming.

4) how to interpret the results of a linear programming study.

Problems

2.1 Swift & Company solved a series of LP problems to identify an optimal production schedule. The first in this series is the scheduling model, which generates a shift-level schedule for a 28-day horizon. The objective is to minimize the difference of the total cost and the revenue. The total cost includes the operating costs and the penalties for shortage and capacity violation. The constraints include carcass availability, production, inventory and demand balance equations, and limits on the production and inventory. The second LP problem solved is that of capable-to-promise models. This is basically the same LP as the first one, but excludes coproduct and inventory. The third type of LP problem arises from the available-to-promise models. The objective is to maximize the total available production subject to production and inventory balance equations.

As a result of this study, the key performance measure, namely the weekly percent-sold position has increased by 22%. The company can now allocate resources to the production of required products rather than wasting them. The inventory resulting from this approach is much lower than what it used to be before. Since the resources are used effectively to satisfy the demand, the production is sold out. The company does not need to offer discounts as often as before. The customers order earlier to make sure that they can get what they want by the time they want. This in turn allows Swift to operate even more efficiently. The temporary storage costs are reduced by 90%. The customers are now more satisfied with Swift. With this study, Swift gained a considerable competitive advantage. The monetary benefits in the first years was $12.74 million, including the increase in the profit from optimizing the product mix, the decrease in the cost of lost sales, in the frequency of discount offers and in the number of lost customers. The main nonfinancial benefits are the increased reliability and a good reputation in the business.

2.2 a)

	A	B	C	D	E	F
1		Doors	Windows			
2	Unit Profit	$600	$300			
3				Hours		Hours
4		Hours Used Per Unit Produced		Used		Available
5	Plant 1	1	0	4	<=	4
6	Plant 2	0	2	6	<=	12
7	Plant 3	3	2	18	<=	18
8						
9		Doors	Windows			Total Profit
10	Units Produced	4	3			$3,300

b) Maximize $P = \$600D + \$300W$,

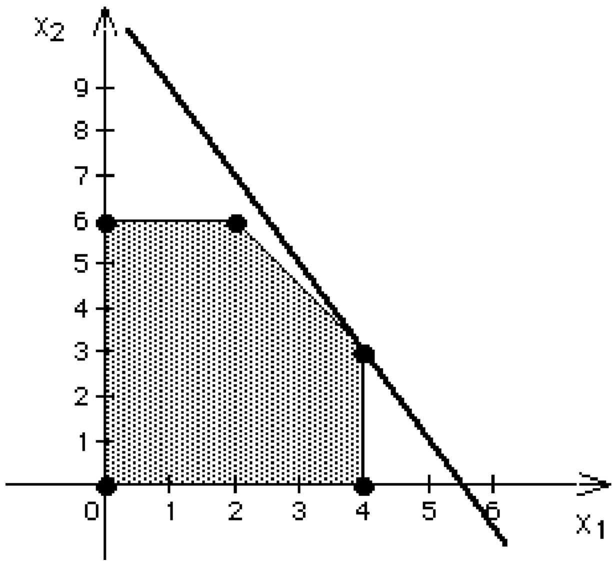
subject to $D \le 4$

$2W \le 12$

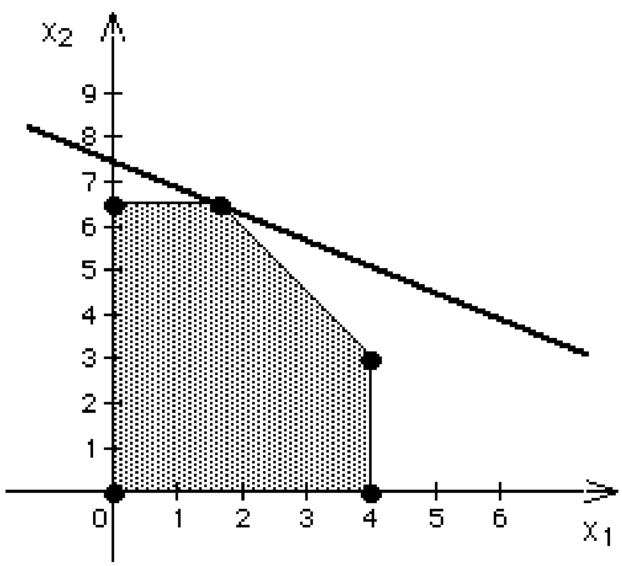
$3D + 2W \le 18$

and $D \ge 0, W \ge 0$.

c) Optimal Solution = $(D, W) = (x_1, x_2) = (4, 3)$. $P = \$3300$.



2.3 a) Optimal Solution: $(D, W) = (x_1, x_2) = (1.67, 6.50)$. $P = \$3750$.



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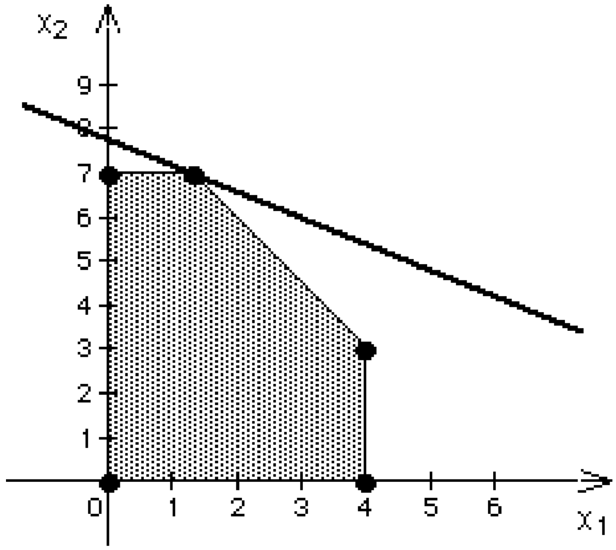
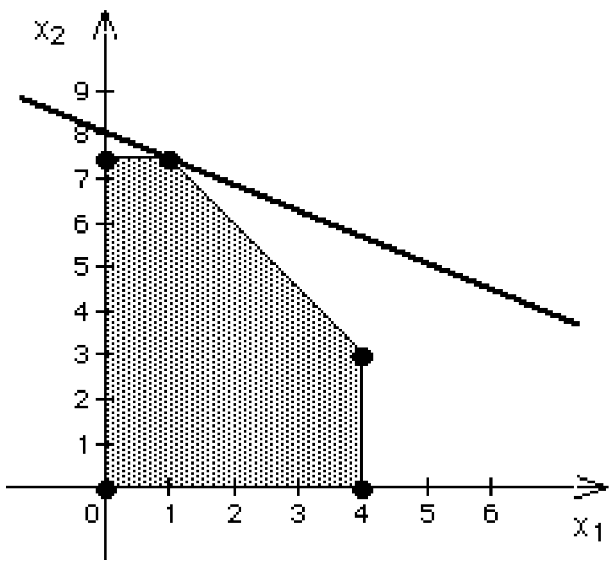
b) Optimal Solution: $(D, W) = (x_1, x_2) = (1.33, 7.00)$. P = $3900.



c) Optimal Solution: $(D, W) = (x_1, x_2) = (1.00, 7.50)$. P = $4050.



d) Each additional hour per week would increase total profit by $150.

2.4 a)

	A	B	C	D	E	F
1		Doors	Windows			
2	Unit Profit	$300	$500			
3				Hours		Hours
4		Hours Used Per Unit Produced		Used		Available
5	Plant 1	1	0	1.67	<=	4
6	Plant 2	0	2	13	<=	13
7	Plant 3	3	2	18	<=	18
8						
9		Doors	Windows			Total Profit
10	Units Produced	1.67	6.50			$3,750

b)

	A	B	C	D	E	F
1		Doors	Windows			
2	Unit Profit	$300	$500			
3				Hours		Hours
4		Hours Used Per Unit Produced		Used		Available
5	Plant 1	1	0	1.33	<=	4
6	Plant 2	0	2	14	<=	14
7	Plant 3	3	2	18	<=	18
8						
9		Doors	Windows			Total Profit
10	Units Produced	1.33	7			$3,900

c)

	A	B	C	D	E	F
1		Doors	Windows			
2	Unit Profit	$300	$500			
3				Hours		Hours
4		Hours Used Per Unit Produced		Used		Available
5	Plant 1	1	0	1	<=	4
6	Plant 2	0	2	15	<=	15
7	Plant 3	3	2	18	<=	18
8						
9		Doors	Windows			Total Profit
10	Units Produced	1	7.50			$4,050

d) Each additional hour per week would increase total profit by $150.

2.5 a)

	A	B	C	D	E	F
1		Product A	Product B			
2	Unit Profit	$3,000	$2,000			
3				Resource		Resource
4		Resource Usage per Unit Produced		Used		Available
5	Resource Q	2	1	2	<=	2
6	Resource R	1	2	2	<=	2
7	Resource S	3	3	4	<=	4
8						
9		Product A	Product B			Total Profit
10	Units Produced	0.667	0.667			$3,333.33

b) Let A = units of product A produced
B = units of product B produced

Maximize $P = \$3{,}000A + \$2{,}000B$,

subject to

$$2A + B \leq 2$$
$$A + 2B \leq 2$$
$$3A + 3B \leq 4$$

and $A \geq 0, B \geq 0.$

2.6 a) As in the Wyndor Glass Co. problem, we want to find the optimal levels of two activities that compete for limited resources.

Let x_1 be the fraction purchased of the partnership in the first friends venture.

Let x_2 be the fraction purchased of the partnership in the second friends venture.

The following table gives the data for the problem:

Resource	Resource Usage per Unit of Activity 1	2	Amount of Resource Available
Fraction of partnership in first friends venture	1	0	1
Fraction of partnership in second friends venture	0	1	1
Money	$10,000	$8,000	$12,000
Summer Work Hours	400	500	600
Unit Profit	$9,000	$9,000	

b) The decisions to be made are how much, if any, to participate in each venture. The constraints on the decisions are that you can't become more than a full partner in either venture, that your money is limited to $12,000, and time is limited to 600 hours. In addition, negative involvement is not possible. The overall measure of performance for the decisions is the profit to be made.

c) First venture: (fraction of 1st) ≤ 1
Second venture: (fraction of 2nd) ≤ 1
Money: 10,000 (fraction of 1st) + 8,000 (fraction of 2nd) ≤ 12,000
Hours: 400 (fraction of 1st) + 500 (fraction of 2nd) ≤ 600
Nonnegativity: (fraction of 1st) ≥ 0, (fraction of 2nd) ≥ 0

Profit = $9,000 (fraction of 1st) + $9,000 (fraction of 2nd)

d)

	A	B	C	D	E	F
1		First Friend	Second Friend			
2	Unit Profit	$9,000	$9,000			
3				Resource		Resource
4		Resource Usage		Used		Available
5	Money	$10,000	$8,000	$12,000	<=	$12,000
6	Work Hours	400	500	600	<=	600
7						
8		First Friend	Second Friend			Total Profit
9	Share	0.667	0.667			$12,000
10		<=	<=			
11		1	1			

Data cells: B2:C2, B5:C6, F5:F6, and B11:C11
Changing cells: B9:C9
Objective cell: F9
Output cells: D5:D6

	D
5	=SUMPRODUCT(B5:C5,$B$9:$C$9)
6	=SUMPRODUCT(B6:C6,$B$9:$C$9)

e) This is a linear programming model because the decisions are represented by changing cells that can have any value that satisfy the constraints. Each constraint has an output cell on the left, a mathematical sign in the middle, and a data cell on the right. The overall level of performance is represented by the objective cell and the objective is to maximize that cell. Also, the Excel equation for each output cell is expressed as a SUMPRODUCT function where each term in the sum is the product of a data cell and a changing cell.

f) Let x_1 = share taken in first friend's venture
x_2 = share taken in second friend's venture

Maximize $P = \$9{,}000x_1 + \$9{,}000x_2$,

subject to
$x_1 \le 1$
$x_2 \le 1$
$\$10{,}000x_1 + \$8{,}000x_2 \le \$12{,}000$
$400x_1 + 500x_2 \le 600$ hours

and $x_1 \ge 0, x_2 \ge 0$.

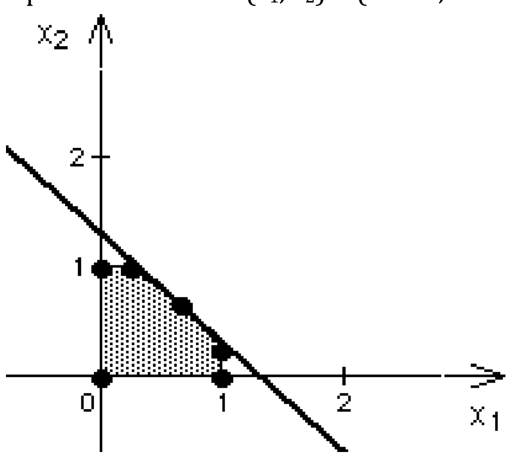
g) Algebraic Version

decision variables:	x_1, x_2
functional constraints:	$x_1 \le 1$
	$x_2 \le 1$
	$\$10{,}000x_1 + \$8{,}000x_2 \le \$12{,}000$
	$400x_1 + 500x_2 \le 600$ hours
objective function:	Maximize $P = \$9{,}000x_1 + \$9{,}000x_2$,
parameters:	all of the numbers in the above algebraic model
nonnegativity constraints:	$x_1 \ge 0, x_2 \ge 0$

Spreadsheet Version

decision variables:	B9:C9
functional constraints:	D4:F7
objective function:	F9
parameters:	B2:C2, B5:C6, F5:F6, and B11:C11
nonnegativity constraints:	"Make Unconstrained Variables Nonnegative" in Solver

h) Optimal solution = $(x_1, x_2) = (0.667, 0.667)$. $P = \$12{,}000$.



2.7 a) objective function $Z = x_1 + 2x_2$

functional constraints $x_1 + x_2 \le 5$
$x_1 + 3x_2 \le 9$

nonnegativity constraints $x_1 \ge 0, x_2 \ge 0$

b & e)

	A	B	C	D	E	F
1		x_1	x_2			
2	Unit Profit	1	2			
3				Resource		Resource
4		Resource Usage		Used		Available
5	Resource 1	1	1	5	<=	5
6	Resource 2	1	3	9	<=	9
7						
8		x_1	x_2			Total Profit
9	Decision	3	2			7

c) Yes.

d) No.

2.8 a) objective function $Z = 3x_1 + 2x_2$

functional constraints $3x_1 + x_2 \leq 9$

$x_1 + 2x_2 \leq 8$

nonnegativity constraints $x_1 \geq 0, x_2 \geq 0$

b & f)

	A	B	C	D	E	F
1		X_1	X_2			
2	Unit Profit	3	2			
3				Resource		Resource
4		Resource Usage		Used		Available
5	Resource 1	3	1	9	<=	9
6	Resource 2	1	2	8	<=	8
7						
8		X_1	X_2			Total Profit
9	Decision	2	3			12

c) Yes.

d) Yes.

e) No.

2.9 a) As in the Wyndor Glass Co. problem, we want to find the optimal levels of two activities that compete for limited resources. We want to find the optimal mix of the two activities.

Let *W* be the number of wood-framed windows to produce.
Let *A* be the number of aluminum-framed windows to produce.

The following table gives the data for the problem:

Resource	Resource Usage per Unit of Activity Wood-framed	Aluminum-framed	Amount of Resource Available
Glass	6	8	48
Aluminum	0	1	4
Wood	1	0	6
Unit Profit	$60	$30	

b) The decisions to be made are how many windows of each type to produce. The constraints on the decisions are the amounts of glass, aluminum and wood available. In addition, negative production levels are not possible. The overall measure of performance for the decisions is the profit to be made.

c) glass: 6 (#wood-framed) + 8 (# aluminum-framed) ≤ 48
aluminum: 1 (# aluminum-framed) ≤ 4
wood: 1 (#wood-framed) ≤ 6
Nonnegativity: (#wood-framed) ≥ 0, (# aluminum-framed) ≥ 0

Profit = $60 (#wood-framed) + $30 (# aluminum-framed)

d)

	A	B	C	D	E	F
1		Wood-framed	Aluminum-framed			
2	Unit Profit	$60	$30			
3						
4		Square-feet Used Per Unit Produced		Used		Available
5	Glass	6	8	48	<=	48
6						
7		Wood-framed	Aluminum-framed			Total Profit
8	Units Produced	6	1.50			$405
9		<=	<=			
10		6	4			

Data cells: B2:C2, B5:C5, F5, B10:C10
Changing cells: B8:C8
Objective cell: F8
Output cells: D5, F8

	D
4	Used
5	=SUMPRODUCT(B5:C5,$B$8:$C$8)

	F
7	Total Profit
8	=SUMPRODUCT(B2:C2,B8:C8)

e) This is a linear programming model because the decisions are represented by changing cells that can have any value that satisfy the constraints. Each constraint has an output cell on the left, a mathematical sign in the middle, and a data cell on the right. The overall level of performance is represented by the objective cell and the objective is to maximize that cell. Also, the Excel equation for each output cell is expressed as a SUMPRODUCT function where each term in the sum is the product of a data cell and a changing cell.

f) Maximize $P = 60W + 30A$
subject to $6W + 8A \leq 48$
$W \leq 6$
$A \leq 4$
and $W \geq 0, A \geq 0.$

g) Algebraic Version

decision variables: W, A

functional constraints: $6W + 8A \leq 48$
$W \leq 6$
$A \leq 4$

objective function: Maximize $P = 60W + 30A$

parameters: all of the numbers in the above algebraic model

nonnegativity constraints: $W \geq 0, A \geq 0$

Spreadsheet Version

decision variables: B8:C8

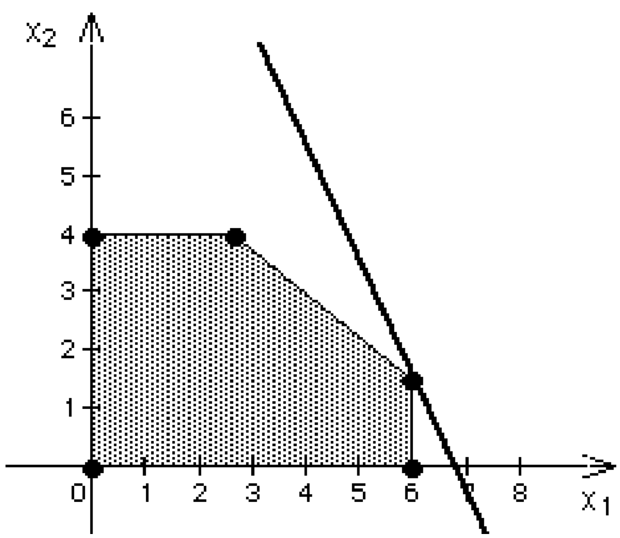
functional constraints: D8:F8, B8:C10

objective function: F8

parameters: B2:C2, B5:C5, F5, B10:C10

nonnegativity constraints: "Assume nonnegativity" in the Options of the Solver

h) Optimal Solution: $(W, A) = (x_1, x_2) = (6, 1.5)$ and $P = \$405$.



i) Solution unchanged when profit per wood-framed window = \$40, with $P = \$285$.
Optimal Solution = $(W, A) = (2.667, 4)$ when the profit per wood-framed window = \$20, with $P = \$173.33$.

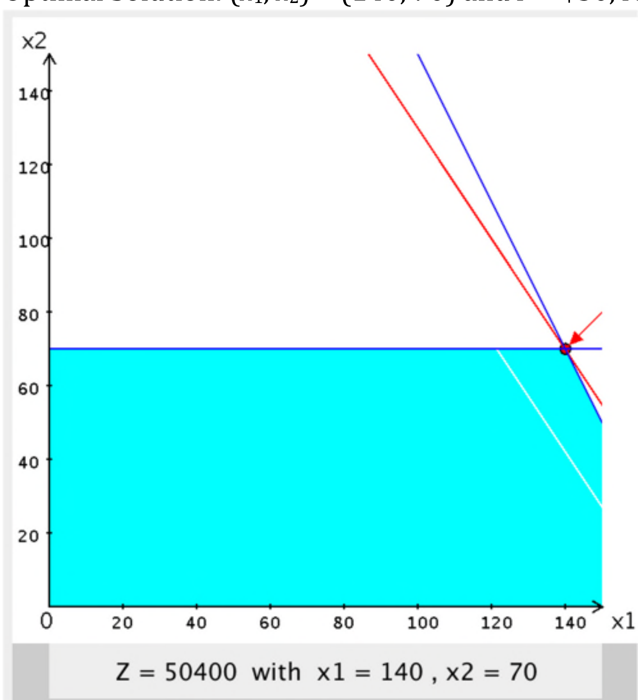
j) Optimal Solution = $(W, A) = (5, 2.25)$ if Doug can only make 5 wood frames per day, with $P = \$367.50$.

2.10 a)

	A	B	C	D	E	F
1		65" Sets	55" Sets			
2	Unit Profit	$270	$180			
3				Hours		Hours
4		Work Hours Per Unit Produced		Used		Available
5	Work Hours	20	10	3500	<=	3500
6						
7		Wood-framed	Aluminum-framed			Total Profit
8	Units Produced	140	70			$50,400
9		<=	<=			
10		280	70			

b) Let x_1 = number of 65" TV sets to be produced per month
Let x_2 = number of 55" TV sets to be produced per month
Maximize $P = \$270x_1 + \$180x_2$,
subject to $20x_1 + 10x_2 \leq 3500$
$x_1 \leq 280$
$x_2 \leq 70$
and $x_1 \geq 0, x_2 \geq 0$.

c) Optimal Solution: $(x_1, x_2) = (140, 70)$ and $P = \$50,400$.



2.11 a) The decisions to be made are how many of each light fixture to produce. The constraints are the amounts of frame parts and electrical components available, and the maximum number of product 2 that can be sold (900 units). In addition, negative production levels are not possible. The overall measure of performance for the decisions is the profit to be made.

b) frame parts: 1 (# product 1) + 3 (# product 2) ≤ 3,000
electrical components: 2 (# product 1) + 2 (# product 2) ≤ 4,500
product 2 max.: 1 (# product 2) ≤ 900
Nonnegativity: (# product 1) ≥ 0, (# product 2) ≥ 0

Profit = $13 (# product 1) + $6 (# product 2)

c)

	A	B	C	D	E	F
1		Product 1	Product 2			
2	Unit Profit	$13	$26			
3				Resource		Resource
4		Resource Usage		Used		Available
5	Frame Parts	1	3	3000	<=	3000
6	Electrical Components	2	2	4500	<=	4500
7						
8		Product 1	Product 2			Total Profit
9	Production	1875	375			$34,125
10			<=			
11			900			

d) Let x_1 = number of units of product 1 to produce
x_2 = number of units of product 2 to produce
Maximize $P = \$13x_1 + \$26x_2$,
subject to $x_1 + 3x_2 \leq 3{,}000$
$2x_1 + 2x_2 \leq 4{,}500$
$x_2 \leq 900$
and $x_1 \geq 0, x_2 \geq 0$.

2.12 a) The decisions to be made are what quotas to establish for the two product lines. The constraints are the amounts of work hours available in underwriting, administration, and claims. In addition, negative levels are not possible. The overall measure of performance for the decisions is the profit to be made.

b) underwriting: 3 (# special risk) + 2 (# mortgage) ≤ 2400
administration: 1 (# mortgage) ≤ 800
claims: 2 (# special risk) ≤ 1200
Nonnegativity: (# special risk) ≥ 0, (# mortgage) ≥ 0

Profit = $5 (# special risk) + $2 (# mortgage)

c)

	A	B	C	D	E	F
1		Special Risk	Mortgage			
2	Unit Profit	$5	$2			
3				Work-Hours		Work-Hours
4		Work-Hours per Unit		Used		Available
5	Underwriting	3	2	2,400	<=	2,400
6	Administration	0	1	300	<=	800
7	Claims	2	0	1,200	<=	1,200
8						
9		Special Risk	Mortgage			Total Profit
10	Sales Quota	600	300			$3,600

d) Let S = units of special risk insurance
M = units of mortgages

Maximize $P = \$5S + \$2M$,

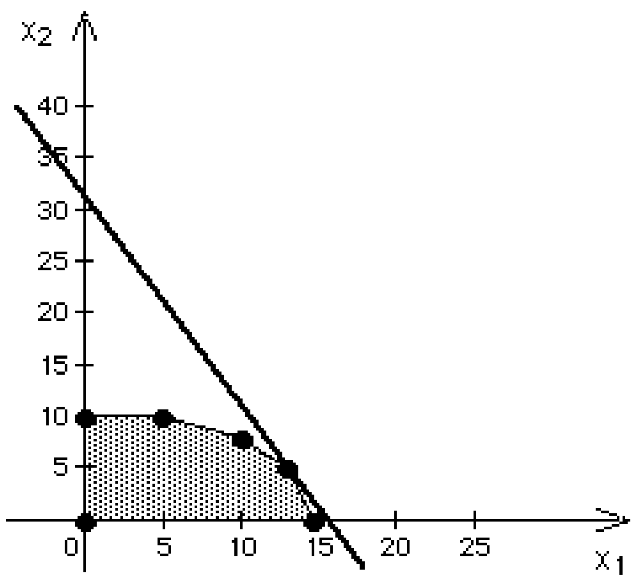
subject to $3S + 2M \leq 2{,}400$

$M \leq 800$

$2S \leq 1{,}200$

and $S \geq 0, M \geq 0$.

2.13 a) Optimal Solution: $(x_1, x_2) = (13, 5)$ and $P = 31$.



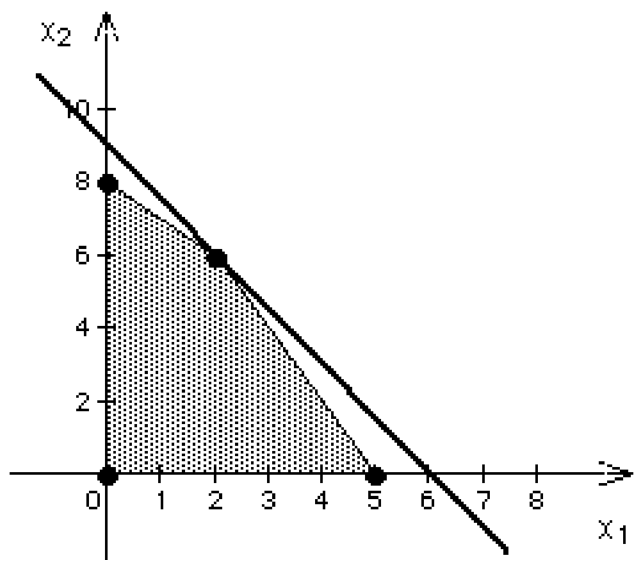
b)

	A	B	C	D	E	F
1		X_1	X_2			
2	Unit Profit	2	1			
3				Resource		Resource
4		Resource Usage		Used		Available
5	Resource 1	0	1	5	<=	10
6	Resource 2	2	5	51	<=	60
7	Resource 3	1	1	18	<=	18
8	Resource 4	3	1	44	<=	44
9						
10		X_1	X_2			Total Profit
11	Decision	13	5			31

2.14

	A	B	C	D	E	F
1		X_1	X_2			
2	Unit Profit	2	1			
3				Resource		Resource
4		Resource Usage		Used		Available
5	Resource 1	0	1	5	<=	10
6	Resource 2	2	5	51	<=	60
7	Resource 3	1	1	18	<=	18
8	Resource 4	3	1	44	<=	44
9						
10		X_1	X_2			Total Profit
11	Decision	13	5			31

2.15 a) Optimal Solution: $(x_1, x_2) = (2, 6)$ and $P = 18$.



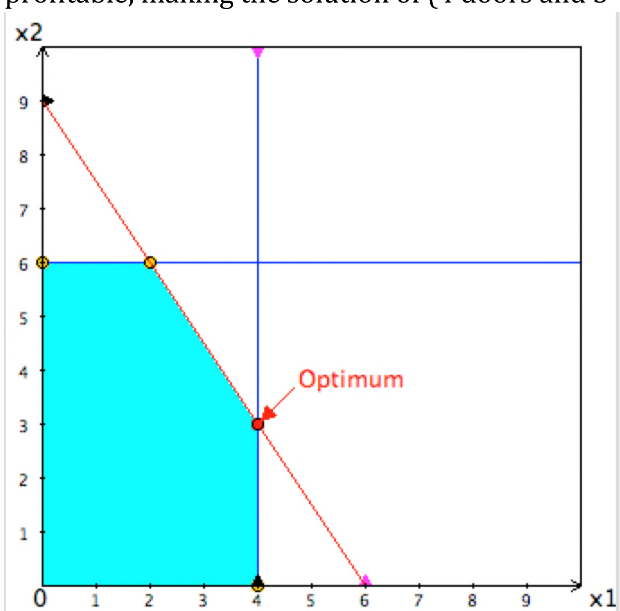
b)

	A	B	C	D	E	F
1		Product 1	Product 2			
2	Unit Profit	3	2			
3				Resource		Resource
4		Resource Usage		Used		Available
5	Resource 1	1	1	8	<=	8
6	Resource 2	2	1	10	<=	10
7						
8		Product 1	Product 2			Total Profit
9	Decision	2	6			18

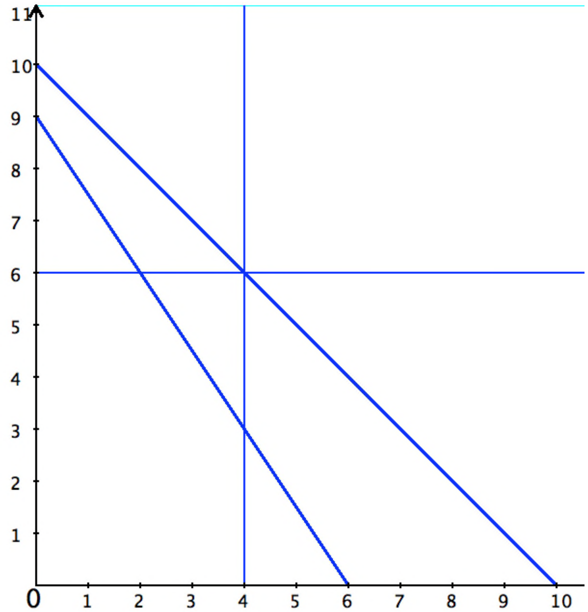
2.16

	A	B	C	D	E	F
1		Product 1	Product 2			
2	Unit Profit	3	2			
3				Resource		Resource
4		Resource Usage		Used		Available
5	Resource 1	1	1	8	<=	8
6	Resource 2	2	1	10	<=	10
7						
8		Product 1	Product 2			Total Profit
9	Decision	2	6			18

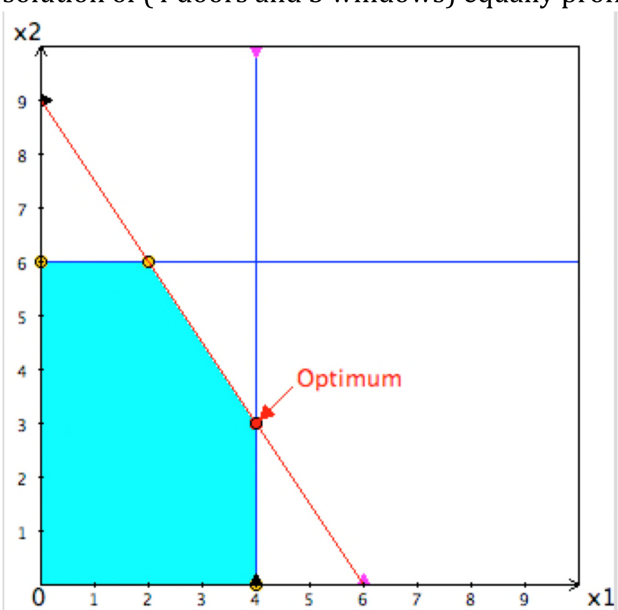
2.17 a) When the unit profit for Windows is $300, there are multiple optima, including (2 doors, 6 windows) and (4 doors and 3 windows) and all points inbetween. It is different than the original unique optimal solution of (2 doors, 6 windows) because windows are now more profitable, making the solution of (4 doors and 3 windows) equally profitable.



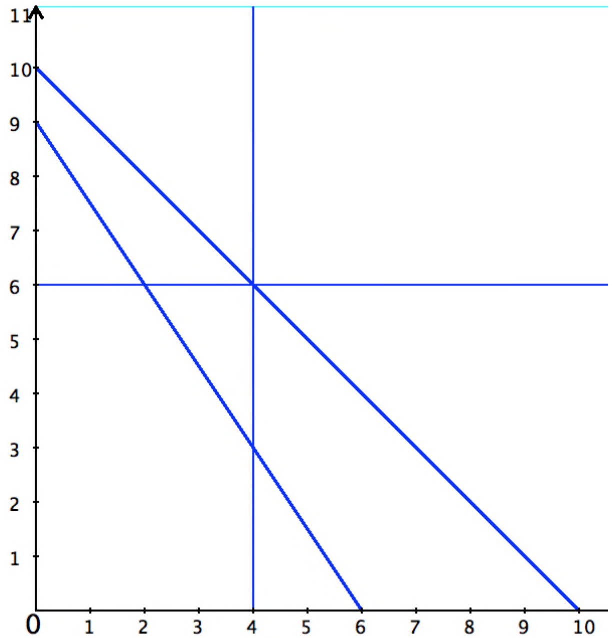
b) There is no feasible solution with the added requirement that there must be a total of 10 doors and/or windows.



c) If the constraints for plant 2 and plant 3 are inadvertently removed, then the solution is unbounded. There is nothing left to prevent making an unbounded number of windows, and hence making an unbounded profit.2.18 a) When the unit profit for Windows is $300, there are multiple optima, including (2 doors, 6 windows) and (4 doors and 3 windows) and all points inbetween. It is different than the original unique optimal solution of (2 doors, 6 windows) because windows are now more profitable, making the solution of (4 doors and 3 windows) equally profitable.



b) There is no feasible solution with the added requirement that there must be a total of 10 doors and/or windows.



c) If the constraints for plant 2 and plant 3 are inadvertently removed, then the solution is unbounded. There is nothing left to prevent making an unbounded number of windows, and hence making an unbounded profit.

2.18 a) The decisions to be made are how many frankfurters and buns should be produced. The constraints are the amounts of flour and pork available, and the hours available to work. In addition, negative production levels are not possible. The overall measure of performance for the decisions is the profit to be made.

b) flour: 0.1 (# buns) ≤ 200
pork: 0.25 (# frankfurters) ≤ 800
work hours: 3 (# frankfurters) + 2 (# buns) ≤ 12,000
Nonnegativity: (# frankfurters) ≥ 0, (# buns) ≥ 0

Profit = $0.40 (# frankfurters) + $0.20 (# buns)

c)

	A	B	C	D	E	F
1		Frankfurters	Buns			
2	Unit Profit	$0.40	$0.20			
3				Resource		Resource
4		Resource Usage		Used		Available
5	Flour	0	0.1	120	<=	200
6	Pork	0.25	0	800	<=	800
7	Work Hours	3	2	12,000	<=	12,000
8						
9		Frankfurters	Buns			Total Profit
10	Decision	3,200	1,200			$1,520

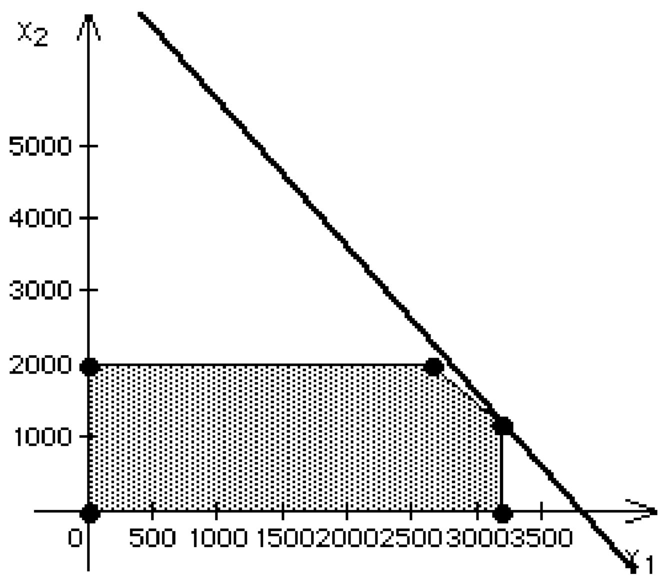
d) Let F = # of frankfurters to produce
B = # of buns to produce

Maximize $P = \$0.40F + \$0.20B$,

subject to
$0.1B \le 200$
$0.25F \le 800$
$3F + 2B \le 12{,}000$

and $F \ge 0, B \ge 0$.

e) Optimal Solution: $(F, B) = (x_1, x_2) = (3200, 1200)$ and $P = \$1520$.



2.19 a)

	A	B	C	D	E	F
1		Tables	Chairs			
2	Unit Profit	$400	$100			
3				Resource		Resource
4		Resource Usage		Used		Available
5	Oak	50	25	2,500	<=	2,500
6	Labor Hours	6	6	450	<=	480
7						
8		Tables	Chairs			Total Profit
9	Decision	25	50			$15,000
10						
11	Chairs	50	>=	50	2	Times Number
12						of Tables

b)

	A	B	C	D	E	F
1		Tables	Chairs			
2	Unit Profit	$400	$100			
3				Resource		Resource
4		Resource Usage		Used		Available
5	Oak	50	25	2,500	<=	2,500
6	Labor Hours	6	6	450	<=	480
7						
8		Tables	Chairs			Total Profit
9	Decision	25	50			$15,000
10						
11	Chairs	50	>=	50	2	Times Number
12						of Tables

c) Let T = # of tables to produce
C = # of chairs to produce

Maximize $P = \$400T + \$100C$

subject to $50T + 25C \leq 2{,}500$
$6T + 6C \leq 480$
$C \geq 2T$

and $T \geq 0, C \geq 0$.

2.20 After the sudden decline of prices at the end of 1995, Samsung Electronics faced the urgent need to improve its noncompetitive cycle times. The project called SLIM (short cycle time and low inventory in manufacturing) was initiated to address this problem. As part of this project, floor-scheduling problem is formulated as a linear programming model. The goal is to identify the optimal values "for the release of new lots into the fab and for the release of initial WIP from every major manufacturing step in discrete periods, such as work days, out to a horizon defined by the user" [p. 71]. Additional variables are included to determine the route of these through alternative machines. The optimal values "minimize back-orders and finished-goods inventory" [p. 71] and satisfy capacity constraints and material flow equations. CPLEX was used to solved the linear programs.

With the implementation of SLIM, Samsung significantly reduced its cycle times and as a result of this increased its revenue by $1 billion (in five years) despite the decrease in selling prices. The market share increased from 18 to 22 percent. The utilization of machines was improved. The reduction in lead times enabled Samsung to forecast sales more accurately and so to carry less inventory. Shorter lead times also meant happier customers and a more efficient feedback mechanism, which allowed Samsung to respond to customer needs. Hence, SLIM did not only help Samsung to survive a crisis that drove many out of the business, but it did also provide a competitive advantage in the business.

2.21 a)

	A	B	C	D	E	F	G	H	I	J	K
1		Beef	Gravy	Peas	Carrots	Roll					
2	Unit Cost	$0.40	$0.35	$0.15	$0.18	$0.10					
3	(per ounce)										
4			Nutritional Data (per ounce)				Total in Diet		Needed		Maximum
5	Calories	54	20	15	8	40	320	>=	280	<=	320
6	Fat Calories	19	15	0	0	10	96				
7	Vitamin A (IU)	0	0	15	350	0	600	>=	600		
8	Vitamin C (mg)	0	1	3	1	0	12.38	>=	10		
9	Protein (g)	8	0	1	1	1	30	>=	30		
10											
11		Beef	Gravy	Peas	Carrots	Roll			Total Cost		
12	Diet (ounces)	2.94	1.47	3.11	1.58	1.82			$2.62		
13		>=									
14	Minimums	2									
15											
16	Fat Calories	96	<=	96	30%	of Total Calories					
17											
18	Gravy	1.47	>=	1.47	50%	of Beef					

b)

	A	B	C	D	E	F	G	H	I	J	K
1		Beef	Gravy	Peas	Carrots	Roll					
2	Unit Cost	$0.40	$0.35	$0.15	$0.18	$0.10					
3	(per ounce)										
4			Nutritional Data (per ounce)				Total in Diet		Needed		Maximum
5	Calories	54	20	15	8	40	320	>=	280	<=	320
6	Fat Calories	19	15	0	0	10	96				
7	Vitamin A (IU)	0	0	15	350	0	600	>=	600		
8	Vitamin C (mg)	0	1	3	1	0	12.38	>=	10		
9	Protein (g)	8	0	1	1	1	30	>=	30		
10											
11		Beef	Gravy	Peas	Carrots	Roll			Total Cost		
12	Diet (ounces)	2.94	1.47	3.11	1.58	1.82			$2.62		
13		>=									
14	Minimums	2									
15											
16	Fat Calories	96	<=	96	30%	of Total Calories					
17											
18	Gravy	1.47	>=	1.47	50%	of Beef					

c) Let B = ounces of beef tips in diet,
G = ounces of gravy in diet,
P = ounces of peas in diet,
C = ounces of carrots in diet,
R = ounces of roll in diet.

Minimize $Z = \$0.40B + \$0.35G + \$0.15P + \$0.18C + \$0.10R$

subject to
$54B + 20G + 15P + 8C + 40R \geq 280$
$54B + 20G + 15P + 8C + 40R \leq 320$
$19B + 15G + 10R \leq 0.3(54B + 20G + 15P + 8C + 40R)$
$15P + 350C \geq 600$
$G + 3P + C \geq 10$
$8B + P + C + R \geq 30$
$B \geq 2$
$G \geq 0.5B$

and $B \geq 0, G \geq 0, P \geq 0, C \geq 0, R \geq 0.$

2.22 a) The decisions to be made are how many servings of steak and potatoes are needed. The constraints are the amounts of carbohydrates, protein, and fat that are needed. In addition, negative levels are not possible. The overall measure of performance for the decisions is the cost.

b) carbohydrates: 5 (# steak) + 15 (# potatoes) ≥ 50
protein: 20 (# steak) + 5 (# potatoes) ≥ 40
fat: 15 (# steak) + 2 (# potatoes) ≤ 60
Nonnegativity: (# steak) ≥0, (# potatoes) ≥ 0

Cost = 4 (# steak) + 2 (# potatoes)

c)

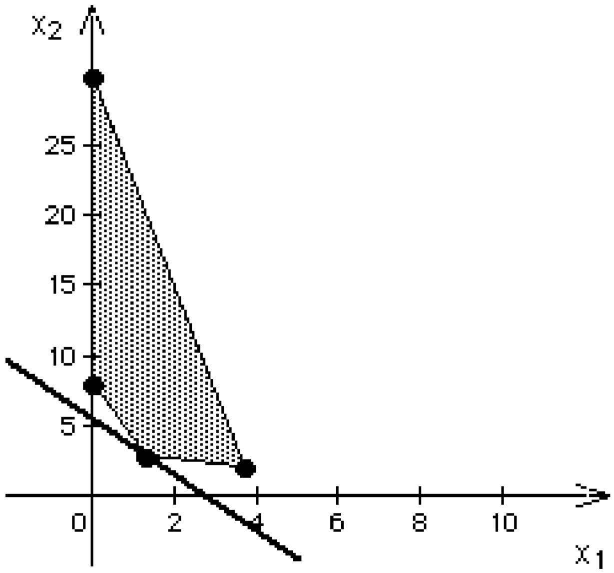
	A	B	C	D	E	F
1		Steak	Potatoes			
2	Unit Cost	$4	$2			
3				Total Nutrition		Daily Requirement
4		Nutritional Info (grams/serving)		(grams)		(grams)
5	Carbohydrates	5	15	50	>=	50
6	Protein	20	5	40	>=	40
7	Fat	15	2	24.91	<=	60
8						
9		Steak	Potatoes			Total Cost
10	Servings	1.27	2.91			$10.91

d) Let S = servings of steak in diet
P = servings of potatoes in the diet

Minimize $C = \$4S + \$2P$,

subject to
$$5S + 15P \geq 50$$
$$20S + 5P \geq 40$$
$$15S + 2P \leq 60$$
and $S \geq 0, P \geq 0$.

e & f) Optimal Solution: $(S, P) = (x_1, x_2) = (1.27, 2.91)$ and $C = \$10.91$.



2.23 a) The decisions to be made are what combination of feed types to use. The constraints are the amounts of calories and vitamins needed, and a maximum level for feed type A. In addition, negative levels are not possible. The overall measure of performance for the decisions is the cost.

b) Calories: 800 (lb. Type A) + 1000 (lb. Type B) ≥ 8000
Vitamins: 140 (lb. Type A) + 70 (lb. Type B) ≥ 700
Type A maximum: (lb. Type A) ≤ 0.333((lb. Type A) + (lb. Type B))
Nonnegativity: (lb. Type A) ≥ 0, (lb. Type B) ≥ 0

Cost = \$0.40 (lb. Type A) + \$0.80 (lb. Type B)

c)

	A	B	C	D	E	F
1		Feed A	Feed B			
2	Unit Cost	$0.40	$0.80			
3	(per pound)			Total		Daily
4		Nutrition (per pound)		Nutrition		Requirement
5	Calories	800	1,000	8,000	>=	8,000
6	Vitamins	140	70	800	>=	700
7						
8		Feed A	Feed B			Total Cost
9	Diet (pounds)	2.86	5.71			$5.71
10		<=				
11		2.86	33.33%	of Total Diet		

d) Let A = pounds of Feed Type A in diet

B = pounds of Feed Type B in diet

Minimize $C = \$0.40A + \$0.80B$,

subject to $800A + 1{,}000B \geq 8{,}000$

$140A + 70B \geq 700$

$A \leq (1/3)(A + B)$

and $A \geq 0, B \geq 0$.

2.24 a)

	A	B	C	D	E	F
1		Television	Print Media			
2	Unit Cost ($millions)	1	2			
3						
4				Increased		Minimum
5		Increase in Sales per Unit of Advertising		Sales		Increase
6	Stain Remover	0%	1.5%	4%	>=	3%
7	Liquid Detergent	3%	4%	18%	>=	18%
8	Powder Detergent	-1%	2%	4%	>=	4%
9						
10						Total Cost
11		Television	Print Media			($millions)
12	Advertising Units	2	3			8

b) Let T = units of television advertising

P = units of print media advertising

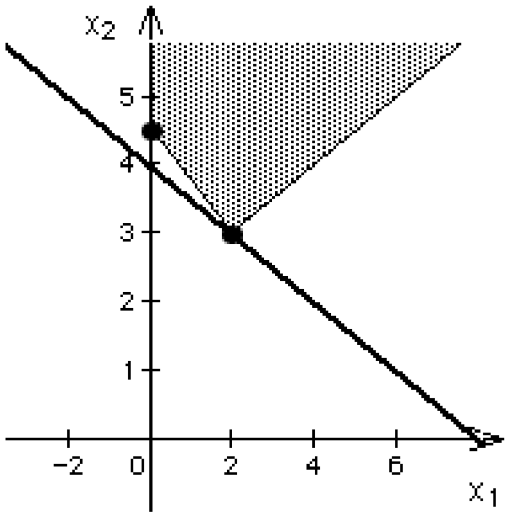
Minimize $C = T + 2P$,

subject to $1.5P \geq 3$

$3T + 4P \geq 18$

$-T + 2P \geq 4$

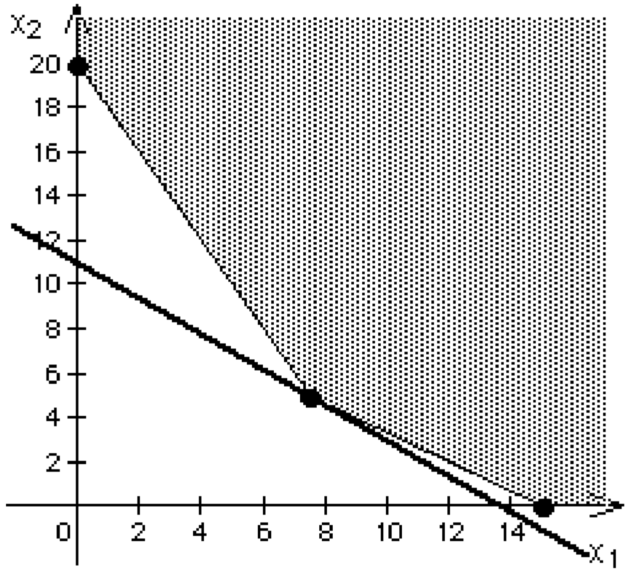
c) Optimal Solution: $(x_1, x_2) = (2, 3)$ and C = $8 million.



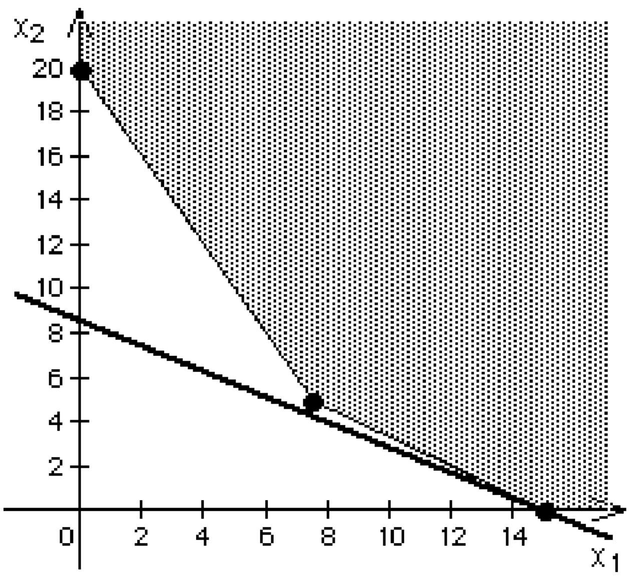
d) Management changed their assessment of how much each type of ad would change sales. For print media, sales will now increase by 1.5% for product 1, 2% for product 2, and 2% for product 3.

e) Given the new data on advertising, I recommend that there be 2 units of advertising on television and 3 units of advertising in the print media. This will minimize cost, with a cost of $8 million, while meeting the minimum increase requirements. Further refining the data may allow us to rework the problem and save even more money while maintaining the desired increases in market share. In addition, when negotiating a decrease in the unit cost of television ads, our new data shows that we should purchase fewer television ads at the current price so they might want to reduce the current price.

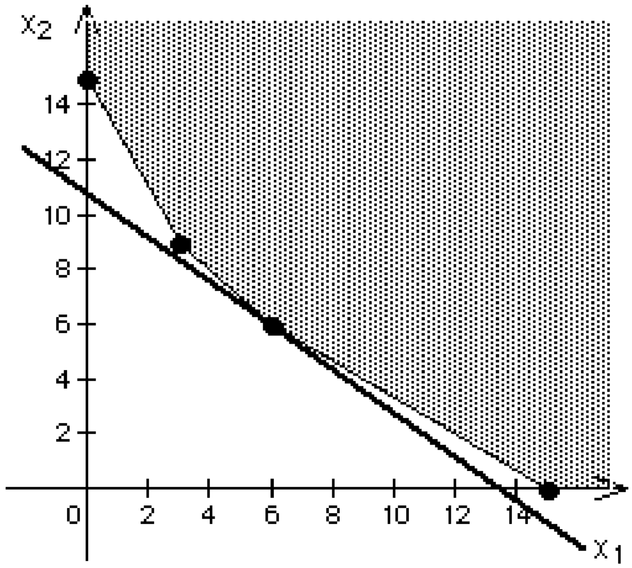
2.25 a) Optimal Solution: $(x_1, x_2) = (7.5, 5)$ and $C = 550$.



b) Optimal Solution: $(x_1, x_2) = (15, 0)$ and $C = 600$.



c) Optimal Solution: $(x_1, x_2) = (6, 6)$ and $C = 540$.



d)

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Cost	40	50			
3						
4				Totals		Limit
5	Constraint 1	2	3	30	>=	30
6	Constraint 2	1	1	12.5	>=	12
7	Constraint 3	2	1	20	>=	20
8						
9		Activity 1	Activity 2			Total Cost
10	Decision	7.5	5			550

e) Part b)

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Cost	40	70			
3						
4				Totals		Limit
5	Constraint 1	2	3	30	>=	30
6	Constraint 2	1	1	15	>=	12
7	Constraint 3	2	1	30	>=	20
8						
9		Activity 1	Activity 2			Total Cost
10	Decision	15	0			600

Part c)

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Cost	40	50			
3						
4				Totals		Limit
5	Constraint 1	2	3	30	>=	30
6	Constraint 2	1	1	12	>=	12
7	Constraint 3	2	1	18	>=	15
8						
9		Activity 1	Activity 2			Total Cost
10	Decision	6	6			540

2.26 a)

	A	B	C	D	E	F	G	H	I	J	K	L
1		Bread	Peanut Butter	Jelly		Milk	Juice					
2		(slice)	(tbsp)	(tbsp)	Apples	(cup)	(cup)					
3	Unit Cost	$0.06	$0.05	$0.08	$0.35	$0.20	$0.40					
4												
5				Nutritional Data				Total in Diet				
6	Calories from Fat	15	80	0	0	60	0	128.46		Needed		Maximum
7	Calories	80	100	70	90	120	110	443.08	>=	300	<=	500
8	Vitamin C (mg)	0	0	4	6	2	80	60	>=	60		
9	Fiber (g)	4	0	3	10	0	1	11.69	>=	10		
10												
11		Bread	Peanut Butter	Jelly		Milk	Juice					
12		(slice)	(tbsp)	(tbsp)	Apples	(cup)	(cup)			Total Cost		
13	Diet (ounces)	2	1	1	0	0.308	0.692			$0.59		
14		>=	>=	>=								
15	Minimums	2	1	1								
16												
17	Fat Calories	128	<=	132.92	30%	of Total Calories						
18												
19	Milk and Juice	1	>=	1								

b) Let B = slices of bread,
P = Tbsp. of peanut butter,
J = Tbsp. of jelly,
A = number of apples,
M = cups of milk,
C = cups of cranberry juice.

Minimize $C = \$0.06B + \$0.05P + \$0.08J + \$0.35A + \$0.20M + \$0.40C$

subject to
$80B + 100P + 70J + 90A + 120M + 110C \geq 300$
$80B + 100P + 70J + 90A + 120M + 110C \leq 500$
$15B + 80P + 60M \leq 0.3(80B + 100P + 70J + 90A + 120M + 110C)$
$4J + 6A + 2M + 80C \geq 60$
$4B + 3J + 10A + C \geq 10$
$B \geq 2$
$P \geq 1$
$J \geq 1$
$M + C \geq 1$

and $B \geq 0, P \geq 0, J \geq 0, A \geq 0, M \geq 0, C \geq 0.$

Cases

2.1 a) In this case, we have two decision variables: the number of Family Thrillseekers we should assemble and the number of Classy Cruisers we should assemble. We also have the following three constraints:

1. The plant has a maximum of 48,000 labor hours.
2. The plant has a maximum of 20,000 doors available.
3. The number of Cruisers we should assemble must be less than or equal to 3,500.

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$3,600	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	6	10.5	48,000	<=	48,000
7	Doors	4	2	20,000	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	3,800	2,400			$26,640,000
12			<=			
13	Demand		3,500			

Range Name	Cells
ClassyCruisers	C11
Demand	C13
Production	B11:C11
ResourcesAvailable	F6:F7
ResourcesUsed	D6:D7
TotalProfit	F11
UnitProfit	B3:C3

	D
4	Resources
5	Used
6	=SUMPRODUCT(B6:C6,Production)
7	=SUMPRODUCT(B7:C7,Production)

	F
10	Total Profit
11	=SUMPRODUCT(UnitProfit,Production)

Solver Parameters

Set Objective Cell: TotalProfit

To: Max

By Changing Variable Cells:

Production

Subject to the Constraints:

ClassyCruisers <= Demand

ResourcesUsed <= Resources Available

Solver Options:

Make Variables Nonnegative

Solving Method: Simplex LP

Rachel's plant should assemble 3,800 Thrillseekers and 2,400 Cruisers to obtain a maximum profit of $26,640,000.

b) In part (a) above, we observed that the Cruiser demand constraint was not binding. Therefore, raising the demand for the Cruiser will not change the optimal solution. The marketing campaign should not be undertaken.

c) The new value of the right-hand side of the labor constraint becomes 48,000 * 1.25 = 60,000 labor hours. All formulas and Solver settings used in part (a) remain the same.

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$3,600	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	6	10.5	56,250	<=	60,000
7	Doors	4	2	20,000	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	3,250	3,500			$30,600,000
12			<=			
13	Demand		3,500			

Rachel's plant should now assemble 3,250 Thrillseekers and 3,500 Cruisers to achieve a maximum profit of $30,600,000.

d) Using overtime labor increases the profit by $30,600,000 – $26,640,000 = $3,960,000. Rachel should therefore be willing to pay at most $3,960,000 extra for overtime labor beyond regular time rates.

e) The value of the right-hand side of the Cruiser demand constraint is 3,500 * 1.20 = 4,200 cars. The value of the right-hand side of the labor hour constraint is 48,000 * 1.25 = 60,000 hours. All formulas and Solver settings used in part (a) remain the same. Ignoring the costs of the advertising campaign and overtime labor,

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$3,600	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	6	10.5	60,000	<=	60,000
7	Doors	4	2	20,000	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	3,000	4,000			$32,400,000
12			<=			
13	Demand		4,200			

Rachel's plant should produce 3,000 Thrillseekers and 4,000 Cruisers for a maximum profit of $32,400,000. This profit excludes the costs of advertising and using overtime labor.

f) The advertising campaign costs $500,000. In the solution to part (e) above, we used the maximum overtime labor available, and the maximum use of overtime labor costs $1,600,000. Thus, our solution in part (e) required an extra $500,000 + $1,600,000 = $2,100,000. We perform the following cost/benefit analysis:

Profit in part (e):	$32,400,000
– Advertising and overtime costs:	$ 2,100,000
	$30,300,000

We compare the $30,300,000 profit with the $26,640,000 profit obtained in part (a) and conclude that the decision to run the advertising campaign and use overtime labor is a very wise, profitable decision.

g) Because we consider this question independently, the values of the right-hand sides for the Cruiser demand constraint and the labor hour constraint are the same as those in part (a). We now change the profit for the Thrillseeker from $3,600 to $2,800 in the problem formulation. All formulas and Solver settings used in part (a) remain the same.

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$2,800	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	6	10.5	48,000	<=	48,000
7	Doors	4	2	14,500	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	1,875	3,500			$24,150,000
12			<=			
13	Demand		3,500			

Rachel's plant should assemble 1,875 Thrillseekers and 3,500 Cruisers to obtain a maximum profit of $24,150,000.

h) Because we consider this question independently, the profit for the Thrillseeker remains the same as the profit specified in part (a). The labor hour constraint changes. Each Thrillseeker now requires 7.5 hours for assembly. All formulas and Solver settings used in part (a) remain the same.

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$3,600	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	7.5	10.5	48,000	<=	48,000
7	Doors	4	2	13,000	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	1,500	3,500			$24,300,000
12			<=			
13	Demand		3,500			

Rachel's plant should assemble 1,500 Thrillseekers and 3,500 Cruisers for a maximum profit of $24,300,000.

i) Because we consider this question independently, we use the problem formulation used in part (a). In this problem, however, the number of Cruisers assembled has to be strictly equal to the total demand. The formulas used in the problem formulation remain the same as those used in part (a).

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$3,600	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	6	10.5	48,000	<=	48,000
7	Doors	4	2	14,500	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	1,875	3,500			$25,650,000
12			=			
13	Demand		3,500			

The new profit is $25,650,000, which is $26,640,000 – $25,650,000 = $990,000 less than the profit obtained in part (a). This decrease in profit is less than $2,000,000, so Rachel should meet the full demand for the Cruiser.

j) We now combine the new considerations described in parts (f), (g), and (h). In part (f), we decided to use both the advertising campaign and the overtime labor. The advertising campaign raises the demand for the Cruiser to 4,200 sedans, and the overtime labor increases the labor hour capacity of the plant to 60,000 labor hours. In part (g), we decreased the profit generated by a Thrillseeker to $2,800. In part (h), we increased the time to assemble a Thrillseeker to 7.5 hours. The formulas and Solver settings used for this problem are the same as those used in part (a).

	A	B	C	D	E	F
1		Family	Classy			
2		Thrillseeker	Cruiser			
3	Unit Profit	$2,800	$5,400			
4				Resources		Resources
5		Resource Requirements		Used		Available
6	Labor Hours	7.5	10.5	60,000	<=	60,000
7	Doors	4	2	16,880	<=	20,000
8						
9		Family	Classy			
10		Thrillseeker	Cruiser			Total Profit
11	Production	2,120	4,200			$28,616,000
12			<=			
13	Demand		4,200			

Rachel's plant should assemble 2,120 Thrillseekers and 4,200 Cruisers for a maximum profit of $28,616,000 – $2,100,000 = $26,516,000.

2.2 a) We want to determine the amount of potatoes and green beans Maria should purchase to minimize ingredient costs. We have two decision variables: the amount (in pounds) of potatoes Maria should purchase and the amount (in pounds) of green beans Maria should purchase. We also have constraints on nutrition, taste, and weight.

Nutrition Constraints

1. We first need to ensure that the dish has 180 grams of protein. We are told that 100 grams of potatoes have 1.5 grams of protein and 10 ounces of green beans have 5.67 grams of protein. Since we have decided to measure our decision variables in pounds, however, we need to determine the grams of protein in one pound of each ingredient.

We perform the following conversion for potatoes:

$$\left(\frac{1.5 \text{ g protein}}{100 \text{ g potatoes}}\right)\left(\frac{28.35 \text{ g}}{1 \text{ oz.}}\right)\left(\frac{16 \text{ oz.}}{1 \text{ lb.}}\right) = \frac{6.804 \text{ g protein}}{1 \text{ lb. of potatoes}}$$

We perform the following conversion for green beans:

$$\left(\frac{5.67 \text{ g protein}}{10 \text{ oz. green beans}}\right)\left(\frac{16 \text{ oz.}}{1 \text{ lb.}}\right) = \frac{9.072 \text{ g protein}}{1 \text{ lb. of green beans}}$$

2. We next need to ensure that the dish has 80 milligrams of iron. We are told that 100 grams of potatoes have 0.3 milligrams of iron and 10 ounces of green beans have 3.402 milligrams of iron. Since we have decided to measure our decision variables in pounds, however, we need to determine the milligrams of iron in one pound of each ingredient.

We perform the following conversion for potatoes:

$$\left(\frac{0.3\text{ mg iron}}{100\text{g potatoes}}\right)\left(\frac{28.35\text{ g}}{1\text{ oz.}}\right)\left(\frac{16\text{ oz.}}{1\text{ lb.}}\right)=\frac{1.361\text{ mg iron}}{1\text{ lb. of potatoes}}$$

We perform the following conversion for green beans:

$$\left(\frac{3.402\text{ mg iron}}{10\text{ oz. green beans}}\right)\left(\frac{16\text{ oz.}}{1\text{ lb.}}\right)=\frac{5.443\text{ mg iron}}{1\text{ lb. of green beans}}$$

3. We next need to ensure that the dish has 1,050 milligrams of vitamin C. We are told that 100 grams of potatoes have 12 milligrams of vitamin C and 10 ounces of green beans have 28.35 milligrams of vitamin C. Since we have decided to measure our decision variables in pounds, however, we need to determine the milligrams of vitamin C in one pound of each ingredient.

We perform the following conversion for potatoes:

$$\left(\frac{12\text{ mg Vitamin C}}{100\text{g potatoes}}\right)\left(\frac{28.35\text{ g}}{1\text{ oz.}}\right)\left(\frac{16\text{ oz.}}{1\text{ lb.}}\right)=\frac{54.432\text{ mg Vitamin C}}{1\text{ lb. of potatoes}}$$

We perform the following conversion for green beans:

$$\left(\frac{28.35\text{ mg Vitamin C}}{10\text{ oz. green beans}}\right)\left(\frac{16\text{ oz.}}{1\text{ lb.}}\right)=\frac{45.36\text{ mg Vitamin C}}{1\text{ lb. of green beans}}$$

Taste Constraint

Edson requires that the casserole contain at least a six to five ratio in the weight of potatoes to green beans. We have:

$$\frac{\text{pounds of potatoes}}{\text{pounds of green beans}} \geq \frac{6}{5}$$

5 (pounds of potatoes) ≥ 6 (pounds of green beans)

Weight Constraint

Finally, Maria requires a minimum of 10 kilograms of potatoes and green beans together. Because we measure potatoes and green beans in pounds, we must perform the following conversion:

$$10 \text{ kg of potatoes and green beans} \left(\frac{1000 \text{ g}}{1 \text{ kg}}\right)\left(\frac{1 \text{ lb}}{453.6 \text{ g}}\right)$$

$$= 22.046 \text{ lb of potatoes and green beans}$$

	A	B	C	D	E	F	G
1			Potatoes	Green Beans			
2		Unit Cost (per lb.)	$0.40	$1.00			
3					Total		Nutritional
4			Nutritional Data (per pound)		Nutrition		Requirement
5		Protein (g)	6.804	9.072	194.87	>=	180
6		Iron (mg)	1.361	5.443	80.00	>=	80
7		Vitamin C (mg)	54.432	45.36	1,251.27	>=	1,050
8							
9			Potatoes	Green Beans	Total Weight		Total Cost
10		Quantity (lb.)	13.57	11.31	25		$16.73
11					>=		
12				Minimum Weight (lb.)	22.046		
13							
14			Taste Constraint:				
15	5	Times Potatoes	67.833	>=	67.833	6	Times Green Beans

	E
3	Total
4	Nutrition
5	=SUMPRODUCT(C5:D5,Quantity)
6	=SUMPRODUCT(C6:D6,Quantity)
7	=SUMPRODUCT(C7:D7,Quantity)
8	
9	Total Weight
10	=SUM(Quantity)

Range Name	Cells
BeanRatio	E15
MinimumWeight	E12
NutritionalRequirement	G5:G7
PotatoRatio	C15
Quantity	C10:D10
TotalCost	G10
TotalNutrition	E5:E7
TotalWeight	E10
UnitCost	C2:D2

	G
9	Total Cost
10	=SUMPRODUCT(UnitCost,Quantity)

	A	B	C	D	E	F	G
14			Taste Constraint:				
15	5	Times Potatoes	=A15*C10	>=	=F15*D10	6	Times Green Beans

Solver Parameters

Set Objective Cell: TotalCost

To: Min

By Changing Variable Cells:

Quantity

Subject to the Constraints:

PotatoRatio >= BeanRation
TotalNutrition >= NutritionalRequirement

TotalWeight <= MinimumWeight

Solver Options:

Make Variables Nonnegative

Solving Method: Simplex LP

Maria should purchase 13.57 lb. of potatoes and 11.31 lb. of green beans to obtain a minimum cost of $16.73.

b) The taste constraint changes. The new constraint is now.

$$\frac{\text{pounds of potatoes}}{\text{pounds of green beans}} \geq \frac{1}{2}$$

2 (pounds of potatoes) ≥ 1 (pounds of green beans)

The formulas and Solver settings used to solve the problem remain the same as part (a).

	A	B	C	D	E	F	G
1			Potatoes	Green Beans			
2		Unit Cost (per lb.)	$0.40	$1.00			
3					Total		Nutritional
4			Nutritional Data (per pound)		Nutrition		Requirement
5		Protein (g)	6.804	9.072	180.00	>=	180
6		Iron (mg)	1.361	5.443	80.00	>=	80
7		Vitamin C (mg)	54.432	45.36	1,110.00	>=	1,050
8							
9			Potatoes	Green Beans	Total Weight		Total Cost
10		Quantity (lb.)	10.29	12.13	22		$16.24
11					>=		
12				Minimum Weight (lb.)	22.046		
13							
14			Taste Constraint:				
15	2	Times Potatoes	20.576	>=	12.125	1	Times Green Beans

Maria should purchase 10.29 lb. of potatoes and 12.13 lb. of green beans to obtain a minimum cost of $16.24.

c) The right-hand side of the iron constraint changes from 80 mg to 65 mg. The formulas and Solver settings used in the problem remain the same as in part (a).

	A	B	C	D	E	F	G
1			Potatoes	Green Beans			
2		Unit Cost (per lb.)	$0.40	$1.00			
3					Total		Nutritional
4			Nutritional Data (per pound)		Nutrition		Requirement
5		Protein (g)	6.804	9.072	180.00	>=	180
6		Iron (mg)	1.361	5.443	65.00	>=	65
7		Vitamin C (mg)	54.432	45.36	1,222.51	>=	1,050
8							
9			Potatoes	Green Beans	Total Weight		Total Cost
10		Quantity (lb.)	15.80	7.99	24		$14.31
11					>=		
12				Minimum Weight (lb.)	22.046		
13							
14			Taste Constraint:				
15	5	Times Potatoes	79.001	>=	47.947	6	Times Green Beans

Maria should purchase 15.80 lb. of potatoes and 7.99 lb. of green beans to obtain a minimum cost of $14.31.

d) The iron requirement remains 65 mg. We need to change the price per pound of green beans from $1.00 per pound to $0.50 per pound. The formulas and Solver settings used in the problem remain the same as in part (a).

	A	B	C	D	E	F	G
1			Potatoes	Green Beans			
2		Unit Cost (per lb.)	$0.40	$0.50			
3					Total		Nutritional
4			Nutritional Data (per pound)		Nutrition		Requirement
5		Protein (g)	6.804	9.072	180.00	>=	180
6		Iron (mg)	1.361	5.443	73.90	>=	65
7		Vitamin C (mg)	54.432	45.36	1,155.79	>=	1,050
8							
9			Potatoes	Green Beans	Total Weight		Total Cost
10		Quantity (lb.)	12.53	10.44	23		$10.23
11					>=		
12				Minimum Weight (lb.)	22.046		
13							
14			Taste Constraint:				
15	5	Times Potatoes	62.657	>=	62.657	6	Times Green Beans

Maria should purchase 12.53 lb. of potatoes and 10.44 lb. of green beans to obtain a minimum cost of $10.23.

e) We still have two decision variables: one variable to represent the amount (in pounds) of potatoes Maria should purchase and one variable to represent the amount (in pounds) of lima beans Maria should purchase. To determine the grams of protein in one pound of lima beans, we perform the following conversion:

$$\left(\frac{22.68\text{ g protein}}{10\text{ oz. lima beens}}\right)\left(\frac{16\text{ oz.}}{1\text{ lb.}}\right)=\frac{36.288\text{ g protein}}{1\text{ lb. of lima beans}}$$

To determine the milligrams of iron in one pound of lima beans, we perform the following conversion:

$$\left(\frac{6.804\text{ mg iron}}{10\text{ oz. lima beans}}\right)\left(\frac{16\text{ oz.}}{1\text{ lb.}}\right)=\frac{10.886\text{ mg iron}}{1\text{ lb. of lima beans}}$$

Lima beans contain no vitamin C, so we do not have to perform a measurement conversion for vitamin C.

We change the decision variable from green beans to lima beans and insert the new parameters for protein, iron, vitamin C, and cost. The formulas and Solver settings used in the problem remain the same as in part (a).

	A	B	C	D	E	F	G
1			Potatoes	Lima Beans			
2		Unit Cost (per lb.)	$0.40	$0.60			
3					Total		Nutritional
4			Nutritional Data (per pound)		Nutrition		Requirement
5		Protein (g)	6.804	36.288	260.41	>=	180
6		Iron (mg)	1.361	10.886	65.00	>=	65
7		Vitamin C (mg)	54.432	0	1,050.00	>=	1,050
8							
9			Potatoes	Lima Beans	Total Weight		Total Cost
10		Quantity (lb.)	19.29	3.56	23		$9.85
11					>=		
12				Minimum Weight (lb.)	22.046		
13							
14			Taste Constraint:				
15	5	Times Potatoes	96.451	>=	21.356	6	Times Lima Beans

Maria should purchase 19.29 lb. of potatoes and 3.56 lb. of lima beans to obtain a minimum cost of $9.85.

f) Edson takes pride in the taste of his casserole, and the optimal solution from above does not seem to preserve the taste of the casserole. First, Maria forces Edson to use lima beans instead of green beans, and lima beans are not an ingredient in Edson's original recipe. Second, although Edson places no upper limit on the ratio of potatoes to beans, the above recipe uses an over five to one ratio of potatoes to beans. This ratio seems unreasonable since such a large amount of potatoes will overpower the taste of beans in the recipe.

g) We only need to change the values on the right-hand side of the iron and vitamin C constraints. The formulas and Solver settings used in the problem remain the same as in part (a). The values used in the new problem formulation and solution follow.

	A	B	C	D	E	F	G
1			Potatoes	Lima Beans			
2		Unit Cost (per lb.)	$0.40	$0.60			
3					Total		Nutritional
4			Nutritional Data (per pound)		Nutrition		Requirement
5		Protein (g)	6.804	36.288	428.58	>=	180
6		Iron (mg)	1.361	10.886	120.00	>=	120
7		Vitamin C (mg)	54.432	0	685.72	>=	500
8							
9			Potatoes	Lima Beans	Total Weight		Total Cost
10		Quantity (lb.)	12.60	9.45	22		$10.71
11					>=		
12				Minimum Weight (lb.)	22.046		
13							
14			Taste Constraint:				
15	5	Times Potatoes	62.988	>=	56.690	6	Times Lima Beans

Maria should purchase 12.60 lb. of potatoes and 9.45 lb. of lima beans to obtain a minimum cost of $10.71.

2.3 a) The number of operators that the hospital needs to staff the call center during each two-hour shift can be found in the following table:

	A	B	C	D	E	F
1			Average	Average	English	Spanish
2		Average	Calls/hour	Calls/hour	Speaking	Speaking
3		Number	from English	from Spanish	Agents	Agents
4	Work Shift	of Calls	Speakers	Speakers	Needed	Needed
5	7am-9am	40	32	8	6	2
6	9am-11am	85	68	17	12	3
7	11am-1pm	70	56	14	10	3
8	1pm-3pm	95	76	19	13	4
9	3pm-5pm	80	64	16	11	3
10	5pm-7pm	35	28	7	5	2
11	7pm-9pm	10	8	2	2	1
12						
13	Percent English Speakers		80%			
14						
15	Calls Handled per hour		6			

For example, the average number of phone calls per hour during the shift from 7am to 9am equals 40. Since, on average, 80% of all phone calls are from English speakers, there is an average number of 32 phone calls per hour from English speakers during that shift. Since one operator takes, on average, 6 phone calls per hour, the hospital needs 32/6 = 5.333 English-speaking operators during that shift. The hospital cannot employ fractions of an operator and so needs 6 English-speaking operators for the shift from 7am to 9am.

b) The problems of determining how many Spanish-speaking operators and English-speaking operators Lenny needs to hire to begin each shift are independent. Therefore we can formulate two smaller linear programming models instead of one large model. We are going to have one model for the scheduling of the Spanish-speaking operators and another one for the scheduling of the English-speaking operators.

Lenny wants to minimize the operating costs while answering all phone calls. For the given scheduling problem we make the assumption that the only operating costs are the wages of the employees for the hours that they answer phone calls. The wages for the hours during which they perform paperwork are paid by other cost centers. Moreover, it does not matter for the callers whether an operator starts his or her work day with phone calls or with paperwork. For example, we do not need to distinguish between operators who start their day answering phone calls at 9am and operators who start their day with paperwork at 7am, because both groups of operators will be answering phone calls at the same time. And only this time matters for the analysis of Lenny's problem.

We define the decision variables according to the time when the employees have their first shift of answering phone calls. For the scheduling problem of the English-speaking operators we have 7 decision variables. First, we have 5 decision variables for full-time employees.

The number of operators having their first shift on the phone from 7am to 9am.
The number of operators having their first shift on the phone from 9am to 11am.
The number of operators having their first shift on the phone from 11am to 1pm.
The number of operators having their first shift on the phone from 1pm to 3pm.
The number of operators having their first shift on the phone from 3pm to 5pm.

In addition, we define 2 decision variables for part-time employees.

The number of part-time operators having their first shift from 3pm to 5pm.
The number of part-time operators having their first shift from 5pm to 7pm.

The unit cost coefficients in the objective function are the wages operators earn while they answer phone calls. All operators who have their first shift on the phone from 7am to 9am, 9am to 11am, or 11am to 1pm finish their work on the phone before 5pm. They earn 4*$15 = $60 during their time answering phone calls. All operators who have their first shift on the phone from 1pm to 3pm or 3pm to 5pm have one shift on the phone before 5pm and another one after 5pm. They earn 2*$15+2*$18 = $66 during their time answering phone calls. The second group of part-time operators, those having their first shift from 5pm to 7pm, earn 4*$18 = $72 during their time answering phone calls.

There are 7 constraints, one for each two-hour shift during which phone calls need to be answered. The right-hand sides for these constraints are the number of operators needed to ensure that all phone calls get answered in a timely manner. On the left-hand side we determine the number of operators on the phone during any given shift. For example, during the 11am to 1pm shift the total number of operators answering phone calls equals the sum of the number of operators who started answering calls at 7am and are currently in their second shift of the day and the number of operators who started answering calls at 11am.

The following spreadsheet describes the entire problem formulation for the English-speaking employees:

	A	B	C	D	E	F	G	H	I	J	K
1	**English**	Full-Time	Full-Time	Full-Time	Full-Time	Full-Time					
2	**Speaking**	on Phone	on Phone	on Phone	on Phone	on Phone	Part-Time	Part-Time			
3		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm	on Phone	on Phone			
4		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm	3pm-7pm	5pm-9pm			
5	Unit Cost	$60	$60	$60	$66	$66	$66	$72			
6									Total		Agents
7	Work Shift?								Working		Needed
8	7am-9am	1	0	0	0	0	0	0	6	>=	6
9	9am-11am	0	1	0	0	0	0	0	12	>=	12
10	11am-1pm	1	0	1	0	0	0	0	11	>=	10
11	1pm-3pm	0	1	0	1	0	0	0	13	>=	13
12	3pm-5pm	0	0	1	0	1	1	0	11	>=	11
13	5pm-7pm	0	0	0	1	0	1	1	5	>=	5
14	7pm-9pm	0	0	0	0	1	0	1	2	>=	2
15											
16		Full-Time	Full-Time	Full-Time	Full-Time	Full-Time					
17		on Phone	on Phone	on Phone	on Phone	on Phone	Part-Time	Part-Time			
18		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm	on Phone	on Phone			
19		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm	3pm-7pm	5pm-9pm			Total Cost
20	Number Working	6	12	5	1	2	4	0			$1,842

	I
6	Total
7	Working
8	=SUMPRODUCT(B8:H8,NumberWorking)
9	=SUMPRODUCT(B9:H9,NumberWorking)
10	=SUMPRODUCT(B10:H10,NumberWorking)
11	=SUMPRODUCT(B11:H11,NumberWorking)
12	=SUMPRODUCT(B12:H12,NumberWorking)
13	=SUMPRODUCT(B13:H13,NumberWorking)
14	=SUMPRODUCT(B14:H14,NumberWorking)

Range Name	Cells
AgentsNeeded	K8:K14
NumberWorking	B20:H20
TotalCost	K20
TotalWorking	I8:I14
UnitCost	B5:H5

	K
19	Total Cost
20	=SUMPRODUCT(UnitCost,NumberWorking)

Solver Parameters

Set Objective Cell: TotalCost

To: Min

By Changing Variable Cells:

NumberWorking

Subject to the Constraints:

TotalWorking >= AgentsNeeded

Solver Options:

Make Variables Nonnegative

Solving Method: Simplex LP

The linear programming model for the Spanish-speaking employees can be developed in a similar fashion.

	A	B	C	D	E	F	G	H	I
1	**Spanish**	Full-Time	Full-Time	Full-Time	Full-Time	Full-Time			
2	**Speaking**	on Phone	on Phone	on Phone	on Phone	on Phone			
3		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm			
4		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm			
5	Unit Cost	$60	$60	$60	$66	$72			
6							Total		Agents
7	Work Shift?						Working		Needed
8	7am-9am	1	0	0	0	0	2	>=	2
9	9am-11am	0	1	0	0	0	3	>=	3
10	11am-1pm	1	0	1	0	0	4	>=	3
11	1pm-3pm	0	1	0	1	0	5	>=	4
12	3pm-5pm	0	0	1	0	1	3	>=	3
13	5pm-7pm	0	0	0	1	0	2	>=	2
14	7pm-9pm	0	0	0	0	1	1	>=	1
15									
16		Full-Time	Full-Time	Full-Time	Full-Time	Full-Time			
17		on Phone	on Phone	on Phone	on Phone	on Phone			
18		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm			
19		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm			Total Cost
20	Number Working	2	3	2	2	1			$624

c) Lenny should hire 27 full-time English-speaking operators. Of these operators, 6 have their first phone shift from 7am to 9am, 12 from 9am to 11am, 5 from 11am to 1pm, 1 from 1pm to 3pm, and 2 from 3pm to 5pm. Lenny should also hire 4 part-time operators who start their work at 3pm. In addition, Lenny should hire 10 Spanish-speaking operators. Of these operators, 2 have their first shift on the phone from 7am to 9am, 3 from 9am to 11am, 2 from 11am to 1pm and 1pm to 3pm, and 1 from 3pm to 5pm. The total (wage) cost of running the calling center equals $2466 per day.

d) The restriction that Lenny can find only one English-speaking operator who wants to start work at 1pm affects only the linear programming model for English-speaking operators. This restriction does not put a bound on the number of operators who start their first phone shift at 1pm because those operators can start work at 11am with paperwork. However, this restriction does put an upper bound on the number of operators having their first phone shift from 3pm to 5pm. The new worksheet appears as follows.

	A	B	C	D	E	F	G	H	I	J	K
1	**English**	Full-Time	Full-Time	Full-Time	Full-Time	Full-Time					
2	**Speaking**	on Phone	on Phone	on Phone	on Phone	on Phone	Part-Time	Part-Time			
3		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm	on Phone	on Phone			
4		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm	3pm-7pm	5pm-9pm			
5	Unit Cost	$60	$60	$60	$66	$66	$66	$72			
6									Total		Agents
7	Work Shift?								Working		Needed
8	7am-9am	1	0	0	0	0	0	0	6	>=	6
9	9am-11am	0	1	0	0	0	0	0	12	>=	12
10	11am-1pm	1	0	1	0	0	0	0	13	>=	10
11	1pm-3pm	0	1	0	1	0	0	0	13	>=	13
12	3pm-5pm	0	0	1	0	1	1	0	11	>=	11
13	5pm-7pm	0	0	0	1	0	1	1	5	>=	5
14	7pm-9pm	0	0	0	0	1	0	1	2	>=	2
15											
16		Full-Time	Full-Time	Full-Time	Full-Time	Full-Time					
17		on Phone	on Phone	on Phone	on Phone	on Phone	Part-Time	Part-Time			
18		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm	on Phone	on Phone			
19		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm	3pm-7pm	5pm-9pm			Total Cost
20	Number Working	6	12	7	1	1	3	1			$1,902
21						<=					
22						1					

Lenny should hire 27 full-time English-speaking operators. Of these operators, 6 have their first phone shift from 7am to 9am, 12 from 9am to 11am, 7 from 11am to 1pm, 1 from 1p-3pm, and 1 from 3pm to 5pm. Lenny should also hire 3 part-time operators who start their work at 3pm and 1 part-time operator starting work at 5pm. The hiring of Spanish-speaking operators is unaffected. The new total (wage) costs equal $2526 per day.

e) For each hour, we need to divide the average number of calls per hour by the average processing speed, which is 6 calls per hour. The number of bilingual operators that the hospital needs to staff the call center during each two-hour shift can be found in the following table:

	A	B	C
1		Average	
2		Number	Agents
3	Work Shift	of Calls	Needed
4	7am-9am	40	7
5	9am-11am	85	15
6	11am-1pm	70	12
7	1pm-3pm	95	16
8	3pm-5pm	80	14
9	5pm-7pm	35	6
10	7pm-9pm	10	2
11			
12	Calls Handled per hour		6

f) The linear programming model for Lenny's scheduling problem can be found in the same way as before, only that now all operators are bilingual. (The formulas and the solver dialog box are identical to those in part (b).)

	A	B	C	D	E	F	G	H	I	J	K
1	Bilingual	Full-Time	Full-Time	Full-Time	Full-Time	Full-Time					
2		on Phone	on Phone	on Phone	on Phone	on Phone	Part-Time	Part-Time			
3		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm	on Phone	on Phone			
4		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm	3pm-7pm	5pm-9pm			
5	Unit Cost	$60	$60	$60	$66	$66	$66	$72			
6									Total		Agents
7	Work Shift?								Working		Needed
8	7am-9am	1	0	0	0	0	0	0	7	>=	7
9	9am-11am	0	1	0	0	0	0	0	15	>=	15
10	11am-1pm	1	0	1	0	0	0	0	14	>=	12
11	1pm-3pm	0	1	0	1	0	0	0	16	>=	16
12	3pm-5pm	0	0	1	0	1	1	0	14	>=	14
13	5pm-7pm	0	0	0	1	0	1	1	6	>=	6
14	7pm-9pm	0	0	0	0	1	0	1	2	>=	2
15											
16		Full-Time	Full-Time	Full-Time	Full-Time	Full-Time					
17		on Phone	on Phone	on Phone	on Phone	on Phone	Part-Time	Part-Time			
18		7am-9am	9am-11am	11am-1pm	1pm-3pm	3pm-5pm	on Phone	on Phone			
19		11am-1pm	1pm-3pm	3pm-5pm	5pm-7pm	7pm-9pm	3pm-7pm	5pm-9pm			Total Cost
20	Number Working	7	15	7	1	2	5	0			$2,268

Lenny should hire 32 full-time bilingual operators. Of these operators, 7 have their first phone shift from 7am to 9am, 15 from 9am to 11am, 7 from 11am to 1pm, 1 from 1pm to 3pm, and 2 from 3pm to 5pm. Lenny should also hire 5 part-time operators who start their work at 3pm. The total (wage) cost of running the calling center equals $2268 per day.

g) The total cost of part (f) is $2268 per day; the total cost of part (b) is $2466. Lenny could pay an additional $2466-$2268 = $198 in total wages to the bilingual operators without increasing the total operating cost beyond those for the scenario with only monolingual operators. The increase of $198 represents a percentage increase of 198/2268 = 8.73%.

h) Creative Chaos Consultants has made the assumption that the number of phone calls is independent of the day of the week. But maybe the number of phone calls is very different on a Monday than it is on a Friday. So instead of using the same number of average phone calls for every day of the week, it might be more appropriate to determine whether the day of the week affects the demand for phone operators. As a result Lenny might need to hire more part-time employees for some days with an increased calling volume.

Similarly, Lenny might want to take a closer look at the length of the shifts he has scheduled. Using shorter shift periods would allow him to “fine tune” his calling centers and make it more responsive to demand fluctuations.

Lenny should investigate why operators are able to answer only 6 phone calls per hour. Maybe additional training of the operators could enable them to answer phone calls quicker and so increase the number of phone calls they are able to answer in an hour.

Finally, Lenny should investigate whether it is possible to have employees switching back and forth between paperwork and answering phone calls. During slow times phone operators could do some paperwork while they are sitting next to a phone, while in times of sudden large call volumes employees who are scheduled to do paperwork could quickly switch to answering phone calls.

Lenny might also want to think about the installation of an automated answering system that gives callers a menu of selections. Depending upon the caller’s selection, the call is routed to an operator who specializes in answering questions about that selection.

SUPPLEMENT TO CHAPTER 2: MORE ABOUT THE GRAPHICAL METHOD FOR LINEAR PROGRAMMING

Section 2.4 introduced the graphical method for solving two-variable linear programming problems by briefly illustrating its application to the Wyndor problem. For those who would like to see a fuller description of this method, we now will go through this same example in much more detail. (This presentation is self-contained and so will include much of what was said in Section 2.4 in this expanded coverage.) We then will present a second example (the Profit & Gambit Co. advertising-mix problem) where the objective is to *minimize total cost* rather than maximize total profit.

To prepare for going through the Wyndor example again, you might find it helpful to review the description of the problem in Section 2.1, the spreadsheet model for this problem in Section 2.2 (see Figure 2.3), and then (of special importance) the algebraic version of this model in Section 2.3. For your easy reference, this algebraic model is summarized below.

Choose the values of D and W so as to *maximize*

$$P = 300D + 500W$$

subject to satisfying all the following constraints:

$$D \leq 4$$

$$2W \leq 12$$

$$3D + 2W \leq 18$$

and

$$D \geq 0 \qquad W \geq 0$$

where

P = Total profit per week from the special new doors and windows (the number in the target cell G12 in Figure 2.3),

D = Production rate for the special new doors (the number in changing cell C12 in Figure 2.3),

W = Production rate for the special new windows (the number in changing cell D12 in Figure 2.3).

The focus of the graphical method is on solving this algebraic model by using a graph to find the feasible values of D and W that maximize P.

Displaying Solutions as Points on a Graph

The key to the graphical method is the fact that possible solutions can be displayed as points on a two-dimensional graph that has a horizontal axis giving the value of D and a vertical axis giving the value of W. Figure 1 shows some sample points.

> *Notation*: Either $(D, W) = (2, 3)$ or just $(2, 3)$ refers to both the solution and the point in the graph where $D = 2$ and $W = 3$. Similarly, $(D, W) = (4, 6)$ means $D = 4$ and $W = 6$, whereas the origin $(0, 0)$ means $D = 0$ and $W = 0$.

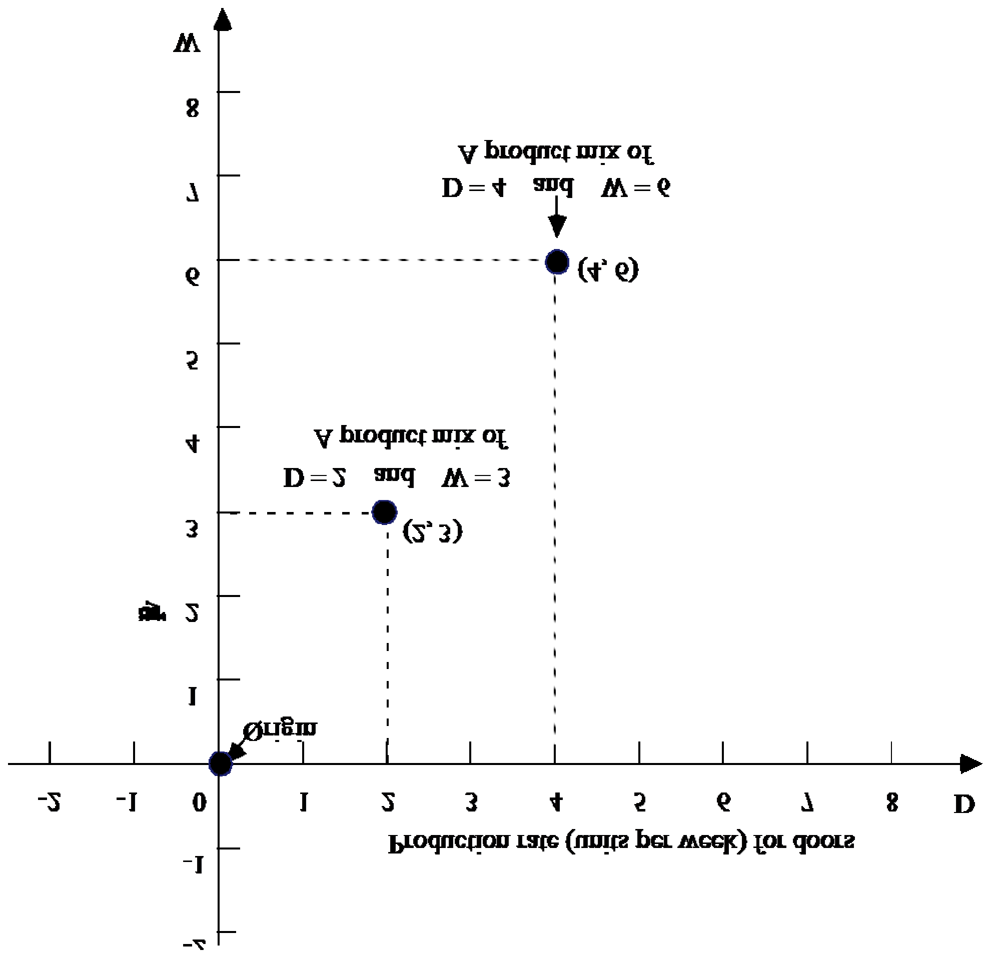


Figure 1 Graph showing the points $(D, W) = (2, 3)$ and $(D, W) = (4, 6)$ for the Wyndor Glass Co. product-mix problem.

To find the optimal solution (the best feasible solution), we first need to display graphically where the feasible solutions are. To do this, we must consider each constraint, identify the solutions graphically that are permitted by that constraint, and then combine this information to identify the solutions permitted by all the constraints.

To begin, the constraint $D \geq 0$ implies that consideration must be limited to points that lie on or to the right of the W axis in Figure 1. Similarly, the constraint, $W \geq 0$ restricts consideration to the points on or above the D axis. Combining these two facts, the region of interest at this juncture is the one shaded in on Figure 2. (This region also includes *larger* values of *D* and *W* than can be shown shaded in the available space.)

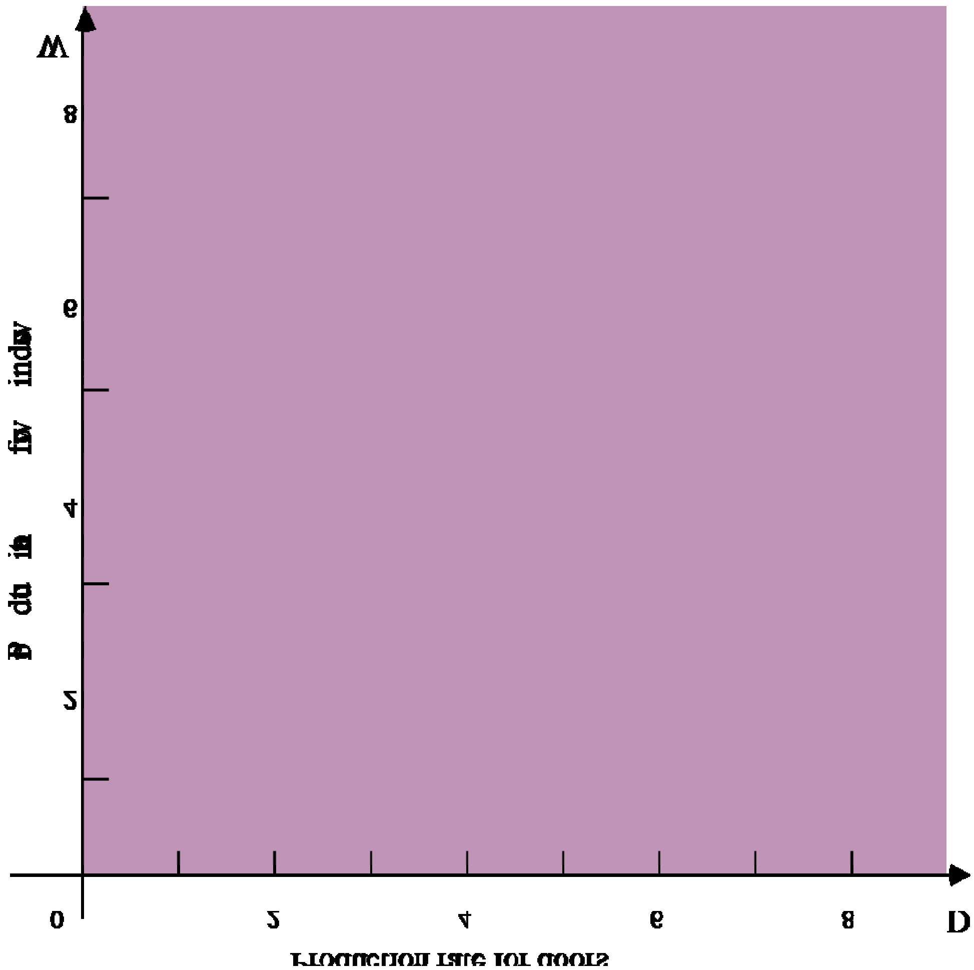


Figure 2 Graph showing that the constraints, $D \geq 0$ and $W \geq 0$, rule out solutions for the Wyndor Glass Co. product-mix problem that are to the left of the vertical axis or under the horizontal axis.

Graphing Nonnegative Solutions Permitted by Each Functional Constraint

We now will look individually at the nonnegative solutions that are permitted by each functional constraint by itself. Later, we will combine all these constraints.

Let us begin with the first functional constraint, $D \leq 4$, which limits the usage of Plant 1 for producing the special new doors to a maximum of 4 hours per week. The solutions permitted by this constraint are those that lie on, or to the left of, the vertical line that intercepts the *D* axis at $D = 4$ (so $D = 4$ is the equation for the line). Combining this permissible region with the one given in Figure 2 yields the shaded region shown in Figure 3.

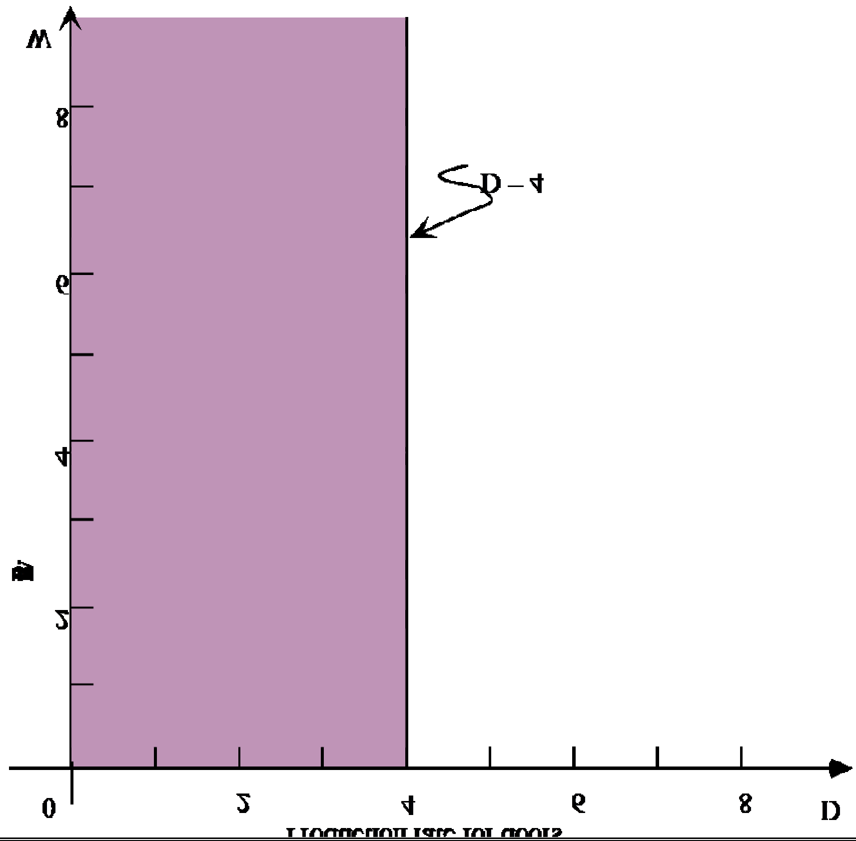


Figure 3 Graph showing that the nonnegative solutions permitted by the constraint $D \leq 4$ lie between the vertical axis and the line where $D = 4$.

The second functional constraint, $2W \leq 12$, has a similar effect, except now the boundary of its permissible region is given by a *horizontal* line with the equation, 2W = 12 (or $W = 6$), as shown in Figure 4. The line forming the boundary of what is permitted by a constraint is sometimes referred to as a **constraint boundary line**, and its equation may be called a **constraint boundary equation**. A *constraint boundary line* is identified by its equation.

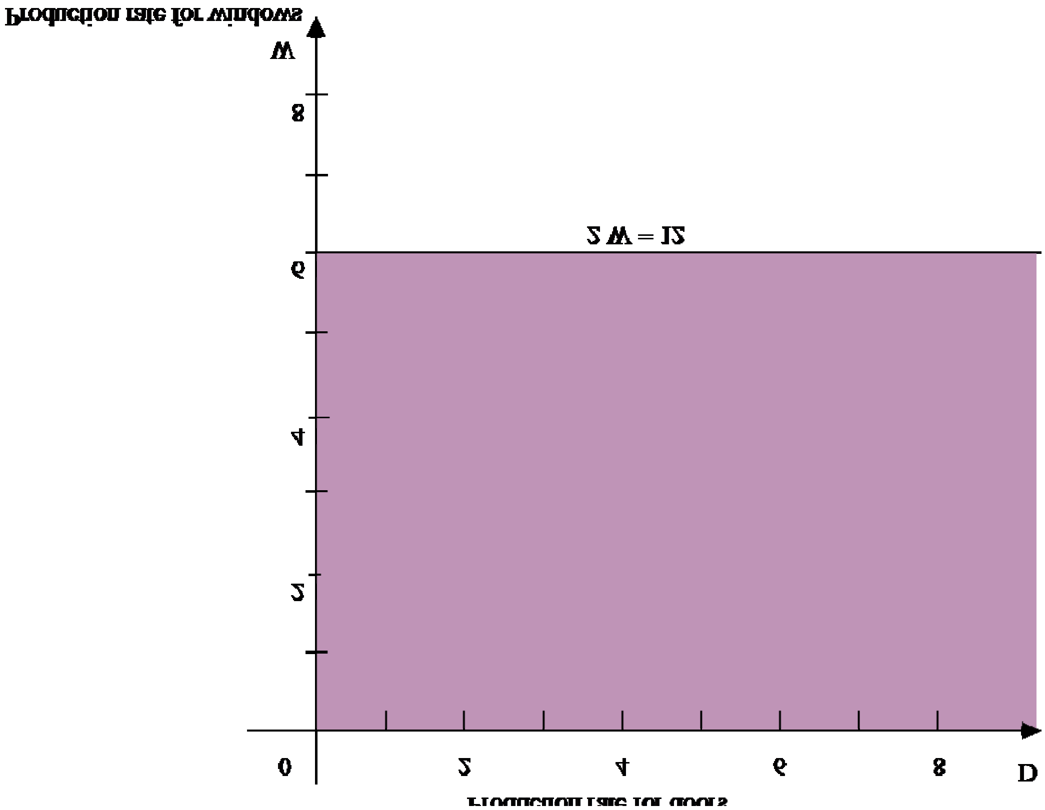


Figure 4 Graph showing that the nonnegative solutions permitted by the constraint $2W \leq 12$ must lie between the horizontal axis and the constraint boundary line whose equation is $2W = 12$.

For each of the first two functional constraints, $D \leq 4$ and $2W \leq 12$, note that the equation for the constraint boundary line ($D = 4$ and $2W = 12$, respectively) is obtained by replacing the inequality sign with an equality sign. For *any* constraint with an inequality sign (whether a functional constraint or a nonnegativity constraint), the general rule for obtaining its constraint boundary equation is to substitute an equality sign for the inequality sign.

We now need to consider one more functional constraint, $3D + 2W \leq 18$. Its constraint boundary equation,

$$3D + 2W = 18,$$

includes both variables, so the boundary line it represents is neither a vertical line nor a horizontal line. Therefore, the boundary line must intercept (cross through) both axes somewhere. But where?

> When a constraint boundary line is neither a vertical line nor a horizontal line, the line *intercepts* the D axis at the point on the line where $W = 0$. Similarly, the line *intercepts* the W axis at the point on the line where $D = 0$.

Hence, the constraint boundary line, $3D + 2W = 18$, intercepts the D axis at the point where $W = 0$.

When $W = 0$, $3D + 2W = 18$ becomes $3D = 18$,
so the intercept with the D axis is at $D = 6$.

Similarly, the line intercepts the W axis where $D = 0$.

When $D = 0$, $3D + 2W = 18$ becomes $2W = 18$,
so the intercept with the D axis is at $W = 9$.

Consequently, the constraint boundary line is the line that passes through these two intercept points, as shown in Figure 5.

Another way to find this constraint boundary line is to change the form of the constraint boundary equation so that it expresses W in terms of D.

$3D + 2W = 18$ implies $2W = -3D + 18$,

so

$$W = -\frac{3}{2}D + 9.$$

This form, $W = -\frac{3}{2}D + 9$, is called the **slope-intercept form** of the constraint boundary equation.

The constant term, 9, automatically is the intercept of the line with the W axis (since $W = 9$ when $D = 0$). The coefficient of D, $-\frac{3}{2}$, is the *slope* of the line.

The **slope** of a line is the change in W on the line when D is increased by 1.

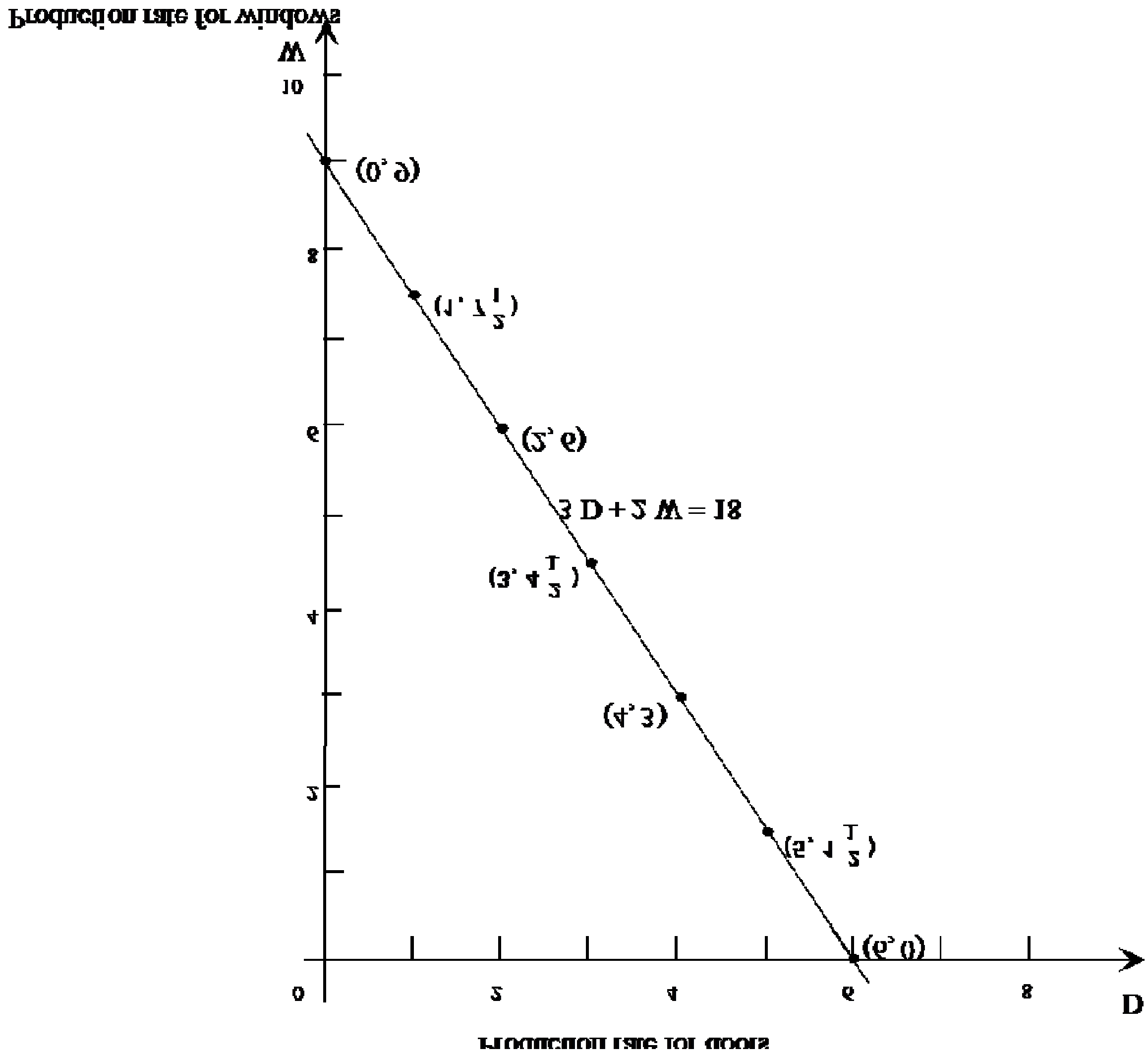


Figure 5 Graph showing that the boundary line for the constraint, $3\,D + 2\,W \le 18$, intercepts the horizontal axis at $D = 6$ and intercepts the vertical axis at $W = 9$.

For example, consider the series of points shown on the constraint boundary line in Figure 5 when moving from (0,9) toward (6,0). Note how *W* changes by the fixed amount, $-\frac{3}{2}$, each time *D* is increased by 1.

This derivation of the *slope-intercept form* demonstrates that the *only* numbers in the equation, $3D + 2W = 18$, that determine the slope of the line are 3 and 2, the coefficients of *D* and *W*. Therefore, if the equation $3D + 2W = 18$ were to be changed *only* by changing the right-hand side (18), the slope of the new line still would be $-\frac{3}{2}$. In other words, the new line would be *parallel* to the original line. To illustrate, suppose that the new equation is $3D + 2W = 12$. Since the original line had an intercept with the

W axis of $\frac{18}{2}$ = 9, the new parallel line has an intercept of $\frac{12}{2}$ = 6, so the new line is closer to the origin, as shown in Figure 6. This figure also shows the parallel line for the equation, $3D + 2W = 24$, which has an intercept with the W axis of $\frac{24}{2}$ = 12, and so is further from the origin than the original line.

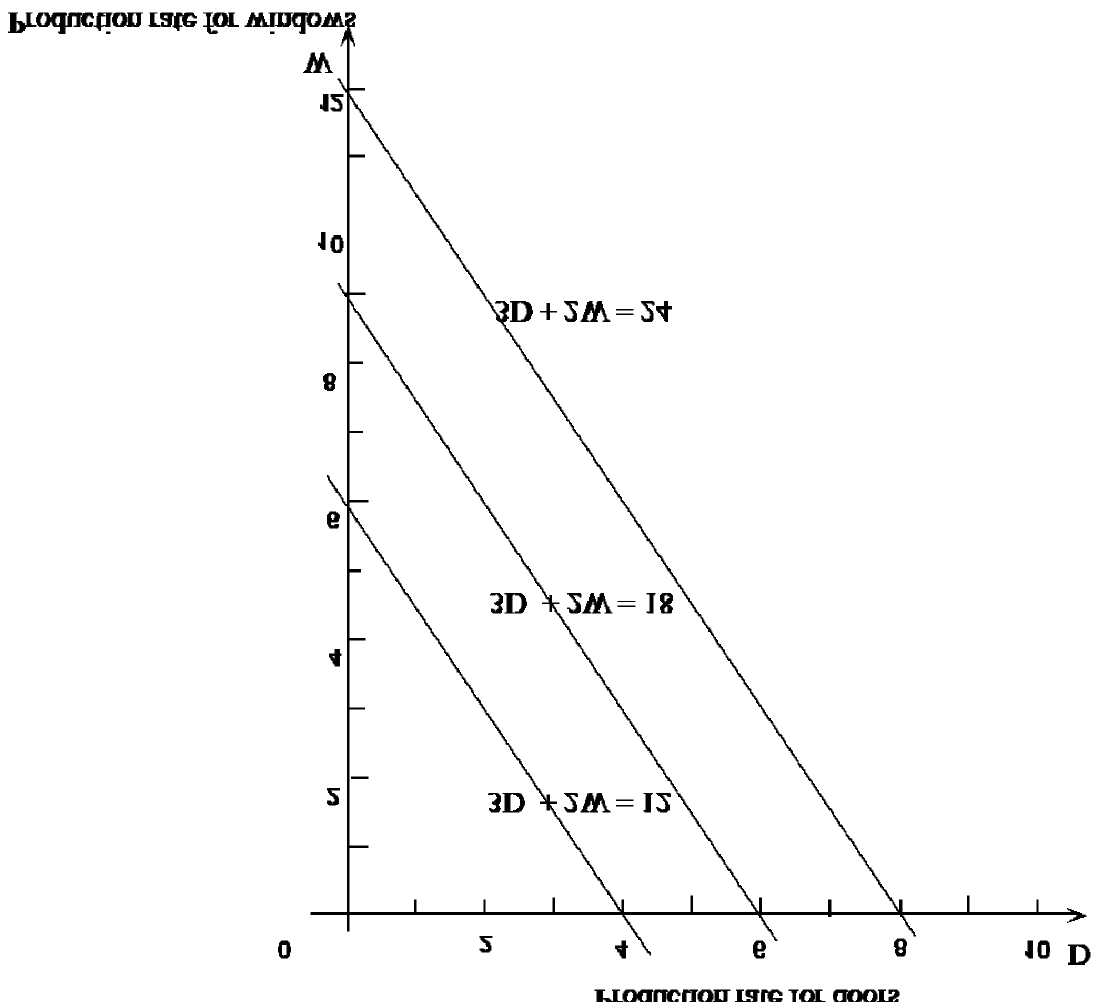


Figure 6 Graph showing that changing only the right-hand side of a constraint (such as $3D + 2W \leq 18$) creates *parallel* constraint boundary lines.

This analysis also shows that the solutions permitted by the constraint, $3D + 2W \leq 18$, are those that lie on the *origin* side of the constraint boundary line,

$3D + 2W = 18$. The easiest way to verify this is to check whether the origin itself, (D, W) = (0,0), satisfies the constraint.[1] If it does, then the permissible region lies on the side of the constraint boundary line where the origin is. Otherwise, it lies on the other side. In this case,

$$3(0) + 2(0) = 0,$$

so $(D, W) = (0, 0)$ satisfies

$$3D + 2W \leq 18.$$

(In fact, the origin satisfies *any* constraint with a $\leq$ sign and a positive right-hand side.) Therefore, the region permitted by this constraint is the one shown in Figure 7.

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[1]The one case where using the origin to help determine the permissible region does *not* work is if the constraint boundary line passes through the origin. In this case, any other point *not* lying on this line can be used just like the origin.

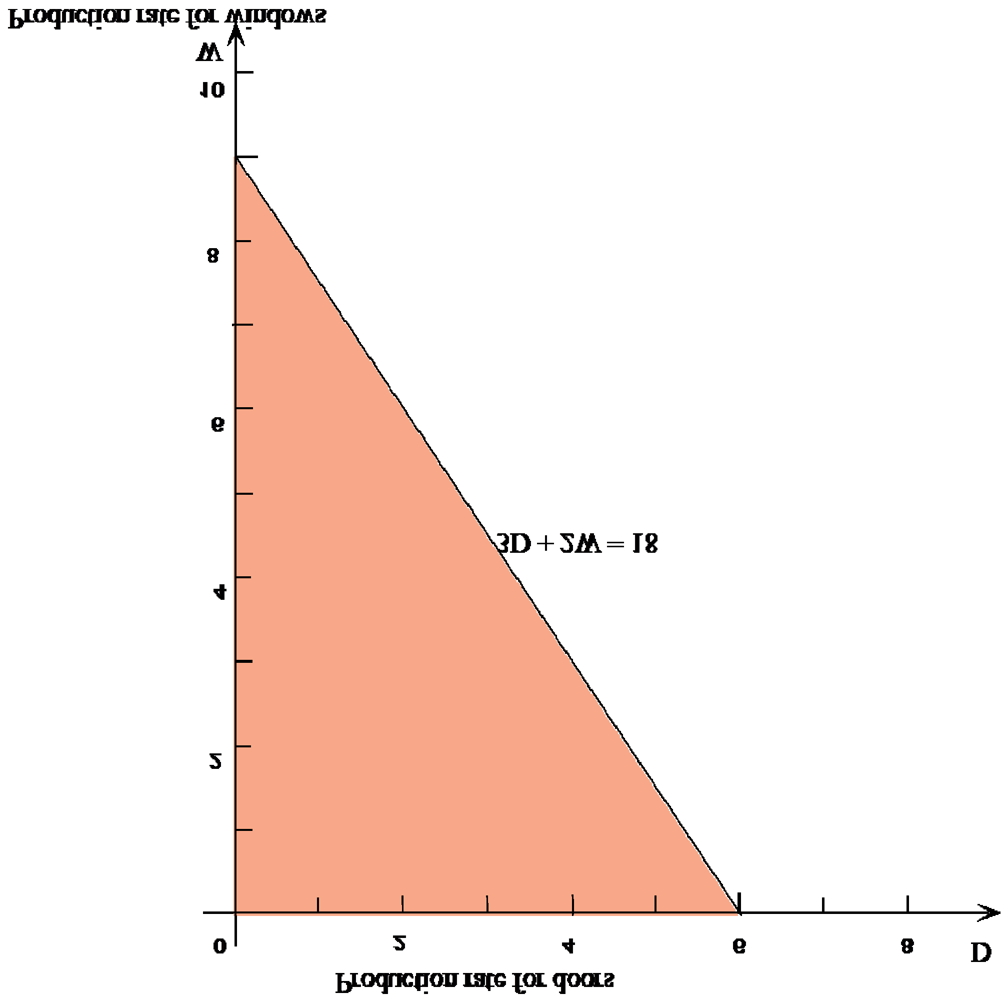


Figure 7 Graph showing that nonnegative solutions permitted by the constraint, $3\,D + 2\,W \leq 18$, lie within the triangle formed by the two axes and this constraint's boundary line, $3D + 2\,W = 18$.

Graphing the Feasible Region

We now have graphed the region where solutions are permitted by the *individual* constraints in Figures 2, 3, 4, and 7. However, a feasible solution for a linear programming problem must satisfy *all* the constraints *simultaneously*. To find where these feasible solutions are located, we need to combine all the constraints in one graph and identify the points representing the solutions that are in *every* constraint's permissible region.

Figure 8 shows the constraint boundary line for each of the three functional constraints. We also have added arrows to each line to show which side of the line is permitted by the corresponding constraint (as identified in the preceding figures). Note that the nonnegative solutions permitted by each of these constraints lie on the side of the constraint boundary line where the origin is (or on the line itself). Therefore, the *feasible solutions* are those that lie nearer to the origin than *all three* constraint boundary lines (or on the line nearest the origin). The resulting region of feasible solutions, called the **feasible region**, is the shaded portion of Figure 8.

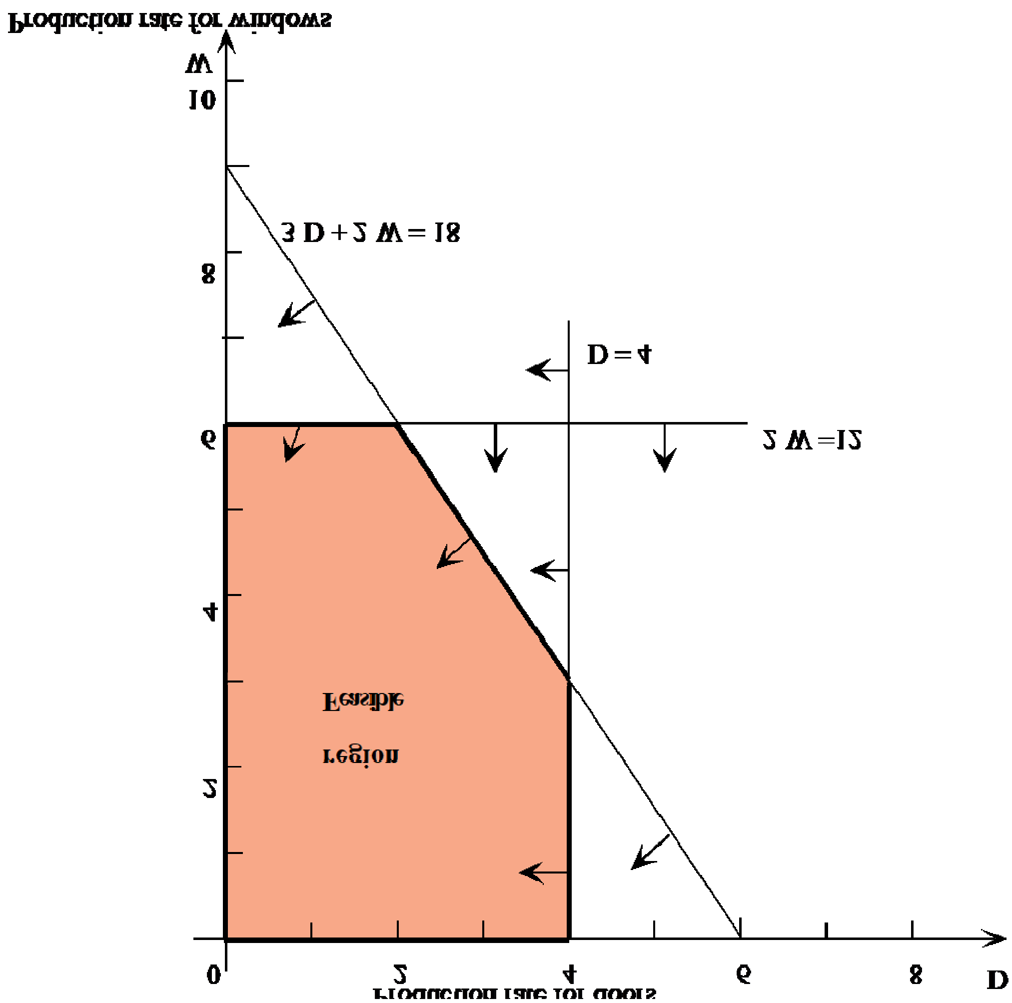


Figure 8 Graph showing how the feasible region is formed by the constraint boundary lines, where the arrows indicate which side of each line is permitted by the corresponding constraint.

Graphing the Objective Function

Having identified the feasible region, the final step is to find which of these feasible solutions is the best one — the *optimal solution*. For the Wyndor problem, the objective happens to be to *maximize* the total profit per week from the two products (denoted by *P*). Therefore, we want to find the feasible solution (*D*, *W*) that makes the value of the objective function,

$$P = 300D + 500W,$$

as large as possible.

To accomplish this, we need to be able to locate all the points (*D*, *W*) on the graph that give a specified value of the objective function. For example, consider a value of *P* = 1,500 for the objective function. Which points (*D*, *W*) give

$$300D + 500W = 1{,}500?$$

This equation is the equation of a *line*. Just as when plotting constraint boundary lines, the location of this line is found by identifying its intercepts with the two axes. When *W* = 0, this equation yields *D* = 5, and similarly, *W* = 3 when *D*= 0, so these are the two intercepts, as shown in Figure 9.

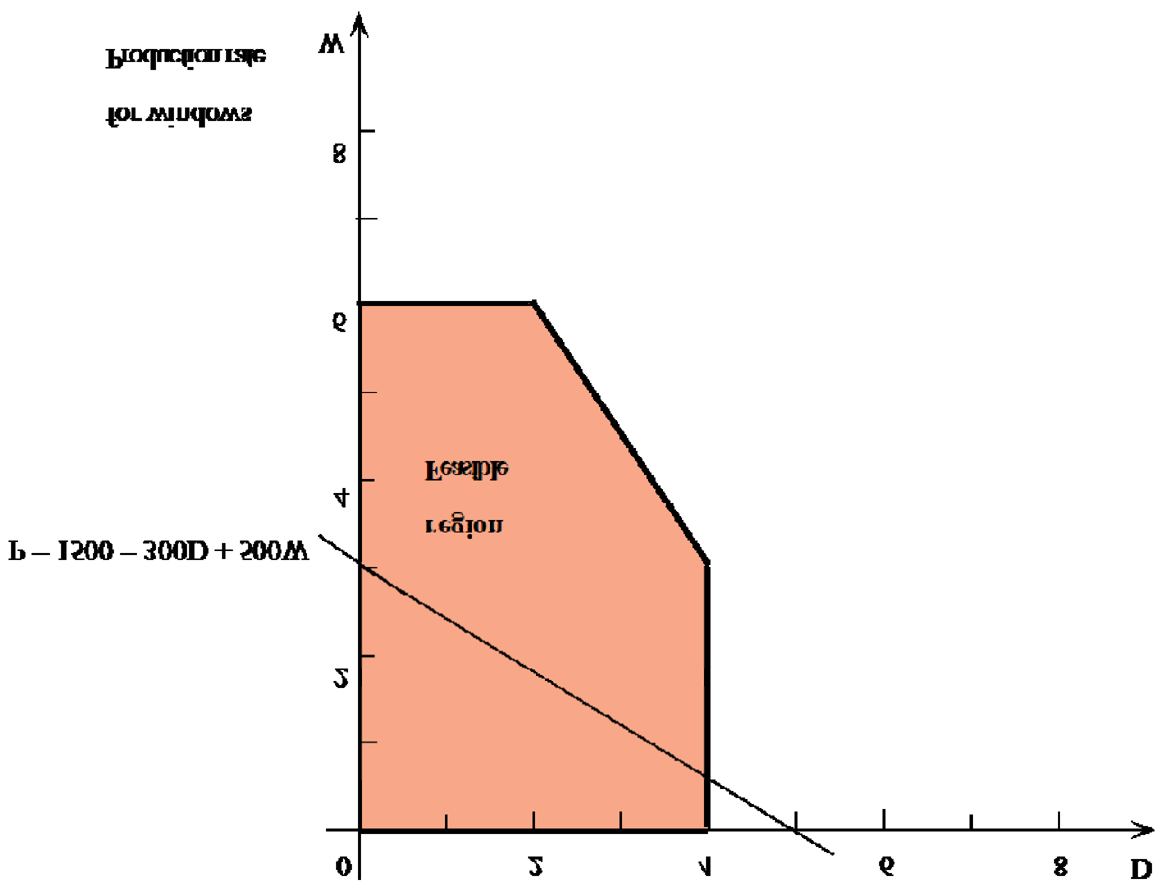


Figure 9 Graph showing the line containing all the points (*D*, W) that give a value of *P* = 1,500 for the objective function.

P = 1,500 is just one sample value of the objective function. For any other specified value of *Z*, the points (*D*, *W*) that give this value of *P* also lie on a line called an *objective function line*.

An **objective function line** is a line whose points all have the same value of the objective function.

For the objective function line in Figure 9, the points on this line that lie in the feasible region provide alternate ways of achieving an objective function value of $Z = 1{,}500$. Can we do better? Let us try doubling the value of Z to $Z = 3{,}000$. The corresponding objective function line,

$$300D + 500W = 3{,}000,$$

is shown as the middle line in Figure 10. (Ignore the top line for the moment.) Once again, this line includes points in the feasible region, so $P = 3{,}000$ is achievable.

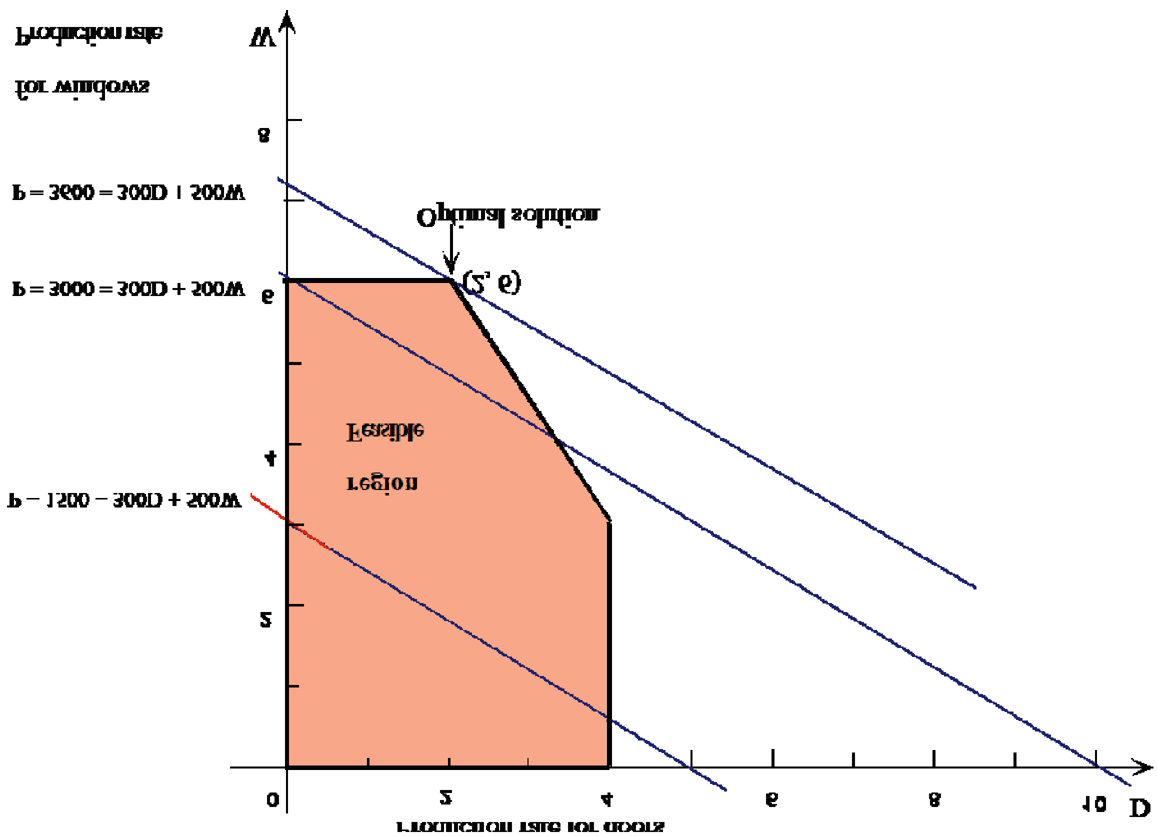


Figure 10 Graph showing three objective function lines for the Wyndor Glass Co. product-mix problem, where the top one passes through the optimal solution.

Let us pause to note two interesting features of these objective function lines for $P = 1{,}500$ and $P = 3{,}000$. First, these lines are *parallel*. Second, *doubling* the value of P from 1,500 to 3,000 also *doubles* the value of W at which the line intercepts the W axis from $W = 3$ to $W = 6$. These features are no coincidence, as indicated by the following properties.

> **Key Properties of Objective Function Lines**: All objective function lines for the same problem are *parallel*. Furthermore, the value of W at which an objective function line intercepts the W axis is *proportional* to the value of P.

To see why these properties hold, look at the *slope-intercept form* of an objective function line for the Wyndor problem:

$$W = -\frac{300}{500}D + \frac{1}{500}P, \quad \text{which reduces to} \quad W = -\frac{3}{5}D + \frac{1}{500}P.$$

This slope-intercept form indicates that the *slope* of the lines for various values of P always is the *same*, $-\frac{3}{5}$, so these lines are parallel. Furthermore, this form indicates that the value of W at which a line intercepts the W axis is $1/500\ P$, so this value of W is *proportional* to P.

These key properties of objective function lines suggest the strategy to follow to find the optimal solution. We already have tried $P = 1{,}500$ and $P = 3{,}000$ in Figure 10 and found that their objective function lines include points in the feasible region. Increasing Z again will generate another parallel objective function line farther from the origin. The objective function line of special interest is the one farthest from the origin that still includes a point in the feasible region. This is the third objective function line in Figure 10. The point on this line that is in the feasible region, $(D, W) = (2, 6)$, is the optimal solution since no other feasible solution has a larger value of P.

Optimal Solution

$D = 2$ (Produce 2 special new doors per week)
$W = 6$ (Produce 6 special new windows per week)

These values of D and W can be substituted into the objective function to find the value of P.

$$P = 300D + 500W = 300(2) + 500(6) = 3{,}600$$

You can graphically implement this strategy for finding the optimal solution by using any straight edge, such as a ruler. Rotate the straight edge in the feasible region until it has the slope of the objective function lines. (You can use any objective function line, such as the $P = 1{,}500$ line in Figure 9, to obtain this slope.) Then push the straight edge with this fixed slope through the feasible region in the direction that increases P. Stop moving the straight edge at the last instant that it still passes through a point in the feasible region. This point is the optimal solution. (It also is possible to have *multiple* optimal solutions because this straight edge is parallel to the line segment on the far edge of the feasible region.)

In addition to finding the optimal solution, another important use of the graphical method is to perform *what-if analysis* to determine what would happen to the optimal solution if any of the numbers (parameters) in the model change. The graphical approach provides key insights for answering a variety of what-if questions described in Chapter 5.

Summary of the Graphical Method

The graphical method can be used to solve any linear programming problem having only two variables. The method uses the following steps:

1. Draw the constraint boundary line for each functional constraint. Use the origin (or any point not on the line) to determine which side of the line is permitted by the constraint.

2. Find the feasible region by determining where all constraints are satisfied simultaneously.

3. Determine the slope of one objective function line. All other objective function lines will have the same slope.

4. Move a straight edge with this slope through the feasible region in the direction of improving values of the objective function. Stop at the last instant that the straight edge still passes through a point in the feasible region. This line given by the straight edge is the optimal objective function line.

5. A feasible point on the optimal objective function line is an optimal solution.

The Profit & Gambit Co. Advertising-Mix Example

To illustrate how the graphical method can be applied to a different kind of linear programming problem, we now will address the Profit & Gambit Co. advertising-mix problem described in Section 2.7. For your easy reference, the algebraic model for this problem is repeated below.

Choose the values of TV and PM so as to *minimize*

$$\text{Cost} = TV + 2PM \quad \text{(in millions of dollars)}$$

subject to satisfying all the following constraints:

$$\begin{aligned} PM &\geq 3 \\ 3TV + 2PM &\geq 18 \\ -TV + 4PM &\geq 4 \end{aligned}$$

and

$$TV \geq 0 \qquad PM \geq 0$$

where

TV = Number of units of advertising on television,
PM = Number of units of advertising in the print media.

Applying the Graphical Method

Since this linear programming model has only two decision variables, it can be solved by the graphical method described above. The interesting new features here are how this method adapts to *minimization* and to functional constraints with a ≥ sign.

Figure 11 shows the feasible region for this model. The three constraint boundary lines are obtained in the manner described above. However, the arrows indicating which side of each line satisfies that constraint now all point away from the origin. The reason is that the origin does not satisfy any functional constraint with a ≥ sign and a positive right-hand side.

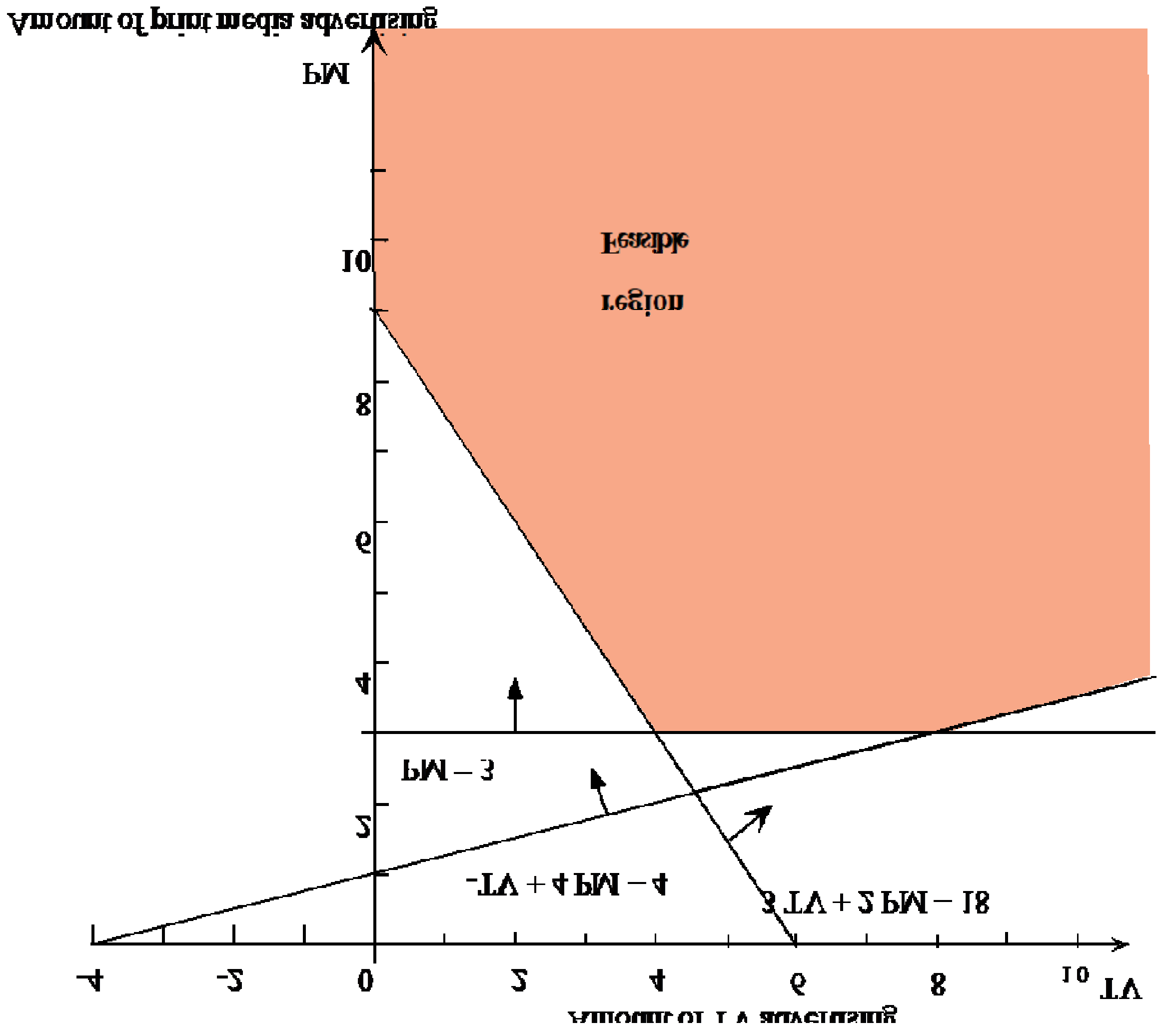


Figure 11 Graph showing the feasible region for the Profit & Gambit Co. advertising-mix problem, where the ≥ functional constraints have moved this region up and away from the origin.

To find the *best* solution in this feasible region (one that minimizes Cost = TV + 2 PM), we first construct a sample objective function line for one specific value of the objective function that appears to be attainable, say, Cost = 15. Figure 12 shows that a large segment of this line passes through the feasible region. Since this is a *minimization* problem, we're looking for the *smallest* value of Cost that provides an objective function line that still passes through a point in the feasible region. The origin automatically has an objective function value of Cost = 0, so objective function lines with a positive value of Cost less than 15 will be closer to the origin than the Cost = 15 line. Therefore, we want to move from the Cost =15 line to objective function lines closer to the origin. Figure 12 shows the objective function line with the smallest value of Cost (10) that still passes through a point in the feasible region. This point, (*TV*, *PM*) = (4, 3), is the optimal solution.

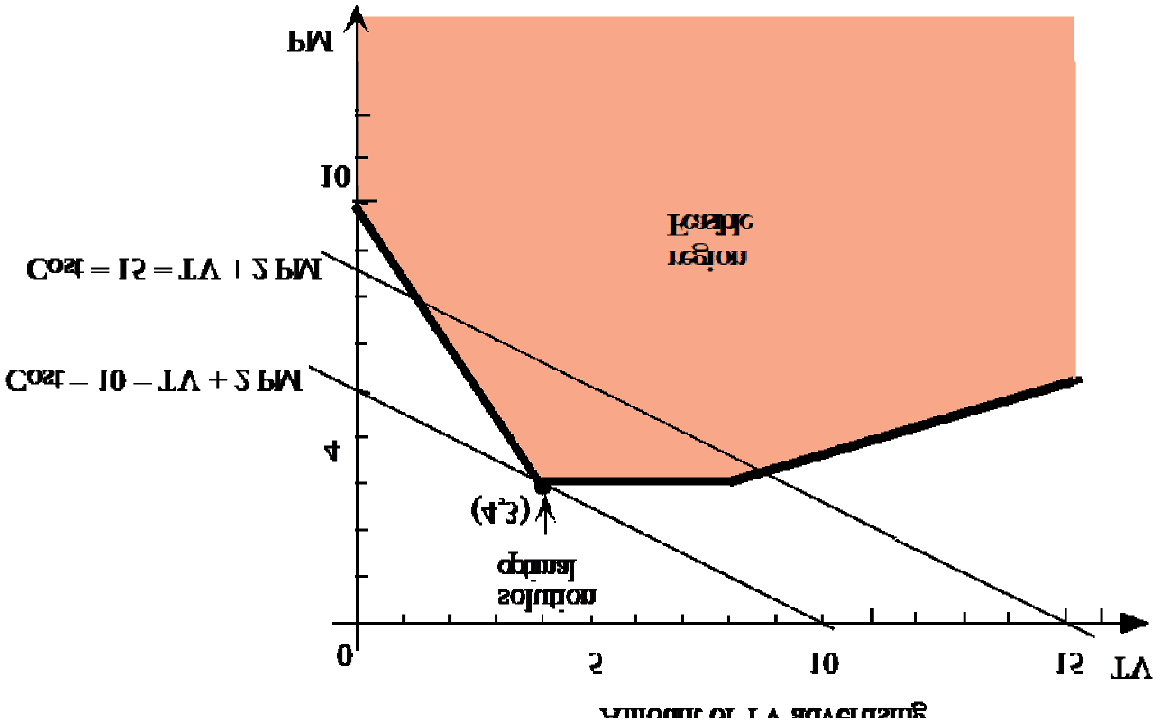


Figure 12 Graph showing two objective function lines for the Profit & Gambit Co. advertising-mix problem, where the bottom one passes through the optimal solution.

Case 5-1 continues this example by performing what-if analysis to see what would happen to the optimal solution and the total advertising cost if management were to change its sales goals for the three products.

Glossary

Constraint boundary equation: The equation for the constraint boundary line.
Constraint boundary line: For linear programming problems with two decision variables, the line forming the boundary of the solutions that are permitted by the constraint.
Feasible region: The geometric region that consists of all the feasible solutions.
Graphical method: A method for solving linear programming problems with two decision variables on a two-dimensional graph.
Objective function line: For a linear programming problem with two decision variables, a line whose points all have the same value of the objective function.
Slope-intercept form: For linear programming problems with two decision variables, the slope-intercept form of a constraint boundary equation displays both the slope of the constraint boundary line and the intercept of this line with the vertical axis.
Slope of a line: For a graph where the horizontal axis represents the variable x and the vertical axis represents y, the slope of a line is the change in y when x is increased by 1.

Problems

2s.1. Reconsider Problem 2.5. Use the graphical method to find the optimal solution.

2s.2. Reconsider the model in Problem 2.7. Use the graphical method to solve this model.

2s.3. Reconsider the model in Problem 2.8. Use the graphical method to solve this model.

2s.4. You are given the following equation for a line:

$$2x_1 + x_2 = 4$$

a. Identify the value of x_1 when $x_2 = 0$. Do the same for x_2 when $x_1 = 0$.
b. Construct a two-dimensional graph with x_1 on the horizontal axis and x_2 on the vertical axis. Then use the information from part *a* to draw the line.
c. Determine the numerical value of the slope of this line.
d. Find the slope-intercept form of this equation. Then use this form to identify both the slope of the line and the intercept of the line with the vertical axis.

2s.5. Follow the instructions of Problem 2s.4 for the following equation of a line.

$$2x_1 + 5x_2 = 10$$

2s.6. Follow the instructions of Problem 2s.4 for the following equation of a line.

$$2x_1 - 3x_2 = 12$$

2s.7. For each of the following constraints on the decision variables x_1 and x_2, draw a separate graph to show the nonnegative solutions that satisfy this constraint.
a. $x_1 + 3x_2 \leq 6$
b. $4x_1 + 3x_2 \leq 12$
c. $4x_1 + x_2 \leq 8$
d. Now combine these constraints into a single graph to show the feasible region for the entire set of functional constraints plus nonnegativity constraints.

2s.8. For each of the following constraints on the decision variables x_1 and x_2, draw a separate graph to show the nonnegative solutions that satisfy this constraint.
a. $10x_1 + 20x_2 \leq 40$
b. $5x_1 + 3x_2 \geq 15$
c. $5x_1 - x_2 \leq 15$

d. Now combine these constraints into a single graph to show the feasible region for the entire set of functional constraints plus nonnegativity constraints.

2s.9. For each of the following constraints on the decision variables x_1 and x_2, draw a separate graph to show the nonnegative solutions that satisfy this constraint.

a. $x_1 - x_2 \leq 2$

b. $-3x_1 + 6x_2 \geq 3$

c. $4x_1 - 3x_2 \geq 1$

d. Now combine these constraints into a single graph to show the feasible region for the entire set of functional constraints plus nonnegativity constraints.

2s.10. Consider the following objective function for a linear programming model with decision variables x_1 and x_2:

$$\text{Maximize} \quad \text{Profit} = 2x_1 + 3x_2$$

a. Draw a graph that shows the corresponding objective function lines for Profit = 6, Profit = 12, and Profit = 18.

b. Find the slope-intercept form of the equation for each of these three objective function lines. Compare the slope for these three lines. Also compare the intercept with the x_2 axis.

2s.11. Using the symbol P to represent total profit, you are given the following objective function for a linear programming model with decision variables x_1 and x_2:

$$\text{Maximize} \quad P = 25x_1 + 10x_2$$

a. Draw a graph that shows the corresponding objective function lines for $P = 100$, $P = 200$, and $P = 300$.

b. Find the slope-intercept form of the equation for each of these three objective function lines. Compare the slope for these three lines. Also compare the intercept with the x_2 axis.

2s.12. Consider the following objective function for a linear programming model with decision variables x_1 and x_2:

$$\text{Minimize} \quad \text{Cost} = 5x_1 - x_2$$

a. Draw a graph that shows the corresponding objective function lines for Cost = 300, Cost = 200, and Cost = 100.

b. Find the slope-intercept form of the equation for each of these three objective function lines. Compare the slope for these three lines. Also compare the intercept with the x_2 axis.

2s.13. Consider the following equation of a line:

$$20x_1 + 40x_2 = 400$$

a. Find the slope-intercept form of this equation.
b. Use this form to identify the slope and the intercept with the x_2 axis for this line.
c. Use the information from part *b* to draw a graph of this line.

2s.14. Find the slope-intercept form of the following equation of a line:

$$8x_1 + x_2 = 40$$

2s.15. Find the slope-intercept form of the following equations of lines:
a. $10x_1 + 5x_2 = 20$
b. $-2x_1 + 3x_2 = 6$
c. $5x_1 - 2x_2 = 10$

2s.16. Consider the following constraint on the decision variables x_1 and x_2:

$$x_1 - 2x_2 \leq 0$$

a. Write the constraint boundary equation for this constraint.
b. Find the slope-intercept form of this equation.
c. Use this form to identify the slope and the intercept with the x_2 axis for the constraint boundary line.
d. Use the information from part *c* to draw a graph of the constraint boundary line.
e. Identify which side of this line is permitted by the constraint.

2s.17. You are given the following linear programming model in algebraic form, where x_1 and x_2 are the decision variables and Z is the value of the overall measure of performance.

$$\text{Maximize} \quad Z = 20x_1 + 10x_2$$

subject to

$$x_1 - x_2 \leq 1$$
$$3x_1 + x_2 \leq 7$$

and

$$x_1 \geq 0 \qquad x_2 \geq 0$$

Use the graphical method to solve this model.

2s.18. You are given the linear programming model in algebraic form shown below, where the objective is to choose the levels of two activities (x_1 and x_2) so as to maximize their total profit, subject to constraints on the amounts of three resources available.

Maximize Profit = $10x_1 + 20x_2$

subject to

$$-x_1 + 2x_2 \leq 15 \quad \text{(resource 1)}$$
$$x_1 + x_2 \leq 12 \quad \text{(resource 2)}$$
$$5x_1 + 3x_2 \leq 45 \quad \text{(resource 3)}$$

and

$$x_1 \geq 0 \qquad x_2 \geq 0$$

Use the graphical method to solve this model.

2s.19. Consider the algebraic form of a linear programming model shown below, where x_1 and x_2 are the decision variables. Use the graphical method to solve this model.

Maximize Profit = $400x_1 + 500x_2$

subject to

$$20x_1 + 10x_2 \leq 100$$
$$5x_1 + 10x_2 \leq 50$$
$$3x_1 - x_2 \leq 10$$
$$-x_1 + 4x_2 \leq 15$$

and

$$x_1 \geq 0 \qquad x_2 \geq 0$$

2s.20. Consider the following algebraic form of a linear programming model, where the value of c_1 has not yet been ascertained.

Maximize $Z = c_1x_1 + x_2$

subject to

$$x_1 + x_2 \leq 6$$
$$x_1 + 2x_2 \leq 10$$

and

$$x_1 \geq 0 \quad x_2 \geq 0$$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of c_1 (both positive and negative).

2s.21. Consider the following algebraic form of a linear programming model, where the value of c_1 has not yet been ascertained.

$$\text{Maximize } Z = c_1x_1 + 2x_2$$

subject to

$$\begin{aligned} 4x_1 + x_2 &\leq 12 \\ x_1 - x_2 &\geq 2 \end{aligned}$$

and

$$x_1 \geq 0 \quad x_2 \geq 0$$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of c_1 (both positive and negative).

2s.22. Consider the following algebraic form of a linear programming model, where the value of k has not yet been ascertained.

$$\text{Maximize } Z = x_1 + 2x_2$$

subject to

$$\begin{aligned} -x_1 + x_2 &\leq 2 \\ x_2 &\leq 3 \\ kx_1 + x_2 &\leq 2k + 3, \quad \text{where } k \geq 0 \end{aligned}$$

and

$$x_1 \geq 0 \quad x_2 \geq 0$$

The solution currently being used is $(x_1, x_2) = (2, 3)$. Use graphical analysis to determine the values of k such that this solution actually is optimal.

2s.23. Your boss has asked you to use your background in management science to determine what the levels of two activities (x_1 and x_2) should be to minimize their total cost while satisfying some constraints. The algebraic form of the model is shown below.

$$\text{Minimize Cost} = 15x_1 + 20x_2$$

subject to

Constraint 1: $x_1 + 2x_2 \geq 10$

Constraint 2: $2x_1 - 3x_2 \leq 6$

Constraint 3: $x_1 + x_2 \geq 6$

and

$$x_1 \geq 0 \quad x_2 \geq 0$$

Use the graphical method to solve this model.

2s.24 . For the following algebraic form of a linear programming model, the objective is to choose the levels of two activities (x_1 and x_2) so as to minimize their total cost while satisfying some constraints.

Minimize $\text{Cost} = 3x_1 + 2x_2$

subject to

Constraint 1: $x_1 + 2x_2 \leq 12$

Constraint 2: $2x_1 + 3x_2 = 12$

Constraint 3: $2x_1 + x_2 \geq 8$

and

$x_1 \geq 0 \qquad x_2 \geq 0$

Use the graphical method to solve this model.

Table of Contents Chapter 2 (Linear Programming: Basic Concepts)

Wyndor Glass Co. Product Mix Problem

- Wyndor has developed the following new products:
  - An 8-foot glass door with aluminum framing.
  - A 4-foot by 6-foot double-hung, wood-framed window.
- The company has three plants
  - Plant 1 produces aluminum frames and hardware.
  - Plant 2 produces wood frames.
  - Plant 3 produces glass and assembles the windows and doors.

Questions:

1. Should they go ahead with launching these two new products?
2. If so, what should be the *product mix*?

Developing a Spreadsheet Model

- Step #1: Data Cells
  - Enter all of the data for the problem on the spreadsheet.
  - Make consistent use of rows and columns.
  - It is a good idea to color code these "data cells" (e.g., light blue).

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			Doors	Windows			
4		Unit Profit	$300	$500			
5							Hours
6			Hours Used Per Unit Produced				Available
7		Plant 1	1	0			4
8		Plant 2	0	2			12
9		Plant 3	3	2			18

Developing a Spreadsheet Model

- Step #2: Changing Cells
  - Add a cell in the spreadsheet for every decision that needs to be made.
  - If you don't have any particular initial values, just enter 0 in each.
  - It is a good idea to color code these "changing cells" (e.g., yellow with border).

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			Doors	Windows			
4		Unit Profit	$300	$500			
5							Hours
6			Hours Used Per Unit Produced				Available
7		Plant 1	1	0			4
8		Plant 2	0	2			12
9		Plant 3	3	2			18
10							
11			Doors	Windows			
12		Units Produced	0	0			

Developing a Spreadsheet Model

- Step #3: Objective Cell
  - Develop an equation that defines the objective of the model.
  - Typically this equation involves the data cells and the changing cells in order to determine a quantity of interest (e.g., total profit or total cost).
  - It is a good idea to color code this cell (e.g., orange with heavy border).

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	$300	$500			
5						Hours
6		Hours Used Per Unit Produced				Available
7	Plant 1	1	0			1
8	Plant 2	0	2			12
9	Plant 3	3	2			18
10						
11		Doors	Windows			Total Profit
12	Units Produced	1	1			$800

	G
11	Total Profit
12	=SUMPRODUCT(UnitProfit,UnitsProduced)

Developing a Spreadsheet Model

- Step #4: Constraints
  - For any resource that is restricted, calculate the amount of that resource used in a cell on the spreadsheet (an output cell).
  - Define the constraint in three consecutive cells. For example, if Quantity A <= Quantity B, put these three items (Quantity A, <=, Quantity B) in consecutive cells.

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			Doors	Windows			
4		Unit Profit	$300	$500			
5					Hours		Hours
6			Hours Used Per Unit Produced		Used		Available
7		Plant 1	1	0	0	<=	4
8		Plant 2	0	2	0	<=	12
9		Plant 3	3	2	0	<=	18
10							
11			Doors	Windows			Total Profit
12		Units Produced	0	0			$0

Formulas for the Spreadsheet Model

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			Doors	Windows			
4		Unit Profit	$300	$500			
5					Hours		Hours
6			Hours Used Per Unit Produced		Used		Available
7		Plant 1	1	0	0	<=	4
8		Plant 2	0	2	0	<=	12
9		Plant 3	3	2	0	<=	18
10							
11			Doors	Windows			Total Profit
12		Units Produced	0	0			$0

Range Name	Cells
HoursAvailable	G7:G9
HoursUsed	E7:E9
HoursUsedPerUnitProduced	C7:D9
TotalProfit	G12
UnitProfit	C4:D4
UnitsProduced	C12:D12

	E
5	Hours
6	Used
7	=SUMPRODUCT(C7:D7,UnitsProduced)
8	=SUMPRODUCT(C8:D8,UnitsProduced)
9	=SUMPRODUCT(C9:D9,UnitsProduced)

	G
11	Total Profit
12	=SUMPRODUCT(UnitProfit,UnitsProduced)

A Trial Solution

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	$300	$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	4	<=	1
8	Plant 2	0	2	6	<=	12
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	4	3			$2,700

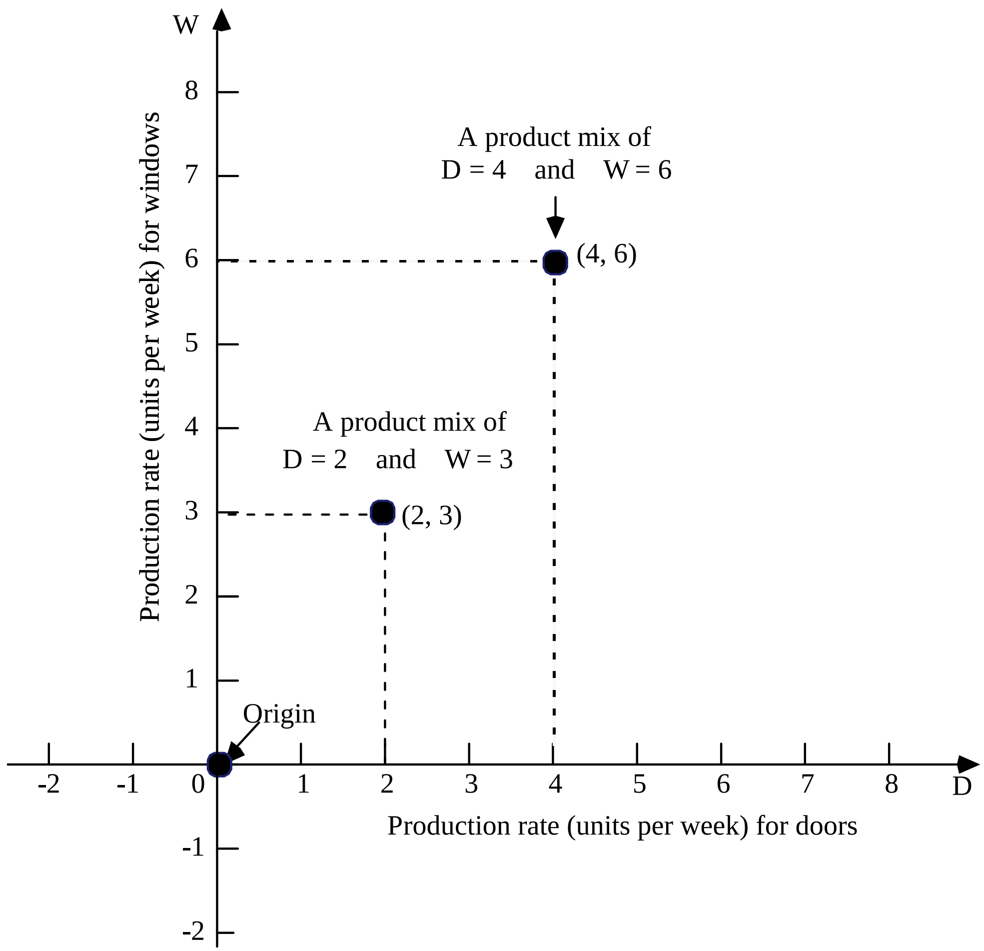
The spreadsheet for the Wyndor problem with a trial solution (4 doors and 3 windows) entered into the changing cells.

Algebraic Model for Wyndor Glass Co.

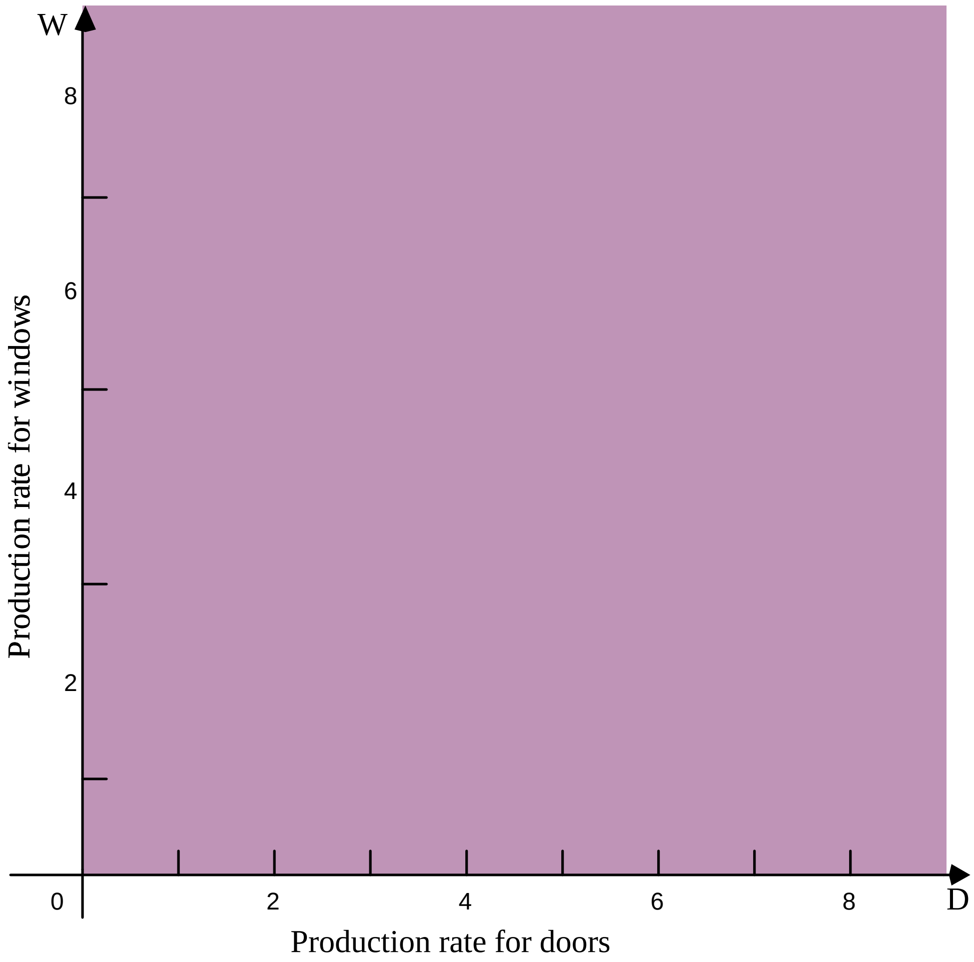
Let D = the number of doors to produce
W = the number of windows to produce

Maximize $P = \$300D + \$500W$
subject to
$D \leq 4$
$2W \leq 12$
$3D + 2W \leq 18$
and
$D \geq 0, W \geq 0.$

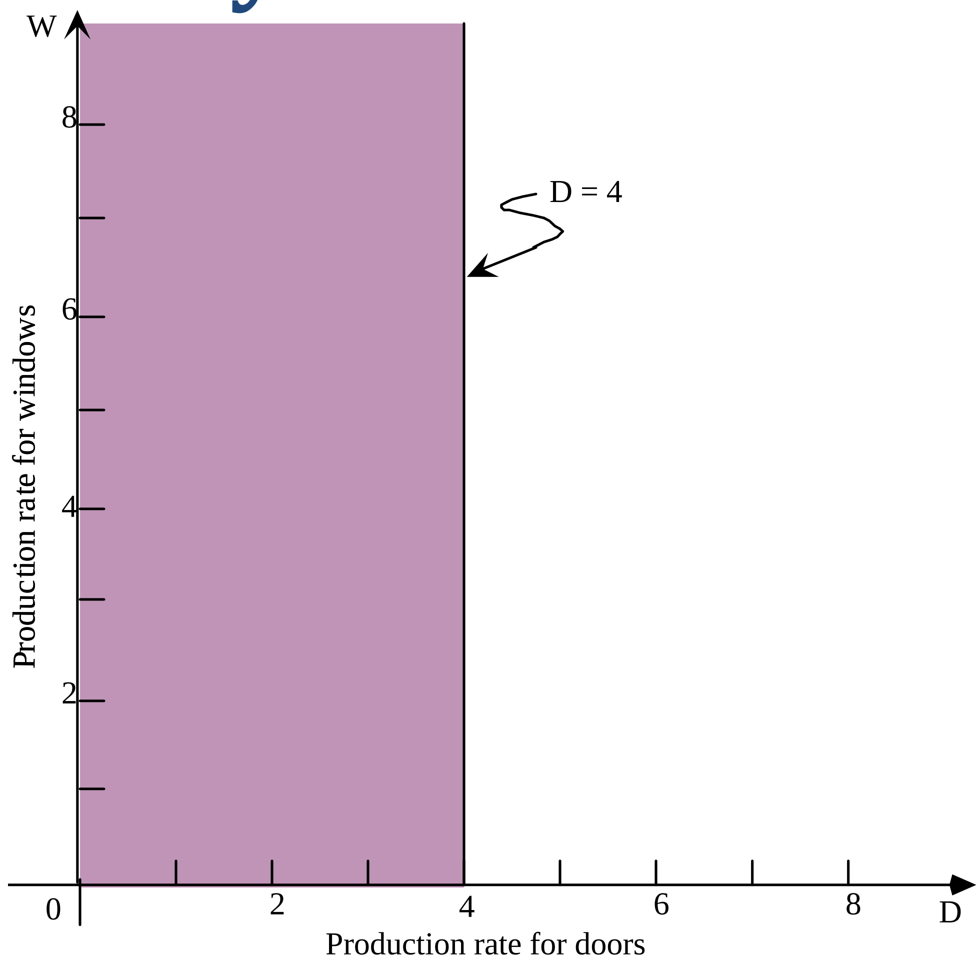
Graphing the Product Mix



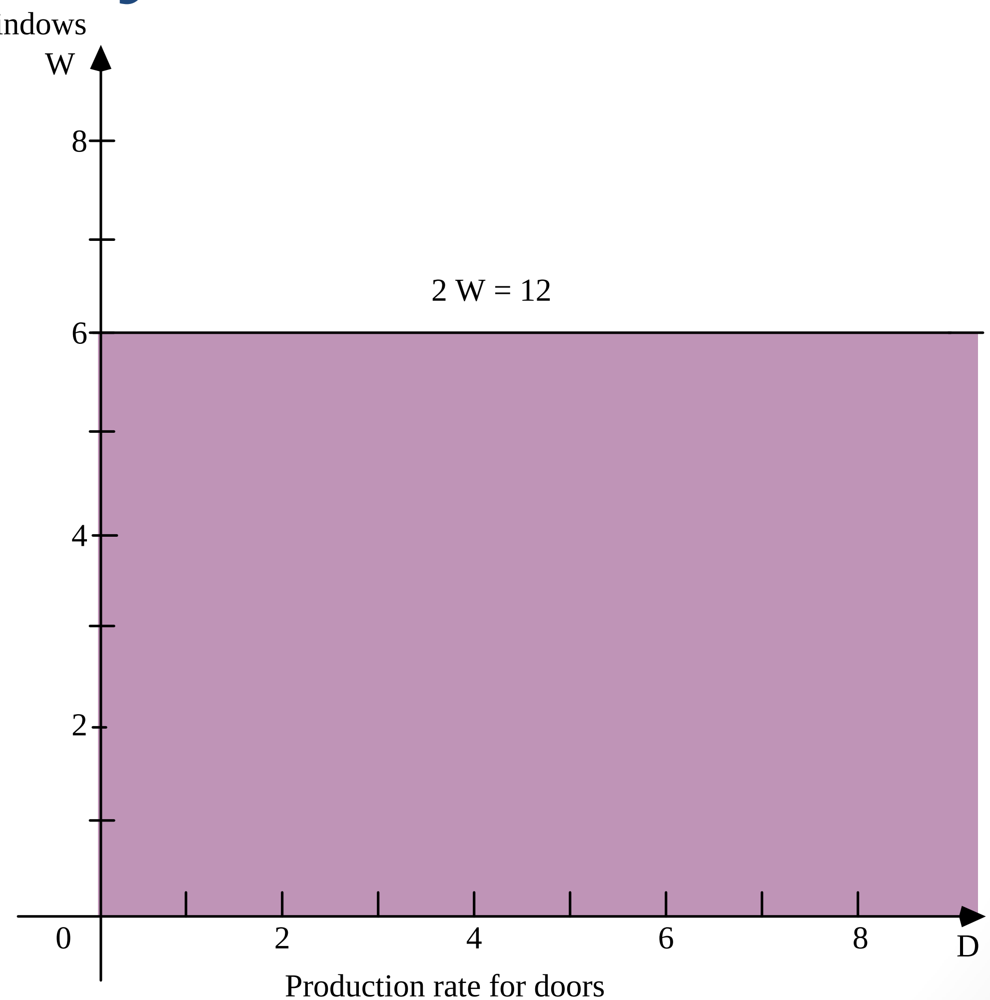
Graph Showing Constraints: $D \geq 0$ and $W \geq 0$



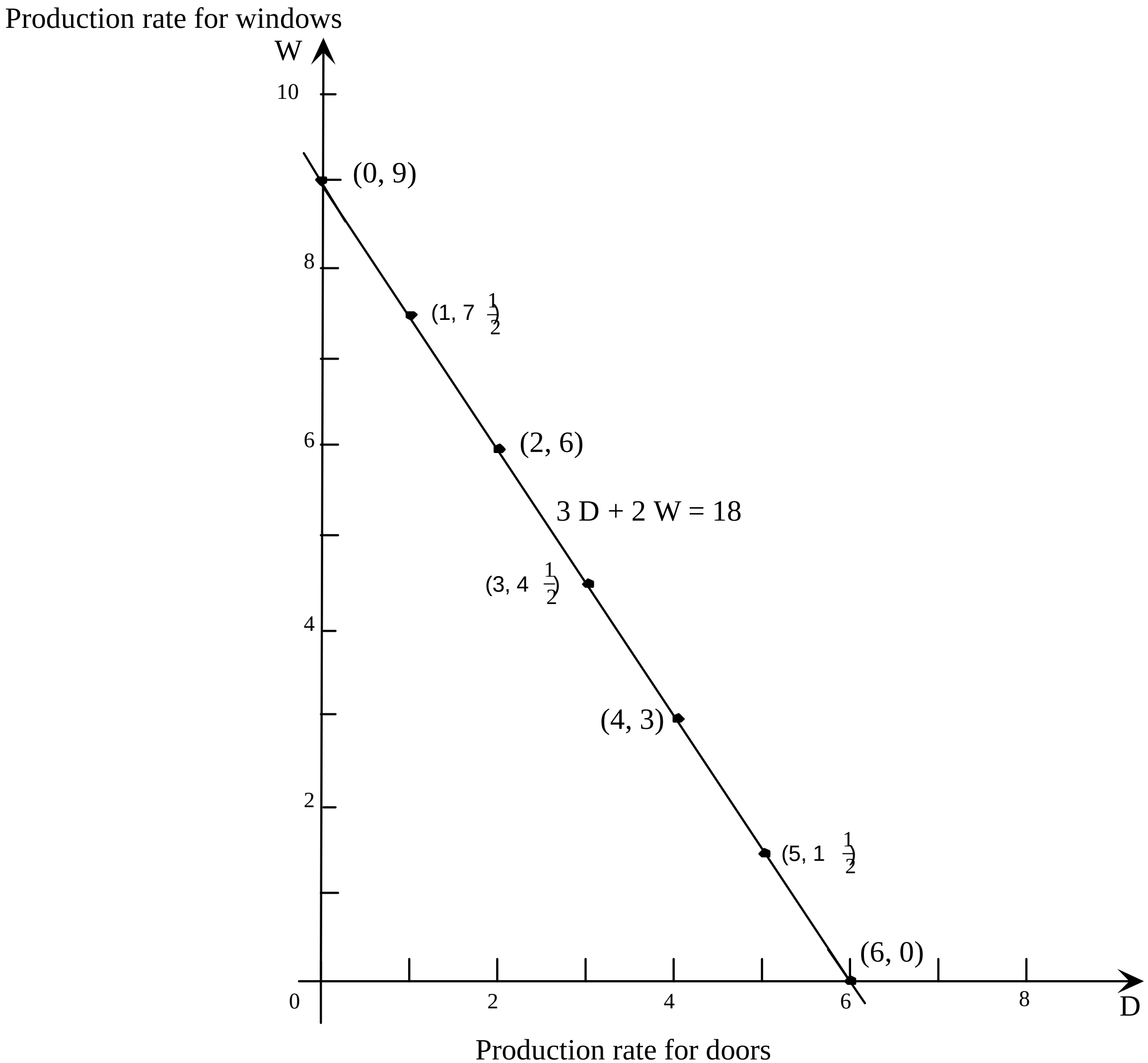
Nonnegative Solutions Permitted by $D \leq 4$



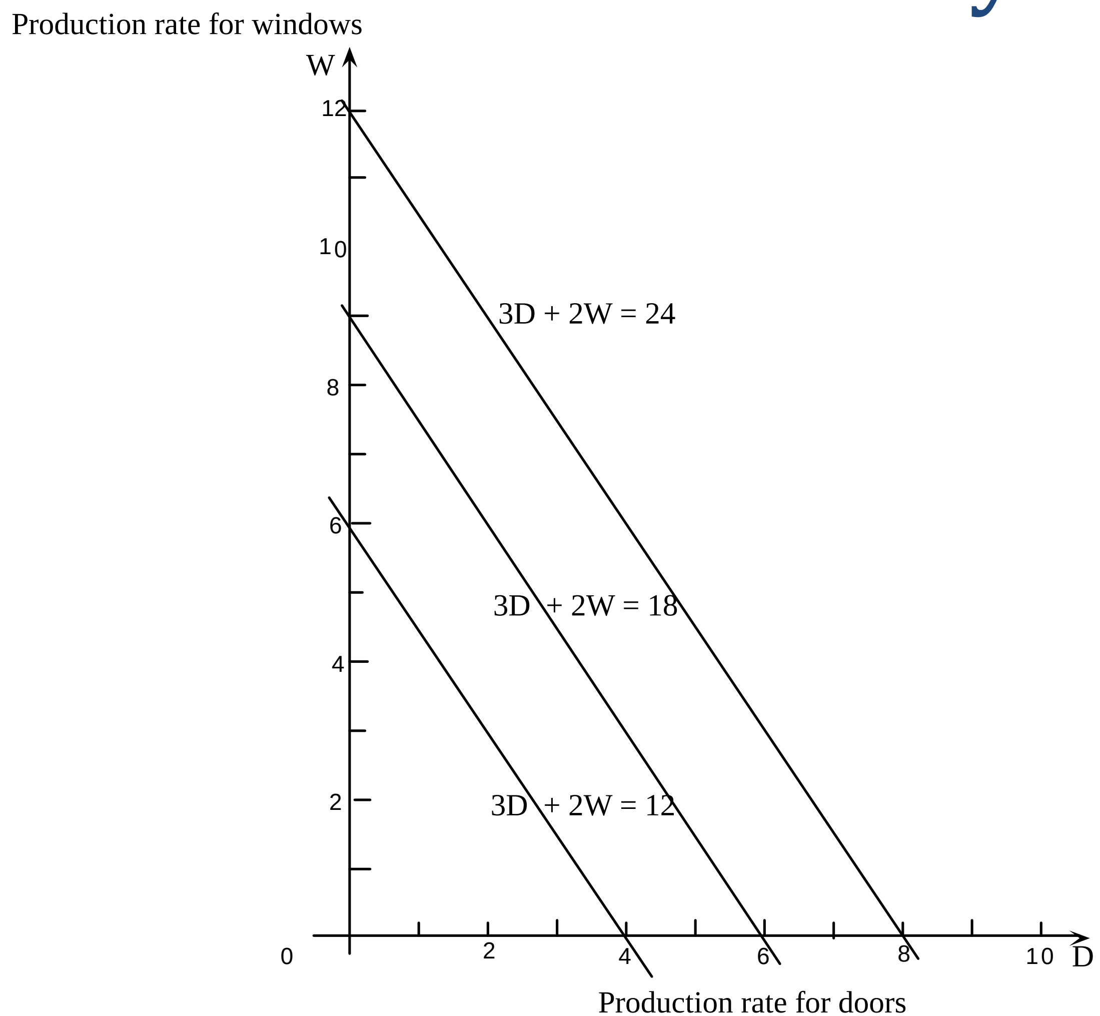
Nonnegative Solutions Permitted by $2W \leq 12$



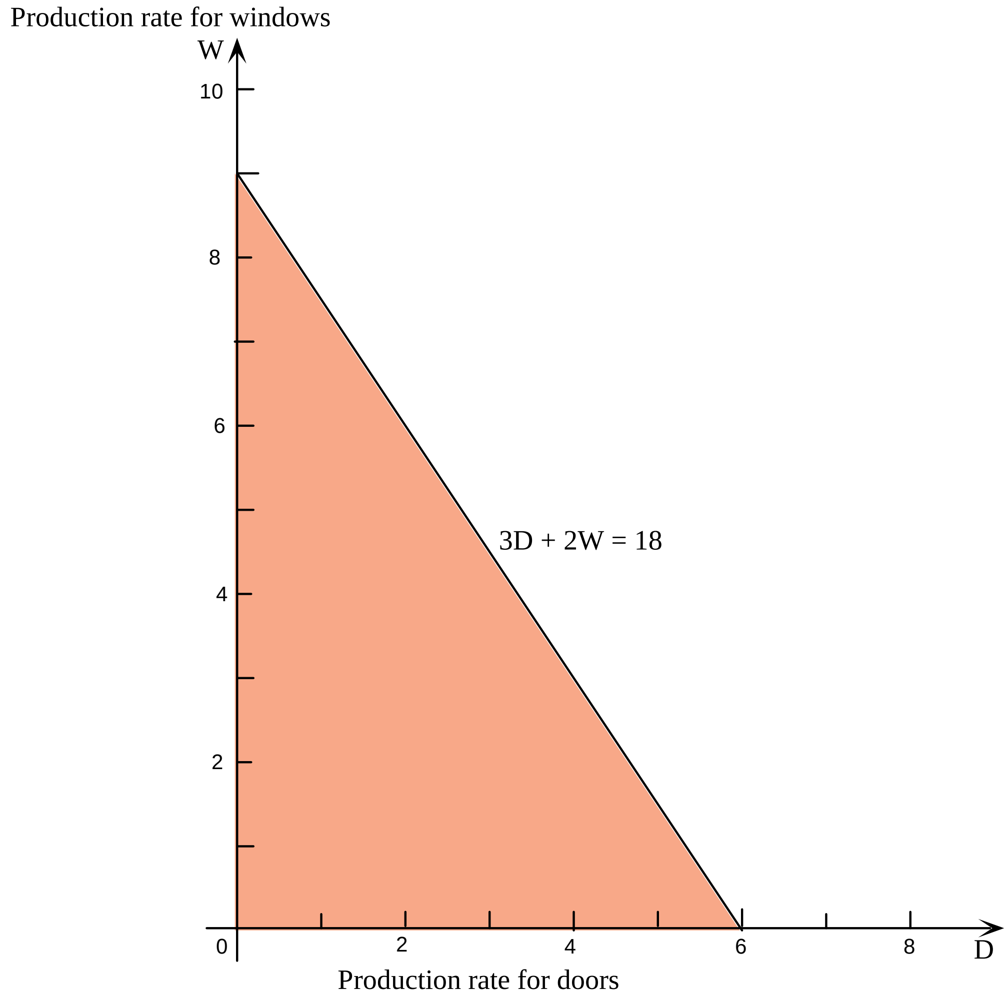
Boundary Line for Constraint $3D + 2W \leq 18$



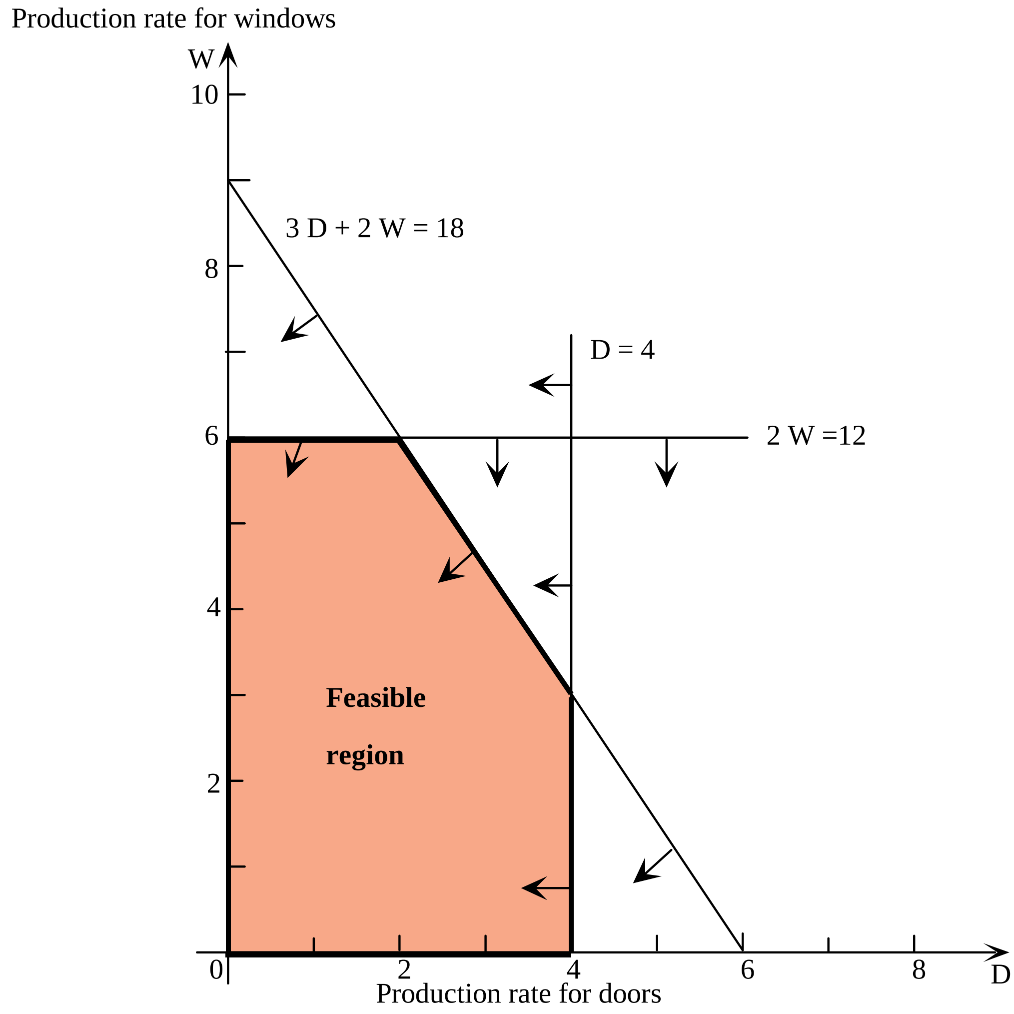
Changing Right-Hand Side Creates *Parallel* Constraint Boundary Lines



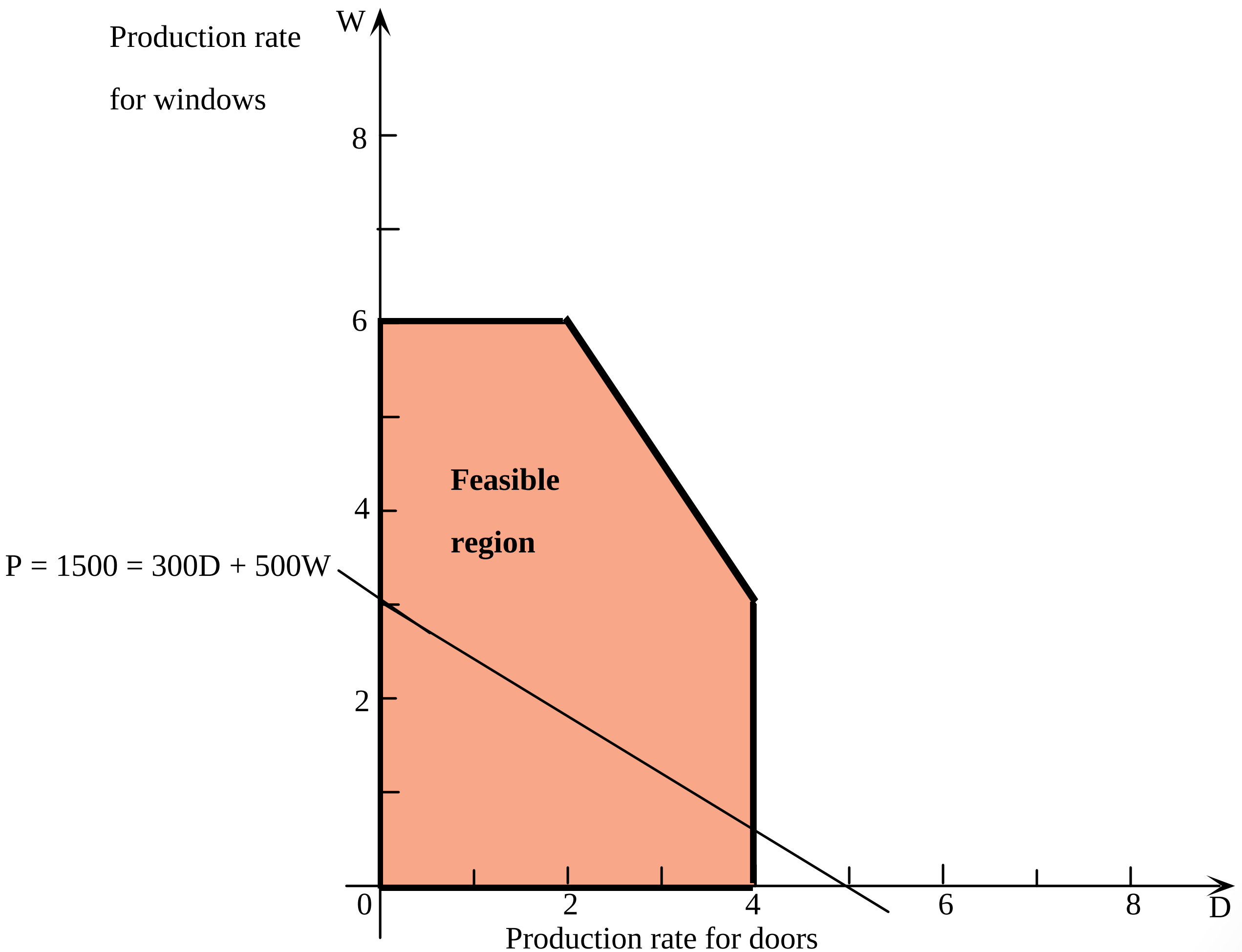
Nonnegative Solutions Permitted by $3D + 2W \leq 18$



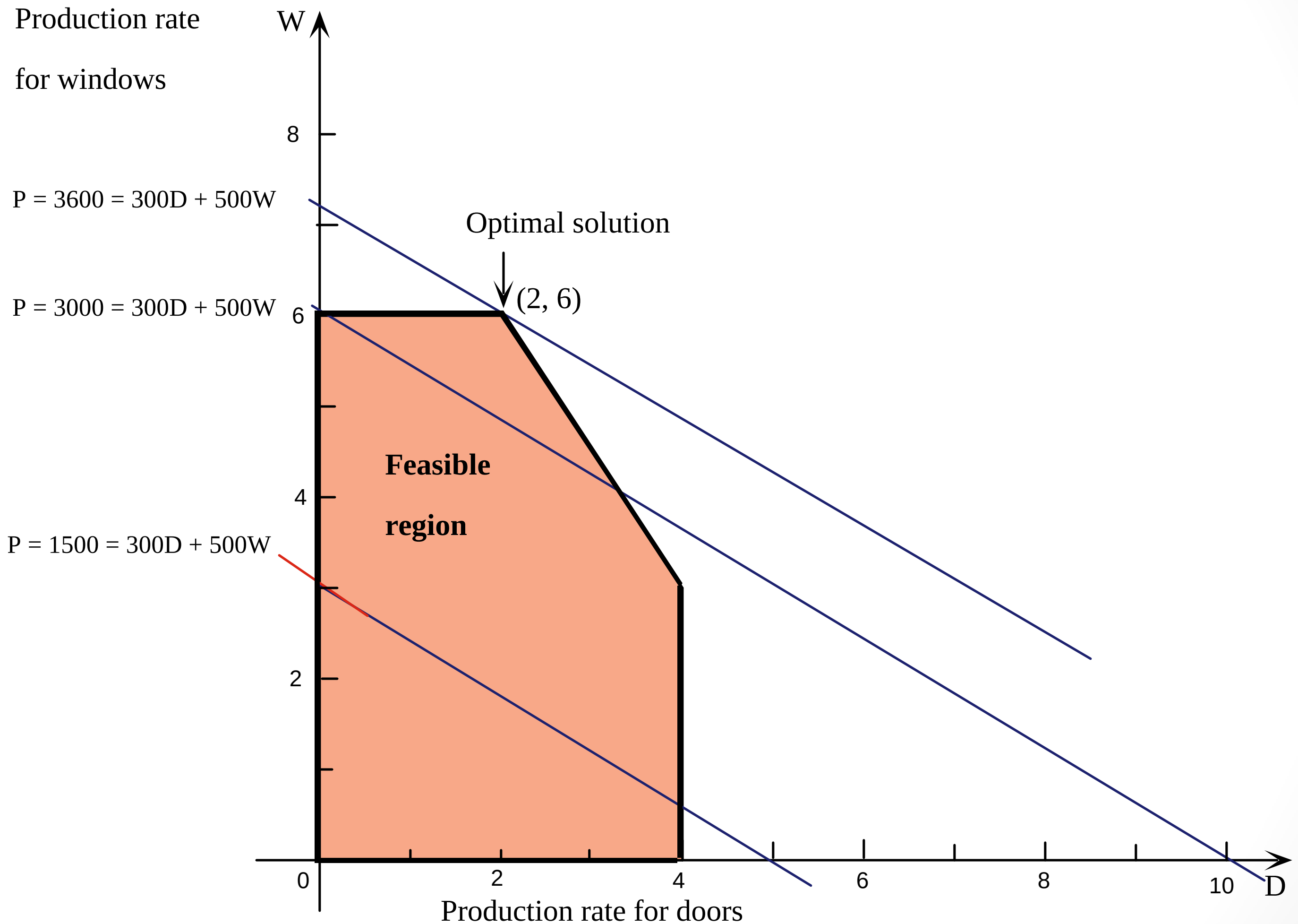
Graph of Feasible Region



Objective Function ($P = 1{,}500$)



Finding the Optimal Solution

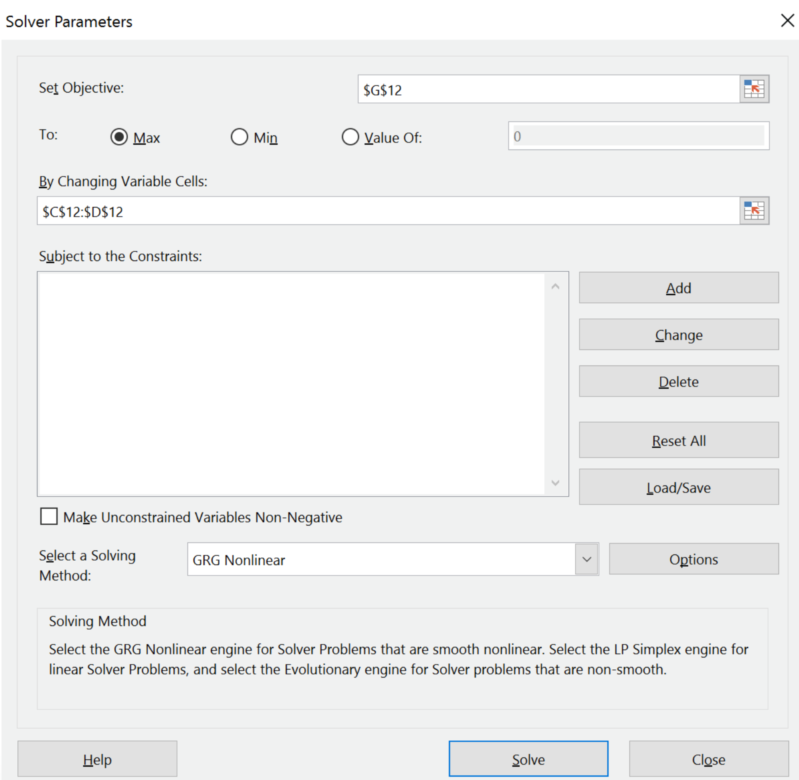


Summary of the Graphical Method

- Draw the constraint boundary line for each constraint. Use the origin (or any point not on the line) to determine which side of the line is permitted by the constraint.
- Find the feasible region by determining where all constraints are satisfied simultaneously.
- Determine the slope of one objective function line. All other objective function lines will have the same slope.
- Move a straight edge with this slope through the feasible region in the direction of improving values of the objective function. Stop at the last instant that the straight edge still passes through a point in the feasible region. This line given by the straight edge is the optimal objective function line.
- A feasible point on the optimal objective function line is an optimal solution.

Identifying the Objective Cell and Changing Cells

- Choose the “Solver” from the Data tab.
- Select the cell you wish to optimize in the “Set Objective” window.
- Choose “Max” or “Min” depending on whether you want to maximize or minimize the objective cell.
- Enter all the changing cells in the “By Changing Variable Cells” window.

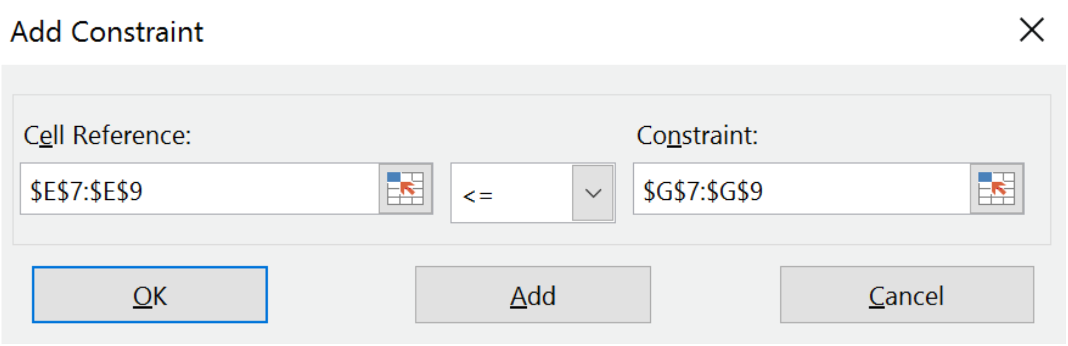


C	D	E	F	G
Doors	Windows			
$300	$500			
		Hours		Hours
Hours Used Per Unit Produced		Used		Available
1	0	1	<=	1
0	2	2	<=	12
3	2	5	<=	18
Doors	Windows			Total Profit
1	1			$800

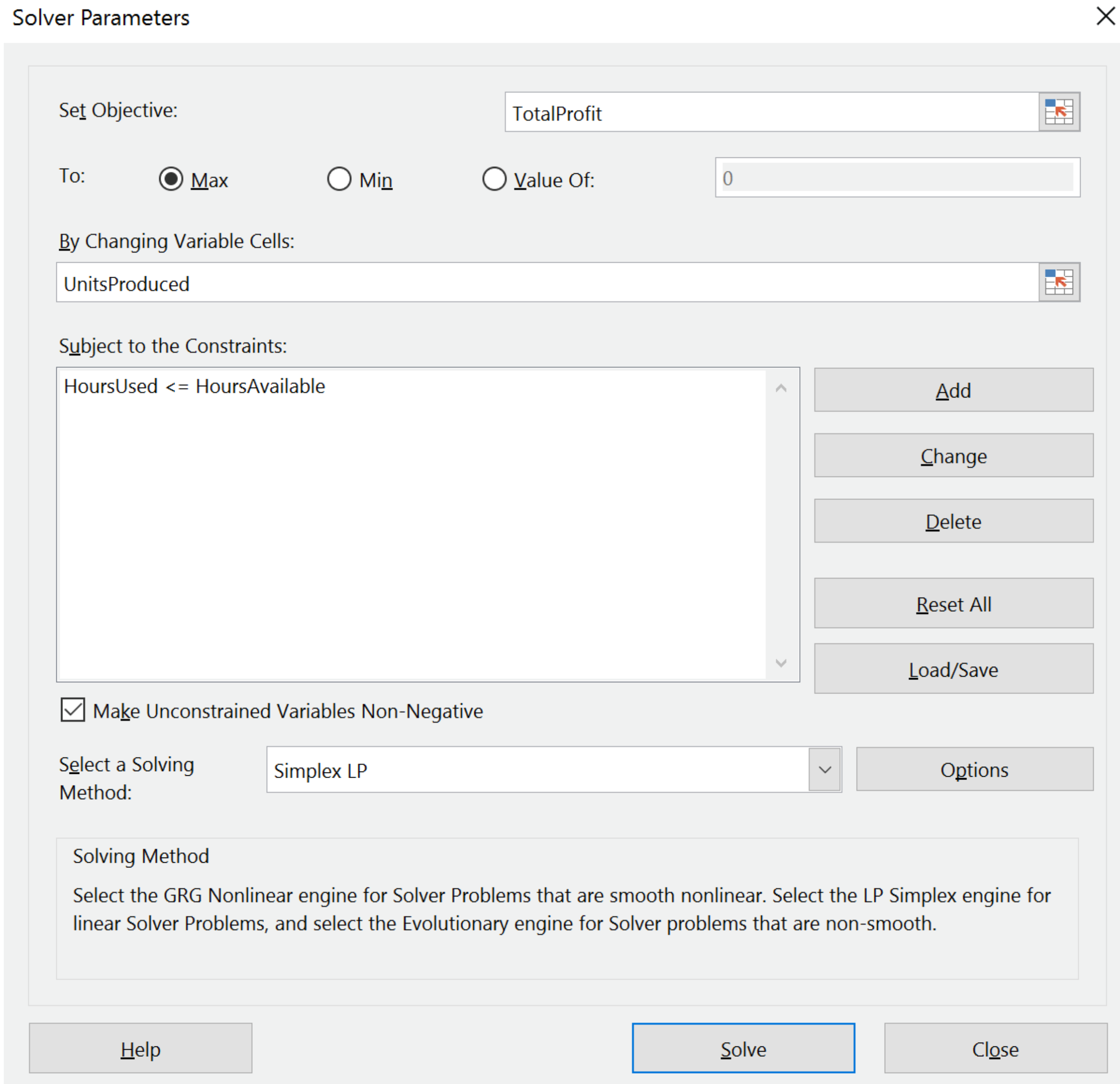
Adding Constraints

- To begin entering constraints, click the "Add" button to the right of the constraints window.
- Fill in the entries in the resulting Add Constraint dialogue box.

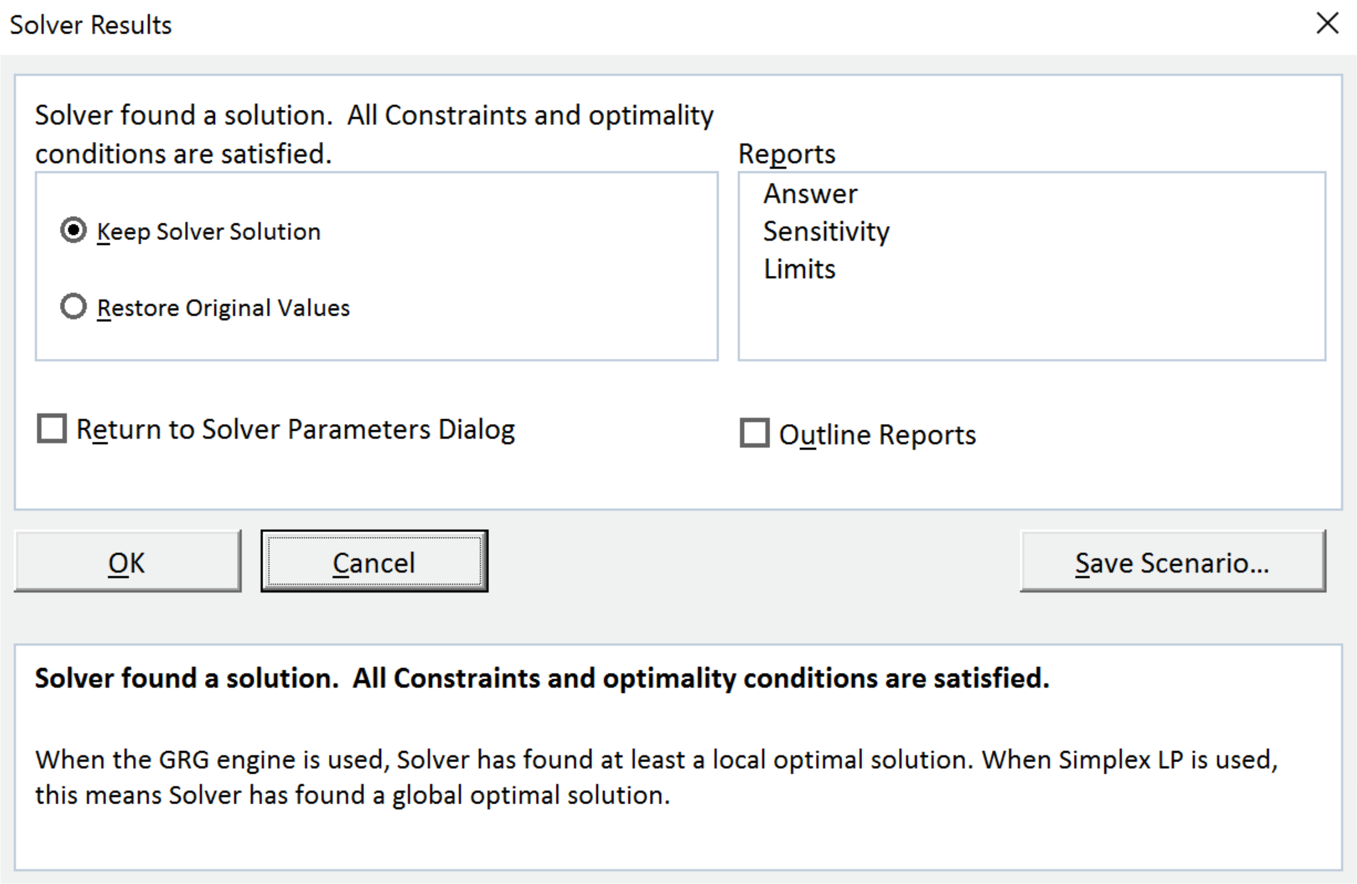
	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	$300	$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	1	<=	1
8	Plant 2	0	2	2	<=	12
9	Plant 3	3	2	5	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	1	1			$800



The Complete Solver Dialogue Box



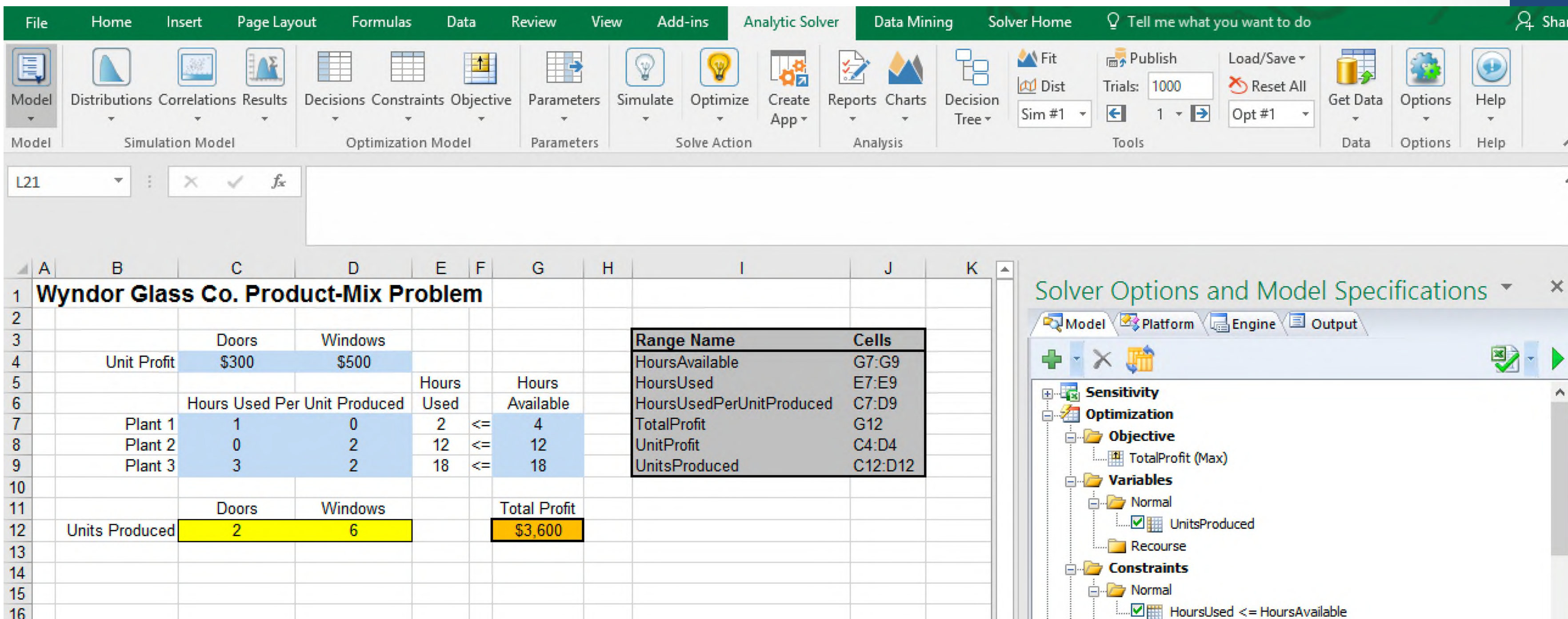
The Solver Results Dialogue Box



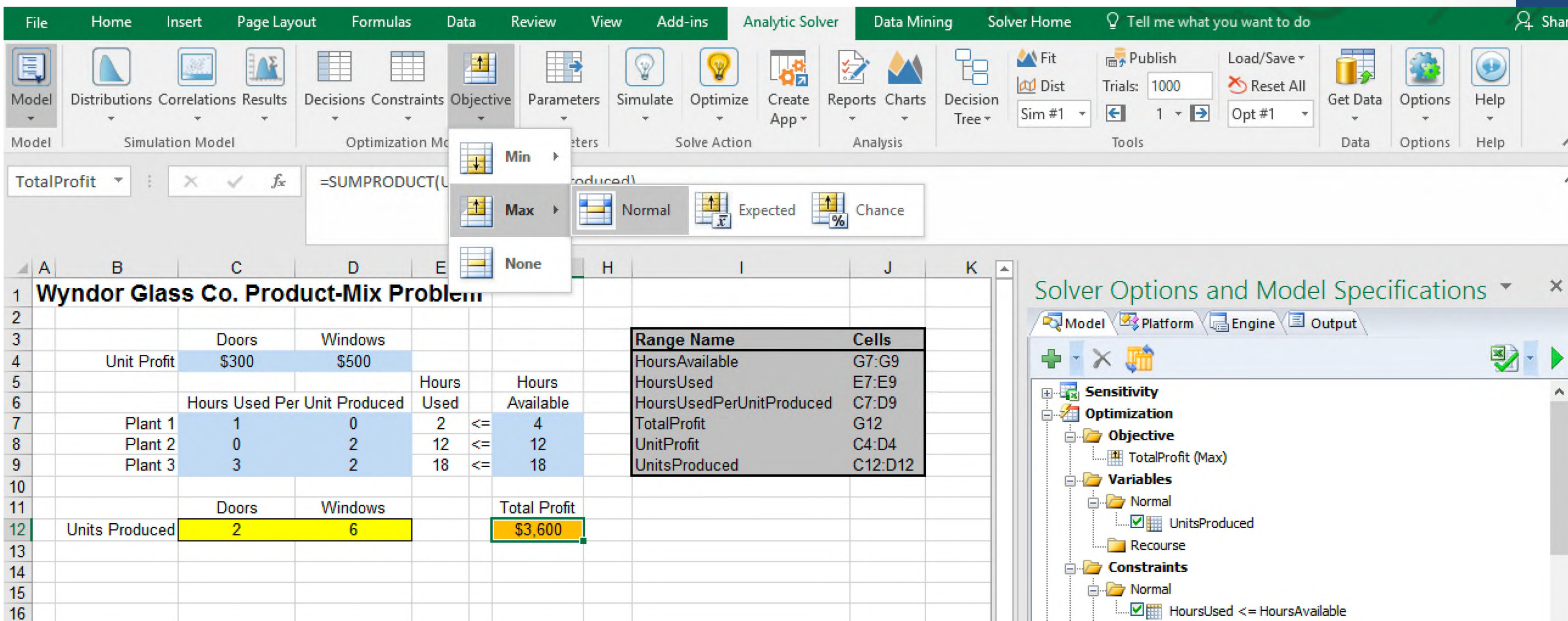
The Optimal Solution

	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	$300	$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	2	<=	1
8	Plant 2	0	2	12	<=	12
9	Plant 3	3	2	18	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	2	6			$3,600

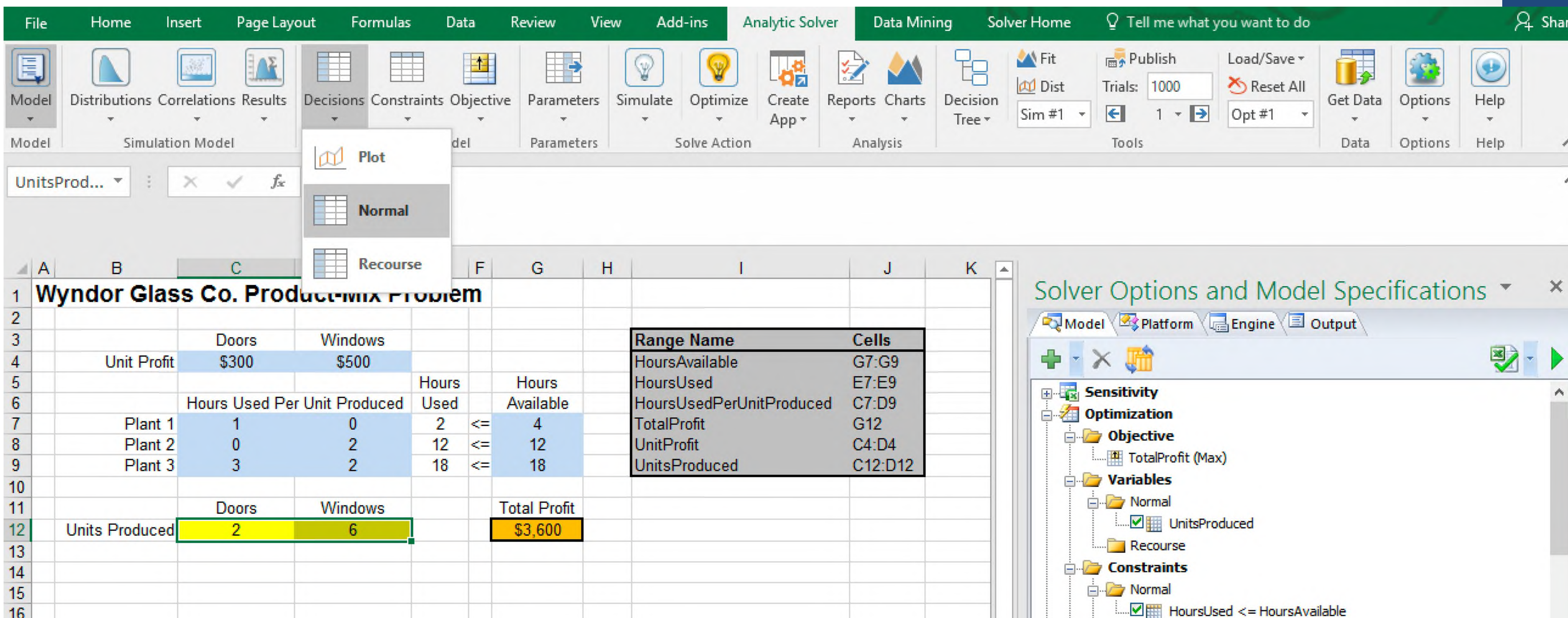
Analytic Solver



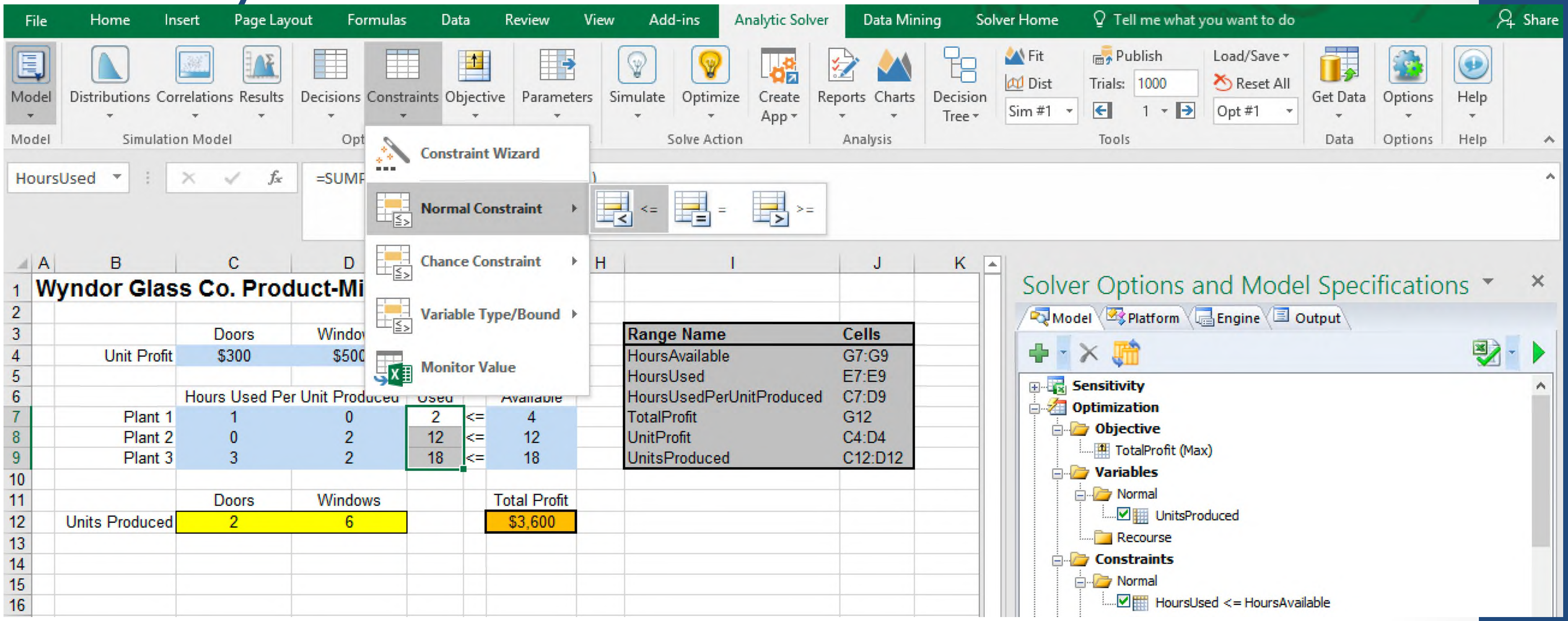
Specifying the Objective Cell with Analytic Solver



Specifying the Changing Cells with Analytic Solver

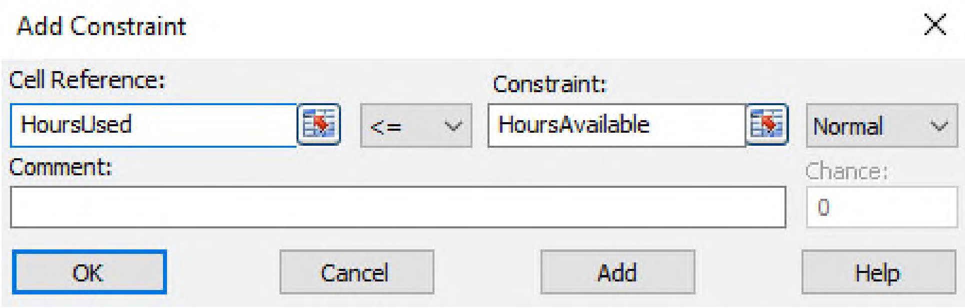


Adding Constraints with Analytic Solver

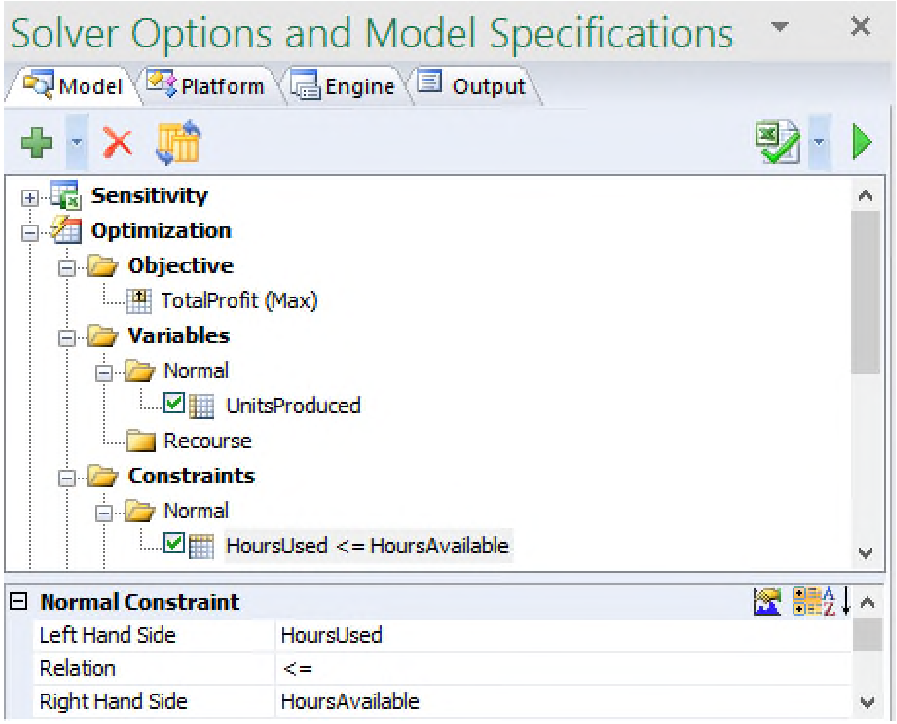


Adding Constraints with Analytic Solver

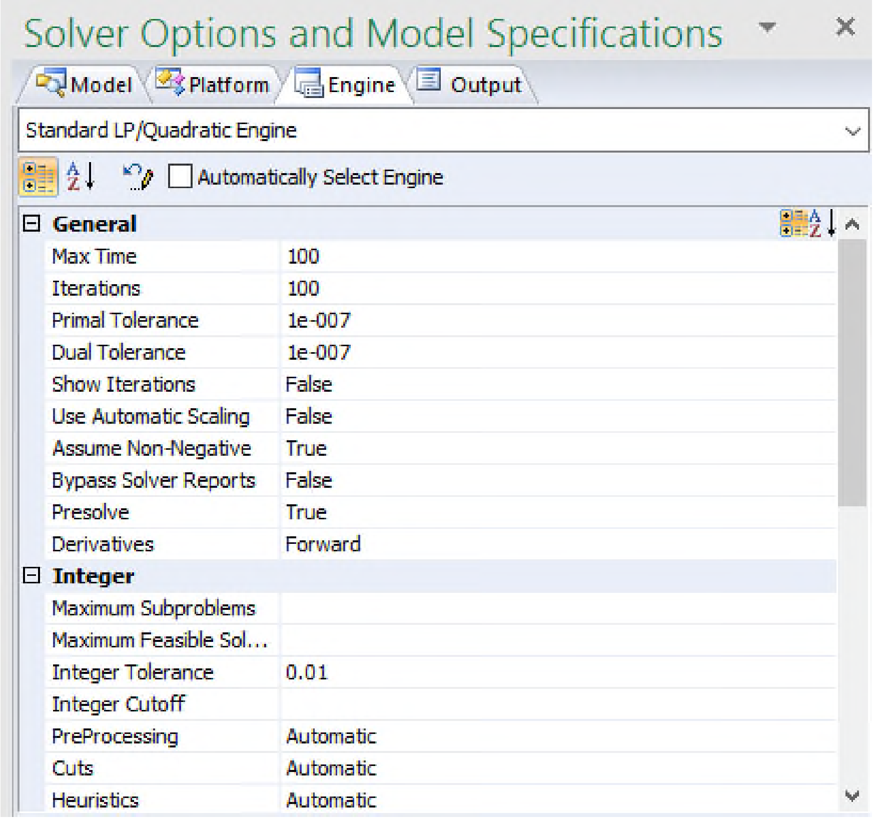
	B	C	D	E	F	G
3		Doors	Windows			
4	Unit Profit	$300	$500			
5				Hours		Hours
6		Hours Used Per Unit Produced		Used		Available
7	Plant 1	1	0	1	<=	1
8	Plant 2	0	2	2	<=	12
9	Plant 3	3	2	5	<=	18
10						
11		Doors	Windows			Total Profit
12	Units Produced	1	1			$800



Making Changes with the Model Pane in Analytic Solver



Specifying Options with Analytic Solver

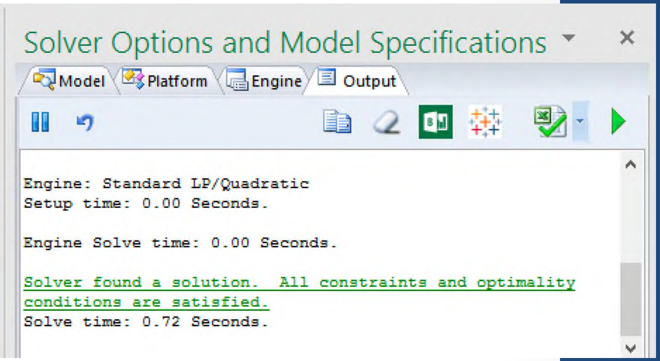


CLICK HERE TO ACCESS THE COMPLETE Solutions

The Optimal Solution with Analytic Solver

	A	B	C	D	E	F	G
1	Wyndor Glass Co. Product-Mix Problem						
2							
3			Doors	Windows			
4		Unit Profit	$300	$500			
5					Hours		Hours
6			Hours Used Per Unit Produced		Used		Available
7		Plant 1	1	0	2	<=	4
8		Plant 2	0	2	12	<=	12
9		Plant 3	3	2	18	<=	18
10							
11			Doors	Windows			Total Profit
12		Units Produced	2	6			$3,600
13							

Range Name	Cells
HoursAvailable	G7:G9
HoursUsed	E7:E9
HoursUsedPerUnitProduced	C7:D9
TotalProfit	G12
UnitProfit	C4:D4
UnitsProduced	C12:D12



The Profit & Gambit Co.

- Management has decided to undertake a major advertising campaign that will focus on the following three key products:
  - A spray prewash stain remover.
  - A liquid laundry detergent.
  - A powder laundry detergent.
- The campaign will use both television and print media
- The general goal is to increase sales of these products.
- Management has set the following goals for the campaign:
  - Sales of the stain remover should increase by at least 3%.
  - Sales of the liquid detergent should increase by at least 18%.
  - Sales of the powder detergent should increase by at least 4%.

Question: how much should they advertise in each medium to meet the sales goals at a minimum total cost?

Profit & Gambit Co. Spreadsheet Model

	B	C	D	E	F	G
3		Television	Print Media			
4	Unit Cost ($millions)	1	2			
5						
6				Increased		Minimum
7		Increase in Sales per Unit of Advertising		Sales		Increase
8	Stain Remover	0%	1%	3%	>=	3%
9	Liquid Detergent	3%	2%	18%	>=	18%
10	Powder Detergent	-1%	4%	8%	>=	4%
11						
12						Total Cost
13		Television	Print Media			($millions)
14	Advertising Units	4	3			10

Algebraic Model for Profit & Gambit

Let TV = the number of units of advertising on television
PM = the number of units of advertising in the print media

Minimize Cost = $TV + 2PM$ (in millions of dollars)

subject to

Stain remover increased sales: $PM \geq 3$

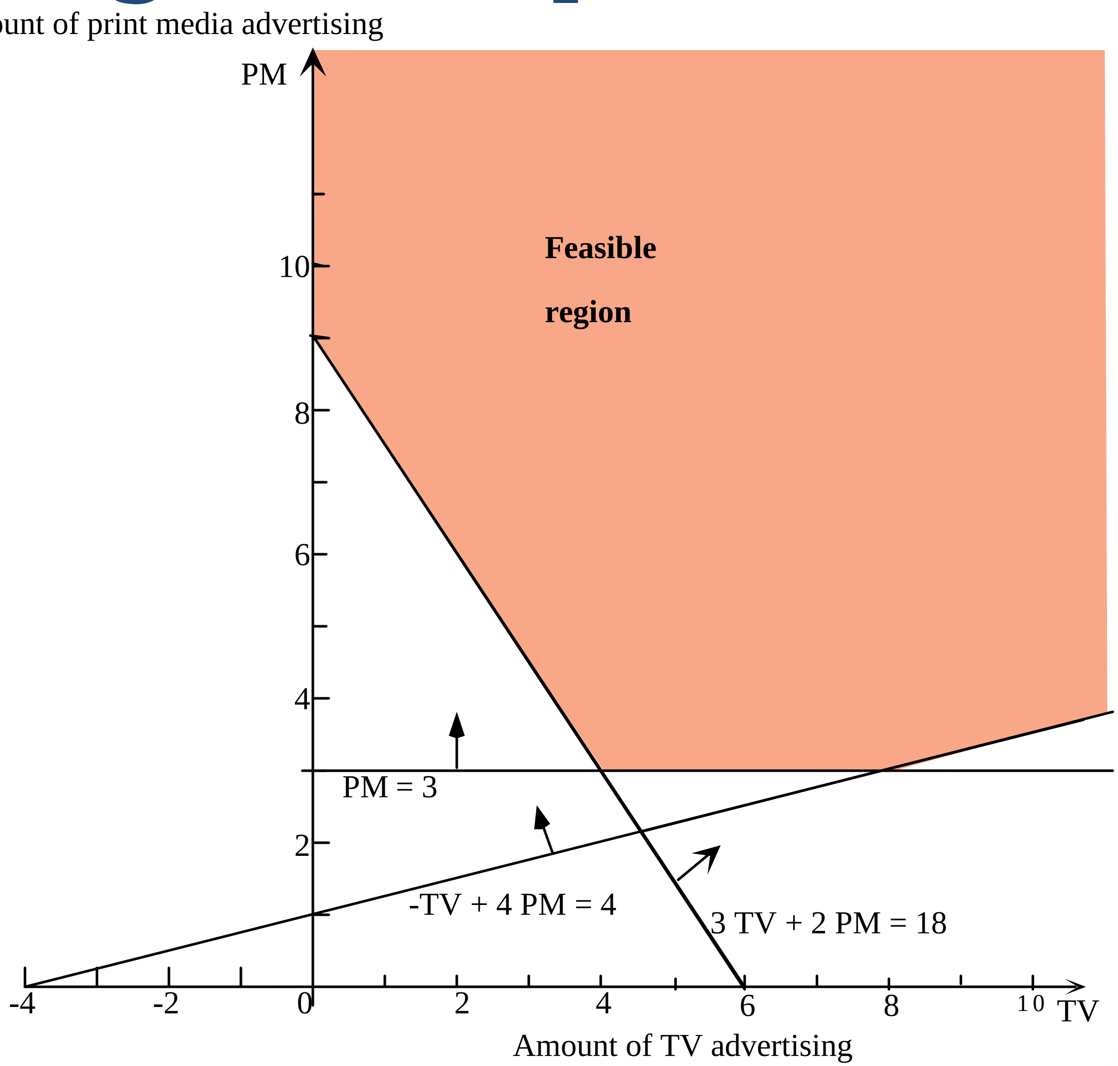
Liquid detergent increased sales: $3TV + 2PM \geq 18$

Powder detergent increased sales: $-TV + 4PM \geq 4$

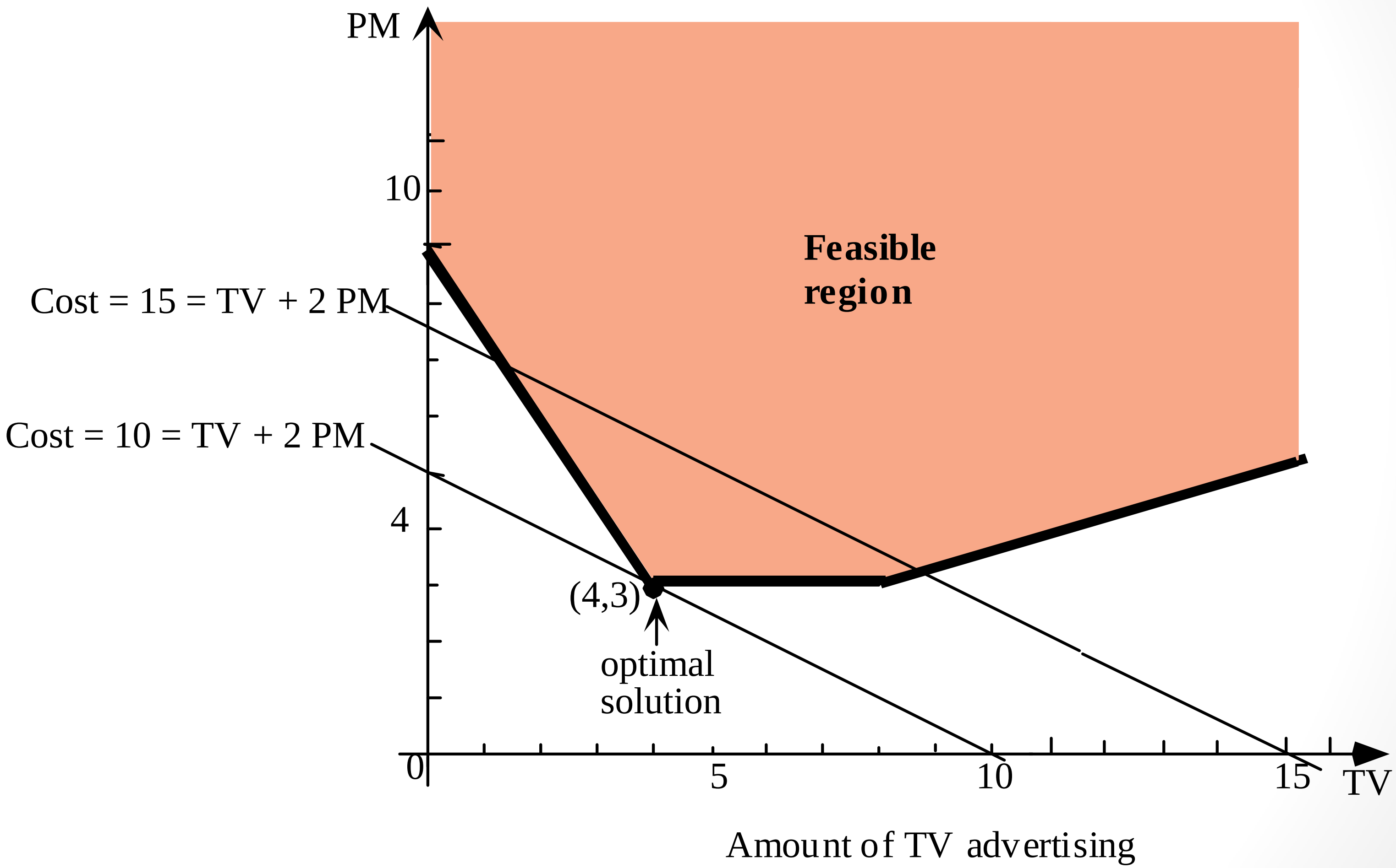
and

$TV \geq 0$, $PM \geq 0$.

Applying the Graphical Method



The Optimal Solution



Summary of the Graphical Method

- Draw the constraint boundary line for each constraint. Use the origin (or any point not on the line) to determine which side of the line is permitted by the constraint.
- Find the feasible region by determining where all constraints are satisfied simultaneously.
- Determine the slope of one objective function line. All other objective function lines will have the same slope.
- Move a straight edge with this slope through the feasible region in the direction of improving values of the objective function. Stop at the last instant that the straight edge still passes through a point in the feasible region. This line given by the straight edge is the optimal objective function line.
- A feasible point on the optimal objective function line is an optimal solution.