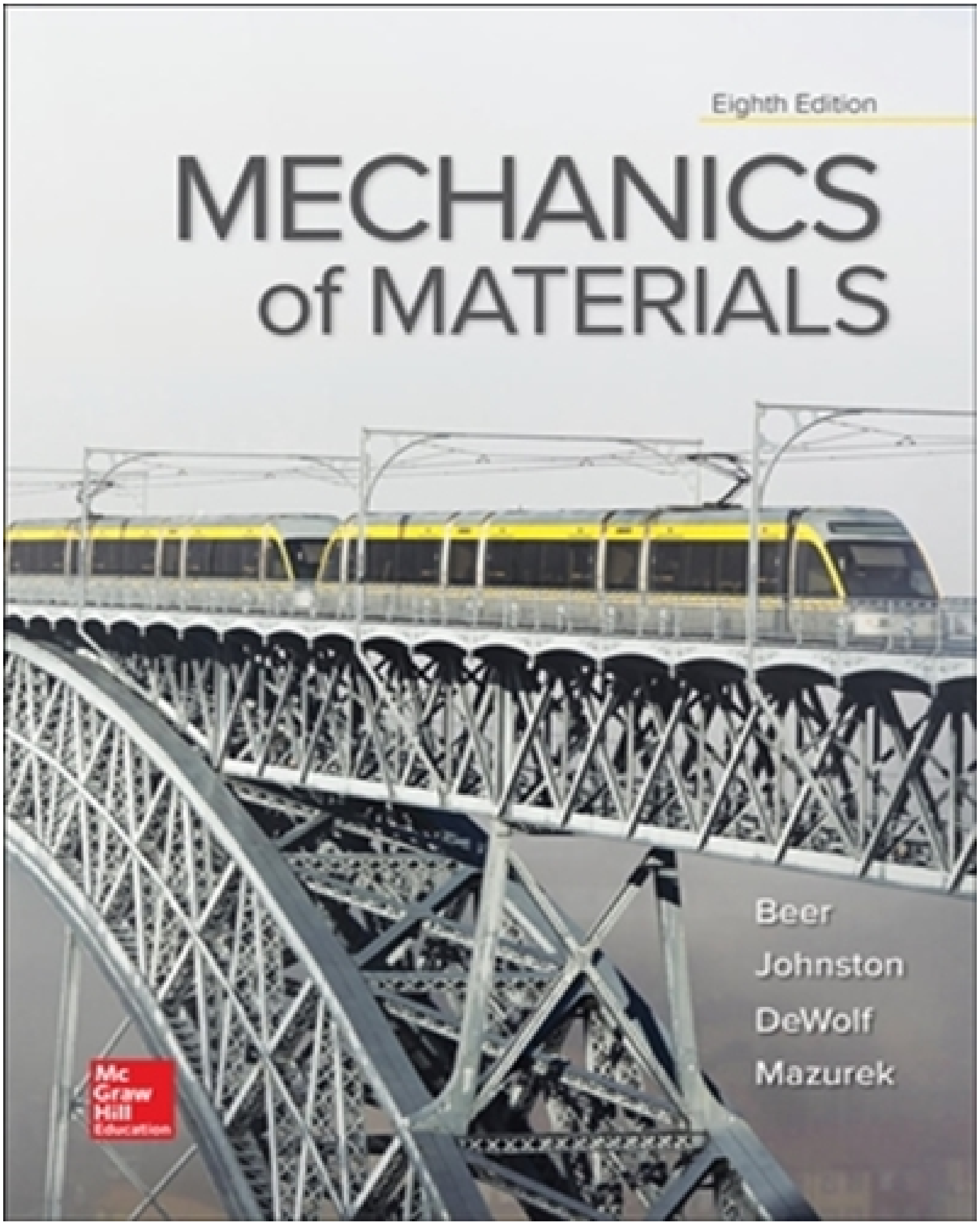
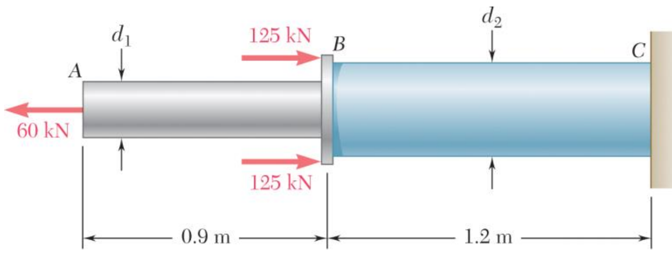
Solutions for Mechanics of Materials 8th Edition by Beer



Solutions

CHAPTER 1



PROBLEM 1.1

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d_1 = 30$ mm and $d_2 = 50$ mm, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

(*a*) Rod *AB*:

Force: $P = 60 \times 10^3 \text{ N}$ tension

Area: $$A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30 \times 10^{-3})^2 = 706.86 \times 10^{-6} \text{ m}^2$$

Normal stress: $$\sigma_{AB} = \frac{P}{A} = \frac{60 \times 10^3}{706.86 \times 10^{-6}} = 84.882 \times 10^6 \text{ Pa}$$

$\sigma_{AB} = 84.9 \text{ MPa}$ ◀

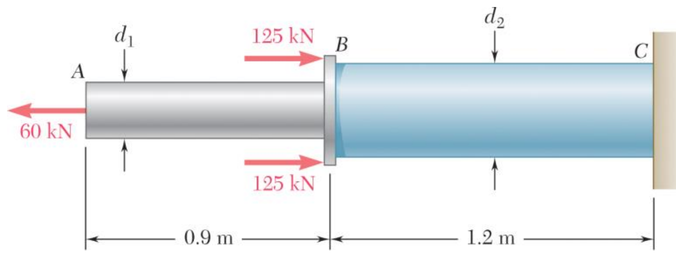
(*b*) Rod *BC*:

Force: $P = 60 \times 10^3 - (2)(125 \times 10^3) = -190 \times 10^3 \text{ N}$

Area: $$A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(50 \times 10^{-3})^2 = 1.96350 \times 10^{-3} \text{ m}^2$$

Normal stress: $$\sigma_{BC} = \frac{P}{A} = \frac{-190 \times 10^3}{1.96350 \times 10^{-3}} = -96.766 \times 10^6 \text{ Pa}$$

$\sigma_{BC} = -96.8 \text{ MPa}$ ◀



PROBLEM 1.2

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the average normal stress must not exceed 150 MPa in either rod, determine the smallest allowable values of the diameters d_1 and d_2.

SOLUTION

(*a*) Rod *AB*:

Force: $P = 60 \times 10^3\ \text{N}$

Stress: $\sigma_{AB} = 150 \times 10^6\ \text{Pa}$

Area: $A = \dfrac{\pi}{4} d_1^2$

$$\sigma_{AB} = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma_{AB}}$$

$$\frac{\pi}{4} d_1^2 = \frac{P}{\sigma_{AB}}$$

$$d_1^2 = \frac{4P}{\pi \sigma_{AB}} = \frac{(4)(60 \times 10^3)}{\pi(150 \times 10^6)} = 509.30 \times 10^{-6}\ \text{m}^2$$

$$d_1 = 22.568 \times 10^{-3}\ \text{m} \qquad d_1 = 22.6\ \text{mm} \blacktriangleleft$$

(*b*) Rod *BC*:

Force: $P = 60 \times 10^3 - (2)(125 \times 10^3) = -190 \times 10^3\ \text{N}$

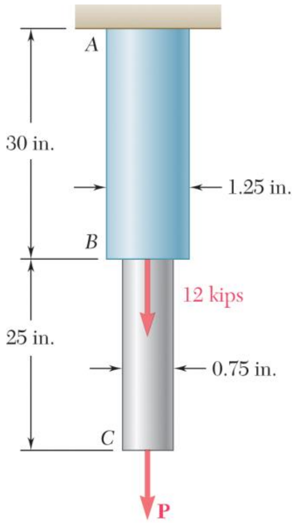
Stress: $\sigma_{BC} = -150 \times 10^6\ \text{Pa}$

Area: $A = \dfrac{\pi}{4} d_2^2$

$$\sigma_{BC} = \frac{P}{A} = \frac{4P}{\pi d_2^2}$$

$$d_2^2 = \frac{4P}{\pi \sigma_{BC}} = \frac{(4)(-190 \times 10^3)}{\pi(-150 \times 10^6)} = 1.61277 \times 10^{-3}\ \text{m}^2$$

$$d_2 = 40.159 \times 10^{-3}\ \text{m} \qquad d_2 = 40.2\ \text{mm} \blacktriangleleft$$



PROBLEM 1.3

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that P = 10 kips, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

(*a*) Rod *AB*:

$$P = 12 + 10 = 22 \text{ kips}$$

$$A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(1.25)^2 = 1.22718 \text{ in}^2$$

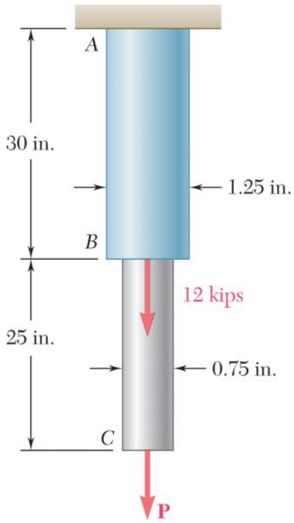
$$\sigma_{AB} = \frac{P}{A} = \frac{22}{1.22718} = 17.927 \text{ ksi} \qquad \sigma_{AB} = 17.93 \text{ ksi} \blacktriangleleft$$

(*b*) Rod *BC*:

$$P = 10 \text{ kips}$$

$$A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{10}{0.44179} = 22.635 \text{ ksi} \qquad \sigma_{AB} = 22.6 \text{ ksi} \blacktriangleleft$$



PROBLEM 1.4

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stresses in rods *AB* and *BC* are equal.

SOLUTION

(*a*) Rod *AB*:

$$P = P + 12 \text{ kips}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4}(1.25 \text{ in.})^2$$

$$A = 1.22718 \text{ in}^2$$

$$\sigma_{AB} = \frac{P + 12 \text{ kips}}{1.22718 \text{ in}^2}$$

(*b*) Rod *BC*:

$$P = P$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.75 \text{ in.})^2$$

$$A = 0.44179 \text{ in}^2$$

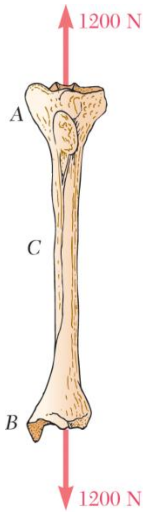
$$\sigma_{BC} = \frac{P}{0.44179 \text{ in}^2}$$

$$\sigma_{AB} = \sigma_{BC}$$

$$\frac{P + 12 \text{ kips}}{1.22718 \text{ in}^2} = \frac{P}{0.44179 \text{ in}^2}$$

$$5.3015 = 0.78539P$$

$P = 6.75$ kips ◀



PROBLEM 1.5

A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.

SOLUTION

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $A = \frac{\pi}{4}(d_1^2 - d_2^2)$

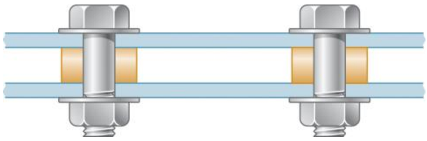
$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25 \times 10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80 \times 10^6)}$$

$$= 222.92 \times 10^{-6}\ \text{m}^2$$

$$d_2 = 14.93 \times 10^{-3}\ \text{m}$$

$d_2 = 14.93$ mm ◀



PROBLEM 1.6

Two steel plates are to be held together by means of 16-mm-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 200 MPa in the bolts and 130 MPa in the spacers, determine the outer diameter of the spacers that yields the most economical and safe design.

SOLUTION

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time, the spacer pushes that plate upward with a compressive force P_s in order to maintain equilibrium.

$$P_b = P_s$$

For the bolt, $\sigma_b = \dfrac{F_b}{A_b} = \dfrac{4P_b}{\pi d_b^2}$ or $P_b = \dfrac{\pi}{4}\sigma_b d_b^2$

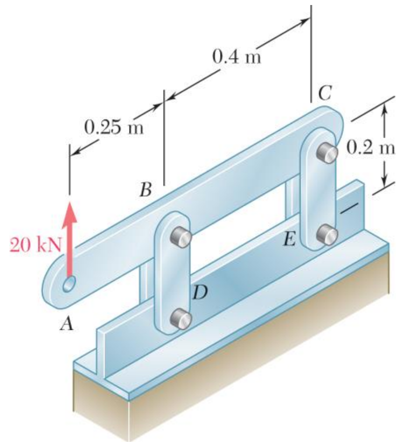
For the spacer, $\sigma_s = \dfrac{P_s}{A_s} = \dfrac{4P_s}{\pi(d_s^2 - d_b^2)}$ or $P_s = \dfrac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

Equating P_b and P_s,

$$\frac{\pi}{4}\sigma_b d_b^2 = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$$

$$d_s = \sqrt{\left(1 + \frac{\sigma_b}{\sigma_s}\right)}\, d_b = \sqrt{\left(1 + \frac{200}{130}\right)}(16)$$

$d_s = 25.2 \text{ mm}$ ◀

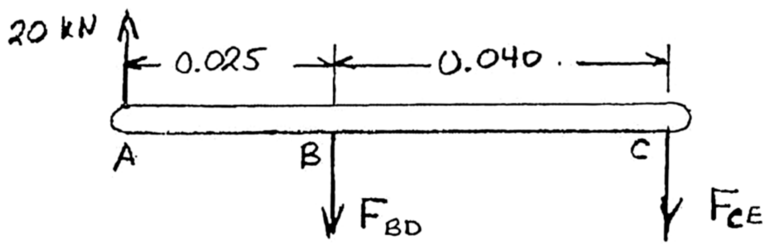


PROBLEM 1.7

Each of the four vertical links has an 8×36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar *ABC* as a free body.



$$\Sigma M_C = 0: \quad (0.040) F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N} \qquad \text{Link } BD \text{ is in tension.}$$

$$\Sigma M_B = 0: \quad -(0.040) F_{CE} - (0.025)(20 \times 10^3) = 0$$

$$F_{CE} = -12.5 \times 10^3 \text{ N} \qquad \text{Link } CE \text{ is in compression.}$$

Net area of one link for tension $= (0.008)(0.036 - 0.016) = 160 \times 10^{-6} \text{ m}^2$

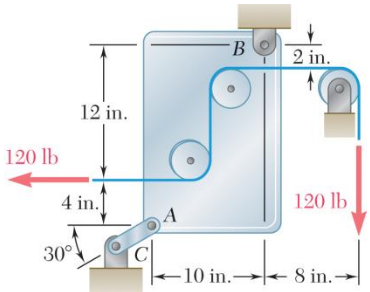
For two parallel links, $A_{\text{net}} = 320 \times 10^{-6} \text{ m}^2$

(*a*) $\sigma_{BD} = \dfrac{F_{BD}}{A_{\text{net}}} = \dfrac{32.5 \times 10^3}{320 \times 10^{-6}} = 101.563 \times 10^6$ $\qquad \sigma_{BD} = 101.6 \text{ MPa}$ ◀

Area for one link in compression $= (0.008)(0.036) = 288 \times 10^{-6} \text{ m}^2$

For two parallel links, $A = 576 \times 10^{-6} \text{ m}^2$

(*b*) $\sigma_{CE} = \dfrac{F_{CE}}{A} = \dfrac{-12.5 \times 10^3}{576 \times 10^{-6}} = -21.701 \times 10^{-6}$ $\qquad \sigma_{CE} = -21.7 \text{ MPa}$ ◀

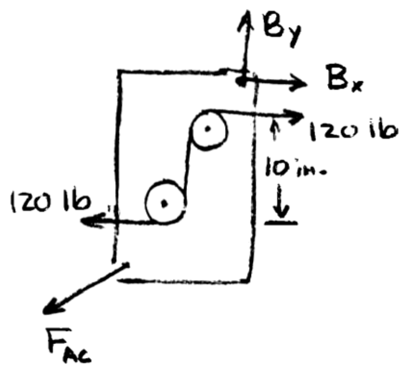


PROBLEM 1.8

Link AC has a uniform rectangular cross section $\frac{1}{8}$ in. thick and 1 in. wide. Determine the normal stress in the central portion of the link.

SOLUTION

Use the plate together with two pulleys as a free body. Note that the cable tension causes at 1200 lb-in. clockwise couple to act on the body.

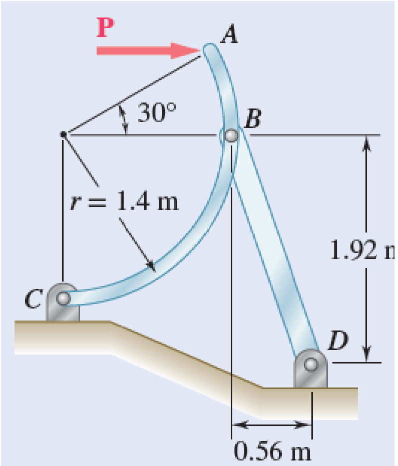


$$+\circlearrowright \Sigma M_B = 0: \quad -(12+4)(F_{AC}\cos 30°) + (10)(F_{AC}\sin 30°) - 1200 \text{ lb} = 0$$

$$F_{AC} = -\frac{1200 \text{ lb}}{16\cos 30° - 10\sin 30°} = -135.500 \text{ lb}$$

Area of link AC: $\quad A = 1 \text{ in.} \times \frac{1}{8} \text{ in.} = 0.125 \text{ in}^2$

Stress in link AC: $\quad \sigma_{AC} = \frac{F_{AC}}{A} = -\frac{135.50}{0.125} = 1084 \text{ psi} = 1.084 \text{ ksi}$ ◀

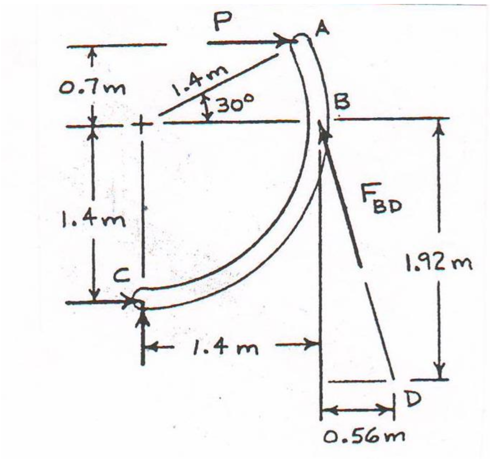


PROBLEM 1.9

Knowing that the central portion of the link *BD* has a uniform cross-sectional area of 800 mm^2, determine the magnitude of the load **P** for which the normal stress in that portion of *BD* is 50 MPa..

SOLUTION

Draw free body diagram of link *AC*.



$$F_{BD} = \sigma A$$

$$= \left(50 \times 10^6 \text{ N/m}^2\right)\left(800 \times 10^{-6} \text{ m}^2\right)$$

$$40 \times 10^3 \text{ N}$$

$$BD = \sqrt{(0.56 \text{ m})^2 + (1.92 \text{ m})^2}$$

$$= 2.00 \text{ m}$$

<u>Free Body *AC*</u>:

$$+\circlearrowleft M_C = 0: \quad \frac{0.56}{2.00}\left(40 \times 10^3\right)(1.4) + \frac{1.92}{2.00}\left(40 \times 10^3\right)(1.4) - P(0.7 + 1.4) = 0$$

$$P = 33.1 \times 10^3 \text{ N}$$

$P = 33.1\,\text{kN}$ ◀

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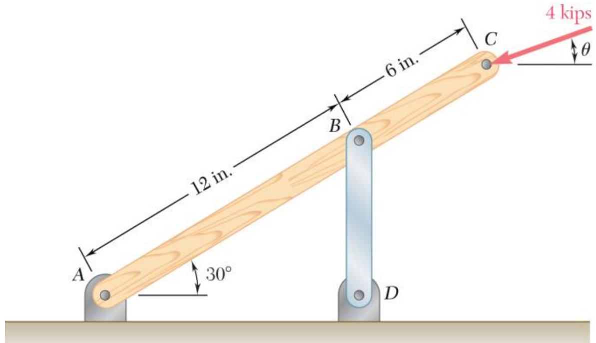
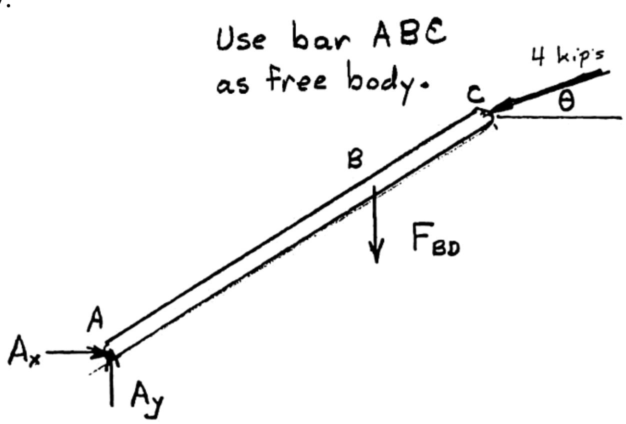


PROBLEM 1.10

Link BD consists of a single bar 1 in. wide and $\frac{1}{2}$ in. thick. Knowing that each pin has a $\frac{3}{8}$-in. diameter, determine the maximum value of the average normal stress in link BD if (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

SOLUTION

Use bar *ABC* as a free body.



(*a*) $\underline{\theta = 0}$.

$+\circlearrowleft \Sigma M_A = 0: \quad (18 \sin 30°)(4) - (12 \cos 30°)F_{BD} = 0$

$F_{BD} = 3.4641 \text{ kips} \quad \text{(tension)}$

Area for tension loading: $A = (b - d)t = \left(1 - \frac{3}{8}\right)\left(\frac{1}{2}\right) = 0.31250 \text{ in}^2$

Stress: $\sigma = \frac{F_{BD}}{A} = \frac{3.4641 \text{ kips}}{0.31250 \text{ in}^2}$ $\qquad \sigma = 11.09 \text{ ksi}$ ◀

(*b*) $\underline{\theta = 90°}$.

$+\circlearrowleft \Sigma M_A = 0: \quad -(18 \cos 30°)(4) - (12 \cos 30°)F_{BD} = 0$

$F_{BD} = -6 \text{ kips} \quad \text{i.e. compression.}$

Area for compression loading: $A = bt = (1)\left(\frac{1}{2}\right) = 0.5 \text{ in}^2$

Stress: $\sigma = \frac{F_{BD}}{A} = \frac{-6 \text{ kips}}{0.5 \text{ in}^2}$ $\qquad \sigma = 12.00 \text{ ksi}$ ◀

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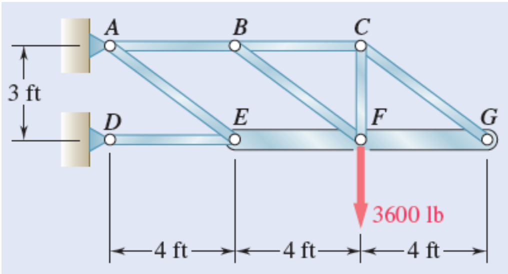
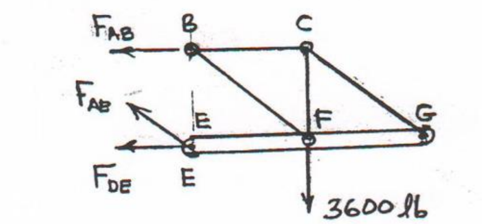
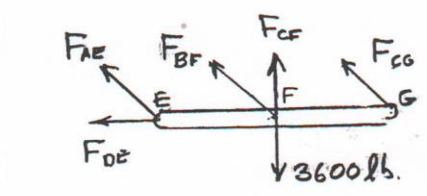


PROBLEM 1.11

The rigid bar *EFG* is supported by the truss system shown. Knowing that the member *CG* is a solid circular rod of 0.75-in. diameter, determine the normal stress in *CG*.

SOLUTION





Using portion *EFGCB* as free body,

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5} F_{AE} - 3600 = 0, \qquad F_{AE} = 6000 \text{ lb}$$

Use beam *EFG* as free body.

$$+\circlearrowleft \Sigma M_F = 0: \quad -(4)\frac{3}{5} F_{AE} + (4)\frac{3}{5} F_{CG} = 0$$

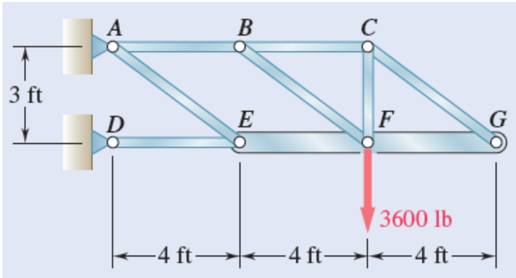
$$F_{AE} = F_{CG} = 6000 \text{ lb}$$

Normal stress in member *CG*

Area: $A = \dfrac{\pi d^2}{4} = 0.44179 \text{ in}^2$

$$\sigma_{CG} = \frac{F}{A} = \frac{6000}{0.44179} = 13580 \text{ psi}$$

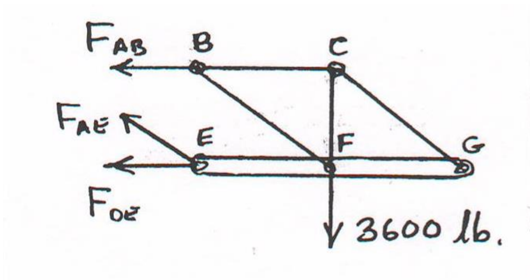
$$\sigma_{CG} = 13.58 \text{ ksi} \blacktriangleleft$$



PROBLEM 1.12

The rigid bar *EFG* is supported by the truss system shown. Determine the cross-sectional area of member *AE* for which the normal stress in the member is 15 ksi.

SOLUTION



Using portion *EFGCB* as free body,

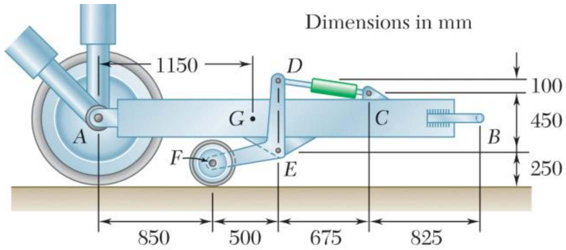
$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{AE} - 3600 = 0, \qquad F_{AE} = 6000 \text{ lb}$$

Normal stress in member *AE* = 15 ksi

$$\sigma_{AE} = \frac{F}{A}$$

$$A = \frac{F}{\sigma_{AE}} = \frac{6.00 \text{ kips}}{15 \text{ ksi}}$$

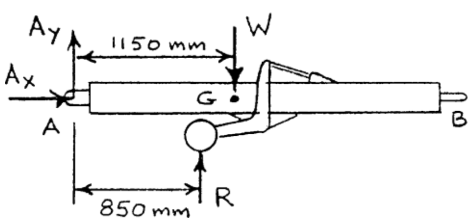
$$A = 0.400 \text{ in}^2 \blacktriangleleft$$



PROBLEM 1.13

An aircraft tow bar is positioned by means of a single hydraulic cylinder connected by a 25-mm-diameter steel rod to two identical arm-and-wheel units *DEF*. The mass of the entire tow bar is 200 kg, and its center of gravity is located at *G*. For the position shown, determine the normal stress in the rod.

SOLUTION

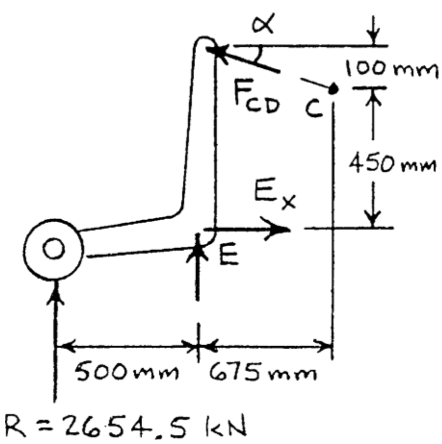


FREE BODY – ENTIRE TOW BAR:

$$W = (200\text{ kg})(9.81\text{ m/s}^2) = 1962.00\text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 850R - 1150(1962.00\text{ N}) = 0$$

$$R = 2654.5\text{ N}$$



FREE BODY – BOTH ARM & WHEEL UNITS:

$$\tan\alpha = \frac{100}{675} \qquad \alpha = 8.4270°$$

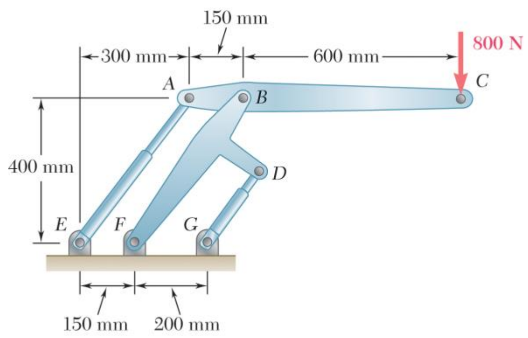
$$+\circlearrowleft \Sigma M_E = 0: \quad (F_{CD}\cos\alpha)(550) - R(500) = 0$$

$$F_{CD} = \frac{500}{550\cos 8.4270°}(2654.5\text{ N})$$

$$= 2439.5\text{ N} \quad \text{(comp.)}$$

$$\sigma_{CD} = -\frac{F_{CD}}{A_{CD}} = -\frac{2439.5\text{ N}}{\pi(0.0125\text{ m})^2}$$

$$= -4.9697\times 10^6\text{ Pa} \qquad \sigma_{CD} = -4.97\text{ MPa} \blacktriangleleft$$

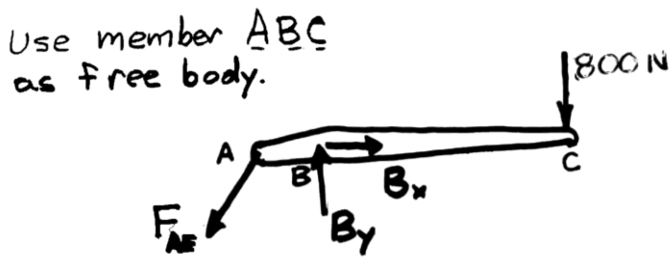


PROBLEM 1.14

Two hydraulic cylinders are used to control the position of the robotic arm *ABC*. Knowing that the control rods attached at *A* and *D* each have a 20-mm diameter and happen to be parallel in the position shown, determine the average normal stress in (*a*) member *AE*, (*b*) member *DG*.

SOLUTION

Use member *ABC* as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad (0.150)\frac{4}{5}F_{AE} - (0.600)(800) = 0$$

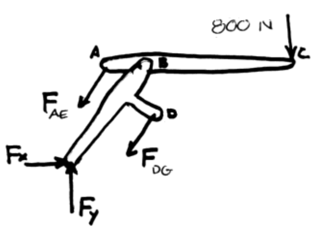
$$F_{AE} = 4 \times 10^3 \text{ N}$$

Area of rod in member *AE* is $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20 \times 10^{-3})^2 = 314.16 \times 10^{-6} \text{m}^2$

Stress in rod *AE*: $\sigma_{AE} = \frac{F_{AE}}{A} = \frac{4 \times 10^3}{314.16 \times 10^{-6}} = 12.7324 \times 10^6 \text{ Pa}$

(*a*) $\sigma_{AE} = 12.73 \text{ MPa}$ ◀

Use combined members *ABC* and *BFD* as free body.



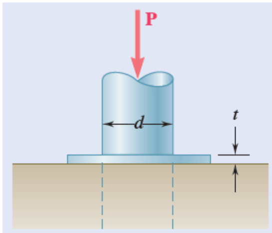
$$+\circlearrowleft \Sigma M_F = 0: \quad (0.150)\left(\frac{4}{5}F_{AE}\right) - (0.200)\left(\frac{4}{5}F_{DG}\right) - (1.050 - 0.350)(800) = 0$$

$$F_{DG} = -1500 \text{ N}$$

Area of rod *DG*: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20 \times 10^{-3})^2 = 314.16 \times 10^{-6} \text{ m}^2$

Stress in rod *DG*: $\sigma_{DG} = \frac{F_{DG}}{A} = \frac{-1500}{3.1416 \times 10^{-6}} = -4.7746 \times 10^6 \text{ Pa}$

(*b*) $\sigma_{DG} = -4.77 \text{ MPa}$ ◀



PROBLEM 1.15

Knowing that a force **P** of magnitude 50 kN is required to punch a hole of diameter d = 20 mm in an aluminum sheet of thickness t = 5 mm, determine the average shearing stress in the aluminum at failure.

SOLUTION

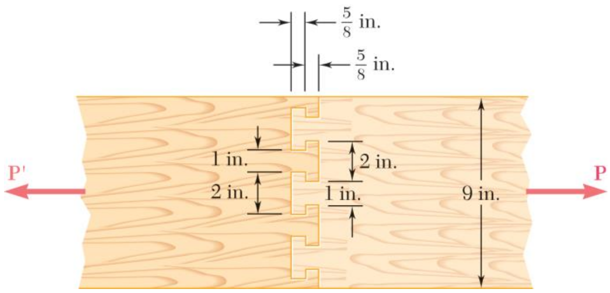
Area of failure in plate:

$$A = \pi dt = \pi(0.020\text{ m})(0.005\text{ m})$$
$$= 3.1416 \times 10^{-4}\text{ m}^2$$

Average shearing stress:

$$\tau_{avg} = \frac{P}{A}$$
$$= \frac{50 \times 10^3\text{N}}{3.1416 \times 10^{-4}\text{ m}^2}$$

$$\tau_{avg} = 159.2\text{ MPa} \blacktriangleleft$$



PROBLEM 1.16

Two wooden planks, each $\frac{1}{2}$ in. thick and 9 in. wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 1.20 ksi, determine the magnitude P of the axial load that will cause the joint to fail.

SOLUTION

Six areas must be sheared off when the joint fails. Each of these areas has dimensions $\frac{5}{8}$ in. $\times$ $\frac{1}{2}$ in., its area being

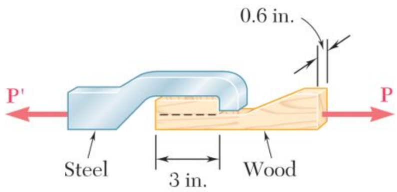
$$A = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16} \text{ in}^2 = 0.3125 \text{ in}^2$$

At failure, the force carried by each area is

$$F = \tau A = (1.20 \text{ ksi})(0.3125 \text{ in}^2) = 0.375 \text{ kips}$$

Since there are six failure areas,

$$P = 6F = (6)(0.375) \qquad P = 2.25 \text{ kips} \blacktriangleleft$$



PROBLEM 1.17

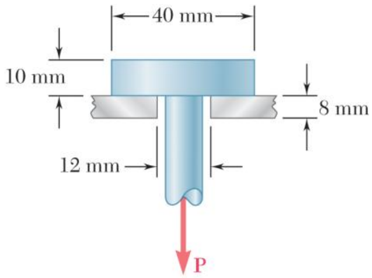
When the force **P** reached 1600 lb, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.

SOLUTION

Area being sheared: $A = 3 \text{ in.} \times 0.6 \text{ in.} = 1.8 \text{ in}^2$

Force: $P = 1600 \text{ lb}$

Shearing stress: $\tau = \dfrac{P}{A} - \dfrac{1600 \text{ lb}}{1.8 \text{ in}^2} = 8.8889 \times 10^2 \text{ psi}$ $\qquad \tau = 889 \text{ psi}$ ◀



PROBLEM 1.18

A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load **P** that can be applied to the rod.

SOLUTION

For steel:

$$A_1 = \pi dt = \pi(0.012 \text{ m})(0.010 \text{ m})$$
$$= 376.99 \times 10^{-6} \text{ m}^2$$

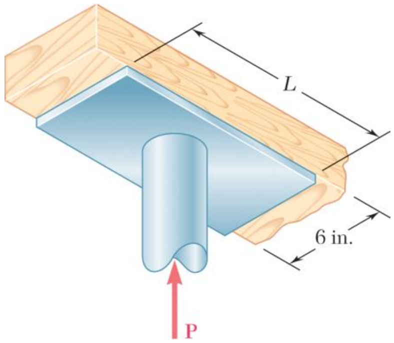
$$\tau_1 = \frac{P}{A} \therefore P = A_1\tau_1 = (376.99 \times 10^{-6} \text{ m}^2)(180 \times 10^6 \text{ Pa})$$
$$= 67.858 \times 10^3 \text{ N}$$

For aluminum:

$$A_2 = \pi dt = \pi(0.040 \text{ m})(0.008 \text{ m}) = 1.00531 \times 10^{-3} \text{ m}^2$$

$$\tau_2 = \frac{P}{A_2} \therefore P = A_2\tau_2 = (1.00531 \times 10^{-3} \text{ m}^2)(70 \times 10^6 \text{ Pa}) = 70.372 \times 10^3 \text{ N}$$

Limiting value of P is the smaller value, so $P = 67.9 \text{ kN}$ ◀



PROBLEM 1.19

The axial force in the column supporting the timber beam shown is $P = 20$ kips. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 400 psi.

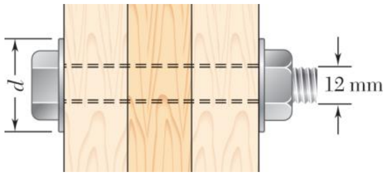
SOLUTION

Bearing area: $A_b = Lw$

$$\sigma_b = \frac{P}{A_b} = \frac{P}{Lw}$$

$$L = \frac{P}{\sigma_b w} = \frac{20 \times 10^3 \text{ lb}}{(400 \text{ psi})(6 \text{ in.})} = 8.33 \text{ in.}$$

$L = 8.33$ in. ◀



PROBLEM 1.20

Three wooden planks are fastened together by a series of bolts to form a column. The diameter of each bolt is 12 mm and the inner diameter of each washer is 16 mm, which is slightly larger than the diameter of the holes in the planks. Determine the smallest allowable outer diameter d of the washers, knowing that the average normal stress in the bolts is 36 MPa and that the bearing stress between the washers and the planks must not exceed 8.5 MPa.

SOLUTION

Bolt: $$A_{\text{Bolt}} = \frac{\pi d^2}{4} = \frac{\pi(0.012\text{ m})^2}{4} = 1.13097 \times 10^{-4}\text{ m}^2$$

Tensile force in bolt: $$\sigma = \frac{P}{A} \quad \Rightarrow \quad P = \sigma A$$

$$= (36 \times 10^6\text{ Pa})(1.13097 \times 10^{-4}\text{ m}^2)$$

$$= 4.0715 \times 10^3\text{ N}$$

Bearing area for washer: $$A_w = \frac{\pi}{4}\left(d_o^2 - d_i^2\right)$$

and $$A_w = \frac{P}{\sigma_{BRG}}$$

Therefore, equating the two expressions for A_w gives

$$\frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{P}{\sigma_{BRG}}$$

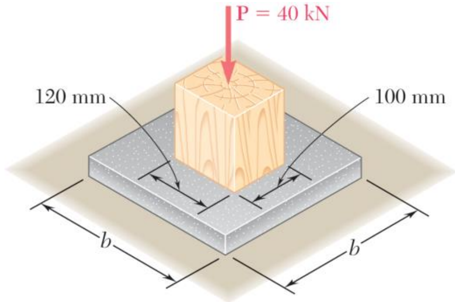
$$d_o^2 = \frac{4P}{\pi\sigma_{BRG}} + d_i^2$$

$$d_o^2 = \frac{4}{\pi}\frac{(4.0715 \times 10^3\text{ N})}{(8.5 \times 10^6\text{ Pa})} + (0.016\text{ m})^2$$

$$d_o^2 = 8.6588 \times 10^{-4}\text{ m}^2$$

$$d_o = 29.426 \times 10^{-3}\text{ m}$$

$$d_o = 29.4\text{ mm} \blacktriangleleft$$



PROBLEM 1.21

A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine (*a*) the maximum bearing stress on the concrete footing, (*b*) the size of the footing for which the average bearing stress in the soil is 145 kPa.

SOLUTION

(*a*) Bearing stress on concrete footing.

$$P = 40 \text{ kN} = 40 \times 10^3 \text{ N}$$

$$A = (100)(120) = 12 \times 10^3 \text{ mm}^2 = 12 \times 10^{-3} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \times 10^3}{12 \times 10^{-3}} = 3.3333 \times 10^6 \text{ Pa}$$

3.33 MPa ◀

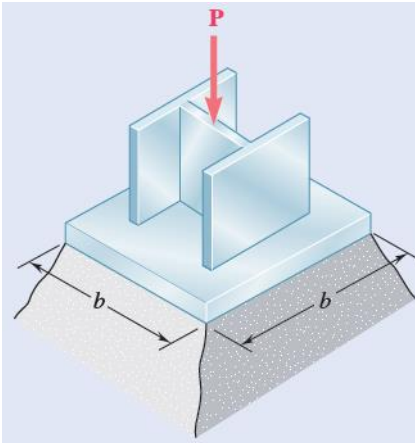
(*b*) Footing area. $P = 40 \times 10^3 \text{ N}$ $\sigma = 145 \text{ kPa} = 45 \times 10^3 \text{ Pa}$

$$\sigma = \frac{P}{A} \quad A = \frac{P}{\sigma} = \frac{40 \times 10^3}{145 \times 10^3} = 0.27586 \text{ m}^2$$

Since the area is square, $A = b^2$

$$b = \sqrt{A} = \sqrt{0.27586} = 0.525 \text{ m}$$

$b = 525$ mm ◀



PROBLEM 1.22

The axial load $P = 240$ kips, supported by a W10 × 45 column, is distributed to a concrete foundation by a square base plate as shown. Determine the size of the base plate for which the average bearing stress on the concrete is 750 psi.

SOLUTION

$$\sigma = \frac{P}{A} \quad \text{or}$$

$$A = \frac{P}{\sigma}$$

$$= \frac{240 \times 10^3 \text{ lb}}{750 \text{ psi}}$$

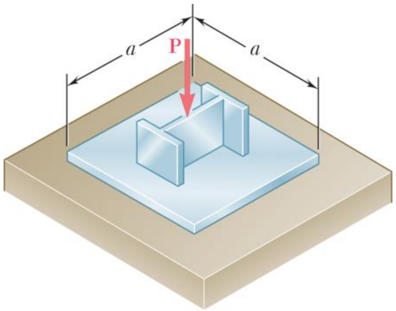
$$= 320 \text{ in}^2$$

Since the plate is square,

$$A = b^2$$

$$b = \sqrt{320 \text{ in}^2}$$

$$b = 17.89 \text{ in.} \blacktriangleleft$$



PROBLEM 1.23

An axial load **P** is supported by a short W8 × 40 column of cross-sectional area $A = 11.7\ \text{in}^2$ and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 30 ksi and that the bearing stress on the concrete foundation must not exceed 3.0 ksi, determine the side *a* of the plate that will provide the most economical and safe design.

SOLUTION

For the column, $\sigma = \dfrac{P}{A}$ or

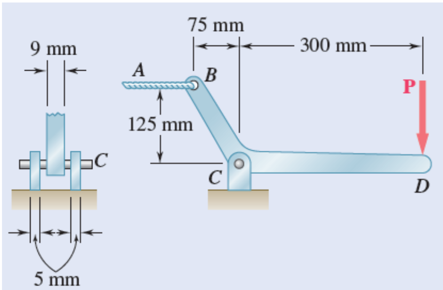
$$P = \sigma A = (30)(11.7) = 351 \text{ kips}$$

For the $a \times a$ plate, $\sigma = 3.0$ ksi

$$A = \frac{P}{\sigma} = \frac{351}{3.0} = 117 \text{ in}^2$$

Since the plate is square, $A = a^2$

$$a = \sqrt{A} = \sqrt{117} \qquad a = 10.82 \text{ in.} ◀$$

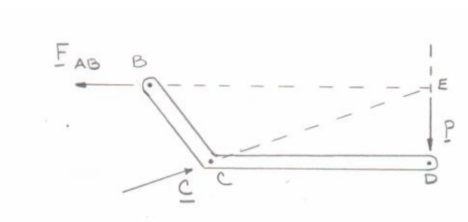


PROBLEM 1.24

A 6-mm-diameter pin is used at connection C of the pedal shown. Knowing that P = 500 N, determine (*a*) the average shearing stress in the pin, (*b*) the nominal bearing stress in the pedal at C, (*c*) the nominal bearing stress in each support bracket at C.

SOLUTION

Since BCD is a 3-force member, the reaction at C is directed toward E, the intersection of the lines of act of the other two forces.



From geometry, $$CE = \sqrt{300^2 + 125^2} = 325 \text{ mm}$$

From the free body diagram of BCD,

$$+\uparrow \Sigma F_y = 0: \frac{125}{325}C - P = 0 \qquad C = 2.6P = 2.6(500 \text{ N}) = 1300 \text{ N}$$

(*a*) $$\tau_{\text{pin}} = \frac{\frac{1}{2}C}{A_P} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} = \frac{2C}{\pi d^2}$$

$$\tau_{\text{pin}} = \frac{2(1300 \text{ N})}{\pi\left(6\times 10^{-3} \text{ m}\right)^2} = 23.0\times 10^6 \text{ Pa}$$

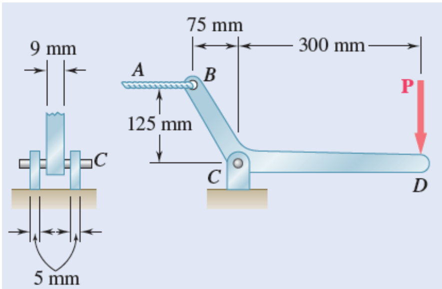
$\tau_{\text{pin}} = 23.0 \text{ MPa}$ ◀

(*b*) $$\sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{(1300)}{\left(6\times 10^{-3}\right)\left(9\times 10^{-3}\right)} = 24.1\times 10^6 \text{ Pa}$$

$\sigma_b = 24.1 \text{ MPa}$ ◀

(*c*) $$\sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{(1300)}{2\left(6\times 10^{-3}\right)\left(9\times 10^{-3}\right)} = 21.7\times 10^6 \text{ Pa}$$

$\sigma_b = 21.7 \text{ MPa}$ ◀

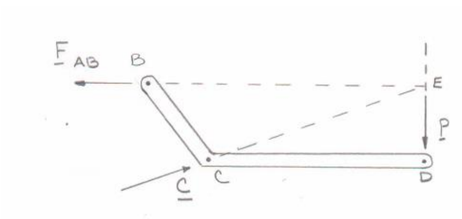


PROBLEM 1.25

Knowing that a force **P** of magnitude 750 N is applied to the pedal shown, determine (*a*) the diameter of the pin at *C* for which the average shearing stress in the pin is 40 MPa, (*b*) the corresponding bearing stress in the pedal at *C*, (*c*) the corresponding bearing stress in each support bracket at *C*.

SOLUTION

Since *BCD* is a 3-force member, the reaction at *C* is directed toward *E*, the intersection of the lines of action of the other two forces.



From geometry, $CE = \sqrt{300^2 + 125^2} = 325 \text{ mm}$

From the free body diagram of *BCD*,

$$+\uparrow \Sigma F_y = 0: \frac{125}{325}C - P = 0 \qquad C = 2.6P = 2.6(750 \text{ N}) = 1950 \text{ N}$$

(*a*)
$$\tau_{pin} = \frac{\frac{1}{2}C}{A_P} = \frac{\frac{1}{2}C}{\frac{\pi}{4}d^2} = \frac{2C}{\pi d^2}$$

$$d = \sqrt{\frac{2C}{\pi \tau_{pin}}} = \sqrt{\frac{2(1950 \text{ N})}{\pi(40 \times 10^6 \text{ Pa})}} = 5.57 \times 10^{-3} \text{ m}$$

$d = 5.57 \text{ mm}$ ◀

(*b*)
$$\sigma_b = \frac{C}{A_b} = \frac{C}{dt} = \frac{(1950)}{(5.57 \times 10^{-3})(9 \times 10^{-3})} = 38.9 \times 10^6 \text{ Pa}$$

$\sigma_b = 38.9 \text{ MPa}$ ◀

(*c*)
$$\sigma_b = \frac{\frac{1}{2}C}{A_b} = \frac{C}{2dt} = \frac{(1950)}{2(5.57 \times 10^{-3})(5 \times 10^{-3})} = 35.0 \times 10^6 \text{ Pa}$$

$\sigma_b = 35.0 \text{ MPa}$ ◀

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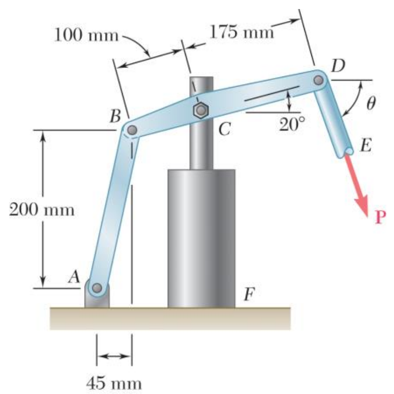
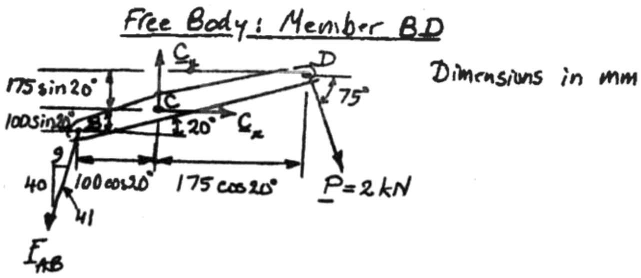


PROBLEM 1.26

The hydraulic cylinder *CF*, which partially controls the position of rod *DE*, has been locked in the position shown. Member *BD* is 15 mm thick and is connected at C to the vertical rod by a 9-mm-diameter bolt. Knowing that $P = 2$ kN and $\theta = 75°$, determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at *C* in member *BD*.

SOLUTION

Free Body: Member *BD*.



$$+\circlearrowleft \Sigma M_c = 0: \quad \frac{40}{41}F_{AB}(100\cos 20°) - \frac{9}{4}F_{AB}(100\sin 20°)$$

$$-(2\text{ kN})\cos 75°(175\sin 20°) - (2\text{ kN})\sin 75°(175\cos 20°) = 0$$

$$\frac{100}{41}F_{AB}(40\cos 20° - 9\sin 20°) = (2\text{ kN})(175)\sin(75° + 20°)$$

$$F_{AB} = 4.1424\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - \frac{9}{41}(4.1424\text{ kN}) + (2\text{ kN})\cos 75° = 0$$

$$C_x = 0.39167\text{ kN}$$

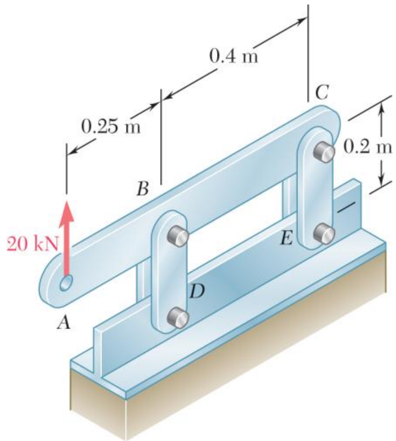
$$+\uparrow \Sigma F_y = 0: \quad C_y - \frac{40}{41}(4.1424\text{ kN}) - (2\text{ kN})\sin 75° = 0$$

$$C_y = 5.9732\text{ kN}$$

$$\mathbf{C} = 5.9860\text{ kN} \measuredangle 86.2°$$

(*a*) $\tau_{\text{ave}} = \dfrac{C}{A} = \dfrac{5.9860 \times 10^3\text{ N}}{\pi(0.0045\text{ m})^2} = 94.1 \times 10^6\text{ Pa} = 94.1\text{ MPa}$ ◀

(*b*) $\tau_b = \dfrac{C}{td} = \dfrac{5.9860 \times 10^3\text{ N}}{(0.015\text{ m})(0.009\text{ m})} = 44.3 \times 10^6\text{ Pa} = 44.3\text{ MPa}$ ◀



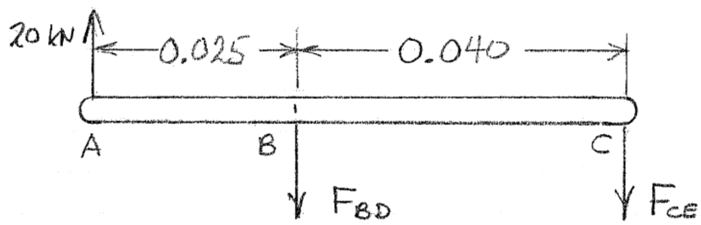
PROBLEM 1.27

For the assembly and loading of Prob. 1.7, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in member *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10 × 50-mm uniform rectangular cross section.

PROBLEM 1.7 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar *ABC* as a free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{N}$$

(*a*) Shear pin at *B*. $\tau = \dfrac{F_{BD}}{2A}$ for double shear

where $A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.822 \times 10^6 \text{ Pa}$$

$\tau = 80.8$ MPa ◀

(*b*) Bearing: link *BD*. $A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{m}^2$

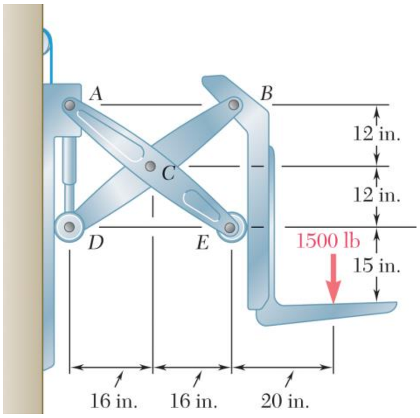
$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \text{ Pa}$$

$\sigma_b = 127.0$ MPa ◀

(*c*) Bearing in *ABC* at *B*. $A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{m}^2$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203.12 \times 10^6 \text{ Pa}$$

$\sigma_b = 203$ MPa ◀

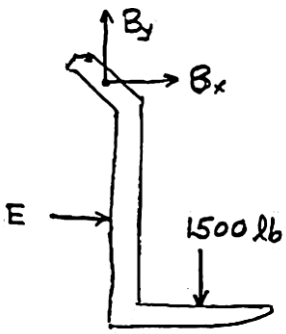


PROBLEM 1.28

Two identical linkage-and-hydraulic-cylinder systems control the position of the forks of a fork-lift truck. The load supported by the one system shown is 1500 lb. Knowing that the thickness of member *BD* is $\frac{5}{8}$ in., determine (*a*) the average shearing stress in the $\frac{1}{2}$-in.-diameter pin at *B*, (*b*) the bearing stress at *B* in member *BD*.

SOLUTION

Use one fork as a free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 24E - (20)(1500) = 0$$

$$E = 1250 \text{ lb} \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E + B_x = 0$$

$$B_x = -E$$

$$B_x = 1250 \text{ lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 1500 = 0 \qquad B_y = 1500 \text{ lb}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{1250^2 + 1500^2} = 1952.56 \text{ lb}$$

(*a*) <u>Shearing stress in pin at *B*.</u>

$$A_{pin} = \frac{\pi}{4} d_{pin}^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.196350 \text{ in}^2$$

$$\tau = \frac{B}{A_{pin}} = \frac{1952.56}{0.196350} = 9.94 \times 10^3 \text{ psi} \qquad \tau = 9.94 \text{ ksi} \blacktriangleleft$$

(*b*) <u>Bearing stress at *B*.</u>

$$\sigma = \frac{B}{dt} = \frac{1952.56}{\left(\frac{1}{2}\right)\left(\frac{5}{8}\right)} = 6.25 \times 10^3 \text{ psi} \qquad \sigma = 6.25 \text{ ksi} \blacktriangleleft$$

CHAPTER 2

PROBLEM 2.1

A 2.2-m-long steel rod must not stretch more than 1.2 mm when it is subjected to a 8.5-kN tension force. Knowing that E = 200 GPa, determine (a) the smallest diameter rod that should be used, (b) the corresponding normal stress in the rod.

SOLUTION

(a) Strain: $\delta = \dfrac{PL}{AE}, \quad A = \dfrac{PL}{E\delta} = \dfrac{(8.5\times 10^3\ \text{N})(2.2\ \text{m})}{(200\times 10^9\ \text{N/m}^2)(1.2\times 10^{-3}\ \text{m})} = 77.917\times 10^{-6}\ \text{m}^2$

Diameter: $A = \dfrac{\pi d^2}{4}, \quad d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(77.917\times 10^{-6}\ \text{m}^2)}{\pi}} = 9.9603\times 10^{-3}\ \text{m}$

$d = 9.96\ \text{mm}$ ◀

(b) Stress: $\sigma = \dfrac{P}{A} = \dfrac{8.5\times 10^3\ \text{N}}{77.917\times 10^{-6}\ \text{m}^2} = 109.1\times 10^6\ \text{Pa}$ $\qquad \sigma = 109.1\ \text{MPa}$ ◀

PROBLEM 2.2

A control rod made of yellow brass must not stretch more than $\frac{1}{8}$ in. when the tension in the wire is 800 lb. Knowing that $E = 15 \times 10^6$ psi and that the maximum allowable normal stress is 32 ksi, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding maximum length of the rod.

SOLUTION

(*a*) Stress: $\sigma = \dfrac{P}{A}$, or $A = \dfrac{P}{\sigma} = \dfrac{800 \text{ lb}}{32 \times 10^3 \text{ lb/in}^2} = 25 \times 10^{-3} \text{ in}^2$

Area: $A = \dfrac{\pi d^2}{4}$, or $d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{4(25 \times 10^{-3} \text{ in}^2)}{\pi}} = 0.178412 \text{ in.}$

$\delta = 0.1784 \text{ in.}$ ◀

(*b*) Strain: $\delta = \dfrac{PL}{AE}$, or $L = \dfrac{AE\delta}{P} = \dfrac{(25 \times 10^{-3} \text{ in}^2)(15 \times 10^6 \text{ lb/in}^2)(0.125 \text{ in.})}{800 \text{ lb}}$

$L = 58.6 \text{ in.}$ ◀

PROBLEM 2.3

A 9-m length of 6-mm-diameter steel wire is to be used in a hanger. It is observed that the wire stretches 18 mm when a tensile force **P** is applied. Knowing that E = 200 GPa, determine (a) the magnitude of the force **P**, (b) the corresponding normal stress in the wire.

SOLUTION

(a) Strain: $\delta = \dfrac{PL}{AE}, \quad \text{or} \quad P = \dfrac{\delta AE}{L}$

with $A = \dfrac{\pi d^2}{4} = \dfrac{\pi(0.006\text{ m})^2}{4} = 28.274 \times 10^{-6}\text{ m}^2$

$$P = \frac{(0.018\text{ m})(28.274\times10^{-6}\text{ m}^2)(200\times10^9\text{ N/m}^2)}{9\text{ m}} = 11.3096\times10^3\text{ N}$$

$P = 11.31$ kN ◀

(b) Stress: $\sigma = \dfrac{P}{A} = \dfrac{11.3096\times10^3\text{ N}}{28.274\times10^{-6}\text{ m}^2} = 400\times10^6\text{ Pa}$

$\sigma = 400$ MPa ◀

PROBLEM 2.4

A cast-iron tube is used to support a compressive load. Knowing that E = 69 GPa and that the maximum allowable change in length is 0.025%, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 7.2 kN if the outside diameter of the tube is 50 mm.

SOLUTION

(*a*)

$$\varepsilon = \frac{\delta}{L_0}$$

$$= \frac{0.00025L}{L} = 0.00025$$

$$\sigma = E\varepsilon$$

$$= (69\times 10^{9}\text{ Pa})(2.5\times 10^{-4}) = 17.25\times 10^{6}\text{ Pa}$$

$\sigma = 17.25$ MPa ◀

(*b*) $A = \dfrac{P}{\sigma} = \dfrac{7.2\times 10^{3}}{17.25\times 10^{6}} = 417.39\times 10^{-6}\text{ m}^2 = 417.39\text{ mm}^2$

$$A = \frac{\pi}{4}\left(d_o^{\,2} - d_i^{\,2}\right),\quad d_i = \sqrt{d_o^{\,2} - \frac{4A}{\pi}} = \sqrt{\left(50^2 - \frac{(4)(417.39)}{\pi}\right)} = 44.368\text{ mm}$$

$$t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(50 - 44.368)$$

t = 2.82 mm ◀

PROBLEM 2.5

An aluminum pipe must not stretch more than 0.05 in. when it is subjected to a tensile load. Knowing that $E = 10.1 \times 10^6$ psi and that the maximum allowable normal stress is 14 ksi, determine (*a*) the maximum allowable length of the pipe, (*b*) the required area of the pipe if the tensile load is 127.5 kips.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}$

Thus, $$L = \frac{EA\delta}{P} = \frac{E\delta}{\sigma} = \frac{(10.1 \times 10^6)(0.05)}{14 \times 10^3}$$

$L = 36.1$ in. ◀

(*b*) $\sigma = \dfrac{P}{A}$

Thus, $$A = \frac{P}{\sigma} = \frac{127.5 \times 10^3}{14 \times 10^3}$$

$A = 9.11\ \text{in}^2$ ◀

PROBLEM 2.6

A 60-m-long steel wire is subjected to a 6-kN tensile load. Knowing that E = 200 GPa and that the length of the rod increases by 48 mm, determine (a) the smallest diameter that can be selected for the wire, (b) the corresponding normal stress.

SOLUTION

(a) $$\delta = \frac{PL}{AE}, \quad A = \frac{PL}{E\delta} = \frac{(6 \times 10^3 \text{ N})(60 \text{ m})}{(200 \times 10^9 \text{ N/m}^2)(48 \times 10^{-3} \text{ m})} = 37.5 \times 10^{-6} \text{ m}^2$$

$$A = \frac{\pi d^2}{4}, \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(37.5 \times 10^{-6} \text{ m}^2)}{\pi}} = 6.91 \times 10^{-3} \text{ m}$$

$d = 6.91$ mm ◀

(b)

$$\sigma = \frac{P}{A} = \frac{6 \times 10^3 \text{ N}}{37.5 \times 10^{-6} \text{ m}^2} = 160 \times 10^6 \text{ Pa}$$

$\sigma = 160.0$ MPa ◀

PROBLEM 2.7

A nylon thread is subjected to a 2-lb tension force. Knowing that $E = 0.5 \times 10^6$ psi and that the maximum allowable normal stress is 6 ksi, determine (*a*) the required diameter of the thread, (*b*) the corresponding percent increase in the length of the thread.

SOLUTION

(*a*) $\sigma = \dfrac{P}{A}, \qquad A = \dfrac{P}{\sigma} = \dfrac{2 \text{ lb}}{6 \times 10^3 \text{ lb/in}^2} = 0.33333 \times 10^{-3} \text{ in}^2$

$$A = \frac{\pi d^2}{4}, \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.33333 \times 10^{-3} \text{ in}^2)}{\pi}} = 0.020601 \text{ in.}$$

$d = 0.0206$ in. ◀

(*b*) $\sigma = E\varepsilon, \qquad \varepsilon = \dfrac{\sigma}{E} = \dfrac{6 \times 10^3 \text{ psi}}{500 \times 10^3 \text{ psi}} = 0.0120$

$\varepsilon = 1.20\%$ ◀

PROBLEM 2.8

Two gage marks are placed exactly 10 in. apart on a $\frac{1}{2}$ -in.-diameter aluminum rod with E = 10.1 × 106 psi and an ultimate strength of 16 ksi. Knowing that the distance between the gage marks is 10.009 in. after a load is applied, determine (a) the stress in the rod, (b) the factor of safety.

SOLUTION

(*a*) $\delta = 10.009 - 10.000 = 0.009 \text{ in.}$

$$\varepsilon = \frac{\delta}{L} = \frac{\sigma}{E} \qquad \sigma = \frac{E\delta}{L} = \frac{(10.1 \times 10^6)(0.009)}{10} = 9.09 \times 10^3 \text{psi}$$

$\sigma = 9.09 \text{ ksi}$ ◀

(*b*) $F.S. = \frac{\sigma_U}{\sigma} = \frac{16}{9.09}$

$F.S. = 1.760$ ◀

PROBLEM 2.9

A 9-kN tensile load will be applied to a 50-m length of steel wire with E = 200 GPa. Determine the smallest diameter wire that can be used, knowing that the normal stress must not exceed 150 MPa and that the increase in length of the wire must not exceed 25 mm.

SOLUTION

Stress:

$$\sigma = \frac{P}{A}$$

$$A = \frac{P}{\sigma} = \frac{9 \times 10^3 \text{ N}}{150 \times 10^6 \text{ Pa}} = 60 \times 10^{-6} \text{m}^2$$

Deformation:

$$\delta = \frac{PL}{AE}$$

$$A = \frac{PL}{E\delta} = \frac{(9 \times 10^3)(50)}{(200 \times 10^9)(25 \times 10^{-3})} = 90 \times 10^{-6} \text{m}^2$$

The larger value of A governs: $A = 90 \text{ mm}^2$

$$A = \frac{\pi}{4} d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(90 \text{ mm}^2)}{\pi}}$$

$d = 10.70 \text{ mm}$ ◀

PROBLEM 2.10

A 1.5-m-long aluminum rod must not stretch more than 1 mm and the normal stress must not exceed 40 MPa when the rod is subjected to a 3-kN axial load. Knowing that E = 70 GPa, determine the required diameter of the rod.

SOLUTION

Stress criterion:

$$\sigma = \frac{P}{A}: \quad A = \frac{P}{\sigma} = \frac{3 \times 10^3 \text{ N}}{40 \times 10^6 \text{ Pa}} = 75.0 \times 10^{-6} \text{m}^2$$

Deformation criterion:

$$\delta = \frac{PL}{AE}:$$

$$A = \frac{PL}{E\delta} = \frac{(3 \times 10^3)(1.5)}{(70 \times 10^9)(1 \times 10^{-3})} = 64.29 \times 10^{-6} \text{m}^2$$

The larger value governs: $A = \frac{\pi}{4} d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(75 \text{ mm}^2)}{\pi}}$ $\quad d = 9.77$ mm ◀

PROBLEM 2.11

A nylon thread is to be subjected to a 2.5-lb tension. Knowing that $E = 0.5 \times 10^6$ psi, that the maximum allowable normal stress is 6 ksi, and that the length of the thread must not increase by more than 1%, determine the required diameter of the thread.

SOLUTION

Stress: $\sigma = \dfrac{P}{A}$

$$A = \frac{P}{\sigma} = \frac{2.5\ \text{lb}}{6 \times 10^3\ \text{lb/in}^2} = 416.67 \times 10^{-6}\text{in}^2$$

Deformation: $\delta = \dfrac{PL}{AE}$

$$A = \frac{P}{E}\left(\frac{L}{\delta}\right) = \frac{(2.5\ \text{lb})}{(0.5 \times 10^6\ \text{lb/in}^2)}(100) = 500 \times 10^{-6}\text{in}^2$$

The larger value of A governs:

$$A = \frac{\pi}{4}d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(500 \times 10^{-6}\ \text{in}^2)}{\pi}}$$

$d = 0.0252\,\text{in.}$ ◀

PROBLEM 2.12

A block of 250-mm length and 50 × 40-mm cross section is to support a centric compressive load **P.** The material to be used is a bronze for which E = 95 GPa. Determine the largest load that can be applied, knowing that the normal stress must not exceed 80 MPa and that the decrease in length of the block should be at most 0.12% of its original length.

SOLUTION

Considering allowable stress:

$$\sigma = \frac{P}{A}$$

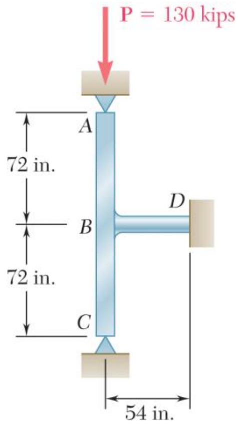
$$P = A\sigma = (0.05\text{ m})(0.04\text{ m})\left(80\times10^6\text{ N/m}^2\right) = 160.0\times10^3\text{ N}$$

Considering allowable deformation:

$$\delta = \frac{PL}{AE}:$$

$$P = AE\left(\frac{\delta}{L}\right) = (2\times10^{-3}\text{ m}^2)(95\times10^9\text{ N/m}^2)(0.0012) = 228\times10^3\text{N}$$

Smaller value governs. $P = 160.0$ kN ◀



PROBLEM 2.13

Rod *BD* is made of steel $(E = 29\times10^6 \text{ psi})$ and is used to brace the axially compressed member *ABC*. The maximum force that can be developed in member *BD* is 0.02*P*. If the stress must not exceed 18 ksi and the maximum change in length of *BD* must not exceed 0.001 times the length of *ABC*, determine the smallest-diameter rod that can be used for member *BD*.

SOLUTION

$$F_{BD} = 0.02P = (0.02)(130) = 2.6 \text{ kips} = 2.6\times10^3 \text{ lb}$$

Considering stress, $\sigma = 18 \text{ ksi} = 18\times10^3 \text{ psi}$

$$\sigma = \frac{F_{BD}}{A} \qquad \therefore \quad A = \frac{F_{BD}}{\sigma} = \frac{2.6}{18} = 0.14444 \text{ in}^2$$

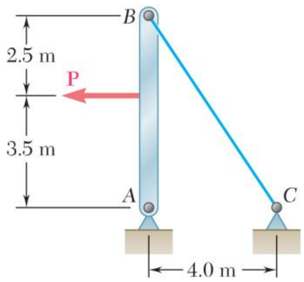
Considering deformation, $\delta = (0.001)(144) = 0.144 \text{ in.}$

$$\delta = \frac{F_{BD}L_{BD}}{AE} \qquad \therefore \quad A = \frac{F_{BD}L_{BD}}{E\delta} = \frac{(2.6\times10^3)(54)}{(29\times10^6)(0.144)} = 0.03362 \text{ in}^2$$

Larger area governs. $A = 0.14444 \text{ in}^2$

$$A = \frac{\pi}{4}d^2 \qquad \therefore \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.14444)}{\pi}}$$

$d = 0.429$ in. ◀



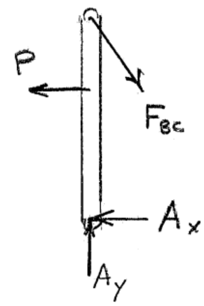
PROBLEM 2.14

The 4-mm-diameter cable *BC* is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.

SOLUTION

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

Use bar *AB* as a free body.



$$+\circlearrowleft \Sigma M_A = 0: \quad 3.5P - (6)\left(\frac{4}{7.2111}F_{BC}\right) = 0$$

$$P = 0.9509 F_{BC}$$

Considering allowable stress, $\sigma = 190 \times 10^6$ Pa

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_{BC}}{A} \quad \therefore \quad F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$$

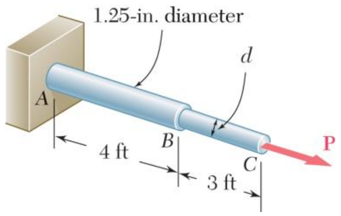
Considering allowable elongation, $\delta = 6 \times 10^{-3}$ m

$$\delta = \frac{F_{BC}L_{BC}}{AE} \quad \therefore \quad F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$$

Smaller value governs. $F_{BC} = 2.091 \times 10^3$ N

$$P = 0.9509 F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N}$$

$P = 1.988$ kN ◀



PROBLEM 2.15

A single axial load of magnitude $P = 15$ kips is applied at end C of the steel rod ABC. Knowing that $E = 30 \times 10^6$ psi, determine the diameter d of portion BC for which the deflection of point C will be 0.05 in.

SOLUTION

$$\delta_C = \sum \frac{PL_i}{A_i E_i} = \left(\frac{PL}{AE}\right)_{AB} + \left(\frac{PL}{AE}\right)_{BC}$$

$$L_{AB} = 4 \text{ ft} = 48 \text{ in.}; \qquad L_{BC} = 3 \text{ ft} = 36 \text{ in.}$$

$$A_{AB} = \frac{\pi d^2}{4} = \frac{\pi(1.25 \text{ in.})^2}{4} = 1.22718 \text{ in}^2$$

Substituting, we have

$$0.05 \text{ in.} = \left(\frac{15 \times 10^3 \text{ lb}}{30 \times 10^6 \text{psi}}\right)\left(\frac{48 \text{ in.}}{1.22718 \text{ in}^2} + \frac{36 \text{ in.}}{A_{BC}}\right)$$

$$A_{BC} = 0.59127 \text{ in}^2$$

$$A_{BC} = \frac{\pi d^2}{4}$$

or $$d = \sqrt{\frac{4A_{BC}}{\pi}}$$

$$d = \sqrt{\frac{4(0.59127 \text{ in}^2)}{\pi}}$$

$$d = 0.86766 \text{ in.}$$

$d = 0.868$ in. ◀

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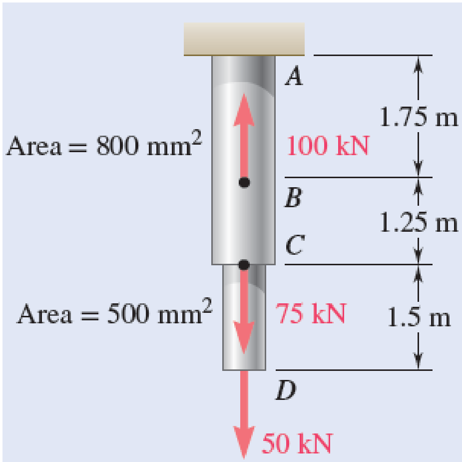


PROBLEM 2.16

The rod *ABCD* is made of an aluminum for which E = 70 GPa. For the loading shown, determine the deflection of (*a*) point *B*, (*b*) point *D*.

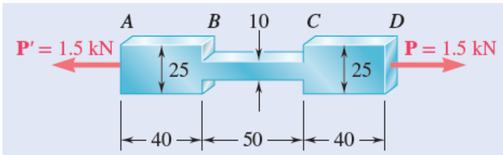
SOLUTION

Portion of rod	P_i	L_i	A_i	$\frac{P_i L_i}{A_i E}$
AB	$+25\times10^3$ N	1.75 m	800×10^{-6} m^2	0.78125 mm
BC	$+125\times10^3$ N	1.25 m	800×10^{-6} m^2	2.7902 mm
CD	$+50\times10^3$ N	1.5 m	500×10^{-6} m^2	2.1429 mm

(*a*) $\delta_B = \delta_{B/A}$ 0.781 mm ↓ ◀

(*b*) $\delta_B = \delta_{B/A} + \delta_{C/B} + \delta_{D/C}$

$\delta_D = 0.78125 + 2.7902 + 2.1429 = 5.7143\text{mm}$ $\delta_D = 5.71$ mm ↓ ◀



PROBLEM 2.17

The specimen shown has been cut from a 5-mm-thick sheet of vinyl (E = 3.10 GPa) and is subjected to a 1.5-kN tensile load. Determine (a) the total deformation of the specimen, (b) the deformation of its central portion BC.

SOLUTION

$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(1.5\times 10^3\ \text{N})(40\times 10^{-3}\ \text{m})}{(3.1\times 10^9\ \text{N/m}^2)(25\times 10^{-3}\ \text{m})(5\times 10^{-3}\ \text{m})} = 154.839\times 10^{-6}\ \text{m}$$

$$\delta_{BC} = \frac{PL_{BC}}{EA_{BC}} = \frac{(1.5\times 10^3\ \text{N})(50\times 10^{-3}\ \text{m})}{(3.1\times 10^9\ \text{N/m}^2)(10\times 10^{-3}\ \text{m})(5\times 10^{-3}\ \text{m})} = 483.87\times 10^{-6}\ \text{m}$$

$$\delta_{CD} = \delta_{AB} = 154.839\times 10^{-6}\ \text{m}$$

(a) Total deformation:

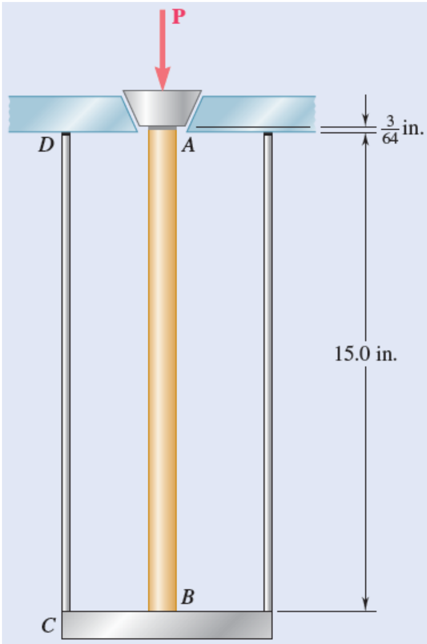
$$\delta = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

$$\delta = 793.55\times 10^{-6}\ \text{m}$$

$$\delta = 0.794\ \text{mm} \blacktriangleleft$$

(b) Deformation of portion BC:

$$\delta_{BC} = 0.484\ \text{mm} \blacktriangleleft$$



PROBLEM 2.18

The brass tube *AB* ($E = 15 \times 10^6$ psi) has a cross-sectional area of 0.22 in^2 and is fitted with a plug at *A*. The tube is attached at *B* to a rigid plate that is itself attached at *C* to the bottom of an aluminum cylinder ($E = 10.4 \times 10^6$ psi) with a cross-sectional area of 0.40 in^2. The cylinder is then hung from a support at *D*. In order to close the cylinder, the plug must move down through $\frac{3}{64}$ in. Determine the force **P** that must be applied to the cylinder.

SOLUTION

Shortening of brass tube *AB*:

$$L_{AB} = 15 + \frac{3}{64} = 15.0469 \text{ in.} \quad A_{AB} = 0.22 \text{ in}^2$$

$$E_{AB} = 15 \times 10^6 \text{ psi}$$

$$\delta_{AB} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{P(15.0469)}{(15 \times 10^6)(0.22)} = 4.5597 \times 10^{-6} P$$

Lengthening of aluminum cylinder *CD*:

$$L_{CD} = 15 \text{ in.} \quad A_{CD} = 0.40 \text{ in}^2 \quad E_{CD} = 10.4 \times 10^6 \text{ psi}$$

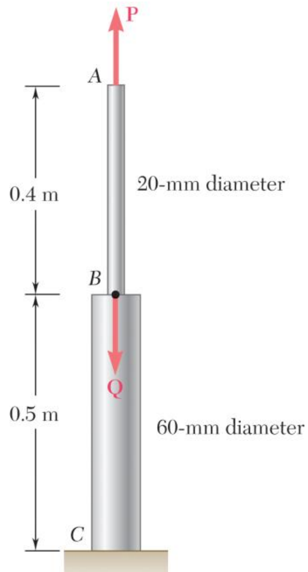
$$\delta_{CD} = \frac{PL_{CD}}{E_{CD}A_{CD}} = \frac{P(15)}{(10.4 \times 10^6)(0.40)} = 3.6058 \times 10^{-6} P$$

Total deflection: $\delta_A = \delta_{AB} + \delta_{CD}$ where $\delta_A = \frac{3}{64}$ in.

$$\frac{3}{64} = (4.5597 \times 10^{-6} + 3.6058 \times 10^{-6})P$$

$$P = 5,740.6 \text{ lb}$$

$P = 5.74$ kips ◀



PROBLEM 2.19

Both portions of the rod ABC are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at A is zero, (*b*) the corresponding deflection of B.

SOLUTION

(*a*) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\,\text{m}^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\,\text{m}^2$

Force in member AB is P tension.

<u>Elongation</u>:

$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(4 \times 10^3)(0.4)}{(70 \times 10^9)(314.16 \times 10^{-6})} = 72.756 \times 10^{-6}\,\text{m}$$

Force in member BC is $Q - P$ compression.

<u>Shortening</u>:

$$\delta_{BC} = \frac{(Q-P)L_{BC}}{EA_{BC}} = \frac{(Q-P)(0.5)}{(70 \times 10^9)(2.8274 \times 10^{-3})} = 2.5263 \times 10^{-9}(Q-P)$$

For zero deflection at A, $\delta_{BC} = \delta_{AB}$

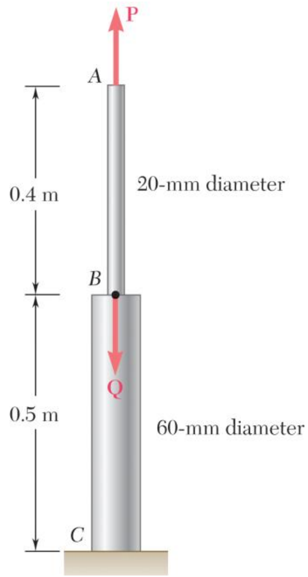
$$2.5263 \times 10^{-9}(Q-P) = 72.756 \times 10^{-6} \quad \therefore \quad Q - P = 28.8 \times 10^3\,\text{N}$$

$$Q = 28.3 \times 10^3 + 4 \times 10^3 = 32.8 \times 10^3\,\text{N}$$

$Q = 32.8$ kN ◀

(*b*) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756 \times 10^{-6}\,\text{m}$

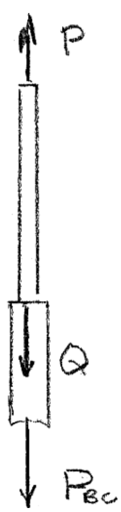
$\delta_{AB} = 0.0728$ mm ↓ ◀



PROBLEM 2.20

The rod *ABC* is made of an aluminum for which $E = 70$ GPa. Knowing that $\mathbf{P} = 6$ kN and $\mathbf{Q} = 42$ kN, determine the deflection of (*a*) point *A*, (*b*) point *B*.

SOLUTION



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\,\text{m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\,\text{m}^2$$

$$P_{AB} = P = 6 \times 10^3\,\text{N}$$

$$P_{BC} = P - Q = 6 \times 10^3 - 42 \times 10^3 = -36 \times 10^3\,\text{N}$$

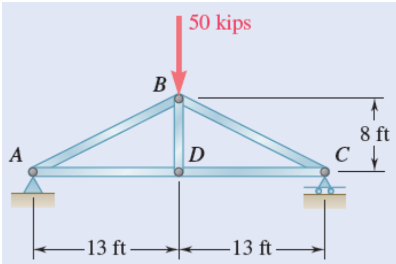
$$L_{AB} = 0.4\text{ m} \quad L_{BC} = 0.5\text{ m}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E_A} = \frac{(6 \times 10^3)(0.4)}{(314.16 \times 10^{-6})(70 \times 10^9)} = 109.135 \times 10^{-6}\,\text{m}$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36 \times 10^3)(0.5)}{(2.8274 \times 10^{-3})(70 \times 10^9)} = -90.947 \times 10^{-6}\,\text{m}$$

(*a*) $\delta_A = \delta_{AB} + \delta_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6}\,\text{m} = 18.19 \times 10^{-6}\,\text{m}$ $\qquad \delta_A = 0.01819\text{ mm} \uparrow$ ◀

(*b*) $\delta_B = \delta_{BC} = -90.9 \times 10^{-6}\,\text{m} = -0.0909\text{ mm}$ or $\delta_B = 0.0909\text{ mm} \downarrow$ ◀



PROBLEM 2.21

For the steel truss ($E = 29 \times 10^6$ psi) and loading shown, determine the deformations of members *AB* and *AD*, knowing that their cross-sectional areas are 4.0 in^2 and 2.8 in^2, respectively.

SOLUTION

Statics: Reactions are 25 kips upward at *A* and *C*.

Member *BD* is a zero force member.

$$L_{AB} = \sqrt{13^2 + 8^2} = 15.2643 \text{ ft} = 183.172 \text{ in.}$$

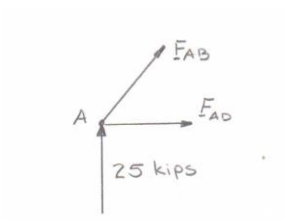
$$L_{AD} = 13 \text{ ft} = 156 \text{ in.}$$

Use joint *A* as a free body.

$$+\uparrow \Sigma F_y = 0: \ 25 + \frac{8}{15.2643} F_{AB} = 0$$

$$F_{AB} = -47.701 \text{ kips}$$

$$\xrightarrow{+} \Sigma F_x = 0: \ F_{AD} + \frac{13}{15.2643} F_{AB} = 0$$



$$F_{AD} = -\frac{(13)(-47.701)}{15.2643} = 40.625 \text{ kips}$$

Member *AB*:

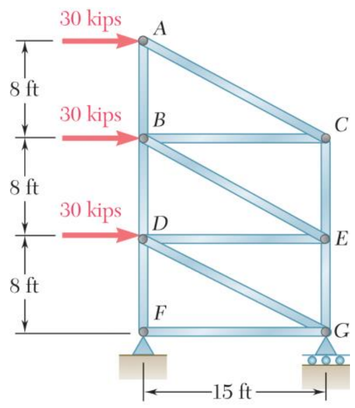
$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(-47.701 \times 10^3)(183.172)}{(29 \times 10^6)(4.0)}$$

$$\delta_{AB} = -0.0753 \text{ in.} \blacktriangleleft$$

Member *AD*:

$$\delta_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}} = \frac{(40.625 \times 10^3)(156)}{(29 \times 10^6)(2.8)}$$

$$\delta_{AD} = 0.0780 \text{ in.} \blacktriangleleft$$

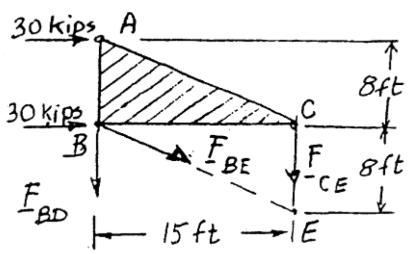


PROBLEM 2.22

For the steel truss $(E = 29\times10^6\ \text{psi})$ and loading shown, determine the deformations of the members *BD* and *DE*, knowing that their cross-sectional areas are 2 in^2 and 3 in^2, respectively.

SOLUTION

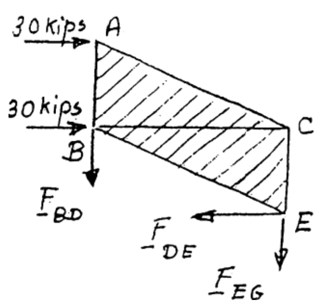
Free body: Portion *ABC* of truss



$$+\circlearrowleft \Sigma M_E = 0:\ F_{BD}(15\ \text{ft}) - (30\ \text{kips})(8\ \text{ft}) - (30\ \text{kips})(16\ \text{ft}) = 0$$

$$F_{BD} = +48.0\ \text{kips}$$

Free body: Portion *ABEC* of truss

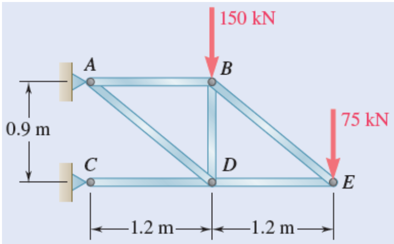


$$\xrightarrow{+} \Sigma F_x = 0:\ 30\ \text{kips} + 30\ \text{kips} - F_{DE} = 0$$

$$F_{DE} = +60.0\ \text{kips}$$

$$\delta_{BD} = \frac{PL}{AE} = \frac{(+48.0\times10^3\ \text{lb})(8\times12\ \text{in.})}{(2\ \text{in}^2)(29\times10^6\ \text{psi})} \qquad \delta_{BD} = +79.4\times10^{-3}\ \text{in.} \blacktriangleleft$$

$$\delta_{DE} = \frac{PL}{AE} = \frac{(+60.0\times10^3\ \text{lb})(15\times12\ \text{in.})}{(3\ \text{in}^2)(29\times10^6\ \text{psi})} \qquad \delta_{DE} = +124.1\times10^{-3}\ \text{in.} \blacktriangleleft$$



PROBLEM 2.23

Members *AB* and *BE* of the truss shown consist of 25-mm-diameter steel rods (E = 200 GPa). For the loading shown, determine the elongation of (*a*) rod *AB*, (*b*) rod *BE*.

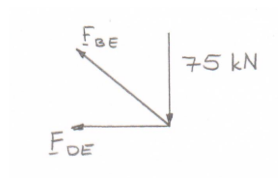
SOLUTION

$$A_{AB} = A_{BE} = \frac{\pi d^2}{4} = \frac{\pi(0.025\text{ m})^2}{4} = 490.87 \times 10^{-6}\text{ m}^2 \qquad L_{BE} = \sqrt{1.2^2 + 0.9^2} = 1.5\text{m}$$

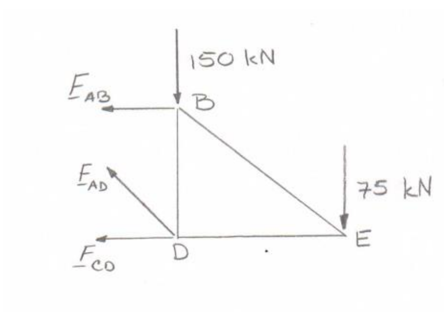
Use joint *E* as a free body.

$$+\uparrow \Sigma F_y = 0: \quad \frac{0.9}{1.5}F_{BE} - 75\text{ kN} = 0$$

$$F_{BE} = 125.0\text{ kN}$$



Use triangle *BDE* as a free body.



$$+\circlearrowleft \Sigma M_D = 0: \; 0.9F_{AB} - (1.2)(75) = 0$$

$$F_{AB} = 100.0\text{ kN}$$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{EA_{AB}}$$

$$= \frac{(100 \times 10^3)(1.2)}{(200 \times 10^9)(490.87 \times 10^{-6})}$$

$$= 1.22232 \times 10^{-6}\text{ m}$$

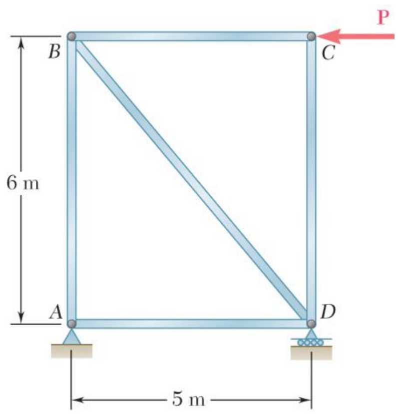
$$\delta_{BE} = \frac{F_{BE}L_{BE}}{EA_{BE}}$$

$$= \frac{(125 \times 10^3)(1.5)}{(200 \times 10^9)(490.87 \times 10^{-6})}$$

$$= 1.90987 \times 10^{-6}\text{ m}$$

(*a*) $\delta_{AB} = 1.222\text{ mm}$ ◀

(*b*) $\delta_{BE} = 1.910\text{ mm}$ ◀



PROBLEM 2.24

The steel frame $(E = 200 \text{ GPa})$ shown has a diagonal brace *BD* with an area of 1920 mm^2. Determine the largest allowable load **P** if the change in length of member *BD* is not to exceed 1.6 mm.

SOLUTION

$$\delta_{BD} = 1.6 \times 10^{-3} \text{ m}, \quad A_{BD} = 1920 \text{ mm}^2 = 1920 \times 10^{-6} \text{ m}^2$$

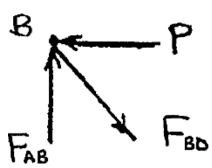
$$L_{BD} = \sqrt{5^2 + 6^2} = 7.810 \text{ m}, \quad E_{BD} = 200 \times 10^9 \text{ Pa}$$

$$\delta_{BD} = \frac{F_{BD} L_{BD}}{E_{BD} A_{BD}}$$

$$F_{BD} = \frac{E_{BD} A_{BD} \delta_{BD}}{L_{BD}} = \frac{(200 \times 10^9)(1920 \times 10^{-6})(1.6 \times 10^{-3})}{7.81}$$

$$= 78.67 \times 10^3 \text{ N}$$

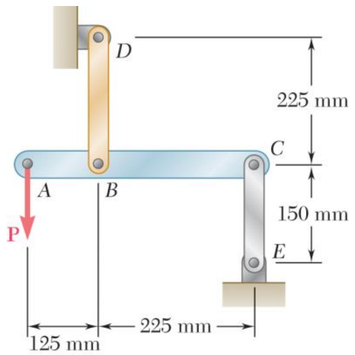
Use joint *B* as a free body. $\xrightarrow{+} \Sigma F_x = 0$:



$$\frac{5}{7.810} F_{BD} - P = 0$$

$$P = \frac{5}{7.810} F_{BD} = \frac{(5)(78.67 \times 10^3)}{7.810}$$

$$= 50.4 \times 10^3 \text{ N} \qquad P = 50.4 \text{ kN} \blacktriangleleft$$

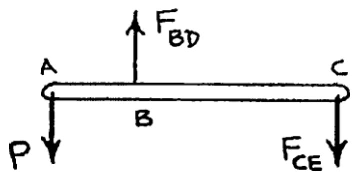


PROBLEM 2.25

Link *BD* is made of brass $(E = 105 \text{ GPa})$ and has a cross-sectional area of 240 mm^2. Link *CE* is made of aluminum $(E = 72 \text{ GPa})$ and has a cross-sectional area of 300 mm^2. Knowing that they support rigid member *ABC*, determine the maximum force **P** that can be applied vertically at point *A* if the deflection of *A* is not to exceed 0.35 mm.

SOLUTION

Free body member *AC*:



$$+\circlearrowleft \Sigma M_C = 0: \ 0.350P - 0.225F_{BD} = 0$$

$$F_{BD} = 1.55556P$$

$$+\circlearrowleft \Sigma M_B = 0: \ 0.125P - 0.225F_{CE} = 0$$

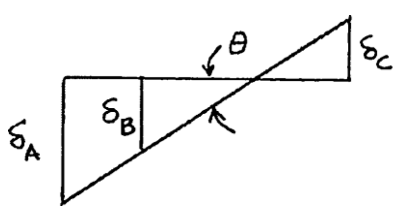
$$F_{CE} = 0.55556P$$

$$\delta_B = \delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}} = \frac{(1.55556P)(0.225)}{(105\times 10^9)(240\times 10^{-6})} = 13.8889\times 10^{-9}P$$

$$\delta_C = \delta_{CE} = \frac{F_{CE}L_{CE}}{E_{CE}A_{CE}} = \frac{(0.55556P)(0.150)}{(72\times 10^9)(300\times 10^{-6})} = 3.8581\times 10^{-9}P$$

<u>Deformation Diagram</u>:

From the deformation diagram,



Slope: $$\theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{17.7470\times 10^{-9}P}{0.225} = 78.876\times 10^{-9}P$$

$$\delta_A = \delta_B + L_{AB}\theta$$

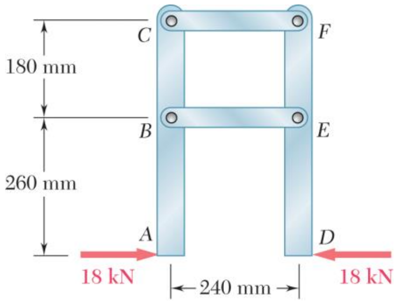
$$= 13.8889\times 10^{-9}P + (0.125)(78.876\times 10^{-9}P)$$

$$= 23.748\times 10^{-9}P$$

Apply displacement limit. $\delta_A = 0.35\times 10^{-3} \text{ m} = 23.748\times 10^{-9}P$

$$P = 14.7381\times 10^3 \text{ N}$$

$P = 14.74$ kN ◀

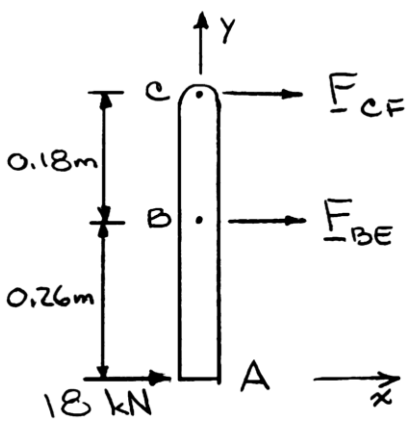


PROBLEM 2.26

Members *ABC* and *DEF* are joined with steel links (E = 200 GPa). Each of the links is made of a pair of 25 × 35-mm plates. Determine the change in length of (*a*) member *BE*, (*b*) member *CF*.

SOLUTION

Free body diagram of Member *ABC*:



$+\circlearrowleft \Sigma M_B = 0:$

$$(0.26 \text{ m})(18 \text{ kN}) - (0.18 \text{ m})F_{CF} = 0$$

$$F_{CF} = 26.0 \text{ kN}$$

$\xrightarrow{+} \Sigma F_x = 0:$

$$18 \text{ kN} + F_{BE} + 26.0 \text{ kN} = 0$$

$$F_{BE} = -44.0 \text{ kN}$$

Area for link made of two plates:

$$A = 2(0.025 \text{ m})(0.035 \text{ m}) = 1.750 \times 10^{-3} \text{ m}^2$$

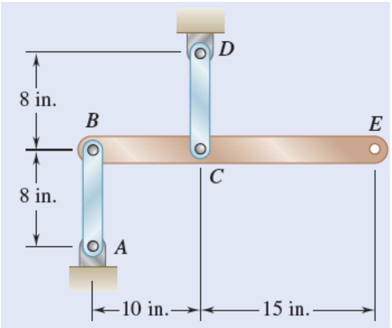
(*a*) $$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-44.0 \times 10^3 \text{ N})(0.240 \text{ m})}{(200 \times 10^9 \text{ Pa})(1.75 \times 10^{-3} \text{ m}^2)} = -30.171 \times 10^{-6} \text{ m}$$

$$\delta_{BE} = -0.0302 \text{ mm} \blacktriangleleft$$

(*b*) $$\delta_{CF} = \frac{F_{BF}L}{EA} = \frac{(26.0 \times 10^3 \text{ N})(0.240 \text{ m})}{(200 \times 10^9 \text{ Pa})(1.75 \times 10^{-3} \text{ m}^2)} = 17.8286 \times 10^{-6} \text{ m}$$

$$\delta_{CF} = 0.01783 \text{ mm} \blacktriangleleft$$

CLICK HERE TO ACCESS THE COMPLETE Solutions

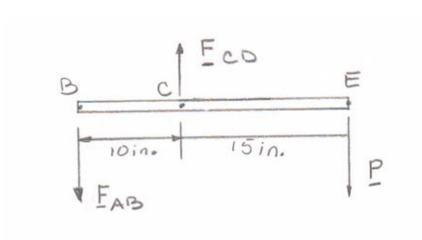


PROBLEM 2.27

Each of the links *AB* and *CD* is made of steel ($E = 29 \times 10^6$ psi) and has a uniform rectangular cross section of $\frac{1}{4} \times 1$ in. Knowing that they support rigid member *BCE*, determine the largest load that can be suspended from point *E* if the deflection of *E* is not to exceed 0.01 in.

SOLUTION

Free body *BCE*:



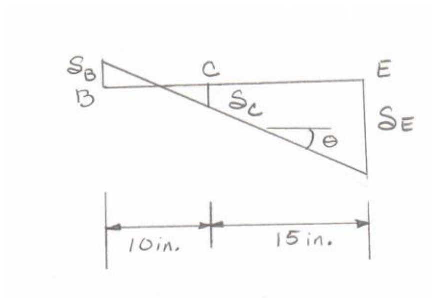
$$+\circlearrowleft \Sigma M_C = 0: \ 10F_{AB} - 15P = 0$$
$$F_{AB} = 1.5P$$

$$+\circlearrowleft \Sigma M_B = 0: \ 10F_{CD} - 25P = 0$$
$$F_{CD} = 2.5P$$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{EA} = \frac{1.5P(8)}{(29\times10^6)(0.25)(1)} = 1.65517\times10^{-6}\,P = \delta_B$$

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA} = \frac{2.5P(8)}{(29\times10^6)(0.25)(1)} = 2.7586\times10^{-6}\,P = \delta_C$$

Deformation diagram:



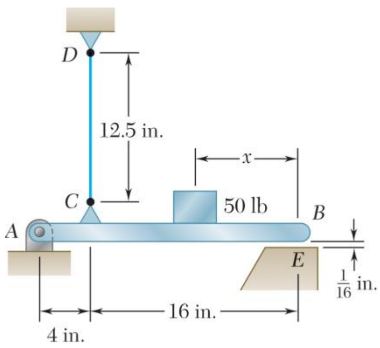
$$\text{Slope } \theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{P\left(1.65517\times10^{-6} + 2.7586\times10^{-6}\right)}{10}$$
$$= 0.44138\times10^{-6}\,P$$

$$\delta_E = \delta_C + L_{CE}\theta$$
$$= 2.7586\times10^{-6}\,P + (15)(0.44138\times10^{-6}\,P)$$
$$= 9.3793\times10^{-6}\,P$$

Limiting the value of $\delta_E = 0.01$ in.

$$0.01 = 9.3793\times10^{-6}\,P$$

$P = 1.066\,\text{kips}$ ◀



PROBLEM 2.28

The length of the $\frac{3}{32}$-in.-diameter steel wire *CD* has been adjusted so that with no load applied, a gap of $\frac{1}{16}$ in. exists between the end *B* of the rigid beam *ACB* and a contact point *E*. Knowing that $E = 29 \times 10^6$ psi, determine where a 50-lb block should be placed on the beam in order to cause contact between *B* and *E*.

SOLUTION

Rigid beam *ACB* rotates through angle θ to close gap.

$$\theta = \frac{1/16}{20} = 3.125 \times 10^{-3} \text{ rad}$$

Point *C* moves downward.

$$\delta_C = 4\theta = 4(3.125 \times 10^{-3}) = 12.5 \times 10^{-3} \text{ in.}$$

$$\delta_{CD} = \delta_C = 12.5 \times 10^{-3} \text{ in.}$$

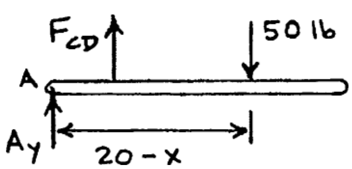
$$A_{CD} = \frac{\pi}{d} d^2 = \frac{\pi}{4}\left(\frac{3}{32}\right)^2 = 6.9029 \times 10^{-3} \text{ in}^2$$

$$\delta_{CD} = \frac{F_{CD} L_{CD}}{E A_{CD}}$$

$$F_{CD} = \frac{E A_{CD} \delta_{CD}}{L_{CD}} = \frac{(29 \times 10^6)(6.9029 \times 10^{-3})(12.5 \times 10^{-3})}{12.5}$$

$$= 200.18 \text{ lb}$$

Free body *ACB*:



$$+\circlearrowleft \Sigma M_A = 0: \quad 4F_{CD} - (50)(20 - x) = 0$$

$$20 - x = \frac{(4)(200.18)}{50} = 16.0144$$

$$x = 3.9856 \text{ in.}$$

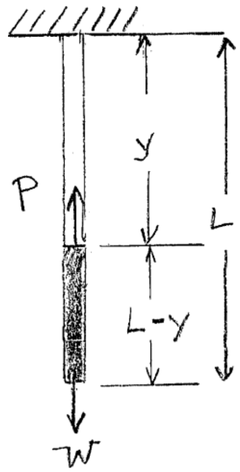
For contact, $x < 3.99$ in. ◀

PROBLEM 2.29

A homogeneous cable of length L and uniform cross section is suspended from one end. (*a*) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (*b*) Show that the same elongation would be obtained if the cable were horizontal and if a force equal to half of its weight were applied at each end.

SOLUTION

(*a*) For element at point identified by coordinate y,



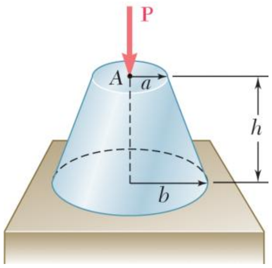
$$P = \text{weight of portion below the point}$$
$$= \rho g A(L-y)$$
$$d\delta = \frac{P\,dy}{EA} = \frac{\rho g A(L-y)dy}{EA} = \frac{\rho g(L-y)}{E}dy$$
$$\delta = \int_0^L \frac{\rho g(L-y)}{E}dy = \frac{\rho g}{E}\left(Ly - \frac{1}{2}y^2\right)\Bigg|_0^L$$
$$= \frac{\rho g}{E}\left(L^2 - \frac{L^2}{2}\right)$$

$$\delta = \frac{1}{2}\frac{\rho g L^2}{E} \blacktriangleleft$$

(*b*) Total weight: $W = \rho g A L$

$$F = \frac{EA\delta}{L} = \frac{EA}{L}\cdot\frac{1}{2}\frac{\rho g L^2}{E} = \frac{1}{2}\rho g A L$$

$$F = \frac{1}{2}W \blacktriangleleft$$

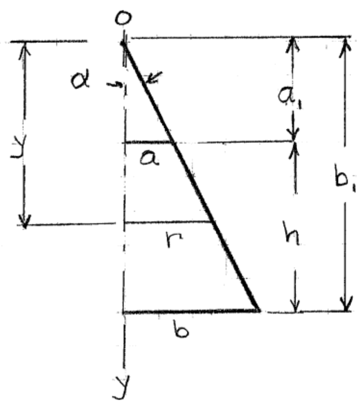


PROBLEM 2.30

A vertical load **P** is applied at the center A of the upper section of a homogeneous frustum of a circular cone of height h, minimum radius a, and maximum radius b. Denoting by E the modulus of elasticity of the material and neglecting the effect of its weight, determine the deflection of point A.

SOLUTION

Extend the slant sides of the cone to meet at a point O and place the origin of the coordinate system there.



From geometry, $\tan\alpha = \dfrac{b-a}{h}$

$$a_1 = \frac{a}{\tan\alpha}, \quad b_1 = \frac{b}{\tan\alpha}, \quad r = y\tan\alpha$$

At coordinate point y, $A = \pi r^2$

Deformation of element of height dy: $d\delta = \dfrac{P\,dy}{AE}$

$$d\delta = \frac{P}{E\pi}\frac{dy}{r^2} = \frac{P}{\pi E\tan^2\alpha}\frac{dy}{y^2}$$

Total deformation:

$$\delta_A = \frac{P}{\pi E\tan^2\alpha}\int_{a_1}^{b_1}\frac{dy}{y^2} = \frac{P}{\pi E\tan^2\alpha}\left(-\frac{1}{y}\right)\Bigg|_{a_1}^{b_1} = \frac{P}{\pi E\tan^2\alpha}\left(\frac{1}{a_1}-\frac{1}{b_1}\right)$$

$$= \frac{P}{\pi E\tan^2\alpha}\frac{b_1-a_1}{a_1 b_1} = \frac{P(b_1-a_1)}{\pi Eab}$$

$$\delta_A = \frac{Ph}{\pi Eab}\downarrow \blacktriangleleft$$

PROBLEM 2.31

Denoting by ε the "engineering strain" in a tensile specimen, show that the true strain is $\varepsilon_t = \ln(1+\varepsilon)$.

SOLUTION

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\frac{L_0+\delta}{L_0} = \ln\left(1+\frac{\delta}{L_0}\right) = \ln(1+\varepsilon)$$

Thus, $\varepsilon_t = \ln(1+\varepsilon)$ ◀

PROBLEM 2.32

The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is d_1, show that when the diameter is d, the true strain is $\varepsilon_t = 2\ln(d_1/d)$.

SOLUTION

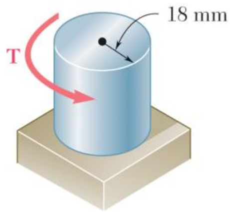
If the volume is constant, $\frac{\pi}{4}d^2 L = \frac{\pi}{4}d_1^2 L_0$

$$\frac{L}{L_0} = \frac{d_1^2}{d^2} = \left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = 2\ln\frac{d_1}{d} \blacktriangleleft$$

CHAPTER 3



PROBLEM 3.1

Determine the torque **T** that causes a maximum shearing stress of 70 MPa in the steel cylindrical shaft shown.

SOLUTION

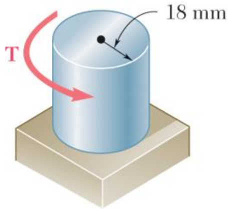
$$\tau_{max} = \frac{Tc}{J}; \quad J = \frac{\pi}{2}c^4$$

$$T = \frac{\pi}{2}c^3\tau_{max}$$

$$= \frac{\pi}{2}(0.018 \text{ m})^3(70 \times 10^6 \text{ Pa})$$

$$= 641.26 \text{ N} \cdot \text{m}$$

$$T = 641 \text{ N} \cdot \text{m} \blacktriangleleft$$



PROBLEM 3.2

For the cylindrical shaft shown, determine the maximum shearing stress caused by a torque of magnitude $T = 800 \text{ N} \cdot \text{m}$.

SOLUTION

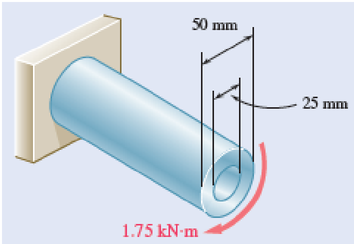
$$\tau_{\max} = \frac{Tc}{J}; \quad J = \frac{\pi}{2}c^4$$

$$\tau_{\max} = \frac{2T}{\pi c^3}$$

$$= \frac{2(800 \text{ N} \cdot \text{m})}{\pi(0.018 \text{ m})^3}$$

$$= 87.328 \times 10^6 \text{ Pa}$$

$$\tau_{\max} = 87.3 \text{ MPa} \blacktriangleleft$$



PROBLEM 3.3

A 1.75-kN · m torque is applied to the solid cylinder shown. Determine (*a*) the maximum shearing stress, (*b*) the percent of the torque carried by the inner 25-mm-diameter core.

SOLUTION

(*a*) Given shaft:

$$\tau_{max} = \frac{Tc}{J}$$

$$\tau_{max} = \frac{(1.75 \times 10^3 \text{ N} \cdot \text{m})(0.025 \text{ m})}{\frac{\pi}{2}(0.025 \text{ m})^4} = 71.301 \times 10^6 \text{ Pa}$$

$\tau_{max} = 71.3 \text{ MPa}$ ◀

(*b*) At surface of core:

$$\tau = \frac{1}{2}\tau_m$$

$$= \frac{1}{2}(71.301 \times 10^6)$$

$$= 35.651 \times 10^6 \text{ Pa}$$

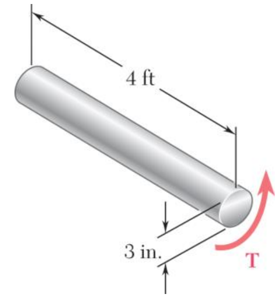
Corresponding torque:

$$T = \frac{\tau J}{c} = \tau\left(\frac{\pi/2\, c^4}{c}\right) = \tau\left(\frac{\pi}{2}c^3\right) = (35.651\times10^6 \text{ Pa})\left(\frac{\pi}{2}\right)(0.0125 \text{ m})^3$$

$$T = 109.376 \text{ N} \cdot \text{m}$$

$$\%T = \frac{109.376 \text{ N} \cdot \text{m}}{1750 \text{ N} \cdot \text{m}}(100)$$

6.25% ◀



PROBLEM 3.4

(*a*) Determine the maximum shearing stress caused by a 40-kip · in. torque **T** in the 3-in.-diameter solid aluminum shaft shown. (*b*) Solve part *a*, assuming that the solid shaft has been replaced by a hollow shaft of the same outer diameter and of 1-in. inner diameter.

SOLUTION

(*a*)

$$\tau = \frac{Tc}{J}$$

$$= \frac{(40\text{ kip}\cdot\text{in.})(1.5\text{ in.})}{\left(\frac{\pi}{2}\right)(1.5\text{ in.})^4}$$

$$= 7.5451\text{ ksi}$$

$$\tau = 7.55\text{ ksi} \blacktriangleleft$$

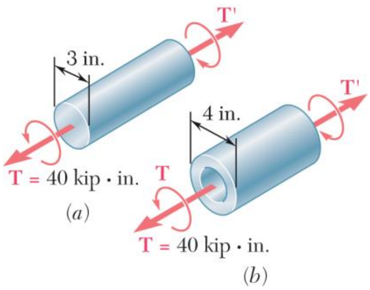
(*b*)

$$\tau = \frac{Tc}{J}$$

$$= \frac{(40\text{ kip}\cdot\text{in.})(1.5\text{ in.})}{\frac{\pi}{2}[(1.5\text{ in.})^4 - (0.5\text{ in.})^4]}$$

$$= 7.6394\text{ ksi}$$

$$\tau = 7.64\text{ ksi} \blacktriangleleft$$



PROBLEM 3.5

(*a*) For the 3-in.-diameter solid cylinder and loading shown, determine the maximum shearing stress. (*b*) Determine the inner diameter of the 4-in.-diameter hollow cylinder shown, for which the maximum stress is the same as in part *a*.

SOLUTION

(*a*) Solid shaft:

$$c = \frac{1}{2}d = \frac{1}{2}(3.0\text{ in.}) = 1.5\text{ in.}$$

$$J = \frac{\pi}{2}c^4$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(40\text{ kip}\cdot\text{in.})}{\pi(1.5\text{ in.})^3} = 7.5451\text{ ksi}$$

$$\tau_{\max} = 7.55\text{ ksi} \blacktriangleleft$$

(*b*) Hollow shaft:

$$c_o = \frac{1}{2}d = \frac{1}{2}(4.0\text{ in.}) = 2.0\text{ in.}$$

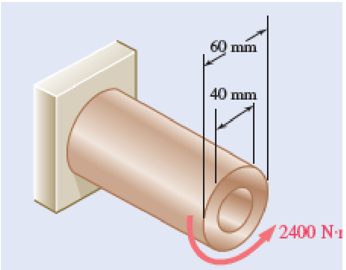
$$\frac{J}{c_o} = \frac{\frac{\pi}{2}\left(c_o^4 - c_i^4\right)}{c_o} = \frac{T}{\tau_{\max}}$$

$$c_i^4 = c_o^4 - \frac{2Tc_o}{\pi\tau_{\max}} = (2.0\text{ in.})^4 - \frac{2(40\text{ kip}\cdot\text{in.})(2.0\text{ in.})}{\pi(7.5451\text{ ksi})}$$

$$= 9.2500\text{ in}^4 \quad \therefore \quad c_i = 1.74395\text{ in.}$$

and $d_i = 2c_i = 3.4879\text{ in.}$

$$d_i = 3.49\text{ in.} \blacktriangleleft$$



PROBLEM 3.6

(*a*) For the hollow shaft and loading shown, determine the maximum shearing stress. (*b*) Determine the diameter of a solid shaft for which the maximum shearing stress under the loading shown is the same as in part a.

SOLUTION

(*a*)

$$c_1 = \frac{1}{2}d_1 = \frac{1}{2}(0.040 \text{ m}) = 0.020 \text{ m}$$

$$c_2 = \frac{1}{2}d_2 = \frac{1}{2}(0.060 \text{ m}) = 0.030 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.030^4 - 0.020^4) = 1.02102 \times 10^{-6} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{(2400 \text{ N} \cdot \text{m})(0.030 \text{ m})}{1.02102 \times 10^{-6} \text{ m}^4} = 70.518 \times 10^6 \text{ Pa}$$

$\tau_{\max} = 70.5 \text{ MPa}$ ◀

(*b*)

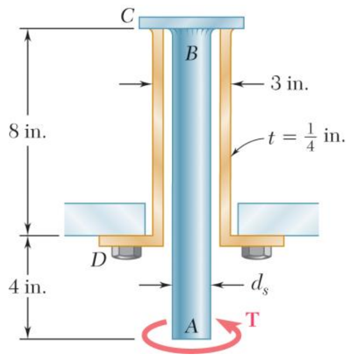
$$J = \frac{\pi}{2}c_3^4, \qquad \tau = \frac{Tc_3}{J} = \left(\frac{2}{\pi}\right)\frac{Tc_3}{c_3^4} = \frac{2T}{\pi c_3^3}$$

$$c^3 = \frac{2T}{\tau\pi} = \frac{2(2400)}{\pi\left(70.518 \times 10^6\right)} = 21.667 \times 10^{-6} \text{ m}^3$$

$$c_3 = 27.878 \times 10^{-3} \text{ m}$$

$$d_3 = 2c_3 = 55.8 \times 10^{-3} \text{ m}$$

$d_3 = 55.8 \text{ mm}$ ◀



PROBLEM 3.7

The solid spindle *AB* is made of a steel with an allowable shearing stress of 12 ksi, and sleeve *CD* is made of a brass with an allowable shearing stress of 7 ksi. Determine (*a*) the largest torque **T** that can be applied at *A* if the allowable shearing stress is not to be exceeded in sleeve CD, (*b*) the corresponding required value of the diameter d_s of spindle *AB*.

SOLUTION

(*a*) Analysis of sleeve *CD*:

$$c_2 = \frac{1}{2}d_o = \frac{1}{2}(3) = 1.5 \text{ in.}$$

$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7\times 10^3)}{1.5} = 19.21\times 10^3 \text{ lb}\cdot\text{in.}$$

$$T = 19.21 \text{ kip}\cdot\text{in.} \blacktriangleleft$$

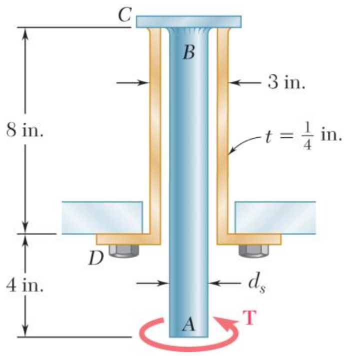
(*b*) Analysis of solid spindle *AB*:

$$\tau = \frac{Tc}{J}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{T}{\tau} = \frac{19.21\times 10^3}{12\times 10^3} = 1.601 \text{ in}^3$$

$$c = \sqrt[3]{\frac{(2)(1.601)}{\pi}} = 1.006 \text{ in.} \quad d_s = 2c$$

$$d = 2.01 \text{ in.} \blacktriangleleft$$



PROBLEM 3.8

The solid spindle AB has a diameter $d_s = 1.5$ in. and is made of a steel with an allowable shearing stress of 12 ksi, while sleeve CD is made of a brass with an allowable shearing stress of 7 ksi. Determine the largest torque **T** that can be applied at A.

SOLUTION

Analysis of solid spindle AB:

$$c = \frac{1}{2} d_s = 0.75 \text{ in.}$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3$$

$$T = \frac{\pi}{2}(12\times 10^3)(0.75)^3 = 7.95\times 10^3 \text{ lb}\cdot\text{in.}$$

Analysis of sleeve CD:

$$c_2 = \frac{1}{2} d_o = \frac{1}{2}(3) = 1.5 \text{ in.}$$

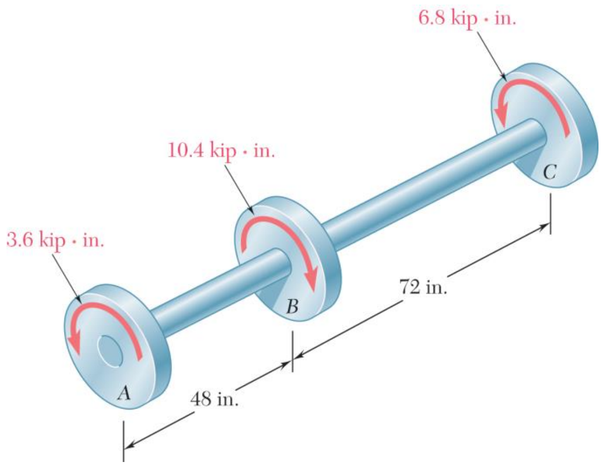
$$c_1 = c_2 - t = 1.5 - 0.25 = 1.25 \text{ in.}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(1.5^4 - 1.25^4) = 4.1172 \text{ in}^4$$

$$T = \frac{J\tau}{c_2} = \frac{(4.1172)(7\times 10^3)}{1.5} = 19.21\times 10^3 \text{ lb}\cdot\text{in.}$$

The smaller torque governs. $T = 7.95\times 10^3 \text{ lb}\cdot\text{in.}$

$T = 7.95 \text{ kip}\cdot\text{in.}$ ◀



PROBLEM 3.9

The torques shown are exerted on pulleys *A*, *B*, and *C*. Knowing that both shafts are solid, determine the maximum shearing stress in (*a*) shaft *AB*, (*b*) shaft *BC*. The diameter of shaft *AB* is 1.3 in. and that of *BC* is 1.8 in.

SOLUTION

(*a*) Shaft *AB*:

$$T_{AB} = 3.6 \times 10^3 \text{ lb}\cdot\text{in.}$$

$$c = \frac{1}{2}d = \frac{1}{2}(1.3) = 0.65 \text{ in.}$$

$$J = \frac{\pi}{2}c^4$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau_{\max} = \frac{(2)(3.6 \times 10^3)}{\pi(0.65)^3} = 8.35 \times 10^3 \text{ psi}$$

$\tau_{\max} = 8.35$ ksi ◀

(*b*) Shaft *BC*:

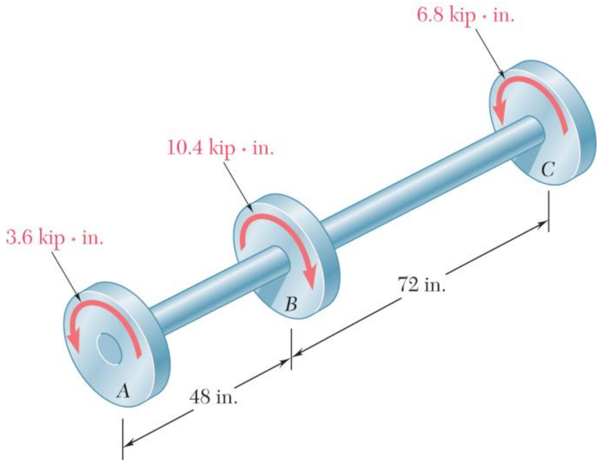
$$T_{BC} = 6.8 \times 10^3 \text{ lb}\cdot\text{in.}$$

$$c = \frac{1}{2}d = \frac{1}{2}(1.8) = 0.9 \text{ in.}$$

$$J = \frac{\pi}{2}c^4$$

$$\tau_{\max} = \frac{2T_{BC}}{\pi c^3} = \frac{(2)(6.8 \times 10^3)}{\pi(0.9)^3} = 5.94 \times 10^3 \text{ psi}$$

$\tau_{\max} = 5.94$ ksi ◀



PROBLEM 3.10

The shafts of the pulley assembly shown are to be redesigned. Knowing that the allowable shearing stress in each shaft is 8.5 ksi, determine the smallest allowable diameter of (*a*) shaft *AB*, (*b*) shaft *BC*.

SOLUTION

(*a*) Shaft *AB*:

$$T_{AB} = 3.6 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\tau_{\max} = 8.5 \text{ ksi} = 8.5 \times 10^3 \text{ psi}$$

$$J = \frac{\pi}{2}c^4 \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c = \sqrt[3]{\frac{2T_{AB}}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(3.6 \times 10^3)}{\pi(8.5 \times 10^3)}} = 0.646 \text{ in.}$$

$$d_{AB} = 2c = 1.292 \text{ in.} \blacktriangleleft$$

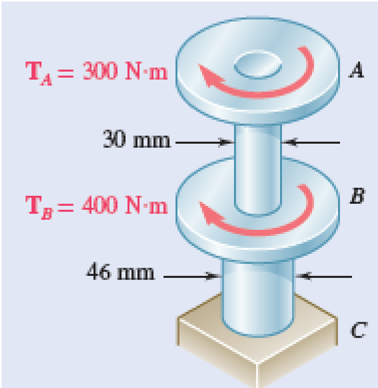
(*b*) Shaft *BC*:

$$T_{BC} = 6.8 \times 10^3 \text{ lb} \cdot \text{in.}$$

$$\tau_{\max} = 8.5 \times 10^3 \text{ psi}$$

$$c = \sqrt[3]{\frac{2T_{BC}}{\pi \tau_{\max}}} = \sqrt[3]{\frac{(2)(6.8 \times 10^3)}{\pi(8.5 \times 10^3)}} = 0.7985 \text{ in.}$$

$$d_{BC} = 2c = 1.597 \text{ in.} \blacktriangleleft$$



PROBLEM 3.11

The torques shown are exerted on pulleys *A* and *B*. Knowing that both shafts are solid, determine the maximum shearing stress in (*a*) in shaft *AB*, (*b*) in shaft *BC*.

SOLUTION

(*a*) Shaft *AB*:

$$T_{AB} = 300 \text{ N}\cdot\text{m}, \; d = 0.030 \text{ m}, \; c = 0.015 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi(0.015)^3}$$

$$= 56.588 \times 10^6 \text{Pa}$$

$\tau_{\max} = 56.6$ MPa ◀

(*b*) Shaft *BC*:

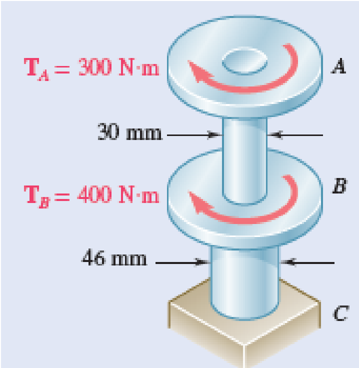
$$T_{BC} = 300 + 400 = 700 \text{ N}\cdot\text{m}$$

$$d = 0.046 \text{ m}, \; c = 0.023 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi(0.023)^3}$$

$$= 36.626 \times 10^6 \text{Pa}$$

$\tau_{\max} = 36.6$ MPa ◀



PROBLEM 3.12

In order to reduce the total mass of the assembly of Prob. 3.11, a new design is being considered in which the diameter of shaft *BC* will be smaller. Determine the smallest diameter of shaft *BC* for which the maximum value of the shearing stress in the assembly will not increase.

SOLUTION

<u>Shaft *AB*</u>: $T_{AB} = 300 \text{ N}\cdot\text{m}, \; d = 0.030 \text{ m}, \; c = 0.015 \text{ m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi(0.015)^3}$$

$$= 56.588 \times 10^6 \text{ Pa} = 56.6 \text{ MPa}$$

<u>Shaft *BC*</u>: $T_{BC} = 300 + 400 = 700 \text{ N}\cdot\text{m}$

$$d = 0.046 \text{ m}, \; c = 0.023 \text{ m}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi(0.023)^3}$$

$$= 36.626 \times 10^6 \text{ Pa} = 36.6 \text{ MPa}$$

The largest stress $(56.588 \times 10^6 \text{ Pa})$ occurs in portion *AB*.

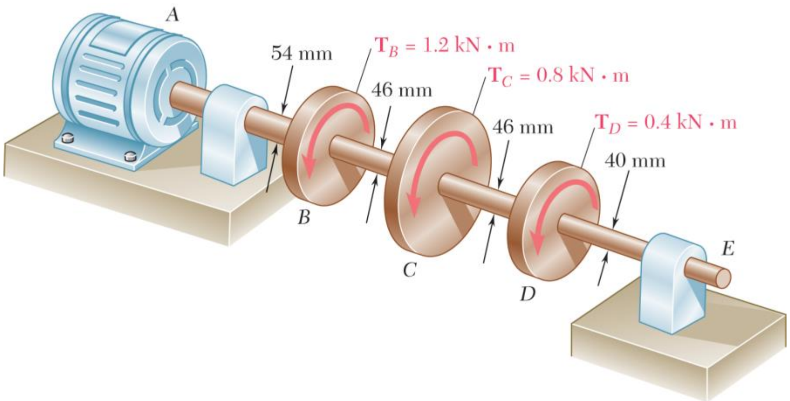
Reduce the diameter of *BC* to provide the same stress.

$$T_{BC} = 700 \text{N}\cdot\text{m} \qquad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi \tau_{max}} = \frac{(2)(700)}{\pi(56.588 \times 10^6)} = 7.875 \times 10^{-6} \text{m}^3$$

$$c = 19.895 \times 10^{-3} \text{m} \qquad d = 2c = 39.79 \times 10^{-3} \text{m}$$

$d = 39.8 \text{ mm}$ ◀



PROBLEM 3.13

Under normal operating conditions, the electric motor exerts a torque of 2.4 kN · m on shaft *AB*. Knowing that each shaft is solid, determine the maximum shearing stress in (*a*) shaft *AB*, (*b*) shaft *BC*, (*c*) shaft *CD*.

SOLUTION

(*a*) Shaft *AB*:

$$T_{AB} = 2.4 \times 10^3 \text{ N} \cdot \text{m}, \quad c = \frac{1}{2}d = 0.027 \text{ m}$$

$$\tau_{AB} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(2.4 \times 10^3)}{\pi (0.027)^3} = 77.625 \times 10^6 \text{ Pa}$$

77.6 MPa ◀

(*b*) Shaft *BC*:

$$T_{BC} = 2.4 \text{ kN} \cdot \text{m} - 1.2 \text{ kN} \cdot \text{m} = 1.2 \text{ kN} \cdot \text{m}, \quad c = \frac{1}{2}d = 0.023 \text{ m}$$

$$\tau_{BC} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1.2 \times 10^3)}{\pi (0.023)^3} = 62.788 \times 10^6 \text{ Pa}$$

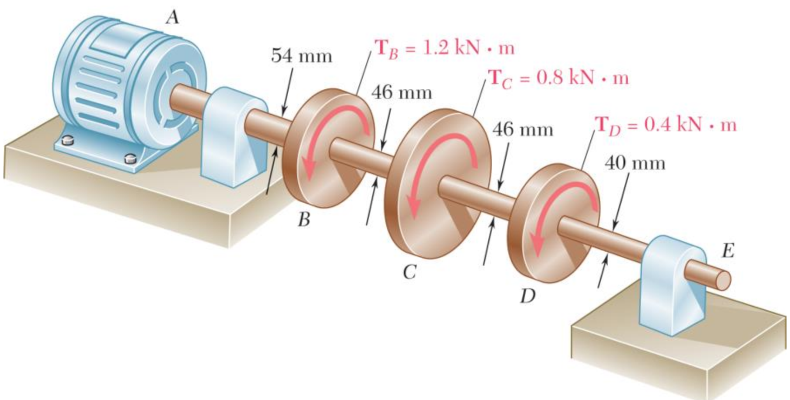
62.8 MPa ◀

(*c*) Shaft *CD*:

$$T_{CD} = 0.4 \times 10^3 \text{ N} \cdot \text{m} \quad c = \frac{1}{2}d = 0.023 \text{ m}$$

$$\tau_{CD} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(0.4 \times 10^3)}{\pi (0.023)^3} = 20.929 \times 10^6 \text{ Pa}$$

20.9 MPa ◀



PROBLEM 3.14

In order to reduce the total mass of the assembly of Prob. 3.13, a new design is being considered in which the diameter of shaft *BC* will be smaller. Determine the smallest diameter of shaft *BC* for which the maximum value of the shearing stress in the assembly will not be increased.

PROBLEM 3.13 Under normal operating conditions, the electric motor exerts a torque of 2.4 kN · m on shaft *AB*. Knowing that each shaft is solid, determine the maximum shearing stress in (*a*) shaft *AB*, (*b*) shaft *BC*, (*c*) shaft *CD*.

SOLUTION

See solution to Problem 3.13 for maximum shearing stresses in portions *AB*, *BC*, and *CD* of the shaft. The largest maximum shearing value is $\tau_{max} = 77.625 \times 10^6$ Pa occurring in *AB*.

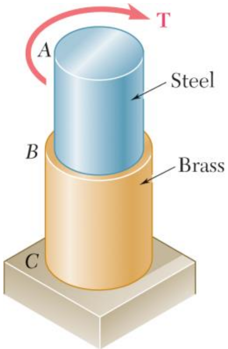
Adjust diameter of *BC* to obtain the same value of stress.

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi \tau} = \frac{(2)(1.2 \times 10^3)}{\pi (77.625 \times 10^6)} = 9.8415 \times 10^{-6}\ \text{m}^3$$

$$c = 21.43 \times 10^{-3}\ \text{m} \quad d = 2c = 42.8 \times 10^{-3}\ \text{m}$$

42.8 mm ◀



PROBLEM 3.15

The allowable shearing stress is 15 ksi in the 1.5-in.-diameter steel rod *AB* and 8 ksi in the 1.8-in.-diameter brass rod *BC*. Neglecting the effect of stress concentrations, determine the largest torque that can be applied at *A*.

SOLUTION

$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}c^4, \quad T = \frac{\pi}{2}c^3\tau_{max}$$

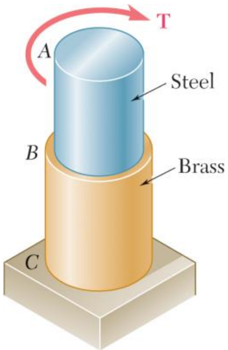
Rod *AB*: $\tau_{max} = 15 \text{ ksi} \qquad c = \frac{1}{2}d = 0.75 \text{ in.}$

$$T = \frac{\pi}{2}(0.75)^3(15) = 9.94 \text{ kip} \cdot \text{in.}$$

Rod *BC*: $\tau_{max} = 8 \text{ ksi} \qquad c = \frac{1}{2}d = 0.90 \text{ in.}$

$$T = \frac{\pi}{2}(0.90)^3(8) = 9.16 \text{ kip} \cdot \text{in.}$$

The allowable torque is the smaller value. $T = 9.16 \text{ kip} \cdot \text{in.}$ ◀



PROBLEM 3.16

The allowable shearing stress is 15 ksi in the steel rod *AB* and 8 ksi in the brass rod *BC*. Knowing that a torque of magnitude $T = 10 \text{ kip} \cdot \text{in.}$ is applied at *A*, determine the required diameter of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}, \quad c^3 = \frac{2T}{\pi\tau_{max}}$$

(*a*) Rod *AB*:

$$T = 10 \text{ kip} \cdot \text{in.} \qquad \tau_{max} = 15 \text{ ksi}$$

$$c^3 = \frac{(2)(10)}{\pi(15)} = 0.4244 \text{ in}^3$$

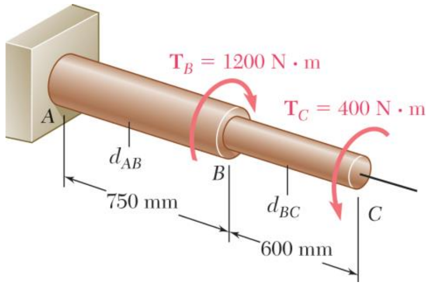
$$c = 0.7515 \text{ in.} \qquad d = 2c = 1.503 \text{ in.} \blacktriangleleft$$

(*b*) Rod *BC*:

$$T = 10 \text{ kip} \cdot \text{in.} \qquad \tau_{max} = 8 \text{ ksi}$$

$$c^3 = \frac{(2)(10)}{\pi(8)} = 0.79577 \text{ in}^2$$

$$c = 0.9267 \text{ in.} \qquad d = 2c = 1.853 \text{ in.} \blacktriangleleft$$



PROBLEM 3.17

The solid shaft shown is formed of a brass for which the allowable shearing stress is 55 MPa. Neglecting the effect of stress concentrations, determine the smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded.

SOLUTION

$$\tau_{max} = 55 \text{ MPa} = 55 \times 10^6 \text{ Pa}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

Shaft *AB*:

$$T_{AB} = 1200 - 400 = 800 \text{ N} \cdot \text{m}$$

$$c = \sqrt[3]{\frac{(2)(800)}{\pi(55 \times 10^6)}} = 21.00 \times 10^{-3} \text{ m} = 21.0 \text{ m}$$

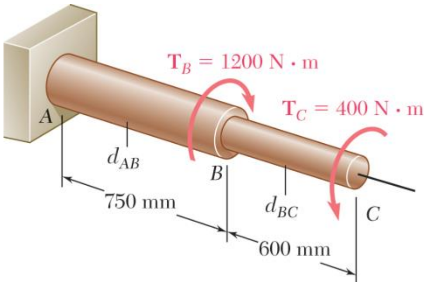
$$\text{minimum } d_{AB} = 2c = 42.0 \text{ mm} \blacktriangleleft$$

Shaft *BC*:

$$T_{BC} = 400 \text{ N} \cdot \text{m}$$

$$c = \sqrt[3]{\frac{(2)(400)}{\pi(55 \times 10^6)}} = 16.667 \times 10^{-3} \text{ m} = 16.67 \text{ mm}$$

$$\text{minimum } d_{BC} = 2c = 33.3 \text{ mm} \blacktriangleleft$$



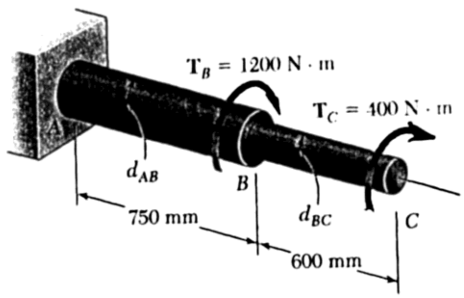
PROBLEM 3.18

Solve Prob. 3.17, assuming that the direction of $\mathbf{T}_C$ is reversed.

PROBLEM 3.17 The solid shaft shown is formed of a brass for which the allowable shearing stress is 55 MPa. Neglecting the effect of stress concentrations, determine the smallest diameters d_{AB} and d_{BC} for which the allowable shearing stress is not exceeded.

SOLUTION

Note that the direction of T_C has been reversed in the figure.



$$\tau_{max} = 55 \text{ MPa} = 55 \times 10^6 \text{ Pa}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

<u>Shaft *AB*</u>: $T_{AB} = 1200 + 400 = 1600 \text{ N} \cdot \text{m}$

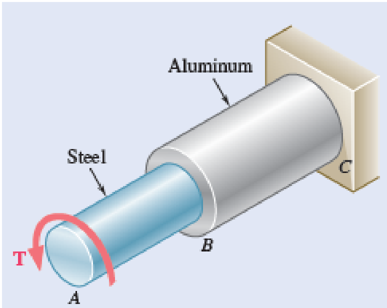
$$c = \sqrt[3]{\frac{(2)(1600)}{\pi(55 \times 10^6)}} = 26.46 \times 10^{-3} \text{ m} = 26.46 \text{ mm}$$

minimum $d_{AB} = 2c = 52.9$ mm ◀

<u>Shaft *BC*</u>: $T_{BC} = 400 \text{ N} \cdot \text{m}$

$$c = \sqrt[3]{\frac{(2)(400)}{\pi(55 \times 10^6)}} = 16.667 \times 10^{-3} \text{ m} = 16.67 \text{ mm}$$

minimum $d_{BC} = 2c = 33.3$ mm ◀



PROBLEM 3.19

Shaft *AB* is made of a steel with an allowable shearing stress of 90 MPa and shaft *BC* is made of an aluminum with an allowable shearing stress of 60 MPa. Knowing that the diameter of shaft *BC* is 50 mm and neglecting the effect of stress concentrations, determine (*a*) the largest torque **T** that can be applied at A if the allowable stress is not to be exceeded in shaft *BC*, (*b*) the corresponding required diameter of shaft *AB*.

SOLUTION

(*a*) Shaft *BC*:

$$\tau_{\text{all}} = 60 \times 10^6 \text{ Pa} \qquad c = \frac{1}{2}d = 0.025 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{Tc}{\left(\frac{\pi}{2}\right)c^4} = \frac{2T}{\pi c^3} \quad \text{thus,} \quad T = \left(\frac{\pi}{2}\right)\tau_{\max} c^3$$

$$T = \left(\frac{\pi}{2}\right)\left(60 \times 10^6 \text{ Pa}\right)\left(0.025 \text{ m}\right)^3 = 1.47262 \times 10^3 \text{ N} \cdot \text{m}$$

1.473 kN · m ◀

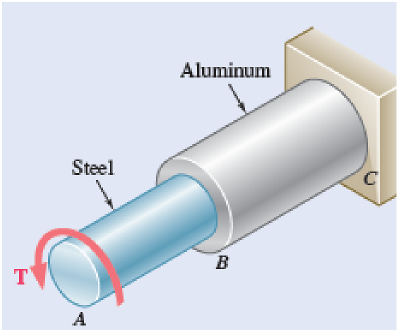
(*b*) Shaft *AB*:

$$\tau_{\text{all}} = 90 \times 10^6 \text{ Pa}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{\max}}} = \sqrt[3]{\frac{2\left(1.47262 \times 10^3 \text{ N} \cdot \text{m}\right)}{\pi\left(90 \times 10^6 \text{ Pa}\right)}} = 21.84 \times 10^{-3} \text{ m}$$

$$d_{AB} = 2c = 43.68 \times 10^{-3} \text{ m}$$

43.7 mm ◀



PROBLEM 3.20

Shaft *AB* has a 30-mm diameter and is made of a steel with an allowable shearing stress of 90 MPa, while shaft *BC* has a 50-mm diameter and is made of an aluminum alloy with an allowable shearing stress of 60 MPa. Neglecting the effect of stress concentrations, determine the largest torque T that can be applied at *A*.

SOLUTION

Shaft *AB*:

$$\tau_{all} = 90 \times 10^6 \text{ Pa} \qquad c = \frac{1}{2}d = 0.015 \text{ m}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{Tc}{\left(\frac{\pi}{2}\right)c^4} = \frac{2T}{\pi c^3} \qquad \text{thus,} \qquad T = \left(\frac{\pi}{2}\right)\tau_{max}c^3$$

$$T = \left(\frac{\pi}{2}\right)(90 \times 10^6 \text{ Pa})(0.015 \text{ m})^3 = 477.13 \text{ N} \cdot \text{m}$$

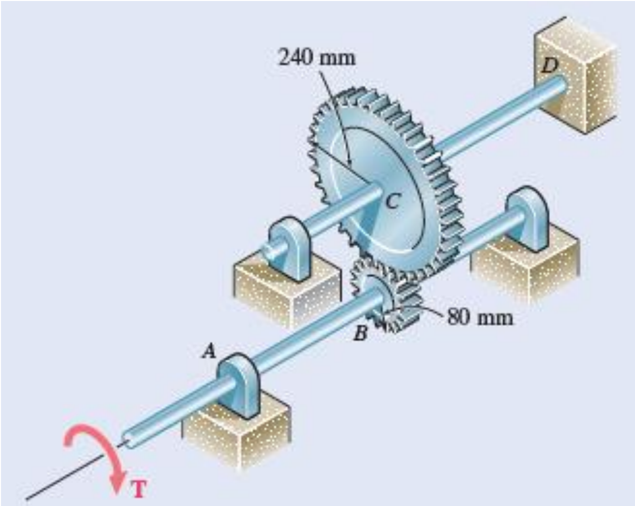
Shaft *BC*:

$$\tau_{all} = 60 \times 10^6 \text{ Pa} \qquad c = \frac{1}{2}d = 0.025 \text{ m}$$

$$T = \left(\frac{\pi}{2}\right)(60 \times 10^6 \text{ Pa})(0.025 \text{ m})^3 = 1472.62 \text{ N} \cdot \text{m}$$

The allowable torque is the smaller value of the two.

477 N · m ◀



PROBLEM 3.21

Two solid steel shafts are connected by the gears shown. A torque of magnitude $T = 900$ N · m is applied to shaft AB. Knowing that the allowable shearing stress is 50 MPa and considering only stresses due to twisting, determine the required diameter of (*a*) shaft AB, (*b*) shaft CD.

SOLUTION

$$T_{AB} = 900\ \text{N}\cdot\text{m}$$

$$T_{CD} = \frac{r_C}{r_B}T_{AB} = \frac{240}{80}(1000) = 2700\ \text{N}\cdot\text{m}$$

(*a*) Shaft AB:

$$\tau_{\text{all}} = 50\times 10^6\ \text{Pa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(900)}{\pi(50\times 10^6)}} = 22.545\times 10^{-3}\ \text{m}^3$$

$$d = 2c = 45.090\times 10^{-3}\ \text{m}$$

45.1 mm ◀

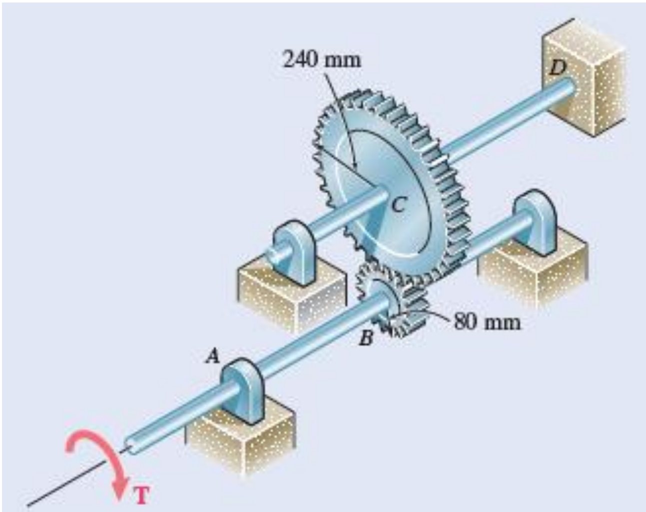
(*b*) Shaft CD:

$$\tau_{\text{all}} = 50\times 10^6\ \text{Pa}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(2700)}{\pi(50\times 10^6)}} = 32.516\times 10^{-3}\ \text{m}^3$$

$$d = 2c = 65.032\times 10^{-3}\ \text{m}$$

65.0 mm ◀



PROBLEM 3.22

Shaft *CD* is made from a 66-mm-diameter rod and is connected to the 48-mm-diameter shaft *AB* as shown. Considering only stresses due to twisting and knowing that the allowable shearing stress is 60 MPa for each shaft, determine the largest torque **T** that can be applied.

SOLUTION

Shaft *AB*:

$$\tau_{\text{all}} = 60 \times 10^6 \text{ Pa} \qquad c = \frac{1}{2}d = 0.024 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{Tc}{\left(\frac{\pi}{2}\right)c^4} = \frac{2T}{\pi c^3} \qquad \text{thus,} \quad T = \left(\frac{\pi}{2}\right)\tau_{\max} c^3$$

$$T = \left(\frac{\pi}{2}\right)\left(60 \times 10^6 \text{ Pa}\right)\left(0.024 \text{ m}\right)^3 = 1302.88 \text{ N} \cdot \text{m}$$

Shaft *BC*:

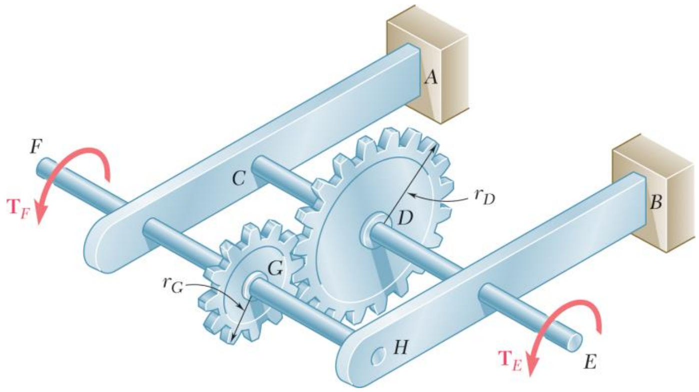
$$\tau_{\text{all}} = 60 \times 10^6 \text{ Pa} \qquad c = \frac{1}{2}d = 0.033 \text{ m}$$

$$T_{CD} = \left(\frac{\pi}{2}\right)\left(60 \times 10^6 \text{ Pa}\right)\left(0.033 \text{ m}\right)^3 = 3387.0 \text{ N} \cdot \text{m}$$

$$T = \frac{r_B}{r_C} T_{CD} = \frac{80}{240}(3387.0) = 1129 \text{ N} \cdot \text{m}$$

The allowable torque is the smaller value of the two.

1.129 kN · m ◀



PROBLEM 3.23

Under normal operating conditions, a motor exerts a torque of magnitude T_F at F. The shafts are made of a steel for which the allowable shearing stress is 12 ksi and have diameters $d_{CDE} = 0.900$ in. and $d_{FGH} = 0.800$ in. Knowing that r_D = 6.5 in. and r_G = 4.5 in., determine the largest allowable value of T_F.

SOLUTION

$$\tau_{\text{all}} = 12 \text{ ksi}$$

Shaft *FG*:

$$c = \frac{1}{2}d = 0.400 \text{ in.}$$

$$T_{F,\text{all}} = \frac{J\tau_{\text{all}}}{c} = \frac{\pi}{2}c^3\tau_{\text{all}}$$

$$= \frac{\pi}{2}(0.400)^3(12) = 1.206 \text{ kip} \cdot \text{in.}$$

Shaft *DE*:

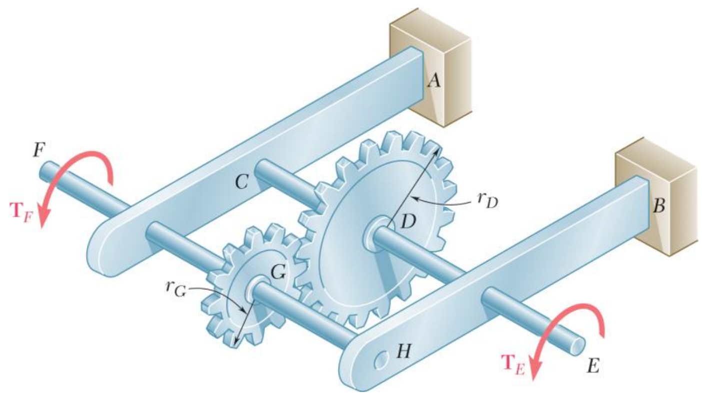
$$c = \frac{1}{2}d = 0.450 \text{ in.}$$

$$T_{E,\text{all}} = \frac{\pi}{2}c^3\tau_{all}$$

$$= \frac{\pi}{2}(0.450)^3(12) = 1.7177 \text{ kip} \cdot \text{in.}$$

$$T_F = \frac{r_G}{r_D}T_E \quad T_{F,\text{ all}} = \frac{4.5}{6.5}(1.7177) = 1.189 \text{ kip} \cdot \text{in.}$$

Allowable value of T_F is the smaller. $T_F = 1.189 \text{ kip} \cdot \text{in.}$ ◀



PROBLEM 3.24

Under normal operating conditions, a motor exerts a torque of magnitude $T_F = 1200\text{ lb}\cdot\text{in.}$ at F. Knowing that $r_D = 8$ in., $r_G = 3$ in., and the allowable shearing stress is 10.5 ksi in each shaft, determine the required diameter of (*a*) shaft *CDE*, (*b*) shaft *FGH*.

SOLUTION

$$T_F = 1200\text{ lb}\cdot\text{in.}$$

$$T_E = \frac{r_D}{r_G} T_F = \frac{8}{3}(1200) = 3200\text{ lb}\cdot\text{in.}$$

$$\tau_{\text{all}} = 10.5\text{ ksi} = 10{,}500\text{ psi}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \text{or} \quad c^3 = \frac{2T}{\pi\tau}$$

(*a*) Shaft *CDE*:

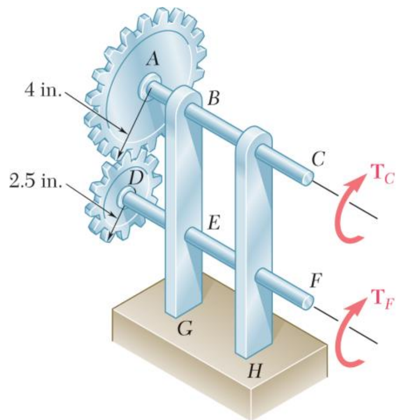
$$c^3 = \frac{(2)(3200)}{\pi(10{,}500)} = 0.194012\text{ in}^3$$

$$c = 0.5789\text{ in.} \qquad d_{DE} = 2c \qquad d_{DE} = 1.158\text{ in.} \blacktriangleleft$$

(*b*) Shaft *FGH*:

$$c^3 = \frac{(2)(1200)}{\pi(10{,}500)} = 0.012757\text{ in}^3$$

$$c = 0.4174\text{ in.} \qquad d_{FG} = 2c \qquad d_{FG} = 0.835\text{ in.} \blacktriangleleft$$



PROBLEM 3.25

The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 7000 psi. Knowing the diameters of the two shafts are, respectively, $d_{BC} = 1.6$ in. and $d_{EF} = 1.25$ in., determine the largest torque $\mathbf{T}_C$ that can be applied at C.

SOLUTION

$$\tau_{max} = 7000 \text{ psi} = 7.0 \text{ ksi}$$

Shaft *BC*:

$$d_{BC} = 1.6 \text{ in.}$$

$$c = \frac{1}{2}d = 0.8 \text{ in.}$$

$$T_C = \frac{J\tau_{max}}{c} = \frac{\pi}{2}\tau_{max}c^3$$

$$= \frac{\pi}{2}(7.0)(0.8)^3 = 5.63 \text{ kip} \cdot \text{in.}$$

Shaft *EF*:

$$d_{EF} = 1.25 \text{ in.}$$

$$c = \frac{1}{2}d = 0.625 \text{ in.}$$

$$T_F = \frac{J\tau_{max}}{c} = \frac{\pi}{2}\tau_{max}c^3$$

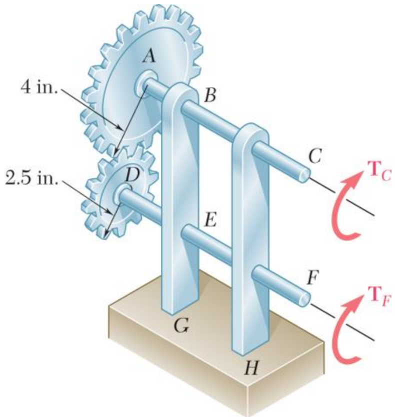
$$= \frac{\pi}{2}(7.0)(0.625)^3 = 2.684 \text{ kip} \cdot \text{in.}$$

By statics,

$$T_C = \frac{r_A}{r_D}T_F = \frac{4}{2.5}(2.684) = 4.30 \text{ kip} \cdot \text{in.}$$

Allowable value of T_C is the smaller.

$$T_C = 4.30 \text{ kip} \cdot \text{in.} \blacktriangleleft$$



PROBLEM 3.26

The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 8500 psi. Knowing that a torque of magnitude $T_C = 5$ kip · in. is applied at C and that the assembly is in equilibrium, determine the required diameter of (*a*) shaft *BC*, (*b*) shaft *EF*.

SOLUTION

$$\tau_{max} = 8500 \text{ psi} = 8.5 \text{ ksi}$$

(*a*) Shaft *BC*:

$$T_C = 5 \text{ kip} \cdot \text{in.}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}}$$

$$c = \sqrt[3]{\frac{(2)(5)}{\pi(8.5)}} = 0.7208 \text{ in.}$$

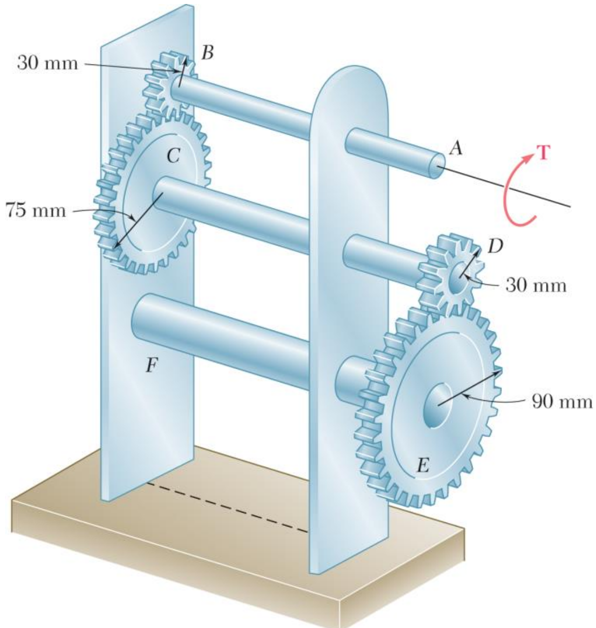
$$d_{BC} = 2c = 1.442 \text{ in.} \blacktriangleleft$$

(*b*) Shaft *EF*:

$$T_F = \frac{r_D}{r_A} T_C = \frac{2.5}{4}(5) = 3.125 \text{ kip} \cdot \text{in.}$$

$$c = \sqrt[3]{\frac{2T}{\pi \tau_{max}}} = \sqrt[3]{\frac{(2)(3.125)}{\pi(8.5)}} = 0.6163 \text{ in.}$$

$$d_{EF} = 2c = 1.233 \text{ in.} \blacktriangleleft$$



PROBLEM 3.27

For the gear train shown, the diameters of the three solid shafts are:

$$d_{AB} = 20\,\text{mm} \quad d_{CD} = 25\,\text{mm} \quad d_{EF} = 40\,\text{mm}$$

Knowing that for each shaft the allowable shearing stress is 60 MPa, determine the largest torque **T** that can be applied.

SOLUTION

Statics: $T_{AB} = T$

$$\frac{T_{CD}}{r_C} = \frac{T_{AB}}{r_B} \qquad T_{CD} = \frac{r_C}{r_B}T_{AB} = \frac{75}{30}T = 2.5T$$

$$\frac{T_{EF}}{r_F} = \frac{T_{CD}}{r_D} \qquad T_{EF} = \frac{r_F}{r_D}T_{CD} = \frac{90}{30}(2.5T) = 7.5T$$

Determine the magnitude of T so that the stress is $60\text{ MPa} = 60 \times 10^6\text{ Pa}$.

$$\tau = \frac{Tc}{J} \qquad T_{\text{shaft}} = \frac{J\tau}{c} = \frac{\pi}{2}\tau c^3$$

<u>Shaft *AB*</u>: $c = \frac{1}{2}d_{AB} = 10\text{ mm} = 0.010\text{ m}$

$$T_{AB} = T = \frac{\pi}{2}(60 \times 10^6)(0.010)^3 \qquad T = 94.2\text{ N}\cdot\text{m}$$

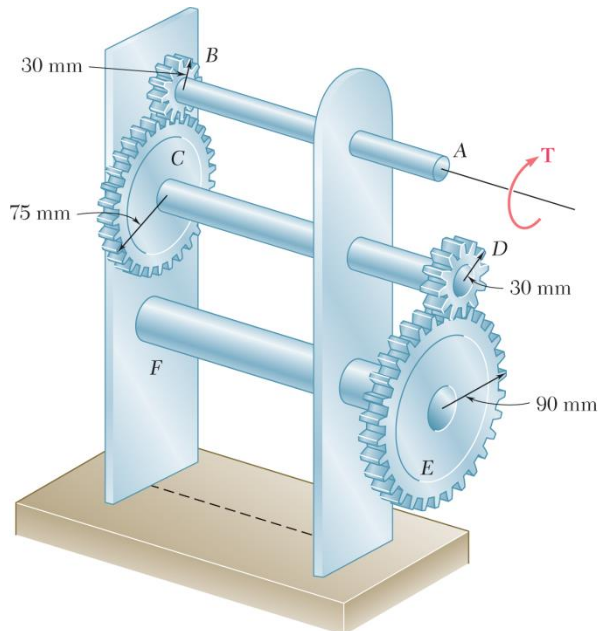
<u>Shaft *CD*</u>: $c = \frac{1}{2}d_{CD} = 12.5\text{ mm} = 0.0125\text{ m}$

$$T_{CD} = 2.5T = \frac{\pi}{2}(60 \times 10^6)(0.0125)^3 \qquad T = 73.6\text{ N}\cdot\text{m}$$

<u>Shaft *EF*</u>: $c = \frac{1}{2}d_{EF} = 20\text{ mm} = 0.020\text{ m}$

$$T_{EF} = 7.5T = \frac{\pi}{2}(60 \times 10^6)(0.020)^3 \qquad T = 100.5\text{ N}\cdot\text{m}$$

The smallest value of **T** is the largest torque that can be applied. $T = 73.6\text{ N}\cdot\text{m}$ ◀



PROBLEM 3.28

A torque $T = 900\ \text{N}\cdot\text{m}$ is applied to shaft AB of the gear train shown. Knowing that the allowable shearing stress is 80 MPa, determine the required diameter of (*a*) shaft AB, (*b*) shaft CD, (*c*) shaft EF.

SOLUTION

From statics,

$$T_{AB} = T$$

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{75\text{ mm}}{30\text{ mm}} T = 2.5T$$

$$T_{EF} = \frac{r_E}{r_D} T_{BC} = \frac{90\text{ mm}}{30\text{ mm}} (2.5T) = 7.5T$$

(*a*) Shaft AB:

$$T_{AB} = T = 900\ \text{N}\cdot\text{m}$$

$$\tau_{\text{all}} = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4}$$

$$c^3 = \frac{2}{\pi}\frac{T}{\tau_{\text{all}}}$$

$$c^3 = \frac{2}{\pi}\frac{T_{AB}}{\tau_{\text{all}}} = \frac{2}{\pi}\frac{900\ \text{N}\cdot\text{m}}{80\text{ MPa}}$$

$$c = 19.2757\text{ mm} \rightarrow d_{AB} = 38.6\text{ mm} \blacktriangleleft$$

(*b*) Shaft CD:

$$T_{CD} = 2.5T = 2.5(900\ \text{N}\cdot\text{m})$$

$$c^3 = \frac{2}{\pi}\frac{T_{CD}}{\tau_{\text{all}}} = \frac{2}{\pi}\frac{2250\ \text{N}\cdot\text{m}}{80\text{ MPa}}$$

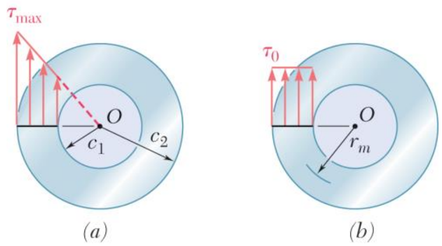
$$c = 26.161\text{ mm} \rightarrow d_{CD} = 52.3\text{ mm} \blacktriangleleft$$

(*c*) Shaft EF:

$$T_{EF} = 7.5T = 7.5(900\ \text{N}\cdot\text{m})$$

$$c^3 = \frac{2}{\pi}\frac{T_{EF}}{\tau_{\text{all}}} = \frac{2}{\pi}\frac{6750\ \text{N}\cdot\text{m}}{80\text{ MPa}}$$

$$c = 37.731\text{ mm} \rightarrow d_{EF} = 75.5\text{ mm} \blacktriangleleft$$



PROBLEM 3.29

While the exact distribution of the shearing stresses in a hollow-cylindrical shaft is as shown in Fig. *a*, an approximate value can be obtained for τ_{max} by assuming that the stresses are uniformly distributed over the area A of the cross section, as shown in Fig. *b*, and then further assuming that all of the elementary shearing forces act at a distance from O equal to the mean radius $\frac{1}{2}(c_1 + c_2)$ of the cross section. This approximate value $\tau_0 = T/Ar_m$, where T is the applied torque. Determine the ratio τ_{max}/τ_0 of the true value of the maximum shearing stress and its approximate value τ_0 for values of c_1/c_2, respectively, equal to 1.00, 0.95, 0.75, 0.50, and 0.

SOLUTION

For a hollow shaft,

$$\tau_{max} = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi\left(c_2^4 - c_1^4\right)} = \frac{2Tc_2}{\pi\left(c_2^2 - c_1^2\right)\left(c_2^2 + c_1^2\right)} = \frac{2Tc_2}{A\left(c_2^2 + c_1^2\right)}$$

By definition,

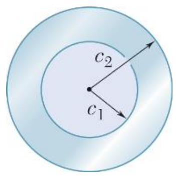
$$\tau_0 = \frac{T}{Ar_m} = \frac{2T}{A(c_2 + c_1)}$$

Dividing,

$$\frac{\tau_{max}}{\tau_0} = \frac{c_2(c_2 + c_1)}{c_2^2 + c_1^2} = \frac{1 + (c_1/c_2)}{1 + (c_1/c_2)^2} \blacktriangleleft$$

c_1/c_2	1.0	0.95	0.75	0.5	0.0
τ_{max}/τ_0	1.0	1.025	1.120	1.200	1.0

◀



PROBLEM 3.30

(a) For a given allowable shearing stress, determine the ratio T/w of the maximum allowable torque T and the weight per unit length w for the hollow shaft shown. (b) Denoting by $(T/w)_0$ the value of this ratio for a solid shaft of the same radius c_2, express the ratio T/w for the hollow shaft in terms of $(T/w)_0$ and c_1/c_2.

SOLUTION

$$w = \text{weight per unit length}$$
$$\rho g = \text{specific weight}$$
$$W = \text{total weight}$$
$$L = \text{length}$$
$$w = \frac{W}{L} = \frac{\rho g L A}{L} = \rho g A = \rho g \pi \left(c_2^2 - c_1^2\right)$$
$$T_{\text{all}} = \frac{J\tau_{\text{all}}}{c_2} = \frac{\pi}{2}\frac{c_2^4 - c_1^4}{c_2}\tau_{\text{all}} = \frac{\pi}{2}\frac{\left(c_2^2 + c_1^2\right)\left(c_2^2 - c_1^2\right)}{c_2}\tau_{\text{all}}$$

(a) $\dfrac{T}{W} = \left(c_1^2 + c_2^2\right)\tau_{\text{all}}$ $\qquad$ $\dfrac{T}{w} = \dfrac{\left(c_1^2 + c_2^2\right)\tau_{\text{all}}}{2\rho g c_2}$ (hollow shaft) ◀

$c_1 = 0$ for solid shaft $\qquad$ $\left(\dfrac{T}{w}\right)_0 = \dfrac{c_2\tau_{\text{all}}}{2\rho g}$ (solid shaft)

(b) $\dfrac{(T/w)_h}{(T/w)_0} = 1 + \dfrac{c_1^2}{c_2^2}$ $\qquad$ $\left(\dfrac{T}{w}\right) = \left(\dfrac{T}{w}\right)_0\left(1 + \dfrac{c_1^2}{c_2^2}\right)$ ◀