Solutions for Mathematics Of Finance 9th Edition by Brown

Mathematics of Finance

Ninth Edition

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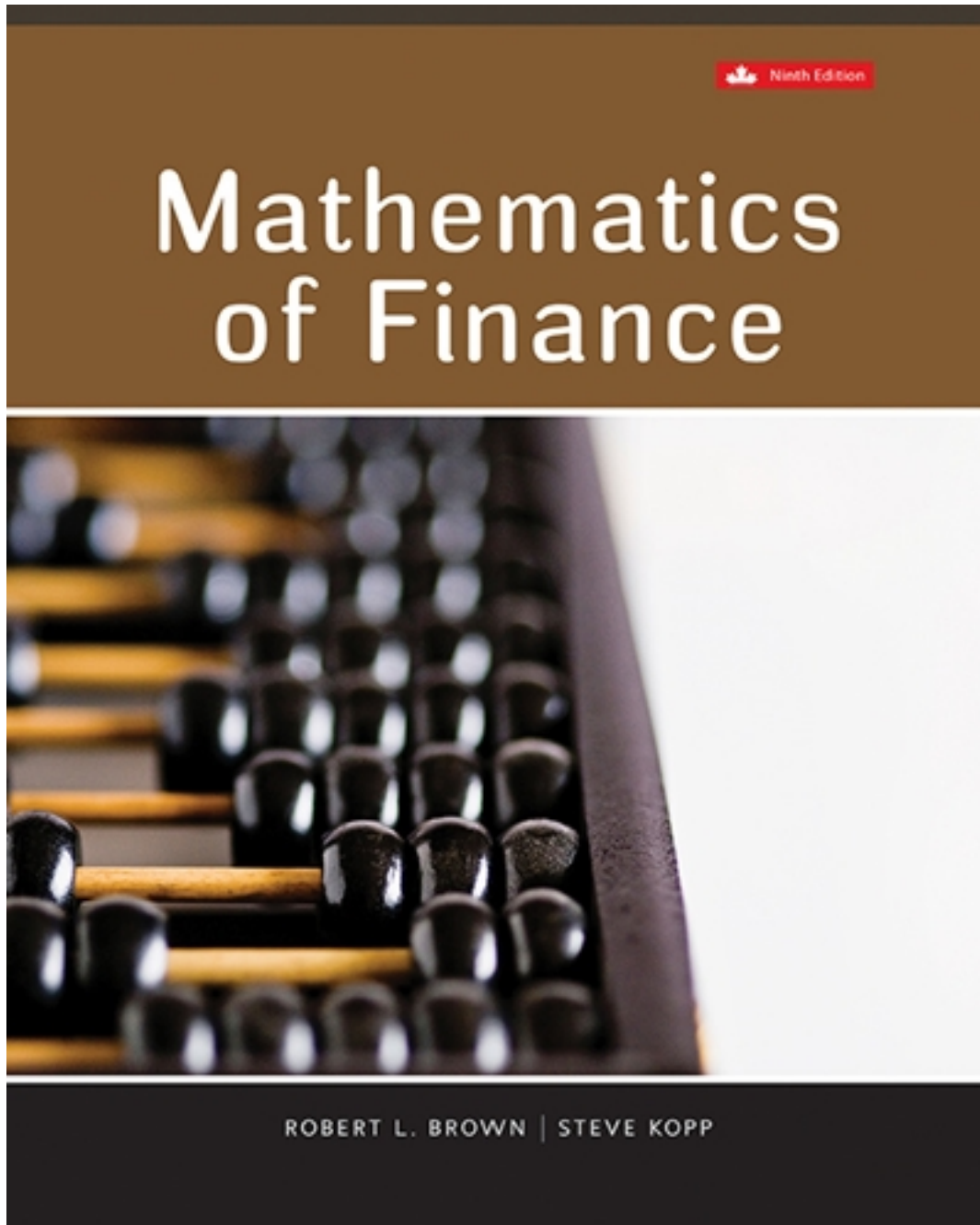
Solutions

Chapter 1

EXERCISE 1.1

CONCEPT QUESTIONS

1. a) 7%; the higher the interest rate, the more interest that will be paid on a loan;

    b) 150 days; the longer is the term of a loan, the more interest that will be paid on a loan; c) ordinary interest

2. 5%; the higher the interest rate, the faster a given investment will grow to a given a ccumulated value after a given amount of time

3. 6%; the lower the interest rate, the more that will be needed to be invested in order to accumulate to a given amount after a given term

EXERCISES

1. a) $S = 2500[1 + (0.12)\left(\frac{18}{12}\right)] = \$2950$

    b) $S = 1200[1 + (0.085)\left(\frac{120}{360}\right)] = \$1234$

    c) $S = 10\,000[1 + (0.07)\left(\frac{64}{365}\right)] = \$10\,122.74$

    d) $S = 500[1 + (0.06)\left(\frac{175}{365}\right)] = \$5143.84$

2. a) $r = 420/(1000 \times 2.5) = 16.8\%$

    b) $r = 1/(1 \times 7) = 14.29\%$

    c) $r = 10/(500 \times \frac{2}{12}) = 12\%$

    d) $r = 50/(800 \times \frac{214}{265}) = 10.66\%$

3. a) $t = \frac{200}{1000(0.055)} = 3.636 \; years \; \doteq 1328 \; days$

    b) $t = \frac{30}{1600(0.035)} = 0.535714 \; years \; \doteq 196 \; days$

    c) $t = \frac{100}{5000(0.09)} = 0.222222 \; years \; \doteq 82 \; days$

4. Ordinary interest $= 5000 \times 0.105 \times \frac{90}{360} = \$131.25$

    Exact interest $= 5000 \times 0.105 \times \frac{90}{365} = \$129.45$

5. $r = 2.50/(50 \times \frac{1}{12}) = 0.60 = 60\%$

6. $P = 5100\left[1 + (0.09)\left(\frac{6}{12}\right)\right]^{-1} = \$4880.38$

7. $P = 580[1 + (0.18)\left(\frac{120}{365}\right)]^{-1} = \$547.59$

8. At ordinary interest $S = 1000[1 + (0.065)\left(\frac{65}{360}\right)] = \$1011.74$

   At exact interest $S = 1000[1 + (0.065)\left(\frac{65}{365}\right)] = \$1011.58$

9. $S = 1000[1 + (0.075)\left(\frac{212}{365}\right)] = \$1042.11$

   Number of Days = Sept. 16(259) − Feb. 16(47) = 212 days

   7 months after Feb. 16 is Sept. 16

10. Exact time = (365 – 138) + 98 = 325 days

    Exact interest $= 2000 \times 0.045 \times \frac{325}{365} = \$80.14$

11. $r = 300/(4000 \times \frac{264}{365}) = 10.37\%$

12.

Dates	# of days	Balance	Interest
January 1 –March 1	59	1000	$1000(0.12)\left(\frac{59}{365}\right) = 19.40$
March 1 – March 31	30	900	$900(0.12)\left(\frac{30}{365}\right) = 8.88$
			March 31 pay $28.28
March 31 –April 17	17	900	$900(0.12)\left(\frac{17}{365}\right) = 5.03$
April 17 – June 30	74	600	$600(0.12)\left(\frac{74}{365}\right) = 14.60$
			June 30 pay $19.63
June 30 – July 12	12	600	$600(0.12)\left(\frac{12}{365}\right) = 2.37$
July 12–August 20	39	400	$400(0.12)\left(\frac{39}{365}\right) = 5.13$
August 20–Sept. 30	41	300	$300(0.12)\left(\frac{41}{365}\right) = 4.04$
			Sept. 30 pay $11.54
Sept. 30-October 18	18	300	$300(0.12)\left(\frac{18}{365}\right) = 1.78$
			October 18 pay $1.78

Total interest paid = 28.28 + 19.63 + 11.54 + 1.78 = $61.23

13.

Interest Period	#of days	Balance	Rate	Interest
Feb.3-Feb.11	8	50 000	6%	$50\,000(0.06)\left(\frac{8}{365}\right) = \underline{67.75}$ Feb. 11 Interest $65.75
Feb.11-Feb.20	9	50 000	6%	$50\,000(0.06)(\frac{9}{365}) = 73.97$
Feb.20-March 11	19	40 000	6%	$40\,000(0.06)(\frac{19}{365}) = \underline{124.93}$ March 11 interest $198.90
March 11-March 20	9	40 000	6%	$40\,000(0.06)(\frac{9}{365}) = 59.18$
March 20-April 1	12	30 000	6%	$30\,000(0.06)(\frac{12}{365}) = 59.18$
April 1 –April 11	10	30 000	7%	$30\,000(0.07)(\frac{10}{365}) = \underline{57.53}$ April 11 interest $175.89
April 11-April 20	9	30 000	7%	$30\,000(0.07)(\frac{9}{365}) = 51.78$
April 20-May 1	11	15 000	7%	$15\,000(0.07)(\frac{11}{365}) = 31.64$
May 1-May11	10	15 000	6.5%	$15\,000(0.065)(\frac{10}{365}) = \underline{26.71}$ May 11 interest $110.13
May 11- May 20	9	15 000	6.5%	$15\,000(0.065)(\frac{9}{365}) = \underline{24.04}$ May 20 interest = $24.04

Total interest paid $= 65.75 + 198.90 + 175.89 + 110.13 + 24.04 =$ $574.71

14. Consider a $100 amount. Then $P = 98, I = 2, t = \frac{20}{365}$ and $r = 2/(98 \times \frac{20}{365}) = 37.24\%$

15. $r = 40/(1960 \times \frac{40}{365}) = 18.62\%$

16. $r = \frac{15}{485 \times \frac{20}{365}} = 56.44\%$

17. a) $r = \frac{50}{2450 \times \frac{45}{365}} = 16.55\%$

b) Interest on loan $= 2450 \times 0.10 \times \frac{45}{365} = \$30.21$

Savings $= 50 - 30.21 = \$19.79$

18. a) $S = 5000 + 5000(0.04)\frac{3}{12} + 5000(0.035)\frac{3}{12} + 5000(0.03)\frac{3}{12} + 5000(0.025)\frac{3}{12}$

$= 5000 + 162.50 = \$5162.50$

b) $S = 5000\left[1 + (0.04)\frac{3}{12}\right]\left[1 + (0.035)\frac{3}{12}\right]\left[1 + (0.03)\frac{3}{12}\right][1 + (0.025)\frac{3}{12}] = \$5164.47$

EXERCISE 1.2

CONCEPT QUESTIONS

1. 9%, the higher the interest rate, the lower the present value

2. 150 days; the larger (smaller) the term, the lower (higher) the present value

3. legal due date = due date + 3 days of grace

4. There is no difference (remember, there are no three days of grace added to regular bank loans, only for promissory notes)

EXERCISES

1. Maturity date is February 6
    Maturity value = $3000.00
    Discount period = 67 days
    $\text{Proceeds} = 3000[1 + (0.1025)\left(\frac{67}{365}\right)]^{-1} = \$2944.60$

2. Maturity date is November 3
    Maturity value = $1200[1 + (0.055)\left(\frac{63}{365}\right)] = \$1211.39$
    Discount period = 27 days
    $\text{Proceeds} = 1211.39[1 + (0.0475)\left(\frac{27}{365}\right)]^{-1} = \$1207.15$

3. Maturity date is June 27
    Maturity value = $500[1 + (0.07)\left(\frac{195}{365}\right)] = \$518.70$
    Discount period = 185 days
    $\text{Proceeds} = 518.70[1 + (0.07)\left(\frac{185}{365}\right)]^{-1} = \$500.93$

4. Maturity date is April 7
    Maturity value = $4000.00
    Discount period = 37 days
    $\text{Proceeds} = 4000[1 + (0.085)\left(\frac{37}{365}\right)]^{-1} = \$3965.83$

5. a) Maturity value = $2000[1 + (0.13)(\frac{93}{365})] = \$2066.25$
    $\text{Proceeds} = 2066.25[1 + (0.12)\left(\frac{42}{365}\right)]^{-1} = \$2038.11$
    b) $r = 38.11/(2000 \times \frac{51}{365}) = 13.64\%$
    c) Bank's profit = $2000[1 + (0.13)\left(\frac{90}{365}\right)] - 2038.11 = \$26.00$
    $r = 26/(2038.11 \times \frac{39}{365}) = 11.94\%$

6. $S = 5000[1 + (0.085)\left(\frac{93}{365}\right)] = \$5108.29$
    $P = 5108.29[1 + (0.08)\left(\frac{93}{365}\right)]^{-1} = \$5006.24$
    Investor's profit = 5006.24 – 5000 = $6.42
    Bank's profit = 5108.29 – 5006.24 = $102.05

7. $S = 800[1 + (0.6)\left(\frac{93}{365}\right)] = \$812.23$

    $P = 812.23[1 + (0.08)\left(\frac{60}{365}\right)]^{-1} = \$801.69$

8. $S = 1000[1 + (0.10)\left(\frac{93}{365}\right)] = \$1025.48$

9. Maturity date is 123 days after October 17, 2020, i.e. February 17, 2021

    Maturity value = $5000[1 + (0.075)\left(\frac{123}{365}\right)] = \$5126.37$

    Proceeds = $5126.37[1 + (0.07)\left(\frac{33}{365}\right)]^{-1} = \$5094.13$

10. Proceeds = $2000[1 + (0.13)\left(\frac{93}{365}\right)]^{-1} = \$1935.88$

    Profit = $\$435.88$

11. Needs 97% of 2000 = $\$1940.00$

    $S = 1940[1 + (0.06)\left(\frac{63}{365}\right)] = \$1960.09$

12. Maturity date is 123 days after June 8, 2020, i.e. October 9, 2020

    Maturity value $S = 1500[1 + (0.065)(\frac{123}{365})] = \$1532.86$

    a) $P = 1532.86[1 + (0.085)\left(\frac{69}{365}\right)]^{-1} = \$1508.62$

    b) $r = \dfrac{8.62}{(1500)\left(\frac{54}{365}\right)} = 3.88\%$

    c) Banks profit = $1500[1 + (0.065)\left(\frac{120}{365}\right)] - 1508.62 = \$23.43$

    $r = 23.43/(1508.62 \times \frac{66}{365}) = 8.59\%$

13. a) $P = 3500[1 + (0.06)\left(\frac{10}{12}\right)]^{-1} = \$3333.33$

    b) $P = 3500[1 + (0.06)\left(\frac{304}{365}\right)]^{-1} = \$3333.42$

    where 10 months after July 31 is May 31, which is 304 days

14. a) Loan 1: $P = 1280[1 + (0.0425)\left(\frac{241}{365}\right)]^{-1} = \$1245.06$

    Loan 2: $P = 1445[1 + (0.0425)\left(\frac{162}{365}\right)]^{-1} = \$1418.25 \rightarrow$ total = $\$2663.31$

    b) Loan 1: $P = 1280[1 + (0.0425)\left(\frac{241}{360}\right)]^{-1} = \$1244.59$

    Loan 2: $P = 1445[1 + (0.0425)\left(\frac{162}{360}\right)]^{-1} = \$1417.88 \rightarrow$ total = $\$2662.47$

EXERCISE 1.3

CONCEPT QUESTIONS

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1. False: $3000 only when $r = 0\%$.

2. a) A dated value is a value of a debt or payment made on a particular date;

   b) An equivalent dated value is the value of a debt or payment that has been accumulated or discounted from the date it is made to another (focal) date;

   c) A focal date is a specific point in time selected to compare the dated values of sets of payments.

3. A debt of $500 due in 6 months means that $500 is due in 6 months and already includes any interest; a debt of $500 due in 6 months with interest means the $500 does not include interest; you have to calculate the accumulated value of the $500 at the end of 6 months by calculating $500[1 + r(6/12)]$.

EXERCISES

1. $X = 1200[1 + (0.035)\left(\frac{80}{365}\right)]^{-1} = \$1190.86$

2. $X = 100[1 + (0.06)\left(\frac{6}{12}\right)]^{-1} + 150[1 + (0.06)(1)]^{-1} = 97.09 + 141.51 = \$238.60$

3. $X = 200\left[1 + (0.08)\left(\frac{3}{12}\right)\right] + 800\left[1 + (0.08)\left(\frac{3}{12}\right)\right]^{-1} = 204 + 784.31 = \$988.31$

4. $500 + X[1 + (0.13)\left(\frac{6}{12}\right)]^{-1} = 600[1 + (0.13)\left(\frac{2}{12}\right)] + 400[1 + (0.13)\left(\frac{3}{12}\right)]^{-1}$
   $$500 + 0.938967136X = 613 + 387.41$$
   $$0.938967136X = 500.41$$
   $$X = \$532.94$$

5. $X[1 + (0.05)\left(\frac{3}{12}\right)] + X = 800[1 + (0.05)\left(\frac{7}{12}\right)]$
   $$1.0125X + X = 823.33$$
   $$2.0125X = 823.33$$
   $$X = \$409.11$$

6. $X + X[1 + (0.11)\left(\frac{60}{365}\right)]^{-1} + X[1 + (0.11)\left(\frac{120}{365}\right)]^{-1} = 2000[1 + (0.11)\left(\frac{300}{365}\right)]$
   $$X + 0.982238967X + 0.965097832X = 2180.82$$
   $$2.947336798X = 2180.82$$
   $$X = \$739.93$$

7. a) $X = 500[1 + (0.07)\left(\frac{4}{12}\right)]^{-1} + 700[1 + (0.07)\left(\frac{9}{12}\right)]^{-1}$
      $= 488.60 + 665.08 = \$1153.68$
   b) $X = 500[1 + (0.07)\left(\frac{2}{12}\right)] + 700[1 + (0.07)\left(\frac{3}{12}\right)]^{-1}$
      $= 505.83 + 687.96 = \$1193.79$

c) $X = 500[1 + (0.07)\left(\frac{8}{12}\right)] + 700[1 + (0.07)\left(\frac{3}{12}\right)]$

$\qquad = 523.33 + 712.25 = \$1235.58$

8. First Debt $= 500\left[1 + (0.06)\left(\frac{3}{12}\right)\right] = \$507.50$

Second Debt $= 200[1 + (0.06)\left(\frac{6}{12}\right)] = \$206.00$

$X + 300[1 + (0.08)(1)] = 507.50[1 + (0.08)\left(\frac{9}{12}\right)] + 206[1 + (0.08)\left(\frac{6}{12}\right)]$

$\qquad X + 324 = 537.95 + 214.221$

$\qquad\qquad X = \$428.19$

9. a) $X[1 + (0.08)\left(\frac{6}{12}\right)] + X = 1000[1 + (0.08)(1)]$

$\qquad\qquad 1.04X + X = 1080$

$\qquad\qquad 2.04X = 1080$

$\qquad\qquad X = \$529.41$

b) $X[1 + (0.08)(6/12)]^{-1} + X[1 + (0.08)(1)]^{-1} = 1000$

$\qquad 0.961538462X + 0.925925926X = 1000$

$\qquad\qquad 1.887464387X = 1000$

$\qquad\qquad X = \$529.81$

10. $2000[1 + (0.06)\left(\frac{247}{365}\right)] + 2000[1 + (0.06)\left(\frac{124}{365}\right)] + X = 5000[1 + (0.06)\left(\frac{359}{365}\right)]$

$\qquad\qquad 2081.21 + 2040.77 + X = 5295.07$

$\qquad\qquad X = \$1173.09$

11. At the end of the 6 months:

$X[1 + (0.12)\left(\frac{3}{12}\right)] + 2X = 200[1 + (0.12)\left(\frac{1}{12}\right)] + 300[1 + (0.12)\left(\frac{4}{12}\right)]^{-1}$

$\qquad\qquad 1.03X + 2X = 202 + 288.46$

$\qquad\qquad 3.03X = 490.46$

$\qquad\qquad X = \$161.87$

At the end of 3 months:

$X + 2X[1 + (0.12)\left(\frac{3}{12}\right)]^{-1} = 200[1 + (0.12)\left(\frac{2}{12}\right)]^{-1} + 300[1 + (0.12)\left(\frac{7}{12}\right)]^{-1}$

$\qquad\qquad X + 1.941747573X = 196.08 + 280.37$

$\qquad\qquad 2.941747573X = 476.45$

$\qquad\qquad X = \$161.96$

12. At the present time:
$$3000 + X[1 + (0.08)(2)]^{-1} = 2000[1 + (0.08)(5)]^{-1} + 8000[1 + (0.08)(10)]^{-1}$$
$$3000 + 0.862068965X = 1428.57 + 4444.44$$
$$0.862068965X = 2873.02$$
$$X = \$3332.70$$

At the end of 2 years
$$3000 + X[1 + (0.08)(2)] + X = 2000[1 + (0.08)(3)]^{-1} + 8000[1 + (0.08)(8)]^{-1}$$
$$3480 + X = 1612.90 + 4878.05$$
$$X = \$3010.95$$

13. Maturity value of loan 1 = $1500\left[1 + (0.072)\left(\frac{150}{365}\right)\right] = \$1544.38$

Maturity value of loan 2 = $2200\left[1 + (0.065)\left(\frac{200}{365}\right)\right] = \$2278.36$

Equivalent single payment made at the end of 180 days:

$X = 1544.38\left[1 + (0.06)\left(\frac{30}{365}\right)\right] + 2278.36\left[1 + (0.06)\left(\frac{20}{365}\right)\right]^{-1} = 1552.00 + 2270.89 =$
$\$3822.89$

14. At the present time:

$$X[1 + (0.10)\left(\tfrac{3}{12}\right)]^{-1} + 2X[1 + (0.10)\left(\tfrac{6}{12}\right)]^{-1} + 4X[1 + (0.10)\left(\tfrac{9}{12}\right)]^{-1} = 800$$
$$6.601301893X = 800$$
$$X = \$121.19$$

At the end of 3 months:
$$X + 2X[1 + (0.10)\left(\tfrac{3}{12}\right)]^{-1} + 4X[1 + (0.10)\left(\tfrac{6}{12}\right)]^{-1} = 800[1 + (0.10)\left(\tfrac{3}{12}\right)]$$
$$6.760743322X = 820$$
$$X = \$121.29$$

At the end of 6 months:
$$X[1 + (0.10)\left(\tfrac{3}{12}\right)] + 2X + 4X[1 + (0.10)\left(\tfrac{3}{12}\right)]^{-1} = 800[1 + (0.10)\left(\tfrac{6}{12}\right)]$$
$$6.927439024X = 840$$
$$X = \$121.26$$

At the end of 9 months:
$$X[1 + (0.10)\left(\tfrac{6}{12}\right)] + 2X[1 + (0.10)\left(\tfrac{3}{12}\right)] + 4X = 800[1 + (0.10)\left(\tfrac{9}{12}\right)]$$
$$7.10X = 860$$
$$X = \$121.13$$

EXERCISE 1.4

1. a) Declining Balance Method:

   End of 3 months balance $= 1000[1 + (0.09)\left(\frac{3}{12}\right)] - 200 = \$822.50$

   End of 7 months balance $= 822.50[1 + (0.09)\left(\frac{4}{12}\right)] - 400 = \$447.18$

   End of year balance $= 447.18\left[1 + (0.09)\left(\frac{5}{12}\right)\right] = \$463.95$

   b) Merchant's Rule:

   Balance due $= 1000(1.09) - 200[1 + (0.09)\left(\frac{9}{12}\right)] - 400[1 + (0.09)\left(\frac{5}{12}\right)]$
   $= 1090 - 213.50 - 415 = \$461.50$

2. a) Declining Balance Method:

   August 17/20 Balance $= 2000[1 + (0.07)\left(\frac{77}{365}\right)] - 800 = \$1229.53$

   November 20/20 Balance $= 1229.53[1 + (0.07)\left(\frac{95}{365}\right)] - 400 = \$851.93$

   February 2/21 Balance $= 851.93[1 + (0.07)\left(\frac{74}{365}\right)] - 500 = \$364.02$

   April 18/21 Balance $= 364.02[1 + (0.07)\left(\frac{75}{365}\right)] = \$369.26$

   b) Merchant's Rule: Balance due on April 18, 2021
   $= 2000[1 + (0.07)\left(\frac{321}{365}\right)] - 800[1 + (0.07)\left(\frac{244}{365}\right)] - 400[1 + (0.07)\left(\frac{149}{365}\right)]$
   $- 500[1 + (0.07)\left(\frac{75}{365}\right)]$
   $= 2123.12 - 837.44 - 411.43 - 507.19 = \$367.07$

3. a) Declining Balance Method:

   End of 2 months balance $= 5000[1 + (0.05)\left(\frac{2}{12}\right)] - 300 = \$2041.67$

   End of 4 months balance $= 2041.67[1 + (0.05)\left(\frac{2}{12}\right)] - 1000 = \$1058.68$

   End of 6 months balance $= 1058.68[1 + (0.05)\left(\frac{2}{12}\right)] = \$1067.50$

   b) Merchant's Rule: Balance due
   $= 5000[1 + (0.05)\left(\frac{6}{12}\right)] - 3000[1 + (0.05)\left(\frac{4}{12}\right)] - 1000[1 + (0.05)\left(\frac{2}{12}\right)]$
   $= 5125 - 3050 - 1008.33 = \$1066.67$

4. a) Declining Balance Method:

    April 12 Balance $= 1000[1 + (0.16)\left(\frac{94}{365}\right)] - 350 = \$691.21$

    Note that interest from April 12 to August 10 is larger than \$20

    $(691.21 \times 0.16 \times \frac{120}{365} = \$36.36)$

    October 3 Balance $= 691.21[1 + (0.16)\left(\frac{174}{365}\right)] - 420 = \$323.93$

    December 15 Balance $= 323.93[1 + (0.16)\left(\frac{73}{365}\right)] = \$334.30$

    b) Merchant's Rule:

    Balance due on December 15

    $= 1000[1 + (0.16)\left(\frac{341}{365}\right)] - 350[1 + (0.16)\left(\frac{247}{365}\right)] - 20[1 + (0.16)\left(\frac{127}{365}\right)]$

    $-400[1 + (0.16)\left(\frac{73}{365}\right)]$

    $= 1149.48 - 387.90 - 21.11 - 412.80 = \$327.67$

5. End of 2 months balance $= 1400[1 + (0.12)\left(\frac{2}{12}\right)] - 400 = \$1028.00$

    Note that interest for the next 4 months and is larger than the partial payment of \$30.00

    $((1028)(0.12)\left(\frac{4}{12}\right) = \$41.12$)

    End of 8 months balance $= 1028[1 + (0.12)\left(\frac{6}{12}\right)] - 630 = \$459.68$

    End of 12 months balance $= \$459.68[1 + (0.12)\left(\frac{4}{12}\right)] = \$478.07$

6. a) Declining Balance Method:

    August 16 balance $= 2500[1 + (0.08)\left(\frac{106}{365}\right)] - 1000 = \$1558.08$

    Note that interest due on Nov 19 is smaller than partial payment of \$40

    $(1558.05(0.08)\left(\frac{85}{365}\right) = \$29.03$)

    Thus, Nov 19 Balance $= (1558.08 + 29.03) - 40 = \$1547.11$

    Jan 9 Balance $= 1547.11[1 + (0.08)\left(\frac{51}{365}\right)] - 350 = \$1214.40$

    Mar 20 Balance $= 1214.40[1 + (0.08)\left(\frac{70}{365}\right)] = \$1233.03$

    b) Merchant's Rule:

    Balance due on March 20

    $= 2500[1 + (0.08)\left(\frac{312}{265}\right)] - 1000[1 + (0.08)\left(\frac{206}{365}\right)] - 40[1 + (0.08)\left(\frac{121}{365}\right)]$

    $-350[1 + (0.08)\left(\frac{70}{365}\right)]$

    $= 2670.96 - 1045.15 - 41.06 - 355.37 = \$1229.35$

EXERCISE 1.5

CONCEPT QUESTIONS

1. Lower

2. Higher

3. a) the loan with the simple interest rate; borrower would receive $2000, while the borrower would receive less than $2000 for the loan with the simple discount rate; b) the loan with the simple discount rate

EXERCISES

1. a) $P = 2000[1 + (0.085)(\frac{130}{365})]^{-1} = \$1941.23$
   b) $P = 2000[1 - (0.085)(\frac{130}{365})] = \$1939.45$

2. a) $P = 5000[1 - (0.045)(\frac{272}{365})] = \$4832.33$
   b) $P = 5000[1 + (0.045)(\frac{272}{365})]^{-1} = \$5173.49$
   c) $r = \frac{0.045}{1-(0.045)(\frac{272}{365})} = 0.0465614 = 4.66\%$

3. $D = 700 \times 0.08 \times \frac{45}{365} = \$6.90$; Thus, $P = 700 - 6.90 = \$693.10$

4. Maturity value $= 2000[1 + (0.07)(\frac{178}{365})] = \$2068.27$
   Discount $= 2068.27 - 2030.00 = \$38.27$
   $d = 38.27/(2068.27 \times \frac{118}{365}) = 5.72\%$

5. $S = 800/[1 + (0.10)(\frac{63}{365})] = \$814.05$

6. Maturity date of the 1st note is August 4
   Interest due on the 1st note $= 50\,000 \times 0.11 \times \frac{95}{365} = \$1431.51$
   Maturity date of the 2nd note is November 7
   $S = 50\,000/[1 - (0.12)(\frac{95}{365})] = \$51\,611.99$

7. $P = \$2869.11 \qquad S = \$3000 \qquad d = 0.065$
   $D = S \times d \times t \quad Thus, \; t = \frac{D}{S \times d} = \frac{130.89}{3000 \times 0.065} = 0.67123 = \frac{days}{365}$
   It is due to be paid in 245 days.

8. $P = \$1500; \; S = \$1580; \; D = \$80; \; t = \frac{200}{365}$; Thus, $d = \frac{D}{S \times t} = \frac{80}{1580 \times (\frac{200}{365})} = 9.24\%$

9. $P = \$1800 \qquad t = 1 \qquad d = 0.087$
   $S = P \times \frac{1}{1-dt} = 1800 \times \frac{1}{1-(0.087)(1)} = \$1971.52$

10. a) Mar 13 to Sep 2 = 173 $days$; $r = 0.10/[1 - (0.12)\left(\frac{173}{365}\right)] = 10.50\%$

    b) $r = 0.065/[1 - (0.065)(\frac{9}{12})] = 6.83\%$

    c) $r = 0.048/[1 - (0.048)\left(\frac{15}{52}\right)] = 4.87\%$

11. a) Canada: $P = 5000[1 + (0.0345)\left(\frac{91}{365}\right)]^{-1} = \$4957.36$

    USA: $P = 5000[1 - (0.0345)\left(\frac{91}{360}\right)] = \$4956.40$

    b) Canada: $P = 5000[1 + (0.0310)\left(\frac{51}{365}\right)]^{-1} = \$4978.44$

    USA: $= 5000[1 - (0.0310)\left(\frac{51}{360}\right)] = \$4978.04$

    c) Canada: $r = (4978.44 - 4957.36)/(4957.36 \times \frac{40}{365}) = 3.88\%$

    USA: $r = (4978.04 - 4956.40)/(4956.40 \times \frac{40}{365}) = 3.98\%$

12. a) $23\ 892.06 = 25000[1 + (r)\left(\frac{364}{365}\right)]^{-1} \rightarrow$ Solves for $r = 4.65\%$

    b) Sept 9, 2020 to Jan 25, 2021 is 138 days and

    Jan 25, 2021 to Sept 8, 2021 is 226 days.

    i) $24\ 102.25 = 25000[1 + (r)\left(\frac{226}{365}\right)]^{-1} \rightarrow$ Solves for $r = 6.02\%$

    ii) $r = (24\ 102.25 - 23\ 892.06)/(23\ 892.06 \times \frac{138}{365}) = 2.33\%$

13. a) Apr 25 to Aug 16 = 113 $days$; $d = 0.05/[1 + (0.05)\left(\frac{113}{365}\right)] = 4.92\%$

    b) $d = 0.04/[1 + (0.04)(\frac{11}{12})] = 3.86\%$

    c) $d = 0.075/[1 + (0.075)\left(\frac{182}{365}\right)] = 7.23\%$

REVIEW EXERCISE 1.6

1. a) $t = \frac{I}{Pr} = \frac{100}{(1000)(0.06)} = 1\left(\frac{2}{3}\right) years = 20\ months$

    b) $t = \frac{I}{Pr} = \frac{200}{(1000)(0.135)} = 1.481481481 years \doteq 541\ days$

2. Ordinary interest: $S = 1000[1 + (0.07)\left(\frac{55}{360}\right)] = \$1010.69$

    $$P = 1000[1 + (0.07)\left(\frac{55}{360}\right)]^{-1} = \$989.42$$

    Exact interest: $S = 1000[1 + (0.07)\left(\frac{55}{365}\right)] = \$1010.55$

    $$P = 1000[1 + (0.07)\left(\frac{55}{365}\right)]^{-1} = \$989.56$$

3. $r = 57/(323 \times \frac{52}{365}) = 123.87\%$

4. a) $r = \frac{I}{Pt} = \frac{240}{(7760)\left(\frac{30}{365}\right)} = 37.63\%$

    b) Interest on loan $= (7760)(0.12)\left(\frac{30}{365}\right) = \$76.54$

    Savings $= 240 - 76.54 = \$163.46$

5. a) $r = \frac{0.03x}{(0.97x)\left(\frac{40}{365}\right)} = 28.22\%$

    b) Interest on loan $= (4850)(0.08)\left(\frac{40}{365}\right) = \$42.52$

    Savings $= 150 - 42.52 = \$107.48$

6. $X[1 + (0.09)\left(\frac{143}{365}\right)] + X[1 + (0.09)\left(\frac{87}{365}\right)] + X[1 + (0.09)\left(\frac{35}{365}\right)] + 500$

    $$= 2500[1 + (0.09)\left(\frac{196}{365}\right)]$$

    $1.035260274X + 1.021452055X + 1.008630137X + 500 = 2620.82$

    $$3.065342466X = 2120.82$$
    $$X = \$691.87$$

7. $X[1 + (0.03)(1.5)] + x[1 + (0.03)(1)] + X[1 + (0.03)(0.5)] + X = 5000[1 + (0.03)(2)]$

    $$1.045X + 1.03X + 1.015X + X = 5300$$
    $$4.09X = 5300$$
    $$X = \$1295.84$$

8. $X[1 + (0.08)\left(\frac{4}{12}\right)]^{-1} + X[1 + (0.08)\left(\frac{8}{12}\right)]^{-1} + X[1 + (0.08)(1)]^{-1} = 2400$

    $$2.849318989X = 2400$$
    $$X = \$842.31$$

9. Merchant's Rule:

$$1250[1 + (0.09)\left(\tfrac{186}{365}\right)] + 2500[1 + (0.09)\left(\tfrac{114}{365}\right)] + X = 4500[1 + (0.09)\left(\tfrac{302}{365}\right)]$$

$$1307.33 + 2570.33 + X = 4835.10$$

$$X = \$957.50$$

Declining Balance Method:

October 27/20 Balance $= 4500[1 + (0.09)\left(\tfrac{116}{365}\right)] - 1250 = \$3378.71$

January 7/21 Balance $= 3378.71[1 + (0.09)\left(\tfrac{72}{365}\right)] - 2500 = \$938.69$

May 1/21 Balance $= 938.69[1 + (0.09)\left(\tfrac{114}{365}\right)] = \$965.08$

10. a) Equation of value at the end of 6 months:

$$X + X[1 + (0.075)\left(\tfrac{3}{12}\right)]^{-1} + 1000[1 + (0.075)\left(\tfrac{3}{12}\right)] = 4000[1 + (0.075)\left(\tfrac{6}{12}\right)]$$

$$X + 0.981595X + 1018.75 = 4150.00$$

$$1.981595X = 3131.25$$

$$X = \$1580.17$$

b) Equation of value at the present time:

$$X[1 + (0.075)\left(\tfrac{6}{12}\right)]^{-1} + X[1 + (0.075)\left(\tfrac{9}{12}\right)]^{-1} + 1000[1 + (0.075)\left(\tfrac{3}{12}\right)]^{-1} = 4000$$

$$0.963855421X + 0.946745562X + 981.60 = 4000$$

$$1.910600984X = 3081.40$$

$$X = \$1579.82$$

11. Merchant's Rule

On October 31:

$$500[1 + (0.08)\left(\tfrac{106}{365}\right)] + 400[1 + (0.08)\left(\tfrac{32}{365}\right)] + X = 1000[1 + (0.08)\left(\tfrac{176}{365}\right)]$$

$$511.6164 + 402.8055 + X = 1038.5753$$

$$X = \$124.15$$

Declining Balance Method:

July 17 balance $= 1000[1 + (0.08)\left(\tfrac{70}{365}\right)] - 500 = \$515.34$

September 29 balance $= 515.34[1 + (0.08)\left(\tfrac{74}{365}\right)] - 400 = \$123.70$

October 31 balance $= 123.70[1 + (0.08)\left(\tfrac{32}{265}\right)] = \$124.57$

12. a) $P = 1000[1 - (0.045)\left(\tfrac{8}{12}\right)] = \$970.00$

b) $S = 1000/[1 - (0.045)\left(\tfrac{8}{12}\right)] = \$1030.93$

c) $r = \dfrac{0.045}{\left[1 - (0.045)\left(\tfrac{8}{12}\right)\right]} = 4.64\%$

13. Maturity date: August 12, 2020

Maturity value $S = 1500[1 + (0.12)\left(\frac{93}{265}\right)] = \$1545.86$

a) $P = 1545.86[1 - (0.13)\left(\frac{41}{365}\right)] = 1523.29$

b) Prasad's profit $= \$23.29, r = \dfrac{23.29}{(1500)\left(\frac{52}{365}\right)} = 10.90\%$

c) Bank's profit $= 1545.86 - 1523.29 = \$22.57$

   $r = \dfrac{22.57}{(1523.29)\left(\frac{41}{365}\right)} = 13.19\%$

d) Bank's profit $= 1500[1 + (0.12)\left(\frac{90}{365}\right)] - 1523.29 = \$21.09$

   $r = \dfrac{21.09}{(1523.29)\left(\frac{41}{365}\right)} = 13.30\%$

   The answer is different from c) since in c), the note was assumed to be paid back on the maturity date, while in d), the note was assumed to be paid back 3 days earlier on the due date. The rate of return is higher despite losing 3 days of interest because getting the note paid back three days earlier is more valuable to the bank than receiving the extra 3 days of interest.

14. Maturity value $S = 2000[1 + (0.07)\left(\frac{183}{365}\right)] = \$2070.19$

a) $P = 2070.19[1 - (0.07)\left(\frac{123}{365}\right)] = \$2021.36$

b) $r = \dfrac{21.36}{(2000)\left(\frac{60}{365}\right)} = 6.50\%$

c) Bank will receive $2000[1 + (0.07)\left(\frac{180}{365}\right)] = \$2069.04$

   Discount $D = 2069.04 - 2021.36 = \$47.68$

   $d = \dfrac{47.68}{(2069.04)\left(\frac{120}{365}\right)} = 7.01\%$

15. Interest due on the first note $= 800(0.06)\left(\frac{93}{365}\right) = \$12.23$

   Face value of the second note $= 800/[1 - (0.07)\left(\frac{63}{365}\right)] = \$809.78$

16. Assume last payment in X days

   $1200[1 + (0.11)\left(\frac{X}{365}\right)] = 500[1 + (0.11)\left(\frac{X-45}{365}\right)] + 300[1 + (0.11)\left(\frac{X-100}{365}\right)] + 436.92$

   $1200 + 0.361643836X$

   $\qquad\qquad = 500 + 0.150684932X - 6.78 + 300 + 0.090410959X - 9.04 + 436.92$

   $0.120547945X = 21.10$

   $\qquad\qquad X = 175 \; days$

17. Exact time $= 365 - 105 + 35 = 295 \; days$

   Banker's Rule: $I = 20\,000 \times 0.09 \times \dfrac{295}{360} = \$1475.00$

   Exact interest: $I = 20\,000 \times 0.09 \times \dfrac{295}{365} = \$1454.79$

   Difference $= 1475.00 - 1454.79 = \$20.21$

18. At present time :

$$X[1 + (0.0375)\left(\tfrac{3}{12}\right)]^{-1} + X[1 + (0.0375)\left(\tfrac{7}{12}\right)]^{-1} + X[1 + (0.0375)\left(\tfrac{12}{12}\right)]^{-1} = 3000$$

$$0.990712074X + 0.978593272X + 0.963855422X = 3000$$

$$2.93316077X = 3000$$

$$X = \$1022.79$$

At the end of 12 months

$$X\left[1 + (0.0375)\left(\tfrac{9}{12}\right)\right] + X\left[1 + (0.0375)\left(\tfrac{5}{12}\right)\right] + X = 3000\left[1 + (0.0375)\left(\tfrac{12}{12}\right)\right]$$

$$1.28125X + 1.015625X + X = 3112.50$$

$$3.04375X = 3112.50$$

$$X = \$1022.59$$

Difference $= 1022.79 - 1022.59 = \$0.20$

19. Maturity Value $= 1500[1 + (0.095)\left(\tfrac{173}{365}\right)] = \$1567.54$

Proceeds $= 1567.54[1 - (0.14)\left(\tfrac{133}{365}\right)] = \$1487.57$

20. a) At the end of 8 months:

$$X[1 + (0.08)\left(\tfrac{2}{12}\right)] + 2X = 1000[1 + (0.08)\left(\tfrac{8}{12}\right)] + 2000[1 + (0.08)\left(\tfrac{4}{12}\right)]$$

$$1.0133333X + 2X = 1053.33 + 2053.33$$

$$3.01333333X = 3106.67$$

$$X = \$1030.97$$

b) At the end of 8 months:

$$X[1 - (0.08)\left(\tfrac{2}{12}\right)]^{-1} + 2X = 1000[1 - (0.08)\left(\tfrac{8}{12}\right)]^{-1} + 2000[1 - (0.08)\left(\tfrac{4}{12}\right)]^{-1}$$

$$1.013513514X + 2X = 1056.34 + 2054.79$$

$$3.013513514X = 3111.13$$

$$X = \$1032.39$$

Case Study 1 – Promissory Notes

a) Maturity Value of note= $400\ 000\left[1 + (0.075)\left(\frac{93}{365}\right)\right] = \$407\ 643.84$

   <u>20 days</u>

   Proceeds $= 407\ 643.84[1 + (0.12)\left(\frac{93-20}{365}\right)]^{-1} = \$398\ 089.69$

   <u>30 days</u>

   Proceeds $= 407\ 643.84[1 + (0.12)\left(\frac{93-30}{365}\right)]^{-1} = \$399\ 371.92$

   <u>40 days</u>

   Proceeds $= 407\ 643.84[1 + (0.12)\left(\frac{93-40}{365}\right)]^{-1} = \$400\ 662.43$

   <u>50 days</u>

   Proceeds $= 407\ 643.84[1 + (0.12)\left(\frac{93-50}{365}\right)]^{-1} = \$401\ 961.32$

   Jerry should sell the note to Bank One sometime between 30 days and 40 days from now.

b) The exact time the note should be sold in order for the proceeds to be $400,000:

   $\$400\ 000 = 407\ 643.84[1 + (0.12)\left(\frac{93-t}{365}\right)]^{-1}$
   Which solves for $t = 34.875$ days

   If Jerry sells the note to Bank One after 35 days, he will receive $400,000 (actually, he will get a bit more than $400,000).

c) If the bank uses $d = 12\%$,

   $\$400\ 000 = 407\ 643.84\left[1 - (0.12)\left(\frac{93-t}{365}\right)\right]$
   Which solves for $t = 35.965$ days

   Jerry should sell the note to Bank One after 36 days.

Case Study 2 – Student Loans

The grace period is May 1 to October 31. From May 1 to September 15 is 258 – 121 = 137 days at 2.5% + 2.5% = 5%, while September 15 to October 31 (inclusive) is 304 – 258 + 1 = 47 days at 3.0% + 2.5% = 5.50%.

The accrued interest at the end of the grace period is

I = 12 500(0.05)(137/365) + 12 500(0.055)(47/365) = $323.12

The loan balance on October 31 is 12 500 + 323.12 = 12 823.12

Date	Days	Interest Rate	Interest ($)	Accrued Interest	Monthly Payment	Principal Portion	Outstanding Balance
Nov 1	--	--	--	--	--	--	12 823.12
Nov 30	30	5.50	57.97	57.97	200.00	142.03	12 681.09
Dec 31	31	5.50	59.24	59.24	200.00	140.76	12 540.33
Jan 19	19	5.50	35.90	35.90			
Jan 31	12	5.75	23.71	59.61	200.00	140.39	12 399.94
Feb 28	28	5.75	54.70	54.70	200.00	145.30	12 254.64

Case Study 3 – Solving for a simple interest rate

a) Set up an equation of value, using the end of 12 months as the focal date:

$220[1 + i\,] \;=\; 20[1 + i\,(11/12)] + 20[1 + i\,(10/12)] + \;...\; + 20[1 + i\,(1/12)] + 20$

$220 + 220i \;=\; 20(12)\; +\; i\,(1 + 2 + \;...\; + 11)/12$

$220 + 220i = 240 + i\,(66/12)$

$i = \dfrac{240-220}{220-5.5} = \dfrac{20}{214.5} = 0.09324 = 9.324\%$

b) Set up an equation of value, using the end of 6 months as the focal date:

$220[1 + i(6/12)\,]\;=\;40[1 + i\,(5/12)] + 40[1 + i\,(4/12)] + \;...\; + 40[1 + i\,(1/12)] + 40$

$220 + 110i \;=\; 40(6)\; +\; i\,(1 + 2 + \;...\; + 5)/12$

$220 + 110i = 240 + i\,(15/12)$

$i = \dfrac{240-220}{110-1.25} = \dfrac{20}{108.75} = 0.1839 = 18.39\%$

Chapter 1

EXERCISE 1.1

1. a) $S = 2500[1 + (0.12)\left(\frac{18}{12}\right)] = \$2950$

    b) $S = 1200[1 + (0.085)\left(\frac{120}{360}\right)] = \$1234$

    c) $S = 10\ 000[1 + (0.07)\left(\frac{64}{365}\right)] = \$10\ 122.74$

    d) $S = 500[1 + (0.06)\left(\frac{175}{365}\right)] = \$5143.84$

3. a) $t = \frac{200}{1000(0.055)} = 3.636\ years \doteq 1328\ days$

    b) $t = \frac{30}{1600(0.035)} = 0.535714\ years \doteq 196\ days$

    c) $t = \frac{100}{5000(0.09)} = 0.222222\ years \doteq 82\ days$

5. $r = 2.50/(50 \times \frac{1}{12}) = 0.60 = 60\%$

7. $P = 580[1 + (0.18)\left(\frac{120}{365}\right)]^{-1} = \$547.59$

9. $S = 1000[1 + (0.075)\left(\frac{212}{365}\right)] = \$1042.11$

    Number of Days $=$ Sept. $16(259) -$ Feb. $16(47) = 212$ days

    7 months after Feb. 16 is Sept. 16

11. $r = 300/(4000 \times \frac{264}{365}) = 10.37\%$

13.

Interest Period	#of days	Balance	Rate	Interest
Feb.3-Feb.11	8	50 000	6%	$50\ 000(0.06)\left(\frac{8}{365}\right) = \underline{67.75}$
				Feb. 11 Interest $65.75
Feb.11-Feb.20	9	50 000	6%	$50\ 000(0.06)(\frac{9}{365}) = 73.97$
Feb.20-March 11	19	40 000	6%	$40\ 000(0.06)(\frac{19}{365}) = \underline{124.93}$
				March 11 interest $198.90
March 11-March 20	9	40 000	6%	$40\ 000(0.06)(\frac{9}{365}) = 59.18$
March 20-April 1	12	30 000	6%	$30\ 000(0.06)(\frac{12}{365}) = 59.18$
April 1 –April 11	10	30 000	7%	$30\ 000(0.07)(\frac{10}{365}) = \underline{57.53}$

<div align="right">April 11 interest $175.89</div>

April 11-April 20	9	30 000	7%	$30\,000(0.07)(\frac{9}{365}) = 51.78$
April 20-May 1	11	15 000	7%	$15\,000(0.07)(\frac{11}{365}) = 31.64$
May 1-May11	10	15 000	6.5%	$15\,000(0.065)(\frac{10}{365}) = \underline{26.71}$

<div align="right">May 11 interest $110.13</div>

May 11- May 20	9	15 000	6.5%	$15\,000(0.065)(\frac{9}{365}) = \underline{24.04}$

<div align="right">May 20 interest = $24.04</div>

Total interest paid $= 65.75 + 198.90 + 175.89 + 110.13 + 24.04 =$ $574.71

15. $r = 40/(1960 \times \frac{40}{365}) = 18.62\%$

17. a) $r = \frac{50}{2450 \times \frac{45}{365}} = 16.55\%$

   b) Interest on loan $= 2450 \times 0.10 \times \frac{45}{365} = \$30.21$

   Savings $= 50 - 30.21 = \$19.79$

EXERCISE 1.2

1. Maturity date is February 6
   Maturity value = $3000.00
   Discount period = 67 days
   Proceeds $= 3000[1 + (0.1025)\left(\frac{67}{365}\right)]^{-1} = \$2944.60$

3. Maturity date is June 27
   Maturity value $= 500[1 + (0.07)\left(\frac{195}{365}\right)] = \$518.70$
   Discount period = 185 days
   Proceeds $= 518.70[1 + (0.07)\left(\frac{185}{365}\right)]^{-1} = \$500.93$

5. a) Maturity value $= 2000[1 + (0.13)(\frac{93}{365})] = \$2066.25$
      Proceeds $= 2066.25[1 + (0.12)\left(\frac{42}{365}\right)]^{-1} = \$2038.11$
   b) $r = 38.11/(2000 \times \frac{51}{365}) = 13.64\%$
   c) Bank's profit $= 2000[1 + (0.13)\left(\frac{90}{365}\right)] - 2038.11 = \$26.00$
      $r = 26/(2038.11 \times \frac{39}{365}) = 11.94\%$

7. $S = 800[1 + (0.6)\left(\frac{93}{365}\right)] = \$812.23$
   $P = 812.23[1 + (0.08)\left(\frac{60}{365}\right)]^{-1} = \$801.69$

9. Maturity date is 123 days after October 17,2020, i.e. February 17, 2021
   Maturity value $= 5000[1 + (0.075)\left(\frac{123}{365}\right)] = \$5126.37$
   Proceeds $= 5126.37[1 + (0.07)\left(\frac{33}{365}\right)]^{-1} = \$5094.13$

11. Needs 97% of 2000 = $1940.00
    $S = 1940[1 + (0.06)\left(\frac{63}{365}\right)] = \$1960.09$

13. a) $P = 3500[1 + (0.06)\left(\frac{10}{12}\right)]^{-1} = \$3333.33$
    b) $P = 3500[1 + (0.06)\left(\frac{304}{365}\right)]^{-1} = \$3333.42$
       where 10 months after July 31 is May 31, which is 304 days

EXERCISE 1.3

1. $X = 1200[1 + (0.035)\left(\frac{80}{365}\right)]^{-1} = \$1190.86$

3. $X = 200\left[1 + (0.08)\left(\frac{3}{12}\right)\right] + 800\left[1 + (0.08)\left(\frac{3}{12}\right)\right]^{-1} = 204 + 784.31 = \$988.31$

5. $X[1 + (0.05)\left(\frac{3}{12}\right)] + X = 800[1 + (0.05)\left(\frac{7}{12}\right)]$

$$1.0125X + X = 823.33$$
$$2.0125X = 823.33$$
$$X = \$409.11$$

7. a) $X = 500[1 + (0.07)\left(\frac{4}{12}\right)]^{-1} + 700[1 + (0.07)\left(\frac{9}{12}\right)]^{-1}$

   $= 488.60 + 665.08 = \$1153.68$

   b) $X = 500[1 + (0.07)\left(\frac{2}{12}\right)] + 700[1 + (0.07)\left(\frac{3}{12}\right)]^{-1}$

   $= 505.83 + 687.96 = \$1193.79$

   c) $X = 500[1 + (0.07)\left(\frac{8}{12}\right)] + 700[1 + (0.07)\left(\frac{3}{12}\right)]$

   $= 523.33 + 712.25 = \$1235.58$

9. a) $X[1 + (0.08)\left(\frac{6}{12}\right)] + X = 1000[1 + (0.08)(1)]$

$$1.04X + X = 1080$$
$$2.04X = 1080$$
$$X = \$529.41$$

   b) $X[1 + (0.08)(6/12)]^{-1} + X[1 + (0.08)(1)]^{-1} = 1000$

$$0.961538462X + 0.925925926X = 1000$$
$$1.887464387X = 1000$$
$$X = \$529.81$$

11. At the end of the 6 months:

$X[1 + (0.12)\left(\frac{3}{12}\right)] + 2X = 200[1 + (0.12)\left(\frac{1}{12}\right)] + 300[1 + (0.12)\left(\frac{4}{12}\right)]^{-1}$

$$1.03X + 2X = 202 + 288.46$$
$$3.03X = 490.46$$
$$X = \$161.87$$

At the end of 3 months:

$X + 2X[1 + (0.12)\left(\frac{3}{12}\right)]^{-1} = 200[1 + (0.12)\left(\frac{2}{12}\right)]^{-1} + 300[1 + (0.12)\left(\frac{7}{12}\right)]^{-1}$

$$X + 1.941747573X = 196.08 + 280.37$$
$$2.941747573X = 476.45$$
$$X = \$161.96$$

13. Maturity value of loan 1 = $1500 \left[1 + (0.072)\left(\frac{150}{365}\right)\right] = \$1544.38$

Maturity value of loan 2 = $2200 \left[1 + (0.065)\left(\frac{200}{365}\right)\right] = \$2278.36$

Equivalent single payment made at the end of 180 days:

$X = 1544.38 \left[1 + (0.06)\left(\frac{30}{365}\right)\right] + 2278.36 \left[1 + (0.06)\left(\frac{20}{365}\right)\right]^{-1} = 1552.00 + 2270.89 =$
$\$3822.89$

EXERCISE 1.4

1. a) Declining Balance Method:

   End of 3 months balance $= 1000[1 + (0.09)\left(\frac{3}{12}\right)] - 200 = \$822.50$

   End of 7 months balance $= 822.50[1 + (0.09)\left(\frac{4}{12}\right)] - 400 = \$447.18$

   End of year balance $= 447.18\left[1 + (0.09)\left(\frac{5}{12}\right)\right] = \$463.95$

   b) Merchant's Rule:

   Balance due $= 1000(1.09) - 200[1 + (0.09)\left(\frac{9}{12}\right)] - 400[1 + (0.09)\left(\frac{5}{12}\right)]$
   $= 1090 - 213.50 - 415 = \$461.50$

3. a) Declining Balance Method:

   End of 2 months balance $= 5000[1 + (0.05)\left(\frac{2}{12}\right)] - 300 = \$2041.67$

   End of 4 months balance $= 2041.67[1 + (0.05)\left(\frac{2}{12}\right)] - 1000 = \$1058.68$

   End of 6 months balance $= 1058.68[1 + (0.05)\left(\frac{2}{12}\right)] = \$1067.50$

   b) Merchant's Rule: Balance due
   $= 5000[1 + (0.05)\left(\frac{6}{12}\right)] - 3000[1 + (0.05)\left(\frac{4}{12}\right)] - 1000[1 + (0.05)\left(\frac{2}{12}\right)]$
   $= 5125 - 3050 - 1008.33 = \$1066.67$

5. End of 2 months balance $= 1400[1 + (0.12)\left(\frac{2}{12}\right)] - 400 = \$1028.00$

   Note that interest for the next 4 months and is larger than the partial payment of \$30.00
   $((1028)(0.12)\left(\frac{4}{12}\right) = \$41.12$)

   End of 8 months balance $= 1028[1 + (0.12)\left(\frac{6}{12}\right)] - 630 = \$459.68$

   End of 12 months balance $= \$459.68[1 + (0.12)\left(\frac{4}{12}\right)] = \$478.07$

EXERCISE 1.5

1. a) $P = 2000[1 + (0.085)(\frac{130}{365})]^{-1} = \$1941.23$

   b) $P = 2000[1 - (0.085)(\frac{130}{365})] = \$1939.45$

3. $D = 700 \times 0.08 \times \frac{45}{365} = \$6.90;$ Thus, $P = 700 - 6.90 = \$693.10$

5. $S = 800/[1 + (0.10)\left(\frac{63}{365}\right)] = \$814.05$

7. $P = \$2869.11 \qquad S = \$3000 \qquad d = 0.065$

   $D = S \times d \times t \quad$ Thus, $t = \frac{D}{S \times d} = \frac{130.89}{3000 \times 0.065} = 0.67123 = \frac{days}{365}$

   It is due to be paid in 245 days.

9. $P = \$1800 \qquad t = 1 \qquad d = 0.087$

   $S = P \times \frac{1}{1-dt} = 1800 \times \frac{1}{1-(0.087)(1)} = \$1971.52$

11. a) Canada: $P = 5000[1 + (0.0345)\left(\frac{91}{365}\right)]^{-1} = \$4957.36$

    USA: $P = 5000[1 - (0.0345)\left(\frac{91}{360}\right)] = \$4956.40$

    b) Canada: $P = 5000[1 + (0.0310)\left(\frac{51}{365}\right)]^{-1} = \$4978.44$

    USA: $= 5000[1 - (0.0310)\left(\frac{51}{360}\right)] = \$4978.04$

    c) Canada: $r = (4978.44 - 4957.36)/(4957.36 \times \frac{40}{365}) = 3.88\%$

    USA: $r = (4978.04 - 4956.40)/(4956.40 \times \frac{40}{365}) = 3.98\%$

13. a) Apr 25 to Aug 16 $= 113\ days; d = 0.05/[1 + (0.05)\left(\frac{113}{365}\right)] = 4.92\%$

    b) $d = 0.04/[1 + (0.04)(\frac{11}{12})] = 3.86\%$

    c) $d = 0.075/[1 + (0.075)\left(\frac{182}{365}\right)] = 7.23\%$

REVIEW EXERCISE 1.6

1. a) $t = \frac{I}{Pr} = \frac{100}{(1000)(0.06)} = 1\left(\frac{2}{3}\right) \, years = 20 \, months$

   b) $t = \frac{I}{Pr} = \frac{200}{(1000)(0.135)} = 1.481481481 \, years \doteq 541 \, days$

3. $r = 57/(323 \times \frac{52}{365}) = 123.87\%$

5. a) $r = \frac{0.03x}{(0.97x)(\frac{40}{365})} = 28.22\%$

   b) Interest on loan $= (4850)(0.08)\left(\frac{40}{365}\right) = \$42.52$

   Savings $= 150 - 42.52 = \$107.48$

7. $X[1 + (0.03)(1.5)] + x[1 + (0.03)(1)] + X[1 + (0.03)(0.5)] + X = 5000[1 + (0.03)(2)]$

   $$1.045X + 1.03X + 1.015X + X = 5300$$
   $$4.09X = 5300$$
   $$X = \$1295.84$$

9. Merchant's Rule:

   $$1250[1 + (0.09)\left(\frac{186}{365}\right)] + 2500[1 + (0.09)\left(\frac{114}{365}\right)] + X = 4500[1 + (0.09)\left(\frac{302}{365}\right)]$$
   $$1307.33 + 2570.33 + X = 4835.10$$
   $$X = \$957.50$$

   Declining Balance Method:

   October 27/20 Balance $= 4500[1 + (0.09)\left(\frac{116}{365}\right)] - 1250 = \$3378.71$

   January 7/21 Balance $= 3378.71[1 + (0.09)\left(\frac{72}{365}\right)] - 2500 = \$938.69$

   May 1/21 Balance $= 938.69[1 + (0.09)\left(\frac{114}{365}\right)] = \$965.08$

11. Merchant's Rule

    On October 31:

    $$500[1 + (0.08)\left(\frac{106}{365}\right)] + 400[1 + (0.08)\left(\frac{32}{365}\right)] + X = 1000[1 + (0.08)\left(\frac{176}{365}\right)]$$
    $$511.6164 + 402.8055 + X = 1038.5753$$
    $$X = \$124.15$$

    Declining Balance Method:

    July 17 balance $= 1000[1 + (0.08)\left(\frac{70}{365}\right)] - 500 = \$515.34$

    September 29 balance $= 515.34[1 + (0.08)\left(\frac{74}{365}\right)] - 400 = \$123.70$

    October 31 balance $= 123.70[1 + (0.08)\left(\frac{32}{265}\right)] = \$124.57$

13. Maturity date: August 12, 2020

    Maturity value $S = 1500[1 + (0.12)\left(\frac{93}{265}\right)] = \$1545.86$

    a) $P = 1545.86[1 - (0.13)\left(\frac{41}{365}\right)] = 1523.29$

b) Prasad's profit = $23.29, $r = \dfrac{23.29}{(1500)\left(\frac{52}{365}\right)} = 10.90\%$

c) Bank's profit = $1545.86 - 1523.29 = \$22.57$

$r = \dfrac{22.57}{(1523.29)\left(\frac{41}{365}\right)} = 13.19\%$

d) Bank's profit = $1500[1 + (0.12)\left(\frac{90}{365}\right)] - 1523.29 = \$21.09$

$r = \dfrac{21.09}{(1523.29)\left(\frac{41}{365}\right)} = 13.30\%$

The answer is different from c) since in c), the note was assumed to be paid back on the maturity date, while in d), the note was assumed to be paid back 3 days earlier on the due date. The rate of return is higher despite losing 3 days of interest because getting the note paid back three days earlier is more valuable to the bank than receiving the extra 3 days of interest.

15. Interest due on the first note = $800(0.06)\left(\frac{93}{365}\right) = \$12.23$

Face value of the second note = $800/[1 - (0.07)\left(\frac{63}{365}\right)] = \$809.78$

17. Exact time = $365 - 105 + 35 = 295\ days$

Banker's Rule: $I = 20\,000 \times 0.09 \times \dfrac{295}{360} = \$1475.00$

Exact interest: $I = 20\,000 \times 0.09 \times \dfrac{295}{365} = \$1454.79$

Difference = $1475.00 - 1454.79 = \$20.21$

19. Maturity Value = $1500[1 + (0.095)\left(\frac{173}{365}\right)] = \$1567.54$

Proceeds = $1567.54[1 - (0.14)\left(\frac{133}{365}\right)] = \$1487.57$