Solutions for Principles of Corporate Finance 14th Edition by Brealey

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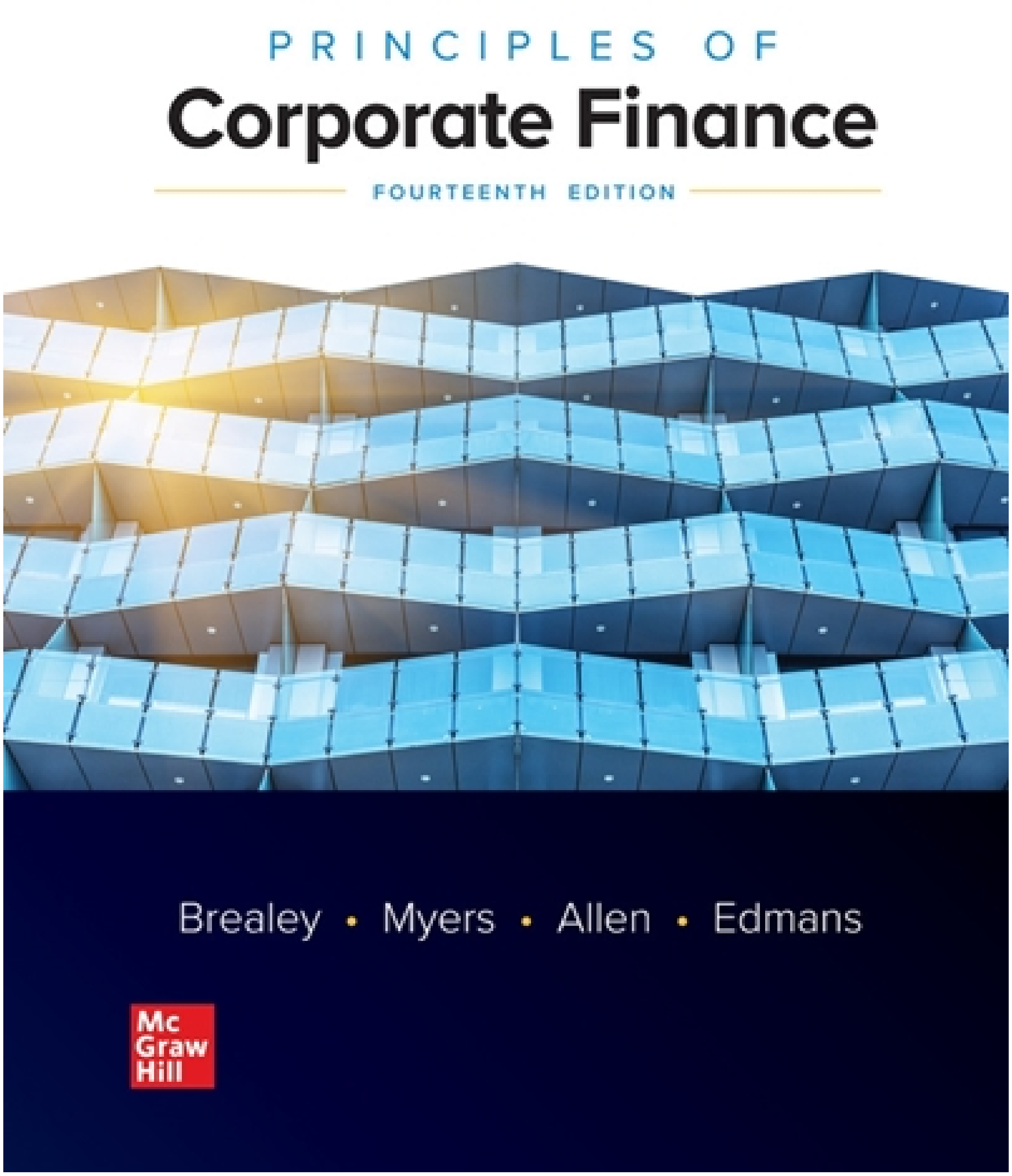


Solutions

Chapter 2 How to Calculate Present Values

OVERVIEW

This chapter introduces the concept of present value and shows why a firm should maximize the market value of the stockholders' stake in it. It describes the mechanics of calculating present values of lump sum amounts, perpetuities, annuities, growing perpetuities, growing annuities, and unequal cash flows. Other related topics like simple interest, frequent compounding, continuous compounding, and nominal and effective interest rates are discussed. The net present value rule and the rate of return rule are explained in great detail.

LEARNING OBJECTIVES

- To learn how to calculate present value of lump sum cash flows.
- To understand and use the formulas associated with the present value of perpetuities, growth perpetuities, annuities, and growing annuities.
- To understand more frequent compounding, including continuous compounding.
- To understand the important difference between nominal and effective interest rates.
- To understand the net present value rule and the rate of return rule.

CHAPTER OUTLINE

Future values and present values

The concepts of future value, present value, net present value (NPV), and the opportunity cost of capital (hurdle rate) are introduced. The authors show, using several numerical examples, that simple projects with rates of return exceeding the opportunity cost of capital have positive net present values. The "net present value rule" and the "rate of return rule" are stated here.

This chapter also extends the concept of discounting to assets, which produce a series of cash flows. Using numerical examples, it shows how to calculate the PV and NPV of a series of cash flows over a number of periods (years).

How do value perpetuities and annuities

This section is devoted to developing formulas for perpetuities and annuities. It explains the difference between an ordinary annuity and an annuity due. It also explains how the future value of an annuity is calculated. The present value of an annuity can be thought of as the difference between two perpetuities beginning at different times. Using this simple idea, the formula for the present value of an annuity is derived. The future value of an annuity formula is also derived. These have numerous applications in pension funds, mortgages, and valuation of financial assets.

How to value growing perpetuities and annuities

Some applications need the present value of a perpetual cash flow growing at a constant rate, as well as annuities that grow at a constant rate. The formula for the present value of a growing perpetuity is derived. The present value of a growing annuity can be thought of as the difference between two growing perpetuities starting at different times. Using this simple idea, the formula for the present value of a growing annuity is also derived. These formulas have many applications in the valuation of assets.

How interest is paid and quoted

This section explains the differences between compound interest and simple interest, as well as the differences between effective annual rates and annual percentage rates. It deals with how each interest rate is used in the marketplace and the math necessary to move between the two kinds of interest rates.

TEACHING TIPS FOR POWERPOINT SLIDES

Slide 1—Title Slide

Slide 2—Topics Covered

Slide 3—Present Value and Future Value

Explain the terms "future value" and "present value." The concept must be emphasized at this point. Consequently, it may be necessary to spend some time explaining real-world examples of how present value and future value relate. A good example to use is retirement planning.

Slide 4—Future Values

$FV = PV \times (1 + r)^t$

Define the terms:

FV = Future value
PV = Present value
r = interest rate
t = number of years (periods)

Explain the time value of money and its importance to financial decision making.

Slide 5—Future Values Continued

Walk through each step in the math process and show how the value increases. If you plan to have your students use a financial calculator, you can skip the details of the basic math. Be aware that students often stumble when doing simple math calculations.

Slide 6—Figure 2.1 Future Values With Compounding

The longer the funds are invested, the greater the advantage with compound interest. Discuss the four examples and be sure to use the phrase "power of compounding."

Slide 7—Present Value

The discount factor (DF) is the present value of $1 expected to be received in the future. Here, it is appropriate to introduce the use of the financial calculator to solve these problems.

Slide 8—Present Value Continued

This slide contains the present value formula.

Slide 9—Present Value Concluded

Here we reverse the future value process from earlier. Show students how they can easily move between future value and present value with the basic formulas.

Slide 10—Figure 2.2 Present Values With Compounding

For visual learners, this graph illustrates the reverse of the future value compounding chart shown earlier. It is downward sloping, which can confuse students; it may be necessary to explain the concept.

Slide 11—Valuing an Investment Opportunity

Explain how the present value concept discussed earlier is useful in valuing assets.

Cost of the building = $700,000
Sale price in Year 1 = $800,000
Opportunity cost of capital = 7%

Slide 12—Valuing an Investment Opportunity Continued

Discount future cash flows at the opportunity cost of capital
PV = $800,000/(1.07) = $747,664

NPV = PV——required investment
= $747,664——$700,000 = $47,664
Explain the difference between PV and NPV. Explain sign conventions for cash flows.

Slide 13—Net Present Value

Explain each variable in the equation. It is easy to tell the students that all present values come at a cost. That cost is the initial investment. This may help transition from present value to net present value.

C_0 = initial investment for the project. Normally it is a cash outflow and has a negative sign (−)
C_1 = cash inflow from the project. Normally it has a positive sign (+)
r = opportunity cost of capital

Positive NPVs increase the value of a firm. Negative NPVs lower the value of a firm.

Slide 14—Figure 2.5 NPV Calculation

This figure illustrates the calculation showing the NPV of the office development example.

Slides 15 and 16—Risk and Present Value and Risk and Net Present Value

The concept of risk is introduced here. Explain the idea of risk (lottery versus bank deposit). Generally, investors do not like risk. In order to induce the investors to invest in risky projects, a higher rate of return is needed. Higher rate of return causes lower PVs. Explain the relationship between discount rates and net present values. The higher the discount rate, the lower the net present value.

NPV at 12%: NPV = $714,286——700,000 = $14,286
NPV at 7%: NPV = $747,664——700,000 = $47,664

Slide 17—Net Present Value Rule

Net Present Value Rule
Accept if NPV > 0: A very powerful financial decision-making rule. It looks simple but can get complicated quickly. This project is acceptable as the NPV > 0. Make sure that students understand this rule clearly.

Slide 18—Rate of Return Rule

This slide explains the rate of return rule.

Slide 19—Calculating Present Values When There Are Multiple Cash Flows

Multiple cash flows occurring at different time periods can be evaluated using the DCF formula. It is a simple extension of the NPV formula but can intimidate students because of the extra equations.

Slide 20—Figure 2.5 NPV Calculation

The graphic presentation of the net present value of multiple cash flows or sequential cash flows is given here. Here, we extend the concept of PV to a series of cash flows by applying the value-additive property of present values. These cash flows can be positive (cash inflows) or negative (cash outflows). We merely add the initial cost to make it NPV.

Slide 21—How to Value Perpetuities

Depending on the type of cash flow, use the formulas to simplify the calculations. There are formulas that can be used for finding the present values for cash flows with a pattern; for example, perpetuities and annuities. Define perpetuity (level cash flow each year forever) and give an example of perpetuity.

Slide 22—Shortcuts

Introduce the perpetuity concept as one in which you earn money forever. In doing so, you can easily demonstrate the return an investor earns.

Slide 23—Shortcuts Continued

Now, manipulate the formula to get the value of the infinite cash flow given a discount rate. Provide the formula for calculating the present value perpetuity. This formula is obtained using an algebraic technique; sum of an infinite geometric series.

Slide 24—Present Values

Present value of $1 billion received forever at 10% is PV = $1/0.1 = $10 billion. Using the formula will simplify the calculations.

Slide 25—Present Values Continued

The same example is used as in the previous slide, except the modification of time is added. Show the students how the value is reduced if you get the money later. This reinforces the time value of money concepts introduced earlier.

Slide 26—How to Value Annuities

This slide provides the formula for the present value of an annuity. Note, an annuity can be thought of as the difference between two perpetuities starting at different times.

Slide 27—Perpetuities and Annuities

This is the PVAF formula. Take some time to explain the variables. If a financial calculator is to be used in class, there is no need to cover the use in detail.

Slide 28—Figure 2.8 Annuity

This slide is a more comprehensive example of an annuity and its relationship to perpetuities.

Slide 29—Figure 2.9 Costing an Installment Plan

An asset that pays a fixed sum each period for a specified number of periods is called an annuity. As an example, the present value of annual payments of $5,000 per year for 5 years is presented. Using a financial calculator, PMT = 5,000; *I* = 7; N = 5; FV = 0; and compute PV = 20,501.

Slide 30—Example 2.3 Paying off a Bank Loan

This is an example of an annuity. In this case we are determining the payment necessary on a loan.

Slide 31—Table 2.1 Amortizing Loan Example

An example of an amortizing loan. If you borrow $1,000 at an interest rate of 10%, you would need to make an annual payment of $315.47 over 4 years to repay that loan with interest.

Slides 32, 33, and 34—Future Value of an Annuity

As was done for the present value of an annuity earlier, these next few slides present the future value of an annuity.

Slide 35—Valuing Annuities Due

This is the formula that reverses the PVAF math for future values.

Slide 36—Growing Perpetuities

This formula is the present value of a perpetuity that is growing at a constant rate, where g is the annual growth rate of the cash flow, and $r > g$. It is useful and should be explained as a formula the students will use often.

Slide 37—Growth Perpetuity Example

The present value of a $1 billion (first payment starting one year from today) perpetuity that is growing at a constant rate of 4% and requires a rate of return of 10% is $PV_0 = C_1/(r - g)$ = $1 billion/(0.1 − 0.04) = 16.667 billion. This is $6.667 billion more than the perpetuity without growth.

Slide 38—How Interest Is Paid and Quoted

Go over each definition: EAR and APR. Students need to know how each is used in the market and why APR is quoted rather than EAR.

Slide 39—EAR and APR Formulas

The basic formulas for APR and EAR are presented. In reality, students are more likely to use the spreadsheet or financial calculator.

Slide 40—Effective Interest Rates

Go over the math of the problem. If students are comfortable with the use of a financial calculator, use the following method as a substitute for the formula. Using the financial calculator: PMT = 0; *I* = 1%; *N* = 12; PV = 1; FV = −1.1268. By dropping the (−1) students will arrive at the answer of 12.68%.

KEY TERMS AND CONCEPTS

Present value, discount factor, discount rate, hurdle rate, opportunity cost of capital, net present value, net present value rule, rate of return rule, discounted cash flow, perpetuity, growing perpetuity, annuity, growing annuity, compound interest, annual percentage rate, and effective annual rate.

CHALLENGE AREAS

The link between financial markets and NPV

ADDITIONAL REFERENCES

Allen, F., & Gale, D. A. (2002, December). Comparative theory of corporate governance, (Working Paper 03–27). Financial Institutions Center, The Wharton School, University of Pennsylvania.

White, M. (2004). *Financial analysis with a calculator* (5th ed.). McGraw Hill/Irwin

Copeland, T. E., Weston, J. F., & Shastri, K. (2005). *Financial theory and corporate policy* (4th ed.). Pearson Addison Wesley.

Benninga, S. (2008). *Financial modeling* (3rd ed.). The MIT Press

CHAPTER 2
How to Calculate Present Values

The values shown in the solutions may be rounded for display purposes. However, the answers were derived using a spreadsheet without any intermediate rounding.

Answers to Problem Sets

1. a. False. The opportunity cost of capital varies with the risks associated with each individual project or investment. The cost of borrowing is unrelated to these risks.

   b. True. The opportunity cost of capital depends on the risks associated with each project and its cash flows.

   c. True. The opportunity cost of capital is dependent on the rates of returns shareholders can earn on the own by investing in projects with similar risks

   d. False. Bank accounts, within FDIC limits, are considered to be risk-free. Unless an investment is also risk-free, its opportunity cost of capital must be adjusted upward to account for the associated risks.

Est time: 01-05

2. a. In the first year, you will earn \$1,000 × 0.04 = \$40.00

   b. In the second year, you will earn \$1,040 × 0.04 = \$41.60

   c. By the end of the ninth year, you will accrue a principle of \$1,040 × (1.04^9) = \$1,423.31. Therefore, in the Tenth year, you will earn \$1,423.31 × 0.04 = \$56.93

Est time: 01-05

3.

$$Transistors_{2019} = Transistors_{1972} \times (1+r)^t$$

$$32,000,000,000 = 2,250 \times (1+r)^{48}$$

$$\Rightarrow$$

$$r = 40.94\% < 59.00\% = r_{\text{Predicted}}$$

Est time: 01-05

4. The "Rule of 72" is a rule of thumb that says with discrete compounding the time it takes for an investment to double in value is roughly 72/interest rate (in percent).
Therefore, without a calculator, the Rule of 72 estimate is:

   Time to double = 72 / *r*

   Time to double = 72 / 4

   Time to double = 18 years, so less than 25 years.

If you did have a calculator handy, this estimate is verified as followed:

C_t = PV × $(1 + r)^t$
t = ln2 / ln1.04
t = 17.67 years
Est time: 01-05

5. a. Using the inflation adjusted 1958 price of $1,060, the real return per annum is:

$\$450,300,000 = \$1,060 \times (1 + r)^{(2017-1958)}$

$r = [\$450,300,000/\$1,060]^{(1/59)} - 1 = 0.2456$ or 24.56% per annum

b. Using the inflation adjusted 1519 price of $575,000, the real return per annum is:

$\$450,300,000 = \$575,000 \times (1 + r)^{(2017-1519)}$

$r = [\$450,300,000/\$575,000]^{(1/498)} - 1 = 0.0135$ or 1.35% per annum
Est time: 01-05

6. C_t = PV × $(1 + r)^t$
C_8 = \$100 × 1.15^8
C_8 = \$305.90
Est time: 01-05

7. a. C_t = PV × $(1 + r)^t$
C_{10} = \$100 × 1.06^{10}
C_{10} = \$179.08

b. C_t = PV × $(1 + r)^t$
C_{20} = \$100 × 1.06^{20}
C_{20} = \$320.71

c. C_t = PV × $(1 + r)^t$
C_{10} = \$100 × 1.04^{10}
C_{10} = \$148.02

d. C_t = PV × $(1 + r)^t$
C_{20} = \$100 × 1.04^{20}
C_{20} = \$219.11
Est time: 01-05

8. a. PV = C_t × DF_t
DF_t = \$125 / \$139
DF_t = .8993

b. $C_t = PV \times (1 + r)^t$
$\$139 = \$125 \times (1+r)^5$
$r = [\$139/\$125]^{(1/5)} - 1 = 0.0215$ or 2.15%

Est time: 01-05

9. $PV = C_t / (1 + r)^t$
$PV = \$374 / 1.09^9$
PV = $172.20

Est time: 01-05

10. $PV = C_1 / (1 + r)^1 + C_2 / (1 + r)^2 + C_3 / (1 + r)^3$
$PV = \$432 / 1.15 + \$137 / 1.15^2 + \$797 / 1.15^3$
PV = $1,003.28

NPV = PV – investment
NPV = $1,003.28 – 1,200
NPV = –$196.72

Est time: 01-05

11. The basic present value formula is: $PV = C / (1 + r)^t$

a. $PV = \$100 / 1.01^{10}$
PV = $90.53

b. $PV = \$100 / 1.13^{10}$
PV = $29.46

c. $PV = \$100 / 1.25^{15}$
PV = $3.52

d. $PV = C_1 / (1 + r) + C_2 / (1 + r)^2 + C_3 / (1 + r)^3$
$PV = \$100 / 1.12 + \$100 / 1.12^2 + \$100 / 1.12^3$
PV = $89.29 + $79.72 + $71.18
PV = $240.18

Est time: 01-05

12. $$NPV = \sum_{t=0}^{10} \frac{C_t}{(1.12)^t}$$

$NPV = -\$380{,}000 + \$50{,}000 / 1.12 + \$57{,}000 / 1.12^2 + \$75{,}000 / 1.12^3 + \$80{,}000 / 1.12^4 + \$85{,}000 / 1.12^5 + \$92{,}000 / 1.12^6 + \$92{,}000 / 1.12^7 + \$80{,}000 / 1.12^8 + \$68{,}000 / 1.12^9 + \$50{,}000 / 1.12^{10}$

NPV = $23,696.15

Est time: 01-05

13. a. NPV = – Investment + C × ((1 / r) – {1 / [r(1 + r)^t]})

NPV = –$800,000 + $170,000 × ((1 / .14) – {1 / [.14(1.14)^{10}]})

NPV = $86,739.66

b. After five years, the factory's value will be the present value of the remaining cash flows:

PV = $170,000 × ((1 / .14) – {1 / [.14(1.14)$^{(10 – 5)}$]})

PV = $583,623.76

Est time: 01-05

14. Use the formula: NPV = $-C_0 + C_1 / (1 + r) + C_2 / (1 + r)^2$

NPV$_{5\%}$ = –$700,000 + $30,000 / 1.05 + $870,000 / 1.05^2

NPV$_{5\%}$ = $117,687.07

NPV$_{10\%}$ = –$700,000 + $30,000 / 1.10 + $870,000 / 1.10^2

NPV$_{10\%}$ = $46,280.99

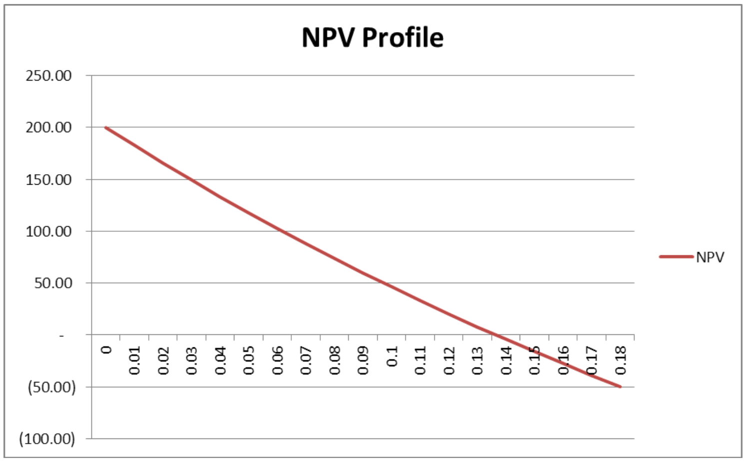
NPV$_{15\%}$ = –$700,000 + $30,000 / 1.15 + $870,000 / 1.15^2

NPV$_{15\%}$ = –$16,068.05

The figure below shows that the project has a zero NPV at about 13.65%.

NPV$_{13.65\%}$ = –$700,000 + $30,000 / 1.1365 + $870,000 / 1.1365^2

NPV$_{13.65\%}$ = –$36.83



Est time: 11-15

15. a. $NPV = –Investment + PVA_{operating\ cash\ flows} – PV_{refits} + PV_{scrap\ value}$

$NPV = –\$8,000,000 + (\$5,000,000 – 4,000,000) × ((1 / .08) – \{1 / [.08(1.08)^{15}]\}) –$
$(\$2,000,000 / 1.08^5 + \$2,000,000 / 1.08^{10}) + \$1,500,000 / 1.08^{15}$

NPV = –$8,000,000 + 8,559,479 – 2,287,553 + 472,863

NPV = –$1,255,212

b. The cost of borrowing does not affect the NPV because the opportunity cost of capital depends on the use of the funds, not the source.

Est time: 06-10

16. NPV = *C* / *r* – investment
NPV = $138 / .09 − $1,548
NPV = −$14.67

Est time: 01-05

17. One way to approach this problem is to solve for the present value of:

(1) $100 per year for 10 years, and
(2) $100 per year in perpetuity, with the first cash flow at year 11.

If this is a fair deal, the present values must be equal, thus solve for the interest rate (*r*).

The present value of $100 per year for 10 years is:

$PV = C × ((1 / r) – \{1 / [r × (1 + r)^t]\})$

$PV = \$100 × ((1 / r) – \{1 / [r × (1 + r)^{10}]\})$

The present value, as of year 0, of $100 per year forever, with the first payment in year 11, is:

$PV = (C / r) / (1 + r)^t$

$PV = (\$100 / r) / (1 + r)^{10}$

Equating these two present values, we have:

$\$100 × ((1 / r) – \{1 / [r × (1 + r)^{10}]\}) = (\$100 / r) / (1 + r)^{10}$

Using trial and error or algebraic solution, *r* = 7.18%.

Est time: 06-10

18. a. $PV = C / r$
$PV = \$1 / .10$
$PV = \$10$

b. $PV_7 = (C_8 / r)$
$PV_{0\ approx} = (C_8 / r) / 2$
$PV_{0\ approx} = (\$1 / .10) / 2$
$PV_{0\ approx} = \$5$

c. A perpetuity paying $1 starting now would be worth $10 (part a), whereas a perpetuity starting in year 8 would be worth roughly $5 (part b). Thus, a payment of $1 for the next seven years would also be worth approximately $5 (= $10 – 5).

d. $PV = C / (r - g)$
$PV = \$10{,}000 / (.10 - .05)$
$PV = \$200{,}000$

Est time: 06-10

19. a. $DF_1 = 1 / (1 + r)$
$r = (1 – .905) / .905$
$r = .1050$, or 10.50%

b. $DF_2 = 1 / (1 + r)^2$
$DF_2 = 1 / 1.105^2$
$DF_2 = .8190$

c. $PVAF_2 = DF_1 + DF_2$
$PVAF_2 = .905 + .819$
$PVAF_2 = 1.7240$

d. $PVA = C \times PVAF_3$
$PVAF_3 = \$24.65 / \$10$
$PVAF_3 = 2.4650$

e. $PVAF_3 = PVAF_2 + DF_3$
$DF_3 = 2.465 – 1.7240$
$DF_3 = .7410$

Est time: 06-10

20. $PV = C_t / (1 + r)^t$

$PV = \$20{,}000 / 1.10^5$

$PV = \$12{,}418.43$

$C = PVA / ((1 / r) – \{1 / [r(1 + r)^t]\})$

$C = \$12{,}418.43 / ((1 / .10) – \{1 / [.10 (1 + .10)^5]\})$

$C = \$3{,}275.95$

Est time: 06-10

21. $C = \text{PVA} / ((1 / r) - \{1 / [r(1 + r)^t]\})$

$C = \$20{,}000 / ((1 / .08) - \{1 / [.08(1 + .08)^{12}]\})$

$C = \$2{,}653.90$

Est time: 01-05

22. a. $\text{PV} = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$

$\text{PV} = (\$9{,}420{,}713 / 19) \times ((1 / .08) - \{1 / [.08(1 + .08)^{19}]\})$

$\text{PV} = \$4{,}761{,}724$

b. $\text{PV} = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$

$\$4{,}200{,}000 = (\$9{,}420{,}713 / 19) \times ((1 / r) - \{1 / [r(1 + r)^t]\})$

Using Excel or a financial calculator, we find that $r = 9.81\%$.

Est time: 06-10

23. a. $\text{PV} = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$

$\text{PV} = \$50{,}000 \times ((1 / .055) - \{1 / [.055(1 + .055)^{12}]\})$

$\text{PV} = \$430{,}925.89$

b. Since the payments now arrive six months earlier than previously:

$\text{PV} = \$430{,}925.89 \times \{1 + [(1 + .055)^{.5} - 1]\}$

$\text{PV} = \$442{,}617.74$

Est time: 06-10

24. $C_t = \text{PV} \times (1 + r)^t$

$C_t = \$1{,}000{,}000 \times (1.035)^3$

$C_t = \$1{,}108{,}718$

Annual retirement shortfall = 12 × (monthly aftertax pension + monthly aftertax Social Security – monthly living expenses)
= 12 × ($7,500 + 1,500 – 15,000)
= –$72,000

The withdrawals are an annuity due, so:

$\text{PV} = C \times ((1 / \text{r}) - \{1 / [r(1 + r)^t]\}) \times (1 + r)$

$\$1{,}108{,}718 = \$72{,}000 \times ((1 / .035) - \{1 / [.035(1 + .035)^t]\}) \times (1 + .035)$

$14.878127 = (1 / .035) - \{1 / [.035(1 + .035)^t]\}$

$13.693302 = 1 / [.035(1 + .035)^t]$

$.073028 / .035 = 1.035t$

$t = \ln 2.086514 / \ln 1.035$

$t = 21.38$ years

Est time: 06-10

25. a. PV = C / r = \$1 billion / .08
PV = \$12.5 billion

b. PV = $C / (r – g)$ = \$1 billion / (.08 – .04)
PV = \$25.0 billion

c. PV = $C \times ((1 / r) – \{1 / [r (1 + r)^t]\})$ = \$1 billion × $((1 / .08) – \{1 / [.08(1 + .08)^{20}]\})$
PV = \$9.818 billion

d. The continuously compounded equivalent to an annually compounded rate of 8% is approximately 7.7%, which is computed as:

Ln(1.08) = .077, or 7.7%

PV = $C \times \{(1 / r) – [1 / (r \times e^{rt})]\}$ = \$1 billion × $\{(1 / .077) – [1 / (.077 – e^{.077 \times 20})]\}$

PV = \$10.206 billion

This result is greater than the answer in Part (c) because the endowment is now earning interest during the entire year.

Est time: 06-10

26. a. PV = $C \times ((1 / r) – \{1 / [r(1 + r)^t]\})$
PV = \$2.0 million × $((1 / .08) – \{1 / [.08(1.08)^{20}]\})$
PV = \$19.64 million

b. If each cashflow arrives one year earlier, then you can simply compound the PV calculated in part a by (1+r)→ \$19.64 million × (1.08) = \$21.21 million

Est time: 01-05

27. a. Start by calculating the present value of an annuity due assuming a price of \$1:
PV = 0.25 + 0.25 × $((1 / .05) – \{1 / [.05(1.05)^3]\})$
PV = 0.93, therefore it is better to pay instantly at a lower cost of 0.90 [= 1 × 0.9]

b. Recalculate, except this time using an ordinary annuity:
PV = 0.25 × $((1 / .05) – \{1 / [.05(1.05)^4]\})$
PV = 0.89, therefore it is better to take the financing deal as it costs less than 0.90.

Est time: 06-10

28. a. Using the annuity formula:
$PV = \$70{,}000 \times ((1 / .08) - \{1 / [.08(1 + .08)^{8}]\})$
PV = $402,264.73

b. The amortization table follows:

Year	Beg Bal.	Payment	Interest (8%)	Loan Red.	Ending Bal.
1	$ 402,265	$ (70,000)	$ (32,181)	$ (37,819)	$ 364,446
2	364,446	(70,000)	(29,156)	(40,844)	323,602
3	323,602	(70,000)	(25,888)	(44,112)	279,490
4	279,490	(70,000)	(22,359)	(47,641)	231,849
5	231,849	(70,000)	(18,548)	(51,452)	180,397
6	180,397	(70,000)	(14,432)	(55,568)	124,829
7	124,829	(70,000)	(9,986)	(60,014)	64,815
8	64,815	(70,000)	(5,185)	(64,815)	-

Est time: 06-10

29. a. $PV = C \times ((1 / r) - \{1 / [r(1 + r)^{t}]\})$
$C = PV / ((1 / r) - \{1 / [r (1 + r)^{t}]\})$
$C = \$200{,}000 / ((1 / .06) - \{1 / [.06(1 + .06)^{20}]\})$
C = $17,436.91

b.

Year	Beg Bal.	Payment	Interest	Loan Red.	Ending Bal.
1	$ 200,000.00	$ (17,436.91)	$ (12,000.00)	$ (5,436.91)	$ 194,563.09
2	194,563.09	(17,436.91)	(11,673.79)	(5,763.13)	188,799.96
3	188,799.96	(17,436.91)	(11,328.00)	(6,108.91)	182,691.05
4	182,691.05	(17,436.91)	(10,961.46)	(6,475.45)	176,215.60
5	176,215.60	(17,436.91)	(10,572.94)	(6,863.98)	169,351.63
6	169,351.63	(17,436.91)	(10,161.10)	(7,275.81)	162,075.81
7	162,075.81	(17,436.91)	(9,724.55)	(7,712.36)	154,363.45
8	154,363.45	(17,436.91)	(9,261.81)	(8,175.10)	146,188.34
9	146,188.34	(17,436.91)	(8,771.30)	(8,665.61)	137,522.73
10	137,522.73	(17,436.91)	(8,251.36)	(9,185.55)	128,337.19
11	128,337.19	(17,436.91)	(7,700.23)	(9,736.68)	118,600.51
12	118,600.51	(17,436.91)	(7,116.03)	(10,320.88)	108,279.62
13	108,279.62	(17,436.91)	(6,496.78)	(10,940.13)	97,339.49
14	97,339.49	(17,436.91)	(5,840.37)	(11,596.54)	85,742.95
15	85,742.95	(17,436.91)	(5,144.58)	(12,292.33)	73,450.61
16	73,450.61	(17,436.91)	(4,407.04)	(13,029.87)	60,420.74
17	60,420.74	(17,436.91)	(3,625.24)	(13,811.67)	46,609.07
18	46,609.07	(17,436.91)	(2,796.54)	(14,640.37)	31,968.71
19	31,968.71	(17,436.91)	(1,918.12)	(15,518.79)	16,449.92
20	16,449.92	(17,436.91)	(986.99)	(16,449.92)	-

c. Interest percent of first payment = Interest_1 / Payment

Interest percent of first payment = (.06 × \$200,000) / \$17,436.91

Interest percent of first payment = .6882, or 68.82%

Interest percent of last payment = Interest_{20} / Payment = \$986.99 / \$17,436.91

Interest percent of last payment = .0566, or 5.66%

Without creating an amortization schedule, the interest percent of the last payment can be computed as:

Interest percent of last payment = 1 – {[Payment / (1 + r)] / Payment}

Interest percent of last payment = 1 – [(\$17,436.91 / 1.06) / \$17,436.91]

Interest percent of last payment = .0566, or 5.66%

After 10 years, the balance is:

$PV_{10} = C \times ((1 + r) - \{1 / [r \times (1 + r)^t]\}) = \$17,436.91 \times \{1.06 - [1 / (.06 \times 1.06^{10})]\}$

PV_{10} = \$128,337.19

Fraction of loan paid off = (Loan amount – PV_{10}) / Loan amount

= (\$200,000 – 128,337.19) / \$200,000

Fraction of loan paid off = .3583, or 35.83%

Though 50% of time has passed, only 35.83% of the loan has been paid off; this is because interest comprises a higher portion of the monthly payments at the beginning of the loan (e.g., Interest percent of first payment > interest percent of last payment).

Est time: 16-20

30. a. $PV = C_t / (1 + r)^t = \$10,000 / 1.05^5$
PV = \$7,835.26

b. $PV = C((1 / r) - \{1 / [r(1 + r)^t]\}) = \$12,000((1 / .08) - \{1 / [.08(1.08)^6]\})$
PV = \$55,474.56

c. $C_t = PV \times (1 + r)^t = (\$60,476 - 55,474.56) \times 1.08^6$
C_t = \$7,936.66

Est time: 06-10

31. $PV_{stock} = \dfrac{\$4.00}{.14 - .04} = \$40.00$

Est time: 01-05

32. a. PV = C × ((1 / r) – {1 / [$r(1 + r)^t$]})
C = \$2,000,000 / ((1 / .08) – {1 / [.08(1 + .08)^{15}]})
C = \$233,659.09

b. r = (1 + R) / (1 + h) – 1 = 1.08 / 1.04 – 1
r = .0385, or 3.85%

PV = C × ((1 / r) – {1 / [$r(1 + r)^t$]})
C = \$2,000,000 / ((1 / .0385) – {1 / [.0385(1 +.0385)^{15}]})
C = \$177,952.49

The retirement expenditure amount will increase by 4% annually.

Est time: 06-10

33. Calculate the present value of a growing annuity for option 1, then compare this amount with the option to pay instantly \$12,750:

PV = C × ([1 / (r – g)] – {(1 + g)^t / [(r – g) × $(1 + r)^t$]})
PV = \$5,000 × ([1 / (.10 – .06)] – {(1 + .06)^3 / [(.10 – .06) × (1 + .10)^3]})
PV = \$13,146.51

Since the \$13,147 present value of the three year growing annual membership dues exceeds the single \$12,750 payment for three years, it is better to pay the lower upfront 3-year dues.

Est time: 06-10

34. a. PV = C_0
PV = \$100,000

b. PV = C_t / $(1 + r)^t$ = \$180,000 / 1.12^5
PV = \$102,136.83

c. PV = C / r = \$11,400 / .12
PV = \$95,000

d. PV = C × ((1 / r) – {1 / – [$r(1 + r)^t$]}) = \$19,000 × ((1 / .12) – {1 / – [.12(1.12)^{10}]})
PV = \$107,354.24

e. PV = C / (r – g) = \$6,500 / (.12 – .05)
PV = \$92,857.14

Prize (d) is the most valuable because it has the highest present value.

Est time: 06-10

35. a. $PV = C / r$
$PV = \$2{,}000{,}000 / .12$
$PV = \$16{,}666{,}667$

b. $PV = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$
$PV = \$2{,}000{,}000 \times ((1 / .12) - \{1 / [.12(1 + .12)^{20}]\})$
$PV = \$14{,}938{,}887$

c. $PV = C / (r - g)$
$PV = \$2{,}000{,}000 / (.12 - .03)$
$PV = \$22{,}222{,}222$

d. $PV = C \times ([1 / (r - g)] - \{(1 + g)^t / [(r - g) \times (1 + r)^t]\})$
$PV = \$2{,}000{,}000 \times ([1 / (.12 - .03)] - \{(1 + .03)^{20} / [(.12 - .03) \times (1 + .12)^{20}]\})$
$PV = \$18{,}061{,}473$

Est time: 06-10

36. First, find the semiannual rate that is equivalent to the annual rate:

$1 + r = (1 + r_{semi})^2$

$1.08 = (1 + r_{semi})^2$

$r_{semi} = 1.08^{.5} - 1$

$r_{semi} = .039230$, or 3.9230%

$PV = C_0 + C \times ((1 / r_{semi}) - \{1 / [r_{semi} \times (1 + r_{semi})^t]\})$

$PV = \$100{,}000 + \$100{,}000 \times ((1 / .039230) - \{1 / [.039230(1 + .039230)^9]\})$

$PV = \$846{,}147.28$

Est time: 06-10

37. a. $C_t = PV \times (1 + r)^t$
$C_1 = \$1 \times 1.12^1 = \$1.1200$
$C_5 = \$1 \times 1.12^5 = \$1.7623$
$C_{10} = \$1 \times 1.12^{10} = \$9.6463$

b. $C_t = PV \times (1 + r / m)^{mt}$
$C_1 = \$1 \times [1 + (.117 / 2)]^{2 \times 1} = \$1.1204$
$C_5 = \$1 \times [1 + (.117 / 2)]^{2 \times 5} = \$1.7657$
$C_{10} = \$1 \times [1 + (.117 / 2)]^{2 \times 20} = \$9.7193$

c. $C_t = PV \times e^{mt}$
$C_1 = \$1 \times e^{(.115 \times 1)} = \$1.1219$
$C_5 = \$1 \times e^{(.115 \times 5)} = \$1.7771$
$C_{10} = \$1 \times e^{(.115 \times 20)} = \$9.9742$

The preferred investment is (c) because it compounds interest faster and produces the highest future value at any point in time.

Est time: 06-10

38.

a. $C_t = \text{PV} \times (1 + r)^t$
$C_t = \$10{,}000{,}000 \text{ x } (1.06)^4$
$C_t = \$12{,}624{,}770$

b. $C_t = \text{PV} \times [1+ (r / m)^{mt}$
$C_t = \$10{,}000{,}000 \times [1 + (.06 / 12)]^{12 \times 4}$
$C_t = \$12{,}704{,}892$

c. $C_t = \text{PV} \times e^{rt}$
$C_t = \$10{,}000{,}000 \times e^{.06 \times 4}$
$C_t = \$12{,}712{,}492$

Est time: 01-05

39. a. $\text{PV}_{\text{end of year}} = C / r$

$\text{PV}_{\text{end of year}} = \$100 / .07$

$\text{PV}_{\text{end of year}} = \$1{,}428.57$

b. $\text{PV}_{\text{beginning of year}} = (C / r) \times (1 + r)$

$\text{PV}_{\text{beginning of year}} = (\$100 / .07) \times (1 + .07)$

$\text{PV}_{\text{beginning of year}} = \$1{,}528.57$

c. To find the present value with payments spread evenly over the year, use the continuously compounded rate that equates to 7% compounded annually. This rate is found using natural logarithms.

$\text{PV}_{\text{CC}} = C / r_{\text{CC}}$

$\text{PV}_{\text{CC}} = \$100 / \ln(1 + .07)$

$\text{PV}_{\text{CC}} = \$1{,}478.01$

[Note: the continuously compounded rate is :Ln(1 + .07) = .0677, or 6.77%]

The sooner payments are received, the more valuable they are.

Est time: 06-10

40. Annual compounding:

$C_t = \text{PV} \times (1 + r)^t$
$C_{20} = \$100 \times 1.15^{20}$
$C_{20} = \$1{,}636.65$

Continuous compounding:

$C_t = PV \times e^{rt}$
$C_{20} = \$100 \times e^{.15 \times 20}$
$C_{20} = \$2,008.55$

Est time: 01-05

41. a. $FV = C \times e^{rt}$
$FV = \$1,000 \times e^{.12 \times 5}$
$FV = \$1,822.12$

b. $PV = C / e^{rt}$
$PV = \$5,000,000 / e^{.12 \times 8}$
$PV = \$1,914,464$

c. $PV = C\,(1 / r \; - 1 / re^{rt})$
$PV = \$2,000\,(1 / .12 \; - 1 / .12e^{.12 \times 15})$
$PV = \$13,911.69$

Est time: 01-05

42. Spreadsheet exercise, answers will vary

Est time: 11-15

43. a. $PV = C / (r - g)$

$PV = \$2,000,000 / [.10 - (-.04)]$

$PV = \$14,285,714$

b. $PV_{20} = C_{21} / (r - g)$
$PV_{20} = \{\$2,000,000 \times [1 + (-.04)]^{20}\} / [.10 - (-.04)]$
$PV_{20} = \$6,314,320$

$PV_{\text{cash flows 1-20}} = PV - PV_{20} / (1 + r)^{20}$
$PV_{\text{cash flows 1-20}} = \$14,285,714 - (\$6,314,320 / 1.10^{20})$
$PV_{\text{cash flows 1-20}} = \$13,347,131$

Est time: 06-10

44. a. Rule of 72 estimate:

Time to double = 72 / r

Time to double = 72 / 12

Time to double = 6 years

Exact time to double:

C_t = PV × (1 + r)^t
t = ln2 / ln1.12
t = 6.12 years

b. With continuous compounding for interest rate r and time period t:

$e^{rt} = 2$

rt = ln2

Solving for t when r is expressed as a decimal:

rt = .693

t = .693 / r

Est time: 06-10