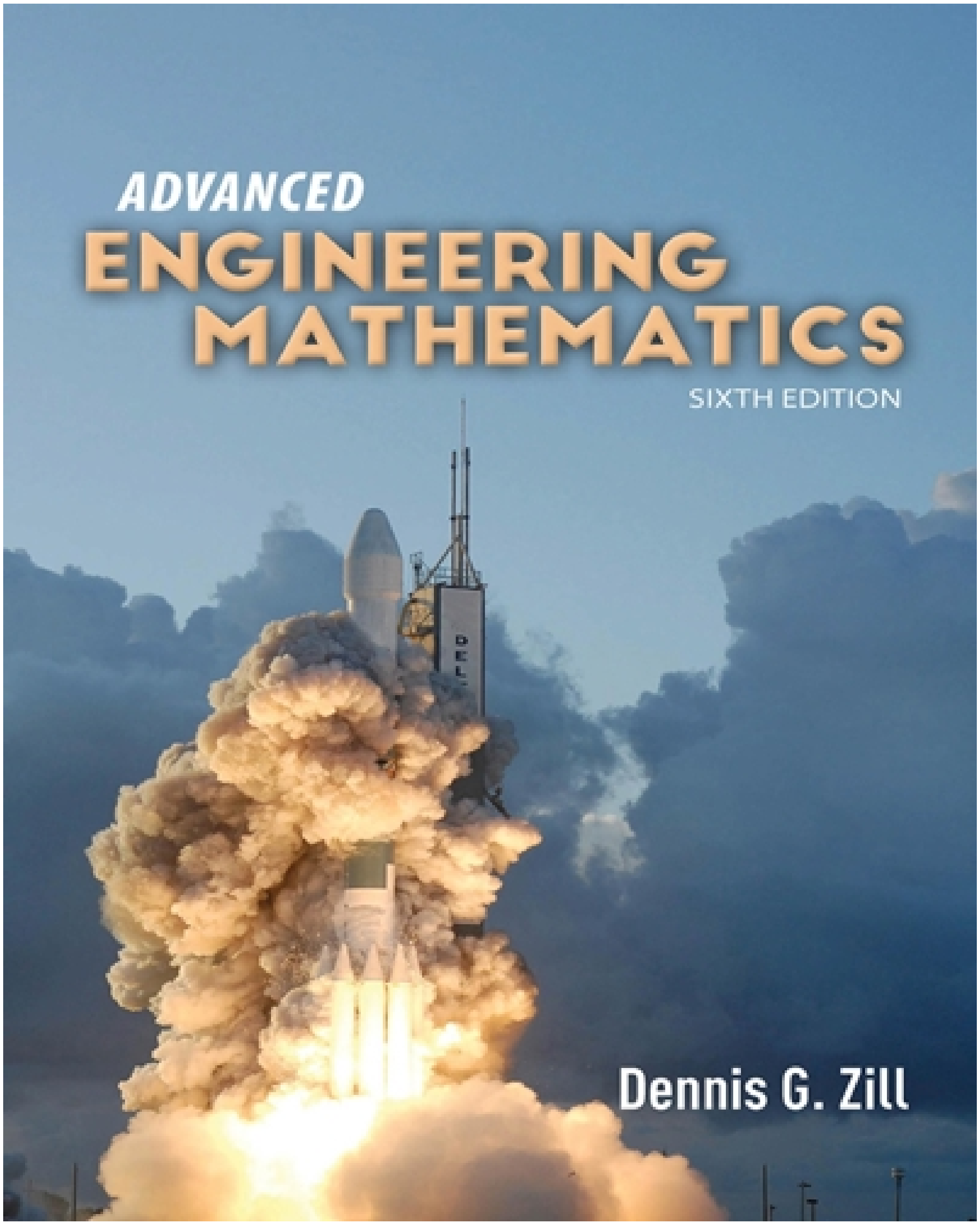
Test Bank for Advanced Engineering Mathematics 6th Edition by Zill



Test Bank

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Chapter 2

1. Which of the following is an example of a separable differential equation?

A. $x\tan^{-1}(y)\dfrac{dy}{dx}=e^{x+y}$
B. $y'=\sin(xy)$
C. $(x^2+1)\dfrac{dy}{dx}=\sqrt{x+y}$
D. $y'=\dfrac{1}{x+y}$

Ans: A

2. The general solution of the differential equation $5\dfrac{dy}{dx}+30y=6$ is

A. $y=Ce^{-6x}+\dfrac{1}{5}$
B. $y=\dfrac{1}{5}e^{6x}+C$
C. $y=Ce^{-6x}+\dfrac{1}{5}e^{6x}$
D. $y=\dfrac{1}{5}e^{12x}+Ce^{6x}$

Ans: A

3. Which of the following is a homogeneous differential equation?

A. $\dfrac{dy}{dx}=\dfrac{2x+7y}{7x+2y}$
B. $(x^2+y^2)dy-x^4dx=0$
C. $\dfrac{dy}{dx}+x^2y=xy^2$
D. $\left(y^2+x\right)dx=\left(x+y^2\right)dy$

Ans: A

4. Which of the following differential equations is a Bernoulli equation?

A. $3dy = \sin(x)\left(y^2 - y\right)dx$

B. $\left(x^2 + y^2\right)dy - 2xydx = 0$

C. $e^y dx = 5 - \sin(y)dy$

D. $\dfrac{dy}{dx} = \ln(y)$

Ans: A

5. Initially, 500 milligrams of a radioactive substance was present. After 3 days, the mass had decreased by 145 milligrams. If the rate of decay is proportional to the amount of the substance present at time t, find the amount remaining after 10 days (rounded to the nearest milligram).

A. 160 mg
B. 7 mg
C. 0 mg
D. 173 mg

Ans: A

6. Suppose that a large mixing tank initially holds 400 gallons of water in which 65 pounds of salt have been dissolved. Another brine solution is pumped into the tank at a rate of 6 gal/min, and when the solution is well stirred, it is pumped out at a slower rate of 5 gal/min. If the concentration of the solution entering is 3 lb/gal, find the amount of salt in the tank after 10 minutes.

A. 95 pounds
B. 85 pounds
C. 90 pounds
D. 100 pounds

Ans: A

7. Suppose that a large mixing tank initially holds 300 gallons of water in which 50 pounds of salt have been dissolved. Another brine solution is pumped into the tank at a rate of 4 gal/min, and when the solution is well stirred, it is pumped out at a faster rate of 6 gal/min. If the concentration of the solution entering is 7 lb/gal, find the amount of salt in the tank after 6 minutes.

A. 202.29 pounds
B. 176.31 pounds
C. 215.78 pounds
D. 264.40 pounds

Ans: A

8. A thermometer reading 65^0 F is placed in an oven heated to a constant temperature. The thermometer reads 125^0 F after 1 minute and 170^0 F after 2 minutes. What is the temperature of the oven?

A. 305^0 F
B. 320^0 F
C. 353^0 F
D. 375^0 F

Ans: A

9. Find the critical point(s) of the differential equation $\frac{dV}{dt} = V\sqrt{c + mP}$ where c and m are nonzero constants.

A. $1, -\frac{c}{m}$

B. $0, \frac{c}{m}$

C. $0, -\frac{c}{m}$

D. $-\frac{c}{m}$

Ans: C

10. Suppose the slope field of a differential equation $\frac{dy}{dx} = f(x, y)$ has a lineal element at the point (2, 3) that slopes up from left to right. What does this tell us about $y'(2)$?

A. $y'(2) = 3$
B. $y'(2) < 0$
C. $y'(2) = 0$
D. $y'(2) > 0$

Ans: D

11. Rewrite the linear differential equation $\cos x\, dy + (\sin x) y\, dx = dx$ in the standard form of a linear differential equation

A. $\frac{dy}{dx} + (\tan x) y = \sec x$
B. $\frac{dy}{dx} + (\sin x) y = \cos x$
C. $\cos x \frac{dy}{dx} + (\sin x) y = 1$
D. $\cos x\, dy = (1 - y \sin x)\, dx$

Ans: A

True/False

12. For the DE $y' = (y-5)^3 (y^2 - 9)^2$, 5 is an unstable critical point while 3 and -3 are semi-stable critical points.

Ans: True

13. The general solution of the differential equation $2\frac{dy}{dx} = -4y + 5$ is $y = Cx^{-2} + \frac{5}{4}$.

Ans: True

14. The integrating factor of the differential equation $-7\left(y + x^5\right)dx + x^2 dy = 0$ is $\frac{-7}{x^2}$.

Ans: False

15. The population of a certain town is known to increase at a rate proportional to the current population. If the population doubles every 15 years, and the current population is 32,000, then the population in 11 years will be 53,199.

Ans: True

16. 0 is a critical number of the differential equation $\frac{dy}{dx} = \frac{1}{x}$.

Ans: False

17. A differential equation can be both separable and linear.

Ans: True

18. An integrating factor is another name for constant in an integral which can be factored out of the integrand.

Ans: False

19. The differential equation $2xy\,dx - \left(x^2 + 1\right)dy = 0$ is exact.

Ans: False

20. A differential equation $M(x)\,dx + N(y)\,dy = 0$ is homogenous if $M(x)$ and $N(y)$ are both polynomials of the same degree.

Ans: True

Short Answer

21. Solve the differential equation $e^{2x}\dfrac{dy}{dx} = e^{-2y} + e^{-2x-2y}$.

Ans: $y = \dfrac{1}{2}\ln\left(-e^{-2x} - \dfrac{1}{2}e^{-4x} + C\right)$

22. Find an implicit solution of the initial value problem $y' = \dfrac{y^2-4}{x^2-9}$, $y(0) = 3$.

Ans: $\dfrac{y-2}{y+2} = \dfrac{1}{5}\left(\dfrac{x-3}{x+3}\right)^{2/3}$

23. Solve the initial value problem $\dfrac{dx}{dt} = t(x-1)$, $x(0) = 7$.

Ans: $x = 1 + 6e^{-t^2/2}$

24. Find the general solution of the differential equation $(x+4)\dfrac{dy}{dx} + (x+5)y = 5xe^{-2x}$ and give the largest interval over which the general solution is defined.

Ans: $y = \dfrac{-5(x+1)e^{-2x}}{x+4} + \dfrac{ce^{-x}}{(x+4)}$, $(-4,\infty)$

25. Solve the following initial value problem:
$\left[2xy^3 - \cos(x)e^y + x\sin(x)e^y + y\cos(xy)\right]dx + \left[3x^2y^2 - x\cos(x)e^y + x\cos(xy)\right]dy = 0,\ y(\pi) = 0$

Ans: $x^2y^3 - x\cos(x)e^y + \sin(xy) = \pi$

26. Find the value of k so that the differential equation
$\left[8yt^5 + kt^2e^{ty}\right]dy = \left[\frac{2}{t} - 20y^2t^4 - ktye^{ty} - 5e^{ty}\right]dt$ is exact.

Ans: $k = 5$

27. Using an appropriate substitution, solve the differential equation $y' = -y + xy^5$.

Ans: $y = \left(x + \frac{1}{4} + ce^{4x}\right)^{-1/4}$

28. Using an appropriate substitution, solve the differential equation
$ydx - 5(x-y)dy = 0$.

Ans: $y^5 = c(5y - 4x)$

29. For the DE $y' = -x - 2y + 2$, use Euler's Method to obtain a three-decimal approximation of $y(2.3)$ if $y(2) = 1$. Use $h = 0.1$.

Ans: 0.484

30. For the DE $y' = y\cos(x)$, use Euler's Method to obtain a four-decimal approximation of $y(0.4)$ if $y(0) = 3$. Use $h = 0.1$.

Ans: 4.3646

31. For the DE $y' = e^{x-y}$, use Euler's Method to obtain a four-decimal approximation of $y(-0.6)$ if $y(-1) = 2$. Use $h = 0.1$.

Ans: 2.0231

32. The population of a certain town is governed by the logistic equation $\frac{dP}{dt} = P\left(0.5 - \frac{0.5}{105,000}P\right)$. What is the carrying capacity for the town? If the current population is 9000, when will the population reach 25,000? What will the population be in 5 years?

Ans: 105,000 people; 2.408 years; 55,983 people

33. Two very large tanks A and B are each partially filled with 200 gallons of brine. Initially, 25 pounds of salt are dissolved in Tank A while 75 pounds are dissolved in Tank B. One interconnecting pipe sends brine from Tank A to Tank B at a rate of 5 gal/min. Another interconnecting pipe sends brine from Tank B to Tank A at a rate of 2 gal/min. An input pipe feeds pure water to Tank A at a rate of 3 gal/min, while an output pipe drains brine from Tank B at a rate of 3 gal/min. Construct a mathematical model for the amount of salt $x_1(t)$ and $x_2(t)$ at time t in Tanks A and B, respectively.

Ans:
$$x_1' = -\frac{x_1}{40} + \frac{x_2}{100}$$
$$x_2' = \frac{x_1}{40} - \frac{x_2}{40}$$
$$x_1(0) = 25, \quad x_2(0) = 75$$

34. Initially, two tanks A and B are each partially filled with 100 gallons of pure water. A feed value lets brine into Tank A at a rate of 4 gal/min with a concentration of 3 lb/gal. An interconnecting pipe pumps brine from Tank A to Tank B at a rate of 2 gal/min, while another pipe pumps brine from Tank B to Tank A at a rate of 1 gal/min. Construct a mathematical model for the amount of salt $x_1(t)$ and $x_2(t)$ at time t in Tanks A and B, respectively.

Ans:
$$x_1' = 12 - \frac{2x_1}{100+3t} + \frac{x_2}{100+t}$$
$$x_2' = \frac{2x_1}{100+3t} - \frac{x_2}{100+t}$$
$$x_1(0) = 0, \quad x_2(0) = 0$$

35. Determine whether $\left[\sin(xy) + xy\cos(xy) + ye^x\right]dx + \left[x^2\cos(xy) + e^x\right]dy = 0$ is exact. If it is exact, solve it.

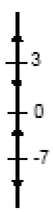
Ans: Yes, it is exact. The solution is $x\sin(xy) + ye^x = c$

36. Determine whether $\left[\pi y + 6xye^y + y^{-2}\right]dx = \left[2xy^{-3} - 3x^2e^y - \pi x - 3x^2ye^y\right]dy$ is exact. If it is exact, solve it

Ans: Yes, it is exact. The solution is $\pi xy + 3x^2ye^y + xy^{-2} = c$

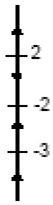
37. Find the critical points and phase portrait of the autonomous first-order differential equation $\frac{dy}{dx} = -2y(y+7)(3-y)$.

Ans: Critical Points: -7, 0, 3

Phase Portrait: 

38. Find the critical points and phase portrait of the autonomous first-order differential equation $\frac{dy}{dx} = (y+3)^2\left(y^2 - 4\right)$.

Ans: Critical Points: -3, -2, 2

Phase Portrait: 

39. Solve implicitly the differential equation $\frac{dy}{dx} = \frac{x}{y^2+1}$.

Ans: $\frac{y^3}{3} + y = \frac{x^2}{2} + C$

40. Find an implicit solution of the initial value problem $\frac{dy}{dx} = \frac{y^3 x}{y+2}, \quad y(1) = -2$

Ans: $-\frac{1}{y} - \frac{1}{y^2} = \frac{x^2}{2} - \frac{1}{4}$

41. Solve the initial-value problem $2\frac{dy}{dx} - 4xy - 3 = 0, \quad y(0) = 4$.

Ans: $y = \frac{3\sqrt{\pi}}{4} e^{x^2} erf(x) + 4e^{x^2}$

42. Solve the differential equation $\frac{dy}{dx} = \frac{4x+6y}{10y-6x}$

Ans: $2x^2 + 6xy - 5y^2 = c$

43. Solve the differential equation $\frac{dy}{dx} + 4y = 2x^2 y^3$

Ans: $y = \left(ce^{8x} + \dfrac{32x^2 + 8x + 1}{64} \right)$

45. Find the general solution of the differential equation $3y' + 2xy = 6x.$.

Ans: $ce^{-x^2/3} + 3$

46. The population of a certain town is known to increase at a rate proportional to the current population. Suppose the population triples every 21 years. If the population is currently 150,000, what was the population 9 years ago?

Ans: 93,672

47. Consider the differential equation $M(x)\,dx + N(y)\,dy = 0$ where $M(x)$ and $N(y)$ are nonzero functions of *x* and *y*, respectively. Determine if this differential equation is separable. Justify your answer.

Ans: The differential equation is separable since it can be written in the form
$\dfrac{dy}{dx} = -M(x) \cdot \dfrac{1}{N(y)}$.

48. Let $y(t) =$ the size of a certain population at time *t*. Suppose that $y(t)$ is described by the initial value problem $y' = \sqrt{y}$, $y(0) = 0$. Find two different solutions to this initial value problem. In this context of this problem, which solution is correct? Briefly explain why.

Ans: By inspection, $y(t) = 0$ is a solution. By separation of variables, $y(t) = t^2/4$ is also a solution. Since $y(t)$ represents a population which starts at 0, it should remain 0 forever. Thus $y(t) = 0$ is the correct solution in this context.

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