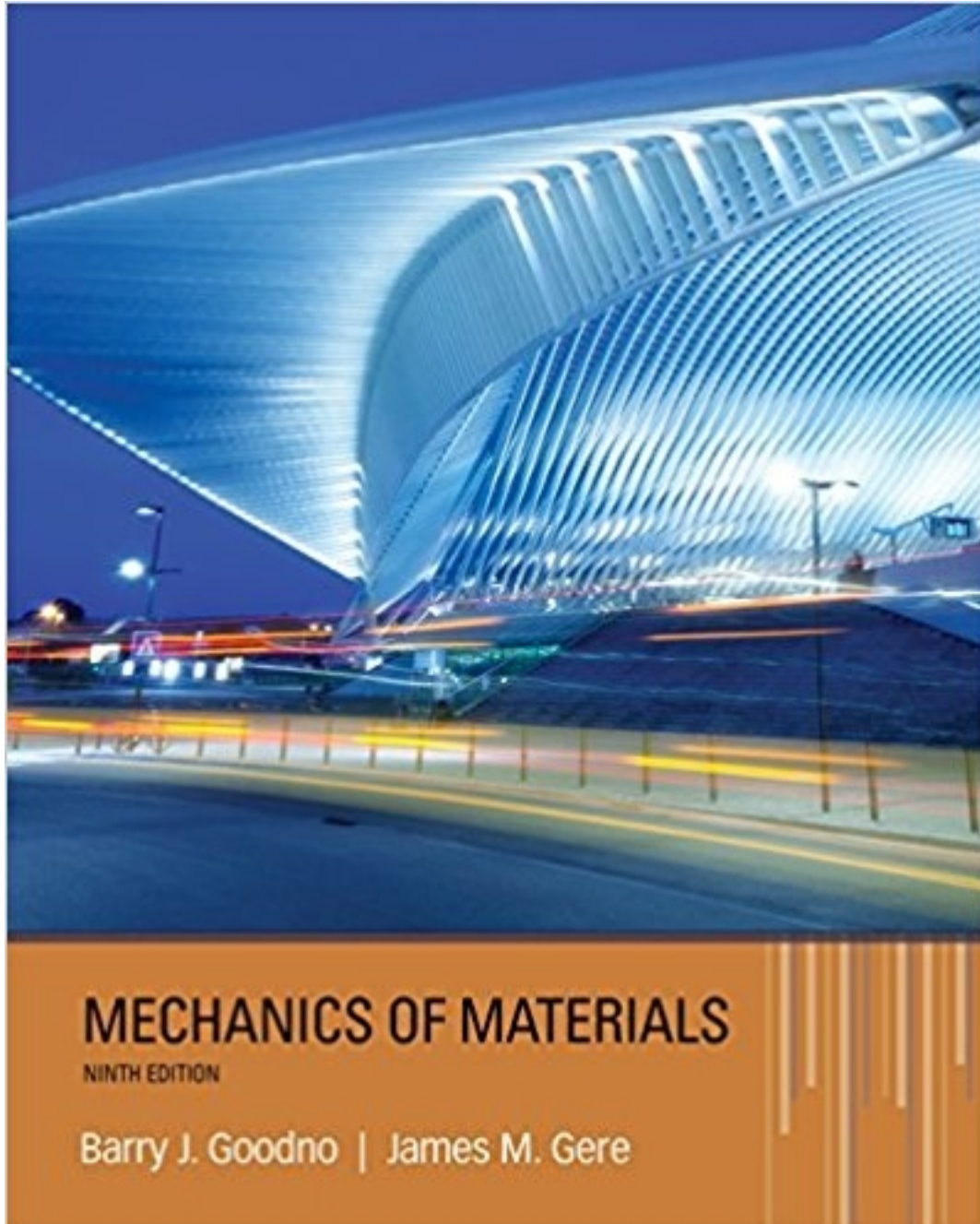


[CLICK HERE TO ACCESS COMPLETE Solutions](#)



# Solutions

# Chapter 2 Solutions

## Problem 2.2-1

$$L = 10\text{ft} \quad q_0 = 2 \cdot \frac{\text{kip}}{\text{ft}} \quad k = 4 \cdot \frac{\text{kip}}{\text{in}}$$

$$\Sigma M_B = 0 \quad A_y = \frac{1}{L} \cdot \left( \frac{1}{2} \cdot q_0 \cdot L \cdot \frac{L}{3} \right) = 3.333 \cdot \text{kip} \quad \delta_A = \frac{A_y}{k} = 0.833 \cdot \text{in} \quad \frac{\delta_A}{L} = 6.944 \times 10^{-3}$$

**Problem 2.2-2**

$$E = 200\text{GPa} \quad d_r = 25\text{mm} \quad q = 5 \frac{\text{kN}}{\text{m}} \quad L_r = 0.75\text{m} \quad P = 10\text{kN} \quad a = 2.5\text{m} \quad b = 0.75\text{m}$$

$$A_r = \frac{\pi}{4} \cdot d_r^2 = 0.761 \cdot \text{in}^2$$

$$\text{Force in rod} \quad \Sigma M_A = 0 \quad F_r = \frac{1}{a} \left[ q \cdot a \cdot \frac{a}{2} + P \cdot (a + b) \right] = 19.25 \cdot \text{kN}$$

$$\text{Change in length of rod} \quad \delta_{\text{rod}} = \frac{F_r \cdot L_r}{E \cdot A_r} = 0.1471 \cdot \text{mm}$$

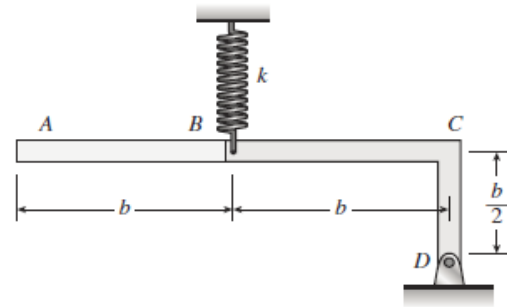
$$\text{Displacement at B using similar triangles} \quad \delta_B = \frac{a + b}{a} \cdot \delta_{\text{rod}} = 0.1912 \cdot \text{mm}$$

**Problem 2.2-3**

(a) SUM MOMENTS ABOUT A

$$\Sigma M_A = 0 \quad \frac{2b}{\frac{5}{2}b} Wb + \frac{\frac{b}{2}}{\frac{5}{2}b} W(2b) = k\delta b$$

$$\delta = \frac{\frac{2b}{\frac{5}{2}b} Wb + \frac{\frac{b}{2}}{\frac{5}{2}b} W(2b)}{kb} = \frac{6W}{5k}$$



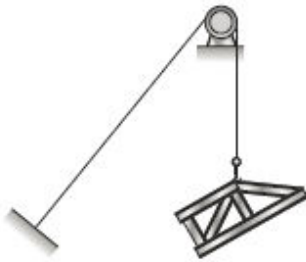
(b)

(b)  $\Sigma M_D = 0 \quad kb\delta = \frac{2b}{\frac{5}{2}b} Wb = \frac{4Wb}{5} \quad \text{so}$

$$\delta = \frac{\frac{2b}{\frac{5}{2}b} Wb}{kb} = \frac{4W}{5k}$$



**Problem 2.2-4**



$$A = 304 \text{ mm}^2 \text{ (from Table 2-1)}$$

$$W = 38 \text{ kN}$$

$$E = 140 \text{ GPa}$$

$$L = 14 \text{ m}$$

**(b) FACTOR OF SAFETY**

$$P_{ULT} = 406 \text{ kN (from Table 2-1)}$$

$$P_{max} = 70 \text{ kN}$$

$$n = \frac{P_{ULT}}{P_{max}} = \frac{406 \text{ kN}}{70 \text{ kN}} = 5.8 \quad \leftarrow$$

**(a) STRETCH OF CABLE**

$$\delta = \frac{WL}{EA} = \frac{(38 \text{ kN})(14 \text{ m})}{(140 \text{ GPa})(304 \text{ mm}^2)}$$

$$= 12.5 \text{ mm} \quad \leftarrow$$

**Problem 2.2-5**

$$(a) \frac{\delta_a}{\delta_s} = \frac{\frac{PL}{E_a A}}{\left(\frac{PL}{E_s A}\right)} \rightarrow \frac{E_s}{E_a}$$

$$E_s = 30,000 \text{ ksi} \quad E_a = 11,000 \text{ ksi}$$

$$\boxed{\frac{E_s}{E_a} = 2.727} \quad \frac{30}{11} = 2.727$$

$$(b) \delta_a = \delta_s \quad \text{so} \quad \frac{PL}{E_a A_a} = \frac{PL}{E_s A_s} \quad \text{so} \quad \frac{A_a}{A_s} = \frac{E_s}{E_a} \quad \text{and} \quad \boxed{\frac{d_a}{d_s} = \sqrt{\frac{E_s}{E_a}} = 1.651}$$

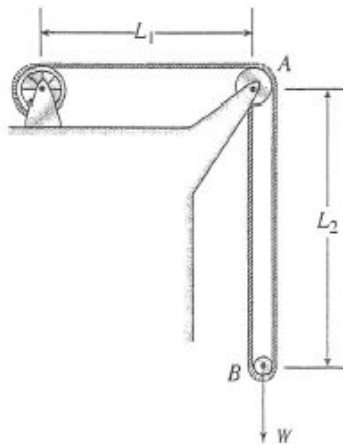
- (c) SAME DIAMETER, SAME LOAD, FIND RATIO OF LENGTH OF ALUMINUM TO STEEL WIRE IF ELONGATION OF ALUMINUM IS 1.5 TIMES THAT OF STEEL WIRE

$$\frac{\delta_a}{\delta_s} = \frac{\frac{PL_a}{E_a A}}{\left(\frac{PL_s}{E_s A}\right)} = \frac{\frac{PL_a}{E_a A}}{\left(\frac{PL_s}{E_s A}\right)} = 1.5 \quad \boxed{\frac{L_a}{L_s} = 1.5 \frac{E_a}{E_s} = 0.55}$$

- (d) SAME DIAMETER, SAME LENGTH, SAME LOAD—BUT WIRE 1 ELONGATION 1.7 TIMES THE STEEL WIRE > WHAT IS WIRE 1 MATERIAL?

$$\frac{\delta_1}{\delta_s} = \frac{\frac{PL}{E_1 A}}{\left(\frac{PL}{E_s A}\right)} = \frac{\frac{PL}{E_1 A}}{\left(\frac{PL}{E_s A}\right)} = 1.7 \quad E_1 = \frac{E_s}{1.7} = 17,647 \text{ ksi} \quad \boxed{< \text{cast iron or copper alloy (see App. I)}}$$

**Problem 2.2-6**



$$\begin{aligned}d_A &= 300 \text{ mm} \\d_B &= 150 \text{ mm} \\L_1 &= 4.6 \text{ m} \\L_2 &= 10.5 \text{ m} \\EA &= 10,700 \text{ kN} \\W &= 22 \text{ kN}\end{aligned}$$

TENSILE FORCE IN CABLE

$$T = \frac{W}{2} = 11 \text{ kN}$$

LENGTH OF CABLE

$$\begin{aligned}L &= L_1 + 2L_2 + \frac{1}{4}(\pi d_A) + \frac{1}{2}(\pi d_B) \\&= 4,600 \text{ mm} + 21,000 \text{ mm} + 236 \text{ mm} + 236 \text{ mm} \\&= 26,072 \text{ mm}\end{aligned}$$

ELONGATION OF CABLE

$$\delta = \frac{TL}{EA} = \frac{(11 \text{ kN})(26,072 \text{ mm})}{(10,700 \text{ kN})} = 26.8 \text{ mm}$$

LOWERING OF THE CAGE

$h$  = distance the cage moves downward

$$h = \frac{1}{2}\delta = 13.4 \text{ mm} \quad \leftarrow$$

**Problem 2.2-7**

$$d_o = 15\text{in} \quad d_i = 14.4\text{in} \quad E = 29000\text{ksi} \quad P = 5\text{kip}$$

$$L_{DC} = \sqrt{(3\text{ft})^2 + (4\text{ft})^2} = 5\text{ft} \quad A_{DC} = \frac{\pi}{4} \cdot (d_o^2 - d_i^2) = 13.854\text{in}^2$$

Find force in DC - use FBD of ACB

$$\Sigma M_A = 0 \quad \frac{3}{5}F_{DC}(4\text{ft}) = P(9\text{ft}) \quad \text{so} \quad F_{DC} = \frac{5}{3} \cdot P \cdot \left(\frac{9}{4}\right) = 18.75\text{kip} \quad \text{compression}$$

Change in length of strut

$$\Delta_{DC} = \frac{F_{DC} \cdot L_{DC}}{E \cdot A_{DC}} = 2.8 \times 10^{-3}\text{in} \quad \text{shortening}$$

Vertical displacement at C (see Example 2-7) and at B

$$\delta_C = \frac{\Delta_{DC}}{\sin(\text{ACD})} \quad \delta_C = \frac{\Delta_{DC}}{\frac{3}{5}} = 4.667 \times 10^{-3}\text{in} \quad \delta_B = \frac{9}{4} \cdot \delta_C = 1.05 \times 10^{-2}\text{in} \quad \text{downward}$$

### Problem 2.2-8

$$L_{BD} = 350\text{mm} \quad L_{CE} = 450\text{mm} \quad A = 720\text{mm}^2 \quad E = 200\text{GPa} \quad P = 20\text{kN}$$

Statics - find axial forces in BD and CE - remove pins at B and E, use FBD of beam ABC - assume beam is rigid

$$\Sigma M_B = 0 \quad CE = \frac{1}{350\text{mm}} \cdot [P \cdot (600\text{mm})] = 34.286\text{ kN} \quad CE \text{ is in tension; force CE acts downward on ABC}$$

$$\Sigma F_y = 0 \quad BD = P + CE = 54.286\text{ kN} \quad BD \text{ is in compression; force BD acts upward on ABC}$$

Use force-displacement relation to find change in lengths of CE and BD and vertical displacements at B and C

$$\delta_{BD} = \frac{BD \cdot L_{BD}}{E \cdot A} = 0.13194\text{ mm} \quad \text{shortening}$$

$$\delta_{CE} = \frac{CE \cdot L_{CE}}{E \cdot A} = 0.10714\text{ mm} \quad \text{elongation}$$

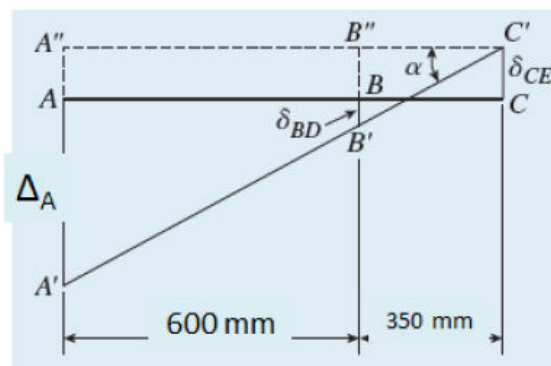
Use geometry to find downward displacement at A

$$\alpha = \text{atan}\left(\frac{|\delta_{BD}| + \delta_{CE}}{350\text{mm}}\right) = 0.03914\text{ deg}$$

$$\Delta_A = 950\text{mm} \cdot \tan(\alpha) - \delta_{CE} = 0.542\text{ mm} \quad \text{downward}$$

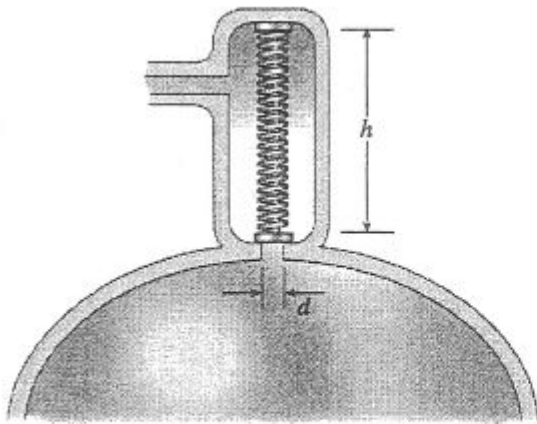
or similar triangles (see figure)  $\frac{\Delta_A + \delta_{CE}}{600 + 350} = \frac{|\delta_{BD}| + \delta_{CE}}{350}$

$$\Delta_A = \left(|\delta_{BD}| + \delta_{CE}\right) \cdot \left(\frac{950}{350}\right) - \delta_{CE} = 0.542\text{ mm} \quad \text{downward}$$



$$\frac{\delta_A + \delta_{CE}}{600 + 350} = \frac{\delta_{BD} + \delta_{CE}}{350}$$

**Problem 2.2-9**



$h$  = height of valve (compressed length of the spring)

$d$  = diameter of discharge hole

$p$  = pressure in tank

$p_{\max}$  = pressure when valve opens

$L$  = natural length of spring ( $L > h$ )

$k$  = stiffness of spring

FORCE IN COMPRESSED SPRING

$$F = k(L - h) \text{ (From Eq. 2-1a)}$$

PRESSURE FORCE ON SPRING

$$P = p_{\max} \left( \frac{\pi d^2}{4} \right)$$

EQUATE FORCES AND SOLVE FOR  $h$ :

$$F = P \quad k(L - h) = \frac{\pi p_{\max} d^2}{4}$$

$$h = L - \frac{\pi p_{\max} d^2}{4k} \quad \leftarrow$$

### Problem 2.2-10

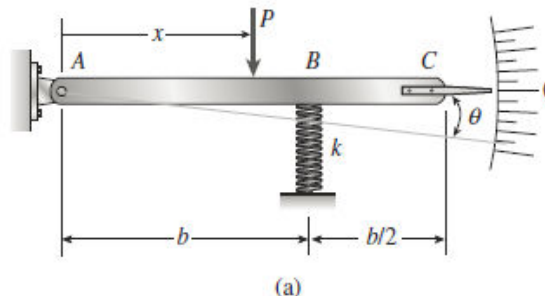
NUMERICAL DATA  $k = 950 \text{ N/m}$   $b = 165 \text{ mm}$   $P = 11 \text{ N}$   $\theta = 2.5^\circ$   $\theta_{\max} = 2^\circ$

$W_p = 3 \text{ N}$   $W_s = 2.75 \text{ N}$

- (a) If the load  $P = 11 \text{ N}$ , at what distance  $x$  should the load be placed so that the pointer will read  $\theta = 2.5^\circ$  on the scale (see Fig. a)?

Sum moments about A, then solve for  $x$ :

$$x = \frac{k\theta b^2}{P} = 102.6 \text{ mm} \quad \boxed{x = 102.6 \text{ mm}}$$

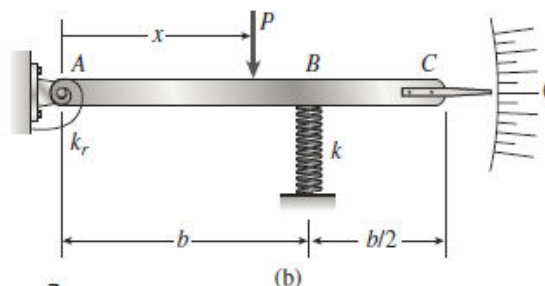


- (b) Repeat (a) if a rotational spring  $k_r = kb^2$  is added at A (see Fig. b).

$$k_r = k b^2 = 25864 \text{ N}\cdot\text{mm}$$

Sum moments about A, then solve for  $x$ :

$$x = \frac{k\theta b^2 + k_r\theta}{P} = 205 \text{ mm} \quad \frac{x}{b} = 1.244 \quad \boxed{x = 205 \text{ mm}}$$



- (c) Now if  $x = 7b/8$ , what is  $P_{\max}$  (N) if  $\theta$  cannot exceed  $2^\circ$ ?  $x = \frac{7}{8}b = 144.375 \text{ mm}$

Sum moments about A, then solve for  $P$ : 
$$P_{\max} = \frac{k\theta_{\max}b^2 + k_r\theta_{\max}}{\frac{7}{8}b} = 12.51 \text{ N} \quad \boxed{P_{\max} = 12.51 \text{ N}}$$

- (d) Now, if the weight of the pointer ABC is known to be  $W_p = 3 \text{ N}$  and the weight of the spring is  $W_s = 2.75 \text{ N}$ , what initial angular position (i.e.,  $\theta$  in degrees) of the pointer will result in a zero reading on the angular scale once the pointer is released from rest? Assume  $P = k_r = 0$ .

Deflection at spring due to  $W_p$ :

$$\delta_{Bp} = \frac{W_p \left( \frac{3}{4}b \right)}{kb} = 2.368 \text{ mm}$$

Deflection at B due to self weight of spring:

$$\delta_{Bk} = \frac{W_s}{2k} = 1.447 \text{ mm}$$

$$\delta_B = \delta_{Bp} + \delta_{Bk} = 3.816 \text{ mm} \quad \theta_{\text{init}} = \frac{\delta_B}{b} = 1.325^\circ$$

$$\text{OR } \theta_{\text{init}} = \arctan \left( \frac{\delta_B}{b} \right) = 1.325^\circ \quad \boxed{\theta_{\text{init}} = 1.325^\circ}$$

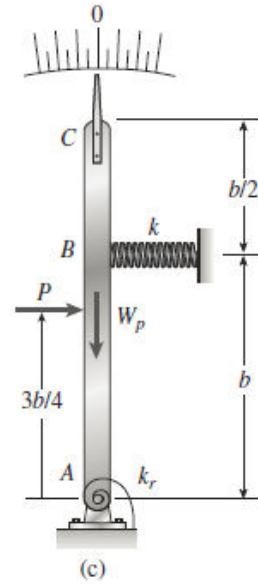
- (e) If the pointer is rotated to a vertical position (figure part c), find the required load  $P$ , applied at mid-height of the pointer that will result in a pointer reading of  $\theta = 2.5^\circ$  on the scale. Consider the weight of the pointer,  $W_p$ , in your analysis.

$$k = 950 \text{ N/m} \quad b = 165 \text{ mm} \quad W_p = 3 \text{ N}$$

$$k_r = kb^2 = 25.864 \text{ N}\cdot\text{m} \quad \theta = 2.5^\circ$$

Sum moments about A to get  $P$ :

$$P = \frac{\theta}{\left(\frac{3b}{4}\right)} \left[ k_r + k \left( \frac{5b^2}{4} \right) - W_p \left( \frac{3b}{4} \right) \right] = 20.388 \text{ N} \quad \boxed{P = 20.4 \text{ N}}$$



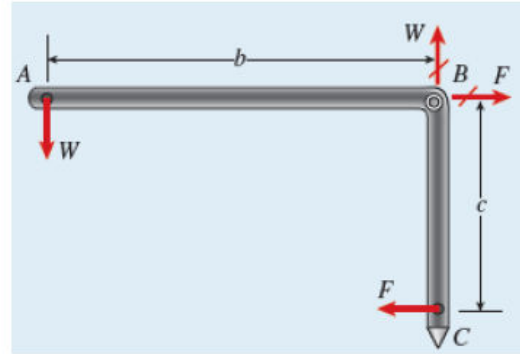


**Problem 2.2-11**

$$b = 10\text{in} \quad c = 7\text{in} \quad k = 5 \frac{\text{lbf}}{\text{in}} \quad p = \frac{1}{16}\text{in} \quad n = 12$$

Use FBD of ABC (pin forces  $B_x = F$  and  $B_y = W$  at B; see fig.); sum moments about B s.t.  $Wb = Fc$ ,  $F$  = force in spring

$$\Sigma M_B = 0 \quad W = F \cdot \frac{c}{b}$$



Force in spring is  $F = k \cdot (n \cdot p) = 3.75 \cdot \text{lbf}$  so  $W = F \cdot \frac{c}{b} = 2.625 \cdot \text{lbf}$

**Problem 2.2-12**

$$b = 30\text{cm} \quad c = 20\text{cm} \quad k = 3650 \frac{\text{N}}{\text{m}} \quad p = 1.5\text{mm} \quad W = 65\text{N}$$

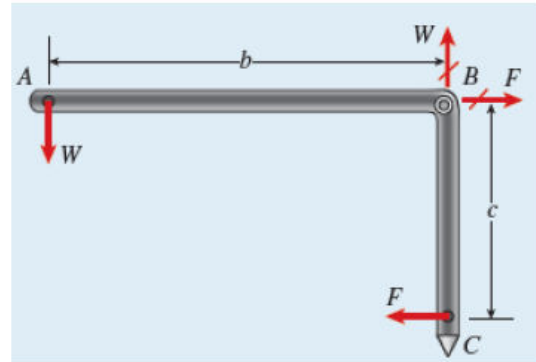
Force in parallel springs is  $F = 2 \cdot k \cdot (n \cdot p)$

Sum moments about B (see FBD) to find F in terms of weight W

$$Wb = Fc \quad \text{so} \quad F = W \cdot \frac{b}{c}$$

Substitute expression for F and solve for n

$$n = \frac{W \cdot \frac{b}{c}}{2 \cdot k \cdot p} = 8.904$$



### Problem 2.2-13

- (a) Derive a formula for the displacement  $\delta_4$  at point 4 when the load  $P$  is applied at joint 3 and moment  $PL$  is applied at joint 1, as shown.

Cut horizontally through both springs to create upper and lower FBD's. Sum moments about joint 1 for upper FBD and also sum moments about joint 6 for lower FBD to get two equations of equilibrium; assume both springs are in tension.

Note that  $\delta_2 = \frac{2}{3} \delta_3$  and  $\delta_5 = \frac{3}{4} \delta_4$

Force in left spring:  $k \left( \delta_4 - \frac{2}{3} \delta_3 \right)$

Force in right spring:  $2k \left( \frac{3}{4} \delta_4 - \delta_3 \right)$

Summing moments about joint 1 (upper FBD) and about joint 6 (lower FBD) then dividing through by  $k$  gives

$$\begin{pmatrix} \frac{-22}{9} & \frac{13}{6} \\ \frac{-26}{9} & \frac{17}{6} \end{pmatrix} \begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \frac{-2P}{k} \\ 0 \end{pmatrix} \quad \begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \frac{-22}{9} & \frac{13}{6} \\ \frac{-26}{9} & \frac{17}{6} \end{pmatrix}^{-1} \begin{pmatrix} \frac{-2P}{k} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{17P}{2k} \\ \frac{26P}{3k} \end{pmatrix} \quad \begin{matrix} \frac{17}{2} = 8.5 \\ \frac{26}{3} = 8.667 \end{matrix} \quad \boxed{\delta_4 = \frac{26P}{3k}}$$

^ deltas are positive downward

- (b) Repeat part (a) if a rotational spring  $k_r = kL^2$  is now added at joint 6. What is the ratio of the deflection  $\delta_4$  in part (a) to that in (b)?

Upper FBD—sum moments about joint 1:

$$k \left( \delta_4 - \frac{2}{3} \delta_3 \right) \frac{2L}{3} + 2k \left( \frac{3}{4} \delta_4 - \delta_3 \right) L = -2PL \quad \text{OR} \quad \left( \frac{22Lk}{9} \right) \delta_3 + \frac{13Lk}{6} \delta_4 = -2PL$$

Lower FBD—sum moments about joint 6:

$$k \left( \delta_4 - \frac{2}{3} \delta_3 \right) \frac{4L}{3} + 2k \left( \frac{3}{4} \delta_4 - \delta_3 \right) L - k_r \theta_6 = 0$$

$$\left[ k \left( \delta_4 - \frac{2}{3} \delta_3 \right) \frac{4L}{3} + 2k \left( \frac{3}{4} \delta_4 - \delta_3 \right) L \right] + (kL^2) \left( \frac{\delta_4}{\frac{4}{3}L} \right) = 0 \quad \text{OR} \quad \left( \frac{26Lk}{9} \right) \delta_3 + \frac{43Lk}{12} \delta_4 = 0$$

Divide matrix equilibrium equations through by  $k$  to get the following displacement equations:

$$\begin{pmatrix} \frac{-22}{9} & \frac{13}{6} \\ \frac{-26}{9} & \frac{43}{6} \end{pmatrix} \begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \frac{-2P}{k} \\ 0 \end{pmatrix} \quad \begin{pmatrix} \delta_3 \\ \delta_4 \end{pmatrix} = \begin{pmatrix} \frac{-22}{9} & \frac{13}{6} \\ \frac{-26}{9} & \frac{43}{6} \end{pmatrix}^{-1} \begin{pmatrix} \frac{-2P}{k} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{43P}{15k} \\ \frac{104P}{45k} \end{pmatrix} \quad \begin{matrix} \frac{43}{15} = 2.867 \\ \frac{104}{45} = 2.311 \end{matrix} \quad \boxed{\delta_4 = \frac{104P}{45k}}$$

^ deltas are positive downward

Ratio of the deflection  $\delta_4$  in part (a) to that in (b):  $\frac{\frac{26}{3}}{\frac{104}{45}} = \frac{15}{4} \quad \boxed{\text{Ratio} = \frac{15}{4} = 3.75}$

### Problem 2.2-14

NUMERICAL DATA

$$A = 3900 \text{ mm}^2 \quad E = 200 \text{ GPa}$$

$$P = 475 \text{ kN} \quad L = 3000 \text{ mm}$$

$$\delta_{B\max} = 1.5 \text{ mm}$$

(a) FIND HORIZONTAL DISPLACEMENT OF JOINT  $B$

STATICS TO FIND SUPPORT REACTIONS AND THEN MEMBER FORCES:

$$\sum M_A = 0 \quad B_y = \frac{1}{L} \left( 2P \frac{L}{2} \right)$$

$$B_y = P$$

$$\sum F_H = 0 \quad A_x = -P$$

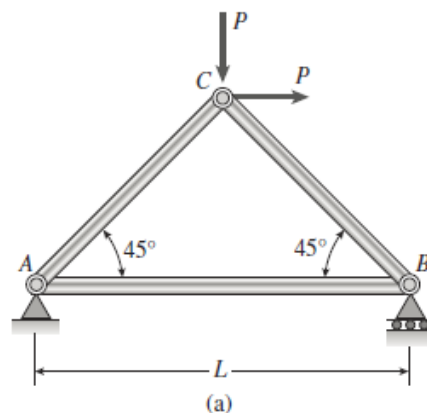
$$\sum F_V = 0 \quad A_y = P - B_y \quad A_y = 0$$

METHOD OF JOINTS:  $AC_V = A_y \quad AC_V = 0 \quad \text{Force in } AC = 0$

$$AB = A_x$$

Force in  $AB$  is  $P$  (tension) so elongation of  $AB$  is the horizontal displacement of joint  $B$ .

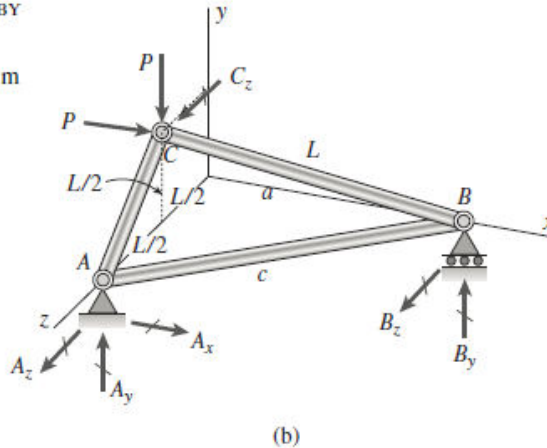
$$\delta_B = \frac{F_{AB}L}{EA} \quad \delta_B = \frac{PL}{EA} \quad \delta_B = 1.82692 \text{ mm} \quad \boxed{\delta_B = 1.827 \text{ mm}}$$



(b) FIND  $P_{\max}$  IF DISPLACEMENT OF JOINT  $B = \delta_{B\max} = 1.5 \text{ mm}$   $P_{\max} = \frac{EA}{L} \delta_{B\max}$   $\boxed{P_{\max} = 390 \text{ kN}}$

(c) REPEAT PARTS (a) AND (b) IF THE PLANE TRUSS IS REPLACED BY A SPACE TRUSS (SEE FIGURE PART b).

FIND MISSING DIMENSIONS  $a$  AND  $c$ ;  $P = 475 \text{ kN} \quad L = 3 \text{ m}$



$$a = \sqrt{L^2 - 2\left(\frac{L}{2}\right)^2} = 2.12132 \text{ m} \quad \frac{a}{L} = 0.707 \quad a = \frac{L}{\sqrt{2}} = 2.12132 \text{ m}$$

$$c = \sqrt{L^2 + a^2} = 3.67423 \text{ m} \quad c = \sqrt{L^2 + \left(\frac{L}{\sqrt{2}}\right)^2} = 3.67423 \text{ m} \quad c = L\sqrt{\frac{3}{2}} = 3.67423 \text{ m}$$

(1) SUM MOMENTS ABOUT A LINE THRU  $A$  WHICH IS PARALLEL TO THE  $y$ -AXIS

$$B_z = -P \frac{L}{a} = -671.751 \text{ kN}$$

(2) SUM MOMENTS ABOUT THE z-AXIS

$$B_y = \frac{P\left(\frac{L}{2}\right)}{a} = 335.876 \text{ kN} \quad \text{SO} \quad A_y = P - B_y = 139.124 \text{ kN}$$

(3) SUM MOMENTS ABOUT THE x-AXIS

$$C_z = \frac{A_y L - P \frac{L}{2}}{\frac{L}{2}} = -196.751 \text{ kN}$$

(4) SUM FORCES IN THE x- AND z-DIRECTIONS  $A_x = -P = -475 \text{ kN}$   $A_z = -C_z - B_z = 868.503 \text{ kN}$

(5) USE METHOD OF JOINTS TO FIND MEMBER FORCES

$$\text{Sum forces in x-direction at joint A: } \frac{a}{c} F_{AB} + A_x = 0 \quad F_{AB} = \frac{-c}{a} A_x = 823 \text{ kN}$$

$$\text{Sum forces in y-direction at joint A: } \frac{\frac{L}{2}}{\sqrt{2} \frac{L}{2}} F_{AC} + A_y = 0 \quad F_{AC} = \sqrt{2} (-A_y) = -196.8 \text{ kN}$$

$$\text{Sum forces in y-direction at joint B: } \frac{\frac{L}{2}}{L} F_{BC} + B_y = 0 \quad F_{BC} = -2 B_y = -672 \text{ kN}$$

(6) FIND DISPLACEMENT ALONG x-AXIS AT JOINT B

Find change in length of member AB then find its projection along x axis:

$$\delta_{AB} = \frac{F_{AB} c}{EA} = 3.875 \text{ mm} \quad \beta = \arctan\left(\frac{L}{a}\right) = 54.736^\circ \quad \delta_{Bx} = \frac{\delta_{AB}}{\cos(\beta)} = 6.713 \text{ mm} \quad \boxed{\delta_{Bx} = 6.71 \text{ mm}}$$

(7) FIND  $P_{\max}$  FOR SPACE TRUSS IF  $\delta_{Bx}$  MUST BE LIMITED TO 1.5 mm

Displacements are linearly related to the loads for this linear elastic small displacement problem, so reduce load variable  $P$  from 475 kN to

$$\frac{1.5}{6.71254} 475 = 106.145 \text{ kN} \quad \boxed{P_{\max} = 106.1 \text{ kN}}$$

Repeat space truss analysis using vector operations  $a = 2.121 \text{ m}$   $L = 3 \text{ m}$   $P = 475 \text{ kN}$

POSITION AND UNIT VECTORS:

$$r_{AB} = \begin{pmatrix} a \\ 0 \\ -L \end{pmatrix} \quad e_{AB} = \frac{r_{AB}}{|r_{AB}|} = \begin{pmatrix} 0.577 \\ 0 \\ -0.816 \end{pmatrix} \quad r_{AC} = \begin{pmatrix} 0 \\ \frac{L}{2} \\ -\frac{L}{2} \end{pmatrix} \quad e_{AC} = \frac{r_{AC}}{|r_{AC}|} = \begin{pmatrix} 0 \\ 0.707 \\ -0.707 \end{pmatrix}$$

FIND MOMENT AT A:

$$M_A = r_{AB} \times R_B + r_{AC} \times R_C$$

$$M_A = r_{AB} \times \begin{pmatrix} 0 \\ RB_y \\ RB_z \end{pmatrix} + r_{AC} \times \begin{pmatrix} 2P \\ -P \\ RC_z \end{pmatrix} = \begin{pmatrix} 3.0 \text{ m } RB_y + 1.5 \text{ m } RC_z - 712.5 \text{ kN}\cdot\text{m} \\ -2.1213 \text{ m } RB_z - 1425.0 \text{ kN}\cdot\text{m} \\ 2.1213 \text{ m } RB_y - 1425.0 \text{ kN}\cdot\text{m} \end{pmatrix}$$

FIND MOMENTS ABOUT LINES OR AXES:

$$M_A e_{AB} = -1.732 \text{ m } RB_y + 1.7321 \text{ m } RB_y + 0.86603 \text{ m } RC_z + 752.15 \text{ kN}\cdot\text{m}$$

$$RC_z = \frac{-244.12}{0.72169} = -338.262 \quad C_z = -196.751 \text{ kN}$$

$$M_A e_{AC} = -1.5 \text{ m } RB_y + -1.5 \text{ m } RB_z \quad \text{So} \quad RB_y = -RB_z$$

$$M_A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = -2.1213 \text{ m } RB_z + -1425.0 \text{ kN}\cdot\text{m} \quad \text{So} \quad RB_z = \frac{462.5}{-1.7678} = -261.625 \quad B_z = -671.75 \text{ kN}$$

$$M_A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2.1213 \text{ m } RB_y + -1425.0 \text{ kN}\cdot\text{m} \quad \text{So} \quad RB_y = -RB_z = 261.625 \quad B_y = -335.876 \text{ kN}$$

$$\Sigma F_y = 0 \quad A_y = P - B_y = 139.124 \text{ kN}$$

Reactions obtained using vector operations agree with those based on scalar operations.

**Problem 2.2-15**

$$d = \frac{1}{10} \text{ in.} \quad L = 12(12) \text{ in.} \quad E = 10,600 \times (10^3) \text{ psi}$$

$$\delta_a = \frac{1}{8} \text{ in.} \quad \sigma_a = 10 \times (10^3) \text{ psi}$$

$$A = \frac{\pi d^2}{4} \quad A = 7.854 \times 10^{-3} \text{ in.}^2$$

$$EA = 8.325 \times 10^4 \text{ lb}$$



Maximum load based on elongation:

$$P_{\max 1} = \frac{EA}{L} \delta_a \quad P_{\max 1} = 72.3 \text{ lb} \quad \leftarrow \text{controls}$$

Maximum load based on stress:

$$P_{\max 2} = \sigma_a A \quad P_{\max 2} = 78.5 \text{ lb}$$

### Problem 2.2-16

#### NUMERICAL DATA

$$W = 25 \text{ N} \quad k_1 = 0.300 \text{ N/mm} \quad L_1 = 250 \text{ mm}$$

$$k_2 = 0.400 \text{ N/mm} \quad L_2 = 200 \text{ mm}$$

$$L = 350 \text{ mm} \quad h = 80 \text{ mm} \quad P = 18 \text{ N}$$

- (a) LOCATION OF LOAD  $P$  TO BRING BAR TO HORIZONTAL POSITION

Use statics to get forces in both springs:

$$\begin{aligned} \sum M_A = 0 \quad F_2 &= \frac{1}{L} \left( W \frac{L}{2} + Px \right) \\ F_2 &= \frac{W}{2} + P \frac{x}{L} \end{aligned}$$

$$\sum F_V = 0 \quad F_1 = W + P - F_2$$

$$F_1 = \frac{W}{2} + P \left( 1 - \frac{x}{L} \right)$$

Use constraint equation to define horizontal position, then solve for location  $x$ :

$$L_1 + \frac{F_1}{k_1} = L_2 + h + \frac{F_2}{k_2}$$

Substitute expressions for  $F_1$  and  $F_2$  above into constraint equilibrium and solve for  $x$ :

$$\begin{aligned} x &= \frac{-2L_1 L k_1 k_2 - k_2 WL - 2k_2 PL + 2L_2 L k_1 k_2 + 2h L k_1 k_2 + k_1 WL}{-2P(k_1 + k_2)} \\ x &= 134.7 \text{ mm} \quad \leftarrow \end{aligned}$$

- (b) NEXT REMOVE  $P$  AND FIND NEW VALUE OF SPRING CONSTANT  $k_1$  SO THAT BAR IS HORIZONTAL UNDER WEIGHT  $W$

$$\text{Now, } F_1 = \frac{W}{2} \quad F_2 = \frac{W}{2} \quad \text{since } P = 0$$

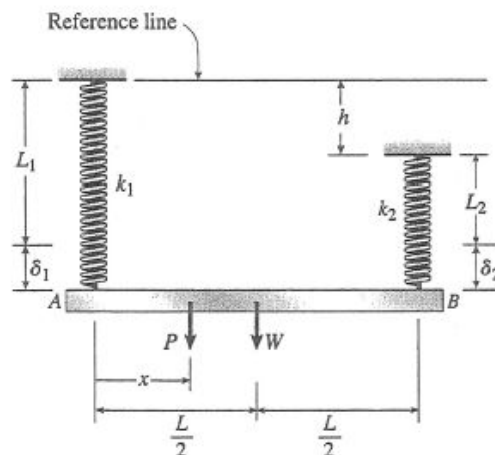
Same constraint equation as above but now  $P = 0$ :

$$L_1 + \frac{W}{2k_1} - (L_2 + h) - \frac{\left(\frac{W}{2}\right)}{k_2} = 0$$

Solve for  $k_1$ :

$$k_1 = \frac{-Wk_2}{[2k_2[L_1 - (L_2 + h)]] - W}$$

$$k_1 = 0.204 \text{ N/mm} \quad \leftarrow$$



PART (c)—CONTINUED (from page below)

STATICS

$$\sum M_{k_1} = 0 \quad F_2 = \frac{W \left( \frac{L}{2} - b \right)}{L - b}$$

$$\sum F_V = 0$$

$$F_1 = W - F_2$$

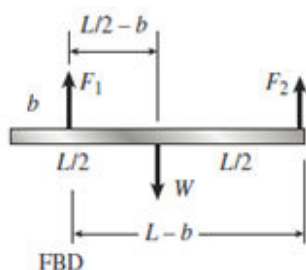
$$F_1 = W - \frac{W \left( \frac{L}{2} - b \right)}{L - b}$$

$$F_1 = \frac{WL}{2(L - b)}$$

Part (c) continued in right column below



- (c) Use  $k_1 = 0.300 \text{ N/mm}$  BUT RELOCATE SPRING  $k_1$  ( $x = b$ ) SO THAT BAR ENDS UP IN HORIZONTAL POSITION UNDER WEIGHT  $W$



$$b = \frac{2L_1k_1k_2L + WLk_2 - 2L_2k_1k_2L - 2hk_1k_2L - Wk_1L}{(2L_1k_1k_2) - 2L_2k_1k_2 - 2hk_1k_2 - 2Wk_1}$$

Part (c) continued on page above

- (d) REPLACE SPRING  $k_1$  WITH SPRINGS IN SERIES:  
 $k_1 = 0.3 \text{ N/mm}$ ,  $L_1/2$ , AND  $k_3$ ,  $L_1/2$ . FIND  $k_3$   
 SO THAT BAR HANGS IN HORIZONTAL POSITION

STATICS  $F_1 = \frac{W}{2}$   $F_2 = \frac{W}{2}$

$$k_3 = \frac{Wk_1k_2}{-2L_1k_1k_2 - Wk_2 + 2L_2k_1k_2 + 2hk_1k_2 + Wk_1}$$

NOTE—equivalent spring constant for series springs:

$$k_e = \frac{k_1k_3}{k_1 + k_3}$$

Constraint equation—substitute above expressions for  $F_1$  and  $F_2$  and solve for  $b$ :

$$L_1 + \frac{F_1}{k_1} - (L_2 + h) - \frac{F_2}{k_2} = 0$$

Use the following data:

$$k_1 = 0.300 \text{ N/mm} \quad k_2 = 0.4 \text{ N/mm} \quad L_1 = 250 \text{ mm}$$

$$L_2 = 200 \text{ mm} \quad L = 350 \text{ mm}$$

$$b = 74.1 \text{ mm} \quad \leftarrow$$

Part (d) continued from left column

New constraint equation; solve for  $k_3$ :

$$L_1 + \frac{F_1}{k_1} + \frac{F_1}{k_3} - (L_2 + h) - \frac{F_2}{k_2} = 0$$

$$L_1 + \frac{W/2}{k_1} + \frac{W/2}{k_3} - (L_2 + h) - \frac{W/2}{k_2} = 0$$

$$k_3 = 0.638 \text{ N/mm} \quad \leftarrow$$

$$k_e = 0.204 \text{ N/mm} \quad \leftarrow \text{ checks—same as (b) above}$$

### Problem 2.2-17

The figure shows a section cut through the pipe, cap, and rod.

#### NUMERICAL DATA

$$E_c = 12000 \text{ ksi} \quad E_b = 14,000 \text{ ksi}$$

$$W = 2 \text{ k} \quad d_c = 6 \text{ in.} \quad d_r = \frac{1}{2} \text{ in.}$$

$$\sigma_a = 5 \text{ ksi} \quad \delta_a = 0.02 \text{ in.}$$

$$\text{Unit weights (see Table I-1): } \gamma_s = 2.836 \times 10^{-4} \text{ k/in.}^3$$

$$\gamma_b = 3.009 \times 10^{-4} \text{ k/in.}^3$$

$$L_c = 48 \text{ in.} \quad L_r = 42 \text{ in.}$$

$$t_s = 1 \text{ in.}$$

#### (a) MINIMUM REQUIRED WALL THICKNESS OF CAST IRON PIPE, $t_{c \min}$

First check allowable stress then allowable shortening:

$$W_{\text{cap}} = \gamma_s \left( \frac{\pi}{4} d_c^2 t_s \right)$$

$$W_{\text{cap}} = 8.018 \times 10^{-3} \text{ k}$$

$$W_{\text{rod}} = \gamma_b \left( \frac{\pi}{4} d_r^2 L_r \right)$$

$$W_{\text{rod}} = 2.482 \times 10^{-3} \text{ k}$$

$$W_t = W + W_{\text{cap}} + W_{\text{rod}} \quad W_t = 2.01 \text{ k}$$

$$A_{\min} = \frac{W_t}{\sigma_a} \quad A_{\min} = 0.402 \text{ in.}^2$$

$$A_{\text{pipe}} = \frac{\pi}{4} [d_c^2 - (d_c - 2t_c)^2]$$

$$A_{\text{pipe}} = \pi t_c (d_c - t_c)$$

$$t_c (d_c - t_c) = \frac{W_t}{\pi \sigma_a}$$

$$\text{Let } \alpha = \frac{W_t}{\pi \sigma_a} \quad \alpha = 0.128:$$

$$t_c^2 - d_c t_c + \alpha = 0$$

$$t_c = \frac{d_c - \sqrt{d_c^2 - 4\alpha}}{2} \quad t_c = 0.021 \text{ in.}$$

^ minimum based on  $\sigma_a$

Now check allowable shortening requirement:

$$\delta_{\text{pipe}} = \frac{W_t L_c}{E_c A_{\min}} \quad A_{\min} = \frac{W_t L_c}{E_c \delta_a}$$

$$A_{\min} = 0.447 \text{ in.}^2 < \text{larger than value based on } \sigma_a \text{ above}$$

$$\pi t_c (d_c - t_c) = \frac{W_t L_c}{E_c \delta_a}$$

$$t_c^2 - d_c t_c + \beta = 0 \quad \beta = \frac{W_t L_c}{\pi E_c \delta_a}$$

$$\beta = 0.142$$

$$t_c = \frac{d_c - \sqrt{d_c^2 - 4\beta}}{2}$$

$$t_c = 0.021 \text{ in.} \quad \leftarrow \text{minimum based on } \delta_a \text{ and } \sigma_a \text{ controls}$$

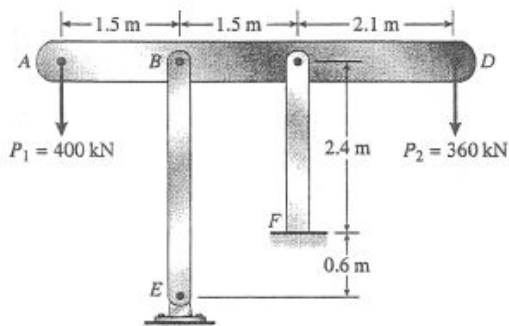
#### (b) ELONGATION OF ROD DUE TO SELF WEIGHT AND ALSO WEIGHT $W$

$$\delta_r = \frac{\left( W + \frac{W_{\text{rod}}}{2} \right) L_r}{E_b \left( \frac{\pi}{4} d_r^2 \right)} \quad \delta_r = 0.031 \text{ in.} \quad \leftarrow$$

#### (c) MINIMUM CLEARANCE $h$

$$h_{\min} = \delta_a + \delta_r \quad h_{\min} = 0.051 \text{ in.} \quad \leftarrow$$

**Problem 2.2-18**



$$A_{BE} = 11,100 \text{ mm}^2$$

$$A_{CF} = 9,280 \text{ mm}^2$$

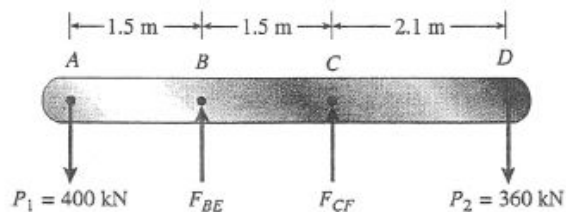
$$E = 200 \text{ GPa}$$

$$L_{BE} = 3.0 \text{ m}$$

$$L_{CF} = 2.4 \text{ m}$$

$$P_1 = 400 \text{ kN}; P_2 = 360 \text{ kN}$$

FREE-BODY DIAGRAM OF BAR ABCD



$$\sum M_B = 0 \quad \curvearrowright$$

$$(400 \text{ kN})(1.5 \text{ m}) + F_{CF}(1.5 \text{ m}) - (360 \text{ kN})(3.6 \text{ m}) = 0$$

$$F_{CF} = 464 \text{ kN}$$

$$\sum M_C = 0$$

$$(400 \text{ kN})(3.0 \text{ m}) - F_{BE}(1.5 \text{ m}) - (360 \text{ kN})(2.1 \text{ m}) = 0$$

$$F_{BE} = 296 \text{ kN}$$

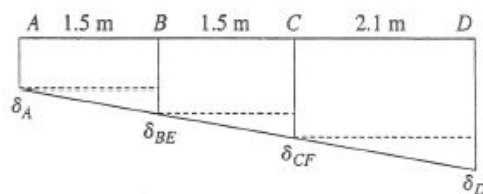
SHORTENING OF BAR BE

$$\delta_{BE} = \frac{F_{BE}L_{BE}}{EA_{BE}} = \frac{(296 \text{ kN})(3.0 \text{ m})}{(200 \text{ GPa})(11,100 \text{ mm}^2)} = 0.400 \text{ mm}$$

SHORTENING OF BAR CF

$$\delta_{CF} = \frac{F_{CF}L_{CF}}{EA_{CF}} = \frac{(464 \text{ kN})(2.4 \text{ m})}{(200 \text{ GPa})(9,280 \text{ mm}^2)} = 0.600 \text{ mm}$$

DISPLACEMENT DIAGRAM



$$\delta_{BE} - \delta_A = \delta_{CF} - \delta_{BE} \text{ or } \delta_A = 2\delta_{BE} - \delta_{CF}$$

$$\delta_A = 2(0.400 \text{ mm}) - 0.600 \text{ mm}$$

$$= 0.200 \text{ mm} \quad \leftarrow$$

(Downward)

$$\delta_D - \delta_{CF} = \frac{2.1}{1.5}(\delta_{CF} - \delta_{BE})$$

$$\begin{aligned} \text{or } \delta_D &= \frac{12}{5}\delta_{CF} - \frac{7}{5}\delta_{BE} \\ &= \frac{12}{5}(0.600 \text{ mm}) - \frac{7}{5}(0.400 \text{ mm}) \\ &= 0.880 \text{ mm} \quad \leftarrow \\ &\quad \text{(Downward)} \end{aligned}$$

### Problem 2.2-19

(a) DISPLACEMENT  $\delta_D$

Use FBD of beam  $BCD$   $\sum M_B = 0$   $R_C = \frac{1}{L} \left[ \left( 2 \frac{P}{L} \right) \left( \frac{3}{4} L \right) \left( \frac{3}{8} L \right) + \frac{P}{4} \left( L + \frac{3}{4} L \right) \right] = P$  <compression force in column  $CF$

$$\sum F_V = 0 \quad R_B = \left( 2 \frac{P}{L} \right) \left( \frac{3}{4} L \right) + \frac{P}{4} - R_C = \frac{3P}{4} \quad \text{<compression force in column } BA$$

Downward displacements at  $B$  and  $C$ :  $\delta_B = R_B f_1 = \frac{3P f_1}{4}$   $\delta_C = R_C f_2 = P f_2$

$$\text{Geometry: } \delta_D = \delta_B + (\delta_C - \delta_B) \left( \frac{L + \frac{3}{4}L}{L} \right) = \frac{7P f_2}{4} - \frac{9P f_1}{16} \quad \delta_D = \frac{7P f_2}{4} - \frac{9P f_1}{16} = \boxed{\frac{P}{16} (28 f_2 - 9 f_1)}$$

(b) DISPLACEMENT TO HORIZONTAL POSITION, SO  $\delta_C = \delta_B$  and  $\frac{3P f_1}{4} = P f_2$  or  $\frac{f_1}{f_2} = \frac{4}{3}$

$$\frac{\frac{L_1}{EA_1}}{\frac{L_2}{EA_2}} = \frac{4}{3} \quad \text{or} \quad \frac{L_1}{L_2} = \frac{4}{3} \left( \frac{A_1}{A_2} \right) \quad \frac{L_1}{L_2} = \frac{4}{3} \left( \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_2^2} \right) = \frac{4 d_1^2}{3 d_2^2} \quad \frac{L_1}{L_2} = \frac{4}{3} \left( \frac{d_1}{d_2} \right)^2 \quad \text{with} \quad \frac{d_1}{d_2} = \frac{9}{8}$$

$$\frac{L_1}{L_2} = \frac{4}{3} \left( \frac{9}{8} \right)^2 = \frac{27}{16} \quad \boxed{\frac{L_1}{L_2} = \frac{27}{16}}$$

(c) IF  $L_1 = 2 L_2$ , FIND THE  $d_1/d_2$  RATIO SO THAT BEAM  $BCD$  DISPLACES DOWNWARD TO A HORIZONTAL POSITION

$$\frac{L_1}{L_2} = 2 \quad \text{and} \quad \delta_C = \delta_B \quad \text{from part (b).} \quad \left( \frac{d_1}{d_2} \right)^2 = \frac{3}{4} \left( \frac{L_1}{L_2} \right) \quad \text{so} \quad \boxed{\frac{d_1}{d_2} = \sqrt{\frac{3}{4} (2)} = 1.225}$$

(d) IF  $d_1 = (9/8) d_2$  AND  $L_1/L_2 = 1.5$ , AT WHAT HORIZONTAL DISTANCE  $x$  FROM  $B$  SHOULD LOAD  $P/4$  AT  $D$  BE PLACED?

$$\text{Given} \quad \frac{d_1}{d_2} = \frac{9}{8} \quad \text{and} \quad \frac{L_1}{L_2} = 1.5 \quad \text{or} \quad \frac{f_1}{f_2} = \frac{L_1}{L_2} \left( \frac{A_2}{A_1} \right) \quad \frac{f_1}{f_2} = \frac{L_1}{L_2} \left( \frac{d_2}{d_1} \right)^2 = \frac{3}{2} \left( \frac{8}{9} \right)^2 = \frac{32}{27}$$

Recompute column forces  $R_B$  and  $R_C$  but now with load  $P/4$  positioned at distance  $x$  from  $B$ .

$$\text{Use FBD of beam } BCD: \quad \sum M_B = 0 \quad R_C = \frac{1}{L} \left[ \left( 2 \frac{P}{L} \right) \left( \frac{3}{4} L \right) \left( \frac{3}{8} L \right) + \frac{P}{4} (x) \right] = \frac{\frac{9LP}{16} + \frac{Px}{4}}{L}$$

$$\sum F_V = 0 \quad R_B = \left( 2 \frac{P}{L} \right) \left( \frac{3}{4} L \right) + \frac{P}{4} - R_C = \frac{7P}{4} - \frac{\frac{9LP}{16} + \frac{Px}{4}}{L}$$

Horizontal displaced position under load  $q$  and load  $P/4$  so  $\delta_C = \delta_B$  or  $R_C f_2 = R_B f_1$ .

$$\left(\frac{\frac{9LP}{16} + \frac{Px}{4}}{L}\right)f_2 = \left(\frac{7P}{4} - \frac{\frac{9LP}{16} + \frac{Px}{4}}{L}\right)f_1 \text{ solve, } x = -\frac{9Lf_2 - 19Lf_1}{4f_1 + 4f_2} = -\frac{L(9f_2 - 19f_1)}{4(f_1 + f_2)}$$

$$x = -\frac{L(9f_2 - 19f_1)}{4(f_1 + f_2)} \text{ or } x = L \left[ \frac{19\frac{f_1}{f_2} - 9}{4\left(\frac{f_1}{f_2} + 1\right)} \right]$$

Now substitute  $f_1/f_2$  ratio from above:

$$x = L \left[ \frac{19\frac{32}{27} - 9}{4\left(\frac{32}{27} + 1\right)} \right] = \frac{365L}{236} \quad \frac{365}{236} = 1.547$$

### Problem 2.2-20

Apply the laws of statics to the structure in its displaced position; also use FBD's of the left and right bars alone (referred to as LHFB and RHFB below).

$$\begin{aligned}\text{OVERALL FBD: } \quad \Sigma F_H = 0 \quad H_A - k_1 \delta = 0 \quad \text{so} \quad H_A = k_1 \delta \\ \Sigma F_V = 0 \quad R_A + R_C = P \\ \Sigma M_A = 0 \quad k_r(\alpha - \theta) - P \frac{L_2}{2} + R_C L_2 = 0 \quad R_C = \frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right]\end{aligned}$$

$$\begin{aligned}\text{LHFB: } \quad \Sigma M_B = 0 \quad H_A h + k \frac{\delta}{2} \left( \frac{h}{2} \right) - R_A \left( \frac{L_2}{2} \right) + k_r(\alpha - \theta) = 0 \\ R_A = \frac{2}{L_2} \left[ k_1 \delta h + k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_r(\alpha - \theta) \right]\end{aligned}$$

$$\text{RHFB: } \quad \Sigma M_B = 0 \quad -k \frac{\delta}{2} \left( \frac{h}{2} \right) - k_1 \delta h + R_C \frac{L_2}{2} = 0 \quad R_C = \frac{2}{L_2} \left[ k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_1 \delta h \right]$$

Equate the two expressions for  $R_C$  then substitute expressions for  $L_2$ ,  $k_r$ ,  $k_1$ ,  $h$  and  $\delta$

$$\begin{aligned}\frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right] &= \frac{2}{L_2} \left[ k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_1 \delta h \right] \quad \text{OR} \\ \frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right] &- \left[ \frac{2}{L_2} \left[ k \frac{2b(\cos(\theta) - \cos(\alpha))}{2} \frac{b \sin(\theta)}{2} + k_1 [2b(\cos(\theta) - \cos(\alpha))] (b \sin(\theta)) \right] \right] = 0\end{aligned}$$

(a) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE  $\theta$  AND DISTANCE INCREASE  $\delta$

$$b = 200 \text{ mm} \quad k = 3.2 \text{ kN/m} \quad \alpha = 45^\circ \quad P = 50 \text{ N} \quad k_1 = 0 \quad k_r = 0$$

$$L_2 = 2b \cos(\theta) \quad L_1 = 2b \cos(\alpha) \quad \delta = L_2 - L_1 \quad \delta = 2b(\cos(\theta) - \cos(\alpha)) \quad h = b \sin(\theta)$$

$$\frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right] - \left[ \frac{2}{L_2} \left[ k \frac{2b(\cos(\theta) - \cos(\alpha))}{2} \frac{b \sin(\theta)}{2} + k_1 [2b(\cos(\theta) - \cos(\alpha))] (b \sin(\theta)) \right] \right] = 0$$

$$\text{Solving above equation numerically gives } \boxed{\theta = 35.1^\circ} \quad \boxed{\delta = 44.6 \text{ mm}}$$

COMPUTE REACTIONS

$$R_C = \frac{2}{L_2} \left[ k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_1 \delta h \right] = 25 \text{ N} \quad R_C = \frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right] = 25 \text{ N}$$

$$R_A = \frac{2}{L_2} \left[ k_1 \delta h + k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_r(\alpha - \theta) \right] = 25 \text{ N} \quad M_A = k_r(\alpha - \theta) = 0$$

$$R_A + R_C = 50 \text{ N} \quad < \text{check}$$

$$\boxed{R_A = 25 \text{ N}}$$

$$\boxed{R_C = 25 \text{ N}}$$

(b) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE  $\theta$  AND DISTANCE INCREASE  $\delta$

$$b = 200 \text{ mm} \quad k = 3.2 \text{ kN/m} \quad \alpha = 45^\circ \quad P = 50 \text{ N} \quad k_1 = \frac{k}{2} \quad k_r = \frac{k}{2} b^2$$

$$L_2 = 2b \cos(\theta) \quad L_1 = 2b \cos(\alpha) \quad \delta = L_2 - L_1 \quad \delta = 2b(\cos(\theta) - \cos(\alpha)) \quad h = b \sin(\theta)$$

$$\frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right] - \left[ \frac{2}{L_2} \left[ k \frac{2b(\cos(\theta) - \cos(\alpha))}{2} \frac{b \sin(\theta)}{2} + k_1 [2b(\cos(\theta) - \cos(\alpha))] (b \sin(\theta)) \right] \right] = 0$$

$$\text{Solving above equation numerically gives } \boxed{\theta = 43.3^\circ} \quad \boxed{\delta = 8.19 \text{ mm}}$$

COMPUTE REACTIONS

$$R_C = \frac{2}{L_2} \left[ k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_1 \delta h \right] = 18.5 \text{ N} \quad R_2 = \frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right] = 18.5 \text{ N}$$

$$R_A = \frac{2}{L_2} \left[ k_1 \delta h + k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_r(\alpha - \theta) \right] = 31.5 \text{ N} \quad M_A = k_r(\alpha - \theta) = 1.882 \text{ N}\cdot\text{m}$$

$$R_A + R_C = 50 \text{ N} \quad < \text{check}$$

$$\boxed{R_A = 31.5 \text{ N}}$$

$$\boxed{R_C = 18.5 \text{ N}}$$

$$\boxed{M_A = 1.882 \text{ N}\cdot\text{m}}$$



### Problem 2.2-21

Apply the laws of statics to the structure in its displaced position; also use FBD's of the left and right bars alone (referred to as LHFB and RHFB below)

OVERALL FBD  $\sum F_H = 0 \quad H_A - k_1 \delta = 0 \quad \text{so} \quad H_A = k_1 \delta$

$\sum F_V = 0 \quad R_A + R_C = P$

$\sum M_A = 0 \quad k_r(\alpha - \theta) - P \frac{L_2}{2} + R_C L_2 = 0 \quad R_C = \frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right]$

LHFB  $\sum M_B = 0 \quad H_A h + k \frac{\delta}{2} \left( \frac{h}{2} \right) - R_A \frac{L_2}{2} + k_r(\alpha - \theta) = 0$

$R_A = \frac{2}{L_2} \left[ k_1 \delta h + k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_r(\alpha - \theta) \right]$

RHFB  $\sum M_B = 0 \quad -k \frac{\delta}{2} \left( \frac{h}{2} \right) - k_1 \delta h + R_C \frac{L_2}{2} = 0 \quad R_C = \frac{2}{L_2} \left[ k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_1 \delta h \right]$

Equate the two expressions above for  $R_C$  then substitute expressions for  $L_2$ ,  $k_r$ ,  $k_1$ ,  $h$ , and  $\delta$

$\frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right] = \frac{2}{L_2} \left[ k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_1 \delta h \right] \quad \text{OR}$

$\frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right] - \left[ \frac{2}{L_2} \left[ k \frac{2b(\cos(\theta) - \cos(\alpha))}{2} \frac{b \sin(\theta)}{2} + k_1 [2b(\cos(\theta) - \cos(\alpha))] (b \sin(\theta)) \right] \right] = 0$

(a) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE  $\theta$  AND DISTANCE INCREASE  $\delta$

$b = 8 \text{ in.} \quad k = 16 \text{ lb/in.} \quad \alpha = 45^\circ \quad P = 101b \quad k_1 = 0 \quad k_r = 0$

$L_2 = 2b \cos(\theta) \quad L_1 = 2b \cos(\alpha) \quad \delta = L_2 - L_1 \quad \delta = 2b(\cos(\theta) - \cos(\alpha)) \quad h = b \sin(\theta)$

$\frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right] - \left[ \frac{2}{L_2} \left[ k \frac{2b(\cos(\theta) - \cos(\alpha))}{2} \frac{b \sin(\theta)}{2} + k_1 [2b(\cos(\theta) - \cos(\alpha))] (b \sin(\theta)) \right] \right] = 0$

Solving above equation numerically gives  $\boxed{\theta = 35.1^\circ} \quad \boxed{\delta = 1.782 \text{ in.}}$

COMPUTE REACTIONS

$R_C = \frac{2}{L_2} \left[ k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_1 \delta h \right] = 5 \text{ lb} \quad R_C = \frac{1}{L_C} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right] = 5 \text{ lb}$

$R_A = \frac{2}{L_2} \left[ k_1 \delta h + k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_r(\alpha - \theta) \right] = 5 \text{ lb} \quad M_A = k_r(\alpha - \theta) = 0$

$R_A + R_C = 10 \text{ lb} \quad < \text{check} \quad \boxed{R_A = 5 \text{ lb}} \quad \boxed{R_C = 5 \text{ lb}}$

(b) SUBSTITUTE NUMERICAL VALUES, THEN SOLVE NUMERICALLY FOR ANGLE  $\theta$  AND DISTANCE INCREASE  $\delta$

$$b = 8 \text{ in.} \quad k = 16 \text{ lb/in.} \quad \alpha = 45^\circ \quad P = 101b \quad k_1 = \frac{k}{2} \quad k_r = \frac{k}{2} b^2$$

$$L_2 = 2b \cos(\theta) \quad L_1 = 2b \cos(\alpha) \quad \delta = L_2 - L_1 \quad \delta = 2b(\cos(\theta) - \cos(\alpha)) \quad h = b \sin(\theta)$$

$$\frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right] - \left[ \frac{2}{L_2} \left[ k \frac{2b(\cos(\theta) - \cos(\alpha))}{2} \frac{b \sin(\theta)}{2} + k_1 [2b(\cos(\theta) - \cos(\alpha))] (b \sin(\theta)) \right] \right] = 0$$

Solving above equation numerically gives  $\theta = 43.3^\circ$   $\delta = 0.327 \text{ in.}$

COMPUTE REACTIONS

$$R_C = \frac{2}{L_2} \left[ k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_1 \delta h \right] = 3.71 \text{ lb} \quad R_C = \frac{1}{L_2} \left[ P \frac{L_2}{2} - k_r(\alpha - \theta) \right] = 3.71 \text{ lb}$$

$$R_A = \frac{2}{L_2} \left[ k_1 \delta h + k \frac{\delta}{2} \left( \frac{h}{2} \right) + k_r(\alpha - \theta) \right] = 6.3 \text{ lb} \quad M_A = k_r(\alpha - \theta) = 1.252 \text{ ft}\cdot\text{lb}$$

$$R_A + R_C = 10.01 \text{ lb} \quad < \text{check} \quad R_A = 6.3 \text{ lb} \quad R_C = 3.71 \text{ lb} \quad M_A = 1.252 \text{ lb}\cdot\text{ft}$$



**Problem 2.3-1**

NUMERICAL DATA

$$P = 3 \text{ k} \quad L_1 = 20 \text{ in.} \quad L_2 = 50 \text{ in.} \quad d_A = 0.5 \text{ in.} \quad d_B = 1 \text{ in.} \quad E = 18000 \text{ ksi}$$

(a) TOTAL ELONGATION

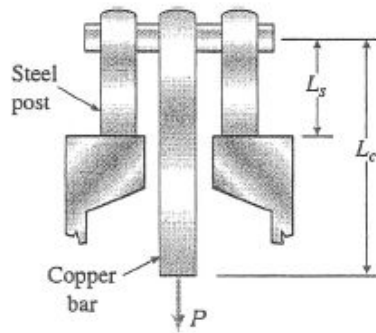
$$\delta_1 = \frac{4PL_1}{\pi E d_A d_B} = 0.00849 \text{ in.} \quad \delta_2 = \frac{PL_2}{E \frac{\pi}{4} d_B^2} = 0.01061 \text{ in.}$$

$$\delta = 2\delta_1 + \delta_2 = 0.0276 \text{ in.} \quad \boxed{\delta = 0.0276 \text{ in.}}$$

(b) FIND NEW DIAMETERS AT *B* AND *C* IF TOTAL ELONGATION CANNOT EXCEED 0.025 in.

$$2\left(\frac{4PL_1}{\pi E d_A d_B}\right) + \frac{PL_2}{E \frac{\pi}{4} d_B^2} = 0.025 \text{ in.} \quad \text{Solving for } d_B: \quad \boxed{d_B = 1.074 \text{ in.}}$$

**Problem 2.3-2**



$$L_c = 2.0 \text{ m}$$

$$A_c = 4800 \text{ mm}^2$$

$$E_c = 120 \text{ GPa}$$

$$L_s = 0.5 \text{ m}$$

$$A_s = 4500 \text{ mm}^2$$

$$E_s = 200 \text{ GPa}$$

(a) DOWNWARD DISPLACEMENT  $\delta$  ( $P = 180 \text{ kN}$ )

$$\delta_c = \frac{PL_c}{E_c A_c} = \frac{(180 \text{ kN})(2.0 \text{ m})}{(120 \text{ GPa})(4800 \text{ mm}^2)} = 0.625 \text{ mm}$$

$$\delta_s = \frac{(P/2)L_s}{E_s A_s} = \frac{(90 \text{ kN})(0.5 \text{ m})}{(200 \text{ GPa})(4500 \text{ mm}^2)} = 0.050 \text{ mm}$$

$$\delta = \delta_c + \delta_s = 0.625 \text{ mm} + 0.050 \text{ mm} = 0.675 \text{ mm} \quad \leftarrow$$

(b) MAXIMUM LOAD  $P_{\max}$  ( $\delta_{\max} = 1.0 \text{ mm}$ )

$$\frac{P_{\max}}{P} = \frac{\delta_{\max}}{\delta} \quad P_{\max} = P \left( \frac{\delta_{\max}}{\delta} \right)$$

$$P_{\max} = (180 \text{ kN}) \left( \frac{1.0 \text{ mm}}{0.675 \text{ mm}} \right) = 267 \text{ kN} \quad \leftarrow$$

### Problem 2.3-3

NUMERICAL DATA

$$A = 0.40 \text{ in.}^2 \quad P_1 = 1700 \text{ lb}$$

$$P_2 = 1200 \text{ lb} \quad P_3 = 1300 \text{ lb}$$

$$E = 10.4 (10^6) \text{ psi}$$

$$a = 60 \text{ in.} \quad b = 24 \text{ in.} \quad c = 36 \text{ in.}$$

(a) TOTAL ELONGATION

$$\delta = \frac{1}{EA} [(P_1 + P_2 - P_3)a + (P_2 - P_3)b + (-P_3)c] = 0.01125 \text{ in.} \quad \boxed{\delta = 0.01125 \text{ in.}} \quad (\text{elongation})$$

(b) INCREASE  $P_3$  SO THAT BAR DOES NOT CHANGE LENGTH

$$\frac{1}{EA} [(P_1 + P_2 - P_3)a + (P_2 - P_3)b + (-P_3)c] = 0 \text{ solve, } P_3 = 1690 \text{ lb}$$

So new value of  $P_3$  is 1690 lb,  
an increase of 390 lb.

(c) NOW CHANGE CROSS-SECTIONAL AREA OF  $AB$  SO THAT BAR DOES NOT CHANGE LENGTH  $P_3 = 1300 \text{ lb}$

$$\frac{1}{E} \left[ (P_1 + P_2 - P_3) \frac{a}{A_{AB}} + (P_2 - P_3) \frac{b}{A} + (-P_3) \frac{c}{A} \right] = 0$$

Solving for  $A_{AB}$ :  $A_{AB} = 0.78 \text{ in.}^2$   $\frac{A_{AB}}{A} = 1.951$

### Problem 2.3-4

$$E = 200 \text{ GPa}$$

$$A_1 = 6000 \text{ mm}^2 \quad A_2 = 5000 \text{ mm}^2 \quad A_3 = 4000 \text{ mm}^2 \quad L_1 = 500 \text{ mm} \quad L_2 = L_1 \quad L_3 = L_1$$

$$P_B = 50 \text{ N} \quad P_C = 250 \text{ N} \quad P_E = 350 \text{ N}$$

Internal forces in each segment (tension +) - cut bar and use lower FBD

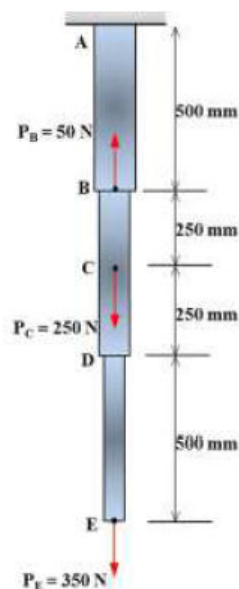
$$N_{AB} = -P_B + P_C + P_E = 550 \text{ N} \quad N_{BC} = P_C + P_E = 600 \text{ N} \quad N_{CD} = P_E = 350 \text{ N} \quad N_{DE} = P_E = 350 \text{ N}$$

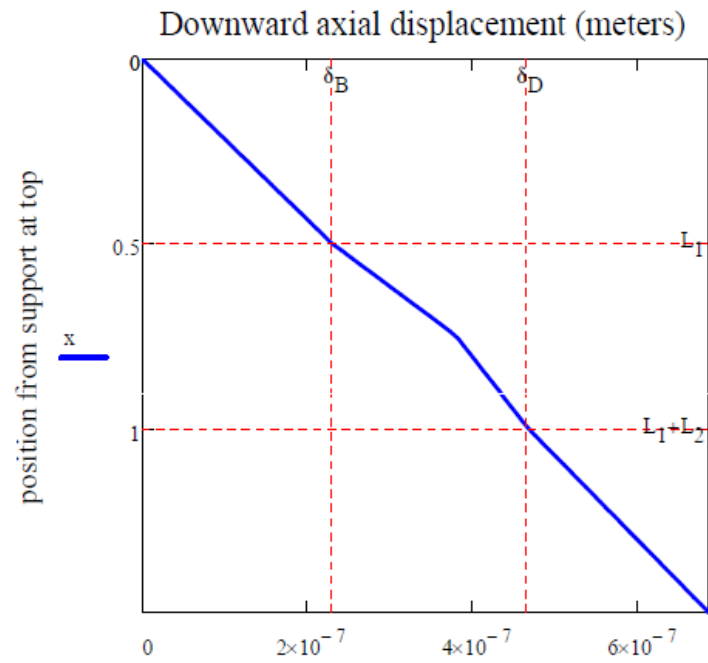
Use force-displacement relation to find segment elongations then sum elongations to find downward displacements

$$\begin{aligned} \delta_B &= \frac{N_{AB} \cdot L_1}{E \cdot A_1} = 2.292 \times 10^{-4} \cdot \text{mm} \quad \text{downward} & \delta_C &= \delta_B + \frac{N_{BC} \cdot \frac{L_2}{2}}{E \cdot A_2} = 3.792 \times 10^{-4} \cdot \text{mm} \\ \delta_D &= \delta_C + \frac{N_{CD} \cdot \frac{L_2}{2}}{E \cdot A_2} = 4.667 \times 10^{-4} \cdot \text{mm} & \delta_E &= \delta_D + \frac{N_{DE} \cdot L_3}{E \cdot A_3} = 6.854 \times 10^{-4} \cdot \text{mm} \end{aligned}$$

Axial displacement diagram - x origin at A, positive downward

$$\delta(x) = \begin{cases} \delta_B \cdot \frac{x}{L_1} & \text{if } x \leq L_1 \\ \delta_B + (\delta_C - \delta_B) \cdot \left( \frac{x - L_1}{\frac{L_2}{2}} \right) & \text{if } L_1 \leq x \leq L_1 + \frac{L_2}{2} \\ \delta_C + (\delta_D - \delta_C) \cdot \left[ \frac{x - \left( L_1 + \frac{L_2}{2} \right)}{\frac{L_2}{2}} \right] & \text{if } L_1 + \frac{L_2}{2} \leq x \leq L_1 + L_2 \\ \delta_D + (\delta_E - \delta_D) \cdot \left[ \frac{x - (L_1 + L_2)}{L_3} \right] & \text{otherwise} \end{cases}$$







### Problem 2.3-6

$$\gamma = 77.0 \frac{\text{kN}}{\text{m}^3} \quad \text{from Table I-1}$$

$$E = 200 \text{ GPa}$$

$$A_1 = 6000 \text{ mm}^2 \quad A_2 = 5000 \text{ mm}^2 \quad A_3 = 4000 \text{ mm}^2 \quad L_1 = 500 \text{ mm} \quad L_2 = L_1 \quad L_3 = L_1$$

$$P_B = 50 \text{ N} \quad P_C = 250 \text{ N} \quad P_E = 350 \text{ N}$$

Internal forces in each segment (tension +) - cut bar and use lower FBD - weight per unit length =  $\gamma A_i$

$$P_{AB} = -P_B + P_C + P_E = 550 \text{ N} \quad P_{BC} = P_C + P_E = 600 \text{ N} \quad P_{CD} = P_E = 350 \text{ N} \quad P_{DE} = P_E = 350 \text{ N}$$

Now add weight per unit length -  $x$  origin at A, positive downward

$$N_{AB}(x) = P_{AB} + \gamma \cdot A_1 \cdot (L_1 - x) + \gamma \cdot A_2 \cdot L_2 + \gamma \cdot A_3 \cdot L_3 \quad N_{BC}(x) = P_{BC} + \gamma \cdot A_2 \cdot \left( L_1 + \frac{L_2}{2} - x \right) + \gamma \cdot A_2 \cdot \frac{L_2}{2} + \gamma \cdot A_3 \cdot L_3$$

$$N_{CD}(x) = P_{CD} + \gamma \cdot A_2 \cdot (L_1 + L_2 - x) + \gamma \cdot A_3 \cdot L_3 \quad N_{DE}(x) = P_{DE} + \gamma \cdot A_3 \cdot (L_1 + L_2 + L_3 - x)$$

$$\text{Note that total bar weight is not small compared to applied loads} \quad W = \gamma \cdot (A_1 \cdot L_1 + A_2 \cdot L_2 + A_3 \cdot L_3) = 577.5 \text{ N}$$

Use force-displacement relation to find segment elongations then sum elongations to find displacements.

$$\Delta_B = \int_0^{L_1} \frac{N_{AB}(x)}{E \cdot A_1} dx = 4.217 \times 10^{-4} \cdot \text{mm}$$

$$\Delta_C = \Delta_B + \int_{L_1}^{L_1 + \frac{L_2}{2}} \frac{N_{BC}(x)}{E \cdot A_2} dx = 6.463 \times 10^{-4} \cdot \text{mm}$$

$$\Delta_D = \Delta_C + \int_{L_1 + \frac{L_2}{2}}^{L_1 + L_2} \frac{N_{CD}(x)}{E \cdot A_2} dx = 7.843 \times 10^{-4} \cdot \text{mm}$$

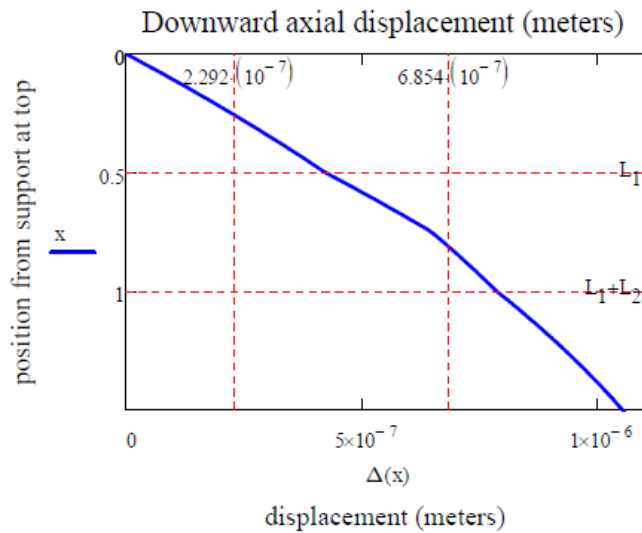
$$\Delta_E = \Delta_D + \int_{L_1 + L_2}^{L_1 + L_2 + L_3} \frac{N_{DE}(x)}{E \cdot A_3} dx = 1.051 \times 10^{-3} \cdot \text{mm}$$

Compare to  
Prob. 2.3-4

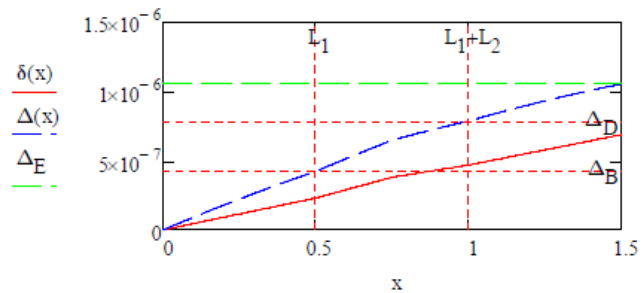
$$\frac{\Delta_B}{2.292 \cdot (10^{-4}) \cdot \text{mm}} = 1.84 \quad \frac{\Delta_C}{3.792 \cdot (10^{-4}) \cdot \text{mm}} = 1.704 \quad \frac{\Delta_D}{4.667 \cdot (10^{-4}) \cdot \text{mm}} = 1.681 \quad \frac{\Delta_E}{6.854 \cdot (10^{-4}) \cdot \text{mm}} = 1.534$$

Axial displacement diagram including weight of bar -  $x$  origin at A, positive downward

$$\Delta(x) = \begin{cases} \int_0^x \frac{N_{AB}(x)}{E \cdot A_1} dx & \text{if } x \leq L_1 \\ \Delta_B + \int_{L_1}^x \frac{N_{BC}(x)}{E \cdot A_2} dx & \text{if } L_1 \leq x \leq L_1 + \frac{L_2}{2} \\ \Delta_C + \int_{L_1 + \frac{L_2}{2}}^x \frac{N_{CD}(x)}{E \cdot A_2} dx & \text{if } L_1 + \frac{L_2}{2} \leq x \leq L_1 + L_2 \\ \Delta_D + \int_{L_1 + L_2}^x \frac{N_{DE}(x)}{E \cdot A_3} dx & \text{if } x \geq L_1 + L_2 \end{cases}$$



Compare ADD's without ( $\delta$ ) & with ( $\Delta$ ) weight of bar (plotted horizontally)







**Problem 2.3-8**

$$(a) \delta = \frac{P}{E} \left( \frac{2 \frac{L}{4}}{bt} + \frac{\frac{L}{2}}{\frac{3}{4}bt} \right) = \frac{7LP}{6Ebt} \quad \boxed{\delta = \frac{7PL}{6Ebt}}$$

$$(b) \text{ NUMERICAL DATA } E = 210 \text{ GPa} \quad L = 750 \text{ mm} \quad \sigma_{\text{mid}} = 160 \text{ MPa}$$

$$\text{so } \sigma_{\text{mid}} = \frac{P}{\frac{3}{4}bt} \quad \text{and} \quad \frac{P}{bt} = \frac{3}{4} \sigma_{\text{mid}}$$

$$\delta = \frac{7LP}{6Ebt} \quad \text{or} \quad \delta = \frac{7L}{6E} \left( \frac{3}{4} \sigma_{\text{mid}} \right) = 0.5 \text{ mm} \quad \boxed{\delta = 0.5 \text{ mm}}$$

$$(c) \delta_{\text{max}} = \frac{P}{E} \left( \frac{L - L_{\text{slot}}}{bt} + \frac{L_{\text{slot}}}{\frac{3}{4}bt} \right) \quad \text{or} \quad \delta_{\text{max}} = \left( \frac{P}{bt} \right) \left( \frac{1}{E} \right) \left( L - L_{\text{slot}} + \frac{4}{3} L_{\text{slot}} \right)$$

$$\text{or} \quad \delta_{\text{max}} = \left( \frac{3}{4} \sigma_{\text{mid}} \right) \left( \frac{1}{E} \right) \left( L + \frac{L_{\text{slot}}}{3} \right) \quad \text{Solving for } L_{\text{slot}} \text{ with } \delta_{\text{max}} = 0.475 \text{ mm}$$

$$L_{\text{slot}} = \frac{4E\delta_{\text{max}} - 3L\sigma_{\text{mid}}}{\sigma_{\text{mid}}} = 244 \text{ mm} \quad \boxed{L_{\text{slot}} = 244 \text{ mm}} \quad \frac{L_{\text{slot}}}{L} = 0.325$$

**Problem 2.3-9**

$$(a) \quad \delta = \frac{P}{E} \left( \frac{2 \frac{L}{4}}{bt} + \frac{\frac{L}{2}}{\frac{3}{4} bt} \right) = \frac{7LP}{6Ebt}$$

$$(b) \quad E = 30,000 \text{ ksi} \quad L = 30 \text{ in.} \quad \sigma_{\text{mid}} = 24 \text{ ksi}$$

$$\text{So} \quad \sigma_{\text{mid}} = \frac{P}{\frac{3}{4} bt} \quad \text{and} \quad \frac{P}{bt} = \frac{3}{4} \sigma_{\text{mid}}$$

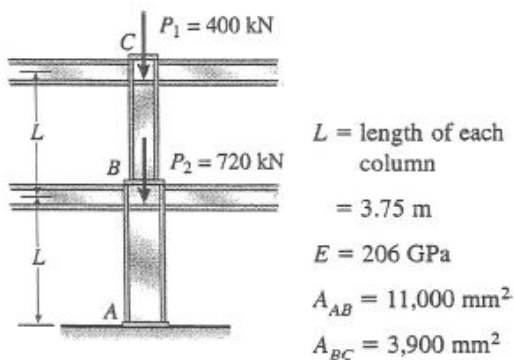
$$\delta = \frac{7LP}{6Ebt} \quad \text{or} \quad \delta = \frac{7L}{6E} \left( \frac{3}{4} \sigma_{\text{mid}} \right) = 0.021 \text{ in.} \quad \boxed{\delta = 0.021 \text{ in.}}$$

$$(c) \quad \delta_{\text{max}} = \frac{P}{E} \left( \frac{L - L_{\text{slot}}}{bt} + \frac{L_{\text{slot}}}{\frac{3}{4} bt} \right) \quad \text{or} \quad \delta_{\text{max}} = \left( \frac{P}{bt} \right) \left( \frac{1}{E} \right) \left( L - L_{\text{slot}} + \frac{4}{3} L_{\text{slot}} \right)$$

$$\text{or} \quad \delta_{\text{max}} = \left( \frac{3}{4} \sigma_{\text{mid}} \right) \left( \frac{1}{E} \right) \left( L + \frac{L_{\text{slot}}}{3} \right) \quad \text{Solving for } L_{\text{slot}} \text{ with } \delta_{\text{max}} = 0.02 \text{ in.}:$$

$$L_{\text{slot}} = \frac{4E\delta_{\text{max}} - 3L\sigma_{\text{mid}}}{\sigma_{\text{mid}}} = 10 \text{ in.} \quad \boxed{L_{\text{slot}} = 10 \text{ in.}} \quad \frac{L_{\text{slot}}}{L} = 0.333$$

### Problem 2.3-10



(a) SHORTENING  $\delta_{AC}$  OF THE TWO COLUMNS

$$\begin{aligned}
 \delta_{AC} &= \sum \frac{N_i L_i}{E_i A_i} = \frac{N_{AB} L}{E A_{AB}} + \frac{N_{BC} L}{E A_{BC}} \\
 &= \frac{(1120 \text{ kN})(3.75 \text{ m})}{(206 \text{ GPa})(11,000 \text{ mm}^2)} + \frac{(400 \text{ kN})(3.75 \text{ m})}{(206 \text{ GPa})(3,900 \text{ mm}^2)} \\
 &= 1.8535 \text{ mm} + 1.8671 \text{ mm} = 3.7206 \text{ mm} \\
 \delta_{AC} &= 3.72 \text{ mm} \quad \leftarrow
 \end{aligned}$$

(b) ADDITIONAL LOAD  $P_0$  AT POINT C

$$(\delta_{AC})_{\max} = 4.0 \text{ mm}$$

$\delta_0$  = additional shortening of the two columns due to the load  $P_0$

$$\begin{aligned}
 \delta_0 &= (\delta_{AC})_{\max} - \delta_{AC} = 4.0 \text{ mm} - 3.7206 \text{ mm} \\
 &= 0.2794 \text{ mm}
 \end{aligned}$$

$$\text{Also, } \delta_0 = \frac{P_0 L}{E A_{AB}} + \frac{P_0 L}{E A_{BC}} = \frac{P_0 L}{E} \left( \frac{1}{A_{AB}} + \frac{1}{A_{BC}} \right)$$

Solve for  $P_0$ :

$$P_0 = \frac{E \delta_0}{L} \left( \frac{A_{AB} A_{BC}}{A_{AB} + A_{BC}} \right)$$

SUBSTITUTE NUMERICAL VALUES:

$$E = 206 \times 10^9 \text{ N/m}^2 \quad \delta_0 = 0.2794 \times 10^{-3} \text{ m}$$

$$L = 3.75 \text{ m} \quad A_{AB} = 11,000 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = 3,900 \times 10^{-6} \text{ m}^2$$

$$P_0 = 44,200 \text{ N} = 44.2 \text{ kN} \quad \leftarrow$$

**Problem 2.3-11**

Numerical data      $E = 30 \cdot (10^6) \text{ psi}$       $P = 5000 \text{ lb}$       $L = 4 \text{ ft}$       $d_1 = 0.75 \text{ in}$       $d_2 = 0.5 \text{ in}$

Part (a)      $\delta_a = \frac{P \cdot L}{E} \cdot \left( \frac{1}{\frac{\pi}{4} \cdot d_1^2} + \frac{1}{\frac{\pi}{4} \cdot d_2^2} \right) = 0.0589 \cdot \text{in}$       $\boxed{\delta_a = 0.0589 \cdot \text{in}}$

Part (b)      $\text{Vol}_a = \left( \frac{\pi}{4} \cdot d_1^2 + \frac{\pi}{4} \cdot d_2^2 \right) \cdot L = 30.631 \cdot \text{in}^3$       $d = \sqrt{\frac{\text{Vol}_a}{\frac{\pi}{4} \cdot (2 \cdot L)}} = 0.637 \cdot \text{in}$       $A = \frac{\pi}{4} \cdot d^2 = 0.31907 \cdot \text{in}^2$

$\delta_b = \frac{P \cdot (2 \cdot L)}{E \cdot A} = 0.0501 \cdot \text{in}$       $\boxed{\delta_b = 0.0501 \cdot \text{in}}$

Part (c)      $q = 1250 \frac{\text{lb}}{\text{ft}}$       $L = 4 \text{ ft}$

$\delta_c = \frac{q \cdot L^2}{2 \cdot E \cdot \left( \frac{\pi}{4} \cdot d_1^2 \right)} + \frac{P \cdot L}{E \cdot \left( \frac{\pi}{4} \cdot d_2^2 \right)} = 0.0498 \cdot \text{in}$       $\boxed{\frac{\delta_c}{\delta_a} = 0.846}$       $\boxed{\frac{\delta_c}{\delta_b} = 0.993}$

### Problem 2.3-12

NUMERICAL DATA

$$d_1 = 100 \text{ mm} \quad d_2 = 60 \text{ mm}$$

$$L = 1200 \text{ mm} \quad E = 4.0 \text{ GPa} \quad P = 110 \text{ kN}$$

$$\delta_a = 8.0 \text{ mm}$$

(a) FIND  $d_{\max}$  IF SHORTENING IS LIMITED TO  $\delta_a$

$$A_1 = \frac{\pi}{4} d_1^2 \quad A_2 = \frac{\pi}{4} d_2^2$$

$$\delta = \frac{P}{E} \left[ \frac{\frac{L}{4}}{\frac{\pi}{4}(d_1^2 - d_{\max}^2)} + \frac{\frac{L}{4}}{A_1} + \frac{\frac{L}{2}}{A_2} \right]$$

Set  $\delta$  to  $\delta_a$ , and solve for  $d_{\max}$ :

$$d_{\max} = d_1 \sqrt{\frac{E\delta_a \pi d_1^2 d_2^2 - 2PLd_2^2 - 2PLd_1^2}{E\delta_a \pi d_1^2 d_2^2 - PLd_2^2 - 2PLd_1^2}}$$

$$d_{\max} = 23.9 \text{ mm} \quad \leftarrow$$

(b) NOW, IF  $d_{\max}$  IS INSTEAD SET AT  $d_2/2$ , AT WHAT DISTANCE  $b$  FROM END  $C$  SHOULD LOAD  $P$  BE APPLIED TO LIMIT THE BAR SHORTENING TO  $\delta_a = 8.0 \text{ mm}$ ?

$$A_0 = \frac{\pi}{4} \left[ d_1^2 - \left( \frac{d_2}{2} \right)^2 \right]$$

$$A_1 = \frac{\pi}{4} d_1^2 \quad A_2 = \frac{\pi}{4} d_2^2$$

$$\delta = \frac{P}{E} \left[ \frac{L}{4A_0} + \frac{L}{4A_1} + \frac{\left( \frac{L}{2} - b \right)}{A_2} \right]$$

No axial force in segment at end of length  $b$ ; set  $\delta = \delta_a$  and solve for  $b$ :

$$b = \left[ \frac{L}{2} - A_2 \left[ \frac{E\delta_a}{P} - \left( \frac{L}{4A_0} + \frac{L}{4A_1} \right) \right] \right]$$

$$b = 4.16 \text{ mm} \quad \leftarrow$$

(c) FINALLY IF LOADS  $P$  ARE APPLIED AT THE ENDS AND  $d_{\max} = d_2/2$ , WHAT IS THE PERMISSIBLE LENGTH  $x$  OF THE HOLE IF SHORTENING IS TO BE LIMITED TO  $\delta_a = 8.0 \text{ mm}$ ?

$$\delta = \frac{P}{E} \left[ \frac{x}{A_0} + \frac{\left( \frac{L}{2} - x \right)}{A_1} + \frac{\left( \frac{L}{2} \right)}{A_2} \right]$$

Set  $\delta = \delta_a$  and solve for  $x$ :

$$x = \frac{\left[ A_0 A_1 \left( \frac{E\delta_a}{P} - \frac{L}{2A_2} \right) \right] - \frac{1}{2} A_0 L}{A_1 - A_0}$$

$$x = 183.3 \text{ mm} \quad \leftarrow$$

### Problem 2.3-13

AFD LINEAR

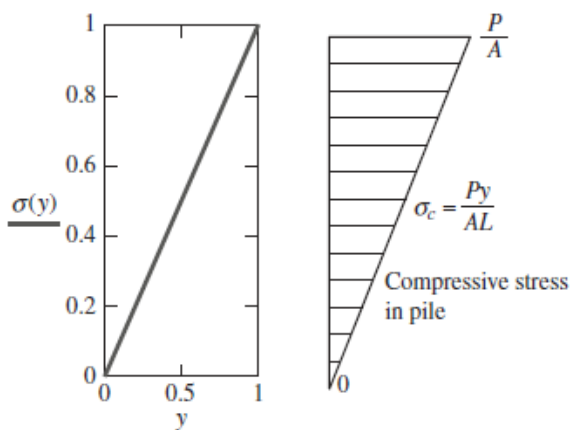
$$(a) \quad N(y) = fy \quad \delta = \int_0^L \frac{(fy)}{EA} dy = \frac{L^2 f}{2AE} \quad \boxed{\delta = \frac{PL}{2EA}}$$

$$(b) \quad \sigma(y) = \frac{N(y)}{A} \quad \sigma(y) = \frac{fy}{A} \quad \sigma(L) = \frac{fL}{A} = \frac{P}{A}$$

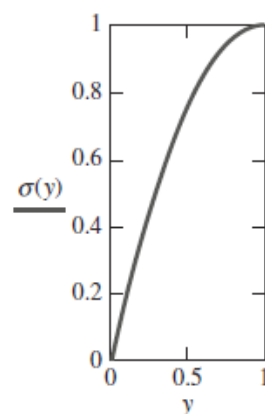
$$\sigma(0) = 0$$

So linear variation, zero at bottom,  $P/A$  at top (i.e., at ground surface)

$$N(L) = f \quad \boxed{\sigma(y) = \frac{P}{A} \left( \frac{y}{L} \right)}$$



$f(y)$  is constant  
and AFD is linear



$f(y)$  is linear and  
AFD quadratic

$$(c) \quad N(y) = f(y)y$$

$$N(y) = \int_0^y f_0 \left( 1 - \frac{\xi}{L} \right) d\xi = \frac{f_0 y (y - 2L)}{2} \quad N(L) = \frac{f_0 L}{2} \quad N(0) = 0$$

$$\delta = \frac{\left( \frac{f_0 L}{2} \right)}{\frac{3}{2} EA} \quad P = \frac{1}{2} f_0 L \quad \boxed{\delta = \frac{PL}{EA} \left( \frac{2}{3} \right)} \quad \boxed{\sigma(y) = \frac{P}{A} \left[ \frac{y}{L} \left( 2 - \frac{y}{L} \right) \right]} \quad \sigma(0) = 0 \quad \sigma(L) = \frac{f_0}{2} = P/A$$

### Problem 2.3-14

#### NUMERICAL DATA

$$P = 5 \text{ kN} \quad E_c = 120 \text{ GPa}$$

$$L_2 = 18 \text{ mm} \quad L_4 = L_2$$

$$L_3 = 40 \text{ mm}$$

$$d_{o3} = 22.2 \text{ mm} \quad t_3 = 1.65 \text{ mm}$$

$$d_{o5} = 18.9 \text{ mm} \quad t_5 = 1.25 \text{ mm}$$

$$\tau_Y = 30 \text{ MPa} \quad \sigma_Y = 200 \text{ MPa}$$

$$\text{FS}_\tau = 2 \quad \text{FS}_\sigma = 1.7$$

$$\tau_a = \frac{\tau_Y}{\text{FS}_\tau} \quad \tau_a = 15 \text{ MPa}$$

$$\sigma_a = \frac{\sigma_Y}{\text{FS}_\sigma} \quad \sigma_a = 117.6 \text{ MPa}$$

#### (a) ELONGATION OF SEGMENT 2-3-4

$$A_2 = \frac{\pi}{4} [d_{o3}^2 - (d_{o5} - 2t_5)^2]$$

$$A_3 = \frac{\pi}{4} [d_{o3}^2 - (d_{o3} - 2t_3)^2]$$

$$A_2 = 175.835 \text{ mm}^2 \quad A_3 = 106.524 \text{ mm}^2$$

$$\delta_{24} = \frac{P}{E_c} \left( \frac{L_2 + L_4}{A_2} + \frac{L_3}{A_3} \right)$$

$$\delta_{24} = 0.024 \text{ mm} \quad \leftarrow$$

#### (b) MAXIMUM LOAD $P_{\max}$ THAT CAN BE APPLIED TO THE JOINT

First check normal stress:

$$A_1 = \frac{\pi}{4} [d_{o5}^2 - (d_{o5} - 2t_5)^2]$$

$$A_1 = 69.311 \text{ mm}^2 < \text{smallest cross-sectional area controls normal stress}$$

$$P_{\max\sigma} = \sigma_a A_1 \quad P_{\max\sigma} = 8.15 \text{ kN} \quad \leftarrow \text{smaller than } P_{\max} \text{ based on shear below so normal stress controls}$$

Next check shear stress in solder joint:

$$A_{\text{sh}} = \pi d_{o5} L_2 \quad A_{\text{sh}} = 1.069 \times 10^3 \text{ mm}^2$$

$$P_{\max\tau} = \tau_a A_{\text{sh}} \quad P_{\max\tau} = 16.03 \text{ kN}$$

#### (c) FIND THE VALUE OF $L_2$ AT WHICH TUBE AND SOLDER CAPACITIES ARE EQUAL

Set  $P_{\max}$  based on shear strength equal to  $P_{\max}$  based on tensile strength and solve for  $L_2$ :

$$L_2 = \frac{\sigma_a A_1}{\tau_a (\pi d_{o5})} \quad L_2 = 9.16 \text{ mm} \quad \leftarrow$$



**Problem 2.3-15**

(a) STATICS  $\sum F_H = 0$   $R_1 = -P - \frac{P}{2}$   
 $R_1 = \frac{-3}{2}P \leftarrow$

(b) DRAW FBD'S CUTTING THROUGH SEGMENT 1 AND AGAIN THROUGH SEGMENT 2

$$N_1 = \frac{3P}{2} < \text{tension} \quad N_2 = \frac{P}{2} < \text{tension}$$

(c) FIND  $x$  REQUIRED TO OBTAIN AXIAL DISPLACEMENT AT JOINT 3 OF  $\delta_3 = PL/EA$

Add axial deformations of segments 1 and 2, then set to  $\delta_3$ ; solve for  $x$ :

$$\frac{N_1 x}{E \frac{3}{4}A} + \frac{N_2 (L - x)}{EA} = \frac{PL}{EA}$$

$$\frac{\frac{3P}{2}x}{E \frac{3}{4}A} + \frac{\frac{P}{2}(L - x)}{EA} = \frac{PL}{EA}$$

$$\frac{3}{2}x = \frac{L}{2} \quad x = \frac{L}{3} \leftarrow$$

(d) WHAT IS THE DISPLACEMENT AT JOINT 2,  $\delta_2$ ?

$$\delta_2 = \frac{N_1 x}{E \frac{3}{4}A} \quad \delta_2 = \frac{\left(\frac{3P}{2}\right)\frac{L}{3}}{E \frac{3}{4}A}$$

$$\delta_2 = \frac{2}{3} \frac{PL}{EA}$$

(e) IF  $x = 2L/3$  AND  $P/2$  AT JOINT 3 IS REPLACED BY  $\beta P$ , FIND  $\beta$  SO THAT  $\delta_3 = PL/EA$

$$N_1 = (1 + \beta)P \quad N_2 = \beta P \quad x = \frac{2L}{3}$$

substitute in axial deformation expression above and solve for  $\beta$

$$\frac{[(1 + \beta)P]\frac{2L}{3}}{E \frac{3}{4}A} + \frac{\beta P \left(L - \frac{2L}{3}\right)}{EA} = \frac{PL}{EA}$$

$$\frac{1}{9} PL \frac{8 + 11\beta}{EA} = \frac{PL}{EA}$$

$$(8 + 11\beta) = 9$$

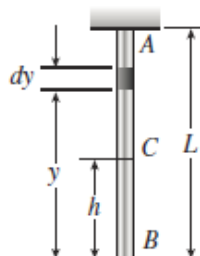
$$\beta = \frac{1}{11} \leftarrow$$

$$\beta = 0.091$$

(f) Draw AFD, ADD—see plots for  $x = \frac{L}{3}$

No plots provided here

**Problem 2.3-16**



$W$  = Weight of bar

(a) DOWNWARD DISPLACEMENT  $\delta_C$   
Consider an element at distance  $y$  from the lower end.

$$N(y) = \frac{Wy}{L} \quad d\delta = \frac{N(y)dy}{EA} = \frac{Wydy}{EAL}$$

$$\delta_C = \int_h^L d\delta = \int_h^L \frac{Wydy}{EAL} = \frac{W}{2EAL} (L^2 - h^2)$$

$$\delta_C = \frac{W}{2EAL} (L^2 - h^2) \quad \leftarrow$$

(b) ELONGATION OF BAR ( $h = 0$ )

$$\delta_B = \frac{WL}{2EA} \quad \leftarrow$$

(c) RATIO OF ELONGATIONS

Elongation of upper half of bar ( $h = \frac{L}{2}$ ):

$$\delta_{\text{upper}} = \frac{3WL}{8EA}$$

Elongation of lower half of bar:

$$\delta_{\text{lower}} = \delta_B - \delta_{\text{upper}} = \frac{WL}{2EA} - \frac{3WL}{8EA} = \frac{WL}{8EA}$$

$$\beta = \frac{\delta_{\text{upper}}}{\delta_{\text{lower}}} = \frac{3/8}{1/8} = 3 \quad \leftarrow$$

(d) NUMERICAL DATA

$$\gamma_s = 77 \text{ kN/m}^3 \quad \gamma_w = 10 \text{ kN/m}^3 \quad L = 1500 \text{ m} \quad A = 0.0157 \text{ m}^2 \quad E = 210 \text{ GPa}$$

In sea water:

$$W = (\gamma_s - \gamma_w)AL = 1577.85 \text{ kN}$$

$$\delta = \frac{WL}{2EA} = 359 \text{ mm}$$

$$\frac{\delta}{L} = 2.393 \times 10^{-4}$$

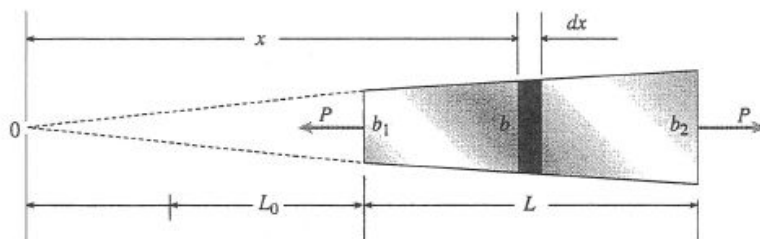
In air:

$$W = (\gamma_s)AL = 1813.35 \text{ kN}$$

$$\delta = \frac{WL}{2EA} = 412 \text{ mm}$$

$$\frac{\delta}{L} = 2.75 \times 10^{-4}$$

**Problem 2.3-17**



$t$  = thickness (constant)

$$b = b_1 \left( \frac{x}{L_0} \right) \quad b_2 = b_1 \left( \frac{L_0 + L}{L_0} \right) \quad (\text{Eq. 1})$$

$$A(x) = bt = b_1 t \left( \frac{x}{L_0} \right)$$

(a) ELONGATION OF THE BAR

$$d\delta = \frac{Pdx}{EA(x)} = \frac{PL_0 dx}{Eb_1 t x}$$

$$\begin{aligned} \delta &= \int_{L_0}^{L_0+L} d\delta = \frac{PL_0}{Eb_1 t} \int_{L_0}^{L_0+L} \frac{dx}{x} \\ &= \frac{PL_0}{Eb_1 t} \ln x \Big|_{L_0}^{L_0+L} = \frac{PL_0}{Eb_1 t} \ln \frac{L_0 + L}{L_0} \quad (\text{Eq. 2}) \end{aligned}$$

$$\text{From Eq. (1): } \frac{L_0 + L}{L_0} = \frac{b_2}{b_1} \quad (\text{Eq. 3})$$

$$\text{Solve Eq. (3) for } L_0: L_0 = L \left( \frac{b_1}{b_2 - b_1} \right) \quad (\text{Eq. 4})$$

Substitute Eqs. (3) and (4) into Eq. (2):

$$\delta = \frac{PL}{Et(b_2 - b_1)} \ln \frac{b_2}{b_1} \quad (\text{Eq. 5})$$

(b) SUBSTITUTE NUMERICAL VALUES:

$$L = 5 \text{ ft} = 60 \text{ in.} \quad t = 10 \text{ in.}$$

$$P = 25 \text{ k} \quad b_1 = 4.0 \text{ in.}$$

$$b_2 = 6.0 \text{ in.} \quad E = 30 \times 10^6 \text{ psi}$$

$$\text{From Eq. (5): } \delta = 0.010 \text{ in.} \quad \leftarrow$$

**Problem 2.3-18**

$$P = 200\text{kN} \quad L = 2\text{m} \quad t = 20\text{mm} \quad b_1 = 100\text{mm} \quad b_2 = 115\text{mm} \quad E = 96\text{GPa}$$

Bar width at B at  $L/2$       $b_B = \frac{b_1 + b_2}{2} = 107.5\text{mm}$

Axial forces in bar segments (use RHFB)      $N_{AB} = 2 \cdot P - P = 200\text{ kN}$       $N_{BC} = 2 \cdot P = 400\text{ kN}$

Axial displacement at B      $\delta_B = \frac{N_{AB} \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_B)} \cdot \ln\left(\frac{b_2}{b_B}\right) = 0.937\text{mm}$

Axial displacement at C      $\delta_C = \delta_B + \frac{N_{BC} \cdot \frac{L}{2}}{E \cdot t \cdot (b_B - b_1)} \cdot \ln\left(\frac{b_B}{b_1}\right) = 2.946\text{mm}$

**Problem 2.3-19**

$$P = 50\text{kip} \quad L = 5\text{ft} \quad t = \frac{3}{8}\text{in} \quad b_1 = 3\text{in} \quad b_2 = 2.75\text{in} \quad E = 16000\text{ksi}$$

$$N_{AB} = 2 \cdot P - P = 50 \cdot \text{kip} \quad N_{BC} = 2 \cdot P = 100 \cdot \text{kip}$$

$$\text{Axial displacement at B} \quad \delta_B = \frac{N_{AB} \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) = 0.087 \cdot \text{in}$$

$$\text{Axial displacement at C} \quad \delta_C = \delta_B + \frac{N_{BC} \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) = 0.261 \cdot \text{in}$$

### Problem 2.3-20

$$E = 72 \text{ GPa} \quad P_2 = 200 \text{ kN} \quad L = 2 \text{ m} \quad t = 20 \text{ mm} \quad b_1 = 100 \text{ mm} \quad b_2 = 115 \text{ mm}$$

$$A_{BC} = b_1 \cdot t = 2 \times 10^3 \cdot \text{mm}^2$$

If only load  $P_2$  is applied at C

$$\delta_B = \frac{P_2 \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) = 1.294 \text{ mm} \quad \delta_C = \delta_B + \frac{P_2 \cdot \frac{L}{2}}{E \cdot A_{BC}} = 2.683 \text{ mm}$$

Now apply both  $P_1$  (to the left) and  $P_2$  at C and solve for  $P_1$  s.t. axial displacement at C = 0

Given

$$\frac{(P_2 - P_1) \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{P_2 \cdot \frac{L}{2}}{E \cdot A_{BC}} = 0 \quad \text{Find}(P_1) = 414.651 \text{ kN}$$

Axial displacement at B with both loads applied as shown

Let  $P_1 = 414.651 \text{ kN}$

Check

$$\delta_B = \frac{(P_2 - P_1) \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) = -1.389 \text{ mm}$$

leftward

$$\frac{(P_2 - P_1) \cdot \frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{P_2 \cdot \frac{L}{2}}{E \cdot A_{BC}} = -0 \text{ m}$$

**Problem 2.3-21**

$$d_A = 4\text{in} \quad d_B = 8\text{in} \quad P = 45\text{kip} \quad \delta_A = 0.02\text{in} \quad d(x) = d_A + \left( \frac{d_B - d_A}{L} \right) \cdot x$$

$$A(x) = \frac{\pi}{4} \cdot d(x)^2 \quad E = 10400\text{ksi}$$

$$\int_0^L \frac{P}{E \cdot A(x)} dx = \delta_A \quad \text{expand integral to obtain following expression} \quad \frac{4 \cdot P \cdot L}{\pi \cdot E \cdot d_A \cdot d_B} = \delta_A$$

$$\text{Solving for } L \quad L = \frac{\pi \cdot E \cdot d_A \cdot d_B}{4 \cdot P} \cdot \delta_A = 9.681 \cdot \text{ft}$$

**Problem 2.3-22**

$$L = 1.8\text{m} \quad r = 36\text{mm} \quad E = 72\text{GPa} \quad a = \frac{r}{8} = 4.5\text{mm} \quad \sigma_2 = 180\text{MPa}$$

$$A_1 = \pi \cdot r^2 = 4071.504\text{mm}^2 \quad \text{Use formulas in **Appendix E, Case 15** for area of slotted segment}$$

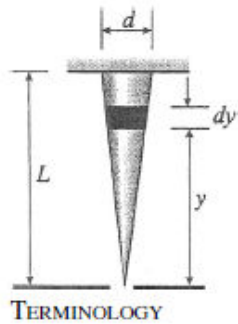
$$\alpha = \arccos\left(\frac{a}{r}\right) = 1.445 \quad b = \sqrt{r^2 - a^2} = 35.718\text{mm} \quad A_2 = 2 \cdot r^2 \cdot \left(\alpha - \frac{a \cdot b}{r^2}\right) = 3425.196\text{mm}^2 \quad \frac{A_2}{A_1} = 0.841$$

$$\text{Stress in middle half is known so use to find force } P \quad P = \sigma_2 \cdot A_2 = 616.535\text{ kN}$$

$$\text{Compute bar elongation now that } P \text{ is known} \quad \delta = 2 \cdot \frac{P \cdot \frac{L}{4}}{E \cdot A_1} + \frac{P \cdot \frac{L}{2}}{E \cdot A_2} = 4.143\text{ mm}$$



**Problem 2.3-23**



$N_y$  = axial force acting on element  $dy$

$A_y$  = cross-sectional area at element  $dy$

$A_B$  = cross-sectional area at base of cone

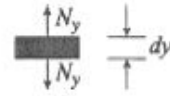
$$= \frac{\pi d^2}{4} \quad V = \text{volume of cone}$$

$$= \frac{1}{3} A_B L \quad V_y = \text{volume of cone below element } dy$$

$$= \frac{1}{3} A_y y \quad W_y = \text{weight of cone below element } dy$$

$$= \frac{V_y}{V} (W) = \frac{A_y y W}{A_B L} \quad N_y = W_y$$

ELEMENT OF BAR



$W$  = weight of cone

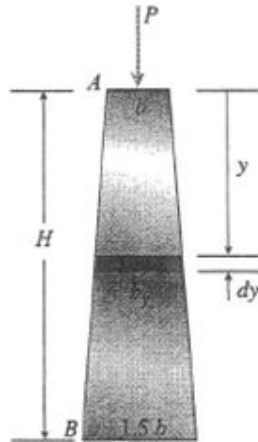
ELONGATION OF ELEMENT  $dy$

$$d\delta = \frac{N_y dy}{E A_y} = \frac{W y dy}{E A_B L} = \frac{4W}{\pi d^2 E L} y dy$$

ELONGATION OF CONICAL BAR

$$\delta = \int d\delta = \frac{4W}{\pi d^2 E L} \int_0^L y dy = \frac{2WL}{\pi d^2 E} \quad \leftarrow$$

**Problem 2.3-24**



Square cross sections:

$b$  = width at A

$1.5b$  = width at B

$b_y$  = width at distance  $y$

$$= b + (1.5b - b) \frac{y}{H}$$

$$= \frac{b}{H}(H + 0.5y)$$

$A_y$  = cross-sectional area at distance  $y$

$$= (b_y)^2 = \frac{b^2}{H^2}(H + 0.5y)^2$$

SHORTENING OF ELEMENT  $dy$

$$d\delta = \frac{Pdy}{EA_y} = \frac{Pdy}{E\left(\frac{b^2}{H^2}\right)(H + 0.5y)^2}$$

SHORTENING OF ENTIRE POST

$$\delta = \int d\delta = \frac{PH^2}{Eb^2} \int_0^H \frac{dy}{(H + 0.5y)^2}$$

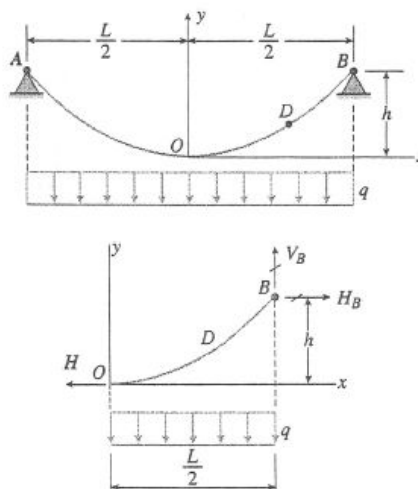
$$\text{From Appendix C: } \int \frac{dx}{(a + bx)^2} = -\frac{1}{b(a + bx)}$$

$$\delta = \frac{PH^2}{Eb^2} \left[ -\frac{1}{(0.5)(H + 0.5y)} \right]_0^H$$

$$= \frac{PH^2}{Eb^2} \left[ -\frac{1}{(0.5)(1.5H)} + \frac{1}{0.5H} \right]$$

$$= \frac{2PH}{3Eb^2} \quad \leftarrow$$

### Problem 2.3-25



Equation of parabolic curve:

$$y = \frac{4hx^2}{L^2}$$

$$\frac{dy}{dx} = \frac{8hx}{L^2}$$

FREE-BODY DIAGRAM OF HALF OF CABLE

$$\Sigma M_B = 0 \quad \curvearrowright$$

$$-Hh + \frac{qL}{2} \left( \frac{L}{4} \right) = 0$$

$$H = \frac{qL^2}{8h}$$

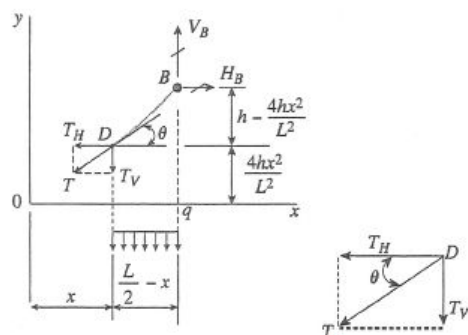
$$\Sigma F_{\text{horizontal}} = 0$$

$$H_B = H = \frac{qL^2}{8h} \quad (\text{Eq. 1})$$

$$\Sigma F_{\text{vertical}} = 0$$

$$V_B = \frac{qL}{2} \quad (\text{Eq. 2})$$

FREE-BODY DIAGRAM OF SEGMENT DB OF CABLE



$$\Sigma F_{\text{horiz}} = 0 \quad T_H = H_B = \frac{qL^2}{8h} \quad (\text{Eq. 3})$$

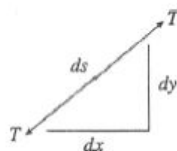
$$\Sigma F_{\text{vert}} = 0 \quad V_B - T_V - q \left( \frac{L}{2} - x \right) = 0$$

$$T_V = V_B - q \left( \frac{L}{2} - x \right) = \frac{qL}{2} - \frac{qL}{2} + qx = qx \quad (\text{Eq. 4})$$

TENSILE FORCE  $T$  IN CABLE

$$T = \sqrt{T_H^2 + T_V^2} = \sqrt{\left( \frac{qL^2}{8h} \right)^2 + (qx)^2} = \frac{qL^2}{8h} \sqrt{1 + \frac{64h^2x^2}{L^4}} \quad (\text{Eq. 5})$$

ELONGATION  $d\delta$  OF AN ELEMENT OF LENGTH  $ds$



$$d\delta = \frac{Tds}{EA}$$

$$\begin{aligned} ds &= \sqrt{(dx)^2 + (dy)^2} = dx \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \\ &= dx \sqrt{1 + \left( \frac{8hx}{L^2} \right)^2} \\ &= dx \sqrt{1 + \frac{64h^2x^2}{L^4}} \end{aligned} \quad (\text{Eq. 6})$$

(a) ELONGATION  $\delta$  OF CABLE AOB

$$\delta = \int d\delta = \int \frac{T ds}{EA}$$

Substitute for  $T$  from Eq. (5) and for  $ds$  from Eq. (6):

$$\delta = \frac{1}{EA} \int \frac{qL^2}{8h} \left( 1 + \frac{64h^2x^2}{L^4} \right) dx$$

For both halves of cable:

$$\begin{aligned} \delta &= \frac{2}{EA} \int_0^{L/2} \frac{qL^2}{8h} \left( 1 + \frac{64h^2x^2}{L^4} \right) dx \\ \delta &= \frac{qL^3}{8hEA} \left( 1 + \frac{16h^2}{3L^4} \right) \quad \leftarrow \quad (\text{Eq. 7}) \end{aligned}$$

(b) GOLDEN GATE BRIDGE CABLE

$$L = 4200 \text{ ft} \quad h = 470 \text{ ft}$$

$$q = 12,700 \text{ lb/ft} \quad E = 28,800,000 \text{ psi}$$

$$27,572 \text{ wires of diameter } d = 0.196 \text{ in.}$$

$$A = (27,572) \left( \frac{\pi}{4} \right) (0.196 \text{ in.})^2 = 831.90 \text{ in.}^2$$

Substitute into Eq. (7):

$$\delta = 133.7 \text{ in} = 11.14 \text{ ft} \quad \leftarrow$$

**Problem 2.3-26**

(a) ELONGATION  $\delta$  FOR CASE OF CONSTANT DIAMETER HOLE

$$d(\zeta) = d_A \left( 1 + \frac{\zeta}{L} \right) \quad A(\zeta) = \frac{\pi}{4} d(\zeta)^2 \quad < \text{solid portion of length } L - x$$

$$A(\zeta) = \frac{\pi}{4} (d(\zeta)^2 - d_A^2) \quad < \text{hollow portion of length } x$$

$$\delta = \frac{P}{E} \left( \int \frac{1}{A(\zeta)} d\zeta \right) \quad \delta = \frac{P}{E} \left[ \int_0^{L-x} \frac{4}{\pi d(\zeta)^2} d\zeta + \int_{L-x}^L \frac{4}{\pi (d(\zeta)^2 - d_A^2)} d\zeta \right]$$

$$\delta = \frac{P}{E} \left[ \int_0^{L-x} \frac{1}{\left[ \frac{\pi}{4} \left[ d_A \left( 1 + \frac{\zeta}{L} \right) \right]^2 \right]} d\zeta + \int_{L-x}^L \frac{1}{\left[ \frac{\pi}{4} \left[ d_A \left( 1 + \frac{\zeta}{L} \right) \right]^2 - d_A^2 \right]} d\zeta \right]$$

$$\delta = \frac{P}{E} \left[ 4 \frac{L^2}{(-2+x)\pi d_A^2} + \left[ \left[ 4 \frac{L}{\pi d_A^2} + \int_{L-x}^L \frac{1}{\left[ \frac{\pi}{4} \left[ d_A \left( 1 + \frac{\zeta}{L} \right) \right]^2 - d_A^2 \right]} d\zeta \right] \right] \right]$$

$$\delta = \frac{P}{E} \left[ 4 \frac{L^2}{(-2+x)\pi d_A^2} + \left( 4 \frac{L}{\pi d_A^2} - 2L \frac{\ln(3)}{\pi d_A^2} + 2L \frac{-\ln(L-x) + \ln(3L-x)}{\pi d_A^2} \right) \right]$$

$$\text{if } x = L/2 \quad \delta = \frac{P}{E} \left( \frac{4}{3} \frac{L}{\pi d_A^2} - 2L \frac{\ln(3)}{\pi d_A^2} + 2L \frac{-\ln\left(\frac{1}{2}L\right) + \ln\left(\frac{5}{2}L\right)}{\pi d_A^2} \right)$$

Substitute numerical data:

$$\delta = 2.18 \text{ mm} \quad \leftarrow$$

(b) ELONGATION  $\delta$  FOR CASE OF VARIABLE DIAMETER HOLE BUT CONSTANT WALL THICKNESS  $t = d_A/20$  OVER SEGMENT  $x$

$$d(\zeta) = d_A \left( 1 + \frac{\zeta}{L} \right) \quad A(\zeta) = \frac{\pi}{4} d(\zeta)^2 \quad < \text{solid portion of length } L - x$$

$$A(\zeta) = \frac{\pi}{4} \left[ d(\zeta)^2 - \left( d(\zeta) - 2 \frac{d_A}{20} \right)^2 \right] \quad < \text{hollow portion of length } x$$

$$\delta = \frac{P}{E} \left( \int \frac{1}{A(\zeta)} d\zeta \right) \quad \delta = \frac{P}{E} \left[ \int_0^{L-x} \frac{4}{\pi d(\zeta)^2} d\zeta + \int_{L-x}^L \frac{4}{\pi \left[ d(\zeta)^2 - \left( d(\zeta) - 2 \frac{d_A}{20} \right)^2 \right]} d\zeta \right]$$

$$\delta = \frac{P}{E} \left[ \int_0^{L-x} \frac{4}{\pi \left[ d_A \left( 1 + \frac{\xi}{L} \right) \right]^2} d\xi + \int_{L-x}^L \frac{4}{\pi \left[ \left[ d_A \left( 1 + \frac{\xi}{L} \right) \right]^2 - \left[ d_A \left( 1 + \frac{\xi}{L} \right) - 2\frac{d_A}{20} \right]^2 \right]} d\xi \right]$$

$$\delta = \frac{P}{E} \left[ 4 \frac{L^2}{(-2L+x)\pi d_A^2} + 4 \frac{L}{\pi d_A^2} + 20L \frac{\ln(3) + \ln(13) + 2\ln(d_A) + \ln(L)}{\pi d_A^2} - 20L \frac{2\ln(d_A) + \ln(39L-20x)}{\pi d_A^2} \right]$$

if  $x = L/2$

$$\delta = \frac{P}{E} \left( \frac{4}{3} \frac{L}{\pi d_A^2} + 20L \frac{\ln(3) + \ln(13) + 2\ln(d_A) + \ln(L)}{\pi d_A^2} - 20L \frac{2\ln(d_A) + \ln(29L)}{\pi d_A^2} \right)$$

Substitute numerical data:

$$\delta = 6.74 \text{ mm} \quad \leftarrow$$

**Problem 2.3-27**

$$P_1 = 2.5\text{kip} \quad P_2 = 1\text{kip} \quad M = 25\text{kip}\cdot\text{in} \quad E = 29000\text{ksi} \quad A_1 = 0.25\text{in}^2 \quad A_2 = 0.15\text{in}^2$$

Find pin force at B - use FBD of bar BDE

$$\Sigma M_D = 0 \quad B_y = \frac{1}{25\text{in}} [P_2 \cdot (25\text{in}) - M] = 0 \cdot \text{kip}$$

No pin force at B so bar ABC is subjected force  $P_1$  at C only

$$\delta_C = \frac{P_1}{E} \left( \frac{20\text{in}}{A_1} + \frac{35\text{in}}{A_2} \right) = 0.027 \cdot \text{in} \quad \text{downward}$$

### Problem 2.3-28

Find pin force at B - use FBD of bar ABC

$$\Sigma M_A = 0$$

$$B_y = \frac{1}{\frac{L}{3}} \cdot (3 \cdot P \cdot L)$$

$$B_y \rightarrow 9P$$

acts upward on ABC  
so downward on  
DBF

Vertical displacements at B and F

$$N_{BD} = P - 9P \rightarrow -8P \quad \delta_B = \frac{N_{BD} \cdot L}{2 \cdot EA} \quad \delta_B \rightarrow -4 \frac{PL}{EA} \quad \text{downward}$$

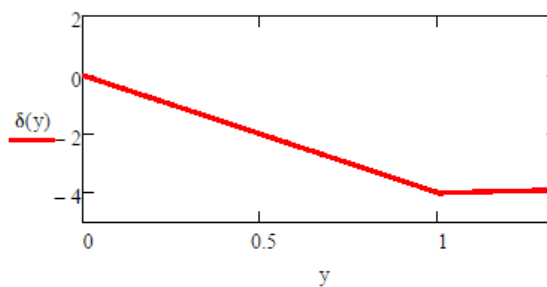
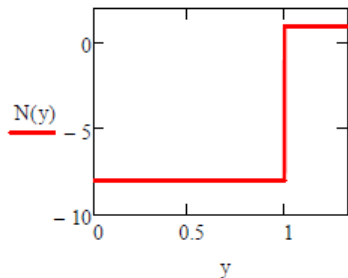
$$N_{BF} = P \quad \delta_C = \delta_B + \frac{N_{BF} \cdot \frac{L}{3}}{EA} \quad \delta_C \rightarrow -\frac{11}{3} \frac{PL}{EA} \quad \text{downward}$$

Axial force ( $N(y)$ ) and displacement ( $\delta(y)$ ) diagrams - origin of  $y$  at D, positive upward (rotated CW to horiz. position below)

$$N(y) = \begin{cases} N_{BD} & \text{if } y \leq L \\ N_{BF} & \text{otherwise} \end{cases} \quad \delta(y) = \begin{cases} \left[ N_{BD} \cdot y \cdot \left( \frac{L}{2 \cdot EA} \right) \right] & \text{if } y \leq L \\ \left[ \delta_B + N_{BF} \cdot (y - L) \cdot \left( \frac{L}{3 \cdot EA} \right) \right] & \text{otherwise} \end{cases}$$

$$\delta(0) \rightarrow 0 \quad \delta(L) \rightarrow -4$$

$$\delta\left(\frac{4L}{3}\right) \rightarrow -\frac{35}{9} = -3.889 \text{ times } PL/EA$$



### Problem 2.3-29

Find pin force at B - use FBD of bar ABC  $\Sigma F_y = 0$   $B_y = 2P$  upward at B on ABC so downward on DBF

Axial forces in column segments (tension is positive)

$$N_{DB} = -P \quad N_{BF} = -P - B_y \rightarrow -3 \cdot P \quad \text{so AFD is constant and compressive over each column segment}$$

Vertical displacements at B and D (positive upward)

$$\delta_B = \frac{N_{BF} \cdot \frac{L}{2}}{2 \cdot EA} \rightarrow \frac{3 \cdot L \cdot P}{4 \cdot EA} \quad \delta_D = \delta_B + \frac{N_{DB} \cdot \frac{L}{2}}{EA} \rightarrow \frac{5 \cdot L \cdot P}{4 \cdot EA} \quad \text{so ADD is linear and downward over each column segment}$$



### Problem 2.3-30

Use FBD of beam ABC - find pin force at B  $\Sigma F_y = 0$   $B_y = 2 \cdot P$  upward on ABC so downward on DBF

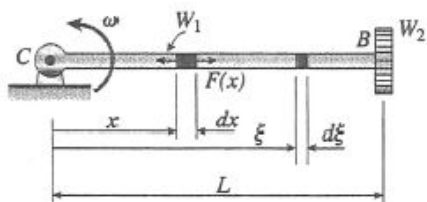
Axial forces in column segments (tension is positive)

$$N_{DB} = 0 \quad N_{BF} = -B_y \rightarrow -2 \cdot P \quad \text{so AFD is 0 over DB and constant and compressive over column segment BF}$$

Vertical displacements at B and D (positive upward)

$$\delta_B = \frac{N_{BF} \cdot \frac{L}{2}}{2 \cdot EA} \rightarrow \frac{L \cdot P}{2 \cdot EA} \quad \delta_D = \delta_B \rightarrow \frac{L \cdot P}{2 \cdot EA} \quad \text{so ADD is linear over BF and constant over column segment DB, both downward}$$

### Problem 2.3-31



$\omega$  = angular speed

$A$  = cross-sectional area

$E$  = modulus of elasticity

$g$  = acceleration of gravity

$F(x)$  = axial force in bar at distance  $x$  from point  $C$

Consider an element of length  $dx$  at distance  $x$  from point  $C$ .

To find the force  $F(x)$  acting on this element, we must find the inertia force of the part of the bar from distance  $x$  to distance  $L$ , plus the inertia force of the weight  $W_2$ .

Since the inertia force varies with distance from point  $C$ , we now must consider an element of length  $d\xi$  at distance  $\xi$ , where  $\xi$  varies from  $x$  to  $L$ .

$$\text{Mass of element } d = \frac{d}{L} \left( \frac{W_1}{g} \right)$$

$$\text{Acceleration of element} = \xi \omega^2$$

Centrifugal force produced by element

$$= (\text{mass})(\text{acceleration}) = \frac{W_1 \omega^2}{gL} d$$

Centrifugal force produced by weight  $W_2$

$$= \left( \frac{W_2}{g} \right) (L \omega^2)$$

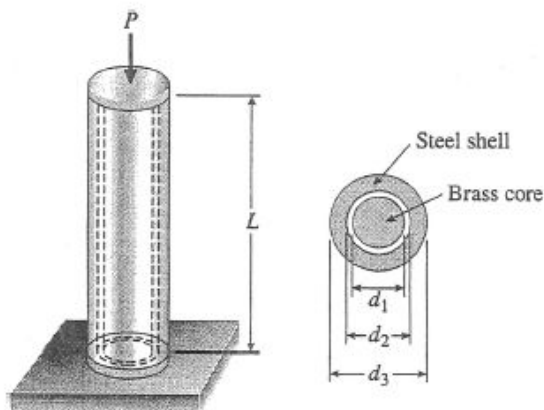
AXIAL FORCE  $F(x)$

$$\begin{aligned} F(x) &= \int_{-x}^L \frac{W_1 \omega^2}{gL} d + \frac{W_2 L \omega^2}{g} \\ &= \frac{W_1 \omega^2}{2gL} (L^2 - x^2) + \frac{W_2 L \omega^2}{g} \end{aligned}$$

ELONGATION OF BAR  $BC$

$$\begin{aligned} \delta &= \int_0^L \frac{F(x) dx}{EA} \\ &= \int_0^L \frac{W_1 \omega^2}{2gL} (L^2 - x^2) dx + \int_0^L \frac{W_2 L \omega^2 dx}{gEA} \\ &= \frac{W_1 L \omega^2}{2gLEA} \left[ \int_0^L L^2 dx - \int_0^L x^2 dx \right] + \frac{W_2 L \omega^2 dx}{gEA} \int_0^L dx \\ &= \frac{W_1 L^2 \omega^2}{3gEA} + \frac{W_2 L^2 \omega^2}{gEA} \\ &= \frac{L^2 \omega^2}{3gEA} + (W_1 + 3W_2) \quad \leftarrow \end{aligned}$$

**Problem 2.4-1**



$$d_1 = 0.25 \text{ in.} \quad E_b = 15 \times 10^6 \text{ psi}$$

$$d_2 = 0.28 \text{ in.} \quad E_s = 30 \times 10^6 \text{ psi}$$

$$d_3 = 0.35 \text{ in.} \quad A_s = \frac{\pi}{4}(d_3^2 - d_2^2) = 0.03464 \text{ in.}^2$$

$$L = 4.0 \text{ in.} \quad A_b = \frac{\pi}{4}d_1^2 = 0.04909 \text{ in.}^2$$

(a) DECREASE IN LENGTH ( $\delta = 0.003 \text{ in.}$ )

Use Eq. (2-18) of Example 2-6.

$$\delta = \frac{PL}{E_s A_s + E_b A_b} \quad \text{or}$$

$$P = (E_s A_s + E_b A_b) \left( \frac{\delta}{L} \right)$$

Substitute numerical values:

$$\begin{aligned} E_s A_s + E_b A_b &= (30 \times 10^6 \text{ psi})(0.03464 \text{ in.}^2) \\ &\quad + (15 \times 10^6 \text{ psi})(0.04909 \text{ in.}^2) \\ &= 1.776 \times 10^6 \text{ lb} \end{aligned}$$

$$\begin{aligned} P &= (1.776 \times 10^6 \text{ lb}) \left( \frac{0.003 \text{ in.}}{4.0 \text{ in.}} \right) \\ &= 1330 \text{ lb} \quad \leftarrow \end{aligned}$$

(b) ALLOWABLE LOAD

$$\sigma_s = 22 \text{ ksi} \quad \sigma_b = 16 \text{ ksi}$$

Use Eqs. (2-17a and b) of Example 2-6.

For steel:

$$\sigma_s = \frac{PE_s}{E_s A_s + E_b A_b} \quad P_s = (E_s A_s + E_b A_b) \frac{\sigma_s}{E_s}$$

$$P_s = (1.776 \times 10^6 \text{ lb}) \left( \frac{22 \text{ ksi}}{30 \times 10^6 \text{ psi}} \right) = 1300 \text{ lb}$$

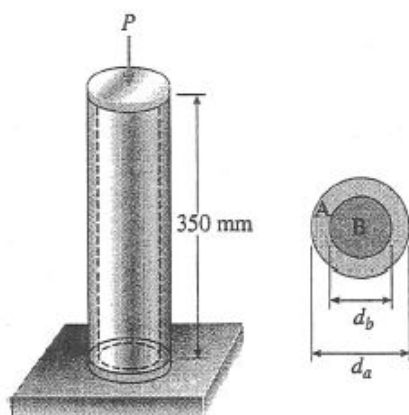
For brass:

$$\sigma_b = \frac{PE_b}{E_s A_s + E_b A_b} \quad P_s = (E_s A_s + E_b A_b) \frac{\sigma_b}{E_b}$$

$$P_s = (1.776 \times 10^6 \text{ lb}) \left( \frac{16 \text{ ksi}}{15 \times 10^6 \text{ psi}} \right) = 1890 \text{ lb}$$

Steel governs.  $P_{\text{allow}} = 1300 \text{ lb} \quad \leftarrow$

**Problem 2.4-2**



A = aluminum

B = brass

$L = 350 \text{ mm}$

$d_a = 40 \text{ mm}$

$d_b = 25 \text{ mm}$

$$A_a = \frac{\pi}{4} (d_a^2 - d_b^2)$$

$$= 765.8 \text{ mm}^2$$

$$E_a = 72 \text{ GPa} \quad E_b = 100 \text{ GPa} \quad A_b = \frac{\pi}{4} d_b^2$$

$$= 490.9 \text{ mm}^2$$

(a) DECREASE IN LENGTH

$$(\delta = 0.1\% \text{ of } L = 0.350 \text{ mm})$$

Use Eq. (2-18) of Example 2-6.

$$\delta = \frac{PL}{E_a A_a + E_b A_b} \quad \text{or}$$

$$P = (E_a A_a + E_b A_b) \left( \frac{\delta}{L} \right)$$

Substitute numerical values:

$$\begin{aligned} E_a A_a + E_b A_b &= (72 \text{ GPa})(765.8 \text{ mm}^2) \\ &\quad + (100 \text{ GPa})(490.9 \text{ mm}^2) \\ &= 55.135 \text{ MN} + 49.090 \text{ MN} \\ &= 104.23 \text{ MN} \end{aligned}$$

$$\begin{aligned} P &= (104.23 \text{ MN}) \left( \frac{0.350 \text{ mm}}{350 \text{ mm}} \right) \\ &= 104.2 \text{ kN} \quad \leftarrow \end{aligned}$$

(b) ALLOWABLE LOAD

$$\sigma_a = 80 \text{ MPa} \quad \sigma_b = 120 \text{ MPa}$$

Use Eqs. (2-17a and b) of Example 2-6.

For aluminum:

$$\sigma_a = \frac{PE_a}{E_a A_a + E_b A_b} \quad P_a = (E_a A_a + E_b A_b) \left( \frac{\sigma_a}{E_a} \right)$$

$$P_a = (104.23 \text{ MN}) \left( \frac{80 \text{ MPa}}{72 \text{ GPa}} \right) = 115.8 \text{ kN}$$

For brass:

$$\sigma_b = \frac{PE_b}{E_a A_a + E_b A_b} \quad P_b = (E_a A_a + E_b A_b) \left( \frac{\sigma_b}{E_b} \right)$$

$$P_b = (104.23 \text{ MN}) \left( \frac{120 \text{ MPa}}{100 \text{ GPa}} \right) = 125.1 \text{ kN}$$

Aluminum governs.  $P_{\max} = 116 \text{ kN} \quad \leftarrow$

### Problem 2.4-3

$$E = 29000 \text{ ksi} \quad A = 8 \text{ in}^2$$

Use superposition - select  $A_y$  as the redundant

Released structure with actual load  $P$  at  $C$        $\delta_{A1} = \frac{2.5 \text{ kip} \cdot (6 \text{ ft})}{E \cdot A} = 7.759 \times 10^{-4} \text{ in}$     upward

Released structure with redundant  $A_y$  applied at  $A$        $\delta_{A2} = A_y \cdot \left( \frac{3 \text{ ft} + 6 \text{ ft}}{E \cdot A} \right) \quad \frac{3 \text{ ft} + 6 \text{ ft}}{E \cdot A} = 4.655 \times 10^{-4} \frac{\text{in}}{\text{kip}}$

Compatibility equation       $\delta_{A1} + \delta_{A2} = 0$       solve for redundant  $A_y$        $A_y = \frac{-\delta_{A1}}{\frac{3 \text{ ft} + 6 \text{ ft}}{E \cdot A}} = -1.667 \text{ kip}$

Statics       $B_y = -(A_y + 2.5 \text{ kip}) = -0.833 \text{ kip}$

Axial displacement at  $C$        $\frac{-B_y \cdot (6 \text{ ft})}{E \cdot A} = 2.586 \times 10^{-4} \text{ in}$     upward ... or       $\frac{-A_y \cdot (3 \text{ ft})}{E \cdot A} = 2.586 \times 10^{-4} \text{ in}$

use either extension of segment  $BC$  or compression of  $AC$  to find upward displ.  $\delta_C$

### Problem 2.4-4

$$P = 10\text{ kN} \quad E = 200\text{ GPa} \quad \sigma_Y = 400\text{ MPa} \quad FS_Y = 2 \quad \sigma_a = \frac{\sigma_Y}{FS_Y} = 200\text{ MPa}$$

Static equilibrium - cut through cables, use lower FBD (see fig.)

$$a = 1.5\text{ m} \quad b = 1.5\text{ m} \quad \alpha_B = \tan^{-1}\left(\frac{a}{b}\right) = 45^\circ$$

$$\alpha_C = \tan^{-1}\left(\frac{a}{2 \cdot b}\right) = 26.565^\circ$$

$$\Sigma M_A = 0$$

$$T_1 \cdot \sin(\alpha_B) + 2 \cdot T_2 \cdot \sin(\alpha_C) = P \cdot (2)$$

Compatibility - from figure, see that  $\Delta_C = 2 \cdot \Delta_B$

Cable elongations

$$\delta_1 = \Delta_B \cdot \sin(\alpha_B) \quad \delta_2 = \Delta_C \cdot \sin(\alpha_C)$$

$$\text{so} \quad \delta_2 = 2 \cdot \left( \frac{\sin(\alpha_C)}{\sin(\alpha_B)} \right) \cdot \delta_1 \quad 2 \cdot \left( \frac{\sin(\alpha_C)}{\sin(\alpha_B)} \right) = 1.26491$$

Force-displacement relations for cables

$$L_1 = \sqrt{a^2 + b^2} = 2.121\text{ m}$$

$$L_2 = \sqrt{a^2 + (2 \cdot b)^2} = 3.354\text{ m}$$

$$\delta_1 = T_1 \cdot f_1 \quad f_1 = \frac{L_1}{E \cdot A_1}$$

$$\delta_2 = T_2 \cdot f_2 \quad f_2 = \frac{L_2}{E \cdot A_2}$$

$$\text{where} \quad T_2 \cdot f_2 = 2 \cdot \left( \frac{\sin(\alpha_C)}{\sin(\alpha_B)} \right) \cdot T_1 \cdot f_1 \quad \text{or} \quad T_2 = 2 \cdot \left( \frac{\sin(\alpha_C)}{\sin(\alpha_B)} \right) \cdot \left( \frac{f_1}{f_2} \right) \cdot T_1 \quad \text{and } A_1 = A_2 \text{ so} \quad \frac{f_1}{f_2} = \frac{L_1}{L_2}$$

Substitute  $T_2$  expression into equilibrium equation and solve for  $T_1$  then solve for  $T_2$

$$T_1 = \left( \frac{2 \cdot f_2 \cdot \sin(\alpha_B)}{f_2 \cdot \sin(\alpha_B)^2 + 4 \cdot f_1 \cdot \sin(\alpha_C)^2} \right) \cdot P \quad \text{or} \quad T_1 = \left[ \frac{2 \cdot \sin(\alpha_B)}{\sin(\alpha_B)^2 + 4 \cdot \left( \frac{L_1}{L_2} \right) \cdot \sin(\alpha_C)^2} \right] \cdot P = 14.058\text{ kN}$$

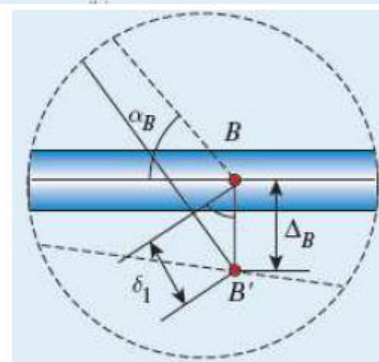
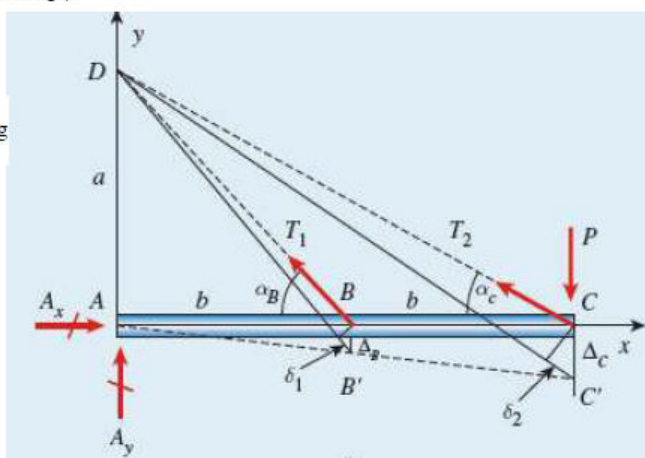
$$\text{and} \quad T_2 = 2 \cdot \left( \frac{\sin(\alpha_C)}{\sin(\alpha_B)} \right) \cdot \left( \frac{L_1}{L_2} \right) \cdot T_1 = 11.247\text{ kN}$$

Use allowable stress  $\sigma_a$  to find minimum required cross sectional area of each cable

$$A_1 = \frac{T_1}{\sigma_a} = 70.291\text{ mm}^2$$

$$A_2 = \frac{T_2}{\sigma_a} = 56.233\text{ mm}^2$$

$$\text{so} \quad A_{\text{reqd}} = 70.3\text{ mm}^2$$



### Problem 2.4-5

$$\sigma_{ys} = 50\text{ksi} \quad \sigma_{yA} = 60\text{ksi} \quad A_s = 12\text{in}^2 \quad A_A = 6\text{in}^2 \quad L = 20\text{in} \quad E_s = 29000\text{ksi} \quad E_A = 10600\text{ksi}$$

Axial stiffnesses of cylinder and tube - treat as springs in parallel

$$k_s = \frac{E_s \cdot A_s}{L} = 1.74 \times 10^4 \cdot \frac{\text{kip}}{\text{in}} \quad k_A = \frac{E_A \cdot A_A}{L} = 3.18 \times 10^3 \cdot \frac{\text{kip}}{\text{in}} \quad k_T = k_s + k_A = 2.058 \times 10^4 \cdot \frac{\text{kip}}{\text{in}}$$

Each "spring" carries a force in proportion to its stiffness

$$P_s(P) = \frac{k_s}{k_T} \cdot P \text{ float}, 5 \rightarrow 0.84548 \cdot P \quad P_A(P) = \frac{k_A}{k_T} \cdot P \text{ float}, 5 \rightarrow 0.15452 \cdot P$$

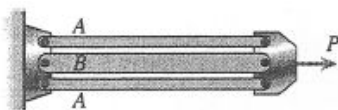
Maximum force in each component is governed by its yield stress

$$\begin{aligned} \sigma_{ys}(P) &= \frac{P_s(P)}{A_s} \text{ float}, 5 \rightarrow \frac{0.070457 \cdot P}{\text{in}^2} & \sigma_{ys}(P) - 50\text{ksi} & \left| \begin{array}{l} \text{solve}, P \\ \text{float}, 5 \end{array} \right. \rightarrow 709.65 \cdot \text{in}^2 \cdot \text{ksi} = 709.65 \cdot \text{kip} \\ \sigma_{yA}(P) &= \frac{P_A(P)}{A_A} \text{ float}, 5 \rightarrow \frac{0.025753 \cdot P}{\text{in}^2} & \sigma_{yA}(P) - 60\text{ksi} & \left| \begin{array}{l} \text{solve}, P \\ \text{float}, 5 \end{array} \right. \rightarrow 2329.8 \cdot \text{in}^2 \cdot \text{ksi} = 2329.8 \cdot \text{kip} \end{aligned}$$

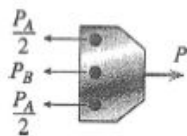
So the allowable load  $P$  is limited by yield stress in steel cylinder  $P_{\text{all}} = 709.655\text{kip}$



**Problem 2.4-6**



FREE-BODY DIAGRAM OF END PLATE



EQUATION OF EQUILIBRIUM

$$\Sigma F_{\text{horiz}} = 0 \quad P_A + P_B - P = 0$$

EQUATION OF COMPATIBILITY

$$\delta_A = \delta_B$$

FORCE-DISPLACEMENT RELATIONS

$A_A$  = total area of both outer bars

$$\delta_A = \frac{P_A L}{E_A A_A} \quad \delta_B = \frac{P_B L}{E_B A_B}$$

Substitute into Eq. (2):

$$\frac{P_A L}{E_A A_A} = \frac{P_B L}{E_B A_B}$$

SOLUTION OF THE EQUATIONS

Solve simultaneously Eqs. (1) and (4):

$$P_A = \frac{E_A A_A P}{E_A A_A + E_B A_B} \quad P_B = \frac{E_B A_B P}{E_A A_A + E_B A_B} \quad (5)$$

Substitute into Eq. (3):

$$\delta = \delta_A = \delta_B = \frac{PL}{E_A A_A + E_B A_B} \quad (6)$$

STRESSES:

$$\sigma_A = \frac{P_A}{A_A} = \frac{E_A P}{E_A A_A + E_B A_B}$$

$$\sigma_B = \frac{P_B}{A_B} = \frac{E_B P}{E_A A_A + E_B A_B} \quad (7)$$

(a) LOAD IN MIDDLE BAR

$$\frac{P_B}{P} = \frac{E_B A_B}{E_A A_A + E_B A_B} = \frac{1}{\frac{E_A A_A}{E_B A_B} + 1}$$

$$\text{Given: } \frac{E_A}{E_B} = 2 \quad \frac{A_A}{A_B} = \frac{1 + 1}{1.5} = \frac{4}{3}$$

$$\therefore \frac{P_B}{P} = \frac{1}{\left(\frac{E_A}{E_B}\right)\left(\frac{A_A}{A_B}\right) + 1} = \frac{1}{\frac{8}{3} + 1} = \frac{3}{11} \quad \leftarrow$$

(b) RATIO OF STRESSES

$$\frac{\sigma_B}{\sigma_A} = \frac{E_B}{E_A} = \frac{1}{2} \quad \leftarrow \quad (4)$$

(c) RATIO OF STRAINS

All bars have the same strain

$$\text{Ratio} = 1 \quad \leftarrow$$



**Problem 2.4-7**

(a) REACTIONS AT  $A$  AND  $B$  DUE TO LOAD  $P$  AT  $L/2$

$$A_{AC} = \frac{\pi}{4} \left[ d^2 - \left( \frac{d}{2} \right)^2 \right] \quad A_{AC} = \frac{3}{16} \pi d^2$$

$$A_{CB} = \frac{\pi}{4} d^2$$

Select  $R_B$  as the redundant; use superposition and a compatibility equation at  $B$ :

$$\text{if } x \leq L/2 \quad \delta_{B1a} = \frac{Px}{EA_{AC}} + \frac{P\left(\frac{L}{2} - x\right)}{EA_{CB}} \quad \delta_{B1a} = \frac{P}{E} \left( \frac{x}{\frac{3}{16} \pi d^2} + \frac{\frac{L}{2} - x}{\frac{\pi}{4} d^2} \right)$$

$$\delta_{B1a} = \frac{2}{3} P \frac{2x + 3L}{E \pi d^2}$$

$$\text{if } x \geq L/2 \quad \delta_{B1b} = \frac{P \frac{L}{2}}{EA_{AC}} \quad \delta_{B1b} = \frac{P \frac{L}{2}}{E \left( \frac{3}{16} \pi d^2 \right)} \quad \delta_{B1b} = \frac{8}{3} \frac{PL}{E \pi d^2}$$

The following expression for  $\delta_{B2}$  is good for all  $x$ :

$$\delta_{B2} = \frac{R_B}{E} \left( \frac{x}{A_{AC}} + \frac{L-x}{A_{CB}} \right) \quad \delta_{B2} = \frac{R_B}{E} \left( \frac{x}{\frac{3}{16} \pi d^2} + \frac{L-x}{\frac{\pi}{4} d^2} \right)$$

$$\delta_{B2} = \frac{R_B}{E} \left( \frac{16}{3} \frac{x}{\pi d^2} + 4 \frac{L-x}{\pi d^2} \right)$$

Solve for  $R_B$  and  $R_A$  assuming that  $x \leq L/2$ :

$$\text{Compatibility:} \quad \delta_{B1a} + \delta_{B2} = 0 \quad R_{Ba} = \frac{-\left(\frac{2}{3} P \frac{2x + 3L}{\pi d^2}\right)}{\left(\frac{16}{3} \frac{x}{\pi d^2} + 4 \frac{L-x}{\pi d^2}\right)} \quad R_{Ba} = \frac{-1}{2} P \frac{2x + 3L}{x + 3L} \quad \leftarrow$$

^ check—if  $x = 0$ ,  $R_B = -P/2$

$$\text{Statics:} \quad R_{Aa} = -P - R_{Ba} \quad R_{Aa} = -P - \frac{-1}{2} P \frac{2x + 3L}{x + 3L} \quad R_{Aa} = \frac{-3}{2} P \frac{L}{x + 3L} \quad \leftarrow$$

^ check—if  $x = 0$ ,  $R_{Aa} = -P/2$

Solve for  $R_B$  and  $R_A$  assuming that  $x \geq L/2$ :

$$\text{Compatibility: } \delta_{B1b} + \delta_{B2} = 0 \quad R_{Bb} = \frac{-\frac{8}{3} \frac{PL}{\pi d^2}}{\left(\frac{16}{3} \frac{x}{\pi d^2} + 4 \frac{L-x}{\pi d^2}\right)} \quad R_{Bb} = \frac{-2PL}{x+3L} \quad \leftarrow$$

^ check—if  $x = L$ ,  $R_B = -P/2$

$$\text{Statics: } R_{Ab} = -P - R_{Bb} \quad R_{Ab} = -P - \left(\frac{-2PL}{x+3L}\right) \quad R_{Ab} = -P \frac{x+L}{x+3L} \quad \leftarrow$$

(b) FIND  $\delta$  AT POINT OF LOAD APPLICATION; AXIAL FORCE FOR SEGMENT 0 TO  $L/2 = -R_A$  AND  $\delta =$  ELONGATION OF THIS SEGMENT

Assume that  $x \leq L/2$ :

$$\delta_a = \frac{-R_{Aa}}{E} \left( \frac{x}{A_{AC}} + \frac{\frac{L}{2} - x}{A_{CB}} \right) \quad \delta_a = \frac{-\left(\frac{-3}{2} P \frac{L}{x+3L}\right) \left( \frac{x}{\frac{3}{16} \pi d^2} + \frac{\frac{L}{2} - x}{\frac{\pi}{4} d^2} \right)}{E}$$

$$\delta_a = PL \frac{2x+3L}{(x+3L)E\pi d^2}$$

$$\text{For } x = L/2, \quad \delta_a = \frac{8}{7} L \frac{P}{E\pi d^2} \quad \leftarrow$$

ASSUME THAT  $x \geq L/2$ :

$$\delta_b = \frac{(-R_{Ab}) \frac{L}{2}}{EA_{AC}} \quad \delta_b = \frac{\left(P \frac{x+L}{x+3L}\right) \frac{L}{2}}{E \left(\frac{3}{16} \pi d^2\right)} \quad \delta_b = \frac{8}{3} P \left(\frac{x+L}{x+3L}\right) \frac{L}{E\pi d^2} \quad \leftarrow$$

$$\text{for } x = L/2 \quad \delta_b = \frac{8}{7} P \frac{L}{E\pi d^2} \quad < \text{ same as } \delta_a \text{ above (OK)}$$

(c) FOR WHAT VALUE OF  $x$  IS  $R_B = (6/5) R_A$ ?

Guess that  $x < L/2$  here and use  $R_{Ba}$  expression above to find  $x$ :

$$\frac{-1}{2} P \frac{2x+3L}{x+3L} - \frac{6}{5} \left( \frac{-3}{2} P \frac{L}{x+3L} \right) = 0 \quad \frac{-1}{10} P \frac{10x-3L}{x+3L} = 0 \quad x = \frac{3L}{10} \quad \leftarrow$$

Now try  $R_{Bb} = (6/5)R_{Ab}$ , assuming that  $x > L/2$

$$\frac{-2PL}{x+3L} - \frac{6}{5} \left( -P \frac{x+L}{x+3L} \right) = 0 \quad \frac{2}{5} P \frac{-2L+3x}{x+3L} = 0 \quad x = \frac{2}{3} L \quad \leftarrow$$

So, there are two solutions for  $x$ .

- (d) FIND REACTIONS IF THE BAR IS NOW ROTATED TO A VERTICAL POSITION, LOAD  $P$  IS REMOVED, AND THE BAR IS HANGING UNDER ITS OWN WEIGHT (ASSUME MASS DENSITY =  $\rho$ ). ASSUME THAT  $x = L/2$ .

$$A_{AC} = \frac{3}{16} \pi d^2 \quad A_{CB} = \frac{\pi}{4} d^2$$

Select  $R_B$  as the redundant; use superposition and a compatibility equation at  $B$

from (a) above. compatibility:  $\delta_{B1} + \delta_{B2} = 0$

$$\delta_{B2} = \frac{R_B}{E} \left( \frac{x}{A_{AC}} + \frac{L-x}{A_{CB}} \right) \quad \text{For } x = L/2, \quad \delta_{B2} = \frac{R_B}{E} \left( \frac{14}{3} \frac{L}{\pi d^2} \right)$$

$$\delta_{B1} = \int_0^{L/2} \frac{N_{AC}}{EA_{AC}} d\zeta + \int_{L/2}^L \frac{N_{CB}}{EA_{CB}} d\zeta$$

Where axial forces in bar due to self weight are  $W_{AC} = \rho g A_{AC} \frac{L}{2}$   $W_{CB} = \rho g A_{CB} \frac{L}{2}$   
(assume  $\zeta$  is measured upward from  $A$ ):

$$N_{AC} = - \left[ \rho g A_{CB} \frac{L}{2} + \rho g A_{AC} \left( \frac{L}{2} - \zeta \right) \right] \quad A_{AC} = \frac{3}{16} \pi d^2 \quad A_{CB} = \frac{\pi}{4} d^2$$

$$N_{CB} = - [\rho g A_{CB} (L - \zeta)]$$

$$N_{AC} = \frac{-1}{8} \rho g \pi d^2 L - \frac{3}{16} \rho g \pi d^2 \left( \frac{1}{2} L - \zeta \right) \quad N_{CB} = - \left[ \frac{1}{4} \rho g \pi d^2 (L - \zeta) \right]$$

$$\delta_{B1} = \int_0^{L/2} \frac{\frac{-1}{8} \rho g \pi d^2 L - \frac{3}{16} \rho g \pi d^2 \left( \frac{1}{2} L - \zeta \right)}{E \left( \frac{3}{16} \pi d^2 \right)} d\zeta + \int_{L/2}^L \frac{- \left[ \frac{1}{4} \rho g \pi d^2 (L - \zeta) \right]}{E \left( \frac{\pi}{4} d^2 \right)} d\zeta$$

$$\delta_{B1} = \left( \frac{-11}{24} \rho g \frac{L^2}{E} + \frac{-1}{8} \rho g \frac{L^2}{E} \right) \quad \delta_{B1} = \frac{-7}{12} \rho g \frac{L^2}{E} \quad \frac{7}{12} = 0.583$$

Compatibility:  $\delta_{B1} + \delta_{B2} = 0$

$$R_B = \frac{- \left( \frac{-7}{12} \rho g \frac{L^2}{E} \right)}{\left( \frac{14}{3} \frac{L}{E \pi d^2} \right)} \quad R_B = \frac{1}{8} \rho g \pi d^2 L \quad \leftarrow$$

Statics:  $R_A = (W_{AC} + W_{CB}) - R_B$

$$R_A = \left[ \left[ \rho g \left( \frac{3}{16} \pi d^2 \right) \frac{L}{2} + \rho g \left( \frac{\pi}{4} d^2 \right) \frac{L}{2} \right] - \frac{1}{8} \rho g \pi d^2 L \right]$$

$$R_A = \frac{3}{32} \rho g \pi d^2 L \quad \leftarrow$$

### Problem 2.4-8

$$P = 200\text{ kN} \quad L = 2\text{ m} \quad t = 20\text{ mm} \quad b_1 = 100\text{ mm} \quad b_2 = 115\text{ mm} \quad E = 96\text{ GPa}$$

Select reaction  $R_C$  as the redundant; use superposition

axial displacement at C due to actual load  $P$  at B

$$\delta_{C1} = \frac{P \cdot \left(\frac{3 \cdot L}{5}\right)}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) = 1.165 \cdot \text{mm}$$

axial displacement at C due to redundant  $R_C$

$$\delta_{C2} = R_C \cdot \left[ \frac{\frac{3 \cdot L}{5} + \frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) \right]$$

$$\frac{\frac{3 \cdot L}{5} + \frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) = 9.706 \times 10^{-3} \cdot \frac{\text{mm}}{\text{kN}}$$

Compatibility equation  $\delta_{C1} + \delta_{C2} = 0$  solve for  $R_C$

$$R_C = \frac{-\delta_{C1}}{\left[ \frac{\frac{3 \cdot L}{5} + \frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) \right]} = -120 \cdot \text{kN}$$

Statics  $\Sigma F = 0$   $R_A = -(P + R_C) = -80 \cdot \text{kN}$

Negative reactions so both act to left

Compute extension of AB or compression of BC to find displ.  $\delta_B$  (to the right)

$$-R_A \cdot \left[ \frac{\left(\frac{3 \cdot L}{5}\right)}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) \right] = 0.466 \cdot \text{mm} \quad \text{or} \quad -R_C \cdot \left[ \frac{\left(\frac{2 \cdot L}{5}\right)}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) \right] = 0.466 \cdot \text{mm}$$

### Problem 2.4-9

$$P = 20 \text{ kip} \quad L = 3 \text{ ft} \quad t = \frac{1}{4} \text{ in} \quad b_1 = 2 \text{ in} \quad b_2 = 2.5 \text{ in} \quad E = 10400 \text{ ksi}$$

$$A_{BC} = b_1 \cdot t = 0.5 \text{ in}^2 \quad b_{ave} = \frac{b_1 + b_2}{2} = 2.25 \text{ in}$$

Select reaction  $R_C$  as the redundant; use superposition

axial displacement at C due to actual load  $P$  at middle of AB

$$\delta_{C1} = \frac{P \left( \frac{L}{4} \right)}{E \cdot t \cdot (b_2 - b_{ave})} \cdot \ln \left( \frac{b_2}{b_{ave}} \right) = 0.029 \text{ in}$$

axial displacement at C due to redundant  $R_C$

$$\delta_{C2} = R_C \left[ \frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln \left( \frac{b_2}{b_1} \right) + \frac{\frac{L}{2}}{E \cdot A_{BC}} \right]$$

flexibility constant for bar

$$\frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln \left( \frac{b_2}{b_1} \right) + \frac{\frac{L}{2}}{E \cdot A_{BC}} = 6.551 \times 10^{-3} \frac{\text{in}}{\text{kip}}$$

Compatibility equation  $\delta_{C1} + \delta_{C2} = 0$  solve for  $R_C$

$$R_C = \frac{-\delta_{C1}}{\left[ \frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln \left( \frac{b_2}{b_1} \right) + \frac{\frac{L}{2}}{E \cdot A_{BC}} \right]} = -4.454 \text{ kip}$$

Statics  $\Sigma F = 0$   $R_A = -(P + R_C) = -15.546 \text{ kip}$

Negative reactions so both act to left

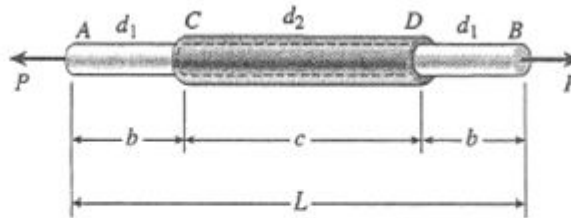
Compute deformations of AB (two terms, more difficult) or deformation of BC (easier) to find displ.  $\delta_B$  (to the right)

$$-R_A \left[ \frac{\left( \frac{L}{4} \right)}{E \cdot t \cdot (b_2 - b_{ave})} \cdot \ln \left( \frac{b_2}{b_{ave}} \right) \right] + \frac{(-R_A - P) \cdot \frac{L}{4}}{E \cdot t \cdot (b_{ave} - b_1)} \cdot \ln \left( \frac{b_{ave}}{b_1} \right) = 1.542 \times 10^{-2} \text{ in}$$

or

$$-R_C \left[ \frac{\frac{L}{2}}{E \cdot A_{BC}} \right] = 1.542 \times 10^{-2} \text{ in}$$

**Problem 2.4-10**



$$P = 12 \text{ kN} \quad d_1 = 30 \text{ mm} \quad b = 100 \text{ mm}$$

$$L = 500 \text{ mm} \quad d_2 = 45 \text{ mm} \quad c = 300 \text{ mm}$$

$$\text{Rod: } E_1 = 3.1 \text{ GPa}$$

$$\text{Sleeve: } E_2 = 2.5 \text{ GPa}$$

$$\text{Rod: } A_1 = \frac{\pi d_1^2}{4} = 706.86 \text{ mm}^2$$

$$\text{Sleeve: } A_2 = \frac{\pi}{4}(d_2^2 - d_1^2) = 883.57 \text{ mm}^2$$

$$E_1 A_1 + E_2 A_2 = 4.400 \text{ MN}$$

(a) ELONGATION OF ROD

$$\text{Part AC: } \delta_{AC} = \frac{Pb}{E_1 A_1} = 0.5476 \text{ mm}$$

$$\begin{aligned} \text{Part CD: } \delta_{CD} &= \frac{Pc}{E_1 A_1 + E_2 A_2} \\ &= 0.81815 \text{ mm} \end{aligned}$$

(From Eq. 2-16 of Example 2-8)

$$\delta = 2\delta_{AC} + \delta_{CD} = 1.91 \text{ mm} \quad \leftarrow$$

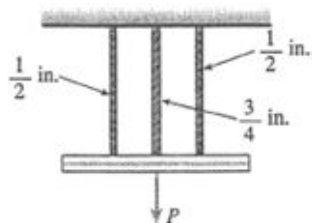
(b) SLEEVE AT FULL LENGTH

$$\begin{aligned} \delta &= \delta_{CD} \left( \frac{L}{c} \right) = (0.81815 \text{ mm}) \left( \frac{500 \text{ mm}}{300 \text{ mm}} \right) \\ &= 1.36 \text{ mm} \quad \leftarrow \end{aligned}$$

(c) SLEEVE REMOVED

$$\delta = \frac{PL}{E_1 A_1} = 2.74 \text{ mm} \quad \leftarrow$$

**Problem 2.4-11**



AREAS OF CABLES (from Table 2-1)

Middle cable:  $A_M = 0.268 \text{ in.}^2$

Outer cables:  $A_O = 0.119 \text{ in.}^2$

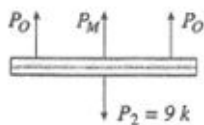
(for each cable)

FIRST LOADING

$$P_1 = 12 \text{ k} \left( \text{Each cable carries } \frac{P_1}{3} \text{ or } 4 \text{ k.} \right)$$

SECOND LOADING

$P_2 = 9 \text{ k}$  (additional load)



EQUATION OF EQUILIBRIUM

$$\Sigma F_{\text{vert}} = 0 \quad 2P_O + P_M - P_2 = 0 \quad (1)$$

EQUATION OF COMPATIBILITY

$$\delta_M = \delta_O \quad (2)$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_M = \frac{P_M L}{EA_M} \quad \delta_O = \frac{P_O L}{EA_O} \quad (3, 4)$$

SUBSTITUTE INTO COMPATIBILITY EQUATION:

$$\frac{P_M L}{EA_M} = \frac{P_O L}{EA_O} \quad \frac{P_M}{A_M} = \frac{P_O}{A_O} \quad (5)$$

SOLVE SIMULTANEOUSLY EQS. (1) AND (5):

$$P_M = P_2 \left( \frac{A_M}{A_M + 2A_O} \right) = (9 \text{ k}) \left( \frac{0.268 \text{ in.}^2}{0.506 \text{ in.}^2} \right) = 4.767 \text{ k}$$

$$P_O = P_2 \left( \frac{A_O}{A_M + 2A_O} \right) = (9 \text{ k}) \left( \frac{0.119 \text{ in.}^2}{0.506 \text{ in.}^2} \right) = 2.117 \text{ k}$$

FORCES IN CABLES

Middle cable: Force =  $4 \text{ k} + 4.767 \text{ k} = 8.767 \text{ k}$

Outer cables: Force =  $4 \text{ k} + 2.117 \text{ k} = 6.117 \text{ k}$

(for each cable)

(a) PERCENT OF TOTAL LOAD CARRIED BY MIDDLE CABLE

$$\text{Percent} = \frac{8.767 \text{ k}}{21 \text{ k}} (100\%) = 41.7\% \quad \leftarrow$$

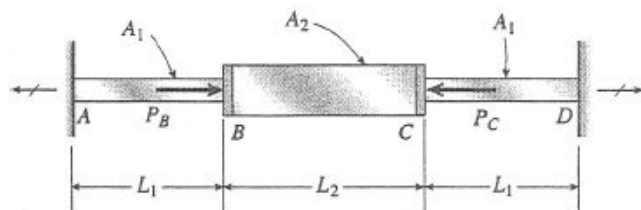
(b) STRESSES IN CABLES ( $\sigma = P/A$ )

$$\text{Middle cable: } \sigma_M = \frac{8.767 \text{ k}}{0.268 \text{ in.}^2} = 32.7 \text{ ksi} \quad \leftarrow$$

$$\text{Outer cables: } \sigma_O = \frac{6.117 \text{ k}}{0.119 \text{ in.}^2} = 51.4 \text{ ksi} \quad \leftarrow$$



**Problem 2.4-12**



$$\begin{aligned} P_B &= 25.5 \text{ kN} & P_C &= 17.0 \text{ kN} \\ L_1 &= 200 \text{ mm} & L_2 &= 250 \text{ mm} \\ A_1 &= 840 \text{ mm}^2 & A_2 &= 1260 \text{ mm}^2 \\ m &= \text{meter} \end{aligned}$$

FREE-BODY DIAGRAM



EQUATION OF EQUILIBRIUM

$$\Sigma F_{\text{horiz}} = 0 \quad \rightarrow \quad \leftarrow$$

$$P_B + R_D - P_C - R_A = 0 \text{ or}$$

$$R_A - R_D = P_B - P_C = 8.5 \text{ kN} \quad (\text{Eq. 1})$$

EQUATION OF COMPATIBILITY

$$\delta_{AD} = \text{elongation of entire bar}$$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 0 \quad (\text{Eq. 2})$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_{AB} = \frac{R_A L_1}{EA_1} = \frac{R_A}{E} \left( 238.05 \frac{1}{\text{m}} \right) \quad (\text{Eq. 3})$$

$$\begin{aligned} \delta_{BC} &= \frac{(R_A - P_B)L_2}{EA_2} \\ &= \frac{R_A}{E} \left( 198.413 \frac{1}{\text{m}} \right) - \frac{P_B}{E} \left( 198.413 \frac{1}{\text{m}} \right) \end{aligned} \quad (\text{Eq. 4})$$

$$\delta_{CD} = \frac{R_D L_1}{EA_1} = \frac{R_D}{E} \left( 238.095 \frac{1}{\text{m}} \right) \quad (\text{Eq. 5})$$

SOLUTION OF EQUATIONS

Substitute Eqs. (3), (4), and (5) into Eq. (2):

$$\begin{aligned} \frac{R_A}{E} \left( 238.095 \frac{1}{\text{m}} \right) + \frac{R_A}{E} \left( 198.413 \frac{1}{\text{m}} \right) \\ - \frac{P_B}{E} \left( 198.413 \frac{1}{\text{m}} \right) + \frac{R_D}{E} \left( 238.095 \frac{1}{\text{m}} \right) = 0 \end{aligned}$$

Simplify and substitute  $P_B = 25.5 \text{ kN}$ :

$$\begin{aligned} R_A \left( 436.508 \frac{1}{\text{m}} \right) + R_D \left( 238.095 \frac{1}{\text{m}} \right) \\ = 5,059.53 \text{ kN/m} \end{aligned} \quad (\text{Eq. 6})$$

(a) REACTIONS  $R_A$  AND  $R_D$

Solve simultaneously Eqs. (1) and (6).

$$\text{From (1): } R_D = R_A - 8.5 \text{ kN}$$

Substitute into (6) and solve for  $R_A$ :

$$R_A \left( 674.603 \frac{1}{\text{m}} \right) = 7083.34 \text{ kN/m}$$

$$R_A = 10.5 \text{ kN} \quad \leftarrow$$

$$R_D = R_A - 8.5 \text{ kN} = 2.0 \text{ kN} \quad \leftarrow$$

(b) COMPRESSIVE AXIAL FORCE  $F_{BC}$

$$F_{BC} = P_B - R_A = P_C - R_D = 15.0 \text{ kN} \quad \leftarrow$$



### Problem 2.4-13

#### NUMERICAL DATA

$$n = 6 \quad d_b = 0.5 \text{ in.} \quad \sigma_a = 14 \text{ ksi} \quad A_b = \frac{\pi}{4} d_b^2 = 0.196 \text{ in.}^2$$

#### (a) FORMULAS FOR REACTIONS $F$

$$\text{Segment } ABC \text{ flexibility: } f_1 = \frac{2\left(\frac{L}{4}\right)}{EA} = \frac{L}{2EA}$$

$$\text{Segment } CDE \text{ flexibility: } f_2 = \frac{2\left(\frac{L}{4}\right)}{\frac{1}{2}EA} = \frac{L}{EA}$$

Loads at points  $B$  and  $D$ :

$$P_B = -2P \quad P_D = 3P$$

(1) Select  $R_E$  as the redundant; find axial displacement  $\delta_1$  = displacement at  $E$  due to loads  $P_B$  and  $P_D$ :

$$\delta_1 = \frac{(P_B + P_D)\frac{L}{4}}{EA} + \frac{P_D\frac{L}{4}}{EA} + \frac{P_D\frac{L}{4}}{\frac{1}{2}EA} = \frac{5LP}{2EA}$$

(2) Next apply redundant  $R_E$  and find axial displacement  $\delta_2$  = displacement at  $E$  due to redundant  $R_E$ :

$$\delta_2 = R_E(f_1 + f_2) = \frac{3LR_E}{2EA}$$

(3) Use compatibility equation to find redundant  $R_E$  then use statics to find  $R_A$ :

$$\delta_1 + \delta_2 = 0 \text{ solve, } R_E = -\frac{5P}{3} \quad R_E = \frac{-5}{3}P$$

$$R_A = -R_E - P_B - P_D = \frac{2P}{3} \quad R_A = \frac{2P}{3} \quad \boxed{R_A = \frac{2P}{3}} \quad \boxed{R_E = \frac{-5P}{3}}$$

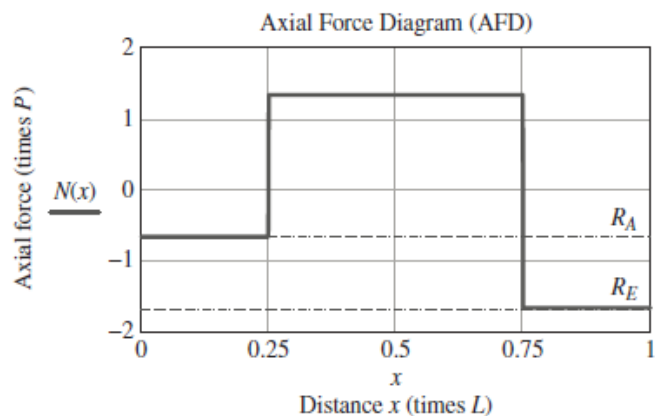
(b) DETERMINE THE AXIAL DISPLACEMENTS  $\delta_B$ ,  $\delta_C$ , AND  $\delta_D$  AT POINTS  $B$ ,  $C$ , AND  $D$ , RESPECTIVELY.

$$\delta_B = \frac{\left(\frac{-2P}{3}\right)\left(\frac{L}{4}\right)}{EA} = \frac{LP}{6EA} \quad \text{leftward} \quad \delta_C = \delta_B + \frac{\left(2P - \frac{2P}{3}\right)\left(\frac{L}{4}\right)}{EA} = \frac{LP}{6EA} \quad \text{to the right} \quad \delta_D = \frac{\left(\frac{5P}{3}\right)\left(\frac{L}{4}\right)}{\frac{EA}{2}} = \frac{5LP}{6EA} \quad \text{to the right}$$

- (c) DRAW AN AXIAL-DISPLACEMENT DIAGRAM (ADD) IN WHICH THE ABCISSA IS THE DISTANCE  $x$  FROM SUPPORT  $A$  TO ANY POINT ON THE BAR AND THE ORDINATE IS THE HORIZONTAL DISPLACEMENT  $\delta$  AT THAT POINT.

AFD for use below in Part (d)

AFD is composed of 4 constant segments, so ADD is linear with zero displacements at supports  $A$  and  $E$ .

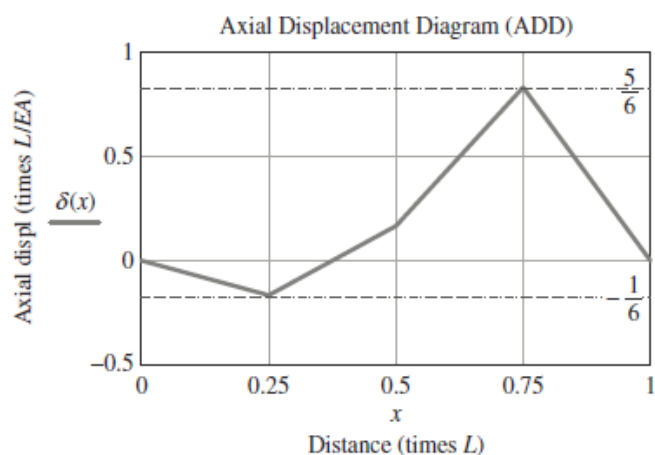


Plot displacements  $\delta_B$ ,  $\delta_C$ , and  $\delta_D$  from part (b) above, then connect points using straight lines showing linear variation of axial displacement between points

$$\delta_{\max} = \delta_D \quad \delta_{\max} = \frac{5LP}{6EA} \quad \text{to the right}$$

Boundary conditions at supports:

$$\delta_A = \delta_E = 0$$



- (d) MAXIMUM PERMISSIBLE VALUE OF LOAD VARIABLE  $P$  BASED ON ALLOWABLE NORMAL STRESS IN FLANGE BOLTS  
FROM AFD, FORCE AT  $L/2$ :

$$F_{\max} = \frac{4}{3}P \quad \text{and} \quad F_{\max} = n\sigma_a A_b = 16.493 \text{ k}$$

$$P_{\max} = \frac{3}{4}F_{\max} = 12.37 \text{ k} \quad \boxed{P_{\max} = 12.37 \text{ k}}$$

### Problem 2.4-14

- (a) STRESSES AND REACTIONS: SELECT  $R_1$  AS REDUNDANT AND DO SUPERPOSITION ANALYSIS (HERE  $q = 0$ ; DEFLECTION POSITIVE UPWARD)

$$d_1 = 50 \text{ mm} \quad d_2 = 60 \text{ mm} \quad d_3 = 57 \text{ mm} \quad d_4 = 64 \text{ mm} \quad A_1 = \frac{\pi}{4} (d_2^2 - d_1^2) = 863.938 \text{ mm}^2$$

$$E = 110 \text{ MPa}$$

$$A_2 = \frac{\pi}{4} (d_4^2 - d_3^2) = 665.232 \text{ mm}^2$$

$$\text{SEGMENT FLEXIBILITIES} \quad L_1 = 2 \text{ m} \quad L_2 = 3 \text{ m}$$

$$f_1 = \frac{L_1}{EA_1} = 0.02105 \text{ mm/N} \quad f_2 = \frac{L_2}{EA_2} = 0.041 \text{ mm/N} \quad \frac{f_1}{f_2} = 0.513$$

$$\text{TENSILE stress } (\sigma_1) \text{ is known in upper segment so } R_1 = \sigma_1 \times A_1 \quad \sigma_1 = 10.5 \text{ MPa} \quad R_1 = \sigma_1 A_1 = 9.07 \text{ kN}$$

$$\delta_{1a} = -Pf_2 \quad \delta_{1b} = R_1(f_1 + f_2) \quad \text{Compatibility: } \delta_{1a} + \delta_{1b} = 0$$

$$\text{Solve for } P: \quad P = R_1 \left( \frac{f_1 + f_2}{f_2} \right) = 13.73 \text{ kN}$$

$$\text{Finally, use statics to find } R_2: \quad R_2 = P - R_1 = 4.66 \text{ kN} \quad \sigma_2 = \frac{R_2}{A_2} = 7 \text{ MPa} \quad < \text{compressive since } R_2 \text{ is positive (upward)}$$

$$P = 13.73 \text{ kN}$$

$$R_1 = 9.07 \text{ kN}$$

$$R_2 = 4.66 \text{ kN}$$

$$\sigma_2 = 7 \text{ MPa}$$

- (b) DISPLACEMENT AT CAP PLATE

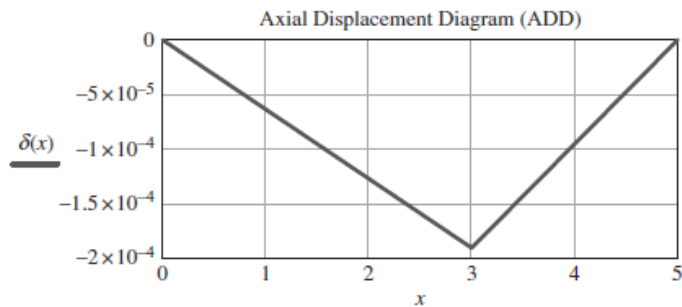
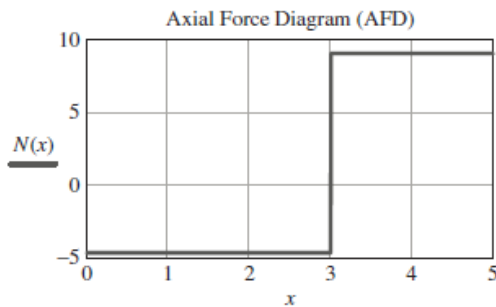
$$\delta_c = R_1 f_1 = 190.909 \text{ mm} < \text{downward} \quad \text{OR} \quad \delta_c = (R_2) f_2 = 190.909 \text{ mm} < \text{downward (neg. } x\text{-direction)}$$

$$\delta_{\text{cap}} = \delta_c = 0.191 \text{ m} \quad \delta_{\text{cap}} = 190.9 \text{ mm}$$

$$\text{AFD and ADD:} \quad R_1 = 9.071 \quad R_2 = 4.657 \quad L_1 = 2 \quad A_1 = 863.938 \quad A_2 = 665.232 \quad E = 110$$

$$L_2 = 3$$

NOTE:  $x$  is measured up from lower support.



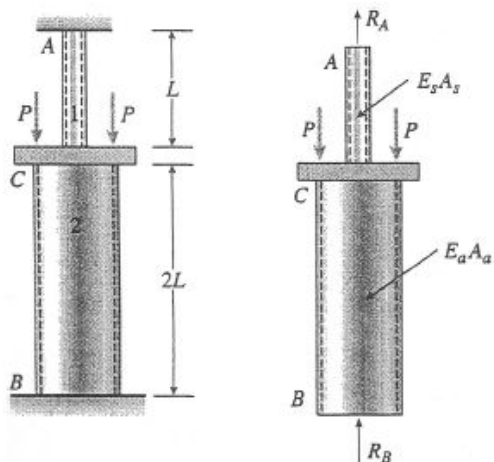
- (c) UNIFORM LOAD  $Q$  ON SEGMENT 2 SUCH THAT  $R_2 = 0$

$$P = 13.728 \text{ kN} \quad R_1 = \sigma_1 A_1 = 9.071 \text{ kN} \quad L_2 = 3 \text{ m}$$

$$\text{Equilibrium: } R_1 + R_2 = P - qL_2 \quad \text{set } R_2 = 0, \text{ solve for req'd } q \quad q = \frac{P - R_1}{L_2} = 1.552 \text{ kN/m}$$

$$q = 1.552 \text{ kN/m}$$

**Problem 2.4-15**



Pipe 1 is steel.  
Pipe 2 is aluminum.

EQUATION OF EQUILIBRIUM

$$\Sigma F_{\text{vert}} = 0 \quad R_A + R_B = 2P \quad (\text{Eq. 1})$$

EQUATION OF COMPATIBILITY

$$\delta_{AB} = \delta_{AC} + \delta_{CB} = 0 \quad (\text{Eq. 2})$$

(A positive value of  $\delta$  means elongation.)

FORCE-DISPLACEMENT RELATIONS

$$\delta_{AC} = \frac{R_A L}{E_s A_s} \quad \delta_{CB} = -\frac{R_B (2L)}{E_a A_a} \quad (\text{Eqs. 3, 4})$$

SOLUTION OF EQUATIONS

Substitute Eqs. (3) and (4) into Eq. (2):

$$\frac{R_A L}{E_s A_s} - \frac{R_B (2L)}{E_a A_a} = 0 \quad (\text{Eq. 5})$$

Solve simultaneously Eqs. (1) and (5):

$$R_A = \frac{4E_s A_s P}{E_a A_a + 2E_s A_s} \quad R_B = \frac{2E_a A_a P}{E_a A_a + 2E_s A_s} \quad (\text{Eqs. 6, 7})$$

(a) AXIAL STRESSES

$$\text{Aluminum: } \sigma_a = \frac{R_B}{A_a} = \frac{2E_a P}{E_a A_a + 2E_s A_s} \quad \leftarrow$$

(compression) (Eq. 8)

$$\text{Steel: } \sigma_s = \frac{R_A}{A_s} = \frac{4E_s P}{E_a A_a + 2E_s A_s} \quad \leftarrow \quad (\text{Eq. 9})$$

(tension)

(b) NUMERICAL RESULTS

$$P = 12 \text{ k} \quad A_a = 8.92 \text{ in.}^2 \quad A_s = 1.03 \text{ in.}^2$$

$$E_a = 10 \times 10^6 \text{ psi} \quad E_s = 29 \times 10^6 \text{ psi}$$

Substitute into Eqs. (8) and (9):

$$\sigma_a = 1,610 \text{ psi (compression)} \quad \leftarrow$$

$$\sigma_s = 9,350 \text{ psi (tension)} \quad \leftarrow$$

### Problem 2.4-16

Numerical data:

$$W = 800 \text{ N} \quad L = 150 \text{ mm}$$

$$a = 50 \text{ mm} \quad d_S = 2 \text{ mm}$$

$$d_A = 4 \text{ mm} \quad E_S = 210 \text{ GPa}$$

$$E_A = 70 \text{ GPa}$$

$$\sigma_{Sa} = 220 \text{ MPa} \quad \sigma_{Aa} = 80 \text{ MPa}$$

$$A_A = \frac{\pi}{4} d_A^2 \quad A_S = \frac{\pi}{4} d_S^2$$

$$A_A = 13 \text{ mm}^2 \quad A_S = 3 \text{ mm}^2$$

(a)  $P_{\text{allow}}$  AT CENTER OF BAR

One-degree statically indeterminate - use reaction ( $R_A$ ) at top of aluminum bar as the redundant

compatibility:  $\delta_1 - \delta_2 = 0$       Statics:  $2R_S + R_A = P + W$

$$\delta_1 = \frac{P+W}{2} \left( \frac{L}{E_S A_S} \right) \quad \text{< downward displacement due to elongation of each steel wire under } P+W \text{ if aluminum wire is cut at top}$$

$$\delta_2 = R_A \left( \frac{L}{2E_S A_S} + \frac{L}{E_A A_A} \right) \quad \text{< upward displ. due to shortening of steel wires and elongation of aluminum wire under redundant } R_A$$

Enforce compatibility and then solve for  $R_A$ :

$$\delta_1 = \delta_2 \quad \text{so} \quad R_A = \frac{\frac{P+W}{2} \left( \frac{L}{E_S A_S} \right)}{\frac{L}{2E_S A_S} + \frac{L}{E_A A_A}} \quad R_A = (P+W) \frac{E_A A_A}{E_A A_A + 2E_S A_S} \quad \text{and} \quad \sigma_{Aa} = \frac{R_A}{A_A}$$

Now use statics to find  $R_S$ :

$$R_S = \frac{P+W-R_A}{2} \quad R_S = \frac{P+W - (P+W) \frac{E_A A_A}{E_A A_A + 2E_S A_S}}{2} \quad R_S = (P+W) \frac{E_S A_S}{E_A A_A + 2E_S A_S}$$

$$\text{and} \quad \sigma_{Sa} = \frac{R_S}{A_S}$$

Compute stresses and apply allowable stress values:

$$\sigma_{Aa} = (P+W) \frac{E_A}{E_A A_A + 2E_S A_S} \quad \sigma_{Sa} = (P+W) \frac{E_S}{E_A A_A + 2E_S A_S}$$

Solve for allowable load  $P$ :

$$P_{Aa} = \sigma_{Aa} \left( \frac{E_A A_A + 2E_S A_S}{E_A} \right) - W \quad P_{Sa} = \sigma_{Sa} \left( \frac{E_A A_A + 2E_S A_S}{E_S} \right) - W \quad (\text{lower value of } P \text{ controls})$$

$$P_{Aa} = 1713 \text{ N}$$

$$P_{Sa} = 1504 \text{ N} \quad \leftarrow P_{\text{allow}} \text{ is controlled by steel wires}$$

(b)  $P_{\text{allow}}$  IF LOAD  $P$  AT  $x = a/2$

Again, cut aluminum wire at top, then compute elongations of left and right steel wires:

$$\delta_{1L} = \left( \frac{3P}{4} + \frac{W}{2} \right) \left( \frac{L}{E_S A_S} \right) \quad \delta_{1R} = \left( \frac{P}{4} + \frac{W}{2} \right) \left( \frac{L}{E_S A_S} \right)$$

$$\delta_1 = \frac{\delta_{1L} + \delta_{1R}}{2} \quad \delta_1 = \frac{P+W}{2} \left( \frac{L}{E_S A_S} \right) \text{ where } \delta_1 = \text{displacement at } x = a$$

Use  $\delta_2$  from part (a):

$$\delta_2 = R_A \left( \frac{L}{2E_S A_S} + \frac{L}{E_A A_A} \right)$$

So equating  $\delta_1$  and  $\delta_2$ , solve for  $R_A$ :  $R_A = (P+W) \frac{E_A A_A}{E_A A_A + 2E_S A_S}$   
 $\wedge$  same as in part (a)

$$R_{SL} = \frac{3P}{4} + \frac{W}{2} - \frac{R_A}{2} < \text{stress in left steel wire exceeds that in right steel wire}$$

$$R_{SL} = \frac{3P}{4} + \frac{W}{2} - \frac{(P+W) \frac{E_A A_A}{E_A A_A + 2E_S A_S}}{2}$$

$$R_{SL} = \frac{PE_A A_A + 6PE_S A_S + 4WE_S A_S}{4E_A A_A + 8E_S A_S} \quad \sigma_{Sa} = \frac{PE_A A_A + 6PE_S A_S + 4WE_S A_S}{4E_A A_A + 8E_S A_S} \left( \frac{1}{A_S} \right)$$

Solve for  $P_{\text{allow}}$  based on allowable stresses in steel and aluminum:

$$P_{Sa} = \frac{\sigma_{Sa}(4A_S E_A A_A + 8E_S A_S^2) - (4WE_S A_S)}{E_A A_A + 6E_S A_S} \quad P_{Aa} = 1713 \text{ N} < \text{same as in part(a)}$$

$$P_{Sa} = 820 \text{ N} \quad \leftarrow \text{steel controls}$$

(c)  $P_{\text{allow}}$  IF WIRES ARE SWITCHED AS SHOWN AND  $x = a/2$

Select  $R_A$  as the redundant; statics on the two released structures:

(1) Cut aluminum wire—apply  $P$  and  $W$ , compute forces in left and right steel wires, then compute displacements at each steel wire:

$$R_{SL} = \frac{P}{2} \quad R_{SR} = \frac{P}{2} + W$$

$$\delta_{1L} = \frac{P}{2} \left( \frac{L}{E_S A_S} \right) \quad \delta_{1R} = \left( \frac{P}{2} + W \right) \left( \frac{L}{E_S A_S} \right)$$

By geometry,  $\delta$  at aluminum wire location at far right is  $\delta_1 = \left(\frac{P}{2} + 2W\right)\left(\frac{L}{E_S A_S}\right)$

(2) Next apply redundant  $R_A$  at right wire, compute wire force and displacement at aluminum wire:

$$R_{SL} = -R_A \quad R_{SR} = 2R_A \quad \delta_2 = R_A \left( \frac{5L}{E_S A_S} + \frac{L}{E_A A_A} \right)$$

(3) Compatibility equate  $\delta_1, \delta_2$  and solve for  $R_A$ , then  $P_{\text{allow}}$  for aluminum wire:

$$R_A = \frac{\left(\frac{P}{2} + 2W\right)\left(\frac{L}{E_S A_S}\right)}{\frac{5L}{E_S A_S} + \frac{L}{E_A A_A}} \quad R_A = \frac{E_A A_A P + 4E_A A_A W}{10E_A A_A + 2E_S A_S} \quad \sigma_{Aa} = \frac{R_A}{A_A}$$

$$\sigma_{Aa} = \frac{E_A P + 4E_A W}{10E_A A_A + 2E_S A_S}$$

$$P_{Aa} = \frac{\sigma_{Aa}(10E_A A_A + 2E_S A_S) - 4E_A W}{E_A} \quad P_{Aa} = 1713 \text{ N}$$

(4) Statics or superposition—find forces in steel wires, then  $P_{\text{allow}}$  for steel wires:

$$R_{SL} = \frac{P}{2} + R_A \quad R_{SL} = \frac{P}{2} + \frac{E_A A_A P + 4E_A A_A W}{10E_A A_A + 2E_S A_S}$$

$$R_{SL} = \frac{6E_A A_A P + P E_S A_S + 4E_A A_A W}{10E_A A_A + 2E_S A_S} \quad < \text{larger than } R_{SR}, \text{ so use in allowable stress calculations}$$

$$R_{SR} = \frac{P}{2} + W - 2R_A \quad R_{SR} = \frac{P}{2} + W - \frac{E_A A_A P + 4E_A A_A W}{5E_A A_A + E_S A_S}$$

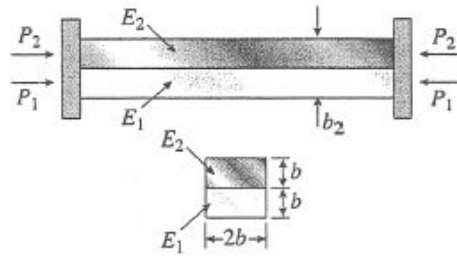
$$R_{SR} = \frac{3E_A A_A P + P E_S A_S + 2E_A A_A W + 2W E_S A_S}{10E_A A_A + 2E_S A_S}$$

$$\sigma_{Sa} = \frac{R_{SL}}{A_S} \quad P_{Sa} = \sigma_{Sa} A_S \left( \frac{10E_A A_A + 2E_S A_S}{6E_A A_A + E_S A_S} \right) - \frac{4E_A A_A W}{6E_A A_A + E_S A_S}$$

$$P_{Sa} = \frac{10\sigma_{Sa} A_S E_A A_A + 2\sigma_{Sa} A_S^2 E_S - 4E_A A_A W}{6E_A A_A + E_S A_S} \quad P_{Sa} = 703 \text{ N} \quad \leftarrow \text{^ steel controls}$$

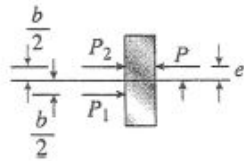


**Problem 2.4-17**



**FREE-BODY DIAGRAM**

(Plate at right-hand end)



**EQUATIONS OF EQUILIBRIUM**

$$\Sigma F = 0 \quad P_1 + P_2 = P \quad (\text{Eq. 1})$$

$$\Sigma M = 0 \quad \curvearrowright \quad Pe + P_1\left(\frac{b}{2}\right) - P_2\left(\frac{b}{2}\right) = 0 \quad (\text{Eq. 2})$$

**EQUATION OF COMPATIBILITY**

$$\delta_2 = \delta_1$$

$$\frac{P_2 L}{E_2 A} = \frac{P_1 L}{E_1 A} \quad \text{or} \quad \frac{P_2}{E_2} = \frac{P_1}{E_1} \quad (\text{Eq. 3})$$

**(a) AXIAL FORCES**

Solve simultaneously Eqs. (1) and (3):

$$P_1 = \frac{PE_1}{E_1 + E_2} \quad P_2 = \frac{PE_2}{E_1 + E_2} \quad \leftarrow$$

**(b) ECCENTRICITY OF LOAD  $P$**

Substitute  $P_1$  and  $P_2$  into Eq. (2) and solve for  $e$ :

$$e = \frac{b(E_2 - E_1)}{2(E_2 + E_1)} \quad \leftarrow$$

**(c) RATIO OF STRESSES**

$$\sigma_1 = \frac{P_1}{A} \quad \sigma_2 = \frac{P_2}{A} \quad \frac{\sigma_1}{\sigma_2} = \frac{P_1}{P_2} = \frac{E_1}{E_2} \quad \leftarrow$$



### Problem 2.4-18

#### NUMERICAL DATA

$$L = 2.5 \text{ m} \quad b = 0.71 \quad L = 1.775 \text{ m} \quad E = 210 \text{ GPa} \quad A = 3500 \text{ mm}^2 \quad P = 185 \text{ kN} \quad \theta_A = 60^\circ$$

$$\sigma_a = 150 \text{ MPa}$$

#### FIND MISSING DIMENSIONS AND ANGLES IN PLANE TRUSS FIGURE

$$x_c = b \cos(\theta_A) = 0.8875 \text{ m} \quad y_c = b \sin(\theta_A) = 1.5372 \text{ m}$$

$$\frac{b}{\sin(\theta_B)} = \frac{L}{\sin(\theta_A)} \quad \text{so} \quad \theta_B = a \sin\left(\frac{b \sin(\theta_A)}{L}\right) = 37.94306^\circ$$

$$\theta_C = 180^\circ - (\theta_A + \theta_B) = 82.05694^\circ$$

$$c = \frac{L}{\sin(\theta_A)} \sin(\theta_C) = 2.85906 \text{ m} \quad \text{or} \quad c = \sqrt{b^2 + L^2 - 2bL \cos(\theta_C)} = 2.85906 \text{ m}$$

- (a) SELECT  $B_x$  AS THE REDUNDANT; PERFORM SUPERPOSITION ANALYSIS TO FIND  $B_x$  THEN USE STATICS TO FIND REMAINING REACTIONS. FINALLY USE METHOD OF JOINTS TO FIND MEMBER FORCES (SEE EXAMPLE 1-1)

$\delta_{Bx1}$  = displacement in  $x$ -direction in released structure acted upon by loads  $P$  and  $2P$  at joint  $C$ :

$$\delta_{Bx1} = 1.2789911 \text{ mm} \quad < \text{this displacement equals force in } AB \text{ divided by flexibility of } AB$$

$$\delta_{Bx2} = \text{displacement in } x\text{-direction in released structure acted upon by redundant } B_x: \quad \delta_{Bx2} = B_x \frac{c}{EA}$$

$$\text{COMPATIBILITY EQUATION:} \quad \delta_{Bx1} + \delta_{Bx2} = 0 \quad \text{so} \quad B_x = \frac{-EA}{c} \delta_{Bx1} = -328.8 \text{ kN}$$

$$\text{STATICS:} \quad \Sigma F_x = 0 \quad A_x = -B_x - 2P = -41.2 \text{ kN}$$

$$\Sigma M_A = 0 \quad B_y = \frac{1}{c} [2P(b \sin(\theta_A)) + P(b \cos(\theta_A))] = 256.361 \text{ kN}$$

$$\Sigma F_y = 0 \quad A_y = P - B_y = -71.361 \text{ kN}$$

#### REACTIONS:

$$\boxed{A_x = -41.2 \text{ kN}} \quad \boxed{A_y = -71.4 \text{ kN}} \quad \boxed{B_x = -329 \text{ kN}} \quad \boxed{B_y = 256 \text{ kN}}$$

- (b) FIND MAXIMUM PERMISSIBLE VALUE OF LOAD VARIABLE  $P$  IF ALLOWABLE NORMAL STRESS IS 150 MPa

- (1) Use reactions and Method of Joints to find member forces in each member for above loading.

$$\text{Results: } F_{AB} = 0 \quad F_{BC} = -416.929 \text{ kN} \quad F_{AC} = 82.40 \text{ kN}$$

- (2) Compute member stresses:

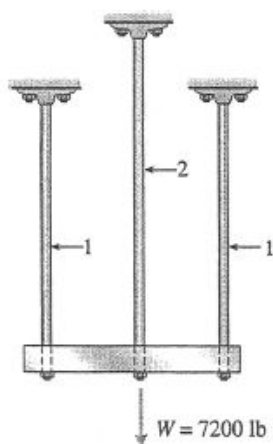
$$\sigma_{AB} = 0 \quad \sigma_{BC} = \frac{-416.93 \text{ kN}}{A} = -119.123 \text{ MPa} \quad \sigma_{AC} = \frac{82.4 \text{ kN}}{A} = 23.543 \text{ MPa}$$

- (3) Maximum stress occurs in member  $BC$ . For linear analysis, the stress is proportional to the load so

$$\boxed{P_{\max} = \frac{\sigma_a}{\sigma_{BC}} P = 233 \text{ kN}}$$

So when downward load  $P = 233 \text{ kN}$  is applied at  $C$  and horizontal load  $2P = 466 \text{ kN}$  is applied to the right at  $C$ , the stress in  $BC$  is 150 MPa

### Problem 2.4-19



BAR 1 ALUMINUM

$$E_1 = 10 \times 10^6 \text{ psi}$$

$$d_1 = 0.4 \text{ in.}$$

$$L_1 = 40 \text{ in.}$$

$$\sigma_1 = 24,000 \text{ psi}$$

BAR 2 MAGNESIUM

$$E_2 = 6.5 \times 10^6 \text{ psi}$$

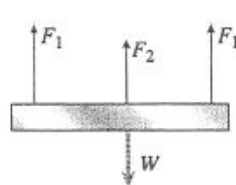
$$d_2 = ? \quad L_2 = ?$$

$$\sigma_2 = 13,000 \text{ psi}$$

$$W = 7200 \text{ lb}$$

FREE-BODY DIAGRAM OF RIGID BAR

EQUATION OF EQUILIBRIUM



$$\Sigma F_{\text{vert}} = 0$$

$$2F_1 + F_2 - W = 0 \quad (\text{Eq. 1})$$

FULLY STRESSED RODS

$$F_1 = \sigma_1 A_1 \quad F_2 = \sigma_2 A_2$$

$$A_1 = \frac{\pi d_1^2}{4} \quad A_2 = \frac{\pi d_2^2}{4}$$

Substitute into Eq. (1):

$$2\sigma_1 \left( \frac{\pi d_1^2}{4} \right) + \sigma_2 \left( \frac{\pi d_2^2}{4} \right) = W$$

Diameter  $d_1$  is known; solve for  $d_2$ :

$$d_2^2 = \frac{4W}{\pi\sigma_2} - \frac{2\sigma_1 d_1^2}{\sigma_2} \quad \leftarrow \quad (\text{Eq. 2})$$

SUBSTITUTE NUMERICAL VALUES:

$$\begin{aligned} d_2^2 &= \frac{4(7200 \text{ lb})}{\pi(13,000 \text{ psi})} - \frac{2(24,000 \text{ psi})(0.4 \text{ in.})^2}{13,000 \text{ psi}} \\ &= 0.70518 \text{ in.}^2 - 0.59077 \text{ in.}^2 = 0.11441 \text{ in.}^2 \\ d_2 &= 0.338 \text{ in.} \quad \leftarrow \end{aligned}$$

EQUATION OF COMPATIBILITY

$$\delta_1 = \delta_2 \quad (\text{Eq. 3})$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_1 = \frac{F_1 L_1}{E_1 A_1} = \sigma_1 \left( \frac{L_1}{E_1} \right) \quad (\text{Eq. 4})$$

$$\delta_2 = \frac{F_2 L_2}{E_2 A_2} = \sigma_2 \left( \frac{L_2}{E_2} \right) \quad (\text{Eq. 5})$$

Substitute (4) and (5) into Eq. (3):

$$\sigma_1 \left( \frac{L_1}{E_1} \right) = \sigma_2 \left( \frac{L_2}{E_2} \right)$$

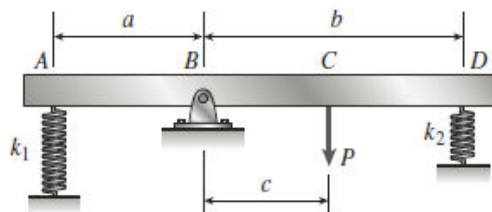
Length  $L_1$  is known; solve for  $L_2$ :

$$L_2 = L_1 \left( \frac{\sigma_1 E_2}{\sigma_2 E_1} \right) \quad \leftarrow \quad (\text{Eq. 6})$$

SUBSTITUTE NUMERICAL VALUES:

$$\begin{aligned} L_2 &= (40 \text{ in.}) \left( \frac{24,000 \text{ psi}}{13,000 \text{ psi}} \right) \left( \frac{6.5 \times 10^6 \text{ psi}}{10 \times 10^6 \text{ psi}} \right) \\ &= 48.0 \text{ in.} \end{aligned}$$

### Problem 2.4-20



#### NUMERICAL DATA

$$a = 250 \text{ mm}$$

$$b = 500 \text{ mm}$$

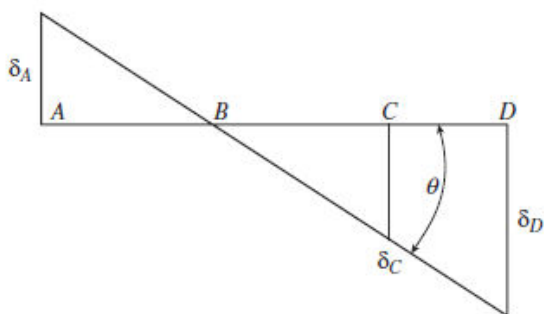
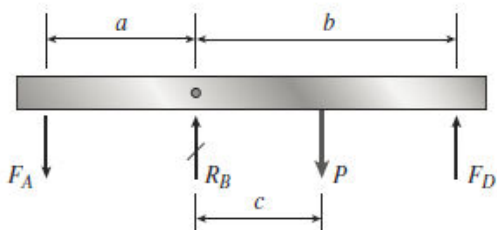
$$c = 200 \text{ mm}$$

$$k_1 = 10 \text{ kN/m}$$

$$k_2 = 25 \text{ kN/m}$$

$$\theta_{\max} = 3^\circ = \frac{\pi}{60} \text{ rad}$$

#### FREE-BODY DIAGRAM AND DISPLACEMENT DIAGRAM



#### EQUATION OF EQUILIBRIUM

$$\Sigma M_B = 0 + -F_A(a) - P(c) + F_D(b) = 0 \quad (\text{Eq. 1})$$

#### EQUATION OF COMPATIBILITY

$$\frac{\delta_A}{a} = \frac{\delta_D}{b} \quad (\text{Eq. 2})$$

#### FORCE-DISPLACEMENT RELATIONS

$$\delta_A = \frac{F_A}{k_1} \quad \delta_D = \frac{F_D}{k_2} \quad (\text{Eqs. 3, 4})$$

#### SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{F_A}{ak_1} = \frac{F_D}{bk_2} \quad (\text{Eq. 5})$$

SOLVE SIMULTANEOUSLY EQS. (1) AND (5):

$$F_A = \frac{ack_1P}{a^2k_1 + b^2k_2} \quad F_D = \frac{bck_2P}{a^2k_1 + b^2k_2}$$

#### ANGLE OF ROTATION

$$\delta_D = \frac{F_D}{k_2} = \frac{bcP}{a^2k_1 + b^2k_2} \quad \theta = \frac{\delta_D}{b} = \frac{cP}{a^2k_1 + b^2k_2}$$

#### MAXIMUM LOAD

$$P = \frac{\theta}{c}(a^2k_1 + b^2k_2)$$

$$P_{\max} = \frac{\theta_{\max}}{c}(a^2k_1 + b^2k_2) \quad \leftarrow$$

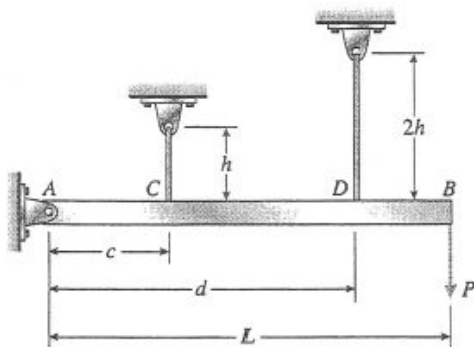
#### SUBSTITUTE NUMERICAL VALUES:

$$P_{\max} = \frac{\pi/60 \text{ rad}}{200 \text{ mm}}[(250 \text{ mm})^2(10 \text{ kN/m})$$

$$+ (500 \text{ mm})^2(25 \text{ kN/m})]$$

$$= 1800 \text{ N} \quad \leftarrow$$

# Problem 2.4-21



$$h = 18 \text{ in.}$$

$$2h = 36 \text{ in.}$$

$$c = 20 \text{ in.}$$

$$d = 50 \text{ in.}$$

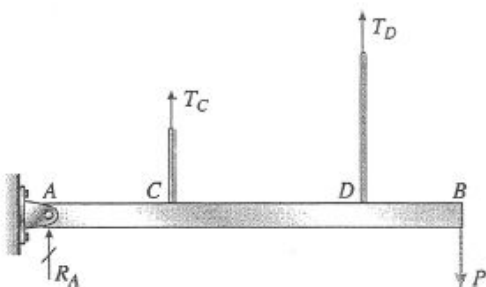
$$L = 66 \text{ in.}$$

$$E = 30 \times 10^6 \text{ psi}$$

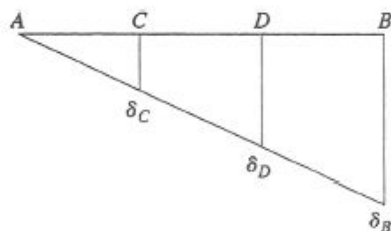
$$A = 0.0272 \text{ in.}^2$$

$$P = 340 \text{ lb}$$

## FREE-BODY DIAGRAM



## DISPLACEMENT DIAGRAM



## EQUATION OF EQUILIBRIUM

$$\sum M_A = 0 \quad \curvearrowright \quad T_C(c) + T_D(d) = PL \quad (\text{Eq. 1})$$

## EQUATION OF COMPATIBILITY

$$\frac{\delta_C}{c} = \frac{\delta_D}{d} \quad (\text{Eq. 2})$$

## FORCE-DISPLACEMENT RELATIONS

$$\delta_C = \frac{T_C h}{EA} \quad \delta_D = \frac{T_D (2h)}{EA} \quad (\text{Eqs. 3, 4})$$

## SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{T_C h}{cEA} = \frac{T_D (2h)}{dEA} \quad \text{or} \quad \frac{T_C}{c} = \frac{2T_D}{d} \quad (\text{Eq. 5})$$

## TENSILE FORCES IN THE WIRES

Solve simultaneously Eqs. (1) and (5):

$$T_C = \frac{2cPL}{2c^2 + d^2} \quad T_D = \frac{dPL}{2c^2 + d^2} \quad (\text{Eqs. 6, 7})$$

## TENSILE STRESSES IN THE WIRES

$$\sigma_C = \frac{T_C}{A} = \frac{2cPL}{A(2c^2 + d^2)} \quad (\text{Eq. 8})$$

$$\sigma_D = \frac{T_D}{A} = \frac{dPL}{A(2c^2 + d^2)} \quad (\text{Eq. 9})$$

## DISPLACEMENT AT END OF BAR

$$\delta_B = \delta_D \left( \frac{L}{d} \right) = \frac{2hT_D}{EA} \left( \frac{L}{d} \right) = \frac{2hPL^2}{EA(2c^2 + d^2)} \quad (\text{Eq. 10})$$

## SUBSTITUTE NUMERICAL VALUES

$$2c^2 + d^2 = 2(20 \text{ in.})^2 + (50 \text{ in.})^2 = 3300 \text{ in.}^2$$

$$\begin{aligned} \text{(a) } \sigma_C &= \frac{2cPL}{A(2c^2 + d^2)} = \frac{2(20 \text{ in.})(340 \text{ lb})(66 \text{ in.})}{(0.0272 \text{ in.}^2)(3300 \text{ in.}^2)} \\ &= 10,000 \text{ psi} \quad \leftarrow \end{aligned}$$

$$\begin{aligned} \sigma_D &= \frac{dPL}{A(2c^2 + d^2)} = \frac{(50 \text{ in.})(340 \text{ lb})(66 \text{ in.})}{(0.0272 \text{ in.}^2)(3300 \text{ in.}^2)} \\ &= 12,500 \text{ psi} \quad \leftarrow \end{aligned}$$

$$\begin{aligned} \text{(b) } \delta_B &= \frac{2hPL^2}{EA(2c^2 + d^2)} \\ &= \frac{2(18 \text{ in.})(340 \text{ lb})(66 \text{ in.})^2}{(30 \times 10^6 \text{ psi})(0.0272 \text{ in.}^2)(3300 \text{ in.}^2)} \\ &= 0.0198 \text{ in.} \quad \leftarrow \end{aligned}$$

### Problem 2.4-22

Remove pin at B; draw separate FBD's of beam and column. Find selected forces using statics

From FBD of column DBF

$$\Sigma M_B = D_x \cdot \frac{L}{2} = 0 \quad D_x = 0$$

$$\Sigma F_x = D_x - B_x = 0 \quad B_x = D_x$$

From FBD of beam ABC

$$\Sigma F_x = A_x + B_x = 0 \quad A_x = 0$$

$$\Sigma M_B = M_A - 2P \cdot \frac{L}{3} = 0 \quad M_A = 2P \cdot \frac{L}{3}$$

$$\Sigma F_y = B_y - 2P = 0 \quad B_y = 2P$$

Remove reaction  $R_F$  to create the release structure; find vertical displacement at F due to actual load  $2P$  at C

$$\delta_{F1} = \frac{B_y \cdot \frac{L}{2}}{EA} \quad \delta_{F1} = \frac{P \cdot L}{EA}$$

Apply redundant  $R_F$  to released structure; find vertical displacement at F

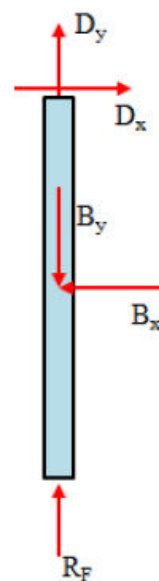
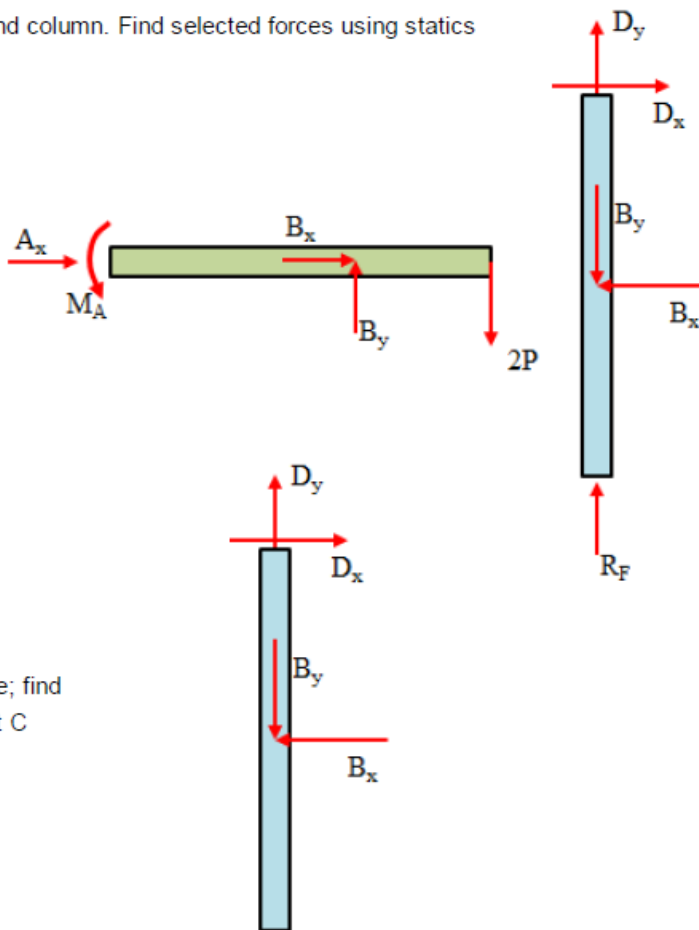
$$B_y = 0 \quad \delta'_{F2} = \frac{-R_F \cdot \frac{L}{2}}{2EA} - \frac{R_F \cdot \frac{L}{2}}{EA} \quad \delta'_{F2} = -R_F \cdot \left( \frac{L}{4 \cdot EA} + \frac{L}{2EA} \right) \quad \delta'_{F2} = -R_F \cdot \frac{3L}{4 \cdot EA}$$

Compatibility equation - solve for  $R_F$

$$\delta_{F1} + \delta'_{F2} = 0 \quad R_F = \frac{\frac{P \cdot L}{EA}}{\left( \frac{3L}{4 \cdot EA} \right)} \quad R_F = \frac{4}{3}P$$

Finally solve for reaction  $D_y$  using FBD of DBF

$$\Sigma F_y = 0 \quad D_y = B_y - R_F \quad D_y = 2P - \frac{4}{3}P \quad D_y = \frac{2}{3}P$$



### Problem 2.4-23

Numerical properties (kips, inches):

$$d_c = 2.25 \text{ in.} \quad d_b = 1.75 \text{ in.} \quad d_s = 1.25 \text{ in.}$$

$$A_s = \frac{\pi}{4} d_s^2$$

$$E_c = 18,000 \text{ ksi} \quad E_b = 16,000 \text{ ksi}$$

$$A_b = \frac{\pi}{4} (d_b^2 - d_s^2)$$

$$E_s = 30,000 \text{ ksi}$$

$$A_c = \frac{\pi}{4} (d_c^2 - d_b^2)$$

$$P = 9 \text{ k}$$

EQUATION OF EQUILIBRIUM

$$\sum F_{\text{vert}} = 0 \quad P_s + P_b + P_c = P \quad (\text{Eq. 1})$$

EQUATIONS OF COMPATIBILITY

$$\delta_s = \delta_b \quad \delta_c = \delta_s \quad (\text{Eqs. 2})$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_s = \frac{P_s L}{E_s A_s} \quad \delta_b = \frac{P_b L}{E_b A_b} \quad \delta_c = \frac{P_c L}{E_c A_c} \quad (\text{Eqs. 3, 4, 5})$$

SOLUTION OF EQUATIONS

Substitute (3), (4), and (5) into Eqs. (2):

$$P_b = P_s \frac{E_b A_b}{E_s A_s} \quad P_c = P_s \frac{E_c A_c}{E_s A_s} \quad (\text{Eqs. 6, 7})$$

SOLVE SIMULTANEOUSLY EQS. (1), (6), AND (7):

$$P_s = P \frac{E_s A_s}{E_s A_s + E_b A_b + E_c A_c} = 3.95 \text{ k}$$

$$P_b = P \frac{E_b A_b}{E_s A_s + E_b A_b + E_c A_c} = 2.02 \text{ k}$$

$$P_c = P \frac{E_c A_c}{E_s A_s + E_b A_b + E_c A_c} = 3.03 \text{ k}$$

$$P_s + P_b + P_c = 9 \quad \text{statics check}$$

COMPRESSIVE STRESSES

$$\text{Let } \sum EA = E_s A_s + E_b A_b + E_c A_c$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{P E_s}{\sum EA} \quad \sigma_s = 3.22 \text{ ksi} \quad \leftarrow$$

$$\sigma_b = \frac{P_b}{A_b} = \frac{P E_b}{\sum EA} \quad \sigma_b = 1.716 \text{ ksi} \quad \leftarrow$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{P E_c}{\sum EA} \quad \sigma_c = 1.93 \text{ ksi} \quad \leftarrow$$



### Problem 2.4-24

Remove  $R_F$  to create released structure; use superposition to find redundant  $R_F = y$ -dir reaction at F

Released structure under actual load; use FBD of ABC to find pin force  $B_y$

$$\Sigma M_A = 0 \quad B_y = \frac{1}{\frac{L}{3}} \cdot (3 \cdot P \cdot L) \quad B_y \rightarrow 9 \cdot P \quad \text{acts upward on ABC so acts downward on DBF}$$

Find vert. displ. of F in released structure under actual loads  $\delta_{F1} = \frac{-B_y \cdot L}{2 \cdot EA} \quad \delta_{F1} \rightarrow \frac{-9 \cdot L \cdot P}{2 \cdot EA} \quad \text{downward}$

Apply redundant  $R_F$  and find vertical displ. at F in released structure  $\delta_{F2} = R_F \left( \frac{\frac{L}{3}}{EA} + \frac{L}{2 \cdot EA} \right) \quad \delta_{F2} \rightarrow \frac{5 \cdot L \cdot R_F}{6 \cdot EA} \quad \text{upward}$

Compatibility equ.  $\delta_{F1} + \delta_{F2} = 0 \quad R_F = \frac{-\delta_{F1}}{\frac{5 \cdot L}{6 \cdot EA}} \quad R_F \rightarrow \frac{27 \cdot P}{5}$

Now use statics to find all remaining reactions FBD of DBF  $\Sigma M_B = 0 \quad \text{so} \quad D_x = 0$

Entire structure  $\Sigma F_x = 0 \quad A_x = 0 \quad \Sigma M_B = 0 \quad A_y = \frac{1}{\frac{L}{3}} \left[ -3 \cdot P \cdot \left( \frac{2 \cdot L}{3} \right) \right] \quad A_y \rightarrow -6 \cdot P$

$$\Sigma F_y = 0 \quad D_y = -R_F + 3 \cdot P - A_y \quad D_y \rightarrow \frac{18 \cdot P}{5}$$

**Problem 2.5-1**

The rails are prevented from expanding because of their great length and lack of expansion joints.

Therefore, each rail is in the same condition as a bar with fixed ends (see Example 2-9).

The compressive stress in the rails may be calculated as follows.

$$\Delta T = 120^{\circ}\text{F} - 60^{\circ}\text{F} = 60^{\circ}\text{F}$$

$$\sigma = E\alpha(\Delta T)$$

$$= (30 \times 10^6 \text{ psi})(6.5 \times 10^{-6}/^{\circ}\text{F})(60^{\circ}\text{F})$$

$$\sigma = 11,700 \text{ psi} \quad \leftarrow$$



### Problem 2.5-2

#### INITIAL CONDITIONS

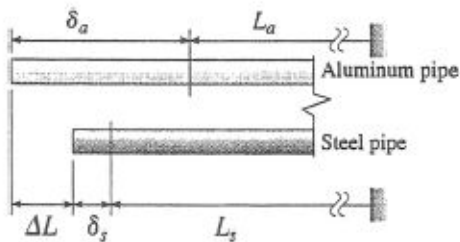
$$\begin{aligned} L_a &= 60 \text{ m} & T_0 &= 10^\circ\text{C} \\ L_s &= 60.005 \text{ m} & T_0 &= 10^\circ\text{C} \\ \alpha_a &= 23 \times 10^{-6}/^\circ\text{C} & \alpha_s &= 12 \times 10^{-6}/^\circ\text{C} \end{aligned}$$

#### FINAL CONDITIONS

Aluminum pipe is longer than the steel pipe by the amount  $\Delta L = 15 \text{ mm}$ .

$\Delta T$  = increase in temperature

$$\delta_a = \alpha_a(\Delta T)L_a \quad \delta_s = \alpha_s(\Delta T)L_s$$



From the figure above:

$$\delta_a + L_a = \Delta L + \delta_s + L_s$$

$$\text{or, } \alpha_a(\Delta T)L_a + L_a = \Delta L + \alpha_s(\Delta T)L_s + L_s$$

Solve for  $\Delta T$ :

$$\Delta T = \frac{\Delta L + (L_s - L_a)}{\alpha_a L_a - \alpha_s L_s} \quad \leftarrow$$

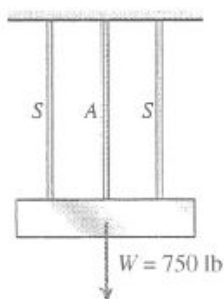
Substitute numerical values:

$$\alpha_a L_a - \alpha_s L_s = 659.9 \times 10^{-6} \text{ m}/^\circ\text{C}$$

$$\Delta T = \frac{15 \text{ mm} + 5 \text{ mm}}{659.9 \times 10^{-6} \text{ m}/^\circ\text{C}} = 30.31^\circ\text{C}$$

$$\begin{aligned} T &= T_0 + \Delta T = 10^\circ\text{C} + 30.31^\circ\text{C} \\ &= 40.3^\circ\text{C} \quad \leftarrow \end{aligned}$$

**Problem 2.5-3**



$S = \text{steel}$      $A = \text{aluminum}$

$W = 750 \text{ lb}$

$$d = \frac{1}{8} \text{ in.}$$

$$A_s = \frac{\pi d^2}{4} = 0.012272 \text{ in.}^2$$

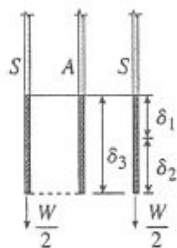
$$E_s = 30 \times 10^6 \text{ psi}$$

$$E_s A_s = 368,155 \text{ lb}$$

$$\alpha_s = 6.5 \times 10^{-6}/^\circ\text{F}$$

$$\alpha_a = 12 \times 10^{-6}/^\circ\text{F}$$

$L = \text{Initial length of wires}$



$\delta_1 = \text{increase in length of a steel wire due to temperature increase } \Delta T$

$$= \alpha_s (\Delta T) L$$

$\delta_2 = \text{increase in length of a steel wire due to load } W/2$

$$= \frac{WL}{2E_s A_s}$$

$\delta_3 = \text{increase in length of aluminum wire due to temperature increase } \Delta T$

$$= \alpha_a (\Delta T) L$$

For no load in the aluminum wire:

$$\delta_1 + \delta_2 = \delta_3$$

$$\alpha_s (\Delta T) L + \frac{WL}{2E_s A_s} = \alpha_a (\Delta T) L$$

or

$$\Delta T = \frac{W}{2E_s A_s (\alpha_a - \alpha_s)} \quad \leftarrow$$

Substitute numerical values:

$$\begin{aligned} \Delta T &= \frac{750 \text{ lb}}{(2)(368,155 \text{ lb})(5.5 \times 10^{-6}/^\circ\text{F})} \\ &= 185^\circ\text{F} \quad \leftarrow \end{aligned}$$

**NOTE:** If the temperature increase is larger than  $\Delta T$ , the aluminum wire would be in compression, which is not possible. Therefore, the steel wires continue to carry all of the load. If the temperature increase is less than  $\Delta T$ , the aluminum wire will be in tension and carry part of the load.

### Problem 2.5-4

#### NUMERICAL PROPERTIES

$$\begin{aligned} d_r &= 15 \text{ mm} & d_b &= 12 \text{ mm} & d_w &= 20 \text{ mm} & t_c &= 10 \text{ mm} & t_{\text{wall}} &= 18 \text{ mm} \\ \tau_b &= 45 \text{ MPa} & \alpha &= 12 (10^{-6}) & E &= 200 \text{ GPa} \end{aligned}$$

(a) TEMPERATURE DROP RESULTING IN BOLT SHEAR STRESS  $\varepsilon = \alpha \Delta T$   $\sigma = E \alpha \Delta T$

$$\text{Rod force} = P = (E \alpha \Delta T) \frac{\pi}{4} d_r^2 \quad \text{and bolt in double shear with shear stress} \quad \tau = \frac{P}{2 A_s} \quad \tau = \frac{P}{2 \frac{\pi}{4} d_b^2}$$

$$\tau_b = \frac{2}{\pi d_b^2} \left[ (E \alpha \Delta T) \frac{\pi}{4} d_r^2 \right] \quad \tau_b = \frac{E \alpha \Delta T}{2} \left( \frac{d_r}{d_b} \right)^2$$

$$\tau_b = 45 \text{ MPa}$$

$$\Delta T = \frac{2 \tau_b}{E (1000) \alpha} \left( \frac{d_b}{d_r} \right)^2 \quad \Delta T = 24^\circ \text{C} \quad P = (E \alpha \Delta T) \frac{\pi}{4} d_r^2 \quad P = 10 \text{ kN}$$

$$\sigma_{\text{rod}} = \frac{P}{\frac{\pi}{4} d_r^2} \quad \boxed{\sigma_{\text{rod}} = 57.6 \text{ MPa}}$$

(b) BEARING STRESSES

$$\text{BOLT AND CLEVIS} \quad \sigma_{bc} = \frac{P}{d_b t_c} \quad \boxed{\sigma_{bc} = 42.4 \text{ MPa}}$$

$$\text{WASHER AT WALL} \quad \sigma_{bw} = \frac{P}{\frac{\pi}{4} (d_w^2 - d_r^2)} \quad \boxed{\sigma_{bw} = 74.1 \text{ MPa}}$$

(c) If the connection to the wall at *B* is changed to an end plate with two bolts (see Fig. b), what is the required diameter  $d_b$  of each bolt if temperature drop  $\Delta T = 38^\circ \text{C}$  and the allowable bolt stress is 90 MPa?

Find force in rod due to temperature drop.

$$\Delta T = 38^\circ \text{C} \quad P = (E \alpha \Delta T) \frac{\pi}{4} d_r^2$$

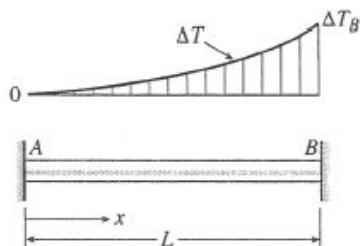
$$P = 200 \text{ GPa} \frac{\pi}{4} (15 \text{ mm})^2 [12 (10^{-6})] (38) = 16116 \text{ N} \quad P = 16.12 \text{ kN}$$

Each bolt carries one half of the force  $P$ :

$$d_b = \sqrt{\frac{\frac{16.12 \text{ kN}}{2}}{\frac{\pi}{4} (90 \text{ MPa})}} = 10.68 \text{ mm} \quad \boxed{d_b = 10.68 \text{ mm}}$$

### Problem 2.5-5

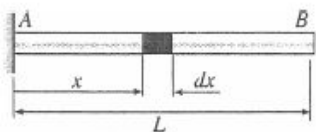
- (a) ONE DEGREE STATICALLY INDETERMINATE—USE SUPERPOSITION SELECT REACTION  $R_B$  AS THE REDUNDANT; FOLLOW PROCEDURE  
Bar with nonuniform temperature change.



At distance  $x$ :

$$\Delta T = \Delta T_B \left( \frac{x^3}{L^3} \right)$$

REMOVE THE SUPPORT AT THE END  $B$  OF THE BAR:



Consider an element  $dx$  at a distance  $x$  from end  $A$ .

$d\delta$  = Elongation of element  $dx$

$$d\delta = \alpha(\Delta T)dx = \alpha(\Delta T_B) \left( \frac{x^3}{L^3} \right) dx$$

$d\delta$  = elongation of bar

$$\delta = \int_0^L d\delta = \int_0^L \alpha(\Delta T_B) \left( \frac{x^3}{L^3} \right) dx = \frac{1}{4} \alpha(\Delta T_B) L$$

COMPRESSIVE FORCE  $P$  REQUIRED TO SHORTEN THE BAR BY THE AMOUNT  $\delta$

$$P = \frac{EA\delta}{L} = \frac{1}{4} EA\alpha(\Delta T_B)$$

COMPRESSIVE STRESS IN THE BAR

$$\sigma_c = \frac{P}{A} = \frac{E\alpha(\Delta T_B)}{4} \leftarrow$$

- (b) ONE DEGREE STATICALLY INDETERMINATE—USE SUPERPOSITION.

Select reaction  $R_B$  as the redundant then compute bar elongations due to  $\Delta T$  and due to  $R_B$

$$\delta_{B1} = \alpha\Delta T_B \frac{L}{4} \quad \text{due to temperature from above}$$

$$\delta_{B2} = R_B \left( \frac{1}{k} + \frac{L}{EA} \right)$$

Compatibility: solve for  $R_B$ :  $\delta_{B1} + \delta_{B2} = 0$

$$R_B = \frac{-\left( \alpha\Delta T_B \frac{L}{4} \right)}{\left( \frac{1}{k} + \frac{L}{EA} \right)}$$

$$R_B = -\alpha\Delta T_B \left[ \frac{EA}{4 \left( \frac{EA}{kL} + 1 \right)} \right]$$

So compressive stress in bar is

$$\sigma_c = \frac{R_B}{A} \quad \sigma_c = \frac{E\alpha(\Delta T_B)}{4 \left( \frac{EA}{kL} + 1 \right)} \leftarrow$$

NOTE:  $\sigma_c$  in part (b) is the same as in part (a) if spring constant  $k$  goes to infinity.

### Problem 2.5-6

$$A = 2 \cdot (1090 \text{ mm}^2) = 2180 \cdot \text{mm}^2 \quad k = 1750 \frac{\text{kN}}{\text{m}} \quad \Delta T = 45 \quad \alpha = 12 \cdot (10^{-6}) \quad L = 3 \text{ m} \quad E = 205 \text{ GPa}$$

Assume that beam and spring are stress free at the start, then apply temperature increase  $\Delta T$ . Select  $R_C$  as the redundant to remove to create the released structure

Apply  $\Delta T$  to beam in released structure  $\delta_{C1} = \alpha \cdot \Delta T \cdot L = 1.62 \cdot \text{mm}$

Apply redundant  $R_C$   $\delta_{C2} = R_C \cdot \left( \frac{L}{E \cdot A} + \frac{1}{k} \right) \quad \frac{L}{E \cdot A} + \frac{1}{k} = 0.578 \cdot \frac{\text{mm}}{\text{kN}}$

Compatibility equation and solution for redundant  $\delta_{C1} + \delta_{C2} = 0 \quad R_C = \frac{-(\alpha \cdot \Delta T \cdot L)}{\left( \frac{L}{E \cdot A} + \frac{1}{k} \right)} = -2.802 \cdot \text{kN}$

Axial normal compressive stress in beam  $\sigma_T = \frac{R_C}{A} = -1.285 \cdot \text{MPa}$

Displacement at B using superposition  $\delta_B = \frac{R_C \cdot L}{E \cdot A} + \alpha \cdot \Delta T \cdot L = 1.601 \cdot \text{mm} \quad \frac{R_C}{k} = -1.601 \cdot \text{mm}$   
elongation of beam is equal to shortening of spring

### Problem 2.5-7

$$E = 29000 \text{ ksi} \quad \alpha = 6.5 \cdot 10^{-6} \quad \Delta T = 20 \quad A = 8.24 \text{ in}^2 \quad L = 10 \text{ ft}$$

Select reaction  $R_B$  as the redundant; remove  $R_B$  to create released structure. Use superposition - apply  $\Delta T$  to released structure, then apply redundant. Solve compatibility equation to find  $R_B$  then use statics to get  $R_A$

$$\delta_{B1} = \alpha \cdot \Delta T \cdot L = 0.016 \text{ in} \quad \delta_{B2} = R_B \cdot \frac{L}{EA}$$

$$\text{Compatibility} \quad \delta_{B1} + \delta_{B2} = 0 \quad \text{solve for } R_B \quad R_B = \frac{-E \cdot A}{L} \cdot (\alpha \cdot \Delta T \cdot L) = -31.065 \cdot \text{kip} \quad \text{negative so } R_B \text{ acts to left}$$

$$\text{Statics} \quad R_A + R_B = 0 \quad \text{so} \quad R_A = -R_B = 31.065 \cdot \text{kip}$$

$$\text{Beam is in uniform axial compression due to temperature change; compressive normal stress is} \quad \sigma_T = \frac{R_B}{A} = -3.77 \cdot \text{ksi}$$

### Problem 2.5-8

#### NUMERICAL DATA

$$d_1 = 50 \text{ mm} \quad d_2 = 75 \text{ mm}$$

$$L_1 = 225 \text{ mm} \quad L_2 = 300 \text{ mm}$$

$$E = 6.0 \text{ GPa} \quad \alpha = 100 \times 10^{-6}/^\circ\text{C}$$

$$\Delta T = 30^\circ\text{C} \quad k = 50 \text{ MN/m}$$

- (a) COMPRESSIVE FORCE  $N$ , MAXIMUM COMPRESSIVE STRESS AND DISPLACEMENT OF PT.  $C$

$$A_1 = \frac{\pi}{4} d_1^2 \quad A_2 = \frac{\pi}{4} d_2^2$$

One-degree statically indeterminate—use  $R_B$  as redundant

$$\delta_{B1} = \alpha \Delta T (L_1 + L_2)$$

$$\delta_{B2} = R_B \left( \frac{L_1}{EA_1} + \frac{L_2}{EA_2} \right)$$

Compatibility:  $\delta_{B1} = \delta_{B2}$ , solve for  $R_B$

$$R_B = \frac{\alpha \Delta T (L_1 + L_2)}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2}} \quad N = R_B$$

$$N = 51.8 \text{ kN} \quad \leftarrow$$

Maximum compressive stress in  $AC$  since it has the smaller area ( $A_1 < A_2$ ):

$$\sigma_{c\max} = \frac{N}{A_1} \quad \sigma_{c\max} = 26.4 \text{ MPa}$$

Displacement  $\delta_C$  of point  $C$  = superposition of displacements in two released structures at  $C$ :

$$\delta_C = \alpha \Delta T (L_1) - R_B \frac{L_1}{EA_1}$$

$$\delta_C = -0.314 \text{ mm} \quad \leftarrow (-) \text{ sign means joint } C \text{ moves left}$$

- (b) COMPRESSIVE FORCE  $N$ , MAXIMUM COMPRESSIVE STRESS AND DISPLACEMENT OF PART  $C$  FOR ELASTIC SUPPORT CASE

Use  $R_B$  as redundant as in part (a):

$$\delta_{B1} = \alpha \Delta T (L_1 + L_2)$$

$$\delta_{B2} = R_B \left( \frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k} \right)$$

Now add effect of elastic support; equate  $\delta_{B1}$  and  $\delta_{B2}$  then solve for  $R_B$ :

$$R_B = \frac{\alpha \Delta T (L_1 + L_2)}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k}} \quad N = R_B$$

$$N = 31.2 \text{ kN} \quad \leftarrow$$

$$\sigma_{c\max} = \frac{N}{A_1} \quad \sigma_{c\max} = 15.91 \text{ MPa} \quad \leftarrow$$

Superposition:

$$\delta_C = \alpha \Delta T (L_1) - R_B \left( \frac{L_1}{EA_1} + \frac{1}{k} \right)$$

$$\delta_C = -0.546 \text{ mm} \quad \leftarrow (-) \text{ sign means joint } C \text{ moves left}$$



### Problem 2.5-9

$$\Delta T = 20 \quad \alpha = 13 \cdot 10^{-6} \quad E = 10400 \text{ ksi} \quad L = 3 \text{ ft} \quad b_1 = 2 \text{ in} \quad b_2 = 2.5 \text{ in} \quad t = \frac{1}{4} \text{ in}$$

Select reaction  $R_C$  as the redundant; remove redundant to create released structure; apply temperature increase and then apply redundant to get displacements at joint C in released structure.

$$\delta_{C1} = \alpha \cdot \Delta T \cdot L = 9.36 \times 10^{-3} \cdot \text{in}$$

$$\delta_{C2} = R_C \cdot \left[ \frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{2}}{E \cdot (b_1 \cdot t)} \right]$$

$$\frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{2}}{E \cdot (b_1 \cdot t)} = 6.551 \times 10^{-3} \cdot \frac{\text{in}}{\text{kip}}$$

Write compatibility equation then solve for  $R_C$

$$\delta_{C1} + \delta_{C2} = 0 \quad R_C = \frac{-(\alpha \cdot \Delta T \cdot L)}{\left[ \frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) + \frac{\frac{L}{2}}{E \cdot (b_1 \cdot t)} \right]} = -1.429 \cdot \text{kip}$$

$$\text{Statics} \quad R_A + R_C = 0 \quad R_A = -R_C = 1.429 \cdot \text{kip}$$

$$\text{Displacement at B using superposition} \quad \delta_B = -R_A \cdot \left[ \frac{\frac{L}{2}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln\left(\frac{b_2}{b_1}\right) \right] + \alpha \cdot \Delta T \cdot \frac{L}{2} = 2.656 \times 10^{-4} \cdot \text{in}$$

joint B moves to right

$$\text{OR} \quad \frac{R_C \cdot \frac{L}{2}}{E \cdot (b_1 \cdot t)} + \alpha \cdot \Delta T \cdot \frac{L}{2} = -2.656 \times 10^{-4} \cdot \text{in}$$

shortening of BC



### Problem 2.5-10

$$\Delta T = 30 \quad \alpha = 19 \cdot 10^{-6} \quad L = 2\text{m} \quad t = 20\text{mm} \quad b_1 = 100\text{mm} \quad b_2 = 115\text{mm} \quad E = 96\text{GPa}$$

Select reaction  $R_C$  as the redundant; remove redundant to create released structure; apply temperature increase and then apply redundant to get displacements at joint C in released structure.

$$\delta_{C1} = \alpha \cdot \Delta T \cdot L = 1.14 \cdot \text{mm} \quad \delta_{C2} = R_C \cdot \left[ \frac{\frac{3 \cdot L}{5} + \frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln \left( \frac{b_2}{b_1} \right) \right] \quad \frac{L}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln \left( \frac{b_2}{b_1} \right) = 9.706 \times 10^{-3} \cdot \frac{\text{mm}}{\text{kN}}$$

Write compatibility equation then solve for  $R_C$

$$\delta_{C1} + \delta_{C2} = 0 \quad R_C = \frac{-(\alpha \cdot \Delta T \cdot L)}{\left[ \frac{L}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln \left( \frac{b_2}{b_1} \right) \right]} = -117.457 \cdot \text{kN}$$

$$\text{Statics} \quad R_A + R_C = 0 \quad R_A = -R_C = 117.457 \cdot \text{kN}$$

$$\text{Displacement at B using superposition} \quad \delta_B = -R_A \cdot \left[ \frac{\frac{3 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln \left( \frac{b_2}{b_1} \right) \right] + \alpha \cdot \Delta T \cdot \frac{3 \cdot L}{5} = 0 \cdot \text{mm}$$

no elongation of AB

$$\text{OR} \quad R_C \cdot \left[ \frac{\frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln \left( \frac{b_2}{b_1} \right) \right] + \alpha \cdot \Delta T \cdot \frac{2 \cdot L}{5} = 0 \cdot \text{mm}$$

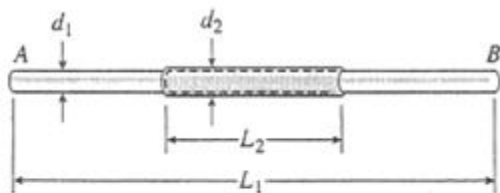
no shortening of BC

Extra - find displ. at  $x = 2L/5$   $b_{2L5} = b_2 - \frac{2}{3} \cdot (b_2 - b_1) \quad b_{2L5} \rightarrow 105 \cdot \text{mm}$

$$\delta_{2L5} = -R_A \cdot \left[ \frac{\frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_{2L5})} \cdot \ln \left( \frac{b_2}{b_{2L5}} \right) \right] + \alpha \cdot \Delta T \cdot \frac{2 \cdot L}{5} = 0.011 \cdot \text{mm}$$

$$\text{OR} \quad R_C \cdot \left[ \frac{\frac{2 \cdot L}{5}}{E \cdot t \cdot (b_2 - b_1)} \cdot \ln \left( \frac{b_2}{b_1} \right) + \frac{\frac{L}{5}}{E \cdot t \cdot (b_{2L5} - b_1)} \cdot \ln \left( \frac{b_{2L5}}{b_1} \right) \right] + \alpha \cdot \Delta T \cdot \frac{3 \cdot L}{5} = -0.011 \cdot \text{mm}$$

**Problem 2.5-11**



$$L_1 = 36 \text{ in.} \quad L_2 = 12 \text{ in.}$$

ELONGATION OF THE TWO OUTER PARTS OF THE BAR

$$\begin{aligned} \delta_1 &= \alpha_s(\Delta T)(L_1 - L_2) \\ &= (6.5 \times 10^{-6}/^\circ\text{F})(500^\circ\text{F})(36 \text{ in.} - 12 \text{ in.}) \\ &= 0.07800 \text{ in.} \end{aligned}$$

ELONGATION OF THE MIDDLE PART OF THE BAR

The steel rod and bronze sleeve lengthen the same amount, so they are in the same condition as the bolt and sleeve of Example 2-10. Thus, we can calculate the elongation from Eq. (2-21):

$$\delta_2 = \frac{(\alpha_s E_s A_s + \alpha_b E_b A_b)(\Delta T)L_2}{E_s A_s + E_b A_b}$$

SUBSTITUTE NUMERICAL VALUES

$$\alpha_s = 6.5 \times 10^{-6}/^\circ\text{F} \quad \alpha_b = 11 \times 10^{-6}/^\circ\text{F}$$

$$E_s = 30 \times 10^6 \text{ psi} \quad E_b = 15 \times 10^6 \text{ psi}$$

$$d_1 = 1.0 \text{ in.}$$

$$A_s = \frac{\pi}{4} d_1^2 = 0.78540 \text{ in.}^2$$

$$d_2 = 1.25 \text{ in.}$$

$$A_b = \frac{\pi}{4} (d_2^2 - d_1^2) = 0.44179 \text{ in.}^2$$

$$\Delta T = 500^\circ\text{F} \quad L_2 = 12.0 \text{ in.}$$

$$\delta_2 = 0.04493 \text{ in.}$$

TOTAL ELONGATION

$$\delta = \delta_1 + \delta_2 = 0.123 \text{ in.} \quad \leftarrow$$

**Problem 2.5-12**

$$\Delta T = 15 \quad \alpha_T = 23 \cdot (10^{-6}) \quad L = 1.8\text{m} \quad r = 36\text{mm} \quad E = 72\text{GPa} \quad a = \frac{r}{8} = 4.5\text{mm}$$

$$A_1 = \pi \cdot r^2 = 4071.504\text{mm}^2 \quad \text{Use formulas in **Appendix E, Case 15** for area of slotted segment}$$

$$\alpha = \arccos\left(\frac{a}{r}\right) = 1.445 \quad b = \sqrt{r^2 - a^2} = 35.718\text{mm} \quad A_2 = 2 \cdot r^2 \cdot \left(\alpha - \frac{a \cdot b}{r^2}\right) = 3425.196\text{mm}^2 \quad \frac{A_2}{A_1} = 0.841$$

Select reaction  $R_C$  as the redundant; remove redundant to create released structure; apply temperature increase and then apply redundant to get displacements at joint C in released structure.

$$\delta_{C1} = \alpha_T \Delta T \cdot L = 0.621\text{mm} \quad \delta_{C2} = R_C \cdot \left( \frac{2 \cdot \frac{L}{4}}{E \cdot A_1} + \frac{\frac{L}{2}}{E \cdot A_2} \right) \quad \frac{2 \cdot \frac{L}{4}}{E \cdot A_1} + \frac{\frac{L}{2}}{E \cdot A_2} = 6.72 \times 10^{-3} \frac{\text{mm}}{\text{kN}}$$

Write compatibility equation then solve for  $R_C$

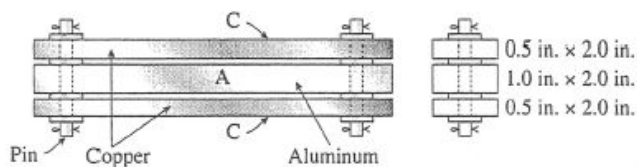
$$\delta_{C1} + \delta_{C2} = 0 \quad R_C = \frac{-(\alpha_T \Delta T \cdot L)}{\left( \frac{2 \cdot \frac{L}{4}}{E \cdot A_1} + \frac{\frac{L}{2}}{E \cdot A_2} \right)} = -92.417\text{kN}$$

$$\text{Statics} \quad R_A + R_C = 0 \quad R_A = -R_C = 92.417\text{kN}$$

$$\text{Thermal compressive stress in solid bar segments} \quad \sigma_{T1} = \frac{R_C}{A_1} = -22.698\text{MPa}$$

$$\text{and in slotted middle segment} \quad \sigma_{T2} = \frac{R_C}{A_2} = -26.982\text{MPa}$$

**Problem 2.5-13**



Diameter of pin:  $d_p = \frac{7}{16} \text{ in.} = 0.4375 \text{ in.}$

Area of pin:  $A_p = \frac{\pi}{4} d_p^2 = 0.15033 \text{ in.}^2$

Copper:  $E_c = 18,000 \text{ ksi}$   $\alpha_c = 9.5 \times 10^{-6}/^\circ\text{F}$

Aluminum:  $E_a = 10,000 \text{ ksi}$

$\alpha_a = 13 \times 10^{-6}/^\circ\text{F}$

Use the results of Example 2-10.

Find the forces  $P_a$  and  $P_c$  in the aluminum bar and copper bar, respectively, from Eq. (2-19).

Replace the subscript "S" in that equation by "a" (for aluminum) and replace the subscript "B" by "c" (for copper), because  $\alpha$  for aluminum is larger than  $\alpha$  for copper.

$$P_a = P_c = \frac{(\alpha_a - \alpha_c)(\Delta T)E_a A_a E_c A_c}{E_a A_a + E_c A_c}$$

Note that  $P_a$  is the compressive force in the aluminum bar and  $P_c$  is the combined tensile force in the two copper bars.

$$P_a = P_c = \frac{(\alpha_a - \alpha_c)(\Delta T)E_c A_c}{1 + \frac{E_c A_c}{E_a A_a}}$$

Area of two copper bars:  $A_c = 2.0 \text{ in.}^2$

Area of aluminum bar:  $A_a = 2.0 \text{ in.}^2$

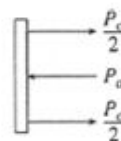
$\Delta T = 100^\circ\text{F}$

SUBSTITUTE NUMERICAL VALUES:

$$P_a = P_c = \frac{(3.5 \times 10^{-6}/^\circ\text{F})(100^\circ\text{F})(18,000 \text{ ksi})(2 \text{ in.}^2)}{1 + \left(\frac{18}{10}\right)\left(\frac{2.0}{2.0}\right)}$$

$$= 4,500 \text{ lb}$$

FREE-BODY DIAGRAM OF PIN AT THE LEFT END



$V$  = shear force in pin

$$= P_c/2$$

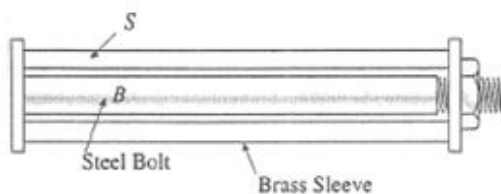
$$= 2,250 \text{ lb}$$

$\tau$  = average shear stress on cross section of pin

$$\tau = \frac{V}{A_p} = \frac{2,250 \text{ lb}}{0.15033 \text{ in.}^2}$$

$$\tau = 15.0 \text{ ksi} \quad \leftarrow$$

**Problem 2.5-14**



Subscript *S* means "sleeve".

Subscript *B* means "bolt".

Use the results of Example 2-10.

$\sigma_S$  = compressive force in sleeve

EQUATION (2-20a):

$$\sigma_S = \frac{(\alpha_S - \alpha_B)(\Delta T)E_S E_B A_B}{E_S A_S + E_B A_B} \text{ (Compression)}$$

SOLVE FOR  $\Delta T$ :

$$\Delta T = \frac{\sigma_S(E_S A_S + E_B A_B)}{(\alpha_S - \alpha_B)E_S E_B A_B}$$

or

$$\Delta T = \frac{\sigma_S}{E_S(\alpha_S - \alpha_B)} \left( 1 + \frac{E_S A_S}{E_B A_B} \right) \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\sigma_S = 25 \text{ MPa}$$

$$d_2 = 36 \text{ mm} \quad d_1 = 26 \text{ mm} \quad d_B = 25 \text{ mm}$$

$$E_S = 100 \text{ GPa} \quad E_B = 200 \text{ GPa}$$

$$\alpha_S = 21 \times 10^{-6}/^\circ\text{C} \quad \alpha_B = 10 \times 10^{-6}/^\circ\text{C}$$

$$A_S = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}(620 \text{ mm}^2)$$

$$A_B = \frac{\pi}{4}(d_B)^2 = \frac{\pi}{4}(625 \text{ mm}^2) \quad 1 + \frac{E_S A_S}{E_B A_B} = 1.496$$

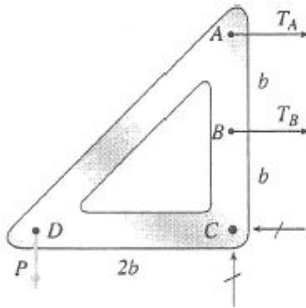
$$\Delta T = \frac{25 \text{ MPa} (1.496)}{(100 \text{ GPa})(11 \times 10^{-6}/^\circ\text{C})}$$

$$\Delta T = 34^\circ\text{C} \quad \leftarrow$$

(Increase in temperature)

### Problem 2.5-15

FREE-BODY DIAGRAM OF FRAME

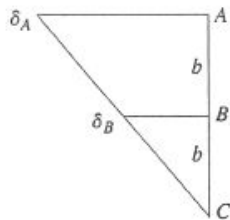


EQUATION OF EQUILIBRIUM

$$\sum M_C = 0 \quad \curvearrowright$$

$$P(2b) - T_A(2b) - T_B(b) = 0 \quad \text{or} \quad 2T_A + T_B = 2P \quad (\text{Eq. 1})$$

DISPLACEMENT DIAGRAM



EQUATION OF COMPATIBILITY

$$\delta_A = 2\delta_B \quad (\text{Eq. 2})$$

(a) LOAD  $P$  ONLY

Force-displacement relations:

$$\delta_A = \frac{T_A L}{EA} \quad \delta_B = \frac{T_B L}{EA} \quad (\text{Eq. 3, 4})$$

( $L$  = length of wires at  $A$  and  $B$ .)

Substitute (3) and (4) into Eq. (2):

$$\frac{T_A L}{EA} = \frac{2T_B L}{EA}$$

or  $T_A = 2T_B \quad (\text{Eq. 5})$

Solve simultaneously Eqs. (1) and (5):

$$T_A = \frac{4P}{5} \quad T_B = \frac{2P}{5} \quad (\text{Eqs. 6, 7})$$

Numerical values:

$$P = 500 \text{ lb}$$

$$\therefore T_A = 400 \text{ lb} \quad T_B = 200 \text{ lb} \quad \leftarrow$$

(b) LOAD  $P$  AND TEMPERATURE INCREASE  $\Delta T$

Force-displacement and temperature-displacement relations:

$$\delta_A = \frac{T_A L}{EA} + \alpha(\Delta T)L \quad (\text{Eq. 8})$$

$$\delta_B = \frac{T_B L}{EA} + \alpha(\Delta T)L \quad (\text{Eq. 9})$$

Substitute (8) and (9) into Eq. (2):

$$\frac{T_A L}{EA} + \alpha(\Delta T)L = \frac{2T_B L}{EA} + 2\alpha(\Delta T)L$$

or  $T_A - 2T_B = EA\alpha(\Delta T) \quad (\text{Eq. 10})$

Solve simultaneously Eqs. (1) and (10):

$$T_A = \frac{1}{5}[4P + EA\alpha(\Delta T)] \quad (\text{Eq. 11})$$

$$T_B = \frac{2}{5}[P - EA\alpha(\Delta T)] \quad (\text{Eq. 12})$$

Substitute numerical values:

$$P = 500 \text{ lb} \quad EA = 120,000 \text{ lb}$$

$$\Delta T = 180^\circ\text{F}$$

$$\alpha = 12.5 \times 10^{-6}/^\circ\text{F}$$

$$T_A = \frac{1}{5}(2000 \text{ lb} + 270 \text{ lb}) = 454 \text{ lb} \quad \leftarrow$$

$$T_B = \frac{2}{5}(500 \text{ lb} - 270 \text{ lb}) = 92 \text{ lb} \quad \leftarrow$$

(c) WIRE  $B$  BECOMES SLACK

Set  $T_B = 0$  in Eq. (12):

$$P = EA\alpha(\Delta T)$$

or

$$\Delta T = \frac{P}{EA\alpha} = \frac{500 \text{ lb}}{(120,000 \text{ lb})(12.5 \times 10^{-6}/^\circ\text{F})}$$

$$= 333.3^\circ\text{F}$$

Further increase in temperature:

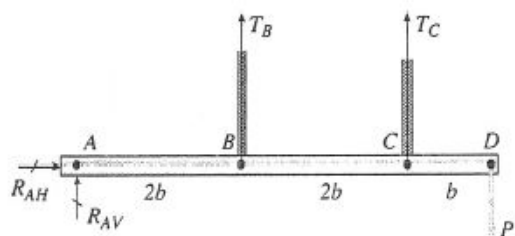
$$\Delta T = 333.3^\circ\text{F} - 180^\circ\text{F}$$

$$= 153^\circ\text{F} \quad \leftarrow$$



# Problem 2.5-16

## FREE-BODY DIAGRAM OF BAR *ABCD*



$T_B$  = force in cable *B*    $T_C$  = force in cable *C*

$d_B = 12 \text{ mm}$     $d_C = 20 \text{ mm}$

From Table 2-1:

$$A_B = 76.7 \text{ mm}^2 \quad E = 140 \text{ GPa}$$

$$\Delta T = 60^\circ\text{C} \quad A_C = 173 \text{ mm}^2$$

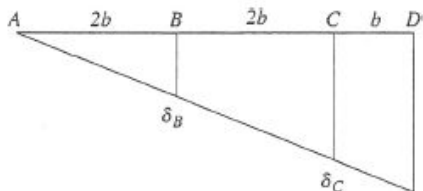
$$\alpha = 12 \times 10^{-6}/^\circ\text{C}$$

EQUATION OF EQUILIBRIUM

$$\Sigma M_A = 0 \quad T_B(2b) + T_C(4b) - P(5b) = 0$$

$$\text{or } 2T_B + 4T_C = 5P \quad (\text{Eq. 1})$$

## DISPLACEMENT DIAGRAM



COMPATIBILITY:

$$\delta_C = 2\delta_B \quad (\text{Eq. 2})$$

FORCE-DISPLACEMENT AND TEMPERATURE-DISPLACEMENT RELATIONS

$$\delta_B = \frac{T_B L}{EA_B} + \alpha(\Delta T)L \quad (\text{Eq. 3})$$

$$\delta_C = \frac{T_C L}{EA_C} + \alpha(\Delta T)L \quad (\text{Eq. 4})$$

SUBSTITUTE EQS. (3) AND (4) INTO EQ. (2):

$$\frac{T_C L}{EA_C} + \alpha(\Delta T)L = \frac{2T_B L}{EA_B} + 2\alpha(\Delta T)L$$

or

$$2T_B A_C - T_C A_B = -E\alpha(\Delta T)A_B A_C \quad (\text{Eq. 5})$$

SUBSTITUTE NUMERICAL VALUES INTO EQ. (5):

$$T_B(346) - T_C(76.7) = -1,338,000 \quad (\text{Eq. 6})$$

in which  $T_B$  and  $T_C$  have units of newtons.

SOLVE SIMULTANEOUSLY EQS. (1) AND (6):

$$T_B = 0.2494 P - 3,480 \quad (\text{Eq. 7})$$

$$T_C = 1.1253 P + 1,740 \quad (\text{Eq. 8})$$

in which  $P$  has units of newtons.

SOLVE EQS. (7) AND (8) FOR THE LOAD  $P$ :

$$P_B = 4.0096 T_B + 13,953 \quad (\text{Eq. 9})$$

$$P_C = 0.8887 T_C - 1,546 \quad (\text{Eq. 10})$$

ALLOWABLE LOADS

From Table 2-1:

$$(T_B)_{\text{ULT}} = 102,000 \text{ N} \quad (T_C)_{\text{ULT}} = 231,000 \text{ N}$$

Factor of safety = 5

$$(T_B)_{\text{allow}} = 20,400 \text{ N} \quad (T_C)_{\text{allow}} = 46,200 \text{ N}$$

$$\text{From Eq. (9): } P_B = (4.0096)(20,400 \text{ N}) + 13,953 \text{ N} = 95,700 \text{ N}$$

$$\text{From Eq. (10): } P_C = (0.8887)(46,200 \text{ N}) - 1,546 \text{ N} = 39,500 \text{ N}$$

Cable *C* governs.

$$P_{\text{allow}} = 39.5 \text{ kN} \quad \leftarrow$$

### Problem 2.5-17

Numerical data:

$$L = 25 \text{ in.} \quad d = 2 \text{ in.} \quad \delta = 0.008 \text{ in.}$$

$$k = 1.2 \times (10^6) \text{ lb/in.} \quad E = 16 \times (10^6) \text{ psi}$$

$$\alpha = 9.6 \times (10^{-6})/^\circ\text{F} \quad \Delta T = 50^\circ\text{F}$$

$$A = \frac{\pi}{4} d^2 \quad A = 3.14159 \text{ in.}^2$$

(a) ONE-DEGREE STATICALLY INDETERMINATE IF GAP CLOSSES

$$\Delta = \alpha \Delta T L \quad \Delta = 0.012 \text{ in.} \quad < \text{exceeds gap}$$

Select  $R_A$  as redundant and do superposition analysis:

$$\delta_{A1} = \Delta \quad \delta_{A2} = R_A \left( \frac{L}{EA} + \frac{1}{k} \right)$$

$$\text{Compatibility: } \delta_{A1} + \delta_{A2} = \delta \quad \delta_{A2} = \delta - \delta_{A1}$$

$$R_A = \frac{\delta - \Delta}{\frac{L}{EA} + \frac{1}{k}} \quad R_A = -3006 \text{ lb}$$

Compressive stress in bar:

$$\sigma = \frac{R_A}{A} \quad \sigma = -957 \text{ psi}$$

(b) FORCE IN SPRING  $F_k = R_C$

STATICS

$$R_A + R_C = 0$$

$$R_C = -R_A$$

$$R_C = 3006 \text{ lb} \quad \leftarrow$$

(c) FIND COMPRESSIVE STRESS IN BAR IF  $k$  GOES TO INFINITY  
FROM EXPRESSION FOR  $R_A$  ABOVE,  $1/k$  GOES TO ZERO

$$R_A = \frac{\delta - \Delta}{\frac{L}{EA}} \quad R_A = -8042 \text{ lb} \quad \sigma = \frac{R_A}{A}$$

$$\sigma = -2560 \text{ psi} \quad \leftarrow$$



**Problem 2.5-18**



Initial prestress:  $\sigma_1 = 42 \text{ MPa}$

Initial temperature:  $T_1 = 20^\circ\text{C}$

$E = 200 \text{ GPa}$

$\alpha = 14 \times 10^{-6}/^\circ\text{C}$

(a) STRESS  $\sigma$  WHEN TEMPERATURE DROPS TO  $0^\circ\text{C}$

$$T_2 = 0^\circ\text{C} \quad \Delta T = 20^\circ\text{C}$$

**NOTE:** Positive  $\Delta T$  means a *decrease* in temperature and an *increase* in the stress in the wire.

Negative  $\Delta T$  means an *increase* in temperature and a *decrease* in the stress.

Stress  $\sigma$  equals the initial stress  $\sigma_1$  plus the additional stress  $\sigma_2$  due to the temperature drop.

$$\sigma_2 = E\alpha(\Delta T)$$

$$\begin{aligned} \sigma &= \sigma_1 + \sigma_2 = \sigma_1 + E\alpha(\Delta T) \\ &= 42 \text{ MPa} + (200 \text{ GPa})(14 \times 10^{-6}/^\circ\text{C})(20^\circ\text{C}) \\ &= 42 \text{ MPa} + 56 \text{ MPa} = 98 \text{ MPa} \quad \leftarrow \end{aligned}$$

(b) TEMPERATURE WHEN STRESS EQUALS ZERO

$$\sigma = \sigma_1 + \sigma_2 = 0 \quad \sigma_1 + E\alpha(\Delta T) = 0$$

$$\Delta T = -\frac{\sigma_1}{E\alpha}$$

(Negative means increase in temp.)

$$\Delta T = -\frac{42 \text{ MPa}}{(200 \text{ GPa})(14 \times 10^{-6}/^\circ\text{C})} = -15^\circ\text{C}$$

$$T = 20^\circ\text{C} + 15^\circ\text{C} = 35^\circ\text{C} \quad \leftarrow$$

### Problem 2.5-19

$$n = 1.5 \quad p = \frac{1}{16} \text{ in} \quad A_s = 0.85 \text{ in}^2 \quad A_A = 4.5 \text{ in}^2$$

$$L = 20 \text{ in} \quad E_s = 29000 \text{ ksi} \quad E_A = 10600 \text{ ksi}$$

Select force in tube as the redundant. Cut through aluminum tube at right end to expose internal force  $F_A$  to create released structure. Apply  $n$  turns of tumbuckles to released structure to find relative displacement between ends of cut tube

$$\delta_1 = 2 \cdot n \cdot p = 0.187 \text{ in} \quad \text{Note that } n \text{ turns of a tumbuckle moves ends together by factor of two}$$

Now apply pair of internal forces  $F_T$  to ends of tube then again find relative displacement. Force  $F_A$  shortens both cables and elongates the tube.

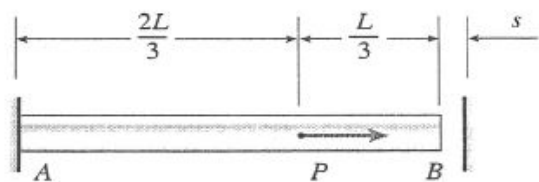
$$\delta_2 = F_A \cdot \left( \frac{L}{E_A \cdot A_A} + \frac{L}{2 \cdot E_s \cdot A_s} \right) \quad \frac{L}{E_A \cdot A_A} + \frac{L}{2 \cdot E_s \cdot A_s} = 8.25 \times 10^{-4} \cdot \frac{\text{in}}{\text{kip}}$$

Compatibility equation  $\delta_1 + \delta_2 = 0$  solve for  $F_A$   $F_A = \frac{-2 \cdot n \cdot p}{\frac{L}{E_A \cdot A_A} + \frac{L}{2 \cdot E_s \cdot A_s}} = -227.282 \cdot \text{kip}$

Statics - force in each cable =  $F_s$   $2 \cdot F_s + F_A = 0$   $F_s = \frac{-F_A}{2} = 113.641 \cdot \text{kip}$

Shortening of aluminum tube  $\delta_A = \frac{F_A \cdot L}{E_A \cdot A_A} = -0.0953 \cdot \text{in}$

**Problem 2.5-20**



$L$  = length of bar

$s$  = size of gap

$EA$  = axial rigidity

Reactions must be equal; find  $s$ .

COMPATIBILITY EQUATION

$$\delta_1 - \delta_2 = s \quad \text{or}$$

$$\frac{2PL}{3EA} - \frac{R_B L}{EA} = s$$

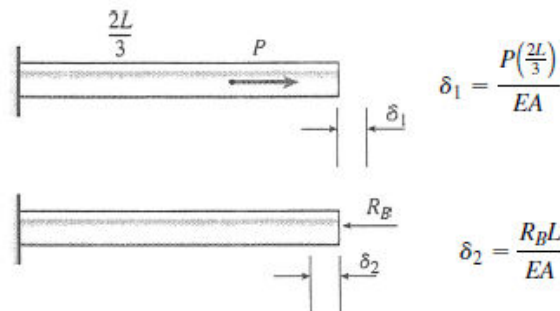
EQUILIBRIUM EQUATION

$R_A$  = reaction at end A (to the left)

$R_B$  = reaction at end B (to the left)

$$P = R_A + R_B$$

FORCE-DISPLACEMENT RELATIONS



$$\delta_1 = \frac{P(2L/3)}{EA}$$

$$\delta_2 = \frac{R_B L}{EA}$$

Reactions must be equal.

$$\therefore R_A = R_B \quad P = 2R_B \quad R_B = \frac{P}{2}$$

Substitute for  $R_B$  in Eq. (1):

$$\frac{2PL}{3EA} - \frac{PL}{2EA} = s \quad \text{or} \quad s = \frac{PL}{6EA} \quad \leftarrow$$

**NOTE:** The gap closes when the load reaches the value  $P/4$ . When the load reaches the value  $P$ , equal to  $6EA s/L$ , the reactions are equal ( $R_A = R_B = P/2$ ). When the load is between  $P/4$  and  $P$ ,  $R_A$  is greater than  $R_B$ . If the load exceeds  $P$ ,  $R_B$  is greater than  $R_A$ .

### Problem 2.5-21

- (a) FIND REACTIONS AT  $A$  AND  $B$  FOR APPLIED FORCE  $P_1$   
First compute  $P_1$ , required to close gap:

$$P_1 = \frac{E_1 A_1}{L_1} s \quad P_1 = 231.4 \text{ k} \quad \leftarrow$$

Statically indeterminate analysis with  $R_B$  as the redundant:

$$\delta_{B1} = -s \quad \delta_{B2} = R_B \left( \frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} \right)$$

$$\text{Compatibility: } \delta_{B1} + \delta_{B2} = 0$$

$$R_B = \frac{s}{\left( \frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} \right)} \quad R_B = 55.2 \text{ k} \quad \leftarrow$$

$$R_A = -R_B \quad \leftarrow$$

- (b) FIND REACTIONS AT  $A$  AND  $B$  FOR APPLIED FORCE  $P_2$

$$P_2 = \frac{E_2 A_2}{\frac{L_2}{2}} s \quad P_2 = 145.1 \text{ k} \quad \leftarrow$$

Analysis after removing  $P_2$  is same as in part (a), so reaction forces are the same

- (c) MAXIMUM SHEAR STRESS IN PIPE 1 OR 2 WHEN EITHER  $P_1$  OR  $P_2$

$$\text{IS APPLIED } \tau_{\max a} = \frac{P_1}{A_1} \quad \tau_{\max a} = 13.39 \text{ ksi} \quad \leftarrow$$

$$\tau_{\max b} = \frac{P_2}{A_2} \quad \tau_{\max b} = 19.44 \text{ ksi} \quad \leftarrow$$

- (d) REQUIRED  $\Delta T$  AND REACTIONS AT  $A$  AND  $B$

$$\Delta T_{\text{reqd}} = \frac{s}{\alpha_1 L_1 + \alpha_2 L_2} \quad \Delta T_{\text{reqd}} = 65.8^\circ \text{F} \quad \leftarrow$$

If pin is inserted but temperature remains at  $\Delta T$  above ambient temperature, reactions are zero.

- (e) IF TEMPERATURE RETURNS TO ORIGINAL AMBIENT TEMPERATURE, FIND REACTIONS AT  $A$  AND  $B$

statically indeterminate analysis with  $R_B$  as the redundant Compatibility:  $\delta_{B1} + \delta_{B2} = 0$

Analysis is the same as in parts (a) and (b) above since gap  $s$  is the same, so reactions are the same.

### Problem 2.5-22

With gap  $s$  closed due to  $\Delta T$ , structure is one-degree statically-indeterminate; select internal force ( $Q$ ) at juncture of bar and spring as the redundant. Use superposition of two released structures in the solution.

$\delta_{\text{rel1}}$  = relative displacement between end of bar at  $C$  and end of spring due to  $\Delta T$

$\delta_{\text{rel1}} = \alpha \Delta T (L_1 + L_2)$   
 $\delta_{\text{rel1}}$  is greater than gap length  $s$

$\delta_{\text{rel2}}$  = relative displacement between ends of bar and spring due to pair of forces  $Q$ , one on end of bar at  $C$  and the other on end of spring

$$\delta_{\text{rel2}} = Q \left( \frac{L_1}{EA_1} + \frac{L_2}{EA_2} \right) + \frac{Q}{k_3}$$

$$\delta_{\text{rel2}} = Q \left( \frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3} \right)$$

Compatibility:  $\delta_{\text{rel1}} + \delta_{\text{rel2}} = s$      $\delta_{\text{rel2}} = s - \delta_{\text{rel1}}$

$$\delta_{\text{rel2}} = s - \alpha \Delta T (L_1 + L_2)$$

$$Q = \frac{s - \alpha \Delta T (L_1 + L_2)}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}}$$

$$Q = \frac{EA_1 A_2 k_3}{L_1 A_2 k_3 + L_2 A_1 k_3 + EA_1 A_2} [s - \alpha \Delta T (L_1 + L_2)]$$

(a) REACTIONS AT  $A$  AND  $D$

Statics:  $R_A = -Q$      $R_D = Q$

$$R_A = \frac{-s + \alpha \Delta T (L_1 + L_2)}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}} \quad \leftarrow$$

$$R_D = -R_A \quad \leftarrow$$

(b) DISPLACEMENTS AT  $B$  AND  $C$

Use superposition of displacements in the two released structures:

$$\delta_B = \alpha \Delta T (L_1) - R_A \left( \frac{L_1}{EA_1} \right) \quad \leftarrow$$

$$\delta_B = \alpha \Delta T (L_1) - \frac{[-s + \alpha \Delta T (L_1 + L_2)]}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}} \left( \frac{L_1}{EA_1} \right)$$

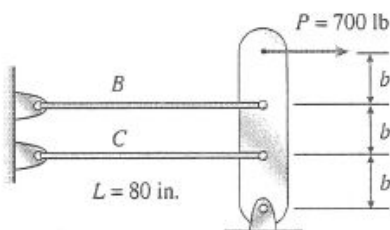
$$\delta_C = \alpha \Delta T (L_1 + L_2) -$$

$$R_A \left( \frac{L_1}{EA_1} + \frac{L_2}{EA_2} \right) \quad \leftarrow$$

$$\delta_C = \alpha \Delta T (L_1 + L_2) -$$

$$\frac{[-s + \alpha \Delta T (L_1 + L_2)]}{\frac{L_1}{EA_1} + \frac{L_2}{EA_2} + \frac{1}{k_3}} \left( \frac{L_1}{EA_1} + \frac{L_2}{EA_2} \right)$$

**Problem 2.5-23**



$$P = 700 \text{ lb}$$

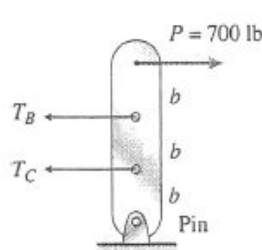
$$A = 0.03 \text{ in.}^2$$

$$E = 30 \times 10^6 \text{ psi}$$

$$L_B = 79.98 \text{ in.}$$

$$L_C = 79.95 \text{ in.}$$

**EQUILIBRIUM EQUATION**



$$\sum M_{\text{pin}} = 0 \quad \leftarrow$$

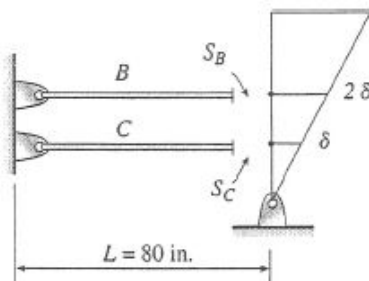
$$T_C(b) + T_B(2b) = P(3b)$$

$$2T_B + T_C = 3P \quad (\text{Eq. 1})$$

**DISPLACEMENT DIAGRAM**

$$S_B = 80 \text{ in.} - L_B = 0.02 \text{ in.}$$

$$S_C = 80 \text{ in.} - L_C = 0.05 \text{ in.}$$



Elongation of wires:

$$\delta_B = S_B + 2\delta \quad (\text{Eq. 2})$$

$$\delta_C = S_C + \delta \quad (\text{Eq. 3})$$

**FORCE-DISPLACEMENT RELATIONS**

$$\delta_B = \frac{T_B L}{EA} \quad \delta_C = \frac{T_C L}{EA} \quad (\text{Eqs. 4, 5})$$

**SOLUTION OF EQUATIONS**

Combine Eqs. (2) and (4):

$$\frac{T_B L}{EA} = S_B + 2\delta \quad (\text{Eq. 6})$$

Combine Eqs. (3) and (5):

$$\frac{T_C L}{EA} = S_C + \delta \quad (\text{Eq. 7})$$

Eliminate  $\delta$  between Eqs. (6) and (7):

$$T_B - 2T_C = \frac{EAS_B}{L} - \frac{2EAS_C}{L} \quad (\text{Eq. 8})$$

Solve simultaneously Eqs. (1) and (8):

$$T_B = \frac{6P}{5} + \frac{EAS_B}{5L} - \frac{2EAS_C}{5L} \quad \leftarrow$$

$$T_C = \frac{3P}{5} - \frac{2EAS_B}{5L} + \frac{4EAS_C}{5L} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

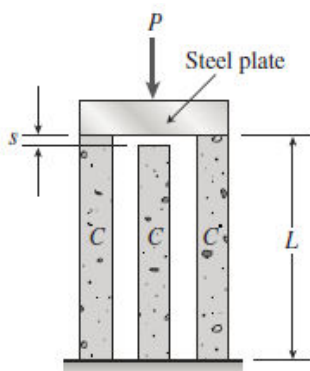
$$\frac{EA}{5L} = 2250 \text{ lb/in.}$$

$$T_B = 840 \text{ lb} + 45 \text{ lb} - 225 \text{ lb} = 660 \text{ lb} \quad \leftarrow$$

$$T_C = 420 \text{ lb} - 90 \text{ lb} + 450 \text{ lb} = 780 \text{ lb} \quad \leftarrow$$

(Both forces are positive, which means tension, as required for wires.)

**Problem 2.5-24**



$s$  = size of gap = 1.0 mm

$L$  = length of posts = 2.0 m

$A$  = 40,000 mm<sup>2</sup>

$\sigma_{\text{allow}}$  = 20 MPa

$E$  = 30 GPa

$C$  = concrete post

DOES THE GAP CLOSE?

Stress in the two outer posts when the gap is just closed:

$$\sigma = E\varepsilon = E\left(\frac{s}{L}\right) = (30 \text{ GPa})\left(\frac{1.0 \text{ mm}}{2.0 \text{ m}}\right)$$

$$= 15 \text{ MPa}$$

Since this stress is less than the allowable stress, the allowable force  $P$  will close the gap.

EQUILIBRIUM EQUATION

$$2P_1 + P_2 = P \quad (\text{Eq. 1})$$

COMPATIBILITY EQUATION

$\delta_1$  = shortening of outer posts

$\delta_2$  = shortening of inner post

$$\delta_1 = \delta_2 + s \quad (\text{Eq. 2})$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_1 = \frac{P_1 L}{EA} \quad \delta_2 = \frac{P_2 L}{EA} \quad (\text{Eqs. 3, 4})$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_1 L}{EA} = \frac{P_2 L}{EA} + s \quad \text{or} \quad P_1 - P_2 = \frac{EAs}{L} \quad (\text{Eq. 5})$$

Solve simultaneously Eqs. (1) and (5):

$$P = 3P_1 - \frac{EAs}{L}$$

By inspection, we know that  $P_1$  is larger than  $P_2$ .

Therefore,  $P_1$  will control and will be equal to  $\sigma_{\text{allow}} A$ .

$$P_{\text{allow}} = 3\sigma_{\text{allow}} A - \frac{EAs}{L}$$

$$= 2400 \text{ kN} - 600 \text{ kN} = 1800 \text{ kN}$$

$$= 1.8 \text{ MN} \quad \leftarrow$$



### Problem 2.5-25

The figure shows a section through the pipe, cap and rod

NUMERICAL PROPERTIES

$$L_{ci} = 48 \text{ in.} \quad E_s = 30000 \text{ ksi} \quad E_b = 14,000 \text{ ksi}$$

$$E_c = 12,000 \text{ ksi} \quad t_c = 1 \text{ in.} \quad p = 52 \times (10^{-3}) \text{ in.} \quad n = \frac{1}{4}$$

$$d_w = \frac{3}{4} \text{ in.} \quad d_r = \frac{1}{2} \text{ in.} \quad d_o = 6 \text{ in.} \quad d_i = 5.625 \text{ in.}$$

#### (a) FORCES AND STRESSES IN PIPE AND ROD

One degree statically indeterminate—cut rod at cap and use force in rod ( $Q$ ) as the redundant:

$\delta_{rel1}$  = relative displacement between cut ends of rod due to 1/4 turn of nut

$$\delta_{rel1} = -np \quad \text{Ends of rod move apart, not together, so this is } (-).$$

$\delta_{rel2}$  = relative displacement between cut ends of rod due pair of forces  $Q$

$$\delta_{rel2} = Q \left( \frac{L + 2t_c}{E_b A_{rod}} + \frac{L_{ci}}{E_c A_{pipe}} \right)$$

$$A_{rod} = \frac{\pi}{4} d_r^2 \quad A_{pipe} = \frac{\pi}{4} (d_o^2 - d_i^2)$$

$$A_{rod} = 0.196 \text{ in.}^2 \quad A_{pipe} = 3.424 \text{ in.}^2$$

$$\text{Compatibility equation: } \delta_{rel1} + \delta_{rel2} = 0$$

$$Q = \frac{np}{\frac{L_{ci} + 2t_c}{E_b A_{rod}} + \frac{L_{ci}}{E_c A_{pipe}}}$$

$$Q = 0.672 \text{ k} \quad F_{rod} = Q$$

$$\text{Statics: } F_{pipe} = -Q$$

$$\text{Stresses: } \sigma_c = \frac{F_{pipe}}{A_{pipe}} \quad \sigma_c = -0.196 \text{ ksi} \quad \leftarrow$$

$$\sigma_b = \frac{F_{rod}}{A_{rod}} \quad \sigma_b = 3.42 \text{ ksi} \quad \leftarrow$$

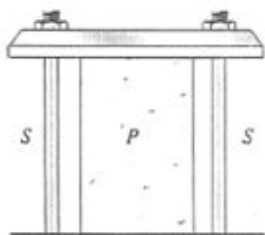
#### (b) BEARING AND SHEAR STRESSES IN STEEL CAP

$$\sigma_b = \frac{F_{rod}}{\frac{\pi}{4} (d_w^2 - d_r^2)} \quad \sigma_b = 2.74 \text{ ksi} \quad \leftarrow$$

$$\tau_c = \frac{F_{rod}}{\pi d_w t_c} \quad \tau_c = 0.285 \text{ ksi} \quad \leftarrow$$



**Problem 2.5-26**



$$L = 200 \text{ mm}$$

$$P = 1.0 \text{ mm}$$

$$E_s = 200 \text{ GPa}$$

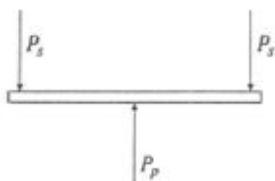
$$A_s = 36.0 \text{ mm}^2 \text{ (for one bolt)}$$

$$E_p = 7.5 \text{ GPa}$$

$$A_p = 960 \text{ mm}^2$$

$$n = 1 \text{ (See Eq. 2-22)}$$

EQUILIBRIUM EQUATION

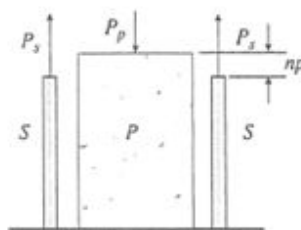


$P_s$  = tensile force in one steel bolt

$P_p$  = compressive force in plastic cylinder

$$P_p = 2P_s \quad (\text{Eq. 1})$$

COMPATIBILITY EQUATION



$\delta_s$  = elongation of steel bolt

$\delta_p$  = shortening of plastic cylinder

$$\delta_s + \delta_p = np \quad (\text{Eq. 2})$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_s = \frac{P_s L}{E_s A_s} \quad \delta_p = \frac{P_p L}{E_p A_p} \quad (\text{Eq. 3, Eq. 4})$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_s L}{E_s A_s} + \frac{P_p L}{E_p A_p} = np \quad (\text{Eq. 5})$$

Solve simultaneously Eqs. (1) and (5):

$$P_p = \frac{2npE_s A_s E_p A_p}{L(E_p A_p + 2E_s A_s)}$$

STRESS IN THE PLASTIC CYLINDER

$$\sigma_p = \frac{P_p}{A_p} = \frac{2np E_s A_s E_p}{L(E_p A_p + 2E_s A_s)}$$

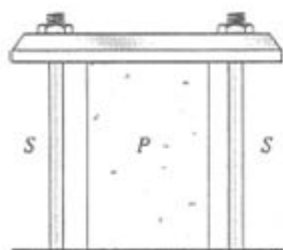
SUBSTITUTE NUMERICAL VALUES:

$$N = E_s A_s E_p = 54.0 \times 10^{15} \text{ N}^2/\text{m}^2$$

$$D = E_p A_p + 2E_s A_s = 21.6 \times 10^6 \text{ N}$$

$$\begin{aligned} \sigma_p &= \frac{2np \left( \frac{N}{D} \right)}{L \left( \frac{N}{D} \right)} = \frac{2(1)(1.0 \text{ mm}) \left( \frac{N}{D} \right)}{200 \text{ mm} \left( \frac{N}{D} \right)} \\ &= 25.0 \text{ MPa} \quad \leftarrow \end{aligned}$$

### Problem 2.5-27



$$\begin{aligned} L &= 10 \text{ in.} \\ p &= 0.058 \text{ in.} \\ E_s &= 30 \times 10^6 \text{ psi} \end{aligned}$$

$$A_s = 0.06 \text{ in.}^2 \text{ (for one bolt)}$$

$$E_p = 500 \text{ ksi}$$

$$A_p = 1.5 \text{ in.}^2$$

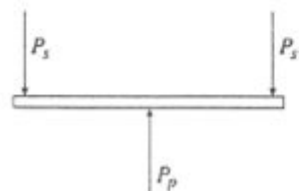
$$n = 1 \text{ (see Eq. 2-22)}$$

EQUILIBRIUM EQUATION

$P_s$  = tensile force in one steel bolt

$P_p$  = compressive force in plastic cylinder

$$P_p = 2P_s \quad (\text{Eq. 1})$$

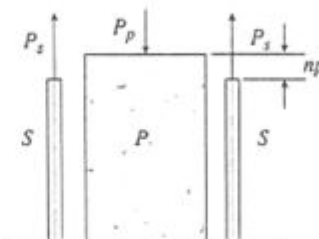


COMPATIBILITY EQUATION

$\delta_s$  = elongation of steel bolt

$\delta_p$  = shortening of plastic cylinder

$$\delta_s + \delta_p = np \quad (\text{Eq. 2})$$



FORCE-DISPLACEMENT RELATIONS

$$\delta_s = \frac{P_s L}{E_s A_s} \quad \delta_p = \frac{P_p L}{E_p A_p} \quad (\text{Eq. 3, Eq. 4})$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_s L}{E_s A_s} + \frac{P_p L}{E_p A_p} = np \quad (\text{Eq. 5})$$

Solve simultaneously Eqs. (1) and (5):

$$P_p = \frac{2npE_sA_sE_pA_p}{L(E_pA_p + 2E_sA_s)}$$

STRESS IN THE PLASTIC CYLINDER

$$\sigma_p = \frac{P_p}{A_p} = \frac{2npE_sA_sE_p}{L(E_pA_p + 2E_sA_s)} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$N = E_sA_sE_p = 900 \times 10^9 \text{ lb}^2/\text{in.}^2$$

$$D = E_pA_p + 2E_sA_s = 4350 \times 10^3 \text{ lb}$$

$$\begin{aligned} \sigma_p &= \frac{2np}{L} \left( \frac{N}{D} \right) = \frac{2(1)(0.058 \text{ in.})}{10 \text{ in.}} \left( \frac{N}{D} \right) \\ &= 2400 \text{ psi} \quad \leftarrow \end{aligned}$$

### Problem 2.5-28

The figure shows a section through the sleeve, cap, and bolt.

NUMERICAL PROPERTIES

$$n = \frac{1}{2} \quad p = 1.0 \text{ mm} \quad \Delta T = 30^\circ\text{C}$$

$$E_c = 120 \text{ GPa} \quad \alpha_c = 17 \times (10^{-6})/^\circ\text{C}$$

$$E_s = 200 \text{ GPa} \quad \alpha_s = 12 \times (10^{-6})/^\circ\text{C}$$

$$\tau_{aj} = 18.5 \text{ MPa} \quad s = 26 \text{ mm} \quad d_b = 5 \text{ mm}$$

$$L_1 = 40 \text{ mm} \quad t_1 = 4 \text{ mm} \quad L_2 = 50 \text{ mm} \quad t_2 = 3 \text{ mm}$$

$$d_1 = 25 \text{ mm} \quad d_1 - 2t_1 = 17 \text{ mm} \quad d_2 = 17 \text{ mm}$$

$$A_b = \frac{\pi}{4} d_b^2 \quad A_1 = \frac{\pi}{4} [d_1^2 - (d_1 - 2t_1)^2]$$

$$A_b = 19.635 \text{ mm}^2 \quad A_1 = 263.894 \text{ mm}^2$$

$$A_2 = \frac{\pi}{4} [d_2^2 - (d_2 - 2t_2)^2] \quad A_2 = 131.947 \text{ mm}^2$$

(a) FORCES IN SLEEVE AND BOLT

One-degree statically indeterminate—cut bolt and use force in bolt ( $P_B$ ) as redundant (see sketches):

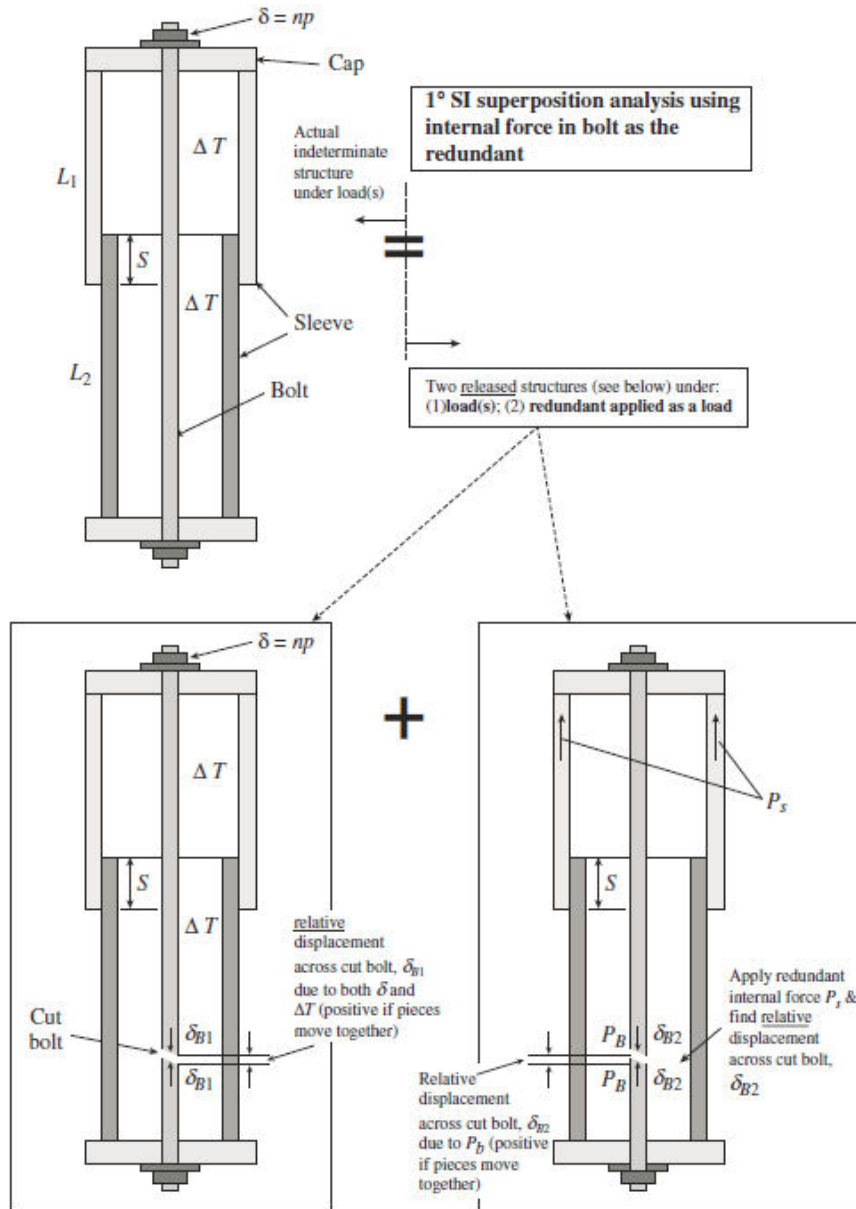
$$\delta_{B1} = -np + \alpha_s \Delta T (L_1 + L_2 - s)$$

$$\delta_{B2} = P_B \left[ \frac{L_1 + L_2 - s}{E_s A_b} + \frac{L_1 - s}{E_c A_1} + \frac{L_2 - s}{E_c A_2} + \frac{s}{E_c (A_1 + A_2)} \right]$$

$$\text{Compatibility:} \quad \delta_{B1} + \delta_{B2} = 0$$

$$P_B = \frac{-[-np + \alpha_s \Delta T (L_1 + L_2 - s)]}{\left[ \frac{L_1 + L_2 - s}{E_s A_b} + \frac{L_1 - s}{E_c A_1} + \frac{L_2 - s}{E_c A_2} + \frac{s}{E_c (A_1 + A_2)} \right]} \quad P_B = 25.4 \text{ kN} \quad \leftarrow \quad P_s = -P_B \quad \leftarrow$$

Sketches illustrating superposition procedure for statically-indeterminate analysis



(b) REQUIRED LENGTH OF SOLDER JOINT≈

$$\tau = \frac{P}{A_s} \quad A_s = \pi d_2 s$$

$$s_{\text{reqd}} = \frac{P_B}{\pi d_2 \tau_{aj}} \quad s_{\text{reqd}} = 25.7 \text{ mm}$$

$$\delta_s = P_s \left[ \frac{L_1 - s}{E_c A_1} + \frac{L_2 - s}{E_c A_2} + \frac{s}{E_c (A_1 + A_2)} \right]$$

$$\delta_s = -0.064 \text{ mm}$$

$$\delta_f = \delta_b + \delta_s \quad \delta_f = 0.35 \text{ mm} \quad \leftarrow$$

(c) FINAL ELONGATION

$\delta_f$  = net of elongation of bolt ( $\delta_b$ ) and shortening of sleeve ( $\delta_s$ )

$$\delta_b = P_B \left( \frac{L_1 + L_2 - s}{E_s A_b} \right) \quad \delta_b = 0.413 \text{ mm}$$

### Problem 2.5-29

The figure shows a section through the tube, cap, and spring.

Properties and dimensions:

$$d_o = 6 \text{ in.} \quad t = \frac{1}{8} \text{ in.} \quad E_t = 100 \text{ ksi}$$

$$A_t = \frac{\pi}{4} [d_o^2 - (d_o - 2t)^2] \quad A_t = 2.307 \text{ in.}^2$$

$$L_1 = 12.125 \text{ in.} > L = 12 \text{ in.} \quad k = 1.5 \text{ k/in.}$$

Spring is 1/8 in. longer than tube

$$\delta = L_1 - L \quad \delta = 0.125 \text{ in.}$$

$$\alpha_k = 6.5(10^{-6})/^{\circ}\text{F} < \alpha_t = 80 \times (10^{-6})/^{\circ}\text{F}$$

$$\Delta T = 0 \quad < \text{note that } Q \text{ result below is for zero temperature (until part(d))}$$

(a) FORCE IN SPRING  $F_k = \text{REDUNDANT } Q$

$$\text{Flexibilities:} \quad f = \frac{1}{k} \quad f_t = \frac{L}{E_t A_t}$$

$\delta_2 = \text{relative displacement across cut spring due to redundant} = Q(f + f_t)$

$\delta_1 = \text{relative displacement across cut spring due to precompression and } \Delta T = \delta + \alpha_k \Delta T L_1 - \alpha_t \Delta T L$

$$\text{Compatibility: } \delta_1 + \delta_2 = 0$$

Solve for redundant  $Q$ :

$$Q = \frac{-\delta + \Delta T(-\alpha_k L_1 + \alpha_t L)}{f + f_t} = F_k$$

$$F_k = -0.174 \text{ k} \quad \leftarrow \text{compressive force in spring (F}_k\text{) and also tensile force in tube}$$

$$(b) F_t = \text{force in tube} = -Q \quad \leftarrow$$

**NOTE:** If tube is rigid,  $F_k = -k\delta = -0.1875 \text{ k}$

(c) FINAL LENGTH OF TUBE

$$L_f = L + \delta_{c1} + \delta_{c2} \quad < \text{i.e., add displacements for the two released structures to initial tube length } L$$

$$L_f = L - Qf_t + \alpha_t(\Delta T)L \quad L_f = 12.01 \text{ in.} \quad \leftarrow$$

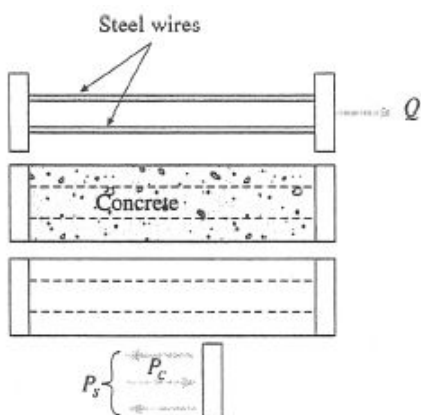
(d) SET  $Q = 0$  TO FIND  $\Delta T$  REQUIRED TO REDUCE SPRING FORCE TO ZERO

$$\Delta T_{\text{reqd}} = \frac{\delta}{(-\alpha_k L_1 + \alpha_t L)}$$

$$\Delta T_{\text{reqd}} = 141.9^{\circ}\text{F}$$

Since  $\alpha_t > \alpha_k$ , a temp. increase is req'd to expand tube so that spring force goes to zero.

### Problem 2.5-30



#### EQUILIBRIUM EQUATION

$$P_s = P_c \quad (\text{Eq. 1})$$

#### COMPATIBILITY EQUATION AND FORCE-DISPLACEMENT RELATIONS

$\delta_1$  = initial elongation of steel wires

$$= \frac{QL}{E_s A_s} = \frac{\sigma_0 L}{E_s}$$

$\delta_2$  = final elongation of steel wires

$$= \frac{P_s L}{E_s A_s}$$

$\delta_3$  = shortening of concrete

$$= \frac{P_c L}{E_c A_c}$$

$\delta_1 - \delta_2 = \delta_3$  or

$$\frac{\sigma_0 L}{E_s} - \frac{P_s L}{E_s A_s} = \frac{P_c L}{E_c A_c} \quad (\text{Eq. 2, Eq. 3})$$

Solve simultaneously Eqs. (1) and (3):

$$P_s = P_c = \frac{\sigma_0 A_s}{1 + \frac{E_s A_s}{E_c A_c}}$$

$L$  = length

$\sigma_0$  = initial stress in wires

$$= \frac{Q}{A_s} = 620 \text{ MPa}$$

$A_s$  = total area of steel wires

$A_c$  = area of concrete

$$= 50 A_s$$

$E_s = 12 E_c$

$P_s$  = final tensile force in steel wires

$P_c$  = final compressive force in concrete

#### STRESSES

$$\sigma_s = \frac{P_s}{A_s} = \frac{\sigma_0}{1 + \frac{E_s A_s}{E_c A_c}} \quad \leftarrow$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{\sigma_0}{\frac{A_c}{A_s} + \frac{E_s}{E_c}} \quad \leftarrow$$

#### SUBSTITUTE NUMERICAL VALUES:

$$\sigma_0 = 620 \text{ MPa} \quad \frac{E_s}{E_c} = 12 \quad \frac{A_s}{A_c} = \frac{1}{50}$$

$$\sigma_s = \frac{620 \text{ MPa}}{1 + \frac{12}{50}} = 500 \text{ MPa (Tension)} \quad \leftarrow$$

$$\sigma_c = \frac{620 \text{ MPa}}{50 + 12} = 10 \text{ MPa (Compression)} \quad \leftarrow$$

### Problem 2.5-31

The figure shows a section through the tube, cap, and spring.

Properties and dimensions:

$$d_o = 6 \text{ in.} \quad t = \frac{1}{8} \text{ in.} \quad E_t = 100 \text{ ksi}$$

$$L = 12 \text{ in.} > L_1 = 11.875 \text{ in.} \quad k = 1.5 \text{ k/in.}$$

$$\alpha_k = 6.5(10^{-6}) < \alpha_t = 80 \times (10^{-6})$$

$$A_t = \frac{\pi}{4} [d_o^2 - (d_o - 2t)^2]$$

$$A_t = 2.307 \text{ in.}^2$$

Pretension and temperature:

Spring is 1/8 in. shorter than tube.

$$\delta = L - L_1 \quad \delta = 0.125 \text{ in.} \quad \Delta T = 0$$

Note that  $Q$  result below is for zero temperature (until part (d)).

$$\text{Flexibilities:} \quad f = \frac{1}{k} \quad f_t = \frac{L}{E_t A_t}$$

(a) FORCE IN SPRING ( $F_k$ ) = REDUNDANT ( $Q$ )

Follow solution procedure outlined in Prob. 2.5-29 solution:

$$Q = \frac{\delta + \Delta T(-\alpha_k L_1 + \alpha_t L)}{f + f_t} = F_k$$

$$F_k = 0.174 \text{ k} \quad \leftarrow \text{also the compressive force in the tube}$$

$$(b) \text{ FORCE IN TUBE } F_t = -Q = -0.174 \text{ k} \quad \leftarrow$$

$$(c) \text{ FINAL LENGTH OF TUBE AND SPRING } L_f = L + \delta_{c1} + \delta_{c2}$$

$$L_f = L - Qf_t + \alpha_t(\Delta T)L \quad L_f = 11.99 \text{ in.} \quad \leftarrow$$

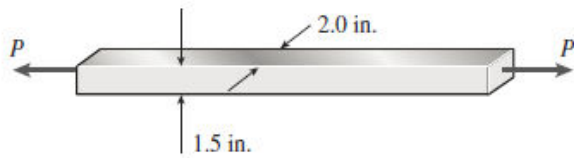
(d) SET  $Q = 0$  TO FIND  $\Delta T$  REQUIRED TO REDUCE SPRING FORCE TO ZERO

$$\Delta T_{\text{reqd}} = \frac{-\delta}{(-\alpha_k L_1 + \alpha_t L)}$$

$$\Delta T_{\text{reqd}} = -141.6^\circ\text{F}$$

Since  $\alpha_t > \alpha_k$ , a temperature drop is required to shrink tube so that spring force goes to zero.

**Problem 2.6-1**



**NUMERICAL DATA**

$$A = 3 \text{ in.}^2 \quad \sigma_a = 14500 \text{ psi}$$

$$\tau_a = 7100 \text{ psi}$$

**MAXIMUM LOAD—TENSION**

$$P_{\max 1} = \sigma_a A \quad P_{\max 1} = 43500 \text{ lbs}$$

**MAXIMUM LOAD—SHEAR**

$$P_{\max 2} = 2\tau_a A \quad P_{\max 2} = 42,600 \text{ lbs}$$

Because  $\tau_{\text{allow}}$  is less than one-half of  $\sigma_{\text{allow}}$ , the shear stress governs.



**Problem 2.6-2**



NUMERICAL DATA     $P = 3.5 \text{ kN}$      $\sigma_a = 118 \text{ MPa}$   
                                   $\tau_a = 48 \text{ MPa}$

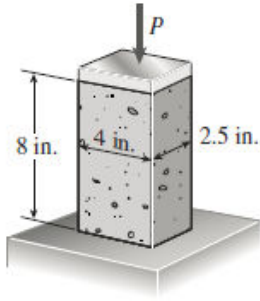
Find  $P_{\max}$  then rod diameter.  
 since  $\tau_a$  is less than  $1/2$  of  $\sigma_a$ , shear governs.

$$P_{\max} = 2\tau_a \left( \frac{\pi}{4} d_{\min}^2 \right)$$

$$d_{\min} = \sqrt{\frac{2}{\pi\tau_a} P}$$

$$d_{\min} = 6.81 \text{ mm} \quad \leftarrow$$

**Problem 2.6-3**



$A = 2.5 \text{ in.} \times 4.0 \text{ in.} = 10.0 \text{ in.}^2$   
Maximum normal stress:

$$\sigma_x = \frac{P}{A}$$

Maximum shear stress:

$$\tau_{\max} = \frac{\sigma_x}{2} = \frac{P}{2A}$$

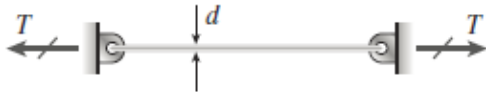
$$\sigma_{\text{ult}} = 3600 \text{ psi} \quad \tau_{\text{ult}} = 1200 \text{ psi}$$

Because  $\tau_{\text{ult}}$  is less than one-half of  $\sigma_{\text{ult}}$ , the shear stress governs.

$$\tau_{\max} = \frac{P}{2A} \quad \text{or} \quad P_{\max} = 2A\tau_{\text{ult}}$$

$$P_{\max} = 2(10.0 \text{ in.}^2)(1200 \text{ psi}) = 24,000 \text{ lb} \quad \leftarrow$$

**Problem 2.6-4**



NUMERICAL DATA

$$d = 2.42 \text{ mm} \quad T = 98 \text{ N}$$

$$\alpha = 19.5 (10^{-6})/^{\circ}\text{C} \quad E = 110 \text{ GPa}$$

(a)  $\Delta T_{\max}$  (DROP IN TEMPERATURE)

$$\sigma = \frac{T}{A} - (E \alpha \Delta T) \quad \tau_{\max} = \frac{\sigma}{2}$$

$$\tau_a = \frac{T}{2A} - \frac{E \alpha \Delta T}{2}$$

$$\tau_a = 60 \text{ MPa} \quad A = \frac{\pi}{4} d^2$$

$$\Delta T_{\max} = \frac{\frac{T}{A} - 2\tau_a}{E \alpha}$$

$$\Delta T_{\max} = -46^{\circ}\text{C} \text{ (drop)}$$

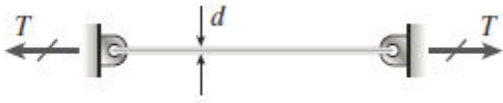
(b)  $\Delta T$  AT WHICH WIRE GOES SLACK

Increase  $\Delta T$  until  $\sigma = 0$ :

$$\Delta T = \frac{T}{E \alpha A}$$

$$\Delta T = 9.93^{\circ}\text{C} \text{ (increase)}$$

**Problem 2.6-5**



NUMERICAL DATA

$$d = \frac{1}{16} \text{ in.} \quad T = 37 \text{ lb} \quad \alpha = 10.6 \times (10^{-6})/^{\circ}\text{F}$$

$$E = 15 \times (10^6) \text{ psi} \quad \Delta T = -60^{\circ}\text{F}$$

$$A = \frac{\pi}{4} d^2$$

(a)  $\tau_{\max}$  (DUE TO DROP IN TEMPERATURE)

$$\tau_{\max} = \frac{\sigma_x}{2} \quad \tau_{\max} = \frac{\frac{T}{A} - (E\alpha\Delta T)}{2}$$

$$\tau_{\max} = 10,800 \text{ psi} \quad \leftarrow$$

(b)  $\Delta T_{\max}$  FOR ALLOWABLE SHEAR STRESS

$$\tau_a = 10000 \text{ psi}$$

$$\Delta T_{\max} = \frac{\frac{T}{A} - 2\tau_a}{E\alpha}$$

$$\Delta T_{\max} = -49.9^{\circ}\text{F} \quad \leftarrow$$

(c)  $\Delta T$  AT WHICH WIRE GOES SLACK

Increase  $\Delta T$  until  $\sigma = 0$ :

$$\Delta T = \frac{T}{E\alpha A}$$

$$\Delta T = 75.9^{\circ}\text{F (increase)} \quad \leftarrow$$

**Problem 2.6-6**

(a)  $d = 12 \text{ mm}$      $P = 9.5 \text{ kN}$      $A = \frac{\pi}{4}d^2 = 1.131 \times 10^{-4} \text{ m}^2$

$$\sigma_x = \frac{P}{A} = 84 \text{ MPa}$$

(b)  $\tau_{\max} = \frac{\sigma_x}{2} = 42 \text{ MPa}$     On plane stress element rotated  $45^\circ$

(c) ROTATED STRESS ELEMENT ( $45^\circ$ ) HAS NORMAL TENSILE STRESS  $\sigma_x/2$  ON ALL FACES,  $-T_{\max}$  (CW) ON  $+x$ -FACE, AND  $+T_{\max}$  (CCW) ON  $+y$ -FACE

$$\tau_{xyl} = \tau_{\max} \quad \sigma_{xl} = \frac{\sigma_x}{2} \quad \sigma_{yl} = \sigma_{xl}$$

On rotated  $x$ -face:  $\sigma_{xl} = 42 \text{ MPa}$      $\tau_{xyl} = 42 \text{ MPa}$

On rotated  $y$ -face:  $\sigma_{yl} = 42 \text{ MPa}$

(d)  $\theta = 22.5^\circ$  < CCW ROTATION OF ELEMENT

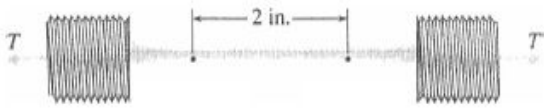
$$\sigma_\theta = \sigma_x \cos^2(\theta) = 71.7 \text{ MPa} \quad < \text{on rotated } x \text{ face} \quad \sigma_y = \sigma_x \cos^2\left(\theta + \frac{\pi}{2}\right) = 12.3 \text{ MPa} \quad < \text{on rotated } y \text{ face}$$

Eq. 2-29b     $\tau_\theta = \frac{-\sigma_x}{2} \sin(2\theta) = -29.7 \text{ MPa} \quad < \text{CW on rotated } x\text{-face}$

On rotated  $x$ -face:  $\sigma_{xl} = 71.7 \text{ MPa}$      $\tau_{xyl} = -29.7 \text{ MPa}$

On rotated  $y$ -face:  $\sigma_{yl} = 12.3 \text{ MPa}$

**Problem 2.6-7**



Elongation:  $\delta = 0.00120$  in.  
(2 in. gage length)

$$\text{Strain: } \varepsilon = \frac{\delta}{L} = \frac{0.00120 \text{ in.}}{2 \text{ in.}} = 0.00060$$

$$\text{Hooke's law: } \sigma_x = E\varepsilon = (30 \times 10^6 \text{ psi})(0.00060) \\ = 18,000 \text{ psi}$$

(a) **MAXIMUM NORMAL STRESS**

$\sigma_x$  is the maximum normal stress.

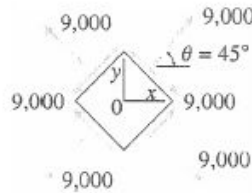
$$\sigma_{\max} = 18,000 \text{ psi} \quad \leftarrow$$

(b) **MAXIMUM SHEAR STRESS**

The maximum shear stress is on a  $45^\circ$  plane and equals  $\sigma_x/2$ .

$$\tau_{\max} = \frac{\sigma_x}{2} = 9,000 \text{ psi} \quad \leftarrow$$

(c) **STRESS ELEMENT AT  $\theta = 45^\circ$**



**NOTE:** All stresses have units of psi.

**Problem 2.6-8**

(a)  $\alpha = 17.5(10^{-6})$      $\Delta T = 50$      $E = 120 \text{ GPa}$

$$\sigma_x = -E\alpha\Delta T = -105 \text{ MPa} \quad \tau_{\max} = \frac{\sigma_x}{2} = -52.5 \text{ MPa} \quad \text{at } \theta = 45^\circ$$

(compression)

Element A:  $\sigma_x = 105 \text{ MPa}$  (compression);  
Element B:  $\tau_{\max} = 52.5 \text{ MPa}$

(b)  $\tau_\theta = 48 \text{ MPa}$

Eq. 2-29b     $\tau_\theta = \frac{-\sigma_x}{2} \sin(2\theta)$

so     $\theta = \frac{1}{2} \arcsin\left(\frac{2\tau_\theta}{-\sigma_x}\right) = 33.1^\circ < \text{CCW rotation of element}$      $\theta = 33.1^\circ$

$$\sigma_\theta = \sigma_x \cos^2(\theta) = -73.8 \text{ MPa} \quad \text{on rotated } x \text{ face}$$

$$\sigma_y = \sigma_x \cos^2\left(\theta + \frac{\pi}{2}\right) = -31.2 \text{ MPa} \quad \text{on rotated } y \text{ face}$$

### Problem 2.6-9

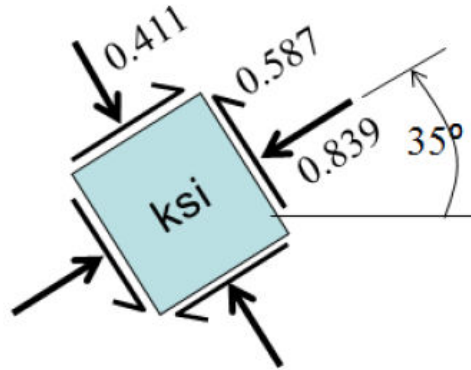
$$P = 10\text{kip} \quad L = 3\text{ft} \quad A = 8\text{in}^2 \quad \theta = 35\text{deg}$$

Normal compressive stress  $\sigma_x = \frac{-P}{A} = -1.25\text{ksi}$

Plane stress transformations  $\sigma_{\theta}(\theta) = \sigma_x \cos^2(\theta) \quad \sigma_{\theta}(\theta) = -0.839\text{ksi} \quad \sigma_{\theta}\left(\theta + \frac{\pi}{2}\right) = -0.411\text{ksi}$

$\tau_{\theta}(\theta) = -\sigma_x \sin(\theta) \cos(\theta) \quad \tau_{\theta}(\theta) = 0.587\text{ksi} \quad \tau_{\theta}\left(\theta + \frac{\pi}{2}\right) = -0.587\text{ksi}$

Rotated stress element





**Problem 2.6-10**

$$L = 1\text{m} \quad A = 1200\text{mm}^2 \quad \Delta T = 25 \quad \alpha = 12 \cdot (10^{-6}) \quad \theta = 45^\circ \quad E = 200\text{GPa}$$

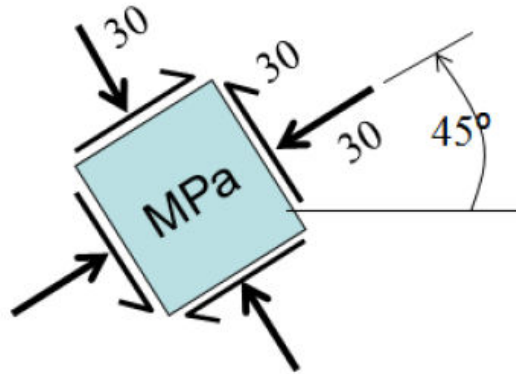
Compressive thermal stress  $\sigma_T = E \cdot \alpha \cdot \Delta T = 60\text{MPa}$

Support reactions  $R_A = \sigma_T \cdot A = 72\text{kN} \quad R_B = -R_A$

Plane stress transformations  $\sigma_x = \frac{R_B}{A} = -60\text{MPa}$

$$\sigma_\theta = \sigma_x \cdot \cos^2(\theta) = -30\text{MPa} \quad \sigma_x \cdot \cos\left(\theta + \frac{\pi}{2}\right)^2 = -30\text{MPa} \quad \tau_\theta = -\sigma_x \cdot \sin(\theta) \cdot \cos(\theta) = 30\text{MPa}$$

Rotated stress element



**Problem 2.6-11**

NUMERICAL DATA

$$L = 10 \text{ ft} \quad b = 0.71 L \quad P = 49 \text{ k} \quad \sigma_a = 14 \text{ ksi} \quad \tau_a = 7.5 \text{ ksi} \quad A = 5.87 \text{ in.}^2$$

(a) FOR LINEAR ANALYSIS, MEMBER FORCES ARE PROPORTIONAL TO LOADING

$$F_{AC} = \frac{P}{35} 15.59 = 21.826 \text{ k} \quad F_{AB} = \frac{P}{35} 62.2 = 87.08 \text{ k}$$

(solution for  $P = 35 \text{ k}$ )

$$F_{BC} = \frac{P}{35} (-78.9) \quad F_{BC} = -110.46 \text{ k}$$

Normal stresses in each member:  $\sigma_{AC} = \frac{F_{AC}}{A} = 3.718 \text{ ksi} \quad \sigma_{AB} = \frac{F_{AB}}{A} = 14.835 \text{ ksi}$

From Eq. 2-31:  $\sigma_{BC} = \frac{F_{BC}}{A} = -18.818 \text{ ksi}$

$$\tau_{\max AC} = \frac{\sigma_{AC}}{2} = 1.859 \text{ ksi}$$

$$\tau_{\max AB} = \frac{\sigma_{AB}}{2} = 7.42 \text{ ksi}$$

$$\tau_{\max BC} = \frac{\sigma_{BC}}{2} = -9.41 \text{ ksi}$$

(b)  $\sigma_a < 2 \times T_a$  so normal stress will control; lowest value governs here

MEMBER AC:  $P_{\max \sigma} = \frac{P}{F_{AC}} (\sigma_a A) = 184.496 \text{ k} \quad P_{\max \tau} = \frac{P}{F_{AC}} (2 \tau_a A) = 197.675 \text{ k}$

MEMBER AB:  $P_{\max \sigma} = \frac{P}{F_{AB}} (\sigma_a A) = 46.243 \text{ k} \quad P_{\max \tau} = \frac{P}{F_{AB}} (2 \tau_a A) = 49.546 \text{ k}$

MEMBER BC:  $P_{\max \sigma} = \left| \frac{P}{F_{BC}} \right| (\sigma_a A) = 36.5 \text{ k} \quad P_{\max \tau} = \left| \frac{P}{F_{BC}} \right| (2 \tau_a A) = 39.059 \text{ k}$

# Problem 2.6-12

## NUMERICAL DATA

$$\begin{aligned} d &= 32 \text{ mm} & A &= \frac{\pi}{4} d^2 \\ P &= 190 \text{ N} & A &= 804.25 \text{ mm}^2 \\ a &= 100 \text{ mm} \\ b &= 300 \text{ mm} \end{aligned}$$

- (a) STATICS—FIND COMPRESSIVE FORCE  $F$  AND STRESSES IN PLASTIC BAR

$$F = \frac{P(a + b)}{a} \quad F = 760 \text{ N}$$

$$\sigma_x = \frac{F}{A} \quad \sigma_x = 0.945 \text{ MPa} \quad \text{or} \quad \sigma_x = 945 \text{ kPa}$$

From (1), (2), and (3) below:

$$\sigma_{\max} = \sigma_x \quad \sigma_{\max} = -945 \text{ kPa}$$

$$\tau_{\max} = 472 \text{ kPa} \quad \frac{\sigma_x}{2} = -472 \text{ kPa}$$

$$(1) \theta = 0^\circ \quad \sigma_x = -945 \text{ kPa} \quad \leftarrow$$

$$(2) \theta = 22.50^\circ$$

On  $+x$ -face:

$$\begin{aligned} \sigma_\theta &= \sigma_x \cos^2(\theta) \\ \sigma_\theta &= -807 \text{ kPa} \quad \leftarrow \end{aligned}$$

$$\begin{aligned} \tau_\theta &= -\sigma_x \sin(\theta) \cos(\theta) \\ \tau_\theta &= 334 \text{ kPa} \quad \leftarrow \end{aligned}$$

$$\text{On } +y\text{-face:} \quad \theta = \theta + \frac{\pi}{2}$$

$$\begin{aligned} \sigma_\theta &= \sigma_x \cos^2(\theta) \\ \sigma_\theta &= -138.39 \text{ kPa} \end{aligned}$$

$$\begin{aligned} \tau_\theta &= -\sigma_x \sin(\theta) \cos(\theta) \\ \tau_\theta &= -334.1 \text{ kPa} \end{aligned}$$

$$(3) \theta = 45^\circ$$

On  $+x$ -face:

$$\begin{aligned} \sigma_\theta &= \sigma_x \cos^2(\theta) \\ \sigma_\theta &= -472 \text{ kPa} \quad \leftarrow \end{aligned}$$

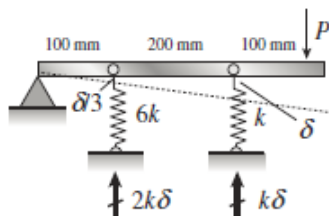
$$\begin{aligned} \tau_\theta &= -\sigma_x \sin(\theta) \cos(\theta) \\ \tau_\theta &= 472 \text{ kPa} \quad \leftarrow \end{aligned}$$

$$\text{On } +y\text{-face:} \quad \theta = \theta + \frac{\pi}{2}$$

$$\sigma_\theta = \sigma_x \cos^2(\theta) \quad \sigma_\theta = -472.49 \text{ kPa}$$

$$\tau_\theta = -\sigma_x \sin(\theta) \cos(\theta) \quad \tau_\theta = -472.49 \text{ kPa}$$

- (b) ADD SPRING—FIND MAXIMUM NORMAL AND SHEAR STRESSES IN PLASTIC BAR



$$\sum M_{\text{pin}} = 0$$

$$P(400) = [2k\delta(100) + k\delta(300)]$$

$$\delta = \frac{4}{5} \frac{P}{k}$$

$$\text{Force in plastic bar:} \quad F = (2k) \left( \frac{4}{5} \frac{P}{k} \right)$$

$$F = \frac{8}{5} P \quad F = 304 \text{ N}$$

Normal and shear stresses in plastic bar:

$$\sigma_x = \frac{F}{A} \quad \sigma_x = 0.38$$

$$\sigma_{\max} = -378 \text{ kPa} \quad \leftarrow$$

$$\tau_{\max} = \frac{\sigma_x}{2} \quad \tau_{\max} = -189 \text{ kPa} \quad \leftarrow$$

### Problem 2.6-13

#### NUMERICAL DATA

$$b = 1.5 \text{ in.} \quad h = 3 \text{ in.} \quad A = bh \quad \Delta T = (160 - 68)^\circ\text{F}$$

$$\Delta T = 92^\circ\text{F}$$

$$A = 4.5 \text{ in.}^2 \quad \sigma_{pq} = -1700 \text{ psi}$$

$$\alpha = 60 \times (10^{-6})/^\circ\text{F}$$

$$E = 450 \times (10^3) \text{ psi}$$

#### (a) SHEAR STRESS ON PLANE $pq$

Statically indeterminate analysis gives,  
for reaction at right support:

$$R = -E\alpha\Delta T \quad R = -11178 \text{ lb}$$

$$\sigma_x = \frac{R}{A} \quad \sigma_x = -2484 \text{ psi}$$

$$\text{Using } \sigma_\theta = \sigma_x \cos(\theta)^2: \quad \cos(\theta)^2 = \frac{\sigma_{pq}}{\sigma_x}$$

$$\theta = \arccos\left(\sqrt{\frac{\sigma_{pq}}{\sigma_x}}\right) \quad \theta = 34.2^\circ$$

Now with  $\theta$ , can find shear stress on plane  $pq$ :

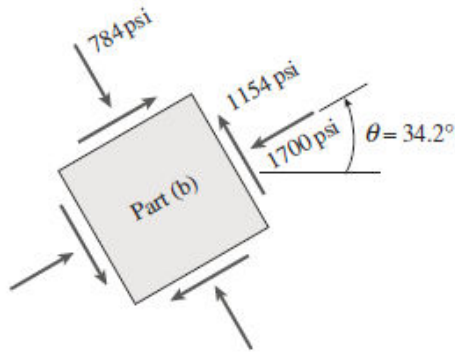
$$\tau_{pq} = -\sigma_x \sin(\theta) \cos(\theta) \quad \tau_{pq} = 1154 \text{ psi} \quad \leftarrow$$

$$\sigma_{pq} = \sigma_x \cos(\theta)^2 \quad \sigma_{pq} = -1700 \text{ psi}$$

Stresses at  $\theta + \pi/2$  (y-face):

$$\sigma_y = \sigma_x \cos\left(\theta + \frac{\pi}{2}\right)^2 \quad \sigma_y = -784 \text{ psi}$$

#### (b) STRESS ELEMENT FOR PLANE $pq$



#### (c) MAXIMUM LOAD AT QUARTER POINT $\sigma_a = 3400 \text{ psi}$

$$\tau_a = 1650 \text{ psi} \quad 2\tau_a = 3300 < \text{less than } \sigma_a, \text{ so shear controls}$$

Statically indeterminate analysis for  $P$  at  $L/4$  gives  
for reactions:

$$R_{R2} = \frac{-P}{4} \quad R_{L2} = \frac{-3}{4}P$$

(tension for 0 to  $L/4$  and compression for rest of bar)

From part (a) (for temperature increase  $\Delta T$ ):

$$R_{R1} = -E\alpha\Delta T \quad R_{L1} = -E\alpha\Delta T$$

Stresses in bar (0 to  $L/4$ ):

$$\sigma_x = -E\alpha\Delta T + \frac{3P}{4A} \quad \tau_{\max} = \frac{\sigma_x}{2}$$

Set  $\tau_{\max} = \tau_a$  and solve for  $P_{\max1}$ :

$$\tau_a = \frac{-E\alpha\Delta T}{2} + \frac{3P}{8A}$$

$$P_{\max1} = \frac{4A}{3}(2\tau_a + E\alpha\Delta T)$$

$$P_{\max1} = 34,704 \text{ lb}$$

$$\tau_{\max} = \frac{-E\alpha\Delta T}{2} + \frac{3P_{\max1}}{8A}$$

$$\tau_{\max} = 1650 \text{ psi} < \text{check}$$

$$\sigma_x = -E\alpha\Delta T + \frac{3P_{\max1}}{4A}$$

$$\sigma_x = 3300 \text{ psi} < \text{less than } \sigma_a$$

Stresses in bar ( $L/4$  to  $L$ ):

$$\sigma_x = -E\alpha\Delta T - \frac{P}{4A} \quad \tau_{\max} = \frac{\sigma_x}{2}$$

Set  $\tau_{\max} = \tau_a$  and solve for  $P_{\max2}$ :

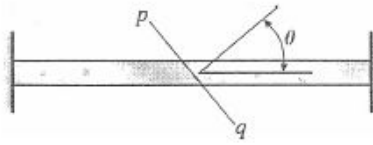
$$P_{\max2} = -4A(-2\tau_a + E\alpha\Delta T)$$

$$P_{\max2} = 14,688 \text{ lb} \quad \leftarrow \text{shear in segment } (L/4 \text{ to } L) \text{ controls}$$

$$\tau_{\max} = \frac{-E\alpha\Delta T}{2} - \frac{P_{\max2}}{8A} \quad \tau_{\max} = -1650 \text{ psi}$$

$$\sigma_x = -E\alpha\Delta T - \frac{P_{\max2}}{4A} \quad \sigma_x = -3300 \text{ psi}$$

**Problem 2.6-14**



**NUMERICAL DATA**

$$\theta = 55 \left( \frac{\pi}{180} \right) \text{ rad}$$

$$b = 18 \text{ mm} \quad h = 40 \text{ mm}$$

$$A = bh \quad A = 720 \text{ mm}^2$$

$$\sigma_{pqa} = 60 \text{ MPa} \quad \tau_{pqa} = 30 \text{ MPa}$$

$$\alpha = 17 \times (10^{-6})/^{\circ}\text{C} \quad E = 120 \text{ GPa}$$

$$\Delta T = 20^{\circ}\text{C} \quad P = 15 \text{ kN}$$

- (a) FIND  $\Delta T_{\max}$  BASED ON ALLOWABLE NORMAL AND SHEAR STRESS VALUES ON PLANE  $pq$

$$\sigma_x = -E\alpha\Delta T_{\max} \quad \Delta T_{\max} = \frac{-\sigma_x}{E\alpha}$$

$$\sigma_{pq} = \sigma_x \cos(\theta)^2 \quad \tau_{pq} = -\sigma_x \sin(\theta) \cos(\theta)$$

Set each equal to corresponding allowable and solve for  $\sigma_x$ :

$$\sigma_{x1} = \frac{\sigma_{pqa}}{\cos(\theta)^2} \quad \sigma_{x1} = 182.38 \text{ MPa}$$

$$\sigma_{x2} = \frac{\tau_{pqa}}{-\sin(\theta) \cos(\theta)} \quad \sigma_{x2} = -63.85 \text{ MPa}$$

Lesser value controls, so allowable shear stress governs.

$$\Delta T_{\max} = \frac{-\sigma_{x2}}{E\alpha} \quad \Delta T_{\max} = 31.3^{\circ}\text{C} \quad \leftarrow$$

- (b) STRESSES ON PLANE  $PQ$  FOR MAXIMUM TEMPERATURE

$$\sigma_x = -E\alpha\Delta T_{\max} \quad \sigma_x = -63.85 \text{ MPa}$$

$$\sigma_{pq} = \sigma_x \cos(\theta)^2 \quad \sigma_{pq} = -21.0 \text{ MPa} \quad \leftarrow$$

$$\tau_{pq} = -\sigma_x \sin(\theta) \cos(\theta) \quad \tau_{pq} = 30 \text{ MPa} \quad \leftarrow$$

- (c) ADD LOAD  $P$  IN  $+x$ -DIRECTION TO TEMPERATURE CHANGE AND FIND LOCATION OF LOAD

$$\Delta T = 28^{\circ}\text{C}$$

$P = 15 \text{ kN}$  from one-degree statically indeterminate analysis, reactions  $R_A$  and  $R_B$  due to load  $P$ :

$$R_A = -(1 - \beta)P \quad R_B = \beta P$$

Now add normal stresses due to  $P$  to thermal stresses due to  $\Delta T$  (tension in segment 0 to  $\beta L$ , compression in segment  $\beta L$  to  $L$ ).

Stresses in bar (0 to  $\beta L$ ):

$$\sigma_x = -E\alpha\Delta T + \frac{R_A}{A} \quad \tau_{\max} = \frac{\sigma_x}{2}$$

Shear controls so set  $\tau_{\max} = \tau_a$  and solve for  $\beta$ :

$$2\tau_a = -E\alpha\Delta T + \frac{(1 - \beta)P}{A}$$

$$\beta = 1 - \frac{A}{P} [2\tau_a + E\alpha\Delta T]$$

$$\beta = -5.1$$

Impossible so evaluate segment ( $\beta L$  to  $L$ ):

Stresses in bar ( $\beta L$  to  $L$ ):

$$\sigma_x = -E\alpha\Delta T - \frac{R_B}{A} \quad \tau_{\max} = \frac{\sigma_x}{2}$$

set  $\tau_{\max} = \tau_a$  and solve for  $P_{\max 2}$

$$2\tau_a = -E\alpha\Delta T - \frac{\beta P}{A}$$

$$\beta = \frac{-A}{P} [-2\tau_a + E\alpha\Delta T]$$

$$\beta = 0.62 \quad \leftarrow$$

### Problem 2.6-15

NUMERICAL DATA

$$P = 5 \text{ k} \quad \alpha = 36^\circ \quad \sigma_a = 13.5 \text{ ksi}$$

$$\tau_a = 6.5 \text{ ksi}$$

$$\theta = \frac{\pi}{2} - \alpha \quad \theta = 54^\circ$$

$$\sigma_{ja} = 6.0 \text{ ksi}$$

$$\tau_{ja} = 3.0 \text{ ksi}$$

Tensile force  $N_{AC}$  using Method of Joints at  $C$ :

$$N_{AC} = \frac{P}{\sin(60^\circ)} \quad (\text{tension})$$

$$N_{AC} = 5.77 \text{ k} \quad \leftarrow$$

Minimum required diameter of bar  $AC$ :

(1) Check tension and shear in bars;  $\tau_a < \sigma_a/2$  so shear

$$\text{controls } \tau_{\max} = \frac{\sigma_x}{2};$$

$$2\tau_a = \frac{N_{AC}}{A} \quad \sigma_x = 2\tau_a = 13 \text{ ksi}$$

$$A_{\text{reqd}} = \frac{N_{AC}}{2\tau_a} \quad A_{\text{reqd}} = 0.44 \text{ in.}^2$$

$$d_{\min} = \sqrt{\frac{4}{\pi} A_{\text{reqd}}} \quad d_{\min} = 0.75 \text{ in.}$$

(2) Check tension and shear on brazed joint:

$$\sigma_x = \frac{N_{AC}}{A} \quad \sigma_x = \frac{N_{AC}}{\frac{\pi}{4} d^2} \quad d_{\text{reqd}} = \sqrt{\frac{4}{\pi} \frac{N_{AC}}{\sigma_x}}$$

Tension on brazed joint:

$$\sigma_\theta = \sigma_x \cos(\theta)^2$$

Set equal to  $\sigma_{ja}$  and solve for  $\sigma_x$ , then  $d_{\text{reqd}}$ :

$$\sigma_x = \frac{\sigma_{ja}}{\cos(\theta)^2} \quad \sigma_x = 17.37 \text{ ksi}$$

$$d_{\text{reqd}} = \sqrt{\frac{4}{\pi} \frac{N_{AC}}{\sigma_x}} \quad d_{\text{reqd}} = 0.65 \text{ in.}$$

Shear on brazed joint:

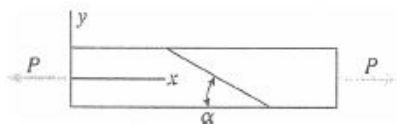
$$\tau_\theta = -\sigma_x \sin(\theta) \cos(\theta)$$

$$\sigma_x = \left| \frac{\tau_{ja}}{-(\sin(\theta) \cos(\theta))} \right| \quad \sigma_x = -6.31 \text{ ksi}$$

$$d_{\text{reqd}} = \sqrt{\frac{4}{\pi} \frac{N_{AC}}{\sigma_x}} \quad d_{\text{reqd}} = 1.08 \text{ in.} \quad \leftarrow$$



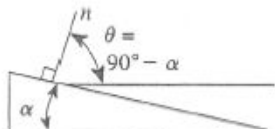
### Problem 2.6-16



$$10^\circ \leq \alpha \leq 40^\circ$$

Due to load  $P$ :  $\sigma_x = 4.9 \text{ MPa}$

(a) STRESSES ON JOINT WHEN  $\alpha = 20^\circ$



$$\theta = 90^\circ - \alpha = 70^\circ$$

$$\begin{aligned}\sigma_\theta &= \sigma_x \cos^2 \theta = (4.9 \text{ MPa})(\cos 70^\circ)^2 \\ &= 0.57 \text{ MPa} \quad \leftarrow\end{aligned}$$

$$\begin{aligned}\tau_\theta &= -\sigma_x \sin \theta \cos \theta \\ &= (-4.9 \text{ MPa})(\sin 70^\circ)(\cos 70^\circ) \\ &= -1.58 \text{ MPa} \quad \leftarrow\end{aligned}$$

(b) LARGEST ANGLE  $\alpha$  IF  $\tau_{\text{allow}} = 2.25 \text{ MPa}$

$$\tau_{\text{allow}} = -\sigma_x \sin \theta \cos \theta$$

The shear stress on the joint has a negative sign. Its numerical value cannot exceed  $\tau_{\text{allow}} = 2.25 \text{ MPa}$ . Therefore,

$$\begin{aligned}-2.25 \text{ MPa} &= -(4.9 \text{ MPa})(\sin \theta)(\cos \theta) \text{ or } \sin \theta \cos \theta \\ \theta &= 0.4592\end{aligned}$$

$$\text{From trigonometry: } \sin \theta \cos \theta = \frac{1}{2} \sin 2\theta$$

$$\text{Therefore: } \sin 2\theta = 2(0.4592) = 0.9184$$

$$\text{Solving: } 2\theta = 66.69^\circ \text{ or } 113.31^\circ$$

$$\theta = 33.34^\circ \text{ or } 56.66^\circ$$

$$\alpha = 90^\circ - \theta \quad \therefore \alpha = 56.66^\circ \text{ or } 33.34^\circ$$

Since  $\alpha$  must be between  $10^\circ$  and  $40^\circ$ , we select

$$\alpha = 33.3^\circ \quad \leftarrow$$

**NOTE:** If  $\alpha$  is between  $10^\circ$  and  $33.3^\circ$ ,

$$|\tau_\theta| < 2.25 \text{ MPa.}$$

If  $\alpha$  is between  $33.3^\circ$  and  $40^\circ$ ,

$$|\tau_\theta| > 2.25 \text{ MPa.}$$

(c) WHAT IS  $\alpha$  IF  $\tau_\theta = 2\sigma_\theta$ ?

Numerical values only:

$$|\tau_\theta| = \sigma_x \sin \theta \cos \theta \quad |\sigma_\theta| = \sigma_x \cos^2 \theta$$

$$\left| \frac{\tau_\theta}{\sigma_\theta} \right| = 2$$

$$\sigma_x \sin \theta \cos \theta = 2\sigma_x \cos^2 \theta$$

$$\sin \theta = 2 \cos \theta \text{ or } \tan \theta = 2$$

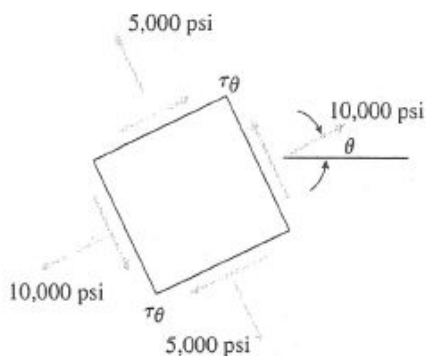
$$\theta = 63.43^\circ \quad \alpha = 90^\circ - \theta$$

$$\alpha = 26.6^\circ \quad \leftarrow$$

**NOTE:** For  $\alpha = 26.6^\circ$  and  $\theta = 63.4^\circ$ , we find  $\sigma_\theta = 0.98 \text{ MPa}$  and  $\tau_\theta = -1.96 \text{ MPa}$ .

$$\text{Thus, } \left| \frac{\tau_\theta}{\sigma_\theta} \right| = 2 \text{ as required.}$$

**Problem 2.6-17**



(a) **ANGLE  $\theta$  AND SHEAR STRESS  $\tau_\theta$**

$$\sigma_\theta = \sigma_x \cos^2 \theta$$

$$\sigma_\theta = 10,000 \text{ psi}$$

$$\sigma_x = \frac{\sigma_\theta}{\cos^2 \theta} = \frac{10,000 \text{ psi}}{\cos^2 \theta}$$

PLANE AT ANGLE  $\theta + 90^\circ$

$$\begin{aligned} \sigma_{\theta + 90^\circ} &= \sigma_x [\cos(\theta + 90^\circ)]^2 = \sigma_x [-\sin \theta]^2 \\ &= \sigma_x \sin^2 \theta \end{aligned}$$

$$\sigma_{\theta + 90^\circ} = 5,000 \text{ psi}$$

$$\sigma_x = \frac{\sigma_{\theta + 90^\circ}}{\sin^2 \theta} = \frac{5,000 \text{ psi}}{\sin^2 \theta}$$

Equate (1) and (2):

$$\frac{10,000 \text{ psi}}{\cos^2 \theta} = \frac{5,000 \text{ psi}}{\sin^2 \theta}$$

$$\tan^2 \theta = \frac{1}{2} \quad \tan \theta = \frac{1}{\sqrt{2}} \quad \theta = 35.26^\circ \quad \leftarrow$$

From Eq. (1) or (2):

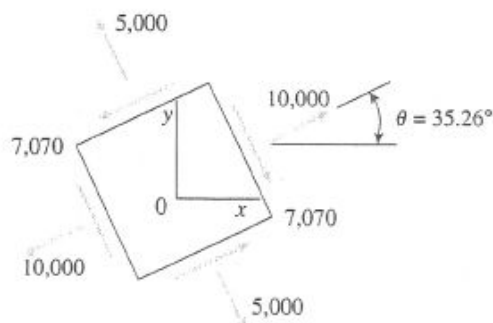
$$\sigma_x = 15,000 \text{ psi}$$

$$\tau_\theta = -\sigma_x \sin \theta \cos \theta$$

$$= (-15,000 \text{ psi})(\sin 35.26^\circ)(\cos 35.26^\circ)$$

$$= -7,070 \text{ psi} \quad \leftarrow$$

Minus sign means that  $\tau_\theta$  acts clockwise on the plane for which  $\theta = 35.26^\circ$ .



**NOTE:** All stresses have units of psi.

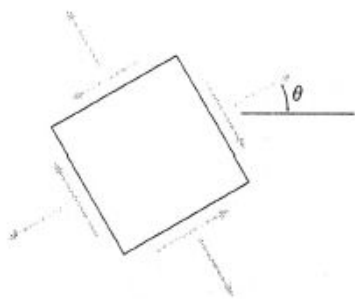
(b) **MAXIMUM NORMAL AND SHEAR STRESSES**

$$\sigma_{\max} = \sigma_x = 15,000 \text{ psi} \quad \leftarrow$$

$$\tau_{\max} = \frac{\sigma_x}{2} = 7,500 \text{ psi} \quad \leftarrow$$



**Problem 2.6-18**



Find  $\theta$  and  $\sigma_x$  for stress state shown in figure.

$$\sigma_\theta = \sigma_x \cos^2(\theta) \quad \cos(\theta) = \sqrt{\frac{\sigma_\theta}{\sigma_x}}$$

$$\text{so} \quad \sin(\theta) = \sqrt{1 - \frac{\sigma_\theta}{\sigma_x}}$$

$$\tau_\theta = -\sigma_x \sin(\theta) \cos(\theta)$$

$$\frac{\tau_\theta}{\sigma_x} = -\sqrt{1 - \frac{\sigma_\theta}{\sigma_x}} \sqrt{\frac{\sigma_\theta}{\sigma_x}}$$

$$\left(\frac{\tau_\theta}{\sigma_x}\right)^2 = \frac{\sigma_\theta}{\sigma_x} - \left(\frac{\sigma_\theta}{\sigma_x}\right)$$

$$\left(\frac{23}{\sigma_x}\right)^2 = \frac{65}{\sigma_x} - \left(\frac{65}{\sigma_x}\right)^2$$

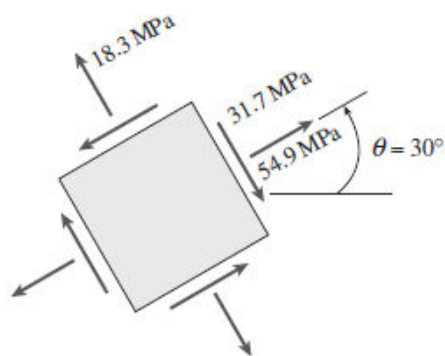
$$\left(\frac{65}{\sigma_x}\right)^2 - \left(\frac{65}{\sigma_x}\right) + \left(\frac{23}{\sigma_x}\right)^2 = 0$$

$$\frac{-(-4754 + 65\sigma_x)}{\sigma_x^2} = 0$$

$$\sigma_x = \frac{4754}{65}$$

$$\sigma_x = 73.1 \text{ MPa} \quad \sigma_\theta = 65 \text{ MPa}$$

$$\theta = \arccos\left(\sqrt{\frac{\sigma_\theta}{\sigma_x}}\right) \quad \theta = 19.5^\circ$$



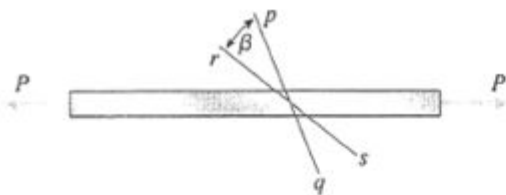
Now find  $\sigma_\theta$  and  $\tau_\theta$  for  $\theta = 30^\circ$ :

$$\sigma_{\theta 1} = \sigma_x \cos^2(\theta) \quad \sigma_{\theta 1} = 54.9 \text{ MPa} \quad \leftarrow$$

$$\tau_\theta = -\sigma_x \sin(\theta) \cos(\theta) \quad \tau_\theta = -31.7 \text{ MPa} \quad \leftarrow$$

$$\sigma_{\theta 2} = \sigma_x \cos^2\left(\theta + \frac{\pi}{2}\right) \quad \sigma_{\theta 2} = 18.3 \text{ MPa} \quad \leftarrow$$

**Problem 2.6-19**



Eq. (2-29a)

$$\sigma_\theta = \sigma_x \cos^2 \theta$$

$$\beta = 30^\circ$$

$$\text{PLANE } pq: \sigma_1 = \sigma_x \cos^2 \theta_1 \quad \sigma_1 = 7500 \text{ psi}$$

$$\text{PLANE } rs: \sigma_2 = \sigma_x \cos^2 (\theta_1 + \beta) \quad \sigma_2 = 2500 \text{ psi}$$

Equate  $\sigma_x$  from  $\sigma_1$  and  $\sigma_2$ :

$$\sigma_x = \frac{\sigma_1}{\cos^2 \theta_1} = \frac{\sigma_2}{\cos^2 (\theta_1 + \beta)} \quad (\text{Eq. 1})$$

or

$$\frac{\cos^2 \theta_1}{\cos^2 (\theta_1 + \beta)} = \frac{\sigma_1}{\sigma_2} \frac{\cos \theta_1}{\cos (\theta_1 + \beta)} = \sqrt{\frac{\sigma_1}{\sigma_2}} \quad (\text{Eq. 2})$$

SUBSTITUTE NUMERICAL VALUES INTO EQ. (2):

$$\frac{\cos \theta_1}{\cos (\theta_1 + 30^\circ)} = \sqrt{\frac{7500 \text{ psi}}{2500 \text{ psi}}} = \sqrt{3} = 1.7321$$

Solve by iteration or a computer program:

$$\theta_1 = 30^\circ$$

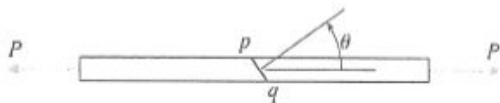
MAXIMUM NORMAL STRESS (FROM EQ. 1)

$$\begin{aligned} \sigma_{\max} = \sigma_x &= \frac{\sigma_1}{\cos^2 \theta_1} = \frac{7500 \text{ psi}}{\cos^2 30^\circ} \\ &= 10,000 \text{ psi} \quad \leftarrow \end{aligned}$$

MAXIMUM SHEAR STRESS

$$\tau_{\max} = \frac{\sigma_x}{2} = 5,000 \text{ psi} \quad \leftarrow$$

**Problem 2.6-20**



$$25^\circ < \theta < 45^\circ$$

$$A = 225 \text{ mm}^2$$

On glued joint:  $\sigma_{\text{allow}} = 5.0 \text{ MPa}$

$$\tau_{\text{allow}} = 3.0 \text{ MPa}$$

ALLOWABLE STRESS  $\sigma_x$  IN TENSION

$$\sigma_\theta = \sigma_x \cos^2 \theta \quad \sigma_x = \frac{\sigma_\theta}{\cos^2 \theta} = \frac{5.0 \text{ MPa}}{\cos^2 \theta} \quad (1)$$

$$\tau_\theta = -\sigma_x \sin \theta \cos \theta$$

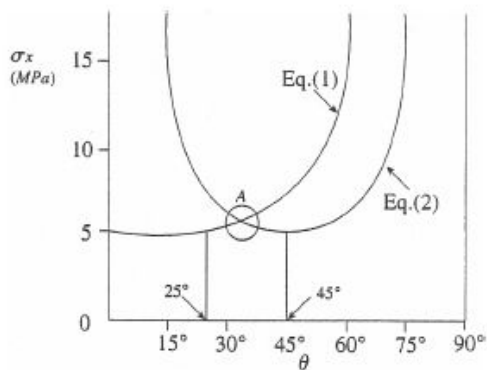
Since the direction of  $\tau_\theta$  is immaterial, we can write:

$$|\tau_\theta| = \sigma_x \sin \theta \cos \theta$$

or

$$\sigma_x = \frac{|\tau_\theta|}{\sin \theta \cos \theta} = \frac{3.0 \text{ MPa}}{\sin \theta \cos \theta} \quad (2)$$

GRAPH OF EQS. (1) AND (2)



(a) DETERMINE ANGLE  $\theta$  FOR LARGEST LOAD

Point A gives the largest value of  $\sigma_x$  and hence the largest load. To determine the angle  $\theta$  corresponding to point A, we equate Eqs. (1) and (2).

$$\frac{5.0 \text{ MPa}}{\cos^2 \theta} = \frac{3.0 \text{ MPa}}{\sin \theta \cos \theta}$$

$$\tan \theta = \frac{3.0}{5.0} \quad \theta = 30.96^\circ \quad \leftarrow$$

(b) DETERMINE THE MAXIMUM LOAD

From Eq. (1) or Eq. (2):

$$\sigma_x = \frac{5.0 \text{ MPa}}{\cos^2 \theta} = \frac{3.0 \text{ MPa}}{\sin \theta \cos \theta} = 6.80 \text{ MPa}$$

$$P_{\text{max}} = \sigma_x A = (6.80 \text{ MPa})(225 \text{ mm}^2) \\ = 1.53 \text{ kN} \quad \leftarrow$$

### Problem 2.6-21

#### NUMERICAL DATA

$$\alpha = 55 (10^{-6}) \quad E = 400 \text{ ksi} \quad L = 2 \text{ ft} \quad \Delta T = 100 \quad k = 18 \text{ k/in.} \quad b = 0.75 \text{ in.} \quad h = 1.5 \text{ in.}$$

$$\sigma_\theta = -950 \text{ psi} \quad \sigma_a = -1000 \text{ psi} \quad \tau_a = -560 \text{ psi} \quad L_\theta = 1.5 \text{ ft} \quad A = bh \quad f = \frac{1}{k} = 5.556 \times 10^{-5} \text{ in./lb}$$

(a) FIND  $\theta$  AND  $T_\theta$

$$R_2 = \text{redundant} \quad R_2 = \frac{-\alpha \Delta T L}{\left(\frac{L}{EA}\right) + f} = -1.212 \times 10^3 \text{ lb} \quad \sigma_x = \frac{R_2}{A} = -1077.551 \text{ psi} \quad \sqrt{\frac{\sigma_\theta}{\sigma_x}} = 0.939$$

$$\theta = \arccos\left(\sqrt{\frac{\sigma_\theta}{\sigma_x}}\right) = 0.351 \quad \cos(2\theta) = 0.763 \quad \theta = 20.124^\circ$$

$$\sigma_x \cos(\theta)^2 = -950 \text{ psi} \quad \text{or} \quad \frac{\sigma_x}{2}(1 + \cos(2\theta)) = -950 \text{ psi} \quad \sigma_y = \sigma_x \cos\left(\theta + \frac{\pi}{2}\right)^2 = -127.551 \text{ psi}$$

$$\theta = 0.351 \quad \theta = 20.124^\circ \quad \sigma_x = -1077.551 \text{ psi} \quad 2\theta = 0.702$$

$$\tau_\theta = -\sigma_x \sin(\theta) \cos(\theta) = 348.1 \text{ psi} \quad \text{or} \quad \tau_\theta = \frac{-\sigma_x}{2} \sin(2\theta) = 348.1 \text{ psi}$$

$$\boxed{\tau_\theta = 348 \text{ psi}} \quad \boxed{\theta = 20.1^\circ}$$

(b) FIND  $\sigma_{x1}$  AND  $\sigma_{y1}$

$$\sigma_{x1} = \sigma_x \cos(\theta)^2 \quad \sigma_{y1} = \sigma_x \cos\left(\theta + \frac{\pi}{2}\right)^2$$

$$\boxed{\sigma_{x1} = -950 \text{ psi}} \quad \boxed{\sigma_{y1} = -127.6 \text{ psi}}$$

(c) GIVEN  $L = 2 \text{ ft}$ , FIND  $k_{\max}$

$$k_{\max 1} = \frac{\sigma_a A}{-\alpha \Delta T L - \sigma_a A \left(\frac{L}{EA}\right)} = 15625 \text{ lb/in.} < \text{controls (based on } \sigma_{\text{allow}})$$

$$\text{or } k_{\max 2} = \frac{2\tau_a A}{-\alpha \Delta T L - 2\tau_a A \left(\frac{L}{EA}\right)} = 19444.444 \text{ lb/in.} < \text{based on allowable shear stress}$$

$$\boxed{k_{\max} = 15625 \text{ lb/in.}}$$

(d) GIVEN ALLOWABLE NORMAL AND SHEAR STRESSES, FIND  $L_{\max}$

$$k = 18000 \text{ lb/in.}$$

$$\sigma_x = \frac{R_2}{A} \quad \sigma_a A = \frac{-\alpha \Delta T L}{\left(\frac{L}{EA}\right) + f} \quad L_{\max 1} = \frac{\sigma_a A(f)}{-\left(\alpha \Delta T + \frac{\sigma_a}{E}\right)} = 1.736 \text{ ft} < \text{controls (based on } \sigma_{\text{allow}})$$

$$\text{or } L_{\max 2} = \frac{2\tau_a A(f)}{-\left(\alpha \Delta T + \frac{2\tau_a}{E}\right)} = 2.16 \text{ ft} < \text{based on } T_{\text{allow}}$$

$$\boxed{L_{\max} = 1.736 \text{ ft}}$$

- (c) FIND  $\Delta T_{\max}$  GIVEN  $L$ ,  $k$ , AND ALLOWABLE STRESSES  $k = 18000 \text{ lb/in.}$   $L = 2 \text{ ft}$   $\sigma_a = -1000 \text{ psi}$   
 $\tau_a = -560 \text{ psi}$

$$\Delta T_{\max 1} = \frac{\left(\frac{L}{EA} + f\right) \sigma_a A}{-\alpha L} = 92.803^\circ\text{F} \quad < \text{based on } \sigma_{\text{allow}} \quad \Delta T = 100$$

$$\Delta T_{\max 2} = \frac{\left(\frac{L}{EA} + f\right) 2 \tau_a A}{-\alpha L} = 103.939^\circ\text{F} < \text{based on } T_{\text{allow}}$$

$$\boxed{\Delta T_{\max} = 92.8^\circ\text{F}}$$

**Problem 2.6-22**

$$b = 50\text{mm} \quad \alpha = 35\text{deg}$$

$$\sigma_a = 11.5\text{MPa} \quad \tau_a = 4.5\text{MPa}$$

$$\sigma_{ga} = 3.5\text{MPa} \quad \tau_{ga} = 1.25\text{MPa}$$

Rotate stress element CW by angle  $\theta$  to align with glue joint (see fig.)

$$\theta = \alpha - 90\text{deg} = -55\text{deg}$$

Plane stress transformations  $\sigma_x = \frac{P}{A} \quad A = b^2 = 2500\text{mm}^2$

$$\sigma_\theta = \sigma_x \cdot \cos(\theta)^2 \quad \tau_\theta = -\sigma_x \cdot \sin(\theta) \cdot \cos(\theta)$$

Equate  $\sigma_\theta$  and  $\tau_\theta$  to allowable values and solve for P - min. P controls

$$\sigma_{\max} = \sigma_x$$

$$P_{\max1} = \sigma_a \cdot A = 28.75\text{ kN}$$

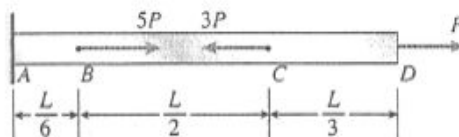
$$\tau_{\max} = -\left(\frac{P}{A}\right) \cdot \sin(45\text{deg}) \cdot \cos(45\text{deg})$$

$$P_{\max2} = \frac{\tau_a}{2} \cdot A = 5.625\text{ kN} \quad < \text{shear in wood controls}$$

$$P_{\max3} = \frac{\sigma_{ga} \cdot A}{\cos(\theta)^2} = 26.597\text{ kN}$$

$$P_{\max4} = \frac{\tau_{ga} \cdot A}{-\sin(\theta) \cdot \cos(\theta)} = 6.651\text{ kN}$$

**Problem 2.7-1**



$$P = 6 \text{ k}$$

$$L = 52 \text{ in.}$$

$$E = 10.4 \times 10^6 \text{ psi}$$

$$A = 2.76 \text{ in.}^2$$

INTERNAL AXIAL FORCES

$$N_{AB} = 3P \quad N_{BC} = -2P \quad N_{CD} = P$$

LENGTHS

$$L_{AB} = \frac{L}{6} \quad L_{BC} = \frac{L}{2} \quad L_{CD} = \frac{L}{3}$$

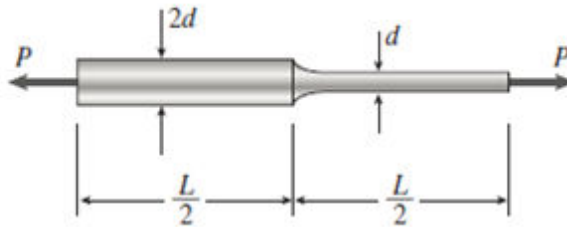
(a) STRAIN ENERGY OF THE BAR (EQ. 2-40)

$$\begin{aligned} U &= \sum \frac{N_i^2 L_i}{2E_i A_i} \\ &= \frac{1}{2EA} \left[ (3P)^2 \left( \frac{L}{6} \right) + (-2P)^2 \left( \frac{L}{2} \right) + (P)^2 \left( \frac{L}{3} \right) \right] \\ &= \frac{P^2 L}{2EA} \left( \frac{23}{6} \right) = \frac{23P^2 L}{12EA} \quad \leftarrow \end{aligned}$$

(b) SUBSTITUTE NUMERICAL VALUES:

$$\begin{aligned} U &= \frac{23(6 \text{ k})^2(52 \text{ in.})}{12(10.4 \times 10^6 \text{ psi})(2.76 \text{ in.}^2)} \\ &= 125 \text{ in.-lb} \quad \leftarrow \end{aligned}$$

**Problem 2.7-2**



(a) STRAIN ENERGY OF THE BAR

Add the strain energies of the two segments of the bar (see Eq. 2-40).

$$U = \sum_{i=1}^2 \frac{N_i^2 L_i}{2 E_i A_i} = \frac{P^2 (L/2)}{2E} \left[ \frac{1}{\frac{\pi}{4}(2d)^2} + \frac{1}{\frac{\pi}{4}(d^2)} \right]$$

$$= \frac{P^2 L}{\pi E} \left( \frac{1}{4d^2} + \frac{1}{d^2} \right) = \frac{5P^2 L}{4\pi E d^2} \quad \leftarrow$$

(b) SUBSTITUTE NUMERICAL VALUES:

$$P = 27 \text{ kN} \quad L = 600 \text{ mm}$$

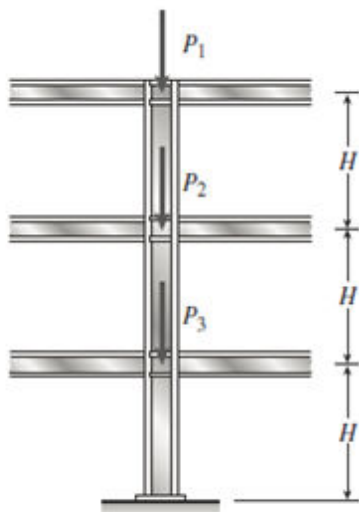
$$d = 40 \text{ mm} \quad E = 105 \text{ GPa}$$

$$U = \frac{5(27 \text{ kN})^2(600 \text{ mm})}{4\pi(105 \text{ GPa})(40 \text{ mm})^2}$$

$$= 1.036 \text{ N} \cdot \text{m} = 1.036 \text{ J} \quad \leftarrow$$



**Problem 2.7-3**



$$H = 10.5 \text{ ft} \quad E = 30 \times 10^6 \text{ psi}$$

$$A = 15.5 \text{ in.}^2 \quad P_1 = 40 \text{ k}$$

$$P_2 = P_3 = 60 \text{ k}$$

To find the strain energy of the column, add the strain energies of the three segments (see Eq. 2-40).

$$\text{Upper segment: } N_1 = -P_1$$

$$\text{Middle segment: } N_2 = -(P_1 + P_2)$$

$$\text{Lower segment: } N_3 = -(P_1 + P_2 + P_3)$$

STRAIN ENERGY

$$U = \sum \frac{N_i^2 L_i}{2E_i A_i}$$

$$= \frac{H}{2EA} [P_1^2 + (P_1 + P_2)^2 + (P_1 + P_2 + P_3)^2]$$

$$= \frac{H}{2EA} [Q]$$

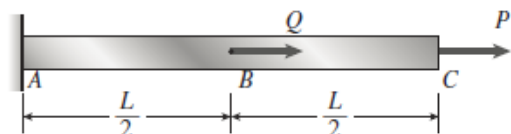
$$[Q] = (40 \text{ k})^2 + (100 \text{ k})^2 + (160 \text{ k})^2 = 37,200 \text{ k}^2$$

$$2EA = 2(30 \times 10^6 \text{ psi})(15.5 \text{ in.}^2) = 930 \times 10^6 \text{ lb}$$

$$U = \frac{(10.5 \text{ ft})(12 \text{ in./ft})}{930 \times 10^6 \text{ lb}} [37,200 \text{ k}^2]$$

$$= 5040 \text{ in.-lb} \quad \leftarrow$$

**Problem 2.7-4**



(a) FORCE  $P$  ACTS ALONE ( $Q = 0$ )

$$U_1 = \frac{P^2 L}{2EA} \quad \leftarrow$$

(b) FORCE  $Q$  ACTS ALONE ( $P = 0$ )

$$U_2 = \frac{Q^2(L/2)}{2EA} = \frac{Q^2 L}{4EA} \quad \leftarrow$$

(c) FORCES  $P$  AND  $Q$  ACT SIMULTANEOUSLY

$$\text{Segment } BC: U_{BC} = \frac{P^2(L/2)}{2EA} = \frac{P^2 L}{4EA}$$

$$\begin{aligned} \text{Segment } AB: U_{AB} &= \frac{(P+Q)^2(L/2)}{2EA} \\ &= \frac{P^2 L}{4EA} + \frac{PQL}{2EA} + \frac{Q^2 L}{4EA} \end{aligned}$$

$$U_3 = U_{BC} + U_{AB} = \frac{P^2 L}{2EA} + \frac{PQL}{2EA} + \frac{Q^2 L}{4EA} \quad \leftarrow$$

(Note that  $U_3$  is *not* equal to  $U_1 + U_2$ . In this case,  $U_3 > U_1 + U_2$ . However, if  $Q$  is reversed in direction,  $U_3 < U_1 + U_2$ . Thus,  $U_3$  may be larger or smaller than  $U_1 + U_2$ .)

**Problem 2.7-5**

**DATA:**

Material	Weight density (lb/in. <sup>3</sup> )	Modulus of elasticity (ksi)	Proportional limit (psi)
Mild steel	0.284	30,000	36,000
Tool steel	0.284	30,000	75,000
Aluminum	0.0984	10,500	60,000
Rubber (soft)	0.0405	0.300	300

**STRAIN ENERGY PER UNIT VOLUME**

$$U = \frac{P^2 L}{2EA} \quad \text{Volume } V = AL$$

$$\text{Stress } \sigma = \frac{P}{A}$$

$$u = \frac{U}{V} = \frac{\sigma^2_{PL}}{2E}$$

At the proportional limit:

$$u = u_R = \text{modulus of resistance}$$

$$u_R = \frac{\sigma^2_{PL}}{2E} \quad (\text{Eq. 1})$$

**STRAIN ENERGY PER UNIT WEIGHT**

$$U = \frac{P^2 L}{2EA} \quad \text{Weight } W = \gamma AL$$

$\gamma$  = weight density

$$u_W = \frac{U}{W} = \frac{\sigma^2}{2\gamma E}$$

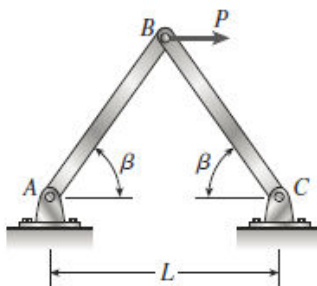
At the proportional limit:

$$u_W = \frac{\sigma^2_{PL}}{2\gamma E} \quad (\text{Eq. 2})$$

**RESULTS**

	$u_R$ (psi)	$u_W$ (in.)
Mild steel	22	76
Tool steel	94	330
Aluminum	171	1740
Rubber (soft)	150	3700

**Problem 2.7-6**



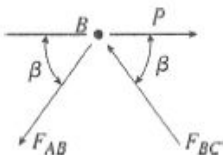
$$\beta = 60^\circ$$

$$L_{AB} = L_{BC} = L$$

$$\sin \beta = \sqrt{3}/2$$

$$\cos \beta = 1/2$$

FREE-BODY DIAGRAM OF JOINT B



$$\Sigma F_{\text{vert}} = 0 \quad \uparrow + \quad \downarrow -$$

$$-F_{AB} \sin \beta + F_{BC} \sin \beta = 0$$

$$F_{AB} = F_{BC} \quad (\text{Eq. 1})$$

$$\Sigma F_{\text{horiz}} = 0 \rightarrow \leftarrow$$

$$-F_{AB} \cos \beta - F_{BC} \cos \beta + P = 0$$

$$F_{AB} = F_{BC} = \frac{P}{2 \cos \beta} = \frac{P}{2(1/2)} = P \quad (\text{Eq. 2})$$

Axial forces:  $N_{AB} = P$  (tension)

$N_{BC} = -P$  (compression)

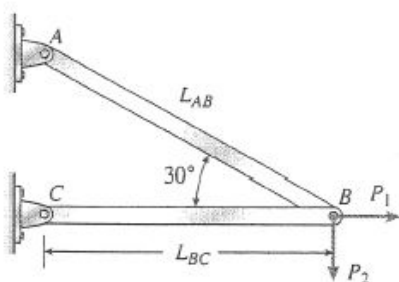
(a) STRAIN ENERGY OF TRUSS (EQ. 2-40)

$$U = \sum \frac{N_i^2 L_i}{2E_i A_i} = \frac{(N_{AB})^2 L}{2EA} + \frac{(N_{BC})^2 L}{2EA} = \frac{P^2 L}{EA} \quad \leftarrow$$

(b) HORIZONTAL DISPLACEMENT OF JOINT B (EQ. 2-42)

$$\delta_B = \frac{2U}{P} = \frac{2}{P} \left( \frac{P^2 L}{EA} \right) = \frac{2PL}{EA} \quad \leftarrow$$

**Problem 2.7-7**



$$P_1 = 300 \text{ lb}$$

$$P_2 = 900 \text{ lb}$$

$$A = 2.4 \text{ in.}^2$$

$$E = 30 \times 10^6 \text{ psi}$$

$$L_{BC} = 60 \text{ in.}$$

$$\beta = 30^\circ$$

$$\sin \beta = \sin 30^\circ = \frac{1}{2}$$

$$\cos \beta = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$L_{AB} = \frac{L_{BC}}{\cos 30^\circ} = \frac{120}{\sqrt{3}} \text{ in.} = 69.282 \text{ in.}$$

$$2EA = 2(30 \times 10^6 \text{ psi})(2.4 \text{ in.}^2) = 144 \times 10^6 \text{ lb}$$

FORCES  $F_{AB}$  AND  $F_{BC}$  IN THE BARS

From equilibrium of joint B:

$$F_{AB} = 2P_2 = 1800 \text{ lb}$$

$$F_{BC} = P_1 - P_2\sqrt{3} = 300 \text{ lb} - 1558.8 \text{ lb}$$

Force	$P_1$ alone	$P_2$ alone	$P_1$ and $P_2$
$F_{AB}$	0	1800 lb	1800 lb
$F_{BC}$	300 lb	-1558.8 lb	-1258.8 lb

(a) LOAD  $P_1$  ACTS ALONE

$$U_1 = \frac{(F_{BC})^2 L_{BC}}{2EA} = \frac{(300 \text{ lb})^2 (60 \text{ in.})}{144 \times 10^6 \text{ lb}}$$

$$= 0.0375 \text{ in.-lb} \quad \leftarrow$$

(b) LOAD  $P_2$  ACTS ALONE

$$U_2 = \frac{1}{2EA} \left[ (F_{AB})^2 L_{AB} + (F_{BC})^2 L_{BC} \right]$$

$$= \frac{1}{2EA} \left[ (1800 \text{ lb})^2 (69.282 \text{ in.}) \right.$$

$$\left. + (-1558.8 \text{ lb})^2 (60 \text{ in.}) \right]$$

$$= \frac{370.265 \times 10^6 \text{ lb}^2 \text{-in.}}{144 \times 10^6 \text{ lb}} = 2.57 \text{ in.-lb} \quad \leftarrow$$

(c) LOADS  $P_1$  AND  $P_2$  ACT SIMULTANEOUSLY

$$U_3 = \frac{1}{2EA} \left[ (F_{AB})^2 L_{AB} + (F_{BC})^2 L_{BC} \right]$$

$$= \frac{1}{2EA} \left[ (1800 \text{ lb})^2 (69.282 \text{ in.}) \right.$$

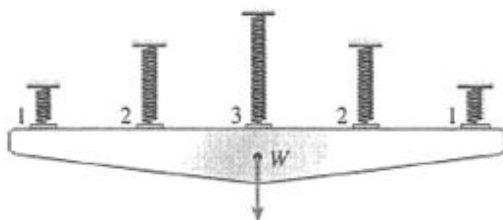
$$\left. + (-1258.8 \text{ lb})^2 (60 \text{ in.}) \right]$$

$$= \frac{319.548 \times 10^6 \text{ lb}^2 \text{-in.}}{144 \times 10^6 \text{ lb}}$$

$$= 2.22 \text{ in.-lb} \quad \leftarrow$$

**NOTE:** The strain energy  $U_3$  is *not* equal to  $U_1 + U_2$ .

**Problem 2.7-8**



$$k_1 = 3k$$

$$k_2 = 1.5k$$

$$k_3 = k$$

$\delta$  = downward displacement of rigid bar

For a spring:  $U = \frac{k\delta^2}{2}$  Eq. (2-38b)

(a) STRAIN ENERGY  $U$  OF ALL SPRINGS

$$U = 2\left(\frac{3k\delta^2}{2}\right) + 2\left(\frac{1.5k\delta^2}{2}\right) + \frac{k\delta^2}{2} = 5k\delta^2 \quad \leftarrow$$

(b) DISPLACEMENT  $\delta$

Work done by the weight  $W$  equals  $\frac{W\delta}{2}$

Strain energy of the springs equals  $5k\delta^2$

$$\therefore \frac{W\delta}{2} = 5k\delta^2 \quad \text{and} \quad \delta = \frac{W}{10k} \quad \leftarrow$$

(c) FORCES IN THE SPRINGS

$$F_1 = 3k\delta = \frac{3W}{10} \quad F_2 = 1.5k\delta = \frac{3W}{20} \quad \leftarrow$$

$$F_3 = k\delta = \frac{W}{10} \quad \leftarrow$$

(d) NUMERICAL VALUES

$$W = 600 \text{ N} \quad k = 7.5 \text{ N/mm} = 7500 \text{ N/mm}$$

$$U = 5k\delta^2 = 5k\left(\frac{W}{10k}\right)^2 = \frac{W^2}{20k}$$

$$= 2.4 \text{ N}\cdot\text{m} = 2.4 \text{ J} \quad \leftarrow$$

$$\delta = \frac{W}{10k} = 8.0 \text{ mm} \quad \leftarrow$$

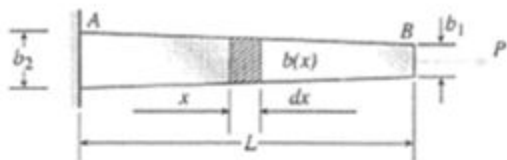
$$F_1 = \frac{3W}{10} = 180 \text{ N} \quad \leftarrow$$

$$F_2 = \frac{3W}{20} = 90 \text{ N} \quad \leftarrow$$

$$F_3 = \frac{W}{10} = 60 \text{ N} \quad \leftarrow$$

NOTE:  $W = 2F_1 + 2F_2 + F_3 = 600 \text{ N}$  (Check)

**Problem 2.7-9**



$$b(x) = b_2 - \frac{(b_2 - b_1)x}{L}$$

$$A(x) = tb(x)$$

$$= t \left[ b_2 - \frac{(b_2 - b_1)x}{L} \right]$$

(a) STRAIN ENERGY OF THE BAR

$$U = \int \frac{[N(x)]^2 dx}{2EA(x)} \quad (\text{Eq. 2-41})$$

$$= \int_0^L \frac{P^2 dx}{2Et b(x)} = \frac{P^2}{2Et} \int_0^L \frac{dx}{b_2 - (b_2 - b_1)\frac{x}{L}} \quad (1)$$

From Appendix C:  $\int \frac{dx}{a + bx} = \frac{1}{b} \ln(a + bx)$

Apply this integration formula to Eq. (1):

$$U = \frac{P^2}{2Et} \left[ \frac{1}{-(b_2 - b_1)(\frac{1}{L})} \ln \left[ b_2 - \frac{(b_2 - b_1)x}{L} \right] \right]_0^L$$

$$= \frac{P^2}{2Et} \left[ \frac{-L}{(b_2 - b_1)} \ln b_1 - \frac{-L}{(b_2 - b_1)} \ln b_2 \right]$$

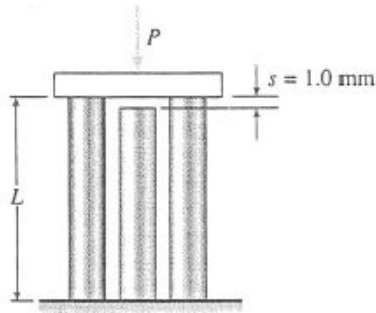
$$U = \frac{P^2 L}{2Et(b_2 - b_1)} \ln \frac{b_2}{b_1} \quad \leftarrow$$

(b) ELONGATION OF THE BAR (Eq. 2-42)

$$\delta = \frac{2U}{P} = \frac{PL}{Et(b_2 - b_1)} \ln \frac{b_2}{b_1} \quad \leftarrow$$

**NOTE:** This result agrees with the formula derived in Prob. 2.3-17.

**Problem 2.7-10**



$$s = 1.0 \text{ mm}$$

$$L = 1.0 \text{ m}$$

For each bar:

$$A = 3000 \text{ mm}^2$$

$$E = 45 \text{ GPa}$$

$$\frac{EA}{L} = 135 \times 10^6 \text{ N/m}$$

(a) LOAD  $P_1$  REQUIRED TO CLOSE THE GAP

$$\text{In general, } \delta = \frac{PL}{EA} \text{ and } P = \frac{EA\delta}{L}$$

For two bars, we obtain:

$$P_1 = 2\left(\frac{EAs}{L}\right) = 2(135 \times 10^6 \text{ N/m})(1.0 \text{ mm})$$

$$P_1 = 270 \text{ kN} \quad \leftarrow$$

(b) DISPLACEMENT  $\delta$  FOR  $P = 400 \text{ kN}$

Since  $P > P_1$ , all three bars are compressed.  
The force  $P$  equals  $P_1$  plus the additional force required to compress all three bars by the amount  $\delta - s$ .

$$P = P_1 + 3\left(\frac{EA}{L}\right)(\delta - s)$$

$$\text{or } 400 \text{ kN} = 270 \text{ kN} + 3(135 \times 10^6 \text{ N/m})(\delta - 0.001 \text{ m})$$

$$\text{Solving, we get } \delta = 1.321 \text{ mm} \quad \leftarrow$$

(c) STRAIN ENERGY  $U$  FOR  $P = 400 \text{ kN}$

$$U = \sum \frac{EA\delta^2}{2L}$$

$$\text{Outer bars: } \delta = 1.321 \text{ mm}$$

$$\begin{aligned} \text{Middle bar: } \delta &= 1.321 \text{ mm} - s \\ &= 0.321 \text{ mm} \end{aligned}$$

$$U = \frac{EA}{2L}[2(1.321 \text{ mm})^2 + (0.321 \text{ mm})^2]$$

$$= \frac{1}{2}(135 \times 10^6 \text{ N/m})(3.593 \text{ mm}^2)$$

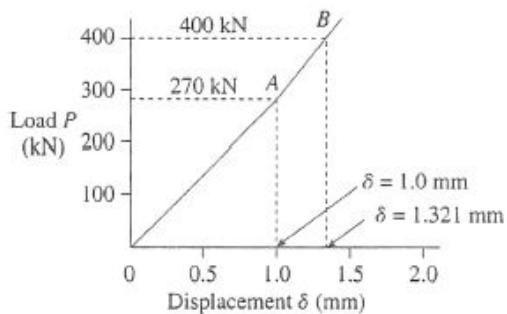
$$= 243 \text{ N}\cdot\text{m} = 243 \text{ J} \quad \leftarrow$$

(d) LOAD-DISPLACEMENT DIAGRAM

$$U = 243 \text{ J} = 243 \text{ N}\cdot\text{m}$$

$$\frac{P\delta}{2} = \frac{1}{2}(400 \text{ kN})(1.321 \text{ mm}) = 264 \text{ N}\cdot\text{m}$$

The strain energy  $U$  is *not* equal to  $\frac{P\delta}{2}$  because the load-displacement relation is not linear.

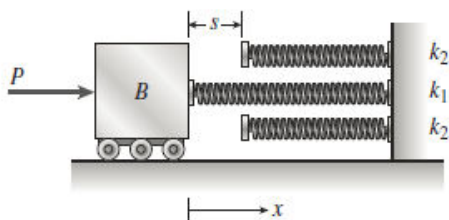


$U = \text{area under line } OAB.$

$\frac{P\delta}{2} = \text{area under a straight line from } O \text{ to } B, \text{ which is larger than } U.$



### Problem 2.7-11



Force  $P_0$  required to close the gap:

$$P_0 = k_1 s \quad (1)$$

FORCE-DISPLACEMENT RELATION BEFORE GAP IS CLOSED

$$P = k_1 x \quad (0 \leq x \leq s) \quad (0 \leq P \leq P_0) \quad (2)$$

FORCE-DISPLACEMENT RELATION AFTER GAP IS CLOSED

All three springs are compressed. Total stiffness equals  $k_1 + 2k_2$ . Additional displacement equals  $x - s$ . Force  $P$  equals  $P_0$  plus the force required to compress all three springs by the amount  $x - s$ .

$$\begin{aligned} P &= P_0 + (k_1 + 2k_2)(x - s) \\ &= k_1 s + (k_1 + 2k_2)x - k_1 s - 2k_2 s \end{aligned}$$

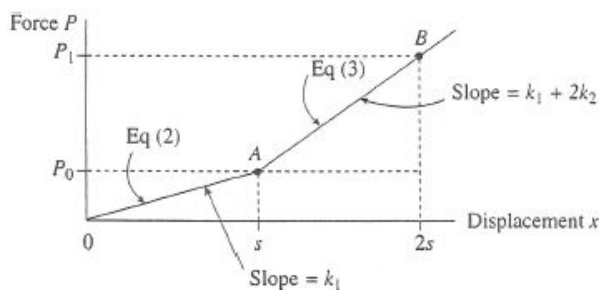
$$P = (k_1 + 2k_2)x - 2k_2 s \quad (x \geq s); (P \geq P_0) \quad (3)$$

$P_1$  = force  $P$  when  $x = 2s$

Substitute  $x = 2s$  into Eq. (3):

$$P_1 = 2(k_1 + k_2)s \quad (4)$$

(a) FORCE-DISPLACEMENT DIAGRAM



(b) STRAIN ENERGY  $U_1$  WHEN  $x = 2s$

$U_1$  = Area below force-displacement curve

$$= \text{triangle} + \text{rectangle} + \text{triangle}$$

$$= \frac{1}{2}P_0 s + P_0 s + \frac{1}{2}(P_1 - P_0)s = P_0 s + \frac{1}{2}P_1 s$$

$$= k_1 s^2 + (k_1 + k_2)s^2$$

$$U_1 = (2k_1 + k_2)s^2 \quad \leftarrow \quad (5)$$

(c) STRAIN ENERGY  $U_1$  IS NOT EQUAL TO  $\frac{P\delta}{2}$

$$\text{For } \delta = 2s: \frac{P\delta}{2} = \frac{1}{2}P_1(2s) = P_1 s = 2(k_1 + k_2)s^2$$

(This quantity is greater than  $U_1$ .)

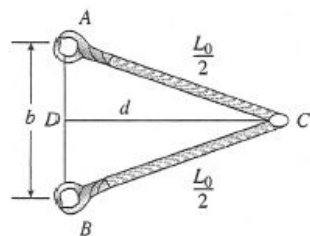
$U_1$  = area under line  $OAB$ .

$\frac{P\delta}{2}$  = area under a straight line from  $O$  to  $B$ , which is larger than  $U_1$ .

Thus,  $\frac{P\delta}{2}$  is *not* equal to the strain energy because the force-displacement relation is not linear.

### Problem 2.7-12

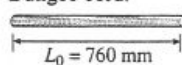
DIMENSIONS BEFORE THE LOAD  $P$  IS APPLIED



$$L_0 = 760 \text{ mm} \quad \frac{L_0}{2} = 380 \text{ mm}$$

$$b = 380 \text{ mm}$$

Bungee cord:

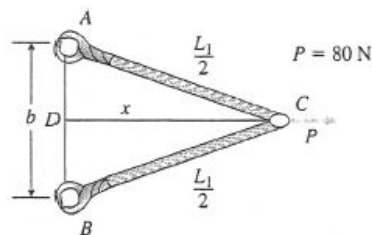


$$k = 140 \text{ N/m}$$

From triangle  $ACD$ :

$$d = \frac{1}{2} \sqrt{L_0^2 - b^2} = 329.09 \text{ mm} \quad (1)$$

DIMENSIONS AFTER THE LOAD  $P$  IS APPLIED



Let  $x$  = distance  $CD$

Let  $L_1$  = stretched length of bungee cord

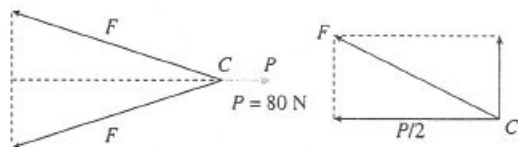
From triangle  $ACD$ :

$$\frac{L_1}{2} = \sqrt{\left(\frac{b}{2}\right)^2 + x^2} \quad (2)$$

$$L_1 = \sqrt{b^2 + 4x^2} \quad (3)$$

EQUILIBRIUM AT POINT  $C$

Let  $F$  = tensile force in bungee cord



$$\begin{aligned} \frac{F}{P/2} &= \frac{L_1/2}{x} \quad F = \left(\frac{P}{2}\right) \left(\frac{L_1}{2}\right) \left(\frac{1}{x}\right) \\ &= \frac{P}{2} \sqrt{1 + \left(\frac{b}{2x}\right)^2} \end{aligned} \quad (4)$$

ELONGATION OF BUNGEE CORD

Let  $\delta$  = elongation of the entire bungee cord

$$\delta = \frac{F}{k} = \frac{P}{2k} \sqrt{1 + \frac{b^2}{4x^2}} \quad (5)$$

Final length of bungee cord = original length +  $\delta$

$$L_1 = L_0 + \delta = L_0 + \frac{P}{2k} \sqrt{1 + \frac{b^2}{4x^2}} \quad (6)$$

SOLUTION OF EQUATIONS

Combine Eqs. (6) and (3):

$$L_1 = L_0 + \frac{P}{2k} \sqrt{1 + \frac{b^2}{4x^2}} = \sqrt{b^2 + 4x^2}$$

$$\text{or } L_1 = L_0 + \frac{P}{4kx} \sqrt{b^2 + 4x^2} = \sqrt{b^2 + 4x^2}$$

$$L_0 = \left(1 - \frac{P}{4kx}\right) \sqrt{b^2 + 4x^2} \quad (7)$$

This equation can be solved for  $x$ .

SUBSTITUTE NUMERICAL VALUES INTO EQ. (7):

$$\begin{aligned} 760 \text{ mm} &= \left[1 - \frac{(80 \text{ N})(1000 \text{ mm/m})}{4(140 \text{ N/m})x}\right] \\ &\quad \times \sqrt{(380 \text{ mm})^2 + 4x^2} \end{aligned} \quad (8)$$

$$760 = \left(1 - \frac{142.857}{x}\right) \sqrt{144,400 + 4x^2} \quad (9)$$

Units:  $x$  is in millimeters

Solve for  $x$  (Use trial-and-error or a computer program):

$$x = 497.88 \text{ mm}$$

(a) STRAIN ENERGY  $U$  OF THE BUNGEE CORD

$$U = \frac{k\delta^2}{2} \quad k = 140 \text{ N/m} \quad P = 80 \text{ N}$$

From Eq. (5):

$$\delta = \frac{P}{2k} \sqrt{1 + \frac{b^2}{4x^2}} = 305.81 \text{ mm}$$

$$U = \frac{1}{2} (140 \text{ N/m}) (305.81 \text{ mm})^2 = 6.55 \text{ N}\cdot\text{m}$$

$$U = 6.55 \text{ J} \quad \leftarrow$$

(b) DISPLACEMENT  $\delta_C$  OF POINT  $C$

$$\begin{aligned} \delta_C &= x - d = 497.88 \text{ mm} - 329.09 \text{ mm} \\ &= 168.8 \text{ mm} \quad \leftarrow \end{aligned}$$

(c) COMPARISON OF STRAIN ENERGY  $U$  WITH THE QUANTITY  $P\delta_C/2$

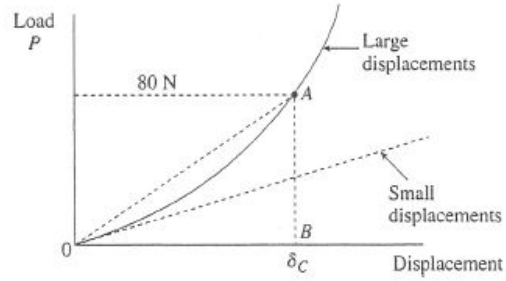
$$U = 6.55 \text{ J}$$

$$\frac{P\delta_C}{2} = \frac{1}{2}(80 \text{ N})(168.8 \text{ mm}) = 6.75 \text{ J}$$

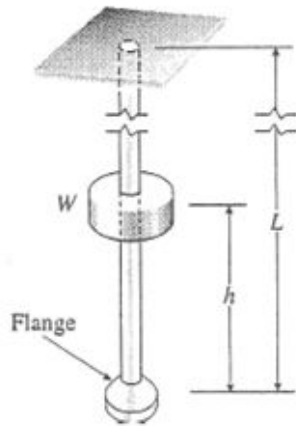
The two quantities are not the same. The work done by the load  $P$  is *not* equal to  $P\delta_C/2$  because the load-displacement relation (see below) is non-linear when the displacements are large. (The *work* done by the load  $P$  is equal to the strain energy because the bungee cord behaves elastically and there are no energy losses.)

$$U = \text{area } OAB \text{ under the curve } OA.$$

$$\frac{P\delta_C}{2} = \text{area of triangle } OAB, \text{ which is greater than } U.$$



**Problem 2.8-1**



$$W = 150 \text{ lb}$$

$$h = 2.0 \text{ in.}$$

$$E = 30 \times 10^6 \text{ psi}$$

$$L = 4.0 \text{ ft} = 48 \text{ in.}$$

$$A = 0.75 \text{ in.}^2$$

(a) DOWNWARD DISPLACEMENT OF FLANGE

$$\delta_{st} = \frac{WL}{EA} = 0.00032 \text{ in.}$$

Eq. (2-53):

$$\begin{aligned} \delta_{\max} &= \delta_{st} \left[ 1 + \left( 1 + \frac{2h}{\delta_{st}} \right)^{1/2} \right] \\ &= 0.0361 \text{ in.} \quad \leftarrow \end{aligned}$$

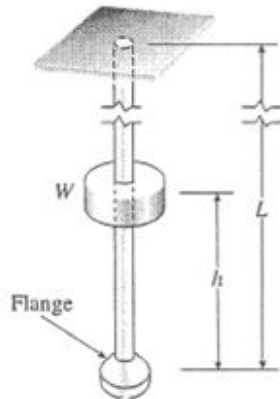
(b) MAXIMUM TENSILE STRESS (EQ. 2-55)

$$\sigma_{\max} = \frac{E\delta_{\max}}{L} = 22,600 \text{ psi} \quad \leftarrow$$

(c) IMPACT FACTOR (EQ. 2-61)

$$\begin{aligned} \text{Impact factor} &= \frac{\delta_{\max}}{\delta_{st}} = \frac{0.0361 \text{ in.}}{0.00032 \text{ in.}} \\ &= 113 \quad \leftarrow \end{aligned}$$

**Problem 2.8-2**



$$M = 80 \text{ kg}$$

$$W = Mg = (80 \text{ kg})(9.81 \text{ m/s}^2) \\ = 784.8 \text{ N}$$

$$h = 0.5 \text{ m} \quad L = 3.0 \text{ m}$$

$$E = 170 \text{ GPa} \quad A = 350 \text{ mm}^2$$

(a) DOWNWARD DISPLACEMENT OF FLANGE

$$\delta_{st} = \frac{WL}{EA} = 0.03957 \text{ mm}$$

$$\text{Eq. (2-53): } \delta_{\max} = \delta_{st} \left[ 1 + \left( 1 + \frac{2h}{\delta_{st}} \right)^{1/2} \right] \\ = 6.33 \text{ mm} \quad \leftarrow$$

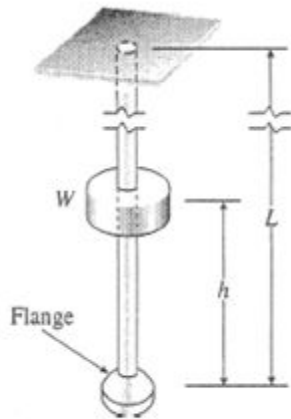
(b) MAXIMUM TENSILE STRESS (Eq. 2-55)

$$\sigma_{\max} = \frac{E\delta_{\max}}{L} = 359 \text{ MPa} \quad \leftarrow$$

(c) IMPACT FACTOR (Eq. 2-61)

$$\text{Impact factor} = \frac{\delta_{\max}}{\delta_{st}} = \frac{6.33 \text{ mm}}{0.03957 \text{ mm}} \\ = 160 \quad \leftarrow$$

**Problem 2.8-3**



$$W = 50 \text{ lb} \quad h = 2.0 \text{ in.}$$

$$L = 3.0 \text{ ft} = 36 \text{ in.}$$

$$E = 30,000 \text{ psi} \quad A = 0.25 \text{ in.}^2$$

(a) DOWNWARD DISPLACEMENT OF FLANGE

$$\delta_{st} = \frac{WL}{EA} = 0.00024 \text{ in.}$$

$$\begin{aligned} \text{Eq. (2-53): } \delta_{\max} &= \delta_{st} \left[ 1 + \left( 1 + \frac{2h}{\delta_{st}} \right)^{1/2} \right] \\ &= 0.0312 \text{ in.} \quad \leftarrow \end{aligned}$$

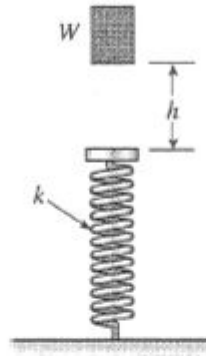
(b) MAXIMUM TENSILE STRESS (EQ. 2-55)

$$\sigma_{\max} = \frac{E\delta_{\max}}{L} = 26,000 \text{ psi} \quad \leftarrow$$

(c) IMPACT FACTOR (EQ. 2-61)

$$\begin{aligned} \text{Impact factor} &= \frac{\delta_{\max}}{\delta_{st}} = \frac{0.0312 \text{ in.}}{0.00024 \text{ in.}} \\ &= 130 \quad \leftarrow \end{aligned}$$

**Problem 2.8-4**



$$W = 5.0 \text{ N} \quad h = 200 \text{ mm} \quad k = 90 \text{ N/m}$$

(a) MAXIMUM SHORTENING OF THE SPRING

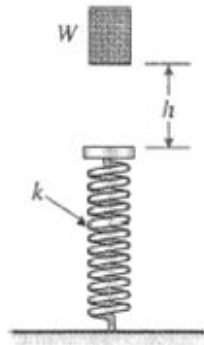
$$\delta_{st} = \frac{W}{k} = \frac{5.0 \text{ N}}{90 \text{ N/m}} = 55.56 \text{ mm}$$

$$\begin{aligned} \text{Eq. (2-53): } \delta_{\max} &= \delta_{st} \left[ 1 + \left( 1 + \frac{2h}{\delta_{st}} \right)^{1/2} \right] \\ &= 215 \text{ mm} \quad \leftarrow \end{aligned}$$

(b) IMPACT FACTOR (EQ. 2-61)

$$\begin{aligned} \text{Impact factor} &= \frac{\delta_{\max}}{\delta_{st}} = \frac{215 \text{ mm}}{55.56 \text{ mm}} \\ &= 3.9 \quad \leftarrow \end{aligned}$$

**Problem 2.8-5**



$$W = 1.0 \text{ lb} \quad h = 12 \text{ in.} \quad k = 0.5 \text{ lb/in.}$$

(a) MAXIMUM SHORTENING OF THE SPRING

$$\delta_{st} = \frac{W}{k} = \frac{1.0 \text{ lb}}{0.5 \text{ lb/in.}} = 2.0 \text{ in.}$$

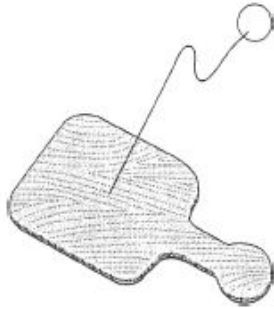
$$\begin{aligned} \text{Eq. (2-53): } \delta_{\max} &= \delta_{st} \left[ 1 + \left( 1 + \frac{2h}{\delta_{st}} \right)^{1/2} \right] \\ &= 9.21 \text{ in.} \quad \leftarrow \end{aligned}$$

(b) IMPACT FACTOR (EQ. 2-61)

$$\begin{aligned} \text{Impact factor} &= \frac{\delta_{\max}}{\delta_{st}} = \frac{9.21 \text{ in.}}{2.0 \text{ in.}} \\ &= 4.6 \quad \leftarrow \end{aligned}$$



**Problem 2.8-6**



$$\begin{aligned} g &= 9.81 \text{ m/s}^2 & E &= 2.0 \text{ MPa} \\ A &= 1.6 \text{ mm}^2 & L_0 &= 200 \text{ mm} \\ L_1 &= 900 \text{ mm} & W &= 450 \text{ mN} \end{aligned}$$

WHEN THE BALL LEAVES THE PADDLE

$$KE = \frac{Wv^2}{2g}$$

WHEN THE RUBBER CORD IS FULLY STRETCHED:

$$U = \frac{EA\delta^2}{2L_0} = \frac{EA}{2L_0}(L_1 - L_0)^2$$

CONSERVATION OF ENERGY

$$KE = U \quad \frac{Wv^2}{2g} = \frac{EA}{2L_0}(L_1 - L_0)^2$$

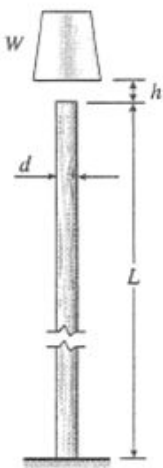
$$v^2 = \frac{gEA}{WL_0}(L_1 - L_0)^2$$

$$v = (L_1 - L_0)\sqrt{\frac{gEA}{WL_0}} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\begin{aligned} v &= (700 \text{ mm})\sqrt{\frac{(9.81 \text{ m/s}^2)(2.0 \text{ MPa})(1.6 \text{ mm}^2)}{(450 \text{ mN})(200 \text{ mm})}} \\ &= 13.1 \text{ m/s} \quad \leftarrow \end{aligned}$$

**Problem 2.8-7**



$$W = 4500 \text{ lb} \quad d = 12 \text{ in.}$$

$$L = 15 \text{ ft} = 180 \text{ in.}$$

$$A = \frac{\pi d^2}{4} = 113.10 \text{ in.}^2$$

$$E = 1.6 \times 10^6 \text{ psi}$$

$$\sigma_{\text{allow}} = 2500 \text{ psi} (= \sigma_{\text{max}})$$

Find  $h_{\text{max}}$

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{4500 \text{ lb}}{113.10 \text{ in.}^2} = 39.79 \text{ psi}$$

MAXIMUM HEIGHT  $h_{\text{max}}$

$$\text{Eq. (2-59): } \sigma_{\text{max}} = \sigma_{st} \left[ 1 + \left( 1 + \frac{2hE}{L\sigma_{st}} \right)^{1/2} \right]$$

or

$$\frac{\sigma_{\text{max}}}{\sigma_{st}} - 1 = \left( 1 + \frac{2hE}{L\sigma_{st}} \right)^{1/2}$$

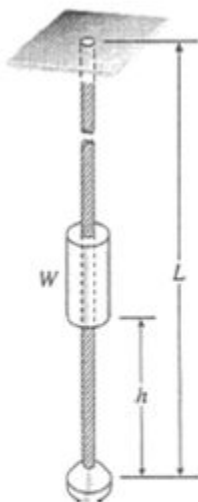
Square both sides and solve for  $h$ :

$$h = h_{\text{max}} = \frac{L\sigma_{\text{max}}}{2E} \left( \frac{\sigma_{\text{max}}}{\sigma_{st}} - 2 \right) \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\begin{aligned} h_{\text{max}} &= \frac{(180 \text{ in.})(2500 \text{ psi})}{2(1.6 \times 10^6 \text{ psi})} \left( \frac{2500 \text{ psi}}{39.79 \text{ psi}} - 2 \right) \\ &= 8.55 \text{ in.} \leftarrow \end{aligned}$$

**Problem 2.8-8**



$$W = Mg = (35 \text{ kg})(9.81 \text{ m/s}^2) = 343.4 \text{ N}$$

$$A = 40 \text{ mm}^2 \quad E = 130 \text{ GPa}$$

$$h = 1.0 \text{ m} \quad \sigma_{\text{allow}} = \sigma_{\text{max}} = 500 \text{ MPa}$$

Find minimum length  $L_{\text{min}}$ .

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{343.4 \text{ N}}{40 \text{ mm}^2} = 8.585 \text{ MPa}$$

MINIMUM LENGTH  $L_{\text{min}}$

$$\text{Eq. (2-59): } \sigma_{\text{max}} = \sigma_{st} \left[ 1 + \left( 1 + \frac{2hE}{L\sigma_{st}} \right)^{1/2} \right]$$

or

$$\frac{\sigma_{\text{max}}}{\sigma_{st}} - 1 = \left( 1 + \frac{2hE}{L\sigma_{st}} \right)^{1/2}$$

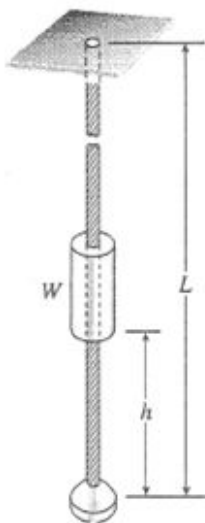
Square both sides and solve for  $L$ :

$$L = L_{\text{min}} = \frac{2Eh\sigma_{st}}{\sigma_{\text{max}}(\sigma_{\text{max}} - 2\sigma_{st})} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$L_{\text{min}} = \frac{2(130 \text{ GPa})(1.0 \text{ m})(8.585 \text{ MPa})}{(500 \text{ MPa})[500 \text{ MPa} - 2(8.585 \text{ MPa})]} \\ = 9.25 \text{ m} \quad \leftarrow$$

**Problem 2.8-9**



$$W = 100 \text{ lb}$$

$$A = 0.080 \text{ in.}^2 \quad E = 21 \times 10^6 \text{ psi}$$

$$h = 45 \text{ in.} \quad \sigma_{\text{allow}} = \sigma_{\text{max}} = 70 \text{ ksi}$$

Find minimum length  $L_{\text{min}}$ .

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{100 \text{ lb}}{0.080 \text{ in.}^2} = 1250 \text{ psi}$$

MINIMUM LENGTH  $L_{\text{min}}$

$$\text{Eq. (2-59): } \sigma_{\text{max}} = \sigma_{st} \left[ 1 + \left( 1 + \frac{2hE}{L\sigma_{st}} \right)^{1/2} \right]$$

or

$$\frac{\sigma_{\text{max}}}{\sigma_{st}} - 1 = \left( 1 + \frac{2hE}{L\sigma_{st}} \right)^{1/2}$$

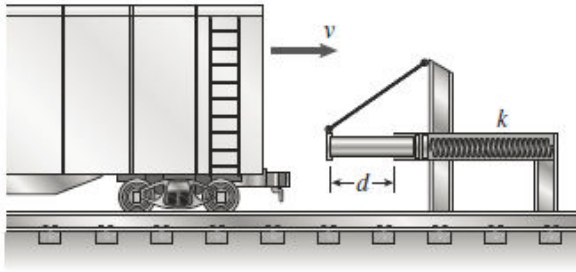
Square both sides and solve for  $L$ :

$$L = L_{\text{min}} = \frac{2Eh\sigma_{st}}{\sigma_{\text{max}}(\sigma_{\text{max}} - 2\sigma_{st})} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\begin{aligned} L_{\text{min}} &= \frac{2(21 \times 10^6 \text{ psi})(45 \text{ in.})(1250 \text{ psi})}{(70,000 \text{ psi})[70,000 \text{ psi} - 2(1250 \text{ psi})]} \\ &= 500 \text{ in.} \quad \leftarrow \end{aligned}$$

**Problem 2.8-10**



$$k = 8.0 \text{ MN/m} \quad W = 545 \text{ kN}$$

$d$  = maximum displacement of spring

$$d = \delta_{\max} = 450 \text{ mm}$$

Find  $v_{\max}$ .

KINETIC ENERGY BEFORE IMPACT

$$KE = \frac{Mv^2}{2} = \frac{Wv^2}{2g}$$

STRAIN ENERGY WHEN SPRING IS COMPRESSED TO THE MAXIMUM ALLOWABLE AMOUNT

$$U = \frac{k\delta_{\max}^2}{2} = \frac{kd^2}{2}$$

CONSERVATION OF ENERGY

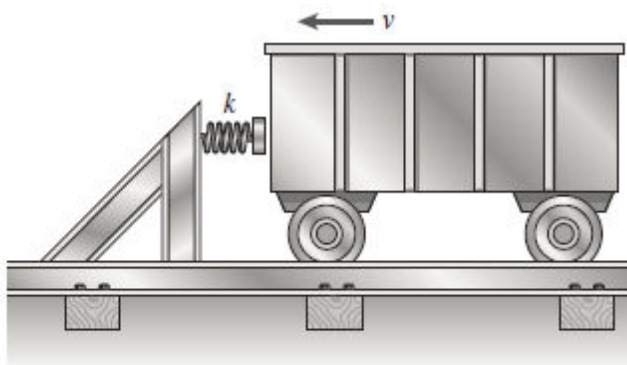
$$KE = U \quad \frac{Wv^2}{2g} = \frac{kd^2}{2} \quad v^2 = \frac{kd^2}{W/g}$$

$$v = v_{\max} = d\sqrt{\frac{k}{W/g}} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\begin{aligned} v_{\max} &= (450 \text{ mm})\sqrt{\frac{8.0 \text{ MN/m}}{(545 \text{ kN})/(9.81 \text{ m/s}^2)}} \\ &= 5400 \text{ mm/s} = 5.4 \text{ m/s} \quad \leftarrow \end{aligned}$$

**Problem 2.8-11**



$$k = 1120 \text{ lb/in.} \quad W = 3450 \text{ lb}$$

$$v = 7 \text{ mph} = 123.2 \text{ in./sec}$$

$$g = 32.2 \text{ ft/sec}^2 = 386.4 \text{ in./sec}^2$$

Find the shortening  $\delta_{\max}$  of the spring.

KINETIC ENERGY JUST BEFORE IMPACT

$$KE = \frac{Mv^2}{2} = \frac{Wv^2}{2g}$$

STRAIN ENERGY WHEN SPRING IS FULLY COMPRESSED

$$U = \frac{k\delta_{\max}^2}{2}$$

Conservation of energy

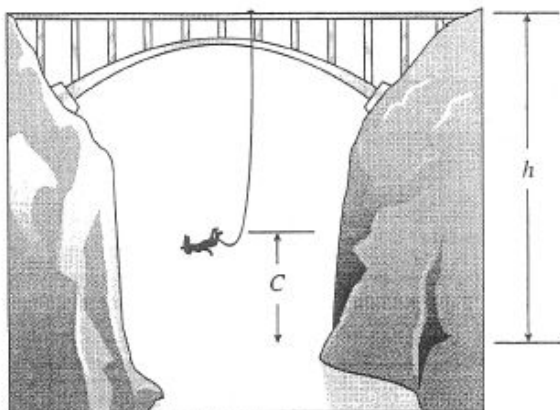
$$KE = U \quad \frac{Wv^2}{2g} = \frac{k\delta_{\max}^2}{2}$$

$$\text{Solve for } \delta_{\max}: \delta_{\max} = \sqrt{\frac{Wv^2}{gk}} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\begin{aligned} \delta_{\max} &= \sqrt{\frac{(3450 \text{ lb})(123.2 \text{ in./sec})^2}{(386.4 \text{ in./sec}^2)(1120 \text{ lb/in.})}} \\ &= 11.0 \text{ in.} \quad \leftarrow \end{aligned}$$

**Problem 2.8-12**



$$W = Mg = (55 \text{ kg})(9.81 \text{ m/s}^2) = 539.55 \text{ N}$$

$$EA = 2.3 \text{ kN}$$

$$\text{Height: } h = 60 \text{ m}$$

$$\text{Clearance: } C = 10 \text{ m}$$

Find length  $L$  of the bungee cord.

$P.E.$  = Potential energy of the jumper at the top of bridge (with respect to lowest position)

$$= W(L + \delta_{\max})$$

$U$  = strain energy of cord at lowest position

$$= \frac{EA\delta_{\max}^2}{2L}$$

CONSERVATION OF ENERGY

$$P.E. = U \quad W(L + \delta_{\max}) = \frac{EA\delta_{\max}^2}{2L}$$

$$\text{or} \quad \delta_{\max}^2 - \frac{2WL}{EA}\delta_{\max} - \frac{2WL^2}{EA} = 0$$

SOLVE QUADRATIC EQUATION FOR  $\delta_{\max}$ :

$$\begin{aligned} \delta_{\max} &= \frac{WL}{EA} + \left[ \left( \frac{WL}{EA} \right)^2 + 2L \left( \frac{WL}{EA} \right) \right]^{1/2} \\ &= \frac{WL}{EA} \left[ 1 + \left( 1 + \frac{2EA}{W} \right)^{1/2} \right] \end{aligned}$$

VERTICAL HEIGHT

$$h = C + L + \delta_{\max}$$

$$h - C = L + \frac{WL}{EA} \left[ 1 + \left( 1 + \frac{2EA}{W} \right)^{1/2} \right]$$

SOLVE FOR  $L$ :

$$L = \frac{h - C}{1 + \frac{W}{EA} \left[ 1 + \left( 1 + \frac{2EA}{W} \right)^{1/2} \right]} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\frac{W}{EA} = \frac{539.55 \text{ N}}{2.3 \text{ kN}} = 0.234587$$

$$\text{Numerator} = h - C = 60 \text{ m} - 10 \text{ m} = 50 \text{ m}$$

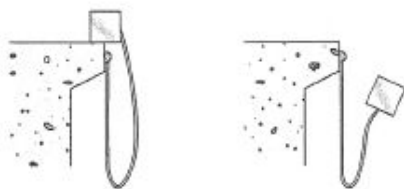
$$\text{Denominator} = 1 + (0.234587)$$

$$\times \left[ 1 + \left( 1 + \frac{2}{0.234587} \right)^{1/2} \right]$$

$$= 1.9586$$

$$L = \frac{50 \text{ m}}{1.9586} = 25.5 \text{ m} \quad \leftarrow$$

### Problem 2.8-13



$W$  = Weight

Properties of elastic cord:

$E$  = modulus of elasticity

$A$  = cross-sectional area

$L$  = original length

$\delta_{\max}$  = elongation of elastic cord

$P.E.$  = potential energy of weight before fall (with respect to lowest position)

$P.E. = W(L + \delta_{\max})$

Let  $U$  = strain energy of cord at lowest position.

$$U = \frac{EA\delta_{\max}^2}{2L}$$

CONSERVATION OF ENERGY

$$P.E. = U \quad W(L + \delta_{\max}) = \frac{EA\delta_{\max}^2}{2L}$$

$$\text{or} \quad \delta_{\max}^2 - \frac{2WL}{EA}\delta_{\max} - \frac{2WL^2}{EA} = 0$$

SOLVE QUADRATIC EQUATION FOR  $\delta_{\max}$ :

$$\delta_{\max} = \frac{WL}{EA} + \left[ \left( \frac{WL}{EA} \right)^2 + 2L \left( \frac{WL}{EA} \right) \right]^{1/2}$$

STATIC ELONGATION

$$\delta_{st} = \frac{WL}{EA}$$

IMPACT FACTOR

$$\frac{\delta_{\max}}{\delta_{st}} = 1 + \left[ 1 + \frac{2EA}{W} \right]^{1/2} \leftarrow$$

NUMERICAL VALUES

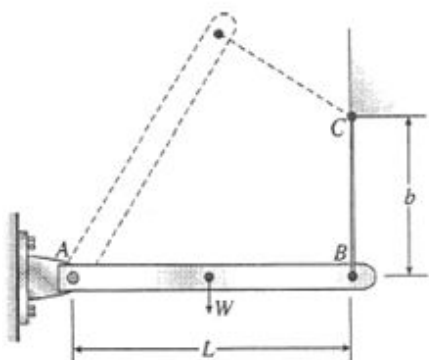
$$\delta_{st} = (2.5\%)(L) = 0.025L$$

$$\delta_{st} = \frac{WL}{EA} \quad \frac{W}{EA} = 0.025 \quad \frac{EA}{W} = 40$$

$$\text{Impact factor} = 1 + [1 + 2(40)]^{1/2} = 10 \leftarrow$$



### Problem 2.8-14



RIGID BAR:

$$W = Mg = (1.0 \text{ kg})(9.81 \text{ m/s}^2) = 9.81 \text{ N}$$

$$L = 0.5 \text{ m}$$

NYLON CORD:

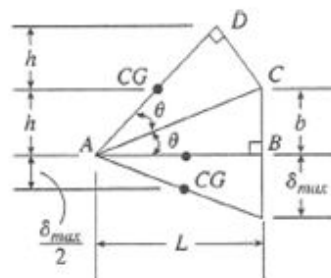
$$A = 30 \text{ mm}^2$$

$$b = 0.25 \text{ m}$$

$$E = 2.1 \text{ GPa}$$

Find maximum stress  $\sigma_{\max}$  in cord BC.

GEOMETRY OF BAR AB AND CORD BC



$$\overline{CD} = \overline{CB} = b$$

$$\overline{AD} = \overline{AB} = L$$

$h$  = height of center of gravity of raised bar AD

$\delta_{\max}$  = elongation of cord

$$\begin{aligned} \text{From triangle } ABC: \sin \theta &= \frac{b}{\sqrt{b^2 + L^2}} \\ \cos \theta &= \frac{L}{\sqrt{b^2 + L^2}} \end{aligned}$$

$$\text{From line } AD: \sin 2\theta = \frac{2h}{AD} = \frac{2h}{L}$$

$$\text{From Appendix C: } \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\therefore \frac{2h}{L} = 2 \left( \frac{b}{\sqrt{b^2 + L^2}} \right) \left( \frac{L}{\sqrt{b^2 + L^2}} \right) = \frac{2bL}{b^2 + L^2}$$

$$\text{and } h = \frac{bL^2}{b^2 + L^2} \quad (\text{Eq. 1})$$

CONSERVATION OF ENERGY

P.E. = potential energy of raised bar AD

$$= W \left( h + \frac{\delta_{\max}}{2} \right)$$

$$U = \text{strain energy of stretched cord} = \frac{EA\delta_{\max}^2}{2b}$$

$$P.E. = U \quad W \left( h + \frac{\delta_{\max}}{2} \right) = \frac{EA\delta_{\max}^2}{2b} \quad (\text{Eq. 2})$$

$$\text{For the cord: } \delta_{\max} = \frac{\sigma_{\max} b}{E}$$

Substitute into Eq. (2) and rearrange:

$$\sigma_{\max}^2 - \frac{W}{A} \sigma_{\max} - \frac{2WhE}{bA} = 0 \quad (\text{Eq. 3})$$

Substitute from Eq. (1) into Eq. (3):

$$\sigma_{\max}^2 - \frac{W}{A} \sigma_{\max} - \frac{2WL^2E}{A(b^2 + L^2)} = 0 \quad (\text{Eq. 4})$$

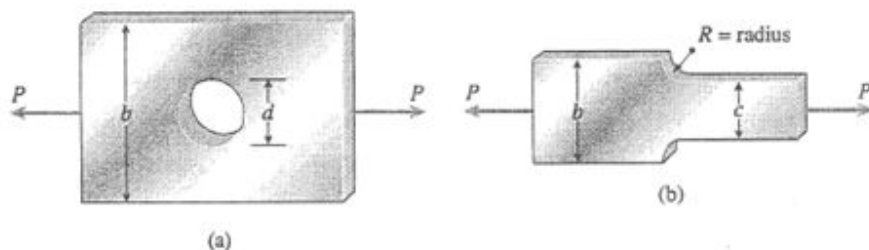
SOLVE FOR  $\sigma_{\max}$ :

$$\sigma_{\max} = \frac{W}{2A} \left[ 1 + \sqrt{1 + \frac{8L^2EA}{W(b^2 + L^2)}} \right] \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$\sigma_{\max} = 33.3 \text{ MPa} \quad \leftarrow$$

**Problem 2.10-1**



$$P = 3.0 \text{ k} \quad t = 0.25 \text{ in.}$$

(a) BAR WITH CIRCULAR HOLE ( $b = 6 \text{ in.}$ )

Obtain  $K$  from Fig. 2-84

FOR  $d = 1 \text{ in.}$ :  $c = b - d = 5 \text{ in.}$

$$\sigma_{\text{nom}} = \frac{P}{ct} = \frac{3.0 \text{ k}}{(5 \text{ in.})(0.25 \text{ in.})} = 2.40 \text{ ksi}$$

$$d/b = \frac{1}{6} \quad K \approx 2.60$$

$$\sigma_{\text{max}} = K\sigma_{\text{nom}} \approx 6.2 \text{ ksi} \quad \leftarrow$$

FOR  $d = 2 \text{ in.}$ :  $c = b - d = 4 \text{ in.}$

$$\sigma_{\text{nom}} = \frac{P}{ct} = \frac{3.0 \text{ k}}{(4 \text{ in.})(0.25 \text{ in.})} = 3.00 \text{ ksi}$$

$$d/b = \frac{1}{3} \quad K \approx 2.31$$

$$\sigma_{\text{max}} = K\sigma_{\text{nom}} \approx 6.9 \text{ ksi} \quad \leftarrow$$

(b) STEPPED BAR WITH SHOULDER FILLETS

$b = 4.0 \text{ in.}$   $c = 2.5 \text{ in.}$ ; Obtain  $k$  from Fig. 2-86

$$\sigma_{\text{nom}} = \frac{P}{ct} = \frac{3.0 \text{ k}}{(2.5 \text{ in.})(0.25 \text{ in.})} = 4.80 \text{ ksi}$$

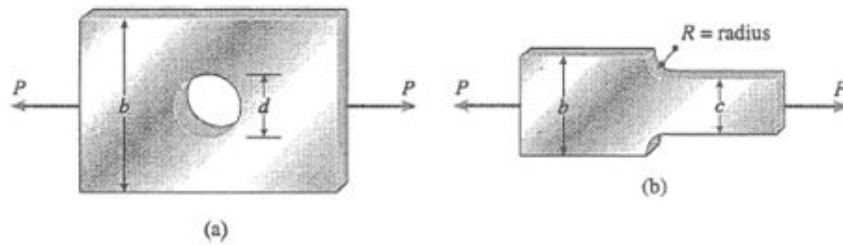
FOR  $R = 0.25 \text{ in.}$ :  $R/c = 0.1$   $b/c = 1.60$

$$k \approx 2.30 \quad \sigma_{\text{max}} = K\sigma_{\text{nom}} \approx 11.0 \text{ ksi} \quad \leftarrow$$

FOR  $R = 0.5 \text{ in.}$ :  $R/c = 0.2$   $b/c = 1.60$

$$K \approx 1.87 \quad \sigma_{\text{max}} = K\sigma_{\text{nom}} \approx 9.0 \text{ ksi} \quad \leftarrow$$

**Problem 2.10-2**



$$P = 2.5 \text{ kN} \quad t = 5.0 \text{ mm}$$

(a) BAR WITH CIRCULAR HOLE ( $b = 60 \text{ mm}$ )

Obtain  $K$  from Fig. 2-84

FOR  $d = 12 \text{ mm}$ :  $c = b - d = 48 \text{ mm}$

$$\sigma_{\text{nom}} = \frac{P}{ct} = \frac{2.5 \text{ kN}}{(48 \text{ mm})(5 \text{ mm})} = 10.42 \text{ MPa}$$

$$d/b = \frac{1}{5} \quad K \approx 2.51$$

$$\sigma_{\text{max}} = K\sigma_{\text{nom}} \approx 26 \text{ MPa} \quad \leftarrow$$

FOR  $d = 20 \text{ mm}$ :  $c = b - d = 40 \text{ mm}$

$$\sigma_{\text{nom}} = \frac{P}{ct} = \frac{2.5 \text{ kN}}{(40 \text{ mm})(5 \text{ mm})} = 12.50 \text{ MPa}$$

$$d/b = \frac{1}{3} \quad K \approx 2.31$$

$$\sigma_{\text{max}} = K\sigma_{\text{nom}} \approx 29 \text{ MPa} \quad \leftarrow$$

(b) STEPPED BAR WITH SHOULDER FILLETS

$$b = 60 \text{ mm} \quad c = 40 \text{ mm};$$

Obtain  $K$  from Fig 2-86

$$\sigma_{\text{nom}} = \frac{P}{ct} = \frac{2.5 \text{ kN}}{(40 \text{ mm})(5 \text{ mm})} = 12.50 \text{ MPa}$$

FOR  $R = 6 \text{ mm}$ :  $R/c = 0.15 \quad b/c = 1.5$

$$K \approx 2.00 \quad \sigma_{\text{max}} = K\sigma_{\text{nom}} \approx 25 \text{ MPa} \quad \leftarrow$$

FOR  $R = 10 \text{ mm}$ :  $R/c = 0.25 \quad b/c = 1.5$

$$K \approx 1.75 \quad \sigma_{\text{max}} = K\sigma_{\text{nom}} \approx 22 \text{ MPa} \quad \leftarrow$$

**Problem 2.10-3**



$t$  = thickness

$\sigma_t$  = allowable tensile stress

Find  $P_{\max}$

Find  $K$  from Fig. 2-84

$$P_{\max} = \sigma_{\text{nom}} ct = \frac{\sigma_{\max}}{K} ct = \frac{\sigma_t}{K} (b - d)t$$

$$= \frac{\sigma_t}{K} bt \left( 1 - \frac{d}{b} \right)$$

Because  $\sigma_t$ ,  $b$ , and  $t$  are constants, we write:

$$P^* = \frac{P_{\max}}{\sigma_t bt} = \frac{1}{K} \left( 1 - \frac{d}{b} \right)$$

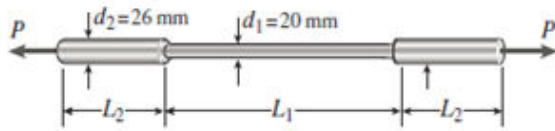
$\frac{d}{b}$	$K$	$P^*$
0	3.00	0.333
0.1	2.73	0.330
0.2	2.50	0.320
0.3	2.35	0.298
0.4	2.24	0.268

We observe that  $P_{\max}$  decreases as  $d/b$  increases. Therefore, the maximum load occurs when the hole becomes very small.

$$\left( \frac{d}{b} \rightarrow 0 \quad \text{and} \quad K \rightarrow 3 \right)$$

$$P_{\max} = \frac{\sigma_t bt}{3} \quad \leftarrow$$

**Problem 2.10-4**



$$E = 100 \text{ GPa}$$

$$\delta = 0.12 \text{ mm}$$

$$L_2 = 0.1 \text{ m}$$

$$L_1 = 0.3 \text{ m}$$

$$R = \text{radius of fillets} = \frac{26 \text{ mm} - 20 \text{ mm}}{2} = 3 \text{ mm}$$

$$\delta = 2 \left( \frac{PL_2}{EA_2} \right) + \frac{PL_1}{EA_1}$$

$$\text{Solve for } P: \quad P = \frac{\delta EA_1 A_2}{2L_2 A_1 + L_1 A_2}$$

Use Fig. 2-87 for the stress-concentration factor:

$$\begin{aligned} \sigma_{\text{nom}} &= \frac{P}{A_1} = \frac{\delta EA_2}{2L_2 A_1 + L_1 A_2} = \frac{\delta E}{2L_2 \left( \frac{A_1}{A_2} \right) + L_1} \\ &= \frac{\delta E}{2L_2 \left( \frac{d_1}{d_2} \right)^2 + L_1} \end{aligned}$$

SUBSTITUTE NUMERICAL VALUES:

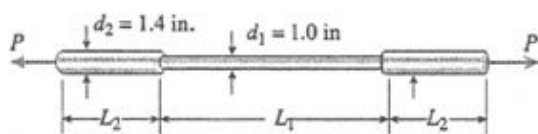
$$\sigma_{\text{nom}} = \frac{(0.12 \text{ mm})(100 \text{ GPa})}{2(0.1 \text{ m}) \left( \frac{20}{26} \right)^2 + 0.3 \text{ m}} = 28.68 \text{ MPa}$$

$$\frac{R}{D_1} = \frac{3 \text{ mm}}{20 \text{ mm}} = 0.15$$

Use the dashed curve in Fig. 2-87.  $K \approx 1.6$

$$\begin{aligned} \sigma_{\text{max}} &= K \sigma_{\text{nom}} \approx (1.6)(28.68 \text{ MPa}) \\ &\approx 46 \text{ MPa} \quad \leftarrow \end{aligned}$$

**Problem 2.10-5**



$$E = 25 \times 10^6 \text{ psi}$$

$$\delta = 0.0040 \text{ in.}$$

$$L_1 = 20 \text{ in.}$$

$$L_2 = 5 \text{ in.}$$

$$R = \text{radius of fillets} \quad R = \frac{1.4 \text{ in.} - 1.0 \text{ in.}}{2} \\ = 0.2 \text{ in.}$$

$$\delta = 2 \left( \frac{PL_2}{EA_2} \right) + \frac{PL_1}{EA_1}$$

$$\text{Solve for } P: P = \frac{\delta EA_1 A_2}{2L_2 A_1 + L_1 A_2}$$

Use Fig. 2-87 for the stress-concentration factor.

$$\sigma_{\text{nom}} = \frac{P}{A_1} = \frac{\delta EA_2}{2L_2 A_1 + L_1 A_2} = \frac{\delta E}{2L_2 \left( \frac{A_1}{A_2} \right) + L_1} \\ = \frac{\delta E}{2L_2 \left( \frac{d_1}{d_2} \right)^2 + L_1}$$

SUBSTITUTE NUMERICAL VALUES:

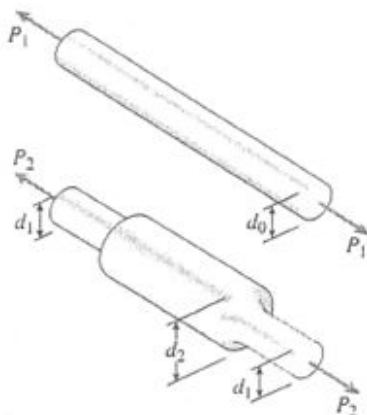
$$\sigma_{\text{nom}} = \frac{(0.0040 \text{ in.})(25 \times 10^6 \text{ psi})}{2(5 \text{ in.}) \left( \frac{1.0}{1.4} \right)^2 + 20 \text{ in.}} = 3,984 \text{ psi}$$

$$\frac{R}{D_1} = \frac{0.2 \text{ in.}}{1.0 \text{ in.}} = 0.2$$

Use the dashed curve in Fig. 2-87.  $K \approx 1.53$

$$\sigma_{\text{max}} = K \sigma_{\text{nom}} \approx (1.53)(3984 \text{ psi}) \\ \approx 6100 \text{ psi} \quad \leftarrow$$

**Problem 2.10-6**



$$d_0 = 20 \text{ mm}$$

$$d_1 = 20 \text{ mm}$$

$$d_2 = 25 \text{ mm}$$

Fillet radius:  $R = 2 \text{ mm}$

Allowable stress:  $\sigma_t = 80 \text{ MPa}$

(a) COMPARISON OF BARS

$$\begin{aligned} \text{Prismatic bar: } P_1 &= \sigma_t A_0 = \sigma_t \left( \frac{\pi d_0^2}{4} \right) \\ &= (80 \text{ MPa}) \left( \frac{\pi}{4} \right) (20 \text{ mm})^2 = 25.1 \text{ kN} \quad \leftarrow \end{aligned}$$

Stepped bar: See Fig. 2-87 for the stress-concentration factor.

$$\begin{aligned} R &= 2.0 \text{ mm} & D_1 &= 20 \text{ mm} & D_2 &= 25 \text{ mm} \\ R/D_1 &= 0.10 & D_2/D_1 &= 1.25 & K &\approx 1.75 \end{aligned}$$

$$\sigma_{\text{nom}} = \frac{P_2}{\frac{\pi}{4} d_1^2} = \frac{P_2}{A_1} \quad \sigma_{\text{nom}} = \frac{\sigma_{\text{max}}}{K}$$

$$\begin{aligned} P_2 &= \sigma_{\text{nom}} A_1 = \frac{\sigma_{\text{max}}}{K} A_1 = \frac{\sigma_t}{K} A_1 \\ &= \left( \frac{80 \text{ MPa}}{1.75} \right) \left( \frac{\pi}{4} \right) (20 \text{ mm})^2 \\ &\approx 14.4 \text{ kN} \quad \leftarrow \end{aligned}$$

Enlarging the bar makes it *weaker*, not stronger. The ratio of loads is  $P_1/P_2 = K = 1.75$

(b) DIAMETER OF PRISMATIC BAR FOR THE SAME ALLOWABLE LOAD

$$\begin{aligned} P_1 &= P_2 \quad \sigma_t \left( \frac{\pi d_0^2}{4} \right) = \frac{\sigma_t}{K} \left( \frac{\pi d_1^2}{4} \right) \quad d_0^2 = \frac{d_1^2}{K} \\ d_0 &= \frac{d_1}{\sqrt{K}} \approx \frac{20 \text{ mm}}{\sqrt{1.75}} \approx 15.1 \text{ mm} \quad \leftarrow \end{aligned}$$

**Problem 2.10-7**



$$b = 2.4 \text{ in.}$$

$$c = 1.6 \text{ in.}$$

$$\text{Fillet radius: } R = 0.2 \text{ in.}$$

Find  $d_{\max}$

BASED UPON FILLETS (Use Fig. 2-86)

$$b = 2.4 \text{ in.} \quad c = 1.6 \text{ in.} \quad R = 0.2 \text{ in.}$$

$$R/c = 0.125 \quad b/c = 1.5 \quad K \approx 2.10$$

$$P_{\max} = \sigma_{\text{nom}} ct = \frac{\sigma_{\max}}{K} ct = \frac{\sigma_{\max}}{K} \left( \frac{c}{b} \right) (bt) \\ \approx 0.317 bt \sigma_{\max}$$

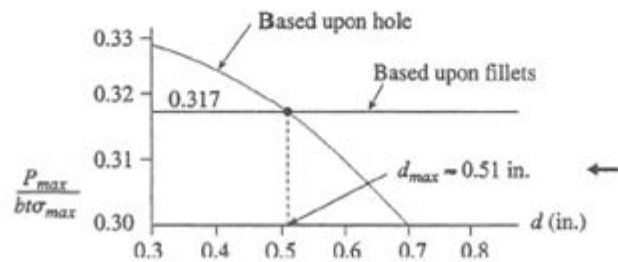
BASED UPON HOLE (Use Fig. 2-84)

$$b = 2.4 \text{ in.} \quad d = \text{diameter of the hole (in.)}$$

$$c_1 = b - d$$

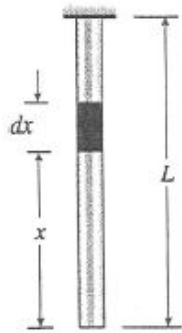
$$P_{\max} = \sigma_{\text{nom}} c_1 t = \frac{\sigma_{\max}}{K} (b - d)t \\ = \frac{1}{K} \left( 1 - \frac{d}{b} \right) bt \sigma_{\max}$$

$d(\text{in.})$	$d/b$	$K$	$P_{\max}/bt\sigma_{\max}$
0.3	0.125	2.66	0.329
0.4	0.167	2.57	0.324
0.5	0.208	2.49	0.318
0.6	0.250	2.41	0.311
0.7	0.292	2.37	0.299





**Problem 2.11-1**



Let  $A$  = cross-sectional area

Let  $N$  = axial force at distance  $x$

$$N = \gamma Ax$$

$$\sigma = \frac{N}{A} = \gamma x$$

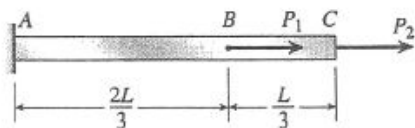
STRAIN AT DISTANCE  $x$

$$\varepsilon = \frac{\sigma}{E} + \frac{\sigma_0 \alpha}{E} \left( \frac{\sigma}{\sigma_0} \right)^m = \frac{\gamma x}{E} + \frac{\sigma_0}{\alpha E} \left( \frac{\gamma x}{\sigma_0} \right)^m$$

ELONGATION OF BAR

$$\begin{aligned} \delta &= \int_0^L \varepsilon dx = \int_0^L \frac{\gamma x}{E} dx + \frac{\sigma_0 \alpha}{E} \int_0^L \left( \frac{\gamma x}{\sigma_0} \right)^m dx \\ &= \frac{\gamma L^2}{2E} + \frac{\sigma_0 \alpha L}{(m+1)E} \left( \frac{\gamma L}{\sigma_0} \right)^m \quad \text{Q.E.D.} \quad \leftarrow \end{aligned}$$

**Problem 2.11-2**



$$L = 1.8 \text{ m} \quad A = 480 \text{ mm}^2$$

$$P_1 = 30 \text{ kN} \quad P_2 = 60 \text{ kN}$$

Ramberg–Osgood equation:

$$\epsilon = \frac{\sigma}{45,000} + \frac{1}{618} \left( \frac{\sigma}{170} \right)^{10} \quad (\sigma = \text{MPa})$$

Find displacement at end of bar.

(a)  $P_1$  ACTS ALONE

$$AB: \sigma = \frac{P_1}{A} = \frac{30 \text{ kN}}{480 \text{ mm}^2} = 62.5 \text{ MPa}$$

$$\epsilon = 0.001389$$

$$\delta_c = \epsilon \left( \frac{2L}{3} \right) = 1.67 \text{ mm} \quad \leftarrow$$

(b)  $P_2$  ACTS ALONE

$$ABC: \sigma = \frac{P_2}{A} = \frac{60 \text{ kN}}{480 \text{ mm}^2} = 125 \text{ MPa}$$

$$\epsilon = 0.002853$$

$$\delta_c = \epsilon L = 5.13 \text{ mm} \quad \leftarrow$$

(c) BOTH  $P_1$  AND  $P_2$  ARE ACTING

$$AB: \sigma = \frac{P_1 + P_2}{A} = \frac{90 \text{ kN}}{480 \text{ mm}^2} = 187.5 \text{ MPa}$$

$$\epsilon = 0.008477$$

$$\delta_{AB} = \epsilon \left( \frac{2L}{3} \right) = 10.17 \text{ mm}$$

$$BC: \sigma = \frac{P_2}{A} = \frac{60 \text{ kN}}{480 \text{ mm}^2} = 125 \text{ MPa}$$

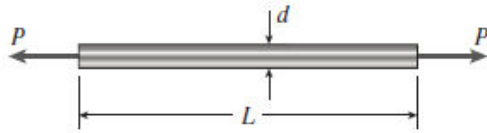
$$\epsilon = 0.002853$$

$$\delta_{BC} = \epsilon \left( \frac{L}{3} \right) = 1.71 \text{ mm}$$

$$\delta_C = \delta_{AB} + \delta_{BC} = 11.88 \text{ mm} \quad \leftarrow$$

(Note that the displacement when both loads act simultaneously is *not* equal to the sum of the displacements when the loads act separately.)

**Problem 2.11-3**

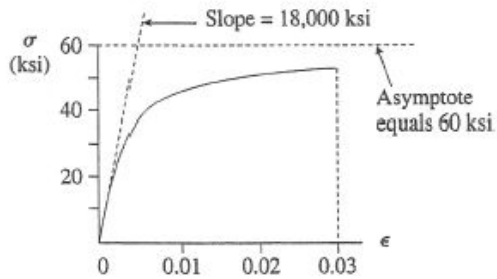


$$L = 32 \text{ in.} \quad d = 0.75 \text{ in.}$$

$$A = \frac{\pi d^2}{4} = 0.4418 \text{ in.}^2$$

(a) STRESS-STRAIN DIAGRAM

$$\sigma = \frac{18,000\varepsilon}{1 + 300\varepsilon} \quad 0 \leq \varepsilon \leq 0.03 \quad (\sigma = \text{ksi})$$



(b) ALLOWABLE LOAD  $P$

$$\text{Maximum elongation } \delta_{\max} = 0.25 \text{ in.}$$

$$\text{Maximum stress } \sigma_{\max} = 40 \text{ ksi}$$

Based upon elongation:

$$\varepsilon_{\max} = \frac{\delta_{\max}}{L} = \frac{0.25 \text{ in.}}{32 \text{ in.}} = 0.007813$$

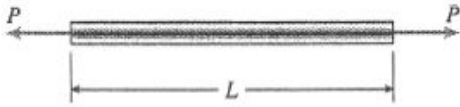
$$\sigma_{\max} = \frac{18,000\varepsilon_{\max}}{1 + 300\varepsilon_{\max}} = 42.06 \text{ ksi}$$

BASED UPON STRESS:

$$\sigma_{\max} = 40 \text{ ksi}$$

$$\text{Stress governs. } P = \sigma_{\max} A = (40 \text{ ksi})(0.4418 \text{ in.}^2) = 17.7 \text{ k} \quad \leftarrow$$

**Problem 2.11-4**



$$L = 2.0 \text{ m}$$

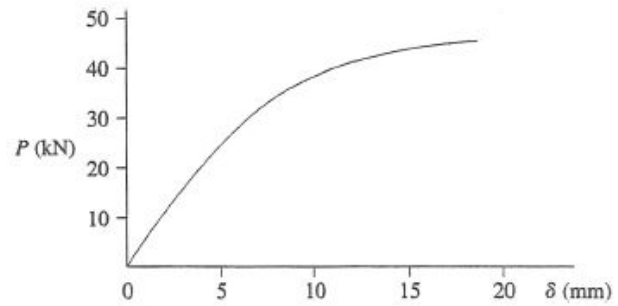
$$A = 249 \text{ mm}^2$$

STRESS-STRAIN DIAGRAM

(See the problem statement for the diagram)

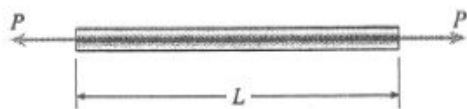
LOAD-DISPLACEMENT DIAGRAM

$P$ (kN)	$\sigma = P/A$ (MPa)	$\epsilon$ (from diagram)	$\delta = \epsilon L$ (mm)
10	40	0.0009	1.8
20	80	0.0018	3.6
30	120	0.0031	6.2
40	161	0.0060	12.0
45	181	0.0081	16.2



**NOTE:** The load-displacement curve has the same shape as the stress-strain curve.

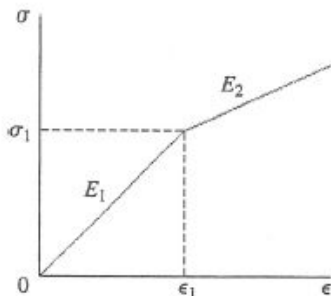
**Problem 2.11-5**



$$L = 150 \text{ in.}$$

$$A = 2.0 \text{ in.}^2$$

**STRESS-STRAIN DIAGRAM**



$$E_1 = 10 \times 10^6 \text{ psi}$$

$$E_2 = 2.4 \times 10^6 \text{ psi}$$

$$\sigma_1 = 12,000 \text{ psi}$$

$$\epsilon_1 = \frac{\sigma_1}{E_1} = \frac{12,000 \text{ psi}}{10 \times 10^6 \text{ psi}} = 0.0012$$

For  $0 \leq \sigma \leq \sigma_1$ :

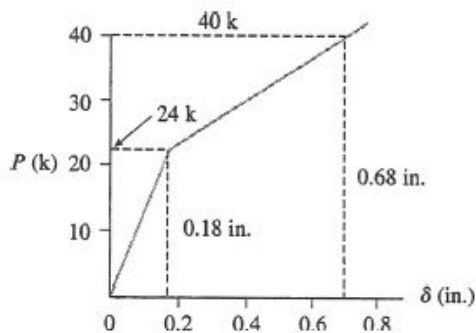
$$\epsilon = \frac{\sigma}{E_1} = \frac{\sigma}{10 \times 10^6 \text{ psi}} \quad (\sigma = \text{psi}) \quad \text{Eq. (1)}$$

For  $\sigma \geq \sigma_1$ :

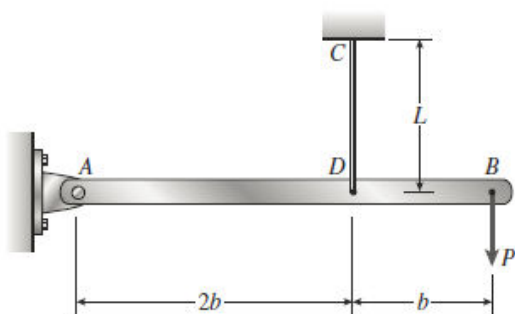
$$\begin{aligned} \epsilon &= \epsilon_1 + \frac{\sigma - \sigma_1}{E_2} = 0.0012 + \frac{\sigma - 12,000}{2.4 \times 10^6} \\ &= \frac{\sigma}{2.4 \times 10^6} - 0.0038 \quad (\sigma = \text{psi}) \quad \text{Eq. (2)} \end{aligned}$$

**LOAD-DISPLACEMENT DIAGRAM**

$P$ (k)	$\sigma = P/A$ (psi)	$\epsilon$ (from Eq. 1 or Eq. 2)	$\delta = \epsilon L$ (in.)
8	4,000	0.00040	0.060
16	8,000	0.00080	0.120
24	12,000	0.00120	0.180
32	16,000	0.00287	0.430
40	20,000	0.00453	0.680



**Problem 2.11-6**



Wire:  $E = 210 \text{ GPa}$

$\sigma_Y = 820 \text{ MPa}$

$L = 1.0 \text{ m}$

$d = 3 \text{ mm}$

$$A = \frac{\pi d^2}{4} = 7.0686 \text{ mm}^2$$

STRESS-STRAIN DIAGRAM

$$\sigma = E\varepsilon \quad (0 \leq \sigma \leq \sigma_Y) \quad (1)$$

$$\sigma = \sigma_Y \left( \frac{E\varepsilon}{\sigma_Y} \right)^n \quad (\sigma \geq \sigma_Y) \quad (n = 0.2) \quad (2)$$

(a) DISPLACEMENT  $\delta_B$  AT END OF BAR

$$\delta = \text{elongation of wire} \quad \delta_B = \frac{3}{2}\delta = \frac{3}{2}E\varepsilon L \quad (3)$$

Obtain  $\varepsilon$  from stress-strain equations:

$$\text{From Eq. (1): } \varepsilon = \frac{\sigma E}{(0 \leq \sigma \leq \sigma_Y)} \quad (4)$$

$$\text{From Eq. (2): } \varepsilon = \frac{\sigma_Y}{E} \left( \frac{\sigma}{\sigma_Y} \right)^{1/n} \quad (5)$$

$$\text{Axial force in wire: } F = \frac{3P}{2}$$

$$\text{Stress in wire: } \sigma = \frac{F}{A} = \frac{3P}{2A} \quad (6)$$

PROCEDURE: Assume a value of  $P$

Calculate  $\sigma$  from Eq. (6)

Calculate  $\varepsilon$  from Eq. (4) or (5)

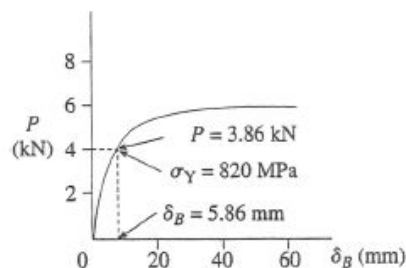
Calculate  $\delta_B$  from Eq. (3)

$P$ (kN)	$\sigma$ (MPa) Eq. (6)	$\varepsilon$ Eq. (4) or (5)	$\delta_B$ (mm) Eq. (3)
2.4	509.3	0.002425	3.64
3.2	679.1	0.003234	4.85
4.0	848.8	0.004640	6.96
4.8	1018.6	0.01155	17.3
5.6	1188.4	0.02497	37.5

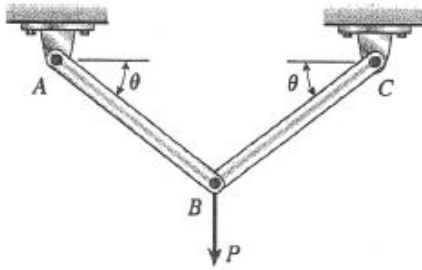
For  $\sigma = \sigma_Y = 820 \text{ MPa}$ :

$$\varepsilon = 0.0039048 \quad P = 3.864 \text{ kN} \quad \delta_B = 5.86 \text{ mm}$$

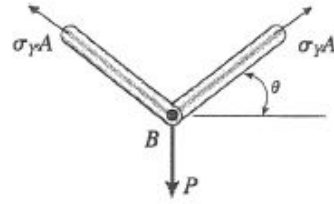
(b) LOAD-DISPLACEMENT DIAGRAM



**Problem 2.12-1**



Structure is statically determinate. The yield load  $P_Y$  and the plastic load  $P_P$  occur at the same time, namely, when both bars reach the yield stress.



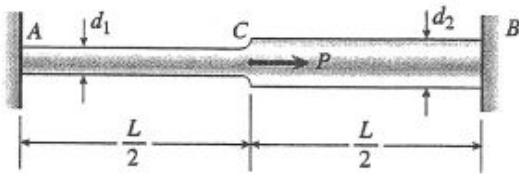
JOINT B

$$\Sigma F_{\text{vert}} = 0$$

$$(2\sigma_Y A) \sin \theta = P$$

$$P_Y = P_P = 2\sigma_Y A \sin \theta \quad \leftarrow$$

**Problem 2.12-2**

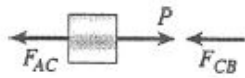


$$d_1 = 20 \text{ mm} \quad d_2 = 25 \text{ mm} \quad \sigma_Y = 250 \text{ MPa}$$

DETERMINE THE PLASTIC LOAD  $P_P$ :

At the plastic load, all parts of the bar are stressed to the yield stress.

Point C:



$$F_{AC} = \sigma_Y A_1 \quad F_{CB} = \sigma_Y A_2$$

$$P = F_{AC} + F_{CB}$$

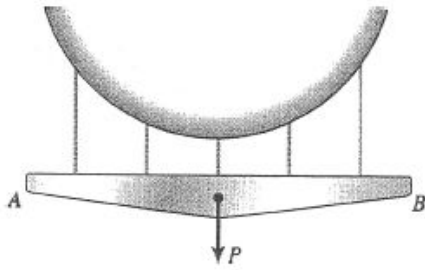
$$P_P = \sigma_Y A_1 + \sigma_Y A_2 = \sigma_Y (A_1 + A_2) \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

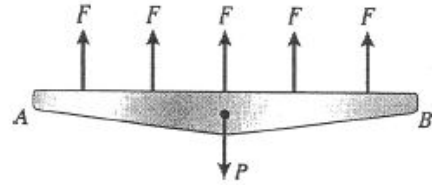
$$\begin{aligned} P_P &= (250 \text{ MPa}) \left( \frac{\pi}{4} \right) (d_1^2 + d_2^2) \\ &= (250 \text{ MPa}) \left( \frac{\pi}{4} \right) [(20 \text{ mm})^2 + (25 \text{ mm})^2] \\ &= 201 \text{ kN} \quad \leftarrow \end{aligned}$$



**Problem 2.12-3**

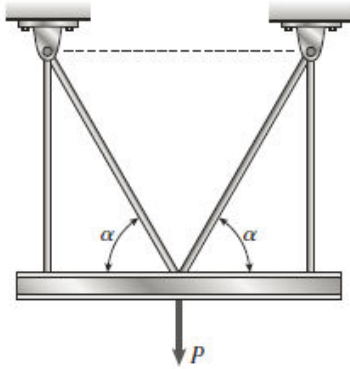


- (a) **PLASTIC LOAD  $P_P$**   
 At the plastic load, each wire is stressed to the yield stress.  $\therefore P_P = 5\sigma_Y A$  ←  
 $F = \sigma_Y A$

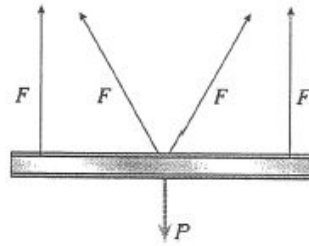


- (b) **BAR AB IS FLEXIBLE**  
 At the plastic load, each wire is stressed to the yield stress, so the plastic load is not changed. ←
- (c) **RADIUS  $R$  IS INCREASED**  
 Again, the forces in the wires are not changed, so the plastic load is not changed. ←

**Problem 2.12-4**



At the plastic load, all four rods are stressed to the yield stress.



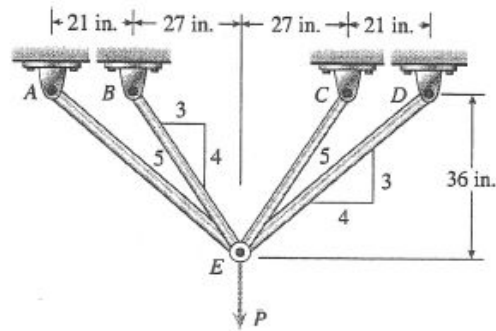
$$F = \sigma_y A$$

Sum forces in the vertical direction and solve for the load:

$$P_P = 2F + 2F \sin \alpha$$

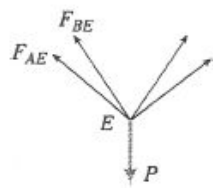
$$P_P = 2\sigma_y A (1 + \sin \alpha) \quad \leftarrow$$

**Problem 2.12-5**



$$L_{AE} = 60 \text{ in.} \quad L_{BE} = 45 \text{ in.}$$

JOINT E



Equilibrium:

$$2F_{AE}\left(\frac{3}{5}\right) + 2F_{BE}\left(\frac{4}{5}\right) = P$$

or

$$P = \frac{6}{5}F_{AE} + \frac{8}{5}F_{BE}$$

PLASTIC LOAD  $P_P$

At the plastic load, all bars are stressed to the yield stress.

$$F_{AE} = \sigma_Y A_{AE} \quad F_{BE} = \sigma_Y A_{BE}$$

$$P_P = \frac{6}{5}\sigma_Y A_{AE} + \frac{8}{5}\sigma_Y A_{BE} \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

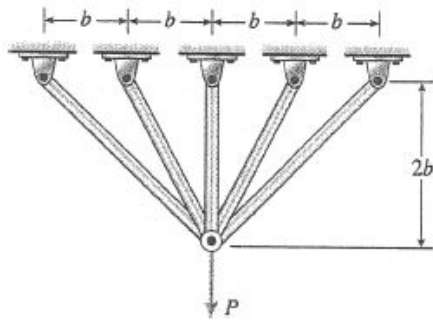
$$A_{AE} = 0.307 \text{ in.}^2 \quad A_{BE} = 0.601 \text{ in.}^2$$

$$\sigma_Y = 36 \text{ ksi}$$

$$P_P = \frac{6}{5}(36 \text{ ksi})(0.307 \text{ in.}^2) + \frac{8}{5}(36 \text{ ksi})(0.601 \text{ in.}^2)$$

$$= 13.26 \text{ k} + 34.62 \text{ k} = 47.9 \text{ k} \quad \leftarrow$$

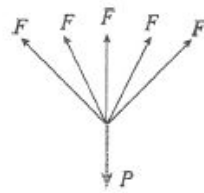
**Problem 2.12-6**



$$d = 10 \text{ mm}$$

$$A = \frac{\pi d^2}{4} = 78.54 \text{ mm}^2$$

$$\sigma_Y = 250 \text{ MPa}$$



At the plastic load, all five bars are stressed to the yield stress

$$F = \sigma_Y A$$

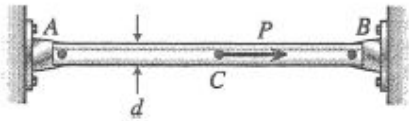
Sum forces in the vertical direction and solve for the load:

$$\begin{aligned} P_P &= 2F \left( \frac{1}{\sqrt{2}} \right) + 2F \left( \frac{2}{\sqrt{5}} \right) + F \\ &= \frac{\sigma_Y A}{5} (5\sqrt{2} + 4\sqrt{5} + 5) \\ &= 4.2031 \sigma_Y A \quad \leftarrow \end{aligned}$$

Substitute numerical values:

$$\begin{aligned} P_P &= (4.2031)(250 \text{ MPa})(78.54 \text{ mm}^2) \\ &= 82.5 \text{ kN} \quad \leftarrow \end{aligned}$$

**Problem 2.12-7**



$$d = 0.6 \text{ in.}$$

$$\sigma_Y = 36 \text{ ksi}$$

Initial tensile stress = 10 ksi

(a) PLASTIC LOAD  $P_P$

The presence of the initial tensile stress does not affect the plastic load. Both parts of the bar must yield in order to reach the plastic load.

POINT C:

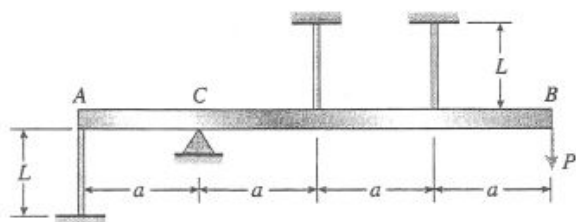
$$\begin{array}{c} \sigma_Y A \quad P \quad \sigma_Y A \\ \leftarrow C \rightarrow \leftarrow \end{array}$$

$$\begin{aligned} P_P &= 2\sigma_Y A = (2)(36 \text{ ksi})\left(\frac{\pi}{4}\right)(0.60 \text{ in.})^2 \\ &= 20.4 \text{ k} \quad \leftarrow \end{aligned}$$

(B) INITIAL TENSILE STRESS IS DOUBLED

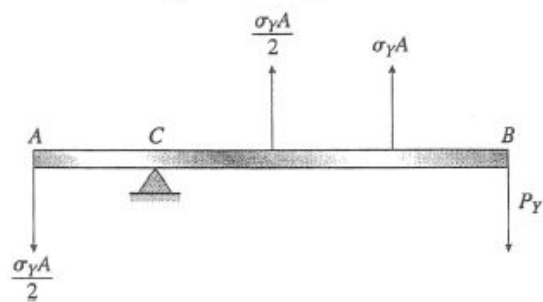
$P_P$  is not changed.  $\leftarrow$

**Problem 2.12-8**



**(a) YIELD LOAD  $P_Y$**

Yielding occurs when the most highly stressed wire reaches the yield stress  $\sigma_Y$



$$\Sigma M_C = 0$$

$$P_Y = \sigma_Y A \quad \leftarrow$$

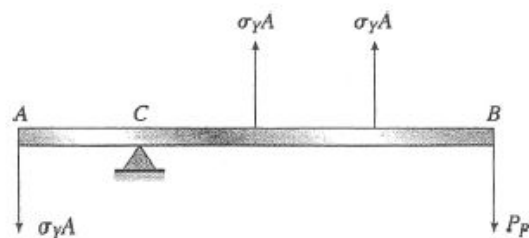
At point A:

$$\delta_A = \left( \frac{\sigma_Y A}{2} \right) \left( \frac{L}{EA} \right) = \frac{\sigma_Y L}{2E}$$

At point B:

$$\delta_B = 3\delta_A = \delta_Y = \frac{3\sigma_Y L}{2E} \quad \leftarrow$$

**(b) PLASTIC LOAD  $P_P$**



At the plastic load, all wires reach the yield stress.

$$\Sigma M_C = 0$$

$$P_P = \frac{4\sigma_Y A}{3} \quad \leftarrow$$

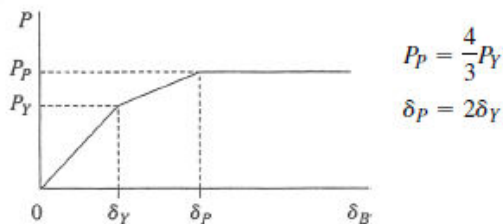
At point A:

$$\delta_A = (\sigma_Y A) \left( \frac{L}{EA} \right) = \frac{\sigma_Y L}{E}$$

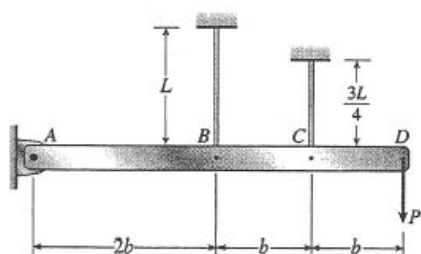
At point B:

$$\delta_B = 3\delta_A = \delta_P = \frac{3\sigma_Y L}{E} \quad \leftarrow$$

**(c) LOAD-DISPLACEMENT DIAGRAM**



### Problem 2.12-9

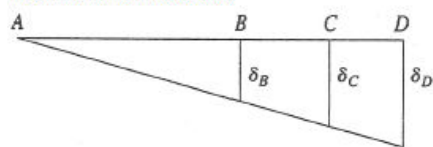


$A$  = cross-sectional area

$\sigma_Y$  = yield stress

$E$  = modulus of elasticity

DISPLACEMENT DIAGRAM

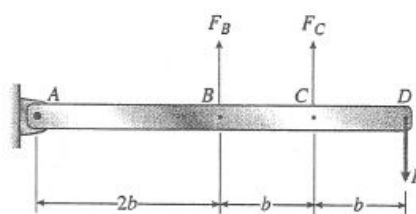


COMPATIBILITY:

$$\delta_C = \frac{3}{2}\delta_B \quad (1)$$

$$\delta_D = 2\delta_B \quad (2)$$

FREE-BODY DIAGRAM



EQUILIBRIUM:

$$\sum M_A = 0 \quad F_B(2b) + F_C(3b) = P(4b) \quad (3)$$

$$2F_B + 3F_C = 4P$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_B = \frac{F_B L}{EA} \quad \delta_C = \frac{F_C \left(\frac{3}{4}L\right)}{EA} \quad (4, 5)$$

Substitute into Eq. (1):

$$\frac{3F_C L}{4EA} = \frac{3F_B L}{2EA} \quad (6)$$

$$F_C = 2F_B$$

STRESSES

$$\sigma_B = \frac{F_B}{A} \quad \sigma_C = \frac{F_C}{A} \quad \sigma_C = 2\sigma_B \quad (7)$$

Wire C has the larger stress. Therefore, it will yield first.

(a) YIELD LOAD

$$\sigma_C = \sigma_Y \quad \sigma_B = \frac{\sigma_C}{2} = \frac{\sigma_Y}{2} \quad (\text{From Eq. 7})$$

$$F_C = \sigma_Y A \quad F_B = \frac{1}{2} \sigma_Y A$$

From Eq. (3):

$$2\left(\frac{1}{2}\sigma_Y A\right) + 3(\sigma_Y A) = 4P$$

$$P = P_Y = \sigma_Y A \quad \leftarrow$$

From Eq. (4):

$$\delta_B = \frac{F_B L}{EA} = \frac{\sigma_Y L}{2E}$$

From Eq. (2):

$$\delta_D = \delta_Y = 2\delta_B = \frac{\sigma_Y L}{E} \quad \leftarrow$$

(b) PLASTIC LOAD

At the plastic load, both wires yield.

$$\sigma_B = \sigma_Y = \sigma_C \quad F_B = F_C = \sigma_Y A$$

From Eq. (3):

$$2(\sigma_Y A) + 3(\sigma_Y A) = 4P$$

$$P = P_P = \frac{5}{4}\sigma_Y A \quad \leftarrow$$

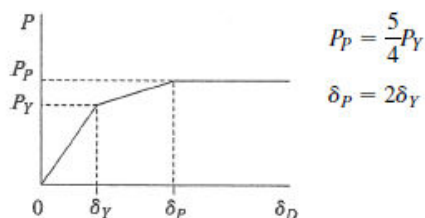
From Eq. (4):

$$\delta_B = \frac{F_B L}{EA} = \frac{\sigma_Y L}{E}$$

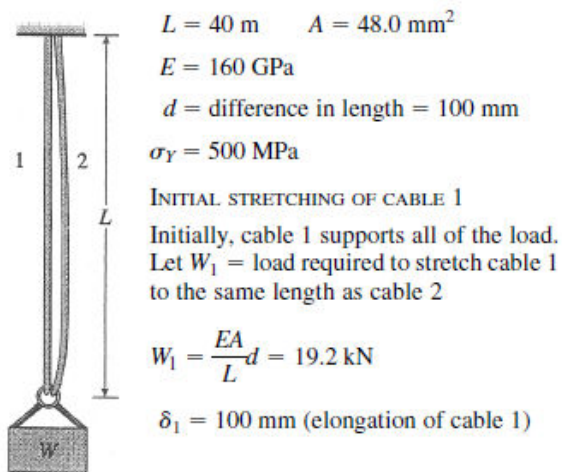
From Eq. (2):

$$\delta_D = \delta_P = 2\delta_B = \frac{2\sigma_Y L}{E} \quad \leftarrow$$

(c) LOAD-DISPLACEMENT DIAGRAM



**Problem 2.12-10**



$$\sigma_1 = \frac{W_1}{A} = \frac{Ed}{L} = 400 \text{ MPa} (\sigma_1 < \sigma_Y \therefore \text{OK})$$

**(a) YIELD LOAD  $W_Y$**

Cable 1 yields first.  $F_1 = \sigma_Y A = 24 \text{ kN}$

$\delta_{1Y} =$  total elongation of cable 1

$\delta_{1Y} =$  total elongation of cable 1

$$\delta_{1Y} = \frac{F_1 L}{EA} = \frac{\sigma_Y L}{E} = 0.125 \text{ m} = 125 \text{ mm}$$

$$\delta_Y = \delta_{1Y} = 125 \text{ mm} \quad \leftarrow$$

$\delta_{2Y} =$  elongation of cable 2

$$= \delta_{1Y} - d = 25 \text{ mm}$$

$$F_2 = \frac{EA}{L} \delta_{2Y} = 4.8 \text{ kN}$$

$$W_Y = F_1 + F_2 = 24 \text{ kN} + 4.8 \text{ kN}$$

$$= 28.8 \text{ kN} \quad \leftarrow$$

**(b) PLASTIC LOAD  $W_P$**

$$F_1 = \sigma_Y A \quad F_2 = \sigma_Y A$$

$$W_P = 2\sigma_Y A = 48 \text{ kN} \quad \leftarrow$$

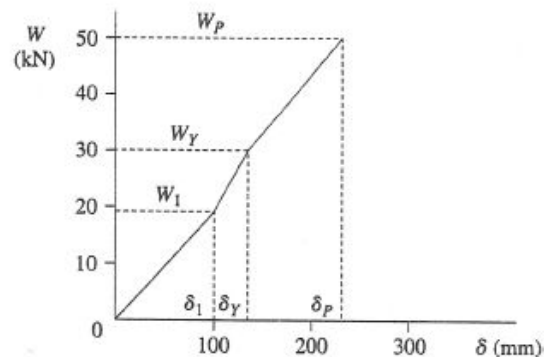
$\delta_{2P} =$  elongation of cable 2

$$= F_2 \left( \frac{L}{EA} \right) = \frac{\sigma_Y L}{E} = 0.125 \text{ m} = 125 \text{ mm}$$

$$\delta_{1P} = \delta_{2P} + d = 225 \text{ mm}$$

$$\delta_P = \delta_{1P} = 225 \text{ mm} \quad \leftarrow$$

**(c) LOAD-DISPLACEMENT DIAGRAM**



$$\frac{W_Y}{W_1} = 1.5 \quad \frac{\delta_Y}{\delta_1} = 1.25$$

$$\frac{W_P}{W_Y} = 1.667 \quad \frac{\delta_P}{\delta_Y} = 1.8$$

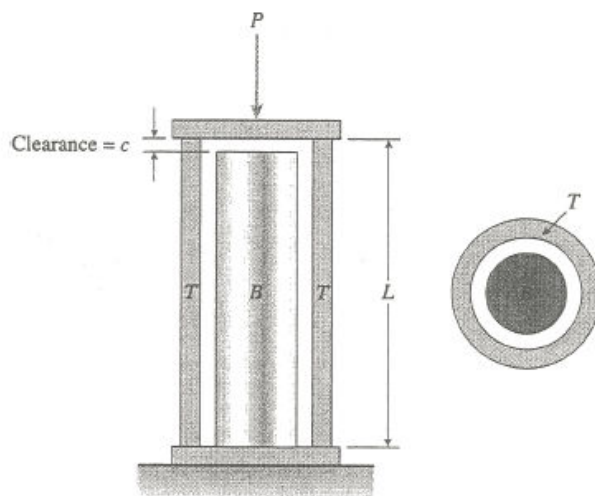
$0 < W < W_1$ : slope = 192,000 N/m

$W_1 < W < W_Y$ : slope = 384,000 N/m

$W_Y < W < W_P$ : slope = 192,000 N/m



**Problem 2.12-11**



$$\begin{aligned} L &= 15 \text{ in.} \\ c &= 0.010 \text{ in.} \\ E &= 29 \times 10^3 \text{ ksi} \\ \sigma_Y &= 36 \text{ ksi} \end{aligned}$$

TUBE:

$$\begin{aligned} d_2 &= 3.0 \text{ in.} \\ d_1 &= 2.75 \text{ in.} \\ A_T &= \frac{\pi}{4} (d_2^2 - d_1^2) = 1.1290 \text{ in.}^2 \end{aligned}$$

BAR

$$\begin{aligned} d &= 1.5 \text{ in.} \\ A_B &= \frac{\pi d^2}{4} = 1.7671 \text{ in.}^2 \end{aligned}$$

INITIAL SHORTENING OF TUBE  $T$

Initially, the tube supports all of the load.

Let  $P_1$  = load required to close the clearance

$$P_1 = \frac{EA_T}{L} c = 21,827 \text{ lb}$$

Let  $\delta_1$  = shortening of tube  $\delta_1 = c = 0.010 \text{ in.}$

$$\sigma_1 = \frac{P_1}{A_T} = 19,330 \text{ psi} \quad (\sigma_1 < \sigma_Y \therefore \text{OK})$$

(a) YIELD LOAD  $P_Y$

Because the tube and bar are made of the same material, and because the strain in the tube is larger than the strain in the bar, the tube will yield first.

$$F_T = \sigma_Y A_T = 40,644 \text{ lb}$$

$\delta_{TY}$  = shortening of tube at the yield stress

$$\sigma_{TY} = \frac{F_T L}{EA_T} = \frac{\sigma_Y L}{E} = 0.018621 \text{ in.}$$

$$\delta_Y = \delta_{TY} = 0.018621 \text{ in.} \quad \leftarrow$$

(b) PLASTIC LOAD  $P_P$

$$\begin{aligned} F_T &= \sigma_Y A_T & F_B &= \sigma_Y A_B \\ P_P &= F_T + F_B = \sigma_Y (A_T + A_B) \\ &= 104,300 \text{ lb} \quad \leftarrow \end{aligned}$$

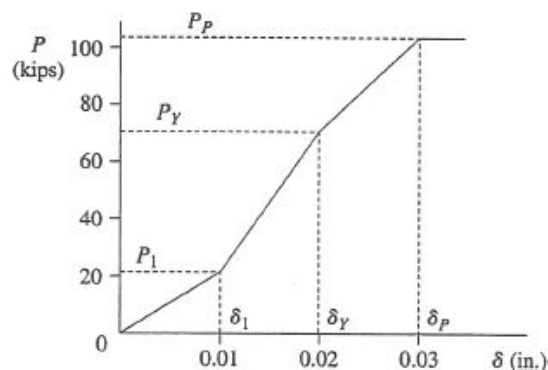
$\delta_{BP}$  = shortening of bar

$$= F_B \left( \frac{L}{EA_B} \right) = \frac{\sigma_Y L}{E} = 0.018621 \text{ in.}$$

$$\delta_{TP} = \delta_{BP} + c = 0.028621 \text{ in.}$$

$$\delta_P = \delta_{TP} = 0.02862 \text{ in.} \quad \leftarrow$$

(c) LOAD-DISPLACEMENT DIAGRAM



Part (c) continued

$\delta_{BY}$  = shortening of bar

$$= \delta_{TY} - c = 0.008621 \text{ in.}$$

$$F_B = \frac{EA_B}{L} \delta_{BY} = 29,453 \text{ lb}$$

$$P_Y = F_T + F_B = 40,644 \text{ lb} + 29,453 \text{ lb} \\ = 70,097 \text{ lb}$$

$$P_Y = 70,100 \text{ lb} \quad \leftarrow$$

$$\frac{P_Y}{P_1} = 3.21 \quad \frac{\delta_Y}{\delta_1} = 1.86$$

$$\frac{P_P}{P_Y} = 1.49 \quad \frac{\delta_P}{\delta_Y} = 1.54$$

$$0 < P < P_1: \text{ slope} = 2180 \text{ k/in.}$$

$$P_1 < P < P_Y: \text{ slope} = 5600 \text{ k/in.}$$

$$P_Y < P < P_P: \text{ slope} = 3420 \text{ k/in.}$$

[CLICK HERE TO ACCESS THE COMPLETE Solutions](#)