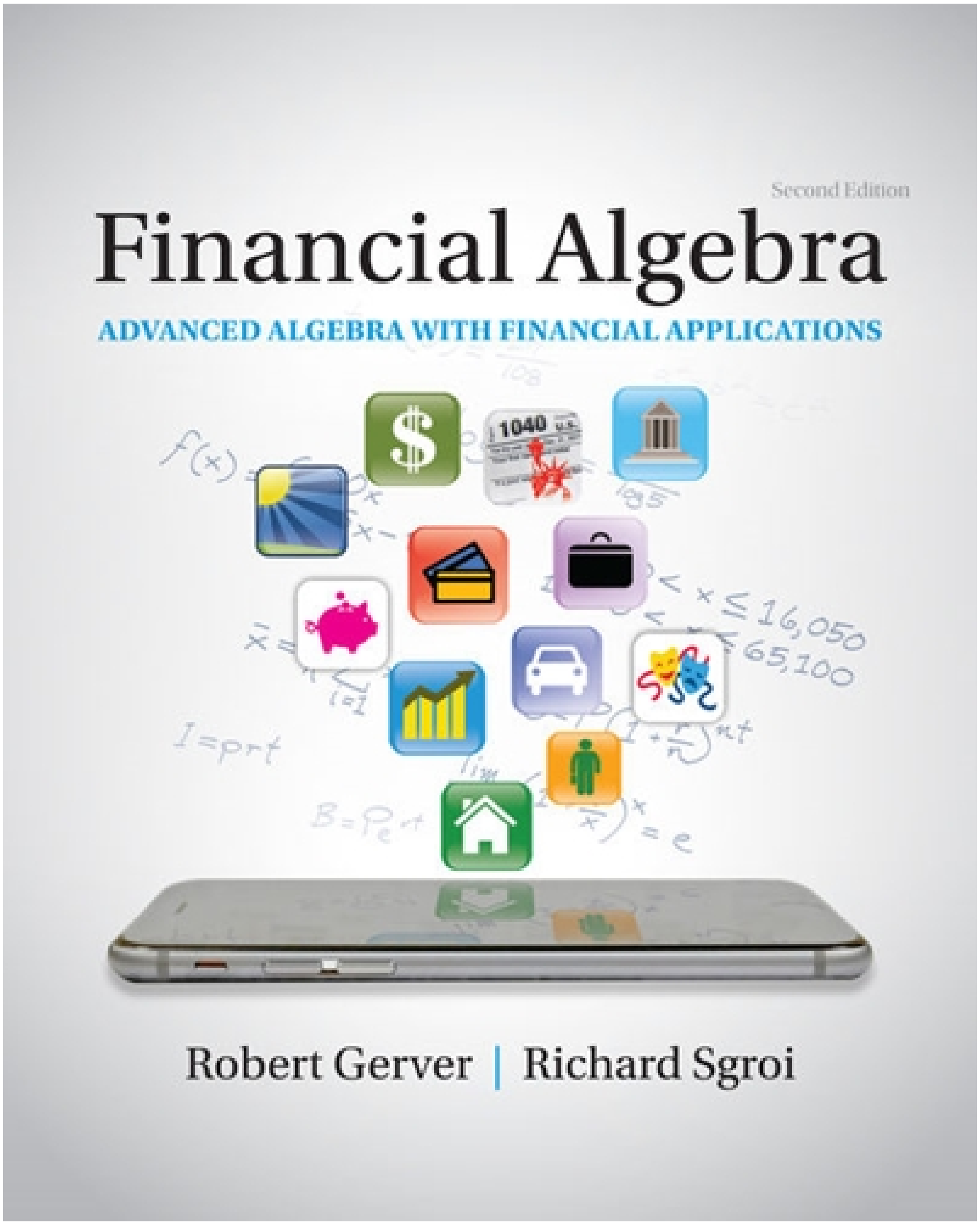
Solutions for Financial Algebra Advanced Algebra with Financial Applications 2nd Edition by Gerver

CLICK HERE TO ACCESS COMPLETE Solutions



Solutions

Chapter 2

Lesson 2-1 Checking Accounts

Check Your Understanding (Example 1)

Begin with the old balance, add the deposits and subtract the withdrawal: $x + b + 2c - d$.

Check Your Understanding (Example 2)

Write the check number: 3273, the date: 5/11, the transaction description: James Sloan, and the payment amount: $150.32. Subtract the payment amount from the old balance: $2,499.90 – $150.32 = $2,349.58.

Extend Your Understanding (Example 2)

The final balance would not change because $2,740.30 – $138.90 = $2,601.40 and $2,601.40 – $101.50 = $2,499.90.

Applications

1. Will Rogers indicates that banking is as essential to your daily life as fire and the wheel.
2. $278.91 + $865.98 + $623 + $60 + $130 = $1,957.89
3. Add the deposits *w*, *h*, and *v*. Subtract the withdrawal *k*. The expression is $t + w + h + v - k$.
4. $630 = 3($50) + 18($20) + *t*($10)

   $630 = $150 + $360 + $10*t*

   $630 = $510 + $10*t*

   $120 = $10*t*

   12 = *t*
5. $9,145.87 + $2,783.71 – $4,871.90 – $12 + $11.15 = $7,056.83
6. $7,859.92;
   5,195.65 + 6,873.22 – *c* – 15 + 6.05 = 4,200.00

   12,059.92 – *c* = 4,200.00

   7,859.92 = *c*
7. 12($13) + 289($0.07) = $156 + $20.23 = $176.23
8. $421.56 + *g*($20) + *k*($0.25) = 421.56 + 20*g* + 0.25*k*
9. $245 = 2($50) + 6($20) + *f*($5)

   $245 = $100 + $120 + $5*f*

   $245 = $220 + $5*f*

   $25 = $5*f*

   5 = *f*
10. $113 = 4($20) + *x*($10) + 3($1)

    $113 = $80 + $10*x* + $3

    $113 = $83 + $10*x*

    $30 = 10*x*

    3 = *x*
11. Let *a* represent the number of 20-dollar bills and *b* represent the number of dollar coins. $20a = 60$, so $a = 3$. This means there are three 20-dollar bills. $b = 4a = 4(3) = 12$, which means there are $12 coins. The total amount of the 20-dollar bills and the dollar coins is $60 + $12 = $72. So Hector has *y* + 72 dollars in his account after the deposit.
12. Speaker Cabinets: $400.00 × 2 = $800.00

    Speaker Cabinets: $611.00 × 2 = $1,222.00

    Horns: $190.00 × 2 = $380.00

    Audio Console: $1,079.00 × 1 = $1,079.00

    Power Amplifier: $416.00 × 5 = $2,080.00

    Microphones: $141.92 × 8 = $1,135.36

    Microphone Stands: $32.50 × 8 = $260.00

    Total: $800.00 + $1,222.00 + $380.00 + $1,079.00 + $2,080.00 + $1,135.36 + $260.00 = $6,956.36

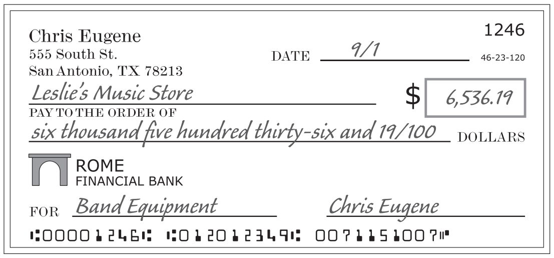
    13% Discount: $6,956.36 × 0.13 = $904.33

    Sales Price: $6,956.36 – $904.33 = $6,052.03

    8% Sales Tax: $6,052.03 × 0.08 = $484.16

    Total Cost: $6,052.03 + $484.16 = $6,536.19

DESCRIPTION	CATALOG NUMBER	LIST PRICE	QUANTITY	TOTAL
Speaker Cabinets	RS101	$400.00	2	$800.00
Speaker Cabinets	RG306	$611.00	2	$1,222.00
Horns	BG42	$190.00	2	$380.00
Audio Console	LS101	$1,079.00	1	$1,079.00
Power Amplifier	NG107	$416.00	5	$2,080.00
Microphones	RKG-1972	$141.92	8	$1,135.36
Microphone Stands	1957-210	$32.50	8	$260.00
			TOTAL	$6,956.36
			13% DISCOUNT	$904.33
			SALE PRICE	$6,052.03
			8% SALES TAX	$484.16
			TOTAL COST	$6,536.19


Chris Eugene
555 South St.
San Antonio, TX 78213
1246
DATE 9/1
46-23-120
Leslie's Music Store
PAY TO THE ORDER OF
$ 6,536.19
six thousand five hundred thirty-six and 19/100 DOLLARS
ROME FINANCIAL BANK
FOR Band Equipment
Chris Eugene
⑆00001246⑆ ⑆012012349⑆ 0071151007⑈

13a. Write the date 10/29 and the balance of $237.47.

13b. Write the check number: 115, the date: 10/29, the transaction description: Fox High School, and the payment amount: $18.00. Subtract: $237.47 – $18.00 = $219.47.

13c. Write the date: 10/30, the transaction description: deposit, and the deposit amount:

$162.75. Add: $219.47 + $162.75 = $382.22.

13d. Write the date: 11/4, the transaction description: deposit, and the deposit amount: $25.00. Add:

$382.22 + $25.00 = $407.22.

13e. Write the date: 11/5, the transaction description: ATM, and the payment amount: $10.00. Subtract: $407.22 – $10.00 = $397.22. There is also an ATM fee of $2.25. Write the date: 11/5, the transaction description: ATM fee, and the payment amount: $2.25. Subtract: $397.22 – $2.25 = $394.97.

13f. Write the check number: 116, the date: 11/7, the transaction description: Credit USA, and the payment amount: $51.16. Subtract: $394.97 – $51.16 = $343.81.

13g. Write the date: 11/10, the transaction description: deposit, and the deposit amount:

$20.00. Add: $343.81 + $20.00 = $363.81.

13h. Write the date: 11/12, the transaction description: ATM, and the payment amount: $20.00. Subtract: $363.81 – $20.00 = $343.81.

There is also an ATM fee of $2.25. Write the date: 11/12, the transaction description: ATM fee, and the payment amount: $2.25. Subtract: $343.81 – $2.25 = $341.56.

13i. Write the date: 11/16, the transaction description: deposit, and the deposit amount: $165.65. Add: $341.56 + $165.65 = $507.21

13j. Write the date: 11/17, the transaction description: deposit, and the deposit amount: $35.00. Add: $507.21 + $35 = $542.21

NUMBER OR CODE	DATE	TRANSACTION DESCRIPTION	PAYMENT AMOUNT		✓	FEE	DEPOSIT AMOUNT		$ BALANCE
	10/29		$						237.47
115	10/29	Fox High School	18	00					– 18.00 219.47
	10/30	Deposit					162	75	+ 162.75 382.22
	11/4	Deposit					25	00	+ 25.00 407.22
	11/5	ATM	15	00					– 15.00 392.22
	11/5	ATM fee	2	25					– 2.25 389.97
116	11/7	Credit USA	51	16					– 51.16 338.81
	11/10	Deposit					20	00	+ 20.00 358.81
	11/12	ATM	25	00					– 25.00 333.81
	11/12	ATM fee	2	25					– 2.25 331.56
	11/16	Deposit					165	65	+ 165.65 497.21
	11/17	Deposit					35	00	+ 35.00 532.21

14. $1,400.00 – $1,380.15 = $19.85

$19.85 – $670 = –$650.15

–$650.15 – $95.67 = –$745.82

–$745.82 – $130 = –$875.82

–$875.82 – $87.60 = –$963.42

There are 4 negative balances, so there are 4 overdraft protection fees: $27 \times 4 = \$108$.

–$963.42 – $108 = –$1,071.42

Nancy owes the bank $1,071.42.

15. $256 – $312 – $33 + $250 – $8 = $153

16a. Write the date 12/15 and the balance of $2,546.50.

16b. Write the check number: 2345, the date: 12/16, the transaction description: Kings Park HSSA, and the payment amount: $54.00. Subtract: $2,546.50 – $54.00 = $2,492.50.

16c. Write the date: 12/17, the transaction description: deposit, and the deposit amount: $324.20. Add: $2,492.50 + $324.20 = $2,816.70.

16d. Write the date: 12/20, the transaction description: deposit, and the deposit amount: \$100.00. Add: \$2,816.70 + \$100.00 = \$2,916.70.

16e. Write the check number: 2346, the date: 12/22, the transaction description: Best Buy, and the payment amount: \$326.89. Subtract: \$2,916.70 – \$326.89 = \$2,589.81. Write the check number: 2347, the date: 12/22, the transaction description: Macy's, and the payment amount: \$231.88. Subtract: \$2,589.81 – \$231.88 = \$2,357.93. Write the check number: 2348, the date: 12/22, the transaction description: Target, and the payment amount: \$123.51. Subtract: \$2,357.93 – \$123.51 = \$2,234.42.

16f. Write the check number: 2349, the date: 12/24, the transaction description: VOID. Write the check number: 2350, the date: 12/24, the transaction description: Apple, and the payment amount: \$301.67. Subtract: \$2,234.42 – \$301.67 = \$1,932.75.

16g. Write the date: 12/26, the transaction description: deposit, and the deposit amount: \$98.00. Add: \$1,932.75 + \$98.00 = \$2,030.75.

16h. Write the code: EFT, the date: 12/28, the transaction description: Allstate, and the payment amount: \$876.00. Subtract: \$2,030.75 – \$876.00 = \$1,154.75

16i. Write the date: 12/29, the transaction description: ATM, and the payment amount: \$200.00. Subtract: \$1,154.75 – \$200.00 = \$954.75. There is also an ATM fee of \$1.50. Write the date: 12/29, the transaction description: ATM fee, and the payment amount: \$1.50. Subtract: \$954.75 – \$1.50 = \$953.25.

NUMBER OR CODE	DATE	TRANSACTION DESCRIPTION	PAYMENT AMOUNT $		✓	FEE	DEPOSIT AMOUNT		$ BALANCE
	12/15								2,546.50
2345	12/16	Kings Park HSSA	54	00					– 54.00 2,492.50
	12/17	Deposit					324	20	+ 324.20 2,816.70
	12/20	Deposit					100	00	+ 100.00 2916.70
2346	12/22	Best Buy	326	89					– 326.89 2,589.81
2347	12/22	Macy's	231	88					– 231.88 2,357.93
2348	12/22	Target	123	51					– 123.51 2,234.42
2349	12/24	VOID							
2350	12/24	Apple	301	67					– 301.67 1,932.75
	12/26	Deposit					98	00	+ 98.00 2,030.75
EFT	12/28	Allstate	876	00					– 876.00 1,154.75
	12/29	ATM	200	00					– 200.00 954.75
	12/29	ATM fee	1	50					– 1.50 953.25

17a. The check number after 622 is 623.

17b. The check number after 628 is 629.

17c. The check number after 629 is 630.

17d. The balance forward shows the amount of check 621 is \$71.10.

17e. Subtract the amount for check 622: –\$500.00.

17f. \$1,792.80 – \$500.00 = \$1,292.80

17g. Subtract the amount for check 623: –\$51.12.

17h. \$1,292.80 – \$51.12 = \$1,241.68

17i . Subtract the amount for check 624: –\$25.00.

17j. \$1,241.68 – \$25.00 = \$1,216.68

17k. Add the deposit amount: \$650.00.

17l. \$1,216.68 + \$650.00 = \$1,866.68

17m. Subtract the amount for check 625: –\$200.00.

17n. \$1,866.68 – \$200.00 = \$1,666.68

17p. Subtract the amount for check 626: –\$90.00.

17q. \$1,666.68 – \$90.00 = \$1,576.68

17r. Subtract the amount for check 627: –\$49.00.

17s. \$1,576.68 – \$49.00 = \$1,527.68

17t. Subtract the amount for check 628: –\$65.00.

17u. \$1,527.68 – \$65.00 = \$1,462.68

17v. Subtract the amount for check 629: –\$300.00.

17w. \$1,462.68 – \$300.00 = \$1,162.68

17x. Add the deposit amount: \$400.00.

17y. \$1,162.68 + \$400.00 = \$1,562.68

17z. \$1,562.68 – \$371.66 = \$1,191.02

Lesson 2-2 Reconcile a Bank Statement

Check Your Understanding (Example 1)

Sample answer: Many people and businesses hold on to checks and do not deposit or cash them immediately. If checks are written toward the end of a cycle, they will probably appear on the next monthly statement.

Check Your Understanding (Example 2)

Yes, Let a = \$885.84, b = \$825, c = \$632.84, r = \$1,078. Then $d = a + b - c$ = \$1,078, so the check register is balanced.

Check Your Understanding (Example 3)

Although formulas vary based on the spreadsheet being used, most spreadsheet programs would use the following formula to determine the sum: = sum (B3:B9).

Applications

1. Errol Flynn used the word reconcile in the same way that it is used in a financial context. With a checking account, the problem lies in reconciling or balancing the check register amount with the revised statement amount. Errol Flynn's problem is in balancing his personal gross (monetary) habits such as making expensive purchases with the money he actually has coming in.
2. Yes. $725.71 + $610.00 – $471.19 = $864.52
3. No. $197.10 + $600.00 – $615.15 = $181.95 for the revised statement balance and $210.10 is the check register balance.
4. Add the deposits and subtract the outstanding checks from the ending balance. This should equal the revised statement balance and the check register balance. $B + D – C = S$ and if $S = R$, the account is reconciled.

5a. 14 + 19 + 23 + 24 = 80
80 ÷ 4 = 20

5b. 20($0.20) + $12.50 = $4

5c. $4 + $12.50 = $16.50

6a. The statement shows that the ending balance is $1,434.19.

6b. The deposit of $700.00 is not on the bank statement, so this is the outstanding deposit amount.

6c. Check numbers 397 and 399 are not on the bank statement. So the outstanding withdrawal amount is $50.00 + $ 39.00 = $89.00.

6d. $1,434.19 + $700.00 – $89.00 = $2,045.19

6e. The check register shows a balance of $2,045.19.

6f. $2,045.19 = $2,045.19, so the account is reconciled.

7. Let x = the number of checks Donna writes each month and let F = the fee charged: $F = 9.75 + 0.15x$.
8. No. adding $75 will correct that he subtracted $75 he should not have subtracted. He will also need to add another $75 for the original deposit.
9. On the statement, you can still see that the balance on 1/27 was $1,371.42 and the check written on 1/30 was for $58.70. Subtract to find the ending balance:
$1,371.42 – $58.70 = $1,312.72.
10. Outstanding deposit: $150; outstanding checks:
$32.00 + $100.00 = $132.00
$1,827.63 – $150.00 + $132.00 = $1,809.63, which reconciles with the statement balance.
11. $728.30 – $75.00 = $653.30
$653.30 – $70.00 = $583.30
$583.30 – $38.50 = $544.80
$544.80 – $28.00 = $516.80
$516.80 – $120.00 = $396.80
$396.80 – $56.73 = $340.07
$340.07 – $100.00 = $240.07
$240.07 – $85.00 = $155.07
$155.07 + $1,000.00 = $1,155.07
$1,155.07 – $80.00 = $1,075.07
$1,075.07 + 950.00 = $2,025.07
outstanding deposit: $950.00
outstanding checks: $38.50 + $100.00 + $85.00 + $80.00 = $303.50
$2,025.07 – $950.00 + $303.50 = $1,378.57, which reconciles with the statement balance.

Checking Account Summary	
Ending Balance	$1,378.57
Deposits	+ $950.00
Checks Outstanding	– $303.50
Revised Statement Balance	
Check Register Balance	$2,025.07

PLEASE BE SURE TO DEDUCT CHANGES THAT AFFECT YOUR ACCOUNT

ITEM NO. FOR TRANSACTION CODE	DATE	DESCRIPTION OF TRANSACTION	SUBTRACTION: AMOUNT OF PAYMENT OR WITHDRAWAL		✓	OTHER	ADDITIONS: AMOUNT OF DEPOSIT OR INTEREST		BALANCE FORWARD	
									728	30
1773	12/28	TO Galaxy Theather	75	00	✓				– 75	00
		FOR Tickets							653	30
1774	12/30	TO American Electric Company	70	00	✓				– 70	00
		FOR Electric Bill							583	30
1775	12/30	TO Hillsdake Water Co.	38	50					– 38	50
		FOR Water Bill							544	80
1776	1/2	TO Barbara's Restaurant	28	00	✓				– 28	00
		FOR Dinner							516	80
1777	1/3	TO Platter Records	120	00	✓				– 120	00
		FOR Compact Disc							396	80
1778	1/9	TO A1 Gas Co.	56	73	✓				– 56	73
		FOR Gas Bill							340	07
1779	1/12	TO Al and Jean Adams	100	00					– 100	00
		FOR Wedding Gift							240	07
1780	1/12	TO Greene College	85	00					– 85	00
		FOR Fees							155	07
	1/14	TO Deposit			✓		1,000	00	+ 1,000	00
		FOR							1,155	07
1780	1/25	TO Rob Gerver	80	00					– 80	00
		FOR Typing Fee							1,075	07
	2/1	TO Deposit					950	00	+ 950	00
		FOR Salary							2,025	07

Yes, Raymond's checking account balances.

12. \$55.65 + \$103.50 + \$25.00 = \$184.15; add the outstanding deposits to the check register balance.

13a. When $d > c$, the amount of the deposits is greater than the amount of the withdrawals. This means there will be more money in the account than the starting balance, so $E >$ \$678.98.

13b. When $d < c$, the amount of the deposits is less than the amount of the withdrawals. This means there will be less money in the account than the starting balance, so $E <$ \$678.98.

Lesson 2-3 Savings Accounts

Check Your Understanding (Example 1)

Answer 172. The 50th term will have had the common difference added to the first term 49 times. Use the formula $a_n = a_1 + (n - 1)d$ and substitute.

$a_{50} = 25 + (50 - 1)\,3 = 172$

Extend Your Understanding (Example 1)

Answer 12. The common difference in an arithmetic sequence can be found by subtracting any term from the next consecutive term. For example,
223 – 211 = 12, so the common difference is 12.

Check Your Understanding (Example 2)

The fractions must be converted to decimals and each percent changed to an equivalent decimal.

5.51% = 0.0551

$5\frac{1}{2}\% = 5.5\% = 0.055$

$5\frac{5}{8}\% = 5.625\% = 0.05625$

5.099% = 0.05099

5.6% = 0.056

0.05625 > 0.056 > 0.0551 > 0.055 > 0.05099, so the list in order from greatest to least is $5\frac{5}{8}\%$, 5.6%, 5.51%, $5\frac{1}{2}\%$, 5.099%.

Check Your Understanding (Example 3)

Because \$891 – \$315 = \$576 is less than \$750, there will be a fee of \$7 each month. The expression to represent the amount of money in the account is $576 - 7x$, where x is the number of months.

Check Your Understanding (Example 4)

Answer \$168. Substitute into the formula $I = Prt$.

$I = 4{,}000(0.012)(3.5) = 168$

Give students additional practice on the board representing fractions of a year in months, and months as fractions of a year.

Check Your Understanding (Example 5)

Answer \$11.24.

This problem extends the fraction of a year notion to include days as the units. In a nonleap year, 300 days is 300/365 of a year. Substitute into the formula $I = Prt$.

$I = 800(0.0171)(300/365) = 11.24$ when rounded to the nearest cent.

Check Your Understanding (Example 6)

Answer \$8,571.43 to the nearest cent. Solve for P in the equation $I = Prt$.

$$P = \frac{I}{rt}$$

$$P = \frac{300}{(0.0175)(2)}$$

$P =$ \$8,571.43 to the nearest cent.

Remind students that there are no additional withdrawals or deposits to this account over the 2 years.

Check Your Understanding (Example 7)

Approximately 9 years. Doubling \$10,000 means earning \$10,000 interest.

$$t = \frac{I}{\text{Pr}}$$

Substitute:

$$t = \frac{10{,}000}{(10{,}000)(0.11)}$$

$t = 9$ when rounded to the nearest year.

Check Your Understanding (Example 8)

4%. Solve for r in the I = Prt formula.

$$r = \frac{I}{Pt}$$

Substitute:

$$r = \frac{50}{(500)(2.5)}$$

r = 4%

4%. Some students may assume that earning $50 means “at least $50,” and can write any percent greater than 4%.

Applications

1. In addition to the pun about “interest,” Greenspan thinks that savings helps in several ways. First, they provide people with a financial cushion. They also give banks more money to lend for people to buy homes, cars, and so on.
2. $256 The 90th term will have had the common difference added to the first term 89 times. Use the formula $a_n = a_1 + (n – 1)d$ and substitute.

   $a_{90} = 78 + (90 – 1)2 = 256$
3. The fractions must be converted to decimals and each percent changed to an equivalent decimal.

   3.4% = 0.034

   3.039% = 0.03039

   $$3\frac{3}{16}\% = 3.1875\% = 0.031875$$

   3.499% = 0.03499

   $$3\frac{1}{2}\% = 3.5\% = 0.035$$

   The list in order from least to greatest is 3.039%,

   $$3\frac{3}{16}\%, 3.4\%, 3.499\%, 3\frac{1}{2}\%.$$
4. 67 The 12th term will have had the common difference added to the first term 11 times. Use the formula $a_n = a_1 + (n – 1)d$ and substitute and solve for the first term.

   $a_{12} = a_1 + (12 – 1)3$

   $100 = a_1 + (12 – 1)3$

   $a_1 = 67$.
5. $843.44. Once he withdraws $300, the balance is $903.44, which is under $1,000. In 6 months he will pay $60 in penalties, so the balance will be $843.44.
6. Let *m* = the number of months for the balance to reach zero. Then the equation to find the number of months is $m = \frac{871.43}{x}$. Substitute 9 for *x*: $m = \frac{871.43}{9} = 96.82\overline{5}$. Although the quotient is $96.82\overline{5}$, it is not until the 97th month that the balance will reach zero.
7. John and George are correct because $6\frac{3}{4}\% = 6.75\% = 0.0675$. When the percent is changed to an equivalent decimal, the percent sign is dropped. So, Paul is incorrect.
8. The advantage of a CD is a higher rate of interest. The disadvantage is the CD has a penalty if the money is withdrawn before maturity.
9. $390.57. Substitute into the formula *I* = *Prt*.

   I = 2,350(0.0277)(6) = 390.57 when rounded to the nearest cent.

10a. $38.44. Substitute into the formula *I* = *Prt*.

I = 775(0.0124)(4) = 38.44 when rounded to the nearest cent.

10b. $813.44. Add the interest of $38.44 to the $775.

10c. $9.61. Substitute into the formula *I* = *Prt*.

I = 775(0.0124)(1) = 9.61 when rounded to the nearest cent.

10d. $9.61. The fourth year interest is computed the same as in part 10c, since the interest is not compounded.

10e. $9.61. Substitute into the formula *I* = *Prt*.

I = 775(0.0124)(1) = 9.61 when rounded to the nearest cent.

10f. $784.61. The interest of $9.61 is added to the $775.

10g. $9.73. Substitute into the formula *I* = *Prt*.

I = 784.61(0.0124)(1) = 9.73 when rounded to the nearest cent. Notice that the principal was higher than the original $775 since it had interest added.

10h. Brian earned more since he got interest on his first year's interest during the second year.

11a. $I = prt$ = ($2,000)(0.0335)(4) = $268

11b. $I = prt = (\$3{,}500)(0.041)\left(\frac{15}{12}\right) = \$179.38$

11c. $I = prt = (\$20{,}100)(0.055)\left(\frac{400}{365}\right) = \$1{,}211.51$

11d. $t = \frac{I}{pr} = \frac{\$100}{(\$700)(0.088)} = 1.62$ years, which is about 20 months.

11e. $t = \frac{I}{pr} = \frac{\$250}{(\$3{,}000)(0.0475)} = 1.75$ years, which is about 22 months.

11f. $r = \frac{I}{pt} = \frac{\$500}{(\$3{,}000)(3)} = 0.0556 = 5.56\%$

11g. $p = \frac{I}{rt} = \frac{\$500}{(0.044)\left(\frac{30}{12}\right)} = \$4{,}545.45$

11h. $t = \frac{I}{pr} = \frac{x}{(p)(0.03)} = \frac{x}{0.03p}$

12. $40.80 Substitute into the formula $I = Prt$.

I = 2,560(0.01125)(17/12)) = 40.80 when rounded to the nearest cent.

13. $t = \frac{I}{pr} = \frac{\$450}{(\$450)(0.14)} = 7.14$, which is about 86 months.

14. $t = \frac{I}{pr} = \frac{\$450}{(\$450)(1)} = 1$ year

15. $r = \frac{I}{pt} = \frac{\$900}{(\$9{,}500)\left(\frac{19}{12}\right)} = 0.0598 = 5.98\%$

16. Slick Bank pays more interest. Substitute:

Bedford Bank: $I = prt$ = ($20,000)(0.01)(5) = $1,000

Slick Bank: $I = prt$ = ($20,000)(0.051)(1) = $1,020

17. Blank Bank: $I = prt$ = (\$$x$)(0.01$r$)($y$) = \$0.01xry

Thank Bank: $I = prt$ = (\$$x$)(0.01$ry$)(1) = \$0.01xry

Both earn the same interest.

18. $I = prt$ = ($3,450)(0.05)(18) = $3,105

$3,450 + $3,105 = $6,555

19. $r = \frac{I}{pt} = \frac{\$310{,}000}{(\$90{,}000)(18)} = 0.19 = 19\%$

20. Less than $352. Two years is 24 months, and at $2 per month, would result in a $48 penalty, which must be subtracted.

21a. $I = prt$, so the formula for cell A2 is = B2*C2*D2.

21b. $p = \frac{I}{rt}$, so the formula for cell B2 is = A2/(C2*D2).

21c. $r = \frac{I}{pt}$, so the formula for cell C2 is = A2/(B2*D2).

21d. $t = \frac{I}{pr}$, so the formula for cell D2 is = A2/(B2*C2).

21e. The time in months is the time in years multiplied by 12, so the formula for cell E2 is = 12*D2.

22. a = 24; b = 36; c = 48; d = 54; e = 60. The common difference is 6, and it is found by subtracting 30 from 42, and dividing by 2, since 42 is the 4th term and 30 is the 2nd term.

Lesson 2-4 Explore Compound Interest

Check Your Understanding (Example 1)

Annual compounding is equivalent to simple interest for the first year. $I = prt$ = (\$$x$)(0.044)(1) = \$0.044x

Check Your Understanding (Example 2)

The amount of interest is $I = prt$ = ($4,000)(0.05)(0.5) = $100.

Add the interest to the principal: $4,000 + $100 = $4,100.

Calculate the interest using the new principal. The amount of interest is $I = prt$ = ($4,100)(0.05)(0.5) = $102.50.

Add the interest to the principal: $4,100 + $102.50 = $4,202.50.

Check Your Understanding (Example 3)

The amount of interest is $I = prt$ = ($3,000)(0.04)(0.25) = $30.

Add the interest to the principal: $3,000 + $30 = $3,030.

Calculate the interest using the new principal. The amount of interest is $I = prt$ = ($3,030)(0.04)(0.25) = $30.30.

Add the interest to the principal: $3,030 + $30.30 = $3,060.30.

Check Your Understanding (Example 4)

$$I = prt = (x)(0.05)\left(\frac{1}{365}\right) = \frac{0.5x}{365}$$

Check Your Understanding (Example 5)

The amount of interest on January 7 is

$I = prt = (\$900)(0.03)\left(\frac{1}{365}\right) = \$0.07$. The balance end of the day on January 7 is $900 + $0.07 = $900.07. Add the deposit: $900.07 + $76.22 = 976.29. The amount of interest on January 8 is

$I = prt = (\$976.29)(0.03)\left(\frac{1}{365}\right) = \$0.08$. The balance at the end of the day on January 8 is $976.29 + $0.08 = $976.37.

Applications

1. Compound interest is better than simple interest but it won't make you rich. Time is very influential in making savings grow, so the earlier an account is started, the longer the money earns interest.
2. $66.60. Annual compounding is equivalent to simple interest for the first year. Substitute into $I = Prt$.

   I = 3,700(0.018)(1) = 66.60
3. $4,084.80. Annual compounding is equivalent to simple interest for the first year. Substitute into $I = Prt$.

   I = 4,000(0.0212)(1) = 84.80. The interest, $84.80, needs to be added to the original $4,000 principal.
4. $63, $9,063. Multiply 9,000 by 0.014 to get 126, which would represent a full year's interest. Divide by 2 to get the semi-annual interest of $63.
5. $I = x(0.022)\left(\frac{1}{4}\right) = 0.0055x$. This interest needs to be added to the principal.

6a. $18.38. Multiply the balance of 3,500 by 0.0105, and divide by 2.

6b. $3,518.38. Add the interest, $18.38, to the principal $3,500.

6c. $18.47. Multiply the balance of 3,518.38 by 0.0105, and divide by 2

6d. $3,536.85. Add the interest of $18.47 to the principal $3,518.38.

6e. $36.85. Add the two interest amounts, $18.38 and $18.47 to get total interest for the year.

6f. $36.75. Use the simple interest formula $I = Prt$.

I = 3,500(0.0105)(1) = $36.75

6g. $0.10

7a. $7. Multiply the principal 3,500 by 0.008, and divide the result by 4.

7b. $3,507. Add the interest of $7 to the $3,500.

7c. $7.01. Multiply the principal 3,507 by 0.008, and divide the result by 4

7d. $3,514.01. Add the interest of $7.01 to the principal of $3,507.

7e. $7.03. Multiply the principal 3,514.01 by 0.008, and divide the result by 4

7f. $3,521.04. Add the interest of $7.03 to $3,514.01.

7g. $7.04. Multiply the principal 3,521.04 by 0.008, and divide the result by 4

7h. $3,528.08. Add the $7.04 interest to the principal $3,521.04.

7i. $28.08. Add the four interest amounts.

8. $0.03. Multiply 720 by 0.014 and divide by 365.
9. $3.27. Multiply 2,000 by 0.0196, and divide the result by 12.

10a. $0. The day started and Jacob had not opened the account yet, so the "balance" is 0.

10b. $4,550.00. This is the amount he deposited.

10c. \$4,550.00. This is the principal used to compute the interest since it is the balance at the end of the day.

10d. \$0.14. The principal \$4,550 is multiplied by 0.011 and the result is divided by 365.

10e. \$4,550.00 + \$0.14 = \$4,550.50. The interest of \$0.14 is added to the principal used to compute the interest, which is \$4,550.

10f. \$4,550.14. The ending balance from the previous day is the opening balance for the next day.

10g. \$300.00. A deposit was made.

10h. \$4,550.14 + \$300.00 = \$4,850.14. The deposit is added to the opening balance.

10i. \$0.15. The principal, \$4,850.14 is multiplied by 0.011 and the result is divided by 365.

10j. \$4,850.29. The \$0.15 interest is added to \$4,850.29

10k. \$4,850.29. The ending balance from the previous day is the opening balance for the next day.

10l. \$900.00. There was a \$900 withdrawal made.

10m. \$3,950.29. The withdrawal is subtracted form the opening balance for August 12.

10n. \$0.12. The principal, \$3,950.29, is multiplied by 0.011 and the result is divided by 365.

10p. \$3,950.41. The interest of \$0.12 is added to the principal of \$3,950.29.

11a. \$0. The day started and Stacy had not opened the account yet, so the “balance” is 0.

11b. \$6,000.00. The account is opened with an initial \$6,000 deposit.

11c. \$0. There is no withdrawal on December 18.

11d. \$6,000.00. The principal used to compute the interest is \$6,000, the initial deposit.

11e. \$0.24. The \$6,000 principal is multiplied by 0.0145 and the result is divided by 366 since it is a leap year.

11f. \$6,000.24. The interest of \$0.24 is added to \$6,000.

11g. \$6,000.24. The ending balance from the previous day is the opening balance for the next day.

11h. \$500.00. There is a deposit of \$500 on December 19.

11i. \$0. There is no withdrawal on December 19.

11j. \$6,500.24. The principal used to compute the interest includes the deposit and the previous day’s interest.

11k. \$0.26. The principal, \$6,500.24, is multiplied by 0.0145, and the result is divided by 366.

11l. \$6,500.50. The interest is added to the principal for December 19.

11m. \$6,500.50. The ending balance from the previous day is the opening balance for the next day.

11n. \$0. There is no deposit on December 20.

11p. \$2,500.00. There is a \$2,500 withdrawal on December 20.

11q. \$4,000.50. The withdrawal is subtracted from \$6,500.50.

11r. \$0.16. The principal, \$4,500.50, is multiplied by 0.0145, and the result is divided by 366

11s. \$4,000.66. The interest is added to the principal for December 20. This is also the opening balance for December 21.

12. $$x+y+\frac{0.013(x+y)}{365}+\frac{0.013\left(x+y+\frac{0.013(x+y)}{365}\right)}{365}$$

The interest for May 30 is based on the ending balance for May 29. On May 29, the principal used to compute the interest is $(x + y)$. On May 30, the principal used to compute the interest is $x+y+\frac{0.013(x+y)}{365}$.

13. $\frac{0.014(d-w)}{52}$ The principal used to compute the interest is $d - w$. This is multiplied by the interest rate, 0.014, and the product is divided by 52 since it is compounded weekly.

14a. $P + D$. The deposit is added to the opening balance.

14b. $\frac{0.02(P+D)}{365}$

The interest is computed by multiplying the principal by the interest rate, 0.02, and the result is divide by 365 since it is compounded daily.

14c. $P + D + \frac{0.02(P+D)}{365}$

The interest is added to the principal for February 2.

14d. $P + D + \frac{0.02(P+D)}{365}$

The opening balance is equal to the previous day's ending balance.

14e. $P + D - W + \frac{0.02(P+D)}{365}$

The February 3 withdrawal is subtracted from the opening balance.

14f. $\dfrac{0.02\left(P + D - W + \frac{0.02(P+D)}{365}\right)}{365}$

The principal used to compute the interest is multiplied by 0.02, and the result is divide by 365 since it is compounded daily.

$$P + D - W + \frac{0.02(P+D)}{365}$$
$$+\frac{0.02\left(P + D - W + \frac{0.02(P+D)}{365}\right)}{365}$$

15. If his opening balance is m, twice that is $2m$, and the principal used to compute the interest is $3m$. The interest is computed by multiplying by 0.0125 and dividing by 12. Then the interest is added to the principal $3m$.

$$3m + \frac{0.0125(3m)}{12}$$

Lesson 2-5 Compound Interest Formula

Check Your Understanding (Example 1)

\$815.06. Use the compound interest formula and substitute.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 800\left(1+\frac{0.0187}{4}\right)^{4}$$

B = \$815.15

Check Your Understanding (Example 2)

\$1,220.56. Use the compound interest formula and substitute.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 1{,}200\left(1+\frac{0.017}{12}\right)^{12}$$

B = \$1,220.56

Extend Your Understanding (Example 2)

1.9%. Use the compound interest formula and substitute for the daily account.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 1{,}200\left(1+\frac{0.0138}{365}\right)^{365}$$

The balance for the daily account is B = \$1,216.67 after 1 year.

Use the compound interest formula and substitute for the quarterly account.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 1{,}200\left(1+\frac{0.019}{4}\right)^{4}$$

The balance for the quarterly account is B = \$1,222.96 after 1 year, so the quarterly account is better.

Check Your Understanding (Example 3)

\$2,482.81. Use the compound interest formula and substitute.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 2{,}350\left(1+\frac{0.011}{12}\right)^{60}$$

B = \$2,482.81

Extend Your Understanding (Example 3)

Use the compound interest formula and substitute.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$2{,}000\left(1+\frac{0.015}{365}\right)^{365k}$$

Check Your Understanding (Example 4)

1. 11%. Use the APY formula and substitute for the quarterly account.

$$\text{APY} = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$\text{APY} = \left(1+\frac{0.011}{365}\right)^{365} - 1$$

APY = 1.11% to the nearest hundredth of a percent.

Extend Your Understanding (Example 4)

Because there are more compounding periods for interest to be earned on already accumulated interest, the balance grows more quickly.

Applications

1. Bank interest on its own will not make you rich; the interest rates are much smaller than possible returns on business investments. However, there is much less risk.
2. $4,880.76. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 4{,}000\left(1+\frac{0.02}{2}\right)^{20}$$

B = $4,880.76

3. $2,739.89. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 2{,}000\left(1+\frac{0.0175}{12}\right)^{(18)(12)}$$

B = $2,739.89

4. $1,551.26. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 1{,}500\left(1+\frac{0.0112}{365}\right)^{(365)(3)}$$

B = $1,551.26

5a. $4,400.04. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 4{,}300\left(1+\frac{0.023}{365}\right)^{(365)}$$

B = $4,400.04

5b. $100.04. Subtract the initial principal, $4,300 from $4,400.04.

5c. 2.33%. Use the APY formula and substitute for the quarterly account.

$$\text{APY} = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$\text{APY} = \left(1+\frac{0.023}{365}\right)^{365} - 1$$

APY = 2.33% to the nearest hundredth of a percent.

6. $5,230.11. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 5{,}000\left(1+\frac{0.015}{52}\right)^{(156)}$$

B = $5,230.11

7a. $1,014.09. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 1{,}000\left(1+\frac{0.014}{12}\right)^{(12)}$$

B = $1,014.09

7b. 1.41%. Use the APY formula and substitute for the quarterly account.

$$\text{APY} = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$\text{APY} = \left(1+\frac{0.014}{12}\right)^{12} - 1$$

APY = 1.41%

8. $2.25. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 1{,}000\left(1+\frac{0.03}{365}\right)^{(8)(365)}$$

When compounded daily, the balance is B = $1,271.24.

$$B = 1{,}000\left(1+\frac{0.03}{2}\right)^{(8)(2)}$$

When compounded semi-annually, the balance is B = $1,268.99.

The difference between the two accounts is $2.25.

9. $3,109.97. Use the compound interest formula and substitute. This must be computed on a calculator. There are 8,760 hours in a year.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 3{,}000\left(1+\frac{0.018}{8760}\right)^{(2)(8760)}$$

B = $3,109.97

10a. No. Use the compound interest formula and substitute. This must be computed on a calculator. Find the interest after 10 and 15 years to see if their money is doubled.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 20{,}000\left(1+\frac{0.0275}{365}\right)^{(10)(365)}$$

Balance after 10 years is $26,330.34, so the money does not double in 10 years.

10b. No.

Substitute using 15 years.

$$B = 20{,}000\left(1+\frac{0.0275}{365}\right)^{(15)(365)}$$

Balance after 15 years is $30,211.32, so the money does not double in 15 years.

11a. Lindsay. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

For Lindsay, $B = 80\left(1+\frac{0.01}{12}\right)^{12} = 80.80.$

For Michele, $B = 60\left(1+\frac{0.02}{52}\right)^{52} = 61.21.$

Lindsay's balance is greater after 1 year.

11b. Lindsay

For Lindsay, $B = 80\left(1+\frac{0.01}{12}\right)^{144} = 90.20.$

For Michele, $B = 60\left(1+\frac{0.02}{52}\right)^{624} = 76.27.$

Lindsay's balance is greater after 12 years

12a. $125. Use the simple interest formula, *I* = *Prt*, and substitute.

I = 5,000(0.025)(1) = $125

12b. $125. Compounding annually for 1 year is the same as simple interest for 1 year.

12c. The interest is the same.

12d. $375. Use the simple interest formula, *I* = *Prt*, and substitute.

I = 5,000(0.025)(3) = $375

12e. $384.45. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 5{,}000\left(1+\frac{0.025}{1}\right)^{3}$$

B = $5,384.45. Subtract $5,000 to find that the interest is $384.45.

12f. The annual compounded interest earned is $9.45 more than the simple interest.

12g. $600. Use the simple interest formula, *I* = *Prt*, and substitute.

I = 5,000(0.02)(6) = $600

12h. \$630.81. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 5{,}000\left(1 + \frac{0.02}{1}\right)^6$$

B = \$630.81

12i. The annual compounded interest earned is \$30.81 more than the simple interest.

12j. No. They are the same for 1 year. For anything longer, compounded interest grows faster than simple interest.

13. Rodney's account balance will always be greater; $P(1 + r) > P(0.5 + 2r)$, for today's common interest rates. If the interest rate is greater than 50%, the inequality is reversed.

14. 1.56%. Use the APY formula and substitute for the quarterly account.

$$\text{APY} = \left(1 + \frac{r}{n}\right)^n - 1$$

$$\text{APY} = \left(1 + \frac{0.0155}{365}\right)^{365} - 1$$

APY = 1.56%

15a. \$212,000. Use the simple interest formula, $I = Prt$, and substitute.

$I = 10{,}000{,}000(0.0212)(1) = \$212{,}000$

15b. \$214,256.88. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 5{,}000\left(1 + \frac{0.0212}{365}\right)^{365}$$

B = \$214,256.88

15c. \$2,256.88. The answers to 15a and 15b need to be subtracted to find this difference.

16a. \$22,251,500. Use the simple interest formula, $I = Prt$, and substitute.

$I = 955{,}000{,}000(0.0233)(1) = \$22{,}251{,}500$

16b. \$22,512,028.20. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 955{,}000{,}000\left(1 + \frac{0.0233}{365}\right)^{365}$$

B = \$22,512,028.20

16c. \$260,528.20. Subtract the interest answers found in 16a and 16b.

16d. 1, since each 4-year (full) scholarship is worth \$244,000.

Lesson 2-6 Continuous Compounding

Check Your Understanding (Example 1)

The value of $g(x)$ will decrease as x increases because of the negative coefficient, –5, of x.

Check Your Understanding (Example 2)

x	*f*(*x*)
10	$\frac{1}{10} = 0.1$
100	$\frac{1}{100} = 0.01$
1,000	$\frac{1}{1{,}000} = 0.001$
10,000	$\frac{1}{10{,}000} = 0.0001$
1,000,000	$\frac{1}{100{,}000} = 0.00001$

The values in the table are approaching 0.

Check Your Understanding (Example 3)

x	*f*(*x*)
10	$1^{10} = 1$
100	$1^{100} = 1$
1,000	$1^{1{,}000} = 1$
10,000	$1^{10{,}000} = 1$
1,000,000	$1^{100{,}000} = 1$

The values in the table are all 1.

Check Your Understanding (Example 4)

x	$f(x)$
10	$\left(1+\frac{0.05}{10}\right)^{10}=1.05114$
100	$\left(1+\frac{0.05}{100}\right)^{100}=1.05126$
1,000	$\left(1+\frac{0.05}{1,000}\right)^{1,000}=1.05127$
10,000	$\left(1+\frac{0.05}{10,000}\right)^{10,000}=1.05127$
1,000,000	$\left(1+\frac{0.05}{1,000,000}\right)^{1,000,000}=1.05127$

The values in the table are approaching 1.05127.

Check Your Understanding (Example 5)

$e^{\pi} - \pi^{e} \approx 23.1406 - 22.4591 \approx 0.6815 \approx 0.682$

Check Your Understanding (Example 6)

$5,229.09. Use the continuous compounding formula, $B = Pe^{rt}$, and substitute.

$B = Pe^{rt}$

$B = 5,000e^{(0.0112)(4)} = \$5,229.09$

Applications

1. People have a difficult time conceiving of infinity. The concept of infinite time, space, and number triggers the imagination and often boggles the mind.

2a. $120. Use the simple interest formula $I = Prt$, ans substitute.

$I = 2,000(0.02)(3) = 120$

2b. $122.42. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 2,000\left(1+\frac{0.02}{1}\right)^{3} = 122.42$$

2c. $123.04. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 2,000\left(1+\frac{0.02}{2}\right)^{6} = 123.04$$

2d. $123.36. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 2,000\left(1+\frac{0.02}{4}\right)^{12} = 123.36$$

2e. $123.57. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 2,000\left(1+\frac{0.02}{12}\right)^{36} = 123.57$$

2f. $123.67 Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 2,000\left(1+\frac{0.02}{365}\right)^{(365)(3)} = 123.67$$

2g. $123.67. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 2,000\left(1+\frac{0.02}{8760}\right)^{(8760)(3)} = 123.67$$

2h. $123.67. There are 525,600 minutes in a year. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 2,000\left(1+\frac{0.02}{525,600}\right)^{(525600)(3)} = 123.67$$

2i. $123.67. Use the continuous compounding formula, $B = Pe^{rt}$, and substitute.

$B = Pe^{rt}$

$B = 2,000e^{(0.02)(3)} = \$123.67$

2j. $3.67. This is the difference between $120 and $123.67.

3a. \$22.006.37. Use the continuous compounding formula, $B = Pe^{rt}$, and substitute.

$B = Pe^{rt}$

$B = 20{,}000e^{(0.0239)(4)} = \$22{,}006.37$

3b. The exponent of *e* is only 0.0539 since the parentheses were closed before the 4 was included. Ed's expression raised *e* to the 0.0539 power, multiplied it by 20,000, and multiplied that result by 4.

4. \$3,491.51. Use the continuous compounding formula, $B = Pe^{rt}$, and substitute.

$B = Pe^{rt}$

$B = 50{,}000e^{(0.0239)(4)} = \$3{,}491.51$

5a. \$385.71. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 9{,}000\left(1+\frac{0.021}{12}\right)^{(12)(2)} = 9{,}385.71$$

5b. \$369.17. Use the continuous compounding formula, $B = Pe^{rt}$, and substitute.

$B = Pe^{rt}$

$B = 9{,}000e^{(0.021)(2)} = \$369.17$

5c. State Bank pays \$16.54 more in interest. This can be found by subtracting the two different interest amounts the banks paid.

5d. Sample answers: How close is the bank to her home? Will she still be living in the same place in 2 years? Does she have other banking business at that bank? What are the hours? How customer-friendly is the service?

6a. $B = pe^{rt} = \$1{,}000e^{(0.16)(5)} = \$2{,}225.54$

6b. \$1,105.17. Use the continuous compounding formula, $B = Pe^{rt}$, and substitute.

$B = Pe^{rt}$

$B = 1{,}000e^{(0.02)(5)} = \$1{,}105.17$

6c. \$1,120.37. This is found by subtracting the answers to 6a and 6b.

7. \$225.85. Use the continuous compounding formula, $B = Pe^{rt}$, and substitute.

$B = Pe^{rt}$

$B = 30{,}000e^{(0.02)(0.5)} = \$225.85$

8a. \$710.76. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 19{,}400\left(1+\frac{0.012}{12}\right)^{(12)(3)} = 710.76$$

8b. \$831.73. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 19{,}400\left(1+\frac{0.012}{12}\right)^{(12)(3.5)} = 831.73$$

8c. \$41.93. The interest on 3 years is from part 8a, and this is \$710.76. So the balance after 3 years is 20,110.76. The interest for the last 5 months (5/12 of a year) can be found using the compound interest formula with the lowered interest rate of 0.5%.

$$B = 20{,}110.76\left(1+\frac{0.005}{12}\right)^{(12)(\frac{5}{12})} = 20{,}152.69$$

The difference between 20,110.76 and 20,152.69, is 41.93.

8d. \$752.69. The interest after 3 years, \$710.76, is added to 41.93.

8e. \$502.69. The \$250 penalty is subtracted from \$752.69.

8f. \$7,902.69. The \$12,000 is subtracted.

8g. \$3.29. The compound interest formulas is used to find the interest for the last month.

$$B = 7{,}902.69\left(1+\frac{0.005}{12}\right)^{(\frac{1}{12})} = 7{,}905.98$$

To find the last month's interest, subtract 7,902.69 from 7,905.98.

8h. \$505.98. The 3.29 is added to 502.69.

8i. Yes. With the penalty and the reduced interest, the 3-year CD would have been better.

9a. \$221.43. The compound interest formulas is used to find the Option 1 interest.

$$B = 4{,}000\left(1+\frac{0.018}{4}\right)^{(12)} = 4{,}221.43.$$

When 4,000 is subtracted from this, the interest remains.

9b. \$184.11. Use the continuous compounding formula, $B = Pe^{rt}$, and substitute.

$B = Pe^{rt}$

$B = 4{,}000e^{(0.015)(3)} = 4{,}184.11$

When 4,000 is subtracted from this, the interest remains.

10. t months $= \frac{t}{12}$ years

$15{,}000e^{0.0175t/12}$

Lesson 2-7 Future Value of Investments

Check Your Understanding (Example 1)

\$6,410.19. Determine the amount that will be in the account after 21 years (\$119,225.08) and subtract that from the answer to Example 1.

Extend Your Understanding (Example 1)

No. Using the order of operations $(1 + 0.0125)^{20}$ will be computed first. 1 is subtracted from that value, not from the exponent of 20.

Check Your Understanding (Example 2)

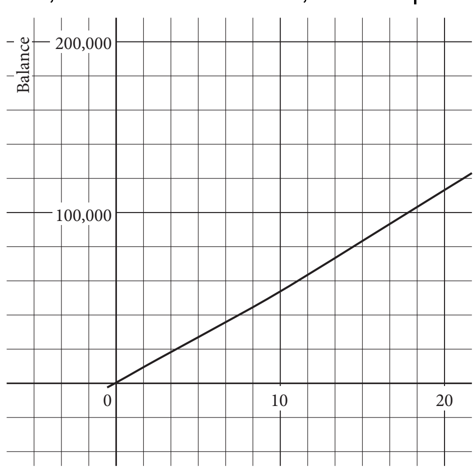
Answer Rich and Laura will earn \$1,410.19 more by retiring 1 year later.

Check Your Understanding (Example 3)

Not necessarily: it depends on the principal, rate, and length of time opened.

Check Your Understanding (Example 4)

Enter the equation in Y_1. The minimum on the x-axis is 0, the maximum is 20. The minimum on the y-axis is 0, the maximum is 200,000. Graph the equation.



Applications

1. Answers will vary but should include some mention of the fact that the longer an amount is in a savings account the more it will accrue in interest. Therefore, the earlier the start of the account, the better.

2. \$1,304.44. $B = 1000\left(1 + \frac{0.0085}{1}\right)^4 \approx \$1034.44$

3. \$513.13. $B = 500\left(1 + \frac{0.013}{2}\right)^4 \approx \$513.13$

4. \$10,428.18

$$B = 10000\left(1 + \frac{0.014}{4}\right)^{12} \approx \$10{,}428.18$$

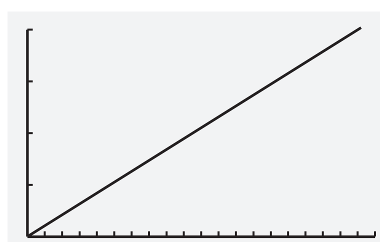
5. \$9,800.07

$$B = \frac{200\left(\left(1 + \frac{0.0105}{12}\right)^{48} - 1\right)}{\frac{0.0105}{12}} \approx \$9{,}800.07$$

6a. $$B = \frac{100\left(\left(1 + \frac{0.024}{52}\right)^{52(3)} - 1\right)}{\frac{0.024}{52}} \approx \$16{,}171.46$$

6b. $$B = \frac{100\left(\left(1 + \frac{0.024}{52}\right)^{x} - 1\right)}{\frac{0.024}{52}}$$

6c. Enter the equation in Y_1. The minimum on the x-axis is 0, the maximum is 260. The minimum on the y-axis is 0, the maximum is 12,000. Graph the equation.



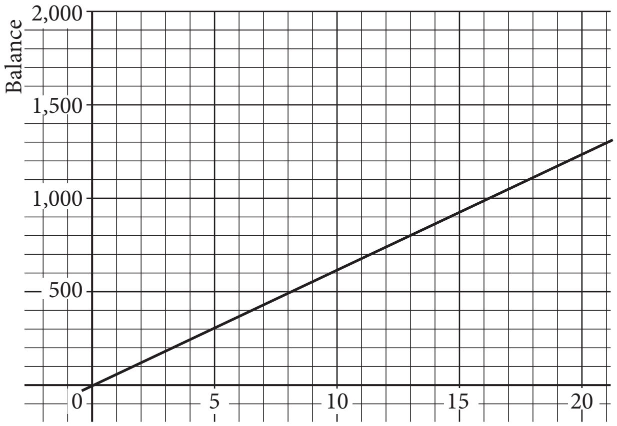
6d. Using the graph, determine approximate y-value when the x-value is 2(52); \$10,600

7a. $12,437.27

$$B = \frac{600\left(\left(1+\frac{0.015}{4}\right)^{4(5)}-1\right)}{\frac{0.015}{4}} \approx \$12{,}437.27$$

7b. $$B = \frac{600\left(\left(1+\frac{0.015}{4}\right)^{x}-1\right)}{\frac{0.015}{4}}$$

7c. Enter the equation in Y_1. The minimum on the *x*-axis is 0, the maximum is 20, and the scale is 1. The minimum on the *y*-axis is 0, the maximum is 10,000, and the scale is 1,000. Graph the equation.



7d. Use the graph, determine the approximate y-value when *x* = 3(4); $7,350.37

8. $2,421.95

$$B = 2{,}000\left(1+\frac{0.012}{2}\right)^{32} \approx \$2{,}421.95$$

9a. $12,488.73

$$B = \frac{100\left(\left(1+\frac{0.008}{12}\right)^{12(10)}-1\right)}{\frac{0.008}{12}} \approx \$12{,}488.73$$

9b. $10,726.94

$$B = \frac{80\left(\left(1+\frac{0.022}{12}\right)^{12(10)}-1\right)}{\frac{0.022}{12}} \approx \$10{,}726.94$$

9c. Sydney had more money.

9d. $40,739.94

$$B = \frac{100\left(\left(1+\frac{0.008}{12}\right)^{12(30)}-1\right)}{\frac{0.008}{12}} \approx \$40{,}672.12$$

9e. $40,672.12

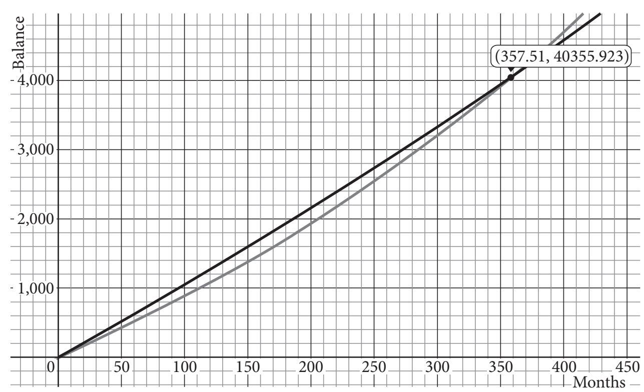
$$B = \frac{80\left(\left(1+\frac{0.022}{12}\right)^{12(30)}-1\right)}{\frac{0.022}{12}} \approx \$40{,}739.94$$

9f. Benny had more money.

9g. $$B = \frac{100\left(\left(1+\frac{0.008}{12}\right)^{x}-1\right)}{\frac{0.008}{12}}$$

9h. $$B = \frac{80\left(\left(1+\frac{0.022}{12}\right)^{x}-1\right)}{\frac{0.022}{12}}$$

9i. Enter Sydney's equation in Y_1. Enter Benny's equation in Y_2. Graph the equation.



9j. Sydney's balance starts out higher than Benny's. Close to the end of the 29th year (around 357 months), Benny's amount overtakes Sydney's.

10. Substitute A8 for *B*, A3 for *P*, A4 for *r*, A6 for *n*, and A5 for *t* in the formula for the future value of a periodic investment:

$$A8 = \frac{A3\left(\left(1+\frac{A4}{A6}\right)^{A6(A5)}-1\right)}{\frac{A4}{A6}}$$

$$B=\frac{A3\left(\left(1+\frac{A4}{A6}\right)^{A6(A5)}-1\right)}{\frac{A4}{A6}}$$

Write the equation as it would appear in a spreadsheet:

= (A3*((1+A4/A6)^(A6*A5)–1))/(A4/A6)

11a. When the balance equals 2 × $2,000 = $4,000 the initial deposit will double.

$$t=25;\ B=2{,}000\left(1+\frac{0.01}{1}\right)^{25}\approx \$2{,}564.86$$

$$t=50;\ B=2{,}000\left(1+\frac{0.01}{1}\right)^{50}\approx \$3{,}289.26$$

$$t=60;\ B=2{,}000\left(1+\frac{0.01}{1}\right)^{60}\approx \$3{,}633.39$$

$$t=70;\ B=2{,}000\left(1+\frac{0.01}{1}\right)^{70}\approx \$4{,}013.53$$

After about 70 years, the initial deposit will double.

11b. When the balance equals 2 × $4,000 = $8,000 the initial deposit will double.

$$t=20;\ B=4{,}000\left(1+\frac{0.02}{1}\right)^{20}\approx \$5{,}943.79$$

$$t=30;\ B=4{,}000\left(1+\frac{0.02}{1}\right)^{30}\approx \$7{,}245.45$$

$$t=35;\ B=4{,}000\left(1+\frac{0.02}{1}\right)^{35}\approx \$7{,}999.56$$

After about 35 years, the initial deposit will double.

11c. When the balance equals 2 × $20,000 = $40,000 the initial deposit will double.

$$t=5;\ B=20{,}000\left(1+\frac{0.06}{1}\right)^{5}\approx \$26{,}764.51$$

$$t=10;\ B=20{,}000\left(1+\frac{0.02}{1}\right)^{10}\approx \$35{,}816.95$$

$$t=12;\ B=20{,}000\left(1+\frac{0.02}{1}\right)^{12}\approx \$40{,}243.93$$

After about 12 years, the initial deposit will double.

11d. $t \times$ percent = 70 × 1 = 70, which is close to 72.

11e. 72 ÷ percent = 72 ÷ 1.75 ≈ 41

41 + 10 = 51 years old

Lesson 2-8 Present Value of Investments

Check Your Understanding (Example 1)

They would have to deposit double the original amount – $36,581.69

Check Your Understanding (Example 2)

Everything remains the same except n = 52.

Check Your Understanding (Example 3)

$$P=\frac{x\left(\frac{r}{200}\right)}{\left(1+\frac{r}{200}\right)^{2y}-1}$$

Check Your Understanding (Example 4)

The higher the interest rate, the lower the present value needed to attain the future goal.

Applications

1. Answers will vary but should include the fact that to look ahead to a future value savings, it is necessary to carefully examine what you can afford to save in the present.

2a. $959.15. $P=\frac{1{,}000}{\left(1+\frac{0.014}{1}\right)^{1(3)}}\approx \$959.15$

2b. $2,374.82. $P=\frac{2{,}500}{\left(1+\frac{0.0103}{2}\right)^{2(5)}}\approx \$2{,}374.82$

2c. $9,094.75. $P=\frac{10{,}000}{\left(1+\frac{0.0095}{4}\right)^{4(10)}}\approx \$9{,}094.75$

2d. $41,770.55.

$$P=\frac{50{,}000}{\left(1+\frac{0.0225}{12}\right)^{12(8)}}\approx \$41{,}770.55$$

3a. $P=\dfrac{50{,}000\left(\dfrac{0.02}{1}\right)}{\left(1+\dfrac{0.02}{1}\right)^{1(8)}-1}\approx \$5{,}825.49$

3b. $P=\dfrac{25{,}000\left(\dfrac{0.015}{2}\right)}{\left(1+\dfrac{0.015}{2}\right)^{2(4)}-1}\approx \$3{,}043.89$

3c. $2,350.00/per quarter

$P=\dfrac{100{,}000\left(\dfrac{0.0125}{4}\right)}{\left(1+\dfrac{0.0125}{4}\right)^{4(10)}-1}\approx \$2{,}350.90$

3d. $3,788.89/per month

$P=\dfrac{1{,}000{,}000\left(\dfrac{0.0094}{12}\right)}{\left(1+\dfrac{0.0094}{12}\right)^{12(20)}-1}\approx \$3{,}788.89$

4. $6,889.80. $P=\dfrac{50{,}000\left(\dfrac{0.012}{1}\right)}{\left(1+\dfrac{0.012}{1}\right)^{1(7)}-1}\approx \$6{,}889.80$

5. $56,389.42

$P=\dfrac{80{,}000}{\left(1+\dfrac{0.0175}{12}\right)^{12(20)}}\approx \$56{,}389.42$

6. $1,658.87

$P=\dfrac{10{,}000\left(\dfrac{0.0225}{12}\right)}{\left(1+\dfrac{0.0225}{12}\right)^{12(0.5)}-1}\approx \$1{,}658.87$

7. $272.75. $P=\dfrac{10{,}000\left(\dfrac{0.0125}{12}\right)}{\left(1+\dfrac{0.0125}{12}\right)^{12(3)}-1}\approx \$272.75$

8. $12,204.45

$P=\dfrac{100{,}000\left(\dfrac{0.009}{2}\right)}{\left(1+\dfrac{0.009}{2}\right)^{2(4)}-1}\approx \$12{,}304.45$

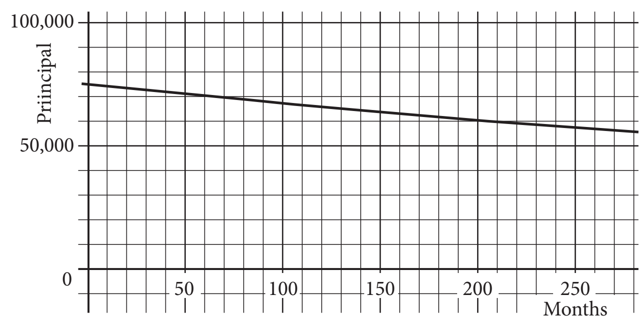
9. $47,186.46 $P=\dfrac{50{,}000}{\left(1+\dfrac{0.0116}{4}\right)^{4(5)}}\approx \$47{,}186.46$

10. Graph the equation: $y=\dfrac{\$75{,}000}{\left(1+\dfrac{0.031}{12}\right)^{x}}$,

Let x represent the number of months.

$y=\dfrac{75{,}000}{\left(1+\dfrac{0.013}{12}\right)^{x}}$

where x is the number of months and y is the principal amount. The display should look similar to the one below.



Lesson 2-9: The Term of a Single Deposit Account

Check Your Understanding (Example 1)

2.4%;

$1.006=1+\dfrac{r}{n}$

$1.006=1+\dfrac{r}{4}$

$0.006=\dfrac{r}{4}$

$0.024=r$

The rate is 2.4%.

Extend Your Understanding (Example 1)

Four times per year

$$1.002 = 1 + \frac{r}{n}$$

$$1.002 = 1 + \frac{0.008}{n}$$

$$0.002 = \frac{0.008}{n}$$

$0.002n = 0.008$

$n = 4$

Check Your Understanding (Example 2)

Here students will be transforming the compound interest formula to solve for t in terms of P, B, r, and n.

$$t = \frac{\log_{\left(1+\frac{r}{n}\right)} \frac{B}{P}}{n}$$

Extend Your Understanding (Example 2)

$a^{5t} = 1000$

Check Your Understanding (Example 3)

$t = 5.$

$$t = \frac{\log_{1.0005} 1.03044}{12}$$

$$12t = \log_{1.0005} 1.03044$$

$$12t = \frac{\log 1.03044}{\log 1.0005}$$

$$t = \frac{\frac{\log 1.03044}{\log 1.0005}}{12} \approx 5$$

Check Your Understanding (Example 4)

Approximately 57.8 years.

$$500\left(1 + \frac{0.012}{4}\right)^{4t} = 1000$$

$$\left(1 + \frac{0.012}{4}\right)^{4t} = 2$$

$$1.0003^{4t} = 2$$

$$\log_{1.003} 2 = 4t$$

$$\frac{\log 2}{\log 1.0003} = 4t$$

$57.8 \approx t$

Extend Your Understanding (Example 4)

Approximately 24.8 years.

$$D\left(1 + \frac{0.028}{4}\right)^{4t} = 2D$$

$$\left(1 + \frac{0.028}{4}\right)^{4t} = 2$$

$$1.007^{4t} = 2$$

$$\log_{1.007} 2 = 4t$$

$$\frac{\log 2}{\log 1.007} = 4t$$

$24.8 \approx t$

Check Your Understanding (Example 5)

Approximately 18.7 years.

$$16000 = 8000e^{0.037t}$$

$$2 = e^{0.037t}$$

$$0.037t = \ln 2$$

$$t = \frac{\ln 2}{0.037} \approx 18.7$$

Applications

1. An alteration is a change. Baldwin draws a similarity to the changes that a dressmaker makes in a dress and the changes that time makes in persons, places, and things. In both cases, the alterations can be minor or major.

2a. $n = 12$, rate = 2.4%

$nt = 12t$ therefore, $n = 12$

$1 + \frac{r}{12} = 1.002$; $r = 0.024$; rate = 2.4%

2b. $n = 2$, rate = 2%

$nt = 2t$ therefore, $n = 2$

$1 + \frac{r}{2} = 1.01$; $r = 0.02$; rate = 2%

2c. n = 6, rate = 3.6%

nt = 6t therefore, n = 6

$1+\frac{r}{6}=1.006$; r = 0.036; rate = 3.6%

2d. n = 12, rate = 4.5%

nt = 12t therefore, n = 12

$1+\frac{r}{12}=1.00375$; r = 0.045; rate = 4.5%

2e. n = 2, rate = 5%

nt = 2t therefore, n = 2

$1+\frac{r}{2}=1.025$; r = 0.05; rate = 5%

3. In each of the equations above, divide both sides by the principal to yield the exponential equation in standard form.

3a. $1.127=1.002^{12t}$

3b. $1.2202=1.01^{2t}$

3c. $1.15=1.006^{6t}$

3d. $1.566993=1.00375^{12t}$

3e. $2.6857=1.025^{2t}$

4a. $t=\frac{\log_{1.009}(1.14375)}{3}$

$5{,}490=4{,}800(1.009)^{3t}$

$\frac{5{,}490}{4{,}800}=(1.009)^{3t}$

$1.14375=(1.009)^{3t}$

$3t=\log_{1.009}1.14375$

$t=\frac{\log_{1.009}1.14375}{3}$

4b. $t=\frac{\log_{1.006}(1.3)}{6}$

$3{,}900=3{,}000(1.006)^{6t}$

$\frac{3{,}900}{3{,}000}=(1.006)^{6t}$

$1.3=(1.006)^{6t}$

$6t=\log_{1.006}1.3$

$t=\frac{\log_{1.006}1.3}{6}$

4c. $t=\frac{\log_{1.0015}(1.46)}{12}$

$1{,}460=1{,}000(1.0015)^{12t}$

$\frac{1{,}460}{1{,}000}=(1.0015)^{12t}$

$1.46=(1.0015)^{12t}$

$12t=\log_{1.0015}1.46$

4d. $t=\frac{\log_{1.00875}(1.19034)}{4}$

$5{,}951.70=5{,}000(1.00875)^{4t}$

$\frac{5{,}951.70}{5{,}000}=(1.00875)^{4t}$

$1.19034=(1.00875)^{4t}$

$4t=\log_{1.00875}1.19034$

$t=\frac{\log_{1.00875}1.19034}{4}$

4e. $t=\frac{\log_{1.008}(1.10034)}{4}$

$33{,}010.20=30{,}000(1.008)^{4t}$

$\frac{33{,}010.20}{30{,}000}=(1.008)^{4t}$

$1.10034=(1.008)^{4t}$

$4t=\log_{1.008}1.10034$

$t=\frac{\log_{1.008}1.10034}{4}$

5. In each of the exercises below, rewrite the logarithm expression in the numerator using the change of base formula with common logs. Use a graphing calculator to evaluate the quotient to solve for t.

5a. t = 1.9

5b. t = 4.0

5c. t = 2.9

5d. t = 5.1

5e. t = 16.6

6a. $n=(\log(1.078)/\log(1.00375))/5\approx 4$

6b. $n = (\log(1.270945)/\log(1.002))/10 \approx 12$

6c. $n = (\log(1.0956)/\log(1.00005))/5 \approx 365$

6d. $n = (\log(1.195)/\log(1.011))/8 \approx 2$

6e. $n = (\log(1.1015)/\log(1.01625))/6 \approx 1$

7. Approximately 7.2 years.

$$225{,}000 = 180{,}000\left(1+\frac{0.0312}{12}\right)^{12t}$$

$$1.25 = \left(1+\frac{0.0312}{12}\right)^{12t}$$

$$1.25 = 1.0026^{12t}$$

$$12t = \log_{1.0026} 1.25$$

$$12t = \frac{\log 1.25}{\log 1.0026}$$

$$t = \frac{\frac{\log 1.25}{\log 1.0026}}{12} \approx 7.2$$

8. Approximately 4.9 years.

$$230{,}000 = 200{,}000\left(1+\frac{0.02875}{4}\right)^{4t}$$

$$1.15 = \left(1+\frac{0.02875}{4}\right)^{4t}$$

$$1.15 = 1.0071875^{4t}$$

$$4t = \log_{1.0071875} 1.15$$

$$4t = \frac{\log 1.15}{\log 1.0071875}$$

$$t = \frac{\frac{\log 1.15}{\log 1.0071875}}{4} \approx 4.9$$

9a. $t = \frac{\log_{1.00003}(1.044775)}{365}$

$$417.91 = 400\left(1+\frac{0.01095}{365}\right)^{365t}$$

$$1.044775 = \left(1+\frac{0.01095}{365}\right)^{365t}$$

$$1.044775 = 1.00003^{365t}$$

$$365t = \log_{1.00003} 1.044775$$

$$t = \frac{\log_{1.00003} 1.044775}{365}$$

9b. $t \approx 4$

$$365t = \frac{\log 1.044775}{\log 1.00003}$$

$$t = \frac{\frac{\log 1.044775}{\log 1.00003}}{365} \approx 4$$

9c. $t \approx 4$

$$365t = \frac{\ln 1.044775}{\ln 1.00003}$$

$$t = \frac{\frac{\ln 1.044775}{\ln 1.00003}}{365} \approx 4$$

9d. The results are the same because the change-of-base formula works regardless of the base chosen.

10. Approximately 3.5 years.

$$B = Pe^{rt}$$

$$10800 = 10000e^{0.022t}$$

$$1.08 = e^{0.022t}$$

$$0.022t = \ln(1.08)$$

$$t = \frac{\ln(1.08)}{0.022} \approx 3.5$$

11a. $t \approx 3$

$$B = Pe^{rt}$$

$$2124 = 2000e^{0.02t}$$

$$1.062 = e^{0.02t}$$

$$0.02t = \ln(1.062)$$

$$t = \frac{\ln(1.062)}{0.02} \approx 3$$

11b. $t \approx 5$

$$B = Pe^{rt}$$

$$957.60 = 900e^{0.0125t}$$

$$1.064 = e^{0.0125t}$$

$$0.0125t = \ln(1.064)$$

$$t = \frac{\ln(1.064)}{0.0125} \approx 5$$

11c. $t \approx 3.1$

$$B = Pe^{rt}$$

$$25{,}700 = 25{,}000e^{0.009t}$$

$$1.028 = e^{0.009t}$$

$$0.009t = \ln(1.028)$$

$$t = \frac{\ln(1.028)}{0.009} \approx 3.1$$

11d. $t \approx 1.9$

$$B = Pe^{rt}$$

$$185.50 = 175e^{0.03t}$$

$$1.06 = e^{0.03t}$$

$$0.03t = \ln(1.06)$$

$$t = \frac{\ln(1.06)}{0.03} \approx 1.9$$

11e. $t \approx 5$

$$B = Pe^{rt}$$

$$1{,}084{,}000 = 1{,}000{,}000e^{0.016t}$$

$$1.084 = e^{0.016t}$$

$$0.016t = \ln(1.084)$$

$$t = \frac{\ln(1.084)}{0.016} \approx 5$$

12. Let $B = 3P$ and $n = 1$ in the compound interest formula

$$B = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$3P = P\left(1 + \frac{r}{1}\right)^{1t}$$

$$3 = (1 + r)^{t}$$

$$t = \log_{1+r} 3$$

$$t = \frac{\log 3}{\log(1 + r)}$$

Lesson 2-10: The Term of a Systematic Account

Check Your Understanding (Example 1)

$$t = \frac{\log_{1.0025}\left(\frac{300}{200}\right)}{4}$$

$$300 = 200\left(1 + \frac{0.01}{4}\right)^{4t}$$

$$\frac{300}{200} = \left(1 + \frac{0.01}{4}\right)^{4t}$$

$$\frac{300}{200} = 1.0025^{4t}$$

$$4t = \log_{1.0025}\left(\frac{300}{200}\right)$$

$$t = \frac{\log_{1.0025}\left(\frac{300}{200}\right)}{4}$$

Check Your Understanding (Example 2)

$t \approx 0.6$

$$3.6 = 1.5^{5t}$$

$$\log(3.6) = \log(1.5)^{5t}$$

$$\log(3.6) = 5t\log(1.5)$$

$$\frac{\log(3.6)}{\log(1.5)} = 5t$$

$$\frac{\frac{\log(3.6)}{\log(1.5)}}{5} = t$$

$$0.6 \approx t$$

Extend Your Understanding (Example 2)

$t \approx 18.6$

$$B = Pe^{rt}$$

$$1{,}000 = 800e^{0.012t}$$

$$1.25 = e^{0.012t}$$

$$\ln(1.25) = \ln\ e^{0.012t}$$

$$\ln(1.25) = 0.012\,t\,\ln e$$

$\ln(1.25) = 0.012t$

$\frac{\ln(1.25)}{0.012} = t$

$18.6 \approx t$

Check Your Understanding (Example 3)

Approximately 2.1 years.

$$5{,}000 = \frac{200\left(\left(1+\frac{0.0126}{12}\right)^{12t}-1\right)}{\frac{0.0126}{12}}$$

$$5.25 = 200\left(\left(1+\frac{0.0126}{12}\right)^{12t}-1\right)$$

$$0.02625 = \left(1+\frac{0.0126}{12}\right)^{12t}-1$$

$$1.02625 = \left(1+\frac{0.0126}{12}\right)^{12t}$$

$$1.02625 = 1.00105^{12t}$$

Method 1:

$12t = \log_{1.00105} 1.02625$

$12t = \frac{\log 1.02625}{\log 1.00105}$

$t = \frac{\frac{\log 1.02625}{\log 1.00105}}{12}$

$t \approx 2.1$

Method 2.

$\log(1.02625) = \log(1.00105)^{12t}$

$\log(1.02625) = 12t \log(1.00105)$

$\frac{\log 1.02625}{\log 1.00105} = 12t$

$\frac{\frac{\log 1.02625}{\log 1.00105}}{12} = t$

$2.1 \approx t$

Check Your Understanding (Example 4)

$t \approx 13.9$ years;

$$40{,}000 = \frac{256\left(1-\left(1+\frac{0.0096}{12}\right)^{-12t}\right)}{\frac{0.0096}{12}}$$

$$32 = 256\left(1-\left(1+\frac{0.0096}{12}\right)^{-12t}\right)$$

$$0.125 = 1-\left(1+\frac{0.0096}{12}\right)^{-12t}$$

$$-0.875 = -\left(1+\frac{0.0096}{12}\right)^{-12t}$$

$$0.875 = (1.0008)^{-12t}$$

$$\log(0.875) = \log(1.0008)^{-12t}$$

$$\log(0.875) = -12t\log(1.0008)$$

$$\frac{\log(0.875)}{\log(1.0008)} = -12t$$

$$\frac{\frac{\log(0.875)}{\log(1.0008)}}{-12} = t$$

$13.9 \approx t$

Students may want to try to substitute 15 into the equation. Ask why replacing t with 15 would not help in answering the question.

Applications

1. Buffet offers a nonmathematical situation in which a present situation is made possible because someone planned and acted in the past. This can be likened to the savings situations covered in this chapter. A saver can reap benefits in the future if careful planning is done ahead of time.

2a. 3; 3 log(10) = 3

2b. $s \log(r)$

2c. $3\log_2 x$

2d. 4

2e. $y \ln(x)$

3a. $\frac{\log 200}{\log 5}$

3b. $\frac{\log y}{\log x}$

3c. $\frac{\log 8}{\log w}$

3d. $\frac{\log z}{\log 9}$

3e. $\frac{\log y}{\log e}$

4a. $t \approx 2.5$

$15 = 3^t$

$\log(15) = \log(3)^t$

$\log(15) = t\log(3)$

$\frac{\log(15)}{\log(3)} = t$

$2.5 \approx t$

4b. $t = 8$

$5^t = 390{,}625$

$\log(5)^t = \log(390{,}625)$

$t\log(5) = \log(390{,}625)$

$t = \frac{\log(390{,}625)}{\log(5)}$

$t = 8$

4c. $t = 1.5$

$8^{2t} = 512$

$\log(8)^{2t} = \log(512)$

$2t\log(8) = \log(512)$

$t = \frac{\log(512)}{2\log(8)}$

$t = 1.5$

4d. $t = 9$

$1.2^{t-3} = 2.985984$

$\log(1.2)^{t-3} = \log(2.985984)$

$(t-3)\log(1.2) = \log(2.985984)$

$t - 3 = \frac{\log(2.985984)}{\log(1.2)}$

$t = \frac{\log(2.985984)}{\log(1.2)} + 3$

$t = 9$

4e. $t = 5.5$

$10^{\frac{t}{2}} - 1 = 549$

$10^{\frac{t}{2}} = 550$

$\log(10)^{\frac{t}{2}} = \log(550)$

$\frac{t}{2}\log(10) = \log(550)$

$\frac{t}{2} = \frac{\log(550)}{\log(10)}$

$t = \frac{2\log(550)}{\log(10)}$

$t \approx 5.5$

5a. $t = \frac{\log_{1.00125} 2.5}{4}$

$1{,}500 = 600\left(1 + \frac{0.005}{4}\right)^{4t}$

$2.5 = \left(1 + \frac{0.005}{4}\right)^{4t}$

$2.5 = (1.00125)^{4t}$

$4t = \log_{1.00125} 2.5$

$t = \frac{\log_{1.00125} 2.5}{4}$

5b. $t = \frac{\log_{1.001} 1.6}{12}$

$800 = 500\left(1 + \frac{0.012}{12}\right)^{12t}$

$1.6 = \left(1 + \frac{0.012}{12}\right)^{12t}$

$1.6 = (1.001)^{12t}$

$12t = \log_{1.001} 1.6$

$t = \frac{\log_{1.001} 1.6}{12}$

5c. $t = \frac{\log_{1.01} 1.25}{2}$

$$1{,}000{,}000 = 800{,}000\left(1+\frac{0.02}{2}\right)^{2t}$$

$$1.25 = \left(1+\frac{0.02}{2}\right)^{2t}$$

$$1.25 = (1.01)^{2t}$$

$$2t = \log_{1.01} 1.25$$

$$t = \frac{\log_{1.01} 1.25}{2}$$

6a. $2{,}400 = 2{,}000\left(1+\frac{0.014}{4}\right)^{4t}$

6b. $2{,}400 = 2{,}000(1.0035)^{4t}$

6c. $1.2 = (1.0035)^{4t}$

6d. $4t = \log_{1.0035} 1.2$

6e. $4t = \frac{\log 1.2}{\log 1.0035}$

6f. $4t = 52.183$

6g. $t = 13.04575$

6h. 13

7a. $2{,}400 = 2{,}000\left(1+\frac{.014}{4}\right)^{4t}$

7b. $2{,}400 = 2{,}000(1.0035)^{4t}$

7c. $1.2 = (1.0035)^{4t}$

7d. $\log 1.2 = \log 1.0035^{4t}$

7e. $\log 1.2 = 4t \log 1.0035$

7f. $t = 13.04575$, $t = 13$ years

8. $t \approx 4$ years

$$5{,}500 = 5{,}000\left(1+\frac{0.024}{12}\right)^{12t}$$

$$1.1 = \left(1+\frac{0.024}{12}\right)^{12t}$$

$$1.1 = 1.002^{12t}$$

Method 1

$$12t = \log_{1.002} 1.1$$

$$12t = \frac{\log 1.1}{\log 1.002}$$

$$t = \frac{\frac{\log 1.1}{\log 1.002}}{12} \approx 4$$

Method 2

$$\log(1.1) = \log(1.002)^{12t}$$

$$\log(1.1) = 12t \log(1.002)$$

$$\frac{\log 1.1}{\log 1.002} = 12t$$

$$\frac{\frac{\log 1.1}{\log 1.002}}{12} \approx 4 = t$$

9. $t \approx 86.7$ years

$$100 = 50\left(1+\frac{0.008}{4}\right)^{4t}$$

$$2 = \left(1+\frac{0.008}{4}\right)^{4t}$$

$$2 = 1.002^{4t}$$

Method 1

$$4t = \log_{1.002} 2$$

$$4t = \frac{\log 2}{\log 1.002}$$

$$t = \frac{\frac{\log 2}{\log 1.002}}{4} \approx 86.7$$

Method 2

$$\log(2) = \log(1.002)^{4t}$$

$$\log(2) = 4t \log(1.002)$$

$$\frac{\log 2}{\log 1.002} = 4t$$

$$\frac{\frac{\log 2}{\log 1.002}}{4} \approx 86.7 = t$$

10. $t \approx 2.1$ years

$$5{,}000 = \frac{200\left(\left(1+\frac{0.015}{12}\right)^{12t} - 1\right)}{\frac{0.015}{12}}$$

$$6.25 = 200\left(\left(1+\frac{0.015}{12}\right)^{12t} - 1\right)$$

$$0.03125 = \left(\left(1+\frac{0.015}{12}\right)^{12t} - 1\right)$$

$$1.03125 = 1.00125^{12t}$$

Method 1

$$12t = \log_{1.00125} 1.03125$$

$$12t = \frac{\log 1.03125}{\log 1.00125}$$

$$t = \frac{\frac{\log 1.03125}{\log 1.00125}}{12} \approx 2.1$$

Method 2

$$\log(1.03125) = 12t \log(1.00125)$$

$$\log(1.03125) = \log(1.00125)^{12t}$$

$$\frac{\log 1.03125}{\log 1.00125} = 12t$$

$$\frac{\frac{\log 1.03125}{\log 1.00125}}{12} \approx 2.1 = t$$

11. $t \approx 2.5$ years

$$10{,}000 = \frac{1{,}000\left(\left(1+\frac{0.014}{4}\right)^{4t} - 1\right)}{\frac{0.014}{4}}$$

$$35 = 1{,}000\left(\left(1+\frac{0.014}{4}\right)^{4t} - 1\right)$$

$$0.035 = \left(\left(1+\frac{0.014}{4}\right)^{4t} - 1\right)$$

$$1.035 = 1.0035^{4t}$$

Method 1

$$4t = \log_{1.0035} 1.035$$

$$4t = \frac{\log 1.035}{\log 1.0035}$$

$$t = \frac{\frac{\log 1.035}{\log 1.0035}}{4} \approx 2.5$$

Method 2

$$\log(1.035) = \log(1.0035)^{4t}$$

$$\log(1.035) = 4t \log(1.0035)$$

$$\frac{\log 1.035}{\log 1.0035} = 4t$$

$$\frac{\frac{\log 1.035}{\log 1.0035}}{4} \approx 2.5 = t$$

12a. $400 = \dfrac{10{,}000 \times \frac{0.0195}{12}}{(1+\frac{.0195}{12})^{12t} - 1}$

12b. $400 = \dfrac{16.25}{(1.001625^{12t} - 1)}$

12c. $400 \times \left(1.001625^{12t} - 1\right) = 16.25$

12d. $(1.001625)^{12t} - 1) = 0.040625$

12e. $(1.001625)^{12t} = 1.040625$

12f. $\log((1.001625)^{12t}) = \log(1.040625)$

12g. $12t = \dfrac{\log(1.040625)}{\log(1.001625)}$; $t = \dfrac{\log(1.040625)}{12\log(1.001625)}$;

$t \approx 2.04$

13a. $3{,}000 = 300\dfrac{1-(1+\frac{0.013}{2})^{-2t}}{\frac{0.013}{2}}$

13b. $3{,}000 = 300\dfrac{1-(1.0065)^{-2t}}{0.0065}$

13c. $10 = \dfrac{1-(1.0065)^{-2t}}{0.0065}$

13d. $0.065 = 1-(1.0065)^{-2t}$

13e. $-0.935 = -(1.0065)^{-2t}$

13f. $0.935 = (1.0065)^{-2t}$

13g. $\log(0.935) = \log((1.0065)^{-2t})$

13h. $\log(0.935) = -2t\log(1.0065)$

13i. $\frac{\log(0.935)}{\log(1.0065)} = -2t$

13j. $\frac{\frac{\log(0.935)}{\log(1.0065)}}{-2} = t$

13k. $5.2 \approx t$

14. 5.3 years

$$50{,}000 = \frac{5{,}000\left(1-\left(1+\frac{0.018}{2}\right)^{-2t}\right)}{\frac{0.018}{2}}$$

$$450 = 5{,}000\left(1-\left(1+\frac{0.018}{2}\right)^{-2t}\right)$$

$$0.09 = 1-\left(1+\frac{0.018}{2}\right)^{-2t}$$

$$-0.91 = -\left(1+\frac{0.018}{2}\right)^{-2t}$$

$$0.91 = (1.009)^{-2t}$$

$$\log(0.91) = \log(1.009)^{-2t}$$

$$\log(0.91) = -2t\log(1.009)$$

$$\frac{\log(0.91)}{\log(1.009)} = -2t$$

$$\frac{\frac{\log(0.91)}{\log(1.009)}}{-2} = t$$

$$5.3 \approx t$$

15. 2 years

$$20{,}000 = \frac{200\left(1-\left(1+\frac{0.0156}{52}\right)^{-52t}\right)}{\frac{0.0156}{52}}$$

$$6 = 200\left(1-\left(1+\frac{0.0156}{52}\right)^{-52t}\right)$$

$$0.03 = 1-\left(1+\frac{0.0156}{52}\right)^{-52t}$$

$$-0.97 = -\left(1+\frac{0.0156}{52}\right)^{-52t}$$

$$0.97 = (1.0003)^{-52t}$$

$$\log(0.97) = \log(1.0003)^{-52t}$$

$$\log(0.97) = -52t\log(1.0003)$$

$$\frac{\log(0.97)}{\log(1.0003)} = -52t$$

$$\frac{\frac{\log(0.97)}{\log(1.0003)}}{-52} = t$$

$$2 \approx t$$

Assessment

Really? Really! Revisited

The data about counterfeit money seizures is for 2015. After students have completed this activity, have them search the internet for data on seizures in the current or past years since 2015. They can use that data to create their own questions similar to the ones offered here. This extension would be an excellent addition to the Reality Check projects.

a. 1,360,000,000,000 x 0.0025 = 3,400,000,000 = $3400M

b. 1,360,000,000,000 x 0.75 = 1,020,000,000,000 = $1,020,000M

c. 100(88,000,000/1,360,000,000,000) ≈ 0.0065%

100(88,000,000/3,400,000,000) ≈ 2.6%

Only about 2.6% of the counterfeit money was removed that year.

d. (20 + 86 + 2 + 2 + 10 + 1.5 + 20 + 4.7 + 0.05 + 1.1)M = $147.35M

e. 100(147,350,000/1,360,000,000,000) ≈ 0.011%

f. $B = 147{,}350{,}000\left(1+\frac{0.01}{12}\right)^{12\left(\frac{1}{12}\right)} \approx$

147,472,791.67

147,472,791.67 – 147,350,000 = $122,791.67

g. There is still a long way to go to get a handle on counterfeit currency.

Applications

1a. Write the date 12/10 and the balance of $3,900.50.

1b. Write the check number: 1223, the date: 12/11, the transaction description: North Shore HS Drama Club, and the payment amount: $84.00. Subtract: $3,900.50 – $84.00 = $3,816.50.

1c. Write the date: 12/12, the transaction description: deposit (paycheck), and the deposit amount: $240.80. Add: $3,816.50 + $240.80 = $4,057.30.

1d. Write the date: 12/13, the transaction description: deposit (birthday check), and the deposit amount: $100.00. Add: $4,057.30 + $100.00 = $4,157.30.

1e. Write the check number: 1224, the date: 12/17, the transaction description: Best Buy, and the payment amount: $480.21. Subtract: $4,157.30 – $480.21 = $3,677.09. Write the check number: 1225, the date: 12/17, the transaction description: Target, and the payment amount: $140.58. Subtract: $3,677.09 – $140.58 = $3,536.51. Write the check number: 1226, the date: 12/17, the transaction description: Aeropostale, and the payment amount: $215.60. Subtract: $3,536.51 – $215.60 = $3,320.91.

1f. Write the check number: 1227, the date: 12/20, the transaction description: VOID. Write the check number: 1228, the date: 12/20, the transaction description: Staples, and the payment amount: $1,250.00. Subtract: $3,320.91 – $1,250.00 = $2,070.91.

1g. Write the date: 12/22, the transaction description: Barnes and Nobles Return, and the deposit amount: $120.00. Add: $2,070.91 + $120.00 = $2,190.91.

1h. Write the date: 12/24, the transaction description: ATM Withdraw, and the payment amount: $300.00. Subtract: $2,190.91 – $300.00 = $1,890.91. There is also an ATM fee of $2.25. Write the date: 12/24, the transaction description: ATM fee, and the payment amount: $1.50. Subtract: $1,890.91 – $1.50 = $1,889.41. There is also a bank fee of $2.50. Write the date: 12/24, the transaction description: bank fee, and the payment amount: $2.50. Subtract: $1,890.91 – $2.50 = $1,886.91.

1i. Write the check number: 1229, the date: 12/28, the transaction description: Len's Auto Body Shop, and the payment amount: $521.00. Subtract: $1,886.91 – $521.00 = $1,365.91.

1j. Write the check number: 1230, the date: 12/29, the transaction description: Amtrak, and the payment amount: $150.80. Subtract: $1,365.91 – $150.80 = $1,215.11.

NUMBER OR CODE	DATE	TRANSACTION DESCRIPTION	PAYMENT AMOUNT		✓	FEE	DEPOSIT AMOUNT		$ BALANCE
	12/10		$						3,900.50
1223	12/11	North Shore HS Drama Club	84	00	✓				– 84.00 3,816.50
	12/12	Deposit (paycheck)			✓		240	80	+ 240.80 4,057.30
	12/13	Deposit (birthday check)			✓		100	00	+ 100.00 4,157.30
1224	12/17	Best Buy	480	21	✓				– 480.21 3,677.09
1225	12/17	Target	140	58	✓				– 140.58 3,536.51
1226	12/17	Aeropostale	215	60	✓				– 215.60 3,320.91
1227	12/20	VOID							
1228	12/20	Staples	1250	00					– 1,250.00 2,070.91
	12/22	Barnes & Nobles Return					120	00	+ 120.00 2,190.91
	12/24	ATM Withdraw	300	00	✓				– 300.00 1,890.91
	12/24	ATM Fee	1	50		✓			– 1.50 1,889.41
	12/24	Bank Fee	2	50		✓			– 2.50 1,886.91
1229	12/28	Len's Auto Body Shop	521	00	✓				– 521.00 1,365.91
1230	12/29	Amtrak	150	80					– 150.80 1,215.11

2a. The statement shows that the ending balance is $2,495.91.

2b. The deposit of $120.00 is not on the bank statement, so this is the outstanding deposit amount.

2c. Check numbers 1228 and 1230 are not on the bank statement. So the outstanding check amount is $1,250.00 + $150.80 = $1,400.80.

2d. $2,495.91 + $120.00 – $1,400.80 = $1,215.11

2e. The check register shows a balance of $1,215.11, so the account is reconciled.

3. $254.30. Use the simple interest formula and substitute:

   $I = Prt$

   $I = 2{,}219(0.0191)(6) = \$254.30$

4. Because $1,722 – $400 = $1,322 is less than $1,500, there will be a fee of $3.50 each month. The balance after 5 months is $1,322 – $3.50(5) = $1,304.50, not including any interest.

5. $7,000 + $224.16 – $1,000 – $250 = $5,974.16

6a. \$47.78. Use the simple interest formula and substitute:

$I = Prt$

$I = 910(0.0175)(3) = \$47.78$

6b. \$957.78. The interest, \$47.78, is added to \$910.

6c. \$15.93. Use the simple interest formula and substitute:

$I = Prt$

$I = 910(0.0175)(1) = \$15.93$

6d. \$15.93. Use the simple interest formula and substitute:

$I = Prt$

$I = 910(0.0175)(1) = \$15.93$

7a. \$74,112.09 + \$77,239.01 = \$151,351.10

7b. All of Matt's money is insured. The FDIC insures up to \$250,000 at one bank, if the accounts are the same type.

8a. \$42. Multiply 5,600 by 0.015, and divided by 2.

8b. \$5,642. Add the \$42 to the original deposit.

8c. \$42.32. Multiply 5,642 by 0.015 and divide by 2.

8d. \$5,684.32. Add 442.32 to \$5,642.

8d. \$84.32. The deposit of \$5,000 needs to be subtracted from the balance of \$5,684.32.

9a. \$0. Since the account is opened March 20, the opening balance the morning of March 20 is 0.

9b. \$5,200.00. The initial deposit was \$5,200.

9c. \$5,200.00. The initial deposit is the principal used to compute the interest since there was no other activity.

9d. $I = prt = (\$5{,}200.00)(0.0399)\left(\frac{1}{365}\right) = \$0.57$

9e. \$5,200.00 + \$0.57 = \$5,200.57

9f. \$5,200.57. The opening balance for March 21 is the same as the March 20 ending balance.

9g. \$700.00. There was a \$700 deposit on March 21.

9h. \$5,200.57 + \$700.00 = \$5,900.57

9i. $I = prt = (\$5{,}900.57)(0.0399)\left(\frac{1}{365}\right) = \$0.65$

9j. \$5,900.57 + \$0.65 = \$5,901.22

9k. \$5,901.22. The opening balance is the same as the previous day's ending balance.

9l. \$500.00. There was a \$500 withdrawal on March 22.

9m. \$5,901.22 – \$500.00 = \$5,401.22

9n. $I = prt = (\$5{,}401.22)(0.0399)\left(\frac{1}{365}\right) = \$0.59$

9p. \$5,401.22 + \$0.59 = \$5,401.81

10. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 3{,}000\left(1+\frac{0.0408}{52}\right)^{(52)(3)} = 3{,}390.46$$

11. Use the compound interest formula and substitute. This must be computed on a calculator. First do the daily compounded account:

$$B = 10{,}000\left(1+\frac{0.0133}{365}\right)^{(365)(3)} = 10{,}407.06$$

Next do the semi-annual compounded account:

$$B = 10{,}000\left(1+\frac{0.0133}{2}\right)^{(2)(3)} = 10{,}405.69$$

The difference is \$1.37.

12a. \$2,302.35. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 2{,}250\left(1+\frac{0.023}{365}\right)^{(365)} = 2{,}302.35$$

12b. \$52.35. Subtract 2,250 from 2,302.35 to find the interest.

12c. 2.33%. Use the APY formula and substitute for the quarterly account.

$$\text{APY} = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$\text{APY} = \left(1+\frac{0.023}{365}\right)^{365} - 1 = 0.0233$$

13. \$2,463.99

14a. II – future value of a periodic investment.

14b. I – future value of a single deposit investment.

14c. III – present value of a periodic deposit investment.

15. \$1,977.42. Use the compound interest formula and substitute. This must be computed on a calculator. Note that 54 months is 4.5 years.

$$B = 1{,}800\left(1+\frac{0.021}{2}\right)^{(2)(4.5)} = \$1{,}977.42$$

16. \$12,321.40

$$B = \frac{150\left(\left(1+\frac{0.016}{12}\right)^{12(6.5)}-1\right)}{\frac{0.016}{12}} \approx \$12{,}321.40$$

17. \$486.00

$$P = \frac{30{,}000\left(\frac{0.0115}{12}\right)}{\left(1+\frac{0.0115}{12}\right)^{12(5)}-1} \approx \$486.00$$

18a.

x	$f(x)$
10	$\frac{9(10)-1}{3(10)-5} = 3.56$
100	$\frac{9(100)-1}{3(100)-5} = 3.047$
1,000	$\frac{9(1{,}000)-1}{3(1{,}000)-5} = 3.005$
10,000	$\frac{9(10{,}000)-1}{3(10{,}000)-5} = 3.000$

The values in the table are approaching 3.

18b.

x	$f(x)$
10	$\frac{3(10)^2+9(10)}{4(10)+1} = 9.512$
100	$\frac{3(100)^2+9(100)}{4(100)+1} = 77.057$
1,000	$\frac{3(1{,}000)^2+9(1{,}000)}{4(1{,}000)+1} = 752.062$
10,000	$\frac{3(10{,}000)^2+9(10{,}000)}{4(10{,}000)+1} = 7{,}502.062$

The values in the table are not approaching a certain number, so the limit is undefined.

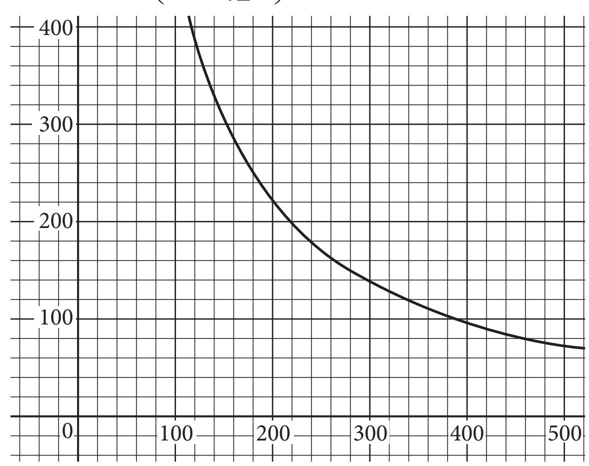
18c.

x	$f(x)$
10	$\frac{7(10)}{(10)^2-41} = 1.186$
100	$\frac{7(100)}{(100)^2-41} = 0.070$
1,000	$\frac{7(1{,}000)}{(1{,}000)^2-41} = 0.007$
10,000	$\frac{7(10{,}000)}{(10{,}000)^2-41} = 0.0007$

The values in the table are approaching 0.

19. x = # of months

$$y = \frac{\left(50{,}000\times\frac{0.0145}{12}\right)}{\left(1+\frac{0.145}{12}\right)^{x}-1}$$



20. Approximately 12.4 years

$$120{,}000 = 96{,}000\left(1+\frac{0.018}{12}\right)^{12t}$$

$$1.25 = \left(1+\frac{0.018}{12}\right)^{12t}$$

$$1.25 = 1.0015^{12t}$$

Method 1

$12t = \log_{1.0015} 1.25$

$12t = \dfrac{\log 1.25}{\log 1.0015}$

$t = \dfrac{\frac{\log 1.25}{\log 1.0015}}{12} \approx 12.4$

Method 2

$\log(1.25) = \log(1.0015)^{12t}$

$\log(1.25) = 12t\log(1.0015)$

$\dfrac{\log 1.25}{\log 1.0015} = 12t$

$\dfrac{\frac{\log 1.25}{\log 1.0015}}{12} \approx 12.4 = t$

21. Approximately 3.7 years.

$$15{,}000 = \frac{1{,}000\left(\left(1+\frac{0.014}{4}\right)^{4t} - 1\right)}{\frac{0.014}{4}}$$

$$52.5 = 1{,}000\left(\left(1+\frac{0.014}{4}\right)^{4t} - 1\right)$$

$$0.0525 = \left(\left(1+\frac{0.014}{4}\right)^{4t} - 1\right)$$

$$1.0525 = 1.0035^{4t}$$

Method 1

$4t = \log_{1.0035} 1.0525$

$4t = \dfrac{\log 1.0525}{\log 1.0035}$

$t = \dfrac{\frac{\log 1.0525}{\log 1.0035}}{4} \approx 3.7$

Method 2

$\log(1.0525) = \log(1.0035)^{4t}$

$\log(1.0525) = 4t\log(1.0035)$

$\dfrac{\log 1.0525}{\log 1.0035} = 4t$

$\dfrac{\frac{\log 1.0525}{\log 1.0035}}{4} \approx 3.7 = t$

22. Approximately 2.11 years.

$$20{,}000 = \frac{800\left(1-\left(1+\frac{0.0126}{12}\right)^{-12t}\right)}{\frac{0.0126}{12}}$$

$$21 = 800\left(1-\left(1+\frac{0.0126}{12}\right)^{-12t}\right)$$

$$0.02625 = 1-\left(1+\frac{0.0126}{12}\right)^{-12t}$$

$$-0.97375 = -\left(1+\frac{0.0126}{12}\right)^{-12t}$$

$$0.97375 = (1.00105)^{-12t}$$

$$\log(0.97375) = \log(1.00105)^{-12t}$$

$$\log(0.97375) = -12t\log(1.00105)$$

$$\frac{\log(0.97375)}{\log(1.00105)} = -12t$$

$$\frac{\frac{\log(0.97375)}{\log(1.00105)}}{-12} = t$$

$$2.11 \approx t$$

Chapter 2

Lesson 2-1 Checking Accounts

Check Your Understanding (Example 1)

Begin with the old balance, add the deposits and subtract the withdrawal: $x + b + 2c - d$.

Check Your Understanding (Example 2)

Write the check number: 3273, the date: 5/11, the transaction description: James Sloan, and the payment amount: $150.32. Subtract the payment amount from the old balance:
$2,499.90 – $150.32 = $2,349.58.

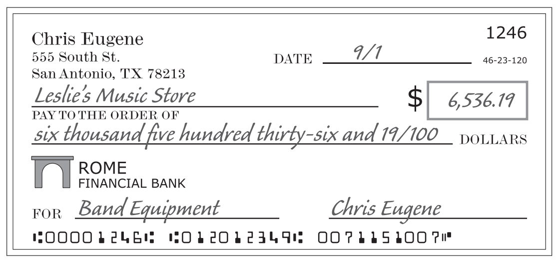
Extend Your Understanding (Example 2)

The final balance would not change because $2,740.30 – $138.90 = $2,601.40 and $2,601.40 – $101.50 = $2,499.90.

Applications

2. $278.91 + $865.98 + $623 + $60 + $130 = $1,957.89
4. $630 = 3($50) + 18($20) + *t*($10)
   $630 = $150 + $360 + $10*t*
   $630 = $510 + $10*t*
   $120 = $10*t*
   12 = *t*
6. $7,859.92;
   5,195.65 + 6,873.22 – *c* – 15 + 6.05 = 4,200.00
   12,059.92 – *c* = 4,200.00
   7,859.92 = *c*
8. $421.56 + *g*($20) + *k*($0.25) = 421.56 + 20*g* + 0.25*k*
10. $113 = 4($20) + *x*($10) + 3($1)
    $113 = $80 + $10*x* + $3
    $113 = $83 + $10*x*
    $30 = 10*x*
    3 = *x*
12. Speaker Cabinets: $400.00 × 2 = $800.00
    Speaker Cabinets: $611.00 × 2 = $1,222.00
    Horns: $190.00 × 2 = $380.00
    Audio Console: $1,079.00 × 1 = $1,079.00
    Power Amplifier: $416.00 × 5 = $2,080.00
    Microphones: $141.92 × 8 = $1,135.36
    Microphone Stands: $32.50 × 8 = $260.00
    Total: $800.00 + $1,222.00 + $380.00 + $1,079.00 + $2,080.00 + $1,135.36 + $260.00 = $6,956.36
    13% Discount: $6,956.36 × 0.13 = $904.33
    Sales Price: $6,956.36 – $904.33 = $6,052.03
    8% Sales Tax: $6,052.03 × 0.08 = $484.16
    Total Cost: $6,052.03 + $484.16 = $6,536.19

DESCRIPTION	CATALOG NUMBER	LIST PRICE	QUANTITY	TOTAL
Speaker Cabinets	RS101	$400.00	2	$800.00
Speaker Cabinets	RG306	$611.00	2	$1,222.00
Horns	BG42	$190.00	2	$380.00
Audio Console	LS101	$1,079.00	1	$1,079.00
Power Amplifier	NG107	$416.00	5	$2,080.00
Microphones	RKG-1972	$141.92	8	$1,135.36
Microphone Stands	1957-210	$32.50	8	$260.00
			TOTAL	$6,956.36
			13% DISCOUNT	$904.33
			SALE PRICE	$6,052.03
			8% SALES TAX	$484.16
			TOTAL COST	$6,536.19


Chris Eugene
555 South St.
San Antonio, TX 78213
1246
DATE 9/1
46-23-120
PAY TO THE ORDER OF Leslie's Music Store $ 6,536.19
six thousand five hundred thirty-six and 19/100 DOLLARS
ROME FINANCIAL BANK
FOR Band Equipment
Chris Eugene
⑆0000 1246⑆ ⑆0 120 12349⑆ 007115 1007⑈

14. $1,400.00 – $1,380.15 = $19.85
    $19.85 – $670 = –$650.15
    –$650.15 – $95.67 = –$745.82
    –$745.82 – $130 = –$875.82
    –$875.82 – $87.60 = –$963.42
    There are 4 negative balances, so there are 4 overdraft protection fees: $27 × 4 = $108.
    –$963.42 – $108 = –$1,071.42
    Nancy owes the bank $1,071.42.

16a. Write the date 12/15 and the balance of $2,546.50.

16b. Write the check number: 2345, the date: 12/16, the transaction description: Kings Park HSSA, and the payment amount: $54.00. Subtract: $2,546.50 – $54.00 = $2,492.50.

16c. Write the date: 12/17, the transaction description: deposit, and the deposit amount: $324.20. Add: $2,492.50 + $324.20 = $2,816.70.

16d. Write the date: 12/20, the transaction description: deposit, and the deposit amount: \$100.00. Add: \$2,816.70 + \$100.00 = \$2,916.70.

16e. Write the check number: 2346, the date: 12/22, the transaction description: Best Buy, and the payment amount: \$326.89. Subtract: \$2,916.70 – \$326.89 = \$2,589.81. Write the check number: 2347, the date: 12/22, the transaction description: Macy's, and the payment amount: \$231.88. Subtract: \$2,589.81 – \$231.88 = \$2,357.93. Write the check number: 2348, the date: 12/22, the transaction description: Target, and the payment amount: \$123.51. Subtract: \$2,357.93 – \$123.51 = \$2,234.42.

16f. Write the check number: 2349, the date: 12/24, the transaction description: VOID. Write the check number: 2350, the date: 12/24, the transaction description: Apple, and the payment amount: \$301.67. Subtract: \$2,234.42 – \$301.67 = \$1,932.75.

16g. Write the date: 12/26, the transaction description: deposit, and the deposit amount: \$98.00. Add: \$1,932.75 + \$98.00 = \$2,030.75.

16h. Write the code: EFT, the date: 12/28, the transaction description: Allstate, and the payment amount: \$876.00. Subtract: \$2,030.75 – \$876.00 = \$1,154.75

16i. Write the date: 12/29, the transaction description: ATM, and the payment amount: \$200.00. Subtract: \$1,154.75 – \$200.00 = \$954.75. There is also an ATM fee of \$1.50. Write the date: 12/29, the transaction description: ATM fee, and the payment amount: \$1.50. Subtract: \$954.75 – \$1.50 = \$953.25.

NUMBER OR CODE	DATE	TRANSACTION DESCRIPTION	PAYMENT AMOUNT $		✓	FEE	DEPOSIT AMOUNT		$ BALANCE
	12/15								2,546.50
2345	12/16	Kings Park HSSA	54	00					– 54.00 2,492.50
	12/17	Deposit					324	20	+ 324.20 2,816.70
	12/20	Deposit					100	00	+ 100.00 2916.70
2346	12/22	Best Buy	326	89					– 326.89 2,589.81
2347	12/22	Macy's	231	88					– 231.88 2,357.93
2348	12/22	Target	123	51					– 123.51 2,234.42
2349	12/24	VOID							
2350	12/24	Apple	301	67					– 301.67 1,932.75
	12/26	Deposit					98	00	+ 98.00 2,030.75
EFT	12/28	Allstate	876	00					– 876.00 1,154.75
	12/29	ATM	200	00					– 200.00 954.75
	12/29	ATM fee	1	50					– 1.50 953.25

Lesson 2-2 Reconcile a Bank Statement

Check Your Understanding (Example 1)

Sample answer: Many people and businesses hold on to checks and do not deposit or cash them immediately. If checks are written toward the end of a cycle, they will probably appear on the next monthly statement.

Check Your Understanding (Example 2)

Yes, Let a = \$885.84, b = \$825, c = \$632.84, r = \$1,078. Then $d = a + b – c$ = \$1,078, so the check register is balanced.

Check Your Understanding (Example 3)

Although formulas vary based on the spreadsheet being used, most spreadsheet programs would use the following formula to determine the sum: = sum (B3:B9).

Applications

2. Yes. \$725.71 + \$610.00 – \$471.19 = \$864.52
4. Add the deposits and subtract the outstanding checks from the ending balance. This should equal the revised statement balance and the check register balance. $B + D – C = S$ and if $S = R$, the account is reconciled.

6a. The statement shows that the ending balance is \$1,434.19.

6b. The deposit of \$700.00 is not on the bank statement, so this is the outstanding deposit amount.

6c. Check numbers 397 and 399 are not on the bank statement. So the outstanding withdrawal amount is \$50.00 + \$ 39.00 = \$89.00.

6d. \$1,434.19 + \$700.00 – \$89.00 = \$2,045.19

6e. The check register shows a balance of \$2,045.19.

6f. \$2,045.19 = \$2,045.19, so the account is reconciled.

8. No. adding \$75 will correct that he subtracted \$75 he should not have subtracted. He will also need to add another \$75 for the original deposit.
10. Outstanding deposit: \$150; outstanding checks:

    \$32.00 + \$100.00 = \$132.00

$1,827.63 – $150.00 + $132.00 = $1,809.63, which reconciles with the statement balance.

12. $55.65 + $103.50 + $25.00 = $184.15; add the outstanding deposits to the check register balance.

Lesson 2-3 Savings Accounts

Check Your Understanding (Example 1)

Answer 172. The 50th term will have had the common difference added to the first term 49 times. Use the formula $a_n = a_1 + (n - 1)d$ and substitute.

$a_{50} = 25 + (50 - 1)\ 3 = 172$

Extend Your Understanding (Example 1)

Answer 12. The common difference in an arithmetic sequence can be found by subtracting any term from the next consecutive term. For example, 223 – 211 = 12, so the common difference is 12.

Check Your Understanding (Example 2)

The fractions must be converted to decimals and each percent changed to an equivalent decimal.

5.51% = 0.0551

$$5\frac{1}{2}\% = 5.5\% = 0.055$$

$$5\frac{5}{8}\% = 5.625\% = 0.05625$$

5.099% = 0.05099

5.6% = 0.056

0.05625 > 0.056 > 0.0551 > 0.055 > 0.05099,

so the list in order from greatest to least is $5\frac{5}{8}\%$, 5.6%, 5.51%, $5\frac{1}{2}\%$, 5.099%.

Check Your Understanding (Example 3)

Because $891 – $315 = $576 is less than $750, there will be a fee of $7 each month. The expression to represent the amount of money in the account is 576 – 7*x,* where *x* is the number of months.

Check Your Understanding (Example 4)

Answer $168. Substitute into the formula $I = Prt$.

$I = 4{,}000(0.012)(3.5) = 168$

Give students additional practice on the board representing fractions of a year in months, and months as fractions of a year.

Check Your Understanding (Example 5)

Answer $11.24.

This problem extends the fraction of a year notion to include days as the units. In a nonleap year, 300 days is 300/365 of a year. Substitute into the formula $I = Prt$.

$I = 800(0.0171)(300/365) = 11.24$ when rounded to the nearest cent.

Check Your Understanding (Example 6)

Answer $8,571.43 to the nearest cent. Solve for P in the equation $I = Prt$.

$$P = \frac{I}{rt}$$

$$P = \frac{300}{(0.0175)(2)}$$

$P =$ $8,571.43 to the nearest cent.

Remind students that there are no additional withdrawals or deposits to this account over the 2 years.

Check Your Understanding (Example 7)

Approximately 9 years. Doubling $10,000 means earning $10,000 interest.

$$t = \frac{I}{\text{Pr}}$$

Substitute:

$$t = \frac{10{,}000}{(10{,}000)(0.11)}$$

$t = 9$ when rounded to the nearest year.

Check Your Understanding (Example 8)

4%. Solve for r in the I = Prt formula.

$$r = \frac{I}{Pt}$$

Substitute:

$$r = \frac{50}{(500)(2.5)}$$

$r = 4\%$

4%. Some students may assume that earning $50 means "at least $50," and can write any percent greater than 4%.

Applications

2. $256 The 90th term will have had the common difference added to the first term 89 times. Use the formula $a_n = a_1 + (n - 1)d$ and substitute.

   $a_{90} = 78 + (90 - 1)2 = 256$

4. 67 The 12th term will have had the common difference added to the first term 11 times. Use the formula $a_n = a_1 + (n - 1)d$ and substitute and solve for the first term.

   $a_{12} = a_1 + (12 - 1)3$

   $100 = a_1 + (12 - 1)3$

   $a_1 = 67$.

6. Let m = the number of months for the balance to reach zero. Then the equation to find the number of months is $m = \frac{871.43}{x}$. Substitute 9 for x: $m = \frac{871.43}{9} = 96.82\overline{5}$. Although the quotient is $96.82\overline{5}$, it is not until the 97th month that the balance will reach zero.

8. The advantage of a CD is a higher rate of interest. The disadvantage is the CD has a penalty if the money is withdrawn before maturity.

10a. $38.44. Substitute into the formula $I = Prt$.

$I = 775(0.0124)(4) = 38.44$ when rounded to the nearest cent.

10b. $813.44. Add the interest of $38.44 to the $775.

10c. $9.61. Substitute into the formula $I = Prt$.

$I = 775(0.0124)(1) = 9.61$ when rounded to the nearest cent.

10d. $9.61. The fourth year interest is computed the same as in part 10c, since the interest is not compounded.

10e. $9.61. Substitute into the formula $I = Prt$.

$I = 775(0.0124)(1) = 9.61$ when rounded to the nearest cent.

10f. $784.61. The interest of $9.61 is added to the $775.

10g. $9.73. Substitute into the formula $I = Prt$.

$I = 784.61(0.0124)(1) = 9.73$ when rounded to the nearest cent. Notice that the principal was higher than the original $775 since it had interest added.

10h. Brian earned more since he got interest on his first year's interest during the second year.

12. $40.80 Substitute into the formula $I = Prt$.

    $I = 2{,}560(0.01125)(17/12)) = 40.80$ when rounded to the nearest cent.

14. $t = \frac{I}{pr} = \frac{\$450}{(\$450)(1)} = 1 \text{ year}$

16. Slick Bank pays more interest. Substitute:

    Bedford Bank: $I = prt = (\$20{,}000)(0.01)(5) = \$1{,}000$

    Slick Bank: $I = prt = (\$20{,}000)(0.051)(1) = \$1{,}020$

18. $I = prt = (\$3{,}450)(0.05)(18) = \$3{,}105$

    $3,450 + $3,105 = $6,555

20. Less than $352. Two years is 24 months, and at $2 per month, would result in a $48 penalty, which must be subtracted.

22. $a = 24$; $b = 36$; $c = 48$; $d = 54$; $e = 60$. The common difference is 6, and it is found by subtracting 30 from 42, and dividing by 2, since 42 is the 4th term and 30 is the 2nd term.

Lesson 2-4 Explore Compound Interest

Check Your Understanding (Example 1)

Annual compounding is equivalent to simple interest for the first year. $I = prt = (\$x)(0.044)(1) = \$0.044x$

Check Your Understanding (Example 2)

The amount of interest is $I = prt = (\$4{,}000)(0.05)(0.5) = \$100$.

Add the interest to the principal: $4,000 + $100 = $4,100.

Calculate the interest using the new principal. The amount of interest is $I = prt$ = ($4,100)(0.05)(0.5) = $102.50.

Add the interest to the principal: $4,100 + $102.50 = $4,202.50.

Check Your Understanding (Example 3)

The amount of interest is $I = prt$ = ($3,000)(0.04)(0.25) = $30.

Add the interest to the principal: $3,000 + $30 = $3,030.

Calculate the interest using the new principal. The amount of interest is $I = prt$ = ($3,030)(0.04)(0.25) = $30.30.

Add the interest to the principal: $3,030 + $30.30 = $3,060.30.

Check Your Understanding (Example 4)

$$I = prt = (x)(0.05)\left(\frac{1}{365}\right) = \frac{0.5x}{365}$$

Check Your Understanding (Example 5)

The amount of interest on January 7 is

$I = prt = (\$900)(0.03)\left(\frac{1}{365}\right) = \$0.07$. The balance end of the day on January 7 is $900 + $0.07 = $900.07. Add the deposit: $900.07 + $76.22 = 976.29. The amount of interest on January 8 is

$I = prt = (\$976.29)(0.03)\left(\frac{1}{365}\right) = \$0.08$. The balance at the end of the day on January 8 is $976.29 + $0.08 = $976.37.

Applications

2. $66.60. Annual compounding is equivalent to simple interest for the first year. Substitute into $I = Prt$.

   I = 3,700(0.018)(1) = 66.60
4. $63, $9,063. Multiply 9,000 by 0.014 to get 126, which would represent a full year's interest. Divide by 2 to get the semi-annual interest of $63.

6a. $18.38. Multiply the balance of 3,500 by 0.0105, and divide by 2.

6b. $3,518.38. Add the interest, $18.38, to the principal $3,500.

6c. $18.47. Multiply the balance of 3,518.38 by 0.0105, and divide by 2

6d. $3,536.85. Add the interest of $18.47 to the principal $3,518.38.

6e. $36.85. Add the two interest amounts, $18.38 and $18.47 to get total interest for the year.

6f. $36.75. Use the simple interest formula $I = Prt$.

I = 3,500(0.0105)(1) = $36.75

6g. $0.10

8. $0.03. Multiply 720 by 0.014 and divide by 365.

10a. $0. The day started and Jacob had not opened the account yet, so the "balance" is 0.

10b. $4,550.00. This is the amount he deposited.

10c. $4,550.00. This is the principal used to compute the interest since it is the balance at the end of the day.

10d. $0.14. The principal $4,550 is multiplied by 0.011 and the result is divided by 365.

10e. $4,550.00 + $0.14 = $4,550.50. The interest of $0.14 is added to the principal used to compute the interest, which is $4,550.

10f. $4,550.14. The ending balance from the previous day is the opening balance for the next day.

10g. $300.00. A deposit was made.

10h. $4,550.14 + $300.00 = $4,850.14. The deposit is added to the opening balance.

10i. $0.15. The principal, $4,850.14 is multiplied by 0.011 and the result is divided by 365.

10j. $4,850.29. The $0.15 interest is added to $4,850.29

10k. $4,850.29. The ending balance from the previous day is the opening balance for the next day.

10l. $900.00. There was a $900 withdrawal made.

10m. $3,950.29. The withdrawal is subtracted form the opening balance for August 12.

10n. $0.12. The principal, $3,950.29, is multiplied by 0.011 and the result is divided by 365.

10p. $3,950.41. The interest of $0.12 is added to the principal of $3,950.29.

12. $$x + y + \frac{0.013(x+y)}{365} + \frac{0.013\left(x + y + \frac{0.013(x+y)}{365}\right)}{365}$$

The interest for May 30 is based on the ending balance for May 29. On May 29, the principal used to compute the interest is (*x* + *y*). On May 30, the principal used to compute the interest is $x+y+\frac{0.013(x+y)}{365}$.

14a. *P* + *D*. The deposit is added to the opening balance.

14b. $\frac{0.02(P+D)}{365}$

The interest is computed by multiplying the principal by the interest rate, 0.02, and the result is divide by 365 since it is compounded daily.

14c. $P+D+\frac{0.02(P+D)}{365}$

The interest is added to the principal for February 2.

14d. $P+D+\frac{0.02(P+D)}{365}$

The opening balance is equal to the previous day's ending balance.

14e. $P+D-W+\frac{0.02(P+D)}{365}$

The February 3 withdrawal is subtracted from the opening balance.

14f. $\frac{0.02\left(P+D-W+\frac{0.02(P+D)}{365}\right)}{365}$

The principal used to compute the interest is multiplied by 0.02, and the result is divide by 365 since it is compounded daily.

$$P+D-W+\frac{0.02(P+D)}{365}+\frac{0.02\left(P+D-W+\frac{0.02(P+D)}{365}\right)}{365}$$

Lesson 2-5 Compound Interest Formula

Check Your Understanding (Example 1)

$815.06. Use the compound interest formula and substitute.

$$B=P\left(1+\frac{r}{n}\right)^{nt}$$

$$B=800\left(1+\frac{0.0187}{4}\right)^{4}$$

B = $815.15

Check Your Understanding (Example 2)

$1,220.56. Use the compound interest formula and substitute.

$$B=P\left(1+\frac{r}{n}\right)^{nt}$$

$$B=1{,}200\left(1+\frac{0.017}{12}\right)^{12}$$

B = $1,220.56

Extend Your Understanding (Example 2)

1.9%. Use the compound interest formula and substitute for the daily account.

$$B=P\left(1+\frac{r}{n}\right)^{nt}$$

$$B=1{,}200\left(1+\frac{0.0138}{365}\right)^{365}$$

The balance for the daily account is B = $1,216.67 after 1 year.

Use the compound interest formula and substitute for the quarterly account.

$$B=P\left(1+\frac{r}{n}\right)^{nt}$$

$$B=1{,}200\left(1+\frac{0.019}{4}\right)^{4}$$

The balance for the quarterly account is *B* = $1,222.96 after 1 year, so the quarterly account is better.

Check Your Understanding (Example 3)

$2,482.81. Use the compound interest formula and substitute.

$$B=P\left(1+\frac{r}{n}\right)^{nt}$$

$$B=2{,}350\left(1+\frac{0.011}{12}\right)^{60}$$

B = $2,482.81

Extend Your Understanding (Example 3)

Use the compound interest formula and substitute.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$2{,}000\left(1+\frac{0.015}{365}\right)^{365k}$$

Check Your Understanding (Example 4)

1. 11%. Use the APY formula and substitute for the quarterly account.

$$\text{APY} = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$\text{APY} = \left(1+\frac{0.011}{365}\right)^{365} - 1$$

APY = 1.11% to the nearest hundredth of a percent.

Extend Your Understanding (Example 4)

Because there are more compounding periods for interest to be earned on already accumulated interest, the balance grows more quickly.

Applications

2. $4,880.76. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 4{,}000\left(1+\frac{0.02}{2}\right)^{20}$$

B = $4,880.76

4. $1,551.26. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 1{,}500\left(1+\frac{0.0112}{365}\right)^{(365)(3)}$$

B = $1,551.26

6. $5,230.11. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 5{,}000\left(1+\frac{0.015}{52}\right)^{(156)}$$

B = $5,230.11

8. $2.25. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 1{,}000\left(1+\frac{0.03}{365}\right)^{(8)(365)}$$

When compounded daily, the balance is B = $1,271.24.

$$B = 1{,}000\left(1+\frac{0.03}{2}\right)^{(8)(2)}$$

When compounded semi-annually, the balance is B = $1,268.99.

The difference between the two accounts is $2.25.

10a. No. Use the compound interest formula and substitute. This must be computed on a calculator. Find the interest after 10 and 15 years to see if their money is doubled.

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$B = 20{,}000\left(1+\frac{0.0275}{365}\right)^{(10)(365)}$$

Balance after 10 years is $26,330.34, so the money does not double in 10 years.

10b. No.

Substitute using 15 years.

$$B = 20{,}000\left(1+\frac{0.0275}{365}\right)^{(15)(365)}$$

Balance after 15 years is $30,211.32, so the money does not double in 15 years.

12a. $125. Use the simple interest formula, *I* = *Prt*, and substitute.

I = 5,000(0.025)(1) = $125

12b. $125. Compounding annually for 1 year is the same as simple interest for 1 year.

12c. The interest is the same.

12d. $375. Use the simple interest formula, *I* = *Prt*, and substitute.

I = 5,000(0.025)(3) = $375

12e. \$384.45. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 5{,}000\left(1+\frac{0.025}{1}\right)^3$$

B = \$5,384.45. Subtract \$5,000 to find that the interest is \$384.45.

12f. The annual compounded interest earned is \$9.45 more than the simple interest.

12g. \$600. Use the simple interest formula, $I = Prt$, and substitute.

I = 5,000(0.02)(6) = \$600

12h. \$630.81. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 5{,}000\left(1+\frac{0.02}{1}\right)^6$$

B = \$630.81

12i. The annual compounded interest earned is \$30.81 more than the simple interest.

12j. No. They are the same for 1 year. For anything longer, compounded interest grows faster than simple interest.

14. 1.56%. Use the APY formula and substitute for the quarterly account.

$$\text{APY} = \left(1+\frac{r}{n}\right)^n - 1$$

$$\text{APY} = \left(1+\frac{0.0155}{365}\right)^{365} - 1$$

APY = 1.56%

16a. \$22,251,500. Use the simple interest formula, $I = Prt$, and substitute.

I = 955,000,000(0.0233)(1) = \$22,251,500

16b. \$22,512,028.20. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 955{,}000{,}000\left(1+\frac{0.0233}{365}\right)^{365}$$

B = \$22,512,028.20

16c. \$260,528.20. Subtract the interest answers found in 16a and 16b.

16d. 1, since each 4-year (full) scholarship is worth \$244,000.

Lesson 2-6 Continuous Compounding

Check Your Understanding (Example 1)

The value of $g(x)$ will decrease as x increases because of the negative coefficient, –5, of x.

Check Your Understanding (Example 2)

x	$f(x)$
10	$\frac{1}{10} = 0.1$
100	$\frac{1}{100} = 0.01$
1,000	$\frac{1}{1{,}000} = 0.001$
10,000	$\frac{1}{10{,}000} = 0.0001$
1,000,000	$\frac{1}{100{,}000} = 0.00001$

The values in the table are approaching 0.

Check Your Understanding (Example 3)

x	$f(x)$
10	$1^{10} = 1$
100	$1^{100} = 1$
1,000	$1^{1{,}000} = 1$
10,000	$1^{10{,}000} = 1$
1,000,000	$1^{100{,}000} = 1$

The values in the table are all 1.

Check Your Understanding (Example 4)

x	$f(x)$
10	$\left(1+\frac{0.05}{10}\right)^{10}=1.05114$
100	$\left(1+\frac{0.05}{100}\right)^{100}=1.05126$
1,000	$\left(1+\frac{0.05}{1{,}000}\right)^{1{,}000}=1.05127$
10,000	$\left(1+\frac{0.05}{10{,}000}\right)^{10{,}000}=1.05127$
1,000,000	$\left(1+\frac{0.05}{1{,}000{,}000}\right)^{1{,}000{,}000}=1.05127$

The values in the table are approaching 1.05127.

Check Your Understanding (Example 5)

$e^{\pi}-\pi^{e}\approx 23.1406-22.4591\approx 0.6815\approx 0.682$

Check Your Understanding (Example 6)

$5,229.09. Use the continuous compounding formula, $B = Pe^{rt}$, and substitute.

$B = Pe^{rt}$

$B = 5{,}000e^{(0.0112)(4)} = \$5{,}229.09$

Applications

2a. $120. Use the simple interest formula $I = Prt$, ans substitute.

$I = 2{,}000(0.02)(3) = 120$

2b. $122.42. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B=2{,}000\left(1+\frac{0.02}{1}\right)^{3}=122.42$$

2c. $123.04. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B=2{,}000\left(1+\frac{0.02}{2}\right)^{6}=123.04$$

2d. $123.36. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B=2{,}000\left(1+\frac{0.02}{4}\right)^{12}=123.36$$

2e. $123.57. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B=2{,}000\left(1+\frac{0.02}{12}\right)^{36}=123.57$$

2f. $123.67 Use the compound interest formula and substitute. This must be computed on a calculator.

$$B=2{,}000\left(1+\frac{0.02}{365}\right)^{(365)(3)}=123.67$$

2g. $123.67. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B=2{,}000\left(1+\frac{0.02}{8760}\right)^{(8760)(3)}=123.67$$

2h. $123.67. There are 525,600 minutes in a year. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B=2{,}000\left(1+\frac{0.02}{525{,}600}\right)^{(525600)(3)}=123.67$$

2i. $123.67. Use the continuous compounding formula, $B = Pe^{rt}$, and substitute.

$B = Pe^{rt}$

$B = 2{,}000e^{(0.02)(3)} = \$123.67$

2j. $3.67. This is the difference between $120 and $123.67.

4. $3,491.51. Use the continuous compounding formula, $B = Pe^{rt}$, and substitute.

$B = Pe^{rt}$

$B = 50{,}000e^{(0.0239)(4)} = \$3{,}491.51$

6a. $B = pe^{rt} = \$1{,}000e^{(0.16)(5)} = \$2{,}225.54$

6b. $1,105.17. Use the continuous compounding formula, $B = Pe^{rt}$, and substitute.

$B = Pe^{rt}$

$B = 1{,}000e^{(0.02)(5)} = \$1{,}105.17$

6c. \$1,120.37. This is found by subtracting the answers to 6a and 6b.

8a. \$710.76. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 19{,}400\left(1+\frac{0.012}{12}\right)^{(12)(3)} = 710.76$$

8b. \$831.73. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 19{,}400\left(1+\frac{0.012}{12}\right)^{(12)(3.5)} = 831.73$$

8c. \$41.93. The interest on 3 years is from part 8a, and this is \$710.76. So the balance after 3 years is 20,110.76. The interest for the last 5 months (5/12 of a year) can be found using the compound interest formula with the lowered interest rate of 0.5%.

$$B = 20{,}110.76\left(1+\frac{0.005}{12}\right)^{(12)(\frac{5}{12})} = 20{,}152.69$$

The difference between 20,110.76 and 20,152.69, is 41.93.

8d. \$752.69. The interest after 3 years, \$710.76, is added to 41.93.

8e. \$502.69. The \$250 penalty is subtracted from \$752.69.

8f. \$7,902.69. The \$12,000 is subtracted.

8g. \$3.29. The compound interest formulas is used to find the interest for the last month.

$$B = 7{,}902.69\left(1+\frac{0.005}{12}\right)^{(\frac{1}{12})} = 7{,}905.98$$

To find the last month's interest, subtract 7,902.69 from 7,905.98.

8h. \$505.98. The 3.29 is added to 502.69.

8i. Yes. With the penalty and the reduced interest, the 3-year CD would have been better.

10. t months $= \frac{t}{12}$ years

$15{,}000e^{0.0175t/12}$

Lesson 2-7 Future Value of Investments

Check Your Understanding (Example 1)

\$6,410.19. Determine the amount that will be in the account after 21 years (\$119,225.08) and subtract that from the answer to Example 1.

Extend Your Understanding (Example 1)

No. Using the order of operations $(1 + 0.0125)^{20}$ will be computed first. 1 is subtracted from that value, not from the exponent of 20.

Check Your Understanding (Example 2)

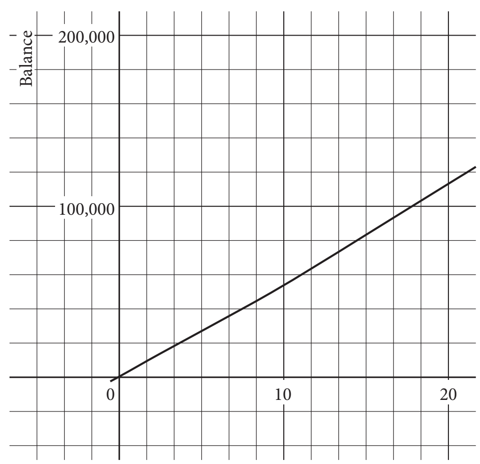
Answer Rich and Laura will earn \$1,410.19 more by retiring 1 year later.

Check Your Understanding (Example 3)

Not necessarily: it depends on the principal, rate, and length of time opened.

Check Your Understanding (Example 4)

Enter the equation in Y_1. The minimum on the x-axis is 0, the maximum is 20. The minimum on the y-axis is 0, the maximum is 200,000. Graph the equation.



Applications

2. \$1,304.44. $B = 1000\left(1+\frac{0.0085}{1}\right)^{4} \approx \$1034.44$

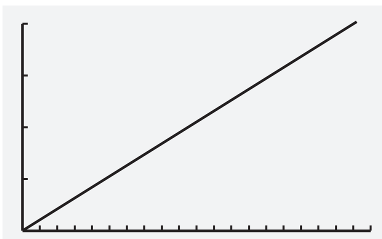
4. \$10,428.18

$$B = 10000\left(1+\frac{0.014}{4}\right)^{12} \approx \$10{,}428.18$$

6a. $B = \dfrac{100\left(\left(1+\dfrac{0.024}{52}\right)^{52(3)}-1\right)}{\dfrac{0.024}{52}} \approx \$16{,}171.46$

6b. $B = \dfrac{100\left(\left(1+\dfrac{0.024}{52}\right)^{x}-1\right)}{\dfrac{0.024}{52}}$

6c. Enter the equation in Y_1. The minimum on the *x*-axis is 0, the maximum is 260. The minimum on the *y*-axis is 0, the maximum is 12,000. Graph the equation.



6d. Using the graph, determine approximate *y*-value when the *x*-value is 2(52); $10,600

8. $2,421.95

$$B = 2{,}000\left(1+\frac{0.012}{2}\right)^{32} \approx \$2{,}421.95$$

10. Substitute A8 for *B*, A3 for *P*, A4 for *r*, A6 for *n*, and A5 for *t* in the formula for the future value of a periodic investment:

$$A8 = \frac{A3\left(\left(1+\dfrac{A4}{A6}\right)^{A6(A5)}-1\right)}{\dfrac{A4}{A6}}$$

$$B = \frac{A3\left(\left(1+\dfrac{A4}{A6}\right)^{A6(A5)}-1\right)}{\dfrac{A4}{A6}}$$

Write the equation as it would appear in a spreadsheet:

= (A3*((1+A4/A6)^(A6*A5)–1))/(A4/A6)

Lesson 2-8 Present Value of Investments

Check Your Understanding (Example 1)

They would have to deposit double the original amount – $36,581.69

Check Your Understanding (Example 2)

Everything remains the same except $n = 52$.

Check Your Understanding (Example 3)

$$P = \frac{x\left(\dfrac{r}{200}\right)}{\left(1+\dfrac{r}{200}\right)^{2y}-1}$$

Check Your Understanding (Example 4)

The higher the interest rate, the lower the present value needed to attain the future goal.

Applications

2a. $959.15. $P = \dfrac{1{,}000}{\left(1+\dfrac{0.014}{1}\right)^{1(3)}} \approx \$959.15$

2b. $2,374.82. $P = \dfrac{2{,}500}{\left(1+\dfrac{0.0103}{2}\right)^{2(5)}} \approx \$2{,}374.82$

2c. $9,094.75. $P = \dfrac{10{,}000}{\left(1+\dfrac{0.0095}{4}\right)^{4(10)}} \approx \$9{,}094.75$

2d. $41,770.55.

$$P = \frac{50{,}000}{\left(1+\dfrac{0.0225}{12}\right)^{12(8)}} \approx \$41{,}770.55$$

4. $6,889.80. $P = \dfrac{50{,}000\left(\dfrac{0.012}{1}\right)}{\left(1+\dfrac{0.012}{1}\right)^{1(7)}-1} \approx \$6{,}889.80$

6. $1,658.87

$$P = \frac{10{,}000\left(\dfrac{0.0225}{12}\right)}{\left(1+\dfrac{0.0225}{12}\right)^{12(0.5)}-1} \approx \$1{,}658.87$$

8. $12,204.45

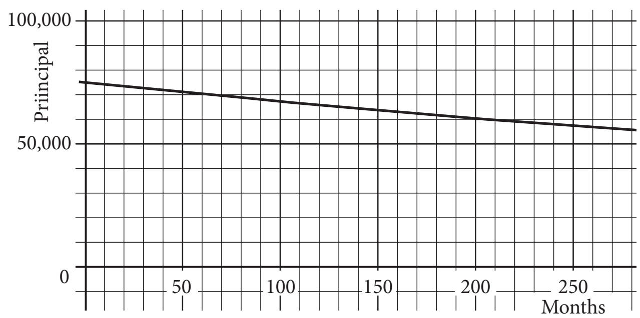
$$P = \frac{100{,}000\left(\frac{0.009}{2}\right)}{\left(1+\frac{0.009}{2}\right)^{2(4)}-1} \approx \$12{,}304.45$$

10. Graph the equation: $y = \frac{\$75{,}000}{\left(1+\frac{0.031}{12}\right)^{x}}$,

Let x represent the number of months.

$$y = \frac{75{,}000}{\left(1+\frac{0.013}{12}\right)^{x}}$$

where x is the number of months and y is the principal amount. The display should look similar to the one below.



Lesson 2-9: The Term of a Single Deposit Account

Check Your Understanding (Example 1)

2.4%;

$$1.006 = 1+\frac{r}{n}$$

$$1.006 = 1+\frac{r}{4}$$

$$0.006 = \frac{r}{4}$$

$0.024 = r$

The rate is 2.4%.

Extend Your Understanding (Example 1)

Four times per year

$$1.002 = 1+\frac{r}{n}$$

$$1.002 = 1+\frac{0.008}{n}$$

$$0.002 = \frac{0.008}{n}$$

$0.002n = 0.008$

$n = 4$

Check Your Understanding (Example 2)

Here students will be transforming the compound interest formula to solve for t in terms of P, B, r, and n.

$$t = \frac{\log_{\left(1+\frac{r}{n}\right)} \frac{B}{P}}{n}$$

Extend Your Understanding (Example 2)

$a^{5t} = 1000$

Check Your Understanding (Example 3)

$t = 5$.

$$t = \frac{\log_{1.0005} 1.03044}{12}$$

$$12t = \log_{1.0005} 1.03044$$

$$12t = \frac{\log 1.03044}{\log 1.0005}$$

$$t = \frac{\frac{\log 1.03044}{\log 1.0005}}{12} \approx 5$$

Check Your Understanding (Example 4)

Approximately 57.8 years.

$$500\left(1+\frac{0.012}{4}\right)^{4t} = 1000$$

$$\left(1+\frac{0.012}{4}\right)^{4t} = 2$$

$1.0003^{4t} = 2$

$\log_{1.003} 2 = 4t$

$$\frac{\log 2}{\log 1.0003} = 4t$$

$57.8 \approx t$

Extend Your Understanding (Example 4)

Approximately 24.8 years.

$$D\left(1+\frac{0.028}{4}\right)^{4t} = 2D$$

$$\left(1+\frac{0.028}{4}\right)^{4t} = 2$$

$$1.007^{4t} = 2$$

$$\log_{1.007} 2 = 4t$$

$$\frac{\log 2}{\log 1.007} = 4t$$

$$24.8 \approx t$$

Check Your Understanding (Example 5)

Approximately 18.7 years.

$$16000 = 8000e^{0.037t}$$

$$2 = e^{0.037t}$$

$$0.037t = \ln 2$$

$$t = \frac{\ln 2}{0.037} \approx 18.7$$

Applications

2a. n = 12, rate = 2.4%

nt = 12t therefore, n = 12

$1+\frac{r}{12} = 1.002$; r = 0.024; rate = 2.4%

2b. n = 2, rate = 2%

nt = 2t therefore, n = 2

$1+\frac{r}{2} = 1.01$; r = 0.02; rate = 2%

2c. n = 6, rate = 3.6%

nt = 6t therefore, n = 6

$1+\frac{r}{6} = 1.006$; r = 0.036; rate = 3.6%

2d. n = 12, rate = 4.5%

nt = 12t therefore, n = 12

$1+\frac{r}{12} = 1.00375$; r = 0.045; rate = 4.5%

2e. n = 2, rate = 5%

nt = 2t therefore, n = 2

$1+\frac{r}{2} = 1.025$; r = 0.05; rate = 5%

4a. $t = \frac{\log_{1.009}(1.14375)}{3}$

$$5{,}490 = 4{,}800(1.009)^{3t}$$

$$\frac{5{,}490}{4{,}800} = (1.009)^{3t}$$

$$1.14375 = (1.009)^{3t}$$

$$3t = \log_{1.009} 1.14375$$

$$t = \frac{\log_{1.009} 1.14375}{3}$$

4b. $t = \frac{\log_{1.006}(1.3)}{6}$

$$3{,}900 = 3{,}000(1.006)^{6t}$$

$$\frac{3{,}900}{3{,}000} = (1.006)^{6t}$$

$$1.3 = (1.006)^{6t}$$

$$6t = \log_{1.006} 1.3$$

$$t = \frac{\log_{1.006} 1.3}{6}$$

4c. $t = \frac{\log_{1.0015}(1.46)}{12}$

$$1{,}460 = 1{,}000(1.0015)^{12t}$$

$$\frac{1{,}460}{1{,}000} = (1.0015)^{12t}$$

$$1.46 = (1.0015)^{12t}$$

$$12t = \log_{1.0015} 1.46$$

4d. $t = \frac{\log_{1.00875}(1.19034)}{4}$

$$5{,}951.70 = 5{,}000(1.00875)^{4t}$$

$$\frac{5{,}951.70}{5{,}000} = (1.00875)^{4t}$$

$$1.19034 = (1.00875)^{4t}$$

$$4t = \log_{1.00875} 1.19034$$

$$t = \frac{\log_{1.00875} 1.19034}{4}$$

4e. $t = \frac{\log_{1.008}(1.10034)}{4}$

$$33{,}010.20 = 30{,}000(1.008)^{4t}$$

$$\frac{33{,}010.20}{30{,}000} = (1.008)^{4t}$$

$$1.10034 = (1.008)^{4t}$$

$$4t = \log_{1.008} 1.10034$$

$$t = \frac{\log_{1.008} 1.10034}{4}$$

6a. $n = (\log(1.078)/\log(1.00375))/5 \approx 4$

6b. $n = (\log(1.270945)/\log(1.002))/10 \approx 12$

6c. $n = (\log(1.0956)/\log(1.00005))/5 \approx 365$

6d. $n = (\log(1.195)/\log(1.011))/8 \approx 2$

6e. $n = (\log(1.1015)/\log(1.01625))/6 \approx 1$

8. Approximately 4.9 years.

$$230{,}000 = 200{,}000\left(1+\frac{0.02875}{4}\right)^{4t}$$

$$1.15 = \left(1+\frac{0.02875}{4}\right)^{4t}$$

$$1.15 = 1.0071875^{4t}$$

$$4t = \log_{1.0071875} 1.15$$

$$4t = \frac{\log 1.15}{\log 1.0071875}$$

$$t = \frac{\frac{\log 1.15}{\log 1.0071875}}{4} \approx 4.9$$

10. Approximately 3.5 years.

$$B = Pe^{rt}$$

$$10800 = 10000e^{0.022t}$$

$$1.08 = e^{0.022t}$$

$$0.022t = \ln(1.08)$$

$$t = \frac{\ln(1.08)}{0.022} \approx 3.5$$

12. Let $B = 3P$ and $n = 1$ in the compound interest formula

$$B = P\left(1+\frac{r}{n}\right)^{nt}$$

$$3P = P\left(1+\frac{r}{1}\right)^{1t}$$

$$3 = (1+r)^t$$

$$t = \log_{1+r} 3$$

$$t = \frac{\log 3}{\log(1+r)}$$

Lesson 2-10: The Term of a Systematic Account

Check Your Understanding (Example 1)

$$t = \frac{\log_{1.0025}\left(\frac{300}{200}\right)}{4}$$

$$300 = 200\left(1+\frac{0.01}{4}\right)^{4t}$$

$$\frac{300}{200} = \left(1+\frac{0.01}{4}\right)^{4t}$$

$$\frac{300}{200} = 1.0025^{4t}$$

$$4t = \log_{1.0025}\left(\frac{300}{200}\right)$$

$$t = \frac{\log_{1.0025}\left(\frac{300}{200}\right)}{4}$$

Check Your Understanding (Example 2)

$$t \approx 0.6$$

$$3.6 = 1.5^{5t}$$

$$\log(3.6) = \log(1.5)^{5t}$$

$$\log(3.6) = 5t\log(1.5)$$

$$\frac{\log(3.6)}{\log(1.5)} = 5t$$

$$\frac{\frac{\log(3.6)}{\log(1.5)}}{5} = t$$

$0.6 \approx t$

Extend Your Understanding (Example 2)

$t \approx 18.6$

$B = Pe^{rt}$

$1{,}000 = 800e^{0.012t}$

$1.25 = e^{0.012t}$

$\ln(1.25) = \ln \ e^{0.012t}$

$\ln(1.25) = 0.012\, t \ln e$

$\ln(1.25) = 0.012t$

$$\frac{\ln(1.25)}{0.012} = t$$

$18.6 \approx \text{t}$

Check Your Understanding (Example 3)

Approximately 2.1 years.

$$5{,}000 = \frac{200\left(\left(1+\frac{0.0126}{12}\right)^{12t} - 1\right)}{\frac{0.0126}{12}}$$

$$5.25 = 200\left(\left(1+\frac{0.0126}{12}\right)^{12t} - 1\right)$$

$$0.02625 = \left(1+\frac{0.0126}{12}\right)^{12t} - 1$$

$$1.02625 = \left(1+\frac{0.0126}{12}\right)^{12t}$$

$1.02625 = 1.00105^{12t}$

Method 1:

$12t = \log_{1.00105} 1.02625$

$$12t = \frac{\log 1.02625}{\log 1.00105}$$

$$t = \frac{\frac{\log 1.02625}{\log 1.00105}}{12}$$

$t \approx 2.1$

Method 2.

$\log(1.02625) = \log(1.00105)^{12t}$

$\log(1.02625) = 12t \log(1.00105)$

$$\frac{\log 1.02625}{\log 1.00105} = 12t$$

$$\frac{\frac{\log 1.02625}{\log 1.00105}}{12} = t$$

$2.1 \approx t$

Check Your Understanding (Example 4)

$t \approx 13.9$ years;

$$40{,}000 = \frac{256\left(1-\left(1+\frac{0.0096}{12}\right)^{-12t}\right)}{\frac{0.0096}{12}}$$

$$32 = 256\left(1-\left(1+\frac{0.0096}{12}\right)^{-12t}\right)$$

$$0.125 = 1-\left(1+\frac{0.0096}{12}\right)^{-12t}$$

$$-0.875 = -\left(1+\frac{0.0096}{12}\right)^{-12t}$$

$0.875 = (1.0008)^{-12t}$

$\log(0.875) = \log(1.0008)^{-12t}$

$\log(0.875) = -12t \log(1.0008)$

$$\frac{\log(0.875)}{\log(1.0008)} = -12t$$

$$\frac{\frac{\log(0.875)}{\log(1.0008)}}{-12} = t$$

$13.9 \approx t$

Students may want to try to substitute 15 into the equation. Ask why replacing *t* with 15 would not help in answering the question.

Applications

2a. 3; 3 log(10) = 3

2b. *s* log (*r*)

2c. $3\log_2 x$

2d. 4

2e. $y\ln(x)$

4a. $t \approx 2.5$

$$15 = 3^t$$
$$\log(15) = \log(3)^t$$
$$\log(15) = t\log(3)$$
$$\frac{\log(15)}{\log(3)} = t$$
$$2.5 \approx t$$

4b. $t = 8$

$$5^t = 390{,}625$$
$$\log(5)^t = \log(390{,}625)$$
$$t\log(5) = \log(390{,}625)$$
$$t = \frac{\log(390{,}625)}{\log(5)}$$
$$t = 8$$

4c. $t = 1.5$

$$8^{2t} = 512$$
$$\log(8)^{2t} = \log(512)$$
$$2t\log(8) = \log(512)$$
$$t = \frac{\log(512)}{2\log(8)}$$
$$t = 1.5$$

4d. $t = 9$

$$1.2^{t-3} = 2.985984$$
$$\log(1.2)^{t-3} = \log(2.985984)$$
$$(t-3)\log(1.2) = \log(2.985984)$$
$$t-3 = \frac{\log(2.985984)}{\log(1.2)}$$
$$t = \frac{\log(2.985984)}{\log(1.2)} + 3$$
$$t = 9$$

4e. $t = 5.5$

$$10^{\frac{t}{2}} - 1 = 549$$
$$10^{\frac{t}{2}} = 550$$
$$\log(10)^{\frac{t}{2}} = \log(550)$$
$$\frac{t}{2}\log(10) = \log(550)$$
$$\frac{t}{2} = \frac{\log(550)}{\log(10)}$$
$$t = \frac{2\log(550)}{\log(10)}$$
$$t \approx 5.5$$

6a. $2{,}400 = 2{,}000\left(1+\frac{0.014}{4}\right)^{4t}$

6b. $2{,}400 = 2{,}000(1.0035)^{4t}$

6c. $1.2 = (1.0035)^{4t}$

6d. $4t = \log_{1.0035} 1.2$

6e. $4t = \frac{\log 1.2}{\log 1.0035}$

6f. $4t = 52.183$

6g. $t = 13.04575$

6h. 13

8. $t \approx 4$ years

$$5{,}500 = 5{,}000\left(1+\frac{0.024}{12}\right)^{12t}$$
$$1.1 = \left(1+\frac{0.024}{12}\right)^{12t}$$
$$1.1 = 1.002^{12t}$$

Method 1

$$12t = \log_{1.002} 1.1$$
$$12t = \frac{\log 1.1}{\log 1.002}$$
$$t = \frac{\frac{\log 1.1}{\log 1.002}}{12} \approx 4$$

Method 2

$$\log(1.1) = \log(1.002)^{12t}$$
$$\log(1.1) = 12t\log(1.002)$$
$$\frac{\log 1.1}{\log 1.002} = 12t$$
$$\frac{\frac{\log 1.1}{\log 1.002}}{12} \approx 4 = t$$

10. $t \approx 2.1$ years

$$5{,}000 = \frac{200\left(\left(1+\frac{0.015}{12}\right)^{12t}-1\right)}{\frac{0.015}{12}}$$

$$6.25 = 200\left(\left(1+\frac{0.015}{12}\right)^{12t}-1\right)$$

$$0.03125 = \left(\left(1+\frac{0.015}{12}\right)^{12t}-1\right)$$

$$1.03125 = 1.00125^{12t}$$

Method 1

$$12t = \log_{1.00125} 1.03125$$

$$12t = \frac{\log 1.03125}{\log 1.00125}$$

$$t = \frac{\frac{\log 1.03125}{\log 1.00125}}{12} \approx 2.1$$

$$\log(1.03125) = 12t\log(1.00125)$$

$$\log(1.03125) = \log(1.00125)^{12t}$$

Method 2

$$\frac{\log 1.03125}{\log 1.00125} = 12t$$

$$\frac{\frac{\log 1.03125}{\log 1.00125}}{12} \approx 2.1 = t$$

12a. $400 = \dfrac{10{,}000 \times \frac{0.0195}{12}}{(1+\frac{.0195}{12})^{12t}-1}$

12b. $400 = \dfrac{16.25}{(1.001625^{12t}-1)}$

12c. $400 \times \left(1.001625^{12t}-1\right) = 16.25$

12d. $(1.001625)^{12t}-1) = 0.040625$

12e. $(1.001625)^{12t} = 1.040625$

12f. $\log((1.001625)^{12t}) = \log(1.040625)$

12g. $12t = \dfrac{\log(1.040625)}{\log(1.001625)}$; $t = \dfrac{\log(1.040625)}{12\log(1.001625)}$;

$t \approx 2.04$

14. 5.3 years

$$50{,}000 = \frac{5{,}000\left(1-\left(1+\frac{0.018}{2}\right)^{-2t}\right)}{\frac{0.018}{2}}$$

$$450 = 5{,}000\left(1-\left(1+\frac{0.018}{2}\right)^{-2t}\right)$$

$$0.09 = 1-\left(1+\frac{0.018}{2}\right)^{-2t}$$

$$-0.91 = -\left(1+\frac{0.018}{2}\right)^{-2t}$$

$$0.91 = (1.009)^{-2t}$$

$$\log(0.91) = \log(1.009)^{-2t}$$

$$\log(0.91) = -2t\log(1.009)$$

$$\frac{\log(0.91)}{\log(1.009)} = -2t$$

$$\frac{\frac{\log(0.91)}{\log(1.009)}}{-2} = t$$

$$5.3 \approx t$$

Assessment

Really? Really! Revisited

The data about counterfeit money seizures is for 2015. After students have completed this activity, have them search the internet for data on seizures in the current or past years since 2015. They can use that data to create their own questions similar to the ones offered here. This extension would be an excellent addition to the Reality Check projects.

a. 1,360,000,000,000 x 0.0025 = 3,400,000,000 = \$3400M

b. 1,360,000,000,000 x 0.75 = 1,020,000,000,000 = \$1,020,000M

c. 100(88,000,000/1,360,000,000,000) ≈ 0.0065%

100(88,000,000/3,400,000,000) ≈ 2.6%

Only about 2.6% of the counterfeit money was removed that year.

d. (20 + 86 + 2 + 2 + 10 + 1.5 + 20 + 4.7 + 0.05 + 1.1)M = $147.35M

e. 100(147,350,000/1,360,000,000,000) ≈ 0.011%

f. $B = 147{,}350{,}000\left(1+\dfrac{0.01}{12}\right)^{12\left(\frac{1}{12}\right)} \approx$

147,472,791.67

147,472,791.67 – 147,350,000 = $122,791.67

g. There is still a long way to go to get a handle on counterfeit currency.

Applications

2a. The statement shows that the ending balance is $2,495.91.

2b. The deposit of $120.00 is not on the bank statement, so this is the outstanding deposit amount.

2c. Check numbers 1228 and 1230 are not on the bank statement. So the outstanding check amount is $1,250.00 + $150.80 = $1,400.80.

2d. $2,495.91 + $120.00 – $1,400.80 = $1,215.11

2e. The check register shows a balance of $1,215.11, so the account is reconciled.

4. Because $1,722 – $400 = $1,322 is less than $1,500, there will be a fee of $3.50 each month. The balance after 5 months is $1,322 – $3.50(5) = $1,304.50, not including any interest.

6a. $47.78. Use the simple interest formula and substitute:

$I = Prt$

$I = 910(0.0175)(3) = \$47.78$

6b. $957.78. The interest, $47.78, is added to $910.

6c. $15.93. Use the simple interest formula and substitute:

$I = Prt$

$I = 910(0.0175)(1) = \$15.93$

6d. $15.93. Use the simple interest formula and substitute:

$I = Prt$

$I = 910(0.0175)(1) = \$15.93$

8a. $42. Multiply 5,600 by 0.015, and divided by 2.

8b. $5,642. Add the $42 to the original deposit.

8c. $42.32. Multiply 5,642 by 0.015 and divide by 2.

8d. $5,684.32. Add 442.32 to $5,642.

8d. $84.32. The deposit of $5,000 needs to be subtracted from the balance of $5,684.32.

10. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 3{,}000\left(1+\frac{0.0408}{52}\right)^{(52)(3)} = 3{,}390.46$$

12a. $2,302.35. Use the compound interest formula and substitute. This must be computed on a calculator.

$$B = 2{,}250\left(1+\frac{0.023}{365}\right)^{(365)} = 2{,}302.35$$

12b. $52.35. Subtract 2,250 from 2,302.35 to find the interest.

12c. 2.33%. Use the APY formula and substitute for the quarterly account.

$$\text{APY} = \left(1+\frac{r}{n}\right)^n - 1$$

$$\text{APY} = \left(1+\frac{0.023}{365}\right)^{365} - 1 = 0.0233$$

14a. II – future value of a periodic investment.

14b. I – future value of a single deposit investment.

14c. III – present value of a periodic deposit investment.

16. $12,321.40

$$B = \frac{150\left(\left(1+\frac{0.016}{12}\right)^{12(6.5)} - 1\right)}{\frac{0.016}{12}} \approx \$12{,}321.40$$

18a.

x	$f(x)$
10	$\dfrac{9(10)-1}{3(10)-5} = 3.56$
100	$\dfrac{9(100)-1}{3(100)-5} = 3.047$
1,000	$\dfrac{9(1{,}000)-1}{3(1{,}000)-5} = 3.005$
10,000	$\dfrac{9(10{,}000)-1}{3(10{,}000)-5} = 3.000$

The values in the table are approaching 3.

18b.

x	$f(x)$
10	$\frac{3(10)^2+9(10)}{4(10)+1}=9.512$
100	$\frac{3(100)^2+9(100)}{4(100)+1}=77.057$
1,000	$\frac{3(1{,}000)^2+9(1{,}000)}{4(1{,}000)+1}=752.062$
10,000	$\frac{3(10{,}000)^2+9(10{,}000)}{4(10{,}000)+1}=7{,}502.062$

The values in the table are not approaching a certain number, so the limit is undefined.

18c.

x	$f(x)$
10	$\frac{7(10)}{(10)^2-41}=1.186$
100	$\frac{7(100)}{(100)^2-41}=0.070$
1,000	$\frac{7(1{,}000)}{(1{,}000)^2-41}=0.007$
10,000	$\frac{7(10{,}000)}{(10{,}000)^2-41}=0.0007$

The values in the table are approaching 0.

20. Approximately 12.4 years

$$120{,}000=96{,}000\left(1+\frac{0.018}{12}\right)^{12t}$$

$$1.25=\left(1+\frac{0.018}{12}\right)^{12t}$$

$$1.25=1.0015^{12t}$$

Method 1

$$12t=\log_{1.0015}1.25$$

$$12t=\frac{\log 1.25}{\log 1.0015}$$

$$t=\frac{\frac{\log 1.25}{\log 1.0015}}{12}\approx 12.4$$

Method 2

$$\log(1.25)=\log(1.0015)^{12t}$$

$$\log(1.25)=12t\log(1.0015)$$

$$\frac{\log 1.25}{\log 1.0015}=12t$$

$$\frac{\frac{\log 1.25}{\log 1.0015}}{12}\approx 12.4=t$$

22. Approximately 2.11 years.

$$20{,}000=\frac{800\left(1-\left(1+\frac{0.0126}{12}\right)^{-12t}\right)}{\frac{0.0126}{12}}$$

$$21=800\left(1-\left(1+\frac{0.0126}{12}\right)^{-12t}\right)$$

$$0.02625=1-\left(1+\frac{0.0126}{12}\right)^{-12t}$$

$$-0.97375=-\left(1+\frac{0.0126}{12}\right)^{-12t}$$

$$0.97375=(1.00105)^{-12t}$$

$$\log(0.97375)=\log(1.00105)^{-12t}$$

$$\log(0.97375)=-12t\log(1.00105)$$

$$\frac{\log(0.97375)}{\log(1.00105)}=-12t$$

$$\frac{\frac{\log(0.97375)}{\log(1.00105)}}{-12}=t$$

$$2.11\approx t$$