Solutions for Finite Mathematics 7th Edition by Waner

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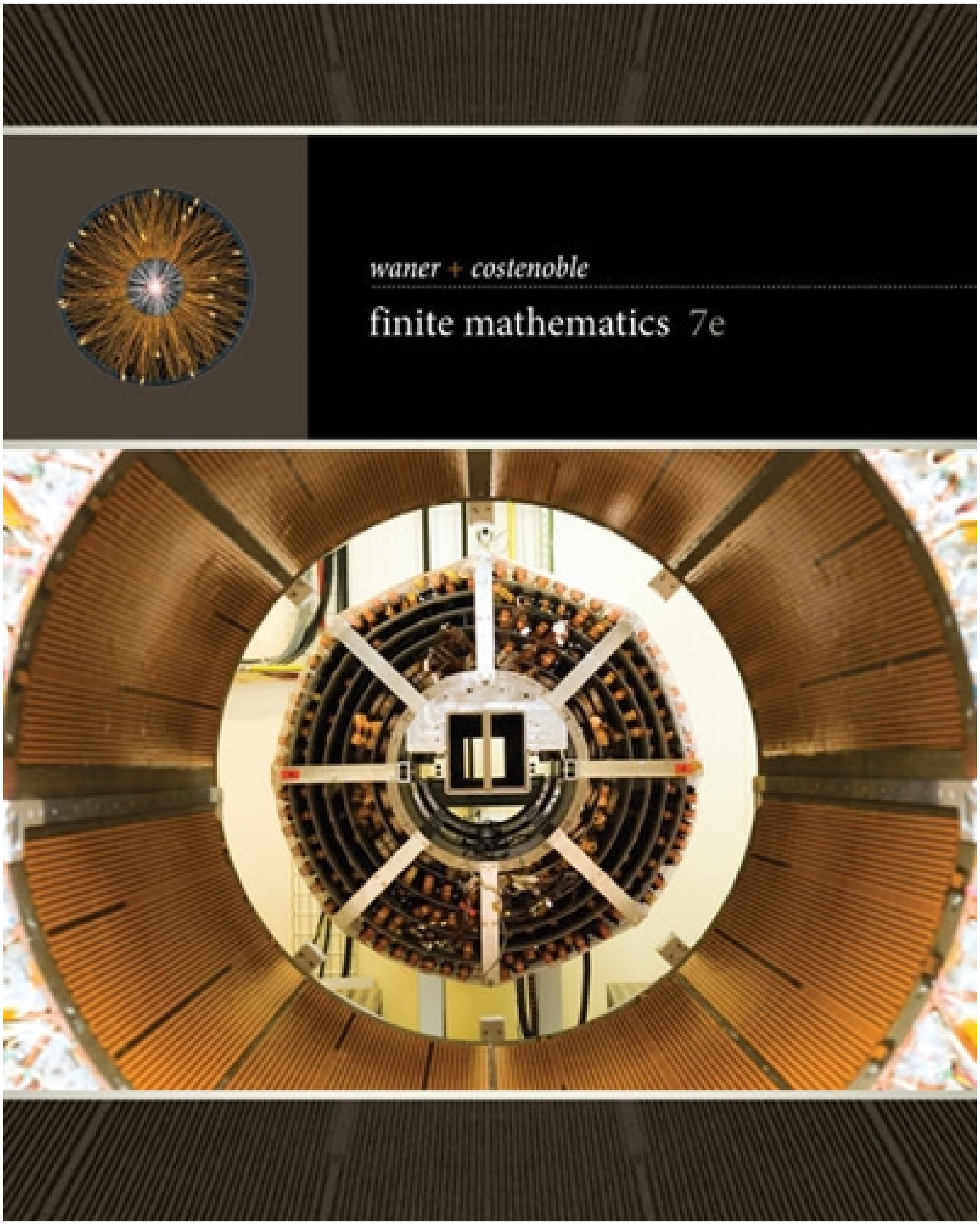


Solutions

Solutions Section 2.1

Section 2.1

1. $PV = 2{,}000,\ r = 0.06,\ t = 1$

$INT = PVrt = 2{,}000(0.06)(1) = \$120$

$FV = PV + INT = 2{,}000 + 120 = \$2{,}120$

2. $PV = 1{,}000,\ r = 0.04,\ t = 10$

$INT = PVrt = 1{,}000(0.04)(10) = \$400$

$FV = PV + INT = 1{,}000 + 400 = \$1{,}400$

3. $PV = 4{,}000,\ i = 0.005,\ n = 8$

$INT = PVin = 4{,}000(0.005)(8) = \$160$

$FV = PV + INT = 4{,}000 + 160 = \$4{,}160$

4. $PV = 2{,}000,\ i = 0.001,\ n = 12$

$INT = PVin = 2{,}000(0.001)(12) = \$24$

$FV = PV + INT = 2{,}000 + 24 = \$2{,}024$

5. $PV = 20{,}200,\ r = 0.05,\ t = \frac{1}{2}$

$INT = PVrt = 20{,}200(0.05)(\frac{1}{2}) = \$505$

$FV = PV + INT = 20{,}200 + 505 = \$20{,}705$

6. $PV = 10{,}100,\ r = 0.11,\ t = \frac{1}{4}$

$INT = PVrt = 10{,}100(0.11)(\frac{1}{4}) = \$277.75$

$FV = PV + INT = 10{,}100 + 277.75 = \$10{,}377.75$

7. $PV = 10{,}000,\ r = 0.03,\ t = 10/12$

$INT = PVrt = 10{,}000(0.03)(10/12) = \$250$

$FV = PV + INT = 10{,}000 + 250 = \$10{,}250$

8. $PV = 6{,}000,\ r = 0.09,\ t = 5/12$

$INT = PVrt = 6{,}000(0.09)(5/12) = \$225$

$FV = PV + INT = 6{,}000 + 225 = \$6{,}225$

9. $PV = 12{,}000,\ i = 0.0005,\ n = 10$

$INT = PVin = 12{,}000(0.0005)(10) = \$60$

$FV = PV + INT = 12{,}000 + 60 = \$12{,}060$

10. $PV = 8{,}000,\ i = 0.0003,\ n = 5$

$INT = PVin = 8{,}000(0.0003)(5) = \$12$

$FV = PV + INT = 8{,}000 + 12 = \$8{,}012$

11. $PV = FV/(1 + rt)$

$= 10{,}000/(1 + 0.02 \times 5) = \$9{,}090.91$

12. $PV = FV/(1 + rt)$

$= 20{,}000/(1 + 0.05 \times 2) = \$18{,}181.82$

13. $PV = FV/(1 + rt)$

$= 1{,}000/(1 + 0.07 \times 0.5) = \$966.18$

14. $PV = FV/(1 + in)$

$= 5{,}000/(1 + 0.01 \times 3) = \$4{,}854.37$

15. $PV = FV/(1 + in)$

$= 15{,}000/(1 + 0.0003 \times 15) = \$14{,}932.80$

16. $PV = FV/(1 + rt)$

$= 30{,}000/(1 + 0.06 \times 20/12) = \$27{,}272.73$

Applications

17. $FV = PV(1 + rt)$

$= 5{,}000(1 + 0.08 \times 0.5) = \$5{,}200$

18. $FV = PV(1 + in)$

$= 10{,}000(1 + 0.01 \times 15) = \$11{,}500$

19. $PV = FV/(1 + in)$

$= 1{,}000/(1 + 0.0002 \times 12) = \$997.61$

20. $PV = FV/(1 + in)$

$= 30{,}360/(1 + 0.095 \times 4) = \$22{,}000$

Solutions Section 2.1

21. We are given the interest and asked to compute r.

$INT = 250,\ PV = 1{,}000,\ t = 5$

$INT = PVrt$

$250 = 1{,}000 \times r \times 5 = 5{,}000r$

$r = \dfrac{250}{5{,}000} = 0.05,\ \text{or } 5\%$

22. We are given the interest and asked to compute r.

$INT = 2{,}800,\ PV = 10{,}000,\ t = 4$

$INT = PVrt$

$2{,}800 = 10{,}000 \times r \times 4 = 40{,}000r$

$r = \dfrac{2{,}800}{40{,}000} = 0.07,\ \text{or } 7\%$

23. Interest every six months: $INT = PVrt = (10{,}000)(0.034)(0.5) = \$170$

Total interest over the 10-year life of the bond: $INT = PVrt = (10{,}000)(0.034)(10) = \$3{,}400$

24. Interest every six months: $INT = PVrt = (5{,}000)(0.04)(0.5) = \$100$

Total interest over the 10-year life of the bond: $INT = PVrt = (5{,}000)(0.04)(10) = \$2{,}000$

25. $PV = \dfrac{FV}{1 + rt} = \dfrac{8{,}840}{1 + (.0525)(2)} = \$8{,}000$

26. $PV = \dfrac{FV}{1 + rt} = \dfrac{5{,}330.25}{1 + (.0615)(3)} = \$4{,}500$

27. Goldman Sachs: $INT = PVrt = (5{,}000)(0.0615)(3) = \$922.50$

Wells Fargo: $INT = PVrt = (5{,}000)(0.035)(7) = \$1{,}225.00$

Wells Fargo would pay the most total interest, \$1,225.00.

28. Bank of America: $INT = PVrt = (2{,}000)(0.04)(10) = \$800$

Verizon: $INT = PVrt = (2{,}000)(0.0515)(8) = \$824$

Verizon would pay the most total interest, \$824.

29. We are given present and future values, and want to compute t.

$FV = 4{,}640,\ PV = 4{,}000,\ r = 0.08$

$FV = PV(1 + rt)$

$4{,}640 = 4{,}000(1 + 0.08t)$

$1 + 0.08t = \dfrac{4{,}640}{4{,}000} = 1.160$

$0.08t = 0.160$

$t = \dfrac{0.160}{0.08} = 2 \text{ years}$

30. We are given PV and INT, and want to compute t.

$PV = 1{,}000,\ INT = 640,\ r = 0.08$

$INT = PVrt$

$640 = 1{,}000 \times 0.08t = 80t$

$t = \dfrac{640}{80} = 8 \text{ years}$

Solutions Section 2.1

31. $INT = PVin \Rightarrow 50 = 1{,}000 \times 4i = 4{,}000i$; $i = 50/4{,}000 = 0.0125$, which is 1.25% weekly interest.

Annual rate = $52 \times 0.0125 = 0.65$, or 65%

32. $INT = PVin \Rightarrow 60 = 1{,}500 \times 3i = 4{,}500i$; $i = 60/4{,}500 = 0.0133$, which is 1.33% weekly interest.

Annual rate = $52 \times 60/4{,}500 \approx 0.6933$, or 69.33%

33. You will pay $5{,}000 \times 0.09 \times 2 = \$900$ in interest on the loan. Adding the \$100 fee, you pay the bank a total of \$1,000 over the two years. To find the effective rate, we solve $1{,}000 = 5{,}000 \times 2r = 10{,}000r$; $r = 1{,}000/10{,}000 = 0.1$, so the rate is 10%.

34. You will pay $7{,}000 \times 0.08 \times 3 = \$1{,}680$ in interest on the loan. Adding the \$100 fee, you pay the bank a total of \$1,780 over the two years. To find the effective rate, we solve $1{,}780 = 7{,}000 \times 3r = 21{,}000r$;

$r = 1{,}780/21{,}000 \approx 0.08476$, so the rate is 8.476%.

35. $5 = 69r/12$; $r = 12 \times 5/69 \approx 0.86957$ or 86.957%

36. $10 = 99r/6$; $r = 6 \times 10/99 \approx 0.60606$ or 60.606%

37. Use $FV = PV(1 + in)$ with

$PV = 255.96$, $FV = 317.44$, $b = 6$ (months)

$317.44 = 255.96(1 + 6i) = 255.96 + 1535.76i$

$i = (317.44 - 255.96)/1535.76 \approx 0.0400$, or 4.00%

38. Use $FV = PV(1 + in)$ with

$PV = 195.46$, $FV = 255.96$, $n = 4$ (months)

$255.96 = 195.46(1 + 4i) = 195.46 + 781.84i$

$i = (255.96 - 195.46)/781.84 \approx 0.0774$, or 7.74%

39. Selling in May would have given you a small loss while selling in any later month of the year would have gotten you a gain. Calculating the monthly returns as in the preceding exercises, we get the following figures:

Jun	Jul	Aug	Sep	Oct	Nov	Dec
4.24%	1.55%	2.56%	1.93%	4.10%	4.91%	4.32%

The largest monthly return would have been 4.91% if you had sold in November 2010.

40. Calculating the monthly returns as in the preceding exercises, we get the following figures:

Jun	Jul	Aug	Sep	Oct	Nov	Dec
8.52%	2.35%	3.42%	2.43%	4.94%	5.74%	4.94%

The largest monthly return would have been 8.52% if you had sold in June 2010.

41. No. Simple interest increase is linear. The graph is visibly not linear in that time period. Further, the slopes of the lines through the successive pairs of marked points are quite different.

42. First calculate the monthly interest rate from March to April: $i = (235.97 - 218.95)/218.95 \approx 0.077735$. Now use the simple interest formula to get the price in June from that in April:

$FV = 235.97(1 + 0.077735 \times 2) = \$272.66.$

Note that if you instead calculate the price in June from that in March, you will find

$FV = 218.95(1 + 0.077735 \times 3) = \$270.01.$

(The discrepancy is due to rounding of prices to the nearest cent.

43. 1950 population: $PV = 500{,}000$; 2000 population: $FV = 2{,}800{,}000$

$INT = FV - PV = 2{,}800{,}000 - 500{,}000 = 2{,}300{,}000$

$INT = PVrt$

$2{,}300{,}000 = 500{,}000r(50) = 25{,}000{,}000r$

$r = \dfrac{2{,}300{,}000}{25{,}000{,}000} \approx 0.092$ or 9.2%

44. 1950 population: $PV = 500{,}000$; 1990 population: $FV = 2{,}500{,}000$

$INT = FV - PV = 2{,}500{,}000 - 500{,}000 = 2{,}000{,}000$

$INT = PVrt$

$2{,}000{,}000 = 500{,}000r(40) = 20{,}000{,}000r$

$r = \dfrac{2{,}000{,}000}{20{,}000{,}000} \approx 0.10$ or 10%

45. We are given: $PV =$ 1950 population $= 500{,}000$; $r = 0.092$ (from Exercise 43); $t = 60$ (years to 2010)

$FV = PV(1 + rt) = 500{,}000(1 + 0.092 \times 60) = 3{,}260{,}000$

46. We are given: $PV =$ 1950 population $= 500{,}000$; $r = 0.10$ (from Exercise 44); $t = 60$ (years to 2010)

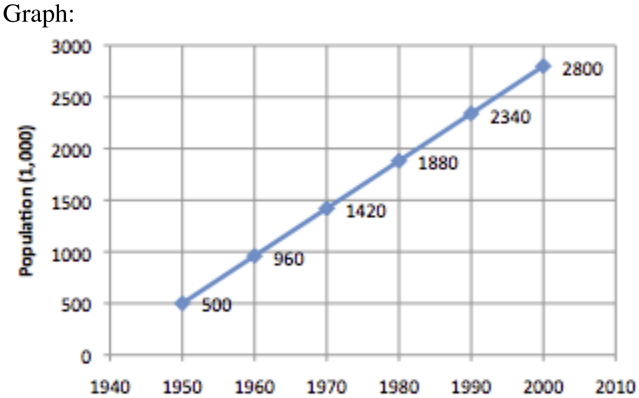
$FV = PV(1 + rt) = 500{,}000(1 + 0.10 \times 60) = 3{,}500{,}000$

47. $PV =$ 1950 population $= 500{,}000$; $r = 0.092$ (from Exercise 43). After t years, the population will be

$FV = PV(1 + rt) = 500{,}000(1 + 0.092t) = 500{,}000 + 46{,}000t$ or $500 + 46t$ thousand

(t = time in years since 1950).

Graph:

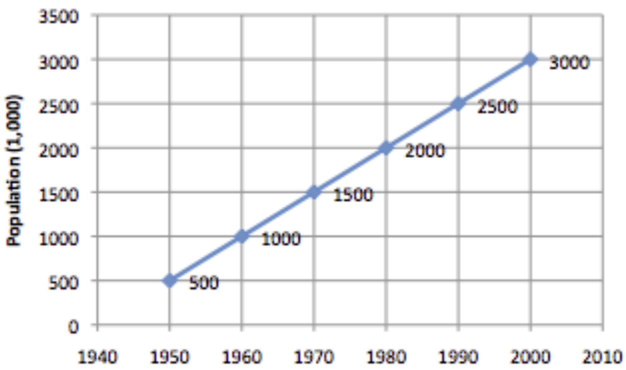


48. $PV =$ 1950 population $= 500{,}000$; $r = 0.10$ (from Exercise 44). After t years, the population will be

$FV = PV(1 + rt) = 500{,}000(1 + 0.10t) = 500{,}000 + 50{,}000t$ or $500 + 50t$ thousand

(t = time in years since 1950).

Graph:



49. Actual discount $= 0.0025/2 = 0.00125$. Selling price $= 5{,}000 - (0.00125)(5{,}000) = \$4{,}993.75$.

$PV = \$4{,}993.75,\ FV = \$5{,}000,\ t = 0.5$

$FV = PV(1 + rt)$

$5{,}000 = 4{,}993.75(1 + 0.5r)$

$1 + 0.5r = 5{,}000/4{,}993.75$

$0.5r = 5{,}000/4{,}993.75 - 1$

$r = 2(5{,}000/4{,}993.75 - 1) \approx 0.002503,$ or 0.2503%

50. Actual discount $= 0.0006/4 = 0.00015$. Selling price $= 15{,}000 - (0.00015)(15{,}000) = \$14{,}997.75$.

$PV = \$14{,}997.75,\ FV = \$15{,}000,\ t = 0.25$

$FV = PV(1 + rt)$

$15{,}000 = 14{,}997.75(1 + 0.25r)$

$1 + 0.25r = 15{,}000/14{,}997.75$

$0.25r = 15{,}000/14{,}997.75 - 1$

$r = 4(15{,}000/14{,}997.75 - 1) \approx 0.00060009,$ or 0.060009%

51. To say that the discount rate is 3.705% means that its selling price (PV) is 3.705% lower than its maturity value (FV). To simplify the calculation, let us use a T-bill with a maturity value of \$10,000:

$PV = 10{,}000 - 10{,}000(0.03705/2) = 9814.75$

$10{,}000 = 9{,}814.75(1 + 0.5r) = 9{,}814.75 + 4{,}907.375r$

$r = (10{,}000 - 9{,}814.75)/4{,}907.375 = 0.03775,$ or 3.775%

52. To say that the discount rate is 3.470% means that its selling price (PV) is 3.470% lower than its maturity value (FV). To simplify the calculation, let us use a T-bill with a maturity value of \$10,000:

$PV = 10{,}000 - 10{,}000(0.03470/4) = 9{,}913.25$

$10{,}000 = 9{,}913.25(1 + 0.25r) = 9{,}913.25 + 2{,}478.3125r$

$r = (10{,}000 - 9{,}913.25)/2{,}478.3125 = 0.03500,$ or 3.500%

Communication and reasoning exercises

53. Graph (A) is the only possible choice, because the equation $FV = PV(1 + rt) = PV + PVrt$ gives the future value as a linear function of time.

54. $FV = PV(1 + in) = PV + (PVi)n$, which is a linear equation with slope PVi. Thus, the slope is

Slope = Interest rate × Present value.

55. $FV = PV(1 + in) = PV + PVin$. Since $FV = 1{,}000 + 0.5n$, we have

$PV = 1{,}000$

$PVin = 1{,}000in = 0.5n$

so

$$i = \frac{0.5}{1{,}000} = 0.0005, \text{ or } 0.05\%$$

56. $FV = PV(1 + rt) = PV + PVrt$. Since $FV = 400 + 5t$, we have

$PV = 400$

$PVrt = 400rt = 5t$

so

$$r = \frac{5}{400} = 0.0125, \text{ or } 1.25\%$$

57. $FV = PV(1 + rt)$;

$r =$ Annual rate = 12 × Monthly rate = $12i$;

$t =$ Number of years $= \frac{n}{12}$

so $$FV = PV(1 + rt) = PV\left(1 + (12i)\frac{n}{12}\right) = PV(1 + in)$$

58. $FV = PV(1 + in)$;

$i =$ Monthly rate $= \frac{r}{12}$

$n =$ Number of months $= 12t$

so $$FV = PV(1 + in) = PV\left(1 + \frac{r}{12}(12t)\right) = PV(1 + rt)$$

59. Wrong. In simple interest growth, the change each year is a fixed percentage of the *starting* value, and not the preceding year's value. (Also see the next exercise.)

60. The economy last year was larger than the year before; 1% of last year's economy is larger than 1% of the year before's.

61. Simple interest is always calculated on a constant amount, PV. If interest is paid into your account, then the amount on which interest is calculated does not remain constant.

Solutions Section 2.1

62. To say that the discount rate is d means that its selling price (PV) is $(1-d)$ times its maturity value (FV) after one year, or

$$PV = (1-d)FV$$

One has, for a single year

$$FV = PV(1+r)$$

$$FV = (1-d)FV(1+r)$$

$$1 = (1-d)(1+r)$$

$$1 + r = \frac{1}{1-d}$$

giving

$$r = \frac{1}{1-d} - 1$$

Solutions Section 2.2

Section 2.2

1. We use $FV = PV(1+i)^n$.

$PV = \$10{,}000,\ i = 0.002,\ n = 15$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 10{,}000(1+0.002)^{15} \\ &= 10{,}000(1.002)^{15} \approx \$10{,}304.24 \end{aligned}$$

Technology: `10000*(1+0.002)^15`

2. We use $FV = PV(1+i)^n$.

$PV = \$10{,}000,\ i = 0.0005,\ n = 6$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 10{,}000(1+0.0005)^{6} \\ &= 10{,}000(1.0005)^{6} \approx \$10{,}030.04 \end{aligned}$$

Technology: `10000*(1+0.0005)^6`

3. We use $FV = PV(1+i)^n$.

$PV = \$10{,}000,\ i = 0.002,\ n = 10 \times 12 = 120$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 10{,}000(1+0.002)^{120} \\ &= 10{,}000(1.002)^{120} \approx \$12{,}709.44 \end{aligned}$$

Technology: `10000*(1+0.002)^120`

4. We use $FV = PV(1+i)^n$.

$PV = \$10{,}000,\ i = 0.0045,\ n = 20 \times 12 = 240$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 10{,}000(1+0.0045)^{240} \\ &= 10{,}000(1.0045)^{240} \approx \$29{,}375.54 \end{aligned}$$

Technology: `10000*(1+0.0045)^240`

5. We use $FV = PV(1+r/m)^{mt}$.

$PV = \$10{,}000,\ r = 0.03,\ m = 1,\ t = 10$

$$\begin{aligned} FV &= PV(1+r/m)^{mt} \\ &= 10{,}000(1+0.03)^{10} \\ &= 10{,}000(1.03)^{10} \approx \$13{,}439.16 \end{aligned}$$

Technology: `10000*(1+0.03)^10`

6. We use $FV = PV(1+r/m)^{mt}$.

$PV = \$10{,}000,\ r = 0.04,\ m = 1,\ t = 8$

$$\begin{aligned} FV &= PV(1+r/m)^{mt} \\ &= 10{,}000(1+0.04)^{8} \\ &= 10{,}000(1.04)^{8} \approx \$13{,}685.69 \end{aligned}$$

Technology: `10000*(1+0.04)^8`

7. We use $FV = PV(1+i)^n$.

$PV = \$10{,}000,\ i = 0.025/4 = 0.00625,$

$n = 4 \times 5 = 20$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 10{,}000(1+0.00625)^{20} \\ &= 10{,}000(1.00625)^{20} \approx \$11{,}327.08 \end{aligned}$$

Technology: `10000*(1+0.025/4)^(4*5)`

8. We use $FV = PV(1+i)^n$.

$PV = \$10{,}000,\ i = 0.015/52,\ n = 52 \times 5 = 260$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 10{,}000\left(1+\frac{0.015}{52}\right)^{260} \approx \$10{,}778.72 \end{aligned}$$

Technology: `10000*(1+0.015/52)^(260)`

9. We use $FV = PV(1+r/m)^{mt}$.

$PV = \$10{,}000,\ r = 0.065,\ m = 365,\ t = 10$

$$\begin{aligned} FV &= PV(1+r/m)^{mt} \\ &= 10{,}000\left(1+\frac{0.065}{365}\right)^{365\times 10} \\ &= 10{,}000\left(1+\frac{0.065}{365}\right)^{3{,}650} \approx \$19{,}154.30 \end{aligned}$$

Technology: `10000*(1+0.065/365)^(365*10)`

10. We use $FV = PV(1+r/m)^{mt}$.

$PV = \$10{,}000,\ r = 0.112,\ m = 12,\ t = 12$

$$\begin{aligned} FV &= PV(1+r/m)^{mt} \\ &= 10{,}000\left(1+\frac{0.112}{12}\right)^{12\times 12} \\ &= 10{,}000\left(1+\frac{0.112}{12}\right)^{144} \approx \$38{,}105.24 \end{aligned}$$

Technology: `10000*(1+0.112/12)^(12*12)`

Solutions Section 2.2

11. We use $PV = FV(1+i)^{-n}$

$FV = \$1{,}000,\ n = 10,\ i = 0.05$

$$\begin{aligned} FV &= FV(1+i)^{-n} \\ &= 1{,}000(1+0.05)^{-10} \\ &= 1{,}000(1.05)^{-10} \approx \$613.91 \end{aligned}$$

Technology: `1000*(1+0.05)^(-10)`

12. We use $PV = FV(1+i)^{-n}$

$FV = \$1{,}000,\ n = 5,\ i = 0.06$

$$\begin{aligned} PV &= FV(1+i)^{-n} \\ &= 1{,}000(1+0.06)^{-5} \\ &= 1{,}000(1.06)^{-5} \approx \$747.26 \end{aligned}$$

Technology: `1000*(1+0.06)^(-5)`

13. We use $PV = \dfrac{FV}{(1+r/m)^{mt}}$

$FV = \$1{,}000,\ t = 5,\ r = 0.042,\ m = 52$

$$\begin{aligned} PV &= \frac{FV}{(1+r/m)^{mt}} \\ &= \frac{1{,}000}{\left(1+\frac{0.042}{52}\right)^{52\times 5}} \\ &= \frac{1{,}000}{\left(1+\frac{0.042}{52}\right)^{260}} \approx \$810.65 \end{aligned}$$

Technology: `1000/(1+0.042/52)^(52*5)`

14. We use $PV = \dfrac{FV}{(1+r/m)^{mt}}$

$FV = \$1{,}000,\ t = 10,\ r = 0.053,\ m = 4$

$$\begin{aligned} PV &= \frac{FV}{(1+r/m)^{mt}} \\ &= \frac{1{,}000}{\left(1+\frac{0.053}{4}\right)^{4\times 10}} \\ &= \frac{1{,}000}{\left(1+\frac{0.053}{4}\right)^{40}} \approx \$590.66 \end{aligned}$$

Technology: `1000/(1+0.053/4)^(4*10)`

15. We use $PV = FV(1+i)^{-n}$

$FV = \$1{,}000,\ n = 4,\ i = -0.05$

$$\begin{aligned} PV &= FV(1+i)^{-n} \\ &= 1{,}000(1-0.05)^{-4} \\ &= 1{,}000(0.95^{-4}) \approx \$1{,}227.74 \end{aligned}$$

Technology: `1000*(1-0.05)^(-4)`

16. We use $PV = FV(1+i)^{-n}$

$FV = \$1{,}000,\ n = 5,\ i = -0.04$

$$\begin{aligned} PV &= FV(1+i)^{-n} \\ &= 1{,}000(1-0.04)^{-5} \\ &= 1{,}000(0.96^{-5}) \approx \$1{,}226.43 \end{aligned}$$

Technology: `1000*(1-0.04)^(-5)`

17. $r_{\text{nom}} = 0.05,\ m = 4$

$$\begin{aligned} r_{\text{eff}} &= (1+r_{\text{nom}}/m)^m - 1 \\ &= \left(1+\frac{0.05}{4}\right)^4 - 1 \\ &\approx 0.0509,\ \text{or } 5.09\% \end{aligned}$$

Technology: `(1+0.05/4)^4-1`

18. $r_{\text{nom}} = 0.05,\ m = 12$

$$\begin{aligned} r_{\text{eff}} &= (1+r_{\text{nom}}/m)^m - 1 \\ &= \left(1+\frac{0.05}{12}\right)^{12} - 1 \\ &\approx 0.0512,\ \text{or } 5.12\% \end{aligned}$$

Technology: `(1+0.05/12)^12-1`

19. $r_{\text{nom}} = 0.10,\ m = 12$

$$\begin{aligned} r_{\text{eff}} &= (1+r_{\text{nom}}/m)^m - 1 \\ &= \left(1+\frac{0.10}{12}\right)^{12} - 1 \\ &\approx 0.1047,\ \text{or } 10.47\% \end{aligned}$$

Technology: `(1+0.10/12)^12-1`

20. $r_{\text{nom}} = 0.10,\ m = 365$

$$\begin{aligned} r_{\text{eff}} &= (1+r_{\text{nom}}/m)^m - 1 \\ &= \left(1+\frac{0.10}{365}\right)^{365} - 1 \\ &\approx 0.1052,\ \text{or } 10.52\% \end{aligned}$$

Technology: `(1+0.10/365)^365-1`

21. $r_{\text{nom}} = 0.10,\ m = 365 \times 24 = 8{,}760$

$$r_{\text{eff}} = (1 + r_{\text{nom}}/m)^m - 1 = \left(1 + \frac{0.10}{8{,}760}\right)^{8{,}760} - 1 \approx 0.1052, \text{ or } 10.52\%$$

Technology: `(1+0.10/8760)^8760-1`

22. $r_{\text{nom}} = 0.10,\ m = 365 \times 24 \times 60 = 525{,}600$

$$r_{\text{eff}} = (1 + r_{\text{nom}}/m)^m - 1 = \left(1 + \frac{0.10}{525{,}600}\right)^{525{,}600} - 1 \approx 0.1052, \text{ or } 10.52\%$$

Technology: `(1+0.10/525600)^525600-1`

Applications

23. $PV = \$1{,}000,\ i = 0.06/4 = 0.015\ n = 4 \times 4 = 16$

$$FV = PV(1+i)^n = 1{,}000(1+0.015)^{16} = 1{,}000(1.015)^{16} \approx \$1{,}268.99$$

The deposit will have grown by
$\$1{,}268.99 - \$1{,}000 = \$268.99.$

Technology: `1000*(1+0.06/4)^(16)`

24. $PV = \$10{,}000,\ i = 0.02/6 = 0.005,\ n = 4 \times 5 = 20$

$$FV = PV(1+i)^n = 1{,}000(1+0.005)^{20} = 1{,}000(1.005)^{20} \approx \$11{,}048.96$$

The deposit will have grown by
$\$11{,}048.96 - \$10{,}000 = \$1{,}048.96$

Technology: `10000*(1+0.02/4)^(20)`

25. $PV = \$3{,}000,\ i = -0.376,\ n = 3$

$$FV = PV(1+i)^n = 3{,}000(1-0.376)^3 \approx \$728.91$$

Technology: `3000*(1-0.376)^3`

26. $PV = \$5{,}000,\ i = -0.42,\ n = 4$

$$FV = PV(1+i)^n = 5{,}000(1-0.42)^4 \approx \$565.82$$

Technology: `5000*(1-0.42)^4`

27. $FV = \$5{,}000,\ i = 0.055,\ n = 10$

$$PV = \frac{FV}{(1+i)^n} = \frac{5{,}000}{(1+0.055)^{10}} = \frac{5{,}000}{1.055^{10}} \approx \$2{,}927.15$$

Technology: `5000/(1+0.055)^10`

28. $FV = \$10{,}000,\ i = 0.0625,\ b = 15$

$$PV = \frac{FV}{(1+i)^n} = \frac{10{,}000}{(1+0.0625)^{15}} = \frac{10{,}000}{1.0625^{15}} \approx \$4{,}027.78$$

Technology: `10000/(1+0.0625)^15`

29. Gold: $PV = \$5{,}000,\ i = 0.10,\ n = 10$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 5{,}000(1+0.10)^{10} \\ &= 5{,}000(1.10)^{10} \approx \$12{,}968.71 \end{aligned}$$

CDs: $PV = \$5{,}000,\ i = 0.05/2 = 0.025,$
$n = 2 \times 10 = 20$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 5{,}000(1+0.025)^{20} \\ &= 5{,}000(1.025)^{20} \approx \$8{,}193.08 \end{aligned}$$

Combined value after 10 years
$= \$12{,}968.71 + \$8{,}193.08 = \$21{,}161.79$

Technology:
```
5000*(1+0.10)^10+5000*
(1+0.05/2)^(2*10)
```

30. Munis: $PV = \$5{,}000,\ i = 0.06,\ n = 10$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 5{,}000(1+0.06)^{10} \\ &= 5{,}000(1.06)^{10} \approx \$8{,}954.24 \end{aligned}$$

CDs: $PV = \$5{,}000,\ i = 0.03/6 = 0.005,\ n = 6 \times 10 = 60$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 5{,}000(1+0.005)^{60} \\ &= 5{,}000(1.005)^{60} \approx \$6{,}774.24 \end{aligned}$$

Combined value after 10 years
$= \$8{,}954.24 + \$6{,}774.24 = \$15{,}698.49$

Technology:
```
5000*(1+0.06)^10+5000*
(1+0.03/6)^(6*10)
```

31. $PV = \$200{,}000,\ i = -0.02,\ n = 10$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 200{,}000(1-0.02)^{10} \\ &= 200{,}000(0.98)^{10} \approx \$163{,}414.56 \end{aligned}$$

Technology: `200000*(1-0.02)^10`

32. $PV = \$40{,}000,\ i = -0.051,\ n = 12$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 40{,}000(1-0.051)^{12} \\ &= 40{,}000(0.949)^{12} \approx \$21{,}342.95 \end{aligned}$$

Technology: `40000*(1-0.051)^12`

33. $FV = \$1{,}000{,}000,\ i = 0.06,\ n = 30$

$$\begin{aligned} PV &= \frac{FV}{(1+i)^n} \\ &= \frac{1{,}000{,}000}{(1+0.06)^{30}} \\ &= \frac{1{,}000{,}000}{1.06^{30}} \approx \$174{,}110 \end{aligned}$$

Technology: `1000000/(1+0.06)^30`

34. $FV = \$1{,}000{,}000,\ i = 0.07,\ n = 40$

$$\begin{aligned} PV &= \frac{FV}{(1+r/m)^{mt}} \\ &= \frac{1{,}000{,}000}{(1+0.07)^{40}} \\ &= \frac{1{,}000{,}000}{1.07^{40}} \approx \$66{,}780 \end{aligned}$$

Technology: `1000000/(1+0.07)^40`

35. $FV = \$297.91,\ n = 6 \times 3 = 18,\ i = -0.05$

$$\begin{aligned} PV &= \frac{FV}{(1+i)^n} \\ &= \frac{297.91}{(1-0.05)^{18}} = \frac{297.91}{0.95^{18}} \approx \$750.00 \end{aligned}$$

Technology: `297.91/(1-0.05)^18`

36. $PV = 100,\ i = 0.06,\ n = 2 \times 5 = 10$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 100(1+0.06)^{10} \\ &= 100(1.06)^{10} \approx 179 \text{ kits per month} \end{aligned}$$

Technology: `100*(1+0.06)^10`

37. $FV = PV(1+i)^n = 8{,}144.64(1+0.0512)^2 = \$9{,}000$

38. $FV = PV(1+i)^n = 8{,}441.50(1+0.0581)^3 = \$10{,}000$

Solutions Section 2.2

39. $PV = \dfrac{FV}{(1+i)^n}$

$$= \frac{10{,}000}{(1+0.0297)^{10}} = \frac{10{,}000}{1.0297^{10}} \approx \$7{,}462.65$$

40. $PV = \dfrac{FV}{(1+i)^n}$

$$= \frac{5{,}000}{(1+0.0342)^{10}} = \frac{5{,}000}{1.0342^{10}} \approx \$3{,}572.11$$

41. Total interest on \$1 during the 8-year life of the bonds is

$$INT = FV - PV = (1+0.0441)^8 - 1 \approx \$0.41233$$

Thus, the monthly simple interest rate would be $\dfrac{0.41233}{8 \times 12} \approx 0.0043$, or 0.43%.

42. Total interest on \$1 during the 7-year life of the bonds is

$$INT = FV - PV = (1+0.0318)^7 - 1 \approx \$0.24500$$

Thus, the biannual simple interest rate would be $\dfrac{0.24500}{7 \times 2} = 0.0175$, or 1.75%.

43. $FV = \$100{,}000,\ i = 0.04,\ n = 15$

$$PV = \frac{FV}{(1+i)^n} = \frac{100{,}000}{(1+0.04)^{15}} = \frac{100{,}000}{1.04^{15}} \approx \$55{,}526.45 \text{ per year}$$

Technology: `100000/(1+0.04)^15`

44. $FV = \$80{,}000,\ i = 0.05,\ n = 10$

$$PV = \frac{FV}{(1+i)^n} = \frac{80{,}000}{\left(1+\frac{0.05}{1}\right)^{10}} = \frac{80{,}000}{1.05^{10}} \approx \$49{,}113.06 \text{ per year}$$

Technology: `80000/(1+0.05)^10`

45. $FV = \$30{,}000,\ n = 5,\ i = 0.02$

$$PV = \frac{FV}{(1+i)^n} = \frac{30{,}000}{(1+0.02)^5} = \frac{30{,}000}{1.02^5} \approx \$27{,}171.92$$

Technology: `30000/(1+0.02)^5`

46. $FV = \$30{,}000,\ n = 5 \times 2 = 10,\ i = 0.01$

$$PV = \frac{FV}{(1+i)^n} = \frac{30{,}000}{(1+0.01)^{10}} = \frac{30{,}000}{1.01^{10}} \approx \$27{,}158.61$$

Technology: `30000/(1+0.01)^10`

47. $FV = \$200{,}000,\ n = 10,\ i = 0.06$

$$PV = \frac{FV}{(1+i)^n} = \frac{200{,}000}{(1+0.06)^{10}} = \frac{200{,}000}{1.06^{10}} \approx \$111{,}678.96$$

Technology: `200000/(1+0.06)^10`

48. $FV = \$200{,}000,\ n = 10 \times 12 = 120,\ i = 0.005$

$$PV = \frac{FV}{(1+i)^n} = \frac{200{,}000}{(1+0.005)^{120}} = \frac{200{,}000}{1.005^{120}} \approx \$109{,}926.55$$

Technology: `200000/(1+0.005)^120`

Solutions Section 2.2

49. Step 1: Calculate the future value of the investment:

$PV = \$1{,}000,\ i = 0.05,\ n = 2$

$$FV = PV(1+i)^n = 1{,}000(1+0.05)^2 = 1{,}000(1.05)^2 \approx \$1{,}102.50$$

Technology: `1000*(1+0.05)^2`

Step 2: Discount this value using inflation:

$FV = \$1{,}102.50,\ i = 0.03,\ n = 2$

$$PV = \frac{FV}{(1+i)^n} = \frac{1{,}102.50}{(1+0.03)^2} = \frac{1{,}102.50}{1.03^2} \approx \$1{,}039.21$$

Technology: `1102.50/(1+0.03)^2`

50. Step 1: Calculate the future value of the investment:

$PV = \$10{,}000,\ r = 0.08,\ m = 12,\ t = 2$

$$FV = PV(1+r/m)^{mt} = 10{,}000\left(1+\frac{0.08}{12}\right)^{12\times 2} \approx \$11{,}728.88$$

Technology: `10000*(1+0.08/12)^24`

Step 2: Discount this value using inflation:

$FV = \$11{,}728.88,\ i = 0.01,\ n = 24$

$$PV = \frac{FV}{(1+i)^n} = \frac{11{,}728.88}{(1+0.01)^{24}} = \frac{11{,}728.88}{1.01^{24}} \approx \$9{,}237.27$$

Technology: `11728.88/(1+0.01)^24`

51. Compare the effective yields of the two investments:

First Investment:

$r_{\text{nom}} = 0.12,\ m = 1$

$r_{\text{eff}} = r_{\text{nom}} = 0.12,\ \text{or } 12\%$

Second Investment:

$r_{\text{nom}} = 0.119,\ m = 12$

$$r_{\text{eff}} = (1 + r_{\text{nom}}/m)^m - 1 = \left(1+\frac{0.119}{12}\right)^{12} - 1 \approx 0.1257,\ \text{or } 12.57\%$$

The better investment is the second.

52. Compare the effective yields of the three investments:

First Investment:

$r_{\text{nom}} = 0.15,\ m = 1$

$r_{\text{eff}} = r_{\text{nom}} = 0.15,\ \text{or } 15\%$

Second Investment:

$r_{\text{nom}} = 0.145,\ m = 4$

$$r_{\text{eff}} = (1 + r_{\text{nom}}/m)^m - 1 = \left(1+\frac{0.145}{4}\right)^{4} - 1 \approx 0.1531,\ \text{or } 15.31\%$$

Technology: `(1+0.145/4)^4-1`

Third Investment:

$r_{\text{nom}} = 0.14,\ m = 12$

$$r_{\text{eff}} = (1 + r_{\text{nom}}/m)^m - 1 = \left(1+\frac{0.14}{12}\right)^{12} - 1 \approx 0.1493,\ \text{or } 14.93\%$$

Technology: `(1+0.14/12)^12-1`

The best investment is the second: 14.5% compounded quarterly.

53. $PV = \$24,\ i = 0.063,\ n = 2015 - 1626 = 389$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 24(1+0.063)^{389} \\ &= 24(1.063)^{389} \approx \$503{,}096 \text{ million} \end{aligned}$$

This is more than the 2015 estimated market value of \$362,524 million. Thus, the Lenape could have bought the island back in 2015.
Technology: `24*(1+0.063)^389`

54. $PV = \$24,\ i = 0.062,\ n = 2015 - 1626 = 389$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 24(1+0.062)^{389} \\ &= 24(1.062)^{389} \approx \$348{,}858 \text{ million} \end{aligned}$$

This is less than the 2015 estimated market value of \$362,524 million. Thus, the Lenape could not have bought the island back in 2015.
Technology: `24*(1+0.062)^389`

55. $PV = 100$ reals, $i = 0.099,\ n = 5$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 100(1+0.099)^5 \\ &= 100(1.099)^5 \approx 160 \text{ reals} \end{aligned}$$

Technology: `100*(1+0.099)^5`

56. $PV = 1{,}000$ pesos, $i = 0.143,\ n = 5$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 1{,}000(1+0.143)^5 \\ &= 1{,}000(1.143)^5 \approx 1{,}951 \text{ pesos} \end{aligned}$$

Technology: `1000*(1+0.143)^5`

57. $FV = 1{,}000$ bolivianos, $i = 0.043,\ n = 10$

$$\begin{aligned} PV &= \frac{FV}{(1+i)^n} \\ &= \frac{1{,}000}{(1+0.043)^{10}} = \frac{1{,}000}{1.043^{10}} \\ &\approx 656 \text{ bolivianos} \end{aligned}$$

Technology: `1000/(1+0.043)^10`

58. $FV = 20{,}000$ pesos, $i = 0.025,\ n = 10$

$$\begin{aligned} PV &= \frac{FV}{(1+i)^n} \\ &= \frac{20{,}000}{(1+0.025)^{10}} = \frac{20{,}000}{1.025^{10}} \\ &\approx 15{,}624 \text{ pesos} \end{aligned}$$

Technology: `20000/(1+0.025)^10`

59. Step 1: Calculate the future value of the investment:

$PV = 1{,}000$ bolivars, $r = 0.08,\ m = 2,\ t = 10$

$$\begin{aligned} FV &= PV(1+r/m)^{mt} \\ &= 1{,}000\left(1+\frac{0.08}{2}\right)^{2\times 10} \\ &= 1{,}000(1.04)^{20} \approx 2{,}191.12 \text{ bolivars} \end{aligned}$$

Technology: `1000*(1+0.08/2)^20`

Step 2: Discount this amount using the inflation rate:

$FV = 2{,}191.12,\ i = 0.685,\ n = 10$

$$\begin{aligned} PV &= \frac{FV}{(1+i)^n} \\ &= \frac{2{,}191.12}{(1+0.685)^{10}} = \frac{2{,}191.12}{1.685^{10}} \approx 12 \text{ bolivars} \end{aligned}$$

Technology: `2191.12/(1+0.685)^10`

60. Step 1: Calculate the future value of the investment:

$PV = 1{,}000$ pesos, $r = 0.08,\ m = 2,\ t = 10$

$$\begin{aligned} FV &= PV(1+r/m)^{mt} \\ &= 1{,}000\left(1+\frac{0.08}{2}\right)^{2\times 10} \\ &= 1{,}000(1.04)^{20} \approx 2{,}191.12 \text{ pesos} \end{aligned}$$

Technology: `1000*(1+0.08/2)^20`

Step 2: Discount this amount using the inflation rate:

$FV = 2{,}191.12,\ i = 0.092,\ n = 10$

$$\begin{aligned} PV &= \frac{FV}{(1+i)^n} \\ &= \frac{2{,}191.12}{(1+0.092)^{10}} = \frac{2{,}191.12}{1.092^{10}} \approx 909 \text{ pesos} \end{aligned}$$

Technology: `2191.12/(1+0.092)^10`

Note: We get the same answer if we round the answer of Step 1 to the nearest peso.

Solutions Section 2.2

61. We compare the future values of 1 unit of the currency for a 1-year period:

Mexico: $PV = 1$ peso, $i = 0.053$, $n = 1$

$$FV = PV(1+i)^n = 1(1+0.053)^1 = 1.053 \text{ pesos}$$

Now discount this using inflation:

$FV = 1.053$, $i = 0.025$, $n = 1$

$$PV = \frac{FV}{(1+i)^n} = \frac{1.053}{(1+0.025)^1} = \frac{1.053}{1.025} \approx 1.027 \text{ pesos}$$

Nicaragua: $PV = 1$ gold cordoba,

$i = 0.06/2 = 0.03$, $n = 2$

$$FV = PV(1+i)^n = 1(1+0.03)^2 = 1.03^2 = 1.0609 \text{ gold cordobas}$$

Now discount this using inflation:

$FV = 1.0609$, $i = 0.03$, $n = 1$

$$PV = \frac{FV}{(1+i)^n} = \frac{1.0609}{(1+0.03)^1} = \frac{1.0609}{1.03} = 1.03 \text{ gold cordobas}$$

The investment in Nicaragua is better.

62. We compare the future values of 1 unit of the currency for a 1-year period:

Brazil: $PV = 1$ real, $r = 0.10$, $m = 1$, $t = 1$

$$FV = PV(1+r/m)^{mt} = 1\left(1+\frac{0.10}{1}\right)^{1\times 1} = 1.10 \text{ reals}$$

Now discount this using inflation:

$FV = 1.10$, $i = 0.099$, $n = 1$

$$PV = \frac{FV}{(1+i)^n} = \frac{1.10}{(1+0.099)^1} = \frac{1.10}{1.099} \approx 1.0009 \text{ reals}$$

Uruguay: $PV = 1$ peso, $r = 0.09$, $m = 2$, $t = 1$

$$FV = PV(1+r/m)^{mt} = 1\left(1+\frac{0.09}{2}\right)^{2\times 1} = 1.045^2 \approx 1.092025 \text{ pesos}$$

Now discount this using inflation:

$FV = 1.092025$, $i = 0.092$, $n = 1$

$$PV = \frac{FV}{(1+i)^n} \approx \frac{1.09203}{(1+0.092)^1} = \frac{1.09203}{1.092} \approx 1.0000 \text{ pesos}$$

The investment in Brazil is better.

63. $PV = 255.96$, $FV = 317.44$, $t = 6$ months $= 1/2$ year

$317.44 = 255.96(1+r)^{1/2}$, so solving for r gives $r = (317.44/255.96)^2 - 1 \approx 0.5381$ or 53.81%

64. $PV = 195.46$, $FV = 255.96$, $t = 4$ months $= 1/3$ year

$255.96 = 195.46(1+r)^{1/3}$

$r = (255.96/195.46)^3 - 1 \approx 1.2457$ or 124.57%

65. Selling in May would have given you a small loss while selling in any later month of the year would have gotten you a gain. Calculating the annual returns as in the preceding exercises, we get the following figures:

Jun	Jul	Aug	Sep	Oct	Nov	Dec
62.89%	19.93%	33.91%	24.78%	55.31%	65.99%	56.03%

The largest annual return would have been 65.99% if you had sold in November 2010.

66. Calculating the annual returns as in the preceding exercises, we get the following figures:

Jun	Jul	Aug	Sep	Oct	Nov	Dec
166.82%	31.71%	47.87%	32.06%	69.79%	80.79%	66.40%

The largest annual return would have been 166.82% if you had sold in June 2010.

67. No. Compound interest increase is exponential, and exponential curves either increase continually (in the case of appreciation) or decrease continually (in the case of depreciation). The graph of the stock price has both increases and decreases during the given period, so the curve cannot model compound interest change.

68. First calculate the monthly interest rate from March to April: $i = 235.97/218.95 - 1 \approx 0.07773$. Now use the compound interest formula to get the price in June: $FV \approx 235.97(1 + 0.07773)^2 \approx \$274.08$.

69. My investment: $PV = \$5{,}000,\ r = 0.054,\ m = 2$

$$FV = PV(1 + r/m)^{mt} = 5{,}000\left(1 + \frac{0.054}{2}\right)^{2t} = 5{,}000(1.027)^{2t}$$

Friend's investment: $PV = \$6{,}000,\ r = 0.048,\ m = 2$

$$FV = PV(1 + r/m)^{mt} = 6{,}000\left(1 + \frac{0.048}{2}\right)^{2t} = 6{,}000(1.024)^{2t}$$

Solution via logarithms:

$$5{,}000(1.027)^{2t} = 6{,}000(1.024)^{2t}$$

$$(1.027)^{2t} = 1.2(1.024)^{2t}$$

$$2t \log\ 1.027 = \log[1.2(1.024)^{2t}] = \log\ 1.2 + 2t \log\ 1.024$$

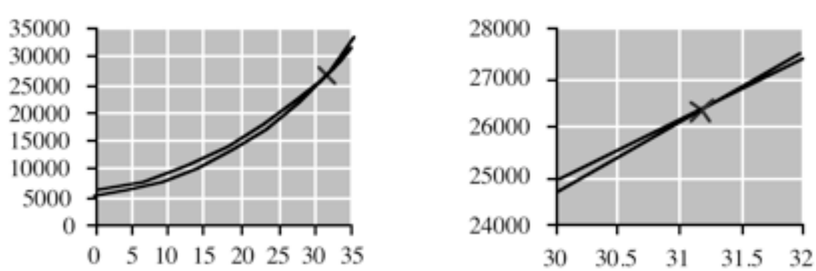
$$2t(\log\ 1.027 - \log\ 1.024) = \log\ 1.2$$

$$t = \frac{\log\ 1.2}{2(\log\ 1.027 - \log\ 1.024)} \approx 31$$

Solution via graphing:
Technology Formulas:

```
Y1 = 5000*1.027^(2*x)      Y2 = 6000*1.024^(2*x)
```

Graph and zoomed-in view:



The graphs cross around $t \approx 31$ years. The value of the investment then is

$$5{,}000(1.027)^{2\times 31} \approx \$26{,}100 \text{ (rounded to 3 significant digits)}$$

70. $PV = \$3{,}000,\ r = 0.05,\ m = 365$

$$FV = PV(1 + r/m)^{mt} = 3{,}000\left(1 + \frac{0.05}{365}\right)^{365t}$$

Logarithms:

$$3{,}000\left(1+\frac{0.05}{365}\right)^{365t} = 10{,}000$$

$$\left(1+\frac{0.05}{365}\right)^{365t} = \frac{10}{3}$$

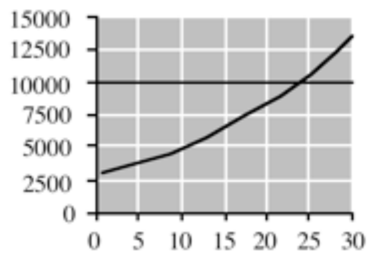
$$365t\log\left(1+\frac{0.05}{365}\right) = \log\frac{10}{3}$$

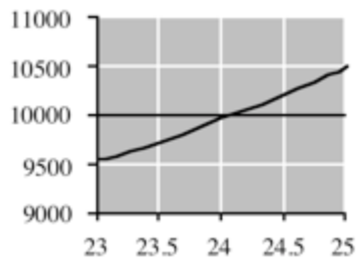
$$t = \frac{\log(10/3)}{365\log(1+0.05/365)} \approx 24$$

Technology:

```
Y1 = 3000*(1+0.05/365)^(365*x)     Y2 = 10000
```

Graph and zoomed-in view:





$t \approx 24$ years

71. $PV = 40{,}000,\ i = 1.0$ (a 100% increase per period)

$$FV = PV(1+i)^n = 40{,}000(1+1.0)^n = 40{,}000(2)^n$$

Logarithms:

$$40{,}000(2)^n = 1{,}000{,}000$$

$$2^n = 25$$

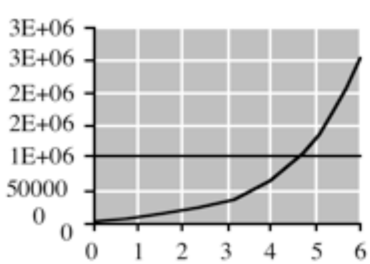
$$n\log 2 = \log 25$$

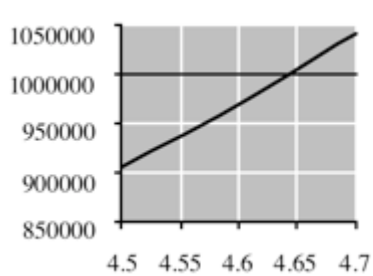
$$n = \frac{\log 25}{\log 2} \approx 4.65 \text{ months}$$

Technology:

```
Y1 = 40000*2^x     Y2 = 1000000
```

Graph and zoomed-in view:





$n \approx 4.65$. Since n measures 6-month periods, this corresponds to $4.65/2 \approx 2.3$ years.

72. $PV = \$4{,}354,\ i = -0.05$

$$FV = PV(1+i)^n = 4{,}354(1-0.05)^n = 4{,}354(0.95)^n$$

Logarithms:

$4{,}354(0.95)^n = 50$

$0.95^n = 50/4{,}354$

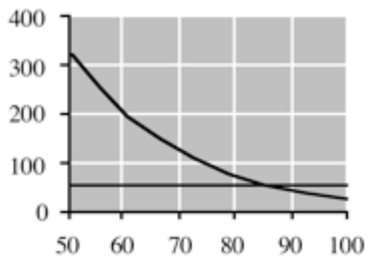
$n \log\ 0.95 = \log(50/4{,}354)$

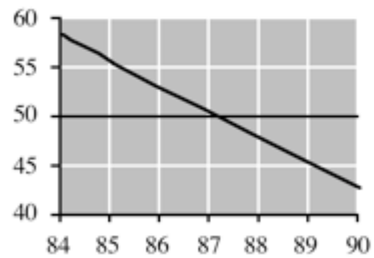
$$n = \frac{\log(50/4{,}354)}{\log\ 0.95} \approx 87.1$$

Technology:

```
Y1 = 4354*0.95^x    Y2 = 50
```

Graph and zoomed-in view:





$n \approx 87.1$. Since n measures 6-month periods, this corresponds to $87.1/2 \approx 44$ years.

73. a. The amount you pay for the bond is its present value.

$FV = \$100{,}000,\ i = 0.15,\ t = 30$

$$PV = \frac{FV}{(1+i)^n} = \frac{100{,}000}{(1+0.15)^{30}} = \frac{100{,}000}{1.15^{30}} \approx \$1{,}510.31$$

b. Because the future value remains fixed at \$100,000, the value of the bond at any time is its present value at that time, given the prevailing interest rate. Since it will pay \$100,000 in 13 years' time, we have:

$FV = \$100{,}000,\ i = 0.0475,\ t = 13$

$$PV = \frac{FV}{(1+i)^n} = \frac{100{,}000}{(1+0.0.475)^{13}} = \frac{100{,}000}{1.0475^{13}} \approx \$54{,}701.29$$

c. By part (a) the bond cost you \$1,510.31 and, by part (b), was worth \$54,701.29 after 17 years.

$PV = \$1{,}510.31,\ FV = \$54{,}701.29$

$FV = PV(1+i)^n$

$54{,}701.29 = 1{,}510.31(1+i)^{17}$

$$\frac{54{,}701.29}{1{,}510.31} = (1+i)^{17}$$

$$1 + i = \left(\frac{54{,}701.29}{1{,}510.31}\right)^{1/17}$$

$$i = \left(\frac{54{,}701.29}{1{,}510.31}\right)^{1/17} - 1 \approx 0.2351, \text{ or } 23.51\%$$

Technology: `(54701.29/1510.31)^(1/17)-1`

74. a. The amount you pay for the bond is its present value.

$FV = \$100{,}000,\ i = 0.05,\ t = 30$

$$PV = \frac{FV}{(1+i)^n} = \frac{100{,}000}{(1+0.05)^{30}} = \frac{100{,}000}{1.05^{30}} \approx \$23{,}137.74$$

b. The value of the bond at any time is its present value at that time, given the prevailing interest rate. Since it will pay \$100,000 in 15 years' time, we have:

$FV = \$100{,}000,\ i = 0.12,\ t = 15$

$$PV = \frac{FV}{(1+i)^n} = \frac{100{,}000}{(1+0.12)^{15}} = \frac{100{,}000}{1.12^{15}} \approx \$18{,}269.63$$

c. By part (a), the bond cost you \$23,137.74 and, by part (b), was worth \$18,269.63 after 15 years.

$PV = \$23{,}137.74,\ FV = \$18{,}269.63$

$FV = PV(1+i)^n$

$18{,}269.63 = 23{,}137.74(1+i)^{15}$

$$\frac{18{,}269.63}{23{,}137.74} = (1+i)^{15}$$

$$1+i = \left(\frac{18{,}269.63}{23{,}137.74}\right)^{1/15}$$

$$i = \left(\frac{18{,}269.63}{23{,}137.74}\right)^{1/15} - 1 \approx -0.0156, \text{ or } -1.56\%$$

You will have lost 1.56% per year.

Technology: `(18269.63/23137.74)^(1/15)-1`

Communication and reasoning exercises

75. The function $y = P(1 + r/m)^{mx}$ is not a linear function of x, but an exponential function. Thus, its graph is not a straight line.

76. Wrong. The second investment earns more, because the second investment pays interest on a larger amount after 6 months.

77. Wrong. Its growth can be modeled by $0.01(1+0.10)^t = 0.01(1.10)^t$. This is an exponential function of t, not a linear one.

78. After one interest period: The calculations are the same for a single interest period.

79. A compound interest investment behaves as though it were being compounded once a year at the effective rate. Thus, if two equal investments have the same effective rates, they grow at the same rate.

80. The effective rate equals the nominal rate when the interest is compounded once a year because, by definition, the effective rate is the annually compounded rate that would result in the same future value.

81. The effective rate exceeds the nominal rate when the interest is compounded more than once a year because then interest is being paid on interest accumulated during each year, resulting in a larger effective rate. Conversely, if the interest is compounded less often than once a year, the effective rate is less than the nominal rate.

82. No, it will not return to the same value it had originally. The simplest way to see this is to note the following lack of symmetry: The interest earned in year 5 is slightly less than the depreciation in year 6, since the depreciation is calculated on a larger value. A direct (two-step) calculation of the future value after 10 years gives

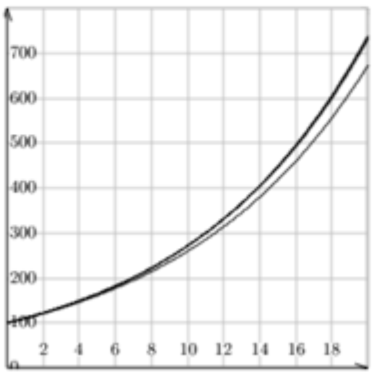
$P(1.1^5)(0.9^5) \approx 0.95099P$, slightly less than P.

83. Compare their future values in constant dollars. That is, compute their future values, and then discount each for inflation. The investment with the larger future value is the better investment.

84. First compute the value in 2011 dollars, using the 2011 inflation rate, then use the 2010 inflation rate to convert the answer to 2010 dollars, and so on, down to 2000 dollars.

85. $PV = 100$, $r = 0.10$, $m = 1, 10, 100, \ldots$

```
Y1 = 100*(1.10)^x
Y2 = 100*(1+ 0.10/10)^(10*x)
Y3 = 100*(1+ 0.10/100)^(100*x)
...
```



The graphs are approaching a particular curve (shown darker) as m gets larger, approximately the curve given by the largest value of m.

86. Since the graph gets arbitrarily close to the t-axis as t gets larger, the future value gets arbitrarily close to zero as time goes on, so it will eventually dip below any value that is set, like $1.

Solutions Section 2.3

Section 2.3

1. $PMT = \$100$, $i =$ interest paid each period $= 0.05/12$, $n =$ total number of periods $= 12 \times 10 = 120$

$$FV = PMT\,\frac{(1+i)^n - 1}{i} = 100\,\frac{(1+0.05/12)^{120} - 1}{0.05/12} \approx \$15{,}528.23$$

Technology: `100*((1+0.05/12)^120-1)/(0.05/12)`

2. $PMT = \$150$, $i = 0.03/12$, $n = 12 \times 20 = 240$

$$FV = PMT\,\frac{(1+i)^n - 1}{i} = 150\,\frac{(1+0.03/12)^{240} - 1}{0.03/12} \approx \$49{,}245.30$$

Technology: `150*((1+0.03/12)^240-1)/(0.03/12)`

3. $PMT = \$1{,}000$, $i = 0.07/4$, $n = 4 \times 20 = 80$

$$FV = PMT\,\frac{(1+i)^n - 1}{i} = 1{,}000\,\frac{(1+0.07/4)^{80} - 1}{0.07/4} \approx \$171{,}793.82$$

Technology: `1000*((1+0.07/4)^80-1)/(0.07/4)`

4. $PMT = \$2{,}000$, $i = 0.07/4$, $n = 4 \times 10 = 40$

$$FV = PMT\,\frac{(1+i)^n - 1}{i} = 2{,}000\,\frac{(1+0.07/4)^{40} - 1}{0.07/4} \approx \$114{,}468.27$$

Technology: `2000*((1+0.07/4)^40-1)/(0.07/4)`

5. $PV = \$5{,}000$, $PMT = \$100$, $i = 0.05/12$, $n = 12 \times 10 = 120$

We need to calculate the sum of $FV = PV(1+i)^n$ and $FV = PMT\,\frac{(1+i)^n - 1}{i}$.

$$FV = 5{,}000(1+0.05/12)^{120} + 100\,\frac{(1+0.05/12)^{120} - 1}{0.05/12} \approx \$23{,}763.28$$

Technology: `5000*(1+0.05/12)^120+100*((1+0.05/12)^120-1)/(0.05/12)`

6. $PV = \$10{,}000$, $PMT = \$150$, $i = 0.03/12$, $n = 12 \times 20 = 240$

We need to calculate the sum of $FV = PV(1+i)^n$ and $FV = PMT\,\frac{(1+i)^n - 1}{i}$.

$$FV = 10{,}000(1+0.03/12)^{240} + 150\,\frac{(1+0.03/12)^{240} - 1}{0.03/12} \approx \$67{,}452.85$$

Technology: `10000*(1+0.03/12)^240+150*((1+0.03/12)^240-1)/(0.03/12)`

7. $FV = \$10{,}000$, $i = 0.05/12$, $n = 12 \times 5 = 60$

$$PMT = FV\,\frac{i}{(1+i)^n - 1} = 10{,}000\,\frac{0.05/12}{(1+0.05/12)^{60} - 1} \approx \$147.05$$

Technology: `10000*0.05/12/((1+0.05/12)^60-1)`

8. $FV = \$20{,}000$, $i = 0.03/12$, $n = 12 \times 10 = 120$

$$PMT = FV\,\frac{i}{(1+i)^n - 1} = 20{,}000\,\frac{0.03/12}{(1+0.03/12)^{120} - 1} \approx \$143.12$$

Technology: `20000*0.03/12/((1+0.03/12)^120-1)`

Solutions Section 2.3

9. $FV = \$75{,}000,\ i = 0.06/4,\ n = 4 \times 20 = 80$

$$PMT = FV \frac{i}{(1+i)^n - 1} = 75{,}000 \frac{0.06/4}{(1+0.06/4)^{80} - 1} \approx \$491.12$$

Technology: `75000*0.06/4/((1+0.06/4)^80-1)`

10. $FV = \$100{,}000,\ i = 0.07/4,\ n = 4 \times 20 = 80$

$$PMT = FV \frac{i}{(1+i)^n - 1} = 100{,}000 \frac{0.07/4}{(1+0.07/4)^{80} - 1} \approx \$582.09$$

Technology: `100000*0.07/4/((1+0.07/4)^80-1)`

11. We first account for the future value of the \$10,000 already in the account: $PV = \$10{,}000,\ i = 0.05/12,$ $n = 12 \times 5 = 60$

$$FV = PV(1+i)^n = 10{,}000(1+0.05/12)^{60}$$

We subtract this from \$20,000 to get the future value of the payments, so:

$$PMT = FV \frac{i}{(1+i)^n - 1}$$

where

$$FV = \$20{,}000 - 10{,}000(1+0.05/12)^{60}$$

$$PMT = \frac{[20{,}000 - 10{,}000(1+0.05/12)^{60}](0.05/12)}{(1+0.05/12)^{60} - 1} \approx \$105.38$$

Technology: `(20000-10000*(1+0.05/12)^60)*0.05/12/((1+0.05/12)^60-1)`

12. We first account for the future value of the \$10,000 already in the account: $PV = \$10{,}000,\ i = 0.03/12,$ $n = 12 \times 10 = 120$

$$FV = PV(1+i)^n = 10{,}000(1+0.03/12)^{120}$$

We subtract this from \$30,000 to get the future value of the payments, so:

$$PMT = FV \frac{i}{(1+i)^n - 1}$$

where

$$FV = \$30{,}000 - 10{,}000(1+0.03/12)^{120}$$

$$PMT = \frac{[30{,}000 - 10{,}000(1+0.03/12)^{120}](0.03/12)}{(1+0.03/12)^{120} - 1} \approx \$118.12$$

Technology: `(30000-10000*(1+0.03/12)^120)*0.03/12/((1+0.03/12)^120-1)`

13. $PMT = \$500,\ i = 0.03/12,\ n = 12 \times 20 = 240$

$$PV = PMT \frac{1-(1+i)^{-n}}{i} = 500 \frac{1-(1+0.03/12)^{-240}}{0.03/12} \approx \$90{,}155.46$$

Technology: `500*(1-(1+0.03/12)^(-240))/(0.03/12)`

14. $PMT = \$1{,}000,\ i = 0.05/12,\ n = 12 \times 15 = 180$

$$PV = PMT \frac{1-(1+i)^{-n}}{i} = 1000 \frac{1-(1+0.05/12)^{-180}}{0.05/12} \approx \$126{,}455.24$$

Technology: `1000*(1-(1+0.05/12)^(-180))/(0.05/12)`

Solutions Section 2.3

15. $PMT = \$1{,}500,\ i = 0.06/4,\ n = 4 \times 20 = 80$

$$PV = PMT\,\frac{1-(1+i)^{-n}}{i} = 1{,}500\,\frac{1-(1+0.06/4)^{-80}}{0.06/4} \approx \$69{,}610.99$$

Technology: `1500*(1-(1+0.06/4)^(80))/(0.06/4)`

16. $PMT = \$2{,}000,\ i = 0.04/4 = 0.01,\ n = 4 \times 20 = 80$

$$PV = PMT\,\frac{1-(1+i)^{-n}}{i} = 2{,}000\,\frac{1-(1+0.01)^{-80}}{0.01} \approx \$109{,}776.41$$

Technology: `2000*(1-(1+0.01)^(-80))/0.01`

17. $FV = \$10{,}000,\ PMT = \$500,\ i = 0.03/12,\ n = 12 \times 20 = 240$

We need to fund both the future value and the payments, so the present value is the sum

$$PV = FV(1+i)^{-n} + PMT\,\frac{1-(1+i)^{-n}}{i}$$

$$= 10{,}000(1+0.03/12)^{-240} + 500\,\frac{1-(1+0.03/12)^{-240}}{0.03/12} \approx \$95{,}647.68$$

Technology: `10000*(1+0.03/12)^(-240)+500*(1-(1+0.03/12)^(-240))/(0.03/12)`

18. $FV = \$10{,}000,\ PMT = \$1{,}000,\ i = 0.05/12,\ n = 12 \times 15 = 180$

We need to fund both the future value and the payments, so the present value is the sum

$$PV = FV(1+i)^{-n} + PMT\,\frac{1-(1+i)^{-n}}{i}$$

$$= 10{,}000(1+0.05/12)^{-180} + 1{,}000\,\frac{1-(1+0.05/12)^{-180}}{0.05/12} \approx \$135{,}917.31$$

Technology: `20000*(1+0.05/12)^(-180)+1000*(1-(1+0.05/12)^(-180))/(0.05/12)`

19. $PV = \$100{,}000,\ i = 0.03/12,\ n = 12 \times 20 = 240$

$$PMT = PV\,\frac{i}{1-(1+i)^{-n}} = 100{,}000\,\frac{0.03/12}{1-(1+0.03/12)^{-240}} \approx \$554.60$$

Technology: `100000*(0.03/12)/(1-(1+0.03/12)^(-240))`

20. $PV = \$150{,}000,\ i = 0.05/12,\ n = 12 \times 15 = 180$

$$PMT = PV\,\frac{i}{1-(1+i)^{-n}} = 150{,}000\,\frac{0.05/12}{1-(1+0.05/12)^{-180}} \approx \$1{,}186.19$$

Technology: `150000*(0.05/12)/(1-(1+0.05/12)^(-180))`

21. $PV = \$75{,}000,\ i = 0.04/4 = 0.01,\ n = 4 \times 20 = 80$

$$PMT = PV\,\frac{i}{1-(1+i)^{-n}} = 75{,}000\,\frac{0.01}{1-(1+0.01)^{-80}} \approx \$1{,}366.41$$

Technology: `75000*0.01/(1-(1+0.01)^(-80))`

22. $PV = \$200{,}000,\ i = 0.06/4 = 0.015,\ n = 4 \times 15 = 60$

$$PMT = PV\,\frac{i}{1-(1+i)^{-n}} = 200{,}000\,\frac{0.015}{1-(1+0.015)^{-60}} \approx \$5{,}078.69$$

Technology: `200000*0.015/(1-(1+0.015)^(-60))`

23. $PV = \$100{,}000,\ FV = \$10{,}000,\ i = 0.03/12,\ n = 12 \times 20 = 240$

Part of the present value has to fund the future value of \$10,000:

$$FV(1+i)^{-n} = 10{,}000(1+0.03/12)^{-240}$$

is the amount required; we subtract this from the present value and use

$$PV = 100{,}000 - 10{,}000(1+0.03/12)^{-240}$$

in the payment formula.

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = \frac{(0.03/12)[100{,}000 - 10{,}000(1+0.03/12)^{-240}]}{1-(1+0.03/12)^{-240}} \approx \$524.14$$

Technology:
`(0.03/12)*(100000-10000*(1+0.03/12)^(-240))/(1-(1+0.03/12)^(-240))`

24. $PV = \$150{,}000,\ FV = \$20{,}000,\ i = 0.05/12,\ n = 12 \times 15 = 180$

Part of the present value has to fund the future value of \$20,000:

$$FV(1+i)^{-n} = 20{,}000(1+0.05/12)^{-180}$$

is the amount required; we subtract this from the present value and use

$$PV = 150{,}000 - 20{,}000(1+0.05/12)^{-180}$$

in the payment formula.

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = \frac{(0.05/12)[150{,}000 - 20{,}000(1+0.05/12)^{-180}]}{1-(1+0.05/12)^{-180}} \approx \$1{,}111.37$$

Technology:
`(0.05/12)*(150000-20000*(1+0.05/12)^(-180))/(1-(1+0.05/12)^(-180))`

25. $PV = \$10{,}000,\ i = 0.09/12 = 0.0075,\ n = 4 \times 12 = 48$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 10{,}000\frac{0.0075}{1-(1+0.0075)^{-48}} \approx \$248.85$$

Technology: `10000*0.0075/(1-(1+0.0075)^(-48))`

26. $PV = \$20{,}000,\ i = 0.08/12,\ n = 12 \times 5 = 60$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 20{,}000\frac{0.08/12}{1-(1+0.08/12)^{-60}} \approx \$405.53$$

Technology: `20000*(0.08/12)/(1-(1+0.08/12)^(-60))`

27. $PV = \$100{,}000,\ i = 0.05/4,\ n = 4 \times 20 = 80$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 100{,}000\frac{0.05/4}{1-(1+0.05/4)^{-80}} \approx \$1{,}984.65$$

Technology: `100000*(0.05/4)/(1-(1+0.05/4)^(-80))`

28. $PV = \$1{,}000{,}000,\ i = 0.04/4,\ n = 4 \times 10 = 40$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 1{,}000{,}000\frac{0.04/4}{1-(1+0.04/4)^{-40}} \approx \$30{,}455.60$$

Technology: `1000000*(0.04/4)/(1-(1+0.04/4)^(-40))`

29. $PV = 100{,}000,\ i = 0.043/12,\ n = 12 \times 30 = 360$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 100{,}000\frac{0.043/12}{1-(1+0.043/12)^{-360}} = \$494.87$$

Technology: `100000*(0.043/12)/(1-(1+0.043/12)^(-360))`

30. $PV = 250{,}000,\ i = 0.062/12,\ n = 12 \times 15 = 180$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 250{,}000\frac{0.062/12}{1-(1+0.062/12)^{-180}} = \$2{,}136.75$$

Technology: `250000*(0.062/12)/(1-(1+0.062/12)^(-180))`

31. $PV = 1{,}000{,}000,\ i = 0.054/12,\ n = 12 \times 30 = 360$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 1{,}000{,}000\frac{0.054/12}{1-(1+0.054/12)^{-360}} = \$5{,}615.31$$

Technology: `1000000*(0.054/12)/(1-(1+0.054/12)^(-360))`

32. $PV = 2{,}000{,}000,\ i = 0.045/12,\ n = 12 \times 15 = 180$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 2{,}000{,}000\frac{0.045/12}{1-(1+0.045/12)^{-180}} = \$15{,}299.87$$

Technology: `2000000*(0.045/12)/(1-(1+0.045/12)^(-180))`

33. We calculated $PMT = \$494.87$ in Exercise 29.

$i = 0.043/12,\ n = 12 \times 30 = 360,\ k = 12 \times 10 = 120,$ so $n - k = 240$

$$PV = PMT\frac{1-(1+i)^{-(n-k)}}{i}$$

$$= 494.87\frac{1-(1+0.043/12)^{-240}}{0.043/12} = \$79{,}573.29$$

Technology: `494.87*(1-(1+0.043/12)^(-240))/(0.043/12)`

34. We calculated $PMT = \$2{,}136.75$ in Exercise 30.

$i = 0.062/12,\ n = 12 \times 15 = 180,\ k = 12 \times 10 = 120,$ so $n - k = 60$

$$PV = PMT\frac{1-(1+i)^{-(n-k)}}{i}$$

$$= 2{,}136.75\frac{1-(1+0.062/12)^{-60}}{0.062/12} = \$109{,}994.70$$

Technology: `2136.75*(1-(1+0.062/12)^(-60))/(0.062/12)`

35. We calculated $PMT = \$5{,}615.31$ in Exercise 31.

$i = 0.054/12,\ n = 12 \times 30 = 360,\ k = 12 \times 5 = 60,$ so $n - k = 300$

$$PV = PMT\frac{1-(1+i)^{-(n-k)}}{i}$$

$$= 5{,}615.31\frac{1-(1+0.054/12)^{-300}}{0.054/12} = \$923{,}373.42$$

Technology: `5615.31*(1-(1+0.054/12)^(-300))/(0.054/12)`

36. We calculated $PMT = \$15{,}299.87$ in Exercise 32.

$i = 0.045/12,\ n = 12 \times 15 = 180,\ k = 12 \times 8 = 96,$ so $n - k = 84$

$$PV = PMT\,\frac{1-(1+i)^{-(n-k)}}{i}$$

$$= 15{,}299.87\,\frac{1-(1+0.045/12)^{-84}}{0.045/12} = \$1{,}100{,}697.30$$

Technology: `15299.87*(1-(1+0.045/12)^(-84))/(0.045/12)`

37. First, we calculate the monthly payments:

$PV = 50{,}000,\ i = 0.085/12,\ n = 12 \times 200 = 2{,}400$

$$PMT = PV\,\frac{i}{1-(1+i)^{-n}} = 50{,}000\,\frac{0.085/12}{1-(1+0.085/12)^{-2{,}400}} = \$354.17$$

Technology: `50000*(0.085/12)/(1-(1+0.085/12)^(-2400))`

Now calculate the outstanding balance based on the above payments:

$k = 12 \times 20 = 240,$ so $n - k = 2{,}160$

$$PV = PMT\,\frac{1-(1+i)^{-(n-k)}}{i}$$

$$= 354.17\,\frac{1-(1+0.085/12)^{-2{,}160}}{0.085/12} = \$50{,}000.46$$

Technology: `354.17*(1-(1+0.085/12)^(-2160))/(0.085/12)`

This is more than the original value of the loan. In a 200-year mortgage, the fraction of the initial payments going toward reducing the principal is so small that the rounding upward of the payment makes it appear that, for many years at the start of the mortgage, more is owed than the original amount borrowed.

38. First, we calculate the monthly payments:

$PV = 100{,}000,\ i = 0.096/12,\ n = 12 \times 200 = 2{,}400$

$$PMT = PV\,\frac{i}{1-(1+i)^{-n}} = 100{,}000\,\frac{0.096/12}{1-(1+0.096/12)^{-2{,}400}} = \$800$$

Technology: `100000*(0.096/12)/(1-(1+0.096/12)^(-2400))`

Now calculate the outstanding balance based on the above payments:

$k = 12 \times 20 = 240,$ so $n - k = 2{,}160$

$$PV = PMT\,\frac{1-(1+i)^{-(n-k)}}{i}$$

$$= 800\,\frac{1-(1+0.096/12)^{-2{,}160}}{0.096/12} = \$100{,}000.00$$

Technology: `800*(1-(1+0.096/12)^(-2160))/(0.096/12)`

The outstanding principal has not changed from the original amount. In a 200-year mortgage, the fraction of the initial payments going toward reducing the principal is so small that rounding the payment amount to the nearest cent erases that contribution in this case.

39. The periodic payments are based on a 4.875% annual payment. For payments twice a year, this is

$$PMT = 1{,}000(0.04875/2) = \$24.375$$

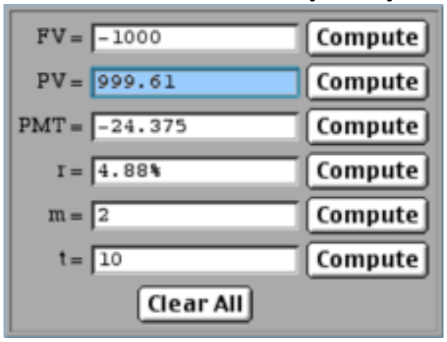
Since the bond yield is 4.880%, $i = 0.0488/2 = 0.0244$; $n = 2 \times 10 = 20$. The present value comes from the future value of \$1,000 and the payments, which we treat like an annuity:

$$PV = FV(1+i)^{-n} + PMT\,\frac{1-(1+i)^{-n}}{i}$$

$$= 1{,}000(1 + 0.0244)^{-20} + 24.375\,\frac{1 - (1 + 0.0244)^{-20}}{0.0244}$$

$$= 1{,}000(1.0244)^{-20} + 24.375\,\frac{1 - 1.0244^{-20}}{0.0244} \approx \$999.61$$

Technology: `1000*1.0244^(-20)+24.375*(1-1.0244^(-20))/0.0244`
Online Time Value of Money Utility:



40. The periodic payments are based on a 5.375% annual payment. For payments twice a year, this is

$$PMT = 1{,}000(0.05375/2) = \$26.875$$

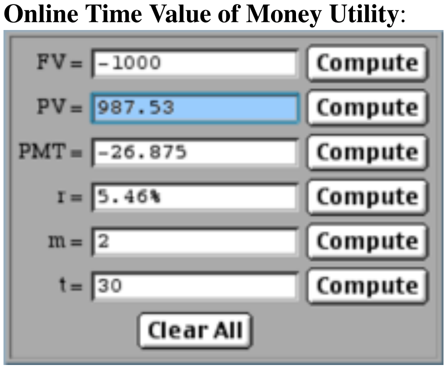
Since the bond yield is 5.460%, $i = 0.0546/2 = 0.0273$; $n = 2 \times 30 = 60$. The present value comes from the future value of \$1,000 and the payments, which we treat like an annuity:

$$PV = FV(1 + i)^{-n} + PMT\,\frac{1 - (1 + i)^{-n}}{i}$$

$$= 1{,}000(1 + 0.0273)^{-60} + 26.875\,\frac{1 - (1 + 0.0273)^{-60}}{0.0273}$$

$$= 1{,}000(1.0273)^{-60} + 26.875\,\frac{1 - 1.0273^{-60}}{0.0273} \approx \$987.53$$

Technology: `1000*1.0273^(-60)+26.875*(1-1.0273^(-60))/0.0273`
Online Time Value of Money Utility:



41. The periodic payments are based on a 3.625% annual payment. For payments twice a year, this is

$$PMT = 1{,}000(0.03625/2) = \$18.125$$

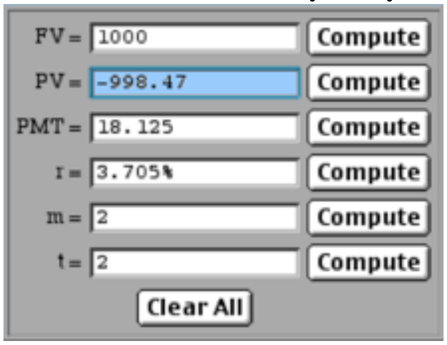
Since the bond yield is 3.705%, $i = 0.03705/2 = 0.018525$; $n = 2 \times 2 = 4$. The present value comes from the future value of \$1,000 and the payments, which we treat like an annuity:

$$PV = FV(1 + i)^{-n} + PMT\,\frac{1 - (1 + i)^{-n}}{i}$$

$$= 1{,}000(1 + 0.018525)^{-4} + 18.125\,\frac{1 - (1 + 0.018525)^{-4}}{0.018525}$$

$$= 1{,}000(1.018525)^{-4} + 18.125\,\frac{1 - 1.018525^{-4}}{0.018525} \approx \$998.47$$

Technology: `1000*1.018525^(-4)+18.125*(1-1.018525^(-4))/0.018525`
Online Time Value of Money Utility:



42. The periodic payments are based on a 4.375% annual payment. For payments twice a year, this is

$PMT = 1{,}000(0.04375/2) = \$21.875$

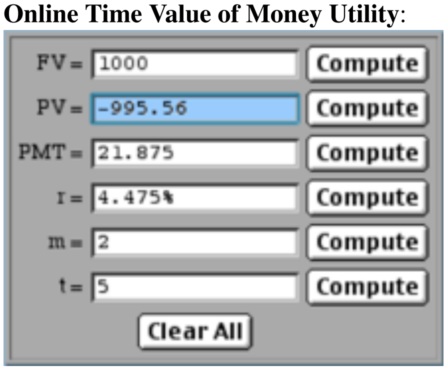
Since the bond yield is 4.475%, $i = 0.04475/2 = 0.022375$; $n = 2 \times 5 = 10$. The present value comes from the future value of \$1,000 and the payments, which we treat like an annuity:

$$PV = FV(1 + i)^{-n} + PMT\,\frac{1 - (1 + i)^{-n}}{i}$$

$$= 1{,}000(1 + 0.022375)^{-10} + 21.875\,\frac{1 - (1 + 0.022375)^{-10}}{0.022375}$$

$$= 1{,}000(1.022375)^{-10} + 21.875\,\frac{1 - 1.022375^{-10}}{0.022375} \approx \$995.56$$

Technology: `1000*1.022375^(-10)+21.875*(1-1.022375^(-10))/0.022375`
Online Time Value of Money Utility:



43. The periodic payments are based on a 5.5% annual payment. For payments twice a year, this is

$PMT = 1{,}000(0.055/2) = \$27.5$

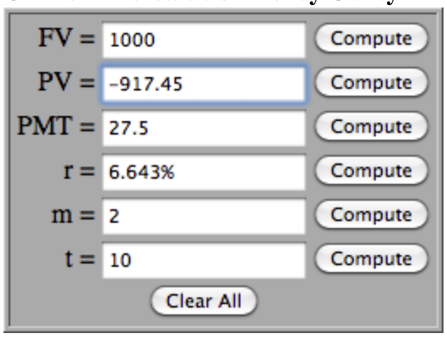
Since the bond yield is 6.643%, $i = 0.06643/2 = 0.033215$; $n = 2 \times 10 = 20$. The present value comes from the future value of \$1,000 and the payments, which we treat like an annuity:

$$PV = FV(1 + i)^{-n} + PMT\,\frac{1 - (1 + i)^{-n}}{i}$$

$$= 1{,}000(1+0.033215)^{-20} + 27.5\,\frac{1-(1+0.033215)^{-20}}{0.033215}$$

$$= 1{,}000(1.033215)^{-20} + 27.5\,\frac{1-1.033215^{-20}}{0.033215} \approx \$917.45$$

Technology: `1000*1.033215^(-20)+27.5*(1-1.033215^(-20))/0.033215`
Online Time Value of Money Utility:



44. The periodic payments are based on a 6.25% annual payment. For payments twice a year, this is

$PMT = 1{,}000(0.0625/2) = \$31.25$

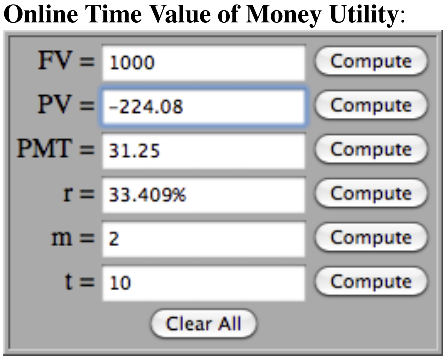
Since the bond yield is 33.409%, $i = 0.33409/2 = 0.167045$; $n = 2 \times 10 = 20$. The present value comes from the future value of \$1,000 and the payments, which we treat like an annuity:

$$PV = FV(1+i)^{-n} + PMT\,\frac{1-(1+i)^{-n}}{i}$$

$$= 1{,}000(1+0.167045)^{-20} + 31.25\,\frac{1-(1+0.167045)^{-20}}{0.167045}$$

$$= 1{,}000(1.167045)^{-20} + 31.25\,\frac{1-1.167045^{-20}}{0.167045} \approx \$224.08$$

Technology: `1000*1.167045^(-20)+31.25*(1-1.167045^(-20))/0.167045`
Online Time Value of Money Utility:



Applications

45. $PMT = \$400,\ i = 0.1483/12,\ n = 12 \times 20 = 240$

$$FV = PMT\,\frac{(1+i)^n - 1}{i} = 400\,\frac{(1+0.1483/12)^{240}-1}{0.1483/12} \approx \$584{,}686.94$$

Technology: `400*((1+0.1483/12)^240-1)/(0.1483/12)`

46. $PMT = \$380,\ i = 0.1325/12,\ n = 12 \times 25 = 300$

$$FV = PMT\frac{(1+i)^n - 1}{i} = 380\frac{(1+0.1325/12)^{300} - 1}{0.1325/12} \approx \$893{,}407.93$$

Technology: `380*((1+0.1325/12)^300-1)/(0.1325/12)`

47. Current value: $PMT = \$500,\ i = 0.0277/12,\ n = 12 \times 15 = 180$

$$FV = PMT\frac{(1+i)^n - 1}{i} = 500\frac{(1+0.0277/12)^{180} - 1}{0.0277/12} \approx \$111{,}422.99$$

Technology: `500*((1+0.0277/12)^180-1)/(0.0277/12)`

Value in 20 years: $PMT = \$500,\ i = 0.0267/12,\ n = 12 \times 20 = 240$

$$FV = PV(1+i)^n + PMT\frac{(1+i)^n - 1}{i}$$

$$= 111{,}422.99(1+0.0267/12)^{240} + 500\frac{(1+0.0267/12)^{240} - 1}{0.0267/12}$$

$$\approx \$348{,}312.44$$

Technology:
```
111422.99*(1+0.0267/12)^240
     + 500*((1+0.0267/12)^240-1)/(0.0267/12)
```

48. Current value: $PMT = \$500,\ i = 0.0267/12,\ n = 12 \times 15 = 180$

$$FV = PMT\frac{(1+i)^n - 1}{i} = 500\frac{(1+0.0267/12)^{180} - 1}{0.0267/12} \approx \$110{,}540.88$$

Technology: `500*((1+0.0267/12)^180-1)/(0.0267/12)`

Value in 20 years: $PMT = \$500,\ i = 0.0277/12,\ n = 12 \times 20 = 240$

$$FV = PV(1+i)^n + PMT\frac{(1+i)^n - 1}{i}$$

$$= 110{,}540.88(1+0.0277/12)^{240} + 500\frac{(1+0.0277/12)^{240} - 1}{0.0277/12}$$

$$\approx \$352{,}332.00$$

Technology:
```
110540.88*(1 + 0.0277/12)^240
     + 500*((1+0.0277/12)^240-1)/(0.0277/12)
```

49. $FV = \$1{,}500{,}000\ i = 0.0277/12,\ n = 12 \times 30 = 360$

$$PMT = FV\frac{i}{(1+i)^n - 1} = 1{,}500{,}000\frac{0.0277/12}{(1+0.0277/12)^{360} - 1} \approx \$2{,}677.02$$

Technology: `1500000*0.0277/12/((1+0.0277/12)^360-1)`

50. $FV = \$1{,}000{,}000\ i = 0.0267/12,\ n = 12 \times 25 = 300$

$$PMT = FV\frac{i}{(1+i)^n - 1} = 1{,}000{,}000\frac{0.0267/12}{(1+0.0267/12)^{300} - 1} \approx \$2{,}347.26$$

Technology: `1000000*0.0267/12/((1+0.0267/12)^300-1)`

51. Take x to be the amount you deposit each month into the stock fund. Then the deposit into the bond fund is four times that amount. The future value of the account is the sum of the future values of two investments: $\$x$ per month at 14.83% for $12 \times 25 = 300$ months, and $\$4x$ per month at 2.67% for 300 months:

$$FV = x\frac{(1+0.1483/12)^{300}-1}{0.1483/12} + 4x\frac{(1+0.0267/12)^{300}-1}{0.0267/12}$$

$$\approx 3{,}142.5166x + 1{,}704.1131x = 4{,}846.6297x$$

We need this amount to equal one million: $4{,}846.6297x = 1{,}000{,}000$, so $x = \frac{1{,}000{,}000}{4{,}846.6297} \approx 206.329$.

Thus, \$206.33 should be invested in the stock fund each month.
The amount in the bond fund is four times that amount: $4 \times 206.33 = \$825.32$ per month.

52. Take x to be the amount you deposit each month into the stock fund. Then the deposit into the bond fund is three times that amount. The future value of the account is the sum of the future values of two investments: \$$x$ per month at 13.25% for $12 \times 25 = 300$ months, and \$$3x$ per month at 2.77% for 300 months:

$$FV = x\frac{(1+0.1325/12)^{300}-1}{0.1325/12} + 3x\frac{(1+0.0277/12)^{300}-1}{0.0277/12}$$

$$\approx 2{,}351.0735x + 1{,}295.8852x = 3{,}646.9587x$$

We need this amount to equal one million: $3{,}646.9587x = 1{,}000{,}000$, so $x = \frac{1{,}000{,}000}{3{,}646.9587} \approx 274.2011$.

The total investment per month is $x + 3x = 4x \approx 4 \times 274.2011 = \$1{,}096.80$ per month.

53. Funding the \$5,000 withdrawals:
$PMT = 5{,}000$, $i = 0.1483/12$, $n = 12 \times 10 = 120$

$$PV = PMT\frac{1-(1+i)^{-n}}{i} = 5{,}000\frac{1-(1+0.1483/12)^{-120}}{0.1483/12} \approx \$311{,}923.95$$

Technology: `5000*(1-(1 + 0.1483/12)^(-120))/(0.1483/12)`
To fund the lump sum of \$30,000 after 10 years, we need the present value of \$30,000 under compound interest:

$$PV = 30{,}000(1+0.1483/12)^{-120}$$

$$= \$6{,}870.84.$$

So, the total in the trust should be
$\$311{,}923.95 + \$6{,}870.84 = \$318{,}794.79.$

54. Funding the \$2,000 withdrawals:
$PMT = 2{,}000$, $i = 0.1325/12$, $n = 12 \times 20 = 240$

$$PV = PMT\frac{1-(1+i)^{-n}}{i} = 2{,}000\frac{1-(1+0.1325/12)^{-240}}{0.1325/12} \approx \$168{,}147.66$$

Technology: `2000*(1-(1 + 0.1325/12)^(-240))/(0.1325/12)`
To fund the lump sum of \$100,000 after 20 years, we need the present value of \$100,000 under compound interest:

$$PV = 100{,}000(1+0.1325/12)^{-240}$$

$$= \$7{,}168.48.$$

So, the total in the trust should be
$\$168{,}147.66 + \$7{,}168.48 = \$175{,}316.14.$

55. This is a two-stage process:
(1) An accumulation stage to build the retirement fund
(2) An amortization stage depleting the fund during retirement

Stage 1: Building the retirement fund. $PMT = \$1{,}200$, $i = 0.04/4 = 0.01$, $n = 4 \times 40 = 160$.

$$FV = PMT\frac{(1+i)^n - 1}{i}$$

$$= 1{,}200\frac{1.01^{160} - 1}{0.01}$$

$$\approx \$469{,}659.16$$

Technology: `1200*(1.01^160-1)/0.01`

Stage 2: Depleting the fund. $PV = \$469{,}659.16,\ i = 0.04/4 = 0.01,\ n = 4 \times 25 = 100.$

$$PMT = PV\frac{i}{1-(1+i)^{-n}}$$

$$= \frac{469{,}659.16 \times 0.01}{1 - 1.01^{-100}}$$

$$\approx \$7{,}451.49$$

Technology: `469659.16*0.01/(1-1.01^(-100))`

56. This is a two-stage process:
(1) An accumulation stage to build the retirement fund
(2) An amortization stage depleting the fund during retirement

Stage 1: Building the retirement fund. $PMT = \$300,\ i = 0.05/12,\ n = 12 \times 45 = 540.$

$$FV = PMT\frac{(1+i)^n - 1}{i}$$

$$= 300\frac{(1+0.05/12)^{540} - 1}{0.05/12}$$

$$\approx \$181{,}630.10$$

Technology: `300*((1+0.05/12)^540-1)/(0.05/12)`

Stage 2: Depleting the fund. $PV = \$607{,}931.19,\ i = 0.05/12,\ n = 12 \times 20 = 240.$

$$PMT = PV\frac{i}{1-(1+i)^{-n}}$$

$$= \frac{607{,}931.19 \times 0.05/12}{1-(1+0.05/12)^{-240}}$$

$$\approx \$4{,}012.08$$

Technology: `607931.19*(0.05/12)/(1-(1+0.05/12)^(-240))`

57. This is a two-stage process:
(1) An accumulation stage to build the retirement fund
(2) An amortization stage depleting the fund during retirement

As in the text, we work backward, starting with Stage 2, where we have $PMT = \$5{,}000,\ i = 0.03/12 = 0.0025,$ and $n = 20 \times 12 = 240,$ and we need to calculate the starting value PV.

$$PV = PMT\frac{1-(1+i)^{-n}}{i}$$

$$= 5{,}000\frac{1-(1+0.0025)^{-240}}{0.0025}$$

$$= 5{,}000\frac{1-(1.0025)^{-240}}{0.0025}$$

$\approx \$901{,}554.57$

Technology: `5000*(1-1.0025^(-240))/0.0025`

Thus, in Stage 1, you need to accumulate \$901,554.57 in the annuity: $FV = \$901{,}554.57,\ i = 0.03/12 = 0.0025,$

$n = 40 \times 12 = 480.$

$$PMT = FV \frac{i}{(1+i)^n - 1}$$

$$= \frac{901{,}554.57 \times 0.0025}{1.0025^{480} - 1}$$

$$\approx \$973.54 \text{ per month}$$

Technology: `901554.57*0.0025/(1.0025^480-1)`

58. This is a two-stage process:
(1) An accumulation stage to build the retirement fund
(2) An amortization stage depleting the fund during retirement

$$PV = PMT \frac{1-(1+i)^{-n}}{i}$$

$$= 12{,}000 \frac{1-(1+0.0125)^{-100}}{0.0125}$$

$$= 12{,}000 \frac{1-(1.0125)^{-100}}{0.0125}$$

$$\approx \$682{,}816.07$$

Technology: `12000*(1-1.0125^(-100))/0.0125`

Thus, in Stage 1, Meg needs to accumulate \$682,816.07 in the annuity: $FV = \$682{,}816.07,\ i = 0.05/4 = 0.0125,$

$n = 4 \times 45 = 180.$

$$PMT = FV \frac{i}{(1+i)^n - 1}$$

$$= \frac{682{,}816.07 \times 0.0125}{1.0125^{180} - 1}$$

$$\approx \$1{,}021.40 \text{ per quarter}$$

Technology: `682816.07*0.0125/(1.0125^180-1)`

59. $FV = \$1{,}000{,}000,\ i = 0.048/12 = 0.004,\ n = 12 \times (87 - 30) = 684$

$$PMT = FV \frac{i}{(1+i)^n - 1} = 1{,}000{,}000 \frac{0.004}{1.004^{684} - 1} \approx \$278.92$$

Technology: `1000000*0.004/(1.004^684-1)`

60. $FV = \$1{,}000{,}000,\ i = 0.048/12 = 0.004,\ n = 12 \times (76 - 30) = 552$

$$PMT = FV \frac{i}{(1+i)^n - 1} = 1{,}000{,}000 \frac{0.004}{1.004^{552} - 1} \approx \$496.43$$

Technology: `1000000*0.004/(1.004^552-1)`

61. Take x to be the current age of an insured person.

$FV = \$500{,}000,\ i = 0.048/12 = 0.004;$

$$PMT = FV\frac{i}{(1+i)^n - 1} = 500{,}000\frac{0.004}{1.004^n - 1}$$

Males: $PMT = FV\frac{i}{(1+i)^n - 1} = 500{,}000\frac{0.004}{1.004^{12(74-x)} - 1}$

Females: $PMT = FV\frac{i}{(1+i)^n - 1} = 500{,}000\frac{0.004}{1.004^{12(77-x)} - 1}$

Technology: `500000*0.004/(1.004^(12*(74-x))-1)`
`500000*0.004/(1.004^(12*(77-x))-1)`

Result:

Age x	Male	Female
30	\$276.62	\$235.24
50	\$927.11	\$756.08
70	\$9,469.40	\$5,020.06

62. Take x to be the current age of an insured person.

$FV = \$800{,}000,\ i = 0.048/12 = 0.004;$

$$PMT = FV\frac{i}{(1+i)^n - 1} = 800{,}000\frac{0.004}{1.004^n - 1}$$

Males: $PMT = FV\frac{i}{(1+i)^n - 1} = 800{,}000\frac{0.004}{1.004^{12(73-x)} - 1}$

Females: $PMT = FV\frac{i}{(1+i)^n - 1} = 800{,}000\frac{0.004}{1.004^{12(79-x)} - 1}$

Technology: `800000*0.004/(1.004^(12*(73-x))-1)`
`800000*0.004/(1.004^(12*(79-x))-1)`

Result:

Age x	Male	Female
20	\$274.30	\$201.46
40	\$829.22	\$584.26
60	\$3,703.46	\$2,155.21

63. While in Mexico:

$FV = \$750{,}000,\ i = 0.048/12 = 0.004,\ n = 12 \times (73 - 22) = 612$

$$PMT = FV\frac{i}{(1+i)^n - 1} = 750{,}000\frac{0.004}{1.004^{612} - 1} \approx \$285.47$$

When he moved to Canada eight years later, $n = 12 \times 8 = 96,$ so the value of the policy was

$$FV = PMT\frac{(1+i)^n - 1}{i} = 285.47\frac{1.004^{96} - 1}{0.004} \approx \$33{,}330.15.$$

To calculate the required payments in Canada, we use an annuity with a present value of \$33,330.15 and want the payments necessary to increase this amount to \$750,000 in (80 – 30) = 50 years ($n = 12 \times 50 = 600$), so we use the formula in the "Before we go on" discussion after after Example 1:

$$FV = PV(1+i)^n + PMT\frac{(1+i)^n - 1}{i}$$ and solve for PMT to obtain

$$PMT = (FV - PV(1+i)^n)\frac{i}{(1+i)^n - 1}$$

$$= \left(750{,}000 - 33{,}330.15(1.004)^{600}\right)\frac{0.004}{1.004^{600} - 1}$$

$\approx \$154.19$.So, the premiums decreased by $285.47 - 154.19 = \$131.28$.

64. While in the U.K.:

$FV = \$800{,}000,\ i = 0.048/12 = 0.004,\ n = 12 \times (83 - 25) = 696$

$$PMT = FV\frac{i}{(1+i)^n - 1} = 800{,}000\frac{0.004}{1.004^{696} - 1} \approx \$212.00$$

When she moved to India ten years later, $n = 12 \times 10 = 120$, so the value of the policy was

$$FV = PMT\frac{(1+i)^n - 1}{i} = 212.00\frac{1.004^{120} - 1}{0.004} \approx \$32{,}569.98.$$

To calculate the required payments in India, we use an annuity with a present value of \$32,569.98 and want the payments necessary to increase this amount to \$800,000 in $(68 - 35) = 33$ years ($n = 12 \times 33 = 396$), so we use the formula in the "Before we go on" discussion after after Example 1:

$$FV = PV(1+i)^n + PMT\frac{(1+i)^n - 1}{i}$$ and solve for PMT to obtain

$$PMT = (FV - PV(1+i)^n)\frac{i}{(1+i)^n - 1}$$

$$= \left(800{,}000 - 32{,}569.98(1.004)^{396}\right)\frac{0.004}{1.004^{396} - 1}$$

$\approx \$665.18$.So, the premiums increased by $665.18 - 212.00 = \$453.18$.

65. We know the payments and need to calculate the present value:

$i = 0.0409/12,\ n = 12 \times 30 = 360,\ PMT = 600$

$$PMT = PV\frac{i}{1 - (1+i)^{-n}}$$

$$600 = PV\frac{0.0409/12}{1 - (1 + 0.0409/12)^{-360}}$$

so $PV = 600\dfrac{1 - (1 + 0.0409/12)^{-360}}{0.0409/12} \approx \$124{,}321.81.$

This is the amount you can afford to finance. Including the down payment gives a total of

$\$124{,}321.81 + 20{,}000 = \$144{,}321.81$

66. We know the payments and need to calculate the present value:

$i = 0.0331/12,\ n = 12 \times 15 = 180,\ PMT = 900$

$$PMT = PV\frac{i}{1 - (1+i)^{-n}}$$

$$900 = PV\frac{0.0331/12}{1 - (1 + 0.0331/12)^{-180}}$$

so $PV = 900\dfrac{1 - (1 + 0.0331/12)^{-180}}{0.0331/12} \approx \$127{,}553.09.$

This is the amount you can afford to finance. Including the down payment gives a total of

$\$127{,}553.09 + 50{,}000 = \$177{,}553.09$

Solutions Section 2.3

67. Your hunch: Wait until December for the price to go down to \$140,000: $PV = \$140{,}000,\ i = 0.0409/12,$

$n = 12 \times 30 = 360$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 140{,}000\frac{0.0409/12}{1-(1+0.0409/12)^{-360}} \approx \$675.67$$

Technology: `140000*(0.0409/12)/(1-(1+0.0409/12)^(-360))`

Broker's suggestion: Buy now for \$150,000: $PV = \$150{,}000,\ i = 0.0393/12,\ n = 12 \times 30 = 360$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 150{,}000\frac{0.0393/12}{1-(1+0.0393/12)^{-360}} \approx \$710.08$$

Technology: `150000*(0.0393/12)/(1-(1+0.0393/12)^(-360))`

Conclusion: Wait until December and pay $710.08 - 675.67 = \$34.41$ less per month.

68. Your friend's hunch: Wait until December for the price to go down to \$297,000: $PV = \$297{,}000,\ i = 0.0334/12,$

$n = 12 \times 15 = 180$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 297{,}000\frac{0.0334/12}{1-(1+0.0334/12)^{-180}} \approx \$2{,}099.94$$

Technology: `297000*(0.0334/12)/(1-(1+0.0334/12)^(-180))`

Broker's suggestion: Buy now for \$300,000: $PV = \$300{,}000,\ i = 0.0314/12,\ n = 12 \times 15 = 180$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 300{,}000\frac{0.0314/12}{1-(1+0.0314/12)^{-180}} \approx \$2{,}092.00$$

Technology: `300000*(0.0314/12)/(1-(1+0.0314/12)^(-180))`

Conclusion: The agent was right; buying now would have saved your friend $2{,}099.94 - 2{,}092.00 = \$7.94$ per month.

69. We first calculate the payments:

$PV = \$150{,}000,\ i = 0.0393/12,\ n = 12 \times 30 = 360$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 150{,}000\frac{0.0393/12}{1-(1+0.0393/12)^{-360}} \approx \$710.08$$

Technology: `150000*(0.0393/12)/(1-(1+0.0393/12)^(-360))`

Now calculate the outstanding principal:

$k = 12 \times 15 = 180,\ n - k = 360 - 180 = 180,\ PMT = 710.08$

$$\text{Outstanding principal } = PMT\frac{1-(1+i)^{-(n-k)}}{i}$$

$$= 710.08\frac{1-(1+0.0393/12)^{-180}}{0.0393/12} \approx \$96{,}454.02$$

Technology: `710.08*(1-(1+0.0393/12)^-180)/(0.0393/12)`

70. We first calculate the payments:

$PV = \$300{,}000,\ i = 0.0314/12,\ n = 12 \times 15 = 180$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 300{,}000\frac{0.0314/12}{1-(1+0.0314/12)^{-180}} \approx \$2{,}092.00$$

Technology: `300000*(0.0314/12)/(1-(1+0.0314/12)^(-180))`

Now calculate the outstanding principal:

$k = 12 \times 8 = 96,\ n - k = 180 - 96 = 84,\ PMT = 2{,}092.00$

$$\text{Outstanding principal } = PMT\frac{1-(1+i)^{-(n-k)}}{i}$$

$$= 2{,}092.00\,\frac{1-(1+0.0314/12)^{-84}}{0.0314/12} \approx \$157{,}571.75$$

Technology: `2092.00*(1-(1+0.0314/12)^-84)/(0.0314/12)`

71. We first calculate the payments on the original mortgage:

$PV = \$250{,}000,\ i = 0.0393/12,\ n = 12\times 30 = 360$

$$PMT = PV\,\frac{i}{1-(1+i)^{-n}} = 250{,}000\,\frac{0.0393/12}{1-(1+0.0393/12)^{-360}} \approx \$1{,}183.47$$

Technology: `250000*(0.0393/12)/(1-(1+0.0393/12)^(-360))`

Now calculate the outstanding principal:

$k = 12\times 10 = 120,\ n-k = 360-120 = 240,\ PMT = 1{,}183.47$

$$\text{Outstanding principal } = PMT\,\frac{1-(1+i)^{-(n-k)}}{i}$$

$$= 1{,}183.47\,\frac{1-(1+0.0393/12)^{-240}}{0.0393/12} \approx \$196{,}492.38$$

Technology: `1183.47*(1-(1+0.0393/12)^-240)/(0.0393/12)`

Now calculate the mortgage payments at the new rate on the the outstanding principal plus prepayment fee:

$PV = \$196{,}492.38\times 1.04 \approx \$204{,}352.08,\ i = (0.0393/2)/12 = 0.0393/24,\ n = 12\times 20 = 240$

$$PMT = PV\,\frac{i}{1-(1+i)^{-n}} = 204{,}352.08\,\frac{0.0393/24}{1-(1+0.0393/24)^{-240}} \approx \$1{,}030.40$$

Technology: `204352.08*(0.0393/24)/(1-(1+0.0393/24)^(-240))`

Saving = \$1,183.47 − 1,030.40 = \$153.07

72. We first calculate the payments on the original mortgage:

$PV = \$500{,}000,\ i = 0.0314/12,\ n = 12\times 15 = 180$

$$PMT = PV\,\frac{i}{1-(1+i)^{-n}} = 500{,}000\,\frac{0.0314/12}{1-(1+0.0314/12)^{-180}} \approx \$3{,}486.67$$

Technology: `500000*(0.0314/12)/(1-(1+0.0314/12)^(-180))`

Now calculate the outstanding principal:

$k = 12\times 5 = 60,\ n-k = 180-60 = 120,\ PMT = 3{,}486.67$

$$\text{Outstanding principal } = PMT\,\frac{1-(1+i)^{-(n-k)}}{i}$$

$$= 3{,}486.67\,\frac{1-(1+0.0314/12)^{-120}}{0.0314/12} \approx \$358{,}680.17$$

Technology: `3486.67*(1-(1+0.0314/12)^-120)/(0.0314/12)`

Now calculate the mortgage payments at the new rate on the the outstanding principal plus prepayment fee:

$PV = \$358{,}680.17\times 1.03 \approx \$369{,}440.58,\ i = 0.0314/24,\ n = 12\times 10 = 120$

$$PMT = PV\,\frac{i}{1-(1+i)^{-n}} = 369{,}440.58\,\frac{0.0314/24}{1-(1+0.0314/24)^{-120}} \approx \$3{,}328.68$$

Technology: `369440.58*(0.0314/24)/(1-(1+0.0314/24)^(-120))`

Saving = \$3,486.67 − 3,328.68 = \$157.99

73. We first calculate the payments:

$PV = \$35{,}000,\ i = 0.0430/12,\ n = 12\times 5 = 60$

$$PMT = PV\,\frac{i}{1-(1+i)^{-n}} = 35{,}000\,\frac{0.0430/12}{1-(1+0.0430/12)^{-60}} \approx \$649.33$$

Technology: `35000*(0.0430/12)/(1-(1+0.0430/12)^(-60))`
Now calculate the amounts you could have financed in November and December:

November: $i = 0.0431/12,\ n = 60,\ PMT = 649.33$

$$PMT = PV\frac{i}{1-(1+i)^{-n}}$$

$$649.33 = PV\frac{0.0431/12}{1-(1+0.0431/12)^{-60}}$$

so $PV = 649.33\frac{1-(1+0.0431/12)^{-60}}{0.0431/12} \approx \$34{,}991.58.$

December: $i = 0.0434/12,\ n = 60,\ PMT = 649.33$

$$PMT = PV\frac{i}{1-(1+i)^{-n}}$$

$$649.33 = PV\frac{0.0434/12}{1-(1+0.0434/12)^{-60}}$$

so $PV = 649.33\frac{1-(1+0.0434/12)^{-60}}{0.0434/12} \approx \$34{,}965.95.$

74. We first calculate the payments:

$PV = \$50{,}000,\ i = 0.0424/12,\ n = 12\times 4 = 48$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 50{,}000\frac{0.0424/12}{1-(1+0.0424/12)^{-48}} \approx \$1{,}134.33$$

Technology: `50000*(0.0424/12)/(1-(1+0.0424/12)^(-48))`
Now calculate the amounts you could have financed in November and December:

November: $i = 0.0426/12,\ n = 48,\ PMT = 1{,}134.33$

$$PMT = PV\frac{i}{1-(1+i)^{-n}}$$

$$1{,}134.33 = PV\frac{0.0426/12}{1-(1+0.0426/12)^{-48}}$$

so $PV = 1{,}134.33\frac{1-(1+0.0426/12)^{-48}}{0.0426/12} \approx \$49{,}980.20.$

December: $i = 0.0429/12,\ n = 48,\ PMT = 1{,}134.33$

$$PMT = PV\frac{i}{1-(1+i)^{-n}}$$

$$1{,}134.33 = PV\frac{0.0429/12}{1-(1+0.0429/12)^{-48}}$$

so $PV = 1{,}134.33\frac{1-(1+0.0429/12)^{-48}}{0.0429/12} \approx \$49{,}950.55.$

75. Treat the card as a loan:

$PV = 5{,}000,\ i = 0.131/12,\ n = 12\times 10 = 120$

$$PMT = PV\frac{i}{1-(1+i)^{-n}}$$

$$= 5{,}000\frac{0.131/12}{1-(1+0.131/12)^{-120}} \approx \$74.95.$$

Technology: `5000*(0.131/12)/(1-(1+0.131/12)^(-120))`

76. Treat the card as a loan:

$PV = 8{,}000,\ i = 0.131/12,\ n = 12$

$$PMT = PV \frac{i}{1-(1+i)^{-n}}$$

$$= 8{,}000 \frac{0.131/12}{1-(1+0.131/12)^{-12}} \approx \$714.91.$$

Technology: `8000*(0.131/12)/(1-(1+0.131/12)^(-12))`

77. Solid Savings & Loan: $PV = \$10{,}000,\ i = 0.09/12 = 0.0075,\ n = 12 \times 4 = 48.$

$$PMT = PV \frac{i}{1-(1+i)^{-n}} = \frac{10{,}000 \times 0.0075}{1-1.0075^{-48}} \approx \$248.85$$

Technology: `10000*0.0075/(1-1.0075^(-48))`

Fifth Federal Bank & Trust: $PV = \$10{,}000,\ i = 0.07/12,\ n = 12 \times 3 = 36.$

$$PMT = PV \frac{i}{1-(1+i)^{-n}} = \frac{10{,}000 \times 0.07/12}{1-(1+0.07/12)^{-36}} \approx \$308.77$$

Technology: `10000*(0.07/12)/(1-(1+0.07/12)^(-36))`

Answer: You should take the loan from Solid Savings & Loan: It will have payments of $248.85 per month. The payments on the other loan would be more than $300 per month.

78. 10% Loan: $PV = \$20{,}000,\ i = 0.10/12,\ n = 12 \times 5 = 60$

$$PMT = PV \frac{i}{1-(1+i)^{-n}} = \frac{20{,}000 \times 0.10/12}{1-(1+0.10/12)^{-60}} \approx \$424.94$$

Technology: `20000*(0.10/12)/(1-(1+0.10/12)^(-60))`

9% Loan: $PV = \$20{,}000,\ i = 0.09/12 = 0.0075,\ n = 12 \times 4 = 48$

$$PMT = PV \frac{i}{1-(1+i)^{-n}} = \frac{20{,}000 \times 0.0075}{1-1.0075^{-48}} \approx \$497.70$$

Technology: `20000*0.0075/(1-1.0075^(-48))`

The first loan will have lower monthly payments.

Total Interest Payments:

10% Loan: You pay a total of $60 \times \$424.94 = \$25{,}496.40$. Of this, $\$25{,}496.40 - 20{,}000 = \$5{,}496.40$ is interest.

9% Loan: You pay a total of $48 \times \$497.70 = \$23{,}889.60$. Of this, $\$23{,}889.60 - 20{,}000 = \$3{,}889.60$ is interest.

Thus, the first loan will have a larger total interest payment.

79. We first calculate the original payments:

$PV = \$96{,}000,\ i = 0.0975/12,\ n = 12 \times 30 = 360$

$$PMT = PV \frac{i}{1-(1+i)^{-n}} = 96{,}000 \frac{0.0975/12}{1-(1+0.0975/12)^{-360}} \approx \$824.79$$

Technology: `96000*(0.0975/12)/(1-(1+0.0975/12)^(-360))`

Now calculate the outstanding principal:

$k = 12 \times 4 = 48,\ n-k = 360-48 = 312,\ PMT = 824.79$

$$\text{Outstanding principal } = PMT \frac{1-(1+i)^{-(n-k)}}{i}$$

$$= 824.79 \frac{1-(1+0.0975/12)^{-312}}{0.0975/12} \approx \$93{,}383.71$$

Technology: `824.79*(1-(1+0.0975/12)^(-312))/(0.0975/12)`

Total Interest Paid During the First Loan = Sum of payments – Reduction in principal

$$= 48 \times 824.79 - (96{,}000 - 93{,}383.71) = \$36{,}973.63$$

Now calculate the mortgage payments at the new rate on the outstanding principal:

$PV = \$93{,}383.71,\ i = 0.06875/12,\ n = 12 \times 30 = 360$

$$PMT = PV \frac{i}{1-(1+i)^{-n}} = 93{,}383.71 \frac{0.06875/12}{1-(1+0.06875/12)^{-360}} \approx \$613.46$$

Technology: `93383.71*(0.06875/12)/(1-(1+0.06875/12)^(-360))`

Total Interest Paid During the New Loan = Sum of payments – Reduction in principal

$= 360 \times 613.46 - 93{,}383.71 = \$127{,}461.89$

Thus, the total interest paid over the duration of the two loans is $36,973.63 + 127,461.89 = $164,435.52.

Had the mortgage not been refinanced, the total interest would have been

Total Interest Paid = Sum of payments – Reduction in principal

$= 360 \times 824.79 - 96{,}000 = \$20{,}0924.40$.Thus, the saving on interest is

$\$200{,}924.40 - 164{,}435.52 = \$36{,}488.88$

80. We first calculate the original payments:

$PV = \$120{,}000,\ i = 0.10/12,\ n = 12 \times 30 = 360$

$$PMT = PV \frac{i}{1-(1+i)^{-n}} = 120{,}000 \frac{0.10/12}{1-(1+0.10/12)^{-360}} \approx \$1{,}053.09$$

Technology: `120000*(0.10/12)/(1-(1+0.10/12)^(-360))`

Now calculate the outstanding principal:

$k = 12 \times 3 = 36,\ n-k = 360 - 36 = 324,\ PMT = 1{,}053.09$

$$\text{Outstanding principal } = PMT \frac{1-(1+i)^{-(n-k)}}{i}$$

$$= 1{,}053.09 \frac{1-(1+0.10/12)^{-324}}{0.10/12} \approx \$117{,}782.44$$

Technology: `1053.09*(1-(1+0.10/12)^(-324))/(0.10/12)`

Total Interest Paid = Sum of payments – Reduction in principal

$= 36 \times 1{,}053.09 - (120{,}000 - 117{,}782.44) = \$35{,}693.68$

Now calculate the mortgage payments at the new rate on the outstanding principal:

$PV = \$117{,}782.44,\ i = 0.065/12,\ n = 12 \times 15 = 180$

$$PMT = PV \frac{i}{1-(1+i)^{-n}} = 117{,}782.44 \frac{0.065/12}{1-(1+0.065/12)^{-180}} \approx \$1{,}026.01$$

Technology: `117,782.44*(0.065/12)/(1-(1+0.065/12)^(-180))`

Total Interest Paid During the New Loan = Sum of payments – Reduction in principal

$= 180 \times 1{,}026.01 - 117{,}782.44 = \$66{,}899.36$

Thus, the total interest paid over the duration of the loan is $35,693.68 + 66,899.36 = $102,593.04.

Had the mortgage not been refinanced, the total interest would have been

Total Interest Paid = Sum of payments – Reduction in principal

$= 360 \times 1{,}053.09 - 120{,}000 = \$259{,}112.40$.Thus, the saving on interest is

$\$259{,}112.40 - 102{,}593.04 = \$156{,}519.36$

Solutions Section 2.3

81. We can construct an amortization table using the technique outlined in the Technology Guides. For example, using Excel, we might set it up as follows:

	A	B	C	D	E	F
1	Month	Outstanding Principal	Payment on Principal	Interest Payment		
2	0	$ 50,000.00			Rate	8%
3	1	=B2-C3	=F$4-D3	=DOLLAR(B2*F$2/12)	Years	15
4	2				Payment	=DOLLAR(-PMT(F2/12,F3*12,B2))
5	3					
6	4					
7	5					
8	6					

Adding the payments on principal (Column C) and interest payments (Column D) for each year will give the following table:

Year	Interest	Payment on Principal
1	$3,934.98	$1,798.98
2	$3,785.69	$1,948.27
3	$3,623.97	$2,109.99
4	$3,448.84	$2,285.12
5	$3,259.19	$2,474.77
6	$3,053.77	$2,680.19
7	$2,831.32	$2,902.64
8	$2,590.39	$3,143.57
9	$2,329.48	$3,404.48
10	$2,046.91	$3,687.05
11	$1,740.88	$3,993.08
12	$1,409.47	$4,324.49
13	$1,050.54	$4,683.42
14	$661.81	$5,072.15
15	$240.84	$5,491.80

82. We can create an amortization table as in Exercise 81. We will get the following:

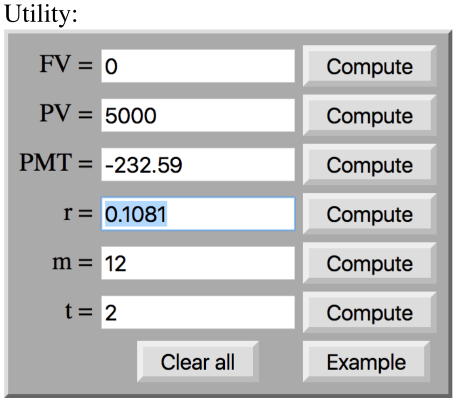
Year	Interest	Payment on Principal
1	$9,238.08	$556.32
2	$9,181.34	$613.06
3	$9,118.83	$675.57
4	$9,049.93	$744.47
5	$8,974.02	$820.38
6	$8,890.36	$904.04
7	$8,798.17	$996.23
8	$8,696.56	$1,097.84
9	$8,584.62	$1,209.78
10	$8,461.26	$1,333.14
11	$8,325.30	$1,469.10
12	$8,175.50	$1,618.90
13	$8,010.39	$1,784.01
14	$7,828.46	$1,965.94
15	$7,627.96	$2,166.44

Adding up the payments on principal gives a total of $17,955.22 paid on principal over the first 15 years, so $\$95{,}000 - 17{,}955.22 = \$77{,}044.78$ is still owed on the mortgage.

83. The payments on the loan, ignoring the fee, are

$$PMT = 5{,}000\frac{0.09/12}{1-(1+0.09/12)^{-24}} = \$228.42.$$

Add to each payment 100/24 = $4.17 to get a new payment of $232.59. Now use technology to find the interest rate being charged on a 2-year $5,000 loan with this payment. For example, we can use the Online Time Value of Money Utility:

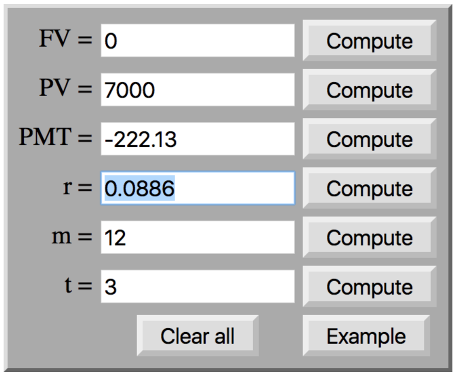


We see that the interest rate is 10.81%.

84. The payments on the loan, ignoring the fee, are

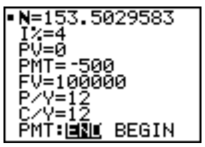
$$PMT = 7{,}000\frac{0.08/12}{1-(1+0.08/12)^{-36}} = \$219.35.$$

Add to each payment 100/36 = $2.78 to get a new payment of $222.13. Now use technology to find the interest rate being charged on a 3-year $7,000 loan with this payment. For example, we can use the Online Time Value of Money Utility:



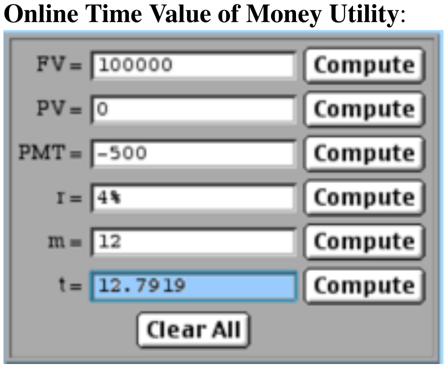
We see that the interest rate is 8.86%.

85. TI-83/84 Plus:

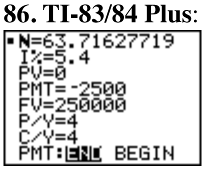


This gives 153.5/12 ≈ 13 years to retirement.

Online Time Value of Money Utility:

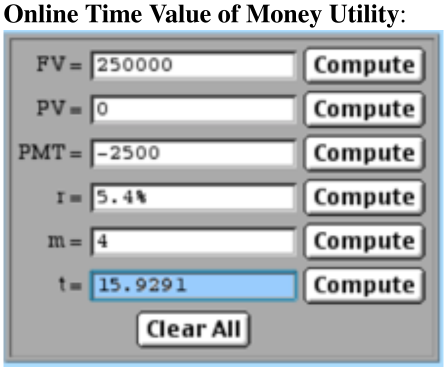


86. TI-83/84 Plus:



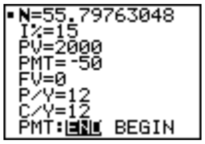
This gives 63.72/4 ≈ 16 years to retirement.

Online Time Value of Money Utility:



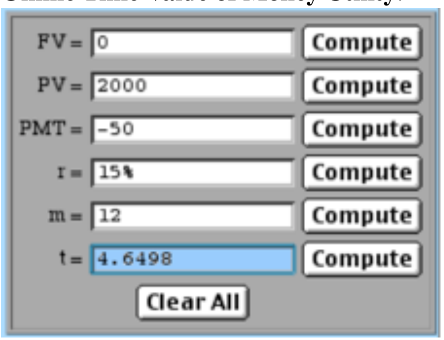
Solutions Section 2.3

87. TI-83/84 Plus:

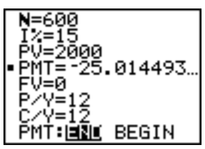


This gives 55.798/12 ≈ 4.5 years to repay the debt.

Online Time Value of Money Utility:

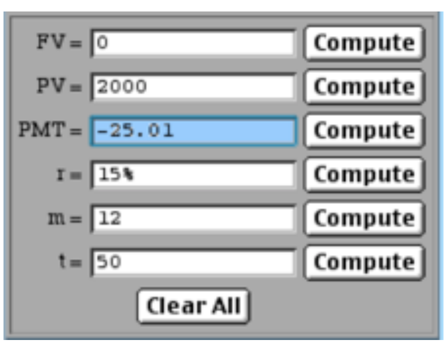


88. Since we are not told what "eventually" means, let us try a 50-year schedule. We get
TI-83/84 Plus:



In other words, if we pay $25.01 it will take us 50 years to pay off the debt. Larger values of n give amounts fractionally larger than $25.00. To be assured of eventually paying off the debt, you should therefore pay $25.01.

Online Time Value of Money Utility:



Another way of seeing this: The monthly interest on $2,000 at 15% is
$\$2000 \times 0.15/12 = \$25.00$.

So, if you pay only $25.00, all you are paying is interest and you are not reducing the $2,000 principal.

89. Graph the future value of both accounts using the formula for the future value of a sinking fund.

Your account: $i = 0.045/12 = 0.00375,\ PMT = \$100$.

$$FV = PMT\frac{(1+i)^n - 1}{i} = 100\frac{1.00375^n - 1}{0.00375}$$

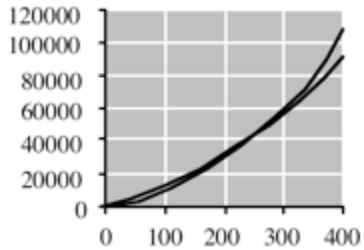
To graph this, use the technology formula `100*(1.00375^x-1)/0.00375`.

Lucinda's account: $i = 0.065/12,\ PMT = \$75$.

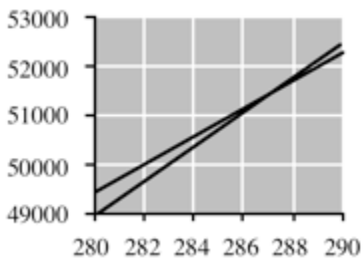
Solutions Section 2.3

$$FV = PMT\frac{(1+i)^n - 1}{i} = 75\frac{(1+0.065/12)^n - 1}{(0.065/12)}$$

To graph this, use the technology formula `75*((1+0.065/12)^x-1)/(0.065/12)`
Graph:

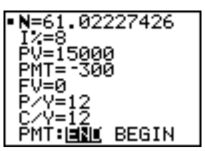


The graphs appear to cross around $n = 300$, so we zoom in there:



The graphs cross around $n = 287$, so the number of years is approximately $t = 287.5/12 \approx 24$ years.

90. If you purchased the car with an 8% per year loan, the length of the loan would be given as follows:
TI-83/84 Plus:



To the nearest year, this is $61/5 \approx 5$ years. Leasing the car for less time would result in lower payments, so the answer is 5 years or less.

Communication and reasoning exercises

91. He is wrong because his estimate ignores the interest that will be earned by your annuity—both while it is increasing and while it is decreasing. Your payments will be considerably smaller (depending on the interest earned).

92. She is incorrect. For example, compare Option 1, making a single payment of $10,000 and letting it earn interest for the next 10 years, with Option 2, making yearly payments of $1,000 for 10 years. All $10,000 earns interest for all 10 years in Option 1, while most of your money earns interest for a shorter period of time (and hence earns less interest) in Option 2.

93. Wrong; the split investment earns more. For instance, after ten years it earns $31,056.46 + $32,775.87 = $63,832.33, which is more than the $63,803.03 earned by the single investment.
(Mathematically, the future value does not depend linearly on the interest rate.)

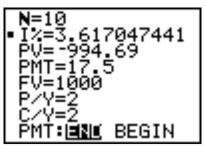
94. Wrong; the combined investments earn net interest. For instance, after 10 years the value is: \$62,112.91 + \$37,833.85 = \$99,946.76, which is more than the accumulated payments: \$96,000.

95. He is not correct. For instance, the payments on a \$100,000 10-year mortgage at 12% are \$1,434.71, while for a 20-year mortgage at the same rate, they are \$1,101.09, which is a lot more than half the 10-year mortgage payment.

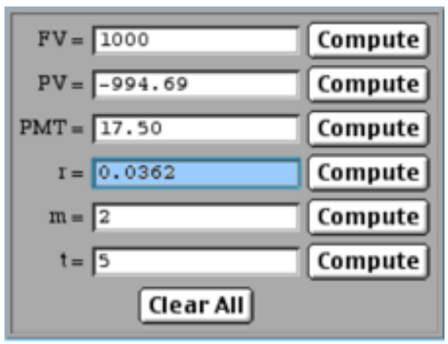
96. He is correct. For instance, the payments on a \$100,000 10-year mortgage at 12% are \$1,434.71, while for a \$200,000 10-year mortgage at the same rate, they are twice that amount: \$2,869.42.

97. $FV =$ maturity value $= \$1{,}000$, $PMT = 1{,}000(0.035/2) = \$17.50$, $PV =$ selling price $= \$994.69$, $n = 2 \times 5 = 10$. Using technology, we compute the interest rate to be approximately 3.617%. (The online utility rounds this to two decimal places.)

TI-83/84 Plus:

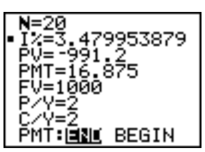


Online Time Value of Money Utility
("Years" mode):

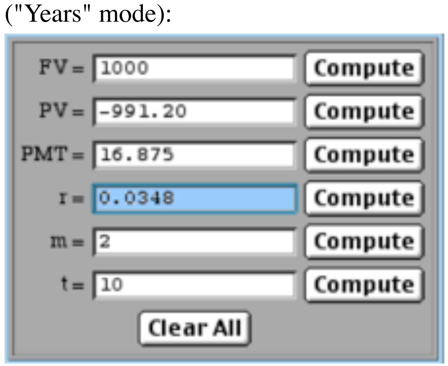


98. $FV = \$1{,}000$, $PMT = 1{,}000(0.03375/2) = \$16.875$, $PV = \$991.20$, $n = 2 \times 10 = 20$. Using technology, we compute the interest rate to be approximately 3.480%. (The online utility rounds this to two decimal places.)

TI-83/84 Plus:



Online Time Value of Money Utility
("Years" mode):



99. $PV = FV(1+i)^{-n} = PMT\dfrac{(1+i)^n - 1}{i}(1+i)^{-n} = PMT\dfrac{1-(1+i)^{-n}}{i}$

100. There are several possible explanations, including the following: Consider two funds with identical interest rates and compounding: The accumulating fund (beginning with no money) and the payment fund, beginning with the present value necessary to fund the payments. Move each payment from the payment fund to the accumulating fund. Since the funds pay the same interest, the net effect is a single account paying compound interest, with no payments or withdrawals. Therefore, the present value of the payment fund must give the future value of the accumulating fund using the future value formula for compound interest.

Solutions Chapter 2 Review

Chapter 2 Review

1. $FV = 6{,}000(1 + 0.0475 \times 5) = \$7{,}425.00$

2. $FV = 10{,}000(1 + 0.0525 \times 2.5) = \$11{,}312.50$

3. $FV = 6{,}000(1 + 0.0475/12)^{60} = \$7{,}604.88$

4. $FV = 10{,}000(1 + 0.0525/2)^{5} = \$11{,}383.24$

5. $FV = 100\dfrac{(1 + 0.0475/12)^{60} - 1}{0.0475/12} = \$6{,}757.41$

6. $FV = 2{,}000\dfrac{(1 + 0.0525/2)^{5} - 1}{0.0525/2} = \$10{,}538.96$

7. $PV = 6{,}000/(1 + 0.0475 \times 5) = \$4{,}848.48$

8. $PV = 10{,}000/(1 + 0.0525 \times 2.5) = \$8{,}839.78$

9. $PV = 6{,}000(1 + 0.0475/12)^{-60} = \$4{,}733.80$

10. $PV = 10{,}000(1 + 0.0525/2)^{-5} = \$8{,}784.85$

11. $PV = 100\dfrac{1 - (1 + 0.0475/12)^{-60}}{0.0475/12} = \$5{,}331.37$

12. $PV = 2{,}000\dfrac{1 - (1 + 0.0525/2)^{-5}}{0.0525/2} = \$9{,}258.32$

13.
$PMT = 12{,}000\dfrac{0.0475/12}{(1 + 0.0475/12)^{60} - 1} = \$177.58$

14. $PMT = 20{,}000\dfrac{0.0525/2}{(1 + 0.0525/2)^{5} - 1} = \$3{,}795.44$

15.
$PMT = 6{,}000\dfrac{0.0475/12}{1 - (1 + 0.0475/12)^{-60}} = \$112.54$

16. $PMT = 10{,}000\dfrac{0.0525/2}{1 - (1 + 0.0525/2)^{-5}} = \$2{,}160.22$

17.
$PMT = 10{,}000\dfrac{0.0475/12}{1 - (1 + 0.0475/12)^{-60}} = \$187.57$

18. $PMT = 15{,}000\dfrac{0.0525/2}{1 - (1 + 0.0525/2)^{-5}} = \$3{,}240.33$

19. $10{,}000 = 6{,}000(1 + 0.0475t) = 6{,}000 + 285t$. $t = (10{,}000 - 6{,}000)/285 = 14.0$ years.

20. $15{,}000 = 10{,}000(1 + 0.0525t) = 10{,}000 + 525t$. $t = (15{,}000 - 10{,}000)/525 = 9.5$ years

21. $10{,}000 = 6{,}000(1 + 0.0475/12)^{12t}$. To solve algebraically requires logarithms:

$$t = \frac{\log(10{,}000/6{,}000)}{12\log(1 + 0.0475/12)} \approx 10.8 \text{ years}$$

We could also find this using, for example, the TI-83/84 Plus TVM Solver.

22. $15{,}000 = 10{,}000(1 + 0.0525/2)^{2t}$. To solve algebraically requires logarithms:

$$t = \frac{\log(15{,}000/10{,}000)}{2\log(1 + 0.0525/2)} \approx 7.8 \text{ years}$$

We could also find this using, for example, the TI-83/84 Plus TVM Solver.

23. $10{,}000 = 100\dfrac{(1 + 0.0475/12)^{12t} - 1}{0.0475/12}$. To solve algebraically requires logarithms:

$$t = \frac{\log[0.0475 \times 10{,}000/(100 \times 12) + 1]}{12\log(1 + 0.0475/12)} \approx 7.0 \text{ years}$$

We could also find this using, for example, the TI-83/84 Plus TVM Solver.

24. $15{,}000 = 2{,}000\dfrac{(1 + 0.0525/2)^{2t} - 1}{0.0525/2}$. To solve algebraically requires logarithms:

$$t = \frac{\log[0.0525 \times 15{,}000/(2{,}000 \times 2) + 1]}{12\log(1 + 0.0525/2)} \approx 3.5 \text{ years}$$

We could also find this using, for example, the TI-83/84 Plus TVM Solver.

25. Each interest payment is $10{,}000 \times 0.06/2 = \$300$. For an annuity earning 7% and paying \$300 every 6 months for 5 years, the present value is

$$PV = 300\frac{1 - (1 + 0.07/2)^{-10}}{0.07/2} = \$2{,}494.98.$$

The present value of the \$10,000 maturity value is

$$PV = 10{,}000(1 + 0.07/2)^{-10} = \$7{,}089.19.$$

The total price is $\$2{,}494.98 + 7{,}089.19 = \$9{,}584.17$.

26. Each interest payment is $10{,}000 \times 0.06/2 = \$300$. For an annuity earning 5% and paying \$300 every 6 months for 5 years, the present value is

$$PV = 300\frac{1 - (1 + 0.05/2)^{-10}}{0.05/2} = \$2{,}625.62.$$

The present value of the \$10,000 maturity value is

$$PV = 10{,}000(1 + 0.05/2)^{-10} = \$7{,}811.98.$$

The total price is $\$2{,}625.62 + 7{,}811.98 = \$10{,}437.60$.

27. 5.346% (using, for example, the TI-83/84 Plus TVM Solver)

28. 4.662% (using, for example, the TI-83/84 Plus TVM Solver)

29. $PV = 3.28,\ FV = 45.74,\ i = r/12,\ n = 9$ (months)

$$45.74 = 3.28(1 + 92r/12) \approx 3.28 + 25.1467r$$

$$r \approx (45.74 - 3.28)/25.1467 \approx 1.6885 = 168.85\%$$

30. $PV = 33.95,\ FV = 12.36,\ i = r/12,\ n = 22$ (months)

$$12.36 = 33.95(1 + 22r/12) \approx 33.95 + 62.2417r$$

$$r \approx (12.36 - 33.95)/62.2417 \approx -0.3469 = -34.69\%$$

31. The only dates on which she would have gotten an increase rather than a decrease were November 2009, February 2010, and August 2010. Calculating the annual returns as in the preceding exercises, we get the following figures:

Jan. 2007–Nov. 2009: 60.45%
Jan. 2007–Feb. 2010: 85.28%
Jan. 2007–Aug. 2010: 75.37%

The largest annual return was 85.28% if she sold in February 2010.

32. The only dates on which he would have gotten a loss rather than an increase were December 2005, January 2007, March 2008, and October 2008. Calculating the annual returns as in the preceding exercises, we get the following figures:

Aug. 2004–Dec. 2005: −40.79%

Aug. 2004–Jan. 2007: −10.02%

Aug. 2004–Mar. 2008: −15.81%

Aug. 2004–Oct. 2008: −7.17%

The largest annual loss was 40.79% if he sold in December 2005.

33. No. Simple interest increase is linear. We can compare slopes between successive points to see whether the slope remained roughly constant: From December 2002 to August 2004 the slope was (16.31 − 3.28)/(20/12) = 7.818, while from August 2004 to March 2005 the slope was (33.95 − 16.31)/(7/12) = 30.24. These slopes are quite different.

34. First calculate the annual interest rate from February 2010 through August 2010:

$r = (45.74 - 44.86)/44.86/(6/12) \approx 0.03923$

Now use the simple interest formula to get the price in December 2011:

$FV \approx 44.86(1 + 0.03923 \times 22/12) \approx 48.09$

35. Use the compound interest formula: $FV = PV(1+i)^n$, where $PV = \$150{,}000,\ i = 0.20,\ n = 1,\ 2, 3, 4, 5.$

2010: $FV = 150{,}000(1.20) = \$180{,}000$

2011: $FV = 150{,}000(1.20)^2 = \$216{,}000$

2012: $FV = 150{,}000(1.20)^3 = \$259{,}200$

2013: $FV = 150{,}000(1.20)^4 = \$311{,}040$

2014: $FV = 150{,}000(1.20)^5 = \$373{,}248$

Revenues first surpass $300,000 in 2013.

36. $PV = \$20{,}000,\ i = -0.0375,\ n = 7$ (quarters)

$FV = PV(1+i)^n = 20{,}000(1 - 0.0375)^7 \approx \$15{,}305.06$

37. After the first day of trading, the value of each share will be $6. Thereafter, the shares appreciate in value by 8% per month for 6 months, and O'Hagan desires a future value of at least $500,000.

$FV = 500{,}000,\ i = 0.08,\ n = 6$

$PV = FV(1+i)^{-n} = 500{,}000 \times 1.08^{-6} \approx \$315{,}084.81$

Therefore, since each share will be worth $6 after the first day, the number of shares they must sell is at least

$\dfrac{315{,}084.81}{6} \approx 52{,}514.14.$

Since they can offer only a whole number of shares, we must round this up to get the minimum desired future value. Thus, they should offer at least 52,515 shares.

38. After the first day of trading, the value of each share will be $3(1 − 0.6) = $1.20. Thereafter, the shares depreciate in value by 10% per week for 5 weeks, and so the future value is

$FV = PV(1+i)^n = 1.20(1 - 0.10)^5 \approx \$0.71$ per share,

so the approximate market value is 600,000 × 0.71 = $426,000.

39. $PV = 250{,}000,\ i = 0.095/12,\ n = 12 \times 10 = 120$

Solutions Chapter 2 Review

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = \frac{250{,}000 \times 0.095/12}{1-(1+0.095/12)^{-120}} \approx \$3{,}234.94$$

40. $PV = 250{,}000,\ i = 0.065/12,\ n = 12 \times 8 = 96$

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = \frac{250{,}000 \times 0.065/12}{1-(1+0.065/12)^{-96}} \approx \$3{,}346.56$$

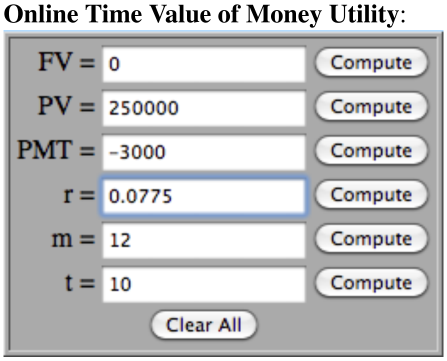
41. $i = 0.095/12,\ PMT = 3{,}000,\ n = 12 \times 10 = 120$

$$PV = PMT\frac{1-(1+i)-n}{i} = 3{,}000\frac{1-(1+0.095/12)^{-120}}{(0.095/12)} \approx \$231{,}844 \text{ (to the nearest dollar)}$$

42. $i = 0.065/12,\ PMT = 3{,}000,\ n = 12 \times 8 = 96$

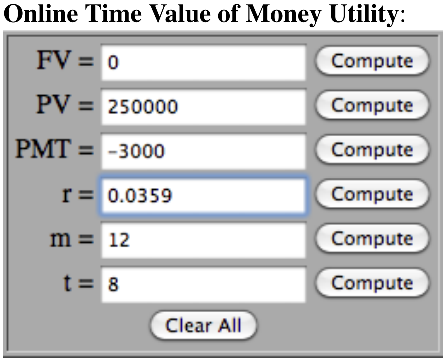
$$PV = PMT\frac{1-(1+i)-n}{i} = 3{,}000\frac{1-(1+0.065/12)^{-96}}{(0.065/12)} \approx \$224{,}111 \text{ (to the nearest dollar)}$$

43. $PV = 250{,}000,\ FV = 0,\ PMT = 3{,}000,\ n = 12 \times 10 = 120,$ and we are seeking the interest rate.
Online Time Value of Money Utility:



The interest rate would be 7.75%.

44. $PV = 250{,}000,\ FV = 0,\ PMT = 3{,}000,\ n = 12 \times 8 = 96,$ and we are seeking the interest rate.
Online Time Value of Money Utility:



The interest rate would be 3.59%.

45. $PV = 50{,}000,\ PMT = 1{,}000 + 800\text{ (company contribution) } = 1{,}800,\ i = 0.073/12,\ n = 12 \times 10 = 120.$
Considering the contribution of the present $50,000 as well as the payments, we get

$$FV = PV(1+i)^n + PMT\frac{(1+i)^n - 1}{i} = 50{,}000(1+0.073/12)^{120} + 1{,}800\frac{(1+0.073/12)^{120}-1}{0.073/12} \approx \$420{,}275$$

(to the nearest dollar).

Technology: `50000*(1+0.073/12)^120+1800*((1+0.073/12)^120-1)/(0.073/12)`

46. $PV = 60{,}000,\ PMT = 950 + 800$ (company contribution) $= 1{,}750,\ i = 0.073/12,\ n = 12 \times 8 = 96.$
Considering the contribution of the present \$60,000 as well as the payments, we get

$$FV = PV(1+i)^n + PMT\frac{(1+i)^n - 1}{i} = 60{,}000(1+0.073/12)^{96} + 1{,}750\frac{(1+0.073/12)^{96} - 1}{0.073/12} \approx \$334{,}670$$

(to the nearest dollar).
Technology: `60000*(1+0.073/12)^96+1750*((1+0.073/12)^96-1)/(0.073/12)`

47. For the company's contribution, take $PMT = 800,\ i = 0.073/12,\ n = 12 \times 10 = 120.$

$$FV = PMT\frac{(1+i)^n - 1}{i} = 800\frac{(1+0.073/12)^{120} - 1}{0.073/12} \approx \$140{,}778$$

(to the nearest dollar).
Technology: `800*((1+0.073/12)^120-1)/(0.073/12)`

48. For the company's contribution, take $PMT = 800,\ i = 0.073/12,\ n = 12 \times (8+5) = 156.$

$$FV = PMT\frac{(1+i)^n - 1}{i} = 800\frac{(1+0.073/12)^{156} - 1}{0.073/12} \approx \$207{,}217$$

(to the nearest dollar).
Technology: `800*((1+0.073/12)^156-1)/(0.073/12)`

49. We first take out the effect of the initial \$50,000: $PV = 50{,}000,\ i = 0.073/12,\ n = 12 \times 10 = 120.$

$$FV = PV(1+i)^n = 50{,}000(1+0.073/12)^{120}$$

Thus, the payments have to result in a future value of

$$FV = 500{,}000 - 50{,}000(1+0.073/12)^{120} \approx 396{,}475.163$$

We can now use the payment formula to determine the necessary payments:

$$PMT = FV\frac{i}{(1+i)^n - 1} \approx \frac{396{,}475.163 \times 0.073/12}{(1+0.073/12)^{120} - 1} \approx \$2{,}253.06$$

Since \$800 of this is contributed by the company, Callahan's payments should be

\$2,253.06 – 800 = \$1,453.06.

50. We first take out the effect of the initial \$60\,000: $PV = 50{,}000,\ i = 0.073/12,\ n = 12 \times 8 = 96.$

$$FV = PV(1+i)^n = 60{,}000(1+0.073/12)^{96}$$

Thus, the payments have to result in a future value of

$$FV = 600{,}000 - 60{,}000(1+0.073/12)^{96} \approx \$492{,}598.3656$$

We can now use the payment formula to determine the necessary payments:

$$PMT = FV\frac{i}{(1+i)^n - 1} \approx \frac{492{,}598.3656 \times 0.073/12}{(1+0.073/12)^{96} - 1} \approx \$3{,}793.08$$

Since \$800 of this is contributed by the company, Egan's payments should be

\$3,793.08 – 800 = \$2,993.08.

51. First compute the amount Callahan needs at the start of retirement: $PMT = 5{,}000,\ i = 0.087/12,$
$n = 12 \times 30 = 360$

$$PV = PMT\frac{1-(1+i)-n}{i} = 5{,}000\frac{1-(1+0.087/12)^{-360}}{0.087/12} \approx \$638{,}461.93$$

Technology: `5000*(1-(1+0.087/12)^(-360))/(0.087/12)`

In order to accumulate this amount, using the information from Exercise 45, we first discount the effect of the current \$50,000 in the account: $PV = 50{,}000$, $FV = 638{,}461.93$, $i = 0.073/12$, $n = 12 \times 10 = 120$. The initial \$50,000 will grow to

$$50{,}000(1 + 0.073/12)^{120}$$

so the payments need to result in a future value of only

$$638{,}461.93 - 50{,}000(1 + 0.073/12)^{120} \approx 534{,}937.093.$$

Now use the payment formula:

$$PMT = FV \frac{i}{(1+i)^n - 1} = \frac{534{,}937.093 \times 0.073/12}{(1 + 0.073/12)^{120} - 1} \approx \$3{,}039.90$$

Since \$800 of this is contributed by the company, Callahan's payments should be

$$\$3{,}039.90 - 800 = \$2{,}239.90.$$

52. First compute the amount Egan needs at the start of retirement: $PMT = 6{,}000$, $i = 0.078/12$, $n = 12 \times 25 = 300$

$$PV = PMT \frac{1 - (1+i) - n}{i} = 6{,}000 \frac{1 - (1 + 0.078/12)^{-300}}{0.078/12} \approx \$790{,}915.68$$

Technology: `6000*(1-(1+0.078/12)^(-300))/(0.078/12)`

In order to accumulate this amount, we first discount the effect of the current \$60,000 in the account: $PV = 60{,}000$, $i = 0.073/12$, $n = 12 \times 8 = 96$.

$$FV = PV(1+i)^n = 60{,}000(1 + 0.073/12)^{96}$$

Thus, the payments have to result in a future value of

$$FV = 790{,}915.68 - 60{,}000(1 + 0.073/12)^{96} \approx \$683{,}514.04.$$

Now use the payment formula:

$$PMT = FV \frac{i}{(1+i)^n - 1} \approx \frac{683{,}514.04 \times 0.073/12}{(1 + 0.073/12)^{96} - 1} \approx \$5{,}263.17$$

Since \$800 of this is contributed by the company, Egan's payments should be

$$\$5{,}263.17 - 800 = \$4{,}463.17.$$

53. The bond will pay interest every 6 months amounting to

$$50{,}000(0.072/2) = \$1{,}800.$$

For someone purchasing this bond after one year, there will be 9 years to maturity. Think of the bond as an investment that will pay the owner \$1,800 every 6 months for 9 years, at which time it will pay \$50,000. This is exactly the behavior of an annuity paired with an investment with future value \$50,000

$$FV = 50{,}000,\ PMT = 1{,}800,\ i = 0.063/2 = 0.0315,\ n = 2 \times 9 = 18$$

The present value has contributions both from the investment and the annuity:

$$PV = FV(1+i)^{-n} + PMT \frac{1 - (1+i)^{-n}}{i} = 50{,}000(1.0315)^{-18} + 1{,}800 \frac{1 - 1.0315^{-180}}{0.0315} \approx \$53{,}055.66$$

Technology: `50000*1.0315^(-18)+1800*(1-1.0315^(-18))/0.0315`

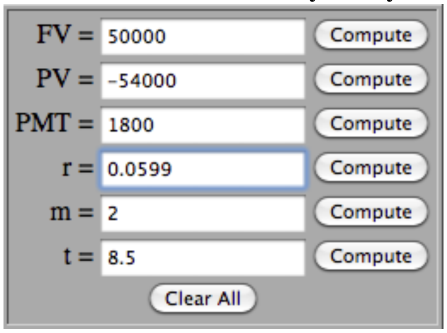
54. This is similar to Exercise 53, but with $FV = 50{,}000$, $PMT = 1{,}800$, $i = 0.06/2 = 0.03$, $n = 2 \times 8.5 = 17$

$$PV = FV(1+i)^{-n} + PMT \frac{1 - (1+i)^{-n}}{i} = 50{,}000(1.03)^{-17} + 1{,}800 \frac{1 - 1.03^{-170}}{0.03} \approx \$53{,}949.84$$

Technology: `50000*1.03^(-17)+1800*(1-1.03^(-17))/0.03`

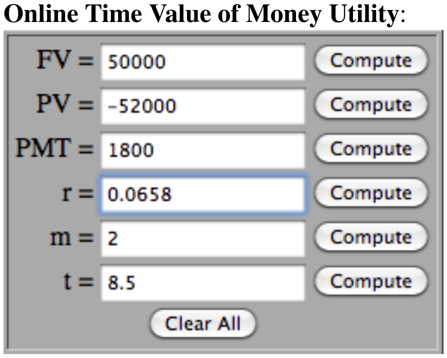
Solutions Chapter 2 Review

55. Here, $FV = 50{,}000$, $PV = 54{,}000$, $PMT = 1{,}800$, $n = 2 \times 8.5 = 17$, and we are seeking the interest rate.
Online Time Value of Money Utility:



The interest rate would have to be 5.99%.

56. Here, $FV = 50{,}000$, $PV = 52{,}000$, $PMT = 1{,}800$, $m = 2$, $t = 8.5$, and we are seeking the interest rate.
Online Time Value of Money Utility:



The interest rate would have to be 6.58%.

Solutions Chapter 2 Case Study

Chapter 2 Case Study

1. The start of the sixth year is the critical time; if the Wongs can afford payments then, they will continue to be able to afford them throughout the loan as one can verify by glancing at the worksheet. To solve with Excel, use one of the three TVM utilities discussed in the book or, in Excel, use

```
=RATE(12*25,-2271.09,343700.55)*12
```

This will return approximately 0.06267892, or 6.27% per year. Since this is 5% above the Fed discount rate, the latter would have to be 1.27%.

2. This is similar to Exercise 1, and we use

```
=RATE(12*25,-2271.09,380000)*12
```

which returns approximately 0.0522, or 5.22%, meaning that the Fed rate would have to be 0.22%.

3. This is similar to Exercise 1, and we use

```
=RATE(12*25,-2271.09,400640.49)*12
```

which returns approximately 0.0469, or 4.69%, meaning that the Fed rate would have to be negative, which is impossible; the Wongs could never make the payments regardless of the Fed rate in this case.

4. Adjusting the price in cell K3 gives affordable payments in the most expensive scenario for a $195,000 mortgage. Adding the $20,000 down payment, the Wongs could afford a $215,000 home.

5. Adjusting the price in cell K3 gives affordable payments in the most expensive scenario for a $170,000 mortgage (to the nearest $5,000). Adding the $20,000 down payment, the Wongs could afford a $190,000 home.

6. Of the three types of mortgage, the hybrid is the most affordable. and the first steep payment (in worst-case scenario 3) is the $4,402.22. 28% of their income first passes that level in year 23 (see the spreadsheet), meaning that they would have to wait until year $23-5=18$, or 17 more years, before being able to afford such a home.

0 Precalculus Review

DreamPictures/Taxi/Getty Images

0.2 Exponents and Radicals

Integer Exponents

Integer Exponents

Positive Integer Exponents

If a is any real number and n is any positive integer, then by a^n we mean the quantity $a \cdot a \cdot \ldots \cdot a$ (n times); thus, $a^1 = a$, $a^2 = a \cdot a$, $a^5 = a \cdot a \cdot a \cdot a \cdot a$. In the expression a^n the number n is called the **exponent**, and the number a is called the **base**.

Quick Example

$3^2 = 9$

Integer Exponents

Negative Integer Exponents

If *a* is any real number *other than zero* and *n* is any positive integer, then we define

$$a^{-n} = \frac{1}{a^n} = \frac{1}{a \cdot a \cdot \ldots \cdot a} \text{ (} n \text{ times).}$$

Quick Example

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Integer Exponents

Zero Exponent

If *a* is any real number other than zero, then we define

$a^0 = 1.$

Quick Examples

$3^0 = 1$

0^0 is not defined

Integer Exponents

When combining exponential expressions, we use the following identities.

Exponent Identity	Quick Examples
1. $a^m a^n = a^{m+n}$	$2^3 2^2 = 2^{3+2} = 2^5 = 32$
2. $\dfrac{a^m}{a^n} = a^{m-n}$ if $a \neq 0$	$\dfrac{4^3}{4^2} = 4^{3-2} = 4^1 = 4$
3. $(a^n)^m = a^{nm}$	$(3^2)^2 = 3^4 = 81$
4. $(ab)^n = a^n b^n$	$(4 \cdot 2)^2 = 4^2 2^2 = 64$
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ if $b \neq 0$	$\left(\dfrac{4}{3}\right)^2 = \dfrac{4^2}{3^2} = \dfrac{16}{9}$

Example 1 – *Combining the Identities*

$$\frac{(x^2)^3}{x^3} = \frac{x^6}{x^3}$$ By identity (3)

$$= x^{6-3}$$ By identity (2)

$$= x^3$$

$$\frac{(x^4y)^3}{y} = \frac{(x^4)^3y^3}{y}$$ By identity (4)

$$= \frac{x^{12}y^3}{y}$$ By identity (3)

$$= x^{12}y^{3-1}$$ By identity (2)

$$= x^{12}y^2$$

Radicals

Radicals

If *a* is any non-negative real number, then its **square root** is the non-negative number whose square is *a*.

For example, the square root of 16 is 4, because $4^2 = 16$. We write the square root of *n* as $\sqrt{n}$. (Roots are also referred to as **radicals**.) It is important to remember that $\sqrt{n}$ is never negative. Thus, for instance, $\sqrt{9}$ is 3 and not –3, even though $(-3)^2 = 9$. If we want to speak of the “negative square root” of 9, we write it as $-\sqrt{9} = -3$. If we want to write both square roots at once, we write $\pm\sqrt{9} = \pm 3$.

Radicals

The **cube root** of a real number *a* is the number whose cube is *a*. The cube root of *a* is written as $\sqrt[3]{a}$ so that, for example, $\sqrt[3]{8} = 2$ (because $2^3 = 8$). Note that we can take the cube root of any number, positive, negative, or zero. For instance, the cube root of –8 is $\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$. Unlike square roots, the cube root of a number may be negative. In fact, the cube root of *a* always has the same sign as *a*.

Higher roots are defined similarly. The **fourth root** of the *nonnegative* number *a* is defined as the nonnegative number whose fourth power is *a*, and written $\sqrt[4]{a}$. The **fifth root** of any number *a* is the number whose fifth power is *a*, and so on.

Example 3 – *nth Roots*

$\sqrt{4} = 2$ Because $2^2 = 4$

$\sqrt{16} = 4$ Because $4^2 = 16$

$\sqrt{1} = 1$ Because $1^2 = 1$

If $x \geq 0$, then $\sqrt{x^2} = x$ Because $x^2 = x^2$

$\sqrt{2} \approx 1.414213562$ $\sqrt{2}$ is not a whole number.

$\sqrt{1+1} = \sqrt{2} \approx 1.414213562$ First add, then take the square root.

$\sqrt{9+16} = \sqrt{25} = 5$ Contrast with $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$.

$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ Multiply top and bottom by $\sqrt{2}$.

Example 3 – *nth Roots*

cont'd

$\sqrt[3]{27} = 3$ — Because $3^3 = 27$

$\sqrt[3]{-64} = -4$ — Because $(-4)^3 = -64$

$\sqrt[4]{16} = 2$ — Because $2^4 = 16$

$\sqrt[4]{-16}$ is not defined — Even-numbered root of a negative number

$\sqrt[5]{-1} = -1$, since $(-1)^5 = -1$. — Odd-numbered root of a negative number

$\sqrt[n]{-1} = -1$ if n is any odd number.

Radicals

Radicals of Products and Quotients

If *a* and *b* are any real numbers (nonnegative in the case of even-numbered roots), then

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$$

Radical of a product = Product of radicals

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad \text{if } b \neq 0$$

Radical of a quotient = Quotient of radicals

Quick Example

$$\sqrt{9 \cdot 4} = \sqrt{9}\sqrt{4} = 3 \times 2 = 6$$

Alternatively, $\sqrt{9 \cdot 4} = \sqrt{36} = 6$.

Rational Exponents

Rational Exponents

We already know what we mean by expressions such as x^4 and a^{-6}. The next step is to make sense of *rational* exponents: exponents of the form p/q with p and q integers as in $a^{1/2}$ and $3^{-2/3}$.

Rational Exponents

Conversion Between Rational Exponents and Radicals

If *a* is any nonnegative number, then

$$a^{p/q} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p.$$

↑ Using exponents ↑ ↑ Using radicals

In particular,

$a^{1/q} = \sqrt[q]{a}$, the *q*th root of *a*.

Quick Example

$$4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$$

Example 4 – *Simplifying Algebraic Expressions*

Simplify the following.

a. $\dfrac{(x^3)^{5/3}}{x^3}$ **b.** $\sqrt[4]{a^6}$ **c.** $\dfrac{(xy)^{-3}y^{-3/2}}{x^{-2}\sqrt{y}}$

Solution:

a. $\dfrac{(x^3)^{5/3}}{x^3} = \dfrac{x^5}{x^3} = x^2$

b. $\sqrt[4]{a^6} = a^{6/4} = a^{3/2} = a \cdot a^{1/2} = a\sqrt{a}$

c. $\dfrac{(xy)^{-3}y^{-3/2}}{x^{-2}\sqrt{y}} = \dfrac{x^{-3}y^{-3}y^{-3/2}}{x^{-2}y^{1/2}} = \dfrac{1}{x^{-2+3}y^{1/2+3+3/2}} = \dfrac{1}{xy^5}$

Radical Form, Positive Exponent Form, and Power Form

Radical Form, Positive Exponent Form, and Power Form

In calculus we must often convert algebraic expressions involving powers of x, such as $\frac{3}{2x^2}$, into expressions in which x does not appear in the denominator, such as $\frac{3}{2}x^{-2}$.

Also, we must often convert expressions with radicals, such as $\frac{1}{\sqrt{1+x^2}}$, into expressions with no radicals and all powers in the numerator, such as $(1 + x^2)^{-1/2}$. In these cases, we are converting from **positive exponent form** or **radical form** to **power form.**

Radical Form, Positive Exponent Form, and Power Form

Radical Form

An expression is in **radical form** if it is written with integer powers and roots only.

Quick Example

$\frac{2}{5\sqrt[3]{x}} + \frac{2}{x}$ is in radical form.

Radical Form, Positive Exponent Form, and Power Form

Positive Exponent Form

An expression is in **positive exponent form** if it is written with positive exponents only.

Quick Example

$\frac{2}{3x^2}$ is in positive exponent form.

Radical Form, Positive Exponent Form, and Power Form

Power Form

An expression is in **power form** if there are no radicals and all powers of unknowns occur in the numerator. We write such expressions as sums or differences of terms of the form

Constant × (Expression with x)^p As in $\frac{1}{3}x^{-3/2}$

Quick Example

$\frac{2}{3}x^4 - 3x^{-1/3}$ is in power form.

Example 5 – *Converting from One Form to Another*

Convert the following to positive exponent form:

a. $\frac{1}{2}x^{-2} + \frac{4}{3}x^{-5}$ **b.** $\frac{2}{\sqrt{x}} - \frac{2}{x^{-4}}$

Convert the following to radical form:

c. $\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-5/4}$ **d.** $\frac{(3+x)^{-1/3}}{5}$

Convert the following to power form:

e. $\frac{3}{4x^2} - \frac{x}{6} + \frac{6}{x} + \frac{4}{3\sqrt{x}}$ **f.** $\frac{2}{(x+1)^2} - \frac{3}{4\sqrt[5]{2x-1}}$

Example 5 – *Solution*

For parts (a) and (b), we eliminate negative exponents

a. $\frac{1}{2}x^{-2} + \frac{4}{3}x^{-5} = \frac{1}{2} \cdot \frac{1}{x^2} + \frac{4}{3} \cdot \frac{1}{x^5} = \frac{1}{2x^2} + \frac{4}{3x^5}$

b. $\frac{2}{\sqrt{x}} - \frac{2}{x^{-4}} = \frac{2}{\sqrt{x}} - 2x^4$

For parts (c) and (d), we rewrite all terms with fractional exponents as radicals:

c. $$\frac{1}{2}x^{-1/2} + \frac{4}{3}x^{-5/4} = \frac{1}{2} \cdot \frac{1}{x^{1/2}} + \frac{4}{3} \cdot \frac{1}{x^{5/4}}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}} + \frac{4}{3} \cdot \frac{1}{\sqrt[4]{x^5}}$$

$$= \frac{1}{2\sqrt{x}} + \frac{4}{3\sqrt[4]{x^5}}$$

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Example 5 – *Solution*

cont'd

d. $$\frac{(3+x)^{-1/3}}{5} = \frac{1}{5(3+x)^{1/3}} = \frac{1}{5\sqrt[3]{3+x}}$$

For (e) and (f), we eliminate any radicals and move all expressions involving *x* to the numerator:

e. $$\frac{3}{4x^2} - \frac{x}{6} + \frac{6}{x} + \frac{4}{3\sqrt{x}} = \frac{3}{4}x^{-2} - \frac{1}{6}x + 6x^{-1} + \frac{4}{3x^{1/2}}$$

$$= \frac{3}{4}x^{-2} - \frac{1}{6}x + 6x^{-1} + \frac{4}{3}x^{-1/2}$$

f. $$\frac{2}{(x+1)^2} - \frac{3}{4\sqrt[5]{2x-1}} = 2(x+1)^{-2} - \frac{3}{4(2x-1)^{1/5}}$$

$$= 2(x+1)^{-2} - \frac{3}{4}(2x-1)^{-1/5}$$

Solving Equations with Exponents

Example 6 – *Solving Equations*

Solve the following equations:

a. $x^3 + 8 = 0$

b. $x^2 - \frac{1}{2} = 0$

c. $x^{3/2} - 64 = 0$

Example 6 – *Solution*

a. Subtracting 8 from both sides gives $x^3 = -8$. Taking the cube root of both sides gives $x = -2$.

b. Adding $\frac{1}{2}$ to both sides gives $x^2 = \frac{1}{2}$.

Thus, $x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$.

c. Adding 64 to both sides gives $x^{3/2} = 64$. Taking the reciprocal (2/3) power of both sides gives

$$(x^{3/2})^{2/3} = 64^{2/3}$$

$$x^1 = (\sqrt[3]{64})^2 = 4^2 = 16$$

so $x = 16$.