Solutions for Introduction to Management Science Quantitative Approach 15th Edition by Anderson

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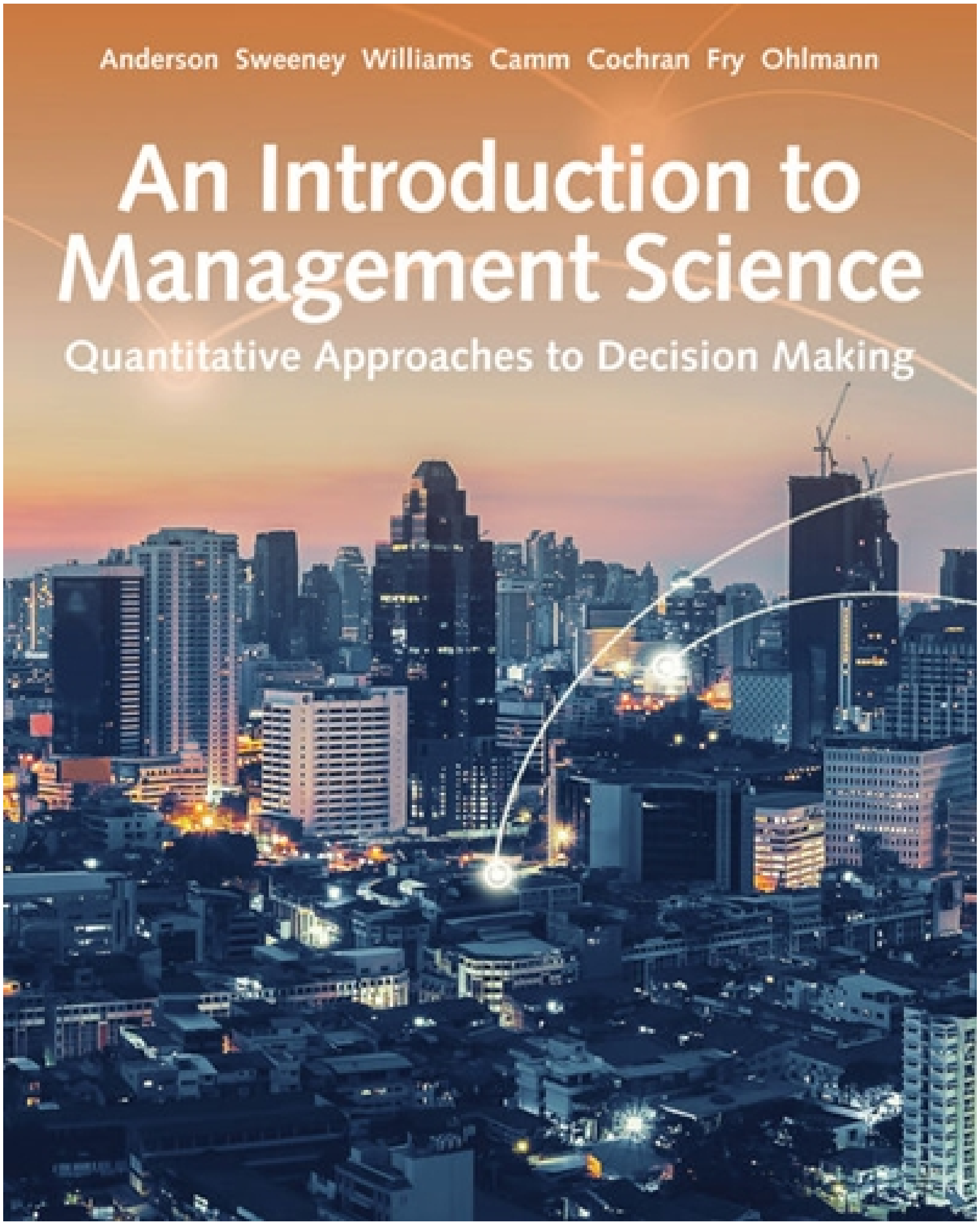


Solutions

Chapter 2
An Introduction to Linear Programming

Learning Objectives

1. Obtain an overview of the kinds of problems linear programming has been used to solve.

2. Learn how to develop linear programming models for simple problems.

3. Be able to identify the special features of a model that make it a linear programming model.

4. Learn how to solve two variable linear programming models by the graphical solution procedure.

5. Understand the importance of extreme points in obtaining the optimal solution.

6. Know the use and interpretation of slack and surplus variables.

7. Be able to interpret the computer solution of a linear programming problem.

8. Understand how alternative optimal solutions, infeasibility and unboundedness can occur in linear programming problems.

9. Understand the following terms:

   problem formulation
   constraint function
   objective function
   solution
   optimal solution
   nonnegativity constraints
   mathematical model
   linear program
   linear functions
   feasible solution
   feasible region
   slack variable
   standard form
   redundant constraint
   extreme point
   surplus variable
   alternative optimal solutions
   infeasibility
   unbounded

Chapter 2

Solutions:

1. a, b, and e, are acceptable linear programming relationships.

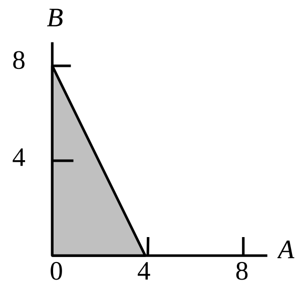
   c is not acceptable because of $-2B^2$

   d is not acceptable because of $3\sqrt{A}$

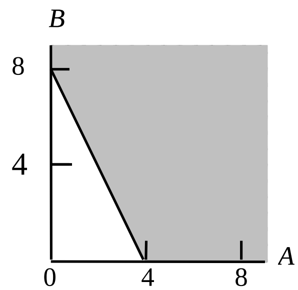
   f is not acceptable because of *1AB*

   c, d, and f could not be found in a linear programming model because they have the above nonlinear terms.

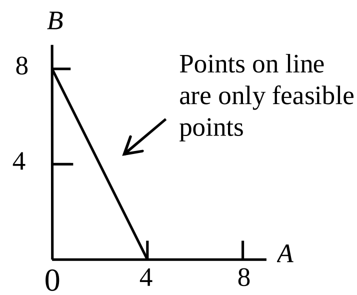
2. a.



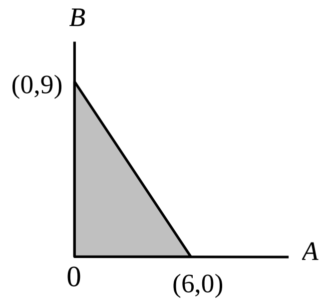
b.



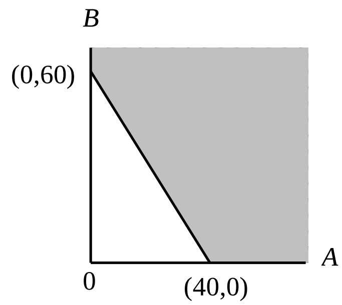
c.



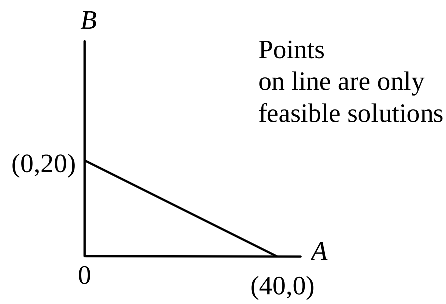
3. a.


B
(0,9)
A
0
(6,0)

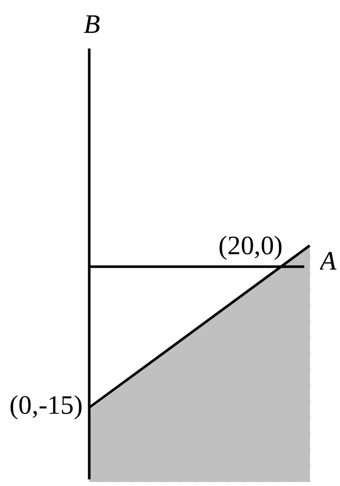
b.


B
(0,60)
A
0
(40,0)

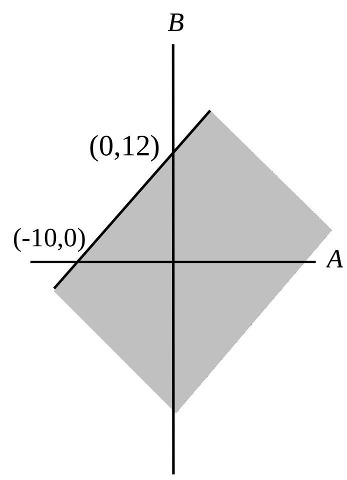
c.


B
Points
on line are only
feasible solutions
(0,20)
A
0
(40,0)

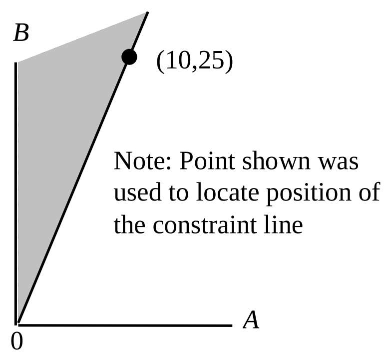
4. a.


B
(20,0)
A
(0,-15)

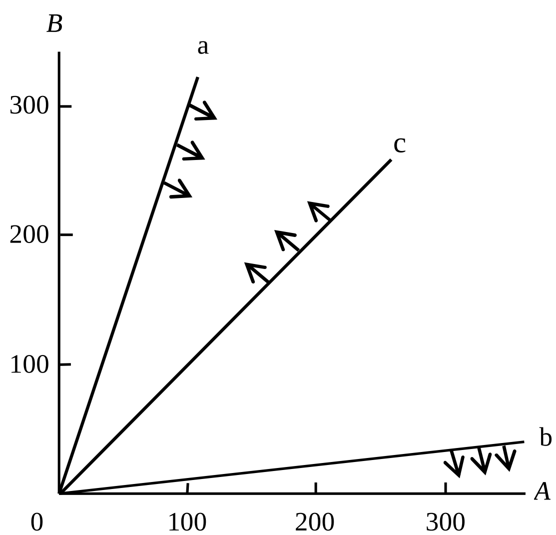
b.


B
(0,12)
(-10,0)
A

c.


B
(10,25)
Note: Point shown was used to locate position of the constraint line
A
0

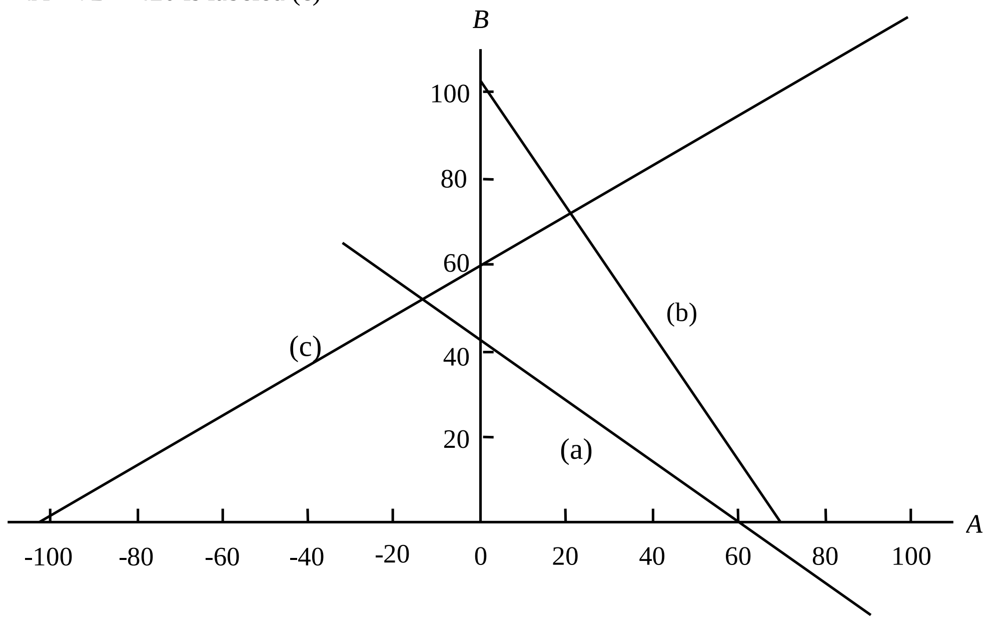
5.


B
a
c
b
A
300
200
100
0
100
200
300

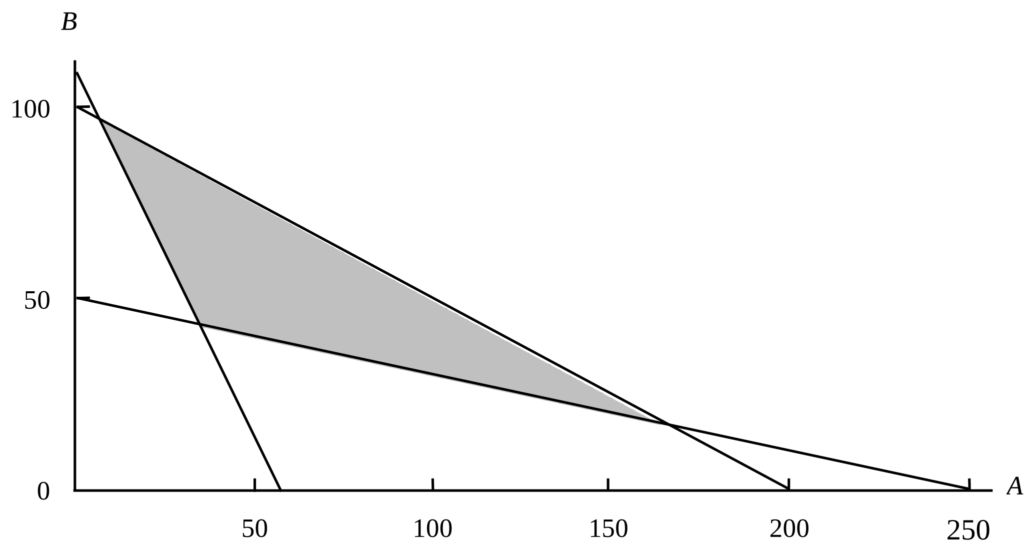
6. $7A + 10B = 420$ is labeled (a)

$6A + 4B = 420$ is labeled (b)

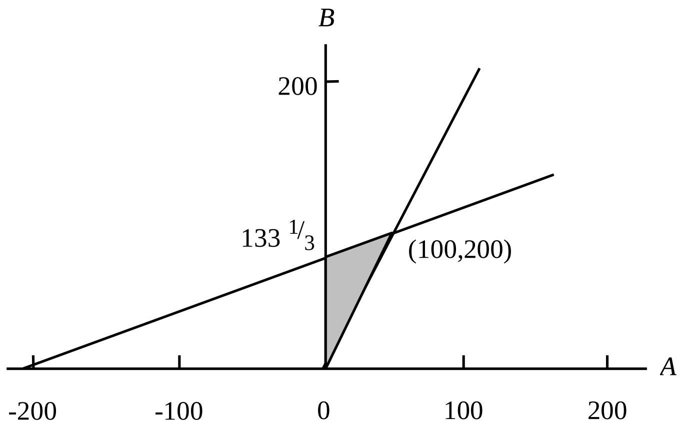
$-4A + 7B = 420$ is labeled (c)



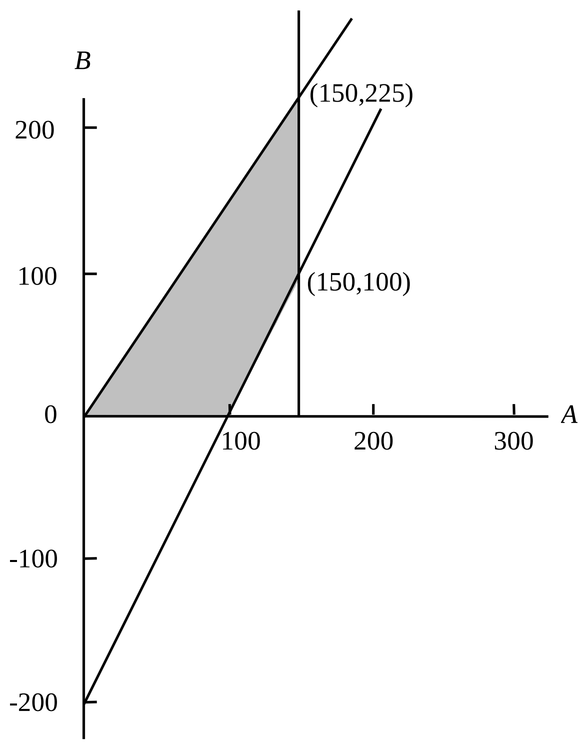
7.



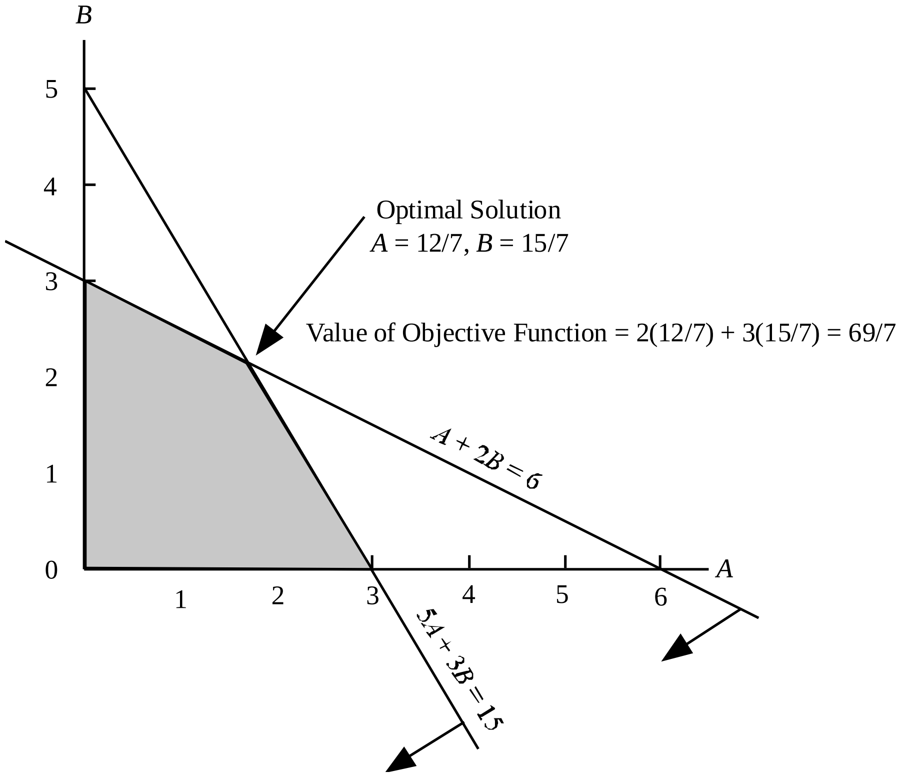
8.


B
200
133 1/3
(100,200)
A
-200
-100
0
100
200

9.


B
200
(150,225)
100
(150,100)
0
A
100
200
300
-100
-200

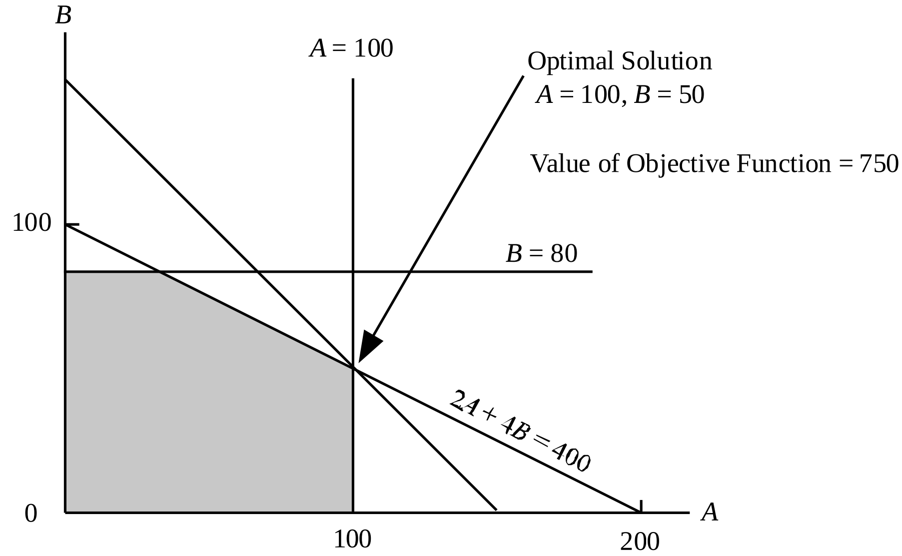
10.



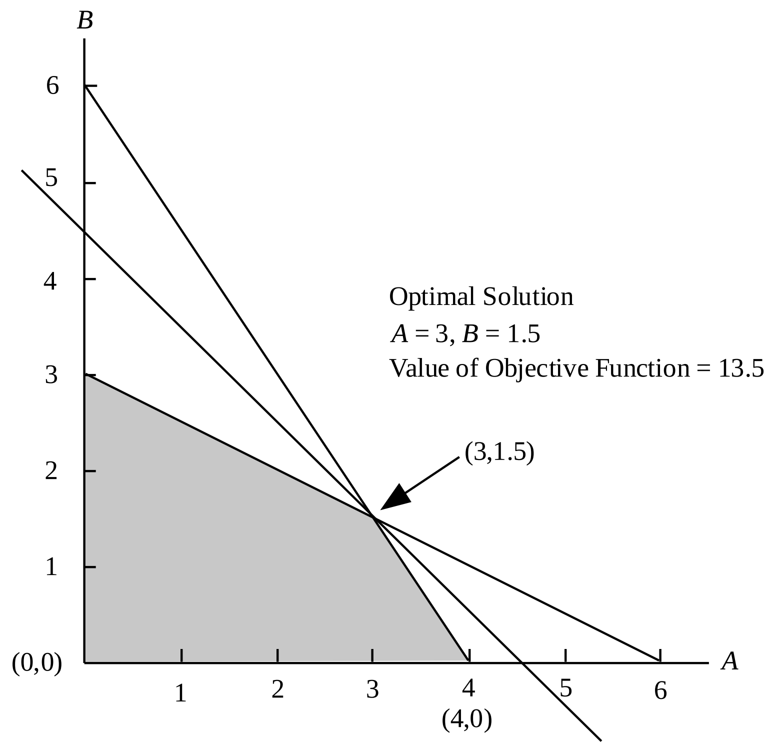
	A	+	$2B$	=	6	(1)
	$5A$	+	$3B$	=	15	(2)
(1) × 5	$5A$	+	$10B$	=	30	(3)
(2) - (3)		-	$7B$	=	-15	
			B	=	15/7	

From (1), $A = 6 - 2(15/7) = 6 - 30/7 = 12/7$

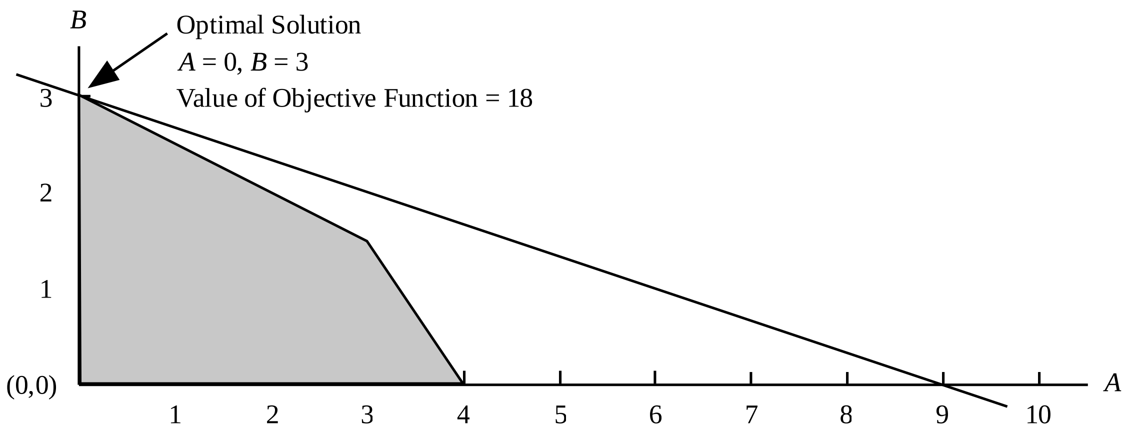
11.


B
A = 100
Optimal Solution
A = 100, B = 50
Value of Objective Function = 750
100
B = 80
2A + 4B = 400
0
100
200
A

12. a.

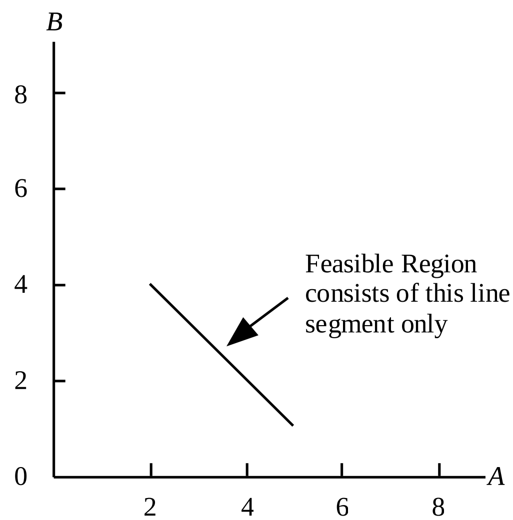

B
6
5
4
3
2
1
Optimal Solution
A = 3, B = 1.5
Value of Objective Function = 13.5
(3,1.5)
(0,0)
1
2
3
4
5
6
A
(4,0)

b.



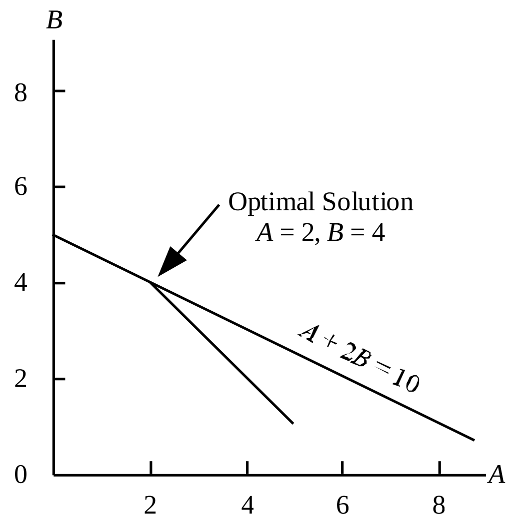
c. There are four extreme points: (0,0), (4,0), (3,1,5), and (0,3).

13. a.



b. The extreme points are (5, 1) and (2, 4).

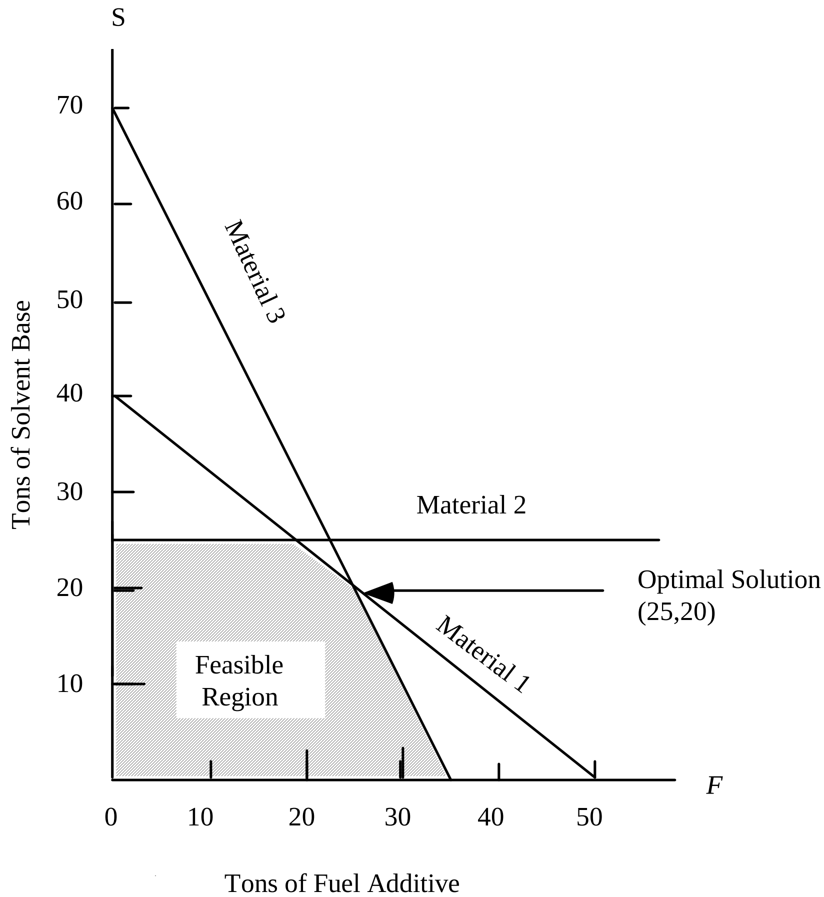
c.



14. a. Let F = number of tons of fuel additive
S = number of tons of solvent base

$$
\begin{aligned}
\text{Max} \quad & 40F + 30S \\
\text{s.t.} \quad & \\
& 2/5F + {}^1/_2\,S \le 200 \quad \text{Material 1} \\
& {}^1/_5\,S \le 5 \quad \text{Material 2} \\
& {}^3/_5\,F + {}^3/_{10}\,S \le 21 \quad \text{Material 3} \\
& F,\ S \ge 0
\end{aligned}
$$

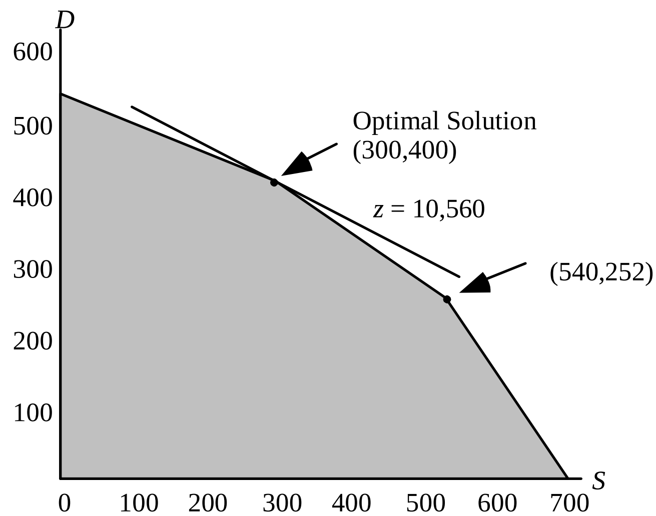
b.



c. Material 2: 4 tons are used, 1 ton is unused.

d. No redundant constraints.

15. a.



b. Similar to part (a): the same feasible region with a different objective function. The optimal solution occurs at (708, 0) with a profit of $z = 20(708) + 9(0) = 14{,}160$.

c. The sewing constraint is redundant. Such a change would not change the optimal solution to the original problem.

16. a. A variety of objective functions with a slope greater than -4/10 (slope of I & P line) will make extreme point (0, 540) the optimal solution. For example, one possibility is $3S + 9D$.

b. Optimal Solution is $S = 0$ and $D = 540$.

c.

Department	Hours Used	Max. Available	Slack
Cutting and Dyeing	1(540) = 540	630	90
Sewing	$^5/_6$(540) = 450	600	150
Finishing	$^2/_3$(540) = 360	708	348
Inspection and Packaging	$^1/_4$(540) = 135	135	0

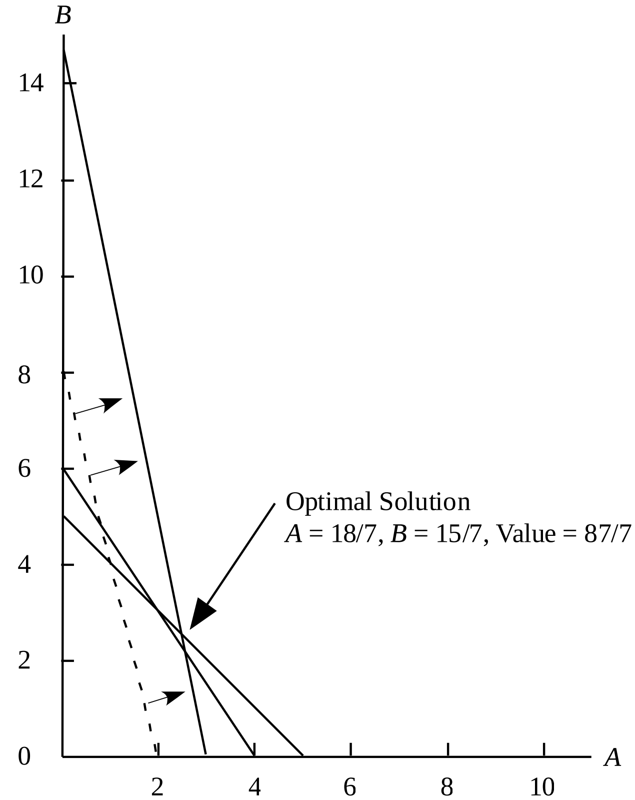
17.

$$\begin{aligned}
\text{Max} \quad & 5A + 2B + 0S_1 + 0S_2 + 0S_3 \\
\text{s.t.} \quad & \\
& 1A - 2B + 1S_1 = 420 \\
& 2A + 3B + 1S_2 = 610 \\
& 6A - 1B + 1S_3 = 125 \\
& A, B, S_1, S_2, S_3 \geq 0
\end{aligned}$$

18. a.

$$\begin{aligned}
\text{Max} \quad & 4A + 1B + 0S_1 + 0S_2 + 0S_3 \\
\text{s.t.} \quad & \\
& 10A + 2B + 1S_1 = 30 \\
& 3A + 2B + 1S_2 = 12 \\
& 2A + 2B + 1S_3 = 10 \\
& A, B, S_1, S_2, S_3 \geq 0
\end{aligned}$$

b.



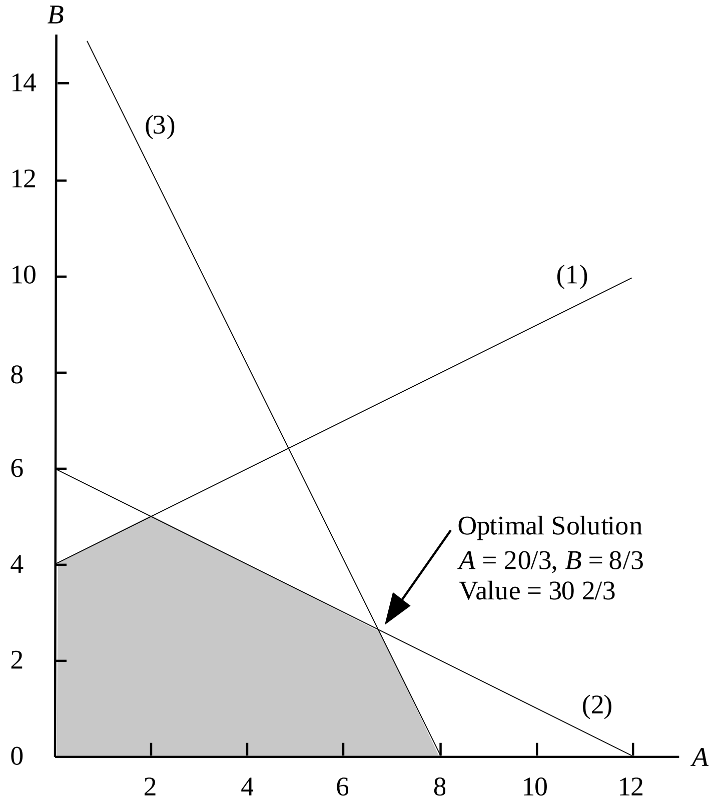
c. $S_1 = 0$, $S_2 = 0$, $S_3 = 4/7$

19. a.

Max	$3A$	+	$4B$	+	$0S_1$	+	$0S_2$	+	$0S_3$			
s.t.												
	$-1A$	+	$2B$	+	$1S_1$					=	8	(1)
	$1A$	+	$2B$			+	$1S_2$			=	12	(2)
	$2A$	+	$1B$					+	$1S_3$	=	16	(3)

$$A, B, S_1, S_2, S_3 \geq 0$$

b.



c. $S_1 = 8 + A - 2B = 8 + 20/3 - 16/3 = 28/3$

$S_2 = 12 - A - 2B = 12 - 20/3 - 16/3 = 0$

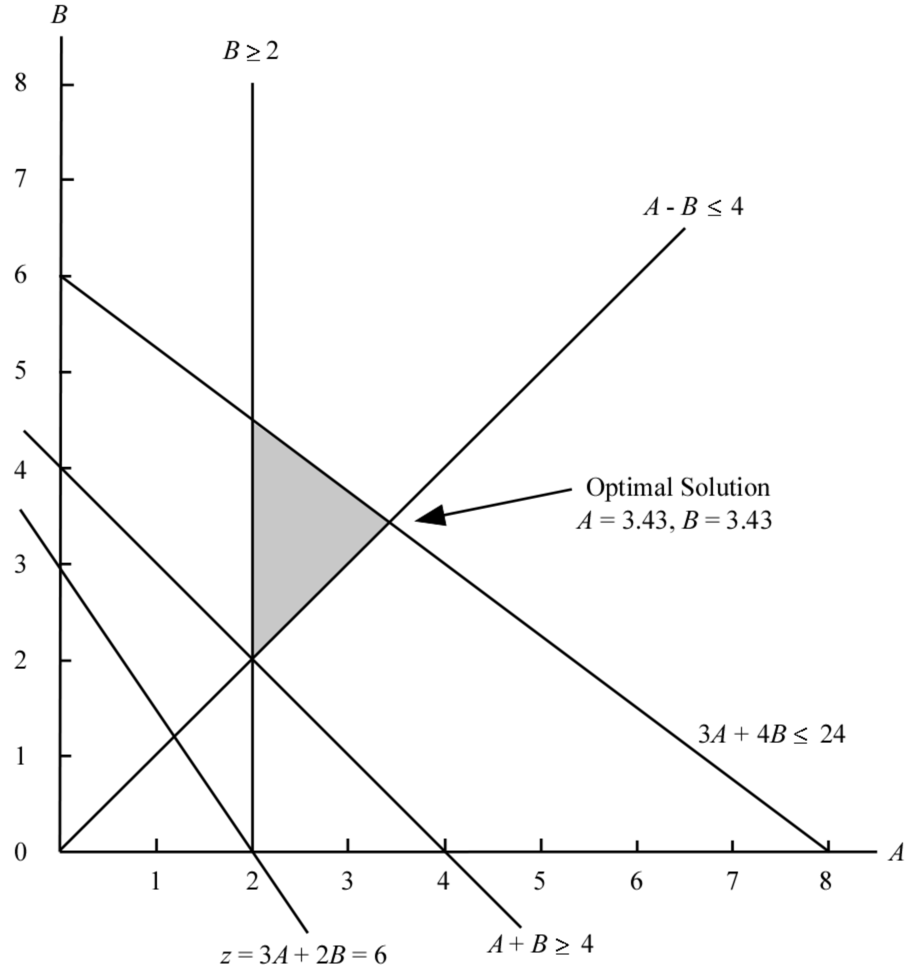
$S_3 = 16 - 2A - B = 16 - 40/3 - 8/3 = 0$

20. a.

$$
\begin{array}{llcllllll}
\text{Max} & 3A & + & 2B & & & & & \\
\text{s.t.} & & & & & & & & \\
& A & + & B & -S_1 & & & & = 4 \\
& 3A & + & 4B & & +S_2 & & & = 24 \\
& A & & & & & -S_3 & & = 2 \\
& A & - & B & & & & -S_4 & = 0 \\
\end{array}
$$

$$A, B, S_1, S_2, S_3, S_4 \geq 0$$

b.



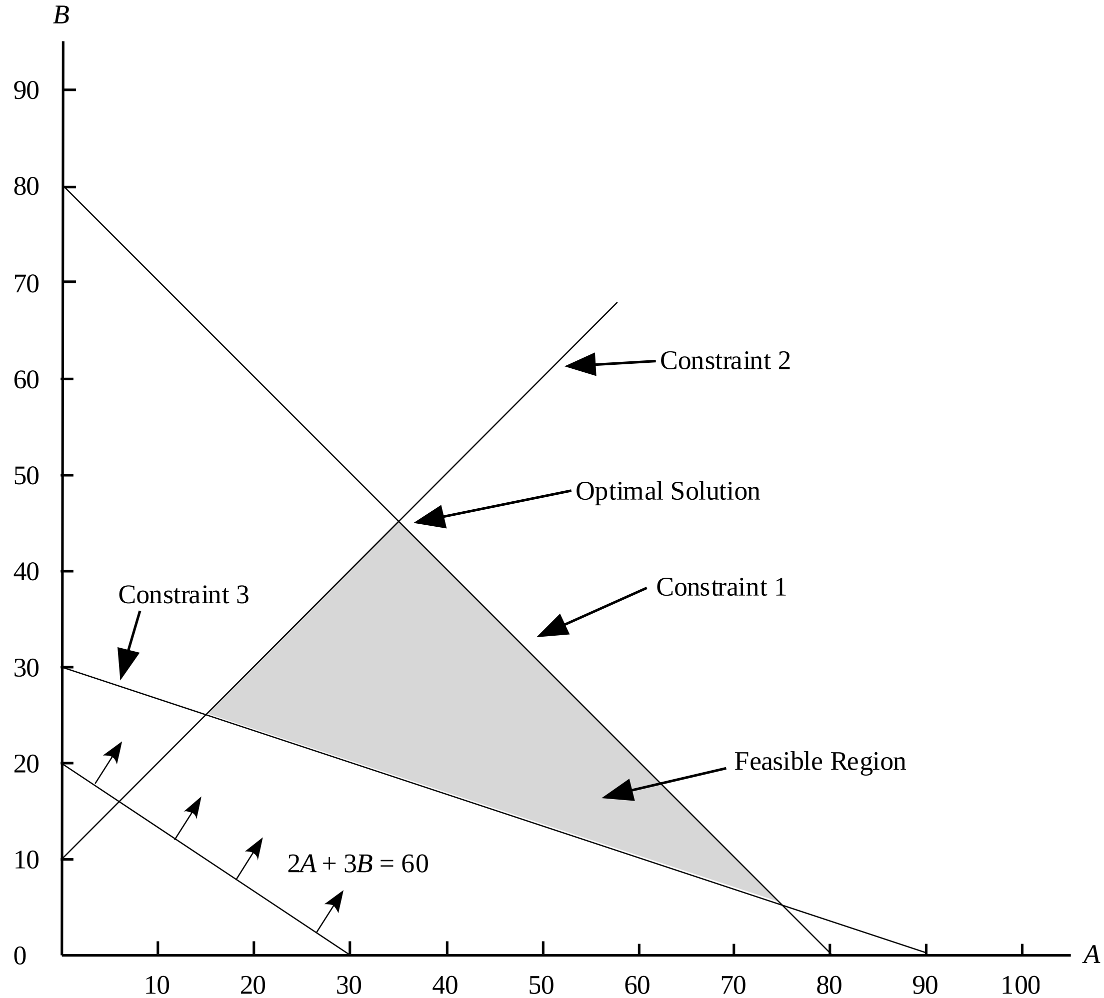
c. $S_1 = (3.43 + 3.43) - 4 = 2.86$

$S_2 = 24 - [3(3.43) + 4(3.43)] = 0$

$S_3 = 3.43 - 2 = 1.43$

$S_4 = 0 - (3.43 - 3.43) = 0$

21. a. and b.



c. Optimal solution occurs at the intersection of constraints 1 and 2. For constraint 2,

$$B = 10 + A$$

Substituting for B in constraint 1 we obtain

$$\begin{aligned} 5A + 5(10 + A) &= 400 \\ 5A + 50 + 5A &= 400 \\ 10A &= 350 \\ A &= 35 \end{aligned}$$

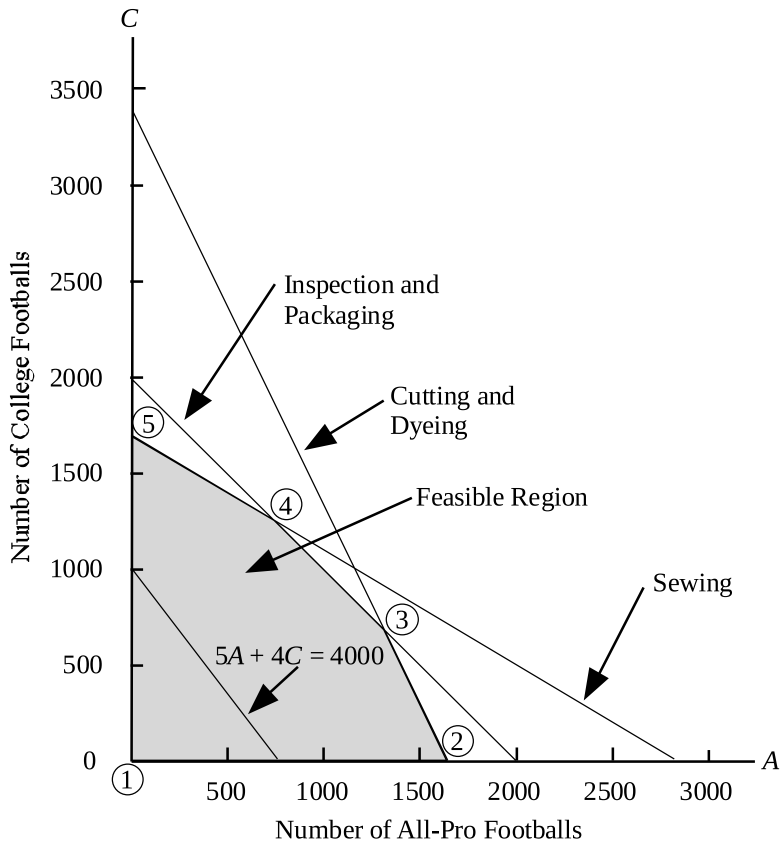
$$B = 10 + A = 10 + 35 = 45$$

Optimal solution is $A = 35$, $B = 45$

d. Because the optimal solution occurs at the intersection of constraints 1 and 2, these are binding constraints.

e. Constraint 3 is the nonbinding constraint. At the optimal solution 1*A* + 3*B* = 1(35) + 3(45) = 170. Because 170 exceeds the right-hand side value of 90 by 80 units, there is a surplus of 80 associated with this constraint.

22. a.



b.

Extreme Point	Coordinates	Profit
1	(0, 0)	5(0) + 4(0) = 0
2	(1700, 0)	5(1700) + 4(0) = 8500
3	(1400, 600)	5(1400) + 4(600) = 9400
4	(800, 1200)	5(800) + 4(1200) = 8800
5	(0, 1680)	5(0) + 4(1680) = 6720

Extreme point 3 generates the highest profit.

c. Optimal solution is $A = 1400$, $C = 600$

d. The optimal solution occurs at the intersection of the cutting and dyeing constraint and the inspection and packaging constraint. Therefore these two constraints are the binding constraints.

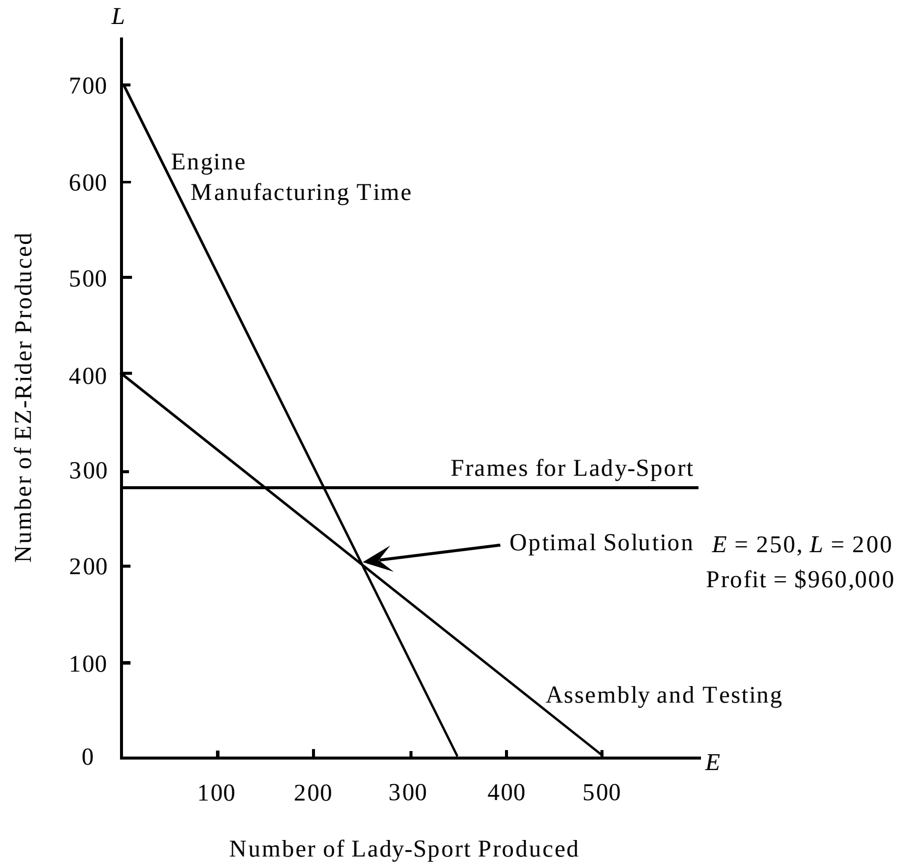
e. New optimal solution is $A = 800$, $C = 1200$

Profit = 4(800) + 5(1200) = 9200

23. a. Let E = number of units of the EZ-Rider produced
L = number of units of the Lady-Sport produced

Max	$2400E$	+	$1800L$			
s.t.						
	$6E$	+	$3L$	$\leq$	2100	Engine time
			L	$\leq$	280	Lady-Sport maximum
	$2E$	+	$2.5L$	$\leq$	1000	Assembly and testing
			$E, L \geq 0$			

b.



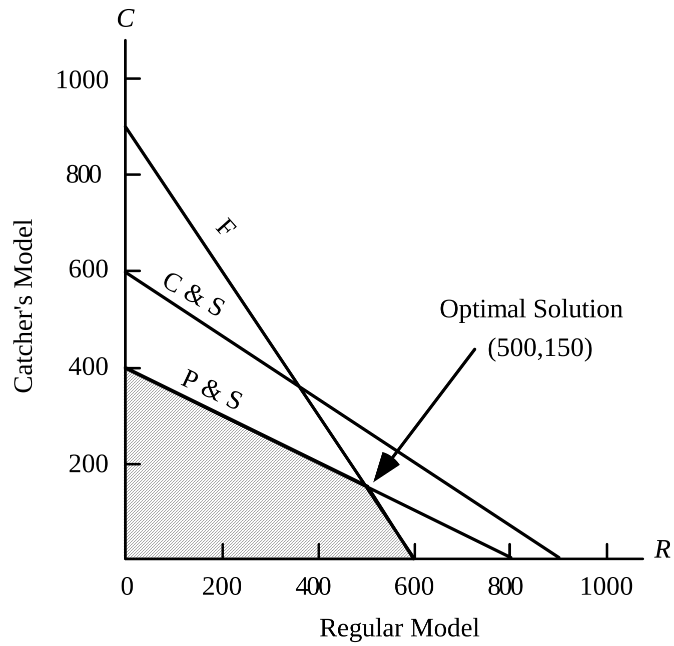
c. The binding constraints are the manufacturing time and the assembly and testing time.

24. a. Let R = number of units of regular model.
C = number of units of catcher's model.

Max	$5R$	+	$8C$			
s.t.						
	$1R$	+	$^{3}/_{2}\,C$	$\leq$	900	Cutting and sewing
	$^{1}/_{2}\,R$	+	$^{1}/_{3}\,C$	$\leq$	300	Finishing
	$^{1}/_{8}\,R$	+	$^{1}/_{4}\,C$	$\leq$	100	Packing and Shipping

$R,\ C \geq 0$

b.



c. 5(500) + 8(150) = $3,700

d. C & S $1(500) + {}^{3}/_{2}(150) = 725$

F $^{1}/_{2}(500) + {}^{1}/_{3}(150) = 300$

P & S $^{1}/_{8}(500) + {}^{1}/_{4}(150) = 100$

e.

Department	Capacity	Usage	Slack
C & S	900	725	175 hours
F	300	300	0 hours
P & S	100	100	0 hours

25. a. Let *B* = percentage of funds invested in the bond fund
S = percentage of funds invested in the stock fund

Max $0.06B + 0.10S$

s.t.

B			$\geq$	0.3	Bond fund minimum
$0.06B$	+	$0.10S$	$\geq$	0.075	Minimum return
B	+	S	=	1	Percentage requirement

b. Optimal solution: $B = 0.3$, $S = 0.7$

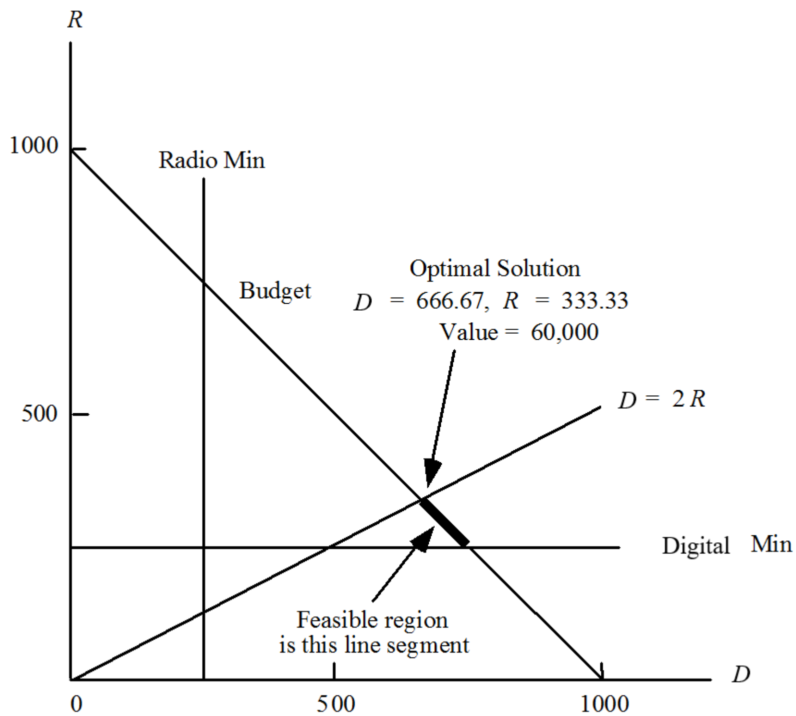
Value of optimal solution is 0.088 or 8.8%

26. a. Let D = amount spent on digital advertising
R = amount spent on radio advertising

$$
\begin{array}{llcrcrl}
\text{Max} & 50D & + & 80R & & & \\
\text{s.t.} & & & & & & \\
 & D & + & R & = & 1000 & \text{Budget} \\
 & D & & & \geq & 250 & \text{Digital min.} \\
 & & & R & \geq & 250 & \text{Radio min.} \\
 & D & & -2R & \geq & 0 & \text{Digital} \geq 2 \text{ Radio}
\end{array}
$$

$D, R \geq 0$

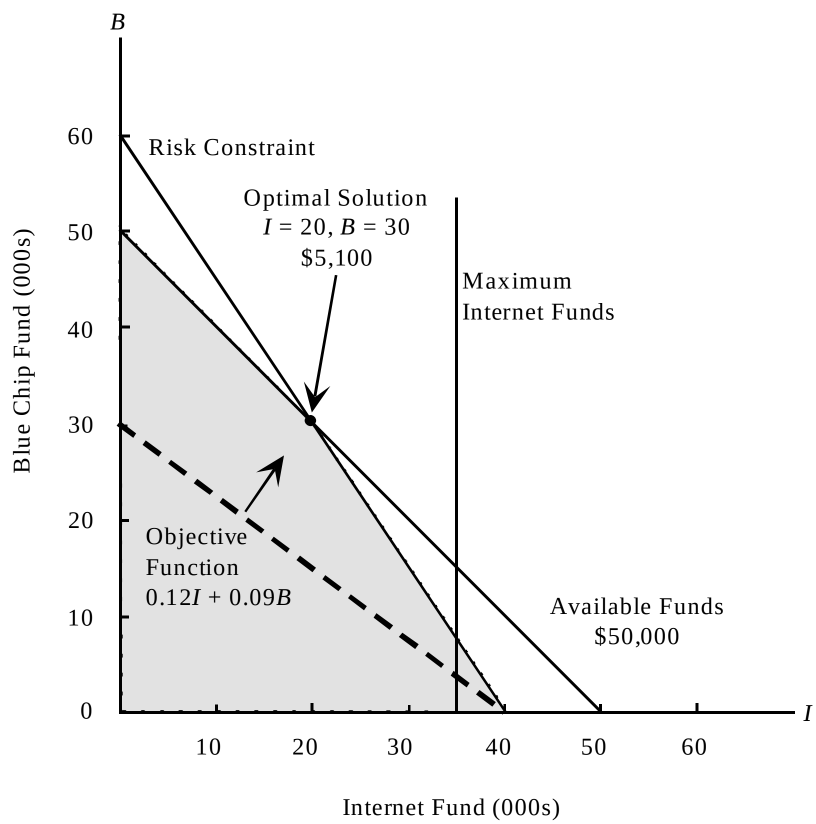
b.



27. Let I = Internet fund investment in thousands
B = Blue Chip fund investment in thousands

$$
\begin{array}{llcrcrl}
\text{Max} & 0.12I & + & 0.09B & & & \\
\text{s.t.} & & & & & & \\
 & 1I & + & 1B & \leq & 50 & \text{Available investment funds} \\
 & 1I & & & \leq & 35 & \text{Maximum investment in the internet fund} \\
 & 6I & + & 4B & \leq & 240 & \text{Maximum risk for a moderate investor}
\end{array}
$$

$I, B \geq 0$



Internet fund	\$20,000
Blue Chip fund	\$30,000
Annual return	\$ 5,100

b. The third constraint for the aggressive investor becomes

$$6I + 4B \leq 320$$

This constraint is redundant; the available funds and the maximum Internet fund investment constraints define the feasible region. The optimal solution is:

Internet fund	\$35,000
Blue Chip fund	\$15,000
Annual return	\$ 5,550

The aggressive investor places as much funds as possible in the high return but high risk Internet fund.

c. The third constraint for the conservative investor becomes

$$6I + 4B \leq 160$$

This constraint becomes a binding constraint. The optimal solution is

Internet fund	\$0
Blue Chip fund	\$40,000
Annual return	\$ 3,600

The slack for constraint 1 is $10,000. This indicates that investing all $50,000 in the Blue Chip fund is still too risky for the conservative investor. $40,000 can be invested in the Blue Chip fund. The remaining $10,000 could be invested in low-risk bonds or certificates of deposit.

28. a. Let W = number of jars of Western Foods Salsa produced
M = number of jars of Mexico City Salsa produced

$$\begin{array}{llllll} \text{Max} & 1W & + & 1.25M & & \\ \text{s.t.} & & & & & \\ & 5W & & 7M & \leq & 4480 \quad \text{Whole tomatoes} \\ & 3W & + & 1M & \leq & 2080 \quad \text{Tomato sauce} \\ & 2W & + & 2M & \leq & 1600 \quad \text{Tomato paste} \\ & W, M \geq 0 & & & & \end{array}$$

Note: units for constraints are ounces

b. Optimal solution: $W = 560$, $M = 240$

Value of optimal solution is 860

29. a. Let B = proportion of Buffalo's time used to produce component 1
D = proportion of Dayton's time used to produce component 1

Maximum Daily Production

	Component 1	Component 2
Buffalo	2000	1000
Dayton	600	1400

Number of units of component 1 produced: $2000B + 600D$

Number of units of component 2 produced: $1000(1 - B) + 600(1 - D)$

For assembly of the ignition systems, the number of units of component 1 produced must equal the number of units of component 2 produced.

Therefore,

$$2000B + 600D = 1000(1 - B) + 1400(1 - D)$$

$$2000B + 600D = 1000 - 1000B + 1400 - 1400D$$

$$3000B + 2000D = 2400$$

Note: Because every ignition system uses 1 unit of component 1 and 1 unit of component 2, we can maximize the number of electronic ignition systems produced by maximizing the number of units of subassembly 1 produced.

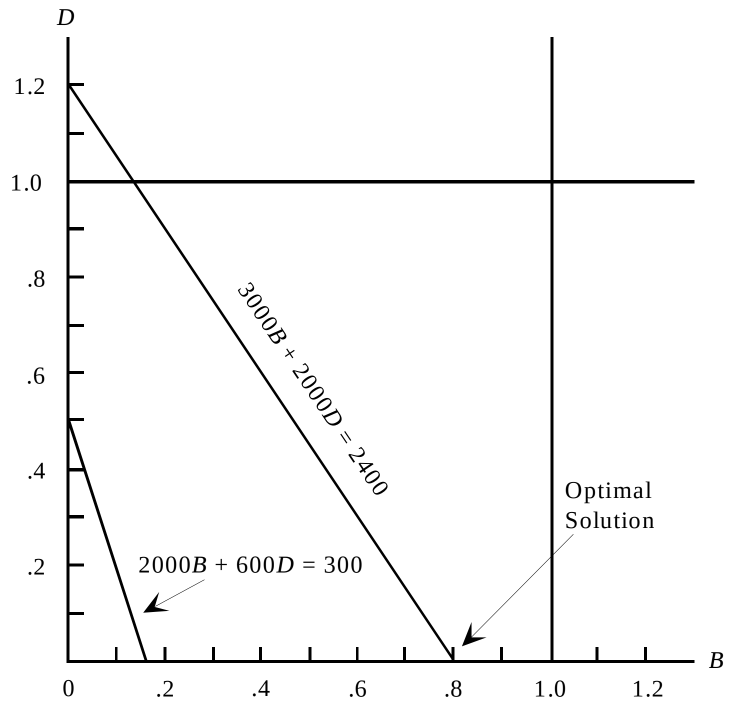
$$\text{Max } 2000B + 600D$$

In addition, $B \leq 1$ and $D \leq 1$.

The linear programming model is:

$$
\begin{array}{lrrl}
\text{Max} & 2000B & + \; 600D & \\
\text{s.t.} & & & \\
 & 3000B & + \; 2000D & = 2400 \\
 & B & & \leq 1 \\
 & & D & \leq 1 \\
 & & B, D & \geq 0
\end{array}
$$

The graphical solution is shown below.



Optimal Solution: B = .8, D = 0

Optimal Production Plan

Buffalo - Component 1	.8(2000) = 1600
Buffalo - Component 2	.2(1000) = 200
Dayton - Component 1	0(600) = 0
Dayton - Component 2	1(1400) = 1400

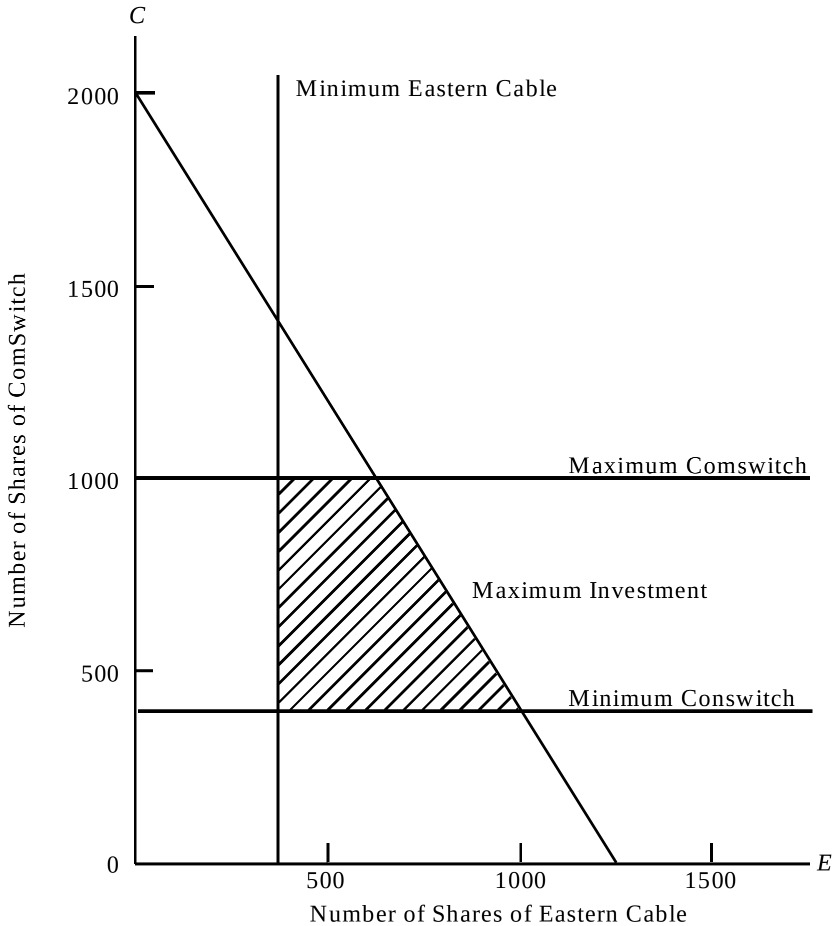
Total units of electronic ignition system = 1600 per day.

30. a. Let E = number of shares of Eastern Cable
C = number of shares of ComSwitch

Max	$15E$	+	$18C$				
s.t.							
	$40E$	+	$25C$	$\leq$	50,000	Maximum Investment
	$40E$			$\geq$	15,000	Eastern Cable Minimum
			$25C$	$\geq$	10,000	ComSwitch Minimum
			$25C$	$\leq$	25,000	ComSwitch Maximum

$E, C \geq 0$

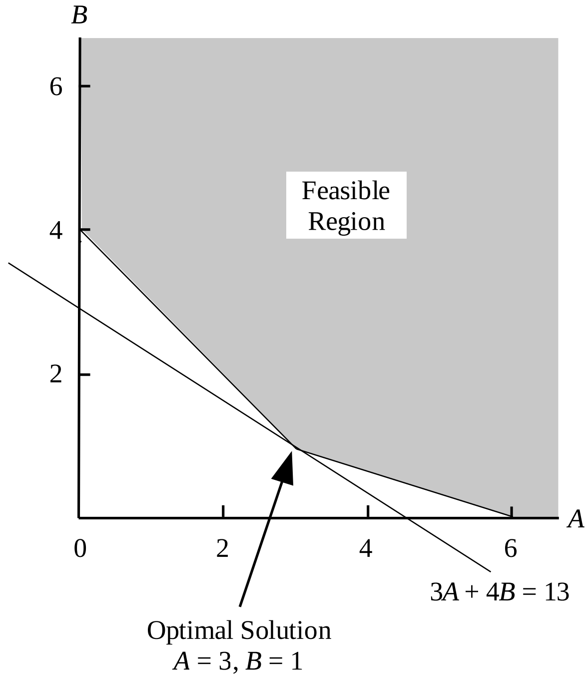
b.



c. There are four extreme points: (375,400); (1000,400);(625,1000); (375,1000)

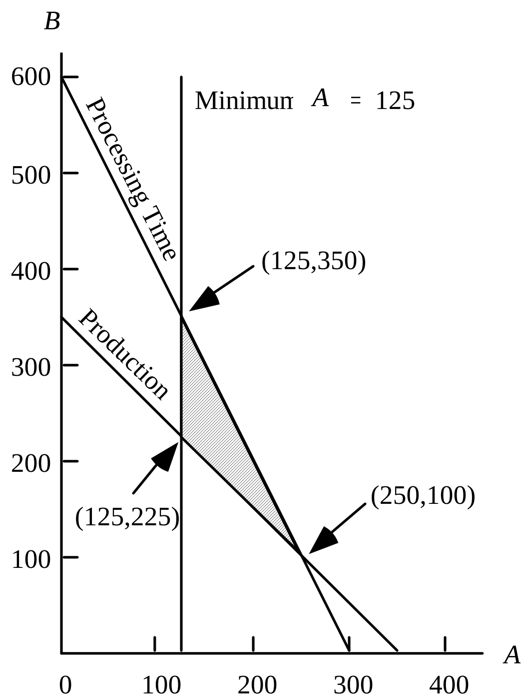
d. Optimal solution is E = 625, C = 1000
Total return = $27,375

31.



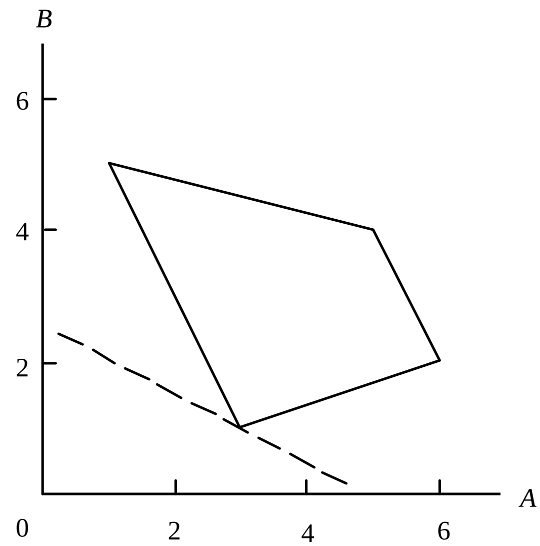
Objective Function Value = 13

32.



Extreme Points	Objective Function Value	Surplus Demand	Surplus Total Production	Slack Processing Time
(*A* = 250, *B* = 100)	800	125	—	—
(*A* = 125, *B* = 225)	925	—	—	125
(*A* = 125, *B* = 350)	1300	—	125	—

33. a.

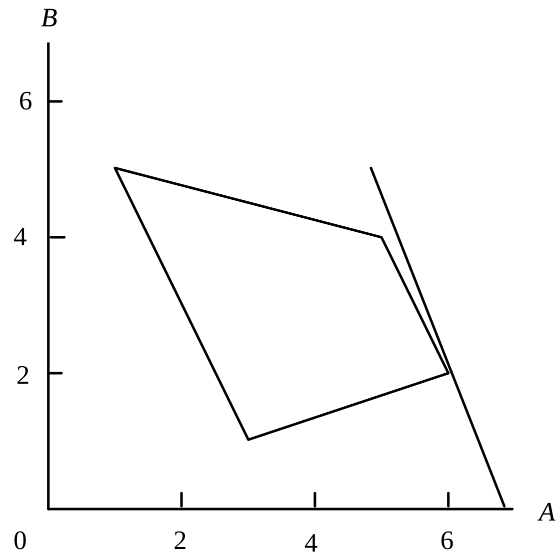


Optimal Solution: *A* = 3, *B* = 1, value = 5

b.

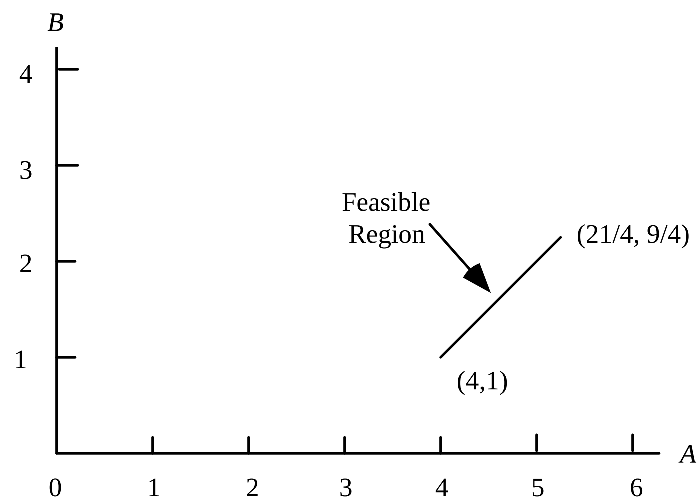
(1)	3 + 4(1) = 7	Slack = 21 - 7 = 14
(2)	2(3) + 1 = 7	Surplus = 7 - 7 = 0
(3)	3(3) + 1.5 = 10.5	Slack = 21 - 10.5 = 10.5
(4)	-2(3) +6(1) = 0	Surplus = 0 - 0 = 0

c.



Optimal Solution: $A = 6$, $B = 2$, value = 34

34. a.



b. There are two extreme points: ($A = 4$, $B = 1$) and ($A = 21/4$, $B = 9/4$)

c. The optimal solution is $A = 4$, $B = 1$

35. a.

$$\begin{array}{lllllllll}
\text{Min} & 6A & + & 4B & + & 0S_1 & + & 0S_2 & + & 0S_3 & & \\
\text{s.t.} & & & & & & & & & & & \\
& 2A & + & 1B & - & S_1 & & & & & = & 12 \\
& 1A & + & 1B & & & - & S_2 & & & = & 10 \\
& & & 1B & & & & & + & S_3 & = & 4
\end{array}$$

$$A, B, S_1, S_2, S_3 \geq 0$$

b. The optimal solution is $A = 6$, $B = 4$.

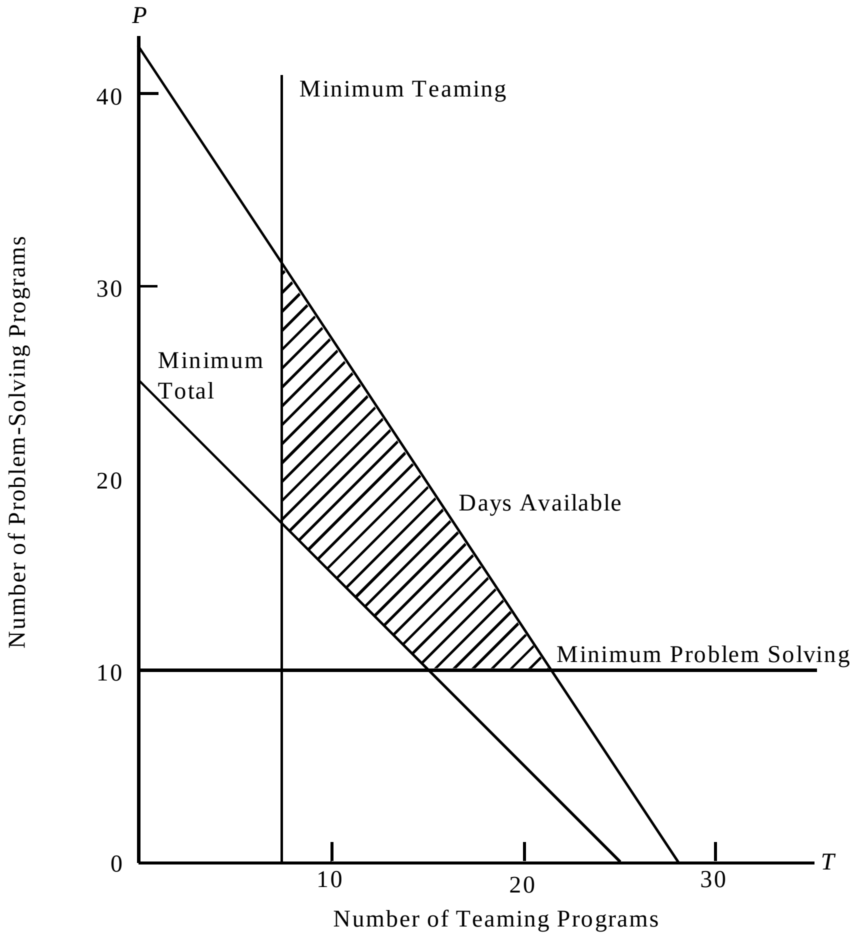
c. $S_1 = 4$, $S_2 = 0$, $S_3 = 0$.

36. a. Let T = number of training programs on teaming
P = number of training programs on problem solving

$$\begin{array}{llllll}
\text{Max} & 10{,}000T & + & 8{,}000P & & \\
\text{s.t.} & & & & & \\
& T & & & \geq 8 & \text{Minimum Teaming} \\
& & & P & \geq 10 & \text{Minimum Problem Solving} \\
& T & + & P & \geq 25 & \text{Minimum Total} \\
& 3T & + & 2P & \leq 84 & \text{Days Available}
\end{array}$$

$$T, P \geq 0$$

b.



c. There are four extreme points: (15,10); (21.33,10); (8,30); (8,17)

d. The minimum cost solution is $T = 8$, $P = 17$
Total cost = $216,000

37.

	Regular	Zesty	
Mild	80%	60%	8100
Extra Sharp	20%	40%	3000

Let R = number of containers of Regular
Z = number of containers of Zesty

Each container holds 12/16 or 0.75 pounds of cheese

Pounds of mild cheese used = 0.80 (0.75) R + 0.60 (0.75) Z
= 0.60 R + 0.45 Z

Pounds of extra sharp cheese used = 0.20 (0.75) R + 0.40 (0.75) Z
= 0.15 R + 0.30 Z

Cost of Cheese = Cost of mild + Cost of extra sharp
= $1.20\,(0.60\,R + 0.45\,Z) + 1.40\,(0.15\,R + 0.30\,Z)$
= $0.72\,R + 0.54\,Z + 0.21\,R + 0.42\,Z$
= $0.93\,R + 0.96\,Z$

Packaging Cost = $0.20\,R + 0.20\,Z$

Total Cost = $(0.93\,R + 0.96\,Z) + (0.20\,R + 0.20\,Z)$
= $1.13\,R + 1.16\,Z$

Revenue = $1.95\,R + 2.20\,Z$

Profit Contribution = Revenue - Total Cost
= $(1.95\,R + 2.20\,Z) - (1.13\,R + 1.16\,Z)$
= $0.82\,R + 1.04\,Z$

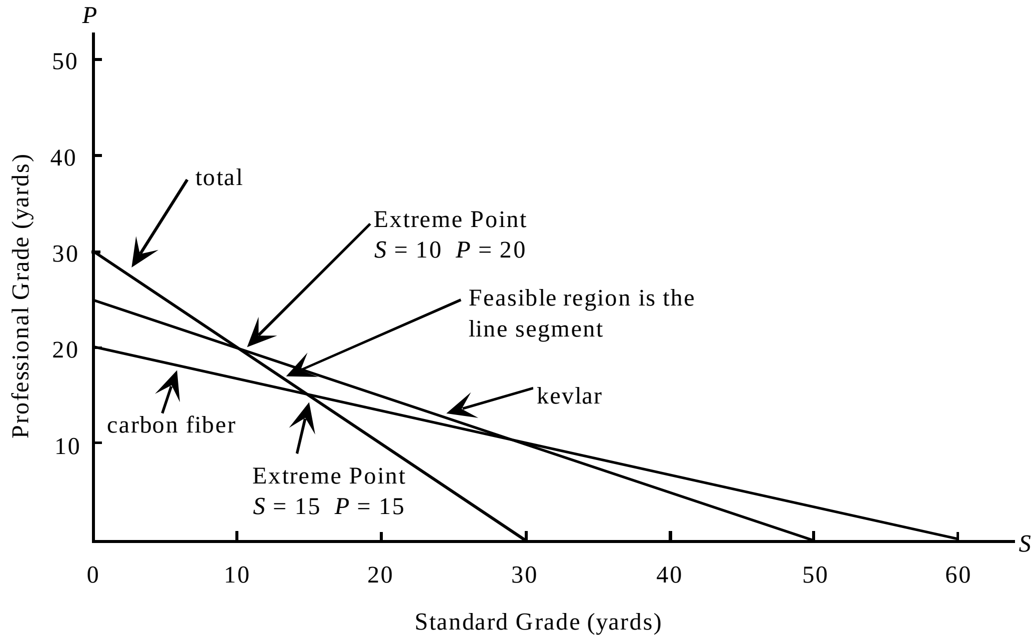
Max	$0.82\,R$	+	$1.04\,Z$			
s.t.						
	$0.60\,R$	+	$0.45\,Z$	$\leq$	8100	Mild
	$0.15\,R$	+	$0.30\,Z$	$\leq$	3000	Extra Sharp
	$R, Z \geq 0$					

Optimal Solution: $R = 9600$, $Z = 5200$, profit = 0.82(9600) + 1.04(5200) = $13,280

38. a. Let S = yards of the standard grade material per frame
P = yards of the professional grade material per frame

Min	$7.50S$	+	$9.00P$			
s.t.						
	$0.10S$	+	$0.30P$	$\geq$	6	carbon fiber (at least 20% of 30 yards)
	$0.06S$	+	$0.12P$	$\leq$	3	kevlar (no more than 10% of 30 yards)
	S	+	P	=	30	total (30 yards)
	$S, P \geq 0$					

b.



c.

Extreme Point	Cost
(15, 15)	7.50(15) + 9.00(15) = 247.50
(10, 20)	7.50(10) + 9.00(20) = 255.00

The optimal solution is $S = 15$, $P = 15$

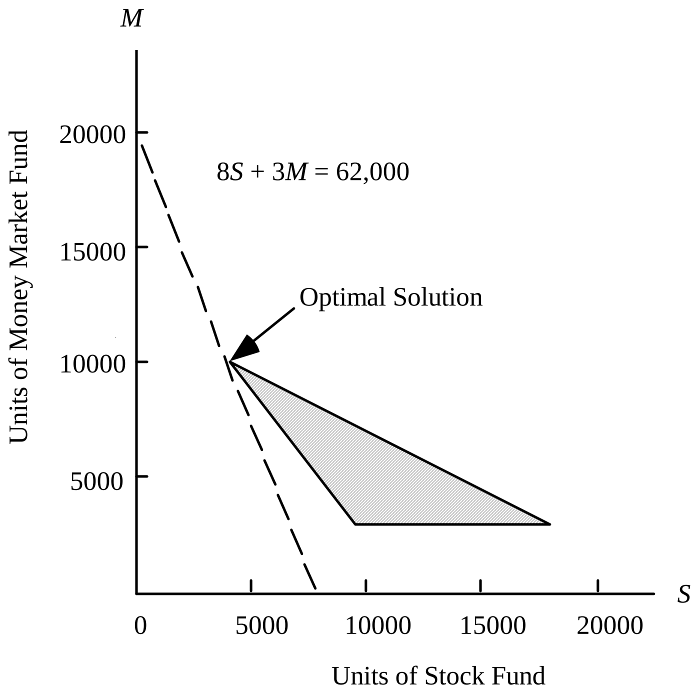
d. Optimal solution does not change: $S = 15$ and $P = 15$. However, the value of the optimal solution is reduced to 7.50(15) + 8(15) = $232.50.

e. At $7.40 per yard, the optimal solution is $S = 10$, $P = 20$. The value of the optimal solution is reduced to 7.50(10) + 7.40(20) = $223.00. A lower price for the professional grade will not change the $S = 10$, $P = 20$ solution because of the requirement for the maximum percentage of kevlar (10%).

39. a. Let S = number of units purchased in the stock fund
M = number of units purchased in the money market fund

$$\begin{aligned}
\text{Min} \quad & 8S + 3M \\
\text{s.t.} \quad & \\
& 50S + 100M \leq 1{,}200{,}000 \quad \text{Funds available} \\
& 5S + 4M \geq 60{,}000 \quad \text{Annual income} \\
& M \geq 3{,}000 \quad \text{Minimum units in money market} \\
& S, M, \geq 0
\end{aligned}$$

CLICK HERE TO ACCESS THE COMPLETE Solutions



Optimal Solution: $S = 4000$, $M = 10000$, value = 62000

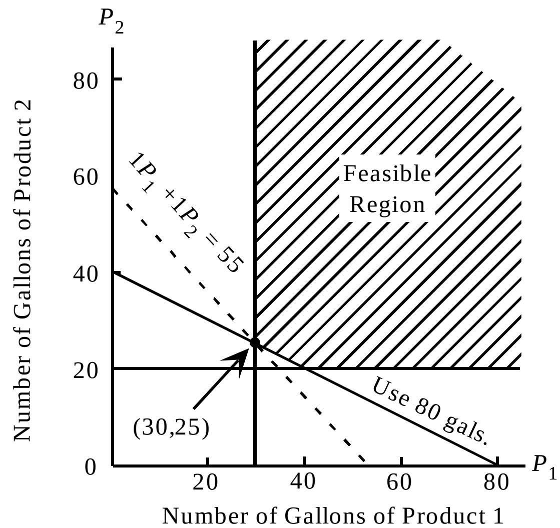
b. Annual income = 5(4000) + 4(10000) = 60,000

c. Invest everything in the stock fund.

40. Let P_1 = gallons of product 1
P_2 = gallons of product 2

Min	$1P_1$	+	$1P_2$			
s.t.						
	$1P_1$	+		$\geq$	30	Product 1 minimum
			$1P_2$	$\geq$	20	Product 2 minimum
	$1P_1$	+	$2P_2$	$\geq$	80	Raw material

$P_1, P_2 \geq 0$



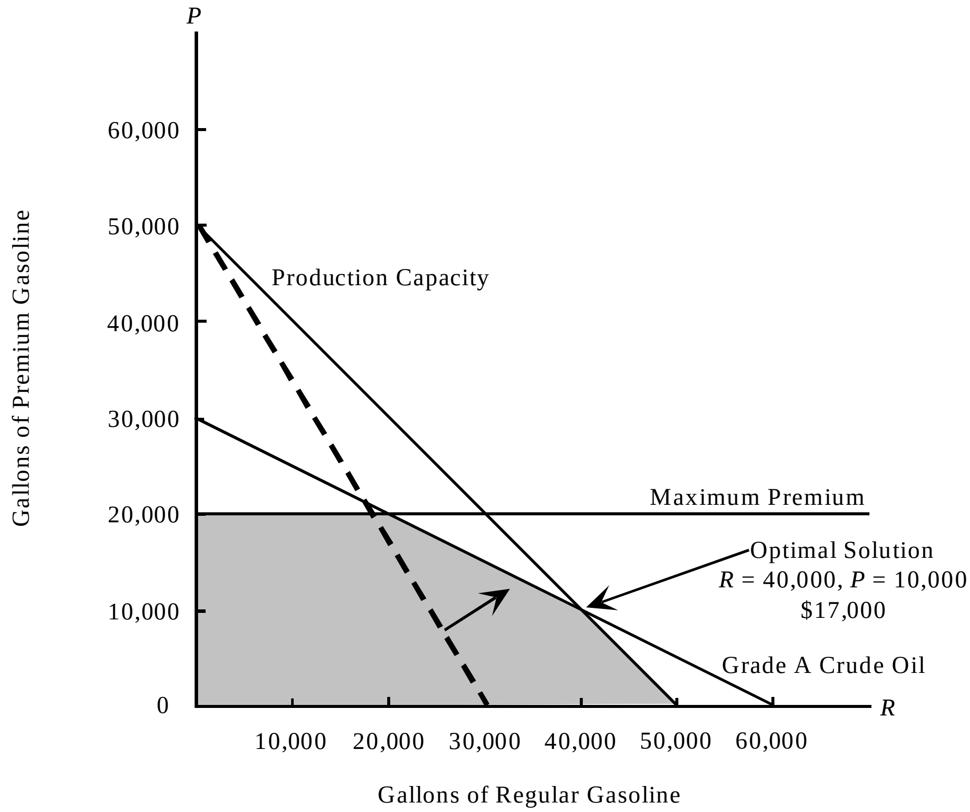
Optimal Solution: $P_1 = 30$, $P_2 = 25$ Cost = $55

41. a. Let R = number of gallons of regular gasoline produced
P = number of gallons of premium gasoline produced

Max	$0.30R$	+	$0.50P$			
s.t.						
	$0.30R$	+	$0.60P$	$\leq$	18,000	Grade A crude oil available
	$1R$	+	$1P$	$\leq$	50,000	Production capacity
			$1P$	$\leq$	20,000	Demand for premium

$R, P \geq 0$

b.



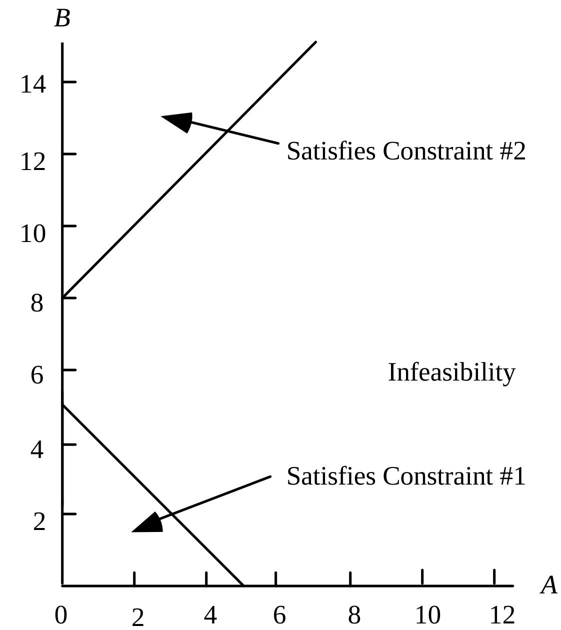
Optimal Solution:
40,000 gallons of regular gasoline
10,000 gallons of premium gasoline
Total profit contribution = $17,000

c.

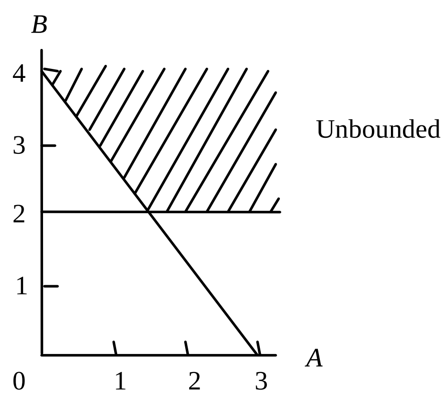
Constraint	Value of Slack Variable	Interpretation
1	0	All available grade A crude oil is used
2	0	Total production capacity is used
3	10,000	Premium gasoline production is 10,000 gallons less than the maximum demand

d. Grade A crude oil and production capacity are the binding constraints.

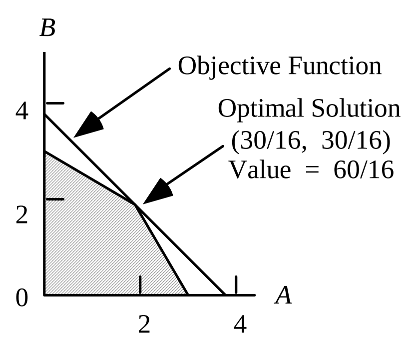
42.



43.

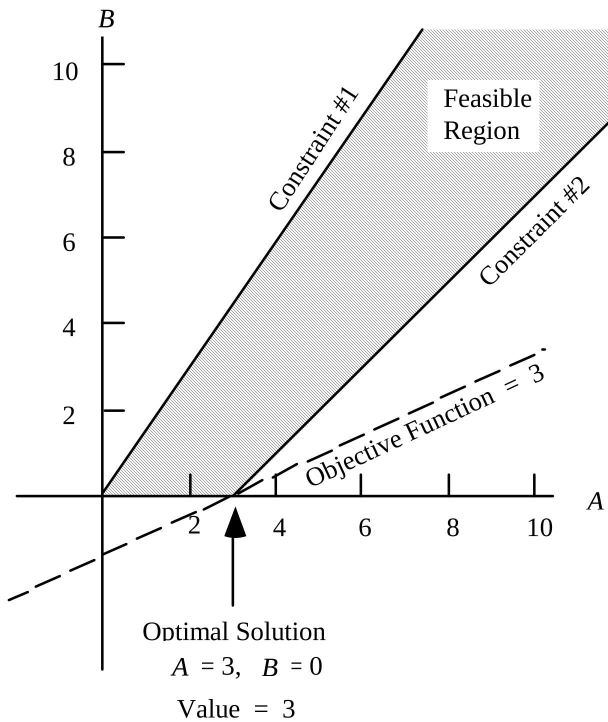


44. a.



b. New optimal solution is $A = 0$, $B = 3$, value = 6.

45. a.



b. Feasible region is unbounded.

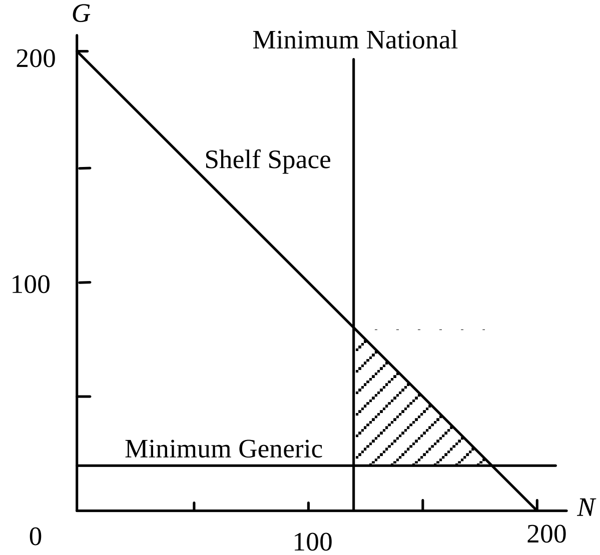
c. Optimal Solution: $A = 3$, $B = 0$, $z = 3$.

d. An unbounded feasible region does not imply the problem is unbounded. This will only be the case when it is unbounded in the direction of improvement for the objective function.

46. Let N = number of sq. ft. for national brands
G = number of sq. ft. for generic brands

Problem Constraints:

N	+	G	$\leq$	200	Space available
N			$\geq$	120	National brands
		G	$\geq$	20	Generic



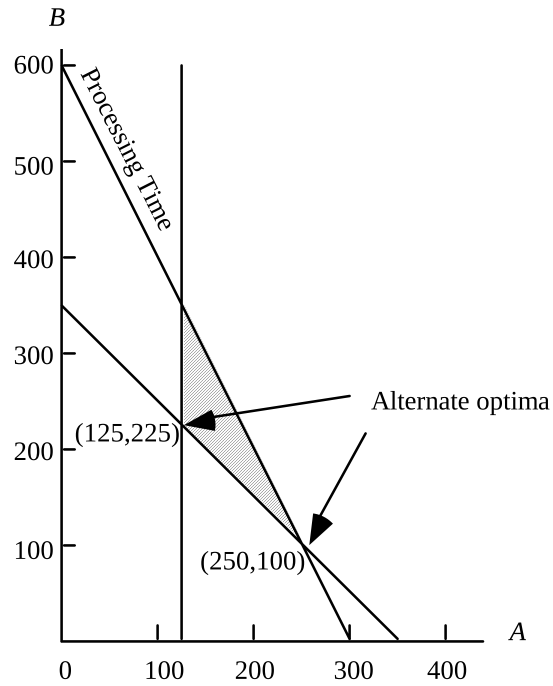
Extreme Point	*N*	*G*
1	120	20
2	180	20
3	120	80

a. Optimal solution is extreme point 2; 180 sq. ft. for the national brand and 20 sq. ft. for the generic brand.

b. Alternative optimal solutions. Any point on the line segment joining extreme point 2 and extreme point 3 is optimal.

c. Optimal solution is extreme point 3; 120 sq. ft. for the national brand and 80 sq. ft. for the generic brand.

47.



Alternative optimal solutions exist at extreme points (*A* = 125, *B* = 225) and (*A* = 250, *B* = 100).

Cost = 3(125) + 3(225) = 1050

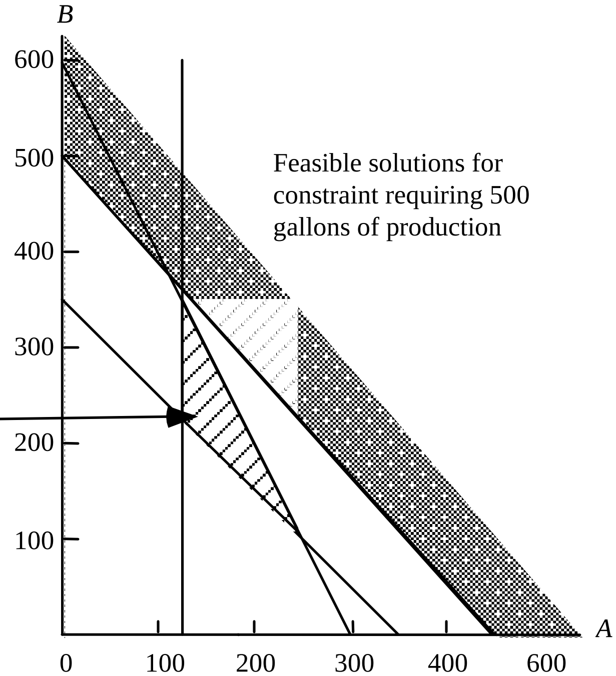
or

Cost = 3(250) + 3(100) = 1050

The solution (*A* = 250, *B* = 100) uses all available processing time. However, the solution (*A* = 125, *B* = 225) uses only 2(125) + 1(225) = 475 hours.

Thus, (*A* = 125, *B* = 225) provides 600 - 475 = 125 hours of slack processing time which may be used for other products.

48.



Possible Actions:

i. Reduce total production to *A* = 125, *B* = 350 on 475 gallons.

ii. Make solution *A* = 125, *B* = 375 which would require 2(125) + 1(375) = 625 hours of processing time. This would involve 25 hours of overtime or extra processing time.

iii. Reduce minimum *A* production to 100, making *A* = 100, *B* = 400 the desired solution.

49. a. Let P = number of full-time equivalent pharmacists
T = number of full-time equivalent physicians

The model and the optimal solution are shown below:

```
MIN 40P+10T

     S.T.
       1)  P+T >=250
       2)  2P-T>=0
       3)  P>=90
```

Optimal Objective Value
5200.00000

Variable	Value	Reduced Cost
P	90.00000	0.00000
T	160.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	10.00000
2	20.00000	0.00000
3	0.00000	30.00000

The optimal solution requires 90 full-time equivalent pharmacists and 160 full-time equivalent technicians. The total cost is $5200 per hour.

b.

	Current Levels	**Attrition**	**Optimal Values**	**New Hires Required**
Pharmacists	85	10	90	15
Technicians	175	30	160	15

The payroll cost using the current levels of 85 pharmacists and 175 technicians is 40(85) + 10(175) = $5150 per hour.

The payroll cost using the optimal solution in part (a) is $5200 per hour.

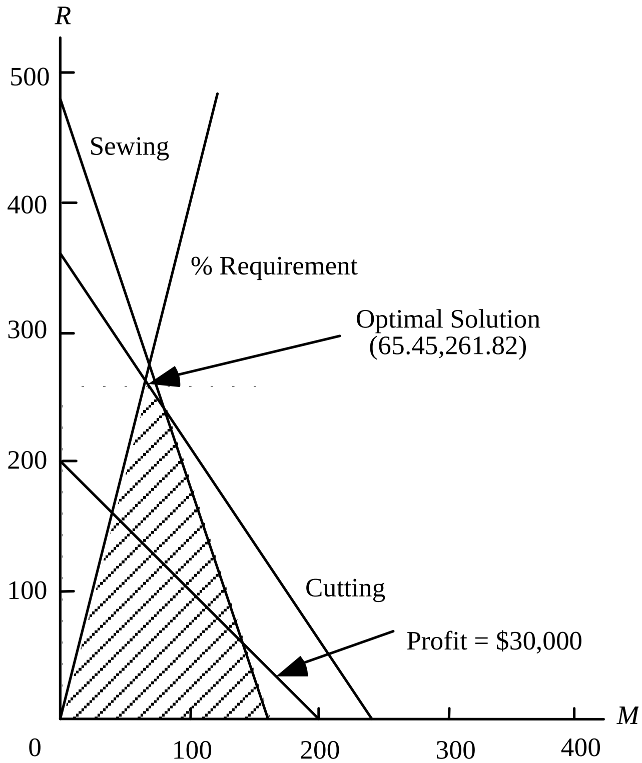
Thus, the payroll cost will go up by $50

50. Let M = number of Mount Everest Parkas
R = number of Rocky Mountain Parkas

Max	$100M$	+	$150R$			
s.t.						
	$30M$	+	$20R$	$\leq$	7200	Cutting time
	$45M$	+	$15R$	$\leq$	7200	Sewing time
	$0.8M$	-	$0.2R$	$\geq$	0	% requirement

Note: Students often have difficulty formulating constraints such as the % requirement constraint. We encourage our students to proceed in a systematic step-by-step fashion when formulating these types of constraints. For example:

M must be at least 20% of total production
$M \geq 0.2$ (total production)
$M \geq 0.2\ (M + R)$
$M \geq 0.2M + 0.2R$
$0.8M - 0.2R \geq 0$



The optimal solution is M = 65.45 and R = 261.82; the value of this solution is z = 100(65.45) + 150(261.82) = $45,818. If we think of this situation as an on-going continuous production process, the fractional values simply represent partially completed products. If this is not the case, we can approximate the optimal solution by rounding down; this yields the solution M = 65 and R = 261 with a corresponding profit of $45,650.

51. Let C = number sent to current customers
N = number sent to new customers

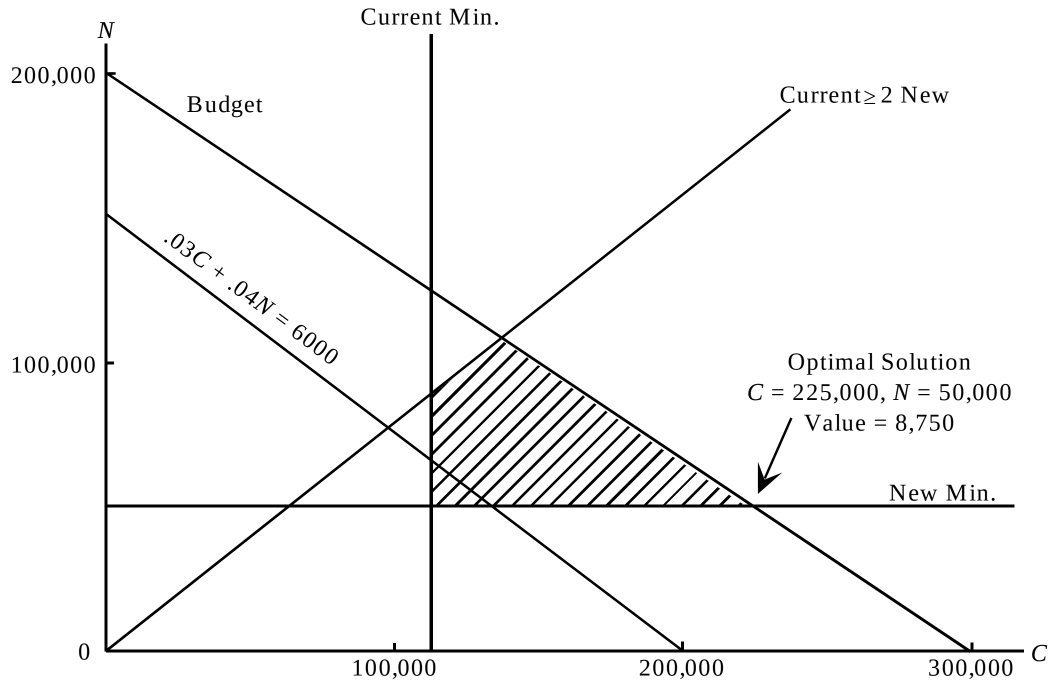
Note:

Number of current customers that test drive = $.25\,C$

Number of new customers that test drive = $.20\,N$

Number sold = $.12\,(.25\,C) + .20\,(.20\,N)$
= $.03\,C + .04\,N$

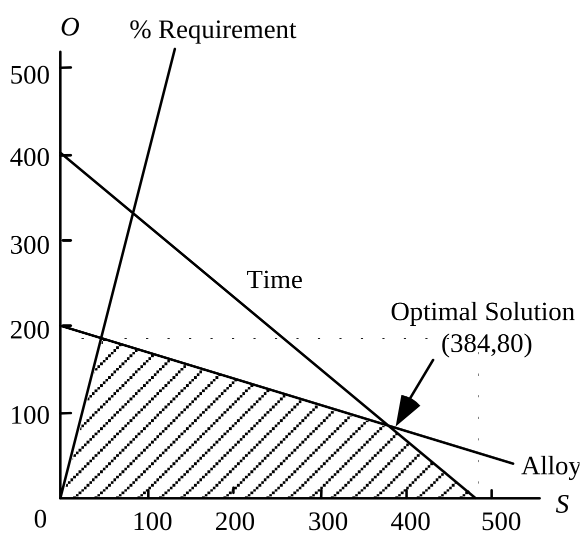
Max	$.03C$	+	$.04N$			
s.t.						
	$.25\,C$			$\geq$	30,000	Current Min
			$.20\,N$	$\geq$	10,000	New Min
	$.25\,C$	-	$.40\,N$	$\geq$	0	Current vs. New
	$4\,C$	+	$6\,N$	$\leq$	1,200,000	Budget
	$C,\ N, \geq 0$					



52. Let S = number of standard size rackets
O = number of oversize size rackets

Max	$10S$	+	$15O$			
s.t.						
	$0.8S$	-	$0.2O$	$\geq$	0	% standard
	$10S$	+	$12O$	$\leq$	4800	Time
	$0.125S$	+	$0.4O$	$\leq$	80	Alloy

$S, O, \geq 0$



53. a. Let R = time allocated to regular customer service
N = time allocated to new customer service

$$\begin{aligned} \text{Max} \quad & 1.2R + N \\ \text{s.t.} \quad & \\ & R + N \leq 80 \\ & 25R + 8N \geq 800 \\ & -0.6R + N \geq 0 \\ & R,\ N,\ \geq 0 \end{aligned}$$

b.

Optimal Objective Value
90.00000

Variable	Value	Reduced Cost
R	50.00000	0.00000
N	30.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	1.12500
2	690.00000	0.00000
3	0.00000	-0.12500

Optimal solution: $R = 50$, $N = 30$, value = 90

HTS should allocate 50 hours to service for regular customers and 30 hours to calling on new customers.

54. a. Let M_1 = number of hours spent on the M-100 machine
M_2 = number of hours spent on the M-200 machine

Total Cost
$6(40)M_1 + 6(50)M_2 + 50M_1 + 75M_2 = 290M_1 + 375M_2$

Total Revenue
$25(18)M_1 + 40(18)M_2 = 450M_1 + 720M_2$

Profit Contribution
$(450 - 290)M_1 + (720 - 375)M_2 = 160M_1 + 345M_2$

Max	$160\,M_1$	+	$345M_2$			
s.t.						
	M_1			$\leq$	15	M-100 maximum
			M_2	$\leq$	10	M-200 maximum
	M_1			$\geq$	5	M-100 minimum
			M_2	$\geq$	5	M-200 minimum
	$40\,M_1$	+	$50\,M_2$	$\leq$	1000	Raw material available

$M_1, \; M_2 \geq 0$

b.

Optimal Objective Value
5450.00000

Variable	Value	Reduced Cost
M1	12.50000	0.00000
M2	10.00000	145.00000

Constraint	Slack/Surplus	Dual Value
1	2.50000	0.00000
2	0.00000	145.00000
3	7.50000	0.00000
4	5.00000	0.00000
5	0.00000	4.00000

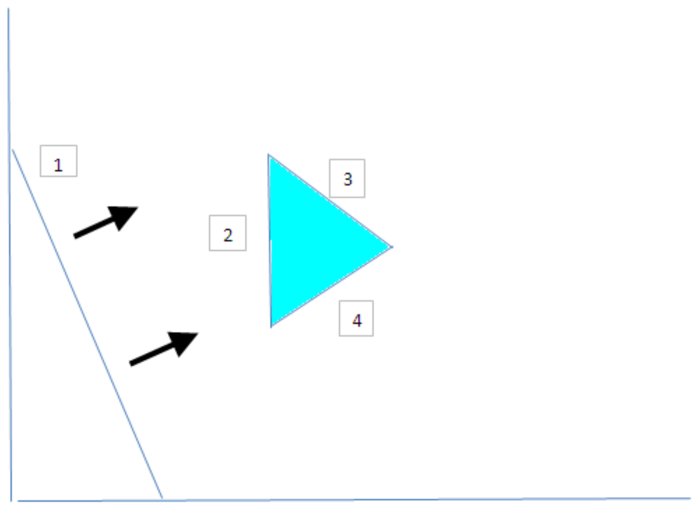
The optimal decision is to schedule 12.5 hours on the M-100 and 10 hours on the M-200.

55. Mr. Krtick's solution cannot be optimal. Every department has unused hours, so there are no binding constraints. With unused hours in every department, clearly some more product can be made.

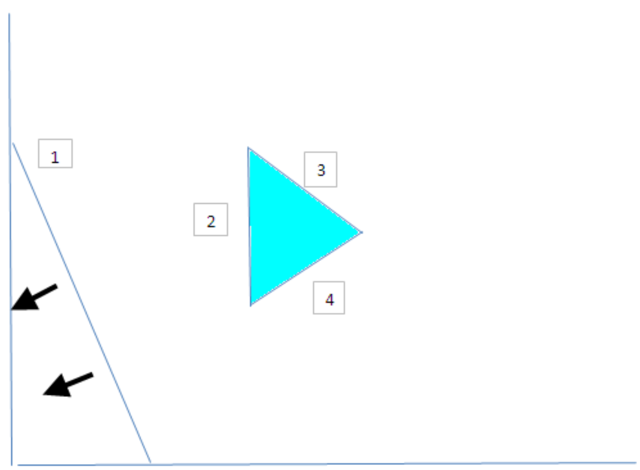
56. No, it is not possible that the problem is now infeasible. Note that the original problem was feasible (it had an optimal solution). Every solution that was feasible is still feasible when we change the constraint to less-than-or-equal-to, since the new constraint is satisfied at equality (as well as inequality). In summary, we have relaxed the constraint so that the previous solutions are feasible (and possibly more satisfying the constraint as strict inequality).

57. Yes, it is possible that the modified problem is infeasible. To see this, consider a redundant greater-than-or-equal to constraint as shown below. Constraints 2,3, and 4 form the feasible region and constraint 1 is redundant. Change constraint 1 to less-than-or-equal-to and the modified problem is infeasible.

Original Problem:



Modified Problem:



58. It makes no sense to add this constraint. The objective of the problem is to minimize the number of products needed so that everyone's top three choices are included. There are only two possible outcomes relative to the boss' new constraint. First, suppose the minimum number of products is <= 15, then there was no need for the new constraint. Second, suppose the minimum number is > 15. Then the new constraint makes the problem infeasible.

Chapter 2
An Introduction to Linear Programming

Case Problem 1: Workload Balancing

1.

	Production Rate (minutes per printer)		
Model	Line 1	Line 2	Profit Contribution ($)
DI-910	3	4	42
DI-950	6	2	87

Capacity: 8 hours $\times$ 60 minutes/hour = 480 minutes per day

Let D_1 = number of units of the DI-910 produced
D_2 = number of units of the DI-950 produced

$$\begin{aligned} \text{Max} \quad & 42D_1 + 87D_2 \\ \text{s.t.} \quad & \\ & 3D_1 + 6D_2 \leq 480 \quad \text{Line 1 Capacity} \\ & 4D_1 + 2D_2 \leq 480 \quad \text{Line 2 Capacity} \\ & D_1, D_2 \geq 0 \end{aligned}$$

The optimal solution is $D_1 = 0$, $D_2 = 80$. The value of the optimal solution is $6960.

Management would not implement this solution because no units of the DI-910 would be produced.

2. Adding the constraint $D_1 \geq D_2$ and resolving the linear program results in the optimal solution D_1 = 53.333, D_2 = 53.333. The value of the optimal solution is $6880.

3. Time spent on Line 1: 3(53.333) + 6(53.333) = 480 minutes

Time spent on Line 2: 4(53.333) + 2(53.333) = 320 minutes

Thus, the solution does not balance the total time spent on Line 1 and the total time spent on Line 2. This might be a concern to management if no other work assignments were available for the employees on Line 2.

4. Let T_1 = total time spent on Line 1
T_2 = total time spent on Line 2

Whatever the value of T_2 is,

$$T_1 \leq T_2 + 30$$
$$T_1 \geq T_2 - 30$$

Thus, with $T_1 = 3D_1 + 6D_2$ and $T_2 = 4D_1 + 2D_2$

$$3D_1 + 6D_2 \leq 4D_1 + 2D_2 + 30$$
$$3D_1 + 6D_2 \geq 4D_1 + 2D_2 - 30$$

Hence,

$-1D_1 + 4D_2 \leq 30$
$-1D_1 + 4D_2 \geq -30$

Rewriting the second constraint by multiplying both sides by -1, we obtain

$-1D_1 + 4D_2 \leq 30$
$1D_1 - 4D_2 \leq 30$

Adding these two constraints to the linear program formulated in part (2) and resolving we obtain the optimal solution $D_1 = 96.667$, $D_2 = 31.667$. The value of the optimal solution is \$6815. Line 1 is scheduled for 480 minutes and Line 2 for 450 minutes. The effect of workload balancing is to reduce the total contribution to profit by \$6880 - \$6815 = \$65 per shift.

5. The optimal solution is $D_1 = 106.667$, $D_2 = 26.667$. The total profit contribution is

$42(106.667) + 87(26.667) = \$6800$

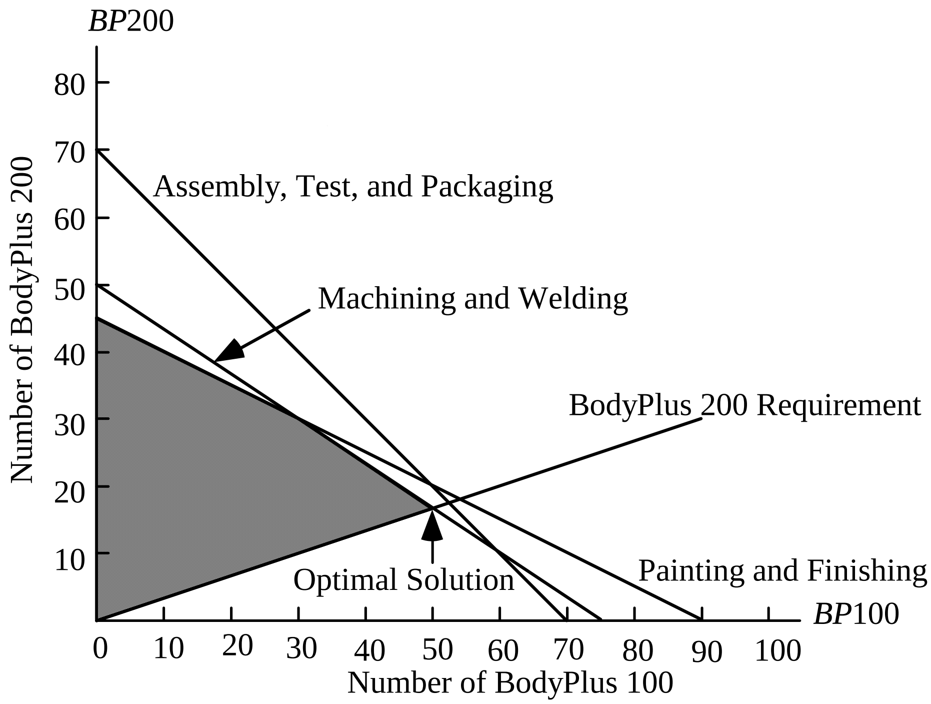
Comparing the solutions to part (4) and part (5), maximizing the number of printers produced (106.667 + 26.667 = 133.33) has increased the production by 133.33 - (96.667 + 31.667) = 5 printers but has reduced profit contribution by \$6815 - \$6800 = \$15. But, this solution results in perfect workload balancing because the total time spent on each line is 480 minutes.

Case Problem 2: Production Strategy

1. Let $BP100$ = the number of BodyPlus 100 machines produced
$BP200$ = the number of BodyPlus 200 machines produced

Max	$371BP100$	+	$461BP200$			
s.t.						
	$8BP100$	+	$12BP200$	$\leq$	600	Machining and Welding
	$5BP100$	+	$10BP200$	$\leq$	450	Painting and Finishing
	$2BP100$	+	$2BP200$	$\leq$	140	Assembly, Test, and Packaging
	$-0.25BP100$	+	$0.75BP200$	$\geq$	0	BodyPlus 200 Requirement

$BP100, BP200 \geq 0$



Optimal solution: *BP*100 = 50, *BP*200 = 50/3, profit = $26,233.33. Note: If the optimal solution is rounded to *BP*100 = 50, *BP*200 = 16.67, the value of the optimal solution will differ from the value shown. The value we show for the optimal solution is the same as the value that will be obtained if the problem is solved using a linear programming software package.

2. In the short run the requirement reduces profits. For instance, if the requirement were reduced to at least 24% of total production, the new optimal solution is *BP*100 = 1425/28, *BP*200 = 225/14, with a total profit of $26,290.18; thus, total profits would increase by $56.85. Note: If the optimal solution is rounded to *BP*100 = 50.89, *BP*200 = 16.07, the value of the optimal solution will differ from the value shown. The value we show for the optimal solution is the same as the value that will be obtained if the problem is solved using a linear programming software package such as Excel Solver.

3. If management really believes that the BodyPlus 200 can help position BFI as one of the leader's in high-end exercise equipment, the constraint requiring that the number of units of the BodyPlus 200 produced be at least 25% of total production should not be changed. Since the optimal solution uses all of the available machining and welding time, management should try to obtain additional hours of this resource.

Case Problem 3: Hart Venture Capital

1. Let S = fraction of the Security Systems project funded by HVC
   M = fraction of the Market Analysis project funded by HVC

$$
\begin{array}{llcllll}
\text{Max} & 1{,}800{,}000S & + & 1{,}600{,}000M & & & \\
\text{s.t.} & & & & & & \\
 & 600{,}000S & + & 500{,}000M & \leq & 800{,}000 & \text{Year 1} \\
 & 600{,}000S & + & 350{,}000M & \leq & 700{,}000 & \text{Year 2} \\
 & 250{,}000S & + & 400{,}000\text{M} & \leq & 500{,}000 & \text{Year 3} \\
 & S & & & \leq & 1 & \text{Maximum for } S \\
 & & & M & \leq & 1 & \text{Maximum for } M \\
 & S,M & \geq & 0 & & &
\end{array}
$$

The solution obtained is shown below:

OPTIMAL SOLUTION

Optimal Objective Value
2486956.52174

Variable	Value	Reduced Cost
S	0.60870	0.00000
M	0.86957	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	2.78261
2	30434.78261	0.00000
3	0.00000	0.52174
4	0.39130	0.00000
5	0.13043	0.00000

Objective Coefficient	Allowable Increase	Allowable Decrease
1800000.00000	120000.00000	800000.00000
1600000.00000	1280000.00000	100000.00000

RHS Value	Allowable Increase	Allowable Decrease
800000.00000	22950.81967	60000.00000
700000.00000	Infinite	30434.78261
500000.00000	25000.00000	38888.88889
1.00000	Infinite	0.39130
1.00000	Infinite	0.13043

Thus, the optimal solution is *S* = 0.609 and *M* = 0.870. In other words, approximately 61% of the Security Systems project should be funded by HVC and 87% of the Market Analysis project should be funded by HVC.

The net present value of the investment is approximately $2,486,957.

2.

	Year 1	**Year 2**	**Year 3**
Security Systems	$365,400	$365,400	$152,250
Market Analysis	$435,000	$304,500	$348,000
Total	$800,400	$669,900	$500,250

Note: The totals for Year 1 and Year 3 are greater than the amounts available. The reason for this is that rounded values for the decision variables were used to compute the amount required in each year.

3. If up to $900,000 is available in year 1 we obtain a new optimal solution with *S* = 0.689 and *M* = 0.820. In other words, approximately 69% of the Security Systems project should be funded by HVC and 82% of the Market Analysis project should be funded by HVC.

The net present value of the investment is approximately $2,550,820.
The solution follows:

```
OPTIMAL SOLUTION
```

Optimal Objective Value
2550819.67213

Variable	Value	Reduced Cost
S	0.68852	0.00000
M	0.81967	0.00000

Constraint	Slack/Surplus	Dual Value
1	77049.18033	0.00000
2	0.00000	2.09836
3	0.00000	2.16393
4	0.31148	0.00000
5	0.18033	0.00000

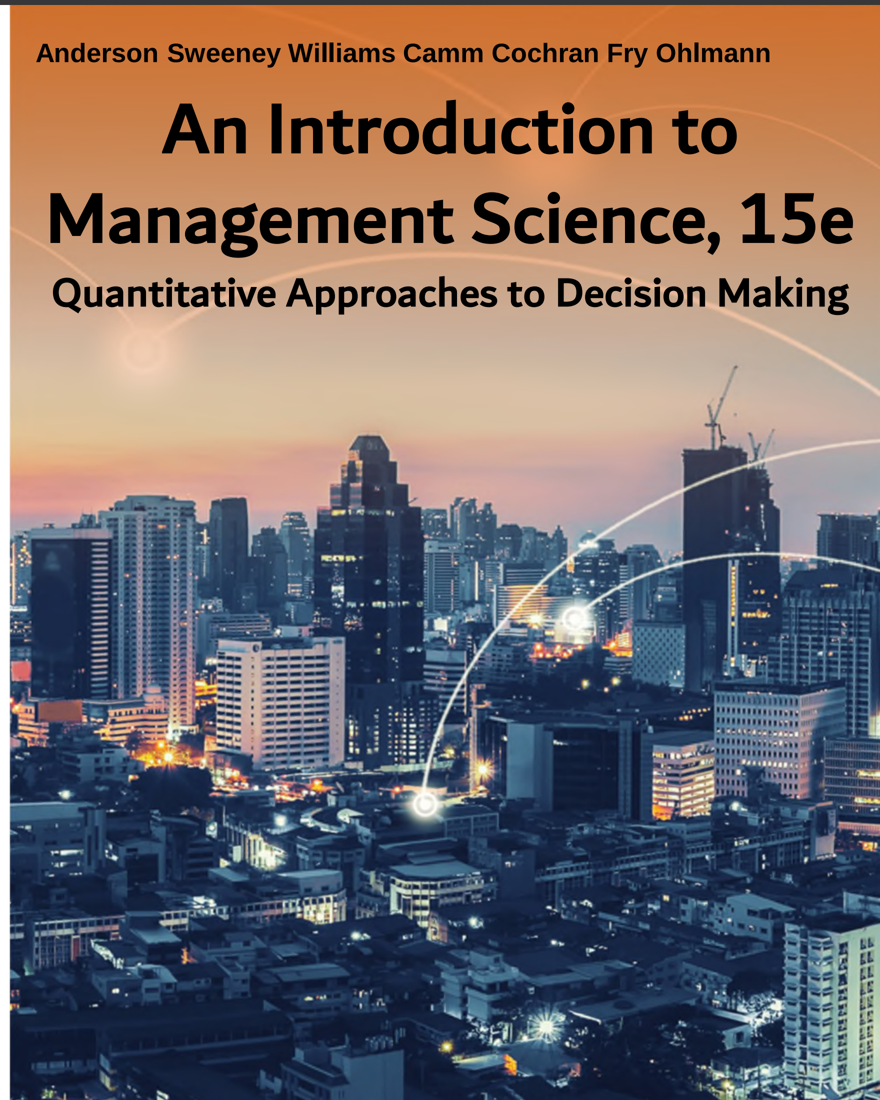
Objective Coefficient	Allowable Increase	Allowable Decrease
1800000.00000	942857.14286	800000.00000
1600000.00000	1280000.00000	550000.00000

RHS Value	Allowable Increase	Allowable Decrease
900000.00000	Infinite	77049.18033
700000.00000	102173.91304	110000.00000
500000.00000	45833.33333	135714.28571
1.00000	Infinite	0.31148
1.00000	Infinite	0.18033

4. If an additional $100,000 is made available, the allocation plan would change as follows:

	Year 1	**Year 2**	**Year 3**
Security Systems	$413,400	$413,400	$172,250
Market Analysis	$410,000	$287,000	$328,000
Total	$823,400	$700,400	$500,250

5. Having additional funds available in year 1 will increase the total net present value. The value of the objective function increases from $2,486,957 to $2,550,820, a difference of $63,863. But, since the allocation plan shows that $823,400 is required in year 1, only $23,400 of the additional $100,00 is required. We can also determine this by looking at the slack variable for constraint 1 in the new solution. This value, 77049.180, shows that at the optimal solution approximately $77,049 of the $900,000 available is not used. Thus, the amount of funds required in year 1 is $900,000 - $77,049 = $822,951. In other words, only $22,951 of the additional $100,000 is required. The differences between the two values, $23,400 and $22,951, is simply due to the fact that the value of $23,400 was computed using rounded values for the decision variables.


Anderson Sweeney Williams Camm Cochran Fry Ohlmann
An Introduction to
Management Science, 15e
Quantitative Approaches to Decision Making

Chapter 2: An Introduction to Linear Programming

Linear Programming (1 of 2)

- Linear programming has nothing to do with computer programming.
- The use of the word "programming" here means "choosing a course of action."
- Linear programming involves choosing a course of action when the mathematical model of the problem contains only linear functions.

Linear Programming (2 of 2)

- The **maximization** or **minimization** of some quantity is the **objective** in all linear programming problems.
- All LP problems have **constraints** that limit the degree to which the objective can be pursued.
- A **feasible solution** satisfies all the problem's constraints.
- An **optimal solution** is a feasible solution that results in the largest possible objective function value when maximizing (or smallest when minimizing).
- A **graphical solution method** can be used to solve a linear program with two variables.

Guidelines for Model Formulation

Problem formulation or modeling is the process of translating a verbal statement of a problem into a mathematical statement.

- Understand the problem thoroughly.
- Describe the objective.
- Describe each constraint.
- Define the decision variables.
- Write the objective in terms of the decision variables.
- Write the constraints in terms of the decision variables.

A Simple Maximization Problem (1 of 4)

Par, Inc., is a small manufacturer of golf equipment and supplies whose management has decided to move into the market for medium- and high-priced golf bags. Par, Inc.'s distributor has agreed to buy all the golf bags Par, Inc., produces over the next three months.

Each golf bag produced will require the following operations:

1. Cutting and dyeing the material
2. Sewing
3. Finishing (inserting umbrella holder, club separators, etc.)
4. Inspection and packaging

A Simple Maximization Problem (2 of 4)

This production information is summarized in this table:

	Production Time (hours)	
Department	**Standard Bag**	**Deluxe Bag**
Cutting and Dyeing	7/10	1
Sewing	1/2	5/6
Finishing	1	2/3
Inspection and Packaging	1/10	1/4

A Simple Maximization Problem (3 of 4)

- Par, Inc.'s production is constrained by a limited number of hours available in each department. The director of manufacturing estimates that 630 hours for cutting and dyeing, 600 hours for sewing, 708 hours for finishing, and 135 hours for inspection and packaging will be available for the production of golf bags during the next three months.
- The accounting department analyzed the production data and arrived at prices for both bags that will result in a profit contribution1 of $10 for every standard bag and $9 for every deluxe bag produced.

A Simple Maximization Problem (4 of 4)

The complete model for the Par, Inc., problem is as follows:

	Production Time (hours)	
Department	**Standard Bag**	**Deluxe Bag**
Cutting and Dyeing	7/10	1
Sewing	1/2	5/6
Finishing	1	2/3
Inspection and Packaging	1/10	1/4

Max $10S + 9D$

subject to (s.t.)

$\frac{7}{10}S + 1D \leq 630$ Cutting and Dyeing

$\frac{1}{2}S + \frac{5}{6}D \leq 600$ Sewing

$1S + \frac{2}{3}D \leq 708$ Finishing

$\frac{1}{10}S + \frac{1}{4}D \leq 135$ Inspection and Packaging

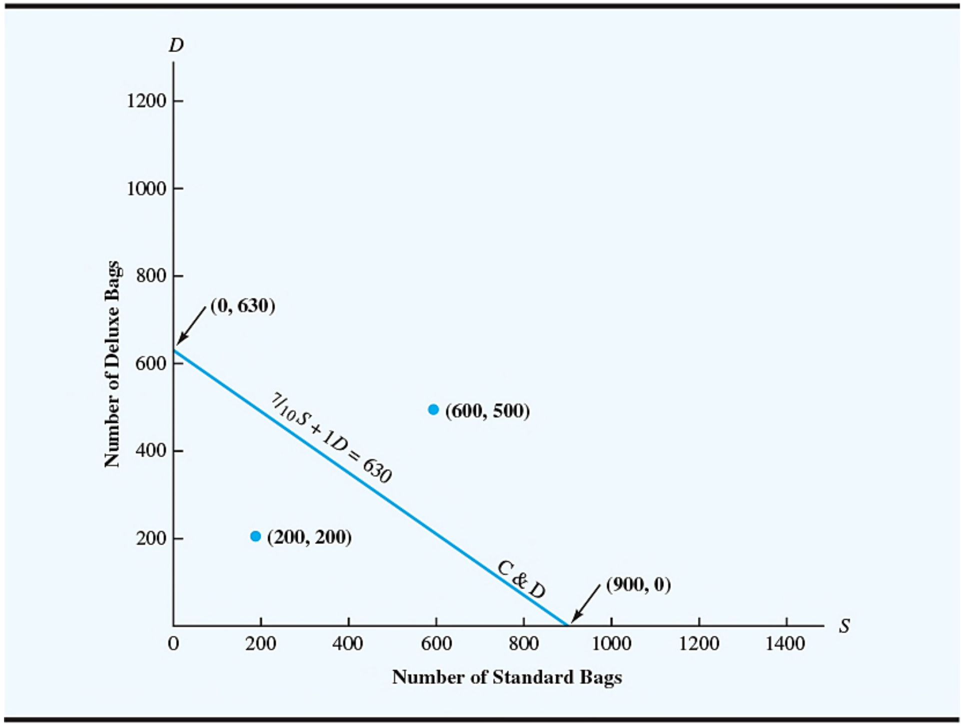
$S, D \geq 0$

Graphical Solution Procedure (1 of 5)

Earlier, we saw that the inequality representing the cutting and dyeing constraint is:

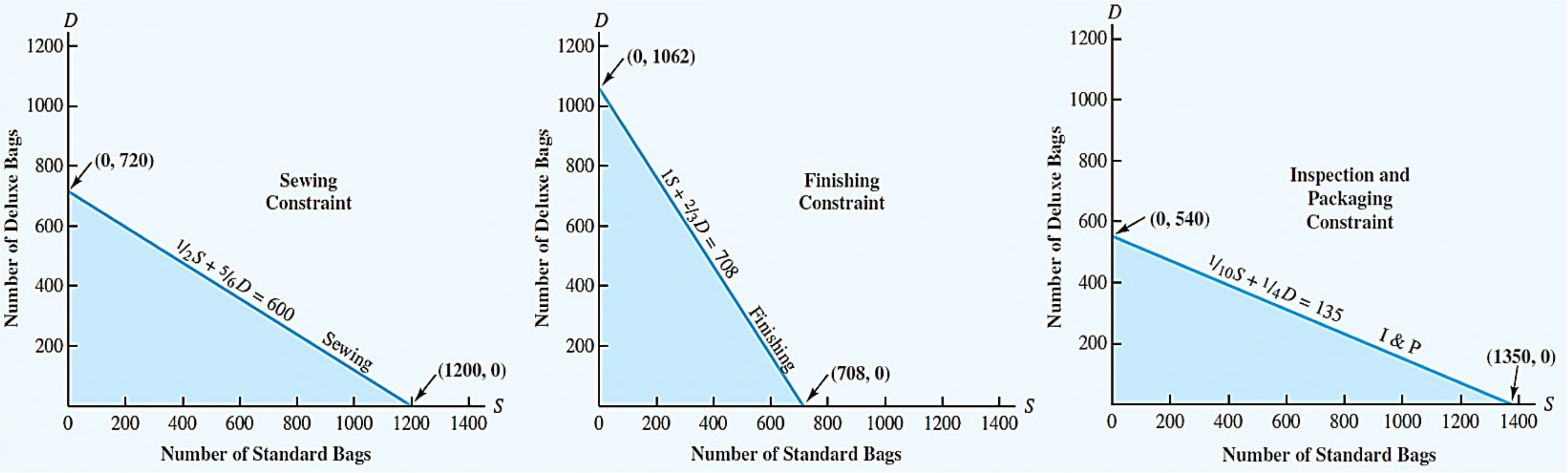
$$\frac{7}{10}S + 1D \leq 630$$

To show all solution points that satisfy this relationship, we start by graphing the solution points satisfying the constraint as an equality.



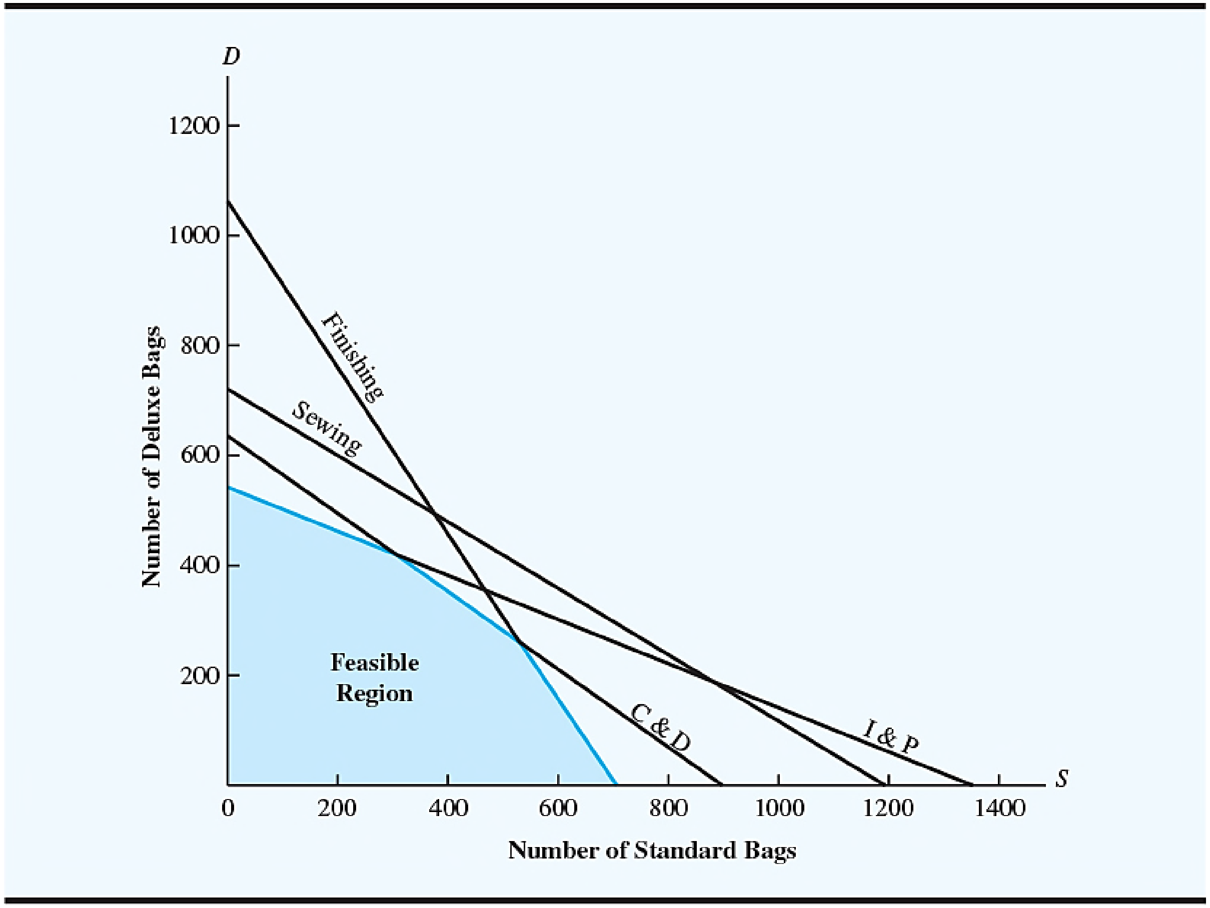
Graphical Solution Procedure (2 of 5)

We continue by identifying the solution points satisfying each of the other three constraints.



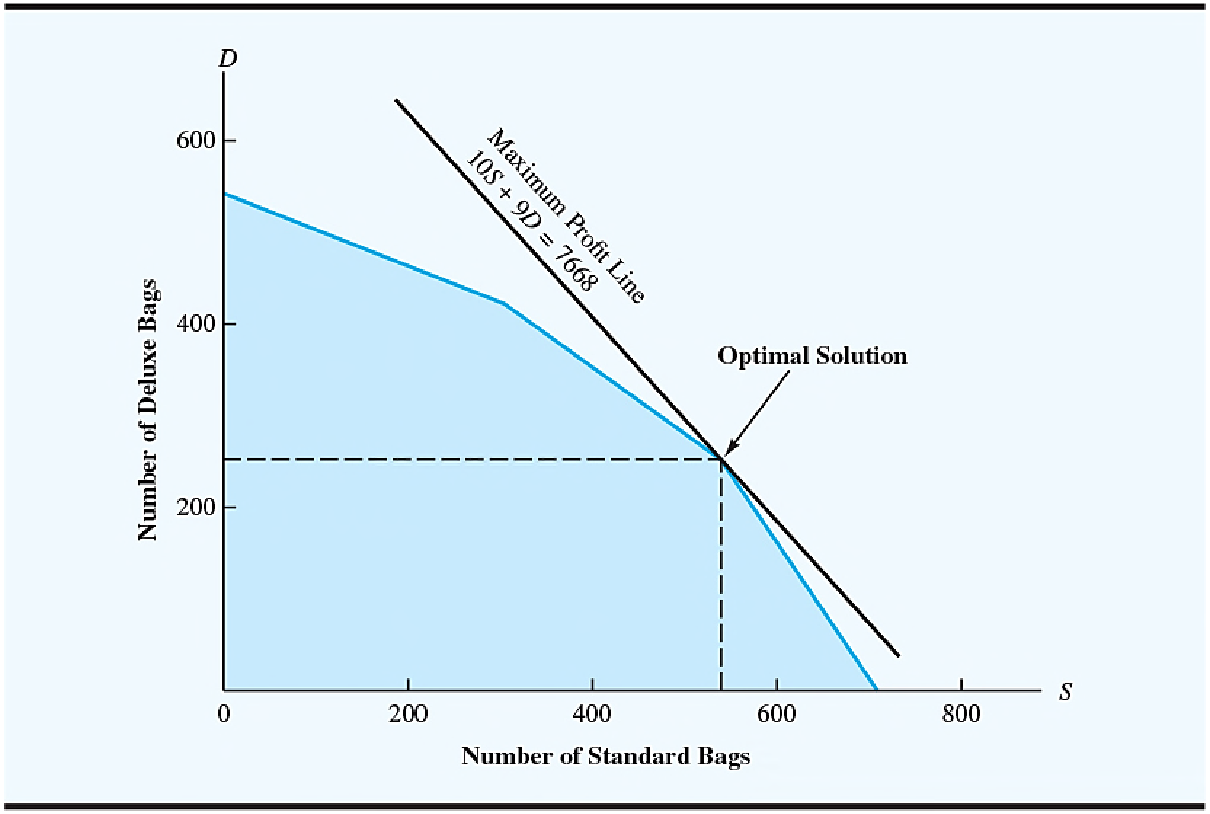
Graphical Solution Procedure (3 of 5)

The graph shown identifies the feasible region:



Graphical Solution Procedure (4 of 5)

The optimal solution point is at the intersection of the cutting and dyeing and the finishing constraint lines.



Graphical Solution Procedure (5 of 5)

The optimal values of the decision variables *S* and *D* must satisfy dyeing and the finishing constraints simultaneously.

$$\frac{7}{10}S + 1D = 630 \quad \text{Dyeing Constraint}$$

$$1S + \frac{2}{3}D = 708 \quad \text{Finishing Constraint}$$

This system of equations can be solved using substitution.

The exact location of the optimal solution point is $S = 540$ and $D = 252$. The optimal production quantities for Par, Inc., are 540 standard bags and 252 deluxe bags, with a resulting profit contribution of 10(540) + 9(252) = $7,668.

Summary of the Graphical Solution Procedure for Maximization Problems

1. Prepare a graph of the feasible solutions for each of the constraints.
2. Determine the feasible region that satisfies all the constraints simultaneously.
3. Draw an objective function line.
4. Move parallel objective function lines toward larger objective function values without entirely leaving the feasible region.
5. Any feasible solution on the objective function line with the largest value is an optimal solution.

Slack and Surplus Variables (1 of 2)

- A linear program in which all the variables are non-negative and all the constraints are equalities is said to be in **standard form**.
- Standard form is attained by adding **slack variables** to "less than or equal to" constraints, and by subtracting **surplus variables** from "greater than or equal to" constraints.
- Slack and surplus variables represent the difference between the left and right sides of the constraints.
- Slack and surplus variables have objective function coefficients equal to 0.

Slack and Surplus Variables (2 of 2)

The complete solution tells management that the production of 540 standard bags and 252 deluxe bags will require all available cutting and dyeing time (630 hours) and all available finishing time (708 hours), while 600 - 480 = 120 hours of sewing time and 135 - 117 = 18 hours of inspection and packaging time will remain unused. The 120 hours of unused sewing time and 18 hours of unused inspection and packaging time are referred to as slack for the two departments.

Constraint	**Hours Required for *S* = 540 and *D* = 252**	**Hours Available**	**Unused Hours**
Cutting and Dyeing	7/10(540) + 1(252) = 630	630	0
Sewing	1/2(540) + 5/6(252) = 480	600	120
Finishing	1(540) + 2/3(252) = 708	708	0
Inspection and Packaging	1/10(540) + 1/4(252) = 117	1/4	18

Slack Variables (1 of 2)

Often slack variables, are added to the formulation of a linear programming problem to represent the slack, or idle capacity. Unused capacity makes no contribution to profit; thus, slack variables have coefficients of zero in the objective function. After the addition of four slack variables, denoted as S_1, S_2, S_3, and S_4, the mathematical model of the Par, Inc., problem becomes

$$\text{Max} \quad 10S + 9D + 0S_1 + 0S_2 + 0S_3 + 0S_4$$

s.t.

$$+ 1D + 1S_1 + \quad + \quad + \quad = 630$$

$$\tfrac{1}{2}S + \tfrac{5}{6}D + \quad + 1S_2 + \quad + \quad = 600$$

$$1S + \tfrac{2}{3}D + \quad + \quad + 1S_3 + \quad = 708$$

$$+ \tfrac{1}{4}D + \quad + \quad + \quad + 1S_4 = 135$$

$$S, D, S_1, S_2, S_3, S_4 \geq 0$$

Slack Variables (2 of 2)

Referring to the standard form of the Par, Inc., problem, we see that at the optimal solution (*S* = 540 and *D* = 252), the values for the slack variables are

Constraint	Value of Slack Variable
Cutting and Dyeing	$S_1 = 0$
Sewing	$S_2 = 120$
Finishing	$S_3 = 0$
Inspection and Packaging	$S_4 = 18$

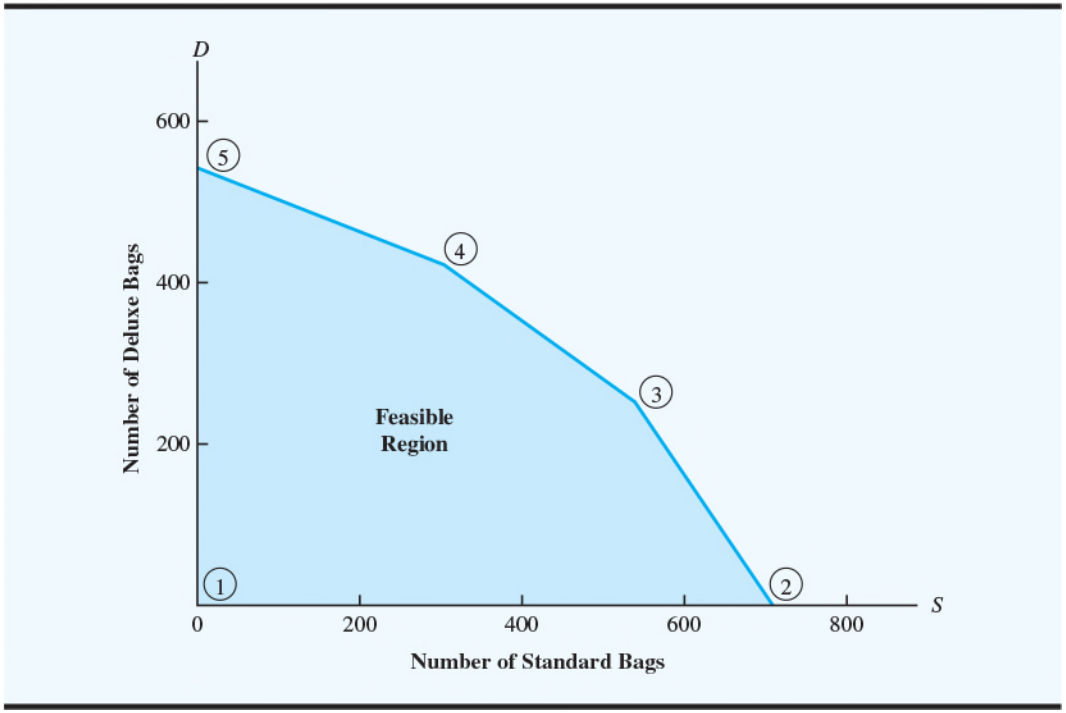
On the other hand, the sewing and the inspection and packaging constraints are not binding the feasible region at the optimal solution, which means we can expect some unused time or slack for these two operations.

Extreme Points and the Optimal Solution (1 of 2)

- The corners or vertices of the feasible region are referred to as the **extreme points**.
- An optimal solution to an LP problem can be found at an extreme point of the feasible region.
- When looking for the optimal solution, you do not have to evaluate all feasible solution points.
- You have to consider only the extreme points of the feasible region.

Extreme Points and the Optimal Solution (2 of 2)

Here are the 5 extreme points of the feasible region for the Par, Inc., Problem:



Computer Solutions (1 of 3)

- LP problems involving 1000s of variables and 1000s of constraints are now routinely solved with computer packages.
- Linear programming solvers are now part of many spreadsheet packages, such as Microsoft Excel.
- Leading commercial packages include CPLEX, LINGO, MOSEK, Xpress-MP, and Premium Solver for Excel.

Computer Solutions (2 of 3)

Here is a computer solution to the Par, Inc., Problem.

Optimal Objective Value = 7668.00000

Variable	Value	Reduced Cost
S	540.00000	0.00000
D	252.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	4.37500
2	120.00000	0.00000
3	0.00000	6.93750
4	18.00000	0.00000

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
S	10.00000	3.50000	3.70000
D	9.00000	5.28571	2.33333

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	630.00000	52.36364	134.40000
2	600.00000	Infinite	120.00000
3	708.00000	192.00000	128.00000
4	135.00000	Infinite	18.00000

Computer Solutions (3 of 3)

1. Prepare a graph of the feasible solutions for each of the constraints.
2. Determine the feasible region that satisfies all the constraints simultaneously.
3. Draw an objective function line.
4. Move parallel objective function lines toward smaller objective function values without entirely leaving the feasible region.
5. Any feasible solution on the objective function line with the smallest value is an optimal solution.

A Simple Minimization Problem (1 of 6)

M&D Chemicals produces two products that are sold as raw materials to companies manufacturing bath soaps and laundry detergents.

- M&D's management specified that the combined production for products A and B must total at least 350 gallons.
- A customer ordered 125 gallons of product A.
- Product A requires 2 hours of processing time per gallon.
- Product B requires 1 hour of processing time per gallon.
- 600 hours of processing time are available.
- M&D's objective is to satisfy these requirements at a minimum total production cost.
- Production costs are $2 per gallon for product A and $3 per gallon for product B.

A Simple Minimization Problem (2 of 6)

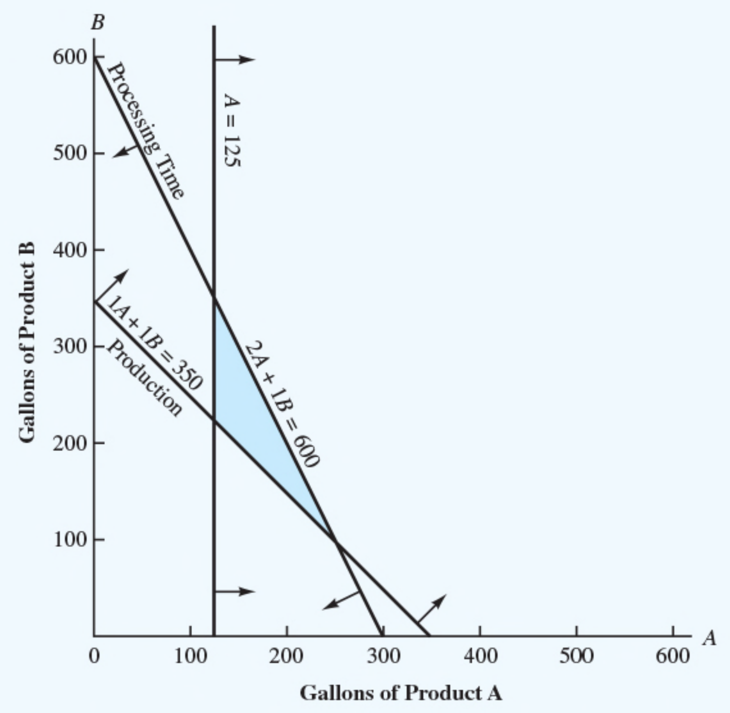
After adding the nonnegativity constraints (A, $B \geq 0$), we arrive at the following linear program for the M&D Chemicals problem:

$$
\begin{array}{lll}
\text{Min} & 2A + 3B & \\
\text{s.t.} & & \\
& 1A \geq 125 & \text{Demand for product A} \\
& 1A + 1B \geq 350 & \text{Total production} \\
& 2A + 1B \leq 600 & \text{Processing time} \\
& A, B \geq 0 &
\end{array}
$$

A Simple Minimization Problem (3 of 6)

Here is the feasible region for the M&D Chemicals problem:

Note that the objective function 2A + 3B = 800 intersects the feasible region at the extreme point A = 250, B = 100.



A Simple Minimization Problem (4 of 6)

The optimal solution to the M&D Chemicals problem shows that the desired total production of A + B = 350 gallons is achieved by using all processing time: 2A + 1B = 2(250) + 1(100) = 600 hours.

Note that the constraint requiring that product A demand be met has been satisfied with A = 250 gallons. In fact, the production of product A exceeds its minimum level by 250 – 125 = 125 gallons.

This excess production for product A is referred to as *surplus.*

A Simple Minimization Problem (5 of 6)

Including two surplus variables, S_1 and S_2, for the ≥ constraints and one slack variable, S_3, for the ≤ constraint, the linear programming model of the M&D Chemicals problem becomes

$$
\begin{aligned}
\text{Min} \quad & 2A + 3B + 0S_1 + 0S_2 + 0S_3 \\
\text{s.t.} \quad & \\
& 1A - 1S_1 = 125 \\
& 1A + 1B - 1S_2 = 350 \\
& 2A + 1B + 1S_3 = 600 \\
& A, B, S_1, S_2, S_3 \geq 0
\end{aligned}
$$

All the constraints are now equalities.

A Simple Minimization Problem (6 of 6)

At the optimal solution of A = 250 and B = 100, the values of the surplus and slack variables are as follows:

Constraint	Value of Surplus or Slack Variables
Demand for product A	$S_1 = 125$
Total production	$S_2 = 0$
Processing time	$S_3 = 0$

Computer Solution

Optimal Objective Value = 800.00000

Variable	Value	Reduced Cost
A	250.00000	0.00000
B	100.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	125.00000	0.00000
2	0.00000	4.00000
3	0.00000	–1.00000

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
A	2.00000	1.00000	Infinite
B	3.00000	Infinite	1.00000

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	125.00000	125.00000	Infinite
2	350.00000	125.00000	50.00000
3	600.00000	100.00000	125.00000

Feasible Region

- The feasible region for a two-variable LP problem can be nonexistent, a single point, a line, a polygon, or an unbounded area.
- Any linear program falls in one of four categories:
  - is infeasible
  - has a unique optimal solution
  - has alternative optimal solutions
  - has an objective function that can be increased without bound
- A feasible region may be unbounded and yet there may be optimal solutions. This is common in minimization problems and is possible in maximization problems.

Special Cases (1 of 5)

Alternative Optimal Solutions

In the graphical method, if the objective function line is parallel to a boundary constraint in the direction of optimization, there are **alternate optimal solutions**, with all points on this line segment being optimal.

Special Cases (2 of 5)

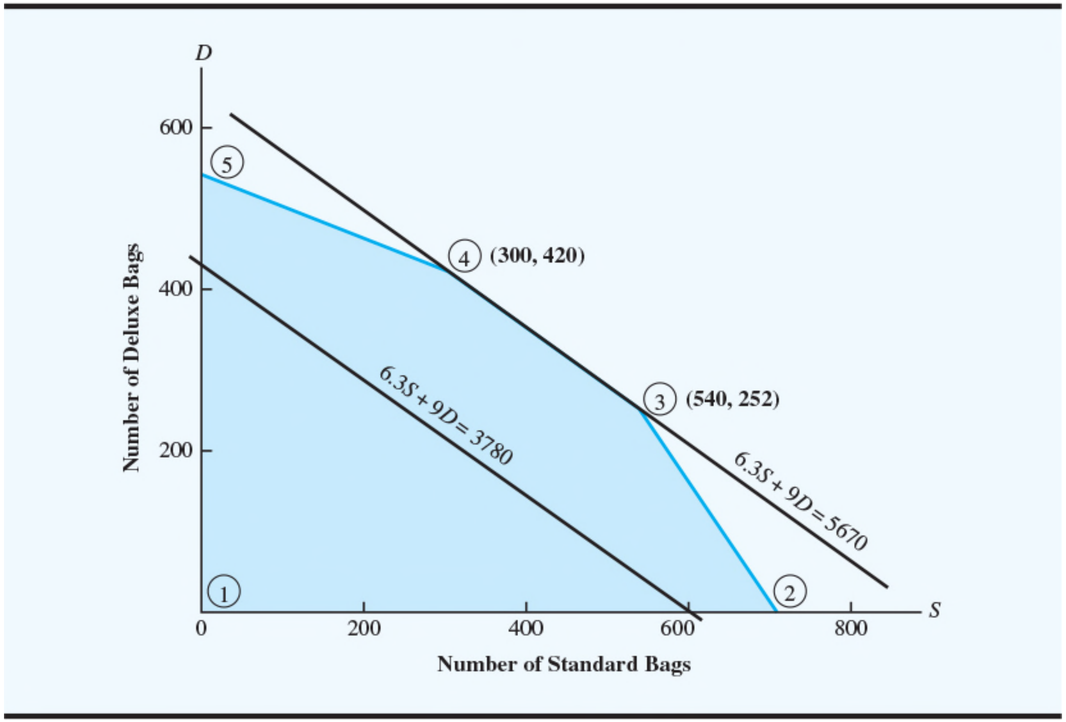
Let's return to the Par, Inc., problem. However, now assume that the profit for the standard golf bag (*S*) has decreased to $6.30. The revised objective function becomes 6.3*S* + 9*D*.

The objective function values at these two extreme points are identical:

6.3S + 9D =
6.3(300)+9(420) = 5670

and

6.3S + 9D =
6.3(540)+9(252) = 5670



Special Cases (3 of 5)

Furthermore, any point on the line connecting the two optimal extreme points also provides an optimal solution.

For example, the solution point (S = 420, D = 336), which is halfway between the two extreme points, also provides the optimal objective function value of $6.3S + 9D = 6.3(420) + 9(336) = 5670$.

A linear programming problem with alternative optimal solutions is generally a good situation for the manager or decision maker. It means that several combinations of the decision variables are optimal and that the manager can select the most desirable optimal solution.

Special Cases (4 of 5)

Infeasibility

- No solution to the LP problem satisfies all the constraints, including the non-negativity conditions.
- Graphically, this means a feasible region does not exist.
- Causes include:
    - A formulation error has been made.
    - Management’s expectations are too high.
    - Too many restrictions have been placed on the problem (i.e. the problem is over-constrained).

Special Cases (5 of 5)

Unbounded

- The solution to a maximization LP problem is unbounded if the value of the solution may be made indefinitely large without violating any of the constraints.
- For real problems, this is the result of improper formulation. (Quite likely, a constraint has been inadvertently omitted.)

General Linear Programming Notation (1 of 3)

We selected decision-variable names of S and D in the Par, Inc., problem and A and B in the M&D Chemicals problem to make it easier to recall what these decision variables represented in the problem.

Although this approach works well for linear programs involving a small number of decision variables, it can become difficult when dealing with problems involving a large number of decision variables.

General Linear Programming Notation (2 of 3)

A more general notation that is often used for linear programs uses the letter *x* with a subscript.

In the Par, Inc., problem, we could have defined the decision variables:

x_1 = number of standard bags

x_2 = number of deluxe bags

In the M&D Chemicals problem, the same variable names would be used, but their definitions would change:

x_1 = number of gallons of product A

x_2 = number of gallons of product B

General Linear Programming Notation (3 of 3)

A disadvantage of using general notation for decision variables is that we are no longer able to easily identify what the decision variables actually represent in the mathematical model.

The advantage of general notation is that formulating a mathematical model for a problem that involves a large number of decision variables is much easier.

End of Presentation: Chapter 2