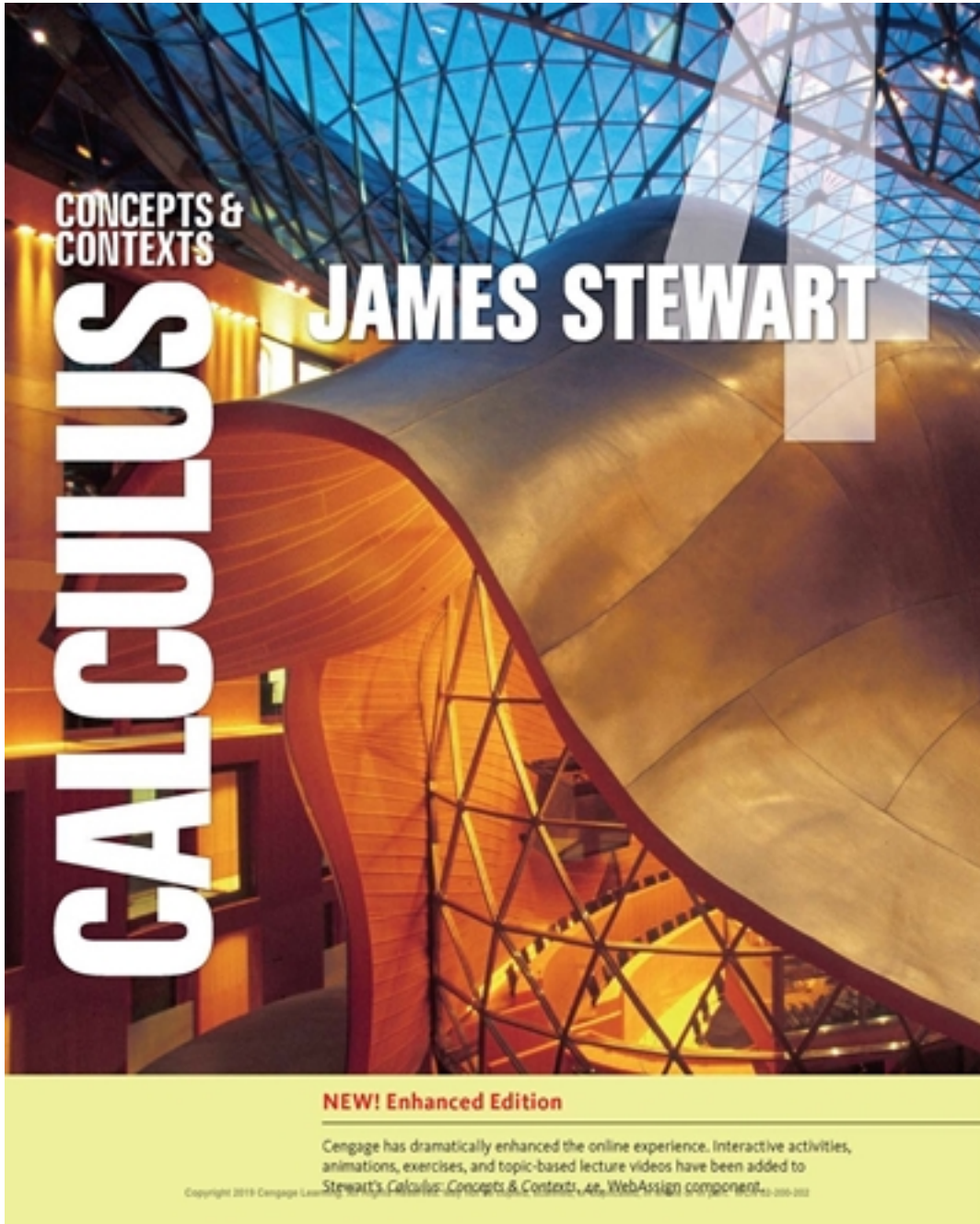


Test Bank for Calculus Concepts and Contexts Enhanced Edition 4th Edition by Stewart

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Test Bank

Test Bank for

Stewart's

Calculus

Concepts and Contexts

FOURTH EDITION

Prepared by

William Tomhave

Concordia College

Xueqi Zeng

Concordia College



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Preface

These test items were designed to accompany *Calculus: Concepts and Contexts, 4th Edition* by James Stewart. As in the previous editions the test items offer both multiple choice and free response formats and approach the Calculus from four viewpoints: Descriptive, Algebraic, Numeric and Graphic. These items are designed to help instructors assess both student manipulative skills and conceptual understanding. It is our hope that you will find enough variety in item difficulty, approach and application areas to allow you substantial flexibility in designing examinations and quizzes that meet the needs of you and your students.

This project could not have been completed without the assistance of several associates. We would especially like to thank Jessie Lenarz for her work related to page layout and design and her careful checking. We would like to thank Jeannine Lawless for her patience, support and encouragement throughout this writing process. As in previous editions we express our sincere thanks to James Stewart for providing us with the opportunity to be part of one of his projects. Finally, we extend our deepest gratitude to Lois and Wentong, two spouses who have been incredibly supportive as we carried out the work involved in a project as time-intensive as this one.

William K. Tomhave
Xueqi Zeng

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1 Functions and Models

1.1 Four Ways to Represent a Function

1. Find the smallest value in the domain of the function $f(x) = \sqrt{2x - 5}$.

(A) 2 (B) $\frac{5}{2}$ (C) 5 (D) $\frac{2}{5}$
 (E) -2 (F) 1 (G) 0 (H) -5

Answer: (B)

2. Find the smallest value in the range of the function $f(x) = 3x^2 + 24x + 40$.

(A) -4 (B) -5 (C) -6 (D) -7
 (E) -8 (F) -16 (G) -24 (H) -40

Answer: (E)

3. The range of the function $f(x) = \sqrt{20 + 8x - x^2}$ is a closed interval $[a, b]$. Find its length $b - a$.

(A) 1 (B) 2 (C) 3 (D) 4
 (E) 5 (F) 6 (G) 7 (H) 9

Answer: (F)

4. Find the smallest value in the range of the function $f(x) = |2x| + |2x + 3|$.

(A) 2 (B) 3 (C) 5 (D) $\frac{1}{2}$
 (E) $\frac{3}{2}$ (F) $\frac{5}{2}$ (G) 0 (H) 1

Answer: (B)

5. Find the largest value in the domain of the function $f(x) = \sqrt{\frac{3 - 2x}{4 + 3x}}$.

(A) $-\frac{3}{2}$ (B) $-\frac{2}{3}$ (C) 0 (D) 2
 (E) $\frac{2}{3}$ (F) $\frac{3}{2}$ (G) 3 (H) No largest value

Answer: (F)

2 1 Functions and Models

6. Find the range of the function $f(x) = \begin{cases} x^2 - 4x & \text{if } x \leq 2 \\ |x - 4| & \text{if } x > 2 \end{cases}$.

- (A) $[0, \infty)$ (B) $(-\infty, 2]$ (C) $[-4, \infty)$ (D) $(-\infty, 0]$
 (E) $[4, \infty)$ (F) $(-\infty, 4]$ (G) $[2, \infty)$ (H) $(-\infty, -4]$

Answer: (C)

7. Find the range of the function $f(x) = |x - 1| + x - 1$.

- (A) $[1, \infty)$ (B) $(1, \infty)$ (C) $[0, \infty)$ (D) $(0, \infty)$
 (E) $[-1, \infty)$ (F) $(-1, \infty)$ (G) $[0, 1]$ (H) \mathbb{R}

Answer: (C)

8. The function $f(x) = \sqrt{\frac{x-1}{x}}$ has as its domain all values of x such that

- (A) $x > 0$ (B) $x \geq 1$ (C) $x \leq 0$ (D) $x \leq 1$
 (E) $0 < x \leq 1$ (F) $x \geq 1$ or $x < 0$ (G) $x \geq -1$ (H) $-1 \leq x < 0$

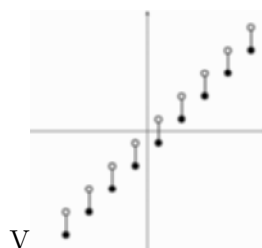
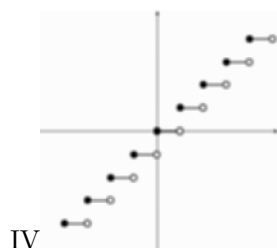
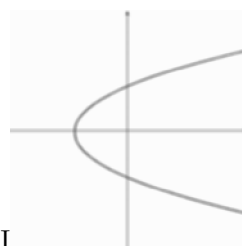
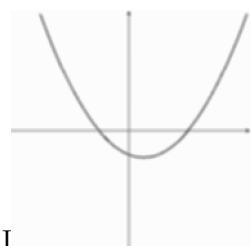
Answer: (F)

9. Find the range of the function $f(x) = \frac{3x+4}{5-2x}$.

- (A) $(-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, \infty)$ (B) $(-\infty, \frac{4}{5}) \cup (\frac{4}{5}, \infty)$
 (C) $(-\infty, \frac{3}{5}) \cup (\frac{3}{5}, \infty)$ (D) $(-\infty, -2) \cup (-2, \infty)$
 (E) $(-\infty, 2) \cup (2, \infty)$ (F) $(-\infty, 3) \cup (3, \infty)$
 (G) $(-\infty, 4) \cup (4, \infty)$ (H) $(-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, \infty)$

Answer: (A)

10. Which of the following are graphs of functions?

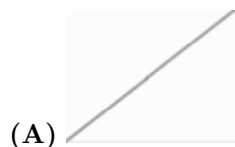


- (A) I only (B) II only (C) III only (D) I and II only
 (E) I and III only (F) I, II, and IV only (G) II and V only (H) I, II, and III only

Answer: (F)

11. Each of the functions in the table below is increasing, but each increases in a different way.
 Select the graph from those given below which best fits each function:

t	1	2	3	4	5	6
$f(t)$	26	34	41	46	48	49
$g(t)$	16	24	32	40	48	56
$h(t)$	36	44	53	64	77	93

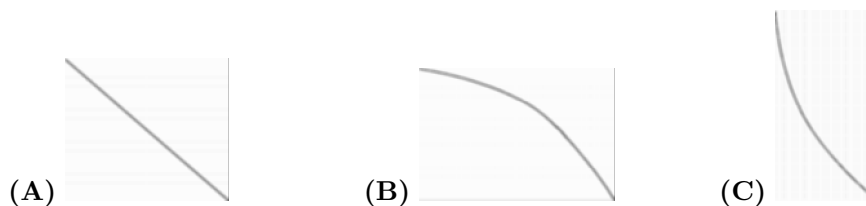


Answer: $f(t)$: (B) $g(t)$: (A) $h(t)$: (C)

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12. Each of the functions in the table below is decreasing, but each decreases in a different way. Select the graph from those given below which best fits each function:

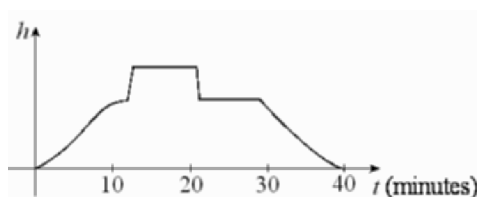
t	1	2	3	4	5	6
$f(t)$	98	91	81	69	54	35
$g(t)$	80	71	63	57	53	52
$h(t)$	78	69	60	51	42	33



Answer: $f(t)$: (B) $g(t)$: (C) $h(t)$: (A)

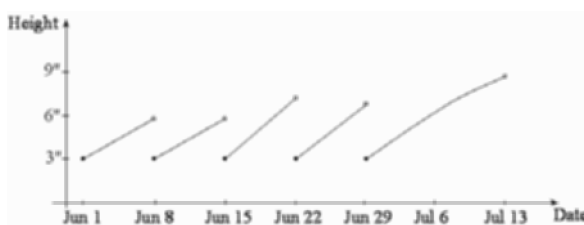
13. Suppose a pet owner decides to wash her dog in the laundry tub. She fills the laundry tub with warm water, puts the dog into the tub and shampoos it, removes the dog from the tub to towel it, then pulls the plug to drain the tub. Let t be the time in minutes, beginning when she starts to fill the tub, and let $h(t)$ be the water level in the tub at time t . If the total time for filling and draining the tub and washing the dog was 40 minutes, sketch a possible graph of $h(t)$.

Answer: (One possible answer — answers will vary.)

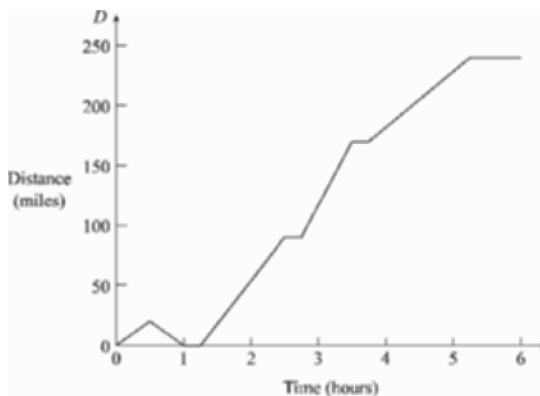


14. A homeowner mowed her lawn on June 1, cutting it to a uniform height of 3". She mowed the lawn at one-week intervals after that until she left for a vacation on June 30. A local lawn service put fertilizer on her lawn shortly after she mowed on June 15, causing the grass to grow more rapidly. She returned from her vacation on July 13 to find that the neighborhood boy whom she had hired to mow the lawn while she was away had indeed mowed on June 22 and on June 29, but had forgotten to mow on July 6. Sketch a possible graph of the height of the grass as a function of time over the time period from June 1 through July 13.

Answer: (One possible answer — answers will vary.)



15. A professor left the college for a professional meeting, a trip that was expected to take 4 hours. The graph below shows the distance $D(t)$ that the professor has traveled from the college as a function of the time t , in hours. Refer to the graph and answer the questions which follow.



- Describe what might have happened at $D(0.5)$.
- Describe what might have happened at $D(1.0)$.
- Describe what might have happened at $D(1.2)$.
- Describe what might have happened at $D(2.5)$.
- Describe what might have happened at $D(3.5)$.
- Describe what might have happened at $D(3.75)$.

6 1 Functions and Models

- (g) Describe what might have happened at $D(4.0)$.
- (h) Describe what might have happened at $D(5.25)$.

Answer:

- (a) He was traveling to the meeting.
- (b) He returned to the college (maybe he forgot something.)
- (c) He left the college for the meeting again.
- (d) He stopped to rest.
- (e) He stopped for a second time after traveling at a relatively high rate of speed, perhaps at the request of a highway patrol officer.
- (f) He continued on his trip but at a substantially lower rate of speed.
- (g) He was traveling to the meeting.
- (h) He arrived at his destination.

16. Let $f(x) = 4 - x^2$. Find

- (a) the domain of f .
- (b) the range of f .

Answer: (a) $(-\infty, \infty)$ (b) $(-\infty, 4]$

17. Let $f(x) = \sqrt{2x + 5}$. Find

- (a) the domain of f .
- (b) the range of f .

Answer: (a) $[-\frac{5}{2}, \infty)$ (b) $[0, \infty)$

18. Let $f(x) = \sqrt{16 - x^2}$. Find

- (a) the domain of f .
- (b) the range of f .

Answer: (a) $[-4, 4]$ (b) $[0, 4]$

19. Let $f(x) = \sqrt{\frac{3-x}{x+2}}$. Find

(a) the domain of f .

(b) the range of f .

Answer: (a) $(-2, 3]$ (b) $(0, \infty)$

20. Express the area A of a circle as a function of its circumference C .

Answer: $A = \frac{C^2}{4\pi}$

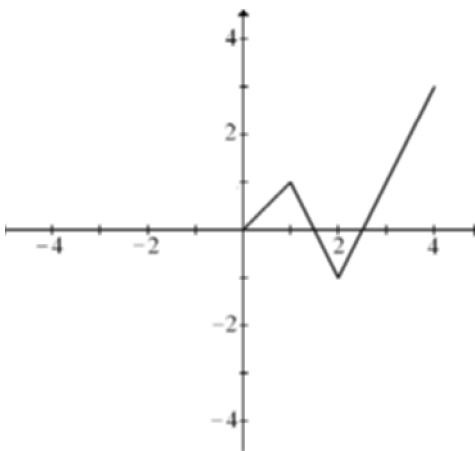
21. Let $f(x) = \begin{cases} x^2 + 3 & \text{if } x \leq -1 \\ \frac{2+3x}{6} & \text{if } x > -1 \end{cases}$ Find

(a) the domain of f .

(b) the range of f .

Answer: (a) $(-\infty, \infty)$ (b) $(-\frac{1}{6}, \infty)$

22. A function has domain $[-4, 4]$ and a portion of its graph is shown.



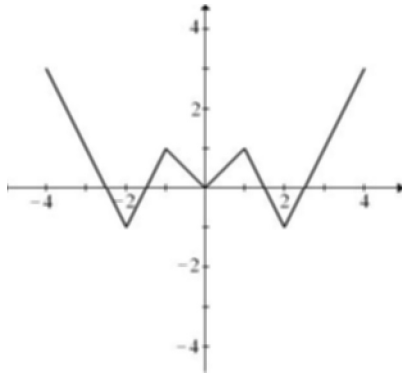
(a) Complete the graph of f if it is known that f is an even function.

(b) Complete the graph of f if it is known that f is an odd function.

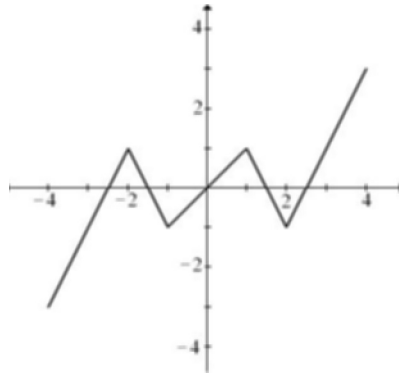
8 1 Functions and Models

Answer:

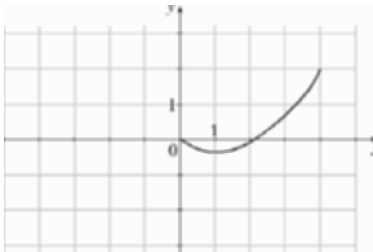
(a)



(b)



23. A function has domain $[-4, 4]$ and a portion of its graph is shown.

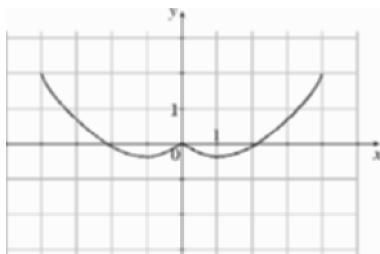


(a) Complete the graph of f if it is known that f is an even function.

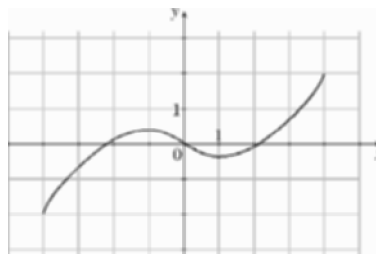
(b) Complete the graph of f if it is known that f is an odd function.

Answer:

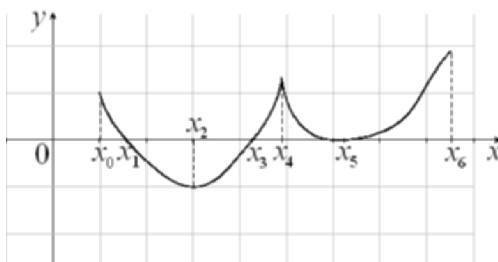
(a)



(b)



24. Given the graph of $y = f(x)$:



Find all values of x where:

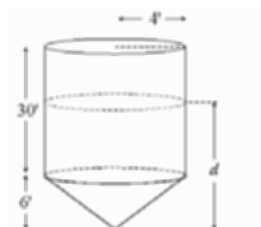
- (a) f is increasing.
 (b) f is decreasing.

Answer: (a) (x_2, x_4) and (x_5, x_6) (b) (x_0, x_2) and (x_4, x_5)

25. An ice cream vendor is stopped on the side of a city street 100 feet from a perpendicular intersection of the street with another straight city street. A bicyclist is riding on the perpendicular street at a rate of 1320 feet/second. If the bicyclist continues to ride straight ahead at the same rate of speed, write a function for the distance, d , between the ice cream vendor and the bicyclist for time t beginning when the bicyclist is in the intersection.

Answer: $d(t) = \sqrt{100^2 + (1320t)^2}$.

26. A tank used for portland cement consists of a cylinder mounted on top of a cone, with its vertex pointing downward. The cylinder is 30 feet high, both the cylinder and the cone have radii of 4 feet, and the cone is 6 feet high.



- (a) Determine the volume of cement contained in the tank as a function of the depth d of the cement.
 (b) What is the domain of this function?

10 1 Functions and Models

Answer:

$$(a) \ V(d) = \begin{cases} \frac{4\pi d^3}{27} & \text{if } 0 \leq d \leq 6 \\ 16\pi d - 64\pi & \text{if } 6 < d \leq 36 \end{cases} \quad (b) \ d \in [0, 36]$$

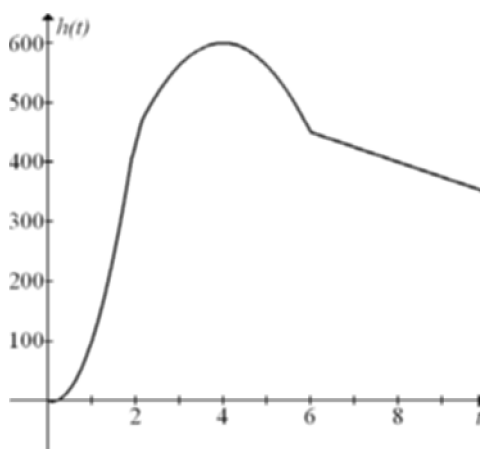
27. A parking lot light is mounted on top of a 20-foot tall lamppost. A person T feet tall is walking away from the lamppost along a straight path. Determine a function which expresses the length of the person's shadow in terms of the person's distance from the lamppost.



Answer: Let L be the length of the person's shadow and x be the person's distance from the lamppost. Then $L = \frac{Tx}{(20 - T)}$.

28. A small model rocket is launched vertically upward on a calm day. The engine delivers its thrust at a constant rate for 2 seconds, at which point the engine burns out. The rocket continues until it begins to fall from its maximum height of 600 feet. Six seconds into the flight a parachute is automatically deployed and the rocket descends at a constant rate of 30 feet per second. Sketch a possible graph of the altitude, $h(t)$, of the rocket at time t for the first 10 seconds of the flight.

Answer: Answers will vary. One possible graph



1.2 Mathematical Models: A Catalog of Essential Functions

1. Classify the function $f(x) = \frac{x^2 + \pi}{x}$.

(A) Power function

(B) Root function

(C) Polynomial function

(D) Rational function

(E) Algebraic function

(F) Trigonometric function

(G) Exponential function

(H) Logarithmic function

Answer: (D)

2. Classify the function $f(x) = \frac{\pi^2 + x^2}{e}$.

(A) Power function

(B) Root function

(C) Polynomial function

(D) Rational function

(E) Algebraic function

(F) Trigonometric function

(G) Exponential function

(H) Logarithmic function

Answer: (C)

3. Classify the function $f(x) = \sin(5)x^2 + \sin(3)x$.

(A) Power function

(B) Root function

(C) Polynomial function

(D) Rational function

(E) Algebraic function

(F) Trigonometric function

(G) Exponential function

(H) Logarithmic function

Answer: (C)

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4. The following time-of-day and temperature ($^{\circ}F$) were gathered during a gorgeous midsummer day in Fargo, North Dakota:

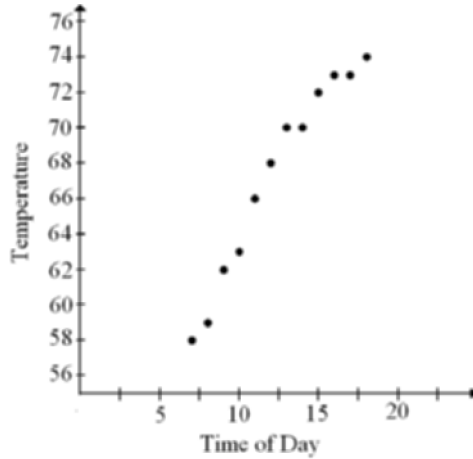
Time of Day	Temperature
18	74
17	73
16	73
15	72
14	70
13	70
12	68
11	66
10	63
9	62
8	59
7	58

Source: National Weather Service website; www.weather.gov

- (a) Make a scatter plot of these data.
- (b) Fit a linear model to the data.
- (c) Fit an exponential model to the data.
- (d) Fit a quadratic model to the data.
- (e) Use your equations to make a table showing the predicted temperature for each model, rounded to the nearest degree.
- (f) The actual temperature at 8:00 P.M. (20 hours) was $70^{\circ}F$. Which model was closest? Which model was second-closest?
- (g) All of the models give values that are too high for each of the times after 6:00 P.M. What is one possible explanation for this?

Answer:

(a)



(b) $y = 1.561x + 47.68$

(c) $y = 49.89802e^{1.023831x}$

(d) $y = -0.09263x^2 + 3.902611x + 33.934$

(e) Linear: 79
Exponential: 80
Quadratic: 75

(f) Closest: quadratic. Second-closest: linear

(g) Answers will vary, but one explanation is that the data only reflect the part of the day when the air is warming and do not take into account cooling that takes place later in the day into evening. The only model that begins to reflect this is the quadratic model.

5. Consider the data below:

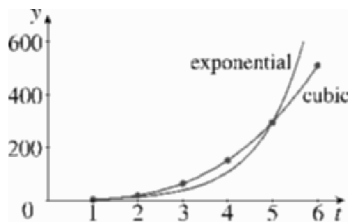
t	1	2	3	4	5	6
y	2.4	19	64	152	295	510

(a) Fit both an exponential curve and a third-degree polynomial to the data.

(b) Which of the models appears to be a better fit? Defend your choice.

Answer:

(a)



(b) A third degree polynomial, for example, $y = 2.40t^3$, appears to be the better fit.

14 1 Functions and Models

6. The following table contains United States population data for the years 1981–1990, as well as estimates based on the 1990 census.

Year	U. S. Population (millions)
1981	229.5
1982	231.6
1983	233.8
1984	235.8
1985	237.9
1986	240.1
1987	242.3
1988	244.4
1989	246.8
1990	249.5

Year	U. S. Population (millions)
1991	252.2
1992	255.0
1993	257.8
1994	260.3
1995	262.8
1996	265.2
1997	267.8
1998	270.2
1999	272.7
2000	275.1

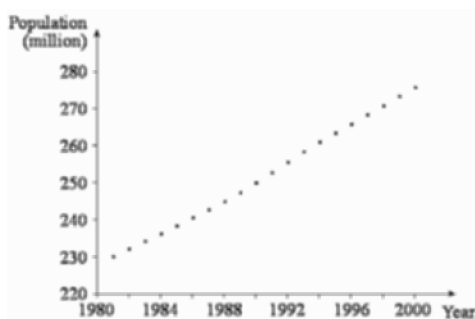
Source: U.S. Census Bureau website

- (a) Make a scatter plot for the data and use your scatter plot to determine a mathematical model of the U.S. population.
- (b) Use your model to predict the U.S. population in 2003.

Answer:

(a)

(b) $P(22) \approx 282.3$



A linear model seems appropriate. Taking $t = 0$ in 1981, we obtain the model $P(t) = 2.4455t + 228.5$.

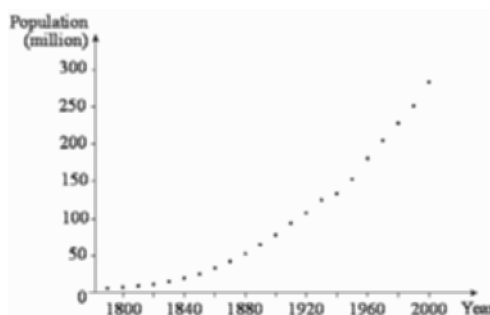
7. The following table contains United States population data for the years 1790–2000 at intervals of 10 years.

Year	Years since 1790	U.S. Population (millions)	Year	Years since 1790	U.S. Population (millions)
1790	0	3.9	1900	110	76.0
1800	10	5.2	1910	120	92.0
1810	20	7.2	1920	130	105.7
1820	30	9.6	1930	140	122.8
1830	40	12.9	1940	150	131.7
1840	50	17.1	1950	160	150.7
1850	60	23.2	1960	170	178.5
1860	70	31.4	1970	180	202.5
1870	80	39.8	1980	190	225.5
1880	90	50.2	1990	200	248.7
1890	100	62.9	2000	210	281.4

- (a) Make a scatter plot for the data and use your scatter plot to determine a mathematical model of the U.S. population.
- (b) Use your model to predict the U.S. population in 2005.

Answer:

- (a)



Answers will vary, but a quadratic or cubic model is most appropriate.

Linear model: $P(t) = 1.28545t - 40.47668$; quadratic model: $P(t) = 0.006666t^2 - 0.1144t + 5.9$; cubic model: $P(t) = (6.6365 \times 10^{-6})t^3 + 0.004575t^2 + 0.057155t + 3.7$; exponential model: $P(t) = 6.04852453 \times 1.020407795^t$

- (b) Linear model: $P(215) \approx 235.9$; quadratic model: $P(215) \approx 289.4$; cubic model: $P(215) \approx 293.4$; exponential model: $P(215) \approx 465.6$

16 1 Functions and Models

8. Refer to your models from Problems 6 and 7 above. Why do the two data sets produce such different models?

Answer: Problem 6 covers a much shorter time span, so its data exhibit local linearity, while Problem 7 shows nonlinear population growth over a longer time span.

9. The following are the winning times for the Olympic Men's 110 Meter Hurdles:

Year	Time
1896	17.6
1900	15.4
1904	16
1906	16.2
1908	15
1912	15.1
1920	14.8
1924	15
1928	14.8

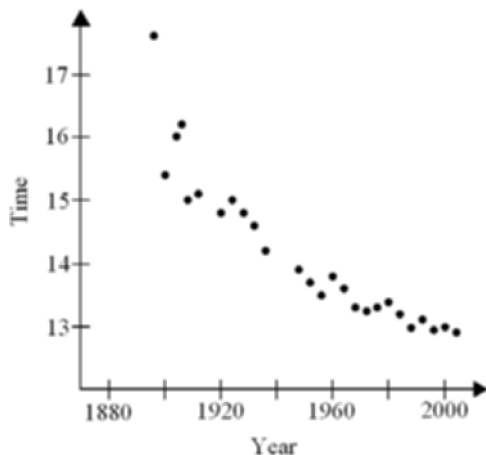
Year	Time
1932	14.6
1936	14.2
1948	13.9
1952	13.7
1956	13.5
1960	13.8
1964	13.6
1968	13.3
1972	13.24

Year	Time
1976	13.3
1980	13.39
1984	13.2
1988	12.98
1992	13.12
1996	12.95
2000	13
2004	12.91

- Make a scatter plot of these data.
- Fit a linear model to the data.
- Fit an exponential model to the data.
- Fit a quadratic model to the data.
- Use your equations to make a table showing the predicted winning time for each model for the 2008 Olympics, rounded to the nearest hundredth of a second.
- The actual time for the 2008 Olympics was 12.93 seconds. Which model was closest? Which model was second-closest?

Answer:

(a)



(b) $y = -0.0320057x + 76.595$

(c) $y = 1053.09176(0.997791842^x)$

(d) $y = 0.000322x^2 - 1.2872778x + 1299.573$

(e) Linear: 12.33
Exponential: 12.44
Quadratic: 13.04

(f) Closest: quadratic. Second-closest: exponential

1.3 New Functions From Old Functions

1. Let $f(x) = x^2 - 3x + 7$, then $f(2x)$ is equal to

(A) $2x^2 - 6x + 7$

(B) $4x^2 - 6x + 7$

(C) $2x^2 - 6x + 14$

(D) $4x^2 - 3x + 7$

(E) $2x^2 + 6x - 7$

(F) $4x^2 + 6x - 7$

(G) $2x^2 - 3x + 7$

(H) $4x^2 - 6x + 14$

Answer: (B)

2. Let $f(x) = \sqrt{x^2 + 4}$ and $g(x) = -\sqrt{x^2 - 4}$. Find the domain of $(g \circ f)(x)$.

(A) $(-\infty, 0]$

(B) $(2, \infty)$

(C) $(-\infty, -2]$

(D) $(-\infty, 2) \cup (2, \infty)$

(E) $[-2, \infty)$

(F) $(-\infty, -2]$

(G) $(-\infty, -2] \cup [2, \infty)$

(H) \mathbb{R}

Answer: (H)

18 1 Functions and Models

3. Let $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{10-x^2}$. Find the domain of $(f \circ g)(x)$.

- | | |
|--|--------------------------------------|
| (A) $[1, \infty)$ | (B) $[-\sqrt{10}, \sqrt{10}]$ |
| (C) $(-\infty, -\sqrt{10}] \cup [\sqrt{10}, \infty)$ | (D) $(-\infty, -3] \cup [3, \infty)$ |
| (E) $[-3, 3]$ | (F) $(-\infty, -1] \cup [1, \infty)$ |
| (G) $(-\infty, -1]$ | (H) $[\sqrt{10}, \infty)$ |

Answer: (E)

4. Let $h(x) = \sin^2 x + 3 \sin x - 4$ and $g(x) = \sin x$. Find $f(x)$ so that $h(x) = (f \circ g)(x)$.

- | | |
|------------------------|---------------------------|
| (A) $f(x) = (3x+2)^2$ | (B) $f(x) = x+3$ |
| (C) $f(x) = 3x^2 - 4$ | (D) $f(x) = x^2 - 3x + 4$ |
| (E) $f(x) = 3x^2 - 4x$ | (F) $f(x) = x^2 + 3x - 4$ |
| (G) $f(x) = x^2 - 4$ | (H) $f(x) = (x-4)^2$ |

Answer: (F)

5. Let $f(x) = 3x - 2$ and $g(x) = 2 - 3x$. Find the value of $(f \circ g)(x)$ when $x = 3$.

- | | | | |
|---------|--------|--------|--------|
| (A) -23 | (B) -9 | (C) -6 | (D) -3 |
| (E) 3 | (F) 6 | (G) 9 | (H) 23 |

Answer: (A)

6. Let $f(x) = 2 - x^3$ and $g(x) = 3 + x$. Find the value of $(f \circ g)(x)$ when $x = -5$.

- | | | | |
|----------|--------|---------|---------|
| (A) -510 | (B) -5 | (C) -2 | (D) 0 |
| (E) 5 | (F) 10 | (G) 127 | (H) 130 |

Answer: (F)

7. Let $f(x) = \frac{1}{2}x$ and $(f \circ g)(x) = x^2$. Find $g(2)$.

- | | | | |
|-------|--------|--------|--------|
| (A) 0 | (B) 1 | (C) 2 | (D) 4 |
| (E) 8 | (F) 16 | (G) 32 | (H) 64 |

Answer: (E)

8. Relative to the graph of $y = x^2 + 2$, the graph of $y = (x - 2)^2 + 2$ is changed in what way?
- (A) Shifted 2 units upward (B) Compressed vertically by a factor of 2
 (C) Compressed horizontally by a factor of 2 (D) Shifted 2 units to the left
 (E) Shifted 2 units to the right (F) Shifted 2 units downward
 (G) Stretched vertically by a factor of 2 (H) Stretched horizontally by a factor of 2

Answer: (E)

9. Relative to the graph of $y = x^2$, the graph of $y = x^2 - 2$ is changed in what way?
- (A) Shifted 2 units downward (B) Stretched horizontally by a factor of 2
 (C) Shifted 2 units to the right (D) stretched vertically by a factor of 2
 (E) Compressed horizontally by a factor of 2 (F) Compressed vertically by a factor of 2
 (G) Stretched vertically by a factor of 2 (H) Stretched horizontally by a factor of 2

Answer: (A)

10. Relative to the graph of $y = x^3$, the graph of $y = \frac{1}{2}x^3$ is changed in what way?
- (A) Compressed horizontally by a factor of 2 (B) Shifted 2 units downward
 (C) Stretched vertically by a factor of 2 (D) Stretched horizontally by a factor of 2
 (E) Shifted 2 units upward (F) Compressed vertically by a factor of 2
 (G) Shifted 2 units to the right (H) Shifted 2 units to the left

Answer: (F)

11. Relative to the graph of $y = x^2 + 2$, the graph of $y = 4x^2 + 2$ is changed in what way?
- (A) Compressed vertically by a factor of 2 (B) Stretched horizontally by a factor of 2
 (C) Compressed horizontally by a factor of 2 (D) Shifted 2 units upward
 (E) Shifted 2 units to the right (F) stretched vertically by a factor of 2
 (G) Shifted 2 units to the left (H) Shifted 2 units downward

Answer: (C)

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12. Relative to the graph of $y = \sin x$, the graph of $y = 3 \sin x$ is changed in what way?

- (A) Compressed horizontally by a factor of 3 (B) Shifted 3 units to the right
 (C) Compressed vertically by a factor of 3 (D) Shifted 3 units upward
 (E) Shifted 3 units to the left (F) Stretched vertically by a factor of 3
 (G) Shifted 3 units downward (H) Stretched horizontally by a factor of 3

Answer: (F)

13. Relative to the graph of $y = e^x$, the graph of $y = e^{x+5}$ is changed in what way?

- (A) Shifted 5 units upward (B) Shifted 5 units downward
 (C) Shifted 5 units to the right (D) Shifted 5 units to the left
 (E) Stretched horizontally by a factor of 5 (F) Stretched vertically by a factor of 5
 (G) Compressed horizontally by a factor of 5 (H) Compressed vertically by a factor of 5

Answer: (D)

14. Relative to the graph of $y = \sin x$, where x is in radians, the graph of $y = \sin x$, where x is in degrees, is changed in what way?

- (A) Compressed horizontally by a factor of $\frac{180}{\pi}$ (B) Stretched vertically by a factor of $\frac{180}{\pi}$
 (C) Compressed horizontally by a factor of $\frac{90}{\pi}$ (D) Stretched horizontally by a factor of $\frac{90}{\pi}$
 (E) Compressed vertically by a factor of $\frac{90}{\pi}$ (F) Stretched vertically by a factor of $\frac{90}{\pi}$
 (G) Stretched horizontally by a factor of $\frac{180}{\pi}$ (H) Compressed vertically by a factor of $\frac{180}{\pi}$

Answer: (G)

15. Let $f(x) = 8 + x^2$. Find each of the following:

- (a) $f(2) + f(-2)$ (b) $f(x+2)$ (c) $[f(x)]^2$ (d) $f(x^2)$

Answer:

- (a) 24 (b) $x^2 + 4x + 12$ (c) $64 + 16x^2 + x^4$ (d) $8 + x^4$

16. Let $f(x) = \sqrt{2x+5}$. Find each of the following:

- (a) $f(0) + f(-2)$ (b) $f(x+2)$ (c) $[f(x)]^2$ (d) $f(x^2)$

Answer:

$$\begin{array}{ll} \text{(a)} f(0) + f(-2) = \sqrt{5} + \sqrt{1} = \sqrt{5} + 1 & \text{(b)} f(x+2) = \sqrt{2(x+2)+5} = \sqrt{2x+9} \\ \text{(c)} [f(x)]^2 = 2x+5, x \geq -\frac{5}{2} & \text{(d)} f(x^2) = \sqrt{2x^2+5} \end{array}$$

17. Let $f(x) = \sqrt{16-x^2}$. Find each of the following:

$$\begin{array}{llll} \text{(a)} f(0) + f(-2) & \text{(b)} f(x+2) & \text{(c)} [f(x)]^2 & \text{(d)} f(x^2) \end{array}$$

Answer:

$$\begin{array}{ll} \text{(a)} f(0) + f(-2) = \sqrt{16} + \sqrt{12} = 4 + 2\sqrt{3} \approx 7.46 & \\ \text{(b)} f(x+2) = \sqrt{16-(x+2)^2} = \sqrt{16-(x^2+4x+4)} = \sqrt{12-4x-x^2}, -6 \leq x \leq 2 & \\ \text{(c)} [f(x)]^2 = 16-x^2, -4 \leq x \leq 4 & \\ \text{(d)} f(x^2) = \sqrt{16-(x^2)^2} = \sqrt{16-x^4}, 0 \leq x \leq 2 & \end{array}$$

18. Let $f(x) = \sqrt{\frac{2}{x+3}}$, $x > -3$. Find each of the following:

$$\begin{array}{llll} \text{(a)} f(-1) - f(-2) & \text{(b)} f(x^2-3) & \text{(c)} f(x^2) - 3 & \text{(d)} [f(x-3)]^2 \end{array}$$

Answer:

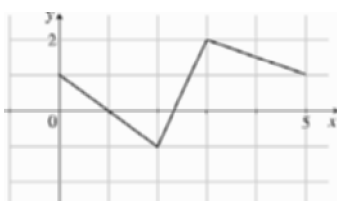
$$\begin{array}{ll} \text{(a)} f(-1) - f(-2) = \sqrt{\frac{2}{2}} - \sqrt{\frac{2}{1}} = 1 - \sqrt{2} & \\ \text{(b)} f(x^2-3) = \sqrt{\frac{2}{(x^2-3)+3}} = \sqrt{\frac{2}{x^2}} = \frac{\sqrt{2}}{|x|}, x \neq 0 & \\ \text{(c)} f(x^2) - 3 = \sqrt{\frac{2}{x^2+3}} - 3 & \\ \text{(d)} [f(x-3)]^2 = \left(\sqrt{\frac{2}{(x-3)+3}} \right)^2 = \frac{2}{x}, x > 0 & \end{array}$$

22 1 Functions and Models

19. Evaluate the difference quotient $\frac{f(x) - f(a)}{x - a}$ for $f(x) = \frac{1}{x^2}$.

Answer: $\frac{f(x) - f(a)}{x - a} = \frac{\frac{1}{x^2} - \frac{1}{a^2}}{x - a} = -\frac{a + x}{a^2x^2}$

20. Given the graph of $y = f(x)$:

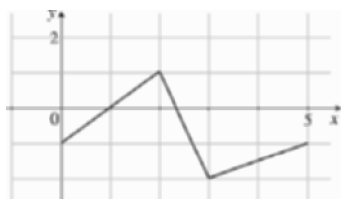


Sketch the graph of each of the following functions:

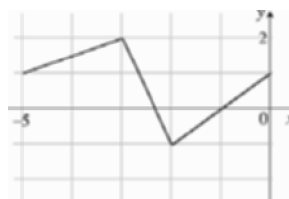
- | | |
|-----------------------|-----------------------|
| (a) $-f(x)$ | (b) $f(-x)$ |
| (c) $f(2x)$ | (d) $2f(x)$ |
| (e) $-f(-x)$ | (f) $f(\frac{1}{2}x)$ |
| (g) $\frac{1}{2}f(x)$ | (h) $f(x + 1)$ |
| (i) $f(x) - 1$ | (j) $1 - f(x)$ |

Answer:

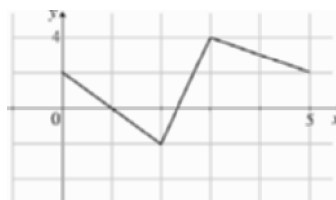
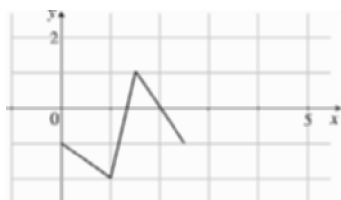
- (a) (b)



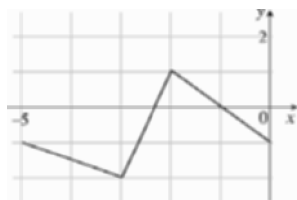
(c)



(d)



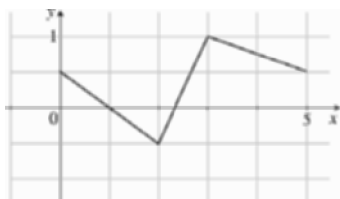
(e)



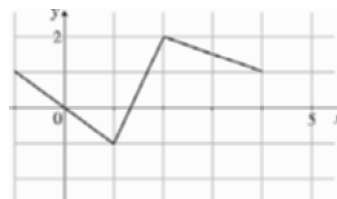
(f)



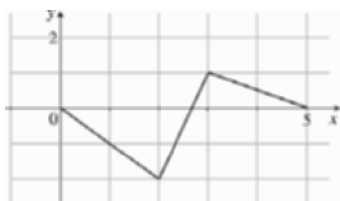
(g)



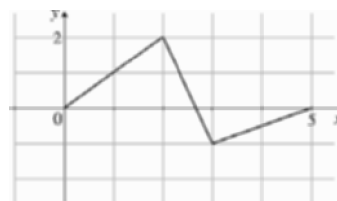
(h)



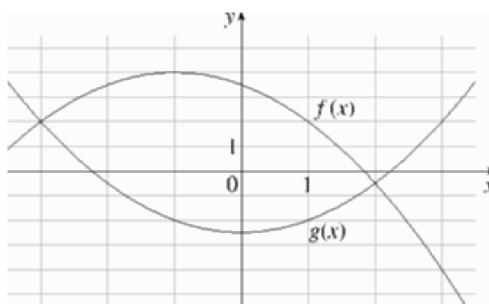
(i)



(j)



21. Use the graphs of f and g given below to estimate the values of $f(g(x))$ for $x = -3, -2, -1, 0, 1, 2$, and 3 , and use these values to sketch a graph of $y = f(g(x))$.



Answer:

x	-3	-2	-1	0	1	2	3
$f(g(x))$	-0.5	3.88	3.50	2.88	3.50	3.88	-0.5

24 1 Functions and Models

22. f and g are functions defined by the following table.

x	-3	-2	-1	0	1	2	3
$f(x)$	-5	-4	-3	-2	-1	-2	-3
$g(x)$	4	1	-1	-2	-1	1	4

Determine the following:

- (a) $(f + g)(2)$ (b) $(f - g)(-1)$ (c) $(f \cdot g)(0)$ (d) $(f/g)(3)$
 (e) $(f \circ g)(-2)$ (f) $(f \circ f)(0)$ (g) $(g \circ f)(-1)$ (h) $(g \circ g)(-2)$

Answer:

- (a) -1 (b) -2 (c) 4 (d) $-\frac{3}{4}$
 (e) -1 (f) -4 (g) 4 (h) -1

23. Find functions f and g such that $F(x) = 1 - 2\cos^2 x = (f \circ g)(x)$.

Answer: $f(x) = 1 - 2x^2$, $g(x) = \cos x$ is one possible answer. Answers will vary.

24. Find functions f and g such that $F(x) = \sqrt{1 - \cos^2 x} = (f \circ g)(x)$.

Answer: $f(x) = \sqrt{x}$, $g(x) = 1 - \cos^2 x$ is one possible answer. Answers will vary.

25. Find functions f and g such that $F(x) = e^{\sin x} = (f \circ g)(x)$.

Answer: $f(x) = e^x$, $g(x) = \sin x$ is one possible answer. Answers will vary.

1.4 Graphing Calculators and Computers

1. Determine an appropriate viewing rectangle for the graph of the function $f(x) = 6x^2 + x - 10$.

- (A) $[-50, 50] \times [-50, 50]$ (B) $[0, 10] \times [0, 10]$
 (C) $[-5, 10] \times [-10, 50]$ (D) $[-5, 5] \times [-5, 5]$
 (E) $[-5, 5] \times [-10, 10]$ (F) $[-50, 10] \times [-10, 100]$
 (G) $[-5, 50] \times [-5, 10]$ (H) $[-10, 10] \times [-5, 5]$

Answer: (E)

2. Determine an appropriate viewing rectangle for the graph of the function $f(x) = (x + 50)^2 + 1000$.

- | | |
|-----------------------------------|---------------------------------------|
| (A) $[-60, -40] \times [40, 60]$ | (B) $[2400, 2600] \times [900, 1200]$ |
| (C) $[40, 60] \times [980, 1020]$ | (D) $[40, 60] \times [-1020, -980]$ |
| (E) $[40, 60] \times [40, 60]$ | (F) $[-60, -40] \times [980, 1020]$ |
| (G) $[-10, 10] \times [-10, 10]$ | (H) $[980, 1020] \times [980, 1020]$ |

Answer: (F)

3. Determine an appropriate viewing rectangle for the graph of the function $f(x) = \frac{3x}{x^4 + 75}$.

- | | |
|------------------------------------|----------------------------------|
| (A) $[-10, 10] \times [-10, 10]$ | (B) $[-1, 1] \times [-1, 1]$ |
| (C) $[-10, 10] \times [-1, 1]$ | (D) $[-100, 100] \times [-1, 1]$ |
| (E) $[-100, 100] \times [-10, 10]$ | (F) $[-5, 5] \times [-5, 5]$ |
| (G) $[-5, 5] \times [-0.5, 0.5]$ | (H) $[-1, 1] \times [-10, 10]$ |

Answer: (C)

4. Determine an appropriate viewing rectangle for the graph of the function $f(x) = 20 \sin(40x) + 10$.

- | | |
|------------------------------------|------------------------------------|
| (A) $[-20, 20] \times [-20, 20]$ | (B) $[-0.2, 0.2] \times [-20, 20]$ |
| (C) $[-20, 20] \times [-0.2, 0.2]$ | (D) $[-0.2, 0.2] \times [-10, 30]$ |
| (E) $[-20, 20] \times [-20, 20]$ | (F) $[-0.2, 0.2] \times [-1, 1]$ |
| (G) $[-10, 10] \times [-10, 10]$ | (H) $[-1, 1] \times [-10, 10]$ |

Answer: (D)

5. Determine an appropriate viewing rectangle for the graph of the function $f(x) = 4x - |3x^2 - 10|$.

- | | |
|----------------------------------|------------------------------------|
| (A) $[-5, 5] \times [-5, 2]$ | (B) $[-50, 20] \times [-50, 10]$ |
| (C) $[-2, 5] \times [-1, 10]$ | (D) $[-10, 10] \times [-500, 500]$ |
| (E) $[-10, 10] \times [-500, 5]$ | (F) $[-2, 5] \times [-15, 10]$ |
| (G) $[-10, 5] \times [1, 10]$ | (H) $[0, 10] \times [-2, 5]$ |

Answer: (F)

26 1 Functions and Models

6. Determine the number of real solutions of the equation $5x^4 - 3x^2 = 10x^5 - 20x^3 + 5$.

- | | | | |
|-------|-------|-------|-------|
| (A) 0 | (B) 1 | (C) 2 | (D) 3 |
| (E) 4 | (F) 5 | (G) 6 | (H) 7 |

Answer: (D)

7. Determine the number of solutions of the equation $5x^4 - 3x^2 = 10x^5 - 20x^6 + 5$.

- | | | | |
|-------|-------|-------|-------|
| (A) 0 | (B) 1 | (C) 2 | (D) 3 |
| (E) 4 | (F) 5 | (G) 6 | (H) 7 |

Answer: (D)

8. Find the difference between the largest and smallest solutions of the equation $-3x^2 - 9x = 2^x$, rounded to two decimal places.

- | | | | |
|----------|----------|----------|----------|
| (A) 0.02 | (B) 0.23 | (C) 0.80 | (D) 1.05 |
| (E) 2.06 | (F) 2.88 | (G) 3.09 | (H) 3.92 |

Answer: (F)

9. Determine an appropriate viewing window for each of the following functions and use it to draw the graph.

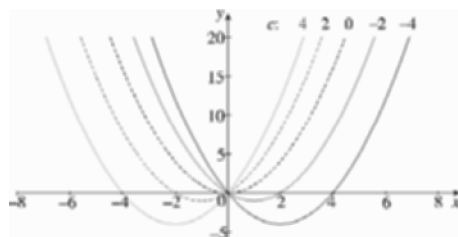
- | | |
|----------------------------------|----------------------------------|
| (a) $f(x) = x^4 - 6x^2$ | (b) $f(x) = \frac{1}{x^4 - 1}$ |
| (c) $f(x) = \sin(x^2)$ | (d) $f(x) = 80x - 5x^3$ |
| (e) $f(x) = -3x^2 + 288x - 6862$ | (f) $f(x) = -3x^2 + 288x + 6862$ |

Answer: (Possible answers — answers will vary.)

- | | |
|---------------------------------|---|
| (a) $[-3, 3] \times [-10, 20]$ | (b) $[-2, 2] \times [-10, 10]$ |
| (c) $[-4, 4] \times [-2, 2]$ | (d) $[-6, 6] \times [-140, 140]$ |
| (e) $[40, 60] \times [-10, 60]$ | (f) $[-30, 130] \times [-5000, 15,000]$ |

10. Consider the family of curves given by $y = x^2 + cx$. Graph the function for values of $c = -4, -2, 0, 2$, and 4 . What characteristics are shared by all of the graphs? How does changing the value of c affect the graph?

Answer:



$f(x) = x^2 + cx = \left(x + \frac{c}{2}\right)^2 - \frac{c^2}{4}$. All of the graphs represent parabolas with vertex $\left(-\frac{c}{2}, -\frac{c^2}{4}\right)$. They have the same shape as $y = x^2$, and they all pass through $(0,0)$. The value of c changes the coordinates of the vertex.

1.5 Exponential Functions

1. The radioactive isotope Bismuth-210 has a half-life of 5 days. How many days does it take for 87.5% of a given amount to decay?

(A) 15 (B) 8 (C) 10 (D) 13
(E) 11 (F) 9 (G) 12 (H) 14

Answer: (A)

2. A bacteria culture starts with 200 bacteria and triples in size every ten minutes. After 1 hour, how many bacteria are there?

(A) 1800 (B) 3600 (C) 5400 (D) 16,200
(E) 48,600 (F) 145,800 (G) 437,400 (H) 1,312,200

Answer: (F)

3. A bacteria culture starts with 500 bacteria and doubles every 2 hours. How many bacteria are there after 6 hours?

(A) 1,000 (B) 1,500 (C) 2,000 (D) 2,500
(E) 3,000 (F) 4,000 (G) 5,000 (H) 10,000

Answer: (F)

4. For what value of x is $3^{4-x} = \sqrt{3}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$
(E) 2 (F) $\frac{5}{2}$ (G) 3 (H) $\frac{7}{2}$

Answer: (H)

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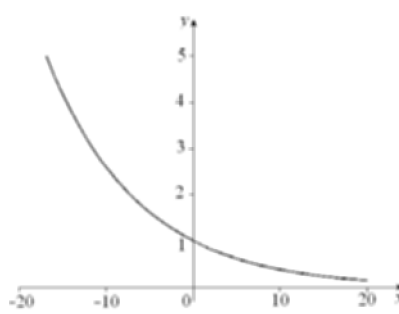
5. Which of the following statements are true about the graph of the function $y = 2^x - 8$?

- | | |
|--------------------------------------|---|
| (i) It has no vertical asymptote. | (iii) It has a y -intercept at -3 . |
| (ii) It has no horizontal asymptote. | (iv) It has an x -intercept at 3 . |
| (A) (i) only | (B) (i), (iii), and (iv) only |
| (C) (ii), (iii), and (iv) only | (D) (iv) only |
| (E) (ii) and (iii) only | (F) (i) and (iv) only |
| (G) (ii) and (iv) only | (H) (i), (ii) and (iv) only |

Answer: (F)

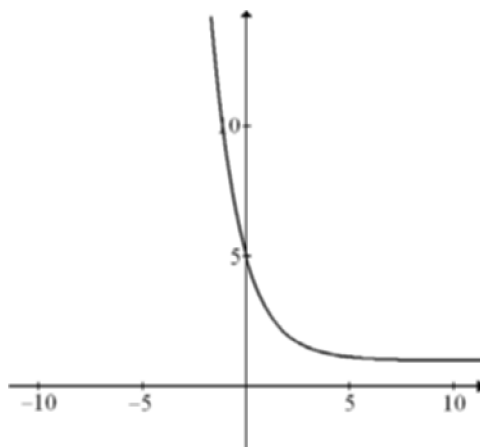
6. Make a rough sketch of the graph of $y = (1.1)^{-x}$. Do not use a calculator.

Answer:



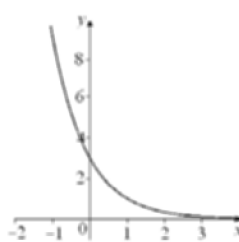
7. Make a rough sketch of the graph of $y = 4^{1-\frac{x}{2}} + 1$. Do not use a calculator.

Answer:



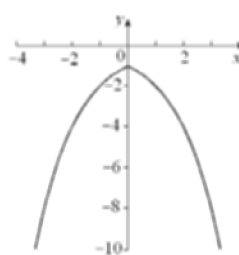
8. Make a rough sketch of the graph of $y = 3^{1-x}$. Do not use a calculator.

Answer:



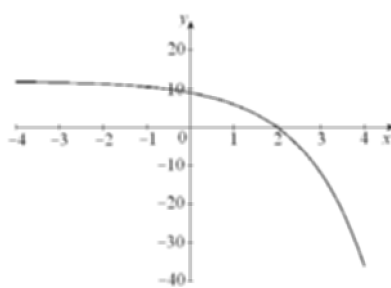
9. Make a rough sketch of the graph of $y = -2^{|x|}$. Do not use a calculator.

Answer:



10. Make a rough sketch of the graph of $y = 3(4 - 2^x)$. Do not use a calculator.

Answer:



11. Given the graph of $y = 2^x$, find an equation of the graph that results from reflecting the given graph about

- (a) the line $x = 0$. (b) the line $y = 3$.
(c) the line $y = -1$. (d) the line $x = 2$.

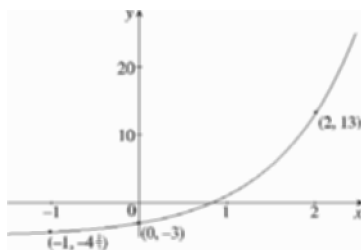
Answer: (a) $y = 2^{-x}$ (b) $y = 6 - 2^x$ (c) $y = -2^x - 2$ (d) $y = 2^{-x+4} = 2^{-(x-4)}$

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12. What single transformation of the graph of $y = e^x$ is the same as shifting the graph of $y = e^x$ four units downward and then reflecting the shifted graph in the x -axis?

Answer: Reflecting the graph about the line $y = 2$.

13. For the exponential function $f(x) = c \cdot a^x + b$ whose graph is given below, determine the values of a , b , and c .



Answer: $a = 3$, $b = -5$, $c = 2$

14. The half-life of a certain radioactive substance is 5 days. The initial size of a sample is 10 grams.
- Find the amount of the substance remaining after 20 days.
 - Find the amount of the substance remaining after t days.
 - Use a graph to estimate, to the nearest 0.01 gram, the amount remaining after 14 days.
 - Use the graph to estimate, to the nearest 0.1 day, the amount of time required for the mass of the substance to be reduced to 0.1 gram.

Answer:

- The half-life is 5 days, so 20 days is 4 half-lives. After 20 days, $y = 10 \left(\frac{1}{2}\right)^4 = \frac{10}{16} = 0.625$ g.
- The half-life is 5 days, so after t days, $y = 10e^{(\ln(1/2)/5)t}$
- When $t = 14$, $y \approx 1.44$ g.
- When $y = 0.1$ gram, $t = \frac{\ln \frac{0.1}{10}}{\ln \frac{1}{2}} \approx 33.2$ days.

15. Match each set of function values in the table with the formula which best fits it. Assume that a , b , and c are constants.

Function A

x	$f(x)$
4	340
5	367
6	400
7	428
8	463

Formula 1

$$y_1 = a(1.04)^x$$

Function B

x	$g(x)$
4	351
5	365
6	380
7	395
8	411

Formula 2

$$y_2 = b(1.08)^x$$

Function C

x	$h(x)$
5	322
6	354
7	390
8	429
9	472

Formula 3

$$y_3 = c(1.10)^x$$

Answer: $f(x)$ generated by $250(1.08)^x$; $y_2 = f(x)$, $b \approx 250$

$g(x)$ generated by $300(1.04)^x$; $y_1 = g(x)$, $a \approx 300$

$h(x)$ generated by $200(1.10)^x$; $y_3 = h(x)$, $c \approx 200$

1.6 Inverse Functions and Logarithms

1. Find the inverse function for $f(x) = \frac{5-2x}{3}$.

- (A) $\frac{5-3x}{2}$ (B) $\frac{2x-5}{3}$ (C) $\frac{2-3x}{5}$ (D) $\frac{3x-2}{5}$
 (E) $\frac{2x+5}{3}$ (F) $\frac{3}{5-2x}$ (G) $\frac{2}{5-3x}$ (H) $3x-5$

Answer: (A)

2. Find the inverse function for $f(x) = \frac{x-1}{x+1}$.

- (A) $\frac{x+1}{x-1}$ (B) $\frac{x}{x+1}$ (C) $\frac{x+1}{x}$ (D) $\frac{1+x}{1-x}$
 (E) $\frac{x+1}{1-x}$ (F) $\frac{x}{x-1}$ (G) $\frac{x-1}{x+1}$ (H) $\frac{x-1}{x}$

Answer: (D)

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3. Find the domain of the inverse for $f(x) = \sqrt{2x - 5}$.

- (A) $(-\infty, -\frac{5}{2}]$ (B) $(-\infty, 0]$ (C) $[-\frac{5}{2}, \frac{5}{2}]$ (D) $(-\infty, \frac{5}{2}]$
 (E) $[-\frac{5}{2}, \infty)$ (F) $[0, \infty)$ (G) $[\frac{2}{5}, \infty)$ (H) $[\frac{5}{2}, \infty)$

Answer: (F)

4. Find the range of the inverse for $f(x) = -\frac{3}{5 + 2x}$.

- (A) $(-\infty, -\frac{5}{2})$ (B) $(-\infty, 0)$ (C) $(-\frac{5}{2}, \frac{5}{2})$ (D) $(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$
 (E) $(-\frac{5}{2}, \infty)$ (F) $(0, \infty)$ (G) $(\frac{5}{2}, \infty)$ (H) $(-\infty, -\frac{5}{2}) \cup (-\frac{5}{2}, \infty)$

Answer: (H)

5. Given the function $\tan x$ with domain $(-\frac{\pi}{2}, \frac{\pi}{2})$, find the domain of its inverse.

- (A) $[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}]$ (B) $[0, \infty)$ (C) $[-\pi, \pi]$ (D) $[-1, 1]$
 (E) $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (F) $[-\frac{1}{2}, \frac{1}{2}]$ (G) $(-\infty, \infty)$ (H) $[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$

Answer: (G)

6. Find the value of $\log_{1/2} 1$.

- (A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) 10^2
 (E) 1 (F) $\frac{1}{2}$ (G) 2 (H) -2

Answer: (C)

7. Find the value of $\log_2 \frac{1}{8}$.

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) 0 (D) 1
 (E) -1 (F) 2 (G) -2 (H) -3

Answer: (H)

8. Find the value of $\log_{16} 8$.

- (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1
 (E) $\frac{3}{2}$ (F) 2 (G) 3 (H) 4

Answer: (C)