Solutions for Batch Distillation Simulation Optimal Design and Control 2nd Edition by Diwekar

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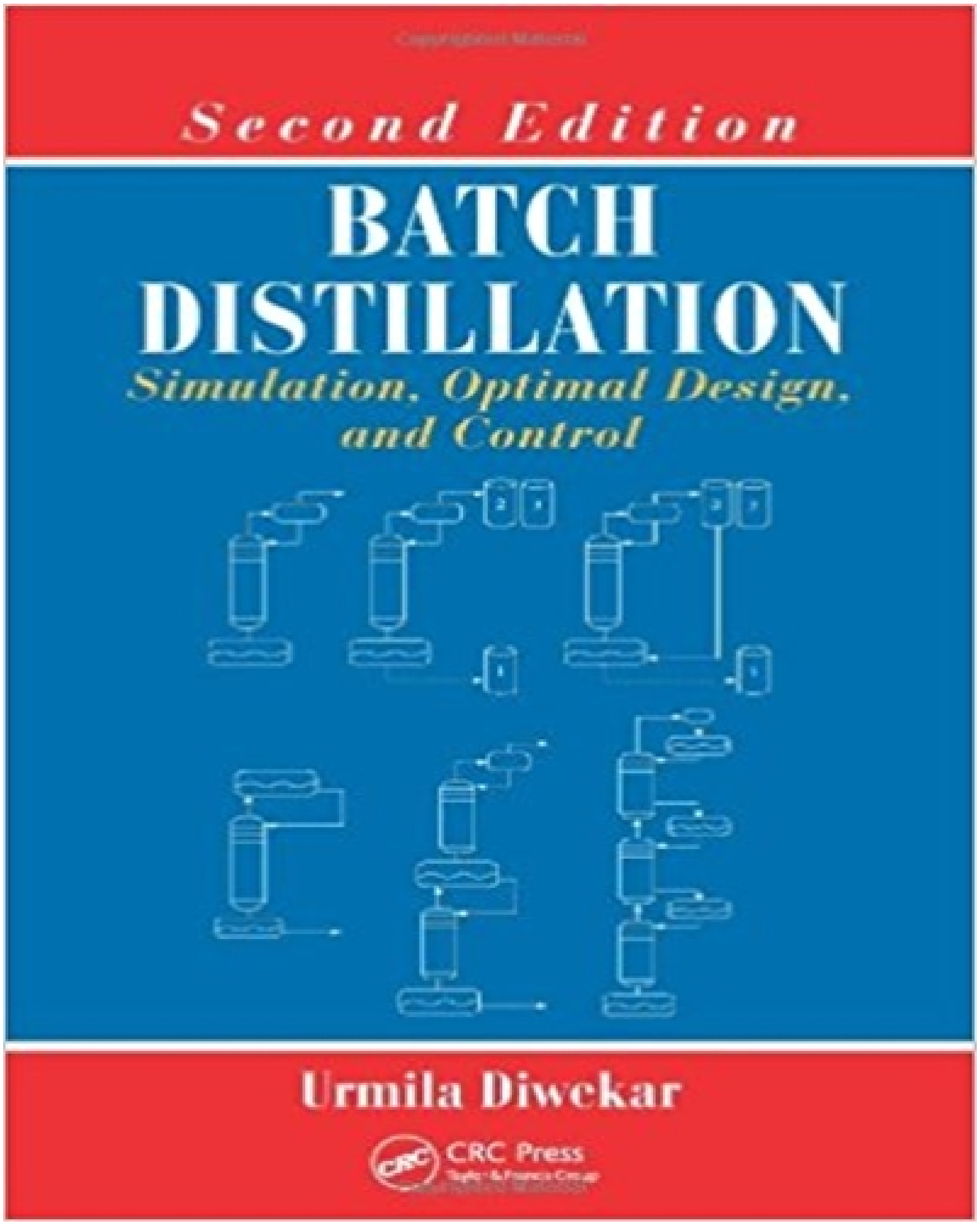


Solutions

2

Basic Modes of Operation

2.1 A mixture containing 48.6 percent benzene and 51.4 percent ethylene chloride is to be distilled in a batch distillation column with 20 theoretical plates. The column is operating under the constant reflux mode at a reflux ratio equal to 9.8. Assume the relative volatility of benzene with respect to ethylene chloride is 1.109.

a) Find the concentration of the product when 95.4 percent of the total mixture is distilled. Assume that the initial distillate composition is approximately 0.722.

b) If the original feed is 100 moles and the vapor boil-up rate is 100 moles/hr, what is the time required to complete the operation?

c) Simulate the variable reflux operation assuming the reflux ratio varies from 5.0 to 20.0. Find the end compositions and the total time.

2.2 A batch distillation column operating at atmospheric pressure is to be designed to separate a mixture containing 15.67 mole percent CS_2 and 84.33 mole percent CCl_4 into an overhead product containing 91 percent CS_2. Assume the column to be operating in the variable reflux mode with an initial reflux ratio of 3.0.

a) How many theoretical plates are required for the process?

b) If the distillate is stopped when the reflux ratio is equal to 15.0, what is the amount of distillate obtained?

c) What is the heat required per kmole of product?

Latent heat for the CS_2 and CCl_4 mixture is 25,900 kJ/kmol and the data for the equilibrium curve is given below.

x_{CS2}	.0296	.0615	.1106	.1435	.2585	.3900	.5318	.6630	.7575	.8604
y_{CS2}	.0823	.1555	.2660	.3320	.4950	.6340	.7470	.8290	.8780	.9320

2.3 If the same system is operating with the constant reflux mode of operation, with the initial product composition of 95 percent and with a reflux ratio of 5.0,

a) How many theoretical plates are required?

b) Simulate the condition and stop the operation when the average distillate composition is 0.90. What is the amount of distillate collected at this stage? What is the batch time if V=100 moles?

c) Find the heat required per kmole of product.

2.4 A multicomponent mixture containing meta, ortho and para mono-nitro-toluene is to be distilled using a distillation column containing eight theoretical plates. The feed composition at the start of operation is 0.6, 0.36, 0.04 of meta,ortho and para-mono-nitro-toluene respectively. The column is operating at a reflux ratio equal to 3.0 and with constant reflux mode of operation. Assume that the product composition varies from 0.94 to 0.85. The relative volatilities of meta, ortho, and para-mono-nitro-toluene can be assumed as 1.7, 1.16 and 1.0 respectively. Use the plate-to-plate calculation method to calculate the relation between the distillate composition and the still composition at any instant.

a) Plot the distillate compositions of meta,ortho and para-mono-nitro-toluene versus the still composition of meta-mono-nitro-toluene.

b) Plot the still compositions of ortho and para-moono-nitro-toluene versus the still composition of meta-mono-nitro-toluene.

c) Find the fraction distilled at the end of the operation.

2.5 The same column in Example 2.4 is operated with the variable reflux mode of operation. The distillate purity of the meta-mono-nitro-toluene is to be maintained at 0.98. Assume that the reflux ratio varies from 10 to 80.

a) Plot the distillate compositions of ortho and para-mono-nitro-toluene versus the still composition of meta-mono-nitro-toluene.

b) Plot the still compositions of ortho and para-mono-nitro-toluene versus the still composition of meta-mono-nitro-toluene.

c) Plot reflux ratio versus the still composition of meta-mono-nitro-toluene.s

c) Find the fraction distilled at the end of the operation.

2.6 Converse and Gross, in 1963, solved the maximum distillate optimal reflux problem given below (Converse and Gross, 1963).

> Maximum Distillate Problem – Maximize the amount of distillate of a specified concentration for a specified time.

Use their system to compare the three modes of operation.

2.7 Find the heat duty of the reboiler for the minimum time problem solved in the section on optimal reflux (Section 2.4).

2.8 Bowman and Cichelli, in 1949, presented a very interesting concept of pole height for a binary batch distillation column. A pole height is defined as the product of the mid-point of the slope of the distillate composition versus material remaining in the still curve and the amount of the material remaining in the still at that time. Figure 2.1 illustrates the concept

FIGURE 2.1
The Pole Height Concept (reproduced from Bowman and Cichelli, 1948)

of pole height. They stated that the pole height is invariant to the initial concentration and provides a good measure for defining sharpness of separation.

Take 100 moles of a binary mixture containing component A & B with a relative volatility 1.5. Use a five theoretical stage batch distillation column and a constant reflux operation with a reflux equal to 5.0. Vary the initial composition from x_A equal to 0.5 to 0.4 and plot the distillate composition versus the amount remaining in the still. Calculate the pole height for each case. Verify the concept.

PROBLEM 2.1

PROBLEM		2.1			
REFLUX RATIO = 9.8					
NUMBER OF PLATES = 20					
		PARTS	A)	AND	B)
RESULTS	FROM	BATCH-DIST			
NO	TIME	B	XB1	XB2	
1	0	100	0.486	0.514	
2	0.4104	96.2	0.4768	0.5232	
3	0.8208	92.4	0.4671	0.5329	
4	1.2312	88.6	0.457	0.543	
5	1.6416	84.8	0.4465	0.5535	
6	2.052	81.0001	0.4355	0.5645	
7	2.4624	77.2001	0.424	0.576	
8	2.8728	73.4001	0.4118	0.5882	
9	3.2832	69.6001	0.3991	0.6009	
10	3.6936	65.8001	0.3858	0.6142	
11	4.104	62.0001	0.3717	0.6283	
12	4.5144	58.2001	0.3569	0.6431	
13	4.9248	54.4001	0.3413	0.6587	
14	5.3352	50.6001	0.3249	0.6751	
15	5.7456	46.8001	0.3074	0.6926	
16	6.156	43.0001	0.289	0.711	
17	6.5664	39.2001	0.2695	0.7305	
18	6.9768	35.4001	0.2489	0.7511	
19	7.3872	31.6001	0.227	0.773	
20	7.7976	27.8001	0.2038	0.7962	
21	8.208	24.0001	0.1793	0.8207	
22	8.6184	20.2001	0.1533	0.8467	
23	9.0288	16.4001	0.1259	0.8741	
24	9.4392	12.6001	0.0971	0.9029	
25	9.8496	8.8001	0.0672	0.9328	
26	10.26	5.0001	0.0367	0.9633	
27	10.26	5	0.0366	0.9634	
NO	TIME	D	XD1	XD2	
1	0	0	0.7241	0.2759	
2	0.4104	3.8	0.7157	0.2843	
3	0.8208	7.6	0.7067	0.2933	
4	1.2312	11.4	0.6971	0.3029	
5	1.6416	15.2	0.6868	0.3132	
6	2.052	18.9999	0.6758	0.3242	
7	2.4624	22.7999	0.664	0.336	
8	2.8728	26.5999	0.6513	0.3487	
9	3.2832	30.3999	0.6377	0.3623	
10	3.6936	34.1999	0.6229	0.3771	
11	4.104	37.9999	0.6069	0.3931	
12	4.5144	41.7999	0.5896	0.4104	
13	4.9248	45.5999	0.5708	0.4292	
14	5.3352	49.3999	0.5503	0.4497	
15	5.7456	53.1999	0.5279	0.4721	
16	6.156	56.9999	0.5033	0.4967	

Initial Distillate Composition
0.7241

17	6.5664	60.7999	0.4764	0.5236
18	6.9768	64.5999	0.4469	0.5531
19	7.3872	68.3999	0.4143	0.5857
20	7.7976	72.1999	0.3784	0.6216
21	8.208	75.9999	0.3389	0.6611
22	8.6184	79.7999	0.2952	0.7048
23	9.0288	83.5999	0.2472	0.7528
24	9.4392	87.3999	0.1945	0.8055
25	9.8496	91.1999	0.1374	0.8626
26	10.26	94.9999	0.0767	0.9233
27	10.26	95	0.0767	0.9233

a) $X_B = 0.0366$

$D = 95$

$B = 5$

$F = 100$

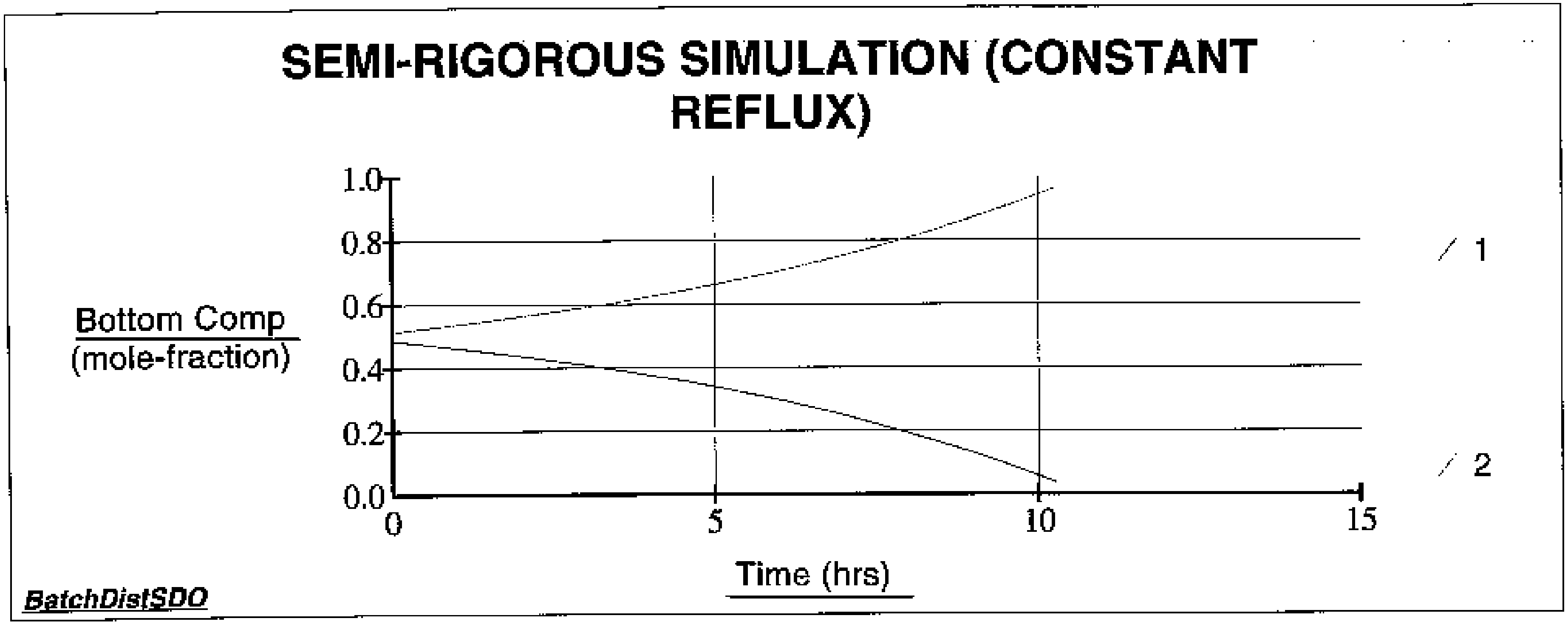
$X_F = 0.486$

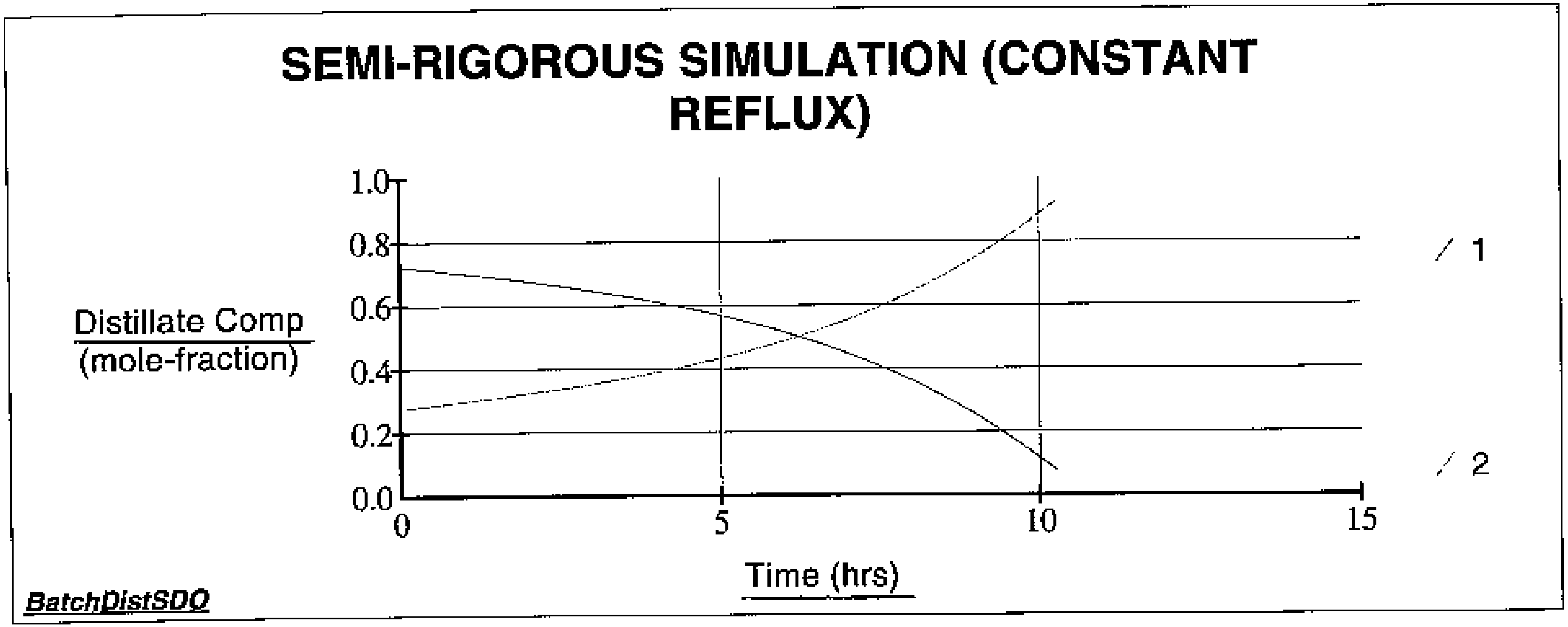
$$X_{D\,av} = \frac{(100)(0.486) - (5)(0.0366)}{95}$$

$X_{D\,av} = 0.51$

b) $T = 10.26$ Hr

2.1 a) b)


SEMI-RIGOROUS SIMULATION (CONSTANT REFLUX)
Bottom Comp (mole-fraction)
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
Time (hrs)
1
2
BatchDistSDO


SEMI-RIGOROUS SIMULATION (CONSTANT REFLUX)
Distillate Comp (mole-fraction)
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
Time (hrs)
1
2
BatchDistSDQ

PROBLEM		2.1	PART	c)	
	RESULTS	FROM	BATCH-DIST		
	NUMBER	OF	PLATES	=	20
	REFLUX	RATIO	=	20.6463	
NO	TIME	B	XB1	XB2	R
1	0	100	0.486	0.514	5
2	0.22	96.4033	0.4803	0.5197	5.2432
3	0.44	92.9513	0.4745	0.5255	5.5073
4	0.66	89.6403	0.4685	0.5315	5.7855
5	0.88	86.4664	0.4623	0.5377	6.0817
6	1.1	83.4262	0.4559	0.5441	6.3964
7	1.32	80.5164	0.4493	0.5507	6.7309
8	1.54	77.7336	0.4426	0.5574	7.0869
9	1.76	75.0743	0.4357	0.5643	7.4664
10	1.98	72.5354	0.4286	0.5714	7.8715
11	2.2	70.1133	0.4214	0.5786	8.3026
12	2.42	67.8048	0.4141	0.5859	8.7659
13	2.64	65.6068	0.4066	0.5934	9.2625
14	2.86	63.5163	0.399	0.601	9.7954
15	3.08	61.5299	0.3913	0.6087	10.368
16	3.3	59.6447	0.3835	0.6165	10.9838
17	3.52	57.8573	0.3756	0.6244	11.647
18	3.74	56.1645	0.3678	0.6322	12.362
19	3.96	54.5633	0.3598	0.6402	13.1337
20	4.18	53.0503	0.3519	0.6481	13.9674
21	4.4	51.6225	0.344	0.656	14.8692
22	4.62	50.2766	0.3362	0.6638	15.8453
23	4.84	49.0094	0.3284	0.6716	16.9029
24	5.06	47.8178	0.3207	0.6793	18.0498
25	5.28	46.6987	0.3131	0.6869	19.2945
26	5.5	45.6487	0.3056	0.6944	20.6463

2.1 c) $T = 5.5$ hrs

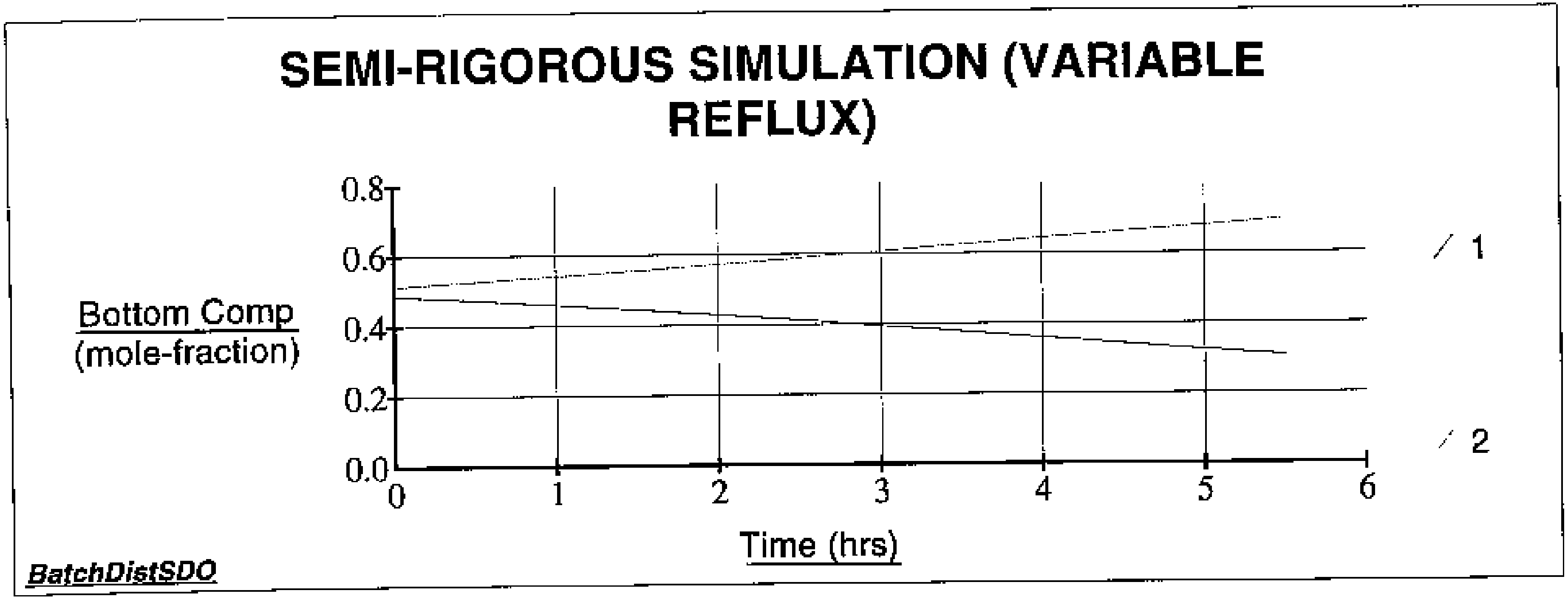
$X_B = 0.3056$

$B = 45.6487$ — R is from 5 to 20.6463

$D = 54.3513$

$X_D = 0.6375$ (Constant)

2.1 c)

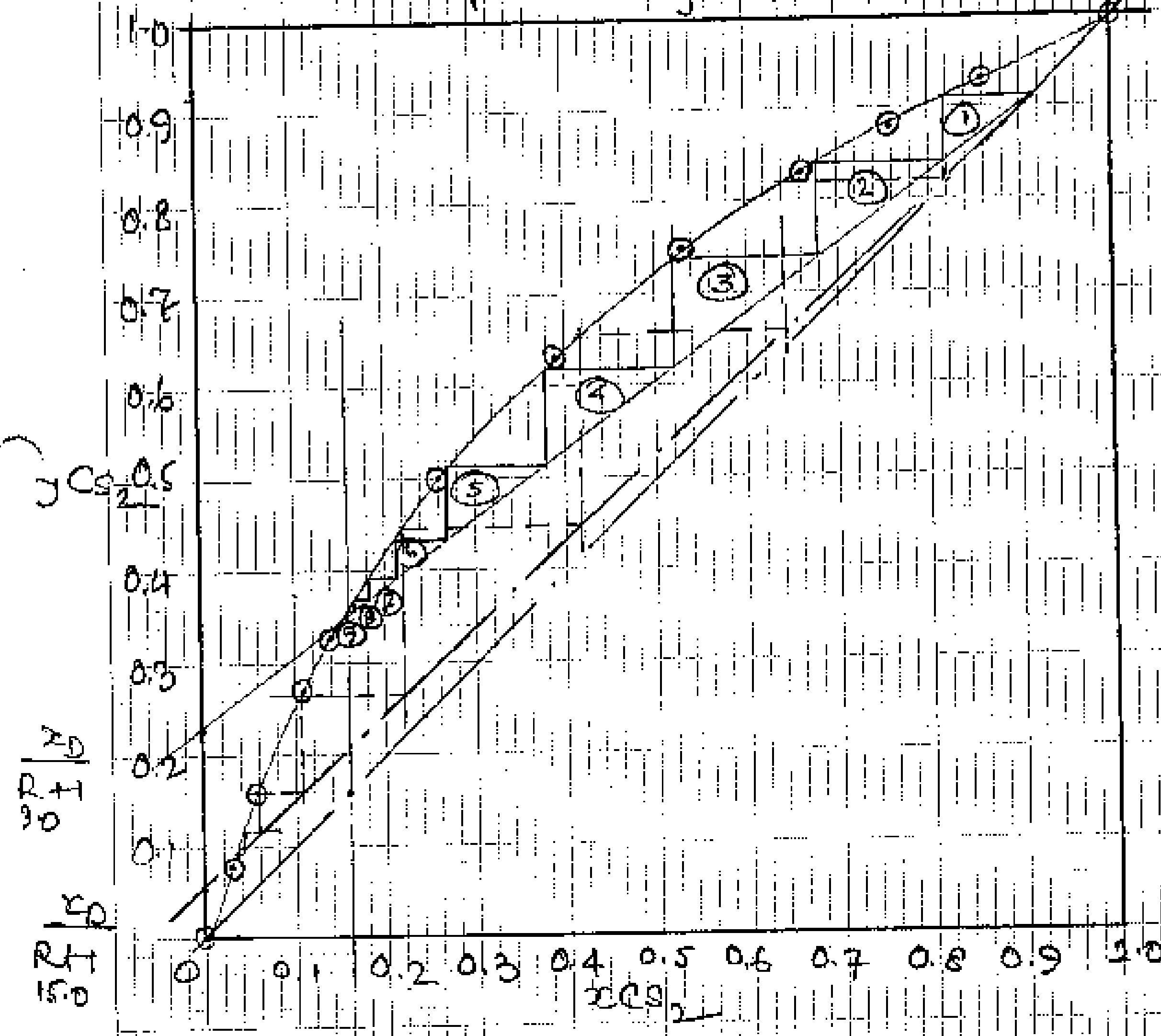


SEMI-RIGOROUS SIMULATION (VARIABLE REFLUX)
0.8
0.6
0.4
0.2
0.0
Bottom Comp
(mole-fraction)
0
1
2
3
4
5
6
Time (hrs)
1
2
BatchDistSDO

PROB 2.2(a)

Equilibrium curve of CS_2–CCl_4 : y_{CS_2} vs x_{CS_2}

Also, shows the number of theoritical stages (9) evaluated from assuming a reflux ratio of 3.0, distillate composition of 0.91 and initial feed composition: 0.1567



Initially,

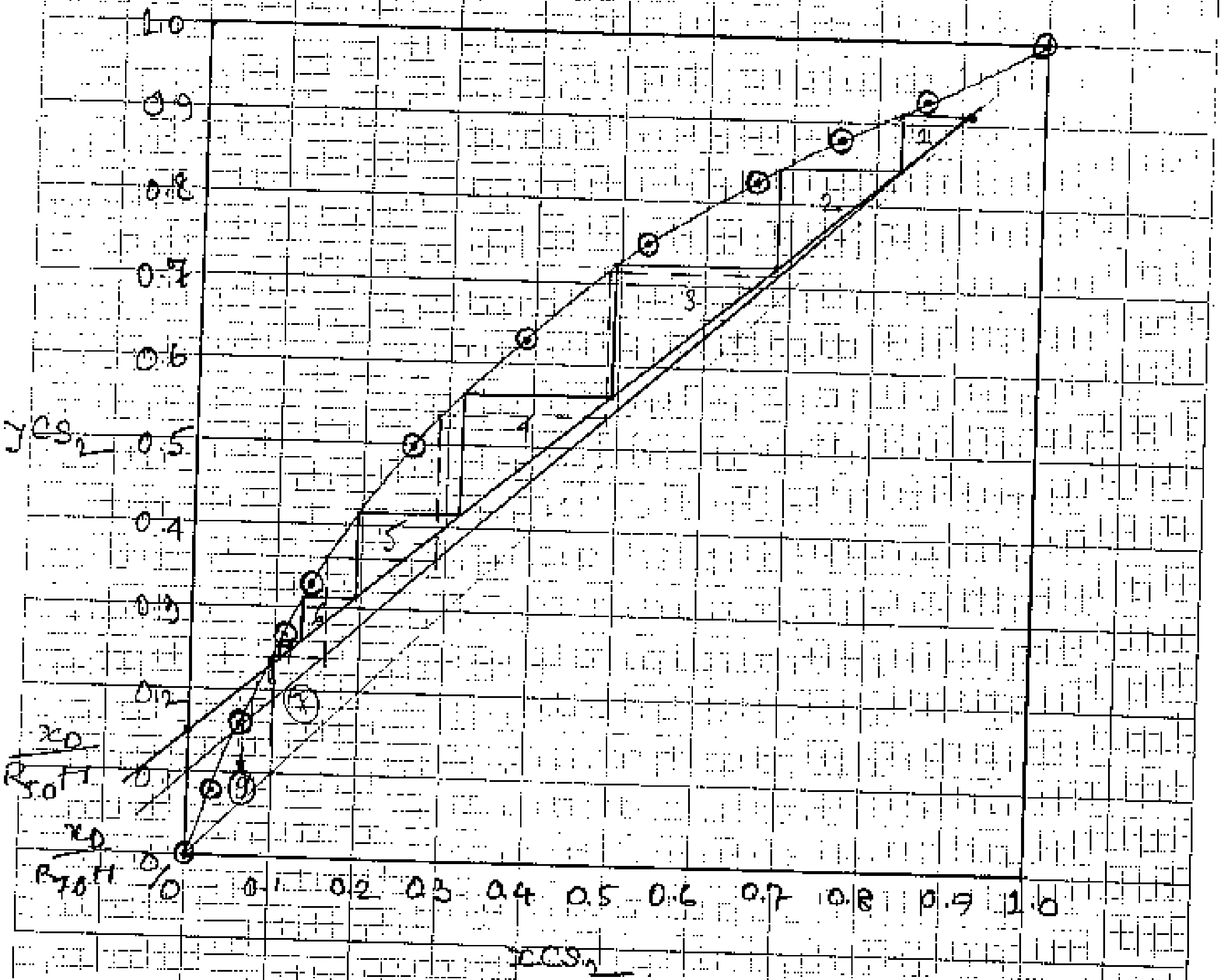
$R = 3.0$; $x_B = 0.1567$

Finally,

$R = 15.0$; $x_B = 0.035$

Reflux ratio 5.0 and 7.0

Equilibrium curve y_{CS_2} vs x_{CS_2}



for $R = 5.0$; $x_B = 0.098$

$R = 7.0$; $x_B = 0.062$

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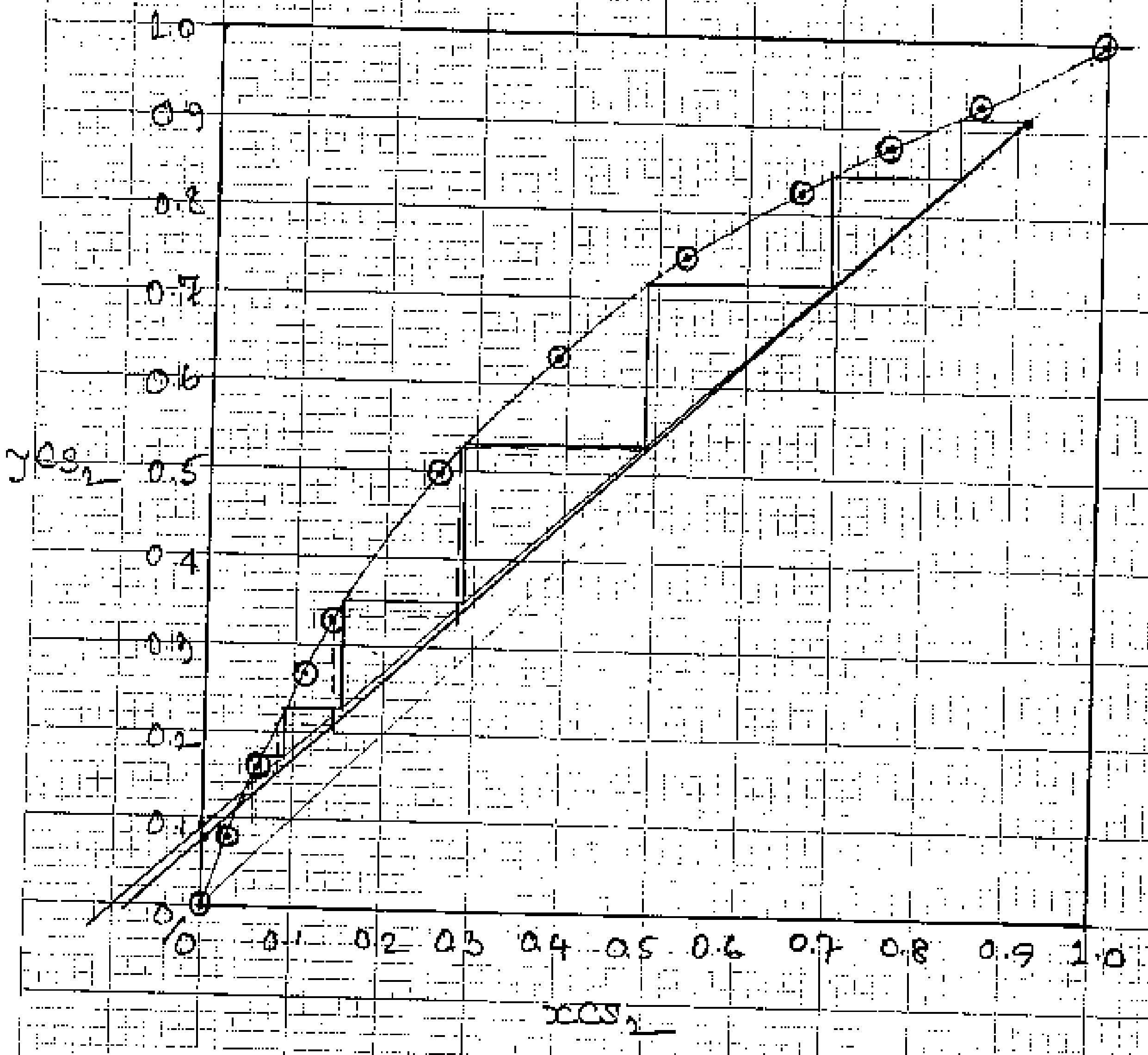
Reflux ratio for 9.0 and 11.0

Equilibrium curve y_{CS_2} vs x_{CS_2}



For $R = 9.0$; $x_{D_S} = 0.05$

$= 11.0$; $x_{D_S} = 0.04$

Squares to the Inch

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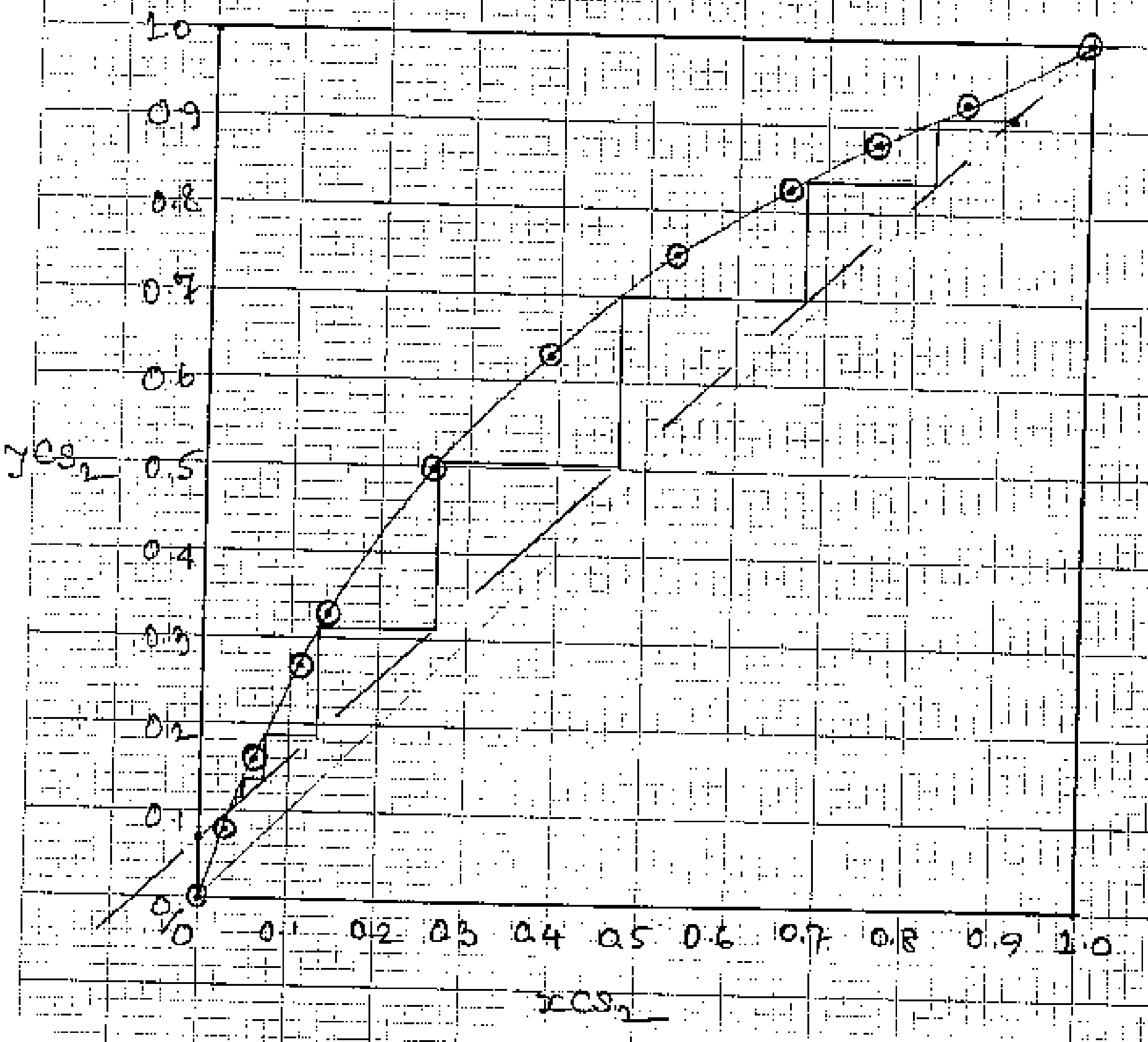
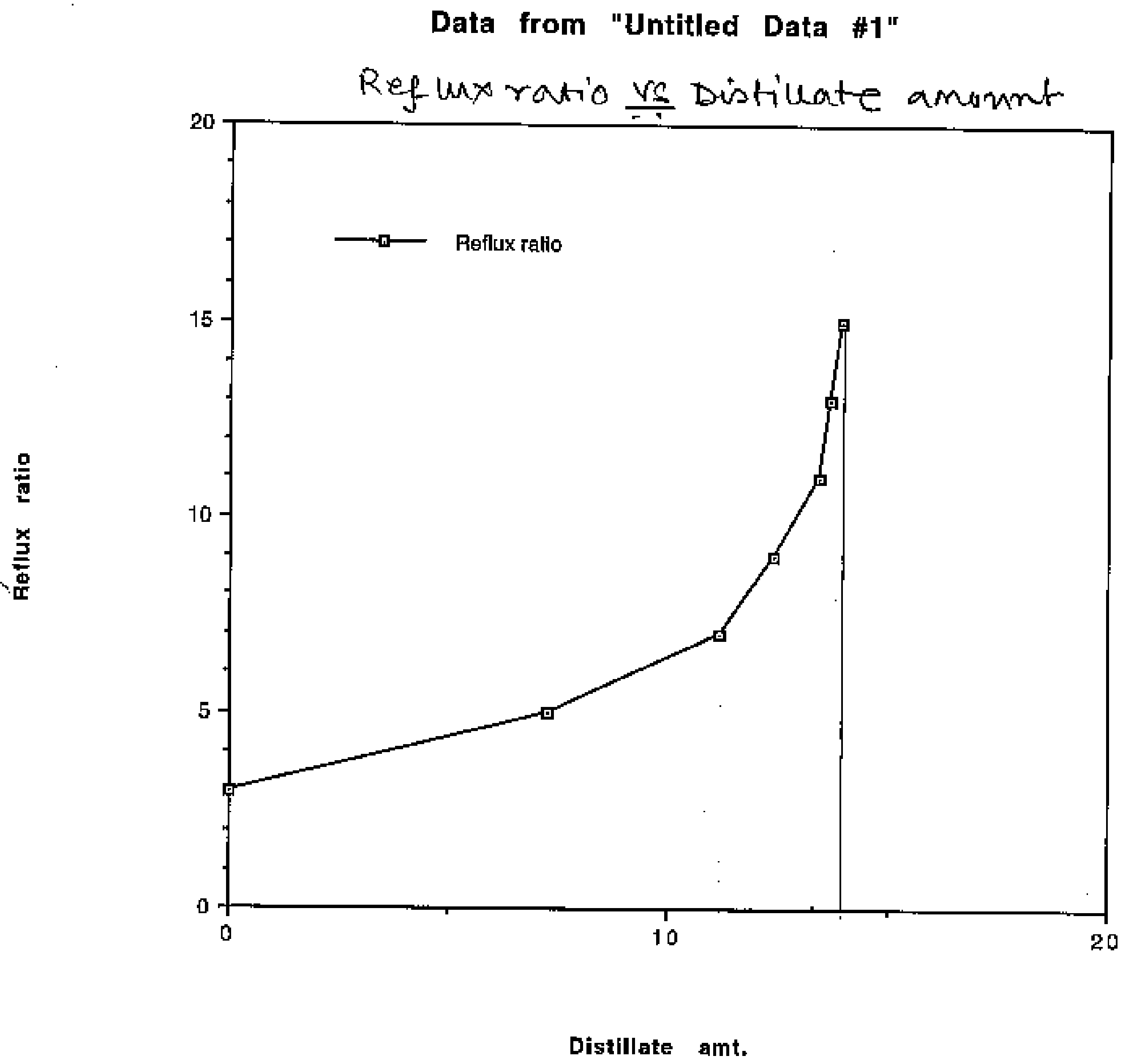
Reflux ratio for 13.0

Equilibrium curve y_{CS_2} vs x_{CS_2}



For R = 13.0 ; $x_B = 0.0375$


Data from "Untitled Data #1"
Reflux ratio vs Distillate amount
Reflux ratio
20
15
10
5
0
0
10
20
Reflux ratio
Distillate amt.

Hence, we have:

Reflux Ratio (R)	x_B	D
3.0	0.1567	—
5.0	0.098	7.23
7.0	0.082	11.17
9.0	0.050	12.41
11.0	0.040	13.41
13.0	0.0375	13.66
15.0	0.0350	13.91

The Distillate amount, when $x_B = 0.0350$ is :-

$$D = F\left(1 - \frac{x_D - x_F}{x_D - x_B}\right)$$

$$= 100\left(1 - \frac{0.91 - 0.1567}{0.91 - 0.035}\right)$$

$$= 13.91 \text{ moles.}$$

(c) The Heat required per K.mole of product is :-

$$Q = \int_0^D \lambda R\, dD = \lambda \int_0^D R\, dD$$

(R vs D)

The area under the curve, is obtained from graphical integration to yield :-

$$Q = \left(25{,}900 \frac{kJ}{1000\, mol} \times 78.98\, mol\right)$$

$$= 25.9\, kJ \times 78.98$$

Average heat required per kmol of distillate:

$$Q = \left(\frac{2045.582}{13.91}\right) \frac{kJ}{mol} = 146.95 \frac{kJ}{mol}$$

PROB 2.3

(a) The number of theoritical stages as obtained from a graphical construction is :- 7.

(b) from material balance calculations:

$$\int_0^D x_D \, dD = \int_F^B x_B \, dB$$

$$x_{DAv} \int_0^D dD = F x_F - B x_B$$

or $$x_{Dav} = \frac{F x_F - B x_B}{D}$$

Also; $$\ln\left(\frac{B}{F}\right) = \int_{x_F}^{x_B} \frac{dx_B}{x_D - x_B}$$

x_D	x_B	$\frac{1}{x_D - x_B}$	$\int \frac{dx_B}{x_D - x_B}$	B	D
0.95	0.1567	1.260	0.047	95.41	4.59
0.90	0.12	1.280	0.033	92.31	7.69
0.85	0.095	1.324	0.0068	91.68	8.32
0.80	0.090	1.408			

18
18

$$x_{Dav} = \frac{15.67 - 91.68(0.09)}{8.32} \simeq 0.90$$

The amount of distillate collected at this stage = 8.32 mol

The batch time assuming a vapor boil-up rate of 100 moles/hr

ie : $\left(\frac{R+1}{V}\right) D. = \left(\frac{6}{100} \times 8.32\right) = 0.5$ hrs

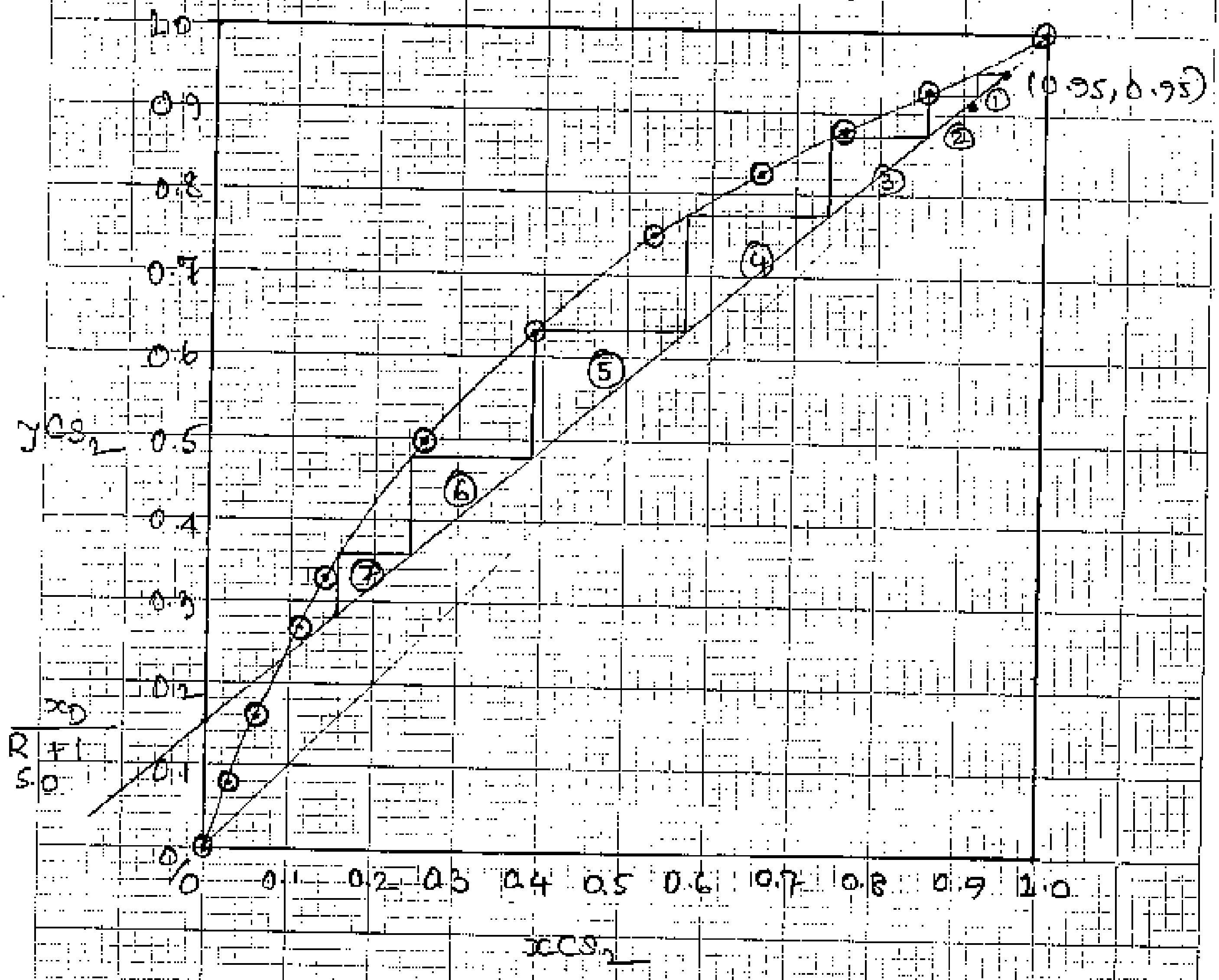
(c) The heat required per Kmol of product;

$$\frac{Q}{D} = \frac{\lambda R D}{D} = \left(25{,}900 \frac{KJ}{Mol} \times 5.0\right)$$

$$= 129.5 \frac{KJ}{Mol}.$$

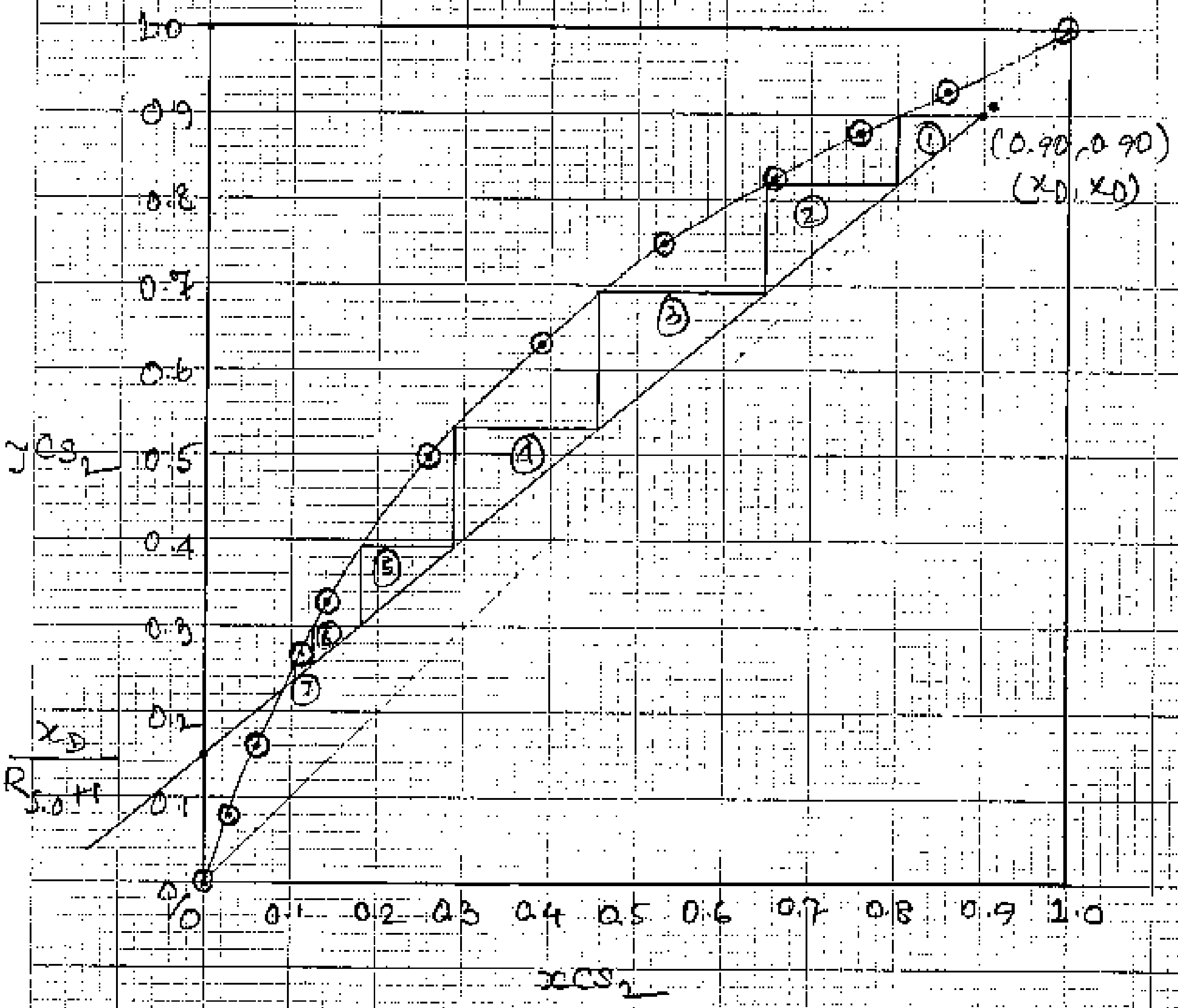
PROB 2.3

Equilibrium curve y_{CS_2} vs x_{CS_2}

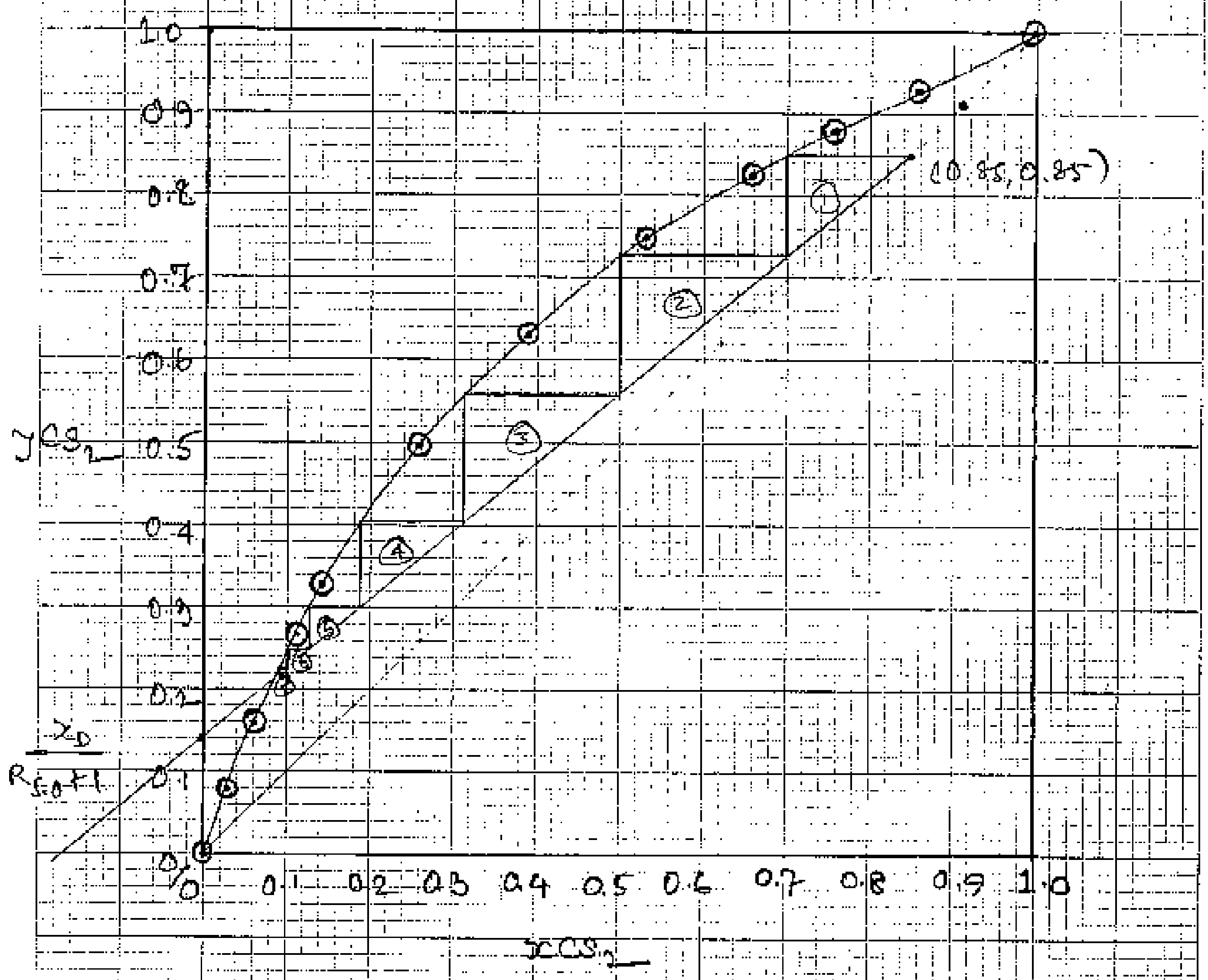


10 Squares to the Inch

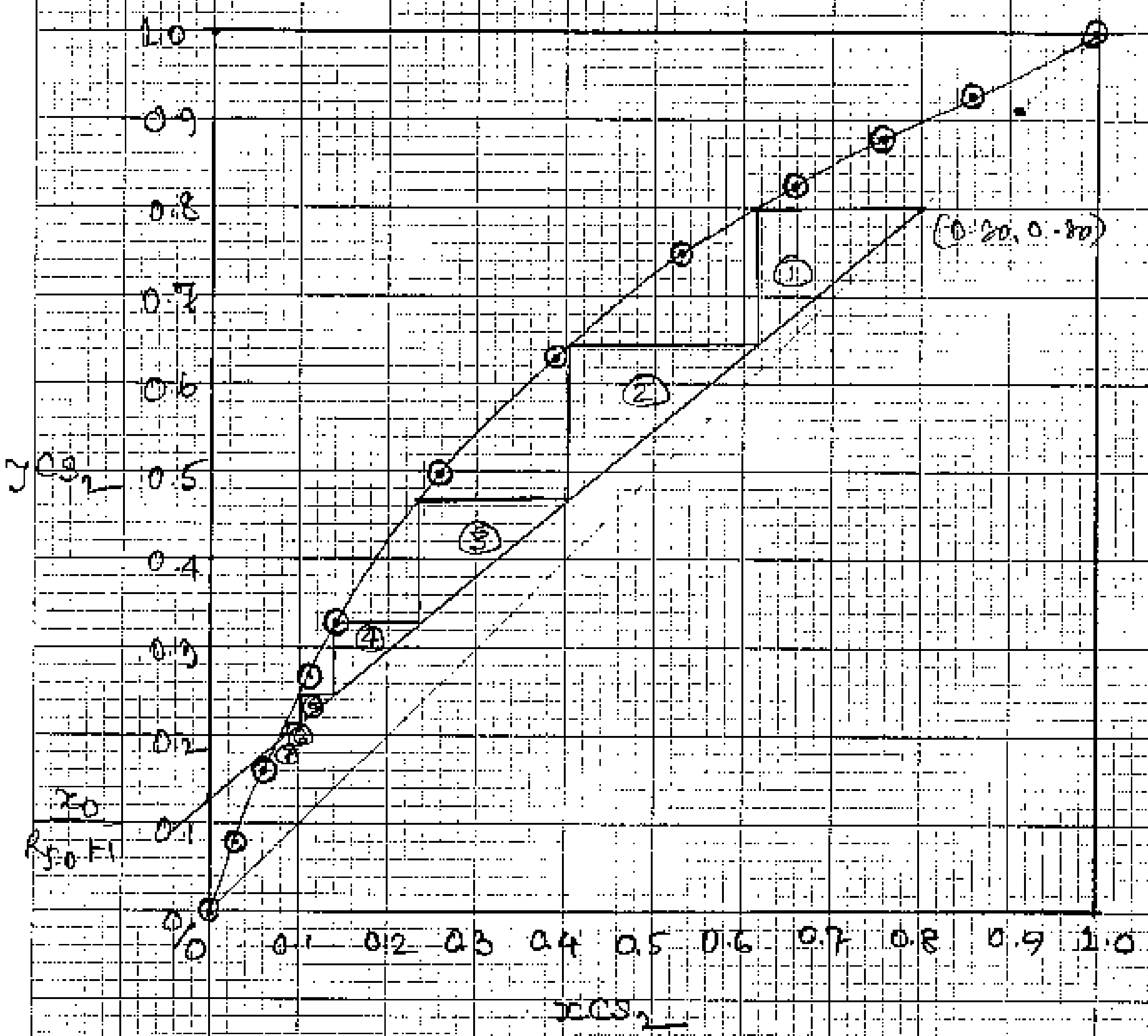
Equilibrium curve y_{CS_2} vs x_{CS_2}



Equilibrium curve y_{CS_2} vs x_{CS_2}



Equilibrium Curve y_{CS_2} vs x_{CS_2}



PROBLEM 2.4					
RESULTS	FROM	BATCH-DIST			
SEMI-RIGOROUS SIMULATION (CONSTANT REFLUX)					
THE COMPONENTS AND RELATIVE VOLATILITIES ARE					
I	XDA(I)	XF(I)	XB(I)	ALPHA(I)	
1	0.8774	0.6	0.5124	1.7	
2	0.1175	0.36	0.4366	1.18	
3	0.0051	0.04	0.051	1	
REFLUX	RATIO	=	3		
NUMBER	OF	PLATES	=	8	
NO	TIME	B	XB1	XB2	XB3
1	0	100	0.6	0.36	0.04
2	0.0384	99.04	0.5971	0.3625	0.0403
3	0.0768	98.08	0.5942	0.3651	0.0407
4	0.1152	97.12	0.5912	0.3677	0.0411
5	0.1536	96.16	0.5882	0.3704	0.0414
6	0.192	95.2	0.5851	0.3731	0.0418
7	0.2304	94.2401	0.582	0.3758	0.0422
8	0.2688	93.2801	0.5788	0.3786	0.0426
9	0.3072	92.3201	0.5756	0.3814	0.043
10	0.3456	91.3601	0.5724	0.3843	0.0434
11	0.384	90.4001	0.569	0.3872	0.0438
12	0.4224	89.4401	0.5657	0.3901	0.0442
13	0.4608	88.4801	0.5623	0.3931	0.0446
14	0.4992	87.5201	0.5588	0.3961	0.0451
15	0.5376	86.5601	0.5553	0.3992	0.0455
16	0.576	85.6001	0.5517	0.4024	0.046
17	0.6144	84.6401	0.548	0.4055	0.0464
18	0.6528	83.6801	0.5443	0.4088	0.0469
19	0.6912	82.7202	0.5406	0.4121	0.0474
20	0.7296	81.7602	0.5367	0.4154	0.0479
21	0.768	80.8002	0.5329	0.4188	0.0484
22	0.8064	79.8402	0.5289	0.4222	0.0489
23	0.8448	78.8802	0.5249	0.4257	0.0494
24	0.8832	77.9202	0.5208	0.4293	0.0499
25	0.9216	76.9602	0.5166	0.4329	0.0505
26	0.96	76.0002	0.5124	0.4366	0.051
27	0.96	76	0.5124	0.4366	0.051
NO	TIME	D	XD1	XD2	XD3
1	0	0	0.8987	0.0972	0.004
2	0.0384	0.96	0.8974	0.0985	0.0041
3	0.0768	1.92	0.896	0.0998	0.0042
4	0.1152	2.88	0.8946	0.1011	0.0042
5	0.1536	3.84	0.8931	0.1025	0.0043
6	0.192	4.8	0.8916	0.104	0.0044
7	0.2304	5.7599	0.8901	0.1055	0.0045

INITIAL $X_{D_{Meth}}$ = 0.8987

8	0.2688	6.7199	0.8885	0.107	0.0045
9	0.3072	7.6799	0.8869	0.1085	0.0046
10	0.3456	8.6399	0.8852	0.1101	0.0047
11	0.384	9.5999	0.8834	0.1118	0.0048
12	0.4224	10.5599	0.8817	0.1135	0.0049
13	0.4608	11.5199	0.8798	0.1152	0.005
14	0.4992	12.4799	0.8779	0.117	0.0051
15	0.5376	13.4399	0.876	0.1189	0.0052
16	0.576	14.3999	0.874	0.1208	0.0053
17	0.6144	15.3599	0.8719	0.1227	0.0054
18	0.6528	16.3199	0.8698	0.1248	0.0055
19	0.6912	17.2798	0.8676	0.1268	0.0056
20	0.7296	18.2398	0.8653	0.129	0.0057
21	0.768	19.1998	0.863	0.1312	0.0058
22	0.8064	20.1598	0.8606	0.1335	0.0059
23	0.8448	21.1198	0.8581	0.1358	0.0061
24	0.8832	22.0798	0.8555	0.1383	0.0062
25	0.9216	23.0398	0.8529	0.1408	0.0063
26	0.96	23.9998	0.8501	0.1434	0.0065
27	0.96	24	0.8501	0.1434	0.0065

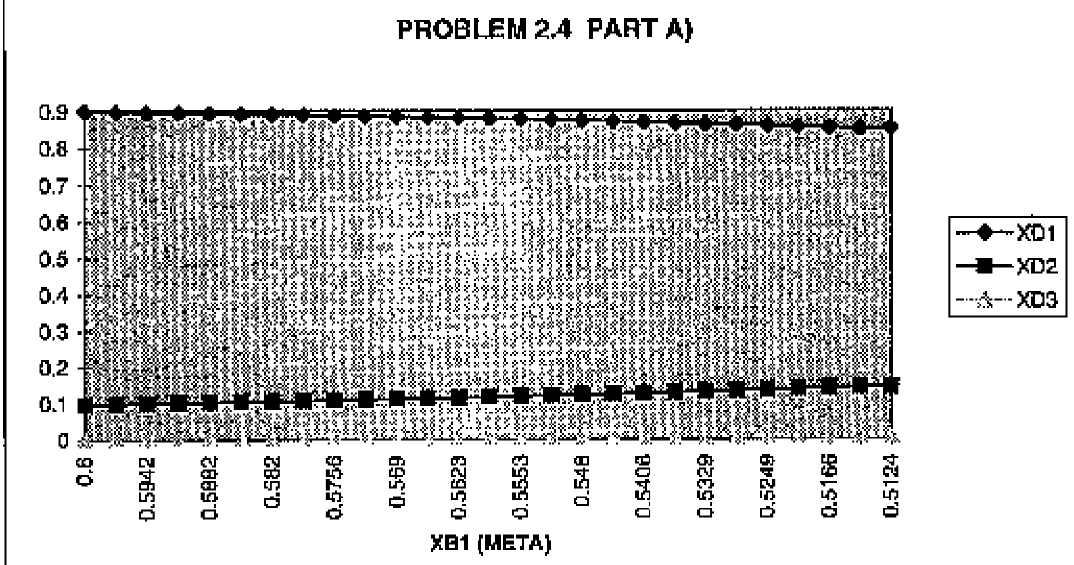
FINAL $X_{D,hex}$ = 0.8501

D = 24

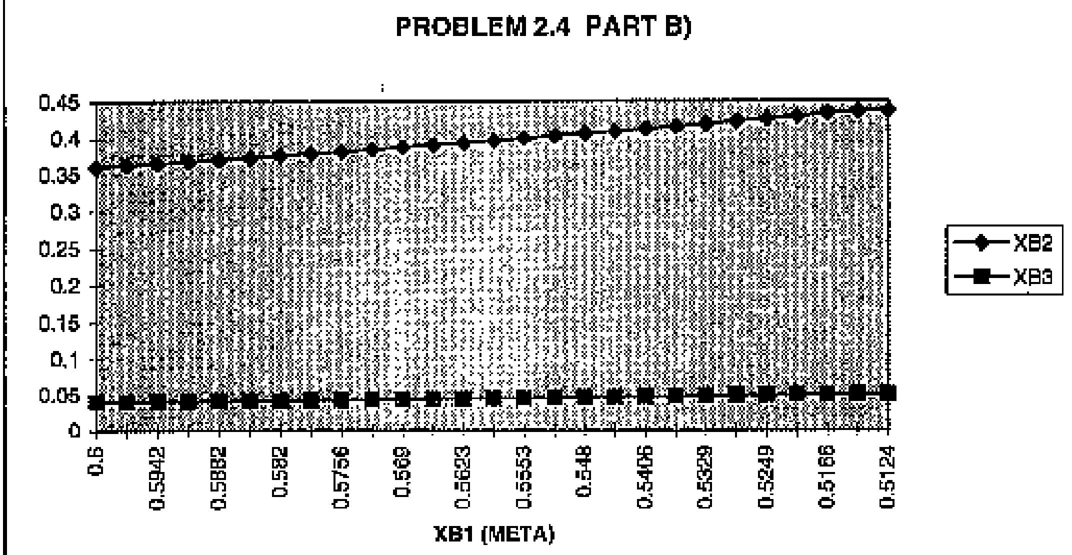
CLICK HERE TO ACCESS THE COMPLETE Solutions

PROBLEM 2.4 (EXCEL)

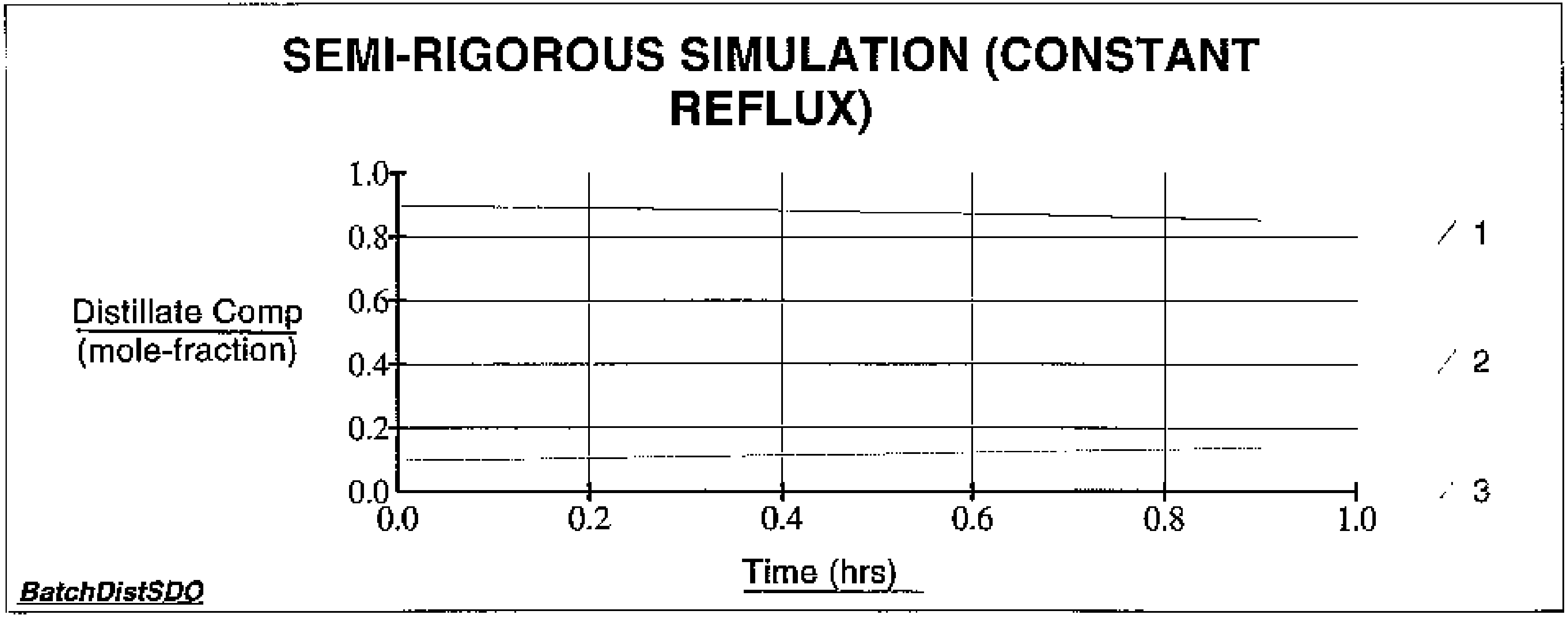
A)



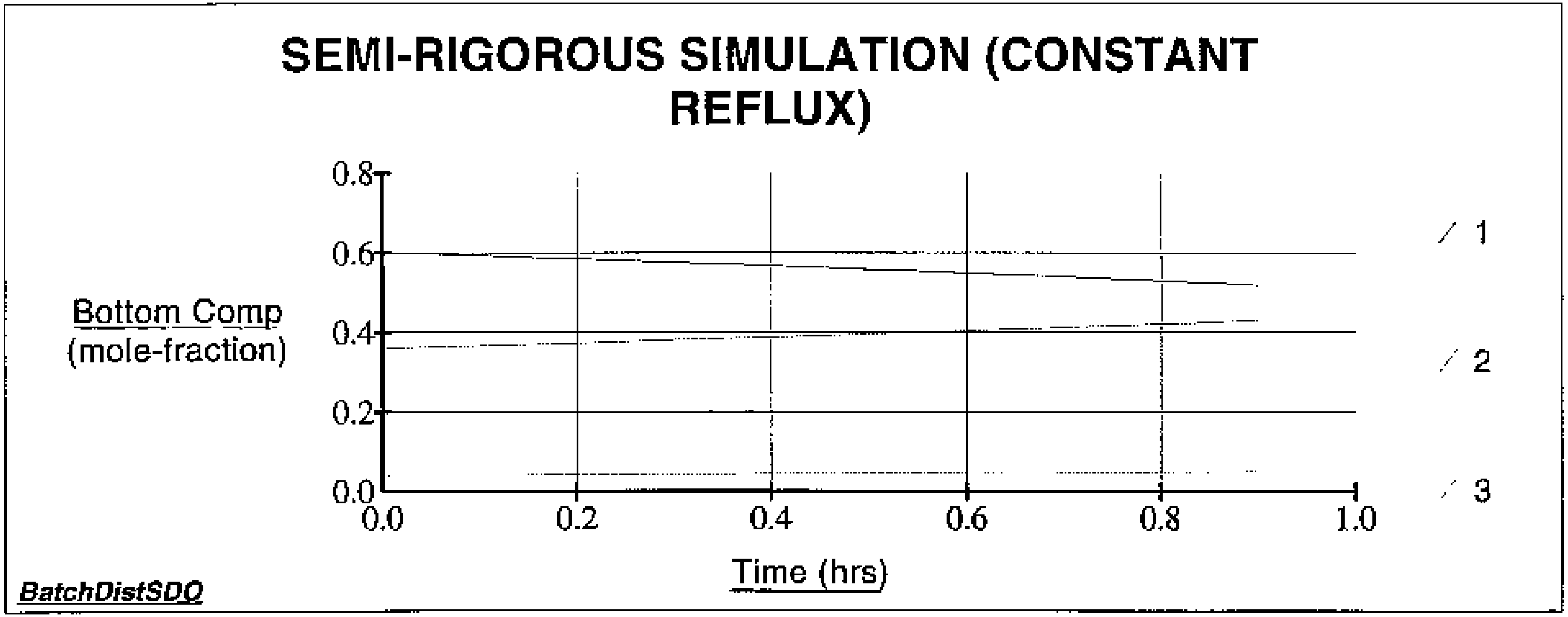
B)



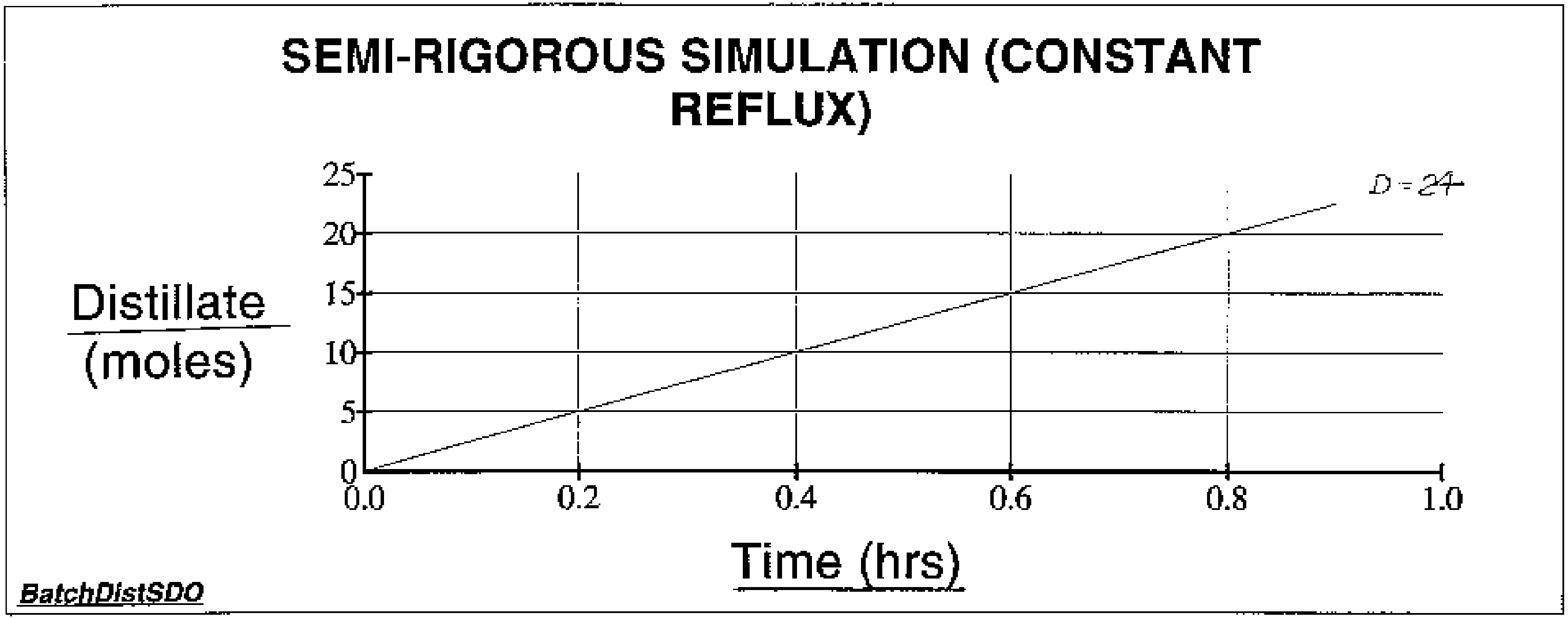
2.4 GRAPHS FROM BATCH-DIST



2.4


SEMI-RIGOROUS SIMULATION (CONSTANT REFLUX)
Bottom Comp (mole-fraction)
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Time (hrs)
1
2
3
BatchDistSDQ

2.4


SEMI-RIGOROUS SIMULATION (CONSTANT REFLUX)
Distillate (moles)
0
5
10
15
20
25
0.0
0.2
0.4
0.6
0.8
1.0
Time (hrs)
D = 24
BatchDistSDO

Problem 2.5 - For the given specifications $N \leq N_{min}$ so the problem could not be solved.

Problem 2.6 - Converse & Gross (1963) have provided a list of table comparing the three modes of operation which is given below.

Table I. Tower Conditions

Relative volatility	2
Number of plates	4
Duration of operation, hour	1
Boil-up rate, moles/hour	110
Specified product purity, y^*	0.90

Table II. Total Distillate Accumulation

(Moles)

Find values

Case No.	x_B ~~x_{B_0}~~ Mole Fraction	B ~~S(T)~~, Moles	Using Optimal Policy	At Constant Overhead Composition	At Constant Reflux Ratio
4	0.303	93.9	13.87	13.22	13.50
3	0.362	78.0	18.01	17.30	17.97
8	0.487	116.1	36.10	35.67	35.63
2	0.498	69.5	29.52	28.46	29.06
7	0.545	101.3	41.26	40.72	40.63
12	0.600	133.0	52.97	52.63	52.19
6	0.636	90.3	50.27	49.90	49.19
11	0.646	117.9	57.94	57.55	56.93
16	0.685	148.6	68.63	68.31	67.50
10	0.713	106.1	66.24	66.18	64.69
1	0.715	74.3	54.33	53.93	53.28
15	0.721	133.0	73.02	72.16	72.00
14	0.770	121.9	81.89	81.30	79.87
5	0.772	91.4	71.51	70.73	69.75
9	0.809	107.1	87.86	87.21	84.38
13	0.836	120.8	100.75	99.13	98.44

Problem 2.7 - Heat duty for minimum time problem

$$Q_R = \int_0^D \lambda R \, dD$$

PROBLEM 2.8

CASE 1 XA = 0.5

SEMI-RIGOROUS SIMULATION (CONSTANT REFLUX)

THE COMPONENTS AND RELATIVE VOLATILITIES ARE

I	XDA(I)	XF(I)	XB(I)	ALPHA(I)
1	0.5549	0.5	0.0059	1.5
2	0.4451	0.5	0.9941	1

REFLUX RATIO = 5

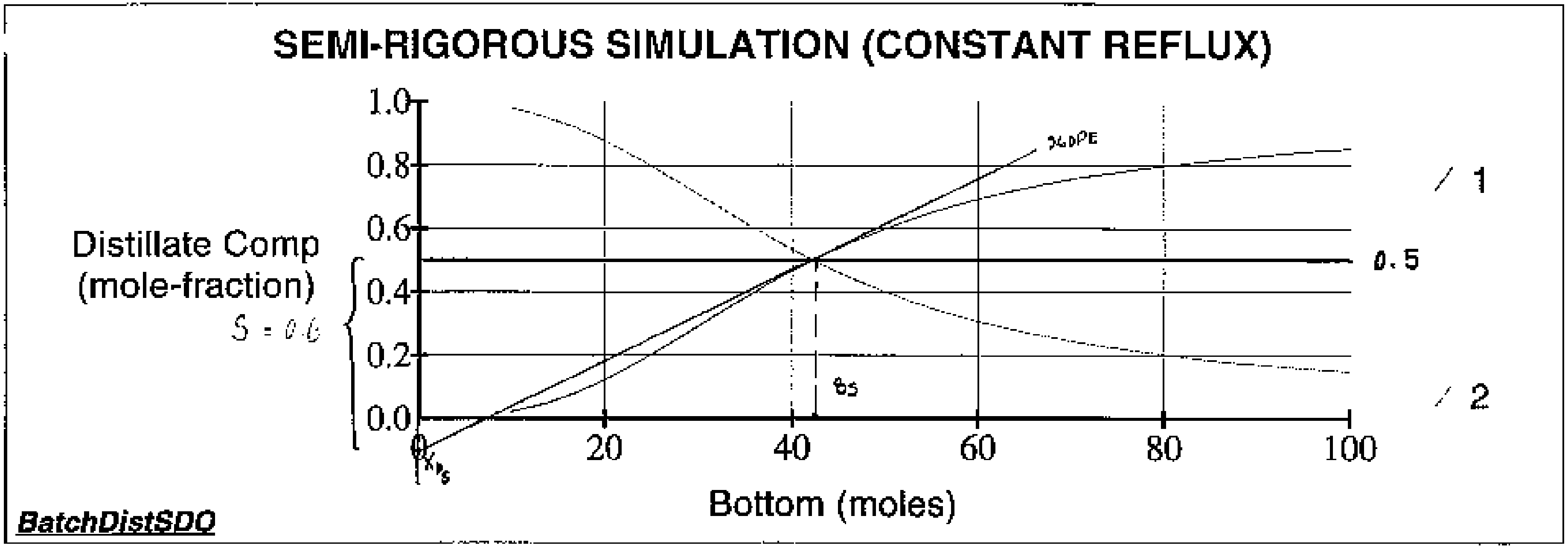
NUMBER OF PLATES = 5

NO	TIME	B	XB1	XB2
1	0	100	0.5	0.5
2	0.216	96.4	0.487	0.513
3	0.432	92.8	0.4733	0.5267
4	0.648	89.2	0.4589	0.5411
5	0.864	85.6	0.4437	0.5563
6	1.08	82	0.4276	0.5724
7	1.296	78.4	0.4106	0.5894
8	1.512	74.8	0.3926	0.6074
9	1.728	71.2	0.3736	0.6264
10	1.944	67.5999	0.3535	0.6465
11	2.16	63.9999	0.3322	0.6678
12	2.376	60.3999	0.3099	0.6901
13	2.592	56.7999	0.2863	0.7137
14	2.808	53.1999	0.2617	0.7383
15	3.024	49.5999	0.2361	0.7639
16	3.24	45.9999	0.2096	0.7904
17	3.456	42.3999	0.1826	0.8174
18	3.672	38.7999	0.1554	0.8446
19	3.888	35.1999	0.1287	0.8713
20	4.104	31.5999	0.1029	0.8971
21	4.32	27.9999	0.0789	0.9211
22	4.536	24.3999	0.0574	0.9426
23	4.752	20.7999	0.039	0.961
24	4.968	17.1999	0.0242	0.9758
25	5.184	13.5999	0.0132	0.9868
26	5.4	10	0.0059	0.9941

NO	TIME	D	XD1	XD2
1	0	0	0.8512	0.1488
2	0.216	3.6	0.8437	0.1563
3	0.432	7.2	0.8354	0.1646
4	0.648	10.8	0.8261	0.1739
5	0.864	14.4	0.8159	0.1841
6	1.08	18	0.8044	0.1956

7	1.296	21.6	0.7914	0.2086	
8	1.512	25.2	0.7768	0.2232	
9	1.728	28.8	0.7603	0.2397	
10	1.944	32.4001	0.7414	0.2586	
11	2.16	36.0001	0.7199	0.2801	
12	2.376	39.6001	0.695	0.305	
13	2.592	43.2001	0.6664	0.3336	
14	2.808	46.8001	0.6335	0.3665	
15	3.024	50.4001	0.5955	0.4045	
16	3.24	54.0001	0.5519	0.4481	
17	3.456	57.6001	0.5024	0.4976	
18	3.672	61.2001	0.4471	0.5529	
19	3.888	64.8001	0.3865	0.6135	
20	4.104	68.4001	0.3223	0.6777	
21	4.32	72.0001	0.2568	0.7432	
22	4.536	75.6001	0.1933	0.8067	
23	4.752	79.2001	0.1355	0.8645	
24	4.968	82.8001	0.086	0.914	
25	5.184	86.4001	0.0477	0.9523	
26	5.4	90	0.0216	0.9784	

$X_{F_1} = 0.5 \quad X_{F_2} = 0.5$



POLE HEIGHT

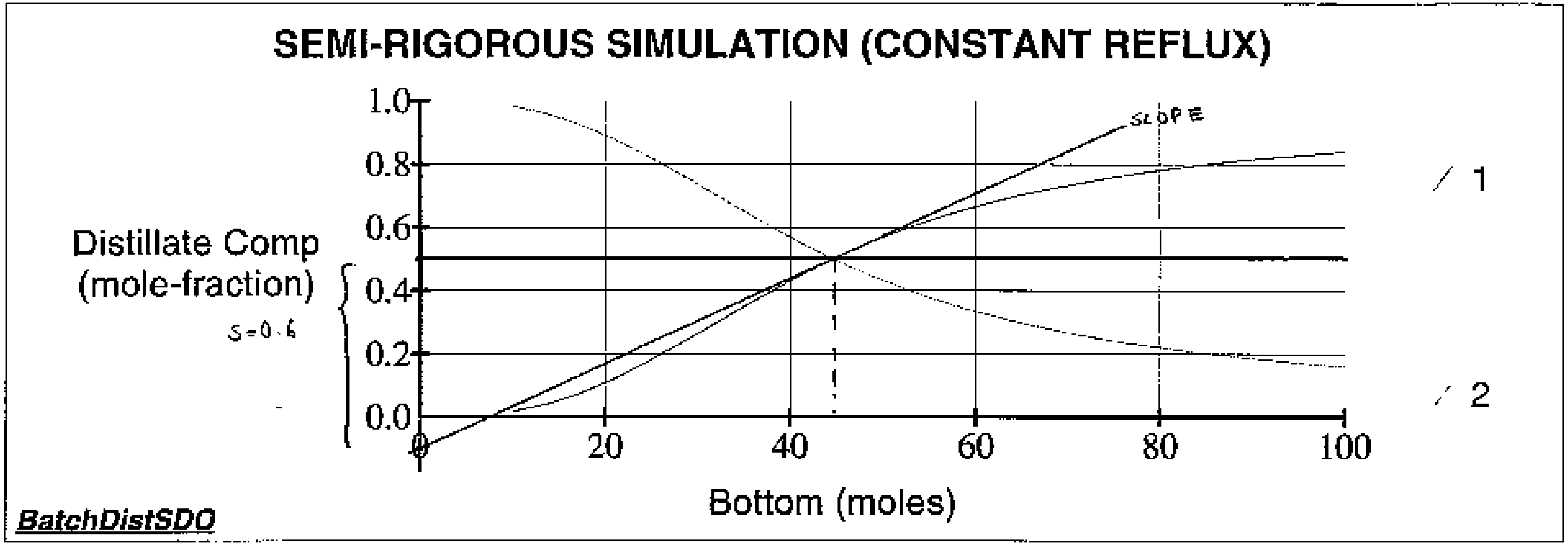
$S = \underbrace{\left(\frac{0.5 - X_{D_S}}{B_S - 0}\right)}_{\text{SLOPE}}$

B_S AMOUNT REMAINING IN THE STILL

$= 0.5 - X_{D_S} = 0.5 - (-0.1) = 0.6$

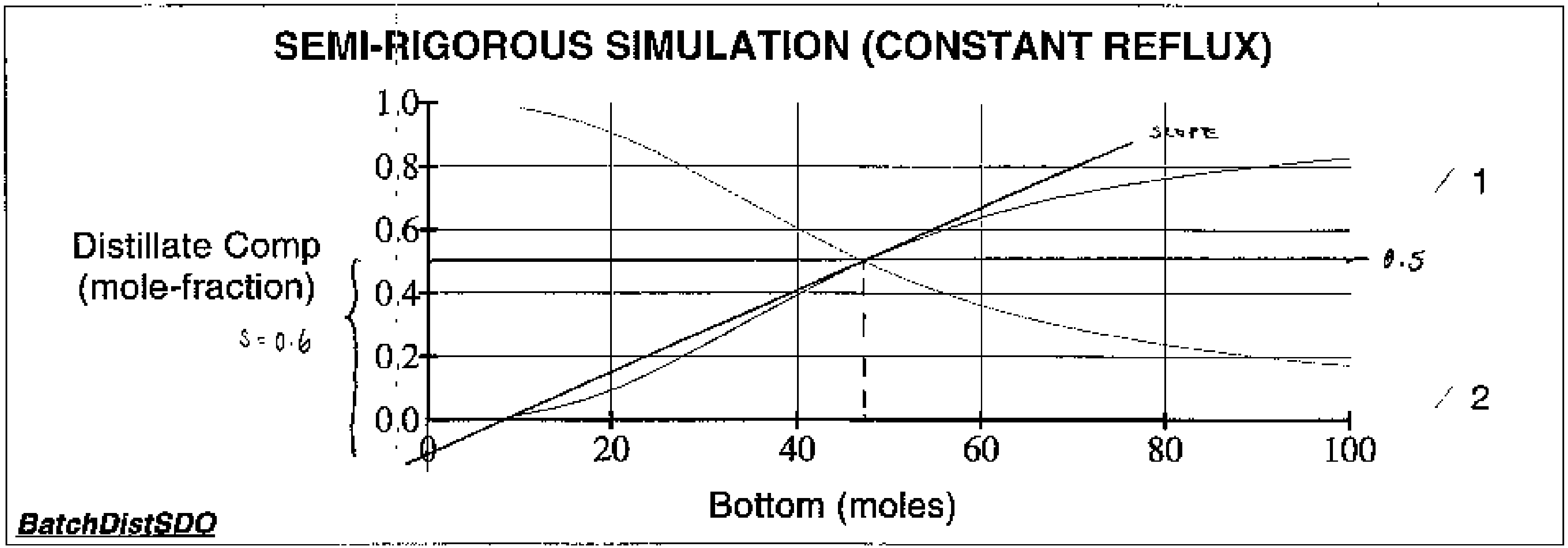
$x_{F_1} = 0.48$

$x_{F_2} = 0.52$



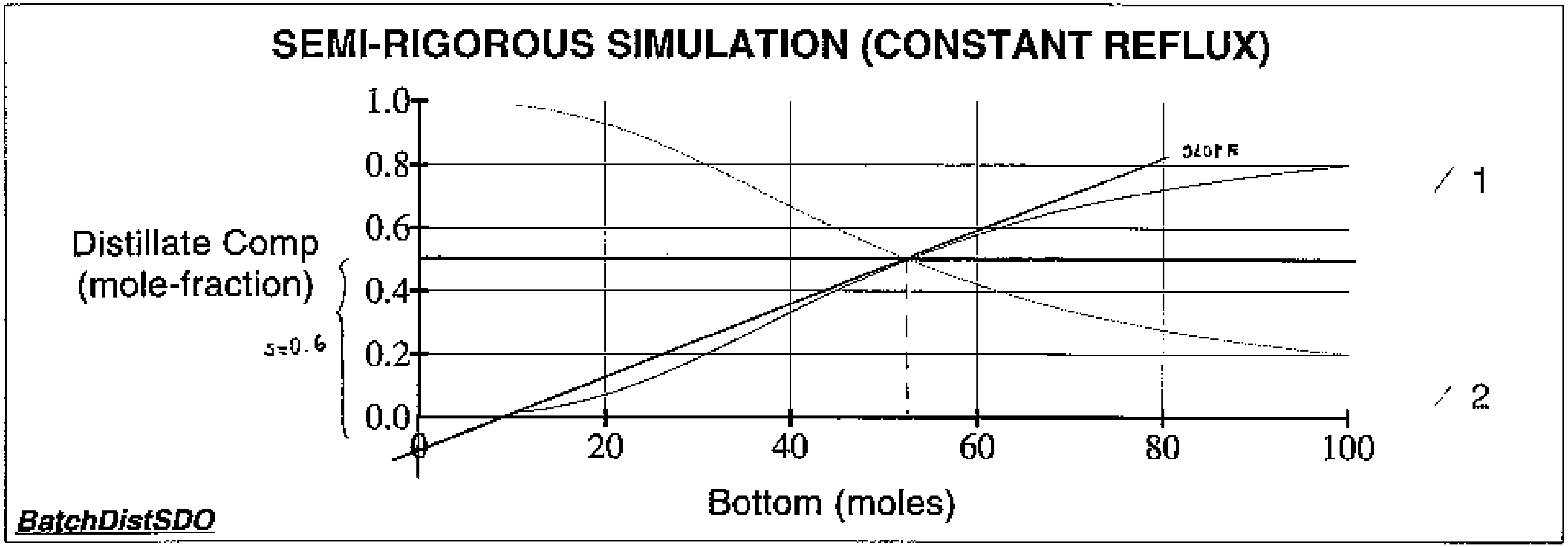
$x_{f_1} = 0.46$

$x_{f_2} = 0.54$

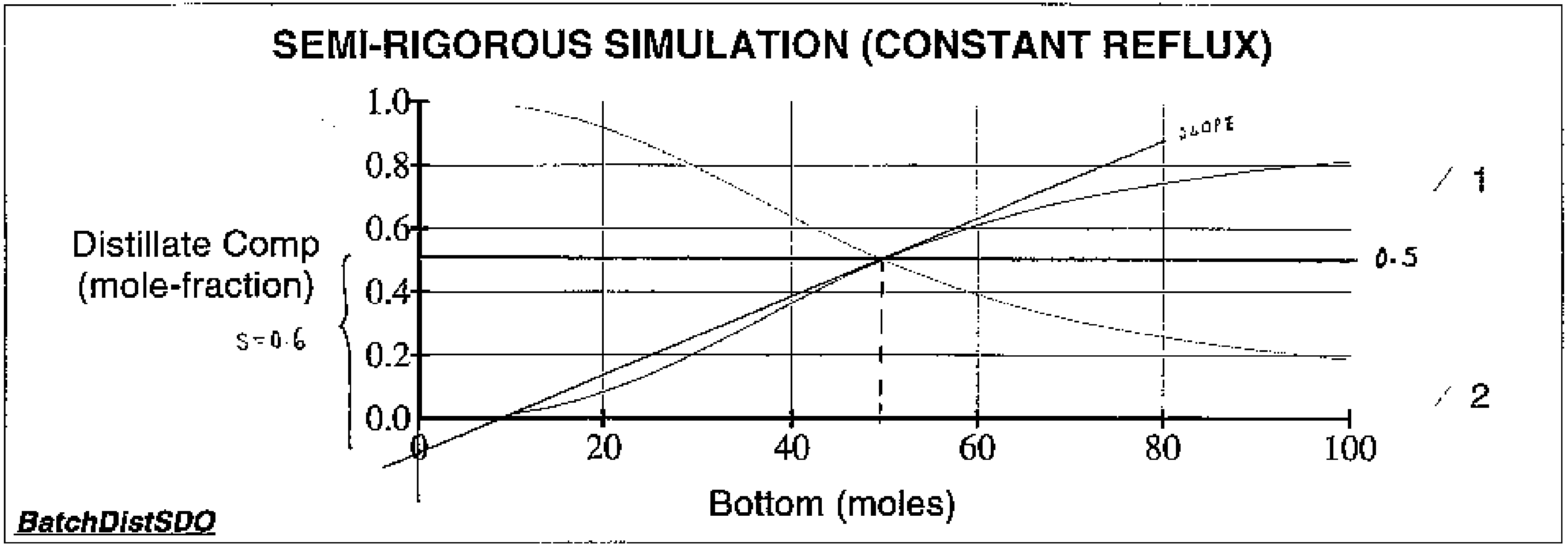


37

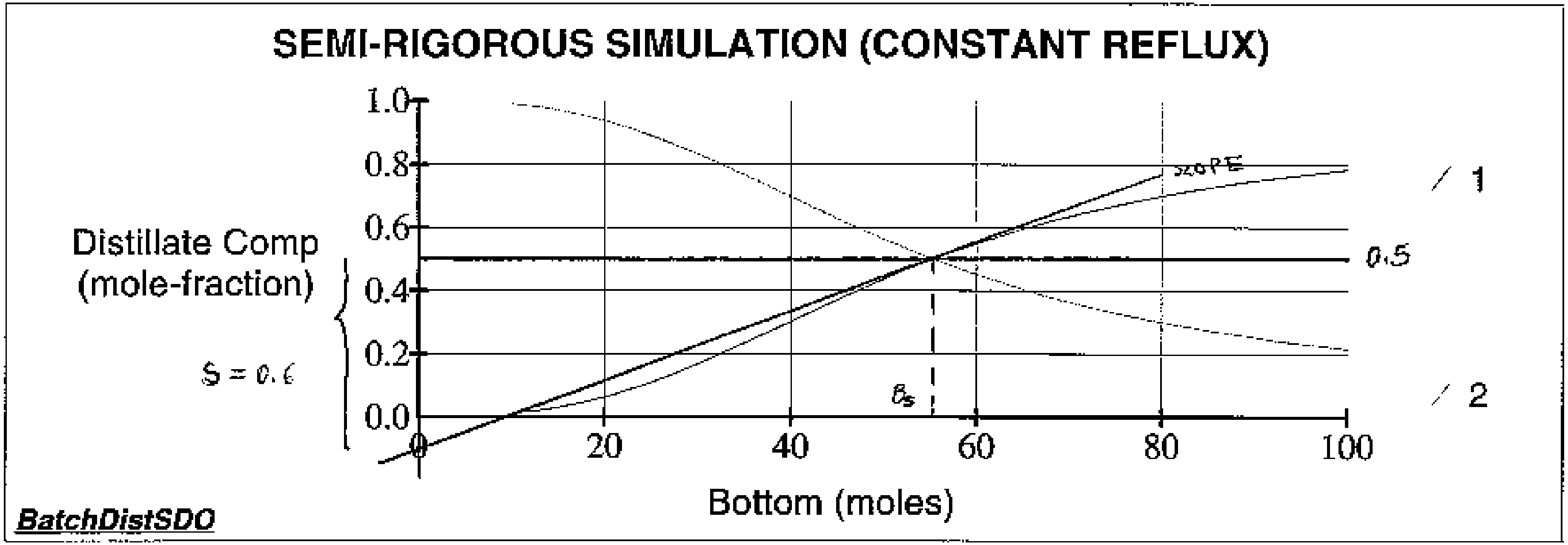
$x_{F_1} = 0.44$ $x_{F_2} = 0.56$



$X_{F_1} = 0.42 \quad X_{F_2} = 0.4$



$X_{F_1} = 0.4$ $X_{F_2} = 0.6$



As we could see, the pole height $S = 0.6$ is invariant to the initial concentration X_F

2.9 Problem 2.8 described the pole height concept proposed by Bowman and Cichelli. Use the above definition to prove that:

a) At a total reflux condition (minimum number of plates), the pole height S is related to the number of plates N by the following relation.

$$S = \frac{\alpha^N}{8F(1 - x_F)}$$

b) At an infinite number of plates (minimum reflux condition) and moderately good separation, the reflux can be expressed as:

$$R = \frac{2S - \alpha}{\alpha - 1}$$

Assume that at moderate separation the quantity of x_B is extremely small.

Solution: A Pole height for binary mixture is defined as

$$S = \frac{dx_D}{dx_B}|_{x_D=0.5} \tag{2.1}$$

a Rayleigh equation gives

$$B\frac{dx_D}{dx_B} = x_D - x_B \tag{2.2}$$

At total reflux

$$N = N_{min} = \frac{1}{\ln\alpha}\frac{x_D(1-x_B)}{x_B(1-x_D)} \tag{2.3}$$

At $x_D = 0.5$ $\alpha^N = \frac{1-x_B}{x_B}$; Hence,

$$x_B = 1/(\alpha^N + 1) \tag{2.4}$$

$$\frac{dx_B}{dB} = \frac{dx_B}{dx_D}\frac{dx_D}{dB} \tag{2.5}$$

From Equations 2.2 and 2.5

$$B\frac{dx_D}{dx_B}\frac{dx_B}{dB} = (x_D - x_B)\frac{dx_D}{dx_B} \tag{2.6}$$

From Equation 2.3

$$\frac{dx_B}{dx_D} = \frac{d}{dx_D}[\frac{x_D}{\alpha^N - (\alpha^N - 1)x_D}] \tag{2.7}$$

$$= \frac{\alpha^N}{(\alpha^N - (\alpha^N - 1)x_D)^2} \tag{2.8}$$

$$\frac{dx_D}{dx_B} = \frac{(\alpha^N - (\alpha^N - 1)x_D)^2}{\alpha^N} \tag{2.9}$$

From Equations 2.4, 2.6, and 2.9

$$B\frac{dx_D}{dx_B}\frac{dx_B}{dB} = \frac{(\alpha^N - (\alpha^N - 1)0.5)2}{\alpha^N}(0.5 - \frac{1}{\alpha^N + 1}) \quad (2.10)$$

$$B \times S = \frac{\alpha^{2N} - 1}{8\alpha^N} \quad (2.11)$$

$$S = \frac{\alpha^{2N} - 1}{8B\alpha^N} \quad (2.12)$$

For sharp separations α^N is large, resulting in $\alpha^{2N} - 1 = \alpha^{2N}$. Further, for sharp separations, one can assume that amount of material remaining in the pot is equal to amount of heavy component in the original charge. Therefore,

$$B = F(1 - x_F) \quad (2.13)$$

Combining Equations 2.12 and pole-8 results in

$$S = \frac{\alpha^N}{8F(1 - x_F)}$$

b At minimum reflux (infinite number of plates), Underwood's equations for binary separation with relative volatility α are given by

$$\sum_{i=1}^{n} \frac{\alpha_i x_B^{(i)}}{\alpha_i - \phi} = 0 \quad (2.14)$$

$$R_{min} + 1 = R + 1 = \sum_{i=1}^{n} \frac{\alpha_i x_D^{(i)}}{\alpha_i - \phi} \quad (2.15)$$

For binary component, from Equation 2.14

$$-\alpha\phi x_B + \alpha - \phi + \phi x_B = 0 \quad (2.16)$$

Therefore;

$$x_B = \frac{-(\alpha - \phi)}{(1 - \alpha)\phi} \quad (2.17)$$

$$\phi = \frac{\alpha}{1 - (\alpha - 1)x_B} \quad (2.18)$$

$$1 - \phi = \frac{(\alpha - 1)(1 - x_B)}{1 - (\alpha - 1)x_B} \quad (2.19)$$

Simplifying Equation 2.15 results in

$$x_D = \frac{-R(\alpha - \phi)}{(\alpha - 1)\phi} + (R + 1)\frac{\phi(\alpha - \phi)}{\alpha - 1} \quad (2.20)$$

Substituting value of x_B

$$x_D = Rx_B + (R+1)\phi x_B \tag{2.21}$$

Replacing ϕ in terms of x_B

$$x_D = -Rx_B + \frac{(R+1)\alpha x_B}{1+(\alpha-1)x_B} \tag{2.22}$$

Rayleigh equation in terms of pole height as shown in (a).

$$S = \frac{dx_D}{dx_B} |_{x_D=0.5} (x_D - x_B) \tag{2.23}$$

From Equation 2.22

$$\frac{dx_D}{dx_B} = -R + \frac{(R+1)\alpha}{(1+(\alpha-1)x_B)^2} \tag{2.24}$$

At $x_D = 0.5.$, $x_B << x_D$, Hence $x_D - x_B = x_D$. Also, since x_B is small $(1+(\alpha-1)x_B) = 1$. Therefore,

$$S = 0.5[-R + (R+1)\alpha] \tag{2.25}$$

Simplifying,

$$R = \frac{2S - \alpha}{\alpha - 1}$$

FIGURE 2.1

Schematic of a Batch Distillation Column

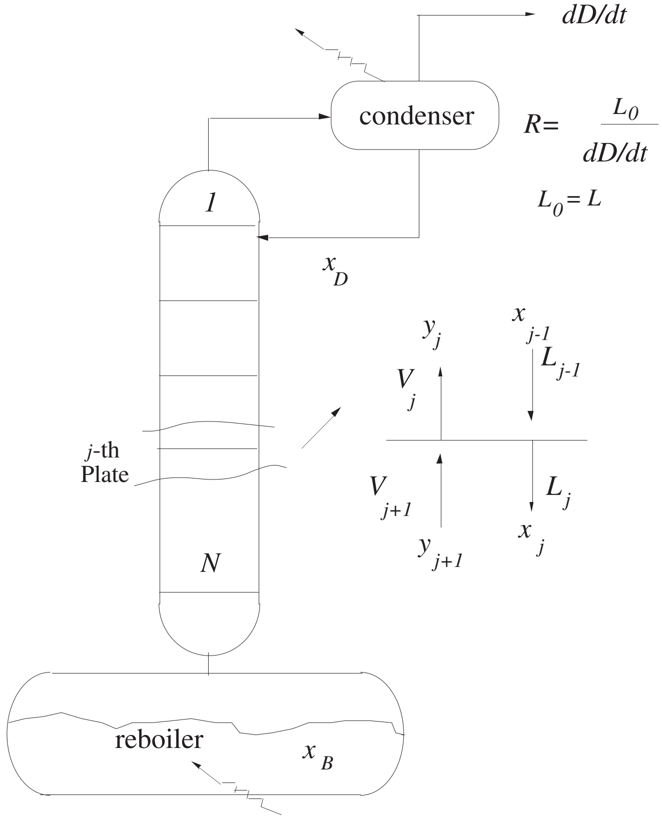


FIGURE 2.2

McCabe–Thiele Method for Plate-to-Plate Calculations

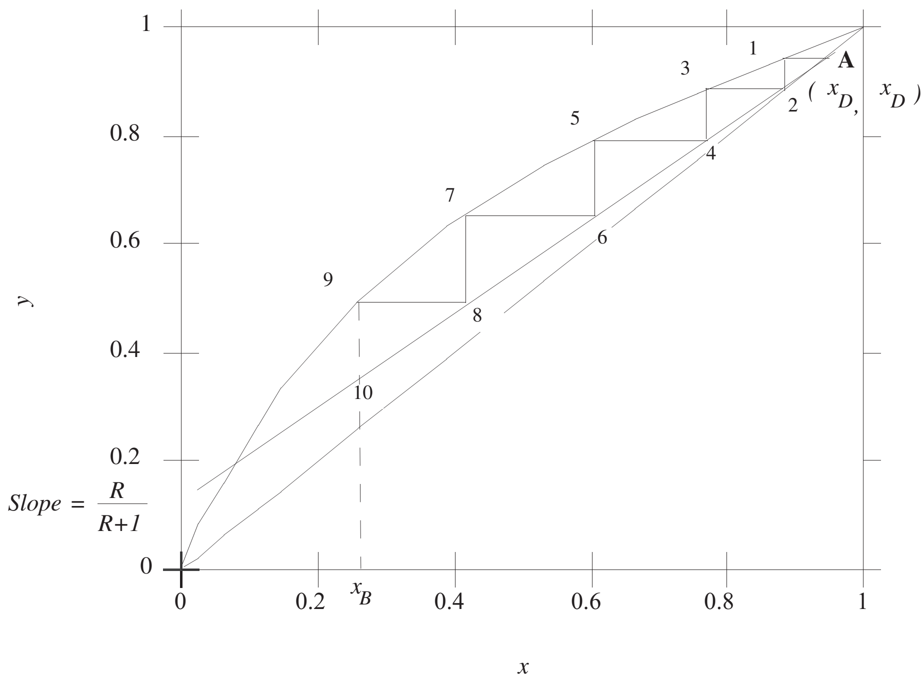


FIGURE 2.3

McCabe–Thiele Method for the Constant Reflux Mode

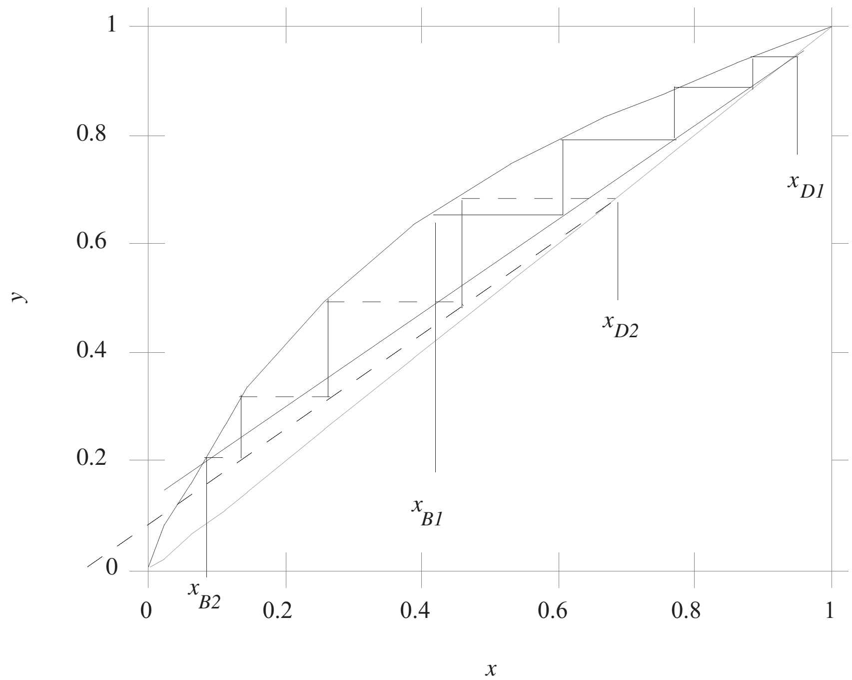


FIGURE 2.4

Graphical Integration for Example 2.1

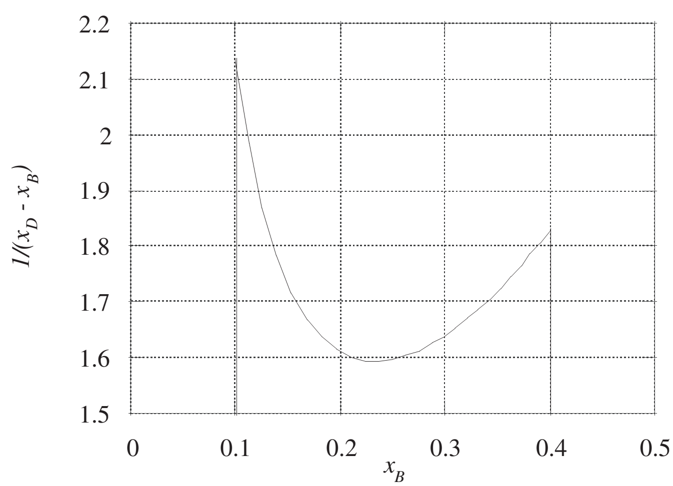


FIGURE 2.5

McCabe–Thiele Method for the Variable Reflux Mode

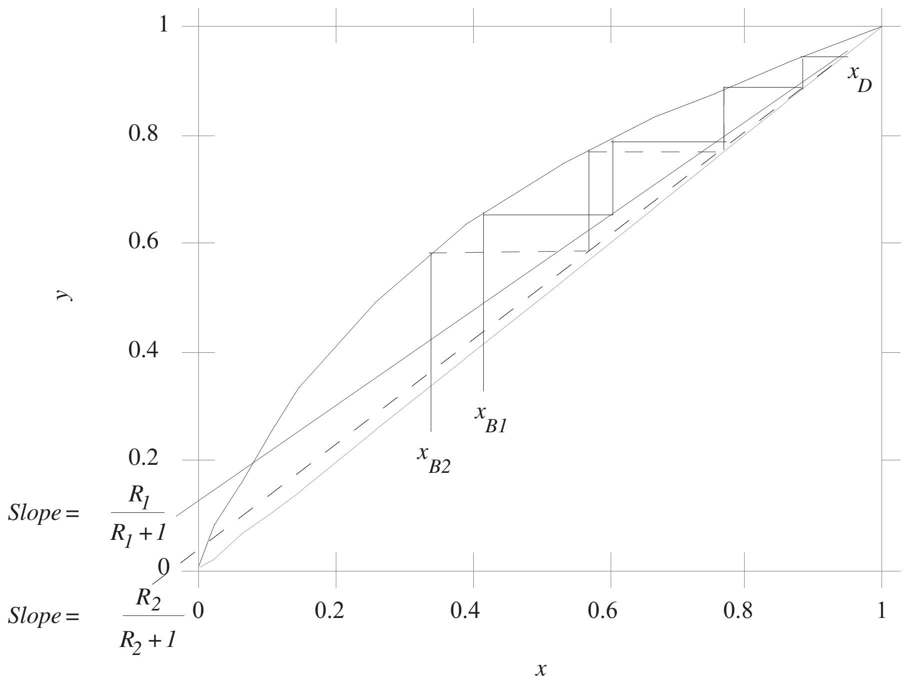


FIGURE 2.6

Graphical Integration for Calculation of Batch Time for Example 2.2

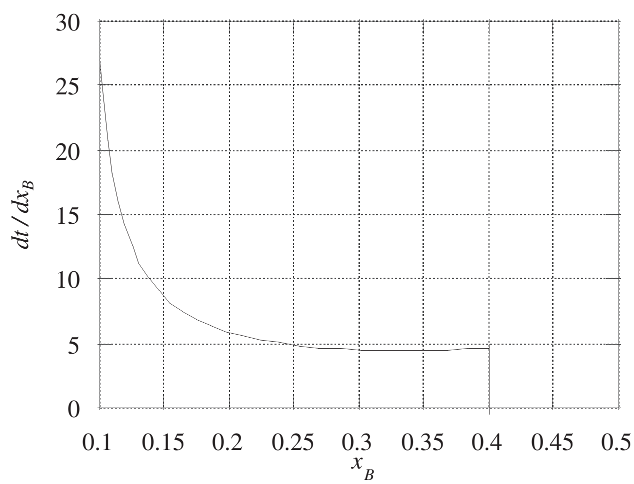


FIGURE 2.7

Graphical Integration for Calculation of Reboiler Heat Duty for Example 2.2

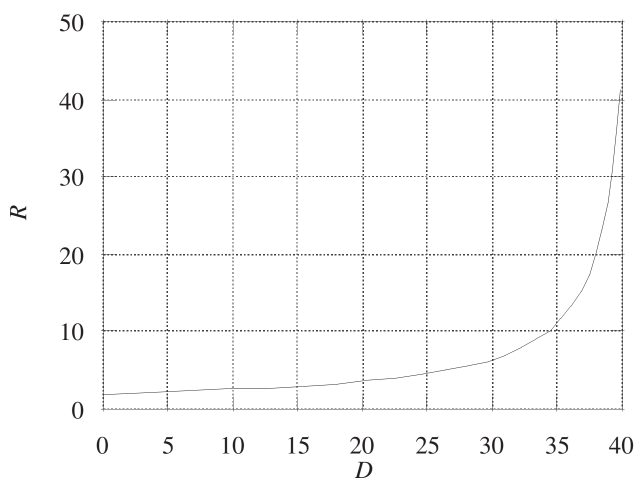


FIGURE 2.8

McCabe–Thiele Procedure for the Third Mode of Operation

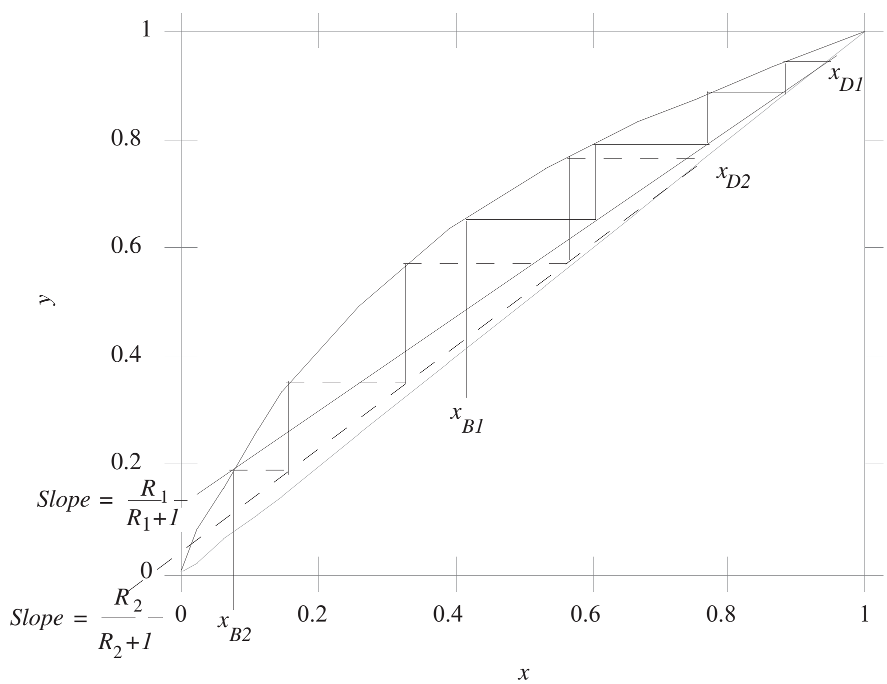


FIGURE 2.9

Graphical Integration for the Rayleigh Equation for the Third Mode of Operation

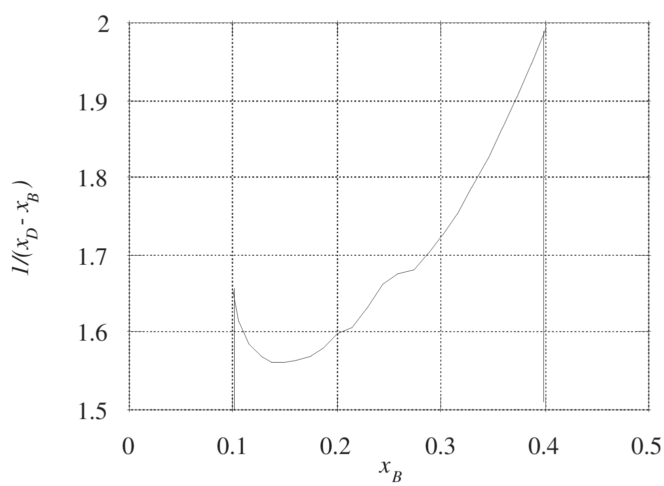


FIGURE 2.10

Graphical Integration for Calculation of Batch Time for the Third Mode of Operation

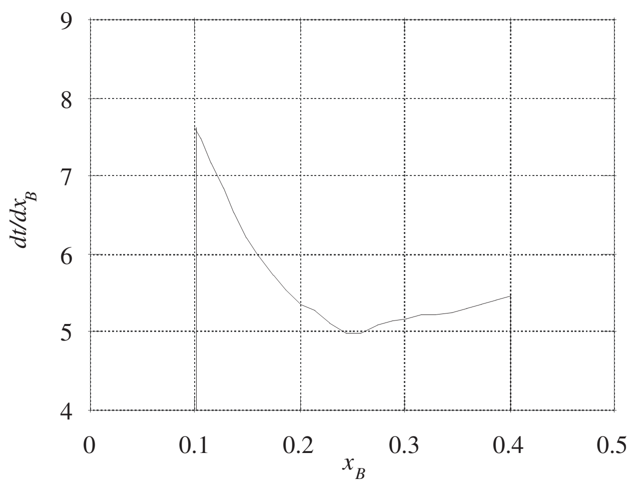


FIGURE 2.11

The Pole Height Concept (Reproduced from Bowman and Cichelli, 1948)