Solutions for Kinetics in Materials Science and Engineering 1st Edition by Readey

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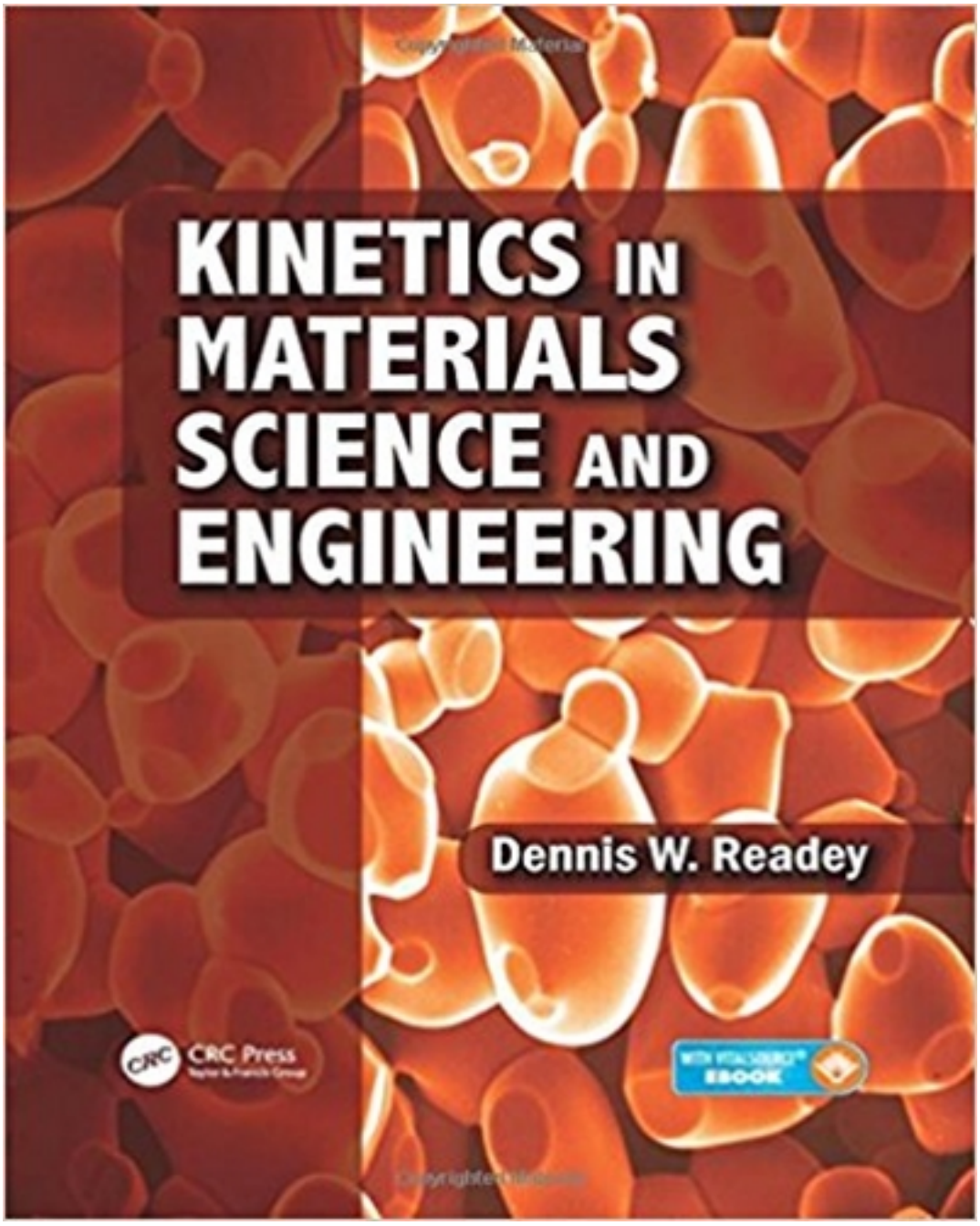

Solutions

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Chapter 2. Introduction to Kinetic Processes in Materials

2.1 The reaction rate constant for a zero order reaction for the decomposition of another liquid dissolved in water is given by $k = k_0 e^{-\frac{Q}{RT}}$ for the reaction

$$A \rightarrow B.$$

a. Write the equation for the reaction rate for the decomposition of *A*.

Solution

$$\frac{d[A]}{dt} = -k$$

b. Give the units of k_0 if the units of the reactant concentration [A] are millimol/cm^3.

Solution

$$\text{units}[k_0] = \text{units}[k] = \text{units}\left[\frac{d[A]}{dt}\right]$$

$$\boxed{\text{units}[k_0] = \frac{\text{millimol}}{\text{cm}^3\text{-s}}}$$

c. If Q = 30 kJ/mole and $k_0 = 10^{-2}$, calculate the reaction rate constant in millimol/liter at 25°F.

Solution

$$25^\circ\text{F} = (25-32)\times\frac{5}{9} = -3.9^\circ\text{C} = 273.15 - 3.9 = \boxed{269.25\text{K}}$$

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and 1 millimol/cm^3 =1000 millimol/ or $\frac{\text{millimol}}{\text{cm}^3}\times\frac{1000\text{cm}^3}{\text{l}}=1000\frac{\text{millimol}}{\text{l}}$

so $k_0=10\frac{\text{millimol}}{\text{l}}$. Therefore, $k=k_0e^{-\frac{Q}{RT}}=10\times e^{-\frac{30{,}000}{8.314\times269.25}}$

or $\boxed{k=1.51\times10^{-5}\text{ millimol/l}}$

d. Calculate how long it takes for the reaction to go to completion if $[A]_0$, the reactant concentration at time zero is one mole per liter.

Solution

For a zero order reaction, $[A]=[A]_0-kt$ and one mole/liter = 1000 millimol/liter

so

$$t=\frac{[A]_0}{k}=\frac{1000\text{ millimol/l}}{1.51\times10^{-5}\text{ millimol/l}\cdot\text{s}}$$

$$\boxed{t=6.62\times10^7\text{ s}}$$

Now the number of seconds in a year is approximately:

$$24\text{ hr/day}\times3600\text{ s/hr}\times365\text{ day/yr}=\boxed{3.15\times10^7\text{ s/yr}}$$

so it would take about two years for the reaction to go to completion.

e. Neglecting any temperature effects on the density of water, which can be assumed to be 1.0 g/cm^3, calculate the number of moles per cm^3 of water in pure water.

Solution

$$\frac{\rho(\text{g/cm}^3)}{M(\text{g/mol})}=\frac{1}{(2\times1.008+15.999)}=\boxed{5.55\times10^{-2}\text{ mol/cm}^3}$$

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f. Assuming that the molar volumes of water and the reactant are the same, calculate the m/o, w/o, and, v/o of *A* in water at 25°F at the start of the reaction if the molecular weight of the reactant is 60 g/mol.

Solution

From part e. above, the molar volume of water is

$$\frac{1}{5.55\times10^{-2}\ \text{mol/cm}^3} = \boxed{18.02\,\text{cm}^3/\text{mol} = \bar{V}_{H_2O}}$$

and the molar volume of *A* is the same, $\bar{V}_A = 18.02\,\text{cm}^3/\text{mol}$. So the total volume stays the same with *A* is solution so

$$1 = n_A\bar{V}_A + n_{H_2O}\bar{V}_{H_2O}$$

and since $n_A = 0.001$, $n_{H_2O} = \frac{1}{18.02} - 0.001 = 0.0545$ moles and the total number of moles is 0.001 + 0.0545 = 0.0555 moles. So,

$$(m/o)_A = \frac{0.001}{0.0555}\times100 = \boxed{1.80}$$
$$(m/o)_{H_2O} = \frac{0.0545}{0.0555}\times100 = \boxed{98.20}$$

The results for the v/o are essentially the same since for 10^{-3} mol/cm^3 of *A* there is 0.018 cm^3 of *A* and 1 – 0.018 = 0.982 cm^3 of H_2O and the total volume is 1.0 cm^3. So,

$$(v/o)_A = \frac{0.018}{1}\times100 = \boxed{1.80}$$
$$(v/o)_{H_2O} = \frac{0.982}{1}\times100 = \boxed{98.20}$$

The results for the w/o are different due to the differences in molecular weights between water and *A*. In one cm^3, there is 0.001×60g/mol = 0.060g. And weight

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of water is the same 0.982 g for a total weight of 1.042 g. Therefore

$$(w/o)_A = \frac{0.060}{1.042} \times 100 = \boxed{5.76}$$

$$(w/o)_{H_2O} = \frac{0.982}{1.042} \times 100 = \boxed{94.24}$$

2.2 a. The reaction rate constant k ($k = k_0 e^{-\frac{Q}{RT}}$) for a first order reaction is $5.00 \times 10^{-3}\ s^{-1}$ at 500 °C and is $10.00\ s^{-1}$ and 1000 °C. Calculate the activation energy for this reaction.

Solution

There are several methods that can be used to solve this problem but, since $k = k_0 e^{-\frac{Q}{RT}}$ a particularly simple method is to take the ratio of the two *k* values to eliminate k_0:

$$\frac{k_1}{k_2} = \frac{k_0}{k_0} \frac{e^{-\frac{Q}{RT_1}}}{e^{-\frac{Q}{RT_2}}} = e^{-\frac{Q}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} .$$

Taking the natural logarithm of both sides of the equation and cancelling the k_0s,

$$\ln\left(\frac{k_1}{k_2}\right) = -\frac{Q}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

and rearranging, gives the result

$$Q = -\frac{R \ln\left(\frac{k_1}{k_2}\right)}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} = -\frac{-8.314 \ln\left(\frac{5 \times 10^{-3}}{10}\right)}{\left(\frac{1}{773} - \frac{1}{1273}\right)}$$

$$\boxed{Q = 124.4 kJ / mole}$$

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b. Calculate the pre-exponential term, k_0, for the reaction rate constant and give its units.

Solution

$$k_0 = k_1 e^{\frac{Q}{RT}} = 5\times10^{-3} e^{\frac{124{,}400}{8.314\times773}}$$

$$\boxed{k_0 = 1.27\times10^{6}\,s^{-1}}$$

or

$$k_0 = k_2 e^{\frac{Q}{RT}} = 10\times e^{\frac{124{,}400}{8.314\times1273}}$$

$$\boxed{k_0 = 1.27\times10^{6}\,s^{-1}}$$

c. Calculate k at 750°C.

Solution

750 °C = 750+273 = 1023 K

$$k = k_0 e^{-\frac{Q}{RT}} = 1.27\times10^{-6} e^{-\frac{124{,}400}{8.314\times1023}}$$

$$\boxed{k = 1.009\,s^{-1}}$$

d. Calculate the relaxation time for this reaction at 750 °C.

Solution

The relaxation time, τ, is defined as 1/k so $\tau = \frac{1}{k} = \frac{1}{1.009} = \boxed{0.991\,s}$

e. Calculate the half-life of this reaction.

Solution

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The "half-life, " $t_{1/2}$, is defined by $\frac{[A]}{[A]_0} = 0.5 = e^{-kt_{1/2}}$ so

$$t_{1/2} = -\frac{\ln 0.5}{k} = \frac{0.6931}{1.009}$$

$$\boxed{t_{1/2} = 0.687 s}$$

f. Make a plot of $\log_{10}$(rate) vs 10^3/T (K^{-1}) from 100 °C to 1200 °C.

Solution

See Figure 2.1.

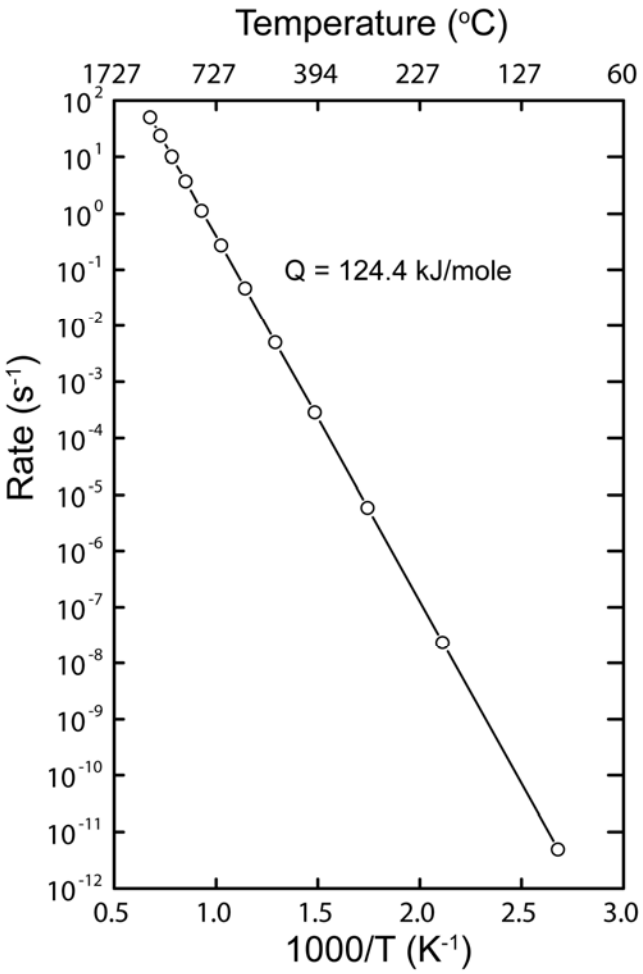

Figure 2.1 Log_{10}(rate) versus 1000/T

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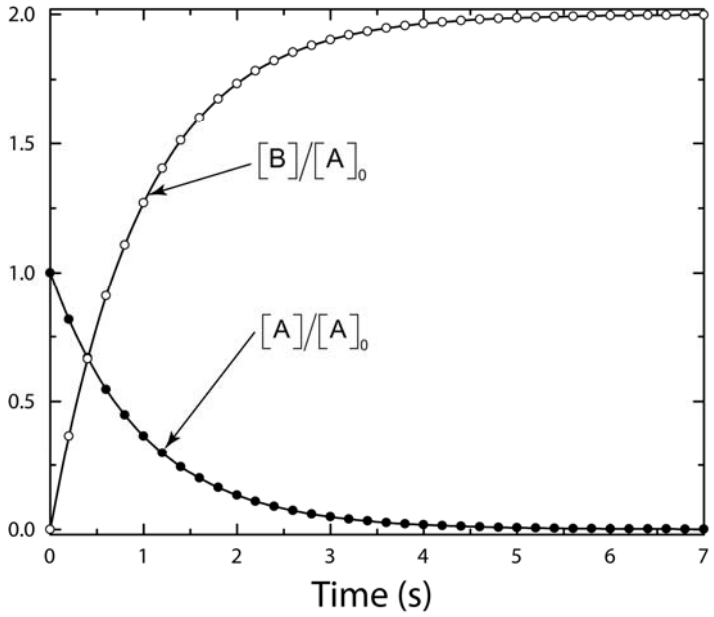

Figure 2.2 The amounts of reactant A and products B for a first order reaction at 750 °C

g. Calculate how long it takes (hours) at 750°C for the unreacted fraction to be 0.001.

Solution

From part e. above,

$$t_{0.001} = -\frac{\ln 0.001}{k} = \frac{6.908}{1.009}$$

$$\boxed{t_{0.001} = 6.846\text{s} = 1.902\times10^{-3}\text{hr}}$$

h. If the reaction is A $\rightarrow$ 2B, plot the concentrations of A and B versus time from time zero up to the time that was calculated in g. above.

Solution

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From the stoichiometry, $[B] = 2([A]_0 - [A]) = 2[A]_0\left(1 - e^{-kt}\right)$. See Figure 2.2 for the plots of [A] and [B].

2.3 A well-known generalization (for back of the envelope calculations) for biochemical first order reactions near room temperature (300 K) is that their rates double for every 10 K rise in temperature.

a. Determine the activation energy (J/mol) for such reactions implied by this rule.

Solution

1. Take $\Delta T = T_1 - T_2 = 310 - 300 = 10$ K. So,

$$\frac{k_1}{k_2} = 2 = \frac{k_0 e^{-\frac{Q}{RT_1}}}{k_0 e^{-\frac{Q}{RT_1}}} = e^{-\frac{Q}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

and

$$Q = -\frac{R \ln 2}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} = -\frac{8.314 \times \ln 2}{\left(\frac{1}{310} - \frac{1}{300}\right)}$$

$$\boxed{Q_1 = 53.6\,\text{kJ/mol}}$$

2. Take $\Delta T = T_1 - T_2 = 300 - 290 = 10\text{K}$. In this case

$$Q = -\frac{8.314 \times \ln 2}{\left(\frac{1}{300} - \frac{1}{290}\right)}$$

$$\boxed{Q_2 = 50.1\,\text{kJ/mol}}$$

3. Finally, take $\Delta T = T_1 - T_2 = 305 - 295 = 10\text{K}$ and

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$$Q = -\frac{8.314 \times \ln 2}{\left(\frac{1}{305} - \frac{1}{295}\right)}$$

$$\boxed{Q_3 = 51.9\,\text{kJ/mol}}$$

Obviously, the value of $\boxed{Q \cong 52\,\text{kJ/mole}}$ is about correct. Why not take $\frac{dk}{dT} = k_0 e^{-\frac{Q}{RT}}\left(\frac{Q}{RT^2}\right) = k\left(\frac{Q}{RT^2}\right) \cong \frac{\Delta k}{\Delta T} = \frac{2k}{10}$? Solving this for Q,

$Q = \frac{2RT^2}{10} = \frac{2 \times 8.314 \times 300^2}{10} \cong 149\,\text{kJ/mol}$ which is about a factor of three too large because Δk is not small. However, if the natural logarithm of *k* were taken,

$$\ln k = \ln k_0 - \frac{Q}{RT}$$

$$\frac{d \ln k}{dT} = \frac{Q}{RT^2} \cong \frac{\Delta \ln k}{\Delta T} = \frac{\ln k_1 - \ln k_2}{\Delta T} = \frac{\ln\left(k_1/k_2\right)}{\Delta T}$$

then,

$$Q = \frac{RT^2 \Delta \ln k}{\Delta T} = \frac{8.314 \times 300^2 \times \ln 2}{10}$$

$$\boxed{Q = 51.9\,\text{kJ/mol}}$$

which is the same as in 1., 2., and 3.

b. Calculate the value of the activation energy in eV/molecule.

Solution

Remember that 1 eV = 1.602×10^{-19} J/atom and $N_A = 6.022 \times 10^{23}$ atoms/mol, so

$$1\text{eV} = 1.602 \times 10^{-19}\left(6.022 \times 10^{23}\right)$$

$$\boxed{1\text{eV} = 96,472\,\text{J/mol}}$$

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and

$$Q \cong \frac{51,900}{96,472}$$

$$\boxed{Q \cong 0.54\text{eV}}$$

roughly one-half of an electron volt.

2.4 a. The half-life for a first order reaction is 1000 seconds. Calculate the reaction rate constant (s^{-1}) for this reaction.

Solution

$$0.5 = e^{-kt_{1/2}} \text{ or } k = -\frac{\ln 0.5}{t_{1/2}} = \frac{0.693}{1000} \text{ and } \boxed{k = 6.93\times10^{-4}\,s^{-1}}$$

b. Calculate the relaxation time (s) for this reaction.

Solution

$$\tau = 1/k = 1/\left(6.93\times10^{-4}\right) = \boxed{1.44\times10^{3}\,s}$$

2.5 a. The half-life for a first order reaction is 1200 seconds. Calculate the reaction rate constant (s^{-1}) for this reaction.

Solution

$$0.5 = e^{-kt_{1/2}} \text{ or } k = -\frac{\ln 0.5}{t_{1/2}} = \frac{0.693}{1200} \text{ and } \boxed{k = 5.77\times10^{-4}\,s^{-1}}$$

b. Calculate the relaxation time (s) for this reaction.

Solution

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$$\tau = 1/k = 1/\left(5.77\times10^{-4}\right) = \boxed{1.73\times10^{3}\,s}$$

c. Plot the fraction unreacted versus time for 0 to 10,000 seconds. Indicate the half-life and the relaxation time on your plot.

Solution

$[A] = [A]_0\, e^{-kt}$ so the fraction unreacted is given by $f_u = \dfrac{[A]}{[A]_0} = e^{-kt} = e^{-5.77\times10^{-4}\,t}$.

The fraction unreacted versus time was calculated with a spreadsheet and is plotted in Figure 2.3.

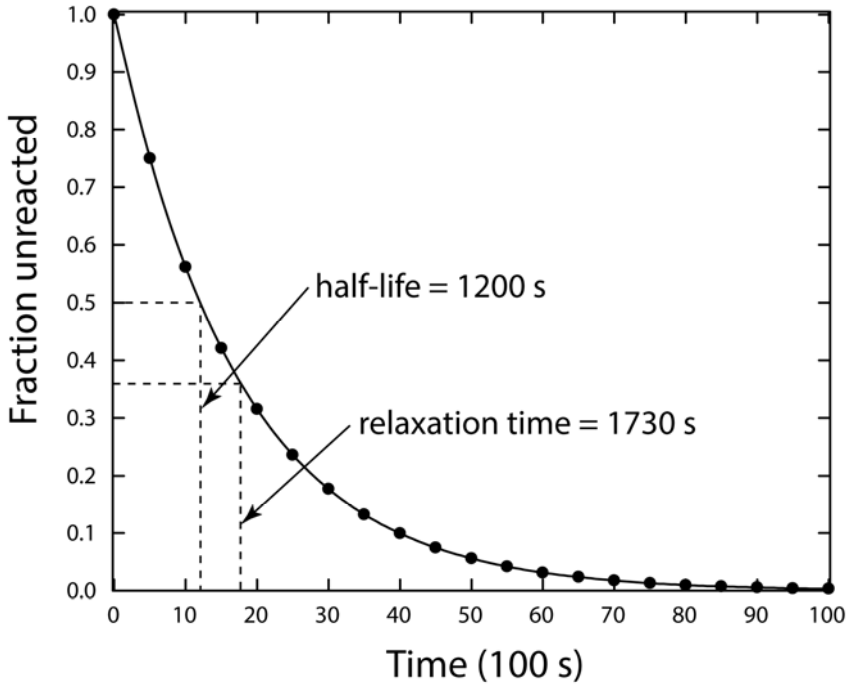

Figure 2.3 Fraction unreacted versus time for a first order reaction showing both the half-life and relaxation time.

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2.6 a. Plutonium-238, $^{238}_{94}\text{Pu}$, spontaneously decays by alpha particle, $^{4}_{2}\text{He}$, emission with a half-life of 86 years. The nuclear disintegration reaction is:

$$^{238}_{94}\text{Pu} \rightarrow {}^{234}_{92}\text{U} + {}^{4}_{2}\text{He} + 5.234\,\text{Mev} \ .$$

This disintegration rate is sufficiently fast to produce enough heat to be used as a power source in satellites. Calculate the first order reaction rate constant for the nuclear disintegration of plutonium-238.

Solution

$k = -\dfrac{\ln 0.5}{t_{1/2}} = -\dfrac{\ln 0.5}{86} = 8.06\times10^{-3}\,\text{yr}^{-1}$ However, the number of seconds per year is $\text{s/yr} = 365\,\text{day/yr} \times 24\,\text{hr/day} \times 3600\,\text{s/hr} = \boxed{3.15\times10^{7}}$. So k is

$$k = \frac{8.06\times10^{-3}\,\text{yr}^{-1}}{3.15\times10^{7}\,\text{s}\cdot\text{yr}^{-1}} = \boxed{2.55\times10^{-10}\,\text{s}^{-1}}$$

b. Calculate the number of becquerels at time = 0 with one mole of pluonium-238.

Solution

1 becquerel = 1 disintegration/s so

$$kN_A = \left(2.55\times10^{-10}\,\text{s}^{-1}\right)\left(6.022\times10^{23}\,\text{atom}\right) = \boxed{1.54\times10^{14}\,\text{Bq}}$$

c. Calculate the number of curies at time zero in one mole of pure $^{238}_{94}\text{Pu}$.

Solution

$1\text{Ci} = 3.7\times10^{10}\,\text{Bq}$ so the number of curies is $\dfrac{1.54\times10^{14}\,\text{Bq}}{3.7\times10^{10}\,\text{Bq/Ci}} = \boxed{4.16\times10^{3}\,\text{Ci}}$

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d. Calculate the rate of energy generated in the plutonium-238 (watts) for one mole of plutonium-238 at time zero.

Solution

The energy per disintegration is 5.592 MeV. The rate of energy generation is therefore $\left(5.592\times10^{6}\,\text{eV}\right)\left(1.602\times10^{-19}\,\text{J/eV}\right)\left(1.54\times10^{14}\,\text{s}^{-1}\right)=\boxed{138\text{W}}$

e. Calculate how long (years) it will take for the rate of energy generation to drop to 10 percent of its initial value.

Solution

The fraction unreacted is given by $f_u = e^{-kt}$ so

$$t=-\frac{\ln 0.9}{\left(2.55\times10^{-10}\,\text{s}^{-1}\right)\left(3.15\times10^{7}\,\text{s}\cdot\text{yr}^{-1}\right)}=\boxed{13.1\text{yr}}$$

f. If the $^{238}_{94}\text{Pu}$ were in the form of PuO_2 rather than pure plutonium, calculate the number of curies in 5 cm^3 of PuO_2 at time zero if the density of PuO_2 is 11.5 g/cm^3.

Solution

The molecular weight of PuO_2 is $M = 238+2\times16 = 270\,\text{g/mol}$. The weight of 5 cm^3 of PuO_2 is $5\,\text{cm}^3\times11.5\,\text{g/cm}^3 = 57.5\text{g}$ which is $\frac{57.5\text{g}}{270\,\text{g/mol}}=0.213\,\text{mol}$. Now there is one mole of Pu per mole of PuO_2 so that, from part b., the number of disintegrations per mole per second is 1.54×10^{14}. So the number of curies in the 5 cm^3 of PuO_2 is $\left(1.54\times10^{14}\,\text{Bq/mol}\right)\left(0.213\text{mol}\right)\left(1/3.7\times10^{10}\,\text{Bq/Ci}\right)=\boxed{886.5\,\text{Ci}}$

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g. Calculate the number of watts that this 5 cm^3 of $^{238}_{94}Pu$ -containing PuO_2 generates.

Solution

From part d., the number of watts per mole is 138 W so the number of watts from 5 cm^3 of PuO_2 is $138\,W/mol \times 0.213 mol = \boxed{29.4 W}$

h. If this 5 cm^3 of PuO_2 were perfectly insulated, calculate how long it takes (hours) for the 5cm^3 of PuO_2 to reach its melting point of 2390°C if its molar heat capacity is 75 J/mole-K.

Solution

$P(W)t(s) = n(mol)C_p(J/mol \cdot K)\Delta T(K)$ Assume that the starting temperature is 25°C = 298 K so that $t = \dfrac{(0.213 mol)(75 J/mol \cdot K)(2390 - 278) K}{(29.4 W)(3600 s/hr)} = \boxed{0.316 hr}$

not much time at all! Of course, heat will be lost by convection, conduction, and radiation and some steady-state temperature will be reached well below the melting point of PuO_2.

2.7 A reaction obeys the stoichiometric equation:

$$A + B = Z$$

The rates of formation of Z at various concentrations of A and B are given in the following table.

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[A] (mol/l)	[B] (mol/l)	rate (mol/l-s)
2.00×10^{-2}	4.20×10^{-2}	3.53×10^{-9}
6.50×10^{-2}	4.80×10^{-2}	4.87×10^{-8}
1.10×10^{-1}	1.22×10^{-1}	9.00×10^{-7}

Determine the values of α, β, and *k* in the rate equation: rate = $k\,[A]^{\alpha}[B]^{\beta}$. Give the units for *k* as well. You may use programs such as Excel, Mathematica, etc., or by hand to get the solution. Just clearly indicate what operations were performed: i.e. copy that part of the Excel spreadsheet or the Mathematica page that shows the calculations.

Solution

There are three nonlinear equations and three unknowns, α, β, and R (R = rate) in the form of $R = k[A]^{\alpha}[B]^{\beta}$. Taking the logarithm of both sides of each equation gives three linear equations $\ln R_i = \ln k + \alpha \ln[A]_i + \beta \ln[B]_i$ with i = 1,2, and 3 and these are

$$\ln\left(3.53\times10^{-9}\right) = \ln k + \alpha \ln\left(2.00\times10^{-2}\right) + \beta \ln\left(4.20\times10^{-2}\right)$$
$$\ln\left(4.87\times10^{-8}\right) = \ln k + \alpha \ln\left(6.50\times10^{-2}\right) + \beta \ln\left(4.80\times10^{-2}\right)$$
$$\ln\left(9.00\times10^{-7}\right) = \ln k + \alpha \ln\left(1.10\times10^{-1}\right) + \beta \ln\left(1.22\times10^{-1}\right)$$

and become the following by taking the logarithms and letting $\gamma = -\ln k$

$$\begin{aligned} 19.462 &= \gamma + 3.912\alpha + 3.170\beta \\ 16.838 &= \gamma + 2.733\alpha + 3.037\beta \\ 13.921 &= \gamma + 2.207\alpha + 2.104\beta \end{aligned} \qquad (2.1)$$

and these three equations can be solved in a number of ways.

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The Hard Way to Solve

Subtracting the second equation from the first gives

$$2.624 = 1.179\alpha + 0.133\beta \qquad (2.2)$$

and the third from the second

$$2.917 = 0.526\alpha + 0.933\beta$$

and multiplying the last equation by 2.624 and dividing by 2.917 gives

$$2.624 = 0.473\alpha + 0.839\beta \qquad (2.3)$$

and subtracting Eq.(2.3) from Eq. (2.2) gives

$$0 = 0.706\alpha - .706\beta$$

or $\boxed{\alpha = \beta}$. Substitution in Eq. (2.2) gives $\boxed{\alpha = \beta = 2.0}$ and substitution in Eq. (2.3) also gives $\boxed{\alpha = \beta = 2.0}$. The value of *k* can be obtained from any one of the three equations in Eq.(2.1)

$$19.462 = \gamma + (3.912 + 3.170)2$$
$$16.838 = \gamma + (2.733 + 3.037)2$$
$$13.921 = \gamma + (2.207 + 2.104)2$$

which give

$$\gamma = 5.298$$
$$\gamma = 5.298$$
$$\gamma = 5.299$$

so $k = e^{-5.298} = \boxed{5.00 \times 10^{-3}\ \text{mol}^3/\text{l}^3 \cdot \text{s}}$.

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Easier Way: Matrix Inversion in Excel®

Solution by Matrix Inversion in Excel

The natural logarithms of the equations must be taken to lineraize:
ie. α ln[A] +β ln[B] + zlnk = ln(rate)

-3.91202	-3.17009	1	-19.4625	2.00	alpha
-2.73337	-3.03655	1	-16.8382	2.00	beta
-2.20727	-2.10373	1	-13.9203	-5.29832	lnk

Determinant 1.029223 $\neq$ 0 so OK

alpha = 2
beta = 2
k = 5.00E-03

Easiest Way: solve with Mathematica®

```
In[1]:= Solve[{-3.912023 α - 3.170086 β + z == -19.46253, -2.733368 α - 3.036554 β + z == -16.83816,
          -2.207275 α - 2.103734 β + z == -13.92034}, {α, β, z}]

Out[1]= {{α → 2., β → 2., z → -5.29834}}

In[2]:= e^(-5.29834)

Out[2]= 0.00499989 = k
```

2.8 In 2012, what were thought to be the bones of King Richard III of England were found under a parking lot in England and later confirmed by DNA evidence in 2014 that the bones were indeed his. The bones were dated by radiocarbon dating and confirmed that the date of death of the skeleton was 1485, the date of death of the king. Calculate the ratio of carbon-14 remaining in the bones.

Solution

For radiocarbon dating,

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$$\frac{\left[{}^{14}_{6}\mathrm{C}\right]}{\left[{}^{14}_{6}\mathrm{C}\right]_0} = e^{-1.21\times10^{-4}\,t}$$

where Units(t) = yr. So $(2016-1485) = 531\text{yr}$. Therefore,

$$\frac{\left[{}^{14}_{6}\mathrm{C}\right]}{\left[{}^{14}_{6}\mathrm{C}\right]_0} = e^{-1.21\times10^{-4}\,x\,531} = \boxed{0.938}$$

2.9 If the number of disintegrations per minute per gram of carbon produced by pyrolyzing a piece of recently cut wood is 12.5 dis/min-g, calculate the fraction of ^{14}C in carbon of age zero if the half-life of ^{14}C is 5730 years. (The wood is assumed to have an age of zero.)

Solution

The number of disintegrations per second is given by,

$$\frac{dm^*}{dt} = km^*$$

where m^* is the amount of radioactive carbon. Now

$$k = \frac{1.21\times10^{-4}\,\text{yr}^{-1}}{(365\,\text{day/yr})(24\,\text{hr/day})(60\,\text{min/hr})} = 2.302\times10^{-10}\,\text{min}^{-1}$$ So

$$m^* = \left(\frac{dm^*}{dt}\right)\left(\frac{1}{k}\right) = \frac{12.5}{2.302\times10^{-10}} = 5.30\times10^{10}$$

^{14}C atoms per gram. Now, M(C) = 12.01 g/mol so 1 gram of carbon contains $1/12.01 = 8.33\times10^{-2}\,\text{mol}$ of carbon or

$$N_A\times8.33\times10^{-2} = \left(6.022\times10^{23}\right)\left(8.33\times10^{-2}\right) = 5.014\times10^{22}\,\text{atom/g}$$

so the fraction of ^{14}C is $\frac{5.30\times10^{10}}{5.014\times10^{22}} = \boxed{1.06\times10^{-12}}$ or about one part per trillion!