Solutions for First Course in Differential Equations Modeling and Simulation 2nd Edition by Smith

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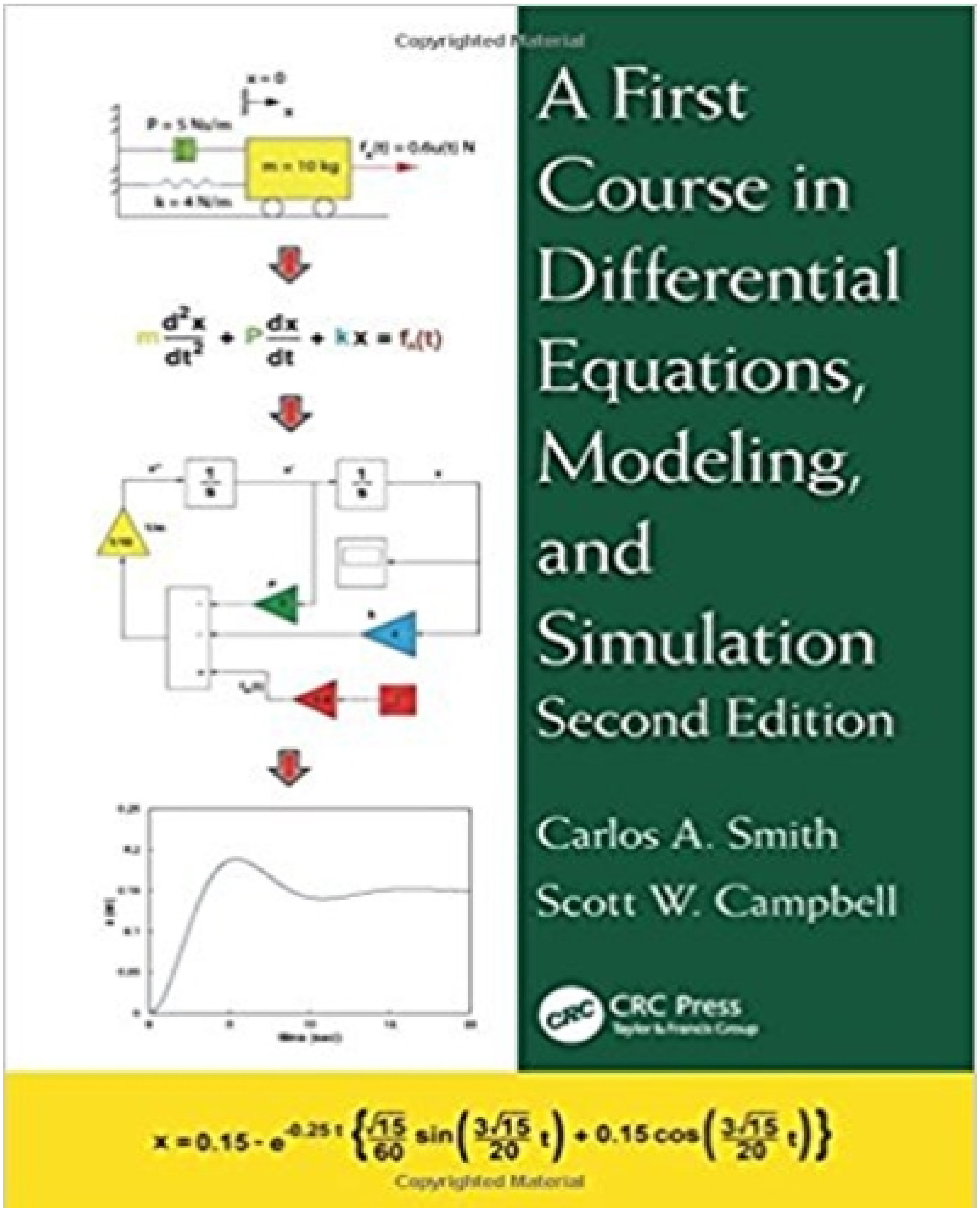


Solutions

CHAPTER 2

Problem 2.1

a) $e^{-2t}\dfrac{dy}{dt} = y^{-1}(1 - e^{-2t});\quad y(0) = 0$

$$ydy = e^{2t}(1 - e^{-2t})dt \;\; =>> \frac{1}{2}y^2|_0^y = \int_0^t (e^{2t} - 1)dt$$

$$\frac{1}{2}y^2 = \left(\frac{1}{2}e^{2t} - t\right)|_0^t$$

$$y = \sqrt{e^{2t} - 2t - 1}$$

b) $\dfrac{dy}{dt} = ycos(t) + y;\qquad y(0) = 2$

$$\int_2^y \frac{dy}{y} = \int_0^t (\cos(t) + 1)dt \quad =>> \quad \ln(y)|_2^y = (\sin(t) + t)|_0^t$$

$$\ln\left(\frac{y}{2}\right) = \sin(t) + t$$

$$y = 2\exp[\sin(t) + t]$$

c) $\frac{dy}{dt} = \frac{t+2}{y};\quad y(0) = 2$

$$\int_2^y ydy = \int_0^t (t + 2)dt \quad =>> \quad \frac{1}{2}y^2|_2^y = \frac{1}{2}t^2 + 2t|_0^t$$

$$y^2 - 4 = t^2 + 4t$$

$$y = \sqrt{t^2 + 4t + 4}$$

d) $\dfrac{d}{dt}\left[t\dfrac{dy}{dt}\right] = 2t;\quad y(1) = 1\;;\; y'(1) = 3$

Let $u = t\frac{dy}{dt}$ with $u(1) = 3$

$$\frac{du}{dt} = 2t \quad =>> \int_3^u du = \int_1^t 2t\, dt \quad =>> \quad u = t^2 + 2$$

$$u = t^2 + 2 = t\frac{dy}{dt} \quad =>> \quad \int_1^y dy = \int_1^t (t + 2t^{-1})\, dt$$

$$y - 1 = \frac{t^2}{2} + 2\ln(t) - \frac{1}{2}$$

$$y = \frac{t^2}{2} + 2\ln(t) + \frac{1}{2}$$

e) $\frac{d^2y}{dt^2} = 32e^{-4t} \quad y(0) = 1 \; ; \; y'(0) = 0$

$$\frac{d}{dt}\left[\frac{dy}{dt}\right] = 32e^{-4t}$$

Let $u = \frac{dy}{dt}$

Then, $\frac{du}{dt} = 32e^{-4t}; \quad u(0) = 0$

$$\int_0^u du = \int_0^t 32e^{-4t}dt \quad =>> \quad u = -\frac{32}{4}e^{-4t}|_0^t = -\frac{32}{4}e^{-4t} + \frac{32}{4}$$

$$u = \frac{dy}{dt} = 8 - 8e^{-4t}$$

$$\int_1^y dy = \int_0^t (8 - 8e^{-4t})dt \quad =>> \quad y - 1 = 8t + 2e^{-4t}|_0^t = 8t + 2e^{-4t} - 2$$

$$y = 8t + 2e^{-4t} - 1$$

f) $\frac{1}{t^2}\frac{dy}{dt} = y; \quad y(0) = 1$

$$\int_1^y \frac{dy}{y} = \int_0^t t^2 dt \quad =>> \quad \ln(y)\,|_1^y = \frac{1}{3}t^3|_0^t \quad =>> \quad \ln(y) = \frac{1}{3}t^3$$

$$y = e^{\frac{t^3}{3}}$$

g) $\frac{dy}{dt} = -y^2e^{2t}; \quad y(0) = 1$

$$\int_1^4 \frac{dy}{y^2} = -\int_0^t e^{2t} dt \quad =>> \quad -\frac{1}{y}\Big|_1^y = -\frac{1}{2}e^{2t}\Big|_0^t \quad =>> \quad -\frac{1}{y} + 1 = -\frac{1}{2}e^{2t} + \frac{1}{2}$$

$$\frac{1}{y} = \frac{1}{2}(e^{2t} + 1) \quad =>> \quad y = \frac{2}{e^{2t}+1}$$

h) $\dfrac{dy}{dt} - (2t+1)y = 0; \qquad y(0) = 2$

$$\frac{dy}{dt} = y(2t+1) \quad =>> \quad \int_2^y \frac{dy}{y} = \int_0^t (2t+1)dt \quad =>> \quad \ln(y)\,\Big|_2^y = (t^2+t)\Big|_0^t$$

$$\ln\left(\frac{y}{2}\right) = t^2 + t \quad =>> \quad y = 2e^{(t^2+t)}$$

i) $\dfrac{dy}{dt} + 4ty^2 = 0; \quad y(0) = 1$

$$\int_1^y \frac{dy}{y^2} = -\int_0^t 4t dt \quad =>> \quad -\frac{1}{y}\Big|_1^y = -2t^2\Big|_0^t$$

$$y = \frac{1}{1+2t^2}$$

j) $\dfrac{d^2y}{dt^2} = \cos\left(\frac{t}{2}\right); \qquad y(0) = 0; \;\; y'(0) = 1$

Let $u = \frac{dy}{dt}$, then

$$\frac{du}{dt} = \cos\left(\frac{t}{2}\right); \quad u(0) = 1$$

$$\int_1^u du = \int_0^t \cos\left(\frac{t}{2}\right) dt$$

$$u = 1 + 2\sin\left(\frac{t}{2}\right)$$

and

$$\frac{dy}{dt} = 1 + 2\sin\left(\frac{t}{2}\right); \quad y(0) = 0$$

$$\int_0^y dy = \int_0^t \left(1 + 2\sin\left(\frac{t}{2}\right)\right) dt$$

$$y = \left(t - 4\cos\left(\frac{t}{2}\right)\right)|_0^t = t - 4\cos\left(\frac{t}{2}\right) + 4$$

$$y = 4 + t - 4\cos\left(\frac{t}{2}\right)$$

Problem 2.2

a) $\frac{du}{dy} = y^2 - 1 \ll= \ linear$

$$du = (y^2 - 1)dy$$
$$u = \frac{1}{3}y^3 - y + C$$

b) $\frac{dT}{dt} = -0.0002(T - 5)$ <<= linear

$$\frac{dT}{T-5} = -0.0002dt \quad =>> \quad \ln(T-5) = -0.0002\text{t} + \text{C}_1$$

$$T = 5 + C\,e^{-0.0002t} \text{ where } C = e^{C_1}$$

c) $\frac{dy(x)}{dx} = e^{2y}$ <<= nonlinear

$$e^{-2y}dy = dx \quad =>> \quad -\frac{1}{2}e^{-2y} = x + C$$

$$y = -\frac{1}{2}\ln(C - 2x)$$

d) $\frac{du}{dt} = u^2\cos(\pi t) \quad u(0) = -\frac{1}{2}$ <<= nonlinear

$$\int_{-\frac{1}{2}}^{u}\frac{du}{u^2} = \int_0^t \cos(\pi t)\,dt \quad =>> \quad \frac{1}{u}|_{-\frac{1}{2}}^{u} = \frac{1}{\pi}\sin(\pi t)\,|_0^t$$

$$u = \frac{1}{-\frac{1}{\pi}\sin(\pi t) - 2}$$

e) $\frac{dy}{dt} = \left(\frac{y^2 + 4y - 5}{y}\right)t$ <<== nonlinear

$$\int \frac{y}{y^2+4y-5}\,dy = \int t\,dt$$

Use Partial Fraction to simplify the first integrand.

$$\frac{y}{y^2+4y-5} = \frac{A_1}{y-1} + \frac{A_2}{y+5}$$

and from here $A_1 = \frac{1}{6}$ and $A_2 = \frac{5}{6}$. Therefore,

$$\int \frac{1/6}{y-1}\,dy + \int \frac{5/6}{y+5}\,dy = \int t\,dt$$

$$\frac{1}{6}\ln(y-1) + \frac{5}{6}\ln(y+5) = \frac{1}{2}t^2 + C$$

f) $\frac{du}{dt} = u^3 + 6u^2 + 11u + 6$ <<== nonlinear

$$\int \frac{1}{u^3+6u^2+11u+6}\,du = \int dt$$

Use Partial Fractions to simplify the first integrand.

$$\frac{1}{u^3+6u^2+11u+6} = \frac{A_1}{u+1} + \frac{A_2}{u+2} + \frac{A_3}{u+3}$$

and from here $A_1 = \frac{1}{2}$; $A_2 = -1$; and $A_3 = \frac{1}{2}$. Therefore,

$$\int \frac{1/2}{u+1}\,du - \int \frac{1}{u+2}\,du + \int \frac{1/2}{u+3}\,du = \int dt$$

$$\frac{1}{2}\ln(u+1) - \ln(u+2) + \frac{1}{2}\ln(u+3) = t + C$$

Problem 2.3

$$-\frac{10C^S}{12+C^S} = \frac{dC^S}{dt}$$

Using Separation of Variables,

$$\int_{13.33}^{C^S} \frac{12+C^S}{10C^S} dC^S = 1.2\int_{13.33}^{C^S} \frac{1}{C^S} dC^S + 0.1\int_{13.33}^{C^S} dC^S = -\int_0^t dt$$

and after evaluating the integrals

$$t = 4.44 - 1.2\ln C^S - 0.1C^S$$

Problem 2.4

$$\tau \frac{dy(t)}{dt} + y(t) = Kx(t)$$

$$\int_{y(0)}^{y(t)} \frac{dy(t)}{Kx(t)-y(t)} = \frac{1}{\tau}\int_0^t dt \quad =>> \quad \ln[Kx(t) - y(t)]\,|_{y(0)}^{y(t)} = -\frac{t}{\tau}$$

$$\ln[Kx(t) - y(t)] - \ln[Kx(t) - y(0)] = -\frac{t}{\tau}$$

$$\frac{Kx(t) - y(t)}{Kx(t) - y(0)} = e^{-\frac{t}{\tau}}$$

$$y(t) = Kx(t) - [Kx(t) - y(0)]e^{-\frac{t}{\tau}}$$

and using $x(t) = x(0) + D$,

$$y(t) = K(x(0) + D) - [K(x(0) + D) - y(0)]e^{-\frac{t}{\tau}}$$

and with $y(0) = K\,x(0)$,

$$y(t) = y(0) + KD - [y(0) + KD - y(0)]e^{-\frac{t}{\tau}}$$

$$y(t) = y(0) + KD\left(1 - e^{-\frac{t}{\tau}}\right)$$

Problem 2.5

$$2\frac{dv}{dt} + 4v = 16u(t) \quad \text{with} \quad v(0) = 0$$

$$\int_0^v \frac{dv}{16-4v} = \int_0^t \frac{1}{2} dt \quad =>> \quad -\frac{1}{4}\ln(16 - 4v)\,|_0^v = \frac{1}{2}t$$

$$\ln(16-4v)-\ln(16)=-2t \quad \text{=>>} \quad \ln\left(\frac{16-4v}{16}\right)=-2t$$

$$v=4(1-e^{-2t})$$

Problem 2.6

$$i_1=1.43\times10^{-4}v_s+0.286i_2 \qquad (1)$$

$$2\times10^{-3}\frac{di_2}{dt}+21428i_2=0.286v_s \qquad (2)$$

From (2): $$9.3\times10^{-8}\frac{di_2}{dt}+i_2=1.33\times10^{-5}v_s$$

where $$\tau=9.3\times10^{-8}sec, \quad and \quad K=1.33\times10^{-5}\frac{V}{A}$$

From Problem 2-5, $$i_2=i_2(0)+KD\left(1-e^{-\frac{t}{\tau}}\right)$$

$$i_2=2.66\times10^{-4}+0.000267\left(1-e^{-\frac{t}{9.3\times10^{-8}}}\right)$$

From (1):

$$i_1=57.2\times10^{-4}+0.76\times10^{-4}+7.64\times10^{-5}\left(1-e^{-\frac{t}{9.3\times10^{-8}}}\right)$$

$$i_1=57.96\times10^{-4}+0.76\times10^{-4}\left(1-e^{-\frac{t}{9.3\times10^{-8}}}\right)$$

Problem 2.7

$$0.3768X\left(1-\frac{X}{18.91}\right)=\frac{dX}{dt}$$

Using Separation of Variables,

$$\frac{dX}{X\left(1-\frac{X}{18.91}\right)}=0.3768\,dt$$

$$\frac{dX}{X\left(18.91-X\right)}=0.02\,dt$$

Using Partial Fractions Expansion,

$$\frac{1}{X\left(18.91-X\right)}=\frac{A_1}{X}+\frac{A_2}{18.91-X}$$

$$\frac{1}{X\left(18.91-X\right)}=\frac{(A_2-A_1)X+18.91A_1}{X\left(18.91-X\right)}$$

Equating equal terms yields,

$$A_1=A_2=0.053$$

Then,

$$0.053\left[\int_{9.52}^{X}\frac{dX}{X}+\int_{9.52}^{X}\frac{dX}{18.91-X}\right]=0.02\int_0^t dt$$

$$\ln\left[\frac{0.986X}{18.91-X}\right]=0.377t$$

$$X=\frac{18.91e^{0.377t}}{0.986+e^{0.377t}}$$

Problem 2.8

$$6.915C_V\sqrt{1000-P(t)}=1.08\frac{dP(t)}{dt}$$

Using Separation of Variables

$$\frac{dP(t)}{\sqrt{1000-P(t)}}=\frac{6.915C_V}{1.08}dt$$

And using the information given in the statement

$$\int_{100}^{500}\frac{dP(t)}{\sqrt{1000-P(t)}}=\frac{6.915C_V}{1.08}\int_0^4 dt$$

$$-2\sqrt{1000-P(t)}\Big|_{100}^{500}=\frac{6.915C_V}{1.08}t\Big|_0^4$$

And solving for C_V, $C_V=0.596$

Problem 2.9

$$\frac{dC^A}{dt} = -85.28\left(C^A\right)^2\left(0.132 + 0.5C^A\right)$$

Using Separation of Variables,

$$\frac{dC^A}{\left(C^A\right)^2\left(0.132 + 0.5C^A\right)} = -85.28\,dt$$

Using partial fractions expansion we factor the term on the left side

$$, \frac{1}{\left(C^A\right)^2\left(0.132 + 0.5C^A\right)} = \frac{A_1}{\left(C^A\right)^2} + \frac{A_2}{C^A} + \frac{A_3}{0.132 + 0.5C^A}$$

From here

$$\frac{1}{\left(C^A\right)^2\left(0.132 + 0.5C^A\right)} = \frac{(A_3 + 0.5A_2)\left(C^A\right)^2 + (0.5A_1 + 0.132A_2)C^A + 0.132A_1}{\left(C^A\right)^2\left(0.132 + 0.5C^A\right)}$$

and equating equal numerator coefficients $A_1 = 7.58$; $A_2 = -28.71$; $A_3 = 14.36$

Then,

$$7.58\int_{1.842}^{C^A} \frac{dC^A}{\left(C^A\right)^2} - 28.71\int_{1.842}^{C^A} \frac{dC^A}{C^A} + 14.36\int_{1.842}^{C^A} \frac{dC^A}{0.132 + 0.5C^A} = -85.28\int_0^t dt$$

And after evaluating the integrals,

$$t = 0.18\,h$$

Problem 2.10

$12.5\,\frac{dx_3^{NaOH}}{dt} + x_3^{NaOH} = 0.73\,x_1^{NaOH}$ with $x_3^{NaOH}(0) = 0.55$ and $x_1^{NaOH} = 0.75 - 0.08\,u(t)$

$$12.5\,\frac{dx_3^{NaOH}}{dt} = 0.73\,x_1^{NaOH} - x_3^{NaOH} = 0.73(0.67) - x_3^{NaOH}$$

or

$$12.5\,\frac{dx_3^{NaOH}}{dt} = [0.4891 - x_3^{NaOH}](1)$$

Using Separation of Variables

$$\int_{0.55}^{x_3^{NaOH}} \frac{dx_3^{NaOH}}{0.4891 - x_3^{NaOH}} = \frac{1}{12.5}\int_0^t dt \quad =>> \quad \ln(0.4891 - x_3^{NaOH})\Big|_{0.55}^{x_3^{NaOH}} = -\frac{t}{12.5}$$

$$\frac{0.4891 - x_3^{NaOH}}{-0.0609} = e^{-\frac{t}{12.5}} \quad =>> \quad x_3^{NaOH} = 0.4891 + 0.0609e^{-\frac{t}{12.5}}$$

and

$$x_3^{NaOH} = 0.4891 + 0.0609e^{-\frac{t}{12.5}} = 0.55 - 0.0609(1 - e^{-\frac{t}{12.5}})$$

Problem 2.11

a)

$$m\frac{dv}{dt} = \sum F_y = F_g + F_d + F_T = -mg - kv + 2u(t)$$

$$0.05\frac{dv}{dt} = -0.05(9.8) - 0.01v + 2u(t)$$

$$0.05\frac{dv}{dt} + 0.01v = -0.49 + 2u(t) \quad \text{with} \quad v_y(0) = 0\,m/s$$

Using Separation of Variables

$$\int_0^{v_y} \frac{1}{1.51 - 0.01v_y}\,dv_y = \int_0^t 20\,dt$$

and

$$v_y(t) = 151(1 - e^{-0.2t}) \quad \rightarrow \quad v_y(2) = 151(1 - e^{-0.4}) = 49.8\,{}^{m}\!/_{s}$$

b)

$$m\frac{dv}{dt} = \sum F_y = F_g + F_d = -mg - kv$$

$$0.05\frac{dv}{dt} + 0.01v = -0.49$$

Using Separation of Variables

$$\int_{49.8}^{0} \frac{1}{-0.49-0.01v_y}\,dv_y = \int_2^t 20\,dt$$

$$-\frac{1}{0.01}\ln\left(-0.49-0.01v_y\right)\Big|_{49.8}^{0} = 20(t-2)$$

and using algebra

$$t = 2-5\ln\left(\frac{-0.49}{-0.49-0.01(49.8)}\right) = 5.5\,s$$

Thus, after the thrusts stop, the rocket continues upward for 3.5 seconds more.

Problem 2.12

a)

$$m\frac{dv}{dt} = -kv \quad \text{with} \quad v(0) = 50\,m/s$$

$$\frac{dx}{dt} = v \quad \text{with} \quad x(0) = 0\,m$$

b)

$$\frac{dv}{dt} = -\frac{100}{500}v = -0.2v \quad \Rightarrow \quad \int_{50}^{v}\frac{dv}{v} = -0.2\int_0^t dt \quad \Rightarrow \quad \ln v\Big|_{50}^{v} = -0.2t$$

$$v = 50e^{-0.2t} \quad \text{and for } v = 1m/s\,,\ t = 19.5\,s$$

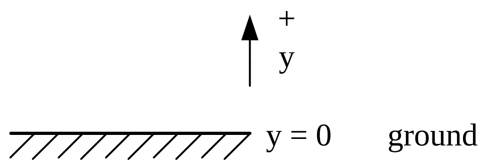
c)

$$\int_0^x dx = \int_0^{19.5} v\,dt = 50\int_0^{19.5} e^{-0.2t}\,dt$$

$$x = -\frac{50}{0.2}e^{-0.2t}\Big|_0^{19.5} = 245\,m$$

Problem 2.13

Going Up



$$\sum F_y = ma_y = m\,\frac{dv_y}{dt} \qquad (1)\ 1\ eq\ 2\ unk\left[\sum F_y\,, v_y\right]$$

Assuming no air resistance: $\sum F_y = F_g = -mg$ $\qquad (2)\ 2\ eq\ 2\ unk$

Substituting (2) into (1) and rearranging,

$$\frac{dv_y}{dt} = -g \quad (3) \quad \text{with} \;\; v_y(0) = 35\; m/s$$

Maximum height is when $v_y = 0\,\frac{m}{s}$

From (3)

$$\int_{35}^{0} dv_y = -g\int_0^t dt \quad \text{=>>} \quad v_y|_{35}^{0} = -gt|_0^t \quad \text{=>>} \quad 0 - 35 = -gt$$

$$t = 3.57\; sec$$

We can also obtain an expression for v_y

$$\int_{35}^{v_y} dv_y = -g\int_0^t dt$$

$$v_y = 35 - gt = 35 - 9.8t$$

For the maximum height,

$$\frac{dy}{dt} = v_y \quad \text{=>>} \quad \int_0^y dy = \int_0^t v_y dt = \int_0^t (35 - 9.8t)dt$$

$$y = 35t - 4.9t^2$$

Then, at $t = 3.57\; s$ $\qquad y_{max} = 35(3.57) - 4.9(3.57)^2$

$$y_{max} = 62.5\ m$$

<u>Going Down</u>

$$\sum F_y = ma_y = m\frac{dv_y}{dt} \quad \text{(4) 1 eq., 2 unk} \left[\sum F_y \quad , v_y\right]$$

Assuming no air resistance,

$$\sum F_y = F_g = -mg \qquad \text{(5) 2 eq., 2 unk.}$$

$$\frac{dy}{dt} = v_y \qquad \text{(6) } 3\ eq\ 3\ unk\ \text{[y]}$$

Substituting (5) into (4) and rearranging,

$$\frac{d\,v_y}{dt} = -g \quad =>> \quad \int_0^{v_y} dv_y = -g\int_0^t dt \quad =>> \quad v_y = -gt \quad (7)$$

Using (7) in (6):

$$\frac{dy}{dt} = -gt \quad =>> \quad \int_{62.5}^{y} dy = -g\int_0^t tdt \quad =>> \quad y = 62.5 - 4.9t^2 \quad (8)$$

For t, $y = 0$

$$t = 3.57\ sec$$

Problem 2.14

$$\sum F_y = ma_y = m\frac{dv_y}{dt} \quad (1)\ 1\ eq., 2\ unk\ [\sum F_y, v_y]$$

Considering air resistance (drag force due to air)

$$\sum F_y = F_g + F_d = -mg - 1.0|v_y|v_y \qquad \text{(2) 2 eq., 2 unk.}$$

Eqs (1) and (2) constitute the model for the velocity. For the position,

$$\frac{dy}{dt} = v_y \qquad (3)\ 3\ eq\ 3\ unk\ [y]$$

Units of 1 are $\frac{N}{m^2/s^2}$.

Problem 2.15

$$\sum F_y = ma_y = m\frac{dv_y}{dt} \quad (1)\ 1\ eq., 3\ unk\ \left[\sum F_y, m, v_y\right]$$

Considering air resistance

$$\sum F_y = F_g + F_d = -mg - 1.0v_y \quad (2) 2\ eq., 3\ unk$$

$$\frac{dy}{dt} = v_y \quad (3)\ 3\ eq., 4\ unk\ [y]$$

We need one more equation; at this time there is still one degree of freedom. We don't really have any more equations but, there is a specification $y(0) = 30\ m$ and after 3 sec, $y(3) = 0\ m$, that we may use. So:

$$y(3) = 0\ m \quad (4)\ \ 4\ eq., 4\ unk$$

Substituting (2) into (1):

$$m\frac{dv_y}{dt} = -mg - 1.0v_y = -1(mg + v_y)$$

and using Separation of Variables,

$$\int_0^{v_y} \frac{dv_y}{mg+v_y} = -\frac{1}{m}\int dt \quad =>> \quad ln(mg + v_y)|_0^{v_y} = -\frac{t}{m}$$

$$\ln(mg + v_y) - \ln(mg) = -\frac{t}{m} \quad =>> \quad \ln\left(\frac{mg+v_y}{mg}\right) = -\frac{t}{m}$$

$$v_y = mg\left(e^{-\frac{t}{m}} - 1\right) \quad (5)$$

Substituting (5) into (3): $\frac{dy}{dt} = mg\left(e^{-\frac{t}{m}} - 1\right)$

$$\int_{30}^{0} dy = mg\int_0^3 \left(e^{-\frac{t}{m}} - 1\right)dt \quad =>> \quad 0 - 30 = \left(-m^2ge^{-\frac{t}{m}} - mgt\right)|_0^3$$

$$-30 = -m^2ge^{-\frac{3}{m}} + m^2g - 3mg$$

By trial and error: $m \approx 2.3\ kg$

Problem 2.16

$$\rho A \frac{dh}{dt} + C_v\sqrt{h} = w_1 + w_2$$

When both inlet flows are shut-off, $w_1 = w_2 = 0.$ $So,$

$$\rho A \frac{dh}{dt} + C_v\sqrt{h} = 0 \quad =>> \quad \int_{3.24}^{h} \frac{dh}{\sqrt{h}} = -\frac{C_v}{\rho A}\int_0^t dt \quad =>> \quad 2\sqrt{h}\Big|_{3.24}^{h} = -\frac{C_v}{\rho A} t\Big|_0^t$$

$$h = (1.8 - 0.571t)^2$$

Drain tank: $h = 0$

$$t = 3.15\ min$$

Problem 2.17

a) For the first part, assume there is no chute that is, no frag force. In that case the model is

$$m\frac{dv}{dt} = -F_{brake} \quad \Rightarrow \quad \frac{dv}{dt} = -F_{brake}/m = -16200/900 = -18\ m/s^2$$

Integrating once:

$$v = v_0 - 18\,t \qquad (1)$$

Integrating again:

$$x = x_0 + v_0 t - 9\,t^2 \qquad (2)$$

To find the stopping distance, set v_0 = 120, v = 0, and x_0 = 0. Equation 1 is rearranged to:

$$t = (v_0 - v)/18 = 120/18 = 6.6666\,\text{sec}$$

This is the stopping time in the absence of a parachute. Substituting for t, v_0 and x_0 in Equation 2 yields the stopping distance:

$$x = x_0 + v_0 t - 9\,t^2 = 0 + 120*(6.6666) - 9*(6.6666)^2 = 400\ m$$

This is twice the desired stopping distance so a parachute is needed.

b) Now the model is

$$m\frac{dv}{dt} = -F_{brake} - \frac{1}{2}\rho C_D A v^2 \qquad v(0) = 120$$

$$\frac{dx}{dt} = v \qquad x(0) = 0$$

The goal is to find the chute area A that will result in a velocity of zero when the position is 200 m. Then the chute diameter d can be found from $A = \pi d^2/4$. This problem can be solved analytically but it is somewhat tedious. Better yet is to solve it using simulation, Chapter 11.

Problem 2.18

a) No parachute

$$m\frac{dv_y}{dt} = F_d = -mg \quad \text{with} \quad v_y(0) = 0\ m/s \qquad (1)$$

1 equation, 1 unknown [v_y]

$$v_y = \frac{dy}{dt} \quad \text{with} \quad y(0) = y_{initial}\ m \qquad (2)$$

2 equations, 2 unknowns [y]

b) After 10 seconds

From Equation 1, using Antidifferentiation,

$$\int_0^{v_y} dv_y = -9.8\int_0^t dt \quad \Rightarrow \quad v_y = -9.8t\ m/s$$

and at $t = 10\,s,\ v_y\big|_{t=10} = -98\,m/s$. From Equation 2,

$$\int_{y_{initial}}^{y} dy = \int_0^t v_y\,dt \quad \Rightarrow \quad y = y_{initial} - 9.8\int_0^t t\,dt = y_{initial} - 4.9t^2$$

and at $t = 10\,s$, the distance covered is $y\big|_{t=10} = y_{initial} - 490\,m$.

c) Open parachute

$$m\frac{dv_y}{dt} = \Sigma F = -mg - Pv_y \quad \text{with} \quad v_y(0) = v_y\big|_{t=10} = -98\,m/s \qquad (3)$$

1 equation, 1 unknown [v_y]

$$v_y = \frac{dy}{dt} \quad \text{with} \quad y(0) = y\big|_{t=10} = y_{initial} - 490\,m \qquad (4)$$

2 equations, 2 unknowns [y]

From Equation 3, and using Separation of Variables,

$$\int_{-98}^{v_y} \frac{dv_y}{mg + Pv_y} = -\frac{1}{m}\int_{10}^{t} dt \quad \Rightarrow \quad \frac{1}{P}\ln\left(\frac{mg + Pv_y}{mg - 98P}\right) = -\frac{1}{m}(t-10) \quad \Rightarrow v_y = \frac{1}{P}\left[\left(mg - 98P\right)e^{-\frac{P}{m}(t-10)} - mg\right]$$

$$v_y = \frac{1}{200}\left[\left(980 - 19600\right)e^{-2(t-10)} - 980\right] = -93.1\,e^{-2(t-10)} - 4.9 \qquad (5)$$

From Equations 4 and 5,

$$\int_{y_{initial}-490}^{y} dy = \int_{10}^{t} v_y\,dt = \int_{10}^{t}\left(-93.1\,e^{-2(t-10)} - 4.9\right)dt$$

$$y = y_{initial} - 490 + \left[46.55e^{-2(t-10)} - 4.9t\right]\Big|_{10}^{t} = y_{initial} - 487.55 + \left[46.55e^{-2(t-10)} - 4.9t\right] \qquad (6)$$

From Equation 5 we obtain the time it takes to reach a velocity of -5 m/s. This time is from the moment the person jumps from the building and equals 13.42 s.

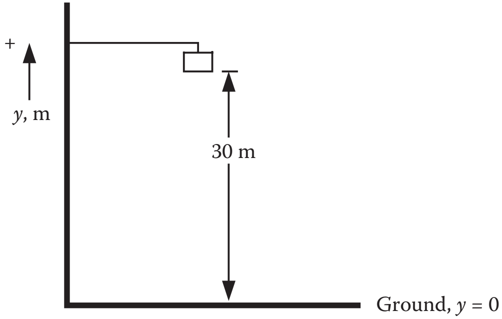


FIGURE 2.1
Object held aboveground.

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