Solutions for Electronic Magnetic and Optical Materials 2nd Edition by Fulay

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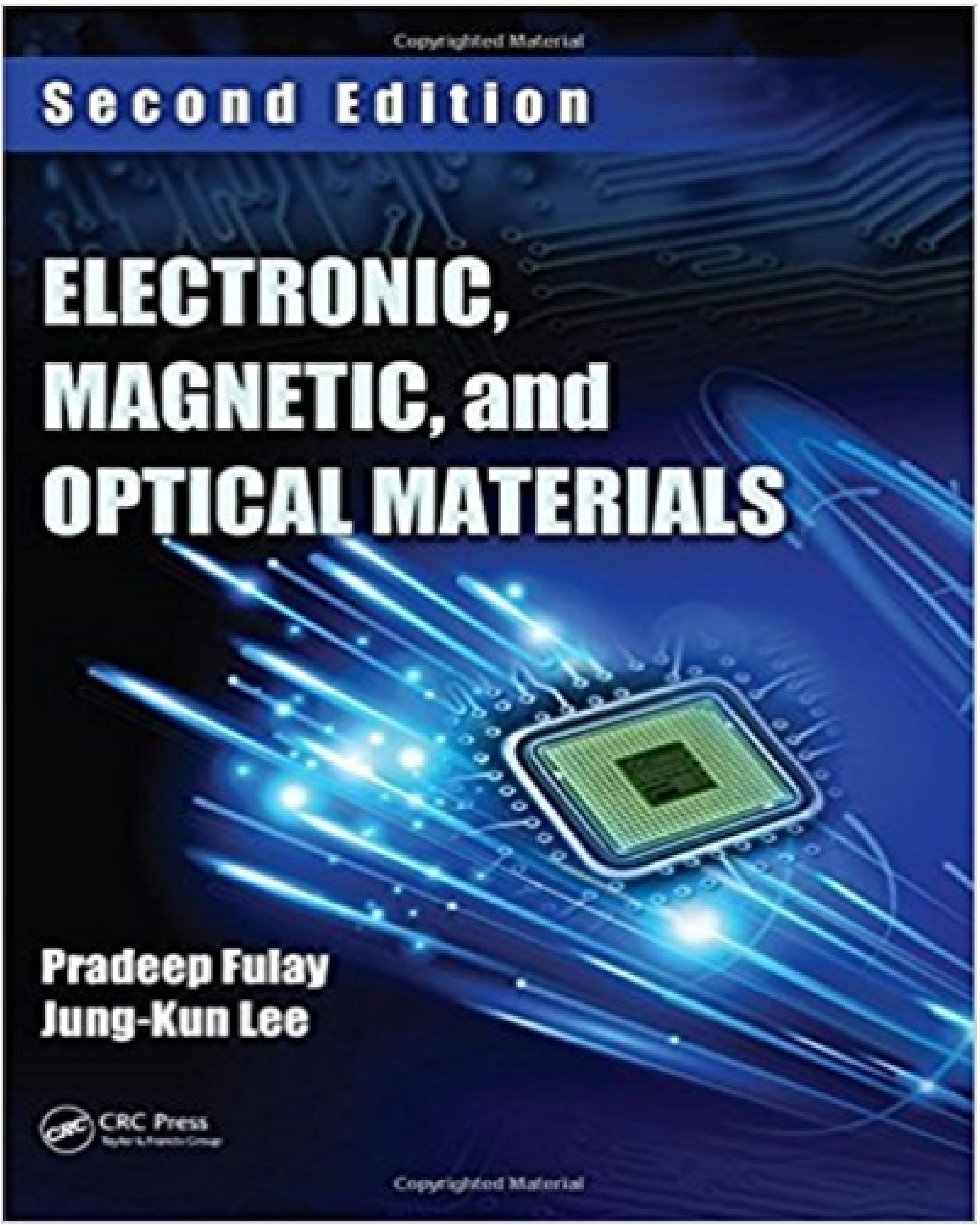


Solutions

Chapter 2

Chapter 2. Problems

Introduction

2.1. What are the typical ranges of resistivity for metals, plastics, and ceramics?

Solution:

Typical ranges of resistivity are as follows:

Metals: 10^{-5} ~ 10^{-6} **Ω**·cm (please note that resistivity of alloys is typically much higher than that of pure metals).

Pure Plastics and Ceramics: 10^{4} - 10^{15} **Ω**·cm (if donor or acceptor impurities are doped, the resistivity of plastics and ceramics could be decreased to the order of 10^{-3} ~ 10^{-4} **Ω**·cm and semiconducting or conducting behavior is observed).

2.2. What is the nature of bonding between atoms for most ceramics and plastics?

Solution:

Most ceramics the bonding is mixed covalent and ionic. Most plastics the bonding is covalent.

2.3 Compared to metals, what is the advantage in using conducting or semiconducting plastics?

Solution:

Low density, flexibility and high optical transparency of conducting polymers offer key primary advantages in several electronic applications including displays. However, the conductivity of impurity doped plastics is still smaller than that of metals, which is a fundamental problem.

2.4 Which one of these elements shows superconductivity—Ag, Au, or Al?

Solution:

At a very low temperature (~ 1.2 K) aluminum becomes superconducting. Silver and gold do not become superconductors even at very low temperatures. This is because aluminum can form superatoms that are clusters of atoms and contain a giant set of electron shells responsible for Cooper pairing. For additional information on superconductivity in elements see, " Buzea, C. and Robbie, K. in Superconducting Technology, Vol. 18, (2005), pages R1-R8" and "Avik Halder, Anthony Liang, and Vitaly V. Kresin, *Nano Letters* **2015** *15* (2), 1410-1413".

Ohm’s Law

2.5 What is the difference between resistance and resistivity?

Solution:

Resistance is not inherence material property, but resistivity is material property. Resistance depends on the geometry of a component i.e. how long how wide etc. and also what a component is made from. In contrast, resistivity depends upon the composition of the material and its microstructure. Resistivity is largely independent of the size (i.e. length, cross-sectional area etc.) of the component.

2.6 Do all materials obey Ohm’s law? Explain.

Solution:

Most materials do obey Ohm’s law. There are a few materials that do not. For example, certain compositions of zinc oxide and other materials have a microstructure in which grain boundaries can show electrical breakdown if the internal electric field across the material is high enough. Under such conditions, these materials show non-ohmic behavior. Also, electric current passing through a junction between heterogeneous materials such as p-/n-Si and semiconductor/metal does not follow Ohm’s law.

2.7. Calculate the resistance of an AWG #20 Cu wire one mile in length.

Solution:

From Table 2.2, the diameter of AWG #20 Cu wire is 0.812 mm, the length is one mile which is 1600 meters.

The resistance R is given by $R = \rho l/A$. Then, use values of resistivity of 1.72×10^{-8} ohm·met (inverse of conductivity of pure copper 5.8×10^{7} S/m), length 1609 m, and area of 5.18×10^{-7} m^2. This gives a resistance of 5.36 ohms.

Copper wires are usually not made from pure copper (because pure copper is too soft) and in this case we can substitute appropriate value of resistivity. For example, if we say the resistivity is 1.678 μΩ·cm, then the value of resistance will be 32.8 ohms for the same diameter and length.

2.8. What is the length of an AWG #16 Cu wire whose resistance is 21 Ω?

Solution:

For 16 gauge wire the diameter is 1.627734 mm (See Table 2.2). This gives an area of 2.08×10^{-6} m^2. This gives a length of 2534 m.

2.9. If the wire in Problem 2.8 carries a current of 5 A, what is the current density?

Solution:

Current density is current per unit area. Area of cross-section is of 2.08 x 10^{-6} m^2. Thus current density will be 2.40 x 10^6 A/m^2.

2.10. Al can handle current densities of 10^5 A/cm^2 at about 150°C (Gupta 2003). What will be the maximum current allowed in an Al wire of AWG #18 operating at 150°C?

Solution:

The wire diameter is (Table 2-2) 1.023696 mm. the area of cross-section is 8.23 x 10^{-7} m^2. The maximum current will be the product of maximum current density and cross-sectional area and this works out to 823.06 A or 0.823 kA.

2.11. A circuit breaker connects an AWG #0000 Cu conductor wire 300 feet in length. What is the resistance of this wire? If the wire carries 150 A, what is the voltage decrease across this wire?

Solution:

From table 2-2, diameter of the wire is 11.684 mm, therefore cross-sectional area is 1.07×10^{-4} m^2. Length is 300 feet or 91.44 m (one foot is 0.3048 m). We use conductivity of copper as 5.8 x 10^7 S/m. Thus, the resistance of this wire will be 0.0147 ohms. From Ohm's law the voltage drop across this will be $150 \times 0.0147 = 2.2$ volts.

2.12. You may know that a conductor carrying an electrical current generates a magnetic field. A long wire carrying a current generates a magnetic field similar to that generated by a bar magnet. This magnet is known as an electromagnet. Consider a meter of magnetic wire AWG #2. Such wires are usually made from high-conductivity soft-drawn electrolytic Cu, and the conductor is coated with a polymer to provide insulation. What will be the electrical resistance (in ohms) of this wire?

Solution:

From Table 2-2, diameter of the AWG # 2 wire is 6.543802 mm, therefore cross-sectional area is 3.36×10^{-5} m^2. Length is 1 meter. We use conductivity of copper as 5.8 x 10^7 S/m. Thus, the resistance of this wire will be 0.0005 ohms.

2.13 Ground rods are used for electrical surge protection and are made from materials such as Au, Au-clad steel, or galvanized mild steel. The resistance of the actual rod itself is small; however, the soil surrounding the rod offers electrical resistance (Paschal 2001). The resistance of a ground rod is given by:

$$R = \frac{\rho}{2L\pi}\ln(\left[\frac{4L}{a}\right] - 1)$$

where *R* is the resistance in ohms, ρ is the resistivity of soil surrounding the ground rod (in Ω -cm), *L* is the length of ground rod in centimeters, and *a* is the diameter of the ground rod. (a) Assuming that the resistivity of a particular soil is 10^4 Ω -cm, the length of the rod is 10 feet, and the diameter is 0.75 inches, what will be the resistance (R) of the ground rod in ohms? (b) Assuming that the ground rod is made from Cu, prove that the resistance of the rod itself is actually very small. (c) What will happen to the resistance of the metallic material as it corrodes over a period of many years?

Solution:

(a) Note that we use soli resistivity which is very high compared to resistivity of copper. The value in ohm-m will be 100 ohm-m. Diameter is 0.75 inches which is 1.91 x 10^{-2} m. Length is 10 feet which is 3.048 meters. The ground resistance then works out to be 33.731 ohms. This resistance is typical of soils that are more like inorganic clays as opposed to sand or gravel mixtures (which are much higher resistivity).

(b) Since resistivity of cooper is orders of magnitude smaller compared to that of soil, we expect the electrical resistance of the rod itself will be very small.

(c) If the metallic rod corrodes, the resistance of the grounding rod will go up and it may not function as intended. This is why copper rods can be electroplated to avoid the risk of oxidation.

2.14. The electrical resistance of pure metals increases with temperature. In many ceramics, the electrical current is carried predominantly by ions (such as oxygen ions in YSZ). Based on the data shown in Figure 2.30, calculate the electrical resistance of a 50-µm-thick YSZ element at 500°C and 800°C. Assume that the cross-sectional area is 1 cm^2.

Solution: $R = \rho l/A$

In Figure 2.30, you can find the straight line that shows variation of conductivity of YSZ, it is the line that is marked as $(ZrO_2)_{0.9}(Y_2O_3)_{0.1}$.

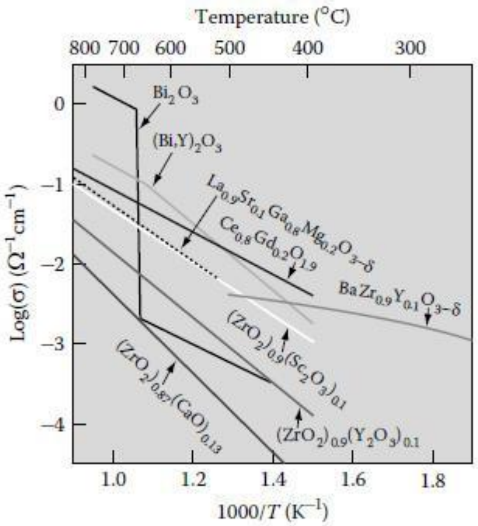


FIGURE 2.30 The conductivity of different materials used as electrolytes in the development of solid oxide fuel cells. (From Haile, S. M. 2003. *Acta Materilia* 51:5981–6000. With permission.)

At 500 C: the log $\sigma = -3$, i.e. $\sigma = 10^{-3}\ \Omega^{-1}\text{cm}^{-1}$. The area is given as 1 cm^2. Using these values we get a resistance of 5 Ω.

At 800 C: From Figure 2-30, log $\sigma = -1.5$ i.e. $\sigma = 10^{-1.5}\ \Omega^{-1}\cdot\text{cm}^{-1}$, since dimensions do not change significantly (at least that's what we assume here), the resistance will drop by 1.5 orders of magnitude. The new resistance will be 0.16 Ω.

2.15. Consider the material calcium oxide–stabilized ZrO_2 (Figure 2.30). Calculate the electrical resistance of a 50-µm-thick YSZ element at 500°C and 800°C. Assume that the cross-sectional area is 1 cm^2.

Solution: From Figure 2-30, you can find the conductivity of calcium oxide stabilized ZrO_2. Compared with YSZ, it has smaller conductivity.

At 500 C: the log $\sigma = -4$, i.e. $\sigma = 10^{-4}\ \Omega^{-1}\text{cm}^{-1}$. The area is given as 1 cm^2. Using these values we get a resistance of 50 Ω.

At 800 C: log $\sigma = -2$ i.e. $\sigma = 10^{-2}\ \Omega^{-1}\cdot\text{cm}^{-1}$, since dimensions do not change significantly (at least that's what we assume here), the resistance will drop by 2 orders of magnitude. The new resistance will be 0.5 Ω.

2.16. If high conductivity at temperatures above 700°C was the only requirement in selecting a material for a solid oxide fuel cell electrolyte, what material would you choose (Figure 2.30)? Besides cost, what additional factors must be considered in the selection of the electrolyte material?

Solution:

Bismuth oxide exhibits the highest electric conductivity above 700 ^{o}C and this will be chosen, if the only consideration were conductivity. Other most important considerations would be the environmental impact, chemical stability and long term availability (i.e. can the electrolytematerial withstand oxidizing and reducing conditions?). In addition, the thermal expansion of the electrolyte material should be comparable to that of other materials such as metal interconnects to reduce the residual thermal stress in the solid oxide fuel cell.

2.17. What is unusual about the change in resistivity as a function of temperature for bismuth oxide (B_2O_3)?

Solution:

Bismuth oxide shows a sharp change in the electric conductivity at around 680 ^{o}C. Often, such large changes in conductivity are associated with some type of phase transformation. As this transformation occurs, there is often a volume change. The volume change leads to the build-up of stress and usually ceramic materials would fracture if such phase transformations occur repeatedly during the operation.

Classical Theory of Electrical Conduction

2.18. Calculate the mobility of the electrons in Zn in cm^2/V·s. Assume that each Zn atom contributes two conduction electrons. The atomic mass of Zn is 65. The resistivity is 5.9 μΩ·cm, and the density is 7.130 g/cm^3.

Solution:

The weight of Zn in a volume of one cubic centimeter is 7.13 grams. One mole i.e. 65 grams of Zn contains an Avogadro's number (i.e. 6.023×10^{23}) of atoms. Therefore, one cm^3 of Zn (i.e. 7.13 grams of Zn) contain 6.607×10^{23} atoms. Each Zn atom is assumed to contribute two valence electrons to a sea of free electrons. Thus, the conduction electron concentration for zinc is $2 \times 6.607 \times 10^{22} = 1.32 \times 10^{23}$ electrons/cm^3. The, resistivity is given to be 5.9 μΩ^{-1}·cm^{-1}, this means conductivity is 1.69×10^5 S/cm.

The conductivity is $\sigma = n \times q \times \mu$ (q: 1.6×10^{-19} C). Substituting the values of conductivity, conduction electron concentration, and electronic charge, we get mobility value 8.03 cm^2/V·s.

2.19. If the mobility of electrons in Au is 31 cm^2/V·s. Assume that the mass of electrons in Au is 9.1×10^{-31} kg. Calculate the mean free path length (λ) of electrons in Au if the average electron speed is 10^6 m/s.

Solution:

Approach 1: In Equation 2-21, the expression for conductivity is

$$J = \left(\frac{nq^2\tau}{m}\right)E$$

For gold, we use (all quantities in SI units) : n=5.91 × 10^{28} electrons/m^3 (Table 2.3, convert from electrons/cm^3), m= 9.1 x 10^{-31} kg, and q=1.6 x 10^{-19} C, and conductivity = 4.25 x 10^7 S/m (Table 2.1, convert to SI units). This gives us a mean free path (λ) of 2.556 x 10^{-8} m or 255.6 Å. This is consistent with values mentioned in Section 2.10.

The time between collisions (τ) can then be calculated as mean free path divided by average velocity.

$$\tau = 2.556 \times 10^{-8} \text{ m}/10^6 \text{ m/s} = 2.56 \text{ x } 10^{-14} \text{ seconds.}$$

Approach 2: We could also have calculated the time between collisions (τ) from the mobility value using equation 2.18. In this we use all SI units,

$$\mu_n = \frac{q\tau}{m}$$

The value of mobility will be 0.0031 m^2/V·s. Use q=1.6 x 10^{-19} C and m: 9.1 x 10^{-31} kg. This gives us a value of τ = 1.76 x 10^{-14} s. This is similar to what we calculated before using conductivity and electron concentration values. Then the mean free path, from Equation 2.19 will be 1.76 x 10^{-8} m or 176 Å. This is similar to the value calculated in approach 1.

2.20. The thermal speed of electrons is about 10^6 m/s. However, the drift velocity is rather small because electrons are scattered off by the vibrations of atoms. Calculate the drift velocity of electrons in Cu for an electric field of 1 V/m. Assume the mobility of electrons in Cu is 32 cm^2/V·s.

Solution:

From Equation 2.17,

$$v_{\text{drift}} = v_{\text{avg}} = \left(\frac{qE}{m}\right)\tau = \left(\frac{q\tau}{m}\right)E$$

Note that the term inside the bracket is mobility. A given mobility is 0.0032 m/s in SI units. If electric field is 1 V/m, the drift velocity is 0.0032 m/s. This is much smaller than thermal velocity, 10^6 m/s, which is due to scattering of electrons during the travel.

2.21. If the density of Ag is 10.5 g/cm^3, what is the concentration of conduction electrons in Ag?

Solution:

Atomic mass of silver is 107.868 (See Table 2.3) and 107.868 grams of silver has 6.023×10^{23} atoms. Therefore, there are 5.862×10^{22} Ag atoms in 10.5 grams or 1 cc of Ag. Since each Ag atom has one valence electron, we assume each atom donates one conduction electron. Then, the concentration of conduction electrons will also be 5.862×10^{22} electrons/cm^3.

2.22. From the information provided in Table 2.3, calculate the expected conductivity of Ag.

Solution:

When the electron concentration in 2.21 is inserted to $\sigma = n \times q \times \mu$ (here, μ is 57 cm^2/V·s), this works out to the conductivity, 53.4×10^4 S/cm and the resistivity, 1.87×10^{-6} Ω·cm. These values are close to what are reported in Table 2.1.

2.23. Au is a face-centered cubic metal with a lattice constant of 4.080 Å. If the atomic mass of Au is 196.9655, calculate the number of conduction electrons per unit volume. Express your answer as number of electrons/cm^3 . Assume that each Au atom donates one conduction electron.

Solution:

For metals such as gold (Au) with face center cubic structure we have a total of four atoms per unit cell. Each of the corner atom counts as 1/8th and each face center atom counts as half.

The volume of the unit cell is $6.7917 \text{ x } 10^{-23}$ cm^3. Thus concentration of atoms in atoms/cm^3 :

$$\frac{6.7917 \text{ x } 10^{-23}}{4} = 5.88 \times 10^{22}$$

For gold, valence is one, thus concentration of conduction electrons will be the same as concentration of atoms since each atom donates only one electron. Thus, concentration of conduction electrons will be $5.88 \text{ x } 10^{22}$ electrons/cm^3. This is close to the value listed in Table 2.3.

2.24 A semiconductor is made so that it carries electrical current primarily from the flow of electrons. If the mobility of electrons (μ_n) is 1350 cm^2/V·s and the conduction electron concentration is 10^{21} cm^{-3} , what is the electrical conductivity of this material?

Solution:

It is stated that the current in this particular semiconductor is predominantly carried by electrons. Then, we use the following equation.

$$\sigma = n \times q \times \mu$$

Using values given we get conductivity of 2.16×10^5 S/cm.

2.25. A heating element for a flat iron is rated at 1000 W. If the iron works at 220 V, what is the resistance of this heating element?

Solution:

Power dissipated is given by = $V \times I = I^2 \times R$ (Equation 2.28). Using this, we get current value of 4.54 Amp. Then, from Ohm's law, the resistance is calculated as 48.4 Ohms.

2.26. Electronic components and devices are often tested at 125°C and −55°C to check the high- and low-temperature performances. They can then be compared with the properties observed at room temperature, 25°C. For example, using 25°C as the reference temperature and +125°C as the other temperature, α_{125} can be written as follows:

$$TCR_{125} = \alpha_{R,125} = \frac{1}{\rho_{125}}\left(\frac{\rho_{125}-\rho_{25}}{125-25}\right) \times 10^6 \ \text{ppm}/^\circ/\text{C}$$

Write an equation to express the temperature coefficient of resistance with $T = -55$°C (note the negative sign) as the other temperature, using 25°C as the base or reference temperature (T_0).

Solution:

$$TCR_{-55} = \alpha_{R,-55} = \frac{1}{\rho_{-55}}\left(\frac{\rho_{-55}-\rho_{25}}{-55-25}\right) \times 10^6 \ \text{ppm}/^\circ/\text{C}$$

Note that the resistivity at lower temperatures will be lower, thus the numerator in the bracketed expression will be negative and so is the denominator.

2.27. In a circuit, the TCR 125 value for a resistor is 100 ppm/°C. If the resistance (R) at 25°C is 1000 Ω, what is the resistance at 125°C?

Solution:

$$TCR_{125} = \alpha_{R,125} = \frac{1}{\rho_{125}}\left(\frac{\rho_{125}-\rho_{25}}{125-25}\right) \times 10^6 \ \text{ppm}/^\circ/\text{C}$$

Since the resistor is the same i.e. geometry remains the same, we can use resistance instead of resistivity. Using values of TCR and R_{25} in the above equation and solving for R_{125} we get:

$$100 = \frac{1}{R_{125}} \left(\frac{R_{125} - 1000}{100} \right) \times 10^6$$

This leads to R_{125} = 1000/0.99=1010 Ω.

2.28. Assume that the bus bar discussed in Example 2.11 is heated due to the Joule losses and now operates at 70°C. Calculate the resistance, power loss, and energy consumption for 24 hours and the total energy costs per year. Ignore the change in the length of the Cu bus bar because of thermal expansion.

Solution:

The copper conductor discussed in Example 2.11 was shown to have a resistance of 2.44×10^{-3} Ω. From the literature, the temperature coefficient of resistance (α_R) for copper is 0.00404/°C, with a reference temperature of 20 °C. The resistance at 70 °C will be given by:

$R = R_0 [1 + \alpha_R (T - T_0)] = 0.00244 [1 + 0.00404(70-20)] = 2.93 \times 10^{-3}$ Ω.

The voltage drop will be V= 1000 Amp x 2.93×10^{-3} ohms = 2.93 volts. Dissipated power is $I^2 \times R$ which is 2933 Watts or 2.933 kW. Energy lost in 24 hours is 70.4 kW·h (2.993 kW × 24 hrs).

Assuming cost of electricity is 4 cents/kW·h (see Example 2.11), the cost per day will be \$2.81/day. In one year, the cost is 365 × \$2.81 = \$1027.68/year.

2.29. Nichrome wire is used for cutting materials such as polystyrene (Styrofoam) into different shapes, including large facades or insulation boards. (a) Calculate the length of an AWG #20 wire that is needed to have a resistance of $R = 8$ Ω. (b) What will be the resistance of this wire if it gets heated to a temperature of 200°C? See Tables 2.2 and 2.5.

Solution:

(a) From Table 2.5 the resistivity of one type of nichrome alloy is 100 μΩ·cm or 0.0001 Ω·cm. The wire gage is 20 and from Table 2.2 the diameter is 0.811814 mm which translates into an area of 0.0051761 cm^2. The resistance is 8 Ohms. Then length of the wire is 414 cm.

(b) If the wire gets heated to 200 °C, the new resistance will be given by:

$R = R_0 [1 + \alpha_R (T - T_0)] = 8 [1 + 400 \times 10^{-6} (200-27)] = 8.5536$ Ω

Note that in Table 2.5 the values of α_R should be listed as 400×10^{-6} and the reference temperature (T_0) of nichrome value is 300 K (i.e. 27 °C).

2.30. In the nanoscale region, why does the resistivity of thin films depend upon thickness?

Solution:

This is because of surface scattering in very thin films (See Section 2.10). If the film thickness is comparable to or smaller than the scattering length (λ: 10~100 nm) of electrons, the scattering of electrons at the surface film becomes significant in addition to the scattering by grain boundary and extended defects (e.g. dislocations). Therefore, a decrease in the film thickness decreases the carrier mobility and increases the resistivity of film.

2.31. What elements most affect the resistivity of high-purity Cu?

Solution:

According to Figure 2.13, addition of Ti into Cu changes the resistivity the most.

2.32. Why does the addition of oxygen in limited concentrations actually *increase* the conductivity of high-purity Cu?

Solution:

As mentioned in Section 2.11.2, an optimum level of oxygen can concentrate the total impurity content in form of small precipitates of copper oxide and thus the conductivity goes up after the removal of impurities in Cu matrix by the incorporation of oxygen.

2.33. Why does the resistivity of pure metals increase with temperature, whereas that of alloys is relatively stable with temperature?

Solution:

In an alloy, the second component is considered as an impurity and the impurity concentration is typically very high. Thus, in alloys, the scattering by impurities is much more significant and the effect of phonon scattering (thermal effect) is not as strong. For pure metals the carrier mobility is high and even a small change in temperature brings in considerable amount of phonon scattering. Therefore, the mobility of pure metals decreases, leading to an increase in the resistivity.

2.34 Nordheim's coefficient for Au dissolved in Cu is $C = 5500$ nΩ· m. If the resistivity of Cu at 300 K is 16.73 nΩ·m, calculate the resistivity of an Au–Cu alloy containing 1 weight % Au.

Solution:

Convert weight fraction (w) of gold, which is an impurity in this case, into atomic fraction using equation shown in Example 2.12. Note that Cu is the matrix.

$$x = \frac{M_{\mathrm{Cu} \times \mathrm{w}}}{(1 - w)M_{\mathrm{Au}} + w\mathrm{M}_{\mathrm{Cu}}}$$

The atomic mass of copper is 63.5, that of gold is 197 and weight fraction of gold (w) is : 0.01. This gives atomic fraction of gold as $x = 0.00324534$.

Using Nordheim's rule (Equation 2.32):

$$\rho_{\mathrm{alloy}} = \rho_{\mathrm{matrix}} + Cx(1 - x)$$

where C (Nordeheim's coefficient) is 5500 nΩ·m, and resistivity of the matrix (Cu, in this case) is 16.73 nΩ·m. Therefore, the resistivity of alloy is 34.52 nΩ·m [=16.73 + 5500 (0.00324534) (1-0.00324534)]

2.35 What is the electronic configuration for an Mg atom ($Z = 12$)?

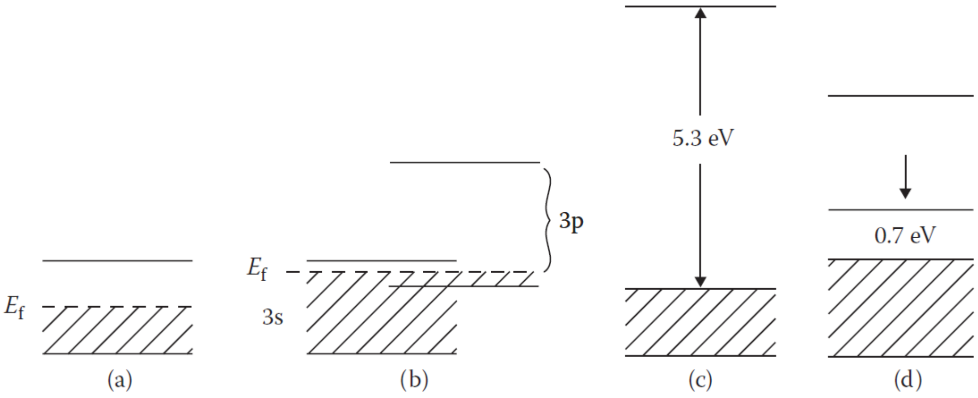
Solution:

$1s^2\ 2s^2\ 2p^6\ 3\ 3s^2$ (refer to Example 2.13 of the textbook)

2.36 Draw a schematic of the band diagrams for a typical metal, a semiconductor, and an insulator.

Solution:

Refer to band diagrams of Figure 2.27.



Chapter 2:
Electrical Conduction
in Metals and Alloys

Key Topics to be Understood

- The fundamentals of what causes a material to be a conductor of electricity.
- Different types of conducting materials based on metals and alloys and their “real-world” technological applications.
- The classical theory of conductivity
- The band theory of solids and its use for examining the differences among conductors, semiconductors, and insulators
- The effects of chemical composition, microstructure, and temperature on the conductivity of metals and alloys
- Real-world applications of conductive materials

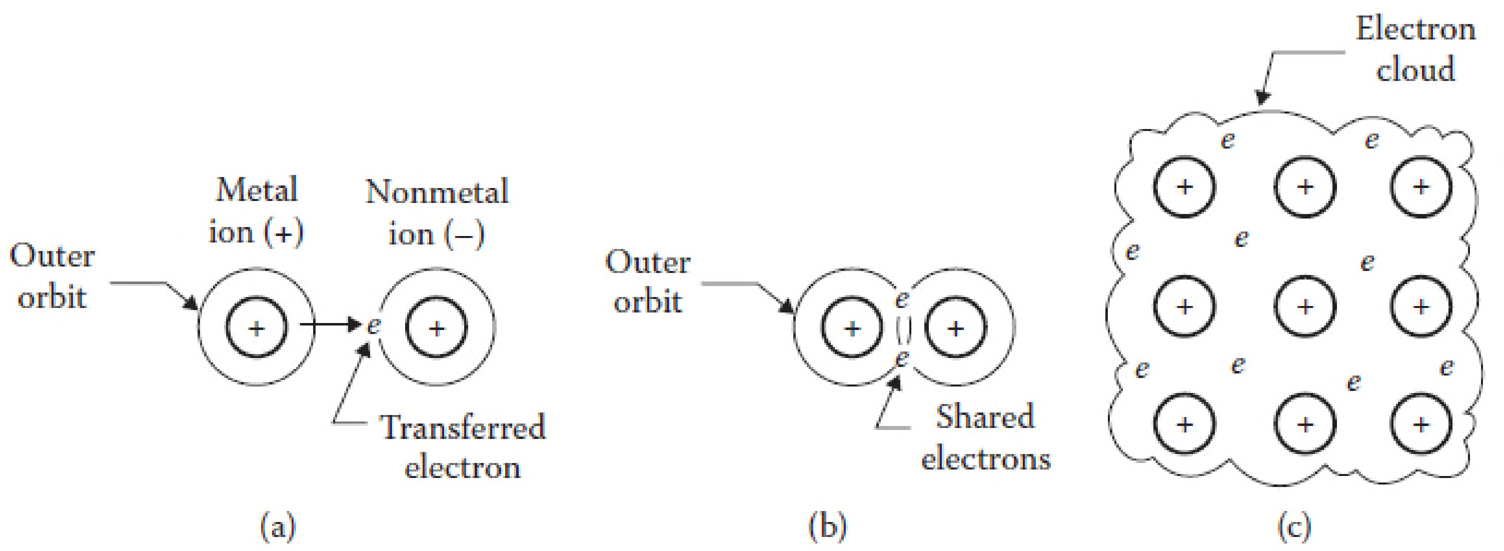


FIGURE 2.1 Illustration of (a) ionic bonds, such as those in sodium chloride and many other ceramics; (b) covalent bonds, such as those in silicon, many ceramics, and polymers; and (c) metallic bonds. (From Groover, M.P., *Fundamentals of Modern Manufacturing: Materials, Processes, and Systems*, Wiley, New York, 2007. With permission.).

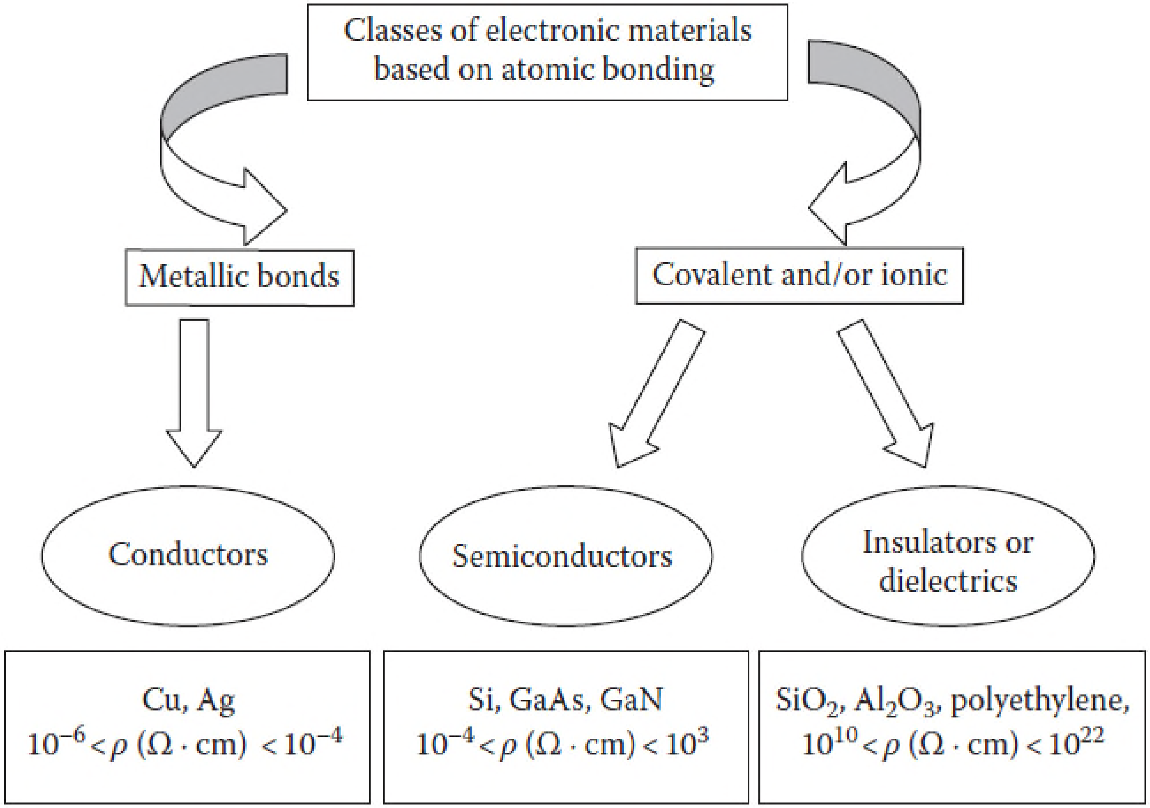


FIGURE 2.2 Classification of materials based on the nature of bonding. Typical ranges of resistivity are also shown.

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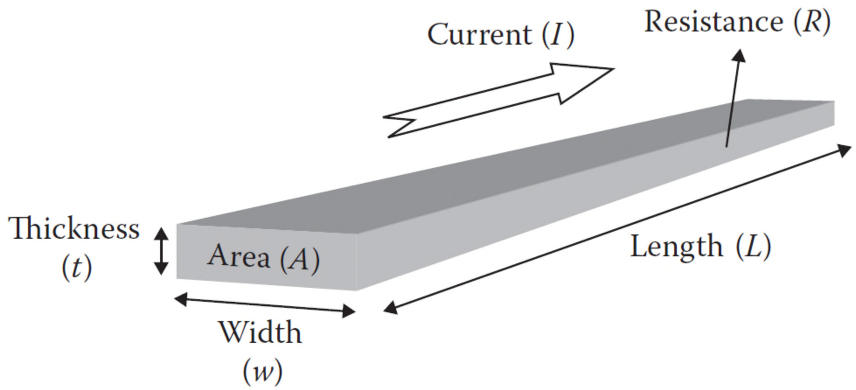

FIGURE 2.3 Geometry of a resistor used in describing Ohm's law.

Unit of electrical resistance (R) is ohm (Ω). The inverse of R is known as *conductance*, which has units of Siemens or Ω^{-1}.

ρ (rho) is defined as the *electrical resistivity*, *bulk resistivity*, or *volume resistivity*. A commonly used unit for resistivity is Ω·cm. However, resistivity values may be listed as Ω·cm, Ω·m, μΩ·cm (micro-ohm-centimeter), nΩ·m (nano-ohm–meter), and so on.

The inverse of ρ is known as *conductivity* (σ) and its common unit is $\Omega^{-1}\cdot\text{cm}^{-1}$.

$$\sigma = \frac{1}{\rho}$$

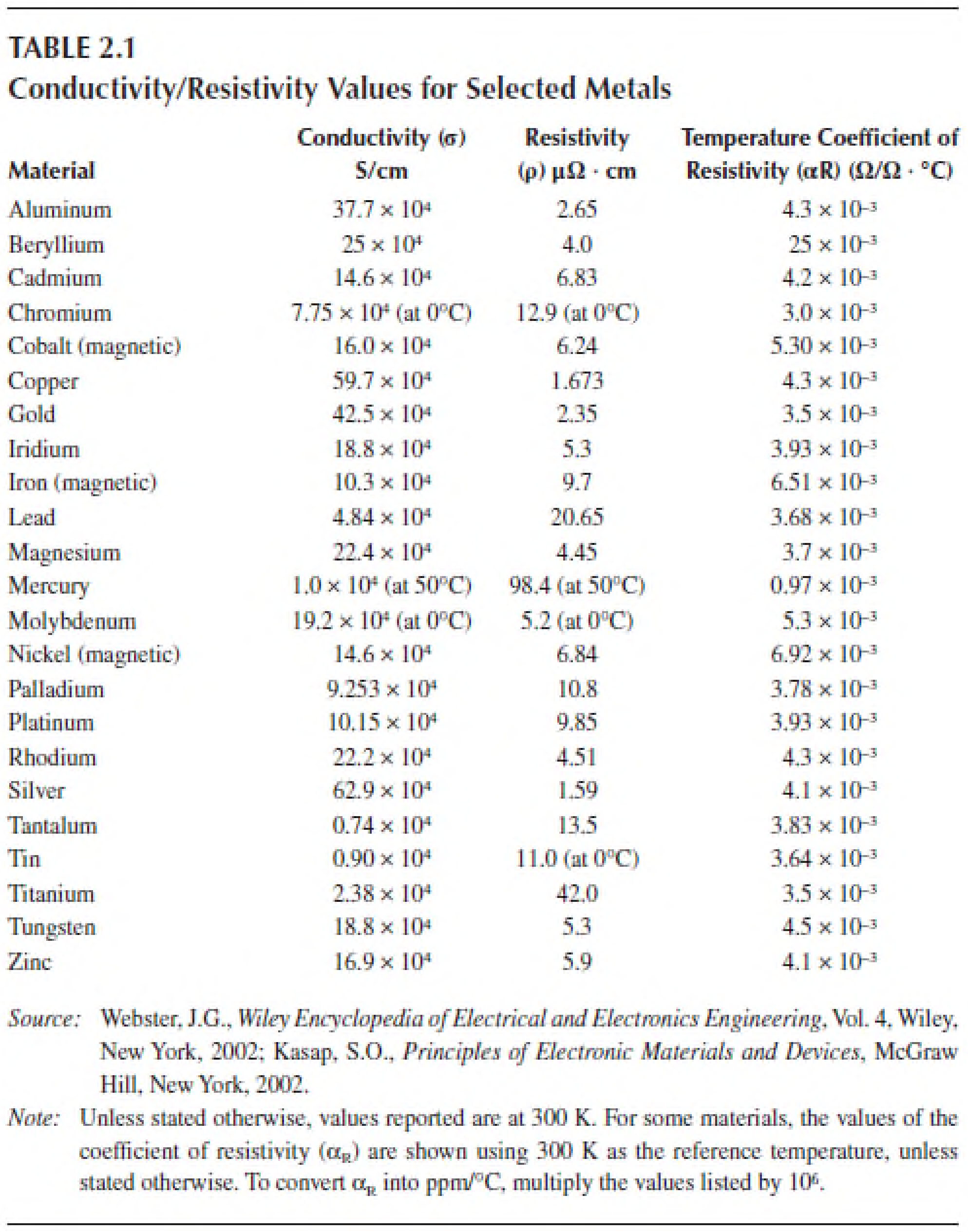

TABLE 2.1

Conductivity/Resistivity Values for Selected Metals

Material	Conductivity (σ) S/cm	Resistivity (ρ) μΩ · cm	Temperature Coefficient of Resistivity (αR) (Ω/Ω · °C)
Aluminum	37.7×10^4	2.65	4.3×10^{-3}
Beryllium	25×10^4	4.0	25×10^{-3}
Cadmium	14.6×10^4	6.83	4.2×10^{-3}
Chromium	7.75×10^4 (at 0°C)	12.9 (at 0°C)	3.0×10^{-3}
Cobalt (magnetic)	16.0×10^4	6.24	5.30×10^{-3}
Copper	59.7×10^4	1.673	4.3×10^{-3}
Gold	42.5×10^4	2.35	3.5×10^{-3}
Iridium	18.8×10^4	5.3	3.93×10^{-3}
Iron (magnetic)	10.3×10^4	9.7	6.51×10^{-3}
Lead	4.84×10^4	20.65	3.68×10^{-3}
Magnesium	22.4×10^4	4.45	3.7×10^{-3}
Mercury	1.0×10^4 (at 50°C)	98.4 (at 50°C)	0.97×10^{-3}
Molybdenum	19.2×10^4 (at 0°C)	5.2 (at 0°C)	5.3×10^{-3}
Nickel (magnetic)	14.6×10^4	6.84	6.92×10^{-3}
Palladium	9.253×10^4	10.8	3.78×10^{-3}
Platinum	10.15×10^4	9.85	3.93×10^{-3}
Rhodium	22.2×10^4	4.51	4.3×10^{-3}
Silver	62.9×10^4	1.59	4.1×10^{-3}
Tantalum	0.74×10^4	13.5	3.83×10^{-3}
Tin	0.90×10^4	11.0 (at 0°C)	3.64×10^{-3}
Titanium	2.38×10^4	42.0	3.5×10^{-3}
Tungsten	18.8×10^4	5.3	4.5×10^{-3}
Zinc	16.9×10^4	5.9	4.1×10^{-3}

Source: Webster, J.G., *Wiley Encyclopedia of Electrical and Electronics Engineering*, Vol. 4, Wiley, New York, 2002; Kasap, S.O., *Principles of Electronic Materials and Devices*, McGraw Hill, New York, 2002.

Note: Unless stated otherwise, values reported are at 300 K. For some materials, the values of the coefficient of resistivity (α_R) are shown using 300 K as the reference temperature, unless stated otherwise. To convert α_R into ppm/°C, multiply the values listed by 10^6.

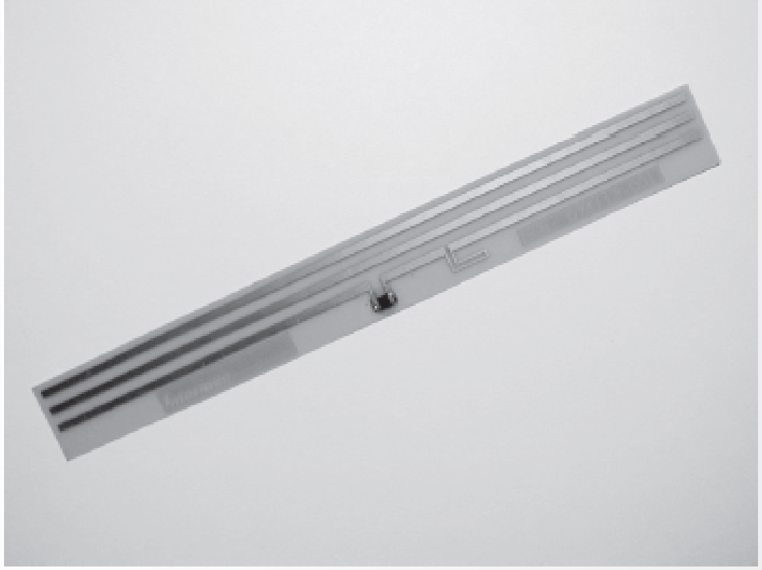

FIGURE 2.4 Typical passive radio frequency–identification (RFID) tag showing the antenna made from copper. (Courtesy of Pavel Nikitin, Intermec Technologies Corporation, Everett, WA.)

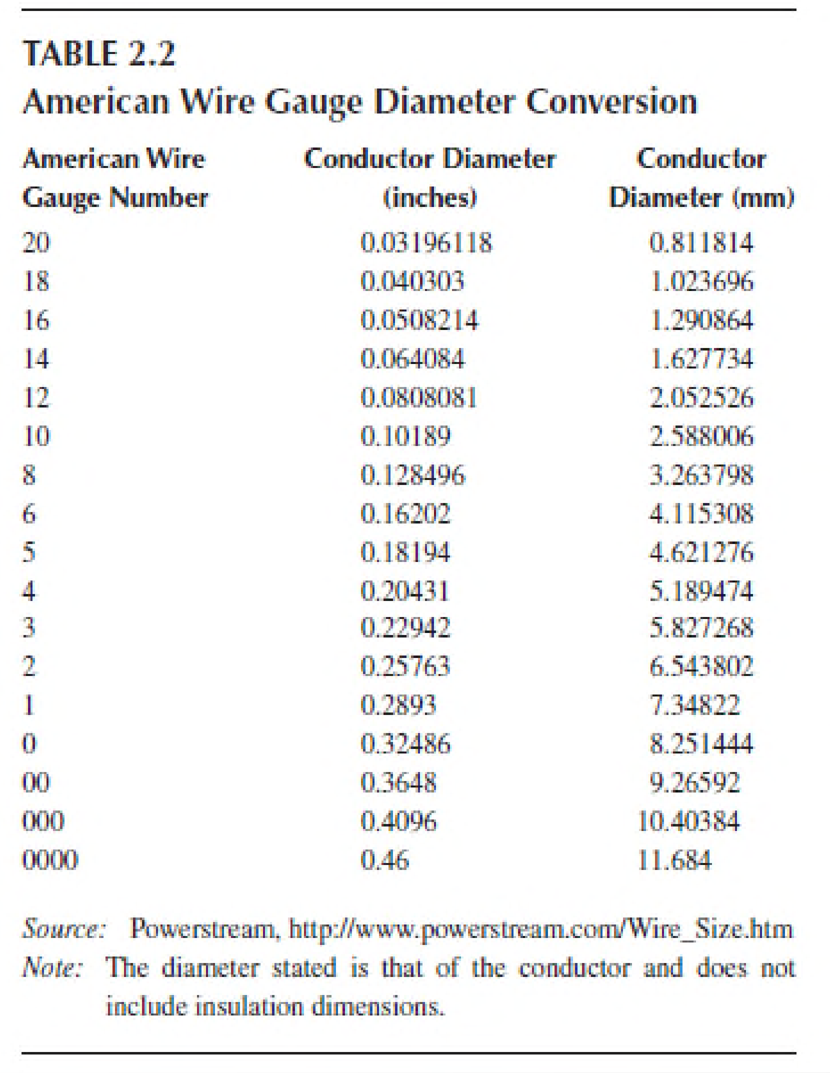

TABLE 2.2

American Wire Gauge Diameter Conversion

American Wire Gauge Number	Conductor Diameter (inches)	Conductor Diameter (mm)
20	0.03196118	0.811814
18	0.040303	1.023696
16	0.0508214	1.290864
14	0.064084	1.627734
12	0.0808081	2.052526
10	0.10189	2.588006
8	0.128496	3.263798
6	0.16202	4.115308
5	0.18194	4.621276
4	0.20431	5.189474
3	0.22942	5.827268
2	0.25763	6.543802
1	0.2893	7.34822
0	0.32486	8.251444
00	0.3648	9.26592
000	0.4096	10.40384
0000	0.46	11.684

Source: Powerstream, http://www.powerstream.com/Wire_Size.htm

Note: The diameter stated is that of the conductor and does not include insulation dimensions.

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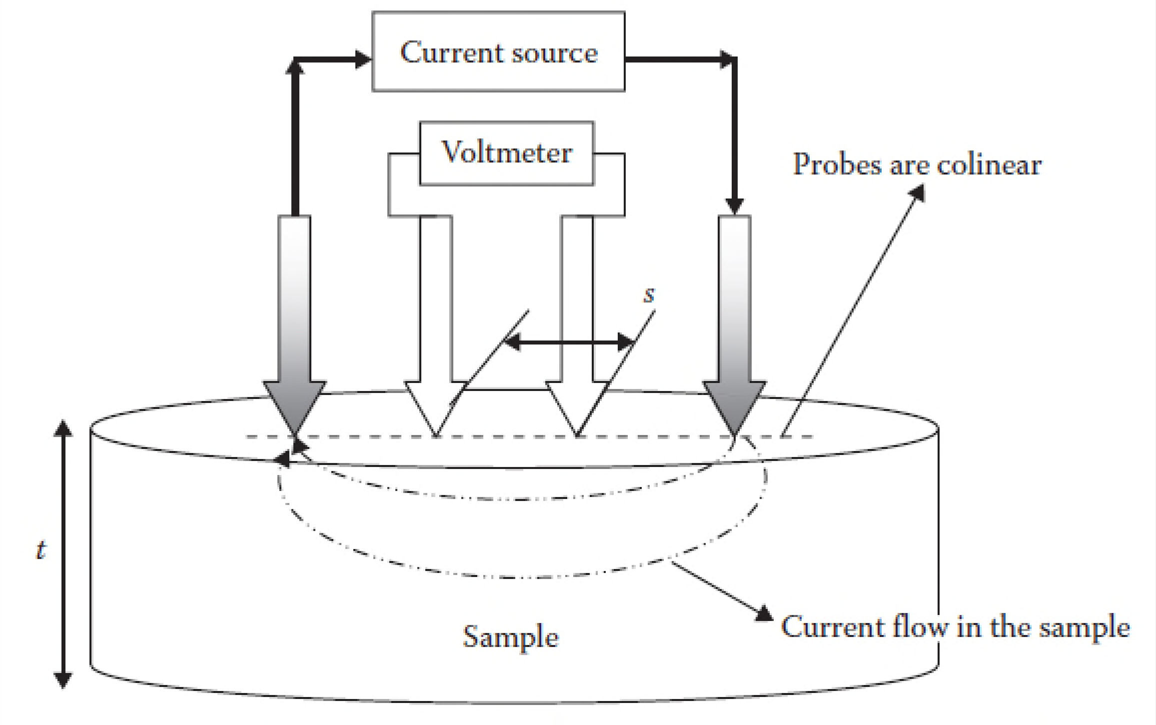


FIGURE 2.5 A schematic of the four-point probe used to measure resistivity. The spacing between probes is *s*, and the sample thickness is *t* (not to scale).

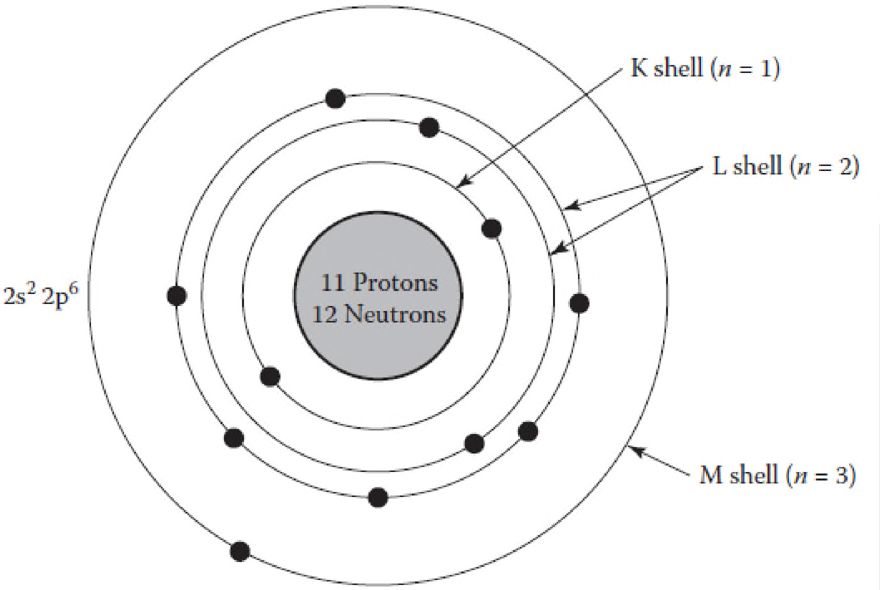


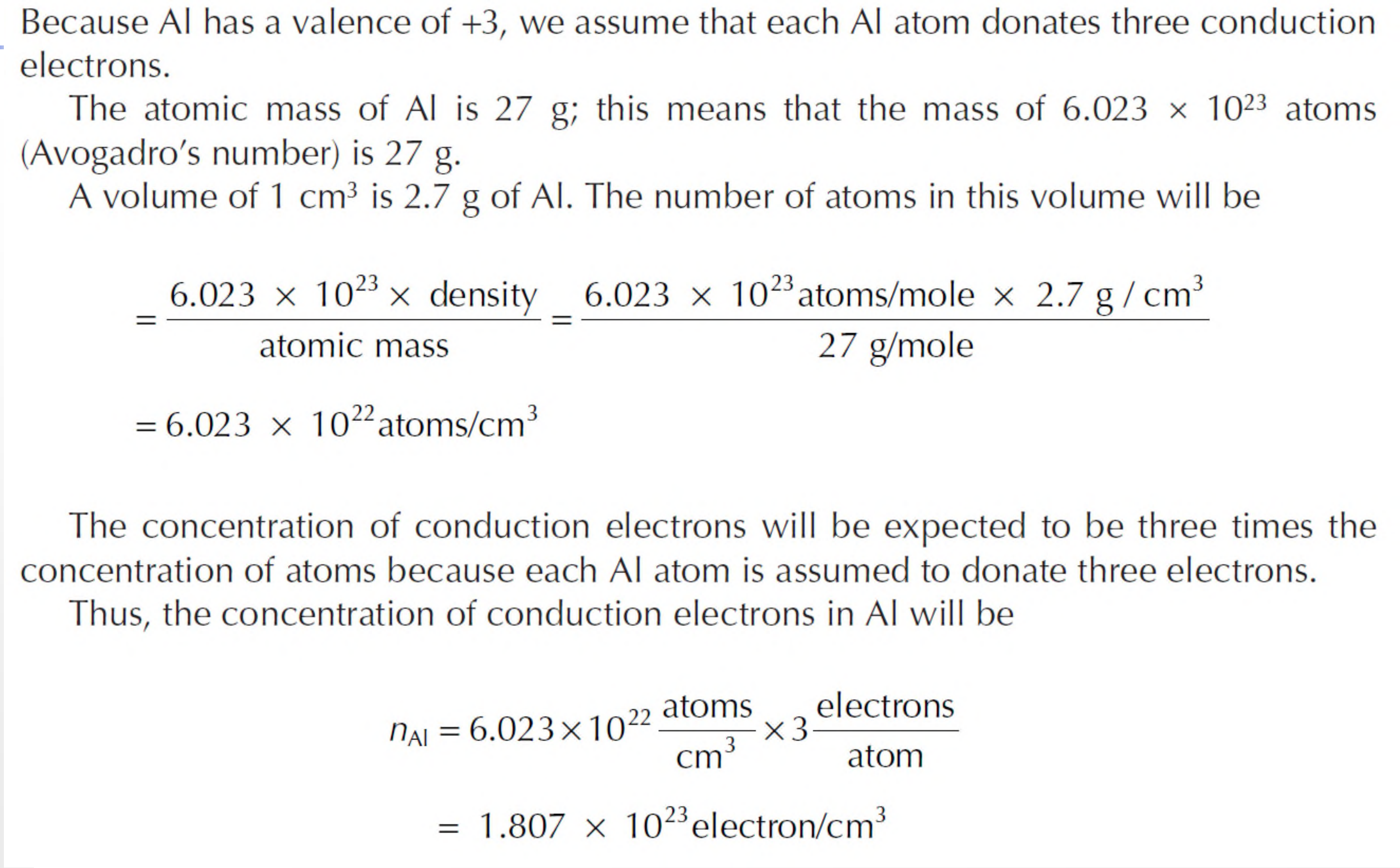
FIGURE 2.6 The structure of a sodium atom (atomic number 11), showing the nucleus surrounded by core electrons and valence electrons.

In metals, a large "sea of electrons" (unbound valence electrons) are the origin of free electrons. They move around within a metallic material under electric field, leading to a higher level of conductivity in metals.

In most solids, the atomic density ranges from 1 × 10^{22}/cm^3 to 1 × 10^{23}/cm^3 . For example, for sodium (Na), there is one valence electron per atom, and each sodium atom donates this valence electron to the sea of electrons. Therefore, the free electron density of Na is the same as the atomic density of Na metal (= density ÷ [atomic mass] × [Avogadro's number]) is 2.54 × 10^{22}/cm^3.

Example) Calculate the concentration of conduction electrons in Al. Assume that the density of Al is 2.73 g/cm^3.

Answer)



Because Al has a valence of +3, we assume that each Al atom donates three conduction electrons.

The atomic mass of Al is 27 g; this means that the mass of 6.023×10^{23} atoms (Avogadro's number) is 27 g.

A volume of 1 cm^3 is 2.7 g of Al. The number of atoms in this volume will be

$$= \frac{6.023 \times 10^{23} \times \text{density}}{\text{atomic mass}} = \frac{6.023 \times 10^{23} \text{atoms/mole} \times 2.7 \text{ g} / \text{cm}^3}{27 \text{ g/mole}}$$

$$= 6.023 \times 10^{22} \text{atoms/cm}^3$$

The concentration of conduction electrons will be expected to be three times the concentration of atoms because each Al atom is assumed to donate three electrons.

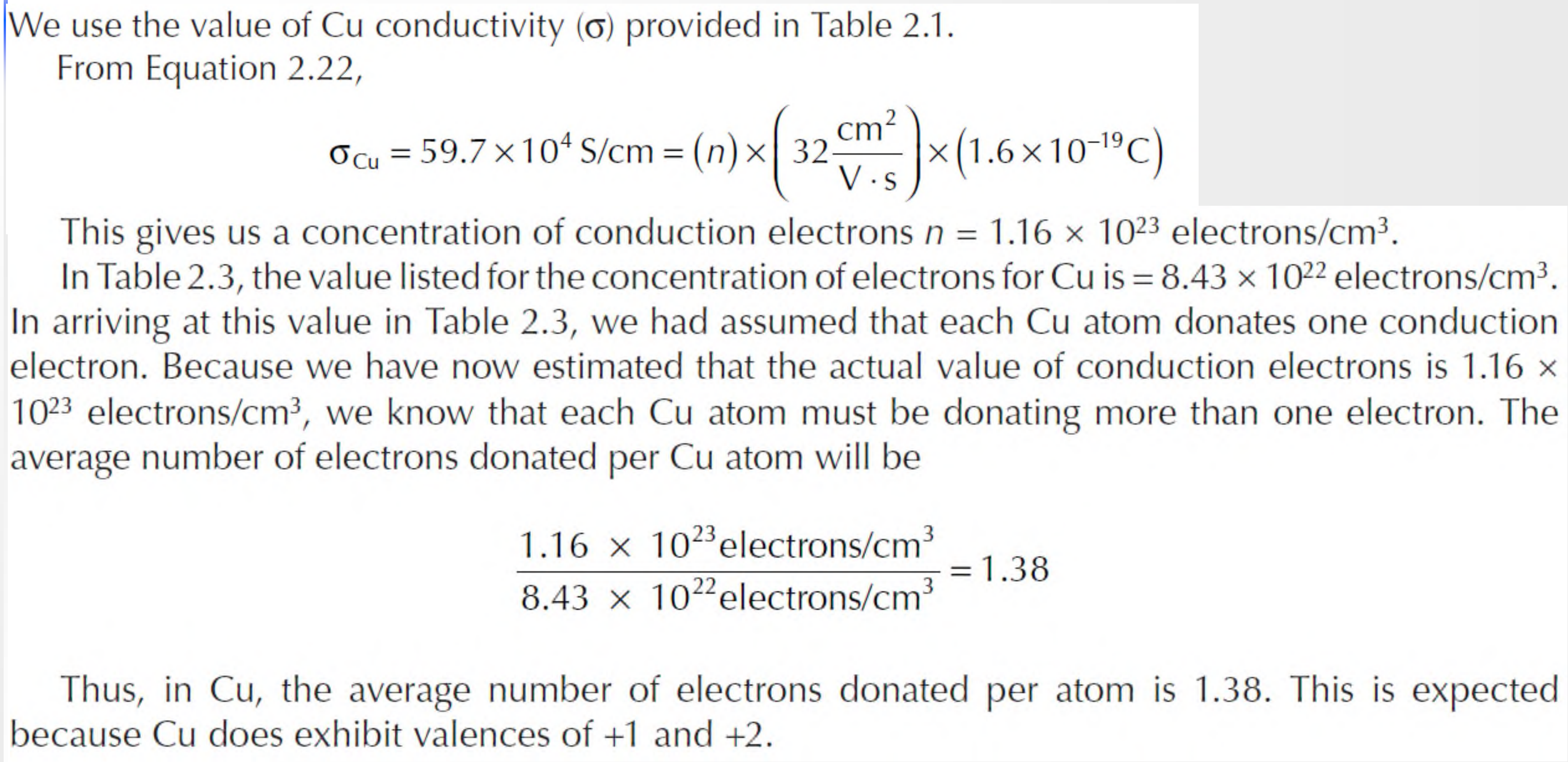
Thus, the concentration of conduction electrons in Al will be

$$n_{\text{Al}} = 6.023 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3} \times 3 \frac{\text{electrons}}{\text{atom}}$$

$$= 1.807 \times 10^{23} \text{electron/cm}^3$$

Example) The experimentally measured mobility of electrons in copper is 32 $cm^2/V \cdot s$. Calculate the carrier concentration (n) for Cu. Compare this with the values listed in Table 2.3. What does this show about the number of conduction electrons contributed per Cu atom?

Answer)



We use the value of Cu conductivity (σ) provided in Table 2.1.

From Equation 2.22,

$$\sigma_{Cu} = 59.7 \times 10^4 \text{ S/cm} = (n) \times \left(32 \frac{\text{cm}^2}{\text{V} \cdot \text{s}}\right) \times \left(1.6 \times 10^{-19} \text{C}\right)$$

This gives us a concentration of conduction electrons $n = 1.16 \times 10^{23}$ electrons/cm^3.

In Table 2.3, the value listed for the concentration of electrons for Cu is $= 8.43 \times 10^{22}$ electrons/cm^3. In arriving at this value in Table 2.3, we had assumed that each Cu atom donates one conduction electron. Because we have now estimated that the actual value of conduction electrons is 1.16×10^{23} electrons/cm^3, we know that each Cu atom must be donating more than one electron. The average number of electrons donated per Cu atom will be

$$\frac{1.16 \times 10^{23} \text{electrons/cm}^3}{8.43 \times 10^{22} \text{electrons/cm}^3} = 1.38$$

Thus, in Cu, the average number of electrons donated per atom is 1.38. This is expected because Cu does exhibit valences of +1 and +2.

cf) electron configuration of Cu: [Ar] $3d^{10}$ $4s^1$

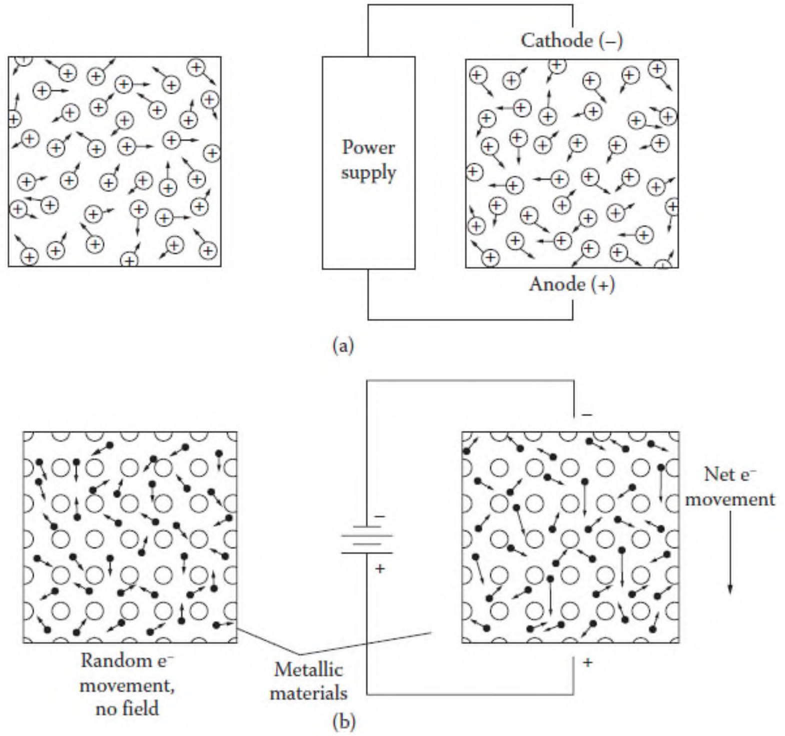


FIGURE 2.7 (a) Random motion of electrons due to thermal energy is similar to the movement of ions in an electroplating solution. (b) The drift of conduction electrons in a pure metal; note that the applied electric field (*E*) and drift are opposite, and overall, the electrons move within the material toward the anode.

Classical view of electric conduction is that 1) E-field causes a net movement of a small portion of charge carriers (free electrons in metals) and 2) charge carrier motion is limited by collisions with nucleus (similar to friction).

How to describe the motion of carriers (i.e. what is μ)?

In classical Drude's model, a sea of electrons in metals is called as "electron gas" or "plasma". Let's think about how free electrons in metals move under E-field and their motion is quantitatively estimated.

Drift of the electrons (motion induced by electric field):

The force (F) exerted on an electron by the electric field (E) is given by

$$F = qE$$

The force results in acceleration a, given by

$$F = ma$$

Thus, the acceleration experienced by the electrons is given by

$$a = qE/m$$

Because of this externally applied driving force (i.e., qE), conduction electrons begin to show a net movement. This motion of charge carriers (electrons, in this case) due to an applied electric field is known as *drift*. The drift of the carriers is characterized by drift velocity ($\boldsymbol{v}$). Note that the electrical current and free electron drift are in opposite directions, because electrons are negatively charged.

Drift velocity

If an electron accelerates and then collides with an atom after time τ, it stops. From Newton's Laws of Motion, we know that $v = u + a\tau$, where v is final velocity, u is initial velocity, a is acceleration, and τ is time. Applying this law here, assuming the initial speed is zero, we get $v = a\tau$. Thus, the average velocity of an electron between collisions is drift velocity (v_{drift}).

If the velocity is linearly proportional to time,

$$v_{\text{avg}} = (1/2)a\tau$$

We should not, however, consider the average time between collisions to calculate the average velocity, because electron velocity gradually decreases. For convenience, we can show that the average velocity or drift velocity (v_{drift}) is given by

$$v_{\text{drift}} = v_{\text{avg}} = a\tau$$

Since $a = qE/m$, $\quad v_{\text{drift}} = v_{\text{avg}} = (qE/m)\tau = (q\tau/m)E$

From $\sigma = Ne\mu$ (N: carrier concentration, μ: carrier mobility),

$$j = \sigma E = \text{Ne}\mu E = \text{Ne}v \text{ } (v\text{: velocity of carriers}) \ \& \ \mu = v/E$$

Mobility is a **proportionality constant** between electron velocity and E-field.

The *mean free-path length* (λ) of conduction electrons is defined as the average distance that the electrons travel before colliding again, and it is obtained by multiplying the average speed (v_{avg}) by the time between collisions (τ) :

$$\lambda = v_{avg} \times \tau \quad \text{(in metals, } \lambda \sim 100\text{nm)}$$

Thus, the current density (J) is given by $J = n \times v_{\text{drift}} \times q$

If velocity is replaced, $J = n \times \left(\frac{q\tau}{m}\right) E \times q$ or $J = \left(\frac{nq^2\tau}{m}\right) E$

Then, the electric conductivity becomes $\sigma = \left(\frac{nq^2\tau}{m}\right)$

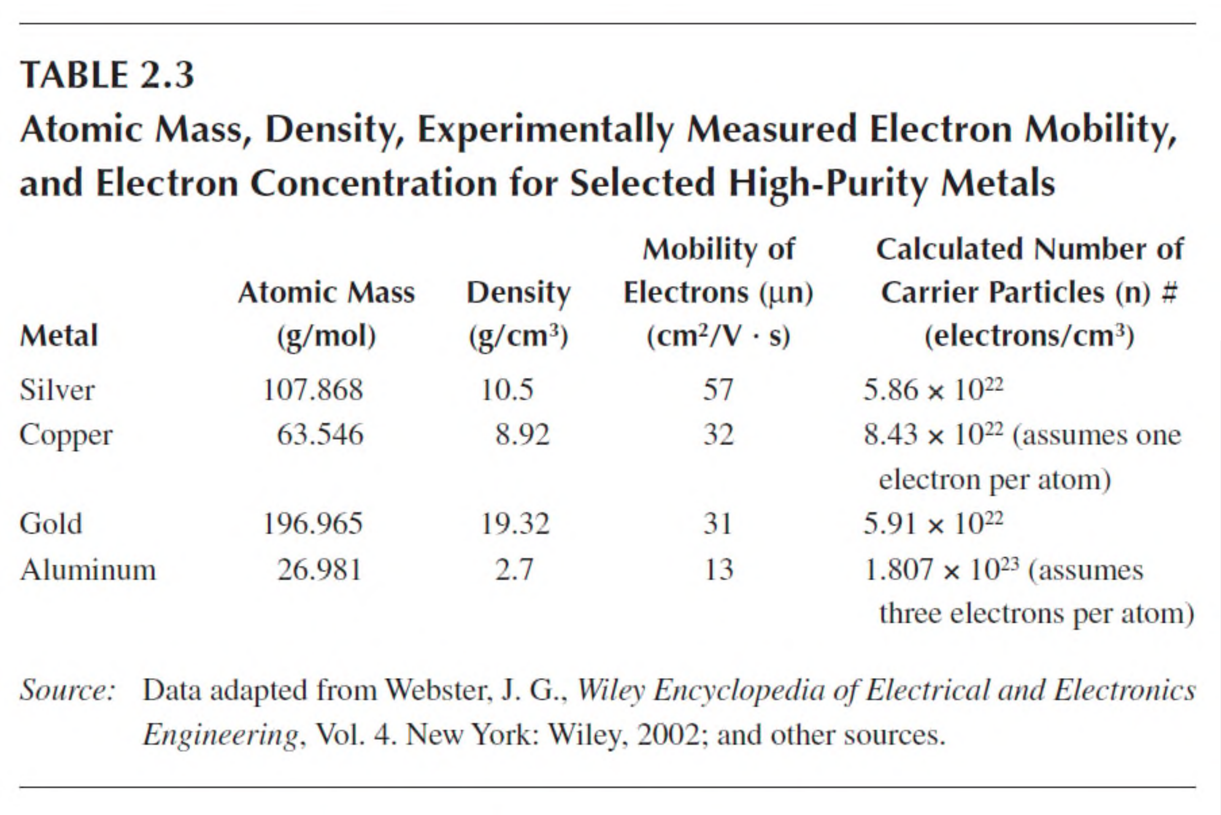


TABLE 2.3

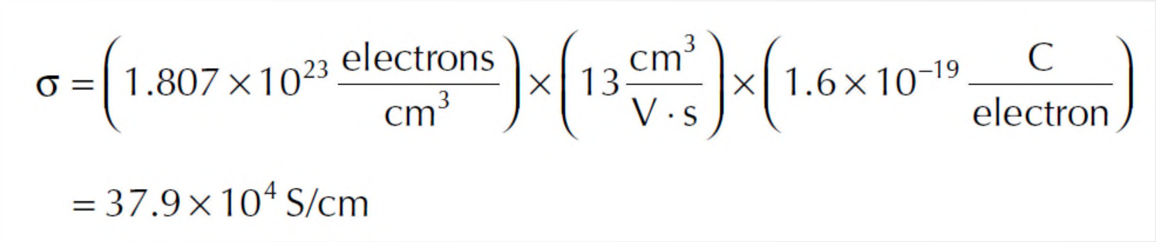
Atomic Mass, Density, Experimentally Measured Electron Mobility, and Electron Concentration for Selected High-Purity Metals

Metal	Atomic Mass (g/mol)	Density (g/cm³)	Mobility of Electrons (µn) (cm²/V · s)	Calculated Number of Carrier Particles (n) # (electrons/cm³)
Silver	107.868	10.5	57	5.86×10^{22}
Copper	63.546	8.92	32	8.43×10^{22} (assumes one electron per atom)
Gold	196.965	19.32	31	5.91×10^{22}
Aluminum	26.981	2.7	13	1.807×10^{23} (assumes three electrons per atom)

Source: Data adapted from Webster, J. G., *Wiley Encyclopedia of Electrical and Electronics Engineering*, Vol. 4. New York: Wiley, 2002; and other sources.

Example) Assume that the mobility of electron in Al is 13 cm^2/V·s. Calculate the expected conductivity of Al.

Answer) The conductivity of Al is given by the expression



$$\sigma = \left(1.807\times10^{23}\,\frac{\text{electrons}}{\text{cm}^3}\right)\times\left(13\,\frac{\text{cm}^3}{\text{V}\cdot\text{s}}\right)\times\left(1.6\times10^{-19}\,\frac{\text{C}}{\text{electron}}\right)$$

$$= 37.9\times10^{4}\ \text{S/cm}$$

cf) Real conductivity of Al: 37.7 × 10^4 S/cm

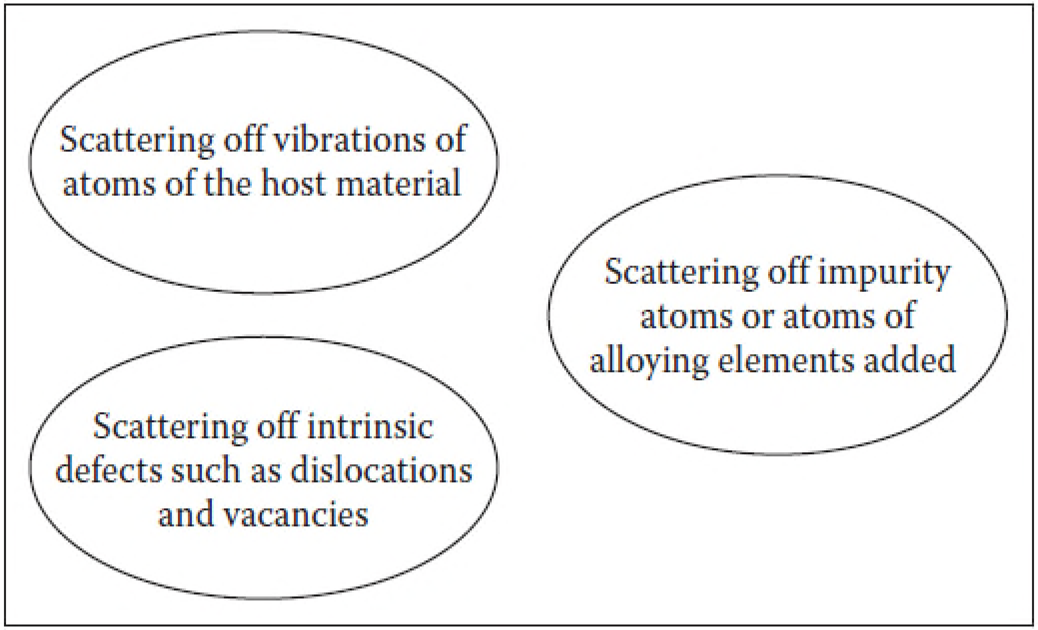


FIGURE 2.8 Schematic showing the sources of the scattering of conduction electrons in a metallic material.

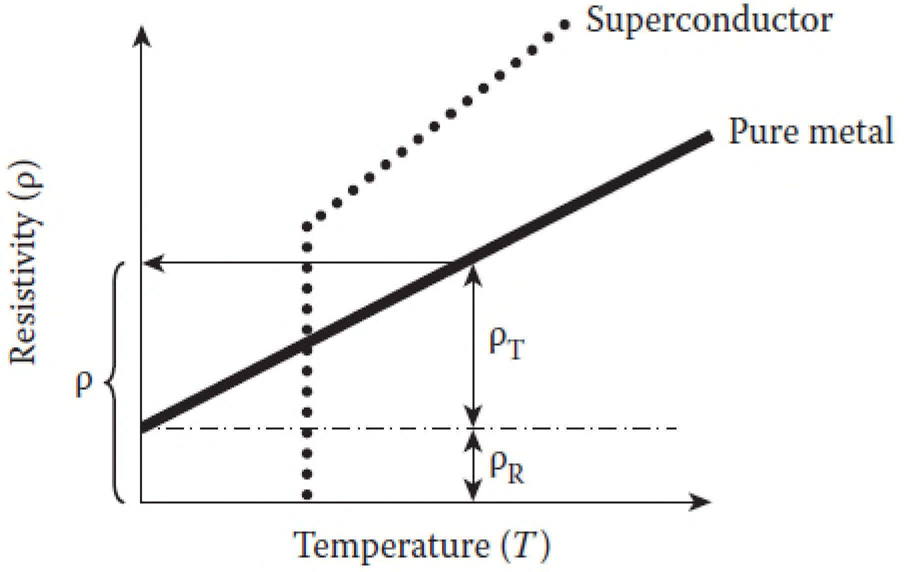


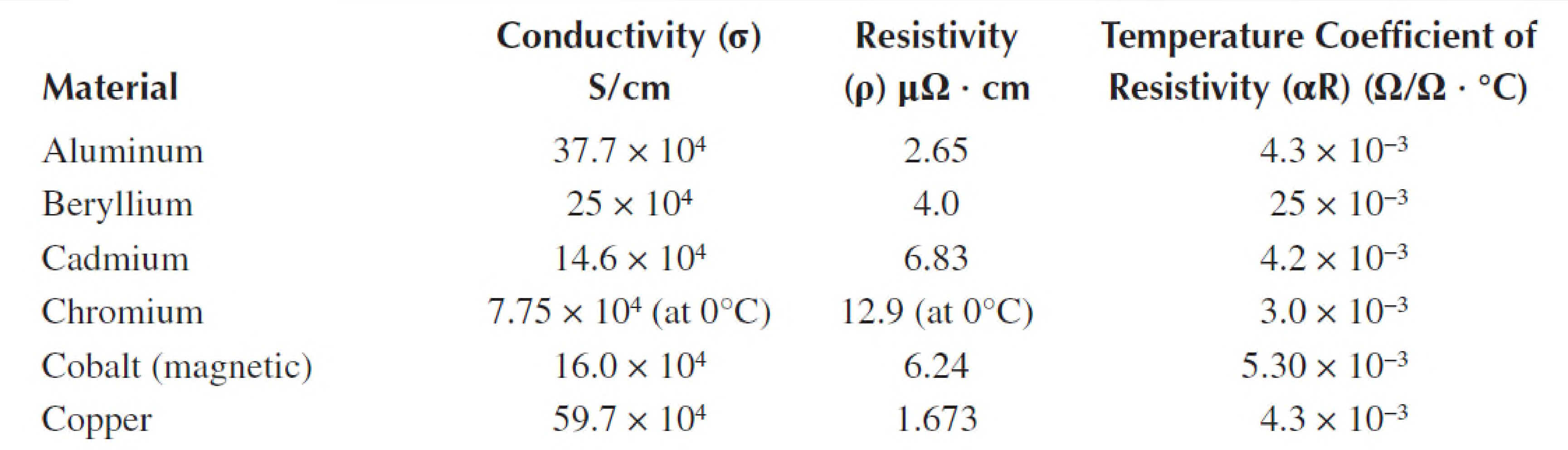
FIGURE 2.9 The temperature dependence of the conductivity of a typical metal and a superconductor.

First, atoms in a metallic material vibrate. We refer to the vibrations of atoms in a material as phonons. The conduction electrons carrying the electrical current bump into or scatter off vibrations of these atoms. The process of electron-scattering of a phonon is temperature-dependent. As the temperature increases, the number of electrons scattering off due to the vibrations of atoms also increases. Thus, the resistivity of a pure metal increases with increasing temperature. This increase in resistivity due to the scattering of electrons by lattice vibrations is linear in general.

The values of a parameter known as the *temperature coefficient of resistivity* (α_R; TCR).

$$\rho = AT + B$$

$$\alpha_R = \frac{1}{\rho_0}\left(\frac{\rho - \rho_0}{T - T_0}\right)$$



Material	Conductivity (σ) S/cm	Resistivity (ρ) μΩ · cm	Temperature Coefficient of Resistivity (αR) (Ω/Ω · °C)
Aluminum	37.7×10^4	2.65	4.3×10^{-3}
Beryllium	25×10^4	4.0	25×10^{-3}
Cadmium	14.6×10^4	6.83	4.2×10^{-3}
Chromium	7.75×10^4 (at 0°C)	12.9 (at 0°C)	3.0×10^{-3}
Cobalt (magnetic)	16.0×10^4	6.24	5.30×10^{-3}
Copper	59.7×10^4	1.673	4.3×10^{-3}

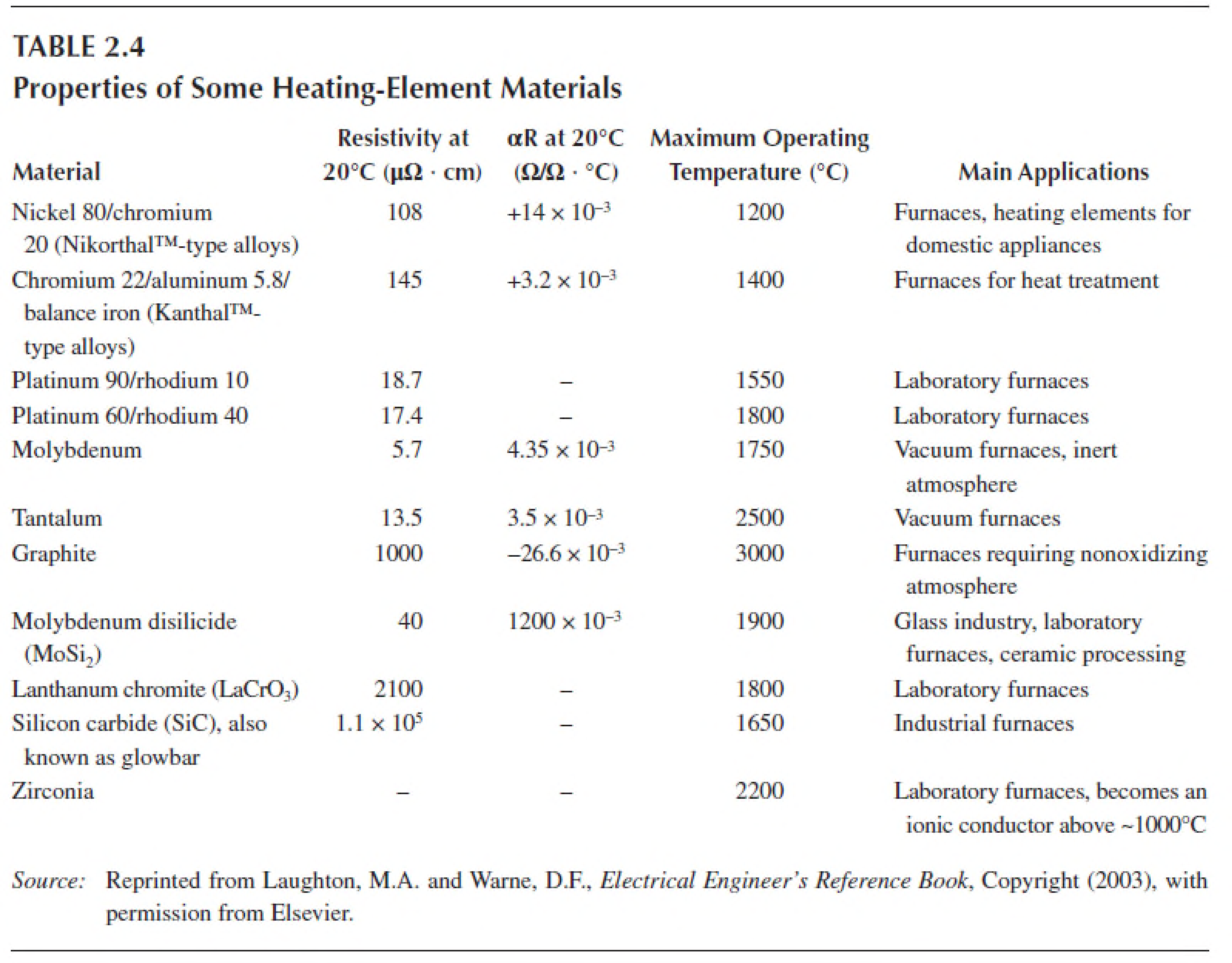


TABLE 2.4
Properties of Some Heating-Element Materials

Material	Resistivity at 20°C (μΩ · cm)	αR at 20°C (Ω/Ω · °C)	Maximum Operating Temperature (°C)	Main Applications
Nickel 80/chromium 20 (Nikorthal™-type alloys)	108	$+14 \times 10^{-3}$	1200	Furnaces, heating elements for domestic appliances
Chromium 22/aluminum 5.8/balance iron (Kanthal™-type alloys)	145	$+3.2 \times 10^{-3}$	1400	Furnaces for heat treatment
Platinum 90/rhodium 10	18.7	–	1550	Laboratory furnaces
Platinum 60/rhodium 40	17.4	–	1800	Laboratory furnaces
Molybdenum	5.7	4.35×10^{-3}	1750	Vacuum furnaces, inert atmosphere
Tantalum	13.5	3.5×10^{-3}	2500	Vacuum furnaces
Graphite	1000	-26.6×10^{-3}	3000	Furnaces requiring nonoxidizing atmosphere
Molybdenum disilicide ($MoSi_2$)	40	1200×10^{-3}	1900	Glass industry, laboratory furnaces, ceramic processing
Lanthanum chromite ($LaCrO_3$)	2100	–	1800	Laboratory furnaces
Silicon carbide (SiC), also known as glowbar	1.1×10^5	–	1650	Industrial furnaces
Zirconia	–	–	2200	Laboratory furnaces, becomes an ionic conductor above ~1000°C

Source: Reprinted from Laughton, M.A. and Warne, D.F., *Electrical Engineer's Reference Book*, Copyright (2003), with permission from Elsevier.

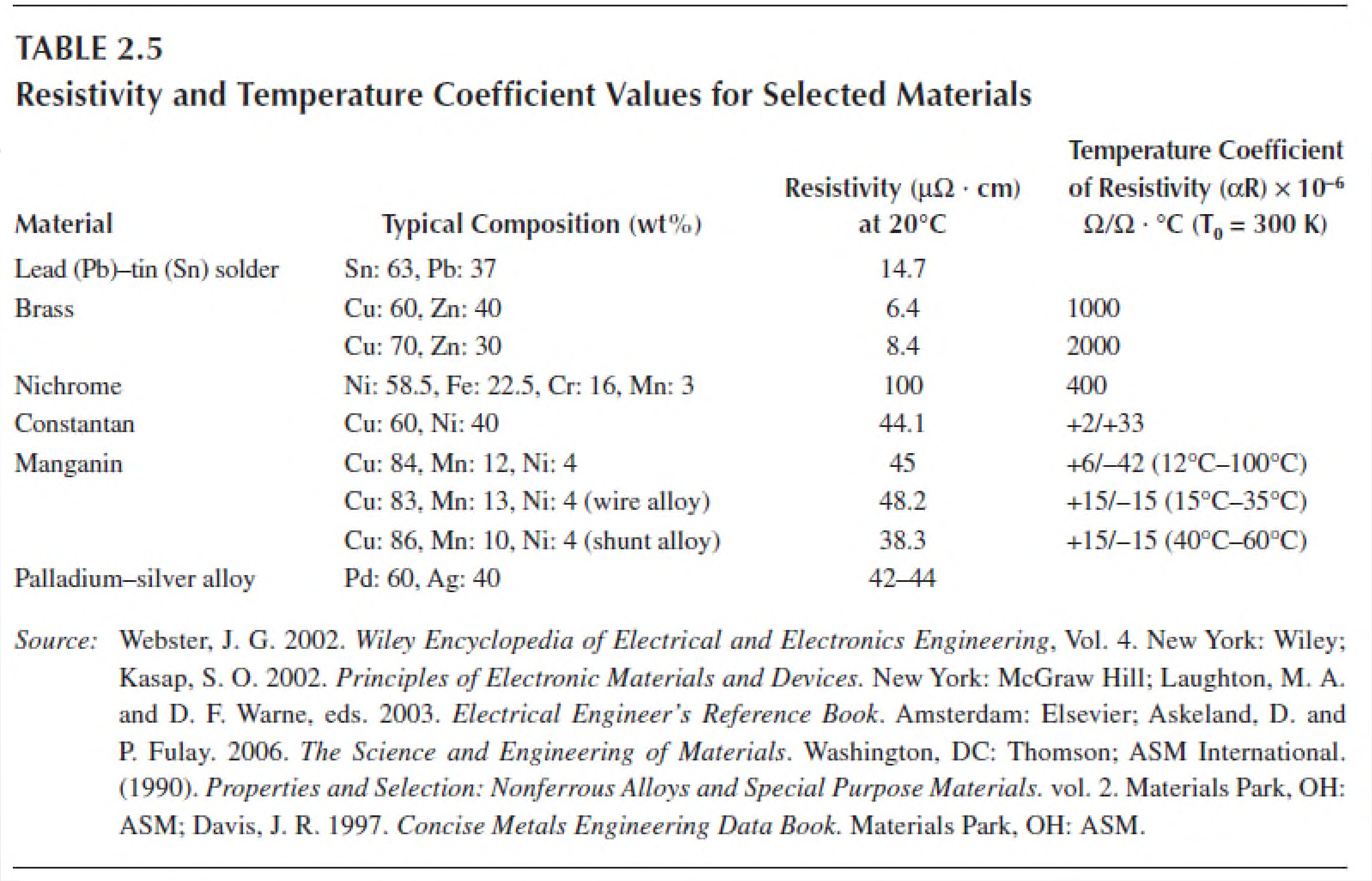


TABLE 2.5

Resistivity and Temperature Coefficient Values for Selected Materials

Material	Typical Composition (wt%)	Resistivity (μΩ · cm) at 20°C	Temperature Coefficient of Resistivity (αR) × 10^{-6} Ω/Ω · °C (T_0 = 300 K)
Lead (Pb)–tin (Sn) solder	Sn: 63, Pb: 37	14.7	
Brass	Cu: 60, Zn: 40	6.4	1000
	Cu: 70, Zn: 30	8.4	2000
Nichrome	Ni: 58.5, Fe: 22.5, Cr: 16, Mn: 3	100	400
Constantan	Cu: 60, Ni: 40	44.1	+2/+33
Manganin	Cu: 84, Mn: 12, Ni: 4	45	+6/–42 (12°C–100°C)
	Cu: 83, Mn: 13, Ni: 4 (wire alloy)	48.2	+15/–15 (15°C–35°C)
	Cu: 86, Mn: 10, Ni: 4 (shunt alloy)	38.3	+15/–15 (40°C–60°C)
Palladium–silver alloy	Pd: 60, Ag: 40	42–44	

Source: Webster, J. G. 2002. *Wiley Encyclopedia of Electrical and Electronics Engineering*, Vol. 4. New York: Wiley; Kasap, S. O. 2002. *Principles of Electronic Materials and Devices*. New York: McGraw Hill; Laughton, M. A. and D. F. Warne, eds. 2003. *Electrical Engineer's Reference Book*. Amsterdam: Elsevier; Askeland, D. and P. Fulay. 2006. *The Science and Engineering of Materials*. Washington, DC: Thomson; ASM International. (1990). *Properties and Selection: Nonferrous Alloys and Special Purpose Materials*. vol. 2. Materials Park, OH: ASM; Davis, J. R. 1997. *Concise Metals Engineering Data Book*. Materials Park, OH: ASM.

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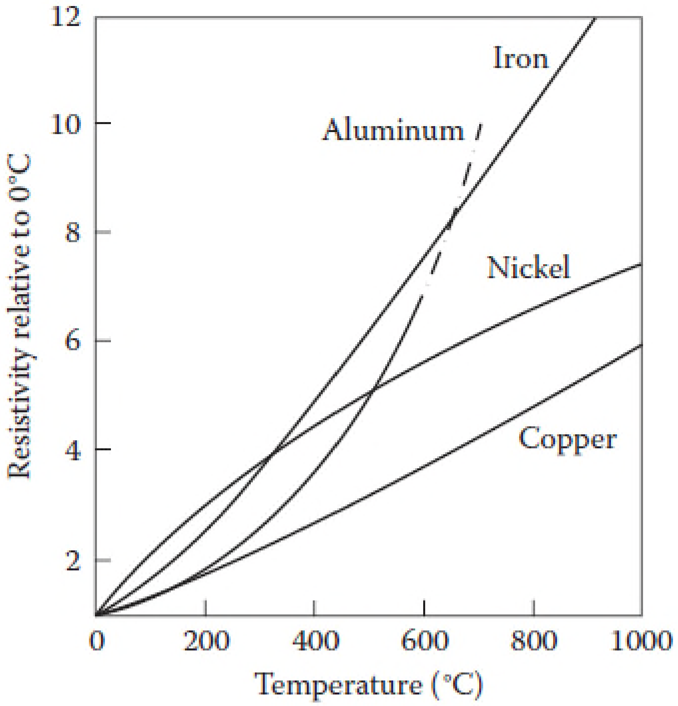

FIGURE 2.10 Relative change in resistivity with temperature (relative to 0°C) for Al, Cu, Ni, and Fe. (Reprinted from *Electrical Engineer's Reference Book*, Laughton, M. A., and D. F. Warne, Copyright 2003, with permission from Elsevier.)

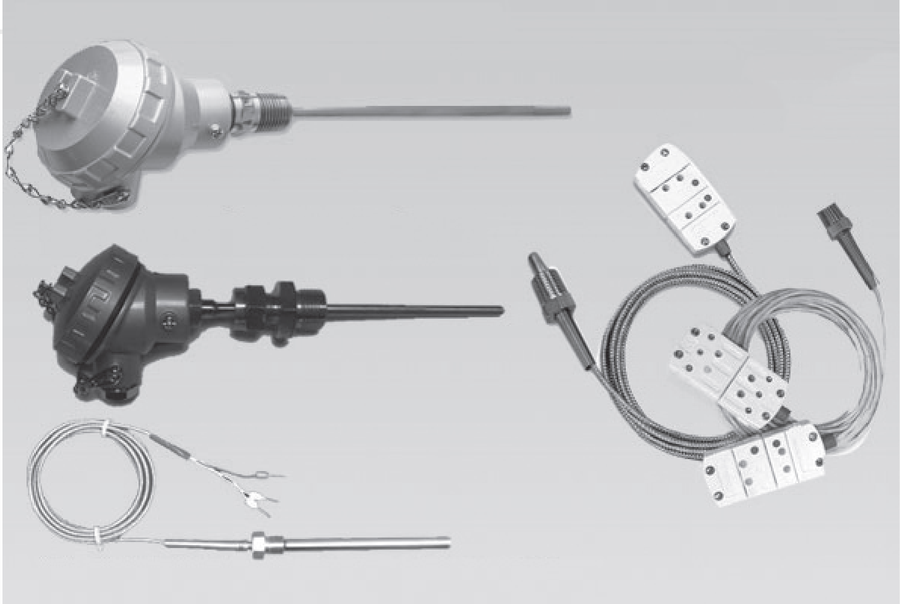

FIGURE 2.11 Commercially available resistance–temperature detector devices. (Courtesy of Omega Corporation, San Diego, CA.)

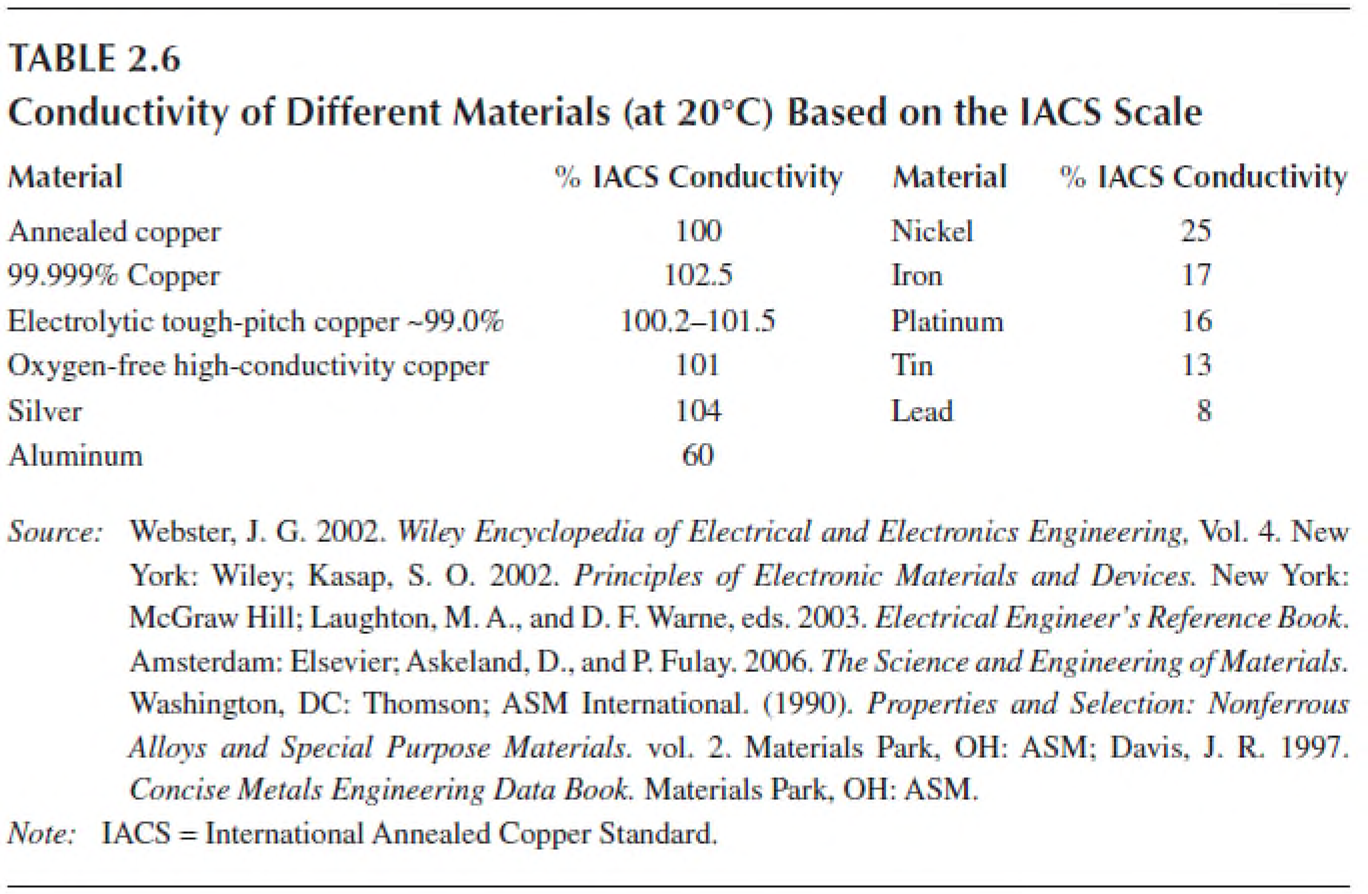

TABLE 2.6

Conductivity of Different Materials (at 20°C) Based on the IACS Scale

Material	% IACS Conductivity	Material	% IACS Conductivity
Annealed copper	100	Nickel	25
99.999% Copper	102.5	Iron	17
Electrolytic tough-pitch copper ~99.0%	100.2–101.5	Platinum	16
Oxygen-free high-conductivity copper	101	Tin	13
Silver	104	Lead	8
Aluminum	60		

Source: Webster, J. G. 2002. *Wiley Encyclopedia of Electrical and Electronics Engineering*, Vol. 4. New York: Wiley; Kasap, S. O. 2002. *Principles of Electronic Materials and Devices*. New York: McGraw Hill; Laughton, M. A., and D. F. Warne, eds. 2003. *Electrical Engineer's Reference Book*. Amsterdam: Elsevier; Askeland, D., and P. Fulay. 2006. *The Science and Engineering of Materials*. Washington, DC: Thomson; ASM International. (1990). *Properties and Selection: Nonferrous Alloys and Special Purpose Materials*. vol. 2. Materials Park, OH: ASM; Davis, J. R. 1997. *Concise Metals Engineering Data Book*. Materials Park, OH: ASM.

Note: IACS = International Annealed Copper Standard.

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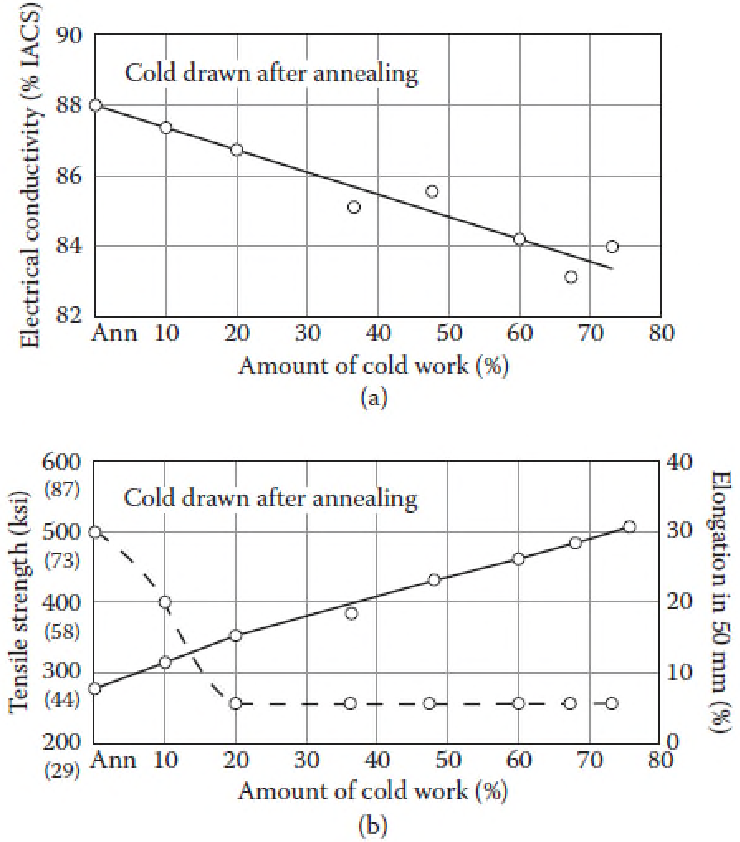

The second factor that affects the mobility of metals is the scattering of electrons off defects such as vacancies, dislocations, and grain boundaries. The concentrations of different defects (i.e. scattering centers) will depend on the microstructure of the material.

FIGURE 2.12 (a) Electrical conductivity of sterling silver (92.5% Ag−7.5% Cu), shown as %IACS, and (b) increase in tensile strength (solid line) and decrease in percentage of elongation (dotted line) as a function of % cold work. The data are for a 2.3-mm wire, which was cold-drawn after annealing. (From ASM International (1990). *Properties and Selection: Nonferrous Alloys and Special Purpose Materials*, Vol. 2, ASM, Materials Park, OH. With permission.)

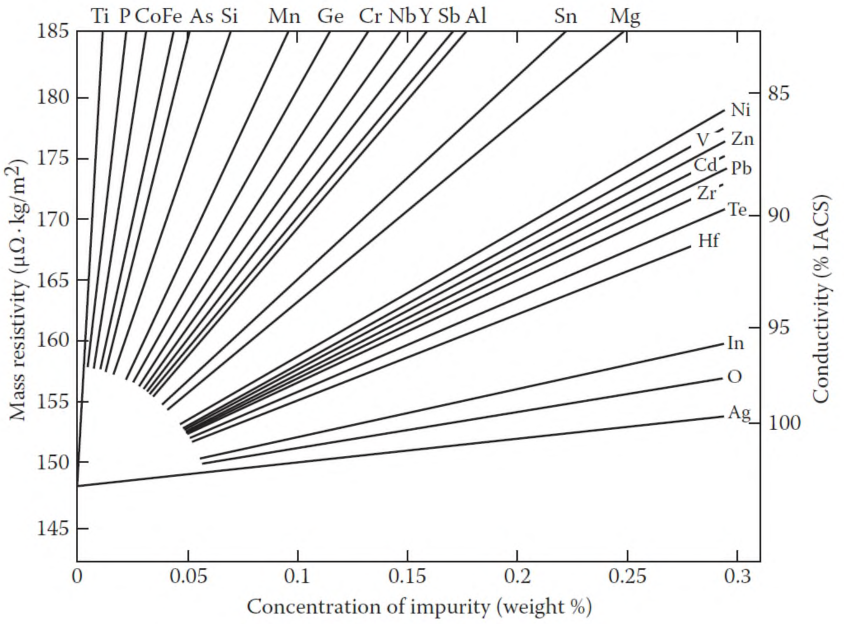

FIGURE 2.13 Effects of different impurities on the conductivity of copper. (Courtesy of Copper Development Association, New York, NY.)

The third factor that influences the resistivity of metallic materials is the presence of impurities or other elements deliberately or accidentally added. Impurities cause an increase in electrical resistivity because the atoms of the added elements act as scattering centers. The effect due to the presence of impurities is generally *not* very sensitive to temperature changes.

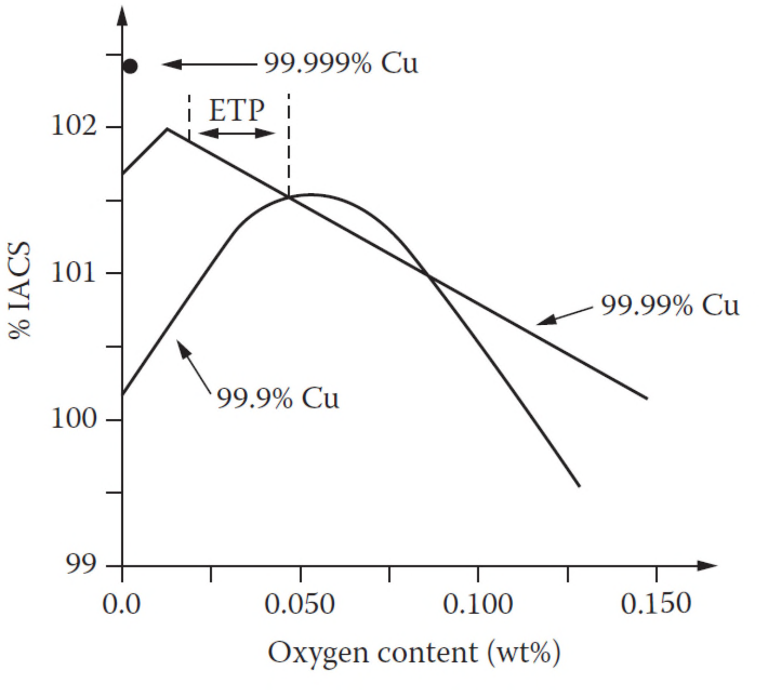

FIGURE 2.14 Effect of oxygen concentration on the conductivity of copper. (Courtesy of Copper Development Association, New York, NY.)

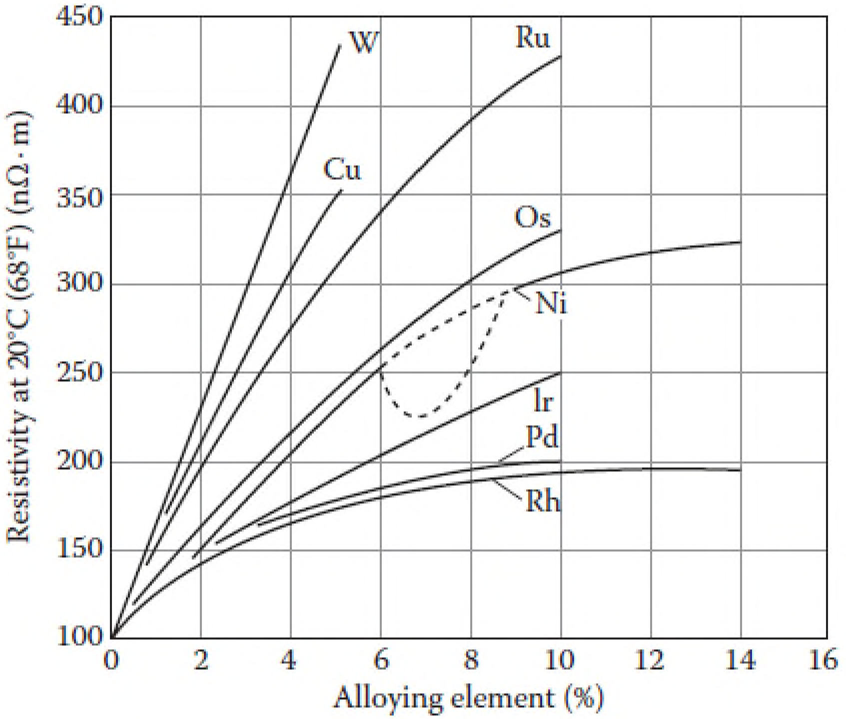

FIGURE 2.15 Effects of additions of various alloying elements on the resistivity of platinum. (From Vines, R.F. and Wise, E.M., *The Platinum Metals and Their Alloys*, International Nickel Co., New York, 1941. With permission.)

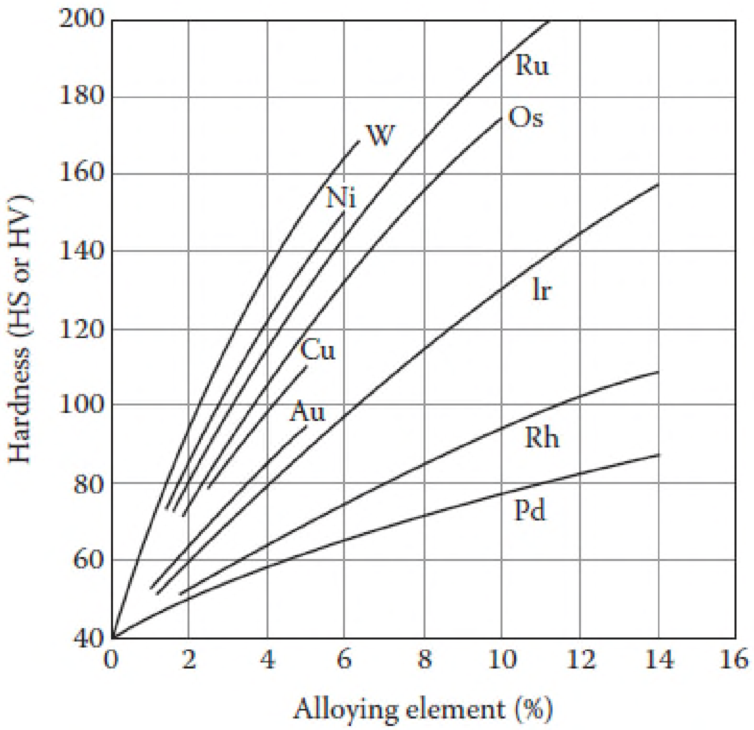

FIGURE 2.16 Increase in the hardness of platinum due to the addition of various alloying elements. (From Lampman, S.R. and Zorc, T.B., eds., *Metals Handbook: Properties and Selection: Nonferrous Alloys and Special Purpose Materials*, ASM, Materials Park, OH, 1990. With permission.)

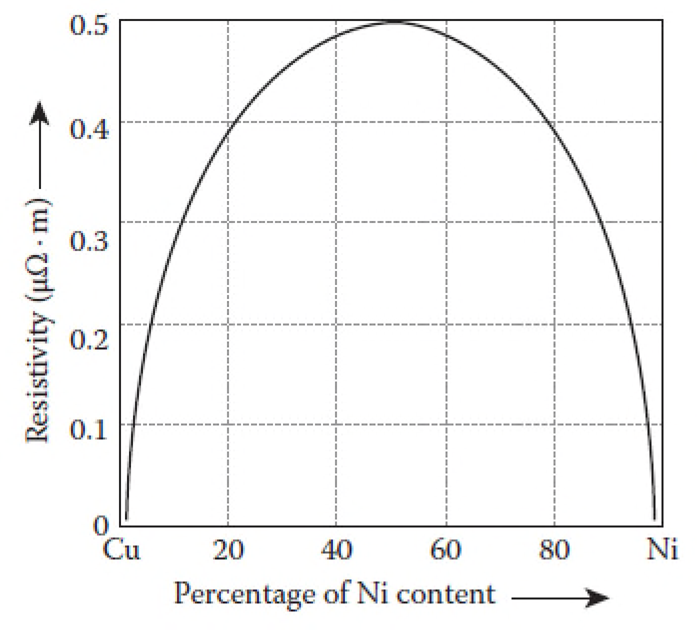

FIGURE 2.17 Variation in the resistivity of copper as a function of nickel concentration. (From Neelkanta, P., *Handbook of Electromagnetic Materials*, CRC Press, Boca Raton, FL, 1995. With permission.)

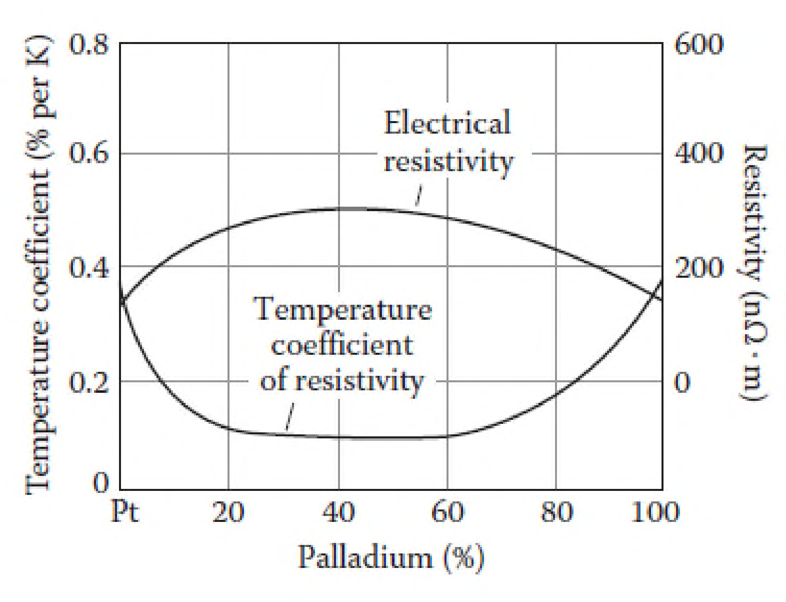

FIGURE 2.18 Electrical resistivity (in nΩ m) and temperature coefficient of resistivity (%/K) for platinum–palladium (Pt–Pd) alloys. (From Lampman, S.R. and Zorc, T.B., eds., *Metals Handbook: Properties and Selection: Nonferrous Alloys and Special Purpose Materials*, ASM, Materials Park, OH, 1990. With permission.)

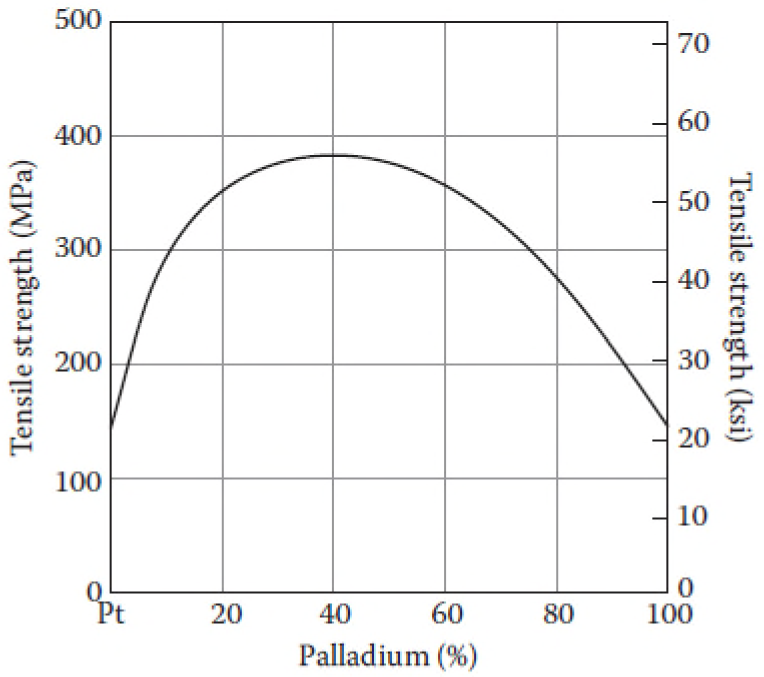

FIGURE 2.19 The change in tensile strength (in MPa on the left *y*-axis and in ksi on the right *y*-axis) of annealed platinum–palladium alloys as a function of palladium concentration. (From Lampman, S.R. and Zorc, T.B., eds., *Metals Handbook: Properties and Selection: Nonferrous Alloys and Special Purpose Materials*, ASM, Materials Park, OH, 1990. With permission.)

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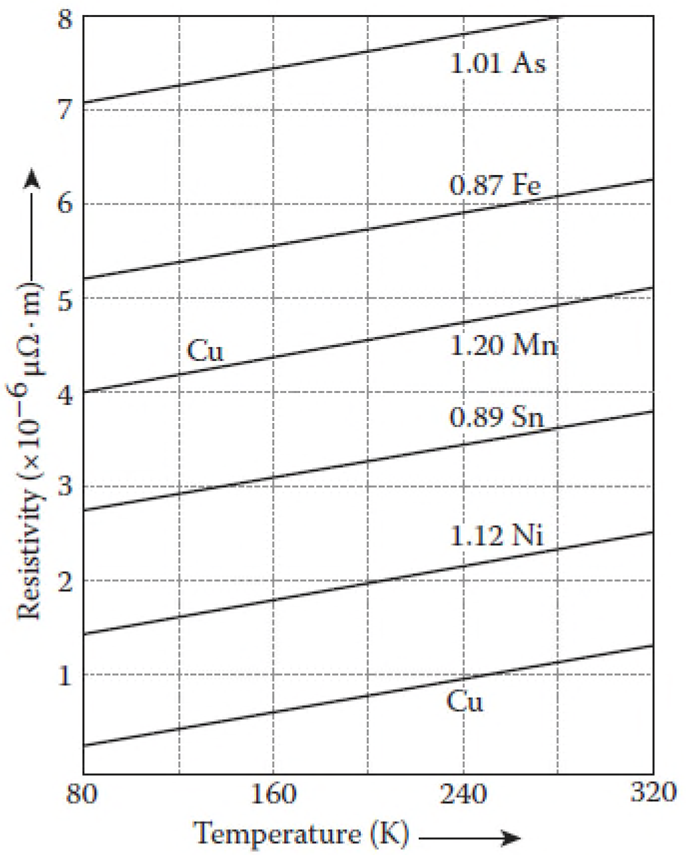


FIGURE 2.20 The resistivity of some copper alloys as a function of temperature. The data for resistivity change of copper is also shown. (From Neelkanta, P., *Handbook of Electromagnetic Materials*, CRC Press, Boca Raton, FL, 1995. With permission.)

Limitation of the Classical Theory of Conductivity

The classical theory based on the delocalization of valence electrons, although useful, does not explain several key features associated with the electrical conductivity of metals. For example, in silicon or diamond (a form of carbon [C]), each atom has four valence electrons, their free charge carrier concentration is much smaller than the valence electron concentration.

Classical theory cannot accurately predict the relationship between the *thermal conductivity* and electrical conductivity of metals. If all electrons in the “sea of electrons” move freely and are responsible for the high electric conductivity of metals, these free electrons should also contribute to the thermal conductivity. However, succeeding quantitative studies show that the thermal conductivity predicted by classical theory is two orders of magnitude lower than the experimentally observed thermal conductivity. Unless the heat capacity of free electrons is two orders of magnitude higher than the measured value, classical theory cannot explain a correlation between the thermal conductivity and the electric conductivity.

Classical model needs to be changed.

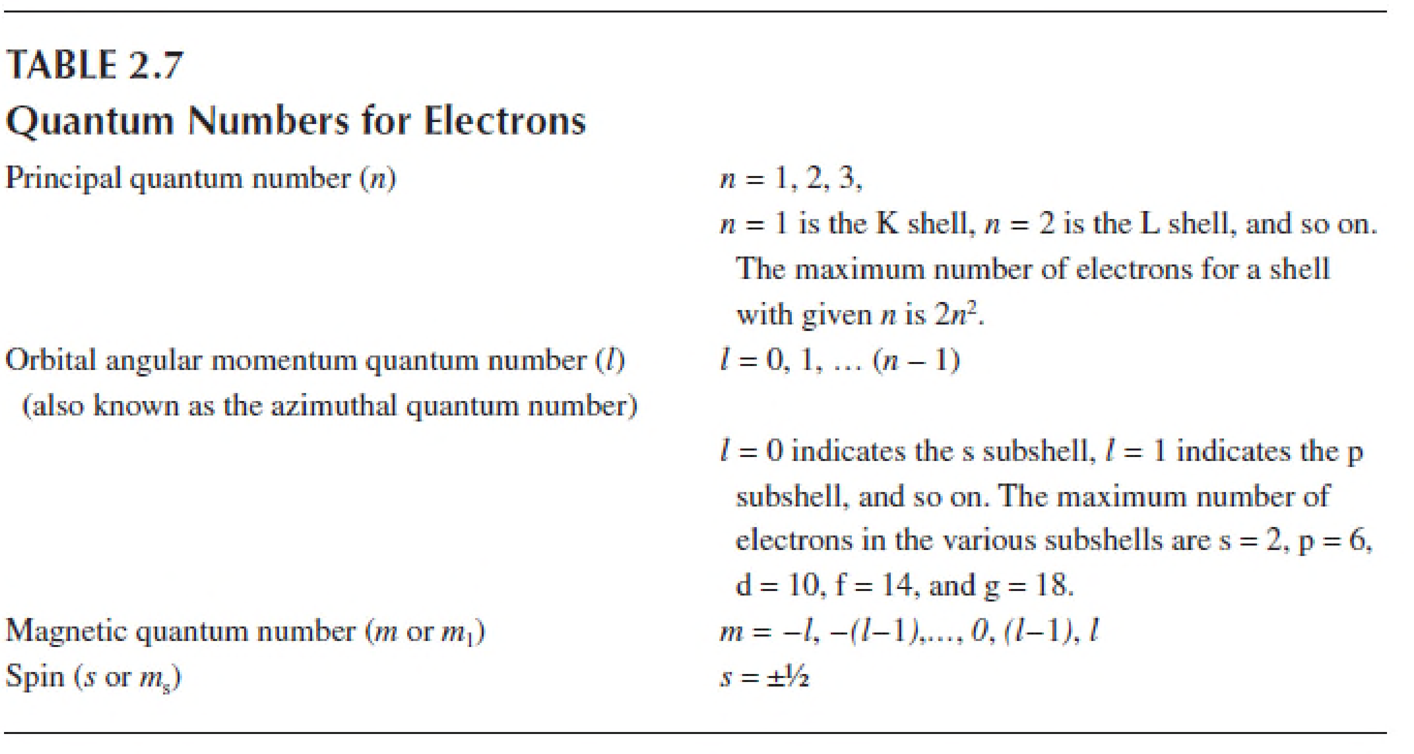


TABLE 2.7

Quantum Numbers for Electrons

Principal quantum number (n)	$n = 1, 2, 3,$ $n = 1$ is the K shell, $n = 2$ is the L shell, and so on. The maximum number of electrons for a shell with given n is $2n^2$.
Orbital angular momentum quantum number (l) (also known as the azimuthal quantum number)	$l = 0, 1, \ldots (n - 1)$ $l = 0$ indicates the s subshell, $l = 1$ indicates the p subshell, and so on. The maximum number of electrons in the various subshells are s = 2, p = 6, d = 10, f = 14, and g = 18.
Magnetic quantum number (m or m_l)	$m = -l, -(l-1), \ldots, 0, (l-1), l$
Spin (s or m_s)	$s = \pm\frac{1}{2}$

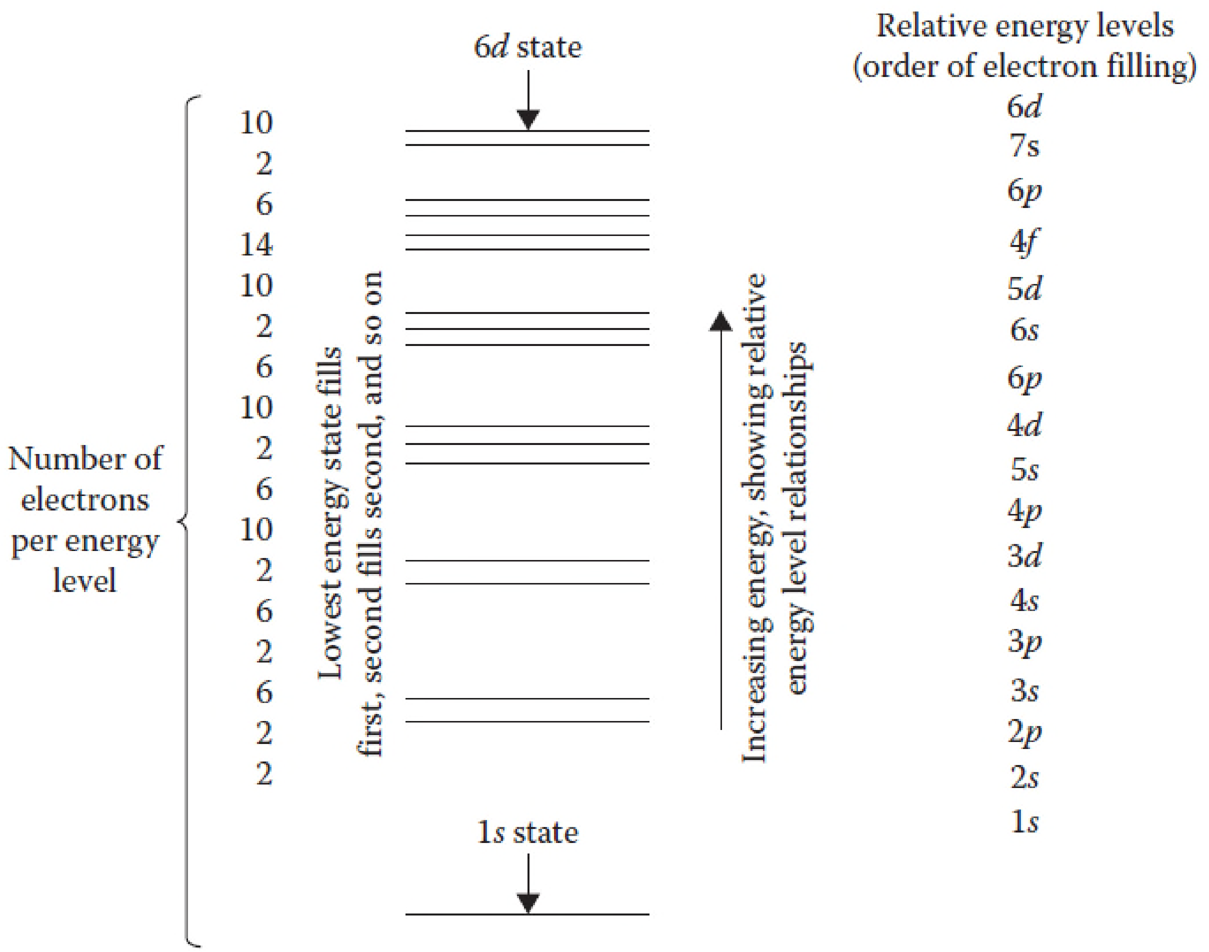


FIGURE 2.21 The order and number of electrons for different elements. (From Minges, M.L., *Electronic Materials Handbook*, Vol. 1, ASM, Materials Park, OH, 1989. With permission.)

Electron configuration of isolated atoms

The electronic configuration of Al (13 electrons): $1s^2 2s^2 2p^6 3s^2 3p^1$
partially empty 3p shell

The electronic configuration of Fe (26 electrons): $1s^2 2s^2 2p^6 3s^2 3p^6 3d^8$
partially empty 3p shell

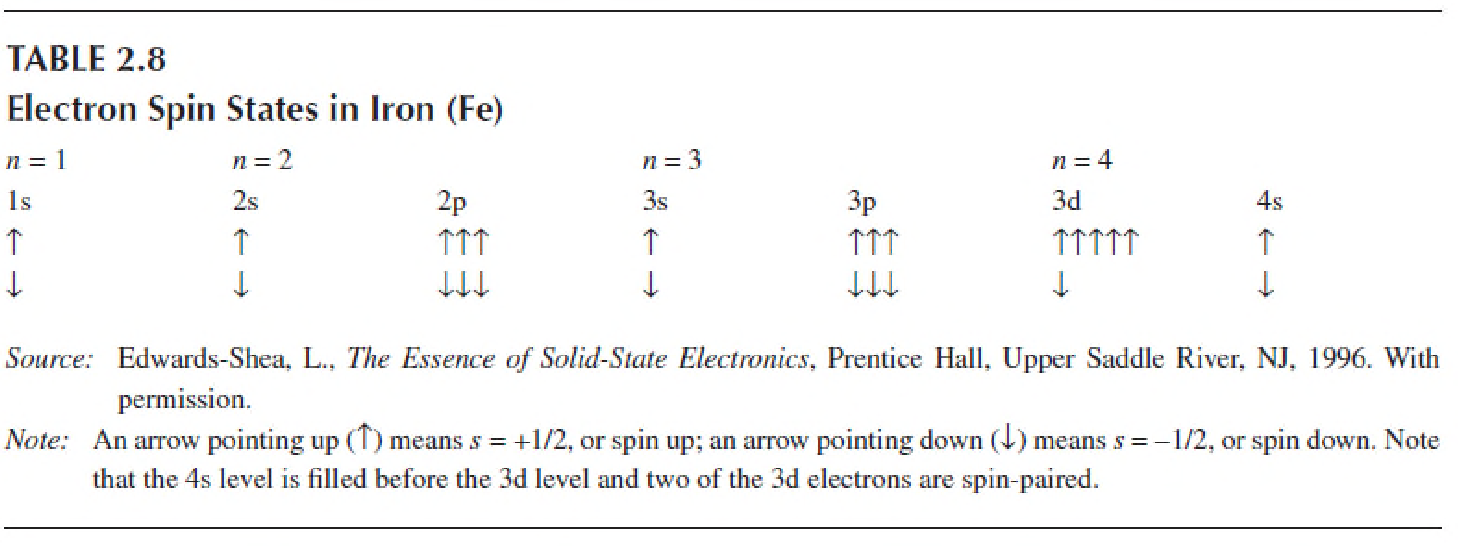


TABLE 2.8

Electron Spin States in Iron (Fe)

n = 1	*n* = 2		*n* = 3		*n* = 4	
1s	2s	2p	3s	3p	3d	4s
↑	↑	↑↑↑	↑	↑↑↑	↑↑↑↑↑	↑
↓	↓	↓↓↓	↓	↓↓↓	↓	↓

Source: Edwards-Shea, L., *The Essence of Solid-State Electronics*, Prentice Hall, Upper Saddle River, NJ, 1996. With permission.

Note: An arrow pointing up (↑) means $s = +1/2$, or spin up; an arrow pointing down (↓) means $s = -1/2$, or spin down. Note that the 4s level is filled before the 3d level and two of the 3d electrons are spin-paired.

A solid material can have billions of atoms. It is important to know not just the electronic structure of individual atoms but also how different atoms interact with one another when they are in close proximity to one another. When individual atoms are brought together to form a solid material, the atomic distance decreases and the electron orbitals begin to overlap.

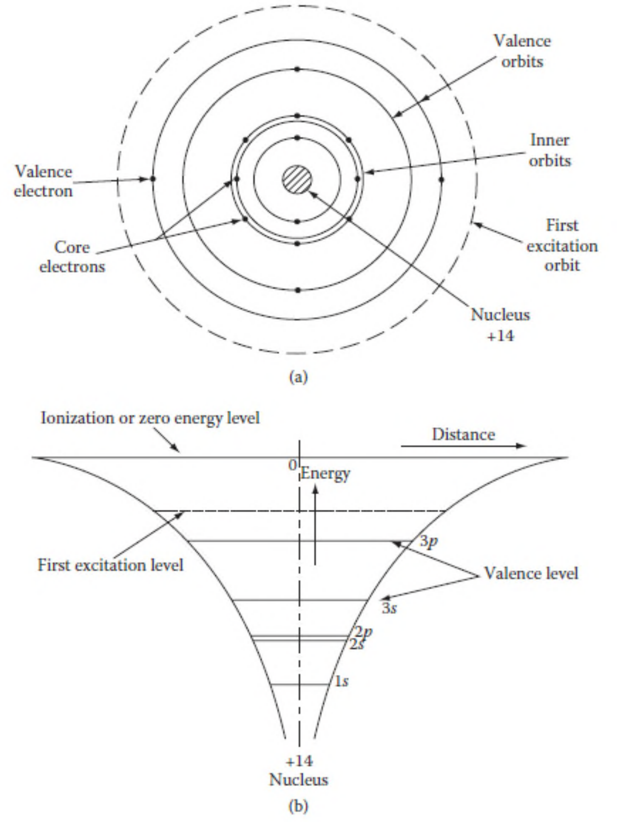


FIGURE 2.22 Electronic structure and energy levels in a Si atom: (a) The orbital model of a Si atom showing the ten core electrons (n = 1 and 2) and the four valence electrons (n = 3); (b) energy levels in the Coulombic potential of the nucleus are also shown schematically. (From Streetman, B.G. and Banerjee, S., *Solid State Electronic Devices*, 5th ed., Prentice Hall, Upper Saddle River, NJ, 2000. With permission.)

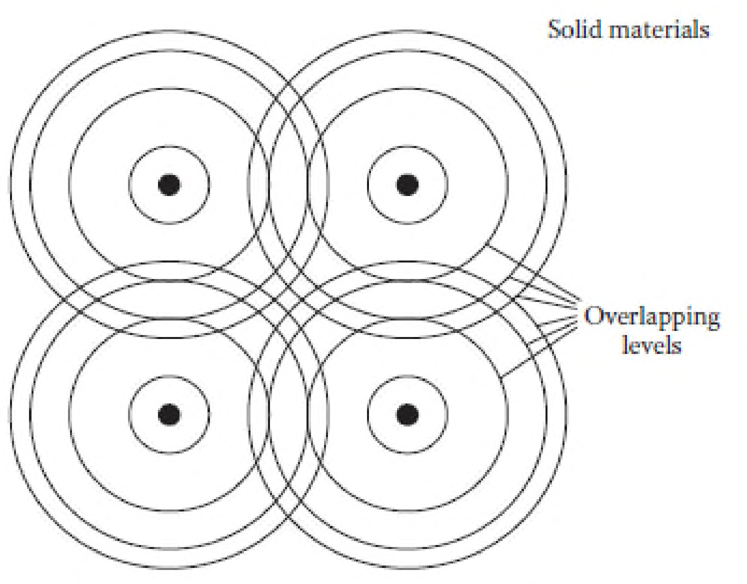


FIGURE 2.23 Overlap of the electron energy levels or wave functions as the atoms come closer. (Adapted from Edwards-Shea, L., *The Essence of Solid-State Electronics*, Prentice Hall, Upper Saddle River, NJ, 1996. With permission.)

Band Structure of Si solid

The overlapping of energy levels of different atoms and the subsequent formation of *energy bands*, which occur in silicon. As the interatomic distance (or lattice spacing) decreases, two discrete energy levels become two separate bands.

Because of the partially filled tetrahedral sites, there are two different valence electron locations.

1) Valence electrons stay near the center of Si–Si bonds, and the Coulombic interactions between valence electrons and the nucleus consequently contributes to strengthening Si–Si bonds. This energy state of valence electrons participating in Si–Si covalent bonding is called the bound state; most valence electrons are in the bound state.

2) The second type of electron state is known as “free state” and only a small portion of the valence electrons take the energy levels in free states. Electrons in free states are found around the empty tetrahedral sites of an Si lattice. Due to the lack of nearby Si atoms, the electrons in the free states are not bound to the atom.

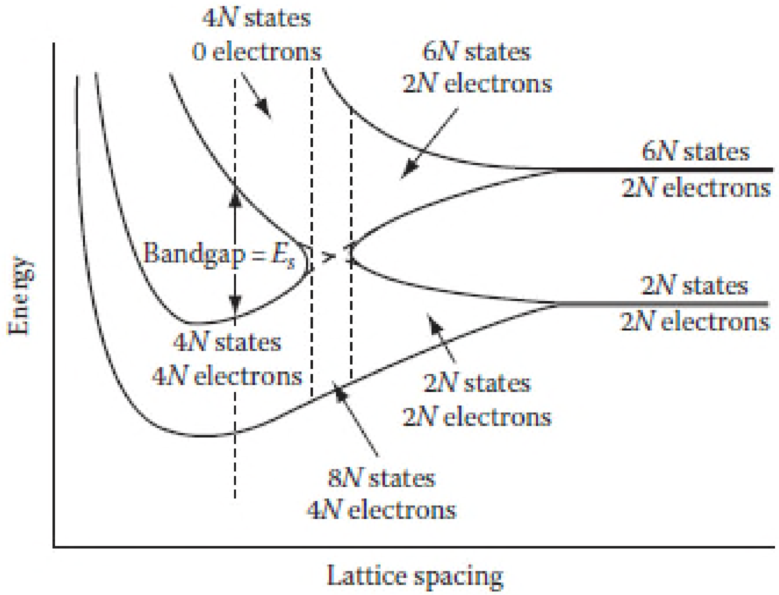
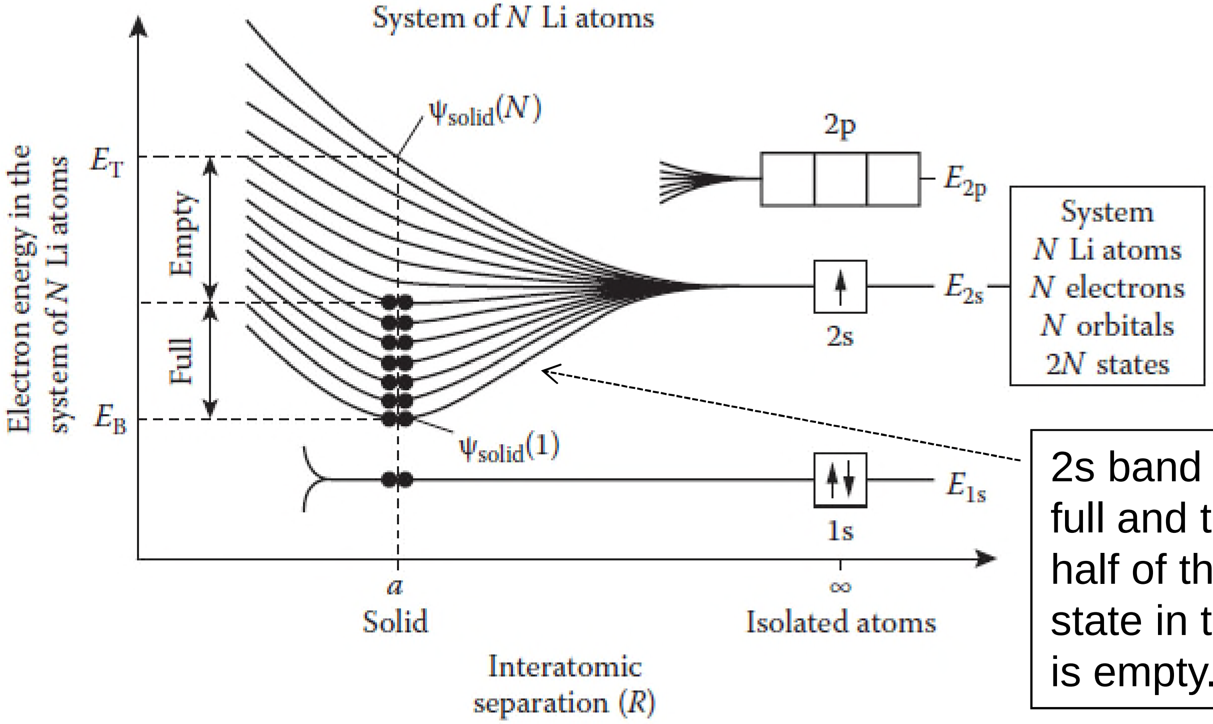


FIGURE 2.24 Formation of energy bands in silicon. (From Streetman, B.G. and Banerjee, S., *Solid State Electronic Devices*, 5th ed., Prentice Hall, Upper Saddle River, NJ, 2000. With permission.)

In Si, the electron energy band corresponding to the bound states is called the valence band, and the electron energy band related to the free states is called the conduction band. The energy gap between the bottom of the conduction band and the top of the valence band is called band gap.

Band Structure of metal solid



2s band is only half-full and the other half of the energy state in the 2s band is empty.

FIGURE 2.25 Formation of energy bands in lithium metal. The 2s band is only half-filled. Note that the 1s level shows very little splitting. (From Kasap, S.O., *Principles of Electronic Materials and Devices*, McGraw Hill, New York, 2002. With permission.)

The band width for electrons of the 1s level does not increase, though the interatomic distance decreases to the equilibrium position. Since the 2s level electrons shield the 1s electrons from outside electrons, the 1s level electrons in the solid are less affected by the presence of electrons of neighboring atoms.

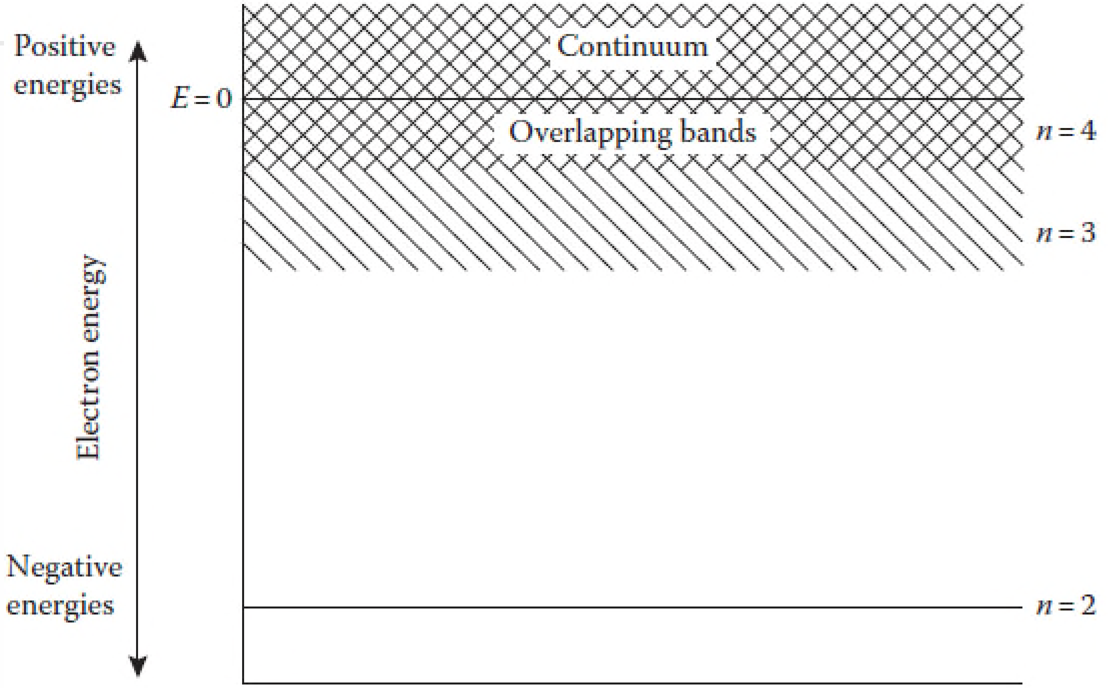


FIGURE 2.26 Schematic of a band diagram for a metal. (From Edwards-Shea, L., *The Essence of Solid-State Electronics*, Prentice Hall, Upper Saddle River, NJ, 1996. With permission.)

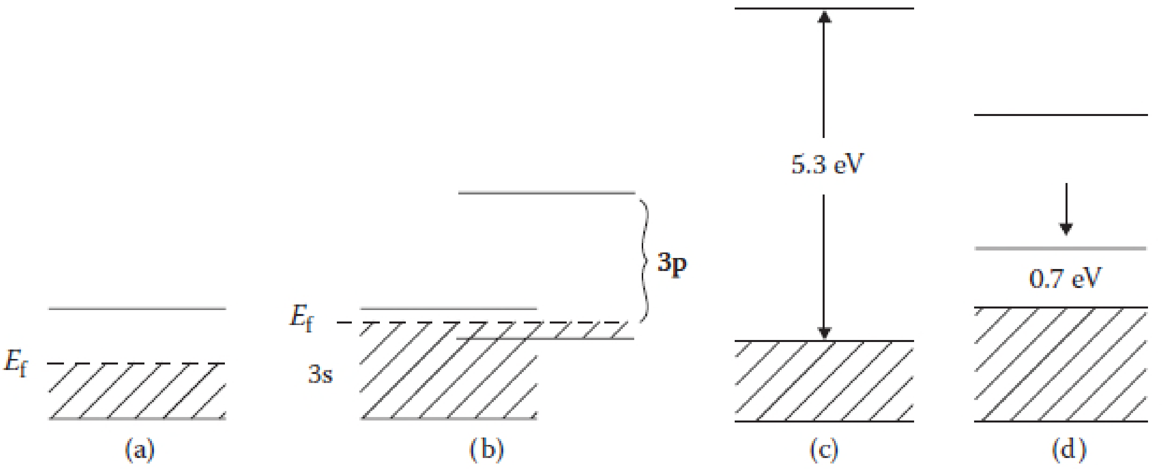


FIGURE 2.27 Band diagrams for conductors: (a) an alkali metal; (b) magnesium (Mg), a bivalent metal; (c) diamond, an insulator; and (d) germanium (Ge), a semiconductor. (From Mahajan, S. and Sree Harsha, K.S., *Principles of Growth and Processing of Semiconductors*, McGraw Hill, New York, 1998. With permission.)

A partially filled valence band is an important feature of metals. In some metals such as magnesium (Mg), the valence band (3s level for Mg) is almost filled, and there is no apparent bandgap. However, the empty 3p bands are overlapped with the almost filled 3s in Mg (Figure 2.27). In this band structure, when an electrical field is applied, the electrons in the almost 3s band accelerate and occupy the empty states in the 3p band and possibly higher bands.

The conducting ability of some materials (e.g., silicon [Si], germanium [Ge], and gallium arsenide [GaAs]) is in between that of insulators and conductors. These materials are known as *semiconductors*.

As shown below, their band gap is marginal and **a considerable number of electrons can jump** from the highest filled band (valence band) to the lowest unfilled band (conduction band) **using thermal energy**.

All the atoms in the crystal vibrates with a distribution of energies (refer to Debye model). As the T increases, the distribution spreads to higher energies. Statistically, **some of the atomic vibrations will be sufficiently energetic to rupture a bond**.

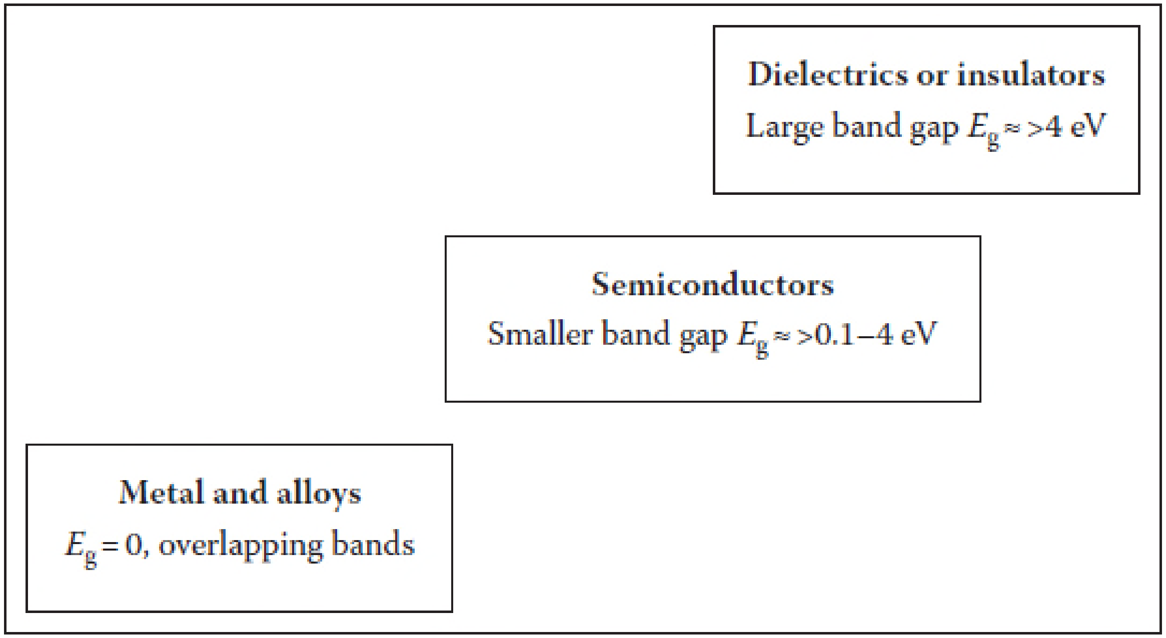


FIGURE 2.28 Classification of materials based on the values of band gap (E_g) in electron volts (eV).

In contrast to metals and semiconductors, most ceramic materials (e.g., silica [SiO_2], zirconia [ZrO_2], alumina [Al_2O_3], and silicon carbide [SiC]) exhibit strong ionic or covalent bonds (or both).

Many polymers (polyethylene, polystyrene, epoxies, etc.) also primarily exhibit covalent bonds *within* the chains of atoms and van der Waals bonds *among* the chains of atoms.

Therefore, ceramics and polymers are usually, but not always, electrical *insulators* and are also referred to as *dielectrics*. The use of the term “insulator” is sometimes preferred when emphasizing the ability of a material to withstand a strong electric field (i.e. *protection of the circuit*) - as opposed to offering only a high electrical resistance. For example, porcelain can be described more appropriately as an insulator rather than as a dielectric.

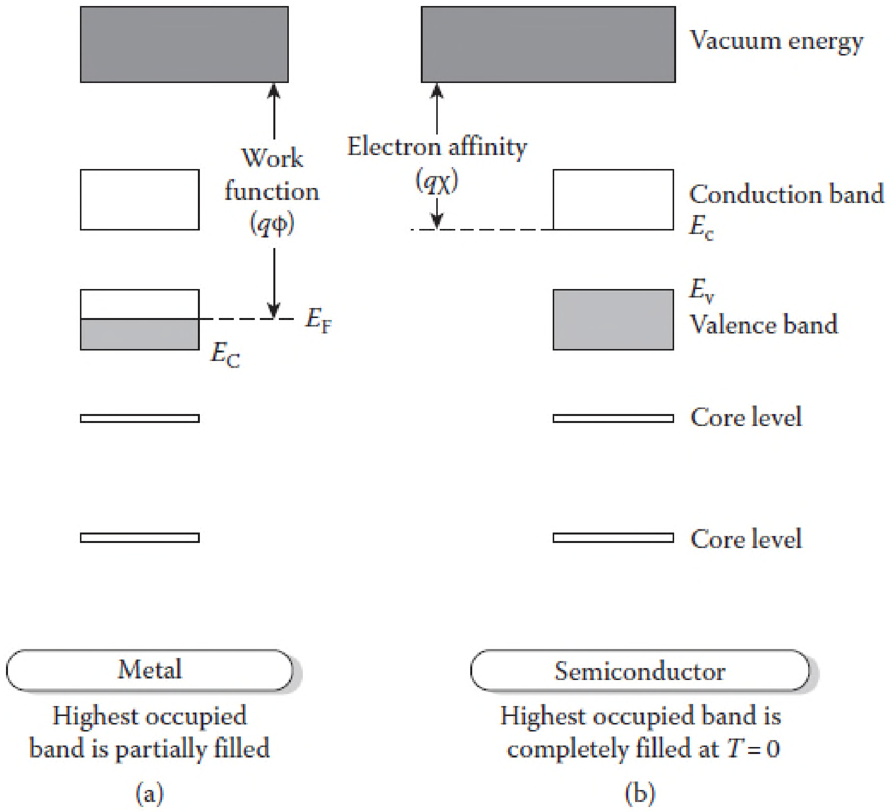


FIGURE 2.29 A schematic of band structure for (a) a typical metal and (b) a semiconductor.

Work function (ϕ) is the energy required to remove the electrons at the Fermi energy level (E_F) to the vacuum level. In addition, the energy needed to free the electron at the bottom of the conduction band to the vacuum level is called electron affinity (χ).

Fermi Energy and Fermi Level

Fermi level - the energy of an electron that is the least tightly bound to the lattice.

Strictly speaking, the Fermi level at 0K is called the Fermi energy; however, Fermi level and Fermi energy are used interchangeably in many cases. For convenience, we will use the term Fermi energy level and display it as E_F.

The Fermi energy level (E_F) can be explained in two different ways.

1)E_F means the highest energy level that electrons occupy at T = 0K. Thus, energy levels below E_F are fully taken, and those above E_F are fully empty at T = 0K.

2)E_F is the energy level that is occupied with a chance of 50% at T > 0K.

(When the temperature is higher than 0K, E_F is not the highest energy level that the electrons occupy. The energy level E_F is partially empty at T > 0K. Since electrons jump sequentially from the top energy levels, the distribution of the partially filled energy states above E_F and that of partially empty energy states below E_F is symmetric.)

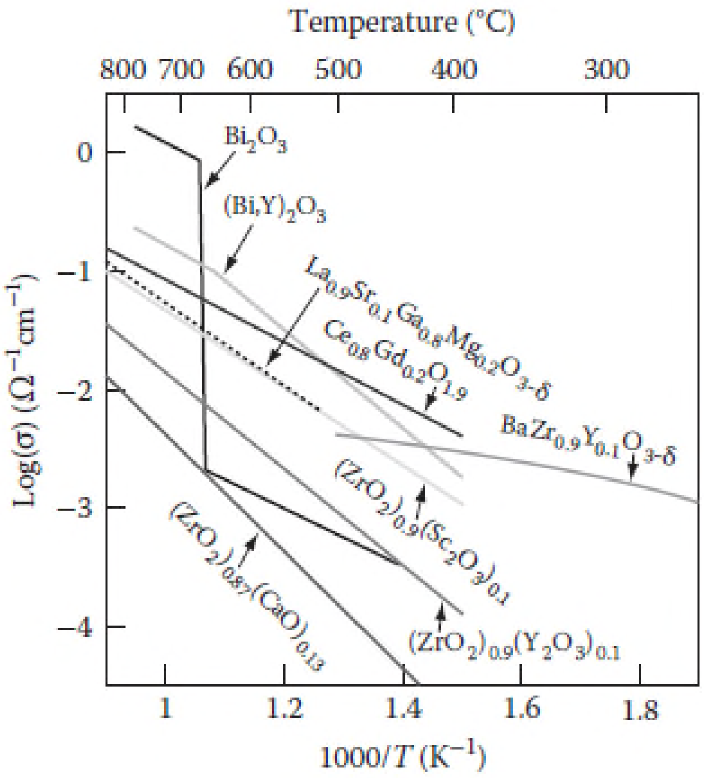


FIGURE 2.30 The conductivity of different materials used as electrolytes in the development of solid oxide fuel cells. (From Haile, S.M., *Acta Mater*., 51, 5981−6000, 2003. With permission.)