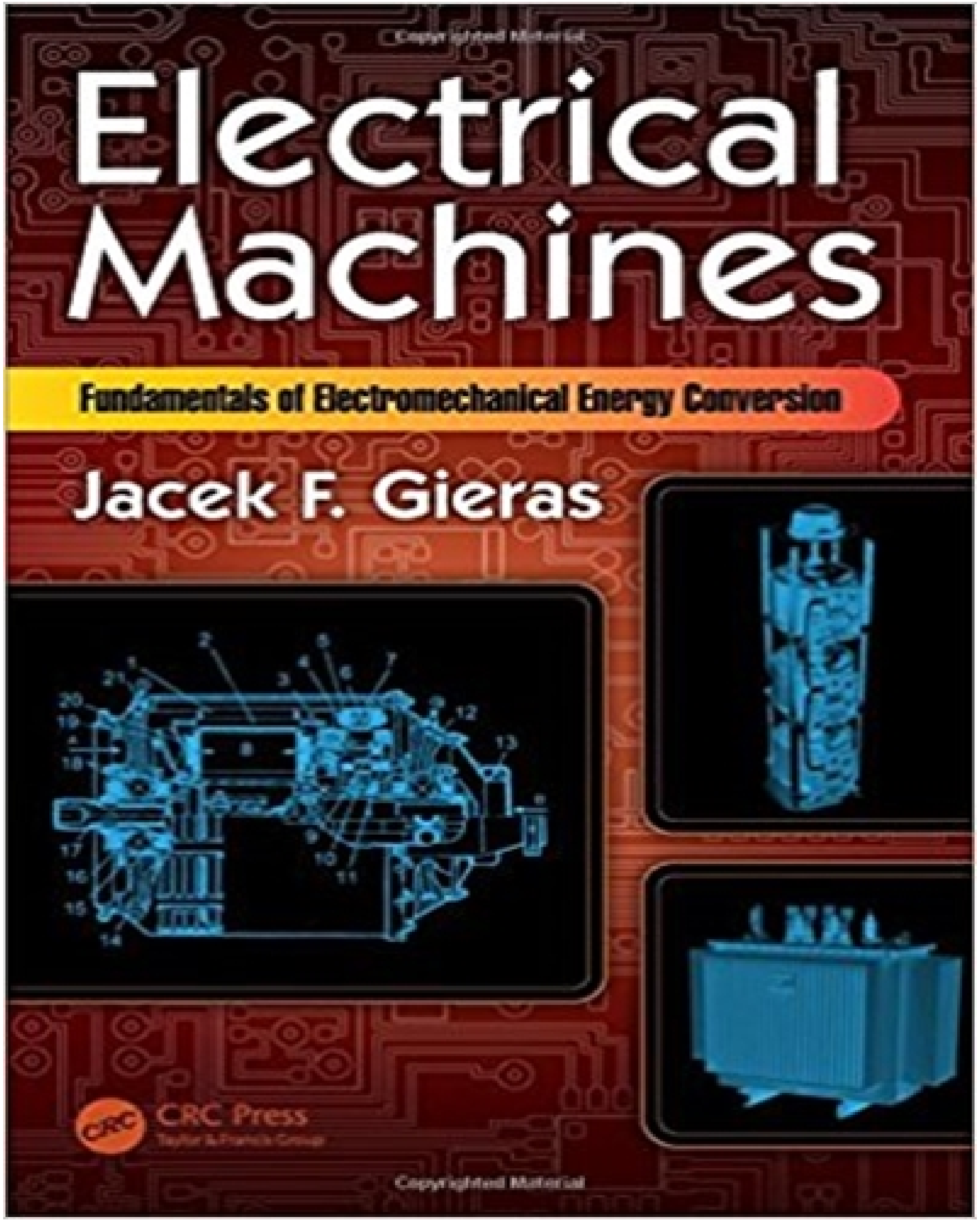
Solutions for Electrical Machines Fundamentals of Electromechanical Energy Conversion 1st Edition by Gieras



Solutions

Chapter 2. Transformers

Problem 2.1 Calculate the no-load parameters of a three-phase transformer with the following rated parameters:

Number of phases	$m_1 := 3$
Apparent power, W	$S_n := 2.0 \cdot 10^6$
Frequency, Hz	$f := 50.0$
Primary line voltage, V	$V_{1n} := 30000$
No-load current, %	$i_{\%} := 5.5$

The winding connection is Yd11 and the no-load power is $P_0 := 8600$

Solution

$V_1 := \frac{V_{1n}}{\sqrt{3}}$ Primary phase voltage, V $V_1 = 17320.5$

$I_{1n} := \frac{S_n}{m_1 \cdot V_1}$ Nominal primary current, A $I_{1n} = 38.49$

$I_{10} := \frac{i_{\%} \cdot I_{1n}}{100}$ No-load phase current, A $I_{10} = 2.12$

$P_{0ph} := \frac{P_0}{m_1}$ No-load power per phase, W $P_{0ph} = 2866.7$

$Z_0 := \frac{V_1}{I_{10}}$ No-load impedance, Ω $Z_0 = 8181.8$

$R_0 := \frac{P_{0ph}}{I_{10}^2}$ No-load resistance, Ω $R_0 = 639.7$

$X_0 := \sqrt{Z_0^2 - R_0^2}$ No-load reactance, Ω $X_0 = 8156.8$

$R_{Fe} := \frac{V_1^2}{P_{0ph}}$ Core loss resistance, Ω $R_{Fe} = 104651.2$

$I_{Fe} := \frac{P_{0ph}}{V_1}$ Active component of vertical branch current, A $I_{Fe} = 0.17$

$I_{\Phi} := \sqrt{I_{10}^2 - I_{Fe}^2}$ Magnetizing current, A $I_{\Phi} = 2.11$

$X_m := \frac{V_1}{I_{\Phi}}$ Vertical branch reactance, Ω $X_m = 8206.9$

Problem 2.2. Calculate the short-circuit parameters of a three-phase transformer with the following rated parameters:

Number of phases	$m_1 := 3$
Apparent power, W	$S_n := 2.4 \cdot 10^6$
Frequency, Hz	$f := 50.0$
Primary voltage, V	$V_{1n} := 35000$
No-load current, %	$i_{\%} := 5.0$
SHort-circuit voltage	$v_{sc\%} := 6.5$

The winding connection is Yd11 and the shortt-circuit power is $P_{sc} := 26000$

Solution

$V_1 := \frac{V_{1n}}{\sqrt{3}}$ Primary phase voltage, V $V_1 = 20207.3$

$I_{1n} := \frac{S_n}{m_1 \cdot V_1}$ Primary phase current, A $I_{1n} = 39.59$

$V_{1sc} := \frac{V_{1n}}{\sqrt{3}} \cdot \frac{v_{sc\%}}{100}$ Short-circuit primary phase voltage, V $V_{1sc} = 1313.5$

$P_{scph} := \frac{P_{sc}}{m_1}$ Short-circuit power per phase, W $P_{scph} = 8666.7$

$Z_{sh} := \frac{V_{1sc}}{I_{1n}}$ Short-circuit impedance, Ω $Z_{sh} = 33.18$

$R_{sh} := \frac{P_{scph}}{I_{1n}^2}$ Short-circuit resistance, Ω $R_{sh} = 5.53$

$X_{sh} := \sqrt{Z_{sh}^2 - R_{sh}^2}$ Short-circuit reactance, Ω $X_{sh} = 32.713$

$R_1 := \frac{1}{2} \cdot R_{sh}$ Primary resistance, Ω $R_1 = 2.76$

$R_{2p} := \frac{1}{2} \cdot R_{sh}$ Secondary resistance referred to the primary winding, Ω $R_{2p} = 2.76$

$X_1 := \frac{1}{2} \cdot X_{sh}$ Primary reactance, Ω $X_1 = 16.36$

$X_{2p} := \frac{1}{2} \cdot X_{sh}$ Secondary reactance referred to the primary winding, Ω $X_{2p} = 16.36$

Problem 2.3. The parameters of the equivalent cicuit of a 150-kVA, 2400-V/240-V transformer are

$R_1 := 0.2$ $R_2 := 0.002$ $X_1 := 0.45$ $X_2 := 0.0045$ $R_{Fe} := 10000.0$

$X_m := 1555$ On the basis of the equivalent circuit shown in Fig. 2.9 calculate:

(a) Voltage regulation;
(b) Efficiency of the transformer operating at rated load with 0.8 lagging power factor.

Solution

$S_n := 150000$ $V_1 := 2400$ $V_2 := 240$ $pf := 0.8$ lagging

(a) Voltage regulation

$\upsilon := \frac{V_1}{V_2}$ Voltage ratio $\upsilon = 10$

$R_{2p} := \upsilon^2 \cdot R_2$ Resistance of the secondary winding referred as to the primaryt side, Ω $R_{2p} = 0.2$

$X_{2p} := \upsilon^2 \cdot X_2$ Reactance of the secondary winding referred as to the primaryt side, Ω $X_{2p} = 0.45$

$I_2 := \frac{S_n}{V_2}$ Secondary current, A $I_2 = 625$

$\theta := \mathrm{acos}(pf)$ Angle between the voltage and current, deg $\mathrm{acos}(pf) \cdot \frac{180}{\pi} = 36.87$

$I_{2p} := \frac{1}{\upsilon} I_2 \cdot e^{-j \cdot \theta}$ Secondary current referred as to the primary side $I_{2p} = 50 - 37.5i$

$V_{2p} := \upsilon \cdot V_2$ Secondary voltage referred as to the primary side, V $V_{2p} = 2400$

$E_1 := V_{2p} + I_{2p} \cdot (R_{2p} + j \cdot X_{2p})$ EMF, V $E_1 = 2426.9 + 15i$

$|E_1| = 2426.9$

$I_{\Phi} := \frac{E_1}{jX_m}$ Magnetizing current, A $I_{\Phi} = 0.0096 - 1.5607i$

$I_{Fe} := \frac{E_1}{R_{Fe}}$ Core loss current, A $I_{Fe} = 0.24 + 0i$

$I_0 := I_{Fe} + I_{\Phi}$ Exciting current, A $I_0 = 0.252 - 1.559i$

$I_1 := I_0 + I_{2p}$ Primary current, A $I_1 = 50.252 - 39.059i$

$V_{10} := E_1 + I_1 \cdot (R_1 + j \cdot X_1)$ Primary voltage at no load, V $V_{10} = 2454.5 + 29.8i$

$V_{20} := |V_{10}|$ Secondary voltage at no load referred as to the primary side, V $|V_{10}| = 2454.7$

$\Delta v_{\%} := \frac{V_{20} - |V_{2p}|}{V_{20}} \cdot 100$ Voltage regulation, % $\Delta v_{\%} = 2.23$

(b) Efficiency of the transformer operating at rated load with 0.8 lagging power factor.

$P_{out} := S_n \cdot pf$ Output power, W $P_{out} = 120000$

$\Delta P_{1w} := (|I_1|)^2 \cdot R_1$ $\Delta P_{1w} = 810.2$

$\Delta P_{2w} := (|I_{2p}|)^2 \cdot R_{2p}$ $\Delta P_{2w} = 781.2$

$\Delta P_{Fe} := (|I_{Fe}|)^2 \cdot R_{Fe}$ $\Delta P_{Fe} = 589$

$\Delta P := \Delta P_{1w} + \Delta P_{2w} + \Delta P_{Fe}$ $\Delta P = 2180.4$

$\eta := \frac{P_{out}}{P_{out} + \Delta P} \cdot 100$ $\eta = 98.2$

Problem 2.4. A single phase transformer has the following nominal parameters:

Apparent power, VA	$S_n := 15000$
Primary voltage, V	$V_{1n} := 600$
Secondary voltage, V	$V_{2n} := 240$
Per unit no-load power, %	$p_{0\%} := 1.2$
No-load current, %	$i_{0\%} := 6.2$
Per unit short-circuit power	$p_{sc\%} := 3.5$
Short-cricuit voltage, %	$v_{sc\%} := 5.5$
Frequency, Hz	$f_n := 50$

The hysteresis losses $\Delta P_h = 3\Delta P_e$ where ΔPe are eddy-current losses.

Calculate:

(a) Parameters of the equivalent circuit

(b) No-load losses, no-load current and and power factor when the primary winding is fed with the voltage of 720 V and frequency 60 Hz

Solution

(a) Parameters of the equivalent circuit

$I_{1n} := \frac{S_n}{V_{1n}}$ Nominal primary current, A $I_{1n} = 25$

$I_{2n} := \frac{S_n}{V_{2n}}$ Nominal secondary current, A $I_{2n} = 62.5$

$I_0 := \frac{i_{0\%}}{100} \cdot I_{1n}$ No-load current, A $I_0 = 1.55$

$V_{1sc} := \frac{v_{sc\%}}{100} \cdot V_{1n}$ Short circuit voltage, V $V_{1sc} = 33$

$P_0 := \frac{p_{0\%}}{100} \cdot S_n$	No-load power (core losses), W	$P_0 = 180$
$P_{sc} := \frac{p_{sc\%}}{100} \cdot S_n$	Short-circuit power (winding losses), W	$P_{sc} = 525$
$\Delta P_w := P_{sc}$	Winding losses, W	$\Delta P_w = 525$
$I_{Fe} := \frac{P_0}{V_{1n}}$	Active component of no-load current, A	$I_{Fe} = 0.3$
$I_\Phi := \sqrt{I_0^2 - I_{Fe}^2}$	Reactive component of no-load current, A	$I_\Phi = 1.5$
$X_m := \frac{V_{1n}}{I_\Phi}$	Reactance of vertical branch (at no-load R1 and X1 can be neglected)	$X_m = 394.6$
$R_{Fe} := \frac{V_{1n}}{I_{Fe}}$	Resistance of vertical branch that represents the core losses, Ω	$R_{Fe} = 2000$
$R_0 := \frac{R_{Fe} \cdot X_m^2}{R_{Fe}^2 + X_m^2}$	Series resistance and reactance of vertical branch, Ω	$R_0 = 74.9$
$X_0 := \frac{R_{Fe}^2 \cdot X_m}{R_{Fe}^2 + X_m^2}$		$X_0 = 379.8$
$R_{sc} := \frac{P_{sc}}{I_{1n}^2}$	Short-circuit resistance, Ω	$R_{sc} = 0.84$
$Z_{sc} := \frac{V_{1sc}}{I_{1n}}$	Short-circuit impedance, Ω	$Z_{sc} = 1.32$
$X_{sc} := \sqrt{Z_{sc}^2 - R_{sc}^2}$	Short-circuit reactance, Ω	$X_{sc} = 1.02$
$R_1 := \frac{R_{sc}}{2}$	Resistance of primary winding, Ω	$R_1 = 0.42$

$X_1 := \frac{X_{sc}}{2}$ Leakage reactance of primary winding, Ω $X_1 = 0.45$

$R_{2p} := \frac{R_{sc}}{2}$ Resistance of the secondary winding referred to the primary winding, Ω $R_{2p} = 0.42$

$X_{2p} := \frac{X_{sc}}{2}$ Leakage reactance of the primary winding referred to the primary winding, Ω $X_{2p} = 0.51$

$\theta := \frac{V_{1n}}{V_{2n}}$ Voltage ratio $\theta = 2.5$

$R_2 := \frac{R_{2p}}{\theta^2}$ Resistance of secondary winding, Ω $R_2 = 0.067$

$X_2 := \frac{X_{2p}}{\theta^2}$ Leakage reactance of secondary winding, Ω $X_2 = 0.081$

(b) No-load losses, no-load current and and power factor when the primary winding is fed with the voltage of $V_1 := 720$ **and frequency** $f_{60} := 60$

Because $\Delta P_h = 3\Delta P_e$ so that

$\Delta P_h := \frac{3}{4} \cdot P_0$ Hysteresis losses at nominal voltage and frequency, W $\Delta P_h = 135$

$\Delta P_e := \frac{1}{4} \cdot P_0$ Edyy-current losses at nominal voltage and frequency, W $\Delta P_e = 45$

Hysteresis losses are proportional to the magnetic flux density square and frequency. Eddy current losses are proportional to the magnetic flux density square and frequency square, i.e.,

$\Delta P_{h60} = (B_{60}/B_n)^2 (f_{60}/f_n) \Delta P_{hn}$

$\Delta Pe_{60} = (B_{60}/Bn)^2 (f_{60}/f_n)^2 \Delta P_{en}$

On the basis of EMF equation, it is also necessary to consider the influence of variable frequency and voltage on the magnetic flux density.

$$\frac{V_1}{V_{1n}} = \frac{E_1}{E_{1n}} = \frac{f_{60}}{f_n} \cdot \frac{B_{60}}{B_n}$$

After a few algebraic steps, the following equation can be obtained

$\Delta P_{h60} := \Delta P_h \cdot \left(\frac{V_1}{V_{1n}}\right)^2 \cdot \frac{f_n}{f_{60}}$ Hysteresis losses, W $\Delta P_{h60} = 162$

$\Delta P_{e60} := \Delta P_e \cdot \left(\frac{V_1}{V_{1n}}\right)^2$ Eddy current loesses, W $\Delta P_{e60} = 64.8$

$P_{060} := \Delta P_{h60} + \Delta P_{e60}$ No-load losses, W $P_{060} = 226.8$

$I_{Fe60} := \frac{P_{060}}{V_1}$ Core loss current at 60 Hz, A $I_{Fe60} = 0.315$

$X_{m60} := \frac{f_{60}}{f_n} \cdot X_m$ Vertical branch reactance, Ω $X_{m60} = 473.5$

$I_{\Phi 60} := \frac{V_1}{X_{m60}}$ Magnetizing current, A $I_{\Phi 60} = 1.521$

$I_{060} := \sqrt{I_{Fe60}^2 + I_{\Phi 60}^2}$ No-load current under new primary voltage and frequency, A $I_{060} = 1.55$

$pf_{60} := \frac{P_{060}}{V_1 \cdot I_{060}}$ Power factor $\cos\phi_0$ at no load $pf_{60} = 0.203$

$V_{20} := \frac{V_1}{\theta}$ Secondary voltage under new primary voltage and frequency, V $V_{20} = 288$

Problem 2.5. A single-phase transformer has the following nomal parameters:

Apparent power, VA	$S_n := 25000$
Primary voltage, V	$V_{1n} := 540$
Secondary voltage, V	$V_{2n} := 115$
Frequency, Hz	$f_n := 60$
No-load power, %	$p_{0\%} := 1.1$
Short-circuit power, %	$p_{sc\%} := 3.5$
No-load current per unit, %	$i_{0\%} := 5.5$
Short circuit voltage, %	$v_{sc\%} := 4.0$

The hysteresis losses $\Delta P_h = 2.5\Delta P_e$ where ΔPe are eddy-current losses. Find:

(a) Parameters of the equivalent circuit;
(b) If the transformer is fed with the voltage $V_1 = 380$ V at frequency $f_1 = 50$ Hz, calculate the primary current I_1 at which the total winding and core losses are the same as under rated conditions.

(c) The copper windings are replaced with aluminum windings with the same number of turns and cross sections. Calculate the new nominal current assuming that the power losses in windings are are the same.

$\sigma_{Cu} := 57 \cdot 10^6$ S/m $\qquad \sigma_{Al} := 32 \cdot 10^6$ S/m

(a) Parametrers of the equivalent circuit

$I_{1n} := \frac{S_n}{V_{1n}}$ Nominal primary current, A $I_{1n} = 46.3$

$I_{2n} := \frac{S_n}{V_{2n}}$ Nominal secondary current, A $I_{2n} = 217.4$

$I_0 := \frac{i_{0\%}}{100} \cdot I_{1n}$ No-load current $I_0 = 2.546$

$V_{1sc} := \frac{v_{sc\%}}{100} \cdot V_{1n}$ Short-circuit voltage, V $V_{1sc} = 33$

$P_0 := \frac{p_{0\%}}{100} \cdot S_n$ No-load power (core losses), W $P_0 = 275$

$P_{sc} := \frac{p_{sc\%}}{100} \cdot S_n$ Short-circuit power (winding losses), W $P_{sc} = 875$

$\Delta P_w := P_{sc}$	Winding losses, W	$\Delta P_w = 875$
$I_{Fe} := \frac{P_0}{V_{1n}}$	Active component of no-load current, A	$I_{Fe} = 0.509$
$I_\Phi := \sqrt{I_0^2 - I_{Fe}^2}$	Reactive component of no-load current, A	$I_\Phi = 2.5$
$X_m := \frac{V_{1n}}{I_\Phi}$	Reactance of vertical branch. At no-load R_1 and X_1 can be neglected, Ω	$X_m = 216.4$
$R_{Fe} := \frac{V_{1n}}{I_{Fe}}$	Core loss resistance (in parallel to Xm), Ω	$R_{Fe} = 1060.4$
$R_0 := \frac{R_{Fe} \cdot X_m^2}{R_{Fe}^2 + X_m^2}$	Series resistance and reactance of vertical branch, Ω	$R_0 = 42.4$
$X_0 := \frac{R_{Fe}^2 \cdot X_m}{R_{Fe}^2 + X_m^2}$		$X_0 = 207.8$
$R_{sc} := \frac{P_{sc}}{I_{1n}^2}$	Short-circuit resistance, Ω	$R_{sc} = 0.41$
$Z_{sc} := \frac{V_{1sc}}{I_{1n}}$	Short-circuit impedance, Ω	$Z_{sc} = 1.32$
$X_{sc} := \sqrt{Z_{sc}^2 - R_{sc}^2}$	Short-circuit reactance, Ω	$X_{sc} = 0.23$
$R_1 := \frac{R_{sc}}{2}$	Primary winding resistance, Ω	$R_1 = 0.2$
$X_1 := \frac{X_{sc}}{2}$	Primary winding leakage reactance, Ω	$X_1 = 0.11$
$R_{2p} := \frac{R_{sc}}{2}$	Secondary winding resistance referred to primary winding, Ω	$R_{2p} = 0.2$
$X_{2p} := \frac{X_{sc}}{2}$	Secondary winding leakage reactance referred to primary winding, Ω	$X_{2p} = 0.11$

(b) Transformer is fed with the voltage $V_1 := 380$ at frequency $f_1 := 50$ **. Calculate the primary current I_1 at which the total winding and core losses are the same as under nominal conditions.**

Since $\Delta P_h = 2.5\Delta P_e$

$\Delta P_e := \frac{P_0}{3.5}$ Eddy current losses under new conditions, W $\Delta P_e = 78.6$

$\Delta P_h := P_0 - \Delta P_e$ Hysteresis losses under new conditions, W $\Delta P_h = 196.4$

Total no-load and short-cicuit losses at nominal voltage and frequency

$P_{0sc} := P_{sc} + P_0$ Total winding and core losses, W $P_{0sc} = 1150$

No-load losses at new voltage and 60 Hz

$\Delta P_{h60} := \Delta P_h \cdot \left(\frac{V_1}{V_{1n}}\right)^2 \frac{f_n}{f_{60}}$ Hysteresis losses, W $\Delta P_{h60} = 162$

$\Delta P_{e60} := \Delta P_e \cdot \left(\frac{V_1}{V_{1n}}\right)^2$ Eddy current loesses, W $\Delta P_{e60} = 64.8$

$P_{060} := \Delta P_{h60} + \Delta P_{e60}$ No-load power, W $P_{060} = 136.2$

$P_{sc1} := P_{0sc} - P_{060}$ Short-circuit power unde new condition, W $P_{sc1} = 1013.8$

Because the short circuit power (winding losses) is proportional to current square

$\frac{P_{sc1}}{P_{sc}} = \left(\frac{I_1}{I_{1n}}\right)^2$ Hence, $I_{1n} = 46.3$

$I_1 := I_{1n} \cdot \sqrt{\frac{P_{sc1}}{P_{sc}}}$ New value of primary current, A $I_1 = 49.8$

(c) The copper windings are replaced with aluminum windings with the same number of turns and cross sections. Find the new nominal current assuming that the power losses in windings are the same.

$\sigma_{Cu} := 57 \cdot 10^6$ S/m $\sigma_{Al} := 32 \cdot 10^6$ S/m

$R_{scAl} := R_{sc} \cdot \frac{\sigma_{Cu}}{\sigma_{Al}}$ Short-circuit resistance in the case of aliminum winding, Ω $R_{scAl} = 0.7$

Since $I_{1nAl}{}^2 \cdot R_{scAl} = I_{1n}{}^2 \cdot R_{sc}{}^2$

$I_{1nAl} := I_{1n} \cdot \sqrt{\frac{R_{sc}}{R_{scAl}}}$ New nominal primary current $I_{1nAl} = 34.7$

$V_{1sc} := I_{1nAl} \cdot Z_{sc}$ Short-circuit voltage, V $V_{1sc} = 16.184$

$v_{sc\%} := \frac{V_{1sc}}{V_{1n}} \cdot 100$ Short-circuit voltage, % $v_{sc\%} = 2.997$

Problem 2. 6. A single phase transformer has the following nominal parameters:

Apparent power, VA $S_n := 45000$

Primary voltage, V $V_{1n} := 460$

Secondary voltage, V $V_{2n} := 6300$

Frequency, Hz $f_n := 60$

Calculated voltage drops across resistance and leakage ractances of windings at nominal current are:

Voltage drop across primary resistance, V $V_{R1n} := 8.0$

Voltage drop across primary leakage reactance, V $V_{X1n} := 18.2$

Voltage drop across secondary resistance, V $V_{R2n} := 102$

Voltage drop across secondary laekage reactance, V $V_{X2n} := 358$

When the primary winding is fed with nominal voltage V_{1n} at open secondary winding terminals, the no-load current and no-load power are:

$I_0 := 9.2$ $P_0 := 384$

If the transformer is fed with nominal voltage V_{1n} from the primary side at open secondary terminals, calculate:

(a) voltage drops across resistances and leakage reactances of windings;
(b) EMF E1 induced in the primary winding;
(c) Percentage difference between the terminal voltage and SEM in the secondary winding.

Solution

(a) Voltage drops across resistances and leakage reactances of windings

At no-load the secondary current is zero, so $V_{R2} := 0$ and $V_{X2} := 0$

$I_{1n} := \frac{S_n}{V_{1n}}$ Nominal primary current, A $I_{1n} = 97.826$

$R_1 := \frac{V_{R1n}}{I_{1n}}$ Resistance of primary winding, Ω $R_1 = 0.0818$

$X_1 := \frac{V_{X1n}}{I_{1n}}$ Leakage rectance of primary winding, Ω $X_1 = 0.186$

Voltage drops and no load are:

$V_{R1} := R_1 \cdot I_0$ across primary resistance, V $V_{R1} = 0.752$

$V_{X1} := X_1 \cdot I_0$ across primary leakage reactance, V $V_{X1} = 1.712$

(b) EMF E1 induced in the primary winding;

$\cos\phi_0 := \frac{P_0}{V_{1n} \cdot I_0}$ No-load power factor when the primary winding is fed with nominal voltage V1n. Hence, $\cos\phi_0 = 0.091$

$\phi_0 := \mathrm{acos}\left(\frac{P_0}{V_{1n} \cdot I_0}\right)$ Angle ϕ in radians $\phi_0 = 1.48$

$\phi_{0d} := \mathrm{acos}\left(\frac{P_0}{V_{1n} \cdot I_0}\right) \cdot \frac{180}{\pi}$ Angle ϕ in degrees $\phi_{0d} = 84.794$

$E_1 := V_{1n} - (R_1 + j \cdot X_1) \cdot I_0 \cdot e^{-j \cdot \phi_0}$ Kirchhoffs voltage law, equivalent circuit $E_1 = 458.227 + 0.594i$

$E_1 := |E_1|$ EMF rms, V $E_1 = 458.2$

(c) Percentage difference between the terminal voltage and SEM in the secondary winding.

Percentage difference, % $\frac{V_{1n} - E_1}{E_1} \cdot 100 = 0.387$

Problem 2.7. A single phase transformer has the following nominal parameters:

Apparent power, VA $S_n := 20000$

Primary voltage, V $V_{1n} := 630$

Secondary voltage, V $V_{2n} := 230$

No-load power, % $p_{0\%} := 0.9$

Short-circuit power, % $p_{sc\%} := 3.2$

No-load current, % $i_{sc\%} := 6.1$

Short-circuit voltage, % $v_{sc\%} := 5.5$

Frequency, Hz $f_n := 50$

Calculate:
(a) Resistances and reactances of equivalent circuit and input voltage to obtain maximum efficiency at primary current $I_1 := 14.5$

(b) Half of the primary winding is fed with a certain input current with frequency $f_{60} := 60$

Calculate the input voltage, if the transformer operates at short circuit and the short circuit current $I_{sc} := I_{1n}$

Solution

(a) Resistances and reactances of equivalent circuit and input voltage to obtain maximum efficiency at primary current $I_1 = 14.5$

$I_{1n} := \frac{S_n}{V_{1n}}$ Nominal primary current, A $I_{1n} = 31.7$

$I_{2n} := \frac{S_n}{V_{2n}}$	Nominal secondary current, A	$I_{2n} = 87$
$I_0 := \frac{i_{0\%}}{100} \cdot I_{1n}$	No-load current, A	$I_0 = 1.746$
$V_{1sc} := \frac{v_{sc\%}}{100} \cdot V_{1n}$	Short-circuit voltage, V	$V_{1sc} = 16.2$
$P_0 := \frac{p_{0\%}}{100} \cdot S_n$	No-load power (core losses), W	$P_0 = 384$
$\Delta P_{Fe} := P_0$	Core losses, W	$\Delta P_{Fe} = 589$
$P_{sc} := \frac{p_{sc\%}}{100} \cdot S_n$	Short circuit power (winding losses), W	$P_{sc} = 875$
$\Delta P_w := P_{sc}$	Winding losses, W	$\Delta P_w = 875$
$I_{Fe} := \frac{P_0}{V_{1n}}$	Active component of no-load current A	$I_{Fe} = 0.286$
$I_\Phi := \sqrt{I_0^2 - I_{Fe}^2}$	Reactive component of no-load current A	$I_\Phi = 1.722$
$X_m := \frac{V_{1n}}{I_\Phi}$	Reactance of vertical branch (R1 and X1 can be beglected at no load), Ω	$X_m = 365.7$
$R_{Fe} := \frac{V_{1n}}{I_{Fe}}$	Core loss resistance in vertical branch (in parallel with Xm), Ω	$R_{Fe} = 2205$
$R_0 := \frac{R_{Fe} \cdot X_m^2}{R_{Fe}^2 + X_m^2}$	Series resistance and ractance in vertical branch, Ω	$R_0 = 59$
$X_0 := \frac{R_{Fe}^2 \cdot X_m}{R_{Fe}^2 + X_m^2}$		$X_0 = 356$
$R_{sc} := \frac{P_{sc}}{I_{1n}^2}$	Short-circuit resistance, Ω	$R_{sc} = 0.635$

$Z_{sc} := \frac{V_{1sc}}{I_{1n}}$ Short-circuit impedance, Ω $Z_{sc} = 1.091$

$X_{sc} := \sqrt{Z_{sc}^2 - R_{sc}^2}$ Short-circuit reactance, Ω $X_{sc} = 0.89$

$R_1 := \frac{R_{sc}}{2}$ Primary winding resistance, Ω $R_1 = 0.318$

$R_{2p} := \frac{R_{sc}}{2}$ Secondary winding resistance referred to the primary winding, Ω $R_{2p} = 0.318$

$X_1 := \frac{X_{sc}}{2}$ Primary winding leakage reactance, Ω $X_1 = 0.444$

$X_{2p} := \frac{X_{sc}}{2}$ Secondary winding resistance referred to the primary winding, Ω $X_{2p} = 0.444$

$\theta := \frac{V_{1n}}{V_{2n}}$ Voltage ratio of a single-phase transformer $\theta = 2.739$

$R_2 := \frac{R_{2p}}{\theta^2}$ Resistance of secondary winding, Ω $R_2 = 0.042$

$X_2 := \frac{X_{2p}}{\theta^2}$ Leakage reactance of secondary winding, Ω $X_2 = 0.059$

Voltage at maximum efficiency to obtain $I_1 = 14.5$

Maximum efficiency is when the short-circuit losses are equal to no-load losses. It can be proved that

$$\Delta P_{\eta max}/\Delta P_w = (I_{2\eta max}/I_{2N})^2 = (S_{\eta max}/S_N)^2 \qquad \Delta P_{\eta max} = P_0$$

Hence $S_n = 20000$

$S_{\eta max} := S_n \cdot \sqrt{\frac{P_0}{\Delta P_w}}$ Apparent power to obtain maximum efficiency, VA $S_{\eta max} = 10606.6$

Such a method is only valid for constant primary and secondary voltage. In this problem, first new winding losses at $I_1 = 14.5$ must be caluclated.

$$\Delta P_{w1} := \Delta P_w \cdot \left(\frac{I_1}{I_{1n}}\right)^2 \qquad \Delta P_{w1} = 133.5$$

Cor losses at constant ferquency are approximately proportional to the magnetic flux density square. I mans, approximately proportional to the voltage square, i.e.,

$P_0/P_{01}=(U_{1n}/U_1)^2$

Assuming $P_{01} = \Delta P_{w1}$ to obtain maximum efficiency, the following voltage corresponds to the current I_1

$$V_1 := \sqrt{\frac{\Delta P_{w1}}{P_0}} \cdot V_{1n}$$ Input voltage to obtain maximum efficiency, V $\qquad V_1 = 380$

(b) Half of the primary winding is fed with current the frequency of which is f_1 = 60 Hz. Calculate the input voltage, if the transformer operates at short circuit and the short circuit current

$I_{sc} := I_{1n} \qquad f_1 := 60$

$$Z_n := \frac{V_{1n}}{I_{1n}}$$ Nominal impedance, Ω $\qquad Z_n = 19.845$

$$\theta := \frac{1}{2} \cdot \frac{V_{1n}}{V_{2n}}$$ New voltage ratio $\qquad \theta = 1.37$

$$R_{sc} := \frac{R_1}{2} + R_2 \cdot \theta^2$$ New short-circuit resistance, Ω $\qquad R_{sc} = 0.635$

$$X_{sc} := \frac{X_1}{2} + X_2 \cdot \theta^2$$ New short-circuit reactance at 50 Hz, Ω $\qquad X_{sc} = 0.888$

$$X_{sc60} := \frac{f_1}{f_n} \cdot X_{sc}$$ New short-circuit reactance at 60 Hz, Ω $\qquad X_{sc60} = 0.399$

$$Z_{sc60} := \sqrt{R_{sc}^2 + X_{sc60}^2}$$ New short-circuit impedance at 60 Hz, Ω $\qquad Z_{sc60} = 0.465$

$$V_{sc60} := Z_{sc60} \cdot I_{sc}$$ Primary voltage, V $\qquad V_{sc60} = 14.76$

Problem 2.8. A three-phase transformer has the following nominal parameters:

apparent power	$S_n := 10000000$
HV voltage	$V_{1n} := 220000$
LV voltage	$V_{2n} := 60000$
frequency	$f_n := 50$
winding losses	$\Delta P_w := 60000$
core losses	$\Delta P_{Fe} := 28000$
short-circuit voltage	$v_{sc\%} := 7.5$
no-load current	$i_{0\%} := 3.0$
connection	Yy0

Calculate:
(a) per unit parameters of the equivalent circuit;
(b) equivalent circuit parameters referred to the HV winding.

All quantities are expressed in International Unit System.

Solution

(a) Per unit parameters of the equivalent circuit

$$I_{1n} := \frac{S_n}{\sqrt{3}\cdot V_{1n}}$$ Nominal current of HV winding, A $I_{1n} = 26.24$

$$I_0 := \frac{i_{0\%}}{100}\cdot I_{1n}$$ No-load current related to HV winding, A $I_0 = 0.787$

$$I_{Fe} := \frac{\Delta P_{Fe}}{\sqrt{3}\cdot V_{1n}}$$ Active component of no-load current, A $I_{Fe} = 0.286$

$$I_\Phi := \sqrt{I_0^{\,2} - I_{Fe}^{\,2}}$$ Magnetizing current, A $I_\Phi = 1.722$

$$Z_n := \frac{V_{1n}}{\sqrt{3}\cdot I_{1n}}$$ Nominal (base) impedance, Ω $Z_n = 4840$

$r_{sc} := \frac{\Delta P_w}{S_n}$ Short-circuit resistance per unit $r_{sc} = 0.006$

$v_{Rsc\%} := 100 \cdot r_{sc}$ Short-circuit resistive voltage drop, % $v_{Rsc\%} = 0.6$

$v_{Xsc\%} := \sqrt{v_{sc\%}^2 - v_{Rsc\%}^2}$ Short-circuit reactive voltage drop, % $v_{Xsc\%} = 7.476$

$x_{sc} := \frac{1}{100} \cdot v_{Xsc\%}$ Short-circuit reactance per unit $x_{sc} = 0.0748$

$v_{sc\%} = 7.5$

(b) Equivalent circuit parameters referred to the HV winding.

$r_{Fe} := \frac{S_n}{\Delta P_{Fe}}$ Core loss resistance (parallel to Xm) per unit $r_{Fe} = 357.143$

$x_m := \frac{I_{1n}}{I_\Phi}$ Magnetizing reactance (parallel to RFe) per unit $x_m = 33.479$

Impedances of equivalen circuit referred to HV winding

$R_{sc} := r_{sc} \cdot Z_n$ Short-circuit resistance, Ω $R_{sc} = 0.24$

$X_{sc} := x_{sc} \cdot Z_n$ Short-circuit reactance, Ω $X_{sc} = 0.33$

$R_{Fe} := R_{Fe} \cdot Z_n$ Core loss resistance (parallel to Xm), Ω $R_{Fe} = 10672200$

$X_m := x_m \cdot Z_n$ Magnetizing reactance (parallel to RFe), Ω $X_m = 162040.7$

Since only the total losses in the windings are known, there is no possibility to calculate separately the resistances of HV and LV side. It is possible to estimate $R_1 = R_2' = 0.5\ R_{sc}$. Similarly, for reactances $X_1 = X_2' = 0.5X_{sc}$.

$R_1 := 0.5 \cdot R_{sc}$ $R_1 = 14.52$

$R_{2p} := 0.5 \cdot R_{sc}$ $R_{2p} = 14.52$

$X_1 := 0.5 \cdot X_{sc}$ $X_1 = 180.918$

$X_{2p} := 0.5 \cdot X_{sc}$ $X_{2p} = 180.918$

Problem 2.9. Two three-phase transformers have the following nominal parameters:

$m_1 = 3$

	Transformator A	Transformator B
Apparent power, W	$S_{nA} := 3.0 \cdot 10^6$	$S_{nB} := 2.5 \cdot 10^6$
Frequency, Hz	$f_n := 50$	$f_n := 50$
Primary (HV) voltage, V	$V_{1n} := 35000$	$V_{1n} := 35000$
Secondary (LV) voltage, V	$V_{2n} := 6600$	$V_{2n} := 6600$
Short-circuit voltage, %	$v_{sh\%A} := 5.0$	$v_{sh\%B} := 6.5$
Windingh losses, W	$\Delta P_{wA} := 26000$	$\Delta P_{wB} := 31000$
Connection	Yd11	Yd11

and operate in parallel. The HV bus bar has the voltage ($V_1 := 30000$ and frequency $f := 50$

Find:
(a) distribution of load between transformers, if the LV bus bars are loaded with current correponding to arithmetic sum of nominal powers of both transformers at power factor of loads $\cos\phi_2 := 0.8$ ind

(b) maximum power the system can deliver without overloading the transformers;
(c) equalizing current I_{2e} when the primary winding tap changer -5% of transformer B and nominal tap changer of transformer A are conencted to HV bus bars.

Solution

(a) Distribution of load between transformers

$$R_{shA} := \frac{\Delta P_{wA} \cdot V_{2n}^2}{S_{nA}^2}$$ Short-circuit resistance of transformer A, Ω $R_{shA} = 0.1258$

$$X_{shA} := \frac{V_{2n}^2}{S_{nA}} \cdot \sqrt{\left(\frac{v_{sh\%A}}{100}\right)^2 - \left(\frac{\Delta P_{wA}}{S_{nA}}\right)^2}$$ Short-circuit reactance of transformer A, Ω $X_{shA} = 0.715$

$$Z_{shA} := \frac{v_{sh\%A} \cdot V_{2n}^2}{100 \cdot S_{nA}}$$ Short-circuit impedance of transformer A, Ω $Z_{shA} = 0.726$

$$Z_{shA} := R_{shA} + j \cdot X_{shA}$$ Short-circuit impedance of transformer A in complex form, Ω $Z_{shA} = 0.126 + 0.715i$

$R_{shB} := \frac{\Delta P_{wB} \cdot V_{2n}^{2}}{S_{nB}^{2}}$ Short-circuit resistance of transformer B, Ω $R_{shB} = 0.2161$

$X_{shB} := \frac{V_{2n}^{2}}{S_{nB}} \cdot \sqrt{\left(\frac{v_{sh\%B}}{100}\right)^{2} - \left(\frac{\Delta P_{wB}}{S_{nB}}\right)^{2}}$ Short-circuit reactance of transformer B, Ω $X_{shB} = 1.112$

$Z_{shB} := \frac{v_{sh\%B} \cdot V_{2n}^{2}}{100 \cdot S_{nB}}$ Short-circuit impedance of transformer B, Ω $Z_{shB} = 1.133$

$Z_{shB} := R_{shB} + j \cdot X_{shB}$ Short-circuit impedance of transformer A in complex form, Ω $Z_{shB} = 0.216 + 1.112i$

$Z_{shA} + Z_{shB} = 0.342 + 1.827i$ Sum of short circuit impedances of transformers A and B, Ω

$I_2 := \frac{S_{nA} + S_{nB}}{\sqrt{3} \cdot V_{2n}} \cdot e^{-j \cdot acos(0.8)}$ Secondary current, A $I_2 = 384.9 - 288.675i$

$I_A := \frac{Z_{shB}}{Z_{shA} + Z_{shB}} \cdot I_2$ Load secondary current of transformer A, A $I_A = 233.333 - 177.538i$

$|I_A| = 293.2$

$I_B := \frac{Z_{shA}}{Z_{shA} + Z_{shB}} \cdot I_2$ Load secondary current of transformer B, A $I_B = 151.567 - 111.137i$

$|I_B| = 187.9$

$I_{nA} := \frac{S_{nA}}{\sqrt{3} \cdot V_{2n}}$ Nominal rms load current of transformer A, A $I_{nA} = 262.432$

$I_{nB} := \frac{S_{nB}}{\sqrt{3} \cdot V_{2n}}$ Nominal rms load current of transformer B, A $I_{nB} = 218.693$

The rms load current - to - rms nominal current ratio of transformer A $\frac{|I_A|}{I_{nA}} = 1.12$

Transformer A is overloaded % $\left(\frac{|I_A|}{I_{nA}} - 1.0\right) \cdot 100 = 11.7$

The rms load current - to - rms nominal current ratio of transformer B $\frac{|I_B|}{I_{nB}} = 0.86$

Transformer A is underloaded % $\left(\frac{|I_B|}{I_{nB}} - 1.0\right)\cdot 100 = -14.1$

(b) the maximum permissible power both transformers can deliver without overloading

The above calcuations show that transformer A is overloaded. Under normal conditions such operation is not permitted. Both transformers can be loaded in such a way, as to not exceed the nominal power and current of the transformer A. In this case the transformer B will be even more underloaded than in (a). The maximum permissible power both transformers can be loaded is calculated from the formula:

$$S_{max} := (S_{nA} + S_{nB})\cdot\frac{I_{nA}}{|I_A|}$$ $$S_{max} = 4.923\times 10^6$$

The coefficient of underload of transformer A $\frac{I_{nA}}{|I_A|} = 0.895$

(c) equalizing current I_{2e} when the primary winding tap changer -5% of transformer B and nominal tap changer of transformer A are conencted to HV bus bars.

Primary winding tap changer -5% will change the voltage ratio of transformer B as compared to transformer A. When the voltage ratios of transformers are different, equalizing current on the secondary side bus bars (LV) will appear.

In this case the equalizing current at LV bus bars can be calculated from the following relationship:

$$I_{2e} := \frac{V_{2n} - 0.95\cdot V_{2n}}{\sqrt{3}\cdot(Z_{shA} + Z_{shB})}$$

Equalizing current at LV bus bars, A $I_{2e} = 18.859 - 100.767i$

The rms equalizing current $|I_{2e}| = 102.5$

Problem 2.10. In a single-phase autotransformer the total number of turns is $N_1 := 420$

The primary winding is connected across $V_1 := 400$ V AC. It has been measured that

the secondary voltaged is $V_2 := 115$ V and the secondary current is $I_2 := 8.5$ A

Calculate the primary current, conduction power, induction power and total apparent power.

Solution

$$\upsilon := \frac{V_1}{V_2}$$ Voltage ratio $\upsilon = 3.48$

$N_2 := \text{round}\left(\frac{N_1}{\upsilon}\right)$ Number of secondary turns $N_2 = 121$

$I_1 := \frac{I_2}{\upsilon}$ Primary current drawn from the power supply, A $I_1 = 2.44$

$S_c := V_1 \cdot I_1 \cdot \frac{1}{\upsilon}$ Conduction power, W $S_c = 281$

$S_i := (V_1 - V_2) I_1$ Induction power, W $S_i = 696.5$

$S_i := V_1 \cdot I_1 \cdot \left(1 - \frac{1}{\upsilon}\right)$ or $S_i = 696.5$

$S := S_c + S_i$ Total apparent power $S = 977.5$

$S := V_1 \cdot I_1$ or $S = 977.5$