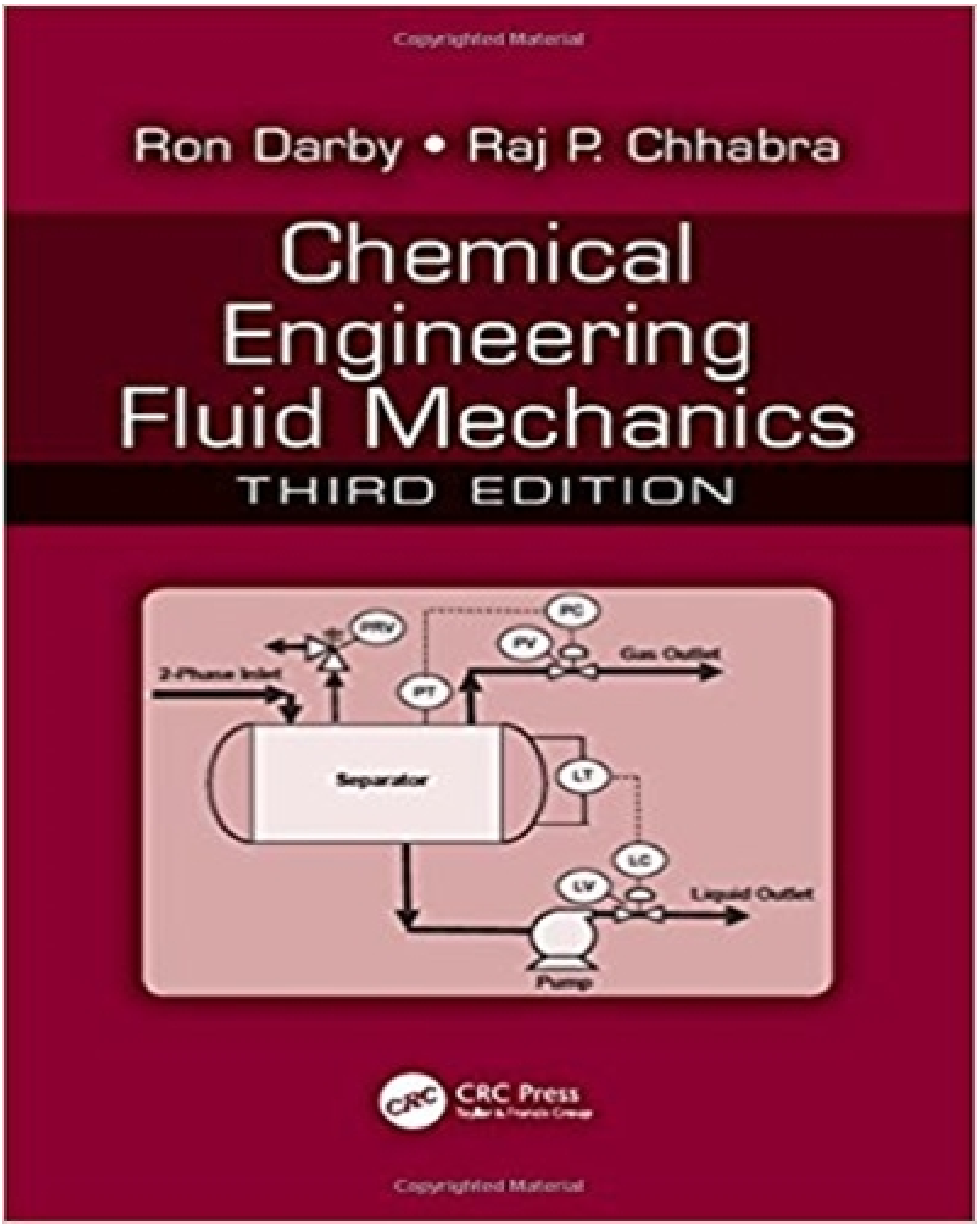
Solutions for Chemical Engineering Fluid Mechanics 3rd Edition by Darby



Solutions

Chapter 2

2-1 Determine the weight of 1 g mass at sea level in units of: (a) dynes; (b) lb_f; (c) g_f ; (d) poundals.

Solution:

Define weight of 1 gm at sea level (L) in units of:

(a) dynes: $W = mg = 1\text{ g }(981\text{ cm/s}^2) = 981\text{ dynes}$

(b) lb_f: $$W = \frac{1\text{g}}{453.6\,\text{g}/\text{lb}_\text{m}}\left(\frac{32.2\frac{\text{ft}}{\text{s}^2}}{32.2\frac{\text{lb}_\text{m}}{\text{lb}_\text{f}}\frac{\text{ft}}{\text{s}^2}}\right) = 0.00220\text{ lb}_\text{f}$$

(c) g_f: $$W = \frac{mg}{g_c} = \frac{1\text{gm}\left(981\frac{\text{cm}}{\text{s}^2}\right)}{981\frac{\text{g}_\text{m}}{\text{g}_\text{f}}\frac{\text{cm}}{\text{s}^2}} = 1\text{g}_\text{f}$$

(d) poundals: $$W = mg = \left(\frac{1\text{gm}}{453.6\frac{\text{g}}{\text{lb}_\text{m}}}\right)\left(32.17\frac{\text{ft}}{\text{s}^2}\right) = 0.0709\text{ poundals}$$

2-2 One cubic foot of water weighs 62.4 lb_f under conditions of standard gravity.

(a) What is its weight in dynes, poundals, and g_f?

(b) What is its density in lb_m/ft^3 and $slugs/ft^3$?

(c) What is its weight on the moon ($g = 5.4\ ft/s^2$) in lb_f?

(d) What is its density on the moon?

Solution:

1 ft^3 H_2O weighs 62.4 lb_f at standard gravity

(a) Find weighs in dynes, Poundals, g_f

$$W = 62.4 lb_f \left(\frac{4.448 \times 10^5\ dyne}{lb_f} \right) = 2.78 \times 10^7\ dyne$$

$$g_c = 32.17 \frac{lb_m}{lb_f} \frac{ft}{s^2} = 1 \frac{lb_m}{poundal} \frac{ft}{s^2} = 32.17 \frac{poundal}{lb_f}$$

$$W = 62.4 lb_f \left(32.17 \frac{poundal}{lb_f} \right) = 2010\ poundals$$

$$W = 62.4\, lb_f \left(453.6 \frac{g_f}{lb_f} \right) = 2.82 \times 10^4\ g_f$$

(b) Find density in lb_m/ft^3 and $slugs/ft^3$:

$$M = \frac{F g_c}{g} = \frac{62.4\, lb_f \left(32.2 \frac{lb_m}{lb_f} \frac{ft}{s^2} \right)}{32.2\ \frac{ft}{s^2}} = 62.4 lb_m \quad \therefore \rho = 62.4 \frac{lb_m}{ft^3}$$

$$\rho = 62.4 \frac{lb_m}{ft^3} \frac{slug}{32.17\, lb_m} = 1.94 \frac{slug}{ft^3}$$

(c) Weight on moon, in lb_f? ($g = 6\ ft/s^2$)

$$W = \frac{mg}{g_c}\ ;\ \frac{W_2}{W_1} = \frac{g_2}{g_1} \quad \text{Since } m,\ g_c = \text{constant}$$

$$W_{moon} = W_{earth} \frac{g_{moon}}{g_{earth}} = 62.4\, lb_f \left(\frac{6\ \frac{ft}{s^2}}{32.17\ \frac{ft}{s^2}} \right) = 11.6\ lb_f$$

(d) Density on moon?

$$\rho = \frac{m}{V} \qquad \text{m, V independent of g}$$

$$\therefore \rho = 62.4 \frac{lb_m}{ft^3}$$

2-3 The acceleration due to gravity on the moon is about 5.4 ft/s^2. If your weight is 150 lb_f on the earth:

(a) What is your mass on the moon in slugs,?

(b) What is your weight on the moon, in SI units?

(c) What is your weight on earth, in poundals?

Solution:

Moon: g = 5.4 ft/s^2. If you weigh 150 lb_f on earth:

(a) Mass on moon in slugs = ? Mass is independent of g, so it is the same as on earth:

$$W = mg$$

$$150\,lb_f = m(\text{slugs})\ 32.2\,{}^{ft}\!/_{s^2}$$

$$M = 4.66\ \text{slugs}$$

(b) Weight on moon, in SI units? (Scientific) W = mg can convert from slugs to kg using:

$$g_c = 1\frac{kg\,m}{N\,s^2} = 1\frac{slug\,ft}{lb_f\,s^2} = 32.2\frac{lb_m\,ft}{lb_f\,s^2}$$

or

$$\frac{W_{moon}}{W_{earth}} = \frac{g_{moon}}{g_{earth}}$$

$$W_{moon} = 150\,lb_f\left(\frac{5.4}{32.2}\right) = 25.155\,lb_f = 25.155\frac{lb_f}{0.2248\frac{lb_f}{N}} = 112\,N$$

(c) Weight on earth in poundals? (Scientific)

Convert from lb_f to poundals:

$$g_c = \frac{lb_m}{poundal}\frac{ft}{s^2} = 32.2\ \frac{lb_m}{lb_f}\frac{ft}{s^2}\quad \text{or}\quad 32.2\ \frac{poundals}{lb_f}$$

$$W_{earth} = 150\,lb_f\left(32.2\frac{poundals}{lb_f}\right) = 4830\,\text{poundals}$$

2-4 You weigh a body with a mass *m* on an electronic scale, which is calibrated with a known mass.

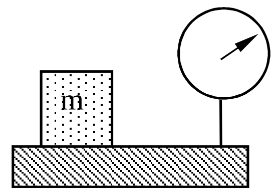
(a) What does the scale actually measure, and what are its dimensions?

(b) If the scale is calibrated in the appropriate system of units, what would the scale reading be if the mass of *m* is (1) 1 slug; (2) 1 lb_m (in scientific units); (3) 1 lb_m (in engineering units); (3) 1 g_m (in scientific units); (4) 1 g_m (in engineering units).

Solution:

Weigh body with mass m on electronic scale

Calculated with known mass



(a) What does scale measure Force due to gravity = Wt

$$[Wt] = F = \frac{ML}{t^2}$$

(b) What is scale reading if m is :

(1) 1 slug : (Scientific, $g_c = 1$)

$$W = mg = 1\text{slug}\ \ 32.2\frac{ft}{s^2} = 32.2\ lb_f$$

(2) 1 lb_m: (Scientific, $g_c = 1$)

$$W = mg = 1 lb_m\ \ 32.2\frac{ft}{s^2} = 32.2\ \text{Poundals}$$

(3) 1 lb_m(Engineering, $g_c = 32.2\frac{lb_m}{lb_f}\frac{ft}{s^2}$)

$$W = \frac{mg}{g_c} = 1\ lb_m \frac{32.2\frac{ft}{s^2}}{32.2\frac{lb_m}{lb_f}\frac{ft}{s^2}} = 1 lb_f$$

(4) 1 gm (Engineering, $g_c = 980\frac{g_m}{g_f}\frac{cm}{s^2}$)

$$W = \frac{mg}{g_c} = 1\ gm \frac{980\frac{cm}{s^2}}{980\frac{g_m}{g_f}\frac{cm}{s^2}} = 1 g_f$$

2-5 Explain why the gravitational "constant' (*g*) is different at Reykjavik, Iceland than it is at La Paz, Bolivia. At which location is it greatest, and why? If you could measure the value of *g* at these two locations, what would this tell you about the earth?

Solution:

Why is g different at Reykjavic than at La Paz ?

Because: $W_{m1} = G\frac{m_1 m_2}{r^2}$ where m_1 = "any mass", Weight W_{m1}

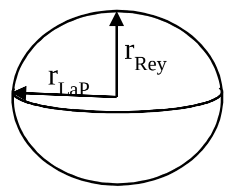
m_2 = earth, r = radius of earth

r is smaller at Reykejavik than at La Paz because earth isn't round but is slightly flat at the poles.

$\therefore \; g_{Rey} > g_{LaP}$ since $r_{Rey} < r_{LaP}$:

Measure of g gives a measure of r (radius of earth)

$$\frac{r_{Rey}}{r_{LaP}} = \left(\frac{g_{LaP}}{g_{Rey}}\right)^{1/2}$$



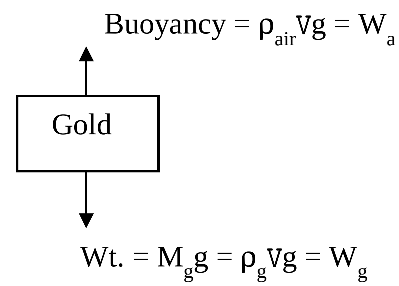
2-6 You have purchased a 5 oz bar of gold (100% pure), at a cost of \$400/oz. Because the bar was weighed in air, you conclude that you got a bargain, because its true mass is greater than 5 oz due to the buoyancy of air. If the true density of the gold is 1.9000 g/cm^3, what is the actual value of the bar based upon its true mass?

Solution:

5 oz gold bar cost \$ 400/oz (weighted in air)

ρ_{gold} = 1.900 g/cc. What is actual value based on true measure?

$$\text{Net Wt.} = W_g - W_a = 5\,oz_f \left(\frac{lb_f}{16\,oz_f}\right) = 0.3125\ lb_f$$



$$W_N = W_g - W_a = \frac{M_g g}{g_c} - \frac{\rho_a V_g g}{g_c}$$

$$V_g = \frac{M_g}{\rho_g}$$

$$W_N = \frac{g}{g_c}\left(M_g - \frac{\rho_a}{\rho_g} M_g\right) = M_g \frac{g}{g_c}\left(1 - \frac{\rho_a}{\rho_g}\right)$$

$$M_g = \left(\frac{W_N\, g_c/g}{1 - \frac{\rho_a}{\rho_g}}\right)$$

Assume, P = 1 atm, T = 70°F

$$\rho_{air} = \frac{\overline{M}}{\overline{V}_{std}} = \frac{29\,\frac{lb_m}{lb\,mole}}{359\,\frac{ft^3 std}{lb\,mole}}\left(\frac{492}{530}\right) = 0.0750\,\frac{lb_m}{ft^3}$$

$$M_g = \frac{0.3125\,lb_f\left(32.2\,\frac{lb_m}{lb_f}\,\frac{ft}{s^2}\right)\Big/\left(32.2\,\frac{ft}{s^2}\right)}{1-\left(\frac{0.0750}{1.90(62.4)}\right)} = 0.3127\,lb_m \quad \text{(True Mass)}$$

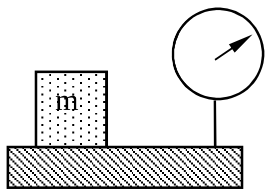
$$\text{True Value} = \frac{400}{oz_{in\,air}}\left(\frac{0.3127\,lb_{m\ true}}{0.3125\ lb_{m\ air}}\right) = \$400.25 \qquad \text{(Made 25¢!)}$$

(Actually, the scale would be calibrated using a std weight in air, so effect of buoyancy cancels out.

2-7 You purchased 5 oz of gold in Quito, Ecuador (g = 977.110 cm/s^2) for $400/oz. You then took the gold and the same spring scale on which you weighed it in Quito to Reykjavik Iceland (g = 983.06 cm/s^2) where you weighed it again and sold it for $400/oz. How much money did you make or lose, or did you break even?

Solution:

Actually, the gold is probably weighed on a balance calibrated with a "standard mass, so that effect of gravity cancels out. However, here we assume the scale was calibrated in Quito, and used again in Rejkjavik without re-calibration.



$M = 5$ oz ; $W = mg/g_c$

Quito: $W_1 = \dfrac{Mg_1}{g_c}$

Iceland: $W_2 = \dfrac{Mg_2}{g_c}$

$$\frac{W_2}{W_1} = \frac{g_2}{g_1} = \frac{983.06}{977.110}$$

Value of gold = \$: $\dfrac{\$_2}{\$_1} = \dfrac{W_2}{W_1} = \dfrac{g_2}{g_1}$

Increase in Value: $\$_2 - \$_1 = \$_1\left(\dfrac{\$_2}{\$_1} - 1\right) = \dfrac{\$400}{\text{oz}}(5\,\text{oz})\left(\dfrac{983.06}{977.11} - 1\right) = \$12.18\,\text{gained}$

2-8 Calculate the pressure at a depth of 2 miles below the surface of the ocean. Explain and justify any assumptions you make. The physical principle that applies to this problem can be described by the equation

$$\Phi = \text{constant}$$

where $\Phi = P + \rho g\, z$ and z is the vertical distance measured upward from any horizontal reference plane. Express your answer in units of (a) atm, (b) psi, (c) Pa, (d) poundal/ft^2, (e) dyn/cm^2, (f) kg$_f$/m^2, (g) N.

Solution:

Calculate P_2:

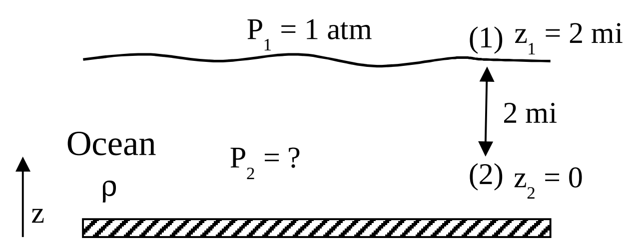
Assume: P_1 = 1 atm (avg)

z_1 = 2 mi (neglect waves)

z_2 = 0

ρ = 64 lb$_m$/ft^3 (sea water) independent of P and z (incompressible).

g is independent of z



(a) Φ = constant = P + ρgz

$P_1 + \rho g z_1 = P_2 + \rho g z_2$

or $P_2 = P_1 + \rho g\,(z_1 - z_2)$

$$= 1\,\text{atm} + \frac{\left(64\,\frac{\text{lb}_\text{m}}{\text{ft}^3}\right)\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)(2\,\text{mi})\left(5290\,\frac{\text{ft}}{\text{mi}}\right)}{\left(32.2\,\frac{\text{lb}_\text{m}}{\text{lb}_\text{f}}\frac{\text{ft}}{\text{s}^2}\right)\left(144\,\frac{\text{in}^2}{\text{ft}^2}\right)\left(14.7\,\frac{\text{lb}_\text{f}}{\text{in}^2\,\text{atm}}\right)} = 320.274\,\text{atm} = 320\text{ atm}$$

(b) P_2 = 320.27 atm (14.7 psi/atm) = 4708 psi = 4710 psi

(c) P_2 = 320.27 atm (1.033×10^5 Pa/atm) = 3.25×10^7 Pa

(d) $P_2 = 320.27\text{atm} \times 14.7 \dfrac{\text{lb}_f}{\text{in}^2\text{atm}} \times 144 \dfrac{\text{in}^2}{\text{ft}^2} \times 32.2 \dfrac{\text{Poundal}}{\text{lb}_f} = 2.18 \times 10^7 \dfrac{\text{Poundal}}{\text{ft}^2}$

(e) $P_2 = 320.27 \left(1.033 \times 10^6 \dfrac{\text{dyn}}{\text{cm}^2\,\text{atm}} \right) = 3.25 \times 10^8 \dfrac{\text{dyn}}{\text{cm}^2}$

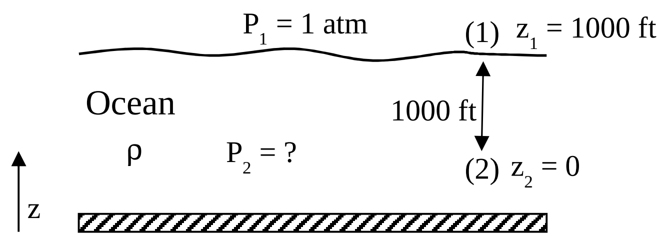
Round off all answers to 3 digits maximum.

2-9 (a) Use the principle in Problem 8 to calculate the pressure at a depth of 1000 ft below the surface of the ocean (in psi, Pa, and atm). Assume that the ocean water density is 64 lb_m/ft^3.

(b) If this ocean were on the moon, what would be the answer to (a)? Use the following information to solve this problem: The diameter of the moon is 2160 miles, the diameter of the earth is 8000 miles, and the density of the earth is 1.6 times that of the moon.

Solution:

Calculate P_2:



Assume: $P_1 = 1$ atm (avg)

$\rho = 64\ lb_m/ft^3$

Φ = constant = $P + \rho g z$

$P_1 + \rho g z_1 = P_2 + \rho g z_2$

or $P_2 = P_1 + \rho g (z_1 - z_2)$

$$P_2 = 1\,atm + \frac{\left(64\dfrac{lb_m}{ft^3}\right)\left(32.2\dfrac{ft}{s^2}\right)(1000\,ft)}{\left(32.2\dfrac{lb_m}{lb_f}\dfrac{ft}{s^2}\right)\left(144\dfrac{in^2}{ft^2}\right)\left(14.7\dfrac{lb_f}{in^2 atm}\right)} = 31.23\,atm = 31.2\,atm$$

P_2 = 31.23 atm (14.7 psi/atm) = 459 psi

P_2 = 31.23atm (1.033×10^5 Pa/atm) = 3.16×10^6 Pa

On moon: D_{moon} = 2160 mi, D_{earth} = 8000 mi, $\rho_{earth} = 1.6\ \rho_{moon}$

Need to find g_{moon}:

$$Wt_{m1} = G\frac{m_1 m_2}{r^2} = gm_1 \qquad m_2 = \text{earth/moon, r = earth/moon}$$

$$g = G\frac{m_2}{r^2}$$

$$\frac{g_{moon}}{g_{earth}} = \frac{m_{moon}\, r^2_{earth}}{m_{earth}\, r^2_{moon}}, \quad m = \rho V = \frac{\rho \pi D^3}{6}$$

$$\frac{g_{moon}}{g_{earth}} = \frac{\left(\rho D^3\right)_m}{\left(\rho D^3\right)_e}\left(\frac{D_e}{D_m}\right)^2 = \frac{(\rho D)_m}{(\rho D)_e} = \frac{1}{1.6}\left(\frac{2160}{8000}\right) = 0.16875$$

$$g_{moon} = 32.2(0.16875)\frac{ft}{s^2} = 5.434\frac{ft}{s^2}$$

Also, on moon $P_1 = 0$ (atm)

$$\therefore P_2 = \rho g_m (z_1 - z_2) = \frac{\left(64\frac{lb_m}{ft^3}\right)\left(5.43\frac{ft}{s^2}\right)(1000\,ft)}{\left(32.2\frac{lb_m}{lb_f}\frac{ft}{s^2}\right)\left(144\frac{in^2}{ft^2}\right)} = 75\,psi$$

$$P_2 = \frac{75\,psi}{14.7\frac{psi}{atm}} = 5.10\,atm$$

$$P_2 = \frac{75\,psi}{14.7}\left(1.033\times10^5\frac{Pa}{atm}\right) = 5.17\times10^5\,Pa$$

2-10 The following formula for the pressure drop through a valve was found in a design manual:

$$h_L = \frac{522\,K\,q^2}{d^4}$$

where h_L = the "head loss" in feet of fluid flowing through the valve, K = dimensionless resistance coefficient for the valve, q = flow rate through the valve, in ft^3/s, and d = diameter of the valve, in inches.

(a) Can this equation be used without changing anything if SI units are used for the variables? Explain.

(b) What are the dimensions of "522" in this equation? What are its units?

(c) Determine the pressure drop through a 2 in. valve with a K of 4 for water at 20° C flowing at a rate of 50 gpm (gal/min), in units of: (1) feet of water, (2) psi, (3) atm, (4) Pa, (5) dyn/cm^2; and (6) inches of mercury.

Solution:

Formula for pressure drop in valve:

$$h_L = \frac{522\,Kq^2}{d^4}$$

h_L = "head loss" (ft of fluid)

K = coefficient [K] = 0

q = ft^3/s

d = in

Head is related to ΔP by $\Delta P = \rho g h_L$

K dimensions: $[L] = \dfrac{[522][0]\left[L^3/t\right]^2}{[L^4]}$

∴ Must have $[522] = t^2/L$

(a) Only dimensionless quantities are independent of scales (i.e., units)

∴ Any other units wouldn't work without converting "522" into the system of interest.

(b) Dimensions of 522 = t^2/L

$$522 = h_L \frac{d^4}{Kq^2} = \frac{ft\,in^4 s^2}{ft^6} = \frac{in^4 s^2}{ft^6}$$

(c) Find ΔP in (1) ft of H_20 (2) psi (3) atm (4) Pa (5) dyn/cm^2 (6) in Hg

For d = 2 in, K = 4, water at 20°C at 50 gpm

$$q = 50\frac{\text{gal}}{\text{min}}\frac{\text{ft}^3\text{min}}{7.48\,\text{gal}\,60\,\text{s}} = 0.1114\frac{\text{ft}^3}{\text{s}}$$

$$h_L = \frac{522(4)(0.114)^2}{2^4} = 1.6195 = 1.62\,\text{ft of } H_2O$$

at 20°C $\rho_w = 998\ \text{kg/m}^3$

at 4°C $\rho_w = 1000\ \text{kg/m}^3$

$$\Delta P = \rho g h_L = 998\frac{\text{kg}}{\text{m}^3}\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left(\frac{1.6195\text{ft}}{3.28\,\text{ft}/\text{m}}\right) = 4834\frac{\text{kg}}{\text{m}\,\text{s}^2} = 4834\,\text{Pa} = 4830\text{Pa} \quad \ldots(4)$$

"ft of H_2O refers to at ρ_w 4°C:

$$h_L = (h_L)\frac{\rho_{20^\circ}}{\rho_{4^\circ}} = 1.6195\,\text{ft}\frac{998}{1000} = 1.616 = 1.62\,\text{ft of } H_2O \quad \ldots(1)$$

$$P_2 = 4834\,\text{Pa}\frac{14.7\,\text{psi}}{1.033\times10^5\,\text{Pa}} = 0.701\text{psi} \quad \ldots(2)$$

$$P_2 = \frac{4834\,\text{Pa}}{1.033\times10^5\,\text{Pa}/\text{atm}} = 0.0477\,\text{atm} \quad \ldots(3)$$

$$P_2 = 0.0477\,\text{atm}\times1.033\times10^5\frac{\text{dyn}}{\text{cm}^2\,\text{atm}} = 4.83\times10^4\frac{\text{dyn}}{\text{cm}^2} \quad \ldots(5)$$

$$P_2 = 0.0477\,\text{atm}\times29.92\frac{\text{in Hg}}{\text{atm}} = 1.43\,\text{in Hg} \quad \ldots(6)$$

2-11 When the energy balance on the fluid in a stream tube is written in the following form it is known as Bernoulli's equation:

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{\alpha}{2}(V_2^2 - V_1^2) + e_f + w = 0$$

where $-w$ is the work done on a unit mass of fluid, e_f is the energy dissipated by friction in the fluid per unit mass, including all thermal energy effects due to heat transfer or internal generation, and α is equal to either 1 or 2 for turbulent or laminar flow, respectively. If P_1 = 25 psig, P_2 = 10 psig, z_1 = 5 m, z_2 = 8 m, V_1 = 20 ft/s, V_2 = 5 ft/s, ρ = 62.4 lb_m/ft^3, α = 1, and w = 0, calculate the value of e_f in each of the following systems of units:

a) SI

(b) mks engineering (e.g., metric engineering)

(c) English engineering

(d) English scientific (with M as a fundamental dimension)

(e) English thermal units (e.g., Btu)

(f) Metric thermal units (e.g., calories)

Solution:

Bernouill's equation (with α =1):

$$\frac{\Delta P}{\rho} + g\Delta z + \frac{1}{2}\Delta V^2 + e_f + w = 0$$

-w = work/mass, e_f = energy lost to friction/mass, $\Delta(\ ) = (\ )_2 - (\ )_1$

P_1= 25 psig, P_2 = 10 psig, z_1 = 5 m, z_2 = 8 m, V_1 = 20 ft/s, V_2 = 5 ft/s

ρ = 62.4 lb_m/ft^3, w = 0

Calculate e_f in various units: (a) SI: since all terms are energy/mass, SI units = Nm/kg

Solution of e_f:

$$e_f = -\frac{\Delta P}{\rho} - g\Delta z - \frac{1}{2}\Delta V^2 - \cancel{w}$$

$$= -\frac{(10-25)\dfrac{lb_m}{in^2}}{\left(62.4\dfrac{lb_m}{ft^3}\right)}\left(144\frac{in^2}{ft^2}\right) - \frac{32.17\dfrac{ft}{s^2}(8-5)\,m}{0.3048\dfrac{m}{ft}\,32.17\dfrac{lb_m}{lb_f}\dfrac{ft}{s^2}} - \frac{\dfrac{1}{2}(5^2-20^2)\dfrac{ft^2}{s^2}}{32.17\dfrac{lb_m}{lb_f}\dfrac{ft}{s^2}}$$

$$= 34.6 - 9.84 + 5.83 = 30.6\frac{\text{ft lb}_f}{\text{lb}_m}$$

Convert to

(a) SI:

$$e_f = 30.6\frac{\text{ft lb}_f}{\text{lb}_m}\left(0.3048\frac{\text{m}}{\text{ft}}\right)\left(4.448\frac{\text{N}}{\text{lb}_f}\right)\left(2.2\frac{\text{lb}_m}{\text{kg}}\right) = 91.3\frac{\text{N m}}{\text{kg}}$$

(b) MKS Engineering: i.e., kg_f, kg_m, m, s, $g_c = 1\frac{\text{kg m}}{\text{N s}^2} = 9.81\frac{\text{kg m}}{\text{kg}_f\ \text{s}^2}$

$$e_f = 30.6\frac{\text{ft lb}_f}{\text{lb}_m}\left(0.3048\frac{\text{m}}{\text{ft}}\right)\left(4.448\frac{\text{N}}{\text{lb}_f}\right)\left(2.2\frac{\text{lb}_m}{\text{kg}_m}\right)\left(\frac{\text{kg}_f}{9.81\,\text{N}}\right) = 9.30\frac{\text{kg}_f\ \text{m}}{\text{kg}_m}$$

(c) English Engineering: i.e., lb_f, lb_m, ft, s (See above): $e_f = 30.6$ ft lb_f/lb_m

(d) English Scientific (M fundamental): i.e., slug, lb_f, ft, s,

$$g_c = 1\frac{\text{slug ft}}{\text{lb}_f\text{s}^2} = 32.2\frac{\text{lb}_m\text{ft}}{\text{lb}_f\ \text{s}^2} = 32.17\frac{\text{lb}_m}{\text{slug}}$$

$$e_f = 30.6\frac{\text{ft lb}_f}{\text{lb}_m}\left(32.17\frac{\text{lb}_m}{\text{slug}}\right) = 984\frac{\text{lb}_f\ \text{ft}}{\text{slug}}$$

(e) English Thermal: (e.g. BTU)

$$e_f = 30.6\frac{\text{ft lb}_f}{\text{lb}_m}\left(\frac{\text{BTU}}{778\ \text{ft lb}_f}\right) = 0.0393\frac{\text{BTU}}{\text{lb}_m}$$

(f) Metric Thermal (Cal, gm)

$$e_f = 0.0393\frac{\text{BTU}}{\text{lb}_m}\left(\frac{0.239\,\text{cal}}{9.48\times10^{-4}\,\text{BTU}}\right)\left(\frac{\text{lb}_m}{453.6\,\text{g}}\right) = 0.0218\frac{\text{cal}}{\text{g}}$$

2-12 Determine the value of the gas constant, *R*, in units of $ft^3atm/\,lbmol\,^oR$, starting with the value of the standard molar volume of a perfect gas..

Solution:

Define R ($ft^3atm/lb_{mol}\,°R$) starting with standard molar volume:

$$V_{sc} = 359\frac{ft^3}{mol}\ \text{at}\ 1\,atm, 32^oF\left(492^oR\right)$$

Ideal Gas:

$$\rho = \frac{PM}{RT} = \frac{M}{V_{sc}}\left(\frac{P}{1\,atm}\right)\left(\frac{492}{T\left({}^oR\right)}\right)$$

or

$$\frac{1}{R} = \frac{492}{V_{sc}},\quad R = \frac{359\dfrac{ft^3atm}{lb_{mol}}}{492^oR} = 0.7297\frac{ft^3atm}{lb_{mol}\,^oR} = 0.73\frac{ft^3atm}{lb_{mol}\,^oR}$$

Define value of R in ($ft^3atm/lb_{mol}\,°R$) starting with standard molar volume for Ideal gas:

Ideal Gas:

$$\rho = \frac{PM}{RT} \rightarrow \frac{M}{\rho} = \frac{RT}{P}\left(\frac{mass/mol}{mass/vol} = \frac{vol}{mol}\right)$$

Standard molar volume: $V_{sc} = 359\dfrac{ft^3}{mol}\ \text{at}\ P = 1\,atm, T = T_s = 492^oR$

$$\frac{vol}{mol} : V_m = V_{sc}\left(\frac{T}{T_s}\right)\left(\frac{P_s}{P}\right) = \frac{RT}{P}$$

$$R = V_{sc}\frac{P_s}{T_s} = \left(359\frac{ft^3}{lb_{mol}}\right)\left(\frac{1\,atm}{492^oR}\right) = 0.7297\frac{ft^3atm}{lb_{mol}\,^oR} = 0.7297\frac{ft^3atm}{lb_{mol}\,^oR} = 0.73\frac{ft^3atm}{lb_{mol}\,^oR}$$

2-13 Calculate the value of the Reynolds number for sodium flowing at a rate of 50 gpm through a tube with a 1/2 in. ID, at 400°F.

Solution:

Calculate N_{Re} for Na at 50 gpm in 1/2 in tube at 400°F.

Appx A-14 Nomograph for Na: X = 16.4 Y = 13.9

μ = 0.45 cP at 400°F

SG = 0.97 at 293 K = 67.4°F

Want ρ at 400°F.

Assume $\rho_{Na}(T) \cong \rho_{Hg}(T)$, From Appx A-4

(both liquid metals)

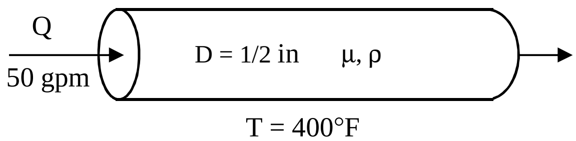
At 67°F $\rho_{Hg} = 845\ lb_m/ft^3$ (interpolate)

At 400°F $\rho_{Hg} = 817\ lb_m/ft^3$

If % change for Na is same:

$$\rho_{Na} = 970\frac{kg}{m^3}\left(1-\frac{845-817}{845}\right) = 938\frac{kg}{m^3}$$

$$N_{Re} = \frac{\rho VD}{\mu} = \frac{4Q\rho}{\pi D\mu} = \frac{4(50\,gpm)\left(63.1\frac{cc/s}{gpm}\right)\left(0.938\frac{g}{cc}\right)}{\pi(0.5\times2.54\,cm)\left(0.0045\frac{g}{cm\,s}\right)} = 6.59\times10^5$$



2-14 The conditions at two different positions along a pipeline (at points 1 and 2) are related by the Bernoulli equation (see Problem 11). For flow in a pipe,

$$e_f = \left(\frac{4fL}{D}\right)\left(\frac{V^2}{2}\right)$$

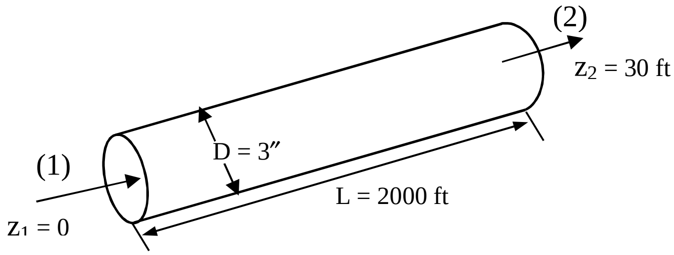
where D is the pipe diameter and L is the pipe length between points 1 and 2. If the flow is laminar ($N_{Re} < 2000$), the value of $\alpha = 2$ and $f = 16/N_{Re}$, but for turbulent flow in a smooth pipe $\alpha = 1$ and $f = 0.0791/N_{Re}^{1/4}$. The work done by a pump on the fluid ($-w$) is related to the power delivered to the fluid (HP) and the mass flow rate of the fluid ($\dot{m}$) by $HP = -w\,\dot{m}$. Consider water (ρ = 1 g/cc, μ = 1 cP) being pumped at a rate of 150 gpm (gal/min) through a 2000 ft long 3 in. diameter pipe. The water is transported from a reservoir (z = 0) at atmospheric pressure, to a condenser at the top of a column which is at an elevation of 30 ft and a pressure of 5 psig. Determine:

(a) The value of the Reynolds number in the pipe;

(b) The value of the friction factor in the pipe (assuming that it is smooth);

(c) The power that the pump must deliver to the water, in horsepower (hp).

Solution:



Bernouill's equation:

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{\alpha}{2}\left(V_2^2 - V_1^2\right) + e_f + w = 0$$

If $N_{Re} < 2000$, $\alpha = 2$, $f = \dfrac{16}{N_{Re}}$

If $N_{Re} > 2000$, $\alpha = 1$, $f = \dfrac{0.0791}{N_{Re}^{1/4}}$

$$\mathbb{P} = -w\dot{m}$$

H_2O: $\rho = 1$ g/cc, $\mu = 1$ cP = 0.01 P

Q = 150 gpm, L = 2000 ft, D = 3 in

$P_1 = 1$ atm, $z_1 = 0$, $z_2 = 30$ ft, $P_2 = 5$ psig

(a) $$N_{Re} = \frac{\rho VD}{\mu} = \frac{4Q\rho}{\pi D\mu} = \frac{4(150\,\text{gpm})\left(63.1\frac{\text{cc/s}}{\text{gpm}}\right)\left(1\frac{\text{g}}{\text{cc}}\right)}{\pi(3\times 2.54\,\text{cm})\left(0.01\frac{\text{g}}{\text{cm s}}\right)} = 1.581\times 10^5 = 1.58\times 10^5$$

(b) f = ? (smooth)

$$\text{Since, } N_{Re} > 2000,\ f = \frac{0.0791}{\left(1.58\times 10^5\right)^{1/4}} = 0.003967 = 0.00397$$

(c) Pump power ($\mathbb{P}$)= ?

$$\dot{m} = \rho Q = 1\frac{\text{g}}{\text{cc}}\left(150\frac{\text{gal}}{\text{min}}\right)63.1\frac{\text{cc/s}}{\text{gpm}} = 9465\frac{\text{g}}{\text{s}}$$

Get -w from Bernoulli's equation ($\alpha = 1$):

$$-w = \frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{1}{2}\left(\cancel{V_2^2} - \cancel{V_1^2}\right) + \frac{4fLV^2}{2D} = 0$$

Assume $V_1 = V_2 = V_{pipe}$

$$\text{Assume } V = \frac{4Q}{\pi D^2} = \frac{4(150\text{gpm})\left(63.1\frac{\text{cc/s}}{\text{gpm}}\right)}{\pi(3\times 2.54)^2} = 207.5\frac{\text{cm}}{\text{s}}$$

$$-w = \frac{(5-0)\frac{\text{lb}_f}{\text{in}^2}\left(1.013\times 10^6\right)\frac{\text{dyn}}{\text{cm}^2}}{1\frac{\text{g}}{\text{cc}}(14.7)\text{psi}} + 980\left(30\text{ft}\times 30.48\frac{\text{cm}}{\text{ft}}\right)\frac{\text{cm}}{\text{s}^2} +$$

$$\frac{4(0.00397)(2000\text{ft})\left(30.48\frac{\text{cm}}{\text{ft}}\right)\left(207.5\frac{\text{cm}}{\text{s}}\right)^2}{2(3\times 2.54)\text{cm}}$$

$$= 3.974\times 10^6\frac{\text{cm}^2}{\text{s}^2} = 4278\frac{\text{ft}^2}{\text{s}^2}$$

$$\mathbb{P} = -w\dot{m} = \frac{9465\,\frac{\text{g}}{\text{s}}\left(4278\,\frac{\text{ft}^2}{\text{s}^2}\right)}{454\,\frac{\text{g}}{\text{lb}_\text{m}}\left(32.2\,\frac{\text{lb}_\text{m}}{\text{lb}_\text{f}}\,\frac{\text{ft}^2}{\text{s}^2}\right)550\,\frac{\text{ft}\,\text{lb}_\text{f}}{\text{hp}\,\text{s}}} = 5.04\ \text{hp}$$

2-15 The Peclet number (N_{Pe}) is defined as

$$N_{\text{Pe}} = N_{\text{Re}} N_{\text{Pr}} = \left(\frac{DV\rho}{\mu}\right)\left(\frac{c_{\text{p}}\mu}{k}\right) = \frac{DGc_{\text{p}}}{\mu}$$

where D = pipe diameter, G = mass flux = ρV, c_p = specific heat, k = thermal conductivity, μ = viscosity. Calculate the value of N_{Pe} for water at 60°F flowing through a 1 cm diameter tube at a rate of 100 lb_m/hr. (Use the most accurate data you can find, and state your answer in the appropriate number of digits consistent with the data you use.)

Solution:

$$N_{Pe} = N_{Re}N_{Pr} = \left(\frac{\rho VD}{\mu}\right)\left(\frac{C_p\mu}{k}\right) = \frac{DGC_p}{\mu}$$

D = pipe diameter, G = mass velocity = ρV, C_p = specific heat

k = thermal conductivity, μ = viscosity

Determine value of N_{pe} for water at 60°F flowing in a 1 cm diameter tube at 100 lb_m/hr.

Use data from Appendix.

Thermal properties not in Apppendix. Must find them elsewhere.

e.g. Eckert gives value of N_{Pr} = 7.7 for H_2O of 60°F.

Appendix. A-1-8: 60°F: ρ = 62.4 lb_m/ft^3, $\mu = 2.36\times10^{-5}$ $lb_f\ s/ft^2$, $\nu = 1.22\times10^{-5}$ ft^2/s

$$N_{Re} = \frac{\rho VD}{\mu} = \frac{\rho V}{\nu} = \frac{4Q\rho}{\pi D\mu}$$

$$= \frac{4\left(100\,\frac{lb_m}{hr}\right)\left(30.48\,\frac{cm}{ft}\right)}{3600\,\frac{s}{hr}\,\pi(1\text{ cm})\,2.36\times10^{-5}\,lb_f\,s\left(32.17\,\frac{lb_m\,ft}{lb_f\,s^2}\right)} = 1.42\times10^3$$

$$N_{Pe} = 1.42\times10^3(7.7) = 1.1\times10^4$$

(Limiting accuracy is $N_{Pr} \rightarrow$ 2 significant figs.)

2-16 The heat transfer coefficient (h) for a vapor bubble rising through a boiling liquid is given by:

$$h = A\left(\frac{kV\rho c_{\mathrm{p}}}{\mathrm{d}}\right)^{1/2} \quad \text{where} \quad V = \left(\frac{\Delta\rho g\sigma}{\rho_{\mathrm{v}}^{2}}\right)^{1/4}$$

where h = heat transfer coefficient [e.g., Btu/(hr ^{o}F ft^{2})],

c_{p} = liquid heat capacity [e.g., cal/(g ^{o}C)],

k = liquid thermal conductivity [e.g,. J/(sKm)],

σ = liquid/vapor surface tension [e.g., dyn/cm],

$\Delta\rho = \rho_{\mathrm{liquid}} - \rho_{\mathrm{vapor}} = \rho_{l} - \rho_{v}$,,

d = bubble diameter , and g = acceleration due to gravity.

(a) What are the fundamental dimensions of V and A?

(b) If the value of h is 1000 Btu/(hr ft^{2} ^{o}F) for a 5 mm diameter steam bubble rising in boiling water at atmospheric pressure, determine the corresponding values of V and A in SI units. You must look up values for the other quantities you need, and be sure to cite the sources you use for these data.

Solution:

Heat Transfer coefficient for a vapor bubble rising through a boiling liquid:

$$\mathrm{h}\left(\frac{\text{energy}}{\text{area, time T}}\right),\ \mathrm{k}\left(\frac{\text{energy}}{\text{t Tdifference}}\right),\ \mathrm{C_p}\left(\frac{\text{energy}}{\text{mass T}}\right)$$

$$h = A\left(\frac{kV\rho C_p}{d}\right)^{1/2} \quad \text{and} \quad V = \left(\frac{\Delta\rho g\sigma}{\rho V^2}\right)^{1/4}$$

$$h\left(\frac{F}{tTL}\right),\ C_p\left(\frac{L^2}{t^2T}\right),\ k\left(\frac{F}{tT}\right)$$

(a) Dimension of V and A ?

$$[\mathrm{V}] = \left[\frac{\Delta\rho g\sigma}{\rho_v^2}\right]^{1/4} = \left[\frac{\cancel{\mathrm{L}}\mathrm{FL}^3}{\mathrm{t}^2\cancel{\mathrm{L}}\mathrm{M}} = \frac{\cancel{\mathrm{M}}\mathrm{L}^4}{\mathrm{t}^4\cancel{\mathrm{M}}}\right]^{1/4} = \frac{\mathrm{L}}{\mathrm{t}}$$

$$[\mathrm{A}] = \left[\frac{\mathrm{hd}^{1/2}}{(\mathrm{kV\rho C_p})^{1/2}}\right] = \left[\frac{\mathrm{FL}^{1/2}\,\mathrm{t}^{1/2}\,\cancel{\mathrm{T}}^{1/2}\,\mathrm{t}^{1/2}\mathrm{L}^{3/2}\,\mathrm{t}\,\cancel{\mathrm{T}}^{1/2}}{\mathrm{t}\,\cancel{\mathrm{T}}\,\mathrm{L}\,\mathrm{F}^{1/2}\mathrm{L}^{1/2}\mathrm{M}^{1/2}\mathrm{L}}\right] = \left[\frac{\mathrm{L}^{1/2}\mathrm{L}^2\,\cancel{\mathrm{t}}}{\cancel{\mathrm{t}}\,\mathrm{L}^{3/2}\,\mathrm{L}}\right] = [0]$$

[A] = 0

(b) h = 1000 BTU/hr ft^2 °F, d = 5 mm in boiling H_2O at 1 atm. Find V and A in SI units,

Look up needed data.

From Handbook of Chemistry and Physics, 43th ed., 1961, H_2O at 100°C:

σ = 58.9 dyn/cm, ρ_V = 0.598 kg/m^3, ρ_l = 7.998 lb$_m$/gal = 0.9587 g/cm^3

k = 0.001598 cal/s °C cm

C_v: 252 cal/BTU 4.185 J/cal 10^7 dyn cm/J

$$V = \left[\frac{\Delta\rho g\sigma}{\rho V^2}\right]^{1/4}$$

$$V = \left(\frac{(0.9587 - 0.000598)\frac{g}{cm^3}\left(980\frac{cm}{s^2}\right)\left(58.9\frac{dyn}{cm}\right)}{\left(0.000598\frac{g}{cm^3}\right)^2}\right)^{1/4}$$

$$= 627\frac{cm}{s} = 6.27\frac{m}{s}$$

$$A = h\left[\frac{d}{(kV\rho C_p)}\right]^{1/2}$$

$$h = 100\frac{BTU}{hr\,ft^2\,{}^oF}\left(252\times4.185\times10^7\frac{dyn\,cm}{BTU}\times1.8\frac{{}^oF}{{}^oC}\times\frac{hr}{3600\,s}\times\left(\frac{ft}{30.48\,cm}\right)^2\right)$$

$$= 567.6\frac{dyn}{s\,cm\,{}^oC}$$

$$C_p = 1\frac{cal}{g\,{}^oC}$$

$$k = 0.001598\frac{cal}{s\,cm\,{}^oC}\left(1.195\frac{J}{cal}\times10^7\frac{dyn\,cm}{J}\right) = 6.688\times10^4\frac{dyn}{s\,{}^oC}$$

$$A = 567.6\frac{dyn}{s\,cm\,{}^oC}\left(\frac{0.5\text{ cm}}{6.688\times10^4\frac{dyn}{s^oC}\times627\frac{cm}{s}\times0.9587\frac{g}{cm^3}\times1\frac{cal}{g^oC}\times4.18\times10^7\frac{dyn\,cm}{cal}}\right)^{1/2}$$

$$A = 9.78\times10^{-6}$$

2-17 Determine the value of the Reynolds number for SAE 10 lube oil at 100°F flowing at a rate of 2000 gpm through a 10 in. Schedule 40 pipe. The oil SG is 0.92, and its viscosity can be found in Appendix A. If the pipe is made of commercial steel (ε = 0.0018 in.), use the Moody diagram (see Fig. 6-4) to determine the friction factor *f* for this system. Estimate the precision of your answer, based upon the information and procedure you used to determine it (i.e., tell what the reasonable upper and lower bounds, or the corresponding percentage variation, should be for the value of *f* based on the information you used).

Solution:

Define N_{Re} for SAE 10 lube oil (100^0 F) at 2000 GPM through a 10 in. sch 40 pipe. S.G. = 0.92, ε = 0.00181. Use moody diagram for *f*. Give range of uncertainty in answer.

$$N_{Re} = \frac{DV\rho}{\mu} \text{ since } V = \frac{4Q}{\pi D^2} \rightarrow N_{Re} = \frac{4Q\rho}{\pi D\mu}$$

ρ = 0.92 g/cm^3, μ = 30 cP (Appx. A-1-1 or A-1-3)

D = 10.025 in (10″ sch 40-Appx. E)

$$N_{Re} = \frac{4Q\rho}{\pi D\mu} = \frac{4(2000\,\text{gal/min})\left(63.1\frac{\text{cc/s}}{\text{gal/min}}\right)(0.92\,\text{g/cc})}{\pi(10.025\,\text{in})(2.54\,\text{cm/in})(30\,\text{cP})\left(0.01\frac{\text{g}}{\text{cms cP}}\right)}$$

= 19,400 (0nly good to 2 significant figures)

From Moody diagram (Fig. 6-4, p149)

$\varepsilon/D = 0.0018/10.025 = 0.000176$ and $N_{Re} = 19,400$

$f = 0.0066 \pm 0.0001$ (due to uncertainty in reading log-log plot)

Regardless of the precision of the data used, the Moody diagram can't be read to better than ±0.0001 precision. This is OK, since data upon which diagram is based is not precise enough to justify it better.

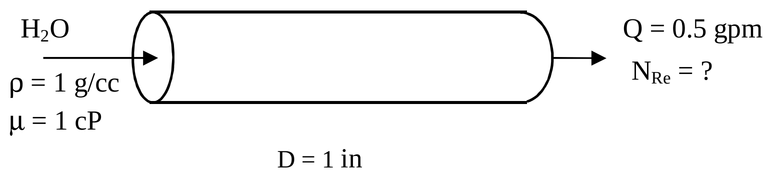
2-18 Determine the value of the Reynolds number for water flowing at a rate of 0.5 gpm through a 1 in. ID pipe. If the diameter of the pipe is doubled at the same flow rate, how much will each of the following change:

(a) The Reynolds number

(b) The pressure drop

(c) The friction factor.

Solution:



$$N_{Re} = \frac{4Q\rho}{\pi D\mu} = \frac{4(0.5\,\text{gpm})\left(63.1\frac{\text{cc/s}}{\text{gpm}}\right)\left(1\frac{\text{g}}{\text{cc}}\right)}{\pi(2.54\,\text{cm})(0.01\,\text{g/cm s})} = 1.58\times10^{3}\ \text{(laminar)}$$

If D is doubled for same Q, what is effect on:

(a) N_{Re} - will be 1/2 , *i.e.*, the new value be 1580/2 = 790

(b) $-\Delta P$ - From Bernoulli: $-\frac{\Delta P}{\rho} = e_f = \frac{4f\,L}{D}\frac{V^2}{2} = \frac{32fLQ^2}{\pi^2 D^5}$

For laminar flow, $f = 16/N_{Re} = \frac{16\mu\pi D}{4Q\rho}$

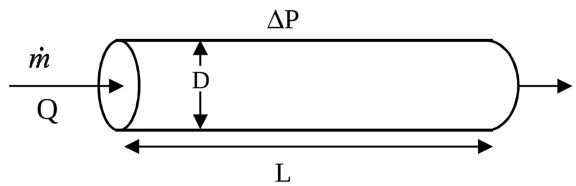
So, $-\Delta P = \frac{32LQ^2}{\pi^2 D^5}\left(\frac{16\mu\pi D}{4Q\rho}\right) = \frac{128}{\pi}\frac{LQ\mu}{D^4\rho}\lim_{x\to\infty}$

So, if D is $\uparrow$ by x 2 then $-\Delta P$ will $\downarrow$ by $\frac{1}{2^4} = \frac{1}{16}$

(c) $f = \frac{16\mu\pi D}{4Q\rho}$ will $\uparrow$ by $\times 2$.

2-19 The pressure drop for a fluid with a viscosity of 5 cP and a density of 0.8 g/cm^3, flowing at a rate of 30 g/s in a 50 ft long 1/4 in. diameter pipe is 10 psi. Use this information to determine the pressure drop for water at 60^{o}F flowing at 0.5 gpm in a 2 in. diameter pipe. What is the value of the Reynolds number for each of these cases?

Solution:



Case a: $-\Delta P = 10\,\text{psi}, \mu = 5\,\text{cP}, \rho = 0.8\,\text{g/cc}, \dot{m} = 30\,\text{g/s},\; L = 50\,\text{ft}, D = 1/4\,\text{in}$

Case b: $-\Delta P = ?\; H_2O:\; Q = 0.5\,\text{gpm}, D = 2\,\text{in}, L = 50\,\text{ft}, \;\text{at } 60^o\,\text{F}: \mu = 1.13\,\text{cP}$

$$N_{Re_a} = \frac{4\dot{m}}{\pi D \mu} = \frac{4\,(30\,\text{g/s})}{\pi\,(0.25\text{x}2.54\,\text{cm})(0.05\,\text{g/cms})} = 1203$$

$$N_{Re_b} = \frac{4Q\rho}{\pi D \mu} = \frac{4\,(0.5\,\text{gpm})\left(63.1\dfrac{\text{cc/s}}{\text{gpm}}\right)\left(1\dfrac{\text{g}}{\text{cc}}\right)}{\pi\,(2\times 2.54\,\text{cm})(0.0113\,\text{g/cms})} = 700$$

For laminar flow of a Newtonian fluid, there is only one dimensionless group:

$$\frac{\Delta P D^2}{L \mu V} = \text{const} = N_{\Delta P}$$

$$\text{So, } N_{\Delta P_a} = N_{\Delta P_b} = \frac{\pi \Delta P D^4}{4 L \mu Q}$$

$$\text{So: } \left(\frac{\not{\pi} \Delta P D^4}{\not{4} \not{L} \mu Q}\right)_a = \left(\frac{\not{\pi} \Delta P D^4}{\not{4} \not{L} \mu Q}\right)_b \qquad Q = \dot{m}_a / \rho_a$$

$$\Delta P_b = \Delta P_a \left(\frac{D_a}{D_b}\right)^4 \left(\frac{\mu_b}{\mu_a}\right)\left(\frac{Q_b \rho_b}{\dot{m}_a}\right)$$

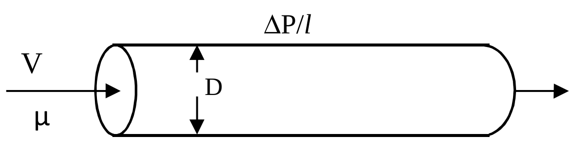
$$= 10\ \text{psi}\left(\frac{0.25}{2}\right)^{4}\left(\frac{1.13\,\text{cP}}{5\,\text{cP}}\right)\left(\frac{0.5\,\frac{\text{gpm}}{\text{cm}}\times 1\,\frac{\text{g}}{\text{cc}}\times 63.1\,\frac{\text{cc/s}}{\text{gpm}}}{30\,\text{g/s}}\right) = 5.8\times 10^{-4}\,\text{psi}$$

2-20 In the steady flow of a Newtonian fluid through a long uniform circular tube, if $N_{Re} <$ 2000 the flow is laminar and the fluid elements move in smooth straight parallel lines. Under these conditions, it is known that the relationship between the flow rate and the pressure drop in the pipe does not depend upon the fluid density or the pipe wall material.

(a) Perform a dimensional analysis of this system to determine the dimensionless groups that apply. Express your result in a form in which the Reynolds number can be identified.

(b) If water is flowing at a rate of 0.5 gpm through a pipe with an ID of 1 in., what is the value of the Reynolds number? If the diameter is doubled at the same flow rate, what will be the effect on the Reynolds number and on the pressure drop?

Solution:



Newtonian, Laminar
Independent of ρ, ε

(a) Dimensional Analysis:

Reference variables: [D] = L $\Rightarrow$ L =[d]

[V] = L/t $\Rightarrow$ t = [D/V]

[μ] = M/Lt $\Rightarrow$ M= [$\mu D^2/V$]

Variable	Dimensions
$\Delta P/l$	$F/L^3 = M/(L^2t^2)$
D	L
V	L/t
μ	M/(Lt)
4 –	3 = 1 group

$$\left[\frac{\Delta P}{l}\right] = \frac{M}{L^2 t^2} = \left[\frac{\mu D^2 V^{\not 2}}{\not V D^2 \not D^2}\right]$$

$$N = \frac{D^2 \, \Delta P/l}{\mu V}$$

(a) Thus $\frac{(\Delta P/l)D^2}{\mu V} = \text{constant} = \left(\frac{DV\rho}{\mu}\right)\left(\frac{\Delta P/l \; D}{\rho V^2}\right) = N_{Re} f = \text{constant}$

b) Water at 0.5 gpm through 1 in. ID pipe, $N_{Re} = ?$

$$N_{Re} = \frac{DV\rho}{\mu} = \frac{4Q\rho}{\pi D\mu} = \frac{4(0.5\,\text{gpm})\left(63.1\frac{cc/s}{gpm}\right)\left(1\frac{g}{cc}\right)}{\pi\left(1\,m \times 2.54\frac{cm}{in}\right)\left(0.01\frac{g}{cm\,s}\right)} = 1580$$

If D is doubled for same Q, what is effect on N_{Re} and ΔP?

$N_{Re} = \frac{DV\rho}{\mu}$ if $D \times 2$ then $N_{Re} \times \frac{1}{2}$; N_{Re} is halved

Since $\frac{\Delta P/l \; D^2}{\mu V} = \text{const}$, $V = \frac{4Q}{\pi D^2}$

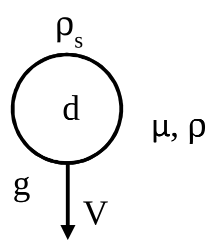
Then, $\frac{\Delta P}{l}\frac{\pi D^4}{4Q\mu} = \text{const}$

If $D \times 2$, the $\Delta P \times (1/2)^4$, i.e., ΔP is reduced by factor of 16.

2-21 Perform a dimensional analysis to determine the groups relating the variables that are important in determining the settling rate of very small solid particles falling in a liquid. Note that the driving force for moving the particles is gravity and the corresponding *net* weight of the particle. At very slow settling velocities, it is known that the velocity is independent of the fluid density. Show that this also requires that the velocity be inversely proportional to the fluid viscosity.

Solution:

Particle Settling:



Net gravity effect is the of ~ $g(\rho_s - \rho)$

Reference variables: $[d] = L \quad \Rightarrow L = [d]$

$[V] = L/t \quad \Rightarrow t = [d/V]$

$[\rho] = M/L^3 \quad \Rightarrow M = [\rho/d^3]$

Variable	Dimensions
d	[L]
$(\rho_s-\rho)g$	$(M/L^3)(L/t^2) = M/(L^2t^2)$
μ	M/(Lt)
ρ	M/L^3
V	L/t
5 –	3 = 2 groups

So,

$$[\mu] = \frac{M}{Lt} = \left[\frac{\rho d^3 V}{d\, d}\right] = [\rho d V] \rightarrow \quad N_1 = \frac{dV\rho}{\mu}$$

$$[\Delta\rho g] = \frac{M}{L^2 t^2} = \left[\frac{\rho d^3 V^2}{d^2\ d^2}\right] = \left[\frac{\rho V^2}{d}\right] \rightarrow \quad N_2 = \frac{\Delta\rho g d}{\rho V^2}$$

At very low velocity, V is independent of ρ (but not gΔρ!)

Eliminating ρ from groups:

$$N_3 = \frac{dV\rho}{\mu} \times \frac{\Delta\rho g d}{\rho V^2} = \frac{\Delta\rho g d^2}{\mu V}, \quad N_2 = \frac{dV\rho}{\mu}$$

Note:

If ρ not important, have 1 less group, no ρ: $N_2 \rightarrow 0$

Then,

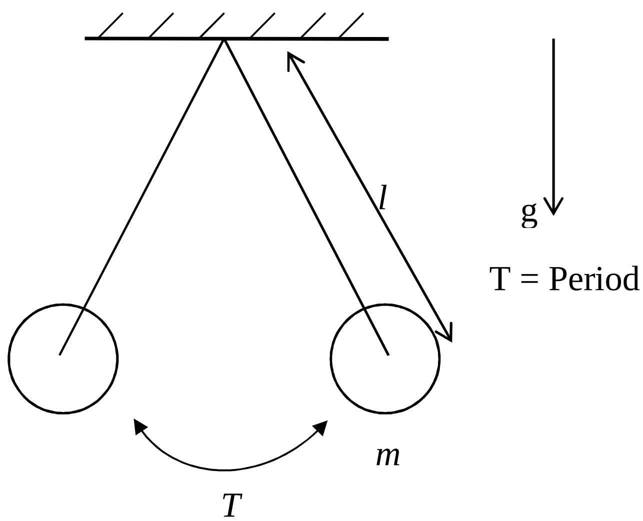
$$N_3 = \frac{\Delta\rho g d^2}{\mu V} = \text{const} \quad \text{or} \quad V = \text{const} \cdot \frac{\Delta\rho g d^2}{\mu}$$

Thus $\quad V \sim \frac{1}{\mu}$

2-22 A simple pendulum consists of a small, heavy ball of mass *m* on the end of a long string of length *L*. The period of the pendulum should depend on these factors, as well as gravity which is the driving force for making it move. What information can you get about the relationship between these variables from a consideration of their dimensions? Suppose you measured the period, T_1, of a pendulum with mass m_1 and length l_1. How could you use this to determine the period of a different pendulum with a different mass and length? What would be the ratio of the pendulum period on the moon to that on the earth? How could you use the pendulum to determine the variation of *g* on the earth's surface?

Solution:

Simple pendulum:



(a) Dimensionless analysis:

Reference variables: [T] = t $\Rightarrow$ L =[d]

[*l*] = L $\Rightarrow$ t = [d/V]

[*m*] = M $\Rightarrow$ M=[ρ/d^3]

Variable	Dimensions
T	t
l	L
m	M
g	L/t^2
V	L/t
4 –	3 = 1 group

$$[g] = \frac{L}{t^2} = \left[\frac{l}{T^2}\right]$$

$\mathrm{N}_1 = \dfrac{gT^2}{l}$ only 1 dimensionless group.

Since there is only 1 group which is independent of mass, this group must be constant (i.e., the same) for all pendulum.

(b) Measure T for given *m*, *l*.

Find T_2 for m_2, l_2:

Assume $g_1 = g_2$: $g\dfrac{T_2^2}{l_2} = g\dfrac{T_1^2}{l_1}$ or $\mathrm{T}_2 = \mathrm{T}_1\sqrt{\dfrac{l_2}{l_1}}$

(c) What is $\mathrm{T}_{\mathrm{moon}}/\mathrm{T}_{\mathrm{earth}}$ $(\mathrm{T}_m/\mathrm{T}_e)$?

Assume $l_m = l_e$, $g_m = g_e/6$;

$$\frac{g_m T_m^2}{l_m} = \frac{g_e T_e^2}{l_e},\quad \frac{T_m}{T_e} = \sqrt{\frac{g_e}{g_m}} = \sqrt{6} = \underline{\underline{2.45}}$$

(d) How to use pendulum to measure *g* at various places on the earth? For a given *l*, measure *T* at each location:

$$\frac{g_1 T_1^2}{l_1} = \frac{g_2 T_2^2}{l_2} \quad \text{or} \quad g_2 = g_1\left(\frac{T_1}{T_2}\right)^2$$

Note: From physics: $T = 2\pi\sqrt{\dfrac{l}{g}}$, or $N_1 = \left(\dfrac{gT^2}{l}\right) = (2\pi)^2 = 39.5$

2-23 An ethylene storage tank in your plant explodes. The distance that the blast wave travels from the blast site (*R*) depends upon the energy released in the blast (*E*), the density of the air (ρ), and time (θ). Use dimensional analysis to determine:

(a) The dimensionless group(s) that can be used to describe the relationship between the variables in the problem

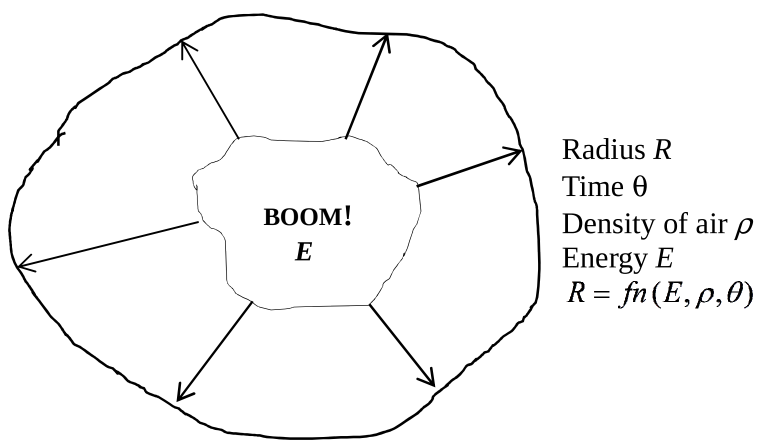
(b) The ratio of the velocity of the blast wave at a distance of 2000 feet from the blast site to the velocity at a distance of 500 feet from the site

The pressure difference across the blast wave (ΔP) also depends upon the blast energy (*E*), the air density (ρ), and time (θ). Use this information to determine:

(c) the ratio of the blast pressure at a distance of 500 ft from the blast site to that at a distance of 2000 ft from the blast site

Solution:

Ethylene Tank Explosion:



Reference variables: $[R] = \mathrm{L}$

$[\theta] = \mathrm{t}$

$[\rho] = \mathrm{M/L^3} = \mathrm{M}/R^3$, i.e., $\mathrm{M} = [\rho R^3]$

Variable	Dimensions
E	$FL = ML^2/t^2$
R	L

θ t
ρ M/L^3
4 – 3 = 1 group

(a) $[E] = \frac{ML^2}{t^2} = \left[\frac{\rho R^3 R^2}{\theta^2}\right]$

$$N_1 = \frac{E\theta^2}{\rho R^5} = \text{constant}$$

(b) $V = {}^{dR}\!/_{dt} \rightarrow$ not independent of R, θ

$$[V] = {}^{[R]}\!/_{[\theta]} \rightarrow [\theta] = \left[\frac{R}{V}\right]$$

$$N_1 = \frac{E}{\rho R^5}\left(\frac{R}{V}\right)^2 = \frac{E}{\rho R^3 V^2}$$

For blast at (R_1, θ_1) and (R_2, θ_2)

$$\left(\frac{E}{\rho R^3 V^2}\right)_1 = \left(\frac{E}{\rho R^3 V^2}\right)_2 \Rightarrow \frac{V_2}{V_1} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{500\,\text{ft}}{2000\,\text{ft}}\right)^{3/2} = 0.125 \text{ or } \frac{1}{8}$$

(c) Since blast energy ~ P$\mathbb{V}$ ~ PR^3, $[E] = [P][R^3]$

(actually, ~ 2% of this E → P$\mathbb{V}$)

Since $(E)_1 = (E)_2$, then

$$\left(PR^3\right)_1 = \left(PR^3\right)_2$$

$$\text{or } \frac{P_2}{P_1} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{500}{2000}\right)^3 = \underline{\underline{0.0156}} \ (= 1/64)$$

(Still only need one group, whether it has V or P or E, since V is defined by (R, θ) and E defined as (P, $\mathbb{V} \sim R^3$).

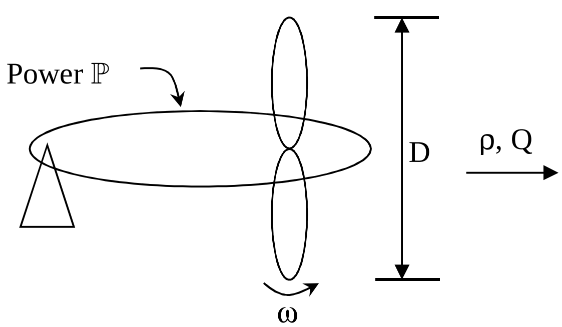
2-24 It is known that the power required to drive a fan depends upon the impeller diameter (*D*), the impeller rotational speed (ω), the fluid density (ρ), and the volume flow rate (*Q*). (Note that the fluid viscosity is not important for gases under normal conditions.)

(a) What is the minimum number of fundamental dimensions required to define all of these variables?

(b) How many dimensionless groups are required to determine the relationship between the power and all the other variables? Find these groups by dimensional analysis, and arrange the results so that the power and the flow rate each appear in only one group.

Solution:

Fan:



(a) Minimum number of dimensions = 3

(b) Number of groups = 5–3 = 2

Want ℙ and Q in only 1 group

Reference variables: [D] = L $\Rightarrow$ L = [D]

[ω] = 1/t $\Rightarrow$ t = [1/ω]

[ρ] = M/L^3 $\Rightarrow$ M = $[\rho D^3]$

Variable	Dimensions

D	L
ρ	M/L^3
Q	L^3/t
ω	$1/t$
ℙ	$FL/t = ML^2/t^3$
5 –	3 = 2 groups

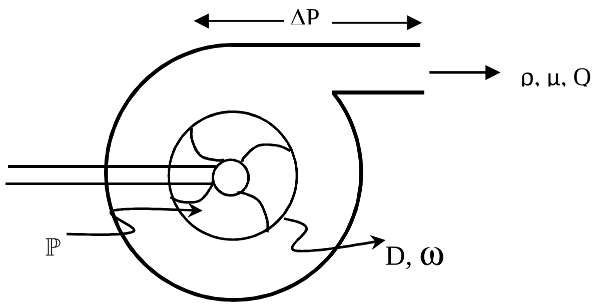
$$[\mathbb{P}] = \frac{ML^2}{t^3} = [\rho D^3 D^2 \omega^3] \quad \text{or} \quad N_1 = \frac{\mathbb{P}}{\rho D^5 \omega^3}$$

$$[Q] = \frac{L^3}{t} = [D^3 \omega] \quad \text{or} \quad N_2 = \frac{Q}{D^3 \omega}$$

2-25 A centrifugal pump with an 8 in. diameter impeller operating at a rotational speed of 1150 rpm requires 1.5 hp to deliver water at a rate of 100 gpm and a pressure of 15 psi. Another pump for water, which is geometrically similar but has an impeller diameter of 13 in., operates at a speed of 1750 rpm. Estimate the pump pressure, flow capacity, and power requirements of this second pump. (Under these conditions, the performance of both pumps is independent of the fluid viscosity.)

Solution:

Centrifugal Pump



Power: $\mathbb{P} = fn\ (D, \omega, Q, \mu, \rho)$ - μ not important if £100 cP

$$\Delta P = \text{fn}\ (D, \omega, Q, \mu, \rho)$$

ΔP, Q and $\mathbb{P}$ are not independent : $\mathbb{P} = Q\Delta P$¬power to fluid

$$\mathbb{P}_m = \mathbb{P}/\eta_c \leftarrow \text{power to motor}, \eta_c = \text{efficiency}$$

$$\eta_c = \mathbb{P}/\mathbb{P}_m = Q\Delta P/\mathbb{P}_m \leftarrow \text{Dimensionless Group}$$

(a)

Reference variables: $[D] = L \Rightarrow L = [D]$

$[\omega] = 1/t \Rightarrow t = [1/\omega]$

$[\rho] = M/L^3 \Rightarrow M = [\rho D^3]$

Variable	Dimensions
D	L
ρ	M/L^3
Q	L^3/t
ω	$1/t$
$\mathbb{P}$	$FL/t = ML^2/t^3$
5 –	3 = 2 groups

$$[\Delta P] = M/Lt^2 = [\frac{\rho D^3\omega^2}{D}] = [\rho D^2\omega^2]$$

$$N_1 = \frac{\mathbb{P}}{\rho D^5\omega^3}$$

$$N_2 = \frac{Q}{D^3\omega}$$

$$[Q] = \frac{L^3}{t} = [D^3\omega]$$

$$[\mathbb{P}] = Q\Delta P = D^3\omega N_2.\rho D^2\omega^2 N_1 = N_1N_2\rho D^5\omega^3$$

$$\text{Also}: \eta_c = \frac{\mathbb{P}_{\text{fluid}}}{\mathbb{P}_{\text{motor}}}$$

$$N_2 = N_1N_2 = \frac{\mathbb{P}}{\rho D^5\omega^3} \text{ - Not indepependent of } N_1, N_2$$

(b) Pump 1: $D_1 = 8$ in, $\Delta P_1 = 15$ psi, $P_1 = 1.5$ hp, $\omega_1 = 1150$ rpm, $Q_1 = 100$ gpm

Pump 2: Similar geometry: $D_2 = 13$ in, $\omega_2 = 1150$ rpm

What is Q for pump 2, Under conditions similar to 1?

$$\left(\frac{Q}{D^3\omega}\right)_2 = \left(\frac{Q}{D^3\omega}\right)_1 \rightarrow Q_2 = Q_1\left(\frac{\omega_2}{\omega_1}\right)\left(\frac{D_2^3}{D_1^3}\right) = 100\left(\frac{1750}{1150}\right)\left(\frac{13}{8}\right)^3$$

$$= 653 \text{ gpm}$$

(c) At this flow rate, what is $(\Delta P)_2$ and $(\mathbb{P})_2$ for water?

$$\left(\frac{\Delta P}{D^2\rho\omega^2}\right)_2 = \left(\frac{\Delta P}{D^2\rho\omega^2}\right)_1 \quad \text{(for same fluid, } \rho_1 = \rho_2\text{)}$$

$$\Delta P_2 = \Delta P_1\left(\frac{D_2}{D_1}\right)^2\left(\frac{\rho_2}{\rho_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2 = 15\,\text{psi}\left(\frac{13}{8}\right)^2\left(\frac{1750}{1150}\right)^2 = 91.7\,\text{psi}$$

Note: Assume $\rho_1 = \rho_2$. Also, note that pump head,

$H = \frac{\Delta P}{\rho g}$ is independent of ρ.

$$\left(\frac{\mathbb{P}}{\rho D^5\omega^3}\right)_2 = \left(\frac{\mathbb{P}}{\rho D^5\omega^3}\right)_1 \quad \text{(for same fluid, } \rho_1 = \rho_2\text{)}$$

$$\mathbb{P}_2 = \mathbb{P}_1\left(\frac{\rho_2}{\rho_1}\right)\left(\frac{D_2}{D_1}\right)^5\left(\frac{\omega_2}{\omega_1}\right)^3 = 1.5\,\text{hp}\left(\frac{13}{8}\right)^5\left(\frac{1750}{1150}\right)^3 = 60\,\text{hp}$$

Note: This is power delivered to fluid. Motor power $= \mathbb{P}/\eta_c$

Also, note that $\mathbb{P}$ is not independent of ρ.

2-26 A gas bubble of diameter D rises with a velocity V in a liquid of density ρ and viscosity μ.

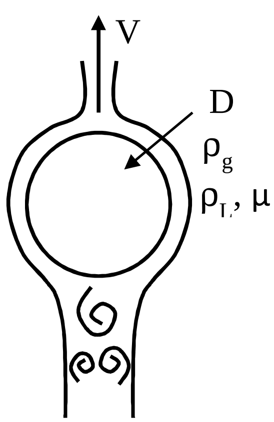
(a) Determine the dimensionless groups that include the effects of all of the significant variables, in such a form that the liquid viscosity appears in only one group. Note that the driving force for the bubble motion is buoyancy, which is equal to the weight of the displaced fluid.

(b) You want to know how fast a 5 mm diameter air bubble will rise in a liquid with a viscosity of 20 cP and a density of 0.85 g/cm^3. You want to simulate this system in the laboratory using water (μ = 1 cP, ρ = 1 g/cm^3) and air bubbles. What size air bubble should you use?

(c) You perform the experiment, and measure the velocity of the air bubble in water (V_m). What is the ratio of the velocity of the 5 mm bubble in the field liquid (V_f) to that in the lab (V_m)?

Solution:

Gas bubble:



a) Find the groups

Note: DF = buoyancy

$$=\left(\rho_L - \rho_g\right)g = \Delta P g$$

Reference variables: [D] = L $\Rightarrow$ L = [D]

[V] = L/t $\Rightarrow$ t = [D/V]

$[\rho_L] = M/L^3 \Rightarrow M = [\rho_L D^3]$

Variable	Dimensions
D	L
V	L/t
ρ_L	M/L^3
$g\Delta\rho$	$(L/t^2)\ (M/L^3) = M/L^2t^2$
μ	M/(Lt)
5 –	3 = 2 groups

$$[g\Delta\rho] = \frac{M}{L^2t^2} = \left[\frac{\rho_L D^3 V^2}{D^2 D^2}\right] = \left[\frac{\rho_L V^2}{D}\right] \to N_1 = \frac{DV\Delta\rho}{\rho_L V^2}$$

$$[\mu] = \frac{M}{Lt} = \left[\frac{\rho_L D^3 V}{D\,D}\right] = [\rho DV] \to N_2 = \frac{DV\rho_L}{\mu}$$

(b) Find:

$V = ?$ for $D = 5\,\text{mm},\ \mu = 20\ \text{cP},\ \rho_2 = 0.85\, {}^{g}\!/_{cc}\ (\Delta\rho \simeq \rho_2)$

Lab Experiment : $\left[\mu = 1\ \text{cP},\ \rho = 1\, {}^{g}\!/_{cc}\right]_M$, air $\left(\rho_{air} << \rho_{H_2O}\right)$

What D_M to use?

$$\text{Unknown V is in both groups}: N_1 N_2^2 = \frac{Dg\Delta\rho}{\rho_L V^2}\left(\frac{DV\rho_L}{\mu}\right)^2 = \frac{D^3 g\Delta\rho\rho_L}{\mu^2}$$

$$\text{Gives } D_m : (\Delta\rho \simeq \rho_2)\ \left(\frac{D^3 g\rho_L^2}{\mu^2}\right)_M = \left(\frac{D^3 g\rho_L^2}{\mu^2}\right)_F$$

$$D_M = D_F \left(\frac{\rho_{L_F}}{\rho_{L_M}} \frac{\mu_M}{\mu_F} \right)^{2/3} = 5\text{mm} \left[\left(\frac{0.85}{1} \right) \left(\frac{1}{20} \right) \right]^{2/3} = 0.609\,\text{mm}$$

(c) Measure V_M. Find V_F/V_M $\quad \left(\frac{DV\rho}{\mu} \right)_F = \left(\frac{DV\rho}{\mu} \right)_M$

$$V_F/V_M = \frac{D_M}{D_F} \frac{\rho_{L_M}}{\rho_{L_F}} \frac{\mu_F}{\mu_M} = \frac{0.609\,\text{mm}}{5\,\text{mm}} \left(\frac{1}{0.85} \right) \left(\frac{20}{1} \right) = 2.87$$

2-27 You must predict the performance of a large industrial mixer under various operating conditions. To obtain the necessary data, you decide to run a laboratory test on a small scale model of the unit. You have deduced that the power (P) required to operate the mixer depends upon the following variables:

Tank diameter D, Impeller diameter d, Impeller rotational speed N, Fluid density ρ, Fluid viscosity μ

(a) Determine the minimum number of fundamental dimensions involved in these variables and the number of dimensionless groups that can be defined by them.

(b) Find an appropriate set of dimensionless groups such that D and N each appear in only one group. If possible, identify one or more of the groups with groups commonly encountered in other systems.

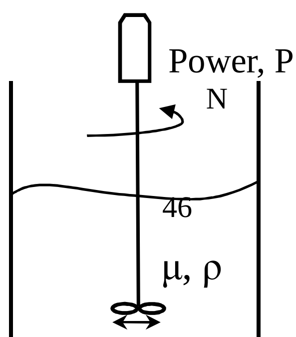
(c) You want to know how much power would be required to run a mixer in a large tank 6 ft in diameter, using an impeller with a diameter of 3 ft operating at a speed of 10 rpm, when the tank contains a fluid with a viscosity of 25 cP and a specific gravity of 0.85. To do this, you run a lab test on a model of the system, using a scale model of the impeller that is 10 in. in diameter. The only appropriate fluid you have in the lab has a viscosity of 15 cp and a specific gravity of 0.75. Can this fluid be used for the test? Explain.

(d) If the preceding lab fluid is used, what size tank should be used in the lab, and how fast should the lab impeller be rotated?

(e) With the lab test properly designed and the proper operating conditions chosen, you run the test and find that it takes 150 W to operate the lab test model. How much power would be required to operate the larger field mixer under the plant operating conditions?

Solution:

Mixer:



Reference variables: $[N] = 1/t \quad \Rightarrow t = [1/N]$

$[d] = L \quad \Rightarrow L = [d]$

$[\rho] = M/L^3 \quad \Rightarrow M = [\rho D^3]$

Variable	Dimensions
P	$FL/t = ML^2/t^3$
N	$1/t$
μ	$M/(Lt)$
ρ	M/L^3
d	L
D	L
6 –	3 = 3 groups

….(a)

$$[\mu] = \frac{M}{Lt} = \left[\frac{\rho d^3 N}{N}\right], \quad [P] = ML^2/t^3 = \left[\rho d^3 d^2 N^3\right], \quad [D] = L = [d]$$

$$N_1 = \frac{P}{\rho d^5 N^3}; \quad N_2 = \frac{d^2 \rho N}{\mu}; \quad N_3 = \frac{d}{D} \qquad \text{…(b)}$$

(c) Large tank (f): $D_f = 6$ ft , $d_f = 3$ ft, $N_f = 10$ RPM, $\mu_f = 25$ cP, $\rho_f = 0.85$ g/cc

Lab (m): $d_m = 10$ in, $\mu_m = 15$ cP , $\rho_m = 0.75$ g/cc

Can you use this fluid?

Number of variables = 12,

Number of knowns: D_f, d_f, N_f, μ_f, ρ_f, d_m, μ_m, ρ_m = 8

Measure P_m makes 9 knowns, 3 equations $\Rightarrow$ 12 total

Yes, can use the specified fluid.

(d) Use this fluid, find D_m, N_m

$$\left(\frac{d}{D}\right)_m = \left(\frac{d}{D}\right)_f$$

$$\therefore\ D_m = D_f\left(\frac{d_m}{d_f}\right) = 6\,\text{ft}\left(\frac{10\,\text{in}}{12\,{}^{\text{in}}\!/\!_{\text{ft}}\ 3\text{ft}}\right) = 1.67\,\text{ft} = 20\,\text{in.}$$

$$\left(\frac{d^2\rho N}{\mu}\right)_m = \left(\frac{d^2\rho N}{\mu}\right)_f$$

$$N_m = N_f\left(\frac{d_f}{d_m}\right)^2\left(\frac{\rho_f}{\rho_m}\right)\left(\frac{\mu_m}{\mu_f}\right) = 10\,\text{rpm}\left(\frac{36}{10}\right)^2\left(\frac{0.85}{0.75}\right)\left(\frac{15}{25}\right) = 88.1\text{ rpm}$$

(e) $P_m = 150$ W, $P_f = ?$

$$\left(\frac{P}{\rho d^5 N^3}\right)_m = \left(\frac{P}{\rho d^5 N^3}\right)_f$$

$$P_f = P_m\left(\frac{N_f}{N_m}\right)^3\left(\frac{d_f}{d_m}\right)^5\left(\frac{\rho_f}{\rho_m}\right) = 150\,\text{W}\left(\frac{10}{88.1}\right)^3\left(\frac{36}{10}\right)^5\left(\frac{0.85}{0.75}\right) = 150\text{ W}$$

2-28 When an open tank with a free surface is stirred with an impeller, a vortex will form around the shaft. It is important to prevent this vortex from reaching the impeller, because entrainment of air in the liquid tends to cause foaming. The shape of the free surface depends upon (among other things) the fluid properties, the speed and size of the impeller, the size of the tank, and the depth of the impeller below the free surface.

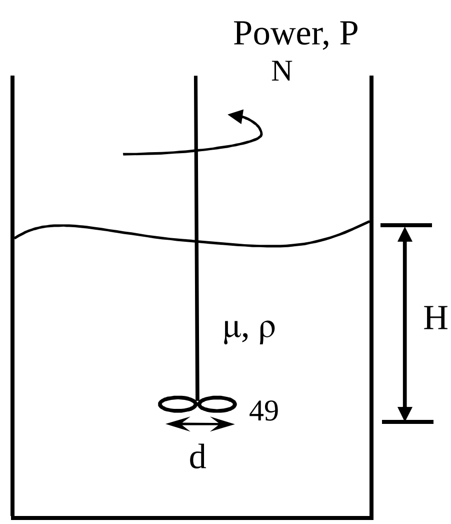
(a) Perform a dimensional analysis of this system to determine an appropriate set of dimensionless groups that can be used to describe the system performance. Arrange the groups so that the impeller speed appears in only one group.

(b) In your plant you have a 10 ft diameter tank containing a liquid that is 8 ft deep. The tank is stirred by an impeller that is 6 ft in diameter and is located 1 ft from the tank bottom. The liquid has a viscosity of 100 cP and a specific gravity of 1.5. You need to know the maximum speed at which the impeller can be rotated without entraining the vortex. To find this out, you design a laboratory test using a scale model of the impeller that is 8 in. in diameter. What, if any, limitations are there on your freedom to select a fluid for use in the lab test?

(c) Select an appropriate fluid for the lab test and determine how large the tank used in the lab should be and where in the tank the impeller should be located.

(d) The lab impeller is run at such a speed that the vortex just reaches the impeller. What is the relation between this speed and that at which entrainment would occur in the tank in the plant?

Solution:



Reference variables: [D] = L = [d]

[H] = L = [d]

[g] = L/t^2 = [$d\omega^2$]

[μ] = M/Lt = [$\rho d^3\omega/\mu$]

Variable	Dimensions
d	L
D	L
H	L
ω	1/t
g	L/t^2
μ	M/(Lt)
ρ	M/L^3
7 –	3 = 4 groups

$$N_1 = d/D; \quad N_2 = H/d; \quad N_3 = \frac{\omega^2 d}{g}; \quad N_4 = \frac{\rho d^3 \omega}{\mu}$$

a) Get groups such that ω is in only one group:

$$\frac{N_3}{N_4^2} = \frac{\omega^2 d}{g}\frac{\mu^2}{\rho^2 d^4 \omega^2} = \frac{\mu^2}{\rho^2 g d^3} = N_1, \ \frac{\rho d^2 \omega}{\mu} = N_2, \frac{d}{D} = N_3, \frac{H}{d} = N_4$$

(b) Plant: $D = 10\,\text{ft},\ H = 8\text{-}1 = 7\,\text{ft},\ d = 6\,\text{ft},\ \mu = 100\,\text{cP},\ \rho = 1.5\,\text{g/cc},\ \omega = ?$ for no vortex

$\text{Lab}: d = 8\,\text{in. Fluid limitations?}$

c) $$\left(\frac{d}{D}\right)_M = \left(\frac{d}{D}\right)_F \rightarrow D_M = D_F \frac{d_M}{d_F} = 10\,\text{ft}\frac{8}{12(6)} = 1.11\,\text{ft} = 13.3\ \text{in}$$

$$\left(\frac{H}{d}\right)_M = \left(\frac{H}{d}\right)_F \rightarrow H_M = H_F \frac{dM}{dF} = 7\,ft \frac{8}{12(6)} = 0.778ft = 9.33\text{ in}$$

(b) Since μ/ρ appear only as a ratio, the only important fluid property is $\upsilon = \frac{\mu}{\rho}$.

$$\left(\frac{\upsilon^2}{gd^3}\right)_M = \left(\frac{\upsilon^2}{gd^3}\right)_F$$

$$\upsilon_M = \upsilon_F \left(\frac{d_M}{d_F}\right)^{3/2} = \frac{100}{1.5}\left(\frac{8}{12(6)}\right)^{3/2} = 2.47\text{ cS (centi Stokes)}$$

Run lab where vortex touches impeller = ω_M

$$\left(\frac{d^2\omega}{\upsilon}\right)_M = \left(\frac{d^2\omega}{\upsilon}\right)_F$$

$$\frac{\omega_F}{\omega_M} = \frac{\upsilon_F}{\upsilon_M}\left(\frac{d_M}{d_F}\right)^2 = \frac{100}{1.5\,(2.47)}\left(\frac{8}{12(6)}\right)^2 = \frac{1}{3}$$

i.e., Impeller in field should turn at $1/3$ the speed of the lab impeller, or less.

2-29 The variables involved in the performance of a centrifugal pump include the fluid properties (μ and ρ), the impeller diameter (*d*), the casing diameter (*D*), the impeller rotational speed (*N*), the volumetric flow rate of the fluid (*Q*), the head (*H*) developed by the pump ($\Delta P = \rho g H$), and the power required to drive the pump (*HP*).

(a) Perform a dimensional analysis of this system to determine an appropriate set of dimensionless groups that would be appropriate to characterize the pump. Arrange the groups so that the fluid viscosity and the pump power each appear in only one group.

(b) You want to know what pressure a pump will develop with a liquid that has a specific gravity of 1.4 and a viscosity of 10 cP, at a flow rate of 300 gpm. The pump has an impeller with a diameter of 12 in., which is driven by a motor running at 1100 rpm. (It is known that the pump performance is independent of fluid viscosity unless the viscosity is greater than about 50 cP). You want to run a lab test that simulates the operation of the larger field pump using a similar (scaled) pump with an impeller that has a diameter of 6 in. and a 3600 rpm motor,. Should you use the same liquid in the lab as in the field, or can you use a different liquid? Why?

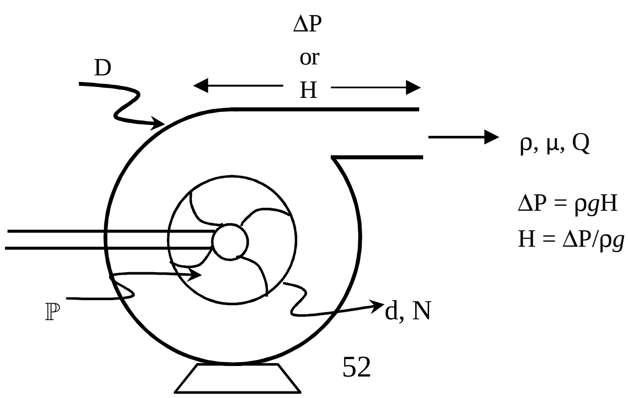
(c) If you use the same liquid, what flow rate should be used in the lab to simulate the operating conditions of the field pump?

(d) If the lab pump develops a pressure of 150 psi at the proper flow rate, what pressure will the field pump develop with the field fluid?

(e) What pressure would the field pump develop with water at a flow rate of 300 gpm?

Solution:

Centrifugal pump:



Reference variables: $[d] = L \quad \Rightarrow L = [d]$

$[\rho] = M/L^3 \quad \Rightarrow M = [\rho D^3]$

$[N] = 1/t \quad \Rightarrow t = [1/N]$

Variable	Dimensions
ΔP	$F/L^2 = M/Lt^2$
Q	$\mathbb{V}/t = L^3/t$
$\mathbb{P}$	$FL/t = ML^2/t^3$
ρ	M/L^3
μ	$M/(Lt)$
D	L
d	L
N	1/t
8 –	3 = 5 groups

Want groups with μ and $\mathbb{P}$ each in only one group N

$$[\Delta P] = M/Lt^2 = \left[\frac{\rho d^3 N^2}{d}\right]$$

$$[\mathbb{P}] = \frac{ML^2}{t^3} = \left[\rho d^3 d^2 N^3\right]$$

$$[\mu] = \frac{M}{Lt} = \left[\frac{\rho d^3 N}{d}\right]$$

$$[Q] = \frac{L^3}{t} = \left[d^3 N\right]$$

$$N_1 = \frac{\Delta P}{\rho d^2 N^2}; \quad N_2 = \frac{D}{d}; \quad N_3 = \frac{\mathbb{P}}{\rho d^5 N^3}; \quad N_4 = \frac{d^2 N \rho}{\mu}; \quad N_5 = \frac{Q}{d^3 N} \qquad \text{… (a)}$$

N_1, N_3 and N_5 are not independent. Use only 2 of the 3.

i.e., $\mathbb{P} = F\,V = \Delta P\,A\,\dfrac{Q}{A} = \Delta P\,Q$

$$N_1.N_5 = \frac{\Delta PQ}{\rho d^5 N^3} = \frac{\Pi}{\rho d^5 N^3} = N_3$$

(b) Given: $SG_1 = 1.4$, $\mu_1 = 10\text{cP}$, $Q_1 = 300$ GPM, $d_1 = 12''$, $N_1 = 1100$ RPM

(μ is not important) – No. variables $= 7 \times 2 = 14$

Lab: $d_l = 6''$, $N_l = 3600$ RPM

Can you use same liquid?

Total variables = 14,

Knowns = 7 + Measured head (ΔP) = 8

Total equations = 4

$\therefore$ Degrees of freedom = 4

$\therefore$ Can use same liquid or any other you want.

(c) If $\mu_\ell = \mu_1$, $\rho_\ell = \rho_1$, Find Q_ℓ using $\left(\dfrac{Q}{d^3 N}\right)_\ell = \left(\dfrac{Q}{d^3 N}\right)_1$

$$\Rightarrow Q_\ell = Q_1\left(\frac{d_\ell}{d_1}\right)^3 \frac{N_\ell}{N_1} = 300\,\text{GPM}\left(\frac{6}{12}\right)^3 \frac{3600}{1100} = 123\,\text{GPM}$$

(d) $(\Delta P)_\ell = 150$ psi at Q_ℓ. Find $(\Delta P)_1$ with $\rho_1 = \rho_\ell$

$$\left(\frac{\Delta P}{\rho d^2 N^2}\right)_1 = \left(\frac{\Delta P}{\rho d^2 N^2}\right)_\ell$$

$$\Delta P_1 = \Delta P_\ell\left(\frac{\rho_1}{\rho_\ell}\right)\left(\frac{d_1}{d_\ell}\right)^2\left(\frac{N_1}{N_\ell}\right)^2 = 150\,\text{psi}\left(\frac{1.4}{1.4}\right)\left(\frac{12}{6}\right)^2\left(\frac{1100}{3600}\right)^2 = 56\,\text{psi}$$

(e) What would ΔP_1 be for water of $Q_1 = 300$ GPM?

Everything is same except for ρ_1 ($S.G._1 = 1$)

$$\Delta P_1 = 150\,\text{psi}\left(\frac{1}{1.4}\right)\left(\frac{12}{6}\right)^2\left(\frac{1100}{3600}\right)^2 = \frac{56}{1.4} = 40\,\text{psi}$$

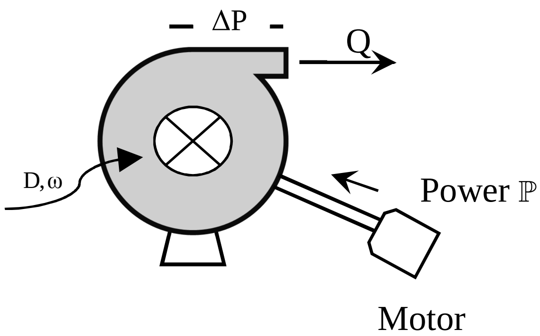
2-30 The purpose of a centrifugal pump is to increase the pressure of a liquid in order to move it through a piping system. The pump is driven by a motor, which must provide sufficient power to operate the pump at the desired conditions. You wish to find the pressure developed by a pump operating at a flow rate of 300 gpm with an oil having a specific gravity (SG) of 0.8 and a viscosity of 20 cP, and the required horsepower for the motor to drive the pump. The pump has an impeller diameter of 10 in., and the motor runs at 1200 rpm.

(a) Determine the dimensionless groups that would be needed to completely describe the performance of the pump.

(b) You want to determine the pump pressure and motor horsepower by measuring these quantities in the lab on a smaller scale model of the pump that has a 3 in. diameter impeller and a 1800 rpm motor, using water as the test fluid. Under the operating conditions for both the lab model and the field pump the value of the Reynolds number is very high, and it is known that the pump performance is independent of the fluid viscosity under these conditions. Determine the proper flow rate at which the lab pump should be tested and the ratio of the pressure developed by the field pump to that of the lab pump operating at this flow rate as well as the ratio of the required motor power in the field to that in the lab.

(c) The pump efficiency (η_e) is the ratio of the power delivered by the pump to the fluid (as determined by the pump pressure and flow rate) to the power delivered to the pump by the motor. Because this is a dimensionless number, it should also have the same value for both the lab and field pumps when they are operating under equivalent conditions. Is this condition satisfied?

Solution:



Fluid: μ, ρ

Pump "Head" = $\Delta P/\rho g$

Represent ΔP (DF), not a "length" variable.

(a) Dimensional analysis:

Reference variables: $[D] = L \quad \Rightarrow L = [D]$

$[\omega] = 1/t \quad \Rightarrow t = [1/\omega]$

$[\rho] = M/L^3 \quad \Rightarrow M = [\rho D^3]$

Variable		Dimensions	
D		L	
ω		1/t	
Q		L^3/t	
ΔP		$F/L^2 = M/Lt^2$	
ℙ		$FL/t = ML^2/t^3$	
μ		M/Lt	
ρ		M/L^3	
7	–	3 =	4 groups

$$[Q] = \frac{L}{t^3} = [D^3\omega] \quad \rightarrow \quad N_1 = \frac{Q}{D^3\omega}$$

$$[\Delta P] = \frac{M}{Lt^2} = \left[\frac{\rho D^3\omega^2}{D}\right] \quad \rightarrow \quad N_2 = \frac{\Delta P}{\rho D^2\omega^2}$$

$$[\mu] = \frac{M}{Lt} = \left[\frac{\rho D^3\omega}{D}\right] \quad \rightarrow \quad N_3 = \frac{\rho D^2\omega}{\mu}$$

$$[\mathbb{P}] = \frac{ML^2}{t^3} = [\rho D^3 D^2\omega^3] \quad \rightarrow \quad N_4 = \frac{\mathbb{P}}{\rho D^5\omega^3}$$

(b) Lab: Measure ΔP, $\mathbb{P}$, D = 3 in, ω = 1800 rpm, ρ = 1 g/cc, μ = 1 cP

Field: Q = 300 gpm, ρ = 0.8 g/cc, μ = 20 cP, D = 10 in, ω = 1200 rpm

Viscosity not important at high N_{Re} – Drop N_3 form the above list

Unknowns: $(Q)_M$, $(\Delta P, \mathbb{P})_F$

Equations: $N_{1_M} = N_{1_F}$, $N_{2_M} = N_{2_F}$, $N_{4_M} = N_{4_F}$

$$\left(\frac{Q}{D^3\omega}\right)_M = \left(\frac{Q}{D^3\omega}\right)_F \quad \rightarrow \quad Q_M = Q_F\left(\frac{D_M}{D_F}\right)^3\left(\frac{\omega_M}{\omega_F}\right)$$

$$= 300\,\text{gpm}\left(\frac{3}{10}\right)^3\left(\frac{1800}{1200}\right) = 12.15 = 12.2\,\text{gpm}$$

$$\left(\frac{\Delta P}{\rho D^2\omega^2}\right)_M = \left(\frac{\Delta P}{\rho D^2\omega^2}\right)_F \quad \rightarrow \quad \frac{\Delta P_F}{\Delta P_M} = \frac{\rho_F}{\rho_M}\left(\frac{D_F}{D_M}\right)^2\left(\frac{\omega_F}{\omega_M}\right)^2$$

$$= \frac{0.8}{1}\left(\frac{10}{3}\right)^2\left(\frac{1200}{1800}\right)^2 = 3.95$$

$$\left(\frac{\mathbb{P}}{\rho D^5\omega^3}\right)_M = \left(\frac{\mathbb{P}}{\rho D^5\omega^3}\right)_F \quad \rightarrow \quad \frac{\mathbb{P}_F}{\mathbb{P}_M} = \frac{\rho_F}{\rho_M}\left(\frac{D_F}{D_M}\right)^5\left(\frac{\omega_F}{\omega_M}\right)^3$$

$$= \frac{0.8}{1}\left(\frac{10}{3}\right)^5\left(\frac{1200}{1800}\right)^3 = 97.6$$

(c) $\eta_e = \dfrac{\Delta P Q}{\mathbb{P}}$

$$\frac{(\eta_e)_M}{(\eta_e)_F} = \frac{\Delta P_M}{\Delta P_F}\frac{Q_M}{Q_F}\frac{\mathbb{P}_F}{\mathbb{P}_M}$$

$$= \frac{1}{3.95}\left(\frac{12.15}{300}\right)97.6 \quad = 1 \qquad \text{(Yes)}$$

$$\left(\text{Power} = \frac{FL}{t} = \left(\frac{F}{L^2}\right)\left(\frac{L^3}{t}\right) \quad \Rightarrow \quad \Delta P Q\right)$$

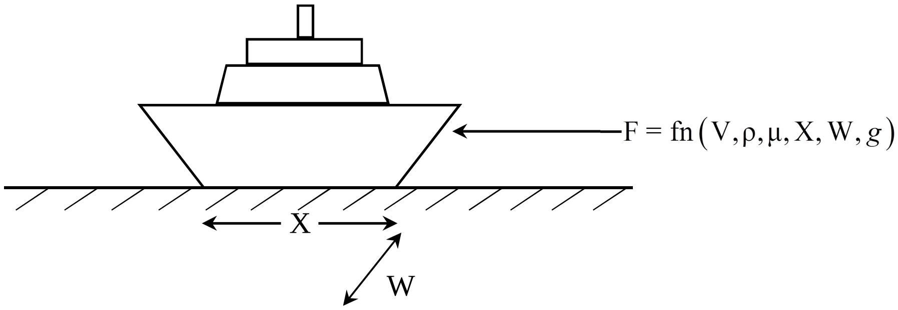
2-31 When a ship moves through the water, it created waves. The energy and momentum in these waves must come from the ship, which is manifested as a "wave drag" force on the ship. It is known that this drag force (*F*) depends upon the ship speed (*V*), the fluid properties (ρ, μ), the length of the waterline (*L*), and the beam width (*W*) as well as the shape of the hull, among other things. (There is at least one important "other thing" that relates to the "wave drag", i.e., the energy required to create and sustain the waves from the bow and the wake. What is this additional variable?) Note that "shape" is a dimensionless parameter, which is implied by the requirement of geometrical similarity. If two geometries have the same shape, the ratio of each corresponding dimension of the two will also be the same.

(a) Perform a dimensional analysis of this system to determine a suitable set of dimensionless groups that could be used to describe the relationship between all of the variables. Arrange the groups such that viscous and gravitational parameters each appear in separate groups.

(b) It is assumed that "wave drag" is independent of viscosity and that "hull drag" is independent of gravity. You wish to determine the drag on a ship having a 500 ft long waterline moving at 30 mph through seawater (SG = 1.1). You can make measurements on a scale model of the ship, 3 ft long, in a towing tank containing fresh water. What speed should be used for the model to simulate the wave drag and the hull drag?

Solution:

Wave drag on ship:



(a) Dimensional analysis such that μ and g appear in separate groups.

Reference variables: [X] = L $\Rightarrow$ L = [X]

[V] = L/t $\Rightarrow$ t = [X/V]

[ρ] = M/L^3 $\Rightarrow$ M = [ρD^3]

Variable	Dimensions
F	ML/t^2
X	L
W	L
V	L/t
μ	M/Lt
ρ	M/L^3
g	L/t^2
7 –	3 = 4 groups

$$[F] = \frac{ML}{t^2} = \left[\frac{\rho X^4 V^2}{X^2}\right] = \left[\rho V^2 X^2\right] \rightarrow \quad N_1 = \frac{F}{\rho V^2 X^2}$$

$$[\mu] = \frac{M}{Lt} = \left[\frac{\rho X^3 V}{X^2}\right] = [\rho VX] \rightarrow \quad N_2 = \frac{\rho VX}{\mu} = N_{Re}$$

$$[g] = \frac{L}{t^2} = \left[\frac{XV^2}{X^2}\right] = \left[\frac{V^2}{X}\right] \rightarrow \quad N_3 = \frac{V^2}{gX}$$

$N_4 = \dfrac{X}{W} \rightarrow$ For geometrically similar system, N_4 is irrelevant.

(b) Wave drag: $F_w \neq f(\mu)$ i.e. $\dfrac{F_w}{\rho V^2 X^2} = f\left(\dfrac{V^2}{gX} \text{ only}\right)$

Hull drag: $F_H \neq (g)$ i.e. $\frac{F_H}{\rho V^2 X^2} = f\left(\frac{\rho V X}{\mu} \text{ only}\right)$

Field: $X_F = 500 \text{ ft}; V_F = 30 \text{ mph}; \rho_F = 1.1 \text{ g/cm}^3$

Model: $X_M = 3 \text{ ft}; \rho_M = 1 \text{ g/cm}^3; V_M = ?$

In order to model wave drag, need $\left(\frac{V^2}{gX}\right)_M = \left(\frac{V^2}{gX}\right)_F$

$$\text{or} \qquad V_M = V_F\sqrt{\frac{X_M}{X_F}} = 30\left(\frac{3}{500}\right)^{1/2} = 2.32 \text{ mph}$$

To model Hull drag, need $\left(\frac{\rho V X}{\mu}\right)_M = \left(\frac{\rho V X}{\mu}\right)_F$, assuming $\mu_F = \mu_M$

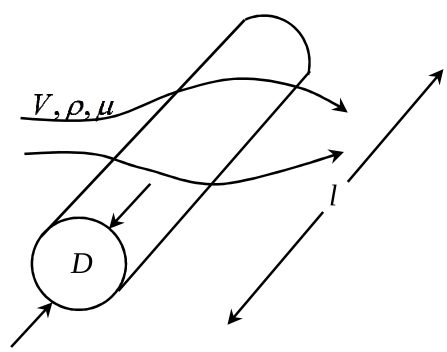
$$V_M = V_F \frac{X_F}{X_M}\frac{\rho_F}{\rho_M} = 30\left(\frac{500}{3}\right)\frac{1.1}{1} = 5500 \text{ mph} \leftarrow \text{ no way !!}$$

$\therefore$ can't model both, unless μ is changed in the lab conditions.

2-32 You want to find the force exerted on an undersea pipeline by a 10 mph current flowing normal to the axis of the pipe. The pipe is 30 in. in diameter, and the density of seawater is 64 lb_m/ft^3 and its viscosity is 1.5 cP. To determine this, you test a 1½ inch diameter model of the pipe in a wind tunnel at 60^oF. What velocity should you use in the wind tunnel in order to scale the measured force to the conditions in the sea? What is the ratio of the force on the pipeline in the sea to that on the model measured in the wind tunnel?

Solution:

Find force of current on pipe:



Assume $L >> D$, so no "end effects" and L/D is

i.e., $\frac{f}{l}$ is all that is needed

$$\frac{f}{l} = \text{fn}(V, \rho, \mu, D)$$

Pipe in sea water: $V = 10 \text{ mph}, \rho = 64 \text{ lb}_m/\text{ft}^3, \mu = 1.5 \text{ cP}, D = 30 \text{ in}$

Test in wind tunnel at $60^{o}\,F$, $V_{wind} = ?, \dfrac{f_r}{f_w} = ?$

$$\rho_{air} = \frac{29}{359}\left(\frac{492}{520}\right) = 0.0764\frac{lb_m}{ft^3},\ \ \mu_{air} = 0.021cP \ \ \text{(Fig A-1)}$$

Reference variables: $[D] = L \quad \Rightarrow L = [D]$

$[V] = L/t \quad \Rightarrow t = [D/V]$

$[\rho] = M/L^3 \quad \Rightarrow M = [\rho D^3]$

Variable	Dimensions
f/l	$F/L = M/t^2$
D	L
V	L/t
μ	M/Lt
ρ	M/L^3
5 –	3 = 2 groups

$$[\mu] = \frac{M}{Lt} = \left[\frac{\rho D^3 V}{D^2}\right] \ \rightarrow N_1 = \frac{DV\rho}{\mu}$$

$$\left[\frac{f}{l}\right] = \frac{M}{t^2} = \left[\frac{\rho D^3 V^2}{D^2}\right] \ \rightarrow N_2 = \frac{f/l}{\rho DV}$$

(b) Wind tunnel: $\left(D = 1.5\,in, \rho = 0.0764\dfrac{lb_m}{ft^3}, \mu = 0.021cP\right)_W$

$$V_w = ? \ \ \frac{f_P}{f_w} = ? \ \ N_{1P} = N_{1W},\ \left(\frac{DV\rho}{\mu}\right)_P = \left(\frac{DV\rho}{\mu}\right)_W$$

or

$$V_w = V_P\left(\frac{\mu_w}{\mu_P}\right)\left(\frac{\rho_P}{\rho_w}\right)\left(\frac{D_P}{D_W}\right)$$

$$= 10\,mph\left(\frac{0.021}{1.5}\right)\left(\frac{64}{0.0764}\right)\left(\frac{30}{1.5}\right) = 2350\,mph$$

$$N_{2p} = N_{2w}, \quad \left(\frac{f/l}{\rho DV}\right)_p = \left(\frac{f/l}{\rho DV}\right)_w$$

$$\frac{(f/l)_p}{(f/l)_w} = \frac{\rho_p}{\rho_w}\frac{D_p}{D_w}\frac{V_p}{V_w} = \left(\frac{64}{0.0764}\right)\left(\frac{30}{1.5}\right)\left(\frac{10}{2350}\right) = 71.4$$

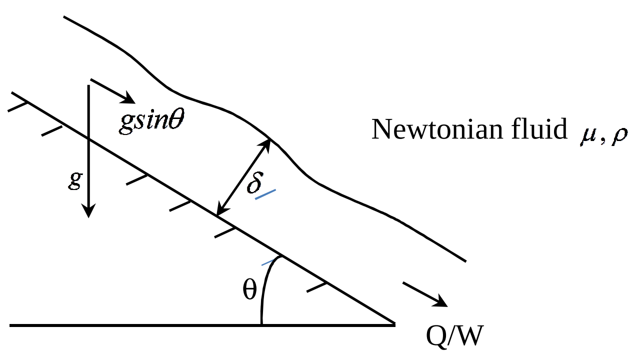
2-33 You want to determine the thickness of the film when a Newtonian fluid flows uniformly down an inclined plane at an angle θ with the horizontal at a specified flow rate. To do this, you design a laboratory experiment from which you can scale up measured values to any other Newtonian fluid under corresponding conditions.

(a) List all the independent variables that are important in this problem, with their dimensions. If there are any variables that are not independent but act only in conjunction with one another, list only the net combination that is important.

(b) Determine an appropriate set of dimensionless groups for this system, in such a way that the fluid viscosity and the plate inclination each appear in only one group.

(c) Decide what variables you would choose for convenience, what variables would be specified by the analysis, and what you would measure in the lab.

Solution:



Want to find δ (W = width).

(a) List all pertinent variables with dimensions.

(Note: g and θ are not independent. It is only $g\sin\theta$, which acts in the direction of flow, that is important).

Also, if plate width (W) is $>> \delta$, then only Q/W is important, not Q and W separately.

Variable	Dimensions
$\frac{Q}{W}$	$\frac{L^3}{tL} = \frac{L^2}{t}$
δ	L
gsinθ	$\frac{L}{t^2}$
μ	M/Lt
ρ	M/L^3
5 –	3 = 2 groups

(b) Find groups with μ and θ each in only 1 group (don't use this as ref variables)

Reference variables: $[\delta] = L \qquad \Rightarrow L = [\delta]$

$$[\rho] = \frac{M}{L^3} \qquad \Rightarrow M = \left[\rho\delta^3\right]$$

$$\left[\frac{Q}{W}\right] = \left[\frac{L^2}{t}\right] \qquad \Rightarrow t = \left[\frac{\delta^2}{Q/W}\right]$$

$$[\mu] = \frac{M}{Lt} = \left[\frac{\rho\delta^3 Q/W}{\delta\delta^2}\right] = \left[\frac{\rho Q/W}{1}\right] \rightarrow N_1 = \frac{\rho Q}{W\mu} = N_{Re}$$

$$[g\sin\theta] = \frac{L}{t^2} = \left[\frac{\delta}{\delta^4}\left(\frac{Q}{W}\right)^2\right] \rightarrow \qquad N_2 = \frac{\delta^3 g\sin\theta}{(Q/W)^2}$$

(c) Tell which variables you would pick, which to measure, and what would be determined.

Variables: $\left(\frac{Q}{W}, \delta, g\sin\theta, \rho, \mu\right)_{l,f} \Rightarrow 10$

Equations: $(N_1)_l = (N_1)_f, (N_2)_l = (N_2)_f \Rightarrow 2$

Knowns: $\left(\frac{Q}{W}, g\sin\theta, \rho, \mu\right)_f, (\delta)_l (\text{measure}) \Rightarrow 5$

Leaves 3 to specify for "convenience"

If you choose the lab fluid: $(\mu,\rho)_l$

This leaves one more to "choose"

Could be $\left(\frac{Q}{W}\right)_l$ or $(g\sin\theta)_l$ i.e. $(\theta)_l$

Easier to specify θ_l

Get $\left(\frac{Q}{W}\right)_l$ from $(N_1)_l = (N_1)_f$

Measure $(\delta)_l$, get $(\delta)_f$ from $(N_2)_l = (N_2)_f$

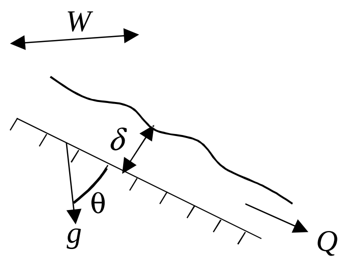
2-34 You would like to know the thickness of a syrup film as it drains at a rate of 1 gpm down a flat surface that is 6 in. wide and is inclined at an angle of 30^o from the vertical. The syrup has a viscosity of 100 cP and a SG of 0.9. In the laboratory, you have a fluid with a viscosity of 70 cP and a SG of 1.0 and a 1 ft wide plane inclined at an angle of 45^o from the vertical.

(a) At what flow rate, in gpm, would the laboratory conditions simulate the specified conditions?

(b) If the thickness of the film in the laboratory is 3 mm at the proper flow rate, what would the thickness of the film be for the 100 cP fluid at the specified conditions?

Solution:

Syrup film:



Field; μ = 100 cP, ρ = 0.9 g/cc

Q = 1 gpm, $\theta = 30^{o}$, W = 6 in

Lab (model): μ = 70 cP, ρ = 1 g/cc, $\theta = 45^{o}$, W = 1 ft

(a) Q_m = ? to simulate field.

Note: Since Q ~ W, Q/W is one variable, Driving force = g cosθ, so g and θ are not separate variables.

Reference variables: $[\delta] = L \qquad \Rightarrow L = [\delta]$

$$[\rho] = \frac{M}{L^3} \qquad \Rightarrow M = [\rho\delta^3]$$

$$\left[\frac{Q}{W}\right] = \left[\frac{L^2}{t}\right] \qquad \Rightarrow t = \left[\frac{\delta^2}{Q/W}\right]$$

Variable	Dimensions
$\frac{Q}{W}$	$\frac{L^3}{tL} = \frac{L^2}{t}$
δ	L
g cosθ	$\frac{L}{t^2}$
μ	M/Lt
ρ	M/L^3
5 –	3 = 2 groups

$$[\mu] = M/Lt = \left[\frac{\rho\delta^3 Q}{\delta\delta^2 W}\right] = \left[\frac{\rho Q}{W}\right] \rightarrow \quad N_1 = \frac{\rho Q/W}{\mu}$$

$$[g\cos\theta] = L/t^2 = \left[\frac{\delta}{\delta^4}\left(\frac{Q}{W}\right)^2\right] \rightarrow \quad N_2 = \left[\frac{(Q/W)^2}{g\cos\theta\,\delta^3}\right]$$

Find Q_m: $\left(\frac{\rho Q/W}{\mu}\right)_m = \left(\frac{\rho Q/W}{\mu_f}\right)_f$, $Q_m = W_m\left(\frac{Q}{W}\right)_f \frac{\rho_f}{\rho_m}\frac{\mu_m}{\mu_f}$

$$Q_m = 1\text{gpm}\left(\frac{1}{0.5}\right)\left(\frac{0.9}{1}\right)\frac{70}{100} = 1.26\text{gpm}$$

(b) If δ_m = 3 mm, δ_f = ?

$$\left(\frac{(Q/W)^2}{\delta^3 \not{g} \cos\theta}\right)_f = \left(\frac{(Q/W)^2}{\delta^3 \not{g} \cos\theta}\right)_m$$

$$\delta_f^{\ 3} = \delta_m^{\ 3}\left(\frac{Q_f}{Q_m}\right)^2\left(\frac{W_m}{W_f}\right)^2\frac{(\cos\theta)_m}{(\cos\theta)_f}$$

$$\delta_f = 3\,\text{mm}\left(\frac{1}{1.26}\right)^{2/3}\left(\frac{1}{0.5}\right)^{2/3}\left(\frac{\cos 45^0}{\cos 30^0}\right)^{1/3} = 3.82\text{mm}$$

Note: Since the driving force is ρgcosθ, there is actually only 1 independent group for this problem: $\left(\frac{(Q/W)}{\delta^3 \mu\rho g\cos\theta}\right) = \left(\frac{N_2}{N_1}\right)$

2-35 The size of liquid droplets produced by a spray nozzle depends upon the nozzle diameter, the fluid velocity and the fluid properties (which may, under some circumstances, include surface tension).

(a) Determine an appropriate set of dimensionless groups for this system.

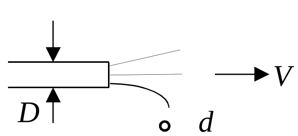
(b) You want to know what size droplets will be generated by a fuel oil nozzle with a diameter of 0.5 mm at an oil velocity of 10 m/s. The oil has a viscosity of 10 cP, a SG of 0.82, and a surface tension of 35 dyn/cm. You have a nozzle in the lab with a nozzle diameter of 0.2 mm that you want to use in a lab experiment to find the answer. Can you use the same fuel oil in the lab test as in the field? If not, why not?

(c) If the only fluid you have is water, tell how you would design the lab experiment. Note: water has a viscosity of 1 cP and a SG of 1, but its surface tension can be varied by adding small amounts of surfactant which does not affect the viscosity or density.

(d) Determine what conditions you would use in the lab, what you would measure, and the relationship between the measured and the unknown droplet diameters.

Solution:

Spray Nozzle



Drops $d = f_n\,(D, V, \rho, \mu, \sigma)$

↑

Surface tension

↖

(a) Defining Dimensionless groups:

Reference variables: $[D] = L \Rightarrow L = [D]$

$[V] = L/t \Rightarrow t = [D/V]$

$[\rho] = M/L^3 \Rightarrow M = [\rho D^3]$

Variable	Dimensions
d	L
D	L
V	L/t
ρ	M/L^3
μ	M/Lt
σ	$F/L= M/t^2$
6 –	3 = 3 groups

$$[d] = L = [D] \rightarrow \qquad N_1 = \frac{d}{D}$$

$$[\mu] = M/Lt = \left[\frac{\rho D^3 V}{D^2}\right] \rightarrow N_2 = \frac{\rho D V}{\mu}$$

$$[\sigma] = M/t^2 = \left[\frac{\rho D^3 V^2}{D^2}\right] \rightarrow N_3 = \frac{\sigma}{\rho D V^2}$$

(b) Want to find d if D = 0.5 mm, V = 10 m/s, μ = 10 cP, σ = 35 dyn/cm, ρ = 0.82 g/cc

Lab: D = 0.2 mm Can you use same fluid?

Unknown: $(V, \mu, \rho, \sigma)_l\, (d)_f \rightarrow 5$ Equations $\rightarrow 3$

Can set 2 variables for convenience.

$\therefore$ No - Since this fires three variables μ, σ, ρ

(c) Use water μ = 10 cP, ρ = 1 g/cc can adgest σ using additives

$$(N_2)_l = (N_2)_f \,;\, V_l = V_f \left(\frac{\mu_l}{\mu_f}\right)\left(\frac{\rho_f}{\rho_l}\right)\left(\frac{D_f}{D_l}\right)$$

$$= 10\,\text{m/s}\left(\frac{1}{10}\right)\left(\frac{0.82}{1}\right)\left(\frac{0.5}{0.2}\right) = 2.05\,\text{m/s}$$

$$(N_3)_l = (N_3)_f;\ \sigma_l = \sigma_f\left(\frac{V_l}{V_f}\right)^2\left(\frac{\rho_l}{\rho_f}\right)\left(\frac{D_l}{D_f}\right)$$

$$= 35\,\text{dyn/cm}\left(\frac{2.05}{10}\right)^2\left(\frac{1}{0.82}\right)\left(\frac{0.2}{0.5}\right) = 0.72\,\text{dyn/cm}$$

You should use water, use a surfactant to lower surface tension to 0.72 dyn/cm, pump it at 2.05 m/s through the nozzle and measure d*l*.

(d) Find $(d)_f$ from $(N_1)_l = (N_1)_f$, with the system operating at the above specified conditions,

$$d_f = d_l\left(\frac{D_f}{D_l}\right) = d_l\left(\frac{0.5}{0.2}\right) = 2.5d_l$$

(This is not valid unless above conditions are satisfied)

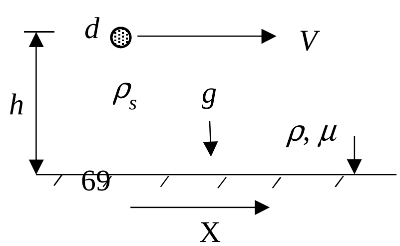
2-36 Small solid particles of diameter d and density ρ_s are carried horizontally by an air stream moving at velocity *V*. The particles are initially at a distance *h* above the ground, and you want to know how far they will be carried horizontally before they settle to the ground. To find this out, you decide to conduct a lab experiment using water as the test fluid.

(a) Determine what variables you must set in the lab and how the value of each of these variables is related to the corresponding variable in the air system. You should note that several forces act on the particle: the drag force due to the moving fluid, which depends on the fluid and solid properties, the size of the particle and the relative velocity; and the gravitational force, which is directly related to the densities of both the solid and the fluid in a known manner.

(b) Is there any reason why this experiment might not be feasible in practice?

Solution:

Entrained falling particle. Find X using experiment with H_2O.



Drag ~ ρ, (Wt-buoyancy) ~ $(\rho_s - \rho) = \Delta\rho$

(a) Reference variables: [d] = L $\Rightarrow$ L = [d]

[V] = L/t $\Rightarrow$ t = [d/V]

$[\rho] = M/L^3 \Rightarrow M = [\rho D^3]$

Variable	Dimensions	
d	L	
h	L	
X	L	
ρ	M/L^3	
μ	M/Lt	
Δρ	M/L^3	
V	L/t	
g	L/t^2	
8 –	3 =	5 groups

$$[X] = L = [d] \rightarrow \qquad N_1 = \frac{X}{d}$$

$$[h] = L = [d] \rightarrow \qquad N_2 = \frac{h}{d}$$

$$[\Delta\rho] = M/L^3 = [\rho] \rightarrow \qquad N_3 = \frac{\Delta\rho}{\rho}$$

$$[\mu] = M/Lt = \left[\frac{\rho d^3 V}{d^2}\right] \rightarrow N_4 = \frac{\rho d V}{\mu}$$

$$[g] = L/t^2 = \left[\frac{dV^2}{d^2}\right] \rightarrow \quad N_5 = \frac{gd}{V^2}$$

(b) Knowns: $(V, d, h, \rho, g, \mu, \Delta\rho)_f, (X)_M, (g)_M = 9$

Unknowns: $(V, d, h, \rho, \mu, \Delta\rho)_M, (X)_F = 7 - 5\,\text{eqns} = 2\text{ to pick}$

Use: Water (μ_M, ρ_M) $\left(\frac{\Delta\rho}{\rho}\right)_M = \left(\frac{\Delta\rho}{\rho}\right)_F \rightarrow (\Delta\rho)_M = (\Delta\rho)_F \left(\frac{\rho_M}{\rho_F}\right)$ or $(\rho_s)_M$

$$= \rho_M + (\Delta\rho)_F \left(\frac{\rho_M}{\rho_F}\right)$$

or $(\rho_s)_M = (\rho_s)_F \left(\frac{\rho_M}{\rho_F}\right)$

$$\left(\frac{N_4}{N_5}\right) = \left(\frac{dVV^2\rho}{\mu g d}\right) = \left(\frac{\rho V^3}{g\mu}\right)_M = \left(\frac{\rho V^3}{g\mu}\right)_F$$

$$V_M = V_F \left(\frac{\mu_M}{\mu_F}\right)^{1/3} \left(\frac{\rho_F}{\rho_M}\right)^{1/3} = V_F \left(\frac{0.076 \times 1}{62.4 \times 0.02}\right)^{1/3} = 0.393 V_F$$

$$\left(\frac{\cancel{g}d}{V^2}\right)_M = \left(\frac{\cancel{g}d}{V^2}\right)_F$$

$$\therefore d_M = d_F \left(\frac{V_M}{V_F}\right)^2 = d_F (0.393)^2 = 0.155 d_F$$

$$(\rho_s)_M = (\rho_s)_M \left(\frac{\rho_M}{\rho_F}\right) = (\rho_s)_F \left(\frac{62.4}{0.076}\right) = 821 (\rho_s)_F \leftarrow \text{Not likely !!}$$

$$\left.\frac{h}{d}\right|_M = \left.\frac{h}{d}\right|_F \quad \Rightarrow h_M = h_F \frac{d_M}{d_F} = 0.155\ h_F$$

$$\left.\frac{X}{d}\right|_M = \left.\frac{X}{d}\right|_F \quad \Rightarrow X_F = X_M \frac{d_F}{d_M} = \frac{X_M}{0.155} = 6.45 X_M$$

2-37 You want to find the wind drag on a new automobile design at various speeds. To do this, you test a 1/30 scale model of the car in the lab. You must design an experiment whereby the drag force measured in the lab can be scaled up directly to find the force on the full-scale car at a given speed.

(a) What is the minimum number of (dimensionless) variables required to completely define the relationship between all the important variables in the problem? Determine the appropriate variables (e.g. the dimensionless groups).

(b) The only fluids you have available in the lab are air and water. Could you use either one of these, if you wanted to? Why (or why not)?

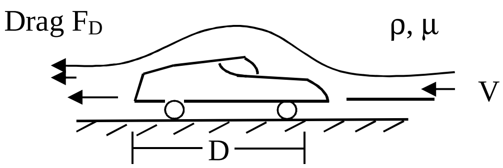
(c) Tell which of these fluids you would use in the lab, and then determine what the velocity of this fluid past the model car would have to be so that the experiment would simulate the drag on the full-scale car at 40 mph. If you decide that it is possible to use either one of the two fluids, determine the answer for each of them.

(d) What is the relationship between the measured drag force on the model and the drag force on the full scale car? If possible, determine this relationship for the other fluid, as well. Repeat this for a speed of 70 mph.

(e) It turns out that for very high values of the Reynolds number, the drag force is independent of the fluid viscosity. Under these conditions, if the speed of the car doubles, by what factor does the power required to overcome wind drag change?

Solution:

Test Car Model:



Lab Model: $D_M = D_F/30$ (all other dimensions are in the same ratio)

Want to find F_D at various V's.

(a) Find Groups:

Reference variables: $[D] = L \Rightarrow L = [D]$

$$[V] = L/t \Rightarrow t = [D/V]$$

$$[\rho] = M/L^3 \Rightarrow M = [\rho D^3]$$

Variable		Dimensions	
D		L	
V		L/t	
μ		M/Lt	
ρ		M/L^3	
F_D		ML/t^2	
5	–	3 =	2 groups

$$[\mu] = \frac{M}{Lt} = \left[\frac{\rho D^3 V}{D\,D}\right] \to \qquad N_1 = \frac{DV\rho}{\mu}$$

$$[F_D] = \frac{ML}{t^2} = \left[\frac{\rho D^3 D}{(D/V)^2}\right] \to N_2 = F_D/\rho V^2 D^2$$

(b) Use water or air or either?

Unknown: $(\rho, \mu, V)_M\ (F_D)_F = 4$ 2 Equations.

$\therefore$ Have 2 "arbitrary" variables. Can pick ρ and μ.

$\therefore$ Can use any fluid.

(c) Use Air: $\rho = 1.2\ kg/m^3 = 0.0012\ g/cc$, $\mu = 1.8\times10^{-5}$ Pas = 0.018 cP

$$\left[\frac{DV\rho}{\mu}\right]_M = \left[\frac{DV\rho}{\mu}\right]_F \qquad V_F = 40 \text{ mph}$$

$$V_M = V_F\left(\frac{D_F}{D_M}\right)\left(\frac{\cancel{\rho_F}}{\cancel{\rho_M}}\right)\left(\frac{\cancel{\mu_M}}{\cancel{\mu_F}}\right) = 40(30) = 1200 \text{ mph in lab. (not practical)}$$

Use H_2O: $\rho = 1$ g/cc, $\mu = 1$ cP

$$V_M = 40 \text{ mph}(30)\left(\frac{0.0012}{1}\right)\left(\frac{1}{0.018}\right) = 80 \text{ mph in lab}$$

$$= 80\frac{5280}{3600} = 117 \text{ft/s}$$

(d) $$\left(\frac{DV\cancel{\rho}}{\cancel{\mu}}\right)_M = \left(\frac{DV\cancel{\rho}}{\cancel{\mu}}\right)_F$$

$$(DV)_M = (DV)_F$$

$\therefore F_{DM} = F_{DF}$ with air

$$\left(\frac{F_D}{\rho V^2 D^2}\right)_M = \left(\frac{F_D}{\rho V^2 D^2}\right)_F$$

$$F_{D_M} = F_{D_F}\left(\frac{\rho_M}{\rho_F}\right)\left(\frac{V_M}{V_F}\right)^2\left(\frac{D_M}{D_F}\right)^2 = F_{D_F}\left(\frac{1200}{40}\right)^2\left(\frac{1}{30}\right)^2 = F_{D_F} \text{ with air}$$

$$= F_{D_F}\left(\frac{1}{0.0012}\right)\left(\frac{80}{40}\right)^2\left(\frac{1}{30}\right)^2 = 3.7F_{D_F} \text{ with } H_2O$$

At 70 mph: $$V_M = V_F\left(\frac{D_F}{D_M}\right)\left(\frac{\rho_F}{\rho_M}\right)\left(\frac{\mu_M}{\mu_F}\right)$$

$$= 70(30) = 2100 \text{ mph with air}$$

$$= 70(30)\left(\frac{0.0012}{1}\right)\left(\frac{1}{0.018}\right) = 140 \text{ mph with } H_2O$$

$$F_{D_M} = F_{D_F}\left(\frac{2100}{70}\right)^2\left(\frac{1}{50}\right)^2 = F_{D_F} \text{ with Air}$$

$$= F_{D_F}\left(\frac{1}{0.0012}\right)\left(\frac{140}{70}\right)^2\left(\frac{1}{30}\right)^2 = 3.7F_{D_F} \text{ with } H_2O$$

(e) At high N_{Re}, F_D is independent of μ. If V doubles, what factor does power increase ?

If μ not a variable, then $\left(\frac{F_D}{\rho V^2 D}\right) = \text{constant}$

or

$F_D = \text{constant} \ \rho V^2 D^2$

Power: $\mathbb{P} = \frac{W}{t} = \frac{FX}{t} = FV$

Given D:

$\therefore \mathbb{P} = \text{constant } \rho V^3$

$$\frac{\mathbb{P}_1}{\mathbb{P}_2} = \frac{V_2^3}{V_1^3} = \left(\frac{2V_1}{V_1}\right)^3 = 8$$

Power increases by factor of 8 when velocity doubles.

at 40 mph: $N_{Re} = \frac{DV\rho}{\mu}$ (Assume $D \cong 6$ ft)

$$= \frac{6(3048)\,\text{cm}\, 40\frac{\text{mi}}{\text{hr}}\left(5280\,{}^{\text{ft}}/_{\text{mi}}\right)30.98\frac{\text{cm}}{\text{ft}}(0.0012)}{3600\,\text{s}/\text{hr}\,(0.00018\text{P})}$$

$$= 2.2 \times 10^6$$

2-38 The power required to drive a centrifugal pump and the pressure that the pump will develop depends upon the size (diameter) and speed (angular velocity) of the impeller, the volumetric flow rate through the pump, and the fluid properties. However, if the fluid is not too viscous (e.g., less than about 100 cP), the pump performance is essentially independent of the fluid viscosity. Under these conditions:

(a) Perform a dimensional analysis to determine the dimensionless groups that would be required to define the pump performance. Arrange the groups so that the power and pump pressure each appear in only one group.

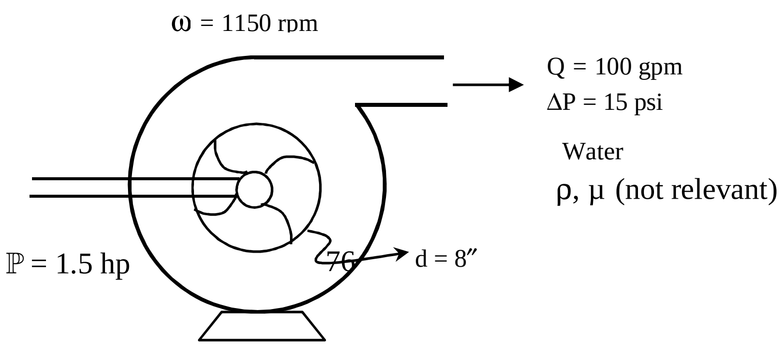
You have a pump with an 8 in. diameter impeller that develops a pressure of 15 psi and requires 1.5 hp to operate when running at 1150 rpm with water at a flow rate of 100 gpm. You also have a similar pump with a 13 in. diameter impeller, driven by a 1750 rpm motor, and you would like to know what pressure this pump would develop with water and what power would be required to drive it.

(b) If the second pump is to be operated under equivalent (similar) conditions to the first one, what should the flow rate be?

(c) If this pump is operated at the proper flow rate, what pressure will it develop, and what power will be required to drive it when pumping water?

Solution:

Centrifugal Pump # 1



Centrifugal Pump # 2 d = 13″, ω = 1750 rpm Find: ΔP, Q, ℙ

Reference variables: [d] = L ⇒ L = [d]

[ω] = 1/t ⇒ t = [1/ω]

[ρ] = M/L^3 ⇒ M = [ρd^3]

Variable	Dimensions
d	L
ω	1/t
Q	L^3/t
ρ	M/L^3
Δℙ	$F/L^2 = M/Lt^2$
ℙ	$FL/t = ML^2/t^3$
6 –	3 = 3 groups

$$[Q] = L^3/t = [d^3\omega] \rightarrow \qquad N_1 = \frac{Q}{d^3\omega}$$

$$\Pi = ML^2/t^2 = [\rho d^3 d^2 \omega^3] \rightarrow \quad N_2 = \mathbb{P}/\rho d^5\omega^3$$

$$[\Delta P] = M/Lt^2 = \left[\rho d^3\omega]^2/d\right] \rightarrow \quad N_3 = \Delta P/\rho d^2\omega^2$$

For similarity $\left(\frac{Q}{d^3\omega}\right)_1 = \left(\frac{Q}{d^3\omega}\right)_2$ $Q_2 = Q_1\left(\frac{d_2}{d_1}\right)^3\left(\frac{\omega_2}{\omega_1}\right)$

$$= 100\,\text{gpm}\left(\frac{13}{8}\right)^3\left(\frac{1750}{1150}\right)$$

$$= 653 \text{ gpm}$$

$$\left(\frac{\mathbb{P}}{\rho d^5 \omega^3}\right)_1 = \left(\frac{\mathbb{P}}{\rho d^5 \omega^3}\right)_2$$

$$\mathbb{P}_2 = \mathbb{P}_1 \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{d_2}{d_1}\right)^5 \left(\frac{\omega_2}{\omega_1}\right)^3 = 1.5\text{hp}\left(\frac{1}{1}\right)\left(\frac{13}{8}\right)^5 \left(\frac{1750}{1150}\right)^3 = 59.9\,\text{hp}$$

$$\left(\frac{\Delta P}{\rho d^2 \omega^2}\right)_1 = \left(\frac{\Delta P}{\rho d^2 \omega^2}\right)_2$$

$$\Delta P_2 = \Delta P_1 \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{d_2}{d_1}\right)^2 \left(\frac{\omega_2}{\omega_1}\right)^2 = 15\text{psi}\left(\frac{1}{1}\right)\left(\frac{13}{8}\right)^2 \left(\frac{1750}{1150}\right)^2 = 91.7\text{psi}$$

2-39 In a distillation column, vapor is bubbled through the liquid to provide good contact between the two phases. The bubbles are formed when the vapor passes upward through a hole (orifice) in a plate (tray) that is in contact with the liquid. The size of the bubbles depends upon the diameter of the orifice, the velocity of the vapor through the orifice, the viscosity and density of the liquid, and the surface tension between the vapor and the liquid.

(a) Determine the dimensionless groups required to completely describe this system, in such a manner that the bubble diameter and the surface tension do not appear in the same group.

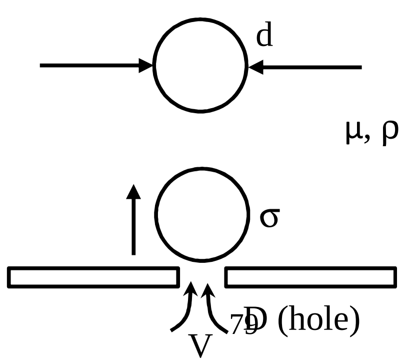
(b) You want to find out what size bubbles would be formed by a hydrocarbon vapor passing through a 1/4 in. orifice at a velocity of 2 ft/s, in contact with a liquid having a viscosity of 4 cP and a density of 0.95 g/cm^3 (the surface tension is 30 dyn/cm). To do this, you run a lab experiment using air and water (with a surface tension of 60 dyn/cm).

(1) What size orifice should you use, and what should the air velocity through the orifice be?

(2) You design and run this experiment and find that the air bubbles are 0.1 in. in diameter. What size would the vapor bubbles be in the organic fluid above the 1/4 in. orifice?

Solution:

Vapor bubbles through tray



Want d and σ in different Groups

(a) Reference variables: [D] = L $\Rightarrow$ L = [D]

[V] = L/t $\Rightarrow$ t = [D/V]

[ρ] = M/L^3 $\Rightarrow$ M = [ρD^3]

Variable	Dimensions
d	L
D	L
V	L/t
ρ	M/L^3
μ	M/Lt
σ	F/L= M/t^2
6 –	3 = 3 groups

$N_1 = d/D$

$$[\sigma]=\frac{M}{t^2}=\left[\frac{\rho D^3V^2}{D^2}\right]=\left[DV^2\rho\right]\rightarrow \; N_2=\frac{\sigma}{DV^2\rho} \text{ or, } \frac{\sigma}{\mu V}$$

$$[\mu]=\frac{M}{Lt}=\left[\frac{\rho D^3 V}{D\,D}\right]\rightarrow \qquad N_3=\frac{DV\rho}{\mu}$$

(b) Field : $D=\frac{1}{4}''$, $V=2\frac{ft}{s}$, $\mu=4\,cP$, $\rho=0.95\ g/cc$, $\sigma=30\frac{dyn}{cm}$, $d=?$

Model : $\rho=1\frac{g}{cc}$, $\mu=1cP$, $\sigma=60\frac{dyn}{cm}$, $d=0.1in$, $V=?$, $D=?$,

Unknown = 3

Constants = 3

Get V in only 1 group: $\left(\frac{\sigma}{\mu V}\right)\left(\frac{DV\rho}{\mu}\right)=\frac{\sigma D\rho}{\mu^2}$

$$\left(\frac{d}{D}\right)_F=\left(\frac{d}{D}\right)_M,\ \left(\frac{DV\rho}{\mu}\right)_F=\left(\frac{DV\rho}{\mu}\right)_M,\ \left(\frac{\sigma D\rho}{\mu^2}\right)_F=\left(\frac{\sigma D\rho}{\mu^2}\right)_M$$

$$D_M = D_F \left(\frac{\mu_M}{\mu_F}\right)^2 \frac{\sigma_F}{\sigma_M}\frac{\rho_F}{\rho_M} = 0.25'' \left(\frac{1}{4}\right)^2 \left(\frac{30}{60}\right)\left(\frac{0.95}{1}\right) = 0.00742\,\text{in}$$

(very small !!)

c) $$V_M = V_F \left(\frac{D_F}{D_M}\right)\left(\frac{\rho_F}{\rho_M}\right)\left(\frac{\mu_M}{\mu_F}\right) = 2\,\text{ft/s}\left(\frac{0.25}{0.00742}\right)\left(\frac{0.95}{1}\right)\left(\frac{1}{4}\right) = 16\,\text{ft/s}$$

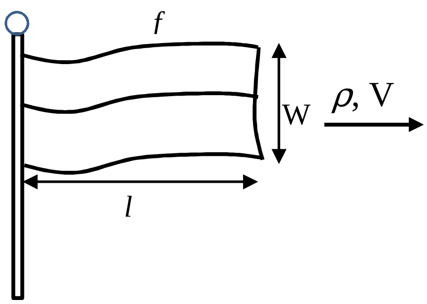
$$d_F = d_M\left(\frac{D_F}{D_M}\right) = 0.1''\left(\frac{0.25}{0.00742}\right) = 3.37\,\text{in}$$ (not practical !)

2-40 A flag will flutter in the wind at a frequency that depends upon the wind speed, the air density, the size of the flag (length and width), gravity, and the "area density" of the cloth (i.e. the mass per unit area). You have a very large flag (40 ft long and 30 ft wide) that weighs 240 lb, and you want to find the frequency at which it will flutter in a wind of 20 mph.

(a) Perform a dimensional analysis to determine an appropriate set of dimensionless groups that which could be used to describe this problem.

(b) To find the flutter frequency you run a test in a wind tunnel (at normal atmospheric temperature and pressure) using a flag made from a cloth that weighs 0.05 lb/ft^2. Determine (1) the size of the flag and the wind speed that you should use in the wind tunnel and (2) the ratio of the flutter frequency of the big flag to that which you observe for the model flag in the wind tunnel.

Solution:



(a) Flag:

Reference variables: $[f] = 1/t \quad \Rightarrow \quad t = [1/f]$

$[l] = L \quad \Rightarrow \quad L = [l]$

$[\rho] = M/L^3 \quad \Rightarrow \quad M = [\rho l^3]$

Variable	Dimensions
f	1/t
ρ	M/L^3
V	L/t
W	L
l	L
M	M/L^2
g	L/t^2
7 –	3 = 4 groups

$$[V] = L/t = [l, f] \rightarrow \quad N_1 = \frac{V}{lf}$$

$$[g] = L/t^2 = [l, f^2] \rightarrow N_2 = \frac{lf^2}{g}$$

$$[W] = L = [l] \rightarrow \quad N_3 = l/W$$

$$[\mu] = \frac{M}{L^2} = \frac{\rho l^3}{l^2} = \rho l \rightarrow N_4 = \frac{M}{\rho l}$$

(b) Field: $V = 20\,\text{ft/s},\ l = 40\,\text{ft}, W = 30\,\text{ft},\ M = \frac{240\,\text{lb}}{30(40)\text{ft}^2} = 0.2\,\frac{\text{lb}}{\text{ft}^2}$

$$\rho = \text{air},\ g = 32.2\frac{\text{ft}}{s^2}$$

Model: $\rho = \text{air}, g = 32.2\,\frac{\text{ft}}{s^2}\ , M = 0.05\,\frac{\text{lb}}{\text{ft}^2}$

Unknown: $(V, l, W)_m, W_f = 4$

Equations = 4

$$\left(\frac{M}{\rho l}\right)_m = \left(\frac{M}{\rho l}\right)_f, l_m = l_f\left(\frac{M_m}{M_f}\right) = 40\,\text{ft}\left(\frac{0.05}{0.2}\right) = 10\,\text{ft}$$

$$\left(\frac{lf^2}{g}\right)_f = \left(\frac{lf^2}{g}\right)_m, \frac{f_f}{f_m} = \left(\frac{l_m}{l_f}\right)^{1/2} = \left(\frac{10}{40}\right)^{1/2} = 0.5$$

$$\left(\frac{V}{lf}\right)_m = \left(\frac{V}{lf}\right)_f, V_m = V_f\left(\frac{l_m}{l_f}\right)\frac{f_m}{f_f} = 20\,\text{mph}\left(\frac{10}{40}\right)(2) = 10\,\text{mph}$$

$$\left(\frac{l}{W}\right)_m = \left(\frac{l}{W}\right)_f, W_m = W_f\left(\frac{l_m}{l_f}\right) = 30\left(\frac{10}{40}\right) = 7.5\,\text{ft}$$

2-41 If the viscosity of the liquid is not too high (e.g. less than about 100 cP), the performance of many centrifugal pumps is not very sensitive to the fluid viscosity. You have a pump with an 8 in. diameter impeller that develops a pressure of 15 psi and consumes 1.5 hp when running at 1150 rpm pumping water at a rate of 100 gpm. You also have a similar pump with a 13 in. diameter impeller, driven by a 1750 rpm motor, and you would like to know what pressure that pump would develop with water, and how much power it would take to drive it.

(a) If the second pump is to be operated under conditions similar to that of the first, what should the flow rate be?

(b) When operated at this flow rate with water, what pressure should it develop and what power would be required to drive it?

Solution:

This problem is similar to Problem 2.25. The relevant dimensional groups are

$$N_1 = \frac{\Delta P}{\rho D^2 \omega^2};\ N_2 = \frac{\mathbb{P}}{\rho D^5 \omega^3};\ N_3 = \frac{Q}{D^3 \omega}$$

$$\left.\frac{Q}{D^3\omega}\right|_1 = \left.\frac{Q}{D^3\omega}\right|_2$$

$$Q_2 = Q_1\left(\frac{D_2}{D_1}\right)^3\left(\frac{\omega_2}{\omega_1}\right)$$

$$= 1000\,\text{gpm}\left(\frac{13}{8}\right)^3\left(\frac{1750}{1150}\right)$$

$$= = 650 \text{ gpm}$$

Note that only 2 of these 3 are independent groups because $\mathbb{P} = Q\,\Delta P$

$$\left.\frac{\Delta P}{\rho D^2 \omega^2}\right|_1 = \left.\frac{\Delta P}{\rho D^2 \omega^2}\right|_2$$

$D_M = 8''$

$D_F = 13''$

$\Delta P_M = 15$ Psi

$\omega_F = 1750$ rpm

$\omega_M = 1150$ rpm

Since water is being used in both: $\rho_M = \rho_F$

$$\Delta P_F = \Delta P_M \left(\frac{D_F}{D_M}\right)^2 \left(\frac{\omega_F}{\omega_M}\right)^2$$

$$= 15\,\text{Psi}\left(\frac{13}{8}\right)^2 \left(\frac{1750}{1150}\right)^2 = 91.7\,\text{Psi}$$

Now matching N2

$$\Delta P_F = \Delta P_M \left(\frac{\rho_F}{\rho_M}\right)\left(\frac{D_F}{D_M}\right)^5 \left(\frac{\omega_F}{\omega_M}\right)^3$$

$$= 15\,\text{hp}\left(\frac{1}{1}\right)\left(\frac{13}{8}\right)^5 \left(\frac{1750}{1150}\right)^3 = 60\,\text{hp}$$

2-42 The pressure developed by a centrifugal pump depends on the fluid density, the diameter of the pump impeller, the rotational speed of the impeller, and the volumetric flow rate through the pump (centrifugal pumps are not recommended for highly viscous fluids, so viscosity is not commonly an important variable). Furthermore, the pressure developed by the pump is commonly expressed as the "pump head", which is the height of a column of the fluid in the pump that exerts the same pressure as the pump pressure.

(a) Perform a dimensional analysis to determine the minimum number of variables required to represent the pump performance characteristic in the most general (dimensionless) form.

(b) The power delivered to the fluid by the pump is also important. Should this be included in the list of important variables, or can it be determined from the original set of variables? Explain.

You have a pump in the field that has a 1.5 ft diameter impeller that is driven by a motor operating at 750 rpm. You want to determine what head the pump will develop when pumping a liquid with density of 50 lb_m/ft^3 at a rate of 1000 gpm. You do this by running a test in the lab on a scale model of the pump that has a 0.5 ft diameter impeller using water (at 70^oF) and a motor that runs at 1200 rpm.

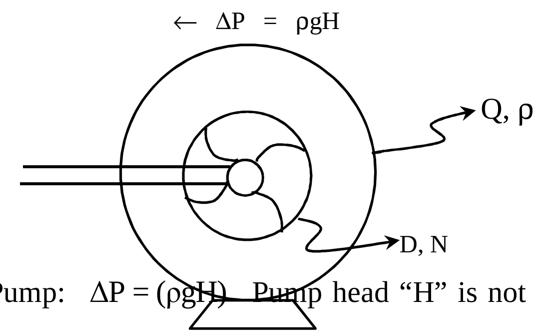
(c) At what flow rate of water should the lab pump be operated (in gpm)?

(d) If the lab pump develops a head of 85 ft at this flow rate, what head would the pump in the field develop with the operating fluid at the specified flow rate?

(e) How much power (in horsepower) is transferred to the fluid in both the lab and the field cases?

(f) The pump efficiency is defined as the ratio of the power delivered to the fluid to the power of the motor that drives the pump. If the lab pump is driven by a 2 hp motor, what is the efficiency of the lab pump? If the efficiency of the field pump is the same as that of the lab pump, what power motor (horsepower) would be required to drive it?

Solution:



Centrifugal Pump: $\Delta P = (\rho g H)$ Pump head "H" is not a length variable, represents ΔP

(a)Reference variables: $[D] = L \quad \Rightarrow L = [D]$

$[N] = 1/t \quad \Rightarrow t = [1/]$

$[\rho] = M/L^3 \quad \Rightarrow M = [\rho D^3]$

Variable	Dimensions
ΔP	$F/L^2 = M/Lt^2$
N	1/t
Q	L^3/t
D	L
ρ	M/L^3
5 –	3 = 2 groups

$$[\Delta P] = \frac{M}{Lt^2} = \left[\frac{\rho D^3 N^2}{D}\right] = [\rho D^2 N^2] \rightarrow \quad N_1 = \frac{\Delta P}{\rho D^2 N^2},\ \Delta P = \rho g H,\ N_1 = \frac{\not{\rho} gH}{\not{\rho} D^2 N^2}$$

$$[Q] = \frac{L^3}{t} = [D^3 N] \rightarrow \ N_2 = \frac{Q}{D^3 N}$$

(b) Power = $\mathbb{P} = \dfrac{FL}{t}$

$$\Delta P = \frac{F}{L^2},\ Q = \frac{L^3}{t}$$

$$\Delta PQ = \frac{FL}{t} = \mathbb{P} \quad \text{not independent of } N_1, N_2$$

$$\text{Field}: D = 1.5\text{ft},\ N = 750\,\text{rpm},\ \rho = 50\ \text{lbm}/\text{ft}^3,\ Q = 1000\,\text{gpm}$$

$$\text{Model}: D = 0.5\,\text{ft}\quad \rho = 62.3\ \text{lbm}/\text{ft}^3,\ N = 1200\,\text{rpm},\ \text{Measure}\ \Delta P$$

$$\text{Unknowns} = 2(\Delta P_f, Q_M)$$

Equations = 2

(c)

$$\left(\frac{Q}{D^3N}\right)_M = \left(\frac{Q}{D^3N}\right)_F \rightarrow Q_M = Q_F\left(\frac{D_M}{D_F}\right)^3\left(\frac{N_M}{N_F}\right)$$

$$= 1000\ \text{gpm}\left(\frac{0.5}{1.5}\right)^3\left(\frac{1200}{750}\right) = 59.3\ \text{gpm}$$

(d)

$$\Delta P = \rho g H,\ H_M = 86\text{ft}\ \ H_F = ?$$

$$\left(\frac{gH}{D^2N^2}\right)_F = \left(\frac{gH}{D^2N^2}\right)_M$$

$$H_F = H_M\left(\frac{D_F}{D_M}\right)^2\left(\frac{N_F}{N_M}\right)^2 = 85\,\text{ft}\left(\frac{1.5}{0.5}\right)^2\left(\frac{750}{1200}\right)^2 = 299\,\text{ft}$$

(e)

$$\mathbb{P} = \Delta P Q = \rho g H Q : \text{Lab}$$

$$\mathbb{P} = \frac{\left(62.3\frac{\text{lbm}}{\text{ft}^3}\right)\left(32.2\frac{\text{ft}}{\text{s}^2}\right)\left(32.2\frac{\text{ft}}{\text{s}^2}\right)(85\text{ft})(59.3\frac{\text{gal}}{\text{min}})}{\left(32.2\frac{\text{lbm}}{\text{lbf}}\frac{\text{ft}}{\text{s}^2}\right)\left(7.98\frac{\text{gal}}{\text{ft}^3}\right)\left(60\frac{\text{s}}{\text{min}}\right)\left(550\frac{\text{ft lb}_f}{\text{hp s}}\right)} = 1.27\text{hp}$$

$$\text{Field}: \mathbb{P} = \frac{50(299)(1000)}{7.48(60)(550)} = 60.6\,\text{hp}$$

(f)

$$\eta_e = \frac{(\mathbb{P})_{\text{fluid}}}{(\mathbb{P})_{\text{motor}}} = \left(\frac{1.27}{2}\right)_{\text{loss}} = 0.635$$

$$\text{Field}: \mathbb{P}_M = \frac{(\mathbb{P})_{\text{fluid}}}{\eta_e} = \frac{60.6}{0.635} = 95.4\ \text{hp}$$

(Probably use 100 hp)