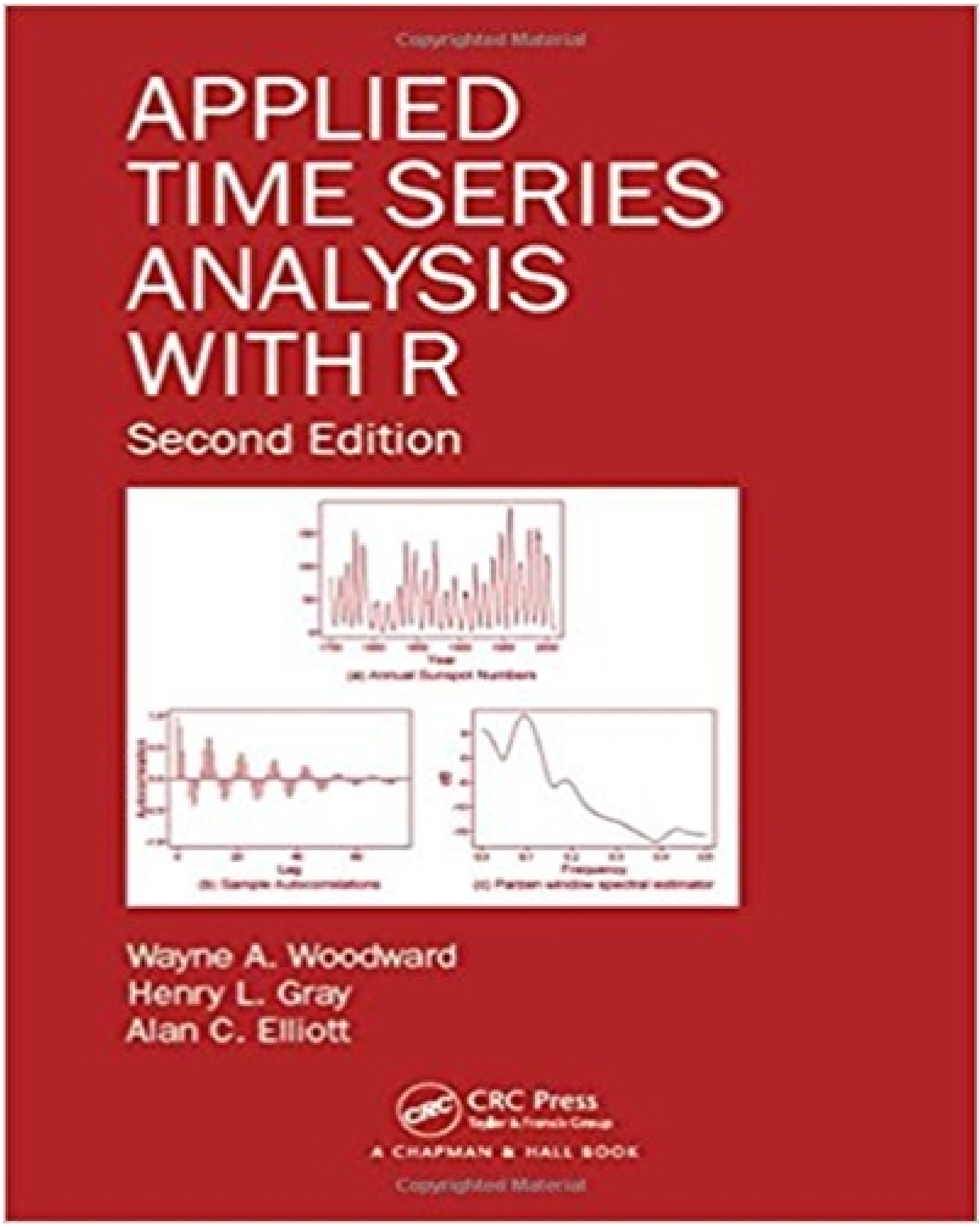
Solutions for Applied Time Series Analysis with R 2nd Edition by Woodward

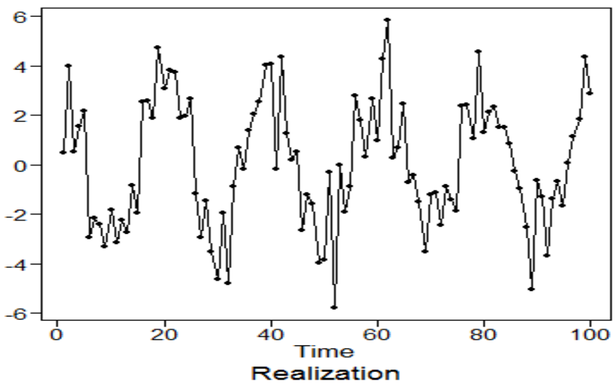


Solutions

CHAPTER 2 – Problem Solutions

Problem 2.1 Below is the plot a data set generated using

```
x=gen.sigplusnoise.wge(100,b0=0,b1=0,coef=c(3,1.5),freq=c(.05,.35),psi=c(0,2))
```



(a) If x is the data set generated above, then the following command produces the low-pass filtered data.

```
xl2=butterworth.wge(x,order=3,type='low',cutoff=.2)
```

(b) If x is the data set generated above, then the following command produces the high-pass filtered data.

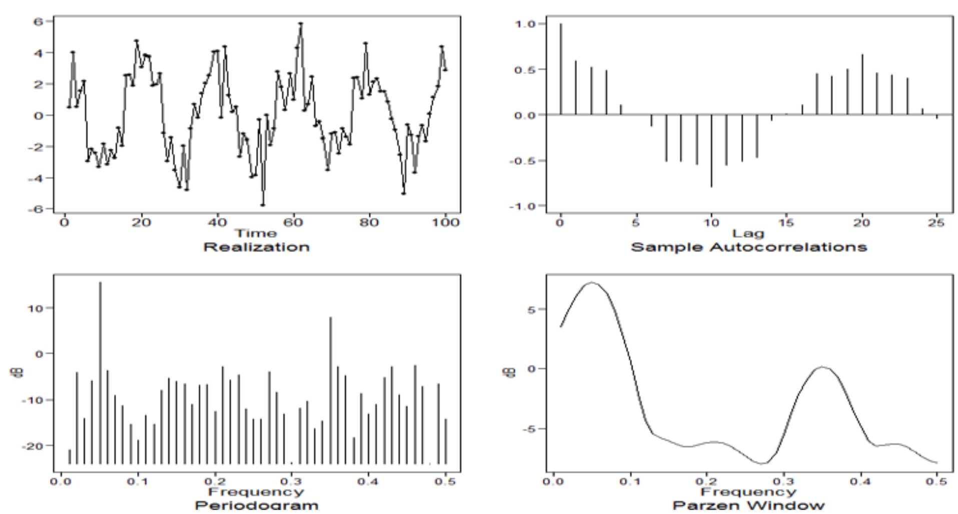
```
xh2=butterworth.wge(x,order=3,type='high',cutoff=.2)
```

(c) If xh2 is the high-pass filtered data in (b) then use the following command:

```
xhl2=butterworth.wge(xh2$x.filt,order=3,type='low',cutoff=.2)
xl2=butterworth.wge(x,oder=3,type='high',cutoff=.2)
```

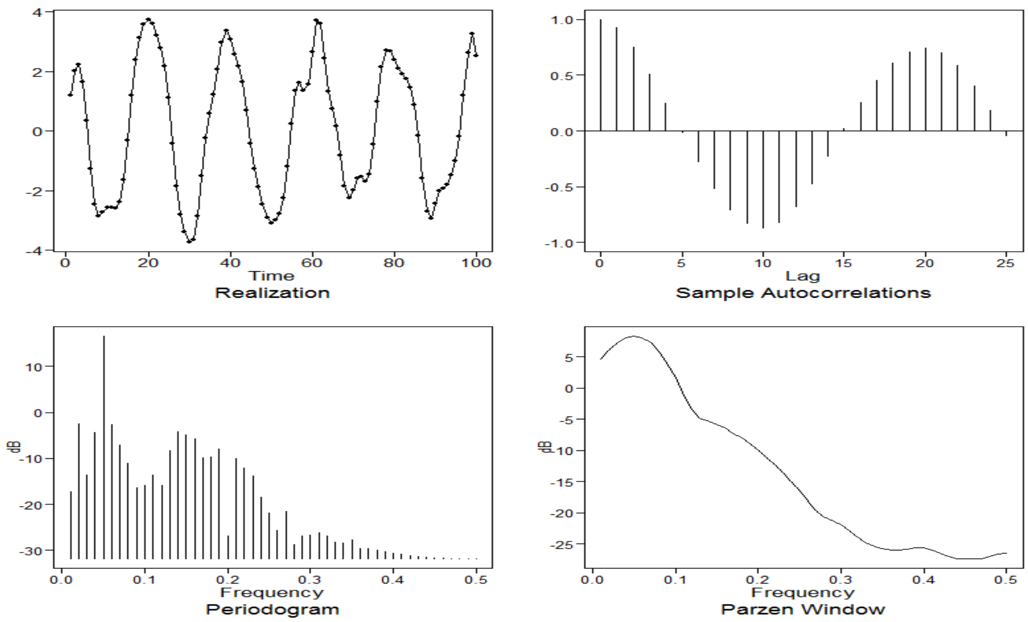
Original data set: command `plotts.sample.wge(x)`

This signal is chacterized by cyclic behavior (with period about 20) along with a high-frequency component. The sample autocorrelations primarily show the cyclic behavior woth period about 20 but may be affected slightly by the high-frequency behavior. The Parzen window (and periodogram) show two peaks at about .05 and .35.



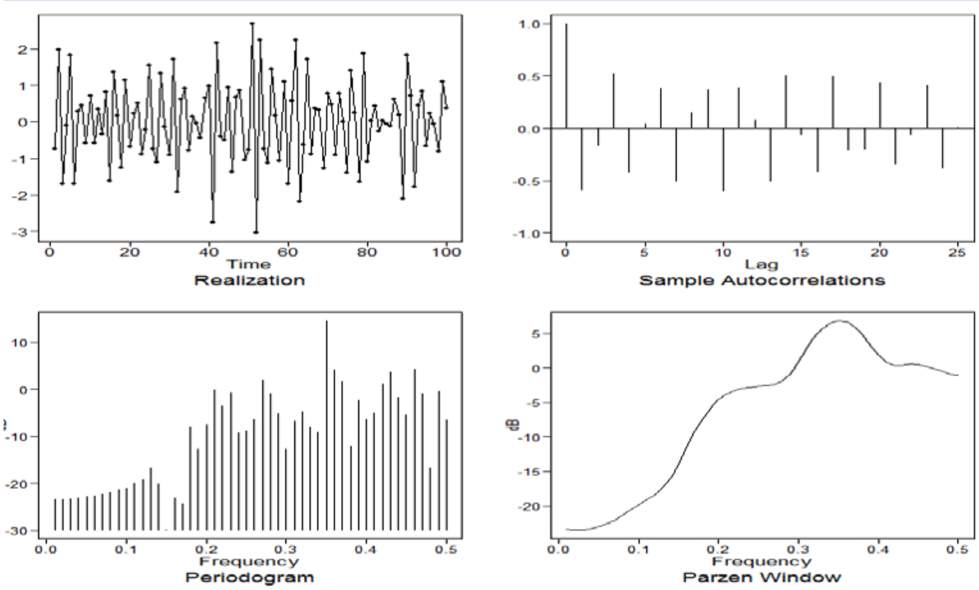
Low-pass filtered data set (`xl2`*)*: command `plotts.sample.wge(xl2)`

This signal is chacterized by a smooth cyclic behavior (with period about 20) with the high-frequency component removed. a high-frequency component. The sample autocorrelations clearly show the cyclic behavior with period about 20. The Parzen window (and periodogram) show a peak at about .05.



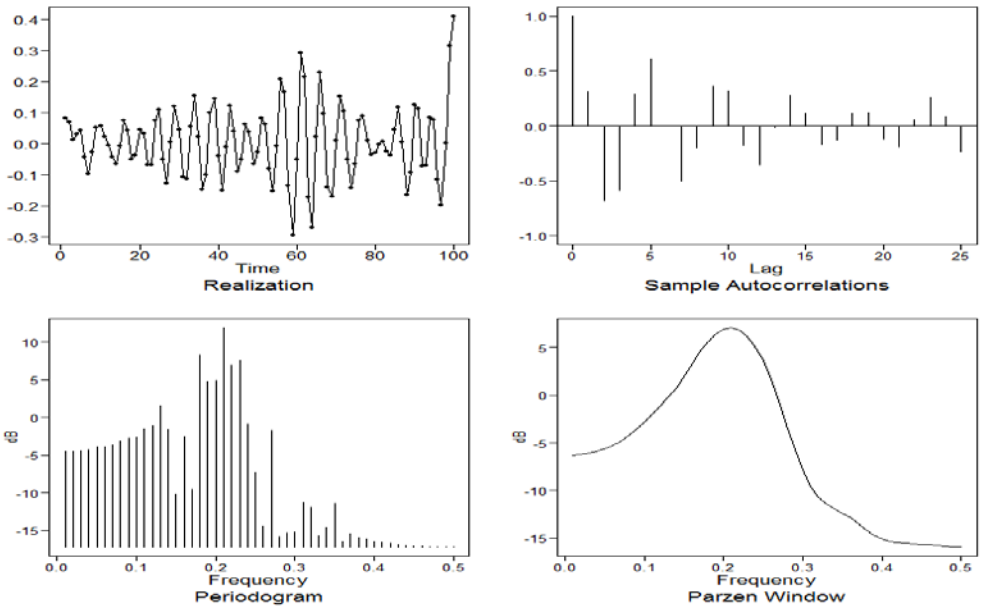
High-pass filtered data set (`xh2$x.filt`*)*: command `xh2=plotts.sample.wge(x)`

This signal is chacterized by a high-frequency (nearly up-and-down) behavior. The sample autocorrelations clearly show the cyclic behavior with period about 3. The Parzen window (and periodogram) show a peak at about .35.



Low-pass filtering the high-pass filtered data: command `xhl2-plotts.sample.wge(xh2$x.filt)`

This signal is very weak. Whereas the original signal went from -6 to 6, the low-pass data went from -4 to 4 and the high-pass data had range -2 to 2, the double filtered data set goes from about -0.4 to 0.4. It would be clearer if all plots were plotted on the same scale. While we might have thought the double-filtered data would be essentially white noise, there does seem to be soime periodic behavior with period about 4-5 as characterized by the data, sample autocorrelations sand spectrum.



Problem 2.2 (a) If x is the data set generated in Problem 2.1, then the following command produces the low-pass filtered data.

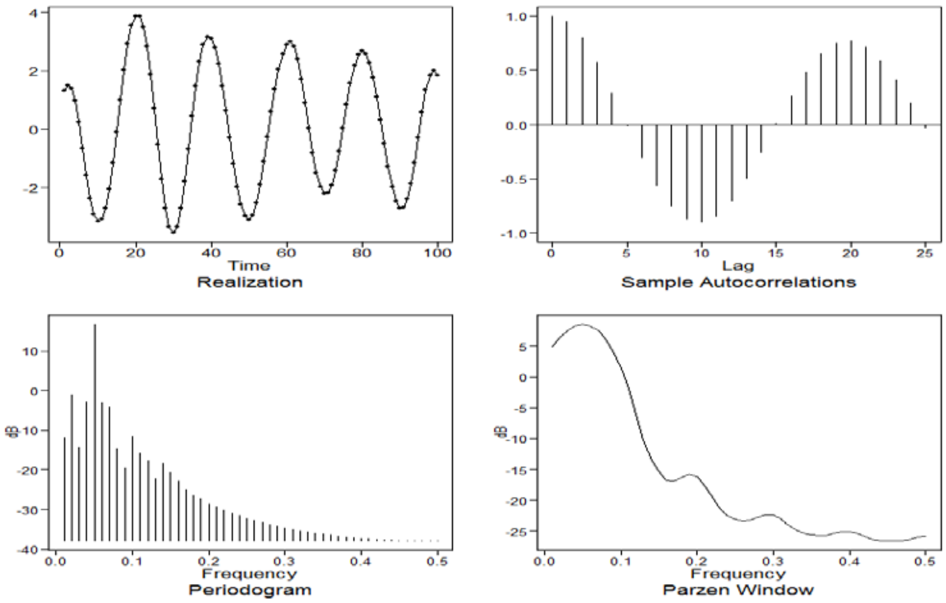
```
xl1=butterworth.wge(x,order=4,type='low',cutoff=.1)
```

(b) If x is the data set generated above, then the following command produces the high-pass filtered data.

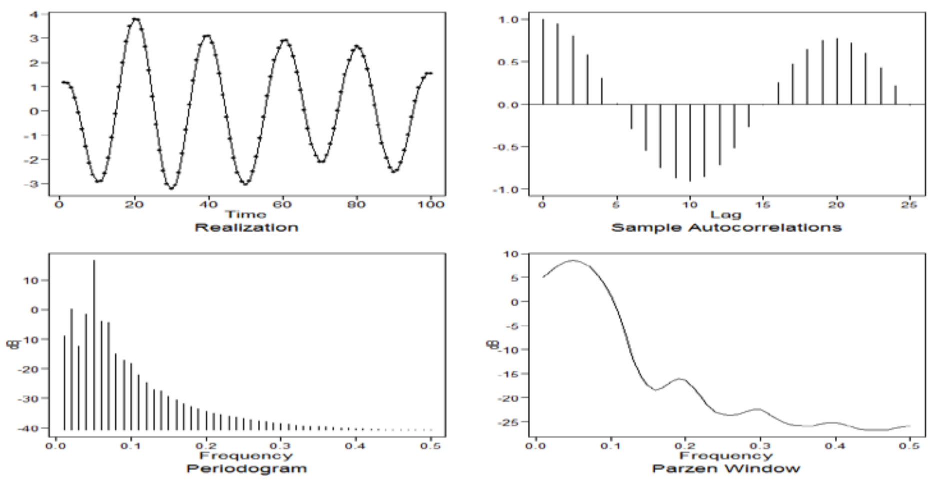
```
xl075=butterworth.wge(x,order=4,type='high',cutoff=.075)
```

The two sets of plots below are very similar to each oter. Com paring the filtered data with the for the low-pass filtered data with cutoff=.2, we see that the data with cutoffs closer to 0.05 are smoother and show no impact of the high-frequency behavior.

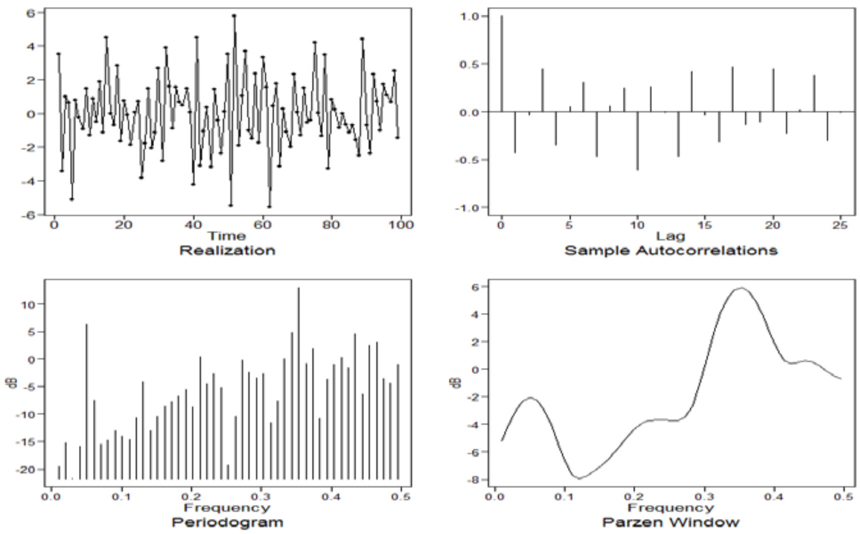
(a) command `plotts.sample.wge(xl1$x.filt)`



(b) command `plotts.sample.wge(xl075$x.filt)`



Problem 2.3 command `xdif=artrans,wge(x,phi.dif=1)`
`plotts.sample.wge(xdif)`



The fitering weakend the frequency component at *f*=0.05 and served as a high-pass filter. As Figure 2.2b shows, the filter allows some frequency behavior as low as 0.05 to leak into the filtered data. It is not as good as the Butterworth filter for filtering out one of the two signals.

Problem 2.4

2-point:

$h_0 = 1/2,\ h_{-1} = 1/2, h = 0,$ elsewhere

$$H(e^{-2\pi if}) = \sum_{j=0}^{1} h_j e^{-2\pi ifj} = 1 + 1 \times e^{-2\pi if}$$
$$= 1/2 + 1/2(\cos(2\pi f) - i\sin(2\pi f))$$
$$=$$

$$\left|H(e^{-2\pi if})\right|^2 = (1/2 + 1/2(\cos(2\pi f))^2 + \sin^2(2\pi f)$$
$$= 1/4 + 1/2\cos(2\pi f) + 1/4\cos^2(2\pi f) + 1/4\sin^2(2\pi f)$$
$$= 1/2(1 + \cos(2\pi f))$$

3-point

$h_0 = 1/3,\ h_{-1} = 1/3, h_1 = 1/3, h = 0,$ elsewhere

$$H(e^{-2\pi if}) = \sum_{j=-1}^{1} h_j e^{-2\pi ifj}$$
$$= 1/3(\cos(2\pi f + i\sin(2\pi f)) + 1/3 + 1/3(\cos(2\pi f) - i\sin(2\pi f))$$
$$= 1/3(1 + 2\cos(2\pi f)$$

$$\left|H(e^{-2\pi if})\right|^2 = 1/9(1 + 2\cos(2\pi f))^2$$

5-point

$$h_0 = h_{-1} = h_{-2} = h_1 = h_2 = 1/5, h_j = 0, \text{elsewhere}$$

$$H(e^{-2\pi i f}) = \sum_{j=-2}^{2} h_j e^{-2\pi i f j}$$

$$= 1/5\left(\cos(4\pi f + i\sin(4\pi f)) + \cos(2\pi f) + i\sin(2\pi f) + 1 + (\cos(2\pi f) - i\sin(2\pi f) + \cos(4\pi f) - i\sin(4\pi f)\right)$$

$$= 1/5(1 + 2\cos(2\pi f) + 2\cos(4\pi f))$$

$$\left|H(e^{-2\pi i f})\right|^2 = 1/9(1 + 2\cos(2\pi f) + 2\cos(4\pi f))^2$$

7-point

$$h_0 = h_{-1} = h_{-2} = h_{-3} = h_1 = h_2 = h_3 = 1/7, h_j = 0, \text{elsewhere}$$

$$H(e^{-2\pi i f}) = \sum_{j=-3}^{3} h_j e^{-2\pi i f j} =$$

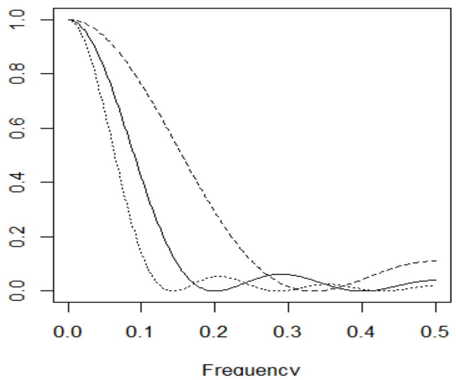
$$= 1/7(\cos(6\pi f) + i\sin(6\pi f) + \cos(4\pi f + i\sin(4\pi f)) + \cos(2\pi f) + i\sin(2\pi f) + 1 + (\cos(2\pi f) - i\sin(2\pi f) + \cos(4\pi f) - i\sin(4$$

$$+ \cos(6\pi f) - i\sin(6\pi f))$$

$$= 1/7(1 + 2\cos(2\pi f) + 2\cos(4\pi f) + 2\cos(6\pi f))$$

$$\left|H(e^{-2\pi i f})\right|^2 = 1/49(1 + 2\cos(2\pi f) + 2\cos(4\pi f) + 2\cos(6\pi f))^2$$

The desired plot of the squared frequency response for a 5-point moving average (solid line). Also shown are the 3-point (dashed line) and 7-point (dotted line). The R code to create thes plots is also shown.




```
ff=1:251
f=(ff-1)/500
H5=(1+2*cos(2*pi*f)+2*cos(4*pi*f))^2/25
plot(f,H5,type='l',xlab='Frequency')
H3=(1+2*cos(2*pi*f))^2/9
lines(f,H3,type='l',lty=2)
H7=(1+2*cos(6*pi*f)+2*cos(4*pi*f)+2*cos(2*pi*f))^2/49
lines(f,H7,type='l',lty=3)
```

Problem 2.5

By Minkowski inequality

$$E[((a_1X_t^{(m)}+b_1Y_t^{(m)})-(a_1X_t+b_1Y_t))^2]\le\left\{[E(a_1X_t^{(m)}-a_1X_t)^2]^{1/2}+[E(b_1Y_t^{(m)}-b_1Y_t)^2]^{1/2}\right\}^2$$

$$=\left\{a_1[E(X_t^{(m)}-X_t)^2]^{1/2}+[(b_1E(Y_t^{(m)}-Y_t)^2]^{1/2}\right\}^2$$

By assumption both $E(X_t^{(m)}-X_t)^2\to 0$ and $E(Y_t^{(m)}-T_t)^2\to 0$ as $m\to\infty$

So, $\lim_{m\to\infty} E[((a_1X_t^{(m)}+b_1Y_t^{(m)})-(a_1X_t+b_1Y_t))^2]$, i.e. $a_1X_t^{(m)}+b_1Y_t^{(m)})\to a_1X_t+b_1Y_t$ in mean square.

Problem 2.6

$(\Rightarrow)$ $Y=\sum_{t=1}^{\infty}X_t$ exists as a limit in mean square with $Y\in L^2$

Define $Y_t^{(m)}=\sum_{t=1}^{m}X_t$ $\qquad E[(Y_t^{(n)}-Y_t^{(m)})^2]\to 0$ as $m,n\to\infty$.

therefore, $E[\sum_{t=m+1}^{n}X_t]^2\to 0$ as $m,n\to\infty$.

Since $\{X_t\}$ is a sequence of independent r.v.'s then $E(X_iX_j)=E(X_i)E(X_j)=0, i\ne j,$ b so

$$E(\sum_{t=m+1}^{n}X_t)^2=E(\sum_{t=m+1}^{n}X_t)^2 \text{ since the } X_t\text{'s a re independent.}$$

$$=\sum_{t=m+1}^{n}\sigma_t^2\to 0 \text{ as } m,n\to\infty \text{ and consequently } \sum_{t=1}^{\infty}\sigma_t^2<\infty.$$

$(\Leftarrow)$ $\sum_{t=1}^{\infty}\sigma_t^2<\infty$ from which it follows that $\sum_{t=m+1}^{n}\sigma_t^2\to 0$ as $m,n\to\infty$. So, from above

$E[(Y_t^{(n)}-Y_t^{(m)})^2]=E[\sum_{t=m+1}^{n}X_t]^2=\sum_{t=m+1}^{n}\sigma_t^2\to 0$ as $m,n\to\infty$. Therefore, $Y_t=\sum_{t=1}^{\infty}X_t$ exists as a limit in mean square and is in L^2.

Problem 2.7 We will show it for k=2 and the proof can be completed by induction.

$$X_t=b_1X_t^1+b_2X_t^2=(b_1\sum_{j=1}^{\infty}h_j^{(1)}+b_2\sum_{j=1}^{\infty}h_i^{(2)})a_{t-j}$$

and since $(|b_1|\sum_{j=1}^{\infty}|h_j^{(1)}|+|b_2|\sum_{j=1}^{\infty}|h_i^{(2)}|)<\infty$, it follows from Theorem 2.3 that X_t is covaraiance stationary.

Problem 2.8 Since X_t is by definition a staionary process (see theorem 2.3)(a)

(a) Note that $\lim_{n\to\infty} |\sum_{k=1}^{n} h_k E(Z_{t-k})| \le \lim_{n\to\infty} \sum_{k=1}^{n} |h_k||E(Z_{t-k})| < \infty$ so the limit exists

(b) $E(X_t Y_t) = E(\sum_{k=0}^{\infty} h_k Z_{t-k})(\sum_{j=0}^{\infty} b_j Z_{t-j}) = \lim_{n\to\infty} E(\sum_{k=0}^{n} h_k Z_{t-k})(\sum_{j=0}^{n} b_j Z_{t-j})$

$= \lim_{n\to\infty} E(\sum_{k=0}^{n}\sum_{j=0}^{n} h_k b_j Z_{t-k} Z_{t-j}) \le \lim_{n\to\infty} \sum_{k=0}^{n}\sum_{j=0}^{n} |h_k||b_j| E|Z_{t-k} Z_{t-j}|$

$= \lim_{n\to\infty} \sum_{k=0}^{n}\sum_{j=0}^{n} |h_k||b_j| E|Z_{t-k} Z_{t-j}| \le \lim_{n\to\infty} \sum_{k=0}^{n} |h_k| Var(Z_{t-k})$ since $|\gamma_h| \le \gamma_0$

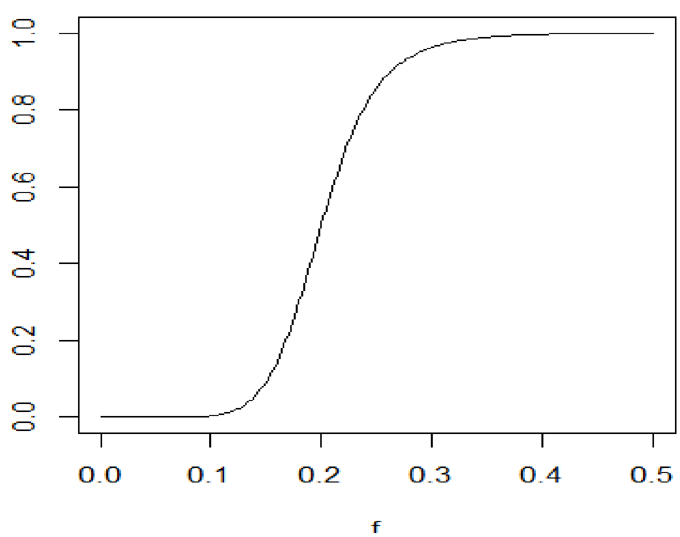
$\le \lim_{n\to\infty} \sum_{k=0}^{n} |h_k| M < \infty.$ Therefore the limit exists and $E(X_t Y_t) = \sum_{k=0}^{n}\sum_{j=0}^{n} h_k b_j E(Z_{t-k} Z_{t-j}).$

Problem 2.9 I will examine the type of filter by plotting $|H(e^{-2\pi i f})|^2$.

(a) Using the code

```
ff=1:251
f=(ff-1)/500
H2.9a=(f/.2)^8/(1+(f/.2)^8)
plot(f,h2.9a,type='l')
```

we obtain the plot



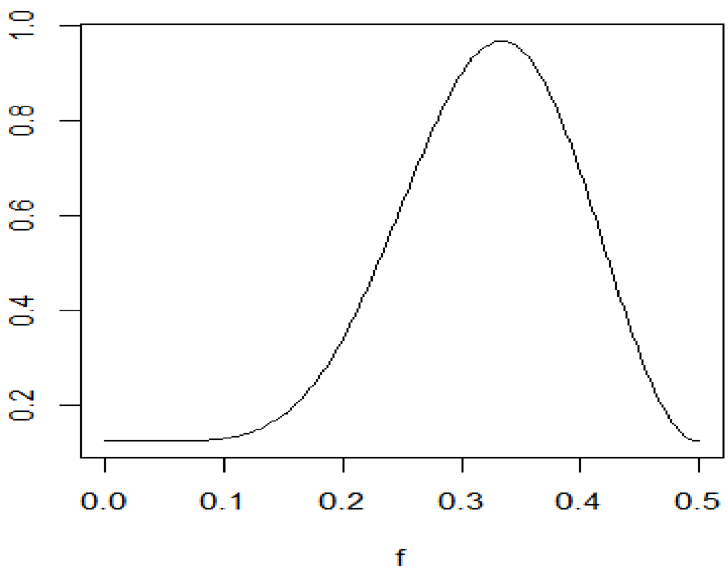
It can be seen that this is a high-pass filter with cutooff at about f=0.2.

(b) It can be seen that $|H(e^{-2\pi i f})|^2 = (1 + (2\sin(2\pi f) - \sin(4\pi f))^2)/8$

Using the code

```
ff=1:251
f=(ff-1)/500
H2.9b=(1+(2*sin(2*pi*f)-sin(4*pi*f))^2)/8
plot(f,H2.9b,type='l')
```

we obtain the plot below wich shows that the filter is band-pass, passing mostly frequencies from *f*=0.25 to *f*=0.45



CHAPTER 3 – Problem Solutions

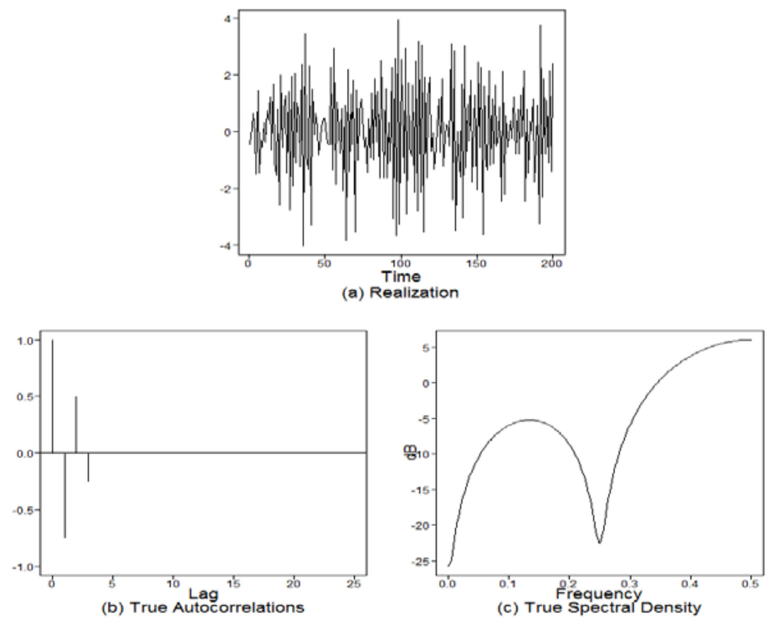
Applied Problems

Problem 3.1

(a,b,c) Using the code

```
x=plotts.true.wge(n=200,theta=c(.95,-.9,.855))
```

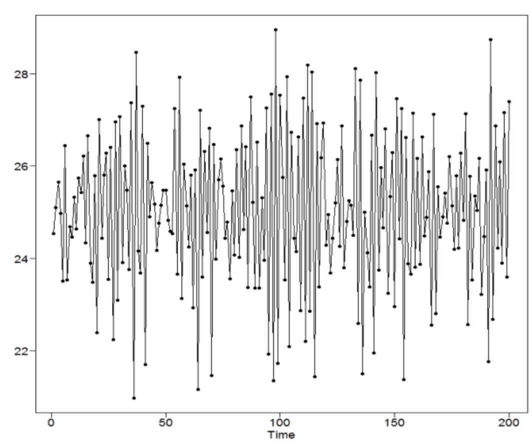
We obtained the plots



To obtain and plot a realization with mean 25 I used thecommands

```
x25=mean(x$data)
plotts.wge(x25)
```

and obtained the following is the data with $\mu = 25$. The other two plots do not change with the mean change.



The realization shows high frequency behavior with oscillation back and forth across the mean of 25. The autocorrelations show only three non-zero autocottelations. The fact that ρ_1 is fairly large negative and ρ_2 is positive, causes the up-and-down oscillatory behavior. The spactreal density shows power at *f*=0.5 which