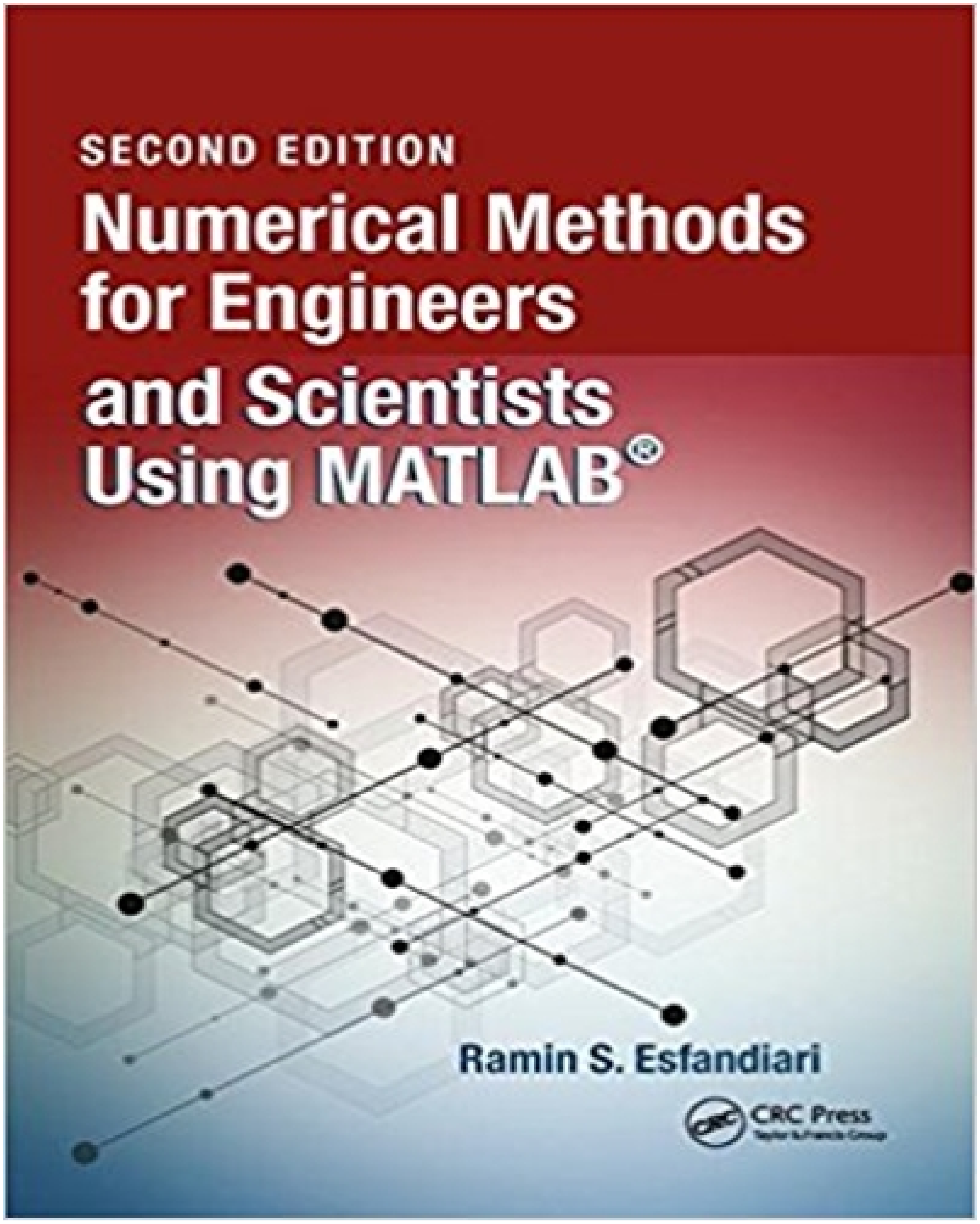
Solutions for Numerical Methods for Engineers and Scientists Using MATLAB 2nd Edition by Esfandiari



Solutions

Problem Set (Chapter 2)

All calculations must be performed in MATLAB.

1 Evaluate the function $g(x,y)=\frac{1}{2}e^{-2x/3}\tan(y+1)$ for $x=0.3, y=-0.7$

(*a*) Using the `subs` command,
(*b*) By conversion into a MATLAB function.

Solution

(*a*)
```
>> g = sym('exp(-2*x/3)*tan(y+1)/2');
>> x = 0.3; y = -0.7; subs(g)

>> double(subs(g))

ans =

    0.1266
```

(*b*)
```
>> G = matlabFunction(g);
>> G(0.3,-0.7)

ans =

    0.1266
```

2 Evaluate the function $h(x,y)=\cos(\frac{1}{3}x-1)\sin(y+\frac{1}{2})$ for $x=\frac{3}{4}, y=1$ using

(*a*) The `subs` command,
(*b*) An anonymous function.

Solution

(*a*)
```
>> h = sym('cos(x/3-1)*sin(y+1/2)');
>> x = 3/4; y = 1; double(subs(h))

ans =

    0.7299
```

(*b*)
```
>> H = @(x,y)(cos(x/3-1)*sin(y+1/2));
>> H(3/4,1)

ans =

    0.7299
```

3 Evaluate the vector function $f(x,y)=\begin{Bmatrix} x-1 \\ 2y+x \end{Bmatrix}$ for $x=2, y=\frac{2}{3}$ using

(*a*) The `subs` command,
(*b*) An anonymous function.

Solution

(*a*)
```
>> f = sym('[x-1;2*y+x]');
>> x = 2; y = 2/3; double(subs(f))
```

```
ans =

    1.0000
    3.3333
```

(*b*)
```
>> F = @(x,y)([x-1;2*y+x]);
>> F(2,2/3)

ans =

    1.0000
    3.3333
```

4 Evaluate the matrix function $f(x,y)=\begin{bmatrix}1-2x & x+y \\ 0 & \cos y\end{bmatrix}$ for $x=1, y=-1$

(*a*) Using the `subs` command,
(*b*) By conversion into a Matlab function.

Solution

(*a*)
```
>> f = sym('[1-2*x x+y;0 cos(y)]');
>> x = 1; y = -1; double(subs(f))

ans =

   -1.0000         0
         0    0.5403
```

(*b*)
```
>> F = matlabFunction(f);
>> F(1,-1)

ans =

   -1.0000         0
         0    0.5403
```

5 Consider $g(t)=t\sin(\frac{1}{2}t)+\ln(t-1)$. Evaluate dg/dt at $t=\frac{4}{3}$

(*a*) Using the `subs` command,
(*b*) By conversion into a Matlab function.

Solution

(*a*)
```
>> g = sym('t*sin(t/2)+log(t-1)');
>> dg = diff(g);
>> t = 4/3;
>> double(subs(dg))

ans =

    4.1423
```

(*b*)
```
>> dG = matlabFunction(dg);
>> dG(4/3)

ans =

    4.1423
```

6 Consider $h(x) = 3^{x-2}\sin x + \frac{2}{3}e^{1-2x}$. Evaluate dh/dx at $x = -0.3$

(*a*) Using the `subs` command,

(*b*) By conversion into a Matlab function.

Solution

(*a*)
```
>> h = sym('3^(x-2)*sin(x)+2*exp(1-2*x)/3');
>> dh = diff(h); x = -0.3; double(subs(dh))

ans =

   -6.5536
```

(*b*)
```
>> dH = matlabFunction(dh); dH(-0.3)

ans =

   -6.5536
```

7 Evaluate $\left[x^2 + e^{-a(x+1)}\right]^{1/3}$ when $a = -1$, $x = 3$ using an anonymous function in another anonymous function.

Solution

```
>> A = @(a,x)(x^2+exp(-a*(x+1)));
>> B = @(a,x)(A(a,x)^(1/3));
>> B(-1,3)

ans =

    3.9916
```

8 Evaluate $\sqrt{\left|x + \ln\left|1 - e^{(a+2)x/3}\right|\right|}$ when $a = -3$, $x = 1$ using an anonymous function in another anonymous function.

Solution

```
>> A = @(a,x)(x+2*log(abs(1-exp((a+2)*x/3))));
>> B = @(a,x)(sqrt(abs(A(a,x))));
>> B(-3,1)

ans =

    1.2334
```

In Problems 9 through 12 write a script file that employs any combination of the *flow control commands* to generate the given matrix.

9 $$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ 2 & 0 & 3 & 0 & -1 & 0 \\ 0 & 2 & 0 & 4 & 0 & -1 \\ 0 & 0 & 2 & 0 & 5 & 0 \\ 0 & 0 & 0 & 2 & 0 & 6 \end{bmatrix}$$

Solution

```
clear
clc
A = zeros(6,6);
for i = 1:6,
  for j = 1:6,
      A(i,i) = i;
    if j == i+2,
      A(i,j) = -1;
elseif i == j+2,
    A(i,j) = 2;
    end
  end
end

>> A

A =
     1     0    -1     0     0     0
     0     2     0    -1     0     0
     2     0     3     0    -1     0
     0     2     0     4     0    -1
     0     0     2     0     5     0
     0     0     0     2     0     6
```

10 $\mathbf{A} = \begin{bmatrix} 4 & 1 & -2 & 3 & 0 & 0 \\ 0 & 4 & -1 & 2 & 3 & 0 \\ 0 & 0 & 4 & 1 & -2 & 3 \\ 0 & 0 & 0 & 4 & -1 & 2 \\ 0 & 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

Solution

```
clear
clc
A = 4*eye(6);
for i = 1:6,
  for j = 1:6,
    if j == i+1,
      A(i,j) = (-1)^(i+1);
elseif j == i+2,
    A(i,j) = 2*(-1)^i;
    elseif j == i+3,
        A(i,j) = 3;
    end
  end
end

>> A

A =
     4     1    -2     3     0     0
     0     4    -1     2     3     0
     0     0     4     1    -2     3
     0     0     0     4    -1     2
     0     0     0     0     4     1
     0     0     0     0     0     4
```

11 $\mathbf{B} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 2 & 0 & 0 & 0 \\ -1 & 0 & 3 & 3 & 0 & 0 \\ 0 & 1 & 0 & -4 & 4 & 0 \\ 0 & 0 & -1 & 0 & 5 & 5 \\ 0 & 0 & 0 & 1 & 0 & -6 \end{bmatrix}$

Solution

```
clear
clc
B = zeros(6,6);
for i = 1:6,
  for j = 1:6,
      B(i,i) = (-1)^(i+1)*i;
    if j == i+1,
      B(i,j) = i;
elseif i == j+2,
    B(i,j) = (-1)^i;
    end
  end
end

>> B

B =

     1     1     0     0     0     0
     0    -2     2     0     0     0
    -1     0     3     3     0     0
     0     1     0    -4     4     0
     0     0    -1     0     5     5
     0     0     0     1     0    -6
```

12 $\mathbf{B} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -2 & 0 \\ 4 & 0 & 2 & 0 & -3 \\ 0 & 5 & 0 & -3 & 0 \\ 0 & 0 & 6 & 0 & 4 \end{bmatrix}$

Solution

```
clear
clc
B = zeros(5,5);
for i = 1:5,
  for j = 1:5,
      B(i,i) = (-1)^(i+1)*(i-1);
    if j == i+2,
      B(i,j) = -i;
elseif i == j+2,
    B(i,j) = i+1;
    end
  end
end
```

```
>> B

B =

     0     0    -1     0     0
     0    -1     0    -2     0
     4     0     2     0    -3
     0     5     0    -3     0
     0     0     6     0     4
```

13 Using any combination of commands `diag`, `triu` and `tril`, construct matrix $\mathbf{B}$ from $\mathbf{A}$.

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 3 & 0 & 4 & 1 \\ 1 & 5 & -1 & 3 \\ 0 & 2 & 6 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 1 & 5 & 0 & 0 \\ 0 & 2 & 6 & 0 \end{bmatrix}$$

Solution

```
>> A = [2 1 -1 2;3 0 4 1;1 5 -1 3;0 2 6 1]

A =

     2     1    -1     2
     3     0     4     1
     1     5    -1     3
     0     2     6     1

>> D = diag(diag(A))

D =

     2     0     0     0
     0     0     0     0
     0     0    -1     0
     0     0     0     1

>> B = tril(A) - D

B =

     0     0     0     0
     3     0     0     0
     1     5     0     0
     0     2     6     0
```

14 Using any combination of commands `diag`, `triu` and `tril`, construct matrix $\mathbf{B}$ from $\mathbf{A}$.

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 3 & 0 & 4 & 1 \\ 1 & 5 & -1 & 3 \\ 0 & 2 & 6 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 1 & -1 & 2 \\ 3 & 0 & 4 & 1 \\ 0 & 5 & 0 & 3 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$

Solution

```
>> A = [2 1 -1 2;3 0 4 1;1 5 -1 3;0 2 6 1];
>> D = diag(diag(A)); C = triu(A) - D;
>> B = C + diag(diag(A,-1),-1)
```

```
B =

     0     1    -1     2
     3     0     4     1
     0     5     0     3
     0     0     6     0
```

15 Plot $\int_1^t e^{t-2x} \sin x dx$ versus $-1 \le t \le 1$, add grid and label.

Solution

```
>> syms t x
>> integ = int(exp(t-2*x)*sin(x),x,1,t);
>> ezplot(integ,[-1,1])
```

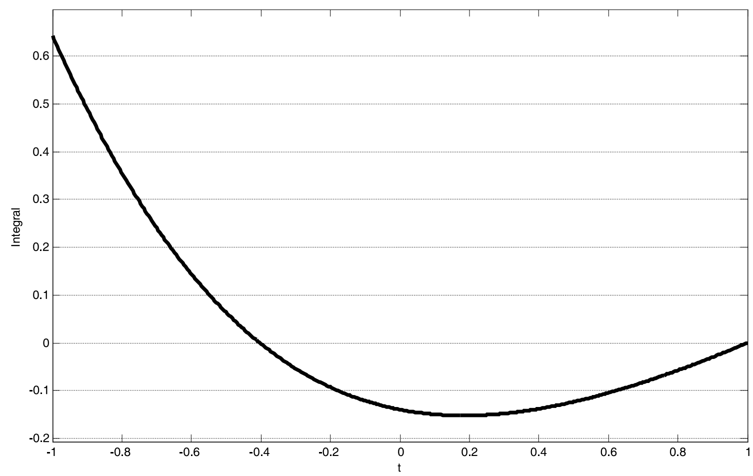


Figure Solution of Problem 15

16 Plot $\int_0^t (x+t)^2 e^{-(t-x)} dx$ versus $-2 \le t \le 1$, add grid and label.

Solution

```
>> syms x t
>> integ = int((x+t)^2*exp(-(t-x)),x,0,t);
>> ezplot(integ,[-2,1])
```

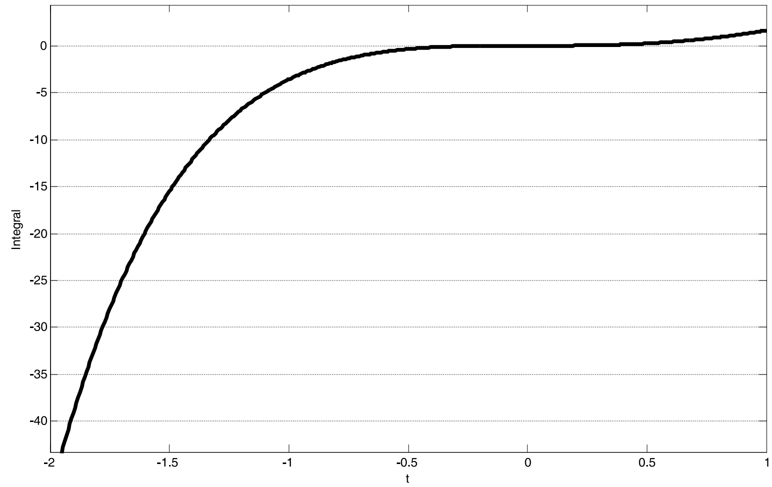


Figure Solution of Problem 16

17 Plot $y_1 = \frac{1}{3}e^{-t}\sin\left(t\sqrt{2}\right)$ and $y_2 = e^{-t/2}$ versus $0 \le t \le 5$ in the same graph. Add grid, and label.

Solution

```
syms t
y1 = sym('(1/3)*exp(-t)*sin(sqrt(2)*t)');
y2 = sym('exp(-t/2)');
ezplot(y1,[0,5])
hold on
ezplot(y2,[0,5])
```

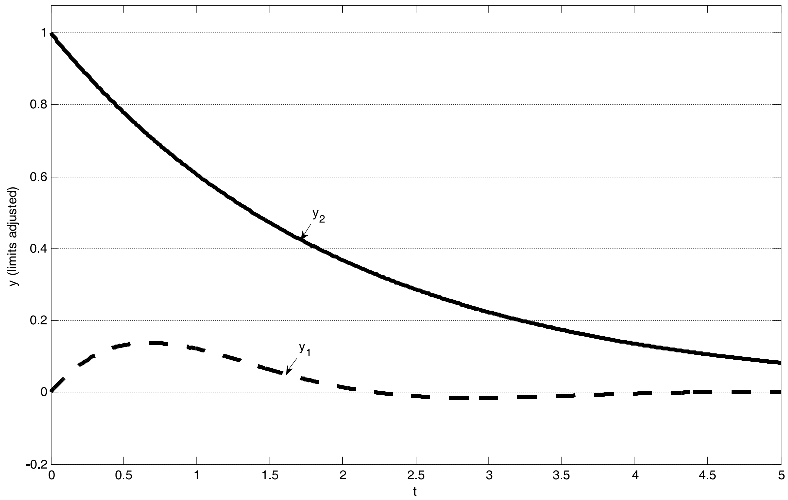


Figure Solution of Problem 17

18 Generate 100 points for each of the two functions in Problem 17 and plot versus $0 \le t \le 5$ in the same graph. Add grid, and label.

Solution

```
y1 = @(t)((1/3)*exp(-t)*sin(sqrt(2)*t));
y2 = @(t)(exp(-t/2));
t = linspace(0,5);
for i = 1:100,
    yy1(i) = y1(t(i));
    yy2(i) = y2(t(i));
end
plot(t,yy1,t,yy2)
```

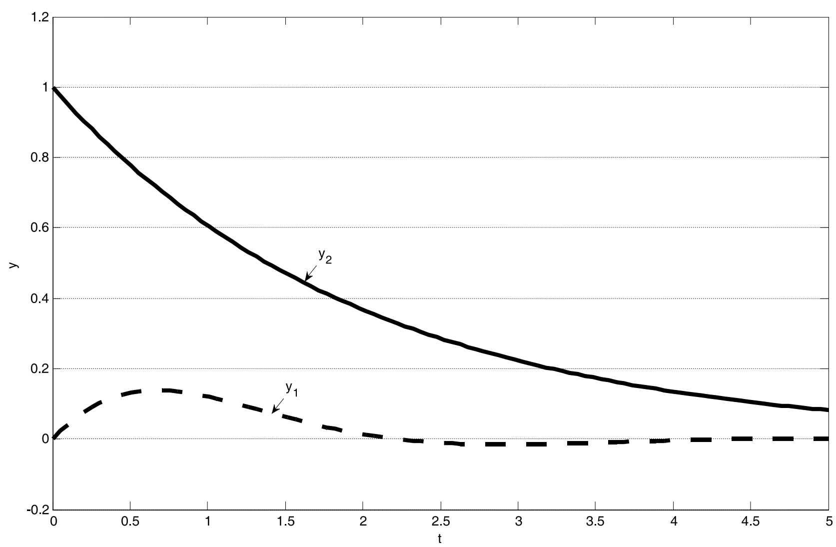


Figure Solution of Problem 18

19 Evaluate $\int_0^\infty \frac{\sin\omega}{\omega} d\omega$.

Solution
```
>> syms w
>> int(sin(w)/w,w,0,inf)

ans =

pi/2
```

20 Plot $u(x,t) = \cos(1.7x)\sin(3.2t)$ versus $0 \le x \le 5$ for four values of $t = 1, 1.5, 2, 2.5$ in a 2×2 tile. Add grid and title.

Solution
```
x = linspace(0,5); t = 1:0.5:2.5;
for i = 1:4,
    for j = 1:100,
    u(j,i) = cos(1.7*x(j))*sin(3.2*t(i));    % Generate 100 values of u for each t
    end
end

% Initiate figure
subplot(2,2,1), plot(x,u(:,1)), title('t = 1')
subplot(2,2,2), plot(x,u(:,2)), title('t = 1.5')
subplot(2,2,3), plot(x,u(:,3)), title('t = 2')
subplot(2,2,4), plot(x,u(:,4)), title('t = 2.5')
```

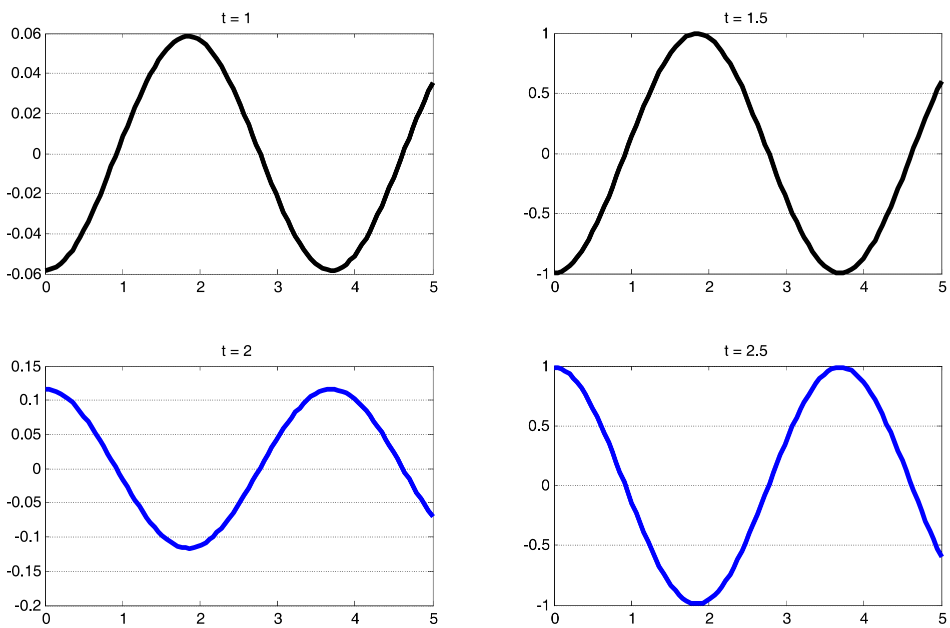


Figure Solution of Problem 20

21 Plot $u(x,t)=(1-\sin x)e^{-(t+1)}$ versus $0\le x\le 5$ for two values of $t=1,3$ in a 1×2 tile. Add grid and title.

Solution

```
x = linspace(0,5); t = [1,3];
for i = 1:2,
    for j = 1:100,
    u(j,i) = (1-sin(x(j)))*exp(-(t(i)+1));    % Generate 100 values of u for each t
    end
end

% Initiate figure
subplot(1,2,1), plot(x,u(:,1)), title('t = 1')
subplot(1,2,2), plot(x,u(:,2)), title('t = 3')
```

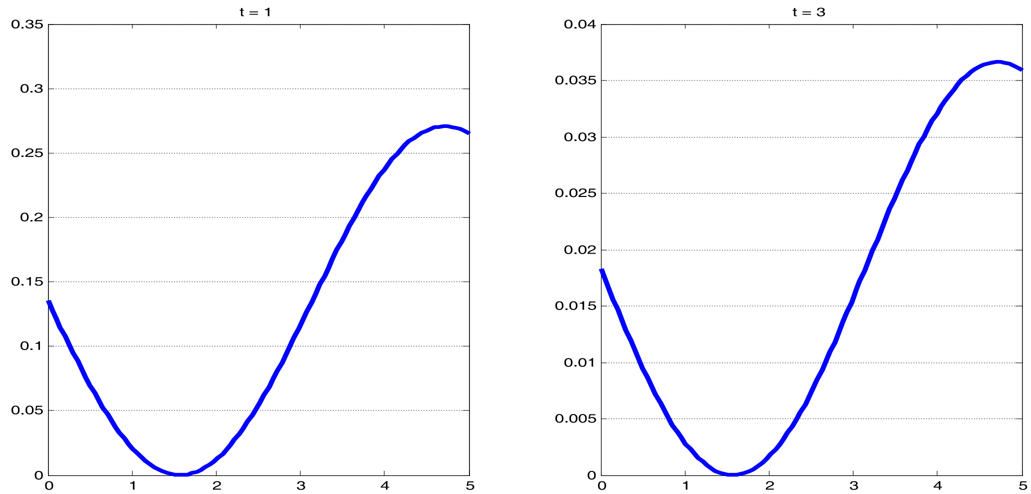


Figure Solution of Problem 21

22 Given that $f(x)=e^{-2x}+\cos(x+1)$, plot $f'(x)$ versus $0\le x\le 8$.

Solution

```
>> f = sym('exp(-2*x)+cos(x+1)');
>> df = matlabFunction(diff(f));
>> ezplot(df,[0,8])
```

Figure Solution of Problem 22

23 Write a user-defined function with function call `val = f_eval(f,a,b)` where f is an anonymous function, and a and b are constants such that $a < b$. The function calculates the midpoint m of the interval $[a,b]$ and returns the value of $f(a)+\frac{1}{2}f(m)+f(b)$. Execute `f_eval` for $f = e^{-x/3}$, $a = -4$, $b = 2$.

Solution

```
function val = f_eval(f,a,b)
m = (a + b)/2;
val = f(a) + f(m)/2 + f(b);
```

```
>> f = @(x)(exp(-x/3));
>> val = f_eval(f,-4,2)

val =

    5.0049
```

24 Write a user-defined function with function call `m = mid_seq(a,b,tol)` where a and b are constants such that $a < b$, and `tol` is a specified tolerance. The function first calculates the midpoint m_1 of the interval $[a,b]$, then the midpoint m_2 of $[a,m_1]$, then the midpoint m_3 of $[a,m_2]$, and so on. The process terminates when two successive midpoints are within `tol` of each other. Allow a maximum of 20 iterations. The output of the function is the sequence $m_1, m_2, m_3, \ldots$. Execute the function for $a = -4$, $b = 10$, $\text{tol} = 10^{-3}$.

Solution

```
function m = mid_seq(a,b,tol)
m = zeros(20);    % Pre-allocate
m(1) = (a + b)/2;
for i = 2:20,
    m(i) = (a + m(i-1))/2;
    if abs(m(i) - m(i-1)) < tol,
        break
    end
end
```

```
>> m = mid_seq(-4,10,1e-3)

m =

  Columns 1 through 8

    3.0000   -0.5000   -2.2500   -3.1250   -3.5625   -3.7813   -3.8906   -3.9453

  Columns 9 through 14

   -3.9727   -3.9863   -3.9932   -3.9966   -3.9983   -3.9991
```

25 Write a user-defined function with function call `C = temp_conv(F)` where F is temperature in Fahrenheit, and C is the corresponding temperature in Celsius. Execute the function for $F = 87$.

Solution

```
function C = temp_conv(F)
C = (F-32)*100/180;
```

```
>> C = temp_conv(87)

C =
   30.5556
```

26 Write a user-defined function with function call `P = partial_eval(f,a)` where f is a function defined symbolically, and a is a constant. The function returns the value of $f' + f''$ at $x = a$. Execute the function for $f = 3x^2 - e^{x/3}$, and $a = 1$.

Solution

```
function P = partial_eval(f,a)
del = diff(f,'x') + diff(f,2,'x');
x = a; P = double(subs(del));
```

```
>> f = sym('3*x^2-exp(x/3)'); P = partial_eval(f,1)

P =
   11.3797
```

27 Write a user-defined function with function call `P = partial_eval2(f,g,a)` where f and g are functions defined symbolically, and a is a constant. The function returns the value of $f' + g'$ at $x = a$. Execute the function for $f = x^2 + e^{-x}$, $g = \sin(0.3x)$, and $a = 0.8$.

Solution

```
function P = partial_eval2(f,g,a)
del = diff(f,'x') + diff(g,'x');
x = a; P = double(subs(del));
```

```
>> f = sym('x^2+exp(-x)'); g = sym('sin(0.3*x)'); P = partial_eval2(f,g,0.8)

P =
    1.4421
```

28 Write a user-defined function with function call `[r, k] = root_finder(f,x0,kmax,tol)` where `f` is an anonymous function, `x0` is a specified value, `kmax` is the maximum number of iterations, and `tol` is a specified tolerance. The function sets $x_1 = x_0$, calculates $|f(x_1)|$, and if it is less than the tolerance, then x_1 approximates the root r. If not, it will increment x_1 by 0.01 to obtain x_2, repeat the procedure, and so on. The process terminates as soon as $|f(x_k)| < \text{tol}$ for some k. The outputs of the function are the approximate root and the number of iterations it took to find it. Execute the function for $f(x) = x^2 - 3.3x + 2.1$, $x_0 = 0.5$, $\text{kmax} = 50$ $\text{tol} = 10^{-2}$.

Solution

```
function [r, k] = root_finder(f,x0,kmax,tol)
x(1) = x0;
if abs(f(x(1)))<tol,
    r = x(1);
end
for k = 2:kmax,
    x(k) = x(k-1) + 0.01;
    if abs(f(x(k)))<tol,
        r = x(k); break, end
end
```

```
>> f = @(x)(x^2-3.3*x+2.1); [r, k] = root_finder(f,0.5,50,1e-2)

r =
    0.8600


k =
    37
```

29 Repeat Problem 28 for $f(x) = 3 + \ln(2x-1) - e^x$, $x_0 = 1$, $\text{kmax} = 25$ $\text{tol} = 10^{-2}$.

Solution

```
function [r, k] = root_finder(f,x0,kmax,tol)
x(1) = x0;
if abs(f(x(1)))<tol,
    r = x(1);
end
for k = 2:kmax,
    x(k) = x(k-1) + 0.01;
    if abs(f(x(k)))<tol,
        r = x(k); break, end
end
```

```
>> f = @(x)(3+log(2*x-1)-exp(x));
>> [r, k] = root_finder(f,1,25,1e-2)

r =
    1.2100

k =
    22
```

30 Write a user-defined function with function call `[opt, k] = opt_finder(fp,x0,kmax,tol)` where fp is the derivative (as a MATLAB function) of a given function f, `x0` is a specified value, `kmax` is the maximum number of iterations, and `tol` is a specified tolerance. The function sets $x_1 = x_0$, calculates $|\text{fp}(x_1)|$, and if it is less than the tolerance, then x_1 approximates the critical point `opt` at which the derivative is near zero. If not, it will increment x_1 by 0.1 to obtain x_2, repeat the procedure, and so on. The process terminates as soon as $\left|\text{fp}\left(x_k\right)\right| < \text{tol}$ for some k. The outputs are the approximate optimal point and the number of iterations it took to find it. Execute the function for $f(x) = x + (x-2)^2$, $x_0 = 1$, $\text{kmax} = 50$ $\text{tol} = 10^{-3}$.

Solution

```
function [opt, k] = opt_finder(fp,x0,kmax,tol)
x = zeros(kmax);  % Pre-allocate
x(1) = x0;
if abs(fp(x(1)))<tol,
    opt = x(1);
end
for k = 2:kmax,
    x(k) = x(k-1) + 0.1;
    if abs(fp(x(k)))<tol,
        opt = x(k); break, end
end
```

```
>> f = sym('x + (x-2)^2');
>> fp = matlabFunction(diff(f));
>> [opt, k] = opt_finder(fp,1,50,1e-3)

opt =

    1.5000

k =
     6
```