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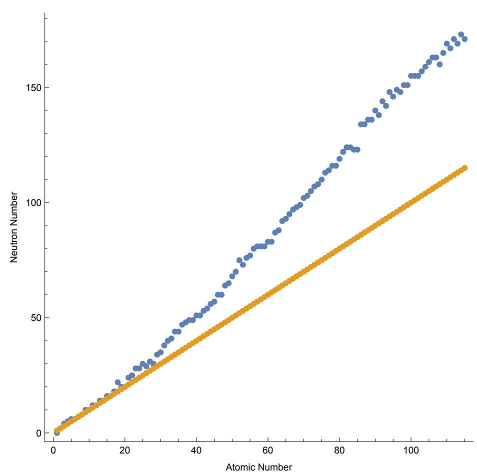
Chapter 1

Interactions and Motion

1.1-Q-01



By inspection, you can see that the number of neutrons increases faster as the atomic number increases.

1.2-Q-01

B, C, D, E, and F show evidence of an interaction. In the case of B, speed changes (and therefore velocity changes). In the case of C through F, direction of motion changes (and therefore so does velocity). In the cases of A, velocity is constant and therefore no net interaction is indicated.

1.2-Q-02

Here is a qualitative description of the diagram. During the first 4 minutes, the dots are evenly spaced since the car's speed is constant. During the next 4 minutes, the dots are successively farther apart since the car's speed increases during each minute. During the next 4 minutes, the dots are evenly spaced (approximately twice as far apart as during the first 4 minutes) since the car's speed is now constant once again (but a different constant than before). During the last 4 minutes, the dots are successively closer together since the car's speed is decreasing. The dots must get closer together faster than they got farther apart when the car first accelerated because the speed is decreasing at a greater rate than it increased before.

1.3-Q-01

Reasons 1, 3, and 4 are true. Reason 2 is irrelevant. Reason 5 is correct only if one assumes that the spaceship is indeed effectively infinitely far away from all other sources of gravitational attraction and is thus really only an approximation, but a very good approximation.

1.3-Q-02

Observers 2, 4 may see something that appears to violate Newton's first law because they are in reference frames that are accelerating relative to Earth. These are not inertial reference frames, and Newton's first law doesn't hold for such noninertial frames. Observers 1, 3, and 5 have constant velocity (magnitude and direction, relative to Earth) and are thus in inertial reference frames so they will see Newton's first law as not being violated.

1.3-Q-03

While you are walking and holding the book, the ball moves with a constant velocity (relative to an observer who is standing at rest). When you stop, the ball continues moving with a constant velocity as it rolls across the book because there is no net force on the ball to change its velocity, until it rolls off the book and then the net force on the ball is the gravitational force by Earth which changes its velocity as it falls.

1.3-Q-04

Because nothing interacts with the spaceship, it will continue in a straight line and at a constant speed of $1 \times 10^4\,\mathrm{m/s}$.

1.4-Q-01

a, c, and d are vectors. b is a scalar.

1.9-Q-01

Statements 1 and 5 are correct. Statements 2, 3, and 4 are incorrect.

1.10-Q-01

(a) γ is a scalar quantity.

(b) The minimum possible value of γ is 1.

(c) The minimum value is reached when the object's speed is low, specifically when it is zero.

(d) There is no maximum value for γ.

(e) γ becomes large when an object's speed is high.

(f) The approximation $\gamma \approx 1$ applies when an object's speed is low.

1.10-Q-02

The approximate formula for momentum may be used for (1), (2), (3) and (5) because in all of these cases, the object or particle is moving with a speed much less than 3×10^8 m/s. In case (5), the electron's speed is one-hundredth the speed of light. If a highly precise calculation is not needed, then even in this case, the approximate formula for momentum may be used. As a rule of thumb, if an object's speed is less than about 10% of the speed of light, then the approximate formula may be used, except in cases where high precision (i.e. many significant figures) is needed.

1.4-P-01

Add the vector components.

$$\begin{aligned} \vec{v}_1 + \vec{v}_2 &= \langle 8, 12, -7 \rangle \text{ m} + \langle -4, 0, 6 \rangle \text{ m} \\ &= \langle 8 + -4, 12 + 0, -7 + 6 \rangle \text{ m} \\ &= \langle 4, 12, -1 \rangle \text{m} \end{aligned}$$

1.4-P-02

(a) The magnitude of a vector is indicated by the length of the arrow representing the vector. The arrows that have the same magnitude as $\vec{a}$ have the same length as $\vec{a}$. Counting gridlines shows that $|\vec{a}| = 10$ units (Note that we don't know what the unit is, and it doesn't matter for answering this question.). So $\vec{b}$, $\vec{c}$, $\vec{d}$, $\vec{e}$, and $\vec{f}$ have the same magnitude as $\vec{a}$. You'll need to use the Pythagorean theorem to prove this for $\vec{b}$ and $\vec{d}$.

(b) Equal vectors must have both the same magnitude and the same direction. So $\vec{a}$, $\vec{c}$, and $\vec{f}$ are the only ones meeting these criteria.

1.4-P-03

$$\begin{aligned} |\vec{v}| &= \sqrt{v_x^2 + v_y^2 + v_z^2} \\ &= \sqrt{\left(8 \times 10^6\right)^2 + (0)^2 + \left(-2 \times 10^7\right)^2} \text{ m/s} \\ &= 2.15 \times 10^7 \text{ m/s} \end{aligned}$$

1.4-P-04

$$\begin{aligned}
\vec{a} &= \langle 5, 3, 0 \rangle \text{ m} \\
\vec{b} &= \langle 6, -9, 0 \rangle \text{ m} \\
\vec{c} &= \langle -10, 3, 0 \rangle \text{ m} \\
|\vec{a}| &= \sqrt{a_x^2 + a_y^2 + a_z^2} \\
&= \sqrt{(5)^2 + (3)^2 + (0)^2} \text{ m} = 5.83 \text{ m} \\
\left|\vec{b}\right| &= \sqrt{b_x^2 + b_y^2 + b_z^2} \\
&= \sqrt{(6)^2 + (-9)^2 + (0)^2} \text{ m} = 10.8 \text{ m} \\
|\vec{c}| &= \sqrt{c_x^2 + c_y^2 + c_z^2} \\
&= \sqrt{(-10)^2 + (3)^2 + (0)^2} \text{ m} = 10.4 \text{ m}
\end{aligned}$$

1.4-P-05

Extract components by counting gridlines.

(a) $\vec{a} = \langle -4, -3, 0 \rangle$

(b) $\vec{b} = \langle -4, -3, 0 \rangle$

(c) The statement is true. $\vec{a}$ and $\vec{b}$ have the same components, so the two vectors must be equivalent.

(d) $\vec{c} = \langle 4, 3, 0 \rangle$

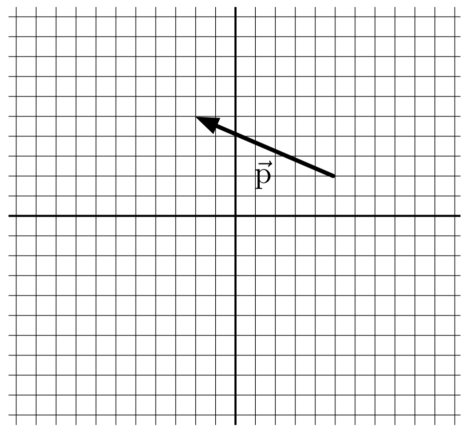
(e) The statement is true. Each component of $\vec{c}$ is the opposite of the corresponding component of $\vec{a}$ so the actual vectors are opposites.

(f) $\vec{d} = \langle -3, 4, 0 \rangle$

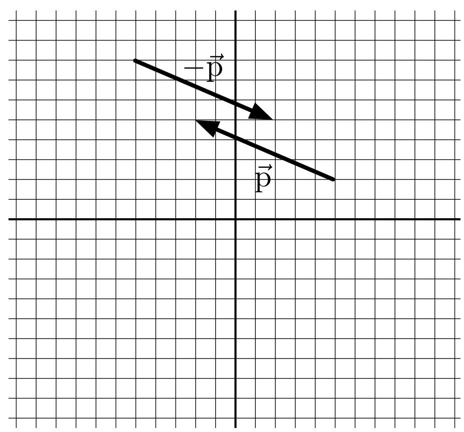
(g) The statement is false because corresponding components of $\vec{c}$ and $\vec{d}$ are not opposites.

1.4-P-06

(a) See drawing.



(b) See drawing.



1.4-P-07

$$
\begin{aligned}
f\,\vec{a} &= (2.0)\,\langle 0.02, -1.7, 30.0\rangle \\
&= \langle 0.04, -3.4, 60.0\rangle
\end{aligned}
$$

1.4-P-08

(a) $\vec{d} = \langle -6, 3, 2\rangle$ m

(b) $\vec{e} = -\vec{d} = -\langle -6, 3, 2\rangle\ \text{m} = \langle +6, -3, -2\rangle$ m

(c) Take the position of the vector's tail and add the vector $\vec{d}$. $\langle -5, -2, 4\rangle\ \text{m} + \langle -6, 3, 2\rangle\ \text{m} = \langle -11, 1, 6\rangle$ m

(d) Take the position of the vector's tail and add the vector $-\vec{d}$. $\langle -1, -1, -1\rangle\ \text{m} + (-\langle -6, 3, 2\rangle\ \text{m}) = \langle 5, -4, -3\rangle$ m

1.4-P-09

Call $\hat{n}$ the direction of an arbitrary vector, then for the first vector we have

$$\begin{aligned}\hat{n} &= \frac{\langle 2,2,2\rangle}{\sqrt{(2)^2+(2)^2+(2)^2}}\\ &= \frac{\langle 2,2,2\rangle}{\sqrt{12}}\\ &= \left\langle \frac{2}{\sqrt{12}}, \frac{2}{\sqrt{12}}, \frac{2}{\sqrt{12}} \right\rangle\\ &\approx \langle 0.58, 0.58, 0.58\rangle\end{aligned}$$

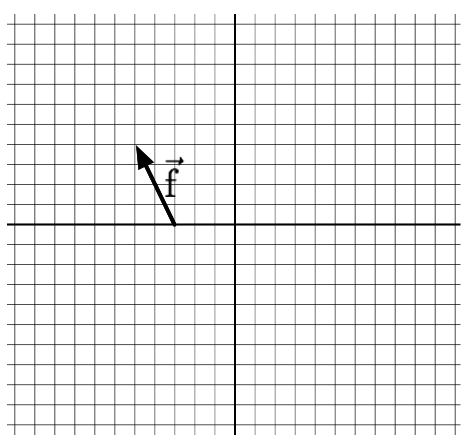
and for the second vector we have the following.

$$\begin{aligned}\hat{n} &= \frac{\langle 3,3,3\rangle}{\sqrt{(3)^2+(3)^2+(3)^2}}\\ &= \frac{\langle 3,3,3\rangle}{\sqrt{27}}\\ &= \left\langle \frac{3}{\sqrt{27}}, \frac{3}{\sqrt{27}}, \frac{3}{\sqrt{27}} \right\rangle\\ &\approx \langle 0.58, 0.58, 0.58\rangle\end{aligned}$$

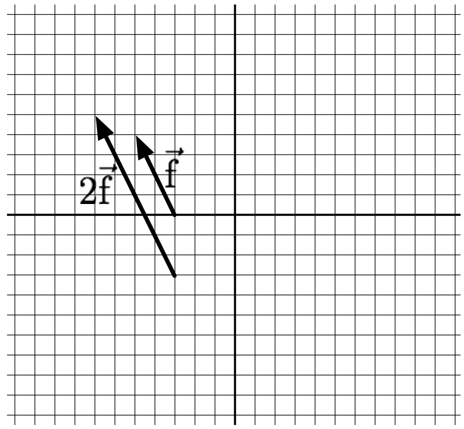
These directions are the same! How can that be? They're the same because one vector is a multiple of the other. $\langle 3,3,3\rangle = \frac{3}{2}\langle 2,2,2\rangle$. Of course you could also write $\langle 2,2,2\rangle = \frac{2}{3}\langle 3,3,3\rangle$. When two vectors are multiples of each other, their directions must be either parallel (if related by a positive multiple) or opposite (if related by a negative multiple).

1.4-P-10

(a) See figure.



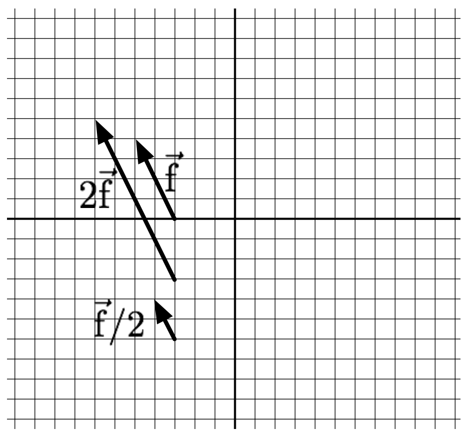
(b) See figure.



(c) The magnitude of $2\vec{f}$ will be twice the magnitude of $\vec{f}$.

(d) The direction of $2\vec{f}$ is the same as that of $\vec{f}$.

(e) See figure.



(f) The magnitude of $\vec{f}/2$ is half that of $\vec{f}$.

(g) The direction of $\vec{f}/2$ is the same as that of $\vec{f}$.

(h) Yes, multiplying a vector by a scalar changes the magnitude, assuming the scalar is neither 0 nor ± 1.

(i)

$$a\vec{f} = -3\vec{f}$$
$$\therefore a = -3$$

Note that you must not attempt to solve for a by dividing both sides by $\vec{f}$ because dividing by a vector is not defined. Instead, what you are really doing here is solving the equation by visual inspection. You may have never thought of this as a legitimate way of solving an equation, but this is a vector equation and the rules of ordinary algebra do not always apply to vector equations. Until you learn how to correctly solve vector equations using the rules of vector algebra (hopefully your instructor will show you), visual inspection is a perfectly legitimate way of solving them.

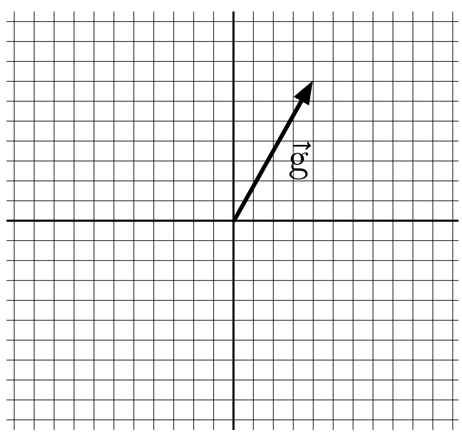
1.4-P-11

The concept of writing a vector as a magnitude multiplying a direction is important and will appear many times in later chapters. It also forces you to think about each part, magnitude and direction, individually.

$$\vec{a} = |\vec{a}| \cdot \hat{a}$$
$$|\vec{a}| = \sqrt{\left(400\ \text{m/s}^2\right)^2 + \left(200\ \text{m/s}^2\right)^2 + \left(-100\ \text{m/s}^2\right)^2}$$
$$|\vec{a}| = 458.3\,\text{m/s}^2$$
$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$
$$= \frac{\langle 400, 200, -100\rangle\,\text{m/s}^2}{458.3\,\text{m/s}^2}$$
$$\therefore \vec{a} = 458.3\,\text{m/s}^2\,\langle 0.873, 0.436, -0.218\rangle$$

1.4-P-12

(a) See figure.



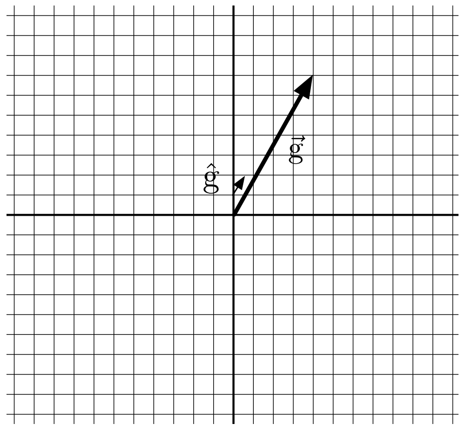
(b)

$$|\vec{g}| = \sqrt{(4)^2 + (7)^2 + (0)^2}\,\text{m}$$
$$= 8.06\,\text{m}$$

(c)

$$\hat{g} = \frac{\langle 4, 7, 0\rangle\,\text{m}}{|\vec{a}|}$$
$$= \frac{\langle 4, 7, 0\rangle\,\text{m}}{8.06\,\text{m}}$$
$$= \langle 0.496, 0.868, 0\rangle$$

(d) See figure.



(e)

$$\begin{aligned}|\vec{g}|\,\hat{g} &= 8.06\,\text{m}\,\langle 0.496, 0.868, 0\rangle \\ &= \langle 4, 7, 0\rangle\,\text{m}\end{aligned}$$

1.4-P-13

(a)

$$\vec{r} = \left\langle 3\times10^{-10}, -3\times10^{-10}, 8\times10^{-10}\right\rangle\,\text{m}$$

(b)

$$\begin{aligned}|\vec{r}| &= \sqrt{\left(3\times10^{-10}\right)^2 + \left(-3\times10^{-10}\right)^2 + \left(8\times10^{-10}\right)^2}\,\text{m} \\ &= 9.1\times10^{-10}\,\text{m}\end{aligned}$$

(c)

$$\begin{aligned}\hat{r} &= \frac{\vec{r}}{|\vec{r}|} \\ &= \frac{\left\langle 3\times10^{-10}, -3\times10^{-10}, 8\times10^{-10}\right\rangle\,\text{m}}{9.1\times10^{-10}\,\text{m}} \\ &= \langle 0.33, -0.33, 0.88\rangle\end{aligned}$$

1.4-P-14

(a)

$$\begin{aligned}\vec{r}_{21} &= \vec{r}_2 - \vec{r}_1 \\ &= \langle 5, 2, 0\rangle\,\text{m} - \langle 3, -2, 0\rangle\,\text{m} \\ &= \langle 2, 4, 0\rangle\,\text{m}\end{aligned}$$

(b)

$$\begin{aligned}\vec{r}_{12} &= \vec{r}_1 - \vec{r}_2 \\ &= \langle 3, -2, 0\rangle\,\text{m} - \langle 5, 2, 0\rangle\,\text{m} \\ &= \langle -2, -4, 0\rangle\,\text{m}\end{aligned}$$

Note that $\vec{r}_{21} = -\vec{r}_{12}$.

1.4-P-15

One way of thinking about the arrow representation of a vector is that the components tell you how to get from the tail to the head. This is equivalent to the position of the head relative to the position of the tail.

(a)

$$\langle 4, -13, 0\rangle\,\text{m} - \langle 9.5, 7, 0\rangle\,\text{m} = \langle -5.5, -20, 0\rangle\,\text{m}$$

(b)

$$\sqrt{(-5.5\text{ m})^2 + (-20\text{ m})^2 + (0\text{ m})^2} = 20.74\,\text{m}$$

1.4-P-16

A helpful hint is to remember that the notation $\vec{r}_{AB}$ is *the position of A relative to B*, which is equivalent to saying *stand at B and tell me how to get to A*. Then you have simply $\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$, with the subtraction done in the order in which the indices appear.

(a)

$$\begin{aligned}\vec{r}_{\text{tree,head}} &= \vec{r}_{\text{tree}} - \vec{r}_{\text{head}} \\ &= \langle -25, 35, 43\rangle\,\text{m} - \langle 12, 30, 13\rangle\,\text{m} \\ &= \langle -37, 5, 30\rangle\,\text{m}\end{aligned}$$

(b)

$$\begin{aligned}\left|\vec{r}_{\text{tree,head}}\right| &= \sqrt{(-37)^2 + (5)^2 + (30)^2}\,\text{m} \\ &= 47.9\,\text{m}\end{aligned}$$

1.4-P-17

A helpful hint is to remember that the notation $\vec{r}_{AB}$ is *the position of A relative to B*, which is equivalent to saying *stand at B and tell me how to get to A*. Then you have simply $\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$, with the subtraction done in the order in which the indices appear.

(a)

$$\begin{aligned}\vec{r}_{\text{planet,star}} &= \vec{r}_{\text{planet}} - \vec{r}_{\text{star}} \\ &= \left\langle -4 \times 10^{10}, -9 \times 10^{10}, 6 \times 10^{10} \right\rangle \text{m} - \left\langle 6 \times 10^{10}, 8 \times 10^{10}, 6 \times 10^{10} \right\rangle \text{m} \\ &= \left\langle -10 \times 10^{10}, -17 \times 10^{10}, 0 \right\rangle \text{m}\end{aligned}$$

(b)

$$\begin{aligned}\vec{r}_{\text{star,planet}} &= -\vec{r}_{\text{planet,sub}} \\ &= \left\langle 10 \times 10^{10}, 17 \times 10^{10}, 0 \right\rangle \text{m}\end{aligned}$$

1.4-P-18

No unit is given, so assume an arbitrary unit in your own calculation.

(a)

$$\begin{aligned}\vec{r}_{\text{planet,star}} &= \vec{r}_{\text{planet}} - \vec{r}_{\text{star}} \\ &= \left\langle -1 \times 10^{10}, 8 \times 10^{10}, -3 \times 10^{10} \right\rangle - \left\langle 6 \times 10^{10}, -5 \times 10^{10}, 1 \times 10^{10} \right\rangle \\ &= \left\langle -7 \times 10^{10}, 13 \times 10^{10}, -4 \times 10^{10} \right\rangle\end{aligned}$$

(b)

$$\begin{aligned}\left|\vec{r}_{\text{planet,star}}\right| &= \sqrt{\left(-7 \times 10^{10}\right)^2 + \left(13 \times 10^{10}\right)^2 + \left(-4 \times 10^{10}\right)^2} \\ &= 1.5 \times 10^{11}\end{aligned}$$

(c)

$$\begin{aligned}\hat{r}_{\text{planet,star}} &= \frac{\sqrt{\left(-7 \times 10^{10}\right)^2 + \left(13 \times 10^{10}\right)^2 + \left(-4 \times 10^{10}\right)^2}}{1.5 \times 10^{11}} \\ &= \langle -0.46, 0.85, -0.26 \rangle\end{aligned}$$

1.4-P-19

$$\begin{aligned}\vec{r}_{\text{proton,electron}} &= \left\langle x_p - x_e, y_p - y_e, z_p - z_e \right\rangle \\ \vec{r}_{\text{electron,proton}} &= \left\langle x_e - x_p, y_e - y_p, z_e - z_p \right\rangle\end{aligned}$$

1.4-P-20

$$\begin{aligned}\hat{r} &= \frac{\langle 3,3,3\rangle \times 10^{-2}\text{m}}{\sqrt{(3)^2+(3)^2+(3)^2}\times 10^{-2}\text{m}}\\ &= \frac{\langle 3,3,3\rangle}{\sqrt{27}}\\ &= \langle 0.577, 0.577, 0.577\rangle\end{aligned}$$

By symmetry, the diagonal makes the same angle with each coordinate axis, so it doesn't matter which one we use. Let's use the y-axis.

$$\begin{aligned}\cos\left(\theta_y\right) &= \text{r}_y\\ &= 0.577\\ \theta_y &= \cos^{\mp 1}(0.577)\\ &= 55\,°\end{aligned}$$

1.6-P-01

(a)

$$\begin{aligned}\vec{v}_{\text{avg}} &= \frac{\Delta\vec{r}}{\Delta t}\\ &= \frac{\vec{r}_{\text{f}} - \vec{r}_{\text{i}}}{\Delta t}\\ &= \frac{\langle -0.202, 0.054, 0.098\rangle\,\text{m} - \langle 0.2, -0.05, 0.1\rangle\,\text{m}}{2\times 10^{-6}\text{s}}\\ &= \frac{\langle -0.402, 0.104, -0.002\rangle\,\text{m}}{2\times 10^{-6}\text{s}}\\ &= \left\langle -2.01\times 10^{5}, 5.2\times 10^{4}, -1\times 10^{3}\right\rangle \text{m/s}\end{aligned}$$

(b) Average speed is not always equal to the magnitude of average velocity unless the motion is linear. We can proceed with this assumption.

$$\begin{aligned}\left|\vec{v}_{\text{avg}}\right| &= \sqrt{\left(-2.01\times 10^{5}\right)^2 + \left(5.2\times 10^{4}\right)^2 + \left(-1\times 10^{3}\right)^2}\ \text{m/s}\\ &= 2.08\times 10^{5}\,\text{m/s}\end{aligned}$$

1.6-P-02

$$\begin{aligned}
\vec{r}_i &= \langle 15, 8, -3 \rangle \text{ m} \\
\vec{r}_f &= \langle 20, 6, -1 \rangle \text{ m} \\
\Delta t &= 0.1 \text{ s} \\
\vec{v}_{avg} &= \frac{\Delta \vec{r}}{\Delta t} \\
&= \frac{\vec{r}_f - \vec{r}_i}{\Delta t} \\
&= \frac{\langle 20, 6, -1 \rangle \text{ m} - \langle 15, 8, -3 \rangle \text{ m}}{0.1 \text{ s}} \\
&= \frac{\langle 5, -2, 2 \rangle \text{ m}}{0.1 \text{ s}} \\
&= \langle 50, -20, 20 \rangle \text{ m/s}
\end{aligned}$$

1.6-P-03

$$\begin{aligned}
\text{At } t = 0: &\quad \vec{r}_i = \langle 0, 0, 0 \rangle \\
\Delta t_1 = 200 \text{ s}: &\quad \hat{v} = \langle 1, 0, 0 \rangle \\
\Delta t_2 = 300 \text{ s}: &\quad \hat{v} = \left\langle \cos\left(45°\right), 0, \cos\left(45°\right) \right\rangle \\
\Delta t_3 = 150 \text{ s}: &\quad \hat{v} = \left\langle \cos\left(60°\right), 0, \cos\left(30°\right) \right\rangle \\
&\quad \vec{v} = |\vec{v}|\,\hat{v} = (2 \text{ m/s})\,\hat{v}
\end{aligned}$$

Use the position update equation for each time interval.

(a)

$$\begin{aligned}
\Delta t_1 = 200 \text{ s}: \vec{r}_f &= \vec{r}_i + \vec{v}_{avg} \Delta t \\
&= \langle 0, 0, 0 \rangle + (2 \text{ m/s}) \langle 1, 0, 0 \rangle (200 \text{ s}) \\
&= \langle 400, 0, 0 \rangle \text{ m}
\end{aligned}$$

$$\begin{aligned}
\Delta t_2 = 300 \text{ s}: \vec{r}_f &= \vec{r}_i + \vec{v}_{avg} \Delta t \\
&= \langle 400, 0, 0 \rangle \text{ m} + (2 \text{ m/s}) \left\langle \cos\left(45°\right), 0, \cos\left(45°\right) \right\rangle (300 \text{ s}) \\
&= \langle 824, 0, 424 \rangle \text{ m}
\end{aligned}$$

$$\begin{aligned}
\Delta t_3 = 150 \text{ s}: \vec{r}_f &= \vec{r}_i + \vec{v}_{avg} \Delta t \\
&= \langle 824, 0, 424 \rangle \text{ m} + (2 \text{ m/s}) \left\langle \cos\left(60°\right), 0, \cos\left(30°\right) \right\rangle (150 \text{ s}) \\
&= \langle 974, 0, 684 \rangle \text{ m}
\end{aligned}$$

(b) The total duration of time is $200\,\text{s} + 300\,\text{s} + 150\,\text{s} = 650\,\text{s}$.

$$\begin{aligned}\vec{v}_{avg} &= \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{\Delta t} \\ &= \frac{\langle 974, 0, 684\rangle\,\text{m} - \langle 0, 0, 0\rangle}{650\,\text{s}} \\ &= \langle 1.50, 0, 1.05\rangle\,\text{m/s}\end{aligned}$$

1.6-P-04

$$\begin{aligned}\vec{r}_i &= \langle 50, 20, 30\rangle\,\text{m} \\ \vec{r}_f &= \langle 53, 18, 31\rangle\,\text{m} \\ \Delta t &= 0.1\,\text{s}\end{aligned}$$

$$\begin{aligned}\vec{v}_{avg} &= \frac{\Delta\vec{r}}{\Delta t} \\ &= \frac{\vec{r}_f - \vec{r}_i}{\Delta t} \\ &= \frac{\langle 53, 18, 31\rangle\,\text{m} - \langle 50, 20, 30\rangle\,\text{m}}{0.1\,\text{s}} \\ &= \frac{\langle 3, -2, 1\rangle\,\text{m}}{0.1\,\text{s}} \\ &= \langle 30, -20, 10\rangle\,\text{m/s}\end{aligned}$$

1.6-P-05

$$\begin{aligned}\vec{r}_i &= \left\langle -3\times 10^3, -4\times 10^3, 8\times 10^3\right\rangle\,\text{m} \\ \vec{r}_f &= \left\langle -1.4\times 10^3, -6.2\times 10^3, 9.7\times 10^3\right\rangle\,\text{m} \\ t_i &= 18.4\,\text{s} \\ t_f &= 21.4\,\text{s} \\ \Delta t &= 21.4\,\text{s} - 18.4\,\text{s} \\ &= 3.0\,\text{s}\end{aligned}$$

$$\begin{aligned}\vec{v}_{avg} &= \frac{\Delta\vec{r}}{\Delta t}\\ &= \frac{\vec{r}_f - \vec{r}_i}{\Delta t}\\ &= \frac{\left\langle -1.4\times10^3, -6.2\times10^3, 9.7\times10^3\right\rangle \text{m} - \left\langle -3\times10^3, -4\times10^3, 8\times10^3\right\rangle \text{m}}{3\text{ s}}\\ &= \frac{\left\langle 1.6\times10^3, -2.2\times10^3, 1.7\times10^3\right\rangle \text{m}}{3\text{ s}}\\ &= \left\langle 5.33\times10^2, -7.33\times10^2, 5.67\times10^2\right\rangle \text{m}\end{aligned}$$

1.6-P-06

(a)

$$\begin{aligned}\vec{v} &= \langle -20, -90, 40\rangle \text{ m/s}\\ \vec{r}_i &= \langle 200, 300, -500\rangle \text{ m}\\ \vec{r}_f &= \langle -380, -2310, 660\rangle \text{ m}\\ \vec{v}_{avg} &= \frac{\Delta\vec{r}}{\Delta t}\end{aligned}$$

You cannot divide vectors, so

$$\Delta t \neq \frac{\Delta\vec{r}}{\vec{v}_{avg}}$$

You may use

$$\Delta t = \frac{\left|\Delta\vec{r}_{avg}\right|}{\left|\vec{v}_{avg}\right|}$$

Or you may write the velocity in component form and use any one of the components. For instance,

$$\Delta t = \frac{\Delta x}{v_{avg\,x}}$$

This method gives

$$\begin{aligned}\Delta t &= \frac{x_f - x_i}{v_{avg\,x}}\\ &= \frac{-380\,\text{m} - 200\,\text{m}}{-20\,\text{m/s}}\\ &= 29\,\text{s}\end{aligned}$$

(b)

$$\begin{aligned}\Delta\vec{r} &= \vec{r}_f - \vec{r}_i\\ &= \langle -380, -2810, 660\rangle\,\text{m} - \langle 200, 300, -500\rangle\,\text{m}\\ &= \langle -580, -2610, 1160\rangle\,\text{m}\end{aligned}$$

$$\begin{aligned}|\Delta\vec{r}| &= \sqrt{(-580)^2 + (-2610)^2 + (1160)^2}\,\text{m}\\ &= 2914\,\text{m}\end{aligned}$$

(c)

$$\begin{aligned}|\vec{v}| &= \frac{|\vec{r}|}{\Delta t}\\ &= \frac{2914\,\text{m}}{29\,\text{s}}\\ &= 100\,\text{m/s}\end{aligned}$$

(d)

$$\begin{aligned}\hat{v} &= \frac{\vec{v}}{|\vec{v}|}\\ &= \frac{\langle -20, -90, 40\rangle\,\text{m/s}}{100.5\,\text{m/s}}\\ &= \langle -0.2, -0.9, 0.4\rangle\end{aligned}$$

1.6-P-07

(a) From $t = 6.3$ s to 6.8 s:

$$\begin{aligned}\Delta t &= 6.8\,\text{s} - 6.3\,\text{s}\\ &= 0.5\,\text{s}\\ \vec{r}_i &= \langle -3.5, 9.4, 0\rangle\,\text{m}\\ \vec{r}_f &= \langle -1.3, 6.2, 0\rangle\,\text{m}\end{aligned}$$

$$\begin{aligned}
\vec{v}_{avg} &= \frac{\Delta \vec{r}}{\Delta t} \\
&= \frac{\langle -1.3, 6.2, 0\rangle\,\text{m} - \langle -3.5, 9.4, 0\rangle\,\text{m}}{0.5\,\text{s}} \\
&= \frac{\langle 2.2, -3.2, 0\rangle\,\text{m}}{0.5\,\text{s}} \\
&= \langle 4.4, -6.4, 0\rangle\,\text{m/s}
\end{aligned}$$

(b) From $t = 6.3\,\text{s}$ to $7.3\,\text{s}$:

$$\begin{aligned}
\Delta t &= 7.3\,\text{s} - 6.3\,\text{s} \\
&= 1.0\,\text{s} \\
\vec{r}_i &= \langle -3.5, 9.4, 0\rangle\,\text{m} \\
\vec{r}_f &= \langle 0.5, 1.7, 0\rangle\,\text{m}
\end{aligned}$$

$$\begin{aligned}
\vec{v}_{avg} &= \frac{\Delta \vec{r}}{\Delta t} \\
&= \frac{\langle 0.5, 1.7, 0\rangle\,\text{m} - \langle -3.5, 9.4, 0\rangle\,\text{m}}{1.0\,\text{s}} \\
&= \langle 4, -7.7, 0\rangle\,\text{m/s}
\end{aligned}$$

(c) The best estimate for $\vec{v}$ at $t = 6.3\,\text{s}$ is the average velocity during the smallest possible time interval that includes $t = 6.3\,\text{s}$. Thus, the time interval from $t = 6.3\,\text{s}$ to $6.8\,\text{s}$ gives the best possible estimate in this case for the instantaneous velocity at $t = 6.3\,\text{s}$.

(d) Assume that the bee's average velocity between $t = 6.3\,\text{s}$ and $6.33\,\text{s}$ is approximately constant. From $t = 6.3\,\text{s}$ to $6.33\,\text{s}$:

$$\begin{aligned}
\Delta t &= 6.33\,\text{s} - 6.3\,\text{s} \\
&= 0.03\,\text{s} \\
\vec{r}_i &= \langle -3.5, 9.4, 0\rangle\,\text{m} \\
\vec{v}_{avg} &\approx \langle 4.4, -6.4, 0\rangle\,\text{m/s}
\end{aligned}$$

$$\begin{aligned}
\vec{v}_{avg} &= \frac{\Delta \vec{r}}{\Delta t} \\
\Delta \vec{r} &= \vec{v}_{avg}\Delta t \\
&= (\langle 4.4, -6.4, 0\rangle\,\text{m/s})\,(0.03\,\text{s}) \\
&= \langle 0.132, -0.192, 0\rangle\,\text{m}
\end{aligned}$$

1.7-P-01

We are given the later (final) position, and we need to calculate the earlier (initial) position. The time interval is 2 s. Use the definition of average velocity.

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$
$$\vec{v}_{avg} = \frac{\vec{r}_f - \vec{r}_i}{\Delta t}$$
$$\vec{r}_i = \vec{r}_f - \vec{v}_{avg}\Delta t$$
$$\vec{r}_i = \langle -9, 11, -5\rangle \text{ m} - (\langle -6, 9.5, -5\rangle \text{ m/s})(2 \text{ s})$$
$$\vec{r}_i = \langle 3, -8, -5\rangle \text{ m}$$

1.7-P-02

$$t_1 = 12 \text{ s}$$
$$\vec{r}_i = \langle 84, 78, 24\rangle \text{ m}$$
$$\vec{v} = \langle 4, 0, -3\rangle \text{ m/s}$$
$$t_2 = 18 \text{ s}$$
$$\Delta t = t_2 - t_1$$
$$= 18 \text{ s} - 12 \text{ s}$$
$$= 6 \text{ s}$$

$$\vec{r}_f = \vec{r}_i + \vec{v}\Delta t$$
$$= \langle 84, 78, 24\rangle \text{ m} + (\langle 4, 0, -3\rangle \text{ m/s})(6 \text{ s})$$
$$= \langle 84, 78, 24\rangle \text{ m} + \langle 24, 0, -18\rangle \text{ m}$$
$$= \langle 108, 78, 6\rangle \text{ m}$$

1.7-P-03

(a)

$$\vec{r}_i = \langle 0.02, 0.04, -0.06\rangle \text{ m}$$
$$\vec{r}_f = \langle 0.02, 1.84, -0.86\rangle \text{ m}$$
$$\Delta t = 2 \times 10^{-6} \text{ s}$$

$$\begin{aligned}\vec{v}_{avg} &= \frac{\Delta\vec{r}}{\Delta t} \\ &= \frac{\vec{r}_f - \vec{r}_i}{\Delta t} \\ &= \frac{\langle 0.02, 1.84, -0.86\rangle\,\text{m} - \langle 0.02, 0.04, -0.06\rangle\,\text{m}}{2\times 10^{-6}\,\text{s}} \\ &= \frac{\langle 0, 1.8, -0.8\rangle\,\text{m}}{2\times 10^{-6}\,\text{s}} \\ &= \left\langle 0, 9\times 10^5, -4\times 10^5\right\rangle \text{m/s}\end{aligned}$$

(b) Now, for this time interval of 5×10^{-6} s, the initial position of the electron is its position at the end of the previous 2×10^{-6} s interval.

$$\begin{aligned}\vec{r}_i &= \langle 0.02, 1.84, -0.86\rangle\,\text{m} \\ \Delta t &= 5\times 10^{-6}\,\text{s} \\ \vec{v} &= \left\langle 0, 9\times 10^5, -4\times 10^5\right\rangle \text{m/s}\end{aligned}$$

$$\begin{aligned}\vec{r}_f &= \vec{r}_i + \vec{v}\Delta t \\ &= \langle 0.02, 1.84, -0.86\rangle\,\text{m} + \left(\left\langle 0, 9\times 10^5, -4\times 10^5\right\rangle \text{m/s}\right)\left(5\times 10^{-6}\,\text{s}\right) \\ &= \langle 0.02, 1.84, -0.86\rangle\,\text{m} + \langle 0, 4.5, -2\rangle\,\text{m} \\ &= \langle 0.02, 6.34, -2.86\rangle\,\text{m}\end{aligned}$$

Another way to solve it is to consider the total time interval of $2\times 10^{-6}\,\text{s} + 5\times 10^{-6}\,\text{s} = 7\times 10^{-6}\,\text{s}$. In this case, $\vec{r}_i$ is the electron's position at the beginning of the 2×10^{-6} s interval.

$$\begin{aligned}\vec{r}_i &= \langle 0.02, 0.04, -0.06\rangle\,\text{m} \\ \Delta t &= 5\times 10^{-6}\,\text{s} \\ \vec{r}_f &= \langle 0.02, 0.04, -0.06\rangle\,\text{m} + \left(\left\langle 0, 9\times 10^5, -4\times 10^5\right\rangle \text{m/s}\right)\left(7\times 10^{-6}\,\text{s}\right) \\ &= \langle 0.02, 0.04, -0.06\rangle\,\text{m} + \langle 0, 6.3, -2.8\rangle\,\text{m} \\ &= \langle 0.02, 6.34, -2.86\rangle\,\text{m}\end{aligned}$$

which agrees with the same answer obtained using the 5×10^{-6} s time interval.

1.7-P-04

(a) Assume that his velocity is in the $+x$ direction. Then

$$\begin{aligned} a_x &= \frac{\Delta v_x}{\Delta t} = \frac{\vec{v}_{fx} - \vec{v}_{ix}}{\Delta t} \\ &= \frac{70\,\text{m/s} - 140\,\text{m/s}}{0.6} = -117\,\text{m/s}^2 \\ |\vec{a}| &\approx -120\,\text{m/s}^2 \end{aligned}$$

(b) Since $g \approx 10\,\text{m/s}^2$, then $|\vec{a}| \approx 120/10 = 12$ g's.

1.7-P-05

$$\begin{aligned} \vec{r}_i &= \langle 7, 21, -17 \rangle\,\text{m} \\ \Delta t &= 3\,\text{s} \\ \vec{v}_{avg} &= \langle -11, 42, 11 \rangle\,\text{m/s} \\ y_f &= ? \end{aligned}$$

$$\begin{aligned} \vec{r}_f &= \vec{r}_i + \vec{v}_{avg}\Delta t \\ &= \langle 7, 21, -17 \rangle\,\text{m} + (\langle -11, 42, 11 \rangle\,\text{m/s})\,(3\,\text{s}) \\ &= \langle 7, 21, -17 \rangle\,\text{m} + \langle -33, 126, 33 \rangle\,\text{m} \\ &= \langle -26, 147, 16 \rangle\,\text{m} \end{aligned}$$

So $y_f = 147\,\text{m}$.

1.7-P-06

$$\begin{aligned} \vec{r}_i &= \langle 0.06, 1.03, 0 \rangle\,\text{m} \\ \vec{v}_{avg} &= \langle 17, 4, 6 \rangle\,\text{m/s} \\ \Delta t &= 0.7\,\text{s} \\ \vec{r}_f &= ? \end{aligned}$$

Use the position update equation.

$$\begin{aligned} \vec{r}_f &= \vec{r}_i + \vec{v}_{avg}\Delta t \\ &= \langle 0.06, 1.03, 0 \rangle\,\text{m} + (\langle 17, 4, 6 \rangle\,\text{m/s})\,(0.7\,\text{s}) \\ &= \langle 11.96, 3.83, 4.2 \rangle\,\text{m} \end{aligned}$$

Thus, the ball's height after a time interval of 0.7 s is 3.83 m.

1.7-P-07

(a)

$$\begin{aligned}\vec{v}_{\text{avg AB}} &= \frac{\Delta \vec{r}}{\Delta t}\\ &= \frac{\vec{r}_B - \vec{r}_A}{\Delta t}\\ &= \frac{\langle 22.3, 26.1, 0\rangle\,\text{m} - \langle 0,0,0\rangle}{1.0\,\text{s} - 0.0\,\text{s}}\\ &= \langle 22.3, 26.1, 0\rangle\,\text{m/s}\end{aligned}$$

(b) From $t = 1.0\,\text{s}$ to $t = 2.0\,\text{s}$, assuming it travels with a constant velocity of $\langle 22.3, 26.1, 0\rangle\,\text{m/s}$,

$$\begin{aligned}\vec{r}_f &= \vec{r}_i + \vec{v}_{\text{avg}}\Delta t\\ &= \langle 22.3, 26.1, 0\rangle\,\text{m} + (\langle 22.3, 26.1, 0\rangle\,\text{m/s})\,(2.0\,\text{s} - 1.0\,\text{s})\\ &= \langle 22.3, 26.1, 0\rangle\,\text{m} + \langle 22.3, 26.1, 0\rangle\,\text{m}\\ &= \langle 44.6, 52.2, 0\rangle\,\text{m}\end{aligned}$$

(c) $\vec{r}$ at point C is $\langle 40.1, 38.1, 0\rangle$ m which is not the same as what we predicted. We assumed constant velocity when making our prediction; however, in reality the velocity was not constant, but was decreasing in both the x and y directions. An approximation of constant velocity is only valid for small time intervals. For this projectile, $\Delta t = 1.0\,\text{s}$ was not a small enough time interval to reasonably assume constant velocity.

1.7-P-08

$$\begin{aligned}t_i &= 6\,\text{s}\\ t_f &= 10\,\text{s}\\ \vec{r}_i &= \langle 6, -3, 10\rangle\,\text{m}\\ \vec{r}_f &= \langle 6.8, -4.2, 11.2\rangle\,\text{m}\\ \vec{r}\text{ at } t = 8.5\,\text{s} &= ?\end{aligned}$$

Assume that the butterfly travels with a constant velocity. Calculate its velocity.

$$\begin{aligned}\vec{v}_{avg} &= \frac{\Delta\vec{r}}{\Delta t}\\ &= \frac{\vec{r}_f - \vec{r}_i}{\Delta t}\\ &= \frac{\langle 6.8, -4.2, 11.2\rangle\ \text{m} - \langle 6, -3, 10\rangle\ \text{m}}{10\ \text{s} - 6\ \text{s}}\\ &= \frac{\langle 0.8, -1.2, 1.2\rangle\ \text{m}}{4\ \text{s}}\\ &= \langle 0.2, -0.3, 0.3\rangle\ \text{m/s}\end{aligned}$$

Now calculate its position at $t = 8.5$ s, if it starts at $t = 6.0$ s.

$$\begin{aligned}\vec{r}_f &= \vec{r}_i + \vec{v}_{avg}\Delta t\\ &= \langle 6, -3, 10\rangle\ \text{m} + (\langle 0.2, -0.3, 0.3\rangle\ \text{m/s})\,(8.5\ \text{s} - 6\ \text{s})\\ &= \langle 6, -3, 10\rangle\ \text{m} + (\langle 0.2, -0.3, 0.3\rangle\ \text{m/s})\,(2.5\ \text{s})\\ &= \langle 6, -3, 10\rangle\ \text{m} + \langle 0.5, -0.75, 0.75\rangle\ \text{m}\\ &= \langle 6.5, -3.75, 10.75\rangle\ \text{m}\end{aligned}$$

1.8-P-01

(a) 3

(b) 6

(c) 1

(d) 5

(e) 2 and 4

1.8-P-02

(a) 4

(b) 1

(c) 6

(d) 3

(e) 7

1.8-P-03

Graph 1

1.8-P-04

Graphs 2 and 4

1.8-P-05

Graphs 2, 3, 4, and 5

1.8-P-06

Graphs 3, 4, and 6

1.9-P-01

$|\vec{v}| << c$ therefore

$$\begin{aligned} |\vec{p}| &= m\,|\vec{v}| \\ &= (0.155\,\text{kg})\,(40\,\text{m/s}) \\ &= 6.2\,\text{kg}\cdot\text{m/s} \end{aligned}$$

1.9-P-02

Note: $|\vec{v}| << c$

$$\begin{aligned} m &= 0.4\,\text{kg} \\ \vec{v} &= \langle 38, 0, -2\rangle\,\text{m/s} \\ \vec{p} &= m\vec{v} \\ &= (0.4\,\text{kg})\,(\langle 38, 0, -2\rangle\,\text{m/s}) \\ &= \langle 15.2, 0, -10.8\rangle\,\text{kg}\cdot\text{m/s} \\ |\vec{p}| &= \sqrt{(15.2)^2 + (0)^2 + (-10.8)^2}\,\text{kg}\cdot\text{m/s} \\ &= 18.6\,\text{kg}\cdot\text{m/s} \end{aligned}$$

1.9-P-03

$$m = 1000\,\text{kg}$$
$$|\vec{v}| = (500\,\text{mph})\left(\frac{1\,\text{m/s}}{2.2369\,\text{mph}}\right)$$
$$= 224\,\text{m/s}$$

Note: $\vec{v} << c$

$$|\vec{p}| = m\,|\vec{v}|$$
$$= (1000\,\text{kg})\,(224\,\text{m/s})$$
$$= 2.24 \times 10^{5}\,\text{kg} \cdot \text{m/s}$$

1.9-P-04

$$m = 155\,\text{g}$$
$$= 0.155\,\text{kg}$$
$$|\vec{v}| = (100\,\text{mph})\left(\frac{1\,\text{m/s}}{2.2369\,\text{mph}}\right)$$
$$= 44.7\,\text{m/s}$$

Note: $\vec{v} << c$

$$|\vec{p}| = m\,|\vec{v}|$$
$$= (0.155\,\text{kg})\,(44.7\,\text{m/s})$$
$$= 6.93\,\text{kg} \cdot \text{m/s}$$

1.9-P-05

$$\vec{p} = \langle 4, -5, 2 \rangle\,\text{kg} \cdot \text{m/s}$$
$$|\vec{p}| = \sqrt{(4)^2 + (-5)^2 + (2)^2}\,\text{kg} \cdot \text{m/s}$$
$$= 6.7\,\text{kg} \cdot \text{m/s}$$

1.9-P-06

$$m = 1.6\,\text{kg}$$
$$\vec{p} = \langle 0, 0, 4 \rangle\,\text{kg}\cdot\text{m/s}$$

(a)

$$|\vec{p}| = 4\,\text{kg}\cdot\text{m/s}$$

(b)

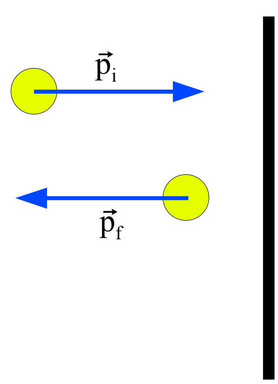
$$\hat{p} = \frac{\vec{p}}{|\vec{p}|} = \langle 0, 0, 1 \rangle$$

(c)

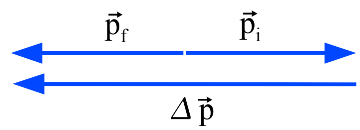
$$|\vec{p}| = m|\vec{v}|$$
$$|\vec{v}| = \frac{|\vec{p}|}{m} = \frac{4\,\text{kg}\cdot\text{m/s}}{1.6\,\text{kg}} = 2.5\,\text{m/s}$$

1.9-P-07

(a) Draw a sketch of the situation, like the one shown in the figure below..



Sketch the change in momentum vector by drawing the initial and final momentum vectors tail to tail and drawing the change in momentum from the head of the initial momentum to the head of the final momentum, as shown in the figure below..



$$\begin{aligned}\vec{v}_i &= \left\langle v_x, 0, 0\right\rangle \\ \vec{v}_f &= \left\langle -v_x, 0, 0\right\rangle \\ \Delta\vec{p} &= \vec{p}_f - \vec{p}_i \\ &= m\left(\vec{v}_f - \vec{v}_i\right) \\ &= m\left(\left\langle -v_x, 0, 0\right\rangle - \left\langle v_x, 0, 0\right\rangle\right) \\ &= m\left\langle -2v_x, 0, 0\right\rangle \\ &= \left\langle -2mv_x, 0, 0\right\rangle\end{aligned}$$

The change in momentum is in the $-x$ direction which is consistent with the picture.

(b)

$$\begin{aligned}\Delta|\vec{p}| &= \left|\vec{p}_f\right| - \left|\vec{p}_i\right| \\ &= m\left|\vec{v}_f\right| - m\left|\vec{v}_i\right| \\ &= mv_x - mv_x \\ &= 0\end{aligned}$$

Note $|\Delta\vec{p}| \neq \Delta|\vec{p}|$.

1.9-P-08

The ball's velocity in the x and z direction is zero before and after the bounce. In the y-direction, $v_{fy} = +5\,\text{m/s}$ and $v_{iy} = -5\,\text{m/s}$. The change in velocity due to the collision is

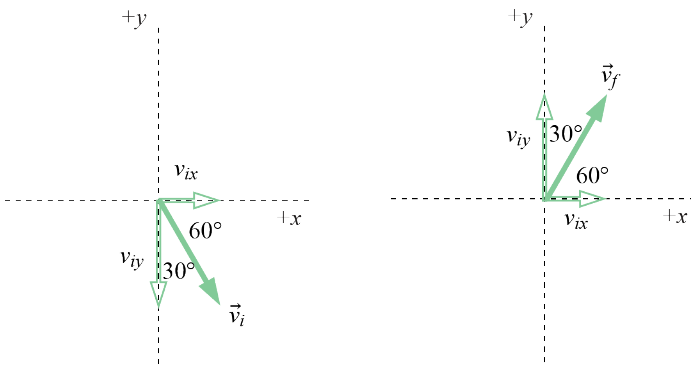
$$\begin{aligned}\Delta\vec{v} &= \vec{v}_f - \vec{v}_i \\ &= \langle 0, 5, 0\rangle\,\text{m/s} - \langle 0, -5, 0\rangle\,\text{m/s} \\ &= \langle 0, 10, 0\rangle\,\text{m/s}\end{aligned}$$

Using the low-speed approximation for momentum, then

$$\begin{aligned}\Delta\vec{\mathrm{p}} &= m\Delta\vec{\mathrm{v}}\\ &= 0.57\,\mathrm{kg}\,(\langle 0, 10, 0\rangle\,\mathrm{m/s})\\ &= \langle 0, 5.7, 0\rangle\,\mathrm{kg}\cdot\mathrm{m/s}\end{aligned}$$

1.9-P-09

It helps to sketch the velocity vector for the basketball before and after it hits the floor.



The angle of the vector with the $+y$ axis is $30\,°$, and the angle with the $+x$ axis is $60\,°$. The vector's components can be easily calculated using the cosine of each of these angles. Thus

$$\begin{aligned}\vec{\mathrm{v}}_{\mathrm{ix}} &= \left|\vec{\mathrm{v}}_{\mathrm{i}}\right|\cos\left(60\,°\right) = 2.5\,\mathrm{m/s}\\ \vec{\mathrm{v}}_{\mathrm{iy}} &= -\left|\vec{\mathrm{v}}_{\mathrm{i}}\right|\cos\left(30\,°\right) = -4.33\,\mathrm{m/s}\end{aligned}$$

After the ball bounces, $\mathrm{v}_{fx} = \mathrm{v}_{ix} = 2.5\,\mathrm{m/s}$ and $\mathrm{v}_{fy} = -\mathrm{v}_{iy} = +4.33\,\mathrm{m/s}$. The change in velocity is

$$\begin{aligned}\Delta\vec{\mathrm{v}} &= \vec{\mathrm{v}}_{\mathrm{f}} - \vec{\mathrm{v}}_{\mathrm{i}}\\ &= \langle 2.5, 4.33, 0\rangle\,\mathrm{m/s} - \langle 2.5, -4.33, 0\rangle\,\mathrm{m/s}\\ &= \langle 0, 8.66, 0\rangle\,\mathrm{m/s}\end{aligned}$$

Using the low speed approximation for momentum, the change in momentum is

$$\begin{aligned}\Delta\vec{\mathrm{p}} &\approx m\Delta\vec{\mathrm{v}}\\ &\approx (0.57\,\mathrm{kg})\,(\langle 0, 8.66, 0\rangle\,\mathrm{m/s})\\ &\approx \langle 0, 4.9, 0\rangle\,\mathrm{kg}\cdot\mathrm{m/s}\end{aligned}$$

1.9-P-10

Acceleration is a vector, $\vec{a} = \Delta\vec{v}/\Delta t = \left(\vec{v}_f - \vec{v}_i\right)/\Delta t$. Note that in general, $\vec{a} \neq \left(\left|\vec{v}_f\right| - \left|\vec{v}_i\right|\right)/\Delta t$. There is a very important difference in these two equations (one of which is correct). Therefore, assume that the rocket is traveling vertically, and express the given speed of the rocket as a velocity vector in the $+y$ direction.

$$
\begin{aligned}
\vec{v}_f &= \langle 0, 2300, 0\rangle \,\text{m/s} \\
\vec{v}_i &= \langle 0, 0, 0\rangle \,\text{m/s} \\
\Delta t &= 170\,\text{s}
\end{aligned}
$$

The acceleration is

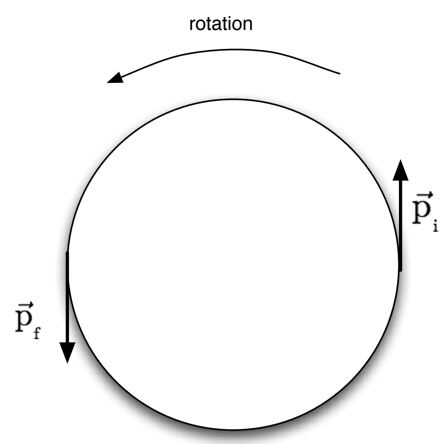
$$
\begin{aligned}
\vec{a} &= \frac{\Delta\vec{v}}{\Delta t} = \frac{\left(\vec{v}_f - \vec{v}_i\right)}{\Delta t} \\
&= \frac{(\langle 0, 2300, 0\rangle \,\text{m/s} - \langle 0, 0, 0\rangle \,\text{m/s})}{170\,\text{s}} \\
&= \langle 0, 13.5, 0\rangle \,\text{m/s}^2 \approx \langle 0, 14, 0\rangle \,\text{m/s}^2 \\
|\vec{a}| &= 14\,\text{m/s}^2
\end{aligned}
$$

Since $g \approx 10\,\text{m/s}^2$, then the acceleration of the rocket is approximately $14/10 = 1.4$ g's.

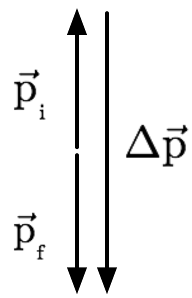
1.9-P-11

In going all the way around, in one revolution, $\vec{p}_f$ is the same as $\vec{p}_i$. Thus, $|\Delta\vec{p}| = 0$.

In going half a revolution (180 °), sketch $\vec{p}_i$ and $\vec{p}_f$ at opposite sides of the circle, as shown in the example in the figure below. (You may choose any two points on the circle, as long as they are on opposite sides of the circle. Also, you may assume either counterclockwise or clockwise rotation. Your choice of points or direction of rotation does not affect the final answer for the magnitude of the change in momentum.)



To find the change in momentum, sketch $\vec{p}_f$ and $\vec{p}_i$ tail to tail. $\Delta\vec{p}$ is the vector from the head of $\vec{p}_i$ to the head of $\vec{p}_f$. See the figure below..



As you can see, $\Delta\vec{p} = 2\vec{p}_f$. Thus,

$$\begin{aligned}|\Delta\vec{p}| &= 2\left|\vec{p}_f\right| \\ &= 2m\left|\vec{v}_f\right| \\ &= 2\,(50\,\mathrm{kg})\,(5\,\mathrm{m/s}) \\ &= 500\,\mathrm{kg\cdot m/s}\end{aligned}$$

1.9-P-12

(a)

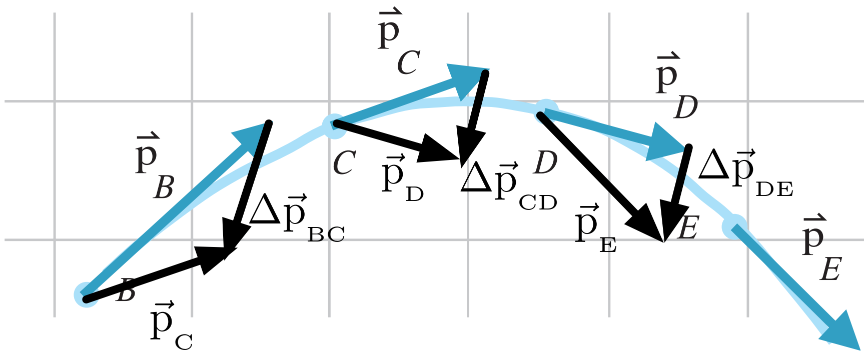
$$\begin{aligned}\Delta\vec{p}_{BC} &= \vec{p}_C - \vec{p}_B \\ &= \langle 2.55, 0.97, 0\rangle\,\mathrm{kg\cdot m/s} - \langle 3.03, 2.83, 0\rangle\,\mathrm{kg\cdot m/s} \\ &= \langle -0.48, -1.86, 0\rangle\,\mathrm{kg\cdot m/s}\end{aligned}$$

$$\begin{aligned}\Delta\vec{p}_{CD} &= \vec{p}_D - \vec{p}_C \\ &= \langle 2.24, -0.57, 0\rangle\,\mathrm{kg\cdot m/s} - \langle 2.55, 0.97, 0\rangle\,\mathrm{kg\cdot m/s} \\ &= \langle -0.31, -1.54, 0\rangle\,\mathrm{kg\cdot m/s}\end{aligned}$$

$$\begin{aligned}\Delta\vec{p}_{DE} &= \vec{p}_E - \vec{p}_D \\ &= \langle 1.97, -1.93, 0\rangle\,\mathrm{kg\cdot m/s} - \langle 2.24, -0.57, 0\rangle\,\mathrm{kg\cdot m/s} \\ &= \langle -0.27, -1.36, 0\rangle\,\mathrm{kg\cdot m/s}\end{aligned}$$

$$\begin{aligned}\Delta\vec{p}_{EF} &= \vec{p}_F - \vec{p}_E \\ &= \langle 1.68, -3.04, 0\rangle\,\mathrm{kg\cdot m/s} - \langle 1.97, -1.93, 0\rangle\,\mathrm{kg\cdot m/s} \\ &= \langle -0.29, -1.11, 0\rangle\,\mathrm{kg\cdot m/s}\end{aligned}$$

(b) To sketch $\Delta\vec{p}_{BC}$, sketch $\vec{p}_C$ tail-to-tail at the location of $\vec{p}_B$ and sketch $\Delta\vec{p}_{BC}$ from the head of $\vec{p}_C$ to the head of $\vec{p}_B$. Do this for each of the other vectors as well. The results are shown in the figure below..



(c) $\left|\Delta\vec{p}_{BC}\right|$ is greatest because both Δp_x and Δp_y are greatest (in magnitude) for the interval from B to C.

1.9-P-13

$$m = 3\,\text{kg}$$
$$\vec{p} = \langle 60, 150, -30\rangle\,\text{kg}\cdot\text{m/s}$$

Since $|\vec{v}| << c$, then $\vec{p} \approx m\vec{v}$.

$$\vec{v} \approx \frac{\vec{p}}{m} = \frac{\langle 60, 150, -30\rangle\,\text{kg}\cdot\text{m/s}}{3\,\text{kg}}$$
$$\approx \langle 20, 50, -10\rangle\,\text{m/s}$$

1.9-P-14

$$m = 1500\,\text{kg}$$
$$\vec{r}_i = \langle 300, 0, 0\rangle\,\text{m}$$
$$\vec{p} = \langle 45000, 0, 0\rangle\,\text{kg}\cdot\text{m/s}$$
$$\Delta t = 10\,\text{s}$$
$$\vec{r}_f = ?$$

Since $|\vec{v}| << c$, then $\vec{p} \approx m\vec{v}$.

$$\vec{v} \approx \frac{\vec{p}}{m} = \frac{\langle 45000, 0, 0\rangle\,\text{kg}\cdot\text{m/s}}{1500\,\text{kg}}$$
$$\approx \langle 600, 0, 0\rangle\,\text{m/s}$$

To find the final position, use the position update equation.

$$\begin{aligned}\vec{r}_f &= \vec{r}_i + \vec{v}\Delta t\\ &= \langle 300, 0, 0\rangle\,\text{m} + (\langle 600, 0, 0\rangle\,\text{m/s})\,(10\,\text{s})\\ &= \langle 600, 0, 0\rangle\,\text{m}\end{aligned}$$

1.9-P-15

$$\begin{aligned}m &= 0.17\,\text{kg}\\ \vec{p} &= \langle 0, 0, -6.3\rangle\,\text{kg}\cdot\text{m/s}\\ \vec{r}_f &= \langle 0, 0, -26\rangle\,\text{m}\\ \Delta t &= 0.4\,\text{s}\\ \vec{r}_i &= ?\end{aligned}$$

Since $|\vec{v}| << c$, then $\vec{p} \approx m\vec{v}$.

$$\begin{aligned}\vec{v} &\approx \frac{\vec{p}}{m} = \frac{\langle 0, 0, -6.3\rangle\,\text{kg}\cdot\text{m/s}}{0.17\,\text{kg}}\\ &\approx \langle 0, 0, -37.06\rangle\,\text{m/s}\end{aligned}$$

In this case we want to find the initial position (i.e. the position before the 0.4 s time interval). Use the position update equation and solve for the initial position.

$$\begin{aligned}\vec{r}_f &= \vec{r}_i + \vec{v}\Delta t\\ \vec{r}_i &= \vec{r}_f - \vec{v}\Delta t\\ &= \langle 0, 0, -26\rangle\,\text{m} - (\langle 0, 0, -37.06\rangle\,\text{m/s})\,(0.4\,\text{s})\\ &= \langle 0, 0, -11\rangle\,\text{m}\end{aligned}$$

Since the velocity was in the z-direction only, then the x-position and y-position did not change.

1.9-P-16

$$\begin{aligned}m &= 400\,\text{kg}\\ \vec{r}_i &= \left\langle 0, 3\times 10^4, -6\times 10^4\right\rangle\,\text{m}\\ \vec{p} &= \left\langle 6\times 10^3, 0, -3.6\times 10^3\right\rangle\,\text{kg}\cdot\text{m/s}\\ \Delta t &= 2\,\text{min} = 120\,\text{s}\\ \vec{r}_f &= ?\end{aligned}$$

Since $|\vec{v}| << c$, then $\vec{p} \approx m\vec{v}$.

$$\vec{v} \approx \frac{\vec{p}}{m} = \frac{\left\langle 6 \times 10^{3}, 0, -3.6 \times 10^{3} \right\rangle \text{kg} \cdot \text{m/s}}{400\,\text{kg}}$$
$$\approx \langle 15, 0, -9 \rangle\ \text{m/s}$$

To find the final position, use the position update equation.

$$\vec{r}_{f} = \vec{r}_{i} + \vec{v}\Delta t$$
$$= \left\langle 0, 3 \times 10^{4}, -6 \times 10^{4} \right\rangle \text{m} + (\langle 15, 0, -9 \rangle\ \text{m/s})\,(120\,\text{s})$$
$$= \left\langle 1.08 \times 10^{3}, 3 \times 10^{4}, -6.11 \times 10^{4} \right\rangle \text{m}$$

1.10-P-01

$$m_{\text{proton}} = 1.67 \times 10^{-27}\,\text{kg}$$
$$|\vec{p}| = \gamma m |\vec{v}|$$
$$= \frac{m|\vec{v}|}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}}$$
$$= \frac{\left(1.67 \times 10^{-27}\,\text{kg}\right)(0.88)\left(3 \times 10^{8}\,\text{m/s}\right)}{\sqrt{1 - \frac{(0.88c)^2}{c^2}}}$$
$$= \frac{\left(1.67 \times 10^{-27}\,\text{kg}\right)(0.88)\left(3 \times 10^{8}\,\text{m/s}\right)}{\sqrt{1 - (0.88)^2}}$$
$$= 9.28 \times 10^{-19}\,\text{kg} \cdot \text{m/s}$$

1.10-P-02

$$m_{\text{electron}} = 9.11 \times 10^{-31}\,\text{kg}$$
$$|\vec{v}| = 0.95c$$

$$
\begin{aligned}
|\vec{p}| &= \gamma m |\vec{v}| \\
&= \frac{m|\vec{v}|}{\sqrt{1-\frac{|\vec{v}|^2}{c^2}}} \\
&= \frac{m(0.95c)}{\sqrt{1-\frac{(0.95c)^2}{c^2}}} \\
&= \frac{m(0.95c)}{\sqrt{1-(0.95)^2}} \\
&= \frac{\left(9.11\times10^{-31}\,\mathrm{kg}\right)(0.95)\left(3\times10^{8}\,\mathrm{m/s}\right)}{\sqrt{1-(0.95)^2}} \\
&= 8.31\times10^{-22}\,\mathrm{kg\cdot m/s}
\end{aligned}
$$

1.10-P-03

$$
\begin{aligned}
m_{\text{proton}} &= 1.67\times10^{-27}\,\mathrm{kg} \\
|\vec{p}| &= \gamma m |\vec{v}| \\
&= \frac{m|\vec{v}|}{\sqrt{1-\frac{|\vec{v}|^2}{c^2}}} \\
&= \frac{m(0.9999c)}{\sqrt{1-\frac{(0.9999c)^2}{c^2}}} \\
&= \frac{m(0.9999c)}{\sqrt{1-(0.9999)^2}} \\
&= \frac{\left(1.67\times10^{-27}\,\mathrm{kg}\right)(0.9999)\left(3\times10^{8}\,\mathrm{m/s}\right)}{\sqrt{1-(0.9999)^2}} \\
&= 3.54\times10^{-17}\,\mathrm{kg\cdot m/s}
\end{aligned}
$$

1.10-P-04

(a)

$$m_{\text{proton}} = 1.67 \times 10^{-27}\,\text{kg}$$
$$|\vec{v}| = 0.99c$$

If $|\vec{p}| \approx m|\vec{v}|$, then

$$|\vec{p}| = \left(1.67 \times 10^{-27}\,\text{kg}\right)(0.99)\left(3 \times 10^{8}\,\text{m/s}\right)$$
$$= 4.96 \times 10^{-19}\,\text{kg} \cdot \text{m/s}$$

(b) Since $|\vec{v}|$ is not small compared to c,

$$\begin{aligned}
|\vec{p}| &= \gamma m|\vec{v}| \\
&= \frac{m|\vec{v}|}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}} \\
&= \frac{m(0.99c)}{\sqrt{1 - \frac{(0.99c)^2}{c^2}}} \\
&= \frac{m(0.99c)}{\sqrt{1 - (0.99)^2}} \\
&= \frac{\left(1.67 \times 10^{-27}\,\text{kg}\right)(0.99)\left(3 \times 10^{8}\,\text{m/s}\right)}{\sqrt{1 - (0.99)^2}} \\
&= 3.52 \times 10^{-18}\,\text{kg} \cdot \text{m/s}
\end{aligned}$$

(c)

$$\frac{\left|\vec{p}_{\text{correct}}\right|}{\left|\vec{p}_{\text{approx}}\right|} = \frac{3.52 \times 10^{-18}\,\text{kg} \cdot \text{m/s}}{4.96 \times 10^{-19}\,\text{kg} \cdot \text{m/s}}$$
$$= 7.1$$

Thus, the approximate calculation for the momentum of the particle is about 7 times too small.

1.10-P-05

$$
\begin{aligned}
|\vec{v}| &= 0.9996c \\
\gamma &= \frac{1}{\sqrt{1-\frac{|\vec{v}|^2}{c^2}}} \\
&= \frac{1}{\sqrt{1-\frac{(0.9996c)^2}{c^2}}} \\
&= \frac{1}{\sqrt{1-(0.9996)^2}} \\
&= 35.4
\end{aligned}
$$

1.10-P-06

$$
\begin{aligned}
m_{\text{electron}} &= 9 \times 10^{-31}\,\text{kg} \\
|\vec{v}| &= 0.996c \\
\hat{v} &= \langle -.655, -0.492, -0.573 \rangle
\end{aligned}
$$

(a)

$$
\begin{aligned}
\gamma &= \frac{1}{\sqrt{1-\frac{|\vec{v}|^2}{c^2}}} \\
&= \frac{1}{1-(0.996)^2} \\
&= 11.2
\end{aligned}
$$

(b)

$$
\begin{aligned}
|\vec{v}| &= 0.996c \\
&= (0.996)\left(3 \times 10^{8}\,\text{m/s}\right) \\
&= 2.988 \times 10^{8}\,\text{m/s}
\end{aligned}
$$

(c)

$$
\begin{aligned}
|\vec{p}| &= \gamma m |\vec{v}| \\
&= (11.2)\left(9 \times 10^{-31}\,\text{kg}\right)\left(2.988 \times 10^{8}\,\text{m/s}\right) \\
&= 3.0 \times 10^{-21}\,\text{kg} \cdot \text{m/s}
\end{aligned}
$$

(d)

$$\vec{p} = |\vec{p}|\,\hat{p}$$

Since $\vec{p}$ is proportional to $\vec{v}$, then their unit vectors $\hat{p}$ and $\hat{v}$ are the same. Thus,

$$\begin{aligned}\vec{p} &= \left(3 \times 10^{-21}\,\text{m/s}\right) \langle 0.655, -0.492, -0.573\rangle \\ &= \left\langle 1.97 \times 10^{-21}, -1.48 \times 10^{-21}, -1.72 \times 10^{-21}\right\rangle \text{kg} \cdot \text{m/s}\end{aligned}$$

1.10-P-07

$$\begin{aligned}|\vec{p}| &= \gamma m |\vec{v}| \\ \frac{|\vec{p}|}{m} &= 0.85c\end{aligned}$$

Thus,

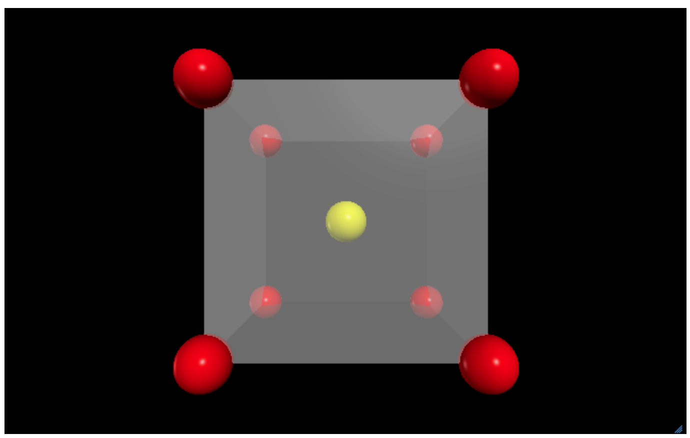
$$\begin{aligned}\gamma |\vec{v}| &= 0.85c \\ \frac{|\vec{v}|}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}} &= 0.85c \\ \frac{|\vec{v}|^2}{1 - \frac{|\vec{v}|^2}{c^2}} &= (0.85c)^2 \\ |\vec{v}|^2 &= (0.85c)^2 \left(1 - \frac{|\vec{v}|^2}{c^2}\right) \\ |\vec{v}|^2 &= (0.85c)^2 - (0.85)^2 |\vec{v}|^2 \\ |\vec{v}|^2 + (0.85)^2 |\vec{v}|^2 &= (0.85c)^2 \\ |\vec{v}|^2 \left(1 + 0.85^2\right) &= (0.85c)^2 \\ |\vec{v}|^2 &= \frac{(0.85c)^2}{1 + (0.85)^2} \\ |\vec{v}| &= \frac{0.85c}{\sqrt{1 + (0.85)^2}} \\ &= 0.65c\end{aligned}$$

1.11-P-01

Lines 8-12 are new lines of code which were added to the code given in Section 1.11.2. The program was written to run in Web VPython.

```
1 Web VPython 3.2
2
3 box(pos=vector(0,0,0),length=2,width=2, height=2, opacity=0.4)
4 sphere(pos=vector(0,0,0), color=color.yellow, radius=0.2)
5 sphere(pos=vector(-1,-1,1), radius=0.2, color=color.red)
6 sphere(pos=vector(-1,-1,-1), radius=0.2, color=color.red)
7 sphere(pos=vector(-1,1,-1), radius=0.2, color=color.red)
8 sphere(pos=vector(-1,1,1), radius=0.2, color=color.red)
9 sphere(pos=vector(1,1,1), radius=0.2, color=color.red)
10 sphere(pos=vector(1,1,-1), radius=0.2, color=color.red)
11 sphere(pos=vector(1,-1,1), radius=0.2, color=color.red)
12 sphere(pos=vector(1,-1,-1), radius=0.2, color=color.red)
```

A screen capture of the output is shown below.

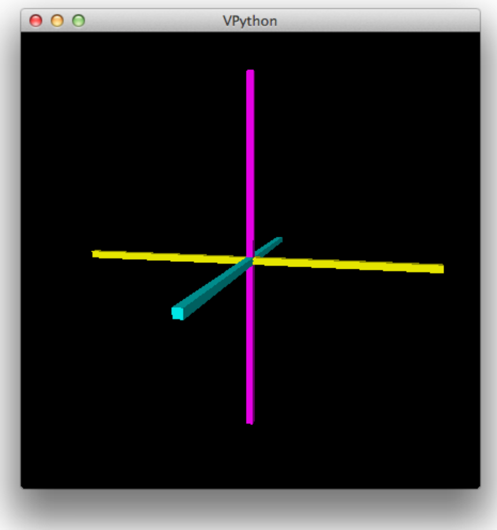


1.11-P-02

This program is one possible solution to the problem.

```
Web VPython 3.2

#length of the box
a=1

#width
w=a/50

#xaxis
box(pos=vector(0,0,0), size=vec(a,w,w), color=color.yellow)
#yaxis
box(pos=vector(0,0,0), size=vec(w,a,w), color=color.magenta)
#zaxis
box(pos=vector(0,0,0), size=vec(w,w,a), color=color.cyan)
```

A screen capture of the output of the program is shown below.



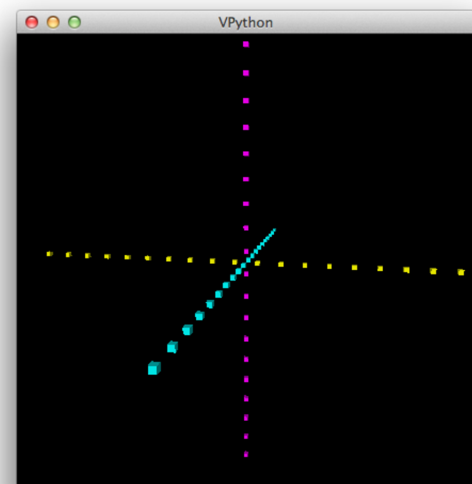
1.11-P-03

Examine the program below. It's useful to define variables for the length of the side of a box, the length of an axis, and the number of boxes along the axis. The space between boxes can be calculated from the length of the axis and the number of boxes along the axis.

```
Web VPython 3.2

#length of the axis
a=1

#width of a box
w=a/100

#Number of boxes on an axis
n=20

#x location of a box
x=-a/2

#space between boxes
dx=a/(n-1)

#boxes on x-axis
while x<a/2:
    box(pos=vec(x,0,0),size=vec(w,w,w),color=color.yellow)
    x=x+dx

```

```
#y location of a box
y=-a/2

#space between boxes
dy=dx

#boxes on y-axis
while y<a/2:
    box(pos=vec(0,y,0),size=vec(w,w,w),color=color.magenta)
    y=y+dy

#z location of a box
z=-a/2

#space between boxes
dz=dx

#boxes on z-axis
while z<a/2:
    box(pos=vec(0,0,z),size=vec(w,w,w),color=color.cyan)
    z=z+dz
```

The output is shown below.



Note: there are many ways to solve this problem. In the sample program below, we draw n boxes on an axis. So for three axes, our `while` loop requires $3n$ iterations. Using an `if` statement, we check to see how many boxes have been drawn, and we use this value to determine the axis on which we will draw the next box. It's important to reset the value of x for the position of the first box whenever starting a new axis.

```
Web VPython 3.2

#length of the axis
a=1
```

```

#width of a box
w=a/100

#Number of boxes on an axis
n=20

#x location of a box
x=-a/2

#space between boxes
dx=a/(n-1)

#There is a total of 3*n boxes.
#After n boxes on the x-axis, then create boxes on the y-axis.
#After another n boxes on the y-axis, then create boxes on the z-axis
boxnum=1
while boxnum<3*n+1:
    #x axis
    if(boxnum<n+1):
        box(pos=vec(x,0,0),size=vec(w,w,w),color=color.yellow)
    #y axis
    elif(boxnum<2*n+1 and boxnum>n):
        box(pos=vec(0,x,0),size=vec(w,w,w),color=color.magenta)
    #z axis
    else:
        box(pos=vec(0,0,x),size=vec(w,w,w),color=color.cyan)
    #have to reset the value of x to -a/2 when starting a new axis
    if(boxnum==n or boxnum==2*n):
        x=-a/2

    x=x+dx
    boxnum=boxnum+1
```

1.11-P-04

(a) Assume that the unit of distance is m and the unit of time is s.

1. The initial velocity of the particle is $\langle 0.5, 0, 0.5\rangle$ m/s.
2. We will assume that "in front" and "behind" refers to the $z-$direction. The box is initially at $z = -1$ m, and the particle is initially at $z = -5$ m. Thus, the particle is initially behind the box.
3. This line updates the position of the particle.

```
particle.pos = particle.pos + v * delta_t
```

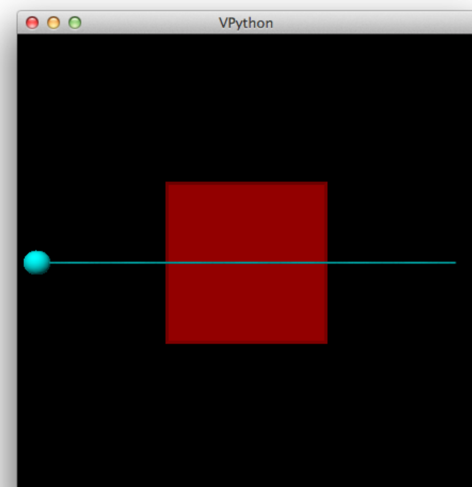
4. The time step is 0.05 s.
5. The particle has a constant velocity. There is no code that causes the velocity of the particle to change.

(b) The answers in part(a) are correct.

(c) Change the initial position of the particle, and change the velocity of the particle. Here's an example program that solves the problem.

```
Web VPython 3.2

box(pos=vector(0,0,-1), size=vec(5,5,0.5),
   color=color.red, opacity = 0.4)
particle = sphere(pos=vector(5,0,2),
    radius=0.3, color=color.cyan,
       make_trail=True)
v = vector(-0.5,0,0)
delta_t = 0.05
t = 0
while t < 20:
      rate(100)
      particle.pos = particle.pos + v * delta_t
      t = t + delta_t
```

Here is the output.



1.11-P-04

(a) Assume that the unit of distance is m and the unit of time is s.

1. The initial velocity of the particle is $\langle 0.5, 0, 0.5\rangle$ m/s.
2. We will assume that "in front" and "behind" refers to the $z-$direction. The box is initially at $z = -1$ m, and the particle is initially at $z = -5$ m. Thus, the particle is initially behind the box.
3. This line updates the position of the particle.

```
particle.pos = particle.pos + v * delta_t
```

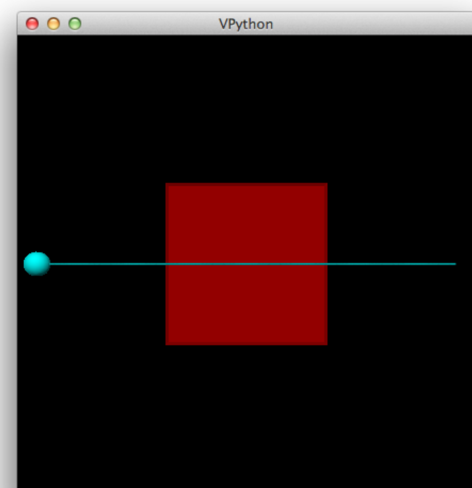
4. The time step is 0.05 s.
5. The particle has a constant velocity. There is no code that causes the velocity of the particle to change.

(b) The answers in part(a) are correct.

(c) Change the initial position of the particle, and change the velocity of the particle. Here's an example program that solves the problem.

```
Web VPython 3.2

box(pos=vector(0,0,-1), size=vec(5,5,0.5),
   color=color.red, opacity = 0.4)
particle = sphere(pos=vector(5,0,2),
     radius=0.3, color=color.cyan,
        make_trail=True)
v = vector(-0.5,0,0)
delta_t = 0.05
t = 0
while t < 20:
      rate(100)
      particle.pos = particle.pos + v * delta_t
      t = t + delta_t
```

Here is the output.



1.11-P-05

Change the velocity to `velocity = vector(-2, 0, 0)`. Here is a WebVPython complete program. Note that the line numbers do not match the line numbers in the code from the textbook since this program begins on line 3.

```
Web VPython 3.2

ball = sphere( pos=vector(0,-10,0))
velocity = vector(-2, 0, 0)
deltat = 0.1
while ball.pos.y < 10:
    rate(60)
    ball.pos = ball.pos + (velocity * deltat)
```

1.11-P-06

When you run the program given in Chapter 1, it prints the velocity of the drone. (After running the program, scroll the textbox to the top in order to see the first line that was printed.) The velocity of the drone is $\langle 1.16667, 0.666667, 0.5\rangle$.

Let's make the drone travel along the x, y, and z directions separately. This is not the only solution to the problem, but it might be interesting to watch it travel along the x-direction, y-direction, and z-direction independently.

First, modify the velocity to be $\langle 1.16667, 0, 0\rangle$, and run the loop while the x-position of the drone is to the right of the x-position of the target.

Then, reset the velocity to be in the y-direction $\langle 0, 0.666667, 0\rangle$, and run a loop while the y-position of the drone is below the y-position of the target.

Then, reset the velocity to be in the z-direction $\langle 0, 0, 0.5\rangle$, and run a loop while the z-position of the drone is "behind" the z-position of the target.

Here's the complete program.

```
Web VPython 3.2

drone = sphere(pos=vector(3,-2,-1), radius=0.3,
    color=color.cyan, make_trail = True)
target = box(pos=vector(-4,2,2), length=1,
    width=1, height=1, opacity=0.4)
light = sphere(pos=vector(-1, 0.7, 0.5),
    radius=0.5, color=color.yellow)
deltar = target.pos - drone.pos
totalTime = 6

#travel along the x-axis
velocity = vec(-1.16667, 0, 0)
print("velocity =", velocity)
deltat = 0.1
while drone.pos.x > target.pos.x:
    rate(100)
    drone.pos = drone.pos + velocity*deltat
    print("position is", drone.pos)

#travel along the y-axis
velocity = vec(0, 0.666667, 0)
while drone.pos.y < target.pos.y:
    rate(100)
    drone.pos = drone.pos + velocity*deltat
    print("position is", drone.pos)

```

```
#travel along the z-axis
velocity = vec(0, 0, 0.5)
while drone.pos.z < target.pos.z:
    rate(100)
    drone.pos = drone.pos + velocity*deltat
    print("position is", drone.pos)
```

Here's a screen capture.